2019

Secondary 4 AMath

1.	Commonwealth Secondary	SA1
2.	Maris Stella High	SA1
3.	Mayflower Secondary	SA1
4.	Nan Chiau High	SA1
5.	St. Margaret's Secondary	SA1
6.	Anglican High	SA2
7.	Fairfield Methodist	SA2
8.	Greendale Secondary	SA2
9.	Nan Hua	SA2
10.	Peicai Secondary	SA2
11.	St Andrew's Secondary	SA2
12.	St. Margaret's Secondary	SA2
13.	West Spring Secondary	SA2

COMMONWEALTH SECONDARY SCHOOL
MID-YEAR EXAMINATION 2019

ADDITIONAL MATHEMATICS
PAPER 1

Name: ______________________________ (   ) Class: __________

SECONDARY FOUR EXPRESS **Wednesday 8 May 2019**
SECONDARY FIVE NORMAL ACADEMIC **08 00 – 10 00**
4047/1 **2h**

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Name of setter: Lee Ying Jie

This paper consists of **5** printed pages including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

1. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formula for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 Find the set of values of x for which $(2+3x)(x-5)>2+3x$. [3]

2 A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=9x+1$. The gradient of the curve at the point $(2,16)$ is 18. Find the equation of the curve. [6]

3 **(i)** On the same axes sketch the curves of $y=2x^{-\frac{2}{3}}$ and $y^3=x$ for $x\geq 0$. [2]

(ii) Find the coordinates of the point of intersection of the two curves. [3]

4 The variables x and y are such that when the values of $\frac{x}{y}$ are plotted against x a straight line graph is obtained. It is given that the line passes through the point $\left(3\sqrt{3},\,4\right)$ and forms an angle of 60° with the x-axis. Express y in terms of x. [4]

5 **(i)** By using long division, divide $2x^3-6x^2+x-3$ by $x-3$. [1]

(ii) Express $\frac{3x^2+21x+5}{2x^3-6x^2+x-3}$ in partial fractions. [5]

6 The equation of a curve is $y=\frac{1}{2}e^{2x}-7e^x+6x$.

(i) Write down expressions for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [2]

(ii) Find the exact x-coordinates of the stationary points on the curve. [4]

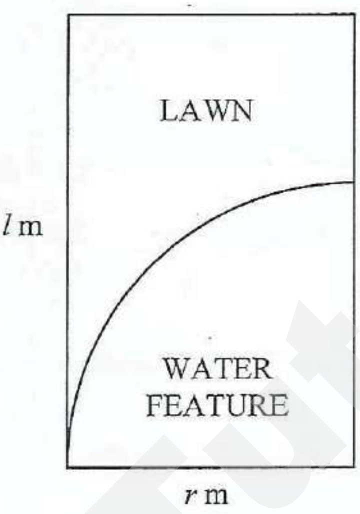
(iii) Determine the nature of each of these stationary points. [3]

7 Find

(i) $\int 3x^4-5\cos 6x\,\mathrm{d}x$, [2]

(ii) $\int \sin x+\sec^2(5+2\pi x)\,\mathrm{d}x$. [2]

8 As part of a garden design, there are plans to put aside a rectangular space which has sides of lengths r m and l m. This rectangular space is to include a quadrant-shaped water feature and a lawn. The area of the lawn is to be 360 m^2.



(i) Show that the perimeter, P m, of the lawn is given by $P = \frac{720}{r} + \pi r$. [4]

A hedge is to be planted along the perimeter of the lawn.

(ii) Given that r can vary, find the dimensions of the rectangular space which can allow the shortest length of hedge to be planted along the perimeter of the lawn. [6]

9 **(i)** Show that $\frac{\tan^2 x - 1}{\tan^2 x + 1} = 1 - 2\cos^2 x$. [3]

(ii) Hence find, for $0 \le x \le 5$, the values of x in radians for which $\frac{\tan^2 x - 1}{\tan^2 x + 1} = \frac{1}{2}$. [3]

10 The roots of the quadratic equation $4x^2 - 6x + 3 = 0$ are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. Find a quadratic equation with roots α^3 and β^3. [6]

11 A rectangular piece of cardboard has the dimensions 18 cm by $5-\sqrt{3}$ cm.

(i) Find the exact value of the square of its diagonal. [2]

A small square piece with sides $1+\sqrt{3}$ cm is cut out from the rectangular piece of cardboard.

(ii) Express in the form $\frac{a+b\sqrt{3}}{c}$, the area of the small square as a fraction of the area of the rectangular piece of cardboard. [3]

12 (i) Given that $y=x\sqrt{5x^2-6}$, find $\frac{dy}{dx}$. [2]

(ii) Hence, evaluate $\int_2^4 \frac{5x^2-3}{\sqrt{5x^2-6}}\,dx$. [3]

13 The equation of the tangent to a circle at the point $A(8,9)$ is given by $4y+3x=60$. The line $y=4x-7$ passes through the centre, P, of the circle.

(i) Find the coordinates of P. [4]

(ii) Find the equation of the circle. [3]

The tangent to the circle at A meets the y-axis at point B.

(iii) Find the equation of another circle with BP as diameter. [4]

END OF PAPER

COMMONWEALTH SECONDARY SCHOOL
MID-YEAR EXAMINATION 2019

ADDITIONAL MATHEMATICS
PAPER 2

Name: ______________________________ ( ) Class:__________

SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC
4047/02

Monday 13 May 2019
08 00 – 10 30
2h 30min

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

Name of setter: Ms Kelly Zhang

This paper consists of **5** printed pages including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

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Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = 3 - \sin x$.

(i) Explain why the curve $y = \mathrm{f}(x)$ has no turning points. [2]

(ii) Given that the curve passes through the origin, find an expression for $\mathrm{f}(x)$. [3]

2 The equation of a curve is $y = \dfrac{2x+1}{x-4}$.

(i) State, with explanation, whether y is an increasing or decreasing function. [4]

(ii) A particle moves along the curve $y = \dfrac{2x+1}{x-4}$ in such a way that the y-coordinate of the particle is increasing at a constant rate of 0.5 units per second.
Find the rate at which the x-coordinate of the particle is changing at the instant when $x = 2$. [3]

3 (i) Using $\sin 3x = \sin(2x + x)$, show that $\sin 3x = 3\sin x - 4\sin^3 x$. [3]

(ii) Find all the values of x between 0 and 2π for which $\sin 3x = \sin^2 x$. [4]

4 (i) Express $\sqrt{3}\sin\theta + \cos\theta$ in the form $R\sin(\theta + \alpha)$, where R and α are constants to be found. [3]

(ii) Using your values of R and α, evaluate $\int_0^{\pi} \sqrt{3}\sin x + \cos x \, \mathrm{d}x$, leaving your answer in exact form. [4]

5 (i) By considering the general term in the binomial expansion of $\left(kx - \dfrac{1}{x^3}\right)^7$, where k is a constant, explain why there are no even powers of x in this expansion. [5]

(ii) Given that the coefficient of the third term is thrice the coefficient of the second term, find the value of k. [3]

6 (i) Find the coordinates of all the points at which the graph of $y = |2x-3| - 4$ meets the coordinates axes. [4]

(ii) Sketch the graph of $y = |2x-3| - 4$. [2]

(iii) Solve the equation $x + 4 = |2x-3|$. [3]

7 It is given that $\mathrm{f}(x) = 2x^3 + 6x^2 + 6x + 5$.

(i) Find the remainder when $\mathrm{f}(x)$ is divided by $(x+1)$. [2]

(ii) Hence, show that $\mathrm{f}(x)$ can be expressed in the form $\mathrm{f}(x) = a(x+1)^3 + b$. [2]

(iii) Find the coordinates of the stationary point(s) and ~~state~~ determine the nature of the stationary point(s). [5]

(iv) Using your answer in part (iii), explain why the graph of $y = \mathrm{f}(x) + k$ will always cut the x-axis only once for all real values of k. [1]

8 **Answer the whole of this question on a piece of graph paper.**

A cuboid box of volume V cm^3 has a height of x cm and a rectangular base of area $(ax^2 + b)$ cm^2. Corresponding values of x and V are shown in the table below.

x	2	4	6	8
V	24	72	168	336

(i) Using suitable variables, draw, on graph paper, a straight line graph and hence estimate the value of each of the constants a and b. [6]

(ii) When a ball is placed in the box, the ball touches every inner side of the box. By drawing a suitable line, find the diameter of the ball. [4]

(iii) Using your values of a and b, calculate the value of x for which the condition in (ii) can be satisfied. [2]

9 (a) Solve the equation $\log_2(2x+1)-\log_4(x+1)=1$. [4]

(b) Given that $(\log_b e)(\log_b a)(\ln a)=16$, express a in terms of b. [4]

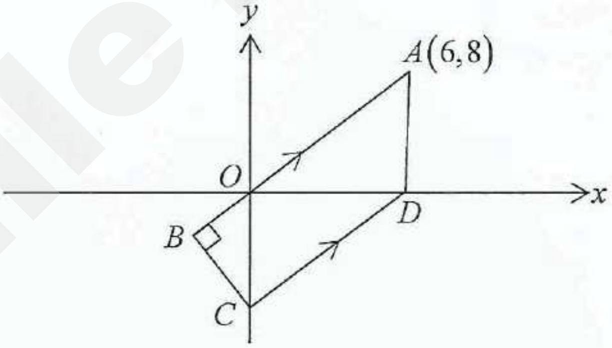
(c) On the same axes, sketch the graphs of $y=e^x$ and $e^y=\frac{1}{x}$.
Hence, determine the number of solutions for $e^x+\ln x=0$. [4]

10 (a) Without using a calculator, find the rational numbers, a and b, for which $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{10}-\sqrt{6}}$ can be expressed as $a\sqrt{2}+b\sqrt{30}$. [4]

(b) (i) Express $5^{2x+3}=5^{x-1}+1$ as a quadratic equation in 5^x and hence find, correct to 2 decimal places, the value of x which satisfies the equation $5^{2x+3}=5^{x-1}+1$. [5]

(ii) Find the range of values of k such that the equation $5^{2x+3}-5^{x-1}+k=0$ has no solution. [3]

11



The diagram shows a trapezium $ABCD$ with vertex $A(6,8)$. The sides AB and DC are parallel. AB cuts through the origin O and its length is given to be 15 units. C and D lies on the y-axis and x-axis respectively.

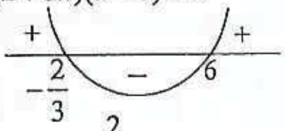
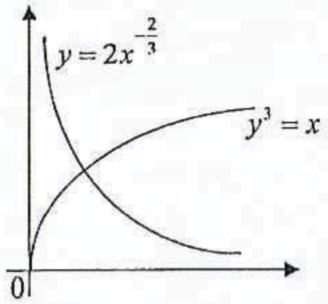
(i) Find the coordinates of vertex B. [4]

(ii) Show that equation of BC is $4y+3x+25=0$. [3]

(iii) Find the coordinates of vertices C and D. [4]

END OF PAPER

2019 4E5N MYE Additional Mathematics Paper 1 Solutions

Qn	Solution
1	$(2+3x)(x-5)>2+3x$ $(2+3x)(x-5)-(2+3x)>0$ $(2+3x)(x-6)>0$  $\therefore x<-\frac{2}{3}$ or $x>6$
2	$\frac{d^2y}{dx^2}=9x+1$ $\frac{dy}{dx}=\int(9x+1)\ dx$ $=\frac{9}{2}x^2+x+c$ When $x=2$, $\frac{dy}{dx}=18$ $\frac{9}{2}(2)^2+2+c=18$ $c=-2$ $\therefore \frac{dy}{dx}=\frac{9}{2}x^2+x-2$ $y=\int\left(\frac{9}{2}x^2+x-2\right)dx$ $=\frac{3}{2}x^3+\frac{1}{2}x^2-2x+d$ When $x=2$, $y=16$, $16=\frac{3}{2}(2)^3+\frac{1}{2}(2)^2-2(2)+d$ $d=6$ $\therefore y=\frac{3}{2}x^3+\frac{1}{2}x^2-2x+6$
3(i)	

3(ii)	$x^{\frac{1}{3}}=2x^{-\frac{2}{3}}$ $x=2$ $y=\sqrt[3]{2}$ $=1.26$ (3s.f.) Coordinates of the point of intersection are $(2,1.26)$
4	$\frac{x}{y}=mx+c------(1)$ $m=\tan 60^\circ=\sqrt{3}$ Sub $m=\sqrt{3}$, $\left(3\sqrt{3},4\right)$ into (1): $4=\sqrt{3}\left(3\sqrt{3}\right)+c$ $c=-5$ $\frac{x}{y}=\sqrt{3}x-5$ $y=\frac{x}{\sqrt{3}x-5}$
5(i)	$\begin{array}{r} 2x^2 \qquad +1 \\ x-3\overline{)2x^3-6x^2+x-3} \\ -\left(2x^3-6x^2\right) \\ \hline x-3 \\ -(x-3) \\ \hline 0 \end{array}$ $\frac{2x^3-6x^2+x-3}{x-3}=2x^2+1$

5(ii)	$\frac{3x^2+21x+5}{2x^3-6x^2+x-3}=\frac{3x^2+21x+5}{(x-3)(2x^2+1)}$ let $\frac{3x^2+21x+5}{(x-3)(2x^2+1)}=\frac{A}{x-3}+\frac{Bx+C}{2x^2+1}$ $3x^2+21x+5=A(2x^2+1)+(Bx+C)(x-3)$ Sub $x=3$: $95=19A$ $A=5$ Sub $x=0$: $5=5-3C$ $C=0$ Sub $x=1$: $29=15-2B$ $B=-7$ $\frac{3x^2+21x+5}{2x^3-6x^2+x-3}=\frac{5}{x-3}-\frac{7x}{2x^2+1}$
6(i)	$y=\frac{1}{2}e^{2x}-7e^x+6x$ $\frac{dy}{dx}=e^{2x}-7e^x+6$ $\frac{d^2y}{dx^2}=2e^{2x}-7e^x$
6(ii)	For stationary points, $\frac{dy}{dx}=0$ $e^{2x}-7e^x+6=0$ let $e^x=u$ $u^2-7u+6=0$ $(u-6)(u-1)=0$ $\Rightarrow u=6$ or $u=1$ $e^x=6$ or $e^x=1$ $x=\ln 6$ or $x=0$
6(iii)	$\left.\frac{d^2y}{dx^2}\right\vert_{x=\ln 6}=2e^{2\ln 6}-7e^{\ln 6}=30>0$ The curve has a minimum point at $x=\ln 6$. $\left.\frac{d^2y}{dx^2}\right\vert_{x=0}=2e^0-7e^0=-5<0$ The curve has a maximum point at $x=0$.
7(i)	$\int(3x^4-5\cos 6x)\,dx=\frac{3}{5}x^5-\frac{5}{6}\sin 6x+c$
7(ii)	$\int(\sin x+\sec^2(5+2\pi x))\,dx=-\cos x+\frac{1}{2\pi}\tan(5+2\pi x)+c$

8(i)	Area of lawn: $lr-\frac{\pi r^2}{4}=360$ $l=\frac{360-\frac{\pi r^2}{4}}{r}$ $l=\frac{360}{r}+\frac{\pi r}{4}$ --------(1) Perimeter of lawn: $P=l+r+(l-r)+\frac{1}{4}(2\pi r)$ $P=2l+\frac{\pi r}{2}$ ------------(2) Sub (1) into (2): $P=2\left(\frac{360}{r}+\frac{\pi r}{4}\right)+\frac{\pi r}{2}$ $P=\frac{720}{r}+\pi r$ (shown)
8(ii)	$\frac{dP}{dr}=-\frac{720}{r^2}+\pi$ $\frac{dP}{dr}=0$ for stationary values $-\frac{720}{r^2}+\pi=0$ $r^2=\frac{720}{\pi}$ $r=15.1$ (3s.f.) (rej. -15.1) $\frac{d^2P}{dr^2}=1440r^{-3}$ $\left.\frac{d^2P}{dr^2}\right\|_{r=15.138}=0.415$ (3s.f.) >0 $\Rightarrow$ Shortest perimeter $l=\frac{360}{15.138}+\frac{\pi(15.138)}{4}$ $=35.7$ (3s.f.) The shortest hedge can be planted when the rectangular space is 15.1cm by 35.7cm.

9(i)	$\dfrac{\tan^2 x-1}{\tan^2 x+1}$ $=\dfrac{\dfrac{\sin^2 x}{\cos^2 x}-1}{\dfrac{\sin^2 x}{\cos^2 x}+1}$ $=\dfrac{\sin^2 x-\cos^2 x}{\sin^2 x+\cos^2 x}$ $=\sin^2 x-\cos^2 x$ $=1-\cos^2 x-\cos^2 x$ $=1-2\cos^2 x$ (shown)
9(ii)	$1-2\cos^2 x=\dfrac{1}{2}$ $\cos^2 x=\dfrac{1}{4}$ $\cos x=\pm\dfrac{1}{2}$ $\alpha=\cos^{-1}\left(\dfrac{1}{2}\right)=\dfrac{\pi}{3}$ $x=\dfrac{\pi}{3},\dfrac{2\pi}{3},\dfrac{4\pi}{3}$

10	$4x^2-6x+3=0$ $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{2}$ ---------(1) $\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)=\frac{3}{4}$ -------(2) From (1): $\frac{\alpha+\beta}{\alpha\beta}=\frac{3}{2}$ ------(3) From (2): $\alpha\beta=\frac{4}{3}$ ----------(4) Sub (3) into (4): $\alpha+\beta=2$ --------(5) $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=(\alpha+\beta)\left((\alpha+\beta)^2-3\alpha\beta\right)$ $=(2)\left((2)^2-3\left(\frac{4}{3}\right)\right)$ $=0$ $(\alpha)^3(\beta)^3=\left(\frac{4}{3}\right)^3$ $=\frac{64}{27}$ The equation is $x^2+\frac{64}{27}=0$.
11(i)	By Pythagora's Theorem, Square of Diagonal $=18^2+(5-\sqrt{3})^2$ $=324+25-10\sqrt{3}+3$ $=352-10\sqrt{3}$
11(ii)	$\frac{\left(1+\sqrt{3}\right)^2}{18\left(5-\sqrt{3}\right)}=\frac{4+2\sqrt{3}}{18\left(5-\sqrt{3}\right)}$ $=\frac{4+2\sqrt{3}}{18\left(5-\sqrt{3}\right)}\left(\frac{5+\sqrt{3}}{5+\sqrt{3}}\right)$ $=\frac{20+10\sqrt{3}+4\sqrt{3}+6}{18(25-3)}$ $=\frac{26+14\sqrt{3}}{396}$ $=\frac{13+7\sqrt{3}}{198}$

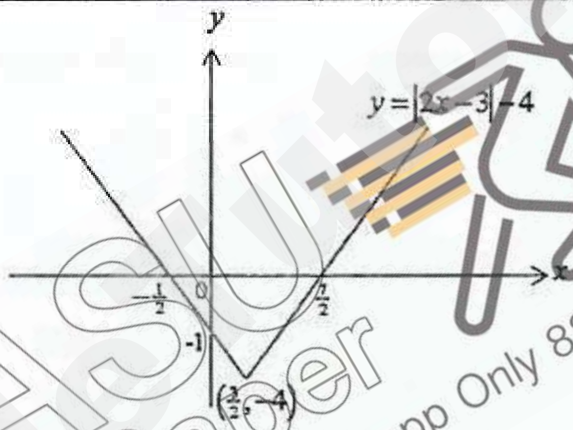
12(i)	$y=x\sqrt{5x^2-6}$ $\frac{dy}{dx}=\left(5x^2-6\right)^{\frac{1}{2}}+x\left(\frac{1}{2}\right)\left(5x^2-6\right)^{-\frac{1}{2}}(10x)$ $=\frac{5x^2-6+5x^2}{\sqrt{5x^2-6}}$ $=\frac{2\left(5x^2-3\right)}{\sqrt{5x^2-6}}$
12(ii)	$\int_2^4\frac{5x^2-3}{\sqrt{5x^2-6}}\,dx$ $=\frac{1}{2}\int_2^4\frac{2\left(5x^2-3\right)}{\sqrt{5x^2-6}}\,dx$ $=\frac{1}{2}\left[x\sqrt{5x^2-6}\right]_2^4$ $=\frac{1}{2}\left[4\sqrt{74}-2\sqrt{14}\right]$ $=13.5$ (3s.f.)
13(i)	$4y+3x=60$ $y=-\frac{3}{4}x+15$ $m_{\text{tangent}}=-\frac{3}{4}$ $m_{\text{normal}}=\frac{4}{3}$ $y-9=\frac{4}{3}(x-8)$ The equation of the normal is $y=\frac{4}{3}x-\frac{5}{3}$. -----(1) $y=4x-7$ -------(2) (1) = (2): $\frac{4}{3}x-\frac{5}{3}=4x-7$ $x=2$ $y=1$ $P(2,1)$
13(ii)	$(x-2)^2+(y-1)^2=r^2$ Sub $(8,9)$: $r^2=100$ Equation of circle is $(x-2)^2+(y-1)^2=100.$

13(iii)	When $x=0,\ 4y+3(0)=60$ $y=15$ $B(0,15)$ Centre of circle $=\left(\frac{2+0}{2},\frac{1+15}{2}\right)$ $=(1,8)$ $BP=\sqrt{(2-0)^2+(1-15)^2}$ $=10\sqrt{2}$ Radius $=5\sqrt{2}$ $(x-1)^2+(y-8)^2=\left(5\sqrt{2}\right)^2$ Equation of circle is $(x-1)^2+(y-8)^2=50$.

CWSS 4E AM 2019 MYE P2 SOLUTION

1(i)	$-1\le \sin x\le 1$ $-1\le -\sin x\le 1$ $-1+3\le -\sin x+3\le 1+3$ $2\le f'(x)\le 4$ Since $f'(x)$ is always more than 0, there are no stationary points on the curve.
1(ii)	$f(x)=\int 3-\sin x\,dx$ $f(x)=3x+\cos x+c$ Since curve passes through origin, $0=3(0)+\cos(0)+c$ $c=-1$ $f(x)=3x+\cos x-1$
2(i)	$y=\frac{2x+1}{x-4}$ $\frac{dy}{dx}=\frac{2(x-4)-1(2x+1)}{(x-4)^2}$ $\frac{dy}{dx}=-\frac{9}{(x-4)^2}$ Since $(x-4)^2\ge 0$ for all real values of x, $\frac{dy}{dx}<0$ for all real values of x. Hence, y is a decreasing function.
2(ii)	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$
	$0.5=-\frac{9}{(2-4)^2}\times\frac{dx}{dt}$
	$\frac{dx}{dt}=-\frac{2}{9}$ units per second
3(i)	$\sin 3x=\sin 2x\cos x+\cos 2x\sin x$ $\sin 3x=2\sin x\cos x\cos x+\left(1-2\sin^2 x\right)\sin x$ $\sin 3x=\sin x\left(2\cos^2 x+1-2\sin^2 x\right)$ $\sin 3x=\sin x\left[2\left(1-\sin^2 x\right)+1-2\sin^2 x\right]$ $\sin 3x=3\sin x-4\sin^3 x$

3(ii)	$\sin 3x = \sin^2 x$ $3\sin x - 4\sin^3 x = \sin^2 x$ $\sin x(4\sin x - 3)(\sin x + 1) = 0$ $\sin x = 0,\ \sin x = \frac{3}{4},\ \sin x = -1$ $x = \pi,\ x = 0.848(3sf),\ x = \frac{3\pi}{2}$
4(i)	$R = \sqrt{(\sqrt{3})^2 + 1^2}$ $R = 2$ $\alpha = \tan^{-1}\frac{1}{\sqrt{3}}$ $\alpha = \frac{\pi}{6}$ $\sqrt{3}\sin\theta + \cos\theta = 2\sin\left(\theta + \frac{\pi}{6}\right)$
4(ii)	$\int_0^\pi \sqrt{3}\sin x + \cos x \, dx$ $= \int_0^\pi 2\sin\left(x + \frac{\pi}{6}\right) dx$ $= 2\left[-\cos\left(x + \frac{\pi}{6}\right)\right]_0^\pi$ $= 2\left\{\left[-\cos\left(\pi + \frac{\pi}{6}\right)\right] - \left[-\cos\left(0 + \frac{\pi}{6}\right)\right]\right\}$ $= 2\left[-\left(-\frac{\sqrt{3}}{2}\right) - \left(-\frac{\sqrt{3}}{2}\right)\right]$ $= 2\sqrt{3}$
5(i)	$\left(kx - \frac{1}{x^3}\right)^7 = \binom{7}{r}(kx)^{7-r}\left(-\frac{1}{x^3}\right)^r + \ldots$ $\left(kx - \frac{1}{x^3}\right)^7 = \binom{7}{r}(k)^{7-r}(-1)^r x^{7-r}(x^{-3})^r + \ldots$ $\left(kx - \frac{1}{x^3}\right)^7 = \binom{7}{r}(k)^{7-r}(-1)^r x^{7-r-3r} + \ldots$ $\left(kx - \frac{1}{x^3}\right)^7 = \binom{7}{r}(k)^{7-r}(-1)^r x^{7-4r} + \ldots$ Since the power of x is $7 - 4r = 2(3 - 2r) + 1$will always be odd, there are no even powers of x for this expansion.

5(ii)	$\binom{7}{2}(k)^{7-2}(-1)^2=3\binom{7}{1}(k)^{7-1}(-1)^1$ $21(k)^5=-3(7)(k)^6$ $k=-1$
6(i)	When $x=0$, $y=\lvert 0-3\rvert-4$ $y=-1$ When $y=0$, $0=\lvert 2x-3\rvert-4$ $\lvert 2x-3\rvert=4$ $2x-3=4$ or $2x-3=-4$ $x=\frac{7}{2}$, $x=-\frac{1}{2}$ Points are $(0,-1)$, $\left(-\frac{1}{2},0\right)$ and $\left(\frac{7}{2},0\right)$.
6(ii)	
6(iii)	$x+4=\lvert 2x-3\rvert$ $x+4=3-2x$ or $x+4=2x-3$ $3x=-1$ or $x=2x-3-4$ $x=-\frac{1}{3}$, $x=7$
7(i)	$f(-1)=2(-1)^3+6(-1)^2+6(-1)+5$ $f(-1)=3$
7(ii)	$f(x)=(x+1)(2x^2+4x+2)+3$ $f(x)=2(x+1)^3+3$

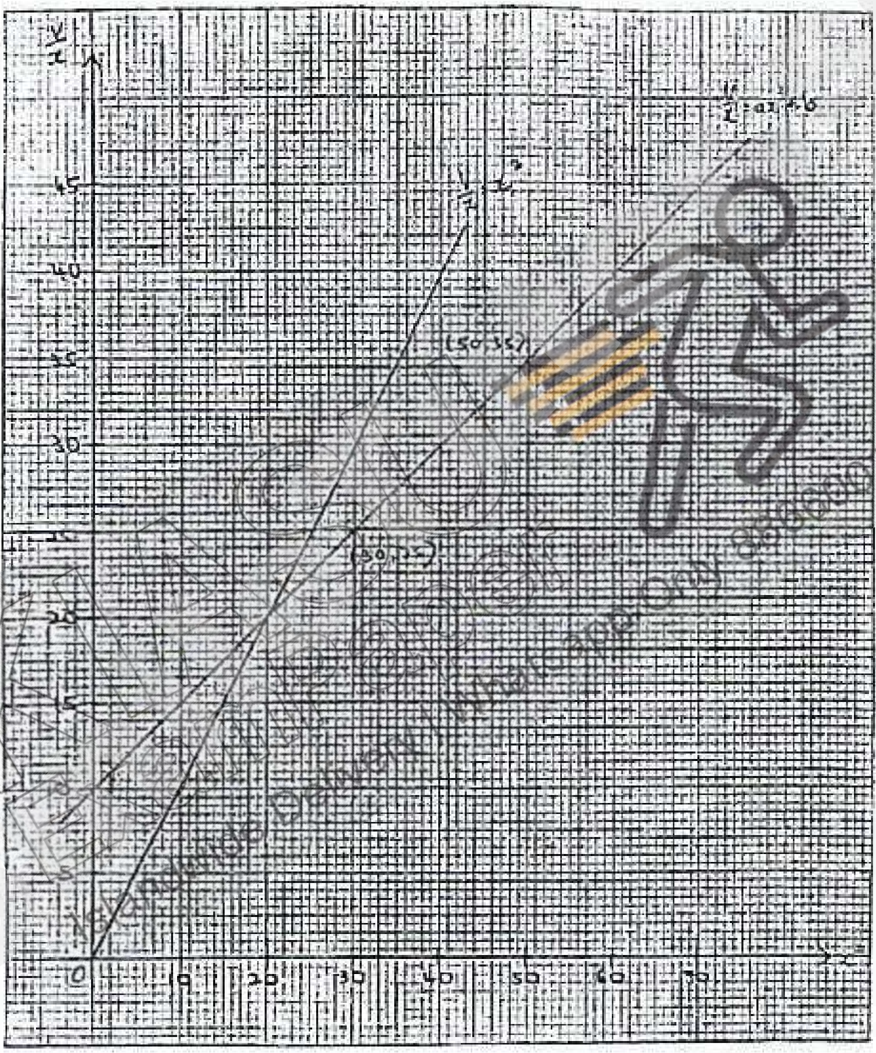
<table>
<tr><td>7(iii)</td><td>$f'(x)=6x^2+12x+6$ or $f'(x)=6(x+1)^2$
$6x^2+12x+6=0$ or $(x+1)^2=0$
$x^2+2x+1=0$
$(x+1)^2=0$
$x=-1$
$y=2(-1+1)^3+3$
$y=3$
By first derivative test,
<table>
<tr><td>x</td><td>-1^-</td><td>-1</td><td>-1^+</td></tr>
<tr><td>$\frac{dy}{dx}$</td><td>positive</td><td>0</td><td>positive</td></tr>
<tr><td>shape</td><td>/</td><td>—</td><td>/</td></tr>
</table>
$(-1,3)$ is a point of inflexion.</td></tr>
<tr><td>7(iv)</td><td>As the graph is an increasing function with no turning points, it will always only cut the x-axis only once.</td></tr>
</table>

8(i)

$V = x\left(ax^2 + b\right)$

$\frac{V}{x} = ax^2 + b$

x^2	4	16	36	64
$\frac{V}{x}$	12	18	28	42

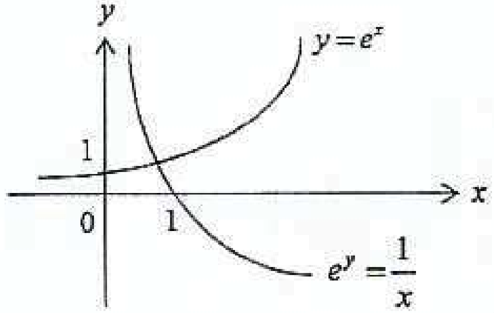


From the graph

$a = \frac{35-25}{50-30}$

$a = \frac{1}{2}$ and $b = 10$

8(ii)	$V=x^3$ Add the line $\frac{V}{x}=x^2$ onto graph From the graph, $x^2=20$ Diameter of the ball $=\sqrt{20}=2\sqrt{5}$ cm
8(iii)	$x^3=x\left(\frac{1}{2}x^2+10\right)$ $\frac{1}{2}x^3-10x=0$ $x=0$ or $x^2=20$ Since $x\neq 0$, the diameter of the ball $=\sqrt{20}=2\sqrt{5}$ cm
9(a)	$\log_2(2x+1)-\log_4(x+1)=1$ $\log_2(2x+1)-\frac{\log_2(x+1)}{\log_2 4}=\log_2 2$ $\log_2(2x+1)-\frac{1}{2}\log_2(x+1)=\log_2 2$ $\log_2\frac{(2x+1)}{\sqrt{x+1}}=\log_2 2$ $2x+1=2\sqrt{x+1}$ $4x^2+4x+1=4x+4$ $4x^2=3$ $x=\frac{\sqrt{3}}{2}$ or $x=-\frac{\sqrt{3}}{2}$ (reject)
9(b)	$(\log_b e)(\log_b a)(\ln a)=16$ $\left(\frac{\ln e}{\ln b}\right)\left(\frac{\ln a}{\ln b}\right)(\ln a)=16$ $(\ln a)^2=16(\ln b)^2$ $\ln a=4\ln b$ or $\ln a=-4\ln b$ $a=b^4$ or $a=\frac{1}{b^4}$

9(c)	$y=e^x$ $e^y=\frac{1}{x}$ $y\ln e=\ln x^{-1}$ $y=-\ln x$  $e^x+\ln x=0$ $e^x=-\ln x$ From the graph, there is only one point of intersection, hence there is only one solution.
10(a)	$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{10}-\sqrt{6}}=\frac{(\sqrt{5}+\sqrt{3})(\sqrt{10}+\sqrt{6})}{(\sqrt{10})^2-(\sqrt{6})^2}$ $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{10}-\sqrt{6}}=\frac{\sqrt{50}+\sqrt{30}+\sqrt{30}+\sqrt{18}}{10-6}$ $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{10}-\sqrt{6}}=\frac{5\sqrt{2}+2\sqrt{30}+3\sqrt{2}}{4}$ $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{10}-\sqrt{6}}=2\sqrt{2}+\frac{1}{2}\sqrt{30}$
10(b)(i)	$5^{2x+3}=5^{x-1}+1$ $5^3(5^x)^2=\frac{5^x}{5}+1$ Let $y=5^x$ $625y^2-y-5=0$ $y=0.090246$ and $y=-0.088646$ $5^x=0.090246$ and $5^x=-0.088646$ (reject) $x=\frac{\ln 0.090246}{\ln 5}$ $x=-1.49$ (2 dp)

10(b)(ii)	Let $y=5^x$ $125y^2-\frac{y}{5}+k=0$ $625y^2-y+5k=0$ $(-1)^2-4(625)(5k)<0$ $1-12500k<0$ $k>\frac{1}{12500}$
11(i)	Length of $AO=\sqrt{6^2+8^2}=10$ Length of $OB=15-10=5$ $\sqrt{x^2+y^2}=5$ and $\frac{y}{x}=\frac{4}{3}$ By substitution or by Pythagoras triplets, $x=-3$ and $y=-4$ $B(-3,-4)$
11(ii)	Gradient of $BC=\frac{-1}{\frac{4}{3}}$ $-4=-\frac{3}{4}(-3)+c$ Equation of BC is $y=-\frac{3}{4}x-\frac{25}{4}$ $4y+3x+25=0$
11(iii)	From equation of BC, C is y-int. Hence, $C\left(0,-\frac{25}{4}\right)$ Equation of CD is $y=\frac{4}{3}x-\frac{25}{4}$ since AB is parallel to DC. $0=\frac{4}{3}x-\frac{25}{4}$ $x=\frac{75}{16}$ $D\left(\frac{75}{16},0\right)$

Class	Index Number	Name

新加坡海星中学

MARIS STELLA HIGH SCHOOL
MID-YEAR EXAMINATION
SECONDARY FOUR

ADDITIONAL MATHEMATICS
Paper 1

4047/01
10 May 2019
2 hours

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use
80

This document consists of **17** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 A triangle has an area of $\left(58+8\sqrt{5}\right)$ cm^2 and a height of $\left(7+3\sqrt{5}\right)$ cm. Without using a calculator, find the exact length of its base, expressing in the form $a+b\sqrt{5}$, where a and b are integers. [4]

2 **(i)** On the same diagram, sketch the curves $y = 9x^{-\frac{1}{2}}$ and $y^2 = 4x$. [2]

(ii) Find the coordinates of the point(s) of intersection of the two curves. [2]

3 The equation of a curve is $y = 2xe^{x-k}$, where k is a constant. The curve passes through the point $(5,10)$.

(i) Find the value of k. [2]

(ii) For what values of x is y an increasing function of x? [3]

4 Express $\frac{16x^2-9x+18}{x^3+3x^2}$ in partial fractions. [5]

5 The function f is given by $f(x) = -3\sin\frac{x}{2} + 2$.

(i) State the amplitude and period of f. [2]

(ii) Sketch the graph of $y = f(x)$ for $0 \le x \le 4\pi$. By drawing a suitable straight line on the same axes, state the number of solutions to the equation $4\pi - x - 6\pi\sin\frac{x}{2} = 0$ for $0 \le x \le 4\pi$. [5]

6 **(i)** Given that $\cos(A+B)=3\cos(A-B)$ and $\tan A=-\frac{5}{2}$, find the value of $\cot B$. [3]

(ii) Prove that $\frac{1+\tan^2 x}{1-\tan^2 x}=\sec 2x$. [3]

7 The roots of the quadratic equation $2x^2 + x + 6 = 0$ are α and β.

(i) Express $\alpha^2 - \alpha\beta + \beta^2$ in terms of $(\alpha + \beta)$ and $\alpha\beta$. [1]

(ii) Form a quadratic equation whose roots are α^3 and β^3. [5]

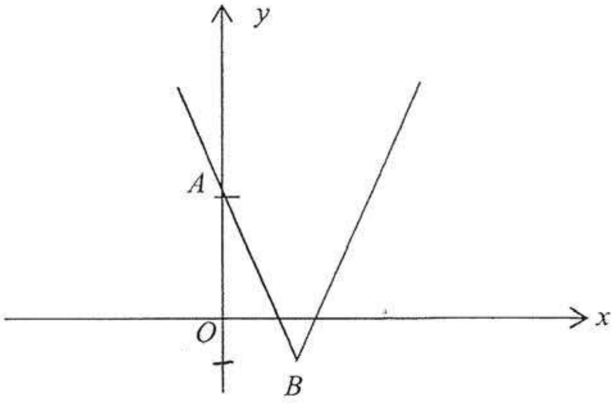
8 An antique grandfather clock manufactured using the finest wood in 1850 had an initial value \$2000. The clock appreciated in its value such that its value $\$V$ can be modelled by the equation $V = 20000 - Ae^{kt}$, where t is the number of years after its manufacture date.

(i) Find the value of A. [2]

(ii) In the year 1880, the clock reached five times its initial value. Show that $k = -0.01959$ correct to 4 significant figures. [3]

(iii) Explain why the value of the clock will not exceed \$20000. [2]

9 The diagram shows the graph of $y = |6 - 2x| - 1$.



(i) Find the coordinates of A and of B. [2]

(ii) By solving the equation $|6 - 2x| = 3x + 1$, find the x-coordinate of the point(s) of intersection between the graphs $y = |6 - 2x| - 1$ and $y = 3x$. [3]

(iii) State the range of values of m for the equation $|6 - 2x| = mx + 1$ to have no solution. [2]

10 A circle passes through the points $P(0,8)$ and $Q(8,12)$. The y-axis is a tangent to the circle at P.

(i) Find the equation of the circle. [5]

The tangent to the circle at Q intersects the x-axis and y-axis at A and B respectively.

(ii) Find the ratio of $AQ:QB$. [3]

11 **(i)** Expand $(1-2x)^9$ in ascending powers of x up to the term in x^3. [2]

(ii) Find the value of k, given that the coefficient of x in the expansion of $\left(3x+\frac{1}{kx^2}\right)(1-2x)^9$ is -53. [3]

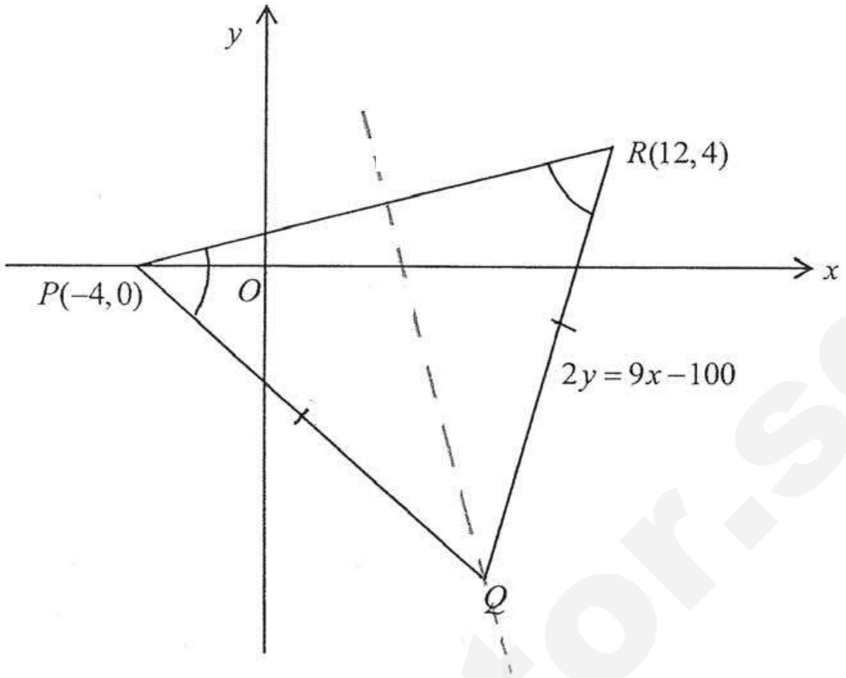
12 The equation of a curve is given by $y = \ln\sqrt{\dfrac{5x}{9x+4}}$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, expressing it as a single fraction. [3]

(ii) Find the rate at which x is changing when the graph crosses the x-axis, given that y is increasing at a rate of 0.3 units per second. [4]

Solutions to this question by accurate drawing will not be accepted.

13



The diagram, which is not drawn to scale, shows a triangle PQR in which $PQ=QR$. The coordinates of the points P and R are $(-4,0)$ and $(12,4)$ respectively.

(i) Find the equation of the perpendicular bisector of PR. [3]

The equation of the line QR is $2y = 9x - 100$.

(ii) Find the coordinates of Q. [2]

(iii) Find the coordinates of S if $PQRS$ forms a rhombus.
Hence, or otherwise, find the area of the rhombus $PQRS$. [4]

End of paper

Class	Index Number	Name

新加坡海星中学
MARIS STELLA HIGH SCHOOL
MID-YEAR EXAMINATION
SECONDARY FOUR

ADDITIONAL MATHEMATICS
Paper 2

4047/02
15 May 2019
2 hours

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
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Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use
80

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Binomial expansion

$$(a+b)^n = a^n + \begin{pmatrix} n \\ 1 \end{pmatrix} a^{n-1}b + \begin{pmatrix} n \\ 2 \end{pmatrix} a^{n-2}b^2 + \ldots + \begin{pmatrix} n \\ r \end{pmatrix} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and

$$\begin{pmatrix} n \\ r \end{pmatrix} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
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Formulae for ΔABC

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$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 The polynomial $f(x)=2x^3+ax^2+bx+8$, where a and b are constants, has a factor $(x+2)$ and leaves a remainder of 10 when divided by $(2x-1)$.

(i) Find the value of a and of b. [4]

(ii) Using the values of *a* and *b* found in part **(i)**, explain why the equation $\mathrm{f}(x)=0$ has only one real root. Find this root. [4]

(iii) Hence, solve $x^3+3x^2+4x+32=0$. [2]

2 **(a)** Find the range of values of k for which $(k-3)x^2+4x+k$ is always positive for all real values of x. [4]

(b) Show that the roots of the equation $6x^2+4(m-1)=2(x+m)$ are real if $m \leq 2\frac{1}{12}$. [3]

Page 6 missing - to copy questions from answers

(c) Solve the equation $\log_3(2x-1)-\frac{1}{2}\log_3(x^2+2)=\log_{25}5$. [5]

4 In a Science experiment, a container of liquid was heated to a temperature of K °C.

It was then left to cool in a chiller such that its temperature, T °C, t minutes after removing the heat, is given by $T = Ke^{-qt}$, where q is a constant.

Measured values of t and T are given in the following table.

t (minutes)	2	4	7	10	12
T °C	71.1	57.0	40.8	29.3	23.4

(i) Using a scale of 1 cm to 1 unit on the t-axis and 4 cm to 1 unit on the ln T- axis, plot ln T against t and draw a straight line graph. [2]

(ii) Use the graph to estimate the value of K and of q. [4]

(iii) Estimate the temperature of the liquid 8 minutes after it was left to cool. [2]

5 **(a)** **(i)** Prove that $\frac{\sin x}{\sec x+1}+\frac{\sin x}{\sec x-1}=2\cot x$. [4]

(ii) Hence find, for $0 \leq x \leq 4$, the exact solutions of the equation

[3]

(b) Given that θ is obtuse and that $\sin\theta = \frac{1}{\sqrt{3}}$, express, **without the use of a calculator**, $\frac{1}{\sin\theta - \cos\theta}$ in the form $\sqrt{a} - \sqrt{b}$ where a and b are integers. [4]

6 The equation of a curve is $y = \frac{a}{x} + bx - 1$, where a and b are constants. The normal to the curve at the point $Q(1, -1)$ is parallel to the line $4y - x = 20$. This normal meets the curve again at point P.

(i) Find the value of a and of b. [5]

(ii) Find the coordinates of point P. [3]

7 The equation of a curve is $y=\frac{x^2}{x-1}$, where $x \neq 1$.

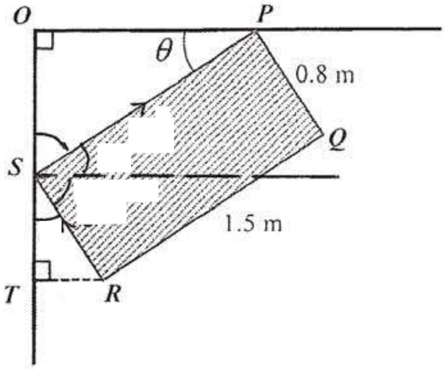
(i) Obtain an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [4]

(ii) Find the coordinates of the stationary points of the curve and determine their nature. [4]

8 **(a)** Differentiate $\cot^4\left(\frac{\pi}{2}-2x\right)$ with respect to x. [3]

(b) Given that a curve has the equation $y=3\sin 2x-\cos x$, find the gradient of the curve when $x=\frac{\pi}{3}$, leaving your answer in exact form. [3]

9



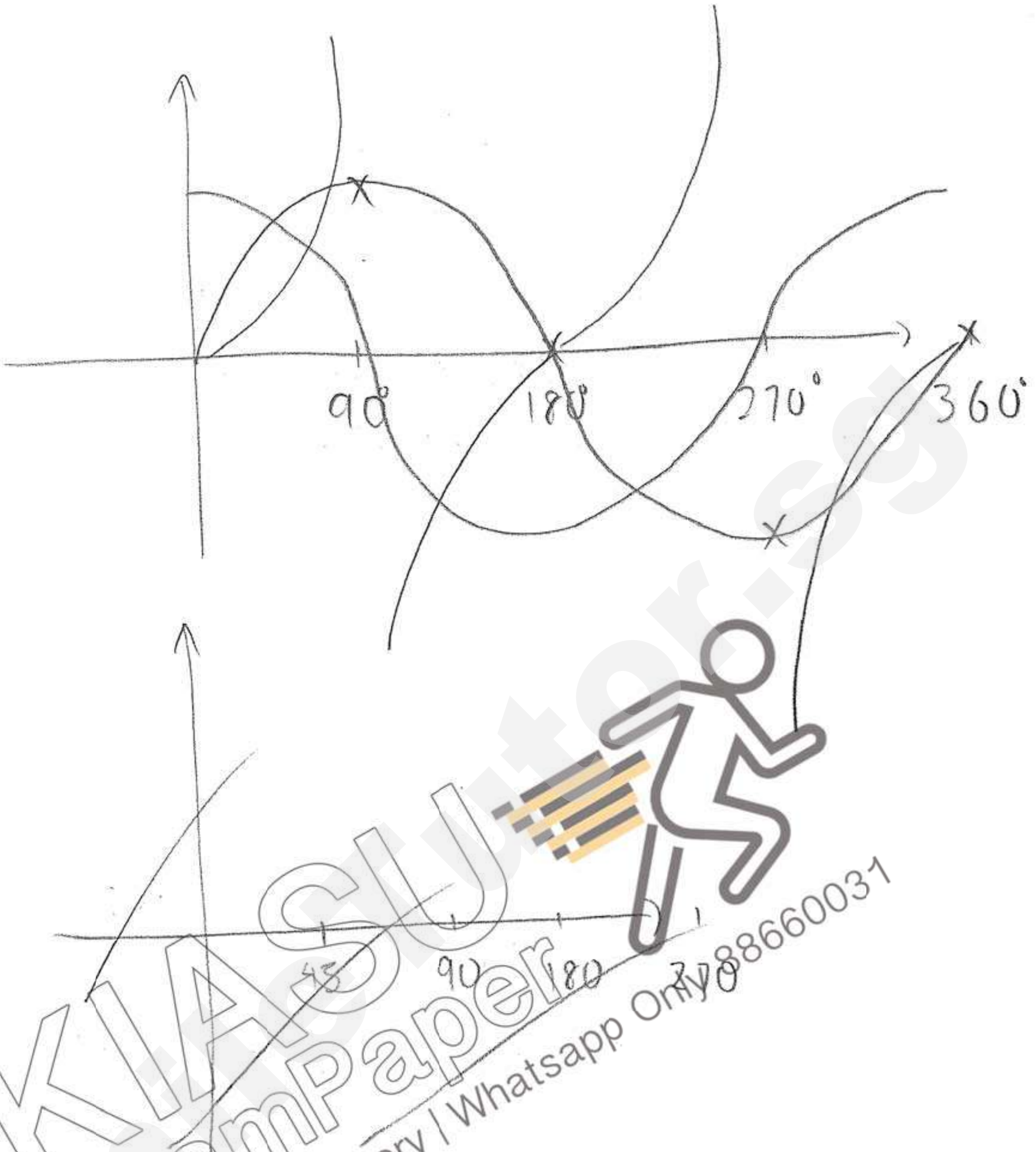
The diagram shows the top view of a rectangular desk, $PQRS$, in a corner of a room. The desk has a length of 1.5 m and width 0.8 m, $\angle POS = \angle STR = 90°$ and $\angle OPS = \theta$.

(i) Show that $OT = (1.5\sin\theta + 0.8\cos\theta)$ m. [3]

(ii) Express OT in the form $R\sin(\theta+\alpha)$, where $R > 0$ and α is acute. [3]

(iii) Given that θ can vary, find the maximum value of OT and the corresponding value of θ. [3]

End of paper


90°
180°
270°
360°
45
90
180
270

2019 Additional Mathematics Paper 1 Sec 4 MYE (Solutions)

1 A triangle has an area of $(58+8\sqrt{5})$ cm^2 and a height of $(7+3\sqrt{5})$ cm. Without using a calculator, find the exact length of its base, expressing in the form $+b\sqrt{5}$, where a and b are integers. [4]

1 Length of the base

$$=\frac{2(58+8\sqrt{5})}{7+3\sqrt{5}}$$

$$=\frac{116+16\sqrt{5}}{7+3\sqrt{5}}\times\frac{7-3\sqrt{5}}{7-3\sqrt{5}}$$

$$=\frac{812-348\sqrt{5}+112\sqrt{5}-240}{49-45}$$

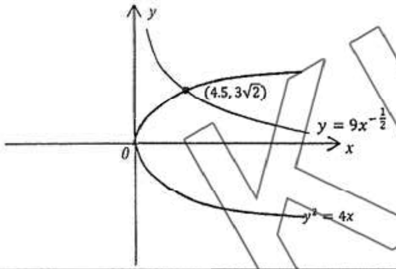
$$=\frac{572-236\sqrt{5}}{4}$$

$$=143-59\sqrt{5}\ \text{cm}$$

2 **(i)** On the same diagram, sketch the curves $y=9x^{-\frac{1}{2}}$ and $y^2=4x$. [2]

(ii) Find the coordinates of the point(s) of intersection of the two curves. [2]

2(i)



(ii)

$y=9x^{-\frac{1}{2}}$ -----(1)

$y^2=4x$ -----(2)

Sub (1) into (2): $(9x^{-\frac{1}{2}})^2=4x$

$81x^{-1}=4x$

$x^2=\frac{81}{4}$

$x=\frac{9}{2}$ or $-\frac{9}{2}$ (reject)

When $x=\frac{9}{2}$, $y=9(\frac{9}{2})^{-\frac{1}{2}}$

$=9\div\frac{\sqrt{9}}{\sqrt{2}}$

$=3\sqrt{2}$

The coordinates of the point of intersection is $(4\frac{1}{2}, 3\sqrt{2})$.

3 The equation of a curve is $y=2xe^{x-k}$, where k is a constant. The curve passes through the point $(5, 10)$.

(i) Find the value of k. [2]

(i) For what values of x is y an increasing function of x. [3]

3(i)

$y=2xe^{x-k}$

When $x=5, y=10,$

$10=2(5)e^{5-k}$

$1=e^{5-k}$

$e^0=e^{5-k}$

$5-k=0$

$k=5$

(ii)

$y=2xe^{x-5}$

$\frac{\mathrm{dy}}{\mathrm{dx}}=2xe^{x-5}+2e^{x-5}$

$=2e^{x-5}(x+1)$

For y to be an increasing function of x,

$\frac{\mathrm{dy}}{\mathrm{dx}}>0$

Since $2e^{x-5}>0$, $x+1>0$

$x>-1$

4 Express $\frac{16x^2-9x+18}{x^3+3x^2}$ in partial fractions. [5]

4

$$\frac{16x^2-9x+18}{x^3+3x^2}=\frac{16x^2-9x+18}{x^2(x+3)}$$

Let $\frac{16x^2-9x+18}{x^3+3x^2}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+3}$

$16x^2-9x+18=Ax(x+3)+B(x+3)+Cx^2$

Let $x=-3$, $16(-3)^2-9(-3)+18=9C$

$9C=189$

$C=21$

Let $x=0$, $18=3B$

$B=6$

Comparing x^2 term, $16x^2=Ax^2+Cx^2$

$A+C=16$

$A+21=16$

$A=-5$

$$\frac{16x^2-9x+18}{x^3+3x^2}=\frac{-5}{x}+\frac{6}{x^2}+\frac{21}{x+3}$$

5 The function f is given by $f(x) = -3\sin\frac{x}{2} + 2$.

(i) State the amplitude and period of f. [2]

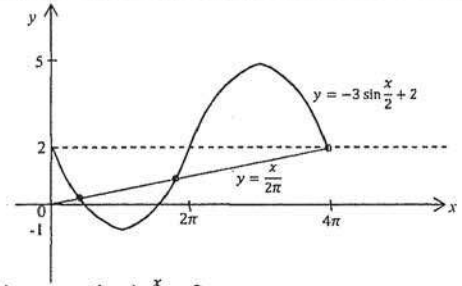
(ii) Sketch the graph of $y = f(x)$ for $0 \le x \le 4\pi$. By drawing a suitable straight line on the same axes, state the number of solutions to the equation $4\pi - x - 6\pi\sin\frac{x}{2} = 0$ for $0 \le x \le 4\pi$. [5]

5(i) Amplitude = 3

Period $= 2\pi \div \frac{1}{2}$

$= 4\pi$

(ii)



$4\pi - x - 6\pi\sin\frac{x}{2} = 0$

$\frac{4\pi - x - 6\pi\sin\frac{x}{2}}{2\pi} = \frac{0}{2\pi}$

$2 - \frac{x}{2\pi} - 3\sin\frac{x}{2} = 0$

$-3\sin\frac{x}{2} + 2 = \frac{x}{2\pi}$

Since there are 3 points of intersection between the graphs $y = -3\sin\frac{x}{2} + 2$ and $y = \frac{x}{2\pi}$, there are 3 solutions.

6 (i) Given that $\cos(A + B) = 3\cos(A - B)$ and $\tan A = -\frac{5}{2}$, find the value of $\cot B$. [3]

(ii) Prove that $\frac{1+\tan^2 x}{1-\tan^2 x} = \sec 2x$. [3]

6(i) $\cos(A + B) = 3\cos(A - B)$

$\cos A\cos B - \sin A\sin B = 3(\cos A\cos B + \sin A\sin B)$

$\cos A\cos B - \sin A\sin B = 3\cos A\cos B + 3\sin A\sin B$

$-4\sin A\sin B = 2\cos A\cos B$

$\frac{-4\sin A\sin B}{\cos A\cos B} = \frac{2\cos A\cos B}{\cos A\cos B}$

$-4\tan A\tan B = 2$

Sub $\tan A = -\frac{5}{2}$,

$-4\left(-\frac{5}{2}\right)\tan B = 2$

$10\tan B = 2$

$\tan B = \frac{1}{5}$

$\cot B = 5$

(ii) $\text{LHS} = \frac{1+\tan^2 x}{1-\tan^2 x} = \frac{1+\frac{\sin^2 x}{\cos^2 x}}{1-\frac{\sin^2 x}{\cos^2 x}}$

$= \frac{1+\frac{\sin^2 x}{\cos^2 x}}{1-\frac{\sin^2 x}{\cos^2 x}} \times \frac{\cos^2 x}{\cos^2 x}$

$= \frac{\cos^2 x+\sin^2 x}{\cos^2 x-\sin^2 x}$

$= \frac{1}{\cos 2x}$

$= \sec 2x$

$= RHS\ (Proven)$

7 The roots of the quadratic equation $2x^2 + x + 6 = 0$ are α and β.

(i) Express $\alpha^2 - \alpha\beta + \beta^2$ in terms of $(\alpha + \beta)$ and $\alpha\beta$. [1]

(ii) Form a quadratic equation whose roots are α^3 and β^3. [5]

7(i) $\alpha^2 - \alpha\beta + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta - \alpha\beta$

$= (\alpha + \beta)^2 - 3\alpha\beta$

(ii) Sum of roots: $\alpha + \beta = -\frac{1}{2}$

Product of roots: $\alpha\beta = \frac{6}{2} = 3$

For an equation whose roots are α^3 and β^3,

Sum of roots: $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$

$= \left(-\frac{1}{2}\right)\left[\left(-\frac{1}{2}\right)^2 - 3(3)\right]$

$= \frac{35}{8}$

Product of roots: $\alpha^3\beta^3 = (3)^3$

$= 27$

The equation is $x^2 - \left(\frac{35}{8}\right)x + 27 = 0$

or $8x^2 - 35x + 216 = 0$

8 An antique grandfather clock manufactured using the finest wood in 1850 was valued at \$2000. The clock appreciated in its value such that its value \$$V$ can be modelled by the equation $V = 20000 - Ae^{kt}$, where t was the number of years after its manufacture date.

(i) Find the value of A. [2]

(ii) In the year 1880, the clock reached five times its initial value. Show that $k = -0.01959$ correct to 4 significant figures. [3]

(iii) Explain why the value of the clock will not exceed \$20000. [2]

8(i) When $t = 0$, $V = 2000$

$2000 = 20000 - Ae^{k(0)}$

$A = 20000 - 2000$

$= 18000$

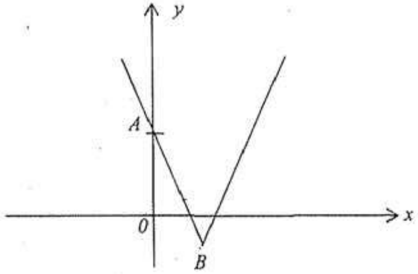
(ii) $V = 20000 - 18000e^{kt}$

In the year 1880, $t = 30$, $V = 5(2000)$

$20000 - 18000e^{30k} = 10000$

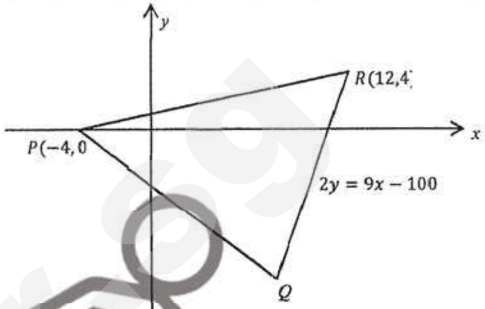
$-18000e^{30k} = -10000$

$e^{30k} = \frac{5}{9}$

	$\ln e^{30k} = \ln\frac{5}{9}$ $30k = \ln\frac{5}{9}$ $k = \frac{\ln\frac{5}{9}}{30}$ $= -0.019592 \ldots$ or 3…. $= -0.01959\ (4\ sf)(shown)$
(iii)	For all values of $t \geq 0$, $e^{-0.01959t} > 0$ $-18000e^{-0.01959t} < 0$ $20000 - 18000e^{-0.01959t} < 20000$ $V < 20000$ Hence the value of the clock will not exceed \$20000.
9	The diagram shows the graph of $y = \|6-2x\| - 1$.  (i) Find the coordinates of A and of B. [2] (ii) By solving the equation $\|6-2x\| = 3x+1$, find the x-coordinate of the point(s) of intersection between the graphs $y = \|6-2x\| - 1$ and $y = 3x$. [3] (iii) State the range of values of m for the equation $\|6-2x\| = mx+1$ to have no solution. [2]
9(i)	When $x = 0$, $y = \|6-2(0)\| - 1$ $= 5$ $A(0,5)$ At B, y is minimum when $6 - 2x = 0$ $x = 3$ $y = -1$ $B(3,-1)$
(ii)	$\|6-2x\| = 3x+1$ $6-2x = 3x+1$ *or* $6-2x = -(3x+1)$ $-5x = -5$ $\quad$ $x = -7$ (reject) $x = 1$
(iii)	For $\|6-2x\| = mx+1$ to have no solutions $-2 \leq m < -\frac{1}{3}$

10	A circle passes through the points $P(0,8)$ and $Q(8,12)$. The y-axisis a tangent to the circle at P. (i) Find the equation of the circle. [5] The tangent to the circle at Q intersects the x-axis and y-axis at A and B respectively. (ii) Find the ratio of $AQ:QB$. [3]
10(i)	$y -$ coordinate of centre of circle $= 8$ Midpoint of $PQ = \left(\frac{0+8}{2}, \frac{8+12}{2}\right)$ $= (4, 10)$ Gradient of $PQ = \frac{12-8}{8-0}$ $= \frac{1}{2}$ Gradient of perpendicular bisector of $PQ = -2$ Equation of perpendicular bisector of PQ: $y - 10 = -2(x-4)$ $y = -2x + 18$ Sub $y = 8$, $\quad 8 = -2x + 18$ $x = 5$ Centre of the circle is $(5, 8)$. $Radius^2 = (5-0)^2$ $= 25$ The equation of the circle is $(x-5)^2 + (y-8)^2 = 25$
(ii)	Gradient of line from Q to centre of circle$= \frac{12-8}{8-5}$ $= \frac{4}{3}$ Equation of tangent at $Q(8,12)$: $y - 12 = -\frac{3}{4}(x-8)$ $y = -\frac{3}{4}x + 18$ When $y = 0$, $x = 24$ $A(24,0)$ When $x = 0$, $y = 18$. $B(0,18)$ For the points $A(24,0)$, $Q(8,12)$ and $B(0,18)$, $AQ:QB = 24-8 : 8-0$ (Comparing difference in x or y-coordinates) $= 2 : 1$
11	(i) Expand $(1-2x)^9$ in ascending powers of x up to the term in x^3. [2] (ii) Find the value of k, given that the coefficient of x in the expansion of $\left(3x + \frac{1}{kx^2}\right)(1-2x)^9$ is -53. [3]
11(i)	$(1-2x)^9 = \binom{9}{0}(-2x)^0 + \binom{9}{1}(-2x)^1 + \binom{9}{2}(-2x)^2$ $+ \binom{9}{3}(-2x)^3 + \cdots$ $= 1 - 18x + 144x^2 - 672x^3 + \cdots$

(ii)	$\left(3x+\frac{1}{kx^2}\right)(1-2x)^9$ $=\left(3x+\frac{1}{kx^2}\right)(1-18x+144x^2-672x^3+\cdots)$ Term in $x = 3x(1)+\frac{1}{kx^2}(-672x^3)$ coefficient of $x = -53$ $3-\frac{672}{k}=-53$ $k=12$
12	The equation of a curve is given by $y=\ln\sqrt{\frac{5x}{9x+4}}$. **(i)** Find $\frac{dy}{dx}$, expressing it as a single fraction. [3] **(ii)** Find the rate at which x is changing when the graph crosses the x-axis, given that y is increasing at a rate of 0.3 units per second. [4]
(i)	$y=\ln\sqrt{\frac{5x}{9x+4}}$ $=\frac{1}{2}[\ln 5x-\ln(9x+4)]$ $\frac{dy}{dx}=\frac{1}{2}\left[\frac{5}{5x}-\frac{9}{9x+4}\right]$ $=\frac{1}{2}\left[\frac{9x+4}{x(9x+4)}-\frac{9x}{x(9x+4)}\right]$ $=\frac{1}{2}\left[\frac{4}{x(9x+4)}\right]$ $=\frac{2}{x(9x+4)}$
(ii)	Let $y=0$, $\ln\sqrt{\frac{5x}{9x+4}}=0$ $\frac{1}{2}[\ln 5x-\ln(9x+4)]=0$ $\ln 5x-\ln(9x+4)=0$ $\ln 5x=\ln(9x+4)$ $5x=9x+4$ $x=-1$ $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $0.3=\frac{2}{x(9x+4)}\times\frac{dx}{dt}$ When $x=-1$, $\frac{dx}{dt}=0.3\div\frac{2}{(-1)(-9+4)}$ $=\frac{3}{4}$ x is increasing at a rate of $\frac{3}{4}$ units per second.
13	**Solutions to this question by accurate drawing will not be accepted.** The diagram, which is not drawn to scale, shows a triangle PQR in which $PQ=QR$. The coordinates of the points P and R are $(-4,0)$ and $(12,4)$ respectively. **(i)** Find the equation of the perpendicular bisector of PR. [3] The equation of the line QR is $2y=9x-100$. **(ii)** Find the coordinates of Q. [2] **(iii)** Find the coordinates of S if $PQRS$ forms a rhombus. Hence, or otherwise, find the area of rhombus $PQRS$. [4]

	
(i)	Midpoint of $PR=\left(\frac{-4+12}{2},\frac{0+4}{2}\right)$ $=(4,2)$ Gradient of $PR=\frac{4-0}{12-(-4)}$ $=\frac{1}{4}$ Gradient of perpendicular bisector of $PR=-4$ Equation of perpendicular bisector of PR: $y-2=-4(x-4)$ $y=-4x+18$
(ii)	$y=-4x+18$ ------(1) $2y=9x-100$-----(2) Sub (1) into (2): $2(-4x+18)=9x-100$ $-8x+36=9x-100$ $x=8$ $y=-14$ The coordinates of Q are $(8,-14)$.
(iii)	Let the coordinates of S be (x_S,y_S). If $PQRS$ forms a rhombus, then Midpoint of QS = Midpoint of PR $\left(\frac{8+x_S}{2},\frac{-14+y_S}{2}\right)=(4,2)$ $\frac{8+x_S}{2}=4$ $x_S=0$ $\frac{-14+y_S}{2}=2$ $y_S=18$ The coordinates of S are $(0,18)$. Area of the rhombus $PQRS$ $=\frac{1}{2}\begin{vmatrix}-4 & 8 & 12 & 0 & -4\\ 0 & -14 & 4 & 18 & 0\end{vmatrix}$ $=\frac{1}{2}\|(56+32+216+0)-(0-168+0-72)\|$ $=\frac{1}{2}\|544\|$ $=272$ unit sq

海 星 中 学 Maris Stella High School

(Q4)

姓 名 ______________________ (   )

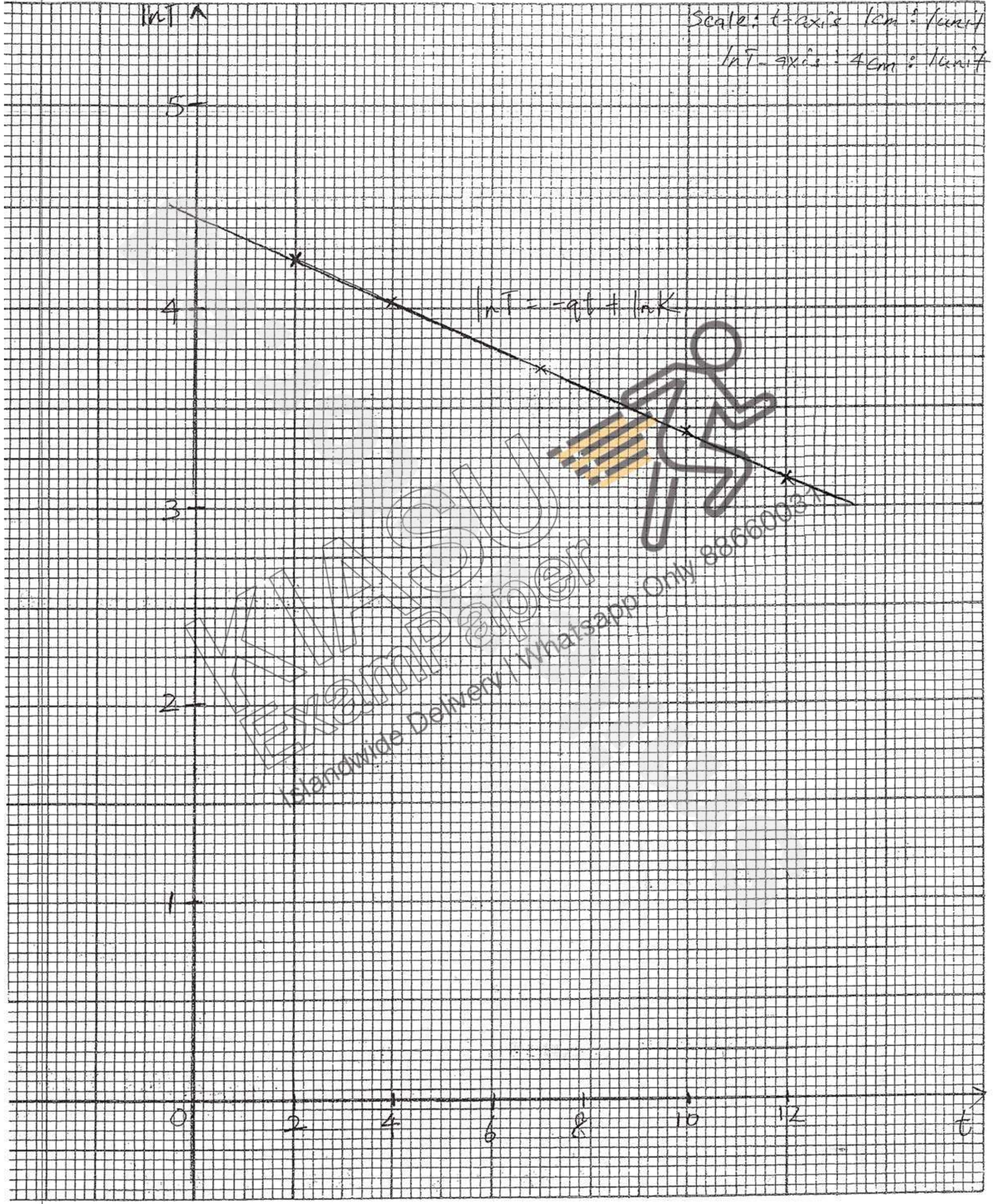
Name: ______________________

科目／Subject: ______________________

班级／Class: ______________________

Scale: t-axis 1cm : 1unit

lnT-axis : 4cm : 1unit



1 The polynomial $f(x)=2x^3+ax^2+bx+8$, where a and b are constants, has a factor $(x+2)$ and leaves a remainder of 10 when divided by $(2x-1)$.

(i) Find the value of a and of b. [4]

(ii) Using the values of a and of b found in part (i), explain why the equation $f(x)=0$ has only one real root. Find this root. [4]

(iii) Hence, solve $x^3+3x^2+4x+32=0$. [2]

1	$f(x)=2x^3+ax^2+bx+8$
(i)	$f(-2)=2(-2)^3+a(-2)^2+b(-2)+8=0$ $4a-2b=8$ --------- Eqn (1)
	$f\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3+a\left(\frac{1}{2}\right)^2+b\left(\frac{1}{2}\right)+8=10$ $a+2b=7$ ------------ Eqn (2)
	Solving the equations, $b=2$, $a=7-2(2)=3$
(ii	$f(x)=2x^3+3x^2+2x+8$ $=(x+2)(2x^2+bx+4)$ Term in x^2: $3x^2=bx^2+4x^2$, $b=-1$ $f(x)=2x^3+3x^2+2x+8$ $=(x+2)(2x^2-x+4)$ [Getting Quadratic factor by long division also allowed]
	For the factor $2x^2-x+4$, Discriminant $=1-4(2)(4)$ $=-31<0$ Hence, the equation $2x^2-x+4=0$ has no real roots. Therefore $f(x)=0$ has only 1 real root. The root is $x=-2$
(iii	$2x^3+3x^2+2x+32=0$ $2\left(\frac{x}{2}\right)^3+3\left(\frac{x}{2}\right)^2+2\left(\frac{x}{2}\right)+8=0$ $\left(\frac{x}{2}+2\right)\left(2\left(\frac{x}{2}\right)^2-\left(\frac{x}{2}\right)+4\right)=0$ $x=-4$

2 (a) Find the range of values of k for which $(k-3)x^2+4x+k$ is always positive for all real values of x. [4]

(b) Show that the roots of the equation $6x^2+4(m-1)=2(x+m)$ are real if $m\le 2\frac{1}{12}$. [3]

2(a)	$(k-3)x^2+4x+k>0$ for all values of x Discriminant <0 $16-4k(k-3)<0$ $4k^2-12k-16>0$ $k^2-3k-4>0$ $(k-4)(k+1)>0$ $k>4$ or $k<-1$ Since $k-3>0$, $k>3$ $\therefore k>4$
(b)	$6x^2+4(m-1)=2(x+m)$ $6x^2-2x+2m-4=0$ Discriminant $=100-48m$ Since $m\le 2\frac{1}{12}$ $25-12m\ge 0$ $100-48m\ge 0$ Since discriminant ≥ 0, $6x^2+4(m-1)=2(x+m)$ has real roots if $m\le 2\frac{1}{12}$

3 (a) Simplify $\frac{9^{x+1}+18(3^{2x})}{3^{2-x}\times 27^{x+1}}$ without the use of a calculator. [4]

(b) Solve the equation $4^{x+1}=18(2^x)-8$. [4]

(c) Solve the equation $\log_3(2x-1)-\frac{1}{2}\log_3(x^2+2)=\log_{25}5$. [5]

3 (a)	$\dfrac{9^{x+1}+18(3^{2x})}{3^{2-x}\times 27^{x+1}}$
	$=\dfrac{3^{2(x+1)}+18(3^{2x})}{3^{2-x}\times 3^{3(x+1)}}$ $=\dfrac{3^{2x}(3^2+18)}{3^{2x+5}}$ $=\dfrac{3^{2x}(3^3)}{3^{2x}(3^5)}$ $=\dfrac{1}{3^2}$ $=\dfrac{1}{9}$
(b)	$4^{x+1}=18(2^x)-8$
	$4\left(2^{2x}\right)=18\left(2^x\right)-8$ $4\left(2^x\right)^2-18\left(2^x\right)+8=0$ Let $2^x=A$, $4A^2-18A+8=0$ $2A^2-9A+4=0$ $(2A-1)(A-4)=0$ $A=\dfrac{1}{2}$ or $A=4$ $2^x=\dfrac{1}{2}$ $\quad 2^x=4$ $x=-1$ or $x=2$
(c)	$\log_3(2x-1)-\dfrac{1}{2}\log_3(x^2+2)=\log_{25}5$
	$2\log_3(2x-1)-\log_3(x^2+2)=1$ $\log_3\left(\dfrac{(2x-1)^2}{x^2+2}\right)=1$ $(2x-1)^2=3(x^2+2)$ $x^2-4x-5=0$ $(x-5)(x+1)=0$ $x=5$ or $x=-1$ (rej)

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4 In a Science experiment, a container of liquid was heated to a temperature of K °C. It was then left to cool in a chiller such that its temperature, T °C, t minutes after removing the heat, is given by $T=Ke^{-qt}$, where q is a constant.

Measured values of t and T are given in the following table.

t (minutes)	2	4	7	10	12
T °C	71.1	57.0	40.8	29.3	23.4

(i) Using a scale of 1 cm to 1 unit on the t-axis and 4 cm to 1 unit on the ln$T$- axis, plot ln T against t and draw a straight line graph. [2]

ln T	4.26	4.04	3.71	3.38	3.15
t	2	4	7	10	12

(ii) Use the graph to estimate the value of K and of q. [4]

(iii) Estimate the temperature of the liquid 8 minutes after it was left to cool. [2]

4(i)	Plot a straight line passing all the points with correct scale etc
(ii)	$T=Ke^{-qt}$ $\ln T=-qt+\ln K$
	Gradient $=\dfrac{3.15-4.45}{12-0}$ $=-\dfrac{13}{120}$ $-q=-\dfrac{13}{120}$ $q\approx\dfrac{13}{120}$
	$\ln K=4.475$ $K=e^{4.475}$ ≈ 87.8
(iii)	$T=87.8e^{-\frac{13}{120}(8)}$ ≈ 36.9 Temperature is 36.9°. Alternatively from graph, $t=8, \ln T=3.6$ $T=e^{3.6}\approx 36.6$ Temperature is 36.6°.

5 **(a)** **(i)** Prove that $\frac{\sin x}{\sec x+1}+\frac{\sin x}{\sec x-1}=2\cot x.$ [4]

(ii) Hence find, for $0 \le x \le 4$, the exact solution of the equation

$$\frac{\sin x}{\sec x+1}+\frac{\sin x}{\sec x-1}=\frac{2\tan x}{3}.$$ [3]

(b) Given that θ is obtuse and that $\sin\theta=\frac{1}{\sqrt{3}}$, express, **without the use of a calculator**, $\frac{1}{\sin\theta-\cos\theta}$ in the form $\sqrt{a}-\sqrt{b}$ where a and b are integers. [4]

5(a)(i)	$LHS=\frac{\sin x}{\sec x+1}+\frac{\sin x}{\sec x-1}$ $=\frac{\sin x(\sec x-1)+\sin x(\sec x+1)}{\sec^2 x-1}$ $=\frac{\tan x-\sin x+\tan x+\sin x}{\sec^2 x-1}$ $=\frac{2\tan x}{\sec^2 x-1}$ $=\frac{2\tan x}{\tan^2 x}$ $=2\cot x$ $=RHS$ (proved)
(ii)	$\frac{\sin x}{\sec x+1}+\frac{\sin x}{\sec x-1}=\frac{2\tan x}{3}$ $2\cot x=\frac{2\tan x}{3}$ $\tan^2 x=3$ $\tan x=\pm\sqrt{3}$ Basic angle $=\frac{\pi}{3}$ For $0 \le x \le 4$, $x=\frac{\pi}{3},\frac{2\pi}{3}$

5b.	$1^2+x^2=\left(\sqrt{3}\right)^2$
	$x=\sqrt{2}$
	$\cos\theta=-\frac{\sqrt{2}}{\sqrt{3}}$
	$\frac{1}{\sin\theta-\cos\theta}=\frac{1}{\frac{1}{\sqrt{3}}+\frac{\sqrt{2}}{\sqrt{3}}}$ $=\frac{\sqrt{3}}{1+\sqrt{2}}\times\frac{1-\sqrt{2}}{1-\sqrt{2}}$ $=\frac{\sqrt{3}-\sqrt{6}}{-1}$ $=\sqrt{6}-\sqrt{3}$

6 The equation of a curve is $y=\frac{a}{x}+bx-1$, where a and b are constants. The normal to the curve at the point $Q(1,-1)$ is parallel to the line $4y-x=20$. This normal meets the curve again at point P.

(i) Find the value of a and of b. [5]

(ii) Find the coordinates of point P. [3]

6 (i)	Equation of line : $y=\frac{1}{4}x+5$ At $x=1$, gradient of normal $=\frac{1}{4}$ Gradient of tangent $=-4$ $\frac{dy}{dx}=-\frac{a}{x^2}+b$ $-a+b=-4$ ------ Eqn (1) sub $(1,-1)$ into $y=\frac{a}{x}+bx-1$ $a+b=0$ --------- Eqn (2) Solving : $a=2, b=-2$

(ii)	$y=\frac{2}{x}-2x-1$ Equation of normal is : $y+1=\frac{1}{4}(x-1)$ $y=\frac{1}{4}x-\frac{5}{4}$ $\frac{2}{x}-2x-1=\frac{1}{4}x-\frac{5}{4}$ $8-8x^2-4x=x^2-5x$ $9x^2-x-8=0$ $(9x+8)(x-1)=0$ $x=-\frac{8}{9}$ or $x=1$ $y=2\left(-\frac{9}{8}\right)-2\left(-\frac{8}{9}\right)-1$ $=-\frac{53}{36}$ Coordinates of P is $\left(-\frac{8}{9},-\frac{53}{36}\right)$

7 The equation of a curve is $y=\frac{x^2}{x-1}$, where $x\neq 1$.

(i) Obtain an expression for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(ii) Find the coordinates of the stationary points of the curve and determine their nature. [4]

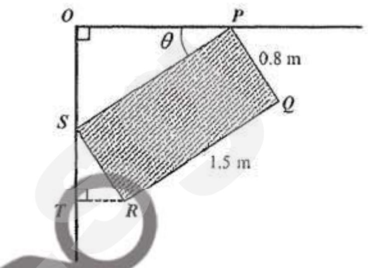
7(i)	$y=\frac{x^2}{x-1}$ $\frac{dy}{dx}=\frac{(x-1)(2x)-x^2(1)}{(x-1)^2}$ $=\frac{x^2-2x}{(x-1)^2}$ $\frac{d^2y}{dx^2}=\frac{(x-1)^2(2x-2)-2(x-1)(x^2-2x)}{(x-1)^4}$ $=\frac{(x-1)(2x^2-4x+2-2x^2+4x)}{(x-1)^4}$ $=\frac{2}{(x-1)^3}$
(ii)	For stationary point, $\frac{dy}{dx}=0$ $\frac{x^2-2x}{(x-1)^2}=0$ $x^2-2x=0$ $x(x-2)=0$ $x=0$ or 2 when $x=0, y=0$ $\frac{d^2y}{dx^2}=\frac{2}{(0-1)^3}$ $=-2<0$ $(0,0)$ is a maximum point. when $x=2, y=4$ $\frac{d^2y}{dx^2}=\frac{2}{(2-1)^3}$ $=2>0$ $(2,4)$ is a minimum point.

8 (a) Differentiate $\cot^4\left(\frac{\pi}{2}-2x\right)$ with respect to x. [3]

(b) Given that the curve has the equation $y=3\sin 2x-\cos x$, find the gradient of the curve when $x=\frac{\pi}{3}$, leaving your answer in exact form. [3]

8 (a)	Let $y=\cot^4\left(\frac{\pi}{2}-2x\right)$
	$y=\frac{1}{\tan^4\left(\frac{\pi}{2}-2x\right)}$ $=\frac{1}{\cot^4(2x)}$ $=\tan^4 2x$ $\frac{dy}{dx}=4\tan^3(2x)\left[2\sec^2(2x)\right]$ $=8\tan^3(2x)\sec^2(2x)$
	OR $y=\frac{1}{\tan^4\left(\frac{\pi}{2}-2x\right)}$ $=\tan^{-4}\left(\frac{\pi}{2}-2x\right)$ $\frac{dy}{dx}=-4\tan^{-5}\left(\frac{\pi}{2}-2x\right)\left[-2\sec^2\left(\frac{\pi}{2}-2x\right)\right]$ $=8\tan^{-5}\left(\frac{\pi}{2}-2x\right)\sec^2\left(\frac{\pi}{2}-2x\right)$
8(b)	$y=3\sin 2x-\cos x,$
	$\frac{dy}{dx}=6\cos 2x+\sin x$ At $x=\frac{\pi}{3}$, Gradient = $6\cos\frac{2\pi}{3}+\sin\frac{\pi}{3}$ $=-3+\frac{\sqrt{3}}{2}$

9



The diagram shows the top view of a rectangular desk, $PQRS$, in a corner of a room. The desk has a length of 1.5 m and width 0.8 m, $\angle POS=\angle STR=90^\circ$ and $\angle OPS=\theta$.

(i) Show that $OT=(1.5\sin\theta+0.8\cos\theta)$ m. [3]

(ii) Express OT in the form $R\sin(\theta+\alpha)$, where $R>0$ and α is acute. [3]

(iii) Given that θ can vary, find the maximum value of OT and the corresponding value of θ. [3]

9(i)	$\angle TSR=\theta$, $\cos\theta=\frac{ST}{0.8}$ $ST=0.8\cos\theta$ $\sin\theta=\frac{OS}{1.5}$ $OS=1.5\sin\theta$ $OT=OS+ST$ $=(1.5\sin\theta+0.8\cos\theta)$ m (Shown)
(ii)	$OT=1.5\sin\theta+0.8\cos\theta=R\sin(\theta+\alpha)$ where $R=\sqrt{1.5^2+0.8^2}=1.7$ $\tan\alpha=\frac{0.8}{1.5},\Rightarrow\alpha=28.072^\circ$ $\therefore\ OT=1.7\sin(\theta+28.1^\circ)$ (correct to 1 d.p.)
(iii)	Maximum value of $OT=1.7m$ when $\sin(\theta+28.072^\circ)=1$ $\theta+28.072^\circ=90^\circ$ $\theta=61.9^\circ$ (1 dp)

Name | Reg. No | Class

OWER SECONDARY SCHOOL **MAYFLOWER** SECONDARY SCHOOL **MAYFLOWER** SECONDARY SCHOOL **MAYFLOWER** SECONDARY SCHOOL **MAYFLOWER** SECONDARY SCHOOL

4E5N

ADDITIONAL MATHEMATICS

4047/01

[80 marks]

SEMESTER ONE EXAMINATION
13 May 2019
2 hours

Additional material: Writing paper

INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.

Write your name, register number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **ALL** questions.

Write your answers on the writing paper provided.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

Write the brand and model of your calculator in the space provided below.

INFORMATION FOR CANDIDATES

You are expected to use an electronic calculator to evaluate explicit numerical expressions.

If the degree of accuracy is not specified in the question, and if the answer is not exact, the answer should be given to **three** significant figures. Answers in degrees should be given to **one** decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

For Examiner's Use

Brand / Model of Calculator

This question paper consists of **7** printed pages, including the cover page.

Setter: Ms Shen Sirui

Vetter: Mr Nara

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \tfrac{1}{2}ab \sin C$$

1 **(i)** On the same diagram sketch the curve $y^2 = 8x$ and $y = 6x^{-2}$. [2]

(ii) Find the coordinates of the point of intersection of the two curves. [3]

2 A particle moves along the curve $y = e^{2x}$ in such a way that the *y*-coordinate of the particle is increasing at a constant rate of 0.3 units per second. Find the *y*-coordinate of the particle at the instant when the *x*-coordinate of the particle is increasing at 0.01 units per second.

[4]

3 The equation of a curve is $y = 3x^2 - kx + 2k - 4$, where k is a constant. Show that the line $y = 2x + 5$ intersects the curve for all real values of *k*. [5]

4 **(a)** Given that $(3^{x+2})(2^{x-2}) = 6^{2x}$, find the value of 6^x. [3]

(b) The side of an equilateral triangle is $6(\sqrt{3}-1)$ cm. **Without using a calculator,** find the exact value of the area of the equilateral triangle in the form $(a + b\sqrt{c})$ cm^2, where a, b and c are integers. [4]

5 Find the range of values of x for which the gradient of the graph $y = x^4 - 3x^3 - 6x^2 + 6$ is increasing. [5]

6 A curve has the equation $y = (2x - 3)^2 - 1$.

(i) Find the coordinates of the points at which the curve intersects the *x*-axis. [2]

(ii) Sketch the graph of $y = |(2x - 3)^2 - 1|$. [3]

(iii) Using your graph, state the range of values of k for which $|(2x - 3)^2 - 1| = k$ has 4 solutions. [1]

7 It is given that $f'(x) = x + \sin 4x$ and $f(0) = \frac{3}{4}$.

Show that $f''(x) + 16f(x) = 8x^2 + 17$. [5]

8 Solve the equation $6\sin^2 x + 5\cos x = 5$ for $0° < x < 360°$. [5]

9 **(a)** Given that the first two non-zero terms in the expansion, in ascending powers of x, of $(1+bx)(1+ax)^6$ are 1 and $-\frac{21}{4}x^2$ and that $a>0$, find the value of a and of b. [5]

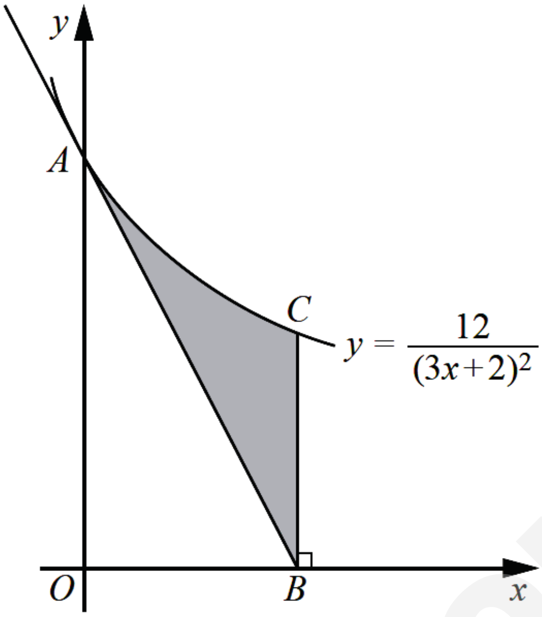
(b) Find the term independent of x in the expansion of $\left(2x+\frac{1}{x^2}\right)^9$. [3]

10 The equation of a curve is $y = \frac{x^2}{2x-1}$.

(i) Find the coordinates of the stationary points of the curve. [4]

(ii) Determine the nature of each of the stationary points of the curve. [4]

11



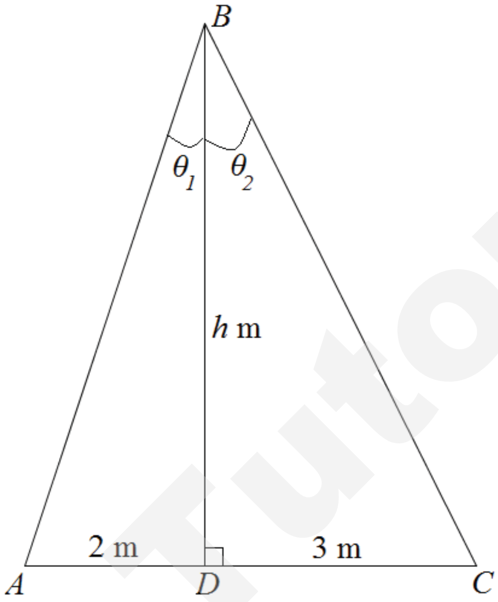
The diagram shows part of the curve $y = \frac{12}{(3x+2)^2}$ meeting the y-axis at point A. The tangent to the curve at A intersects the x-axis at point B. Point C lies on the curve such that BC is parallel to the y-axis. Find

(i) the equation of AB, [4]

(ii) the area of the shaded region. [5]

12 **(a)** State the values between which the principal value of $\tan^{-1} x$ must lie. Give your answer in terms of π . [1]

(b) The diagram below shows triangle *ABC* where *AD* = 2 m, *DC* = 3 m and *BD* = h m. *BD* is perpendicular to *AC* and $\theta_1 + \theta_2 = 45°$.



By using a suitable formula for $\tan(\theta_1 + \theta_2)$, find the value of h. [5]

13 The Ultraviolet Index describes the level of solar radiation on the earth's surface. The Ultraviolet Index, U, measured from the top of a building is given by $U = 6 - 5\cos qt$, where t is the time in hours, $0 \leq t \leq 20$, from the lowest value of Ultraviolet Index and q is a constant. It takes 10 hours for the Ultraviolet Index to reach its lowest value again.

(i) Explain why it is impossible to measure a Ultraviolet Index of 12. [1]

(ii) Show that $q = \frac{\pi}{5}$. [1]

(iii) The top of the building is equipped with solar panels that supply power to the building when the Ultraviolet Index is at least 3.5. Find the duration, in hours and minutes, that the building is supplied with power by the solar panels. [5]

END OF PAPER

Name	Reg. No	Class

MAYFLOWER SECONDARY SCHOOL **MAYFLOWER** SECONDARY SCHOOL **MAYFLOWER** SECONDARY SCHOOL **MAYFLOWER** SECONDARY SCHOOL

4E/5N

ADDITIONAL MATHEMATICS

4047/02

Paper 2 [100 marks]

SEMESTER ONE EXAMINATION

May 2019

2 hours 30 minutes

Candidates answer on the question paper.

INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.
Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **ALL** questions.
Write your answers in the spaces provided on the question paper.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Write the brand and model of your calculator in the space provided below.

INFORMATION FOR CANDIDATES

You are expected to use a scientific calculator to evaluate explicit numerical expressions.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to **three** significant figures. Give answers in degrees to **one** decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **100**.

Brand / Model of Calculator

For Examiner's Use	
Total	100

This question paper consists of **15** printed pages.

Setter: Mr. Gabriel Cheow

Vetter: Mr. Narayanan

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The roots of the quadratic equation $2x^2 - 8x + 9 = 0$ are α and β.

(i) Show that the value of $\alpha^3 + \beta^3$ is 10. [3]

(ii) Find a quadratic equation whose roots are $\frac{1}{\alpha^2 + \beta}$ and $\frac{1}{\alpha + \beta^2}$. [4]

2 The function $f(x)=6x^3+ax^2+bx-12$, where a and b are constants, is exactly divisible by $x+2$ and leaves a remainder of 5 when divided by $x+1$.

(i) Find the value of a and of b. [4]

(ii) By showing your working clearly, factorise $f(x)$. [3]

(iii) Hence, solve the equation $6(2^{3y})+2^{2y+3}-2^{y+4}-12=2^{2y}$ [4]

3 **(i)** Express $\dfrac{2x+16}{(x^2+4)(2x-1)}$ in partial fractions. [5]

(ii) Differentiate $\ln(x^2+4)$ with respect to *x*. [2]

(iii) Hence, using your results in **(i)** and **(ii)**, find $\displaystyle\int \frac{x+8}{(x^2+4)(2x-1)}\,\mathrm{d}x$. [4]

4 Prove the following identities.

(a) $(\sec x - \tan x)(\operatorname{cosec} x + 1) = \cot x$ [3]

$LHS = (\sec x \; - \tan x)(cosec\ x + 1)$

(b) $\dfrac{1 - \cos 2x + \sin x}{\sin 2x + \cos x} = \tan x$ [3]

5 The lines $y = 8$ and $4x + 3y = 30$ are tangent to a circle C at the points $(-1, 8)$ and $(3, 6)$ respectively.

(i) Show that the equation of C is $x^2 + y^2 + 2x - 6y - 15 = 0$. [5]

(ii) Explain whether or not the x-axis is tangent to C. [3]

(iii) The points Q and R also lie on the circle, and the length of the chord QR is 2 units. Calculate the shortest distance from the center of C to the chord QR. [2]

6 The table shows experimental values of two variables x and y, which are known to be connected by the equation $yx^n = A$, where n and A are constants.

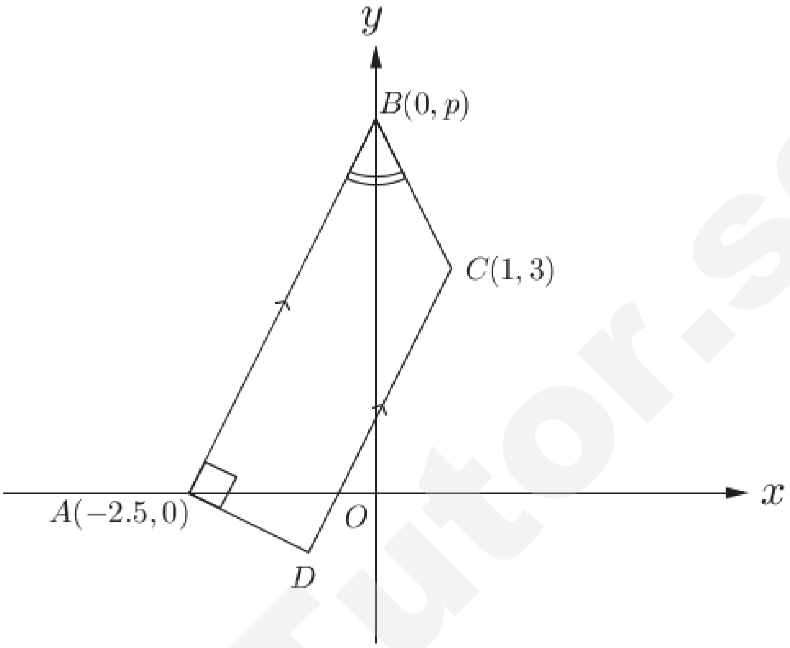
x	1.0	1.5	2.0	2.5	3.0
y	22.0	13.0	8.9	6.9	5.3

(i) Plot lg y against lg x and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of A and of n. [4]

(iii) On the same diagram, draw the line representing the equation $y = x^2$ and hence find the value of x which satisfies the equation $x^{n+2} = A$. [2]

7 The diagram shows a trapezium with vertices $A(-2.5, 0)$, $B(0, p)$, $C(1, 3)$ and D.
The sides AB and DC are parallel and the angle DAB is 90°.
Angle ABO is equal to angle CBO.



(i) Express the gradients of the lines AB and CB in terms of p and hence, or otherwise, show that $p = 5$. [3]

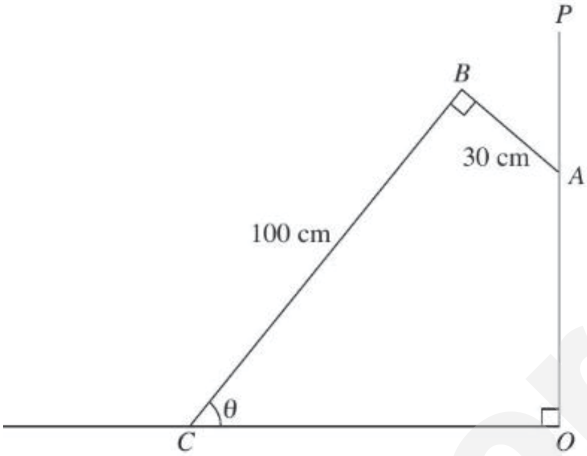
(ii) Find the coordinates of *D*. [4]

(iii) Find the area of the trapezium *ABCD*. [2]

8 **(a)** Solve the equation $3\log_x 3 = 8 - 4\log_3 x$. [5]

(b) It is given that $\log_a x = p$ and $\log_a y = q$.
Express $\log_y ax^2y^3$ in terms of p and q. [3]

9 The figure shows a stage prop *ABC* used by a member of the theatre, leaning against a vertical wall *OP*. It is given that $AB = 30$ cm, $BC = 100$ cm, $\angle ABC = \angle AOC = 90°$ and $\angle BCO = \theta$.



(i) Show that $OC = (100\cos\theta + 30\sin\theta)$ cm. [2]

Let *D* be foot of *B* on *OC*, let *E* be foot of *A* on *BD*.

(ii) Express *OC* in terms of $R\cos(\theta - \alpha)$, where *R* is a positive constant and α is an acute angle. [3]

(iii) State the maximum value of *OC* and the corresponding value of θ. [2]

(iv) Find the value of θ for which $OC = 80$ cm. [3]

For Examiner's Use

For Examiner's Use

10 Given that $y = a + b\cos 4x$, where a and b are integers, and x is in radians,

(i) state the period of y. [1]

Given that the maximum and minimum values of y are 3 and – 5 respectively, find

(ii) the amplitude of y, [1]

[2]

Using the values of a and b found in part **(iii)**,

(iv) sketch the graph of $y = a + b\cos 4x$ for $0 \le x \le \pi$. [3]

(v) On the same set of axes, sketch the graph of $y = |4\sin 3x|$, and hence state the number of solutions of $a + b\cos 4x = |4\sin 3x|$. [3]

11 The dimensions of a cuboid are $3x$ cm by $2x$ cm by h cm and its total surface area is 312 cm^2. The volume of the cuboid is V cm^3.

(i) Express h in terms of x. [2]

(ii) Show that $V = \frac{36}{5}x(26 - x^2)$. [2]

(iii) Find the maximum volume of the cuboid as x varies, giving your answer to the nearest cm^3. [5]

– End of Paper –

Name | Reg. No | Class

4E5N

ADDITIONAL MATHEMATICS 4047/01

[80 marks]

SEMESTER ONE EXAMINATION
13 May 2019
2 hours

Additional material: Writing paper

INSTRUCTIONS TO CANDIDATES

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Write your name, register number and class on all the work you hand in.

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The total number of marks for this paper is **80**.

For Examiner's Use

Brand / Model of Calculator

This question paper consists of **7** printed pages, including the cover page.

Setter: Ms Shen Sirui

Vetter: Mr Nara

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For the equation $ax^2 + bx + c = 0$,

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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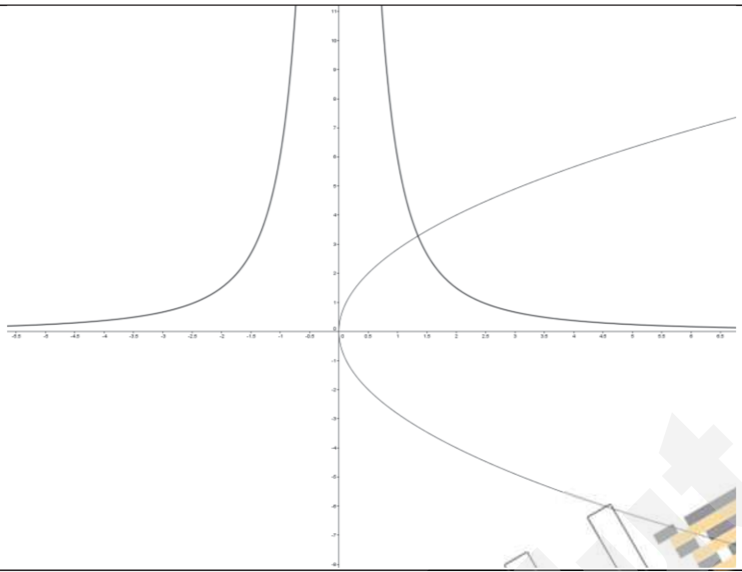
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \tfrac{1}{2} ab \sin C$$

1 **(i)** On the same diagram sketch the curve $y^2 = 8x$ and $y = 6x^{-2}$. [2]

(ii) Find the coordinates of the point of intersection of the two curves. [3]

Qn	Solution	Mark
i		B1 for $y^2 = 8x$ B1 for $y = 6x^{-2}$
ii	$y^2 = 8x - - - (1)$ $y = 6x^{-2} - - - (2)$ Sub (2) into (1): $(6x^{-2})^2 = 8x$ $\frac{36}{x^4} = 8x$ $x^5 = 4.5$ $x = 1.3509$ $y = 3.2877$ Intersection: $(1.35, 3.29)$	M1 for substitution M1 for value of x or y A1

2 A particle moves along the curve $y = e^{2x}$ in such a way that the y-coordinate of the particle is increasing at a constant rate of 0.3 units per second. Find the y-coordinate of the particle at the instant when the x-coordinate of the particle is increasing at 0.01 units per second. [4]

Qn	Solution	Mark
	$y = e^{2x}$ $\frac{dy}{dx} = 2e^{2x}$	M1 for dy/dx
	$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ $2e^{2x} = 0.3 \div 0.01$	M1 for sub into equation connecting dy/dx, dy/dt, dx/dt
	$e^{2x} = 15$ $x = \frac{\ln 15}{2}$	M1 for $x = \frac{\ln 15}{2}$ or $e^{2x} = 15$
	Sub $x = \frac{\ln 15}{2}$, $y = e^{2\left(\frac{\ln 15}{2}\right)} = 15$	A1

3 The equation of a curve is $y = 3x^2 - kx + 2k - 4$, where k is a constant. Show that the line $y = 2x + 5$ intersects the curve for all real values of k. [5]

Qn	Solution	Mark
	$y = 3x^2 - kx + 2k - 4 - - - (1)$ $y = 2x + 5 - - - (2)$ $(1) = (2): 3x^2 - kx + 2k - 4 = 2x + 5$	M1 for combining equations
	$3x^2 - kx - 2x + 2k - 9 = 0$ $3x^2 - (k+2)x + 2k - 9 = 0$	M1 for $ax^2 + bx + c = 0$
	$b^2 - 4ac = [-(k+2)]^2 - 4(3)(2k-9)$ $= k^2 + 4k + 4 - 24k + 108$ $= k^2 - 20k + 111$	M1 for subs into $b^2 - 4ac$
	$= (k-10)^2 - 10^2 + 112$ $= (k-10)^2 + 12$	M1 for $(k-10)^2 + 12$
	Since $(k-10)^2 + 12 > 0, b^2 - 4ac > 0$ and line intersects the curve for all real values of k.	A1 for conclusion

4 **(a)** Given that $(3^{x+2})(2^{x-2}) = 6^{2x}$, find the value of 6^x . [3]

(b) The side of an equilateral triangle is $6(\sqrt{3}-1)$ cm. **Without using a calculator,** find the exact value of the area of the equilateral triangle in the form $(a+b\sqrt{c})$ cm^2, where a, b and c are integers. [4]

Qn	Solution	Mark
a	$(3^{x+2})(2^{x-2}) = 6^{2x}$ $3^x(3^2)(2^x)(2^{-2}) = 6^{2x}$ $6^x\left(\frac{9}{4}\right) = 6^{2x}$ $6^x = \frac{9}{4}$	M1 for $3^x(3^2)$ or $(2^x)(2^{-2})$ M1 for $6^x\left(\frac{9}{4}\right)$ A1
b	Area $= \frac{1}{2}\left[6(\sqrt{3}-1)\right]^2 \sin 60$ $= \frac{1}{2}(36)(3-2\sqrt{3}+1)\left(\frac{\sqrt{3}}{2}\right)$ $= 9\sqrt{3}(4-2\sqrt{3})$ $= 36\sqrt{3}-54$ $= -54+36\sqrt{3}$	M1 M1 for $(3-2\sqrt{3}+1)$ M1 for $\left(\frac{\sqrt{3}}{2}\right)$ A1

5 Find the range of values of x for which the gradient of the graph $y = x^4 - 3x^3 - 6x^2 + 6$ is increasing. [5]

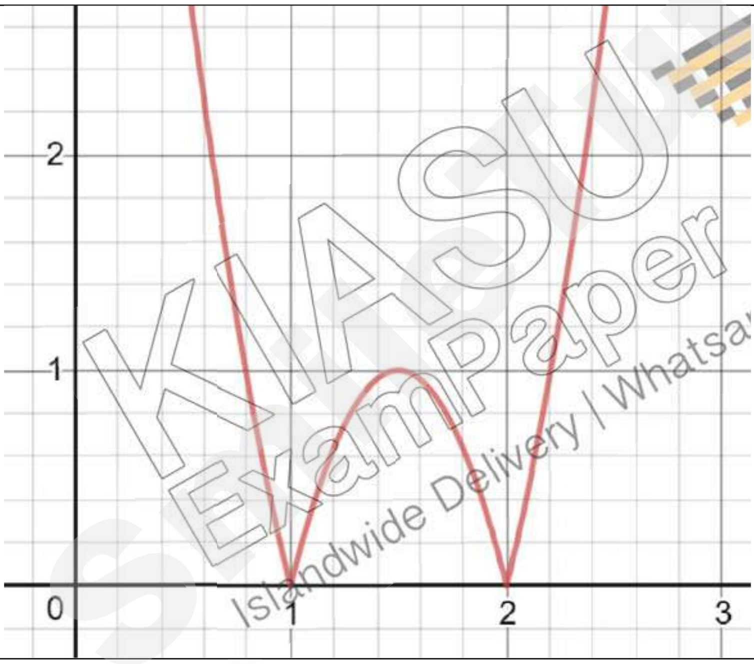
Qn	Solution	Mark
	$y = x^4 - 3x^3 - 6x^2 + 6$ $\frac{dy}{dx} = 4x^3 - 9x^2 - 12x$ $\frac{d^2y}{dx^2} = 12x^2 - 18x - 12$ $12x^2 - 18x - 12 > 0$ $2x^2 - 3x - 2 > 0$ $(2x+1)(x-2) > 0$ $x < -\frac{1}{2}, x > 2$	M1 for $\frac{dy}{dx}$ M1 for $\frac{d^2y}{dx^2}$ M1 for $\frac{d^2y}{dx^2} > 0$ M1 for factorised form A1

6 A curve has the equation $y = (2x-3)^2 - 1$.

(i) Find the coordinates of the points at which the curve intersects the x-axis. [2]

(ii) Sketch the graph of $y = |(2x-3)^2 - 1|$. [3]

(iii) Using your graph, state the range of values of k for which $|(2x-3)^2 - 1| = k$ has 4 solutions. [1]

Qn	Solution	Mark
i	$(2x-3)^2 - 1 = 0$ $2x - 3 = \pm 1$ $x = 1, \quad x = 2$ $(1, 0) \quad (2, 0)$	M1 A1 or B2
ii		T1 for turning point (1.5, 1) P1 for (1, 0) and (2, 0) C1 for shape of graph
iii	$0 < k < 1$	B1 (no mark if students got part ii wrong)

7 It is given that $f'(x) = x + \sin 4x$ and $f(0) = \frac{3}{4}$.

Show that $f''(x) + 16f(x) = 8x^2 + 17$. [5]

Qn	Solution	Mark
	$f(x) = \frac{x^2}{2} - \frac{\cos 4x}{4} + c$	M1 for $\frac{x^2}{2} - \frac{\cos 4x}{4}$
	$\frac{3}{4} = 0 - \frac{1}{4} + c$	
	$c = 1$	
	$f(x) = \frac{x^2}{2} - \frac{\cos 4x}{4} + 1$	M1 for $f(x) = \frac{x^2}{2} - \frac{\cos 4x}{4} + 1$
	$f''(x) = 1 + 4\cos 4x$	M1 for $1 + 4\cos 4x$
	$f''(x) + 16f(x) = 1 + 4\cos 4x + 16(\frac{x^2}{2} - \frac{\cos 4x}{4} + 1)$	M1 for sub into $f''(x) + 16f(x)$
	$= 1 + 4\cos 4x + 8x^2 - 4\cos 4x + 16$	A1
	$= 8x^2 + 17$	

8 Solve the equation $6\sin^2 x + 5\cos x = 5$ for $0° < x < 360°$. [5]

Qn	Solution	Mark
	$6(1 - \cos^2 x) + 5\cos x = 5$	M1 for $1 - \cos^2 x$
	$6 - 6\cos^2 x + 5\cos x - 5 = 0$	
	$6\cos^2 x - 5\cos x - 1 = 0$	M1 for equation
	$(6\cos x + 1)(\cos x - 1) = 0$	
	$\cos x = -\frac{1}{6}$, $\cos x = 1$	M1 for $\cos x = -\frac{1}{6}$
	$\alpha = 80.405$ (Rej)	M1 for basic angle
	$x = 180 - \alpha, 180 + \alpha$	
	$x = 99.6°, 260.4°$	A1 for both answers Ignore if students do not reject $\cos x = 1$

9 **(a)** Given that the first two non-zero terms in the expansion, in ascending powers of x, of $(1+bx)(1+ax)^6$ are 1 and $-\frac{21}{4}x^2$ and that $a>0$, find the value of a and of b. [5]

(b) Find the term independent of x in the expansion of $\left(2x+\frac{1}{x^2}\right)^9$. [3]

Qn	Solution	Mark
a	$(1+ax)^6 = 1+\binom{6}{1}(1)^5(ax)^1+\binom{6}{2}(1)^4(ax)^2+\cdots$ $= 1+6ax+15a^2x^2+\cdots$ $(1+bx)(1+ax)^6 = (1+bx)(1+6ax+15a^2x^2+\cdots)$ $= 1+6ax+bx+15a^2x^2+6abx^2+\cdots$ $6a+b=0$ $b=-6a---(1)$ $15a^2+6ab=-\frac{21}{4}---(2)$ sub (1) into (2): $15a^2+6a(-6a)=-\frac{21}{4}$ $21a^2=\frac{21}{4}$ $a^2=\frac{1}{4}$ $a=\frac{1}{2}$ $b=-3$	M1 for $1+6ax+15a^2x^2$ M1 for $6a+b=0$ M1 for $15a^2+6ab=-\frac{21}{4}$ A1 A1
b	$T_{r+1}=\binom{9}{r}(2x)^{9-r}\left(\frac{1}{x^2}\right)^r$ For x^0, $x^{9-r}(x)^{-2r}=x^0$ $r=3$ $T_{3+1}=\binom{9}{3}(2x)^{9-3}\left(\frac{1}{x^2}\right)^3$ $=84(2x)^6(x)^{-6}$ $=5376$	M1 for $\binom{9}{r}(2x)^{9-r}\left(\frac{1}{x^2}\right)^r$ M1 for $r=3$ A1

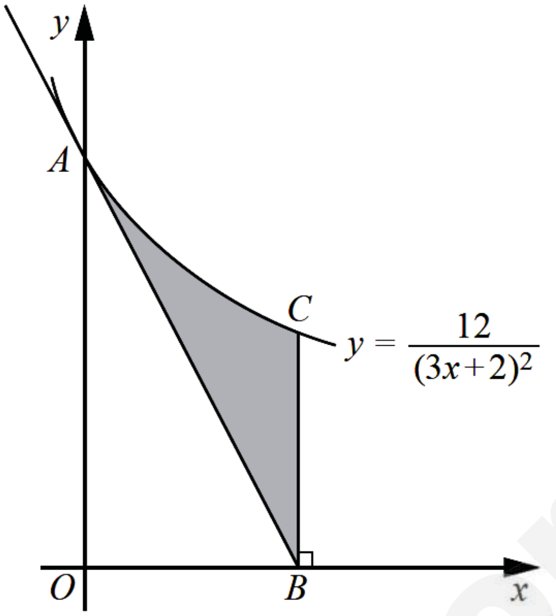
10 The equation of a curve is $y = \frac{x^2}{2x-1}$.

(i) Find the coordinates of the stationary points of the curve. [4]

(ii) Determine the nature of each of the stationary points of the curve. [4]

<table>
<tr><th>Qn</th><th>Solution</th><th>Mark</th></tr>
<tr><td>i</td><td>$y = \frac{x^2}{2x-1}$
$\frac{dy}{dx} = \frac{2x(2x-1)-2x^2}{(2x-1)^2}$
$= \frac{2x^2-2x}{(2x-1)^2}$
when $\frac{dy}{dx} = 0, \frac{2x^2-2x}{(2x-1)^2} = 0$
$2x(x-1) = 0$
$x = 0, x = 1$
$y = 0, y = 1$
Stationary points: $(0,0)$ and $(1,1)$</td><td>M1 for quotient or product rule
M1 for $\frac{2x^2-2x}{(2x-1)^2} = 0$
M1 for both x
A1 for both coordinates</td></tr>
<tr><td>ii</td><td>$\frac{d^2y}{dx^2} = \frac{(4x-2)(2x-1)^2-4(2x-1)(2x^2-2x)}{(2x-1)^4}$
when $x = 0, \frac{d^2y}{dx^2} = -2 < 0$
$(0,0)$ is maximum point.
when $x = 1, \frac{d^2y}{dx^2} = 2 > 0$
$(1,1)$ is minimum point.
OR
<table><tr><td>x</td><td>-0.1</td><td>0</td><td>0.1</td></tr><tr><td>$\frac{dy}{dx}$</td><td>> 0</td><td>0</td><td>< 0</td></tr></table>(0, 0) is maximum point.
<table><tr><td>x</td><td>0.9</td><td>1</td><td>1.1</td></tr><tr><td>$\frac{dy}{dx}$</td><td>< 0</td><td>0</td><td>> 0</td></tr></table>(1, 1) is minimum point.</td><td>M1 for $\frac{d^2y}{dx^2}$
M1 for sub either $x = 0$ or $x = 1$ into $\frac{d^2y}{dx^2}$
A1 for (0, 0) max pt
A1 for (1, 1) min pt
M1 for 1st derivative test
A1 for (0, 0) max pt
M1 for 1st derivative test
A1 for (1, 1) min pt</td></tr>
</table>

11



The diagram shows part of the curve $y = \frac{12}{(3x+2)^2}$ meeting the y-axis at point A. The tangent to the curve at A intersects the x-axis at point B. Point C lies on the curve such that BC is parallel to the y-axis. Find

(i) the equation of AB, [4]

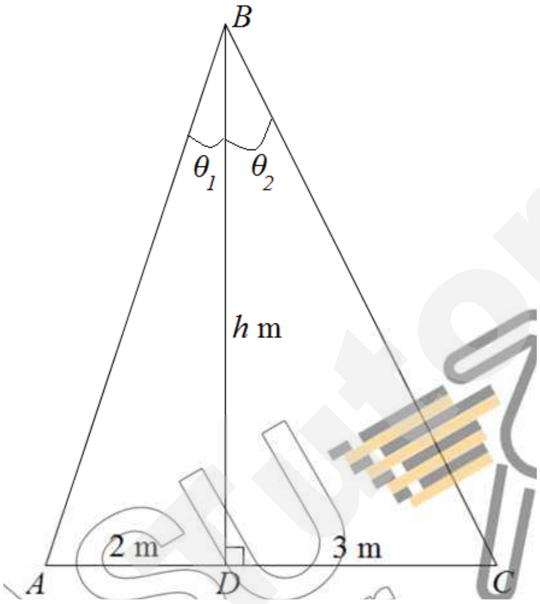
(ii) the area of the shaded region. [5]

Qn	Solution	Mark
i	$y = \frac{12}{(3x+2)^2}$ $\frac{dy}{dx} = -24(3x+2)^{-3}(3)$ $= -\frac{72}{(3x+2)^3}$ when $x = 0, \frac{dy}{dx} = -9$ when $x = 0, y = 3$ Line AB: $y = -9x + 3$	M1 for dy/dx M1 for dy/dx at A M1 for $y = 3$ A1
ii	sub $y = 0, 0 = -9x + 3$ $x = \frac{1}{3}$ $B = \left(\frac{1}{3}, 0\right)$	M1 for x-coordinate of B

	Area of $OACB = \int_0^{\frac{1}{3}} 12(3x+2)^{-2}\,dx$ $= \left[\frac{12(3x+2)^{-1}}{-1(3)}\right]_0^{\frac{1}{3}}$ $= \left[-\frac{4}{3x+2}\right]_0^{\frac{1}{3}}$ $= -\frac{4}{3\left(\frac{1}{3}\right)+2} - \left(-\frac{4}{3(0)+2}\right)$ $= \frac{2}{3}$ Area of $\Delta OAB = \frac{1}{2}\left(\frac{1}{3}\right)(3) = \frac{1}{2}$ Area of shaded region $= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$ unit^2	M1 for $-\frac{4}{3x+2}$ (independent of limits) M1 for area of *OACB* M1 for area of tri *OAB* A1

12 **(a)** State the values between which the principal value of $\tan^{-1} x$ must lie. Give your answer in terms of π . [1]

(b) The diagram below shows triangle *ABC* where *AD* = 2 m, *DC* = 3 m and *BD* = *h* m. *BD* is perpendicular to *AC* and $\theta_1 + \theta_2 = 45°$.



By using a suitable formula for $\tan(\theta_1 + \theta_2)$, find the value of *h*. [5]

Qn	Solution	Mark
a	$-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$	B1
b	$\tan(\theta_1 + \theta_2) = \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2}$ $\tan 45 = \frac{\frac{2}{h} + \frac{3}{h}}{1 - \left(\frac{2}{h}\right)\left(\frac{3}{h}\right)}$ $1 = \frac{\frac{2}{h} + \frac{3}{h}}{1 - \left(\frac{2}{h}\right)\left(\frac{3}{h}\right)}$ $1 - \frac{6}{h^2} = \frac{5}{h}$ $h^2 - 5h - 6 = 0$ $(h-6)(h+1) = 0$ $h = 6, \quad h = -1$ (rej)	M1 for tan addition formula M1 for either $\tan\theta_1 = \frac{2}{h}$ or $\tan\theta_2 = \frac{3}{h}$ M1 for tan 45 = 1 M1 for $h^2 - 5h - 6 = 0$ A1

13 The Ultraviolet Index describes the level of solar radiation on the earth's surface. The Ultraviolet Index, U, measured from the top of a building is given by $U = 6 - 5\cos qt$, where t is the time in hours, $0 \le t \le 20$, from the lowest value of Ultraviolet Index and q is a constant. It takes 10 hours for the Ultraviolet Index to reach its lowest value again.

(i) Explain why it is impossible to measure a Ultraviolet Index of 12. [1]

(ii) Show that $q = \frac{\pi}{5}$. [1]

(iii) The top of the building is equipped with solar panels that supply power to the building when the Ultraviolet Index is at least 3.5. Find the duration, in hours and minutes, that the building is supplied with power by the solar panels. [5]

Qn	Solution	Mark
i	Max $U = 6 + 5 = 11$ Since max value of $U = 11$, we cannot measure a Ultraviolet Index of 12.	B1 for stating max value of U
ii	$10 = \frac{2\pi}{q}$ $q = \frac{2\pi}{10} = \frac{\pi}{5}$	B1 for $q = \frac{2\pi}{10}$
iii	$6 - 5\cos\frac{\pi}{5}t = 3.5$ $\cos\frac{\pi}{5}t = \frac{1}{2}$ $\alpha = \frac{\pi}{3}$ $\frac{\pi}{5}t = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}, \frac{\pi}{3} + 2\pi, 2\pi - \frac{\pi}{3} + 2\pi$ $t = 1.6666, 8.3333, 11.66, 18.33$ Duration $= (8.3333 - 1.6666) + (18.33 - 11.66)$ $= 13.3367$ $=$ 13 hours 20 mins	M1 for forming equation M1 for basic angle M1 for $\frac{\pi}{3}, 2\pi - \frac{\pi}{3}$ M1 for all 4 values A1

END OF PAPER

Name | Reg. No | Class

MARK SCHEME

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4E/5N

ADDITIONAL MATHEMATICS

4047/02

Paper 2 [100 marks]

SEMESTER ONE EXAMINATION

May 2019

2 hours 30 minutes

Candidates answer on the question paper.

INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.
Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **ALL** questions.
Write your answers in the spaces provided on the question paper.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Write the brand and model of your calculator in the space provided below.

INFORMATION FOR CANDIDATES

You are expected to use a scientific calculator to evaluate explicit numerical expressions.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to **three** significant figures. Give answers in degrees to **one** decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **100**.

Brand / Model of Calculator

For Examiner's Use	
Total	100

This question paper consists of **15** printed pages.

Setter: Mr. Gabriel Cheow

Vetter: Mr. Narayanan

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

1 The roots of the quadratic equation $2x^2 - 8x + 9 = 0$ are α and β.

(i) Show that the value of $\alpha^3 + \beta^3$ is 10. [3]

$$\alpha + \beta = 4\,,\, \alpha\beta = \frac{9}{2}$$

M1 – sum & pdt

$$\begin{aligned}\alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2\alpha\beta \\ &= 4^2 - 9 \\ &= 7\end{aligned}$$

M1

$$\begin{aligned}\alpha^3 + \beta^3 &= (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) \\ &= (4)\left(7 - \frac{9}{2}\right) \\ &= 10\ (shown)\end{aligned}$$

A1

(ii) Find a quadratic equation whose roots are $\dfrac{1}{\alpha^2 + \beta}$ and $\dfrac{1}{\alpha + \beta^2}$. [4]

$$New\ sum:\ \frac{1}{\alpha^2 + \beta} + \frac{1}{\alpha + \beta^2} = \frac{\alpha + \beta^2 + \alpha^2 + \beta}{(\alpha^2 + \beta)(\alpha + \beta^2)}$$

Allow for ECF

$$= \frac{\alpha + \beta^2 + \alpha^2 + \beta}{\alpha^3 + \beta\alpha + \alpha^2\beta^2 + \beta^3}$$

$$= \frac{4 + 7}{10 + \frac{9}{2} + \left(\frac{9}{2}\right)^2}$$

M1

$$= \frac{44}{139}$$

M1

$$New\ pdt:\ \frac{1}{\alpha^2 + \beta} \times \frac{1}{\alpha + \beta^2} = \frac{1}{(\alpha^2 + \beta)(\alpha + \beta^2)}$$

$$= \frac{1}{10 + \frac{9}{2} + \left(\frac{9}{2}\right)^2}$$

$$= \frac{4}{139}$$

M1

$$New\ eqn:\ x^2 - \frac{44}{139}x + \frac{4}{139} = 0$$

$$139x^2 - 44x + 4 = 0$$

A1

2 The function $f(x) = 6x^3 + ax^2 + bx - 12$, where a and b are constants, is exactly divisible by $x + 2$ and leaves a remainder of 5 when divided by $x + 1$.

(i) Find the value of a and of b. [4]

$$f(-2) = 0$$
$$-48 + 4a - 2b - 12 = 0 \quad \textbf{M1}$$
$$2a - b = 30 \cdots\cdots Eqn\ 1$$

$$f(-1) = 5$$
$$-6 + a - b - 12 = 5 \quad \textbf{M1}$$
$$a - b = 23 \cdots\cdots Eqn\ 2$$

$Eqn\ 1 - Eqn\ 2 : a = 7$ **A1**

$Sub\ into\ Eqn\ 1 : b = -16$ **A1**

(ii) By showing your working clearly, factorise $f(x)$. [3]

$$6x^3 + 7x^2 - 16x - 12 = (x+2)(Ax^2 + Bx + C)$$

$By\ observation: A = 6, C = -6$

$$\Rightarrow 6x^3 + 7x^2 - 16x - 12 = (x+2)(6x^2 + Bx - 6) \quad \textbf{M1}$$

$Let\ x = 1:$

$$6 + 7 - 16 - 12 = (3)(6 + B - 6)$$
$$-15 = 3B$$
$$B = -5$$

$$6x^3 + 7x^2 - 16x - 12 = (x+2)(6x^2 - 5x - 6) \quad \textbf{M1}$$
$$= (x+2)(3x+2)(2x-3) \quad \textbf{A1}$$

(iii) Hence, solve the equation $6(2^{3y}) + 2^{2y+3} - 2^{y+4} - 12 = 2^{2y}$ [4]

$$6(2^{3y}) + 2^{2y+3} - 2^{y+4} - 12 = 2^{2y}$$
$$6(2^{3y}) + 8(2^{2y}) - 16(2^y) - 12 = 2^{2y}$$
$$6(2^y)^3 + 7(2^y)^2 - 16(2^y) - 12 = 0 \quad \textbf{M1}$$

$Let\ x = 2^y$

$$\Rightarrow (x+2)(3x+2)(2x-3) = 0 \quad \textbf{M1}$$
$$x = -2, -\frac{2}{3}, \frac{3}{2}$$
$$2^y = -2\ (rej.), -\frac{2}{3}(rej.), \frac{3}{2} \quad \textbf{M1}$$
$$y \ln 2 = \ln \frac{3}{2}$$
$$y = \frac{\ln 1.5}{\ln 2} = 0.585\ (3sf) \quad \textbf{A1}$$

3 **(i)** Express $\frac{2x+16}{(x^2+4)(2x-1)}$ in partial fractions. [5]

$$\frac{2x+16}{(x^2+4)(2x-1)} = \frac{Ax+B}{x^2+4} + \frac{C}{2x-1}$$

$$2x+16 = (Ax+B)(2x-1) + C(x^2+4) \quad \boxed{\text{M1}}$$

$Let\ x = 0.5$:

$17 = C\left(\frac{17}{4}\right)$

$C = 4$ $\boxed{\text{M1}}$

$Let\ x = 0$:

$16 = B(-1) + 4(4)$

$B = 0$ $\boxed{\text{M1}}$

$Let\ x = -1$:

$14 = -A(-3) + 20$

$3A = -6$

$A = -2$ $\boxed{\text{M1}}$

$$\therefore \frac{2x+16}{(x^2+4)(2x-1)} = \frac{-2x}{x^2+4} + \frac{4}{2x-1} \quad \boxed{\text{A1}}$$

(ii) Differentiate $\ln(x^2+4)$ with respect to x. [2]

$$\frac{d}{dx}[\ln(x^2+4)] = \frac{2x}{x^2+4} \quad \boxed{\text{B2}}$$

(iii) Hence, using your results in **(i)** and **(ii)**, find $\int \frac{x+8}{(x^2+4)(2x-1)}\,dx$. [4]

$$\int \frac{x+8}{(x^2+4)(2x-1)}\,dx = \frac{1}{2}\int \frac{2x+16}{(x^2+4)(2x-1)}\,dx$$

$$= \frac{1}{2}\int \left(\frac{-2x}{x^2+4} + \frac{4}{2x-1}\right) dx \quad \boxed{\text{M1 partial frac}}$$

$$= -\frac{1}{2}\int \frac{2x}{x^2+4}\,dx + \underline{\frac{1}{2}(2\ln(2x-1)+c_1)} \quad \boxed{\text{M1}}$$

$$= \underline{-\frac{1}{2}\ln(x^2+4)} + c_2 + \ln(2x-1) + \frac{1}{2}c_1 \quad \boxed{\text{M1}}$$

$$= \ln(2x-1) - \frac{1}{2}\ln(x^2+4) + c \quad \boxed{\text{A1}}$$

4 Prove the following identities.

(a) $(\sec x - \tan x)(\operatorname{cosec} x + 1) = \cot x$ [3]

$$LHS = (\sec x - \tan x)(cosec\ x + 1)$$

$$= \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x} + \frac{\sin x}{\sin x}\right)$$

M1 sin and cos only

$$= \frac{(1-\sin x)(1+\sin x)}{\cos x \sin x}$$

M1 single fraction

$$= \frac{1-\sin^2 x}{\cos x \sin x}$$

$$= \frac{\cos^2 x}{\cos x \sin x}$$

M1 cos^2 + sin^2 = 1

$$= \frac{\cos x}{\sin x}$$

$$= \cot x$$

$$= RHS\ (proven)$$

(b) $\dfrac{1-\cos 2x + \sin x}{\sin 2x + \cos x} = \tan x$ [3]

$$LHS = \frac{1-\cos 2x + \sin x}{\sin 2x + \cos x}$$

$$= \frac{1-(1-2\sin^2 x) + \sin x}{2\sin x \cos x + \cos x}$$

M1 cosine double angle

M1 sine double angle

$$= \frac{2\sin^2 x + \sin x}{\cos x\,(2\sin x + 1)}$$

$$= \frac{\sin x\,(2\sin x + 1)}{\cos x\,(2\sin x + 1)}$$

M1 factorise and cancel

$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$

$$= RHS\ (proven)$$

5 The lines $y=8$ and $4x+3y=30$ are tangent to a circle C at the points $(-1,8)$ and $(3,6)$ respectively.

(i) Show that the equation of C is $x^2+y^2+2x-6y-15=0$. [5]

Let centre of circle be O.
Horizontal tangent at $(-1,8)$ means that O is on the line $x = -1$. **M1**

To find normal of circle at $(3,6)$:

$$4x+3y=30$$
$$y=-\frac{4}{3}x+10$$
$$\therefore m_{normal}=\frac{3}{4}$$

$eqn\ of\ normal: y-6=\frac{3}{4}(x-3)$ **M1**

$When\ x=-1, y=3. \Rightarrow O(-1,3)$ **M1**

Horizontal tangent is $y = 8$. Hence radius is 5! **M1**

$$(x+1)^2+(y-3)^2=5^2$$ **M1**
$$x^2+2x+1+y^2-6x+9=25$$
$$x^2+y^2+2x-6y-15=0\ (shown)$$

(ii) Explain whether or not the x-axis is tangent to C. [3]

C has centre $(-1,3)$ and radius 5.
Hence its horizontal tangents are $y=3\pm5 \Rightarrow y=8\ or\ y=-2$ **M1**

x-axis is $y=0$, which is between the two horizontal tangents. **M1**
Hence the x-axis will cut through C at two points.
Hence the x-axis is **not** tangent to C. **A1**

Alternative solution: Sub $y=0$ into eqn of C, show that $b^2-4ac\neq 0$.

(iii) The points Q and R also lie on the circle, and the length of the chord QR is 2 units. Calculate the shortest distance from the center of C to the chord QR. [2]

Let M be midpoint of QR. Hence OM perpendicular to QR.
Hence, OM is shortest distance from C to chord QR.
Consider right-angled triangle OMR.
By Pythagoras Theorem,

$$OM=\sqrt{5^2-\left(\frac{2}{2}\right)^2}$$ **M1**
$$=\sqrt{24}=2\sqrt{6}$$
$$=4.90\ (3sf)$$ **A1**

6 The table shows experimental values of two variables x and y, which are known to be connected by the equation $yx^n = A$, where n and A are constants.

x	1.0	1.5	2.0	2.5	3.0
y	22.0	13.0	8.9	6.9	5.3

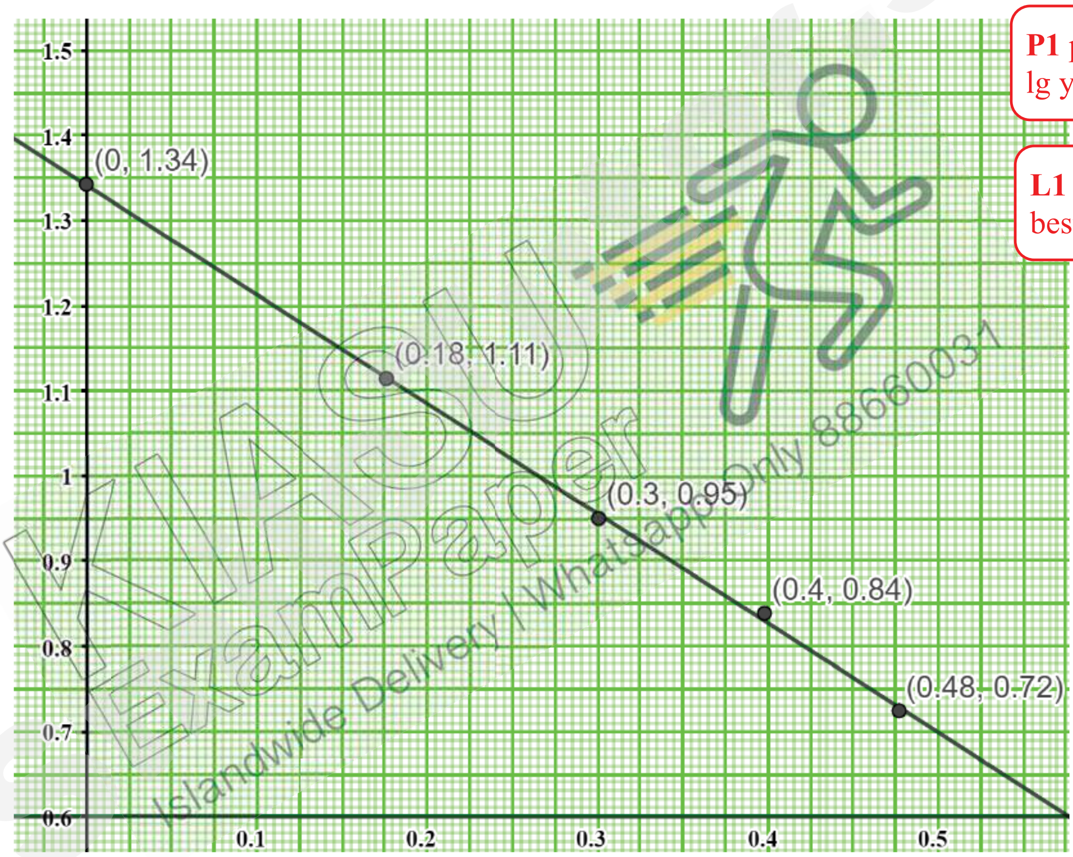
(i) Plot lg y against lg x and draw a straight line graph. [3]

lg x	0	0.176	0.301	0.398	0.477
lg y	1.34	1.11	0.949	0.839	0.724

T1 table of values

P1 plot of lg y / lg x

L1 scale & best fit line



Scale: 4 cm to 0.1 units on X-axis, 2 cm to 0.1 units on Y-axis.
Scale used must be appropriate in order to award L1.

(ii) Use your graph to estimate the value of A and of n. [4]

$$yx^n = A$$

$\lg y + n \lg x = \lg A$ **M1**

$$\lg y = -n \lg x + \lg A$$

$$Y = mX + c$$

$$\Rightarrow m = -n, c = \lg A$$

$m = \dfrac{0.7 - 1.34}{0.5 - 0}$ **M1**

$= -1.28$

$n = 1.28$ **A1**

$$c = 1.34$$

$$\lg A = 1.34$$

$$A = 10^{1.34}$$

$= 21.9$ **A1**

(iii) On the same diagram, draw the line representing the equation $y = x^2$ and hence find the value of x which satisfies the equation $x^{n+2} = A$. [2]

$$y = x^2$$

$$\lg y = 2 \lg x$$

$Draw: Y = 2X$ **M1 for drawing line**

$$x^{n+2} = A$$

$$(n+2) \lg x = \lg A$$

$$2 \lg x = -n \lg x + \lg A$$

Let graph 1 be $\lg y = 2 \lg x$, and
Let graph 2 be $\lg y = -n \lg x + \lg A$

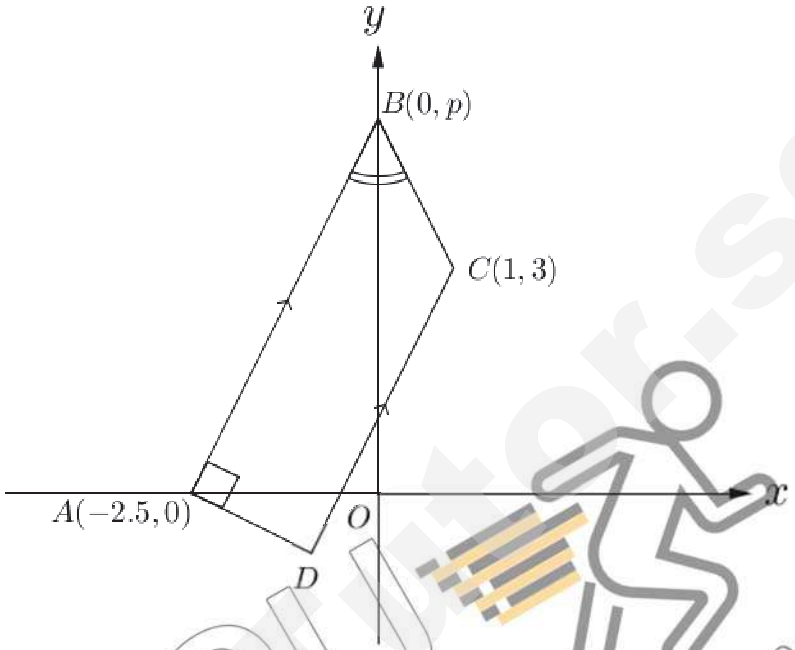
From graph, let intersection be (X, Y).
$(X, Y) = (0.41, 0.82)$

$$\lg x = 0.41$$

$$x = 10^{0.41}$$

$= 2.57$ **A1**

7 The diagram shows a trapezium with vertices $A(-2.5, 0)$, $B(0, p)$, $C(1, 3)$ and D.
The sides AB and DC are parallel and the angle DAB is 90°.
Angle ABO is equal to angle CBO.



(i) Express the gradients of the lines AB and CB in terms of p and hence, or otherwise, show that $p = 5$. [3]

$$m_{AB} = \frac{p}{2.5}$$

$$= \frac{2p}{5} \quad \boxed{\text{M1}}$$

$$m_{BC} = \frac{3-p}{1} \quad \boxed{\text{M1}}$$

$$m_{AB} = -m_{BC}$$

$$\frac{2p}{5} = \frac{p-3}{1} \quad \boxed{\text{M1}}$$

$$2p = 5p - 15$$

$$3p = 15$$

$$p = 5\ (shown)$$

(ii) Find the coordinates of D. [4]

$m_{CD} = m_{AB} = 2$

$m_{AD} = -\frac{1}{2} \because AD \perp CD$ **M1** $m_1m_2 = -1$

$Let\ D(k, h)$

$m_{CD} = \frac{3-h}{1-k}$

$2 = \frac{3-h}{1-k}$

$3 - h = 2 - 2k$

$h = 2k + 1 \cdots\cdots Eqn\ 1$ **M1** form eqn of k, h

$m_{AD} = \frac{h-0}{k+2.5}$

$-\frac{1}{2} = \frac{h-0}{k+2.5}$

$2h = -k - 2.5 \cdots\cdots Eqn\ 2$

$Eqn\ 1\ in\ Eqn\ 2: \quad 2(2k+1) = -k - 2.5$

$5k = -2 - 2.5$

$k = -0.9$ **M1** solving either unknown

$Eqn\ 1 + 2 \times Eqn\ 2: \ 5h = -4$

$h = -0.8$

$\therefore\ D(-0.9, -0.8)$ **A1**

Alternative method: finding eqn of line *AD* and eqn of line *CD*.

(iii) Find the area of the trapezium *ABCD*. [2]

$$Area = \frac{1}{2}\begin{vmatrix} 1 & 0 & -2.5 & -0.9 & 1 \\ 3 & 5 & 0 & -0.8 & 3 \end{vmatrix}$$

M1 shoelace method
Or attempt to cut up trapezium

$$= \frac{1}{2}\left[\left(5 + 2 - \frac{2}{7}\right) - (-12.5 - 0.8)\right]$$

$= 21\ units^2$ **A1**

8 **(a)** Solve the equation $3\log_x 3 = 8 - 4\log_3 x$. [5]

$$3\log_x 3 = 8 - 4\log_3 x$$

$$\frac{3}{\log_3 x} = 8 - 4\log_3 x$$ **M1** common log base 3 eqn

$$Let\ y = \log_3 x$$

$$\frac{3}{y} = 8 - 4y$$

$$3 = 8y - 4y^2$$

$$4y^2 - 8y + 3 = 0$$ **M1** simplify to quad eqn

$$(2y-3)(2y-1) = 0$$

$$y = 1.5\ or\ 0.5$$ **M1**

$$x = 3^{1.5}\ or\ 3^{0.5}$$

$$= \sqrt{27}\ or\ \sqrt{3}$$ **A1, A1**

(b) It is given that $\log_a x = p$ and $\log_a y = q$.

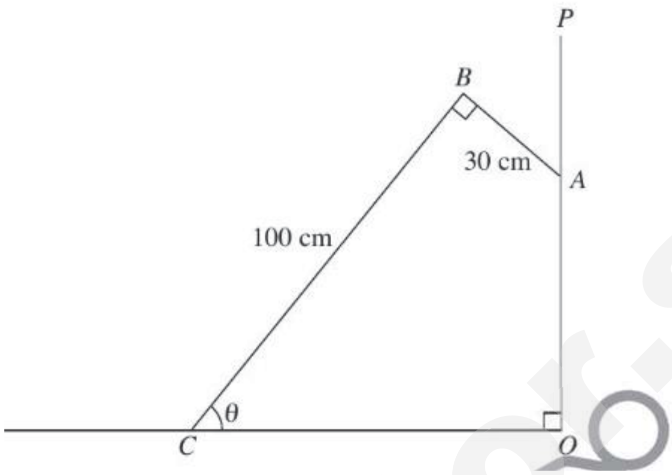
Express $\log_y ax^2y^3$ in terms of p and q. [3]

$$\log_y ax^2y^3 = \log_y a + 2\log_y x + 3\log_y y$$ **M1** splitting of logs

$$= \frac{1}{\log_a y} + 2 \cdot \frac{\log_a x}{\log_a y} + 3$$ **M1**

$$= \frac{1}{q} + \frac{2p}{q} + 3$$ **A1**

9 The figure shows a stage prop *ABC* used by a member of the theatre, leaning against a vertical wall *OP*. It is given that $AB = 30$ cm, $BC = 100$ cm, $\angle ABC = \angle AOC = 90°$ and $\angle BCO = \theta$.



(i) Show that $OC = (100\cos\theta + 30\sin\theta)$ cm. [2]

Let *D* be foot of *B* on *OC*, let *E* be foot of *A* on *BD*.

$\cos\theta = \frac{CD}{100} \Rightarrow CD = 100\cos\theta$ **M1**

$\sin\theta = \frac{AE}{30} \Rightarrow AE = 30\sin\theta$ **M1**

$OC = CD + AE = 100\cos\theta + 30\sin\theta$

(ii) Express *OC* in terms of $R\cos(\theta - \alpha)$, where *R* is a positive constant and α is an acute angle. [3]

$R = \sqrt{100^2 + 30^2}$ **M1 for R**

$= 100\sqrt{109}$

$\alpha = \tan^{-1}\left(\frac{30}{100}\right)$ **M1 for alpha**

$= 16.7°(1dp)$

$\therefore OC = 10\sqrt{109}\cos(\theta - 16.7°)$ **A1**

(iii) State the maximum value of *OC* and the corresponding value of θ. [2]

$OC_{max} = 10\sqrt{109}$ **B1**

$\theta = 16.7°$ **B1**

(iv) Find the value of θ for which $OC = 80$ cm. [3]

$80 = 10\sqrt{109}\cos(\theta - 16.7°)$

$\cos(\theta - 16.7°) = \frac{8}{\sqrt{109}}$ **M1**

$\theta - 16.7° = 39.98°$ $(\theta\ is\ acute)$ **M1**

$\theta = 56.7°$ **A1**

10 Given that $y = a + b\cos 4x$, where a and b are integers, and x is in radians,

(i) state the period of y. [1]

$\frac{\pi}{2}$ **B1**

Given that the maximum and minimum values of y are 3 and – 5 respectively, find

(ii) the amplitude of y, [1]

$$amplitude = \frac{3-(-5)}{2}$$
$$= 4$$ **B1**

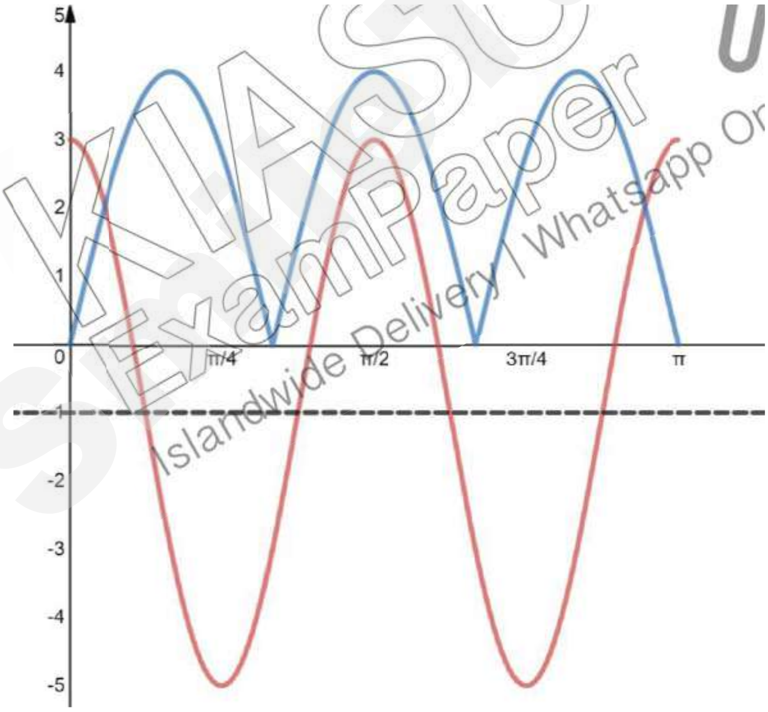
(iii) the value of a and of b. [2]

$b = 4$ **B1**

$a = -1$ **B1**

Using the values of a and b found in part **(iii)**,

(iv) sketch the graph of $y = a + b\cos 4x$ for $0 \le x \le \pi$. [3]



C1 shape (cos)

P1 two periods

I1 impt coords

ECF max 2 marks

S1 shape (sin)

M1 modulus

(v) On the same set of axes, sketch the graph of $y = |4\sin 3x|$, and hence state the number of solutions of $a + b\cos 4x = |4\sin 3x|$. [3]

Number of solutions = 2 **A1**

11 The dimensions of a cuboid are $3x$ cm by $2x$ cm by h cm and its total surface area is 312 cm^2. The volume of the cuboid is V cm^3.

(i) Express h in terms of x. [2]

$$2[3x(2x) + 3xh + 2xh] = 312$$
$$6x^2 + 5xh = 156$$
$$h = \frac{156 - 6x^2}{5x}$$

M1 form eqn of total SA

A1

(ii) Show that $V = \frac{36}{5}x(26 - x^2)$. [2]

$$V = (3x)(2x)\left(\frac{156 - 6x^2}{5x}\right) \quad \textbf{M1}$$
$$= 6x\left(\frac{156 - 6x^2}{5}\right) \quad \textbf{M1}$$
$$= \frac{36}{5}x(26 - x^2)$$

(iii) Find the maximum volume of the cuboid as x varies, giving your answer to the nearest cm^3. [5]

$$\frac{dV}{dx} = \frac{36}{5}[(26 - x^2) + x(-2x)]$$
$$= \frac{36}{5}[-3x^2 + 26]$$

M1 differentiate

$$\frac{dV}{dx} = 0$$
$$3x^2 - 26 = 0$$
$$x^2 = \frac{26}{3}$$
$$x = \pm\sqrt{\frac{26}{3}} \; (rej. -ve \because x > 0)$$

M1 solve for x

$$\frac{d^2V}{dx^2} = \frac{36}{5}(-6x)$$
$$\left.\frac{d^2V}{dx^2}\right|_{x=\sqrt{\frac{26}{3}}} = \frac{36}{5}(-6)\left(\sqrt{\frac{26}{3}}\right) < 0 \Rightarrow max$$

M1 2nd deriv. test

$$V = \frac{36}{5}\left(\sqrt{\frac{26}{3}}\right)\left(26 - \frac{26}{3}\right) \quad \textbf{M1}$$
$$= 367.4 \ldots$$
$$= 367 \; cm^3 \quad \textbf{A1}$$

– **End of Paper** –

Name: **Register Number:** **Class:**

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南僑中學

NAN CHIAU HIGH SCHOOL

MID-YEAR EXAMINATION 2019
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS
Paper 1

4047/01
14 May 2019, Tuesday

2 hours

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

Setter: Mdm Chua Seow Ling and Mdm Siak Chock Kwun

This document consists of **13** printed pages including the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$cosec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Given that $\sqrt{64^x} = \frac{8^{x+1}}{32^{1-2x}}$, find the value of x. [3]

2 Prove the identity $(\tan x + \sec x)^2 = \frac{1+\sin x}{1-\sin x}$. [2]

3 The equation of a curve is $y = \frac{2x^2}{1-3x}$.

(i) Find an expression for $\frac{dy}{dx}$. [2]

(ii) Given that x is changing at a constant rate of 0.05 units per second, find the rate of change of y when $x = 3$. [2]

4 Express $\frac{-4x^3+11x^2-16x+9}{x(2x-1)(x^2+3)}$ in partial fractions. [7]

5 **(i)** Express $\left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)^2$ in the form $a + b\sqrt{15}$, where a and b are integers. [4]

(ii) Given that $y = 2x^2 - px + 8$ and that $y < 0$ only when $(\sqrt{3} - 1) < x < k$, find the exact value of p and of k. [5]

6 **(i)** Sketch the graph of $y = 2\sqrt{x^3}$ for $x \geq 0$. [1]

(ii) Find the coordinates of the point~~s~~ of intersection of the curve $y = 2\sqrt{x^3}$ and the line $y = -2x + 4$. [5]

7 Solve each of the following equations.

(i) $10^{\log_5 x} = 5$ [3]

(ii) $\log_2(6 - x) - \log_2(x - 2) = 3 - \log_2(2x + 1)$ [4]

8 A curve is such that $\frac{dy}{dx} = 2x^2 - x - 10$. The curve has a maximum y value of 13. Find the equation of the curve. [6]

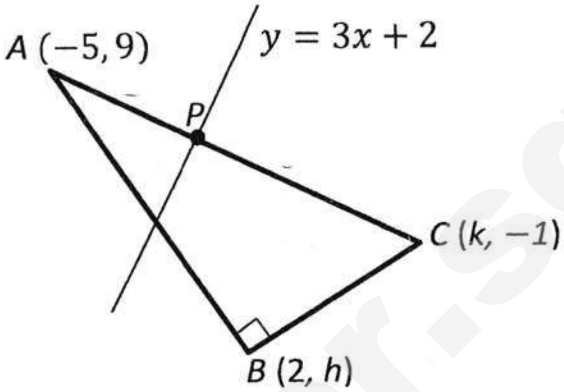
9 It is given that $y = -\frac{1}{9}\ln(3x-2) - 2x + 3$ for $x > \frac{2}{3}$.

(i) Determine, with appropriate workings, whether y is increasing or decreasing. [5]

(ii) Find the range of values of x for which $\frac{dy}{dx}$ is increasing. [2]

10 ***Solutions to this question by accurate drawing will not be accepted.***

The diagram shows a right-angled triangle ABC such that $\angle ABC = 90°$. Given that the coordinates of A, B and C are $(-5, 9)$, $(2, h)$ and $(k, -1)$ respectively where h and k are integers. The line $y = 3x + 2$ meets AC at P such that $5AP = 2AC$.



Find

(i) the coordinates of P, [2]

(ii) the value of k and of h, [4]

(iii) the area of the triangle ABC. [2]

11 **(i)** Find the range of values of x for which $|2x - 3| > 7$. [3]

(ii) Given that the coordinates of the maximum point of the graph $y = a|bx - 3| + c$ is $\left(\frac{3}{4}, 5\right)$, where a, b and c are integers. The y-intercept of the graph is -4.

(a) Find the value of a, of b and of c. [3]

(b) Find the coordinates of the x-intercepts. [4]

12 It is given that $y = 2\cos^2 x - 4\sin^2 x$ for $0 \leq x \leq 2\pi$.

(i) Express y in the form $a + b\cos 2x$, where a and b are integers. [3]

(ii) Hence, state the period and amplitude of y. [2]

(iii) Sketch the graph of $y = 2\cos^2 x - 4\sin^2 x$ for $0 \leq x \leq 2\pi$. [3]

(iv) On the same axes, draw a suitable straight line to find the number of solutions that satisfy the equation $x = 2\pi - 3\pi\cos 2x$ for $0 \leq x \leq 2\pi$. [3]

--- End of Paper ---

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Name: **Register Number:** **Class:**

南僑中學

NAN CHIAU HIGH SCHOOL

MID-YEAR EXAMINATION 2019
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS **4047/02**

Paper 2 **15 May 2019, Wednesday**

2 hours 30 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

Setter: Mrs Sim Hwee Mung and Ms Doris Toh

This document consists of **20** printed pages including the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive intege and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\cdots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1. **(a)** State the values between which each of the following must lie:

(i) the principal value of $\tan^{-1} x$, [1]

(ii) the principal value of $\cos^{-1} x$. [1]

(b) Without using a calculator, find the **exact** value of $\tan 105°$. [3]

2. A curve is such that $\frac{d\,^2y}{dx^2} = \frac{36}{(1-2x)^3}$. The gradient of the tangent at the point $(-1\,,3)$ is $\frac{1}{2}$. Find the equation of the curve. [5]

3. The roots of the equation $x^2 + mx + n = 0$ are α and β, where α β > 0.

Given that $\alpha^2 - \beta^2 = 13$, $\alpha - \beta = -1$ and $2\beta^2 = 72$, find the value of *m* and of *n*. [7]

4. Given that $y = (k-2)x^2 - kx + k - x - 2$, find the range of values of k for which y is always positive. [7]

5. An object is heated until it reaches a temperature of T_0 °C. It is then allowed to cool. Its temperature, T °C, when it has been cooled for n minutes, is given by the equation $T = 33 + 12e^{-\frac{3}{4}n}$.

(i) Find the value of T_0. [1]

(ii) Find the value of n when $T = 37$ °C. [1]

(iii) Find the value of n at which the rate of decrease of temperature is 0.67 °C/minute. [2]

(iv) Explain why the temperature of the object is always greater than 33 °C. [1]

(v) Sketch the graph of $T = 33 + 12e^{-\frac{3}{4}n}$. [2]

6. The polynomial $f(x) = ax^3 + x^2 + bx + 6$ has a factor of $(x + 2)$ and leaves a remainder of 18 when divided by $(x - 1)$.

(i) Find the value of a and of b. [4]

(ii) Factorise $f(x) = ax^3 + x^2 + bx + 6$ completely. [2]

(iii) **Hence**, using the values of a and b found in (i), solve the equation $a(y-1)^3 + (y-1)^2 + b(y-1) + 6 = 0$. [2]

7. **(i)** Differentiate $sin^3 2x$ with respect to x. [2]

(ii) Hence evaluate the following.

(a) $\int_0^{\frac{\pi}{8}} sin^2 2xcos2x\, dx$ [2]

(b) $\int_0^{\frac{\pi}{8}} cos^3 2x\, dx$ [4]

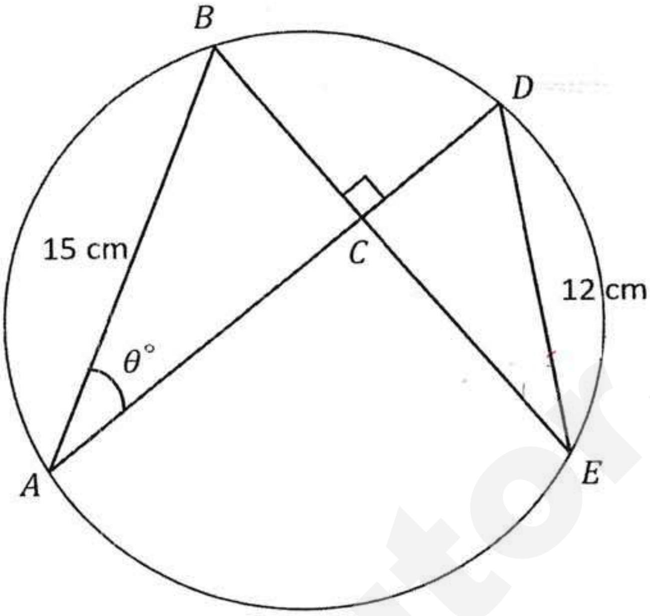
8. In the expansion of $(3+5x)^n$, the value obtained when coefficient of x^2 is divided by coefficient of x^3 is 0.3.

(i) Find the value of n. [4]

(ii) Hence, find the term independent of x in the expansion of $(3 + 5x)^n\ (1 - \frac{2}{x})^2$. [5]

9. In the diagram below, BE is perpendicular to AD.

Given that $\angle BAC = \theta^{\circ}$, where θ is an acute angle, $AB = 15$ cm and $DE = 12$ cm.



(i) Express ~~the~~ AD in the form $R\cos(\theta - \alpha)$, where R is positive and α is an acute angle. [4]

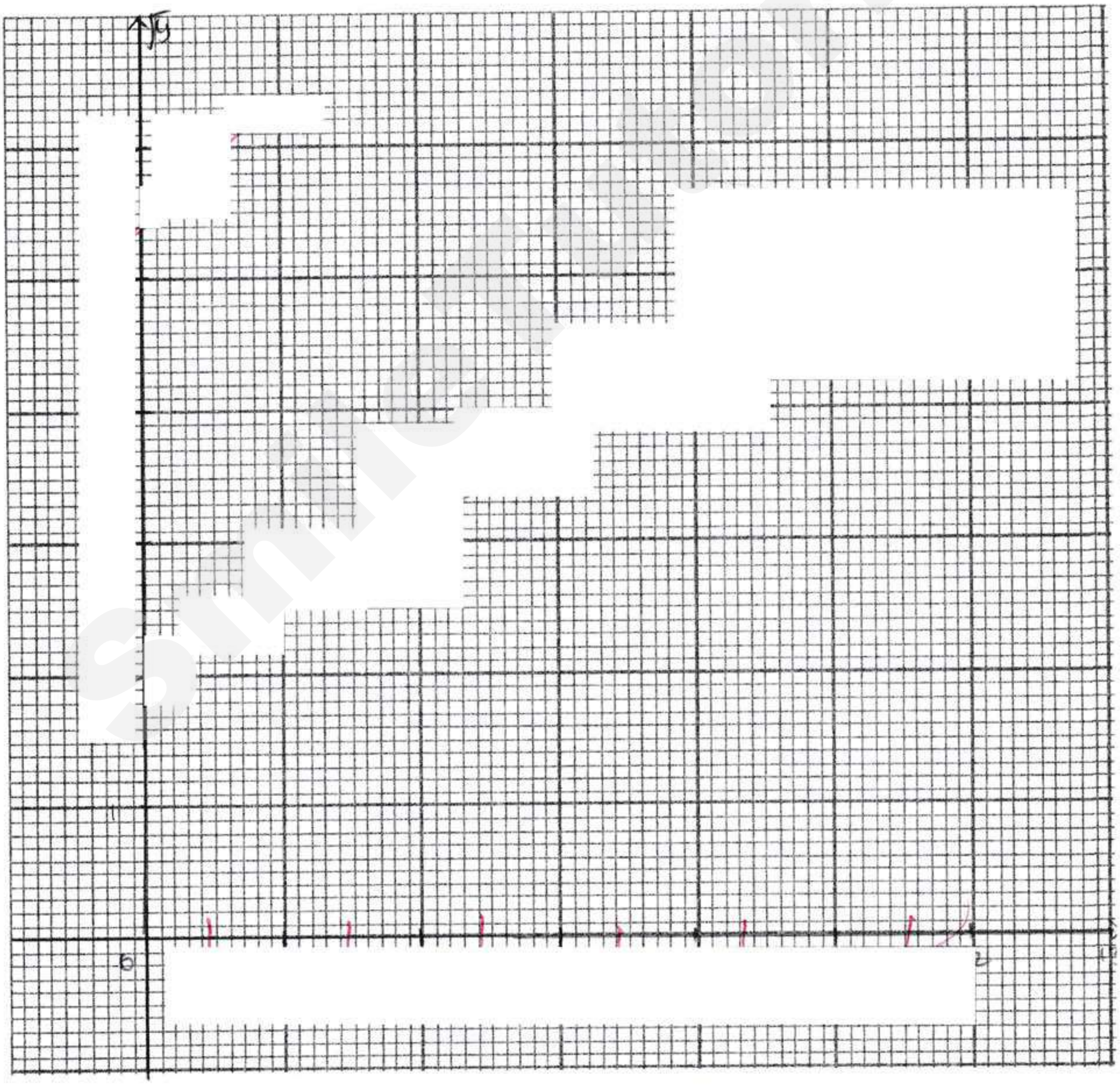
(ii) Find the value of θ for which $AD = 16.5$ cm. [3]

(iii) Given that AD is the diameter, find the length of AD and the corresponding value of θ. [3]

10. The table shows experimental values of two variables x and y. The two variables are related by the equation $b\sqrt{y} = ab + ax^2$, where a and b are non-zero constants. One of the y values have been misprinted.

x	1	1.5	2	2.5	3	3.5
y	5.23	6.98	7.88	14.3	20.9	30.3

(i) Using a scale of 1 cm to 1 unit on the x^2 axis and 2 cm to 1 unit on the $\sqrt{y}$ axis, plot x^2 against $\sqrt{y}$ and draw a straight line graph on the grid provided. [3]



(ii) Use your graph to estimate the value of a and of b. [4]

(iii) Using your graph, identify the abnormal reading and estimate its correct value. [3]

11 **(a)** Find the exact coordinates of the stationary points of the curve $y = 5x^2e^{-3x}$ and determine the nature of the stationary points. [6]

(b) A curve has the equation $y = \frac{x^3+2}{x^2}$. Find the value of k for which the line $y + \frac{27}{23}x = k$ is a normal to the curve. [6]

12. A circle, C_1, has equation $2x^2 - 3x + 2y^2 - \frac{1}{2}(4y - 3) = 0$.

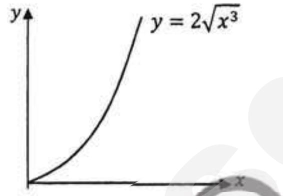
(i) Find the coordinates of the centre and the radius of C_1. [3]

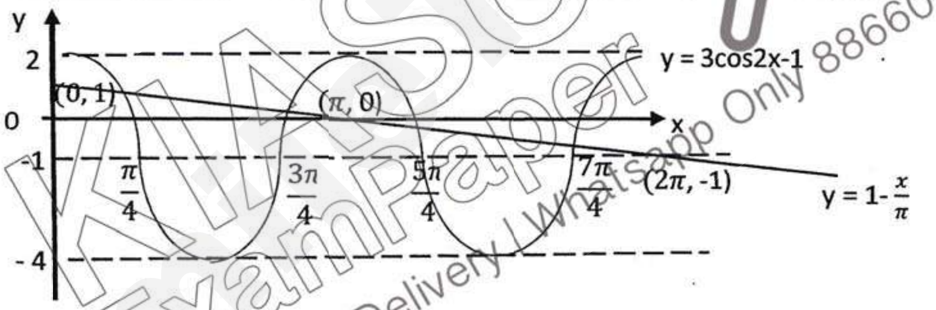
(ii) Show your working clearly whether the point $P\ (-1\,,2)$ lies inside or outside C_1. [2]

(iii) Find the equation of another circle, C_2, which is a reflection of C_1 in the line $y - x - 3 = 0$. [7]

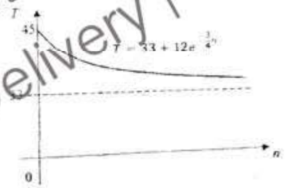
- End of Paper -

NCHS Sec 4 Mid-Year Exam 2019
Additional Mathematics Paper 1

1	$\sqrt{64^x} = \frac{8^{x+1}}{32^{1-2x}}$ $2^{6x} = \left(\frac{2^{3x+3}}{2^{5-10x}}\right)^2$ $2^{6x} = \frac{2^{6x+6}}{2^{10-20x}}$ $6x = 6x + 6 - (10 - 20x)$ $20x = 4$ $x = \frac{1}{5}$ or 0.2
2	$(\tan x + \sec x)^2$ $= \left(\frac{\sin x}{\cos x} + \frac{1}{\cos x}\right)^2$ $= \left(\frac{\sin x+1}{\cos x}\right)^2$ $= \frac{(\sin x + 1)^2}{\cos^2 x}$ $= \frac{(\sin x + 1)^2}{1-\sin^2 x}$ $= \frac{(\sin x + 1)^2}{(1-\sin x)(1+\sin x)}$ $= \frac{1+\sin x}{1-\sin x}$
3i	$y = \frac{2x^2}{1-3x}$ $\frac{dy}{dx} = \frac{4x(1-3x)-2x^2(-3)}{(1-3x)^2}$ $= \frac{4x-12x^2+6x^2}{(1-3x)^2}$ $= \frac{4x-6x^2}{(1-3x)^2}$ $= \frac{2x(2-3x)}{(1-3x)^2}$
3ii	$\frac{dx}{dt} = 0.05$ unit/s , $x = 3$ $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $= \frac{2(3)(2-9)}{(1-9)^2} \times 0.05$ $= -\frac{21}{640}$ or -0.0328 units/s
4	$\frac{-4x^3+11x^2-16x+9}{x(2x-1)(x^2+3)} = \frac{A}{x} + \frac{B}{2x-1} + \frac{Cx+D}{x^2+3}$ $-4x^3 + 11x^2 - 16x + 9$ $= A(2x-1)(x^2+3) + Bx(x^2+3) + x(Cx+D)(2x-1)$ Let $x = 0$, $9 = -3A$ $A = -3$ Let $x = \frac{1}{2}$, $\frac{13}{8}B = \frac{13}{4}$ $B = 2$ Compare x^3; $-4 = 2A + B + 2C$ $C = 0$ Let $x = 1$, $D = 4$ $\therefore$ $\frac{-3}{x} + \frac{2}{2x-1} + \frac{4}{x^2+3}$
5i	$\left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)^2$ $= \frac{5+2\sqrt{15}+3}{5-2\sqrt{15}+3}$ $= \frac{8+2\sqrt{15}}{8-2\sqrt{15}} \times \frac{8+2\sqrt{15}}{8+2\sqrt{15}}$ $= \frac{64+32\sqrt{15}+60}{64-60}$ $= \frac{124+32\sqrt{15}}{4} = 31 + 8\sqrt{15}$
5ii	$2x^2 - px + 8 < 0$ $2[x - (\sqrt{3} - 1)](x - k) < 0$ $2x^2 - 2kx - 2\sqrt{3}x + 2x + 2\sqrt{3}k - 2k < 0$ $2x^2 - (2k + 2\sqrt{3} - 2)x + 2\sqrt{3}k - 2k < 0$ Compare with $2x^2 - px + 8 < 0$, $2\sqrt{3}k - 2k = 8$ $k(2)(\sqrt{3} - 1) = 8$ $k = \frac{4}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = 2\sqrt{3} + 2$ $p = (2k + 2\sqrt{3} - 2)$ $= 2(2\sqrt{3} + 2) + 2\sqrt{3} - 2$ $= 6\sqrt{3} + 2$
6i	
6ii	$2\sqrt{x^3} = -2x + 4$ Square both sides, $4x^3 = (-2x + 4)^2$ $4x^3 = 4x^2 - 16x + 16$ $4x^3 - 4x^2 + 16x - 16 = 0$ Let $f(x) = 4x^3 - 4x^2 + 16x - 16$ $f(1) = 4 - 4 + 16 - 16 = 0$ $x - 1$ is a factor of $f(x)$ By inspection, $f(x) = (x - 1)(4x^2 + bx + 16)$ where b is a constant. Compare x^2, $-4 + b = -4$ $b = 0$ $4x^3 - 4x^2 + 16x - 16 = (x - 1)(4x^2 + 16) = 0$ $x - 1 = 0$ or $4x^2 + 16 = 0$ $x = 1$ (rejected) When $x = 1$, $y = -2 + 4 = 2$ Coordinates $= (1, 2)$
7i	$10^{\log_5 x} = 5$ $\log_5 x = \log_{10} 5$ $x = 5^{\lg 5}$ $= 3.08$
7ii	$\log_2(6 - x) - \log_2(x - 2) = 3 - \log_2(2x + 1)$ $\log_2(6 - x) - \log_2(x - 2) + \log_2(2x + 1) = 3$ $\log_2 \frac{(6-x)}{(x-2)} \times (2x + 1) = 3$ $\frac{(6-x)(2x+1)}{(x-2)} = 2^3$ $12x + 6 - 2x^2 - x = 8(x - 2)$ $2x^2 - 3x - 22 = 0$ $x = \frac{3\pm\sqrt{(-3)^2-4(2)(-22)}}{2(2)}$ $= \frac{3+\sqrt{185}}{4}$ (4.15) or $\frac{3-\sqrt{185}}{4}$ (−2.65)(reject)
8	$2x^2 - x - 10 = 0$ $(x + 2)(2x - 5) = 0$ $x = -2$ or 2.5 $\frac{d^2y}{dx^2} = 4x - 1$ $\frac{d^2y}{dx^2} = 4(2.5) - 1 = 9 > 0$, min y when x =2.5 $\frac{d^2y}{dx^2} = 4(-2) - 1 = -9 < 0$, max y when x = −2 $y = \int 2x^2 - x - 10\,dx$ $y = \frac{2x^3}{3} - \frac{x^2}{2} - 10x + c$ $13 = \frac{2(-2)^3}{3} - \frac{(-2)^2}{2} - 10(-2) + c$ $c = \frac{1}{3}$ $y = \frac{2}{3}x^3 - \frac{1}{2}x^2 - 10x + \frac{1}{3}$
9i	$\frac{dy}{dx} = \frac{1}{9}\left(\frac{1}{3x-2}\right)(3) - 2$ $= -\frac{1}{3(3x-2)} - 2$ Since $3x - 2 > 0$, $-\frac{1}{3(3x-2)} < 0$, $\frac{dy}{dx} < 0$ or $-\frac{1}{3(3x-2)} - 2 < 0$ Therefore y is decreasing for $x > \frac{2}{3}$.
9ii	$\frac{d^2y}{dx^2} = -\frac{1}{3}(-1)(3x-2)^{-2}(3)$ $= \frac{1}{(3x-2)^2}$ $\frac{1}{(3x-2)^2} > 0$ Therefore $x > \frac{2}{3}$.
10i	$y_P = -1 + \frac{10}{5}(3) = 5$ $5 = 3x + 2$ $x = 1$ P (1, 5)
10ii	$k = \frac{1-(-5)}{2}(5) - 5 = 10$ $\left(\frac{9-h}{-5-2}\right)\left(\frac{-1-h}{10-2}\right) = -1$ $(9 - h)(1 + h) = -56$ $h^2 - 8h - 65 = 0$ $h = 13\,(rej)\ or -5$

10iii	$A=\frac{1}{2}\begin{vmatrix}-5 & 2 & 10 & -5\\ 9 & -5 & -1 & 9\end{vmatrix}$ $=70$	11iib	$0=-3\lvert 4x-3\rvert+5$ $\lvert 4x-3\rvert=\frac{5}{3}$ $4x-3=\frac{5}{3}$ or $4x-3=-\frac{5}{3}$ $x=\frac{7}{6}$ or $x=\frac{1}{3}$ $\left(\frac{7}{6},\ 0\right)$ and $\left(\frac{1}{3},\ 0\right)$
11i	$(2x-3)^2>49$ $x^2-3x-10>0$ $(x-5)(x+2)>0$ $x<-2\ \ or\ \ x>5$ OR $2x-3>7\ \ or\ \ 2x-3<-7$ $x>5\ \ or\ \ x<-2$	12i	$y=2cos^2x-4(1-cos^2x)$ $y=6cos^2x-4$ $y=3(cos2x+1)-4$ $y=3cos2x-1$
11iia	$y=a\lvert 4x-3\rvert+5$ $-4=a\lvert -3\rvert+5$ $a=-3,\quad b=4,\quad c=5$	12ii	Period = π Amplitude = 3
12iii			
12iv	$y=3cos2x-1$ $\frac{x}{\pi}=2-3cos2x$ $3\ cos2x=2-\frac{x}{\pi}$ $3\ cos2x-1=1-\frac{x}{\pi}$ $y=1-\frac{x}{\pi}$		

CHS Sec 4 Mid-Year Exam 2019
dditional Mathematics Paper 2

1ai	$-90^\circ < tan^{-1}x < 90^\circ$ or $-\frac{\pi}{2} < tan^{-1}x < \frac{\pi}{2}$
1aii	$0^\circ \le cos^{-1}x \le 180^\circ$ or $0 \le cos^{-1}x \le \pi$
1b	$\tan 105^\circ$ $= \tan(60^\circ + 45^\circ)$ $= \frac{tan60^\circ + tan45^\circ}{1 - tan60^\circ tan45^\circ}$ $= \frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$ $= \frac{\sqrt{3}+3+1+\sqrt{3}}{1-3}$ $= \frac{2(\sqrt{3}+2)}{-2}$ $= -\sqrt{3} - 2$
2	$\frac{d^2y}{dx^2} = \frac{36}{(1-2x)^3}$ $\frac{dy}{dx} = \int 36(1-2x)^{-3}\,dx$ $= \frac{36(1-2x)^{-2}}{(-2)(-2)} + c$ $= \frac{9}{(1-2x)^2} + c$ Sub $x = -1, \frac{dy}{dx} = \frac{1}{2}$, $c = -\frac{1}{2}$
	$\frac{dy}{dx} = \frac{9}{(1-2x)^2} - \frac{1}{2}$ $y = \int 9(1-2x)^{-2} - \frac{1}{2}dx$ $y = \frac{9}{2(1-2x)} - \frac{1}{2}x + d$ Sub $x = -1, y = 3$, $3 = \frac{9}{2(3)} + \frac{1}{2} + d$ $d = 1$ $y = \frac{9}{2(1-2x)} - \frac{1}{2}x + 1$
3	$\alpha + \beta = -m$ $\alpha\beta = n$ $(\alpha+\beta)(\alpha-\beta) = \alpha^2 - \beta^2$ $(\alpha+\beta)(-1) = 13$ $\therefore m = 13$ $2\beta^2 = 72$ $\beta = 6 \; or -6$ Since $\alpha - \beta = -1$, $\alpha = 5$ (rejected) or $\alpha = -7$ $n = \alpha\beta = (-7)(-6) = 42$
4	$y = (k-2)x^2 - (k+1)x + (k-2)$ $b^2 - 4ac < 0$ $[-(k+1)]^2 - 4(k-2)(k-2) < 0$ $k^2 + 2k + 1 - 4(k^2 - 4k + 4) < 0$ $-3k^2 + 18k - 15 < 0$ $-k^2 + 6k - 5 < 0$ $(k-5)(k-1) > 0$ $k < 1 \; or \; 5 < k$ $k - 2 > 0$ $k > 2$ Answer: $\therefore k > 5$
5i	Sub $n = 0$, $T_0 = 45^\circ$
5ii	$37 = 33 + 12e^{-\frac{3}{4}n}$ $n = 1.46$
5iii	$\frac{dT}{dn} = 12(-\frac{3}{4})e^{-\frac{3}{4}}$ $-0.67 = -9e^{-\frac{3}{4}n}$ $n = 3.46$
5iv	$12e^{-\frac{3}{4}n} > 0$ $33 + 12e^{-\frac{3}{4}n} > 33$ $T > 33$°C (shown)
5v	
6i	$f(-2) = 0$ $-8a - 2b + 10 = 0$ $8a + 2b = 10$ $4a + b = 5$ ------ {1} $f(1) = 18$ $a + b = 11$ ----- {2}
	{1} - {2} : $3a = -6$ $a = -2$ $-2 + b = 11$ $b = 13$
6ii	$f(x) = -2x^3 + x^2 + 13x + 6$ $= (x+2)(-2x^2 + 5x + 3)$ $= (x+2)(3-x)(2x+1)$
6iii	Let $x = y - 1$ $-(y - 1 + 2)(y - 1 + 3)(2(y - 1) + 1) = 0$ $-(y+1)(y-4)(2y-1) = 0$ $y = -1, 4, 0.5$
7i	$\frac{d}{dx}(sin^3 2x) = 3sin^2 2x(2cos2x)$ $\frac{d}{dx}(sin^3 2x) = 6sin^2 2xcos2x$
7iia	$\int_0^{\frac{\pi}{8}} sin^2 2xcos2x\,dx$ $= \frac{1}{6}\int_0^{\frac{\pi}{8}} \frac{d}{dx}(sin^3 2x)dx$ $= \frac{1}{6}[sin^3 2x]_0^{\frac{\pi}{8}} = 0.0589$ (3sf)
7iib	$\int_0^{\frac{\pi}{8}} cos^3 2x\,dx$ $= \int_0^{\frac{\pi}{8}} cos^2 2xcos2x\,dx$ $= \int_0^{\frac{\pi}{8}}(1 - sin^2 2x)cos2x\,dx$ $= \int_0^{\frac{\pi}{8}} cos2x\,dx - \frac{1}{6}\int_0^{\frac{\pi}{8}} 6\,sin^2 2xcos2x\,dx$ $= [\frac{1}{2}sin2x - \frac{1}{6}sin^3 2x]_0^{\frac{\pi}{8}}$ $= 0.295$ (3sf)
8i	$\binom{n}{2}(3)^{n-2}(5x)^2 = \frac{n(n-1)}{2}3^{n-2}(25x^2)$ $\binom{n}{3}(3)^{n-3}(5x)^3 = \frac{n(n-1)(n-2)}{6}3^{n-3}(125x^3)$ $\frac{\frac{n(n-1)}{2}3^{n-2}(25x^2)}{\frac{n(n-1)(n-2)}{6}3^{n-3}(125x^3)} = \frac{3}{10}$ $\frac{3}{\frac{5(n-2)}{3}} = \frac{3}{10}$ $30 = 5(n-2)$ $6 = n - 2$ $n = 8$
8ii	$(3+5x)^8(1 - \frac{2}{x})^2$ $= (3+5x)^8(1 - \frac{4}{x} + \frac{4}{x^2})$ x term : $\binom{8}{1}(3)^7(5x)$ $= 87480\,x$ x^2 term: $\binom{8}{2}(3)^6(5x)^2$ $= 510300\,x^2$ Term independent of x $= 3^8(1) - 4\,(87480) + 4\,(510300)$ $= 1697841$

9i

angle $DEC = \theta$ (angles in same segment)

$15\cos\theta + 12\sin\theta$

$= \sqrt{15^2 + 12^2}\cos(\theta - \tan^{-1}\left(\frac{12}{15}\right))$

$= \sqrt{369}\cos(\theta - 38.65981°)$

$= 3\sqrt{41}\cos(\theta - 38.7°)$

$= 19.2\cos(\theta - 38.7°)$

9ii

$16.5 = 3\sqrt{41}\cos(\theta - 38.65981)$

$\cos(\theta - 38.65981) = 0.85896$

Basic angle = 30.80°

$\theta = -30.80°, 69.46028°$

$= 7.9°, 69.5°$

Full mark was given even though students missed out 7.9°.

9iii

$-1 \le \cos(\theta - 38.65981) \le 1$

Max value = $3\sqrt{41}$

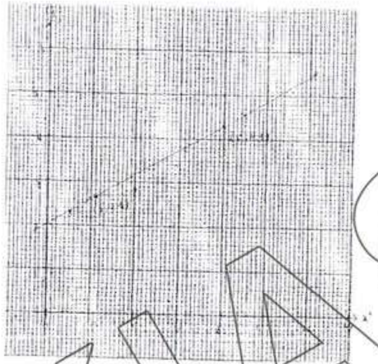
$\cos(\theta - 38.65981) = 1$

$\theta - 38.65981 = 0$

$\theta = 38.7°$

10i

x^2	1	2.25	4	6.25	9	5.50
$\sqrt{y}$	2.29	2.64	2.81	3.78	4.57	12.25



10ii

$b\sqrt{y} = ab + ax^2$,

$\sqrt{y} = \frac{a}{b}x^2 + a$

$m = \frac{4.3-2.6}{8-2} = \frac{17}{60} = 0.28333 = \frac{a}{b}$

Y-intercept = $a = 2$

$\frac{17}{60} = \frac{2}{b}$

b = 7.0588

= 7.06

Accept 6.9 - 7.1

10iii

Abnormal reading when $x^2 = 4$, $\sqrt{y} = 2.81$

Correct $\sqrt{y}$ should be 3.15, $y = 9.92$ (3.s.f)

Accept $\sqrt{y}$ - 3.05 – 3.15

Accept y = 9.3025 – 10.5625

11a

$y = 5x^2e^{-3x}$ ....(1)

$\frac{dy}{dx} = 10xe^{-3x} + 5x^2e^{-3x}(-3)$

$= 5xe^{-3x}(2 - 3x)$

At stationary point, $\frac{dy}{dx} = 0$

$5xe^{-3x}(2 - 3x) = 0$

$x = 0$, $2 - 3x = 0$ or $e^{-3x} = 0$ (rejected)

$x = 0$, $x = \frac{2}{3}$

Sub $x = 0$ into (1),

$y = 0$

Sub $x = \frac{2}{3}$ into (1),

$y = \frac{20}{9e^2}$

x	-0.1	0	0.1
Sign of $\frac{dy}{dx}$	–	0	+
Sketch of tangent			

(0, 0) is a minimum point

x	0.57	0.67	0.77
Sign of $\frac{dy}{dx}$	+	0	–
Sketch of tangent			

$(\frac{2}{3}, \frac{20}{9e^2})$ is a maximum point

11b

$y = \frac{x^3+2}{x^2}$ ....(1)

$= x + 2x^{-2}$

$\frac{dy}{dx} = 1 - \frac{4}{x^3}$

Equation of normal: $y + \frac{27}{23}x = k$

$y = -\frac{27}{23}x + k$ ....(2)

Gradient of normal = $-\frac{27}{23}$

Gradient of tangent = $\frac{23}{27}$

$\frac{23}{27} = 1 - \frac{4}{x^3}$

$x = 3$

Sub $x = 3$ into (1),

$y = 3\frac{2}{9}$

Sub $(3, 3\frac{2}{9})$ into (2)

$3\frac{2}{9} = -\frac{27}{23}(3) + k$

$k = \frac{1396}{207}$ or $6\frac{154}{207}$

12i

$2x^2 - 3x + 2y^2 - \frac{1}{2}(4y - 3) = 0$

$x^2 + y^2 - 1.5x - y + 0.75 = 0$

Centre = $(\frac{-1.5}{-2}, \frac{-1}{-2}) = (0.75, 0.5)$

Radius = $\sqrt{0.75^2 + 0.5^2 - 0.75}$

$= 0.25$

12ii

Distance of P from centre

$= \sqrt{(0.75 + 1)^2 + (0.5 - 2)^2}$

$= 2.3049 >$ radius

Hence, P lies outside the circle.

12iii

$y = x + 3$

Gradient = 1

Perpendicular gradient = -1

Equation of the line joining the two centres:

$y - 0.5 = -(x - 0.75)$

$y = -x + 1.25$ ...... (1)

$y = x + 3$............. (2)

Sub (2) into (1),

$x + 3 = -x + 1.25$

$x = -\frac{7}{8}$

Sub $x = -\frac{7}{8}$ into (2),

$y = 2\frac{1}{8}$

Let centre of C_2 be (x, y),

$(-\frac{7}{8}, 2\frac{1}{8}) = (\frac{x+0.75}{2}, \frac{y+0.5}{2})$

$x = -2.5$

$y = 3\frac{3}{4}$

Equation of C_2:

$(x + 2.5)^2 + \left(y - 3\frac{3}{4}\right)^2 = \frac{1}{16}$

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ST. MARGARET'S SECONDARY SCHOOL

Mid-Year Examinations 2019

CANDIDATE NAME ☐

CLASS ☐☐☐ REGISTER NUMBER ☐☐

ADDITIONAL MATHEMATICS **4047/01**

Paper 1 **7 May 2019**

Secondary 4 Express **2 hours**

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of 17 printed pages and a blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$, $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where *n* is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

[Turn over

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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Identities

$$\sin^2 A + \cos^2 A = 1$$

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$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

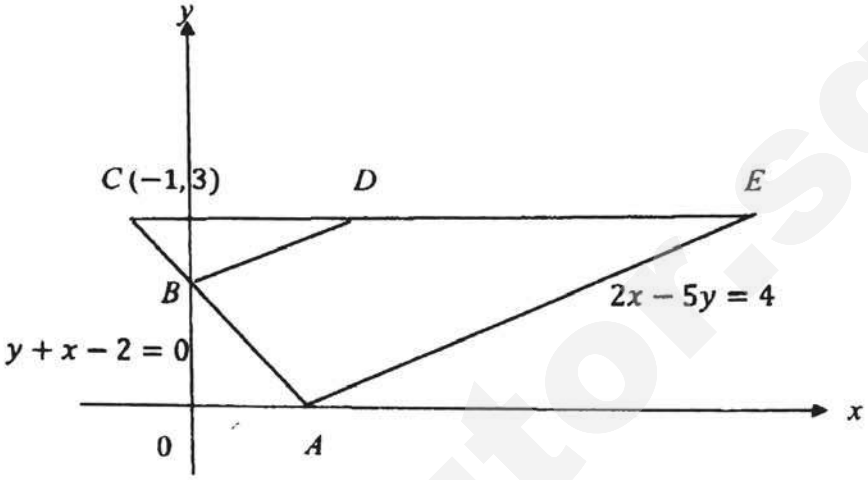
$$\Delta = \frac{1}{2}bc \sin A$$

2 **(i)** A prism with a trapezium base has a volume of $24 + 17\sqrt{2}$ cm^3. The trapezium has a height of $\sqrt{2}$ cm and its parallel sides are $3\sqrt{2} + 2$ cm and 2 cm respectively. Find the height of the prism, leaving your answer in the form $a\sqrt{2} + b$ cm, where a and b are integers. [3]

(ii) Simplify $\frac{5}{\sqrt{2}} + 2\sqrt{50} - \frac{2}{\sqrt{8}}$. [2]

3 Variables x and y are related by the equation $y = \frac{11x-1}{9-x}$. Given that x and y are functions of t and that y increases from an initial value of 2.9 at a constant rate of 0.005 units/s, find the corresponding rate of change of x after 20 seconds. [5]

4 The diagram shows a trapezium *BDEA* in which *BD* is parallel to *AE*. The side *ED* is parallel to the *x*-axis. It is extended to meet at point *C* which has coordinates $(-1, 3)$. The equation of *AE* is $2x - 5y = 4$ and the equation of *AC* is $y + x - 2 = 0$.



Find

(i) the coordinates of *A*, *E* and *D*. [4]

(ii) the ratio of area of triangle BCD to area of trapezium $BDEA$. [1]

5 **(i)** Differentiate $x^2 \ln x$ with respect to x. [2]

(ii) Hence find $\int x \ln x \ dx$. [3]

6 Given that $\sin A = \frac{24}{25}$ where A is acute, $\tan B = \frac{3}{4}$ and that A and B are in different quadrants, find, without evaluating A or B, the value of

(i) $\sin(A + B)$, [3]

(ii) $\cos\left(\frac{B}{2}\right)$. [2]

7 It is given that a curve has equation $= f(x)$, where $f(x) = (2x + 3)(x - 2)^2$.

(i) Find the coordinates of the stationary points of the curve. [4]

(ii) Hence, determine the nature of these stationary points. [3]

(iii) Sketch the graph of $f'(x)$ against x. [2]

8 (i) The quadratic function is defined by $y = 2x^2 - 8x - 15$, where x is real.

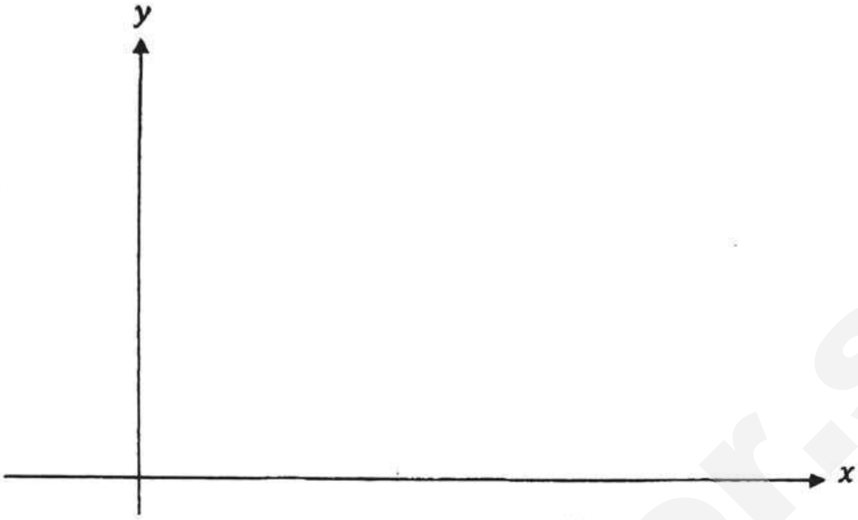
(a) Find the set of values of x for which $y \leq 3x^2$. [3]

(b) Find the set of values of k for which the equation $y = kx - 23$ has no real roots. [3]

(ii) Show that the line $y = \frac{x}{p} + \frac{p}{2}$ is a tangent to the curve $y^2 = 2x$ for all real values of p. [3]

[Turn over

9 **(i)** Sketch the graph of = $3\sqrt{x}$. [1]



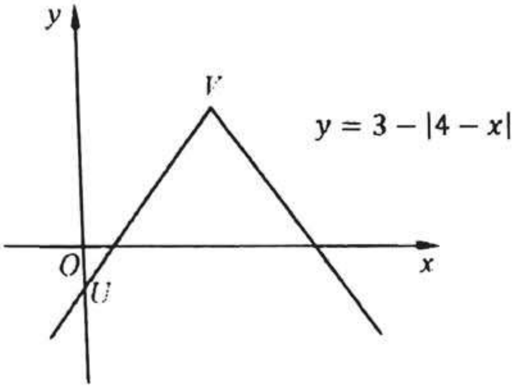
(ii) On the same axes, sketch the graph of $y = \frac{12}{\sqrt{x^3}}$, $x > 0$. [1]

(iii) Calculate the x co-ordinate of the point of intersection of your graphs in exact form. [2]

(iv) Determine, with explanation, whether the tangents to the graphs at the point of intersection are perpendicular. [4]

[Turn over

10



The diagram shows part of the graph $y = 3 - |4 - x|$ intersecting the y-axis at U.

V is the highest point on the graph.

(i) Find the coordinates of U and V. [2]

The equation of a line is $y = mx + 3$, where m is a constant.

(ii) In the case where $m = -2$, find the coordinates of any point of intersection of the line and the graph of $y = 3 - |4 - x|$. [3]

(iii) Determine the range of values of m for which the line intersects the graph of $y = 3 - |4 - x|$ in two points. [2]

11 The roots of the quadratic equation $2x^2 + 5x - 4 = 0$ are α and β.

Find

(i) the value of $\alpha^3 + \beta^3$, [4]

(ii) a quadratic equation with roots $\dfrac{1}{\alpha^3}$ and $\dfrac{1}{\beta^3}$. [4]

12 **(i)** Show that $\frac{4\cos 2x}{1+\cos 2x}$ can be written as $+bsec^2x$, where a and b are integers. **[4]**

(ii) Solve, for $0^o < x < 180^o$, the equation $\frac{4\cos 2x}{1+\cos 2x} = 4\tan x - 5$. [4]

(iii) State the number of solutions of the equation $\frac{4\cos 2x}{1+\cos 2x} = 4\tan x - 5$ in the range $-360^o < x < 360^o$. [1]

ST. MARGARET'S SECONDARY SCHOOL.

Mid-Year Examinations 2019

CANDIDATE NAME

CLASS

REGISTER NUMBER

ADDITIONAL MATHEMATICS **4047/02**

Paper 2 **10 May 2019**

Secondary 4 Express **2 hours 30 minutes**

Candidates answer on the Question Paper

Additional Materials: Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of **18** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 A curve has the equation $y = x^2 e^{2-3x}$.

Find the range of values of x for which y is an increasing function of x. [3]

2 Find all the exact values of x which satisfies the equation

$8 \cos x - 2 \sin x \cos x + 4 - \sin x = 0$ for $0 \leq x \leq 9$. [3]

3 Differentiate xe^{5x} with respect to x. Hence find $\int xe^{5x} \, dx$. [4]

4 Evaluate the following definite integrals, giving your answer in exact form.

(a) $\int_0^1 \frac{3e^{5x}-7}{e^{2x}} \, dx$, [3]

(b) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{4-\cos^2 2x}{1-\sin^2 2x} \, dx$. [3]

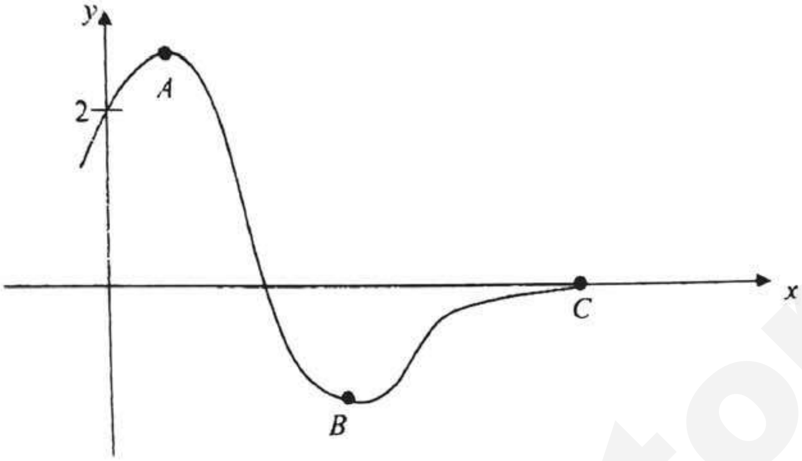
5 Express $\frac{x^3}{x^2+3x+2}$ in the form $Ax+B+\frac{C}{x+2}+\frac{D}{x+1}$, where A, B, C and D are constants. Hence evaluate $\int \frac{x^3}{x^2+3x+2}\ \mathrm{d}x$. [6]

6 (i) **Find the number of real roots of the equation $3x^3 + 2x^2 + 10 = 3x$.** [4]

(ii) Hence solve the equation $3 + 2y - 3y^2 + 10y^3 = 0$. [2]

7 The diagram shows part of a graph whose gradient function is given by

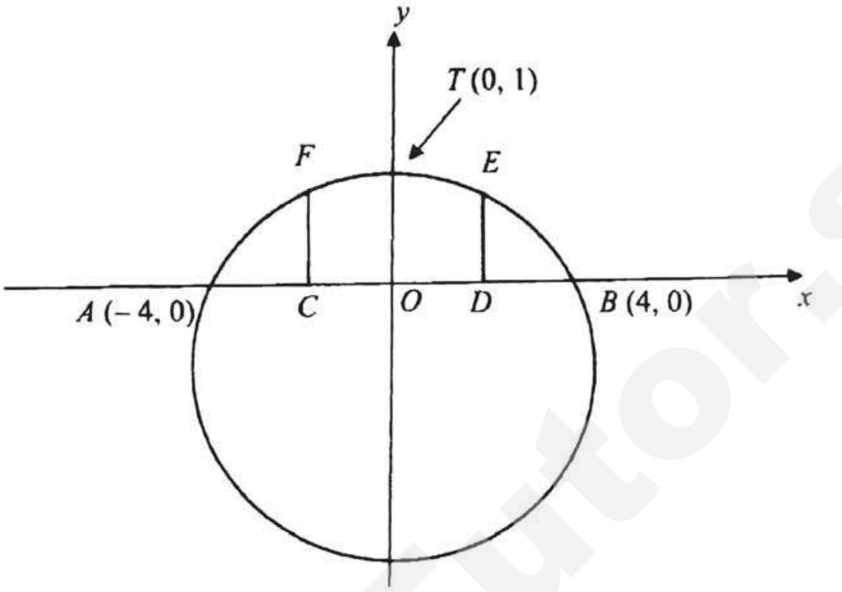
$\frac{dy}{dx} = 2\cos 2x - 2\sin x$. A, B and C are stationary points on the graph.



(i) Show that C is a point of inflexion. [4]

(ii) Find the equation of the curve. [2]

8 The diagram shows the arch *AFTEB* of a stone bridge. The bridge forms an arc of a circle and the length *AB* forms a chord of the circle. *AB* is 8 m and the top of the bridge *T* is 1 m vertically above *AB*. *C* and *D* are the midpoints of *OA* and *OB*. *CF* and *DE* are two vertical pillars supporting the arch.



(i) Show that the equation of the circle is $x^2 + y^2 + 15y - 16 = 0$. [4]

(ii) Find the height of the pillar *CF*. [2]

9 (i) Given that $y = \cos^3 x$, show that $\frac{dy}{dx} = 3\sin^3 x - 3\sin x$. [2]

(ii) Hence evaluate $\int_0^{\frac{\pi}{3}} \sin^3 x \, dx$. [5]

10 (a) Solve the equation $\ln(2x + e) = 1 + \frac{1}{\log_x e}$, leaving your answer in the exact form. [3]

10 (b) Without using a calculator, find the exact value of y if $(5y)^{\ln 5} = (2y)^{\ln 2}$. [5]

11 **Answer the whole question on a sheet of graph paper.**

The table below shows the experimental values of the variables x and y.

x	0.5	1.5	3	4.5	5.5	6
y	4.43	6.24	7.44	11.4	15.7	18.7

It is known that x and y are related by an equation of the form $y = e + ab^x$.

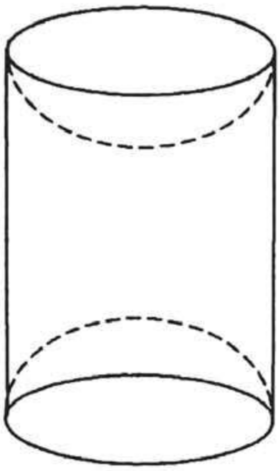
One of the y values is incorrect.

(i) Plot a straight line graph of ln $(y - e)$ against x. [4]

(ii) Use your graph to identify the abnormal reading and estimate its correct value. [2]

(iii) Use the graph to estimate the value of a and of b. [2]

12 A solid right circular cylinder with base radius r cm and height h cm has a hemisphere hollowed out from each end as shown in the diagram.



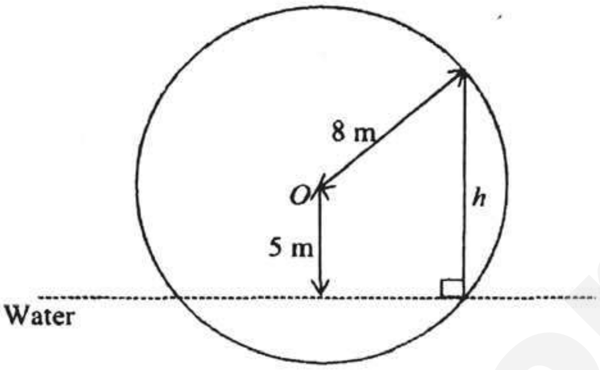
Given that the surface area is 128π cm^2,

(i) show that the volume of the solid, V cm^3, is given by $V = \frac{2\pi r}{3}\left(96 - 5r^2\right)$. [3]

(ii) find the value of r for which V is stationary, [2]

(iii) find the corresponding value of V and determine whether it is a maximum or a minimum value. [3]

13 A waterwheel rotates 5 revolutions anticlockwise in 1 minute. A bucket B is attached to the waterwheel. Tammy starts a stopwatch when the bucket B is at its highest height above water level. The radius of the waterwheel is 8 m and its centre is 5 m above the water level.



The height of the bucket B above water level is given by $h = a \cos bt + c$, where t is the time, in seconds, since Tammy started the stopwatch.

(i) Determine the value of each of the constant a, b, and c. [5]

(ii) For how long in each revolution is $h < 0$? [3]

(iii) Explain what does the answer in (ii) mean. [1]

14 (a) In the expansion of $\left(x^9 - \frac{1}{3x}\right)^{10}$, determine if there is a x^9 term. [3]

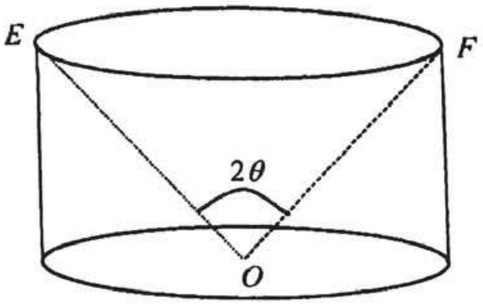
(b)(i) Find the first three terms in the expansion of $\left(2 - \frac{1}{x}\right)^8$ in descending powers of x. [2]

(ii) Hence find the values of a, b and c given that the first three terms in the expansion of $(a + bx)\left(2 - \frac{1}{x}\right)^8$ are $128x$, -256 and $\frac{c}{x}$ respectively. [5]

15 An open cylindrical tank with O as the centre of the base is shown in the diagram.

It is given that $\angle EOF = 2\theta$ where $0° < \theta < 90°$ and $OF = 2$ cm.

The external total surface area of the cylindrical tank is S cm^2.



(i) Show that $S = 2\pi\,(2\sin 2\theta - \cos 2\theta + 1)$. [4]

(ii) Express $S = 2\pi(2\sin 2\theta - \cos 2\theta + 1)$ in the form $2\pi[R\sin(2\theta - \alpha) + 1]$

where $R > 0$ and $0° < \theta < 90°$. [3]

(iii) Find the maximum possible value of S and the corresponding value of θ. [3]

Anwer

1 $0 < x < \frac{2}{3}$

2 $\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$

3 $\frac{1}{5}\left(xe^{5x} - \frac{1}{5}e^{5x}\right) + C_2$

4(a) $e^3 + \frac{7}{2e^2} - \frac{9}{2}$ (b) $2\sqrt{3} - \frac{\pi}{6}$

5 $\frac{x^2}{2} - 3x + 8\ln(x+2) - \ln(x+1) + C$

6(i) 1 real root (ii) -0.5

7(ii) $y = \sin 2x + 2\cos x$

8(ii) 0.761 m, – 15.8 (reject)

9(ii) $\frac{5}{24}$

10(a) $x = \frac{e}{e-2}$ (b) 0.1

11(ii) abnormal reading $y = 6.24$, correct $y = 5.30$

(iii) $a = 1.42$, $b = 1.50$

12(ii) $r = 2.53$ (iii) $V = 339$ cm^3, V is maximum

13(i) $a = 8$, $c = 5$, $b = \frac{\pi}{6}$ (ii) 3.42 s

(iii) It is th duration of time that bucket is in the water.

14(a) no x^9 term (b)(i) $256 - \frac{1024}{x} + \frac{1792}{x^2} + \cdots$

(c) $a = 1$, $b = 0.5$, $c = -128$

15(ii) $S = 2\pi[\sqrt{5}sin(2\theta - 26.6°) + 1]$

(iii) max $s = 20.3$ cm^2 when $\theta = 58.3^0$

________________________ (   )   CLASS: 4 (   )

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2019
ADDITIONAL MATHEMATICS PAPER 1 [4047/01]

S4

03 September 2019 Tuesday **2 hours**

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a 2B pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiners' Use

Question	Marks	Question	Marks	Question	Marks
1		7		**Table of Penalties**	
2		8			
3		9			
4		10		**Units**	
5		11		**Presentation**	
6				**Accuracy**	
Parent's Name & Signature: **Date:**		**Total:**			**80**

This document consists of **17** printed pages and **1** blank page.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

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$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

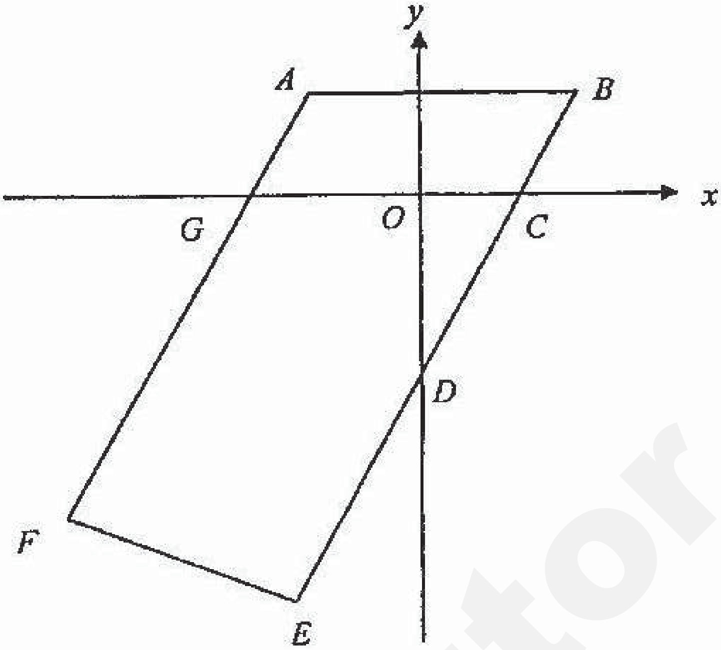
$$\text{Area of } \Delta = \frac{1}{2} ab \sin C$$

1 Rashid bought a hot bowl of soup and he left it to cool on the table. The temperature of the soup, $T°$ C, at time, t minutes, is given by Newton's Law of Cooling formula, $T = Ae^{-kt} + T_a$, where A and k are positive constants and T_a is the ambient temperature or the temperature of the surroundings. When the temperature of the soup was first taken, its temperature was $81°C$ and the ambient temperature was $31°C$. After 10 minutes, the temperature of the soup was $51°C$, with no change in the ambient temperature.

(i) Calculate the value of A and of k . [3]

(ii) If Rashid wants the soup to be at most $35°C$ when he drinks it, determine the minimum number of minutes he has to wait, assuming no change to the ambient temperature. [2]

2 **Solutions to this question by accurate drawing will not be accepted.**



In the diagram, O is the origin. The points, C and G, lie on the x-axis. The line $BCDE$ is parallel to the line AGF and perpendicular to the line EF. The coordinates of A, B and D are $(-2,3)$, $(3,k)$ and $(0,-3)$ respectively. The length of BD is $\sqrt{45}$, and $\dfrac{BD}{AF}=\dfrac{2}{3}$.

(i) Find the value of k. [3]

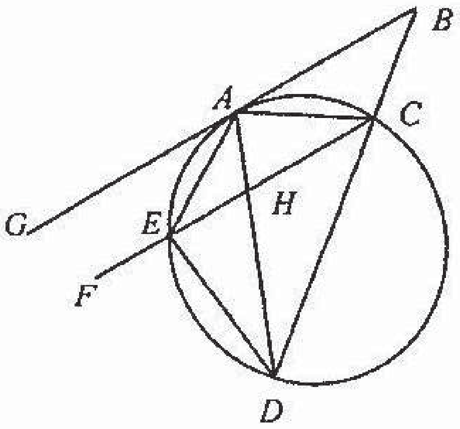
2

(ii) Show that the coordinates of F are $\left(-\frac{13}{2},-6\right)$. [2]

(iii) Find the coordinates of E. [3]

[Turn over

3 In the diagram, *ACDE* is a cyclic quadrilateral. Lines *GAB* and *FEHC* are parallel, and line *GAB* is a tangent to the circle at *A*. Lines *AD* and *EC* meet at *H*.



Prove that

(i) triangle *ABD* and triangle *CBA* are similar, [2]

(ii) triangle *ACH* and triangle *ADC* are similar, [2]

3

(iii) AD bisects angle CDE, [1]

(iv) $AB \times AH = AC \times BC$. [2]

4 (i) Express $12\sin\theta\cos\theta - 8\cos^2\theta + 7$ in the form $A\sin 2\theta + B\cos 2\theta + C$, where A, B and C are constants. [2]

(ii) Solve $12\sin\theta\cos\theta - 8\cos^2\theta + 7 = 0$ for $0^\circ < \theta < 180^\circ$. [5]

4

5 **(a)** Given that $y = \dfrac{x^2}{e^x}$, find the range of values of x for which y is an increasing function. [4]

(b) The equation of a curve is $y = (x-1)\ln(1-x)$. Find the exact x-coordinate of the point at which the normal is parallel to the y-axis. [4]

6 A particle P leaves a fixed point O and moves in a straight line so that, t seconds after leaving O, its velocity v cm s^{-1} is given by $v = t^2 - 14t + 48$. Calculate

(i) the minimum velocity of P, [2]

(ii) the values of t when P is instantaneously at rest, [2]

5

(iii) the distance travelled by P in the first 10 seconds. [4]

(iv) Show that the particle will not return to O. [1]

7 A right circular cone of depth 40 cm and radius 10 cm is held with vertex downwards. It contains water which leaks out through a hole at a rate of 8 cm^3s^{-1}. Find the rate at which the water level is decreasing when the radius of the surface of the water is 4 cm. [6]

6

8 Determine the number of solutions for the equation

$$2+6\log_8 x=\frac{\log_5(9x-15)}{\log_5 2}.$$

[5]

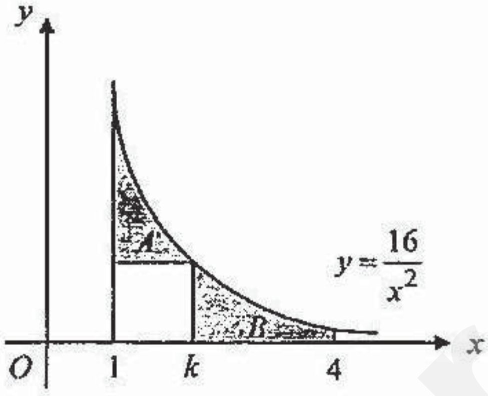
[Turn over

9 Find the range of values of x such that the curve $y=2x-x^2$ lies above the curve $y=2x^2+17x-42$. [4]

10 (a) Find, in terms of π, the value of $\int_0^{\pi} \sin^2 x \cos^2 x \, dx$. [4]

7

(b) The diagram shows part of the curve $y=\frac{16}{x^2}$. Also shown are lines perpendicular to the x-axis at the points with x-coordinates 1, k and 4.



Given that the areas of the regions marked A and B are equal, find the value of k.

[7]

11 The table shows experimental values of two variables, x and y, which are connected by an equation of the form $y^m x = k$, where k and m are constants.

x	2	4	6	8	10
y	6.25	1.56	0.694	0.391	0.250

(i) Plot $\ln y$ against $\ln x$, and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of k and of m. [4]

(iii) By adding a suitable straight line to the same diagram, find the solution to the pair of simultaneous equations $y^m x = k$ and $y = \sqrt[3]{x}$. [3]

End of Paper

Q

1 The roots of the equation $3x^2-6x+5=0$ are α and β. Given that the roots of $x^2+px+q=0$ are $\dfrac{\beta^2}{\alpha}$ and $\dfrac{\alpha^2}{\beta}$, find the value of p and of q. [5]

9

2 (i) Show that $-x^2+x-(1+h^2)$ is always negative for all values of h. [2]

(ii) Find the possible values of k for which the line $y=2x+k$ is tangent to the curve $y^2=1-2x^2$. [3]

(iii) Find the range of values of x which satisfies $x+2 \leq x^2 < 16$. [4]

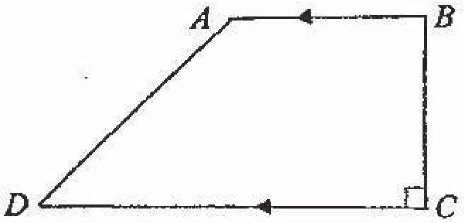
3 **(i)** Solve the equation $x-\sqrt{1-2x}=-7$. [3]

10

(ii) Solve the simultaneous equations

$$\frac{3^x}{9^{1-y}} = 81 \quad \text{and} \quad 4^x(2^{3y}) = \frac{1}{\sqrt{8}}.$$ [5]

(iii) The trapezium *ABCD*, where *AB* is parallel to *DC*, and has an area of $12+6\sqrt{10}$ cm^2.



Given that the length of *AB* is $\sqrt{2}+\sqrt{5}$ cm and the length of *DC* is 2 times of *AB*, find,

(a) the height, *BC* of the trapezium in the form $a\sqrt{b}$, where a and b are integers. [4]

(11)

(iii) **(b)** the exact value of AD^2 in the form $c+d\sqrt{10}$, where c and d are integers. [2]

4 **(i)** The function $f(x)=x^3+ax^2-2x-36$ is divisible by $x-2$. Find the value of a. [1]

(ii) Given that $x^3+5x^2-2x-24=(x+1)^2(x+b)+cx-27$ for all values of x, find the

value of b and of c.
[3]

(iii) The graph of a cubic polynomial expression, $y=f(x)$ has a coefficient of w for x^3.

This graph cuts the x-axis at $(-3, 0)$, $(-1, 0)$, $(4, 0)$ and the y-axis at $(0, 24)$.

Find an expression for $f(x)$.
[3]

12

5 (i) Sketch the graph of $y=\left|x^2+4x\right|$ showing the x-intercepts and the coordinates of the turning point.

[Turn over

[3]

(ii) State the number of solutions to the equation $\left|x^2+4x\right|=x+4$ by drawing a suitable

line on the same axes.
[2]

6 (i) Find the middle term of $\left(x-\frac{1}{2x^2}\right)^8$.

[2]

(ii) The first 3 terms of $(2a+x)(1-3x)^n$ is $4-59x+bx^2$. Find the value of a, of n and

of b.

[5]

13

7. The points $A(5, -7)$ and $B(6, 0)$ lie on a circle, with centre C.

Given that the point C lies on the line $y = 5x - 13$.

(i) Find the equation of the circle.
[7]

A second circle with radius r units and centre P, also passes through the points A and B.

(ii) Find the exact smallest possible value of r.

[2]

(iii) State the coordinates of centre P.

[1]

14

8 **(i)** Show that $\cot\theta + \tan\theta = 2\operatorname{cosec} 2\theta$.
[4]

(ii) Hence, solve the equation $\cot\frac{x}{2} + \tan\frac{x}{2} = \operatorname{cosec}^2 x - 3$, giving your answers in radians and within the principal range.
[5]

9 (i) Sketch, on the same axes, the graphs of $y = 2\sin x - 3$ and $y = 4|\cos x|$ for

$-\pi \leq x \leq \pi$.

Hence, deduce the value of m for which the equation $2\sin x - 3 = 4|\cos x| + m$ has

1 solution in this range.

[5]

15

(ii) A student reasoned that since the range of values of y for both equations $y = \sin x$ and $y = \cos x$ are between -1 and 1 inclusive, then the range of values of the expression $8 \sin x + 5 \cos x$ can be obtain as follow

$$-8 \leq 8 \sin x \leq 8$$

$$-5 \leq 5 \cos x \leq 5$$

$$\Rightarrow \quad -13 \leq 8 \sin x + 5 \cos x \leq 13$$

(a) Without performing any calculations, explain why this reasoning is incorrect.

[1]

(b) Find the range of values of $8 \sin x + 5 \cos x$.

[4]

10 (i) Express $\dfrac{2x-18}{x^3+6x^2+9x}$ in partial fraction.

[5]

16

(ii) Hence, or otherwise, find the equation of the curve where the gradient is

$\dfrac{x-9}{x^3+6x^2+9x}$ and passes through the point (3, ln 2).

[5]

11 Given that $y=3(x-1)^4-4(x-1)^3+5$, find and determine the nature of the stationary points.

[6]

17

12 (i) Sketch the graph of $y = 3\ \ln (x - 2)$, clearly showing the asymptote and the point where the curve crosses the x-axis.

[3]

(ii) Find $\frac{d}{dx}\left(\cos^3\frac{x}{2}\right)$ and hence evaluate $\int_0^\pi \cos^2\frac{x}{2}\sin\frac{x}{2}\ dx$

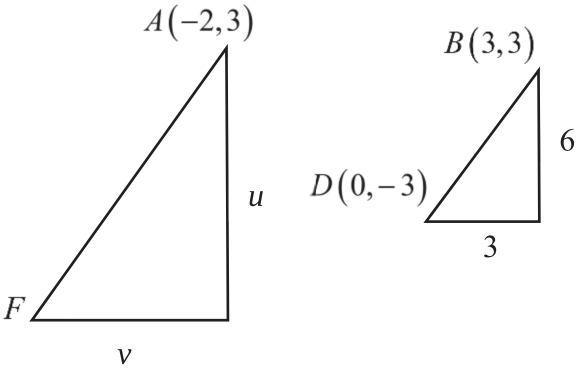
[5]

18

AM-2019-AHS-Prelim-P1 – Marking Scheme

1(i)	$T = A\,e^{-kt} + T_a$ $t = 0$, $T_a = 31$, $T = 81$ $81 = A\,e^{-k\times 0} + 31$ $A = 50$ $t = 10$, $T_a = 31$, $T = 51$ $51 = 50\,e^{-k\times 10} + 31$ $e^{-10k} = \frac{20}{50}$ $k = -\frac{1}{10}\times\ln\left(\frac{2}{5}\right)$ $= 0.091629$ $= 0.0916$
(ii)	$T = 50\,e^{-0.091629t} + 31$ $35 = 50\,e^{-0.091629t} + 31$ $e^{-0.091629t} = \frac{4}{50}$ $-0.091629t = \ln\left(\frac{2}{25}\right)$ $t = \frac{1}{-0.091629}\times\ln\left(\frac{2}{25}\right)$ $= 27.565$ $= 27.6$ Rashid must wait for at least 27.6 minutes before he can drink the soup.

2(i)	$BD = \sqrt{45}$ $\sqrt{(3-0)^2 + (k-(-3))^2} = \sqrt{45}$ $9 + k^2 + 6k + 9 = 45$ $k^2 + 6k - 27 = 0$ $(k+9)(k-3) = 0$ $k = -9(\text{NA})$, $k = 3$
(ii)	$A(-2,3)$, $B(3,3)$, $D(0,-3)$

	 $\frac{AF}{BD}=\frac{3}{2}$ $\frac{u}{6}=\frac{3}{2}$ $u=9$ $\frac{AF}{BD}=\frac{3}{2}$ $\frac{v}{3}=\frac{3}{2}$ $v=\frac{9}{2}$ Coordinates of $F=\left(-2-\frac{9}{2},\ 3-9\right)=\left(-\frac{13}{2},-6\right)$
(iii)	Gradient of $BCDE=\frac{3-(-3)}{3-0}=2$ Equation of line $BCDE$ is $y=2x-3$ Gradient of $EF=-\frac{1}{2}$ Equation of line EF $y-(-6)=-\frac{1}{2}\left(x-\left(-\frac{13}{2}\right)\right)$ $y=-\frac{1}{2}x-\frac{37}{4}$ Solving the pair of simultaneous equations $2x-3=-\frac{1}{2}x-\frac{37}{4}$ $\frac{5}{2}x=-\frac{37}{4}+3$ $x=-\frac{5}{2}$ $y=2\left(-\frac{5}{2}\right)-3$ $=-8$ Coordinates of $E=\left(-\frac{5}{2},-8\right)$

3(i)	$\angle CAB = \angle CDA$ (Alternate Segment Theorem) And $\angle BDA = \angle CDA$ (same angle) $\angle ABC = \angle ABD$ (Common angle) Triangle *ABD* is similar to triangle *CBA*. (AA)
(ii)	$\angle CAB = \angle CDA$ (Alternate Segment Theorem) $\angle CAB = \angle ACH$ (Alternate angles, *GAB*//*FEHC*) Hence $\angle ACH = \angle CDA$ $\angle HAC = \angle DAC$ (Common angle) Triangle *ACH* is similar to triangle *ADC*. (AA)
(iii)	From (ii), $\angle ACH = \angle CDA$ $\angle ACH = \angle ADE$ (Angles in the same segment) Hence $\angle ADE = \angle CDA$ Therefore *AD* bisects angle *CDE*.
(iv)	Triangle *ABD* is similar to triangle *CBA*. $\frac{AB}{BC} = \frac{AD}{AC}$ Triangle *ACH* is similar to triangle *ADC*. $\frac{AC}{AH} = \frac{AD}{AC}$ Hence $\frac{AB}{BC} = \frac{AC}{AH}$ $AB \times AH = AC \times BC$

4(i)	$12\sin\theta\cos\theta - 8\cos^2\theta + 7 = 6\left(2\sin\theta\cos\theta\right) - 8\cos^2\theta + 7$ $= 6\sin 2\theta - 8\left(\frac{\cos 2\theta + 1}{2}\right) + 7$ $= 6\sin 2\theta - 4\cos 2\theta + 3$
(ii)	$6\sin 2\theta - 8\cos^2\theta + 7 = 0$ $6\sin 2\theta - 4\cos 2\theta + 3 = 0$ Let $6\sin 2\theta - 4\cos 2\theta = R\sin\left(2\theta - \alpha\right)$ $R = \sqrt{6^2 + 4^2} = \sqrt{52}$ $\tan\alpha = \frac{4}{6}$ $\alpha = 33.690°$ $\sqrt{52}\sin\left(2\theta - 33.690^{\circ}\right) + 3 = 0$

	$\sin\left(2\theta-33.690^{\circ}\right)=-\frac{3}{\sqrt{52}}$ basic angle = 24.583° $2\theta-33.690^{\circ}=-24.583^{\circ}$ or $2\theta-33.690^{\circ}=180^{\circ}+24.583^{\circ}$ $\theta=4.553^{\circ}$ $\theta=119.137^{\circ}$ $\theta=4.6^{\circ}$ $=119.1^{\circ}$

5(a)	$y=\frac{x^2}{e^x}$ $\frac{dy}{dx}=\frac{e^x(2x)-x^2e^x}{\left(e^x\right)^2}$ $\frac{dy}{dx}=\frac{2x-x^2}{e^x}$ Since y is an increasing function, $\frac{dy}{dx}>0$ $\frac{2x-x^2}{e^x}>0$ Since $e^x>0$, $2x-x^2>0$ $x(2-x)>0$ $0<x<2$
(b)	$y=(x-1)\ln(1-x)$ $\frac{dy}{dx}=(x-1)\frac{1}{1-x}(-1)+(1)\ln(1-x)$ $\frac{dy}{dx}=1+\ln(1-x)$ Given that normal is parallel to the y-axis,

	$\frac{dy}{dx}=0$ $1+\ln(1-x)=0$ $\ln(1-x)=-1$ $1-x=e^{-1}$ $x=1-\frac{1}{e}$ $x=\frac{e-1}{e}$

6(i)	$v=t^2-14t+48$ $\frac{dv}{dt}=2t-14$ For minimium velocity, $\frac{dv}{dt}=0$ $2t-14=0$ $t=7$ Minimum $v=7^2-14(7)+48=-1\text{ m / s}$
(ii)	$v=t^2-14t+48$ When P is instantaneous at rest, $v=0$ $t^2-14t+48=0$ $t=6$ or $t=8$

(iii)	$v = t^2 - 14t + 48$ $s = \int t^2 - 14t + 48 \, dt$ $s = \frac{t^3}{3} - 7t^2 + 48t + c$ When $t = 0$, $s = 0$, $c = 0$ $s = \frac{t^3}{3} - 7t^2 + 48t$ When $t = 6$, $s = 108$ When $t = 8$, $s = 106\frac{2}{3}$ When $t = 10$, $s = 113\frac{1}{3}$ Total distance $= 108 + \left(108 - 106\frac{2}{3}\right) + \left(113\frac{1}{3} - 106\frac{2}{3}\right) = 116$ m
(iv)	$s = \frac{t^3}{3} - 7t^2 + 48t$ When P returns to O, $s = 0$ $\frac{t^3}{3} - 7t^2 + 48t = 0$ $\frac{t}{3}\left(t^2 - 21t + 144\right) = 0$ $t = 0$ or $t^2 - 21t + 144 = 0$ Discriminant $= (-21)^2 - 4(1)(144)$ $= -135 < 0$ The particle does not return to O.

7 By similar triangles,

$$\frac{10}{40}=\frac{r}{h}$$

$$r=\frac{h}{4}$$

$$V=\frac{1}{3}\pi r^2 h$$

$$=\frac{1}{3}\pi\left(\frac{h}{4}\right)^2 h$$

$$=\frac{1}{48}\pi h^3$$

$$\frac{dV}{dh}=\frac{1}{16}\pi h^2$$

When r = 4, h = 16 cm.

Rate at which the volume is decreasing, $\frac{dV}{dt}=-8$

Using chain rule,

$$\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}$$

$$-8=\frac{1}{16}\pi(16)^2\times\frac{dh}{dt}$$

$$\frac{dh}{dt}=-\frac{1}{2\pi}$$

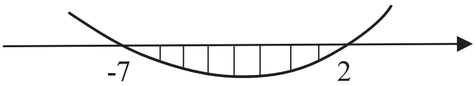
Rate at which the water level is decreasing is $\frac{1}{2\pi}$ cm s^{-1}.

8 Determine the number of solutions for the equation

$$2+6\log_8 x=\frac{\log_5(9x-15)}{\log_5 2}$$ [5]

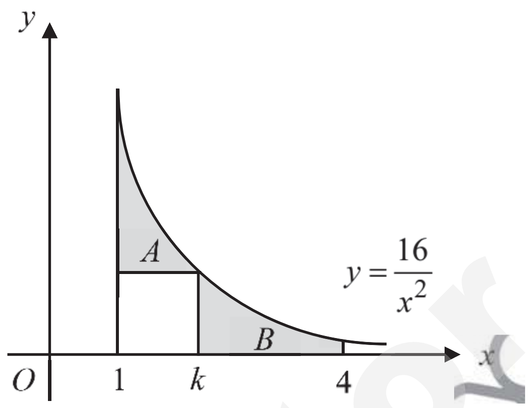
8	$2+6\log_8 x=\frac{\log_5(9x-15)}{\log_5 2}$ $2+\frac{6\log_2 x}{\log_2 8}=\log_2(9x-15)$ $2+\frac{6\log_2 x}{\log_2 2^3}=\log_2(9x-15)$ $2+\frac{6\log_2 x}{3\log_2 2}=\log_2(9x-15)$ $2+2\log_2 x=\log_2(9x-15)$	**M2 – apply rules of logarithms and change of base to obtain the quadratic equation**
	$2+2\log_2 x=\log_2(9x-15)$ $2+\log_2 x^2=\log_2(9x-15)$ $\log_2(9x-15)-\log_2 x^2=2$ $\log_2\frac{(9x-15)}{x^2}=2$ $\frac{(9x-15)}{x^2}=2^2$ $9x-15=4x^2$	**A1 – quadratic equation**
	$9x-15=4x^2$ $4x^2-9x+15=0$ $\text{Discriminant}=(-9)^2-4(4)(15)$	**M1**
	$=-159<0$ The equation has no real roots, hence there are zero solutions.	**A1**

9 Find the range of values of x such that the curve $y = 2x - x^2$ lies above the curve $y = 2x^2 + 17x - 42$. [4]

9	$2x - x^2 > 2x^2 + 17x - 42$ $2x - x^2 - 2x^2 - 17x + 42 > 0$ $-3x^2 - 15x + 42 > 0$ $3x^2 + 15x - 42 < 0$ $x^2 + 5x - 14 < 0$ $(x-2)(x+7) < 0$  $-7 < x < 2$	M1 M1 M1 A1

10 **(a)** Find, in terms of π, the value of $\int_0^{\pi} \sin^2 x \cos^2 x \, dx$. [4]

(b) The diagram shows part of the curve $y = \frac{16}{x^2}$. Also shown are lines perpendicular to the x-axis at the points with x-coordinates 1, k and 4.



Given that the areas of the regions marked A and B are equal , find the value of k. [7]

10(a)	$\int_0^{\pi} \sin^2 x \cos^2 x \, dx$ $= \int_0^{\pi} (\sin x \cos x)^2 \, dx$ $= \int_0^{\pi} \left(\frac{1}{2}\sin 2x\right)^2 dx$ $= \frac{1}{4}\int_0^{\pi} \sin^2 2x \, dx$ $= \frac{1}{4}\int_0^{\pi} \frac{1-\cos 4x}{2} dx$ $= \frac{1}{8}\left[x - \frac{\sin 4x}{4}\right]_0^{\pi}$ $= \frac{\pi}{8}$	M1 M1 M1 A1

(b)	When $x = k,\ y = \dfrac{16}{k^2}$	
	Area of A	
	$= \int_1^k \dfrac{16}{x^2}\,dx - (k-1)\left(\dfrac{16}{k^2}\right)$	M1
	$= \left[-\dfrac{16}{x}\right]_1^k - \left(\dfrac{16}{k} - \dfrac{16}{k^2}\right)$	
	$= -\dfrac{16}{k} - \left(-\dfrac{16}{1}\right) - \dfrac{16}{k} + \dfrac{16}{k^2}$	
	$= -\dfrac{32}{k} + 16 + \dfrac{16}{k^2}$	A1
	Area of B	
	$= \int_k^4 \dfrac{16}{x^2}\,dx$	M1
	$= \left[-\dfrac{16}{x}\right]_k^4$	
	$= -\dfrac{16}{4} - \left(-\dfrac{16}{k}\right)$	
	$= -4 + \dfrac{16}{k}$	A1
	Area of A = Area of B	
	$-\dfrac{32}{k} + 16 + \dfrac{16}{k^2} = -4 + \dfrac{16}{k}$	
	$\dfrac{16}{k^2} - \dfrac{48}{k} + 20 = 0$	
	$16 - 48k + 20k^2 = 0$	M1
	$5k^2 - 12k + 4 = 0$	
	$(k-2)(5k-2) = 0$	M1
	$k = 2$ or $k = 0.4$ (N.A.)	A1

11 The table shows experimental values of two variables, x and y, which are connected by an equation of the form $y^m x = k$, where k and m are constants.

x	2	4	6	8	10
y	6.25	1.56	0.694	0.391	0.250

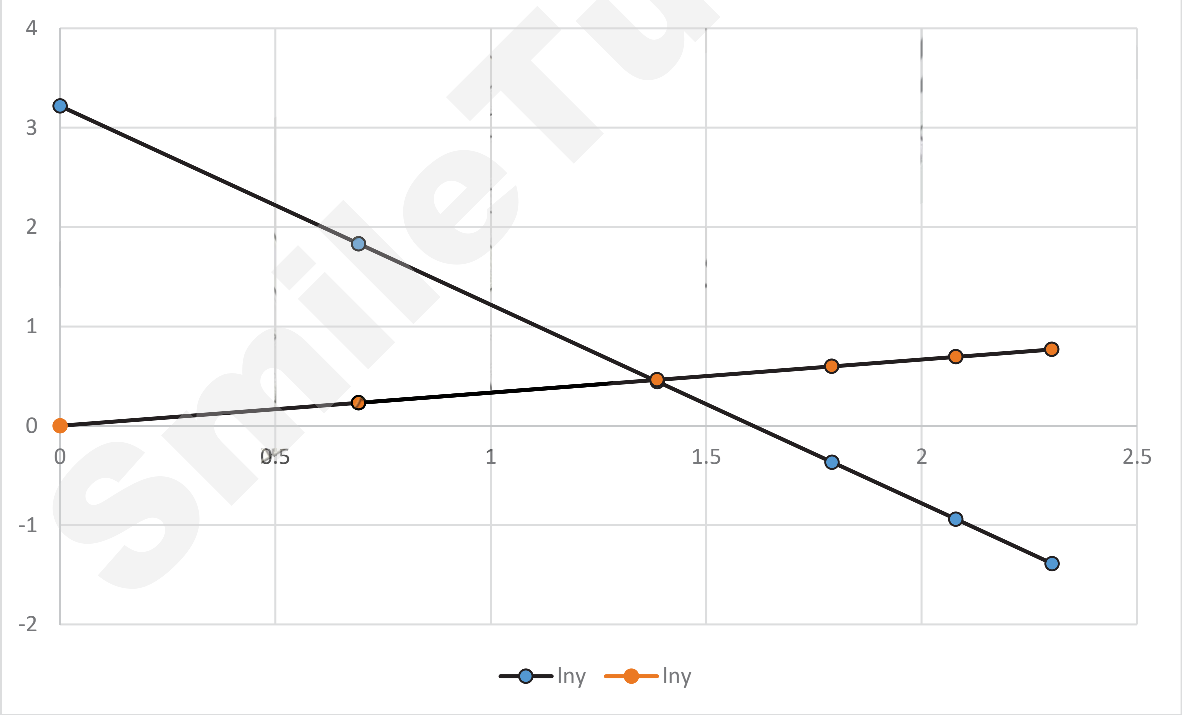
(i) Plot $\ln y$ against $\ln x$, and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of k and of m. [4]

(iii) By adding a suitable straight line to the same diagram, find the solution to the pair of simultaneous equations $y^m x = k$ and $y = \sqrt[3]{x}$. [3]

11(i)

$\ln x$	0.693	1.386	1.792	2.079	2.303
$\ln y$	1.833	0.445	-0.365	-0.939	-1.386



(ii)

$$y^m x = k$$

$$\ln\left(y^m x\right) = \ln k$$

$$\ln y^m + \ln x = \ln k$$

$$m \ln y = -\ln x + \ln k$$

$$\ln y = -\frac{1}{m}\ln x + \frac{1}{m}\ln k$$

Gradient of the line $= -\dfrac{1}{m}$

Y-intercept $= \dfrac{1}{m}\ln k$

Gradient of the line $= -2 \ (\text{Accept } -1.7 \text{ to } -2.3)$

$$-\frac{1}{m} = -2$$

$$m = 0.5 \ (\text{Accept } 0.435 \text{ to } 0.588)$$

Y-intercept $=3.217859 \quad (\text{Accept } 3.1 \text{ to } 3.3)$

$$\frac{1}{m}\ln k = 3.217589$$

$$\frac{1}{0.5}\ln k = 3.217589$$

$$\ln k = 0.5 \times 3.217589$$

$$k = e^{0.5\times 3.217589}$$

$$= 5 \ (\text{Accept } 3.8 \text{ to } 6.2)$$

(iii)

$$y = \sqrt[3]{x}$$

$$y = x^{\frac{1}{3}}$$

$$\ln y = \frac{1}{3}\ln x$$

$\ln x$	0	1	2
$\ln y$	0	0.333	0.667

$$\ln x = 1.2 \ (\text{Accept } 1.0 \text{ to } 1.4)$$

At intersection point, $x = e^{1.2}$

$$= 3.32 \quad (\text{Accept } 2.7 \text{ to } 4.1)$$

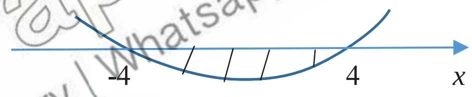
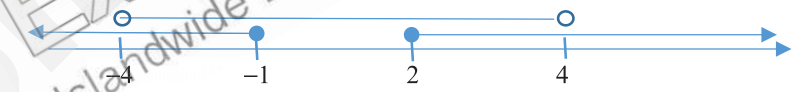
AM-2019-AHS-SA2-P2-Marking Scheme

1	$\alpha+\beta=\frac{6}{3}$ $\qquad\qquad$ $\alpha\beta=\frac{5}{3}$ $\qquad=2$ Sum of roots : $\frac{\beta^2}{\alpha}+\frac{\alpha^2}{\beta}=-p$ $-p=\frac{\alpha^3+\beta^3}{\alpha\beta}$ $-p=\frac{(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)}{\alpha\beta}$ $-p=\frac{(\alpha+\beta)\left[(\alpha+\beta)^2-2\alpha\beta-\alpha\beta\right]}{\alpha\beta}$ $-p=\frac{2\left[(2)^2-3\times\frac{5}{3}\right]}{\frac{5}{3}}$ $p=\frac{6}{5}$ $\qquad\left[\text{or } \ 1\frac{1}{5} \ \text{ or } \ 1.2\right]$ Product of roots: $\frac{\beta^2}{\alpha}\times\frac{\alpha^2}{\beta}=q$ $q=\frac{\alpha^2\beta^2}{\alpha\beta}$ $q=\alpha\beta$ $q=\frac{5}{3}$ $\quad\left[\text{or } \ 1\frac{2}{3}\right]$

2(i)	$D=b^2-4ac\ of\ -x^2+x-(1+h^2)$ $=1^2-4(-1)(-1-h^2)$ $=1-4-4h^2$ $=-3-4h^2$ <0	**alternative** $-x^2+x-(1+h^2)$ $=-(x^2-x)-(1+h^2)$ $=-\left[(x-\frac{1}{2})^2-\frac{1}{4}\right]-(1+h^2)$ $=-(x-\frac{1}{2})^2+\frac{1}{4}-(1+h^2)$ $=-(x-\frac{1}{2})^2-(\frac{3}{4}+h^2)$ <0

	Since D (b^2- $4ac$) is always negative for all values of h and coefficient of x^2 is -1, then $-x^2+x-h^2-1$ is always negative.

2(ii)	Subst $y=2x+k$ into $y^2=1-2x^2$ $(2x+k)^2=1-2x^2$ $4x^2+4kx+k^2=1-2x^2$ $6x^2+4kx+k^2-1=0$ For line to be tangent to curve, discriminant =0 $(4k)^2-4(6)(k^2-1)=0$ $16k^2-24k^2+24=0$ $8k^2=24$ $k=\pm\sqrt{3}$

2(iii)	$x+2\le x^2<16$ $x+2\le x^2$ $\qquad$ $x^2<16$ $x^2-x-2\ge 0$ $\qquad$ $x^2-16<0$ $(x-2)(x+1)\ge 0$ $\qquad$ $(x+4)(x-4)<0$   $x\le -1$ or $x\ge 2$ $\qquad$ $-4<x<4$  Therefore the range of values of x are $-4<x\le -1$ or $2\le x<4$

3(i)	$x-\sqrt{1-2x}=-7$ $x+7=\sqrt{1-2x}$ Square both sides $x^2+14x+49=1-2x$ $x^2+16x+48=0$ $(x+4)(x+12)=0$ $x=-4 \quad or\ x=-12(reject)$

3(ii)	$\frac{3^x}{9^{1-y}}=81$ and $4^x(2^{3y})=\frac{1}{\sqrt{8}}$ $\frac{3^x}{9^{1-y}}=81$ -------------(1) $4^x(2^{3y})=\frac{1}{\sqrt{8}}$ -------------(2) Eq(1) $3^x=9^{1-y}(81)$ $3^x=3^{2-2y}(3^4)$ $\therefore\ x=6-2y$ ------------(3) Eq(2) $2^x(2^{3y})=2^{-\frac{3}{2}}$ $2^{2x+3y}=2^{-\frac{3}{2}}$ $\therefore\ 2x+3y=-\frac{3}{2}$ ------------(4) Sub Eq(3) into Eq(4) $2(6-2y)+3y=-\frac{3}{2}$ $12-4y+3y=-\frac{3}{2}$ $y=\frac{27}{2}$ $\left[\text{or } 13\frac{1}{2} \text{ or } 13.5\right]$ Sub $y=\frac{27}{2}$ into Eq(3) $x=6-2(\frac{27}{2})$ $=-21$

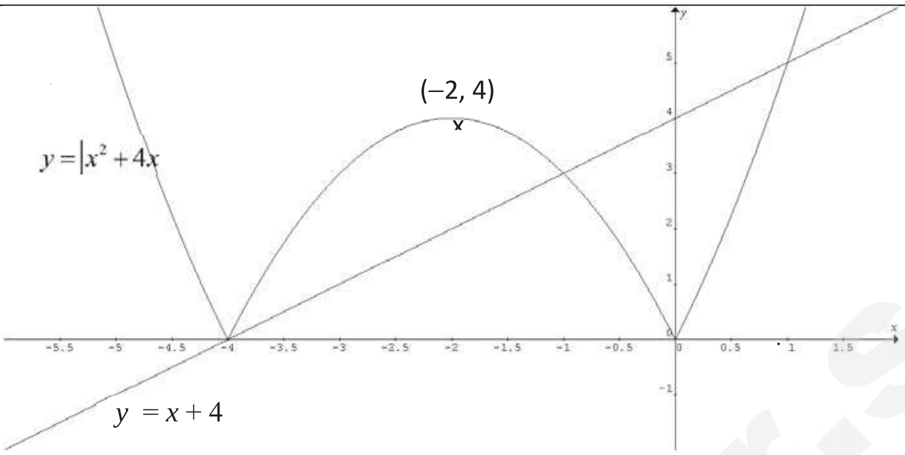
3(iii)	Area of Trapezium $=12+6\sqrt{10}$ $\frac{1}{2}\left(AB+CD\right)Height=12+6\sqrt{10}$
(a)	$Height=\frac{2\left(12+6\sqrt{10}\right)}{3\left(\sqrt{2}+\sqrt{5}\right)}$ $=\frac{\left(24+12\sqrt{10}\right)}{3\left(\sqrt{2}+\sqrt{5}\right)}\times\frac{\sqrt{2}-\sqrt{5}}{\sqrt{2}-\sqrt{5}}$ $=\frac{24\sqrt{2}+12\sqrt{20}-24\sqrt{5}-12\sqrt{50}}{3(2-5)}$ $=\frac{24\sqrt{2}+12(2\sqrt{5})-24\sqrt{5}-12(5\sqrt{2})}{-9}$ $=\frac{-36\sqrt{2}}{-9}$ $=4\sqrt{2}cm$

(b)	$AD^2=\left(\sqrt{2}+\sqrt{5}\right)^2+\left(4\sqrt{2}\right)^2$ $=2+2\left(\sqrt{2}\right)\left(\sqrt{5}\right)+25+16(2)$ $=59+2\sqrt{10}$

4(i)	$f(2)=0$ $(2)^3+a(2)^2-2(2)-36=0$ $8+4a-4-36=0$ $4a=32$ $a=8$

(ii)	$x^3+5x^2-2x-24=(x+1)^2(x+b)+cx-27$ $Let\ x=-1$ $-1+5+2-24=-c-27$ $c=-27+1-5-2+24$ $c=-9$ $Let\ x=0$ $-24=(1)^2b-27$ $b=3$ ~~~~~~~~~~~~~~~~~~~**Alternatively** ~~~~~~~~~~~~~~~ Using comparison method, $x^3+5x^2-2x-24=(x^2+2x+1)(x+b)+cx-27$ $x^3+5x^2-2x-24=x^3+bx^2+2x^2+2bx+x+b+cx-27$ $x^3+5x^2-2x-24=x^3+(b+2)x^2+(2b+1+c)x+b-27$ By comparing constant $\therefore -24=b-27$ $b=3$ By comparing coefficient of x $-2=2(3)+1+c$ $c=-9$

4(iii)	$f(x)=w(x+3)(x+1)(x-4)$ At (0, 24) $24=w(3)(1)(-4)$ $w=-2$ $\therefore\ f(x)=-2(x+3)(x+1)(x-4)$

5(i)	 Deduct 1 mark if axes and graphs are not labelled. G1 for correct shape of $y=\|x^2+4x\|$. G1 for indicating -4 and 0 as x-intercepts along the x-axis. G1 for indicating coordinates of turning point (-2,4) on the graph.
5(ii)	G1 for correct sketch of $y=x+4$ B1 for stating **3 solutions. No B1 marks if graph is not drawn.**

6(i)	Middle term of $\left(x-\frac{1}{2x^2}\right)^8$ is $T_5=\binom{8}{4}x^4\left(-\frac{1}{2x^2}\right)^4$ $=\frac{35}{8x^4}$

6(ii)	$(2a+x)(1-3x)^n$ $=(2a+x)[1^n+\binom{n}{1}1^{n-1}(-3x)+\binom{n}{2}1^{n-2}(-3x)^2+...]$ $=(2a+x)[1-3nx+\frac{9n(n-1)x^2}{2}+...]$ $=2a-6anx+9an(n-1)x^2+x-3nx^2+...$ $=2a+(1-6an)x+[9an(n-1)-3n]x^2$ Comparing expression with $(4-59x+bx^2)$, Constant: $2a=4$

$a = 2$

Coeff. of x: $1 - 6an = -59$

$1 - 6(2)n = -59$

$n = 5$

Coeff. of x^2: $9an(n-1) - 3n = b$

$9(2)(5)(5-1) - 3(5) = b$

$b = 345$

7(i)

$$\text{Mid-point of } AB = \left(\frac{5+6}{2}, \frac{-7+0}{2}\right)$$

$$= \left(\frac{11}{2}, -\frac{7}{2}\right)$$

$$\text{Gradient of } AB = \frac{-7-0}{5-6}$$

$$= 7$$

Equation of the perpendicular bisector of AB

$$y - \left(-\frac{7}{2}\right) = -\frac{1}{7}\left(x - \frac{11}{2}\right)$$

$$y = -\frac{1}{7}x - \frac{19}{7}$$

$$y = -\frac{1}{7}x - \frac{19}{7} \quad \ldots\ldots\ldots\ldots..(1)$$

$$y = 5x - 13 \quad \ldots\ldots\ldots\ldots (2)$$

(1)=(2),

$$-\frac{1}{7}x - \frac{19}{7} = 5x - 13$$

$$36x = 72$$

$$x = 2$$

sub $x = 2$ into (2),

$$y = 5(2) - 13$$

$$= -3$$

Centre $(2, -3)$

$$\text{Radius} = \sqrt{(2-6)^2 + (-3-0)^2}$$

$= 5$ units

Equation of the circle:

$$(x-2)^2 + (y+3)^2 = 25$$

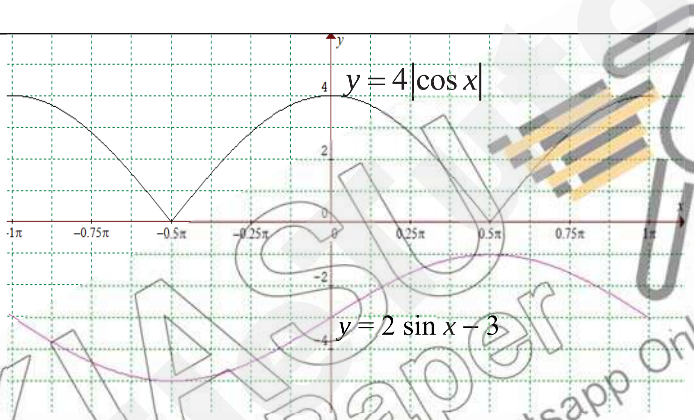
[OR $x^2 + y^2 - 4x + 6y - 12 = 0$]

(ii)	The exact smallest possible value of r $= \frac{1}{2} \times \sqrt{(5-6)^2 + (-7-0)^2}$ $= \frac{\sqrt{50}}{2}$ $= \frac{5\sqrt{2}}{2}$

(iii)	Coordinates of the centre P are $(5.5, -3.5)$ [OR $\left(\frac{11}{2}, -\frac{7}{2}\right)$]	B1

8(i)	Method 1 LHS = $\cot\theta + \tan\theta$ $= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$ $= \frac{\cos^2\theta + sin^2\theta}{\sin\theta\cos\theta}$ $= \frac{1}{\sin\theta\cos\theta}$ $= \frac{2}{2\sin\theta\cos\theta}$ $= \frac{2}{\sin 2\theta}$ $= 2 \text{ cosec } 2\theta$ $= RHS$ (shown)		Method 2 LHS = $\cot\theta + \tan\theta$ $= \frac{1}{\tan\theta} + \tan\theta$ $= \frac{1 + tan^2\theta}{\tan\theta}$ $= \frac{sec^2\theta}{\tan\theta}$ $= \frac{1}{cos^2\theta} \times \frac{\cos\theta}{\sin\theta}$ $= \frac{1}{\sin\theta\cos\theta}$ $= \frac{2}{2\sin\theta\cos\theta}$ $= \frac{2}{\sin 2\theta}$ $= 2 \text{ cosec } 2\theta$ $= RHS$ (Shown)

(ii)	$\cot\frac{x}{2}+\tan\frac{x}{2}=\operatorname{cosec}^2 x-3$ $2\operatorname{cosec} x=\operatorname{cosec}^2 x-3$ $\operatorname{cosec}^2 x-2\operatorname{cosec} x-3=0$ $(\operatorname{cosec} x-3)(\operatorname{cosec} x+1)=0$ $\operatorname{cosec} x=3$ or $\operatorname{cosec} x=-1$ $\frac{1}{\sin x}=3$ $\qquad$ $\frac{1}{\sin x}=-1$ $\sin x=\frac{1}{3}$ $\qquad$ $\sin x=-1$ $x=0.33983$ $\qquad$ $x=-\frac{\pi}{2}$ (or -1.5707) Ans: $x\approx 0.340,\ -\frac{\pi}{2}$ (or -1.57)	M[1] -- QE M[1] – factorization / formula M[1] – in terms of sine A[1, 1] --- deduct 1 mk if ans in degree.



9(i)

For $2\sin x-3=4|\cos x|+m$ to have 1 solution in the range $-\pi\le x\le\pi$.

$m=-1$

9(iia)	The maximum & minimum values of each curve do not occur at the same value of *x*. [max & min pts of each curves do not occur simultaneously.] **Accept solution where students sketch the graphs to explain.

9 (ii) Method 1		Method 2	
$\sqrt{8^2+5^2}=\sqrt{89}$, $\tan\alpha=\frac{5}{8}$ $\alpha=32.005°$ $8\sin x+5\cos x$ $=\sqrt{89}\sin(x-32.0°)$ $\therefore$ The range of $8\sin x+5\cos x$ $-\sqrt{89}\le 8\sin x+5\cos x\le\sqrt{89}$		Let $y=8\sin x+5\cos x$ $\frac{dy}{dx}=8\cos x-5\cos x$ let $\frac{dy}{dx}=0$, $8\cos x-5\sin x=0$ $\tan x=\frac{8}{5}$ $x=57.994°, 237.994°$	

Or $-9.4339 \le 8\sin x + 5\cos x \le 9.4339$ $-9.43 \le 8\sin x + 5\cos x \le 9.43$		when $x = 57.994^{o}$, $8\sin 57.994^{o} + 5\cos 57.994^{o}$ $= 9.4339$ when $x = 237.994^{o}$, $8\sin 237.994^{o} + 5\cos 237.994^{o}$ $= -9.4339$ $\therefore -9.43 \le 8\sin x + 5\cos x \le 9.43$	

10(i)	$\frac{2x-18}{x^3+6x^2+9x}=\frac{2x-18}{x(x+3)^2}$ Let $\frac{2x-18}{x(x+3)^2}=\frac{A}{x}+\frac{B}{x+3}+\frac{C}{(x+3)^2}$ $2x-18 \equiv A(x+3)^2+Bx(x+3)+Cx$ let $x=0$, $-18=A(9)$ $A=-2$ let $x=-3$, $2(-3)-18=C(-3)$ $C=8$ let $x=1$, $2(1)-18=(-2)(4)^2+B(1)(4)+(8)(1)$ $B=2$ $\therefore \frac{2x-18}{x(x+3)^2}=-\frac{2}{x}+\frac{2}{x+3}+\frac{8}{(x+3)^2}$

(ii)	Given that $\frac{dy}{dx}=\frac{x-9}{x^3+6x^2+9x}$ $y=\frac{1}{2}\int\frac{2x-18}{x(x+3)^2}\ dx$ $=\frac{1}{2}\int(-\frac{2}{x}+\frac{2}{x+3}+\frac{8}{\left(x+3\right)^2})\ dx$ $=\frac{1}{2}\left[-2\ln x+2\ln(x+3)+\frac{8(x+3)^{-1}}{(-1)(1)}\right]+c$ $=\frac{1}{2}\left[-2\ln x+2\ln(x+3)-\frac{8}{(x+3)}\right]+c$ $=-\ln x+\ln(x+3)-\frac{4}{(x+3)}+c$ $=\ln\frac{x+3}{x}-\frac{4}{(x+3)}+c$ Sub (3, ln 2) into the above eqn $\ln 2=\ln\frac{3+3}{3}-\frac{4}{3+3}+c$ $c=\frac{2}{3}$ Hence the equation of the curve is $y=\ln\frac{x+3}{x}-\frac{4}{(x+3)}+\frac{2}{3}$ OR $y=\ln(x+3)-\ln x-\frac{4}{(x+3)}+\frac{2}{3}$

11

$y = 3(x-1)^4 - 4(x-1)^3 + 5$

$$\frac{dy}{dx} = 12(x-1)^3 - 12(x-1)^2$$

Let $\frac{dy}{dx} = 0$,

$$12(x-1)^3 - 12(x-1)^2 = 0$$
$$12(x-1)^2[(x-1)-1)] = 0$$
$$(x-1)^2 = 0 \quad \text{or} \quad (x-2) = 0$$
$$x = 1 \qquad\qquad x = 2$$

when $x = 1$, $y = 3(1-1)^4 - 4(1-1)^3 + 5$
$= 5$

x	< 1	$= 1$	> 1
Sketch of tangent	╲	—	╲

$\therefore$ (1, 5) is a point of inflexion

when $x = 2$, $y = 3(2-1)^4 - 4(2-1)^3 + 5$
$= 4$

x	< 2	$= 2$	> 2
Sketch of tangent	╲	—	╱

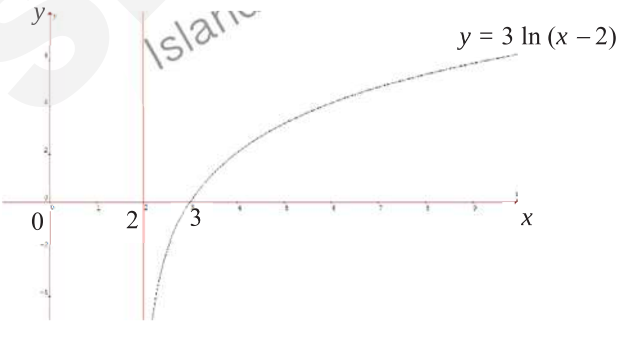
$\therefore$ (2, 4) is a minimum point

12(i)

$y = 3 \ln (x-2)$

The range of value of x is $x > 2$.

The curve crosses the x-axis at the point (3, 0)



12(ii)	$\frac{d}{dx}\left(\cos^3\frac{x}{2}\right)$ $=(3\cos^2\frac{x}{2})(-\sin\frac{x}{2})(\frac{1}{2})$ $=-\frac{3}{2}\cos^2\frac{x}{2}\sin\frac{x}{2}$ $\int_0^{\pi}\cos^2\frac{x}{2}\sin\frac{x}{2}\ dx$ $=-\frac{2}{3}\int_0^{\pi}\left(-\frac{3}{2}\right)\cos^2\frac{x}{2}\sin\frac{x}{2}\ dx$ $=-\frac{2}{3}\left[\cos^3\frac{x}{2}\right]_0^{\pi}$ $=-\frac{2}{3}\left[\cos^3\frac{\pi}{2}-\cos^3 0\right]$ $=-\frac{2}{3}\left[(0)^3-(1)^3\right]$ $=\frac{2}{3}$

NAME: ______________________________ (   )   CLASS: _______

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2019
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS

4047/01

Paper 1

Date: 30 August 2019

Duration: 2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
Write your answers on the space provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use	
Paper 1	/ 80

Setter: Ms Lim Chee Chin and Mr Wilson Ho

This paper consists of 22 printed pages.

Name: ______________________________ (   )   Class: __________

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

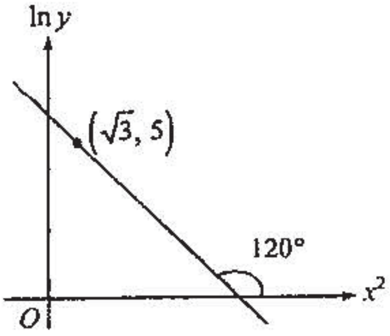
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

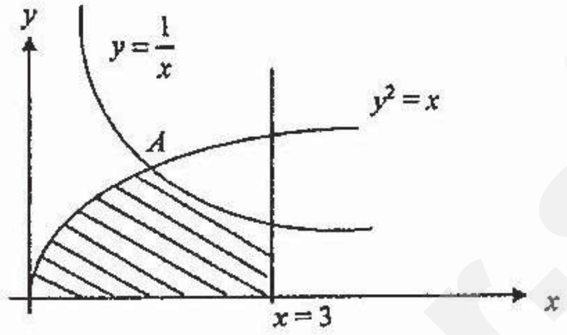
Name: ______________________________ (   )   Class: __________

1 Solve the equation $5\sin y\cos y-3\cos^2 y=0$ for $0° < y < 360°$. [4]

2 The figure shows part of a straight line graph obtained by plotting ln y against x^2, together with the coordinates of a point ($\sqrt{3}$,5) on the line. Express y as a function of x. [4]

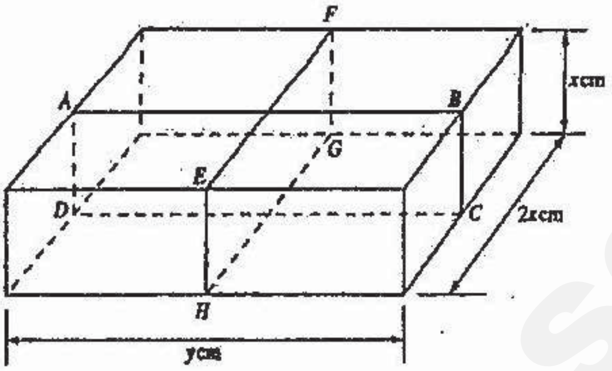


3 The diagram shows part of the curve $y = \frac{1}{x}$ $(x > 0)$ and $y^2 = x$ which intersect at A. Calculate the area of the shaded region. [5]



Name: ______________________________ (   )   Class: __________

4



The diagram shows a package in the shape of a rectangular block whose sides are of length x cm, $2x$ cm and y cm. The package is secured by two pieces of ribbon, $ABCDA$ and $EFGHE$, whose total length is 312 cm. The volume of the package is V cm^3.

(i) Show that $V = 312x^2 - 8x^3$. [2]

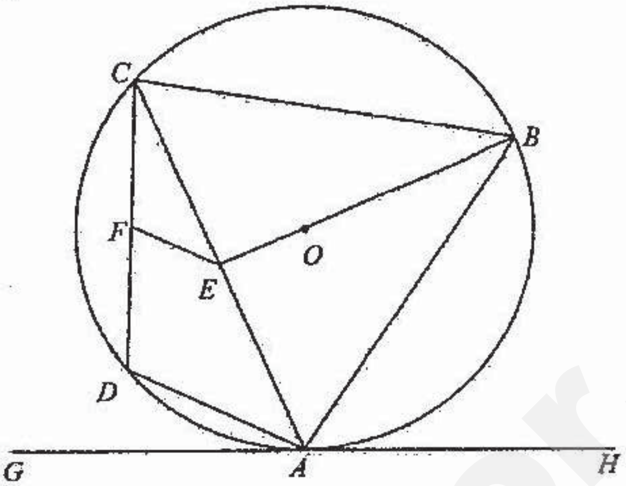
3

Name: ______________________________ (   )   Class: ________

4 (ii) Given that x can vary, find the dimensions of the block that make V a maximum. [3]

Name: ______________________________ (   )   Class: __________

5



The diagram shows a circle with centre O, passing through points A, B, C and D. GAH is a tangent to the circle at point A. It is given that $BA = BC$ and $FC = FD$. It is also given that E is a point on AC such that BE is perpendicular to AC and BE passes through O.

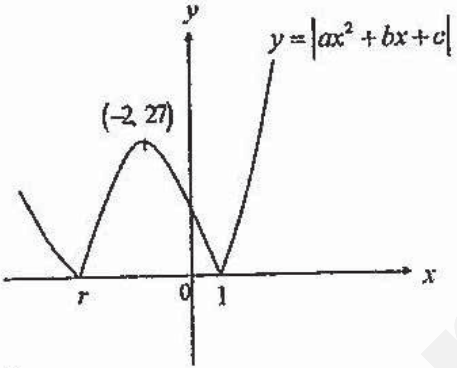
(i) Show that AB bisects $\angle CAH$. [3]

(4)

5 **(ii)** Show that FE is parallel to DA. [3]

Name: ______________________________ (   )   Class: __________

6 The diagram shows part of the curve $y=|ax^2+bx+c|$, where $a>0$. The curve touches the x-axis at $A(r,0)$ and $B(1,0)$ and has a maximum point at $(-2,27)$.



(i) Show that $r=-5$. [1]

(ii) Determine the value of a, b and c. [4]

(iii) State the value of q for which the line $y=q$ intersects the curve at exactly 3 points. [1]

5

Name: ______________________________ (   )   Class: __________

7 (a) Without using a calculator, show that cosec $105° = \sqrt{2}\left(\sqrt{3}-1\right)$. [3]

7 **(b)** Prove that $\tan x + \cot x = 2 \operatorname{cosec} 2x$. [4]

6

Name: ______________________________ (   )   Class: __________

8 The curve $y = e^{\frac{1}{2}x} + ke^{-\frac{1}{2}x}$, where k is a constant has a stationary point at $x = \ln 3$.

(i) Show that the value of $k = 3$. [3]

Name: ______________________________ (   )   Class: _________

8 (ii) Hence, find the y-coordinate of the stationary point in the form $a\sqrt{b}$ where a and b are integers, and determine the nature of the stationary point. [4]

7

Name: ______________________________ (   )   Class: __________

9 (a) The coefficient of x^{-2} in the expansion of $\left(1-\frac{3}{x}\right)^n$, where n is a positive integer, is 819.

Find the value of n. [4]

Name: ______________________________ (   )   Class: ________

9 **(b)** Write down, and simplify, the first 4 terms in the expansion of $(1+p)^9$ in ascending powers of p. Hence, find the coefficient of x^3 in the expansion of $(1-x-2x^2)^9$. [4]

8

Name: ______________________________ (   )   Class: _________

10 The equation of the curve C is $2y = x^2 + 4$. The equation of the line L is $y = 3x - k$, where k is an integer.

(i) Find the largest value of the integer k for which L intersects C. [4]

Name: ______________________________ (   )   Class: __________

10 (ii) In the case where $k = -2$, show that the line joining the points of intersection of L and C is bisected by the line $y = 2x + 5$. [4]

9

Name: ______________________________ (   )   Class: ________

11 (a) A curve is such that $\frac{dy}{dx} = \frac{3k}{(x-4)^2}$. The equation of the normal to the curve at the point where the curve crosses the x-axis is given by $y = 2x + 4$.

Find the value of k and hence find the equation of the curve. [5]

Name: ________________________________ (   )   Class: __________

11 **(b)** Show that $\frac{d}{dx}\left(\cos^3 x - 3\cos x\right) = 3\sin^3 x$.

Hence, evaluate $\int_{0.5}^{1}(3\sin^3 x - 2\sin x)\,dx$. [5]

10

Name: ______________________________ (   )   Class: __________

12 The line $x = 17$ is a tangent to a circle and the points $A(1, 9)$ and $B(1, -7)$ lie on the circle.

(i) Show that the radius of the circle is 10 units. [4]

12 **(ii)** State the coordinates of the centre of the circle. [1]

(iii) Write down the equation of the circle in the form $x^2 + y^2 + px + qy + r = 0$. [2]

(iv) The circle is reflected along the line $y = -1$. Determine whether the point (3, 10) lies on or inside or outside the reflected circle. [3]

~ End of Paper ~

Name: ______________________________ (   )   Class: __________

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Name: ______________________________ (   )   Class: __________

1 A cuboid has a square base of side $\left(5-\sqrt{12}\right)$ cm and a volume of $\left(14-\sqrt{27}\right)$ cm^3. Find the height of the cuboid in the form $\left(a+b\sqrt{3}\right)$ cm, where a and b are integers.

[4]

Name: ______________________________ (   )   Class: __________

2 A flock of 20 Yellow-crested Cockatoos was introduced to an island where it is not a native species. The population of Cockatoos is predicted to increase so that after a period of t years, the population, P, is given by $P = \frac{k}{1+4e^{-0.14t}}$, where k is a constant.

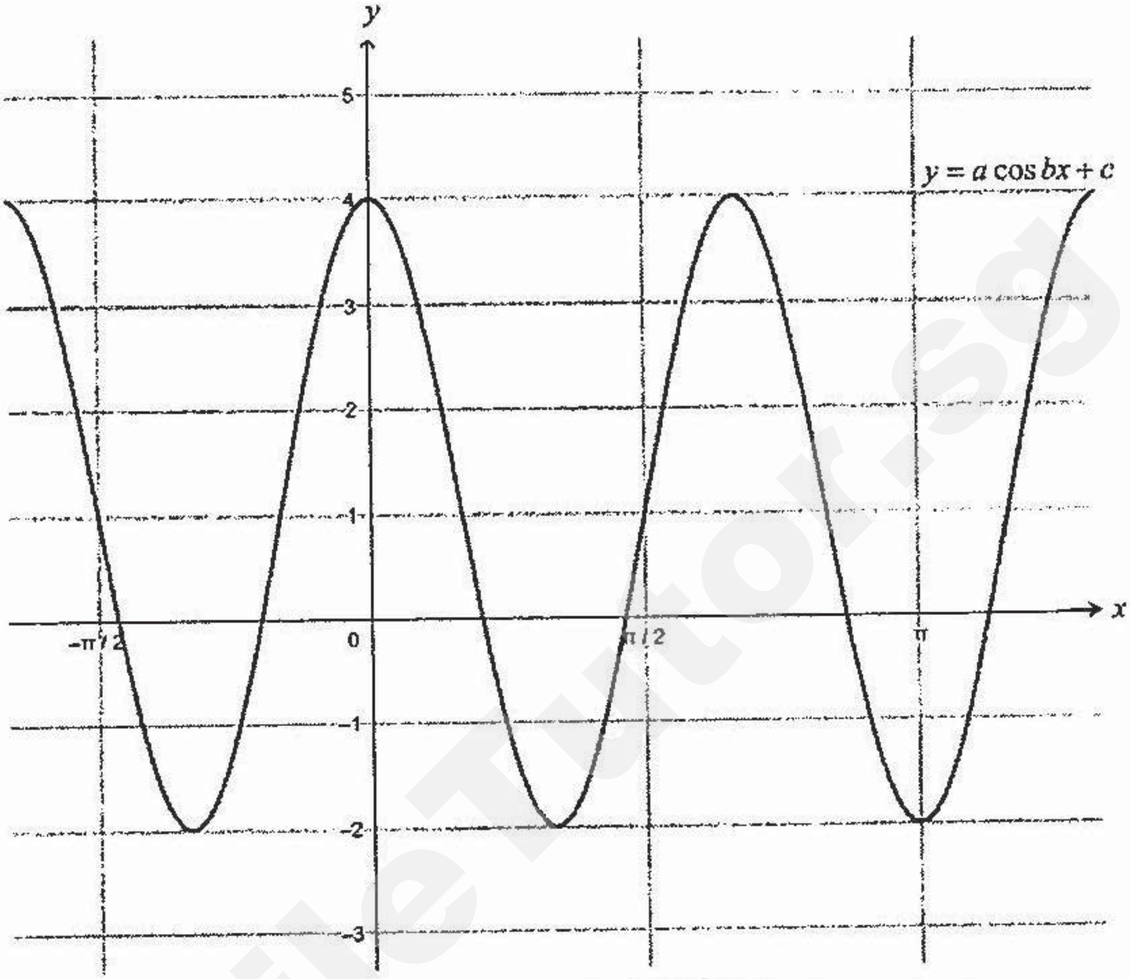
(i) Show that $k = 100$.
[1]

(ii) Find the number of Cockatoos present after 10 years. Give your answer correct to the nearest integer.
[2]

(iii) Find the value of P as t becomes very large. Explain the significance of this value. [2]

Name: ______________________________ (   )   Class: __________

3



The diagram below shows the graph of $y = a\cos bx + c$, where a, b and c are integers.

Name: ______________________________ (   )   Class: __________

(i) From the graph above, find the values of a, b and c.

[3]

(ii) Sketch on the same diagram, the graph of $y=1-2\sin x$ for the interval $0\leq x\leq\pi$.

State the number of solutions in the interval $0\leq x\leq\pi$ of the equation

$a\cos bx+c=1-2\sin x$.

[3]

Name: ______________________ (   )   Class: ________

4 A curve has the equation $y = \ln\left(\dfrac{6+2x}{5x-3}\right)$.

(i) Express $\dfrac{dy}{dx}$ in the form $\dfrac{k}{(3+x)(5x-3)}$, where k is a constant.

[4]

4 **(ii)** Show that y is a decreasing function for

$x > \dfrac{3}{5}$.   [2]

4

Name: ________________________________ (   )   Class: __________

5 The roots of the equation $x^2 - 2x + 7 = 0$ are α and β.

(i) *Show that*

$\alpha^3 + \beta^3 = -34$.

[4]

Name: ______________________________ (   )   Class: __________

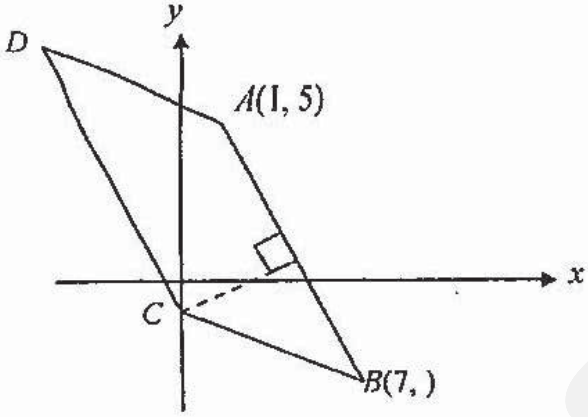
5 **(ii)** Given that the roots of $x^2+ax+b=0$ are $\frac{1}{\alpha^3}+2$ and $\frac{1}{\beta^3}+2$, find the values of a and b where a and b are constants.

[4]

3

Name: ________________________________ (   )   Class: __________

6 **Solution to this question by accurate drawing will not be accepted.**



The figure above shows a sketch of a quadrilateral *ABCD*. The coordinates of *A* and *B* are (1, 5) and (7, –3) respectively.

(i) Find the gradient of the line perpendicular to *AB*.
[2]

(ii) If the perpendicular bisector of *AB* cuts the *y*-axis at *C*, find the coordinates of *C*.
[
3]

Name: ______________________________ (   )   Class: _________

6 **(iii)** Find the coordinates of D if $ABCD$ is a parallelogram.

[1]

(iv) Calculate the area of the parallelogram $ABCD$.

[2]

5

7 **(a)** Given that $\frac{a^{y+1}}{b^{5-x}} \times \frac{b^{2y}}{a^{2x-2}} = ab^6$, find the value of x and of y.

[4]

Name: ______________________________ (   )   Class: _________

7 (b) Solve the equation $3^{2x-2} - 6(3^{x-1}) + 5 = 0$.

[4]

6

7 **(c)** Given that $\log_3 a = p$, $\log_{27} b = q$ and $\frac{a}{b} = 3^c$, express c in terms of p and q.

[4]

Name: ________________________________ (   )   Class: __________

8 The cubic polynomial $f(x)=6x^3+hx^2+kx-18$ is exactly divisible by $3x-2$ and leaves a remainder of 11 when it is divided by $x-1$.

(i) Show that $h=-4$ and $k=27$. [4]

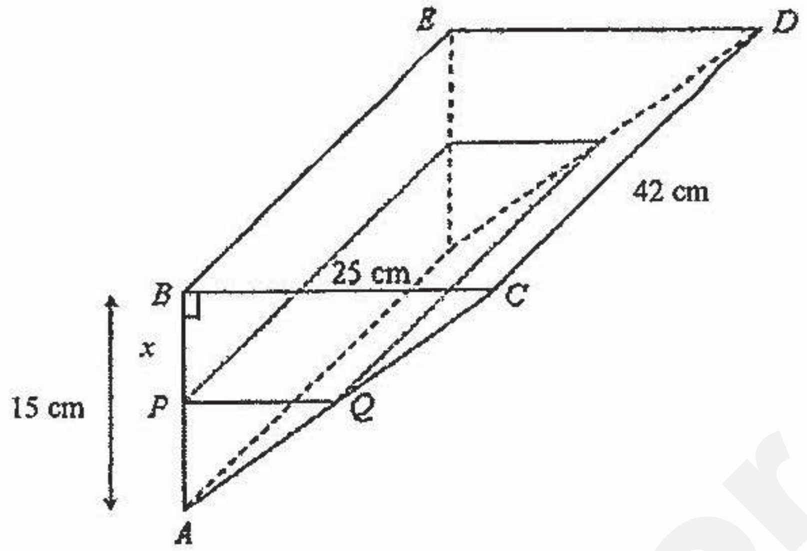
(8)

Name: ______________________________ (   )   Class: ________

8 **(ii)** Hence, factorise *f(x)* completely. [2]

8 **(iii)** Using the results in **(ii)**, express $\dfrac{-8x^2-7x+28}{f(x)}$ in partial fractions. [4]

9



A trough has the shape of a triangular prism as shown in the diagram. The cross-section is a right-angled triangle with a height of 15 cm. The open top *BCDE* is horizontal and rectangular in shape with $BC = 25$ cm and $BE = 42$ cm.

The trough is being filled with water. At time t seconds after filling starts, the surface of water is x cm from the open top (i.e. $BP = x$ cm), with *PQ* indicating the level reached. Given that the water is flowing into the trough at a rate of 45 cm^3/s,

(i) find *PQ* in terms of x,

[2]

9 (ii) show that the volume, V, of water in the trough at time t is $35(15-x)^2$,

[2]

⑦

Name: ______________________________ (   ) Class: __________

(iii) find the rate at which x is changing when $x = 8$.

[3]

10 (i) Express $\frac{4x}{2x-1}$ in the form $a+\frac{b}{2x-1}$, where a and b are integers.

[2]

(ii) Differentiate $2x\ln(2x-1)$ with respect to x.

[3]

Name: ______________________________ (   )   Class: __________

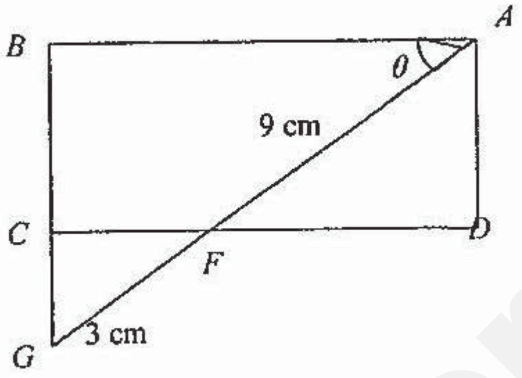
10 **(iii)** Using the results in part (i) and part (ii), determine $\int \ln(2x-1)\ dx$.

[4]

9

Name: ______________________________ (   )   Class: __________

11 In the diagram below, $ABCD$ is a rectangle. A line through A makes an angle of θ with AB and intersects DC and BC produced at F and G respectively. $AF = 9$ cm, $FG = 3$ cm and θ is acute.



(i) Show that the perimeter, P cm, of the rectangle $ABCD$ is given by

$P = 24\cos\theta + 18\sin\theta$.

[3]

(ii) Express P in the form $P = R\cos(\theta - \alpha)$ where R is positive and α is acute.

[4]

Name: ______________________________ (   )   Class: __________

11 **(iii)** Given that θ varies, state the maximum value of P and the corresponding value of θ.

[2]

(iv) Find the values of θ for which $P = 28$ cm.

[4]

(10)

Name: ______________________________ (   )   Class: __________

12 A particle starts from point O and moves in a straight line with a velocity, v ms^{-1}, given by $v = 5e^{-t} - \frac{1}{5}$ where t is the time in seconds after leaving O. Calculate the

(i) initial velocity of the particle, [1]

(ii) value of t when the particle is instantaneously at rest, [3]

12 **(iii)** acceleration of the particle when it is instantaneously at rest,

[3]

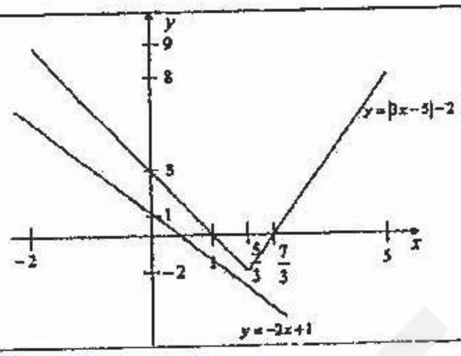
(iv) distance travelled from $t = 0$ to $t = 5$.

[5]

12

Name: ______________________________ (   )   Class: __________

Sec 4/5 Preliminary Examination 2019
Additional Mathematics Paper 1
Answer Key

1(i)	$k > \frac{25}{12}$	1(ii)	For values of $k > \frac{25}{12}$, $3x^2 - 5x + k > 0$. $(x-2)^2 \geq 0$ for all real values of x. Therefore $\frac{(x-2)^2}{3x^2 - 5x + k} \geq 0$
2	$x = 16$	3	$-\frac{621}{200}$ or $-3\frac{21}{200}$ or -3.105.
4(i)		4(ii) 4(iii)	$y = -2x + 1$ 0 solutions
5	20	6(ii)	$x = -23.6°, -156,4°, 203.6°, 336.4°$
7(i)	12 cm	7(ii)	$\frac{1}{4}$ cm²/s
7(iii)	The rate will decrease. Since $\frac{dr}{dt} = \frac{3}{r}$ (as $r \uparrow$, $\frac{dr}{dt} \downarrow$)	8(i)	$2x - 3 - \frac{2}{3x+2} + \frac{1}{2x-1}$
8(ii)	$x^2 - 3x - \frac{3}{2}\ln\|3x+2\| + \frac{1}{2}\ln\|2x-1\| + c$	9	$y = \frac{3}{2}x^3 - 3x^2 + \frac{3}{2}x + 4$
10(ii)(a)	$p = 2.0$ (Accept 1.9 – 2.1) $q = 3.0$ (Accept 2.5 – 3.1)	10(ii)(b)	$x = 4.39$ (Accept 4.36 – 4.49)
11(i)	$y = x - 3$	11(ii)	$Q(9, 0)$ S(3, 6)
11(iii)	$T(-15, 24)$	11(iv)	96 units²
11(v)	18.8 units	12(iii)	$x = \frac{2}{3}$ Maximum $V = 9.93$ cm³

~ End of Paper ~

Sec 4 Express Prelim Examination 2019
Additional Mathematics Paper 2
Answer Key

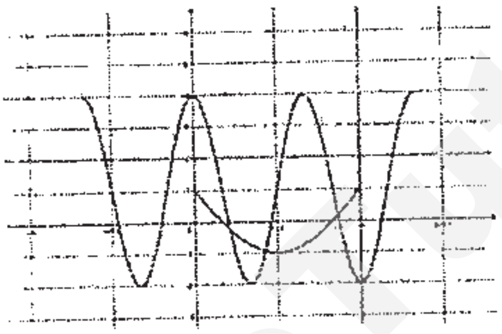
1. $h=2+\sqrt{3}$

2. (ii) 50

(iii) As t becomes very large, $e^{-0.14t}$ approaches zero.
Therefore, P approaches <u>100</u>.
This means that the <u>population of Cockatoos will not exceed 100.</u>

3 (i) $a=3, b=3, c=1$

(ii)



4(i) $\frac{dy}{dx}=-\frac{18}{(3+x)(5x-3)}$

4(ii) For $x>\frac{3}{5}$, $5x-3>0$.

Since $(3+x)>0$ and $(5x-3)>0$ for $x>\frac{3}{5}$,

$\Rightarrow (3+x)(5x-3)>0$

$\therefore -\frac{18}{(3+x)(5x-3)}<0$ for $x>\frac{3}{5}$,

$\Rightarrow \frac{dy}{dx}<0$

Thus, y is a decreasing function for $x>\frac{3}{5}$ (Shown)

5(ii) $\therefore a=-3\frac{309}{343}, b=3\frac{276}{343}$

6(i) $\frac{3}{4}$

6(ii) $C(0, -2)$

6(iii) $D(-6,6)$

6(iv) 50 units^2

7(a) $x=3$, $y=4$

7(b) $x=1$ or 2.46 (to 3 s.f.)

7(c) $c=p-3q$

8(ii) $f(x)=(3x-2)(2x^2+9)$

8(iii) $\frac{2}{3x-2}-\frac{4x+5}{2x^2+9}$

9(i) $PQ=\frac{5}{3}(15-x)$

9(iii) −0.0918 (to 3 s.f.) or $-\frac{9}{98}$ cm/s

10(i) $2+\frac{2}{2x-1}$

10(ii) $\frac{4x}{2x-1}+2\ln(2x-1)$

10(iii) $x\ln(2x-1)-x-\frac{1}{2}\ln(2x-1)+c$

11(ii) $P=30\cos(\theta-36.9)^\circ$ (to 1 d.p.)

11(iii) Max. $P=30$ when $\theta=36.9^\circ$

11(iv) $\theta=15.8^\circ, 57.9^\circ$

12(i) $4\frac{4}{5}$ m/s

12(ii) ln 25 or 3.22 (to 3 s.f.)

12(iii) −0.2 m/s^2

12(iv) 4.35 m (to 3 s.f.)

Name:______________________________ (   )   Class: Sec ________

80

SERVE-LEAD-EXCEL

GREENDALE SECONDARY SCHOOL
Preliminary Examination 2019

Additional Mathematics **4047/01**

Paper 1 18 September 2019

Secondary 4 Express / 5 Normal Academic **2 hours**

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

Question	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12
Marks												

No of additional bookets/ writing paper used		**No of additional graph paper used**	

This document consists of 15 printed pages including this cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

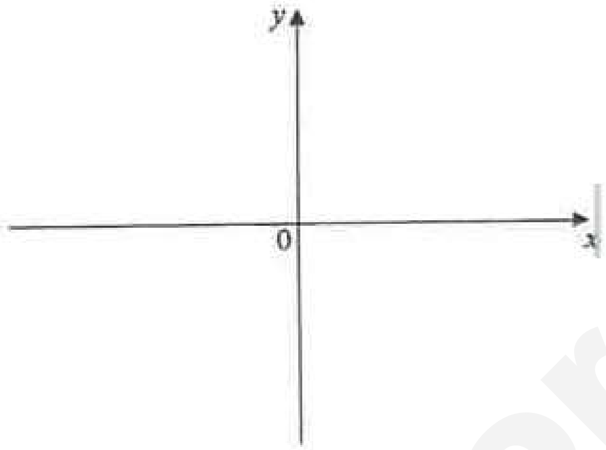
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$Area\ of\ \Delta = \frac{1}{2}bc\sin A$$

Answer all the questions.

1 Express $\frac{10x^2-7x+10}{(3x-2)(x^2+2)}$ in partial fractions. [5]

2 (i) Sketch the graph of $y^2 = 3x$. [1]



(ii) Find the coordinates of the points of intersection of the curve $y^2 = 3x$ and the line $3y = 6x - 5$. [4]

3 The variables x and y are such that when the values of xy are plotted against $\sqrt{x}$, a straight line is obtained.

It is given that $y=\frac{1}{2}$ when $x=1$, and that $y=-\frac{1}{4}$ when $x=4$.

(i) Express y in terms of x. [4]

(ii) Find the value of y when $x=16$. [1]

4 (i) Show that $\frac{\cos 2x - \cos 4x}{2\sin^2 x} = 1 + 2\cos 2x$. [3]

(ii) Hence find, for $0^\circ < x < 360^\circ$, the values of x for which $\frac{\cos 2x - \cos 4x}{2\sin^2 x} = 2$. [3]

5 The roots of a quadratic equation $4x^2 - 37x + 9 = 0$ are α^2 and β^2, where $\alpha < 0 < \beta$ and $\beta < |\alpha|$.

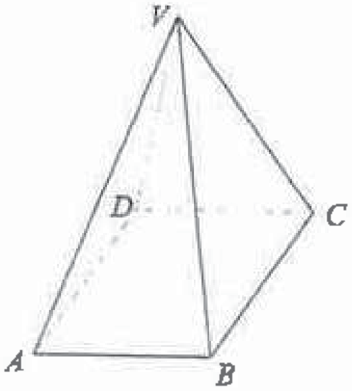
(i) Show that $\alpha\beta = -\frac{3}{2}$ and find the value of $\alpha + \beta$. [4]

(ii) Find a quadratic equation whose roots are $\frac{\alpha}{\alpha+\beta}$ and $\frac{\beta}{\alpha+\beta}$. [2]

6 A curve is such that $\frac{d^2 y}{d x^2} = 1 - \frac{4}{(2x+5)^2}$ and has a stationary point at $P(-2, 5)$.

Find the equation of the curve. [5]

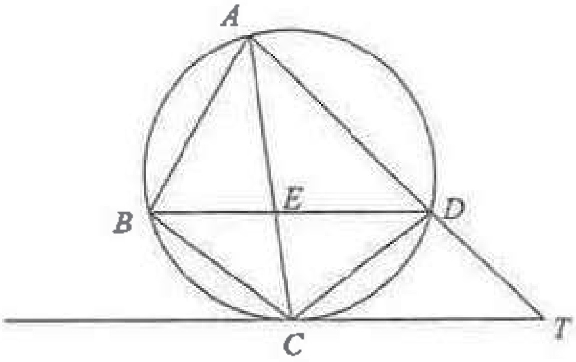
7 *VABCD* is a right pyramid with a square base *ABCD*, as shown in the diagram. The volume of the pyramid is $\left(6\sqrt{3}-8\right)$ cm^3 and the height is $\left(1+2\sqrt{3}\right)$ cm.



(i) Show that $AB^2 = 12 - 6\sqrt{3}$. **[3]**

(ii) Find the value of VA^2, giving your answer in the form $p + q\sqrt{3}$ where p and q are rational numbers. **[4]**

8 The diagram shown is not drawn to scale.

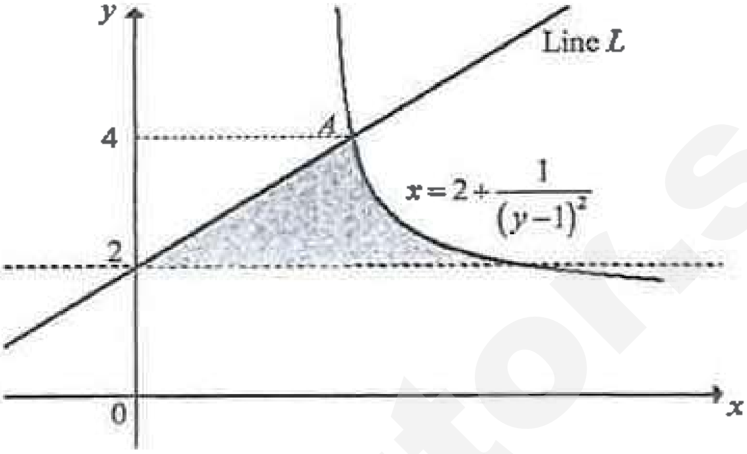


A, B, C and D are four points on the circle such that $CB = CD$. The chords AC and BD meet at E. The tangent to the circle at C meets AD extended at T.

(i) Prove that BD is parallel to CT. [3]

(ii) Show that $CT^2 = AT \times DT$ [4]

9 The diagram shows part of the curve $x = 2 + \frac{1}{(y-1)^2}$, $y \neq 1$. A line L intersects the curve at A, where $y = 4$, and cuts the y-axis at $y = 2$.



(i) Find the equation of line L. [3]

(ii) Find the area of the shaded region bounded by the line L, the line $y = 2$ and the curve $x = 2 + \frac{1}{(y-1)^2}$. [4]

10 An experiment to measure the growth of bacteria was conducted.

At 0900 on Monday, 1000 bacteria were introduced to the culture. At 1700 on the same day, the number of bacteria had grown to 1492. It is known that the number of bacteria, N, at t hours from the start of the experiment, is given by $N = pe^{kt}$, where p and k are constants.

(i) Find the value of p and of k. [3]

(ii) Calculate the number of bacteria at 0900 on Tuesday. [2]

(iii) Determine the earliest day and time (to the whole hour) at which there is at least 20 000 bacteria. [3]

11 The equation of a circle C_1 is $x^2+6x+y^2-16y+24=0$, and its centre is P.

(i) Find the coordinates of P and the radius of C_1. **[2]**

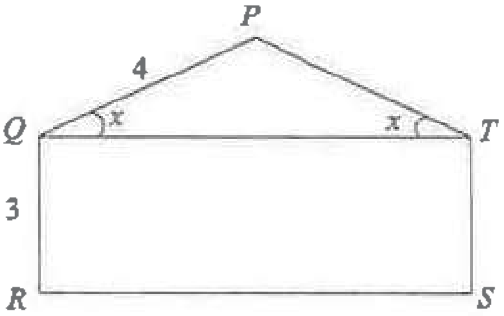
AB is a chord of C_1 and M is the midpoint of AB, where $M(-1,12)$.

(ii) Find the equation of the chord AB. [3]

A second circle C_2 with centre Q also passes through A and B.

(iii) Given that $PM:MQ=1:2$, show that one possible point for Q is $(3,20)$ and find the coordinates of another point. [4]

12 $PQRST$ is a pentagon as shown in the diagram. $QRST$ is a rectangle with $QR = 3$ cm. PQT is a triangle with $PQ = 4$ cm and $\angle PQT = \angle PTQ = x$ radians.



(i) Show that $QT = k\cos x$, where k is a positive integer to be found. [2]

(ii) Show that the area of the pentagon, A cm^2 is given by $A = 8\sin 2x + 24\cos x$. [2]

[Question 12 continues on next page]

[Question 12 continues]

(iii) Find the stationary value of A and determine whether it is a maximum or a minimum. **[6]**

END-OF-PAPER

Name:______________________________ (   ) Class:S

/ 100

SERVE·LEAD·EXCEL

GREENDALE SECONDARY SCHOOL
Preliminary Examination 2019

Additional Mathematics Paper 2

4047/2

17 September 2019

Secondary 4 Express / 5 Normal Academic

2 hours 30 mins

Candidates answer on the Question Paper.
No Additional Materials are required.

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The total number of marks for this paper is **100**.

Q	1	2	3	4	5	6	7	8	9	10	11
M											

No of additional booklets/ writing paper used		**No of additional graph paper used**	

Target Before :

Target After :

This document consists of **16** printed pages including this cover page.

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$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 A curve has the equation $y=(ax-3)\ln x$, where $x>0$, $x \neq \frac{3}{a}$ and a is a positive constant. The normal to the curve at the point where the curve crosses the x-axis is parallel to the line $x+5y-4=0$. Find the value of a. [7]

2 **(a)** Differentiate the following with respect to x.

(i) $\ln(\cos 2x)$ **[2]**

(ii) $\frac{x}{2}\tan 2x$ **[2]**

(b) Using your results from part **(a)**, find $\int 2x\sec^2 2x\,dx$. **[4]**

3 **(i)** Given that the constant term in the binomial expansion of $\left(\frac{2}{x}-\frac{x^2}{k}\right)^6$ is 60, find the value of the positive constant k. **[4]**

(ii) Using the value of k found in part **(i)**, find the term independent of x in the expansion of $\left(1+x^3\right)\left(\frac{2}{x}-\frac{x^2}{k}\right)^6$. **[4]**

4 (a) A particle moves along the curve $y = 3x^2 - 2x + 5$. At the point P, the x-coordinate of the particle is increasing at a rate of 0.002 units/sec and the y-coordinate is increasing at 0.02 units/sec.
Find the coordinates of P. [4]

(b) The equation of a curve is $y = x^3 + 5x^2 - 8x + k$, where k a constant.
Find the set of values of x for which y is decreasing. [4]

5 **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\ln 2x}{x^3}\right)=\frac{1}{x^4}-\frac{3\ln 2x}{x^4}$. **[4]**

(ii) Hence, integrate $\frac{\ln 2x}{x^4}$ with respect to x. **[3]**

(iii) Given that the curve $y=\mathrm{f}(x)$ passes through the point $\left(1,\frac{8}{9}\right)$ and is such that $\mathrm{f}'(x)=\frac{\ln 2x}{x^4}$, find f($x$). **[2]**

6 Mr Tan drives his car along a straight road. As he passes a point *A* he applies the brake and his car slows down, coming to a rest at point *B*. For the journey from *A* to *B*, the distance, *s* meters, of the car from *A*, *t* seconds after passing *A*, is given by

$$s = 600\left(1 - e^{\quad 6}\right) - 12t$$

(i) Find an expression, in terms of *t*, for the velocity of the car during the journey from *A* to *B*. [2]

(ii) Find the velocity of the car at *A*. [1]

(iii) Find the time taken for the journey from *A* to *B*. [3]

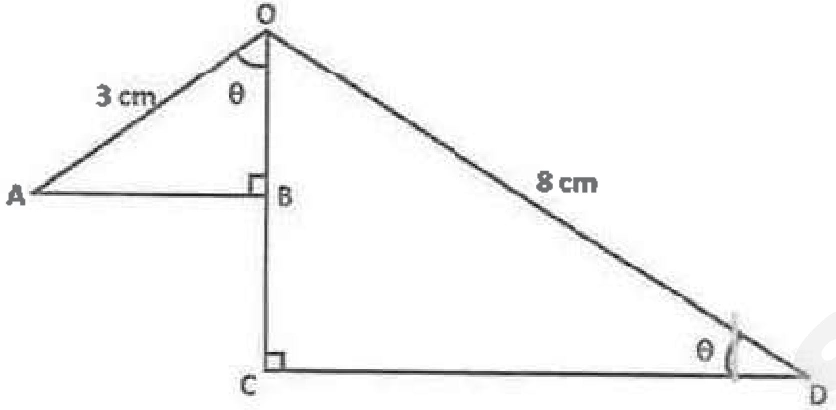
(iv) Find the average speed of the car for the journey from *A* to *B*. [3]

7 Solve each of the following equations.

(i) $e^{2\ln x} + \ln e^{2x} = 8$ **[5]**

(ii) $\log_5 50 + 4\log_{25} x - \log_5(2x+4) = 2$ **[5]**

8 In the diagram, triangles OAB and OCD are right-angled triangles.
Angle AOB = angle $ODC = \theta$, $OA = 3$ cm and $OD = 8$ cm.



(i) Show that the length of $AB + CD = 3\sin\theta + 8\cos\theta$ **[1]**

(ii) Express $3\sin\theta + 8\cos\theta$ in the form $R\sin(\theta + \alpha)$ where $R > 0$ and α is acute. **[4]**

8 **(iii)** Find the maximum length of $AB + CD$ and the corresponding value of θ. **[3]**

(iv) Find the value of θ, if B is the midpoint of OC. **[2]**

9 The function f is defined by $f(x)=4\cos 2x-3$.

(i) State the amplitude of f. [1]

(ii) State the period of f in terms of π. [1]

The equation of a curve is $y=4\cos 2x-3$ for $0\leq x\leq \pi$

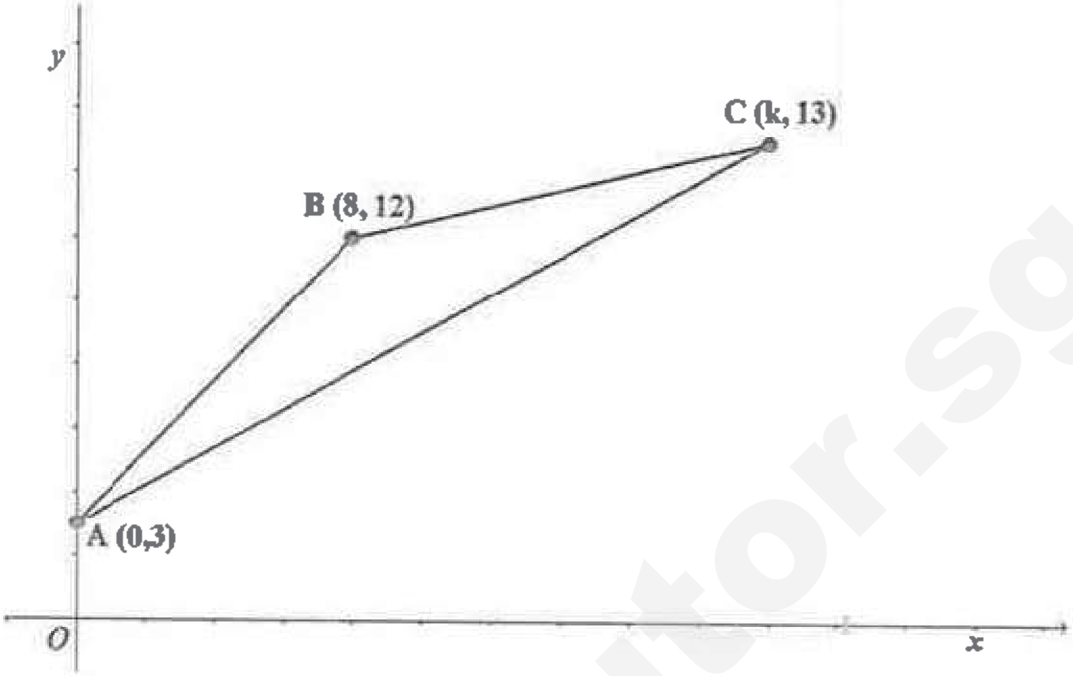
(iii) Find the minimum value of the curve. [1]

(iv) Find the x-coordinates of the points where the curve meets the x-axis. [3]

9 (v) Sketch the graph of $y=|4\cos 2x-3|$ for $0\le x\le \pi$. [3]

(vi) Hence, find the range of values of c, for which $|4\cos 2x-3|=c$ has exactly two solutions only. [1]

10 The diagram shows a triangle ABC with vertices at $A(0, 3)$, $B(8, 12)$ and $C(k, 13)$.



Given that $AB = BC$,

(i) find the value of k. **[4]**

10 A line is drawn from B to meet the x-axis at D such that $AD = CD$.

(ii) Name the quadrilateral $ABCD$. [1]

(iii) Find the equation of BD and the coordinates of D. [4]

(iv) Find the area of the triangle ABC. [2]

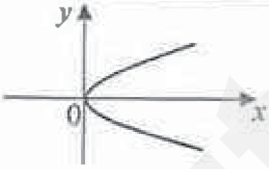
11 **(a)** **(i)** Find the range of values of x for which $x^2 - 8x +$ [illegible] 0 **[2]**

(ii) Hence, find the range of values of x for which
$(x+2)^2 - 8x - 1 < 0$. **[3]**

(b) Show that $my = x^2 - 4(x-1)$ meets the curve $y = x^2 - 3x + 2$ at two distinct points for all real values of m, except $m = 0$ and $m = 1$. **[5]**

End of Paper

2019 PRELIMINARY EXAMINATION
SECONDARY 4E5N
AMATH PAPER 1 – MARK SCHEME

1		Let $\frac{10x^2-7x+10}{(3x-2)(x^2+2)}=\frac{A}{3x-2}+\frac{Bx+C}{x^2+2}$ $10x^2-7x+10=A(x^2+2)+(Bx+C)(3x-2)$ Sub $x=\frac{2}{3}$ to get $A=4$ Sub $x=0$ to get $C=-1$ Sub $x=1$ (or any other value) to get $B=2$ $\frac{10x^2-7x+10}{(3x-2)(x^2+2)}=\frac{4}{3x-2}+\frac{2x-1}{x^2+2}$	
			TOTAL: 5m
2	(i)		
	(ii)	Equate both equations to reduce to one variable $2y^3-3y-5=0$ $(2y-5)(y+1)=0$ Solve for x and y The points of intersection are $\left(\frac{25}{12},\frac{5}{2}\right)$ and $\left(\frac{1}{3},-1\right)$	
			TOTAL: 5m
3	(i)	Coordinates of two points on the straight line are $\left(1,\frac{1}{2}\right)$ and $(2,-1)$ Gradient of line $=\frac{\frac{1}{2}-(-1)}{1-2}=-\frac{3}{2}$ Equation of curve is $\frac{xy-(-1)}{\sqrt{x}-2}=-\frac{3}{2}$ $y=\frac{4-3\sqrt{x}}{2x}$	
	(ii)	$y=-\frac{1}{4}$	
			TOTAL: 5m

4	(i)	$\frac{\cos 2x-\cos 4x}{2\sin^2 x}=\frac{\cos 2x-\left(2\cos^2 2x-1\right)}{2\sin^2 x}$ $=\frac{(1+2\cos 2x)(1-\cos 2x)}{2\sin^2 x}$ $=\frac{(1+2\cos 2x)\left(1-1+2\sin^2 x\right)}{2\sin^2 x}$ $=1+2\cos 2x$ (shown)
	(ii)	$1+2\cos 2x=2$ $\cos 2x=\frac{1}{2}$ $2x=60^\circ,300^\circ,420^\circ,660^\circ$ $x=30^\circ,150^\circ,210^\circ,330^\circ$
		TOTAL: 6m
5	(i)	$\alpha^2\beta^2=\frac{9}{4}$ $\alpha<0<\beta \quad \Rightarrow \quad \alpha\beta<0$ Hence $\alpha\beta=-\sqrt{\frac{9}{4}}=-\frac{3}{2}$ (shown) $\alpha^2+\beta^2=\frac{37}{4} \Rightarrow (\alpha+\beta)^2=\alpha^2+2\alpha\beta+\beta^2=\frac{25}{4}$ $\beta<\lvert\alpha\rvert$ and $\alpha<0<\beta \quad \Rightarrow \quad \alpha+\beta<0$ Hence $\alpha+\beta=-\sqrt{\frac{25}{4}}=-\frac{5}{2}$
	(ii)	SOR: $\frac{\alpha}{\alpha+\beta}+\frac{\beta}{\alpha+\beta}=\frac{\alpha+\beta}{\alpha+\beta}=1$ POR: $\frac{\alpha}{\alpha+\beta}\times\frac{\beta}{\alpha+\beta}=\frac{\alpha\beta}{(\alpha+\beta)^2}=\frac{-\frac{3}{2}}{\left(-\frac{5}{2}\right)^2}=-\frac{6}{25}$ The equation is $x^2-x-\frac{6}{25}=0$ (or $25x^2-25x-6=0$)
		TOTAL: 6m

6		$\frac{\mathrm{d}y}{\mathrm{d}x}=\int\left[1-\frac{4}{(2x+5)^2}\right]\mathrm{d}x=x+\frac{2}{2x+5}+c$ Sub $x=-2$, $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ to get $c=0$, so $\frac{\mathrm{d}y}{\mathrm{d}x}=x+\frac{2}{2x+5}$ $y=\int\left[x+\frac{2}{2x+5}\right]\mathrm{d}x=\frac{1}{2}x^2+\ln(2x+5)+k$ Sub $x=-2$, $y=5$ to get $k=3$ $y=\frac{1}{2}x^2+\ln(2x+5)+3$	
			TOTAL: 5m
7	(i)	$\frac{1}{3}(AB)^2\left(1+2\sqrt{3}\right)=6\sqrt{3}-8$ $AB^2=\frac{18\sqrt{3}-24}{1+2\sqrt{3}}\times\frac{1-2\sqrt{3}}{1-2\sqrt{3}}$ $=\frac{18\sqrt{3}-108-24+48\sqrt{3}}{1-12}$ $=\frac{66\sqrt{3}-132}{-11}=12-6\sqrt{3}$ (shown)	
	(ii)	Let M be midpoint of AC. By Pythagoras Theorem, $AC^2=AB^2+BC^2=2AB^2$, so $AM^2=\left(\frac{1}{2}AC\right)^2=\frac{1}{2}AB^2=6-3\sqrt{3}$ $VA^2=AM^2+VM^2$ $=\left(6-3\sqrt{3}\right)+\left(1+2\sqrt{3}\right)^2=19+\sqrt{3}$	
			TOTAL: 7m
8	(i)	$\angle BDC=\angle CBD$ (base angles in isos $\triangle$) $=\angle TCD$ (alternate segment theorem) By the alternate-angle property, BD is parallel to CT.	
	(ii)	$\angle TCD=\angle TAC$ (alternate segment theorem) $\angle CTD=\angle ATC$ (common angle) Hence ΔTCD is similar to ΔTAC (AA-test) $\frac{CT}{AT}=\frac{DT}{CT}\quad\Rightarrow\quad CT^2=AT\times DT$ (shown)	
			TOTAL: 7m

9	(i)	When $y=4$, $x=2+\frac{1}{(4-1)^2}=\frac{19}{9}$ Gradient of line L $=\frac{4-2}{\frac{19}{9}-0}=\frac{18}{19}$ Equation of L is $y=\frac{18}{19}x+2$
	(ii)	Area $=\int_2^4\left[2+\frac{1}{(y-1)^2}\right]\mathrm{d}y-\frac{1}{2}(2)\left(\frac{19}{9}\right)$ $=\left[2y-\frac{1}{y-1}\right]_2^4-\frac{19}{9}$ $=\left[\left(8-\frac{1}{3}\right)-(4-1)\right]-\frac{19}{9}=\frac{23}{9}$ sq units
		TOTAL: 7m
10	(i)	$p=1000$ Sub $t=8$, $N=1492$ and $p=1000$ (found value) $1492=1000e^{8k}$ $8k=\ln\left(\frac{1492}{1000}\right)$ $k=0.05001\approx0.05$
	(ii)	Sub $t=24$ $N=1000e^{0.05(24)}=3320.1\approx3320$
	(iii)	$1000e^{0.05t}\geq20000$ $t\geq\frac{\ln\left(\frac{20000}{1000}\right)}{0.05}$ $t\geq59.9$ hours = 2 days 11.9 hours On Wednesday 2100 (or 9pm)
		TOTAL: 8m
11	(i)	$(x+3)^2+(y-8)^2=49$ Centre $P=(-3,8)$, Radius $=7$
	(ii)	Gradient of PM= $\frac{8-12}{-3-(-1)}=2$ Gradient of chord AB = $-\frac{1}{2}$ (*AB* perpendicular to *PM*) Equation of chord AB is

		$\frac{y-12}{x-(-1)}=-\frac{1}{2}$ $2y=-x+23$
	(iii)	Note that P, M and Q lie on a straight line. Case 1: M is between P and Q $x_Q=x_M+2(x_M-x_P)=-1+2(-1-(-3))=3$ $y_Q=y_M+2(y_M-y_P)=12+2(12-8)=20$ So coordinate of Q is $(3,20)$ (shown) Case 2: P is the midpoint of Q and M $x_P=\frac{x_Q+x_M}{2} \Rightarrow x_Q=2x_P-x_M=2(-3)-(-1)=-5$ $y_P=\frac{y_Q+y_M}{2} \Rightarrow y_Q=2y_P-y_M=2(8)-(12)=4$ So coordinate of Q is $(-5,4)$.
		TOTAL: 9m
12	(i)	Using cosine rule, $QT^2=4^2+4^2-2(4)(4)\cos(\pi-2x)$ $=32+32\cos 2x$ $=32+32(2\cos^2 x-1)$ $=64\cos^2 x$ $QT=\sqrt{64\cos^2 x}=8\cos x$
	(ii)	$A=\frac{1}{2}(4)(4)\sin(\pi-2x)+3(8\cos x)$ $=8\sin 2x+24\cos x$ (shown)
	(iii)	$\frac{dA}{dx}=16\cos 2x-24\sin x=0$ $16(1-2\sin^2 x)-24\sin x=0$ $4\sin^2 x+3\sin x-2=0$ $\sin x=0.4253$ or -1.175 (rejected) For stationary point, $x=0.4392$, $A=27.8799\approx 27.9\text{ cm}^2$ $\frac{d^2A}{dx^2}=-32\sin 2x-24\cos x$ When $x=0.4392$, $\frac{d^2A}{dx^2}=-46.35<0$, so $A=27.9\text{ cm}^2$ is a maximum area.
		TOTAL: 10m

Marking Scheme

1 A curve has the equation $y=(ax-3)\ln x$, where $x>0$, $x\neq\dfrac{3}{a}$ and a is a positive constant. The normal to the curve at the point where the curve crosses the x-axis is parallel to the line $x+5y-4=0$. Find the value of a. **[7]**

$(ax-3)\ln x=0$	M1
$\ln x=0$	
$x=1$	M1
$x+5y-4=0$	
$y=-\dfrac{1}{5}x+\dfrac{4}{5}$	
$m_{line}=-\dfrac{1}{5}$	M1
$m_{\perp}=5$	M1
$y=(ax-3)\ln x$	
$\dfrac{dy}{dx}=\dfrac{(ax-3)}{x}+a\ln x$	M1
$@\,x=1,\quad m_{tan}=\dfrac{a-3}{1}+a\ln 1$	M1
$=a-3$	
$\therefore\quad a-3=5$	
$a=8$	A1

2a Differentiate the following with respect to x,

(i) $\ln(\cos 2x)$ **[2]**

$y=\ln(\cos 2x)$	
$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\cos 2x}\cdot -\sin 2x\cdot 2$	M1
$=-2\tan 2x$	A1

(ii) $\frac{x}{2}\tan 2x$ **[2]**

$y=\frac{x}{2}\tan 2x$	
$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x}{2}\cdot\sec^2 2x\cdot 2+\tan 2x\cdot\frac{1}{2}$	M1
$=x\sec^2 2x+\frac{1}{2}\tan 2x$	A1

b Using your results from part (a) find $\int 2x\sec^2 2x\,\mathrm{d}x$. **[4]**

$\int x\sec^2 2x+\frac{1}{2}\tan 2x\,\mathrm{d}x=\frac{x}{2}\tan 2x$	M1
$\int x\sec^2 2x\,\mathrm{d}x=\frac{x}{2}\tan 2x-\frac{1}{2}\int\tan 2x\,\mathrm{d}x$	M1
$2\int x\sec^2 2x\,\mathrm{d}x=x\tan 2x-\int\tan 2x\,\mathrm{d}x$	M1
$\int 2x\sec^2 2x\,\mathrm{d}x=x\tan 2x+\frac{1}{2}\ln\cos 2x+c$	A1

3 (i) Given that the constant term in the binomial expansion of $\left(\frac{2}{x}-\frac{x^2}{k}\right)^6$ is 60, find the value of the positive constant k. [4]

$T_{r+1}=\left(\frac{2}{x}\right)^{6-r}C_r^6\left(\frac{x^2}{k}\right)^r$	M1
$\rightarrow x^{-6+r}\times x^{2r}$	M1
$\therefore 3r-6=0$	
$r=2$	M1
$T_3=\left(\frac{2}{x}\right)^4 C_2^6\left(\frac{x^2}{k}\right)^2$	
$\frac{240}{k^2}=60$	
$k=2,\quad -2(NA)$	A1

(ii) Using the value of k found in part (i), find the term independent of x in the expression $\left(1+x^3\right)\left(\frac{2}{x}-\frac{x^2}{k}\right)^6$. [4]

$\left(1+x^3\right)\left(\frac{2}{x}-\frac{x^2}{k}\right)^6$	
$=\left(1+x^3\right)\left[\left(\frac{2}{x}\right)^6+\left(\frac{2}{x}\right)^5C_1^6\left(-\frac{x^2}{2}\right)^1+\left(\frac{2}{x}\right)^4C_2^6\left(-\frac{x^2}{2}\right)^2+\ldots\right]$	M2
$=\left(1+x^3\right)\left[\ldots-6\left(\frac{2^{5-1}}{x^{5-2}}\right)+15\left(\frac{2^{4-2}}{x^{4-4}}\right)+\ldots\right]$	M1
$=-96+60$	
$=-36$	A1

4a A particle moves along the curve $y=3x^2-2x+5$. At the point P, the x-coordinate of the particle is increasing at a rate of 0.002 units/sec and the y-coordinate is increasing at 0.02 units/sec. Find the coordinates of P. [4]

$y=3x^2-2x+5$	
$\frac{dy}{dx}=6x-2$	M1
$\frac{dx}{dt}=0.002\ u/s$ $\quad\frac{dy}{dt}=0.02\ u/s$	Both seen M1
$\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$	
$6x-2=\frac{0.02}{0.002}$	M1
$x=2$	A1
$y=3(2)^2-2(2)+5$	
$=13$	
$P(2,13)$	

b The equation of a curve is $y=x^3+5x^2-8x+k$, where k is a constant. Find the set of values of x for which y is decreasing. [4]

$y=x^3+5x^2-8x+k$	
$\frac{dy}{dx}=3x^2+10x-8$	M1
For decreasing function, $\frac{dy}{dx}<0$	
$3x^2+10x-8<0$	M1
$(3x-2)(x+4)<0$	M1
$-4<x<\frac{2}{3}$	A1

5 **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\ln 2x}{x^3}\right)=\frac{1}{x^4}-\frac{3\ln 2x}{x^4}$ **[4]**

$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\ln 2x}{x^3}\right)=\frac{x^3\cdot\frac{1}{2x}\cdot 2-\ln 2x\cdot\left(3x^2\right)}{\left(x^3\right)^2}$	M1, M1
$=\frac{x^2}{x^6}-\frac{3x^2\ln 2x}{x^6}$	M1
$=\frac{1}{x^4}-\frac{3\ln 2x}{x^4}$	A1

(ii) Hence, integrate $\frac{\ln 2x}{x^4}$ with respect to x. **[3]**

$\int\frac{1}{x^4}-\frac{3\ln 2x}{x^4}\,\mathrm{d}x=\frac{\ln 2x}{x^3}$	M1
$\int\frac{3\ln 2x}{x^4}\,\mathrm{d}x=\int\frac{1}{x^4}\,\mathrm{d}x-\frac{\ln 2x}{x^3}$	M1
$3\int\frac{\ln 2x}{x^4}\,\mathrm{d}x=\frac{x^{-3}}{-3}-\frac{\ln 2x}{x^3}+c$	
$\int\frac{\ln 2x}{x^4}\,\mathrm{d}x=-\frac{1}{9x^3}-\frac{\ln 2x}{3x^3}+c$	A1

(iii) Given that the curve $y=\mathrm{f}(x)$ passes through the point $\left(1,\frac{8}{9}\right)$ and is such that $\mathrm{f}'(x)=\frac{\ln 2x}{x^4}$, find f($x$). **[2]**

$\mathrm{f}(x)=\int\frac{\ln 2x}{x^4}\,\mathrm{d}x$	
$y=-\frac{1}{9x^3}\left[1+3\ln 2x\right]+c$	M1
$\frac{8}{9}=-\frac{1}{9}\left[1+3\ln 1\right]+c$	
$c=1$	A1
$f(x)=-\frac{1}{9x^3}\left[1+3\ln 2x\right]+1$	

6 Mr Tan drives his car along a straight road. As he passes a point A he applies the brake and his car slows down, coming to a rest at point B.
For the journey from A to B, the distance, s meters, of the car from A, t seconds after passing A, is given by

$$s = 600\left(1 - e^{-\frac{t}{6}}\right) - 12t$$

(i) Find an expression, in terms of t, for the velocity of the car during the journey from A to B. [2]

$s = 600 - 600e^{-\frac{t}{6}} - 12t$ $\frac{ds}{dt} = -600 \cdot e^{-\frac{t}{6}} \cdot \left(-\frac{1}{6}\right) - 12$ $v = 100e^{-\frac{t}{6}} - 12$	 M1 A1

(ii) Find the velocity of the car at A. [1]

$v = 100e^{-\frac{t}{6}} - 12$ $= 100 - 12$ $= 88\ m/s$	 B1

(iii) Find the time taken for the journey from A to B. [3]

$0 = 100e^{-\frac{t}{6}} - 12$ $100e^{-\frac{t}{6}} = 12$ $-\frac{t}{6} = \ln\left(\frac{12}{100}\right)$ $t = 12.72\ s$	M1 M1 A1

(iv) Find the average speed of the car for the journey from A to B. [3]

$\text{Ave speed} = \frac{\text{tot dist}}{\text{tot time}}$ $= \frac{600\left(1 - e^{-\frac{12.72}{6}}\right) - 12(12.72)}{12.72}$ $= 29.5\ m/s$	 M1 (num) M1 (den) A1

7 Solve each of the following equations.

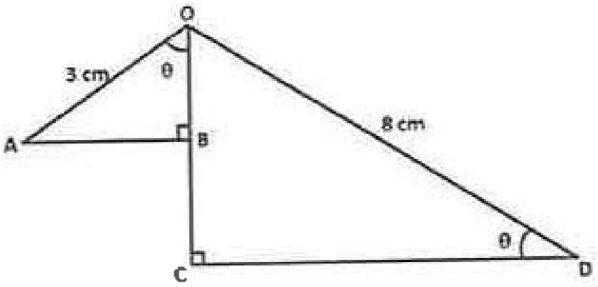
(i) $e^{2\ln x}+\ln e^{2x}=8$ **[5]**

$e^{2\ln x}+\ln e^{2x}=8$	
$e^{\ln x^2}+2x\ln e=8$	M1, M1
$x^2+2x-8=0$	M1
$(x+4)(x-2)=0$	
$x=2,\quad or\ x=-4\ (NA)$	A1, A1

(ii) $\log_5 50+4\log_{25}x-\log_5(2x+4)=2$ **[5]**

$\log_5 50+4\log_{25}x-\log_5(2x+4)=2$	
$\log_5 25\times 2+\dfrac{4\log_5 x}{\log_5 25}-\log_5(2x+4)=2$	M1, M1
$\log_5 5^3+\log_5 2+\dfrac{4\log_5 x}{\log_5 5^2}-\log_5 2(x+2)=2$	
$2+\log_5 2+\dfrac{4\log_5 x}{2}-[\log_5 2+\log_5(x+2)]=2$	M1
$2+\log_5 2+2\log_5 x-\log_5 2-\log_5(x+2)=2$	
$\log_5(x+2)=2\log_5 x$	
$x+2=x^2$	
$x^2-x-2=0$	M1
$(x-2)(x+1)=0$	
$x=2,\quad or\ x=-1\ (NA)$	A1

8 In the diagram, triangles OAB and OCD are right-angled triangles.



Angle AOB = angle $ODC = \theta$., $OA = 3$ cm and $OD = 8$ cm.

(i) Show that the length of $AB + CD = 3\sin\theta + 8\cos\theta$. [1]

$AB + CD = 3\sin\theta + 8\cos\theta$	B1

(ii) Express $3\sin\theta + 8\cos\theta$ in the form $R\sin(\theta+\alpha)$ where $R > 0$ and α is acute. [4]

$3\sin\theta + 8\cos\theta = R\sin(\theta+\alpha)$ $= R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$	
$R\cos\alpha = 3$ $R\sin\alpha = 8$	M1 both
$\tan\alpha = \frac{8}{3}$ $\alpha = 69.44^\circ$	M1
$R = \sqrt{73}$	M1
$3\sin\theta + 8\cos\theta = \sqrt{73}\sin(\theta + 69.44^\circ)$	A1

(iii) Find the maximum length of $AB + CD$ and the corresponding value of θ. [3]

Max = $\sqrt{73}$ *or* 8.544	B1
$\sin(\theta + 69.44^\circ) = 1$	M1
$\theta = 90^\circ - 69.44^\circ$ $\theta = 20.56^\circ$	A1

(iv) Find the value of θ, if B is the midpoint of OC. [2]

$2OB = OC$ $2(3\cos\theta) = 8\sin\theta$	M1
$\tan\theta = \frac{6}{8}$ $\theta = 36.87^\circ$	A1

9 The function f is defined by $f(x) = 4\cos 2x - 3$.

(i) State the amplitude of f. [1]

Amplitude = 4	B1

(ii) State the period of f in terms of π. [1]

$\text{Period} = \frac{2\pi}{2}$ $= \pi$	B1

The equation of a curve is $y = 4\cos 2x - 3$ for $0 \le x \le \pi$.

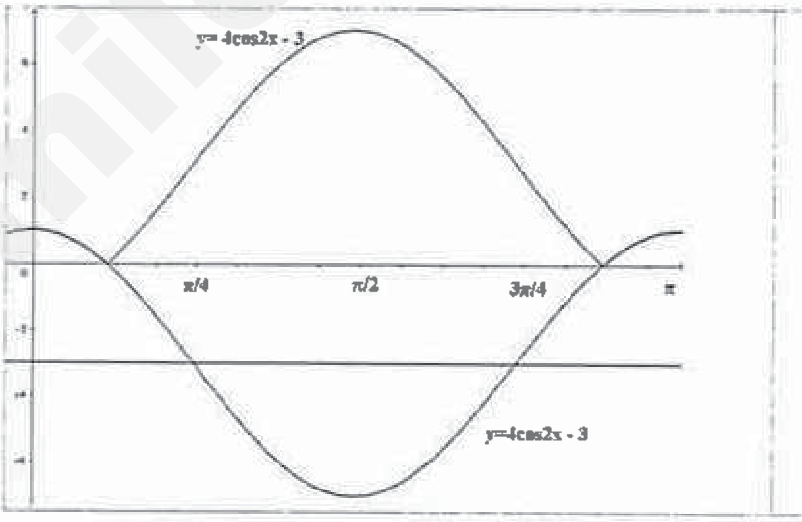
(iii) Find the minimum value of the curve. [1]

$\text{Min} = -4 - 3$ $= -7$	B1

(iv) Find the x-coordinates of the points where the curve meets the x-axis. [3]

$4\cos 2x - 3 = 0 \qquad 0 \le x \le \pi$	M1
$\cos 2x = \frac{3}{4} \qquad 0 \le 2x \le 2\pi$	M1
$2x = 0.7227, \quad 5.560$	
$x = 0.3614, \quad 2.780$	A1

(v) Sketch the graph of $y = \left|3\cos 2x - 4\right|$ for $0 \le x \le \pi$. [3]

	Shape B1 Correct keypoints: x-axis B1 y-axis B1

(vi) Hence, find the range of values of c, for which $\left|3\cos 2x - 4\right| = c$ has exactly two solutions only. [1]

$1 < c < 7$	B1

10 The diagram shows a triangle ABC with vertices at $A(0, 3)$, $B(8, 12)$ and $C(k, 13)$.

(i) Given that $AB = BC$, find the value of k. [4]

$AB^2 = BC^2$	
$(k-8)^2+(13-12)^2=(8-0)^2+(12-3)^2$	M1
$(k-8)^2=64+81-1$	
$(k-8)^2-144=0$	M1
$(k-8+12)(k-8-12)=0$	
$(k+4)(k-20)=0$	M1
$k=20, \quad k=-4(NA)$	A1

A line is drawn from B to meet the x-axis at D such that $AD = CD$.

(ii) Name the quadrilateral $ABCD$. [1]

Kite	B1

(iii) Find the equation of BD and the coordinates of D. [4]

Property of Kite △. Diagonals intersect at 90°	
$m_{AC}=\dfrac{13-3}{20-0}$	M1
$=\dfrac{1}{2}$	
$m_{BD}=-2$	M1
$12=-2(8)+c$	
$c=28$	
$y=-2x+28$	A1
$0=-2x+28$	
$x=14$	
$D(14,0)$	A1

(iv) Find the area of the triangle ABC. [2]

$A=\dfrac{1}{2}\begin{vmatrix} 0 & 8 & 20 & 0 \\ 3 & 12 & 13 & 3 \end{vmatrix}$	M1
$=\dfrac{1}{2}[(264)-(104)]$	
$=80\ units^2$	A1

11a **(i)** Find the range of values of x for which $x^2-8x+15\geq 0$ **[2]**

$x^2-8x+15\geq 0$ $(x-5)(x-3)\geq 0$	M1
$x\leq 3 \quad or\ x\geq 5$	A1

(ii) Hence, find the range of values of x for which $(x+2)^2-8x-1<0$ **[3]**

$(x+2)^2-8(x+2)+16-1<0$ $(x+2)^2-8(x+2)+15<0$	M1
$[(x+2)-5][(x+2)-3]<0$	M1
$(x-3)(x-1)<0$ $1<x<3$	A1

b Show that $my=x^2-4(x-1)$ meets the curve $y=x^2-3x+2$ at two distinct points for all real values of m, except $m=0$ and $m=1$. **[5]**

$my=x^2-4(x-1)$ $y=\dfrac{x^2-4x+4}{m}$ $y=x^2-3x+2$	M1
$\dfrac{x^2-4x+4}{m}=x^2-3x+2$ $x^2-4x+4=mx^2-3mx+2m$	M1
$(m-1)x^2+(4-3m)x+(2m-4)=0$	M1
$b^2-4ac=(4-3m)^2-4(m-1)\cdot 2(m-2)$ $=16-24m+9m^2-8(m^2-3m+2)$ $=16-24m+9m^2-8m^2+24m-16$ $=m^2$	M1
$m^2>0$ $\therefore b^2-4ac>0$ $\therefore$ 2 distinct roots	A1

Name		( )	Class	

南 华 中 学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2019

Subject : **Additional Mathematics**
Paper : **4047/01**
Level : **Secondary Four Express**
Date : **29 August 2019**
Duration : **2 hours**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correcting fluid / tape.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiner's Use

This paper consists of 26 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

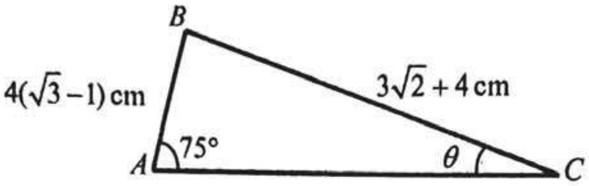
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1



The diagram shows a triangle ABC in which $AB = 4(\sqrt{3}-1)$ cm, $BC = 3\sqrt{2}+4$ cm, $\angle BAC = 75°$ and $\angle BCA = \theta$. Given that $\sin 75° = \frac{\sqrt{2}}{4}\left(\sqrt{3}+1\right)$, find, **without using a calculator**, the value of $\sin\theta$ in the form of $a + b\sqrt{2}$ where a and b are integers. [4]

2 Evaluate, **without using a calculator,** $\tan\left[\cos^{-1}\left(-\frac{8}{17}\right)\right]$. [3]

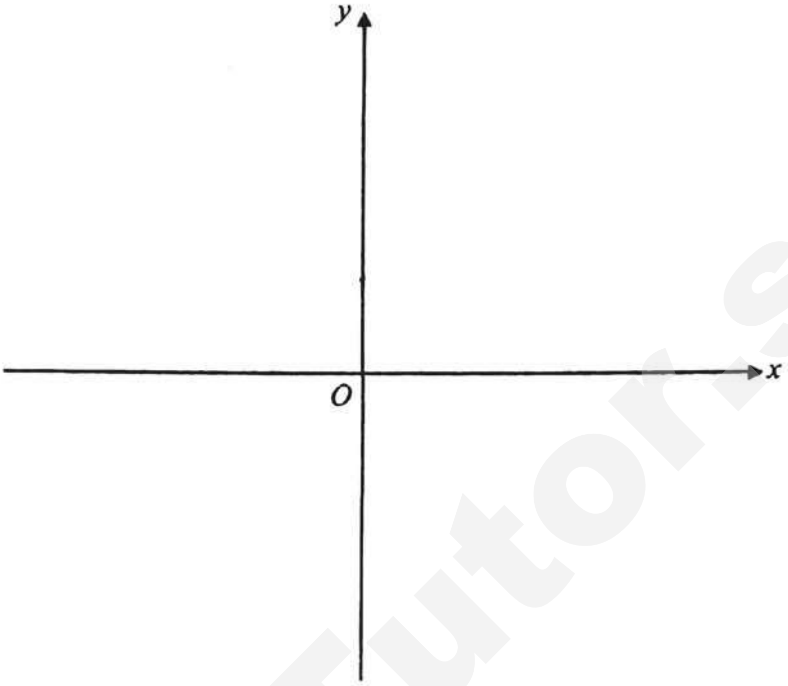
3 A function is defined by the equation $y = \frac{\sin x}{1 - \cos x}$ for $0 \le x \le 2\pi$ where $x \ne a$.

(i) State the exact values of a. [1]

(ii) Explain, with reasons, whether the function is increasing or decreasing. [5]

4 A curve has equation $y = e^{2x-1}$.

(i) Sketch the graph of $y = e^{2x-1}$. [2]



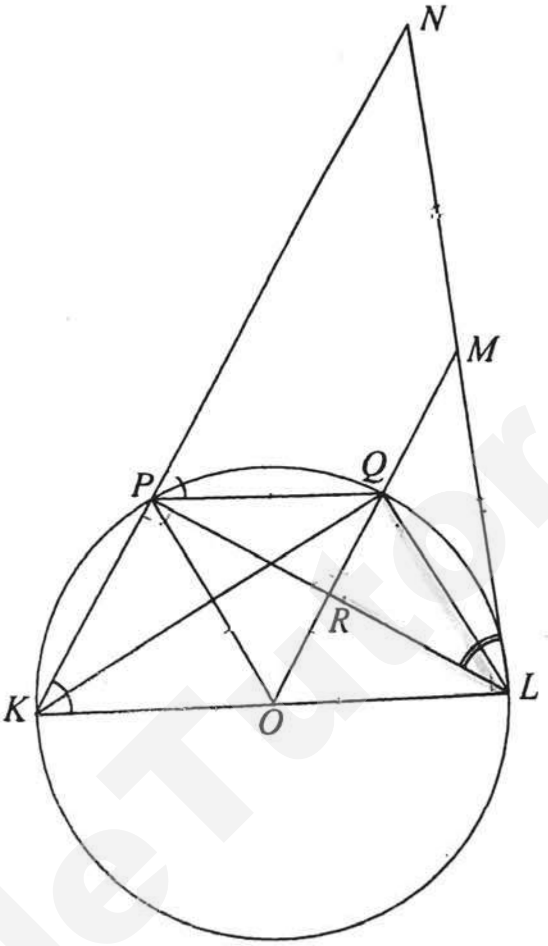
(ii) The curve $y = e^{2x-1}$ and $y = e^{k-x}$ meet at point R where $x = 1$. Find the value of k. [2]

5 (a) Find the range of values of n for which $9x^2+8nx+2n^2>8$ for all real values of x. [3]

(b) A curve has equation $y=(x+3)(x^2-3x+6)$. Explain why $y=(x+3)(x^2-3x+6)$ is always positive for $x>-3$. [3]

6 The coefficient of x^3 in the cubic polynomial $g(x)$ is a, where $a > 0$. The repeated roots of the equation $g(x) = 0$ are 2. Find the value of a if $g(x)$ has a remainder of $-\frac{9}{2}$ and 28 when divided by $(x+1)$ and $(x-4)$ respectively. [4]

7

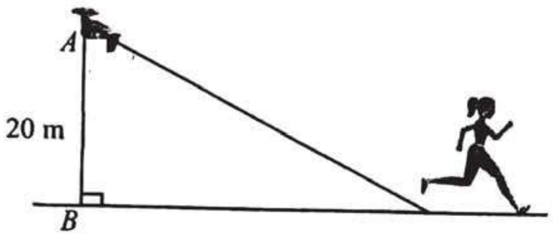


The diagram shows a circle with centre O, diameter KL. NML is a tangent to the circle at L and M is the midpoint of NL. The lines KN and OM cut the circle at P and Q respectively. The lines PL and OQ intersect at R. The line LQ bisects $\angle RLM$ and $\angle NPQ = \angle NKL$.

(i) Prove that *OKPQ* is a rhombus. [3]

(ii) Prove that $KQ \times RQ = LQ \times LR$. [3]

8



In the diagram, a surveillance camera is mounted at a point A that is 20 m above a point B. A runner runs from point B along a straight course at a speed of 4 m/s The surveillance camera tracks the motion of the runner by panning upwards and downwards at point A. Find the rate of change of the angle that the surveillance camera makes with AB when the runner is 15 m from B. Give your answer in radians per second. [5]

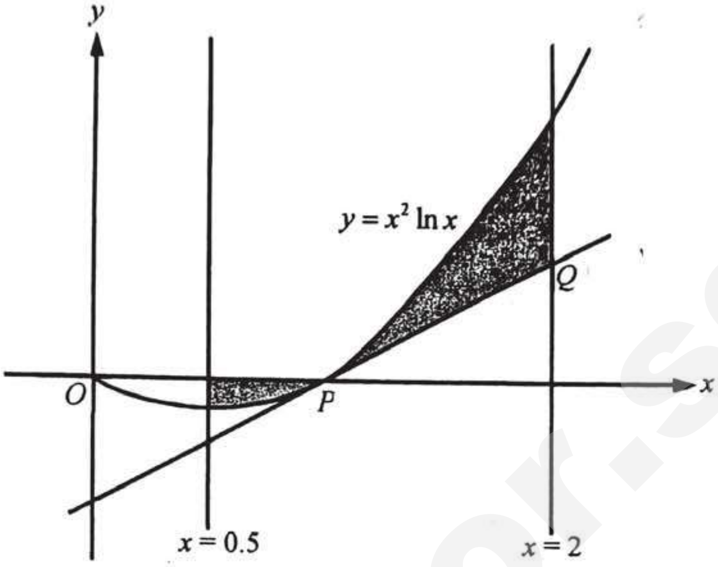
9 The equation $2x^2+x-4=0$ has roots α and β. The equation $16x^2+21x+p=0$ has roots $\frac{1+q\beta^2}{\alpha}$ and $\frac{1+q\alpha^2}{\beta}$. Without finding the values of α and β, find the values of p and q.

[6]

10 **(i)** Find $\frac{d}{dx}\left(x^3 \ln x\right)$. [2]

(ii) Hence find $\int x^2 \ln x \, dx$. [2]

(iii)



The diagram shows the lines $x = 0.5$, $x = 2$ and part of the curve $y = x^2 \ln x$. The curve intersects the x-axis at the point P and the tangent to the curve at P meets the line $x = 2$ at point Q. Find the total area of the shaded region. [6]

11 The table shows experimental values of two variables x and y.

x	0.1	0.5	1	1.5	2
y	–5.95	1.63	0.83	0.61	0.5

It is known that x and y are related by the equation $\frac{\sqrt{x}}{y} = ax + \frac{b\sqrt{x}}{a}$, where a and b are constants.

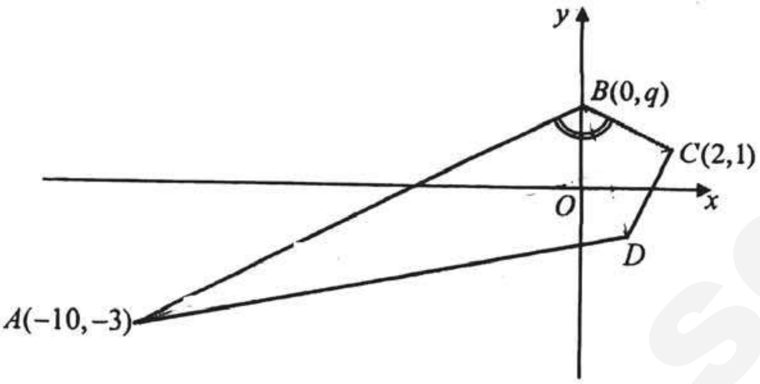
(i) Plot $\frac{1}{y}$ against $\sqrt{x}$ to obtain a straight line graph. [2]

11(i)

(ii) Use your graph to estimate the values of a and b. [4]

(iii) If, instead, a straight line is obtained by plotting $\frac{1}{y\sqrt{x}}$ against $\frac{1}{\sqrt{x}}$, find the gradient of the line. [2]

12



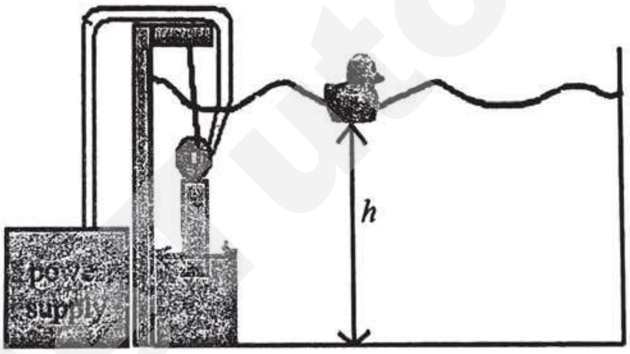
The diagram shows a kite with vertices $A(-10,-3)$, $B(0,q)$, $C(2,1)$ and D. It is given that angle ABO is equal to angle OBC.

(i) Show that $q = 2$. [4]

(ii) Find the coordinates of the point D. **[4]**

(iii) Find the area of the kite *ABCD*. [2]

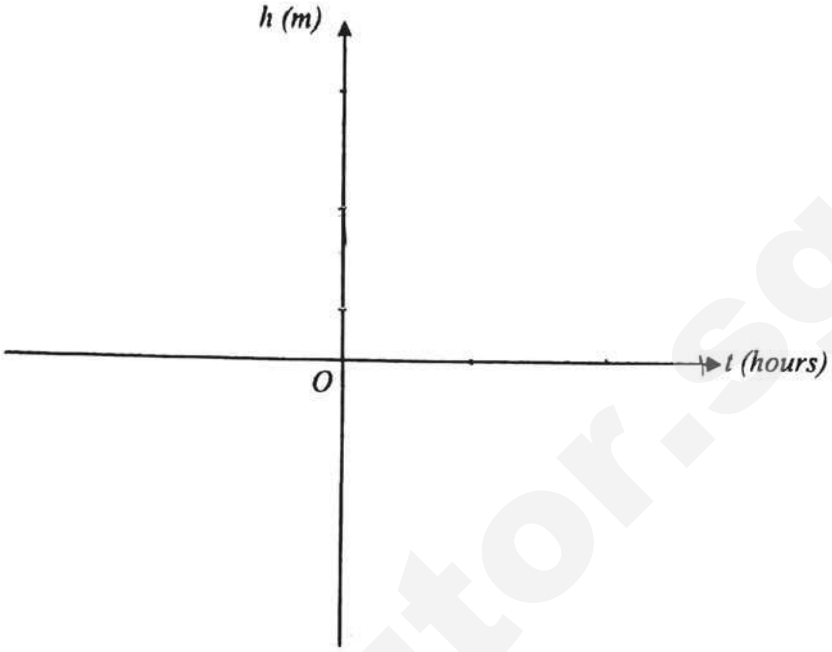
13



To study the effects of water waves, a wave generator and a rubber duck were placed in a water tank as shown in the diagram above. The height, *h* metres, from the bottom of water tank was modelled by $h = a\sin(kt) + b$, where t is the time in hours after midnight and a, b and k are constants. The motion of the rubber duck was observed for 36 hours. The minimum height of 1.5 m from bottom of water tank was first recorded at 06 00. The maximum height of 2.5 m was first recorded at 18 00.

(i) Find the values of a, b and k. [3]

(ii) Using the values found in (i), sketch the graph of $h = a\sin(kt) + b$ for $0 \le t \le 36$. [2]



(iii) Find the range of values of t such that the rubber duck is above 2.1 m. [3]

~ **End of Paper** ~

Name	( )	Class	

南 华 中 学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2019

Subject : **Additional Mathematics**
Paper : **4047/02**
Level : **Secondary Four Express**
Date : **2 September 2019**
Duration : **2 hours 30 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correcting fluid / tape.

Answer **all** the questions.
Write your answers on the space provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use

This paper consists of 24 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

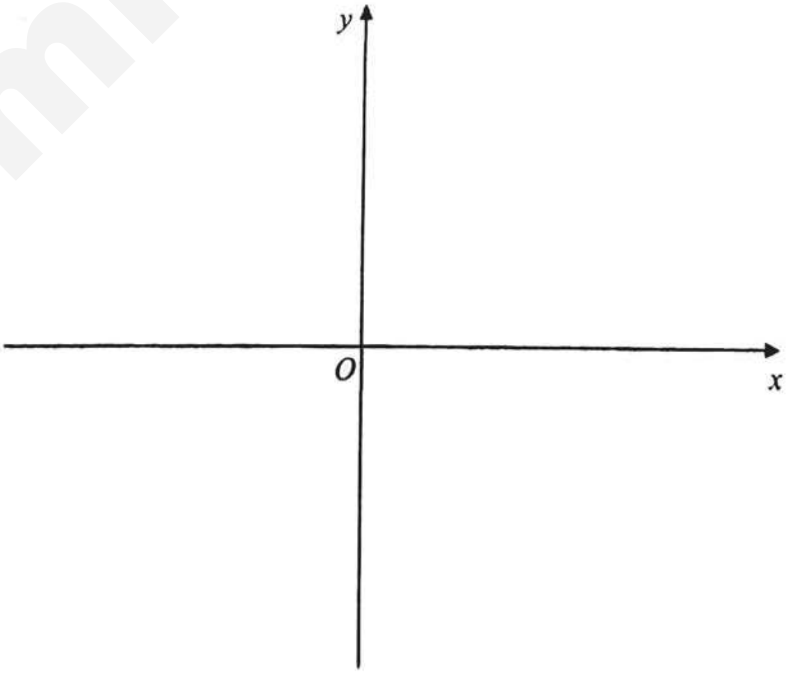
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta ABC = \frac{1}{2} bc \sin A$$

1 (a) Without using a calculator, find the value of 9^x, given that $\frac{1}{3}\left(5^x\right)\left(3^{2x}+81\right)=45^x$. [4]

(b) Sketch the graph of $y=2\log_4 x-1$ for $x>0$. [2]



(c) Solve the equation $\log_3 \frac{1}{9}\sqrt{x} = 1 + 2\log_x 81$. [5]

2 In a natural habitat, the population of a certain species of snails is given by $P = 0.8\left(Ae^{kt} + 500\right)$, where A and k are constants and t is the time in years starting from 1 January 2010. Over a period of 8 years from 1 January 2010 to 31 December 2017, the population decreased from 50 000 to 19 000.

(i) Calculate the values of A and of k. [3]

(ii) Calculate the year in which the population is 30% more compared to 31 Dec 2017. [3]

(iii) Explain, with justification, the expected population of the snails over a long period of time. [2]

3 (i) Express $\dfrac{4x^4-4x^3+23x^2-24x+5}{x(2x-1)^2}$ in partial fractions. [5]

(ii) Hence find $\int \frac{4x^4 - 4x^3 + 23x^2 - 24x + 5}{5x(2x-1)^2} dx$. [4]

4 (a) (i) Write down the general term in the binomial expansion of $\left(\frac{6}{x^2} - \frac{x}{2}\right)^{15}$. [1]

(ii) Write down the power of x in this general term. [1]

(iii) Hence, determine the coefficient of x^{-9} in the expansion of $\left(\frac{6}{x^2}-\frac{x}{2}\right)^{15}$. [2]

(iv) Hence determine the coefficient of x^{-9} in $\left(\frac{2}{x^2}-\frac{x}{6}\right)^{15}$. [2]

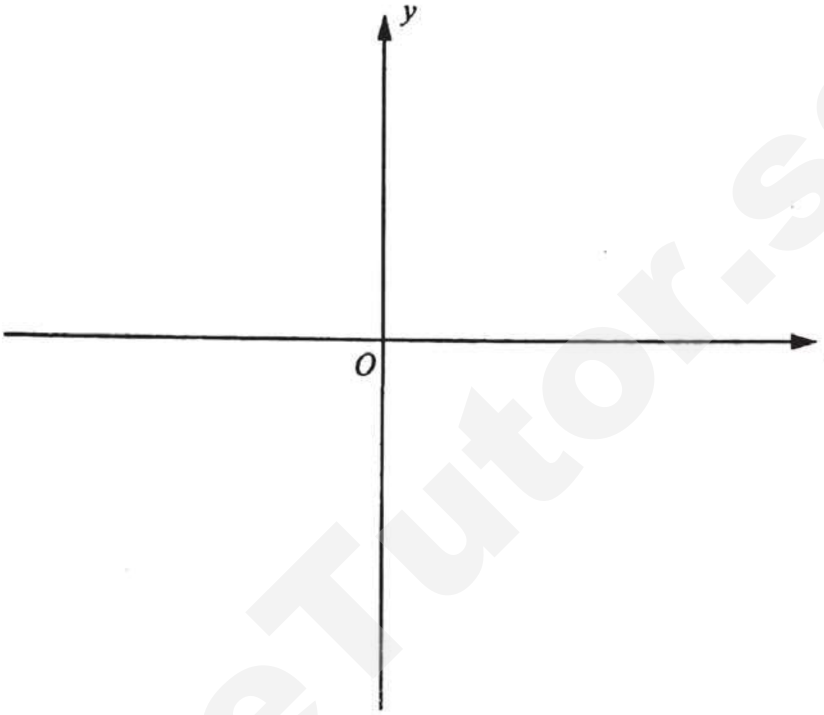
(b) The coefficient of x^2 in the expansion, in ascending powers of x, of $(1+x)^n(5-2x)^3$ is 3210. Find the value of n, where n is a positive integer. [5]

5 (a) Solve the equation $4-|2x+3|=x$. [3]

(b) A curve has the equation $y=(2x-1)^2-9$.

(i) Explain why the lowest point on the curve has coordinates $\left(\frac{1}{2},-9\right)$. [1]

(ii) Sketch the graph of $y=\left|(2x-1)^2-9\right|$. [3]



(iii) Determine the set of values of *m* such that $\left|(2x-1)^2-9\right|=mx-2$ has no solution. [2]

6 The coordinates of the points A, B and C are $(0,7)$, $(-1,0)$ and $(6,-1)$ respectively.

(i) Show that AB is perpendicular to BC. [2]

(ii) Explain why A, B and C lie on the circumference of a circle, C_1 with diameter AC. [1]

(iii) Find the centre of C_1. [1]

(iv) **The tangent to C_1 at point B is also a tangent to another circle, C_2. Given that the centre of C_2, lies on both the y-axis and the perpendicular bisector of BC, find the equation of C_2.**

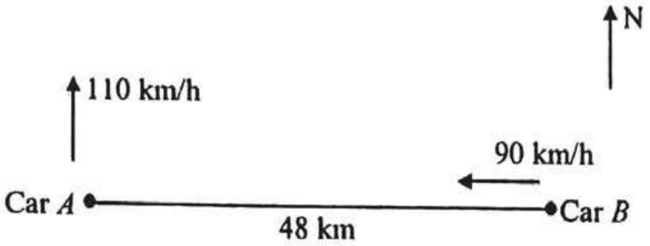
[8]

7 **(i)** **Prove that** $\sec 3x\left(\sin 3x - 2\sin^3 3x\right) = \tan 3x \cos 6x$. **[4]**

(ii) Hence find, for $0 \le x \le \frac{\pi}{3}$, the values of x in radians for which [6]

$-2\sin\frac{3}{2}x\cos\frac{3}{2}x = \sec 3x\left(\sin 3x - 2\sin^3 3x\right)$.

8



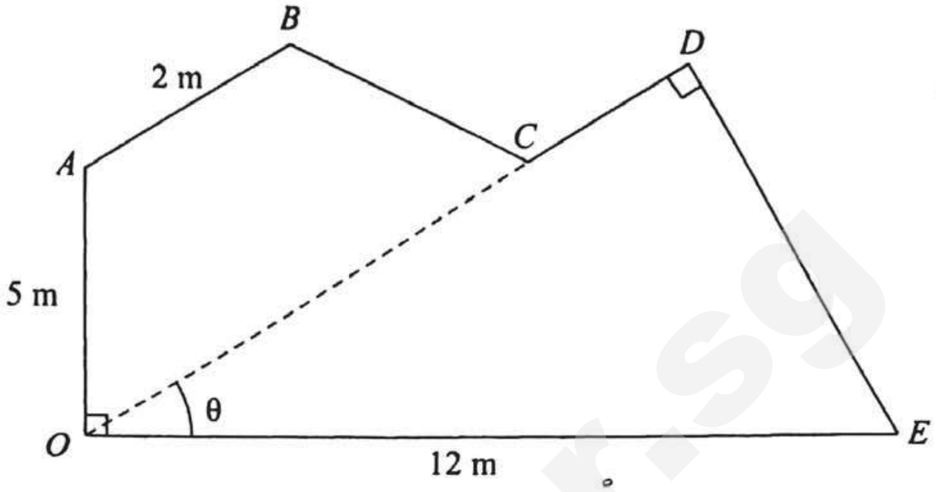
The diagram shows Car B, which is 48 km due east of Car A. Both cars start moving at the same time. Car A travels due north at a constant speed of 110 km/h while Car B travels due west at a constant speed of 90 km/h.

(i) The distance between Car A and Car B at time t hours after the cars started moving is denoted by L km. Express L in the form of $\sqrt{pt^2+(q-rt)^2}$ where p, q and r are constants. [3]

(ii) Given that t can vary, find the stationary value of L. [5]

(iii) Determine whether this value stationary value of L gives the maximum or minimum distance between Car A and Car B. [1]

9



For a theatre production, a panel is constructed by joining an isosceles trapezium and a right- angled triangle together.
It is given that *OA* = 5 m, *OE* = 12 m and *OA* is perpendicular to the base *OE*. *OCD* is perpendicular to *DE* and makes an angle θ with the base *OE*. *AB* and *OC* are the parallel sides of the trapezium *OABC* and *AB* = 2 m.
The total length of the edges of the panel *OABCDE* is represented by *S*.

(i) Show that $S = 12\cos\theta + 2\sin\theta + 22$. [3]

(ii) Express S in the form of $R\cos(\theta-\alpha)+Q$, where Q is a constant, $R>0$ and $0°<\alpha<90°$. [4]

(iii) A 30 m LED strip is placed along all the edges of the panel $OABCDE$. Triangle ODE of the panel is to be painted in black. Calculate the area to paint. [4]

10 A particle X, moves in a straight line with velocity, v m/s, given by $v = 2t^2 + kt + 63$, where k is a constant and t is the time in seconds, measured from the start of the motion. Its initial displacement from a fixed point O is -8 m. The minimum velocity of X occurs at $t = 5.75$.

(i) Find the minimum velocity of X. [2]

(ii) Find the values of t for which the particle is at instantaneous rest. [2]

(iii) Find the distance travelled by particle X when $t = 7$. [3]

Another particle Y starts its motion at the same time as particle X and moves in a straight line with an initial velocity of 6 m/s from O. Its acceleration, a m/s^2, is given by $a = \frac{3}{5}t$.

(iv) Show that particle Y will not change its direction of motion. [3]

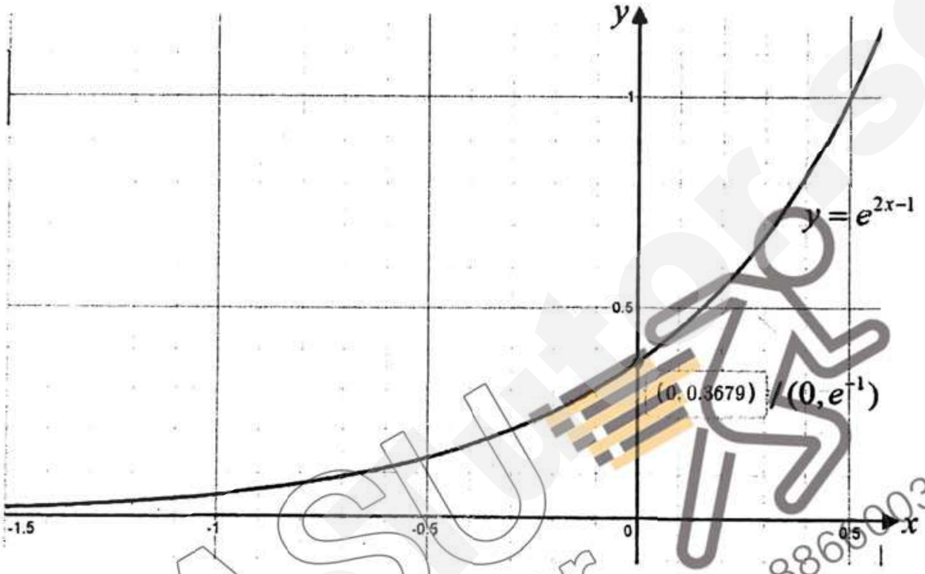
~End of Paper~

Answer Key

1 $6-4\sqrt{2}$ **2** $-\frac{15}{8}$

3(i) $a=0,\ 2\pi$ **3(ii)** Decreasing function

4(i)



4(ii) $k=2$ **5(a)** $n<-6$ or $n>6$

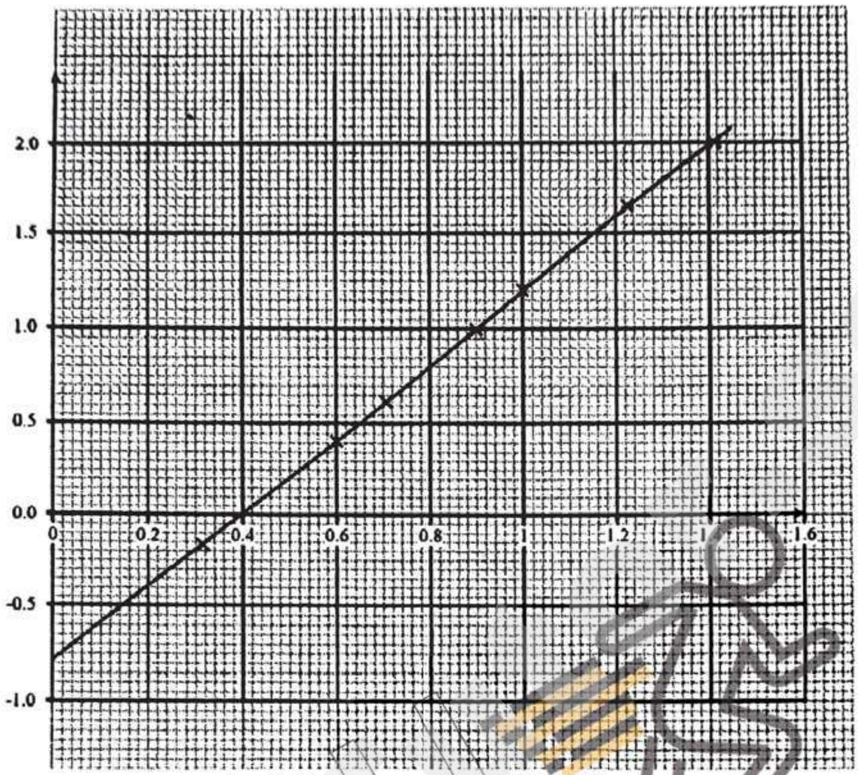
6 $a=\frac{3}{2}$ **8** 0.128 rad / s

9 $p=-6,\ q=-1$ **10(i)** $x^2+3x^2\ln x$

10(ii) $\int x^2\ln x\,dx=\frac{1}{3}x^3\ln x-\frac{1}{9}x^3+c_2$, where c_2 is an arbitrary constant

10(iii) 0.639 unit^2 (3 s.f.)

11(i)

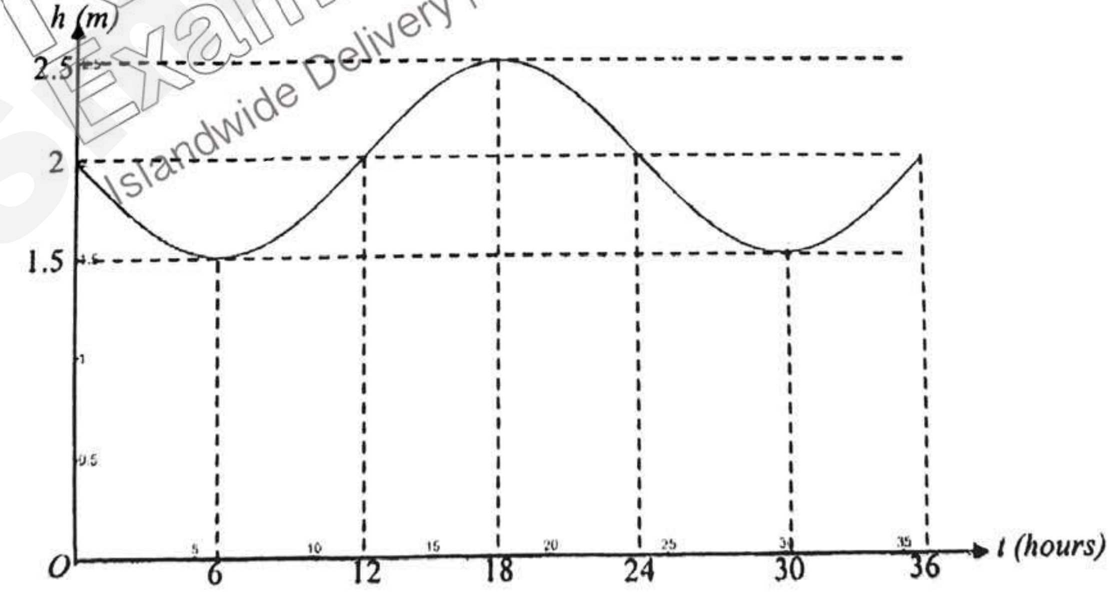


11(ii) $a = 2$ [1.5 to 2.5], $b = -1.6$ [-2.125 to -1.125] **11(iii)** -0.8

12(ii) $D(1, -1)$ **12(iii)** 20 units^2

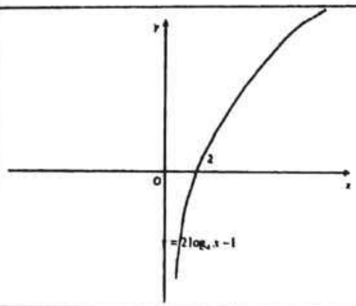
13(i) $a = -0.5$, $b = 2$, $k = \frac{\pi}{12}$

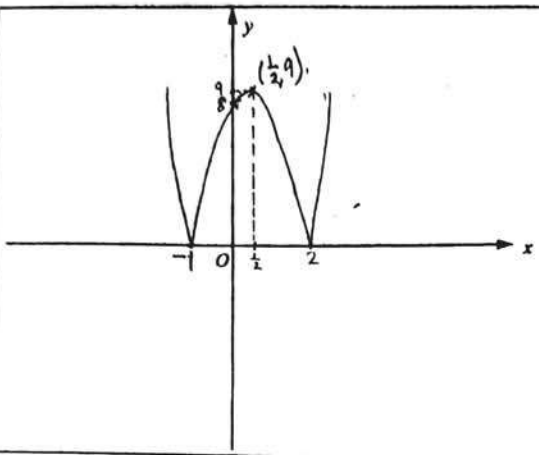
13(ii)



13(iii) $12.8 < t < 23.2$

Answer Key:

1a	40.5
b	
c	6561 or $\frac{1}{9}$
2(i)	-0.123
(ii)	t = 5.81967 (must have). In year 2015
(iii)	As t becomes very large (over a long period of time), $e^{-0.122604t}$ approaches 0. Then expected population is $P = 0.8\ (0 + 500)$ $= 400$
3(i)	$x+\frac{5}{x}+\frac{1}{2x-1}-\frac{3}{(2x-1)^2}$
(ii)	$\frac{1}{10}x^2+\ln x+\frac{1}{10}\ln(2x-1)+\frac{3}{10(2x-1)}+c,$
4a(i)	$\binom{15}{r}(6)^{15-r}\left(-\frac{1}{2}\right)^r\left(x^{-30+3r}\right)$
a(ii)	Power of $x = -30+3r$
a(iii)	-84440070
a(iv)	$-\frac{1430}{243}$
b	9
5a	$x=\frac{1}{3}$ or -7
b(i)	For $x \in$ □ , $(2x-1)^2 \geq 0$ $(2x-1)^2-9 \geq -9$ $y \geq -9$ At $y=-9, x=\frac{1}{2}$. Hence lowest point is $\left(\frac{1}{2},-9\right)$

b(ii)	
b(iii)	$-2 < m < 1$
6(i)	Product of the gradients of AB and BC $= (7)\times(-\frac{1}{7}) = -1$ $\therefore$ AB is perpendicular to BC.
(ii)	From (i), AB is perpendicular to BC implies that $\angle ABC = 90°$. Due to $\angle$ in a semi-circle, AC is the diameter of the circle, and A, B and C are points on the circumference of the same circle.
(iii)	(3,3)
(iv)	$x^2 + (y+18)^2 = 100.$
7(ii)	$0, \frac{\pi}{3}, \frac{\pi}{9}$
8(i)	$L = \sqrt{12100t^2 + (48-90t)^2}$
(ii)	37.1
(iii)	L is minimum
9(i)	$S = 12\cos\theta + 2\sin\theta + 22$
(ii)	$S = 12.2\cos(\theta - 9.5°) + 22$
(iii)	32.2 m^2
10(i)	-3.125 m/s
(ii)	7 or 4.5
(iii)	117 m
(iv)	For all values of t, since $\frac{3}{10}t^2 > 0$, $v > 0$. Particle will not change its direction of motion since velocity is always positive.

PEICAI SECONDARY SCHOOL
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC
PRELIMINARY EXAMINATION 2019

CANDIDATE NAME

CLASS ☐☐☐ REGISTER NUMBER ☐☐

ADDITIONAL MATHEMATICS

4047/01

27 August 2019

2 hours

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your register number, class and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

This document consists of **19** printed pages and **1** blank page.

Setter: Mrs Ariel Leong

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

(1)

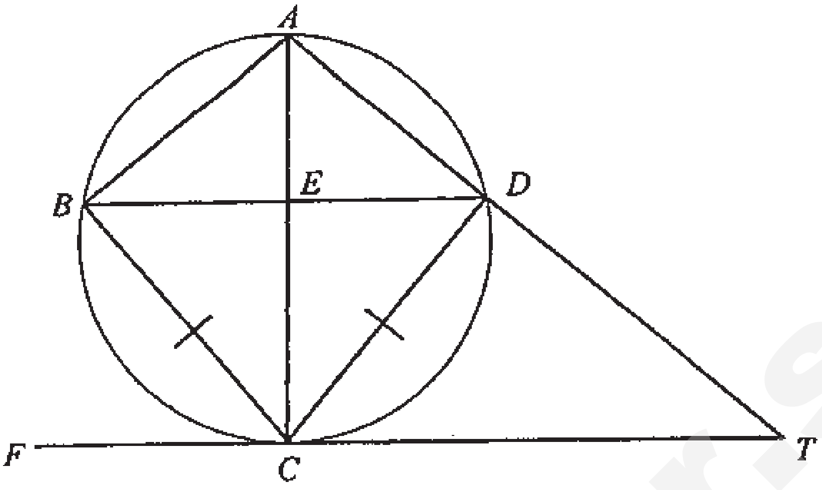
1 Find the range of values of the constant p for which the line $y = px - 1$ intersects the curve $y = 2x^2 - 5x + 1$ at two points. [4]

2 Show that $\frac{36^{x+1}+15(6^{2x})}{3^{2x+1}}$ is divisible by 17, where x is a positive integer. [4]

2

3 Find the coordinates of the stationary points of the curve $y = 4x + \frac{25}{x}$, and determine the nature of these stationary points. [6]

4



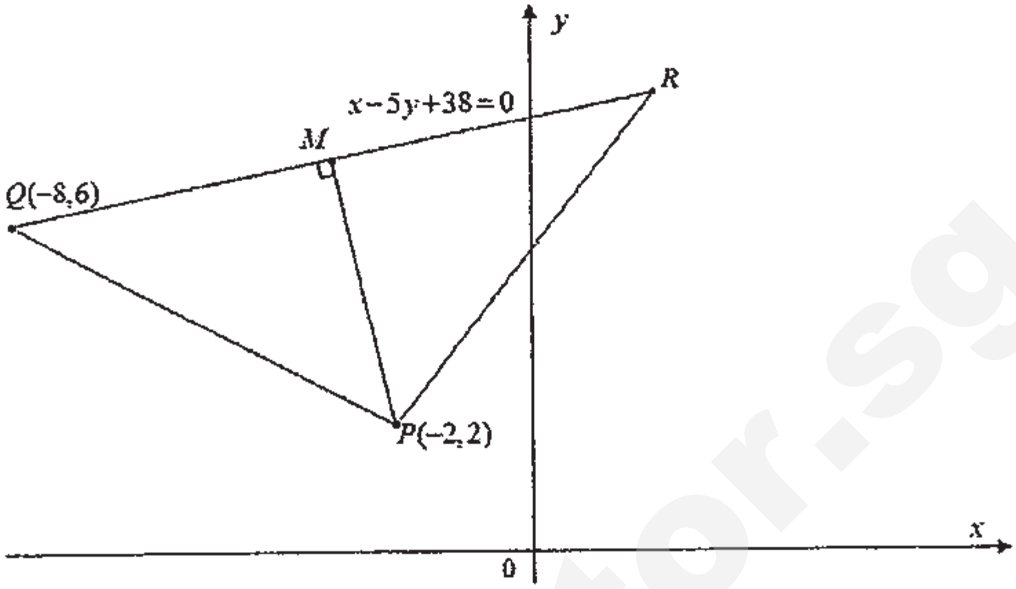
A, *B*, *C* and *D* are four points on the circle. *CT* is the tangent to the circle at point *C*. The diagonals of *BD* and *AC* intersect at point *E* and *BC* = *CD*.

(i) Prove that *BD* is parallel to *FT*. [3]

(ii) Show that $(CT)^2 = TD \times TA$. [3]

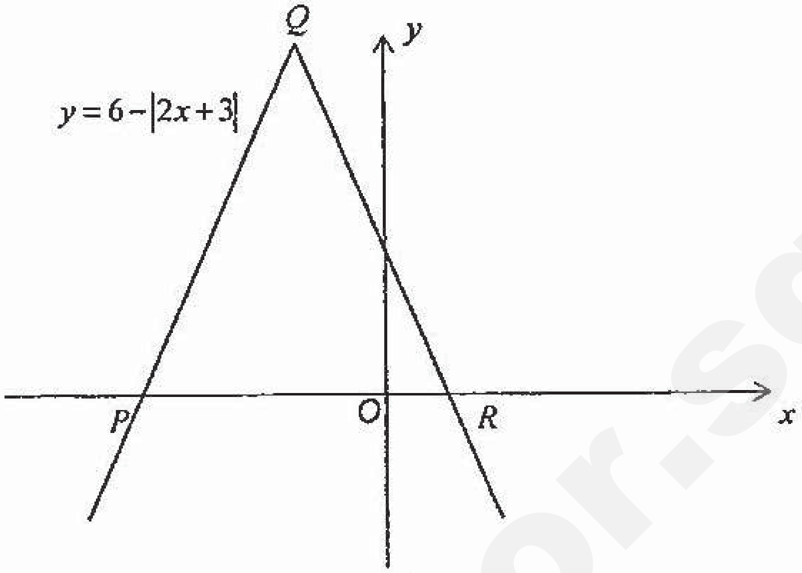
3

5



The diagram shows an isosceles triangle PQR with vertices $P(-2,2)$ and $Q(-8,6)$. $QP=PR$. The line QR with equation $x-5y+38=0$ passes through M, the mid-point of QR.
Find the area of triangle PQR. [6]

6



The diagram shows part of the graph $y = 6 - |2x + 3|$.

(a) Find the coordinates of P, Q and R. [4]

4

(b) In each of the following cases determine the number solutions of the equation $6-|2x+3|=mx-1$. Justify your answer.

(i) $m=2$, [2]

(ii) $m=-\frac{1}{2}$. [2]

7 The roots of the quadratic equation $2x^2 - 4x + 3 = 0$ are α and β.

(i) Show that $\alpha^3 + \beta^3 = -1$. [3]

(ii) Find a quadratic equation whose roots are $\alpha^3 + 1$ and $\beta^3 + 1$. [4]

8 Solve the equation $\sin\theta \tan\theta + 2\sin\theta = 3\cos\theta$, where $\cos\theta \neq 0$, for $0° < \theta < 180°$. [6]

9 A particle moves in a straight line such that, at time t seconds after leaving a fixed point O, its velocity v m/s is given by $v = t^2\ (t - 4)$. Find

(i) the acceleration of the particle when $t = 3$, [2]

(ii) the value of t when the particle comes to an instantaneous rest, [2]

(iii) the time taken for the particle to return to the point O, [4]

(iv) the total distance travelled by the particle in the interval $t = 0$ to $t = 5$. [3]

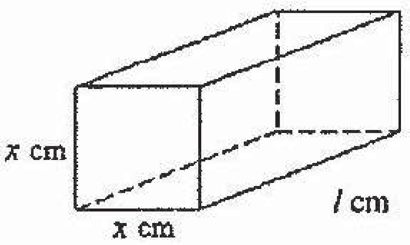
10 Variables x and y are connected by the equation $y = a^{x+b}$, where a and b are constants.
When a graph of $\lg y$ is plotted against x, a straight line passing through the points (3, 1) and (6, 4) is obtained. Find

(i) the value of a and of b, [4]

7

(ii) the coordinates of the point on the line at which $\lg y = 2x - 4$. [3]

11



The diagram shows a rectangular block of ice, x cm by x cm by l cm.
Volume of the ice is 1 litres.

(i) Show that the total surface area, A cm^2, is given by $A = 2x^2 + \frac{4000}{x}$. [2]

8

(ii) The block of ice is melting such that the total surface area is changing at a constant rate of 3 cm^2/s. Find the rate of decrease of x when $x = 5$. [4]

12 On a particular day, the water level at the beach of Tanjung Rima first reached a maximum of 2 m at 8 am. The lowest water level was forecast to be at 6 pm. The depth of water at the beach may be modelled by the equation

$$h = 0.8\cos(kt) + c$$

where h is the water level in metres and t is the number of hours after 8 am.

(i) Explain why this model suggests that the minimum water level will be 0.4 m. [1]

(ii) Show that $c = 1.2$. [1]

(iii) Show that the value of k is $\frac{\pi}{10}$. [2]

The corals at the beach of Tanjung Rima are visible at low tide when the water level is less than 0.45 m.

(iv) Between what times in the daylight hours will corals be visible at Tanjung Rima? [5]

PEICAI SECONDARY SCHOOL
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC
PRELIMINARY EXAMINATION 2019

CANDIDATE NAME ☐

CLASS ☐☐☐ REGISTER NUMBER ☐☐

ADDITIONAL MATHEMATICS
Paper 2

4047/02
28 August 2019

2 hours 30 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your register number, class and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

This document consists of **21** printed pages and **3** blank pages.

Setter: Mrs Ho Thuk Lan

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

①

1 **(i)** In the expansion of $(1+4x)^n$ the coefficient of x^2 is 8 times the coefficient of x. Show that $n=5$. [4]

(ii) Using your answer to part (i), find, in terms of p, the coefficient of x^2 in the expansion of $(1+px+3x^2)(1+4x)^n$. [1]

(iii) If the coefficient of x^2 in the expansion of $(1+px+3x^2)(1+4x)^n$ is 263, find the value of the constant p. [2]

2 **(i)** Differentiate $x\cos 3x$ with respect to x. [3]

(ii) Using your answer to part (i), find $\int x\sin 3x \, dx$. [3]

②

(iii) Hence show that $\int_0^{\frac{\pi}{3}} x\sin 3x \, dx = \frac{\pi}{9}$. [2]

3 The equation of a curve is $\dfrac{x^2-1}{x^2+1}$.

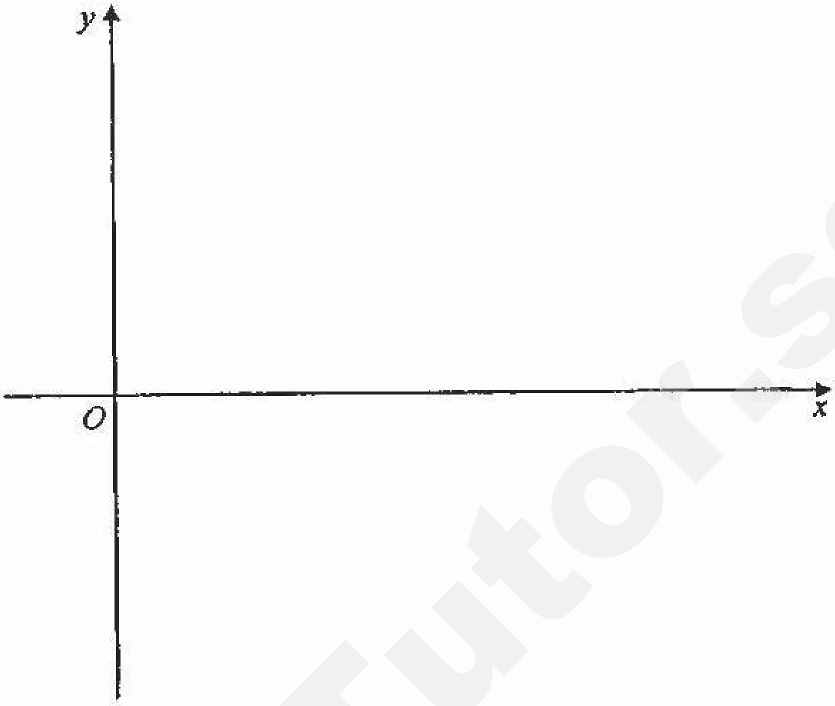
(i) Obtain an expression for $\mathrm{f}'(x)$. [2]

(ii) Find the equations of the tangent to the curve at the points where the curve meets the x – axis. [3]

2

(iii) Show that $f(x)$ is increasing when $x > 0$. [2]

4 **(i)** Sketch the graph of $y=\frac{1}{4}x^{\frac{2}{3}}$ for $x>0$. [1]



(ii) On the same diagram sketch the graph of $y=-\frac{1}{4}x^{\frac{2}{3}}$ for $x>0$. [1]

(iii) What is the relationship between the graphs of $y=\frac{1}{4}x^{\frac{2}{3}}$ and $y=-\frac{1}{4}x^{\frac{2}{3}}$? [1]

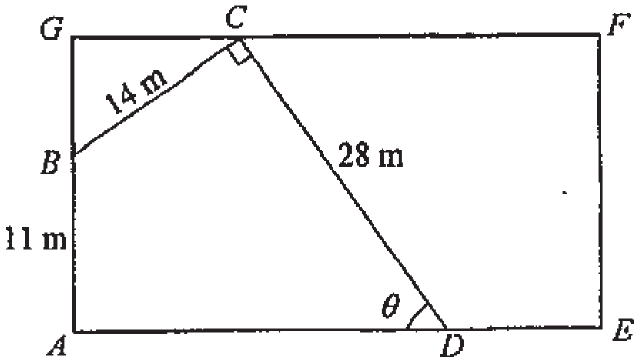
4

(iv) Calculate the coordinates of the point of intersection of $y=\frac{1}{4}x^{\frac{2}{3}}$ and $y=4x^{-\frac{2}{3}}$ for $x>0$. [2]

(v) On the same diagram sketch the graph of $y=4x^{-\frac{2}{3}}$ for $x>0$. [1]

(vi) Determine, with explanation, whether the tangents to the graphs of $y=\frac{1}{4}x^{\frac{2}{3}}$ and $y=4x^{-\frac{2}{3}}$ for $x>0$ at the point of intersection are perpendicular. [3]

5



The diagram shows a rectangular basketball court, *AEFG*.
From a point *A* on the court, players are to run along the straight paths *AB*, *BC*, *CD* and *DA*.
The lengths of *AB*, *BC* and *CD* are 11m, 14 metres and 28 metres respectively.
Angle *ADC* is θ, where $0^0 < \theta < 90°$.
The total distance covered by each player is *T* metres.

(i) Show that *T* can be expressed as $p+q\cos\theta+r\sin\theta$ where *p*, *q* and *r* are constants to be found. [3]

(ii) Express *T* in the form $p+R\cos(\theta-\alpha)$, where $R > 0$ and α is an acute angle. [4]

5

Given that the total distance is found to be 78 metres.

(iii) Find the value of θ. [2]

Given that the length of DE is $\left(5+2\sqrt{5}\right)$ m and the area of triangle DEF is $\left(45+42\sqrt{5}\right)$.

(iv) Find the length of EF in the form $a\sqrt{5}-b$. [2]

6 A circle, C_1, has centre $A(4,2)$ and radius $\sqrt{13}$.

(i) Write down the equation of circle, C_1. [1]

(ii) Determine whether the point $R(8, 3)$ lies inside or outside of circle, C_1. [2]

Circle, C_1 intersects the $x-$axis at points P and Q.

(iii) Find the mid-point of PQ. [4]

6

A second circle, C_2, with centre B and radius $\sqrt{18}$ also passes through P and Q.

(iv) *State the x–coordinate of B.* [1]

(v) Given that the y–coordinate of B is positive, find the centre of circle C_2. [3]

7 **(a)** Prove the identity $\frac{1}{1+\tan^2 x}=(1+\sin x)(1-\sin x)$. [3]

(b) Find all the angles between 0° and 360° that satisfy the equation $8\tan x = 3\cos x$. [4]

⑦

(c) Solve the equation $2\cos 2y - 5\cos y = 4$ for $0 \le y \le 2\pi$, giving your answers in radians. Correct your answers to 2 decimal places. [4]

8 A prism has a volume of $\left(6x^2 - 21x + 25\right)$ cm^3 and a base area of $\left(2x^2 - 5x\right)$ cm^2.

(i) Find an expression for the height, h(x), of the prism. [1]

(ii) Using your answer to part **(i)**, express the height in partial fractions. [5]

(iii) Differentiate $\ln(2x-5)$ with respect to x. [1]

(iv) Using your answers to part (ii) and (iii), find $\int_3^4 h(x)\,\mathrm{d}x$ in the form $a+\ln\frac{b}{c}$ where a, b and c are integers. [5]

9 (a) Given that $\log_3 p = m$, $\log_{27} q = n$ and $\frac{p}{q} = 3^r$, express r in terms of m and n.

[3]

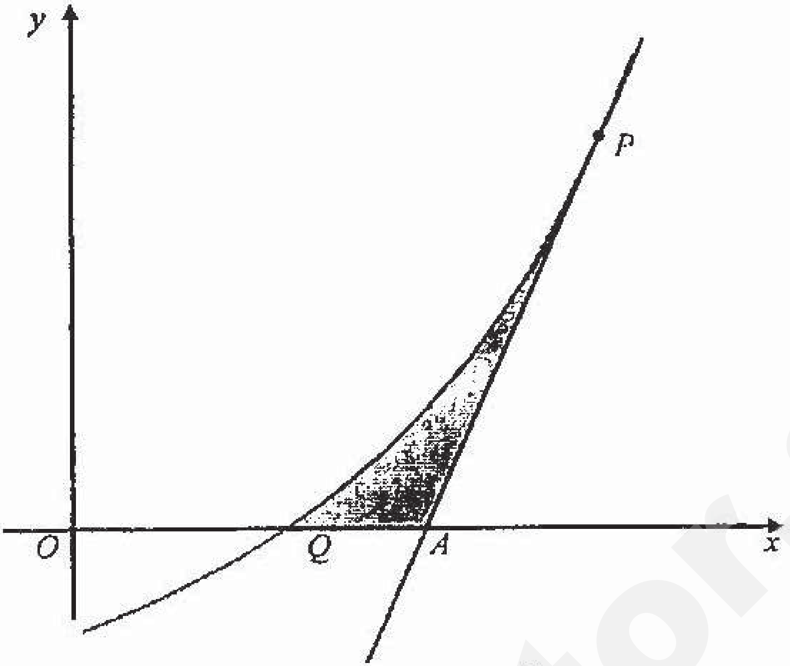
(b) Solve the equation

(i) $\log_3(3x-7) = \log_3(2x-3) - 2$, [4]

9

(ii) $3\log_5 y = 2 + \log_y 5$. [5]

10



The diagram shows part of the curve $y = \dfrac{9}{(7-x)^2} - 1$, cutting the $x-$axis at Q. The tangent at the point P on the curve cuts the $x-$axis at A. Given that the gradient of this tangent is $\dfrac{9}{4}$, calculate

(i) the coordinates of P, [5]

10

(ii) the area of the shaded region *PQA*. [7]

The End

Answer Key

1 $p < -9$, $p > -1$

2 Simplify to $17(4^x)$

Since 17 is a factor, the expression is divisible by 17

3 $\frac{d^2y}{dx^2} = 50x^{-2}$

(2.5, 20) is a min point, (–2.5, –20) is a max point

5 Area = 26 sq units

6a) P(–4.5, 0) Q(–1.5, 6) R(1.5, 0)

6b(i) One Solution. The line $y = 2x - 1$ is parallel to one part of the graph $y = 6 - |2x + 3|$ and the y-intercept of $y = 2x - 1$ is below the max point of $y = 6 - |2x + 3|$. Hence, the two graphs will only interest at one point.

6b(ii) Two solutions.

The line $y = -½\, x - 1$ is not parallel to both parts of the graph $y = 6 - |2x + 3|$ and the y-intercept of $y = = -½\, x - 1$ is below the max point of $y = 6 - |2x + 3|$. Hence, the two graphs will interest at two points.

7(i) –1 (ii) $x^2 - x + \frac{27}{8} = 0$

8 45°, 108.4°

9(i) $a = 3t^2 - 8t$; 3 m/s^2

9(ii) at rest, $v = 0$, $t = 4$ (iii) $5\frac{1}{3}$ s

9(iv) Total distance = 32.25 m

10(i) $a = 10$, $b = -2$

10(iii) (2, 0)

11(i) $A = 2x^2 + 4x(1000/x^2)$

11(ii) 3/140 cm/s

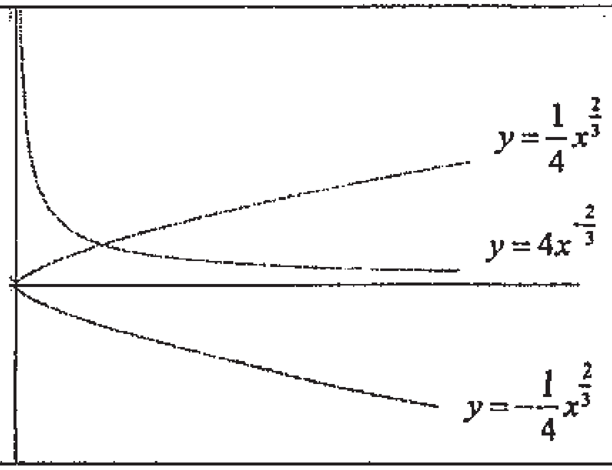
12(i) Min level = 2 – 2(0.8) = 0.4 m (ii) $c = 2 - 0.8$ or $c = ½\,(2+0.4)$

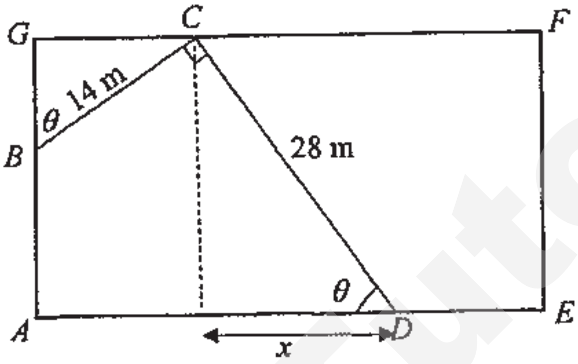
12(iii) $k = \pi/10$ (shown) (iv) $\frac{\pi}{10}t = \pi - 0.35542, \quad \pi + 0.35542$

$t = 8.869, \quad 11.131$

Between 16 53 to 19 08

Peicai Secondary School
Preliminary Exam 2019

1i	$4nx$
	$\frac{n(n-1)}{2}(16x^2)$
	$8(4n)=\frac{n(n-1)}{2}(16)$
	$n(n-5)=0$ $n=0$ (rejected)
1ii	$163+20p$
1iii	$163+20p=263$
	$p=5$
2i	$(\cos 3x)\left(\frac{d}{dx}(x)\right)+(x)\left(\frac{d}{dx}(\cos 3x)\right)$
	$\cos 3x = -3x\sin 3x$
2ii	$\frac{1}{3}\int(\cos 3x-x\cos 3x)dx$
	$\frac{\sin 3x}{3}$
	$\frac{1}{9}\sin 3x-\frac{1}{3}x\cos 3x+c$
2iii	$\left(\frac{1}{9}\sin\pi-\frac{1}{3}\left(\frac{\pi}{3}\right)\cos\pi\right)-\left(\frac{1}{9}\sin 0-\frac{1}{3}(0)\cos 0\right)=\left(0-\left(\frac{\pi}{9}\right)(-1)\right)-(0)=\frac{\pi}{9}$
3i	$\frac{(x^2+1)(2x)-(x^2-1)(2x)}{(x^2+1)^2} = (x^2+1)(-1)(x^2+1)^{-1-1}(2x)+(x^2+1)^{-1}(2x)$
	$=\frac{4x}{(x^2+1)^2}$
3ii	$x=-1$ or $x=1$
	$y=x-1$
	$y=-x-1$
3iii	$4x>0$, $(x^2+1)^2>0$ → $f'(x)>0$ Hence f(x) is increasing
4i,ii,v	
4iii	Reflection in the x – axis

4iv	$\frac{1}{4}x^{\frac{2}{3}}=4x^{-\frac{2}{3}}$ $\quad x^{\frac{4}{3}}=16 \quad$ (8, 1)
4vi	$\frac{1}{6}x^{-\frac{1}{3}} \;=\; -\frac{8}{3}x^{-\frac{5}{3}}$
	When $x=8$, Product of the gradients = $\frac{1}{12}\times-\frac{1}{12}=-\frac{1}{144}$ Since $\quad m_t m_n \neq -1$, Tangents are NOT perpendicular.
5	
5i	$x=28\cos\theta$
	$GC=14\sin\theta$
	$AD = 14\sin\theta+28\cos\theta$
	$T = 53+14\sin\theta+28\cos\theta$
5ii	$\sqrt{28^2+14^2} = \sqrt{980}$ or $14\sqrt{5}$
	$\tan^{-1}\frac{14}{28} = 26.6°$
	$T = 53+14\sqrt{5}\cos(\theta-26.6°)$
5iii	$\cos(\theta-26.565°)=\frac{25}{14\sqrt{5}}$ $\quad$, θ = 63.6° $\theta-26.565°=37.0037°$
5iv	$\frac{1}{2}\times EF\times(5+2\sqrt{5})=45+42\sqrt{5}$
	$EF=\frac{2(45+42\sqrt{5})}{(5+2\sqrt{5})}\times\frac{5-2\sqrt{5}}{5-2\sqrt{5}}$
	$EF = 48\sqrt{5}-78$
6i	$(x-4)^2+(y-2)^2=13$
6ii	$AR=\sqrt{17}$
	Since AR > $\sqrt{13}$, Point R lies OUTSIDE circle
6iii	$(x-4)^2+(0-2)^2=13$
	$x=7, x=1$ $\quad \left(\frac{7+1}{2},0\right) \quad$ (4, 0)

6iv	4
6v	$(4-7)^2+(y-0)^2=18$ or $(4-1)^2+(y-0)^2=18$
	$y=3$ or $y=-3$(rejected) Centre = (4, 3)
7b	19.5°, 160.5°
7c	2.42, 3.86
8i	$h(x)=\dfrac{6x^2-21x+25}{2x^2-5x}$
8ii	By long division $6x^2-21x+25\div\left(2x^2-5x\right)=3$ Remainder $-6x+25$ or $a(2x^2-5x)+bx+c=6x^2-21x+25$
	$\dfrac{-6x+25}{2x^2-5x}=\dfrac{A}{x}+\dfrac{B}{2x-5}$ A= –5, B = 4
	$3-\dfrac{5}{x}+\dfrac{4}{2x-5}$
8iii	$\dfrac{2}{2x-5}$
8iv	$3+\ln\dfrac{2187}{1024}$
9a	$r=m-3n$
9bi	$x=2\dfrac{2}{5}$
9bii	$y=0.585$ or $y=5$
10i	$P=(5,\dfrac{5}{4})$
10ii	$\dfrac{9}{(7-x)^2}-1=0$
	$x=4$ or $x=10$ (rejected) $Q=(4,0)$
	Equation of AP is $y=\dfrac{9}{4}x-10$
	$x=\dfrac{40}{9}$ $A=(\dfrac{40}{9},0)$
	Area of triangle = $\dfrac{25}{72}$
	Area of shaded portion = $\int_4^5\left(\dfrac{9}{(x-x)^2}-1\right)dx-\dfrac{25}{72}$
	$\dfrac{11}{72}$

Candidate Name ______________________________	Class	Index Number

ST ANDREW'S SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2019
SECONDARY 4 EXPRESS & 5 NORMAL ACADEMIC

ADDITIONAL MATHEMATICS
Paper 1

4047/01

WEDNESDAY **28 August 2019** **2 hours**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This question paper consists of 15 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}.$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\cdots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

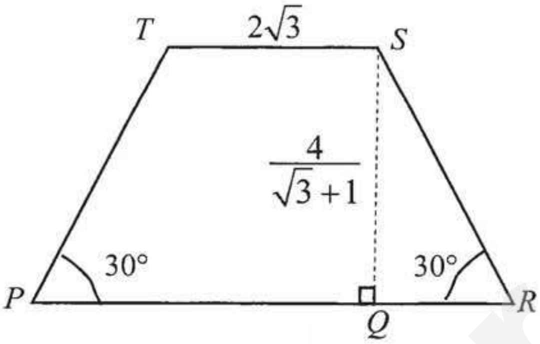
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

1 In the diagram below, $PQRST$ is a trapezium where angle QRS = angle $TPR = 30°$. SQ is the height of the trapezium and the length of SQ is $\frac{4}{\sqrt{3}+1}$ cm. The length of TS is $2\sqrt{3}$ cm. Find the area of the trapezium $PQRST$ in the form $\left(a\sqrt{3}-12\right)\text{cm}^2$, where a is an integer. [5]

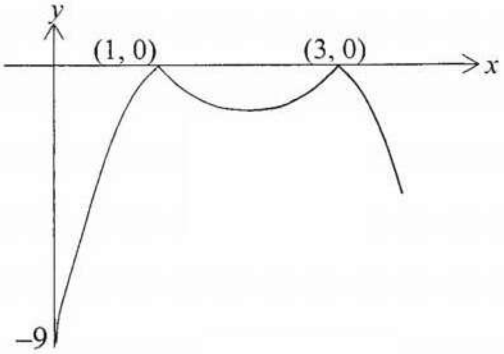


2 Express $\dfrac{4x^3+x^2+6}{(x-2)\left(x^2+2\right)}$ in partial fractions. [5]

3 **(a)** Factorise $8x^3-(x-1)^3$ completely. [3]

(b) Without finding the solution, explain why the equation $8x^3-(x-1)^3=0$ has only one real root. [2]

4 The diagram shows part of the curve $y = -\left|a(x-h)^2 + k\right|$, where $a > 0$. The curve touches the x-axis at (1, 0) and (3, 0) and has a minimum point at (h, k). The curve also cuts the y-axis at -9.

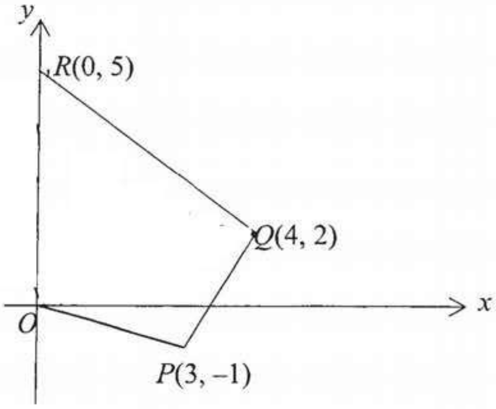


(i) Explain why $h = 2$. [1]

(ii) Determine the value of a and of k. [3]

(iii) Find the set of values of m for which the line $y = mx$ intersects the curve at four distinct points. [2]

5 In the diagram, the coordinates of P, Q and R are $(3, -1)$, $(4, 2)$ and $(0, 5)$ respectively.



(i) Find the equation of the perpendicular bisector of OQ. [3]

(ii) Name the quadrilateral $OPQR$. Justify your answer. [2]

(iii) Given that T is a point on PR such that $OPQT$ is a rhombus, find the coordinates of T. [2]

6 The roots of the quadratic equation $4x^2 + px + q = 0$ are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

(i) Given that $\alpha + \beta = 5$ and $\alpha\beta = 2$, find the value of p and q.

(ii) Find the quadratic equation whose roots are $\frac{2\alpha^2}{\beta}$ and $\frac{2\beta^2}{\alpha}$. [5]

7 **(a)** Find the range of values of a for which $x^2 + ax + 2(a-1)$ is greater than 1. [3]

(b) The equation of a curve is $y = 3x^2 + 4x + 6$.

(i) Find the set of values of x for which the curve is above the line $y = 6$.

(ii) Show that the line $y = -8x - 6$ is a tangent to the curve. [2]

8 **(a)** Find the minimum gradient of $y = 2x^3 - 9x^2 - 1$. [3]

(b) The curve $y = x^3 - 6x^2 + k$ touches the positive x-axis at point A.

(i) Find the coordinates of point A. [2]

(ii) Find the value of k

(iii) Find the value of $\frac{d^2y}{dx^2}$ at A and hence the nature of this point. [2]

9 **(a)** Show that $\frac{5-10\cos^2 A}{\sin A - \cos A}$ can be written as $k(\sin A + \cos A)$ and state the value of k.

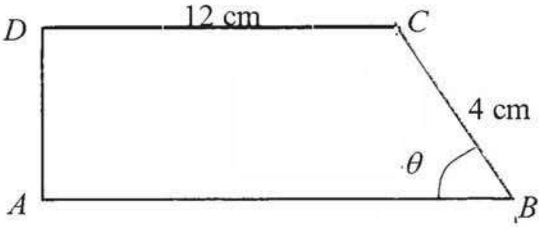
(b) Given that $\sin A = -p$ and $\cos B = -q$, where A and B are in the same quadrant and p and q are positive constants, find the value of

(i) $\sin(-A)$, [1]

(ii) $\tan(45° - A)$,

(iii) $\sec(2B)$. [2]

10 The diagram shows a trapezium $ABCD$ in which $CD = 12$ cm, $BC = 4$ cm and angle $ABC = \theta$ radians, where θ is acute.

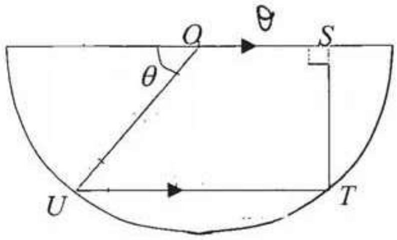


(i) Show that the area, A cm^2, of the trapezium $ABCD$ is given by
$A = 48\sin\theta + 4\sin 2\theta$. [3]

(ii) Given that θ can vary, find the value of θ for which the area of the trapezium A is maximum. [5]

(iii) Hence find the maximum value of A. [1]

11



The diagram shows a trapezium $OSTU$ inscribed in a semi-circle of centre O and radius 10 cm. OU makes an angle θ with the diameter. UT is parallel to the diameter and ST is perpendicular to the OS. The perimeter of the trapezium is L cm.

(i) Show that $L = 10 + 30\cos\theta + 10\sin\theta$. [3]

(ii) Express L in the form $a + R\cos(\theta - \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) Anthony claims that the perimeter of $OSTU$ is 50 cm. Is his claim reasonable? Justify your answer. [2]

(iv) Find the value of θ for which $L = 35$. [2]

Class	Index Number

Candidate Name ____________________

ST ANDREW'S SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2019
SECONDARY 4 EXPRESS & 5 NORMAL ACADEMIC

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ADDITIONAL MATHEMATICS 4047/02
Paper 2

THURSDAY **29 August 2019** **2 hours 30 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This question paper consists of 20 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}.$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\cdots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

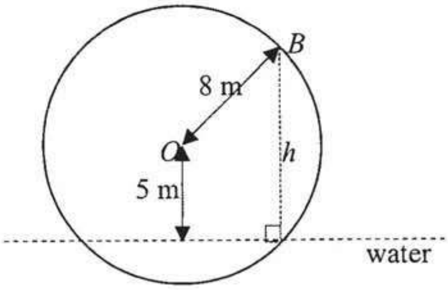
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

1 A curve is such that $\frac{d^2y}{dx^2} = 6x - 2$ and $P(2, -8)$ is a point on the curve. The gradient of the normal at P is $-\frac{1}{2}$. Find the equation of the curve. [6]

2 **(i)** On the same axes, sketch the graphs $y = \sqrt{288x}$ and $y = 3x^3$ for $x > 0$. [2]

(ii) The tangent to the curve $y = 3x^3$ at point A is parallel to the line passing through the two points of intersection of the curves drawn in **(i)**.
Find the x-coordinate of A. [4]

3



A waterwheel rotates 5 revolutions anticlockwise in 1 minute. Tom starts a stopwatch when the bucket B is at its highest height above water level. The radius of the waterwheel is 8 m and its centre is 5 m above the water level.

The height of bucket B above water level is given by $h = a \cos bt + c$, where t is the time, in seconds, since Tom started the stopwatch.

(i) Determine the value of each of the constant a, b and c. [3]

(ii) For how long is $h < 0$? [3]

4 In the binomial expansion of $x\left(2x+\frac{k}{x}\right)^8$, where k is a positive constant, the coefficient of x^3 is 28.

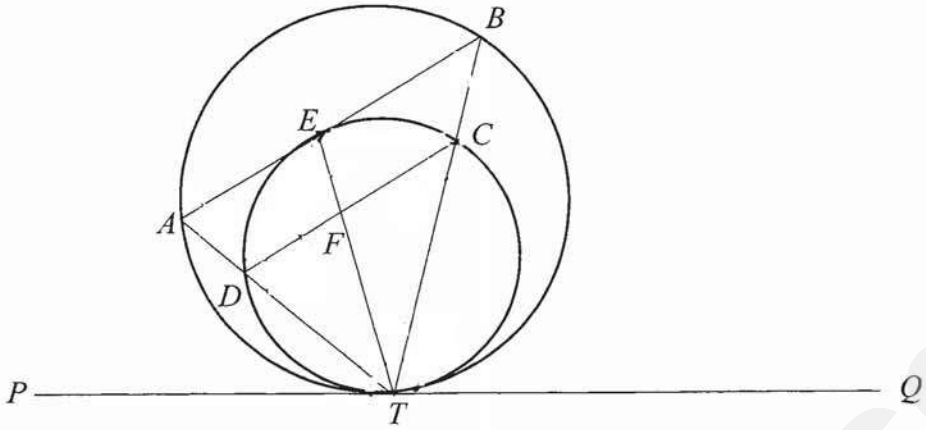
(i) Show that $k=\frac{1}{4}$. [4]

(ii) Explain why there is no constant term in the expansion of $x\left(2x+\frac{k}{x}\right)^8$. [1]

(iii) Hence find the coefficient of x^3 in the expansion of

$$x\left(2x+\frac{k}{x}\right)^8+k\left(1-x\right)^{10}.$$ [2]

5



In the diagram, the two circles touch each other at T and PTQ is their common tangent. AB is a tangent to the smaller circle at E. AT and BT cut the smaller circle at D and C respectively. ET and CD intersect at F. Prove that

(i) AB is parallel to DC, [2]

(ii) the line TE bisects angle ATB, [3]

(iii) triangle DFT is congruent to triangle EFC if $DF = EF$. [2]

6 The variables x and y are related by an equation of the form $y - x = \frac{b}{a}x^2 + b$.

Corresponding values of x and y are shown in the table below.

x	1	2	3	4
y	2.73	7.5	14.75	24.5

(i) Using suitable variables, draw on the graph paper, a straight line graph. [3]

(ii) Using the graph, estimate the value of each of constants a and b. [2]

(ii) By drawing a suitable straight line on your graph, estimate the value of x and y when $y = \frac{1}{2}x^2 + x + 2$. [2]

7 The term containing the highest power of x and the term independent of x in the polynomial $\mathrm{P}(x)$ are $2x^4$ and -3 respectively. It is given that $\left(2x^2+x-1\right)$ is a quadratic factor of $\mathrm{P}(x)$ and the remainder when $\mathrm{P}(x)$ is divided by $(x-1)$ is 4.

(i) Find the polynomial $\mathrm{P}(x)$ and factorise it completely. [4]

(ii) Solve $\mathrm{P}(x)=0$. [1]

(iii) Find the values of x that satisfy the equation $\mathrm{P}(1-x)=0$. [2]

8 **(a)** Given that $y = x^2 e^{3x}$, find the range of values of x for which y is an increasing function. [4]

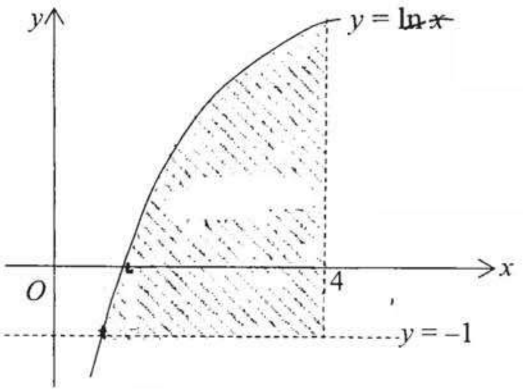
(b) It is given that $y = \dfrac{x}{\sqrt{2x^2 - 1}}$, where $x > 0$. Find the exact value of x when the rate of decrease of y is $\dfrac{9}{8}$ times the rate of increase of x.

9 Given that $y = x \ln x - x$,

(i) show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(x \ln x - x\right) = \ln x$, [2]

(ii) hence show that $\int_a^b (\ln x + 1)\, \mathrm{d}x = b \ln b - a \ln a$. [3]

(iii)



find the area of the shaded region. [3]

4 ln 4 − 3

10 **(a)** Given that $0 < x < \pi$, find the values of x such that

$$\cos\left(\frac{3x}{2}\right) = -\cos\frac{\pi}{10},$$ giving your answers in terms of π. [3]

(b) Prove the identity $\frac{1}{1-\sin x}-\frac{1}{1+\sin x}=2\tan x\sec x$. [3]

(c) Solve the equation $\sin 4x+3\sin 2x=0$ for $-180°\leq x\leq 180°$. [3]

11 A particle travelling in a straight line passes through a fixed point O with a speed of -10 m/s. The acceleration, a m/s^2, of the particle, t s after passing through O, is given by $a = \frac{24}{(2t+1)^2}$. The particle comes to instantaneous rest at the point P.

(i) Find the time when the particle reaches P. [4]

(ii) Calculate the distance travelled by the particle in the first 3 sec. [3]

(iii) Show that the particle is again at O at some instant during the ninth second after first passing through O. [2]

12 A circle C has a diameter AB where A and B are $(-2, 5)$ and $(12, 11)$ respectively.

(i) Find the equation of the circle C. [3]

The line AB produced intersects another line l which touches the circle C at point $D(8, k)$, where $k > 1$.

(ii) Find the value of k. [1]

(iii) Find the equation of line *l*. [3]

A chord in the circle C has a midpoint (12, 8).

(iv) Find the coordinates of the points of the intersection of the chord with the circle C. [2]

13 **(a)** Solve the equation $9^x + 8 = 3^{x+2}$. [4]

(b) Without using a calculator, find the value of 20^p given that

$$40^{2p-1} = 5^{2-p}.$$ [3]

(c) Find the value(s) of y that satisfy the equation

$$\log_4(2y) = \log_{16}(y-3) + 3\log_9 3\,,$$ [4]

2019 Additional Mathematics Prelim Paper 1 Solutions

1 ΔSQR: $\tan 30° = \dfrac{\frac{4}{\sqrt{3}+1}}{QR} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{\frac{4}{\sqrt{3}+1}}{QR}$

$$\Rightarrow QR = \frac{4\sqrt{3}}{\sqrt{3}+1}$$
$$= \frac{4\sqrt{3}}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$
$$= \frac{4\sqrt{3}\left(\sqrt{3}-1\right)}{3-1}$$
$$= 2\left(3-\sqrt{3}\right)$$
$$\therefore\ PR = 2\sqrt{3} + 4\left(3-\sqrt{3}\right)$$
$$= 12 - 2\sqrt{3}$$

$$\therefore \text{Area of trapezium} = \frac{1}{2}\left[2\sqrt{3} + 12 - 2\sqrt{3}\right] \times \frac{4}{\sqrt{3}+1}$$
$$= \frac{24}{\sqrt{3}+1}$$
$$= \frac{24\left(\sqrt{3}-1\right)}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}$$
$$= \frac{24\left(\sqrt{3}-1\right)}{3-1}$$
$$= \underline{\underline{12\left(\sqrt{3}-1\right)\ \text{units}^2}}$$

[5]

2 $$\frac{4x^3+x^2+6}{(x-2)\left(x^2+2\right)} = 4 + \frac{A}{x-2} + \frac{Bx+C}{x^2+2}$$

Multiplying by $(x-2)\left(x^2+2\right)$, we obtain

$4x^3 + x^2 + 6 = 4(x-2)(x^2+2) + A(x^2+2) + (Bx+C)(x-2)$

Sub $x = 2$: $4 \times 8 + 4 + 6 = A(4+2)$

$42 = 6A \Rightarrow A = 7$

Sub $x = 0$: $6 = -16 + 2(7) + C(-2)$

$-2C = 8 \Rightarrow C = -4$

Compare x^2: $1 = -8 + 7 + B \Rightarrow B = 2$

$$\therefore\ \frac{4x^3+x^2+6}{(x-2)\left(x^2+2\right)} = \underline{\underline{4 + \frac{7}{x-2} + \frac{2-4x}{x^2+2}}}.$$

[5]

3 **(i)** $8x^3 - (x-1)^3 = (2x)^3 - (x-1)^3$

$$= \left[2x-(x-1)\right]\left[(2x)^2 + 2x(x-1) + (x-1)^2\right]$$
$$= (x+1)(4x^2 + 2x^2 - 2x + x^2 - 2x + 1)$$
$$= (x+1)(7x^2 - 4x + 1)$$

[3]

(ii) $8x^3 - (x-1)^3 = 0$

$\Rightarrow (x+1)(7x^2 - 4x + 1) = 0$

$\Rightarrow x = -1$ since for $7x^2 - 4x + 1 = 0$,

$D = (-4)^2 - 4(7) = -12 < 0$

$\therefore\ 7x^2 - 4x + 1 = 0$ has no real roots.

Thus $8x^3 - (x-1)^3 = 0$ has only one real root, -1. [2]

4 **(i)** The minimum point (h, k) lies on the line of symmetry: $x = \dfrac{1+3}{2} = 2$.

$\therefore\ h = 2$. [1]

(ii) $y = -\left|a(x-h)^2 + k\right|$ where $a > 0$

For (1, 0): $-|a+k| = 0 \Rightarrow a + k = 0$

$\Rightarrow k = -a\ \ \ldots(1)$

For (0, −9): $-|4a+k| = -9 \Rightarrow |3a| = 9$

$\Rightarrow\ 3a = 9$

$a = 3$

$k = -3$

(iii) We find the value of m where the line $y = mx$ is tangent to the curve.

$$\begin{cases} y = mx & \ldots(1) \\ y = 3(x-2)^2 - 3 & \ldots(2) \end{cases}$$

(1) = (2): $mx = 3x^2 - 12x + 9$

$3x^2 - (12+m)x + 9 = 0$

Since the line is tangent to the curve, then $D = 0$,

i.e. $\left[-(12+m)\right]^2 - 4 \times 3 \times 9 = 0$

$(12+m)^2 = 108$

$12 + m = \pm\sqrt{108}$

$= \pm\, 6\sqrt{3}$

$\therefore\ m = -12 \pm 6\sqrt{3}$

$\therefore\ m = -12 + 6\sqrt{3}$ since $m > -3$

Thus, the line intersects the curve at four distinct points when $\underline{\underline{-12 + 6\sqrt{3} < m < 0}}$. [2]

5 **(i)** gradient of $OQ = \dfrac{2-0}{4-0} = \dfrac{1}{2}$

$\therefore$ gradient of ⊥ bisector of $OQ = -2$

Midpoint of $OQ = \left(\dfrac{4+0}{2}, \dfrac{2+0}{2}\right)$

$= (2, 1)$

Thus, equation of the perpendicular bisector of OQ is $y - 1 = -2(x-2)$

$\underline{\underline{y = -2x + 5}}$ [3]

(ii) When $x = 0,\ y = 5$.

When $x = 3,\ y = -1$.

These results show that R and P lie on the perpendicular bisector of OQ.

i.e., RP is the perpendicular bisector of OQ.

Thus, the quadrilateral $OPQR$ is a <u>kite</u>. [2]

(iii) Let $T = (a, b)$

Since $OPQT$ is a rhombus, then midpoint of OQ is the midpoint of RP.

$\therefore\ \left(\dfrac{a+3}{2}, \dfrac{b-1}{2}\right) = (2, 1)$

$\Rightarrow\ a = 1,\ b = 3$

$\therefore\ \underline{T = (1, 3)}$. [2]

6 $4x^2 + px + q = 0$

(i) Since the roots are $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$, then

$$\frac{1}{\alpha} + \frac{1}{\beta} = -\frac{p}{4} \Rightarrow \frac{\beta+\alpha}{\alpha\beta} = -\frac{p}{4}$$
$$\Rightarrow \frac{5}{2} = -\frac{p}{4}$$
$$\therefore\ p = -10$$
$$\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{q}{4} \Rightarrow \frac{1}{2} = \frac{q}{4}$$
$$\therefore\ q = 2$$

$\underline{p = -10,\ q = 2}$ [4]

(ii) For the new equation, the roots are $\frac{2\alpha^2}{\beta}$ and $\frac{2\beta^2}{\alpha}$.

$$\text{Sum of roots} = \frac{2\alpha^2}{\beta} + \frac{2\beta^2}{\alpha}$$
$$= \frac{2\left(\alpha^3+\beta^3\right)}{\alpha\beta}$$
$$= \frac{2(\alpha+\beta)\left[\alpha^2-\alpha\beta+\beta^2\right]}{\alpha\beta}$$
$$= \frac{2(\alpha+\beta)\left[(\alpha+\beta)^2-3\alpha\beta\right]}{\alpha\beta}$$
$$= \frac{2(5)\left[5^2-3\times 2\right]}{2}$$
$$= 95$$

$$\text{Product of roots} = \left(\frac{2\alpha^2}{\beta}\right)\left(\frac{2\beta^2}{\alpha}\right)$$
$$= 4\alpha\beta$$
$$= 4 \times 2$$
$$= 8$$

Thus the equation is $\underline{x^2 - 95x + 8 = 0}$. [5]

7 **(a)** Given: $x^2 + ax + 2(a-1) > 1$

$x^2 + ax + (2a-3) > 0$

For a positive quadratic function, $D < 0$

$\therefore\ a^2 - 4(2a-3) < 0$

$\therefore\ a^2 - 8a + 12 < 0$

$\therefore\ (a-2)(a-6) < 0$

$\therefore\ \underline{2 < a < 6}$ [3]

(b) $y = 3x^2 + 4x + 6$

(i) $y > 6 \Rightarrow 3x^2 + 4x + 6 > 6$

$3x^2 + 4x > 0$

$x(3x+4) > 0$

$\therefore\ \underline{\underline{x < -\frac{4}{3} \text{ or } x > 0}}$ [2]

(ii) $y = 3x^2 + 4x + 6$

$y = -8x - 6$

$\therefore\ 3x^2 + 4x + 6 = -8x - 6$

$3x^2 + 12x + 12 = 0$

$3(x+2)^2 = 0$

The equation has equal roots $x = -2$

So the line is tangent to the curve. [2]

8 **(a)** Given: $y = 2x^3 - 9x^2 - 1$

gradient: $\frac{dy}{dx} = 6x^2 - 18x$

$\frac{d^2y}{dx^2} = 12x - 18$

For minimum gradient, $\frac{d^2y}{dx^2} = 0$,

i.e., $12x - 18 = 0 \Rightarrow x = \frac{3}{2}$

Since $\frac{d^3y}{dx^3} = 12\ (> 0)$, gradient is minimum at $x = \frac{3}{2}$.

$\therefore$ Minimum gradient $= 6\left(\frac{3}{2}\right)^2 - 18\left(\frac{3}{2}\right)$

$= \underline{-13.5}$ [3]

(b) When $y = x^3 - 6x^2 + k$ touches the positive x-axis at A

[This means that the x-axis is tangent to the curve at A and so A is a stationary point.]

(i) $\frac{dy}{dx} = 3x^2 - 12x$

At A, $\frac{dy}{dx} = 0 \Rightarrow 3x(x-4) = 0$

$\therefore\ x = 0$ or $x = 4$

For A, $x > 0$, $\therefore\ \underline{A = (4, 0)}$ [2]

(ii) A also lies on the curve,

$\therefore\ 0 = 4^3 - 6 \times 4^2 + k$

$\therefore\ \underline{k = 32}$ [1]

(iii) $\frac{d^2y}{dx^2} = 6x - 12$

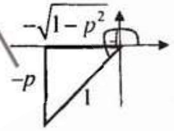
At A, $\frac{d^2y}{dx^2} = 6(4) - 12$

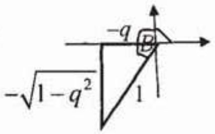
$= 12\ (> 0)$

Thus A is a minimum point. [2]

9 **(a)**
$$\frac{5-10\cos^2 A}{\sin A - \cos A} = \frac{5\left(\sin^2 A + \cos^2 A\right) - 10\cos^2 A}{\sin A - \cos A}$$
$$= \frac{5\left(\sin^2 A - \cos^2 A\right)}{\sin A - \cos A}$$
$$= \frac{5(\sin A + \cos A)(\sin A - \cos A)}{\sin A - \cos A}$$
$$= 5(\sin A + \cos A)$$
[3]

(b) Given: $\sin A = -p$ $\cos B = -q$ and A and B lie in the same quadrant, then they both lie in 3rd quadrant.





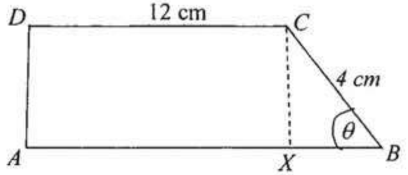
(i) $\sin(-A) = -\sin A$

$= \underline{p}$ [1]

(ii)
$$\tan(45° - A) = \frac{\tan 45° - \tan A}{1 + \tan 45° \tan A}$$
$$= \frac{1 - \tan A}{1 + \tan A} \text{ since } \tan 45° = 1$$
$$= \frac{1 - \frac{-p}{-\sqrt{1-p^2}}}{1 + \frac{-p}{-\sqrt{1-p^2}}}$$
$$= \underline{\underline{\frac{\sqrt{1-p^2} - p}{\sqrt{1-p^2} + p}}}$$
[3]

(iii)
$$\sec 2B = \frac{1}{\cos 2B}$$
$$= \frac{1}{2\cos^2 B - 1}$$
$$= \frac{1}{2(-q)^2 - 1}$$
$$= \underline{\underline{\frac{1}{2q^2 - 1}}}.$$
[2]

10 **(i)**



$$\sin\theta = \frac{CX}{4} \quad \Rightarrow \quad CX = 4\sin\theta$$

$$\cos\theta = \frac{BX}{4} \quad \Rightarrow BX = 4\cos\theta$$

$$\text{Area of trapezium} = \frac{1}{2}[12+12+4\cos\theta]\times 4\sin\theta$$
$$= (12 + 2\cos\theta)\, 4\sin\theta$$
$$= 48\sin\theta + 4(2\sin\theta\cos\theta)$$
$$= 48\sin\theta + 4\sin 2\theta \quad \text{(shown)}$$

[3]

(ii) $\frac{dA}{d\theta} = 48\cos\theta + 8\cos 2\theta$

For maximum A, $\frac{dA}{d\theta} = 0$.

$\therefore 48\cos\theta + 8\cos 2\theta = 0$

$48\cos\theta + 8(2\cos^2\theta - 1) = 0$

$8(2\cos^2\theta + 6\cos\theta - 1) = 0$

$$\cos\theta = \frac{-6\pm\sqrt{6^2-4\times2\times(-1)}}{4}$$

$$\cos\theta = \frac{-6\pm\sqrt{44}}{4}$$

$$\cos\theta = \frac{-6\pm2\sqrt{11}}{4}$$

$\cos\theta = \frac{-3+\sqrt{11}}{4}$ since θ is acute.

$\therefore \theta = 1.491\ 55$

$$\frac{d^2A}{d\theta^2} = -48\sin\theta - 16\sin 2\theta$$

When $\theta = 1.491\ 55$, $\frac{d^2A}{d\theta^2} < 0$,

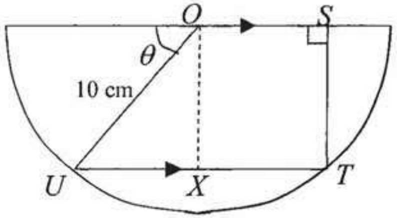
Thus A is maximum when $\underline{\underline{\theta = 1.49}}$. [5]

(iii) Maximum A

$= 48\sin(1.491\ 55) + 4\sin 2(1.491\ 55)$

$= \underline{\underline{53.3 \text{ sq units}}}$ [1]

11 **(i)**



Consider Δ OUX,

$\angle OUX = \theta$ (alternate angles)

$\sin\theta = \frac{OX}{10} \Rightarrow OX = 10\sin\theta$

$\cos\theta = \frac{UX}{10} \Rightarrow UX = 10\cos\theta$

$L = 10 + UT + ST + OS$

$= 10 + 2\times 10\cos\theta + 10\sin\theta + 10\cos\theta$

$= 10 + 30\cos\theta + 10\sin\theta$ (shown) [3]

(ii) Let $30\cos\theta + 10\sin\theta = R\cos(\theta - \alpha)$

Then $R = \sqrt{30^2+10^2} = 10\sqrt{10}$

$\tan\alpha = \frac{10}{30} \Rightarrow \alpha = 18.434°$

$\therefore \underline{\underline{L = 10 + 10\sqrt{10}\cos(\theta - 18.4°)}}$ [3]

(iii) Since max $L = 10 + 10\sqrt{10} = 41.6$, it is impossible for the perimeter of $OSTU$ to be 50 cm. [1]

(iv) Given: $L = 35$

$10 + 10\sqrt{10}\cos(\theta - 18.434°) = 35$

$$\cos(\theta - 18.434°) = \frac{35-10}{10\sqrt{10}} = \frac{5}{2\sqrt{10}}$$

Basic angle = 37.761°

$\therefore \theta - 18.434° = 37.761°$

$\therefore \underline{\underline{\theta = 56.2°}}$ [3]

1 Given: $\dfrac{d^2y}{dx^2} = 6x - 2$

$\therefore$ Integrating, $\dfrac{dy}{dx} = 3x^2 - 2x + c$

Given that gradient of normal at $P(2, -8) = -\dfrac{1}{2}$, this means that gradient of tangent at $P = 2$, i.e. when $x = 2$, $\dfrac{dy}{dx} = 2$

$\therefore\ 2 = 3(2)^2 - 2(2) + c$

$\Rightarrow c = -6$

$\therefore\ \dfrac{dy}{dx} = 3x^2 - 2x - 6$

$\therefore$ Integrating, $y = x^3 - x^2 - 6x + c_2$

Since $P(2, -8)$ lies on the curve, i.e. when $x = 2$, $y = -8$,

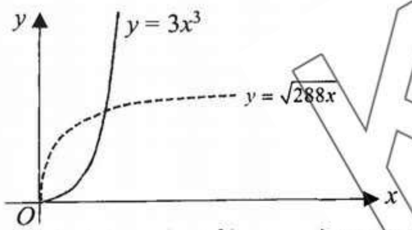
$\therefore\ -8 = 2^3 - 2^2 - 6(2) + c_2$

$\Rightarrow c_2 = 0$

Thus the equation of the curve is

$\underline{\underline{y = x^3 - x^2 - 6x}}$ [6]

2 **(i)**



[2]

(ii) To find the point of intersection, we solve the equations simultaneously:

$y = 3x^3$ … (1)

$y = \sqrt{288x}$ … (2)

(1) = (2): $3x^3 = \sqrt{288x}$

$9x^6 = 288x$

$9x^6 - 288x = 0$ **[Do not divide by variable x]**

$9x\,(x^5 - 32) = 0$

$\therefore x = 0$ or $x = 2$

$y = 0$ or $x = 24$

So the 2 points of intersection are (0, 0) and (2, 24).

Gradient of line joining these 2 points $= \dfrac{24-0}{2-0}$

$= 12$

$y = 3x^3$

$\dfrac{dy}{dx} = 9x^2$

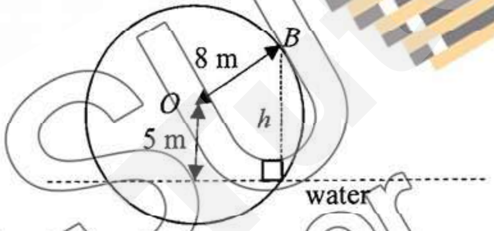
Since gradient of tangent to the curve at A is parallel to the line passing through the 2 points of intersection,

$\therefore 9x^2 = 12$

$x^2 = \dfrac{4}{3}$

$\therefore\ \underline{\underline{x = \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}}}$ since $x \geq 0$. [4]

3



Given: $h = a\cos bt + c$

(i) Starting point is when B is at its highest point, i.e., when $t = 0$, $h = 13$

$\therefore 13 = a\,(1) + c \Rightarrow a + c = 13$ … (1)

and lowest point is when B is 3 m below water level.

$\therefore -3 = a\,(-1) + c \Rightarrow -a + c = -3$ … (2)

Solving (1) and (2),

$\underline{\underline{a = 8, c = 5}}$

Given: 5 revolutions take 1 minute

$\therefore$ 1 revolution takes $\dfrac{1}{5}$ minute (= 12 seconds)

So, period: $\dfrac{2\pi}{b} = 12 \Rightarrow \underline{\underline{b = \dfrac{\pi}{6}}}$ [3]

$\therefore\ h = 8\cos\left(\dfrac{\pi}{6}t\right) + 5$

(ii) $h < 0 \Rightarrow 8\cos\left(\dfrac{\pi}{6}t\right) + 5 < 0$

When $h = 0$, $8\cos\left(\dfrac{\pi}{6}t\right) + 5 = 0$

$\cos\left(\dfrac{\pi}{6}t\right) = -\dfrac{5}{8}$

Basic angle $= \cos^{-1}\left(\dfrac{5}{8}\right) = 0.895\ 66$

The variable angle $\dfrac{\pi}{6}t$ lies in the 2nd and 3rd quadrants,

$\therefore \dfrac{\pi}{6}t = \pi - 0.895\ 66$ or $\pi + 0.895\ 66$ in the 1st revolution

$t = 4.289$ or 7.710

so duration $= 7.710 - 4.289$

$= \underline{3.42 \text{ seconds}}$ [3]

4 **(i)** For $x\left(2x + \dfrac{k}{x}\right)^8$, $T_{r+1} = x\dbinom{8}{r}(2x)^{8-r}\left(\dfrac{k}{x}\right)^r$

Power of $x = 1 + (8 - r) + (-r)$

$= 9 - 2r$

For term in x^3, power of $x = 3$

$\therefore 9 - 2r = 3$

$\Rightarrow r = 3$

$\therefore$ Term in $x^3 = x\dbinom{8}{3}(2x)^5\left(\dfrac{k}{x}\right)^3$

$= \ldots$

$= 1\ 792\ k^3$

Since coefficient of $x^3 = 28$,

$1\ 792\ k^3 = 28$

$\Rightarrow k = \dfrac{1}{4}$ (shown) [4]

(ii) For constant term, power of $x = 0$,

$\therefore 9 - 2r = 0$

$\Rightarrow r = \dfrac{9}{2}$

Since r is not a <u>whole number</u> (or <u>positive integer</u>), then we can conclude that there is no constant term. [1]

(iii) In the expansion of $x\left(2x + \dfrac{k}{x}\right)^8 + k(1 - x)^{10}$,

Term in $x^3 = 28x^3 + \dfrac{1}{4}\times\dbinom{10}{3}(-x)^3$

$= 28x^3 + \dfrac{1}{4}\times\left(-120x^3\right)$

$= -2x^3$

$\therefore$ $\underline{\text{coefficient of } x^3 = -2}$ [2]

5 **(i)** To show: AB is parallel to DC

This means that we need only to show that there are 2 equal angles that satisfy either alternate angles, corresponding angles or interior angles

Here we can show $\angle CDT = \angle BAT$]

$\angle CDT = \angle DTP$ (alternate segment theorem)

$= \angle ATP$ (common angle)

$= \angle BAT$ (alternate segment theorem)

Since this result satisfies the properties of corresponding angles, then $AB \parallel DC$. (shown) [2]

(ii) To show: the line TE bisects $\angle ATB$

[This means that we need to show: $\angle ATE = \angle ETB$]

$\angle ATE = \angle DTE$ (common angle)

$= \angle DCE$ (angle in the same segment)

$= \angle CEB$ (alternate angles since $AB/\!/DC$)

$= \angle ETB$ (alternate segment theorem)

Thus, TE bisects $\angle ATB$. (show) [3]

(iii) To show: $\Delta DFT \equiv \Delta EFC$ if $DF = EF$

From (ii), $\angle DTE = \angle DCE$ (angle in same segment)

i.e. $\angle DTF = \angle ECF$ (common angles) **A**

$DF = EF$ (given) **S**

and $\angle DFT = \angle EFC$ (vertically opp angles) **A**

Thus, $\Delta DFT \equiv \Delta EFC$ (AAS) [2]

6 Given: $\underbrace{y-x}_{Y} = \underbrace{\frac{b}{a}}_{m}\underbrace{x^2}_{X} + \underbrace{b}_{c}$ **[note that this is already in linear form]**

(i) Plotting $y-x$ against x^2 will give a straight line.

x^2	1	4	9	16
$y-x$	1.73	5.5	11.75	20.5

(ii) $b = Y-$ intercept

$= 0.5$

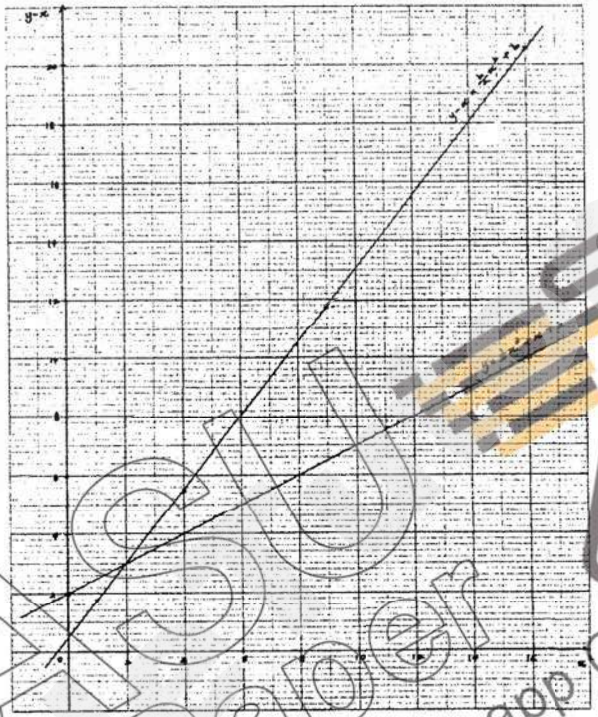
$\frac{b}{a}$ = gradient of line

$= \frac{20.5-0.5}{16-0}$

$= 1.25$

$\therefore \frac{0.5}{a} = 1.25 \Rightarrow a = 0.4$

$\therefore \underline{a = 0.4,\ b = 0.5}$ [2]



[3]

(iii) $y = \frac{1}{2}x^2 + x + 2 \Rightarrow y - x = \frac{1}{2}x^2 + 2$

Using the same axes, $Y = y - x$ and $X = x^2$,

Then we need to draw the line $Y = \frac{1}{2}X + 2$.

When $X = 0$, $Y = 2$ and when $X = 16$, $Y = 10$

So the line passes through (0, 2) and (16, 10)

The point of intersection between the 2 lines is approximately (2, 3).

i.e. $X = 2 \Rightarrow x^2 = 2$, $\therefore x = 1.41$ (3 sf)

$Y = 3 \Rightarrow y - x = 3$, $\therefore y = 3 + 1.41 = 4.41$

$\underline{x = 1.41 \text{ and } y = 4.41}$ [2]

7 **(i)** Since $(2x^2 + x - 1)$ is a factor of P(x), then we can write P(x) = $(2x^2 + x - 1)\,Q(x)$. However, we know that $Q(x)$ is a polynomial of degree 2.

So, P(x) = $(2x^2 + x - 1)(ax^2 + bx + c)$.

However, we know that for P(x), coeff of x^2 = and constant term = –3.

This tells us that $a = 1$ and $c = 3$.

So, P(x) = $(2x^2 + x - 1)(x^2 + bx + 3)$.

[Writing P(x) in this form, we only need to find one unknown, i.e. b.]

Given: when divided by $(x - 1)$, P(x) has remainder 4, i.e. P(1) = 4.

$\therefore 4 = (2 + 1 - 1)(1 + b + 3)$

$= 2(b + 4)$

$b + 4 = 2 \Rightarrow b = -2.$

$\therefore$ P(x) = $(2x^2 + x - 1)(x^2 - 2x + 3)$

$= \underline{(2x - 1)(x + 1)(x^2 - 2x + 3)}$ [4]

(ii) P(x) = 0

$\Rightarrow \underline{\underline{x = \frac{1}{2} \text{ or } -1}}$ since $x^2 - 2x + 3 = 0$ has no real roots as D = $(-2)^2 - 4\times3 = -8 < 0$. [1]

(iii) P($1 - x$) = 0

$\Rightarrow 1 - x = \frac{1}{2}$ or -1

$\therefore \underline{\underline{x = \frac{1}{2} \text{ or } 2}}$ [2]

8 **(a)** $y = x^2e^{3x}$

$\therefore \frac{dy}{dx} = 2xe^{3x} + 3x^2e^{3x}$

$= xe^{3x}(2 + 3x)$

For increasing function, $\frac{dy}{dx} > 0$

$\therefore xe^{3x}(2 + 3x) > 0$

Since $e^{3x} > 0$ for all real values of x.

$\therefore x(2 + 3x) > 0$

$\therefore \underline{\underline{x < -\frac{2}{3} \text{ or } x > 0}}$ [4]

(b) $y = \dfrac{x}{\sqrt{2x^2-1}}$ where $x > 0$

$$\frac{dy}{dx} = \frac{\sqrt{2x^2-1} - x\times\frac{1}{2}\times\frac{4x}{\sqrt{2x^2-1}}}{2x^2-1}$$

$$= \frac{\sqrt{2x^2-1} - x\times\frac{1}{2}\times\frac{4x}{\sqrt{2x^2-1}}}{2x^2-1}\times\frac{\sqrt{2x^2-1}}{\sqrt{2x^2-1}}$$

$$= \frac{(2x^2-1)-2x^2}{\sqrt{(2x^2-1)^3}}$$

$$= -\frac{1}{\sqrt{(2x^2-1)^3}}$$

Given: $\dfrac{dy}{dt} = -\dfrac{9}{8}\dfrac{dx}{dt}$ $\Rightarrow$ $\dfrac{dy}{dx} = -\dfrac{9}{8}$

$\therefore\ -\dfrac{1}{\sqrt{(2x^2-1)^3}} = -\dfrac{9}{8}$

$\therefore\ \sqrt{(2x^2-1)^3} = \dfrac{8}{9}$

$(2x^2-1)^3 = \dfrac{64}{81}$

$2x^2-1 = \sqrt[3]{\dfrac{64}{81}}$

$x^2 = \dfrac{1}{2}\left(1+\dfrac{4}{\sqrt[3]{81}}\right)$

$\therefore\ x = \sqrt{\dfrac{1}{2}+\dfrac{2}{\sqrt[3]{81}}}$ since $x > 0$

$= 0.981$ (3 sf) [4]

9 Given: $y = x\ln x - x$

(i) $\dfrac{dy}{dx} = \left(\ln x + x\times\dfrac{1}{x}\right) - 1$

$= (\ln x + 1) - 1$

$= \ln x$ (shown) [2]

(ii) From (i), we know that $\int \ln x\, dx = x\ln x - x + c$

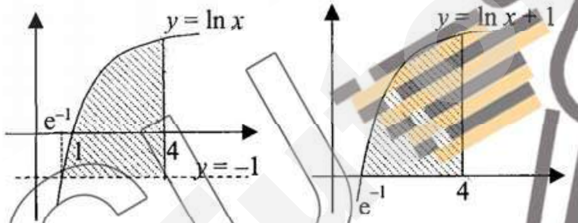
$\int_a^b (\ln x + 1)\,dx = \int_a^b \ln x\,dx + \int_a^b 1\,dx$

$= [x\ln x - x]_a^b + [x]_a^b$

$= (b\ln b - b) - (a\ln a - a) + (b - a)$

$= b\ln b - a\ln a$ (shown) [3]

(iii) The shaded area is the same in both diagrams:



When $\ln x = -1$, $x = e^{-1} = \dfrac{1}{e}$.

Shaded area $= \int_{\frac{1}{e}}^{4} (\ln x + 1)\,dx$

$= = 4\ln 4 - \dfrac{1}{e}\ln\dfrac{1}{e}$ [using (ii)]

$= \underline{5.91 \text{ units}^2}$ [3]

10 **(a)** $\cos\left(\dfrac{3x}{2}\right) = -\cos\dfrac{\pi}{10}$ for $0 < x < \pi$

basic angle $= \dfrac{\pi}{10}$

The variable angle $\dfrac{3x}{2}$ lies in 2nd and 3rd quadrants.

$\therefore\ \dfrac{3x}{2} = \pi - \dfrac{\pi}{10}$ or $\pi + \dfrac{\pi}{10}$

$= \dfrac{9\pi}{10}$ or $\dfrac{11\pi}{10}$

$\therefore\ \underline{\underline{x = \dfrac{3\pi}{5} \text{ or } \dfrac{11\pi}{15}}}$ [3]

(b) $\dfrac{1}{1-\sin x} - \dfrac{1}{1+\sin x} = \dfrac{(1+\sin x)-(1-\sin x)}{(1-\sin x)(1+\sin x)}$

$= \dfrac{2\sin x}{1-\sin^2 x}$

$= \dfrac{2\sin x}{\cos^2 x}$

$= 2\dfrac{\sin x}{\cos x}\times\dfrac{1}{\cos x}$

$= 2\tan x\sec x$ (shown) [3]

(c) $\sin 4x + 3\sin 2x = 0$ for $-180° \le x \le 180°$

$2\sin 2x\cos 2x + 3\sin 2x = 0$

$\sin 2x\,(2\cos 2x + 3) = 0$

$\therefore\ \sin 2x = 0$ or $\cos 2x = -\dfrac{3}{2}$ (no solution)

basic angle $= 0°$

$\therefore\ 2x = -360°, -180°, 0°, 180°$ or $360°$

$\underline{x = -180°, -90°, 0°, 90° \text{ or } 180°}$ [3]

11 Given: $a = \dfrac{24}{(2t+1)^2}$

(i) $v = \dfrac{24(2t+1)^{-1}}{2(-1)} + c$

$= -\dfrac{12}{2t+1} + c$

when $t = 0$, $v = -10$ m/s

$\therefore\ c = 2$

$\therefore\ v = 2 - \dfrac{12}{2t+1}$

At P, $v = 0 \Rightarrow 2 - \dfrac{12}{2t+1} = 0$

$\Rightarrow \underline{t = 2.5 \text{ s}}$ [4]

(ii) $s = 2t - 12\dfrac{\ln(2t+1)}{2} + c_1$

$= 2t - 6\ln(2t+1) + c_1$

when $t = 0$, $s = 0$, $\therefore\ c_1 = 0$

$\therefore\ s = 2t - 6\ln(2t+1)$

$t = 0$, $s = 0$

$t = 2.5$, $s = 2(2.5) - 6\ln 6 = -5.750\,5$

$t = 3$, $s = 2(3) - 6\ln 7 = -5.675\,4$

Distance travelled $= 5.750\,5 + (5.750\,5 - 5.675\,4)$

$= \underline{5.83 \text{ m}}$ (3 sf) [3]

(iii) **[9th second means from $t = 8$ s to $t = 9$ s]**

When $t = 8$, $s = 2(8) - 6\ln 17 = -0.999\ 28$ m

When $t = 9$, $s = 2(9) - 6\ln 19 = +0.333\ 36$ m

$\therefore s = 0$ for $8 < t < 9$

i.e. The particle is again at O during the 9th sec. [2]

---

12 **(i)** centre, X = midpoint of AB

$$= \left(\frac{-2+12}{2}, \frac{5+11}{2}\right)$$

$$= (5, 8)$$

diameter = AB

$$= \sqrt{(12+2)^2+(11-5)^2}$$

$$= \sqrt{232}$$

$\therefore$ Equation of circle is

$$(x-5)^2+(y-8)^2 = \left(\frac{\sqrt{232}}{2}\right)^2$$

$$\underline{\underline{(x-5)^2+(y-8)^2 = 58}}$$ [3]

(ii) Since $D(8, k)$ lies on the circle, then

$(8-5)^2 + (k-8)^2 = 58$

$(k-8)^2 = 49$

$\therefore k - 8 = 7$ or -7

$k = 15$ or 1

Since $k > 1$, $\therefore \underline{k = 15}$. [1]

(iii) gradient of $DX = \dfrac{15-8}{8-5} = \dfrac{7}{3}$

Since line l is perpendicular to radius DX,

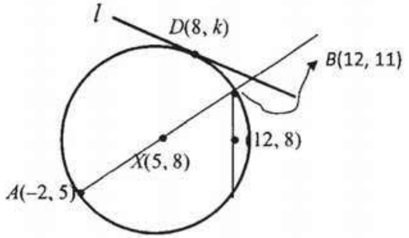
$\therefore$ gradient of $l = -\dfrac{3}{7}$

So, equation of line l is

$$y - 15 = -\frac{3}{7}(x-8)$$

$$y = -\frac{3}{7}x + \frac{24}{7} + 15$$

$$\underline{\underline{y = -\frac{3}{7}x + \frac{129}{7}}}$$ [3]



(iv) Given that (12, 8) is the midpoint of the chord, and since $B(12, 11)$ is vertically above (12, 8), then the chord is a vertical line with $x = 12$.

The other end-point of the chord is (12, 5)

Thus the points of intersection are

$\underline{(12, 5) \text{ and } (12, 11)}$. [2]

---

13 **(a)** Given: $9^x + 8 = 3^{x+2}$

$3^{2x} + 8 = 3^x \times 3^2$

Let $u = 3^x$, then the equation becomes

$u^2 + 8 = 9u$

$u^2 - 9u + 8 = 0$

$(u-1)(u-8) = 0$

$u = 1$ or $u = 8$

$3^x = 1$ or $3^x = 8$

$\therefore x = 0$ or $x = \log_3 8$

$$= \frac{\lg 8}{\lg 3}$$

$= 1.89$ (3 sf) [4]

(b) $40^{2p-1} = 5^{2-p} \Rightarrow \dfrac{40^{2p}}{40} = \dfrac{5^2}{5^p}$

$\therefore 40^{2p} \times 5^p = 5^2 \times 40$

$(40^2 \times 5)^p = 1000$

$(8000)^p = 1000$

$(20)^{3p} = 10^3$

$\therefore \underline{20^p = 10}$ [3]

(c) $\log_4(2y) = \log_{16}(y-3) + 3\log_9 3$

$$\log_4(2y) = \frac{\log_4(y-3)}{\log_4 16} + 3\frac{\log_3 3}{\log_3 9}$$

$$\log_4(2y) = \frac{\log_4(y-3)}{2} + \frac{3}{2}$$

$2\log_4(2y) = \log_4(y-3) + 3$

$\log_4(2y)^2 - \log_4(y-3) = 3$

$$\log_4\frac{(2y)^2}{y-3} = 3$$

$$\therefore \frac{4y^2}{y-3} = 4^3$$

$4y^2 = 64(y-3)$

$y^2 = 16(y-3)$

$y^2 - 16y + 48 = 0$

$(y-4)(y-12) = 0$

$\therefore \underline{y = 4 \text{ or } y = 12}$. [4]

ST. MARGARET'S SECONDARY SCHOOL

Preliminary Examinations 2019

CANDIDATE NAME

CLASS

REGISTER NUMBER

ADDITIONAL MATHEMATICS **4047/01**

Paper 1

26 August 2019

Secondary 4 Express / 5 Normal (Academic)

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **17** printed pages and a blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$, $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\dfrac{n!}{r!(n-r)!}=\dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC,

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}ab\sin C$$

1 The equation of a curve is $y = \frac{2x + 5}{x + 6}$, for $x \neq -6$.

Explain, with working, whether the curve has turning points. [3]

2 Find the values of the integers a and b for which $a + \sqrt{b}$ is a solution to the equation $x\sqrt{27} - 2x\sqrt{2} = x\sqrt{75} - \sqrt{8}$. [4]

3 **(i)** Sketch the graph of $y^2 = \frac{1}{4}x$, for $x \leq 16$. [1]

(ii) Find the coordinates of the points of intersection of the curve $y^2 = \frac{1}{4}x$ and the line $6y - 4x + 10 = 0$. [4]

4 A particle is travelling in a straight line with a velocity of $v = 8t - \frac{t^2}{4}$ cm/s where t is the time in seconds after leaving a fixed point O.

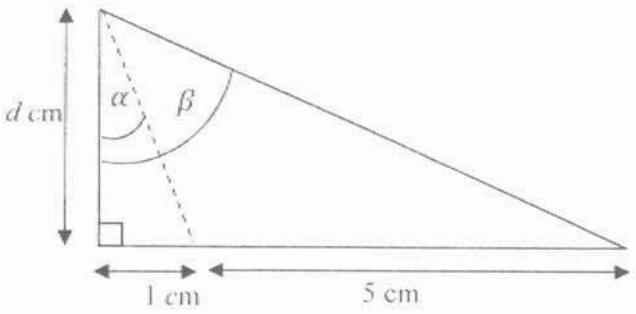
Calculate,

(i) its acceleration when the particle is at instantaneous rest. [3]

(ii) the value of t when the particle returns to O. [2]

5 The curve $y = \mathrm{f}(x)$ has a gradient of -1 at the point $(2, 8)$. If $\mathrm{f}''(x) = 6 - 6x$, find the equation of the curve. [4]

6



Find, in terms of d, an expression for

(i) $\tan\alpha$, [1]

(ii) $\tan\beta$, [1]

where α, β and d are shown in the diagram.

Hence obtain, in terms of d, an expression for

(iii) $\tan(\beta - \alpha)$. [2]

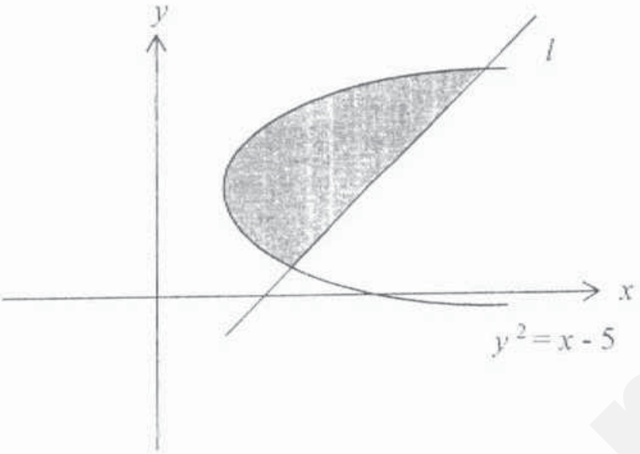
Given that $\beta - \alpha = 45°$, find the values of d. [2]

7 The quadratic equation $4x^2 - 44x + 1 = 0$ has roots $\alpha^2 - 1$ and $\beta^2 - 1$. Find the quadratic equation whose roots are α and β, where α and β are positive. [6]

8 **(i)** By using long division, divide $2x^3 - 11x^2 + 12x + 9$ by $2x + 1$. [2]

(ii) Express $\frac{13x^2-52x+32}{2x^3-11x^2+12x+9}$ in partial fractions. [5]

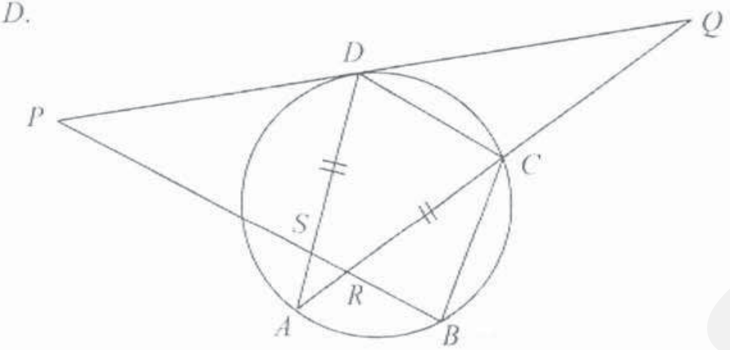
9 The diagram shows part of the curve $y^2 = x - 5$ and the line l. The equation of the line l is $4y + 2 = x$. Calculate the area of the shaded region. [6]



10 **(i)** Prove $(\sin x + \cos x)\left(1 - \frac{1}{2}\sin 2x\right) = \sin^3 x + \cos^3 x$. [4]

(ii) Hence solve $(\sin x + \cos x)\left(1 - \frac{1}{2}\sin 2x\right) = 0$ for $-2\pi < x < 2\pi$. [3]

11 In the diagram below, A, B, C and D are points on the circle and QDP is a tangent to the circle at D.



Given that $AD = AC$, prove that

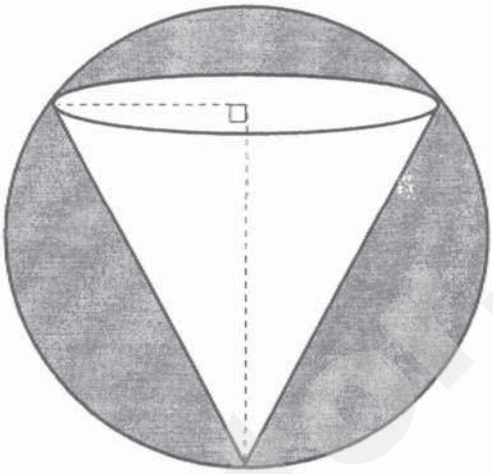
(i) ΔQCD is similar to ΔQDA. [2]

(ii) $QD^2 - QC^2 = QC \times DA$. [4]

[Turn over

12 The diagram below shows a pendant in the shape of a sphere of radius 3 cm.

A right inverted circular cone of base radius r cm and height $(x + 3)$ cm is being removed from the solid sphere. [Volume of sphere $= \frac{4}{3}\pi r^3$; Volume of cone $= \frac{1}{3}\pi r^2 h$]



(i) Show that $r = (9 - x^2)^{\frac{1}{2}}$. [2]

(ii) Show that the volume of the cone is $V = \frac{1}{3}\pi\ (27 + 9x - 3x^2 - x^3)$. [2]

(iii) Given that x can vary, find the maximum value of V.

Hence, find the least amount of solid left in the pendant. [6]

13 The points $A(0, 2)$ and $B(8, 2)$ lie on the circumference of a circle C_1.

The line $x = -1$ is a tangent to C_1.

(i) Find the radius and the coordinates of the centre of C_1, given that the centre of the circle lies below the x-axis. [4]

(ii) Express the equation of C_1 in the form $x^2 + y^2 + 2px + 2qy + r = 0$, where p, q, r are integers. [2]

(iii) Find the equations of the tangents to C_1, which are parallel to x - axis. [2]

(iv) Another circle C_2 has its centre at B. Given that the area of C_2 is one-quarter that of C_1, find the equation of C_2. [3]

St. Margaret's Secondary School

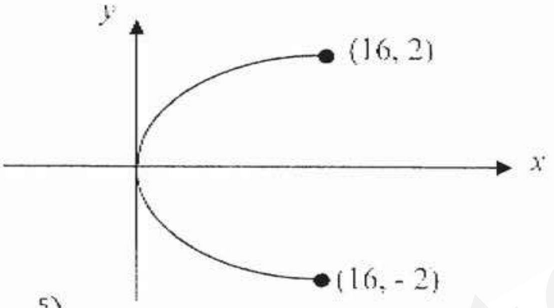
Sec 4E5N Preliminary Examinations

Additional Mathematics Paper 1

1. $\frac{dy}{dx} = \frac{7}{(x+2)^2}$. the curve has no turning point.

2. $x = -2 + \sqrt{6}$. $a = -2, b = 6$

3(i)



3(ii) (4, 1), $\left(\frac{25}{16}, -\frac{5}{8}\right)$

4(i) acceleration = -8 m/s^2 (ii) time = 48 s

5. $f(x) = 3x^2 - x^3 - x + 6$

6(i) $\frac{1}{d}$ (ii) $\frac{6}{d}$ (iii) $\frac{5d}{6+d^2}$; $d = 2$ or 3

7. $2x^2 - 4\sqrt{5}x + 7 = 0$

8(i)

$$\begin{array}{r|l} & x^2 - 6x + 9 \\ \hline 2x+1 & 2x^3 - 11x^2 + 12x + 9 \\ & \vdots \end{array}$$

(ii) $\frac{5}{(2x+1)} + \frac{4}{(x-3)} - \frac{1}{(x-3)^2}$

9. $\frac{4}{3}$ units^2

10 (ii) $x = \frac{3}{4}\pi, \frac{7}{4}\pi, -\frac{\pi}{4}, -\frac{5}{4}\pi$

12(iii) $x = -3$ (rej) or 1; $x = 1$, $\frac{d^2y}{dx^2} < 0$, ∴ max point;

Volume of cone = 33.5 cm^3; least volume of solid left = 79.6 cm^3

13(i) radius = 5 units; Centre(4, -1) (ii) $x^2 + y^2 - 8x + 2y - 8 = 0$

(iii) $y = 4$ & $y = -6$ (iv) $(x-8)^2 + (y-2)^2 = \frac{25}{4}$

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 (i) Sketch the [illegible]h of $y = -|3x-5|$, indicating the intercepts clearly. [2]

(ii) Explain why the equation $-2 = |3x-5|$ has no real roots using a graphical approach. [1]

2 It is known that $\log_2 b = y$ and $\log_b y = 3$.

(i) Find an expression for $\log_{\sqrt{2}} b - \log_2 2b$ in terms of y.

(ii) By considering a pair of simultaneous equations, show that $\lg b = b^3 \lg 2$.

3 **(i)** On the same diagram, sketch the graphs of $\frac{y^2}{25} = x$ and $y = -3x^{\frac{3}{2}}$. [3]

(ii) Find the coordinates of the points of intersection of the two curves. [3]

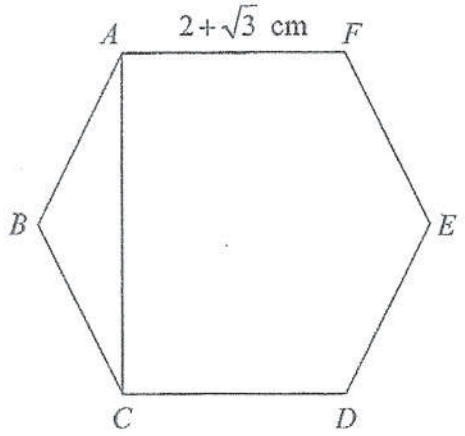
4 **(i)** Factorise $8x^3 + 27$. [

(ii) Express $\dfrac{8x^3 + 27}{(2x^2 + 3x)(x-1)^2}$ in partial fractions. [

5 **(i)** Given that $u = 2^{2y}$, solve the equation $8^y = 7(2^y) + \frac{8}{2^y}$. [4]

(ii) Hence, solve the equation $64^{-x} = 7(4^{-x}) + \frac{8}{4^{-x}}$. [2]

6 A regular hexagon *ABCDEF* with sides $\left(2+\sqrt{3}\right)$ cm is shown below.



(i) Show that $AC=\left(2\sqrt{3}+3\right)$ cm.

(ii) If the line segment *AC* has length $\left(\sqrt{27}-\sqrt{3}\right)r$ cm, find the value of r, expressing answer in the form $a+b\sqrt{3}$, where a and b are rational numbers.

7 The roots of $x^2 - 7x + 4 = 0$ are α^2 and β^2. Given that α and β are opposite in sign,

(i) Find two possible values of $\alpha + \beta$. [4]

(ii) Find two non-equivalent quadratic equations whose roots are α and β. [2]

8 (i) Show that $\frac{\mathrm{d}}{\mathrm{d}x}(2x\sin^2 x) = 2\sin^2 x + 4x\sin x\cos x$.

(ii) Hence find $\int x\sin 2x \, \mathrm{d}x$.

9 **(i)** Using $\tan 3x = \tan(2x + x)$, show that $\tan 3x$ may be expressed as $\dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$. [3]

(ii) Find all the values of x between 0 and π for which $\tan 3x = -5\tan x$. [5]

10 **(i)** Show that the curve $y = \frac{1}{4}x^2 - hx + 4$ meets the line $y = 2x - (2h-1)$ for any real val

(ii) State the value of h if the line $y = 2x - (2h-1)$ is a tangent to the curve $y = \frac{1}{4}x^2 - hx$

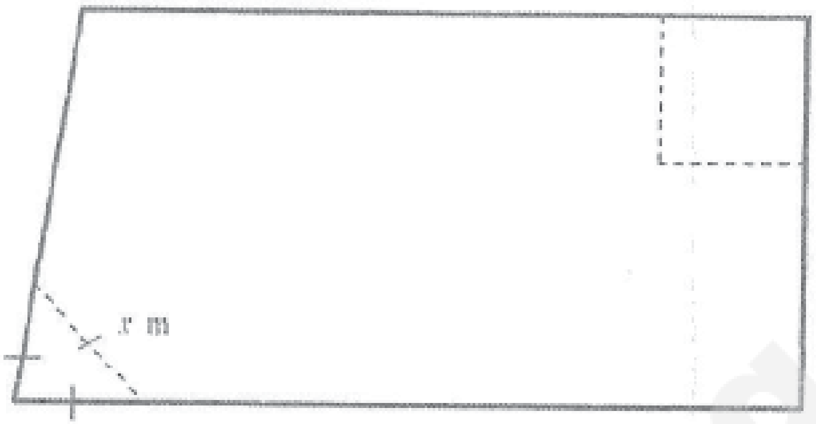
(iii) Using your answer from part (ii), find the values of p and q if the line $y = 2x$ intersects the line $y = px + q$ at an infinite number of points.

11 A semi-circle has radius r m, area A m^2 and perimeter P m. At the instant when its radius is a m, its area is increasing at a rate of 2π m^2/s.

(i) Find an expression in terms of a, for the rate of increase of the radius at this instant. [3]

(ii) Find an expression in terms of a and π, for the rate of increase of the perimeter at the same instant. [2]

12



The diagram shows a room (crime scene) surrounded by solid walls. Two bodies are found at extreme corners of the room. To facilitate forensic work, a tape of total length p m is used to off these two extreme corners. The dotted lines represent the length of tape used. One corne room is an equilateral triangle of side x m and the other corner is a square.

(i) Show that the area of the triangle is $\frac{\sqrt{3}}{4}x^2$.

(ii) Show that the total area, A m^2, of the two cordoned-off corners is given by

$$A = \frac{p^2}{4} - \frac{p}{2}x + \frac{\sqrt{3}+1}{4}x^2.$$

(iii) Given that x can vary depending on how taut the tape is, find an expression for x in terms of p for which the area A is stationary. [2]

(iv) Write down, but do not simplify, an expression for the stationary value of A in terms of p and determine the nature of this stationary value. [3]

(v) In the case where $p = \sqrt{3} + 1$, sketch a graph of A against x, indicating the turning point and the y-intercept clearly. [3]

End of Paper 1

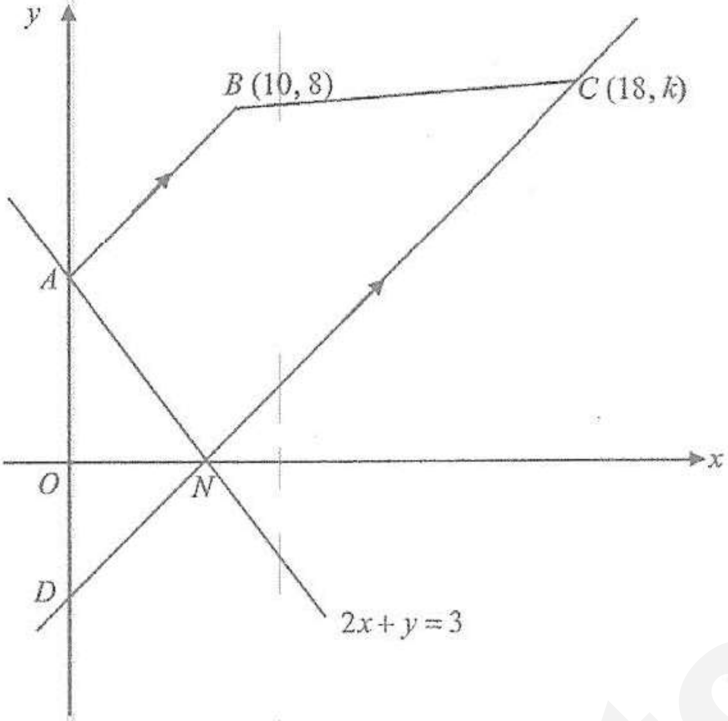
Paper 2

1 **(i)** Find the remainder when the polynomial $f(x) = 4x^3 - x + 3$ is divided by $2x - 1$. [

(ii) Given that $f(x) - 5 = (2x - 1)A(x) + b$, where $A(x)$ is a polynomial, state the value of b the degree of $A(x)$. [

(iii) A constant c is added to $f(x)$ to make it divisible by $2x - 1$. State the value of c. [

2 **Solutions to this qu[illegible]n by accurate drawing will not be accepted.**



The diagram shows a trapezium *ABCD* in which *AB* is parallel to *DC*. The coordinates of the points *B* and *C* are (10, 8) and (18, *k*) respectively. The line with equation $2x + y = 3$ intersects the *x*-axis and *y*-axis at *N* and *A* respectively. The line *CN* intersects the *y*-axis at *D*.

(i) Determine whether $\angle ANC = 90^\circ$. [4]

(ii) Write down the equation of the line CD.

(iii) Find the value of k and hence, find the area of the trapezium $ABCD$.

3 The curve $y = f(x)$ is such that $f''(x) = x^2 - \frac{1}{2}$.

(i) Find the range of values of x for which $f'(x)$ is a decreasing function. [3]

The point $P(3, 10)$ lies on the curve. The gradient of the curve at P is $\frac{5}{2}$.

(ii) Find the equation of the curve. [5]

4 (i) Given that the coefficient of the $\frac{1}{x^2}$ term in the binomial expansion of $\left(\frac{1}{x^3} - kx^2\right)^9$ is 4032, show that $k = -2$.

(ii) Find the constant term in the expansion of $\left(x^{12} + x^7\right)\left(\frac{1}{x^3} - kx^2\right)^9$.

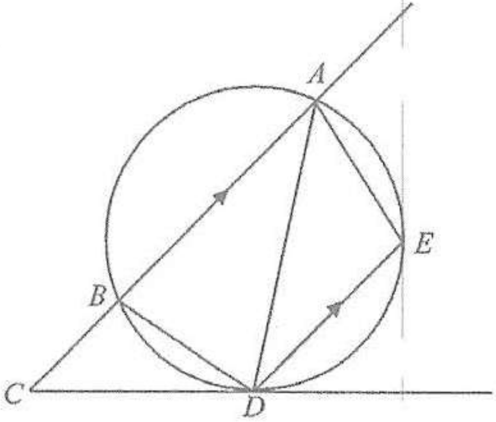
5 A curve C has equation $y = \ln(1-2x)$, $x < \frac{1}{2}$. The point P on C has coordinates $\left(\frac{1}{3}, \ln\frac{1}{3}\right)$. The tangent and normal to C at P meets the y-axis at Q and R respectively.

(i) Find the equations of the tangent and normal to the curve C at P. [4]

(ii) Find the coordinates of Q and of R [2]

(ii) Find the area of triangle OPR. [2]

6



In the diagram, *ABDE* is a cyclic quadrilateral in which *BA* is parallel to *DE*. The tang
circle at *D* meets *AB* produced at *C*. The chord *AD* bisects angle *BAE*.

(i) Prove that angle $BCD = \frac{1}{2}$ angle *BAE*.

(ii) If *AD* is the diameter of the circle,
(a) prove that ΔABD is similar to ΔDEA,

(b) state the name of the geometric shape given to the quadrilateral *ABDE*.

7 A boy runs along the coastline of a beach and passes a fixed point A. The velocity, v m/s, that he runs in t seconds after he passes A is given by

$$v = 20e^{-0.9t} - 3.$$

(i) Find the distance that the boy ran 60 seconds after he passes A. [5]

(ii) Find the boy's acceleration when he is instantaneously at rest. [4]

(iii) Explain what the sign of the acceleration indicates. [1]

(iv) Explain whether the boy will be running at his maximum velocity at any point of time of his run. [2]

The function $f(x) = 2x^3 + bx^2 - 14x - 20$ has a factor of $a(x+5)(x+1)$, where a and b are positive constants.

(i) Find the value of b. [2]

(ii) If $(x-2)$ is also a factor of $f(x)$, state the value of a. [1]

(iii) Hence, deduce another function $g(x)$ whose coefficient of x^3 is 3 and values of the roots of $g(x)=0$ are twice the values of the roots of $f(x)=0$. [1]

9 The function $f(x) = a\cos bx + 2$ has a period of 6π and a range of $-2 \leq f(x) \leq 6$.

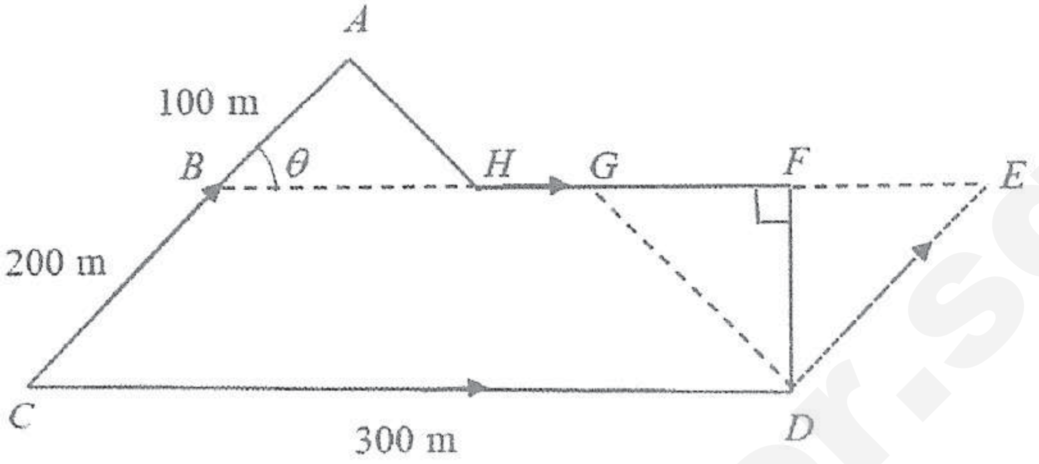
(i) State the value of a and of b.

(ii) Sketch the graph of $y = f(x)$ for $0 \leq x \leq 9\pi$.

(iii) On the same diagram, sketch $g(x)$ where $g(x) = -f(x)$ for $0 \leq x \leq 9\pi$.

(iv) State the equation of the line of symmetry between f(x) and g(x).

10



The diagram shows the running path ($A \rightarrow B \rightarrow C \rightarrow D \rightarrow F \rightarrow G \rightarrow H \rightarrow A$) of Ali. ΔABH and ΔDEG are isosceles triangles. CA is parallel to DE and CD is parallel to HF. $AB = 100$ m, $BC = 200$ m and $CD = 300$ m. It is also given that angle $GFD = 90°$ and angle $ABH = \theta°$, where $0° < \theta < 90°$.

(i) Given that Ali runs at a uniform speed of 10 m/s throughout, show that the time taken t s for the run can be expressed as $100 + 20\sin\theta - 40\cos\theta$. [5]

(ii) Express t in the form of $100 + R\sin(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [3]

(iii) Using your answer from part **(ii)**, justify with working whether Ali can complete his run in the shortest possible time, assuming that Ali completes his run. [2]

A circle, C_1, has equation $x^2 + y^2 - 4x + 6y = 12$.

(i) Find the radius and coordinates of the centre of the circle C_1. [3]

(ii) Determine whether $(3,1)$ lies inside, outside or on the circle. [2]

Another circle, C_2, has centre $(2,-8)$ and the same radius as C_1.

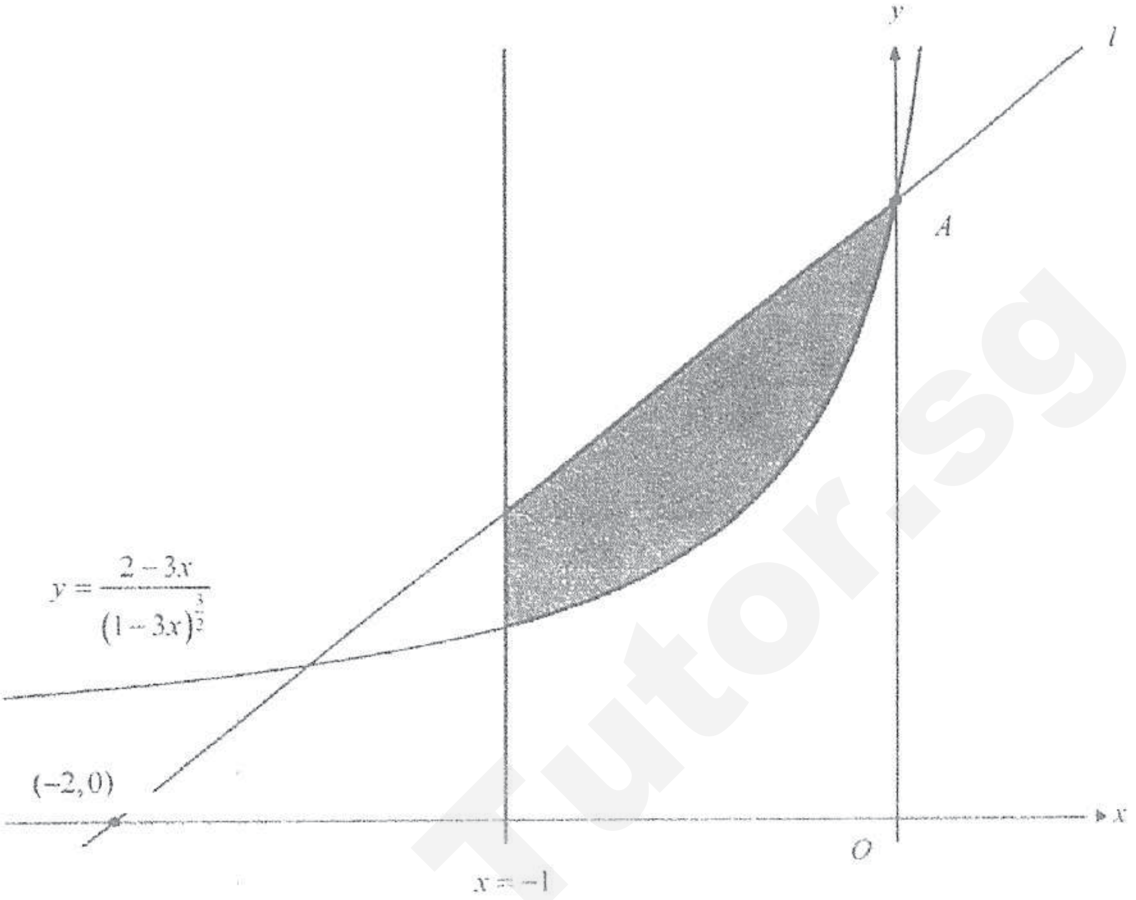
(iii) State the equation of C_2. [1]

The line which is parallel to the x-axis and passes through $(2,-8)$ intersects C_2 at A and B.

(iv) State the equations of the tangents to C_2 at A and B. [2]

12 (a) Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{2x}{\sqrt{1-3x}}\right)=\frac{2-3x}{(1-3x)^{\frac{3}{2}}}$.

(b)



The diagram shows the line $x = -1$ and part of the curve $y = \frac{2-3x}{(1-3x)^{\frac{3}{2}}}$. The curve intersects the y-axis at point A. The line l through A intersects the x-axis at $(-2, 0)$.

(i) Determine the area of the shaded region bounded by the curve, the line $x = -1$ and the line l. [4]

(ii) The area bounded by the x-axis, the curve, the line $x = -1$ and the line $x = a$ is four times the area of the shaded region in part (i), where $a > 0$. Find the value of a. [3]

3 The population, P, in millions of a country on 1st January has been increasing every year from 1960 to 2000. The increase is exponential and so can be modelled by an equation of the form

$$P = P_0 e^{kt} \ ,$$

where P_0 and k are constants and t is the time in years since 1st January 1960. The table below gives values of P and t for some of the years 1960 to 2000.

Year	1960	1970	1980	1990	2000
t years	0	10	20	30	40
P	5.75	9.97	17.3	30.0	51.9

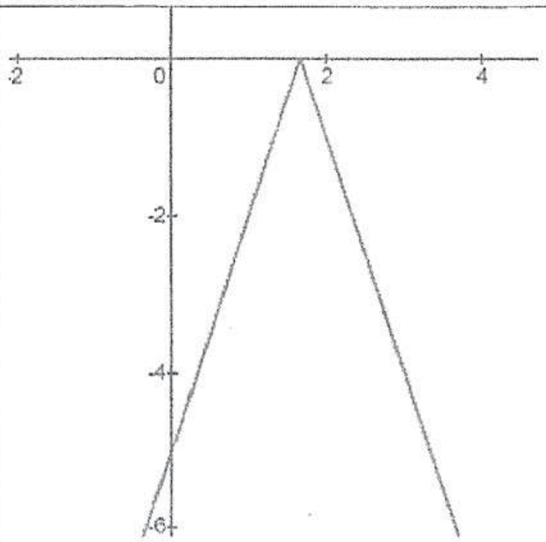
(i) Plot a suitable straight line graph to show that the model is valid for years 1960 to 2000. [3]

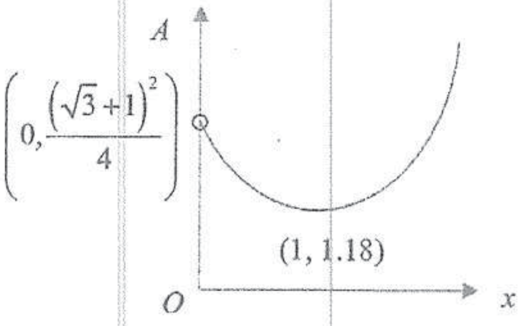
(ii) Estimate the value of k. [2]

(iii) Assuming that the model is still appropriate, use your graph to estimate the population in 1st January 2005. [2]

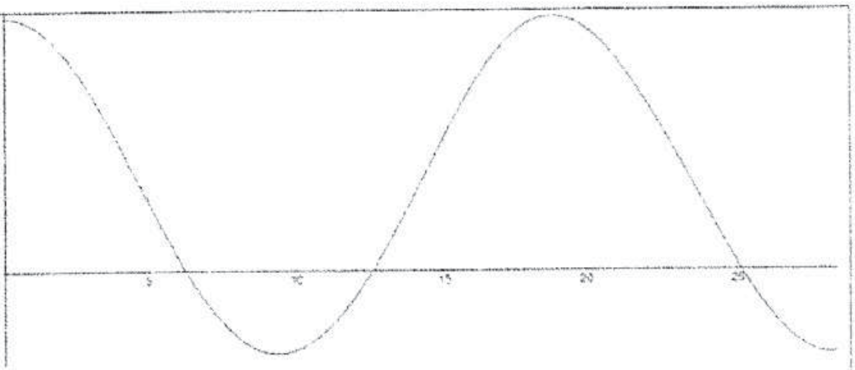
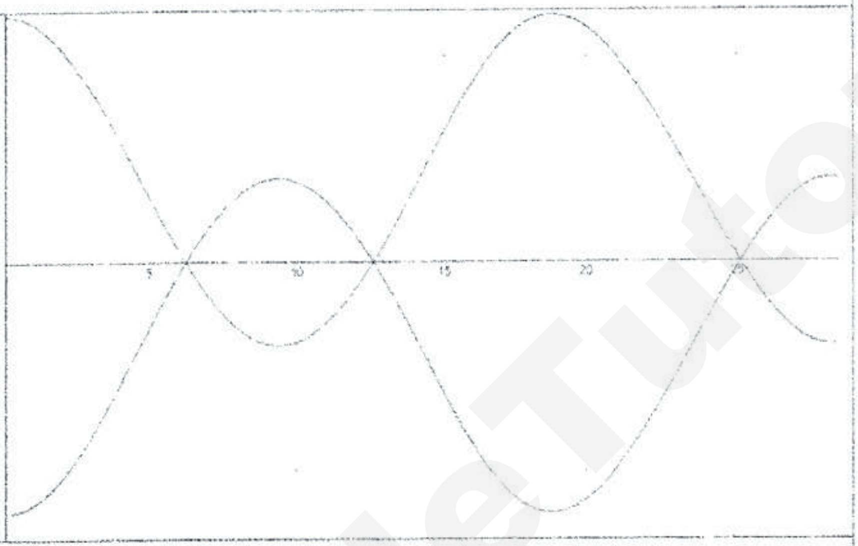
End of Paper 2

2019 4E5N PRELIM AM P1 ANSWER KEY

Q	Solution
1i	
1ii	Since the line y = 2 does not intersect the graph, there are no real roots for the equation $-2=\|3x-5\|$.
2i	$y-1$
3i	
3ii	$(0,0)$ and $(\frac{5}{3},-6.45)$
4i	$8x^3+27=(2x+3)(4x^2-6x+9)$
4ii	$\frac{8x^3+27}{(2x^2+3x)(x-1)^2}=\frac{9}{x}-\frac{5}{x-1}+\frac{7}{(x-1)^2}$
5i	$y=\frac{3}{2}$
5ii	$x=-\frac{3}{4}$
6i	$AC=2\sqrt{3}+3$
6ii	$r=1+\frac{1}{2}\sqrt{3}$
7i	$\alpha+\beta=\sqrt{3}$ or $\alpha+\beta=-\sqrt{3}$

7ii	$x^2-\sqrt{3}x-2=0$ or $x^2+\sqrt{3}x-2=0$
8i	$2\sin^2 x+4x(\sin x)(\cos x)$
8ii	$\int x(\sin 2x)\mathrm{d}x = x\sin^2 x-\frac{x}{2}+\frac{1}{4}\sin 2x+C_3$
9i	$\frac{3\tan x-\tan^3 x}{1-3\tan^2 x}$
9ii	$x=0.615,\ 2.53$
10i	Since $(h+1)^2\geq 0$ for all values of h, there will always intersections between the line and curve, hence $y=\frac{1}{4}x^2-hx+4$ and $y=2x-(2h-1)$ meet.
10ii	$h=-1$
10iii	$p=2$ $q=3$
11i	$\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{2}{a}$
11ii	$\frac{4+2\pi}{a}$
12i	$\left(\frac{\sqrt{3}}{4}\right)x^2$
12ii	$\frac{p^2}{4}-\frac{p}{2}x+\frac{\sqrt{3}+1}{4}x^2$
12iii	$x=\frac{p}{\sqrt{3}+1}$
12iv	$\frac{\mathrm{d}^2A}{\mathrm{d}x^2}=\frac{\sqrt{3}+1}{2}>0$ $\therefore A$ is minimum
12v	When $p=\sqrt{3}+1$, $x=1$ $A=1.18$ 

Qns	Answer Key
1i	3
1ii	$b = -2$ Degree = 2
1iii	$c = -3$
2i	$m_{AN} \times m_{CN} = -2 \times \frac{1}{2} = -1$, therefore $\angle ANC = 90^\circ$
2ii	Equation of line *CD* $y = \frac{1}{2}x - \frac{3}{4}$
2iii	52.5 units^2
3i	$-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$ (Accept: $-0.707 < x < 0.707$)
3ii	$\mathrm{f}(x) = \frac{x^4}{12} - \frac{1}{4}x^2 - 5x + 20.5$
4i	$r = 5$ $k = -2$
4ii	2688
5i	$y = \frac{1}{6}x - \frac{1}{18} + \ln\frac{1}{3}$
5ii	$R(0,\ \ln\frac{1}{3} - \frac{1}{18})$ [Accept: R (0, -1.15)
5iii	0.192 units^2 (3 s.f)
6iib	Rectangle
7i	Distance = 158m
7ii	$-2.70\ \mathrm{m/s^2}$
7iii	It means that his velocity is decreasing/ boy is slowing down
7iv	$\frac{\mathrm{d}v}{\mathrm{d}t} = -18e^{-0.9t}$ For $t \geq 0$, $-18 < 0$ and $e^{-0.9t} > 0$, $\therefore \frac{\mathrm{d}v}{\mathrm{d}t} = -18e^{-0.9t} < 0, \therefore \frac{\mathrm{d}v}{\mathrm{d}t} \neq 0$. Hence, the boy will be **NOT** running at his maximum velocity at any point of time of his run.
8i	$b = 8$
8ii	$\mathrm{f}(x) = 2x^3 + 8x^2 - 14x - 20 = a(x+5)(x+1)(x-2)$ $\therefore a = 2$
8iii	$\mathrm{g}(x) = 3(x+10)(x+2)(x-4)$
9i	$a = 4,\ b = \frac{1}{3}$

9ii	
9iii	
9iv	$y = 0$
10i	$100 + 20\sin\theta - 40\cos\theta$
10ii	$t = 100 + \sqrt{2000}\sin(\theta - 63.4^\circ)$
10iii	Since no θ within the range of $0^\circ < \theta < 90^\circ$ can be found, Ali cannot complete his run in the shortest possible time.
11i	Radius $r = \sqrt{(-2)^2 + (3)^2 - (-12)} = 5$
11ii	Distance of (3,1) to centre $= \sqrt{(3-2)^2 + (1+3)^2} = \sqrt{17} < 5$ Since distance of (3,1) to centre is smaller than the radius of the circle, (3,1) lies inside the circle.
11iii	$(x-2)^2 + (y+8)^2 = 25$
11iv	$x = -3, x = 7$
12a	$\dfrac{2-3x}{(1-3x)^{\frac{3}{2}}}$
12bi	$\dfrac{1}{2}$ square units
12bii	$a = 0.25$ or $a = -1$ (rejected since $a > 0$)
13ii	$k = 0.055$ (Allow $0.05 \le k \le 0.06$)
13iii	Population is 67.4 **millions**