2019

Secondary 4 EMath

1.	Ang Mo Kio Secondary	SA2
2.	Commonwealth Secondary	SA2
3.	Nan Chiau High	SA2
4.	St. Margaret's Secondary	SA2
5.	Beatty Secondary	SA2
6.	Bowen Secondary	SA2
7.	CHIJ St. Theresa's	SA2
8.	Fairfield Methodist	SA2
9.	Hillgrove Secondary	SA2
10.	Mayflower Secondary	SA2
11.	West Spring Secondary	SA2
12.	Yuying Secondary	SA2

Class	Index Number	Candidate Name

ANG MO KIO SECONDARY SCHOOL
MID-YEAR EXAMINATION 2019
SECONDARY FOUR EXPRESS / FIVE NORMAL ACADEMIC

MATHEMATICS **4048/01**
Paper 1

Setter: Mrs Seah Kwan Chaet

Wednesday **15 May 2019** **2 hours**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use
80

This document consists of **17** printed pages and **1** blank page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curve surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector Area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** Simplify $x+7-x(2-3x)$.

Answer [2]

(b) Factorise completely $4ax+12a-x-3$.

Answer [2]

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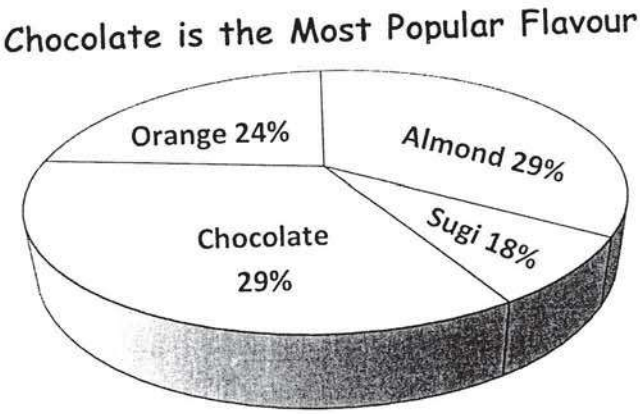
2 The pie chart shows the sales for 4 different flavours of cookies.

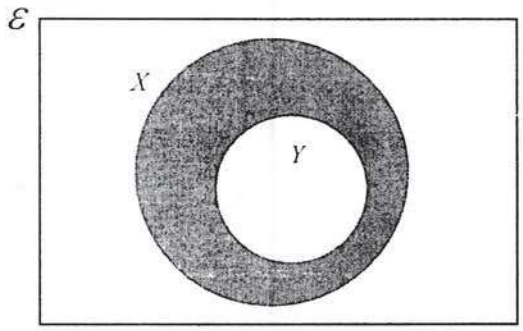


State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer

[2]

3 **(a)** Write down the set notation for the set represented by the shaded region in the Venn diagram below.



Answer [1]

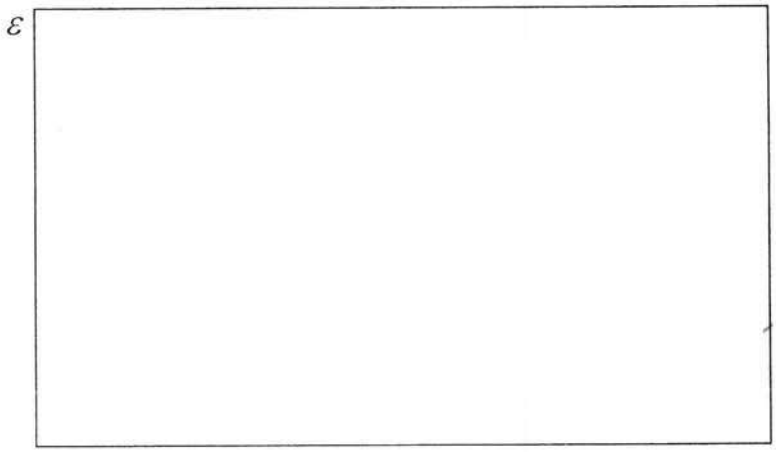
(b) $\mathcal{E} = \{x : x \text{ is an integer, } 1 \leq x \leq 9\}$

$A = \{x : x \text{ is an odd number}\}$

$B = \{x : x \text{ is a factor of } 6\}$

(i) Draw a Venn diagram in the box below to illustrate the above information.

Answer



[1]

(ii) List the elements in the set $(A \cup B)'$.

Answer [1]

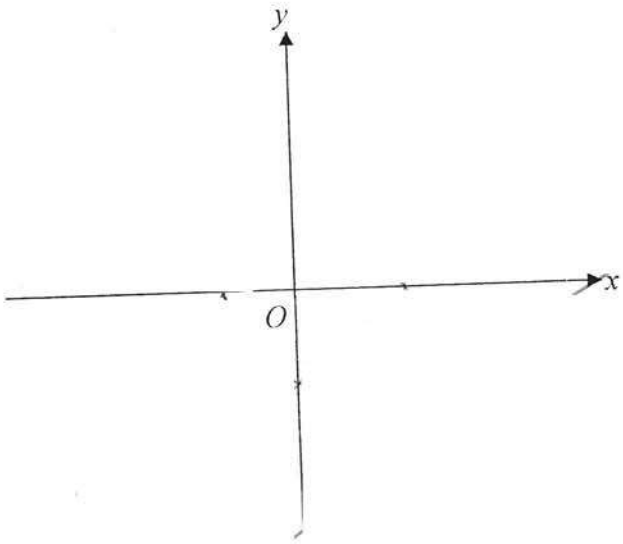
4 (a) Express $x^2 - 2x - 2$ in the form $p + (x+q)^2$.

Answ

[2]

(b) Sketch the graph of $y = x^2 - 2x - 2$ on the axes below.
Indicate clearly the values where the graph crosses the x– and y– axes.

Answer



[2]

5 The exchange rate between Singapore dollars ($) and Japanese Yen (¥) was $1 = ¥81.7339. Mr Toshi bought a watch priced at $1550. The shop charged 1.8% commission as he chose to pay in Japanese Yen using his credit card. What was the total cost of the watch in Japanese Yen?

Answer ¥

[2]

6 Write as a single fraction in its simplest form $\frac{4}{3-x}+\frac{1}{x^2-3x}$.

Answer [2]

---

7 A map is drawn to a scale of 1 : 500 000.

(a) The length of a runway at an airport is 0.6 cm on the map. Calculate the actual length of the runway in kilometres.

Answer km [1]

(b) The airport has an area of 5 km^2. Calculate the area, in square centimetres, of the airport on the map.

Answer cm^2 [2]

---

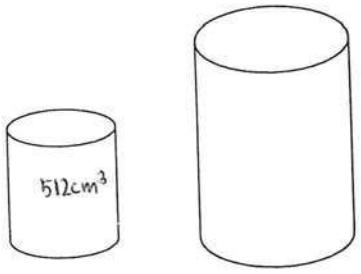
8 The monthly telephone charges offered by a telecommunications company is obtained by adding a fixed charge of \$28 and the total usage for the month as shown below.

Usage Per Month	Rate (cents per minute)
First 5 hours	Free
Next 20 hours	3.5
Exceed 20 hours	10

If the total usage for the month of February was 31 hours, calculate the telephone charges for that month.

Answer \$ [2]

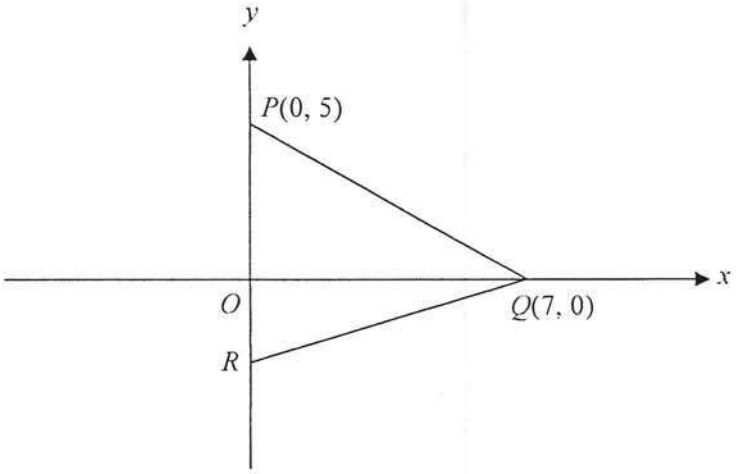
9 Two pails, X and Y, are geometrically similar. The volume of pail X is $512\ \text{cm}^3$. Given that the base area of pail Y is four times the base area of pail X, find the volume of pail Y.



Pail X Pail Y

Answer cm^3 [3]

10 The diagram below shows a triangle PQR with coordinates $P(0, 5)$ and $Q(7, 0)$. The area of the triangle is 28 cm^2.



Find

(a) the coordinates of R,

Answer R (..... .....) [2]

(b) the equation of the line QR.

Answer .. [2]

---

11 If the length of a rectangle is increased by 30% and its breadth is decreased by 20%, find the percentage change in its area.

Answer % [2]

---

12 Written as the product of its prime factors, $126 = 2 \times 3^2 \times 7$.

(a) Express 720 as a product of its prime factors.

Answer [2]

(b) Find the largest integer which is a factor of both 126 and 720.

Answer [1]

(c) Find the smallest possible value of m if $720m$ is both a perfect square and a perfect cube.

Answer $m =$ [1]

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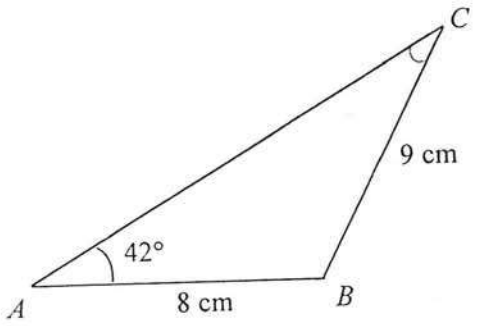
13 In the diagram below, $AB = 8$ cm, $BC = 9$ cm and $\angle BAC = 42°$. Find $\angle ACB$.



Answer ° [2]

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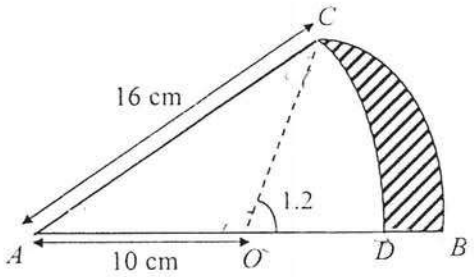
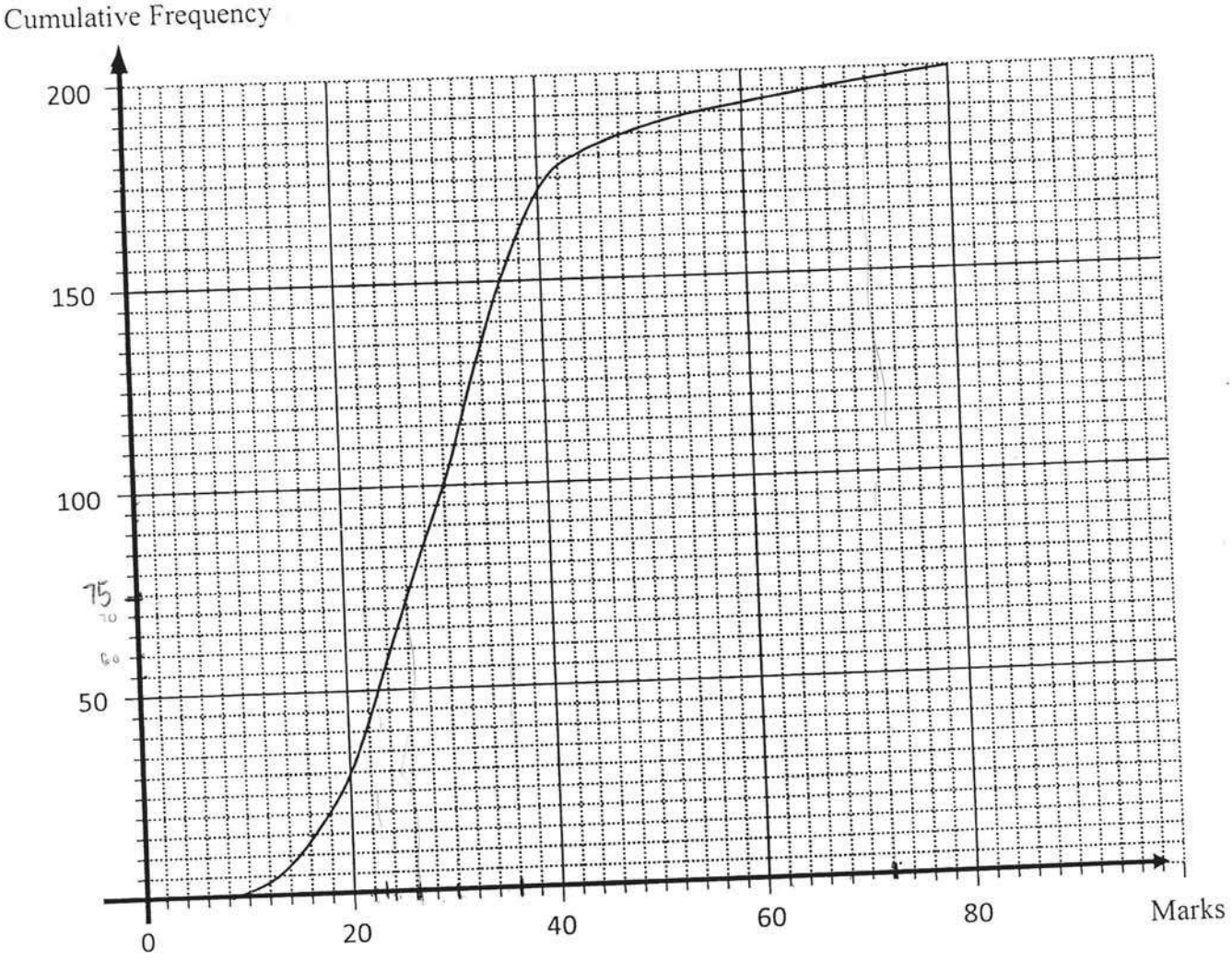
14 The diagram below shows part of a circle ABC with centre O and radius 10 cm. The radius OC makes an angle of 1.2 radians with the radius OB. ACD is a sector with centre A and radius 16 cm.



Find the area of the shaded region BCD.

Answer ..... cm^2 [5]

15 The cumulative frequency curve shows the marks obtained by 200 pupils in a particular Science examination. The maximum mark is 80.



Find the

(a) (i) median,

Answer [1]

(ii) interquartile range.

Answer [2]

(b) Find the passing mark if 36% of the pupils passed the examination.

Answer [1]

16 The sum of a series of numbers is given below

$$S_n = 1+2+3+\ldots+n = \frac{n(n+1)}{2}.$$

(a) Find the value of S_{35}.

Answer [1]

(b) Find the value of n given that $S_n = 1378$.

Answer [2]

(c) If $T = 101 + 102 + 103 + \ldots.. + 199$, find the value of T.

Answer [2]

(d) Given the sum of even integers is $P = 2 + 4 + 6 + \ldots\ldots 100$, find the value of P.

Answer [2]

(e) Using your answer in part **(d)**, find the sum of all the odd integers between 0 and 100.

Answer [1]

17 A drinks vending machine takes 50 cent coins and \$1 coins. A drink costs \$1.50. The probability that the machine will accept a particular 50 cent coin is 0.9 and that it will accept a particular \$1 coin is 0.85.

(a) What is the probability that the machine will not accept a particular 50 cent coin?

Answer [1]

(b) John put one 50 cent coin and one \$1 coin into the machine. Calculate the probability that

(i) the machine will not accept both coins,

Answer [1]

(ii) John will get a drink only when he inserts another \$1 coin.

Answer [1]

(c) Peter only has three 50 cent coins. Calculate the probability that

(i) the machine will accept all three coins,

Answer [1]

(ii) Peter will not get a drink.

Answer [1]

18 $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$ and B is the point (7, 0). Find

(a) $|\overrightarrow{AB}|$,

Answer [1]

(b) the coordinates of the point C.

Answer C (,) [2]

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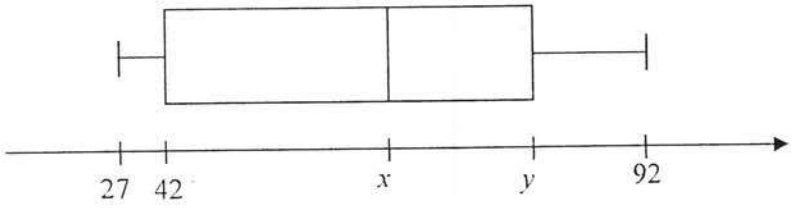
19 The marks scored by 12 students from a Sec 4 class in a test are listed below.

72, 27, 38, 85, 54, 32, 76, 46, 68, 56, 64, 92.

The diagram below shows a box-and-whisker plot representing their results.



(a) Find the value of x and of y.

Answer $x =$, $y =$ [2]

(b) A new student later joined the group and sat for the same test. If the median mark remains unchanged, find the mark scored by the new student.

Answer [1]

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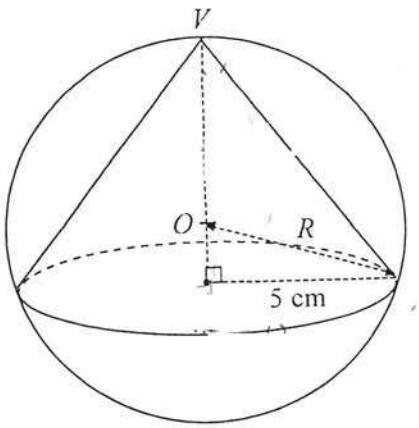
20 The diagram below shows a cone fitted inside a sphere. V is the vertex of the cone and O is the centre of the sphere. Given that the cone has a height of 10 cm and a base radius of 5 cm, find the radius of the sphere, R.



Answer cm [3]

21 (a) Simplify each of the following, giving your answer as a positive index.

(i) $\left(x^{-3}y\right)^{-5}$

Answer [1]

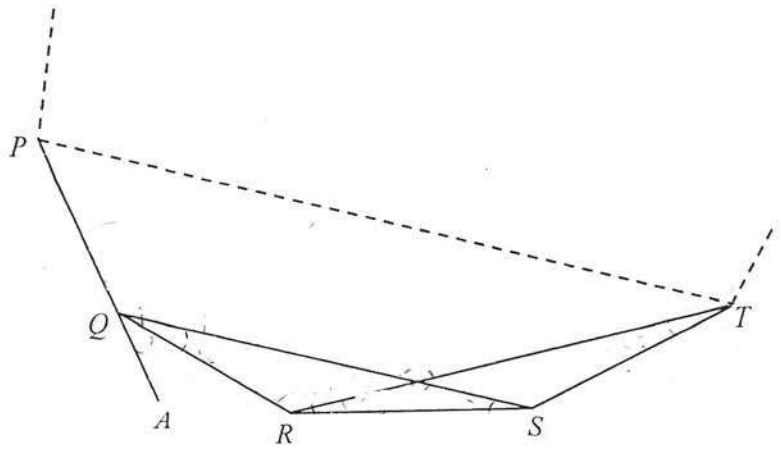
(ii) $\dfrac{18x^{\frac{1}{3}}}{4\sqrt{x}}$

Answer [2]

(b) Solve the equation $2^{5x-1} = 3^{5x-1}$.

Answer $x =$ [2]

22



The diagram shows part of a regular n-sided polygon, $PQRST...$.

(a) Explain why triangles QRS and TSR are congruent.

Answer

[2]

(b) It is given that angle $AQR = 30°$. Find

(i) the value of n,

Answer [1]

(ii) angle RTS,

QRS

Answer [2]

(iii) angle QPT.

Answer [2]

END OF PAPER

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Class	Index Number	Candidate Name

ANG MO KIO SECONDARY SCHOOL
MID-YEAR EXAMINATION 2019
SECONDARY FOUR EXPRESS / FIVE NORMAL ACADEMIC

MATHEMATICS **4048/02**
Paper 2

Setter: Mdm Kwa Leng Leng

Monday **13 May 2019** **2 hours 30 minutes**

Candidates answer on the Question Paper.
Additional Materials: Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **100**.

For Examiner's Use
100

This document consists of **22** printed pages and **1** insert.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curve surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

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$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector Area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** It is given that $y+z=\frac{4y-6z}{7x}$.

(i) Find x when $y=2$ and $z=-1$. [1]

(ii) Express z in term of x and y. [2]

(b) Solve the equation $\frac{2a+1}{3}+\frac{a-2}{4}=2$. [2]

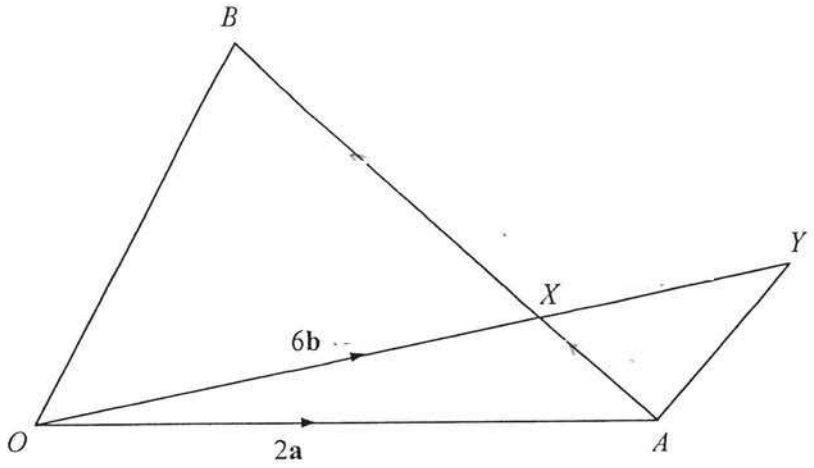
(c) Solve these simultaneous equations.

$$4p + 3q = 2$$
$$5p - 4q = 49$$

[3]

(d) Simplify $\dfrac{4h^2 - 36}{4h^2 - 5h - 21}$. [3]

2



(a) In ΔOAB, the point X on AB is such that $AX : XB = 1 : 3$. OXY is a straight line and $XY = \frac{1}{4}OY$.

Given that $\overrightarrow{OA} = 2\mathbf{a}$, $\overrightarrow{OX} = 6\mathbf{b}$, write each of the following in terms of **a** and **b**. Give your answers in their simplest form.

(i) $\overrightarrow{AX}$, [1]

(ii) $\overrightarrow{OB}$, [2]

(iii) $\overrightarrow{AY}$. [2]

(b) State 2 facts about $\overrightarrow{AY}$ and $\overrightarrow{OB}$. [2]

3 Famous Factory produces pastries and delivers them to two outlets every day that each outlet is open. There are 3 deliveries for each day. The number of pastries supplied in a single delivery is given by the matrix **P**.

$$\mathbf{P} = \begin{array}{c} \begin{array}{cc} \text{Outlet 1} & \text{Outlet 2} \end{array} \\ \begin{pmatrix} 60 & 80 \\ 30 & 50 \\ 80 & 100 \end{pmatrix} \end{array} \begin{array}{l} \\ \text{Chicken Pie} \\ \text{Tuna Puff} \\ \text{Cream Puff} \end{array}$$

(a) Evaluate the matrix $\mathbf{D} = 3\mathbf{P}$. [1]

(b) A chicken pie is sold at \$2. A tuna puff is sold at \$1.80. A cream puff is sold at \$1.40. Represent these prices in a 1×3 matrix **C**. [1]

(c) Evaluate the matrix $\mathbf{Q} = \mathbf{CD}$. [2]

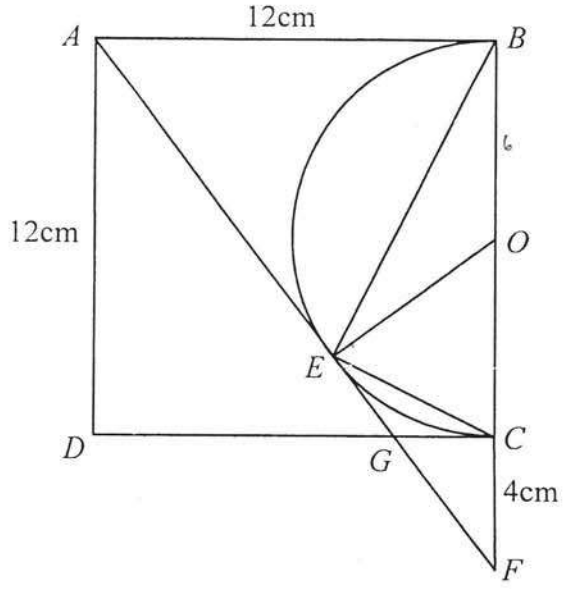
(d) State what the elements of **Q** represent. [1]

(e) In a particular month, Outlet 1 was opened for 22 days and Outlet 2 was opened for 30 days. Write down a 2×2 matrix **E** such that the matrix $\mathbf{F} = \mathbf{QE}$, where the elements of **F** represent the amount of money collected from selling all the pastries in each outlet in that month.

Hence evaluate **F**. [2]

(f) There are two promotion schemes proposed. Scheme A is price reduction of 10% on all pastries and Scheme B is price reduction of 20% on chicken pies only. Supposing all pastries at each outlet are sold at the end of a day, calculate the daily amount of money collected at each outlet from the sale of pastries under each promotion schemes. Hence, propose the suitable scheme for each outlet. [2]

4 $ABCD$ is a square with sides 12cm and BCE is a semi-circle with centre O. AF is a tangent to the semi-circle at point E and $BOCF$ is a straight line.



(a) Name a triangle similar to ΔADG. [1]

(b) Show that ΔOEF is similar to ΔABF. [2]

(c) Find the length of EF. [2]

(d) Find the ratio

(i) $\dfrac{\text{Area of } \Delta EFC}{\text{Area of } \Delta ECB}$, [1]

(ii) $\dfrac{\text{Area of } \Delta OEF}{\text{Area of quadrilateral } ABOE}$. [2]

5 Xavier and Yves were running on a 400m track in Bishan Stadium. Both of them started from the same point. Xavier started running at a speed of x m/s. At the same time, Yves also started running in the opposite direction at speed of $(x+3)$ m/s and he met Xavier on the track again after t s.

(a) Write down two expressions that each represents the distance travelled by Xavier and Yves after t s. [2]

(b) Using the two expressions of distances in part **(a)**, show that $t = \dfrac{400}{2x+3}$. [1]

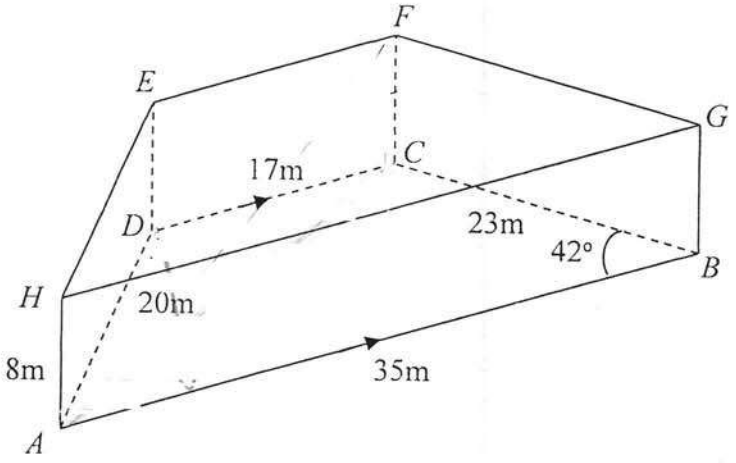
(c) Zed started off together with the 2 boys and he ran in the same direction as Yves. However, he ran at a speed of $(x - 1)$ m/s. Find, in terms of x, the time taken when Xavier and Zed met each other on the track again. [1]

(d) Given that Xavier met Zed 20 seconds after passing Yves, form an equation in terms of x and show that it simplifies to $4x^2 + 4x - 83 = 0$. [3]

(e) Solve the equation $4x^2 + 4x - 83 = 0$. [3]

(f) Find the time taken by Xavier to complete running one round around the track. [1]

6 The diagram shows a solid trapezoidal prism. The base $ABCD$ is on flat ground and $ABGH$, $BCFG$, $CDEF$ and $ADEH$ are vertical rectangular planes. $AB = 35$m, $BC = 23$m, $CD = 17$m, $AH = 8$m, $AD = 20$m, $\angle ABC = 42°$ and AB is parallel to DC.



Calculate

(a) the length of AC, [3]

(b) the angle of depression of A from F, [2]

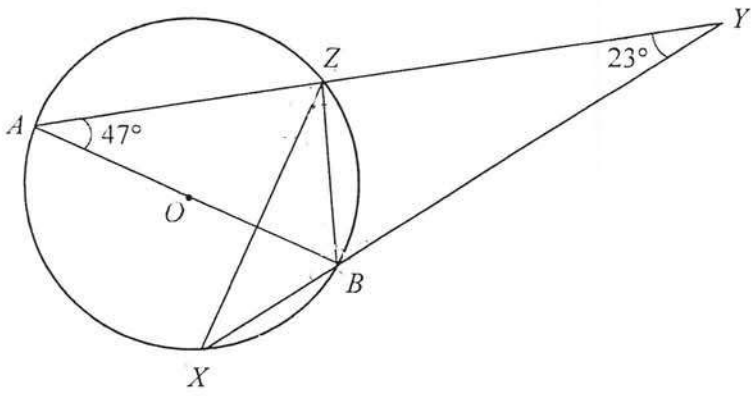
(c) the area of triangle ABC. [2]

(d) the perpendicular distance from point D to the line AB, [2]

(e) the surface area of the prism. [3]

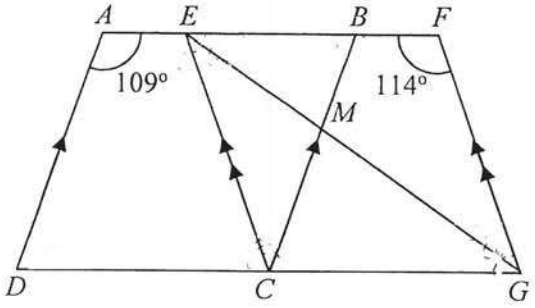
7 **(a)** In the figure below, AB is a diameter of the circle with centre O. AZY and XBY are straight lines. $\angle BAZ = 47°$ and $\angle BYZ = 23°$.

Stating your reasons clearly, find $\angle BZX$.



[3]

(b) The above diagram shows two rhombuses $ABCD$ and $EFGC$. $\angle BFG = 114°$ and $\angle DAE = 109°$. Stating your reasons clearly, calculate



(i) $\angle FBC$, [1]

(ii) $\angle FGE$, [2]

(iii) $\angle EMB$. [2]

8 (a) Dice A has the numbers 1, 2, 3 and 4 engraved on it while dice B has the numbers 2, 3, 5 and 7 engraved on it. They are rolled one after another and the sum of the two rolls is then recorded on a possibility diagram.

(i) Complete the possibility diagram. [2]

		Dice A			
Dice B	+	1	2	3	4
	2		4	5	
	3	4	5		7
	5	6	7		9
	7		9	10	

(ii) Find the probability that both dice show odd numbers. [1]

(iii) Find the probability that the sum of the two numbers is prime. [1]

(iv) Find the probability that the sum of the two numbers is at most 9. [1]

8 (b) A group of 23 basketball players were asked how many points they scored in a season of matches. The results are presented in the following stem-and-leaf diagram.

4	3	5	6	7	7	9	
5	1	2	4	6	8		
6	0	3	4	6	7	8	9
7	1	1	2	3			
8	9						

Key: 4 | 3 means 43 points

(i) Find the range. [1]

(ii) Find the median score. [1]

(iii) A special award is given to players who scored more than 75 points in a season. Find the percentage of players who attained this award. [1]

(iv) A moderation has to be done and 2 points are to be added across all the scores. Explain how the median score is affected by this moderation. [1]

(v) Find the standard deviation of the scores. [2]

(vi) The standard deviation of the scores of another group of players was 7.96 points. Use this information to comment on one difference between the two groups. [1]

9 The variables x and y are connected by the equation

$$y = 2x + \frac{18}{x^2}.$$

Some corresponding values of x and y are given in the following table.

X	1	1.5	2	3	4	5	6
Y	20	11	a	8	9.1	10.7	12.5

(a) Find the value of a. [1]

(b) Draw the graph on the graph paper provided and staple it at the end of this question paper.

Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $0 \le x \le 6$.

Using a scale of 1 cm to 1 unit, draw a vertical y-axis for $0 \le y \le 22$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to solve $2x + \frac{18}{x^2} = 12$ in the range $0 \le x \le 6$. [2]

(d) By drawing a tangent, find the gradient of the graph at the point where $x = 1.5$. [2]

(e) On the same axes, draw the graph of $y = x + 10$ for $0 \leq x \leq 6$. [1]

(i) Write down the x-coordinate of the point where the two graphs intersect. [1]

(ii) Given that this value of x is a solution to the equation

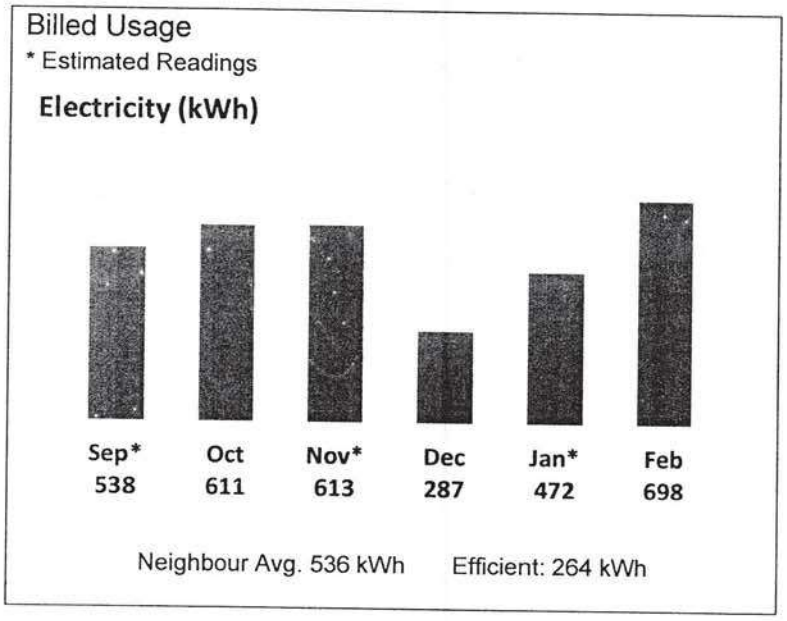
$$x^3 + Ax^2 + Bx + 18 = 0,$$

find the value of A and the value of B. [2]

10 The Open Electricity Market is an initiative by the Energy Market Authority (EMA) that allows households to buy electricity from a retailer of their choice to benefit from their competitive pricing or continue to buy from SP Group at the regulated tariff rate, which is reviewed every quarter. There are two types of standard price plans:

- **Fixed Price Plans** are for consumers who prefer a constant electricity rate throughout the contract duration. The rate may be higher or lower than the regulated tariff rate during the contract duration.
- **Discount off the Regulated Tariff Plans** provide a fixed discount off the regulated tariff rate, but the electricity rates will change when the regulated tariff rates changes every quarter.

Jimmy is reviewing some price plans of two retailers. He stays in a private condominium and a part of his utilities bills is given below.



(a) Calculate the mean value of the electricity usage over the 6 months. Explain why this mean may not be an appropriate average to assist Jimmy in reviewing his options. [2]

(b) **The fact sheets of two of the retailers are shown in the INSERT.**

(i) The electricity retailers charge a fee if you terminate the contract before its expiry date. If Jimmy signs a **12-Month Fixed Price Plan** contract with **Sembcorp Power Pte Ltd** starting on 1 January 2019 but considers to switch over to **iSwitch Pte Ltd** from 1 March 2019 onwards, calculate the early termination charge that he as to pay. [2]

(ii) Jimmy forgets to pay his bill for the electricity usage in the month of February 2019 before the payment due date in March. Considering a **12-Month Fixed Price Plan** contract, which company imposes a higher late payment charge? Justify your answer and show your calculations clearly. [2]

(iii) By considering the period of 1 January to 28 February 2019, suggest the plan in **iSwitch Pte Ltd** that Jimmy should choose. Justify your suggestion and show your calculations clearly. [4]

END OF PAPER

INSERT FOR QUESTION 10

The fact sheets of two of the retailers are shown below. All fees and charges stated in these fact sheets are inclusive of Goods and Services Tax (GST).

Name of retailer	**iSwitch Pte Ltd**	
Price Plan	**'Chope' the Rate (12 Months)**	**Super Saver Discount (12 Months)**
Electricity Rate	17.62 cents/kWh	22.80% off the regulated tariff
Late Payment Charge	1% of the outstanding amount per month	
Early Termination Charge	• $100 if terminated within 3 days from sign-up • 30% of months left × average of latest 2 months bill if terminated after 3 days from sign-up	

Name of retailer	**Sembcorp Power Pte Ltd**	
Price Plan	**12-Month Fixed Price Plan**	**12-Month Discount off Regulated Tariff Plan**
Electricity Rate	18.65 cents/kWh	20.50% off the regulated tariff
Late Payment Charge	$5.35 per bill	
Early Termination Charge	Termination Rate × Unexpired Months Termination Rate per month: HDB 1-Room $10.70 / HDB 2-Room $16.05 / HDB 3-Room $21.40 / HDB 4-Room $32.10 / HDB 5-Room $37.45 / HDB Executive & Private Condominium $42.80 / Terrace $74.90 / Semi-Detached $101.65 / Bungalow $214	

(Adapted from source: https://www.ema.gov.sg/openelectricitymarket.aspx)

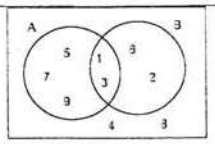
The historical electricity tariff rate of SP Group is shown below.

SP Group Electricity Tariff Rates (cents/kWh) are exclusive of 7% Goods and Services Tax (GST)					
January 2018	April 2018	July 2018	October 2018	January 2019	April 2019
21.56	22.15	23.65	24.13	21.85	22.79

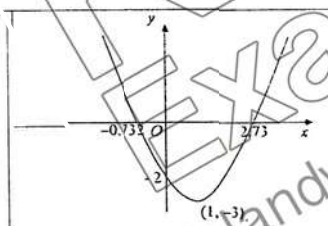
(Adapted from source: https://www.spgroup.com.sg)

AMKSS MID YEAR EXAMINATION 2019

4E/5N ELEMENTARY MATHEMATICS PAPER 1

NO	SOLUTIONS	MARKS
1a	$x+7-x(2-3x)$ $=x+7-2x+3x^2$ $=3x^2-x+7$	 M1 A1
1b	$4ax+12a-x-3$ $=4a(x+3)-(x+3)$ $=(4a-1)(x+3)$	 M1 A1
	Misleading feature: Chocolate and Almond flavours have equal percentage but the size of the sectors on the pie chart do not look equal. Effect of this feature on the graph It mislead readers into believing that Chocolate cookies is selling better than Almond cookies.	 B1 B1
OR	Misleading feature: The title is biased. Effect of this feature on the graph It does not allow readers to make their own judgement.	B1 B1
3a	$X \cap Y'$	B1
3bi		 B1
3bii	4, 8	B1
4a	x^2-2x-2 $=x^2-2x+1-2-1$ $=(x-1)^2-3$	 B2

11	$1.3\times0.8=1.04$ % change $=104-100=4\%$	M1 A1
12a	$2^3\times3^2\times5$ Accept $2\times2\times2\times3\times3\times5$	B2
12b	H.C.F. $=2\times3^2=18$	B1
12c	$m=2^2\times3^4\times5^3=1012500$	B1
13	$\frac{9}{\sin 42}=\frac{8}{\sin\angle ACB}$ $\sin\angle ACB=\frac{8\sin 42}{9}$ $\angle ACB=36.49714719=36.5^\circ$	 M1 A1
14	Area of sector $OCB=\frac{1}{2}\times10^2\times1.2=60$ cm^2 $\angle CAD=\frac{1.2}{2}=0.6$ rad Area of sector $ACD=\frac{1}{2}\times16^2\times0.6=76.8$ cm^2 $\Delta OAC=\frac{1}{2}\times10\times6\times\sin 0.6=45.17139787$ cm^2 $\wedge CD$ = sector $ACD-\Delta OAC=31.62860213$ cm^2 Shaded area = sector $OCB-\wedge CD$ $=28.37139787=28.4$ cm^2	M1 M1 M1 A1
15ai	30	B1
15aii	$36-23$ $=13$	M1 A1
15b	33 (≥ 11)	B1
16a	630	B1
16b	$\frac{n(n+1)}{2}=1378$ $n^2+n-2756=0$ $(n+53)(n-52)=0$ $n=-53$ or 52 $n=52$	 M1 A1 or B2
16c	$T=(1+2+3+\ldots+99)+99(100)$ $=\frac{99(99+1)}{2}+9900$ $=14850$	 M1 A1

4bi		shape of slope B1 for x intercepts B1 for y intercept and correct shape
5	$1550\times81.7339\times1.018$ =¥128967.92	M1 A1
6	$\frac{4}{3-x}+\frac{1}{x^2-3x}$ $=\frac{-4}{x-3}+\frac{1}{x(x-3)}$ $=\frac{1-4x}{x(x-3)}$ or $\frac{4x-1}{x(3-x)}$	
7a	$5\times0.6=3$ km	B1
7b	1 cm^2 : 25 km^2 $\frac{5}{25}$ $=0.2$ km^2	 M1 A1
8	$28+(60\times20\times0.035)+(60\times6\times0.1)$ $=28+42+36$ =$106	M1 A1
9	$\frac{A_y}{A_x}=\frac{4}{1}=\left(\frac{2}{1}\right)^2$ $\frac{V_y}{V_x}=\left(\frac{2}{1}\right)^3=\frac{8}{1}$ $V_y=8\times512=4096$ cm^3	M1 M1 A1
10a	$PR=\frac{28\times2}{7}=8$ units $R=(0,\ -3)$	M1 A1
10b	Gradient $=\frac{3}{7}$ $y=\frac{3}{7}x-3$	M1 A1

16d	$P=2(1+2+3+\ldots+50)$ $=2\times\left(\frac{50\times51}{2}\right)$ $=2550$	M1 A1
16e	$\frac{100(101)}{2}-2550=2500$	B1
17a	0.1	B1
17bi	0.015 Accept $\frac{3}{200}$	B1
17bii	$0.9\times0.15\times0.85=0.11475$ Accept $\frac{459}{4000}$	B1
17c(i)	$(0.9)^3=0.729$ Accept $\frac{729}{1000}$	B1
17c(ii)	$1-0.729=0.271$ Accept $\frac{271}{1000}$	B1
18a	$\sqrt{4^2+(-5)^2}=6.403124237$ $=6.40$ (3sf)	B1
18b	$\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}$ $\overrightarrow{OC}=\begin{pmatrix}-5\\2\end{pmatrix}+\begin{pmatrix}7\\0\end{pmatrix}=\begin{pmatrix}2\\2\end{pmatrix}$ $C=(2,2)$	 M1 A1
19a	$x=\frac{56+64}{2}=60$ $y=\frac{72+76}{2}=74$	B1 B1
19b	60	B1
20	$R^2=(10-R)^2+5^2$ $R^2=100-20R+R^2+25$ $20R=125$ $R=6.25$	M1 M1 A1

Solutions to 4E EM P2 2019

Qn	Solutions	Marks
1ai	$y+z=\frac{4y-6z}{7x}$ $7x-1=\frac{8+6}{7x}$ $1=\frac{14}{7x}$ $7x=14$ $x=2$	B1
1aii	$y+z=\frac{4y-6z}{7x}$ $7xy+7xz=4y-6z$ $7xz+6z=4y-7xy$ $z(7x+6)=4y-7xy$ $z=\frac{4y-7xy}{7x+6}$	M1 for expansion A1
1b	$\frac{2a-1}{3}+\frac{a-2}{4}=2$ $\frac{4(2a-1)+3(a-2)}{12}=2$ $8a-4+3a-6=24$ $11a-2=24$ $11a=26$ $a=2\frac{4}{11}$	M1 for single fraction A1
1c	$\times 5,\ 20p+15q=10$ ---(1) $\times 4,\ 20p-16q=196$ ---(2) (1) - (2), $31q=-186$ $q=-6$ $p=5$	M1 for $20p$ or $15q$ for both eq A1 A1
1d	$\frac{4h^2-36}{4h^2-5h-21}$ $=\frac{4(h+3)(h-3)}{(4h+7)(h-3)}$ $=\frac{4(h+3)}{4h+7}$	M1 factorise numerator M1 factorise denominator A1

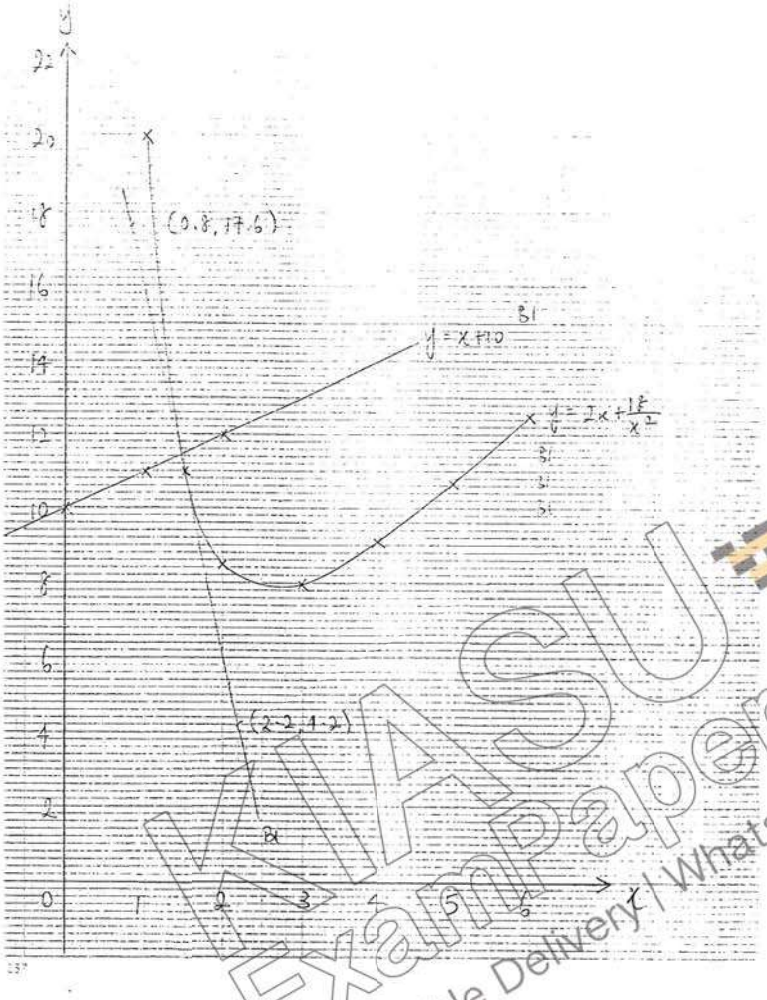
21ai	$(x^{-1}y)^3=\frac{x^{-3}}{y^3}$	B1
21aii	$\frac{18x^{\frac{1}{2}}}{4\sqrt{x}}=\frac{18x^{\frac{1}{2}}}{4x^{\frac{1}{2}}}$ $=\frac{9}{2x^{\frac{1}{2}}}$	M1 for rewriting as fractional index A1
21b	$3^{5x+1}=3^{4x+1}$ $\left(\frac{2}{3}\right)^{5x+1}=1$ $\left(\frac{2}{3}\right)^{5x+1}=\left(\frac{2}{3}\right)^0$ $5x+1=0$ $x=\frac{1}{5}$	M1 A1
22a	$QR=TS$ (regular polygon) RS is common $\angle QRS=\angle TSR$ (interior $\angle$ of regular polygon) $\Delta QRS\equiv\Delta TSR$ (SAS)	M1 M1
22bi	$n=\frac{360}{30}=12$	B1
22bii	interior $\angle=180-30=150$ $\angle RTS=\frac{180-150}{2}=15°$ (base $\angle$s of isos Δ)	M1 A1
22biii	Sum of interior $\angle$s of $PQRST=(5-2)\times180=540°$ $\angle QPT=\frac{540-3(150)}{2}=45°$	M1 A1
OR	$\angle AQS=15+30=45°$ $\angle QPT=\angle AQS=45°$ (corresponding $\angle$s)	* must state corresponding $\angle$s

2ai	$-2\mathbf{a}+6\mathbf{b}$	B1
2aii	$\overrightarrow{OB}=\overrightarrow{OX}+\overrightarrow{XB}$ $=6\mathbf{b}+3(-2\mathbf{a}+6\mathbf{b})$ $=-6\mathbf{a}+24\mathbf{b}$	M1 for triangle law A1
2aiii	$\overrightarrow{AY}=\overrightarrow{AO}+\overrightarrow{OY}$ $=-2\mathbf{a}+(6\mathbf{b}-2\mathbf{b})$ $=-2\mathbf{a}+8\mathbf{b}$	M1 for triangle law A1
2b	$\overrightarrow{OB}=-6\mathbf{a}+24\mathbf{b}$ $=3(-2\mathbf{a}+8\mathbf{b})$ $=3\overrightarrow{AY}$ OB is parallel to AY and $OB=3AY$	1 mark for presentation M1 for vector expression A1 for both facts
3a	$\mathbf{D}=\begin{pmatrix}180 & 240\\ 90 & 150\\ 240 & 300\end{pmatrix}$	B1
3b	$\mathbf{C}=(2\quad 1.8\quad 1.4)$	B1
3c	$\mathbf{Q}=(2\quad 1.8\quad 1.4)\begin{pmatrix}180 & 240\\ 90 & 150\\ 240 & 300\end{pmatrix}$ $=(858\quad 1170)$	M1 A1
3d	Elements of Q represent the money collected by each outlet from selling all the pastries in a day.	B1
3e	$\mathbf{E}=\begin{pmatrix}22 & 0\\ 0 & 30\end{pmatrix}$ $\mathbf{F}=(858\quad 1170)\begin{pmatrix}22 & 0\\ 0 & 30\end{pmatrix}$ $=(18876\quad 35100)$	B1 B1

3f	Scheme A: $0.8 \times Q = 0.8 \times (358 \quad 1170) = (772.2 \quad 1053)$ $= (686.4 \quad 936)$ Scheme B: $(0.8\times2 \quad 1.8 \quad 1.4)\begin{pmatrix}180 & 240\\ 90 & 150\\ 240 & 300\end{pmatrix}$ $= (786 \quad 1074)$ Scheme B allows both outlets to have a larger amount of money collected from the sale of all pastries.	B1 for the money collected from each outlet for each scheme B1
4a	ΔFCG or ΔFBA	B1
4b	$\angle OEF = 90^\circ$ (rad $\perp$ tan) $\angle ABF = 90^\circ$ (angle in a square) $\angle OEF = \angle ABF$ $\angle F$ is common. $\angle EOF = \angle BAF$ ($\angle$ sum of Δ) Since all corresponding angles are equal, ΔOEF is similar to ΔABF.	M1, M1 for each angle
4c	$\frac{EF}{BF} = \frac{OE}{AB}$ $\frac{EF}{12+4} = \frac{6}{12}$ $EF = 8\ cm$	M1 A1
4di	Triangles share same height. $\frac{4}{12} = \frac{1}{3}$	B1
4dii	$\frac{Area\ \Delta OEF}{Area\ \Delta ABF} = \left(\frac{OE}{AB}\right)^2 = \left(\frac{6}{12}\right)^2 = \frac{1}{4}$ $\frac{Area\ \Delta OEF}{Area\ ABOE} = \frac{1}{3}$	M1 A1 or B2
5a	Xavier $= xt$ m Yves $= (x+3)t$ m	B1 B1
5b	$xt + (x+3)t = 400$ $xt + xt + 3t = 400$ $2xt + 3t = 400$ $t(2x+3) = 400$	M1 equate sum of distances to 400 and expand
5c	Yves $= xT$ m Zed $= (x-1)T$ m $xT + (x-1)T = 400$ $xT + xT - T = 400$ $T = \frac{400}{2x-1}$	B1
5d	$T - t = 20$ $\frac{400}{2x-1} - \frac{400}{2x+3} = 20$ $\frac{400(2x+3) - 400(2x-1)}{(2x-1)(2x+3)} = 20$ $\frac{800x + 1200 - 800x + 400}{4x^2 + 6x - 2x - 3} = 20$ $1600 = 20(4x^2 + 4x - 3)$ $1600 = 80x^2 + 80x - 60$ $80x^2 + 80x - 1660 = 0$ $\div 20,\ 4x^2 + 4x - 83 = 0$	M1 for correct difference of time M1 for single fraction M1 for expansion and cross multiply
5e	$x = \frac{-4 \pm \sqrt{16 + 1328}}{8} = \frac{-4 \pm \sqrt{1344}}{8}$ $= 4.082575695$ or -5.082575695 $= 4.08$ or -5.08	M1 A1, A1 for each answer
5f	$\frac{400}{4.082575695}$ $= 97.97736279$ $= 98.0s$	B1
6a	$AC^2 = 23^2 + 35^2 - 2(23)(35)\cos 42^\circ$ $= 557.536831$ $AC = 23.61221783 = 23.6m$	M1 M1 A1
6b	$\tan\theta = \frac{8}{AC}$ $\theta = \tan^{-1}\frac{8}{23.61221783} = 18.71677335 = 18.7^\circ$	M1 A1
6c	Area $= \frac{1}{2}(23)(35)\sin 42^\circ$ $= 269.3250691 = 269$ m	M1 A1
6d	Area of triangle ABC = Area of triangle ABD $269.3250691 = \frac{1}{2} \times 35 \times d$ $d = 15.39000395 = 15.4m$	M1 A1

6e	Top + Bottom Areas = $2 \times \frac{1}{2} \times 15.39000395 \times (17+35) = 800.2802052m^2$ Vertical sides $= (20+17+23+35) \times 8 = 760m^2$ Total surface area $= 1560.2802052 = 1560\ m^2$	M1 M1 A1
7a	$\angle BXZ = 47^\circ$ (angles in same segment) $\angle BZA = 90^\circ$ (angle in semi-circle) $\angle BZY = 180^\circ - 90^\circ$ (angles on a straight line) $\angle BZX = 180^\circ - 47^\circ - 90^\circ - 23^\circ = 20^\circ$ (angle sum of Δ)	M1 M1 Minus 1mark if any above reasons missing A1
7bi	109° (corresponding angle)	Minus 1mark from entire qn if any underlined reasons missing B1
7bii	$\frac{180 - 114}{2} = 33^\circ$ (EF = FG as sides of rhombus and base angles of isosceles triangle)	M1 A1
7biii	$\angle EBM = 180 - 109 = 71^\circ$ (interior angles) $\angle EMB = 180 - 71 - 33 = 76^\circ$ (angle sum of Δ)	M1 A1
8ai	3, 6, 6, 8, 8, 11	B1 for any 3 B1 for next 3
8aii	$\frac{6}{16} = \frac{3}{8}$	B1
8aiii	$\frac{6}{16} = \frac{3}{8}$	B1
8aiv	$\frac{14}{16} = \frac{7}{8}$	B1
8bi	$89 - 43 = 46$	B1
8bii	60	B1
8biii	$\frac{1}{23} \times 100\% = 4.35\%$	B1
8biv	Add 2 points across all score => median increased by 2 or median changed to 62.	B1
8bv	11.46062545=11.5 (3sf)	B2
8bvi	The group with the lower standard deviation of 7.96 points has players with more consistent scores.	B1
9a	$a = 8.5$	B1
9b	Refer to graph (3m)	
9c	$x =$ 1.4 and 5.7 (accept 1.3 to 1.5 and 5.6 to 5.8) (calculator 1.399 and 5.73)	B1 each

9d	Refer to graph (1m) Gradient $= \frac{4.2 - 17.6}{2.2 - 0.8} = -9.57$ (accept -9.5 to -11) (calculator -8.667)	A1
9e	Refer to graph (1m)	
9ei	$x = 1.45$ (accept 1.35 to 1.55) (calculator 1.45)	A1
9eii	$x + 10 = 2x + \frac{18}{x^2}$ $x^3 + 10x^2 = 2x^3 + 18$ $0 = x^3 - 10x^2 + 18$ $A = -10, B = 0$	M1 for sub eq A1 for both answer
10a	Mean $= \frac{538 + 611 + 613 + 287 + 472 + 698}{6} = 536.5$ kWh There is an extreme lower value in Dec as compared to the other 5 months' figures so this will make the mean value inaccurate.	A1 A1
10bi	42.80×10 $= \$428$	M1 A1
10bii	Feb – 698 kWh iSwitch: $\frac{17.62 \times 698}{100} \times \frac{1}{100}$ $= \$1.23$ Sembcorp: \$5.35 Sembcorp is more expensive	M1 to calc iSwitch A1 for correct comparison
10biii	Usage = 1170 kWh Chope the rate: $\frac{17.62 \times 1170}{100} = \$206.15$ Super Saver Discount: $\frac{100 - 22.80}{100} \times \frac{21.85}{100} \times 1170 = \$197.35794$ Add 7% GST $= 1.07 \times \$197.35794 = \$211.17$ He should choose the fixed rate Chope the rate scheme as it is cheaper and he can save \$5.05	M1 for fixed rate M1 for discount off M1 to include GST A1 only for correct accurate comparison



COMMONWEALTH SECONDARY SCHOOL
MID-YEAR EXAMINATION 2019

MATHEMATICS
PAPER 1

Name: _____ ( ) Class: _____

SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC

Friday 03 May 2019
10 30 – 12 30
2 h

4048/1

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Name of setter: Mr Toinh Long Teng

For Examiner's Use	
Presentation	
Accuracy	
Total	80

Parent's Signature: _____

This paper consists of **17** printed pages including the cover page.

[Turn over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 Write the following in order of size, in descending order.

$$\pi,\ 3.\dot{1}4\dot{2},\ \frac{22}{7},\ 3.14\dot{2}$$

Answer . [2]

---

2 Factorise completely $16p^4 - 81q^4$.

Answer ..... [2]

---

3 Given that $\frac{1}{27} = 9^k$, find k.

Answer $k =$ ..... [2]

---

4 The stem-and-leaf diagram shows the times, in minutes, taken by some students to complete a task.

3	2	4	5	6	
4	0	1	1	9	9

Key: 3 | 4 represents 34 minutes

For these times, find

(a) the lower quartile,

Answer . minutes [1]

(b) the mean,

Answer minutes [1]

(c) the standard deviation,

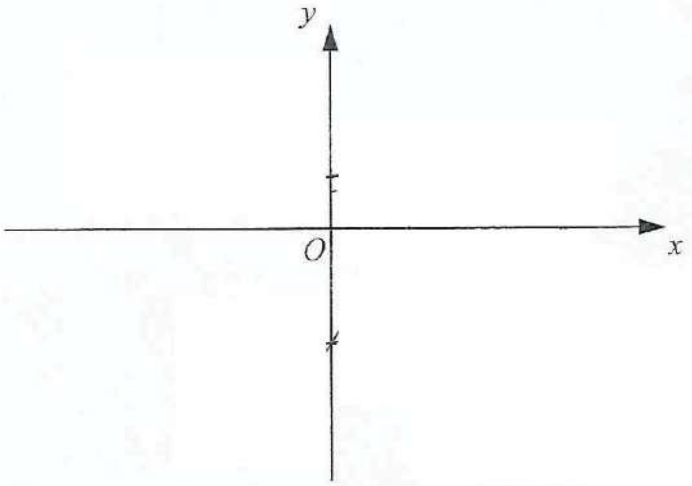
$\sqrt{\frac{x^2}{n} - \bar{x}^2} = 5.81$

Answer minutes [1]

---

5 **Sketch** the graph of $y = -(2-x)^2 + 1$ on the axes below.

Indicate clearly the values where the graph crosses the x- and y-axes and the coordinates of any turning points.



[2]

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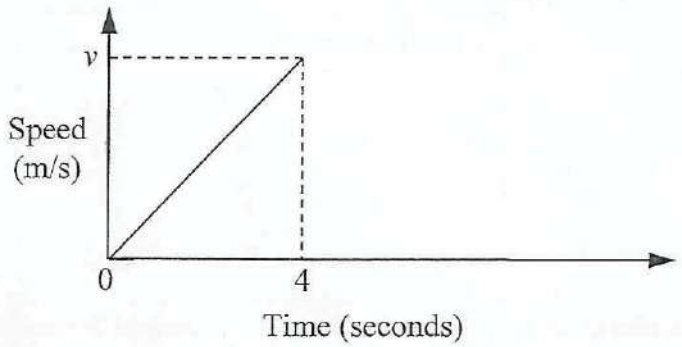
6 The diagram shows the speed-time graph for part of a car's journey between two sets of traffic lights.



The distance travelled in the first 4 seconds is 20 metres.

(a) Calculate the value of v.

Answer $v =$ .. [1]

The car then decelerated at twice the rate of the acceleration.

(b) Calculate the average speed of the car for the entire journey.

Answer . m/s [2]

---

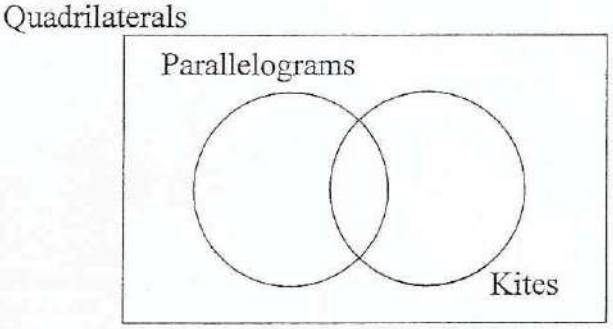
7 A club is made up of members that are either a child, an adult or a senior citizen. There are 19 children and 14 senior citizens. The club wishes to maintain that at most three-fifths of the members are adults.

By forming an inequality, find the maximum number of adults possible.

Answer . [3]

---

8 The Venn diagram illustrates the relationship between two different types of quadrilaterals.



(a) What special shape is represented by the intersection of the sets representing Parallelograms and Kites?

Answer [1]

(b) Using an appropriate symbol, complete the statement:
{Parallelograms}.....{Trapeziums}

Answer [1]

---

9 A sum of money was divided between A, B and C in the ratio 2 : 3 : 4.
If the money had been divided equally between them, A would have received an extra \$40.
What was the total sum of money?

Answer [2]

---

10 The probability that a phone will ring in a thirty-minute interval is $\frac{1}{3}$. What is the probability that the phone will not ring in a one-hour interval?

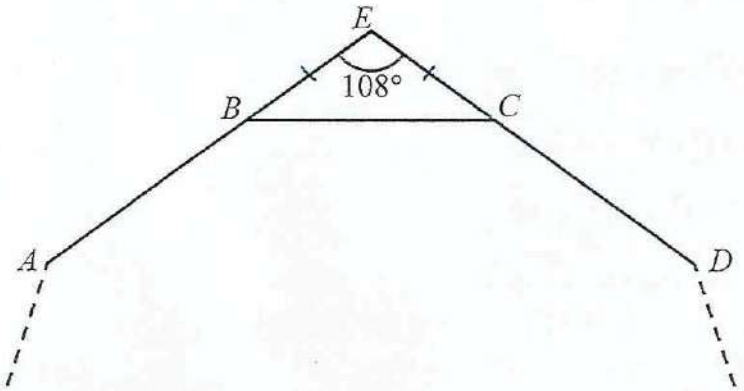
Answer [2]

---

11 Adam can lay 70 bricks in 30 minutes.
Charlie can lay 80 bricks in 45 minutes.
Adam and Charlie work together to lay a total of 1000 bricks.
If they continue to lay bricks at the same rate, how long will it take them to lay the 1000 bricks? Give your answer in hours and minutes, to the nearest minute.

Answer [3]

12 The diagram shows three of the sides, AB, BC and CD of a regular polygon. AB produced and DC produced meets at E. Angle $BEC = 108°$



Find the number of sides of the regular polygon.

Answer [3]

13 Solve $\frac{5x}{14}-\frac{2x-3}{21}=1$.

Answer $x=$.[2]

14 Solve $x^2+4x-11=0$ by using completing the square.

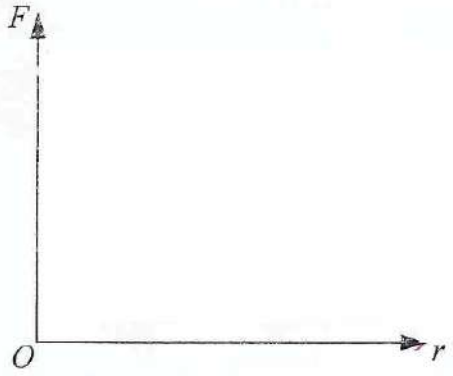
Answer $x=$ or $x=$ [3]

15 Given that the value of $\frac{1}{x^2}+\frac{1}{y^2}=3$ and $xy=4$, find the value of $(x+y)^2$.

Answer .. [3]

16 The attractive force, F newtons, between two stars, is inversely proportional to the square of the distance between the centres of the two stars, r km.

(a) Sketch the graph of F against r.



[1]

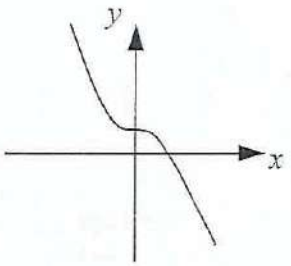
(b) The distance between the centres of star A and star B is 150% larger than the distance between the centres of star C and star D. Find the ratio of the attractive force between star A and star B to the attractive force between star C and star D.

Answer [3]

17 **(a)** Write down a possible equation for each of the sketch graphs below.
In each case select one of the equations from the box below.

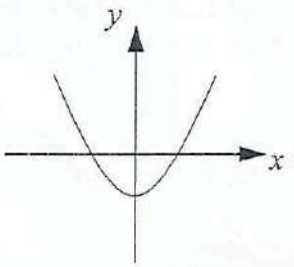
$y = x^2 - 3$	$y = -x^2 + 3$	$y = 3^x$
$y = -x^3 + 3$	$y = 3^{-x}$	$y = x^3 + 3$

(i)



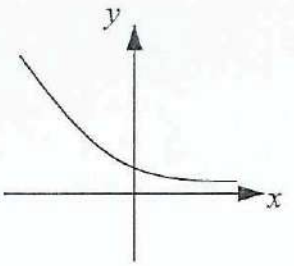
Answer [1]

(ii)



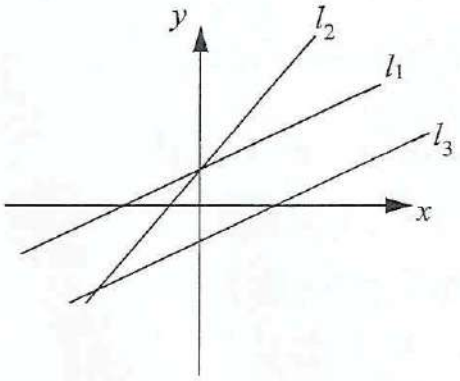
Answer [1]

(iii)



Answer [1]

(b) The diagram shows three lines l_1, l_2 and l_3.



The equation of l_1 is $y = 2x + 1$.

(i) l_1 and l_2 have the same y-intercept. State a possible value of the gradient of l_2.

Answer . [1]

(ii) l_1 and l_3 have the same gradient. State a possible equation of l_3.

Answer .. [1]

---

18 A solid cylinder has radius r cm and height h cm. A solid hemisphere has radius r cm. The total surface area of the cylinder and hemisphere are equal.

Work out, in terms of r, the volume of the cylinder.

Answer . cm^3 [3]

---

19 Daniel invested a sum of money in an account paying compound interest at r% per year. After 3 years, the money had earned **total interest** of 20% of the principal.

Find the value of r, correct to one decimal place.

Answer [3]

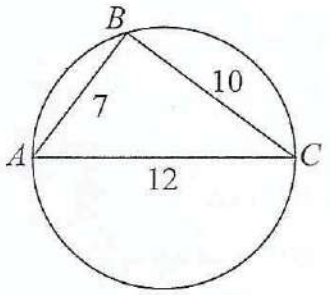
---

20 Two glasses are geometrically similar. The height of the small glass is 9 cm and the height of the large glass is 15 cm.

If a big tank of fruit juice can fill up 125 small glasses, how many large glasses could be filled from the same big tank?

Answer [3]

21



In the diagram, A, B and C are points on a circle. $AB = 7$ cm, $BC = 10$ cm and $AC = 12$ cm. Explain, with reasons, whether AC is a diameter of the circle.

Answer

[3]

---

22 It is given that $T = 2\pi\sqrt{\frac{L}{g}}$.

(a) Calculate the value of T when $L = 1.2$ and $g = 9.81$. Write your answer correct to three decimal places.

Answer .. [1]

(b) Rearrange the formula to make g the subject.

Answer $g =$. [3]

---

23 The Singapore River is 3.2 kilometres long. It is represented on a map with a distance of 2 cm.

(a) Express the scale of the map in the form $1 : n$.

Answer [2]

(b) The length of the Ayer Rajah Expressway is 27 km, corrected to the nearest kilometre. Find the greatest possible distance of the expressway on the map.

Answer . cm [1]

(c) The Jurong Lake District is represented by an area of 1.4 square centimetres on the map. Find the actual area, in square kilometres, of the district.

Answer km^2 [2]

24 (a) Express 84 as a product of its prime factors, leaving your answer in index notation.

Answer .. [1]

(b) Find the highest common factor of 84 and $2\times3^3\times5$.

Answer [1]

(c) The lowest common multiple of 84 and x is $2^3\times3\times7^2$. Find the smallest possible value of x.

Answer . [2]

25 The nth term of a sequence is given by $n(n+3)$.

(a) Find the fifth term.

Answer [1]

(b) One term in the sequence is 154. Find the value of n for this term.

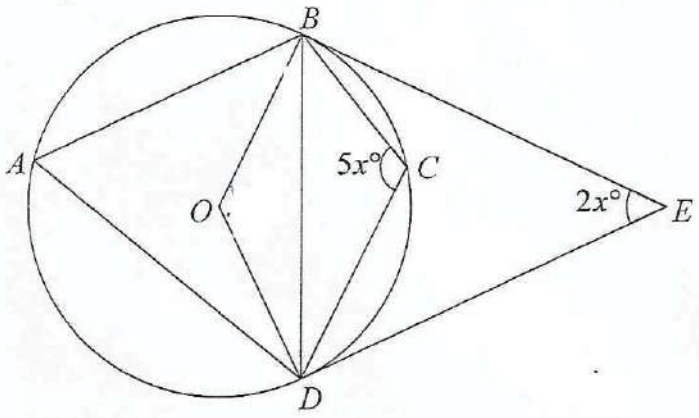
Answer $n =$ [2]

(c) Explain why every term in the sequence is an even number.

Answer

[2]

26



In the diagram, A, B, C and D are points on a circle, centre O. BE and DE are tangents to the circle at points B and D respectively.
Angle $BCD = 5x°$ and angle $BED = 2x°$.

Find x.
Showing your working clearly and give reasons.

Answer $x =$ [4]

END OF PAPER

COMMONWEALTH SECONDARY SCHOOL
MID YEAR EXAMINATION 2019

MATHEMATICS
PAPER 2

Name: _____ ( ) Class: _____

SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC
4048/2

Monday 6 May 2019
08 00 – 10 30
2h 30 min

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Name of setter: Mrs Ang YM
Mr Koh HY

For Examiner's Use	
Total	100

Parent's Signature: _____

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{ , where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{ , where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1. **(a)** Solve the inequality $\frac{3-x}{5} < 1 + \frac{2x+1}{4}$. [2]

(b) Express as a single fraction in its simplest form $\frac{4y}{3-2y} - \frac{y}{(2y-3)^2}$. [2]

(c) Simplify $\frac{(-3h)^2}{8h^3j^5} \div \frac{27h^4j^2}{4j^3}$. [2]

(d) Simplify $\left(\frac{256p^{16}}{q^{20}r^{-4}}\right)^{-\frac{1}{4}}$. [2]

(e) Solve the equation $\frac{10}{x^2-9} - \frac{3}{x+3} = 1$. [3]

---

2. One astronomical unit (1 au) is a unit of length defined as 149597870700 metres, which is roughly the average distance between the earth and the sun.

(a) Express one astronomical unit in metres, correct to 3 significant figures in standard form. [1]

(b) The speed of sound is 343 m/s. How long, in seconds, does sound take to travel a distance of 1 au? Give your answer in standard form, correct to 3 significant figures. [2]

(c) The average distance between the earth and the moon is 384400 km. Express this distance as a percentage of 1 au. [2]

(d) A rocket travels a distance of one metre in 8000 nanoseconds (ns) and $1\text{ns} = 10^{-9}$ s. How long, in seconds, does the rocket take to travel 1 au. Give your answer in standard form, correct to 3 significant figures. [2]

---

3. Answer the whole of this question on a sheet of graph paper.

A stone was projected directly up a slope. Its distance, y metres, from the bottom of the slope, t seconds after it was projected, is given in the table below.

t	0	0.5	1	2	3	4	5	6
y	0	1.80	3.10	5.50	6.85	7.75	8.30	8.40

(a) Using a scale of 2 cm to represent 1 second, draw a horizontal t-axis for $0 \le t \le 6$. Using a scale of 2 cm to represent 1 metre, draw a vertical y-axis for $0 \le y \le 9$.
On your axes, draw the graph of y against t. [3]

(b) Use your graph to find the distance of the stone from the bottom of the slope when $t = 2.5$. [1]

(c) **(i)** By drawing a tangent, find the gradient of the curve at $t = 3$. [2]

(ii) State briefly what this gradient represents. [1]

(d) At the instance the stone was projected, an object was released down the slope from a point 8 metres from the bottom. This object moved directly down the slope at a constant speed of 2 m/s.

(i) On the same axes, draw the graph representing the motion of this object. [1]

(ii) Use your graphs to find when the stone and the object passed each other. [1]

---

4. A is the point $(-2, 7)$ and B is the point $(6, -4)$.

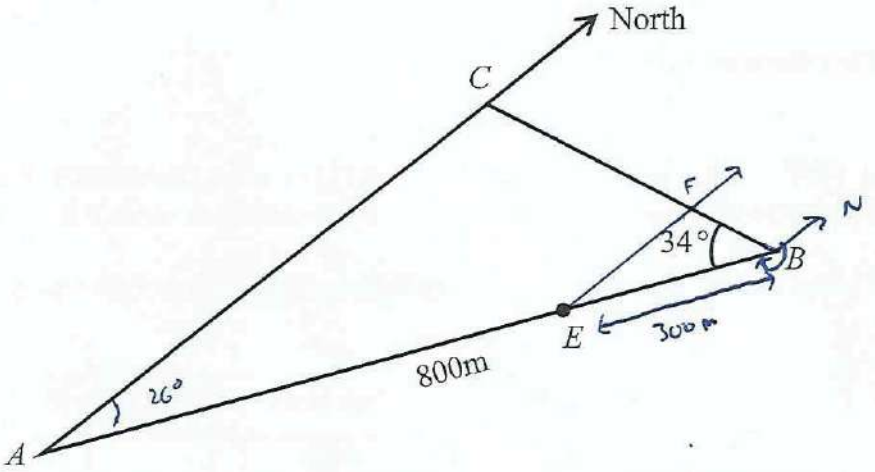
(a) Find the length of the line AB. [2]

(b) Find the equation of the line AB. [2]

(c) The equation of another line l is $8y + 11x = -16$.
Show how you can tell that the line l is parallel to the line AB. [2]

(d) The coordinates of C is $(2, k)$. Given that the points A, B and C are collinear, find the value of k. [2]

5.



Three buoys, A, B and C, are positioned in a lake to provide a course for a water sports event. $AB = 800$ m and A is due south of C. $\angle ABC = 34°$ and the bearing of B from A is $026°$. E is a point on AB which is 300 m from B.

(a) Calculate AC. [2]

(b) Find the area of triangle ABC. [2]

(c) Calculate the bearing of A from B. [2]

(d) A helicopter, H, is hovering at a point vertically above E.

(i) The angle of elevation of the helicopter from B is $11°$. Calculate the vertical height of the helicopter above E. [2]

(ii) F is a point on BC which is nearest to the helicopter. Calculate the angle of depression of F from the helicopter. [3]

6. (a) Given that $\mathbf{P} = \begin{pmatrix} w & 1 \\ 0 & 2 \end{pmatrix}$ and $\mathbf{P}^2 = \begin{pmatrix} 6w-9 & w+2 \\ 0 & 4 \end{pmatrix}$, where w is a constant.

Find the value(s) of w. [3]

(b) A waffle maker produces three different types of waffles: Red bean, Chocolate and Peanut, for distribution to its outlets at various locations.

The table below shows the quantity delivered to each location each time.

	Chocolate	Red bean	Peanut
Outlet 1	225	140	125
Outlet 2	265	115	245
Outlet 3	245	125	175

(i) Represent the data in the above table by a 3×3 matrix A. [1]

(ii) Hence, find, by matrix multiplication, the total number of waffles delivered to each outlet. [2]

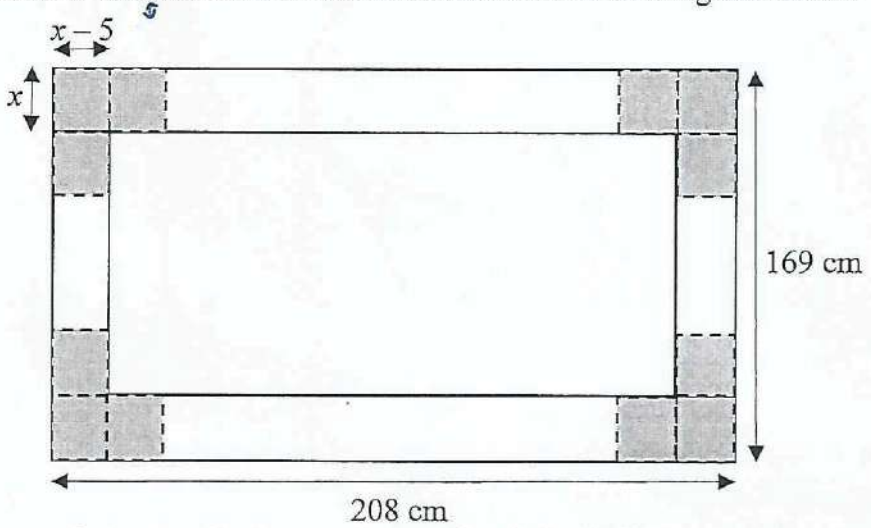
(c) The following table shows the selling price and the cost price of 1 unit of each type of waffle.

	Chocolate	Red Bean	Peanut
Selling Price ($)	1.20	0.80	1.00
Cost Price ($)	0.60	0.50	0.40

(i) Represent the data in the above table by a matrix C such that AC gives the total selling price and total cost price of each outlet. Hence, evaluate AC. [3]

(ii) Find the profit earned by outlet 2. [1]

7. A rectangular wall is 208 cm by 169 cm. The border around the wall is to be covered with tiles. The tiles measure x cm by $(x-5)$ cm. Each tile is placed so that its longer side is vertical. Some of the tiles are shown in the diagram below.



(a) Express, in terms of x, the number of tiles that will fit

(i) across the top row of the border, [1]

(ii) along one vertical side of the border. [1]

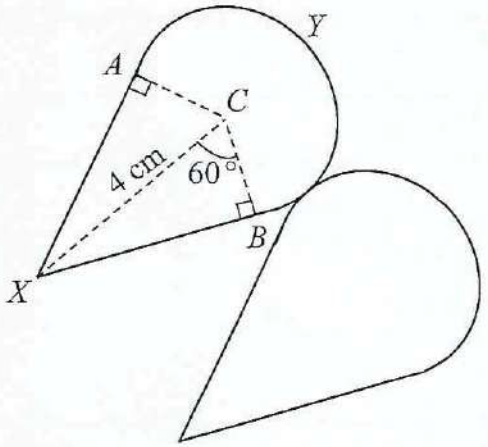
(b) Given that 74 tiles are required to fill the whole border around the wall, form an equation and show that it reduces to $3x^2 - 44x + 65 = 0$. [3]

(c) Solve the equation $3x^2 - 44x + 65 = 0$. [2]

(d) Explain why one of the solutions in part **(c)** must be rejected as the length of the tile. [1]

(e) Find the area of the wall that will be filled with tiles. [2]

8. The diagram shows the cross-section design of the medal for a sports event. It is made up of two identical shapes.



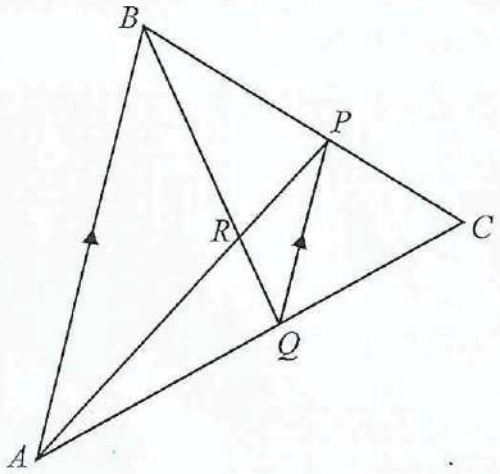
AYB is an arc of a circle with centre C.
XA and XB are tangents to the circle.
$XC = 4$ cm and $\angle XCB = 60°$.

(a) Calculate the length of BC. [2]

(b) Find the total cross-sectional area of the medal. [3]

(c) If the medal has a thickness of 0.7 cm, find the volume of this piece of medal. [2]

9. In the diagram, AB is parallel to QP.



(a) Prove that triangle PQR is similar to triangle ABR. [3]

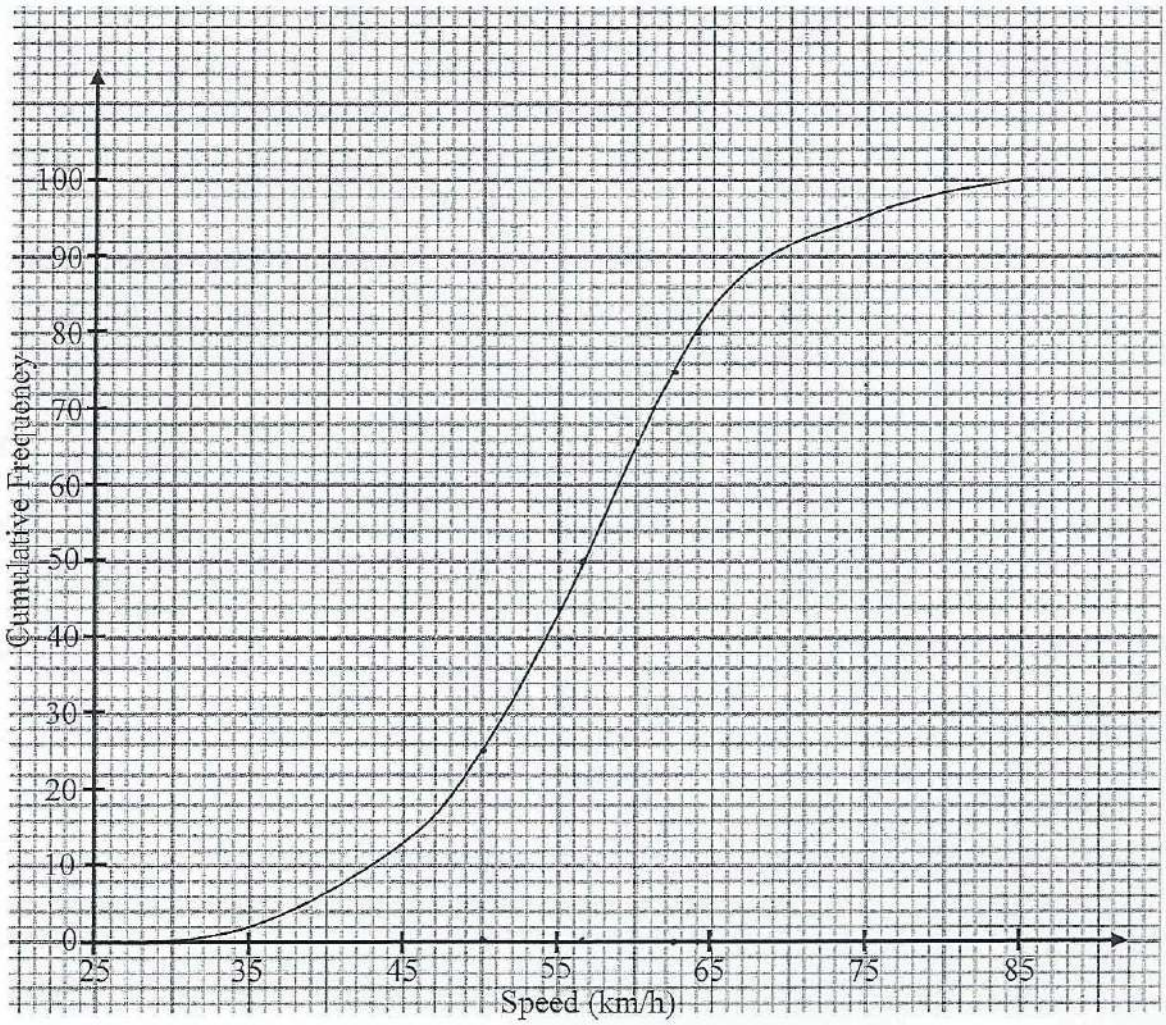
(b) State another pair of similar triangles. [1]

(c) Given that $PQ = 5$ cm, $PR = 4$ cm and $AB = 12$ cm, calculate the length of AR using similar triangles. [2]

(d) Find the numerical value of the ratio of $\dfrac{\text{Area of triangle } PQR}{\text{Area of triangle } ABR}$. [1]

(e) Given that the area of triangle CPQ is 15 cm^2, find the area of $ABPQ$. [2]

10. **(a)** In the afternoon, on a certain stretch of road, the speeds of 100 vehicles were recorded. The cumulative frequency curve shows the distribution of the speeds of the vehicles.



(i) Use the curve to estimate

(a) the median speed, [1]

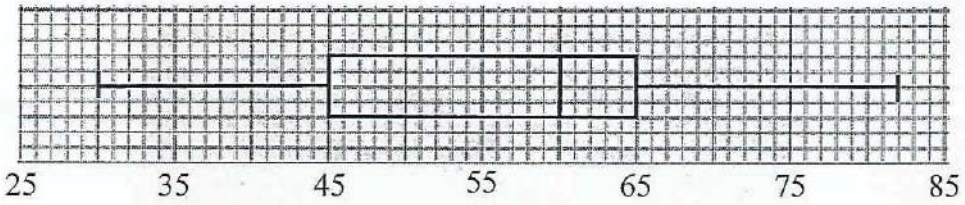
(b) the interquartile range of the speeds, [1]

(c) the 80^{th} percentile, [1]

(d) the percentage of vehicles that travels within the speed limit if the speed limit on the road is 60 km/h. [1]

(ii) From the curve, find the probability that at least one of the two vehicles chosen at random travels at a speed more than 55 km/h. [2]

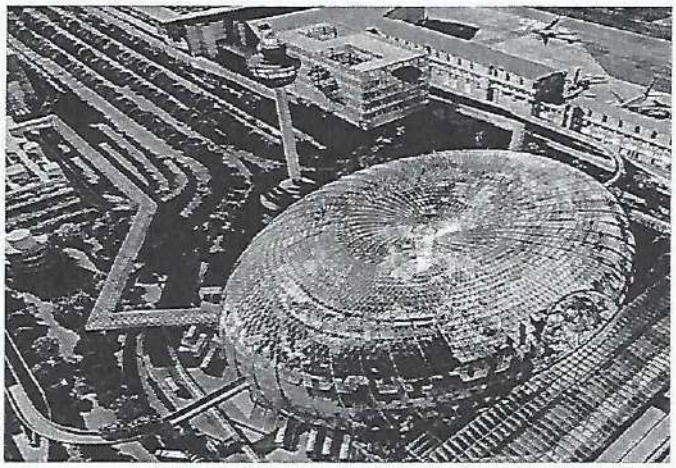
(b) The speeds of 100 vehicles passing the same stretch of road at night were also recorded. The box-and-whisker plot shows the distribution of the speeds.



(i) Make two comments comparing the speeds of the vehicles in the afternoon and at night. [2]

(ii) Find the range of the speeds of the vehicles at night. [1]

11. Jewel Changi Airport is set to become an iconic landmark of Singapore and a key tourist attraction. The building occupies an area of about 134 000 m^2 on the ground.



(a) A map has a scale of 1 : 7500. Calculate the area of the building on the map. [2]

(b) The diagram below shows the view of the building from the top. A Skytrain Bridge runs through the building at about 18 m above the ground of Forest Valley.

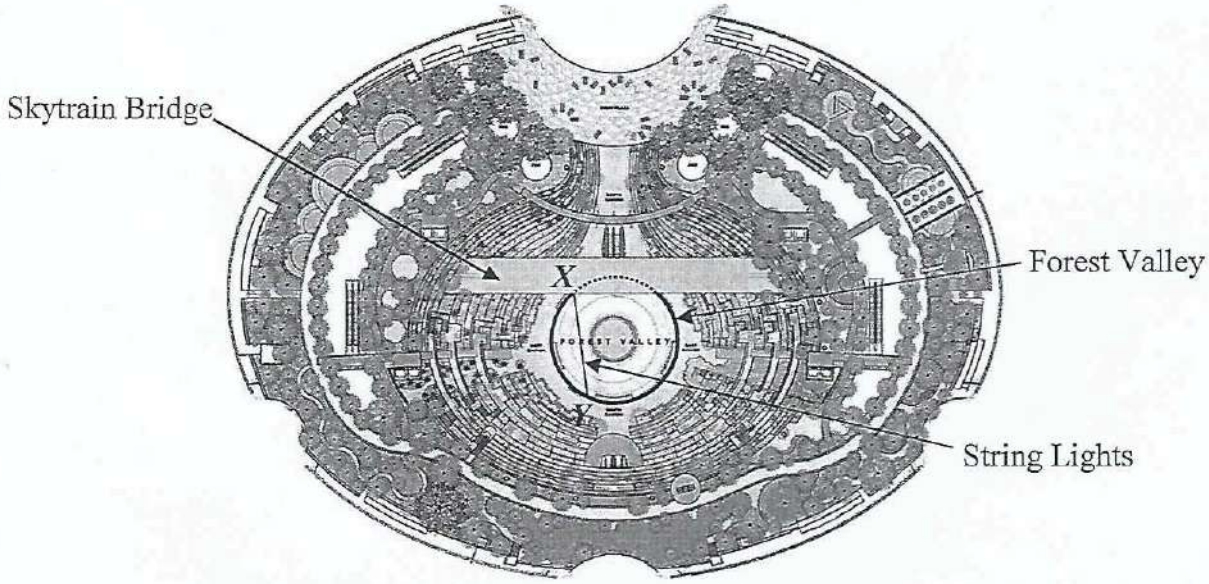
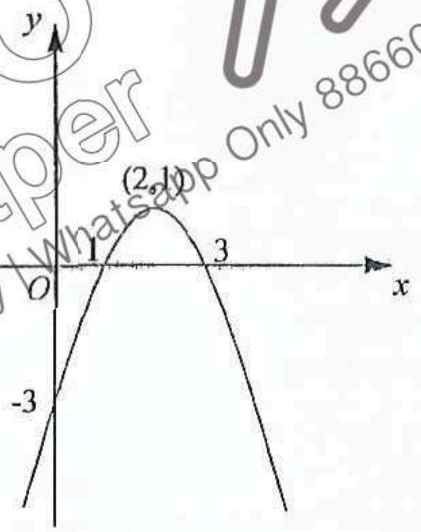


Diagram 1 [Source: Jewel Changi Development & Straits Times Graphic]

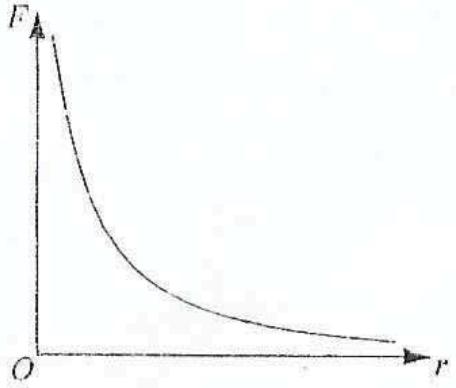
(i) The actual circumference of the whole circular Forest Valley is about 105 m. Find the actual radius of the whole circular Forest Valley in terms of π, in metres. [1]

(ii) Mr Yap, a lighting designer, plans to hang string lights from point X on the skytrain bridge, down to point Y on the ground, which is on the circumference of the Forest Valley as illustrated in diagram 1.
Assuming XY is taut, he claims that he will need at least 34 m of string lights.
Given that Z is another point on the ground, which is also on the circumference of the Forest Valley, vertically below point X, and the minor arc length ZY is 35 m, verify if his claim is true or false, by showing your working clearly. [6]

END OF PAPER

1	$\frac{22}{7}$, $3.14\dot{2}$, $3.\dot{1}4\dot{2}$, π
2	$16p^4-81q^4$ $=\left(4p^2\right)^2-\left(9q^2\right)^2$ $=\left(4p^2-9q^2\right)\left(4p^2+9q^2\right)$ $=(2p-3q)(2p+3q)\left(4p^2+9q^2\right)$
3	$\frac{1}{27}=9^k$ $3^{2k}=3^{-3}$ $k=-1.5$
4a	$\frac{34+35}{2}=34.5$
4b	$39\frac{2}{3}$ or 39.7
4c	$\sqrt{\frac{14465}{9}-\left(39\frac{2}{3}\right)^2}=5.81$ (3 s.f.)
5	
6a	$\frac{1}{2}(4)(v)=20 \Rightarrow v=10$
6b	$t=6$ total distance $=\frac{1}{2}(6)(10)=30$ avg speed $=\frac{30}{6}=5$ m/s

7	Let x be the number of adults. $\frac{x}{x+19+14} \le \frac{3}{5}$ $5x \le 3x+99$ $x \le 49.5$ Therefore, the maximum number of adults is 49.
8a	Rhombus
8b	$\subset$ or $\subseteq$
9	1 unit - \$40 Total sum of money = $40 \times 9 = \$360$
10	$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$
11	Adam's rate $= \frac{70}{30}$ Charlie's rate $= \frac{80}{45}$ Total rate $= \frac{70}{30} + \frac{80}{45} = \frac{37}{9}$ $\text{time} = \frac{1000}{\frac{37}{9}} = 243.24 \text{ min } (5 \text{ s.f.}) = 4 \text{ hours } 3.24 \text{ min}$ Time taken is 4 hours 4 minutes.
12	$\angle EBC = \frac{180° - 108°}{2} = 36° \Rightarrow \text{no. of sides} = \frac{360°}{36} = 10$
13	$\frac{5x}{14} - \frac{2x-3}{21} = 1$ $15x - 2(2x-3) = 42$ $15x - 4x + 6 = 42$ $11x = 36 \Rightarrow x = \frac{36}{11} = 3\frac{3}{11}$
14	$(x+2)^2 - 4 - 11 = 0$ $(x+2)^2 = 15$ $x+2 = \pm\sqrt{15}$ $x = -2 \pm \sqrt{15} \Rightarrow x = 1.87 \text{ or } -5.87 \ (3 \text{ s.f.})$
15	$\frac{1}{x^2} + \frac{1}{y^2} = 3$ $x^2 + y^2 = 3x^2y^2$ $(x+y)^2 = 3x^2y^2 + 2xy$ $= 3(4)^2 + 2(4)$ $= 56$

16a	
16b	$r_{AB} = \frac{5}{2} r_{CD}$ $r_{AB}^2 = \frac{25}{4} r_{CD}^2$ $\frac{F_{AB}}{F_{CD}} = \frac{r_{CD}^2}{r_{AB}^2}$ $= \frac{4}{25}$ Answer: 4 : 25
17ai	$y = -x^3 + 3$
aii	$y = x^2 - 3$
aiii	$y = 3^{-x}$
bi	Any value greater than 2
bii	$y = 2x + c$, where $c < 0$
18	$2\pi r^2 + 2\pi rh = 2\pi r^2 + \pi r^2$ $h = \frac{r}{2}$ vol. of cylinder $= \pi r^2 h$ $= \frac{1}{2}\pi r^3$
19	$P\left(1 + \frac{r}{100}\right)^3 = 1.2P$ $1 + \frac{r}{100} = \sqrt[3]{1.2}$ $r = 6.3$ (1 d.p.)

20	$\frac{l_1}{l_2}=\frac{9}{15}=\frac{3}{5}$ $\frac{V_1}{V_2}=\left(\frac{3}{5}\right)^3=\frac{27}{125}$ $V_1=\frac{27}{125}V_2$ $125V_1=27V_2$ Therefore, 27 large glasses could be filled.
21	$7^2+10^2\neq 12^2$ $\therefore AB^2+BC^2\neq AC^2$ By Pythagoras Theorem, $\angle ABC\neq 90°$ By Right-Angle in Semicircle, AC is not a diameter.
22a	2.198
22b	$\frac{T}{2\pi}=\sqrt{\frac{L}{g}}\Rightarrow\frac{T^2}{4\pi^2}=\frac{L}{g}\Rightarrow g=\frac{4\pi^2L}{T^2}$
23a	$2:3.2\times1000\times100$ $1:160000$
23b	$\frac{27.5}{3.2}\times2=17.1875\text{ cm}$
23c	4 cm^2 represents 10.24 km^2 1.4 cm^2 represents 3.584 km^2
24a	$2^2\times3\times7$
24b	$2\times3=6$
24c	$2^3\times1\times7^2=392$
25a	$5(5+3)=40$
25b	$n(n+3)=154$ $n^2+3n-154=0$ $(n-11)(n+14)=0$ $n=11$ or $n=-14$(rejected)
25c	When n is odd, $n+3$ is even. When n is even, $n+3$ is odd. Product of odd and even is always even.
26	$\angle BAD=180°-5x°$ (angles in opposite segments) $\angle BOD=2\times\angle BAD$ (angle at centre $=2\times$ angle at circumference) $=360°-10x°$ $360°-10x°+2x°+90°\times2=360°$ (tangent $\perp$ radius, total interior angle of quadrilateral) $x=22.5$

2019 Sec 4E5N MYE EM P2 Solutions

Qn No	Solutions
1a	$\frac{3-x}{5}<1+\frac{2x+1}{4}$
	$\frac{3-x}{5}<\frac{4+2x+1}{4}$
	$4(3-x)<5(5+2x)$
	$12-4x<25+10x$
	$-14x<13$
	$x>-\frac{13}{14}$
1b	$\frac{4y}{3-2y}-\frac{y}{(2y-3)^2}$
	$=\frac{-4y}{2y-3}-\frac{y}{(2y-3)^2}$ or $=\frac{4y}{3-2y}-\frac{y}{(3-2y)^2}$
	$=\frac{-4y(2y-3)-y}{(2y-3)^2}$ $=\frac{4y(3-2y)-y}{(3-2y)^2}$
	$=\frac{11y-8y^2}{(2y-3)^2}$ $=\frac{11y-8y^2}{(3-2y)^2}$
	$=\frac{y(11-8y)}{(2y-3)^2}$ $=\frac{y(11-8y)}{(3-2y)^2}$
1c	$\frac{(-3h)^2}{8h^3j^5}\div\frac{27h^4j^2}{4j^3}$
	$=\frac{9h^2}{8h^3j^5}\times\frac{4j^3}{27h^4j^2}$
	$=\frac{1}{6h^5j^4}$
1d	$\left(\frac{256p^{16}}{q^{20}r^{-4}}\right)^{-\frac{1}{4}}$
	$=\left(\frac{q^{20}r^{-4}}{256p^{16}}\right)^{\frac{1}{4}}$
	$=\frac{q^5r^{-1}}{4p^4}$
	$=\frac{q^5}{4p^4r}$ or $\frac{q^5r^{-1}}{4p^4}$ or $\frac{1}{4}p^{-4}q^5r^{-1}$

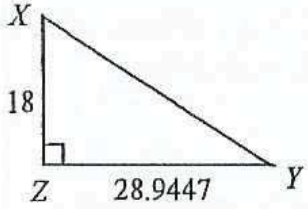
1e	$\frac{10}{x^2-9}-\frac{3}{x+3}=1$
	$10-3(x-3)=x^2-9$
	$10-3x+9=x^2-9$
	$x^2+3x-28=0$
	$(x+7)(x-4)=0$
	$x=-7 \quad \text{or} \quad x=4$
2a	1.50×10^{11} m (3 sig. fig)
2b	Time taken for sound to travel 1 au $=\frac{1.4959\times10^{11}}{343}$
	$=4.36\times10^{8}$ s (3 sig. fig)
2c	$\frac{384400\times10^{3}}{1.4959\times10^{11}}\times100$
	$=0.257\%$ (3 sig. fig)
2d	Time taken $=1.4959\times10^{11}\times8000\times10^{-9}$
	$=1.20\times10^{6}$ s (3 sig.fig)
3	Answers on the last page
4a	$AB=\sqrt{(-2-6)^2+(7-(-4))^2}$
	$=\sqrt{64+121}$
	$=\sqrt{185}=13.6$ units
4b	Gradient of $AB=\frac{7-(-4)}{-2-6}=-\frac{11}{8}$
	Equation of AB :
	$y-7=-\frac{11}{8}(x+2) \quad \text{or} \quad 7=-\frac{11}{8}(-2)+c$
	$c=\frac{17}{4}$
	$y=-\frac{11}{8}x+\frac{17}{4} \qquad y=-\frac{11}{8}x+\frac{17}{4}$
	$8y=-11x+34 \qquad 8y=-11x+34$
4c	$8y+11x=-16$
	$8y=-11x-16$
	$y=-\frac{11}{8}x-2$
	Since they have the same gradient, they are parallel.

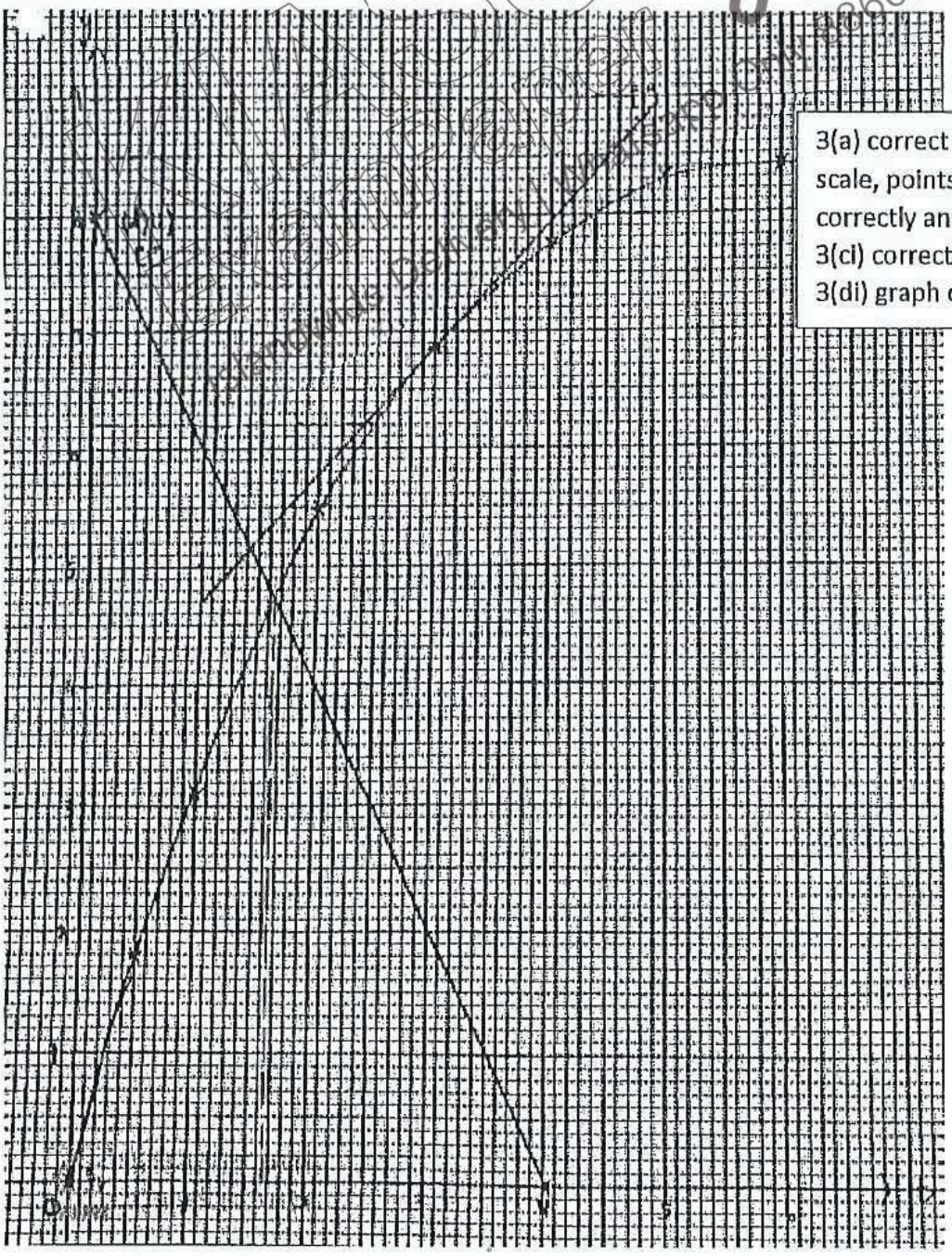
4d	$8k=-11(2)+34$ or $\frac{7-k}{-2-2}=-\frac{11}{8}$
	$k=1\frac{1}{2}$ $k=1\frac{1}{2}$
5a	Using Sine Rule,
	$\frac{\sin 120°}{800}=\frac{\sin 34°}{AC}$
	$AC=516.560$
	$=517\text{m}$ (3 sig. fig)
5b	Area of triangle $ABC=\frac{1}{2}(800)(516.560)\sin 26°$
	$=90577.9$
	$=90600\text{m}^2$ (3 sig. fig)
5c	Bearing of A from $B=360°-120°-34°$ or $180°+26°$
	$=206°$
5d	(i) Let the vertical height of the helicopter above E be h
	$\tan 11°=\frac{h}{300}$
	$h=300\tan 11°$
	$h=58.314$
	$h=58.3\text{m}$ (3 sig. fig)
	(ii) $\sin 34°=\frac{EF}{300}$
	$EF=300\sin 34°$
	Let the angle of depression of F from the helicopter be θ
	$\tan\theta=\frac{58.314}{300\sin 34°}$
	$\theta=19.167$
	$\theta=19.2°$ (1 d.p.)
6a	$\begin{pmatrix}w&1\\0&2\end{pmatrix}\begin{pmatrix}w&1\\0&2\end{pmatrix}=\begin{pmatrix}6w-9&w+2\\0&4\end{pmatrix}$
	$\begin{pmatrix}w^2&w+2\\0&4\end{pmatrix}=\begin{pmatrix}6w-9&w+2\\0&4\end{pmatrix}$
	$w^2=6w-9$
	$w^2-6w+9=0$
	$(w-3)^2=0$
	$w=3$

6b	(i) $A=\begin{pmatrix}225 & 140 & 125\\ 265 & 115 & 245\\ 245 & 125 & 175\end{pmatrix}$
	(ii) $\begin{pmatrix}225 & 140 & 125\\ 265 & 115 & 245\\ 245 & 125 & 175\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix}$
	$=\begin{pmatrix}490\\625\\545\end{pmatrix}$
	Total number of waffles delivered to outlet 1 is 490, outlet 2 is 625 and outlet 3 is 545.
6c	(i) $C=\begin{pmatrix}1.20 & 0.60\\ 0.80 & 0.50\\ 1.00 & 0.40\end{pmatrix}$ or $C=\begin{pmatrix}0.60 & 1.20\\ 0.50 & 0.80\\ 0.40 & 1.00\end{pmatrix}$
	$AC=\begin{pmatrix}225 & 140 & 125\\ 265 & 115 & 245\\ 245 & 125 & 175\end{pmatrix}\begin{pmatrix}1.20 & 0.60\\ 0.80 & 0.50\\ 1.00 & 0.40\end{pmatrix}$ or $AC=\begin{pmatrix}225 & 140 & 125\\ 265 & 115 & 245\\ 245 & 125 & 175\end{pmatrix}\begin{pmatrix}0.60 & 1.20\\ 0.50 & 0.80\\ 0.40 & 1.00\end{pmatrix}$
	$=\begin{pmatrix}507 & 255\\ 655 & 314.50\\ 569 & 279.50\end{pmatrix}$ or $\begin{pmatrix}255 & 507\\ 314.50 & 655\\ 279.50 & 569\end{pmatrix}$
	(ii) Profit earned by outlet 2 = \$ 340.50
7a	(i) $\frac{208}{x-5}$ tiles
	(ii) $\frac{169}{x}$ tiles
7b	$2\left(\frac{208}{x-5}\right)+2\left(\frac{169}{x}\right)-4=74$
	$\frac{416}{x-5}+\frac{338}{x}=78$
	$416x+338(x-5)=78x(x-5)$
	$416x+338x-1690=78x^2-390x$
	$78x^2-1144x+1690=0$
	$3x^2-44x+65=0$ (shown)

7c	$(3x-5)(x-13)=0$
	$x=\frac{5}{3}=1\frac{2}{3}$ or $x=13$
7d	If $x=1\frac{2}{3}$, the width of the tile will be negative.
7e	Taking $x=13$,
	area of the wall that will be filled with tiles $=13(13-5)\times 74$
	$=7696\text{ cm}^2$
8a	$\cos 60°=\frac{BC}{4}$
	$BC=2\text{ cm}$
8b	Total cross-sectional area of medal
	$=2\left[\frac{1}{2}(2^2)\left(\frac{4\pi}{3}\right)+2\left(\frac{1}{2}\right)(4)(2)\sin 60°\right]$
	$=30.6115$
	$=30.6\text{ cm}^2$ (3 sig. fig)
8c	Volume of the medal $=30.6115\times 0.7$
	$=21.428$
	$=21.4\text{ cm}^3$ (3 sig. fig)
9a	$\angle BAR=\angle QPR$ (alternate angles)
	$\angle ABR=\angle PQR$ (alternate angles)
	$\angle ARB=\angle PRQ$ (vertically opposite angles)
	Since all corresponding angles are equal, triangle PQR is similar to triangle ABR.
9b	Triangle CPQ and Triangle CBA
9c	Using similar triangles,
	$\frac{AR}{PR}=\frac{AB}{PQ}$
	$\frac{AR}{4}=\frac{12}{5}$
	$PB=9.6\text{ cm}$ or $9\frac{3}{5}\text{ cm}$
9d	$\frac{\text{Area of triangle } PQR}{\text{Area of triangle } ABR}=\left(\frac{5}{12}\right)^2=\frac{25}{144}$
9e	$\left(\frac{5}{12}\right)^2=\frac{15}{\text{Area of triangle } CBA}$
	Area of triangle $CBA=86.4\text{ cm}^2$

	Area of $ABPQ = 71.4\ \text{cm}^2$
10a	(i) (a) median = 56.5 km/h
	(b) Interquartile range = 62.5 – 50
	= 12.5 km/h
	(c) 80^{th} percentile = 64 km/h
	(d) 65%
	(ii) Probability $= 1-\left(\frac{43}{100}\times\frac{42}{99}\right)$
	$=\frac{1349}{1650}$
10b	(i) The median speed in the afternoon is 56.5 km/h while the median speed at night is 60 km/h. Vehicles at night travel faster.
	The interquartile range in the afternoon is 12.5 km/h while the interquartile range at night is 20 km/h. The speeds of the vehicles in the afternoon are less widespread (or more consistent) than the speeds at night due to smaller interquartile range in the afternoon.
	(ii) Range = 82 – 30 = 52 km/h
11a	1 cm : 75 m
	$1\ \text{cm}^2 : 5625\ \text{m}^2$
	Area of building on the map $=\frac{134000}{5625}$
	$= 23.8\ \text{cm}^2$ (3 sig. fig)
11b	(i) actual circumference = 105 m
	$2\pi r = 105$
	Actual radius $=\frac{105}{2\pi}$ m
	(ii) Let the centre of forest valley be O and angle ZOY be θ radian
	$\left(\frac{105}{2\pi}\right)\theta = 35$
	$\theta = \frac{2\pi}{3}$
	Using Cosine Rule,
	$YZ^2 = \left(\frac{105}{2\pi}\right)^2 + \left(\frac{105}{2\pi}\right)^2 - 2\left(\frac{105}{2\pi}\right)\left(\frac{105}{2\pi}\right)\cos\frac{2\pi}{3}$
	$YZ = 28.9447$ m

	 By Pythagoras Theorem, $XY^2 = 18^2 + YZ^2$ $XY = 34.1$ m (3 sig. fig)
	His claim is false.
3b	$y = 6.2$
3c	(i) gradient $= \frac{8.5-5}{4.5-1.3} = 1.09375$
	(ii) The gradient represents the speed of the stone at $t = 3$.
3d	(ii) At $t = 1.65$



3(a) correct labelled axes & scale, points plotted correctly and smooth curve
3(ci) correct tangent
3(di) graph of object

Name: Register Number: Class:

For Marker's Use

南僑中學

NAN CHIAU HIGH SCHOOL

MID–YEAR EXAMINATION 2019
SECONDARY FOUR EXPRESS

MATHEMATICS **4048/01**

Paper 1 **7 May 2019, Tuesday**

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a 2B pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 80.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer all the questions.

1 Given that $64^k \times 16 = 1$, find the value of k.

Answer $k =$ ..... [1]

---

2 The stem-and-leaf diagram shows the Science quiz marks scored by a group of 15 students.

0	7						
1	2						
2	4	X					
3	0	4	5	6			
4	3	5	6	6	6	7	9

Key: 0 | 7 represents 7 marks

Given that the median mark is twice the interquartile range, find the value of X.

Answer $X =$ ..... [2]

---

3 The LCM and HCF of p and q are 60 and 6 respectively. Given that both p and q are between 6 and 60 and $p < q$, find the value of p and of q.

Answer $p =$ ....., $q =$ ..... [2]

---

4 A, B and C are three non-empty sets satisfying the following conditions:

$$A \subset B,\ A \cap C \neq \emptyset,\ A \not\subset C \text{ and } C \not\subset B.$$

Draw a clearly labelled Venn diagram to illustrate the above information.

Answer

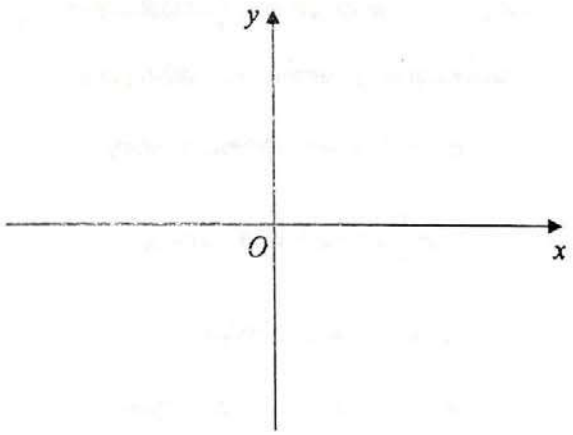
[2]

---

5 Sketch the graph of $y = (x + 2)(4 - x)$ on the axes below.

Indicate clearly the x-intercepts, y-intercept and the coordinates of the turning point.

Answer



[2]

---

6 Tide detergent is having its monthly promotion at MAMA Supermarket.

Show that the cost of the detergent is directly proportional to its volume.

Answer

[2]

---

7 The employees of a company are offered a wage increment which is calculated according to one of the following schemes:

Scheme A: An increment of 5% of their present wages.
Scheme B: An increment of 3% of their present wages plus additional $8 per week.

Employees earning $\$x$ per week at present will receive higher increment if they have chosen *Scheme A*. Find the range of values of x.

Answer ..... [2]

---

8 Solve $\frac{1}{x} - \frac{3}{2x+1} = 1$.

Answer $x =$ ..... [3]

---

9 Rearrange $y = \sqrt{\frac{2x+y}{3x-5}}$ to express x in terms of y.

Answer $x =$ ..... [3]

---

10 Mr Lim invested \$10 000 in a fund that pays a compound interest of 2.75% per annum, compounded quarterly. Find the interest earned, correct to the nearest dollar, at the end of 3 years.

Answer \$..... [3]

---

11 The diagram shows a container in the shape of a prism with a triangular cross-section. Water is poured into the container until the depth of water is $\frac{3}{5}$ the height of the container.

If the volume of the container is 200 ml, find the volume of water in the container.

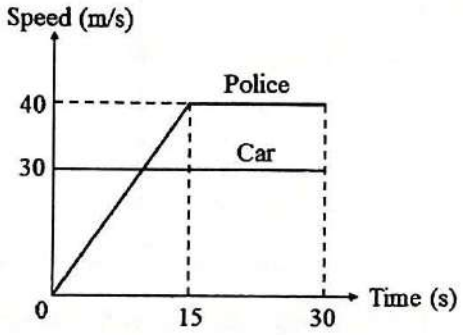
Answer ..... ml [3]

---

12 **(a)** Convert 80 km/h to m/s.

Answer ..... m/s [1]

(b) In Marina Coastal Expressway (MCE), the speed limit of all vehicles is 80 km/h. A car is travelling at a constant speed of 30 m/s and passes a stationary traffic police motorcycle. The traffic police immediately gives chase, accelerating uniformly to reach a maximum speed of 40 m/s and continues with this speed until it overtakes the car.

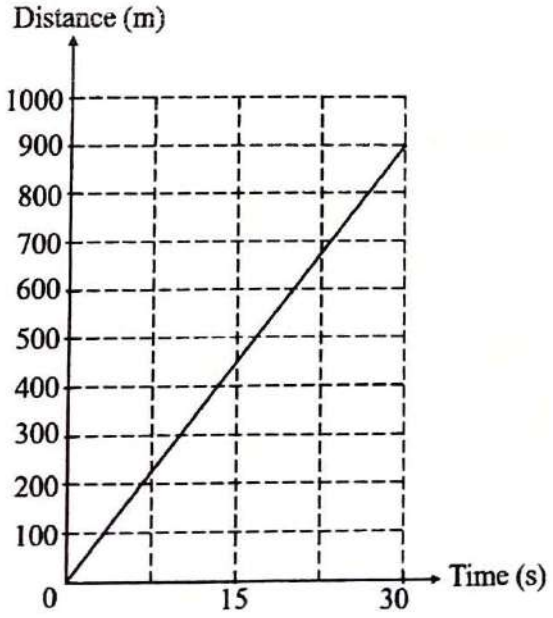


(i) Calculate the acceleration of the police motorcycle.

Answer ..... m/s^2 [1]

(ii) In the axes below, draw the distance–time graph for the police motorcycle for the first 30 seconds. The distance–time graph for the car has been drawn for you.

Answer



[1]

13 $PQRS$ is a parallelogram.

$\overrightarrow{PQ} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$, $\overrightarrow{PS} = \begin{pmatrix} 12 \\ 5 \end{pmatrix}$.

Calculate $|\overrightarrow{PR}|$.

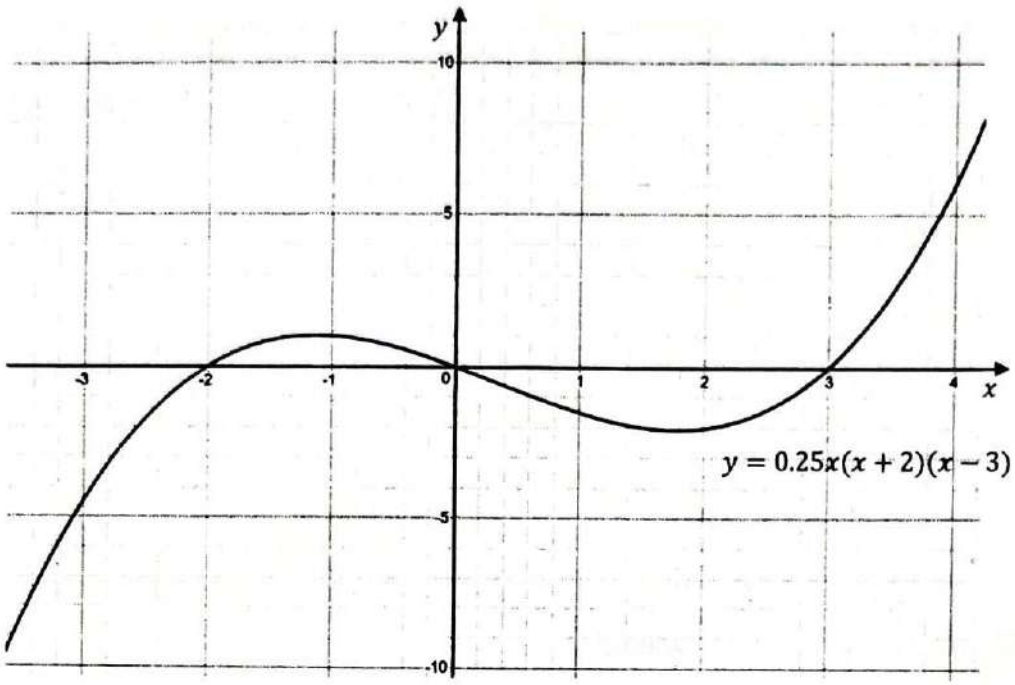
Answer ..... units [3]

---

14 By observing the pattern in the last digit of 3^n and of 8^n, where $n > 0$, find the last digit in the subtraction $3^{31} - 8^{16}$.

Answer ..... [3]

15



The graph of $y = 0.25x(x + 2)(x - 3)$ was drawn.

By adding a suitable straight line on the graph above, solve the equation $x(x + 2)(x - 3) = 6x$.

Answer $x =$ ..... [3]

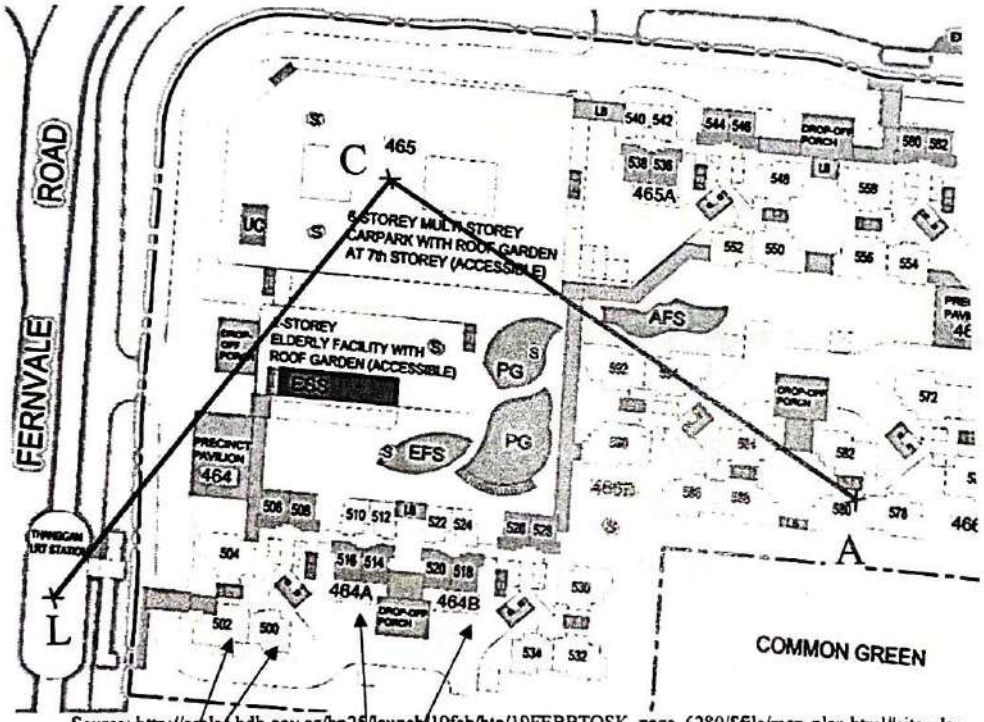
16 The diagram below shows part of a site map of the newly launched Build-To-Order (BTO) flats at Fernvale Road.

Andy and his parents applied for this BTO project under the Multi-Generation Priority Scheme which allows married children to make a joint application with their parents for two units in the same project.

Their application has gotten them a good queue number in the selection of units. Andy has chosen the unit 580, marked as *A*.

State the unit which Andy's parents should choose if they wish to satisfy the following criteria:

- equidistant from the multi storey carpark, *C*, and the LRT station, *L*, and
- equidistant from the line *LC* and *AC*.



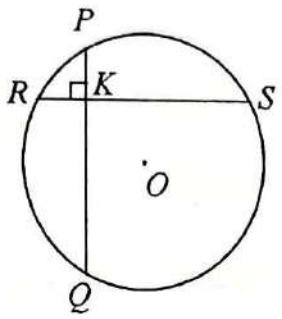
Source: http://esales.hdb.gov.sg/bp25/launch/19feb/bto/19FEBBTOSK_page_6280/$file/map-plan.html#site-plan

Unit numbers 502 and 500
i.e. #02–**502**,
#02–**500**

Block numbers 464A and 464B
i.e. Blk **464A**,
Blk **464B**

Answer ..... [3]

17 In the diagram, O is the centre of the circle with radius 13 cm. PQ and RS are perpendicular equal chords of length 24 cm and intersecting at K.



Calculate the length of OK.
Show your workings and give reasons.

Answer ..... cm [3]

18 For a n–sided polygon, the largest interior angle is 100° and the smallest interior angle is 20°. Find the value(s) of n.

Answer $n =$ ..... [4]

---

19 **(a)** Factorise $(2x - 7)(x - 2) - 9$ completely.

Answer ..... [2]

(b) Hence, solve $2(y - 1)^2 - 11y + 16 = 0$.

Answer ..... [2]

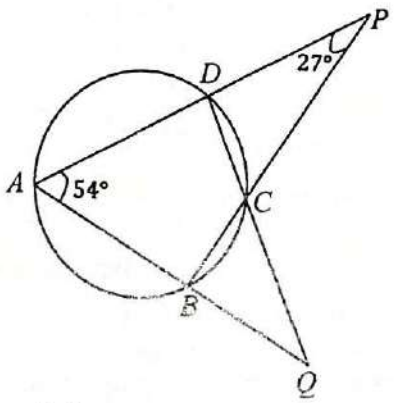
---

20 When $x + 8$ is divided by y, the result is 4. When x is divided by y, its quotient is 2 and the remainder is 6. By forming two equations in terms of x and of y, solve for x and y.

Answer $x =$ ....., $y =$..... [4]

---

21 In the diagram, A, B, C and D lie on the circumference of the circle. ADP, ABQ, BCP and DCQ are straight lines. $\angle BAD = 54°$ and $\angle CPD = 27°$.

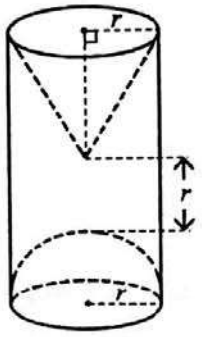


Find $\angle AQD$.
Show your working and give reasons.

Answer $\angle AQD =$ .....° [4]

---

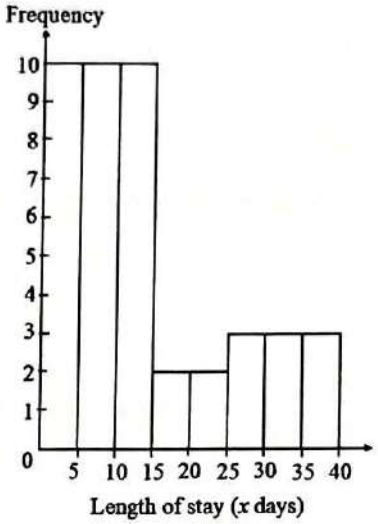
22 A carpenter wants to create a wooden toy for his son.
He removed a right-angled cone and a hemisphere from a uniform cylindrical wood. The radius of the base of the cone and hemisphere are r cm. The distance between the top of the hemisphere and the vertex of the cone is also r cm.



Given that the curved surface area of the cone is equal to the curved surface area of the hemisphere, work out the volume of the toy, in terms of r.

Answer ..... cm^3 [4]

23 The histogram illustrates the length of stay (in days) in Australia for a group of Singaporean tourists last year.



(a) Complete the following table.

Answer

Length of stay (days)	Numbers of tourists
$0 < x \leq 5$	
$5 < x \leq 10$	
$10 < x \leq 15$	
$15 < x \leq 20$	
$20 < x \leq 25$	
$25 < x \leq 30$	
$30 < x \leq 35$	
$35 < x \leq 40$	

[1]

(b) Calculate the fraction of the Singaporean tourists who stayed in Australia longer than 15 days.

Answer ..... [1]

(c) Calculate an estimate for the mean and standard deviation of the length of stay.

Answer Mean = ...days

Standard deviation = ...days [2]

24 **(a)** Explain why $(x-2)^4(x^2-4x+4)$ is both a perfect square and a perfect cube when x is an integer.

Answer

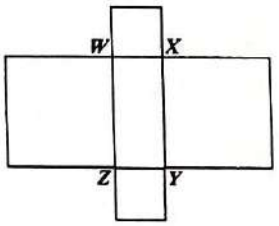
.....

.....

.....

..... [2]

(b) The figure is made up of 2 big squares, 2 small squares and a rectangle *WXYZ*. The perimeter of rectangle *WXYZ* is 20 cm. The total area of all the four squares is 140 cm^2.



Find the area of *WXYZ*.

Answer ..... cm^2 [4]

25 Mrs Tan sells three different types of cakes. The table below shows the number of cakes bought by Childcare Centre *A* and *B* for Children's Day Celebration.

	Sponge	Chocolate	Cheese
Childcare Centre *A*	32	27	20
Childcare Centre *B*	44	45	38

(a) Represent this information in a 2×3 matrix, **P**.

Answer $\mathbf{P} = \begin{pmatrix} & & \\ & & \end{pmatrix}$ [1]

(b) The selling price for each sponge cake, chocolate cake and cheese cake is \$3.20, \$4.50 and \$4.80 respectively.

The information can be represented as $\mathbf{Q} = \begin{pmatrix} 3.2 \\ 4.5 \\ 4.8 \end{pmatrix}$.

Evaluate the matrix **PQ**.

Answer $\mathbf{PQ} = \begin{pmatrix} & \\ & \end{pmatrix}$ [1]

(c) Mrs Tan has a special promotion for cakes order for Children's Day celebration.

- Get the second sponge cake at half price.
- Buy three chocolate cakes for the price of two.
- Cheese cakes 1–for–1!

Write down a 3×3 matrix, **R**, such that **RQ** will give the discounted price for each type of cakes sold to Childcare Centre *A* and *B*.

Answer $\mathbf{R} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$ [2]

25 (d) Evaluate matrix $\mathbf{S} = \mathbf{PRQ}$.

Answer $\mathbf{S} = \begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(e) Describe what the elements in matrix **S** represents.

Answer

.....

.....

..... [1]

– End –

Name: Register Number: Class:

南僑中學

For Marker's Use

NAN CHIAU HIGH SCHOOL

MID-YEAR EXAMINATION 2019
SECONDARY FOUR EXPRESS

MATHEMATICS **4048/02**

Paper 2 **9 May 2019, Thursday**

Candidates answer on the Question Paper. **2 hours 30 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a 2B pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 100.

This document consists of **25** printed pages including the cover page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1. **(a)** Simplify $\frac{4b^3}{3a} \div \frac{(-4ab^3)^2}{2a^{-4}b^2}$. Give your answer in positive indices. [2]

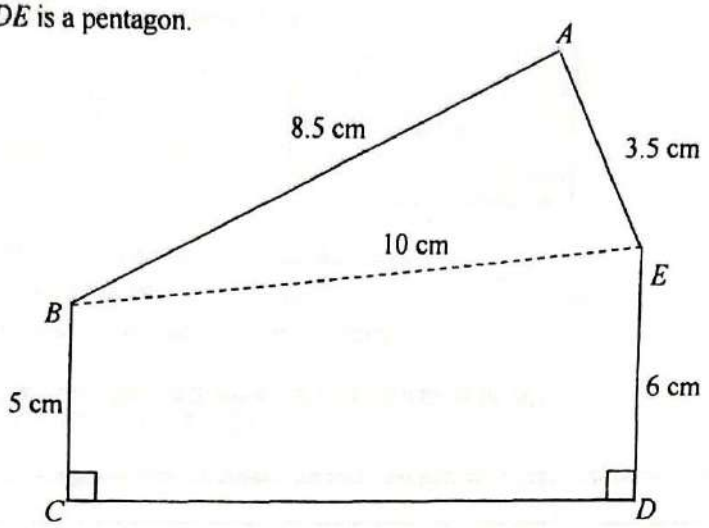
(b) Factorise $14xb + 3ay - 2xy - 21ab$ completely. [2]

(c) Solve the inequality $\frac{x}{5} < \frac{x+2}{3} \leq \frac{3-2x}{4}$. [3]

(d) Simplify $\frac{2x(1-6x)}{4x^2-1} - \frac{2}{2x+1}$. [4]

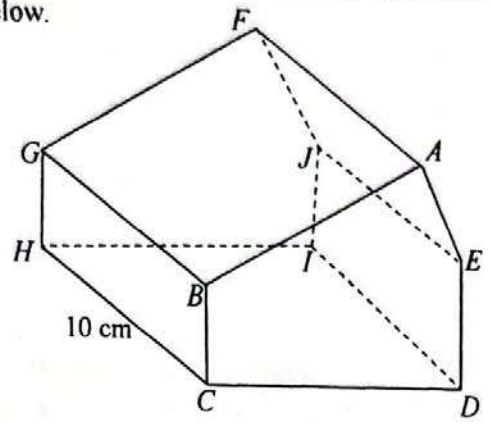
(e) Express $2x^2 + 6x - 15$ in the form $a(x+b)^2 + c$, where a, b and c are constants.
Hence, solve the equation $2x^2 + 6x - 15 = 0$, giving your answers correct to four decimal places. [4]

2. *ABCDE* is a pentagon.



(a) Calculate the area of the pentagon *ABCDE*. [5]

(b) The pentagon $ABCDE$ is a cross-section of a prism which is a model of a house as shown below.



(i) Calculate the total surface area of the model, including the base. [2]

(ii) Calculate the angle of depression of H from E. [3]

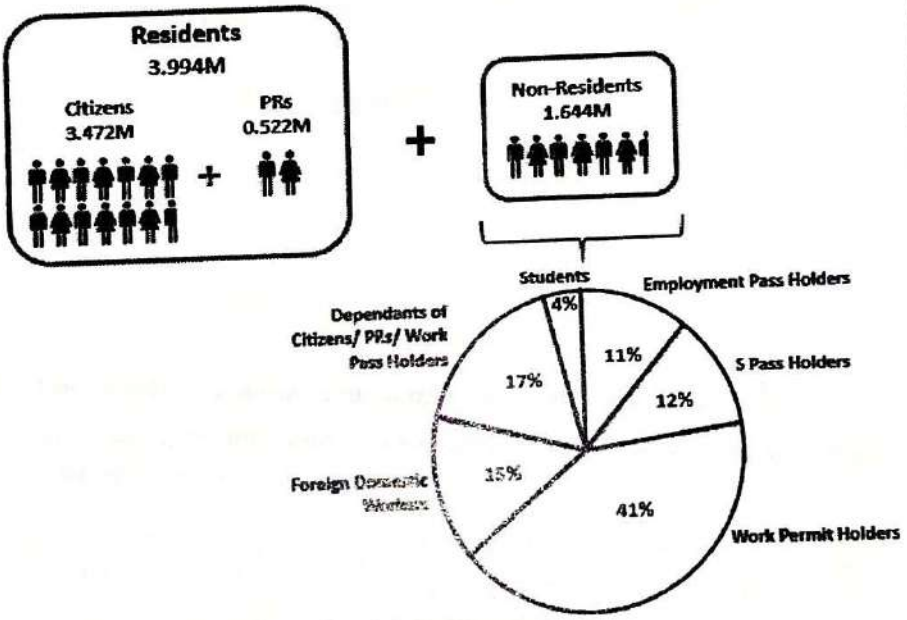
3. There was a newspaper article on Singapore's population published in 2018.

Singapore's population grows slightly to 5.638 million, with non-resident numbers stable.

Singapore's population rose 0.5 percent to 5.638 million from 2017 to 2018.

The slight increase over the past year was due to stable growth in citizen population, which rose 1 percent to 3.472 million citizens as of June 2018.

The 1 percent growth in citizens was due to citizen births and immigration.



Source:
https://www.channelnewsasia.com/news/singapore/singapore-s-population-grows-slightly-to-5-64m-with-non-resident-10763132

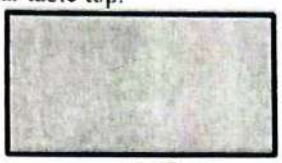
(a) Calculate the Singapore's total population in 2017. [2]

(b) Express the number of Foreign Domestic Workers as a percentage of the Singapore's total population in 2018. [2]

(c) Based on the information shown in this article, do you agree that Singapore's total population will reach 6.3 million by 2030?
Support your answer with appropriate workings and state an assumption that you have made in your calculation. [3]

(d) State one aspect of the diagram that may be misleading and explain how this may lead to a misinterpretation of information. [2]

4. The diagram shows a rectangular table top.



The area of the table top is 5 square metres.
The length of the table top is x metres.
The perimeter of the table top is y metres.

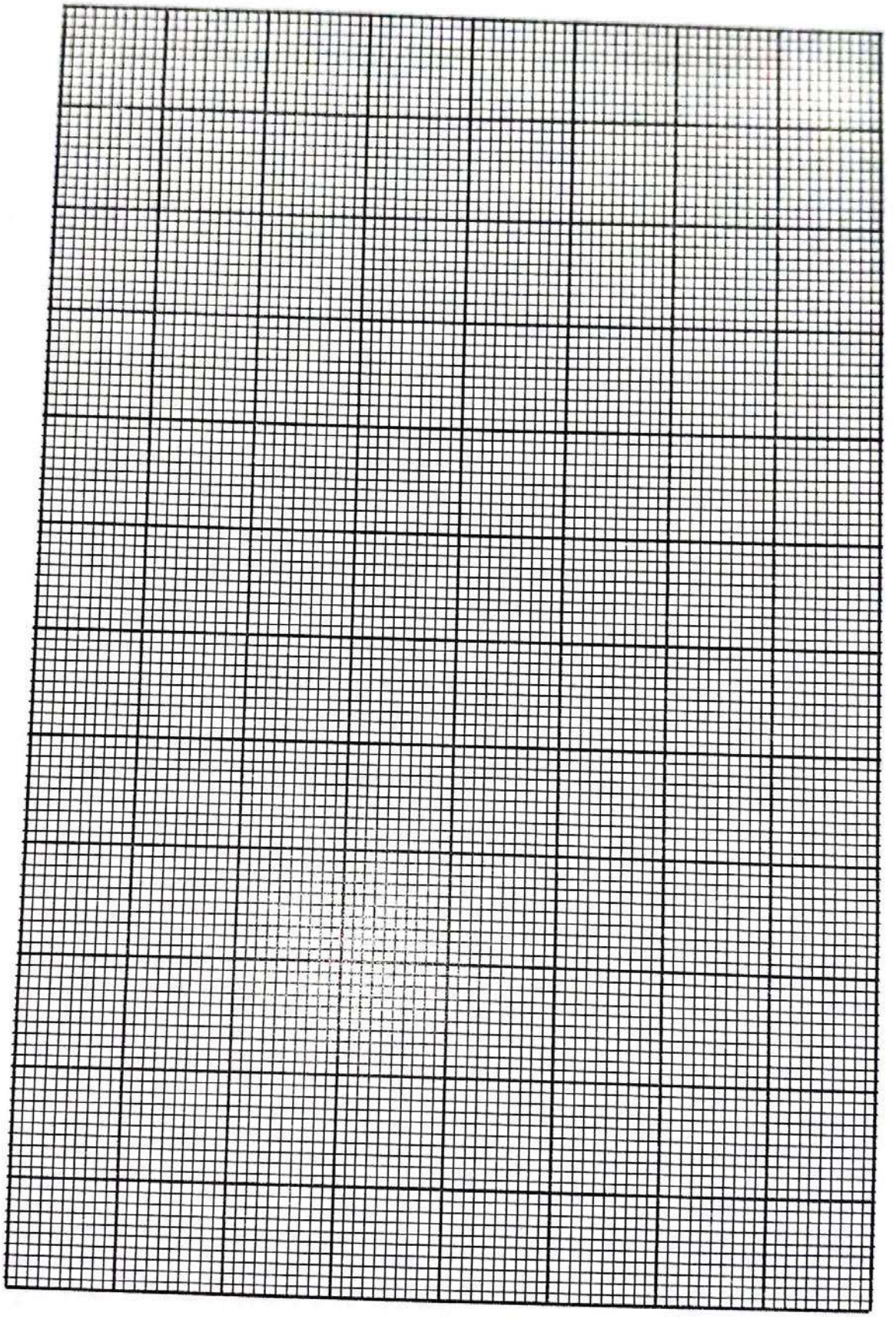
(a) Show that $y = 2x + \frac{10}{x}$. [2]

(b) The variables x and y are connected by the equation $y = 2x + \frac{10}{x}$.
Some corresponding values of x and y, correct to one decimal place, are given in the table below.

x	0.5	1	1.5	2	4	6	8
y	p	12	q	9	10.5	13.7	17.3

(i) Find the value of p and of q. [1]

(ii) Using a scale of 1 cm to represent 1 m, draw a horizontal x-axis for $0 \leq x \leq 8$.
Using a scale of 1 cm to represent 1 m, draw a vertical y-axis for $0 \leq y \leq 22$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]



(iii) Use your graph to find the length of the table top if the perimeter of the table top is 16 m. [2]

(iv) By drawing a suitable straight line, find the x coordinate of the point at which the gradient of the curve is $\frac{1}{3}$. [2]

5. Given equation of a line L is $3x - 2y = 8$.

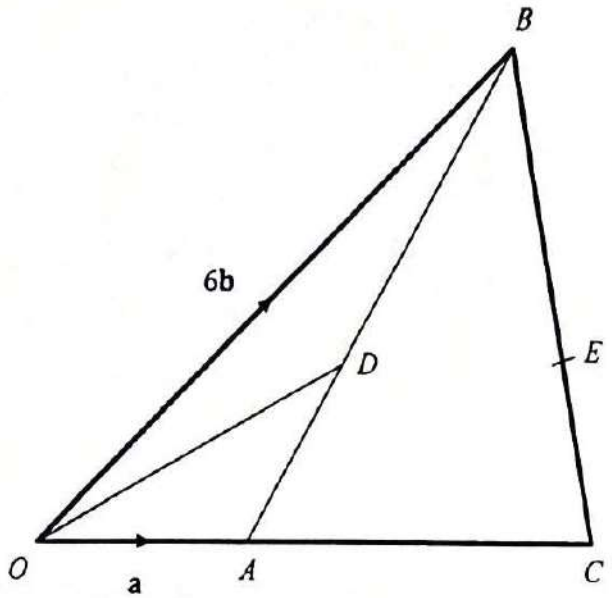
(a) State the gradient of line L. [1]

(b) If $P\ (k, -2)$ is a point on the line L, find the value of k. [1]

(c) Find the equation of another line that is parallel to the x-axis and passes through P. [1]

(d) Calculate the perpendicular distance from the origin, O, to the line L. [4]

6.



In the diagram, $\overrightarrow{OA} = \boldsymbol{a}$, $\overrightarrow{OB} = 6\boldsymbol{b}$ and $\overrightarrow{OA} = \frac{1}{3}\overrightarrow{OC}$.

D is a point on AB such that $3\overrightarrow{AD} = 2\overrightarrow{DB}$ and E is a point on BC such that $\overrightarrow{CE} : \overrightarrow{EB} = 4 : 5$.

(a) Express, as simply as possible, in terms of **a** and **b**,

(i) $\overrightarrow{BA}$, [1]

(ii) $\overrightarrow{OD}$, [1]

(iii) $\overrightarrow{CB}$, [1]

(iv) $\overrightarrow{AE}$. [2]

(b) Write down the relationship between OD and AE. Explain your answer. [2]

(c) Find the ratio of

(i) area of triangle CAE : area of triangle AOD , [2]

(ii) area of triangle CAE : area of triangle AOB . [2]

7. **(a)** There are two boxes of sweets containing toffees and chocolates.
Box A contains 8 toffees and 4 chocolates, whereas box B contains 7 toffees and 3 chocolates.

Jolin loves chocolates.
One of the boxes is chosen at random and a sweet is taken out.
If she gets a chocolate, she will consume it.
If she did not get a chocolate from the first selection, she will place the sweet into the other box and she will select again from the other box.
Jolin limits herself to two selections.

Find, as a fraction in its simplest form, the probability that

(i) Jolin will have her favourite chocolate from the first selection, [3]

(ii) Jolin will have her favourite chocolate. [3]

(b) Your friend, Kenton gives you a chance to win $1000 by playing a game of "Guess the number". There are two options of the game that he allows you to choose.

Option A

He uses a random number generator to choose a number from 1 to 8.
If you guess it correctly, you win.

Option B

You flip a fair coin.
If the coin lands on head, Kenton will roll a fair 6-sided die. If you manage to guess what it rolled, you win.
If the coin lands on tail, Kenton will use a random number generator to choose a random number from 1 to 8. If you guess the number correctly, you win.

Which option will you choose? Explain your answer. [3]

8.

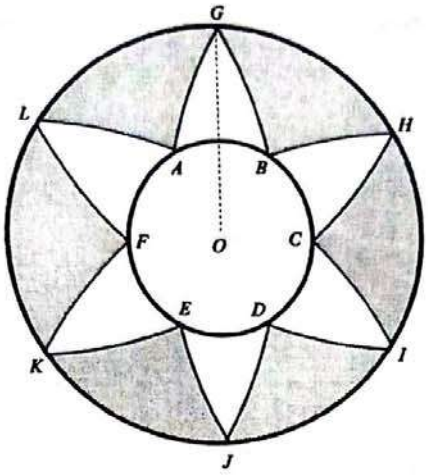


Figure 1

Figure 1 shows a dreamcatcher-inspired ornament which is made up of wire.
The ornament consists of an inner circle *ABCDEF* and an outer circle *GHIJKL*, both with centre *O*, and 6 identical "petal" designs such as *AGB* and *BHC*.

Team Hō'ola will be making these ornaments to raise funds for their Voluntary Welfare Organisation.

To estimate the amount of wire to be purchased, Head of Fundraising team, Janice modelled the following:

- The arc *AG* is an arc of another circle with centre *X*, radius 18 cm and $\angle AXG = \frac{\pi}{6}$ radians.

  This information is illustrated in Figure 2.

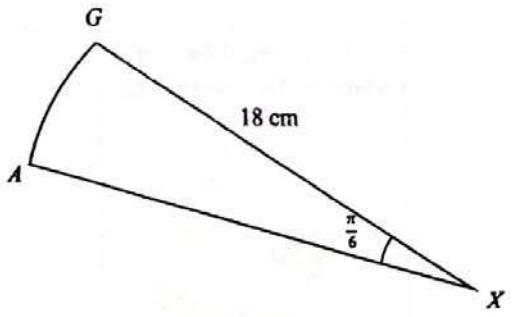


Figure 2

- A regular hexagon forms within the inner circle $ABCDEF$ with $OB = 5$ cm.
  This information is illustrated in Figure 3.

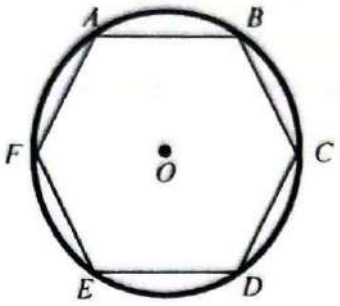


Figure 3

(a) Show that the radius of the outer circle is 13.31 cm, correct to 2 decimal places. [6]

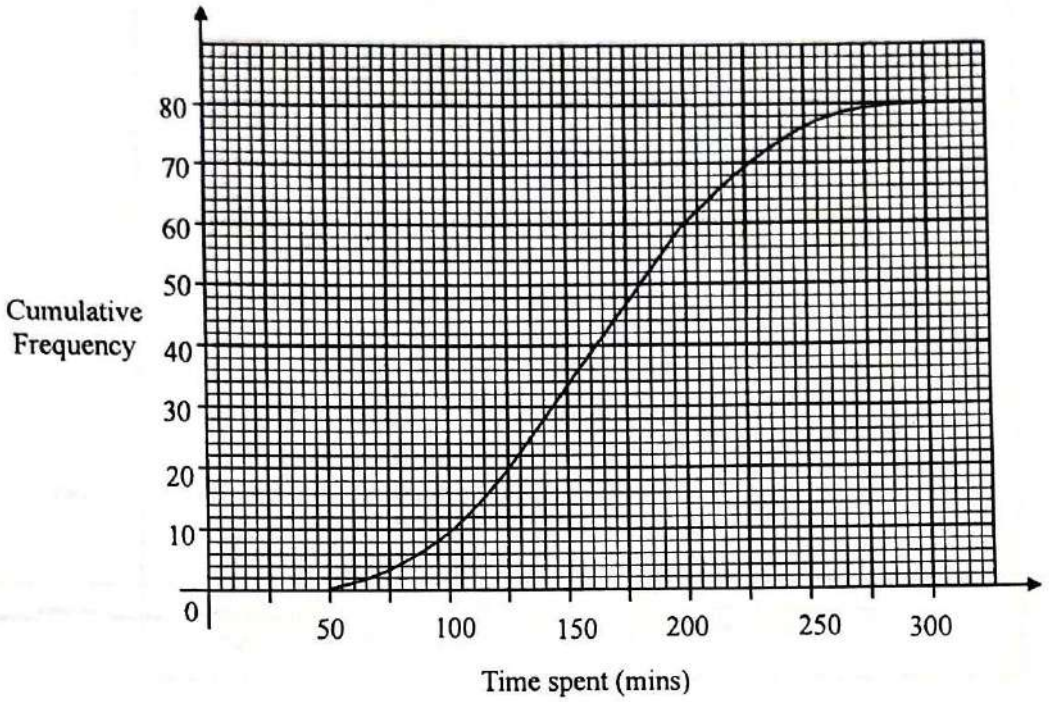
(b) Team Hō'ola decided to make 50 dreamcatcher-inspired ornaments and Janice chanced upon the following promotion.

10 m silver craft bendable DIY wire (2mm thick)
$9.28

What is the estimated cost to make these ornaments? [5]

(c) Find the area of the shaded region as shown in Figure 1. [5]

9. **(a)** The cumulative frequency curve below shows the time spent in minutes by a group of 80 teenagers on Instagram (a social media platform) on a particular day.



Use the curve to estimate

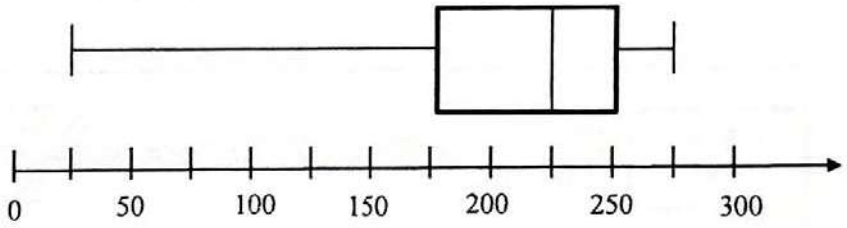
(i) the median, [1]

(ii) the interquartile range. [2]

(b) Through a market research, it was found out that the time spent on Facebook (another social media platform) is less popular and less consistent among the same group of 80 teenagers. A second cumulative frequency curve for the same group of 80 teenagers spending their time on Facebook is drawn.

Describe how the second cumulative frequency curve may differ from the curve for Instagram. [2]

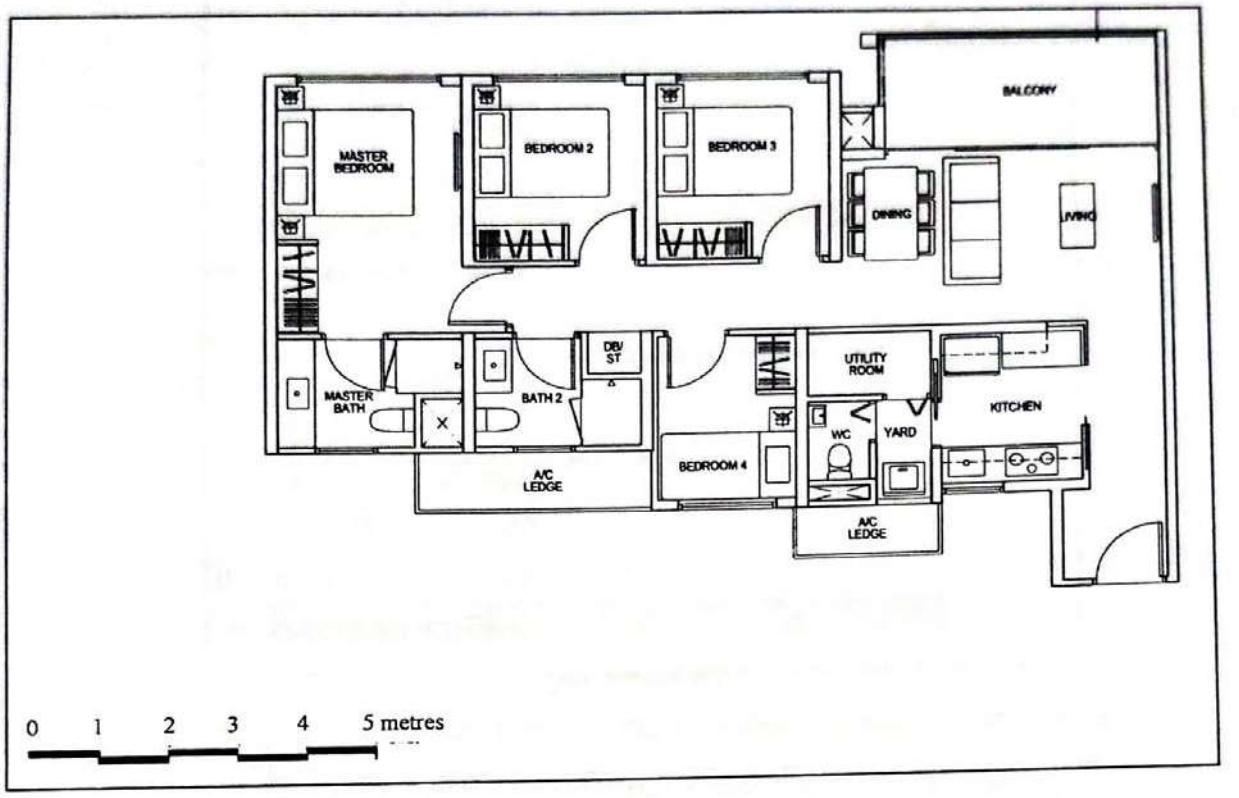
(c) The box-and-whisker plot represents the distribution of the time spent for the same group of 80 teenagers on SnapChat (another social media platform).



For this group of 80 teenagers, which of the social media platforms - Instagram, Facebook or Snapchat, is the most popular?

Support your answer with an appropriate statistical measure. [2]

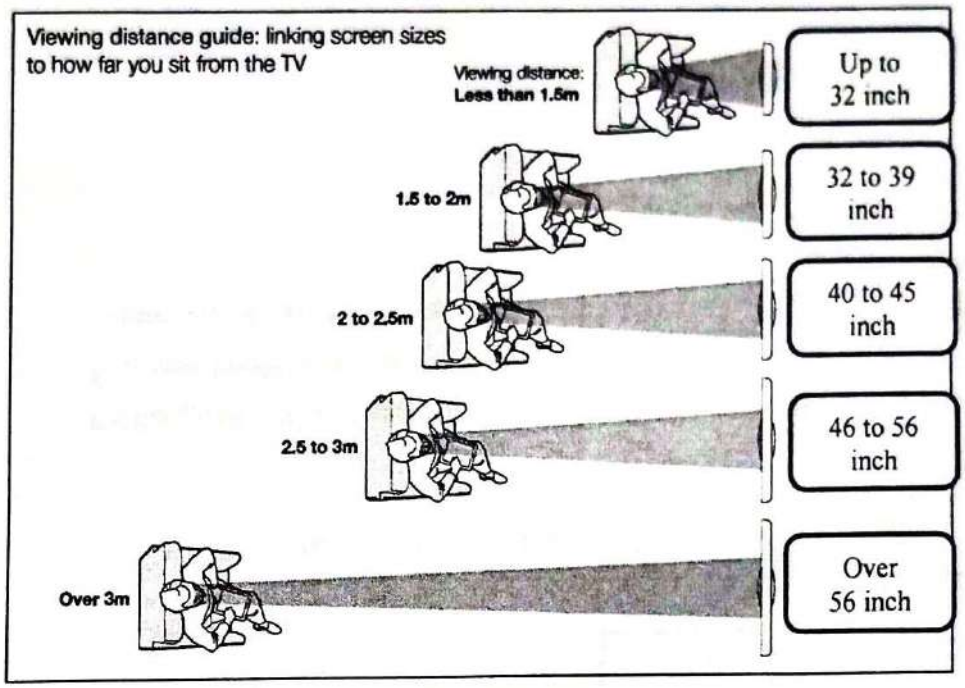
10. Ms Tan got her new home recently and the layout of her house is shown in the diagram below. The layout is drawn to scale.



(a) Express the scale of the map in the form of $1:n$. [1]

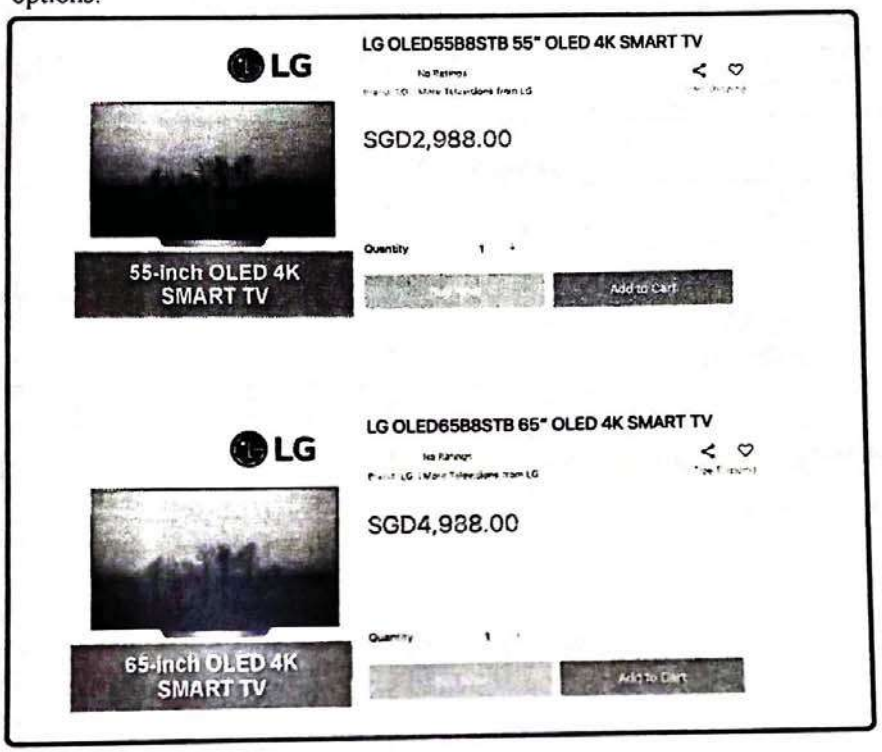
(b) Ms Tan decides to get a television set for her living room.

She found the following infographic online.



What is the range of television size which Ms Tan should get for her living room? [2]

(c) Ms Tan decided to shop for her television set online and she saw the following options:



Ms Tan pays a downpayment of $2 000 for her television set as suggested in (b). She pays the remaining amount over 3 months, with a simple interest rate of 7 % per annum.

Calculate her monthly instalment. [3]

- End of Paper -

Answers

1

$4^{3k} \times 4^2 = 4^0$

$3k + 2 = 0$

$k = -\frac{2}{3}$ **exact only!**

2

Median = 36

Interquartile Range = 18

$Q_1 = 46 - 18$

$= 28$

$X = 8$ → digit in the ones place.

3

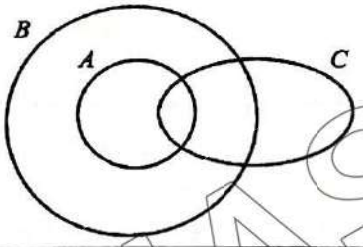
HCF $= 6 = 2 \times 3$

LCM $= 60 = 2^2 \times 3 \times 5$

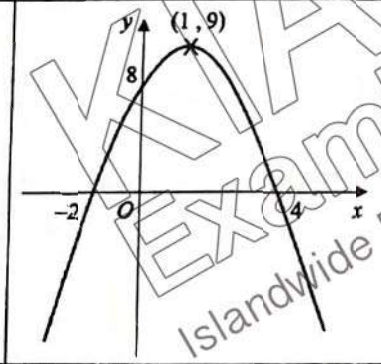
$p = 2^2 \times 3 \quad = 12$

$q = 2 \times 3 \times 5 \quad = 30$

4



5



6

$\frac{1.79}{10} = \frac{179}{1000}$,

$\frac{17.9}{2\times50} = \frac{179}{1000}$,

$\frac{26.85}{150} = \frac{179}{1000}$.

Must show for all three!

Since $\frac{\text{cost}}{\text{volume}}$ = **constant**, the cost is directly proportional to its volume.

7

$0.05x > 0.03x + 8$ or $1.05x > 1.03x + 8$

$0.02x > 8$

$x > 400$

8

$\frac{2x+1-3x}{x(2x+1)} = 1$

$1 - x = 2x^2 + x$

$2x^2 + 2x - 1 = 0$

$x = \frac{-2\pm\sqrt{(2)^2-4(2)(-1)}}{2(2)} = 0.366 \text{ or } -1.37$

Must show quad formula with values subbed in.

9

$y^2 = \frac{2x+y}{3x-5}$

$3xy^2 - 5y^2 = 2x + y$

$3xy^2 - 2x = 5y^2 + y$

$x(3y^2 - 2) = 5y^2 + y$

$x = \frac{y(5y+1)}{3y^2-2}$ or $\frac{5y^2+y}{3y^2-2}$ or $-\frac{5y^2+y}{2-3y^2}$

10

$10\,000\left(1 + \frac{2.75\div4}{100}\right)^{3\times4} - 10\,000$

$= 10\,856.92 - 10\,000$

$= 856.92$

$= \$857$

Quarterly is 4 times!

11

$\left(\frac{3}{5}\right)^2 = \frac{9}{25}$

$\frac{9}{25} \times 200 = 72$ ml

Not cube because one of the component is constant!!

12a

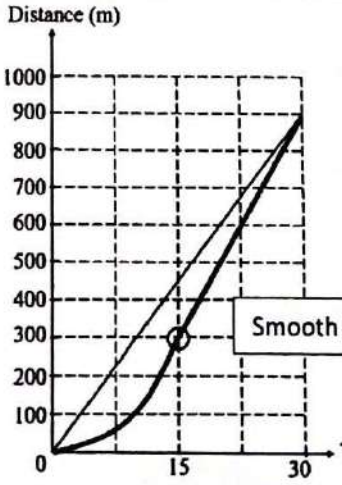
$\frac{80\text{km}}{1\text{h}} = \frac{80\times1000\text{ m}}{3600\text{ s}} = \frac{200}{9} = 22\frac{2}{9}$

12b

$\frac{40}{15} = 2\frac{2}{3}$

Mixed number or 3sf only!!

12c



Smooth curve only!

13

$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = \begin{pmatrix}-3\\4\end{pmatrix} + \begin{pmatrix}12\\5\end{pmatrix} = \begin{pmatrix}9\\9\end{pmatrix}$

$|\overrightarrow{PR}| = \sqrt{9^2 + 9^2} = \sqrt{162} = 12.7$ units

PQ is not perpendicular to PS!

Length PR ≠ length PQ + length QR!

14

$3^1 = 3$ $\quad 8^1 = 8$

$3^2 = 9$ $\quad 8^2 = 64$

$3^3 = 27$ $\quad 8^3 = 512$

$3^4 = 81$ $\quad 8^4 = 4096$

$3^5 = 243$ $\quad 8^5 = 32768$

Last digit for 3^n is 3, 9, 7, 1

Last digit for 8^n is 8, 4, 2, 6

$\frac{31}{4} = 7\frac{3}{4}$ → Remainder = 3 → Last digit = 7

$\frac{16}{4} = 4$ → Remainder = 0 → Last digit = 6

Last digit of $3^{31} - 8^{16} = 7 - 6 = 1$

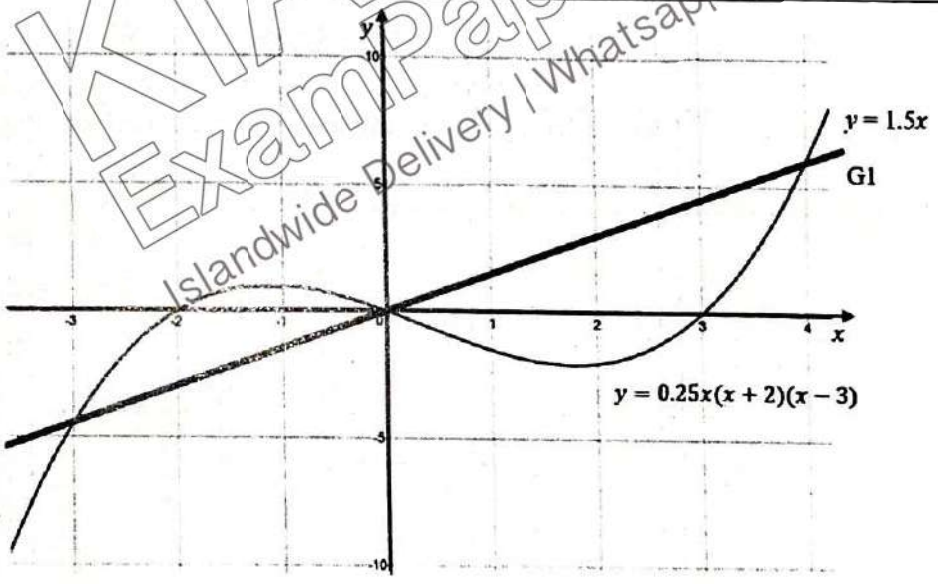
We must see that you had chosen the digit "7" and "6".

15

$\frac{1}{4}x(x+2)(x-3) = \frac{6}{4}x$

Plot $y = \frac{3}{2}x$

Equation of line must be seen in this page. Or else, we will take it as you use calculator mode 3, 4.



$x = -3$ or 0 or 4 → only accept **exact** answers.

16

Draw perpendicular bisector of LC.
Draw angle bisector of AC.
Unit 518.

17

Let M be the midpoint of RS.

$OM = \sqrt{13^2 - 12^2} = 5$ (Perpendicular bisector of chord)

$OK = \sqrt{5^2 + 5^2} = 7.07$ (Equal chords)

Leave answer in 3sf as you are finding length!

18

$$20 \leq \frac{(n-2)\times 180 - 100 - 20}{n-2} \leq 100$$

$$20(n-2) \leq (n-2)\times 180 - 120 \leq 100(n-2)$$

$$20n - 40 \leq 180n - 360 - 120 \leq 100n - 200$$

$20n - 40 \leq 180n - 480$ or $180n - 480 \leq 100n - 200$

$440 \leq 160n$ or $80n \leq 280$

$2.75 \leq n$ or $n \leq 3.5$

$2.75 \leq n \leq 3.5$

$n = 3$

Guess & check allowed for this qn.

19a

$2x^2 - 4x - 7x + 14 - 9 = 2x^2 - 11x + 5 = (2x - 1)(x - 5)$

Factorise, not completing the square! Read carefully.

19b

$2(y-1)^2 - 11(y-1) + 5 = 0$

$x = y - 1$ → **must be seen! Read question. It says "Hence".**

$y - 1 = \frac{1}{2}$ or 5

$y = 1.5$ or 6

20

$\frac{x+8}{y} = 4$

$x + 8 = 4y$

$2y + 6 + 8 = 4y$

$2y = 14$

$y = 7$

$x = 20$

$\frac{x}{y} = 2 + \frac{6}{y}$

$x = 2y + 6$

Do not write mixed number for algebra!!

Recall: dividend = (divisor)(quotient) + remainder

21

$\angle BCD = 180 - 54 = 126$ ($\angle$s in opp segment)

$\angle DCP = 180 - 126 = 54$ (adj $\angle$ on a straight line)

$\angle ADC = 27 + 54 = 81$ (ext $\angle$ of Δ)

$\angle AQD = 180 - 54 - 81 = 45$ ($\angle$ sum of Δ)

Easier method:
$\angle ABP = 180 - 54 - 27 = 99$ ($\angle$ sum of Δ)
$\angle ADQ = 180 - 99 = 81$ ($\angle$s in opposite segment)
$\angle AQD = 180 - 54 - 81 = 45$ ($\angle$ sum of Δ)

22

$\pi rl = 2\pi r^2$

$l = 2r$

$h = \sqrt{(2r)^2 - r^2} = \sqrt{3}r$

Vertical height and slanted length are different!

$$\text{Volume} = \pi r^2(\sqrt{3}r + 2r) - \frac{1}{3}\pi r^2(\sqrt{3}r) - \frac{2}{3}\pi r^3$$

$$= \pi r^3\left(\sqrt{3} + 2 - \frac{\sqrt{3}}{3} - \frac{2}{3}\right)$$

$$= 7.82r^3$$

Refer to cover page if you had left your answers in π.

23a

Length of stay (days)	Numbers of tourists
$0 < x \leq 5$	10
$5 < x \leq 10$	10
$10 < x \leq 15$	10
$15 < x \leq 20$	2
$20 < x \leq 25$	2
$25 < x \leq 30$	3
$30 < x \leq 35$	3
$35 < x \leq 40$	3

23b

$\frac{13}{43}$

23c

Mean $= 13\frac{77}{86}$ or 13.9

SD = 11.0

3sf only! Do not round to nearest integer!

24a

$(x-2)^4(x^2-4x+4) = (x-2)^4(x-2)^2 = (x-2)^6$ → **must be seen!**

Since the power 6 is both a **multiple of 2 and of 3**, it is both a perfect cube and a perfect square.

24b

$2x + 2y = 20$ $\quad 2x^2 + 2y^2 = 140$

$x + y = 10$ $\quad x^2 + y^2 = 70$

$(x+y)^2 = x^2 + y^2 + 2xy$

$2xy = 100 - 70$

$xy = 15$

Alternative method is to solve of x and y to get 8.16 or 1.84 and find xy = 15.0 cm².

25a

$\mathbf{P} = \begin{pmatrix} 32 & 27 & 20 \\ 44 & 45 & 38 \end{pmatrix}$

25b

$\mathbf{PQ} = \begin{pmatrix} 319.9 \\ 525.7 \end{pmatrix}$

25c

$\mathbf{R} = \begin{pmatrix} \frac{3}{4} & 0 & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$

Work with exact! This is percentage, not money hence, it should not be 2dp!

Only accept exact or 3sf.

25d

$$\mathbf{S} = \begin{pmatrix} 32 & 27 & 20 \\ 44 & 45 & 38 \end{pmatrix} \begin{pmatrix} \frac{3}{4} & 0 & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 3.2 \\ 4.5 \\ 4.8 \end{pmatrix} = \begin{pmatrix} 24 & 18 & 10 \\ 33 & 30 & 19 \end{pmatrix} \begin{pmatrix} 3.2 \\ 4.5 \\ 4.8 \end{pmatrix} = \begin{pmatrix} 205.8 \\ 331.8 \end{pmatrix}$$

25e

They represent the **total money collected from Childcare Centre A and B respectively** for the sales of cakes **after discount**.

Money collected from sales is not money earned! The word "earned/earnings" relates to profit.

The End

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南僑中學

NAN CHIAU HIGH SCHOOL

MID-YEAR EXAMINATION 2019
SECONDARY FOUR EXPRESS

MARKING SCHEME

MATHEMATICS **4048/02**

Paper 2 **9 May 2019, Thursday**

Candidates answer on the Question Paper. **2 hours 30 minutes**

2019 Sec 4 MYE EM P2 Solution

1. (a) Simplify $\frac{4b^3}{3a} \div \frac{(-4ab^3)^2}{2a^{-4}b^2}$. Give your answer in positive indices. [2]

$\frac{4b^3}{3a} \div \frac{(-4ab^3)^2}{2a^{-4}b^2}$

$= \frac{4b^3}{3a} \times \frac{2a^{-4}b^2}{16a^2b^6}$ -------- [M1] convert expression to multiplication & use of power

$= \frac{b^3}{3a} \times \frac{b^2}{2a^6b^6}$

$= \frac{1}{6a^7b}$ -------- [A1]

(b) Factorise $14xb + 3ay - 2xy - 21ab$ completely. [2]

$14xb + 3ay - 2xy - 21ab$

$= 14xb - 2xy - 21ab + 3ay$

$= 2x(7b - y) - 3a(7b - y)$ -------- [M1] factorise common terms

$= (7b - y)(2x - 3a)$ -------- [A1]

(c) Solve the inequality $\frac{x}{5} < \frac{x+2}{3} \leq \frac{3-2x}{4}$. [3]

$\frac{3-2x}{4} \geq \frac{x+2}{3}$	*and*	$\frac{x+2}{3} > \frac{x}{5}$
$3(3-2x) \geq 4(x+2)$	*and*	$5(x+2) > 3x$
$9 - 6x \geq 4x + 8$	*and*	$5x + 10 > 3x$
$1 \geq 10x$	*and*	$2x > -10$
$x \leq \frac{1}{10}$ -------- [M1]	*and*	$x > -5$ -------- [M1]

$-5 < x \leq \frac{1}{10}$ -------- [A1]

(d) Simplify $\frac{2x(1-6x)}{4x^2-1} - \frac{2}{2x+1}$. [3]

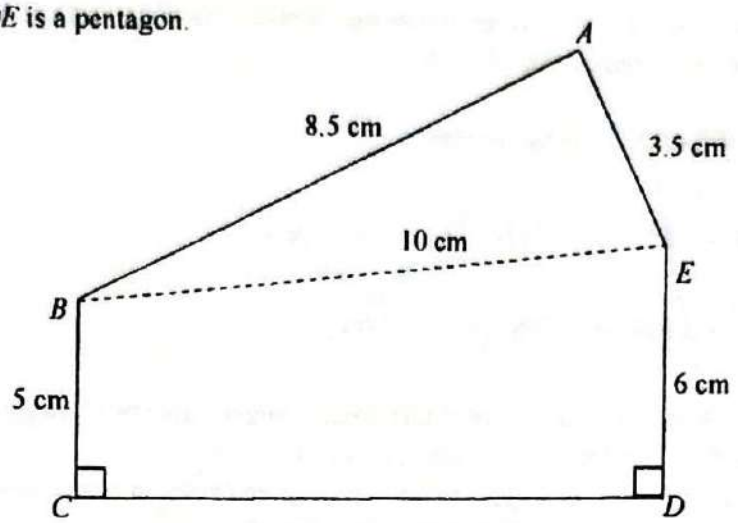
$$\frac{2x(1-6x)}{4x^2-1} - \frac{2}{2x+1}$$
$$= \frac{2x(1-6x)}{(2x+1)(2x-1)} - \frac{2(2x-1)}{(2x+1)(2x-1)} \quad \text{-------- [M1] common denominator}$$
$$= \frac{2x-12x^2-4x+2}{(2x+1)(2x-1)}$$
$$= \frac{-12x^2-2x+2}{(2x+1)(2x-1)} \quad \text{-------- [M1] simplify numerator}$$
$$= \frac{-2(6x^2+x-1)}{(2x+1)(2x-1)}$$
$$= \frac{-2(3x-1)(2x+1)}{(2x+1)(2x-1)} \quad \text{-------- [M1] factorise numerator}$$
$$= \frac{-2(3x-1)}{2x-1} \quad \text{-------- [A1]}$$

(e) Express $2x^2 + 6x - 15$ in the form $a(x+b)^2 + c$, where a, b and c are constants.
Hence, solve the equation $2x^2 + 6x - 15 = 0$, giving your answers correct to four decimal places. [4]

$$2x^2 + 6x - 15$$
$$= 2(x^2 + 3x - \frac{15}{2})$$
$$= 2[(x+\frac{3}{2})^2 - \frac{9}{4} - \frac{15}{2}] \quad \text{-------- [M1] ability to do completing the square}$$
$$= 2[(x+\frac{3}{2})^2 - \frac{39}{4}]$$
$$= 2(x+\frac{3}{2})^2 - \frac{39}{2} \quad \text{-------- [A1]}$$

$$2x^2 + 6x - 15 = 0$$
$$2(x+\frac{3}{2})^2 - \frac{39}{2} = 0$$
$$(x+\frac{3}{2})^2 = \frac{39}{4} \quad \text{-------- [M1] solving using completing the square}$$
$$x + \frac{3}{2} = \pm\sqrt{\frac{39}{4}}$$
$$x = 1.6225 \quad \text{or} \quad x = -4.6225 \quad \text{-------- [A1]}$$

2. *ABCDE* is a pentagon.



(a) Calculate the area of the pentagon *ABCDE*. [5]

$10^2 = 8.5^2 + 3.5^2 - 2(8.5)(3.5)cosB\hat{A}E$ -------- [M1] use of cosine rule

$$cosB\hat{A}E = \frac{8.5^2 + 3.5^2 - 10^2}{2(8.5)(3.5)}$$

$cosB\hat{A}E = -0.260504$

$\alpha = 74.90°$

$B\hat{A}E = 105.1°$ *or* $285.1°$ *(rej)* -------- [M1] ability to find angle *BAE*

Or

$A\hat{B}E = 19.75°$ *or* $340.25°$ *(rej)*

$B\hat{E}A = 55.15°$ *or* $304.85°$ *(rej)*

area of $\Delta ABE = \frac{1}{2}(8.5)(3.5)sin105.1°$ --------[M1] finding area of triangle

$= 14.36\ cm^2$

$BE = \sqrt{10^2 - 1^2}$

$= \sqrt{99}\ cm$

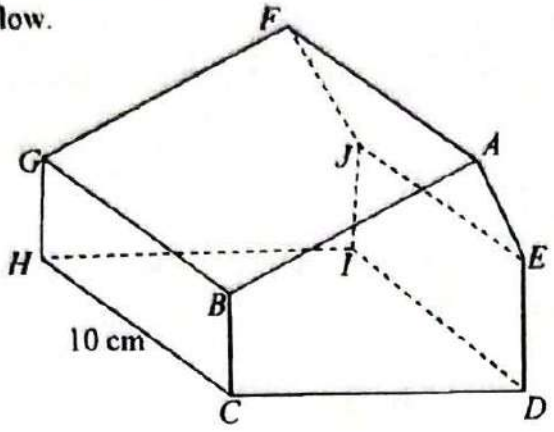
Area of *BCDE* $= \frac{1}{2}(5 + 6)(\sqrt{99})$ -------- [M1] finding area of trapezium

$= 54.72\ cm^2$

Area of *ABCDE* = 14.36 + 54.72

$= 69.1\ cm^2$ -------- [A1]

(b) The pentagon $ABCDE$ is a cross-section of a prism which is a model of a house as shown below.



(i) Calculate the total surface area of the model, including the base. [2]

total surface area

$= 2(69.08) + \underbrace{10(5 + 8.5 + 3.5 + 6 + \sqrt{99})}$

[M1] area of all surface area (excluding cross-sectional area)

$= 468\ \text{cm}^2$ -------- [A1]

(ii) Calculate the angle of depression of H from E. [3]

Let the angle of depression be θ

Student use
(i) angle of depression = angle of elevation or
(ii) angle of depression = $90^o - 57.5^o$

$HD = \sqrt{10^2 + 99}$ -------- [M1] finding length HD

$= \sqrt{199}$ cm

$\tan\theta = \frac{6}{\sqrt{199}}$ -------- [M1] finding relevant angle (either angle EHD or HED)

$\theta = 23.0^o$ -------- [A1]

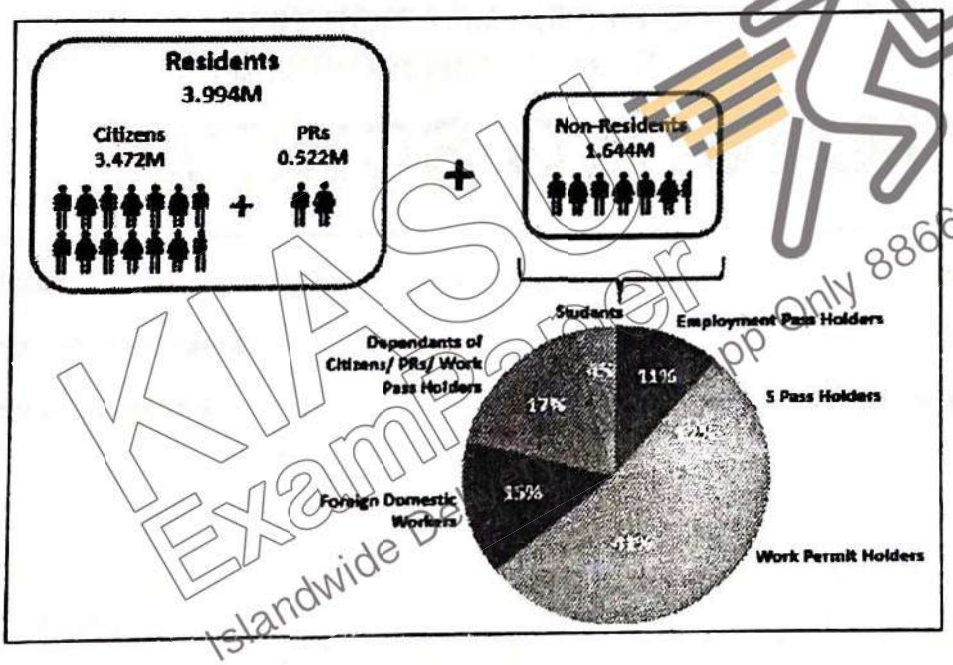
3. There was a newspaper article on Singapore's population published in 2018.

Singapore's population grows slightly to 5.638 million, with non-resident numbers stable.

Singapore's population rose 0.5 percent to 5.638 million from 2017 to 2018.

The slight increase over the past year was due to stable growth in citizen population, which rose 1 percent to 3.472 million citizens as of June 2018.

The 1 percent growth in citizens was due to citizen births and immigration.



Source:
https://www.channelnewsasia.com/news/singapore/singapore-s-population-grows-slightly-to-5-64m-with-non-resident-10763132

(a) Calculate the Singapore's total population in 2017. [2]

100.5% → 5.638 million

100% → $\frac{100}{100.5} \times 5.638$ million -------- [M1] showing $\frac{100}{100.5} \times 5.638$

$= 5.61 \times 10^6$ -------- [A1] or 5.61 million

(b) Express the number of Foreign Domestic Workers as a percentage of the Singapore's total population in 2018. [2]

Number of foreign domestic workers
$= \frac{15}{100} \times 1.644 \times 10^6$
$= 0.2466 \times 10^6$ -------- [M1] must in 10^6 or million

$\frac{0.2466 \times 10^6}{5.638 \times 10^6} \times 100\% = 4.37\%$ -------- [A1]

(c) Based on the information shown in this article, do you agree that "Singapore's total population will reach 6.3 million by 2030?"
Support your answer with appropriate workings and state an assumption that you have made in your calculation. [3]

2030 – 2018 = 12 years

Population of citizen by 2030
$= (1.01)^{12} \times 3.472 \times 10^6$ -------- [M1] finding the increase of citizen only
$= 3.912 \times 10^6$

total population by 2030
$= (3.912 + 0.522 + 1.644) \times 10^6$
$= 6.08 \times 10^6$ -------- [A1]

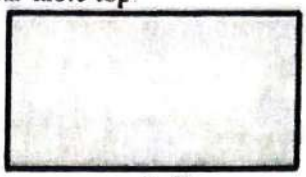
Assumption:
There is no increase for PRs and Non-Residents or
There is a constant growth of citizens at 1% per year -------- [M1]

(d) State one aspect of the diagram that may be misleading and explain how this may lead to a misinterpretation. [2]

Accept any logical answer
(i) Pictogram used alternate of a male and female figure --- [B1]
[Reason] misleading readers that there is an equal number of male and female population. --- [B1]

(ii) Inaccurate use of ½ pictogram "figure" for "non-resident" & ¾ in "citizen" --- [B1]
[Reason]
in PRs : if 1 figure → 0.261 M
in citizen : 3.472 M should be represented by 13.3 figures
not 13 + ¾ figure --- [B1]

4. The diagram shows a rectangular table top.



The area of the table top is 5 square metres.
The length of the table top is x metres.
The perimeter of the table top is y metres.

(a) Show that $y = 2x + \frac{10}{x}$. [2]

Width of table top $= \frac{5}{x}$ -------- [M1]

Perimeter, $y = x + \frac{5}{x} + x + \frac{5}{x}$ -------- [M1]

$y = 2x + \frac{10}{x}$

(b) The variables x and y are connected by the equation $y = 2x + \frac{10}{x}$.

x	0.5	1	1.5	2	4	6	8
y	p	12	q	9	10.5	13.7	17.3

Find the value of p and of q. [1]

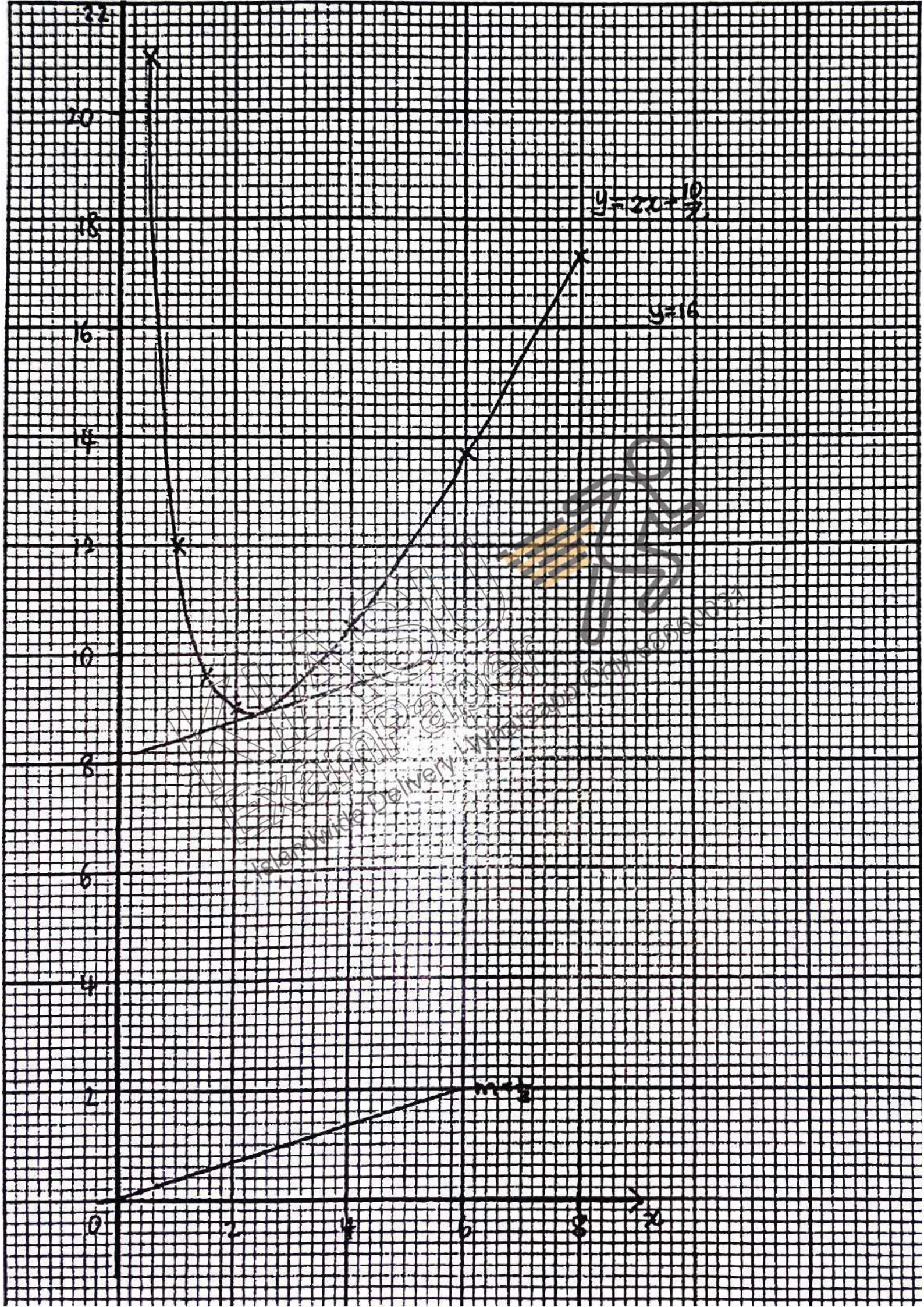
$p = 21$

$q = 9.7$ (accept also 9.67) -------- [B1] for both answers

(c) Using a scale of 1 cm to represent 1 unit, draw a horizontal x-axis for $0 \leq x \leq 8$.

Using a scale of 1 cm to represent 1 unit, draw a vertical y-axis for $0 \leq y \leq 22$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]


y
22
20
18
16
14
12
10
8
6
4
2
0
2
4
6
8
x
$y = 2x + \frac{10}{x}$
y=16
m=3

(d) Use your graph to find the possible width of the table top if the perimeter of the table top is 16 m. [2]

When $P = 16$,

$x = 0.7$ (± 0.1) ------ [B1] answer must be rejected as length is longer

or $x = 7.3$ (± 0.1) -------- [B1]

(e) By drawing a suitable straight line, find the x-coordinate of the point at which gradient of the curve is $\frac{1}{3}$. [2]

Draw a line with grad $= \frac{1}{3}$ on the graph -------- [M1]

x-coordinate $= 2.3$ (± 0.1) -------- [A1]

5. Given equation of a line L is $3x - 2y = 8$.

(a) State the gradient of line L. [1]

$2y = 3x - 8$

$y = \frac{3}{2}x - 4$

Gradient $= \frac{3}{2}$ -------- [B1]

(b) If $P\ (k, -2)$ is a point on the line L, find the value of k. [1]

$3k - 2(-2) = 8$

$k = \frac{4}{3}$ -------- [B1]

(c) Find the equation of another line that is parallel to the x-axis and passes through P. [1]

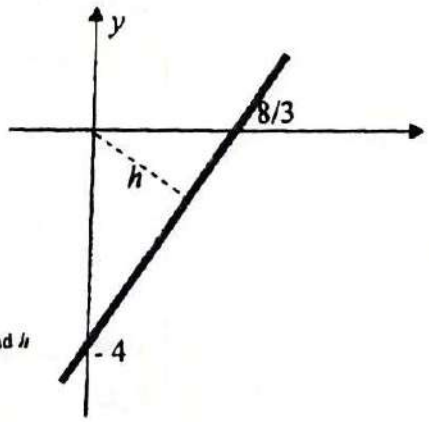
$y = -2$ -------- [B1]

(d) Calculate the perpendicular distance from the origin, O, to the line L. [4]

$3x - 2y = 8$

When $x = 0$, $y = -4$ $(0, -4)$

When $y = 0$, $x = \frac{8}{3}$ $(\frac{8}{3}, 0)$ -------- [M1] finding relevant coordinates



Distance between 2 points

$= \sqrt{\left(\frac{8}{3}\right)^2 + (4)^2}$ -------- [M1]

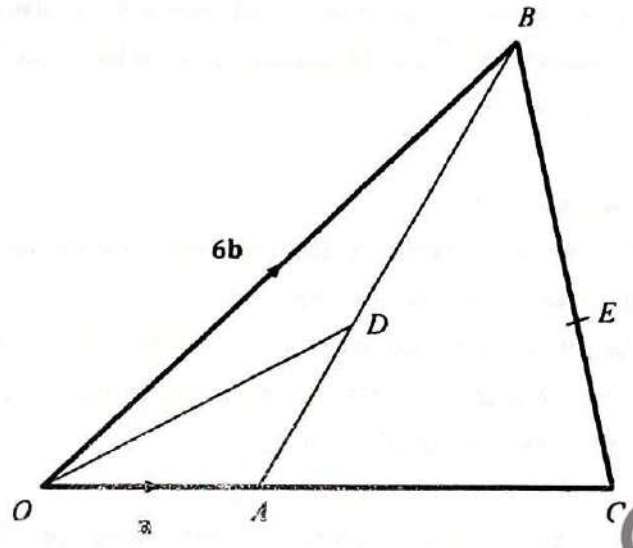
$= 4.8074$ units

Let the perpendicular distance be h.

$\frac{1}{2}(4)\left(\frac{8}{3}\right) = \frac{1}{2}(h)(4.8074)$ -------- [M1] suitable method to find h

$h = 2.22$ units -------- [A1] accept h = 2.21 units

6.



In the diagram, $\overrightarrow{OA} = \boldsymbol{a}$, $\overrightarrow{OB} = 6\boldsymbol{b}$ and $\overrightarrow{OA} = \frac{1}{3}\overrightarrow{OC}$.

D is a point on AB such that $3\overrightarrow{AD} = 2\overrightarrow{DB}$ and E is a point on BC such that $\overrightarrow{CE} : \overrightarrow{EB} = 4 : 5$.

(a) Express, as simply as possible, in terms of **a** and **b**,

(i) $\overrightarrow{BA}$, [1]

$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$

$= \boldsymbol{a} - 6\boldsymbol{b}$ B1

(ii) $\overrightarrow{OD}$, [1]

$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$

$= \overrightarrow{OA} - \frac{2}{5}\overrightarrow{BA}$

$= \boldsymbol{a} - \frac{2}{5}(\boldsymbol{a} - 6\boldsymbol{b})$

$= \frac{3}{5}(\boldsymbol{a} + 4\boldsymbol{b})$ B1

(iii) $\overrightarrow{CB}$. [1]

$\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC}$

$= 6\boldsymbol{b} - 3\boldsymbol{a}$

$= 3(2\boldsymbol{b} - \boldsymbol{a})$ B1

(iv) $\overrightarrow{AE}$. [2]

$\overrightarrow{AE} = \overrightarrow{AC} + \overrightarrow{CE}$

$= 2\boldsymbol{a} + \frac{4}{9}\overrightarrow{CB}$

$= 2\boldsymbol{a} + \frac{4}{9}(6\boldsymbol{b} - 3\boldsymbol{a})$

$= \frac{2}{3}(\boldsymbol{a} + 4\boldsymbol{b})$ B1

(b) Write down the relationship between OD and AE. Explain your answer. [2]

$\overrightarrow{OD} = \frac{3}{5}(\boldsymbol{a} + 4\boldsymbol{b})$

$\overrightarrow{AE} = \frac{2}{3}(\boldsymbol{a} + 4\boldsymbol{b})$

Since

$|\overrightarrow{OD}| = \frac{9}{10}|\overrightarrow{AE}|$, M1 – Relationship between OD and AE

OD and AE are parallel lines. A1 – Parallel lines

(c) Find the ratio of

(i) area of triangle CAE : area of triangle AOD, [2]

$$\frac{\text{area of triangle } CAE}{\text{area of triangle } AOD} = \frac{\frac{1}{2}(CA)(AE)\sin\theta}{\frac{1}{2}(OA)(OD)\sin\theta}$$

$$= \frac{AE}{OD} \times \frac{CA}{OA}$$

$$= \frac{10}{9} \times \frac{2}{1}$$

$$= \frac{20}{9}$$

M1 – With correct ratio of corresponding sides

20: 9 A1

(ii) area of triangle CAE : area of triangle AOB. [2]

$$\frac{\text{area of triangle } CAE}{\text{area of triangle } AOB}$$

$$= \frac{\text{area of triangle } CAE}{\text{area of triangle } AOD} \times \frac{\text{area of triangle AOD}}{\text{area of triangle } AOB}$$

$$= \frac{20}{9} \times \frac{\frac{1}{2}(h)(AD)}{\frac{1}{2}(h)(AB)}$$

$$= \frac{20}{9} \times \frac{2}{5}$$

$$= \frac{8}{9}$$

M1 – Common height relationship

8: 9 A1

7. (a) There are two boxes of sweets containing toffees and chocolates.
Box A contains 8 toffees and 4 chocolates, whereas box B contains 7 toffees and 3 chocolates.

Jolin loves chocolates.
One of the boxes is chosen at random and a sweet is taken out.
If she gets a chocolate, she will consume it.
If she did not get a chocolate from the first selection, she will place the sweet into the other box and she will select again from the other box.
Jolin limits herself to two selections.

Find, as a fraction in its simplest form, the probability that

(i) Jolin will have her favourite chocolate from the first selection, [3]

$$P(\text{first selection}) = P(\text{Box A}) + P(\text{Box B})$$
$$= \left(\frac{1}{2}\right)\left(\frac{4}{12}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{10}\right)$$
$$= \frac{19}{60}$$

M1 – with ½
M1 – 4/12 + 3/10 (without ½)
A1

(ii) Jolin will have her favourite chocolate. [3]

$$P(\text{favourite})$$
$$= P(\text{at first}) + P(\text{at second})$$
$$= \frac{19}{60} + P(\text{no A, yes B}) + P(\text{yes A, no B})$$
$$= \frac{19}{60} + \left(\frac{1}{2}\right)\left(\frac{8}{12}\right)\left(\frac{3}{11}\right) + \left(\frac{1}{2}\right)\left(\frac{7}{10}\right)\left(\frac{4}{13}\right)$$
$$= \frac{4421}{8580}$$

M1 – from (i)
M1 – with ½ and 2 cases for P(at second) correct
A1

(b) Your friend, Kenton gives you a chance to win $1000 by playing a game of "Guess the number". There are two options of the game that he allows you to choose.

Option A

He uses a random number generator to choose a number from 1 to 8.

If you guess it correctly, you win.

Option B

You flip a fair coin.

If the coin lands on head, Kenton will roll a fair 6-sided die. If you manage to guess what it rolled, you win.

If the coin lands on tail, Kenton will use a random number generator to choose a random number from 1 to 8. If you guess the number correctly, you win.

Which option will you choose? Explain your answer. [3]

Option A

$P(\text{win}) = \frac{1}{8}$ B1

Option B

$P(\text{win}) = P(\text{H, die}) + P(\text{T, No. C+C})$

$= \left(\frac{1}{2}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{8}\right)$ M1

$= \frac{7}{48} \left(> \frac{1}{8}\right)$

Since Option B has a higher probability, Option B should be chosen. A1 – with both options' probability correctly calculated.

- A regular hexagon forms within the inner circle $ABCDEF$ with $OB = 5$ cm.
  This information is illustrated in Figure 3.

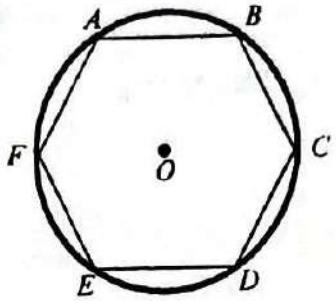


Figure 3

(a) Show that the radius of the outer circle is 13.31 cm, correct to 2 decimal places. **[6]**

Let M be the midpoint of AB.

Consider ΔAGX,

Using Cosine Rule,

$AG^2 = 18^2 + 18^2 - 2(18)(18)\cos\frac{\pi}{6}$ — M1 – finding line segment AG

$AG = 9.31749$ cm

Consider ΔAMO,

$\angle AOM = 360° \div 12$ — M1 – finding angle AOX or AOB = 60°

$= 30°$

$AM = 5\sin 30°$ — M1 – AX

$MO = 5\cos 30°$ — M1 – XO

Consider ΔAGM,

By Pythagoras theorem,

$GM = \sqrt{AG^2 - AM^2}$

$= \sqrt{9.31749^2 - (5\sin 30°)^2}$

$= 8.9758$ cm

Radius $= GM + MO$

$= 8.9758 + 5\cos 30°$ — M1 - $GX + XO$

= 13.31 cm

(b) Team Hō'ola decided to make 50 dreamcatcher-inspired ornaments and Janice chanced upon the following promotion.

10 m silver craft bendable DIY wire (2mm thick) $9.28

What is the estimated cost to make these ornaments? [5]

arc $AG = (18)\left(\frac{\pi}{6}\right)$ M1

$= 3\pi$ cm

Petals $= 12 \times 3\pi$

$= 36\pi$ cm

Circumference of small circle $= 2\pi(5)$

$= 10\pi$

Circumference of big circle $= 2\pi(13.30597)$

$= 26.6119\pi$

M1 – Both circumference found.

Amount of wire for 50 dreamcatchers

$= (26.6119\pi + 10\pi + 36\pi) \times 50$

$= 3630.59\pi$ cm

$= 36.3059\pi$ m

M1 – Total circumference multiply by 50

Rolls of wire needed

$= \frac{36.3059\pi}{10}$

$= 11.4058$

$= 12$ (rounded up)

M1 – Conversion from cm to m and rounded up.

Cost $= 12 \times \$9.28$

= $111.36 (to 2dp) A1

(c) Find the area of the shaded region as shown in Figure 1. [5]

area of AG segment

$= \frac{1}{2}(18)^2\left(\frac{\pi}{6}\right) - \frac{1}{2}(18)(18)\sin\frac{\pi}{6}$ M1

$= 3.823\ \text{cm}^2$

area of $\Delta AGX = \frac{1}{2}(8.9758)(5\sin 30°)$ M1 – area of right angle triangle AGX.

$= 11.21975\ \text{cm}^2$

area of $\Delta OXA = \frac{1}{2}(5\cos 30°)(5\sin 30°)$ M1 – area of right angle triangle OXA.

$= 5.41266\ \text{cm}^2$

area of shaded region

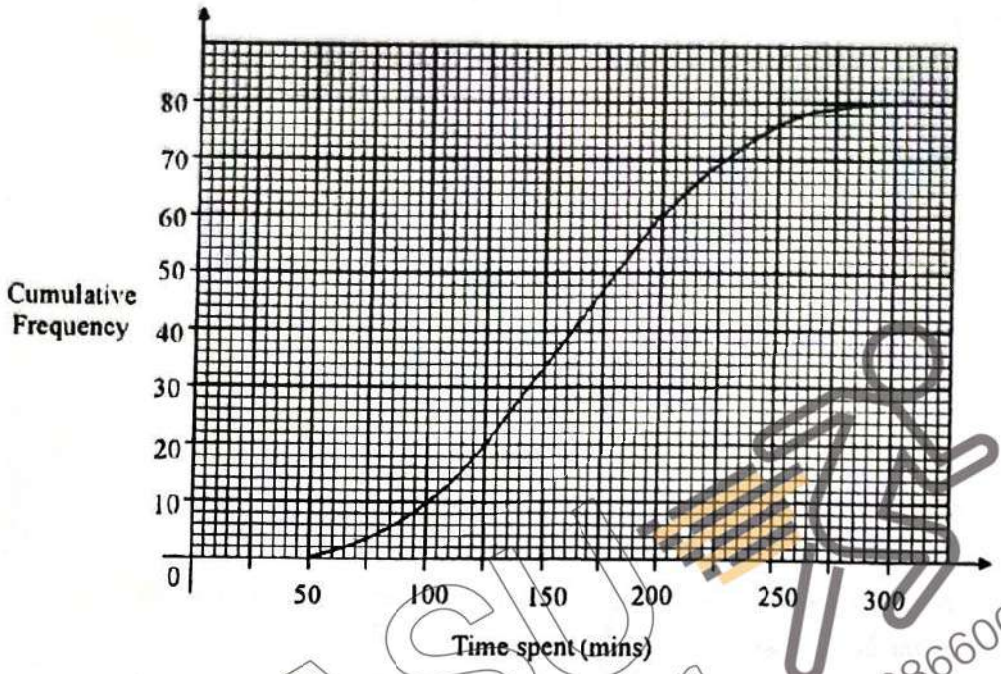
$= \pi(13.30597)^2 - 5.41266 \times 12$ M1 – big circle area

$- (3.823 + 11.21975) \times 12$

$= 310.7504\ \text{cm}^2$

$= 311\ \text{cm}^2$ A1

9. (a) The cumulative frequency curve below shows the time spent in minutes by a group of 80 teenagers on Instagram (a social media platform) on a particular day.



Use the curve to estimate

(i) the median, [1]

$\frac{N}{2} = 40$

median = 162.5 min B1

(ii) the interquartile range. [2]

$\frac{N}{4} = 20$ $\frac{3N}{4} = 60$

Q1 = 125 min Q3 = 200 min B1 – both Q1 and Q3 correct

Interquartile Range = Q3 − Q1

= 200 − 125

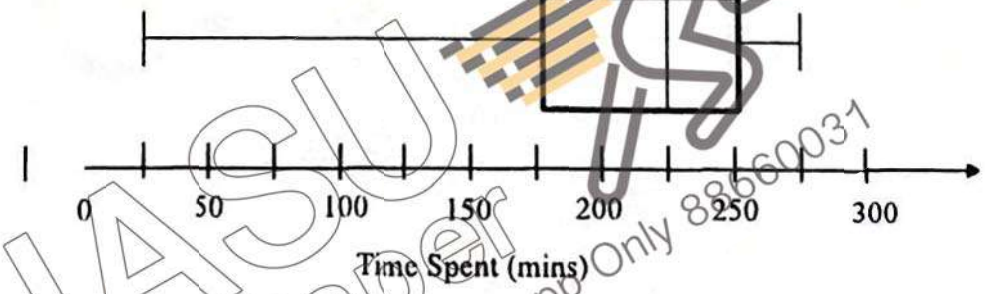
= 75 min A1

(b) Through a market research, it was found out that the time spent on Facebook (another social media platform) is less popular and less consistent among the same group of 80 teenagers. A second cumulative frequency curve for the same group of 80 teenagers spending their time on Facebook is drawn.

Describe how the second cumulative frequency curve may differ from the curve for Instagram. [2]

The curve will <u>shift to the left</u>	B1 – shift to left
and has a <u>gentler slope</u>	B1 – Gentler slope
as compared to the given curve.	

(c) The box-and-whisker plot represents the distribution of the time spent for the same group of 80 teenagers on SnapChat (another social media platform).

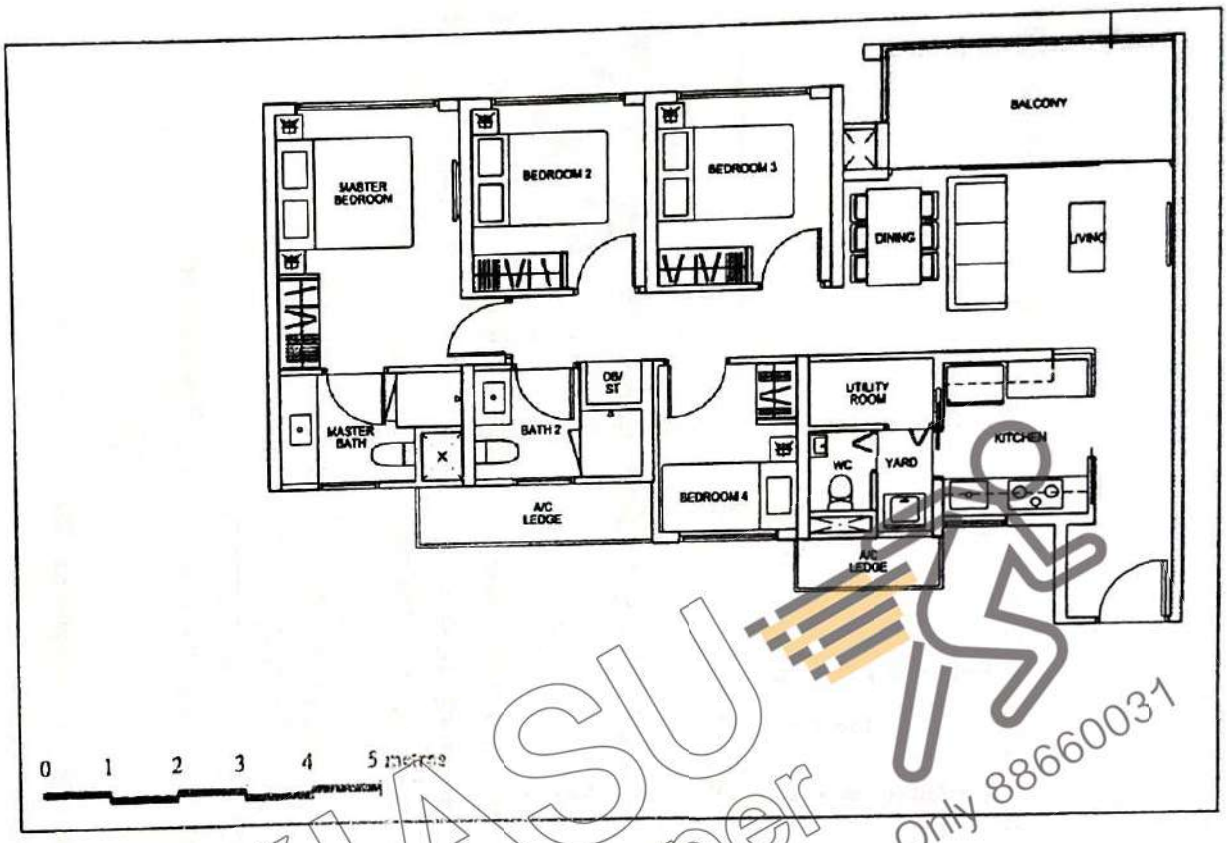


For this group of 80 teenagers, which of the social media platforms - Instagram, Facebook or Snapchat, is the most popular?

Support your answer with an appropriate statistical measure. [2]

Median (Instagram) = 162.5 min Median (Facebook) < 162.5 min Median (SnapChat) = 225 min	M1 – Evidence for median especially for SnapChat
Since <u>Snapchat has the highest median</u>, it is the most popular.	A1 – Median as statistical comparison
*If students show no data evidence but mentioned about highest median, award 1 mark.	

10. Ms Tan got her new home recently and the layout of her house is shown in the diagram below. The layout is drawn to scale.



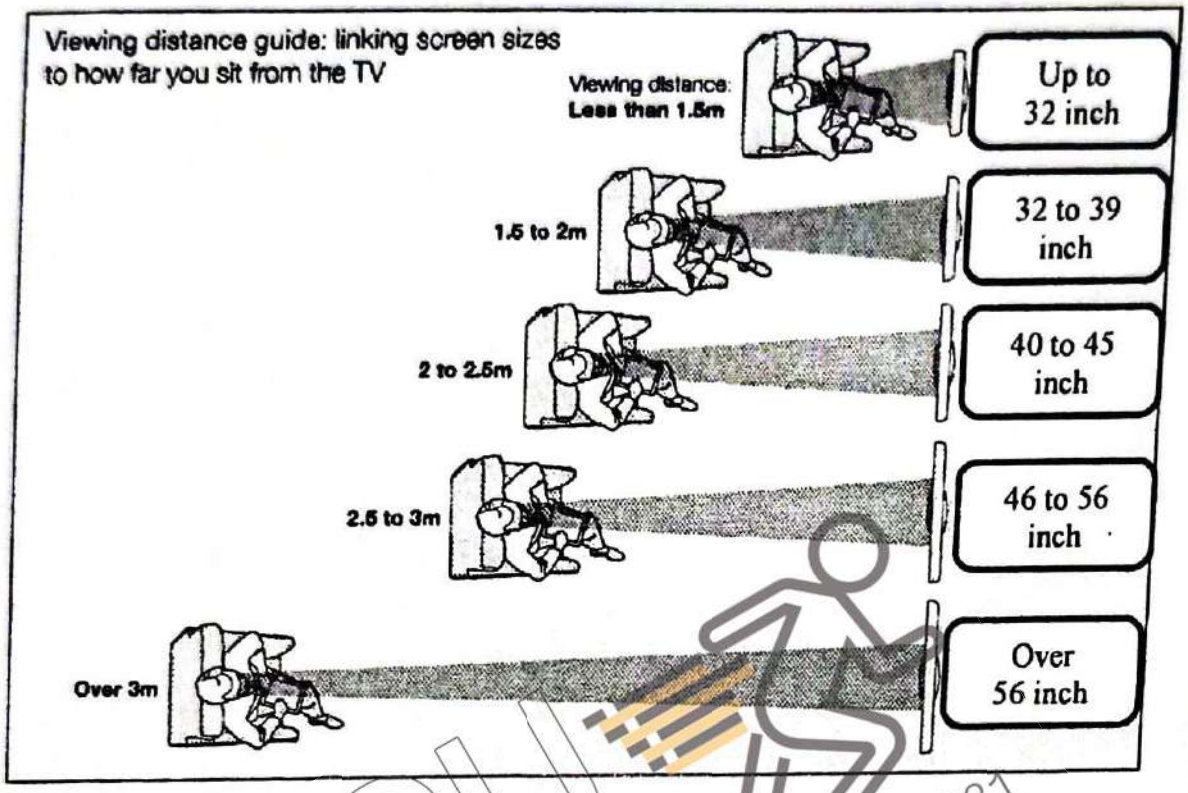
(a) Express the scale of the map in the form of 1 : n. [1]

1cm = 1m
1cm = 100 cm

1 : 100 B1

(b) Ms Tan decides to get a television set for her living room.

She found the following infographic online.



What is the range of television size which Ms Tan should get for her living room? [2]

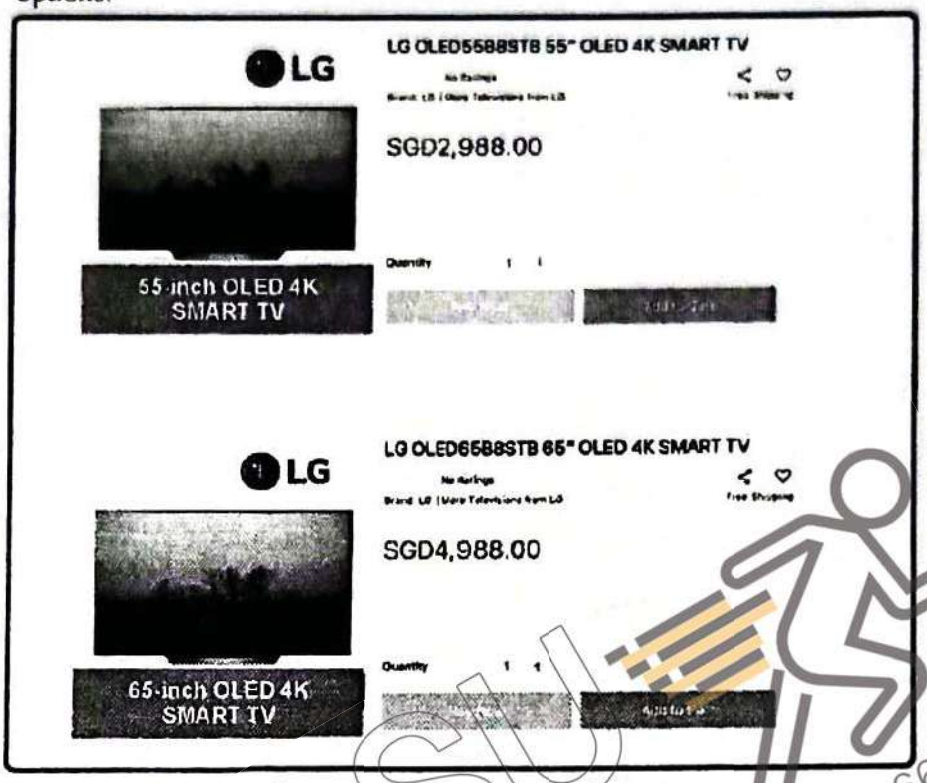
Distance of Sofa from TV = 3 cm × 100 cm
= 3 m

M1 - Measured distance can range from 2.3 cm to 3.1 cm

Range – 46 to 56 inch

A1

(c) Ms Tan decided to shop for her television set online and she saw the following options:



Ms Tan pays a downpayment of $2 000 for her television set as suggested in (b). She pays the remaining amount over 3 months, with a simple interest rate of 7 % per annum.

Calculate her monthly instalment. [3]

Ms Tan to purchase 55 inch TV.

Remaining amount $= \$2988 - \$2000$ M1

$= \$988$

$$\text{Interest} = \frac{988 \times 7 \times \frac{3}{12}}{100}$$ M1 – with correct $988 and 3/12

$= \$17.29$

$$\text{Monthly instalment} = \frac{\$988 + \$17.29}{3}$$

$= \$335.10$ (to 2dp) A1

ST. MARGARET'S SECONDARY SCHOOL

Mid-Year Examinations 2019

CANDIDATE NAME					
CLASS				REGISTER NUMBER	

MATHEMATICS **4048/01**

Paper 1 **13 May 2019**

Secondary 4 Express **2 hours**

Additional Materials: Nil

READ THESE INSTRUCTIONS FIRST

Write your name, registration number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use

This document consists of **18** printed pages

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 (a) Expand and simplify $(4x - 1)^2 - (8x + 1)(2x - 1)$.

*Answer (a)*_____ [2]

(b) Simplify $\frac{(2x^2)^3}{5\sqrt{x}} \div 4x^{-2}$, giving your answer in the form ax^n, where a and n are rational numbers.

*Answer (b)*_____ [2]

2 (a) The area of Singapore is about 710 km^2. Express the area in square metres, giving your answer in standard form.

Answer (a) _____ [2]

(b) $2019.04 = 2\times 10^3 + 1\times 10 + 9\times 10^m + 4\times 10^n$, where m and n are integers.
Write down the value of m and n.

Answer (b) $m =$ _____ ,$n =$ _____ [2]

3 (a) Solve $\frac{2}{m^2} - 3 = \frac{1}{m}$.

*Answer (a)*_____[2]

(b) Hence solve $\frac{2}{(3y-1)^2} - 3 = \frac{1}{(3y-1)}$.

*Answer (b)*_____[2]

4 An interior angle of a regular hexagon is three times the size of the exterior angle of another *n*-sided regular polygon, Find the value of *n*.

Answer _____[2]

5 The number 2200, written as the product of their prime factors, are $2^3 \times 5^2 \times 11$.

(a) Express 5880 as the product of its prime factors.

*Answer (a)*_____[2]

(b) Hence write down the greatest integer that will divide both 2200 and 5880 exactly.

*Answer (b)*_____[1]

(c) Write down the smallest integer k, such that $\sqrt{\frac{2200}{k}}$ will give a whole number.

*Answer (c)*_____[1]

6 Four points lie on a line segment such that $AB : BC = 1 : 2$ and $BC : CD = 8 : 5$.

(a) Find $AB : BD$.

*Answer (a)*_____[1]

(b) If A represents the number -7 and D represents the number 98.4, find the number that is represented by C.

*Answer (c)*_____[2]

7 (a) Factorise $4x^2 - 28xy + 49y^2$ completely.

*Answer (a)*_____[2]

(b) Given that $4x^2 - 28xy + 49y^2 = 0$, express x in terms of y.

*Answer (b)*_____[1]

8 The number of days some students were late for school was recorded in the table as follows.

Number of days late	0	1	2	3	4	5
Number of students	13	9	7	x	8	4

(a) Write down the largest possible value of x if the mode is 0 days.

*Answer (a)*_____[1]

(b) Find the smallest possible value of x if the median is 3 days.

*Answer (b)*_____[1]

(c) Find the value of x if mean is 2.

*Answer (c)*_____[2]

9 An open field has an area of 112.5 km^2. It is represented by an area of 18 cm^2 on map X.

(a) Find the scale of the map in the form 1 : n.

*Answer (a)*_____[2]

(b) Map Y has a scale of 1 : 400 000. A road is measured 2.4 cm on Map X.
Find, in centimetres, the length representing this road on Map Y.

*Answer (b)*_____[2]

10 The volume of cylinder A of radius r cm and height h cm is 360 cm^3.

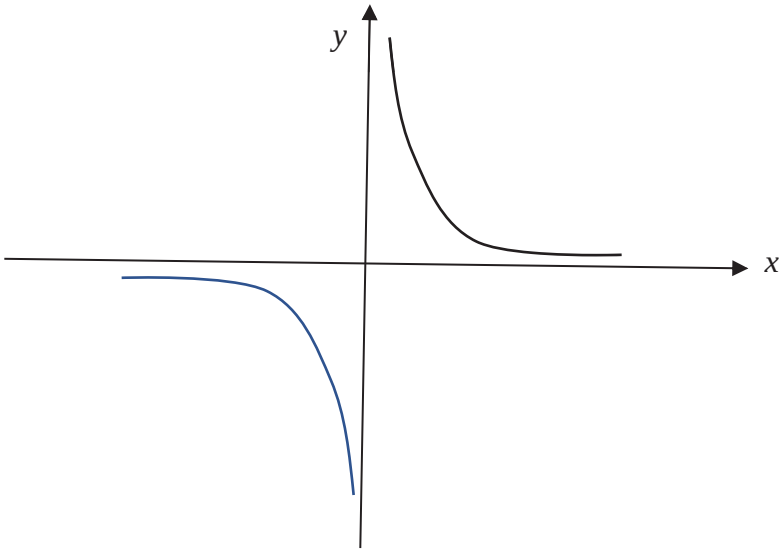
(a) Find the volume of cylinder B of radius $2r$ and height $\frac{1}{3}h$ cm.

*Answer (a)*_____[2]

(b) Cylinder C is similar to cylinder A. If the radius of cylinder C is $0.5r$ cm, find the volume of cylinder C.

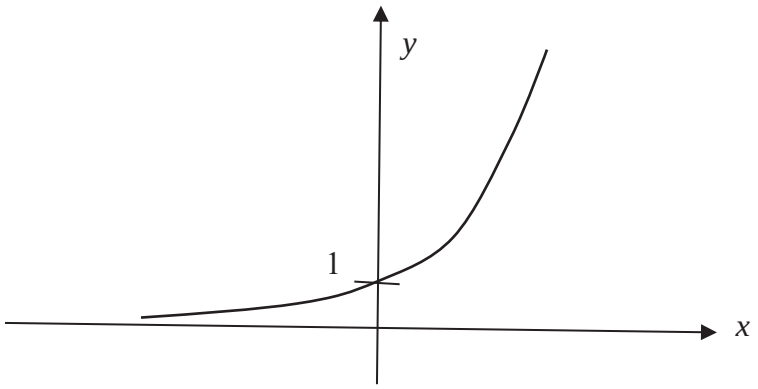
*Answer (b)*_____[2]

11 (a) The sketch represents the graph of $y = x^n$. Write down a possible value of n.



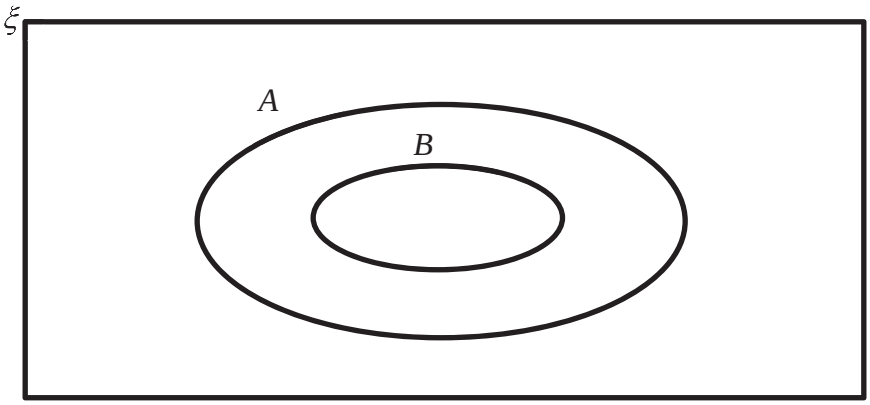
*Answer (a)*_____[1]

(b) Write down a possible equation for this graph.



*Answer (b)*_____[1]

12 (a) On the Venn diagram, shade the set $A \cap B'$. [1]



(b) $\varepsilon = \{x : 0 < x \leq 30\}$
$M = \{x : x \text{ is a perfect cube}\}$
$N = \{x : x = 2k + 1, k \text{ integer}\}$
Find $(M \cap N)$.

Answer _____[2]

13 A bookshelf contains 20 fiction and 5 non-fiction books.

(a) Write down the probability that a book drawn at random from the bookshelf will be non-fiction.

*Answer (a)*_____[1]

(b) Given that p non-fiction books are added to the bookshelf such that the probability of drawing a non-fiction book is 0.5, find the value of p.

*Answer (b)*_____[2]

14 Regina exercises on Tuesdays and Fridays.

On Tuesdays, she jogs for 40 minutes and sprints for 15 minutes.

On Fridays, she jogs 10 more minutes and sprints 5 minutes less.

This information can be represented by the matrix $\boldsymbol{R} = \begin{pmatrix} 40 & 15 \\ 10 & -5 \end{pmatrix}$.

(a) Regina's jogging speed is 8 km/h and her sprinting speed is 10 km/h.
Represent these speeds in a 2×1 column matrix $\boldsymbol{S}$.

Answer (a) $S =$ _____[1]

(b) Evaluate the matrix $\boldsymbol{P} = \frac{1}{60}\boldsymbol{RS}$.

Answer (b) $P =$ _____[2]

(c) What is the distance covered on Fridays?

*Answer (c)*_____[2]

15 Theresa weighed 7 oranges in a supermarket.

The mean mass of the oranges was 138 grams.

The standard deviation of the masses of the oranges was 4.29 grams.

The scales in the supermarket was faulty.

The correct mass of each orange was 15 grams more than what Theresa has recorded.

Write down the correct values for the mean and standard deviation of each orange.

Answer mean = _____ [1]

Standard Deviation = _____ [1]

16 The number of goals scored by 2 soccer players, Fati and Bala, in 6 matches are listed below.

Fati	2	1	3	2	3	1
Bala	5	2	2	0	0	3

One of them has to be selected for the national team. Who should be selected?

Justify your answer with clear working.

*Answer*_____

_____

_____[2]

17 Given that $\mathbf{u} = \begin{pmatrix} 8 \\ -2 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 16 \\ p \end{pmatrix}$. Find

(a) $|\mathbf{u} - \mathbf{v}|$,

*Answer (a)*_____[2]

(b) $2\mathbf{v} + \mathbf{u}$.

*Answer (b)*_____[1]

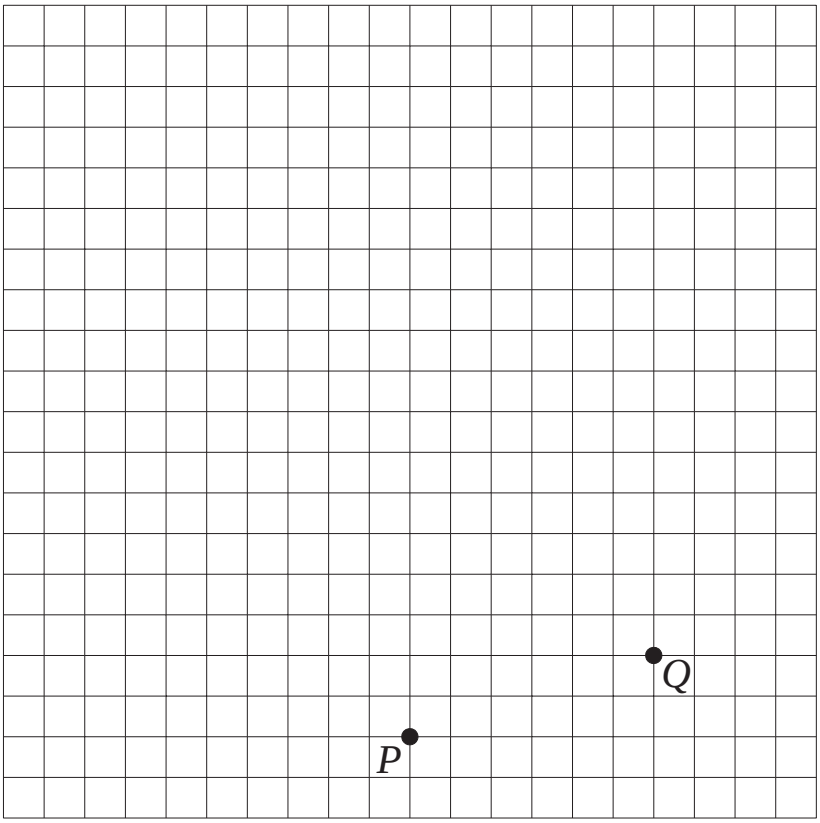
18 The force (F units) between two particles is inversely proportional to the square of the distance (x units) between them.

When the distance between two particles is x, the force is F. When the distance is reduced to $0.5x$, what is the ratio of the force to the original force?

Answer _____ [2]

19 The point R is such that $\overrightarrow{QR} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$. It is given that $\overrightarrow{RS} = \begin{pmatrix} -12 \\ h \end{pmatrix}$.

Find the two possible values of h which will make $PQRS$ a trapezium.



Answer _____ [2]

20 (a) Solve the linear inequalities $\frac{4}{3}x - 14 \leq 3(x-4) < 7.5$ and represent their solution on the number line provided. [3]

(b) Hence, write down the smallest integer which satisfies $\frac{4}{3}x - 14 \leq 3(x-4) < 7.5$.

*Answer (b)*_____[1]

21 A man invested \$7600 in a fund that pays 2.8% compound interest per annum, compounded every half-yearly. Calculate the total amount of money he has at the end of 4 years, giving your answer correct to the nearest 10 cents.

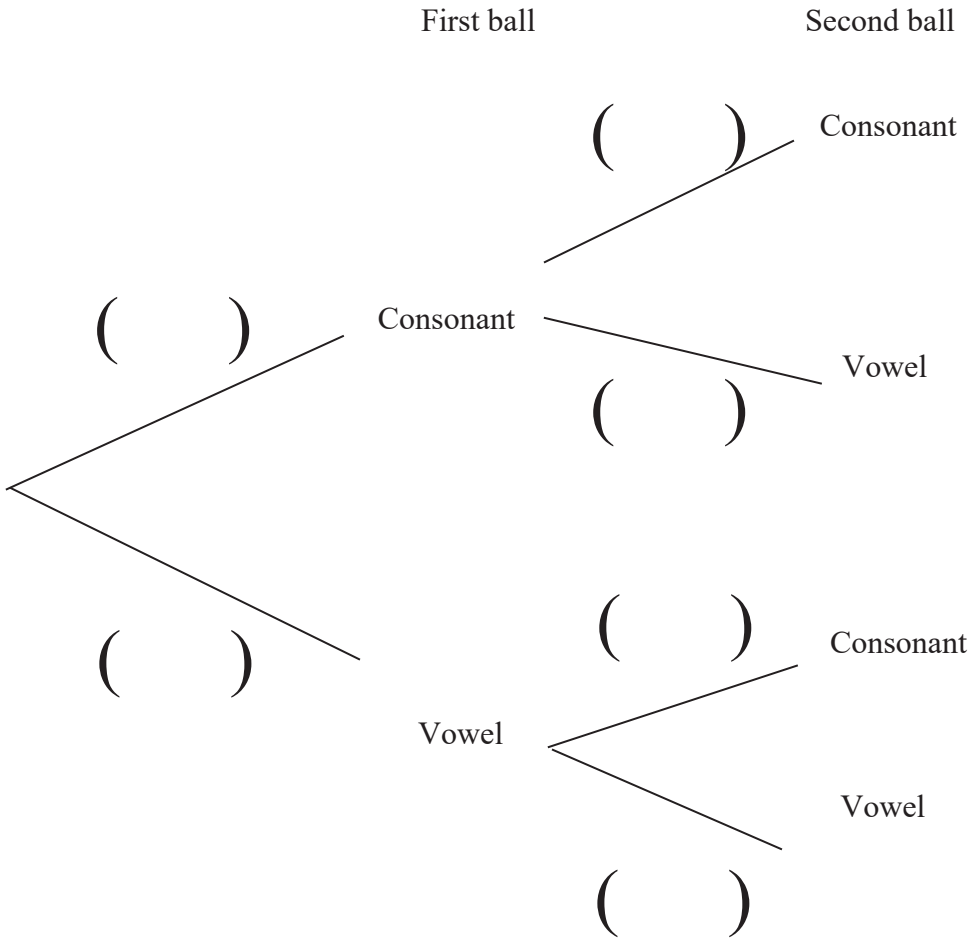
Answer _____ [2]

22 The letters of the word ‘RESILIENCE PACKAGE’ are pasted onto 17 plastic balls, one letter on each ball and they are then put into a box. A ball is drawn at random.

If it is a ball with a consonant, it is put back into the box and a new ball is drawn.

If it is a ball with a vowel, it is NOT put back into the box and a second ball is then drawn.

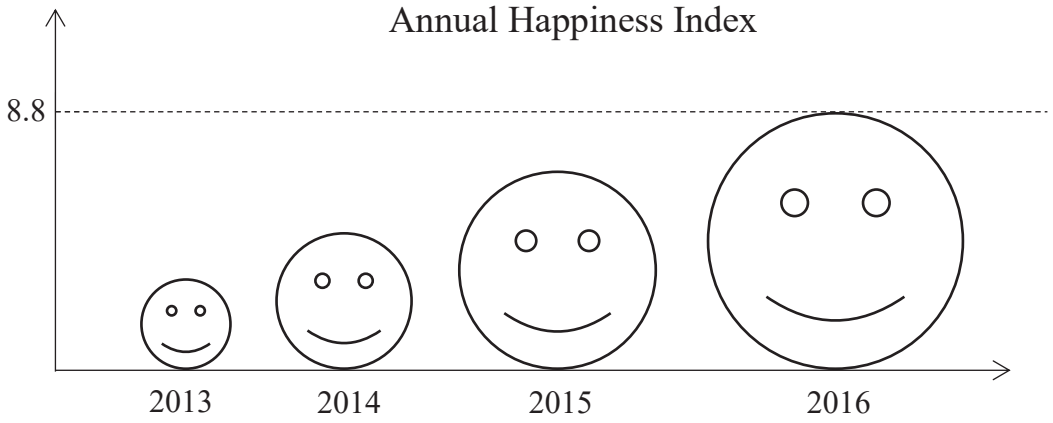
(a) Complete the tree diagram. [2]



(b) Calculate the probability that two vowels are drawn.

Answer (*b*)_____[1]

23 Ada draws this graph to show the happiness index of her country for the last 4 years.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

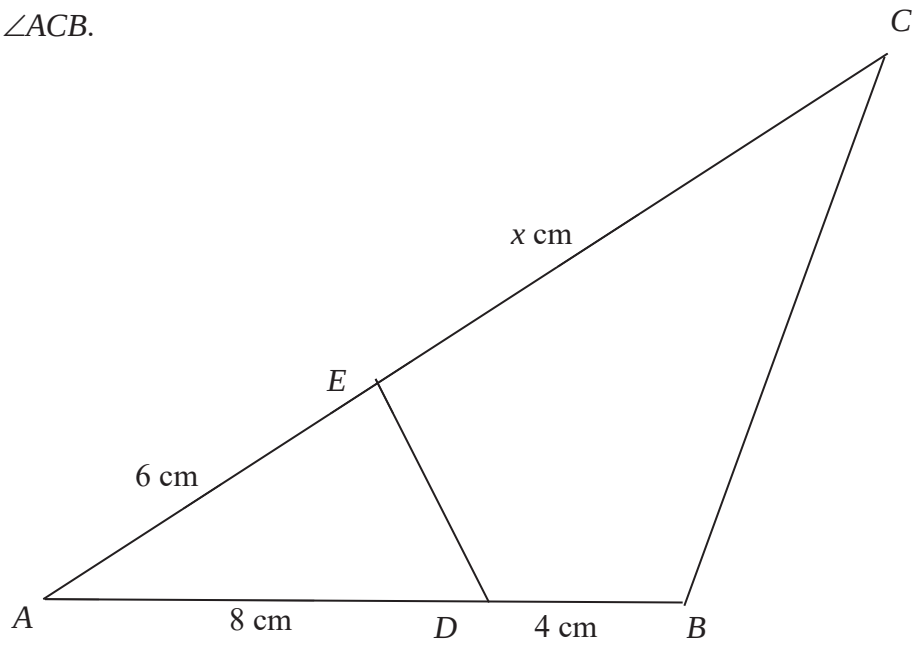
*Answer*_____

_____

_____

_____[2]

24 In the triangle ABC, D and E are points on AB and AC respectively such that $\angle ADE = \angle ACB$.



(a) Show that triangle AED and triangle ABC are similar.

_____

_____

_____ [2]

Given that $AD = 8$ cm, $DB = 4$ cm, $AE = 6$ cm and $EC = x$ cm, find

(b) the value of x,

*Answer (b)*_____[1]

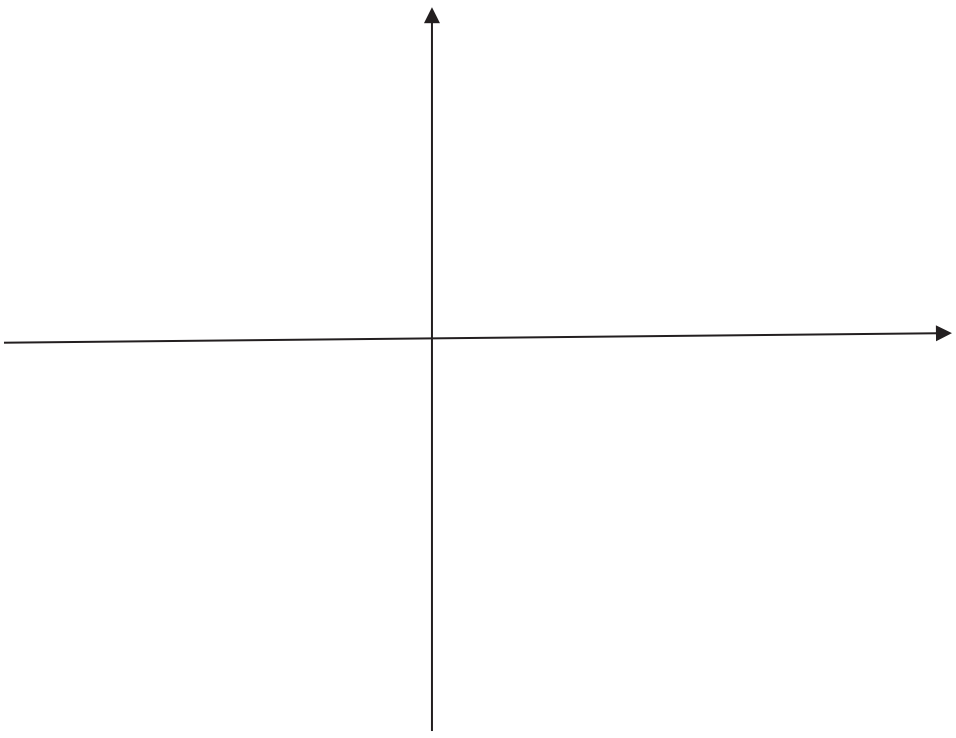
(c) the area of triangle ADE : area of quadrilateral $BCED$.

*Answer (c)*_____[2]

25 (a) Express $y = x^2 - 6x + 4$ in the form $y = (x - a)^2 + b$.

*Answer (a)*_____ [2]

(b) Hence sketch the curve $y = x^2 - 6x + 4$, indicating clearly, the points of intersection with the y axis and the turning point. [2]



ST. MARGARET'S SECONDARY SCHOOL

Mid-Year Examinations 2019

CANDIDATE NAME

CLASS | | | | REGISTER NUMBER | | |

MATHEMATICS **4048/02**

Paper 2 **6 May 2019**

Secondary 4 Express **2 hours 30 minutes**

Additional Materials: Writing papers
Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, registration number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

This document consists of 22 printed pages

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 The graph $y = (x - 1)(x + 3)$ cuts the x-axis at A and B and the y-axis at C.

(a) Find the coordinates of C. [1]

(b) Find the coordinates of M, the minimum point of the curve. [2]

Hence,

(c) (i) find the equation of the line MC, and [2]

(ii) the length of the line joining M to C. [2]

2 (a) Factorise completely

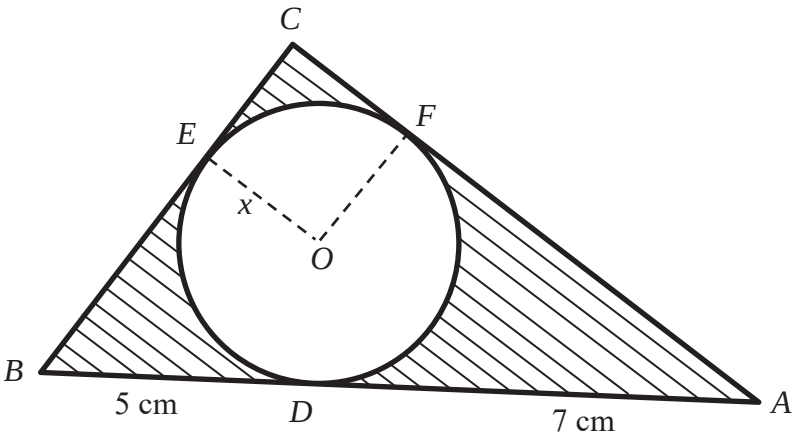
[2]

[2]

(b) Express $\frac{y-1}{y^2-1} - \frac{2}{1-y}$ as a single fraction in its simplest form. [3]

(c) Given that $b = a + \frac{c}{x}$ and $d = \frac{c}{b}$, express x in terms of a, c and d. [3]

3 In the diagram, the circle of radius x cm with centre O is touching the sides of the right angled triangle ABC at D, E, F, where angle ACB is 90°.



(a) Find the length of CE in terms of x. State the circle properties used in your working. [2]

(b) Given also that AD = 7 cm and BD = 5 cm, form an equation in x, and show that it simplifies to $x^2 + 12x - 35 = 0$. [3]

(c) Solve the equation $x^2 + 12x - 35 = 0$, giving both answers correct to 2 decimal places. [3]

(d) Hence find the shaded area. [2]

4 4 toothpicks are arranged to form a square. The diagram shows the first three of a sequence of figures that are formed. All the squares are of the same size.

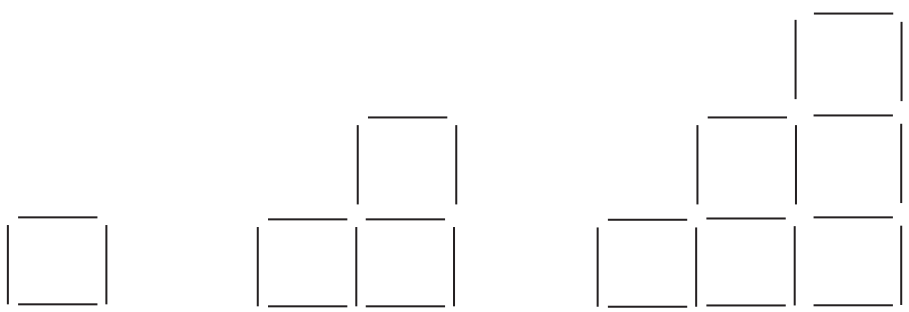


Figure 1 Figure 2 Figure 3

Figure Number (n)	Number of vertical toothpicks (V)	Total number of toothpicks (S)
1	2	4
2	5	10
3	9	18
⋮	⋮	⋮
7	v	s

The number of vertical sides (V) and the total number of sides (S) are recorded in the table above.

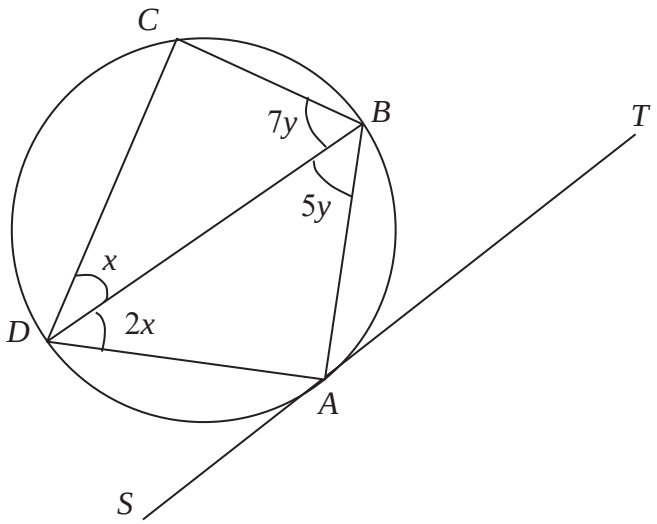
(a) Find the value of v and of s. [2]

(b) Write down a formula that shows the relationship between S and n. [1]

(c) If the total number of sides is 700, find the value of n. [2]

(d) Give a reason why the value of S cannot be a prime number. [1]

5 In the diagram, DB is a diameter of the circle and ST is a tangent to the circle at A. $\angle BDA = 2x$, $\angle BDC = x$, $\angle CBD = 7y$ and $\angle DBA = 5y$.



(a) Explain why angle $BCD = 90°$. [1]

(b) Find the value of x and of y. [4]

(c) Given that M is the mid-point of BD,

(i) determine if $\angle MAT$ is a right angle. Explain your answer. [1]

(ii) Find $\angle DMA$. [2]

6 (a) Find the values of the unknown in each of the following.

[3]

(b) If $\mathbf{P} = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} -2 & 1 \\ 5 & 9 \end{pmatrix}$, find the matrix $\mathbf{P^2} + 3\mathbf{Q}$. [3]

(c) The following table shows the working hours of 3 clerks in a week.

	Regular Working Hours	Overtime
Fatimah	40	4
Vani	45	3
June	38	7

The hourly wages of these clerks are \$20 for regular working hours and \$30 for overtime.

(i) Represent the working hours of the clerks by a matrix **A** and the hourly wage rates by a matrix **B** such that **AB** exists. [2]

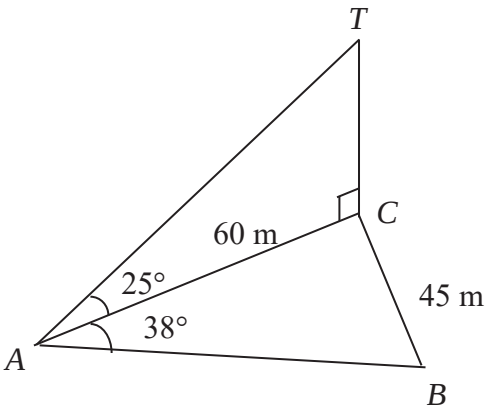
(ii) Find **AB**. [2]

(iii) Explain what your answer in (ii) represents. [1]

The elements in **AB** represent the wages earned by Fatimah, Vani and June respectively.

(iv) The hourly wage for regular working hours increased by 20% and overtime increased by 15% respectively, but the working hours remain the same. Calculate the increase in the company's expense in wages paid to the three clerks using matrix multiplication. [3]

7 (a)



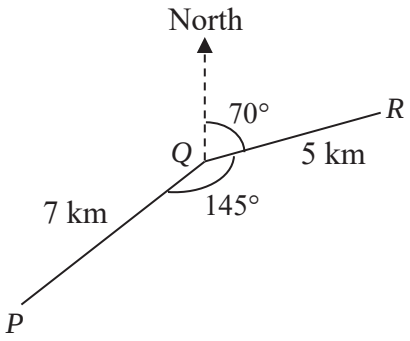
In the above diagram, C represents the foot of a vertical tower CT. The points A, B and C are on horizontal ground, where angle $CAB = 38°$, $AC = 60$ m and $BC = 45$ m.
Given that the angle of elevation of T from A is 25°, calculate

(i) the height of the tower, [2]

(ii) angle ABC, [2]

(iii) the angle of depression of B from T. [2]

(b)



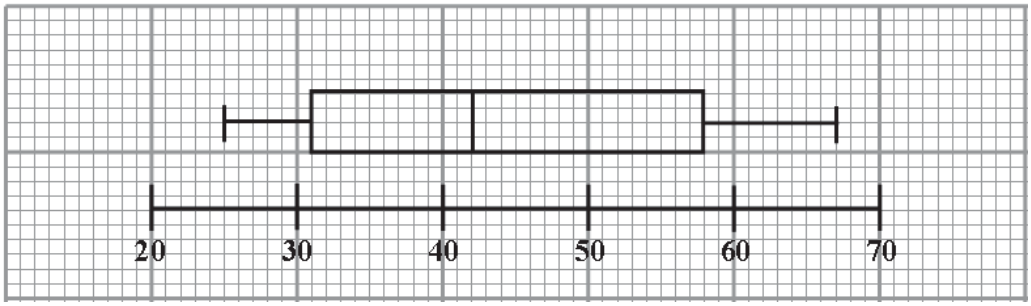
A ship sails 7 km from P to Q. It then sails 5 km from Q to R on a bearing of 070°. Given that angle $PQR = 145°$, calculate

(i) the bearing of Q from P, [2]

(ii) how far Q is east of P, [2]

(iii) the distance PR. [2]

8 (a) The box-and-whisker plot below shows the distribution of the ages (in years) of 50 members from the Rainbow Country Club.



Age (years)

(i) State the median and find the interquartile range. [3]

(ii) The table below shows the ages of 50 members from Sunshine Country Club.

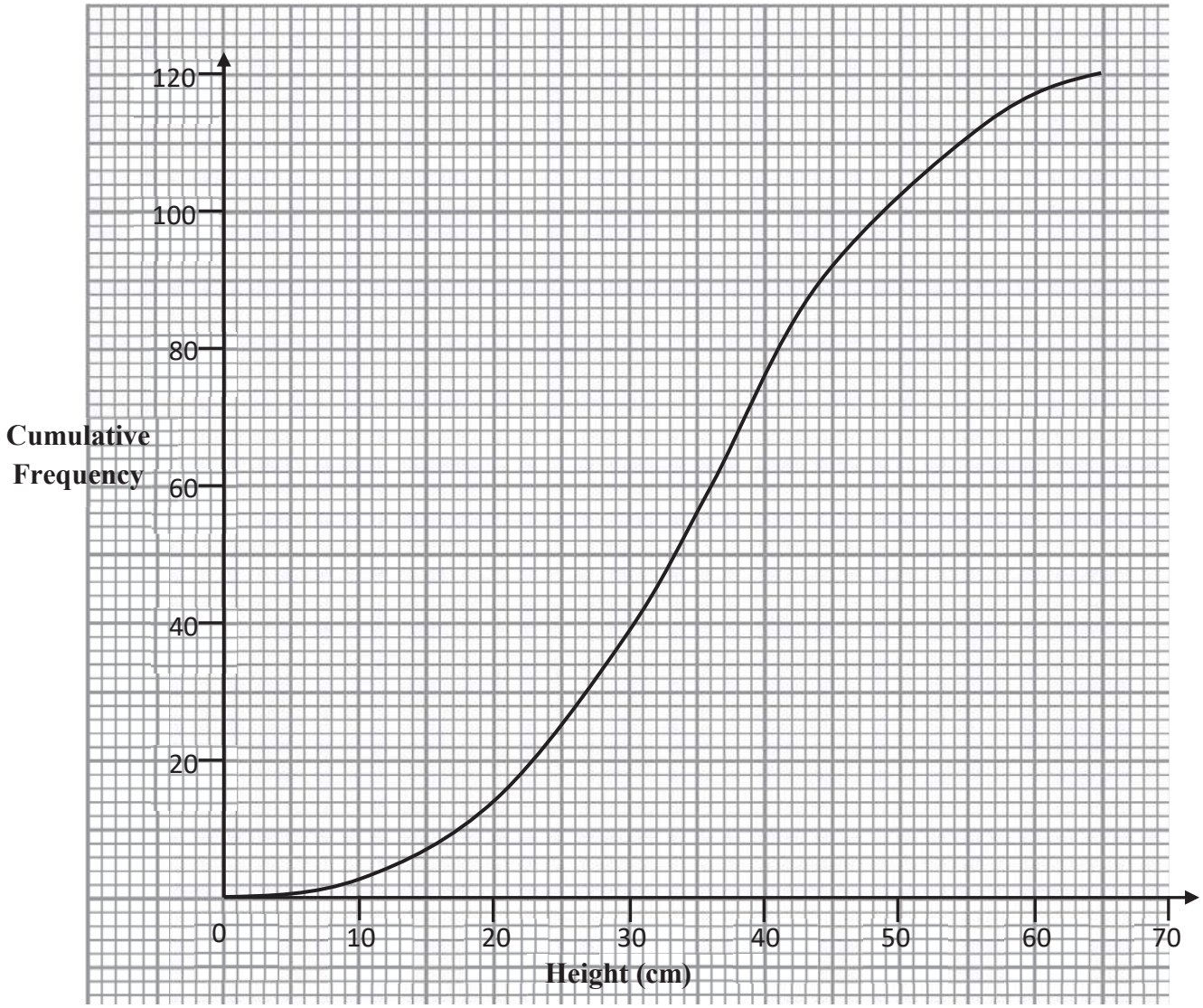
Age (years)	30	35	40	45	50	55	60
Number of Members	8	7	15	8	6	4	2

[3]

(iii) Sunshine Club claims that their members are generally younger than those from Rainbow Club. Do you agree? Give a reason for your answer. [1]

(iv) Which country club has a wider spread of ages among its members? Give a reason for your answer. [1]

8 (b) A school has two gardens, Famous Garden and Eco-Garden.
The heights of 120 plants from the Eco-Garden were measured.
The cumulative frequency diagram below shows the distribution of these heights.



Use the graph to estimate the

(i) 40^{th} percentile, [1]

(ii) percentage of the plants whose height is at least 45 cm. [2]

The heights of 120 plants from the Famous Garden were also measured.
It was found that the height measured has the same median but a smaller interquartile range compared to that of the Eco-Garden.

(iii) Describe how the cumulative frequency curve of the Famous Garden may differ from that of the Eco-Garden given above. [2]

9 There are 120 commercial buildings in the Sunflower City. A commercial building generates solid waste at a rate of 2.75 m^3 per day.

The solid waste of commercial buildings to that of the entire city is in the ratio 4 : 10.

The Seashell landfill has a capacity of 1.2×10^7 m^3. Currently, 50% of the landfill is filled.

(a) Calculate the volume of solid waste the city generates per week. (Assume 5 working days in a week) [2]

(b) Burning the solid waste in incinerators reduces its volume by 90%. However, only 60% of the solid waste can be burned. Determine how long more, in years, will the Seashell landfill be completely filled.

[6]

(c) Norman commented that incineration is a good way of getting rid of rubbish. Do you agree? Give a reason for your answer. [2]

10 Answer the whole of this question on a sheet of graph paper.

The following is a table of values for the graph of $y = x + \frac{6}{x}$.

x	1	1.5	2	2.5	3	4	5	6	8
y	7	5.5	5	4.9	5	p	6.2	7	8.8

(a) Calculate the value of p. [1]

(b) Using a scale of 2 cm to 1 unit on each axis, draw the graph of $y = x + \frac{6}{x}$ for $1 \leq x \leq 8$. [3]

(c) Use your graph to find the values of x for which $y = 6.5$. [2]

(d) By drawing a suitable tangent to your curve, find the coordinates of the point at which the gradient of the tangent is equal to -2. [2]

(e) By drawing another suitable line on the same axes, use your graph to find the solutions of the equation $\frac{3x}{2} + \frac{6}{x} = 7$. [3]

Answer:

1(a) $-2x + 2$ (b) $\frac{2}{5}x^{\frac{15}{2}}$

2(a) 7.1×10^8 m^2 (b) $m = 0, n = -2$

3(a) $\frac{2}{3}, -1$ (b) $\frac{5}{9}, 0$

4 9

5(a) $2^3 \times 3 \times 5 \times 7^2$ (b) 40 (c) 22

6(a) 4 : 13 (b) 67.4

7(a) $(2x - 7y)^2$ (b) $x = \frac{7y}{2}$

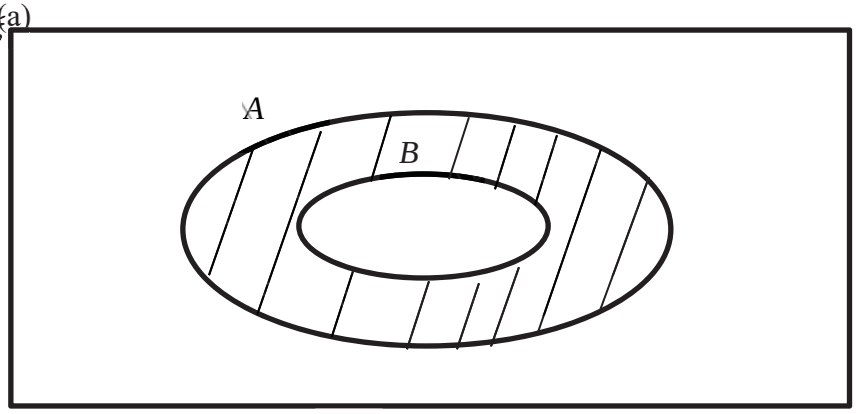
8(a) 12 (b) 18 (c) 7

9(a) 1 : 250 000 (b) 1.5 cm

10(a) 480 cm^3 (b) 45 cm^3

11(a) any negative odd integer (b) any exponential equation

12(a)



(b) {1, 27}

13(a) $\frac{1}{5}$ (b) 15

14(a) $\begin{pmatrix} 8 \\ 10 \end{pmatrix}$ (b) $\begin{pmatrix} \frac{47}{6} \\ \frac{1}{2} \end{pmatrix}$ (c) $8\frac{1}{3}$ km

15 Mean = 153 g, SD = 4.29 g

16 SD of Fati = 0.816, SD of Bala = 1.73, Mean of Fati = Mean of Bala = 2

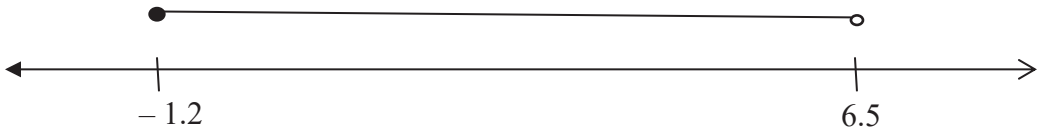
Select Fati as he is more consistent in his score.

17(a) 10 units (b) $\begin{pmatrix}12\\10\end{pmatrix}$

18 4 : 1

19 10, – 4

20(a)



(b) – 1

21 \$8494.10

22(a) $\frac{9}{17}, \frac{8}{17}, \frac{9}{17}, \frac{8}{17}, \frac{9}{16}, \frac{7}{16}$ (b) $\frac{7}{34}$

23 Scale was not state in the question.

Height of similey face or area of similey face could be used to represent the readings from each year which may lead to misinterpretation of data.

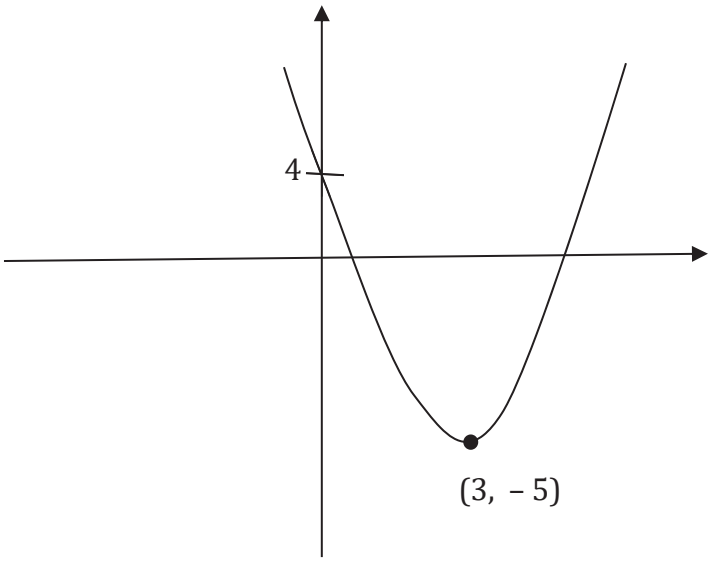
24(a) $\angle EAD = \angle BAC$ (common angle)

$\angle ADE = \angle ACB$ (given)

Triangle *ADE* is similar to triangle *ACB* (*AA* similarity test)

(b) 10 (c) 1 : 3

25(a) $(x-3)^2 - 5$ (b)



ST. MARGARET'S SECONDARY SCHOOL

Mid-Year Examinations 2019

CANDIDATE NAME

CLASS | REGISTER NUMBER

MATHEMATICS **4048/02**

Paper 2 **6 May 2019**

Secondary 4 Express **2 hours 30 minutes**

Additional Materials: Writing papers
Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, registration number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

This document consists of 22 printed pages

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 The graph $y = (x - 1)(x + 3)$ cuts the x-axis at A and B and the y-axis at C.

(a) Find the coordinates of C. [1]

$y = (x - 1)(x + 3)$
At C, $x = 0$,
$y = (0 - 1)(0 + 3)$
$= -3$ $\therefore$ Coordinates of C are $(0, -3)$.

(b) Find the coordinates of M, the minimum point of the curve. [2]

When $y = 0$, $x = -3, 1$
x- coordinate of $M = \frac{-3+1}{2}$
$= -1$
y - coordinate of $M = (-1 - 1)(-1 + 3)$
$= -4$
$\therefore$ Coordinates of M are $(-1, -4)$

Hence,

(c) (i) find the equation of the line MC, and [2]

$M(-1, -4)$, $C(0, -3)$

Gradient $MC = \frac{-4+3}{-1-0}$
$= 1$
Equation of MC : $y = x - 3$

(ii) the length of the line joining M to C. [2]

$M(-1, -4)$, $C(0, -3)$

Length of $MC = \sqrt{(0 + 1)^2 + (-3 + 4)^2}$
$= \sqrt{2}$
= 1.41 units

2 (a) Factorise completely

(i) $2p^2 + 6p - pq - 3q$, [2]

$2p^2 + 6p - pq - 3q$
$= 2p(p + 3) - q(p + 3)$
$= (p + 3)(2p - q)$

(ii) $24 - 54x^2$. [2]

$24 - 54x^2$
$= 6(4 - 9x^2)$
$= 6(2 + 3x)(2 - 3x)$

(b) Express $\frac{y-1}{y^2-1} - \frac{2}{1-y}$ as a single fraction in its simplest form. [3]

$$\frac{y-1}{y^2-1} - \frac{2}{1-y}$$
$$= \frac{y-1}{(y+1)(y-1)} + \frac{2}{y-1}$$
$$= \frac{y-1+2(y+1)}{(y+1)(y-1)}$$
$$= \frac{3y+1}{(y+1)(y-1)}$$

(c) Given that $b = a + \frac{c}{x}$ and $d = \frac{c}{b}$, express x in terms of a, c and d. [3]

$b = a + \frac{c}{x}$ ---------- (1)

$d = \frac{c}{b}$ ---------- (2)

From (2) $b = \frac{c}{d}$

Substitute into (1)

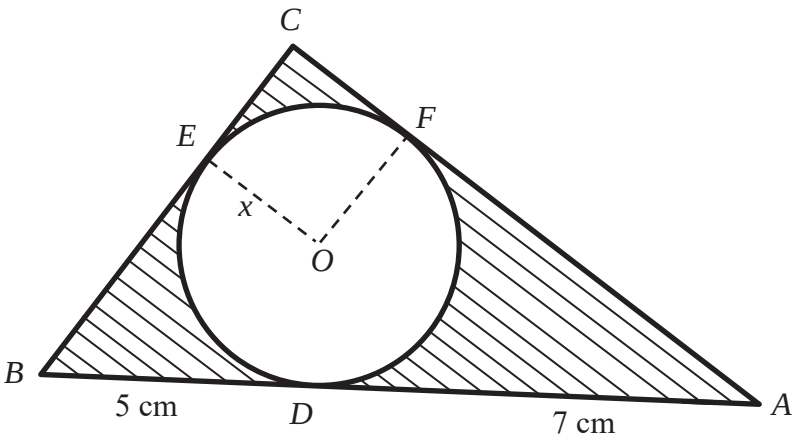
$\frac{c}{d} = a + \frac{c}{x}$

$\frac{c}{x} = \frac{c}{d} - a$

$\frac{c}{x} = \frac{c-ad}{d}$

$x = \frac{cd}{c-ad}$

3 In the diagram, the circle of radius x cm with centre O is touching the sides of the right angled triangle ABC at D, E, F, where angle ACB is 90°.



(a) Find the length of CE in terms of x. State the circle properties used in your working. [2]

$\angle ACB = 90°$ (given)
$\Rightarrow \angle ECO = 45°$ (tangents rom external point)
$\angle CEO = 90°$ (tangent is perpendicular to radius)
$\Rightarrow \angle EOC = 45°$ (angle sum of triangle)

$\angle ECO = \angle EOC \Rightarrow$ triangle CEO is isosceles

Hence $CE = OE$
$= x$

(b) Given also that $AD = 7$ cm and $BD = 5$ cm, form an equation in x, and show that it simplifies to $x^2 + 12x - 35 = 0$. [3]

$CF = CE$ (tangents from external point)
$= x$
By Pythagoras' Theorem
$(5 + x)^2 + (7 + x)^2 = (5 + 7)^2$
$25 + 10x + x^2 + 49 + 14x + x^2 = 144$
$2x^2 + 24x - 70 = 0$
$x^2 + 12x - 35 = 0$ (shown)

(c) Solve the equation $x^2 + 12x - 35 = 0$, giving both answers correct to 2 decimal places. [3]

$x^2 + 12x - 35 = 0$

$x = \frac{-12 \pm \sqrt{12^2 - 4(1)(-35)}}{2(1)}$

$= \frac{-12 \pm \sqrt{284}}{2}$

$= 2.426$ or -14.426

$= 2.43$ or -14.43 (to 2 d.p.)

(d) Hence find the shaded area. [2]

Since $x > 0$, $\therefore x = 2.426$

Shaded area $= \frac{(5+2.426)(7+2.426)}{2} - \pi(2.426)^2$

$= 16.5$ cm^2 (to 3 s.f.)

4 4 toothpicks are arranged to form a square. The diagram shows the first three of a sequence of figures that are formed. All the squares are of the same size.

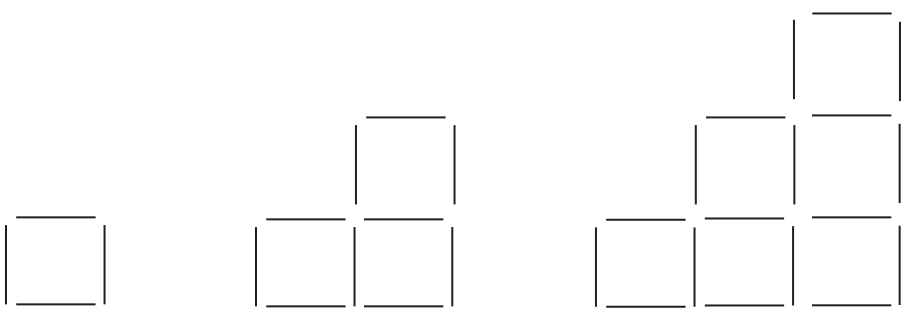


Figure 1 Figure 2 Figure 3

Figure Number (n)	Number of vertical toothpicks (V)	Total number of toothpicks (S)
1	2	4
2	5	10
3	9	18
⋮	⋮	⋮
7	v	s

The number of vertical sides (V) and the total number of sides (S) are recorded in the table above.

(a) Find the value of v and of s. [2]

2 5 9 14 20 27 35

3 4 5 6 7 8

4 10 18 28 40 54 70

6 8 10 12 14 16

$\therefore v = 35, \quad s = 70$

(b) Write down a formula that shows the relationship between S and n. [1]

$n = 1,\ s = (1)(1+3) = 4; \qquad n = 2,\ s = (2)(2+3) = 10$

$\therefore\ s = n(n+3)$

(c) If the total number of sides is 700, find the value of n. [2]

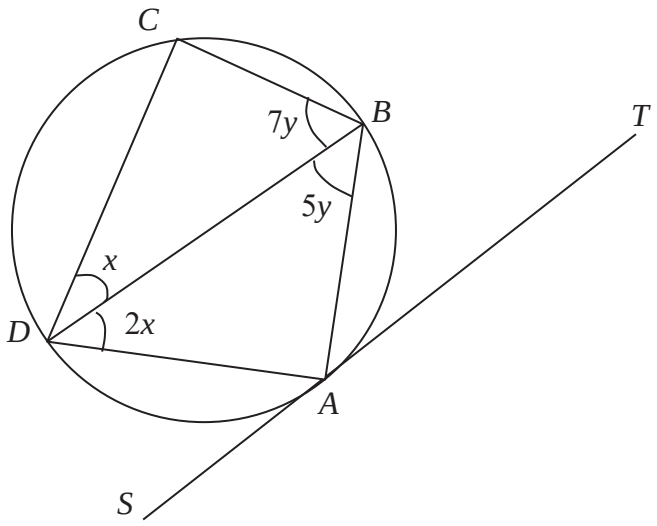
$n(n+3) = 700$
$n^2 + 3n - 700 = 0$
$(n+28)(n-25) = 0$

$n = -28$ is rejected. $\therefore\ n = 25$

(d) Give a reason why the value of S cannot be a prime number. [1]

Values of S start from 4 and are even numbers, hence it cannot be a prime number.

5 In the diagram, DB is a diameter of the circle and ST is a tangent to the circle at A. $\angle BDA = 2x$, $\angle BDC = x$, $\angle CBD = 7y$ and $\angle DBA = 5y$.



(a) Explain why angle $BCD = 90°$. [1]

Since BD is a diameter, by "right angle in a semi-circle" property, $\angle BCD = 90°$.

(b) Find the value of x and of y. [4]

$x + 2x + 7y + 5y = 180°$ (angles in opposite segments)
$\Rightarrow \quad x + 4y = 60°$ -------- (1)
$x + 7y + 90° = 180°$ (angle sum of triangle)
$\Rightarrow \quad x + 7y = 90°$ -------- (2)

(2) – (1) $\quad 3y = 30°$
$y = 10°$

Substitute into (1) $\quad x = 60° - 4(10°)$

$= 20°$

(c) Given that M is the mid-point of BD,

(i) determine if $\angle MAT$ is a right angle. Explain your answer. [1]

M is the mid-point of BD
$\Rightarrow$ M is the centre of the circle
$\Rightarrow MA$ is a radius of the circle
Since ST is a tangent, by "tangent is perpendicular to radius" property
$\angle MAT$ is a right angle.

(ii) Find $\angle DMA$. [2]

ΔDMA is isosceles, hence
$\angle MAD = \angle MDA$
$= 2(20°)$
$= 40°$
$\angle DMA = 180° - 2 \times 40°$ (angle sum of triangle)
$= 100°$

6 (a) Find the values of the unknown in each of the following.

$$\begin{pmatrix} 1 & 0 & 4 \\ 5 & 2 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ a \end{pmatrix} = \begin{pmatrix} 8 \\ b \end{pmatrix}. \quad [3]$$

$$\begin{pmatrix} 1+0+4a \\ 5+4-3a \end{pmatrix} = \begin{pmatrix} 8 \\ b \end{pmatrix}$$

$1 + 4a = 8$
$a = 1.75$

$b = 5 + 4 - 3(1.75)$
$= 3.75$

(b) If $\mathbf{P} = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} -2 & 1 \\ 5 & 9 \end{pmatrix}$, find the matrix $\mathbf{P^2 + 3Q}$. [3]

$\mathbf{P^2 + 3Q}$

$= \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} + 3 \begin{pmatrix} -2 & 1 \\ 5 & 9 \end{pmatrix}$

$= \begin{pmatrix} 9 & 8 \\ 16 & 17 \end{pmatrix} + \begin{pmatrix} -6 & 3 \\ 15 & 27 \end{pmatrix}$

$= \begin{pmatrix} 3 & 11 \\ 31 & 44 \end{pmatrix}$

(c) The following table shows the working hours of 3 clerks in a week.

	Regular Working Hours	Overtime
Fatimah	40	4
Vani	45	3
June	38	7

The hourly wages of these clerks are \$20 for regular working hours and \$30 for overtime.

(i) Represent the working hours of the clerks by a matrix **A** and the hourly wage rates by a matrix **B** such that **AB** exists. [2]

$$\mathbf{A} = \begin{pmatrix} 40 & 4 \\ 45 & 3 \\ 38 & 7 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 20 \\ 30 \end{pmatrix}$$

(ii) Find **AB**. [2]

$$\begin{pmatrix} 40 & 4 \\ 45 & 3 \\ 38 & 7 \end{pmatrix}\begin{pmatrix} 20 \\ 30 \end{pmatrix} = \begin{pmatrix} 800 + 120 \\ 900 + 90 \\ 760 + 210 \end{pmatrix} = \begin{pmatrix} 920 \\ 990 \\ 970 \end{pmatrix}$$

(iii) Explain what your answer in (ii) represents. [1]

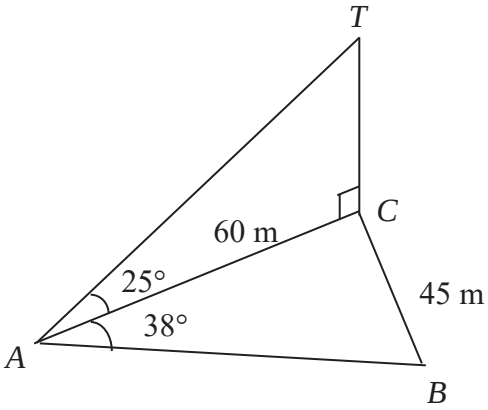
The elements in **AB** represent the wages earned by Fatimah, Vani and June respectively.

(iv) The hourly wage for regular working hours increased by 20% and overtime increased by 15% respectively, but the working hours remain the same. Calculate the increase in the company's expense in wages paid to the three clerks using matrix multiplication. [3]

$$(1 \quad 1 \quad 1)\begin{pmatrix} 40 & 4 \\ 45 & 3 \\ 38 & 7 \end{pmatrix}\begin{pmatrix} 4 \\ 4.5 \end{pmatrix}$$
$$= (123 \quad 14)\begin{pmatrix} 4 \\ 4.5 \end{pmatrix}$$
$$= (555)$$

The increase in wages paid is \$555.

7 (a)



In the above diagram, C represents the foot of a vertical tower CT. The points A, B and C are on horizontal ground, where angle $CAB = 38°$, $AC = 60$ m and $BC = 45$ m.
Given that the angle of elevation of T from A is 25°, calculate

(i) the height of the tower, [2]

$$\tan 25° = \frac{CT}{60}$$

$$CT = 60 \tan 25°$$
$$= 27.978$$
$$= 28.0 \text{ (to 3 s.f.)}$$ Height of tower is 28.0 m

(ii) angle ABC, [2]

$$\frac{\sin \angle ABC}{60} = \frac{\sin 38°}{45}$$
$$\angle ABC = \sin^{-1} \frac{60 \sin 38°}{45}$$
$$= 55.173°$$
$$= 55.2° \text{ (to 1 d.p.)}$$

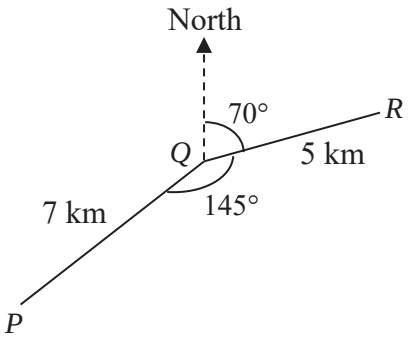
(iii) the angle of depression of B from T. [2]

Let θ be the angle of depression

$$\tan \theta = \frac{27.978}{45}$$

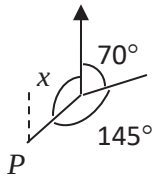
$$\theta = \tan^{-1} \frac{27.978}{45}$$
$$= 31.870°$$
$$= 31.9° \text{ (to 1 d.p.)}$$

(b)



A ship sails 7 km from P to Q. It then sails 5 km from Q to R on a bearing of 070°. Given that angle $PQR = 145°$, calculate

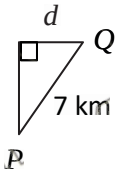
(i) the bearing of Q from P, [2]



$x = 360° - 70° - 145°$
$= 145°$

Bearing of Q from P
$= 180° - 145°$
$= 035°$

(ii) how far Q is east of P, [2]



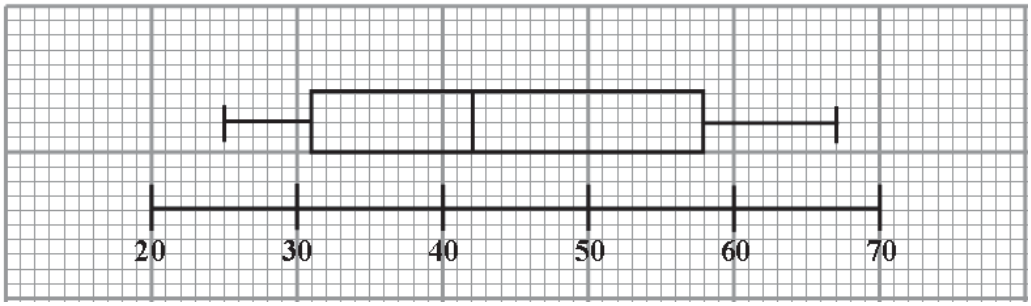
$\sin 35° = \frac{d}{7}$
$d = 7 \sin 35°$
$= 4.01503$
$= 4.02$ km

Ans: Q is 4.20 km east of P.

(iii) the distance PR. [2]

$PR^2 = 7^2 + 5^2 - 2(7)(5)\cos 145°$
$PR = \sqrt{74 - 70\cos 145°}$
$= 11.5$ km (to 3 s.f.)

8 (a) The box-and-whisker plot below shows the distribution of the ages (in years) of 50 members from the Rainbow Country Club.



Age (years)

(i) State the median and find the interquartile range. [3]

Median = 42 years

Interquartile range = 58 – 31
= 27 years

(ii) The table below shows the ages of 50 members from Sunshine Country Club.

Age (years)	30	35	40	45	50	55	60
Number of Members	8	7	15	8	6	4	2

Find the median and the interquartile range. [3]

Median = 40 years

Interquartile range = 45 – 35
= 10 years

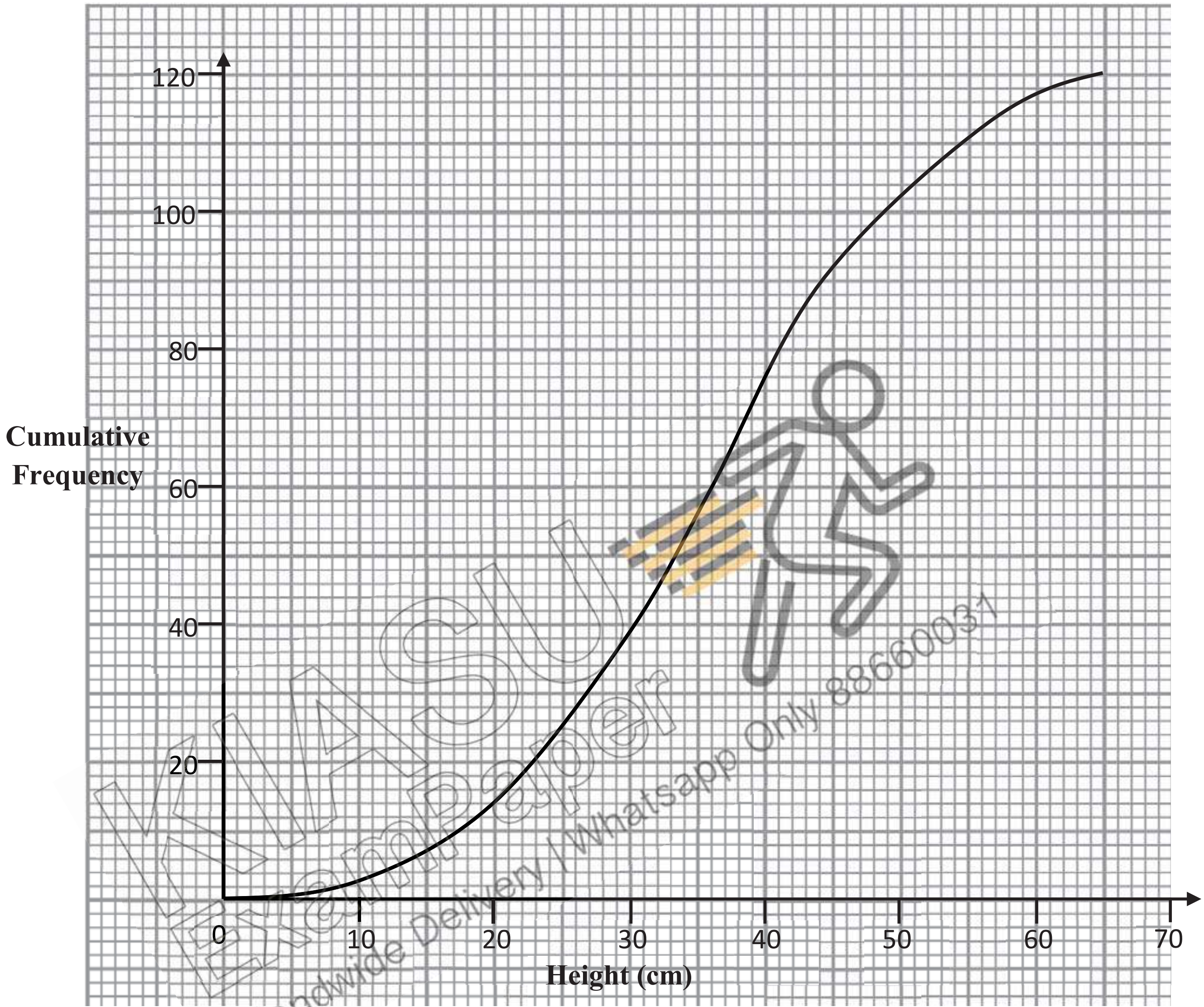
(iii) Sunshine Club claims that their members are generally younger than those from Rainbow Club. Do you agree? Give a reason for your answer. [1]

I agree that members of Sunshine Club are generally younger than those from Rainbow Club because the median age of Sunshine Club (40 years) is lower that the median age of Rainbow Club (42 years).

(iv) Which country club has a wider spread of ages among its members? Give a reason for your answer. [1]

Rainbow Club has a wider spread of ages among its members because its interquartile range (27 years) is bigger/larger than that of Sunshine Club (10 years).

8 (b) A school has two gardens, Famous Garden and Eco-Garden.
The heights of 120 plants from the Eco-Garden were measured.
The cumulative frequency diagram below shows the distribution of these heights.



Use the graph to estimate the

(i) 40th percentile, [1]

$120 \times 0.4 = 48$

From the graph, 40th percentile = 33 cm.

(ii) percentage of the plants whose height is at least 45 cm. [2]

No. of plants with length at least 45 cm = 120 – 92
= 28

Percentage of plants whose height is at least 45 cm
$= \frac{28}{120} \times 100\%$
$= 23\frac{1}{3}\ \%$

The heights of 120 plants from the Famous Garden were also measured.
It was found that the height measured has the same median but a smaller interquartile range compared to that of the Eco-Garden.

(iii) Describe how the cumulative frequency curve of the Famous Garden may differ from that of the Eco-Garden given above. [2]

The cumulative frequency curve of the Famous Garden will be steeper and it will intersect the graph of the Eco-Garden at the median height.

9 There are 120 commercial buildings in the Sunflower City. A commercial building generates solid waste at a rate of 2.75 m^3 per day.

The solid waste of commercial buildings to that of the entire city is in the ratio 4 : 10.

The Seashell landfill has a capacity of 1.2×10^7 m^3. Currently, 50% of the landfill is filled.

(a) Calculate the volume of solid waste the city generates per week. (Assume 5 working days in a week) [2]

Volume of waste generated by a commercial building per week
$= 120 \times 5 \times 2.75$
$= 1650$ m^3

Volume of solid waste generated by the city per week
$= \frac{10}{4} \times 1650$
$= 4125$ m^3

(b) Burning the solid waste in incinerators reduces its volume by 90%. However, only 60% of the solid waste can be burned. Determine how long more, in years, will the Seashell landfill be completely filled. [6]

60% of solid waste that can be burned
$= 0.6 \times 4125$
$= 2475$ m^3

Remain of solid waste after incineration
$= 0.1 \times 2475$
$= 247.5$ m^3

40% of waste that cannot be burned
$= 0.4 \times 4125$
$= 1650$ m^3

50% of landfill that is not used
$= 0.5 \times 1.2 \times 10^7$
$= 6 \times 10^6$ m^3

No. of years left where the landfill will be completely filled
$= 6 \times 10^6 \div (1650 + 247.5) \div 52$
$= 3162.055 \div 52$
$= 60.8$ years

The landfill will be completely filled in another 60.8 years.

(c) Norman commented that incineration is a good way of getting rid of rubbish. Do you agree? Give a reason for your answer. [2]

Agree.
Reason: It is a more efficient use of space.
OR
The process of incineration can produce electricity that can be used.

Disagree.
Reason: Incineration facilities are costly.
OR
The process of incineration emits / produces hazardous pollutants that are harmful.

10 Answer the whole of this question on a sheet of graph paper.

The following is a table of values for the graph of $y = x + \frac{6}{x}$.

x	1	1.5	2	2.5	3	4	5	6	8
y	7	5.5	5	4.9	5	p	6.2	7	8.8

(a) Calculate the value of p. [1]

(b) Using a scale of 2 cm to 1 unit on each axis, draw the graph of $y = x + \frac{6}{x}$ for $1 \leq x \leq 8$. [3]

(c) Use your graph to find the values of x for which $y = 6.5$. [2]

(d) By drawing a suitable tangent to your curve, find the coordinates of the point at which the gradient of the tangent is equal to -2. [2]

(e) By drawing another suitable line on the same axes, use your graph to find the solutions of the equation $\frac{3x}{2} + \frac{6}{x} = 7$. [3]

BEATTY SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2019

O

SUBJECT : Mathematics
LEVEL : Sec 4E/5N

PAPER : 4048 / 01
DURATION : 2 hours

SETTER : Mr Leong S C
DATE : 26 Aug 2019

CLASS :	NAME :	REG NO :

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READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use
80

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1. The first 4 terms of a sequence of numbers T_1, T_2, T_3 and T_4 are given below:

$T_1 = 1^2 + 2^2 = 5$

$T_2 = 2^2 + 3^2 = 13$

$T_3 = 3^2 + 4^2 = 25$

$T_4 = 4^2 + 5^2 = 41$

(a) Write down an expression in terms of n, for the n^{th} term T_n of the sequence.

Answer ..... [1]

(b) Explain why T_n is always odd.

Answer: [1]

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2. Simplify $\frac{a^2b}{2} \times \left(2a^0b^{-2}\right)^3$, leaving your answer in positive index form.

Answer ..... [2]

3. $x = \frac{4}{3}$ is a solution to the equation $18x^2 - kx - 20 = 0$, where k is a constant.

(a) Find the value of k.

Answer $k =$ ..... [1]

(b) Hence, find the other solution to the equation $18x^2 - kx - 20 = 0$.

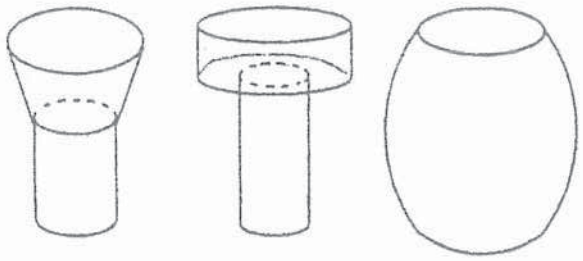
Answer ..... [1]

---

4. Given that the size of an exterior angle of a regular polygon is one-fifth its interior angle, find the number of sides of the polygon.

Answer ..... [2]

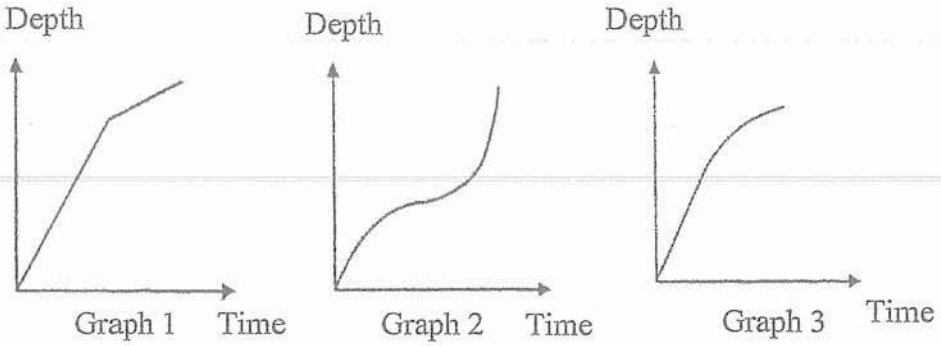
5. Water is poured at a constant rate into each of the containers shown below.



A B C

The graphs show the depth of water in the containers as they are being filled.

Match the graphs with the appropriate container and complete the table below.



Answer: [2]

Graph	Container
1	
2	
3	

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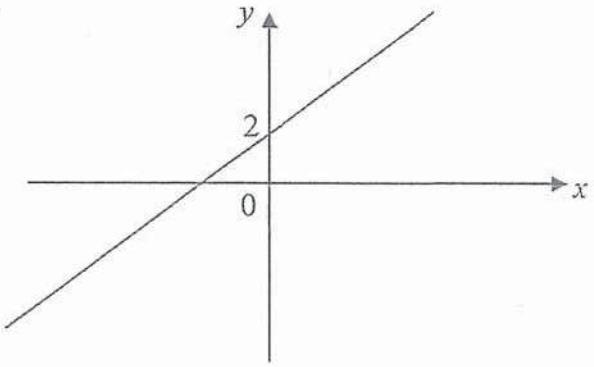
6. (a) The graph below can be represented by the equation, $y = ax^n + b$.
State the value of b, of n and the range of a.

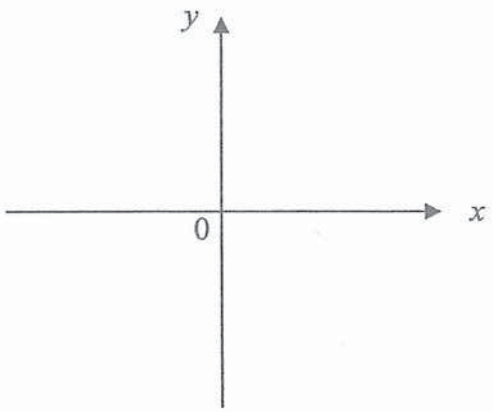


Answer b = ... n =..... [1]

..... [1]

(b) On the axes in the answer space, sketch the graph of $y = ax^n$, where $a < 0$ and $n = 2$. [1]

Answer



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7. Find the largest prime number that satisfy the inequality $3 + y \leq 15 - 2y < y + 10$.

Answer ..... [3]

8. Solve the simultaneous equations

$$8x+3y=4$$
$$4x=y+4$$

Answer $x =$ ..... [3]

$y =$ .....

---

9. It is given that $\mathbf{p}=\begin{pmatrix}6\\5\end{pmatrix}$, $\mathbf{q}=\begin{pmatrix}-4\\3\end{pmatrix}$ and $\mathbf{r}=\begin{pmatrix}8\\-6\end{pmatrix}$.

(a) Find $|\mathbf{q}-2\mathbf{p}|$.

Answer ..... [2]

(b) Use vectors to explain whether $\mathbf{r}$ is parallel to $\mathbf{q}$.

Answer [1]

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10. Simplify and express $\frac{9m^2 - 12mn + 4n^2}{m^2 - n^2} \div \frac{3m - 2n}{m - n}$ as a single fraction.

Answer ..... [3]

---

11. The ground area of a scale model is 1 : 100 of the actual ground area.

(a) Given that the length of the model is 3 m, calculate the length of the house in metre.

Answer .....m [1]

(b) Given that the actual volume of a storeroom in the house is 15 000 000 cm^3, calculate the volume of the storeroom in the model in cm^3.

Answer .....cm^3 [2]

12. A bag contains a total of 80 green, red and blue balls. There are 18 red balls in the bag.

(a) If a ball is picked randomly, the probability of picking a green ball is $\frac{3}{8}$. How many blue balls are there?

Answer ..... [1]

(b) Two balls are picked randomly from the bag. Find the probability of

(i) picking a green ball and a red ball,

Answer ..... [2]

(ii) picking at least one red ball.

Answer ..... [1]

13. Find the equation of the straight line

(a) passing through the point (–2, 5) and parallel to the y-axis,

Answer ..... [1]

(b) which passes through the point (2, –5) and its gradient is the same as the line $y - 3x - 5 = 0$.

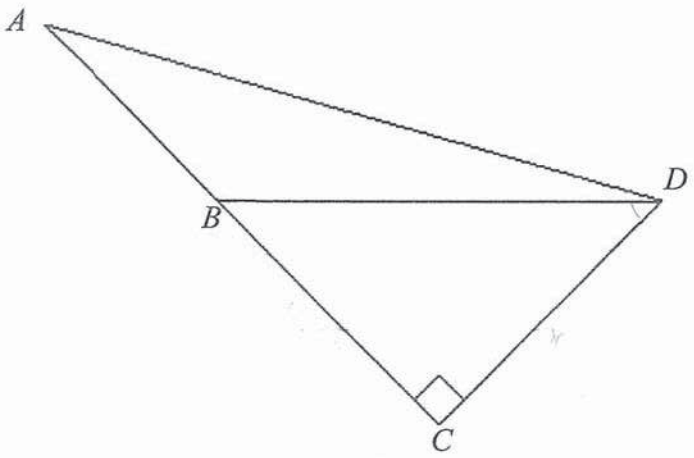
Answer ..... [2]

---

14. In the diagram triangle ACD is right-angled, and B is on AC such that $\sin \angle ABD = \frac{x}{y}$.

Find in terms of x and y.

(a) $\tan \angle BDC$,



Answer ..... [2]

(b) $\cos \angle ABD$.

Answer ..... [1]

15. (a) An insurance agent is paid a basic monthly salary of \$1200 and a commission of 8% on all the sales if the monthly sales made by the insurance agent exceed \$7500. The insurance agent's salary for a particular month was \$3694. Calculate the total sales made by the insurance agent.

Answer ..... [2]

(b) Two taps X and Y are turned on at the same time to fill up an empty tank. The rate at which Tap X fills up the tank per minute is twice as fast as that of Tap Y. When used together, both taps can fill the tank with 300 cm^3 of water in 5 minutes. Find the rate of flow of Tap Y in cm^3/minute.

Answer .....cm^3/minute [2]

16. The table below shows the frequency of visits to the library by some students in Betta School in a week.

Number of visits	0	1	2	3	4
Number of students	3	5	4	x	2

(a) If the mode is 3, write down the smallest possible value of x.

Answer $x =$ ..... [1]

(b) If the median is 2, write down the largest possible value of x.

Answer $x =$ ..... [1]

(c) Given that the mean is $2\frac{1}{22}$, calculate the value of x.

Answer $x =$ ..... [2]

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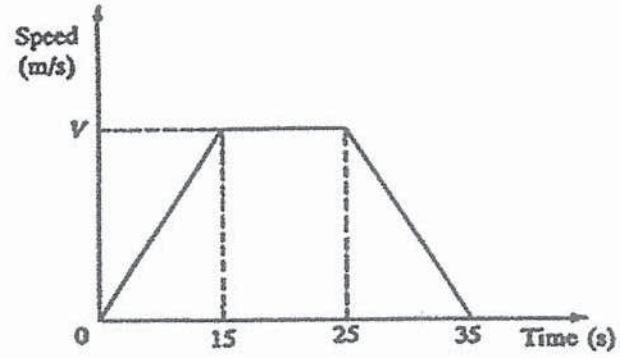
17. The diagram shows a speed-time graph of a car. The total distance travelled in 35 s is 540 m.



(a) Calculate the maximum speed V m/s.

Answer $V =$ .....m/s [2]

(b) Calculate the speed of the car at 28 seconds.

Answer ..... 16.8 .....m/s [2]

---

18. The base areas of two geometrically similar pyramids are 98 cm^2 and 242 cm^2.

(a) Find the ratio of the height of the larger pyramid to the height of the smaller pyramid.

Answer .....:..... [1]

(b) Given the total surface area of the smaller pyramid is 343 cm^2. Find the total surface area of the larger pyramid.

Answer .....cm^2 [1]

(c) Find the percentage difference in volumes of the two pyramids in terms of the larger pyramid.

Answer ..... [2]

---

19. **(a)** s is directly proportional to the square of v. It is known that $s = 36$ for a particular value of v.

Find the value of s when this value of v is halved.

Answer $s =$ ..... [2]

(b) y is inversely proportional to x and $y = 4$ when $x = 6$. Find the value of y when $x = 9$.

Answer $y =$ ..... [2]

20. Written as a product of its prime factors,

$$p = 3^x \times 5^y \times 7^3 \text{ and } q = 5 \times 7^2 \times 11$$

(a) Find the smallest value of x and of y for which p is a multiple of 35.

Answer $x =$ ..... $y =$ ..... [2]

(b) Explain why $55q$ is a perfect square.

Answer: [1]

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(c) Show your workings and state, with reason (s), if the product of p and q is an odd or even number.

Answer: [1]

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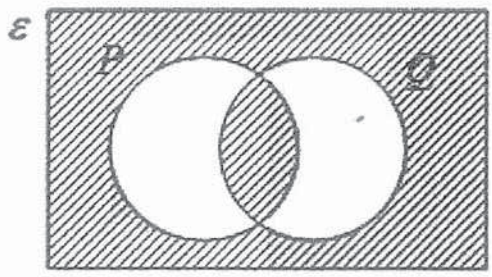
21. **(a)** Describe the shaded region in set notation.



Answer ..... [1]

(b) Given that

$\xi = \{x \text{ is a positive integer and } 0 < x < 10\}$

$A = \{x : x \text{ is a prime number}\}$

$B = \{x : x \text{ is a factor of } 12\}$

$C = \{x : \frac{50}{x} < 8\}$

(i) Find $B \cap C$.

Answer ..... [1]

(ii) List the elements of $(A \cup B)'$.

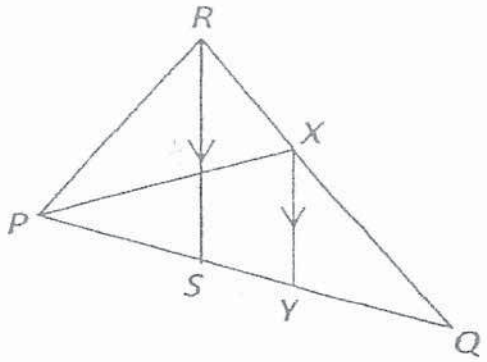
Answer ..... [1]

(ii) Draw a Venn diagram to illustrate the given information. [2]

Answer:

ε

22. In triangle PQR, X is a point on QR such that $QX : XR$ is 3 : 2 and RS is parallel to XY.



(a) Show that triangle QXY is similar to triangle QRS.

Answer: [2]

.....

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(b) Given that the area of triangles QXY and PXR are 27 cm^2 and 46 cm^2 respectively. Find the area of

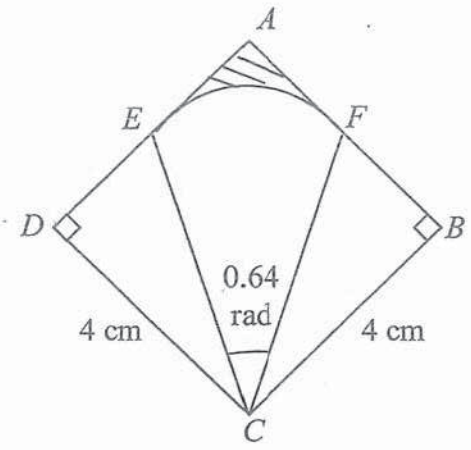
(i) triangle QRS,

Answer .....cm^2 [2]

(ii) triangle PQR.

Answer .....cm^2 [2]

23.



The sector CEF with centre at C is inscribed in the square $ABCD$ of side 4 cm.
Given that $AF = FB$, $DE = EA$ and angle $ECF = 0.64$ radian.

(a) Calculate the perimeter of the sector CEF.

Answer ..... cm [3]

(b) Find the area of the shaded region.

Answer .....cm^2 [4]

End of Paper

BEATTY SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2019

SUBJECT :	Mathematics	LEVEL :	Sec 4 Express Sec 5 Normal (Academic)
PAPER :	4048 / 02	DURATION :	2 hours 30 minutes
SETTER :	Mr Teo CK Miss Chong HY	DATE :	29 August 2019

CLASS :	NAME :	REG NO :

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

For Examiner's Use
100

This paper consists of 23 printed pages (including this cover page)

[Turn over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P(1+\frac{r}{100})^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 (a) Simplify $\left(\frac{64v^9}{t^{12}}\right)^{-\frac{1}{3}}$. [2]

(b) Express as a single fraction in its simplest form.

(i) $\frac{20x^2}{9} \div \frac{4xy^3}{15y}$. [1]

(ii) $\frac{4}{3x-1} - \frac{5}{2+x}$. [2]

(c) **(i)** Express $x^2 - 5x - 8$ in the form $(x+a)^2 + b$. [1]

(ii) **Hence,** solve the equation $x^2 - 5x - 8 = 0$, giving your answers correct to two decimal places. [3]

2 An ice-cream shop sells 4 different flavours of ice-cream. The table below shows the number of cups of ice-cream sold by the shop over 3 days.

	Chocolate	Vanilla	Strawberry	Durian
Friday	252	168	84	105
Saturday	305	158	115	152
Sunday	316	191	134	167

(a) Represent the above information using a 3×4 matrix **A**. [1]

(b) The selling price of each cup of ice-cream is shown in the table below.

	Chocolate	Vanilla	Strawberry	Durian
Selling Price	\$2.10	\$1.80	\$2.05	\$2.55

Write down a matrix **P** such that the product **AP** represents the total sales of the ice-cream for each day respectively. Evaluate **AP**. [2]

(c) Evaluate $\begin{pmatrix}1 & 1 & 1\end{pmatrix}\mathbf{A}$ and state what the elements of the product represents. [2]

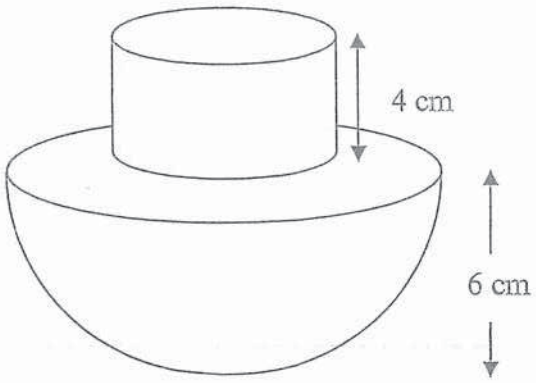
(d) The cost price of each cup of ice-cream is shown in the table below.

	Chocolate	**Vanilla**	**Strawberry**	**Durian**
Cost Price	\$0.40	\$0.45	\$0.55	\$0.80

Write down two matrices such that the elements of their product represent the total profit received by the shop over the 3 days for the sale of each flavor of ice-cream respectively.
Evaluate this product. [2]

3 (a) The diagram shows a toy which is made up of a cylindrical part and a hemispherical part.

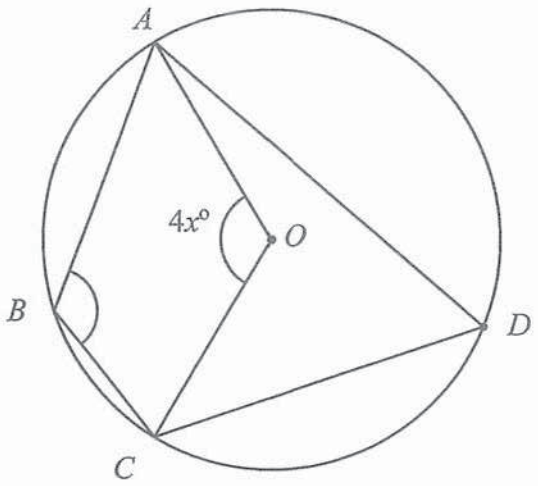
The height of the cylindrical part is 4 cm and the radius of the hemispherical part is 6 cm.



(i) Find the radius of the cylindrical part, given that the volume of the cylindrical part is 150 cm^3. [2]

(ii) Calculate total surface area of the toy. [3]

(b) In the diagram, A, B, C and D lie on a circle with centre O. Angle $AOC = 4x°$.



(i) Find angle ADC, giving your answer in terms of $x°$. [2]

State a reason for your answer.

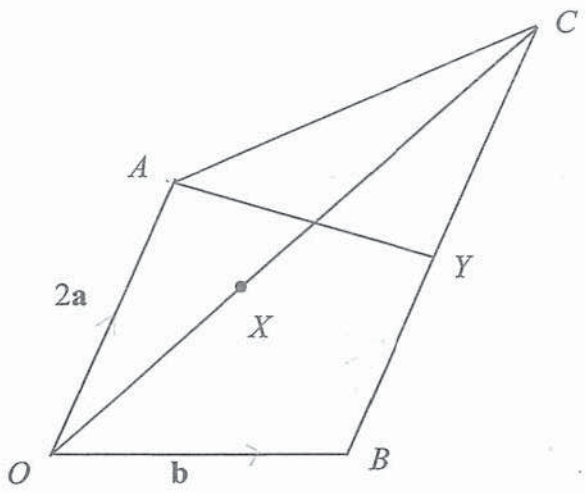
(ii) Given further that angle $ABC = 5x° + 12°$, find the value of x. [2]

4 (a) Construct a triangle ABC such that $AB = 13$ cm, $AC = 12$ cm, angle $ABC = 55°$, and angle ACB is acute.
The line AB has been drawn for you. [2]

(b) Construct the angle bisector of angle BAC such that it intersects the line BC, and label the point of intersection D. [2]

(c) Construct the perpendicular bisector of AD such that it intersects the line AC, and label the point of intersection E. Measure and write down the value of DE. [2]

5



In the diagram, $\overrightarrow{OA} = 2\mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$. BC is parallel to OA and $BC = \frac{3}{2}OA$. X is a point on OC such that $OX = \frac{2}{3}XC$. Y is the midpoint of BC.

(a) Express in terms of **a** and/or **b**, as simply as possible,

(i) $\overrightarrow{AB}$, [1]

(ii) $\overrightarrow{OC}$, [1]

(iii) $\overrightarrow{OX}$, [1]

(iv) $\overrightarrow{AX}$. [1]

(b) What can you deduce about the points A, X and B ?
Justify your answer. [2]

(c) (i) AY produced meets OB produced at a point Z. Given that $\overrightarrow{AZ} = h\overrightarrow{AY}$, express $\overrightarrow{AZ}$ in terms of a, b and h. [3]

(ii) Given also that $\overrightarrow{OZ} = k\overrightarrow{OB}$, express $\overrightarrow{AZ}$ in terms of a, b,and k. [1]

(iii) Hence, show that $h = 4$ and $k = 4$. [2]

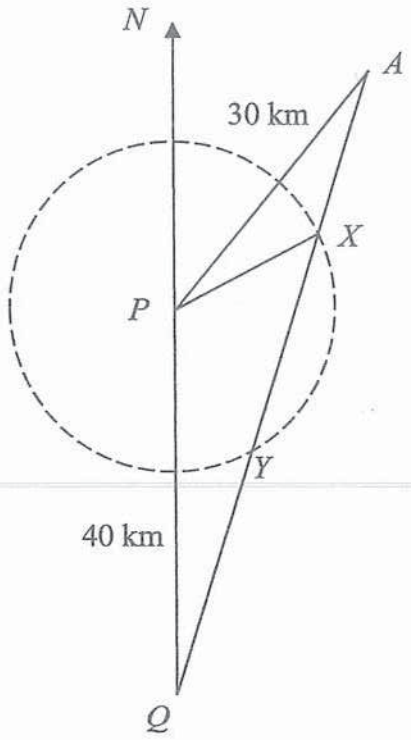
(d) Find the value of

(i) $\dfrac{\text{area of } \Delta OAX}{\text{area of } \Delta OAC}$, [1]

(ii) $\dfrac{\text{area of } \Delta OBX}{\text{area of } \Delta ABC}$. [1]

6 In the diagram, P and Q are the bases of two lighthouses such that P is located 40 km due north of Q.

A is a boat 30 km from P and on a bearing of 055° from P.



(a) Find the distance AQ. [3]

(b) Find the bearing of A from Q. [2]

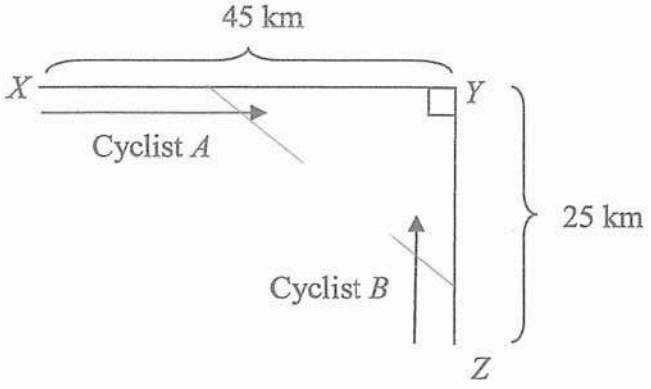
Light from P can be seen within 20 km radius of P, beyond which the light becomes too faint to be seen. When the boat moves from A to Q, X and Y are the positions on the boat's journey which are 20 km from P.

(c) Calculate

(i) the shortest distance of the boat from P. [2]

(ii) the **smallest** angle of elevation of the lighthouse P from the boat as it travels from X to Y, given that the height of lighthouse P is 600m. [2]

7 The diagram shows two roads XY and YZ meeting at point Y. The roads are perpendicular to each other. $YX = 45$ km and $YZ = 25$ km.



Cyclist A is travelling from point X towards point Y at a constant speed of 20 km/h.
Cyclist B is travelling from point Z towards point Y at a constant speed of 10 km/h.

(a) Write down an expression, in terms of t, for the distance in kilometres

(i) between cyclist A and point Y, after t hours, [1]

(ii) between cyclist B and point Y, after t hours. [1]

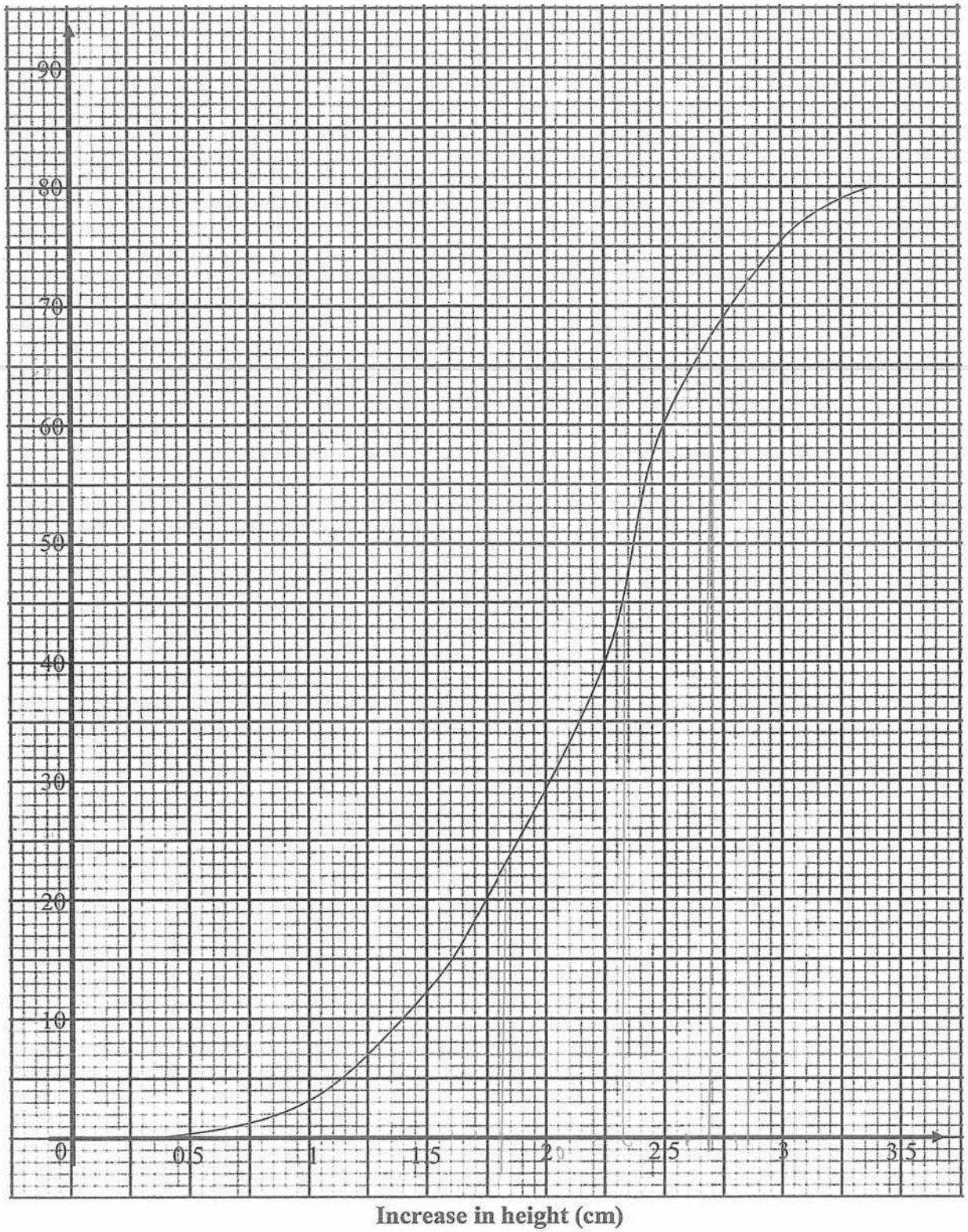
(b) Form an expression, in terms of t, for the shortest distance, d, between the two cyclists and show that it reduces to $\sqrt{500t^2 - 2300t + 2650}$.
Hence find the shortest distance between the two cyclists after 15 minutes. [4]

(c) The two cyclists are 10 km apart at a certain instant, t hours.
Form a quadratic equation in terms of t and show that it reduces to $10t^2 - 46t + 51 = 0$. [2]

(d) Given that $t < 2$, find the time, in minutes, when the two cyclists are 10 km apart.
Correct your answer to 3 significant figures. [3]

8 A scientist wanted to test the effect of different music on the growth of plants.
80 *Rosa chinensis* plants were exposed to Beethoven's Ninth Symphony, and their growth was observed over a period of 10 days.
The cumulative frequency curve below shows the increase in the height of the plants at the end of 10 days.

Increase in heights of plants



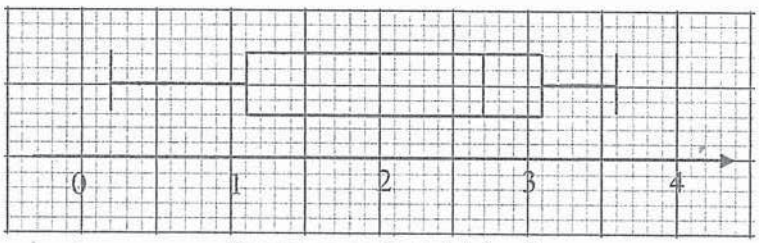
(a) Find

(i) the median increase in the height of the plants, [1]

(ii) the interquartile range, [2]

(iii) the 90^{th} percentile. [2]

(b) Another group of 80 *Rosa chinensis* plants were exposed to Bach's Goldberg Variations, and their growth were also observed over a period of 10 days.
The box-and-whisker plot below shows the increase in the heights of the plants after 10 days.



Increase in height (cm)

(i) Describe how the cumulative frequency graph of the growth of the 80 plants exposed to Bach's Goldberg Variations will differ from that of those exposed to Beethoven's Ninth Symphony. [1]

(ii) Make two comparisons between the growths of the plants under the two conditions. [2]

9 The variables x and y are connected by the equation $y = \frac{1}{2x^2} + 0.1x^2 - 3$. The table below shows some values of x and the corresponding values of y, correct to 2 decimal places.

x	0.5	1	1.5	2	3	4	5	6
y	−0.98	−2.40	−2.55	−2.48	−2.04	−1.37	p	0.61

(a) Find the value of p. [1]

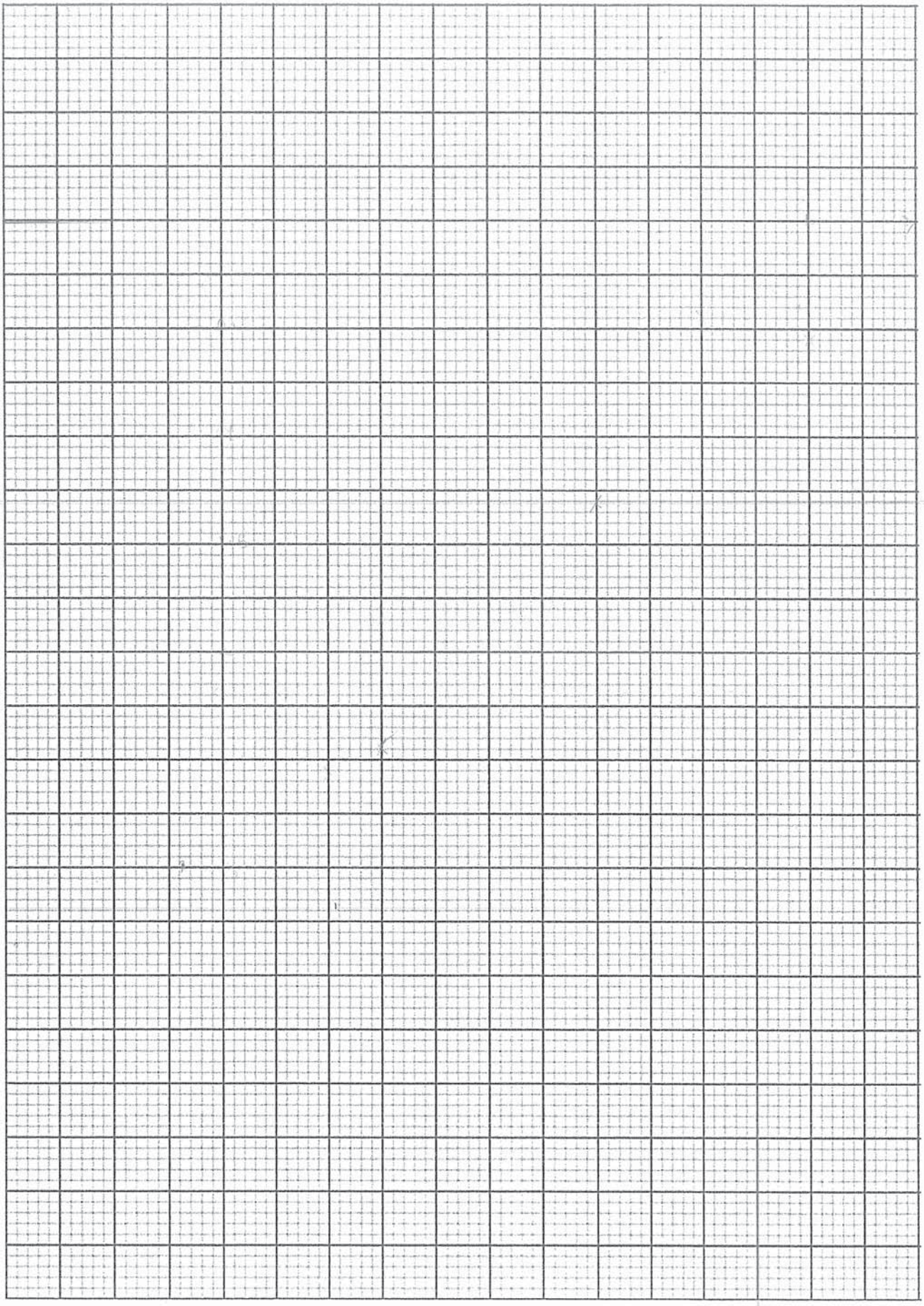
-0·48

(b) In the space provided on the next page, using a scale of 2 cm to represent 1 unit on the horizontal axis and 2 cm to represent 0.5 units on the vertical axis, draw the graph of $y = \frac{1}{2x^2} + 0.1x^2 - 3$ for $0.5 \leq x \leq 6$. [3]

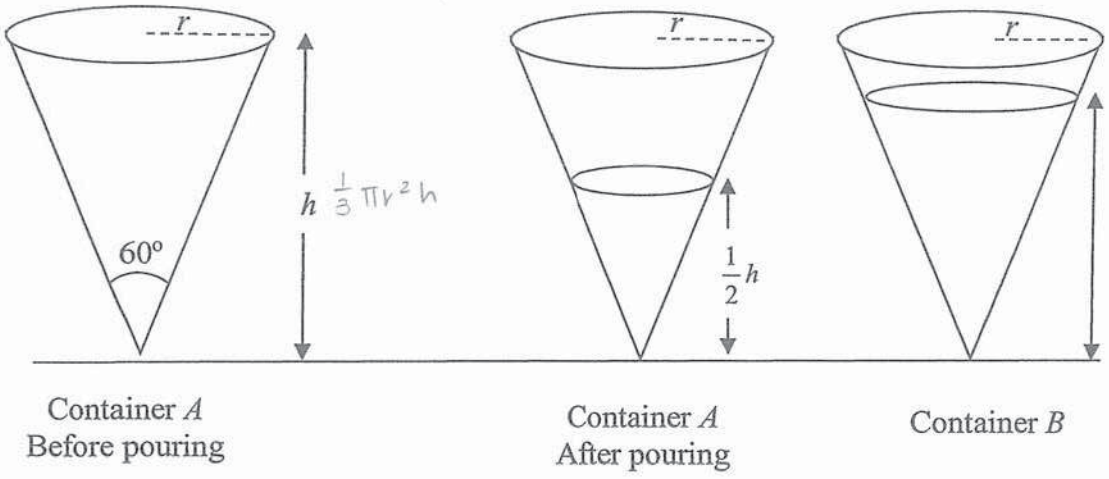
(c) Using your graph, write down the solution(s) to the equation $\frac{1}{2x^2} + 0.1x^2 - 1.5 = 0$. [2]

(d) By drawing a tangent, find the gradient of the curve at $x = 4$. [2]

(e) By drawing a suitable straight line on the same axes, solve the equation $\frac{1}{2x^2} + 0.1x^2 - 0.5x = 0$. [3]



10 A right conical container of capacity 24π cm^3 and vertical angle 60° is completely filled with water. The height of the container is h cm and the base radius is r cm.



Container A
Before pouring

Container A
After pouring

Container B

(a) Find the value of r and of h. [5]

The water in the container is poured into another identical container B so that the depth of water in container A is $\frac{1}{2}h$ cm.

(b) Find the volume of water in container B in terms of π. [2]

11 A class of students plans to sell breakfast sets during the National Day carnival in order to raise funds for a charity.

Each breakfast set consists of 2 scrambled eggs, 2 slices of toast, 2 sausages, and a cup of coffee.

The students estimate that they will sell 250 breakfast sets.

Item	Description	Unit cost
Eggs	Pasar Fresh Eggs (10 per pack) Pasar Fresh Eggs (30 per pack)	\$1.80 \$4.35
Bread	Sunshine Enriched Soft White Bread (14 slices) FairPrice Wholemeal Bread (12 slices)	\$1.80 \$1.55
Sausages	Tierney's Chicken Hot Dog (10 per pack) FairPrice Sausages (6 per pack) (\$0.35 off per 2 packs)	\$5.25 \$3.20
Coffee	Nescafe 3 in 1 Instant Coffee (35 per pack) (Buy 5 get 1 free) Gold Roast 3 in 1 Coffeemix (25 per pack)	\$6.15 \$3.95

(a) Find the lowest possible total cost of the ingredients for the breakfast sets. [5]

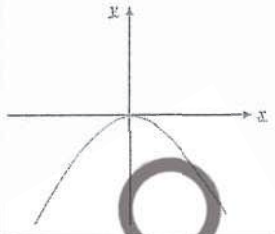
(b) The school provides \$200 in funding for the students, and up to 30% of the sales can be used to cover for their expenses, while the remaining goes to the charity. The students also wish to raise at least \$600 for charity,
Find the minimum they must charge for each breakfast set (to the nearest ten cents).
Justify your answer, showing all necessary workings clearly.
State an assumption you have made in your calculations. [4]

End of Paper

Beatty Secondary School

4E5N Preliminary Examination 2019

1a	$n^2+(n+1)^2$ or $2n^2+2n+1$	B1
1b	For any two consecutive numbers, one would be even and the other one odd. Since the square of an even number is even and the square of an odd number is odd, one of the squares of the two consecutive numbers will be even, and the other will be odd. Hence, their sum will be odd.	B1
2	$\frac{a^2b}{2}\times\left(2a^0b^{-2}\right)^3=\frac{a^2b}{2}\times 8b^{-6}$ $=\frac{4a^2}{b^5}$	M1 A1
3a	$18\left(\frac{4}{3}\right)^2-k\left(\frac{4}{3}\right)-20=0$ $k=9$	[B1]
3b	$18x^2-9x-20=0$ $(3x-4)(6x+5)=0$ $x=\frac{4}{3}$ *or* $x=-\frac{5}{6}$ *(the other solution)*	[B1]
4	$x+5x=180$ $x=30$ $\frac{360}{30}=12$ Number of sides=12	M1 A1
5	Graph 1 – Container B Graph 2 – Container C Graph 3 – Container A	B1 – 1 correct B2 – All correct
6a	$b=2$ $n=1$ Range of a: $a>0$	B1 for both answers correct B1

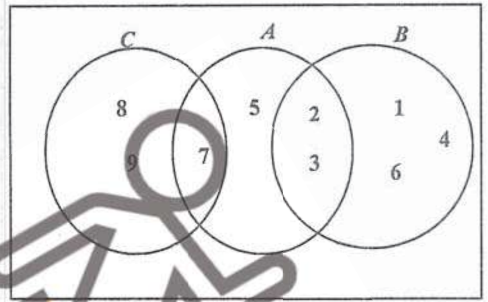
6b		B1
7	$3+y\le 15-2y$ *and* $15-2y<y+10$ $3y\le 12$ $\quad 5<3y$ $y\le 4$ $\quad y>\frac{5}{3}$ Largest prime number is 3.	M1 for each correct answer A1
8	$8x+3y=4$ ---------- (1) $4x=y+4$ ---------- (2) *From* (2), $y=4x-4$ ---- (3) *Sub* (3) in (1), $8x+3(4x-4)=4$ $20x=16$ $x=\frac{4}{5}$ $y=4\times\frac{4}{5}-4$ $=-\frac{4}{5}$	 M1 A1 A1
9a	$\lvert q-2p\rvert$ $=\left\lvert\begin{pmatrix}-4\\3\end{pmatrix}-2\begin{pmatrix}6\\5\end{pmatrix}\right\rvert$ $=\left\lvert\begin{pmatrix}-4\\3\end{pmatrix}-\begin{pmatrix}12\\10\end{pmatrix}\right\rvert$ $=\left\lvert\begin{pmatrix}-16\\-7\end{pmatrix}\right\rvert$ $=\left\lvert\sqrt{(-16)^2+(-7)^2}\right\rvert$ $=\lvert\sqrt{305}\rvert$ $=17.464$ units (5 *s.f.*) $=17.5$ units (3 *s.f.*)	 M1 A1

9b	If **r** is parallel to **q**, **r** = k**q** Now, $\begin{pmatrix} 8 \\ -6 \end{pmatrix} = -2\begin{pmatrix} -4 \\ 3 \end{pmatrix}$, i.e. **r** = -2**q** or **q** = $-\frac{1}{2}$**r** Hence, **r** is parallel to **q**.	B1
10	$\frac{9m^2-12mn+4n^2}{m^2-n^2} \div \frac{3m-2n}{m-n}$ $= \frac{(3m-2n)(3m-2n)}{(m-n)(m+n)} \div \frac{3m-2n}{m-n}$ $= \frac{(3m-2n)(3m-2n)}{(m-n)(m+n)} \times \frac{m-n}{3m-2n}$ $= \frac{3m-2n}{m+n}$	M1 – correct factorisation M1 – for changing divide to multiplication and interchange the numerator with the denominator of the second fraction. A1
11a	Area Scale 1 : 100 Linear Scale 1 : 10 Length of the house = 3 x 10 = 30 m	B1
11b	Volume of the model = $\left(\frac{1}{10}\right)^3 \times 15000000$ $= 15000\ cm^3$	M1 A1
12a	$\frac{3}{8} = \frac{30}{80}$ $80 - 30 - 18 = 32$	M1 A1
12bi	$\frac{30}{80} \times \frac{18}{79} + \frac{18}{80} \times \frac{30}{79}$ $= \frac{27}{158}$	M1 A1
12bii	P(at least 1 red ball) = 1 – P(no red ball) $1 - \left(\frac{62}{80}\right)\left(\frac{61}{79}\right)$ $= \frac{1269}{3160}$ OR	B1
	P(at least 1 red ball) =1 – P(GG) – P(GB) – P(BG) – P(BB) $= 1 - \left(\frac{30}{80}\right)\left(\frac{29}{79}\right) - \left(\frac{30}{80}\right)\left(\frac{32}{79}\right) - \left(\frac{32}{80}\right)\left(\frac{30}{79}\right) - \left(\frac{32}{80}\right)\left(\frac{31}{79}\right)$ $= \frac{1269}{3160}$ OR P(at least 1 red ball) =P(GR) + P(RG) + P(RR) + P(RB) + P(BR) $\left(\frac{30}{80}\right)\left(\frac{18}{79}\right) + \left(\frac{18}{80}\right)\left(\frac{30}{79}\right) + \left(\frac{18}{80}\right)\left(\frac{17}{79}\right) + \left(\frac{18}{80}\right)\left(\frac{32}{79}\right) + \left(\frac{32}{80}\right)\left(\frac{18}{79}\right)$ $= \frac{1269}{3160}$	
13a	$x = -2$	B1
13b	$y = 3x + c$ *Sub* (2, –5), $-5 = 3(2) + c$ $c = -11$ $y = 3x - 11$	M1 A1
14a	$BC = \sqrt{y^2 - x^2}$ $\tan \angle BDC = \frac{\sqrt{y^2 - x^2}}{x}$	M1 A1
14b	$\cos \angle ABD = -\cos \angle DBC = -\frac{\sqrt{y^2 - x^2}}{y}$	B1
15a	*Total sales* $= (3694 - 1200) \times \frac{100}{8}$ = \$31175	M1 A1
15b	Tap *X*: 2*r* cm³/min Tap *Y*: *r* cm³/min In 5 min, $5(2r) + 5r = 300$ $r = 20$ Rate of Tap *Y* = 20 cm³/min OR	M1 A1

	$V_X : V_Y : 2 : 1$ Volume of water in 5 min $= \frac{300}{3}$ $= 100\ cm^3$ Rate $= \frac{100}{5}$ $= 20\ cm^3/min$	 M1 A1
16a	6	B1
16b	$3+5+4-1 = x+2$ $x = 9$	 B1
16c	$\frac{3(0)+5(1)+4(2)+3x+2(4)}{14+x} = 2\frac{1}{22}$ $\frac{21+3x}{14+x} = \frac{45}{22}$ $462+66x = 630+45x$ $21x = 168$ $x = 8$	M1 A1
17a	$Total\ dist = \frac{1}{2}v(10+35)$ $540 = 22.5v$ $v = 24m/s$	M1 A1
17b	$a = \frac{24}{10}$ $= 2.4\ m/s^2$ Speed $= 24 - 3(2.4)$ $= 16.8$ m/s	M1 A1
18a	$\frac{H_L}{H_S} = \sqrt{\frac{242}{98}}$ $= \frac{11}{7}$	 B1
18b	$\frac{A_S}{A_L} = \left(\frac{7}{11}\right)^2$ $\frac{343}{A_L} = \frac{49}{121}$ $A_L = \frac{121}{49} \times 343$ $A_L = 847\ cm^2$	 B1

18c	$\frac{V_S}{V_L} = \left(\frac{7}{11}\right)^3$ $\frac{V_S}{V_L} = \frac{343}{1331}$ $\%\ difference = \frac{1331-343}{1331} \times 100\%$ $= 74.229\%\ (5\ s.f)$ $= 74.2\%\ (3\ s.f)$	 M1 A1

19a	$s = kv^2$ $36 = kv^2$ $s = k(\frac{v}{2})^2$ $s = 9$	M1 A1
19b	$y = \frac{k}{x}$ $k = 24$ $y = \frac{24}{9}$ $y = 2\frac{2}{3}$	M1 A1
20a	35 = 5 x 7 $3^x \times 5^y \times 7^3$ needs to have a factor of 5 and 7. Therefore, $x = 0$, $y = 1$	B1 – if 1 correct B2 – If 2 correct
20b	$55q$ $= 5 \times 11 \times q$ $= 5 \times 11 \times 5 \times 7^2 \times 11$ $= 5^2 \times 7^2 \times 11^2$ Since the indices of the prime factors are multiples of 2, $55q$ is a perfect square.	B1
20c	$p \times q = 3^x \times 5^y \times 7^3 \times 5 \times 7^2 \times 11$ $= 3^x \times 5^{y+1} \times 7^5 \times 11$ Since p x q is a product of prime factors that are odd, p x q is an odd number.	B1
21a	$(P \cup Q)' \cup (P \cap Q)$ or $(P' \cap Q') \cup (P \cap Q)$	B1
21bi	$B \cap C = \varnothing$ or $\{\}$	B1
21bii	$(A \cup B)' = \{8, 9\}$	B1

21biii		B1 – 5 out of 9 correct B2 – All correct
22a	$\angle QXY = \angle QRS$ (corr. $\angle s$, $XY // RS$) $\angle QYX = \angle QSR$ (corr. $\angle s$, $XY // RS$) $\angle RQS = \angle XQY$ (common $\angle s$) ΔQXY is similar to ΔQRS. (AA)	B1- Either 2 of these reasons A1
22bi	Since ΔQXY is similar ΔQRS, $\frac{Area\ of\ \Delta QRS}{Area\ of\ \Delta QXY} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$ $Area\ of\ \Delta QRS = \frac{25}{9} \times 27 = 75\ cm^2$	M1, A1
22ii	Consider ΔPQR and ΔPXR: $\frac{Area\ of\ \Delta PQR}{Area\ of\ \Delta PXR} = \frac{\frac{1}{2} \times QR \times h}{\frac{1}{2} \times XR \times h} = \frac{QR}{XR} = \frac{5}{2}$ $Area\ of\ \Delta PQR = \frac{5}{2} \times 46 = 115\ cm^2$	M1, A1

23a	$EC = \sqrt{4^2 + 2^2}$ $= \sqrt{20}$ Perimeter of sector *CEF* $= 2\sqrt{20} + \sqrt{20}(0.64)$ $\approx 11.8\ cm$	M1 M1 A1
23b	Area of sector *CEF* $= \frac{1}{2}\left(\sqrt{20}\right)^2(0.64)$ $= 6.4\ cm^2$ Area of shaded region $= 4^2 - 2\left(\frac{1}{2} \times 4 \times 2\right) - 6.4$ $= 1.6\ cm^2$	 M1 A1 M1 A1

Answer Key

1(a)	$\frac{t^4}{4v^3}$
1(b)(i)	$\frac{25x}{3y^2}$
1(b)(ii)	$\frac{13-11x}{(3x-1)(2+x)}$
1(c)(i)	$\left(x-\frac{5}{2}\right)^2-\frac{57}{4}$
1(c)(ii)	$x=6.27$ or -1.27
2(a)	$\mathbf{A}=\begin{pmatrix}252 & 168 & 84 & 105\\ 305 & 158 & 115 & 152\\ 316 & 191 & 134 & 167\end{pmatrix}$
2(b)	$\mathbf{P}=\begin{pmatrix}2.10\\ 1.80\\ 2.05\\ 2.55\end{pmatrix}$ $\mathbf{AP}=\begin{pmatrix}1271.55\\ 1548.25\\ 1707.95\end{pmatrix}$
2(c)	The elements represents the total number of cups of ice-cream sold over the 3 days of each flavor respectively.
2(d)	(1484.10 697.95 499.50 742)
3(a)(i)	$r=3.45$ cm
3(a)(ii)	426 cm^2
3(b)(i)	$2x^o$
3(b)(ii)	$x^o=24^o$
4(c)	6.3 cm
5(a)(i)	$\mathbf{b}-2\mathbf{a}$
5(a)(ii)	$\mathbf{b}+3\mathbf{a}$
5(a)(iii)	$\frac{2}{5}(\mathbf{b}+3\mathbf{a})$
5(a)(iv)	$-\frac{4}{5}\mathbf{a}+\frac{2}{5}\mathbf{b}$
5(b)	$\overrightarrow{AB}=\frac{5}{2}\overrightarrow{AX}$ A, X, and B are collinear.
5(c)(i)	$\overrightarrow{AZ}=h\mathbf{b}-\frac{1}{2}h\mathbf{a}$
5(c)(ii)	$\overrightarrow{AZ}=-2\mathbf{a}+k\mathbf{b}$
5(c)(iii)	*(AG)*

5(d)(i)	$\frac{2}{5}$
5(d)(ii)	$\frac{2}{5}$
6(a)	62.3 km
6(b)	023.2°
6(c)	15.8 km
6(d)	1.7^{o}
7(a)(i)	$(45-20t)$ km
7(a)(ii)	$(25-10t)$ km
7(b)	45.9 km
7(c)	*(AG)*
7(d)	112 minutes
8(a)(i)	2.25 cm
8(a)(ii)	0.75 cm
8(a)(iii)	2.85 cm
8(b)(i)	The cumulative frequency graph of the growth of plants exposed to Bach will be **less steep in the middle/steeper at the upper quartile/wider range/broader range** compared to that of plants exposed to Beethoven. **OR** The middle of the cumulative frequency graph of the growth of plants exposed to Bach is **shifted to the right** compared to that of plants exposed to Beethoven.
8(b)(ii)	The plants grow **better** when exposed to Bach **on average** compared to when exposed to Beethoven as the **median increase in heights** of the plants when exposed to Bach **is higher** than those exposed to Beethoven. The increase in heights of the plants are **more spread** when exposed to Bach than when exposed to Beethoven as the **interquartile range is larger** when exposed to Bach than when exposed to Beethoven.
9(a)	−0.48
9(b)	*(graph)*
9(c)	0.6 or 3.8
9(d)	0.792
9(e)	1.1 or 4.95
10(a)	$h = 6$ cm $r = 3.46$ cm
10(b)	21π
11(a)	$428.65
11(b)	Any value above $3.3146 Assume no cost incurred for cooking. (or any reasonable)

BEATTY SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2019

MARKING SCHEME

SUBJECT :	Mathematics	LEVEL :	Sec 4 Express Sec 5 Normal (Academic)
PAPER :	4048 / 02	DURATION :	2 hours 30 minutes
SETTER :	Mr Teo CK Miss Chong HY	DATE :	29 August 2019

CLASS :	NAME :	REG NO :

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

This paper consists of 23 printed pages (including this cover page)

[Turn over

Mathematical Formulae

Compound Interest

Total amount = $P(1+\frac{r}{100})^n$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where is in radians

Sector area = $\frac{1}{2}r^2\theta$, where is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Simplify $\left(\frac{64v^9}{t^{12}}\right)^{-\frac{1}{3}}$. [2]

$$\left(\frac{64v^9}{t^{12}}\right)^{-\frac{1}{3}}$$

$$=\left(\frac{t^{12}}{64v^9}\right)^{\frac{1}{3}} \text{----}M1$$

$$=\frac{t^4}{4v^3} \text{----}A1$$

(b) Express as a single fraction in its simplest form.

(i) $\frac{20x^2}{9} \div \frac{4xy^3}{15y}$. [1]

$$\frac{20x^2}{9} \div \frac{4xy^3}{15y}$$

$$=\frac{20x^2}{9} \times \frac{15y}{4xy^3}$$

$$=\frac{25x}{3y^2} \text{--------}B1$$

(ii) $\frac{4}{3x-1} - \frac{5}{2+x}$. [2]

$$\frac{4}{3x-1} - \frac{5}{2+x}$$

$$=\frac{4(2+x)-5(3x-1)}{(3x-1)(2+x)} \text{----}M1$$

$$=\frac{13-11x}{(3x-1)(2+x)} \text{--------}A1$$

(c) **(i)** Express x^2-5x-8 in the form $(x+a)^2+b$. [1]

$$x^2-5x-8$$

$$=\left(x-\frac{5}{2}\right)^2-\left(-\frac{5}{2}\right)^2-8$$

$$=\left(x-\frac{5}{2}\right)^2-\frac{57}{4} \text{----}B1$$

(ii) **Hence,** solve the equation $x^2-5x-8=0$, giving your answers correct to two decimal places. [3]

$$x^2-5x-8=0$$

$$\left(x-\frac{5}{2}\right)^2-\frac{57}{4}=0$$

$$\left(x-\frac{5}{2}\right)^2=\frac{57}{4} \text{----}M1 \text{ (ECF their (ci))}$$

$$x-\frac{5}{2}=\pm\sqrt{\frac{57}{4}}$$

$$x=6.27 \text{ or } -1.27 \text{----}A2$$

2 An ice-cream shop sells 4 different flavours of ice-cream. The table below shows the number of cups of ice-cream sold by the shop over 3 days.

	Chocolate	Vanilla	Strawberry	Durian
Friday	252	168	84	105
Saturday	305	158	115	152
Sunday	316	191	134	167

(a) Represent the above information using a 3×4 matrix A. [1]

$$A=\begin{pmatrix}252 & 168 & 84 & 105\\ 305 & 158 & 115 & 152\\ 316 & 191 & 134 & 167\end{pmatrix}$$ --- B1

(b) The selling price of each cup of ice-cream is shown in the table below.

	Chocolate	Vanilla	Strawberry	Durian
Selling Price	\$2.10	\$1.80	\$2.05	\$2.55

Write down a matrix P such that the product AP represents the total sales of all the ice-cream for each day respectively. Evaluate AP. [2]

$$P=\begin{pmatrix}2.10\\ 1.80\\ 2.05\\ 2.55\end{pmatrix}$$ --- B1

$$AP=\begin{pmatrix}252 & 168 & 84 & 105\\ 305 & 158 & 115 & 152\\ 316 & 191 & 134 & 167\end{pmatrix}\begin{pmatrix}2.10\\ 1.80\\ 2.05\\ 2.55\end{pmatrix}$$

$$=\begin{pmatrix}1271.55\\ 1548.25\\ 1707.95\end{pmatrix}$$ --- B1

(for P given as row matrix, B0)

(c) Evaluate $\begin{pmatrix}1 & 1 & 1\end{pmatrix}A$ and state what the elements of the product represents. [2]

$$\begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}252 & 168 & 84 & 105\\ 305 & 158 & 115 & 152\\ 316 & 191 & 134 & 167\end{pmatrix}=\begin{pmatrix}873 & 517 & 333 & 424\end{pmatrix}$$ --- B1

The elements represents the total number of cups of ice-cream sold over the 3 days of each flavor respectively. --- B1

(d) The cost price of each cup of ice-cream is shown in the table below.

	Chocolate	Vanilla	Strawberry	Durian
Cost Price	\$0.40	\$0.45	\$0.55	\$0.80

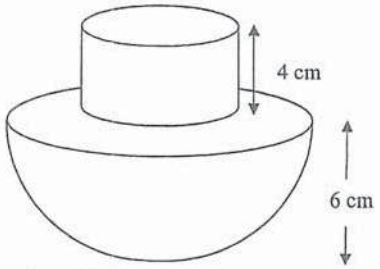
Write down two matrices such that the elements of their product represent the total profit received by the shop over the 3 days for the sale of each flavor of ice-cream respectively.
Evaluate this product. [2]

$$\begin{pmatrix}873 & 517 & 333 & 424\end{pmatrix}\begin{pmatrix}1.70 & 0 & 0 & 0\\ 0 & 1.35 & 0 & 0\\ 0 & 0 & 1.50 & 0\\ 0 & 0 & 0 & 1.75\end{pmatrix}$$ --- M1 for correct 4×4 matrix

$$=\begin{pmatrix}1484.10 & 697.95 & 499.50 & 742\end{pmatrix}$$ --- A1

3 (a) The diagram shows a toy which is made up of a cylindrical part and a hemispherical part.

The height of the cylindrical part is 4 cm and the radius of the hemispherical part is 6 cm.



(i) Find the radius of the cylindrical part, given that the volume of the cylindrical part is 150 cm^3. [2]

$\pi \times r^2 \times 4 = 150$ ----$M1$

$r^2 = \dfrac{150}{\pi \times 4}$

$r = 3.4549...$

$r = 3.45$ cm (3sf) ----$A1$

(ii) Calculate total surface area of the toy. [3]

SA of hemisphere $= 2 \times \pi \times 6^2 + \pi \times 6^2 = 108\pi$ ---- M1 (ECF their (i))

Total SA $= 108\pi + 2 \times \pi \times 3.4549... \times 4$ ---- M1 (ECF their (i))

$= 426.178... = 426$ cm^2 ----$A1$

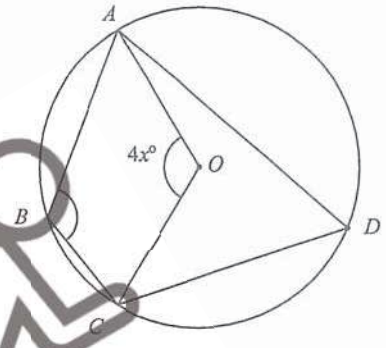
OR

SA of hemisphere $= 3\pi(6)^2 = 108\pi$ --- M1

SA of cylinder $= \pi(3.4549)^2 + 2\pi(3.4549)(4) = 39.575\pi$ --- M1

TSA $= 108\pi + 39.575\pi - \pi(3.4549)^2 = 426$ cm^2 --- A1

(b) In the diagram, A, B, C and D lie on a circle with centre O. Angle $AOC = 4x^o$.



(i) Find angle ADC, giving your answer in terms of x^o. [2]

State a reason for your answer.

angle ADC=2*x* (angle at centre = 2 angle at circumference) ----$B2$

(ii) Given further that angle $ABC = 5x^o + 12°$, find the value of x. [2]

$5x^o + 12^o = 180^o - 2x^o$ ----$M1$

$7x^o = 168^o$

$x^o = 24^o$ ----$A1$

OR

$2(5x+12) = 360 - 4x$ --- M1

$10x + 24 = 360 - 4x$

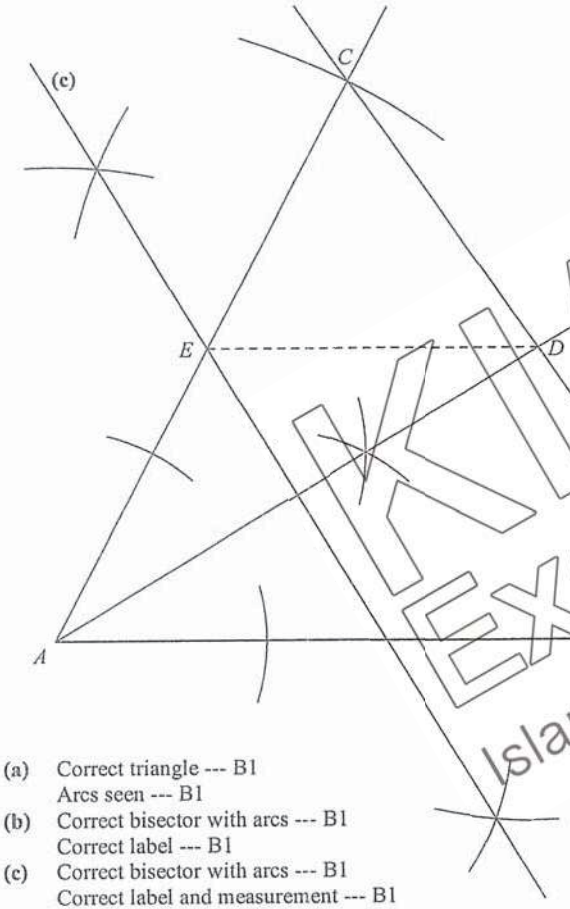
$14x = 336$

$x = 24$ --- A1

4 (a) Construct a triangle ABC such that $AB = 13\,\text{cm}$, $AC = 12\,\text{cm}$, angle $ABC = 55°$, and angle ACB is acute.
The line AB has been drawn for you. [2]

(b) Construct the angle bisector of angle BAC such that it intersects the line BC, and label the point of intersection D. [2]

(c) Construct the perpendicular bisector of AD such that it intersects the line AC, and label the point of intersection E. Measure and write down the value of DE. [2]



(a) Correct triangle --- B1
Arcs seen --- B1

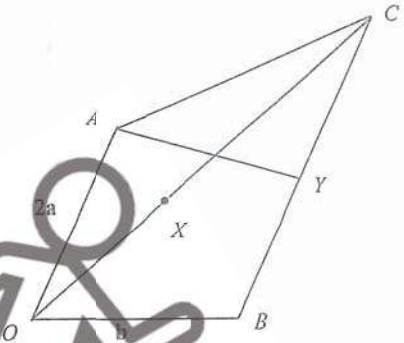
(b) Correct bisector with arcs --- B1
Correct label --- B1

(c) Correct bisector with arcs --- B1
Correct label and measurement --- B1

No question labels = minus 1 overall

(c) 6.3 cm (±0.1)
(actual = 6.24 cm

5



In the diagram, $\overrightarrow{OA} = 2\mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$. BC is parallel to OA and $BC = \frac{3}{2}OA$. X is a point on OC such that $OX = \frac{2}{3}XC$. Y is the midpoint of BC.

(a) Express in terms of a and/or b, as simply as possible,

(i) $\overrightarrow{AB}$, [1]

$\overrightarrow{AB} = \mathbf{b} - 2\mathbf{a}$ ------B1

(ii) $\overrightarrow{OC}$ [1]

$\overrightarrow{OC} = \mathbf{b} + 3\mathbf{a}$ ------B1

(iii) $\overrightarrow{OX}$, [1]

$\overrightarrow{OX} = \frac{2}{5}(\mathbf{b} + 3\mathbf{a})$ oe------B1

(iv) $\overrightarrow{AX}$. [1]

$\overrightarrow{AX} = \overrightarrow{AO} + \overrightarrow{OX}$

$= -2\mathbf{a} + \frac{2}{5}\mathbf{b} + \frac{6}{5}\mathbf{a}$

$= -\frac{4}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$ -------M1

(b) Write down two facts about the points A, X and B. [2]

$\overrightarrow{AB} = \frac{5}{2}\left(\frac{2}{5}\mathbf{b} - \frac{4}{5}\mathbf{a}\right)$

$= \frac{5}{2}\overrightarrow{AX}$ ------B1

A, X, and B are collinear. --- B1

(c) (i) AY produced meets OB produced at a point Z. Given that $\overrightarrow{AZ} = h\overrightarrow{AY}$, express $\overrightarrow{AZ}$ in terms of a, b and h. [3]

$\overrightarrow{AZ} = h\overrightarrow{AY}$

$\overrightarrow{AY} = \overrightarrow{AB} + \overrightarrow{BY}$ ------$M1$

$= \mathbf{b} - \frac{1}{2}\mathbf{a}$ -------$M1$

$\overrightarrow{AZ} = h\mathbf{b} - \frac{1}{2}h\mathbf{a}$ ----$A1$

(ii) Given also that $\overrightarrow{OZ} = k\overrightarrow{OB}$, express $\overrightarrow{AZ}$ in terms of a, b,and k. [1]

$\overrightarrow{AZ} = -2\mathbf{a} + k\mathbf{b}$ ----B1

(iii) Hence, show that $h = 4$ and $k = 4$. [2]

$h\mathbf{b} - \frac{1}{2}h\mathbf{a} = 2\mathbf{a} + k\mathbf{b}$

Comparing

$h = 4, k = 4$ ---------A1, A1

(d) Find the value of

(i) $\frac{\text{area of } \Delta OAX}{\text{area of } \Delta OAC}$, [1]

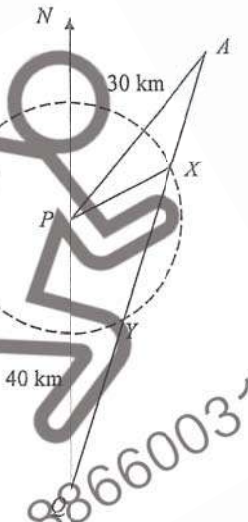
$\frac{\text{area of } \Delta OAX}{\text{area of } \Delta OAC} = \frac{2}{5}$ ----$B1$

(ii) $\frac{\text{area of } \Delta OBX}{\text{area of } \Delta ABC}$. [1]

$\frac{\text{area of } \Delta OBX}{\text{area of } \Delta ABC} = \frac{3}{5} \times \frac{2}{3} = \frac{2}{5}$ ---$B1$

6 In the diagram, P and Q are the bases of two lighthouses such that P is located 40 km due north of Q.

A is a boat 30 km from P and on a bearing of 055° from P.



(a) Find the distance AQ. [3]

$AQ^2 = 30^2 + 40^2 - 2(30)(40)\cos 125^\circ$ ----$M1M1$

$= 2500 - 2400\cos 125^\circ$

$AQ = 62.26221525...$

$AQ = 62.3$ km(3 sf)------$A1$

(b) Find the bearing of A from Q. [2]

$\frac{\sin \angle PQA}{30} = \frac{\sin \angle APQ}{62.26221525}$

$\sin \angle PQA = \frac{30 \times \sin 125^\circ}{62.26221525}$

$\angle PQA = 23.2469296^\circ$

Bearing of A from $Q = 023.2^\circ$.

Light from P can be seen within 20 km radius of P, beyond which the light becomes too faint to be seen. When the boat moves from A to Q, X and Y are the positions on the boat's journey which are 20 km from P.

(c) Calculate

(i) the shortest distance of the boat from P. [2]

$\angle PQX = \angle PQA = 23.2469296^o$

$\sin \angle PQX = \frac{h}{40}$ ------ $M1$

$h = 15.78778476$ km

$h = 15.8$ km ----- $A1$

(ii) the **smallest** angle of elevation of the lighthouse P from the boat as it travels from X to Y, given that the height of lighthouse P is 600m. [2]

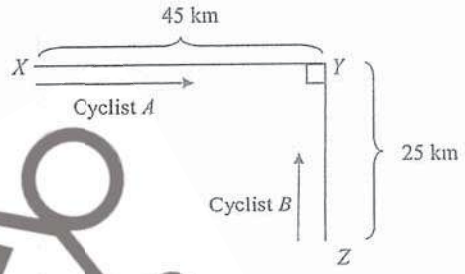
smallest angle of elevation

$= \tan^{-1}\left(\frac{600}{20000}\right)$ ----- $M1$

$= 1.7183...$

$= 1.7^o$ ------ $A1$

7 The diagram shows two roads XY and YZ meeting at point Y. The roads are perpendicular to each other. $YX = 45$ km and $YZ = 25$ km.



Cyclist A is travelling from point X towards point Y at a constant speed of 20 km/h.
Cyclist B is travelling from point Z towards point Y at a constant speed of 10 km/h.

(a) Write down an expression, in terms of t, for the distance in kilometres

(i) between cyclist A and point Y, after t hours, [1]

$(45 - 20t)$ km

(ii) between cyclist B and point Y, after t hours. [1]

$(25 - 10t)$ km

(b) Form an expression, in terms of t, for the shortest distance, d, between the two cyclists and show that it reduces to $\sqrt{10t^2 - 46t + 53}$.
Hence find the shortest distance between the two cyclists after 15 minutes. [4]

$d^2 = (45 - 20t)^2 + (25 - 10t)^2$ ---------- $M1$

$d = \sqrt{2025 - 1800t + 400t^2 + 625 - 500t + 100t^2}$

$= \sqrt{500t^2 - 2300t + 2650}$ ---------- $A1$

At $t = 0.25$ hour,

$d = \sqrt{500(0.25)^2 - 2300(0.25) + 2650}$ ------- $M1$

$= 45.893...$

$= 45.9$ km ------- $A1$

(c) The two cyclists are 10 km apart at a certain instant, t hours.
Form a quadratic equation in terms of t and show that it reduces to $10t^2 - 46t + 51 = 0$. [2]

$10 = \sqrt{500t^2 - 2300t + 2650}$ ---------M1

$100 = 500t^2 - 2300t + 2650$

$500t^2 - 2300t + 2550 = 0$

$20t^2 - 92t + 102 = 0$

$10t^2 - 46t + 51 = 0$ -----B1

(d) Given that $t < 2$, find the time, in minutes, when the two cyclists are 10 km apart.
Correct your answer to 3 significant figures. [3]

$10t^2 - 46t + 51 = 0$

$t = \dfrac{-(-46) \pm \sqrt{(-46)^2 - 4(10)(51)}}{2(10)}$ ----M1

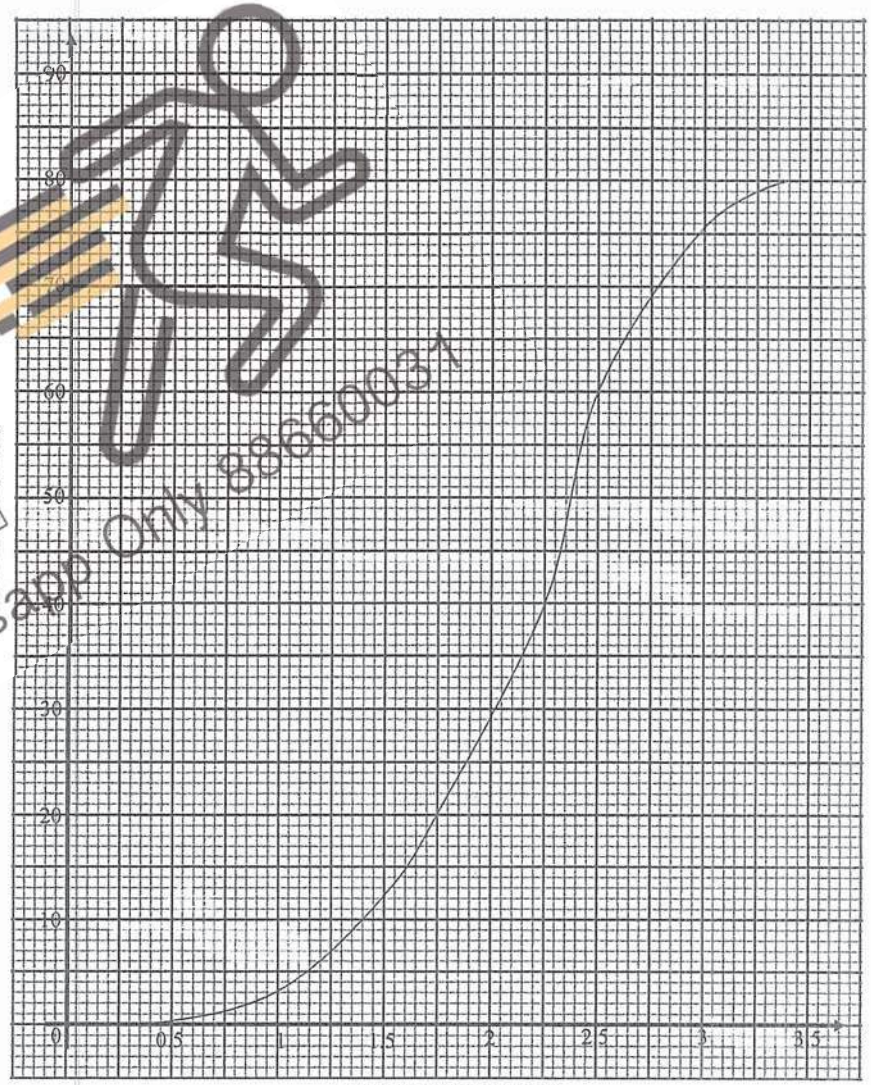
$= \dfrac{46 \pm \sqrt{76}}{20}$

$= 2.73588...$ *or* $1.8641...$ ------A1

Hence $t = 1.8641...$ = 112 minutes (3 sf). ----------A1

8 A scientist wanted to test the effect of different music on the growth of plants.
80 *Rosa chinensis* plants were exposed to Beethoven's Ninth Symphony, and their growth were observed over a period of 10 days.
The cumulative frequency curve below shows the increase in the height of the plants at the end of 10 days.

Increase in heights of plants



(a) Find

(i) the median increase in the height of the plants, [1]

2.25 cm --- B1

(ii) the interquartile range, [2]

$2.5 - 1.75 = 0.75$ cm --- M1, A1

(iii) the 90th percentile. [2]

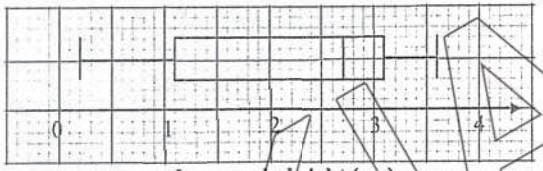
90^{th} percent $= \frac{90}{100} \times 80$ --- M1

$= 72$

90^{th} percentile $= 2.85$ cm --- A1

(all answers ±0.025)

(b) Another group of 80 *Rosa chinensis* plants were exposed to Bach's Goldberg Variations, and their growth were also observed over a period of 10 days.
The box-and-whisker plot below shows the increase in the heights of the plants after 10 days.



Increase in height (cm)

(i) Describe how the cumulative frequency graph of the growth of the 80 plants exposed to Bach's Goldberg Variations will differ from that of those exposed to Beethoven's Ninth Symphony. [1]

The cumulative frequency graph of the growth of plants exposed to Bach will be **less steep in the middle/steeper at the upper quartile/wider range/broader range** compared to that of plants exposed to Beethoven. --- B1

OR

The middle of the cumulative frequency graph of the growth of plants exposed to Bach is **shifted to the right** compared to that of plants exposed to Beethoven.- B1

(ii) Make two comparisons between the growths of the plants under the two conditions. [2]

The plants grow **better** when exposed to Bach **on average** compared to when exposed to Beethoven as the **median increase in heights** of the plants when exposed to Bach **is higher** than those exposed to Beethoven. --- B1

The increase in heights of the plants are **more spread** when exposed to Bach than when exposed to Beethoven as the **interquartile range is larger** when exposed to Bach than when exposed to Beethoven. --- B1

9 The variables x and y are connected by the equation $y = \frac{1}{2x^2} + 0.1x^2 - 3$. The table below shows some values of x and the corresponding values of y, correct to 2 decimal places.

x	0.5	1	1.5	2	3	4	5	6
y	−0.98	−2.40	−2.55	−2.48	−2.04	−1.37	p	0.61

(a) Find the value of p. [1]

$p = -0.48$ --- B1

(b) In the space provided on the next page, using a scale of 2 cm to represent 1 unit on the horizontal axis and 2 cm to represent 0.5 units on the vertical axis, draw the graph of $y = \frac{1}{2x^2} + 0.1x^2 - 3$ for $0.5 \le x \le 6$. [3]

Refer to attached

(minus 1 if the graph turns to the left or is vertical at $x = 0.5$)

(c) Using your graph, write down the solution(s) to the equation $\frac{1}{2x^2} + 0.1x^2 - 1.5 = 0$. [2]

$x = 0.6$ or 3.8 ± 0.1 --- B2

(d) By drawing a tangent, find the gradient of the curve at $x = 4$. [2]

Tangent drawn. --- M1

$\text{gradient} = \frac{-0.25 - (-2.15)}{5.4 - 3}$

$= 0.792$ $(0.7 - 0.85)$ $(\text{actual} = 0.784375)$ --- A1

(e) By drawing a suitable straight line on the same axes, solve the equation $\frac{1}{2x^2} + 0.1x^2 - 0.5x = 0$ [3]

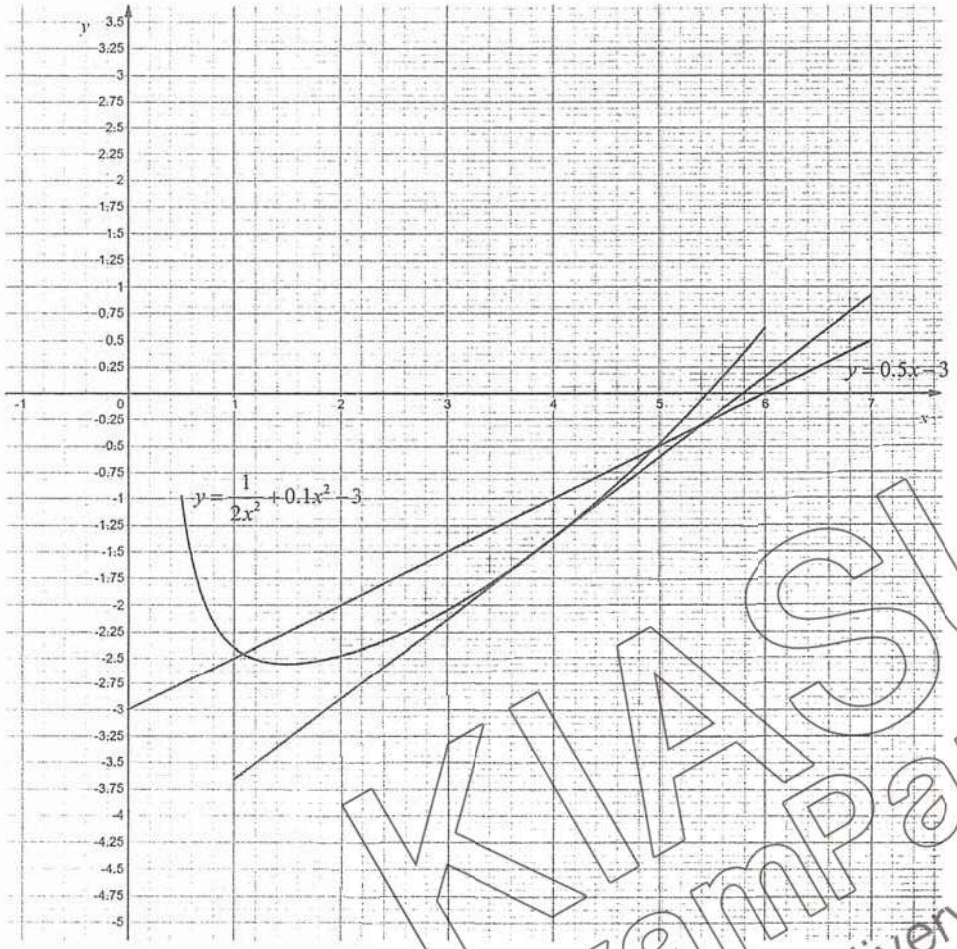
$\frac{1}{2x^2} + 0.1x^2 - 0.5x = 0$

$\frac{1}{2x^2} + 0.1x^2 - 3 = 0.5x - 3$

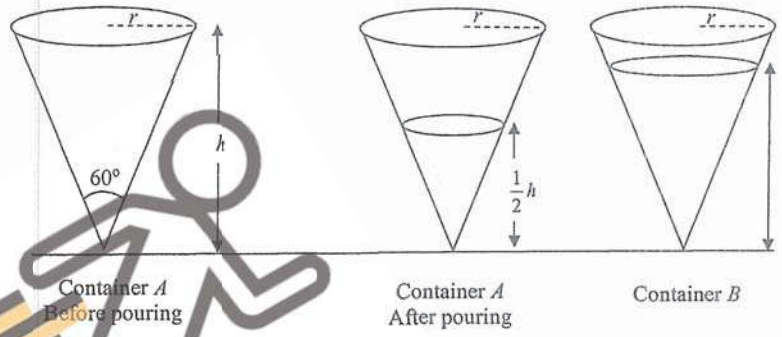
$\therefore$ Draw $y = 0.5x - 3$ --- M1

Correctly drawn graph --- B1

$x = 1.1$ or 4.95 ± 0.1 --- B1



10 A right conical container of capacity 24π cm^3 and vertical angle 60° is completely filled with water. The height of the container is h cm and the base radius is r cm.



(a) Find the value of r and of h. [5]

$\frac{1}{3}\pi r^2 h = 24\pi$ ------ $M1$

$r^2 h = 72$

$\tan 30^o = \frac{r}{h}$ ------ $M1$

$h = \frac{r}{\tan 30^o}$

Hence, $r^3 = 72\tan 30^o$ ------- $M1$

$r = \sqrt[3]{72\tan 30^o} = 3.4610... = 3.46$ cm ---- $A1$

$h = 6$ cm -------- $A1$

OR

$r^2 + h^2 = (2r)^2$

$r^2 = \frac{h^2}{3}$ --- (1)

$\frac{1}{3}\pi r^2 h = 24\pi$ --- (2)

sub (1) into (2):

$\frac{1}{3}\pi\left(\frac{h^2}{3}\right)h = 24\pi$

$h^3 = 216$

$h = 6$ cm

$r = \sqrt{\frac{6^2}{3}}$

$= 3.46$ cm

The water in the container is poured into another identical container B so that the depth of water in container A is $\frac{1}{2}h$ cm.

(b) Find the volume of water in container B in terms of π. [2]

$$\frac{V_A}{original\ volume} = \left(\frac{1}{2}\right)^3 = \frac{1}{8} \text{----} M1$$

Volume of water in container $B = \frac{7}{8} \times 24\pi = 21\pi \text{ cm}^3$ ----$A1$

OR

$$\text{volume remaining } = \frac{1}{3}\pi\left(\frac{\sqrt{12}}{2}\right)^2\left(\frac{6}{2}\right) = 3\pi$$

$$\text{volume in } B = 24\pi - 3\pi = 21\pi \text{ cm}^3$$

11 A class of students plan to sell breakfast sets during the National Day carnival in order to raise funds for Food Bank Singapore.

Each breakfast set consists of 2 scrambled eggs, 2 slices of toast, 2 sausages, and a cup of coffee.

The students estimate that they will sell 250 breakfast sets.

Item	Description	Unit cost
Eggs	Pasar Fresh Eggs (10 per pack)	\$1.80
	Pasar Fresh Eggs (30 per pack)	\$4.35
Bread	Sunshine Enriched Soft White Bread (14 slices)	\$1.80
	FairPrice Wholemeal Bread (12 slices)	\$1.55
Sausages	Tierney's Chicken Hot Dog (10 per pack)	\$5.25
	FairPrice Sausages (6 per pack) (\$0.35 off per 2 packs)	\$3.20
Coffee	Nescafe 3 in 1 Instant Coffee (35 per pack) (Buy 5 get 1 free)	\$6.15
	Gold Roast 3 in 1 Coffeemix (25 per pack)	\$3.95

(a) Find the lowest possible total cost of the ingredients for the breakfast sets. [5]

	Cost		
Eggs	10 per pack $(50)(1.80)$ $= 90$	30 per pack $(17)(4.35) = 73.95$	Mix $(16)(4.35)+(2)(1.80) = 73.20$
Bread	Sunshine $(36)(1.80)$ $= 64.80$	FairPrice $(42)(1.55) = 65.10$	Mix $(35)(1.80)+(1)(1.55) = 64.55$ $(28)(1.80)+(9)(1.55) = 64.35$ $(34)(1.80)+(2)(1.55) = 64.30$ $(10)(1.80)+(30)(1.55) = 64.50$
Sausages	Tierney's $(50)(5.25)$ $= 262.50$	FairPrice $(84)(3.20)-(42)(0.35)$ $= 254.10$	Mix $(82)(3.20)-(41)(0.35)+(1)(5.25)$ $= 253.30$ $(80)(3.20)-(40)(0.35)+(2)(5.25)$ $= 252.5$
Coffee	Nescafe $(7)(6.15)$ $= 43.05$	Gold Roast $(10)(3.95) = 39.50$	Mix $(5)(6.15)+(2)(3.95) = 38.65$

B1 for each correct choice for each item.

total cost = $73.20 + 64.30 + 252.50 + 38.65 = \$428.65$ --- A1

(not lowest cost per item used: M1 for each correct item, no A1)
(insufficient quantities purchased: M0)

(b) The school provides \$200 in funding for the students, and up to 30% of the sales can be used to cover for their expenses, while the remaining goes to the charity.
The students also wish to raise at least \$600 for charity,
Find the minimum they must charge for each breakfast set (to the nearest ten cents).
Justify your answer, showing all necessary workings clearly.
State an assumption you have made in your calculations. [4]

$$\text{min charge to raise \$600} = \frac{(428.65-200)+600}{250} = \$3.3146 \quad \text{--- M1 (ECF their (a))}$$

$$\left(\frac{30}{100}\right)(3.3146)(250) = 248.595 > 431.60-200 \quad \text{--- M1 (ECF their (a))}$$

$\therefore$ min charge = \$3.40 --- A1

Assume no cost incurred for cooking. (or any reasonable) --- B1

OR

$$\text{min charge to raise \$600} = \frac{(428.65-200)+600}{250} = \$3.3146 \quad \text{--- M1 (ECF their (a))}$$

$$\text{min charge to cover expenses} = \left[\frac{(428.65-200)}{30}\times 100\right] \div 250 = \$3.048 \quad \text{--- M1 (ECF their (a))}$$

$\therefore$ min charge = \$3.40 --- A1

Assume no cost incurred for cooking. (or any reasonable) --- B1

Alternative assumptions:
1) No accidents that result in wastage during the event
2) No free samples were given out
3) All 250 sets were sold

Calculator Model :

Class	Full Name	Index Number

bowen SECONDARY
I believe, therefore I am

PRELIMINARY EXAMINATION 2019

O

4048/01

MATHEMATICS
Paper 1

Secondary 4 Express/ 4A1/ 5 Normal Academic
30 August 2019 **2 hours**

Additional Materials: Nil

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any questions it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 80.

DO NOT OPEN THIS PAPER UNTIL YOU ARE TOLD TO DO SO

For Examiner's use
80

Setter: Mr Alvis Mazon Tan

This document consists of **20** printed pages, including this cover page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curve surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector Area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions.

1 Write down the following in ascending order.

$$\frac{25}{38} \qquad \sqrt{0.49} \qquad 0.60^{\frac{2}{3}} \qquad 0.701$$

Answer ……. , ……. , ……. , ……. [1]

---

2 (a) Expand and simplify $(2x-1)(2-3x)-3x(2x-5)$.

Answer (a) ……..…….. [2]

(b) Factorise completely $24ab-4ac+pc-6pb$.

Answer (b) ……..…….. [2]

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3 Calculate $\frac{13.5^3}{6.48-2.57}$, giving your answers corrected to 2 significant figures.

Answer …….. [1]

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4 If the radius of a sphere increases by 10%, find the percentage increase in its volume.

Answer ……..% [2]

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5 On a certain day the exchange rate between the pounds (£) and the Singapore dollars was S$1.684 = £1.

(a) Calculate the amount of pounds that Renee can buy with S$1263.

Answer (a) £= …….. [1]

(b) After four weeks, she realized she has too much pounds and she now wants to change £200 back to Singapore dollars.
If the loss by this transaction is S$6, what is the current exchange rate? Leave your answers corrected to 4 decimal places.

Answer (b) £1 = S$…….. [2]

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6 Integers P and Q, written as products of their prime factors, are

$P = 2^2 \times 3 \times k^2$ and $Q = 2^3 \times 7 \times k$, where k is a prime number.

(a) Express, in terms of k and as a product of its prime factors, the smallest possible integer which is exactly divisible by both P and Q.

Answer (a) ……..…….. [1]

(b) Find the smallest integer, n, such that 27 kn is a multiple of P. Give your answer in terms of k if necessary.

Answer (b) $\boldsymbol{n}$ = …….. [2]

7 Kai Xuan has written down seven numbers.
The mean of these numbers is 8, the median is 7 and the mode is 11.
The smallest number is an even prime number and the largest number is eight times the smallest number.
The second and third numbers are consecutive numbers.

Find the seven numbers.

Answer = ……. , ……. , ……. , ……. , ……. , ……. , ……. [2]

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8 Rearrange the formula $v = \frac{-u^2+5}{u^2-a}$ and make u the subject of the formula.

Answer u= ……..…….. [3]

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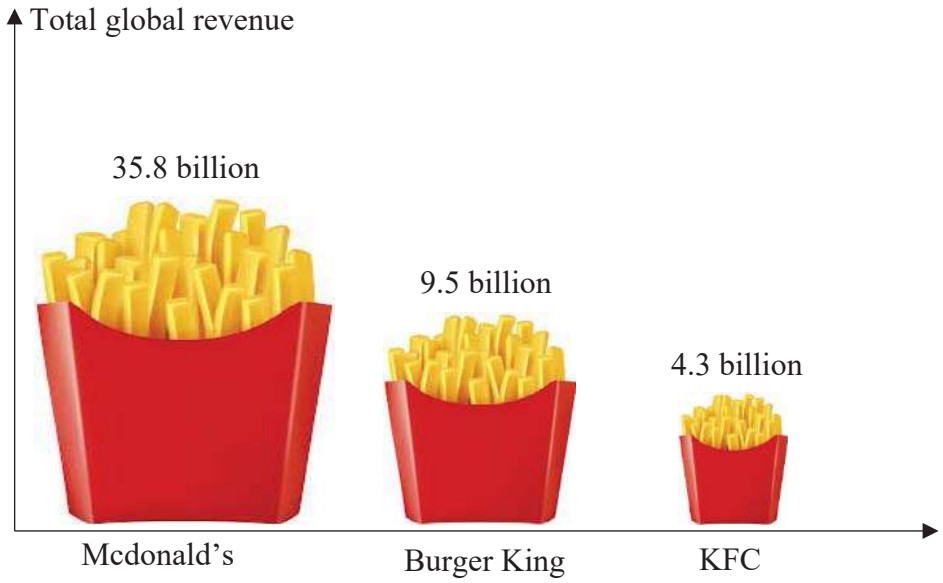
9 The graph shows the total revenue, in billion dollars, of three different fast food chain.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer .....

.....

.....

.....

..... [2]

10 (a) Solve the inequalities $8 + x < 10 + \frac{3}{2}x \leq 15.5 - 2x$.

Answer (a) $x =$ …….. [3]

(b) Hence, write down the largest rational number that satisfies

$$8 + x < 10 + \frac{3}{2}x \leq 15.5 - 2x$$

Answer (b) $x =$ …… [1]

11 The first four terms in a sequence of numbers T_1, T_2, T_3, T_4 are as follow

$$T_1 = \frac{2}{1} - \frac{3}{2}$$

$$T_2 = \frac{3}{2} - \frac{4}{2^2}$$

$$T_3 = \frac{4}{2^2} - \frac{5}{2^3}$$

$$T_4 = \frac{5}{2^3} - \frac{6}{2^4}$$

$$\vdots$$

(a) Write down the n^{th} line and show that it can be expressed as $T_n = \frac{n}{2^n}$.

Answer(a) ……..…… [3]

(b) Hence or otherwise, evaluate the following sum and leave your answer as a fraction.

$$T_1 + T_2 + T_3 + \ldots\ldots\ldots\ldots\ldots\ldots T_{11}$$

Answer (b) ……..…… [2]

12 $A = \{\text{points lying on the line } 2x + y = 8\}$
$B = \{\text{points lying on the line } 3x - 4y = 12\}$
$C = \{\text{points lying on the line } mx - 4y = c\}$

(a) Is (-1,6) $\in A$? Explain your answer clearly.

.....

.....

..... [1]

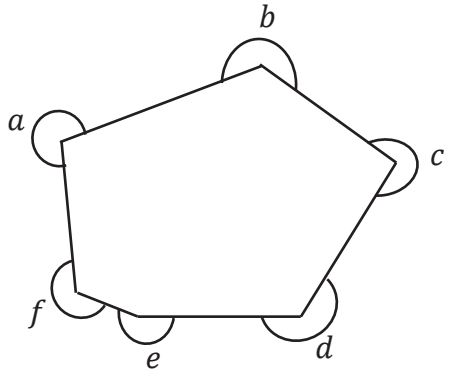
(b) Find the element p such that $p \in (A \cap B)$.

Answer (b) p =……..…….. [2]

(c) Write down a possible value of m and of c such that $B \cap C = \emptyset$.

Answer (c) m =……..…….. [1]
c =……..…….. [1]

13 The diagram below shows an irregular hexagon .
Calculate the value of $a + b + c + d + e + f$.



Answer ……..……..° [2]

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14 Jia Lung invested some money in the savings account for 4 years . The rate of compound interest was fixed at 4% per annum compounded anually.
At the end of 4 years, there was $8436.48 in her account.

How much did Jia Lung invest in the account?

Answer $ …… [3]

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15 Akshay jogs at a speed of 10 km/h.
One evening he jogged around his neighborhood for 1 hour 30 minutes.

(a) Calculate the distance that Akshay covered.

Answer (a) =……..……..km [1]

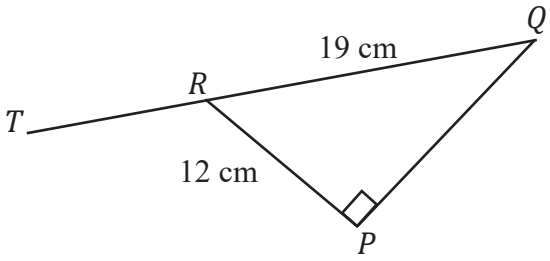
(b) Given that the scale of the neighbourhood is 1: 25000, find in cm, the map distance that he covered.

Answer (b) = …….. cm [2]

(c) A reservoir located in his neighbourhood occupies a total area of 1.70 cm^2 on the map. What is the actual area, in m^2, of the reservoir?

Answer (c)……..m^2 [2]

16 PQR is a right-angled triangle. QRT is a straight line.
$PR = 12$ cm and $QR = 19$ cm.



Find the values of the following, giving your answer to two decimal places where necessary.

(a) $\tan \angle PQR$

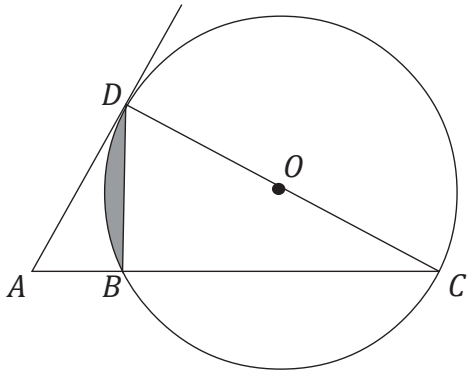
Answer (a) …….. [2]

(b) $\cos \angle TRP$

Answer (b)…….. [2]

17 In the diagram, *O* is the centre of the circle *BCD* with radius 20 cm and *CD* is the diameter of the circle. The ratio of the length AB to the length AD is 0.5.

A is a point on *BC* produced such that *AD* is a tangent to the circle at *D*.



Calculate the area of the shaded region.

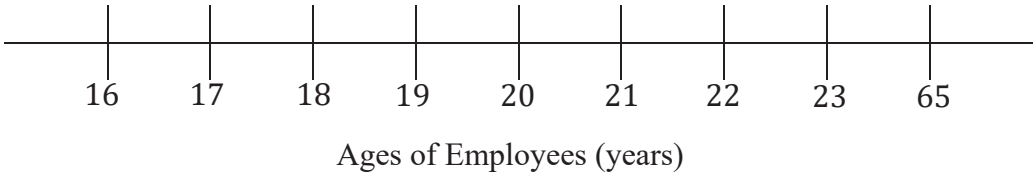
Answer (b) ……cm^2 [5]

18 The table below shows the ages of 16 employees who work part-time at a cafe.

20	21	16	23
22	17	19	65
23	22	17	22
23	19	19	18

(a) Complete the dot diagram to show the distribution of the ages of the employees.

Answer (a) [1]



Ages of Employees (years)

(b) Find the median of the distribution of ages.

Answer (b) Median = …… years [1]

(c) Calculate the mean age of the employees.

Answer (c) Mean = …… years [1]

(d) Pranav made the following statement:
"The mean is the most accurate way to determine the average age of the employees."
Validate if Pranav statement is true.

.....

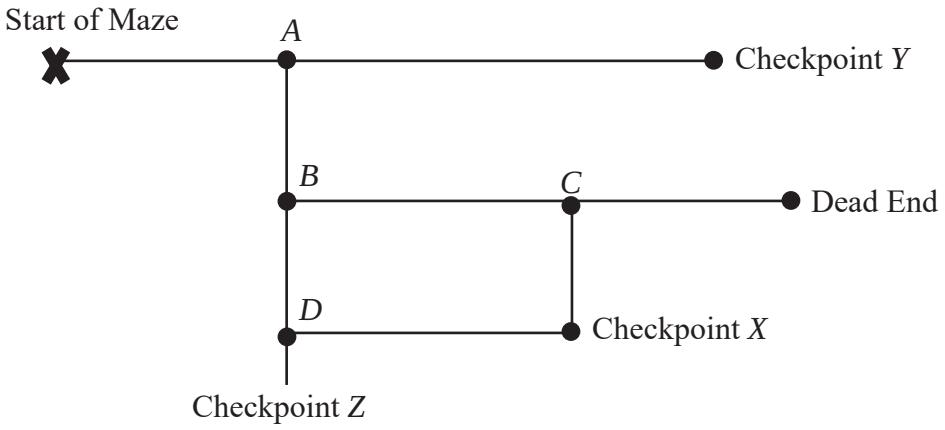
.....

.....

..... [2]

19 Allyson participates in a game show. In order to win a prize, she has to navigate through a maze.
The prize is located at checkpoint *X*. There are no prizes awarded at checkpoint *Y* and *Z*.
The diagram below shows four junctions *A, B, C* and *D* in the maze.
Once Allyson runs pass a junction, she is not able to make a turnaround.

The probability that Allyson goes straight, without changing direction, at every junction is $\frac{3}{7}$.



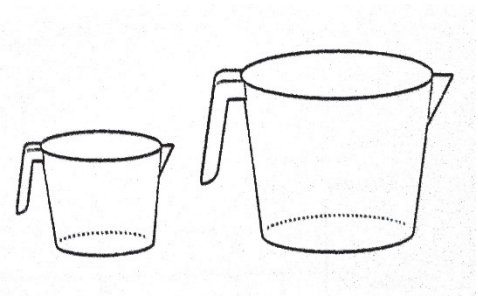
(a) Find the probability that Allyson hits the dead end.

Answer (a) ..... [2]

(b) Find the probability that Allyson wins a prize.

Answer (b) ..... [3]

20 Two similar jugs have base area of 45 cm^2 and 125 cm^2.



(a) Find the ratio of the height of the smaller jug to the ratio of the height of the larger jug.

Answer (a) ..... **:** ..... [1]

(b) The curved surface area of the smaller jug is 63 cm^2.
Find the curved surface area of the larger jug.

Answer (b) ..... cm^2 [2]

(c) The capacity of the larger jug is 2.5 litres. Find the capacity of the smaller jug.
Give your answer in cubic centimetres.

Answer (c) ..... cm^3 [2]

21 *OPQR* is a parallelogram such that $\overrightarrow{PQ} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ and P is the point (3,2).

(a) Express $\overrightarrow{RP}$ as a column vector.

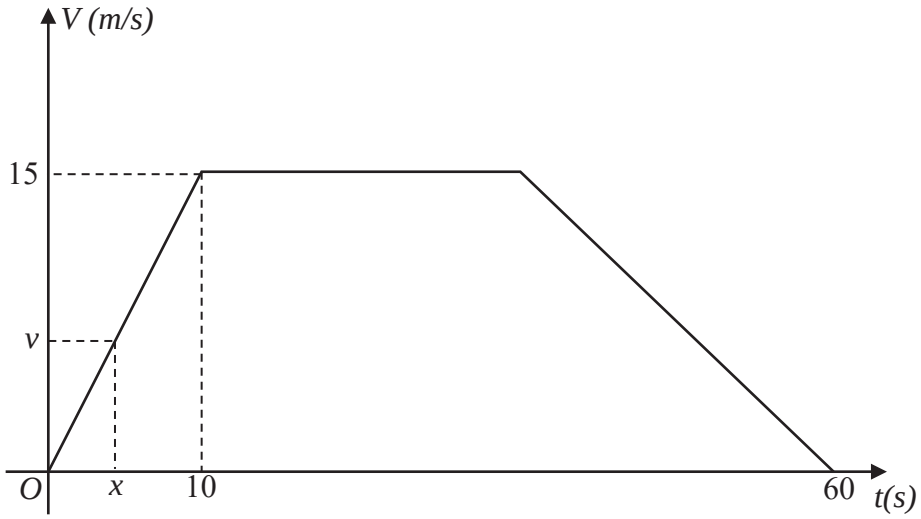
Answer (a) = ... [2]

(b) The point J lies on $\overrightarrow{RP}$ produced such that $\overrightarrow{PJ} = m\overrightarrow{RP}$

Show that $\overrightarrow{OJ} = \begin{pmatrix} 3+m \\ 2-2m \end{pmatrix}$

[2]

22 The diagram shows the speed time graph of a car. The car starts from rest and accelerates uniformly to a speed of 15 m/s in 10 seconds.
The car then travels at a constant speed for some time before it decelerates uniformly at 0.75 m/s^2 until it comes to rest.
The whole journey takes one minute.



(a) Given that the speed of the car after x seconds is v m/s, express v in terms of x.

Answer v =..... [1]

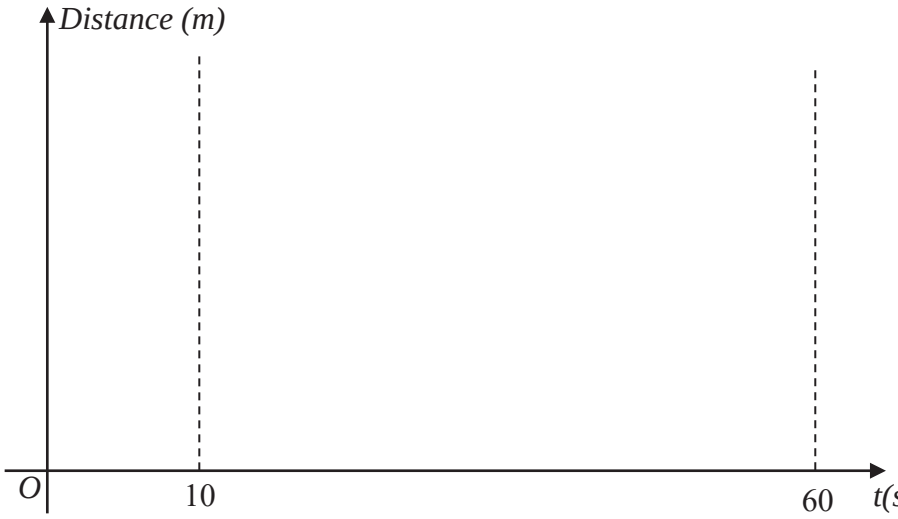
(b) For how long does the car travel at the maximum speed?

Answer ..... [2]

(c) Calculate the total distance travelled by the car during this 1 minute journey.

Answer ..... m [2]

(d) Hence, sketch the distance time graph for the whole journey, indicating all relevant values in your sketch.



[2]

~ End of Paper ~

Calculator Model :

Class	Full Name	Index Number

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PRELIMINARY EXAMINATION 2019

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4048/02

MATHEMATICS
Paper 2

Secondary 4 Express / 5 Normal Aacdemic / 4A1
3 September 2019 **2 hours 30 minutes**

Candidates answer on the Question Paper.
Additional Materials: Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
You may use a pencil for any diagrams or graphs.
Do not use staples, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any questions it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 100.

DO NOT OPEN THIS PAPER UNTIL YOU ARE TOLD TO DO SO

For Examiner's use
100

Setter: Mrs Jane Cheng

This document consists of **21** printed pages, including this cover page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curve surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector Area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Solve the inequality $\frac{x+2}{3} \geq \frac{4-x}{7}$. [2]

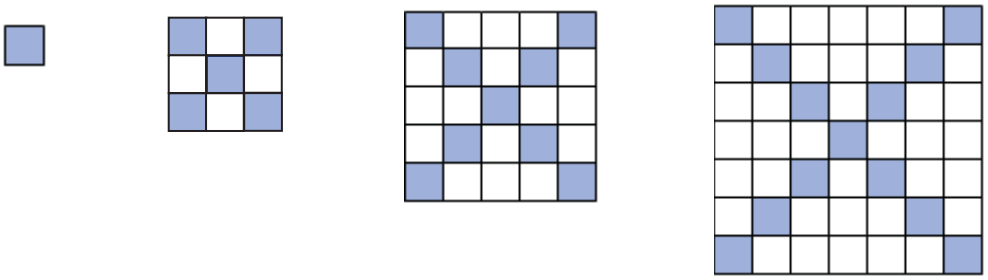
(b) Express as a single fraction in its simplest form $\frac{2x}{(3x-5)^2} + \frac{x}{5-3x}$. [2]

(c) Simplify $\left(\frac{27a^6}{b^{12}}\right)^{-\frac{1}{3}}$. [2]

(d) Simplify $\frac{24p^3q^2}{5r^3} \div \frac{8p^4r}{15q^3}$. [2]

(e) Solve the equation $\frac{15}{x+2} = 2x + 3$ [3]

2 A series of diagrams of shaded and unshaded small squares is shown below.
The shaded squares are those which lie on the diagonals of the diagram.



(a) Copy and complete the table below. [2]

Diagram, ***n***	1	2	3	4	5
Number of shaded squares, ***S***	1	5	9		
Number of unshaded squares, ***U***	0	4	16		
Total number of squares, ***T***	1	9	25		

(b) By observing the number patterns, without drawing further diagrams,
(i) write down the total number of squares in diagram 12, [1]

(ii) find an expression, in terms of n, for the total number of squares, ***T***. [1]

(c) **(i)** Find an expression, in terms of n, for the number of shaded squares, ***S***. [1]

(ii) Write down the number of the diagram that has 41 shaded squares. [1]

(d) Hence, or otherwise, find an expression, in terms of n, for the number of unshaded squares, ***U***. [2]

3 P is the point $(-5, 12)$ and Q is the point $(5, -4)$

(a) Find the length of PQ. [2]

(b) Find the equation of the line PQ. [2]

(c) The equation of the line l_1 is $8x + 5y + 10 = 0$.

(i) Show how you can decide whether the line l_1 does or does not intersect the line PQ? [2]

(ii) The equation of line l_2 is $3y = 4x - 39$.
Find the coordinates of the point of intersection of the line l_1 and the line l_2. [3]

3 Mrs Tan is a Korean Language teacher.
She conducts classes for basic and advaced students on weekdays and weekends.
Each student has a 15-week block of lessons with one lesson per week.
The matrix K shows the number of students she teaches each week in one 15-week block.

$$\mathbf{K} = \begin{matrix} \text{Basic} & \text{Advanced} \\ \begin{pmatrix} 12 & 3 \\ 5 & 8 \end{pmatrix} & \begin{matrix} \text{Weekday} \\ \text{Weekend} \end{matrix} \end{matrix}$$

(a) Evaluate the matrix $\mathbf{P} = 15\mathbf{K}$. [1]

(b) Mrs Tan charges \$20 for each basic lesson and \$32 for each advanced lesson.
Represent the lesson charges in a 2×1 matrix **L**. [1]

(c) Evaluate the matrix $\mathbf{T} = \mathbf{PL}$. [2]

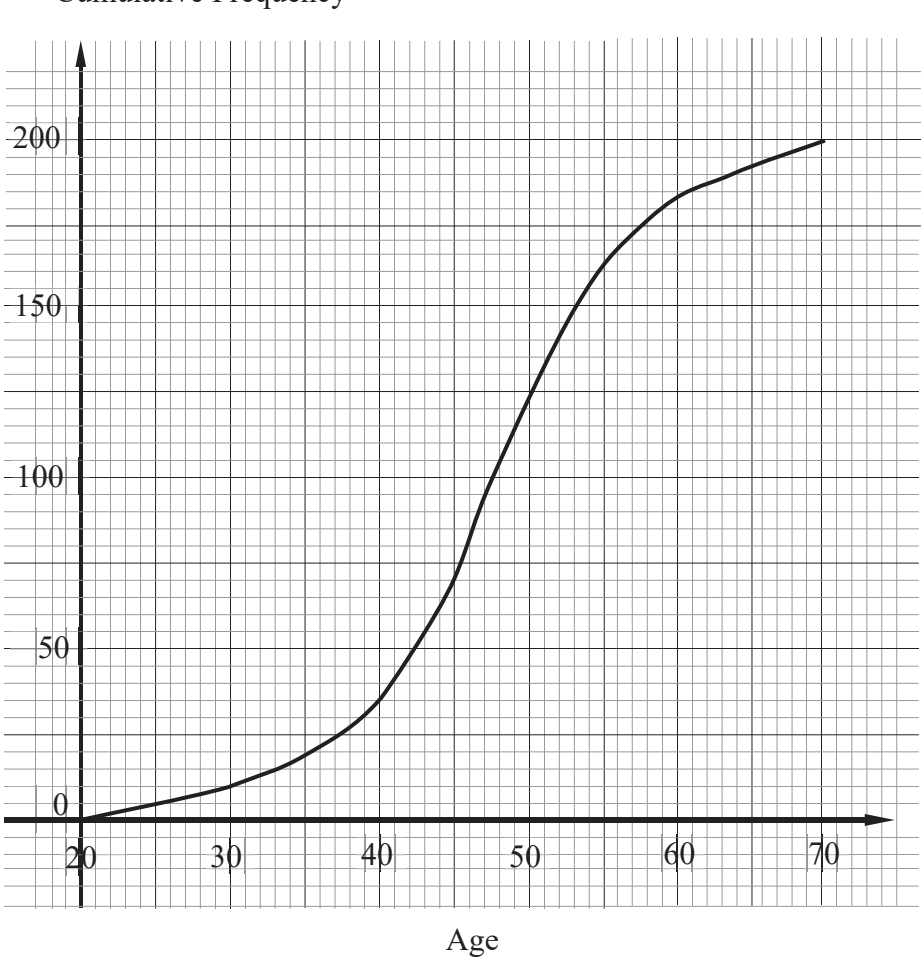
(d) State what the elements of **T** represent. [1]

(e) Mrs Tan wants to attract more students, so in the next 15-week block she reduces her prices by 10%.
For this block of lessons, on weekdays she has 15 basic students and 5 advanced students.
On weekends she has 7 basic students and 6 advanced students.

Calculate the total amount of money she earns for this 15-week block of lessons. [3]

5 The cumulative frequency graph shows the distribution of the age groups of the *Fitness First* club.

(a)



(i) Complete the grouped frequency table for the ages of the members. [1]

Age (x)	$20 \leq x < 30$	$30 \leq x < 40$	$40 \leq x < 50$	$50 \leq x < 60$	$60 \leq x < 70$
Frequency	10				15

(ii) Calculate the mean age of each member. [2]

(iii) Calculate the standard deviation. [2]

(iv) Find the percentage of members whose age is 30 years old and above but less than 60 years old. [1]

(v) A magazine article stated that citizens aged 50 and above are less active than those aged below 40.
Comment on whether the data from the *Fitness First* club supports this claim. [2]

(b) The table below gives information about the ages of the members in the *Any Time Fitness* club.

	Members aged under 50	Members aged 50 or over
Male	50	34
Female	36	30

(i) One of these members is selected at random.
Find, as a fraction in its lowest terms, the probability that he or she is under 50. [1]

(ii) Two of the members are selected at random.
Find the probability that
(a) both members are female, [2]

(b) they are both aged 50 or over, but only one is a male member. [2]

6 A litre of *95-octane* unleaded petrol cost *$x* in *January 2019*.

(a) Mr Ang paid $ 85.50 for his petrol. Write down in terms of *x*, the amount of petrol bought. [1]

Mr Bala paid $100 for his *98-octane* unleaded petrol which cost 25 cents more per *litre*.

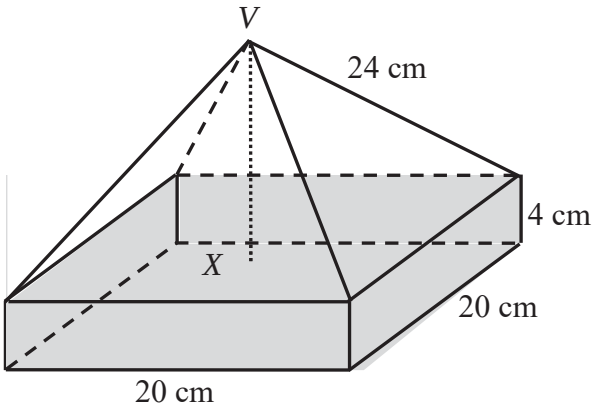
(b) Write down in terms of *x*, the amount of petrol bought by Mr Bala. [1]

(c) If Mr Ang received 2 *litres* less petrol than Mr Bala, write down an equation to represent this information and show that it can reduce to $16x^2 - 112x + 171 = 0$. [3]

(d) Solve the equation $16x^2 - 112x + 171 = 0$. [2]

(e) The price of the 98-octane unleaded petrol in *January 2019* was a reduction of 7% on the price in *December 2018*. [2]
Find the price of the 98-octane unleaded petrol in December 2018 if it cost less than \$3 for a litre of 95-octane unleaded petrol in January 2019.

7 The diagram shows a container consisting of a square bottom with rectangular sides, each 20 cm by 4 cm, and a regular pyramid on top with perpendicular height given by VX. Water is poured into the container till the brim of the cuboid.

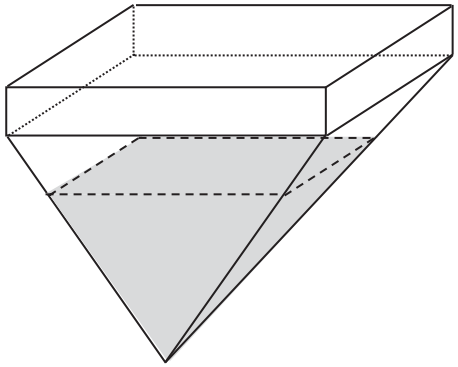


(a) Find the height VX of the pyramid. [2]

(b) Calculate the total surface area of the container. [2]

(c) Find the volume of water in the cuboid. [1]

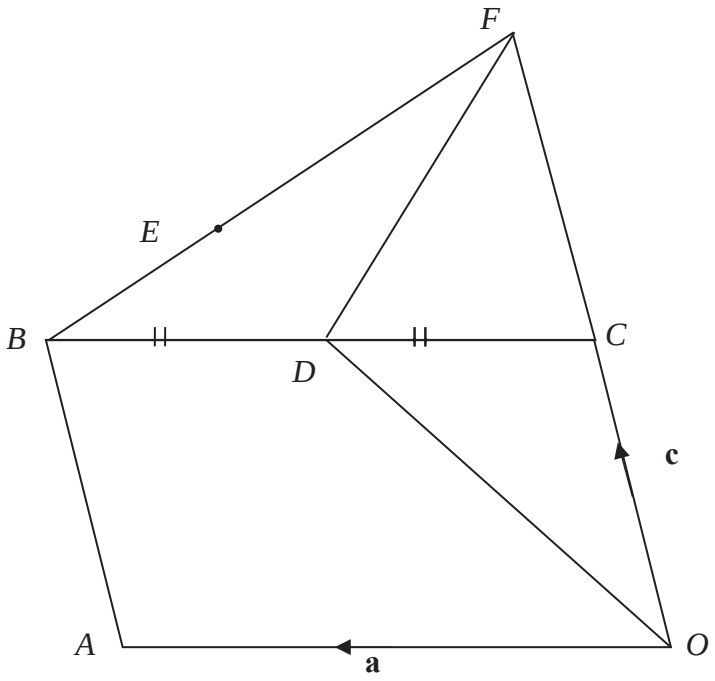
The container is now inverted as shown in the diagram below.



(d) Calculate the depth of the water in the pyramid when inverted. [3]

(e) Another smaller container, which is geometrically similar, has a square base of 225 cm^2. Both containers are made of the same material. [2]
Find the mass of the smaller container in grams, given that the mass of the empty larger container is 1.28 kg.

8 In the diagram, $OABC$ is a parallelogram and D is the midpoint of BC. BE and OC produced intersect at the point F. $BE : BF = 1 : 3$ and $OC : OF = 1 : 2$.
Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.



(a) Express and simply the following vectors in terms of **a** and **c**.

(i) $\overrightarrow{AC}$ [1]

(ii) $\overrightarrow{BF}$ [1]

(iii) $\overrightarrow{OD}$ [1]

(iv) $\overrightarrow{OE}$ [2]

(b) State two facts about the vectors $\overrightarrow{OD}$ and $\overrightarrow{OE}$ from the results in **(a)**. [2]

(c) Find the ratio of the areas of

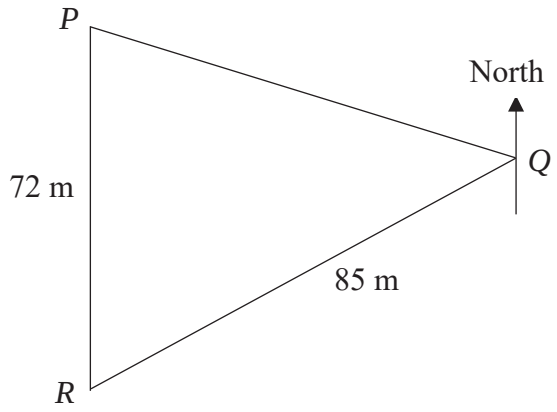
(i) ΔODF and ΔOEF, [1]

(ii) ΔOCD and $OABC$, [1]

(iii) ΔOCD and $OABF$. [2]

9 Two school teams, *Novotel* and *Temasek*, are participating in an Amazing Race in Bishan Park. The diagram shows the paths in the park.
The teams assemble at *P* before heading to *Q* to start the race.
P is due north of *R*.
The bearing of R from Q is 241°.
The distance *PR* is 72 metres and the distance *RQ* is 85 metres.

(a) Find the distance *PQ*. [3]



(b) The final station of the race is at R, each team is required to find a ***clue*** that is hidden at point S before completing the race at R. [3]
The bearing of S from R is 099° and QS is 54 metres.
Given that there are **two** possible locations for S, find the two possible values of angle RSQ.

(c) Both teams manage to find the ***clue*** at the same time and team *Novotel* is closer to *R* than team *Temasak*. [5]

Team *Novotel* claims that they are the winner.

Given that the speed of team *Novotel* is 30% less than the speed of team *Temask* when they travel from *S* to *R*.

Do you agree with team *Novotel* that they will win the race?

Justify your answer with clear working in your calculations.

10 ***Answer the whole of this question on a sheet of graph paper.***

The table below gives some values of x and the corresponding values of y for $y = x(1 + x)(5 – x)$.

x	– 2	– 1	– 0.5	1	2	3	4	5
y	14	0	– 1.375	8	18	p	20	0

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $-2 \leq x \leq 5$.
Using a scale of 2 cm to represent 5 units, draw a vertical y-axis for $-5 \leq y \leq 25$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) By drawing a tangent, find the gradient of the curve where $x = 4$. [2]

(d) **(i)** On the same axes, draw the line $2x + y = 12$ for $-2 \leq x \leq 5$. [1]

(ii) Write down the x-coordinates of the points where this line intersects the curve. [2]

(iii) The x-coordinates of the points where the two graphs intersect are solutions of the equation $x^3 + Ax^2 + Bx + 12 = 0$. Find the value of A and the value of B. [2]

~~~ End of Paper ~~~

Calculator Model :

Class	Full Name	Index Number

bowen
SECONDARY
I believe, therefore I am

PRELIMINARY EXAMINATION 2019

O

4048/01

MATHEMATICS
Paper 1

Secondary 4 Express/ 4A1/ 5 Normal Academic
30 August 2019 **2 hours**

Additional Materials: Nil

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any questions it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 80.

DO NOT OPEN THIS PAPER UNTIL YOU ARE TOLD TO DO SO

For Examiner's use
80

Setter: Mr Alvis Mazon Tan

This document consists of **20** printed pages, including this cover page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curve surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector Area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions.

1 Write down the following in ascending order.

$\frac{25}{38}$ $\sqrt{0.49}$ $0.60^{\frac{2}{3}}$ 0.701

Answer $\frac{25}{38}$, $\sqrt{0.49}$, 0.701, $0.60^{\frac{2}{3}}$ [1]

---

2 (a) Expand and simplify $(2x-1)(2-3x)-3x(2x-5)$.

$(2x-1)(2-3x)-3x(2x-5)$

$= 4x - 6x^2 - 2 + 3x - 6x^2 + 15x$ [M1]

$= -12x^2 + 22x - 2$

$= -2(6x^2 - 11x + 1)$ [A1]

Answer (a) $-2(6x^2-11x+1)$ [2]

(b) Factorise completely $24ab - 4ac + pc - 6pb$.

$24ab - 4ac + pc - 6pb$

$= 4a(6b - c) + pc - 6pb$ [M1]

$= 4a(6b - c) - p(6b - c)$

$= (4a - p)(6b - c)$ [A1]

Answer (b) ..... [2]

---

3 Calculate $\frac{13.5^3}{6.48-2.57}$, giving your answers corrected to 2 significant figures.

Answer 630 [1]

---

4 If the radius of a sphere increases by 10%, find the percentage increase in its volume.

$$\frac{\frac{4}{3}\pi r^3(1.1)^3 - \frac{4}{3}\pi r^3}{\frac{4}{3}\pi r^3} \times 100\% \quad [M1]$$

$= 33.1\%$ [A1]

Answer 33.1 ..... % [2]

---

5 On a certain day the exchange rate between the pounds (£) and the Singapore dollars was S$1.684 = £1.

(a) Calculate the amount of pounds that Renee can buy with S$1263.

$\frac{\$1263}{\$1.684} = £750$ [M1]

Answer (a) £= 750 ..... [1]

(b) After four weeks, she realized she has too much pounds and she now wants to change £200 back to Singapore dollars.
If the loss by this transaction is S$6, what is the current exchange rate? Leave your answers corrected to 4 decimal places.

With reference to the original exchange rate

£200 : S$1.684 × 200

= S$336.80

S$336.80 − S$6 = S$330.8

New exchange rate:

$\frac{S\$330.8}{£200} = S\$1.654$ [A1]

[M1]

Answer (b) £1 = S$ 1.654 ..... [2]

---

6 Integers P and Q, written as products of their prime factors, are

$P = 2^2 \times 3 \times k^2$ and $Q = 2^3 \times 7 \times k$, where k is a prime number.

(a) Express, in terms of k and as a product of its prime factors, the smallest possible integer which is exactly divisible by both P and Q.

L.C.M $= 2^3 \times 3 \times 7 \times k^2$ [B1]

Answer (a) $2^3 \times 3 \times 7 \times k^2$ [1]

(b) Find the smallest integer, n, such that 27 kn is a multiple of P. Give your answer in terms of k if necessary.

$P = 2^2 \times 3 \times k^2$

$27kn = 3^3 kn$

$= 3^2 (3k)(2^2 \times k)$ [M1]

$= 4k$ [A1]

Answer (b) $n =$ $4k$ [2]

7 Kai Xuan has written down seven numbers.
The mean of these numbers is 8, the median is 7 and the mode is 11.
The smallest number is an even prime number and the largest number is eight times the smallest number.
The second and third numbers are consecutive numbers.

Find the seven numbers.

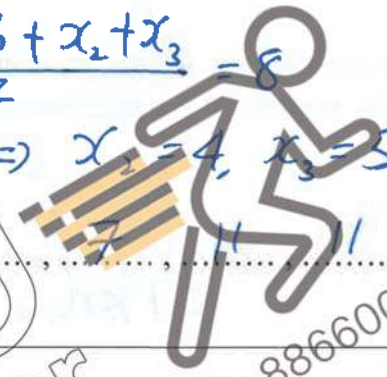
[M1] $x_1, x_2, x_3, x_4, x_5, x_6, x_7$

$x_1 = 2$, $x_4 = 7$, $x_5 = 11$, $x_6 = 11$, $x_7 = 16$

$$\frac{2+7+11+11+16+x_2+x_3}{7} = 8$$

$\therefore x_2 + x_3 = 9 \Rightarrow x_2 = 4, x_3 = 5.$

[A1] *Answer* = 2, 4, 5, 7, 11, 11, 16 [2]



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8 Rearrange the formula $v = \frac{-u^2+5}{u^2-a}$ and make u the subject of the formula.

$v(u^2 - a) = -u^2 + 5$

$vu^2 - av = -u^2 + 5$ [M1]

$vu^2 + u^2 = av + 5$

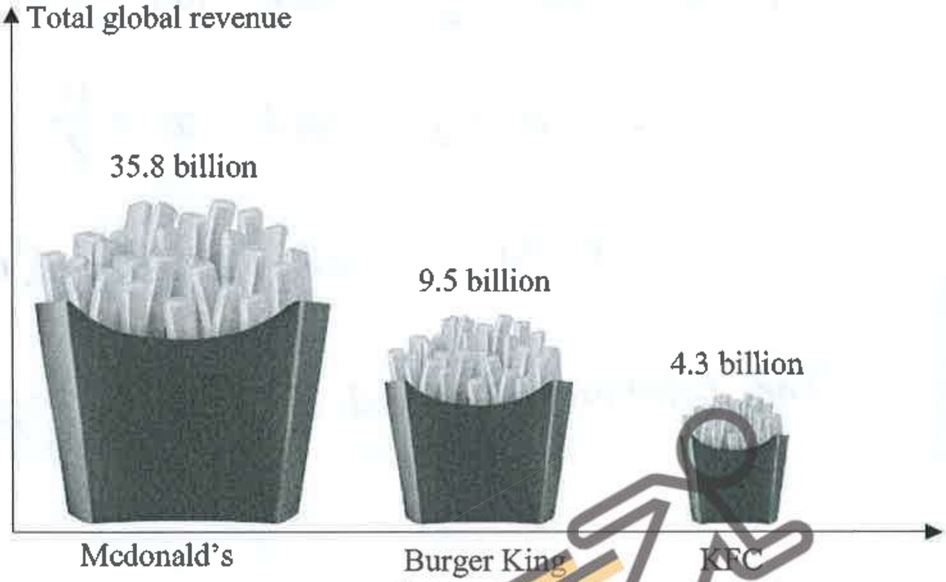
$u^2(v+1) = av + 5$ [M1]

$u = \pm\sqrt{\frac{av+5}{(v+1)}}$ [A1]

Answer u = $\pm\sqrt{\frac{av+5}{(v+1)}}$ [3]

---

9 The graph shows the total revenue, in billion dollars, of three different fast food chain.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer ①The size of the fries and hence the area was used to compare the global revenue. [B1]

② Total area for Mcdonald's appears to be twice that of Burger King but in fact the revenue is more than 3 times of the latter [B1] [2]

10 (a) Solve the inequalities $8 + x < 10 + \frac{3}{2}x \leq 15.5 - 2x$.

$8 + x < 10 + \frac{3}{2}x$ and $10 + \frac{3}{2}x \leq 15.5 - 2x$ [M1]

$-\frac{1}{2}x < 2$ and $x \leq \frac{11}{7}$

$\therefore x > -4$ and $x \leq \frac{11}{7}$ [M1]

The solution is $-4 < x \leq \frac{11}{7}$ [A1]

Answer (a) $x =$ $-4 < x \leq \frac{11}{7}$ [3]

(b) Hence, write down the largest rational number that satisfies

$$8 + x < 10 + \frac{3}{2}x \leq 15.5 - 2x$$

Answer (b) $x =$ $\frac{11}{7}$ [1]

11 The first four terms in a sequence of numbers T_1, T_2, T_3, T_4 are as follow

$$T_1 = \frac{2}{1} - \frac{3}{2}$$

$$T_2 = \frac{3}{2} - \frac{4}{2^2}$$

$$T_3 = \frac{4}{2^2} - \frac{5}{2^3}$$

$$T_4 = \frac{5}{2^3} - \frac{6}{2^4}$$

$$\vdots$$

(a) Write down the n^{th} line and show that it can be expressed as $T_n = \frac{n}{2^n}$.

$T_n = \frac{n+1}{2^{n-1}} - \frac{n+2}{2^n}$ [M1]

$= \frac{n+1}{2^n(2^1)} - \frac{n+2}{2^n}$

$= \frac{2n+2-n-2}{2^n}$ [A1]

$= \frac{n}{2^n}$ #

Answer(a) ..... [3]

(b) Hence or otherwise, evaluate the following sum and leave your answer as a fraction.

$$T_1 + T_2 + T_3 + \ldots\ldots\ldots\ldots\ldots\ldots T_{11}$$

[M1] $T_1 + T_2 + \cdots T_{11} = \left(\frac{2}{1} - \frac{3}{2}\right) + \left(\frac{3}{2} - \frac{4}{2^2}\right) + \left(\frac{4}{2^2} - \frac{5}{2^3}\right)$

$\vdots$

$\left(\frac{11}{2^9} - \frac{12}{2^{10}}\right) + \left(\frac{12}{2^{10}} - \frac{13}{2^{11}}\right)$

$= \frac{2}{1} - \frac{13}{2^{11}}$

$= \frac{4083}{2048}$

[A1]

Answer (b) $\frac{4083}{2048}$ ..... [2]

12 A = {points lying on the line $2x + y = 8$}
B = {points lying on the line $3x - 4y = 12$}
C = {points lying on the line $mx - 4y = c$}

(a) Is $(-1,6) \in A$? Explain your answer clearly.

Sub $(-1,6)$ into "A"

L.H.S $= 2(-1)+6 = 4$

R.H.S $= 8$, Since L.H.S $\neq$ R.H.S, $(-1,6) \notin A$ // [B1] [1]

(b) Find the element p such that $p \in (A \cap B)$.

$2x + y = 8$ — ①

$3x - 4y = 12$ — ②

Sub ① into ②

$3x - 4(8-2x) = 12$ [M1]

$11x = 44$

$x = 4$

$y = 0$

$p = (4, 0)$ [A1]

Answer (b) $p =$ (4, 0) [2]

(c) Write down a possible value of m and of c such that $B \cap C = \emptyset$.

$B \cap C = \phi \Rightarrow$ No Solution.

The line $y = \frac{3}{4}x - 3$ and $mx - 4y = c$ must be parallel but cannot have the same y-intercept.

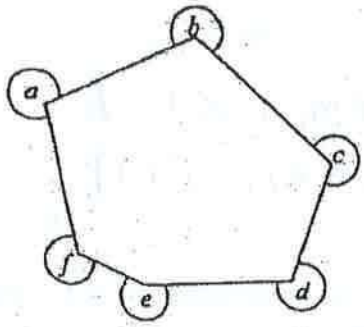
$\therefore m = 3$ [B1]

$c \neq 12$ [B1]

Answer (c) $m =$ 3 [1]

$c =$ Any Value except 12. [1]

13 The diagram below shows an irregular hexagon .
Calculate the value of $a + b + c + d + e + f$.



Sum of interior ∠ in a hexagon: $(6-2) \times 180°$
$= 720°$

$360° \times 6 = 2160°$

Sum of reflex ∠'s : $a+b+c+d+e+f = 2160° - 720°$ [M1]
$= 1440°$ [A1]

Answer 1440 ° [2]

---

14 Jia Lung invested some money in the savings account for 4 years . The rate of compound interest was fixed at 4% per annum compounded anually.
At the end of 4 years, there was $8436.48 in her account.

How much did Jia Lung invest in the account?

$A = P\left(1 + \frac{r}{100}\right)^n$

$8436.48 = P\left(1 + \frac{4}{100}\right)^4$ [M1]

$8436.48 = P(1.04)^4$

$P = \frac{8436.48}{(1.04)^4} = \$7211.54$ [A1]

Answer $ 7211.54 [3]

---

15 Akshay jogs at a speed of 10 km/h.
One evening he jogged around his neighborhood for 1 hour 30 minutes.

(a) Calculate the distance that Akshay covered.

10km/h × 1.5h
= 15 km [B1]

Answer (a) = 15 km [1]

(b) Given that the scale of the neighbourhood is 1: 25000, find in cm, the map distance that he covered.

1 cm : 25000 cm
1 cm : 0.25 km
15 km ⇒ 15 km / 0.25 km = 60 cm [A1]
[M1]

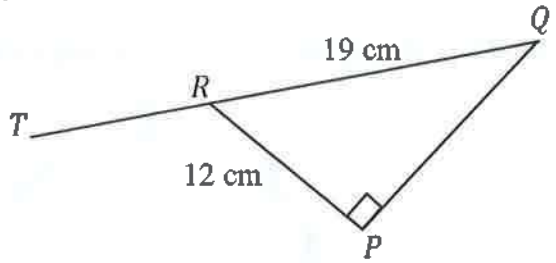
Answer (b) = 60 cm [2]

(c) A reservoir located in his neighbourhood occupies a total area of 1.70 cm^2 on the map. What is the actual area, in m^2, of the reservoir?

$1 cm^2$: 625 000 000 cm^2 [M1]
$1 cm^2$: 6250 0 m^2
$1.70 cm^2$: 1.70 × 6250 m^2
= 106250 m^2 [A1]

Answer (c) 10625 m^2 [2]

16 PQR is a right-angled triangle. QRT is a straight line.
$PR = 12$ cm and $QR = 19$ cm.



Find the values of the following, giving your answer to two decimal places where necessary.

(a) $\tan \angle PQR$

$QP = \sqrt{19^2 - 12^2} = \sqrt{217}$

$\tan \angle PQR = \frac{12}{\sqrt{217}}$ [M1]

$= 0.81$ [A1]

Answer (a) 0.81 [2]

(b) $\cos \angle TRP$

$\cos \angle TRP = -\cos(180° - \angle TRP)$

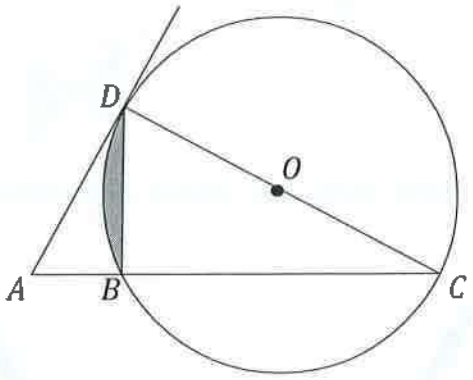
$= -\cos(\angle QRP)$ [M1]

$= -\frac{12}{19}$ [A1]

Answer (b) $-\frac{12}{19}$ [2]

17 In the diagram, O is the centre of the circle BCD with radius 20 cm and CD is the diameter of the circle. The ratio of the length AB to the length AD is 0.5.

A is a point on BC produced such that AD is a tangent to the circle at D.



Calculate the area of the shaded region.

$\angle ADB = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ rad

$\angle BDC = \frac{\pi}{2}$ rad $- \frac{\pi}{6}$ rad $= \frac{\pi}{3}$ rad (tangent at a point) [M1]

$\angle BCD = \pi$ rad $- \frac{\pi}{2}$ rad $- \frac{\pi}{3}$ rad $= \frac{\pi}{6}$ rad [A1]

$\angle BOD = 2 \times \angle BCD$ (At $\odot$ = 2 × ∠ at circumference)

$\angle BOD = 2 \times \frac{\pi}{6}$ rad $= \frac{\pi}{3}$ rad [M1]

Area of shaded region = Area of sector OBD − Area of $\triangle OBD$

$= \frac{1}{2}(20)^2\left(\frac{\pi}{3}\right) - \frac{1}{2}(20)^2 \sin\left(\frac{\pi}{3}\right)$ [M1]

$= 36.2\ cm^2$

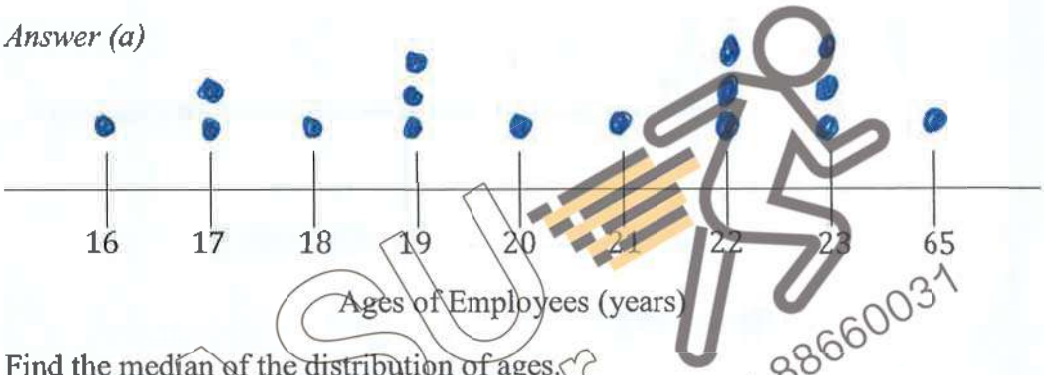
Answer (b) .....cm^2 [5]

18 The table below shows the ages of 16 employees who work part-time at a cafe.

20	21	16	23
22	17	19	65
23	22	17	22
23	19	19	18

(a) Complete the dot diagram to show the distribution of the ages of the employees.

Answer (a) [1]



(b) Find the median of the distribution of ages.

$\frac{20+21}{2} = 20.5$ [B1]

Answer (b) Median = 20.5 years [1]

(c) Calculate the mean age of the employees.

$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{366}{16} = 22.9$ [B1]

Answer (c) Mean = 22.9 years [1]

(d) Pranav made the following statement:

"The mean is the most accurate way to determine the average age of the employees" Validate if Pranav statement is true.

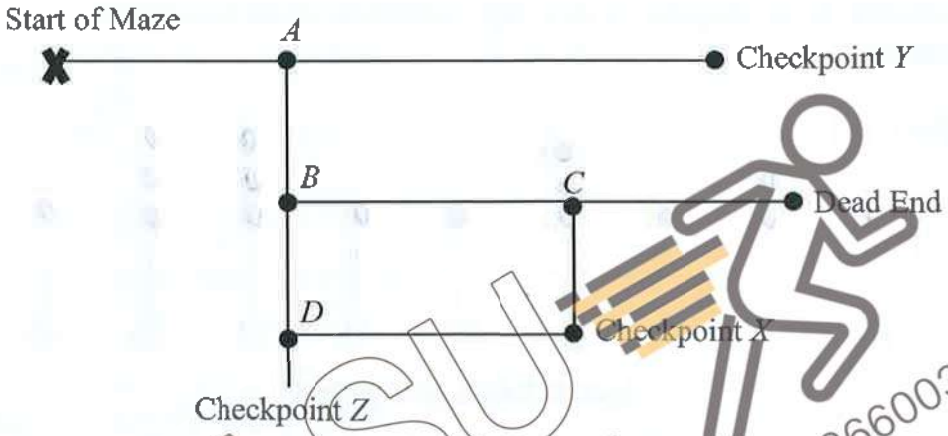
① Not Valid [B1]

② Mean is affected by extremal points in the data set. [B1]

[2]

19 Allyson participates in a game show. In order to win a prize, she has to navigate through a maze.
The prize is located at checkpoint X. There are no prizes awarded at checkpoint Y and Z.
The diagram below shows four junctions A, B, C and D in the maze.
Once Allyson runs pass a junction, she is not able to make a turnaround.

The probability that Allyson goes straight, without changing direction, at every junction is $\frac{3}{7}$.



(a) Find the probability that Allyson hits the dead end.

P(Allyson hits the dead end)
$= \left(\frac{4}{7} \times \frac{4}{7} \times \frac{3}{7}\right)$ [M1]
$= \frac{48}{343}$ [A1]

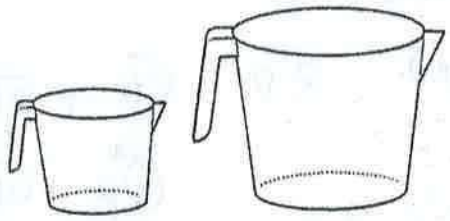
Answer (a) $\frac{148}{343}$ [2]

(b) Find the probability that Allyson wins a prize.

P(Allyson wins a prize)
$= \left(\frac{4}{7} \times \frac{4}{7} \times \frac{4}{7}\right) + \left(\frac{4}{7} \times \frac{3}{7} \times \frac{4}{7}\right)$ [M2]
$= \frac{64}{343} + \frac{48}{343}$
$= \frac{112}{343}$ [A1]

Answer (b) $\frac{112}{343}$ [3]

20 Two similar jugs have base area of 45 cm^2 and 125 cm^2.



(a) Find the ratio of the height of the smaller jug to the ratio of the height of the larger jug.

$\left(\frac{h_{small}}{h_{large}}\right)^2 = \frac{45}{125} \Rightarrow \frac{h_{small}}{h_{large}} = \sqrt{\frac{45}{125}} = \frac{3}{5}$ [B1]

Answer (a) 3 : 5 [1]

(b) The curved surface area ~~of the bottom~~ of the smaller jug is 63 cm^2.
Find the surface area ~~of the bottom~~ of the larger jug.

$\left(\frac{3}{5}\right)^2 = \frac{63}{\text{Curved S.A of larger jug}}$ [M1]

Curved surface area of larger jug $= \frac{63 \times 25}{9} = 175\,cm^2$ [A1]

Answer (b) 175 cm^2 [2]

(c) The capacity of the larger jug is 2.5 litres. Find the capacity of the smaller jug.
Give your answer in cubic centimetres.

$\left(\frac{3}{5}\right)^3 = \frac{V_{small}}{2.5\,l}$ [M1]

$V_{small} = \frac{27}{125} \times 2.5\,l$

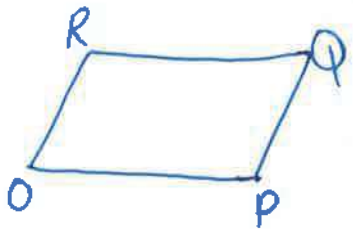
$= 0.54\,l$

$= 540\,cm^3$ [A1]

Answer (c) 540 cm^3 [2]

21 $OPQR$ is a parallelogram such that $\overrightarrow{PQ} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ and P is the point (3,2).

(a) Express $\overrightarrow{RP}$ as a column vector.



$\overrightarrow{RP} = \overrightarrow{RO} + \overrightarrow{OP}$

$= \overrightarrow{QP} + \overrightarrow{OP}$

$= -\overrightarrow{PQ} + \overrightarrow{OP}$ [M1]

$= \begin{pmatrix} -2 \\ -4 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ [A1]

Answer (a) = $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ [2]

(b) The point J lies on $\overrightarrow{RP}$ produced such that $\overrightarrow{PJ} = m\overrightarrow{RP}$

Show that $\overrightarrow{OJ} = \begin{pmatrix} 3+m \\ 2-2m \end{pmatrix}$

$\overrightarrow{PJ} = m\overrightarrow{RP}$

$\overrightarrow{OJ} - \overrightarrow{OP} = m(\overrightarrow{RP})$ [M1]

$\overrightarrow{OJ} = m(\overrightarrow{RP}) + \overrightarrow{OP}$

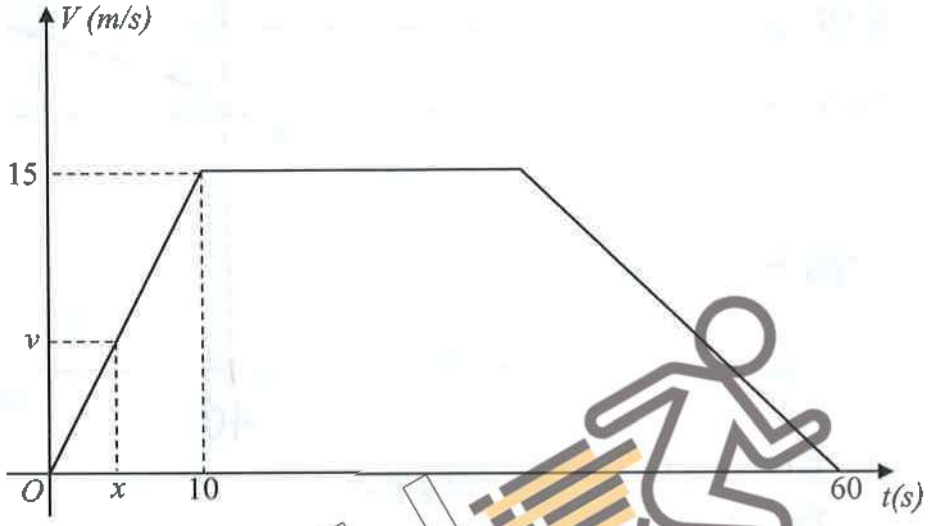
$= m\begin{pmatrix} 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} m \\ -2m \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 3+m \\ 2-2m \end{pmatrix}$ // (shown) [A1]

[2]

22 The diagram shows the speed time graph of a car. The car starts from rest and accelerates uniformly to a speed of 15 m/s in 10 seconds.
The car then travels at a constant speed for some time before it decelerates uniformly at 0.75 m/s^2 until it comes to rest.
The whole journey takes one minute.



(a) Given that the speed of the car after x seconds is v m/s, express v in terms of x.

By similar Δs
$\frac{15}{10} = \frac{v}{x}$
$v = 1.5x$ [M1]

Answer $v =$ $1.5x$ [1]

(b) For how long does the car travel at the maximum speed?

Let the time where he ends at constant speed be t.
$\therefore \frac{(0-15)\text{m/s}}{(60-t)\text{s}} = -0.75\text{m/s}^2$ [M1]
$\therefore t = 40\text{s}$

Answer 30 [2]

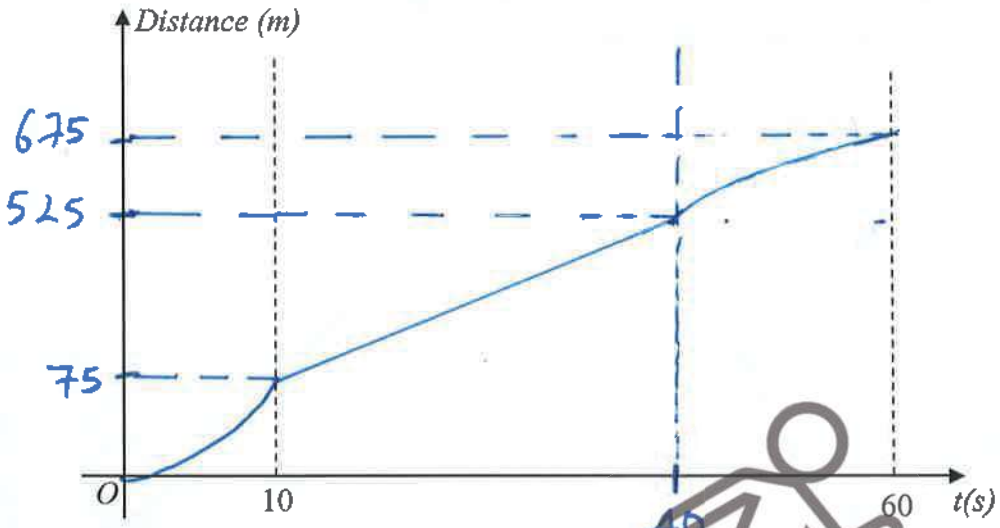
Duration: $40\text{s} - 10\text{s} = 30\text{s}$ [A1]

(c) Calculate the total distance travelled by the car during this 1 minute journey.

$\frac{1}{2}(60+30) \times 15\text{m/s}$ [M1]
$= 675\text{m}$ [A1]

Answer 675m m [2]

(d) Hence, sketch the distance time graph for the whole journey, indicating all relevant values in your sketch.



[2]

[B1] : Correct shape.
[B1] : Correct values.

~ End of Paper ~

Calculator Model :

Class	Full Name	Index Number

bowen
SECONDARY
I believe, therefore I am

Marking Scheme
PRELIMINARY EXAMINATION 2019

O

4048/02

MATHEMATICS
Paper 2

Secondary 4 Express / Secondary 5 Normal (Academic)
3 September 2019 **2 hours 30 minutes**

Candidates answer on the Question Paper.
Additional Materials: Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is required for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give your answer to three significant figures. Give answer in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is **100**.

DO NOT OPEN THIS PAPER UNTIL YOU ARE TOLD TO DO SO

For Examiner's Use
100

Setter: Mrs Jane Cheng

Answer **all** the questions.

1 (a) Solve the inequality $\frac{x+2}{3} \geq \frac{4-x}{7}$. [2]

$7(x+2) \geq 3(4-x)$

$7x + 14 \geq 12 - 3x$ (M1)

$10x \geq -2$

$x = -\frac{1}{5}$ (A1)

(b) Express as a single fraction in its simplest form $\frac{2x}{(3x-5)^2} + \frac{x}{5-3x}$. [2]

$\frac{2x}{(3x-5)^2} - \frac{x}{3x-5}$

$= \frac{2x - x(3x-5)}{(3x-5)^2}$ (M1)

$= \frac{7x - 3x^2}{(3x-5)^2} = \frac{x(7-3x)}{(3x-5)^2}$ (A1)

(c) Simplify $\left(\frac{27a^6}{b^{12}}\right)^{-\frac{1}{3}}$. [2]

$\left(\frac{27a^6}{b^{12}}\right)^{-\frac{1}{3}}$

$= \left(\frac{b^{12}}{27a^6}\right)^{\frac{1}{3}}$ (M1)

$= \frac{b^4}{3a^2}$ (A1)

(d) Simplify $\frac{24p^3q^2}{5r^3} \div \frac{8p^4r}{15q^3}$. [2]

$= \frac{24p^3q^2}{5r^3} \times \frac{15q^3}{8p^4r}$ (m1)

$= \frac{9q^{2+3}\ p^{3-4}}{r^{3+1}}$

$= \frac{9q^5}{pr^4}$ (A1)

(e) Solve the equation $\frac{15}{x+2} = 2x + 3$ [3]

$15 = (2x+3)(x+2)$

$15 = 2x^2 + 7x + 6$

$2x^2 + 7x + 6 - 15 = 0$

$2x^2 + 7x - 9 = 0$ (m1)

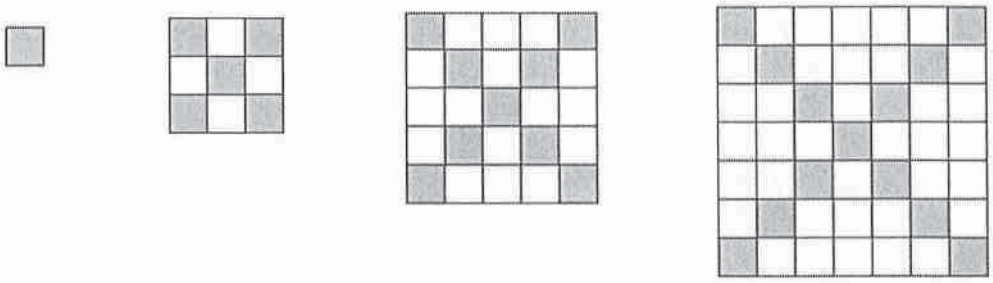
$(2x + 9)(x - 1) = 0$

$2x + 9 = 0$ or $x - 1 = 0$

$x = -4\frac{1}{2}$ $x = 1$

(A1) (A1)

2 A series of diagrams of shaded and unshaded small squares is shown below.
The shaded squares are those which lie on the diagonals of the diagram.



(a) Copy and complete the table below. [2]

Diagram, n	1	2	3	4	5
Number of shaded squares, S	1	5	9	13	17
Number of unshaded squares, U	0	4	16	36	64
Total number of squares, T	1	9	25	$7^2=49$	$9^2=81$

$(2n-1)^2$

(b) By observing the number patterns, without drawing further diagrams,
(i) write down the total number of squares in diagram 12, [1]

$n=12 \quad (2n-1)^2=(2\times12-1)^2=23^2=529$ (A1)

(ii) find an expression, in terms of n, for the total number of squares, T. [1]

$T=(2n-1)^2$
$=4n^2-4n+1$ (A1)

(c) **(i)** Find an expression, in terms of n, for the number of shaded squares, S. [1]

$S=4n-3$ (A1)

(ii) Write down the number of the diagram that has 41 shaded squares. [1]

$4n-3=41$
$4n=44$
$n=11$ (A1)

(d) Hence, or otherwise, find an expression, in terms of n, for the number of unshaded squares, U. [2]

$U=T-S$
$=(2n-1)^2-(4n-3)$ (m1)
$=4n^2-4n+1-4n+3$
$=4n^2-8n+4$
$=4(n^2-2n+1)$ (A1)

3 P is the point $(-5, 12)$ and Q is the point $(5, -4)$

(a) Find the length of PQ. [2]

$$l = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$= \sqrt{(12+4)^2 + (-5-5)^2}$$

$$= \sqrt{356}$$

$$= 18.867$$

$$= \underline{18.9 \text{ units}} \quad (3 \text{ s.f.})$$

(b) Find the equation of the line PQ. [2]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{12+4}{-5-5}$$

$$= -\frac{8}{5}$$

Equation:

$$y = -\frac{8}{5}x + 4 \quad (A1)$$

OR

$$5y = -8x + 20$$

$$y = mx + c$$

$$-4 = -\frac{8}{5}(5) + c$$

$$c = 4 \quad (m1)$$

(c) The equation of the line l_1 is $8x + 5y + 10 = 0$.

(i) Show how you can decide whether the line l_1 does or does not intersect the line PQ? [2]

l_1 $5y = -8x - 10$ $m = -8/5$ (M1)

$y = -8/5 x - 2$

Both equations have the same gradient, they are parallel to each other.

Hence, the line l_1 does not intersect with PQ. (A1)

(ii) The equation of line l_2 is $3y = 4x - 39$.

Find the coordinates of the point of intersection of the line l_1 and the line l_2. [3]

l_1 $8x + 5y = -10$ ①

l_2 $4x - 3y = 39$ ②

③

(M1)

eqn ② × 2 $8x - 6y = 78$

eqn ① − ③ $13y = -88$

$11y = -88$

$y = -8$ (A1)

sub $y = -8$ into eqn ①

$8x + 5(-8) = -10$

$8x = -10 + 40$

$8x = 30$

$x = 3\frac{3}{4}$

$x = 3.75$ (A1)

3 Mrs Tan is a Korean Language teacher.
She conducts classes for basic and advaced students on weekdays and weekends.
Each student has a 15-week block of lessons with one lesson per week.
The matrix K shows the number of students she teaches each week in one 15-week block.

$$\mathbf{K} = \begin{matrix} \text{Basic} & \text{Advanced} \\ \begin{pmatrix} 12 & 3 \\ 5 & 8 \end{pmatrix} & \begin{matrix} \text{Weekday} \\ \text{Weekend} \end{matrix} \end{matrix}$$

(a) Evaluate the matrix $\mathbf{P} = 15\mathbf{K}$. [1]

$P = 15 \begin{pmatrix} 12 & 3 \\ 5 & 8 \end{pmatrix}$

$= \begin{pmatrix} 180 & 45 \\ 75 & 120 \end{pmatrix}$ (A1)

(b) Mrs Tan charges \$20 for each basic lesson and \$32 for each advanced lesson.
Represent the lesson charges in a 2×1 matrix $\mathbf{L}$. [1]

$L = \begin{pmatrix} 20 \\ 32 \end{pmatrix}$ (M1)

(c) Evaluate the matrix $\mathbf{T} = \mathbf{PL}$. [2]

$T = \begin{pmatrix} 180 & 45 \\ 75 & 120 \end{pmatrix} \begin{pmatrix} 20 \\ 32 \end{pmatrix}$ (M1)

$= \begin{pmatrix} 5040 \\ 5340 \end{pmatrix}$ (A1)

(d) State what the elements of **T** represent. [1]

The elements of T represent the total amount of money Mrs Tan collects for a 15-week block of lessons on weekdays and weekends respectively.

(e) Mrs Tan wants to attract more students, so in the next 15-week block she reduces her prices by 10%.
For this block of lessons, on weekdays she has 15 basic students and 5 advanced students.
On weekends she has 7 basic students and 6 advanced students.

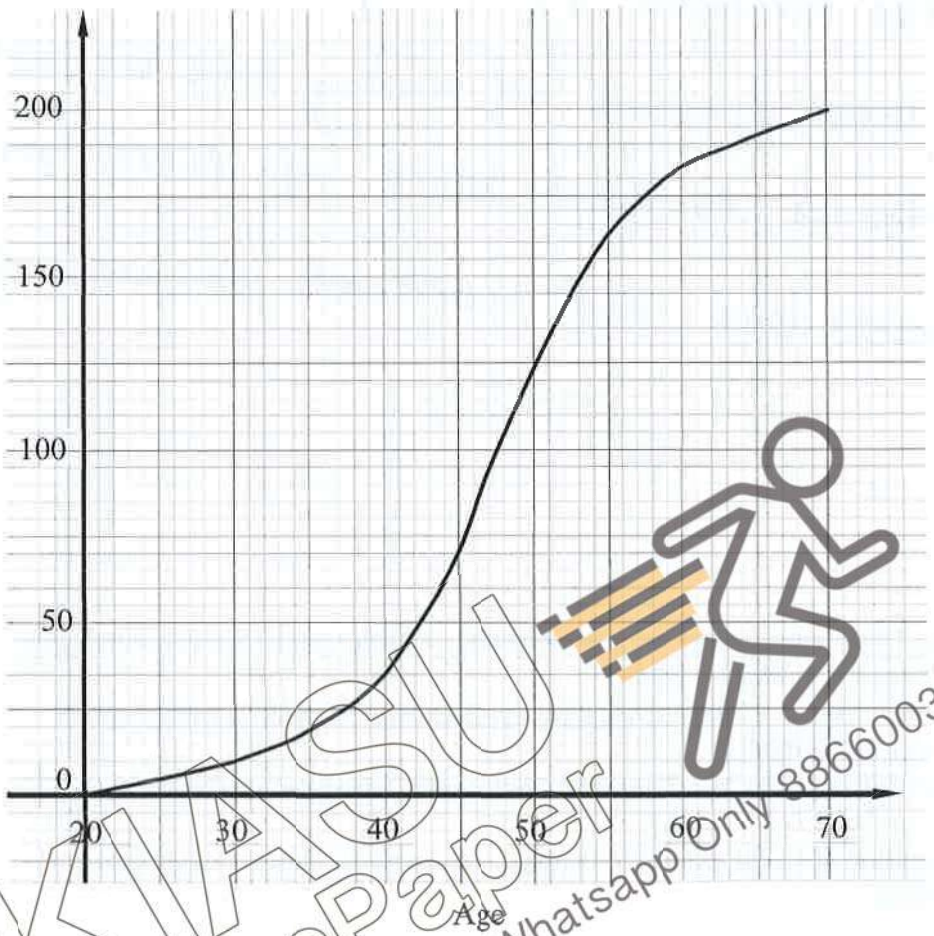
Calculate the total amount of money she earns for this 15-week block of lessons. [3]

$0.9 \times [(15 + 7) \times 20 + (5 + 6) \times 32)] \times 15$ (M1) (M1)

$= \$ 10692$

5 The cumulative frequency graph shows the distribution of the age groups of the *Fitness First* club.

(a)



(i) Complete the grouped frequency table for the ages of the members. [1]

Age (x)	$20 \leq x < 30$	$30 \leq x < 40$	$40 \leq x < 50$	$50 \leq x < 60$	$60 \leq x < 70$
Frequency	10	25	90	60	15

(ii) Calculate the mean age of each member. [1]

$\bar{x} = (10 \times 25 + 25 \times 35 + 90 \times 45 + 60 \times 55 + 15 \times 65) \div 200$

$= 47.25$ years old (A1)

[2]

(iii) Calculate the standard deviation.

$SD = \sqrt{\frac{\Sigma fx^2}{N} - \left(\frac{\Sigma fx}{N}\right)^2} = \sqrt{\frac{464000}{200} - (47.25)^2}$ (M1)

$= 9.3508$

$= 9.35$ (3 s.f.) (A1)

(iv) Find the percentage of members whose age is 30 years old and above but less than 60 years old. [1]

No. of members = 25 + 90 + 60 = 175

$\frac{175}{200} \times 100\% = 87.5\%$ (A1)

(v) A magazine article stated that citizens aged 50 and above are less active than those aged below 40. [2]
Comment on whether the data from the *Fitness First* club supports this claim.

Accept any other reasonable answer.

{ percentage of members ≥ 50 $\frac{75}{100} \times 100\% = 37.5\%$
percentage of members < 40 $\frac{35}{100} \times 100\% = 17.5\%$ } (M1)

It is not true that citizens ≥ 50 yrs old are less active (A1)

(b) The table below gives information about the ages of the members in the *Any Time Fitness* club.

	Members aged under 50	Members aged 50 or over	
Male	50	34	84
Female	36	30	66
	86	64	

(i) One of these members is selected at random. [1]
Find, as a fraction in its lowest terms, the probability that he or she is under 50.

P(a member < 50) $= \frac{86}{150} = \frac{43}{75}$ (A1)

(ii) Two of the members are selected at random. [2]
Find the probability that
(a) both members are female,

No. of female members = 66

P(both are female) $= \frac{66}{150} \times \frac{65}{149} = \frac{143}{745}$ (A1)

(M1)

[2]

(b) they are both aged 50 or over, but only one is a male member.

No. of members ≥ 50 yrs old male : 34 female : 30

P(both ≥ 50 yrs old, only one is a male member)

$= \frac{34}{150} \times \frac{30}{149} + \frac{30}{150} \times \frac{34}{149} = \frac{69}{745}$ (A1)

(M1)

6 A litre of *95-octane* unleaded petrol cost $\$x$ in *January 2019*.

(a) Mr Ang paid \$ 85.50 for his petrol. Write down in terms of x, the amount of petrol bought. [1]

Mr Ang: $\frac{85.5}{x}$ litre (A1)

Mr Bala paid \$100 for his *98-octane* unleaded petrol which cost 25 cents more per *litre*.

(b) Write down in terms of x, the amount of petrol bought by Mr Bala. [1]

Mr Bala: $\frac{100}{x+0.25}$ litre (A1)

(c) If Mr Ang received 2 *litres* less petrol than Mr Bala, write down an equation to represent this information and show that it can reduce to $16x^2 - 112x + 171 = 0$. [3]

$\frac{100}{x+0.25} - \frac{85.5}{x} = 2$ (M1)

$100x - 85.5(x+0.25) = 2x(x+0.25)$ (M1)

$\frac{29}{2}x - \frac{171}{8} = 2x^2 + \frac{1}{2}x$

$2x^2 + (\frac{1}{2} - \frac{29}{2}x) + 21\frac{3}{8} = 0$

$2x^2 - \frac{28}{2}x + \frac{171}{8} = 0$

$16x^2 - 112x + 171 = 0$ shown (A1)

(d) Solve the equation $16x^2 - 112x + 171 = 0$. [2]

$a = 16 \quad b = -112 \quad c = 171$

$$x = \frac{-(-112) \pm \sqrt{(-112)^2 - 4(16)(171)}}{2 \times 16}$$ (M1)

$x = 4.75$ (2 d.p) or $x = 2.25$

(A1)

(e) The price of the 98-octane unleaded petrol in *January 2019* was a reduction of 7% on the price in *December 2018*. [2]
Find the price of the 98-octane unleaded petrol in December 2018 if it cost less than \$3 for a litre of 95-octane unleaded petrol in January 2019.

95-octane : $x = \$2.25$ (Jan 2019)

98-octane : $2.25 + 0.25 = \$2.50$ per litre in Jan 2019

\$2.50 — 93% of Dec 2018 price (M1)

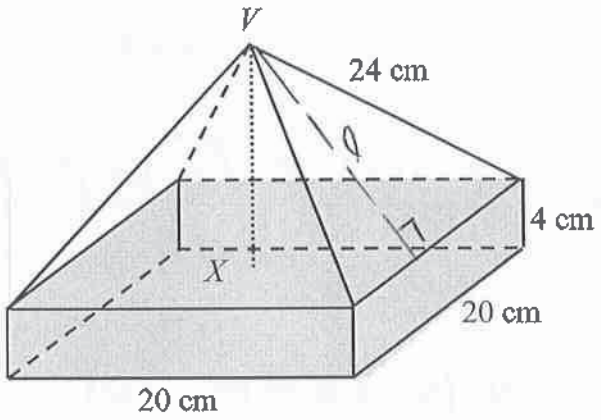
$\frac{2.50}{93} \times 100\% = 2.68817$

$= \$2.69$ (2 d.p)

(A1)

Price of 98-octane in Dec 2018 was \$2.69

7 The diagram shows a container consisting of a square bottom with rectangular sides, each 20 cm by 4 cm, and a regular pyramid on top with perpendicular height given by VX. Water is poured into the container till the brim of the cuboid.



(a) Find the height VX of the pyramid. [2]

Slant height $l = \sqrt{24^2 - 10^2} = \sqrt{476}$ cm (M1)

$VX^2 = (\sqrt{476})^2 - 10^2$

$VX = \sqrt{376}$

$= 19.3907$

$= 19.4$ cm (3 s.f) (A1)

(b) Calculate the total surface area of the container. [2]

Total surface area

$= 4(20 \times 4) + 20 \times 20 + 4 \times \frac{1}{2} \times 20 \times \sqrt{476}$ (M1)

$= 320 + 400 + 872.697$

$= 1592.697$

$= 1590 \text{ cm}^2$ (3 s.f)

(c) Find the volume of water in the cuboid. [1]

Volume of water in the cuboid

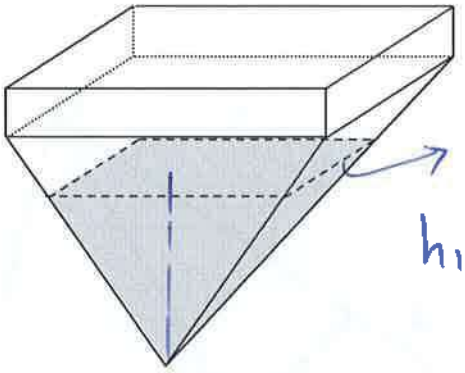
$= 20 \times 20 \times 4$

$= 1600 \text{ cm}^3$

The container is now inverted as shown in the diagram below.

$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$

Let $\frac{h_1}{h_2} = x$



$h_1 = 19.39x$

(d) Calculate the depth of the water in the pyramid when inverted. [3]

Let x be the ratio of ht of water to the ht of the pyramid ($V_1 : V_2$)

$\frac{1}{3}(20x)(20x)(19.39x) = 1600$ (M1)

$19.39x^3 = 12$

$x^3 = \frac{12}{19.39}$ (M1)

$x = 0.8522$

Height of water $= 0.8522 \times 19.39$

$= 16.5$ cm (3 s.f.) (A1)

(e) Another smaller container, which is geometrically similar, has a square base of 225 cm^2. Both containers are made of the same material. [2]
Find the mass of the smaller container in grams, given that the mass of the empty larger container is 1.28 kg.

$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

$\frac{l_1}{l_2} = \sqrt{\frac{225}{400}}$

$= 3/4$

$\frac{V_1}{V_2} = \left(\frac{3}{4}\right)^3$

$= \frac{27}{64}$ (M1)

Mass of the smaller container

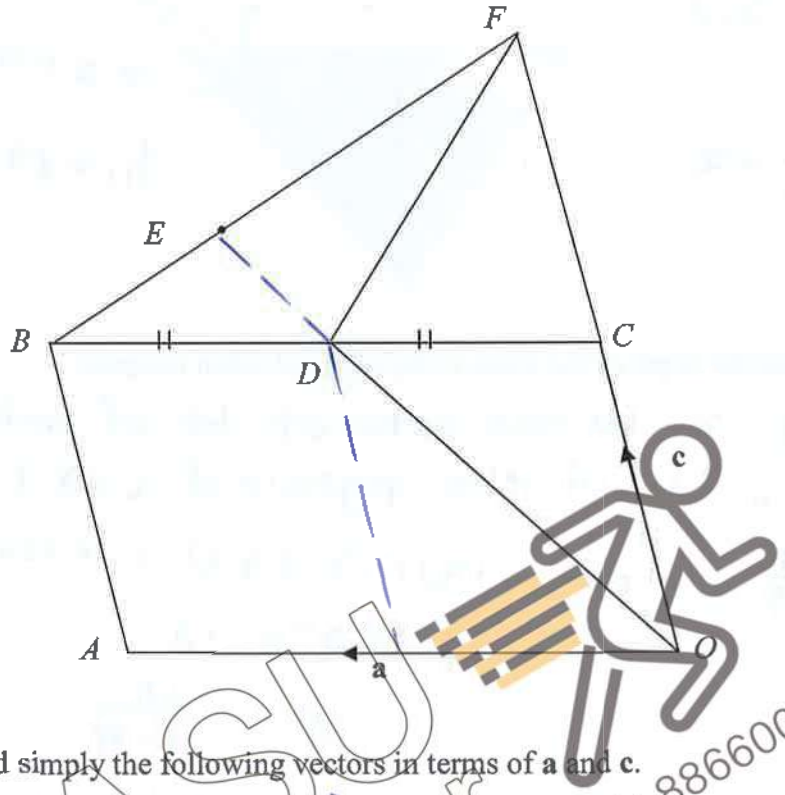
$= \frac{27}{64} \times 1280$ g

$= 27 \times 20$ g

$= 540$ g (A1)

8 In the diagram, $OABC$ is a parallelogram and D is the midpoint of BC. BE and OC produced intersect at the point F. $BE : BF = 1 : 3$ and $OC : OF = 1 : 2$.

Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.



(a) Express and simply the following vectors in terms of **a** and **c**.

(i) $\overrightarrow{AC}$

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$= -\underset{\sim}{a} + \underset{\sim}{c}$ (A1)

[1]

(ii) $\overrightarrow{BF}$

$\overrightarrow{BF} = \overrightarrow{BC} + \overrightarrow{CF}$

$= \overrightarrow{AO} + \overrightarrow{OC}$

$= -\underset{\sim}{a} + \underset{\sim}{c}$ (A1)

[1]

(iii) $\overrightarrow{OD}$

$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD}$

$= \underset{\sim}{c} + \frac{1}{2}\underset{\sim}{a}$ (A1)

[1]

(iv) $\overrightarrow{OE}$ [2]

$\overrightarrow{OE} = \overrightarrow{OF} + \overrightarrow{FE}$

$= 2\underset{\sim}{c} + \frac{2}{3}\overrightarrow{FB}$ (M1)

$= 2\underset{\sim}{c} - \frac{2}{3}(-\underset{\sim}{a} + \underset{\sim}{c})$

$= \frac{4}{3}\underset{\sim}{c} + \frac{2}{3}\underset{\sim}{a}$

$= \frac{4}{3}(\underset{\sim}{c} + \frac{1}{2}\underset{\sim}{a})$ (A1)

(b) State two facts about the vectors $\overrightarrow{OD}$ and $\overrightarrow{OE}$ from the results in (a). [2]

$\overrightarrow{OE} = \frac{4}{3}\overrightarrow{OD}$

$\frac{\overrightarrow{OE}}{\overrightarrow{OD}} = 4:3$

1) $\overrightarrow{OE}$ is parallel to $\overrightarrow{OD}$ $\Rightarrow$ O, D and E are collinear. (A1)

2) $OE : OD = 4:3$ (A1)

(c) Find the ratio of the areas of

(i) ΔODF and ΔOEF, [1]

Area of ΔODF : Area of ΔOEF

$= 3$ (A1)

(ii) ΔOCD and $OABC$, [1]

Area of ΔOCD : Area of $OABC$

$= 1 : 4$

(iii) ΔOCD and $OABF$. Areas of ΔOBC : $\Delta BCF = 1:1$ [2]

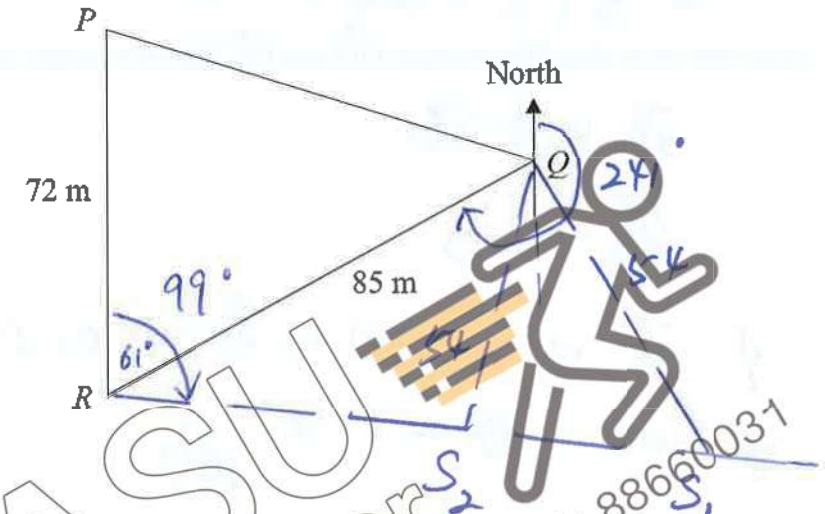
ΔOCD	$OABC$	ΔBCF
1	4	
	2	1
1	4	2

(M1)

Area of ΔOCD : Area of $OABF = 1:6$ (A1)

9 Two school teams, *Novotel* and *Temasek*, are participating in an Amazing Race in Bishan Park. The diagram shows the paths in the park.
The teams assemble at P before heading to Q to start the race.
P is due north of R.
The bearing of R from Q is 241°.
The distance PR is 72 metres and the distance RQ is 85 metres.

(a) Find the distance PQ. [3]



Bearing of Q from R is 241°

$\angle PRQ = 241° - 180° = 61°$ (alt ∠s) (M1)

$PQ^2 = 72^2 + 85^2 - 2 \times 72 \times 85 \cos 61°$ (M1)

$PQ = 80.467$

$= \underline{80.5 \text{ m}}$ (3 s.f) (A1)

(b) The final station of the race is at R, each team is required to find a ***clue*** that is hidden at point S before completing the race at R. [3]
The bearing of S from R is 099° and QS is 54 metres.
Given that there are **two** possible locations for S, find the two possible values of angle RSQ.

$\angle QRS = 99° - 61° = 38°$

In $\triangle RQS$

$\frac{\sin \angle RSQ}{84} = \frac{\sin 38°}{54}$ (M1)

$\sin \angle RSQ = \frac{84 \sin 38°}{54}$

$\angle RSQ = 75.7188°$

$= 75.7°$ (1 d.p) (A1)

OR $\angle RSQ = 180° - 75.7188°$

$= 104.2812°$

$= 104.3°$ (1 d.p) (A1)

(c) Both teams manage to find the ***clue*** at the same time and team *Novotel* is closer to *R* than team *Temasak*. [5]
Team *Novotel* claims that they are the winner.
Given that the speed of team *Novotel* is 30% less than the speed of team *Temask* when they travel from *S* to *R*.
Do you agree with team *Novotel* that they will win the race?
Justify your answer with clear working in your calculations.

Novotel — S_2 Temasak — S_1

Team Novotel
$\angle RQS_2 = 180° - 38° - 104.2812°$
$= 37.7188°$
$= 37.7°$

Team Temasak
$\angle RQS_1 = 180° - 38° - 77.7188°$
$= 66.2812°$
$= 66.3°$ (M1)

Novotel $\frac{RS_2}{\sin 37.7188°} = \frac{54}{\sin 38°}$

$RS_2 = 53.66\text{m}$
$= 53.7\text{ m}$ (M1)

Temasak $\frac{RS_1}{\sin 66.2812°} = \frac{54}{\sin 38°}$

$RS_1 = 80.3017$
$= 80.3\text{ m}$ (M1)

Let the speed of team Temasak be x

$\text{Time}_{\text{Novotel}} = \frac{53.66}{0.7x} = \frac{76.657}{x}$

$\text{Time}_{\text{Temasak}} = \frac{80.3017}{x}$ (A1)

Time taken by Team Novotel is less than time taken by Team Temasak.

Yes, I agree that Team Novotel will win the race. (A1)

10 ***Answer the whole of this question on a sheet of graph paper.***

The table below gives some values of x and the corresponding values of y for $y = x(1 + x)(5 - x)$.

x	-2	-1	-0.5	1	2	3	4	5
y	14	0	-1.375	8	18	p	20	0

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $-2 \leq x \leq 5$.
Using a scale of 2 cm to represent 5 units, draw a vertical y-axis for $-5 \leq y \leq 25$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) By drawing a tangent, find the gradient of the curve where $x = 4$. [2]

(d) **(i)** On the same axes, draw the line $2x + y = 12$ for $-2 \leq x \leq 5$. [1]

(ii) Write down the x-coordinates of the points where this line intersects the curve. [1] [2]

(iii) The x-coordinates of the points where the two graphs intersect are solutions of the equation $x^3 + Ax^2 + Bx + 12 = 0$. Find the value of A and the value of B. [2]

d(ii) $y = -2x + 12$

x	-2	0	4
y	16	12	4

d(iii) $-2x + 12$

$y = x(5 + 4x - x^2)$

$= -x^3 + 4x^2 + 5x$

$-x^3 + 4x^2 + 5x = -2x + 12$

$-x^3 + 4x^2 + 7x - 12 = 0$

$x^3 - 4x^2 - 7x + 12 = 0$

$A = -4$ (A1)

$B = -7$ (A1)

~~~ End of Paper ~~~

Name _____ Index No. _____

Class _____ Date _____

Q10

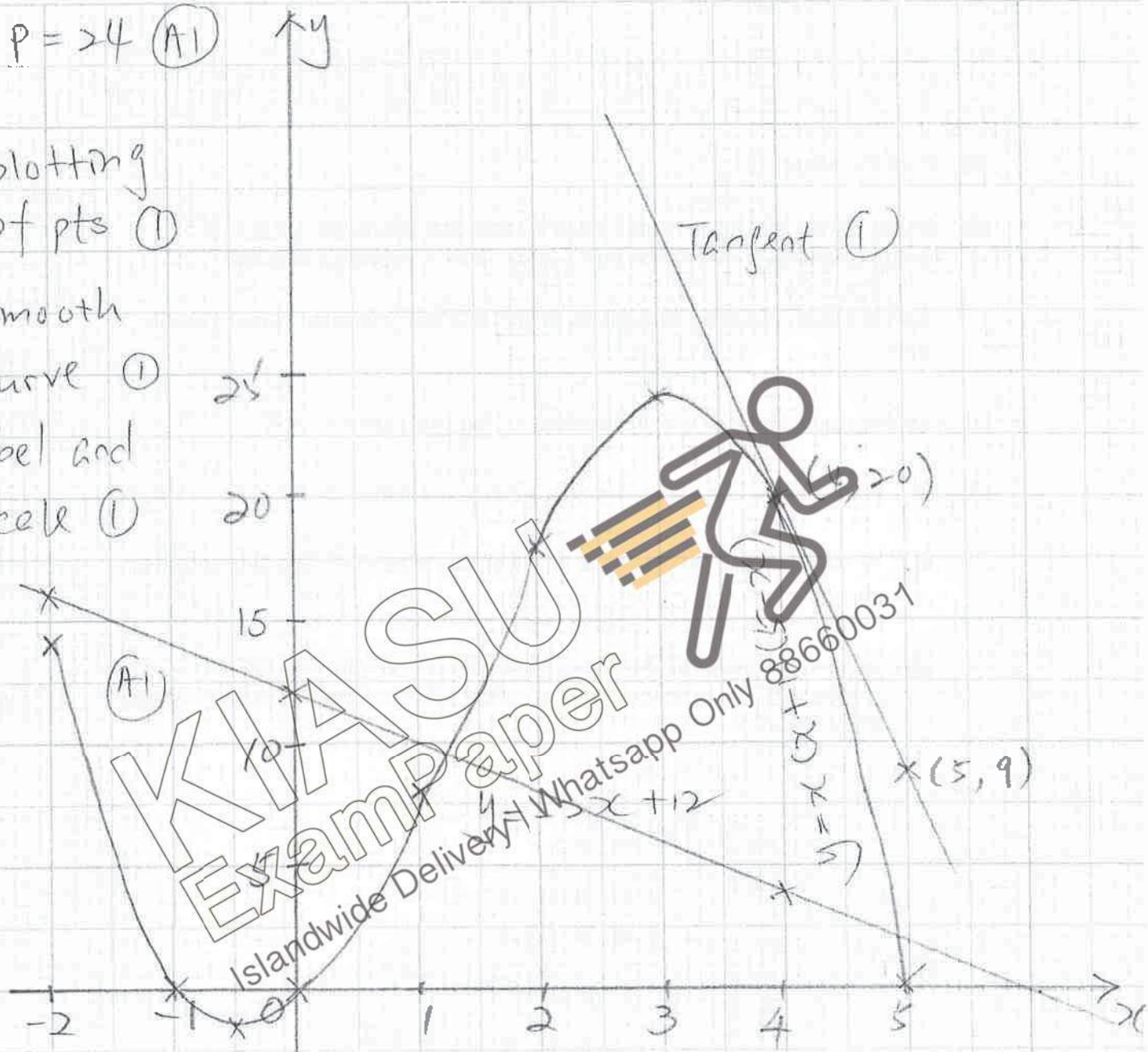
a) $P = >4$ (A1)

b) plotting of pts (1)

Smooth curve (1)

label and scale (1)

Tangent (1)



(A1)

c) $m = \frac{9-20}{5-4} = -11$ (±1) (A1)

d(i) $y = +27x + 12$

d(ii) $x = 1.2 \pm 0.1$ or $x = 4.9 \pm 0.1$ (A2)

d(iii) $A = -4$ (A1) $B = -7$ (A1)

CHIJ ST. THERESA'S CONVENT

PRELIMINARY EXAMINATION 2019

SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

CANDIDATE NAME			
CLASS		INDEX NUMBER	

MATHEMATICS **4048/1**

Paper 1 **05 September 2019**

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your index number, and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answers in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

ANSWERS TO QUESTIONS 1 TO 15 MUST BE HANDED IN SEPARATELY FROM ANSWERS TO QUESTIONS 16 TO 24.

This document consists of **17** printed pages.

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Solve $\frac{x}{4}+13=6$.

Answer $x =$ …… [1]

---

2 Peter boards a bus at bus stop A at 6.50 am. The bus travels to bus stop B at an average speed of 60 km/h to school. He reaches bus stop B at 7.15 am.

What is the distance between the 2 bus stops?

Answer ……..… km [2]

---

3 John wrote down five numbers.
The mean of these numbers is 8, the median is 6 and the mode is 5.
The largest number is three times the smallest number.

Find the five numbers.

Answer ……..… , ….…… , ……..… , ……..… , ……..… [2]

---

4 A field which has an area of 1800 m^2, is used to plant sunflowers.
It is known that an acre of land, which is about 4047 m^2, can grow an average of 20 000 sunflower plants.
Each sunflower plant has an average seeding rate of 1500 seeds.

Calculate an estimate of the total number of seeds that can be harvested from the field, leaving your answer in standard form, correct to 3 significant figures.

Answer ……..… seeds [2]

---

5 Simplify $\frac{3x}{5} - \frac{4(2-3x)}{7}$.

Answer ……...……. [2]

---

6 $\xi = \{x : x \text{ is a positive integer and } x < 25\}$

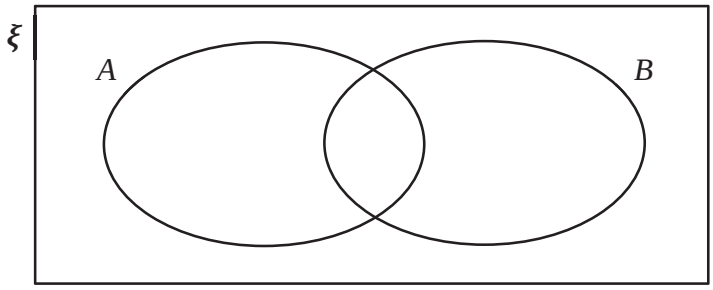
$A = \{x : x \text{ is a perfect square}\}$

$B = \{x : x \text{ is an odd number}\}$

(a) Find $n(A \cap B)$.

Answer …… [1]

(b) On the Venn diagram, shade the region which represents $(A \cap B')'$.



[1]

---

7 Factorise fully $6ac + 9ad - 12bd - 8bc$.

Answer ……...……. [2]

---

8 A is the point $(-2,\ 5)$ and $\overrightarrow{BA} = \begin{pmatrix} -4 \\ 8 \end{pmatrix}$.

(a) Find the coordinates of point B.

Answer B (……..…, ……..…..…) [1]

(b) Calculate $\left|\overrightarrow{BA}\right|$.

Answer …… [1]

---

9 Andrew weighed ten large watermelons from his farm.

The mean mass of the watermelons was 9070 grams.
The standard deviation of the masses of the watermelons was 362.8 grams.

The scales used by Andrew were found to be inaccurate.
The correct mass of each watermelon turns out to be 1650 grams more than Andrew recorded.

Write down the correct values for the mean and standard deviation (SD).

Answer Mean = ..…….. g

SD = ……... g [2]

---

10 A man purchased a painting in 2016.
The value of the painting increased by 8% in 2017.
In 2018, the value of the painting was $73288.80, an increase of 20% as compared to 2017.
Express the cost of the painting in 2016 as a percentage of the cost of the painting in 2018.

Answer ..…….…… % [3]

---

11 **(a)** David makes a mocktail drink for his kids.
He uses apple juice, lime juice and sparkling water in the ratio 3 : 2 : 7 respectively.
He uses 2.1 litres of sparkling water.

(i) How much apple juice does he use?

Answer ……... litres [1]

(ii) How much mocktail drink does he make altogether?

Answer ……... litres [1]

(b) Wen Ni makes a mocktail drink using lemon juice, syrup and coconut water.
The ratio lemon juice : syrup is $\frac{1}{2} : \frac{1}{3}$.
The ratio syrup : coconut water is 1 : 8.

Find the ratio lemon juice : syrup : coconut water.

Answer ……..…. : …… : ……..…. [2]

---

12 Jane draws a triangle.
The first angle is 20º bigger than the second angle
The third angle is three times the size of the first angle.

Form an equation and solve it to find the angles of the triangle.

Answer ……..... º, ……. º, ……..….. º [3]

---

13 It is given that x and y are in direct proportion.
The difference in the value of y, when x is 5 and when x is 17, is 54.

(a) Find an equation connecting x and y.

Answer …… [3]

(b) Find the value of x when $y = 84$.

Answer …… [1]

---

14 When written as the product of their prime factors,

m is $2^2 \times 5^2 \times 11$
n is $2^3 \times 3 \times 5 \times 7$

Find

(a) the largest integer which is a factor of m and n,

Answer ..... [1]

(c) the smallest integer p such that mp is a perfect cube,

Answer ..... [1]

(d) the smallest positive integer value of q such that $\sqrt[3]{\frac{mn}{q}}$ is an integer.

Answer ..... [1]

---

15 **(a)** Solve the inequalities $-15 \le 8 - 3x < 2$.

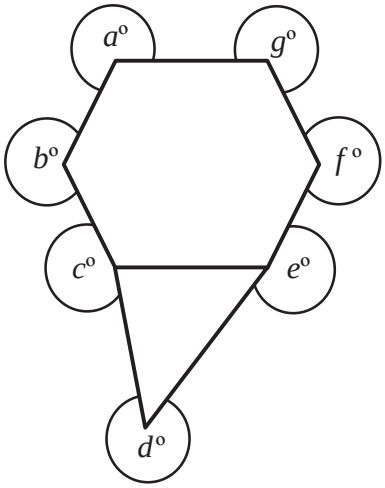
Answer ..... [2]

(b) Write down all the integers that satisfy $-15 \le 8 - 3x < 2$.

Answer ..... [1]

Name: _____( ) Class: _____

16 The diagram shows a regular hexagon and a triangle.



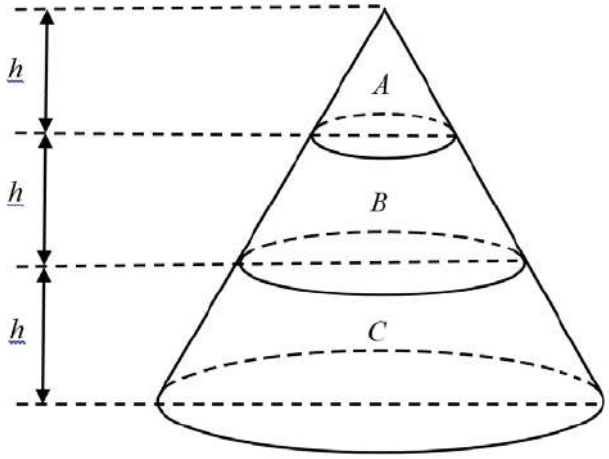
(a) Calculate the sum of the interior angles of the regular hexagon.

Answer ……...° [1]

(b) Calculate the sum of the angles a, b, c, d, e, f and g.

Answer ……...° [2]

17 The diagram shows a right circular cone which is divided into 3 parts *A*, *B* and *C* by planes parallel to the base as shown. *B* and *C* are frustums of a cone. The height of each part is *h* cm.



The curved surface area of *A* is 200 cm^2

(a) Show that $A_A : A_{A+B}$ is 1 : 4, where A_A is the curved surface area of *A* and A_{A+B} is the combined curved surface areas of *A* and *B*.

[1]

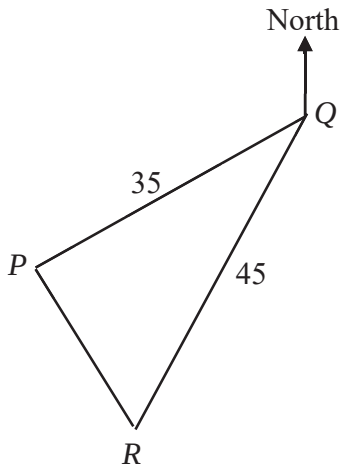
(b) Find A_B, the curved surface area of *B*.

Answer …… cm^2 [1]

(c) Find the ratio of the volume of *B* to the volume of *C*.

Answer …….. [3]

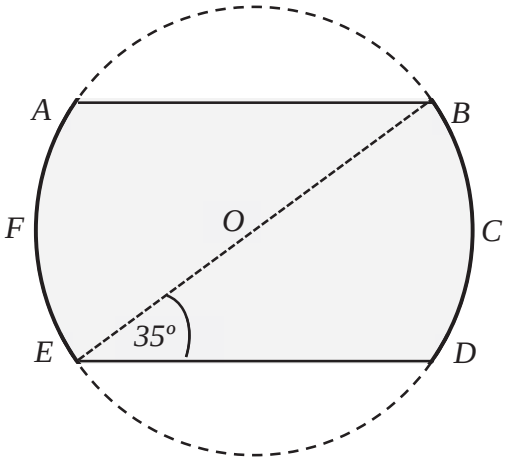
18 The diagram shows the positions of three towns P, Q and R.
$PQ = 35$ km and $QR = 45$ km.
The bearing of P from Q is 235º and the bearing of R from Q is 205º.



Calculate the distance between towns P and R.

Answer ……..… km [3]

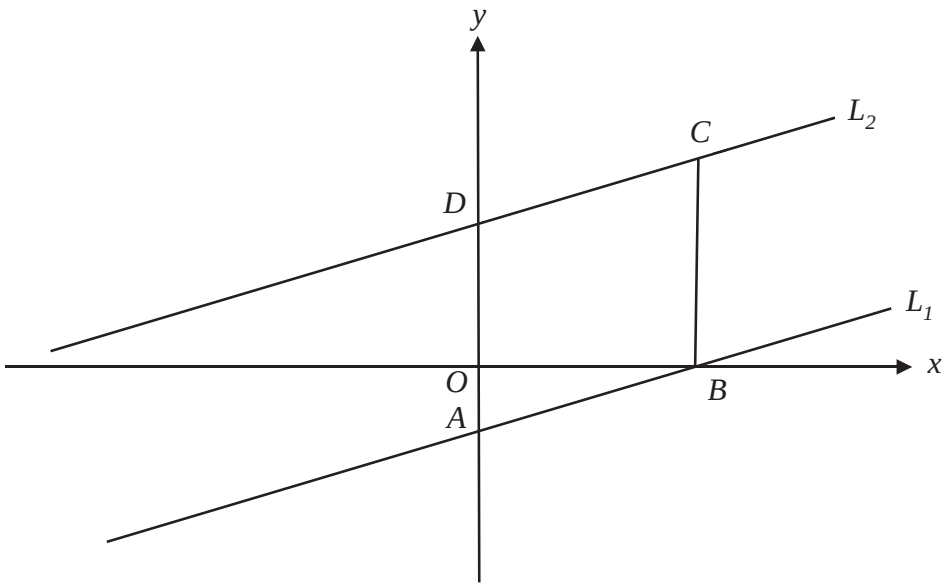
19 The diagram shows six points A, B, C, D, E and F on the circumference of a circle with centre at O.
It is given that the diameter BE of the circle is 14 cm, $\angle BED = 35^\circ$ and $AB = ED$.



Calculate the area of the shaded region.

Answer ..... cm^2 [5]

20 In the diagram, the equation of the line L_1 is $2y = 3x - 5$.
The line L_2 has the same gradient as the line L_1.
A and D lie on the y-axis. BC is parallel to the y-axis and $BC = 8$ units.



(a) Find the equation of the line L_2.

Answer …… [2]

(b) What is the area of quadrilateral $ABCD$?

Answer ……..… units^2 [1]

(c) Find the value of tan $\angle ABO$.

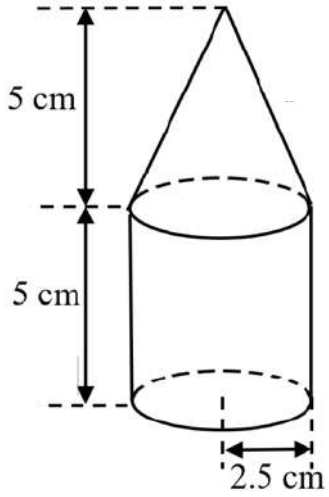
Answer …… [1]

21 **(a)** The period of oscillation of a pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$.

Rearrange the formula to make l the subject.

Answer …… [2]

(b) A solid is made from a cylinder and a cone.
Both the cylinder and cone have radius 2.5 cm and height 5 cm.



Calculate the total surface area of the solid.

Answer …… cm^2 [3]

22 A gift company sells three hamper packages containing packets of biscuits, bars of chocolates and bottles of wine.
The cost of one packet of biscuit, one bar of chocolate and one bottle of wine are \$7.20, \$10.80 and \$32.00 respectively.

	Biscuit	Chocolate	Wine
Hamper *A*	6 packets	5 bars	3 bottles
Hamper *B*	9 packets	7 bars	4 bottles
Hamper *C*	10 packets	8 bars	2 bottles

The quantity of the items packed into each of the three types of hamper packages is shown in the table above.

These information can be represented by the matrices $\mathbf{H} = \begin{pmatrix} 6 & 5 & 3 \\ 9 & 7 & 4 \\ 10 & 8 & 2 \end{pmatrix}$ and $\mathbf{P} = \begin{pmatrix} 7.20 \\ 10.80 \\ 32.00 \end{pmatrix}$.

(a) Evaluate the matrix **R = HP**.

Answer **R** = ... [2]

(b) State what the elements of **R** represent.

..... [1]

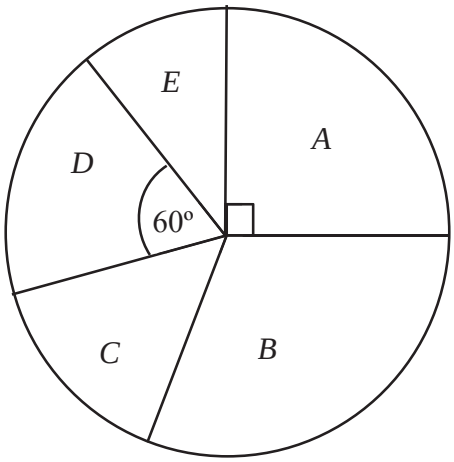
(c) A total of 20 Hamper *A*, 25 Hamper *B* and 30 Hamper *C* are sold.
Represent the total number of hamper packages sold in a 1×3 matrix **T**.

Answer *T* = ... [1]

(d) Using matrix multiplication, find the total amount of money obtained by the gift company from the sales of the hamper packages *A*, *B* and *C*.

Answer \$ ..... [2]

23 Some boys are placed into five groups, *A*, *B*, *C*, *D* and *E*, based on their heights.
The pie chart shows the proportion of boys in each group.
Group *A* comprises $\frac{1}{4}$ of the boys, Group *B* 30% of the boys and Group *C* has 18 boys.
Group *D* is represented by a $60°$ sector.



(a) Find the percentage of the boys who are in Group *D*.

Answer ……... % [1]

(b) Given that the number of boys in group *B* is 36, find the total number of boys.

Answer ……..…. boys [2]

(c) Calculate the number of boys in group *E*.

Answer ……..…. boys [2]

24 A bag contains 20 marbles, n of which are red and the rest are yellow.
A marble is chosen at random and not replaced.

(a) Write down, in terms of n, the probability that the marble is yellow.

Answer ..... [1]

A second marble is chosen at random.

(b) Find, in terms of n, the probability that both marbles are yellow.

Answer ..... [1]

(c) **(i)** The probability that both marbles are yellow is $\frac{39}{95}$.
Show that $n^2 - 39n + 224 = 0$

[2]

(ii) Solve the equation $n^2 - 39n + 224 = 0$ to find the number of yellow marbles in the bag.

Answer ..... [3]

~~ End of Paper 1 ~~~

CHIJ ST. THERESA'S CONVENT

PRELIMINARY EXAMINATION 2019

SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

CANDIDATE NAME			
CLASS		INDEX NUMBER	

MATHEMATICS **4048/02**

Paper 2 **29 Aug 2019**

Additional Material: Graph paper (1 sheet) **2 hours 30 minutes**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.
Answers to Questions 1 to 6 must be handed in separately from answers to Questions 7 to 10.

This document consists of **23** printed pages.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Factorise $27a^4 - 3$.

Answer ……..... [2]

(b) Express as a single fraction in its simplest form

(i) $\dfrac{2(x-1)^2}{4y^3} \div \dfrac{6y(x-1)}{8y^2}$,

Answer ……..... [1]

(ii) $\dfrac{3}{m-2} - \dfrac{2}{3m-1}$.

Answer ……..... [2]

(c) Solve the equation $2^{2-x} = \dfrac{1}{\sqrt[3]{2^{5x+1}}}$.

Answer ……..... [2]

(d) **(i)** Express $x^2 - 8x - 6$ in the form $(x-b)^2 - c$.

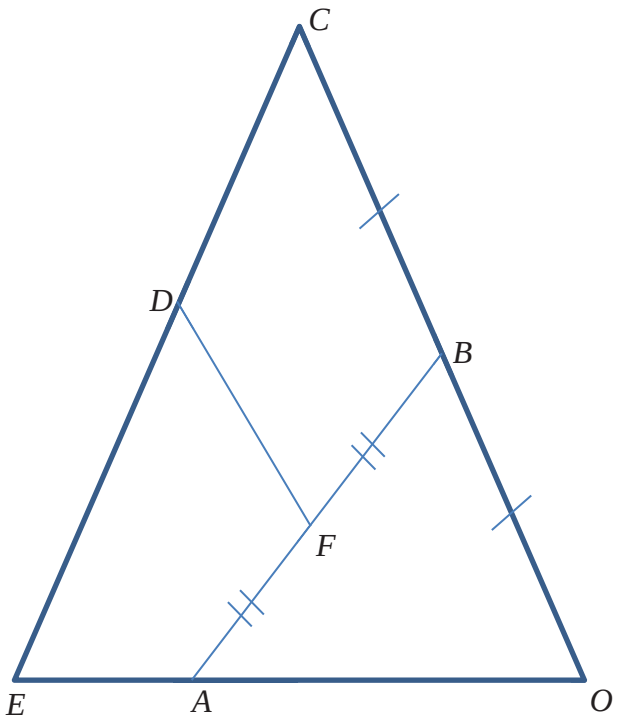
Answer ……..... [1]

(ii) **Hence** solve the equation $x^2 - 8x - 6 = 0$, giving your answers correct to one decimal place.

Answer $x =$ ..... or .....[3]

---

2 In the diagram below, $OB = BC$ and $AF = FB$. It is given that $OA : AE = 2 : 1$ and $ED : DC = 4 : 3$. $\overrightarrow{OA} = 2\mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.



(a) Express, as simply as possible, in terms of **a** and/or **b**,

(i) $\overrightarrow{CE}$,

Answer ……..... [1]

(ii) $\overrightarrow{CD}$,

Answer ……..... [1]

(iii) $\overrightarrow{BA}$,

Answer ……..... [1]

(iv) $\overrightarrow{OF}$,

Answer ……..... [2]

(v) $\overrightarrow{FD}$.

Answer ……..... [2]

(b) Find

(i) $\dfrac{\text{Area of } \Delta OBA}{\text{Area of } \Delta OBE}$,

Answer ……..... [1]

(ii) $\dfrac{\text{Area of } \Delta OBA}{\text{Area of } \Delta OCE}$.

Answer ……..... [2]

3 **(a)** Celine wishes to buy a new car. The price of the car is $98 000.

(i) The car dealer requires Celine to make a downpayment of $19 600. Express this amount as a percentage of the price of the car.

Answer .....% [1]

(ii) After making the downpayment, Celine decides to take a bank loan for the rest of the money to be paid to the car dealer.
Bank OCC charges an interest rate of 2.78% per annum compounded half yearly.
Bank DBB charges a simple interest rate of 2.99% per annum.
If Celine decides to take a seven year loan, which bank should she loan from? Justify your answer.

Answer…….

……... [4]

(b) Celine visits the petrol station weekly to refill petrol, refilling 51 litres of petrol each and every time. In order to save on petrol costs, she decides to drive to Johor Bahru, Malaysia, weekly to refill the petrol.

(i) How much does she need to pay weekly to refill petrol in Singapore if the price of petrol is $2.25 per litre?

Answer S$..... [1]

(ii) How much does she need to pay to refill petrol weekly in Johor Bahru if the price of petrol is RM 2.08 per litre?

Answer RM..... [1]

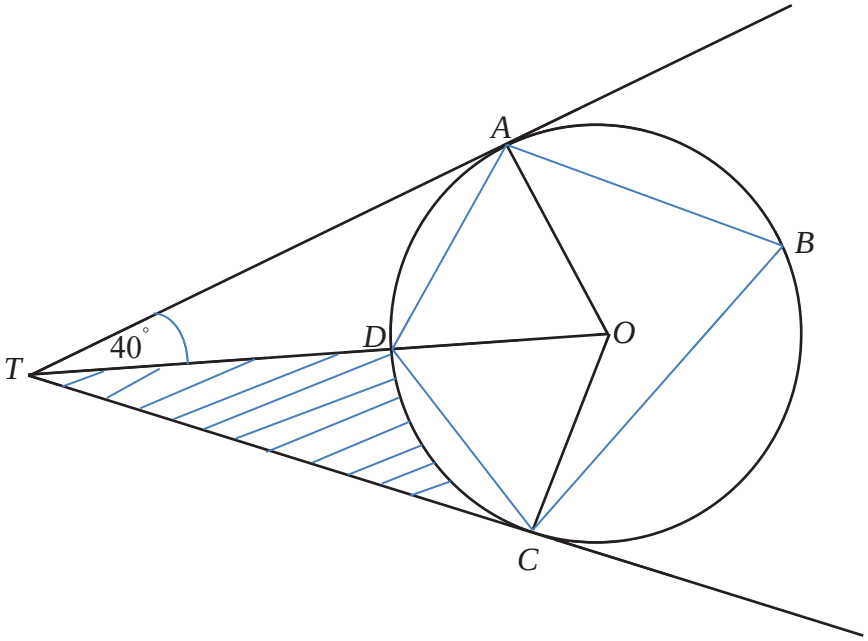
(iii) (a) How much is Celine able to save weekly if she refills the petrol in Malaysia? Give your answer in terms of Singapore dollars. (Given the exchange rate is S$1 = RM 3.05)

Answer S$..... [2]

(b) Express the amount of savings as a percentage of the cost of refilling petrol weekly in Singapore.

Answer .....% [1]

4 The diagram below shows a circle $ABCD$ with centre O. AT and CT are tangents to the circle and angle $ATO = 40^\circ$.



(a) Find, giving reasons for each answer,

(i) angle AOD,

Answer ……° [2]

(ii) angle ABC,

Answer ……° [2]

(iii) angle ADC,

Answer ……° [1]

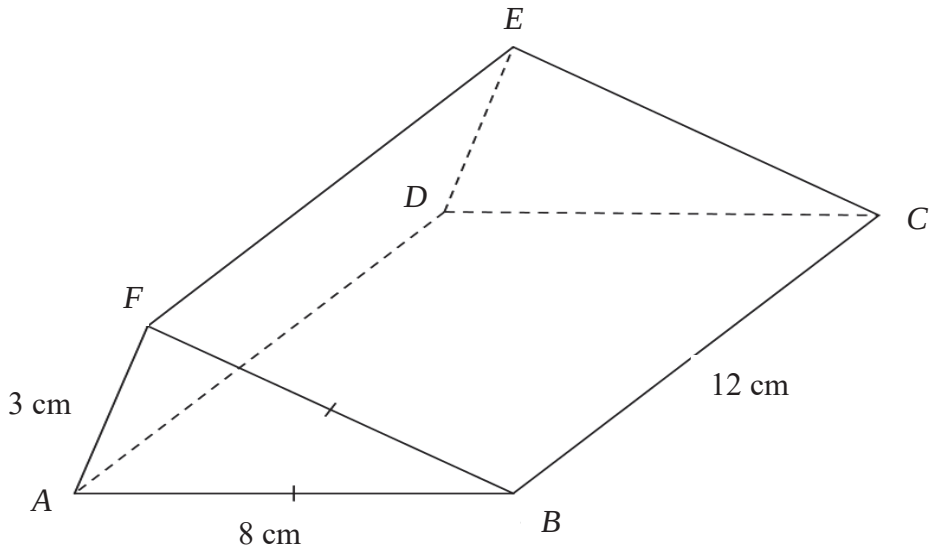
(iv) angle OCD.

Answer ……° [1]

(b) Calculate the area of the shaded region, given that the radius of the circle is 5 cm.

Answer ….…..cm^2 [4]

5



The diagram above shows a prism *ABCDEF* whose cross section is an isosceles triangle.

It is given that $AB = 8$ cm, $BC = 12$ cm, and $AF = 3$ cm.

(a) Show that $\angle ABF = 21.6^\circ$, correct to 1 decimal place. [3]

(b) Find the angle of elevation of *E* from *B*.

Answer ……..$^\circ$ [3]

(c) **(i)** Calculate the volume of the prism *ABCDEF*.

Answer ….…..cm^3 [2]

(ii) The prism is melted and moulded into a hemisphere. Calculate the radius of the hemisphere.

Answer ….…..cm [2]

6 **Answer the whole of this question on the sheet of graph paper.**

The variables x and y are connected by the equation

$$y = \frac{x^2}{3} + \frac{2}{x} - 3.$$

Some corresponding values of x and y, correct to two decimal places, are given in the table below.

x	0.5	1	1.5	2	2.5	3	4	5
y	p	–0.67	–0.92	–0.67	–0.12	0.67	2.83	q

(a) Find the values of p and q.

Answer p =…….....

q=……..... [2]

(b) Using a scale of 2 cm to represent 1 unit on each axis, draw a horizontal x-axis for $0 \le x \le 5$ and a vertical y-axis for $-2 \le y \le 6$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the solutions to the equation $\frac{x^2}{3} + \frac{2}{x} = 3$.

Answer x =…… or ….....[2]

(d) By drawing a tangent, find the gradient of the curve at $(3, 0.67)$.

Answer ..... [2]

(e) By drawing a suitable line on the graph, solve the equation

$$\frac{x^2}{3}+\frac{2}{x}+2x-6=0$$

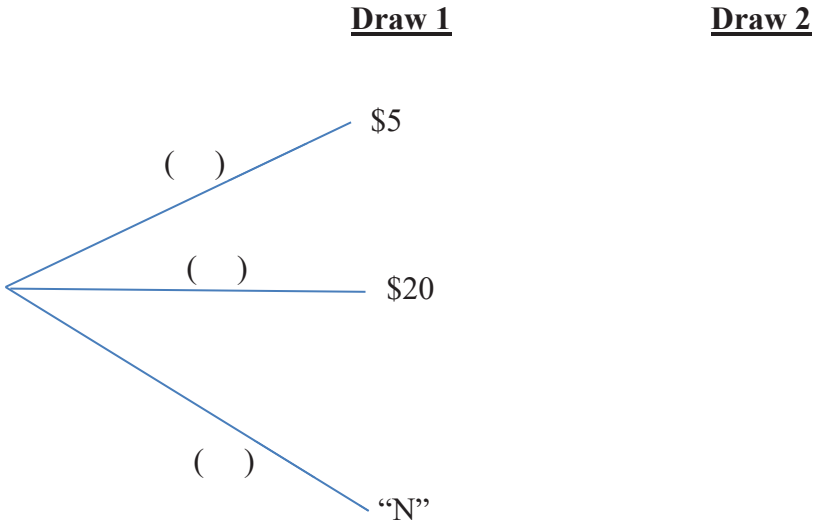
Answer Equation of line $y =$ .....[1]

$x =$……..... [1]

Name: ……..( ) Class: ……..

7 **(a)** In the lucky draw, there are 3 types of tickets to be drawn from a box: \$5 tickets, \$20 tickets and "N" tickets. If a customer draws a \$5 or a \$20 ticket, the ticket will be exchanged for a \$5 or \$20 shopping voucher respectively. However, if the customer draws an "N" ticket in the first draw, the "N" ticket will be placed back into the box and the customer will draw a ticket a second time. The second draw is the final one.

Given $\frac{3}{5}$ of the tickets are \$5 tickets, $\frac{1}{10}$ of the tickets are \$20 tickets and the rest are "N" tickets, complete the tree diagram below.



[2]

(i) Find the probability that a customer will draw an "N" ticket.

Answer ……..... [1]

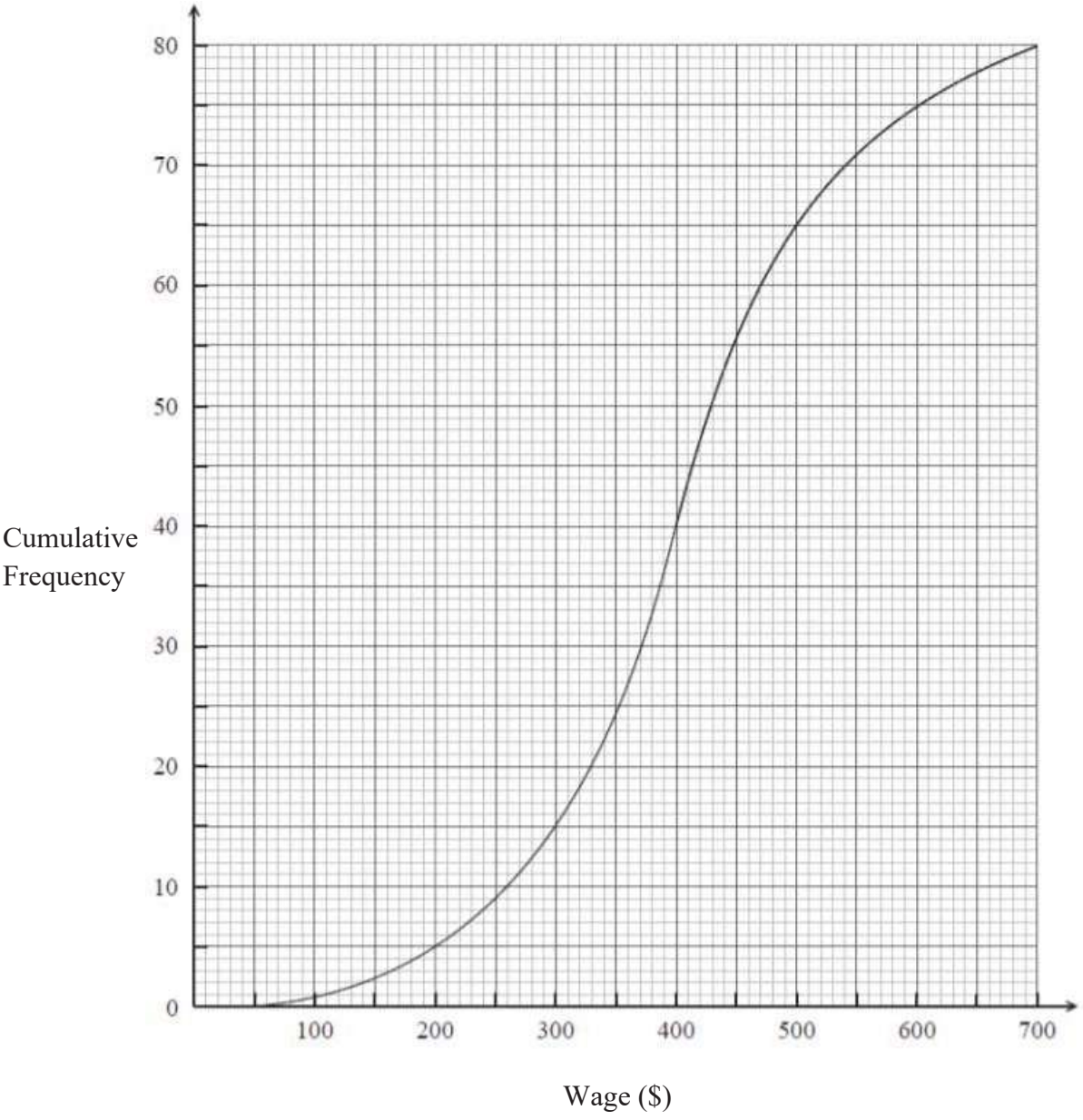
(ii) Find the probability that a customer will win a cash voucher.

Answer ……..... [2]

(iii) If both Mary and Peter take part in the lucky draw, what is the probability that at least one of them will win a cash voucher?

Answer ……..... [2]

(b) The cumulative frequency curve below shows the distribution of the weekly wages of 80 employees.



Use the curve to estimate

(i) the median wage,

Answer $…..... [1]

(ii) the interquartile range of the wages,

Answer $…..... [1]

(iii) the percentage of the employees who earned at least $500.

Answer ….....% [2]

8 **(a)** The first four terms in a sequence are $\frac{1}{3}, -\frac{1}{6}, \frac{1}{12}, -\frac{1}{24}$

Find the 5th term .

Answer ..... [1]

(b) The *n*th term of a sequence is given by $T_n = \frac{1}{n^2+1}$. Find the sum of the 4th and 5th terms.

Answer ..... [3]

(c) The first four terms in a sequence of numbers are given below

$$T_1 = 1^3 + 5 = 6$$
$$T_2 = 2^3 + 7 = 15$$
$$T_3 = 3^3 + 9 = 36$$
$$T_4 = 4^3 + 11 = 75$$

(i) Find T_5.

Answer ..... [1]

(ii) Express T_n in the form of $an^3 + bn^2 + cn + d$, where a, b, c and d are integers to be determined.

Answer $a =$ .....

$b =$ .....

$c =$ .....

$d =$ .....[2]

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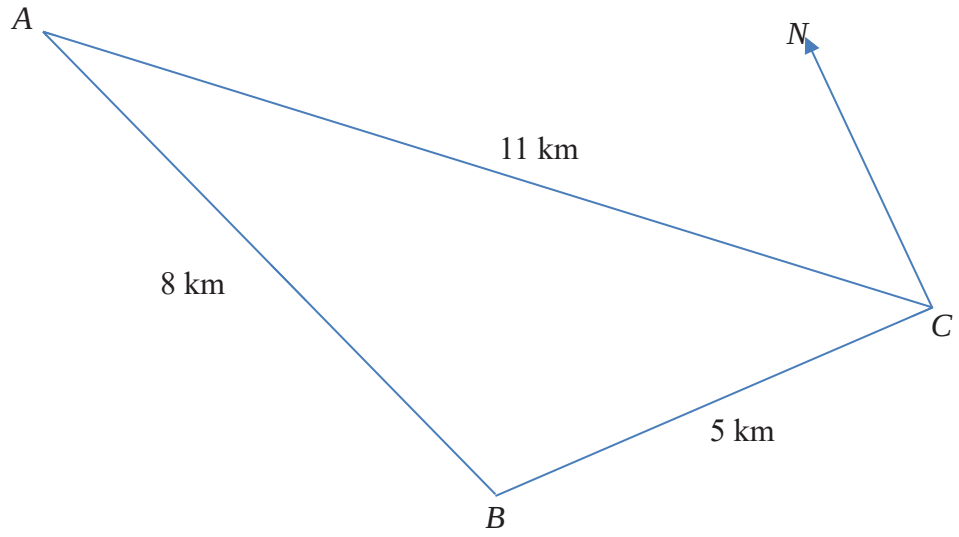
9 *ABC* represents a triangular plot of land on horizontal ground.
AB = 8 km, *BC* = 5 km, *AC* = 11 km and *B* is due west of *C*.



(a) Calculate

(i) the bearing of *A* from *C*,

Answer .....° [3]

(ii) the bearing of *C* from *A*,

Answer .....° [2]

(iii) the reflex angle *ABC*,

Answer ..... ° [2]

(iv) the area of the triangular plot of land *ABC*.

Answer ..... km^2 [2]

(b) A vertical lamp post is erected at point *A*. State with a reason, whether the angle of depression of *B* or of *C* is larger from the top of the lamp post.

Answer……..

…….[2]

10 Mr Wong is thinking of applying for a credit card that gives the most savings in terms of dining, grocery and petrol. His gross monthly expenses (before any discounts) are listed in the table below:

Type of Expenses	Amount ($)
Petrol	350
Grocery	100
Dining	400

After doing some research, he decided to narrow his options to the three cards listed below:

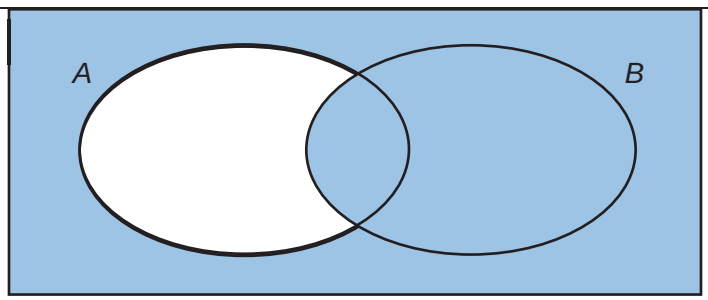
Credit Card	Savings on Petrol	Savings on Dining	Savings on Grocery
CBSH Card	• 14% upfront discount plus 5% cash rebate (on monthly petrol expenses) with monthly minimum spending of $600 on the CBSH card	• 5% cash rebate on dining expenses with monthly minimum spending of $600 on the CBSH card	• 5% cash rebate on grocery expenses with monthly minimum spending of $600 on the CBSH card
BSOP Card	• 15% upfront discount plus 6% cash rebate (on monthly petrol expenses) with minimum spending of $1000 on BSOP Card	• 5% cash rebate on all dining expenses with minimum monthly spending of $1000 on the BSOP Card	• 5% cash rebate on grocery expenses
CBCO Card	• 14% upfront discount plus 4.3% cash rebate (on monthly petrol expenses) with monthly minimum spending of $800 on the CBCO Card plus 2.1% cash rebate (on monthly petrol expenses) with monthly minimum spending of $400 on petrol	• 5% cash rebate on all dining expenses with monthly minimum spending of $800 on the CBCO Card	• 5% cash rebate on grocery expenses with monthly minimum spending of $800 on the CBCO Card

Which credit card should Mr Wong apply for so as to maximise his savings, given that he can only apply for one card and this card is to be used only for these three types of expenses? Show your working clearly. [9]

~ End of Paper 2~

Remember to check your work carefully

Marking Scheme for Sec 4E/5NA EMath P1

Qn	Solutions	Marks
1	$\frac{x}{4}+13=6$ $\frac{x}{4}=-7$ $x=-28$	B1
2	$25\text{ min}=\frac{25}{60}=\frac{5}{12}hrs$ $D=S\times T$ $D=60\times\frac{5}{12}=25km$	M1 A1
3	5, 5, 6, 9, 15	B1 (1st 3 numbers), B1 (last 2 numbers)
4	4047 m^2 = 1 Acre grows 20 000 sunflower plants $=20000\times1500$ seeds $=3\times10^7$ seeds $1\text{ m}^2=\frac{3\times10^7}{4047}$ seeds $1800/\text{m}^2=\frac{3\times10^7}{4047}\times1800$ seeds $=1.33\times10^7$	M1 A1
5	$\frac{3x}{5}-\frac{4(2-3x)}{7}=\frac{21x}{35}-\frac{20(2-3x)}{35}$ $=\frac{21x-20(2-3x)}{35}$ $=\frac{21x-40+60x}{35}$ $=\frac{81x-40}{35}$	M1 A1
6a	2	B1
6b		B1

7	$6ac+9ad-12bd-8bc$ $=3a(2c+3d)-4b(3d+2c)$ $=(3a-4b)(2c+3d)$	 M1 A1
8a	$\overrightarrow{OB}=\overrightarrow{OA}-\overrightarrow{BA}$ $\overrightarrow{OB}=\begin{pmatrix}-2\\5\end{pmatrix}-\begin{pmatrix}-4\\8\end{pmatrix}=\begin{pmatrix}2\\-3\end{pmatrix}$ $B=(2,\ -3)$	 B1
8b	$\left\lvert\overrightarrow{AB}\right\rvert=\sqrt{(-4)^2+(8)^2}=8.94$	B1
9	Mean = 10 720 g SD = 362.8 g	B1 B1
10	Cost of painting in 2017 = $\frac{73288.8}{1.2}=\$61074$ Cost of painting in 2016 = $\frac{61704}{1.08}=\$56550$ Percentage = $\frac{56550}{73288.8}\times100\%=77.2\%$	M1 M1 A1
11a	7 units represent 2.1*l* 1 units represent 0.3*l* 3 units represent 0.9*l*	 B1
11b	1 units represent 0.3*l* 12 units represent 3.6*l*	 B1
11c	Lemon : Simply Syrup : Coconut water $\frac{1}{2}$: $\frac{1}{3}$ 1 : 8 $\frac{1}{2}\times6$: $\frac{1}{3}\times6$ 1×2 : 8×2 3 : 2 : 16	 M1 A1
12	$x+(x-20)+3x=180$ $5x-20=180$ $5x=200$ $x=40°$ 40°, 20°, 120° **or** $(x+20)+x+3(x+20)=180$ $5x+80=180$ $5x=100$ $x=20°$ 40°, 20°, 120°	M1 M1 A1

13a	$y = kx$ **or** $x = ky$ $y = 5k$ ---------- (1) $\quad$ $5 = ky$ ---------- (1) $y = 17k$ ---------- (2) $\quad$ $17 = ky$ ---------- (2) (2) – (1); $\quad 17k - 5k = 54$ $\quad$ $\frac{17}{k} - \frac{5}{k} = 54$ $12k = 54$ $\quad$ $17 - 5 = 54k$ $k = 4.5$ $\quad$ $k = \frac{2}{9}$ $y = 4.5x$ $\quad$ $x = \frac{2}{9}y$	M1 M1 A1
13b	$y = 4.5x$ $84 = 4.5x$ $x = 18\frac{2}{3}$	B1
14a	20	B1
14b	1210	B1
14c	924	B1
15a	$-15 \le 8 - 3x$ $\quad$ $8 - 3x < 2$ $3x \le 23$ $\quad$ and $\quad$ $6 < 3x$ $x \le \frac{23}{3}$ $\quad$ $2 < x$ $2 < x \le \frac{23}{3}$	M1 (Either 1) A1
15b	3, 4, 5, 6, 7	B1
16a	Sum of interior angles = $(6-2)\times 180 = 720°$	B1
16b	Sum of angles $= 7 \times$ angles at a point $-$ [Sum of interior angle of hexagon + sum of interior angle of triangle] $= 7 \times 360 - [720 + 180]$ $= 1620°$ Or $\quad \angle a + \angle g + \angle f + \angle b = 4 \times (360 - 120) = 960°$ $\angle d + \angle c + \angle e = (3 \times 360) - 2(120) - 180 = 660°$ Sum of angles = $960° + 660° = 1620°$	M1 A1 M1 A1

17a	$\frac{A_A}{A_{A+B}} = (\frac{l_A}{l_{A+B}})^2$ $= (\frac{h}{2h})^2$ $= (\frac{1}{4})$	B1
17b	$\frac{200}{200+B} = (\frac{1}{4})$ $B = 600$	B1
17c	$\frac{V_A}{V_{A+B+C}} = (\frac{l_A}{l_{A+B+C}})^3$ $= (\frac{h}{3h})^3$ $= (\frac{1}{27})$ $\frac{V_A}{V_{A+B}} = (\frac{l_A}{l_{A+B}})^3$ $= (\frac{h}{2h})^3$ $= (\frac{1}{8})$ Vol of B : Vol of C = 7 : 19	M1 M1 A1
18	$\angle PQR = 235 - 205 = 30°$ $PR^2 = 35^2 + 45^2 - 2(35)(45)\cos 30°$ $= 522.0199781$ $PR = \sqrt{522.0199781}$ $= 22.8$ km	M1 M1 A1

19	$\angle DOE = 180 - 35 - 35 = 110°$ Area of segment = area of sector DOC – area of triangle DOC $= \frac{110}{360} \times \pi \times (7)^2 - \frac{1}{2}(7)(7)\sin 110°$ $= 24.01415413$ Area of section of circle = area of circle – 2 x area of segment $= \pi(7)^2 - 2(24.01415413)$ $= 106 cm^2$ OR $\angle DOE = 180 - 35 - 35 = 110°$ 2 x (Area of sector BOD + Area of triangle DOE) = 2 x ($\frac{180-110}{360} \times \pi \times (7)^2 + \frac{1}{2}(7)(7)\sin 110°$) = 2 (29.932396 + 23.224692) $= 106 cm^2$	M1 M1, M1 M1 A1 M1 M1 M1 M1,M1 A1
20a	Line L_1: $2y = 3x + 5$ Gradient = 1.5 Line L_2: $y = 1.5x + 5.5$ or $2y = 3x + 11$	 B1 B1
20b	$area = \frac{5}{3} \times 8 = 13\frac{1}{3}$ units^2	B1
20c	$\tan \angle ABO = \frac{\frac{5}{2}}{\frac{5}{3}} = 1.5$	B1
21a	$T = 2\pi\sqrt{\frac{l}{g}}$ $\frac{T}{2\pi} = \sqrt{\frac{l}{g}}$ $\left(\frac{T}{2\pi}\right)^2 = \frac{l}{g}$ $l = \frac{T^2 g}{4\pi^2}$	 M1 A1

21b	$l = \sqrt{5^2 + 2.5^2} = 5.59016994$ Total surface area = curved SA of cone + curved surface area of cylinder + base area $= \pi(2.5)(\sqrt{31.25}) + 2\pi(2.5)(5) + \pi(2.5)^2$ = 142 cm^2 (3sf)	 M1, M1 (any 2) A1
22a	$R = \begin{pmatrix} 6 & 5 & 3 \\ 9 & 7 & 4 \\ 10 & 8 & 2 \end{pmatrix} \begin{pmatrix} 7.20 \\ 10.80 \\ 32 \end{pmatrix}$ $= \begin{pmatrix} 193.20 \\ 268.40 \\ 222.40 \end{pmatrix}$	M1 A1
22b	R represent the cost of each hamper.	B1
22c	$T = \begin{pmatrix} 20 & 25 & 30 \end{pmatrix}$	B1
22d	$\begin{pmatrix} 20 & 25 & 30 \end{pmatrix} \begin{pmatrix} 193.20 \\ 268.40 \\ 222.40 \end{pmatrix}$ = \$ 17 246	M1 A1
23a	$\frac{60}{360} \times 100\% = 16.7\%$	B1
23b	30% rep 108° represent 36 boys 1° represent $\frac{36}{108}$ boys 360° represent $\frac{36}{108} \times 360 = 120$ boys	 M1 A1
23c	No of boys in group E = Total boys – boys in ($A + B + C + D$) = 120 – 30 – 36 – 18 – 20 = 16 boys	 M1 A1
24a	$\frac{20-n}{20}$	B1
24b	$\frac{20-n}{20} \times \frac{19-n}{19}$ $= \frac{(20-n)(19-n)}{380}$ or $= \frac{380-39n+n^2}{380}$	 B1

24c	$\frac{(20-n)(19-n)}{380}=\frac{39}{95}$ $(20-n)(19-n)=156$ $380-20n-19n+n^2=156$ $n^2-39n+224=0$ (Shown)	M1 A1
24d	$n^2-39n+224=0$ $(n-32)(n-7)=0$ Either $n=32$ or $n=7$ No of yellow marbles = 20 – 7 = 13	M1 M1 A1

2019 4E5N Prelims EMath P2 Mark Scheme		
Qn	Solution	Marks
1a	$27a^4-3=3\left(9a^4-1\right)$ $=3\left[\left(3a^2\right)^2-1\right]$ $=3\left(3a^2-1\right)\left(3a^2+1\right)$ Comments: Students did not factorise $(9a^4-1)$	M1 A1
1b	(i) $\frac{2(x-1)^2}{4y^3}\div\frac{6y(x-1)}{8y^2}=\frac{2(x-1)^2}{4y^3}\times\frac{8y^2}{6y(x-1)}$ $=\frac{2(x-1)}{3y^2}$ Comments: Students made careless mistake as they cancelled the powers instead of applying the indices rules	 B1
	(ii) $\frac{3}{m-2}-\frac{2}{3m-1}=\frac{3(3m-1)-2(m-2)}{(m-2)(3m-1)}$ $=\frac{7m+1}{(m-2)(3m-1)}$ Comments: Students made mistake when they expand $-2(m-2)$.	M1 A1

1c	$2^{2-x} = \frac{1}{\sqrt[3]{2^{5x+1}}}$ $2^{2-x} = 2^{-\frac{1}{3}(5x+1)}$ $2-x = -\frac{5}{3}x - \frac{1}{3}$ $\frac{2}{3}x = -\frac{7}{3}$ $x = -\frac{7}{2}$ Comments: Students did not apply the indices rules $\frac{1}{a^{-m}} = a^m$, $\sqrt[3]{a} = a^{\frac{1}{3}}$ and $1 = a^0$	M1 A1
1d	(i) $x^2 - 8x - 6 = (x-4)^2 - 16 - 6$ $= (x-4)^2 - 22$ Comments: Majority of the students did it correctly. (ii) $(x-4)^2 - 22 = 0$ $(x-4)^2 = 22$ $x = 4 \pm \sqrt{22}$ $x = -0.7$ *or* $x = 8.7$ (to 1 dp) Comments: Many students did not follow the instruction and use the requested method. Some of them did not correct the answers to one decimal place.	B1 M1 A1, A1
2	(a)(i) 3a - 2b (ii) $\frac{3}{7}$ (3a - 2b) (iii) 2a - b (iv) a + $\frac{1}{2}$ b	B1 B1 B1 M1A1

	(v) $\overrightarrow{FD} = \overrightarrow{FB} + \overrightarrow{BC} + \overrightarrow{CD}$ $= -\frac{1}{2}(2\mathbf{a} - \mathbf{b}) + \frac{1}{2}(2\mathbf{b}) + \frac{3}{7}(3\mathbf{a} - 2\mathbf{b})$ $= \frac{2}{7}\mathbf{a} + \frac{9}{14}\mathbf{b}$	M1 A1
	(b)(i) $\frac{2}{3}$	B1
	(ii) $\frac{\text{Area of } \Delta OBA}{\text{Area of } \Delta OCE} = \frac{\frac{1}{2} \times OB \times OA \times \sin BOA}{\frac{1}{2} \times OC \times OE \times \sin BOA}$ $= \frac{\frac{1}{2} \times 1 \times 2 \times \sin BOA}{\frac{1}{2} \times 2 \times 3 \times \sin BOA}$ $= \frac{1}{3}$	M1 A1
	Comments: Badly done. Students did not consider the direction of the vectors, answers without vector notation. Could not find the ratio of areas, answers given with units.	
3a	(i) $\frac{19600}{98000} \times 100\% = 20\%$	B1
	(ii) **Bank OCC** $A = 78400\left(1 + \frac{\frac{2.78}{2}}{100}\right)^{14} = \$95114.73$ Interest paid = $95114.73 - 78400 = \$16714.73$	B1 B1
	Bank DBB	

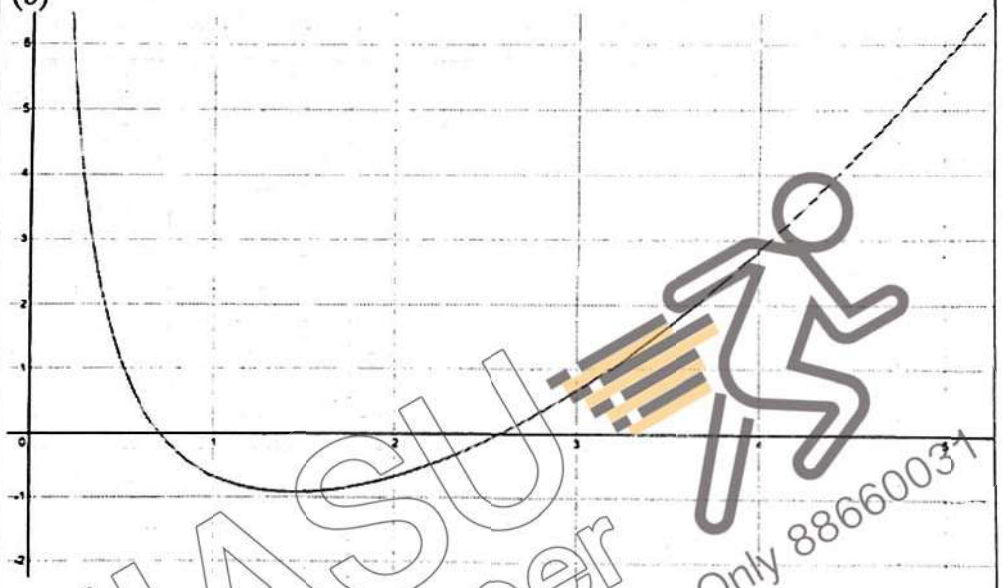
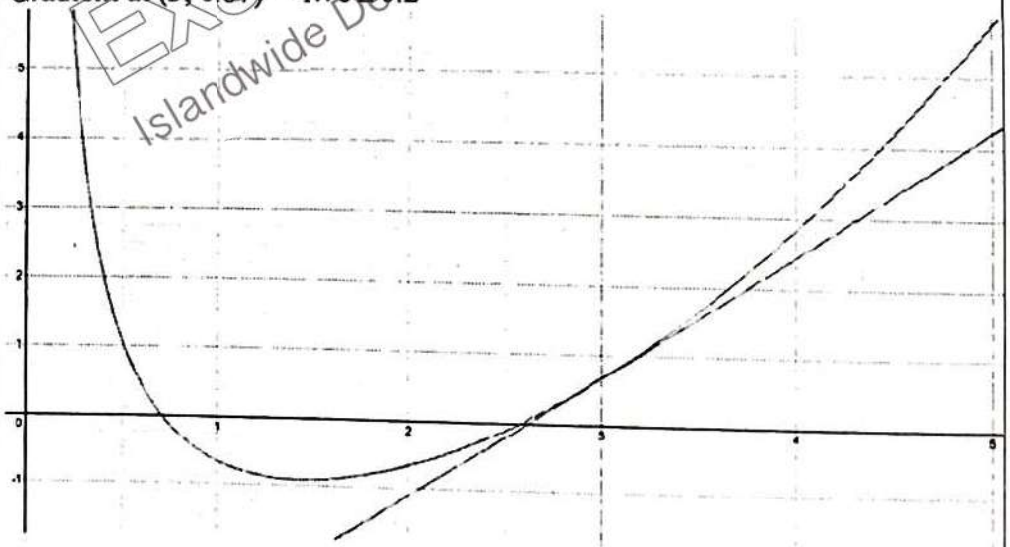
	$I = 78400 \times \frac{2.99}{100} \times 7 = \$16409.12$ Choose Bank DBB as lesser interest charged. Comments: Students thought that the bank with more interest is to be chosen. Forgot that this is a loan.	B1 B1
3b	(i) $2.25 \times 51 = \$114.75$ (ii) $2.08 \times 51 = RM106.08$ (iii)(a) Converting to Singapore dollars, Celine paid $\frac{106.08}{3.05} = S\$34.78$ She saves S$79.97 weekly (iii)(b) $\frac{79.97}{114.75} \times 100\% = 69.691\% \approx 69.7\%$ Comments: Students did not give answers correct to 2 decimal places.	B1 B1 M1 A1 B1
4	(a) (i) Angle DAT = 90° (tangent perpendicular to radius Angle AOD = 90° − 40° = 50° (sum of angles in a triangle) (ii) Angle AOC = 50° × 2 = 100° Angle ABC = 100° ÷ 2 = 50° (angle at centre = 2 times angle at Circumference) (iii) Angle ADC = 180° − 50° = 130° (angles in opp segments) (iv) Angle OCD = $\frac{180° - 50°}{2} = 65°$ (Base angles of isosceles triangle) Comments: Students did not write the angle properties properly.	B1 B1 M1 A1 B1 B1

	$I = 78400 \times \frac{2.99}{100} \times 7 = \$16409.12$ Choose Bank DBB as lesser interest charged. Comments: Students thought that the bank with more interest is to be chosen. Forgot that this is a loan.	B1 B1
3b	(i) $2.25 \times 51 = \$114.75$ (ii) $2.08 \times 51 = RM106.08$ (iii)(a) Converting to Singapore dollars, Celine paid $\frac{106.08}{3.05} = \$34.78$ She saves S$79.97 weekly (iii)(b) $\frac{79.97}{114.75} \times 100\% = 69.691\% \approx 69.7\%$ Comments: Students did not give answers correct to 2 decimal places.	B1 B1 M1 A1 B1
4	(a) (i) Angle DAT = 90° (tangent perpendicular to radius Angle AOD = 90° − 40° = 50° (sum of angles in a triangle) (ii) Angle AOC = 50° × 2 = 100° Angle ABC = 100° ÷ 2 = 50° (angle at centre = 2 times angle at Circumference) (iii) Angle ADC = 180° − 50° = 130° (angles in opp segments) (iv) Angle OCD = $\frac{180° - 50°}{2}$ = 65° (Base angles of isosceles triangle) Comments: Students did not write the angle properties properly.	B1 B1 M1 A1 B1 B1

2019 4E5N Prelims EMath P2 Mark Scheme		
Qn	Solution	Marks
1a	$27a^4 - 3 = 3\left(9a^4 - 1\right)$ $= 3\left[\left(3a^2\right)^2 - 1\right]$ $= 3\left(3a^2 - 1\right)\left(3a^2 + 1\right)$ Comments: Students did not factorise $(9a^4 - 1)$	M1 A1
1b	(i) $\frac{2(x-1)^2}{4y^3} \div \frac{6y(x-1)}{8y^2} = \frac{2(x-1)^2}{4y^3} \times \frac{8y^2}{6y(x-1)}$ $= \frac{2(x-1)}{3y^2}$ Comments: Students made careless mistake as they cancelled the powers instead of applying the indices rules (ii) $\frac{3}{m-2} - \frac{2}{3m-1} = \frac{3(3m-1) - 2(m-2)}{(m-2)(3m-1)}$ $= \frac{7m+1}{(m-2)(3m-1)}$ Comments: Students made mistake when they expand $-2(m-2)$.	B1 M1 A1

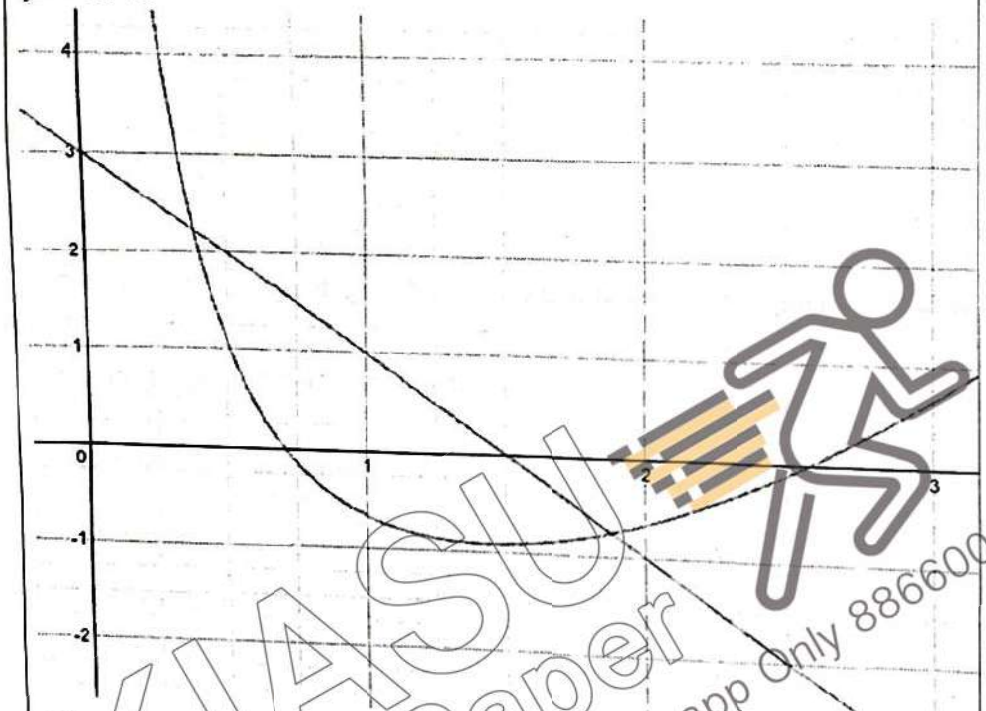
1c	$2^{2-x} = \frac{1}{\sqrt[3]{2^{5x+1}}}$ $2^{2-x} = 2^{-\frac{1}{3}(5x+1)}$ $2-x = -\frac{5}{3}x - \frac{1}{3}$ $\frac{2}{3}x = -\frac{7}{3}$ $x = -\frac{7}{2}$ Comments: Students did not apply the indices rules $\frac{1}{a^{-m}} = a^m$, $\sqrt[3]{a} = a^{\frac{1}{3}}$ and $1 = a^0$.	M1 A1
1d	(i) $x^2 - 8x - 6 = (x-4)^2 - 16 - 6$ $= (x-4)^2 - 22$ Comments: Majority of the students did it correctly. (ii) $(x-4)^2 - 22 = 0$ $(x-4)^2 = 22$ $x = 4 \pm \sqrt{22}$ $x = -0.7$ *or* $x = 8.7$ (to 1 dp) Comments: Many students did not follow the instruction and use the requested method. Some of them did not correct the answers to one decimal place.	B1 M1 A1, A1
2	(a)(i) 3a - 2b (ii) $\frac{3}{7}$ (3a - 2b) (iii) 2a - b (iv) $a + \frac{1}{2}b$	B1 B1 B1 M1A1

	(b) $\tan 40^\circ = \frac{5}{TC}$ $TC = 5.9588 cm$ Area of $\Delta OTC = \frac{1}{2} \times 5.9588 \times 5 = 14.897 cm^2$ Area of sector $ODC = \frac{1}{2} \times (5)^2 \times \frac{50\pi}{180} = 10.908\ cm^2$ Area of shaded region $= 14.897 - 10.908 = 3.989 \approx 3.99 cm^2$ Comments: Some students could not find the area of sector correctly. Did not convert the angle from degrees to radians correctly or choose the right formula for area of sector.	M1 M1 M1 A1
5	(a) $3^2 = 8^2 + 8^2 - 2(8)(8)\cos ABF$ $\angle ABF = \cos^{-1}\left(\frac{119}{128}\right)$ $= 21.61384575^\circ$ $= 21.6^\circ$ (shown) (b) Length of BE $= \sqrt{12^2 + 8^2} = 14.422 cm$ Area of triangle DCE $= \frac{1}{2} \times 8 \times 8 \times \sin 21.614^\circ = 11.787 cm^2$ Perpendicular height from E to CD $= \frac{2 \times 11.787}{8} = 2.94675 cm$ Angle of elevation $= \sin^{-1}\left(\frac{2.94675}{14.422}\right) = 11.8^\circ$ Comments: Students used the wrong triangle EDB to find angle of elevation of E from B and assumed that $\angle EDB = 90^\circ$. (c)(i) Volume of prism $= 11.787 \times 12 = 141.447 cm^3 \approx 141 cm^3$	B1, B1 B1 M1 M1 A1 M1 A1

	(c)(ii)Since volume of prism equals to volume of hemisphere, Radius $= \sqrt[3]{\frac{141.447 \times 3}{2\pi}} = 4.0723\,cm \approx 4.07\,cm$ <u>Comments:</u> Students did not apply the formula to find volume of hemisphere.	M1 A1
6	(a) p = 1.08, q = 5.73 (b)  (c) $x = 0.7 \pm 0.1$ or $x = 2.6 \pm 0.1$ (d) Gradient at (3, 0.67) = 1.78 ± 0.2 	B1, B1 B1: correct plot B1: correct shape B1: correct scale and axes B1 B1 B1: draw the tangent line on graph B1 for the answer

(e)

$y = -2x + 3$



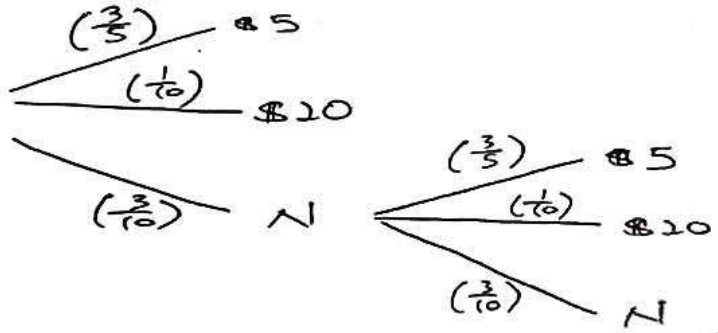
B1

$x = 1.88$ (accept $x = 1.85$ to 1.9)

B1

Comments:

Many students did not answer the part: equation of the line.

7	(a) 	B1: draw 1 B1: draw 2
	(a)(i) $\frac{3}{10}$	B1
	(ii) $\frac{3}{5}+\frac{1}{10}+\left(\frac{3}{10}\times\frac{3}{5}\right)+\left(\frac{3}{10}\times\frac{1}{10}\right)=\frac{91}{100}$	M1 A1
	(iii) $\left(\frac{91}{100}\right)^2+2\left(\frac{91}{100}\times\frac{9}{100}\right)=\frac{9919}{10000}$	M1 A1
	(b)(i) \$400	
	(ii) $470-330=\$140$	B1
	(iii) $\frac{15}{80}\times100\%=18.75\%$	B1 M1 A1
	Comments: 1. For part (a), many students drew the tree diagram for the 2nd draw for the cash vouchers side. This would not have happened if they understood the question that once the customer drew a cash voucher, they will not be given a chance to draw again. 2. For part (aiii), many students did not know that they should use the answer to part (aii) to help them find the answer. They went on to use other ways to find the answer which was wrong.	

	3. For part (biii), many left their answers as 18.8 (correct to 3 sf), which is not right as the answer is 18.75 which is an exact answer, hence had to penalize students who rounded their answers to 3 sf.	
8	(a) $T_5 = \frac{1}{48}$ (b) $T_4 = \frac{1}{17}$ $T_5 = \frac{1}{26}$ $Sum = \frac{1}{17} + \frac{1}{26} = \frac{43}{442}$ (c)(i) $T_5 = 5^3 + 13 = 138$ (ii) $T_n = n^3 + 2n + 3$ $a = 1,\ b = 0,\ c = 2,\ d = 3$ Comments: This question is ok and most students are able to get full marks. Those who did not made mistakes/gave up the last part which they should not have as it was just an expansion of algebraic expression.	B1 B1 B1 B1 B1 B2: all correct B1: any 1 correct
9	(a)(i) $\text{Bearing} = 270^\circ + \cos^{-1}\left(\frac{11^2 + 5^2 - 8^2}{2 \times 11 \times 5}\right) = 270^\circ + 41.801^\circ \approx 311.8^\circ$ (ii) $\text{Bearing} = 180^\circ - \left(90^\circ - 41.8^\circ\right) = 131.8^\circ$ (iii) $\frac{8}{\sin 41.8} = \frac{11}{\sin ABC}$ $\angle ABC\,(\text{acute}) = 66.4^\circ$ However, angle ABC is obtuse (seen from the diagram), hence actual angle ABC $= 180^\circ - 66.4^\circ = 113.6^\circ$ Reflex $\angle ABC = 360^\circ - 113.6^\circ = 246.4^\circ$ OR	M2 A1 M1 A1 M1 A1 M1 A1 B1 B1

	$\cos ABC = \frac{8^2 + 5^2 - 11^2}{2 \times 5 \times 8}$ Angle $ABC = 113.5782^\circ$ Reflex $\angle ABC = 360^\circ - 113.5782^\circ = 246.4^\circ$ (to 1 dp) (iv) Area $= \frac{1}{2} \times 11 \times 5 \times \sin 41.8 = 18.3\, km^2$ (b) Point B. Point B is nearer to point A than point C <u>Comments:</u> 1. Students lost marks in part (iii), especially those who used sine rule to get the angle ABC. Many did not find the obtuse angle ABC and used the acute angle ABC instead as they have forgotten that $\sin\theta = \sin(180^\circ - \theta)$. 2. Many students leave their answers to 3 sf for angles which is incorrect as it should be to 1 dp. Pls take note of this small but important detail.	
10	<u>CBSH Card</u> Petrol savings $= 0.14 \times 350 + 0.05 \times 350 = \$66.50$ Dining Savings $= 0.05 \times 400 = 20$ Grocery savings $= 0.05 \times 100 = 5$ Total savings = \$91.50 <u>BSOP Card</u> Petrol savings $= 0.15 \times 350 = \$52.50$ Dining Savings $= 0$ (as minimum monthly spending on the card is less than \$1000) Grocery savings $= 0.05 \times 100 = 5$ Total savings = \$57.50 <u>CBCO Card</u> Petrol savings $= 0.14 \times 350 + 0.043 \times 350 = \$64.05$ Dining Savings $= 0.05 \times 400 = 20$ Grocery savings $= 0.05 \times 100 = 5$ Total savings = \$89.05 He should apply for the CBSH card	B1 B1 B1 B1 B1 B1 B1 B1 B1

Comments:

1. Some students use the amount that was listed in the criteria for the rebates/discount to be used: example, students used $1000 for calculation of savings for BSOP card rather than the expenses of Mr Wong which was given in the question. This is a result of misinterpreting the question.

2. Students cannot understand the term 'upfront discount' which means regardless of the amount spent, the discount will be given the moment the customer presents the card. Quite a number of students lost marks here.

NAME: _____ ( ) CLASS: _____

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2019
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

MATHEMATICS

4048/01

Paper 1

Date: 27 August 2019

Duration: 2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to 3 significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use	
Paper 1	/80
Paper 2	/100
Total	%

Setter: Miss Shamsiah Zainalabidin

This question paper consists of 19 printed pages including the cover page.

Name: _____ ( ) Class: _____

Mathematical Formulae

Compound interest

$$Total\ amount = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of a triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Name: _____ ( ) Class: _____

Answer **all** the questions.

1 (a) Express 3780 as a product of its prime factors.

Answer (a) ..... [1]

(b) **Hence**, find the smallest integer by which 3780 must be multiplied to obtain a perfect square.

Answer (b) ..... [1]

---

2 Given that $\mathbf{A} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -3 & 9 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}$, find

(a) $\frac{1}{2}\mathbf{BA}$,

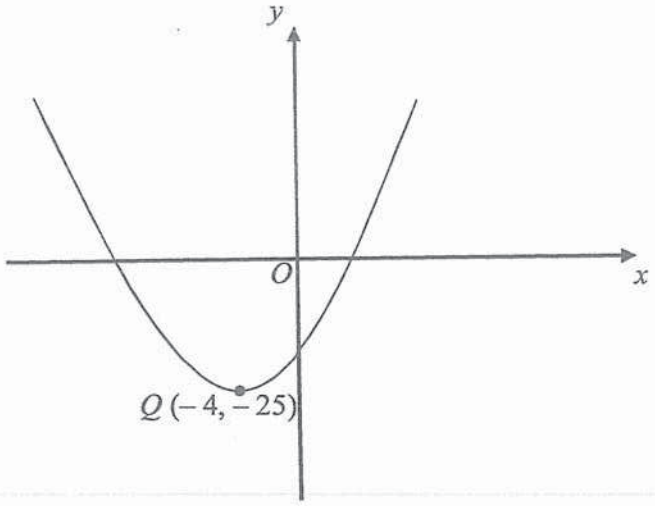
Answer (a) ..... [1]

(b) $\mathbf{C}^2$.

Answer (b) ..... [1]

Name: _____ ( ) Class: _____

3 The curve $y = (x - 1)(x + k)$ has a minimum point Q as shown.



(a) Write down the equation of the line of symmetry of this curve.

Answer (a) ..... [1]

(b) Write down the value of k.

Answer (b) k =..... [1]

---

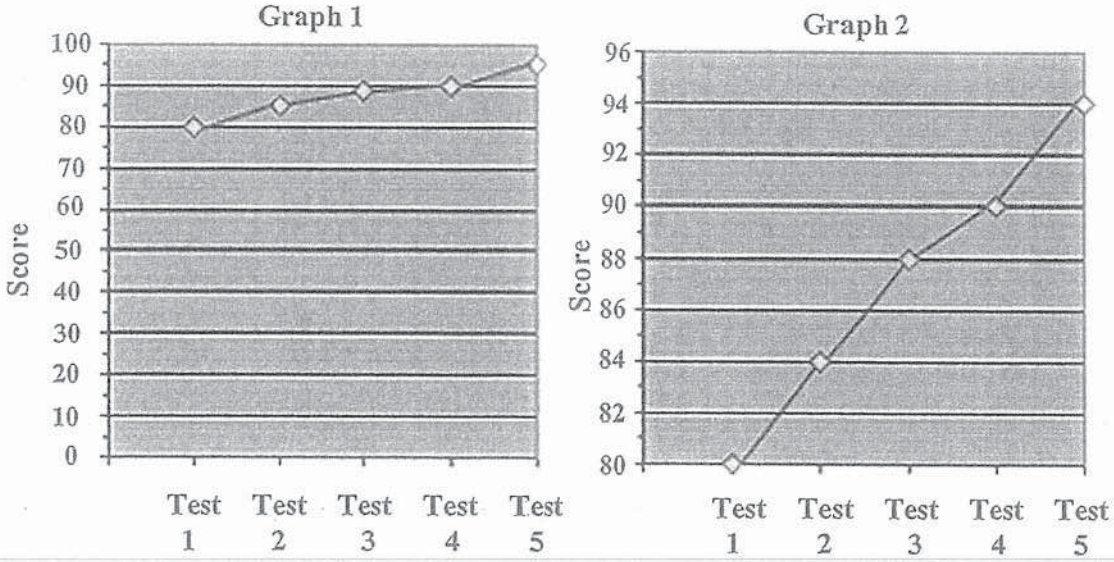
4 The volume, V cm^3, of a cylinder is directly proportional to r^2h, where r is the radius and h is the height of the cylinder. Find the percentage change in the volume when the radius is increased by 50% and the height is decreased by 20%.

Answer... ..... % [2]

Name: _____( ) Class: _____

5 Charlene wanted to impress her parents by showing the rapid increase in her marks in 5 tests.

Suggest, with reason, which graph she should use to impress her parents.



Answer.....

.....

.....

..... [2]

---

6 The estimated atomic mass of 12 billion nitrogen atoms is 2.80×10^{-9} grams.

(a) Express the mass of 1 nitrogen atom in picograms, leaving your answer in standard form.

[1 pico = 10^{-12}]

Answer (a) ..... picograms[1]

(b) The atomic mass of a helium atom is 6.684×10^{-24} g. Express the ratio of the mass of a helium atom to a nitrogen atom in the form of n : 1, leaving your answer in standard form.

Answer (b) ..... : 1 [2]

Name: _____ ( ) Class: _____

7 The height of Mount Kiki is 4.2 km and the temperature at the foot of the mountain is 31°C. The temperature decreases constantly at a rate of 8 °C for every 700 m.

(a) Calculate the temperature at the peak of the mountain.

Answer (a).....°C [1]

(b) Calculate the height from the foot of the mountain at which the temperature is 7 °C.

Answer (b)..... m [1]

(c) A man took 6 hours 43 minutes to climb from the foot of the mountain to the peak. Given that he reached the peak at 12 25, at what time did he begin his climb?

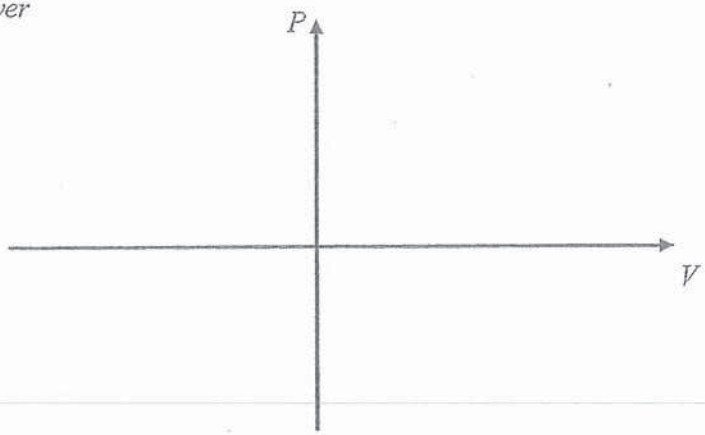
Answer (c)..... [1]

Name: _____ ( ) Class: _____

8 The pressure, P, of a fixed mass of gas at a constant temperature is inversely proportional to the volume, V, of the gas.

(a) Sketch the graph of P against V on the axis provided.

Answer [1]



(b) When the pressure of the gas is 4 Nm^{-2}, the volume is 8 m^3. Find P when $V = 12$ m^3.

Answer (b) ..... Nm^{-2} [1]

---

9 A map is drawn to a scale of 1 : 120 000.

(a) Calculate the actual distance, in km, represented by 6.3 cm on the map.

Answer (a) ..... km [1]

(b) A lake has an actual area of 3.9 km^2. Find the area of the lake on the map, in square centimetres.

Answer (b) ..... cm^2 [2]

Name: _____ ( ) Class: _____

10 Consider the number pattern:

$$1 + 3 = 4 = 2 \times 2$$
$$1 + 3 + 5 = 9 = 3 \times 3$$
$$1 + 3 + 5 + 7 = 16 = 4 \times 4$$
$$1 + 3 + 5 + 7 + 9 = 25 = 5 \times 5$$

(a) Write down the sixth line in the pattern.

Answer (a) ..... [1]

(b) Using the above number pattern, find the sum $1 + 3 + 5 + 7 + 9 + \ldots + 81$

Answer (b) ..... [1]

---

11 John deposited \$10 000 in a bank paying an interest of 10% per annum, compounded half yearly. Calculate the amount of interest he would receive after 2 years.

Answer \$ ..... [3]

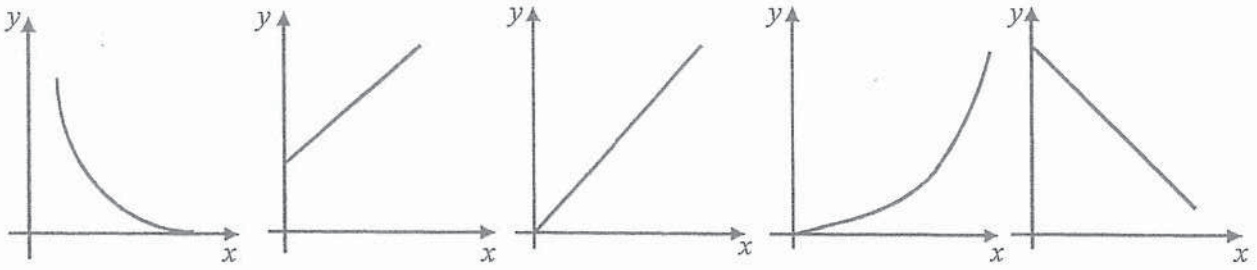
Name: _____ ( ) Class: _____

12 Match the correct graphs A to E, found below to represent each of the following statements.

(a) The cost, y, of taxi fare which consists of a fixed charge plus an amount proportional to the distance travelled, x.

(b) The volume, y, of a sphere is proportional to the cube of the radius x.

(c) The distance travelled by an object, y, varies directly with the time taken, x.



A B C D E

Answer (a) ..... [1]

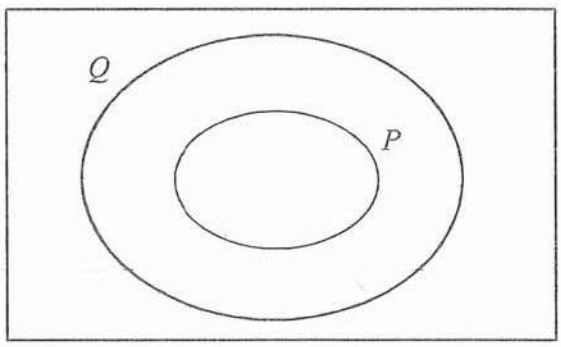
(b) ..... [1]

(c) ..... [1]

---

13 (a) On the Venn diagram in the answer space, shade the region which represents $(P \cup Q')'$.

Answer



[1]

(b) $\varepsilon = \{ x: x \text{ is an integer between 0 and 21} \}$

$A = \{ x: x \text{ is a multiple of 5} \}$

$B = \{ x: x \text{ is not a prime number} \}$

(i) List the elements contained in the set $(A \cup B)'$.

Answer(b)(i)..... [1]

(ii) Find $n(A \cap B)$.

Answer(b)(ii)..... [1]

Name: _____ ( ) Class: _____

14 (a) Solve the inequality $4-3x \le \frac{2-x}{3} < \frac{4+x}{5}$.

Answer (a) ..... [2]

(b) Hence, represent the solution on the number line below. [1]

Answer

---

15 (a) Express $x^2-9x+45$ in the form $(x-p)^2+q$.

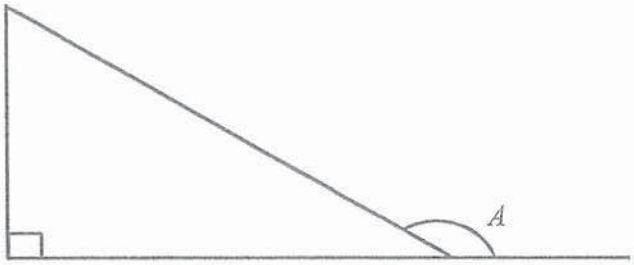
Answer(a) ..... [1]

(b) Hence, solve the equation $x^2-9x+45=50$, giving your answers correct to two decimal places.

Answer(b) $x =$ ..... or ..... [2]

Name: _____ ( ) Class: _____

16 In the diagram, $\sin A = \frac{5}{13}$ and angle A is an obtuse angle.



Leaving your answer, as a fraction, find the value of

(a) $\sin A - \cos A$,

Answer (a) ..... [2]

(b) $\cos(180° - A) + \tan(A - 90°)$.

Answer (b)..... [2]

Name: _____ ( ) Class: _____

17 **(a)** Factorise completely $4a^2 + 2ab - 14xb - 28ax$.

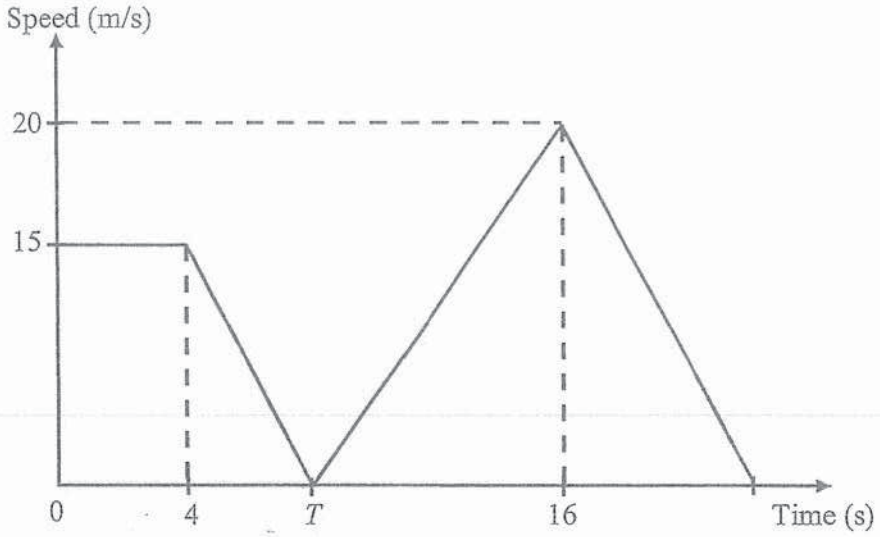
Answer(a) ..... [2]

(b) Solve the equation $\frac{3x+1}{7} = -\frac{3-x}{4}$.

Answer(b) $x =$ .....[2]

Name: _____ ( ) Class: _____

18 The graph below shows the speed-time graph of a moving object.



(a) Describe the motion of the object for the first 4 seconds.

Answer(a).....

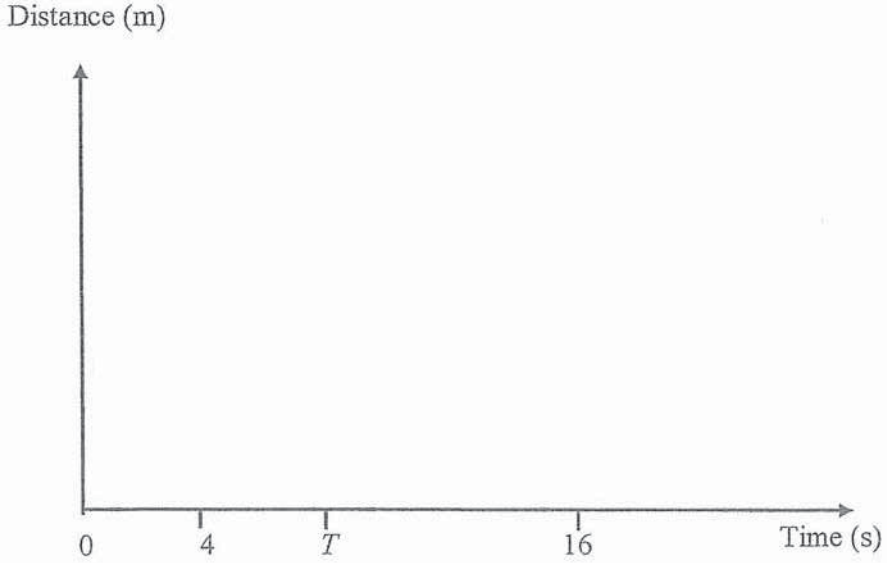
.....[1]

(b) Given that after 4 seconds, the object started to decelerate at a rate of 5 m/s^2, find the value of T.

Answer(b) ..... [1]

(c) Sketch the distance time graph of the object for the first 16 seconds.

Answer(c) [2]



Name: _____( ) Class: _____

19 Construct a quadrilateral $PQRS$ such that $PQ = 10$ cm, $QS = QR = 9$ cm, $RS = 5$ cm and $\angle PQR = 120°$. PQ has already been drawn. [2]

(a) Construct the perpendicular bisector of PQ. [1]

(b) The perpendicular bisector meets PS at T. Hence, measure and write down the length RT.

Answer (a)

P Q

Answer(b) RT = ... cm [1]

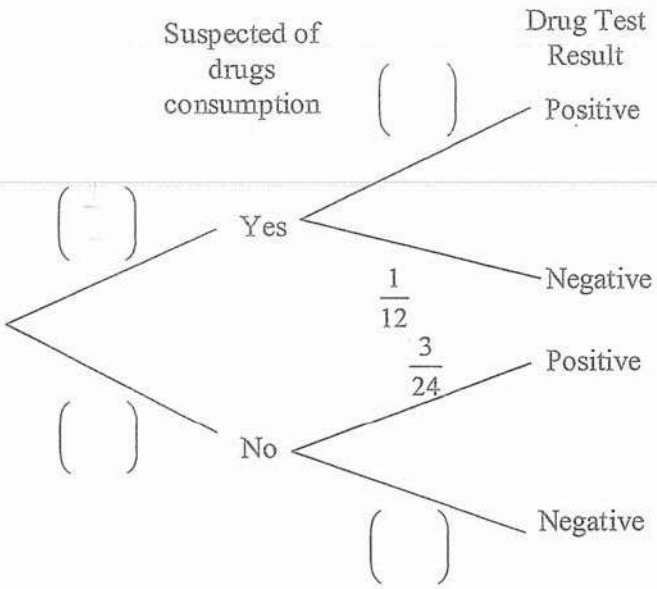
Name: _____ ( ) Class: _____

20 The table below shows the drug testing results of 36 athletes for 2018 Olympic Games.

		Drug test Results	
		Positive	**Negative**
Suspected of drugs consumption	**Yes**	11	1
	No	3	21

(a) Present the results in the probability tree diagram below.

Answer(a) [2]



(b) Find the probability that an athlete

(i) is suspected of taking drugs and tested positive,

Answer(b)(i)..... [1]

(ii) receives a negative test result.

Answer(b)(ii)..... [2]

Name: _____ ( ) Class: _____

21 (a) Simplify $\left(\frac{a^4 - a^3}{a^3}\right) - 3(a)^0$.

Answer (a) ..... [2]

(b) Given that $\frac{1}{9^{1-3x}} = 243^{\frac{x}{2}-1}$, find the value of x.

Answer (b) x = ... [3]

Name: _____ ( ) Class: _____

22 The numbers 4, 6, 7, 9, 2, 5, 9, 12, 2, x and y have a mean of 7 and a mode of 9. Find

(a) the values of the two numbers x and y, given that $x < y$,

Answer (a) $x =$ ....., $y =$ ..... [2]

(b) the median,

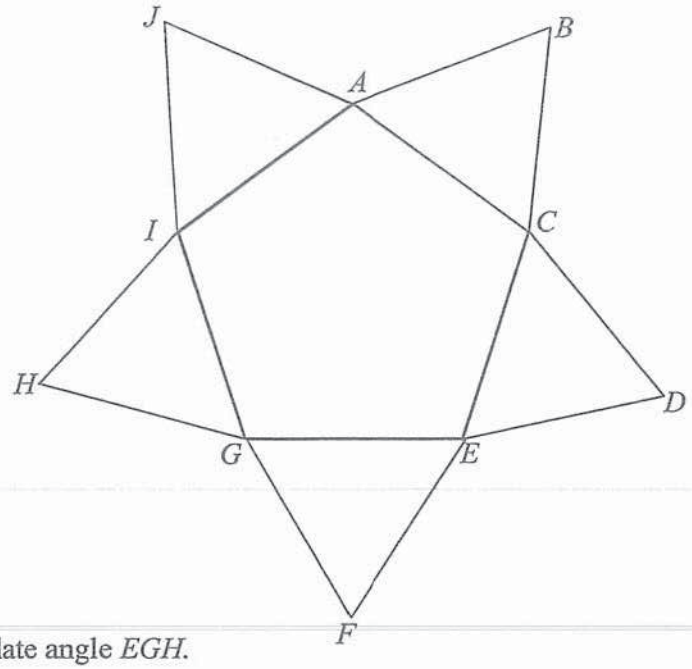
Answer (b) ..... [1]

(c) the standard deviation of this set of eleven numbers.

Answer (c) ..... [2]

Name: _____ ( ) Class: _____

23 The figure below consists of a pentagon and five identical equilateral triangles.



(a) Calculate angle *EGH*.

Answer(a).....° [2]

(b) Explain why $AI = AB$.

Answer.....

.....

.....

..... [1]

(c) Calculate angle *AEI*.

Answer(c).....° [2]

Name: _____ ( ) Class: _____

24 (a) Expand and simplify $y(y-2)+12y^2-6y$.

Answer(a) ..... [2]

(b) Express $\frac{6}{3y+7}-\frac{1}{49-9y^2}$ as a single fraction in its simplest form.

Answer(b)..... [3]

End of paper

NAME: _____ ( ) CLASS: _____

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2019
SECONDARY 4 EXPRESS/ 5 NORMAL (ACADEMIC)

MATHEMATICS 4048/02

Paper 2

Date: 28 August 2019 Duration: 2 hours 30 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

For Examiner's Use	
Paper 2	/ 100

Setter: Miss Lee CP

This paper consists of 30 printed pages including 4 blank pages.

Name: _____ ( ) Class: _____

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of a triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Name: _____ ( ) Class: _____

Answer all the questions.

1 (a) Express $\frac{3x}{x-3}+\frac{2}{x+4}$ as a single fraction. [2]

(b) Using factorisation, simplify fully $(x^2 + 5)^2 - (x^2 - 3)^2$. [2]

Name: _____ ( ) Class: _____

1 (c) Solve $2x^3 - 13x^2 - 24x = 0$. [3]

(d) n is an integer. Showing your working clearly, explain why the sum of $\frac{1}{2}n(n+1)$ and $\frac{1}{2}(n+1)(n+2)$ is always a square number. [2]

Name: _____ ( ) Class: _____

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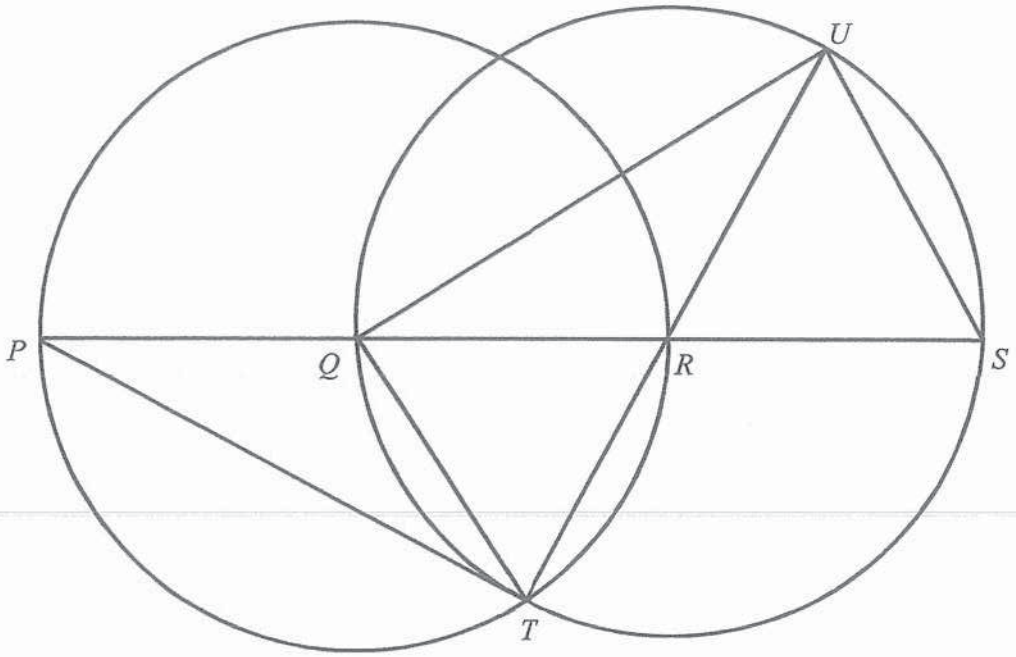
Name: _____ ( ) Class: _____

2 The diagram shows two circles with equal radii.
P, T and R are points on the circle with centre Q.
Q, T, S and U are points on the circle with centre R.
$PQRS$ is a straight line.



(i) Show that the triangles PTR and UQT are congruent. [3]

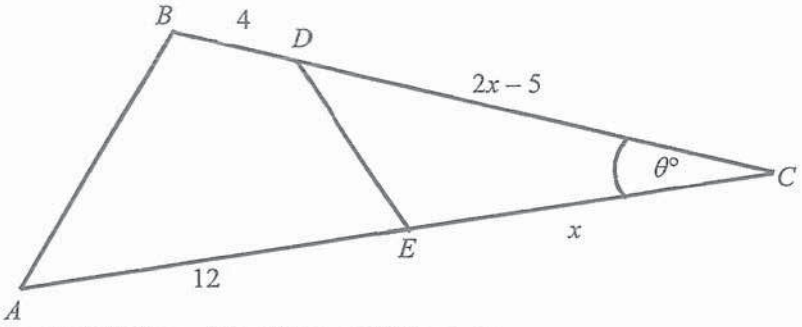
2 (ii) Name another triangle that is congruent to PTR. [1]

(iii) Explain why TQ is parallel to SU. [1]

(iv) Stating the reasons clearly, find the value of angle UQR. [1]

Name: _____ ( ) Class: _____

3



AEC and BDC are straight lines. $AE = 12$ cm and $BD = 4$ cm.

$CE = x$ cm and $CD = (2x - 5)$ cm. Angle $ACB = \theta°$.

(a) Show that $\frac{Area\ of\ triangle\ CDE}{Area\ of\ triangle\ ABC} = \frac{CE \times CD}{AC \times BC}$. [2]

(b) It is given that $\frac{Area\ of\ triangle\ CDE}{Area\ of\ triangle\ ABC} = \frac{1}{3}$.

Using the result from part **(a)**, form an equation in x and show that it simplifies to $2x^2 - 19x + 6 = 0$. [3]

Name: _____ ( ) Class: _____

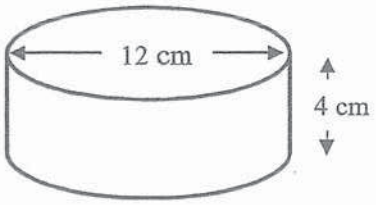
3 (c) (i) Solve the equation $2x^2 - 19x + 6 = 0$, giving your answers correct to 2 decimal places. [3]

(ii) State, with a reason, which of these solutions does not apply to triangle CDE. [1]

(d) Given that $\theta = 25$, calculate DE. [3]

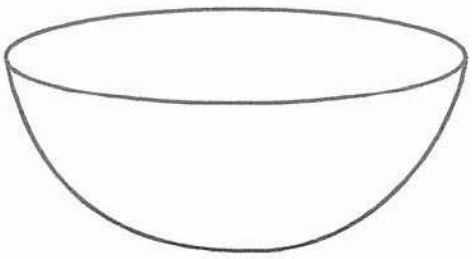
Name: _____( ) Class: _____

4 The diagram below shows an open cylindrical container with a diameter of 12 cm and height of 4 cm.



Container

(a) Assuming the thickness of the container is negligible, calculate the area of material needed to make one container. Give your answer correct to the nearest square centimetre. [3]



Pan

(b) A hemispherical pan is completely filled with 13 litres of soup. As many containers as possible are completely filled with the soup from the pan.

(i) Calculate the number of containers which are filled. [3]

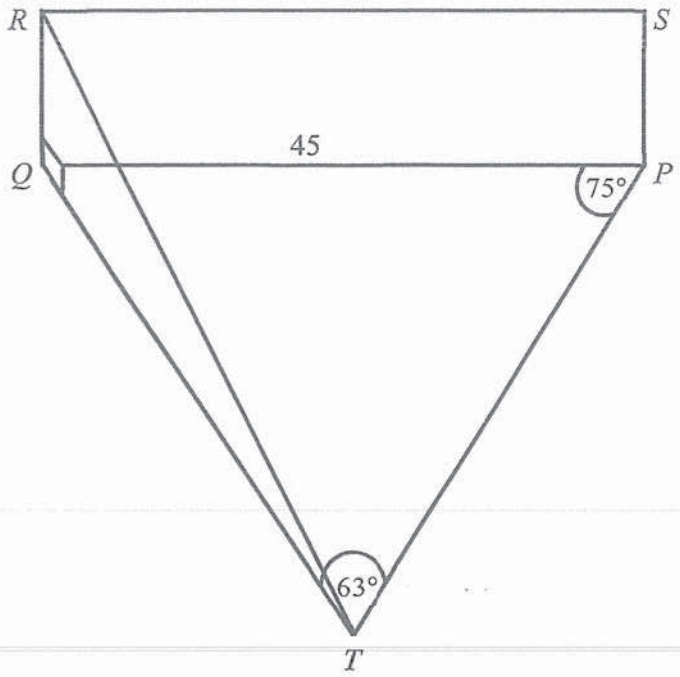
4 (b) (ii) Calculate the volume of soup which is left in the pan, giving your answer in cubic centimetres. [2]

(iii) Calculate the radius of the hemispherical pan, giving your answer correct to the nearest millimetre. [2]

(c) Peter has two different containers, which are geometrically similar to each other. The heights of the containers are in the ratio of 2 : 3. Write down the ratio of the volumes of soup these containers hold when full. [1]

Name: _____ ( ) Class: _____

5



In the diagram, the rectangle $PQRS$ represents a vertical cliff face.
The foot of the cliff, PQ, runs from East to West, and is at sea level.
A ship is in the sea at T.
Angle $QPT = 75°$, angle $PTQ = 63°$ and $PQ = 45$ m.

(a) Find the bearing of T from Q. [2]

Name: _____ ( ) Class: _____

5 (b) Show that $QT = 48.8$ m, correct to three significant figures. [2]

(c) Calculate the shortest distance from the ship to the cliff. [2]

5 (d) The angle of depression of the ship when viewed from R is $16°$.

(i) Find the height of the cliff. [2]

(ii) Calculate the greatest possible value of the angle of elevation of the top of the cliff when viewed from the ship. [2]

Name: _____ ( ) Class: _____

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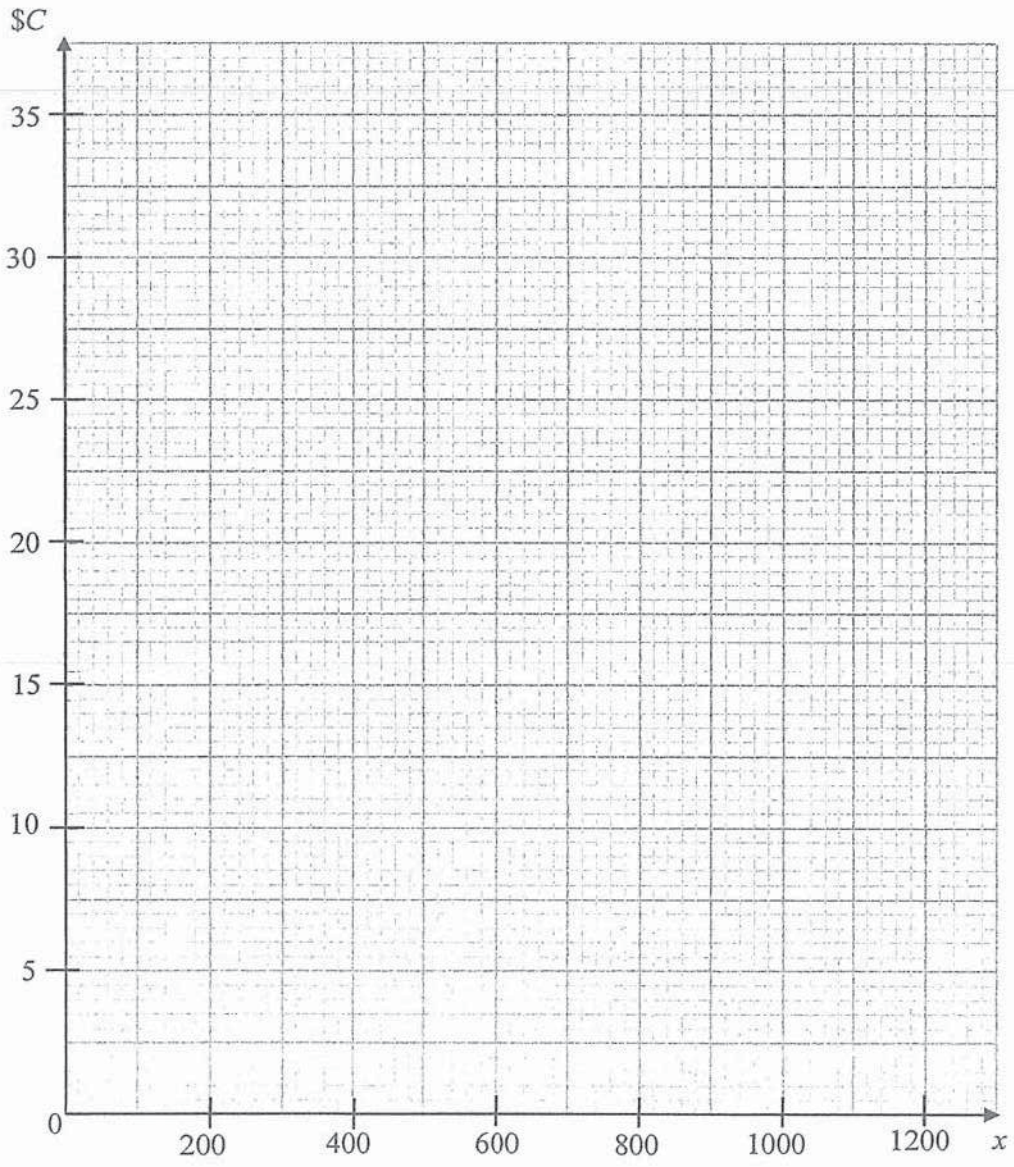
6 When x copies of a book are printed, the cost \$$C$ of each copy is given by the formula $C = 10 + \frac{2400}{x}$.

(a) The table gives some values of x and the corresponding values of C.

x	100	200	300	400	600	800	1200
C	34	22	18	16	14	13	p

(i) Find the value of p. [1]

(ii) On the grid, plot the points given in the table and join them with a smooth curve. [3]



6 (b) Use your graph to estimate the number of books to be printed if the cost of printing each book is \$15. [1]

(c) (i) By drawing a tangent, find the gradient of the curve at the point where $x = 300$. [2]

(ii) Describe briefly what this gradient represents. [1]

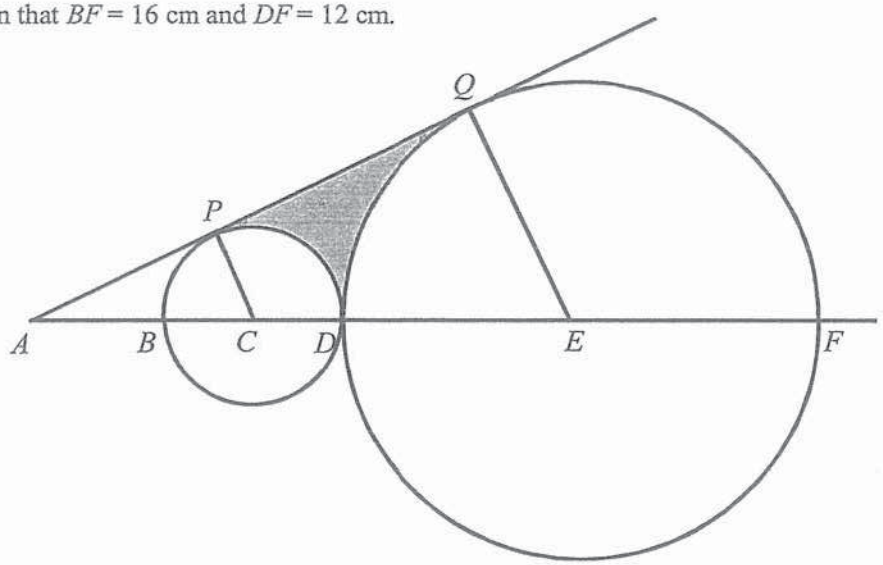
(d) In order to sell x books, the selling price of each book must be $\$\left(25-\frac{x}{60}\right)$.

(i) On the axes, used in part (a), draw the graph of $C = 25-\frac{x}{60}$ for the values of x from 0 to 1200. [2]

(ii) Use your graphs to find the range of the number of books that should be printed if no loss to be incurred. [1]

Name: _____ ( ) Class: _____

7 The diagram shows a tangent APQ to two circles with centre C and E.
The points A, B, C, D, E and F lie on the same straight line.
It is given that $BF = 16$ cm and $DF = 12$ cm.



(a) **(i)** Show that the triangles APC and AQE are similar. [2]

(ii) Hence, find the length of AB. [2]

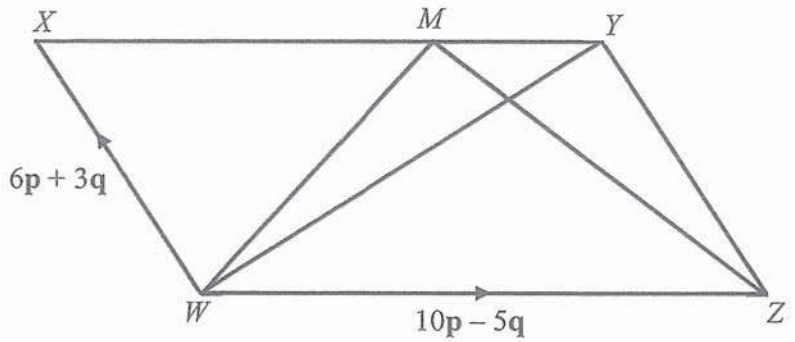
Name: _____ ( ) Class: _____

7 (b) Show that angle EAQ is $\frac{\pi}{6}$. [1]

(c) Calculate the perimeter of the shaded region. [4]

Name: _____ ( ) Class: _____

8 (a) In the diagram, $WXYZ$ is a parallelogram.
M is a point on XY such that $XM : MY = 3 : 2$, $\overrightarrow{WX} = 6\mathbf{p} + 3\mathbf{q}$ and $\overrightarrow{WZ} = 10\mathbf{p} - 5\mathbf{q}$.



(i) Find, in terms of $\mathbf{p}$ and/or $\mathbf{q}$,

(a) $\overrightarrow{WM}$, [1]

(b) $\overrightarrow{ZM}$. [1]

Name: _____ ( ) Class: _____

8 (a) (ii) (a) Find area of triangle WMX : area of $WXYZ$. [1]

(b) The area of triangle WMX is 8 units^2.
Hence, calculate the area of $WXYZ$. [1]

(iii) Given that N is on WX produced such that ZMN is a straight line.
Express $\overrightarrow{WN}$ in terms of p and q. [1]

8 (b) Coordinates of A and B are (–3, 3) and (7, –13) respectively.

(i) Write $\overrightarrow{AB}$ as a column vector. [1]

(ii) Find the acute angle formed by the line AB with the horizontal axis. [2]

(iii) If the gradient of $AB = -\frac{2m}{n}$, express $\overrightarrow{AB}$ in terms of m and n. [1]

Another vector $\overrightarrow{CD}$ is parallel to $\overrightarrow{AB}$ and has the magnitude thrice that of $\overrightarrow{AB}$.

(iv) Write down the possible vectors of $\overrightarrow{CD}$. [2]

Name: _____ ( ) Class: _____

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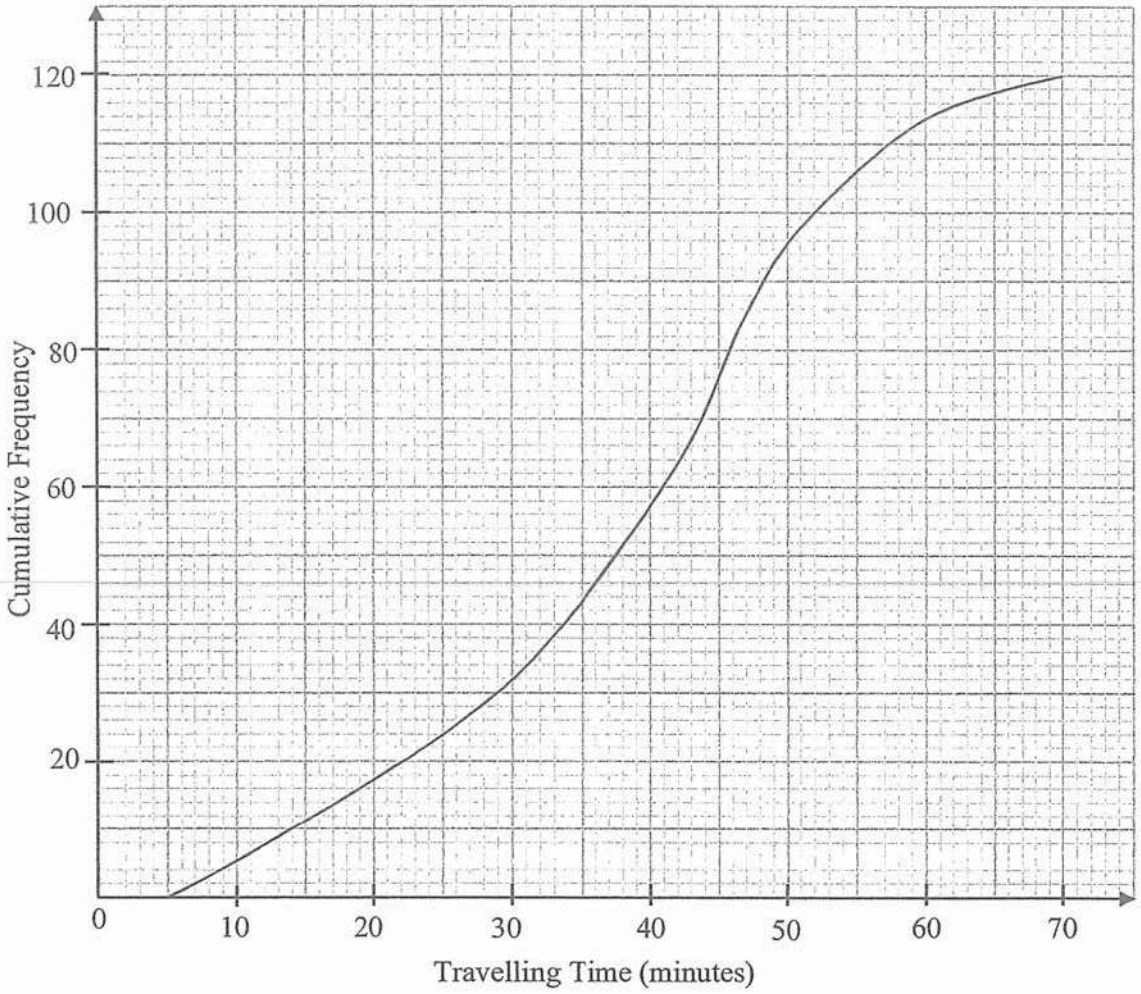
Name: _____ ( ) Class: _____

9 The cumulative frequency curve below shows the travelling time of 120 working adults travelling to work daily by train.



(a) Use the graph to estimate

(i) the median of travelling time, [1]

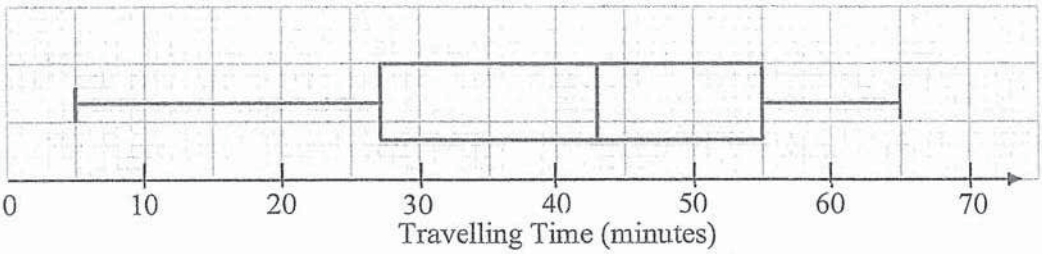
(ii) the 20th percentile of travelling time, [1]

Name: _____ ( ) Class: _____

9 (a) (iii) the interquartile range of travelling time, [2]

(iv) the percentage of the total number of adults who spend more than 45 minutes travelling to work every day. [1]

(b) Another 120 working adults travelled to work by bus. The travelling time is illustrated in the box and whisker diagram below.



Find the median travelling time and the interquartile range. Hence, compare and comment on the travelling times by train and bus in two different ways. [3]

9 (c) One working adult is chosen at random. Assume that the travelling times between the train and bus are independent. The working adult makes the first trip on Monday by train and the second trip on Tuesday by bus.
Expressing each answer as a fraction in its lowest terms, calculate the probability that the working adult took

(i) more than 55 minutes on both trips, [1]

(ii) more than 55 minutes on one trip, but not the other. [2]

Name: _____ ( ) Class: _____

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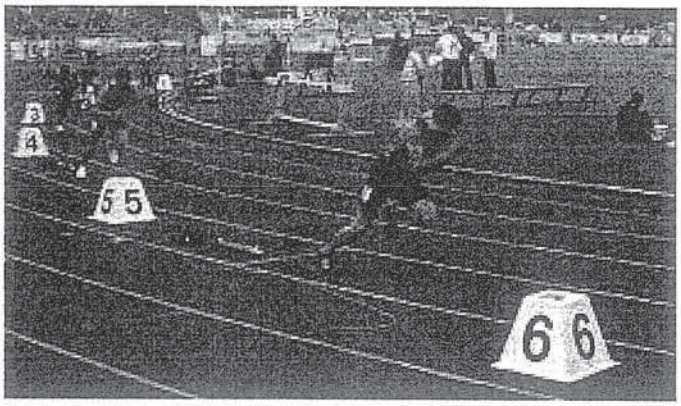
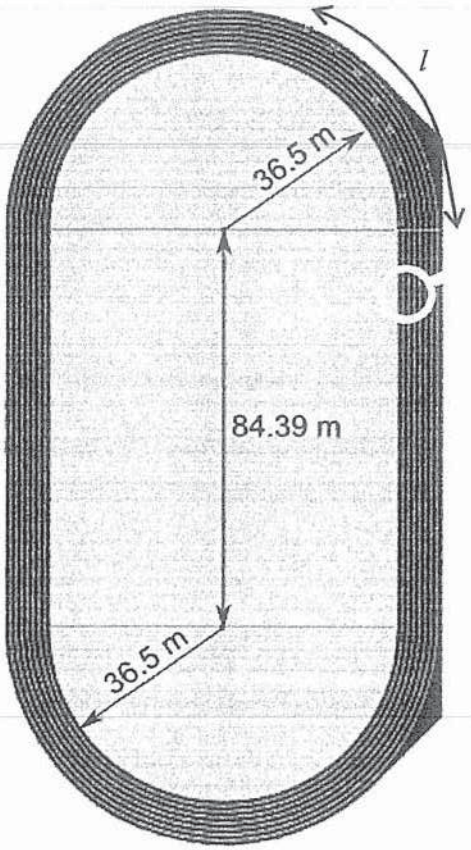
Name: _____ ( ) Class: _____

10 The diagram below shows a race in the Olympic Games. For certain races, the athletes do not all start from the same part of the track. This is called "staggered start".





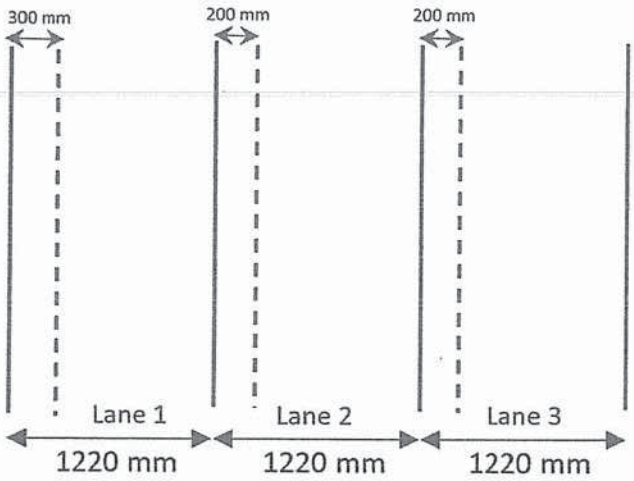


Figure 1

The grass field comprises two semi-circular ends of radius 36.5 m and two straight lengths of 84.39 m each. The field is surrounded by a running track of 8 lanes, each of width 1220 mm. The route along which the running distance is measured for each lane is as below:

<u>Lane 1</u> 300 mm from inner edge of the lane <u>Lanes 2 to 8</u> 200 mm from inner edge of the lane

Name: _____ ( ) Class: _____

10 (a) (i) Show that the total distance that an athlete in **Lane 1** would have to run to complete one lap of the track is 400 m. [2]

(ii) Show that the staggered start line for **Lane 8** is 53.03 m from the start line for **Lane 1** (distance of l) as seen in Figure 1. [3]

(iii) Explain why a "staggered start" is needed for each runner in **Lane 1** to **Lane 8** to complete one lap of the track. [1]

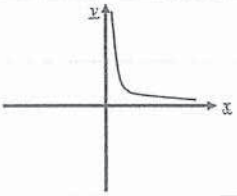
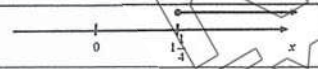
10 (b) An athlete wants to incorporate in his training a minimum of 150 minutes of brisk walking weekly, at the an average speed of 6.8 km/h. He claims that he needs to walk briskly around the track in **Lane 8** five rounds daily to hit his target.
Justify whether his claims is true or false. Show your working clearly. [4]

Reference: http://www.mathisfun.com/activity-atheletics-track.html

End of Paper

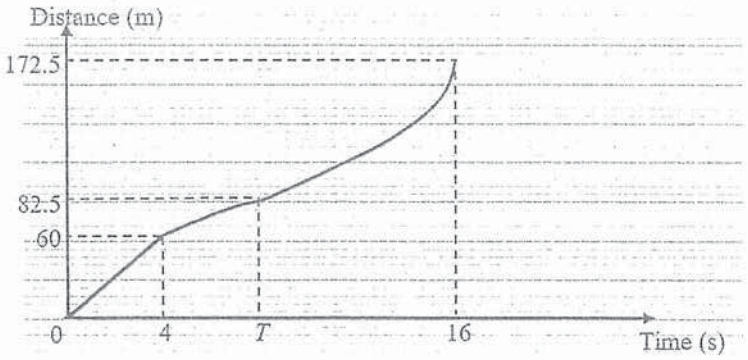
Name: _____ ( ) Class: _____

2019 Sec 4Exp/5NA Preliminary Examination
Mathematics Paper 1
Answer Key

1a	$2^2 \times 3^3 \times 5 \times 7$	**1b**	105	**2a**	(28.5)
2b	$\begin{pmatrix} 7 & 14 \\ -7 & -14 \end{pmatrix}$	**3a**	x = -4	**3b**	k = 0
4	80%	**5a**	Charlene should use **Graph 2** as the **scale for the vertical axis is bigger and does not start from zero,** making the **difference in marks** between each test looks **bigger.**	**6a**	2.325×10^{-7} picograms
6b	2.86×10^{-5} : 1	**7a**	– 17 °C	**7b**	2100 m
7c	05 42	**8a**		**8b**	$P=\frac{32}{12}=2.67Nm^{-2}or2\frac{2}{3}Nm^{-2}$
9a	7.56 km	**9b**	2.71 cm^2 (3 s.f.)	**10a**	1 + 3 + 5 + 7 + 9 +11+ 13 = 49 = 7 × 7
10b	1 + 3 + 5 + 7 + 9 + …+ 81= 41 × 41=1681	**11**	\$2155.06 (2 d.p)	**12a**	B
12b	D	**12c**	C	**13a**	
13bi	{2,3,7,11,13,17,19}	**13bii**	3	**14a**	$x\geq1\frac{1}{4}$
14b		**15a**	$(x-4.5)^2-24.75$	**15b**	25.28 or 15.23 (2 d.p.)
16a	$\frac{17}{13}$	**16b**	$\frac{216}{75}$	**17a**	$2(a-7x)(2a+b)$
17b	x = - 5	**18a**	The object is moving at **constant speed of 5 ms^{-1} with zero acceleration** for the first 4 sec	**18b**	7 sec
19c	10.0 ±0.1 cm	**20bi**	$\frac{11}{36}$	**20bii**	$\frac{7}{12}$
21a	a – 4	**21b**	$x=\frac{6}{7}$	**22a**	$y=12$
22b	7	**22c**	3.55	**23a**	168°
23c	36°	**24a**	$13y^2-8y$	**24b**	$\frac{41-18y}{(7+3y)(7-3y)}$

Name: _____ ( ) Class: _____

18c

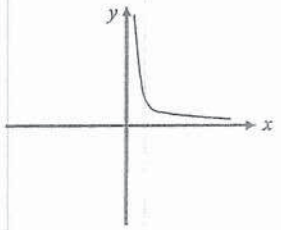


Name: _____ ( ) Class: _____

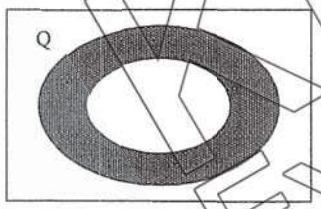
Marking Scheme for Sec 4 Exp/ 5NA Mathematics P1 2019

Qn	Solution	Marking Scheme
1a	$3780 = 2^2 \times 3^3 \times 5 \times 7$	B1
1b	$2^2 \times 3^3 \times 5 \times 7 \times (3 \times 5 \times 7)$ = perfect sq. Therefore, $3 \times 5 \times 7 = 105$	B1
2a	$\frac{1}{2}BA$ $= \frac{1}{2}\begin{pmatrix}-3 & 9\end{pmatrix}\begin{pmatrix}2\\7\end{pmatrix}$ $= \frac{1}{2}(57)$ $= (28.5)$	B1
2b	C^2 $= \begin{pmatrix}3 & 2\\-1 & 4\end{pmatrix}\begin{pmatrix}3 & 2\\-1 & 4\end{pmatrix}$ $= \begin{pmatrix}7 & 14\\-7 & -14\end{pmatrix}$	B1
3a	$x = -4$	B1
3b	$\frac{1+(-k)}{2} = -4$ $1-k = -8$ $-k = -8-1$ $k = 9$	B1
4	$V = kr^2h$ new radius, $r_1 = 1.5r$, new height, $h_1 = 0.8h$ $V = kr^2h$ $V_{new} = k(1.5r)^2(0.8h)$ $V_{new} = \frac{V}{r^2h}(1.5r)^2(0.8h)$ $\therefore V_{new} = 1.8V$ % change in V $= \frac{1.8-1}{1} \times 100\%$ $= 80\%$	M1 B1 or B2
5	Charlene should use **Graph 2** as the **scale for the vertical axis is bigger and does not start from zero**, making the **difference in marks** between each test looks **bigger**.	A1 B1

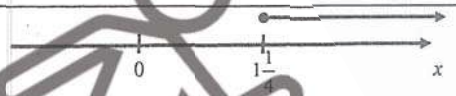
Name: _____ ( ) Class: _____

6a	Mass of 12 billion nitrogen atoms $= 2.80\times10^{-9}$ g $= 2.80\times10^{3}$ picograms Mass of 1 nitrogen atom $= \frac{2.80\times10^{3}}{12\times10^{9}}$ $= 0.2325\times10^{-6}$ $= 2.325\times10^{-7}$ picograms	B1
6b	Helium : nitrogen 6.684×10^{-24} : $\frac{2.80\times10^{-9}}{12\times10^{3}}$ 6.684×10^{-24} : 0.2333×10^{-18} 28.6×10^{-6} : 1 2.86×10^{-5} : 1	M1 A1
7a	No. of times temp will decrease $= 4200 \div 700 = 6$ Amt. of decrease In temp. $= 8 \times 6 = 48$°C Temp. at the peak of mountain $= 31 - 48 = -17$ °C	B1
7b	Amt. of decrease in temp. $= 31 - 7 = 24$ °C Height of mountain $= 24 \div 8 \times 700$ m $= 2100$ m	B1
7c	Time he begins his climb $= 12\ 25$ $-\ 6\ 43$ $05\ \ 42$	B1
8a		B1 (Negative region of the reciprocal graph should not be shown as its not applicable)
8b	$P = \frac{k}{V}$ $k = PV$ $= 4(8) = 32$ $\therefore P = \frac{32}{12} = 2.67 Nm^{-2}$ or $2\frac{2}{3} Nm^{-2}$	B1

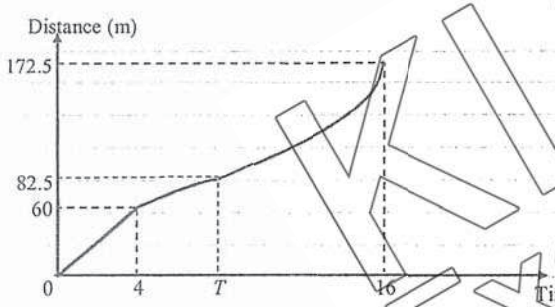
Name: ( ) Class:

9a	1 cm : 120 000 cm 1 cm : $\frac{120000}{1000\times100}=1.2$km 6.3 cm : 6.3×1.2 km $= 7.56$ km	B1
9b	$(1\text{ cm})^2$: $(1.2\text{ km})^2$ 1 cm^2 : 1.44 km^2 $\frac{3.9}{1.44}\times1$: 3.9 km^2 $2.71\text{ cm}^2=$ (3 s.f.)	M1 A1
10a	$1+3+5+7+9+11+13=49=7\times7$	B1
10b	$1+3+5+7+9+\ldots+81=41\times41=1681$	B1
11	$T=P(1+\frac{r}{100})^n$ $=10000(1+\frac{\frac{10}{2}}{100})^{2\times2}$ $=\$12155.0625$ *Interest* $=\$12155.0625-\$10000$ $=\$2155.06(2d.p.)$	M1 M1 A1
12a	B	B1
12b	D	B1
12c	C	B1
13a	*Answer* ε 	B1
13bi	{2,3,7,11,13,17,19}	B1
13bii	3	B1

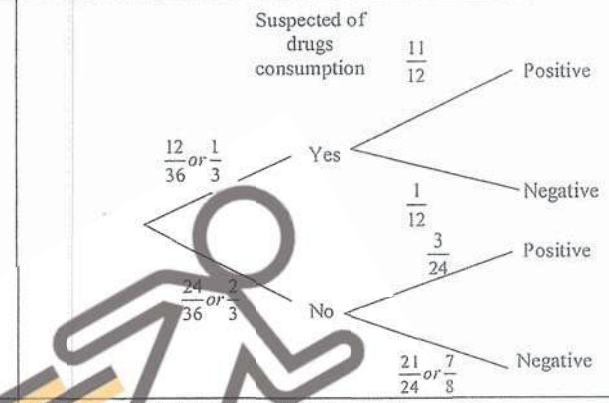
Name: ( ) Class:

14a	$4-3x\le\frac{2-x}{3}<\frac{4+x}{5}$ $4-3x\le\frac{2-x}{3}$ or $\frac{2-x}{3}<\frac{4+x}{5}$ $-9x+x\le2-12$ $-5x-3x<12-10$ $-8x\le-10$ $-8x<2$ $x\ge1\frac{1}{4}$ $x>-\frac{1}{4}$ $\therefore x\ge1\frac{1}{4}$	M1 A1
14b		B1
15a	$x^2-9x+45$ $=x^2-9x+\left(\frac{9}{2}\right)^2-\left(\frac{9}{2}\right)^2+45$ $=(x-4.5)^2+24.75$	B1
15b	$(x-4.5)^2=\pm\sqrt{50-24.75}$ $x-20.25=\pm5.024938$ $x=5.024938+20.25$ or $-5.024938+20.25$ $=25.28$ (2 d.p.) or 15.28 (2 d.p)	M1 A1 for both correct answers
16a	Length of base of triangle $=\sqrt{13^2-5^2}=12$ units $\sin\angle A-\cos\angle A$ $=\sin\angle A-(-\cos(180°-\angle A))$ $=\frac{5}{13}-\left(-\frac{12}{13}\right)$ $=\frac{17}{13}$	 M1 A1
16b	$\cos(180°-\angle A)+\tan(\angle A-90°)$ $=\frac{12}{13}+\tan[180°-(180°-\angle A)-90°]$ $=\frac{12}{13}+\frac{12}{5}$ $=\frac{216}{75}$	 M1 A1

Name: ( ) Class:

17a	$4a^2+2ab-14xb-28ax$ $=2a(2a+b)-14x(b+2a)$ $=(2a-14x)(2a+b)$ $=2(a-7x)(2a+b)$	M1 A1
17b	$\frac{3x+1}{7}=-\frac{3-x}{4}$ $4(3x+1)=-7(3-x)$ $12x+4=-21+7x$ $12x-7x=-21-4$ $5x=-25$ $x=-5$	M1 A1
18a	The object is moving at **constant speed of 5 ms^{-1} with zero acceleration** for the first 4 sec	B1
18b	Gradient = 5 $\frac{15}{Time}=5$ $Time=\frac{15}{5}$ $=3\text{sec}$ $T=4+3=7\text{sec}$	 B1
18c		All 3 parts of correct graph shape = B2 1 wrong shape = deduct B1 2 or more wrong shape = 0 marks
19a,b,	Refer to last page for the details	[2M1]
19c	10.0 ± 0.1 cm	B1

Name: ( ) Class:

20a		Any 2 correct ans = B1 × 2 = B2
20bi	P(suspected of taking drugs and tested positive) $=\frac{12}{36}\times\frac{11}{12}$ $=\frac{11}{36}$	 B1
20bii	P(received negative results) $=(\frac{12}{36}\times\frac{1}{12})+(\frac{24}{36}\times\frac{21}{24})$ $=\frac{1}{36}+\frac{7}{12}$ $=\frac{21}{36}$ $=\frac{7}{12}$	M1 A1
21a	$\left(\frac{a^4-a^3}{a^3}\right)-3(a)^0$ $=a^3\left(\frac{a-1}{a^3}\right)-3$ $=(a-1)-3$ $=a-4$	 M1 A1

Name: ( ) Class:

21b	$\frac{1}{(3^2)^{1-3x}} = (3^5)^{\frac{x}{2}-1}$ $(3^2)^{3x-1} = 3^{\frac{5x}{2}-5}$ $6x-2 = \frac{5x}{2}-5$ $6x-\frac{5x}{2} = -5+2$ $3.5x = -3$ $x = \frac{-3}{3.5}$ $x = \frac{6}{7}$	M1 M1 A1
22a	Let x = 9 as mode is 9 $\text{mean} = \frac{5+6+7+9+2+4+9+12+2+9+y}{11}$ $7 = \frac{65+y}{11}$ $77 = 65+y$ $y = 77-65$ $y = 12$	B1 B1
22b	Median position $= \frac{11+1}{2} = 6th$ Arranging the values in ascending order, 2, 2, 4, 5, 6, 7, 9, 9, 9, 12, 12 Hence the median at 6th position is 7	B1
22c	Standard deviation = 3.55	B2
23a	Int angle *IGH* $= \frac{180°}{3}$ (angle in equilateral triangle) $= 60°$ Angle in pentagon $= \frac{(5-2)\times 180°}{5}$ $= 108°$ Angle EDH = 60° + 108° = 168°	M1 A1
23b	Since the triangle is equilateral and all triangles are identical, AB = BC = AC = AI. Hence AI = AB	B1

Name: ( ) Class:

23c	Angle GEI = Angle CEA $= \frac{180°-108}{2}$ (base angles in isos.triangle) $= 36°$ $\therefore \angle AEI = 108° - (36°\times 2)$ $= 36°$	M1 A1
24a	$y(y-2)+12y^2-6y$ $= y^2-2y+12y^2-6y$ $= 13y^2-8y$	M1 for correct expansion A1
24b	$\frac{6}{3y+7} - \frac{1}{49-9y^2}$ $= \frac{6}{3y+7} - \frac{1}{7^2-(3y)^2}$ $= \frac{6}{3y+7} - \frac{1}{(7+3y)(7-3y)}$ $= \frac{6(7-3y)-1}{(7+3y)(7-3y)}$ $= \frac{42-18y-1}{(7+3y)(7-3y)}$ $= \frac{41-18y}{(7+3y)(7-3y)}$	M1 M1 A1

Name: _____ ( ) Class: _____

Fairfield Methodist School (Secondary)
2019 Sec 4Exp/5NA Preliminary Examination
Answer Key for Mathematics Paper 1

Answer Key			
1(a)	$\frac{3x^2+14x-6}{(x-3)(x+4)}$	1(b)	$16(x^2+1)$ or $16x^2+16$
1(c)	$x=0$ or $x=-1.5$ or $x=8$	1(d)	Since $(n+1)^2=(n+1)(n+1)$, therefore, it is a square number. Or $(n+1)^2$ has a repeated same factor, therefore it is a square number.
2(i)	Statement with reasons for SAA/RHS/SAS/SSS	2(ii)	Triangle QUS
2(iii)	Angle TQS = Angle SUT (Angle in same segment) = 60° (mentioned in (i) that triangle QTR is equilateral triangle) Angle QTR = 60° (triangle QTR is an equilateral triangle). Since angle QTR = Angle SUR, the form alternate angle, therefore TQ is parallel to SU. Or Angle TQU = Angle SUQ = 90° (Angle in a semi-circle) Since they are the same and they form interior angle, therefore, TQ is parallel to SU. Or by converse of property of interior angle, therefore, TQ is parallel to SU.		
2(iv)	30°	3(c)(i)	$x=9.17$ or $x=0.33$ (2 d.p.)
3(c)(ii)	When $x=0.327$, $DC=2x-5=-4.356<0$. Therefore, x cannot be 0.33.	3(d)	6.35 (3 s.f.)
4(a)	264 cm^2 (nearest cm^2)	4(b)(i)	28
4(b)(ii)	333 cm^3 (nearest cm^3)	4(b)(iii)	184 mm (nearest mm)
4(c)	8: 27	5(a)	132°
5(b)	48.8 m (3 s.f.)	5(c)	$h=32.6$ m (3 s.f.)
5(d)(i)	14.0 m (3 s.f.)	5(d)(ii)	23.2°
6(a)	$p=12$	6(c)(i)	Accept answers: -0.023 to – 0.03
6(c)(ii)	The gradient represent the rate of decrease of the cost of printing each book when number of books is 300. Or The gradient represent the rate at which the cost of printing each book is decreasing.	6(d)(i)	Plot points (0, 25), (600, 15) and (1200, 5) Draw a line
6(d)(ii)	$210 \le x \le 700$ (accept 180 – 220; 680 – 730, with interval of 10)	7(a)(i)	Angle APC = 90° (tangent perpendicular radius) Angle AQE = 90°(tangent perpendicular radius) Therefore, angle APC = angle AQE Angle PAC = Angle QAE (common angle) Since there are two pairs of corresponding angles are equal, triangle APC and triangle AQE are similar.
7(a)(ii)	$AB=2$	7(b)	$\angle CAP=\sin^{-1}\left(\frac{1}{2}\right)=30°=\frac{\pi}{6}$
7(c)	17.4 cm (3 s.f.)	8(a)(i)(a)	$12p$

Name: _____ ( ) Class: _____

Answer Key			
8(a)(i)(b)	$2\boldsymbol{p} + 5\boldsymbol{q}$	8(a)(ii)(a)	3 : 10
8(a)(ii)(b)	$\frac{80}{3}$ units^2 or $26\frac{2}{3}$ or 26.7 (3 s.f)	8(a)(iii)	$\frac{15}{2}(2\boldsymbol{p} + \boldsymbol{q})$
8(b)(i)	$\overrightarrow{AB} = \begin{pmatrix} 10 \\ -16 \end{pmatrix}$	8(b)(ii)	58.0° (1 d.p.)
8(b)(iii)	$\overrightarrow{AB} = k\begin{pmatrix} n \\ -2m \end{pmatrix}$ or $\overrightarrow{AB} = k\begin{pmatrix} -n \\ 2m \end{pmatrix}$	8(b)(iv)	$\begin{pmatrix} -30 \\ 48 \end{pmatrix}$
9(a)(i)	41 minutes	9(a)(ii)	25 minutes
9(a)(iii)	IQR = 48 – 29 = 19 mins	9(a)(iv)	$36\frac{2}{3}\%$ *or* $\frac{110}{3}\%$ or 36.7% (3 s.f.)
9(b)	The travelling time using train is much shorter (faster) than using a bus as the median time travelling with a train < median time travelling with a bus. The travelling time using train is less spread out (more consistent) than using travelling using a bus as the interquartile range of travelling time of a train < interquartile range of travelling time of a bus.	9(c)(i)	$\frac{7}{240}$
9(c)(ii)	$\frac{37}{120}$	10(a)(ii)	53.03 m (2 d.p.)
10(a)(iii)	Using staggered start, each runner runs exactly 400 meters or same distance.	10(b)	His claim in false.

Preliminary Examination 2019
Mathematics Paper 2
Marking Scheme

No	Description	Marks	Total
1(a)	$\frac{3x}{x-3}+\frac{2}{x+4}$ $=\frac{3x(x+4)+2(x-3)}{(x-3)(x+4)}$ $=\frac{3x^2+12x+2x-6}{(x-3)(x+4)}$ $=\frac{3x^2+14x-6}{(x-3)(x+4)}$	M1 A1	2
1(b)	$(x^2+5)^2-(x^2-3)^2$ $=[x^2+5-(x^2-3)][x^2+5+(x^2-3)]$ $=(8)(2x^2+2)$ $=16(x^2+1)$ or $16x^2+16$	M1 A1	2
1(c)	$2x^3-13x^2-24x=0$ $x(2x^2-13x-24)=0$ $x(2x+3)(x-8)=0$ $x=0$ or $x=-1.5$ or $x=8$	M1 A1 (two correct), A1	3
1(d)	$\frac{1}{2}n(n+1)+\frac{1}{2}(n+1)(n+2)$ $=\frac{1}{2}(n+1)[n+n+2]$ $=\frac{1}{2}(n+1)[2n+2]$ $=\frac{1}{2}(n+1)[2(n+1)]$ $=(n+1)^2$ Since $(n+1)^2=(n+1)(n+1)$, therefore, it is a square number. Or $(n+1)^2$ has a repeated same factor, therefore it is a square number.	B1 B1	2
			8
2(i)	$QR=QT=RT$ (radius of circles centred at Q and R) Therefore, triangle QRT is an equilateral triangle. Angle Q = Angle R = Angle $T=60°$ $\angle QUT=30°$ ($\angle$ at centre $=2\times\angle$ at circumference) $\angle TPR=30°$ ($\angle$ at centre $=2\times\angle$ at circumference) Therefore, angle QUT = angle TPR Therefore, by SAA / AAS, triangle PTR and UQT are congruent.	B1 B1 B1	3

No	Description	Marks	Total
2(i)	Angle $PTR=90°$ (right angle in semi-circle) Angle $UQT=90°$ (right angle in semi-circle) Therefore, angle PTR = angle UQT $PR=UT$ (as both are diameter of the circle centred at Q and R respectively) $QR=QT=RT$ (radius of circles centred at Q and R) Therefore, by RHS congruency test, triangle PTR and UQT are congruent.	B1 B1 B1	
	$PR=UT$ (as both are diameter of the circle centred at Q and R respectively) $QR=QT=RT$ (radius of circles centred at Q and R) Therefore, triangle QRT is an equilateral triangle. Angle Q = Angle R = Angle $T=60°$ Therefore, by SAS congruency test, triangle PTR and UQT are congruent.	B1 B1 B1	
	Angle $PTR=90°$ (right angle in semi-circle) Angle $UQT=90°$ (right angle in semi-circle) Therefore, angle PTR = angle UQT $PR=UT$ (as both are diameter of the circle centred at Q and R respectively) $QR=QT=RT$ (radius of circles centred at Q and R) Therefore, triangle QRT is an equilateral triangle. Angle Q = Angle R = Angle $T=60°$ Angle $PRT=60°$ (int angle of equilateral triangle) Angle $UTQ=60°$ (int angle of equilateral triangle) Therefore, angle PRT = angle UTQ Therefore, by SAA / AAS congruency test, triangle PTR and UQT are congruent.	B1 B1 B1	
2(ii)	Triangle QUS	B1	1
2(iii)	Angle TQS = Angle SUT (Angle in same segment) = 60° (mentioned in (i) that triangle QTR is equilateral triangle) Angle $QTR=60°$ (triangle QTR is an equilateral triangle). Since angle QTR = Angle SUR, the form alternate angle, therefore TQ is parallel to SU. Since angle QTR = Angle SUR, by converse property of alternate angle, TQ is parallel to SU.	B1	
	Angle TQU = Angle $SUQ=90°$ (right angle in a semi-circle) Since they are the same and they form interior angle, therefore, TQ is parallel to SU. Since angle TQU = Angle SUQ, by converse property of interior angle, TQ is parallel to SU.	B1	1
2(iv)	Angle $UQR=30°$ (angle at circumference $=\frac{1}{2}$ angle at centre) Or Angle $UQR=30°$ (complementary angle and angle in semicircle)	B1	1
			6

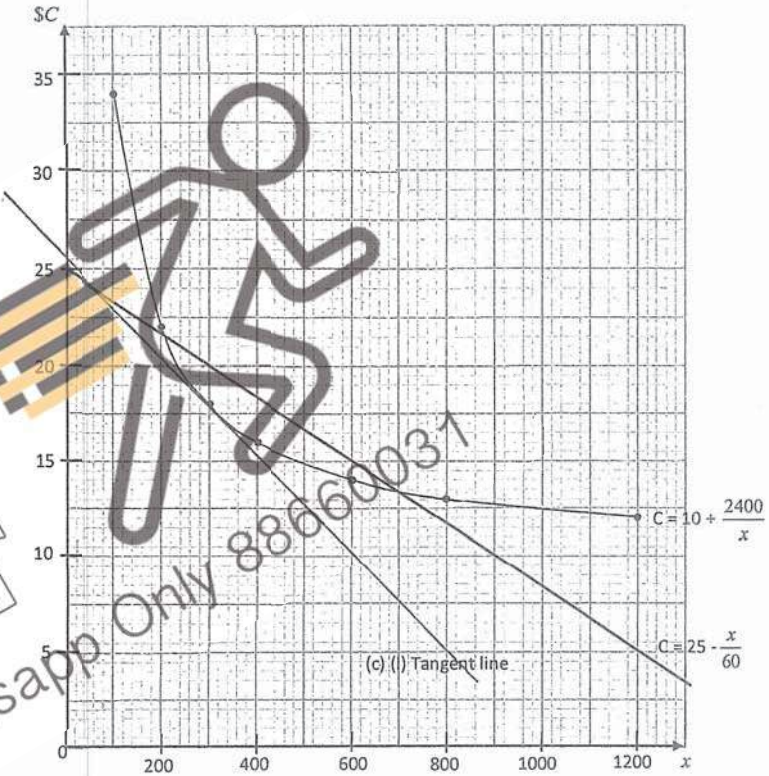
No	Description	Marks	Total
3(a)	$\frac{Area\ of\ triangle\ CDE}{Area\ of\ triangle\ ABC}$ $=\frac{\frac{1}{2}\times CD\times CE\times sin\theta}{\frac{1}{2}\times AC\times BC\times sin\theta}$ $=\frac{CD\times CE}{AC\times BC}$	M1 AG1	2
3(b)	$\frac{CD\times CE}{AC\times BC}=\frac{1}{3}$ $\frac{(2x-5)\times x}{(12+x)\times(4+2x-5)}=\frac{1}{3}$ $3x(2x-5)=(12+x)(2x-1)$ $6x^2-15x=24x-12+2x^2-x$ $4x^2-15x-24x+x+12=0$ $4x^2-38x+12=0$ $2(2x^2-19x+6)=0$ $2x^2-19x+6=0$	M1 M1 (Expand and simplify) AG1	3
3(c)(i)	$2x^2-19x+6=0$ $x=\frac{19\pm\sqrt{(-19)^2-4(2)(6)}}{2(2)}$ $x=\frac{19\pm\sqrt{313}}{4}$ $x=9.1729$ or $x=0.3270484$ $x=9.17$ or $x=0.33$ (2 d.p.)	M1 A1, A1	3
3(c)(ii)	When $x=0.327$, $DC=2x-5=-4.356<0$. Therefore, x cannot be 0.33.	B1	1
3(d)	When $x=9.1729$, $2x-5=13.3458$ $DE=\sqrt{(13.3458)^2+9.1729^2-2(13.3458)(9.1729)\cos 25°}$ $=\sqrt{40.3526}$ =6.35237 =6.35 (3 s.f.)	M2 A1	3
			12
4(a)	Total surface area of bowl $=\pi\times6^2+2\times\pi\times6\times4$ = 263.893 =264 cm^2 (nearest cm^2)	M1, M1 A1	3
4(b)(i)	Number of bowls completely filled $=\frac{13l}{(\pi\times6^2\times4)cm^3}$ $=\frac{13\times1000cm^3}{(\pi\times6^2\times4)cm^3}$ =28.7363 =28	M1 M1 A1	3
4(b)(ii)	Volume of soup left $=13l-28\times(\pi\times6^2\times4)$ = 333.0984	M1 A1	

No	Description	Marks	Total
	= 333 cm^3 (nearest cm^3)		
Or 4(b)(ii)	Volume of soup left $=0.7363\times(\pi\times6^2\times4)$ = 333.094 = 333 cm^3 (nearest cm^3)	M1 A1	2
4(b)(iii)	Volume of hemisphere pan = 13litres = 13000 cm^3 $\frac{2}{3}\pi r^3=13000$ $r=\sqrt[3]{\frac{13000\times3}{2\times\pi}}$ = 18.377 cm = 184 mm (nearest mm) or 18.4 cm (nearest mm)	M1 A1	2
4(c)	$\frac{V_2}{V_1}=\left(\frac{2}{3}\right)^3=\frac{8}{27}$ The ratio is 8: 27	B1	1
			11
5(a)	Angle $PQT=180°-63°-75°=42°$ Bearing of S from $Q=90°+42°=132°$	B1 B1	2
5(b)	$\frac{QT}{sin75°}=\frac{45}{sin63°}$ $QT=\frac{45}{sin63°}\times sin75°$ = 48.783775 = 48.8 m (3 s.f.)	M1 AG1	2
5(c)	Let shortest distance from ship to cliff be h. $\frac{1}{2}\times h\times45=\frac{1}{2}\times45\times48.783776\times sin42°$ $h=48.783776\times sin42°$ $h=32.642715$ $h=32.6$ m (3 s.f.)	M1 A1	2
Or 5(c)	$\sin 42°=\frac{h}{48.8}$ $h=\sin 42°\times48.8$ = 32.654 = 32.7 m (3 s.f.)	M1 A1	
5(d)(i)	Let the height of the cliff be H. $\tan 16°=\frac{H}{48.783755}$ $H=\tan 16°\times48.783755$ = 13.9883 = 14. 0 m (3 s.f.)	Note : if H = tan 16° × 48.8 =13.993 = 14.0m M1 A1	2
5(d)(ii)	Angle of elevation = $\tan^{-1}\left(\frac{13.9883}{32.642715}\right)=23.196°$ = 23.2°	M1 note can be FT1 A1	2
			12
6(a)(i)	$p=12$	B1	1
6(a)(ii)	Plot the points Draw a smooth curve	P2 C1	3
6(b)	490 (accept answer: 460 – 500)	B1	1

No	Description	Marks	Total
6(c)(i)	Draw a tangent line Gradient of tangent $= -\frac{20.1}{800} = -0.025125$ $= -0.0251$ (3 s.f.) Accept answers: (-0.023 to – 0.03) Actual answer: –0.02666 = –0.0267	B1 B1 * answer must be a decimal number	2
6(c)(ii)	The gradient represent the rate of decrease of the cost of printing each book when number of books is 300. Or The gradient represent the rate at which the cost of printing each book is decreasing.	B1	1
6(d)(i)	Plot points (0, 25), (600, 15) and (1200, 5) Draw a line	P1 L1	2
6(d)(ii)	$210 \le x \le 700$ (accept 180 – 220; 680 – 730, with interval of 10)	B1	1
7(a)(i)	Angle $APC = 90°$ (tangent perpendicular radius) Angle $AQE = 90°$(tangent perpendicular radius) Therefore, angle APC = angle AQE Angle PAC = Angle QAE (common angle) Since there are two pairs of corresponding angles are equal, triangle APC and triangle AQE are similar.	B1 B1	2
7(a)(ii)	$\frac{AC}{AE} = \frac{PC}{QE}$ $\frac{AB+2}{AB+4+6} = \frac{2}{6}$ $\frac{AB+2}{AB+10} = \frac{1}{3}$ $3(AB+2) = AB+10$ $3AB+6 = AB+10$ $2AB = 4$ $AB = 2$	M1 A1	2
7(b)	Angle EAQ = Angle CAP $\sin\angle CAP = \frac{PC}{AC} = \frac{2}{4} = \frac{1}{2}$ $\angle CAP = \sin^{-1}\left(\frac{1}{2}\right) = 30° = \frac{\pi}{6}$	AG1 (for $\sin^{-1}\left(\frac{1}{2}\right)$)	1
7(c)	Perimeter of the shaded region = Arc PD + PQ + Arc QD $= \frac{120}{360}\times 2\times\pi\times 2 + \sqrt{12^2-6^2} - \sqrt{4^2-2^2} + \frac{60}{360}\times 2\times\pi\times 6$ =4.18879+ 6.92820 + 6.28318 =17.40017 =17.4 cm (3 s.f.) or 17.5 cm (3 s.f.) (if rounded off to 5 s.f. for working)	M1 (Arc $PD=\frac{4}{3}\pi$) M1 ($PQ=4\sqrt{3}$) M1 (Arc $QD = 2\pi$) A1	4
8(a)(i)(a)	$\overrightarrow{WM} = \overrightarrow{WX} + \overrightarrow{XM} = 6p + 3q + \frac{3}{5}(10p - 5q)$ $\overrightarrow{WM} = 12p$	B1	1
8(a)(i)(b)	$\overrightarrow{ZM} = \overrightarrow{ZW} + \overrightarrow{WM}$ $\overrightarrow{ZM} = -10p + 5q + 12p$	Note: If notation is wrong, minus 1 marks for overall (a)(i)	

No	Description	Marks	Total
8(a)(i)(b) OR	$\overrightarrow{ZM} = 2p + 5q$	B1	
	$\overrightarrow{ZM} = \overrightarrow{ZY} + \overrightarrow{YM} = 6p + 3q + \frac{2}{5}(-10p + 5q)$ $\overrightarrow{ZM} = 2p + 5q$	B1	1
8(a)(ii)(a)	Area of triangle WMX :area of $WXYZ$ $= \frac{Area\ of\ triangle\ WMX}{Area\ of\ triangle\ WXY} \times \frac{Area\ of\ triangle\ WXY}{Area\ of\ WXYZ}$ $= \frac{3}{5}\times\frac{1}{2} = \frac{3}{10}$ Area of triangle WMX : area of $WXYZ$ = 3 : 10	B1	1
8(a)(ii)(b)	Area of $WXYZ = 8\times\frac{10}{3} = \frac{80}{3}$ units^2 or $26\frac{2}{3}$ or 26.7 (3 s.f)	B1	1
8(a)(iii)	$\overrightarrow{WN} = \frac{5}{7}(6p + 3q) = \frac{15}{7}(2p + q)$	B1	1
8(b)(i)	$\overrightarrow{AB} = \begin{pmatrix}3\\-3\end{pmatrix} + \begin{pmatrix}7\\-13\end{pmatrix} = \begin{pmatrix}10\\-16\end{pmatrix}$	B1	1
8(b)(ii)	Acute angle $= \tan^{-1}\left(\frac{16}{10}\right) = 57.994 = 58.0°$ (1 d.p.)	M1 (tan ratio), A1	2
8(b)(iii)	$\overrightarrow{AB} = k\begin{pmatrix}n\\-2m\end{pmatrix}$ or $\overrightarrow{AB} = k\begin{pmatrix}-n\\2m\end{pmatrix}$	B1	1
8(b)(iv)	$\overrightarrow{CD} = 3\overrightarrow{AB} = 3\begin{pmatrix}10\\-16\end{pmatrix} = \begin{pmatrix}30\\-48\end{pmatrix}$ $\overrightarrow{CD} = -3\overrightarrow{AB} = -3\begin{pmatrix}10\\-16\end{pmatrix} = \begin{pmatrix}-30\\48\end{pmatrix}$	B1 B1	2
			9
9(a)(i)	41 minutes	B1	1
9(a)(ii)	25 minutes	B1	1
9(a)(iii)	Q_1 = 29 mins; Q_3 = 48 mins IQR = 48 – 29 = 19 mins	M1, A1 or B2	2
9(a)(iv)	Percentage of adults who spend more than 40 minutes travelling to work everyday $= \frac{120-76}{120}\times 100\% = \frac{44}{120}\times 100\%$ $= 36\frac{2}{3}\%$ *or* $\frac{110}{3}\%$ or 36.7% (3 s.f.)	B1	1
9(b)	Median for bus = 43 mins IQR for bus = 55 – 27 = 28 mins The travelling time using train is much shorter (faster) than using a bus as the median time travelling with a train < median time travelling with a bus. The travelling time using train is less spread out (more consistent) than using travelling using a bus as the interquartile range of travelling time of a train < interquartile range of travelling time of a bus.	B1 for both median and IQR of bus B1 B1	3
9(c)(i)	P(more than 55 minutes) $= \frac{120-106}{120}\times\frac{1}{4} = \frac{14}{120}\times\frac{1}{4} = \frac{7}{240}$	B1	1
9(c)(ii)	P(more than 55 minutes on one trip) $= \frac{120-106}{120}\times\frac{3}{4} + \frac{106}{120}\times\frac{1}{4}$ $= \frac{14}{120}\times\frac{3}{4} + \frac{106}{120}\times\frac{1}{4} = \frac{37}{120}$	M1 A1	2

No	Description	Marks	Total
			11
10(a)(i)	Total distance for Lane 1 $= 2\times\pi\times(36.5+0.3)+(84.39\times2)$ $= 400.001$ m $= 400$ m (3 s.f.)	M1 for $36.5 + 0.3$ AG1	2
10(a)(ii)	Total distance for Lane 8 $= 2\times\pi\times(36.5+0.2+1.22\times7)+(84.39\times2)$ $= 453.031$ Staggered start $= 453.031 - 400$ $= 53.03$ m (2 d.p.)	M1 for 1.22×7 M1 for the formula AG1	3
10(a)(ii)	Total distance for Lane 8 =		
10(a)(iii)	Using staggered start, each runner runs exactly 400 meters or run the same distance.	B1	1
10(b)	Speed = 6.8 km/h $= \frac{6.8\times1000}{1\times60} m/\min$ $= 113\frac{1}{3} m/\min$ Time taken to complete one round in Lane 8 $= \frac{453.031m}{6.8km/h} = \frac{453.031m}{113\frac{1}{3} m/\min}$ =3.99733 mins Time taken to complete five rounds in Lane 8 $= 3.99733\times5$ = 19.9866 mins Time taken to complete five rounds in Lane 8 in 1 week $= 19.98666\times7$ =138.906 mins (< 150 minutes) His claim in false.	B1 (convert speeds) M1 (time for 1 round) M1 (time taken for 1 week) A1	4
Or 10b	Total distance for average speed of 6.8 km/h = 6.8 km/h × 150 mins = 6.8 km/h × 2.5 hours = 17 km = 17000 m Total distance covered for 5 round for 1 week $= 453.03\times5\times7$ =15856.05 m Since 15856.05 m < 17000 m, his claim is false.	B1 M1 for × 5, M1 for × 7 A1	
Or 10b	Total distance for average speed of 6.8 km/h = 6.8 km/h × 150 mins = 6.8 km/h × 2.5 hours = 17 km = 17000 m No. of rounds covered in 1 week $= \frac{17000}{453.03} = 37.525$ rounds No. of round covered in 1 week if he ran 5 rounds in one week = 5× 7 = 35 rounds. Since 35 rounds < 37.525 rounds, his claim is false.	B1 M1 M1 A1	
			10



HILLGROVE SECONDARY SCHOOL

HILLGROVE SECONDARY SCHOOL PRELIMINARY EXAMINATION 2019 SECONDARY 4 (EXPRESS) / 5 (NORMAL ACADEMIC)

CANDIDATE NAME	( )	CLASS	-
CENTRE NUMBER	S	INDEX NUMBER	

MATHEMATICS 4048/02

Paper 2 — 26 August 2019

Candidates answer on the Question Paper. — 2 hours 30 minutes

No Additional Materials are required. — 8.00 a.m. to 10.30 a.m.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use	
TOTAL	100

Parent's/ Guardian's Signature:

_____

Setter: Miss Li Ziyi

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 (a) Solve the inequality $\frac{2+3x}{5} \leq \frac{3-x}{2}$. [2]

(b) Simplify $\frac{16x^2 - y^2}{4x^2 + 9xy + 2y^2}$. [3]

(c) Given that $x=-\frac{1}{2}$ is a solution of $(m-1)x^2+(m-2)x=2m-15$, find the value of m and the second possible value of x. [3]

(d) Solve the equation $x+5=\frac{15x-1}{3x-1}$. [3]

2 $\overrightarrow{AB} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$, where O is the origin.

(a) Find the length of AB. [1]

(b) Find the equation of line AB. [2]

(c) C is a point on $x-$axis such that angle ACB is a right angle. [2]

Find the tangent of angle ABC.

(d) The equation of the line m is $3y+2x=6$. Line m intersects the $x-$axis at D and $y-$axis at E.

(i) Show how you can tell that the line m does **not** intersect the line AB. [2]

(ii) Find $\overline{ED}$ [2]

(iii) What type of quadrilateral is $ABDE$? Explain your answer. [3]

3 Rashidi wants to make a closed rectangular box to contain his toys. The box has a rectangular base of sides $3x$ cm and x cm and its height is h cm.

(a) (i) The volume of the box is 90 cm^3. [1]

Express h in terms of x.

(ii) Show that the total surface area of the box A cm^2, is given by $A = \frac{240}{x} + 6x^2$. [2]

The table shows some corresponding values of x and A, correct to the nearest integer.

x	1	2	3	4	5	6	7	8
A	246	144	134	156	b	256	328	414

(a) Find the value of b. [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $0 \le x \le 8$.
Using a scale of 2 cm to represent 50 units, draw a vertical A-axis for $0 \le A \le 500$.

On your axes, plot the points given in the table and join them with a smooth curve on the grid in page 9. [3]

(c) By drawing a tangent, find the gradient of the curve at $(2,\ 144)$. [2]

(d) Rashidi wants to make a box with a total surface area of 100 cm^2 without changing the volume of the box. Is it possible to do so? Explain your answer. [2]

Replace this page with graph paper.

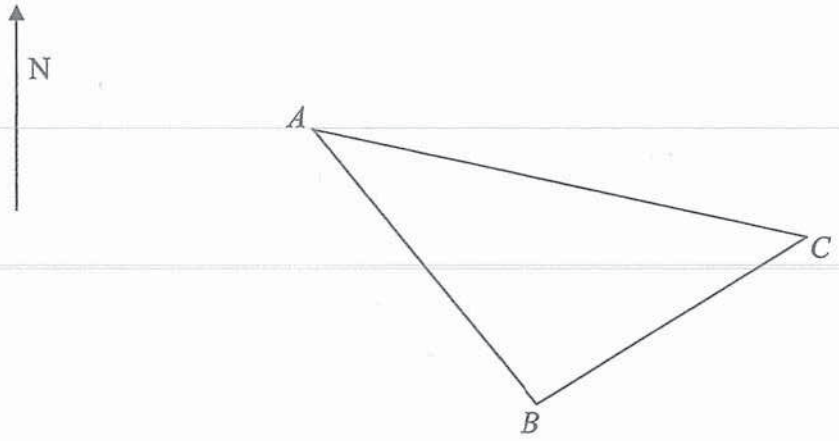
4 A, B and C represent three islands.

Both boats X and Y left from island A.

The bearings of B and C from A is 160° and 100° respectively.

Boat X sailed towards B at a speed of x km/h. Boat Y sailed towards C at a speed of $(x+32)$ km/h.

Boat X and Y took 2 hours to reach B and C respectively.



(a) Write an expression, in terms of x, for the distance travelled by Boat X. [1]

(b) B and C are 150 km apart. [4]

Write down an equation in x to represent this information and show that it reduces to

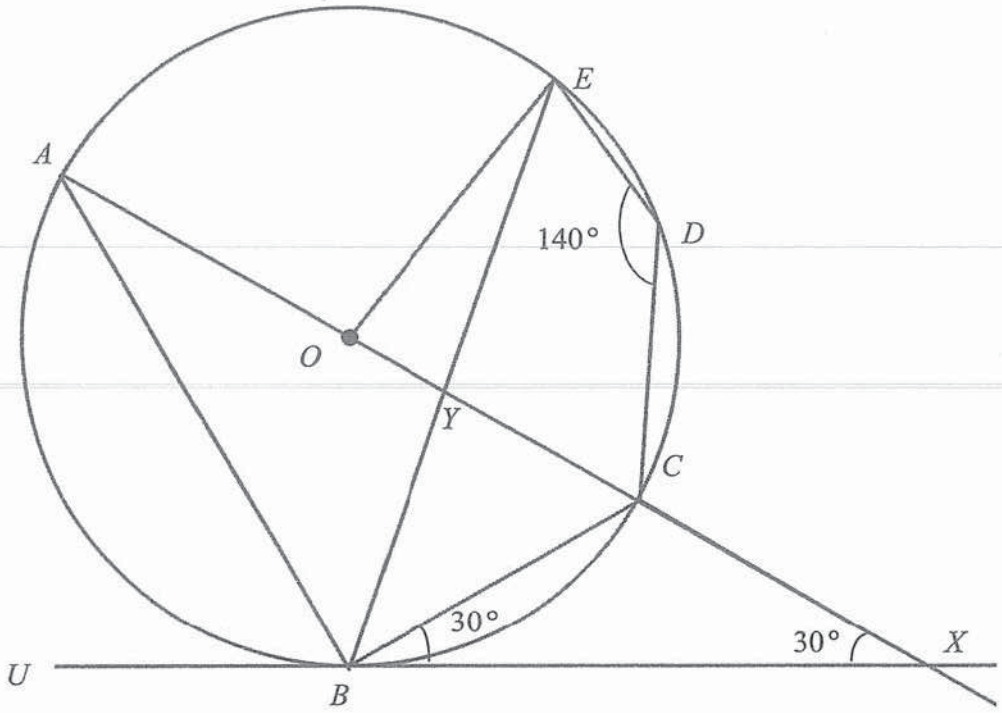
$$x^2 + 32x - 4601 = 0.$$

(b) Solve the equation $x^2 + 32x - 4601 = 0$. [3]

(c) Boat X returned from B to A and took 30 more minutes than the onward journey. [2]

Find the speed of Boat X for the return journey.

5 O is the centre of the circle passing through A, B, C, D and E. UBX is the tangent to the circle at B. ACX is a straight line which passes through O. $\angle CBX = \angle CXB = 30°$ and $\angle EDC = 140°$.



(a) Stating your reasons clearly, find,

(i) $\angle BAC$, [2]

(ii) $\angle CEB$, [1]

(iii) $\angle CBE$, [1]

(iv) $\angle OBY$. [2]

(b) T is a point such that angle ATE is 60° and on the same side as point B. [3]

State whether point T lies in the circle.

Explain your answer.

6 Angela and Ruth went on a free-and-easy trip to Sydney.

(a) They both exchanged S$1600 at a money changer in Singapore. The money changer used an exchange rate between Singapore dollars (S$) and Australian dollars (A$) of A$ 1 = S$0.95. [2]

Calculate the amount of Australian dollars they received.

(b) They booked their accommodation through the Waterbnb application. [2]

Each night at the accommodation costs A$110.25. However, there was a 10% service charge. An additional 5% daily cleaning fee was imposed **only** on the cost of the accommodation.

They booked a total of 8 nights at the accommodation.

Calculate the total cost of their accommodation in Australian dollars.

(c) Ruth also purchased a watch in Sydney. [3]

To pay for the watch, she borrowed S$ 2340 for 2 years at an interest of 5.6% per annum compounded half-yearly.

Calculate the amount of interest she paid for the watch.

(c) Sydney has a population of 4.627 million and Darwin has a population of 132 000. [2]

Calculate how many more people live in Sydney than in Darwin, giving your answer in standard form.

7 Hillgrove organised an annual school carnival to raise funds for a children's home.

Two different classes sold three different flavours of Bubble Tea to raise funds.

The table below shows the number of Bubble Tea sold by each class and the price of the Bubble Tea.

Flavours	Class 4-10	Class 4-11	Price per cup
Milk Oolong	30	20	\$1.90
Cheese Peach	20	40	\$3.20
Brown Sugar	15	10	\$2.50

(a) Represent the number of different flavours of Bubble Tea sold by each class in a 3×2 matrix, **F**. [1]

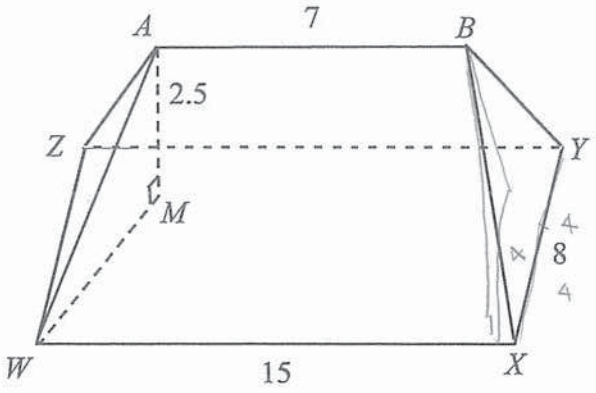
(b) Represent the prices of Bubble Tea in a 1×3 matrix, **P**. [1]

(c) Evaluate the matrix $\mathbf{R} = \mathbf{PF}$. [2]

(d) Explain what each element in matrix $\mathbf{R}$ means. [1]

(e) $\mathbf{M}$ is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Evaluate $\mathbf{A} = \mathbf{RM}$ and explain what the element in $\mathbf{A}$ means. [2]

8



The diagram represents the roof of a house. The ridge AB is horizontal and is built centrally above the rectangular plane $WXYZ$. AM is the height of the ridge above $WXYZ$. It is given that $WX = 15$ m, $XY = 8$ m, $AB = 7$ m and $AM = 2.5$ m.

(a) Show that WM is 5.657 m, correct to 4 significant figures. [2]

(b) Find

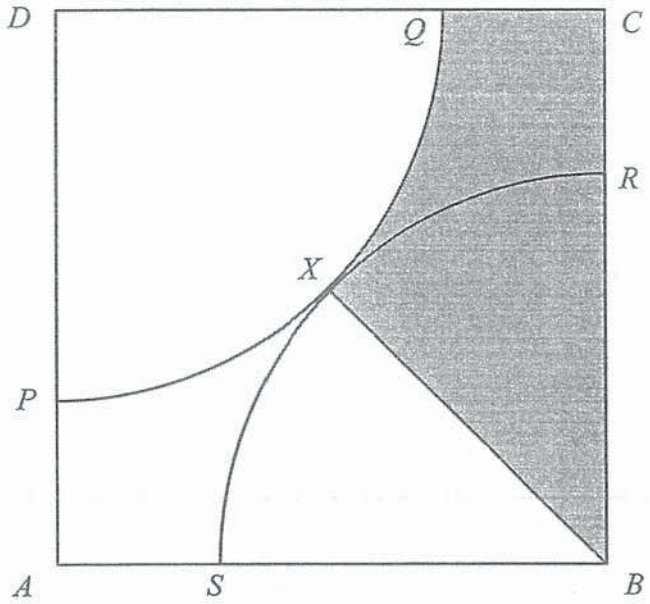
(i) angle AWM, [2]

(ii) AW, [2]

(iii) AX. [4]

(c) Find the **smallest** angle of elevation of A from a point along WX. [2]

9



ABCD is a square of length 12 cm. *DQP* and *BRS* are two identical quadrants and arcs *QP* and *RS* meet at point *X*. *DCX* is a straight line.

(a) Find

(i) the perimeter of the shaded region, [4]

(ii) the area of QRX. [Note: QRX is not a triangle] [3]

(b) Show that triangle DAB is similar to triangle RCQ. [3]

10 There was a hit and run car accident along the Pan-Island Expressway (PIE). It was raining the day before and the road was wet. PIE has a speed limit of 80 km/h. The police needed to find out who the driver was and there were some items found at the scene.

Exhibit A: slip of paper with driver's telephone number

Exhibit B: driver's shoes print –25 cm in length

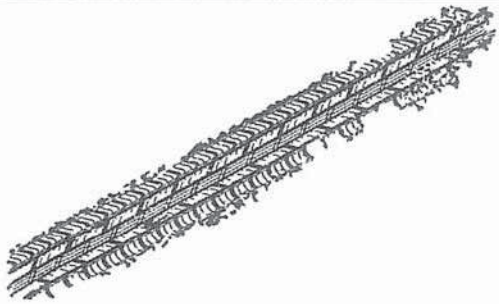
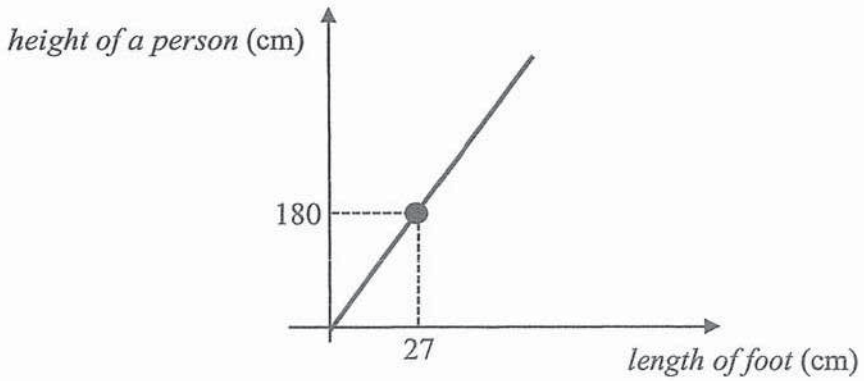
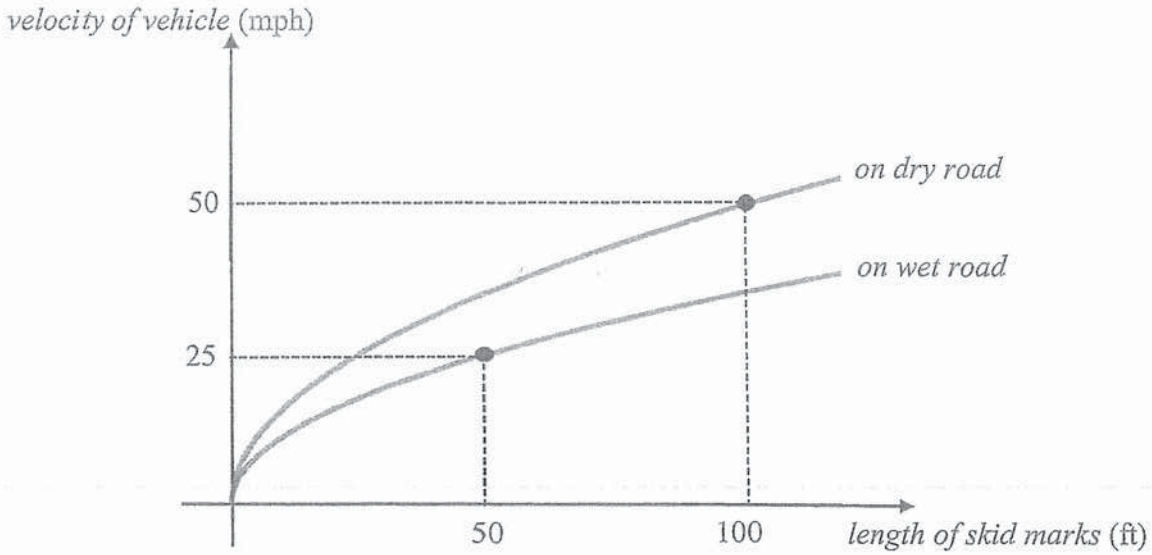


Exhibit C: driver's vehicle skid marks – 350 ft. in length
It is given that the velocity of the car is directly proportional to the square root of the length of skid marks.

The police also uses graphs below to help them to solve crimes.





Length Conversion	
1 ft	0.3048 m
1 mile	1.609 km
ft – feet *mph* – miles per hour	

Note that the graphs only give predicted estimated values.

(a) Given that there are 8-digits in the driver's telephone number, find the total number of possible telephone numbers. [2]

The police then narrowed to three suspects with their details below.

	Daniel Lee	Jacob Yap	Samuel Wong
Height	1.78 m	1.66 m	1.82 m
Brand of vehicle they own	Kia	Toyota	Volkswagen

Speed limit	Composition Fine	
	Light Vehicle *(Examples: motor cars, motorcycles, light goods van, etc.)*	Heavy Vehicle *(Examples: tractors, buses, trailers, etc.)*
Exceeding road speed limit by 1 – 20 km/h	$150	$200
Exceeding road speed limit by 21 – 30 km/h	$200	$250
Exceeding road speed limit by 31 – 40 km/h	$300	$400
Exceeding 41 km/h	prosecution in court	

(b) Find the driver who was involved in the accident and determine the appropriate fine for speeding ticket. [7]

Justify your decision with calculations.

HILLGROVE SECONDARY SCHOOL

HILLGROVE SECONDARY SCHOOL PRELIMINARY EXAMINATION 2019 SECONDARY 4 (EXPRESS) / 5 (NORMAL ACADEMIC)

CANDIDATE NAME	( )	CLASS	-
CENTRE NUMBER	S	INDEX NUMBER	

MATHEMATICS 4048/01

Paper 1 22 August 2019

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

8.00 a.m. to 10.00 a.m.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use	
TOTAL	80

Parent's/ Guardian's Signature:

_____

Setter: Miss Li Ziyi

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 Write the following numbers in order of size, starting with the **largest**.

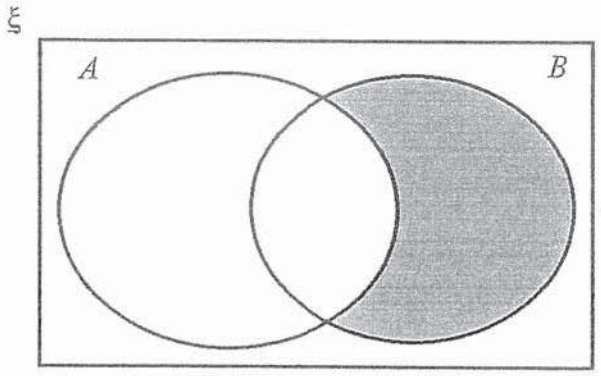
$$\frac{37}{100}, \quad 0.3\dot{7}, \quad 0.\dot{3}\dot{7}, \quad \left(\frac{37}{100}\right)^2$$

Answer _____, _____, _____, _____ [1]

---

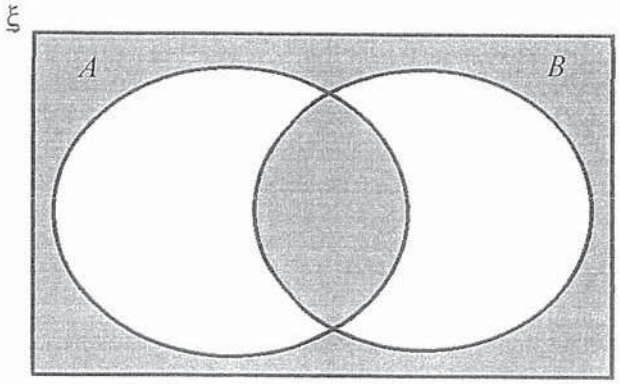
2 Write down the sets represented by the following shaded regions.

(a)



Answer _____ [1]

(b)



Answer _____ [1]

---

3 Show that $(1+5n)^2 - 6$ is a multiple of 5 for all integer values of n.

Answer

[2]

---

4 **(a)** Express $x^2 - 6x + 5$ in the form of $(x-a)^2 + b$.

Answer _____ [1]

(b) Hence, solve $x^2 - 6x + 5 = 0$.

Answer _____ or _____ [1]

---

5 Nithya runs a photography club.

Her club currently has 62 boys and 39 girls.

Her aim is to have at least 45 % of members to be girls.

Find the **smallest** number of girls she needs to recruit to achieve this.

Answer _____ [2]

---

6 A pizza is divided in the ratio of 3 : 7 : 5 for Adrian, Ben and Carl respectively. Ben then divides his share in the ratio of 9 : 7 and give it to Adrian and Carl respectively.

Carl says that both he and Adrian have the same amount of pizza now.

Do you agree with Carl? Explain your answer.

Answer

[2]

---

7 It is given that $y = \dfrac{3x+5b}{x-2c}$.

(a) Find y when $x=1,\ b=2,\ c=4$.

Answer _____ [1]

(b) Express x in terms of y, b and c.

Answer _____ [2]

8 Given that $\cos x = \dfrac{21}{29}$, where x is an acute angle, find the value of $\sin x + \cos(180^\circ - x) - \tan x$ without the use of calculator.

Answer _____ [3]

9 (a) The product of three prime numbers a, b and c is m, where m is an even number.

The smallest and the largest number out of the three prime numbers are a and c respectively.

State the value of a and explain your answer.

Answer _____

_____

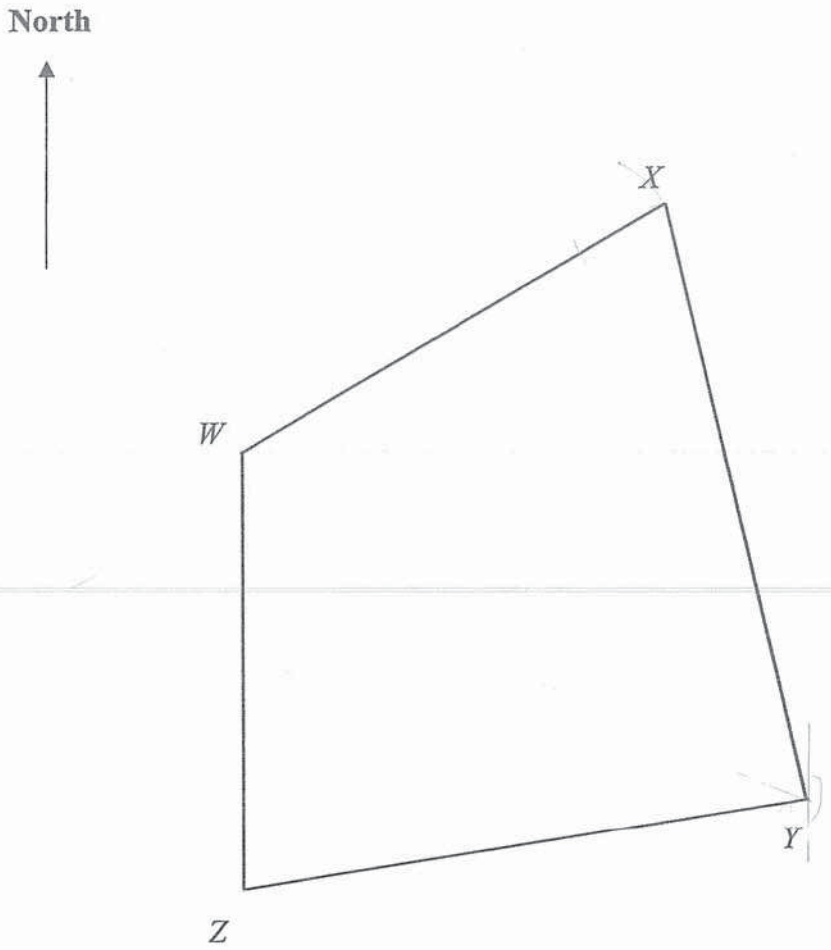
_____

_____ [2]

(b) Given that $b = 23$ and m is less than 670, find the largest possible value of c.

Answer 13 [2]

10 The diagram shows a plot of land labelled $WXYZ$.



(a) Construct the bisector of angle WZY. [1]

(b) Construct the perpendicular bisector of WZ. [1]

(c) **(i)** The government is planning to build a playground that is equidistant from W and Z and WZ and YZ. Mark and label the location of the playground as P. [1]

(ii) Hence, find the bearing of P from Y.

Answer _____ [1]

11 (a) Given that $4(x+2y)=5x-3y$, find the ratio of $x : y$.

Answer _____ [2]

(b) Solve $\frac{27^b}{9}=81^{b+1}$.

Answer _____ [2]

12 Winnie is drawing an n-sided regular polygon.

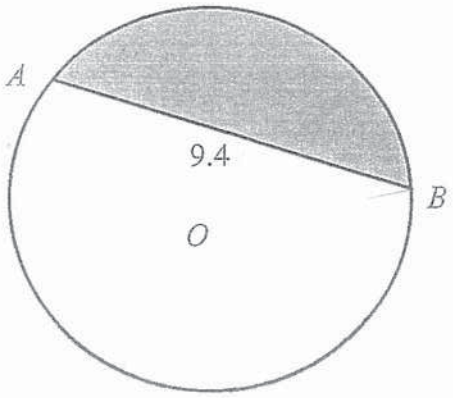
The total sum of the interior angles is $h°$.

Given that $h+n=1631$, find the values of h and n.

Answer $h=$ _____

$n=$ _____ [4]

13

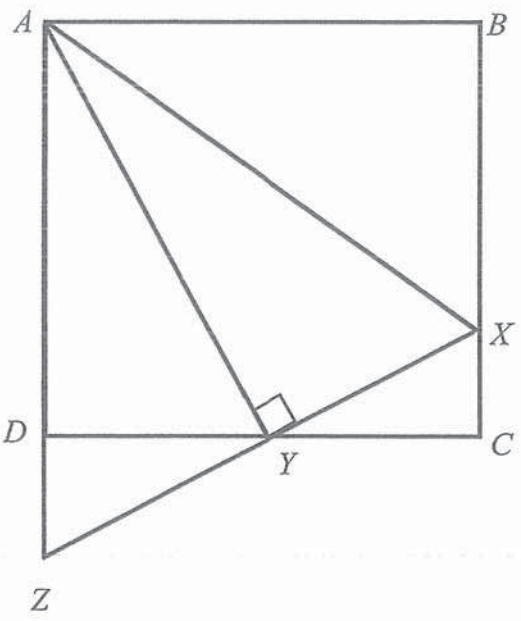


In the diagram, A and B are points on a circle, centre O. The radius of the circle is 5 cm and length of AB is 9.4 cm.

Find the area of the minor segment AB.

Answer _____ [4]

14



$ABCD$ is a square and AY is perpendicular to XZ. XYZ and ADZ are straight lines. Y is the midpoint of CD.

(a) Show that triangle DYZ is congruent to triangle CYX.

_____

_____

_____

_____ [2]

(ii) Show that $AZ = AX$.

_____

_____

_____

_____ [2]

15 Serene has two boxes of cards.
One box contains cards that has either a circle or a square.
The other box contain cards that are green or blue in colour.

Serene picks a card from each box at random.
The probability that she picks a circle card is c.
The probability that she picks a green card is g.

(a) Complete the table for the card that Serene picks, writing each probability in terms of c and g.

Event	Probability
Circle and Green	
Square and Green	
Circle and Blue	
Square and Blue	

[2]

(b) The probability that she picks a circle card is $\frac{2}{7}$.

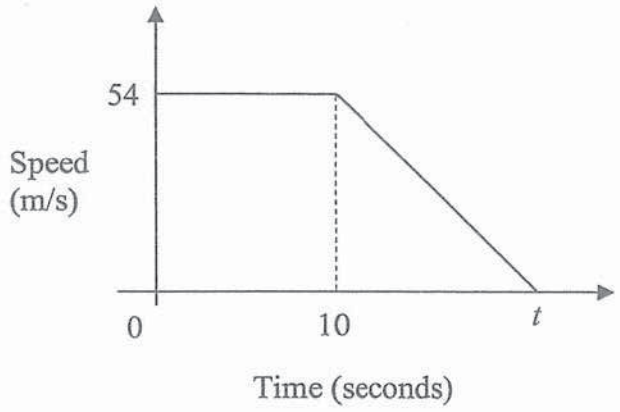
The probability that she picks a circle card and green card is $\frac{1}{5}$.

Hence, or otherwise, find the total number of green cards given that the total number of blue and green cards is 30.

Answer _____ [3]

16 The diagram shows the speed – time graph for a truck's journey.

The distance travelled for the first t seconds is 756 m.



(a) Find t.

Answer _____ [2]

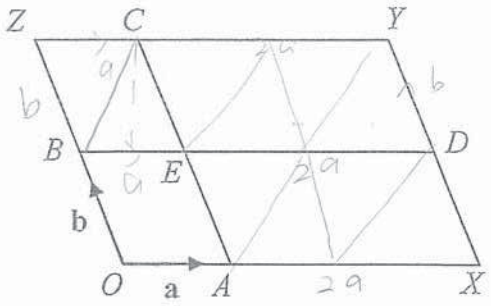
(b) Calculate the deceleration at the 15^{th} second of the journey.

Answer _____ [2]

(c) Find the average speed in km/h.

Answer _____ [2]

17 The diagram shows a parallelogram $OXYZ$.



$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. BED and AEC are lines parallel to the sides of the parallelogram. $\overrightarrow{OX} = 3\overrightarrow{OA}$ and $\overrightarrow{OZ} = 2\overrightarrow{OB}$.

(a) Express $\overrightarrow{EY}$ in terms of **a** and **b**, as simply as possible.

Answer _____ [2]

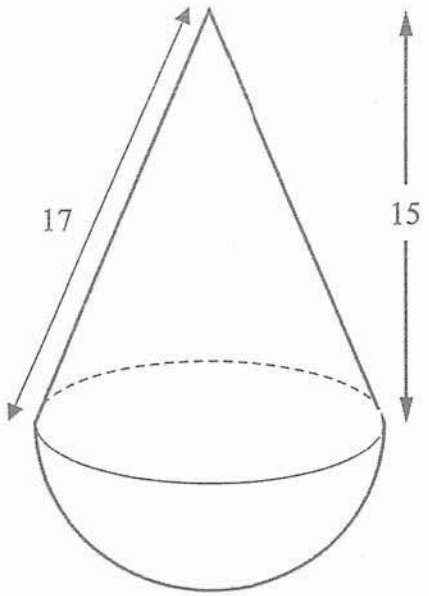
(b) Write down two vectors that can be written as $\mathbf{b} - 3\mathbf{a}$.

Answer _____ and _____ [2]

(c) Find the ratio of the area of triangle BCE to the area of parallelogram $OXYZ$.

Answer _____ [2]

18



The diagram shows a toy that combines a cone and a hemisphere. The slant height of the cone is 17 cm and the perpendicular height of the cone is 15 cm.

(a) Show that the radius of the hemisphere is 8 cm.

[2]

(b) Calculate the surface area of the toy.

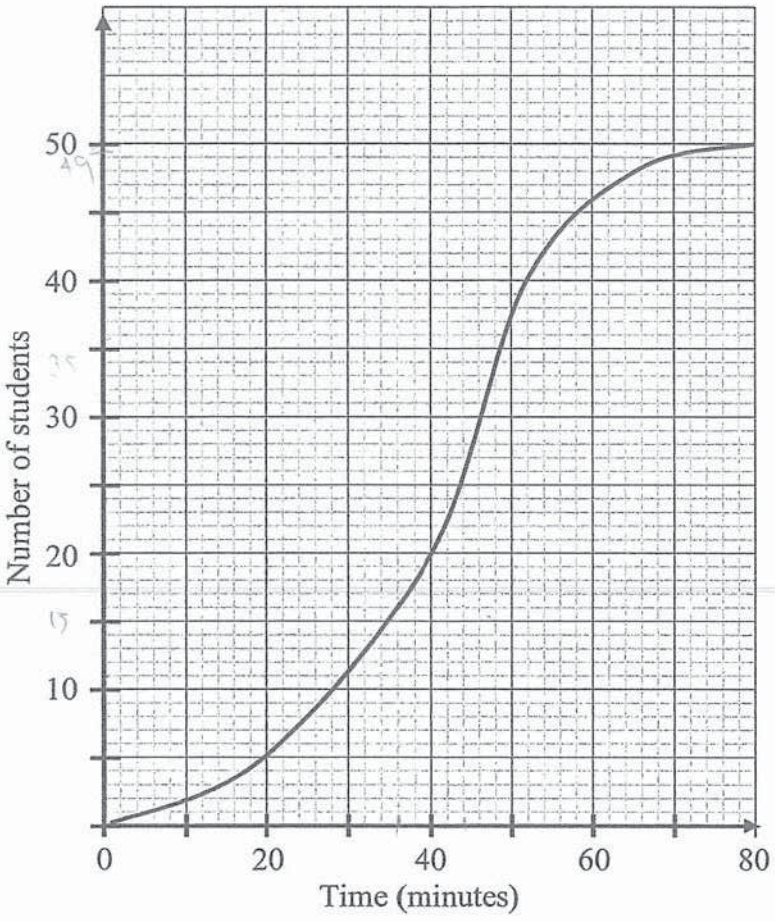
Answer _____ [3]

(c) The company wants to make a miniature version of the toy such that the volume of the miniature toy is $\frac{1}{6}$ of the volume of the actual toy.

Calculate the radius of the miniature toy.

Answer _____ [2]

19 The travelling times taken by 50 students from home to school are recorded.
The cumulative frequency curve below shows the distribution of their times.



(a) Use your graph to estimate

(i) the median

Answer _____ [1]

(ii) the interquartile range

Answer _____ [2]

(b) Find the probability that two students, chosen at random, take less than 70 minutes to travel to school.

Answer _____ [2]

(c) A student states that 32% of the students took at least 36 minutes to travel to school.

Comment on whether the data from the school supports this claim.

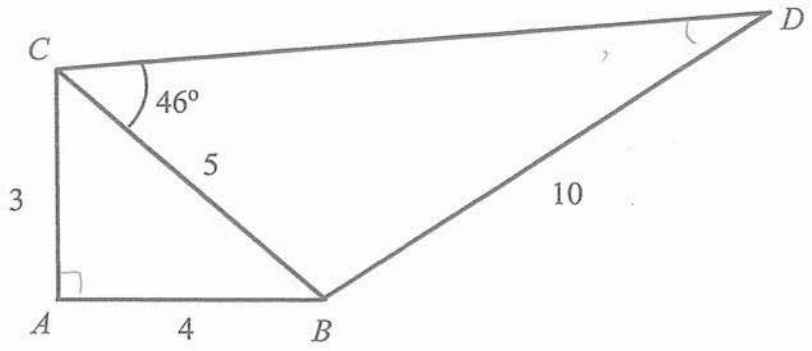
Answer _____

_____

_____

_____ [2]

20 The diagram shows two triangles ABC and BDC.



$AC = 3$ cm, $AB = 4$ cm, $BC = 5$ cm, $BD = 10$ cm and $\angle CBD = 113°$.

(a) Show that $\angle CAB$ is a right angle.

[2]

(b) Find $\angle BDC$.

Answer _____ [2]

(c) Find the area of $ABDC$.

Answer _____ [2]

(d) A circle is drawn through the points A, B and C. State the length of the diameter of the circle. Explain your answer.

_____

_____

_____

_____ [2]

HILLGROVE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2019
SECONDARY 4 (EXPRESS) / 5 (NORMAL ACADEMIC)

CANDIDATE NAME ( ) CLASS -

CENTRE NUMBER S INDEX NUMBER

MATHEMATICS **4048/02**

Paper 2 **26 August 2019**

Candidates answer on the Question Paper. **2 hours 30 minutes**

No Additional Materials are required. **8.00 a.m. to 10.30 a.m.**

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
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Answer **all** questions
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Omission of essential working will result in loss of marks.
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For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Parent's/ Guardian's Signature:

For Examiner's Use	
TOTAL	100

Setter: Miss Li Ziyi

This document consists of **25** printed pages

Mathematical Formulae

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Total amount = $P\left(1+\frac{r}{100}\right)^n$

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Curved surface area of a cone = πrl

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Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle ABC = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f}-\left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer all the questions.

1 (a) Solve the inequality $\frac{2+3x}{5} \leq \frac{3-x}{2}$. [2]

$\frac{2+3x}{5} \leq \frac{3-x}{2}$

$4+6x \leq 15-5x$ [M1]

$11x \leq 11$

$x \leq 1$ [A1]

(b) Simplify $\frac{16x^2-y^2}{4x^2+9xy+2y^2}$. [3]

$\frac{16x^2-y^2}{4x^2+9xy+2y^2} = \frac{(4x+y)(4x-y)}{(4x+y)(x+2y)}$ [M1] [M1]

$\frac{4x-y}{x+2y}$ [A1]

$4x^2 + 9xy + 2y^2$

$3x \quad -y$

$3x \quad -4$

$x \quad +1$

(c) Given that $x = -\frac{1}{2}$ is a solution of $(m-1)x^2 + (m-2)x = 2m-15$, find the value of m and the second possible value of x. [3]

since $x = -\frac{1}{2}$ is a solution,

$(m-1)(-\frac{1}{2})^2 + (m-2)(-\frac{1}{2}) = 2m-15$ [M1]

$\frac{1}{4}m - \frac{1}{4} - \frac{1}{2}m + 1 = 2m - 15$

$\frac{63}{4} = \frac{9}{4}m$

$m = 7.$ [A1]

$6x^2 + 5x = -1$

$6x^2 + 5x + 1 = 0$

$(3x+1)(2x+1) = 0$

$\therefore x = -\frac{1}{3}$ is the second possible value. [A1]

(d) Solve the equation $x+5 = \frac{15x-1}{3x-1}$. [3]

$(x+5)(3x-1) = 15x-1$ [M1]

$3x^2 - x + 15x - 5 = 15x - 1$

$3x^2 - x - 4 = 0$

$(3x-4)(x+1) = 0$ [M1]

$\therefore x = -1$ or $1\frac{1}{3}$ [A1]

2 $\dot{AB} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\dot{OB} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$, where O is the origin.

(a) Find the length of AB. [1]

$AB = \sqrt{3^2 + (-2)^2}$

$= \sqrt{13}$

$= 3.605551275$

$= 3.61$ units. [B1]

(b) Find the equation of line AB. [2]

grad of $AB = \frac{-2}{3}$

subst. $m = -\frac{2}{3}$ and $(4, 0)$ into $y = mx + c$.

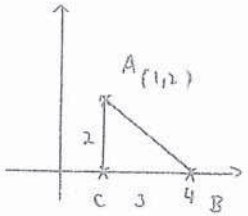
$0 = -\frac{2}{3}(4) + c$ [M1]

$c = 2\frac{2}{3}$

Eqn of AB: $y = -\frac{2}{3}x + 2\frac{2}{3}$ [A1]

(c) C is a point on x -- axis such that angle ACB is a right angle. [2]

Find the tangent of angle ABC.



$\vec{CA} = \vec{CB} + \vec{BA}$

$\begin{pmatrix} 4 \\ 0 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ [M1]

$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

tan $\angle ABC = \frac{2}{3}$ [A1]

(d) The equation of the line m is $3y + 2x = 6$. Line m intersects the x – axis at D and y – axis at E.

(i) Show how you can tell that the line m does not intersect the line AB. [2]

line m: $3y + 2x = 6$

$y = -\frac{2}{3}x + 2$

grad of $m = -\frac{2}{3}$ [M1] : find grad of m.

since line m is parallel to line AB and has a different y-intercept, they will not intersect one another. [B1]

(ii) Find $\vec{ED}$ [2]

For D, at $y = 0$, $x = 3$.

For E, at $x = 0$, $y = 2$.

$\vec{ED} = \vec{EO} + \vec{OD}$

$= \begin{pmatrix} 0 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

(iii) What type of quadrilateral is $ABDE$? Explain your answer. [3]

$\vec{DE}$ is parallel to $\vec{AB}$ because $\vec{ED} = \vec{AB}$. [B1]

$\vec{BD} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$\vec{AE} = \begin{pmatrix} -1 \\ -2 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$\vec{AE}$ is also parallel to $\vec{BD}$ because $\vec{AE} = \vec{BD}$ [M1]

∴ ABDE is a parallelogram because it has 2 pairs of parallel lines. [B1]

3 Rashidi wants to make a closed rectangular box to contain his toys. The box has a rectangular base of sides $3x$ cm and x cm and its height is h cm.

(a) (i) The volume of the box is 90 cm^3. [1]

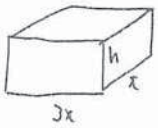
Express h in terms of x.

$90 = (3x)(x)(h)$

$h = \frac{90}{3x^2}$

$= \frac{30}{x^2}$ [A1]

(ii) Show that the total surface area of the box A cm^2, is given by $A = \frac{240}{x} + 6x^2$. [2]



$A = 2hx + 2(3x)(h) + 2(3x)(x)$ [M1]

$= 8hx + 6x^2$

$= 8\left(\frac{30}{x^2}\right)x + 6x^2$

$= \frac{240}{x} + 6x^2$ [A1]

The table shows some corresponding values of x and A, correct to the nearest integer.

x	1	2	3	4	5	6	7	8
A	246	144	134	156	b	256	328	414

198.

(a) Find the value of b. [1]

$b = \frac{240}{5} + 6(5)^2$

$= 198$ [B1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $0 \le x \le 8$.
Using a scale of 2 cm to represent 50 units, draw a vertical A-axis for $0 \le A \le 500$.

On your axes, plot the points given in the table and join them with a smooth curve on the grid in page 9. [3]

(c) By drawing a tangent, find the gradient of the curve at (2, 144). [2]

(1, 190) and (6, 0)

$\text{grad} = \frac{180 - 0}{1 - 6}$

$= -36$

[M1]: draw tangent at correct point.

[A1]: $-40.5 \le \text{grad} \le -31.9$

(d) Rashidi wants to make a box with a total surface area of 100 cm^2 without changing the volume of the box. Is it possible to do so? Explain your answer. [2]

[A1]

It is not possible to do so because the box with the smallest possible surface area is approximately 185 cm^2. [M1]

Name _____ Index No. _____

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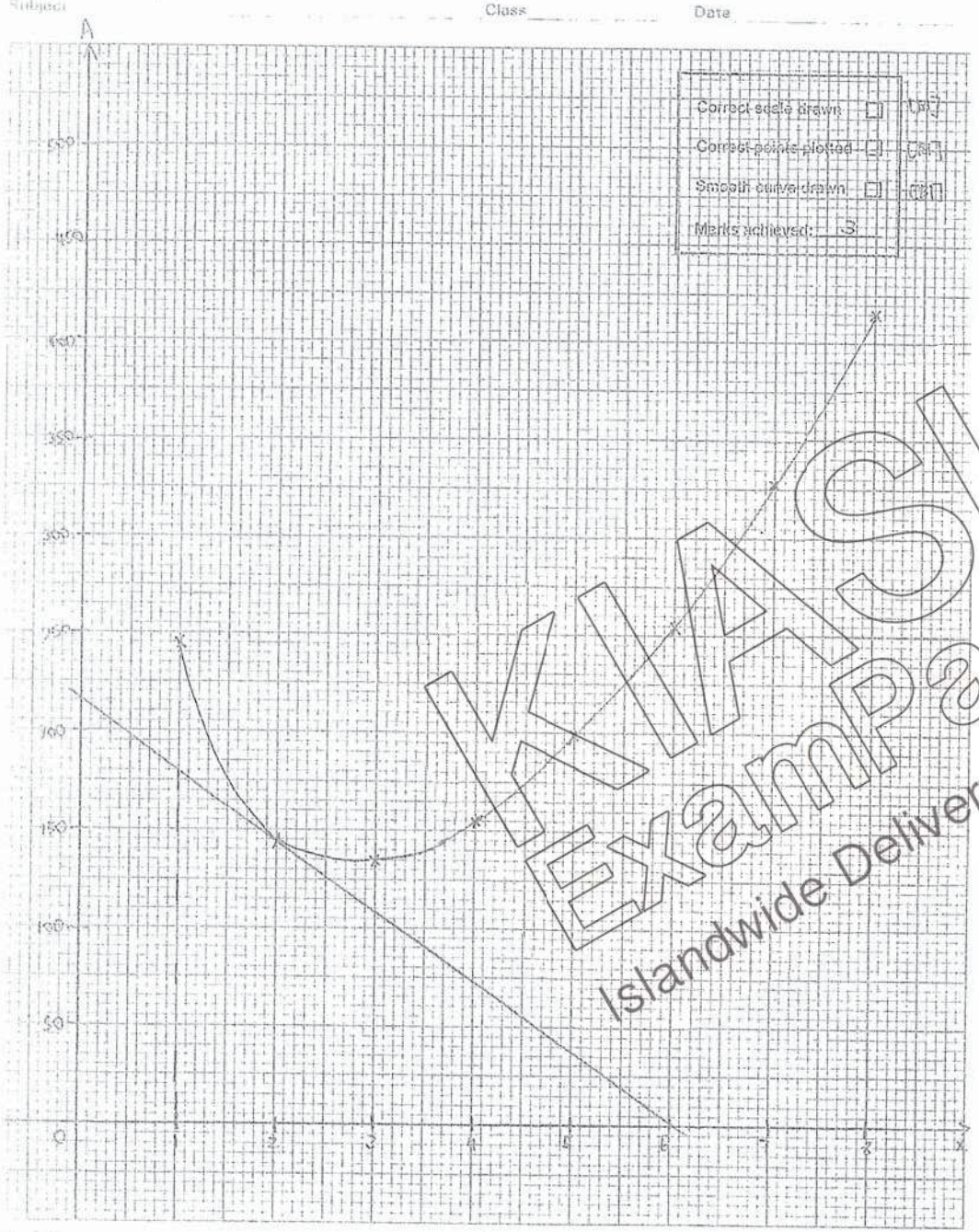


9

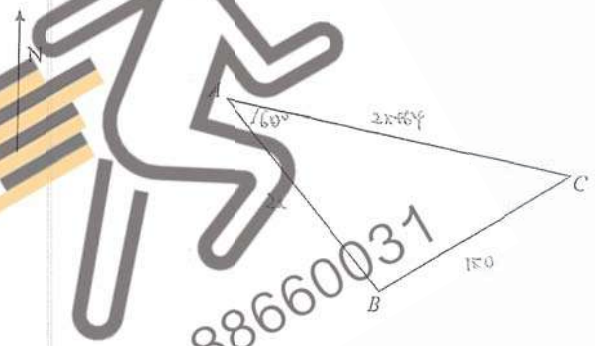
4 A, B and C represent three islands.

Both boats X and Y left from island A.

The bearings of B and C from A is 160° and 100° respectively.

Boat X sailed towards B at a speed of x km/h. Boat Y sailed towards C at a speed of $(x+32)$ km/h.

Boat X and Y took 2 hours to reach B and C respectively.



(a) Write an expression, in terms of x, for the distance travelled by Boat X. [1]

Dist. = 2 × x

= 2x [1]

(b) B and C are 150 km apart. [4]

Write down an equation in x to represent this information and show that it reduces to

$$x^2 + 32x - 4601 = 0.$$

∠BAC = 160° − 100° [M1]

= 60°

$150^2 = (2x)^2 + (2x+64)^2 - 2(2x)(2x+64)\cos 60°$ [M1]

$22500 = 4x^2 + 4x^2 + 256x + 4096 - 4x^2 - 128x$ [M1]

$4x^2 + 128x - 18404 = 0$

$x^2 + 32x - 4601 = 0$ [A1]

(b) Solve the equation $x^2 + 32x - 4601 = 0$. [3]

$$x = \frac{-32 \pm \sqrt{(32)^2 - 4(1)(-4601)}}{2} \quad [M1]$$

$$= \frac{-32 \pm \sqrt{19428}}{2}$$

$$= 53.692\,18034 \text{ or } -85.692\,18034$$

$$= 53.7 \text{ or } -85.7 \quad [A1], [A1]$$

(c) Boat X returned from B to A and took 30 more minutes than the onward journey. [2]

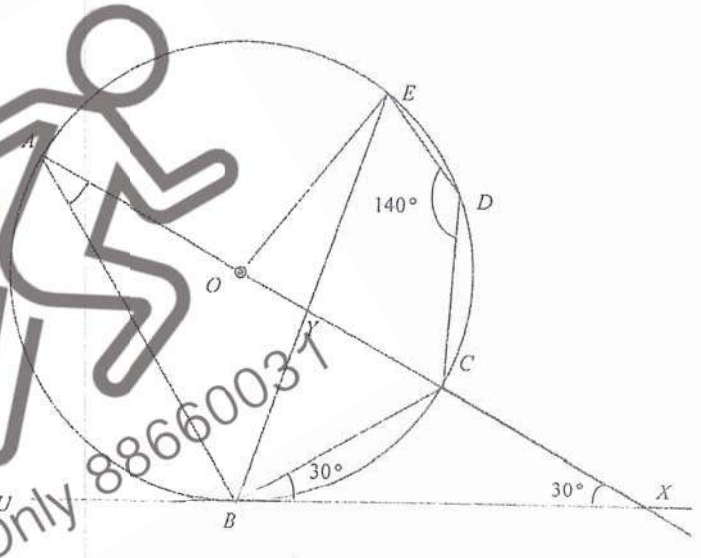
Find the speed of Boat X for the return journey.

$$\text{speed} = \frac{2 \times 53.692\,18034}{2.5} \quad [M1]$$

$$= 42.953\,744\,27$$

$$= 43.0 \text{ km/h} \quad [A1]$$

5 O is the centre of the circle passing through A, B, C, D and E. UBX is the tangent to the circle at B. ACX is a straight line which passes through O. $\angle CBX = \angle CXB = 30°$ and $\angle EDC = 140°$.



(a) Stating your reasons clearly, find,

(i) $\angle BAC$, [2]

$\angle BCA = 30° + 30°$ [M1] (ext. $\angle$ in a triangle)

$= 60°$

$\angle ABC = 90°$ (rt. $\angle$ in a semicircle).

$\angle BAC = 180° - 60° - 90°$ (sum of $\angle$ in a triangle)

$= 30°$, [A1]

Note:
Alternative solution

$\angle BAC = 30°$ ($\angle$ in alt. segment] → [B2]

(ii) $\angle CEB$, [1]

$\angle CEB = 30°$ [B1] ($\angle$s in the same segment).

(iii) $\angle CBE$, [1]

$\angle CBE = 180° - 140°$ ($\angle$s in opp. segment)

$= 40°$ [A1]

(iv) $\angle OBY$. [2]

$\angle OBX = 90°$ (tan $\perp$ rad)

$\angle OBY = 90° - 40° - 30°$

$= 20°$ [A1]

(b) T is a point such that angle ATE is 60°. and on the same side as B. [3]

State whether point T lies in the circle.

Explain your answer.

$\angle ABE = 20° + 30°$ (base $\angle$s of isos. $\triangle$) [M1]

$= 50°$ [M1]

$\therefore$ T lies inside the circle because $\angle ATE > 50°$. [A1]

6 Angela and Ruth went on a free-and-easy trip to Sydney.

(a) They both exchanged S$1600 at a money changer in Singapore. The money changer used an exchange rate between Singapore dollars (S$) and Australian dollars (A$) of A$ 1 = S$0.95. [2]

Calculate the amount of Australian dollars they received.

Amount of A$

$= \frac{1600}{0.95}$ [M1]

$= A\$1684.210526$

$= A\$1684.21$ [A1]

(b) They booked their accommodation through the Waterbnb application. [2]

Each night at the accommodation costs A$110.25. However, there was a 10% service charge. An additional 5% daily cleaning fee was imposed only on the cost of the accommodation.

They booked a total of 8 nights at the accommodation.

Calculate the total cost of their accommodation in Australian dollars.

Total cost. $= (8 \times 110.25) \times \frac{110}{100} + \frac{5}{100} \times (8 \times 110.25)$ [M1]

$= A\$1014.30$ [A1]

(c) Ruth also purchased a watch in Sydney. [3]

To pay for the watch, she borrowed S$ 2340 for 2 years at an interest of 5.6% per annum compounded half-yearly.

Calculate the amount of interest she paid for the watch.

Amount of interest [M1]

$= 2340\left(1+\frac{5.6}{\frac{2}{100}}\right)^{F} - 2340$

$= S\$\ 273.294269$

$= S\$\ 273.29$ [A1]

(d) Sydney has a population of 4.627 million and Darwin has a population of 132 000. [2]

Calculate how many more people live in Sydney than in Darwin, giving your answer in standard form.

$4.627 \times 10^6 - 132000$ [M1]

$= 4495000$

$= 4.495 \times 10^6$ more people [A1]

7 Hillgrove organised an annual school carnival to raise funds for a children's home.

Two different classes sold three different flavours of Bubble Tea to raise funds.

The table below shows the number of Bubble Tea sold by each class and the price of the Bubble Tea.

Flavours	Class 4-10	Class 4-11	Price per cup
Milk Oolong	30	20	$1.90
Cheese Peach	20	40	$3.20
Brown Sugar	15	10	$2.50

(a) Represent the number of different flavours of Bubble Tea sold by each class in a 3×2 matrix, F. [1]

$F = \begin{pmatrix} 30 & 20 \\ 20 & 40 \\ 15 & 10 \end{pmatrix}$ [B1]

(b) Represent the prices of Bubble Tea in a 1×3 matrix, P. [1]

$P = (1.9 \quad 3.2 \quad 2.5)$ [B1]

(c) Evaluate the matrix $R = PF$. [2]

$R = PF$

$= (1.9 \quad 3.2 \quad 2.5)_{1\times3} \begin{pmatrix} 30 & 20 \\ 20 & 40 \\ 15 & 10 \end{pmatrix}_{3\times2}$ [M1]

$= (159.5 \quad 191)$ [A1]

(d) Explain what each element in matrix R means. [1]

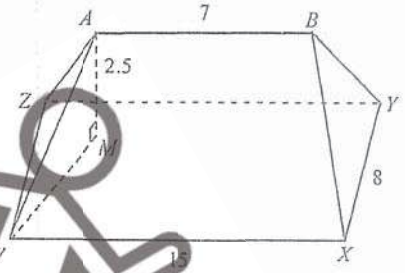
The elements in R represents the total amount of money 4-10 and 4-11 collected respectively. [B1]

(e) M is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Evaluate A = RM and explain what the element in A means. [2]

$A = \begin{pmatrix} 158 \cdot 5 & 191 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 349 \cdot 5 \end{pmatrix}$ [A1]

This element represents the total amount of money collected from both 4-10 and 4-11. [B1]



The diagram represents the roof of a house. The ridge AB is horizontal and is built centrally above the rectangular plane $WXYZ$. AM is the height of the ridge above $WXYZ$. It is given that $WX = 15$ m, $XY = 8$ m, $AB = 7$ m and $AM = 2.5$ m.

(a) Show that WM is 5.657 m, correct to 4 significant figures. [2]

$WM = \sqrt{4^2 + 4^2}$ [M1]

$= 5.656854249$

$= 5.657$ m [A1]

(b) Find

(i) angle AWM, [2]

$\tan \angle AWM = \frac{2.5}{\sqrt{32}}$ [M1]

$\angle AWM = 23.84263602$

$= 23.8°$ [A1]

(ii) AW, [2]

$AW = \sqrt{2.5^2 + 5.656854249^2}$ [M1]

$= 6.184858439$

$= 6.18$ m [A1]

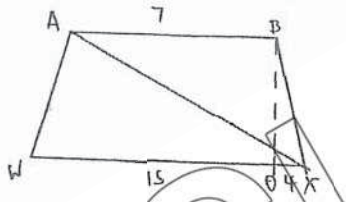
(iii) AX. [4]

$\sin \angle OBX = \frac{4}{6.184858439}$ [M1]

$\angle OBX = 40.29631692°$

$\angle ABX = 90° + 40.29631692°$ [M1]

$= 130.2963169°$



$AX^2 = 7^2 + 6.184858439^2 - 2(7)(6.184858439)\cos 130.2963169°$ [M1]

$AX = 11.96881255$

$= 12.0$ m [A1]

(c) Find the **smallest** angle of elevation of A from a point along WX. [2]

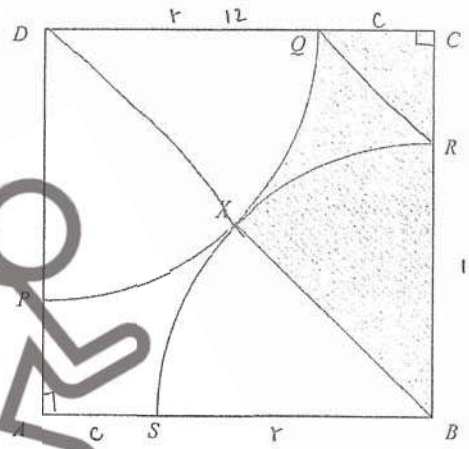
smallest $\angle$ of elevation $= \angle AXM$

$\sin \angle AXM = \frac{2.5}{11.96881255}$ [M1]

$\angle AXM = 12.05650442$

$\therefore$ smallest $\angle$

$= 12.1°$ (to 1 d.p.) [A1]



$ABCD$ is a square of length 12 cm. DQP and BRS are two identical quadrants and arcs QP and RS meet at point X. BXD is a straight line.

(a) Find

(i) the perimeter of the shaded region, [4]

$DB = \sqrt{12^2 + 12^2}$ [M1]

$= \sqrt{288}$ cm.

radius of circle $= \frac{1}{2}\sqrt{288}$ cm.

Arc length $AS = r\theta$

$= \sqrt{288} \times \frac{\pi}{2}$ [M1]

$= \frac{\pi\sqrt{288}}{2}$

Perimeter $= \frac{\pi\sqrt{288}}{4} + [12 - \frac{1}{2}\sqrt{288}] + 12. + \frac{1}{2}\sqrt{288}$ (find QC: [M1])

$= 37.32384831$

$= 37.3$ cm [A1]

(ii) the area of QRX. [QRX is not a triangle] [3]

Area of QXRC

$= \frac{(12\times12)}{2} - \left[\frac{1}{2}\left(\frac{\sqrt{288}}{2}\right)^2 \times \frac{\pi}{4}\right] \times 2$ [M1]

$= 15.45133224$ cm^2

Area of QRX $= 15.45133224 - \frac{1}{2} \times [12 - \frac{1}{2}\sqrt{288}]$ [M1]

$= 9.274704728$

$= 9.27$ cm^2 [A1]

(b) Show that triangle DAB is similar to triangle RCQ. [3]

$\angle DAB = \angle RCQ = 90°$ (square) [M1]

$\angle ADB = 45°$ (base $\angle$s of isos. triangle)

$\angle CRQ = 45°$ (base $\angle$s of isos. triangle) [M1]

$= \angle ADB$

$\therefore \triangle DAB$ is similar to $\triangle RCQ$ because their corresponding angles are equal. [A1]

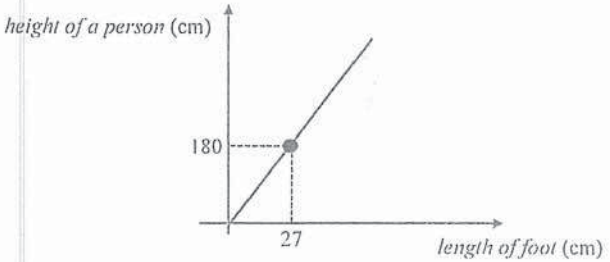
10 There was a hit and run car accident along the Pan-Island Expressway (PIE). It was raining the day before and the road was wet. PIE has a speed limit of 80 km/h. The police needed to find out who the driver was and there were some items found at the scene.

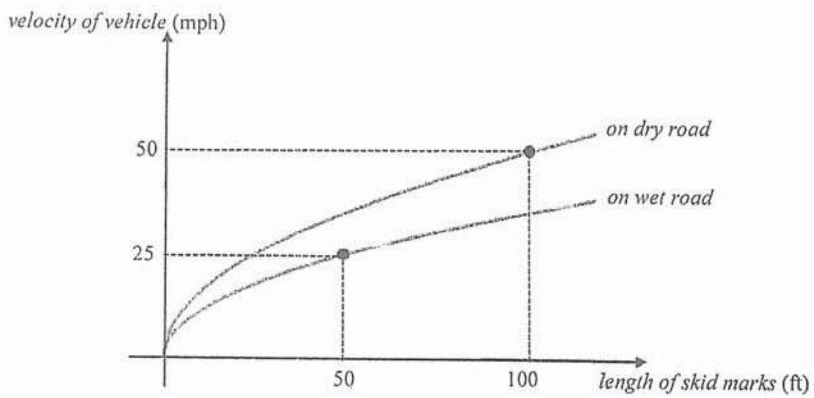
Exhibit A: slip of paper with driver's telephone number

Exhibit B: driver's shoes print – 25 cm in length

Exhibit C: driver's vehicle skid marks – 350 ft. in length
It is given that the velocity of the car is directly proportional to the square root of the length of skid marks.

The police also uses graphs below to help them to solve crimes.





Length Conversion	
1 ft	0.3048 m
1 mile	1.609 km
ft – feet *mph* – miles per hour	

Note that the graphs only give predicted estimated values.

(a) Given that there are 8-digits in the driver's telephone number, find the total number of possible telephone numbers. [2]

Total no. = 10 X10 [M1]
= 100 [A1]

The police then narrowed to three suspects with their details below.

	Daniel Lee	Jacob Yap	Samuel Wong
Height	1.78 m	1.66 m	1.82 m
Brand of vehicle they own	Kia	Toyota	Volkswagen

Speed limit	Composition Fine	
	Light Vehicle *(Examples: motor cars, motorcycles, light goods van, etc.)*	Heavy Vehicle *(Examples: tractors, buses, trailers, etc.)*
Exceeding road speed limit by 1 – 20 km/h	$150	$200
Exceeding road speed limit by 21 – 30 km/h	$200	$250
Exceeding road speed limit by 31 – 40 km/h	$300	$400
Exceeding 41 km/h	prosecution in court	

(b) Find the driver who was involved in the accident and determine the appropriate fine for speeding ticket. [7]

Justify your decision with calculations.

(b) let the height be h.

$$\frac{h}{25} = \frac{190}{27} \quad [M1]$$

$$h = 166\frac{2}{3} \text{ cm}.$$

since velocity of car is directly proportional to square root of length of skid mark,

$$V = k\sqrt{L}, \text{ where } k \text{ is a constant.}$$

$$25 = k\sqrt{50} \quad [M1]$$

$$k = \frac{25}{\sqrt{50}}$$

$$V = \frac{25}{\sqrt{50}}\sqrt{L}$$

to find velocity at that instant,

$$V = \frac{25}{\sqrt{50}} \times \sqrt{350} \quad [M1]$$

$$= 66.14378178 \text{ mph}$$

$$= 66.14378278 \times 1.609 \quad [M1]$$

$$= 106.4253485 \text{ km/h}.$$

Exceed = 106.4253485 − 80

$$= 26.4253485$$

$$= 26.4 \text{ km/h}. \quad [A1]$$

∴ The driver is Jacob Yap [A1]

and he has to pay $200 fine. [A1]

HILLGROVE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2019
SECONDARY 4 (EXPRESS) / 5 (NORMAL ACADEMIC)

CANDIDATE NAME: Marking Scheme. ( ) CLASS: [ ] - [ ]

CENTRE NUMBER: S [ ][ ][ ][ ] INDEX NUMBER: [ ][ ][ ][ ]

MATHEMATICS 4048/01

Paper 1 **22 August 2019**

2 hours

Candidates answer on the Question Paper. **8.00 a.m. to 10.00 a.m.**

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use	
TOTAL	/80

Parent's/ Guardian's Signature:

_____

Setter: Miss Li Ziyi

This document consists of **21** printed pages.

Mathematical Formulae

Compound interest

Total amount = $P\left(1+\frac{r}{100}\right)^n$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle ABC = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f}-\left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 Write the following numbers in order of size, starting with the **largest**.

$$\frac{37}{100}, \quad 0.3\dot{7}, \quad 0.\dot{3}\dot{7}, \quad \left(\frac{37}{100}\right)^2$$

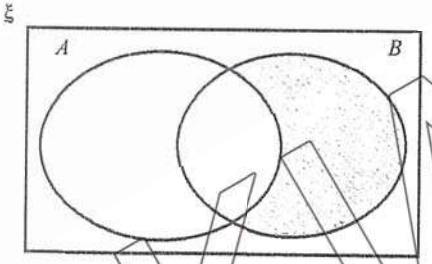
$\frac{37}{100} = 0.37$, $\left(\frac{37}{100}\right)^2 = 0.1369$.

Answer $0.3\dot{7}$, $0.\dot{3}\dot{7}$, $\frac{37}{100}$, $\left(\frac{37}{100}\right)^2$ [1]

[B1]

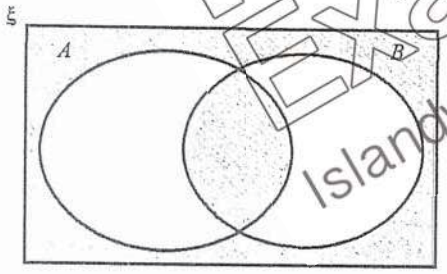
2 Write down the sets represented by the following shaded regions.

(a)



Answer $A' \cap B$ [B1] [1]

(b)



Answer $(A \cup B)' \cup (A \cap B)$ [B1] [1]

3 Show that $(1+5n)^2 - 6$ is a multiple of 5 for all integer values of n.

Answer

$(1+5n)^2 - 6 = 1 + 10n + 25n^2 - 6$

$= 25n^2 + 10n - 5$ [M1]

$= 5(5n^2 + 2n - 1)$

since $(1+5n)^2 - 6$ has a factor of 5, [A1]

$(1+5n)^2 - 6$ is a multiple of 6.

[2]

4 (a) Express $x^2 - 6x + 5$ in the form of $(x-a)^2 + b$.

$x^2 - 6x + 5 = x^2 - 6x + \left(\frac{-6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 5$

$= (x-3)^2 - 4$

Answer $(x-3)^2 - 4$ [A1] [1]

(b) Hence, solve $x^2 - 6x + 5 = 0$.

$(x-3)^2 - 4 = 0$

$(x-3)^2 = 4$

$x - 3 = \pm 2$

$x = 5$ or 1

together

Answer 5 or 1 [A1] [1]

5 Nithya runs a photography club.

Her club currently has 62 boys and 39 girls.

Her aim is to have at least 45 % of members to be girls.

Find the **smallest** number of girls ~~required to join her club.~~ she needs to recruit to achieve this.

Alternative solution:

$\frac{39+x}{62+39+x} \times 100\% \geq 45\%$. [M1]

$\frac{39+x}{101+x} \geq 0.45$

$39+x \geq 0.45(101+x)$

$0.55x \geq 6.45$

$x \geq 11\frac{8}{11}$

smallest no. = 12. [A1]

Let the smallest no. of girls be x.

$\frac{39+x}{62+39+x} \times 100\% = 45\%$. [M1]

$\frac{39+x}{101+x} = 0.45$

$39+x = 0.45(101+x)$

$x - 0.45x = 45.45 - 39$

$0.55x = 6.45$

$x = 11\frac{2}{11}$

$\therefore$ smallest no. = 12.

Answer 12. [A1] [2]

6 A pizza is divided in the ratio of 3 : 7 : 5 for Adrian, Ben and Carl respectively. Ben then divides his share in the ratio of 9 : 7 and give it to Adrian and Carl respectively.

Carl says that both he and Adrian have the same amount of pizza now.

Do you agree with Carl? Explain your answer.

Answer

A	:	B	:	C
3	:	7	:	5
$\frac{3}{7}$	:	1	:	$\frac{5}{7}$
$\frac{49}{7}$		16	:	$\frac{80}{7}$

No, Carl has more pizza than Adrian. [A1]

No. of units of pizza Adrian has $= \frac{49}{7} \times \frac{9}{16}$

$= 15\frac{5}{7}$

No. of units of pizza Carl has $= \frac{80}{7} \times \frac{7}{16}$

$= 18\frac{3}{7}$

[M1] : finding Adrian's / Carl's pizza. [2]

7 It is given that $y = \frac{3x+5b}{x-2c}$.

(a) Find y when $x = 1$, $b = 2$, $c = 4$.

$y = \frac{3(1)+5(2)}{1-2(4)}$

$= -1\frac{6}{7}$

Answer $-1\frac{6}{7}$ [A1] [1]

(b) Express x in terms of y, b and c.

$y = \frac{3x+5b}{x-2c}$

$xy - 2cy = 3x + 5b$

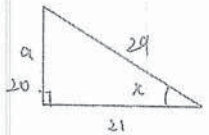
$xy - 3x = 2cy + 5b$

$x(y-3) = 2cy + 5b$ [M1]

$x = \frac{2cy+5b}{y-3}$

Answer $x = \frac{2cy+5b}{y-3}$ [A1] [2]

8 Given that $\cos x = \frac{21}{29}$, where x is an acute angle, find the value of $\sin x + \cos(180° - x) - \tan x$ without the use of calculator.



$a = \sqrt{29^2 - 21^2}$ [M1]

$= 20$.

$\sin x + \cos(180° - x) - \tan x$

$= \frac{20}{29} - \frac{21}{29} - \frac{20}{21}$ [M1]

$= -\frac{601}{609}$

Answer $-\frac{601}{609}$ [A1] [3]

9 (a) The product of three prime numbers a, b and c is m, where m is an even number.

The smallest ~~and the largest number out~~ of the three prime numbers are a and ~~c respectively.~~ $c > a$.

State the value of a and explain your answer.

Answer a must be 2. [B1] for m to be an even number, one of the three prime numbers must be even. 2 is the only even prime number. [B1]

[2]

(b) Given that $b = 23$ and m is less than 670, find the largest possible value of c.

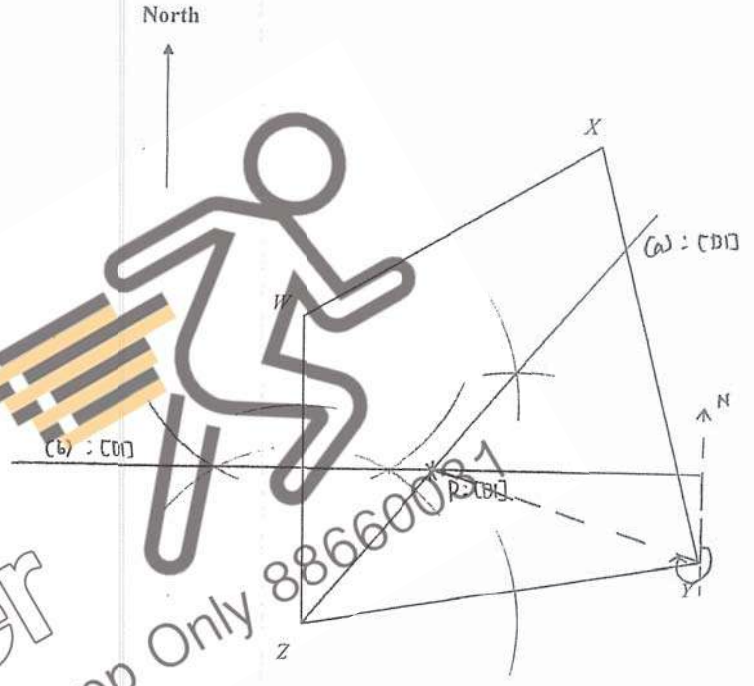
$2 \times 23 \times c < 670$. [M1]

$c < 14\frac{13}{23}$

∴ largest $c = 13$. [A1]

Answer 13 [2]

10 The diagram shows a plot of land labelled $WXYZ$.



(a) Construct the bisector of angle WZY. [1]

(b) Construct the perpendicular bisector of WZ. [1]

(c) (i) The government is planning to build a playground that is equidistant from W and Z and WZ and YZ. Mark and label the location of the playground as P. [1]

(ii) Hence, find the bearing of P from Y.

Answer 237.5° ± 1° [1] [D1]

11 (a) Given that $4(x+2y)=5x-3y$, find the ratio of $x:y$.

$4x+8y = 5x-3y$

$11y = x.$

$\frac{x}{y} = \frac{11}{1}$ [M1]

$\therefore x : y \Rightarrow 11:1$

Answer 11 : 1 [A1] [2]

(b) Solve $\frac{27^b}{9}=81^{b+1}$.

$\frac{27^b}{9} = 81^{b+1}$

$\frac{3^{3b}}{3^2} = (3^4)^{(b+1)}$

$3^{3b-2} = 3^{4b+4}$

$3b-2 = 4b+4$ [M1]

$-b = 6$ [A1]

$b = -6.$

Answer $b =$ -6 [2]

12 Winnie is drawing an n-sided regular polygon.

The total sum of the interior angles is $h°$.

Given that $h+n=1631$, find the values of h and n.

$(n-2)\times 180° = h.$ [M1] : form this eqn.

$180n - 360 = h.$ — (1)

$h+n = 1631$ — (2)

subst (1) into (2).

$180n - 360 + n = 1631$ [M1]

$181n = 1991$

$n = 11.$ [A1]

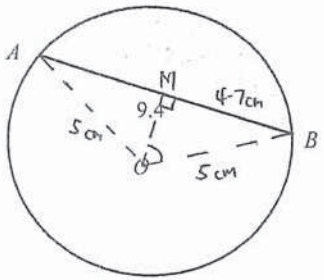
$h = 1631 - 11$

$= 1620.$ [A1]

Answer $h =$ 1620

$n =$ 11 [4]

13



In the diagram, A and B are points on a circle, centre O. The radius of the circle is 5 cm and length of AB is 9.4 cm.

Find the area of the minor segment AB.

$\sin \angle MOD = \frac{4.7}{5}$ [M1]: find $\angle AOB$ (any method)

$\angle AOB = 2 \times \sin^{-1}\left(\frac{4.7}{5}\right)$

$= 140.103\,1128°$

Area of minor segment AB — [M1]: find sector area

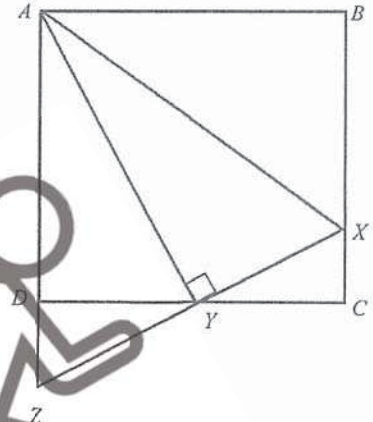
$= \frac{140.103129°}{360} \times \pi(5)^2 - \frac{1}{2}(5)^2 \sin 140.103129°$ [M1]: sector − triangle

$= 22.548\,15924$

$= 22.5\ cm^2$ [A1]

Answer 22.5 cm^2 [4]

14



$ABCD$ is a square and AY is perpendicular to XZ. XYZ and ADZ are straight lines. Y is the midpoint of CD.

(a) Show that triangle DYZ is congruent to triangle CYX.

$\angle DYZ = \angle CYX$ (vert. opp. $\angle$).

$DY = CY$ (Y is the midpoint) } [M1]

$\angle YDZ = 180° - 90° = 90°$ (adj $\angle$s on str. line)

$= \angle YCX$ (square). [2]

$\therefore \Delta DYZ$ is congruent to ΔCYX. (AAS) [A1]

(ii) Show that $AZ = AX$.

$AY = AY$ (common)

$YZ = YX$ (because ΔDYZ is congruent to ΔCYX)

$\angle AYZ = \angle AYX = 90°$ (adj $\angle$ on a str. line). } [M1]

ΔAYZ is congruent to ΔAYX. (SAS). [2]

$\therefore AZ = AX$. [A1]

15 Serene has two boxes of cards.
One box contains cards that has either a circle or a square.
The other box contain cards that are green or blue in colour.

Serene picks a card from each box at random.
The probability that she picks a circle card is c.
The probability that she picks a green card is g.

(a) Complete the table for the card that Serene picks, writing each probability in terms of c and g.

Event	Probability
Circle and Green	cg
Square and Green	$(1-c)g$
Circle and Blue	$c(1-g)$
Square and Blue	$(1-c)(1-g)$

[B1] : any 2 correct
[B2] ; all 4 correct

[2]

(b) The probability that she picks a circle card is $\frac{2}{7}$.

The probability that she picks a circle card and green card is $\frac{1}{5}$.

Hence, or otherwise, find the total number of green cards given that the total number of blue and green cards is 30.

$\therefore c = \frac{2}{7}$

$cg = \frac{1}{5}$

$(\frac{2}{7})g = \frac{1}{5}$ [M1] ; subst. $c = \frac{2}{7}$

$g = \frac{7}{10}$

$\therefore$ total no. of green cards

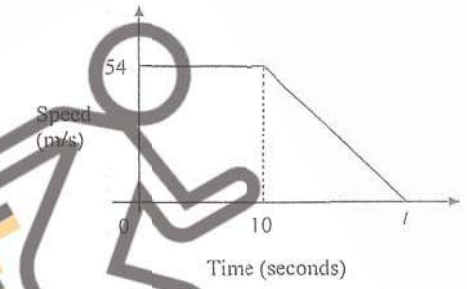
$= \frac{7}{10} \times 30$ M1

$= 21.$ [A1]

Answer 21 [3]

16 The diagram shows the speed – time graph for a truck's journey.

The distance travelled for the first t seconds is 756 m.



(a) Find t.

$\frac{1}{2} \times (10 + t) \times 54 = 756$ [M1]

$10 + t = 28$

$t = 18$ [A1]

Answer $t = 18$ [2]

(b) Calculate the deceleration at the 15th second of the journey.

$\text{acceleration} = \frac{54 - 0}{10 - 18}$ [M1]

$= -6\frac{3}{4}$ m/s^2

$\therefore$ deceleration $= 6\frac{3}{4}$ m/s^2 [A1]

Answer $6\frac{3}{4}$ m/s^2 or 6.75 m/s^2 [2]

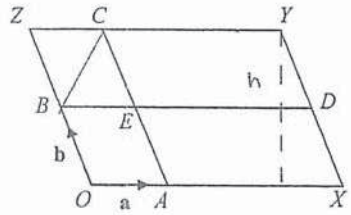
(c) Find the average speed in km/h.

$\text{Average speed} = \frac{756 \div 1000}{18 \div 3600}$ [M1]

$= 151.2$ km/h [A1]

Answer 151.2 km/h or $151\frac{1}{5}$ km/h [2]

17 The diagram shows a parallelogram *OXYZ*.



$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. *BED* and *AEC* are lines parallel to the sides of the parallelogram.
$\overrightarrow{OX} = 3\overrightarrow{OA}$ and $\overrightarrow{OZ} = 2\overrightarrow{OB}$.

(a) Express $\overrightarrow{EY}$ in terms of **a** and **b**, as simply as possible.

$\frac{OX}{OA} = \frac{3}{1}$, $\frac{OZ}{OB} = \frac{2}{1}$ $\overrightarrow{EY} = \overrightarrow{AD}$ [M1]
$= \overrightarrow{AX} + \overrightarrow{XD}$
$= 2\underset{\sim}{a} + \underset{\sim}{b}$ [A1] or [B2]

Answer $2\underset{\sim}{a} + \underset{\sim}{b}$ [2]

(b) Write down two vectors that can be written as $\mathbf{b} - 3\mathbf{a}$.

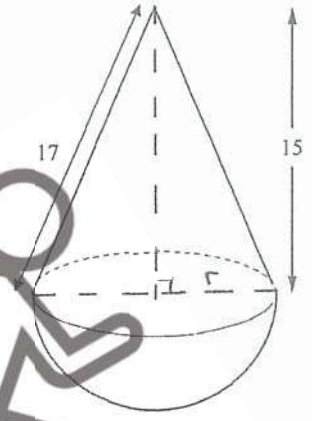
Answer $\overrightarrow{XB}$ [B1] and $\overrightarrow{DZ}$ [B1] [2]

(c) Find the ratio of the area of triangle *BCE* to the area of parallelogram *OXYZ*.

let h be the height of OXYZ. BCE : OXYZ
AOBE : OXYZ
$\frac{h}{2} \times 1$: $h \times 3$ [M1]
$\frac{1}{2}$: 3
1 : 6 [A1]
1 : 12 [B2]

Answer 1 : 12 [2]

18



The diagram shows a toy that combines a cone and a hemisphere. The slant height of the cone is 17 cm and the perpendicular height of the cone is 15 cm.

(a) Show that the radius of the hemisphere is 8 cm.

let radius be r.
$r = \sqrt{17^2 - 15^2}$ [M1]
$= 8$ cm.
∴ radius = 8 cm [A1]

[2]

(b) Calculate the surface area of the toy.

surface area $= \pi rl + 2\pi r^2$
$= \pi(8)(17) + 2\pi(8)^2$ [M1] : add curved s.a of cone + s.a of hemisphere
$= 829.3804605$
$= 829$ cm^2 [A1]
[M1] : find curved s.a of cone

Answer 829 cm^2 [3]

(c) The company wants to make a miniature version of the toy such that the volume of the miniature toy is $\frac{1}{6}$ of the volume of the actual toy.

Calculate the radius of the miniature toy.

r_s be radius of miniature toy.

$\left(\frac{r_s}{8}\right)^3 = \frac{1}{6}$ [M1]

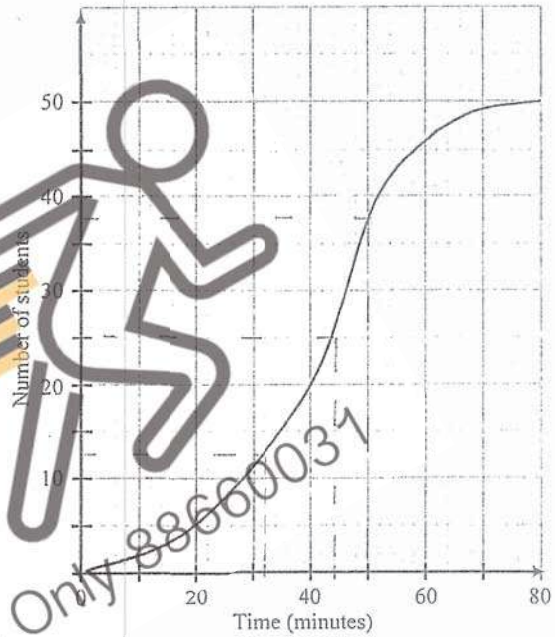
$r_s^3 = 85\frac{1}{3}$

$r_s = 4.40256$

$= 4.40$ cm [A1]

Answer 4.40 cm [2]

19 The travelling times taken by 50 students from home to school are recorded. The cumulative frequency curve below shows the distribution of their times.



(a) Use your graph to estimate

(i) the median

$Q_2 = 44$ mins

Answer 44 mins [B1] [1]

(ii) the interquartile range

$Q_1 = 32$, $Q_3 = 50$

IQR = 50 − 32 [M1]

= 18 [A1]

Answer 18 [2]

(b) Find the probability that two students, chosen at random, take less than 70 minutes to travel to school.

P(2 students took less than 70 mins) = $\frac{49}{50} \times \frac{48}{49}$ [M1]

$= \frac{24}{25}$ [A1]

Answer $\frac{24}{25}$ [2]

(c) A student states that 32% of the students took at least 36 minutes to travel to school.

Comment on whether the data from the school supports this claim.

Answer No. of students who took at least 36 mins = 50 − 16 [M1]

= 34.

$\frac{34}{50} \times 100\% = 68\%$ took at least 36 mins, not 32%. [A1]

[2]

or

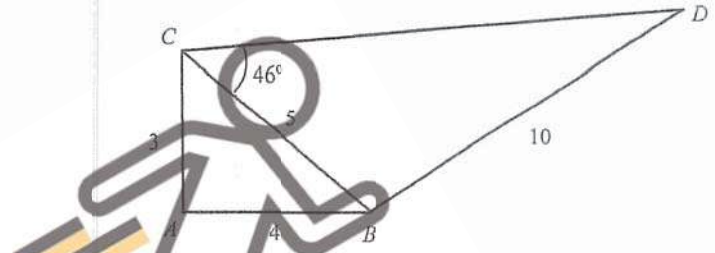
32% — $\frac{32}{100} \times 50$

= 16 students.

16 students took less than 36 mins, not at least 36 mins.

[B2]

20 The diagram shows two triangles ABC and BDC.



$AC = 3$ cm, $AB = 4$ cm, $BC = 5$ cm, $BD = 10$ cm and $\angle CBD = 113°$.

(a) Show that $\angle CAB$ is a right angle.

$AC^2 + AB^2 = 3^2 + 4^2$
$= 25$
$BC^2 = 5^2$
$= 25$ [M1]

Since $AC^2 + AB^2 = BC^2$, by converse of Pythagoras' theorem, $\angle CAB$ is a right angle. [A1]

[2]

(b) Find $\angle BDC$.

$\frac{\sin \angle BDC}{5} = \frac{\sin 46°}{10}$ [M1]

$\angle BDC = 21.079924$

$= 21.1°$ [A1]

$\angle BDC$
$= 180° - 46° - 113°$ [M1]
$= 21°$ [A1]

Answer 21.1° [2]

(c) Find the area of $ABDC$.

Area of ABDC

$= \left[\frac{1}{2} \times 3 \times 4\right] + \left[\frac{1}{2} \times 5 \times 10 \times \sin 113°\right]$ [M1]

$= 29.0126$

$= 29.0 \text{ cm}^2$ [A1]

Answer 29.0 cm^2 [2]

(d) A circle is drawn through the points A, B and C. State the length of the diameter of the circle. Explain your answer.

Diameter = 5 cm. [B1] Since $\angle CAB = 90°$, BC must be the diameter due to the property of right angle in a semicircle. [B1]

[2]

Name | Reg. No | Class

4E5N

ADDITIONAL MATHEMATICS

4047/01

[80 marks]

SEMESTER ONE EXAMINATION

13 May 2019

2 hours

Additional material: Writing paper

INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.

Write your name, register number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **ALL** questions.

Write your answers on the writing paper provided.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

Write the brand and model of your calculator in the space provided below.

INFORMATION FOR CANDIDATES

You are expected to use an electronic calculator to evaluate explicit numerical expressions.

If the degree of accuracy is not specified in the question, and if the answer is not exact, the answer should be given to **three** significant figures. Answers in degrees should be given to **one** decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

For Examiner's Use

Brand / Model of Calculator

This question paper consists of **7** printed pages, including the cover page.

Setter: Ms Shen Sirui | Vetter: Mr Nara

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\text{Area of } \Delta = \frac{1}{2}ab\sin C$$

1 **(i)** On the same diagram sketch the curve $y^2 = 8x$ and $y = 6x^{-2}$. [2]

(ii) Find the coordinates of the point of intersection of the two curves. [3]

2 A particle moves along the curve $y = e^{2x}$ in such a way that the *y*-coordinate of the particle is increasing at a constant rate of 0.3 units per second. Find the *y*-coordinate of the particle at the instant when the *x*-coordinate of the particle is increasing at 0.01 units per second.

[4]

3 The equation of a curve is $y = 3x^2 - kx + 2k - 4$, where k is a constant. Show that the line $y = 2x + 5$ intersects the curve for all real values of k. [5]

4 **(a)** Given that $(3^{x+2})(2^{x-2}) = 6^{2x}$, find the value of 6^x. [3]

(b) The side of an equilateral triangle is $6(\sqrt{3}-1)$ cm. **Without using a calculator,** find the exact value of the area of the equilateral triangle in the form $(a + b\sqrt{c})$ cm^2, where a, b and c are integers. [4]

5 Find the range of values of x for which the gradient of the graph $y = x^4 - 3x^3 - 6x^2 + 6$ is increasing. [5]

6 A curve has the equation $y = (2x - 3)^2 - 1$.

(i) Find the coordinates of the points at which the curve intersects the *x*-axis. [2]

(ii) Sketch the graph of $y = |(2x - 3)^2 - 1|$. [3]

(iii) Using your graph, state the range of values of k for which $|(2x - 3)^2 - 1| = k$ has 4 solutions. [1]

7 It is given that $f'(x) = x + \sin 4x$ and $f(0) = \frac{3}{4}$.

Show that $f''(x) + 16f(x) = 8x^2 + 17$. [5]

8 Solve the equation $6\sin^2 x + 5\cos x = 5$ for $0° < x < 360°$. [5]

9 **(a)** Given that the first two non-zero terms in the expansion, in ascending powers of x, of $(1 + bx)(1 + ax)^6$ are 1 and $-\frac{21}{4}x^2$ and that $a > 0$, find the value of a and of b. [5]

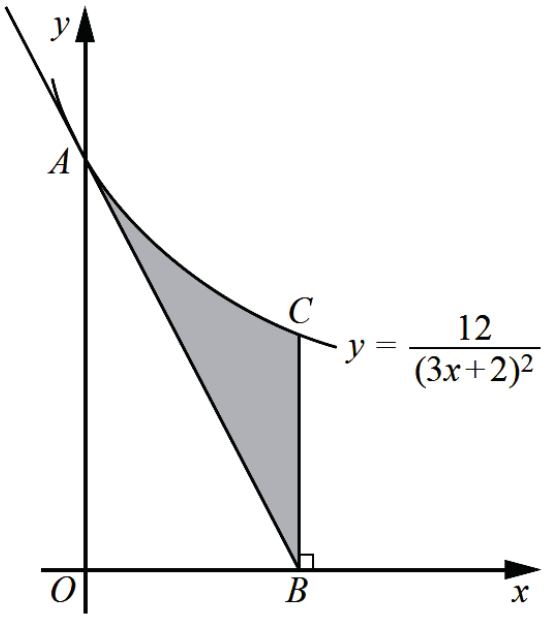
(b) Find the term independent of x in the expansion of $\left(2x + \frac{1}{x^2}\right)^9$. [3]

10 The equation of a curve is $y = \frac{x^2}{2x-1}$.

(i) Find the coordinates of the stationary points of the curve. [4]

(ii) Determine the nature of each of the stationary points of the curve. [4]

11



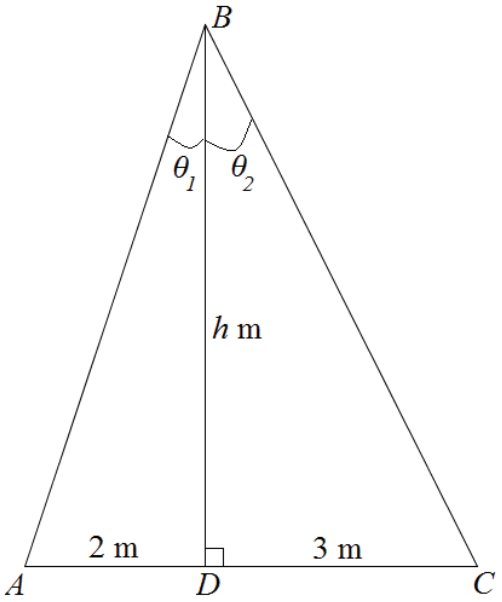
The diagram shows part of the curve $y = \frac{12}{(3x+2)^2}$ meeting the y-axis at point A. The tangent to the curve at A intersects the x-axis at point B. Point C lies on the curve such that BC is parallel to the y-axis. Find

(i) the equation of AB, [4]

(ii) the area of the shaded region. [5]

12 **(a)** State the values between which the principal value of $\tan^{-1} x$ must lie. Give your answer in terms of π. [1]

(b) The diagram below shows triangle *ABC* where *AD* = 2 m, *DC* = 3 m and *BD* = *h* m. *BD* is perpendicular to *AC* and $\theta_1 + \theta_2 = 45°$.



By using a suitable formula for $\tan(\theta_1 + \theta_2)$, find the value of *h*. [5]

13 The Ultraviolet Index describes the level of solar radiation on the earth's surface. The Ultraviolet Index, U, measured from the top of a building is given by $U = 6 - 5\cos qt$, where t is the time in hours, $0 \leq t \leq 20$, from the lowest value of Ultraviolet Index and q is a constant. It takes 10 hours for the Ultraviolet Index to reach its lowest value again.

(i) Explain why it is impossible to measure a Ultraviolet Index of 12. [1]

(ii) Show that $q = \frac{\pi}{5}$. [1]

(iii) The top of the building is equipped with solar panels that supply power to the building when the Ultraviolet Index is at least 3.5. Find the duration, in hours and minutes, that the building is supplied with power by the solar panels. [5]

END OF PAPER

Name	Reg. No	Class

4E/5N

ADDITIONAL MATHEMATICS

4047/02

Paper 2 [100 marks]

SEMESTER ONE EXAMINATION

May 2019

2 hours 30 minutes

Candidates answer on the question paper.

INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.
Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **ALL** questions.
Write your answers in the spaces provided on the question paper.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Write the brand and model of your calculator in the space provided below.

INFORMATION FOR CANDIDATES

You are expected to use a scientific calculator to evaluate explicit numerical expressions.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to **three** significant figures. Give answers in degrees to **one** decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **100**.

Brand / Model of Calculator

For Examiner's Use	
Total	/ 100

This question paper consists of **15** printed pages.

Setter: Mr. Gabriel Cheow
Vetter: Mr. Narayanan

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

For Examiner's Use

1 The roots of the quadratic equation $2x^2 - 8x + 9 = 0$ are α and β.

(i) Show that the value of $\alpha^3 + \beta^3$ is 10. [3]

(ii) Find a quadratic equation whose roots are $\frac{1}{\alpha^2 + \beta}$ and $\frac{1}{\alpha + \beta^2}$. [4]

For Examiner's Use

For Examiner's Use

2 The function $f(x) = 6x^3 + ax^2 + bx - 12$, where a and b are constants, is exactly divisible by $x + 2$ and leaves a remainder of 5 when divided by $x + 1$.

(i) Find the value of a and of b. [4]

(ii) By showing your working clearly, factorise $f(x)$. [3]

(iii) Hence, solve the equation $6(2^{3y}) + 2^{2y+3} - 2^{y+4} - 12 = 2^{2y}$ [4]

For Examiner's Use

3 **(i)** Express $\dfrac{2x+16}{(x^2+4)(2x-1)}$ in partial fractions. [5]

For Examiner's Use

(ii) Differentiate $\ln(x^2+4)$ with respect to x. [2]

(iii) Hence, using your results in **(i)** and **(ii)**, find $\int \dfrac{x+8}{(x^2+4)(2x-1)}\,\mathrm{d}x$. [4]

For Examiner's Use

4 Prove the following identities.

(a) $(\sec x - \tan x)(\operatorname{cosec} x + 1) = \cot x$ [3]

$LHS = (\sec x - \tan x)(cosec\ x + 1)$

(b) $\dfrac{1 - \cos 2x + \sin x}{\sin 2x + \cos x} = \tan x$ [3]

For Examiner's Use

For Examiner's Use

5 The lines $y = 8$ and $4x + 3y = 30$ are tangent to a circle C at the points $(-1, 8)$ and $(3, 6)$ respectively.

(i) Show that the equation of C is $x^2 + y^2 + 2x - 6y - 15 = 0$. [5]

(ii) Explain whether or not the x-axis is tangent to C. [3]

(iii) The points Q and R also lie on the circle, and the length of the chord QR is 2 units. Calculate the shortest distance from the center of C to the chord QR. [2]

For Examiner's Use

6 The table shows experimental values of two variables x and y, which are known to be connected by the equation $yx^n = A$, where n and A are constants.

x	1.0	1.5	2.0	2.5	3.0
y	22.0	13.0	8.9	6.9	5.3

(i) Plot lg y against lg x and draw a straight line graph. [3]

For Examiner's Use

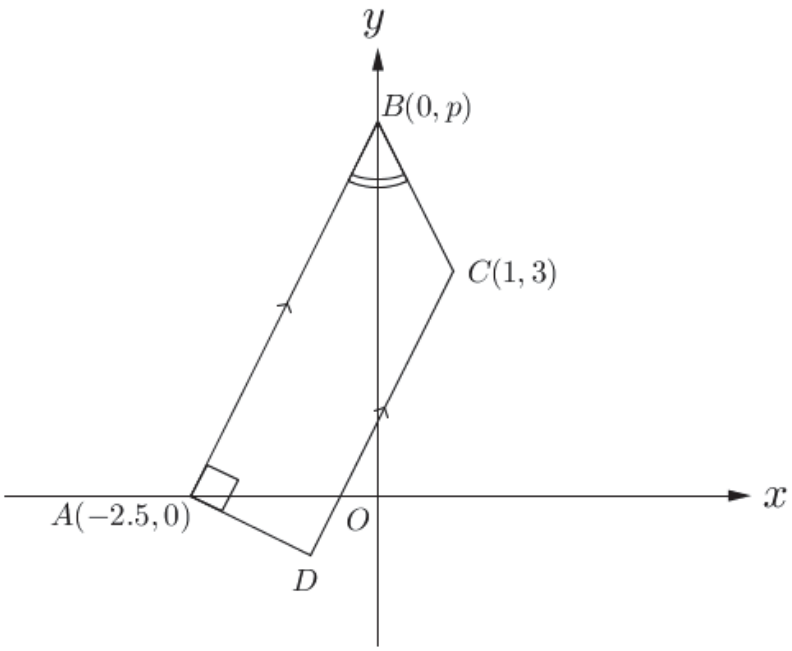
For Examiner's Use

(ii) Use your graph to estimate the value of A and of n. [4]

(iii) On the same diagram, draw the line representing the equation $y = x^2$ and hence find the value of x which satisfies the equation $x^{n+2} = A$. [2]

For Examiner's Use

7 The diagram shows a trapezium with vertices $A(-2.5, 0)$, $B(0, p)$, $C(1, 3)$ and D.
The sides AB and DC are parallel and the angle DAB is 90°.
Angle ABO is equal to angle CBO.



(i) Express the gradients of the lines AB and CB in terms of p and hence, or otherwise, show that $p = 5$. [3]

For Examiner's Use

For Examiner's Use

For Examiner's Use

(ii) Find the coordinates of D. [4]

(iii) Find the area of the trapezium $ABCD$. [2]

For Examiner's Use

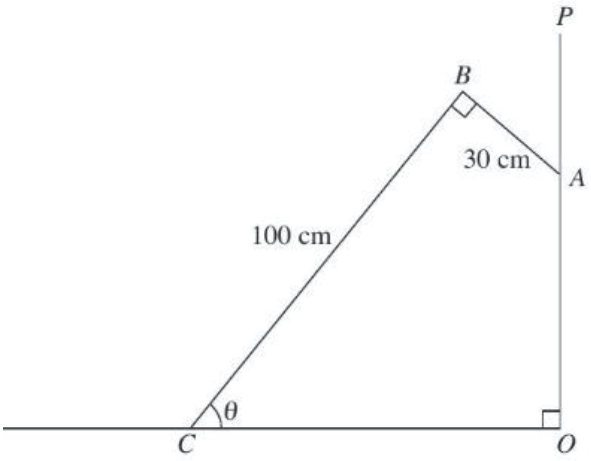
8 **(a)** Solve the equation $3\log_x 3 = 8 - 4\log_3 x$. [5]

For Examiner's Use

(b) It is given that $\log_a x = p$ and $\log_a y = q$.
Express $\log_y ax^2y^3$ in terms of p and q. [3]

For Examiner's Use

9 The figure shows a stage prop ABC used by a member of the theatre, leaning against a vertical wall OP. It is given that $AB = 30$ cm, $BC = 100$ cm, $\angle ABC = \angle AOC = 90°$ and $\angle BCO = \theta$.



(i) Show that $OC = (100\cos\theta + 30\sin\theta)$ cm. [2]

Let D be foot of B on OC, let E be foot of A on BD.

(ii) Express OC in terms of $R\cos(\theta - \alpha)$, where R is a positive constant and α is an acute angle. [3]

(iii) State the maximum value of OC and the corresponding value of θ. [2]

(iv) Find the value of θ for which $OC = 80$ cm. [3]

For Examiner's Use

10 Given that $y = a + b\cos 4x$, where a and b are integers, and x is in radians,

(i) state the period of y. [1]

Given that the maximum and minimum values of y are 3 and – 5 respectively, find

(ii) the amplitude of y, [1]

[2]

Using the values of a and b found in part **(iii)**,

(iv) sketch the graph of $y = a + b\cos 4x$ for $0 \le x \le \pi$. [3]

(v) On the same set of axes, sketch the graph of $y = |4\sin 3x|$, and hence state the number of solutions of $a + b\cos 4x = |4\sin 3x|$. [3]

For Examiner's Use

11 The dimensions of a cuboid are $3x$ cm by $2x$ cm by h cm and its total surface area is 312 cm^2. The volume of the cuboid is V cm^3.

(i) Express h in terms of x. [2]

(ii) Show that $V = \frac{36}{5}x(26 - x^2)$. [2]

(iii) Find the maximum volume of the cuboid as x varies, giving your answer to the nearest cm^3. [5]

– End of Paper –

Name | Reg. No | Class

OWER SECONDARY SCHOOL **MAYFLOWER** SECONDARY SCHOOL **MAYFLOWER** SECONDARY SCHOOL **MAYFLOWER** SECONDARY SCHOOL **MAYFLOWER** SECONDARY SCHOOL

4E5N

ADDITIONAL MATHEMATICS

4047/01

[80 marks]

SEMESTER ONE EXAMINATION

13 May 2019

2 hours

Additional material: Writing paper

INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.

Write your name, register number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **ALL** questions.

Write your answers on the writing paper provided.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

Write the brand and model of your calculator in the space provided below.

INFORMATION FOR CANDIDATES

You are expected to use an electronic calculator to evaluate explicit numerical expressions.

If the degree of accuracy is not specified in the question, and if the answer is not exact, the answer should be given to **three** significant figures. Answers in degrees should be given to **one** decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

For Examiner's Use

Brand / Model of Calculator

This question paper consists of **7** printed pages, including the cover page.

Setter: Ms Shen Sirui | Vetter: Mr Nara

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

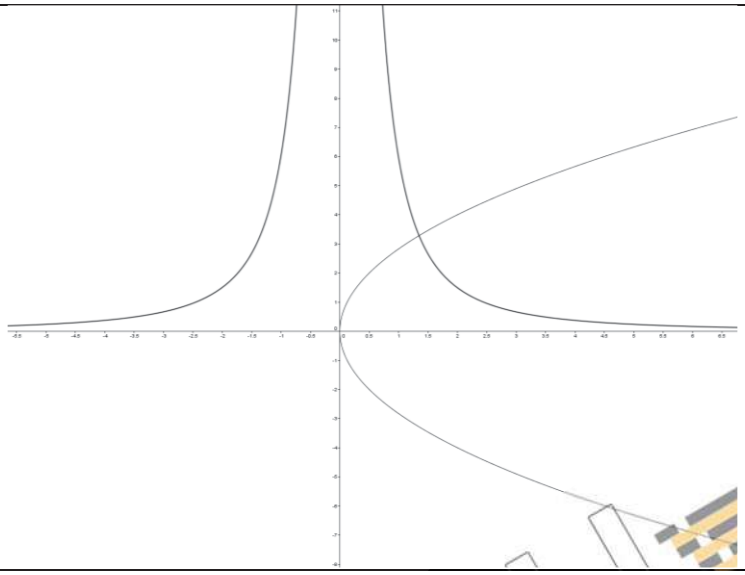
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\text{Area of } \Delta = \frac{1}{2}ab\sin C$$

1 **(i)** On the same diagram sketch the curve $y^2 = 8x$ and $y = 6x^{-2}$. [2]

(ii) Find the coordinates of the point of intersection of the two curves. [3]

Qn	Solution	Mark
i		B1 for $y^2 = 8x$ B1 for $y = 6x^{-2}$
ii	$y^2 = 8x - - - (1)$ $y = 6x^{-2} - - - (2)$ Sub (2) into (1): $(6x^{-2})^2 = 8x$ $\frac{36}{x^4} = 8x$ $x^5 = 4.5$ $x = 1.3509$ $y = 3.2877$ Intersection: (1.35, 3.29)	M1 for substitution M1 for value of *x* or *y* A1

2 A particle moves along the curve $y = e^{2x}$ in such a way that the *y*-coordinate of the particle is increasing at a constant rate of 0.3 units per second. Find the *y*-coordinate of the particle at the instant when the *x*-coordinate of the particle is increasing at 0.01 units per second. [4]

Qn	Solution	Mark
	$y = e^{2x}$ $\frac{dy}{dx} = 2e^{2x}$ $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ $2e^{2x} = 0.3 \div 0.01$ $e^{2x} = 15$ $x = \frac{\ln 15}{2}$ Sub $x = \frac{\ln 15}{2}$, $y = e^{2(\frac{\ln 15}{2})} = 15$	M1 for dy/dx M1 for sub into equation connecting dy/dx, dy/dt, dx/dt M1 for $x = \frac{\ln 15}{2}$ or $e^{2x} = 15$ A1

3 The equation of a curve is $y = 3x^2 - kx + 2k - 4$, where *k* is a constant. Show that the line $y = 2x + 5$ intersects the curve for all real values of *k*. [5]

Qn	Solution	Mark
	$y = 3x^2 - kx + 2k - 4 - - - (1)$ $y = 2x + 5 - - - (2)$ $(1) = (2)$: $3x^2 - kx + 2k - 4 = 2x + 5$ $3x^2 - kx - 2x + 2k - 9 = 0$ $3x^2 - (k+2)x + 2k - 9 = 0$ $b^2 - 4ac = [-(k+2)]^2 - 4(3)(2k-9)$ $= k^2 + 4k + 4 - 24k + 108$ $= k^2 - 20k + 111$ $= (k-10)^2 - 10^2 + 112$ $= (k-10)^2 + 12$ Since $(k-10)^2 + 12 > 0, b^2 - 4ac > 0$ and line intersects the curve for all real values of *k*.	M1 for combining equations M1 for $ax^2 + bx + c = 0$ M1 for subs into $b^2 - 4ac$ M1 for $(k-10)^2 + 12$ A1 for conclusion

4 **(a)** Given that $(3^{x+2})(2^{x-2}) = 6^{2x}$, find the value of 6^x. [3]

(b) The side of an equilateral triangle is $6(\sqrt{3}-1)$ cm. **Without using a calculator,** find the exact value of the area of the equilateral triangle in the form $(a + b\sqrt{c})$ cm^2, where a, b and c are integers. [4]

Qn	Solution	Mark
a	$(3^{x+2})(2^{x-2}) = 6^{2x}$ $3^x(3^2)(2^x)(2^{-2}) = 6^{2x}$ $6^x\left(\frac{9}{4}\right) = 6^{2x}$ $6^x = \frac{9}{4}$	M1 for $3^x(3^2)$ or $(2^x)(2^{-2})$ M1 for $6^x\left(\frac{9}{4}\right)$ A1
b	$\text{Area} = \frac{1}{2}\left[6(\sqrt{3}-1)\right]^2 \sin 60$ $= \frac{1}{2}(36)(3-2\sqrt{3}+1)\left(\frac{\sqrt{3}}{2}\right)$ $= 9\sqrt{3}(4-2\sqrt{3})$ $= 36\sqrt{3} - 54$ $= -54 + 36\sqrt{3}$	M1 M1 for $(3-2\sqrt{3}+1)$ M1 for $\left(\frac{\sqrt{3}}{2}\right)$ A1

5 Find the range of values of x for which the gradient of the graph $y = x^4 - 3x^3 - 6x^2 + 6$ is increasing. [5]

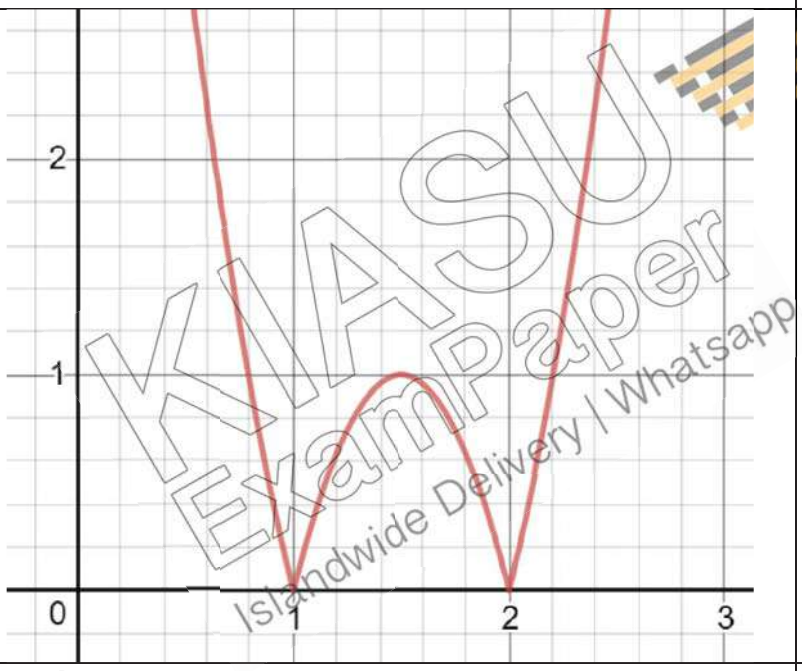
Qn	Solution	Mark
	$y = x^4 - 3x^3 - 6x^2 + 6$ $\frac{dy}{dx} = 4x^3 - 9x^2 - 12x$ $\frac{d^2y}{dx^2} = 12x^2 - 18x - 12$ $12x^2 - 18x - 12 > 0$ $2x^2 - 3x - 2 > 0$ $(2x+1)(x-2) > 0$ $x < -\frac{1}{2}, x > 2$	M1 for $\frac{dy}{dx}$ M1 for $\frac{d^2y}{dx^2}$ M1 for $\frac{d^2y}{dx^2} > 0$ M1 for factorised form A1

6 A curve has the equation $y = (2x - 3)^2 - 1$.

(i) Find the coordinates of the points at which the curve intersects the *x*-axis. [2]

(ii) Sketch the graph of $y = |(2x - 3)^2 - 1|$. [3]

(iii) Using your graph, state the range of values of k for which $|(2x - 3)^2 - 1| = k$ has 4 solutions. [1]

Qn	Solution	Mark
i	$(2x - 3)^2 - 1 = 0$ $2x - 3 = \pm 1$ $x = 1, \quad x = 2$ $(1, 0) \quad (2, 0)$	M1 A1 or B2
ii		T1 for turning point (1.5, 1) P1 for (1, 0) and (2, 0) C1 for shape of graph
iii	$0 < k < 1$	B1 (no mark if students got part ii wrong)

7 It is given that $f'(x) = x + \sin 4x$ and $f(0) = \frac{3}{4}$.

Show that $f''(x) + 16f(x) = 8x^2 + 17$. [5]

Qn	Solution	Mark
	$f(x) = \frac{x^2}{2} - \frac{\cos 4x}{4} + c$ $\frac{3}{4} = 0 - \frac{1}{4} + c$ $c = 1$ $f(x) = \frac{x^2}{2} - \frac{\cos 4x}{4} + 1$ $f''(x) = 1 + 4\cos 4x$ $f''(x) + 16f(x) = 1 + 4\cos 4x + 16(\frac{x^2}{2} - \frac{\cos 4x}{4} + 1)$ $= 1 + 4\cos 4x + 8x^2 - 4\cos 4x + 16$ $= 8x^2 + 17$	M1 for $\frac{x^2}{2} - \frac{\cos 4x}{4}$ M1 for $f(x) = \frac{x^2}{2} - \frac{\cos 4x}{4} + 1$ M1 for $1 + 4\cos 4x$ M1 for sub into $f''(x) + 16f(x)$ A1

8 Solve the equation $6\sin^2 x + 5\cos x = 5$ for $0° < x < 360°$. [5]

Qn	Solution	Mark
	$6(1 - \cos^2 x) + 5\cos x = 5$ $6 - 6\cos^2 x + 5\cos x - 5 = 0$ $6\cos^2 x - 5\cos x - 1 = 0$ $(6\cos x + 1)(\cos x - 1) = 0$ $\cos x = -\frac{1}{6}$, $\cos x = 1$ $\alpha = 80.405$ (Rej) $x = 180 - \alpha, 180 + \alpha$ $x = 99.6°, 260.4°$	M1 for $1 - \cos^2 x$ M1 for equation M1 for $\cos x = -\frac{1}{6}$ M1 for basic angle A1 for both answers Ignore if students do not reject $\cos x = 1$

9 **(a)** Given that the first two non-zero terms in the expansion, in ascending powers of x, of $(1+bx)(1+ax)^6$ are 1 and $-\frac{21}{4}x^2$ and that $a > 0$, find the value of a and of b. [5]

(b) Find the term independent of x in the expansion of $\left(2x+\frac{1}{x^2}\right)^9$. [3]

Qn	Solution	Mark
a	$(1+ax)^6 = 1+\binom{6}{1}(1)^5(ax)^1+\binom{6}{2}(1)^4(ax)^2+\cdots$ $= 1+6ax+15a^2x^2+\cdots$ $(1+bx)(1+ax)^6 = (1+bx)(1+6ax+15a^2x^2+\cdots)$ $= 1+6ax+bx+15a^2x^2+6abx^2+\cdots$ $6a+b=0$ $b=-6a---(1)$ $15a^2+6ab=-\frac{21}{4}---(2)$ sub (1) into (2): $15a^2+6a(-6a)=-\frac{21}{4}$ $21a^2=\frac{21}{4}$ $a^2=\frac{1}{4}$ $a=\frac{1}{2}$ $b=-3$	M1 for $1+6ax+15a^2x^2$ M1 for $6a+b=0$ M1 for $15a^2+6ab=-\frac{21}{4}$ A1 A1
b	$T_{r+1}=\binom{9}{r}(2x)^{9-r}\left(\frac{1}{x^2}\right)^r$ For x^0, $x^{9-r}(x)^{-2r}=x^0$ $r=3$ $T_{3+1}=\binom{9}{3}(2x)^{9-3}\left(\frac{1}{x^2}\right)^3$ $=84(2x)^6(x)^{-6}$ $=5376$	M1 for $\binom{9}{r}(2x)^{9-r}\left(\frac{1}{x^2}\right)^r$ M1 for $r=3$ A1

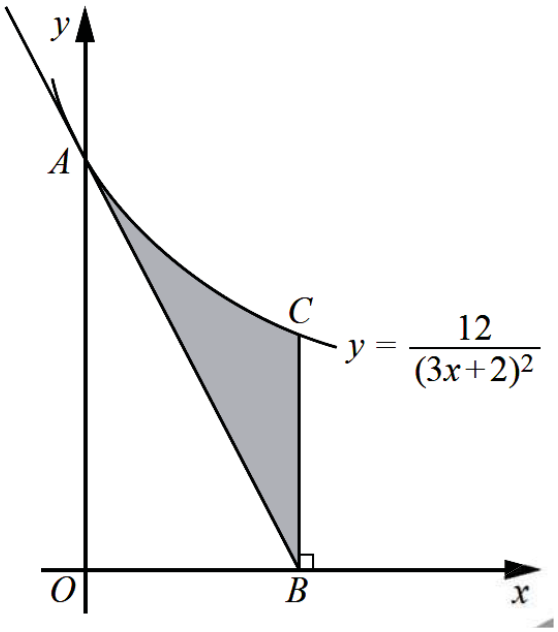
10 The equation of a curve is $y = \frac{x^2}{2x-1}$.

(i) Find the coordinates of the stationary points of the curve. [4]

(ii) Determine the nature of each of the stationary points of the curve. [4]

Qn	Solution	Mark
i	$y = \frac{x^2}{2x-1}$ $\frac{dy}{dx} = \frac{2x(2x-1) - 2x^2}{(2x-1)^2}$ $= \frac{2x^2 - 2x}{(2x-1)^2}$ when $\frac{dy}{dx} = 0, \frac{2x^2-2x}{(2x-1)^2} = 0$ $2x(x-1) = 0$ $x = 0, x = 1$ $y = 0, y = 1$ Stationary points: $(0, 0)$ and $(1, 1)$	M1 for quotient or product rule M1 for $\frac{2x^2-2x}{(2x-1)^2} = 0$ M1 for both x A1 for both coordinates
ii	$\frac{d^2y}{dx^2} = \frac{(4x-2)(2x-1)^2 - 4(2x-1)(2x^2-2x)}{(2x-1)^4}$ when $x = 0, \frac{d^2y}{dx^2} = -2 < 0$ $(0, 0)$ is maximum point. when $x = 1, \frac{d^2y}{dx^2} = 2 > 0$ $(1, 1)$ is minimum point. OR x | -0.1 | 0 | 0.1 $\frac{dy}{dx}$ | > 0 | 0 | < 0 $(0, 0)$ is maximum point. x | 0.9 | 1 | 1.1 $\frac{dy}{dx}$ | < 0 | 0 | > 0 $(1, 1)$ is minimum point.	M1 for $\frac{d^2y}{dx^2}$ M1 for sub either $x = 0$ or $x = 1$ into $\frac{d^2y}{dx^2}$ A1 for $(0, 0)$ max pt A1 for $(1, 1)$ min pt M1 for 1st derivative test A1 for $(0, 0)$ max pt M1 for 1st derivative test A1 for $(1, 1)$ min pt

11



The diagram shows part of the curve $y = \frac{12}{(3x+2)^2}$ meeting the y-axis at point A. The tangent to the curve at A intersects the x-axis at point B. Point C lies on the curve such that BC is parallel to the y-axis. Find

(i) the equation of AB, [4]

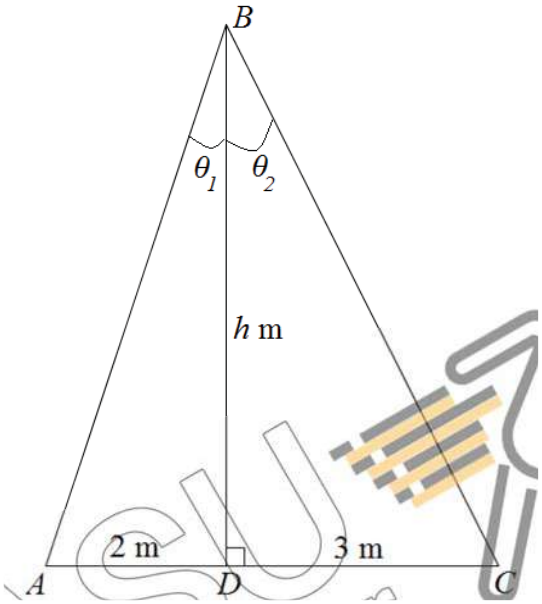
(ii) the area of the shaded region. [5]

Qn	Solution	Mark
i	$y = \frac{12}{(3x+2)^2}$ $\frac{dy}{dx} = -24(3x+2)^{-3}(3)$ $= -\frac{72}{(3x+2)^3}$ when $x = 0, \frac{dy}{dx} = -9$ when $x = 0, y = 3$ Line AB: $y = -9x + 3$	M1 for dy/dx M1 for dy/dx at A M1 for $y = 3$ A1
ii	sub $y = 0, 0 = -9x + 3$ $x = \frac{1}{3}$ $B = \left(\frac{1}{3}, 0\right)$	M1 for x-coordinate of B

	Area of $OACB = \int_0^{\frac{1}{3}} 12(3x+2)^{-2}\,dx$ $= \left[\frac{12(3x+2)^{-1}}{-1(3)}\right]_0^{\frac{1}{3}}$ $= \left[-\frac{4}{3x+2}\right]_0^{\frac{1}{3}}$ $= -\frac{4}{3\left(\frac{1}{3}\right)+2} - \left(-\frac{4}{3(0)+2}\right)$ $= \frac{2}{3}$ Area of $\Delta OAB = \frac{1}{2}\left(\frac{1}{3}\right)(3) = \frac{1}{2}$ Area of shaded region $= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$ unit^2	M1 for $-\frac{4}{3x+2}$ (independent of limits) M1 for area of *OACB* M1 for area of tri *OAB* A1

12 **(a)** State the values between which the principal value of $\tan^{-1} x$ must lie. Give your answer in terms of π. [1]

(b) The diagram below shows triangle *ABC* where *AD* = 2 m, *DC* = 3 m and *BD* = *h* m. *BD* is perpendicular to *AC* and $\theta_1 + \theta_2 = 45°$.



By using a suitable formula for $\tan(\theta_1 + \theta_2)$, find the value of *h*. [5]

Qn	Solution	Mark
a	$-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$	B1
b	$\tan(\theta_1 + \theta_2) = \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2}$ $\tan 45 = \frac{\frac{2}{h} + \frac{3}{h}}{1 - \left(\frac{2}{h}\right)\left(\frac{3}{h}\right)}$ $1 = \frac{\frac{2}{h} + \frac{3}{h}}{1 - \left(\frac{2}{h}\right)\left(\frac{3}{h}\right)}$ $1 - \frac{6}{h^2} = \frac{5}{h}$ $h^2 - 5h - 6 = 0$ $(h-6)(h+1) = 0$ $h = 6, \quad h = -1$ (rej)	M1 for tan addition formula M1 for either $\tan\theta_1 = \frac{2}{h}$ or $\tan\theta_2 = \frac{3}{h}$ M1 for tan 45 = 1 M1 for $h^2 - 5h - 6 = 0$ A1

13 The Ultraviolet Index describes the level of solar radiation on the earth's surface. The Ultraviolet Index, U, measured from the top of a building is given by $U = 6 - 5\cos qt$, where t is the time in hours, $0 \leq t \leq 20$, from the lowest value of Ultraviolet Index and q is a constant. It takes 10 hours for the Ultraviolet Index to reach its lowest value again.

(i) Explain why it is impossible to measure a Ultraviolet Index of 12. [1]

(ii) Show that $q = \frac{\pi}{5}$. [1]

(iii) The top of the building is equipped with solar panels that supply power to the building when the Ultraviolet Index is at least 3.5. Find the duration, in hours and minutes, that the building is supplied with power by the solar panels. [5]

Qn	Solution	Mark
i	Max $U = 6 + 5 = 11$ Since max value of $U = 11$, we cannot measure a Ultraviolet Index of 12.	B1 for stating max value of U
ii	$10 = \frac{2\pi}{q}$ $q = \frac{2\pi}{10} = \frac{\pi}{5}$	B1 for $q = \frac{2\pi}{10}$
iii	$6 - 5\cos\frac{\pi}{5}t = 3.5$ $\cos\frac{\pi}{5}t = \frac{1}{2}$ $\alpha = \frac{\pi}{3}$ $\frac{\pi}{5}t = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}, \frac{\pi}{3} + 2\pi, 2\pi - \frac{\pi}{3} + 2\pi$ $t = 1.6666, 8.3333, 11.66, 18.33$ Duration $= (8.3333 - 1.6666) + (18.33 - 11.66)$ $= 13.3367$ $=$ 13 hours 20 mins	M1 for forming equation M1 for basic angle M1 for $\frac{\pi}{3}, 2\pi - \frac{\pi}{3}$ M1 for all 4 values A1

END OF PAPER

Name **MARK SCHEME** Reg. No Class

4E/5N

ADDITIONAL MATHEMATICS 4047/02

Paper 2 [100 marks]

SEMESTER ONE EXAMINATION

May 2019

2 hours 30 minutes

Candidates answer on the question paper.

INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.
Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **ALL** questions.
Write your answers in the spaces provided on the question paper.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Write the brand and model of your calculator in the space provided below.

INFORMATION FOR CANDIDATES

You are expected to use a scientific calculator to evaluate explicit numerical expressions.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to **three** significant figures. Give answers in degrees to **one** decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **100**.

Brand / Model of Calculator		**For Examiner's Use**	
		Total	**100**

This question paper consists of **15** printed pages.

Setter: Mr. Gabriel Cheow Vetter: Mr. Narayanan

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

For Examiner's Use

1 The roots of the quadratic equation $2x^2 - 8x + 9 = 0$ are α and β.

(i) Show that the value of $\alpha^3 + \beta^3$ is 10. [3]

$\alpha + \beta = 4, \alpha\beta = \frac{9}{2}$ **M1 – sum & pdt**

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$= 4^2 - 9$
$= 7$ **M1**

$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$
$= (4)\left(7 - \frac{9}{2}\right)$
$= 10\ (shown)$ **A1**

(ii) Find a quadratic equation whose roots are $\frac{1}{\alpha^2 + \beta}$ and $\frac{1}{\alpha + \beta^2}$. [4]

New sum: $\frac{1}{\alpha^2 + \beta} + \frac{1}{\alpha + \beta^2} = \frac{\alpha + \beta^2 + \alpha^2 + \beta}{(\alpha^2 + \beta)(\alpha + \beta^2)}$ **Allow for ECF**

$= \frac{\alpha + \beta^2 + \alpha^2 + \beta}{\alpha^3 + \beta\alpha + \alpha^2\beta^2 + \beta^3}$

$= \frac{4 + 7}{10 + \frac{9}{2} + \left(\frac{9}{2}\right)^2}$ **M1**

$= \frac{44}{139}$ **M1**

New pdt: $\frac{1}{\alpha^2 + \beta} \times \frac{1}{\alpha + \beta^2} = \frac{1}{(\alpha^2 + \beta)(\alpha + \beta^2)}$

$= \frac{1}{10 + \frac{9}{2} + \left(\frac{9}{2}\right)^2}$

$= \frac{4}{139}$ **M1**

New eqn: $x^2 - \frac{44}{139}x + \frac{4}{139} = 0$ **A1**

$139x^2 - 44x + 4 = 0$

For Examiner's Use

2 The function $f(x) = 6x^3 + ax^2 + bx - 12$, where a and b are constants, is exactly divisible by $x + 2$ and leaves a remainder of 5 when divided by $x + 1$.

(i) Find the value of a and of b. [4]

$f(-2) = 0$
$-48 + 4a - 2b - 12 = 0$ — **M1**
$2a - b = 30 \cdots\cdots Eqn\ 1$

$f(-1) = 5$
$-6 + a - b - 12 = 5$ — **M1**
$a - b = 23 \cdots\cdots Eqn\ 2$

A1

$Eqn\ 1 - Eqn\ 2: a = 7$ **A1**
$Sub\ into\ Eqn\ 1: b = -16$

(ii) By showing your working clearly, factorise $f(x)$. [3]

$6x^3 + 7x^2 - 16x - 12 = (x + 2)(Ax^2 + Bx + C)$

$By\ observation: A = 6, C = -6$

$\Rightarrow 6x^3 + 7x^2 - 16x - 12 = (x + 2)(6x^2 + Bx - 6)$ **M1**

$Let\ x = 1:$
$6 + 7 - 16 - 12 = (3)(6 + B - 6)$
$-15 = 3B$
$B = -5$

M1

$6x^3 + 7x^2 - 16x - 12 = (x + 2)(6x^2 - 5x - 6)$
$= (x + 2)(3x + 2)(2x - 3)$ **A1**

(iii) Hence, solve the equation $6(2^{3y}) + 2^{2y+3} - 2^{y+4} - 12 = 2^{2y}$ [4]

$6(2^{3y}) + 2^{2y+3} - 2^{y+4} - 12 = 2^{2y}$
$6(2^{3y}) + 8(2^{2y}) - 16(2^y) - 12 = 2^{2y}$
$6(2^y)^3 + 7(2^y)^2 - 16(2^y) - 12 = 0$ **M1**

$Let\ x = 2^y$

$\Rightarrow (x + 2)(3x + 2)(2x - 3) = 0$ **M1**
$x = -2, -\frac{2}{3}, \frac{3}{2}$
$2^y = -2\ (rej.), -\frac{2}{3}(rej.), \frac{3}{2}$ **M1**
$y \ln 2 = \ln \frac{3}{2}$
$y = \frac{\ln 1.5}{\ln 2} = 0.585\ (3sf)$ **A1**

For Examiner's Use

3 **(i)** Express $\frac{2x+16}{(x^2+4)(2x-1)}$ in partial fractions. [5]

$$\frac{2x+16}{(x^2+4)(2x-1)} = \frac{Ax+B}{x^2+4} + \frac{C}{2x-1}$$
$$2x+16 = (Ax+B)(2x-1) + C(x^2+4) \quad \textbf{M1}$$

Let $x = 0.5$:
$17 = C\left(\frac{17}{4}\right)$
$C = 4$ **M1**

Let $x = 0$:
$16 = B(-1) + 4(4)$
$B = 0$ **M1**

Let $x = -1$:
$14 = -A(-3) + 20$
$3A = -6$
$A = -2$ **M1**

$$\therefore \frac{2x+16}{(x^2+4)(2x-1)} = \frac{-2x}{x^2+4} + \frac{4}{2x-1} \quad \textbf{A1}$$

(ii) Differentiate $\ln(x^2+4)$ with respect to x. [2]

$$\frac{d}{dx}[\ln(x^2+4)] = \frac{2x}{x^2+4} \quad \textbf{B2}$$

(iii) Hence, using your results in **(i)** and **(ii)**, find $\int \frac{x+8}{(x^2+4)(2x-1)}\,dx$. [4]

$$\int \frac{x+8}{(x^2+4)(2x-1)}\,dx = \frac{1}{2}\int \frac{2x+16}{(x^2+4)(2x-1)}\,dx$$
$$= \frac{1}{2}\int \left(\frac{-2x}{x^2+4} + \frac{4}{2x-1}\right)dx \quad \textbf{M1 partial frac}$$
$$= -\frac{1}{2}\int \frac{2x}{x^2+4}\,dx + \frac{1}{2}(2\ln(2x-1) + c_1) \quad \textbf{M1}$$
$$= -\frac{1}{2}\ln(x^2+4) + c_2 + \ln(2x-1) + \frac{1}{2}c_1 \quad \textbf{M1}$$
$$= \ln(2x-1) - \frac{1}{2}\ln(x^2+4) + c \quad \textbf{A1}$$

For Examiner's Use

4 Prove the following identities.

(a) $(\sec x - \tan x)(\operatorname{cosec} x + 1) = \cot x$ [3]

$LHS = (\sec x - \tan x)(cosec\ x + 1)$

$= \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x} + \frac{\sin x}{\sin x}\right)$ **M1** sin and cos only

$= \frac{(1 - \sin x)(1 + \sin x)}{\cos x \sin x}$ **M1** single fraction

$= \frac{1 - \sin^2 x}{\cos x \sin x}$

$= \frac{\cos^2 x}{\cos x \sin x}$ **M1** cos^2 + sin^2 = 1

$= \frac{\cos x}{\sin x}$

$= \cot x$

$= RHS\ (proven)$

(b) $\frac{1 - \cos 2x + \sin x}{\sin 2x + \cos x} = \tan x$ [3]

$LHS = \frac{1 - \cos 2x + \sin x}{\sin 2x + \cos x}$ **M1** cosine double angle

$= \frac{1 - (1 - 2\sin^2 x) + \sin x}{2 \sin x \cos x + \cos x}$ **M1** sine double angle

$= \frac{2 \sin^2 x + \sin x}{\cos x\,(2 \sin x + 1)}$

$= \frac{\sin x\,(2 \sin x + 1)}{\cos x\,(2 \sin x + 1)}$ **M1** factorise and cancel

$= \frac{\sin x}{\cos x}$

$= \tan x$

$= RHS\ (proven)$

For Examiner's Use

5 The lines $y = 8$ and $4x + 3y = 30$ are tangent to a circle C at the points $(-1, 8)$ and $(3, 6)$ respectively.

(i) Show that the equation of C is $x^2 + y^2 + 2x - 6y - 15 = 0$. [5]

Let centre of circle be O.
Horizontal tangent at $(-1, 8)$ means that O is on the line $x = -1$. **M1**

To find normal of circle at $(3, 6)$:

$4x + 3y = 30$

$y = -\frac{4}{3}x + 10$

$\therefore m_{normal} = \frac{3}{4}$

$eqn\ of\ normal: y - 6 = \frac{3}{4}(x - 3)$ **M1**

$When\ x = -1, y = 3. \Rightarrow O(-1, 3)$ **M1**

Horizontal tangent is $y = 8$. Hence radius is 5. **M1**

$(x + 1)^2 + (y - 3)^2 = 5^2$ **M1**

$x^2 + 2x + 1 + y^2 - 6x + 9 = 25$

$x^2 + y^2 + 2x - 6y - 15 = 0\ (shown)$

(ii) Explain whether or not the x-axis is tangent to C. [3]

C has centre $(-1, 3)$ and radius 5.
Hence its horizontal tangents are $y = 3 \pm 5 \Rightarrow y = 8\ or\ y = -2$ **M1**

x-axis is $y = 0$, which is between the two horizontal tangents. **M1**
Hence the x-axis will cut through C at two points.
Hence the x-axis is **not** tangent to C. **A1**

Alternative solution: Sub $y = 0$ into eqn of C, show that $b^2 - 4ac \neq 0$.

(iii) The points Q and R also lie on the circle, and the length of the chord QR is 2 units. Calculate the shortest distance from the center of C to the chord QR. [2]

Let M be midpoint of QR. Hence OM perpendicular to QR.
Hence, OM is shortest distance from C to chord QR.
Consider right-angled triangle OMR.
By Pythagoras Theorem,

$OM = \sqrt{5^2 - \left(\frac{2}{2}\right)^2}$ **M1**

$= \sqrt{24} = 2\sqrt{6}$

$= 4.90\ (3sf)$ **A1**

For Examiner's Use

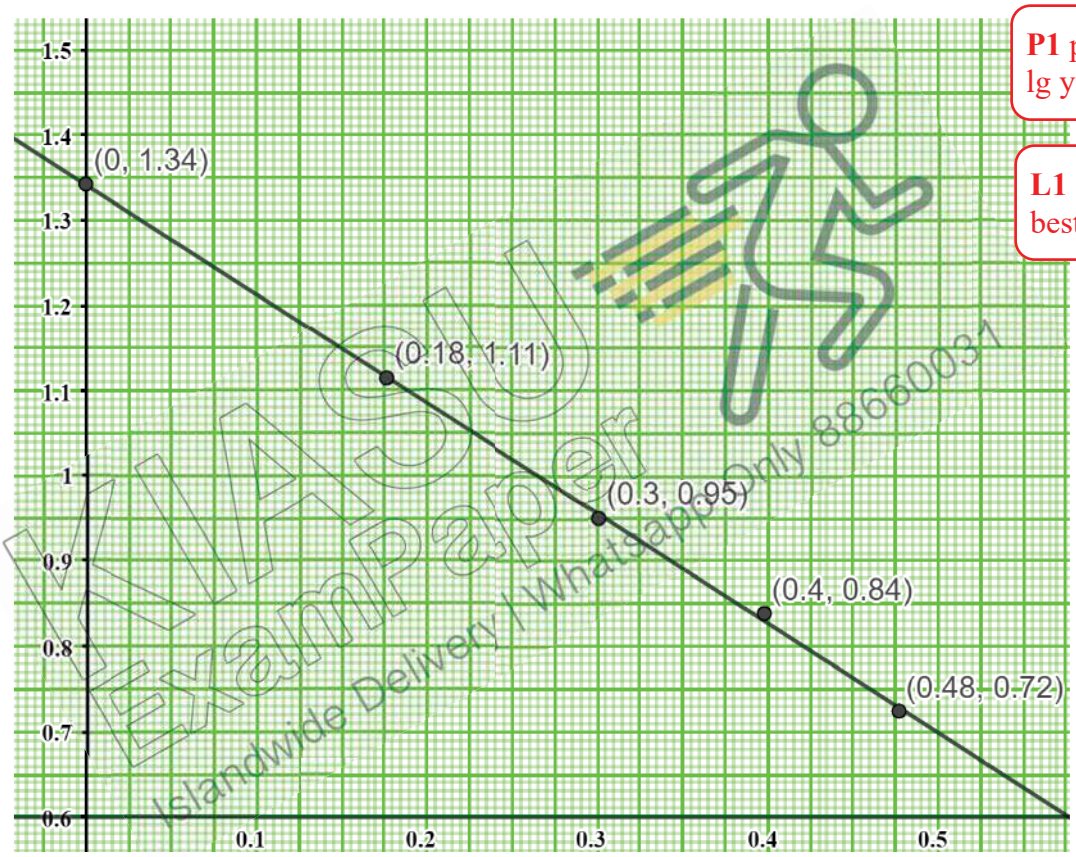
6 The table shows experimental values of two variables x and y, which are known to be connected by the equation $yx^n = A$, where n and A are constants.

x	1.0	1.5	2.0	2.5	3.0
y	22.0	13.0	8.9	6.9	5.3

(i) Plot lg y against lg x and draw a straight line graph. [3]

lg x	0	0.176	0.301	0.398	0.477
lg y	1.34	1.11	0.949	0.839	0.724

T1 table of values



P1 plot of lg y / lg x

L1 scale & best fit line

Scale: 4 cm to 0.1 units on X-axis, 2 cm to 0.1 units on Y-axis.
Scale used must be appropriate in order to award L1.

For Examiner's Use

For Examiner's Use

(ii) Use your graph to estimate the value of A and of n. [4]

$$yx^n = A$$
$$\lg y + n\lg x = \lg A \quad \textbf{M1}$$
$$\lg y = -n\lg x + \lg A$$
$$Y = mX + c$$
$$\Rightarrow m = -n, c = \lg A$$

$$m = \frac{0.7 - 1.34}{0.5 - 0} \quad \textbf{M1}$$
$$= -1.28$$
$$n = 1.28 \quad \textbf{A1}$$

$$c = 1.34$$
$$\lg A = 1.34$$
$$A = 10^{1.34}$$
$$= 21.9 \quad \textbf{A1}$$

(iii) On the same diagram, draw the line representing the equation $y = x^2$ and hence find the value of x which satisfies the equation $x^{n+2} = A$. [2]

$$y = x^2$$
$$\lg y = 2\lg x$$
$$Draw: Y = 2X \quad \textbf{M1 for drawing line}$$

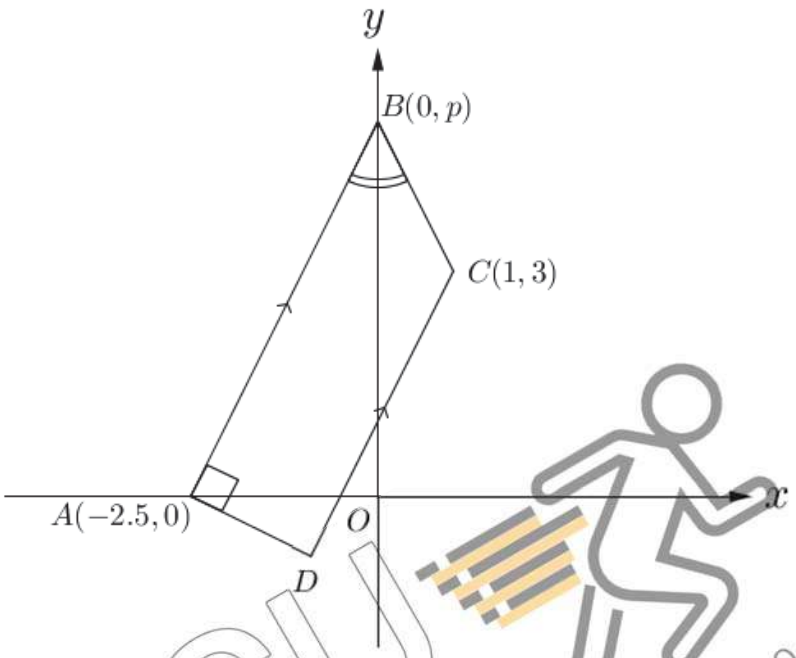
$$x^{n+2} = A$$
$$(n+2)\lg x = \lg A$$
$$2\lg x = -n\lg x + \lg A$$

Let graph 1 be $\lg y = 2\lg x$, and
Let graph 2 be $\lg y = -n\lg x + \lg A$

From graph, let intersection be (X, Y).
$$(X, Y) = (0.41, 0.82)$$
$$\lg x = 0.41$$
$$x = 10^{0.41}$$
$$= 2.57 \quad \textbf{A1}$$

For Examiner's Use

7 The diagram shows a trapezium with vertices $A(-2.5, 0)$, $B(0, p)$, $C(1, 3)$ and D.
The sides AB and DC are parallel and the angle DAB is 90°.
Angle ABO is equal to angle CBO.



(i) Express the gradients of the lines AB and CB in terms of p and hence, or otherwise, show that $p = 5$. [3]

$m_{AB} = \frac{p}{2.5}$

$= \frac{2p}{5}$ **M1**

$m_{BC} = \frac{3-p}{1}$ **M1**

$m_{AB} = -m_{BC}$

$\frac{2p}{5} = \frac{p-3}{1}$ **M1**

$2p = 5p - 15$

$3p = 15$

$p = 5$ *(shown)*

For Examiner's Use

(ii) Find the coordinates of *D*. [4]

$m_{CD} = m_{AB} = 2$

$m_{AD} = -\frac{1}{2} \because AD \perp CD$ **M1** $m_1m_2 = -1$

$Let\ D(k, h)$

$m_{CD} = \frac{3-h}{1-k}$

$2 = \frac{3-h}{1-k}$

$3 - h = 2 - 2k$

$h = 2k + 1 \cdots\cdots Eqn\ 1$ **M1** form eqn of k, h

$m_{AD} = \frac{h-0}{k+2.5}$

$-\frac{1}{2} = \frac{h-0}{k+2.5}$

$2h = -k - 2.5 \cdots\cdots Eqn\ 2$

$Eqn\ 1\ in\ Eqn\ 2: \quad 2(2k+1) = -k - 2.5$

$5k = -2 - 2.5$

$k = -0.9$ **M1** solving either unknown

$Eqn\ 1 + 2 \times Eqn\ 2: \quad 5h = -4$

$h = -0.8$

$\therefore D(-0.9, -0.8)$ **A1**

Alternative method: finding eqn of line *AD* and eqn of line *CD*.

(iii) Find the area of the trapezium *ABCD*. [2]

$Area = \frac{1}{2}\left|\begin{matrix} 1 & 0 & -2.5 & -0.9 & 1 \\ 3 & 5 & 0 & -0.8 & 3 \end{matrix}\right|$ **M1** shoelace method
Or attempt to cut up trapezium

$= \frac{1}{2}\left[\left(5 + 2 - \frac{2}{7}\right) - (-12.5 - 0.8)\right]$

$= 21\ units^2$ **A1**

For Examiner's Use

8 **(a)** Solve the equation $3\log_x 3 = 8 - 4\log_3 x$. [5]

$3\log_x 3 = 8 - 4\log_3 x$

$\frac{3}{\log_3 x} = 8 - 4\log_3 x$ **M1** common log base 3 eqn

$Let\ y = \log_3 x$

$\frac{3}{y} = 8 - 4y$

$3 = 8y - 4y^2$

$4y^2 - 8y + 3 = 0$ **M1** simplify to quad eqn

$(2y - 3)(2y - 1) = 0$

$y = 1.5\ or\ 0.5$ **M1**

$x = 3^{1.5}\ or\ 3^{0.5}$

$= \sqrt{27}\ or\ \sqrt{3}$ **A1, A1**

(b) It is given that $\log_a x = p$ and $\log_a y = q$.

Express $\log_y ax^2y^3$ in terms of p and q. [3]

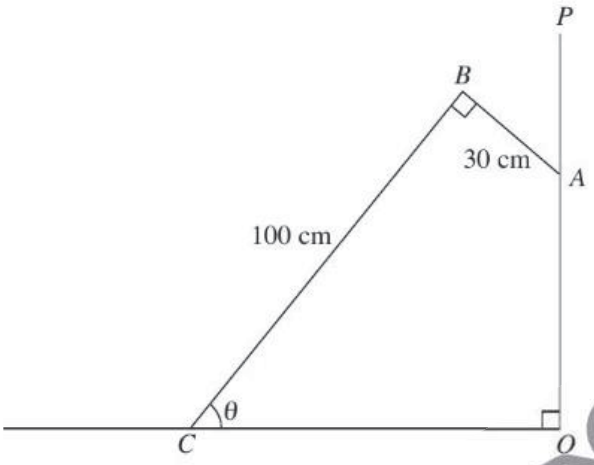
$\log_y ax^2y^3 = \log_y a + 2\log_y x + 3\log_y y$ **M1** splitting of logs

$= \frac{1}{\log_a y} + 2 \cdot \frac{\log_a x}{\log_a y} + 3$ **M1**

$= \frac{1}{q} + \frac{2p}{q} + 3$ **A1**

For Examiner's Use

9 The figure shows a stage prop ABC used by a member of the theatre, leaning against a vertical wall OP. It is given that $AB = 30$ cm, $BC = 100$ cm, $\angle ABC = \angle AOC = 90°$ and $\angle BCO = \theta$.



(i) Show that $OC = (100\cos\theta + 30\sin\theta)$ cm. [2]

Let D be foot of B on OC, let E be foot of A on BD.

$\cos\theta = \frac{CD}{100} \Rightarrow CD = 100\cos\theta$ **M1**

$\sin\theta = \frac{AE}{30} \Rightarrow AE = 30\sin\theta$ **M1**

$OC = CD + AE = 100\cos\theta + 30\sin\theta$

(ii) Express OC in terms of $R\cos(\theta - \alpha)$, where R is a positive constant and α is an acute angle. [3]

$R = \sqrt{100^2 + 30^2}$
$= 100\sqrt{109}$ **M1 for R**

$\alpha = \tan^{-1}\left(\frac{30}{100}\right)$
$= 16.7°\,(1dp)$ **M1 for alpha**

$\therefore OC = 10\sqrt{109}\cos(\theta - 16.7°)$ **A1**

(iii) State the maximum value of OC and the corresponding value of θ. [2]

$OC_{max} = 10\sqrt{109}$ **B1**

$\theta = 16.7°$ **B1**

(iv) Find the value of θ for which $OC = 80$ cm. [3]

$80 = 10\sqrt{109}\cos(\theta - 16.7°)$

$\cos(\theta - 16.7°) = \frac{8}{\sqrt{109}}$ **M1**

$\theta - 16.7° = 39.98°\ (\theta\ is\ acute)$ **M1**

$\theta = 56.7°$ **A1**

For Examiner's Use

10 Given that $y = a + b\cos 4x$, where a and b are integers, and x is in radians,

(i) state the period of y. [1]

$\frac{\pi}{2}$ **B1**

Given that the maximum and minimum values of y are 3 and – 5 respectively, find

(ii) the amplitude of y, [1]

$amplitude = \frac{3-(-5)}{2}$

$= 4$ **B1**

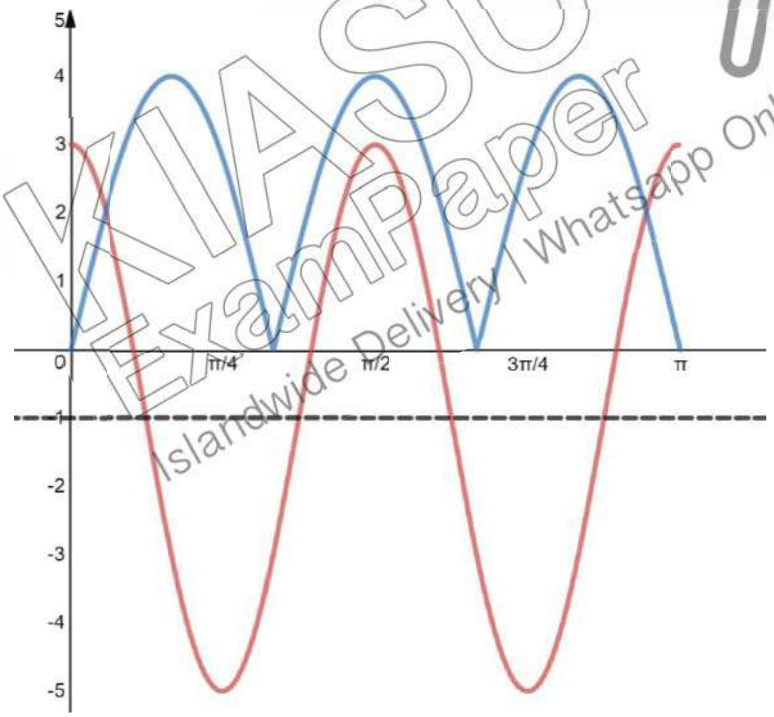
(iii) the value of a and of b. [2]

$b = 4$ **B1**

$a = -1$ **B1**

Using the values of a and b found in part **(iii)**,

(iv) sketch the graph of $y = a + b\cos 4x$ for $0 \le x \le \pi$. [3]



C1 shape (cos)

P1 two periods

I1 impt coords

ECF max 2 marks

S1 shape (sin)

M1 modulus

(v) On the same set of axes, sketch the graph of $y = |4\sin 3x|$, and hence state the number of solutions of $a + b\cos 4x = |4\sin 3x|$. [3]

Number of solutions = 2 **A1**

For Examiner's Use

11 The dimensions of a cuboid are $3x$ cm by $2x$ cm by h cm and its total surface area is 312 cm^2. The volume of the cuboid is V cm^3.

(i) Express h in terms of x. [2]

$2[3x(2x) + 3xh + 2xh] = 312$ **M1** form eqn of total SA

$6x^2 + 5xh = 156$

$h = \dfrac{156 - 6x^2}{5x}$ **A1**

(ii) Show that $V = \dfrac{36}{5}x(26 - x^2)$. [2]

$V = (3x)(2x)\left(\dfrac{156 - 6x^2}{5x}\right)$ **M1**

$= 6x\left(\dfrac{156 - 6x^2}{5}\right)$ **M1**

$= \dfrac{36}{5}x(26 - x^2)$

(iii) Find the maximum volume of the cuboid as x varies, giving your answer to the nearest cm^3. [5]

$\dfrac{dV}{dx} = \dfrac{36}{5}[(26 - x^2) + x(-2x)]$

$= \dfrac{36}{5}[-3x^2 + 26]$ **M1** differentiate

$\dfrac{dV}{dx} = 0$

$3x^2 - 26 = 0$

$x^2 = \dfrac{26}{3}$

$x = \pm\sqrt{\dfrac{26}{3}}\ (rej. -ve \because x > 0)$ **M1** solve for x

$\dfrac{d^2V}{dx^2} = \dfrac{36}{5}(-6x)$

$\left.\dfrac{d^2V}{dx^2}\right|_{x=\sqrt{\frac{26}{3}}} = \dfrac{36}{5}(-6)\left(\sqrt{\dfrac{26}{3}}\right) < 0 \Rightarrow max$ **M1** 2nd deriv. test

$V = \dfrac{36}{5}\left(\sqrt{\dfrac{26}{3}}\right)\left(26 - \dfrac{26}{3}\right)$ **M1**

$= 367.4\ldots$

$= 367\ cm^3$ **A1**

West Spring Secondary School

PRELIMINARY EXAMINATION 2019

Mathematics Paper 1 **4048/01**

Secondary 4 Express/4 Normal Academic (O)/5 Normal Academic

Name _____ ( ) **Date** **2 September 2019**

Class _____ **Duration** **2 hours**

Candidates answer on the question paper
READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
The number of marks is given in [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

FOR EXAMINER'S USE	
	/80

This document consists of **21** printed pages including this cover page.

Setter **Mr Kok Yeong Haur** [Turn over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Solve $\frac{x}{3}+11=7$

Answer $x=$ ..... [1]

---

2 Given that $\sqrt[3]{2^{18}}=\left(\frac{1}{k}\right)^6$, find k.

Answer $k=$ ..... [2]

---

3 A set of five numbers is shown below.

7 5 18 2 7

(a) Write down the median.

Answer ..... [1]

(b) When one of the number is removed from the set, the median and the range do not change. Which number was removed?

Answer ..... [1]

---

4 n is a positive integer.

Show that $(5n+2)^2-(5n-2)^2$ is a multiple of 8.

Answer

[2]

5 Factorise completely $4ax+15by-20ay-3bx$.

Answer ..... [2]

6 The frequency, f Hz, of a note produced by a string is proportional to the square root of the tension, T newtons, of the string.
For two identical strings, the ratio of the frequencies of the notes produced is 3 : 1.

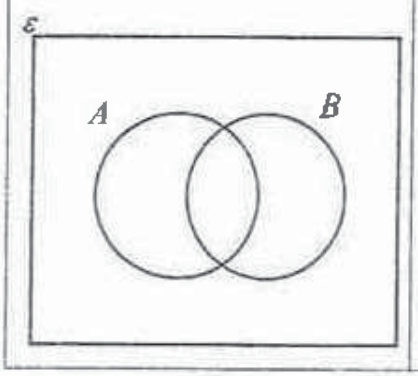
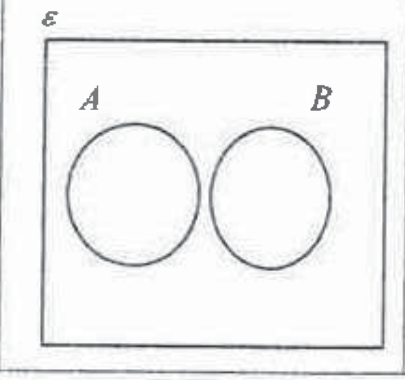
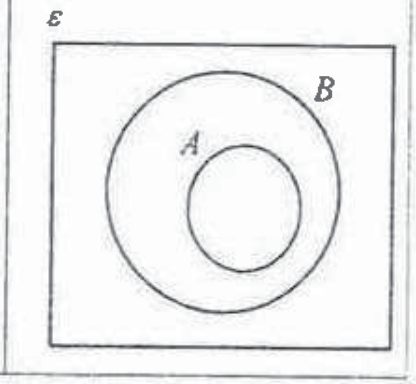
Find the ratio of the tensions in the strings.

Answer ..... : ..... [2]

7 A village of 120 people has two newspapers, the Arirang and the Busan.
35% of the villagers read the Arirang, 60% read the Busan, and 15% read neither.

ε = {all people in the village}
A = {people who read the Arirang}
B = {people who read the Busan}

(a) State which Venn Diagram represents the village.

Diagram 1	Diagram 2	Diagram 3
		

Answer Diagram ..... [1]

(b) In the diagram you have selected in (a), shade the region that represents the people in the village who read Arirang but not Busan. [1]

(c) Find the percentage of the villagers who read both newspapers.

Answer ..... [1]

8 A model of an auditorium is built using a scale of 1 : 250.
The interior volume of the model is 125 000 cm^3.

Find the actual interior volume, in m^3, of the auditorium.
Give your answer to 3 significant figures in standard form.

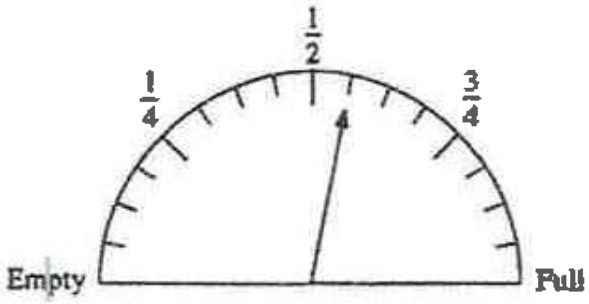
Answer ..... m^3 [3]

9 John can either buy or rent a particular laptop.
The cost of buying is \$1288.
The cost of renting is 25% of the price for the first year, and a monthly rental fee of \$16.50 after the first year.

If x is the number of months after the first year, use the information to form an inequality in x and calculate, in years and months, when it becomes more expensive to rent than to buy the laptop.

Answer ..... years ..... months [3]

10 The diagram shows the fuel gauge of Kumar's car.
The fuel gauge indicates the amount of petrol in the car.



(a) Find the fraction of the car tank that is not filled with petrol.

Answer ..... [1]

The fuel tank can store a maximum of 50 litres of petrol.
For cars travelling into Johor Bahru, their fuel tanks must be at least $\frac{3}{4}$ full.

(b) Calculate how much fuel must Kumar top up in Singapore before he can enter Johor Bahru.

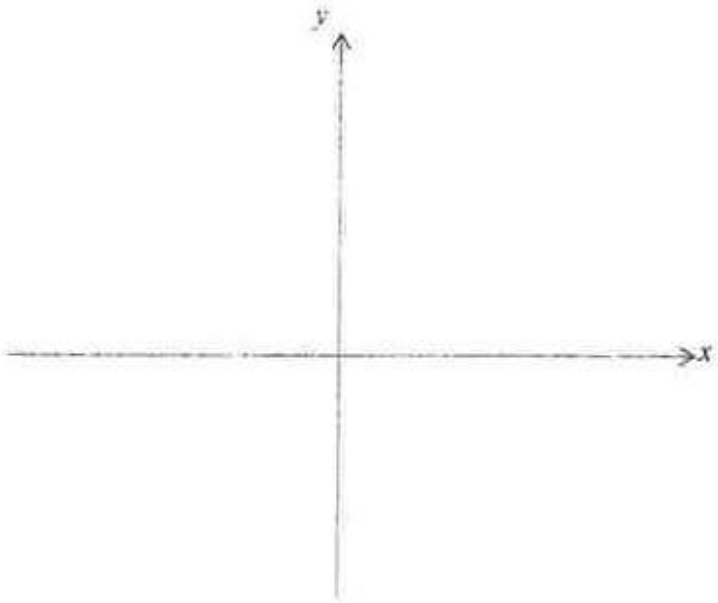
Answer ..... litres [2]

11 (a) Express $x^2 - 8x + 19$ in the form $(x-a)^2 + b$.

Answer ..... [1]

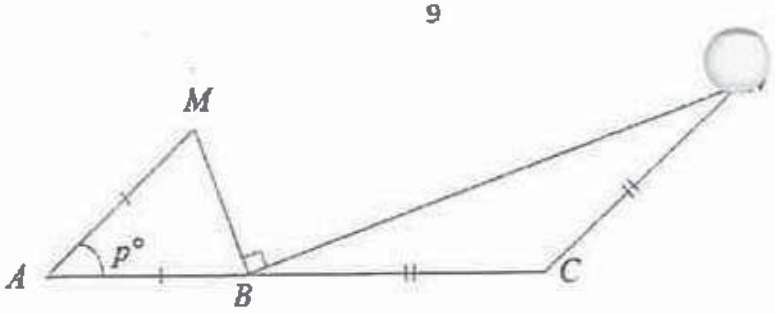
(b) Hence, sketch the graph of $y = x^2 - 8x + 19$ on the axes below, labelling clearly the turning point and intercept(s).

Answer



[2]

12



ABC is a straight line.
$AB = AM$ and $CB = CN$.
Angle $MAB = p^\circ$ and angle $MBN = 90^\circ$.

(a) Find angle CBN in terms of p.

Answer ..... [1]

(b) Explain with workings if AM is parallel to CN.

.....

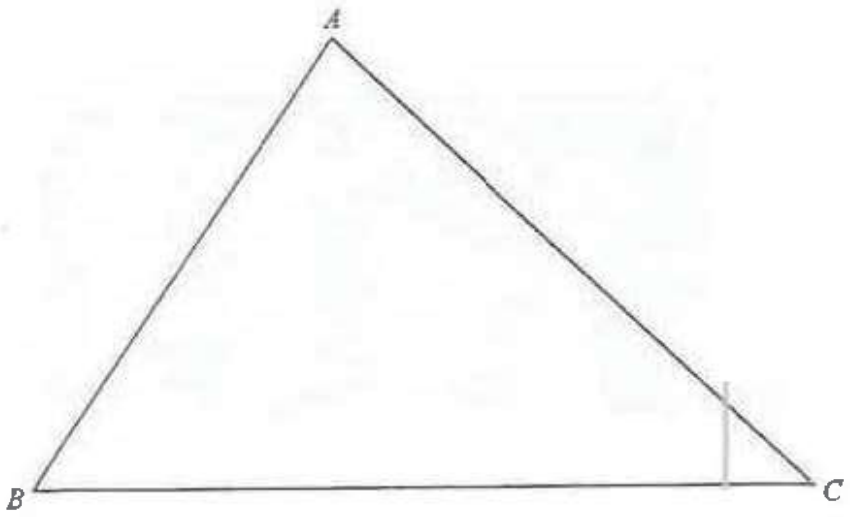
.....

.....

.....

[2]

13 The diagram represents a park ABC.



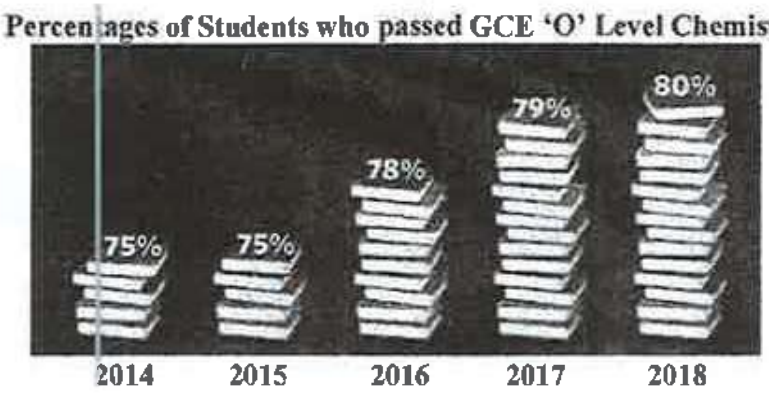
(a) Construct the perpendicular bisector of BC. [1]

(b) Construct the bisector of angle ABC. [1]

(c) A café is to be built in the park, nearer to B than to C and nearer to AB than to BC.

Shade the region where the café is to be built. [1]

14 Sam draws this graph to show the percentages of his students that passed Chemistry exam for the last four years.



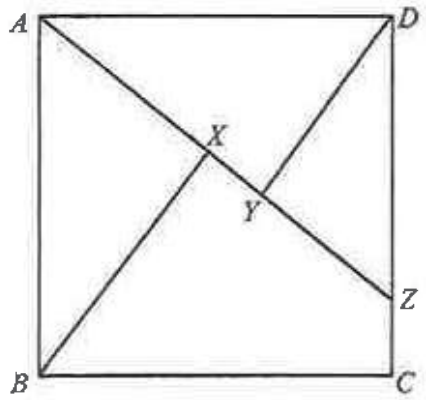
(a) State and explain one aspect of the graph that may be misleading.

.....
.....
.....
.....
..... [2]

(b) Based on the statistic, explain if Sam can also claim that the number of students passing Chemistry has increased?

.....
.....
.....
.....
..... [1]

15



ABCD is a square.
Point *Z* lies on *CD* such that *A*, *X*, *Y* and *Z* form a straight line.
Angle *AXB* = angle *DYA* = 90°.

By considering angle *DAY* = θ, prove that triangles *ABX* and *DAY* are congruent.

Answer

[3]

16 The table shows the times taken by 140 girls to complete the West Spring Cross Country 2019.

Time (in minutes)	$10 \le x < 20$	$20 \le x < 30$	$30 \le x < 40$	$40 \le x < 50$
Number of girls	25	39	62	14

(a) Calculate an estimate of

(i) the mean time,

Answer ..... minutes [1]

(ii) the standard deviation.

Answer ..... minutes [1]

(b) The mean time for the boys to complete the run was 23.8 minutes and the standard deviation was 10.4 minutes.
Make two comments comparing the times of the girls and the boys.

1. .....

.....

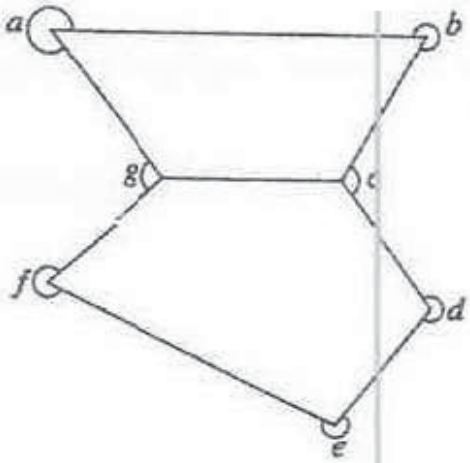
2. .....

..... [2]

17 (a) Calculate the sum of interior angles of a pentagon.

Answer ..... ° [2]

(b) Calculate the sum of the angles a, b, c, d, e, f and g in the diagram.



Answer ..... ° [2]

18 The table shows the travel times in minutes between some stations on an MRT route.

Admiralty	Sembawang	Yishun	Khatib	Yio Chu Kang	Ang Mo Kio
3					
9	6				
13	10	4			
a	15	9	5		
20	17	b	7	2	

(a) Find the values of a and b.

Answer $a =$ ..... , $b =$ ..... [2]

(b) A train leaves Sembawang MRT station and reaches Ang Mo Kio MRT station at 20 08.

(i) Calculate the time when the train leaves Sembawang MRT station.

Answer ..... [1]

(ii) Given that the distance between Sembawang MRT and Ang Mo Kio MRT stations is 12.5 km, find the average speed of the train in km/h, between these two stations.

Answer ..... km/h [1]

19 (a) Express 600 as the product of its prime factors.

Answer ..... [1]

(b) The number $600k$ is a perfect cube.

Find the smallest positive integer value of k.

Answer $k =$ ..... [1]

(c) x is a number between 950 and 1000.
The highest common factor of x and 600 is 20.

Find the smallest possible value of x.

Answer $x =$ ..... [2]

20 A factory makes wooden tables and chairs.

A table requires 8 hours of labour (L), 9 planks of wood (W) and 3 tins of paint (P).
A chair requires x hours of labour (L), 2 planks of wood (W) and 1 tin of paint (P).

(a) Represent this information in a 2 x 3 matrix, **P**.

$$\mathbf{P} = \begin{array}{c} \begin{matrix} \text{L} & \text{W} & \text{P} \end{matrix} \\ \begin{pmatrix} & & \\ & & \end{pmatrix} \end{array} \begin{matrix} \text{Table} \\ \text{Chair} \end{matrix}$$

[1]

(b) The cost of labour is \$10 per hour, the cost of wood is \$20 per plank and the cost of paint is \$4 per tin.

Find, in terms of x, the matrix $\mathbf{R} = \mathbf{P}\begin{pmatrix} 10 \\ 20 \\ 4 \end{pmatrix}$.

Answer $\mathbf{R} = \begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(c) Explain what each element in matrix **R** represents.

.....

..... [1]

(d) The cost of a table is four times the cost of a chair.
Calculate x.

Answer $x =$ ..... [1]

21 The diagram shows part of the speed-time graph of an object over a period of 50 seconds.
The object accelerates uniformly from 10 m/s to v m/s in 20 seconds.
It then decelerates uniformly for the next 15 seconds.
Thereafter it maintains a constant speed of 10 m/s.
The object travelled 450 m in the first 20 seconds.

(a) Calculate the value of v.

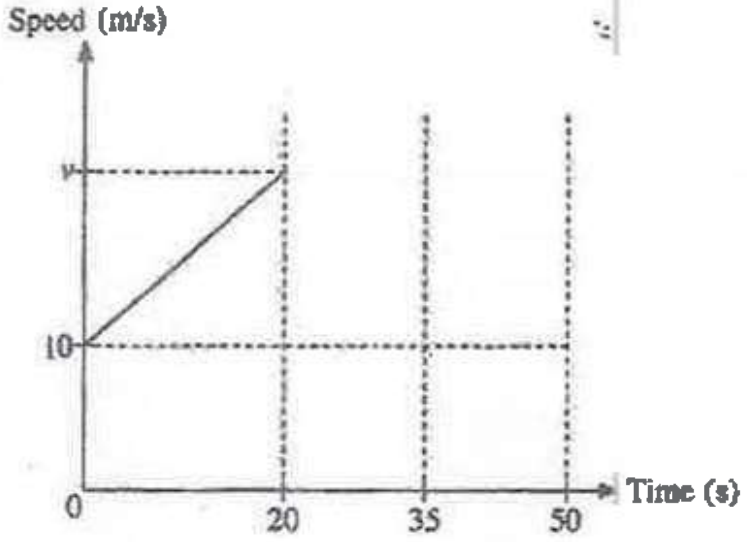
Answer $v =$ ..... m/s [2]

(b) Find the acceleration of the object after 7 seconds.

Answer ..... m/s^2 [1]

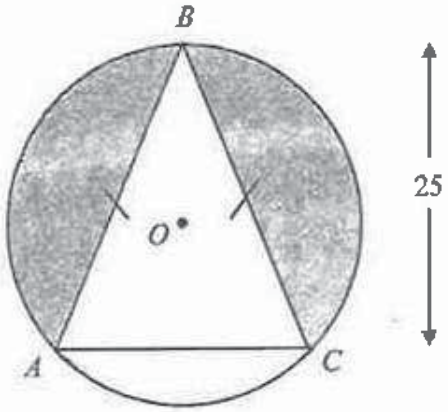
(c) Complete the speed-time graph.

Answer



[2]

22



ABC is an isosceles triangle with vertices on a circle with centre *O* and radius 15 cm. The height of the triangle *ABC* is 25 cm.

Calculate the area of the shaded region.

Answer ..... [5]

23 A company sells two sizes of the same brand of drink.

1.5 litre
$3.40

6.5 litres
$16.80

(a) Show that the cost of the drink is **not** directly proportional to the volume of the drink.

Answer

[2]

(b) The bottles are all geometrically similar.
The height of the 1.5 litre is 18 cm.

Calculate the height of the 6.5 litres bottle.

Answer ..... cm [3]

24 A is the point $(1, 1)$.

$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$.

D divides BC such that $BD : DC = 1 : 1$.

(a) Find $\overrightarrow{BC}$.

Answer $\overrightarrow{BC} = \begin{pmatrix} \quad \\ \quad \end{pmatrix}$ [2]

(b) Find $\left|\overrightarrow{AD}\right|$

Answer $\left|\overrightarrow{AD}\right| =$ ..... [2]

(c) P is the point $(3, 9)$.
Use vectors to show whether or not $ABPC$ is a parallelogram.

Answer

[2]

End of Paper 1

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Simplify $\frac{9a^2}{4b^5} \div \frac{15a^4}{12ab^3}$ [1]

(b) Express as a single fraction in its simplest form $\frac{5}{p+2} - \frac{2}{2p-3}$ [2]

(c) Solve the inequality $7-2x \geq 3x-8$ [2]

(d) It is given that $a = \frac{1}{2}\sqrt{\frac{\pi\left(b^2-6\right)}{c}}$.

Express b in terms of a and c. [2]

2 The diagram show patterns of grey and white squares.

Diagram 1 Diagram 2 Diagram 3 Diagram 4

Diagram	Number of grey squares (G)	Number of white squares (W)	Total number of squares (T)
1	1	0	1
2	1	8	9
3	17	8	25
4	17	32	49
5	⋮	⋮	⋮
6	49	x	y

(a) Find the values of x and y. [2]

(b) **(i)** Write down an expression, in terms of n, for the total number of squares (T) for Diagram n. [1]

(ii) Explain why it is not possible to have a diagram with a total of 226 squares. [1]

(c) Write an equation connecting G, W and T. [1]

(d) If there are 161 grey squares in Diagram 10, calculate the number of white squares. [2]

3 (a) One litre of petrol costs $2.25 in a particular petrol station.
During National Day, there was a promotion of 7% fuel discount.
Members are entitled to an additional 10% discount on their fuel purchase.
Karen, who is a member, paid $61.58 for her petrol on National Day.

Calculate to 1 decimal place, the number of litres she pumped. [2]

(b) The Singapore Savings Bonds (SSB) for the month of Aug pays the following interest rate for the first three years. The interest is paid out twice a year for every year.

Rate	1.95%	2.00%	2.10%
Year	1st	2nd	3rd

Sam bought $20 000 of the SSB for Aug.

(i) Calculate the interest Sam will receive in the first half of the first year. [1]

(ii) Calculate the total interest earned after three years. [1]

Sam also invested $20 000 in a savings account of a bank with a rate of compound interest of 1.98% per year. He leaves the money in the account for 3 years.

(iii) Calculate the total amount of interest he will earn after 3 years.
Give your answer to the nearest cent. [2]

(iv) Based on the given information, give one possible reason why Sam would prefer to invest more in SSB than in the bank. [1]

(c) Din booked a hotel in China using his credit card.
The hotel costs RMB 390 per night. Din booked the hotel for 4 nights and has two payment options:

Option A: The hotel charge in Singapore dollars using the hotel's exchange rate.
Option B: The hotel charge in RMB, after which Din's credit card company will convert to Singapore dollars using the company's exchange rate. There is a fee of 0.5% charged by the credit card company for the currency conversion.

Din found the currency exchange rates for the hotel and the credit card company

Hotel:	S$ 1 = RMB 4.97
Credit Card:	S$1 = RMB 5.06

Explain with workings, which payment option Din should choose. [2]

4 **(a)** The equation of line p is $2x+3y=12$.
The line cuts the x-axis at point A and the y-axis at point B.

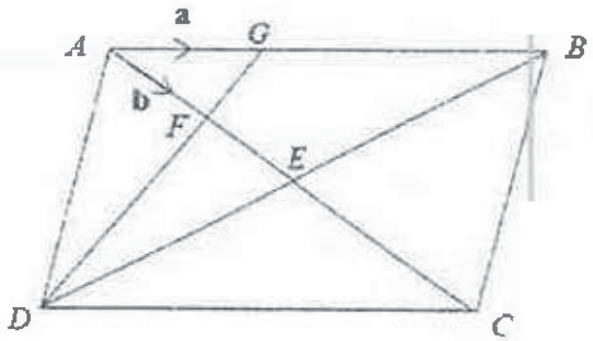
(i) Find the length of AB, [3]

A point C lies on the line p such that it is equidistant from the coordinate axes.

(ii) Show that the coordinates of point C is (2.4, 2.4). [2]

(iii) Write down the equation of the line which passes through C and is parallel to the y-axis. [1]

(b) The diagram shows a parallelogram $ABCD$.
AC and BD intersects at E.
G is a point on AB such that $2AG = GB$.
$AF : AE = 1 : 2$.
$\overrightarrow{AG} = \boldsymbol{a}$ and $\overrightarrow{AF} = \boldsymbol{b}$.



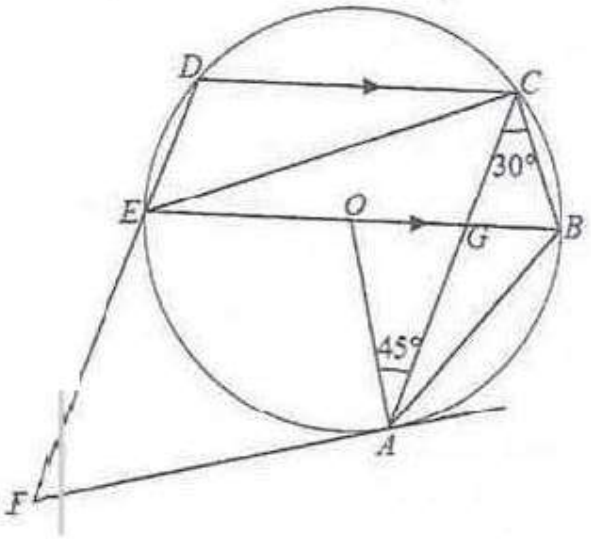
(i) Use vectors to determine if D, F and G lie on a straight line. [3]

(ii) Find the ratio of

(a) $\dfrac{\text{the area of } \Delta AFG}{\text{the area of } \Delta DFC}$, [1]

(b) $\dfrac{\text{the area of } \Delta AFG}{\text{the area of } \Delta DEC}$. [1]

5 (a)



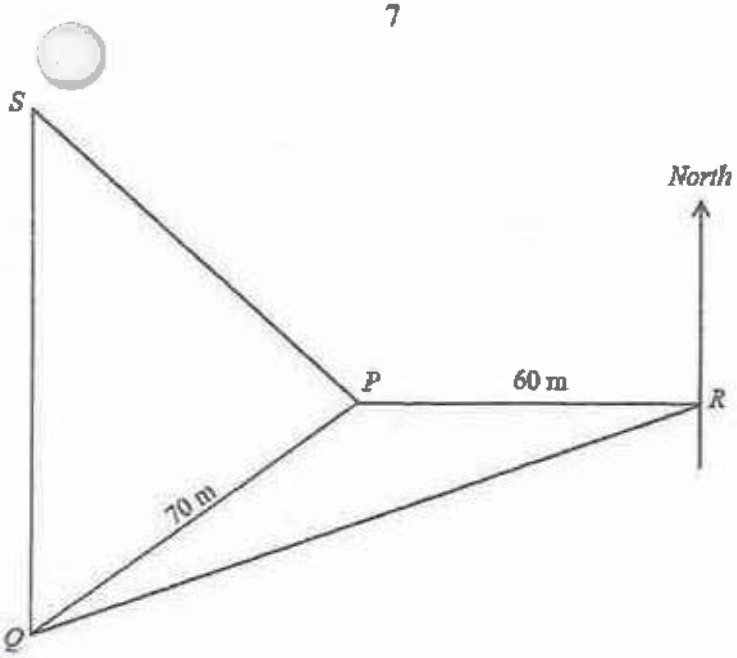
The diagram shows a circle, centre O.
AF is a tangent to the circle.
The line CD is parallel to the diameter of the circle BE.
DEF is a straight line.
The lines AC and BE intersect at G.
Angle $OAC = 45^\circ$ and angle $ACB = 30^\circ$

(i) Find, giving reasons for each answer,
(a) angle ECG, [1]
(b) angle CAB, [2]
(c) angle EFA. [3]

(ii) Determine with workings if triangles GAB and DCE are similar [2]

(b) A sector has radius 8 cm and angle 0.873 radians.
It is then formed into a cone by joining the two radii together
Calculate the perpendicular height of the cone. [3]

6



Points P, S, Q and R are at ground level.
Q is on a bearing of 240° from R.
R is 60 m due East of P.
S is due North of Q.
$PQ = 70$ m.

(a) Find angle PQR. [2]

(b) Calculate the bearing of P from Q. [1]

(c) Calculate QR. [3]

(d) An engineer, X, walks along a straight line from S to Q.

Calculate the shortest distance of X from P during this journey. [2]

(e) S is the base of a vertical tower.
T is the point on top of the tower vertically above S.
The angle of depression of R from T is 27°.

Calculate the height of the tower. [2]

7 The variables x and y are connected by the equation

$$y = x + \frac{4}{x} - 5.2.$$

Some corresponding values of x and y, correct to 2 decimal places, are given in the table below.

x	0.7	1.0	1.5	2.0	3.0	4.0	5.0	7.0	7.5
y	1.21	–0.20	–1.03	–1.2	p	–0.20	0.60	2.37	2.83

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $0 < x \leq 8$.
Using a scale of 4 cm to represent 1 unit, draw a vertical y-axis for $-2 \leq y \leq 3$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to solve the equation $x^2 - 4x + 4 = 0$. [2]

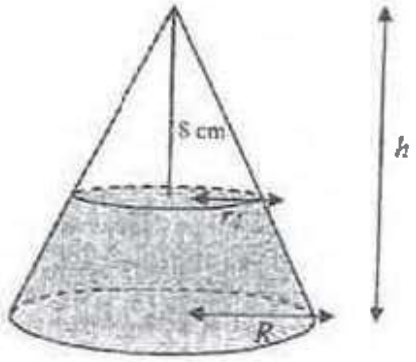
(d) By drawing a tangent, find the value of x where the gradient of the curve is 0.75. [2]

(e) **(i)** On the same axes, draw the line $y = \frac{3}{2}x - 2$, for $0 < x \leq 8$. [1]

(ii) Write down the x-coordinate of the point where this line intersects the curve. [1]

(iii) This value of x is a solution of the equation $x^2 + Ax + B = 0$.
Find the value of A and the value of B. [2]

8 The diagram shows a conical bottle of height h and radius R that is filled with water. When rests on its base, the water in the bottle is 8 cm from its vertex.

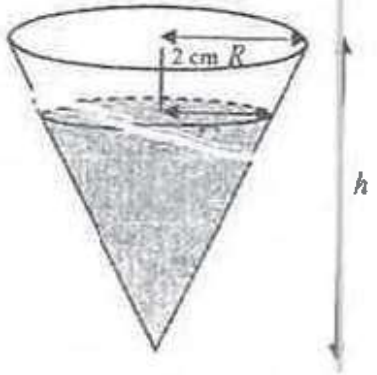


(a) Express r in terms of R and h.

Hence show that the volume of the water can be expressed as $= \frac{1}{3}\pi R^2\left(h - \frac{512}{h^2}\right)$

[3]

When the same conical bottle is turned upside down, the water level is 2 cm from its base.



(b) Show that the volume of water $= \frac{1}{3}\pi R^2 \frac{(h-2)^3}{h^2}$. [2]

(c) Using your answers from part **(a)** and **(b)**, or otherwise, write down an equation in h and show that it reduces to

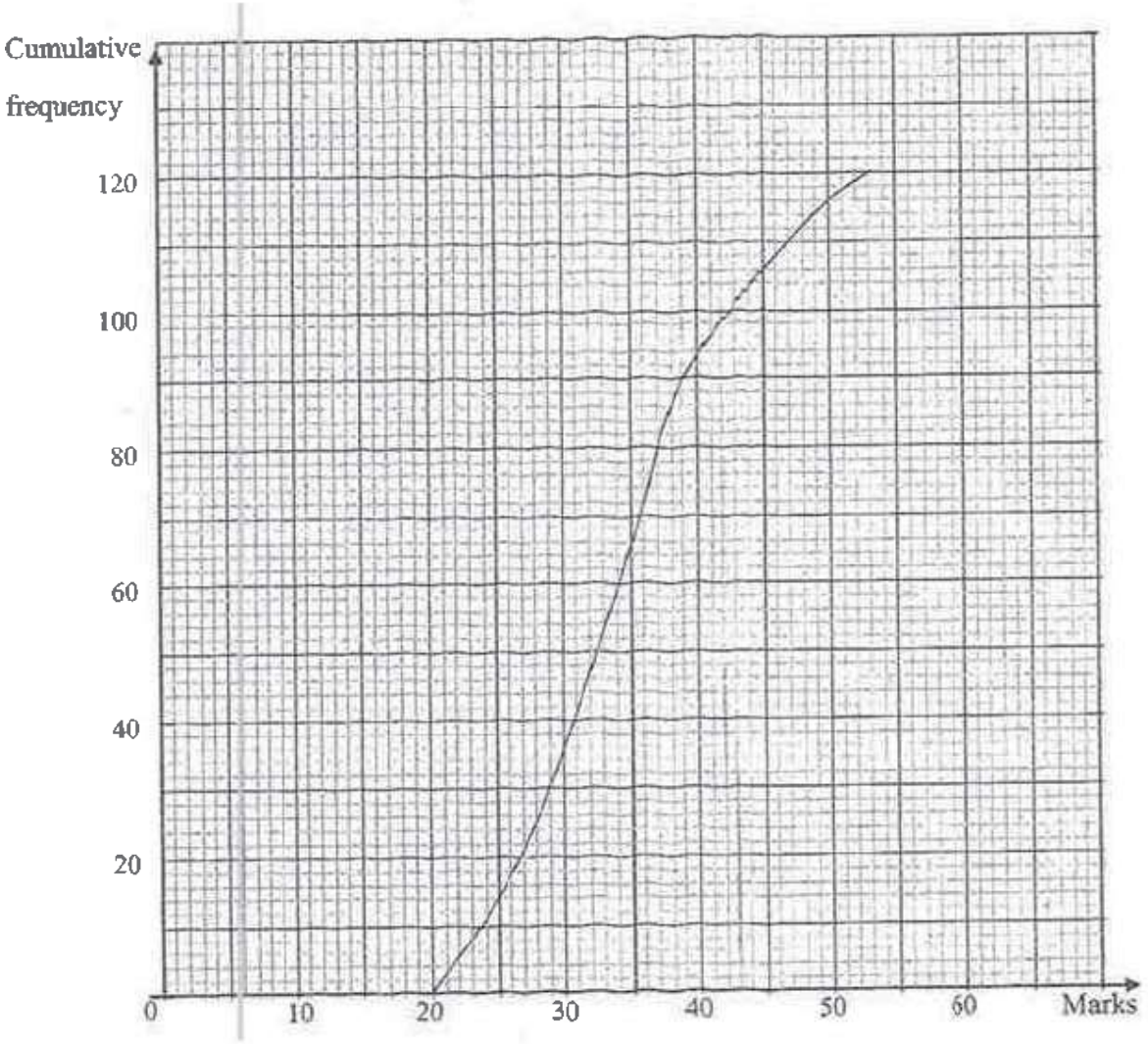
$$h^2 - 2h - 84 = 0.$$

[2]

(d) Solve the equation $h^2 - 2h - 84 = 0$, giving your solutions correct to one decimal place. [3]

(e) Calculate the volume of water in the conical bottle if $R = 7$ cm. [2]

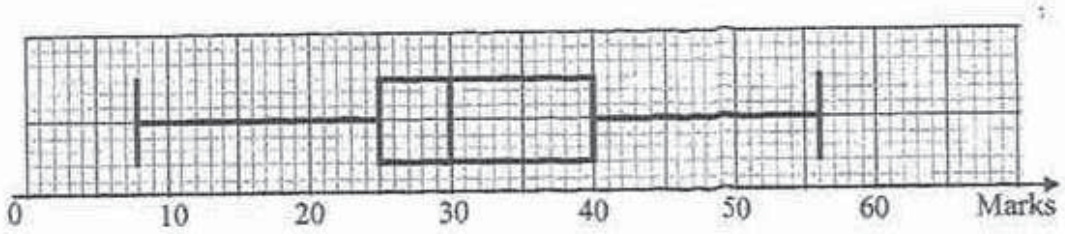
9 (a) The marks of 120 students in a Physics test were recorded.
The cumulative frequency curve below shows the distribution of the marks.



(i) Use the curve to estimate
(a) the median mark, [1]
(b) the interquartile range of the marks. [2]

(ii) The criteria for distinction is 45 marks.
Estimate the percentage of students who scored distinction. [2]

(iii) marks of the same 120 students in a Chemistry test were also recorded. The box-and-whisker plot shows the distribution of the marks.



Make two comments comparing the marks of the students for Physics and for Chemistry. [2]

(b) The table summarises the number of practice papers each student did before taking the Physics test.

Number of practice papers	0	1	2	3	4
Number of students	23	40	19	26	12

(i) One student is selected at random.
Find the probability that the student did not do any practice papers. [1]

(ii) Two students are selected at random.
Find, as a fraction in its simplest form, the probability that
(a) they both did three practice papers, [2]
(b) one had done more than two practice papers and the other had done fewer than two practice papers. [2]

10 Zander runs a restaurant in a shopping mall for the month of June.

His full-service, non-corner, 24-hour restaurant is made up of the following sections:

Percentage of Restauran Floor Area	Purpose
60%	Dining Area
30%	Kitchen
10%	Others (e.g. cashier, dish washing, receiving, storage etc)

The restaurant has 70 seats in the dining area.
Full-service restaurants typically have about one seat per 12 square feet (sqf).

(a) Estimate the floor area of the restaurant in sqf. [1]

(b) The monthly rental of the restaurant is calculated in dollars based on the table.

Location in mall	Monthly Rent in Dollars per square feet (S/psf		
	< 1000	1000 to 2000	> 2000
Corner	50	40	30
Non-Corner	40	30	20

The mall management also charge a "Maintenance & Advertising" cost every month. This cost is based on the floor area of the tenant, and is $200 per 100 sqf.

Use the table and information provided to calculate the monthly rental cost, inclusive of the "Maintenance & Advertising" cost. [2]

(c) In addition to the monthly rental cost calculated in part (b), Zander estimates that he will these costs each month

- Food Raw Materials & Ingredients $14,400*
- Utilities $21,600**
- Other administrative cost (fixed) $2,000

* Dependent on hours of operation; based on 24-hour and $20 per hour
** Dependent on hours of operation; based on 24-hour and $30 per hour

The shopping mall requires all restaurant tenants to open at least 12 hours each day, and at least till 1 am.

Zander requires 20 workers who are paid according to their working hours (Table 1).

To help understand and improve his business, Zander also collected information on his restaurant revenue at different times of the day (Table 2).

Table 1: Workers' Wage

Regular Hours (8 am to 12am)	Irregular Hours (12 am to 8am)
$12/h	$18/h

Table 2: Estimated Revenue at different hours in a day

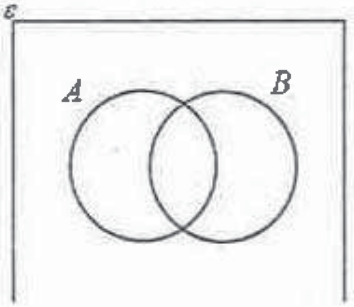
Hours	Sales
12 pm to 2 pm (Lunch)	$800 / h
7 pm to 11 pm (Dinner)	$900 / h
Other hours	$250 / h

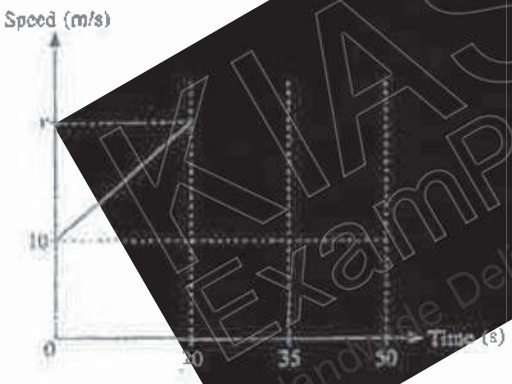
Zander needs a monthly profit of at least $7000 for his repayment for the loan he took for his restaurant business.

(i) Determine with workings, if Zander is able to repay his loan in June. [4]

(ii) Suggest a sensible opening hours for Zander's restaurant in July that will allow him to pay his loans. Justify your decision and show all calculations clearly. [2]

End of Paper ☺

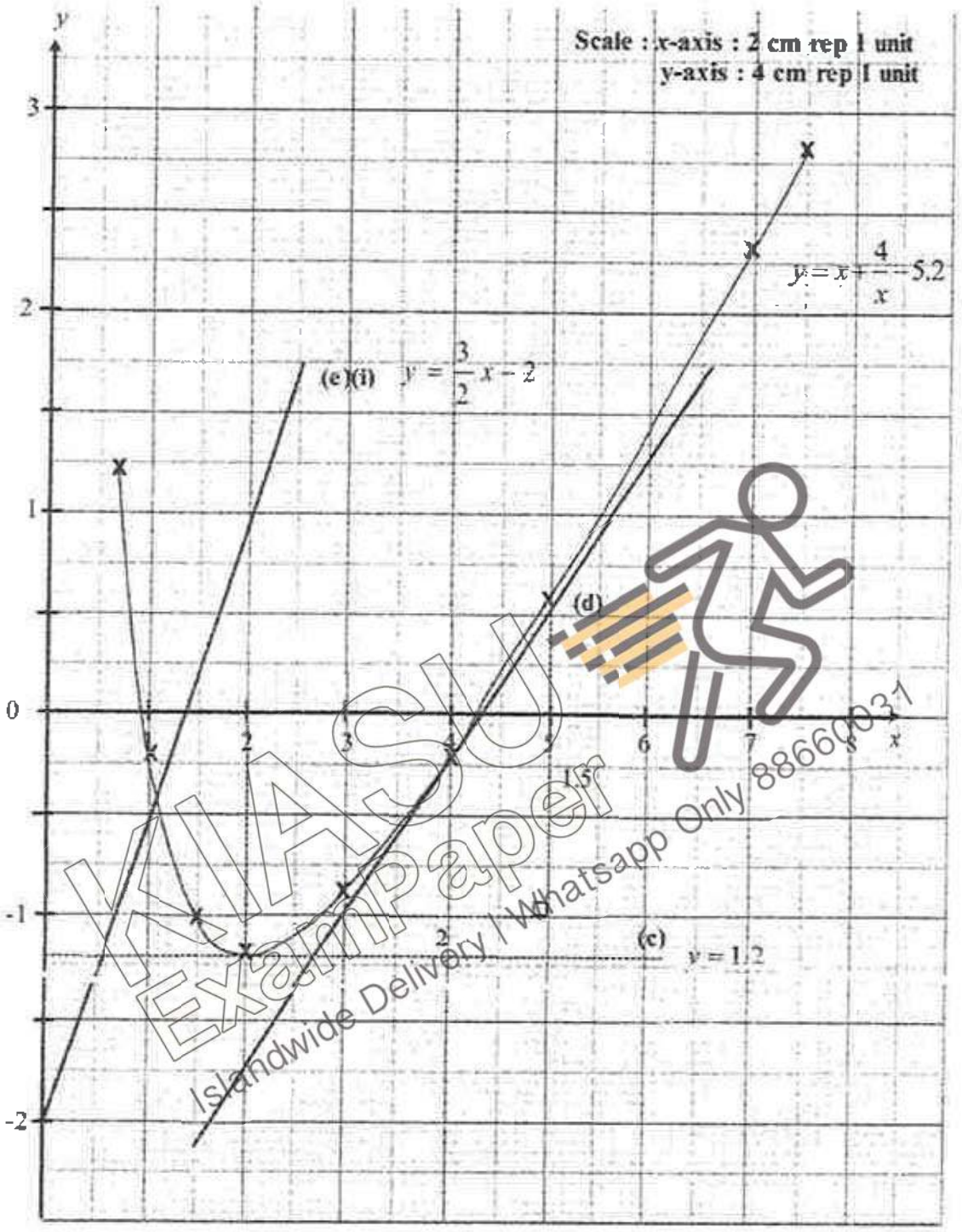
Q	Solution
1	$x = -12$
2	$k = \frac{1}{2}$
3a	7
3b	5
5	$(4a-3b)(x-5y)$
6	9 : 1
7a	Diagram 1
7b	
7c	10%
8	1.95×10^6
9	It is more expensive to rent after 5 years 11 months
10a	$\frac{7}{16}$
10b	$9\frac{3}{8}$ litres
11a	$(x-4)^2+3$
12a	$\frac{p}{2}$
13	Construction
14a	Misleading feature: the percentage is not proportional to the number of books drawn. How it is misleading: gives the impression there is huge increase in the percentage of students passing Chemistry.
14b	No, Sam cannot claim that the number of students passing has increased. This is because the number of students taking the subject may not the same over the years
15	Method 1 (AAS) Let $\angle DAY = \theta$. $\angle AXB = \angle DYA = 90°$ (given) $\angle BAX = 90° - \theta$ (complementary angles) $= 90° - (180° - 90° - \angle ADY)$ ($\angle$sum of Δ) $= \angle ADY$ $AB = DA$ (sides of a square are equal) ΔABX and ΔDAY are congruent (AAS)
16ai	Mean = 29.6 min (3sf)
16aii	SD = 8.98 min (3 sf)
16b	Two comments:

	time is shorter. 2) Girls' timing are more consistent as their SD is smaller.
17a	540°
17b	1620°
18a	$a = 18$ $b = 11$
18bi	1951
18bii	Speed 44.1 km/h
19a	$2^3 \times 3 \times 5^2$
19b	45
19c	980
20a	$P = \begin{pmatrix} 8 & 9 & 3 \\ x & 2 & 1 \end{pmatrix}$
20b	$P\begin{pmatrix} 10 \\ 20 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 & 9 & 3 \\ x & 2 & 1 \end{pmatrix}\begin{pmatrix} 10 \\ 20 \\ 4 \end{pmatrix}$ $= \begin{pmatrix} 272 \\ 10x + 44 \end{pmatrix}$
20c	The cost of making a table and a chair respectively.
20d	$x = 2.4$
21a	$v = 35$
21b	Acceleration $= \frac{35 - 10}{20} = 1.25\ m/s^2$
21c	Diagram 
22	350 cm^2
23a	Since the ratio of cost/volume are not the same, the cost is not directly proportional to the volume.
23b	$h_1 = 29.3\,cm$
24a	$\begin{pmatrix} 6 \\ 2 \end{pmatrix}$
24b	4.12
24c	Parallelogram

2019 Prelim 4E5N Math West Spring Secondary P2 Answer Key

Qns	Solution
1a	$\frac{9}{5ab^2}$
1b	$\frac{8p-19}{(p+2)(2p-3)}$
1c	$x \leq 3$
1d	$b = \pm\sqrt{\frac{4a^2c+6\pi}{\pi}}$
2a	$y = 121, x = 72$
2bi	$T = (2n-1)^2$
2bii	Column (T) consists of perfect squares only but 226 is not a perfect square.
2c	$T = G + W$
2d	200
3a	$N = 32.7$ litres
3bi	Interest = \$195
3bii	\$1210
3biii	\$1211.68
3biv	One possible reason – interest earned is comparable but SSB pays interest twice every year compared to the bank, which pays the interest after 3 years.
3c	Din should choose Option B.
4ai	7.21
4aii	$C\left(\frac{12}{5}, \frac{12}{5}\right)$
4aiii	$x = \frac{12}{}$
4bi	D, F and G lie on a straight line
4biia	$\frac{1}{9}$
4biib	$\frac{1}{12}$
5aia	60°
5aib	15°
5aic	45°
5b	7.92 cm
6a	25.4°
6b	034.6°
6c	115m
6d	39.8 m
6e	50.8 m
7a	$p = 3 + \frac{4}{3} - 5.2 = -0.87$
7b	(see graph below)
	All points plotted correctly

Right scale, axes labelled and graphs labelled correctly
Smooth curve through all the points



7c $x = 2$

7d Draw a tangent with gradient = 0.75 on the graph
$x = 4$

7ei See graph

7eii $x = 1.1$ $\{1.05 \le x \le 1.15\}$

7eiii $\therefore A = 6.4$ and $B = -8$

8a $\frac{1}{3}\pi R^2\left(h - \frac{512}{h^2}\right)$

8b $\frac{1}{3}\pi R^2 \frac{(h-2)^3}{h^2}$

8d $h = 10.21954446 \simeq 10.2$ or $h = -8.22$

8e $2790\ cm^3$

9aia Median = 34 marks

9aib IQR = 39 – 29 = 10 marks

9aii Percentage with distinction 11.7%

9aiii 2 comments:

1) On average, students scored higher in Physics than in Chemistry, as Physics median of 34 is higher than Chemistry median of 30.
2) Students' marks in Physics more consistent than Chemistry as Physics IQR of 10 is lower than Chemistry IQR of 15.

9bi $\frac{23}{120}$

9biia $\frac{65}{1428}$

9biib $\frac{57}{170}$

10a 1400sqf

10b 44800

10ci As amount is less than $7000, Zander will not be able to meet his repayment.

10cii Any sensible operating hours with relevant workings to prove that profit earned is more than $7000

Operating hours must be at least 12 hours
Operating hours must be till 1 am
Calculations for Food Raw Materials and Utilities must be based on hours of operation

One possible solution of operating hours: 8 am to 1am

YUYING SECONDARY SCHOOL
PRELIMINARY EXAMINATION
Secondary 4 Express/ 5 Normal (Academic) / 4N1 'O'

NAME			
CLASS		**REG. NO**	

MATHEMATICS 4048/01

Paper 1

26 August 2019

2 hours

Candidates answer on the Question Paper. **Setter: Mr Tai Kay Seng**

READ THESE INSTRUCTIONS FIRST

Write your name, class and class register number on the work that you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use	
Total	/80

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions

For Examiner's Use

1 Write the following numbers in order of size, starting with the **smallest**.

$$0.85, \quad \frac{8}{13}, \quad (0.75)^2, \quad \sqrt{0.49}$$

Answer , , , [1]

2 Leo is travelling to Johor Bahru from Singapore. He wants to change 400 Singapore Dollars into Malaysian Ringgit.
In Singapore, the exchange rate is 1 Singapore Dollar = 3.02 Malaysian Ringgit.
In Johor Bahru, the exchange rate is 1 Malaysian Ringgit = 0.328 Singapore Dollars.

Where should he change his money to get more Malaysian Ringgit and by how much more?

Answer ,

Ringgit [2]

3 Show that $(5n-1)^2+4$ is a multiple of 5.

Answer

[2]

[Turn Over

4 Given that $5 \times 5^{2n-2} = 125$, find the value of n.

For Examiner's Use

Answer $n =$ [2]

---

5 The first five terms of a sequence are as follows:

78, a, b, 69, c, ...

(a) Find the value of a, b, and c.

Answer (*a*) $a =$

$b =$

$c =$ [2]

(b) Write down an expression, in terms of n, for the nth term of the sequence.

Answer (*b*) [1]

(c) Explain why 32 is not a term of this sequence.

Answer (*c*)

[1]

---

6 The angles, in degrees, of a quadrilateral $ABCD$ are represented by these expressions:
Angle $A = (80 - 4x)°$, angle $B = (10 + 3x)°$, angle $C = (5x + 90)°$ and
angle $D = (15x - 10)°$.

(a) Calculate the value of x.

Answer (a) [1]

(b) What is the name of the quadrilateral?

Answer (b) [1]

---

7 Two boxes are geometrically similar. The base area of the smaller box is 64 cm^2 while the base area of the larger box is 81 cm^2.

Calculate the volume of the smaller box as a percentage of the volume of the bigger box.

For Examiner's Use

Answer % [2]

---

8 **(a)** Express 324 as a product of its prime factors.

Answer (a) [1]

(b) A number has exactly 8 factors. Two of the factors are 4 and 18.
List all the factors of the number.

Answer (b) [2]

---

9 The length of a rectangle exceeds its width by 2 cm.
(a) If the diagonal is 10 cm, find the width of the rectangle.

Answer (*a*) cm [2]

(b) How many squares of side 3 cm can be cut out from this rectangle?

Answer (*b*) [1]

10 Kenneth invests \$2000 for 3 years at a fixed rate of compound interest.
At the end of the first year there is \$2100 in his account.
(a) What is the rate of compound interest?

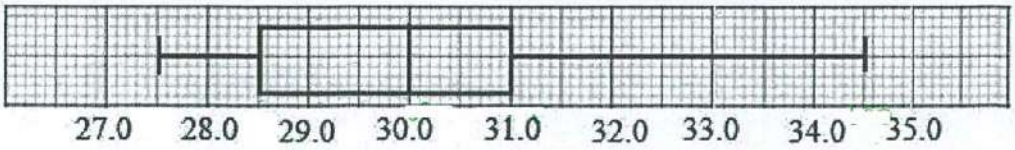
For Examiner's Use

Answer (*a*) % [2]

(b) How much does Kenneth have in his account at the end of 3 years?

Answer (*b*) \$ [2]

11 The diagram shows the box-and-whisker plot for the power, in kilowatts(kW), supplied to an electrical circuit.



(a) Find the median of the power supplied.

Answer (a) kW [1]

(b) Find the interquartile range.

Answer (b) kW [1]

---

12 A is the point $(-1, 4)$, B is the point $(2, -5)$ and O is the origin.

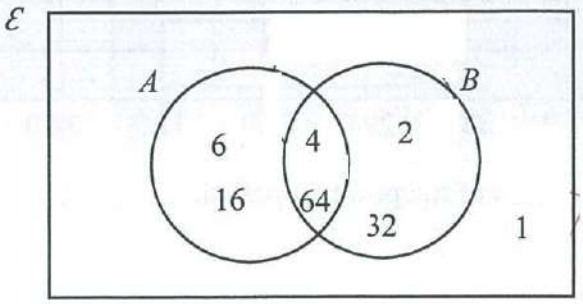
(a) Find the gradient of the line OA.

Answer (a) [1]

(b) Find the equation of the line through B parallel to OA.

Answer (b) [2]

13 The Venn Diagram represents the sets A and B.



(a) List the elements of $(A \cup B)'$.

Answer (*b*) [1]

(b) Find $n(A' \cap B)$.

Answer (*b*) [1]

(c) Set C is defined as the first two multiples of 32. Insert set C in the Venn diagram above. [1]

---

14 Simplify

(a) $27x^3y^{-2} \div 18xy^3$

Answer (*a*) [1]

(b) $\dfrac{2}{(x-3)^2} - \dfrac{1}{3-x}$

Answer (*b*) [3]

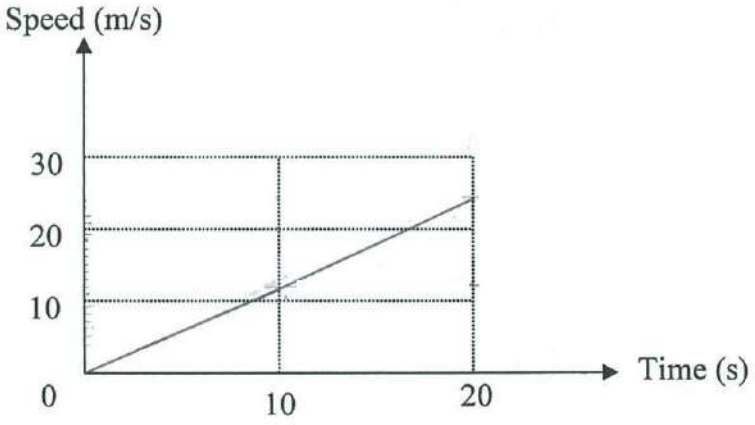
For Examiner's Use

15 A bicycle accelerates from rest at a constant rate to a certain speed in 10 seconds. It maintains at this speed for the next 10 seconds. The total distance travelled by the bicycle in the 20 seconds is 240 metres.

(a) Calculate the speed of the bicycle in the tenth second and

Answer (a) _____ m/s [2]

(b) draw the speed-time graph of the bicycle journey for the first 20 seconds.



[2]

16 Factorise completely

(a) $4x^2 - 10x - 6$,

Answer (a) [2]

(b) $10x^2y - 5xy + 2x - 1$.

Answer (b) [2]

[Turn Over

For Examiner's Use

17 A two digit number is formed using the digits 4, 8 and 9. Repetition of the digit is allowed.

(a) List the sample space.

Answer (a) [1]

(b) Find the probability that a number selected at random is

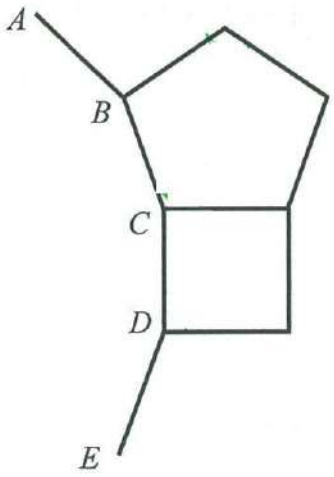
(i) a prime number,

Answer (b)(i) [1]

(ii) divisible by 5.

Answer (b)(ii) [1]

18



The diagram above is made up of a square, a regular pentagon and an incomplete regular polygon *ABCDE* of n sides. Find the value of n.

Answer $n =$ [3]

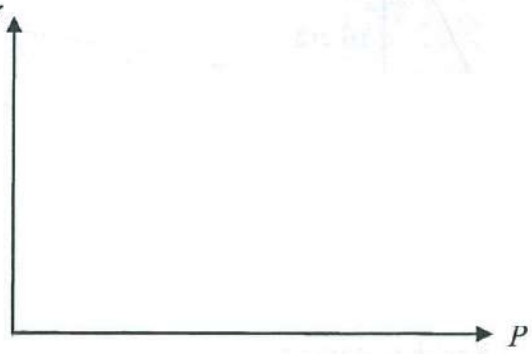
---

19 The volume, V, of a given mass of gas, is inversely proportional to the pressure, P.

For Examiner's Use

(a) Sketch a volume-pressure graph for the mass of gas.

Answer (a)



[1]

When the volume is 3 m^3, the pressure of the gas is 200 N/m^2.

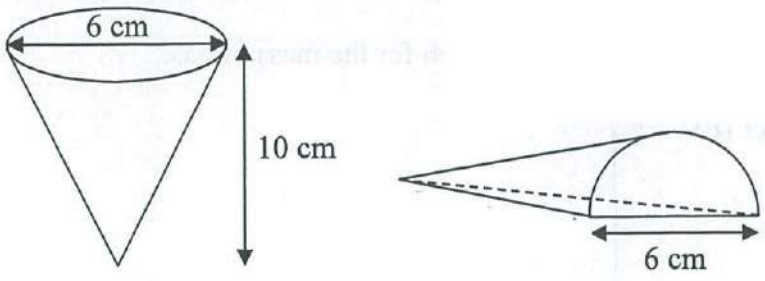
(b) Find the equation for V in terms of P.

Answer (b) $V =$ _____ [2]

(c) Calculate the pressure when the volume is 5 m^3.

Answer (c) _____ N/m^2 [1]

20



A rubber cone of diameter 6 cm and height 10 cm is cut in half to make two rubber door stoppers. Find

(a) the volume of a rubber stopper,

Answer (a) cm^3 [1]

(b) the total surface area of a rubber stopper.

Answer (b) cm^2 [3]

21 The following shows the Formula 1 track where the turns along the track are numbered 01 to 23.

For Examiner's Use

(a) Estimate the actual length of the track from turn 04 to turn 07.

Answer (a) _m [1]

(b) A racer finished the 309.316 km race in 1 h 45.599 min. Calculate the average speed in km/h.

Answer (b) km/h [1]

(c) Suggest and explain a possible speed when the racer
(i) went past the grandstand,

Answer (i)

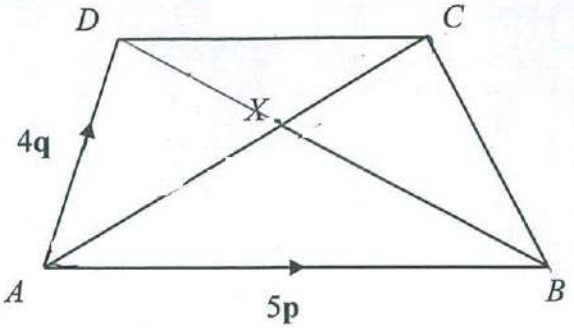
[1]

(ii) was at turn 05.

Answer (ii)

..... [1]

22



$ABCD$ is a quadrilateral.
$\overrightarrow{AB} = 5\,\mathbf{p}$, $\overrightarrow{AD} = 4\,\mathbf{q}$, $DC : AB = 3 : 5$, $AX : AC = 5 : 8$.

(a) Write each of the following in terms of **p** and **q**.

(i) $\overrightarrow{AC}$

Answer (a)(i) [1]

(ii) $\overrightarrow{BX}$

Answer (a)(ii) [1]

(iii) $\overrightarrow{XD}$

Answer (a)(iii) [1]

(iv) Explain why B, X and D lie in a straight line.

Answer ....

..... [1]

23 A particular restaurant offers 3 different dinner set menu Deluxe, Superior and Economy Set Package. The following table shows the orders for the three set packages on three days of a particular week.

For Examiner Use

	Deluxe	Superior	Economy
Friday	35	45	60
Saturday	70	85	150
Sunday	90	130	180

(a) Represent the number of orders for each type of set package on the three days by a 3×3 matrix A.

Answer (a) $A =$ [1]

(b) Given that each Deluxe, Superior and Economy Set Package costs \$188, \$88 and \$38 respectively, write down a 3×1 matrix B showing the price for each type of the set packages.

Answer (b) $B =$ [1]

(c) Evaluate the matrix $C = AB$.

Answer (c) $C =$ [1]

(d) State what the elements of C represent.

Answer (d)

[1]

For Examiner's Use

(e) For Mothers' Day, the restaurant gives a discount of 20% for Deluxe Set, 15% for Superior Set and 10% for Economy Set.

Matrix N is a 3×1 matrix that represents the price for each type of the set packages after the respective discount.

$M = (20 \quad 30 \quad 45)$ represents the order for these set packages on Mothers' Day.
Evaluate $Q = MN$ and state what the element of Q represents.

Answer (e) $Q =$ [2]

Answer (e)

[1]

---

24 Density, d kg/m^3, of a material is the mass, m kg, per unit volume, v m^3, in which

$$d = \frac{m}{v}.$$

An alloy is a mixture of metals.

If 0.0002 m^3 of copper is mixed with 0.0008 m^3 of tin, 7.62 kg of the alloy is formed.

If 0.0005 m^3 of copper is mixed with 0.0005 m^3 of tin, 8.1 kg of the alloy is formed.

Calculate the density of each of the two metals.

Answer $d_{copper} =$ _____ kg/m^3, $d_{tin} =$ _____ kg/m^3 [5]

~ End of Paper ~

YUYING SECONDARY SCHOOL
PRELIMINARY EXAMINATION
Secondary 4 Express / 5 Normal (Academic) / 4N1

NAME _____

CLASS _____ **REG. NO** _____

MATHEMATICS 4048/02

Paper 2 **28 Aug 2019**

2 hours 30 minutes

Candidates answer on the Question Paper.
Additional Materials: Graph Paper (1 sheet)

Setters: Mr Lee Mun Tat
Ms Wee Li Hui

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use	
Total	/100

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 **(a)** It is given that $V = \frac{1}{3}ax^2 + by$.

(i) Find V when $x = 2$, $y = -1$, $a = 3$ and $b = 5$. [1]

(ii) Express x in terms of V, a, b and y. [2]

(b) Simplify $\frac{6ab + 15b}{4a^2 - 25}$. [2]

(c) Solve the equation $2=\frac{3}{x+1}+\frac{1}{x(x+1)}$. [3]

(d) Solve the inequality $\frac{x}{3}+\frac{5}{6}<\frac{5x}{3}$ and state the smallest possible prime number which satisfies the inequality $\frac{x}{3}+\frac{5}{6}<\frac{5x}{3}$. [3]

2 The table shows the number of lightning flashes, in billions, on Earth, from year 2016 to 2018.

Year 2016	Year 2017	Year 2018
3.21	2.98	3.11

(a) The number of lightning flashes in 2016 can be expressed as k millions. Find k. [1]

(b) Assuming there are 365 days in a year, find the average number of lightning flashes that can be seen in a day in 2018.
Give your answer in standard form, correct to 3 significant figures. [2]

(c) Express the lightning flashes in 2016 as a percentage of the total lightning flashes from 2016 to 2018. [2]

(d) The number of lightning flashes in 2015 is 3.65 billions.

Calculate the percentage decrease in the number of lightning flashes from 2015 to 2018. [2]

(e) It is predicted that the number of lightning flashes will increase by 25% from 2018 to 2019.

Calculate the predicted number of lightning flashes in 2019, giving your answer to the nearest billion. [2]

3 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation $y = \frac{x}{10}(15 - x^2)$.

Some corresponding values of x and y are given in the table below.

x	−3	−2	−1	0	1	2	2.5	3	4	5
y	−1.8	−2.2	−1.4	0	1.4	2.2	2.2	q	−0.4	−5

(a) Calculate the value of q. [1]

(b) Using a scale of 2 cm to represent 1 unit on each axis, draw a horizontal x-axis for $-3 \le x \le 5$ and a vertical y-axis for $-5 \le y \le 3$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the solutions of $\frac{x}{10}(15 - x^2) = -1$ for $-3 \le x \le 5$. [2]

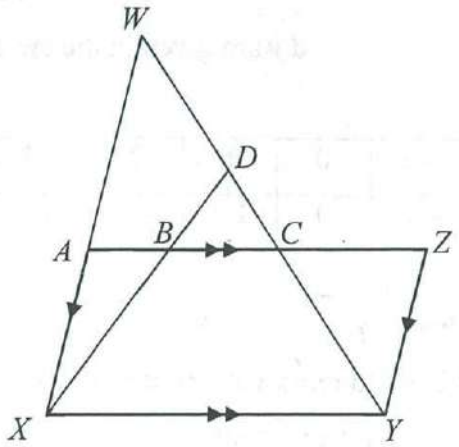
(d) By drawing a tangent, find the gradient of the curve at $x = 3.5$. [2]

(e) **(i)** On the same axes, draw the line with gradient $\frac{1}{2}$ that passes through the coordinates $(-3, -1)$. [1]

(ii) Write down the equation of the line. [1]

(iii) Write down the coordinates of the points where the line intersects the curve. [2]

4



In the diagram above, AZ is parallel to XY and WX is parallel to ZY.

(a) State a triangle that is similar to triangle DBC. [1]

(b) Show that triangle WAC is similar to triangle YZC. [2]

(c) It is given that $AB = 6$ cm, $BC = 8$ cm, $CZ = 10$ cm and $ZY = 12$ cm.

(i) If the area of triangle DBC is 48 cm^2, find the area of triangle DXY. [1]

(ii) Find the ratio $WX : ZY$. [2]

(iii) Find $\dfrac{\text{area of trapezium } ACYX}{\text{area of triangle } CZY}$. [2]

5 **(a)** The position vector of point A is $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$ and the position vector of point B is $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

(i) Find the column vector $\overrightarrow{AB}$. [1]

(ii) Find $\left|\overrightarrow{AB}\right|$. [2]

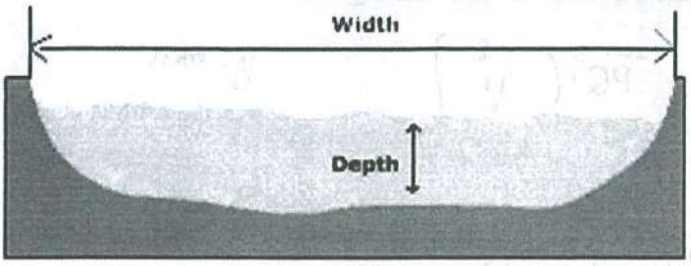
(iii) Given that $\overrightarrow{AC} = 3\overrightarrow{AB}$, find the coordinates of C. [2]

(b) The point P has coordinates $(4,-2)$ and $\overrightarrow{PQ} = \begin{pmatrix} -8 \\ 12 \end{pmatrix}$.

(i) Find the equation of the line PQ. [2]

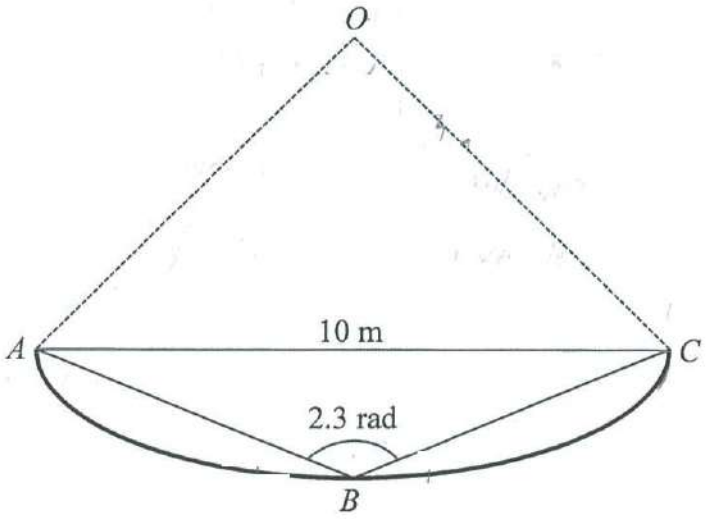
(ii) The equation of another line is $3x+2y=11$. Show how you can tell that this line does **not** intersect the line PQ. [2]

6 The diagram below shows a cross-sectional view of a river.



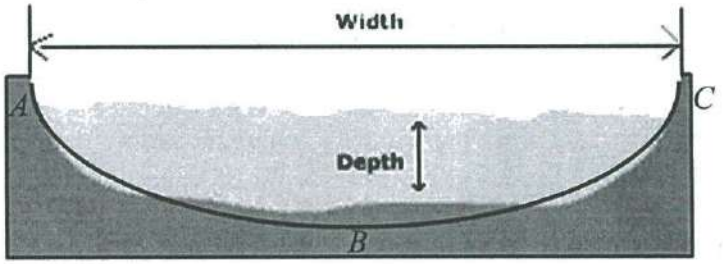
The profile of the river is modelled by the arc ABC as shown in the diagram below.
The arc ABC is part of a sector with centre O.
Given $AC = 10$ m and angle $ABC = 2.3$ *rad* ,

(a) (i) show that $AO = 6.703$ m, correct to 4 significant figures. [4]



(ii) find that the length of arc ABC. [1]

(b) The cross-section of the river with the arc ABC superimposed on it is shown in the diagram below. The volume of rain flowing down a 100 m stretch of the river is 1700 m^3. Determine if the rain will overflow from the river and cause flooding to this stretch of the river. [5]



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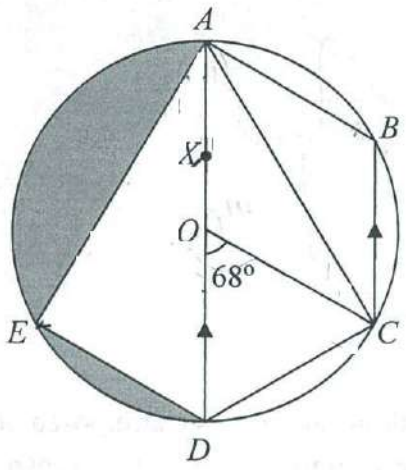


The diagram shows a circle with centre O and passing through the points A, B, C, D and E. AD is a diameter of the circle and $\angle COD = 68°$.
OA is parallel to BC.

(a) Find, giving reasons for each answer,

(i) angle OAC, [1]

(ii) angle ODC, [1]

(iii) angle ABC, [2]

(iv) angle CAB. [2]

(b) X is a point on AD such that $AX = \frac{1}{3}AD$.

Given that the area of $\Delta AED = 30\ \text{cm}^2$, find the area of ΔAXE. [2]

(c) Given that the shaded area of the segments is $36.375\ \text{cm}^2$,

find the radius of the circle. [2]

8 A water tank has a capacity of 1500 litres.

(a) The water is pumped into the tank at x litres per minute.
Write down an expression, in terms of x, for the time taken, in minutes, for an empty water tank to be completely full. [1]

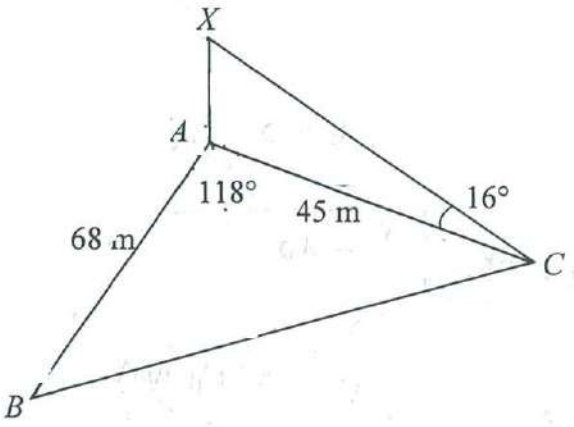
(b) If the rate of pumping water is increased by 3 litres per minute, write down an expression, in terms of x, for the time taken, in minutes, for an empty water tank to be completely full. [1]

(c) Given that the difference in the time taken is 28 minutes, form an equation in x and show that it simplifies to $7x^2 + 21x - 1125 = 0$. [3]

(d) Solve the equation $7x^2 + 21x - 1125 = 0$, giving your solutions correct to 3 decimal places. [3]

(e) Hence, find the time taken for the tank to be completely full when the water is pumped in at a rate of x litres per minute.
Give your answer in hours and minutes, correct to the nearest minute. [2]

9



In the diagram, ABC is a horizontal triangular field in which $AB = 68$ m, $AC = 45$ m and $\angle BAC = 118°$.

(a) Calculate

(i) the length of BC, [2]

(ii) the area of triangle ABC, [2]

(b) Find the shortest distance from A to BC. [2]

(c) A vertical tower XA stands at A. The angle of elevation of the top of the tower from C is 16°. Calculate

(i) the height of the tower. [2]

(ii) the greatest angle of elevation of the top of the tower when viewed from any point along BC. [2]

10 The table below shows the weight of 40 students from class 4A.

Weight (kg)	Frequency
$40 < x \leq 50$	13
$50 < x \leq 60$	16
$60 < x \leq 70$	8
$70 < x \leq 80$	3

(a) Calculate an estimate of

(i) the mean weight, [2]

(ii) the standard deviation. [2]

(b) The mean and standard deviation weight of class 4B are shown below.

Mean	60.5 kg
Standard Deviation	12.5 kg

Make two comparisons between the weight of the two classes. [2]

(c) A student has seven 50 cents coins and five 20 cents coins in his wallet.
He takes 2 coins out of the wallet, at random, one after the other.
The coins taken out are not replaced.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

(ii) Find the probability that the total value of the two coins taken out is

(a) 40 cents, [1]

(b) 70 cents. [2]

~~~ End of Paper ~~~
~~~

Answer all the questions

For Examiner's Use

1 Write the following numbers in order of size, starting with the smallest.

$$0.85,\quad \frac{8}{13},\quad (0.75)^2,\quad \sqrt{0.49}$$

Answer $(0.75)^2$, $\frac{8}{13}$, $\sqrt{0.49}$, 0.85 [1]

2 Leo is travelling to Johor Bahru from Singapore. He wants to change 400 Singapore Dollars into Malaysian Ringgit.
In Singapore, the exchange rate is 1 Singapore Dollar = 3.02 Malaysian Ringgit.
In Johor Bahru, the exchange rate is 1 Malaysian Ringgit = 0.328 Singapore Dollars.

Where should he change his money to get more Malaysian Ringgit and by how much more?

S'pore - $400 \times 302 = 1208$ Ringgit
JB - $400 \div 0.328 = 1219.51$ Ringgit

Difference = $1219.51 - 1208 = 11.51$ Ringgit

Answer Johor Bahru, 11.51 Ringgit [2]

3 Show that $(5n-1)^2+4$ is a multiple of 5.

Answer

$(5n-1)^2+4 = 25n^2-10n+5$ [M1]
$= 5(5n^2-2n+1)$ [A1]

[2]

For Examiner's Use

4 Given that $5 \times 5^{2n-2} = 125$, find the value of n.

$5 \times 5^{2n-2} = 5^3$
$5^{1+2n-2} = 5^3$
$1+2n-2 = 3$ [M1]
$2n = 4$
$n = 2$ [A1]

Answer $n =$ [2]

5 The first five terms of a sequence are as follows:

78, a, b, 69, c, ...

(a) Find the value of a, b, and c.

$\frac{78-69}{3} = 3$

Answer (a) $a =$ 75
$b =$ 72
$c =$ 66 [2]

(b) Write down an expression, in terms of n, for the nth term of the sequence.

Answer (b) $81-3n$ [1]

(c) Explain why 32 is not a term of this expression.

Answer (c) ... $81-3n = 32$, $n = 16.3$. n is not an integer. .....

Hence 32 is not a term of this expression. ..... [1]

6 The angles, in degrees, of a quadrilateral $ABCD$ are represented by these expressions:
Angle $A = (80-4x)°$, angle $B = (10+3x)°$, angle $C = (5x+90)°$ and
angle $D = (15x-10)°$.
(a) Calculate the value of x.

$80-4x+10+3x+5x+90+15x-10 = 360$
$170+19x = 360$
$x = 10$

Answer (a) $x =$ 10 [1]

(b) What is the name of the quadrilateral?

Answer (b) Trapezium [1]

7 Two boxes are geometrically similar. The base area of the smaller box is 64 cm^2 while the base area of the larger box is 81 cm^2.

For Examiner's Use

Calculate the volume of the smaller box as a percentage of the volume of the bigger box.

$$\frac{A_1}{A_2}=\left(\frac{l_1}{l_2}\right)^2=\frac{64}{81}$$

$$\frac{l_1}{l_2}=\frac{8}{9} \qquad [M1]$$

$$\frac{V_1}{V_2}=\left(\frac{l_1}{l_2}\right)^3=\frac{512}{729}$$

$$\frac{V_1}{V_2}\times 100\%=\frac{512}{729}\times 100\%$$

$$=70.2\% \qquad [A1]$$

Answer _____% [2]

8 (a) Express 324 as a product of its prime factors.

Answer (a) ____$2^2\times 3^4$____ [1]

(b) A number has exactly 9 factors. Two of the factors are 4 and 18.
List all the factors of the number.

$4=2^2$

$18=2\times 3^2$

Number $=2^2\times 3^2=36$ [M1]

Factors: 1, 2, 3, 4, 6, 9, 12, 18, 36 [A1]

Answer (b) _____ [2]

9 The length of a rectangle exceeds its width by 2 cm.
(a) If the diagonal is 10 cm, find the width of the rectangle.

$x^2+(x+2)^2=10^2$ [M1]

$2x^2+4x-96=0$

$x^2+2x-48=0$

$(x+8)(x-6)=0$

$x=-8(rej)$ or 6 [A1]

Answer (a) _____cm [2]

(b) How many squares of side 3 cm can be cut out from this rectangle?

Answer (b) ____4____ [1]

[Turn Over

For Examiner's Use

10 Kenneth invests $2000 for 3 years at a fixed rate of compound interest. At the end of the first year there is $2100 in his account.
(a) What is the rate of compound interest?

$$2100=2000\left(1+\frac{r}{100}\right)^1 \qquad [M1]$$

$$r=\left(\frac{2100}{2000}-1\right)\times 100=5 \qquad [A1]$$

Answer (a) _____% [2]

(b) How much does Kenneth have in his account at the end of 3 years?

Sum at the end of 3 years $=2000\left(1+\frac{5}{100}\right)^3$ [M1]

$=2315.25$ [A1]

Answer (b) $ _____ [2]

11 The diagram shows the box-and-whisker plot for the power, in kilowatts(kW), supplied to an electrical circuit.

(a) Find the median of the power supplied.

Answer (a) ____30.0____ kW [1]

(b) Find the interquartile range.

Answer (b) ____2.5____ kW [1]

12 A is the point $(-1, 4)$, B is the point $(2, -5)$ and O is the origin.
(a) Find the gradient of the line OA.

$$\frac{4-0}{-1-0}=-4$$

Answer (a) _____−4_____ [1]

(b) Find the equation of the line through B parallel to OA.

$y=-4x+c$ [M1]

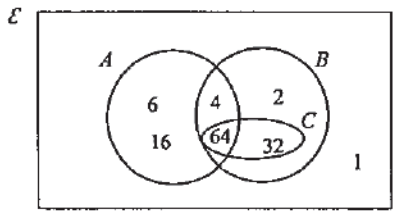
Sub $(2, -5)$ into eqn

$c=3$

Answer (b) __$y=-4x+3$ [A1]_ [2]

13 The Venn Diagram represents the sets A and B.

For Examiner's Use



(a) List the elements of $(A \cup B)'$.

Answer (b) {1} [1]

(b) Find $n(A' \cap B)$.

Answer (b) 2 [1]

(c) Set C is defined as the first two multiples of 32. Insert set C in the Venn diagram above. [1]

14 Simplify

(a) $27x^3y^{-2} \div 18xy^3$

$\frac{27x^3y^{-2}}{18xy^3} = \frac{3}{2}x^{3-1}y^{-2-3}$

$= \frac{3}{2}x^2y^{-5}$ [A1]

Answer (a) _____ [1]

(b) $\frac{2}{(x-3)^2} - \frac{1}{3-x}$

$= \frac{2}{(x-3)^2} + \frac{1}{x-3}$ [M1]

$= \frac{2+(x-3)}{(x-3)^2}$ [M1]

$= \frac{x-1}{(x-3)^2}$ [A1]

Answer (b) _____ [3]

15 A bicycle accelerates from rest at a constant rate to a certain speed in 10 seconds. It maintains at this speed for the next 10 seconds. The total distance travelled by the bicycle in the 20 seconds is 240 metres.

For Examiner's Use

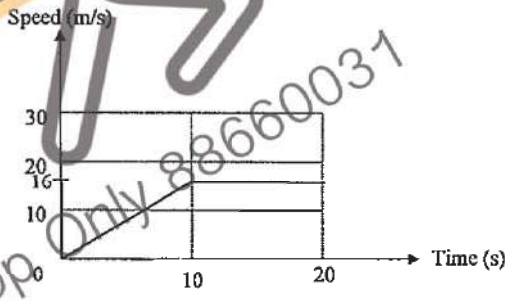
(a) Calculate the speed of the bicycle in the tenth second and

$\frac{1}{2}(x)(10) + (x)(10) = 240$ [M1]

$x = 16$ [A1]

Answer (a) _____ m/s [2]

(b) draw the speed-time graph of the car's journey for the first 20 seconds,



[2]

16 Factorise

(a) $4x^2 - 10x - 6$,

$= 2(2x^2 - 5x - 3)$ [M1]

$= 2(2x+1)(x-3)$ [A1]

Answer (a) _____ [2]

(b) $10x^2y - 5xy + 2x - 1$.

$= 5xy(2x-1) + 1(2x-1)$ [M1]

$= (5xy+1)(2x-1)$ [A1]

Answer (b) _____ [2]

17 A two digit number is formed using the digits 4, 8 and 9. Repetition of the digit is allowed.

(a) List the sample space.

Answer (*a*) {44, 48, 49, 84, 88, 89, 94, 98, 99} [1]

(b) Find the probability that a number selected at random is

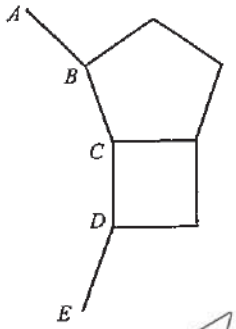
(i) a prime number,

Answer (*b*)(*i*) $\frac{1}{9}$ [1]

(ii) divisible by 5.

Answer (*b*)(*ii*) 0 [1]

18



The diagram below is made up of a square, a regular pentagon and an incomplete regular polygon *ABCDE* of *n* sides. Find the value of *n*.

Int. angle of pentagon $= \frac{(5-2)\times 180°}{5} = 108°$ [*M*1]

Int. angle of polygon $= 360° - 108° - 90° = 162°$ [*M*1]

Ext. angle of polygon $= 180° - 162° = 18°$ [*M*1]

$n = \frac{360°}{18°} = 20$ [*A*1]

Answer $n =$ [4]

19 The volume, *V*, of a given mass of gas, is inversely proportional to the pressure, *P*.

(a) Sketch a volume-pressure graph for the mass of gas.

Answer (*a*)

[1]

When the volume is 3 m^2, the pressure of the gas is 200 N/m^2.

(b) Find the equation for *V* in terms of *P*.

$V = \frac{k}{P}$ [*M*1]

$3 = \frac{k}{200}, k = 600$

$V = \frac{600}{P}$ [*A*1]

Answer (*b*) $V =$ [2]

(c) Calculate the pressure when the volume is 5 m^2.

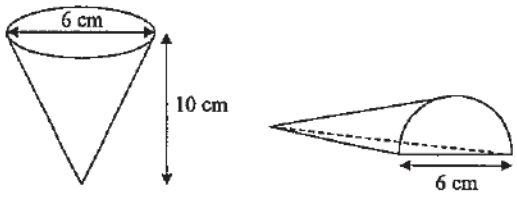
$5 = \frac{600}{P}$

$P = \frac{600}{5}$

$= 120$ [*A*1]

Answer (*c*) N/m^2 [1]

20



A rubber cone of diameter 6 cm and height 10 cm is cut in half to make two rubber door stoppers. Find

(a) the volume of a rubber stopper,

$$V = \frac{1}{2}\left[\frac{1}{3}\pi(3)^2(10)\right]$$
$$= 47.1 \qquad [A1]$$

Answer (a) _____ cm^3 [1]

(b) the total surface area of a rubber stopper.

Slant height $= \sqrt{10^2 + 3^2} = \sqrt{109} cm$ [M1]

$$A = \frac{1}{2} \times \left[\pi(3)^2\right] + \frac{1}{2} \times 6 \times 10 + \frac{1}{2} \times \pi \times 3 \times \sqrt{109} \quad [M1]$$
$$= 93.3 \qquad [A1]$$

Answer (b) _____ cm^2 [3]

For Examiner's Use

21 The following shows the Formula 1 track where the turns along the track are numbered 01 to 23.

(a) Estimate the actual length of the track from turn 04 to turn 07.

Answer (a) _1200 t0 1400___ m [1]

(b) A racer finished the 309.316 km race in 1 h 45.599 min. Calculate the average speed in km/h.

1 h 45.599 min = 1.759983 h

$$\text{Av. Speed} = \frac{309.316}{1.759983} = 176 \qquad [A1]$$

Answer (b) _____ km/h [1]

(c) Suggest and explain a possible speed when the racer
(i) went past the grandstand,

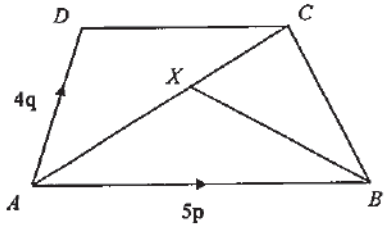
Answer (i) When the racer passed the grandstand, the speed should be higher than the average speed as the course is relatively straight. Speed should be around 200 km/h..... [1]

(ii) was at turn 05.

Answer (ii) The racer should slow down at turn 05 and the speed is lower than the average speed. Speed should be around 150 km/h. ..... [1]

For Examiner's Use

22



$ABCD$ is a quadrilateral.
$\overrightarrow{AB} = 5\,\mathbf{p}$, $\overrightarrow{AD} = 4\,\mathbf{q}$, $DC : AB = 3 : 5$, $AX : AC = 5 : 8$.

(a) Write each of the following in terms of **p** and **q**.

(i) $\overrightarrow{AC}$,

$$\begin{aligned}\overrightarrow{AC} &= \overrightarrow{AD} + \overrightarrow{DC} \\ &= 4q + \frac{3}{5}\overrightarrow{AB} \\ &= 4q + \frac{3}{5}(5p) \\ &= 4q + 3p \qquad [A1]\end{aligned}$$

Answer (a)(i) _____ [1]

(ii) $\overrightarrow{BX}$

$$\begin{aligned}\overrightarrow{BX} &= \overrightarrow{AX} - \overrightarrow{AB} \\ &= \frac{5}{8}\overrightarrow{AC} - 5p \\ &= \frac{5}{8}(4q + 3p) - 5p \\ &= \frac{5}{2}q - \frac{25}{8}p \qquad [A1]\end{aligned}$$

Answer (a)(ii) _____ [1]

(iii) $\overrightarrow{XD}$

$$\begin{aligned}\overrightarrow{XD} &= \overrightarrow{AD} - \overrightarrow{AX} \\ &= 4q - \frac{5}{8}\overrightarrow{AC} \\ &= 4q - \frac{5}{8}(4q + 3p) \\ &= \frac{3}{2}q - \frac{15}{8}p \qquad [A1]\end{aligned}$$

Answer (a)(iii) _____ [1]

(iv) Explain why B, X and D lie in a straight line.

Answer ... $\overrightarrow{BX} = \frac{5}{8}(4q - 5p)$ *and* $\overrightarrow{XD} = \frac{3}{8}(4q - 5p)$, hence $BX = \frac{5}{3}XD$

... B, X and D lies in a straight line..... [1]

[Turn Over

23 A particular restaurant offers 3 different dinner set menu Deluxe, Superior and Economy Set Package. The following table shows the orders for the three set packages on three days of a particular week.

	Deluxe	Superior	Economy
Friday	35	45	60
Saturday	70	85	150
Sunday	90	130	180

(a) Represent the number of orders for each type of set package on the three days by a 3×3 matrix A.

$$\begin{pmatrix} 35 & 45 & 60 \\ 70 & 85 & 150 \\ 90 & 130 & 180 \end{pmatrix}$$

Answer (a) $A =$ _____ [1]

(b) Given that each Deluxe, Superior and Economy Set Package costs \$188, \$88 and \$38 respectively, write down a 3×1 matrix B showing the price for each type of the set packages.

$$\begin{pmatrix} 188 \\ 88 \\ 38 \end{pmatrix}$$

Answer (b) $B =$ _____ [1]

(c) Evaluate the matrix $C = AB$.

$$C = \begin{pmatrix} 35 & 45 & 60 \\ 70 & 85 & 150 \\ 90 & 130 & 180 \end{pmatrix}\begin{pmatrix} 188 \\ 88 \\ 38 \end{pmatrix}$$

$$= \begin{pmatrix} 12820 \\ 33840 \\ 35200 \end{pmatrix} \qquad [A1]$$

Answer (c) $C =$ _____ [1]

(d) State what the elements of C represents.

Answer (d) ...It represents the restaurant takings from the three set packages on each day. ..... [1]

(c) For Mothers' Day, the restaurant give a discount of 20% for Deluxe Set, 15% for Superior Set and 10% for Economy Set.
Matrix N is a 3 x 1 matrix that represents the price for each type of the set packages after the respective discount.
$M = (20 \quad 30 \quad 45)$ represents the order for these set Packages on Mothers' Day.
Evaluate $Q = MN$ and state what the element of Q represnt.

$$N = \begin{pmatrix} 150.4 \\ 74.8 \\ 34.2 \end{pmatrix} \qquad [M1]$$

$$Q = (20 \quad 30 \quad 45)\begin{pmatrix} 150.4 \\ 74.8 \\ 34.2 \end{pmatrix}$$

$$= (6791) \qquad [A1]$$

Answer (e) $Q =$ _____ [2]

Answer (e) ...The restaurant total takings from the three set packages on

...Mothers' day..... [1]

24 Density, d kg/m^3, of a material is the mass, m kg, per unit volume, v m^3, in which

$$d = \frac{m}{v}.$$

An alloy is a mixture of metals.

If 0.0002 m^3 of copper is mixed with 0.0008 m^3 of tin, 7.62 kg of the alloy is formed.

If 0.0005 m^3 of copper is mixed with 0.0005 m^3 of tin, 8.1 kg of the alloy is formed.

Calculate the density of each of the two metals.

$7.62 = d_c \times 0.0002 + d_t \times 0.0008$

$38100 = d_c + 4d_t$ ----------(1) [M1]

$8.1 = d_c \times 0.0005 + d_t \times 0.0005$

$16200 = d_c + d_t$ ----------(2) [M1]

(1) – (2) [M1]

$3d_t = 21900$

$d_t = 7300$ [A1]

Sub $d_t = 7300$ *into* (2)

$d_c = 8900$ [A1]

Answer d_{copper} = _____ kg/m^3, d_{tin} = _____ kg/m^3 [5]

~ End of Paper ~

Marking Scheme

Sec 4E5N/4N1-Maths P2-Prelim-2019

1 (a) (i)	$V=\frac{1}{3}(2)^2(3)+(-1)(5)$ $=-1$
1 (a) (ii)	$V=\frac{1}{3}ax^2+by$ $V-by=\frac{1}{3}ax^2$ $3(V-by)=ax^2$ $\frac{3(V-by)}{a}=x^2$ $x=\pm\sqrt{\frac{3(V-by)}{a}}$
1 (b)	$\frac{6ab+15b}{4a^2-25}=\frac{3b(2a+5)}{(2a+5)(2a-5)}$ $=\frac{3b}{2a-5}$
1 (c)	$2=\frac{3}{x+1}+\frac{1}{x(x+1)}$ $2=\frac{3x}{x(x+1)}+\frac{1}{x(x+1)}$ $2=\frac{3x+1}{x^2+x}$ $2x^2+2x=3x+1$ $2x^2-x-1=0$ $(2x+1)(x-1)=0$ $x=-0.5$ or $x=1$
1 (d)	$\frac{x}{3}+\frac{5}{6}<\frac{5x}{3}$ $\frac{2x+5}{6}<\frac{10x}{6}$ $2x+5<10x$ $5<8x$ $8x>5$ $x>\frac{5}{8}$

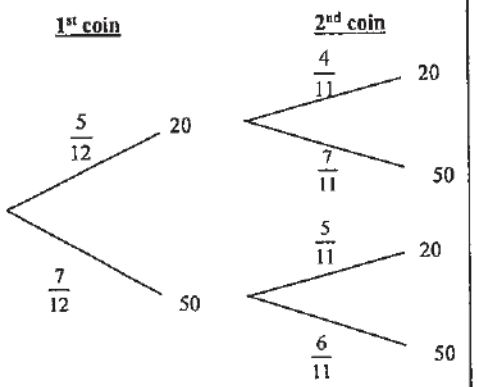
	smallest prime number = 2
2 (a)	3210
2 (b)	$\frac{3.11\times10^9}{365}=8520547.945$ $=8.52\times10^6$ (3 s.f.)
2 (c)	$\frac{3.21\times10^9}{3.21\times10^9+2.98\times10^8+3.11\times10^9}\times100$ $=34.5\%$ (3 s.f.)
2 (d)	$\frac{3.65\times10^9-3.11\times10^9}{3.65\times10^9}\times100$ $=14.8\%$ (3 s.f.)
2 (e)	$\frac{125}{100}\times3.11\times10^9$ = 4 billions (nearest billion) or 4 000 000 000 (nearest billion) or 4×10^9 (nearest billion)
3	Refer to graph paper

4 (a)	Triangle DXY
4 (b)	$\angle ACW = \angle ZCY$ (vert. opp. angles) $\angle WAC = \angle YZC$ ($WA // ZY$, alt. angles) By AA similarity, triangle WAC is similar to triangle YZC.
4 (c) (i)	$\frac{\text{area of triangle } DBC}{\text{area of triangle } DXY} = \left(\frac{8}{24}\right)^2$ $= \frac{1}{9}$ Area of triangle $DXY = 48 \times 9$ $= 432 \text{ cm}^2$
4 (c) (ii)	By similarity, $\frac{WA}{YZ} = \frac{AC}{ZC}$ $\frac{WA}{12} = \frac{14}{10}$ $WA = \frac{14}{10} \times 12$ $= 16.8$ $\frac{WX}{ZY} = \frac{16.8+12}{12}$ $= \frac{12}{5}$ $WX : ZY = 12 : 5$
4 (c) (iii)	$\frac{\text{area of } ACXY}{\text{area of triangle } CZY} = \frac{\frac{1}{2}(6+8+24)(\text{ht})}{\frac{1}{2}(10)(\text{ht})}$ $= \frac{19}{5}$
5(a)(i)	$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$ $= \begin{pmatrix} 3 \\ -8 \end{pmatrix}$
(ii)	$\sqrt{3^2+8^2}$ $= 8.54$ units (3 s.f.)

(iii)	$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$ $= \begin{pmatrix} -1 \\ 5 \end{pmatrix} + 3\overrightarrow{AB}$ $= \begin{pmatrix} -1 \\ 5 \end{pmatrix} + 3\begin{pmatrix} 3 \\ -8 \end{pmatrix}$ $= \begin{pmatrix} 8 \\ -19 \end{pmatrix}$ C (8, -19)
(b)(i)	Gradient of $PQ = \frac{12}{-8} = -\frac{3}{2}$ $y = -\frac{3}{2}x + c$ $-2 = -\frac{3}{2}(4) + c$ $c = 4$ $y = -\frac{3}{2}x + 4$
(ii)	$3x + 2y = 11$ $y = -\frac{3}{2}x + 5.5$ Both lines have the same gradient so they are parallel and will not intersect
6(a)(i)	Reflex $AOC = 4.6$ rad (angle at centre = 2X angle at circum) Angle $AOC = 2\pi - 4.6 = 1.684$ $AO = \frac{5}{\sin 0.842}$ $= 6.703$ m (shown)
(ii)	Arc ABC $= 6.703 \times 1.684$ $= 11.3$ m
(b)	Cross sectional area = $\frac{1}{2} \times (6.703)^2 \times 1.684 - \frac{1}{2}(6.703)^2 \sin 1.684$ $= 15.5099$ $= 15.5 \text{ m}^2$

	Volume $= 15.5099 \times 100$ $= 1551\ m^3$ The rain will overflow from the drain as the rain flowing is more than the capacity of the drain
7(a)(i)	$34°$ (ext angle of triangle / angle at centre = 2 X angle at circum)
(ii)	$56°$ (angle sum of isos triangle)
(iii)	$= \frac{180+68}{2}$ $= 124°$ (angle at centre = 2 X angle at circum)
(iv)	$= 180 - 34 - 124$ $= 22°$ (int angle, OA parallel to BC)
(b)	Both triangles have common height $= \frac{1}{3}(30)$ $= 10\ cm^2$
(c)	Area of semicircle $= 30 + 36.375 = 66.375$ $\frac{1}{2}\pi r^2 = 66.375$ $r = 6.50$ cm
8(a)	$\frac{1500}{x}$
(b)	$\frac{1500}{x+3}$
(c)	$\frac{1500}{x} - \frac{1500}{x+3} = 28$ $4500 = 28x(x+3)$ $28x^2 + 84x - 4500 = 0$ $7x^2 + 21x - 1125 = 0$ (shown)

(d)	$x = \frac{-21 \pm \sqrt{21^2 - 4(-1125)(7)}}{2(7)}$ $= 11.266$ *or* -14.266
(e)	$t = \frac{1500}{11.266}$ $= 133$ mins $= 2$ h 13 mins
9(a)(i)	$BC^2 = 68^2 + 45^2 - 2(68)(45)\cos 118$ $BC = 97.582$ $= 97.6m$ (3 s.f.)
(ii)	$= \frac{1}{2}(68)(45)\sin 118$ $= 1350m^2$ (3 s.f.)
(b)	$\frac{1}{2}(97.582)h = 1350.9$ $h = 27.7m$ (3 s.f.)
(c)(i)	$h = 45\tan 16$ $= 12.9m$ (3 s.f.)
(ii)	$\tan\theta = \frac{12.904}{27.688}$ $\theta = 25.0°$ (1 d.p.)
10(a)(i)	$= \frac{45\times13 + 55\times16 + 65\times8 + 75\times3}{40}$ $= 55.25$ kg
(ii)	$\sqrt{\frac{125400}{40} - 55.25^2}$ $= 9.08$ kg (3 s.f.)
(b)	Class 4B is heavier than 4A as the mean is higher. Class 4A weight is more consistent as the S.D. is smaller.
(c)(i)	

	
ii(a)	$\frac{5}{12}\times\frac{4}{11}$ $=\frac{5}{33}$
(b)	$\frac{5}{12}\times\frac{7}{11}+\frac{5}{11}\times\frac{7}{12}$ $=\frac{35}{66}$