Answer all the questions

1 The equation of a curve is $y = 2x^2 - 6x + k$, where k is a constant.

(i) In the case when $k = -20$, find the set of values of x for which $y < 0$. [2]

(ii) In the case when $k = 10$, show that the line $y + 2x = 8$ is a tangent to the curve. [3]

2 (i) Given that $u = 2^x$, express $4^x - 2^{x+1} = 3$ as an equation in u. [2]

(ii) Hence find the value of x, correct to 2 decimal places. [3]

(iii) Explain why the equation $4^x - 2^{x+1} = k$ has no solution if $k < -1$. [2]

3 The equation $3x^2 - x + 5 = 0$ has roots α and β.

(i) Find the value of $\alpha^3 + \beta^3$. [5]

(ii) Find a quadratic equation with integer coefficient whose roots are $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$. [3]

4 (i) Express $\dfrac{1 - \sin x \cos x + 2\cos^2 x}{\sin^2 x}$ as a quadratic expression in $\cot x$. [3]

(ii) Hence, using (i) solve the equation $1 + 2\cos^2 x = \sin x(3\sin x + \cos x)$ for $0° < x < 360°$. [4]

5 A freshly baked bread, with an initial temperature of $75^\circ C$, is left to cool on the rack. The temperature, $T^\circ C$ of the bread, t minutes after it has been placed on the rack is given by $T = 27 + ke^{-nt}$, where k and n are positive constants.

(i) Calculate the value of k. [1]

(ii) If the bread has cooled down by $20^\circ C$ after 10 minutes, find the value of n. [2]

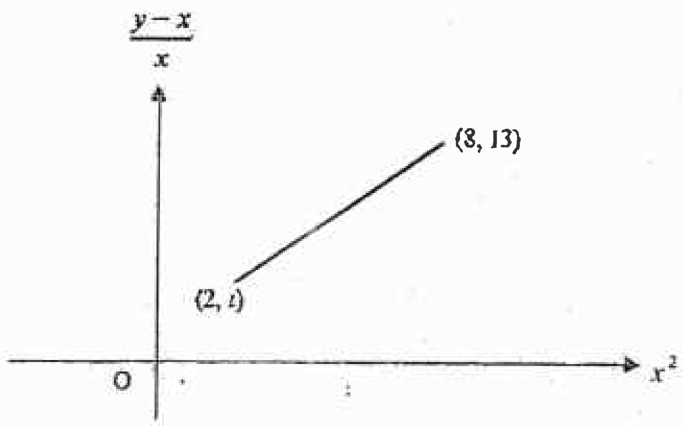
(iii) Find the least time taken, to the nearest minute, for the bread to have a temperature of less than $30^\circ C$. [3]

(iv) Explain with the sketch for $T = 27 + ke^{-nt}$, why the temperature of the bread can never reach $27^\circ C$. [1]

6

The diagram shows a trapezium $ABCD$ in which side $AB = 4\sqrt{3} - 2\sqrt{2}$ cm and angle $BAD = 30^\circ$. Given that the length of AD is twice the length of BC and that area of the trapezium is $25 + 5\sqrt{6}$ cm^2, find without the use of a calculator, the length of AD in the form $a\sqrt{2} + b\sqrt{3}$. [4]

7 The diagram shows part of a straight line, passing through $(2, t)$ and $(8, 13)$, drawn to represent the equation $3y = 4x^3 + ax$, where a and t are constants. Find the value of a and t.



[4]

8 (a) Given that the first 3 terms in the expansion of $(a-x)(1+3x)^n$ in ascending powers of x is

$$2+29x+bx^2+\ldots$$

Find the values of the constants a, b and n. [5]

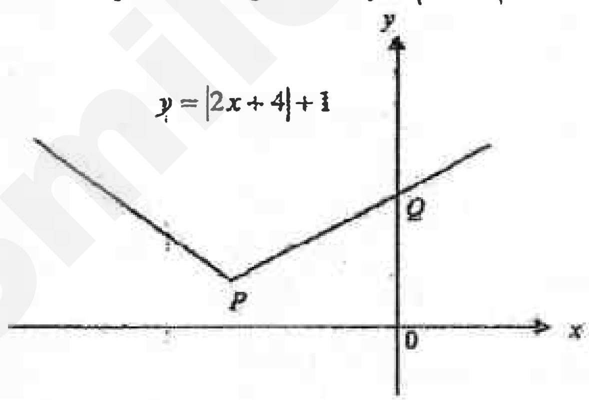
(b) (i) Write down the general term in the binomial expansion of $(x^2+\frac{1}{2x^3})^{10}$. [1]

(ii) Write down the power of x in this general term. [1]

(iii) Hence, or otherwise, determine the coefficient of x^{-15} in the binomial expansion of $(x^2+\frac{1}{2x^3})^{10}$. [2]

9 A curve has equation $y=\ln\left(\frac{3x-1}{5-2x}\right)$. The normal to the curve at the point (a, b) is parallel to the line $13y+5x-26=0$. Given that where $a>1$, find the values of a and b. [5]

10 The diagram shows part of the graph of $y=|2x+4|+1$.



(i) Find the coordinates of P and of Q. [3]

A line of gradient m passes through the point (0,3).

(ii) In the case where $m=-1$, find the x-coordinates of the points of intersection between the line and the graph of $y=|2x+4|+1$ [3]

(iii) Determine the set of values of m for which the line intersects the graph of $y=|2x+4|+1$ at two points. [2]

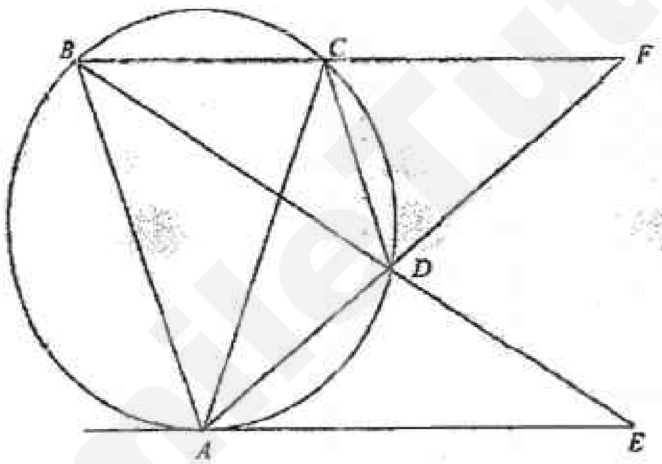
11 A particle starts from rest and moves in a straight line, so that t seconds after leaving O, its velocity, v m/s is given by $v = 28 - 2e^{\frac{-t}{3}}$.

(i) Calculate the initial acceleration of the particle. [2]

(ii) Calculate, to 2 decimal places, the displacement of the particle from O when $t = 10$. [4]

(iii) Determine, with explanation, whether the particle will return to O. [1]

12 In the diagram below, the line AE is tangent to the circle at A. The line EB is the angle bisector of angle ABC and cuts the circle at D. The chord AC is the angle bisector of angle BAD and cuts the circle at C. The chords BC and AD are produced to meet at F. The line segments AD and FD are equal.



(i) Prove that angle DAC = angle DAE. [3]

(ii) Prove that triangles ADE and BAE are similar. [2]

(iii) Prove that C is a mid-point of BF. [2]

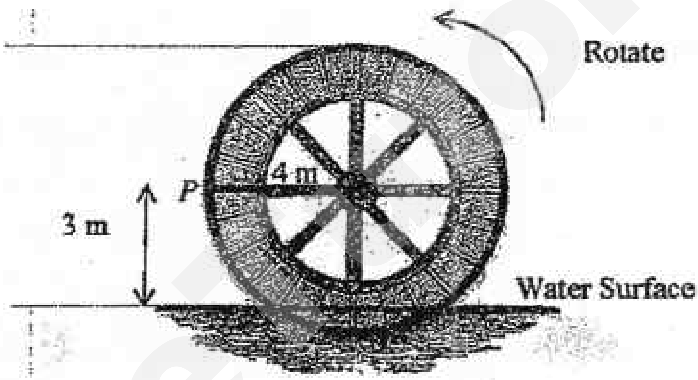
(iv) Hence, using (ii) and (iii), show that $AF \times AE = 4CD \times DE$. [2]

Answer all the questions

1 (a) Given that $\lg(3y+2)-4x^2=2$, express y in terms of x. [3]

(b) Solve the equation $2\log_4(8-2x)-\log_2(x-2)=3-\log_2(1+x)$ [5]

2 The diagram shows a water wheel which rotates at 3 revolutions per minute in an anticlockwise direction.At the start of the revolution, a point P on the rim of the wheel is at the height of 3 m above the surface of the water. The radius of the water wheel is 4 m.



The height, h m, of point P above the water surface is given as $h=a\sin(\frac{\pi}{b}t)+c$, where t is the time in seconds.

(i) State the values of a, b and c. [3]

(ii) Find the time, t, where point P first emerge from the water. [3]

3 (i) Show that $\frac{d}{dx}[e^{2x}(2x+1)]=e^{2x}(4x+4)$. [2]

(ii) Hence, or otherwise, evaluate $\int_0^1 2xe^{2x}\,dx$. [4]

4 The cubic polynomial f(x) is such that the coefficient of x^3 is 4 and the constant term is -3. When $x+1$ is a factor of f(x), the quadratic factor is px^2+qx+r. It is given that f(x) leaves a remainder of -20 when divided by $x-1$.

(i) Find the values of p, q and r. [3]

(ii) Solve f(x) =0. [2]

(iii) Hence solve the equation f(-x) = 0. [1]

5 It is given that $\dfrac{x^3-x^2-4x+1}{x^2-4}=ax+b+\dfrac{c}{x^2-4}$

(i) Find the values of a, b and c. [3]

(ii) Hence, using partial fractions and the values of a, b and c obtained in part (i), find $\int \dfrac{x^3-x^2-4x+1}{x^2-4}\,dx$. [6]

6 (a) The curves $y=a(\sqrt[3]{x})$ and $y=\dfrac{2a}{k}\left(\dfrac{1}{x^2}\right)$ meets at the point (1,5) where a and k are constants.

(i) Find the value of a and of k. [2]

(ii) On the same axes, sketch the two curves, for $x>0$. [2]

(b) Given that $y=(2x+\tan x)^2$ and that $\dfrac{dy}{dx}=a\pi+b\sqrt{3}$ when $x=\dfrac{\pi}{3}$, find the value of a and of b. [4]

7 A curve has the equation $y = f(x)$, where $f(x) = \dfrac{3(x-1)}{5x+3}$ for $x > 0$.

(i) Find an expression for $f'(x)$. [2]

(ii) Explain why the curve has no stationary points. [1]

(iii) Show with full workings, determine whether the gradient function of the curve is an increasing or decreasing function for $x > 0$. [2]

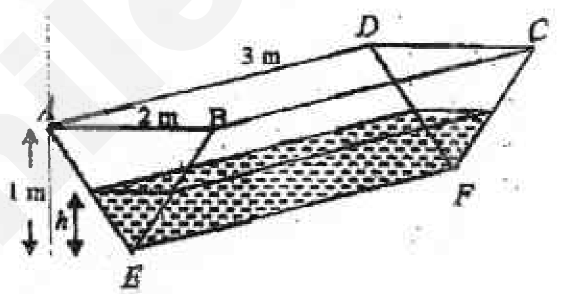
8 The points (1, 10) and (7, 10) are on the circumference of a circle whose centre, C, lies above the x-axis. The line $y = 1$ is tangent to the circle.

(i) Find the coordinates of C. [3]

(ii) Find the equation of the circle in the form $x^2 + y^2 + px + qy + r = 0$, where p, q and r are integers. [2]

(iii) Find the equations of the tangents to the circle parallel to the y-axis. [2]

9

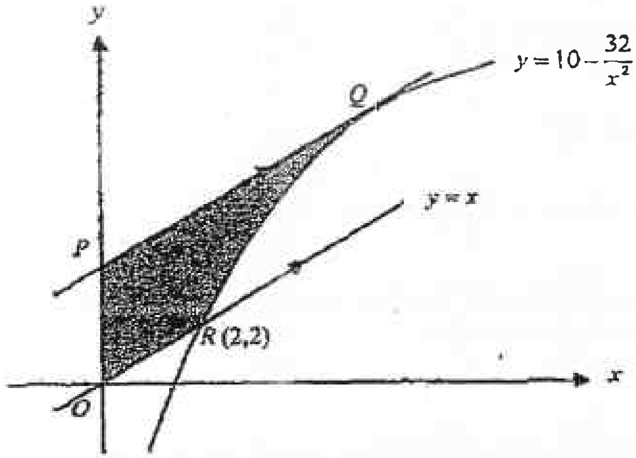


The above diagram shows a trough of 3m long and 1m deep with $ABCD$ horizontal. Its cross section is an isosceles triangle of base 2m with its vertex downwards. The empty trough is filled with water at the rate of 0.03 m^3/s.

(i) If the depth of the water at time t seconds is h m, show that the volume of water is $3h^2$ m^3. [1]

(ii) Hence, find the rate at which the water level is rising after the water has been running for 25s. [4]

10

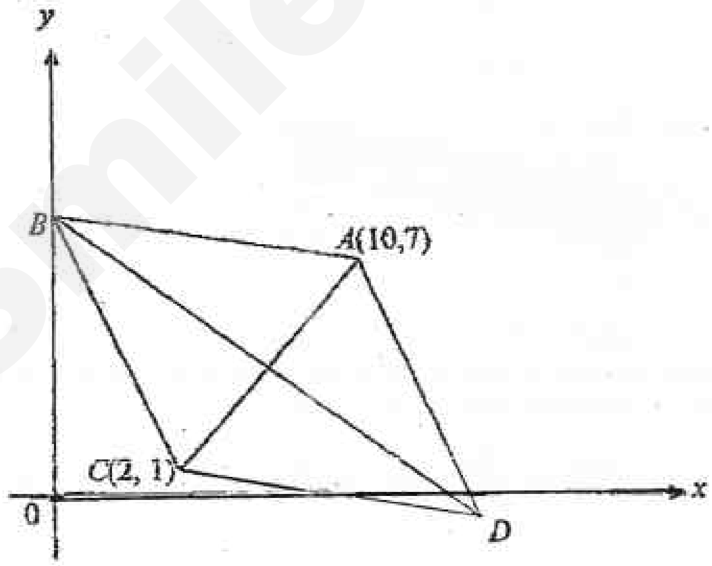


The diagram shows part of the curve and two parallel lines OR and PQ.
The line OR intersects the curve at the point $R(2,2)$ and the line PQ is a tangent to the curve at the point Q.

(i) Find the coordinates of P and of Q. [4]

(ii) Find the area of the shaded region $OPQR$. [6]

11 The diagram shows a rhombus $ABCD$ where $A(10,7)$ and $C(2, 1)$. B is a point on the y-axis and is equidistant from A and C.

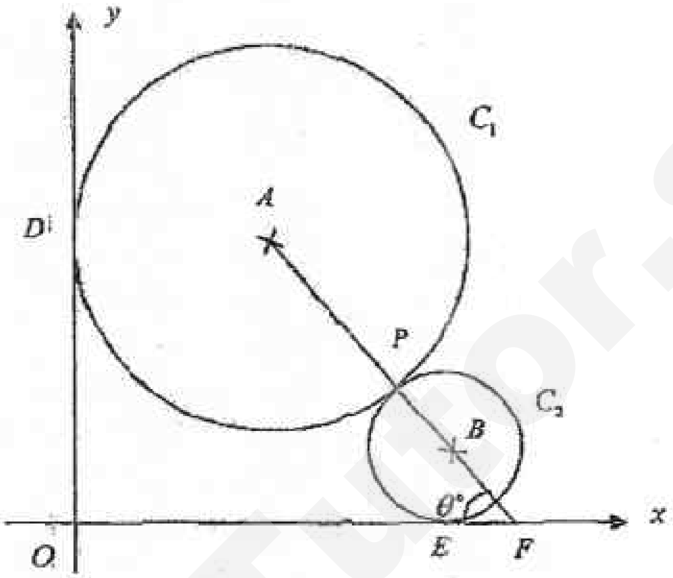


(i) Find the coordinates of B and D. [4]

(ii) Find the area of $ABCD$. [2]

(iii) If point P lies on AC extended such that $PC : PA = 1 : 5$. Find the coordinates of P. [3]

12 In the diagram, two circles C_1 and C_2 whose centres are A and B respectively, touch each other at P. The radii of C_1 and C_2 are 4 units and 1 unit respectively. C_1 touches the y-axis at D and C_2 touches the x-axis at E. The line AB joining the centres of C_1 and C_2 meets the x-axis at F and $\angle BFO = \theta°$.



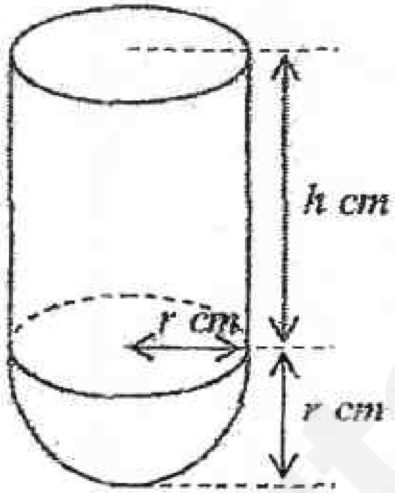
(i) Express OD and OE in terms of θ. [2]

(ii) Hence, or otherwise, show that $DE^2 = 40\cos\theta + 10\sin\theta + 42$. [3]

(iii) Express DE^2 in the form $R\cos(\theta - \alpha) + 42$ where $R > 0$ and α is acute. [3]

(iii) Find the maximum value of DE and the value of θ at which the maximum value occurs. [3]

13 The diagram below shows an open container. It consists of a hemisphere of radius r cm, and a cylinder of radius r cm and height h cm. The hemisphere is fixed to the end of the cylinder and the volume of the container is 800 cm^3.



(i) Show that $h = \dfrac{2400 - 2\pi r^3}{3\pi r^2}$. [2]

The container is made of some thin metal sheets. The cost of metal sheets for the cylindrical surface is \$1.50 per cm^2 and the cost of metal sheets for hemispherical surface is \$3.20 per cm^2.

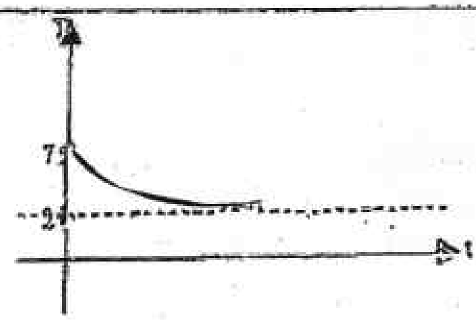
(ii) Let \$$C$ be the cost of making the container. Show that $C = 4.4\pi r^2 + \dfrac{2400}{r}$. [3]

(iii) Find the value of h and r such that the total cost of constructing the container is minimum. [5]

Anglo-Chinese School (Barker) Sec 4 Prelim 2017 Add Math Paper 1

		Answers	
1	(i)	When $k=-20$	
		$y=2x^2-6x-20$	
		$2x^2-6x-20<0$	
		$(x-5)(x+2)<0$	
		$-2<x<5$	
	(ii)	When $k=10$	
		$y=2x^2-6x+10$	
		$y=-2x+8$	
		$2x^2-6x+10=-2x+8$	
		$2x^2-4x+2=0$	
		$x^2-2x+1=0$	
		b^2-4ac	
		$=4-4(1)(1)$	
		$=0$ (shown)	
		The line is tangent to curve	
2	(i)	$(2^x)^2-2^x\times2=3$	
		$u^2-2u-3=0$	
	(ii)	$(u+1)(u-3)=0$	
		$u=3$ or $u=-1$	
		$2^x=3$ or $2^x=-1$(NA)	
		$x=\frac{\lg 3}{\lg 2}$	
		$x=1.58$	
	(iii)	$u^2-2u-k=0$	
		For no solution, $b^2-4ac<0$	
		$(-2)^2-4(1)(-k)<0$	
		$4+4k<0$	
		$1+k<0$	
		$k<-1$	
		The equation has no solution if $k<-1$	

3		$\alpha+\beta=\frac{1}{3} \qquad \alpha\beta=\frac{5}{3}$	
		$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$	
		$=\left(\frac{1}{3}\right)^2-2\left(\frac{5}{3}\right)$	
		$=-\frac{29}{9}$	
		$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$	
		$\alpha^3+\beta^3=\left(\frac{1}{3}\right)\left(-\frac{29}{9}-\frac{5}{3}\right)$	
		$=-\frac{44}{27}$	
	(iii)	$\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\beta^3+\alpha^3}{\alpha^3\beta^3}$	
		$=-\frac{44}{27}\div\left(\frac{5}{3}\right)^3$	
		$=-\frac{44}{125}$	
		$\left(\frac{1}{\alpha^3}\right)\left(\frac{1}{\beta^3}\right)=\frac{27}{125}$	
		$x^2+\frac{44}{125}x+\frac{27}{125}=0$	
		$125x^2+44x+27=0$	
4	(i)	$\frac{1}{\sin^2 x}-\frac{\cos x}{\sin x}+\frac{2\cos^2 x}{\sin^2 x}$	
		$\operatorname{cosec}^2 x-\cot x+2\cot^2 x$	
		$(\cot^2 x+1)-\cot x+2\cot^2 x$	
		$3\cot^2 x-\cot x+1$	
	(ii)	$1+2\cos^2 x=3\sin^2 x+\sin x\cos x$	
		$1+2\cos^2 x-\sin x\cos x=3\sin^2 x$	
		$\frac{1-\sin x\cos x+2\cos^2 x}{\sin^2 x}=3$	
		$3\cot^2 x-\cot x+1=3$	
		$3\cot^2 x-\cot x-2=0$	
		$(3\cot x+2)(\cot x-1)=0$	

		$\cot x = -\frac{2}{3}$ or $\cot x = 1$	
		$\tan x = -\frac{3}{2}$ or $\tan x = 1$	
		$x = 180^0 - 56.3^0, 360^0 - 56.3^0, 45^0, 225^0$	
		$x = 45^0, 123.7^0, 225^0, 303.7^0$ (1 d.p)	
5	(i)	when $t = 0$, $T = 75$	
		$75 = 27 + k$	
		$k = 48$	
	(ii)	when $t = 10$, $T = 55$	
		$55 = 27 + 48e^{-10n}$	
		$28 = 48e^{-10n}$	
		$\frac{7}{12} = e^{-10n}$	
		$-10n = \ln\frac{7}{12}$	
		$n = 0.0539$ (3sf)	
	(iii)	$27 + 48e^{-0.0539t} < 30$	
		$48e^{-0.0539t} < 3$	
		$e^{-0.0539t} < \frac{1}{16}$	
		$-0.0539t < \ln\frac{1}{16}$	
		$t > \frac{\ln\frac{1}{16}}{-0.0539}$	
		$t > 51.439$	
		Least time taken = 52 minutes	
	(iv)		
			
		The temperature of the bread can never reach 27^0C	

6		Let $AD = 2x \Rightarrow BC = x$	
		$\sin 30^\circ = \dfrac{h}{4\sqrt{3}-2\sqrt{2}} \Rightarrow h = 2\sqrt{3}-\sqrt{2}$	
		$\dfrac{1}{2}(x+2x)(2\sqrt{3}-\sqrt{2}) = 25+5\sqrt{6}$	
		$\dfrac{3x}{2} = \dfrac{25+5\sqrt{6}}{2\sqrt{3}-\sqrt{2}} \times \dfrac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}+2\sqrt{2}}$	
		$x = \dfrac{2}{3} \times \dfrac{50\sqrt{3}+25\sqrt{2}+10\sqrt{18}+5\sqrt{12}}{12-2}$	
		$x = \dfrac{2(50\sqrt{3}+25\sqrt{2}+30\sqrt{2}+10\sqrt{3})}{30}$	
		$x = \dfrac{(60\sqrt{3}+55\sqrt{2})}{15}$	
		$AD = \dfrac{22}{3}\sqrt{2}+8\sqrt{3}$	

7		$y = \dfrac{4}{3}x^3 + \dfrac{a}{3}x$	
		$y - x = \dfrac{4}{3}x^3 + \dfrac{a}{3}x - x$	
		$\dfrac{y-x}{x} = \dfrac{4}{3}x^2 + \dfrac{a}{3} - 1$ ---(1)	
		$\dfrac{y-x}{x} = \dfrac{13-t}{6}x^2 + C$ ---(2)	
		$\dfrac{13-t}{6} = \dfrac{4}{3}$	
		$t = 5$	
		Sub $(8,13)$ into (2)	
		$13 = \dfrac{13-t}{6}(8) + C$	
		Sub $t = 5$	
		$13 = \dfrac{13-5}{6}(8) + C$	
		$C = \dfrac{7}{3}$	

		$\frac{a}{3}-1=\frac{7}{3}$	
		$a=10$	
8	(a)	$(a-x)\left(1^n+\binom{n}{1}(1)(3x)+\binom{n}{2}(1)(3x)^2+\ldots\right)$	
		By comparing like terms	
		$a=2$	
		$-x+3anx=29x$	
		$-1+6n=29$	
		$6n=30$	
		$n=5$	
		$a\left(\frac{n(n-1)}{2}\right)(1)(9x^2)-\binom{n}{1}(1)(3x^2)$	
		Sub $a=2,\ n=5$	
		$2\left(\frac{5(4)}{2}\right)(1)(9x^2)-\binom{5}{1}(1)(3x)$	
		$180x^2-15x^2=bx^2$	
		$b=165$	
	(b)(i)	$\binom{10}{r}(x^2)^{10-r}\left(\frac{1}{2x^3}\right)^r$	
	(ii)	x^{20-5r}	
	(iii)	$20-5r=-15$	
		$r=7$	
		$\binom{10}{r}(2^{-r})$	
		$\binom{10}{7}(2^{-7})$	
		$=\frac{15}{16}$	
9		$y=\ln(3x-1)-\ln(5-2x)$	
		$\frac{dy}{dx}=\frac{3}{3x-1}+\frac{2}{5-2x}$	
		$y=-\frac{5}{13}x+2$	
		Gradient of normal $=-\frac{5}{13}$	

		Gradient of tangent = $\frac{13}{5}$	
		$\frac{13}{5}=\frac{3}{3x-1}+\frac{2}{5-2x}$	
		$\frac{13}{5}=\frac{3(5-2x)+2(3x-1)}{17x-5-6x^2}$	
		$\frac{13}{5}=\frac{13}{17x-5-6x^2}$	
		$17x-5-6x^2=5$	
		$6x^2-17x+10=0$	
		$(6x-5)(x-2)=0$	
		$x=\frac{5}{6}$ or $x=2$	
		since $x=a$	
		$a=\frac{5}{6}$ or $a=2$	
		since $a>1$,	
		$a=2$	
		$b=\ln\frac{5}{1}\approx 1.61$	
10	(i)	At $P=(x,1)$	
		$y=\lvert 2x+4\rvert+1$	
		$1=\lvert 2x+4\rvert+1$	
		$\lvert 2x+4\rvert=0$	
		$2x+4=0$	
		$x=-2$	
		$P=(-2,1)$	
		At $Q=(0,y)$	
		$y=\lvert 4\rvert+1$	
		$y=5$	
		$Q=(0,5)$	
	(ii)	$\lvert 2x+4\rvert+1=-x+3$	
		$\lvert 2x+4\rvert=-x+2$	
		$2x+4=-x+2$ or $2x+4=x-2$	
		$x=-\frac{2}{3}$ $x=-6$	

	(iii)	Max $= \dfrac{3-1}{0-(-2)} = 1$	
		Min $= -2$	
		$-2 < m < 1$	
11	(i)	$a = \frac{2}{3}e^{-\frac{t}{3}}$	
		When $t = 0$, $a = \frac{2}{3}$	
	(ii)	$s = \int 28 - 2e^{-\frac{t}{3}} dt$	
		$s = 28t + 6e^{-\frac{t}{3}} + c$	
		$s = 0, t = 0$	
		$c = -6$	
		$s = 28t + 6e^{-\frac{t}{3}} - 6$	
		$t = 10$	
		$s = 28t + 6e^{-\frac{10}{3}} - 6$	
		$s = 274.21$	
	(iii)	No.	
		For $t \geq 0$, $2e^{-\frac{t}{3}} \leq 2$,	
		$v = 28 - 2e^{-\frac{t}{3}} \leq 26$	
		$\therefore v > 0$	
12	(i)	Let $\angle DAC = a$	
		$\angle DBC = a$ (angles in the same segment)	
		$\angle DBA = a$ (BD is angle bisector of $\angle ABC$)	
		$\angle DAE = \angle DBA = a$ (tangent chord theorem)	
		▯ $\angle DAC = \angle DAE$ (proven)	
	(ii)	$\angle DEA = \angle BEA$ (common angle)	
		$\angle DAE = \angle ABE = a$ (tangent chord theorem)	
		ΔADE and ΔBAE are similar (AA property)	
	(iii)	$\angle DCA = \angle DBA = a$ (angles in the same segment)	
		$\angle DCA = \angle CAB = a$	
		$\angle DCA, \angle CAB$ are alternate angles $\therefore CD // BA$	
		$CD // BA$ & D is the midpoint of AF	

		$\therefore C$ is a midpoint of BF (By midpoint theorem)	
	(iv)	ΔADE and ΔBAE are similar	
		$\frac{AE}{BE}=\frac{AD}{AB}=\frac{DE}{AE}$	
		$AD\times CE=BC\times DE$	
		ΔGCD and ΔGBA are similar	
		$\frac{AD}{AB}=\frac{DE}{AE}$	
		$\frac{\frac{1}{2}AF}{2CD}=\frac{DE}{AE}$	
		$AF\times AE=4CD\times DE$ (shown)	

Anglo-Chinese (Barker) Prelim 2017 4E Add Math P2 Answer

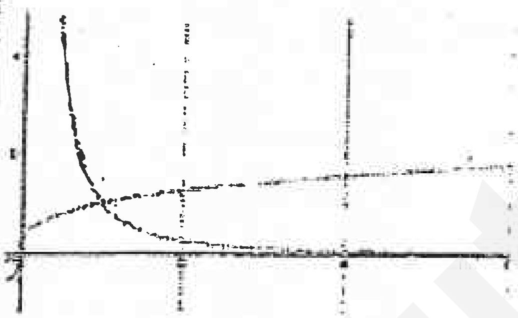
1(a)	$\lg(3y+2)-4x^2=2$	
	$\lg(3y+2)=4x^2+2$	
	$10^{2+4x^2}=3y+2$	
	$3y=10^{2+4x^2}-2$	
	$y=\frac{1}{3}\left(10^{2+4x^2}-2\right)$	
(b)	$2\log_4(8-2x)-\log_2(x-2)=3-\log_2(1+x)$	
	$2\left[\frac{\log_2(8-2x)}{\log_2 2^2}\right]-\log_2(x-2)=\log_2 2^3-\log_2(1+x)$	
	$\log_2(8-2x)-\log_2(x-2)=\log_2\frac{8}{(1+x)}$	
	$\log_2\frac{8-2x}{x-2}=\log_2\frac{8}{(1+x)}$	
	$\frac{8-2x}{x-2}=\frac{8}{(1+x)}$	
	$(8-2x)(1+x)=8(x-2)$	
	$8+8x-2x-2x^2=8x-16$	
	$-2x^2-2x+24=0$	
	$x^2+x-12=0$	
	$(x-3)(x+4)=0$	
	$x=3\quad x=-4$(NA)	
	$\therefore x=3$	

2(i)	$a=-4$	
	$b=10$	
	$c=3$	
(ii)	$h=-4\sin(\frac{\pi}{10}t)+3$	
	$h=0$	
	$-4\sin(\frac{\pi}{10}t)+3=0$	
	$\sin(\frac{\pi}{10}t)=\frac{3}{4}$	
	$\frac{\pi}{10}t=\sin^{-1}\frac{3}{4}$	
	$\frac{\pi}{10}t=0.84806, \pi-0.84806$	
	$t=\frac{10(\pi-0.84806)}{\pi}$	
	$t=7.30$ (3sf)	

3(i)	$\frac{d}{dx}\left(e^{2x}(2x+1)\right)$	
	$=2e^{2x}+(2x+1)(2e^{2x})$	
	$=2e^{2x}(2+4x+2)$	
	$=2e^{2x}(4x+4)$ (shown)	
(ii)	$\int_0^1 4xe^{2x}+4e^{2x}dx=\left[e^{2x}(2x+1)\right]$	
	$2\int_0^1 2xe^{2x}dx=\left[e^{2x}(2x+1)\right]_0^1-\int_0^1 4e^{2x}dx$	
	$\int_0^1 2xe^{2x}dx=\frac{1}{2}\left[e^{2x}(2x+1)\right]_0^1-\frac{1}{2}\int_0^1 4e^{2x}dx$	
	$=\frac{1}{2}\left[e^{2x}(2x+1)\right]_0^1-\frac{1}{2}\left[2e^{2x}\right]_0^1$	
	$=\frac{1}{2}\left[3e^2-1\right]-\frac{1}{2}\left[2e^2-2\right]$	
	$=\frac{1}{2}\left[3e^2-2e^2-1+2\right]$	
	$=\frac{1}{2}\left[e^2+1\right]$	
	$=4.19$	

4(i)	$f(x)=(x+1)(px^2+qx+r)$	
	$p=4$	
	$r=-3$	
	$f(1)=-20$	
	$(2)(p+q+r)=-20$	
	$(2)(4+q-3)=-20$	
	$(2)(q+1)=-20$	
	$q=-11$	
(ii)	$f(x)=(x+1)(4x^2-11x-3)$	
	$f(x)=0$	
	$(x+1)(4x+1)(x-3)=0$	
	$x=-1,-\frac{1}{4},3$	
(iii)	$(-x+1)(-4x+1)(-x-3)=0$	
	$x=1,\frac{1}{4},-3$	

5(i)	By long division, $\frac{x^3-x^2-4x+1}{x^2-4} = x-1-\frac{3}{x^2-4}$	
	$a=1, b=-1, c=-3$	
(ii)	$-\frac{3}{x^2-4} = \frac{A}{x+2} + \frac{B}{x-2}$	
	$-3 = A(x-2) + B(x+2)$	
	Let $x=-2, A=\frac{3}{4}$	
	Let $x=2, B=-\frac{3}{4}$	
	$\int x-1+\frac{3}{4(x+2)}-\frac{3}{4(x-2)}\,dx$	
	$=\frac{1}{2}x^2-x+\frac{3}{4}\ln(x+2)-\frac{3}{4}\ln(x-2)+c$	

6a(i)	sub $x=1$, $y=5$ into $y=a\sqrt[3]{x}$	
	$a=5$ sub $x=1$, $y=5$, $a=5$ into $y=\frac{2a}{kx^2}$	
	$k=2$	
6a(ii)		
6b	$\frac{dy}{dx}=2(2x+\tan x)(2+\sec^2 x)$	
	When $x=\frac{\pi}{3}$	
	$\frac{dy}{dx}=2(2\left(\frac{\pi}{3}\right)+\tan\frac{\pi}{3})(2+\sec^2\frac{\pi}{3})$	
	$\frac{dy}{dx}=2\left(\frac{2\pi}{3}+\sqrt{3}\right)\left(2+\frac{1}{\left(\frac{1}{2}\right)^2}\right)$	
	$\frac{dy}{dx}=\left(\frac{4\pi}{3}+2\sqrt{3}\right)(6)$	
	$\frac{dy}{dx}=8\pi+12\sqrt{3} \Rightarrow \quad a=8, b=12$	

7(i)	$f'(x)=\frac{3(5x+3)-3(x-1)(5)}{(5x+3)^2}$	
	$=\frac{9+15}{(5x+3)^2}$	
(ii)	$f'(x)>0$ since $(5x+3)^2>0, x>0$	
	$f'(x)\neq 0$, the curve has no stationary points	
(ii)	$f''(x)=-48(5x+3)^{-3}(5)$	
	$=\frac{-240}{(5x+3)^3}$	
	For $x>0$, $f''(x)<0$, the gradient function is a decreasing function.	

8(i)		
	Midpoint = $\left(\frac{1+9}{2}, \frac{10+10}{2}\right)$	
	$=(4,10)$	
	Let the centre be $(4, y)$	
	$(4-4)^2+(y-1)^2=(4-1)^2+(y-10)^2$	
	$y^2-2y+1=9+y^2-20y+100$	
	$18y=108$	
	$y=6$	
	Centre $(4,6)$	
(ii)	Radius = 5 units	
	$(x-4)^2+(y-6)^2=25$	
	$x^2+y^2-8x-12y+27=0$	
(iv)	$x=-1$	
	$x=9$	

9(i)	Let the base of the water be x	
	$\frac{x}{2}=\frac{h}{1}$	
	$x=2h$	
	$V=\frac{1}{2}(2h)(h)(3)$	
	$V=3h^2$ (shown)	
(ii)	$\frac{dV}{dt}=0.03$	
	$\frac{dV}{dh}=6h$	
	When $t=25$	
	$V=25\times0.03=0.75m^3$	
	$3h^2=0.75$	
	$h=0.5$	
	$\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}$	
	$=\frac{1}{6h}\times0.03$	
	$=\frac{1}{6(0.5)}\times0.03$	
	$\frac{dh}{dt}=0.01m/s$	

10(i)	$y=10-32x^{-2}$	
	$\frac{dy}{dx}=64x^{-3}$	
	$\frac{64}{x^3}=1$	
	$x^3=64$	
	$x=4$	
	When $x=4, y=8$	
	$Q=(4,8)$	
	$P=(0,y)$	
	$\frac{4-0}{8-y}=1$	
	$4=8-y$	
	$y=4$	
	$\therefore P=(0,4)$	
	Area of trapezium $=\frac{1}{2}(4+8)(4)=24$	
	Area of triangle $=\frac{1}{2}(2)(2)=2$	
	Area of under curve $=\int_2^4 10-32x^{-2}\,dx$	
	$=\left[10x-\frac{32x^{-1}}{-1}\right]_2^4$	
	$=\left[10x+\frac{32}{x}\right]_2^4$	

	$= 48 - 36$	
	$= 12$	
	Shaded area $= 24 - 12 - 2$	
	$= 10$	

11(i)	Since D is equidistant from A and C ∴ DB is perpendicular bisector of AC.	
	Midpoint of AC $=\left(\frac{10+2}{2},\frac{7+1}{2}\right)$	
	$=(6,4)$	
	Gradient of $AC=\frac{1-7}{2-10}$	
	$=\frac{3}{4}$	
	Gradient of $BD=-\frac{4}{3}$	
	Equation of BD, $y-4=-\frac{4}{3}(x-6)$	
	$y=-\frac{4}{3}x+12$	
	At the y-axis, $x=0$	
	$y=12$	
	$\therefore B\,(0,12)$	
	Midpoint of DB = Midpoint of A $=(6,4)$	
	$D\,(0,12)$, $B(x,y)$	
	$\left(\frac{0+x}{2},\frac{12+y}{2}\right)=(6,4)$	
	$\therefore\frac{x}{2}=6$ $\qquad$ $\therefore\frac{12+y}{2}=4$	
	$x=12$ $\qquad$ $y=-4$	
	$\therefore D\,(12,-4)$	

9(i)	Let the base of the water be x	
	$\frac{x}{2}=\frac{h}{1}$	
	$x=2h$	
	$V=\frac{1}{2}(2h)(h)(3)$	
	$V=3h^2$ (shown)	
(ii)	$\frac{dV}{dt}=0.03$	
	$\frac{dV}{dh}=6h$	
	When $t=25$	
	$V=25\times 0.03=0.75\text{m}^3$	
	$3h^2=0.75$	
	$h=0.5$	
	$\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}$	
	$=\frac{1}{6h}\times 0.03$	
	$=\frac{1}{6(0.5)}\times 0.03$	
	$\frac{dh}{dt}=0.01\text{m/s}$	

10(i)	$y = 10 - 32x^{-2}$	
	$\frac{dy}{dx} = 64x^{-3}$	
	$\frac{64}{x^3} = 1$	
	$x^3 = 64$	
	$x = 4$	
	When $x = 4, y = 8$	
	$Q = (4,8)$	
	$P = (0, y)$	
	$\frac{4-0}{8-y} = 1$	
	$4 = 8 - y$	
	$y = 4$	
	$\therefore P = (0,4)$	
	Area of trapezium $= \frac{1}{2}(4+8)(4) = 24$	
	Area of triangle $= \frac{1}{2}(2)(2) = 2$	
	Area of under curve $= \int_2^4 10 - 32x^{-2}\,dx$	
	$= \left[10x - \frac{32x^{-1}}{-1}\right]_2^4$	
	$= \left[10x + \frac{32}{x}\right]_2^4$	

	= 48 − 36	
	= 12	
	Shaded area = 24 − 12 − 2	
	= 10	

11(i)	Since D is equidistant from A and C ∴ DB is perpendicular bisector of AC.	
	Midpoint of AC $=\left(\frac{10+2}{2},\frac{7+1}{2}\right)$	
	$=(6,4)$	
	Gradient of $AC=\frac{1-7}{2-10}$	
	$=\frac{3}{4}$	
	Gradient of $BD=-\frac{4}{3}$	
	Equation of BD, $y-4=-\frac{4}{3}(x-6)$	
	$y=-\frac{4}{3}x+12$	
	At the y-axis, $x=0$	
	$y=12$	
	$\therefore B\,(0,12)$	
	Midpoint of DB = Midpoint of A = (6,4)	
	$D\,(0,12)$, $B(x,y)$	
	$\left(\frac{0+x}{2},\frac{12+y}{2}\right)=(6,4)$	
	$\therefore \frac{x}{2}=6$ $\qquad \therefore \frac{12+y}{2}=4$	
	$x=12$ $\qquad y=-4$	
	$\therefore D\,(12,-4)$	

Register No.	Class

Name : ______________________

DATE : **21 August 2017**
DURATION : **2 Hours**
TOTAL : **80 Marks**

ADDITIONAL MATERIALS
Cover Page (1 Sheet)
Answer Paper (7 sheets)

READ THESE INSTRUCTIONS FIRST
Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a 2B pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** questions.
All the diagrams in this paper are **not** drawn to scale.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

FOR EXAMINER'S USE
80

This document consists of **6** printed pages including this cover page.

[Turn over

MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1.$$
$$\sec^2 A=1+\tan^2 A.$$
$$\operatorname{cosec}^2 A=1+\cot^2 A$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}.$$
$$a^2=b^2+c^2-2bc\cos A.$$
$$\Delta=\frac{1}{2}bc\sin A.$$

Answer **ALL** questions. Omission of essential working will result in the loss of marks.
Write your answers clearly and neatly on foolscap paper.
Begin each question on a fresh page.

1. Solve the equation $\log_3 x^5 - \log_x 3 = 4$. [4]

---

2. (i) Find $\frac{d}{dx}(x^2 \ln 2x)$. [2]

(ii) Hence, find $\int x \ln 2x \, dx$. [3]

---

3. Points $A(36, 4)$, $B(q, -2)$ and $C(1, r)$ lies on the graph of $y = \log_p x^2$.

(i) Determine the value of p, of q and of r. [3]

(ii) Sketch the graph of $y = \log_p x^2$ [2]

---

4. It is given that $y = f(x)$ such that $f(x) = 3e^x - \frac{1}{4}e^{-2x} - \frac{3}{4}$.

(i) Explain why the curve $y = f(x)$ has no stationary point. [2]

(ii) Find the equation of the normal to the curve at the point $x = 0$. [3]

---

5. The term containing the highest power of x in the polynomial $f(x)$ is $2x^3$. Given that the quadratic factor of $f(x)$ is $x^2 - 4x + 2$ and $x = -1$ is a solution to the equation $f(x) = 0$, find

(i) an expression for $f(x)$ in descending power of x, [2]

(ii) the number of real roots of the equation $f(x) = 0$, justifying your answers, [2]

(iii) the remainder when $f(x)$ is divided by $x - 3$. [2]

---

6. The solution to the inequality $-ax^2+bx-1>0$, where a and b are constants is $\frac{1}{4}<x<1$

(i) Find the value of a and of b. [3]

(ii) Using the values of a and b found in part (i), find the set of values of x which the curve, $f(x)=-ax^2+bx-1$, lies completely below the line $y=1-4x$. [3]

---

7. (i) Express $\frac{3x^2+10x+15}{(2+x)^2(3-2x)}$ in partial fractions. [3]

(ii) Hence evaluate $\int_0^1 \frac{3x^2+10x+15}{(2+x)^2(3-2x)}\,dx$ [3]

---

8. Without using a calculator, show that :

(a) $\tan 105° = -\left(2+\sqrt{3}\right)$ [4]

(b) $\sin^2 75° = \frac{1}{4}\left(2+\sqrt{3}\right)$ [3]

---

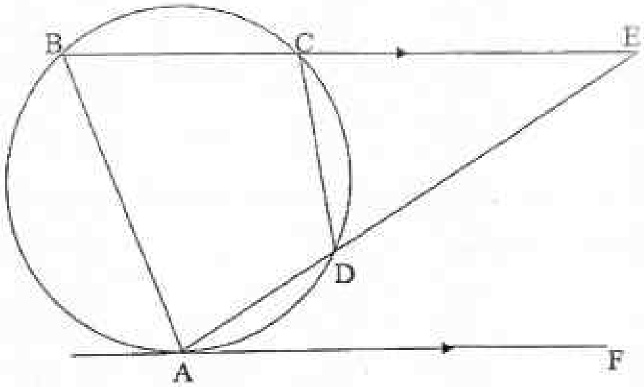
9. The roots of the quadratic equation $2x^2+px+1=0$, where p is a positive constant are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. The roots of the equation $2x^2-qx+10=0$, where q is a positive constant are $\frac{\alpha}{\beta}+2$ and $\frac{\beta}{\alpha}+2$.

(i) Find the value of p and of q. [6]

(ii) Show that the value of $\alpha^3+\beta^3$ is 4. [2]

---

10. In the diagram, AF is a tangent to the circle at A. ADE and BCE are straight lines. AF is parallel to BE and $AB = CE$.



Prove that

(i) $\angle ABD = \angle CED$. [2]

(ii) ΔABD is congruent to ΔCED. [3]

(iii) $\frac{1}{2}\angle ABC = \angle DAF$ [3]

---

11. The equation of a curve is $y = f(x)$, where $f(x) = -\frac{49}{x} - x + 12$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(ii) Find the range of values of x for which $f(x)$ is an increasing function. [2]

(iii) Determine the nature of each of the stationary points of the curve. [2]

(iv) A particle moves alone the curve $y = -\frac{49}{x} - x + 12$. At the point $x = 4$, the y-coordinate is changing at a constant rate of 0.625 units per second. Find the rate of change of the x - coordinate. [2]

---

12. The function $f(x) = a\cos 2x + b$ is defined for $-\pi \le x \le \pi$, where a and b are positive constants.

(i) Given that the greatest and the least value of $f(x)$ are 8 and -2 respectively, find the value of a and of b. [2]

(ii) State the range of values between which the principal value of x must lie and find the principal value of x for which $f(x) = 0$. [3]

(iii) Sketch the graph of $y = a\cos 2x + b$ for $-\pi \le x \le \pi$. [3]

(iv) Hence, state the number of solutions to the equation $\frac{8}{\pi}x = a\cos 2x + b$. [2]

End of Paper

Register No.	Class

Name : ____________________

DATE : **25 August 2017**
DURATION : **2 Hours 30 Min**
TOTAL : **100 Marks**

ADDITIONAL MATERIALS
Cover Page (1 Sheet)
Graph Paper (1 Sheet)
Answer Paper (7 sheets)

READ THESE INSTRUCTIONS FIRST
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FOR EXAMINER'S USE
100

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[Turn over

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$$\Delta=\frac{1}{2}bc\sin A.$$

Answer **ALL** questions. Omission of essential working will result in the loss of marks.
Write your answers clearly and neatly on foolscap paper.
Begin each question on a fresh page.

1. A point P lies on the curve $y = x^2 + 4x - 8$. The normal to curve is parallel to the line $2y - \frac{x}{3} = 1$. Find the coordinates of P. [3]

---

2. (a) Given that $2^{x-1} \times 3^{x+2} = 8^{x-1} \times 3^{2x}$, evaluate 12^x. [3]

(b) Solve the equation $e^y(5 - e^y) + 14 = 0$. [3]

---

3. Given that $\int_0^6 f(x)\,dx = 5$ and $\int_2^6 f(x)\,dx = 2$, find

(i) $\int_6^2 f(x)\,dx$. [1]

(ii) $\int_0^2 f(x)\,dx$. [2]

(iii) the value of k for which $\int_0^2 f(x) - kx\,dx = 15$. [3]

---

4. (a) Show that the binomial expansion $\left(x - \frac{1}{2x^3}\right)^{15}$ does not have an independent term. [3]

(b) In the binomial expansion of $(1+kx)^n$, where $n \geq 3$ and k is a constant, the coefficient of x^3 and x^4 are equal . Express k in terms of n . [4]

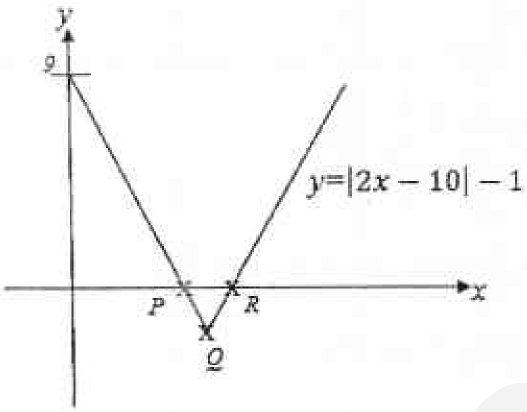
---

5. (i) Prove that $\cos 3A = 4\cos^3 A - 3\cos A$. [4]

(ii) Hence , find in terms π, the solution to the equation $1 = 8\cos^3 A - 6\cos A$ for $0 < \theta < \pi$. [3]

---

6. The diagram shows part of the graph of $y=|2x-10|-1$.



(a) Find the coordinates of P, Q and R. [4]

(b) In the case when $mx+c=|2x-10|-1$, find

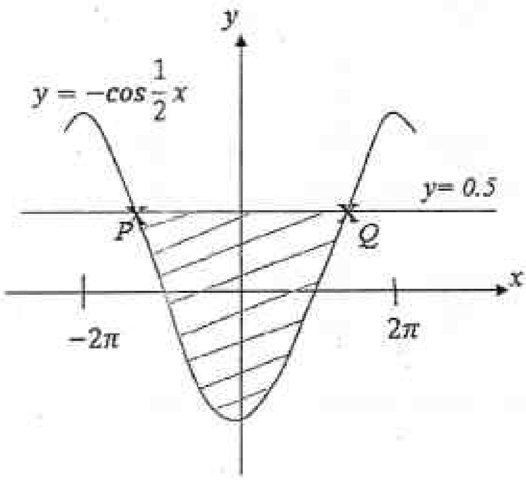
(i) the range of values of c when $m=-2$ where there is only **1** solution. [1]

(ii) the range of values of m when $c=-1$ where there are **2** solutions. [2]

7. The diagram shows part of the graph $y=-\cos\frac{1}{2}x$ for $-2\pi \le x \le 2\pi$. The line $y=\frac{1}{2}$ intersects the curve at P and at Q.

(i) Find the coordinates of P and of Q. [2]

(ii) Find the area bounded by the curve and the line $y=\frac{1}{2}$. [5]



8. The value, $\$V$, of a house is related to t, the number of years after it was built in year 2008. The variables are related by the formula $V = ae^{kt}$, where a and k are constants. The table below gives the value of the house in year 2010, 2012, 2014 and 2016.

Year	2010	2012	2014	2016
t	2	4	6	8
V($)	517 600	595 400	684 800	787 800

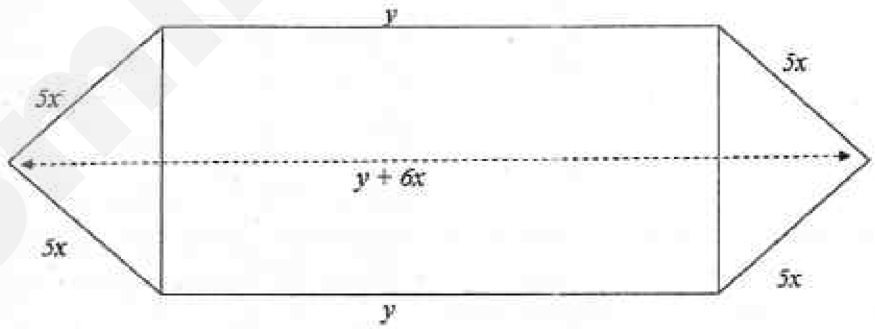
(i) On the graph paper, plot $\ln V$ against t and draw a suitable straight line. The vertical axis should start from 13.0 and have a scale of 2 cm to 0.1 [2]

(ii) Use the graph from part (i) to estimate the value of a and of k. [4]

(iii) Estimate the value of the house in 2015. [2]

---

9. An area is fenced up to enclose a landscape. The shape of the landscape is shown below. It is made up of a rectangle of length y cm and two isosceles triangles of sides $5x$ cm. The perimeter of the landscape is 420 cm and the length from one end to the other end is $y + 6x$ cm.



(i) Show that the area of the landscape, A cm^2, is given by $1680x - 56x^2$. [4]

(ii) Given that x can vary, find the stationary value of A. [2]

(iii) Determine whether this stationary value is a maximum or minimum. [2]

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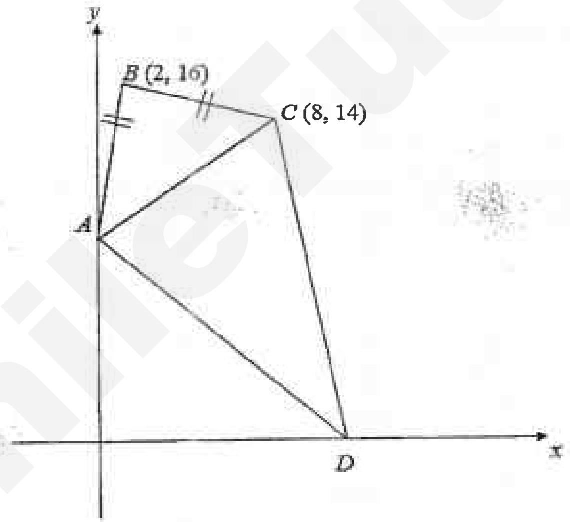
10. A particle travels in a straight line from a fixed point O where the distance S in meters is given by $S = \frac{4}{3}t^3 + kt^2 + qt$ where t is the time in seconds after passing O. k and q are constants. The velocity of the particle is 20 m/s when it passes O and at $t = 3s$, its acceleration is 0 m/s^2.

(i) Find the value of k and of q. [4]

(ii) Find the value(s) of t when the particle is instantaneously at rest. [2]

(iii) Find the total distance travelled during the first 8 seconds. [3]

11. **[Solution to this question by accurate drawing will not be accepted]**



The diagram which is not drawn to scale shows a quadrilateral $ABCD$. The point B is (2, 16) and the point C is (8, 14). Triangle ABC is an isosceles triangle and point A and point D lies the y – axis and x – axis respectively.

(i) Find the coordinates of A. [3]

(ii) Given that the ratio of area of ΔABC : area of ΔACD is 1 : 3 , find the coordinates of D. [4]

(iii) Show that $ABCD$ is a kite. [3]

12. The line $x = 17$ is a tangent to a circle and the points A (1, 9) and B (1, – 7) are on the circumference of the circle.

(i) Show that the radius of the circle is 10 units. [4]

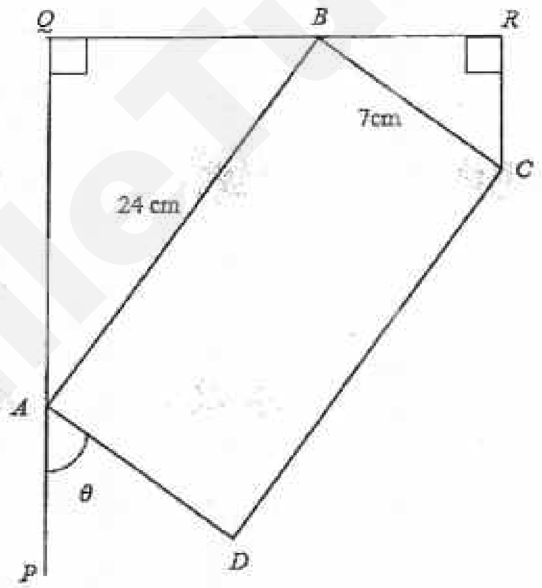
(ii) State the coordinates of the centre of the circle. [1]

(iii) Write down the equation of the circle in the form $x^2 + y^2 + px + qy + r = 0$ [2]

(iv) The circle is reflected along the line $y = -1$, show that the point (3, 10) does not lie on the reflected circle. [3]

---

13. In the diagram below, $ABCD$ is a rectangle. The line QR is perpendicular to the lines PQ and CR. Points A and B lie on the lines PQ and QR respectively and angle $PAD = \theta$. AB is 24 cm and BC is 7 cm.



(i) Show that the length of QR is $24\cos\theta + 7\sin\theta$. [4]

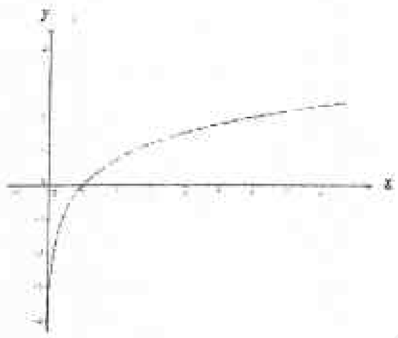
(ii) Express $24\cos\theta + 7\sin\theta$ in the form of $R\cos(\theta - \alpha)$ where $R > 0$ and α is acute. [3]

(iii) Find the value of θ when QR is 17 cm. [2]

(iv) Find the maximum length of QR and state the corresponding value of θ. [3]

End of Paper

ANSWER KEY

1	$x=0.803$ or $x=3$	2	(i) $\frac{d}{dx}(x^2\ln 2x)=x+2x\ln 2x$ (ii) $\int x\ln 2x\,dx=\frac{2x^2\ln 2x-x^2}{4}$
3	(i) $p=6$ $q=\frac{1}{6}$ $r=0$ (ii) 	4	(ii) $y=-\frac{2}{7}x+2$
5	(i) $f(x)=2x^3-6x^2-4x+4$ (ii) 3 real roots (iii) -8	6	(i) $a=4, b=5$ (ii) $x<\frac{1}{4}$ or $x>2$
7	(i) $\frac{3x^2+10x+15}{(2+x)^2(3-2x)}=\frac{1}{(2+x)^2}+\frac{3}{3-2x}$ (ii) 1.81	8	
9	$p=2$, $q=8$	10	
11	(i) $\frac{dy}{dx}=\frac{49}{x^2}-1$ $\frac{d^2y}{dx^2}=-\frac{98}{x^3}$ (ii) $-7<x<7$ (iii) $x=7$ is a maximum point $x=-7$ is a minimum point (iv) $\frac{dx}{dt}=\frac{10}{33}$	12	$a=5$ $b=3$ $x=1.11$ 3 solutions

Solution

1. Solve the equation $\log_3 x^5 - \log_x 3 = 4$. [4]

$$5\log_3 x - \frac{\log_3 3}{\log_3 x} = 4$$ -M1

Let $y = \log_3 x$

$$5y - \frac{1}{y} = 4$$

$$5y^2 - 4y - 1 = 0$$ -M1

$$(5y+1)(y-1) = 0$$

$$\log_3 x = -\frac{1}{5} \quad \text{or} \quad \log_3 x = 1$$ - M1

$$x = 3^{-\frac{1}{5}} \quad \text{or} \quad x = 3$$

$$x = 0.803$$ -A1

---

2. (i) Find $\frac{d}{dx}(x^2 \ln 2x)$. [2]

(ii) Hence, find $\int x \ln 2x \, dx$. [3]

(i) $$\frac{d}{dx} x^2 \ln 2x = x^2 \frac{1}{x} + 2x \ln 2x$$ - M1

$$= x + 2x\ln 2x$$ -A1

(ii) $$\int x + 2x\ln 2x \, dx = x^2 \ln 2x$$

$$\int 2x \ln 2x \, dx = x^2 \ln 2x - \int x dx$$ -M1

$$\int 2x \ln 2x \, dx = x^2 \ln 2x - \frac{x^2}{2}$$ - M1

$$\int x \ln 2x \, dx = \frac{x^2 \ln 2x}{2} - \frac{x^2}{4} \quad \text{or} \quad \int x \ln 2x \, dx = \frac{2x^2 \ln 2x - x^2}{4}$$ - A1

3 Points $A(36, 4)$, $B(q, -2)$ and $C(1, r)$ lies on the graph of $y = \log_p x^2$.

(i) Determine the value of p, of q and of r. [3]

(ii) Sketch the graph of $y = \log_p x^2$ [2]

$4 = \log_p 36^2$

$2 = \log_p 36$

$p^2 = 36$

$p = 6$ - B1

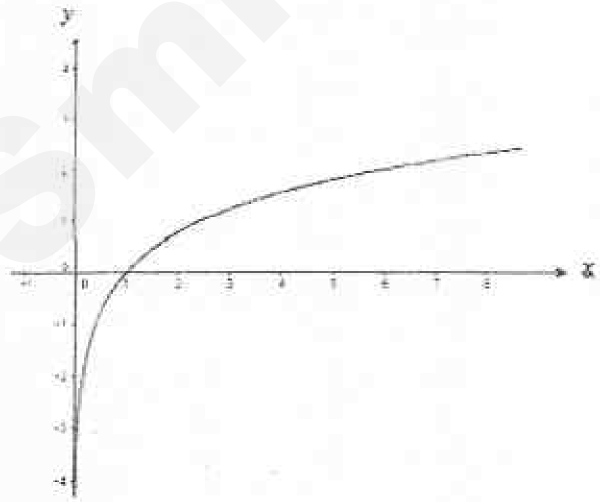
$6^{\frac{-2}{2}} = q$

$q = \frac{1}{6}$ - B1

$6^{\frac{r}{2}} = 1$

$\frac{r}{2} = 0$

$r = 0$ -B1



Shape – M1

Showing that it cuts at (1,0) – A1

4. It is given that $y = f(x)$ such that $f(x) = 3e^x - \frac{1}{4}e^{-2x} - \frac{3}{4}$.

(i) Explain why the curve $y = f(x)$ has no stationary point. [2]

(ii) Find the equation of the normal to the curve at the point $x = 0$. [3]

(i) $f'(x) = 3e^x + \frac{1}{2}e^{-2x}$

If $3e^x + \frac{1}{2}e^{-2x} = 0$ - M1

$3e^x = -\frac{1}{2}e^{-2x}$

$e^{3x} = -\frac{1}{6}$ (N.A)

Since $f'(x) = 0$ has no solution, $f(x)$ has no stationary point - A1

(ii) When $x = 0$, $y = 3 - \frac{1}{4} - \frac{3}{4}$

$= 2$

$f'(x) = 3 + \frac{1}{2}$

$= \frac{7}{2}$ -M1

$y - 2 = -\frac{2}{7}(x - 0)$ -M1

$y = -\frac{2}{7}x + 2$ - A1

5. The term containing the highest power of x in the polynomial $f(x)$ is $2x^3$. Given that the quadratic factor of $f(x)$ is x^2-4x+2 and that $x=-1$ is a solution to the equation $f(x)=0$, find

(i) an expression for $f(x)$ in descending power of x, [2]

(ii) the number of real roots of the equation $f(x)=0$, justifying your answers, [2]

(iii) the remainder when $f(x)$ is divided by $x-3$. [2]

(i) $2(x+1)(x^2-4x+2)=2x^3-8x^2+4x+2x^2-8x+4$ - M1

$f(x)=2x^3-6x^2-4x+4$ - A1

(ii) For x^2-4x+2, $(-4)^2-4(1)(2)=8$

>0 - M1

Therefore $f(x)$ has 3 real roots - A1

(iii) $f(3)=2x^3-6x^2-4x+4$ -M1

$=-8$ - A1

---

6. The solution to the inequality $-ax^2+bx-1>0$, where a and b are constants is

$\frac{1}{4}<x<1$

(i) Find the value of a and of b. [3]

(ii) Using the values of a and b found in part (i), find the set of values of x which the curve, $f(x)=-ax^2+bx-1$, lies completely below the line $y=1-4x$. [3]

(i) $(x-\frac{1}{4})(x-1)=0$ - M1

$x^2-x-\frac{1}{4}x+\frac{1}{4}=0$

$x^2-\frac{5}{4}x+\frac{1}{4}=0$ - M1

$-4x^2+5x-1=0$

$a=4, b=5$ - A1

(ii) $-4x^2+5x-1<1-4x$ - M1

$t-1$

$(4x-1)(x-2)>0$ - M1

$x<\frac{1}{4}$ or $x>2$ - A1

---

7. (i) Express $\frac{3x^2+10x+15}{(2+x)^2(3-2x)}$ in partial fractions. [3]

(ii) Hence evaluate $\int_0^1 \frac{3x^2+10x+15}{(2+x)^2(3-2x)}\,dx$ [3]

$$\frac{3x^2+10x+15}{(2+x)^2(3-2x)}=\frac{A}{(2+x)^2}+\frac{B}{2+x}+\frac{C}{3-2x}$$

$3x^2+10x+15=A(3-2x)+B(2+x)(3-2x)+C(2+x)^2$ - M1

Let $x=-2$

$7=7A$

$A=1$ - M1

$3=-2B+C$

$C=3-2B$

Compare coeff x

$10=-2A-B+4C$

$10=-2-B+4C$

Subst $C=3-2B$ into $10=-2-B+4C$

$10=-2-B+12-8B$

$B=0$ - M1

$C=3$ - M1

$$\frac{3x^2+10x+15}{(2+x)^2(3-2x)}=\frac{1}{(2+x)^2}+\frac{3}{3-2x}$$ - A1

(ii) $\int_0^1 \frac{3x^2+10x+15}{(2+x)^2(3-2x)}\,dx$

$$\int_0^1 \frac{3x^2+10x+15}{(2+x)^2(3-2x)}\,\mathrm{dx}=\int_0^1 \frac{1}{(2+x)^2}+\frac{3}{3-2x}\,dx$$

$$=\left[\frac{-1}{(2+x)}\right]_0^1-\frac{3}{2}\left[\ln(3-2x)\right]_0^1$$

$$=\frac{1}{6}-\left(\frac{3}{2}(\ln 1-\ln 3)\right)$$

$$=1.81$$

8. Without using a calculator, show that : [3]

(a) $\tan 105° = -\left(2+\sqrt{3}\right)$

[3]

(b) $\sin^2 75° = \frac{1}{4}(2+\sqrt{3})$

$$\tan 105° = \frac{\tan 60° + \tan 45°}{1-\tan 60° \tan 45°}$$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}} \quad \text{-M1}$$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} \quad \text{-M1}$$

$$= \frac{\left(\sqrt{3}+1\right)^2}{1-3}$$

$$= \frac{3+2\sqrt{3}+1}{1-\sqrt{3}}$$

$$= -[2+\sqrt{3}] \quad \text{-A1}$$

$$\text{(ii)} \sin^2 75° = \frac{1-\cos 2(75°)}{2}$$

$$= \frac{1-\cos 150°}{2}$$

$$= \frac{1-(-\frac{\sqrt{3}}{2})}{2} \quad \text{- M1}$$

$$= \frac{2+\sqrt{3}}{2} \times \frac{1}{2} \quad \text{-M1}$$

$$= \frac{1}{4}[2+\sqrt{3}] \quad \text{- A1}$$

---

9. The roots of the quadratic equation $2x^2 + px + 1 = 0$, where p is a positive

const 1 1 pts of the equation $2x^2 - qx + 10 = 0$, where

q is a positive constant are $\frac{\alpha}{\beta}+2$ and $\frac{\beta}{\alpha}+2$.

[6]

(i) Find the value of p and of q.

(iii) Show that the value of $\alpha^3+\beta^3$ is 4. [2]

$\frac{1}{\alpha}+\frac{1}{\beta}=-\frac{p}{2}$

$\frac{\beta+\alpha}{\alpha\beta}=-\frac{p}{2}$

$\beta+\alpha=-p$ - M1

$\frac{1}{\alpha\beta}=\frac{1}{2}$

$\alpha\beta=2$ - M1

$\frac{\alpha}{\beta}+2+\frac{\beta}{\alpha}+2=\frac{q}{2}$ - eqn 1

$\left(\frac{\alpha}{\beta}+2\right)\left(\frac{\beta}{\alpha}+2\right)=\frac{10}{2}$

$\frac{\alpha\beta}{\beta\alpha}+2\frac{\alpha}{\beta}+2\frac{\beta}{\alpha}+4=5$

$1+2\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)+4=5$

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=0$ - eqn 2 -M1

Subst eqn 1 into eqn 2

$4=\frac{q}{2}$

$q=8$ - A1

Fm eqn 2 : $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=0$

$\frac{\alpha^2+\beta^2}{\alpha\beta}=0$

$(\alpha+\beta)^2-2\alpha\beta=0$ -M1

$(\alpha+\beta)^2=2(2)$

$(\alpha+\beta)=2$ (NA)or -2 (p is positive)

$p=2$ - A1

$\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$

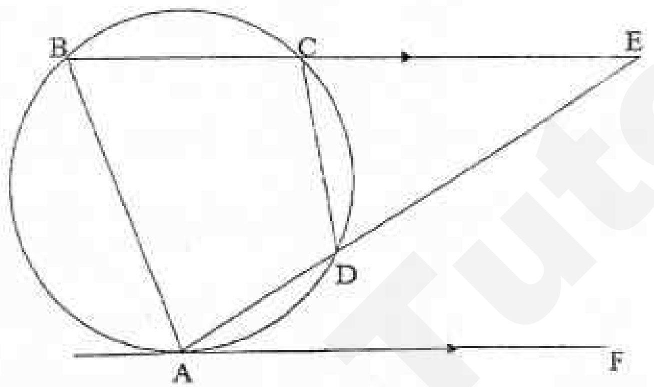
$\alpha^3 + \beta^3 = (\alpha+\beta)((\alpha+\beta)^2 - 2\alpha\beta - \alpha\beta)$ - M1

$= (-2)[(-2)^2 - 3(2)]$

$= 4$ - A1

---

10. In the diagram, AF is a tangent to the circle at A. ADE and BCE are straight lines. AF is parallel to BE and $AB = CE$.



Prove that

(i) $\angle ABD = \angle CED$. [2]

(ii) ΔABD is congruent to ΔCED. [3]

(iii) $\frac{1}{2}\angle ABC = \angle DAF$ [3]

(i) $\angle ABD = \angle DAF$ (alt seg. Thm) - M1
$\angle CED = \angle DAF$ (alt $\angle$, BE // AF)
$\angle ABD = \angle CED$ (Shown) - A1

(ii) Let $\angle BAD = a$
$\angle BCD = 180° - a$ ($\angle$ in opp segment)
$\angle ECD = a$ (adj $\angle$ on a st line)
$\angle BAD = \angle ECD$ - M1
$AB = CE$ (given)
$\angle ABD = \angle CED$ (part (i) - M1
$\Delta ABD \equiv \Delta CED$ (ASA test) - A1

(i ΔCED)
ase $\angle$ of isos Δ) - M1

$\angle CED = \angle DBE$ ($\angle BED = \angle CED$)
$\angle ABC = \angle ABD + \angle DBE$ - M1
$\angle ABC = \angle CED + \angle CED$ ($\angle ABD = \angle CED$ fm part (i))

$\frac{1}{2}\angle ABC = \angle CED$ - A1

---

11. The equation of a curve is $y = f(x)$, where $f(x) = -\frac{49}{x} - x + 12$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(ii) Find the range of values of x for which $f(x)$ is an increasing function. [2]

(iii) Determine the nature of each of the stationary points of the curve. [2]

(iv) A particle moves alone the curve $y = -\frac{49}{x} - x + 12$. At the point $x = 4$, y-coordinate is changing at a constant rate of 0.625 units per second. Find the rate of change of the x-coordinate. [2]

(i) $y = -49x^{-1} - x + 12$

$\frac{dy}{dx} = \frac{49}{x^2} - 1$

$\frac{d^2y}{dx^2} = (-2)49x^{-3}$

$= \frac{-98}{x^3}$

(ii) $\frac{49}{x^2} - 1 > 0$

$(7 + x)(7 - x) > 0$

The solution is $-7 < x < 7$

(iii) Stationary points are $x = -7$ or $x = 7$.

At $x = -7$

$\frac{d^2y}{dx^2} = -\frac{98}{(-3)^3} > 0$

um point

At $x = 7$

$$\frac{d^2y}{dx^2} = -\frac{98}{(7)^3} < 0$$

$\therefore x = 7$ is a maximum point

(iv) $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\frac{49}{4^2} - 1 = 0.625 \times \frac{dt}{dx}$$

$$\frac{33}{16} = 0.625 \times \frac{dt}{dx}$$

$$\frac{dx}{dt} = \frac{10}{33}$$

---

12. The function $f(x) = a\cos 2x + b$ is defined for $-\pi \le x \le \pi$, where a and b are positive constants.

(i) Given that the greatest and the least value of $f(x)$ are 8 and -2 respectively, find the value of a and of b. [2]

(ii) State the range of values between which the principal value of x must lie and find the principal value of x for which $f(x) = 0$. [3]

(iii) Sketch the graph of $y = a\cos 2x + b$. [3]

(v) Hence, state the number of solutions to the equation $\frac{8}{\pi}x = a\cos 2x + b$. [2]

(i) $a = 5$ - B1

$b = 3$ - B1

(c) Range of values $= 0 \le x \le \frac{\pi}{2}$ - B1

$\cos 2x = -\frac{3}{5}$ - M1

$2x = 2.21$

$x = 1.11$ - A1

ANSWER KEY

1	P = (− 5, − 3)	**2**	$12^x = 36$ $y = 1.95$
3	$\int_6^2 f(x)\,dx = -2$ $\int_0^2 f(x)\,dx = 3$ $k = -6$	**4**	$k = \frac{4}{n-3}$
5	$A = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$	**6**	$R = \left(5\frac{1}{2}, 0\right), Q = (5, -1), P = \left(4\frac{1}{2}, 0\right)$ $c > 9$ $0 < m < 2$
7	$P = (-\frac{4}{3}\pi, \frac{1}{2})$, $Q = (\frac{4}{3}\pi, \frac{1}{2})$ 7.65 units^2	**8**	
9	$A = 12600 cm^2$ A is a maximum value	**10**	(i) $k = -12$, $q = 20$ (ii) $t = 1$ *or* 5 (iii) 160 m
11	$A(0,10)$ $D(10,0)$	**12**	Centre of circle = $(7,1)$
13	$24\cos\theta + 7\sin\theta = 25\cos(\theta - 16.3°)$ $\theta = 63.5°$ Max length $QR = 25$ $\theta = 16.3°$		

Solution

1. A point P lies on the curve $y = x^2 + 4x - 8$. The normal to curve is parallel to the line $2y - \frac{x}{3} = 1$. Find the coordinates of P. [3]

$\frac{dy}{dx} = 2x + 4$

$\frac{-1}{2x+4} = \frac{1}{6}$ - M1

$-6 = 2x + 4$

$x = -5$ - M1

$y = -3$

$P = (-5, -3)$ - A1

2. (a) Given that $2^{x-1} \times 3^{x+2} = 8^{x-1} \times 3^{2x}$, evaluate 12^x. [3]

(b) Solve the equation $e^y(5 - e^y) + 14 = 0$. [3]

(a) $2^x \times \frac{1}{2} \times 3^x \times 9 = 2^{3x} \times \frac{1}{8} \times 3^{2x}$ - M1

$\frac{1}{2} \times 9 \times 8 = \frac{2^{3x} \times 3^x}{2^x \times 3^x}$ - M1

$36 = (2^2)^x \times 3^x$

$12^x = 36$ - A1

(b) $-e^{2y} + 5e^y + 14 = 0$ - M1

$e^{2y} - 5e^y - 14 = 0$

$(e^y - 7)(e^y + 2) = 0$ - M1

$e^y = 7$ or $e^y = -2$ (N.A)

$y = \ln 7$

$y = 1.95$ - A1

3. Given that $\int_0^6 f(x)\,dx = 5$ and $\int_2^6 f(x)\,dx = 2$, find

(i) $\int_6^2 f(x)\,dx$. [1]

(ii) $\int_0^2 f(x)\,dx$. [2]

(iii) the value of k for which $\int_0^2 f(x) - kx\,dx = 15$. [3]

(i) $\int_6^2 f(x)\,dx = -2$ - B1

(ii) $\int_0^2 f(x)\,dx = \int_0^6 f(x)dx - \int_2^6 f(x)dx$ - M1

$= 5 - 2$

$= 3$ - A1

(iii) $\int_0^2 f(x) - kx\,dx = 15$

$\int_0^2 f(x)\,\mathrm{dx} - \int_0^2 kx\,dx = 15$ - M1

$3 - 15 = k\left[\frac{x^2}{2}\right]_0^2$ - M1

$3 - 15 = k[2 - 0]$

$k = -6$ - A1

4.

(a) Show that the binomial expansion $\left(x - \frac{1}{2x^3}\right)^{15}$ does not have an independent term. [3]

(b) In the binomial expansion of $(1+kx)^n$, where $n \geq 3$ and k is a constant, the coefficient of x^3 and x^4 are equal. Express k in terms of n. [4]

(a) $T_{r+1} = 15C_r(x)^{15-r}\left(-\frac{1}{2x^3}\right)^r$

$= 15C_r(-\frac{1}{2})^r(x)^{15-r-3r}$ - M1

$15 - 4r = 0$

r - M1

Since r $\neq$ integer, the binomial expansion does not have an independent term. –A1

(b) Coeff of $x^3 = \dfrac{n(n-1)(n-2)}{6}k^3$ - M1

Coeff of $x^4 = \dfrac{n(n-1)(n-2)(n-3)}{24}k^4$ -M1

$\dfrac{n(n-1)(n-2)}{6}k^3 = \dfrac{n(n-1)(n-2)(n-3)}{24}k^4$ - M1

$4=(n-3)k$

$k=\dfrac{4}{n-3}$ - A1

---

5. (i) Prove that $\cos 3A = 4\cos^3 A - 3\cos A$. [4]

(ii) Hence , find in terms π, the solution to the equation $1 = 8\cos^3 A - 6\cos A$ for $0 < \theta < \pi$. [3]

(i) $\cos 3A = \dfrac{5}{2}\cos^3 A - \dfrac{3}{2}\cos A$

$\cos 3A = \cos(2A + A)$

$= \cos 2A \cos A - \sin 2A \sin A$ - M1

$= (2\cos^2 A - 1)\cos A - (2\sin A \cos A)\sin A$ -M1

$= 2\cos^3 A - \cos A - 2\sin^2 A \cos A$

$= 2\cos^3 A - \cos A - 2(1-\cos^2 A)\cos A$ - M1

$= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A$

$= 4\cos^3 A - 3\cos A$ - A1

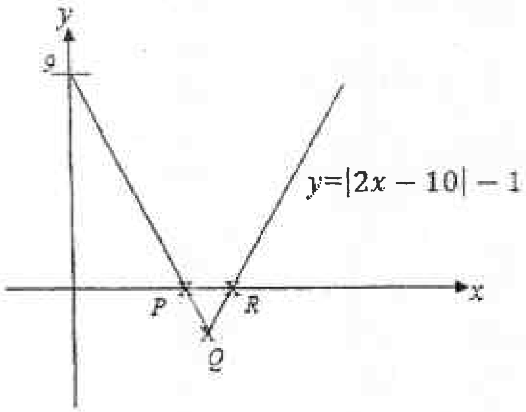
(ii) $1 = 8\cos^3 A - 6\cos A$

$\dfrac{1}{2} = \cos 3A$ - M1

$3A = \dfrac{\pi}{3}, \dfrac{5\pi}{3}, \dfrac{7\pi}{3}$ -M1

$A = \dfrac{\pi}{9}, \dfrac{5\pi}{9}, \dfrac{7\pi}{9}$ -A1

6. The diagram shows part of the graph of $y = |2x-10|-1$.



(a) Find the coordinates of P , Q and R . [4]

(b) In the case when $mx+c=|2x-10|-1$, find

(i) the range of values of c when $m=-2$ where there is only **1** solution. [1]

(ii) the range of values of m when $c=-1$ where there are **2** solutions. [2]

(a) $|2x-10|-1=0$

$2x-10=1$ or $2x-10=-1$

$x=5\frac{1}{2}$ or $x=4\frac{1}{2}$

$P=\left(4\frac{1}{2},0\right)$ or $R=\left(5\frac{1}{2},0\right)$ - B2

x - coordinates of Q = $\frac{4.5+5.5}{2}=5$ - M1

y- coordinates of Q = -1

$Q=(5,-1)$ - A1

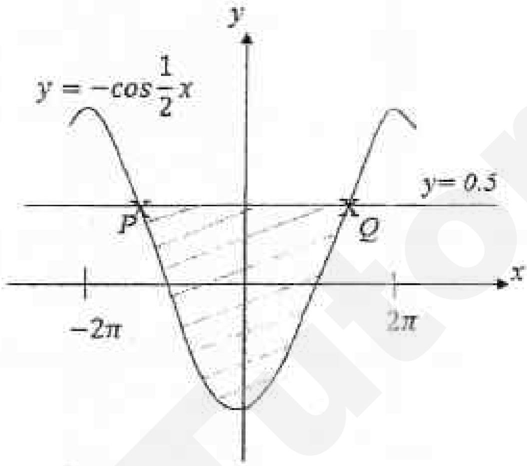
b(i) c > 9 - A1

(iii) $0<m<2$ - A2

7. The diagram shows part of the graph $y=-\cos\frac{1}{2}x$ for $-2\pi \le x \le 2\pi$. The line $y=\frac{1}{2}$ intersects the curve at P and at Q.

(i) Find the coordinates of P and of Q. [2]

(ii) Find the area bounded by the curve and the line $y=\frac{1}{2}$. [5]



(i) $-\cos\frac{1}{2}x=\frac{1}{2}$

$\cos\frac{1}{2}x=-\frac{1}{2}$

$\frac{1}{2}x=\frac{2}{3}\pi,-\frac{2}{3}\pi$

$x=\frac{4}{3}\pi,-\frac{4}{3}\pi$ -M1

$P=(-\frac{4}{3}\pi,\frac{1}{2})$, $Q=(\frac{4}{3}\pi,\frac{1}{2})$ - A2

(ii) $\int_{-\frac{4}{3}\pi}^{\frac{4}{3}\pi}\left|-\cos\frac{1}{2}x-\frac{1}{2}\right|dx$. - M1

$=\left|\left[-2\sin\frac{1}{2}x-\frac{1}{2}x\right]_{-\frac{4}{3}\pi}^{\frac{4}{3}\pi}\right|$ - M1

$=\left|\left[2\sin\frac{2}{3}\pi-\frac{2}{3}\pi\right]-\left[-2\sin-\frac{2}{3}\pi+\frac{2}{3}\pi\right]\right|$ -M1

$=\left|\left[\ 2\sqrt{3}\quad 2\ \right]\ \left[2\sqrt{3}\quad \frac{2}{3}\pi\right]\right|$ - M1

$= 7.65 \text{ units}^2$ – A1

8. The value , \$$V$, of a house is related to t, the number of years after it was built in year 2008.The variables are related by the formula $V = ae^{kt}$, where a and k are constants. The table below gives the value of the house in year 2010, 2012, 2014 and 2016.

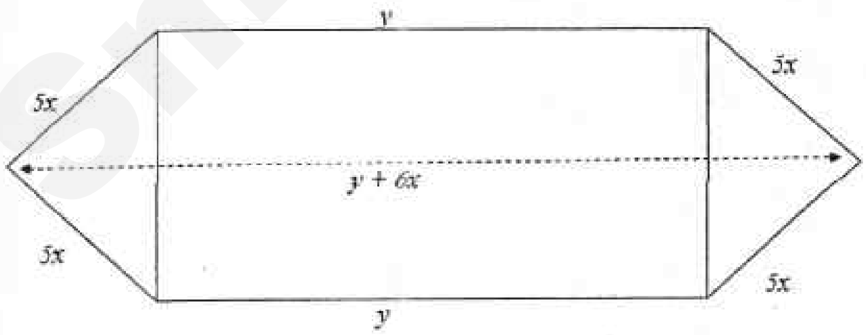
Year	2010	2012	2014	2016
t	2	4	6	8
V(\$)	517 600	595 400	684 800	787 800

(iv) On the graph paper, plot $\ln V$ against t and draw a suitable straight line. The vertical axis should start from 13.0 and have a scale of 2 cm to 0.1. [2]

(v) Use the graph from part (i) to estimate the value of a and of k. [3]

(vi) Estimate the value of the house in 2015. [3]

9. An area is fenced up to enclose a landscape. The shape of the landscape is as shown below. The shape is made up of a rectangle of length y cm and two isosceles triangles of sides
$5x$ cm. The perimeter of the landscape is 420 cm and the length from one end to the other end is $y + 6x$ cm.



(i) Show that the area of the landscape , A cm^2 , is given by $1680x - 56x^2$. [4]

(ii) Given that x can vary, find the stationary value of A. [2]

(iii) ... stationary value is a maximum or minimum. [2]

(i)

$y = \frac{420-20x}{2}$

$y = 210-10x$ - M1

Length of rect = $8x$ - M1

Area of isosceles triangle = $24x^2$

$A = y(8x)+24x^2$

$= (210-10x)8x+24x^2$ - M1

$= 1680x-56x^2$ - A1

(ii) $\frac{dA}{dx} = 1680-112x$

$1680-112x=0$ - M1

$x=15$ - M1

$A = 12600cm^2$ - A1

(iii) $\frac{d^2A}{dx^2} = -112$ - M1

$\frac{d^2A}{dx^2} < 0$, A is a maximum value. – A1

---

10. A particle travels in a straight line from a fixed point O where the distance S in meters is given by $S = \frac{4}{3}t^3 + kt^2 + qt$ where t is the time in seconds after passing O. k and q are constants. The velocity of the particle is 20 m/s when it passes O and at $t = 3s$, its acceleration is 0 m/s^2.

(i) Find the value of k and of Q. [4]

(ii) Find the value(s) of t when the particle is instantaneously at rest. [2]

(iii) Find the total distance travelled during the first 8 seconds. [3]

$V = 4t^2 + 2kt + q$ - M1

$20 = 4(0)^2 + 2k(0) + q$

$q = 20$ - A1

$a = 8t + 2k$ - M1

$0 = 8(t) + 2k$

$k = -12$

(ii) $V = 4t^2 - 24t + 20$

$4t^2 - 24t + 20 = 0$

$4(t-5)(t-1) = 0$ - M1

$t = 1 \text{ or } 5$ -A1

(iii) $S = \frac{4}{3}t^3 - 12t^2 + 20t$

When $t = 1$

$S = 9\frac{1}{3}m$

When $t = 5$

$S = -33\frac{1}{3}m$ - M1

When $t = 8$

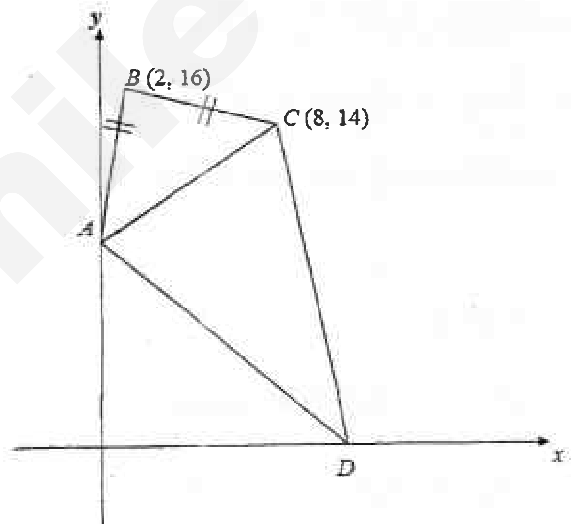
$S = 74\frac{2}{3}m$ -M1

Total distance travelled during the 1st 8 s

$S = 9\frac{1}{3} + 9\frac{1}{3} + 33\frac{1}{3} + 33\frac{1}{3} + 74\frac{2}{3}m$

$= 160$ m -A1

11. [Solution to this question by accurate drawing will not be accepted]



The diagram which is not drawn to scale shows a quadrilateral $ABCD$. The point B is (2, 16) and the point C is (8, 14). Triangle ABC is an isosceles triangle and point A and point D lies the y – axis and x – axis respectively.

(i) Find the coordinates of A. [3]

(i) Given tł | C : area of ΔACD is 1 : 3 , find the coordinates of D. [4]

(ii) Show that $ABCD$ is a kite. [3]

(i) Let A be (0,y)

$\sqrt{2^2+(16-y)^2}=\sqrt{36+4}$

$\sqrt{4+256-32y+y^2}=\sqrt{40}$ - M1

$y^2-32y+220=0$

$(y-10)(y-22)=0$ -M1

$y=10$ or $y=22$ (reject)

$A(0,10)$ - A1

(ii) Area of $\Delta ABC=\frac{1}{2}\begin{vmatrix}2 & 0 & 8 & 2\\ 16 & 10 & 14 & 16\end{vmatrix}$ - M1

$= 20$ units

Area of $\Delta ACD=\frac{1}{2}\begin{vmatrix}0 & x & 8 & 0\\ 10 & 0 & 14 & 10\end{vmatrix}$ - M1

$120 = 14x+80-10x$

$x=10$ -M1

$D(10,0)$ - A1

(iii) $AD=\sqrt{10^2+10^2}$

$AD=\sqrt{200}$

$CD=\sqrt{14^2+2^2}$

$CD=\sqrt{200}$ - M1

Grad of $AC=\frac{4}{8}$

$=\frac{1}{2}$

Grad of $BD=-\frac{16}{8}$

$=-2$ - M1

Since grad. of AC x grad. $BD=-1$ and length of AD = length of CD, $ABCD$ is a kite.
– A1

12. The line $x = 17$ is a tangent to a circle and the points A (1, 9) and B (1, – 7) are on the circumference of the circle.

(i) Show that the radius of the circle is 10 units. [4]

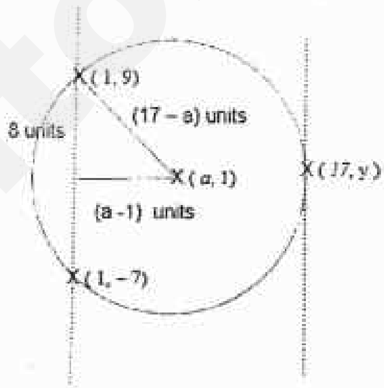
(ii) State the coordinates of the centre of the circle. [1]

(iii) Write down the equation of the circle in the form $x^2 + y^2 + px + qy + r = 0$ [2]

(iv) The circle is reflected along the line $y = -1$, show that the point (3, 10) does not lie on the reflected circle. [3]

(i) $y-$ coordinates of the centre of the circle

$= \dfrac{9+(-7)}{2} = 1$ -M1



Let the x coordinates of the centre of the circle be a

$(17-a)^2 = (a-1)^2 + 8^2$ - M1

$289 - 34a + a^2 = a^2 - 2a + 1 + 64$

$224 = 32a$

$a = 7$ -M1

Radius $= 17 - 7 = 10$ units (Shown) - A1

(ii) Centre of circle = (7,1) - B1

(iii) $(x-7)^2 + (y-1)^2 = 100$ - M1

$x^2 + y^2 - 14x - 2y - 50 = 0$ - A1

(iv) Center of the reflected circle is (7,–3) - M1

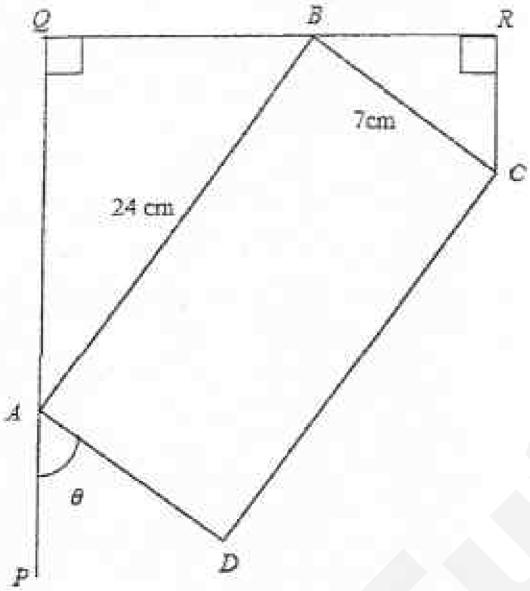
Distance $= \sqrt{(3-7)^2 + (10+3)^2}$

$= \sqrt{16+169}$

$= \sqrt{185}$ -M1

$= 13.6 > 10$ -A1

13. In the diagram below, $ABCD$ is a rectangle. The line QR is perpendicular to the lines PQ and CR. Points A and B lie on the lines PQ and QR respectively and angle $PAD = \theta$. AB is 24 cm and BC is 7 cm.



(i) Show that the length of QR is $24\cos\theta + 7\sin\theta$. [4]

(ii) Express $24\cos\theta + 7\sin\theta$ in the form of $R\cos(\theta - \alpha)$ where $R > 0$ and α is acute. [3]

(iii) Find the value of θ when QR is 17 cm. [2]

(iv) Find the maximum length of QR and state the corresponding value of θ [3]

(i) $\angle QAB = 90° - \theta$
$\angle QBA = \theta$ - M1
$QB = 24\cos\theta$ - M1
$\angle RBC = 90° - \theta$
$\angle BCR = \theta$
$BR = 7\sin\theta$ - M1
$QR = QB + BR$
$QR = 24\cos\theta + 7\sin\theta$ - A1

(ii) $24\cos\theta + 7\sin\theta = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$24 = R\cos\alpha$
$7 = R\sin\alpha$
$R = 25$ - M1
$\tan\alpha = \dfrac{7}{24}$

$\alpha =$ -M1

$24\cos\theta + 7\sin\theta = 25\cos(\theta - 16.3°)$ - A1

(iii) $17 = 5\cos(\theta - 16.3°)$

$\cos(\theta - 16.3°) = \frac{17}{25}$ -M1

$\theta = 63.5^0$ -A1

(iv) $QR = 25\cos(\theta - 16.3°)$

Max length of $QR = 25$ -A1

$\cos(\theta - 16.3°) = 1$ -M1

$\theta = 16.3°$ -A1

Answer **all** the questions.

1 A curve has the equation $y = 2x^3 \ln x$, where $x > 0$.

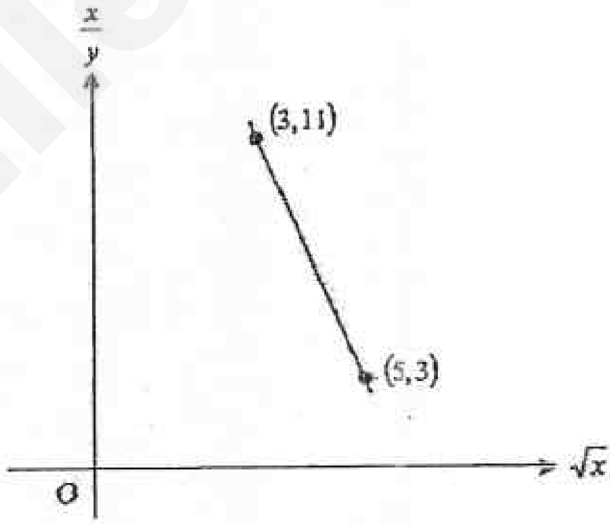
(i) Find $\dfrac{dy}{dx}$. [2]

(ii) Show that x-coordinate of the turning point is $\dfrac{1}{\sqrt[3]{e}}$ and determine whether the turning point is a maximum ~~or a minimum.~~ [4]

2 (a) Find the range of values of p for which the expression $(p+6)x^2 - 8x + p$ is always positive for all real values of x. [4]

(b) Show that the line $y = \dfrac{x}{k} + \dfrac{k}{4}$ is a tangent to the curve $y^2 = x$ for all real values of k. [3]

3 The diagram shows part of a straight line graph drawn to represent the equation $y = \dfrac{x}{b\sqrt{x} - a}$, where a and b are constants.



Given that the line passes through $(3,11)$ and $(5,3)$, find the values of a and of b. [4]

4 (a) Using an appropriate substitution, or otherwise,

solve $\left(\sqrt{9}\right)^{2x} - 3^{x+2} = 6\left(3^x\right) - 54$. [6]

(b) **Without using a calculator**, find the value of 10^x, given that $2^{2x+5} \times 5^{x-2} = 5^{2x} \times 8^{x+1}$. [3]

5 (a) (i) State the values between which the principal value of $\cos^{-1} x$ must lie. [1]

(ii) Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ in radians. [1]

(b) It is given that $\cos A = \frac{4}{5}$ where $270° < A < 360°$

Without the use of calculator, find the exact value of each of the following.

(i) $\cot 2A$. [3]

(ii) $\sin \frac{1}{2} A$. [2]

6 (a) Evaluate $\int_1^9 \left(\sqrt{x} + 2 - \frac{4}{\sqrt{x}}\right) dx$. [3]

(b) The gradient of a curve is $\frac{12}{(4x-1)^2}$. Given that the curve passes through the points $\left(\frac{1}{2}, 5\right)$ and $(-2, k)$, find the value of k. [4]

Questions 7 to 11 must be handed in separately from Questions 1 to 6.

Begin your answer to question 6 on a fresh sheet of paper.

7 It is given that $3x+4y=k$ and $(6x-20)^2+(4y+3)^2=200$.

(i) If $k=8$, find the solutions of these simultaneous equations. [4]

(ii) If $k=-10$, show that there are no solutions without solving the equations. [2]

(iii) Explain why there cannot be more than 2 solutions for all values of k. [2]

8 Express $\dfrac{2x^3-5x^2-11x+44}{x^2-x-6}$ in partial fractions. [5]

9 (a) Find the equation of the normal to the curve $y=\sin 4x-3\cos 2x$ at the point where $x=\dfrac{\pi}{12}$. [7]

(b) Singapore has two high tides and two low tides a day. The tidal movement on East Coast beach during a particular day can be modelled by the curve $y=\sin x$.
If the four tides occur at 5 am, 11 am, 5 pm and 11 pm respectively, at what time will the flow of the water onto East Coast beach be the fastest? [1]

10 (a) Given that the coefficient of x^2 in the expansion of $(1-3x)^2(1-kx)^8$ is 117, find the two possible values of the constant k. [5]

(b) Find the term independent of x in the expansion of $\left(\dfrac{1}{x}-\dfrac{x^3}{4}\right)^{24}$. [4]

11 A curve has the equation $y=\left(\frac{x}{2}+1\right)^2-4$.

(i) Explain why the lowest point on the curve has the coordinates $(-2,-4)$. [2]

(ii) Find the x–coordinates of the points at which the curve intersects the x–axis. [2]

(iii) Sketch the graph of $y=\left|\left(\frac{x}{2}+1\right)^2-4\right|$, indicating clearly the coordinates of the turning point and the points where the curve meets the axes. [3]

(iv) State the set of the values of k for which the line $y=k$ intersects the curve

(a) at 2 distinct points, [2]

(b) at 4 distinct points. [1]

End of Paper 1

Answer all the questions.

1 A particular species of fish living in a fish farm is being studied. After t years, its population P is given by $P = 300(2+5e^{-kt})$ where k is a constant.

(a) Find the initial population of the fish in the farm. [1]

The population of the fish in the farm after 3 years is predicted to be 2400.

(b) Find the value of k. [2]

The fish farm owner has to replenish the supply of fish in the farm when the population drops below 1000.

(c) Using the k value obtained in part (b), determine, with working, whether the fish farm owner needs to replenish the fish supply after 5 years. [2]

2 Given that $f(x) = 3x^4 + x^3 - mx^2 - nx + 36$,

(a) find the values of m and n when $(x^2 - 9)$ is a factor of $f(x)$, [4]

(b) hence solve the equation $f(x) = 0$. [3]

3 The quadratic equation $2x^2 + 4x - 7 = 0$ has roots α and β.

(a) State the values of $(\alpha + \beta)$ and $\alpha\beta$. [2]

(b) Find the value of $\alpha^2 + \beta^2$. [2]

(c) Hence, form a quadratic equation whose roots are $\frac{2}{\alpha^2}$ and $\frac{2}{\beta^2}$. [3]

4 (a) Solve the equation $\log_2(x-3) - 6\log_{x-3}(2) = 1$. [5]

(b) Given that $w = \log_5 a$, find, in terms of w,

(i) $\log_5 \frac{5}{a}$, [1]

(ii) $(\log_5 a)^4$ [1]

(iii) $\log_5 125a^2$ [2]

5 (a) (i) Prove that $\dfrac{1+\cos x}{1-\cos x} \equiv \cot^2 \dfrac{x}{2}$. [3]

(ii) Hence prove that $\cot^2 15^\circ = 7 + 4\sqrt{3}$. [3]

(b) Solve the equation $\dfrac{4\sec x}{1+\sec^2 x} = -1$ for $0 \le x \le 360^\circ$, giving your answers correct to 2 decimal places. [5]

6 The area of a triangle is $\left(3+\sqrt{15}\right)$ cm^2.

(a) In the case whereby the triangle is a right angle triangle with height $\left(\sqrt{5}-\sqrt{3}\right)$ cm, find, without using a calculator, the length of the base of this triangle in the form $\left(a\sqrt{5}+b\sqrt{3}\right)$ cm. [4]

(b) In the case whereby the triangle is an equilateral triangle with the length of each side w cm, find the value of w^2, giving your answer in the form $a\left(\sqrt{3}+\sqrt{5}\right)$. [4]

Begin Question 7 on a fresh sheet of Answer Paper.

7 A curve has the equation $y = x(3-x)^3$. Points A and B are the two stationary points on the curve.

(a) Find the coordinates of points A and B. [4]

(b) Determine the nature of these 2 stationary points on the curve. [4]

(c) (i) Find the values of x for which y is increasing. [1]

(ii) Find the values of x for which y is decreasing. [1]

8 **Answer the whole of this question on the graph paper provided.**

The table below shows the experimental values of x and y which are related by the equation $y = b^{a-x}$. One value of y has been recorded wrongly.

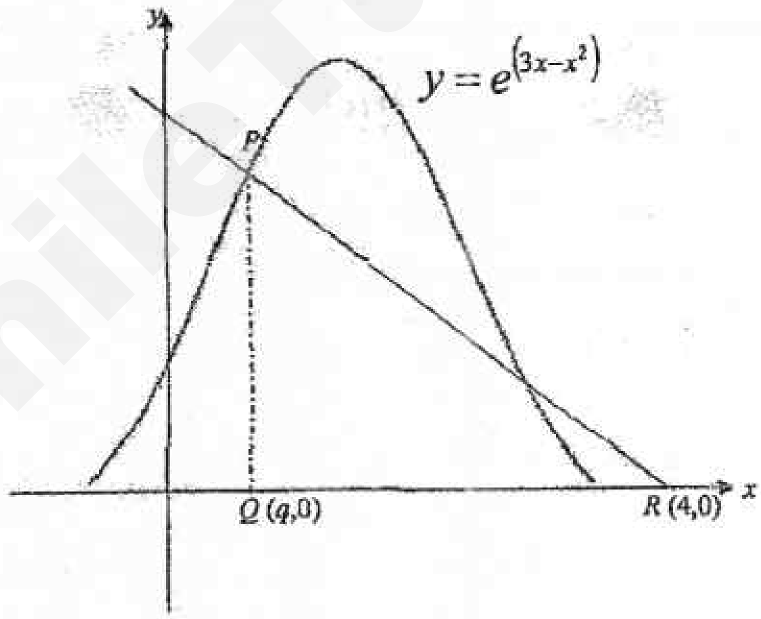
x	1	1.5	2	2.5	3
y	4	2.21	2	1.41	1

(a) Plot $\lg y$ against x and draw a straight line graph. [4]

(b) Use your graph to estimate the values of a and b. [4]

(c) Determine which value of y is incorrect and estimate the correct value of y. [2]

9 The diagram below shows the graph of $y = e^{(3x-x^2)}$. Points Q and R lie on the x-axis such that their coordinates are $(q, 0)$ and $(4, 0)$ respectively. P is a point on the curve such that PQ is parallel to the y-axis.

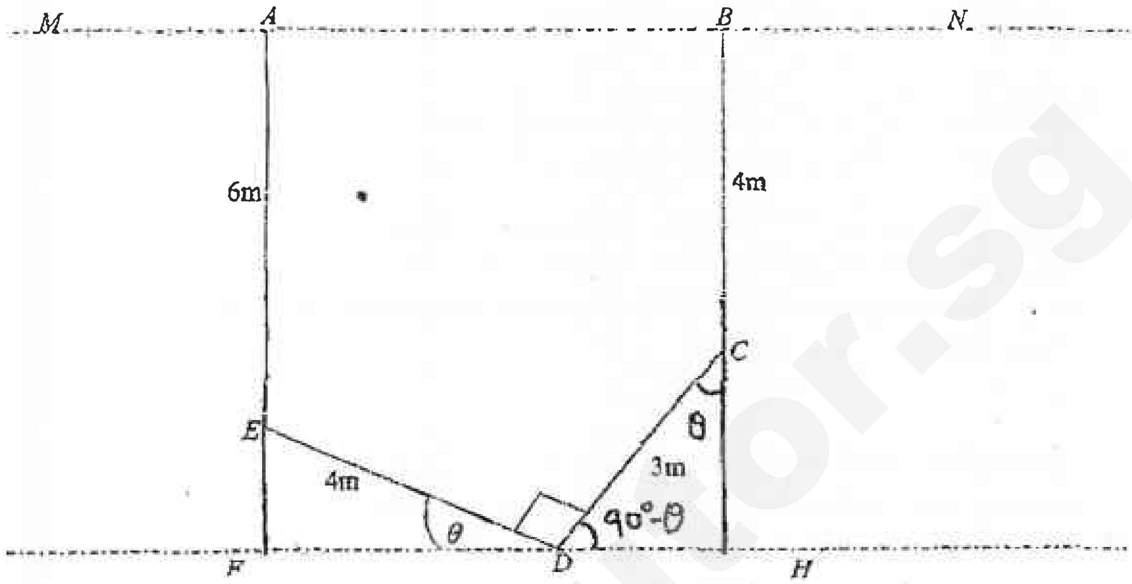


(a) Express the coordinates of P in terms of q. [1]

(b) Show that the area of triangle PQR, $A = \left(2 - \frac{q}{2}\right)\left(e^{3q-q^2}\right)$. [2]

(c) If q is decreasing at a rate of 2 units per second, find the rate at which the area of triangle PQR is changing at the instant when $\boxed{q = 2}$. [4]

10



In the diagram above, $AE = 6$ m, $BC = DE = 4$ m and $CD = 3$ m, $\angle CDE = 90°$ $\angle EDF = \theta$, AE and BC are both perpendicular to the line MN. MN is parallel to FH.

(a) Show that $AB = 4\cos\theta + 3\sin\theta$. [2]

(b) Express AB in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0 < \alpha < 90°$. [3]

(c) Find the maximum perimeter, P, of the figure and the corresponding value of θ. [4]

11 The function f is defined, for $x \geq 0$, by $f(x) = p\cos\left(\frac{x}{2}\right) - q$.

(a) State the period of f(x). [1]

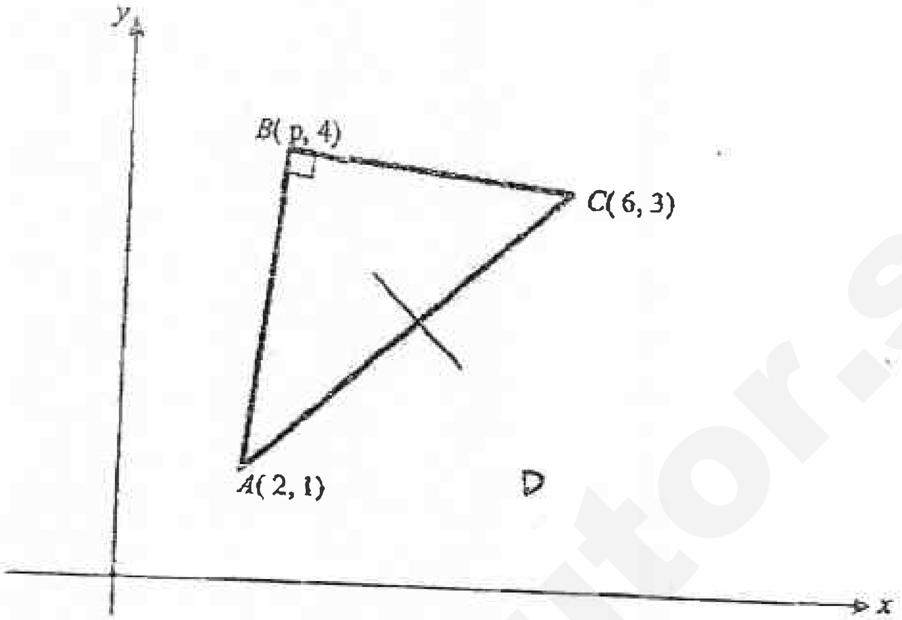
Given that the maximum and minimum values of f(x) are 1 and -5 respectively, find

(b) the amplitude of f, [1]

(c) the values of p and q, [2]

(d) Using the values of p and q found in part (iii), sketch the graph of $f(x)$ for $0 \leq x \leq 3\pi$. [3]

12 **Solutions to this question by accurate drawing will not be accepted.**



The diagram, which is not drawn to scale, shows a triangle ABC in which point A is (2, 1), point B is (p, 4) and point C is (6, 3). The line AB is perpendicular to the line BC.

Find

(a) the value of p, where $p < 4$, [3]

(b) the equation of the perpendicular bisector of AC. [3]

The point D is such that $ABCD$ is a square. Find, using the value of p from part (i),

(c) the coordinates of D, [2]

(d) the area of $ABCD$. [2]

~ End of Paper 2 ~

Paper 1 Answer

Q1

SEC 4 EXP A-MATHS SA1 PAPER 1 (2017)

Qn 1 (i) $y = 2x^3 \ln x,\ x > 0$

$\frac{dy}{dx} = 2x^3\left(\frac{1}{x}\right) + (\ln x)(6x^2)$ $\boxed{\text{does not accept } \ln x \quad 6x^2}$

$= 2x^2 + 6x^2 \ln x$
[B1] [B1]

1(ii) when $\frac{dy}{dx} = 0$, $2x^2 + 6x^2 \ln x = 0$ [B1]

Does not know how to solve the equation by factorisation $2x^2 + 6x^2 \ln x = 0$

$2x^2(1 + 3\ln x) = 0$

Since $x \neq 0$, $\therefore 1 + 3\ln x = 0$

$\ln x = -\frac{1}{3}$

$\therefore x = e^{-\frac{1}{3}}$

$x = \frac{1}{\sqrt[3]{e}}$ (shown) [M1]

$\frac{d^2y}{dx^2} = 4x + 6x^2\left(\frac{1}{x}\right) + (\ln x)(12x)$

$= 10x + 12x \ln x$ [B1]

When $x = \frac{1}{\sqrt[3]{e}}$, $\frac{d^2y}{dx^2} = 10\left(\frac{1}{\sqrt[3]{e}}\right) + 12\left(\frac{1}{\sqrt[3]{e}}\right)\ln\left(\frac{1}{\sqrt[3]{e}}\right)$

$= 4.299 > 0$ (Min). [B1]

$\therefore$ the turning pt is a Minimum point.

Q2

Q2(a): $b^2-4ac<0$ and $(p+6)>0$

$(-8)^2-4(p+6)(p)<0$ [M1] $\quad p>-6$ [M1]

$64-4p^2-24p<0$

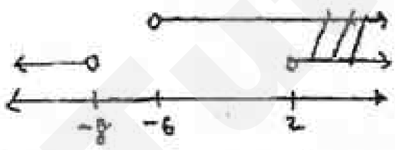
$4p^2+24p-64>0$

$p^2+6p-16>0$

$(p+8)(p-2)>0$

$p<-8$ or $p>2$ [M1]

Mistake made:
(1) Used $b^2-4ac>0$
(2) " not indicate $p+6>0$
(3) Did not write down the final answer



Ans: $p>2$ [A1]

Q2(b): $\left(\frac{x}{k}+\frac{k}{4}\right)\left(\frac{x}{k}+\frac{k}{4}\right)=x$ [B1]

$\frac{x^2}{k^2}+\frac{x}{2}+\frac{k^2}{16}=x$

$\frac{x^2}{k^2}-\frac{x}{2}+\frac{k^2}{16}=0$

$16x^2-8k^2x+k^4=0$

$b^2-4ac=(-8k^2)^2-4(16)(k^4)$ [B1]

$=64k^4-64k^4$

$=0$

Since $b^2-4ac=0$ for all values of k, the line $y=\frac{x}{k}+\frac{k}{4}$ is a tangent to the curve $y^2=x$ (shown) [M1]

Alternate solution:

$y=\frac{y^2}{k}+\frac{k}{4}$

$4ky=4y^2+k^2$

$4y^2-4ky+k^2=0$

$(2y-k)^2=0$

$y=\frac{k}{2}$

there is only one solution

$\therefore y=\frac{x}{k}+\frac{k}{4}$ is a tangent to the curve

Students did not present the working correctly.

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Q3

Q3:
$$y = \frac{x}{b\sqrt{x} - a}$$

$$b\sqrt{x} - a = \frac{x}{y}$$

$$\frac{x}{y} = b\sqrt{x} - a \qquad [M1]$$

Let $Y = \frac{x}{y}$, $X = \sqrt{x}$.

then grad = b and vertical intercept = $-a$.

$$\text{Grad} = b = \frac{11-3}{3-5} = -4 \qquad [A1]$$

$$Y = -4X + c$$

When $X = 5$, $Y = 3$, we have,

$$3 = -4(5) + c$$

$$c = 23. \qquad [M1]$$

$$\therefore -a = 23$$

$$\therefore a = -23 \qquad [A1]$$

OR $X = 3$, $Y = 11$ (3, 11)

$$11 = -4(3) + c$$

$$c = 23$$

Q 4

Q4(a) $(\sqrt{9})^{2x} - 3^{x+2} = 6(3^x) - 54$

$(3^x)^2 - 9(3^x) = 6(3^x) - 54$

$(3^x)^2 - 15(3^x) + 54 = 0$ [M1]

Let $u = 3^x$.

$\therefore u^2 - 15u + 54 = 0$

$(u-6)(u-9) = 0$ [M1]

$\therefore u = 6$ or $u = 9$. [B1]

$3^x = 6$ $\qquad$ $3^x = 9$

$x \lg 3 = \lg 6$ $\qquad$ $3^x = 3^2$

$\therefore x = \frac{\lg 6}{\lg 3}$ [M1] $\Rightarrow x = 2$ # [A1]

$= 1.63$ (3s.f) [A1]

(1) Students gave answers $u=6$ or $u=9$. Did not solve the equations to find values of x.

(2) Did not know how to solve $3^x = 6$

4(b) $2^{2x+5} \times 5^{x-2} = 5^{2x} \times 8^{x+1}$

$32(2^{2x}) \times \frac{5^x}{25} = 5^{2x} \times 8(2^{3x})$ [M1]

$\frac{32}{8 \times 25} = \frac{5^{2x} \times 2^{3x}}{5^x \times 2^{2x}}$ [M1]

$5^x \times 2^x = \frac{4}{25}$

$\Rightarrow 10^x = \frac{4}{25}$ #. [A1]

Students did not change 8^{x+1} to 2^{3x+3}

Q5

5 (a) (i) Let the principal value of $\cos^{-1} x$ be θ.

$\therefore\ 0 \le \theta \le \pi$ # [B1]

Most students did not know what principal value is.

5(a) (ii) Let $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \theta$, $0 \le \theta \le \pi$

$\therefore \cos\theta = -\frac{1}{\sqrt{2}}$

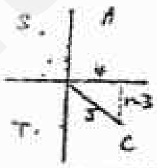
basic $\angle\ \alpha = \frac{\pi}{4}$

$\therefore\ \theta = \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$ # [B1]

Did not know that $\cos^{-1} x$ is an angle

5(b) (i) $\cos A = \frac{4}{5}$, $270° < A < 360°$.



$\tan A = -\frac{3}{4}$ [B1]

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

$= \frac{2\left(-\frac{3}{4}\right)}{1-\left(-\frac{3}{4}\right)^2}$ [M1]

$= -\frac{24}{7}$

$\therefore \cot 2A = -\frac{7}{24}$ # [A1]

Did not know that $\sin A = -\frac{3}{5}$, $\tan A = -\frac{3}{4}$

5(b)(ii) $\cos 2A = 1 - 2\sin^2 A$

$\cos A = 1 - 2\sin^2 \frac{A}{2}$

$\sin^2 \frac{A}{2} = \frac{1-\cos A}{2}$

$= \frac{1-\frac{4}{5}}{2}$ [M1]

$= \frac{1}{10}$

$\therefore \sin\frac{1}{2}A = \frac{1}{\sqrt{10}}$ [A1] # or $-\frac{1}{\sqrt{10}}$ (N.A.)

Most students tried to use $\sin 2A = 2\sin A\cos A$ formula

Q6

Q6 (a) $\int_1^9 \left(\sqrt{x} + 2 - \frac{4}{\sqrt{x}}\right) dx$

$= \int_1^9 \left(x^{\frac{1}{2}} + 2 - 4x^{-\frac{1}{2}}\right) dx$

$= \left[\underbrace{\frac{2x^{\frac{3}{2}}}{3}}_{[B1]} + \underbrace{2x - 8x^{\frac{1}{2}}}_{[B1]}\right]_1^9$

$= \left[\frac{2(9)^{\frac{3}{2}}}{3} + 2(9) - 8(9)^{\frac{1}{2}}\right] - \left[\frac{2}{3} + 2 - 8\right]$

$= 12 - \left(-5\frac{1}{3}\right)$

$= 17\frac{1}{3}$ *. [B1]

6(b) $\frac{dy}{dx} = \frac{12}{(4x-1)^2} = 12(4x-1)^{-2}$

$\therefore y = \frac{12(4x-1)^{-1}}{(-1)(4)} + c = \underbrace{\frac{-3}{4x-1}}_{[B1]} + c$

When $x = \frac{1}{2}$, $y = 5$,

$5 = \frac{-3}{4(\frac{1}{2})-1} + c$

$c = 8$ [B1]

$\therefore y = \frac{-3}{4x-1} + 8$

Some students tried to use $\therefore m = \frac{y_1 - y_2}{x_1 - x_2}$ formula

When $x = -2$,

$k = \frac{-3}{4(-2)-1} + 8$ [M1]

$k = 8\frac{1}{3}$ *. [A1]

Q7

$3x + 4y = 8$

$4y = 8 - 3x$ —— ①

$(6x-20)^2 + (4y+3)^2 = 200$ —— ②

Subst. ① into ②, $(6x-20)^2 + (8-3x+3)^2 = 200$ [M1]

$36x^2 - 240x + 400 + (-3x+11)^2 = 200$

$36x^2 - 240x + 400 + 9x^2 - 66x + 121 - 200 = 0$

$45x^2 - 306x + 321 = 0$

$\therefore x = \frac{(306) \pm \sqrt{(-306)^2 - 4(45)(321)}}{2(45)}$ [M1]

$= \frac{306 \pm \sqrt{35856}}{90}$

$= 5.50$ * (3 s.f.) or 1.30 * (3 s.f.) [A1]

$\therefore y = -2.13$ * (3 s.f.) or 1.03 (3 s.f.) * [A1]

*many students did not show working to how they solved for x or y in the quad eqn.

*many students forgot to solve for y.

7(ii) If $k = -10$, then $4y = -10 - 3x$.

$\therefore (6x-20)^2 + (-10-3x+3)^2 = 200$

$36x^2 - 240x + 400 + (-3x-7)^2 = 200$

$36x^2 - 240x + 400 + 9x^2 + 42x + 49 - 200 = 0$

$45x^2 - 198x + 249 = 0$ [B1]

$b^2 - 4ac = (-198)^2 - 4(45)(249)$

$= -5616 < 0$ [B1]

Since $b^2 - 4ac < 0 \Rightarrow$ no real roots $\Rightarrow$ no sol^ns *.

7(iii) Since $4y = k - 3x$,

$(6x-20)^2 + (k-3x+3)^2 = 200$

*many students thought that $(6x-20)^2 + (4y+3)^2 = 200$ is a quadratic curve.

Addition of two quadratic expressions in this eq^n will result in a quadratic equation. [B1]

And the max. no. of roots in a quad eq^n is 2 *. [B1]

Q8

Q^2 8 :

$$
\begin{array}{r}
2x - 3 \\
x^2-x-6 \overline{\big) \, 2x^3 - 5x^2 - 11x + 44} \\
-(2x^3 - 2x^2 - 12x) \\
\hline
-3x^2 + x + 44 \\
-(-3x^2 + 3x + 18) \\
\hline
-2x + 26
\end{array}
$$

* Many students did not find the quotient. They went straight into express the fract^n as partial fractions.

[B1]

$$\therefore \frac{2x^3-5x^2-11x+44}{x^2-x-6} = 2x-3 + \frac{-2x+26}{x^2-x-6}$$

$$\frac{-2x+26}{x^2-x-6} = \frac{-2x+26}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2} \quad [B1]$$

$$-2x+26 = A(x+2) + B(x-3)$$

When $x=-2$, $-2(-2)+26 = B(-2-3)$

$$-5B = 30$$

$$B = -6 \quad [B1]$$

When $x=3$, $-2(3)+26 = A(3+2)$

$$5A = 20$$

$$A = 4 \quad [B1]$$

$$\therefore \frac{2x^3-5x^2-11x+44}{x^2-x-6} = 2x-3+\frac{4}{x-3}-\frac{6}{x+2} \quad [A1]$$

Q9

Q^{n} 9(a) $y = \sin 4x - 3\cos 2x$

$$\frac{dy}{dx} = \underbrace{4\cos 4x}_{[B1]} + \underbrace{6\sin 2x}_{[B1]}$$

When $x = \frac{\pi}{12}$, $\frac{dy}{dx} = 4\cos 4\left(\frac{\pi}{12}\right) + 6\sin 2\left(\frac{\pi}{12}\right)$

$= 2 + 6\left(\frac{1}{2}\right)$

$= 5$ [B1]

$\therefore$ Grad of normal $= -\frac{1}{5}$. [B1]

missing students cannot differentiate the trigo terms.

* Students were found working on degree mode (calculator) when this is in radian.

* Students should learn to give answer in exact value.

When $x = \frac{\pi}{12}$, $y = \sin 4\left(\frac{\pi}{12}\right) - 3\cos 2\left(\frac{\pi}{12}\right)$

$= \frac{\sqrt{3}}{2} - 3\left(\frac{\sqrt{3}}{2}\right)$

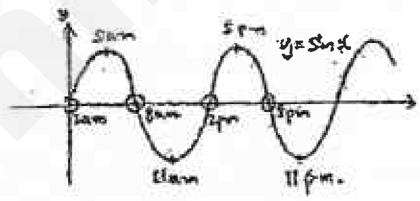
$= \frac{\sqrt{3} - 3\sqrt{3}}{2} = -\sqrt{3}$ [B1]

$\therefore y = -\frac{1}{5}x + c$ $\quad -\sqrt{3} = -\frac{\pi}{60} + c$

When $x = \frac{\pi}{12}$, $y = -\sqrt{3}$, $c = \frac{\pi}{60} - \sqrt{3}$ [B1]

$\therefore$ eq^{n} of normal is $y = -\frac{1}{5}x + \frac{\pi}{60} - \sqrt{3}$ # [B1]

9(b)



From the graph, the timings of the slowest water flow are

2 am, 8 am, 2 pm, 8 pm #. [B1]

Q10

Q 10 (a) $(1-3x)^2(1-kx)^8$

$= (1-6x+9x^2)(1-8kx+28k^2x^2+\cdots)$ [B1]

$\Rightarrow 28k^2+48k+9=117$ [B1]

$28k^2+48k-108=0$

$7k^2+12k-27=0$

$(7k-9)(k+3)=0$ [M1]

$\therefore k=\frac{9}{7}$ * or $k=-3$ * [A1]

[A1]

10(b) $T_{r+1}={}^{24}C_r\left(\frac{1}{x}\right)^{24-r}\left(-\frac{x^3}{4}\right)^r$ [M1]

$={}^{24}C_r\left(-\frac{1}{4}\right)^r x^{-24+r+3r}$

$={}^{24}C_r\left(-\frac{1}{4}\right)^r x^{4r-24}$

$\Rightarrow 4r-24=0$ [M1]

$4r=24$

$r=6$ [B1]

$\therefore T_7={}^{24}C_6\left(-\frac{1}{4}\right)^6 x^{4(6)-24}$

$=\frac{33649}{1024}$ * [A1]

* After finding $r=6$, Many students claimed that the 7^{th} term was the answer.

* Many students thought $\left(\frac{1}{x}-\frac{x^3}{4}\right)^{24}={}^{24}C_r\left(\frac{1}{x}\right)^{24-r}\left(\frac{x^3}{4}\right)^r$

Q11

11 (i) $y = \left(\frac{x}{2}+1\right)^2 - 4$

When $\frac{x}{2}+1=0$

$\frac{x}{2} = -1$

$x = -2$ [B1]

When $x=-2$, $y = \left(\frac{-2}{2}+1\right)^2 - 4 = -4$

Since the coeff of x^2 is +ve, ∴ the lowest pt of the graph is $(-2, -4)$ ※. [B1]

11 (ii) When $y=0$, $\left(\frac{x}{2}+1\right)^2 - 4 = 0$

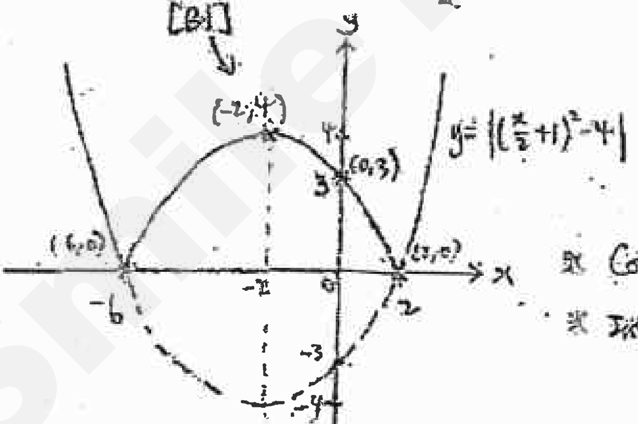
$\left(\frac{x}{2}+1\right)^2 = 4$

$\frac{x}{2}+1 = 2$ or -2 [M1]

$\frac{x}{2} = 1$ or -3

$x = 2$ ※ or -6 ※ [A1]

11 (iii)



When $x=0$, $y = \left(\frac{0}{2}+1\right)^2 - 4$

$y = -3$.

※ Correct shape. [B1]

※ Intercepts clearly indicated. [B1]

11 (iv)(a) $k=0$ [B1] and $k>4$ ※. [B1]

11 (iv)(b) $0<k<4$ ※. [B1]

Q1

Marking Scheme

(1) $P = 300(2 + 5e^{-kt})$

(a) When $t = 0$: $P = 300[2 + 5] = 2100$ [B1]

(b) $2400 = 300(2 + 5e^{-3k})$

$\frac{2400}{300} = 2 + 5e^{-3k}$

$8 = 2 + 5e^{-3k}$

$6 = 5e^{-3k}$ [M1]

$e^{-3k} = \frac{6}{5}$

$-3k = \ln(\frac{6}{5})$

$k = -0.0608$ [A1]

(c) $P = 300(2 + 5e^{0.061t})$

When $t = 5$: $P = 300[2 + 5e^{0.061(5)}]$ [M1]

$= 2634.94 > 1000$

hence no need to replenish. [B1]

Q2

2(b) Hence $f(x) = 3x^4 + x^3 - 31x^2 - 9x + 36$

$$
\begin{array}{r|l}
 & 3x^2 + x - 4 \\
\hline
x^2 - 9 & 3x^4 + x^3 - 31x^2 - 9x + 36 \\
 & -(3x^4 - 27x^2) \\
\hline
 & x^3 - 4x^2 - 9x \\
 & -(x^3 - 9x) \\
\hline
 & -4x^2 + 36 \\
 & -(-4x^2 + 36) \\
\hline
 & 0
\end{array}
$$

Hence $(x^2-9)(3x^2+x-4) = 0$ [M1]

$x^2 - 9 = 0$ or $3x^2 + x - 4 = 0$

$x = \pm 3$ [A1] $(3x+4)(x-1) = 0$

$\therefore x = -\frac{4}{3}$ or $x = 1$ [A1], [A1].

---

3) $\alpha + \beta = -\frac{4}{2} = -2$ [B1] $\alpha\beta = -\frac{7}{2}$ [B1]

(b) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ [M1]

$= (-2)^2 - 2(-\frac{7}{2}) = 4 + 7 = 11$ [A1]

(c) $\frac{2}{\alpha^2} + \frac{2}{\beta^2} = \frac{2\beta^2 + 2\alpha^2}{\alpha^2\beta^2}$

$= \frac{2(\alpha^2+\beta^2)}{(\alpha\beta)^2}$

$= \frac{2(11)}{(-\frac{7}{2})^2} = \cancel{\frac{22}{\frac{49}{4}}} = -\frac{88}{49}$ [B1]

wrong!

$\left(\frac{2}{\alpha^2}\right)\left(\frac{2}{\beta^2}\right) = \frac{4}{(\alpha\beta)^2}$

$= \frac{4}{(-\frac{7}{2})^2}$

$= \frac{4}{\frac{49}{4}}$

$= \frac{16}{49}$ [B1].

Hence $x^2 - (-\frac{88}{49})x + \frac{16}{49} = 0$

$49x^2 + 88x + 16 = 0$. [B1]

er

Q4

$\log_2(x-3) - 6\left[\frac{\log_2 2}{\log_2(x-3)}\right] = 1$ [M1] (change of base)

$\log_2(x-3) - \frac{6}{\log_2(x-3)} = 1$

Let $X = \log_2(x-3)$: $X - \frac{6}{X} = 1$ [M1].

$X^2 - 6 = X$

$X^2 - X - 6 = 0$

$(X-3)(X+2) = 0$

$X = 3$ or $X = -2$

$\log_2(x-3) = 3$ $\quad\quad$ $\log_2(x-3) = -2$

$x - 3 = 2^3$ $\quad\quad$ $x - 3 = 2^{-2}$ [M1]

$x = 5$ [B1] or $x = 3\frac{1}{4}$. [B1]

(b)(i) $\log_5\left(\frac{5}{a}\right) = \log_5 5 - \log_5 a$

$= 1 - w$ [B1]

(ii) $(\log_5 a)^4 = w^4$ [B1]

(iii) $\log_5 125a^2 = \log_5 125 + \log_5 a^2$ [M1]

$= \log_5 (5)^3 + 2\log_5 a$

$= 3 + 2w$ [A1].

Q5

5(a)(i) LHS: $\dfrac{1+\cos x}{1-\cos x} = \dfrac{1+[2\cos^2\frac{x}{2}-1]}{1-[1-2\sin^2\frac{x}{2}]}$ [B1] [B1]

$= \dfrac{2\cos^2\frac{x}{2}}{2\sin^2\frac{x}{2}} = \cot^2\left(\frac{x}{2}\right)$ (proven). [B1]

(ii) Let $x = 30°$: $\dfrac{1+\cos 30°}{1-\cos 30°} = \cot^2(15°)$ [B1].

$\cot^2 15° = \dfrac{1+\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}}$

$= \dfrac{2+\sqrt{3}}{2} \div \dfrac{2-\sqrt{3}}{2}$

$= \dfrac{2+\sqrt{3}}{2-\sqrt{3}} \times \dfrac{2+\sqrt{3}}{2+\sqrt{3}}$ [M1]

$= \dfrac{4+4\sqrt{3}+3}{1} = 7+4\sqrt{3}$. [A1].

(b) $4\sec x = -1-\sec^2 x$

$\sec^2 x + 4\sec x + 1 = 0$

$\sec x = \dfrac{-4 \pm \sqrt{16-4}}{2}$ [M1].

$= \dfrac{-4 \pm \sqrt{12}}{2}$

$\sec x = -0.26795$ or $\sec x = -3.7321$ [M1].

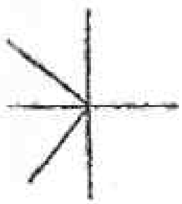
$\cos x = \dfrac{1}{-0.26795}$

$= -3.73$ (NA) [A1]

$\cos x = \dfrac{1}{-3.7321}$

$\cos x = -0.26795$

Basic $\angle = 74.46°$



$\therefore x = 105.54°$ or $254.46°$
[A1] [A1]

Q6

(b)(i) $\frac{1}{2}(b)(\sqrt{5}-\sqrt{3}) = (3+\sqrt{15})$ [B1].

$b = \frac{2(3+\sqrt{15})}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ [M1].

$= \frac{(6+2\sqrt{15})(\sqrt{5}+\sqrt{3})}{5-3}$

$= \frac{6\sqrt{5}+6\sqrt{3}+2\sqrt{75}+2\sqrt{45}}{2}$ [M1]

$= 3\sqrt{5}+3\sqrt{3}+\sqrt{75}+\sqrt{45}$

$= 3\sqrt{5}+3\sqrt{3}+5\sqrt{3}+3\sqrt{5}$

$= 6\sqrt{5}+8\sqrt{3}$ [A1]

(ii) $\frac{1}{2}w^2(\sin 60°) = 3+\sqrt{15}$ [B1].

$\frac{1}{2}w^2\left(\frac{\sqrt{3}}{2}\right) = 3+\sqrt{15}$

$w^2\left(\frac{\sqrt{3}}{2}\right) = 6+2\sqrt{15}$

$w^2 = (6+2\sqrt{15})\left(\frac{2}{\sqrt{3}}\right)$

$= \frac{12+4\sqrt{15}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ [M1].

$= \frac{12\sqrt{3}+4\sqrt{45}}{3}$

$= \frac{12\sqrt{3}+4(3\sqrt{5})}{3}$ [M1].

$= \frac{12\sqrt{3}+12\sqrt{5}}{3}$

$= 4(\sqrt{3}+\sqrt{5})$. [A1].

Q7

(7) (i) $y = x(3-x)^3$

$\frac{dy}{dx} = (x)[3(3-x)^2(-1)] + (3-x)^3$ [B1].

At stat. pts: $(-3x)(3-x)^2 + (3-x)^3 = 0$

$(3-x)^2[-3x+3-x] = 0$ [M1].

$(3-x)^2 = 0$ or $-4x+3=0$

$\therefore x = 3$ or $x = \frac{3}{4}$

$y = 0$ [A1] $\quad y = \frac{2187}{256}$ or $y = 8.54$ [A1]

(ii) Using the first derivative test,

x	3^-	3	3^+
$\frac{dy}{dx}$	-ve	0	-ve
	\	—	\

[B1]

x	$\frac{3}{4}^-$	$\frac{3}{4}$	$\frac{3}{4}^+$
$\frac{dy}{dx}$	+ve	0	-ve
	/	—	\

[B1].

(3,0) is a pt of inflexion [B1] $\quad$ (0.75, 8.54) is a max point [B1].

(iii)(a) For y increasing, $x < 0.75$ [B1]

For y decreasing, $x > 0.75$. [B1].

Alt. Solution

(ii) $\frac{d^2y}{dx^2} = (3-x)^2[-4] + (3-4x)[2(3-x)(-1)]$

$= -4(3-x)^2 - 2(3-4x)(3-x)$

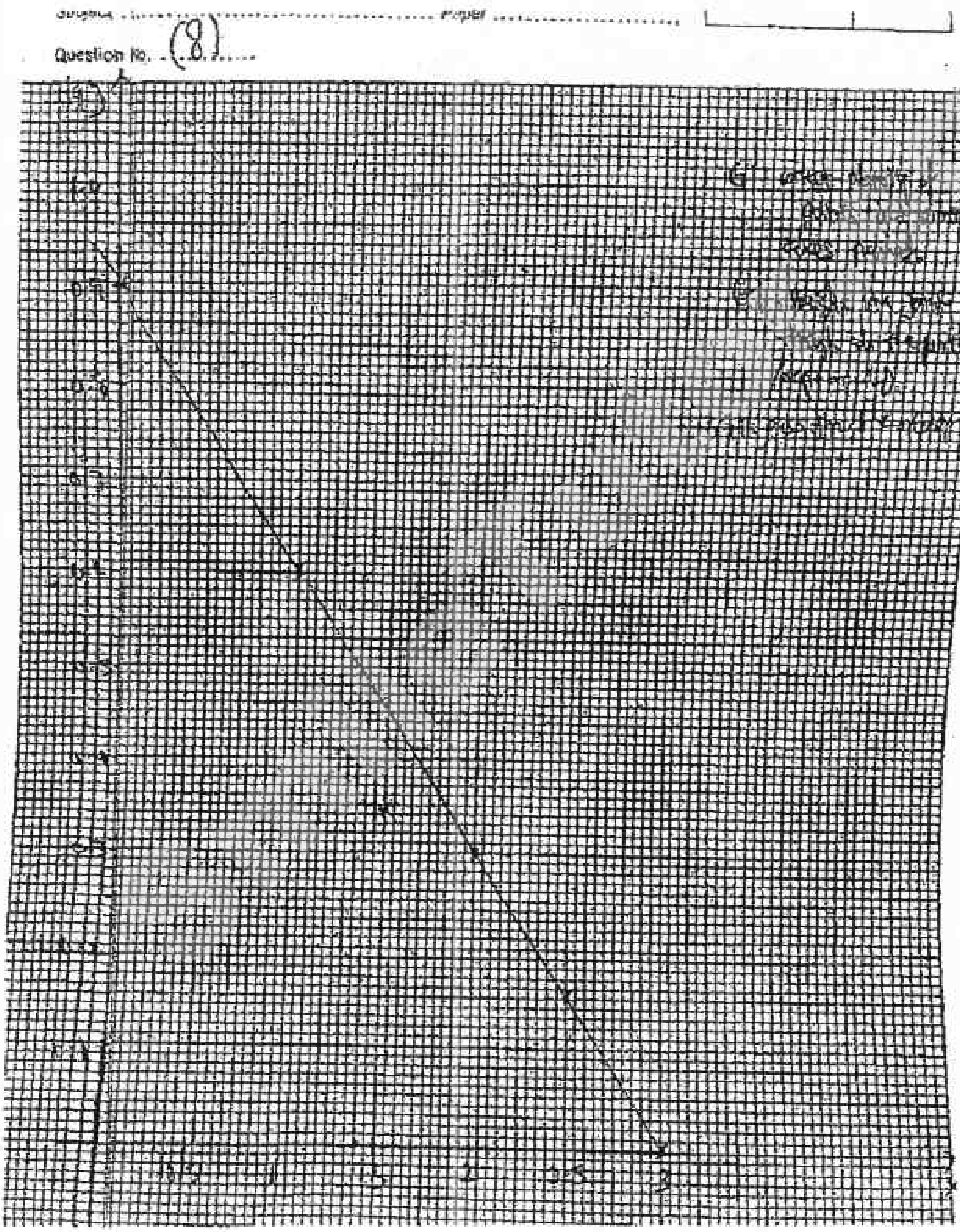
When $x = 3$, $\frac{d^2y}{dx^2} = 0$ B1 (pt of inflexion) B1

$x = \frac{3}{4}$, $\frac{d^2y}{dx^2} < 0$ B1 (max pt). B1

$\therefore$ (3, 0) is a pt of inflexion

(0.75 $\quad$ point.

Q8



Q9

9(i) Co-ord of $P = (q, e^{3q-q^2})$ [B1]

(ii) $A = \frac{1}{2}(4-q)(e^{3q-q^2})$ [B1, B1].

$= (2 - \frac{q}{2})(e^{3q-q^2})$.

(iii) $\frac{dq}{dt} = -2$ units/sec

$\frac{dA}{dt} = \left(\frac{dA}{dq}\right)\left(\frac{dq}{dt}\right)$

$= \left(-\frac{3}{2}e^2\right)(-2)$ [M1]

$= 3e^2 = 22.2$ units²/sec.
[A1]

$\frac{dA}{dq} = \left(2 - \frac{q}{2}\right)\left(e^{3q-q^2}\right)(3-2q) +$
$\left(e^{3q-q^2}\right)\left(-\frac{1}{2}\right)$ [B1]

When $q=2$: $\frac{dA}{dq} = \left(2 - \frac{2}{2}\right)\left(e^{6-4}\right)(3-4) + (e^2)\left(-\frac{1}{2}\right)$

$= -e^2 - \frac{1}{2}e^2$

$= -\frac{3}{2}e^2$ [B1]
(-11.08358)

Q10

10(a) $\angle CDH = 180° - 90° - \theta = 90° - \theta$

$DF = 4\cos\theta$ [B1]

$DH = 3\cos(90° - \theta)$
$= 3\sin\theta$ [B1].

$\therefore AB = 4\cos\theta + 3\sin\theta$

(b) $R = \sqrt{3^2 + 4^2} = 5$ [B1], $\alpha = \tan^{-1}\left(\frac{3}{4}\right) = 36.9°$ [B1]

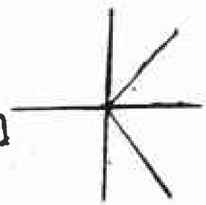
$\therefore AB = 5\cos(\theta - 36.9°)$ [B1].

(c) Max AB is when $\cos(\theta - 36.9°) = 1$ [B1]

$\therefore$ max. perimeter $= 6 + 4 + 3 + 4 + 5$
$= 22$ m. [B1].

When $\cos(\theta - 36.9°) = 1$

$\theta - 36.9° = 0°, 360°$ [M1]



$\therefore \theta = 36.9°, 396.9°$
[A1]. (NA).

Q11

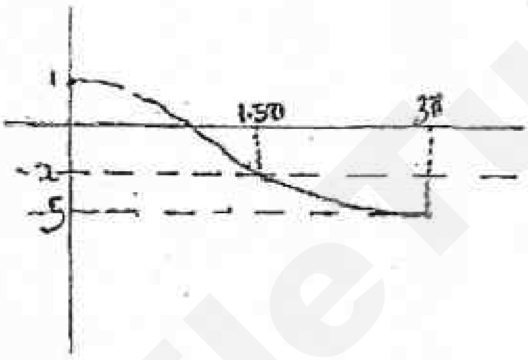
(11) $f(x) = p\cos\left(\frac{x}{3}\right) - q$

(i) Period $= \frac{2\pi}{\frac{1}{3}} = 6\pi$ [B1].

(ii) Amplitude $= 3$ [B1].

(iii) $p = 3$, $q = 2$. [B1; B1].

(iv)



$f(x) = 3\cos\left(\frac{x}{3}\right) - 2$.

G1 : correct shape

G1 : correct y-intercept

G1 : correct values of y at $x = 1.5\pi$ and $x = 3\pi$.

$y = b^{a-x}$

$\lg y = \lg b^{a-x}$

$= (a-x)\lg b$

$= a\lg b - x\lg b$

$\therefore \lg y = (-\lg b)x + a\lg b$

x	1	1.5	2	2.5	3
y	4	2.21	2	1.41	1
$\lg y$	0.602	0.344	0.301	0.149	0

[B1]

Grad of graph $= \frac{0.75 - 0.45}{0.5 - 1.5} = -0.3$ [A1]

Hence $-\lg b = -0.3$ and $a\lg b = 0.9$

$b = 1.995 \approx 2.0$ [B1]

$a = \frac{0.9}{\lg b} = 3$ [B1].

Incorrect value of y is $y = 2.21$ [B1]

Correct value of $\lg y = 0.45$

hence correct $y = 2.82$. [B1].

Q12

Grad of $AB = \frac{4-1}{p-2} = \frac{3}{p-2}$

Since AB ⊥ BC: $\frac{1}{p-6} = \frac{p-2}{-3}$ [M1]

$-3 = (p-6)(p-2)$

$-3 = p^2 - 8p + 12$

$p^2 - 8p + 15 = 0$

$(p-3)(p-5) = 0$ [M1]

$\therefore p = 3$ or $p = 5$ (NA) [A1].

(ii) Midpt AC $= \left(\frac{6+2}{2}, \frac{1+3}{2}\right)$

$= (4, 2)$

Grad of $AC = \frac{3-1}{6-2} = \frac{2}{4} = \frac{1}{2}$ [B1]

Grad of ⊥ bisector of AC $= -2$

$\therefore y = -2x + c$

at $(4,2) \Rightarrow c = y + 2x$ [M1]

$= 2 + 2(4)$

$= 10$, hence $y = -2x + 10$ [A1].

(iii) Midpt of AC $= \left(\frac{6+2}{2}, \frac{3+1}{2}\right) = (4, 2)$

Let D have a coord of (x, y).

$\frac{3+x}{2} = 4$ and $\frac{4+y}{2} = 2$ [M1]

$x = 5$ $\quad y = 0$

(iv) Area $= \frac{1}{2}\begin{vmatrix} 3 & 6 & 5 & 2 & 3 \\ 4 & 3 & 0 & 1 & 4 \end{vmatrix}$ [M1]

$= \frac{1}{2}|22 - 42|$

$= \frac{1}{2}(20) = 10$ units^2 [A1].

CONVENT OF THE HOLY INFANT JESUS SECONDARY
Semestral Assessment 2 in preparation for
the General Certificate of Education Ordinary Level 2017

ADDITIONAL MATHEMATICS

4047/01

Paper 1

23 Aug 2017

2 hours

Additional Materials: Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

sists of **6** printed pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1) \ldots (n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 (i) Write down and simplify the fourth term in the binomial expansion of $\left(x - \frac{p}{x}\right)^n$, where $n > 0$. [2]

(ii) Given that the fourth term is equal to 160, find the value of n. [2]

(iii) With this value of n found in part **(ii)**, calculate the value of p. [2]

2 (i) Express $\frac{3}{3x^2 - 4x + 1}$ in partial fractions. [3]

(ii) Find the second derivative of $\frac{3}{3x^2 - 4x + 1}$ with respect to x and express your answer in the form $\frac{a}{(3x-1)^3} + \frac{b}{(x-1)^3}$, where a and b are integers. [4]

3 (a) Find, correct to 2 decimal places, the value of x which satisfies the equation $3^{4x} = 7^{3x-1}$. [3]

(b) **Without using a calculator**, express $\log_{25} h$ in terms of x, given that $h^x = 125$. [4]

4 It is given that $f(x) = (2 - 3p)x^2 + (4 - p)x + 2$, where p is an integer.

(i) Find the range of values of p for which $f(x) = 0$ has no real roots. [3]

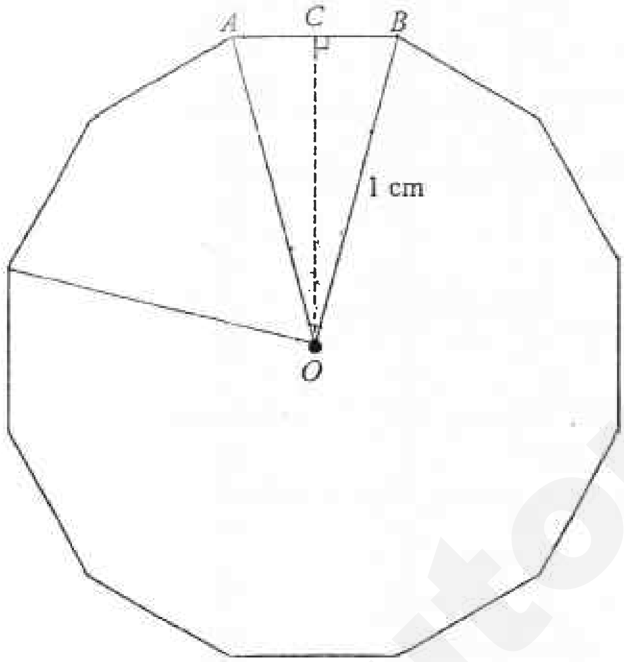
(ii) By considering the result of part **(i)**, explain whether the coefficient of x^2 is positive or negative. [1]

5 The equation of a curve is $y = \frac{x}{e^{2x-1}}$, where $x > \frac{1}{2}$.

(i) Using the **Quotient Rule**, find the gradient of the curve where $x = 2$. [4]

(ii) Given that x is increasing at a rate of e^3 units per second, find the rate of decrease of y when $x = 2$. [2]

6 **Diagram is not drawn to scale**



The diagram shows a regular 12-sided polygon with centre O. AB is one side of the polygon, C is the midpoint of AB and $OB = 1$ cm.

(i) Show that $AB = 2\sin 15°$. [2]

(ii) Express $\cos 30°$ in terms of $\sin 15°$ and show that $\sin 15° = \frac{1}{2}\sqrt{2-\sqrt{3}}$. [3]

7 (i) Prove that $\tan x + \cot x = \frac{2}{\sin 2x}$. [3]

(ii) Find all the angles between 0 and 4 which satisfy the equation

$$\frac{\tan x + \cot x}{4} = \frac{1}{\sqrt{3}}.$$ [4]

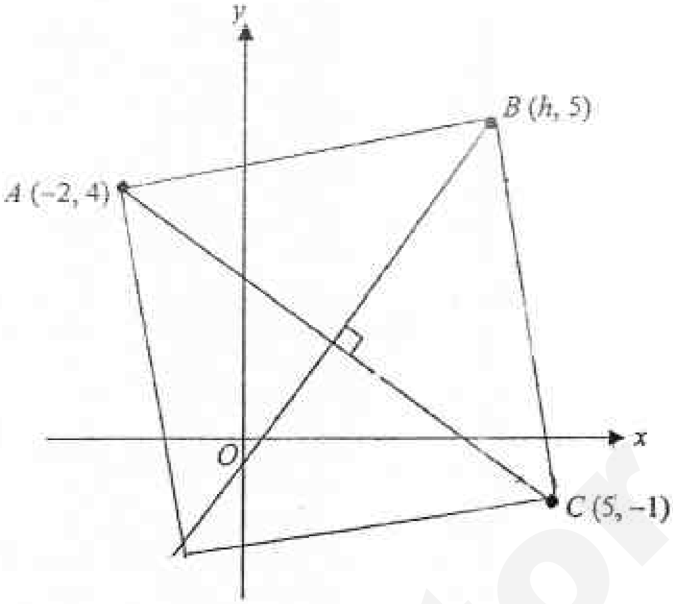
8 It is given that $y = e^{2x}(A\sin 3x + B\cos 3x)$, where A and B are constant.

(i) Obtain an expression for $\frac{dy}{dx}$. [2]

(ii) Find the constants A and B such that $\frac{dy}{dx} = 13e^{2x}\sin 3x$. [4]

(iii) Hence fir [2]

9



In the diagram, the point A is $(-2, 4)$, the point B is $(h, 5)$ and the point C is $(5, -1)$. The point B lies on the perpendicular bisector of AC. Find

(i) the equation of the perpendicular bisector of AC, [3]

(ii) the value of h. [1]

The point D is such that $ABCD$ is a rhombus.

(iii) Find the coordinates of D. [2]

10 In an experiment, the mass, x grams, of a substance is measured at various times, t seconds. The two variables, x and t are related by a law of the form $2x - 10 = ab^{(t-3)}$, where a and b are constants. The table below shows some measured values of x and t and it is believed that one value of x does not conform to this law.

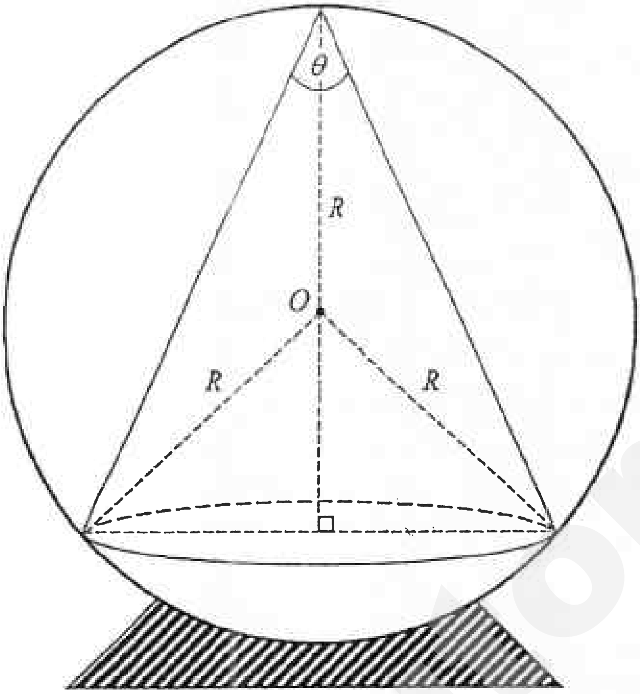
t (seconds)	7	11	15	19
x (grams)	7.4	8.5	10.2	15.0

(i) On graph paper, plot $\lg(2x-10)$ against $(t-3)$ and draw a straight line graph. The vertical $\lg(2x-10)$ - axis should start at 0.4 and have a scale of 2 cm to 0.1. [2]

(ii) Use the graph to estimate the value of a and of b. [3]

(iii) Explain whether it is sensible to use these measured values to predict the mass when $t = 23$. [1]

11



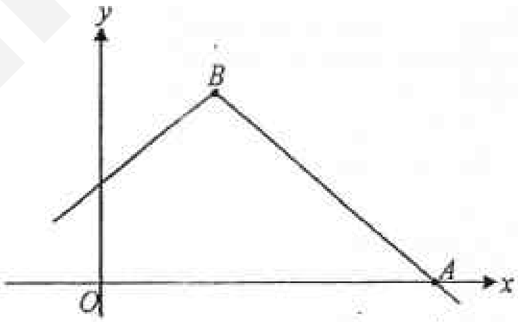
A snow globe is being designed to include a right circular cone inside a hollow sphere of fixed radius R cm and centre O, as shown in the diagram above. The vertical angle of the cone is θ.

(i) Given that the volume of the cone is V cm^3, show that

$$V = \frac{\pi}{3}R^3(1+\cos\theta)\sin^2\theta.$$ [3]

(ii) Given that θ can vary, show that V has a stationary value when $\theta = \cos^{-1}\frac{1}{3}$. [5]

12 **(a)**



The diagram shows part of the graph of $y = 4 - |2x-3|$. Find the coordinates of A and of B. [2]

(b) **(i)** Sketch the graph of $y = |2x^2 - 5x - 3|$ for $-1.5 \le x \le 4$, indicating on your graph the coordinates of the stationary point and of the points where the graph meets the vertical and horizontal axes. [4]

(ii) Calculate the values of x for which $|2x^2 - 5x - 3| = 3$, giving your answers to 2 de riate. [4]

--- End of Paper 1 ---

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CONVENT OF THE HOLY INFANT JESUS SECONDARY
Semestral Assessment 2 in preparation for
the General Certificate of Education Ordinary Level 2017

ADDITIONAL MATHEMATICS

4047/02

Paper 2

12 September 2017

2 hours and 30 minutes

Additional Materials: Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of **6** printed pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

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$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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$$\sin^2 A + \cos^2 A = 1$$

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$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Peter estimates that his score s marks in a class test is given by the formula

$$s = 95 - 58(1.8)^{-0.6t},$$

where t is the number of hours used to prepare for the test.

(i) How many marks can Peter expect to score if he did not prepare for the test? [1]

2½ h

(ii) If Peter spends 2 hours 30 minutes to prepare for the test, what will his expected marks be? Give your answer correct to the nearest whole number. [1]

(iii) Peter's mother expects him to score at least 78 marks, how many hours must he put in in order not to disappoint his mother? Give your answer correct to the nearest half an hour. [4]

2 It is given that $2\log_5(2x-3)+\log_5(x+1)=3$.

(i) Show that $4x^3-8x^2-3x-116=0$. [3]

(ii) Factorise $4x^3-8x^2-3x-116$ completely. [3]

(iii) Hence, solve the equation $2\log_5(2x-3)+\log_5(x+1)=3$. Explain why there is only one real solution to the equation. [2]

3 The area of ΔABC is $11+7\sqrt{3}$ cm^2. $AC = 5+3\sqrt{3}$ cm and $\angle ACB = \frac{\pi}{3}$ radian. Find the length of BC in the form $a+b\sqrt{3}$ where a and b are rational numbers.

[4]

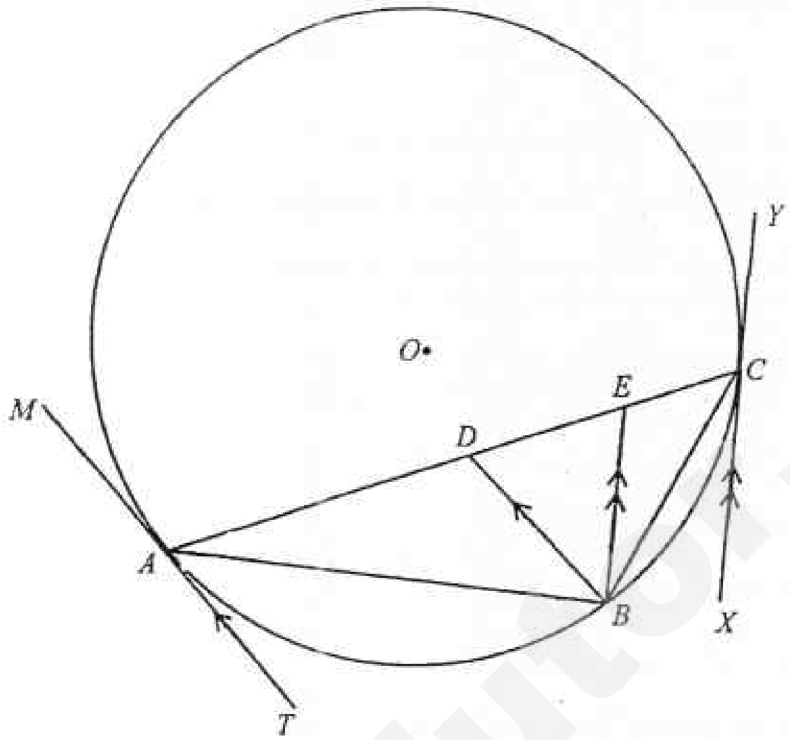
4 Given that α and β are the roots of the equation $3x^2-5x+4=0$, find

(i) the numerical value of $\alpha^3+\beta^3$, [3]

(ii) a quadratic equation whose roots are $\dfrac{\alpha}{\beta+2}$ and $\dfrac{\beta}{\alpha+2}$. [5]

5 Calculate the maximum value of $3uv^2$ given that u and v are two variables such that $u+v=15$. [5]

6



In the diagram, O is the centre of the circle. TAM is the tangent to the circle at A and XCY is the tangent to the circle at C. D and E are points on the chord AC where BD is parallel to TAM and BE is parallel to XCY. Prove that

(i) ΔABD and ΔBCE are similar, [4]

(ii) ΔBDE is isosceles, [2]

(iii) $AD \times CE = BE^2$. [2]

7 A particle moves in a straight line so that, at time t seconds after leaving a fixed point O, its displacement, s m, is given by $s = 8 - 8e^{-2t} - \frac{1}{8}t$. Calculate the

(i) initial velocity of the particle, [2]

(ii) time t when the particle is instantaneously at rest, [3]

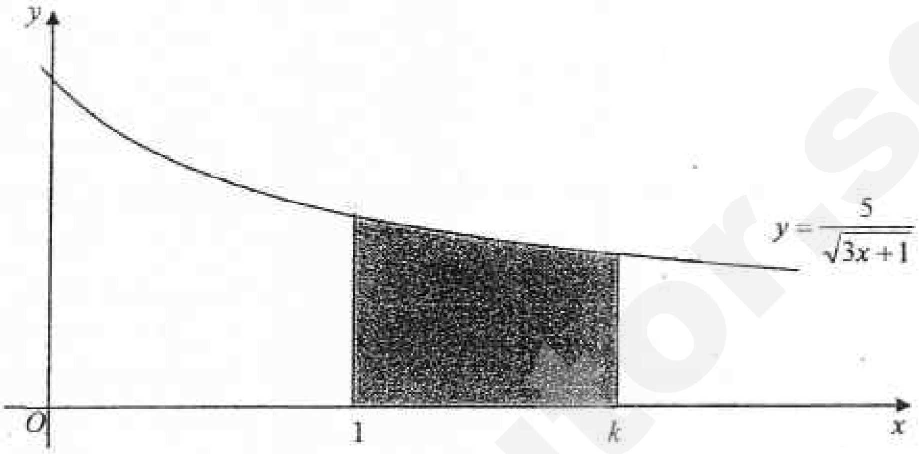
(iii) acceleration of the particle at the time $t = 2$ seconds, [2]

(iv) total distance travelled by the particle in the first 3 seconds. [4]

8 (a) Find $\int e^{\frac{x}{3}} \, dx$. [2]

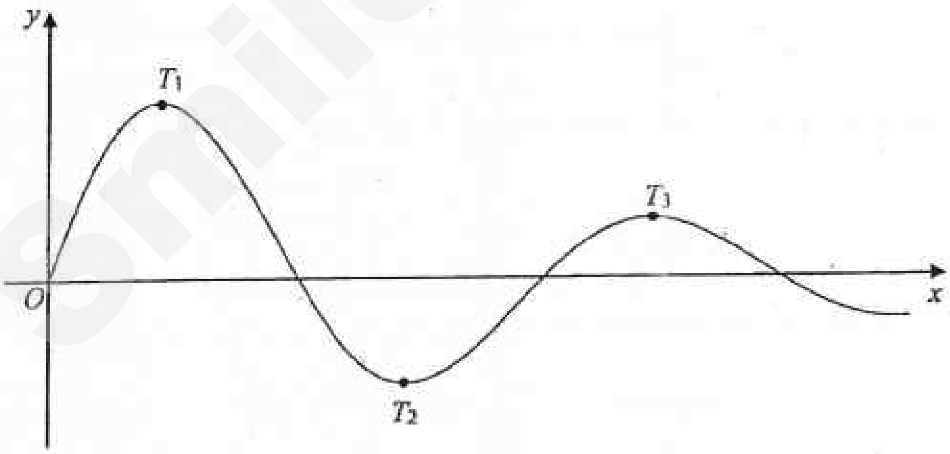
(b) Evaluate $\int_1^e \left(\frac{3}{2x-1} + \frac{9e}{x^2} \right) dx$. [4]

(c)



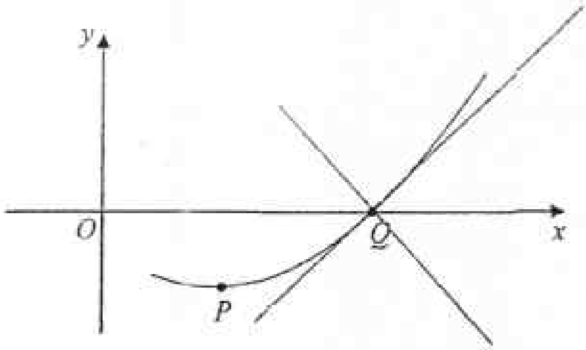
The shaded area bounded by the curve $y = \frac{5}{\sqrt{3x+1}}$, the x-axis and the lines $x = 1$ and $x = k$ is 5 units, find the value of k. [6]

9



The diagram shows the graph of $y = 6e^{-\left(\frac{x}{2}\right)} \sin 3x$ for $x \geq 0$. The first three stationary points are labelled T_1, T_2 and T_3. Find the x-coordinate of T_1 and of T_2, giving your answers correct to 3 decimal places. [6]

10



The diagram shows part of the curve $y = x^2 \ln x$ which intersects the x-axis at Q and has a minimum point at P.

(i) Find the equation of the normal to the curve at Q. [4]

(ii) Show that the x-coordinate of P is $\frac{1}{\sqrt{k}}$, where k is a constant and hence state the exact value of k. [4]

11 (i) Express $3\cos x + 4\sin x$ in the form $R\cos(x-\theta)$ where R is positive and $0° < \theta < 90°$. [4]

(ii) State the minimum value of $(3\cos x + 4\sin x)^2$ and the angles of x between $0°$ and $360°$ which give the minimum value. [2]

(iii) Find the maximum value of $3\cos x + 4\sin x - 2$ and the value of x between $0°$ and $90°$ which gives the maximum value. [3]

(iv) Find the principal value of $6\cos x + 8\sin x = 3$ for $0° < x < 360°$. [3]

12 A circle, C_1, has equation $x^2 + y^2 - 4x - 21 = 0$.

(i) Find the radius and the coordinates of the centre of C_1. [3]

(ii) Find the equation of the tangent to the circle C_1 at the point $(-1,4)$. [3]

The equation of another circle C_2 is $(x+5)^2 + (y-2)^2 = 49$.

(iii) The tangent to the circle C_1 at the point $(-1,4)$ cuts the circle C_2 at points P and Q. Find the x-coordinate of P and of Q, leaving your answers correct to 2 decimal places. [4]

(iv) Determine whether the point $(0,-4)$ is inside, outside or on the circle C_2. [2]

--- **End of Paper 2** ---

CHIJ Sec Toa Payoh 2017 SA2 Paper 2 Numerical solutions

1 (i) 37 (ii) 71 (iii) 3.5 hours or 3h 30 min

2 (ii) $(4x^2+8x+29)(x-4)$

(iii) $x=4$, as $4x^2+8x+29=0$ has no real solutions as $b^2-4ac<0$

3 $-4+\frac{16}{3}\sqrt{3}$

4 (i) $-2\frac{1}{27}$ (ii) $x^2-\frac{31}{78}x+\frac{2}{13}=0$ or $78x^2-31x+12=0$

5 1500

6. **(i)** Let $T\hat{A}B=\theta=A\hat{C}B$ ($\angle$ in alt. segment)

$T\hat{A}B=A\hat{B}D=\theta$ (alt $\angle$, $TAM \parallel BD$)

Let $X\hat{C}B=\alpha=B\hat{A}C$ ($\angle$ in alt. segment)

$XCB=\alpha=C\hat{B}E$ (alt $\angle$, $XCY \parallel BE$)

Since two angles of the triangles are equal, ΔABD and ΔBCE are similar (AA similarity test) A1

(ii) From (i) $B\hat{E}C=A\hat{D}B=\beta$

$\therefore B\hat{D}E=180^\circ-\beta=B\hat{E}D$

$\therefore \Delta BDE$ is isosceles

(iii) From (i) $\frac{AB}{BC}=\frac{BD}{CE}=\frac{AD}{BE}$

$AD\times CE=BE\times BD$

$\therefore AD\times CE=BE^2$ ($\because BD=BE$ from (ii))

7 (i) $15\frac{7}{8}$ ms^{-1} (ii) 2.43 s (iii) -0.586 ms^{-2} (iv) 7.66 m

8 (a) $3e^{\frac{x}{3}}+c$ (b) 25.5 (c) k=3.75

9 0.469 or 1.516

10 (i) $y=-x+1$ (ii) k=e

11 (i) $5\cos(x-53.13^\circ)$ (ii) 0, $x=143.1^\circ$ or 323.1° (iii) 3, $x=53.1^\circ$ (iv) PV of $x=125.7^\circ$

12 (i) r=5, (2,0) (ii) $4y=3x+19$ (iii) 1.04 or -10.08 (iv) outside

CHIJ Sec Toa Payoh 2017 SA2 Paper 1 Numerical solutions

1 (i) [illegible] $n=6$ (iii) $p=-2$

2 [illegible]

3 (a) $x = 1.348 = 1.35$ (to 2 dp) (b) $\therefore \log_{15} h = \dfrac{3}{2x}$

4 (i) $-16 < p < 0$ (ii) The coefficient of x^2 must be positive, because $(2-3p) > 0$ when $-16 < p < 0$

5 (i) $= \dfrac{-3}{e^3}$ or $-0.14936 = -0.149$ (ii) y is decreasing at a rate of 3 units per second

6 (i) $\angle BOC = \dfrac{360°}{24} = 15°$, $\sin \angle BOC = \dfrac{BC}{1}$, $\sin 15° = BC$, $AB = 2 \times \sin 15°$ (Shown)

(ii) $\cos 30° = 1 - 2\sin^2 15°$, $2\sin^2 15° = 1 - \dfrac{\sqrt{3}}{2}$ $\sin^2 15° = \dfrac{2-\sqrt{3}}{4}$, $\sin 15° = \dfrac{1}{2}\sqrt{2-\sqrt{3}}$

7 (i) $\tan x + \cot x = \dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x} = \dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \dfrac{1}{\frac{1}{2}\sin 2x} = \dfrac{2}{\sin 2x}$ (ii) $x = \dfrac{\pi}{6}, \dfrac{\pi}{3}, \dfrac{7\pi}{6}$

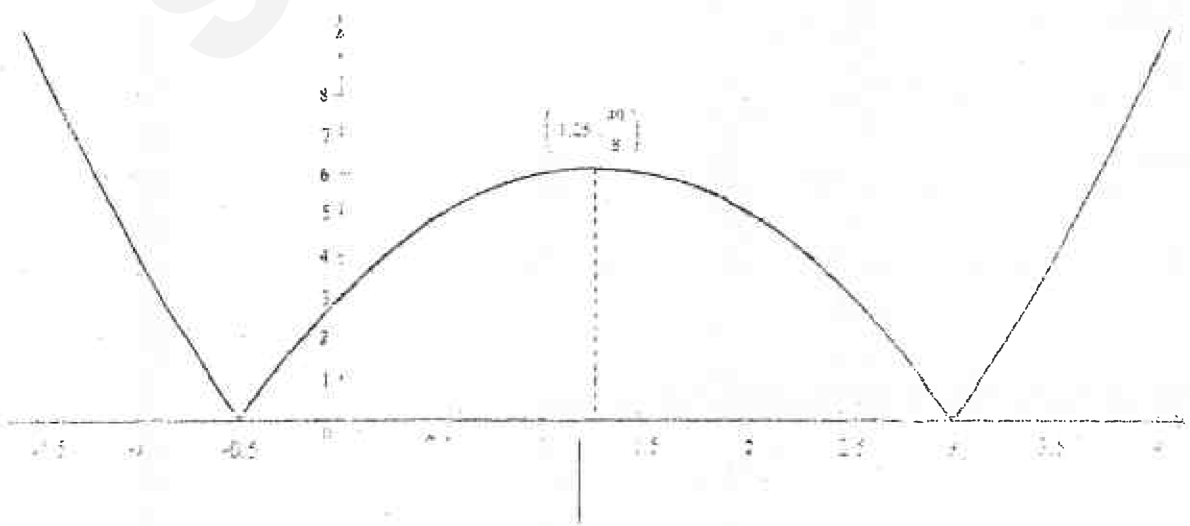
8 (i) $\dfrac{dy}{dx} = 3Ae^{2x}\cos 3x - 3Be^{2x}\sin 3x + 2Ae^{2x}\sin 3x + 2Be^{2x}\cos 3x$

(ii) A=2, B=-3 (iii) $\int (e^{2x}\sin 3x)dx = \dfrac{1}{13}e^{2x}(2\sin 3x - 3\cos 3x) + c$ $y = e^{2x}(A\sin 3x + B\cos 3x)$

9 (i) $y = \dfrac{7}{5}x - \dfrac{3}{5}$ or $5y = 7x - 3$ (ii) $h = 4$ (iii) $\therefore D = (-1, -2)$

10 (ii) a= 3.24 (3 sf) b= 1.10 (3 sf) (iii) From the graph, the relationship between t and x applies for $7 \le t \le 15$. It clearly does not apply to $x = 19$, so it is not sensible to use the graph to predict the mass when $t = 23$.

12 (a) $A\left(\dfrac{7}{2}, 0\right)$, [redacted] $x = 3.39$ (2 dp) or $x = -0.89$ (2 dp)



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GAN ENG SENG SCHOOL
Preliminary 1 Examination 2017

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS
Paper 1

4047/01

9 May 2017
2 hours

Sec 4 Express/ 5 Normal (Academic)

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

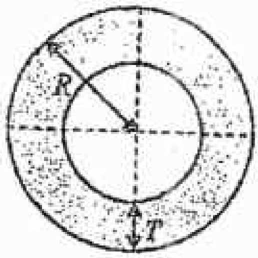
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

	For Examiner's Use
Total	80

This pa ges including the cover page. Turn over

Answer all questions.

1 A hollow copper pipe with an external radius, $R = (4\sqrt{3} - 1)$ cm has a thickness, $T = \sqrt{3}$ cm. The volume of copper needed to make the pipe is $(521\sqrt{3} - 108)\pi$ cm^3.



Find

(i) the cross sectional area of the pipe, in the form $\pi(a + b\sqrt{3})$, where a and b are integers. [2]

(ii) the length of the pipe in the form $(c + d\sqrt{3})$, where c and d are integers. [3]

2 Given that $\int_1^4 px^2\,dx = 6$, where p is a constant,

(i) express $\int_1^4 \left(px^2 + \frac{3k}{2}\right)dx$ in terms of constant k, [2]

(ii) determine the value of p and find the value of $\int_2^8 px^2\,dx$. [3]

3 (i) Find the range of values of k for which the equation $x^2 + 6x + k = 2kx - 9$ has no real roots. [3]

(ii) Hence, deduce by giving a reason, whether the line $y = 18x - 9$ intersects the curve $y = (x+3)^2$. [2]

4 (a) Solve each of the following equations.

(i) $10^x = e^{3x+1}$, [2]

(ii) $\lg|2x+3| = 0$. [2]

(b) Express $2\log_3 15 - (\log_a 5)(\log_3 a)$, where $a > 1$, as a single logarithm to base 3. [3]

5 The equation of a curve is $y = 7 - 5x + 6x^2 - 3x^3$ and $x + y = k$ is a tangent to the curve.

(i) Find the value of k. [4]

(ii) Show that $y = 7 - 5x + 6x^2 - 3x^3$ decreases as x increases. [2]

6 (i) Express $\frac{3x^2 - 6x + 2}{(x-1)^2(x-2)}$ in partial fractions. [5]

(ii) Hence find $\int \frac{9x^2 - 18x + 6}{(x-1)^2(x-2)} dx$. [2]

7 A piece of metal is heated to 100°C and allowed to cool to room temperature. The temperature of metal, θ after it has been cooled for t minutes is given by the equation $\theta = 26 + 74e^{-0.5t}$. Find

(i) its temperature at $t = 3$, [1]

(ii) the time needed to the nearest minute for the temperature to drop to 50°C, [2]

(iii) the rate at which θ is decreasing when $t = 5$, [2]

(iv) the expected room temperature. [1]

8 The number of hours of daylight in a city is given by $L(t) = -3.5\cos\left(\frac{\pi}{6}t\right) + 12.75$, where t is an integer ranging from 1 to 12 inclusive, which represents the month of January to December.

(i) Find the number of hours of daylight in the month of September. [1]

(ii) Find the month which has the highest number of hours of daylight. [3]

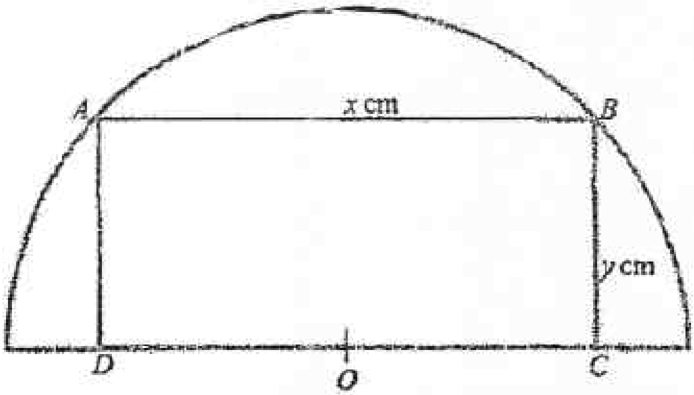
(iii) When should you plant in a garden if you want to do it during the month where there are 11 hours of daylight? [2]

9 (i) Given that $\sin(x + \alpha) = \lambda\cos(x - \alpha)$, show that $\tan x = \frac{\lambda - \tan\alpha}{1 - \lambda\tan\alpha}$. [4]

(ii) Hence solve, for $0 < x < 2\pi$, the equation [3]

$$\sin\left(x + \frac{\pi}{3}\right) = \frac{2\sqrt{3}}{3}\cos\left(x - \frac{\pi}{3}\right).$$

10



In the figure above, $ABCD$ is a rectangle with $AB = x$ cm and $BC = y$ cm. It fits inside a semicircle of radius 10 cm and centre O.

(i) Express y in terms of x. [1]

(ii) Show that A cm^2, the area of the rectangle, is given by $A = \frac{x}{2}\sqrt{400 - x^2}$. [1]

(iii) Given that x can vary, find the value of x for which the area of the rectangle is stationary. [4]

(iv) Explain why this value of x gives the rectangle the largest possible area. [1]

11 The equation of a curve is $y = 3x^2 \ln x$. The tangent to the curve at the point $x = e^2$ meets the x-axis at A and the y-axis at B.

(i) Show that the coordinates of B are $(0, -9e^4)$. [5]

(ii) Calculate the area of triangle AOB in terms of e. [2]

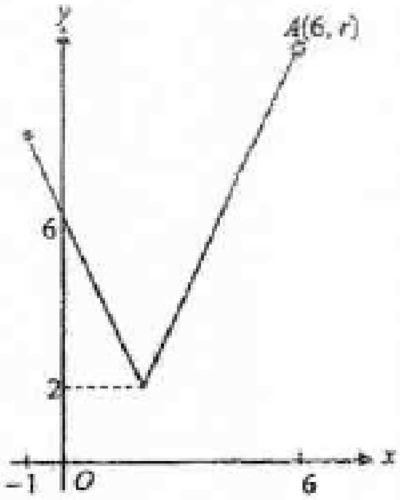
12 It is given that the graph of the function $f(x) = 1 - 3\sin nx$ between the interval $0 \le x \le \frac{\pi}{2}$, where n is a positive integer, intersects the x axis at 2 points.

(i) State the value of n. [1]

(ii) Sketch the graph of $f(x) = 1 - 3\sin nx$, given the value of n in (i) [2]

(iii) By sketching an additional linear graph on the same axes, find the number of solution that satisfy the equation $\sin nx - 1 = \frac{-x}{2\pi}$. [2]

13



The diagram above shows part of the graph of $y=|2x-p|+q$, where p and q are constants for $-1 \le x < 6$. The y-intercept of the graph is 6 and its minimum y-value is 2. A point $A(6, r)$ lies on the graph.

(i) State the value of p and of q. [2]

(ii) Show that $r = 10$. [1]

(iii) A line, $y = mx + c$ is added onto the same axes in the diagram for $-1 \le x < 6$.

(a) In the case where $m = 0$, write down the greatest integer value of c such that the line intersects the graph of $y=|2x-p|+q$ at exactly one point. [1]

(b) In the case where $c = 1$, find the range of values of m such that the line intersects the graph of $y=|2x-p|+q$ at exactly two points. [3]

End-of-paper

e 6

Answer ALL questions

1. (i) Given that $2x+1$ is a factor of the expression $2x^3+5x^2+kx-6$, solve the equation $2x^3+5x^2+kx-6=0$, giving non-integer solutions in the form $a \pm b\sqrt{7}$. [6]

(ii) The roots of the equation $x^2-6x+3=0$ are $2\alpha+\beta$ and $\alpha+2\beta$

Find

(a) the value of $\alpha^3+\beta^3$, [2]

(b) the quadratic equation whose roots are $\dfrac{\alpha^2}{\beta}$ and $\dfrac{\beta^2}{\alpha}$. [5]

2. (i) Solve the equation

$$9^x-5(3^{x+1})+50=0$$ [4]

(ii) Solve the simultaneous equations

$$64^x \times 8^y = 2^{x+1}$$

$$81^{5-x} \div 27^{y+1} = \frac{1}{729^y}$$ [5]

3. (i) The expansion of $(1+ax+bx^2)^8$ in ascending powers of x is given by $1-40x+748x^2+....$
Find the value of a and of b. [4]

(ii) Evaluate the term independent of x in the binomial expansion of $\left(x^2-\dfrac{1}{2x^6}\right)^{16}$. [2]

(iii) In the binomial expansion of $\left(x+\dfrac{k}{x}\right)^9$, where k is a positive constant, the coefficient of x and x^3 are equal.

(a) Find the value of k.

(b) Use the value of k found in part (a) to find the coefficient of x^3 in the expansion of $(1-3x^2)\left(x+\dfrac{k}{x}\right)^9$. [4]

4. A curve is such that $\dfrac{d^2y}{dx^2}=6x-6$ and the gradient of the curve at $(2,-40)$ is -24.
Find the coordinates of the stationary points of the curve and determine their nature. [7]

(4)

5. One night when the street lights were switched on, Jovan was walking towards a lamppost at 1.2 m/s. The lighted lamp on the lamppost was 6 m above the ground and Jovan was 1.5 m tall.

(i) At what rate was the length of his shadow decreasing? [3]

(ii) At what rate was the distance from the end of his shadow to the lamppost decreasing? [3]

6. It is given that $f(x)$ is such that $f'(x) = \cos 2x - \sin x$.

Given that $f\left(\frac{\pi}{2}\right) = 1$, show that $f''(x) + 4f(x) = 3\cos x + 4$. [4]

7. (i) Given that $A = \cos^{-1} p$ and A is acute, calculate in terms of p,

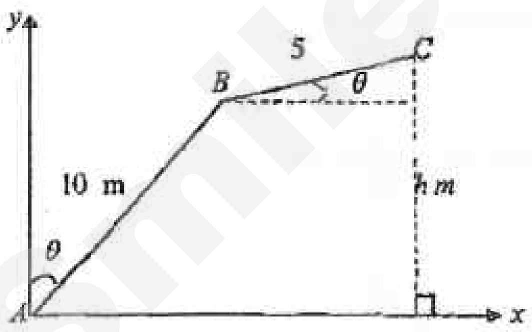
(a) $\sin 2A$ [2]

(b) $\cos 4A$ [2]

(c) $\sin \frac{A}{2}$ [2]

(ii) Prove that $2\operatorname{cosec}^2 2x + 2\cot 2x \operatorname{cosec} 2x = \operatorname{cosec}^2 x$. [3]

8.



The diagram shows an extended mobile crane made up of a movable boom AB and a movable jib BC. At a certain time, the crane is in a vertical plane. Ax is horizontal and Ay is vertical. The boom makes an angle of θ with the vertical and the jib makes an angle of θ with the horizontal. The lengths of AB and BC are 10 m and 5 m respectively. Given that C is h m above Ax,

(i) Find the values of the integers a and b for which $h = a\cos\theta + b\sin\theta$. [2]

Using the value of a and of b found in part (i),

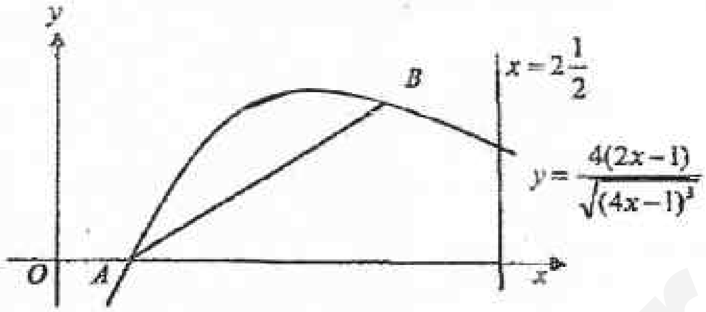
(ii) express h in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) Hence state the maximum value of h and find the corresponding value of θ. [2]

(iv) Find the values of θ when $h = 10.5$ m. [2]

9. (i) Show that $\frac{d}{dx}\left(\frac{2x}{\sqrt{4x-1}}\right)=\frac{4x-2}{\sqrt{(4x-1)^3}}$. [2]

(ii)



The diagram shows the line $x=2\frac{1}{2}$ and part of the curve $y=\frac{4(2x-1)}{\sqrt{(4x-1)^3}}$. The curve intersects the x-axis at A. The line through A with gradient 1 intersects the curve again at B.

(a) Verify that the y-coordinate of B is $\frac{3}{4}$. [5]

(b) Find the area enclosed by the curve $y=\frac{4(2x-1)}{\sqrt{(4x-1)^3}}$, the lines AB, $x=2\frac{1}{2}$ and the x-axis, giving your answer correct to three decimal places. [4]

10. The equation of a circle with centre A is $x^2+y^2-24x-16y+108=0$.

(i) Find the coordinates of A and the radius of the circle. [3]

(ii) Show that $y=-2$ is a tangent to the circle. [2]

A tangent to the circle at B passes through the point $P(-8,-2)$.

(iii) Find the coordinates of B. [4]

(iv) Find the equation of the tangent PB. [2]

11. The variables x and y are related by the equation $y = e^{-A}b^x$, where A and b are constants. The table below shows values of x and corresponding values of y.

x	5	10	15	20	25
y	0.14	1.06	8.02	60.9	462.5

(i) By drawing a straight line graph of $\ln y$ against x, estimate the value of A and of b. [6]

(ii) Use your graph to estimate the value of x when $y = 15$. [2]

(iii) On the same diagram, draw the line representing $y^3 = e^{-x}$ and hence find the value of x for which

$$e^{A-\frac{x}{3}} = b^x.$$ [3]

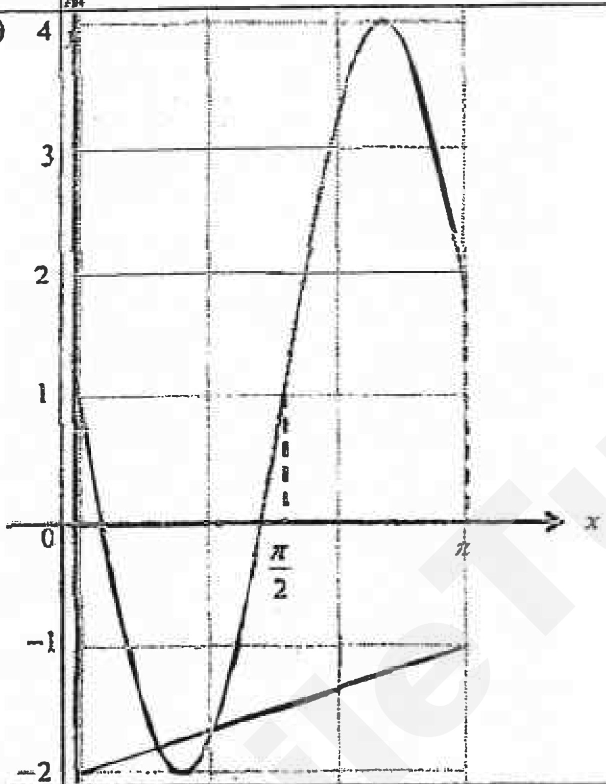
END of PAPER

GAN ENG SENG SCHOOL
PRELIM I 2017 S4EXP/S5NA AMI/OLP

Answers

2017 Sec 4 Express 5 Normal (A) AM Paper 1

1	(i)	Cross-sectional area = $\left(21-2\sqrt{3}\right)\pi$ cm^2
	(ii)	$25\sqrt{3}+2$ cm
2	(i)	$6+\frac{9k}{2}$
	(ii)	48
3	(i)	$0<k<7$
	(ii)	The line $y=18x-9$ will intersect the curve.
4	(a)(i)	-1.43
	(ii)	$x=-1$ or $x=-2$
	(b)	$\log_3 45$
5	(i)	$k=6\frac{1}{9}$
6	(i)	$\frac{3x^2-6x+2}{(x-1)^2(x-2)}=\frac{1}{x-1}+\frac{1}{(x-1)^2}+\frac{2}{x-2}$
	(ii)	$\int\frac{9x^2-18x+6}{(x-1)^2(x-2)}dx=3\int\frac{3x^2-6x+2}{(x-1)^2(x-2)}dx$ $=3\ln(x-1)-\frac{3}{(x-1)}+6\ln(x-2)+c$, where c is a constant
7	(i)	42.5°C
	(ii)	3 minutes (nearest min)
	(iii)	The rate at which θ is decreasing = 3.04 °C/min
	(iv)	The room temperature is 26 °C
8	(i)	12.75 hours
	(ii)	June has the highest number of hours of daylight.
	(iii)	You should plant in February or October.
9	(ii)	$x=\frac{\pi}{6},\frac{7\pi}{6}$
10	(i)	$\left(\frac{x}{2}\right)^2+y^2=10^2$ $y=\sqrt{100-\frac{x^2}{4}}$
	(iii)	$\frac{dA}{dx}=\frac{-2x^2+400}{2\sqrt{400-x^2}}$

		$x = 10\sqrt{2}$ or 14.1 cm
11	(ii)	Area of triangle Triangle AOB = $\frac{27}{10}e^{6}$ units^2
12	(i)	[illegible] $= 2$
	(ii)	
	(iii)	No of solutions = 2
13	(i)	$q = 2$ $p = 4$
	(iii)(a)	$c = 9$
	(b)	$\frac{1}{2} < m < \frac{3}{2}$

GAN ENG SENG SCHOOL
PRELIM 1 2017 S4EXP/S5NA AM1/OLP
Marking Scheme
2017 Sec 4 Express 5 Normal (A) Paper 1

1	(i)	Cross-sectional area $= \pi(4\sqrt{3}-1)^2 - \pi(3\sqrt{3}-1)^2$ $= \pi(16(3)-8\sqrt{3}+1) - \pi(9(3)-6\sqrt{3}+1)$ $= (21-2\sqrt{3})\pi$ cm^2
	(ii)	Give that volume $= (521\sqrt{3}-108)\pi$ $\pi(21-2\sqrt{3})(c+d\sqrt{3}) = (521\sqrt{3}-108)\pi$ $(c+d\sqrt{3}) = \frac{(521\sqrt{3}-108)\pi}{(21-2\sqrt{3})\pi}$ $= \frac{(521\sqrt{3}-108)\pi}{(21-2\sqrt{3})\pi} \times \frac{21+2\sqrt{3}}{21+2\sqrt{3}}$ $= \frac{10941\sqrt{3}+3126-2268-216\sqrt{3}}{441-12}$ $= \frac{10725\sqrt{3}+858}{429}$ $= 25\sqrt{3}+2$ cm
2	(i)	$\int_1^4 \left(px^2 + \frac{3k}{2}\right) dx$ $= \int_1^4 px^2 dx + \frac{3k}{2}[x]_1^4$ $= 6 + \frac{9k}{2}$
	(ii)	$\int_2^8 px^2 dx$ $= \left[\frac{px^3}{3}\right]_2^8$ $= p\left[\frac{512}{3} - \frac{8}{3}\right]$ $= 168p$ Given $\int_1^4 px^2 dx = 6$ $\left[\frac{px^3}{3}\right]_1^4 = 6$ $p\left[\frac{64}{3} - \frac{1}{3}\right] = 6$ $\frac{63}{3}p = 6$ p

		$\int_1^4 px^2 dx = 168\left(\frac{2}{7}\right)$ $= 48$
	(ii)	$\int_1^4 \left(px^2 + \frac{3k}{2}\right)dx$ $= \int_1^4 px^2 dx + \frac{3k}{2}[x]_1^4$ $= 6 + \frac{9k}{2}$
3	(i)	$x^2 + 6x + k = 2kx - 9$ $x^2 + 6x + k - 2kx + 9 = 0$ ------------------ (1) $a = 1, b = 6 - 2k, c = k + 9$ Since equation has no real roots, $D < 0$ $(6-2k)^2 - 4(1)(k+9) < 0$ $36 - 24k + 4k^2 - 4k - 36 < 0$ $4k^2 - 28k < 0$ $4k(k-7) < 0$ $0 < k < 7$
	(ii)	$18x - 9 = (x+3)^2$ $18x - 9 = x^2 + 6x + 9$ $x^2 - 12x + 18 = 0$ ------------------ (2) Compare (1) and (20 $k = 9$ The line $y = 18x - 9$ will intersect the curve.
4	(a)(i)	$10^x = e^{3x+1}$ Take ln on both sides, $x\ln 10 = 3x + 1$ $x(\ln 10 - 3) = 1$ $x = \frac{1}{\ln 10 - 3}$ $= -1.43$
	(ii)	$\lg\lvert 2x+3\rvert = 0$ $\lvert 2x+3\rvert = 1$ $2x + 3 = 1$ or $2x + 3 = -1$ $x = -1$ or $x = -2$
	(b)	$2\log_3 15 - (\log_a 5)(\log_3 a)$ $= 2\log_3 15 - \left(\frac{\log_3 5}{\log_3 a}\right)(\log_3 a)$ $= 2\log_3 15$

10

		$=\log_3 15^2-\log_3 5$ $=\log_3 \frac{15^2}{5}$ $=\log_3 45$
5	(i)	$y=7-5x+6x^2-3x^3$ $\frac{dy}{dx}=-5+12x-9x^2$ $y+x=k$ $y=-x+k$ Gradient of tangent $=-1$ $-5+12x-9x^2=-1$ $9x^2-12x+4=0$ $(3x-2)^2=0$ $x=\frac{2}{3}$ At $x=\frac{2}{3}$, $y=7-5\left(\frac{2}{3}\right)+6\left(\frac{2}{3}\right)^2-3\left(\frac{2}{3}\right)^3$ $=\frac{49}{9}$ Sub $x=\frac{2}{3}$, $y=\frac{49}{9}$ into $y+x=k$ $k=\frac{49}{9}+\frac{2}{3}$ $=\frac{55}{9}$ $=6\frac{1}{9}$
	(ii)	$\frac{dy}{dx}=-5+12x-9x^2$ If y is a decreasing function, $\frac{dy}{dx}<0$ $-5+12x-9x^2$ $=-9\left(x^2-\frac{12}{9}x\right)-5$ $=-9\left[\left(x-\frac{6}{9}\right)^2-\frac{36}{81}\right]-5$ $=-9\left(x-\frac{2}{3}\right)^2+\frac{36}{9}-5$ $=-9\left(x-\frac{2}{3}\right)^2-1$

Since $\left(x-\frac{2}{3}\right)^2 > 0$

$$-9\left(x-\frac{2}{3}\right)^2 < 0$$

$$-9\left(x-\frac{2}{3}\right)^2 - 1 < 0$$

Since $\frac{dy}{dx} < 0$, y decreases as x increases for all real values of x.

6 (i)

$$\frac{3x^2-6x+2}{(x-1)^2(x-2)}$$

$$= \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2}$$

$$= \frac{A(x-1)(x-2)+B(x-2)+C(x-1)^2}{(x-1)^2(x-2)}$$

$$3x^2-6x+2 \equiv A(x-1)(x-2)+B(x-2)+C(x-1)^2$$

Sub $x=1$,

$-B = 3-6+2$

$B = 1$

Sub $x=2$,

$C = 3(2)^2 - 6(2) + 2$

$C = 2$

Compare coefficient of x^2

$A + C = 3$

$A + 2 = 3$

$A = 1$

$$\frac{3x^2-6x+2}{(x-1)^2(x-2)} = \frac{1}{x-1} + \frac{1}{(x-1)^2} + \frac{2}{x-2}$$

(ii)

$$\int \frac{9x^2-18x+6}{(x-1)^2(x-2)}dx = 3\int \frac{3x^2-6x+2}{(x-1)^2(x-2)}dx$$

$$= 3\ln(x-1) - \frac{3}{(x-1)} + 6\ln(x-2) + c$$, where c is a constant

7 (i)

At $t = 3$,

$\theta = 26 + 74e^{-0.5(3)}$

$= 42.5°C$

(ii)

$26 + 74e^{-0.5(t)} = 50$

$74e^{-0.5(t)} = 24$

$e^{-0.5(t)} = \frac{24}{74}$

		$-0.5t = \ln\frac{24}{74}$ $t = 2.2520$ $= 3$ minutes (nearest min)
	(iii)	$\theta = 26 + 74e^{-0.5(t)}$ $\frac{d\theta}{dt} = 74(-0.5)e^{-0.5t}$ At $t = 5$, $\frac{d\theta}{dt} = 74(-0.5)e^{-0.5(5)}$ $= -3.0371$ $= -3.04\,^{\circ}C/min$ The rate at which θ is decreasing $= 3.04\,^{\circ}C/min$
	(iv)	As t becomes larger, $e^{-0.5t}$ approaches 0 $74e^{-0.5t}$ approaches 0 $\theta = 26 + 74e^{-0.5(t)}$ approaches 26 The room temperature is $26\,^{\circ}C$
8	(i)	$L(t) = -3.5\cos\left(\frac{\pi}{6}t\right) + 12.75$ At $t = 9$, $L(9) = -3.5\cos\left(\frac{\pi}{6}\right)(9) + 12.75$ $= 12.75$ hours
	(ii)	For greatest value of $L(t)$, $\cos\left(\frac{\pi}{6}t\right) = -1$ $\frac{\pi}{6}t = \pi, 3\pi$ $t = 6$ June has the highest number of hours of daylight.
	(iii)	$-3.5\cos\left(\frac{\pi}{6}t\right) + 12.75 = 11$ $-3.5\cos\left(\frac{\pi}{6}t\right) = 11 - 12.75$ $\cos\left(\frac{\pi}{6}t\right) = \frac{1}{2}$ $\frac{\pi}{6}t = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$ $t = 2, 10$ You should plan

9	(i)	$\sin(x+\alpha)=\lambda\cos(x-\alpha)$ $\sin x\cos\alpha+\cos x\sin\alpha=\lambda[\cos x\cos\alpha+\sin x\sin\alpha]$ $\sin x\cos\alpha-\lambda\sin x\sin\alpha=\lambda\cos x\cos\alpha-\cos x\sin\alpha$ $\div\cos x$ throughout $\tan x\cos\alpha-\lambda\tan x\sin\alpha=\lambda\cos\alpha-\sin\alpha$ $\tan x[\cos\alpha-\lambda\sin\alpha]=\lambda\cos\alpha-\sin\alpha$ $\tan x=\frac{(\lambda\cos\alpha-\sin\alpha)\div\cos\alpha}{(\cos\alpha-\lambda\sin\alpha)\div\cos\alpha}$ $=\frac{\lambda-\tan\alpha}{1-\lambda\tan\alpha}$ (shown)
	(ii)	Sub $\alpha=\frac{\pi}{3}$ and $\lambda=\frac{2\sqrt{3}}{3}$ $\tan x=\frac{\frac{2\sqrt{3}}{3}-\tan\frac{\pi}{3}}{1-\frac{2\sqrt{3}}{3}\tan\frac{\pi}{3}}$ $=\frac{\frac{2\sqrt{3}}{3}-\sqrt{3}}{1-\left(\frac{2\sqrt{3}}{3}\right)(\sqrt{3})}$ $=\frac{\frac{-\sqrt{3}}{3}}{-1}$ $=\frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$ $x=\frac{\pi}{6},\frac{7\pi}{6}$
10	(i)	$\left(\frac{x}{2}\right)^2+y^2=10^2$ $y=\sqrt{100-\frac{x^2}{4}}$
	(ii)	$A=xy$ $=x\sqrt{100-\frac{x^2}{4}}$ $=x\sqrt{\frac{400-x^2}{4}}$ $=\frac{x}{2}\sqrt{400-x^2}$
	(iii)	$\frac{dA}{dx}=\frac{x}{2}\times\frac{d}{dx}\sqrt{400-x^2}+\sqrt{400-x^2}\times\frac{d}{dx}\left(\frac{x}{2}\right)$ $=\frac{x}{2}\times\frac{1}{2}(400-x^2)^{\frac{-1}{2}}(-2x)+\frac{1}{2}\sqrt{400-x^2}$

$$= \frac{-x^2}{2\sqrt{400-x^2}} + \frac{\sqrt{400-x^2}}{2}$$

$$= \frac{-x^2+400-x^2}{2\sqrt{400-x^2}}$$

$$= \frac{-2x^2+400}{2\sqrt{400-x^2}}$$

Let $\frac{dA}{dx} = 0$

$$\frac{-2x^2+400}{2\sqrt{400-x^2}} = 0$$

$$400-2x^2 = 0$$

$x = 10\sqrt{2}$ or 14.14 cm

(iv)

x	14	$10\sqrt{2}$	14.2
$\frac{dA}{dx}$	0.280	0	-0.116
Sketch of gradient			

Using <u>1st derivative test,</u> there is a change of sign of $\frac{dA}{dx}$ from positive to negative as x increases through $x = 10\sqrt{2}$, thus at $x = 10\sqrt{2}$, the area of the rectangle, A is the largest possible.

11 (i)

$y = 3x^2 \ln x$

$$\frac{dy}{dx} = 3x^2\left(\frac{1}{x}\right) + 6x\ln x$$

$$= 3x + 6x\ln x$$

At $x = e^2$

$$\frac{dy}{dx} = 3e^2 + 6e^2 \ln e^2$$

$$= 3e^2 + 12e^2$$

$$= 15e^2$$

At $x = e^2$

$$y = 3(e^2)^2 \ln e^2$$

$$y = 6e^4$$

Equation of tangent:

$$y - 6e^4 = 15e^2(x - e^2)$$

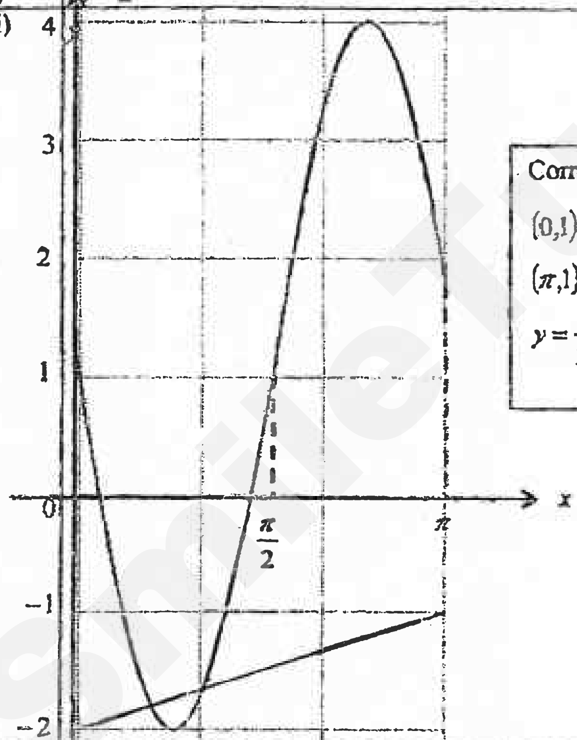
$$y = 15e^2x - 9e^4$$

Intersection with y-axis, let $x = 0$,

$$y = 15e^2(0) - 9e^4$$

$$y = -9e^4$$

B $(0, -9e^4)$

	(ii)	Intersection with x-axis, let $y=0$, $15e^2x-9e^4=0$ $15e^2x=9e^4$ $x=\frac{3}{5}e^2$ B $(\frac{3}{5}e^2, 0)$ Area of triangle Triangle AOB $=\frac{1}{2}\times\frac{3}{5}e^2\times 9e^4$ $=\frac{27}{10}e^6$ units^2
12	(i)	$a=2$
	(ii)	 Correct Shape $(0,1)$, $\left(\frac{\pi}{4},-2\right)$, $\left(\frac{\pi}{2},1\right)$, $\left(\frac{3\pi}{4},4\right)$, $(\pi,1)$ seen $y=\frac{3x}{2\pi}-2$ drawn
	(iii)	$\sin nx-1=\frac{-x}{2\pi}$ $-3\sin nx+3=\frac{3x}{2\pi}$ $-3\sin nx+1=\frac{3x}{2\pi}-2$ To draw $y=\frac{3x}{2\pi}-2$ No of solutions $=2$
13	(i)	$q=2$

		$p=4$
	(ii)	$y=\lvert 2x-4\rvert+2$ Sub in $x=6$. $y=\lvert 2(6)-4\rvert+2$ $y=10$
	(iii)(a)	$c=9$
	(b)	Consider $(0,1)$ and $(2,2)$ Gradient $=\dfrac{2-1}{2-0}$ $=\dfrac{1}{2}$ Consider $(0,1)$ and $(6,10)$ Gradient $=\dfrac{10-1}{6-0}$ $=\dfrac{3}{2}$ $\dfrac{1}{2}<m<\dfrac{3}{2}$

GAN ENG SENG SCHOOL
PRELIMINARY EXAMINATION 1 – 2017
SECONDARY 4 EXPRESS/SECONDARY 5 NORMAL
ADDITIONAL MATHEMATICS 2

1. (i) Let $f(x)=2x^3+5x^2+kx-6$

Since $2x+1$ is a factor,

$$f(-\frac{1}{2})=2\left(-\frac{1}{2}\right)^3+5\left(-\frac{1}{2}\right)^2-\frac{1}{2}k-6=0$$

$$-\frac{1}{4}+\frac{5}{4}-\frac{1}{2}k-6=0$$

$$-\frac{1}{2}k=6-1$$

$$k=-10$$

$$\begin{array}{r} x^2+2x-6 \\ 2x+1\overline{)2x^3+5x^2-10x-6} \\ -\underline{(2x^3+x^2)} \\ 4x^2-10x \\ -\underline{(4x^2+2x)} \\ -12x-6 \\ -\underline{(12x-6)} \\ 0 \end{array}$$

$$(2x+1)(x^2+2x-6)=0$$

$$x=-\frac{1}{2} \text{ or } x^2+2x-6=0$$

If $x^2+2x-6=0$

$$x=\frac{-2\pm\sqrt{(-2)^2-(4)(1)(-6)}}{2}$$

$$=\frac{-2\pm\sqrt{28}}{2}$$

$$=\frac{-2\pm2\sqrt{7}}{2}$$

$$=-1\pm\sqrt{7}$$

$$x=-\frac{1}{2} \text{ or } x=-1\pm\sqrt{7}$$

1.	(ii)		$x^2-6x+3=0$ Sum of roots = $2\alpha+\beta+\alpha+2\beta=6$ $3\alpha+3\beta=6$ $\alpha+\beta=2$ Product of roots = $(2\alpha+\beta)(\alpha+2\beta)$ $2\alpha^2+5\alpha\beta+2\beta^2=3$ $2\alpha^2+2\beta^2+5\alpha\beta=3$ $2(\alpha^2+\beta^2)+5\alpha\beta=3$ $2[(\alpha+\beta)^2-2\alpha\beta]+5\alpha\beta=3$ $2(2^2)-4\alpha\beta+5\alpha\beta=3$ $\alpha\beta=3-8$ $\alpha\beta=-5$
		(a)	$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=2[(\alpha+\beta)^2-2\alpha\beta-\alpha\beta]$ $=2[(\alpha+\beta)^2-3\alpha\beta]$ $=2[2^2-3(-5)]$ $=2[4+15]$ $=38$
		(b)	Sum of roots = $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$ $=\frac{\alpha^3+\beta^3}{\alpha\beta}$ $=\frac{38}{-5}$ Product of roots = $\frac{\alpha^2}{\beta}\times\frac{\beta^2}{\alpha}$ $=\alpha\beta$ $=-5$ The quadratic equation is $x+\frac{38x}{5}-5=0$ $5x^2+38x-25=0$ Ans

2	(i)	$9^x-5(3^{x+1})+50=0$ $3^{2x}-5(3^x\times 3)+50=0$ $3^{2x}-15(3^x)+50=0$ Let $u=3^x$ $u^2-15u+50=0$ $(u-10)(u-5)=0$ $u=$ [illegible] If $u=1$ $3^x=$

		$\lg 3^x = \lg 10$
		$x\lg 3 = 1$ $x = \dfrac{1}{\lg 3} = 2.0959$ $x \approx 2.10$ If $u = 5$ $3^x = 5$ $\lg 3^x = \lg 5$ $x\lg 3 = \lg 5$ $x = \dfrac{\lg 5}{\lg 3}$ $x \approx 1.464$ $x \approx 1.46$

	(ii)	$64^x \times 8^y = 2^{x+1}$ $2^{6x} \times 2^{3y} = 2^{x+1}$ $6x + 3y = x + 1$ $5x + 3y = 1$ .... (1) $81^{5-x} \div 27^{y+1} = \dfrac{1}{729^y}$ $3^{4(5-x)} \div 3^{3(y+1)} = \dfrac{1}{3^{6y}}$ $3^{20-4x-3y-3} = 3^{-6y}$ $20 - 4x - 3y - 3 = -6y$ $-4x - 3y + 6y = -17$ $4x - 3y = 17$ ....(2) Eqn (1) + Eqn (2) $9x = 18.$ $x = 2$ Ans If $x = 2$, $5(2) + 3y = 1$ $3y = 1 - 10$ $3y = -9$ $y = -3$ Ans

3.	(i)	$(1 + ax + bx^2)^8 = [1 + (ax + bx^2)]^8$ $= 1^8 + \binom{8}{1}(1^7)(ax + bx^2)^1 + \binom{8}{2}(1^6)(ax + bx^2)^2 + \ldots\ldots$ $= 1 + 8(ax + bx^2) + 28(a^2x^2 + \ldots) + \ldots$ $= 1 + 8ax + 8bx^2 + 28a^2x^2 + \ldots\ldots$ $= 1 - 40x + 748x^2 + \ldots$ Compare terms in x, $8ax = -40x$ $8a = -40$ $a = -\dfrac{40}{8}$ $a =$

		Compare terms in x^2 $8bx^2+28a^2x^2=748x^2$ $8b+28a^2=748$ $8b=748-28(-5)^2$ $b=\dfrac{748-28(-5)}{8}$ $b=6$
3.	(ii)	$\left(x^2-\dfrac{1}{2x^6}\right)^{16}=(x^2)^{16}+\binom{16}{1}(x^2)^{15}\left(-\dfrac{1}{2x^6}\right)^1+\binom{16}{2}(x^2)^{14}\left(-\dfrac{1}{2x^6}\right)^3+$ $\binom{16}{3}(x^2)^{13}\left(-\dfrac{1}{2x^6}\right)^4+\binom{16}{4}(x^2)^{12}\left(-\dfrac{1}{2x^6}\right)^4+\ldots\ldots$ Term independent of x is $113\frac{3}{4}$ Ans

	(iii)	(a)	$\left(x+\dfrac{k}{x}\right)^9=x^9+\binom{9}{1}(x^8)\left(\dfrac{k}{x}\right)+\binom{9}{2}(x^7)\left(\dfrac{k}{x}\right)^2+\binom{9}{3}(x^6)\left(\dfrac{k}{x}\right)^3+\binom{9}{4}(x^5)\left(\dfrac{k}{x}\right)^4$ $\binom{9}{3}k^3=\binom{9}{4}k^4$ $\dfrac{9\cdot 8\cdot 7k^3}{1\cdot 2\cdot 3}=\dfrac{9\cdot 8\cdot 7\cdot 6k^4}{1\cdot 2\cdot 3\cdot 4}$ $1=\dfrac{6k}{4}$ $k=\dfrac{2}{3}$
		(b)	$\left(x+\dfrac{k}{x}\right)^9=\ldots\ldots\binom{9}{3}(x^6)\left(\dfrac{2}{3x}\right)^3+\binom{9}{4}(x^5)\left(\dfrac{2}{3x}\right)^4+\ldots$ $=\ldots\ldots+84\times\dfrac{8x^3}{27}+126\times\dfrac{16x}{81}+\ldots\ldots$ $(1-3x^2)\left(\ldots+\dfrac{224x^3}{9}+\dfrac{224x}{9}+\ldots\right)$ Term with $x^3=\dfrac{224x^3}{9}-\dfrac{3\times 224x^3}{9}$ Coefficient of $x^3=-\dfrac{2\times 224}{9}=-\dfrac{448}{9}$

4.	$\dfrac{d^2y}{dx^2}=6x-6$ $\dfrac{dy}{dx}=\int(6x-6)d$ $=3x^2-6x+c$

At $x = 2$, $\frac{dy}{dx} = -24$

$$-24 = 3\times2^2 - 6\times2 + c$$
$$c = -24 - 12 + 12$$
$$c = -24$$

Therefore $\frac{dy}{dx} = 3x^2 - 6x - 24$

$$y = \int(3x^2 - 6x - 24)dx$$

$y = x^3 - \frac{6x^2}{2} - 24x + p$ where p is a constant.

At $x = 2, y = -40$

$$-40 = 2^3 - 3\times2^2 - 24\times2 + p$$
$$p = -40 - 8 + 12 + 48$$
$$p = 12$$

therefore $y = x^3 - 3x^2 - 24x + 12$

When $\frac{dy}{dx} = 0$, $3x^2 - 6x - 24 = 0$

$$x^2 - 2x - 8 = 0$$
$$(x - 4)(x + 2) = 0$$
$$x = 4 \text{ or } x = -2$$

At $x = 4$, $y = 4^3 - 3\times4^2 - 24\times4 + 12$

$$y = -68$$

At $x = 4$, $\frac{d^2y}{dx^2} = 6\times4 - 6 = 18 > 0$

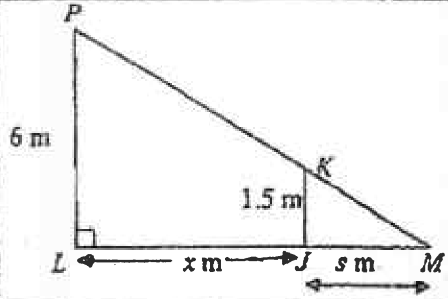
Therefore $(4, -68)$ is a minimum point.

At $x = -2$, $\frac{d^2y}{dx^2} = 6(-2) - 6 = -18 < 0$

At $x = -2$, $y = (-2)^3 - 3(-2)^2 - 24(-2) + 12$

$$y = 40$$

Therefore $(-2, 40)$ is a maximum point.

5.	(i)		Let PM be the lamppost, JK be Jovan, JM be the shadow of s m. Let LJ be the distance of Jovan from the lamppost of x m.

By similar triangles,

$$\frac{s}{s+x}=\frac{1.5}{6}$$

$$s=\frac{1}{4}(x+s)$$

$$s-\frac{1}{4}s=\frac{x}{4}$$

$$\frac{3}{4}s=\frac{x}{4}$$

$$s=\frac{x}{3}$$

$$\frac{ds}{dx}=\frac{1}{3}$$

By the chain rule, $\frac{ds}{dt}=\frac{ds}{dx}\times\frac{dx}{dt}$

$$\frac{ds}{dt}=\frac{1}{3}(-1.2)$$

$$=-0.4 \text{ m/s}$$

Shadow is decreasing at 0.4 m/s.

(ii) Let $LM=y$ m.

By similar triangles

$$\frac{y-x}{y}=\frac{1.5}{6}$$

$$\frac{y-x}{y}=\frac{1}{4}$$

$$4y-4x=y$$

$$4y-y=4x$$

$$3y=4x$$

$$y=\frac{4x}{3}$$

$$\frac{dy}{dx}=\frac{4}{3}$$

By the chain rule, $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$

$$=\frac{4}{3}(-1.2)$$

Shadow is decr

At $x=2$, $\frac{dy}{dx}=-24$

$$-24=3\times2^2-6\times2+c$$
$$c=-24-12+12$$
$$c=-24$$

Therefore $\frac{dy}{dx}=3x^2-6x-24$

$y=\int(3x^2-6x-24)dx$

$y=x^3-\frac{6x^2}{2}-24x+p$ where p is a constant.

At $x=2, y=-\ 40$

$-\ 40=2^3-3\times2^2-24\times2+p$

$p=-\ 40-8+12+48$

$p=12$

therefore $y=x^3-3x^2-24x+12$

When $\frac{dy}{dx}=0$, $3x^2-6x-24=0$

$$x^2-2x-8=0$$
$$(x-4)(x+2)=0$$
$$x=4 \text{ or } x=-2$$

At $x=4$, $y=4^3-3\times4^2-24\times4+12$

$y=-\ 68$

At $x=4$, $\frac{d^2y}{dx^2}=6\times4-6=18>0$

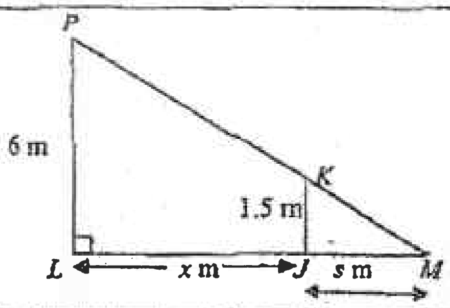
Therefore $(4,\ -68)$ is a minimum point.

At $x=-2$, $\frac{d^2y}{dx^2}=6(-2)-6=-18<0$

At $x=-2$, $y=(-2)^3-3(-2)^2-24(-2)+12$

$y=40$

Therefore $(-2, 40)$ is a maximum point.

5.	(i)		Let PM be the lamppost, JK be Jovan, JM be the shadow of s m. Let LJ be the distance of Jovan from the lamppost of x m.

By similar triangles,

$$\frac{s}{s+x}=\frac{1.5}{6}$$

$$s=\frac{1}{4}(x+s)$$

$$s-\frac{1}{4}s=\frac{x}{4}$$

$$\frac{3}{4}s=\frac{x}{4}$$

$$s=\frac{x}{3}$$

$$\frac{ds}{dx}=\frac{1}{3}$$

By the chain rule, $\frac{ds}{dt}=\frac{ds}{dx}\times\frac{dx}{dt}$

$$\frac{ds}{dt}=\frac{1}{3}(-1.2)$$

$$=-0.4 \text{ m/s}$$

Shadow is decreasing at 0.4 m/s.

(ii) Let $LM=y$ m.

By similar triangles

$$\frac{y-x}{y}=\frac{1.5}{6}$$

$$\frac{y-x}{y}=\frac{1}{4}$$

$$4y-4x=y$$

$$4y-y=4x$$

$$3y=4x$$

$$y=\frac{4x}{3}$$

$$\frac{dy}{dx}=\frac{4}{3}$$

By the chain rule, $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$

$$=\frac{4}{3}(-1.2)$$

Shadow is decreas

			$=\sqrt{125}\sin(\theta+63.4^\circ)$

8.	(iii)	Max value of $h=\sqrt{125}$ m when $\sin(\theta+63.4^\circ)=1$. $\theta+63.4^\circ=90^\circ$ $\theta=90^\circ-63.4^\circ$ $=26.6^\circ$
	(iv)	When $h=10.5$m $\sqrt{125}\sin(\theta+63.4^\circ)=10.5$ $\sin(\theta+63.4^\circ)=\frac{10.5}{\sqrt{125}}$ $\theta+63.4^\circ=69.9^\circ$ or 110.1° $\theta=6.5^\circ$ or 46.7°

9.	(i)	$\frac{d}{dx}\left(\frac{2x}{\sqrt{4x-1}}\right)=\frac{(4x-1)^{\frac{1}{2}}\frac{d}{dx}(2x)-2x\frac{d}{dx}\left(\sqrt{4x-1}\right)}{4x-1}$ $=\frac{2(4x-1)^{\frac{1}{2}}-2x\left(\frac{1}{2}\right)(4x-1)^{\frac{-1}{2}}(4)}{4x-1}$ $=\frac{2(4x-1)^{\frac{1}{2}}-\frac{4x}{(4x-1)^{\frac{1}{2}}}}{4x-1}$ $=\frac{2(4x-1)-4x}{(4x-1)^{\frac{3}{2}}}$ $=\frac{8x-2-4x}{\sqrt{(4x-1)^3}}$ $=\frac{4x-2}{\sqrt{(4x-1)^2}}$
	(ii)	If $y=\frac{4(2x-1)}{\sqrt{(4x-1)^3}}$ meets $y=0$ $4(2x-1)=0$ $2x-1=0$ $x=\frac{1}{2}$ $A\left(\frac{1}{2},0\right)$ Equation of AB is $y-0=1(x-\frac{1}{2})$. $y=x-\frac{1}{2}$

If $y = x - \frac{1}{2}$ meets $y = \frac{4(2x-1)}{\sqrt{(4x-1)^3}}$,

$$\frac{4(2x-1)}{\sqrt{(4x-1)^3}} = x - \frac{1}{2}$$

$$(2x-1)\left(\frac{4}{\sqrt{(4x-1)^3}} - \frac{1}{2}\right) = 0$$

If $\frac{4}{\sqrt{(4x-1)^3}} = \frac{1}{2}$

$$(4x-1)^{\frac{3}{2}} = 8$$

$$4x-1 = 8^{\frac{2}{3}}$$

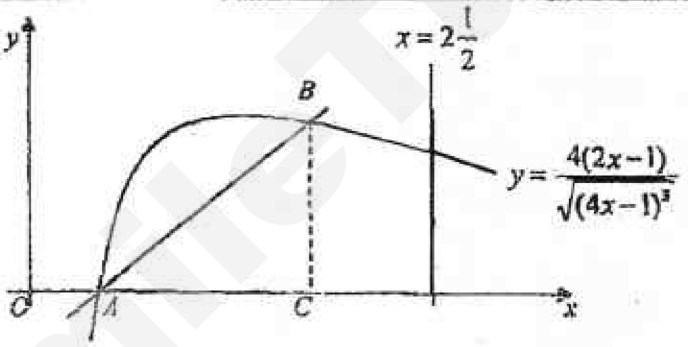
$$4x-1 = 4$$

$$4x = 5$$

$$x = \frac{5}{4}$$

If $x = \frac{5}{4}$, $y = \frac{5}{4} - \frac{1}{2} = \frac{3}{4}$ (proven)

(ii) (b)



Area of triangle ABC $= \frac{1}{2}\left(\frac{5}{4} - \frac{1}{2}\right) \times \frac{3}{4} = \frac{9}{32}$ units^2.

Area enclosed by $x = 2\frac{1}{2}$, $x = \frac{5}{4}$, the curve and the x-axis

$$= \int_{\frac{3}{4}}^{2\frac{1}{2}} \frac{4(2x-1)}{(4x-1)^{\frac{3}{2}}} dx = 2\int_{\frac{5}{4}}^{2\frac{1}{2}} \frac{4x-2}{(4x-1)^{\frac{3}{2}}} dx$$

$$= 2\left[\frac{2x}{\sqrt{4x-1}}\right]_{1.25}^{2.5}$$

$$= 2\left[\frac{2\times 2\frac{1}{2}}{\sqrt{4\times 2.5-1}} - \frac{2\times 1.25}{\sqrt{4\times 1.25-1}}\right]$$

$$= 2\left[\frac{5}{3}-\frac{\frac{5}{2}}{2}\right] = 2\times5\left[\frac{1}{3}-\frac{1}{4}\right]$$

$$= \frac{10}{12}\text{ units}^2$$

Area enclosed by the curve, the line AB, $x = 2\frac{1}{2}$, and the x-axis $= \frac{9}{32}+\frac{10}{12}$

$$= 1\frac{11}{42} \quad \text{or} \quad 1.11\text{ units}^2$$

10. (i)

$x^2+y^2-24x-16y+108=0$

$(x^2-24x)+(y^2-16y)=-108$

$x^2-24x+144+y^2-16y+64=-108+144+64$

$(x-12)^2+(y-8)^2=100$

Coordinates of A is (12, 8)

Radius of circle $= \sqrt{100} = 10$ units

(ii)

If $y=-2$

$x^2+(-2)^2-24x-16(-2)+108=0$

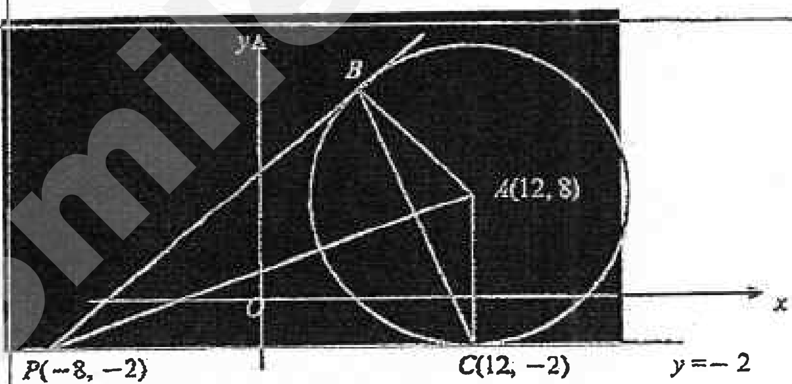
$x^2-24x+144=0$

$(x-12)^2=0$

$x=12$

Since there is only point of contact, $y=-2$ is a tangent.

(iii)



Let C be at (12, −2).

Gradient of $AP = \frac{8+2}{12+8} = \frac{1}{2}$

Gradient of $BC = -2$.

Equation of BC is $y+2=-2(x-12)$.

$y=-2x+24-2$

$y=-2x+22$

If $y=-2x+22$ meets $x^2+y^2-24x-16y+108=0$

$x^2+(-2x+22)^2-24x-16(-2x+22)+108=0$

$x^2+4x^2-88x+484-24x+32x-352+108=0$

0

		$x^2 - 16x + 48 = 0$ $(x-12)(x-4) = 0$ $x = 12$ (rejected) or $x = 4$ If $x = 4$, $y = -2\times4+22$ $y = 14$ $B(4, 14)$ Ans

10.	(iv)	Gradient of $BP = \dfrac{14+2}{4+8} = \dfrac{4}{3}$ Equation of tangent PB is $y-14=\dfrac{4}{3}(x-4)$ $3y-42 = 4x-16$ $3y = 4x+26$ Ans

11.	(i)	$y = e^{-A}b^x$ $\ln y = \ln e^{-A} + \ln b^x$ $\ln y = -A + x\ln b$
	(i)	From the graph, $-A = -4$ $A = 4$ (accept 4 ± 0.2) $\ln b \approx \dfrac{4+2}{15}$ $b \approx 1.49$ (accept ±0.1)
	(ii)	If $y = 15$, $\ln 15 = 2.70$ From the graph $x \approx 17.2$
	(iii)	$y^3 = e^{-x}$ $y = e^{\frac{-x}{3}}$ $\ln y = -\dfrac{x}{3}$

x	5	10	15	20	25
$\ln y$	-1.97	0.058	2.08	4.11	6.14

x	0	12
lny	0	-4

$e^{A-\frac{x}{3}} = b^x$

$e^{-\frac{x}{3}} = \dfrac{b^x}{e^A}$

$e^{-\frac{x}{3}} = e^{-A}b^x$

From the graph $x \approx 5.5$.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

1 Show that the equation $x^2 + (2-k)x + k = 3$ has real roots for all real values of k. [4]

2 (i) Sketch the graph of $y = |4-x|$. [2]

(ii) Determine the number of intersections of the line $y = \frac{1}{2}x$ with $y = |4-x|$, justifying your answer. [2]

3 Express $\dfrac{3x^2+2x-28}{x^2-5x+6}$ in partial fractions. [5]

4 A rectangular block has a square base. The length of each side of the base is $(2\sqrt{2}-\sqrt{3})$ m and the volume of the block is $(21\sqrt{2}-13\sqrt{3})$ m^3. Find, **without using a calculator**, the height of the block in the form $(a\sqrt{2}+b\sqrt{3})$ m, where a and b are integers. [5]

5 Find the range of values of x which satisfy both of the inequalities

$$3-2x<5,$$
$$2x^2+3x<2.$$ [4]

6 Given that $\log_b(x^3y) = p$ and $\log_b\left(\dfrac{y}{x^2}\right) = q$, express $\log_b(xy)$ in terms of p and q. [4]

7 Solve the equation $e^{2x} = e^x + 12$. [5]

8 Given that the expansion of $(a+x)(1-3x)^n$ in ascending powers of x is $2-47x+bx^2+\ldots$ find the values of the constants n, a and b. [6]

[Turn over

9 A collector bought a painting in the beginning of 1990. The value V dollars of the painting is given by the formula $V = 2800\ e^{kt}$, where t is the time in years since the beginning of 1990 and k is a constant.

(i) Find the value of the painting when the collector bought it. [1]

The value of the painting in the beginning of 2010 was 10000 dollars.

(ii) Find the expected value of the painting in the beginning of 2020. [3]

(iii) Find the year in which the expected value of the painting first crosses 40000 dollars. [2]

10 Solve $\log_{16}(3x-1) = \log_4(3x) - \frac{1}{2}$. [5]

11 The expression $ax^3 + 2ax^2 - 15x + b$ is exactly divisible by $x + 3$ but leaves a remainder of -12 when divided by $x - 1$.

(i) Find the value of a and of b. [4]

(ii) Using the values of a and b found in part (i), factorise the expression completely and hence solve the equation $ax^3 + 2ax^2 - 15x + b = 0$ [4]

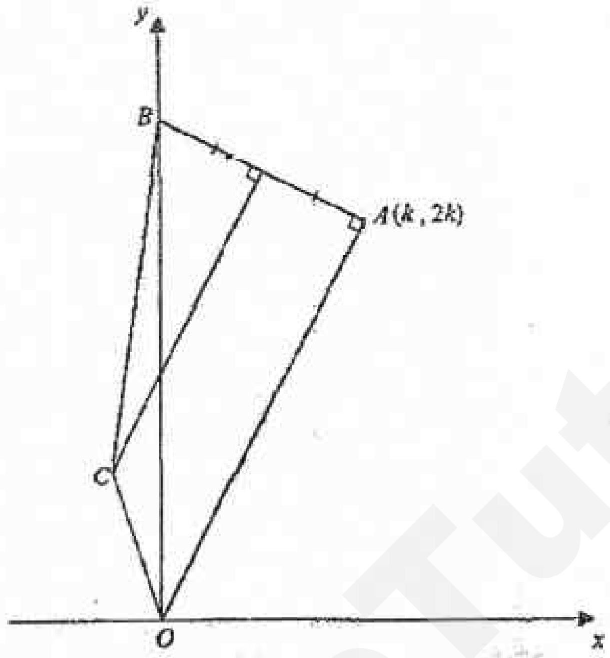
12 The roots of the equation $2x^2 - 3x + 4 = 0$ are α and β.

(i) Form a quadratic equation whose roots are $\alpha - 2\beta$ and $\beta - 2\alpha$. [6]

(ii) Show that $4\alpha^3 = \alpha - 12$. [3]

(iii) Find the value of $\alpha^3 + \beta^3$. [3]

13



The diagram shows the quadrilateral $OABC$.
The coordinates of A are $(k, 2k)$, where $k > 0$, and the length of OA is $\sqrt{80}$ units.

(i) Show that $k = 4$. [2]

AB is perpendicular to OA and B lies on the y-axis.

(ii) Find the equation of AB and the coordinates of B. [4]

The point C lies on the line through O parallel to $y + 3x = 5$ and also on the perpendicular bisector of AB.

(iii) Calculate the coordinates of C. [4]

(iv) Calculate the area of the quadrilateral $OABC$. [2]

1 A curve is such that $\frac{dy}{dx} = \frac{4}{(2x+3)^2}$. Given that the curve passes through the point $(1, -2)$, find the coordinates of the point where the curve crosses the x-axis. [4]

2 The radius, r cm, of a sphere is increasing at a constant rate of 0.5 cm/s. Find, in terms of π, the rate at which the volume is increasing at the instant when the volume is 972π cm^3. [4]

[Volume of sphere, $V = \frac{4}{3}\pi r^3$]

3 An experiment on the topic of Optics in Physics was carried out to find the focal length, f cm, of a certain type of lens. The experiment requires the student to place an object at a distance, u cm, from the lens and to record the distance, v cm, at which the image can be seen on the other side of the lens. The data below shows some of the tabulated experimental results.

u	15	20	30	40	50	55
v	52.5	27.8	18.9	18.3	15.2	14.7

It is known that u, v and f are related by the equation $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$.
It is believed that an error was made in recording one of the values of v.

(i) Plot $\frac{1}{v}$ against $\frac{1}{u}$ and draw a straight line graph which represents the experimental values in the table above. [2]

(ii) Determine which value of v is the incorrect reading.
Use the straight line graph obtained in part (i) to estimate the correct value that should replace the incorrect value of v. [2]

(iii) Estimate the focal length of the lens, f cm, from the graph. [2]

(iv) Verify the accuracy of the straight line graph drawn by evaluating the gradient. [1]

4 A polynomia $= 12x + 4$, $f'(-1) = 1$ and $f(2) = 19$.

(i) Using integration, show that $f(x) = 2x^3 + 2x^2 - x - 3$. [4]

(ii) Show that $x = 1$ is the only solution to $f(x) = 0$ [3]

5 Variables x and y are connected by the equation $y = a^{b+x}$, where a and b are constants. Using experimental values of x and y, a graph was drawn in which $\lg y$ was plotted on the vertical axis against x on the horizontal axis. The straight line which was obtained passed through the points $(0.48, 0.7)$ and $(0.6, 0.82)$. Find

(i) the values of a and b, [4]

(ii) the coordinates of the point on the line at which $y = 0.1^x$. [3]

6 A curve has the equation $y = (x-1)\sqrt{2x+1}$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{kx}{\sqrt{2x+1}}$, where k is a constant and state the value of k. [4]

(ii) Hence, evaluate $\displaystyle\int_0^{12} \frac{6x}{5\sqrt{2x+1}}\,\mathrm{d}x$. [4]

7 The point $A(-1, 2)$ lies on a circle with centre $(3, -1)$.

(i) Given that AB is the diameter of the circle, find the coordinates of B. [2]

(ii) Find the radius of the circle and hence, state its equation. [2]

Another point $C(3, 4)$ lies on the circle.

A line which passes through the point A, cuts the circle at point D and is parallel to BC.

(iii) Find the equation of the straight line BD. [4]

8 The equation of a curve is $y = x^2(x-2)^2$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 4x(x-1)(x-2)$ and hence state the number of stationary points of the curve. [3]

(ii) Find the coordinates of the stationary points of the curve. [3]

(iii) Find an exp $\mathrm{d}x^2$ ce determine the nature of these stationary points. [4]

9 The curve $y = f(x)$ is such that $f(x) = \dfrac{2x+6}{x+1}$ where $x \neq a$.

(i) State the value of a. [1]

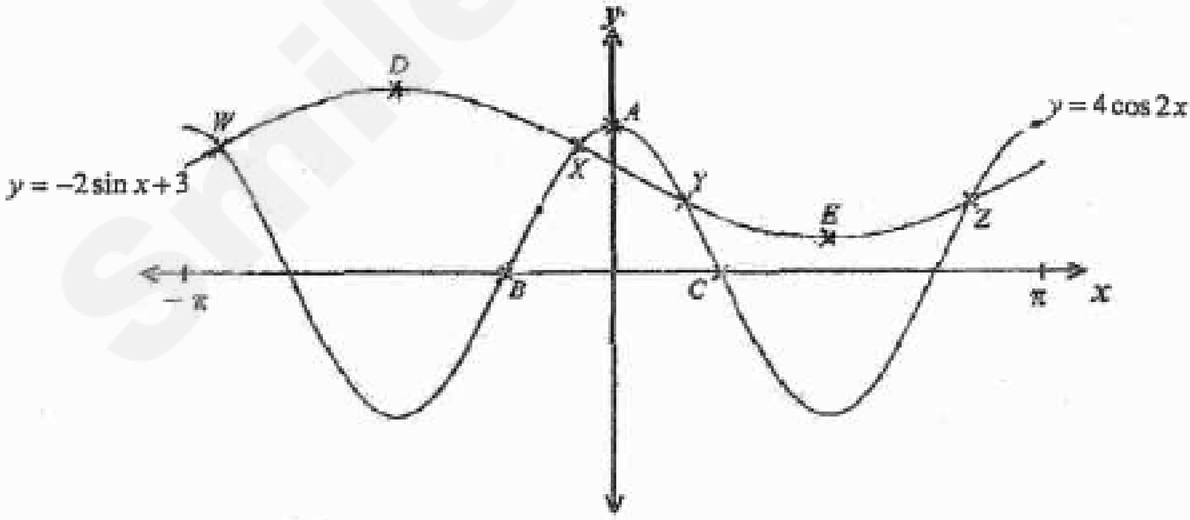
(ii) Find $f'(x)$ and explain why the curve $y = f(x)$ is a decreasing function. [4]

The curve intersects the $x-$axis at the point A.

(iii) Find the equation of the tangent at A. [3]

(iv) If the normal to the curve at A meets the $y-$axis at B, show that the area of ΔAOB, where O is the origin, is 4.5 square units. [3]

10 The diagram shows the curves $y = 4\cos 2x$ and $y = -2\sin x + 3$ for $-\pi \leq x \leq \pi$ radians.
A, B and C are the axes intercepts of the curve $y = 4\cos 2x$.
D and E are the turning points of the curve $y = -2\sin x + 3$.
The curves intersect at the points W, X, Y and Z.



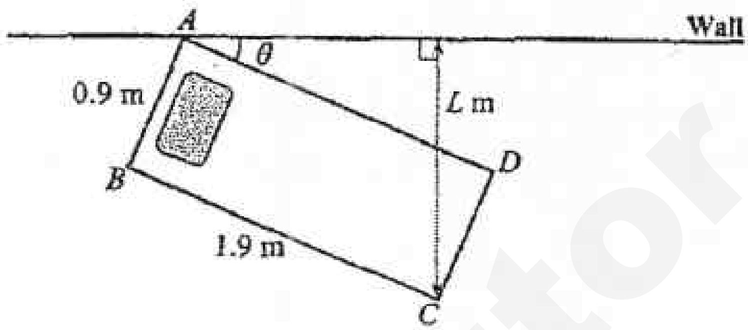
(i) State the coordinates of A, B, C, D and E. [5]

(ii) Show that the equation $4\cos 2x = -2\sin x + 3$ can be expressed as

$$\sin x - 1 = 0.$$ [2]

(iii) Hence, find in radians, the $x-$coordinate of W, X, Y and Z. [4]

11 The diagram shows a rectangular single bed with wheels, $ABCD$, which is hinged to the wall at A. It is given that the dimensions of the bed is 1.9 m by 0.9 m and L m is the perpendicular distance from the wall to C. The bed can be rolled such that the angle between the wall and the side, AD, of the bed is θ and that $0° \le \theta < 90°$.



(i) Show that the length, L m, can be expressed as $L = 1.9\sin\theta + 0.9\cos\theta$. [3]

(ii) Express L in the form $R\sin(\theta+\alpha)$ where $R>0$ and α is an acute angle. [3]

(iii) Hence, find the maximum value of L and the corresponding value of θ. [3]

(iv) Find the value of θ when $L = 1.3$ m. [2]

12 (i) Prove the identity $\cos^4 x - \sin^4 x + 2\cos^2 x - 1 = 2\cos 2x$. [3]

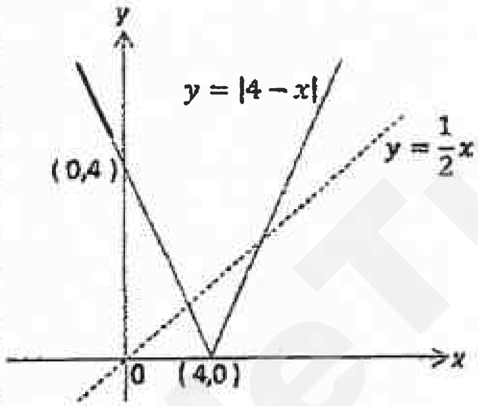
(ii) Solve the equation $2\cos^4 x - 2\sin^4 x = \sqrt{2}$, for $0 < x < \pi$, giving your answers in terms of π. [4]

(iii) Given that $3\cos^4 x - 3\sin^4 x = \sqrt{3}\sin 2x$, and **without using a calculator**,

(a) deduce that $\tan 2x = \sqrt{3}$, [2]

(b) find the possible values of $\tan x$. [3]

ADDITIONAL MATHEMATICS Paper 1 (4047/01) – Marking Scheme

1	$x^2 + (2-k)x + k - 3 = 0$ $b^2 - 4ac = (2-k)^2 - 4(1)(k-3)$ $= 4 - 4k + k^2 - 4k + 12$ $= k^2 - 8k + 16$ $= (k-4)^2$ ≥ 0 $\therefore$ The equation has real roots for all real values of k.	M1 M1A1 A1	[4]	> 0 is not acceptable
2(i)		B2l0		
(ii)	No. of solutions = 2 Justification: Correct sketch/ Explanation/ Solving	A1 A1	[4]	
3	$x^2 - 5x + 6 \overline{)\ 3x^2 + 2x - 28\ }$ quotient 3 $3x^2 - 15x + 18$ $17x - 46$ $x^2 - 5x + 6 = (x-3)(x+2)$ Let $\frac{17x-46}{(x-3)(x-2)} = \frac{A}{x-2} + \frac{B}{x-3}$ $17x - 46 = A(x-3) + B(x-2)$ $34 - 46 = -A$ (Taking $x = 2$) $A = 12$ [illegible] (Taking $x = 3$) $\therefore \frac{[illegible]}{x^2-5x+6} = 3 + \frac{12}{x-2} + \frac{5}{x-3}$	B1 B1 M1 M1A1	[5]	

4	Height $= \frac{21\sqrt{2}-13\sqrt{3}}{(2\sqrt{2}-\sqrt{3})^2}$	B1		
	$= \frac{21\sqrt{2}-13\sqrt{3}}{8-4\sqrt{6}+3}$			
	$= \frac{21\sqrt{2}-13\sqrt{3}}{11-4\sqrt{6}}$	M1		
	$= \frac{(21\sqrt{2}-13\sqrt{3})}{(11-4\sqrt{6})} \times \frac{(11+4\sqrt{6})}{(11+4\sqrt{6})}$	M1		
	$= \frac{231\sqrt{2}+84\sqrt{12}-143\sqrt{3}-52\sqrt{18}}{121-96}$	M1		
	$= \frac{75\sqrt{2}+25\sqrt{3}}{25}$			
	$= (3\sqrt{2}+\sqrt{3})$ m	A1	[5]	
5	$3-2x<5$ $-2x<2$ $x>-1$	B1		
	$2x^2+3x-2<0$ $(2x-1)(x+2)<0$ $-2<x<\frac{1}{2}$	M1A1		
	$\therefore -1<x<\frac{1}{2}$	A1	[4]	
6	$\log_b(x^3y)=p$ $3\log_b x+\log_b y=p$ ……….(1)	M1		
	$\log_b\left(\frac{y}{x^2}\right)=q$ $\log_b y-2\log_b x=q$ ……..(2)	M1		
	$(1)-(2) \Rightarrow 5\log_b x=p-q$ $\log_b x=\frac{p-q}{5}$ $\log_b y=q+2\frac{(p-q)}{5}$ $\log_b xy=\log_b x+\log_b y$ $=\frac{p-q}{5}+q+2\frac{(p-q)}{5}$	M1A1	[4]	

7	$e^{2x} - e^x - 12 = 0$ $let\ y = e^x$ $y^2 - y - 12 = 0$ $(y-4)(y+3) = 0$ $y = 0$ or $y = -3$ (N.A) $e^x = 4$ $x = \ln 4$ $= 1.39$ (3 s.f.)	M1 M1A1 M1A1	[5]	
8	$(a+x)(1-3x)^n$ $= (a+x)\left[1 - \binom{n}{1}(3x) + \binom{n}{2}(3x)^2 + \cdots\right]$ $= (a+x)\left[1 - 3nx + \binom{n}{2}9x^2 + \cdots\right]$ $= a - 3anx + \binom{n}{2}9ax^2 + x - 3nx^2 + \cdots$ $= a + (1-3an)x + \left[\binom{n}{2}9a - 3n\right]x^2 + \cdots$ By comparison, $a = 2$ $1 - 3 \times 2n = -47$ $-6n = -48$ $n = 8$ $b = \binom{8}{2} \times 9 \times 2 - 3 \times 8 = 480$	M1A1 M1 A1 A1 A1	[6]	
9	(i) when $t = 0$ V = $2800 ii) $t = 20$ $10000 = 2800e^{20k}$ $e^{20k} = \frac{10000}{2800}$ $k = \frac{\ln\left(\frac{10000}{2800}\right)}{20} = 0.063648$ In the beginning of 2020, $t = 30$ $V = 2800e^{30 \times 0.063648}$ $= \$18898.06$ (Accept) $= \$18900$ (iii) $40000 = 2800e^{0.063648t}$ $t = \frac{\ln\left(\frac{40000}{2800}\right)}{0.063648}$ [illegible]31.	B1 M1A1 A1 M1 A1	[6]	

10		$\log_{16}(3x-1)=\log_4(3x)-\frac{1}{2}$ $\frac{\log_4(3x-1)}{\log_4 16}=\log_4 3x-\log_4 2$ $\frac{\log_4(3x-1)}{2}=\log_4\left(\frac{3x}{2}\right)$ $\log_4(3x-1)=2\log_4\left(\frac{3x}{2}\right)$ $\log_4(3x-1)=\log_4\left(\frac{3x}{2}\right)^2$ $3x-1=\frac{9x^2}{4}$ $9x^2-12x+4=0$ $(3x-2)^2=0$ $\Rightarrow 3x-2=0$ $x=\frac{2}{3}$	B1 M1 M1A1 A1	[5]	B1 for correct application of change of base
11	(i)	Let $f(x)=ax^3+2ax^2-15x+b$ $f(-3)=0$ $-27a+18a+45+b=0$ $-9a+b=-45$ ——— (1) $f(1)=-12$ $a+2a-15+b=-12$ $3a+b=3$ ——— (2) (1)−(2) $\Rightarrow -12a=-48$ $a=4$ $12+b=3$ $b=-9$ $\therefore a=4,\ b=-9$	M1 M1 A2		
	(ii)	$$\begin{array}{r\|l} & 4x^2-4x-3 \\ \hline x+3 & 4x^3+8x^2-15x-9 \\ & 4x^3+12x^2 \\ \hline & -4x^2-15x \\ & -4x^2-12x \\ \hline & -3x-9 \\ & -3x-9 \\ \hline & 0 \end{array}$$ $(x+3)(4x^2-4x-3)=0$ $(x+3)(2x-3)(2x+1)=0$ $\therefore x=-3,-\frac{1}{2},\frac{3}{2}$	M1 M1A1 A1	[8]	

12	(i) $\alpha+\beta=\frac{3}{2}$, $\alpha\beta=2$ $\alpha-2\beta+\beta-2\alpha=-(\alpha+\beta)=-\frac{3}{2}$ $(\alpha-2\beta)(\beta-2\alpha)$ $=\alpha\beta-2\alpha^2-2\beta^2+4\alpha\beta$ $=5\alpha\beta-2(\alpha^2+\beta^2)$ $=5\alpha\beta-2[(\alpha+\beta)^2-$ $2\alpha\beta]$ $=9\alpha\beta-2(\alpha+\beta)^2$ $=9\times2-2\times\left(\frac{3}{2}\right)^2$ $=\frac{27}{2}$ $\therefore$ The equation is $x^2+\frac{3}{2}x+\frac{27}{2}=0$ *ie,* $2x^2+3x+27=0$	B1B1 A1 M1A1 A1		
	(ii) Since α is a root, $2\alpha^2-3\alpha+4=0$ $2\alpha^2=3\alpha-4$ $4\alpha^3=6\alpha^2-8\alpha$ $=3(2\alpha^2)-8\alpha$ $=3(3\alpha-4)-8\alpha$ $=\alpha-12$ (shown)	B1 M1A1		
	(iii) $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$ $=\frac{3}{2}[(\alpha+\beta)^2-3\alpha\beta]$ $=\frac{3}{2}\left[\left(\frac{3}{2}\right)^2-3\times2\right]$ $=-\frac{45}{8}$	B1 M1A1	[12]	
13.	1. (i) $(2k)^2+k^2=80$ $5k^2=80$ $k^2=16$ $k=4$	M1A1		
	(ii) $A(4,8)$ $M_{OA}=2$ $M_{AB}=-\frac{1}{2}$ Equation of AB is $y-8=-\frac{1}{2}(x-4)$ $\therefore B(0,10)$	M1 M1A1 A1		

	(iii) Equation of OC is $y = -3x$ ———(1) Midpoint of AB is (2,9) Equation of the perpendicular of AB is $y - 9 = 2(x - 2)$ $y = 2x + 5$ ———(2) Solving (1) and (2), C is (-1, 3) (iv) Area $= \frac{1}{2}\begin{vmatrix} 0 & 4 & 0 & -1 & 0 \\ 0 & 8 & 10 & 3 & 0 \end{vmatrix}$ $= \frac{1}{2}[40 + 10]$ $= 25 \text{ units}^2$	B1 M1 M1A1 M1A1	[12]	

Geylang Methodist School (Secondary)
Mid – Year Examination 2017

ANSWER SCHEME

ADDITIONAL MATHEMATICS 4047/02
Paper 2

Express / Normal (Academic)

Additional Materials provided: Cover Sheet (1 sheet)
Writing Paper (2 sheets)
Graph Paper (1 sheet)

2 hours 30 minutes

Setter: Ms Naimah Ismail

Monday, 8 May 2017

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the Writing Papers provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

This document consists of 19 printed pages including the cover page and 1 blank page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}ab\sin C$$

1 A curve is such that $\frac{dy}{dx}=\frac{4}{(2x+3)^2}$. Given that the curve passes through the point $(1,-2)$, find the coordinates of the point where the curve crosses the x-axis. [4]

1 Given: $\frac{dy}{dx}=\frac{4}{(2x+3)^2}$ and $(1,-2)$

$$y=\int\frac{4}{(2x+3)^2}\,dx=\int 4(2x+3)^{-2}\,dx=4\int(2x+3)^{-2}\,dx$$

$$=4\left[\frac{(2x+3)^{-2+1}}{(-2+1)(2)}\right]+c=4\left[\frac{(2x+3)^{-1}}{(-1)(2)}\right]+c$$

$$y=-2\left[\frac{1}{(2x+3)}\right]+c$$ [M1]

At $(1,-2)$, $(x=1, y=-2)$

$$-2=-2\left[\frac{1}{(2(1)+3)}\right]+c$$

$$-2=-2\left[\frac{1}{5}\right]+c$$

$$-2+\frac{2}{5}=c$$

$$c=-\frac{8}{5}$$

$$\Rightarrow y=-\frac{2}{(2x+3)}-\frac{8}{5}$$ [M1]

Crosses the x-axis $\Rightarrow y=0$

$$0=-\frac{2}{(2x+3)}-\frac{8}{5}$$ [M1]

$$\frac{2}{(2x+3)}=-\frac{8}{5}$$

$$(2)(5)=-8(2x+3)$$

$$10=-16x-24$$

$$16x=-24-10$$

$$16x=-34$$

$$x=-\frac{34}{16}=-\frac{17}{8}=-2\frac{1}{8}=-2.125$$

$\therefore$ Coordinates: $\left(-\frac{17}{8},0\right)$; $\left(-2\frac{1}{8},0\right)$; $(-2.125,0)$ [A1]

2 The radius, r cm, of a sphere is increasing at a constant rate of 0.5 cm/s.

Find, in terms of π, the rate at which the volume is increasing at the instant when the volume is 972π cm^3. [4]

[Volume of sphere, $V = \frac{4}{3}\pi r^3$]

---

2 Given: $V = \frac{4}{3}\pi r^3$; $V = 972\pi$ cm^3 ; $\frac{dr}{dt} = 0.5$ cm s^{-1}

$V = \frac{4}{3}\pi r^3$

$972\pi = \frac{4}{3}\pi r^3$

$972 = \frac{4}{3}r^3$

$972 \div \frac{4}{3} = r^3$

$r^3 = 972 \times \frac{3}{4}$

$r^3 = 729$

$r = 9$ cm [M1]

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = \frac{4}{3}(3)\pi r^{3-1}$

$\frac{dV}{dr} = 4\pi r^2$ [M1]

$\left.\frac{dV}{dr}\right|_{r=9} = 4\pi(9)^2$

$\left.\frac{dV}{dr}\right|_{r=9} = 4 \times 81\pi$

$\left.\frac{dV}{dr}\right|_{r=9} = 324\pi$ [M1]

$\left(\frac{dV}{dr} = 324\pi \ ; \ \frac{dr}{dt} = 0.5 \text{ cm s}^{-1}\right)$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$\frac{dV}{dt} = 324\pi \times 0.5$

$\therefore \frac{dV}{dt} = 162\pi$ [A1]

3 An experiment on the topic of Optics in Physics was carried out to find the focal length, f cm, of a certain type of lens. The experiment requires the student to place an object at a distance, u cm, from the lens and to record the distance, v cm, at which the image can be seen on the other side of the lens. The data below shows some of the tabulated experimental results.

u	15	20	30	40	50	55
v	52.5	27.8	18.9	18.3	15.2	14.7

It is known that u, v and f are related by the equation $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$.

It is believed that an error was made in recording one of the values of v.

(i) Plot $\frac{1}{v}$ against $\frac{1}{u}$ and draw a straight line graph which represents the experimental values in the table above. [2]

(ii) Determine which value of v is the incorrect reading. Use the straight line graph obtained in part (i) to estimate the correct value that should replace the incorrect value of v. [2]

(iii) Estimate the focal length of the lens, f cm, from the graph. [2]

(iv) Verify the accuracy of the straight line graph drawn by evaluating the gradient. [1]

3 (i) Given: $\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \Rightarrow \frac{1}{v}=-\frac{1}{u}+\frac{1}{f} \Rightarrow \frac{1}{v}=(-1)\frac{1}{u}+\frac{1}{f}$

$Y = m\,X + c$

Y vs $X \Rightarrow \frac{1}{v}$ vs $\frac{1}{u}$; Gradient, $m=-1$; y-intercept, $c=\frac{1}{f}$

	u	15	20	30	40	50	55
X	$\frac{1}{u}$	0.067	0.050	0.033	0.025	0.020	0.018
	v	52.5	27.8	18.9	18.3	15.2	14.7
Y	$\frac{1}{v}$	0.019	0.036	0.053	0.055	0.066	0.068

Correct Plot / X, Y – axes / Label (graph, coordinates etc) → [M1]
Suitable Scale / Best Straight Line / Cuts y-axis (need not cut x axis) → [A1]

(ii) Incorrect value: $\frac{1}{v}=0.055 \Rightarrow v=18.3$ [M1]

Correct value (from Straight-Line Graph): $\frac{1}{v}=0.061 \Rightarrow v=16.4$ (3sf) [A1]

(Accept: $\frac{1}{v}=0.061 \pm 0.001$; $v=16.4 \pm 0.3$)

(iii) From graph,

y-intercept, $c=\frac{1}{f}=0.086$ [M1]

(Accept: $c=0.086 \pm 0.001$)(3sf)

$f=\frac{1}{0.086} \Rightarrow f=11.6279$ cm [A1]

(Accept:

(iv) Comparing gradient, $m=-1$ (from equation)
From graph: $(0.067, 0.019)$; $(0, 0.086)$

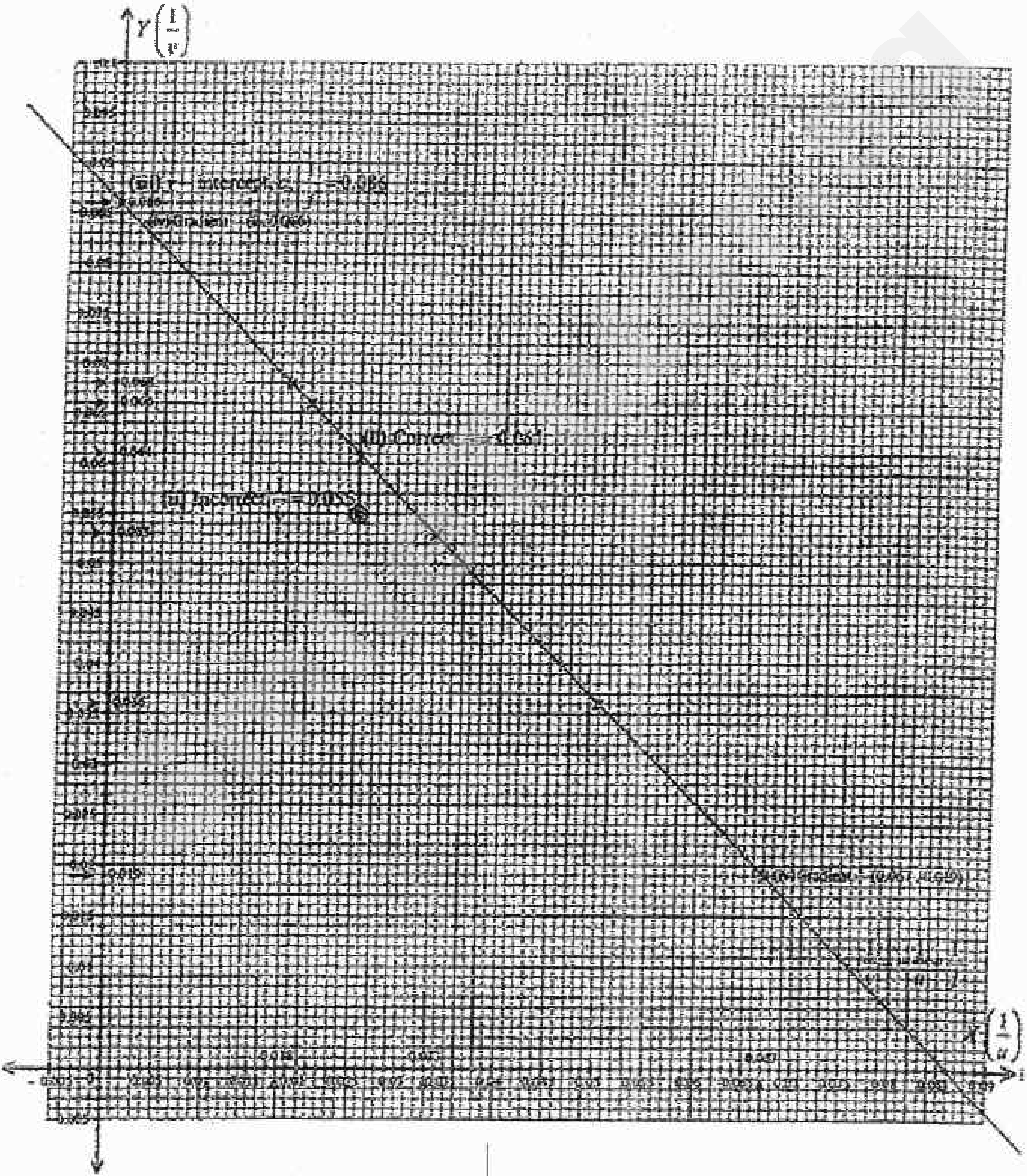
$m=\frac{y_2-y_1}{x_2-x_1}=\frac{0.086-0.019}{0-0.067}=\frac{0.067}{-0.067}$

$m=-1$ $(=-1)$ (Graph is accurate) [B1]

(Graph gentler/steeper than what is required depending on student's answer)

[Turn over

(i) Graph of $\frac{1}{v} = -\frac{1}{u} + \frac{1}{f}$ → *Scale*: Horizontal & Vertical axis, 2 cm to 0.01 units



4 A polynomial $f(x)$ is such that $f''(x)=12x+4$, $f'(-1)=1$ and $f(2)=19$.

(i) Using integration, show that $f(x)=2x^3+2x^2-x-3$. [4]

(ii) Show that $x=1$ is the only solution to $f(x)=0$. [3]

4 (i) Given: $f''(x)=12x+4$, $f'(-1)=1$

$f'(x)=\int f''(x)\,dx$

$f'(x)=\int(12x+4)\,dx$

$=12\left[\frac{x^{1+1}}{1+1}\right]+4\left[\frac{x^{0+1}}{0+1}\right]+c$

$f'(x)=12\left[\frac{x^2}{2}\right]+4\left[\frac{x^1}{1}\right]+c$

$f'(x)=6x^2+4x+c$ [M1]

$f'(-1)=1$,

$f'(-1)=6(-1)^2+4(-1)+c$

$1=6-4+c$

$c=1-6+4$

$c=-1$

$\Rightarrow f'(x)=6x^2+4x-1$ [M1]

Since: $f'(x)=6x^2+4x-1$, $f(2)=19$

$f(x)=\int f'(x)\,dx$

$f(x)=\int(6x^2+4x-1)\,dx$

$=6\left[\frac{x^{2+1}}{2+1}\right]+4\left[\frac{x^{1+1}}{1+1}\right]-1\left[\frac{x^{0+1}}{0+1}\right]+c$

$=6\left[\frac{x^3}{3}\right]+4\left[\frac{x^2}{2}\right]-1\left[\frac{x^1}{1}\right]+c$

$f(x)=2x^3+2x^2-x+c$ [M1]

$f(2)=19$,

$f(2)=2(2)^3+2(2)^2-(2)+c$

$19=16+8-2+c$

$c=19-16-8+2$

$c=-3$

$\therefore f(x)=2x^3+2x^2-x-3$ (Shown) [A1]

(ii) Show: $x=1$ is the <u>only</u> solution to $f(x)=0$

$(x-1)$ is a factor of $f(x)=2x^3+2x^2-x-3$.

Try: $f(1)=2(1)^3+2(1)^2-(1)-3$

$=2+2-1-3$

$=0$

$\Rightarrow f(1)=0 \Rightarrow x=1$ is a solution to $f(x)=2x^3+2x^2-x-3=0$ [B1]

$f(x)=2x^3+2x^2-x-3=0$

$\rightarrow 2x^3+2x^2-x-3=0$

$\Rightarrow (x-1)(ax^2+bx+c)=0$

$(x-1)(2x^2+4x+3)=0$

$(x-1)=0$; $(2x^2+4x+3)=0$

$\therefore x=1$; $(a=2, b=4, c=3)$

$b^2-4ac=4^2-4(2)(3)$

$=16-24$

$b^2-4ac=-8<0$ [M1]

No solution for:

$(2x^2+4x+3)=0$

Hen[illegible] to $f(x)=0$

$$\begin{array}{r}
2x^2+4x+3 \\
x-1\,\big)\,\overline{2x^3+2x^2-x-3} \\
-(2x^3-2x^2)\quad\downarrow\quad\downarrow \\
\hline
4x^2-x\quad\downarrow \\
-(4x^2-4x)\ \downarrow \\
\hline
3x-3 \\
-(3x-3) \\
\hline
0
\end{array}$$ [M1]

<u>OR</u> By Synthetic Division:

	[illegible]	[illegible]	[illegible]	[illegible]
	2	2	−1	−3
1		2	4	3
	2	4	3	0
	[illegible]	[illegible]	[illegible]	

5 Variables x and y are connected by the equation $y=a^{b+x}$, where a and b are constants. Using experimental values of x and y, a graph was drawn in which $\lg y$ was plotted on the vertical axis against x on the horizontal axis. The straight line which was obtained passed through the points $(0.48, 0.7)$ and $(0.6, 0.82)$. Find

(i) the values of a and b, [4]

(ii) the coordinates of the point on the line at which $y=0.1^x$. [3]

5 (i) Given: $y=a^{b+x}$

$\lg y=\lg a^{b+x}$

$\lg y=(b+x)\lg a$

$\lg y=b\lg a+x\lg a$

$\lg y=(\lg a)x+b\lg a$ [M1]

$Y = mX + c$

Y vs $X \Rightarrow \lg y$ vs x

Gradient, $m=\lg a$

y-intercept, $c=b\lg a$

$(0.48, 0.7)$ and $(0.6, 0.82)$

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$\lg a=\frac{0.82-0.7}{0.6-0.48}=\frac{0.12}{0.12}$$

$\lg a=1$ [M1]

$\log_{10} a=1$

$a=10^1$

$\therefore a=10$ [M1]

$(x, \lg y)\Rightarrow(0.48, 0.7)$, $\lg a=1$

$\lg y=(\lg a)\ x+b\ \lg a$

$0.7=(1)(0.48)+b(1)$

$b=0.7-0.48$

$\therefore b=0.22$ [A1]

Hence, Straight-Line equation:

$\lg y=(1)x+0.22$

$Y=mX+c$

(ii) $y=a^{b+x}$, $(a=10, b=0.22)$

$y=10^{0.22+x}\rightarrow(1)$

$y=0.1^x\rightarrow(2)$

$(1)=(2)$: $10^{0.22+x}=0.1^x$ [M1]

$10^{0.22+x}=\left(\frac{1}{10}\right)^x$

$10^{0.22+x}=(10^{-1})^x$

$10^{0.22+x}=10^{-x}$

$\Rightarrow 0.22+x=-x$

$x+x=-0.22$

$2x=-0.22$

$\therefore x=-0.11$

Sub $x=-0.11$ in Straight-Line equation:

$\Rightarrow \lg y=(1)\ x+0.22$

$\lg y=(1)(-0.11)+0.22$ [M1]

$\lg y=-0.11+0.22$

$\therefore \lg y=0.11$

Hence, point on the straight line

$\Rightarrow \lg y=x+0.22$

$(x, \lg y)\rightarrow(-0.11, 0.11)$ [A1]

6 A curve has the equation $y=(x-1)\sqrt{2x+1}$

(i) Show that $\frac{dy}{dx}=\frac{kx}{\sqrt{2x+1}}$, where k is a constant and state the value of k. [4]

(ii) Hence, evaluate $\int_0^{12}\frac{6x}{5\sqrt{2x+1}}\,dx$. [4]

6 (i) Given: $y=(x-1)\sqrt{2x+1}$

Let: $u=x-1$

$\frac{du}{dx}=1$

$v=\sqrt{2x+1}$

$v=(2x+1)^{\frac{1}{2}}$

$\frac{dv}{dx}=\frac{1}{2}(2x+1)^{\frac{1}{2}-1}(2)$

$\frac{dv}{dx}=(2x+1)^{-\frac{1}{2}}$

$\frac{dv}{dx}=\frac{1}{(2x+1)^{\frac{1}{2}}}$

$\frac{dv}{dx}=\frac{1}{\sqrt{2x+1}}$

$$\frac{dy}{dx}=u\,dv+v\,du$$

$$=(x-1)\left(\frac{1}{\sqrt{2x+1}}\right)+\left(\sqrt{2x+1}\right)(1)$$

[M1] [M1]

$$=\frac{x-1}{\sqrt{2x+1}}+\sqrt{2x+1}$$

$$=\frac{x-1}{\sqrt{2x+1}}+\frac{\left(\sqrt{2x+1}\right)\left(\sqrt{2x+1}\right)}{\sqrt{2x+1}}\quad\text{[M1]}$$

$$=\frac{x-1}{\sqrt{2x+1}}+\frac{(2x+1)}{\sqrt{2x+1}}$$

$$=\frac{x-1+(2x+1)}{\sqrt{2x+1}}$$

$$\therefore\frac{dy}{dx}=\frac{3x}{\sqrt{2x+1}}\quad\left(\frac{kx}{\sqrt{2x+1}};\ k=3\right)\quad\text{[A1]}$$

(ii) From (i):

$$\int\frac{3x}{\sqrt{2x+1}}\,dx=(x-1)\sqrt{2x+1}+c\quad\rightarrow\quad(*)$$

$$\int\frac{6x}{5\sqrt{2x+1}}\,dx=\int\left(\frac{2}{5}\right)\frac{3x}{\sqrt{2x+1}}\,dx$$

$$=\left(\frac{2}{5}\right)\int\frac{3x}{\sqrt{2x+1}}\,dx\quad\text{[M1]}$$

$$=\left(\frac{2}{5}\right)\left[(x-1)\sqrt{2x+1}+c\right]\quad\rightarrow\quad\text{From }(*)\quad\text{[M1]}$$

$$\int_0^{12}\frac{6x}{5\sqrt{2x+1}}\,dx=\left(\frac{2}{5}\right)\left[(x-1)\sqrt{2x+1}\right]_0^{12}$$

$$=\left(\frac{2}{5}\right)\left[\left((12-1)\sqrt{2(12)+1}\right)-\left((0-1)\sqrt{2(0)+1}\right)\right]\quad\text{[M1]}$$

$$=\left(\frac{2}{5}\right)\left[(11\sqrt{25})-(-\sqrt{1})\right]=\left(\frac{2}{5}\right)\left[(11)(5)-(-1)\right]$$

$$=\left(\frac{2}{5}\right)\left[55+1\right]=\frac{2\times56}{5}$$

$$\therefore\int_0^{12}\frac{6x}{5\sqrt{2x+1}}\,dx=\frac{112}{5};\ 22\frac{2}{5};\ 22.4\quad\text{[A1]}$$

7 The point $A(-1, 2)$ lies on a circle with centre $(3, -1)$.

(i) Given that AB is the diameter of the circle, find the coordinates of B. [2]

(ii) Find the radius of the circle and hence, state its equation. [2]

Another point $C(3, 4)$ lies on the circle.

A line which passes through the point A, cuts the circle at point D and is parallel to BC.

(iii) Find the equation of the straight line BD. [4]

7 (i) Given: Centre $(3, -1)$; $A(-1, 2)$

Let: $B(x, y)$

Midpoint of AB is centre $(3, -1)$ { AB is diameter}

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (3, -1)$$

$$\left(\frac{-1+x}{2}, \frac{2+y}{2}\right) = (3, -1)$$ [M1]

Comparing $x-$ component.

$$\Rightarrow \frac{-1+x}{2} = 3$$
$$-1+x=6$$
$$x=7$$

Comparing $y-$ component.

$$\Rightarrow \frac{2+y}{2} = -1$$
$$2+y=-2$$
$$y=-4$$

$\therefore B(7, -4)$ [A1]

(ii) Centre $(3, -1)$; $A(-1, 2)$

$$\text{Radius} = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$
$$= \sqrt{(3-(-1))^2+(-1-2)^2} = \sqrt{(3+1)^2+(-3)^2} = \sqrt{(4)^2+(-3)^2}$$
$$= \sqrt{16+9} = \sqrt{25}$$

Radius = 5 units [A1]

Equation of circle: Centre $(3, -1)$; Radius = 5 units

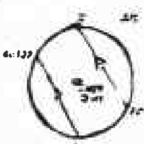
Standard form: $(x-a)^2+(y-b)^2 = r^2$

$$(x-3)^2+(y-(-1))^2 = 5^2$$
$$(x-3)^2+(y+1)^2 = 5^2$$
$$(x-3)^2+(y+1)^2 = 25$$ [B1]

General form: $x^2+y^2+2gx+2fy+c=0$

$$(x-3)^2+(y+1)^2 = 25$$
$$x^2-6x+9+y^2+2y+1-25=0$$
$$x^2+y^2-6x+2y-15=0$$ [B1]

(iii)



$$m_{BC} = \frac{y_2-y_1}{x_2-x_1} = \frac{-4-4}{7-3}$$
$$m_{BC} = \frac{-8}{4} = -2$$ [M1]

$m_{AD} = m_{BC} = -2$ { $AD // BC$ }

AD is diameter, $\angle ACB = \angle ADB = 90°$

($\angle$ in semicircle)

($BD \perp BC$), $(m_{BD})(m_{BC}) = -1$

$$(m_{BD})(-2) = -1$$
$$m_{BD} = \frac{-1}{-2} = \frac{1}{2}$$ [M1]

Equation of BD: $B(7, -4)$, $m_{BD} = \frac{1}{2}$

$$y - y_1 = m(x - x_1)$$
$$y-(-4) = \frac{1}{2}(x-7)$$ [M1]
$$y+4 = \frac{1}{2}(x-7)$$
$$y = \frac{1}{2}x - \frac{7}{2} - 4$$
$$\therefore y = \frac{1}{2}x - 7\frac{1}{2}$$ [A1]

10

8 The equation of a curve is $y = x^2(x-2)^2$.

(i) Show that $\frac{dy}{dx} = 4x(x-1)(x-2)$ and hence state the number of stationary points of the curve. [3]

(ii) Find the coordinates of the stationary points of the curve. [3]

(iii) Find an expression for $\frac{d^2y}{dx^2}$ and hence determine the nature of these stationary points. [4]

---

8 (i) Given: $y = x^2(x-2)^2$

Let: $u = x^2$, $\frac{du}{dx} = 2x$

$v = (x-2)^2$, $\frac{dv}{dx} = 2(x-2)(1)$, $\frac{dv}{dx} = 2(x-2)$

$$\frac{dy}{dx} = u\,dv + v\,du$$

$$= (x^2)(2)(x-2) + (x-2)^2(2x) \quad \boxed{M1}$$

$$= (2x)(x-2)[(x)+(x-2)]$$

$$= (2x)(x-2)[2x-2] = (2x)(x-2)(2)(x-1)$$

$$\therefore \frac{dy}{dx} = 4x(x-1)(x-2) \quad \text{(Shown)} \quad \boxed{A1}$$

Stationary points: $\frac{dy}{dx} = 0$

$4x(x-1)(x-2) = 0$

3 solutions for x for $\frac{dy}{dx} = 0$

$\therefore$ Number of stationary points: 3 (Shown) $\boxed{A1}$

(ii) From (i):

$4x(x-1)(x-2) = 0$

$\rightarrow 4x = 0 \Rightarrow x = 0$ $\quad\rightarrow (x-1) = 0 \Rightarrow x = 1$ $\quad\rightarrow (x-2) = 0 \Rightarrow x = 2$

Substitute in $y = x^2(x-2)^2$,

$\rightarrow y = 0^2(0-2)^2 \Rightarrow y = 0$ $\quad\rightarrow y = 1^2(1-2)^2 \Rightarrow y = 1$ $\quad\rightarrow y = 2^2(2-2)^2 \Rightarrow y = 0$

$\therefore$ Coordinates of stationary points:

$(0,0)$ $\boxed{B1}$ $\quad(1,1)$ $\boxed{B1}$ $\quad(2,0)$ $\boxed{B1}$

(iii) $\frac{d^2y}{dx^2} = 4x(x-1)(x-2) = 4x(x^2-3x+2) = 4x^3 - 12x^2 + 8x$

$$\frac{d^2y}{dx^2} = 4(3)x^{3-1} - 12(2)x^{2-1} + 8 = 12x^2 - 24x + 8 \quad \boxed{A1}$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=0} = 12(0)^2 - 24(0) + 8 = 8 \quad \left(\frac{d^2y}{dx^2} > 0\right) \rightarrow (0,0) \text{ (Min. pt.)} \quad \boxed{B1}$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=1} = 12(1)^2 - 24(1) + 8 = 12 - 24 + 8 = -4 \quad \left(\frac{d^2y}{dx^2} < 0\right) \rightarrow (1,1) \text{ (Max. pt.)} \quad \boxed{B1}$$

$$\frac{d^2y}{dx^2} = 12(2)^2 - 24(2) + 8 = 48 - 48 + 8 = 8 \quad \left(\frac{d^2y}{dx^2} > 0\right) \rightarrow (2,0) \text{ (Min. pt.)} \quad \boxed{B1}$$

9 The curve $y=f(x)$ is such that $f(x)=\dfrac{2x+6}{x+1}$ where $x \neq a$.

(i) State the value of a. [1]

(ii) Find $f'(x)$ and explain why the curve $y=f(x)$ is a decreasing function. [4]

The curve intersects the x-axis at the point A.

(iii) Find the equation of the tangent at A. [3]

(iv) If the normal to the curve at A meets the y-axis at B, show that the area of ΔAOB, where O is the origin, is 4.5 square units. [3]

---

9 (i) $f(x)$ is not defined when $x+1=0$

Hence, $x+1 \neq 0 \rightarrow x \neq -1 \rightarrow x \neq a$

Value of a is $\underline{-1}$ **[B1]**

(ii) $f(x) = \dfrac{2x+6}{x+1}$

$f'(x) = \dfrac{v\,du - u\,dv}{v^2}$

$= \dfrac{(x+1)(2)-(2x+6)(1)}{(x+1)^2} = \dfrac{2x+2-2x-6}{(x+1)^2}$ **[M1]**

$f'(x) = \dfrac{-4}{(x+1)^2}$ **[A1]**

Let: $u = 2x+6$

$\dfrac{du}{dx} = 2$

$v = x+1$

$\dfrac{dv}{dx} = 1$

$(x+1)^2 > 0 \quad (x \neq -1)$

$\dfrac{-4}{(x+1)^2} < 0$ **[M1]**

$\therefore f'(x) < 0$

$\therefore$ Hence $y=f(x)$ is a decreasing function **[A1]**

(iii) Curve intersects the x-axis $\Rightarrow y=0$

$y=f(x)=0$

$f(x)=\dfrac{2x+6}{x+1}$

$0=\dfrac{2x+6}{x+1}$

$2x+6=0$

$2x=-6$

$x=-3$

$A(-3,0)$ **[M1]**

m_{tan} at $x=-3$

$f'(x) = \dfrac{-4}{(x+1)^2}$

$f'(-3) = \dfrac{-4}{(-3+1)^2} = \dfrac{-4}{(-2)^2}$

$f'(-3) = -1$

$m_{tan} = -1$ at $x=-3$ **[M1]**

Equation of tangent:

$A(-3,0)$; $m_{tan}=-1$

$y-y_1 = m_{tan}(x-x_1)$

$y-0 = -1(x-(-3))$

$y = -1(x+3)$

$\therefore y = -x-3$ **[A1]**

(iv) $(m_{tan})(m_{nor})=-1$

$(-1)(m_{nor})=-1$

$(m_{nor})=1$

Equation of normal:

$A(-3,0)$; $m_{nor}=1$

$y-y_1 = m_{nor}(x-x_1)$

$y-0$

$y=x$

Meets the y-axis:

$x=0$

$y=0+3$

$y=3$

$B(0,3)$ **[M1]**

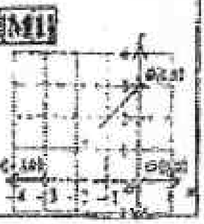
$\therefore$ Area of ΔAOB

$=(0.5)(\text{base})(\perp \text{height})$

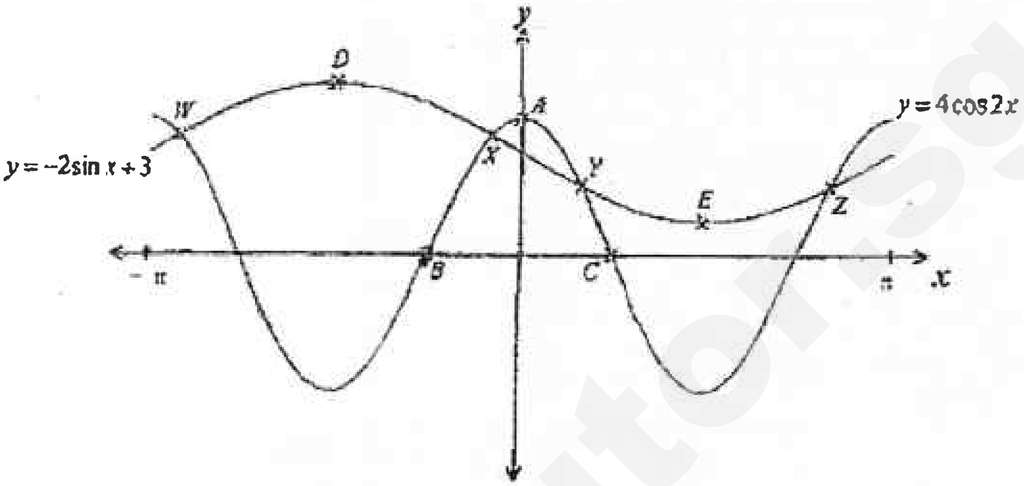
$=(0.5)(3)(3)$

$=4.5$ square units **[M1]**

(Shown)



10 The diagram shows the curves $y = 4\cos 2x$ and $y = -2\sin x + 3$ for $-\pi \le x \le \pi$ radians.
A, B and C are the axes intercepts of the curve $y = 4\cos 2x$.
D and E are the turning points of the curve $y = -2\sin x + 3$.
The curves intersect at the points W, X, Y and Z.



(i) State the coordinates of A, B, C, D and E. [5]

(ii) Show that the equation $4\cos 2x = -2\sin x + 3$ can be expressed as

$$8\sin^2 x - 2\sin x - 1 = 0.$$ [2]

(iii) Hence, find in radians, the x-coordinate of W, X, Y and Z. [4]

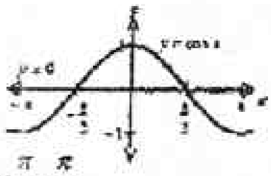
10 (i) $A \rightarrow y-$ intercept of $y = 4\cos 2x \quad \rightarrow x = 0$

$y = 4\cos 2(0) = 4\cos 0 = 4(1) = 4$ $\quad \therefore A(0, 4)$ [B1]

$B, C \rightarrow x-$ intercepts of $y = 4\cos 2x \quad \rightarrow y = 0$

$0 = 4\cos 2x$

$\cos 2x = 0$



$2x = -\frac{\pi}{2}, \frac{\pi}{2}$ $\quad \therefore B\left(-\frac{\pi}{4}, 0\right)$ [B1]

$x = \left(-\frac{\pi}{4} \div 2\right), \left(\frac{\pi}{4} \div 2\right) = -\frac{\pi}{4}, \frac{\pi}{4}$ $\quad \therefore C\left(\frac{\pi}{4}, 0\right)$ [B1]

$D, E \rightarrow$ turning points of $y = -2\sin x + 3$

$x-$coordinate of $D = -\frac{\pi}{2}$; $y-$coordinate of $D = 2 + 3 = 5$ $\quad \therefore D\left(-\frac{\pi}{2}, 5\right)$ [B1]

$x-$coordinate of $E = \frac{\pi}{2}$; $y-$coordinate of $E = -2 + 3 = 1$ $\quad \therefore E\left(\frac{\pi}{2}, 1\right)$ [B1]

(ii) $4\cos 2x = -2\sin x + 3$ $\quad (\cos 2x = 1 - 2\sin^2 x)$

$4(1 - 2\sin^2 x) = -2\sin x + 3$ [M1]

$4 - 8\sin^2 x = -2\sin x + 3$

$0 = -2\sin x + 3 - 4 + 8\sin^2 x$

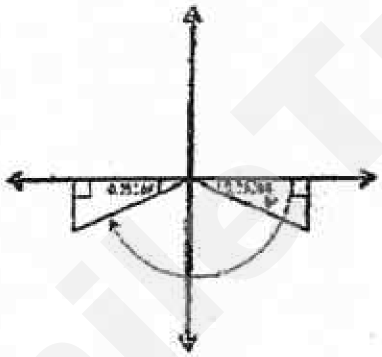
$8\sin$ … wn) [A1]

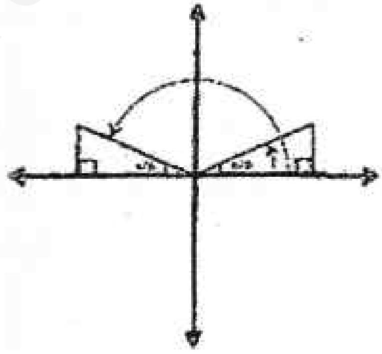
(iii) $y = 4\cos 2x \rightarrow (1)$
$y = -2\sin x + 3 \rightarrow (2)$
(Intersection points)
$(1) = (2)$:
$4\cos 2x = -2\sin x + 3$
$8\sin^2 x - 2\sin x - 1 = 0$ [From (ii)]
$(4\sin x + 1)(2\sin x - 1) = 0$

$4\sin x$	$+1$	$+2\sin x$
$2\sin x$	-1	$-4\sin x$
$8\sin^2 x$	-1	$-2\sin x$

$(4\sin x + 1) = 0$
$4\sin x = -1$
$\sin x = -\frac{1}{4}$
[$\sin x$ is negative → Quadrants 3 and 4]
Basic angle, $\alpha = \sin^{-1}\left(\frac{1}{4}\right) \Rightarrow \alpha = 0.25268$

$(2\sin x - 1) = 0$
$2\sin x = 1$
$\sin x = \frac{1}{2}$
[$\sin x$ is positive → Quadrants 1 and 2]
Basic angle, $\alpha = \sin^{-1}\left(\frac{1}{2}\right) \Rightarrow \alpha = \frac{\pi}{6}$





$\Rightarrow \quad x = -0.25268 \ , \ -(\pi - 0.25268) \qquad \Rightarrow \quad x = \frac{\pi}{6} \ , \ \left(\pi - \frac{\pi}{6}\right)$

$\Rightarrow \quad x = -0.253 \ , \ -2.89$ (3sf) $\qquad \Rightarrow \quad x = \frac{\pi}{6} \ , \ \frac{5\pi}{6}$

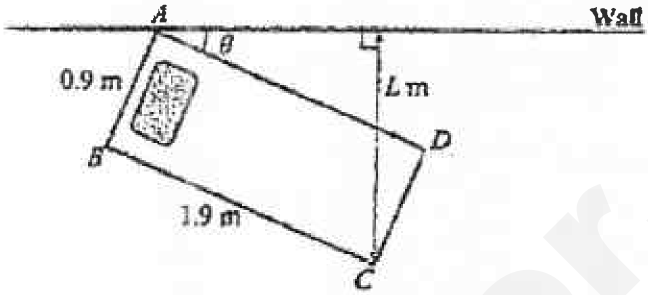
$\therefore$ x – coordinate of $W = -2.89$ [B1]

$\therefore$ x – coordinate of $X = -0.253$ [B1]

$\therefore$ x – coordinate of $Y = \frac{\pi}{6}$ [B1]

$\therefore$ x – coordinate of $Z = \frac{5\pi}{6}$ [B1]

11 The diagram shows a rectangular single bed with wheels, $ABCD$, which is hinged to the wall at A. It is given that the dimensions of the bed is 1.9 m by 0.9 m and L m is the perpendicular distance from the wall to C. The bed can be rolled such that the angle between the wall and the side, AD, of the bed is θ and that $0° \le \theta < 90°$.



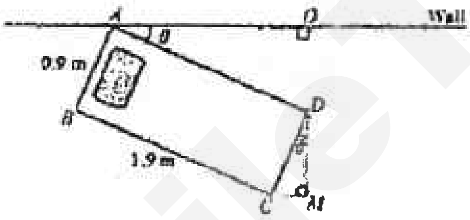
(i) Show that the length, L m, can be expressed as $L = 1.9\sin\theta + 0.9\cos\theta$. [3]

(ii) Express L in the form $R\sin(\theta+\alpha)$ where $R > 0$ and α is an acute angle. [3]

(iii) Hence, find the maximum value of L and the corresponding value of θ. [3]

(iv) Find the value of θ when $L = 1.3$ m. [2]

11 (i)



- In ΔOAD, $\angle OAD = \theta$
- $\angle ODA = 180° - 90° - \theta$
  $\angle ODA = 90° - \theta$
- $\angle ADC = 90°$
- $\angle CDM = 180° - \angle ADC - \angle ODA$
  $= 180° - 90° - (90° - \theta)$
  $\angle CDM = \theta$

In ΔODA,

$\sin\theta = \dfrac{OD}{AD}$

$\sin\theta = \dfrac{OD}{1.9}$

$OD = 1.9\sin\theta$ [M1]

In ΔCDM,

$\cos\theta = \dfrac{DM}{DC}$

$\cos\theta = \dfrac{DM}{0.9}$

$DM = 0.9\cos\theta$ [M1]

$L = OD + DM$

$\therefore L = 1.9\sin\theta + 0.9\cos\theta$ [A1]

(Shown)

(ii) The R-Formula:

$(a\sin\theta + b\cos\theta) = R\sin(\theta+\alpha)$, $R = \sqrt{a^2+b^2}$ and $\tan\alpha = \dfrac{b}{a}$

$(1.9\sin\theta + 0.9\cos\theta) \Rightarrow a = 1.9, b = 0.9$

$R = \sqrt{a^2+b^2} = \sqrt{(1.9)^2+(0.9)^2} = \sqrt{4.42}$ [M1]

$\tan\alpha = \dfrac{b}{a} = \dfrac{0.9}{1.9} \Rightarrow \alpha = \tan^{-1}\left(\dfrac{0.9}{1.9}\right) \Rightarrow \alpha = 25.34618°$ [M1]

$\therefore$ $\sqrt{4.42}\sin(\theta + 25.346°)$ [A1]

(iii) From (ii): $L=(1.9\sin\theta+0.9\cos\theta)=\sqrt{4.42}\sin(\theta+25.346^\circ)$

Maximum value of L occurs when:

$\sin(\theta+25.346^\circ)=1$

Hence, Maximum value of $L=\sqrt{4.42}\ (1)$

$\therefore L=2.10\text{ m}$ [A1]

This occurs when

$$\begin{aligned}
\sin(\theta+25.346^\circ) &= 1 \quad \text{[M1]}\\
(\theta+25.346^\circ) &= \sin^{-1}1\\
(\theta+25.346^\circ) &= 90^\circ\\
\theta &= 90^\circ-25.346^\circ\\
\Rightarrow \theta &= 64.654^\circ\\
\therefore \theta &= 64.7^\circ\ \text{(1dp)} \quad \text{[A1]}
\end{aligned}$$

(iv) When $L=1.3\text{ m}$,

$L=(1.9\sin\theta+0.9\cos\theta)=\sqrt{4.42}\sin(\theta+25.346^\circ)$

$1.3=\sqrt{4.42}\sin(\theta+25.346^\circ)$

$\frac{1.3}{\sqrt{4.42}}=\sin(\theta+25.346^\circ)$

$(\theta+25.346^\circ)=\sin^{-1}\left(\frac{1.3}{\sqrt{4.42}}\right)$ [M1]

$\theta=\sin^{-1}\left(\frac{3}{\sqrt{4.42}}\right)-25.346^\circ$

$\theta=38.19551968^\circ-25.34618^\circ$

$\theta=12.84933968^\circ$

$\therefore\theta=12.8^\circ$ (1dp) [A1]

12 (i) Prove the identity $\cos^4 x - \sin^4 x + 2\cos^2 x - 1 = 2\cos 2x$ [3]

(ii) Solve the equation $2\cos^4 x - 2\sin^4 x = \sqrt{2}$, for $0 < x < \pi$, giving your answers in terms of π. [4]

(iii) Given that $3\cos^4 x - 3\sin^4 x = \sqrt{3}\sin 2x$, and without using a calculator,

(a) deduce that $\tan 2x = \sqrt{3}$. [2]

(b) find the possible values of $\tan x$. [3]

---

12 (i) Given: $\cos^4 x - \sin^4 x + 2\cos^2 x - 1 = 2\cos 2x$.

$LHS = \cos^4 x - \sin^4 x + 2\cos^2 x - 1$

$= (\cos^2 x)^2 - (\sin^2 x)^2 + (2\cos^2 x - 1)$ [M1] $\{\cos 2x = 2\cos^2 x - 1\}$ $\{a^2 - b^2 = (a+b)(a-b)\}$

$= (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) + (\cos 2x)$ [M1] $\{\cos 2x = \cos^2 x - \sin^2 x\}$ $\{\cos^2 x + \sin^2 x = 1\}$

$= (1)(\cos 2x) + (\cos 2x)$

$= 2\cos 2x$ [A1]

$= RHS$

(ii) Solve: $2\cos^4 x - 2\sin^4 x = \sqrt{2}$, for $0 < x < \pi$

$2(\cos^4 x - \sin^4 x) = \sqrt{2}$

$2((\cos^2 x)^2 - (\sin^2 x)^2) = \sqrt{2}$

$(\cos^2 x)^2 - (\sin^2 x)^2 = \dfrac{\sqrt{2}}{2}$ [M1]

$(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) = \dfrac{\sqrt{2}}{\sqrt{2}\times\sqrt{2}}$

$(1)(\cos 2x) = \dfrac{1}{\sqrt{2}}$

$\cos 2x = \dfrac{1}{\sqrt{2}}$ {$\cos 2x$ is positive → Quadrants 1 and 4} [M1]

Basic angle, $\alpha = \cos^{-1}\left(\dfrac{1}{\sqrt{2}}\right) \Rightarrow \alpha = \dfrac{\pi}{4}$

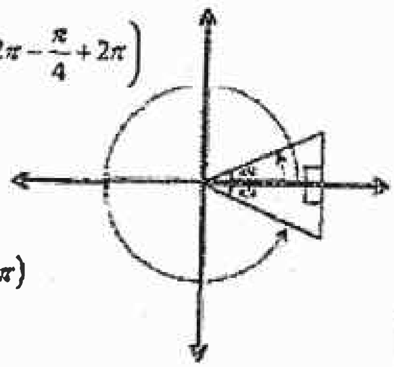
$\Rightarrow \quad 2x = \dfrac{\pi}{4}, \left(2\pi - \dfrac{\pi}{4}\right), \left(\dfrac{\pi}{4} + 2\pi\right), \left(2\pi - \dfrac{\pi}{4} + 2\pi\right)$

$\Rightarrow \quad 2x = \dfrac{\pi}{4}, \dfrac{7\pi}{4}, \dfrac{9\pi}{4}, \dfrac{15\pi}{4}$

$\Rightarrow \quad x = \dfrac{\pi}{8}, \dfrac{7\pi}{8}, \dfrac{9\pi}{8}, \dfrac{15\pi}{8}$

$\therefore \quad x = \dfrac{\pi}{8}, \dfrac{7\pi}{8} \quad (0 < x < \pi)$

[A1] [A1]



[Turn over

(iii) (a) Given:

$$3\cos^4 x - 3\sin^4 x = \sqrt{3}\sin 2x$$
$$3\{\cos^4 x - \sin^4 x\} = \sqrt{3}\sin 2x$$
$$3\{(\cos^2 x)^2 - (\sin^2 x)^2\} = \sqrt{3}\sin 2x$$
$$3\{(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)\} = \sqrt{3}\sin 2x$$
$$3\{(1)(\cos 2x)\} = \sqrt{3}\sin 2x$$
$$3\cos 2x = \sqrt{3}\sin 2x$$
$$\frac{3}{\sqrt{3}} = \frac{\sin 2x}{\cos 2x} \quad \text{[M1]}$$
$$\tan 2x = \frac{\sqrt{3}\sqrt{3}}{\sqrt{3}}$$
$$\therefore \tan 2x = \sqrt{3} \quad \text{(Deduced)} \quad \text{[A1]}$$

(b) From (iii)(a): $\tan 2x = \sqrt{3} \rightarrow (1)$

And, $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x} \rightarrow (2)$

$\Rightarrow (1) = (2)$: $\sqrt{3} = \dfrac{2\tan x}{1-\tan^2 x}$ [M1]

$$\sqrt{3}(1-\tan^2 x) = 2\tan x$$
$$\sqrt{3} - \sqrt{3}\tan^2 x = 2\tan x$$
$$0 = \sqrt{3}\tan^2 x + 2\tan x - \sqrt{3}$$
$$\sqrt{3}\tan^2 x + 2\tan x - \sqrt{3} = 0$$
$$(\sqrt{3}\tan x - 1)(\tan x + \sqrt{3}) = 0$$

$\sqrt{3}\tan x$	-1	$-\tan x$
$\tan x$	$+\sqrt{3}$	$+3\tan x$
$\sqrt{3}\tan^2 x$	$-\sqrt{3}$	$2\tan x$

$\rightarrow (\sqrt{3}\tan x - 1) = 0$

$\sqrt{3}\tan x = 1$

$\therefore \tan x = \dfrac{1}{\sqrt{3}}$ [A1]

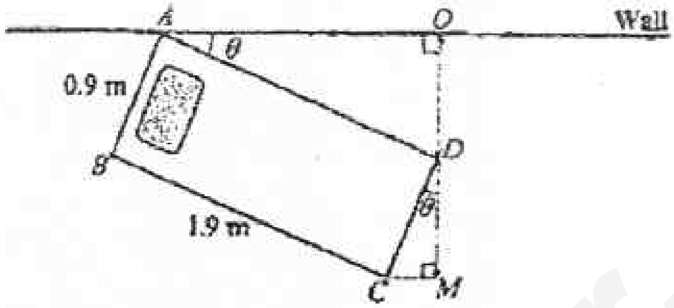
$\rightarrow (\tan x + \sqrt{3}) = 0$

$\therefore \tan x = -\sqrt{3}$ [A1]

- End Of Paper -

GMS(S) 2017 MYE 4E #5N(A) A-Maths Paper 2 [redacted] Answers

1	$\left(-\frac{17}{8},0\right)$; $\left(-2\frac{1}{8},0\right)$; $(-2.125,0)$			
2	$\frac{dV}{dt}=162\pi\ \text{cm}^3\ \text{s}^{-1}=162\pi\ \text{cm}^3/\text{s}$			
3	(i) On graph (ii) Incorrect value: $\frac{1}{v}=0.055 \Rightarrow v=18.3$, Correct value: $\frac{1}{v}=0.061 \Rightarrow v=16.4$ (3sf) $\left(\text{Accept: } \frac{1}{v}=0.061 \pm 0.001\ ;\ v=16.4 \pm 0.3\right)$ (iii) y-intercept, $c=\frac{1}{f}=0.086$ (Accept: $c \approx 0.086 \pm 0.001$) (3sf) $f=\frac{1}{0.086} \Rightarrow f \approx 11.6279$ cm (Accept: $f=11.6 \pm 0.2$) (3sf) (iv) Comparing gradient, $m=-1$ (from equation) (Graph gentler/steeper than what is required depending on student's answer)			
4	(i) Use integration	(ii) Show: $b^2-4ac<0$		
5	(i) $a \approx 10$, $b=0.22$	(ii) $(x, \lg y) \to (-0.11, 0.11)$		
6	(i) $\frac{dy}{dx}=\frac{3x}{\sqrt{2x+1}}$ $\left(\frac{kx}{\sqrt{2x+1}}\ ;\ k=3\right)$	(ii) $\frac{112}{5}$; $22\frac{2}{5}$; 22.4		
7	(i) $B(7,-4)$	(ii) Radius = 5 units $(x-3)^2+(y+1)^2=5^2$ OR $x^2+y^2-6x+2y-15=0$	(iii) $y=\frac{1}{2}x-7\frac{1}{2}$	
8	(i) Use differentiation (Product rule)	(ii) $(0,0)$, $(1,1)$, $(2,0)$	(iii) $\frac{d^2y}{dx^2}=12x^2-24x+8$ $(0,0)$ (Minimum point) $(1,1)$ (Maximum point) $(2,0)$ (Minimum point)	
9	(i) $a=-1$	(ii) $f'(x)=\frac{-4}{(x+1)^2}$ Show: $f'(x)<0$	(iii) $y=-x-3$	(iv) Show: Area = 4.5 units^2
10	(i) $A(0,4)$; $B\left(-\frac{\pi}{4},0\right)$; $C\left(\frac{\pi}{4},0\right)$ $D\left(-\frac{\pi}{2},5\right)$; $E\left(\frac{\pi}{2},1\right)$	(ii) Use: $(\cos 2x=1-2\sin^2 x)$	(iii) x-coordinate of: $x_R=-2.89$; $x_X=-0.253$ $x_Y=\frac{\pi}{6}$; $x_Z=\frac{5\pi}{6}$	
11	(i) Show: $L=OD+DM$	(ii) $\sqrt{4.42}\sin(\theta+25.346^\circ)$	(iii) Max $L=2.10$ m $\theta=64.7^\circ$ (1dp)	(iv) $L=1.3$ m $\theta=12.8^\circ$ (1dp)
12	(ii) $x=\frac{\pi}{\ }$ $\frac{7\pi}{\ }$	(iii)(b) $\tan x=\frac{1}{\sqrt{3}}$; $\tan x=-\sqrt{3}$		


A
O
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Name: Register Number: Class:

南僑中學

NAN CHIAU HIGH SCHOOL

PRELIMINARY EXAMINATION (2) 2017
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS **4047/01**
Paper 1 **12 September 2017, Tuesday**

Additional Materials : Writing Papers (8 sheets) **2 hours**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on the separate writing papers provided.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

Setter: Ms Ting Shi Yun

This paper consists of 7 printed pages including the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\mathrm{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

Answer ALL Questions.

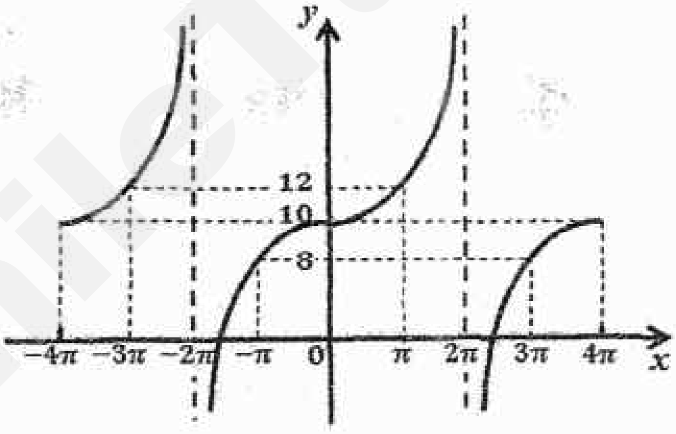
1 (a) Find the range of values of p which satisfy the inequality $px^2 + 8x + p > 6$. [4]

(b) Show that the line $y + qx = q$ will intersect the curve $y = (q+1)x^2 + qx - 1$ at two distinct points for all real values of q, where $q \neq -1$. [3]

2 A right circular cone has a vertical height of $(2\sqrt{3} - \sqrt{2})$ cm and slanted height of l cm. The volume of the cone is $(\sqrt{48} + \sqrt{18})\pi\ \text{cm}^3$. **Without using a calculator,** show that l^2 can be expressed as $a - b\sqrt{6}$, where a and b are integers. [5]

3 (a) State the values between which the principal value of $\tan^{-1} x$ must lie. [1]

(b)

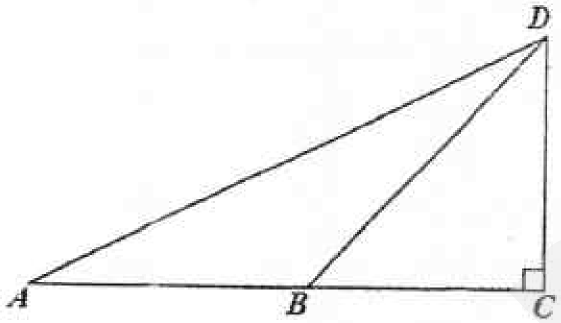


The figure shows part of the graph $y = a\tan bx + c$.
Find the value of each of the constants a, b and c. [3]

4 (i) On the same axes, sketch the curves $y = \sqrt{288x}$ and $y = 3x^3$ for $x > 0$. [2]

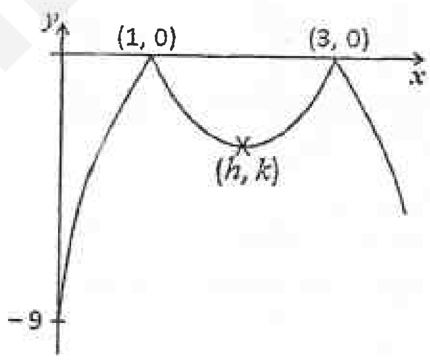
(ii) The tangent to the curve $y = 3x^3$ at the point A is parallel to the line passing through **the two** points of intersection of the curves drawn in **(i)**. Find the x–coordinate of A. [4]

5 The diagram shows a right-angled ΔACD such that $\cos\angle ADC = \frac{5}{13}$.
B is a point on the line AC such that $\tan\angle ABD = -1$.



Without finding the values of any angles, show that $\sin\angle ADB = \frac{7\sqrt{2}}{26}$. [4]

6 The diagram shows part of the curve $y = -\left|a(x-h)^2 + k\right|$, where $a > 0$.
The curve touches the x-axis at $(1, 0)$ and $(3, 0)$ and has a minimum point at (h, k). The curve also cuts the y-axis at -9.

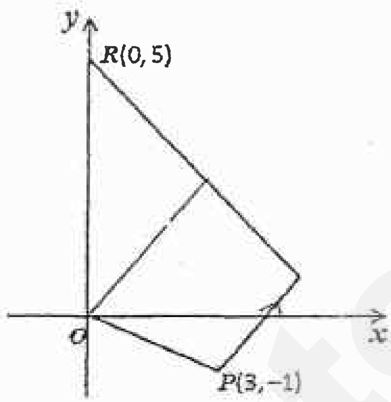


(i) Explain why $h = 2$. [1]

(ii) Determine the value of a and of k. [3]

(iii) State the set of values of m for which the line $y = m$ intersects the curve at four distinct points. [1]

7 The equations of the line PQ and QR are $y = 3x - 10$ and $4y + 3x = 20$ respectively. The coordinates of P and R are $(3, -1)$ and $(0, 5)$ respectively.



(i) Find the coordinates of Q. [2]

(ii) Name the quadrilateral $OPQR$.
Justify your answers with appropriate workings. [3]

(iii) Given that T is a point on PR such that $OPQT$ is a rhombus, find the coordinates of T. [2]

(iv) Find the ratio of the area of ΔOTP to the area of ΔOTR. [2]

8 (a) Calculate the minimum gradient of $y = 2x^3 - 9x^2 - 1$. [4]

(b) Given that $f(x) = e^{-x}(x^2 - 3x + 1)$,

(i) Find the range of values of x for which $f(x)$ is an increasing function. [4]

(ii) Hence, state the coordinates and nature of all the stationary points of $f(x)$. [3]

9 (i) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(x^2+4\right)\right]$. [1]

(ii) Express $\dfrac{4x+24}{(3x-2)(x^2+4)}$ in partial fractions. [3]

(iii) Hence, find $\int \dfrac{x+6}{(3x-2)(x^2+4)}\mathrm{d}x$. [3]

10 At 8 a.m., Ship A is 100 km due North of Ship B.
Ship A is sailing due South at 20 km/h.
Ship B is sailing at a bearing of 120° at 10 km/h.

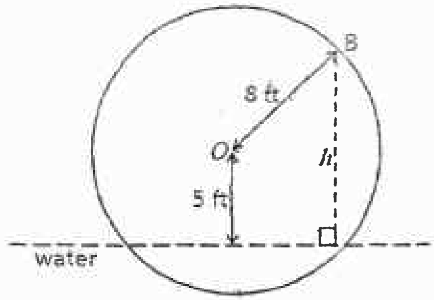
(i) Show that the distance between the two ships after t hours is given by $AB = 10\sqrt{3t^2 - 30t + 100}$. [3]

(ii) At what time is Ship A closest to Ship B? [3]

11 (i) Prove $\sin^2 2\theta\left(\cot^2\theta - \tan^2\theta\right) = 4\cos 2\theta$. [4]

(ii) Hence, solve $\sin^2 2\theta\left(\cot^2\theta - \tan^2\theta\right) = 4\sin 2\theta + 2\sqrt{2}$ for $0° \le \theta \le 360°$. [5]

12



A waterwheel rotates 5 revolutions anticlockwise in 1 minute. Tom starts a stopwatch when the bucket B is at its highest height above water level. The radius of the waterwheel is 8 ft and its centre is 5 ft above the water level.

The height of bucket B above water level is given by $h = a\cos bt + c$, where t is the time, in seconds, since Tom started the stopwatch.

(i) Determine the value of each of the constant a, b and c. [3]

(ii) For how long is $h < 0$? [3]

(iii) Explain what does the answer in (ii) mean. [1]

~~~End of paper~~~
Do check your work. All the best!
~~~

2017 Prelim 2 AM P1 Solutions

1a) $px^2+8x+p>6$

$px^2+8x+p-6>0$

$\therefore p>0$ and $(8)^2-4(p)(p-6)<0$

$-4p^2+24p+64<0$

$p^2-6p-\cancel{8}\,16>0$

$(p+2)(p-8)>0$

$\therefore p<-2$ or $p>8$

$\therefore p>8$

1b) $(q+1)x^2+qx-1=-qx+q$

$(q+1)x^2+2qx-1-q=0$

Method 1

Discriminant $=(2q)^2-4(q+1)(-1-q)$

$=(2q)^2+4(q+1)^2$

Since $(2q)^2\geq 0$ and $4(q+1)^2>0$,

$\therefore$ Discriminant >0 (Shown)

Method 2

Discriminant $=(2q)^2-4(q+1)(-1-q)$

$=8q^2+8q+4$

$=8\left(q+\frac{1}{2}\right)^2+4-8\left(\frac{1}{2}\right)^2$

$=8\left(q+\frac{1}{2}\right)^2+2>0$ (Shown)

2) $\frac{1}{3}\pi r^2(2\sqrt{3}-\sqrt{2})=(\sqrt{48}+\sqrt{18})\pi$

$r^2=\dfrac{3(4\sqrt{3}+3\sqrt{2})}{2\sqrt{3}-\sqrt{2}}$

$=\dfrac{3(4\sqrt{3}+3\sqrt{2})}{2\sqrt{3}-\sqrt{2}}\times\dfrac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}+\sqrt{2}}$

$=\dfrac{72+18\sqrt{6}+18+12\sqrt{6}}{12-2}$

$=\dfrac{90+30\sqrt{6}}{10}$

$=9+3\sqrt{6}$

$l^2=9+3\sqrt{6}+(2\sqrt{3}-\sqrt{2})^2$

$=23-\sqrt{6}$

(Do not need to specify $a=23$, $b=1$. Read On!)

3a) $-90^\circ<\tan^{-1}x<90^\circ$

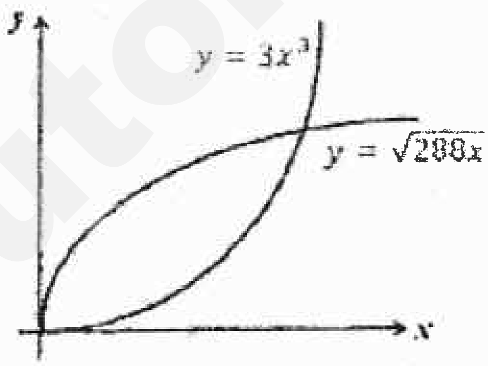
or $-\frac{\pi}{2}<\tan^{-1}x<\frac{\pi}{2}$

3b) Period $=4\pi$, $b=\frac{1}{4}$

$c=10$

$a=2$

4a) **Must label graphs!**



4b) $\sqrt{288x}=3x^3$

$288x=9x^6$

$9x(32-x^5)=0$

$x=0$ or $x=2$

$y=0$ or $y=24$

$\therefore$ gradient $=\frac{24}{2}=12$

$\frac{dy}{dx}=9x^2$

$9x^2=12$

$x^2=\frac{4}{3}$

$x=\frac{2\sqrt{3}}{3}$ (exact only because it's x-coord!)

5) Method [illegible]

$\sin\angle ADB$ [illegible]

$= \sin(\angle ADC - \angle BDC)$

$= \sin\angle ADC \cos\angle BDC - \cos\angle ADC \sin\angle BDC$

$= \left(\frac{12}{13}\right)\left(\frac{5}{5\sqrt{2}}\right) - \left(\frac{5}{13}\right)\left(\frac{5}{5\sqrt{2}}\right)$

$= \frac{12}{13\sqrt{2}} - \frac{5}{13\sqrt{2}}$

$= \frac{7}{13\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{7\sqrt{2}}{26}$ (Shown)

Method 2 *(The ratio happen to be length)*

$\tan\angle ADB = -\tan\angle DBC = 1$

$\sin\angle ABD = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\frac{\sin\angle ADB}{7} = \frac{\sin\angle ABD}{13}$

$\sin\angle ADB = \frac{7\sqrt{2}}{26}$ (Shown)

Method 3 *(The ratio happen to be length)*

Area of $\Delta ADB = \frac{1}{2} \times 7 \times 5 = 17.5$

$\therefore 17.5 = \frac{1}{2}(13)(5\sqrt{2})\sin\angle ADB$

$\sin\angle ADB = \frac{7\sqrt{2}}{26}$ (Shown)

6i) The curve is symmetrical.

6ii) $(x-1)(x-3) = 0$

$x^2 - 4x + 3 = 0$

$3(x^2 - 4x + 3) = 0$

$3[(x-2)^2 + 3 - 2^2] = 0$

$3(x-2)^2 - 3 = 0$

$a = 3$

$k = -3$

6iii) $-3 < m < 0$

7i) $4(3x - 10) + 3x = 20$

$15x = 60$

$x = 4$

$y = 2$

$Q(4, 2)$

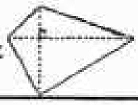
7ii) [illegible]

$OQ = \sqrt{[illegible] + (5-2)^2} = 5$

$OP = \sqrt{3^2 + (-1)^2} = \sqrt{10}$

$PQ = \sqrt{(4-3)^2 + (2+1)^2} = \sqrt{10}$

Since $OR = OQ$ and $OP = PQ$, $OPQR$ is a kite.

> Do not accept if students **only** find the grad of diagonals and conclude that they are perpendicular.
> Counter example:  which is not a kite!

> Students can prove diagonals are perpendicular **AND** one pair of adjacent sides is equal in length.

> Students can also show that the midpoint of *OQ* lies on the diagonal *PR*.

7iii) Midpoint of $OQ = (2, 1)$

Let T be (x, y)

$\left(\frac{x+3}{2}, \frac{y-1}{2}\right) = (2, 1)$

$x = 1$ and $y = 3$

$T(1, 3)$

7iv) $TR = \sqrt{1^2 + 2^2} = \sqrt{5}$

$PT = \sqrt{2^2 + 4^2} = 2\sqrt{5}$

area of ΔOTP : area of ΔOTR

$2\sqrt{5} : \sqrt{5}$

$2 : 1$

> Students can also use area shoelace method.
> Area of OTP = 5
> Area of OTR = 2.5

8a) **Method 1**

$\frac{dy}{dx} = 6x^2 - 18x \rightarrow \frac{d^2y}{dx^2} = 12x - 18$

When $12x - 18 = 0$, $x = 1.5$

Gradient $= 6(1.5)^2 - 18(1.5) = -13.5$

$\frac{d^3y}{dx^3} = 12 > 0 \quad \therefore$ minimum gradient

Method 2

$\frac{dy}{dx} = 6x^2 - 18x$

$= 6(x - 1.5)^2 - 6(1.5)^2$

$= 6(x - 1.5)^2 - 13.5$

$\therefore$ min gradient $= -13.5$

10ii) $\dfrac{dAB}{dt} = \dfrac{10\left(\frac{1}{2}\right)(6t-30)}{\sqrt{3t^2-30t+100}}$

When $\dfrac{dAB}{dt}=0$, $\quad 6t=30$, $\quad \therefore t=5$

Method 1

t	4.9	5	5.1
$\dfrac{dAB}{dt}$	$-$	0	$+$
Sketch	\	—	/

$\therefore AB$ is a minimum.

Method 2

$$\frac{d^2AB}{dt^2} = \frac{30}{\sqrt{3t^2-30t+100}} + \frac{(6t-30)(75-15t)}{\left(3t^2-30t+100\right)^{\frac{3}{2}}}$$

When $t=5$, $\dfrac{d^2AB}{dt^2}=6 \quad >0$

$\therefore AB$ is a minimum.

Time $= 0800 + 5\text{h} = 1300$ or 1pm

11i) $\sin^2 2\theta\left(\cot^2\theta - \tan^2\theta\right)$

$$= (2\sin\theta\cos\theta)^2\left(\frac{\cos^2\theta}{\sin^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}\right)$$

$$= 4\sin^2\theta\cos^2\theta\left(\frac{\cos^4\theta-\sin^4\theta}{\sin^2\theta\cos^2\theta}\right)$$

$$= 4\,(\cos^2\theta-\sin^2\theta)(\cos^2\theta+\sin^2\theta)$$

$$= 4\,(\cos 2\theta)(1)$$

$$= 4\cos 2\theta$$

11ii) $4\cos 2\theta = 4\sin 2\theta + 2\sqrt{2}$

$$4\cos 2\theta - 4\sin 2\theta = 2\sqrt{2}$$

$$\cos 2\theta - \sin 2\theta = \frac{\sqrt{2}}{2}$$

$$\sqrt{2}\cos(2\theta+45^\circ) = \frac{\sqrt{2}}{2}$$

$$\cos(2\theta+45^\circ) = 0.5$$

Basic angle $= 60^\circ$

$2\theta + 45^\circ = 60, 300, 420, 660$

$\theta = 7.5^\circ, 127.5^\circ, 187.5^\circ, 307.5^\circ$

12i) Period $=$ 12 seconds

$\dfrac{2\pi}{b} = 12 \rightarrow b = \dfrac{\pi}{6}$, $\quad a=8$, $\quad c=5$

12ii) $8\cos\dfrac{\pi}{6}t + 5 = 0$

$\cos\dfrac{\pi}{6}t = -\dfrac{5}{8}$

Basic angle $= 0.89566$

$\dfrac{\pi}{6}t = 2.2459, \quad 4.03726$

$t = 4.28935, \quad 7.7106$

Duration $= 7.7106 - 4.28935$

$= 3.42125$

$=3.42$ seconds

12iii) It is the duration of the bucket when it is in the water.

(accept "below water level", "submerged under water")

8bi) $f'(x) = -e^{-x}(x^2 - 3x + 1) + e^{-x}(2x - 3)$

$= e^{-x}(-x^2 + 5x - 4)$

For f(x) to be increasing, $f'(x) > 0$.

Since $e^{-x} > 0$, $-x^2 + 5x - 4 > 0$

$x^2 - 5x + 4 < 0$

$(x-1)(x-4) < 0$

$1 < x < 4$

8bii) When $f'(x) = 0$, $x = 1$ and $x = 4$

$\left(1, -\frac{1}{e}\right)$ min point and $\left(4, \frac{5}{e^4}\right)$ max point

(Exact value of coords only!)

9i) $\frac{d}{dx}\left[\ln\left(x^2+4\right)\right] = \frac{2x}{x^2+4}$

9ii) $\frac{4x+24}{(3x-2)(x^2+4)} = \frac{A}{3x-2} + \frac{Bx+C}{x^2+4}$

$4x + 24 = A(x^2+4) + (Bx+C)(3x-2)$

When $x = \frac{2}{3}$, $\frac{80}{3} = \frac{40}{9}A$ $\rightarrow A = 6$

Comparing coeff of x^2: $0 = A + 3B$ $\rightarrow B = -2$

Comparing coeff of x^0: $24 = 4A - 2C \rightarrow C = 0$

$\frac{4x+24}{(3x-2)(x^2+4)} = \frac{6}{3x-2} - \frac{2x}{x^2+4}$

9iii) **Method 1**

$\int \frac{x+6}{(3x-2)(x^2+4)}\,dx$

$= \frac{1}{4}\int \frac{4x+24}{(3x-2)(x^2+4)}\,dx$

$= \frac{1}{4}\int \frac{6}{3x-2} - \frac{2x}{x^2+4}\,dx$

$= \frac{1}{4}\left[\frac{6\ln(3x-2)}{3} - \ln(x^2+4)\right] + c$

$= \frac{1}{2}\ln(3x-2) - \frac{1}{4}\ln(x^2+4) + c$

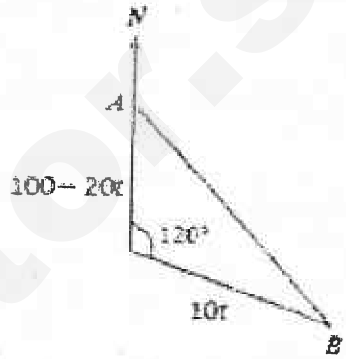
9iii) **Method 2**

$4\int \frac{x+6}{(3x-2)(x^2+4)}\,dx = \int \frac{6}{3x-2} - \frac{2x}{x^2+4}\,dx$

$\int \frac{x+6}{(3x-2)(x^2+4)}\,dx = \frac{1}{4}\left[\frac{6\ln(3x-2)}{3} - \ln(x^2+4)\right] + c$

$= \frac{1}{2}\ln(3x-2) - \frac{1}{4}\ln(x^2+4) + c$

10i) **Method 1**



$AB^2 = (100-20t)^2 + (10t)^2 - 2(100-20t)(10t)\cos 120°$

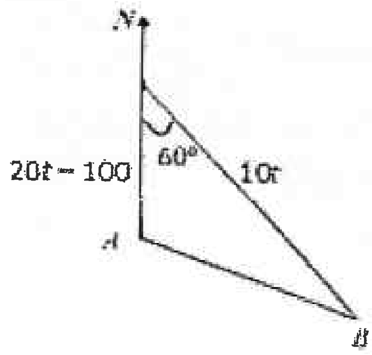
$AB^2 = 10000 - 4000t + 400t^2 + 100t^2 + 1000t - 200t^2$

$AB^2 = 300t^2 - 3000t + 10000$

$AB^2 = 100(3t^2 - 30t + 100)$

$AB = 10\sqrt{3t^2 - 30t + 100}$ (Shown)

Method 2



$AB^2 = (20t-100)^2 + (10t)^2 - 2(20t-100)(10t)\cos 60°$

$AB^2 = 300t^2 - 3000t + 10000$

$AB = 10\sqrt{3t^2 - 30t + 100}$ (Shown)

2017 4E AM Naval Base Prelim

Answer all the questions.

1 Given that $\tan A = \frac{5}{12}$, $\cos B = -\frac{4}{5}$ and that A and B are in the same quadrant, calculate without the use of calculator, the value of

(i) $\cos(A+B)$, [2]

(ii) $\cos\frac{A}{2}$. [2]

2 (i) Find the equation of a curve with $\frac{dy}{dx} = \frac{6}{(2x-1)^2}$ and which passes through the point $(1, 7)$. [2]

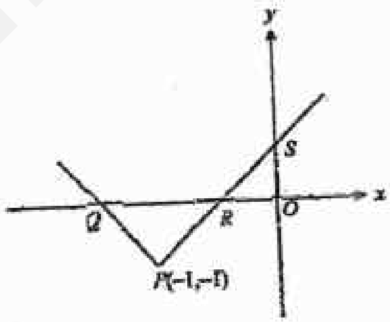
(ii) The curve for which $\frac{dy}{dx} = 2x + k$, where k is a constant, has a turning point at $(2, -9)$.

(a) Show that $k = -4$. [1]

(b) Hence, find the equation of the curve. [2]

3 Given that $\sqrt{a + b\sqrt{3}} = \frac{2\sqrt{3}}{3-\sqrt{3}}$, where a and b are integers, find, without using a calculator, the value of a and b. [5]

4



The diagram shows part of the graph of $y = |kx - 2| - 1$ where $P(-1, -1)$ is the minimum point of the graph.

(i) Show that the value of k is -2. [1]

(ii) Find the coordinates of the points Q, R and S. [3]

(iii) Hence, write down the range of values of x for which y is positive. [1]

[Turn over

5 (i) Write down the principal value, in radians as a multiple of π, of

(a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$, [1]

(b) $\tan^{-1}(-1)$. [1]

(ii) Solve the equation $2\sin 2x = -1$, for $0 \le x \le 2\pi$. [3]

6 (i) Find the range of values of k for which the line $y = k - 2kx$ does not intersect the curve $y = x^2 + 2$. [4]

(ii) Hence, describe the relationship between the curve $y = x^2 + 2$ and the line $y = 1 - 2x$. [1]

7 Determine the nature of each of the stationary points of the curve $y = \frac{x^2 + 3}{x - 1}$. [6]

8 (i) Prove the identity $\frac{1 + \cos A + \cos 2A}{\sin A + \sin 2A} = \cot A$. [3]

(ii) Sketch the graph of $y = 2\sin 2x - 3$ for $-180° \le x \le 180°$. [3]

9 (i) Given that $y = 2xe^{\frac{x}{2}}$, show that $\frac{dy}{dx} = xe^{\frac{x}{2}} + 2e^{\frac{x}{2}}$. [3]

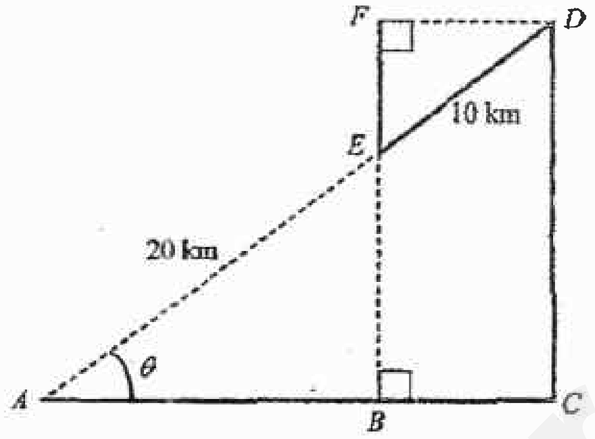
(ii) Hence, find the exact value of $\int_0^2 xe^{\frac{x}{2}}\,dx$. [3]

10 (i) Express $\frac{2x^3 + 5x^2 + 3}{2x^2 + x - 1}$ in partial fractions. [5]

(ii) Hence, find $\int \frac{2x^3 + 5x^2 + 3}{2x^2 + x - 1}\,dx$. [2]

[Turn over

11 The diagram shows a proposed route $ABCDEF$ travelled by a group of cyclist.



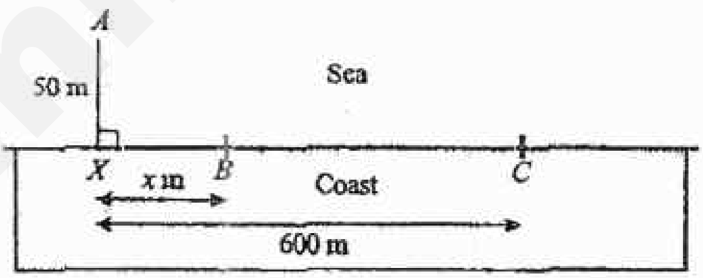
$BCDF$ is a rectangle. ABC is a straight line and the line AD intersects BF at E.
$AE = 20$ km, $ED = 10$ km and angle EAB is θ, where $0° < \theta < 90°$.
The total distance of the route is S km.

(i) Show that the total distance, S, is given by $S = 10 + 30\cos\theta + 40\sin\theta$. [2]

(ii) Express $30\cos\theta + 40\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [4]

(iii) Given that the total distance of the route is 60 km, find the value of θ. [2]

12 A man is sitting on a boat at point A, 50 metres from a point X on a straight coast. He wishes to get as quickly as possible to a point C on the coast 600 metres from X. From A, he rows the boat at 40 m/min to point B, and then cycles along the coast at 50 m/min to point C.



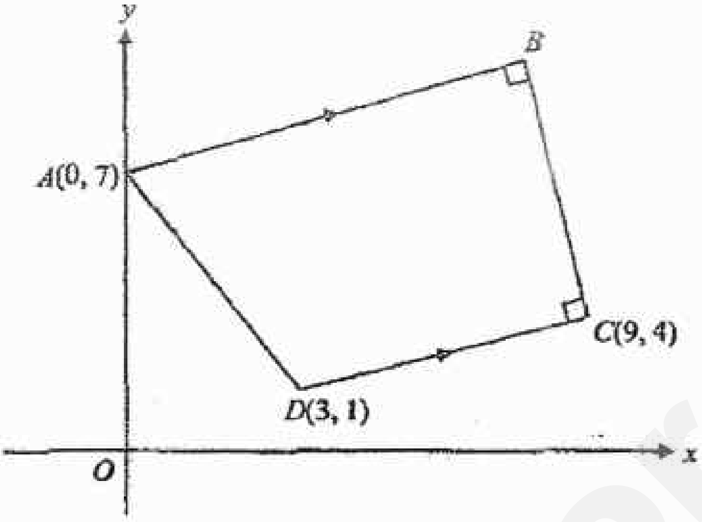
(i) Express the time taken for the man to travel from A to B and B to C in terms of x, and show that the total time taken, T min, for the man to travel from A to C is given by $T = \dfrac{\sqrt{x^2 + 2500}}{40} - \dfrac{x}{50} + 12$. [3]

(ii) Obtain an expression for $\dfrac{dT}{dx}$. [2]

(iii) Find the distance from X where the man should land so that he can get to C in the least possible time. (Note: Proof of minimum value is not required) [3]

[Turn over

13



$ABCD$ is a trapezium where AB is parallel to DC, and angle ABC = angle $BCD = 90°$.

The coordinates of A, C and D are (0, 7), (9, 4) and (3, 1) respectively.

(i) Calculate the gradient of DC. [1]

(ii) Find the equation of the line BC. [2]

(iii) Show that the coordinates of B is (6, 10). [3]

(iv) Calculate the area of the trapezium $ABCD$. [2]

(v) If B is the midpoint of a line segment DE, find the coordinates of E. [2]

---------END OF PAPER---------

1 Given that $2^{3(x-1)} = 27^{2-x}$, find the value of 6^x without using a calculator. [3]

2 The expression $ax^3 + 4x^2 + bx - 1$ is exactly divisible by $2x - 1$ and has a remainder of -6 when divided by $x + 1$. Find the value of each of the constants a and of b. [4]

3 The function f is defined, for all values of x, by $\mathrm{f}(x) = \dfrac{x}{x^2 + 9}$. Find the values of x for which f is a decreasing function. [4]

4 Solve the equation $x^3 - 4x^2 - 8x + 8 = 0$, expressing non-integer solutions in the form $a \pm \sqrt{b}$, where a and b are integers. [5]

5 Solve the equation $5\log_2 y = 4 + \log_y 2$. [5]

6 The curve $y = x^3 - 6x^2 + k$ touches the positive x-axis at point A.

(i) Find the coordinates of A. [2]

(ii) Find the value of k for and the value of $\dfrac{d^2y}{dx^2}$ at A. [3]

7 The mass, m grams, of a radioactive substance is modelled by an equation in the form

$$m = m_0 e^{-kt},$$

where m_0 and k are constants and t is the time in days after the mass was first recorded. The table below gives values of m and t for the days recorded from 10 to 40 days.

t days	10	20	30	40
m grams	40.3	27.0	18.1	12.2

(i) Plot ln m against t for the given data and draw straight line graph. [2]

(ii) Estimate the value of m_0 and k. [3]

(iii) Assuming that the radioactive substance decayed at a constant rate, estimate the number of grams of substance at 5 days. [2]

[Turn over

8 (i) Find, in ascending powers of x, the first 3 terms in the expansion of $(2-3x)^5$. [3]

The first 3 terms in the expansion of $(a+bx)(2-3x)^5$ in ascending powers of x are $32-176x+cx^2$.

(ii) Find the value of a, of b and of c. [5]

9 (i) Using $\sin 3x = \sin(2x+x)$, show that $\sin 3x$ may be expressed as $\sin x(4\cos^2 x-1)$. [3]

(ii) Find all the values between $0°$ and $360°$ for which $3\sin 3x = 16\sin x\cos x$. [5]

10 The roots of the quadratic equation $x^2+5x+2=0$ are α and β. Find

(i) the value of $\alpha^3+\beta^3$, [5]

(ii) a quadratic equation with roots $\dfrac{\alpha}{\beta^2}$ and $\dfrac{\beta}{\alpha^2}$. [3]

11 The equation of a curve is $y = x^3+6x^2-15x+k$, where k is a constant.

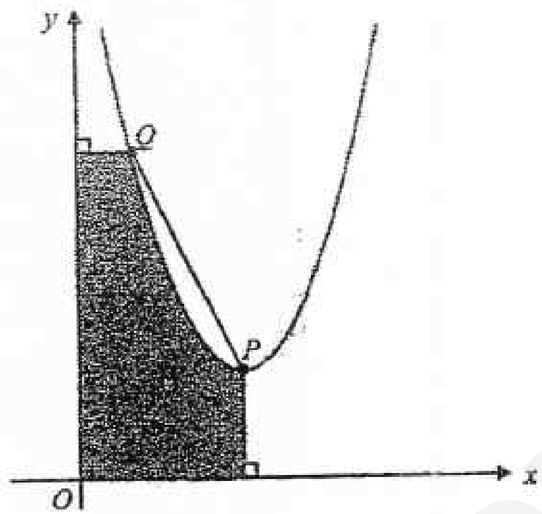
(i) Find the set of values of x for which y is a decreasing function, [4]

(ii) Find the positive value of k for which the x-axis is a tangent to the curve. [3]

The variables of x and y increase in such a way that, when $x = 2$, y is increasing at a rate of 0.5 units per second.

(iii) Find the rate of change of x, when $x = 2$. [2]

12



The diagram shows part of the graph of $y = x^2 - 6x + 11$ passing through the points P and Q. The curve has a minimum point at P.

(i) Find the coordinates of P. [3]

The gradient of the line PQ is -2.

(ii) Show that the coordinates of Q is $(1, 6)$. [4]

(iii) Showing all your working, find the total area of the shaded region. [3]

13 (i) Given that $u = 2^x$, express $2^{2x} = 2^{x+3} - 7$ as an equation in u. [3]

(ii) Hence, find the values of x for which $2^{2x} = 2^{x+3} - 7$, giving your answer, where appropriate, to 1 decimal place. [4]

(iii) Explain why the equation $2^{2x} = 2^{x+3} - k$ has no solution if $k > 16$. [3]

14 The equation of a circle, C_1, with centre A, is $x^2 + y^2 - 12x - 6y + 35 = 0$.

(i) Find the coordinates of A and the radius of C_1. [3]

(ii) Show that the point $P(3, 4)$ lies on C_1. [1]

(iii) Find the equation of the tangent to C_1 at P. [3]

A second circle, C_2, has a diameter EF. The point E has coordinates $(-2, 2)$ and the equation of the tangent to C_2 at F is $4y = -3x - 48$.

(iv) Find the equation of the diameter EF and hence the coordinates of F. [4]

(v) Find the coordinates of its centre and radius of C_2. [3]

——End of Paper——

Sec 4ESA Add Mathematics
Prelim 1, 2017 (Solutions)
Paper 1

1(i) Quadrant 3

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$= \left(-\frac{12}{13}\right)\left(-\frac{4}{5}\right) - \left(-\frac{5}{13}\right)\left(-\frac{3}{5}\right)$ M1

$= \frac{33}{65}$ A1

(ii) $\cos\frac{A}{2} = -\sqrt{\frac{\cos A + 1}{2}}$

$= -\sqrt{\frac{\left(-\frac{12}{13}\right)+1}{2}}$ M1

$= -\frac{1}{\sqrt{26}}$ A1

2(i) $\frac{dy}{dx} = 6(2x-1)^{-2}$

$y = \frac{6(2x-1)^{-1}}{(-1)(2)} + c$

$= \frac{-3}{2x-1} + c$ M1

Subst (1, 7) into equation,

$7 = \frac{-3}{2(1)-1} + c$

$c = 10$

Equation is $y = \frac{-3}{2x-1} + 10$ A1

(ii)(a) $\frac{dy}{dx} = 2x + k$

At $x = 2$, $\frac{dy}{dx} = 0$

$0 = 2(2) + k$

$k = -4$ B1

(b) $\frac{dy}{dx} = 2x - 4$

$y = \frac{2x^2}{2} - 4x + c$

Subst (2, −9) into equation,

$-9 = (2)^2 - 4(2) + c$

$-5 = c$

Equation is $y = x^2 - 4x - 5$ A1

3. $\sqrt{a+b\sqrt{3}} = \frac{2\sqrt{3}}{3-\sqrt{3}} \frac{\times(3+\sqrt{3})}{\times(3+\sqrt{3})}$ M1

$\sqrt{a+b\sqrt{3}} = \frac{6\sqrt{3}+2(3)}{9-3}$

$\sqrt{a+b\sqrt{3}} = \sqrt{3}+1$ M1

$a+b\sqrt{3} = (\sqrt{3}+1)^2$

$a+b\sqrt{3} = 3+2\sqrt{3}+1$ M1

$a+b\sqrt{3} = 4+2\sqrt{3}$

$a = 4,\ b = 2$ A1, A1

4(i) $y = |kx-2|-1$

Subst (−1, −1)

$-1 = |-k-2|-1$

$0 = |-k-2|$

$0 = -k-2$

$k = -2$ B1

(ii) $y = |-2x-2|-1$

Let $x = 0$,

$y = |-2|-1 = 1$ $S(0, 1)$ B1

Let $y = 0$,

$0 = |-2x-2|-1$

$1 = |-2x-2|$

$-2x-2 = 1$ or $-2x-2 = -1$

$x = -1.5$ $x = -0.5$

$Q(-1.5, 0)$ $R(-0.5, 0)$ B1, B1

(iii) $x < -1.5$ or $x > -0.5$ B1

5(i)(a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{3}$ B1

(b) $\tan^{-1}(-1)=-\frac{\pi}{4}$ B1

(ii) $2\sin 2x=-1$

$\sin 2x=\frac{-1}{2}$

(Q3, Q4)

Basic angle $=\sin^{-1}\frac{1}{2}=0.52360$ (or $\frac{\pi}{6}$) M1

New range: $0\le x\le 4\pi$

$Q3=\pi+0.52360$, $Q4=2\pi-0.52360$ M1

$=3.6652$ $=5.7596$

$2x=3.6652, 5.7596, 9.9484, 12.0428$

$x=1.83, 2.88, 4.97, 6.02$ A1

6(i) $y=k-2kx$ ---(1)

$y=x^2+2$ ---(2)

Subst (1) into (2)

$k-2kx=x^2+2$

$x^2+2kx+(2-k)=0$ M1

$b^2-4ac<0$

$(2k)^2-4(1)(2-k)<0$ M1

$4k^2+4k-8<0$

$k^2+k-2<0$ M1

$(k-1)(k+2)<0$

$-2<k<1$ A1

(ii) $y=1-2x$ $y=k-2kx$

Comparing, $k=1$

$b^2-4ac=k^2+k-2$

$=(1)^2+(1)-2$

$=0$

Hence, $y=1-2x$ is a tangent to the curve $y=x^2+2$. B1

7. $y=\dfrac{x^2+3}{x-1}$

$\dfrac{dy}{dx}=\dfrac{(x-1)(2x)-(x^2+3)(1)}{(x-1)^2}$ M1

(Quotient Rule)

$=\dfrac{x^2-2x-3}{(x-1)^2}$

Let $\dfrac{dy}{dx}=0$

$\dfrac{x^2-2x-3}{(x-1)^2}=0$ M1

$(x-3)(x+1)=0$

$x=3$ or -1 M1

$y=6$ or -2

$\dfrac{dy}{dx}=\dfrac{x^2-2x-3}{(x-1)^2}$

$\dfrac{d^2y}{dx^2}=\dfrac{(x-1)^2(2x-2)-(x^2-2x-3)2(x-1)}{(x-1)^4}$ (Or 1st derivative test)

$=\dfrac{(x-1)[(x-1)(2x-2)-(x^2-2x-3)2]}{(x-1)^4}$

$=\dfrac{2x^2-2x-2x+2-2x^2+4x+6}{(x-1)^3}$

$=\dfrac{8}{(x-1)^3}$ M1

At $x=3$,

$\dfrac{d^2y}{dx^2}=1>0 \rightarrow (3,6)$ is a minimum point A1

At $x=-1$,

$\dfrac{d^2y}{dx^2}=-1<0 \rightarrow (-1,-2)$ is a maximum point A1

8(i) $\dfrac{1+\cos A+\cos 2A}{\sin A+\sin 2A}=\cot A$

LHS $=\dfrac{1+\cos A+\cos 2A}{\sin A+\sin 2A}$

$=\dfrac{1+\cos A+(2\cos^2 A-1)}{\sin A+(2\sin A\cos A)}$ M1

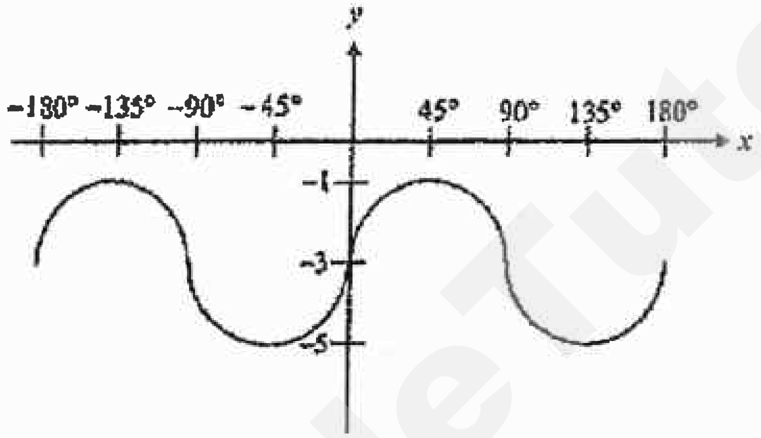
$=\dfrac{\cos A+2\cos^2 A}{\sin A+2\sin A\cos A}$

$=\dfrac{\cos A(1+2\cos A)}{\sin A(1+2\cos A)}$ M1

$=\dfrac{\cos A}{\sin A}$

$=\cot A=$ RHS A1

(ii)



B1 – intervals
B1 – shape of sine graph
B1 – position on y-axis

9(i) $y = 2xe^{\frac{x}{2}}$

Let $u = 2x$ $\quad v = e^{\frac{x}{2}}$

$\frac{du}{dx} = 2$ $\quad \frac{dv}{dx} = \frac{1}{2}e^{\frac{x}{2}}$ M1

$\frac{dy}{dx} = (2x)\left(\frac{1}{2}e^{\frac{x}{2}}\right) + \left(e^{\frac{x}{2}}\right)(2)$ M1

$= xe^{\frac{x}{2}} + 2e^{\frac{x}{2}}$ A1

(ii) From (i),

$\int_0^2 xe^{\frac{x}{2}} + 2e^{\frac{x}{2}}\ dx = \left[2xe^{\frac{x}{2}}\right]_0^2$ M1

$\int_0^2 xe^{\frac{x}{2}}\ dx + \int_0^2 2e^{\frac{x}{2}}\ dx = \left[2xe^{\frac{x}{2}}\right]_0^2$

$\int_0^2 xe^{\frac{x}{2}}\ dx = \left[2xe^{\frac{x}{2}}\right]_0^2 - \int_0^2 2e^{\frac{x}{2}}\ dx$

$= \left[2xe^{\frac{x}{2}}\right]_0^2 - \left[\frac{2e^{\frac{x}{2}}}{\left(\frac{1}{2}\right)}\right]_0^2$ M1

$= \left[2(2)e^{\frac{2}{2}} - 2(0)e^{\frac{0}{2}}\right] - \left[4e^{\frac{2}{2}} - 4e^{\frac{0}{2}}\right]$

$= 4e - (4e - 4) = 4$ A1

$$\begin{array}{r|l} & \quad x+2 \\ \hline 2x^2+x-1 & 2x^3+5x^2+0x+3 \\ & \underline{2x^3+x^2-x} \\ & \qquad 4x^2+x+3 \\ & \qquad \underline{4x^2+2x-2} \\ & \qquad\quad -x+5 \end{array}$$ M1

$$\frac{-x+5}{(2x-1)(x+1)}=\frac{A}{2x-1}+\frac{B}{x+1}$$ M1

$-x+5=A(x+1)+B(2x-1)$

Let $x=-1$ M1

$-(-1)+5=B(-2-1) \rightarrow B=-2$ A1

Let $x=0.5$

$-0.5+5=A(1.5) \rightarrow A=3$ A1

$$\frac{2x^3+5x^2+3}{2x^2+x-1}=x+2+\frac{3}{2x-1}-\frac{2}{x+1}$$

$$\int\frac{2x^3+5x^2+3}{2x^2+x-1}\,dx = \int x+2+\frac{3}{2x-1}-\frac{2}{x+1}\,dx$$

$$=\frac{x^2}{2}+2x+\frac{3\ln(2x-1)}{2}-2\ln(x+1)+c$$ B2

11(i) $\sin\theta = \dfrac{BE}{20} \rightarrow BE = 20\sin\theta$

$\cos\theta = \dfrac{AB}{20} \rightarrow AB = 20\cos\theta$

$\sin\theta = \dfrac{EF}{10} \rightarrow EF = 10\sin\theta$

$\cos\theta = \dfrac{FD}{10} \rightarrow FD = 10\cos\theta$ M1

$S = AB + BC + CD + DE + EF$
$= 20\cos\theta + 10\cos\theta + (20\sin\theta + 10\sin\theta) + 10 + 10\sin\theta$
$= 10 + 30\cos\theta + 40\sin\theta$ A1

(ii) $30\cos\theta + 40\sin\theta = R\cos(\theta - \alpha)$

$30\cos\theta + 40\sin\theta = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

By comparison,

$30 = R\cos\alpha$ ---- (1) and $40 = R\sin\alpha$ ---- (2)

$(1)^2 + (2)^2$, $30^2 + 40^2 = R^2(\cos^2\alpha + \sin^2\alpha)$ M1

$2500 = R^2$

$R = 50$ A1

$\dfrac{(2)}{(1)}$, $\tan\alpha = \dfrac{40}{30}$ M1

$\alpha = \tan^{-1}\dfrac{4}{3} = 53.1301°$ A1

Thus, $30\cos\theta + 40\sin\theta = 50\cos(\theta - 53.1°)$

(iii) $10 + 30\cos\theta + 40\sin\theta = 60$
$50\cos(\theta - 53.1301°) = 50$
$\cos(\theta - 53.1301°) = 1$ M1
$\theta - 53.1301° = 0°$ (Q1, reject Q4)
$\theta = 53.1°$ A1

12(i) Distance from A to $B = \sqrt{x^2 + 50^2}$

Time taken from A to $B = \dfrac{\sqrt{x^2 + 2500}}{40}$ min M1

Time taken from B to $C = \dfrac{600 - x}{50}$ min M1

Total time, $T = \dfrac{\sqrt{x^2 + 2500}}{40} + \dfrac{600 - x}{50}$

$= \dfrac{\sqrt{x^2 + 2500}}{40} - \dfrac{x}{50} + 12$ min A1

(ii) $T = \dfrac{(x^2 + 2500)^{\frac{1}{2}}}{40} - \dfrac{x}{50} + 12$

$\dfrac{dT}{dx} = \dfrac{\frac{1}{2}(x^2 + 2500)^{-\frac{1}{2}}(2x)}{40} - \dfrac{1}{50}$

$= \dfrac{x}{40\sqrt{x^2 + 2500}} - \dfrac{1}{50}$ B2

(iii) Let $\dfrac{dT}{dx} = 0$

$\dfrac{x}{40\sqrt{x^2 + 2500}} - \dfrac{1}{50} = 0$ M1

$\dfrac{x}{40\sqrt{x^2 + 2500}} = \dfrac{1}{50}$

$50x = 40\sqrt{x^2 + 2500}$

$2500x^2 = 1600(x^2 + 2500)$ M1

$900x^2 = 4000000$

$x^2 = \dfrac{40000}{9}$

$x = 66.7\text{ m}$ or $x = -66.7\text{ m}$ (reject) A1

13(i) Gradient of $DC = \frac{4-1}{9-3} = \frac{1}{2}$ B1

(ii) Gradient of $BC = -2$

$y = -2x + c$ M1

Subst (9, 4) into equation

$4 = -2(9) + c$

$c = 22$

Equation of BC is $y = -2x + 22$ A1

(iii) Gradient of $AB = \frac{1}{2}$

$y = \frac{1}{2}x + c$

Subst (0, 7) into equation

$7 = \frac{1}{2}(0) + c$

$c = 7$

Equation of AB is $y = \frac{1}{2}x + 7$ ---(1) M1

$y = -2x + 22$ ---(2)

Subst (1) into (2),

$\frac{1}{2}x + 7 = -2x + 22$ M1

$\frac{5}{2}x = 15$

$x = 6$

$y = 10$ $B\,(6, 10)$ A1

(iv) Area $= \frac{1}{2}\begin{vmatrix} 0 & 3 & 9 & 6 & 0 \\ 7 & 1 & 4 & 10 & 7 \end{vmatrix}$

$= \frac{1}{2}[0 + 12 + 90 + 42 - 21 - 9 - 24 - 0]$ M1

$= 45 \text{ units}^2$ A1

(v) $(6, 10) = \left(\frac{x+3}{2}, \frac{y+1}{2}\right)$

Comparing,

$x =$ M1, A1

4E5N AMath Prelim 1 Paper 2, 2017
Answer Scheme

Qn	Answer
1	$2^{3(x-1)} = 27^{2-x}$ $\frac{2^{3x}}{2^3} = \frac{3^6}{3^{3x}}$ $6^{3x} = 5832$ $6^x = 18$
2	$f(\frac{1}{2}) = ax^3 + 4x^2 + bx - 1 = 0$ $a(\frac{1}{2})^3 + 4(\frac{1}{2})^2 + b(\frac{1}{2}) - 1 = 0$ $a = -4b - - - - - (1)$ $f(-1) = ax^3 + 4x^2 + bx - 1 = 0$ $a(-1)^3 + 4(-1)^2 + b(-1) - 1 = 0$ $a + b = 9 - - - - - - - - - (2)$ Sub (1) into (2) $-4b + b = 9$ $b = -3$ $\therefore a = -12$
3	$f(x) = \frac{x}{x^2+9}$ $f'(x) = \frac{(x^2+9)(1) - (x)(2x)}{(x^2+9)^2}$ $= \frac{x^2 + 9 - 2x^2}{(x^2+9)^2}$ $= \frac{9 - x^2}{(x^2+9)^2}$ Since $(x^2+9)^2 > 0$, and the function f is a decreasing function, $9 - x^2 < 0$ $x^2 - 9 > 0$ $(x+3)(x-3) > 0$ $x < -3$ or $x > 3$

4	$x^3-4x^2-8x+8=0$ By trial and error, $(x+2)$ is a factor $(x+2)(x^2-6x+4)=0$ $x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(4)}}{2(1)}$ $x=\frac{6\pm\sqrt{36-16}}{2}$ $x=2$ or $x=\frac{6\pm\sqrt{20}}{2}$ $x=\frac{6\pm 2\sqrt{5}}{2}$ $x=3\pm\sqrt{5}$
5	$5\log_2 y=4+\log_y 2$ $5\log_2 y-\log_y 2-4=0$ $5\log_2 y-\frac{1}{\log_2 y}-4=0$ $5(\log_2 y)^2-4\log_2 y-1=0$ $(\log_2 y-1)(5\log_3 y+1)=0$ $\log_2 y=1$ *or* $\log_2 y=-\frac{1}{5}$ $y=2$ $y=0.871$
6i	Since the curve touches the x-axis, point A is a stationery point $\frac{dy}{dx}=0$ $y=x^3-6x^2+k$ $\frac{dy}{dx}=3x^2-12x$ $3x^2-12x=0$ $3x(x-4)=0$ $x=0$ *or* $x=4$ Coordinates of A is (4, 0)
6ii	At point A(4, 0) $y=x^3-6x^2+k$ $0=4^3-6(4)^2+k$ $k=32$ $\frac{dy}{dx}=3x^2-12x$ At $x=4$,

	$\frac{d^2y}{dx^2}=6(4)-12$ $=12$
7i	<table><tr><td>t</td><td>10</td><td>20</td><td>30</td><td>40</td></tr><tr><td>$\ln m$</td><td>3.7</td><td>3.3</td><td>2.9</td><td>2.5</td></tr></table> Refer to graph attached behind
7ii	$-k=\frac{3.3-2.9}{20-30}$ $k=0.04(\pm0.01)$ $\ln m_0=4.1$ $m_0=60.3$
7iii	$\ln m=3.9$ $m=49.4$
8i	$(2-3x)^5$ $=2^5+\binom{5}{1}(2)^4(-3x)+\binom{5}{2}(2)^3(-3x)^2+...$ $=32-240x+720x^2+...$
8ii	$(a+bx)(2-3x)^5$ $=32a-240ax+720ax^2+32bx-240bx^2+...$ $=32a+(32b-240a)x+(720a-240b)x^2+....$ By comparing terms, $32a=32$ $a=1$ $-240a+32b=-176$ $b=2$ $720a-240b=c$ $c=240$
9i	$\sin(2x+x)$ $=\sin 2x\cos x+\cos 2x\sin x$ $=2\sin x\cos^2 x+\cos 2x\sin x$ $=2\sin x\cos^2 x+(2\cos^2 x-1)\sin x$ $=\sin x(4\cos^2 x-1)$ (shown)

9ii	$3\sin 3x = 16\sin x\cos x$ $3\sin x(4\cos^2 x-1)-16\sin x\cos x=0$ $\sin x(12\cos^2 x-3-16\cos x)=0$ $\sin x=0$ $x=0°, 180°$ $12\cos^2 x-16\cos x-3=0$ $(2\cos x-3)(6\cos x+1)=0$ $\cos x=1.5(\textit{reject})$ *or* $\cos x=-\frac{1}{6}$ $x=180-80.41, \quad 180+80.41$ $x=99.6°, 260.4°$
10i	$x^2+5x+2=0$ Sum of roots: $\alpha+\beta=-5$ Product of roots: $\alpha\beta=2$ $\alpha^3+\beta^3$ $=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=(\alpha+\beta)[(\alpha+\beta)^2-2\alpha\beta-\alpha\beta]$ $=(-5)[(-5)^2-3(2)]$ $=-95$
10ii	Sum of roots: $\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}$ $=\frac{\alpha^3+\beta^3}{\alpha^2\beta^2}$ $=\frac{-95}{(2)^2}$ $=\frac{-95}{4}$ Product of roots: $\frac{\alpha}{\beta^2}\times\frac{\beta}{\alpha^2}$ $=\frac{1}{\alpha\beta}$ $=\frac{1}{2}$

	Quadratic equation: $x^2 - \left(-\frac{95}{4}\right)x + \frac{1}{2} = 0$ $x^2 + \frac{95}{4}x + \frac{1}{2} = 0$
11i	$y = x^3 + 6x^2 - 15x + k$ $\frac{dy}{dx} = 3x^2 + 12x - 15$ $3x^2 + 12x - 15 < 0$ $x^2 + 4x - 5 < 0$ $(x+5)(x-1) < 0$ $-5 < x < 1$
11ii	$\frac{dy}{dx} = 3x^2 + 12x - 15 = 0$ $x^2 + 4x - 5 = 0$ $(x+5)(x-1) < 0$ $x = -5 \text{ or } x = 1$ When $x = 1$, $0 = 1^3 + 6(1)^2 - 15(1) + k$ $k = 8$ When $x = -5$, $0 = (-5)^3 + 6(-5)^2 - 15(-5) + k$ $k = -100 (reject)$
11iii	$\frac{dy}{dx} = 3x^2 + 12x - 15$ $\frac{dy}{dx} = 3(2)^2 + 12(2) - 15$ $= 21$ $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $0.5 = 21 \times \frac{dx}{dt}$ $\frac{dx}{dt} = \frac{1}{42}$
12i	$y = x^2 - 6x + 11$ $\frac{dy}{dx} = 2x - 6 = 0$ $x = 3$ $y = 3^2 - 6(3) + 11$ $y = 2$ Coordinates of P is (3, 2)

12ii	Let coordinates of Q be $(a, a^2 - 6a + 11)$. Gradient PQ = -2 $\frac{a^2 - 6a + 11 - 2}{a - 3} = -2$ $a^2 - 6a + 9 = -2a + 6$ $a^2 - 4a + 3 = 0$ $(a-1)(a-3) = 0$ $a = 1$ or $a = 3$ (reject as this is the value of x for point P) y − coordinate of Q $1^2 - 6(1) + 11 = 6$ $Q(1,6)$
12iii	Total area of shaded region $= (1 \times 6) + \int_1^3 x^2 - 6x + 11\, dx$ $= 6 + \left[\frac{x^3}{3} - 3x^2 + 11x\right]_1^3$ $= 6 + \left[(9 - 27 + 33) - (\frac{1}{3} - 3 + 11)\right]$ $= 12\frac{2}{3}$
13i	$2^{2x} = 2^{x+3} - 7$ $(2^x)^2 - 8.2^x + 7 = 0$ $u^2 - 8u + 7 = 0$
13ii	$u^2 - 8u + 7 = 0$ $(u-7)(u-1) = 0$ $u = 7$ or $u = 1$ $2^x = 7$ or $2^x = 2^0$ $x = \frac{\ln 7}{\ln 2}$ or $x = 0$ $x = 2.8(1dp)$
13iii	$u^2 - 8u + k = 0$ $(-8)^2 - 4(1)(k) < 0$ $64 - 4k < 0$ $k > 16$ Therefore, the equation has no solution if $k > 16$.
14i	$x^2 + y^2 - 12x - 6y + 35 = 0$ [illegible] $6^2 + 3^2$ Centre, $A = (6,3)$ radius $= \sqrt{10}$

14ii	$x^2+y^2-12x-6y+35$ $=3^2+4^2-12(3)-6(4)+35$ $=0$ Therefore the point P lies on C_1.
14iii	Gradient AP $=\frac{4-3}{3-6}$ $=-\frac{1}{3}$ Gradient of tangent: 3 Equation of tangent is $y=mx+c$ $3=3(4)+c$ $c=-9$ $y=3x-9$
14iv	$4y=-3x-48$ $y=\frac{-3x-48}{4}$ $Gradient=-\frac{3}{4}$ Gradient of diameter EF(tangent) $=\frac{4}{3}$ $y=\frac{4x}{3}+c$ $2=\frac{4(-2)}{3}+c$ $c=\frac{14}{3}$ $y=\frac{4x}{3}+\frac{14}{3}$ $\frac{-3x-48}{4}=\frac{4x}{3}+\frac{14}{3}$ $-9x-144=16x+56$ $x=-8$ $\therefore y=\frac{4(-8)}{3}+\frac{14}{3}$ $y=-6$
14v	$\frac{F}{C}$ $=\left(\frac{-2+(-8)}{2},\frac{2+(-6)}{2}\right)$ $=(-5,-2)$

	Radius $=\sqrt{(-8+5)^2+(-6+2)^2}$ $=5$
	Total

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PRELIMINARY EXAMINATION 2017

CANDIDATE NAME

CLASS REGISTER NUMBER

ADDITIONAL MATHEMATICS 4047/01
Paper 1 23 August 2017
Secondary 4 Express 2 hours
1200 – 1400

Additional Materials: Writing Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and class register number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

Areas for Improvement		
Error	**Penalty**	**Qn. No.(s)**
Accuracy of non-exact answers	– 1	
Missing/ wrong units	– 1	
Presentation/ Not using ink	– 1	

Name/Signature of Parent/Guardian Date

FOR MARKER'S USE

80

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Setter: Mr Ng HJ Vetter: Ms Tay HY

SGS/A Maths/4Exp/2017/PRELIMS/4047/P1/QP

MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

Answer all the questions.

1 Find the range of values of k for which the line $y = kx - 2$ meets the curve $y^2 = 4x - x^2$. [3]

Hence, describe the relationship between the line and the curve if $k = 1$. [1]

2 (i) Sketch, on the same diagram, the graphs of $y = |x| - 1$ and $y = |x^2 - 2x|$, including all the important features of the graphs and the intersections with the x- and y-axes. [4]

(ii) Hence, determine the value of a such that the equation

$$|x| - |x^2 - 2x| = a + 1$$

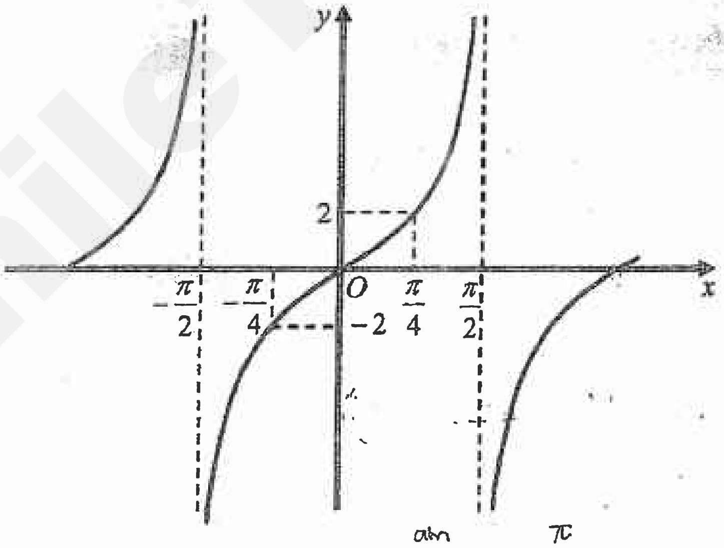
has exactly one solution. [1]

3 (a) State the values between which each of the following must lie:

(i) the principal value of $\sin^{-1} x$, [1]

(ii) the principal value of $\tan^{-1} x$. [1]

(b)



The diagram shows part of the graph of $y = a\tan(bx)$.

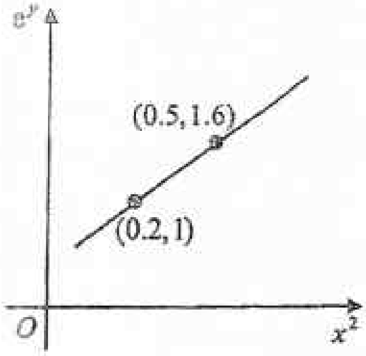
(i) Find the value of each of the constants a and b. [2]

(ii) Find the gradient of the curve at $x = \frac{\pi}{4}$. [2]

4 The quadratic equation $x^2 + mx + 2m = 0$, where m is a non-zero constant, has roots α anc tion with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [4]

[Turn over

5 Variables x and y are such that, when e^y is plotted against x^2, a straight line passing through the points $(0.2, 1)$ and $(0.5, 1.6)$ is obtained.



(i) Find the value of e^y when $x = 0$. [2]

(ii) Express y in terms of x. [1]

6 (a) (i) For what values of x is $\log_x \sqrt{(x+1)(2-x)}$ defined? [2]

(ii) Differentiate $\ln\sqrt{(x+1)(2-x)}$ with respect to x. [2]

(b) Solve the equations $9^y + 5(3^y - 10) = 0$. [3]

(c) If $x^2 + y^2 = 11xy$, show that $\lg(x-y) = a\lg x + b\lg y + \lg c$, where a, b and c are constants to be determined. [5]

7 A circle has equation $x^2 + y^2 - 4x - 8y = 25$.

(i) Show that the radius of the circle is $3\sqrt{5}$ units and state the coordinates of the centre of the circle. [4]

(ii) Determine whether the point $(8, 8)$ lies inside or outside the circle. [2]

(iii) C and D are the points where the line $y + 2x = 8$ crosses the circle.

(a) Find the coordinates of C and D. [3]

(b) Show that CD is a diameter of the circle. [1]

8 (i) Prove the identity

$$\sin^2\theta\cos^2\theta = \frac{1}{8}(1-\cos 4\theta).$$ [3]

(ii) Hence,

(a) show that

$$\int_0^{\frac{\pi}{3}} \sin^2\theta\cos^2\theta \, d\theta = \frac{1}{8}\left(\frac{\pi}{3}+\frac{\sqrt{3}}{8}\right),$$ [3]

(b) solve, for $0^\circ \le \theta \le 180^\circ$, the equation

$$\sin^2\theta\cos^2\theta = \frac{1}{10}.$$ [4]

9 A particle moves in a straight line, so that, t seconds after passing a fixed point A on the line, its velocity, v m/s, is given by

$$v = pt^2 + qt + 24,$$

where p and q are constants. When $t=1$, the acceleration of the particle is -4 m/s^2. It comes to rest at a point B when $t=4$.

(i) Find the value of p and of q. [4]

(ii) Find the distance AB. [3]

10 The equation of a curve is $y = f(x)$, where $f(x) = \dfrac{3x+1}{(x+2)(x-3)}$.

(i) Express $f(x)$ in partial fractions. [2]

(ii) Hence find $f'(x)$ and determine if $y = f(x)$ is increasing or decreasing. [3]

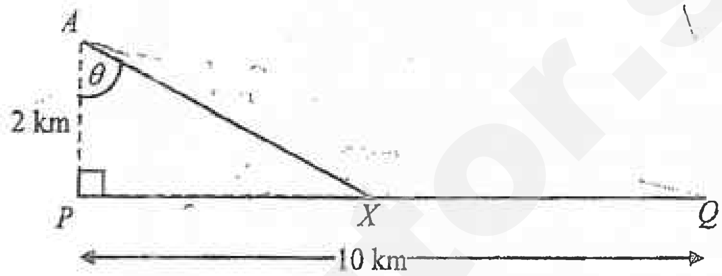
(iii) Find $\displaystyle\int_4^6 \frac{3x+1}{(x+2)(x-3)}\,dx$. [3]

[Turn over

11 (i) By considering $\sec\theta$ as $(\cos\theta)^{-1}$, show that

$$\frac{\mathrm{d}}{\mathrm{d}\theta}(\sec\theta)=\frac{\sin\theta}{\cos^2\theta}.$$ [2]

(ii) The diagram shows a main straight road joining two towns, P and Q, 10 km part. An ambulance is at point A, where AP is perpendicular to PQ and AP is 2 km. The ambulance wishes to reach the hospital at Q as quickly as possible and travels in a straight line along a rocky road to meet the road at point X, where angle $\angle PAX=\theta$ radians.



The ambulance travels along AX at a speed of 10 kmh^{-1} but on reaching the main road, it travels at a speed of 60 kmh^{-1} along XQ.

(a) Given that the ambulance takes T hours to travel from A to Q, show that

$$T=\frac{\sec\theta}{5}+\frac{1}{6}-\frac{\tan\theta}{30}.$$ [4]

(b) Given that θ can vary, find the distance PX for which T has a stationary value. [5]

END OF PAPER

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PRELIMINARY EXAMINATION 2017

CANDIDATE NAME

CLASS | REGISTER NUMBER

ADDITIONAL MATHEMATICS **4047/02**
Paper 2 **24 August 2017**
2 hours 30 minutes
Secondary 4 Express **1000 - 1230**

Materials needed: Writing Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and class register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Writing Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

Areas for Improvement		
Error	**Penalty**	**Qn. No.(s)**
Accuracy of non-exact answers	–1	
Missing/ wrong units (for Paper 2 only)	–1	
Presentation/ Not using ink	–1	

Name/Signature of Parent/Guardian Date

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100

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SGS/Add. Mathematics/4Exp/2017/PRELIM/4047/P2/QP [Turn over

MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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Identities

$$\sin^2 A + \cos^2 A = 1.$$
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$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for Δ ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$
$$a^2 = b^2 + c^2 - 2bc\cos A.$$
$$\Delta = \frac{1}{2}bc\sin A.$$

Answer all the questions.

1 It is given that $f(x) = 2x^3 + ax^2 + bx + 6$ has a factor of $(x+2)$ and leaves a reminder of 15 when divided by $(2x-6)$.

(i) Find the value of a and of b. [4]

(ii) Solve $f(x) = 0$, leaving your answers in exact value. [3]

(iii) Hence, solve the equation $8y^3 - 4y^2 - 9y + 3 = 0$, leaving your answers in exact value. [2]

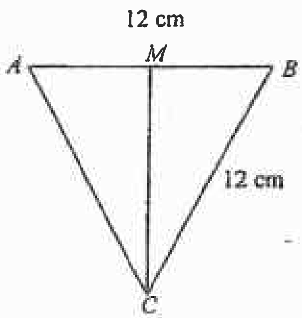
2 (i) In the binomial expansion of $\left(x + \frac{k}{x}\right)^7$, where k is a negative constant, the ratio of the coefficients of $\frac{1}{x}$ and x^3 is 15: 1.

(a) Show that $k = -3$. [3]

(b) Hence, find the coefficient of x in the expansion of $\left(1 - \frac{1}{3}x^2\right)\left(x + \frac{k}{x}\right)^7$. [2]

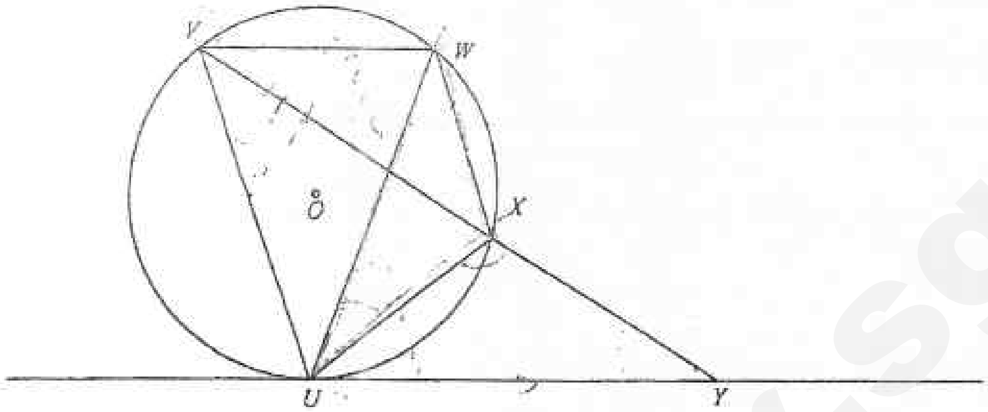
(ii) In the binomial expansion of $(1 + bx)^n$, the first three terms are $1 + \frac{9}{4}x + \frac{9}{4}x^2 + \ldots$ Calculate the value of n and of b. [6]

3 (a) The diagram below shows a conical cup with slant height and diameter being 12 cm each. There is a tiny spider at C. Given that the spider climbs at a constant speed of $\frac{6-3\sqrt{3}}{4}$ cm/s, find the time, in seconds, taken by the spider to climb up along CM, giving your answer in the form $a\sqrt{3}+b$ where a and b are integers. You may assume that the spider is of negligible size. [4]



(b) Find the value of k, given that $125^k = \sqrt[3]{25\sqrt{5}}$ and k is a fraction. [3]

4 In the figure below, $UV = UW$ and the line UY is a tangent to the circle at the point U. VX is produced to meet the tangent at point Y.



Prove that

(i) VW is parallel to UY, [3]

(ii) ΔVUY is similar to ΔWXU, [2]

(iii) $VU^2 = WX \times VY$. [2]

5 (i) Find $\int \frac{1}{\sqrt{(4x-1)^3}} dx$. [2]

(ii) Show that $\frac{d}{dx}\left[\frac{8x+4}{\sqrt{4x-1}}\right] = \frac{16(x-1)}{\sqrt{(4x-1)^3}}$. [3]

(iii) Hence, evaluate $\int_1^2 \frac{x}{\sqrt{(4x-1)^3}} dx$, giving your answer correct to 4 significant figures. [5]

6 (i) It is given that a curve has an equation $y = (x+2)^3(x-k)$, where k is a positive constant. Find the x - coordinates of the stationary points of the curve, leaving your answers in terms of k where necessary. [4]

(ii) Determine the nature of each of the stationary points found in (i), showing your working clearly. [5]

7 The table below shows the experimental values of x and y which are known to be related by the equation $ya^x = b+1$, where a and b are constants. It is known that one value of y has been incorrectly recorded.

x	1	1.5	2	2.5	3	3.5	4
y	3.8	2.9	2.2	1.5	1.3	1	0.77

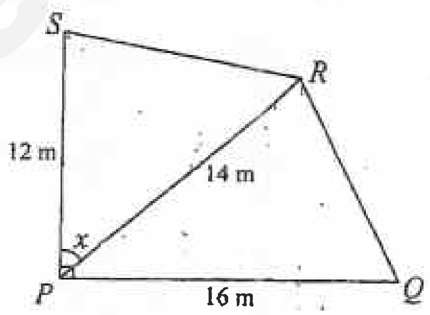
(i) On graph paper, plot $\lg y$ against x and draw a straight line graph. [3]

(ii) Use your graph to
(a) estimate the value of a and of b, [4]

(b) identify the abnormal value of y and estimate the correct value of it. [2]

(iii) On the same graph paper, draw the straight line representing the equation $y = 10^{0.4x-0.1}$ and hence find the value of x for which the two lines intersect. [3]

8 A playground $PQRS$ is formed by two triangles, ΔPQR and ΔPRS, where $PQ = 16$ m, $PR = 14$ m, $PS = 12$ m, $\angle RPS = x$ radians, $x < \pi$ and $\angle SPQ = \frac{\pi}{2}$.
The area of the playground is A m^2.



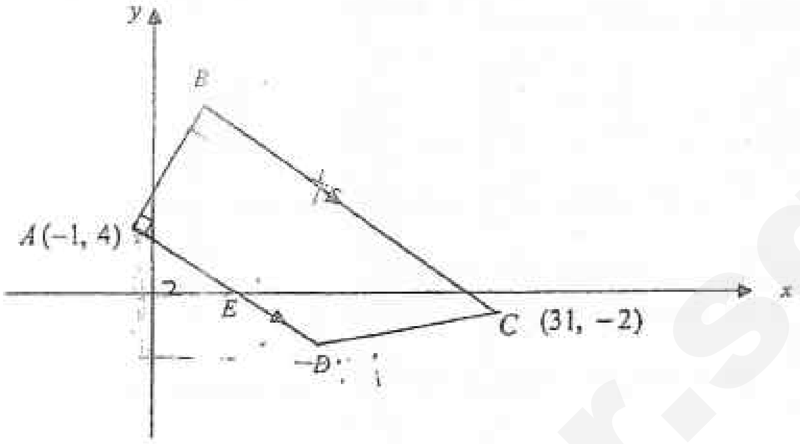
(i) Show clearly that $A = 112\cos x + 84\sin x$. [3]

(ii) Express A in the form $R\cos(x-\alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [4]

(iii) There are two contractors who worked on estimating the area of the playground. Contractor A concluded that the area of the playground was more than 160 m^2 but Contractor B disagreed. Explain whether you agree with Contractor B, stating your reason clearly. [2]

(iv) Find the [illegible] rea of the playground was 130 m^2. [2]

9 The diagram below, not drawn to scale, shows a trapezium $ABCD$ in which AD is parallel to BC and AB is perpendicular to BC and AD. The coordinates of A and C are $(-1,\ 4)$ and $(31,\ -2)$ respectively. AD cuts the x-axis at E. The gradient of AB is 2.



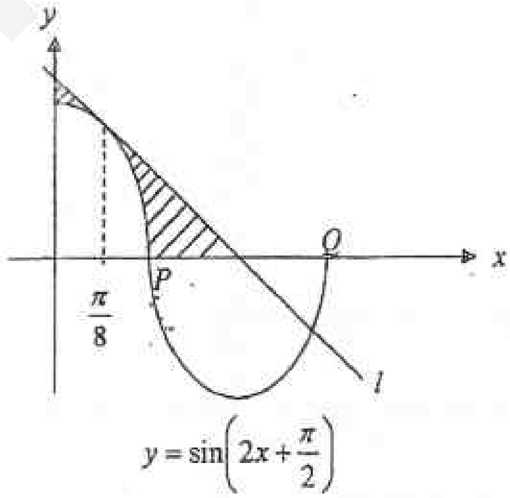
(i) Find the coordinates of B and E. [5]

(ii) Given that $AE:ED$ is $2:3$, find the coordinates of D. [2]

(iii) Find the area of trapezium $ABCD$. [2]

(iv) F is a point on the line BC such that $ABFE$ is a rhombus. Find the coordinates of F. [3]

10 The diagram below shows part of the curve $y = \sin\left(2x + \frac{\pi}{2}\right)$. The straight line, l, is a tangent to the curve at $x = \frac{\pi}{8}$. The points P and Q are on the x-axis.



Find the

(i) coordinates of P and Q, [3]

(ii) equation of the line l, [4]

(iii) sum of the areas of the shaded regions. [5]

END OF PAPER

Serangoon Garden Secondary School
Prelim Exam 2017
Sec 4E Add Maths (Paper 2)

Qn	Solution	Marks
1(i)	$f(x)=2x^3+ax^2+bx+6$	
	Since $(x+2)$ is a factor, $f(-2)=2(-2)^3+a(-2)^2+b(-2)+6=0$	
	$-16+4a-2b+6=0$	
	$4a-2b=10$--------------- (1)	M1
	$f(3)=2(3)^3+a(3)^2+b(3)+6=15$	
	$54+9a+3b+6=15$	
	$9a+3b=-45$	M1
	$a=-2$ and $b=-9$	A1, A1
1(ii)	$f(x)=2x^3-2x^2-9x+6=0$	
	$(x+2)(2x^2+kx+3)=0$	
	$kx^2+4x^2=-2x^2$	
	$k=-6$	
	$(x+2)(2x^2-6x+3)=0$	M1
	$x=-2$ or $x=\dfrac{-(-6)\pm\sqrt{(-6)^2-4(2)(3)}}{2(2)}$ 24.0	
	$x=-2$ or $x=\dfrac{3\pm\sqrt{3}}{2}$	A2 for all three answers
1(iii)	$8y^3-4y^2-9y+3=0$	
	$16y^3-8y^2-18y+6=0$	
	$2(2y)^3-2(2y)^2-9(2y)+6=0$	
	Consider $2y=x$	M1
	$2y=-2$ or $2y=\dfrac{3\pm\sqrt{3}}{2}$	
	$y=-1$ or $y=\dfrac{3\pm\sqrt{3}}{4}$	A1 for all three answers
	Total for Q1	**9m**

2(i)(a)	$\left(x+\frac{k}{x}\right)^7$	
	$=x^7+{}^7C_1(x)^6\left(\frac{k}{x}\right)^1+{}^7C_2(x)^5\left(\frac{k}{x}\right)^2+{}^7C_3(x)^4\left(\frac{k}{x}\right)^3+{}^7C_4(x)^3\left(\frac{k}{x}\right)^4+\ldots$	M1
	$=x^7+\ldots+21k^2x^3+\ldots+35k^4\frac{1}{x}+\ldots$	
	$\frac{35k^4}{21k^2}=15$	M1
	$k=3$ (reject) or $k=-3$	A1
2(i)(b)	$\left(1-\frac{1}{3}x^2\right)\left(x-\frac{3}{x}\right)^7$	
	$=\left(1-\frac{1}{3}x^2\right)(x^7-21x^5+189x^3-945x+2835\frac{1}{x}+\ldots)$	M1
	$=-945x-945x$	
	$=-1890x$	
	Therefore, coefficient of x is -1890.	A1
2(ii)	$(1+bx)^n=1+\binom{n}{1}(bx)^1+\binom{n}{2}(bx)^2+\ldots$	M1
	$=1+nbx+\frac{n(n-1)}{1\times 2}(bx)^2+\ldots$	M1
	$=1+\frac{9}{4}x+\frac{9}{4}x^2+\ldots$	
	Hence, $bn=\frac{9}{4}$ -------- (1)	
	$\frac{n^2-n}{2}b^2=\frac{9}{4}$ -------- (2)	
	From (1), $b=\frac{9}{4n}$ --------- (3)	M1
	Sub (3) into (2) $\frac{n^2-n}{2}\left(\frac{9}{4n}\right)^2=\frac{9}{4}$	M1
	$\frac{n^2-n}{2}\left(\frac{9}{4n^2}\right)=1$	
	$9n^2-9n=8n^2$	
	$n^2-9n=0$	
	$n(n-9)=0$	
	$n=0$ (reject) or $n=9$	A1
	Hen	A1
	Total for Q2	11m

3(a)	$CM=\sqrt{12^2-6^2}$	
	$CM=\sqrt{108}$	M1
	$CM=6\sqrt{3}$	
	Time taken by spider $=6\sqrt{3}\div\frac{6-3\sqrt{3}}{4}$	M1
	$=\frac{24\sqrt{3}}{6-3\sqrt{3}}\times\frac{6+3\sqrt{3}}{6+3\sqrt{3}}$	M1
	$=\frac{144\sqrt{3}+216}{(6)^2-(3\sqrt{3})^2}$	
	$=\frac{144\sqrt{3}+216}{9}$	
	$=16\sqrt{3}+24$ seconds	A1
3(b)	$125^k=\sqrt[3]{25\sqrt{5}}$	
	$5^{3k}=\sqrt[3]{5^2\sqrt{5}}$	M1
	$5^{3k}=\sqrt[3]{5^2(5)^{0.5}}$	
	$5^{3k}=\sqrt[3]{5^{2.5}}$	M1
	$5^{3k}=(5^{2.5})^{\frac{1}{3}}$	
	$3k=\frac{5}{6}$	
	$k=\frac{5}{18}$	A1
	Total for Q3	7 m

4(i)	$\angle WUY = \angle UVW$ (Alt Segment Theorem)	M1
	$\angle UVW = \angle VWU$ ($UV = UW$)	
	$\angle WUY = \angle VWU$	M1
	$\therefore VW$ is parallel to UY (Alternate Angles)	A1
4(ii)	$\angle UVY = \angle XWU$ ($\angle$ in same segment)	M1
	$\angle VYU = \angle WVX$ (alternate $\angle$)	
	$\angle WVX = \angle WUX$ ($\angle$ in same segment)	
	$\angle VYU = \angle WUX$	M1
	$\therefore \Delta VUY$ is similar to ΔWXU by AA test.	
4(iii)	Since ΔVUY is similar to ΔWXU, $\frac{VU}{WX} = \frac{VY}{WU}$	M1
	$\frac{VU}{WX} = \frac{VY}{VU}$ as $UV = UW$	
	Hence, $VU^2 = WX \times VY$.	A1
	Total for Q4	7 m

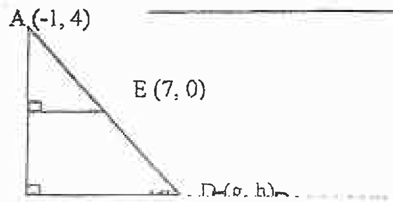
5(i)	$\int \frac{1}{\sqrt{(4x-1)^3}}dx = \int (4x-1)^{-1.5}dx$	
	$= \frac{(4x-1)^{-0.5}}{-0.5} \times \frac{1}{4} + c$	M1
	$= -\frac{1}{2\sqrt{(4x-1)}} + c$	A1
5(ii)	$\frac{d}{dx}\left[\frac{8x+4}{\sqrt{4x-1}}\right] = \frac{8(4x-1)^{0.5} - (8x+4)\times\frac{1}{2}(4x-1)^{-0.5}\times 4}{(4x-1)}$	M2
	$= \frac{(4x-1)^{-0.5}[8(4x-1) - 2(8x+4)]}{(4x-1)}$	
	$= \frac{16(x-1)}{\sqrt{(4x-1)^3}}$	A1
5(iii)	$\int_1^2 \frac{16(x-1)}{\sqrt{(4x-1)^3}}dx = \left[\frac{8x+4}{\sqrt{(4x-1)}}\right]_1^2$	M1
	$\int_1^2 \frac{16x}{\sqrt{(4x-1)^3}}dx - \int_1^2 \frac{16}{\sqrt{(4x-1)^3}}dx = \left[\frac{8x+4}{\sqrt{(4x-1)}}\right]_1^2$	M1
	$16\int_1^2 \frac{x}{\sqrt{(4x-1)^3}}dx - 16\int_1^2 \frac{1}{\sqrt{(4x-1)^3}}dx = \left[\frac{8x+4}{\sqrt{(4x-1)}}\right]_1^2$	
	$16\int_1^2 \frac{x}{\sqrt{(4x-1)^3}}dx - 16\left[-\frac{1}{2\sqrt{4x-1}}\right]_1^2 = \left[\frac{8x+4}{\sqrt{(4x-1)}}\right]_1^2$	M1
	$16\int_1^2 \frac{x}{\sqrt{(4x-1)^3}}dx = \left[\frac{8x+4}{\sqrt{(4x-1)}}\right]_1^2 - \left[\frac{16}{2\sqrt{4x-1}}\right]_1^2$	
	$\int_1^2 \frac{x}{\sqrt{(4x-1)^3}}dx = \frac{1}{16}\left\{\left[\frac{20}{\sqrt{7}} - \frac{12}{\sqrt{3}}\right] - \left[\frac{8}{\sqrt{7}} - \frac{8}{\sqrt{3}}\right]\right\}$	M1
	$\int_1^2 \frac{x}{\sqrt{(4x-1)^3}}dx = 0.1391$ (correct to 4 s.f)	A1
	Total for Q5	**10m**

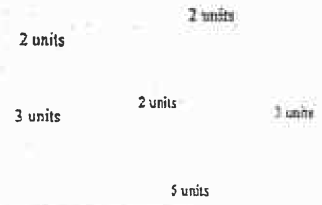
6(i)	$y=(x+2)^3(x-k)$	
	$\frac{dy}{dx}=3(x+2)^2(x-k)+(x+2)^3$	M1
	$\frac{dy}{dx}=(x+2)^2[3(x-k)+(x+2)]$	
	$\frac{dy}{dx}=(x+2)^2(4x-3k+2)$	
	$\frac{dy}{dx}=(x+2)^2[4x-(3k-2)]$	
	To find stationary point, let $\frac{dy}{dx}=0$	
	$(x+2)^2[4x-(3k-2)]=0$	M1
	$x=-2$ or $x=\frac{3k-2}{4}$	A2
6(ii)	$\frac{d^2y}{dx^2}=2(x+2)(4x-3k+2)+(x+2)^2(4)$	M1
	$\frac{d^2y}{dx^2}=(x+2)[12x-6k+12]$	
	$\frac{d^2y}{dx^2}=6(x+2)[2x-k+2]$	
	Sub $x=\frac{3k-2}{4}$, $\frac{d^2y}{dx^2}=6(\frac{3k-2}{4}+2)\left[2\left(\frac{3k-2}{4}\right)-k+2\right]$	M1
	$\frac{d^2y}{dx^2}=6(\frac{3k-2+8}{4})\left[\frac{6k-4-4k+8}{4}\right]$	
	$\frac{d^2y}{dx^2}=6(\frac{3k+6}{4})\left[\frac{2k+4}{4}\right]>0$ since $k>0$	
	Hence, the stationary point at $x=\frac{3k-2}{4}$ is a minimum point.	A1
	Sub $x=-2$, $\frac{d^2y}{dx^2}=6(x+2)[2x-k+2]$	
	$\frac{d^2y}{dx^2}=0$	
	Hence, 2nd derivative test fails when $x=-2$.	
	Therefore, use 1st derivative test for $x=-2$.	
	Since $k>0$, $3k>0$, $3k-2>-2$, $-(3k-2)<2$.	
	(table below)	M1
	Hen[illegible] $=-2$ is an inflexion point.	A1
	Total for Q6	9m

x	-2^- e.g. -2.01	-2	-2^+ e.g. -1.99
$\frac{dy}{dx}$	-ve since $(x+2)^2>0$ and $4x-(3k-2)<0$	0	-ve since $(x+2)^2>0$ and $4x-(3k-2)<0$
Slope	$\searrow$	—	$\searrow$

8(i)	$A =$ Area of ΔPRS + Area of ΔPQR	
	$A=\frac{1}{2}(12)(14)\sin x+\frac{1}{2}(14)(16)\sin\left(\frac{\pi}{2}-x\right)$	M1, M1
	$A=84\sin x+112\sin\left(\frac{\pi}{2}-x\right)$	
	$A=84\sin x+112\cos x$ since $\sin\left(\frac{\pi}{2}-x\right)=\cos x$	
	$A=112\cos x+84\sin x$ (shown)	A1
	Note: Students must state the result $\sin\left(\frac{\pi}{2}-x\right)=\cos x$ to get A1.	
8(ii)	$R=\sqrt{112^2+84^2}$	
	$R=140$	M1
	$\tan\alpha=\frac{84}{112}$	M1
	$\alpha=0.64350$	
	$\alpha=0.644$	M1
	$A=140\cos(x-0.644)$	A1
8(iii)	Since $-1\le\cos(x-0.644)\le 1$,	B1
	the maximum area of the playground is 140 m^2.	B1
	Hence, I agree with Contractor B that Contractor A is wrong.	
	[Note: If student does not state whether he agrees or disagrees with Contractor B, minus 1 mark]	
8(iv)	$140\cos(x-0.64350)=130$	
	$\cos(x-0.64350)=\frac{13}{14}$	
	$x-0.64350=0.38025$	M1
	$x=1.02375$	
	$x=1.02$ (Given that $x<\pi$)	A1
	Total for Q8	11m

9(i)	Equation of AB is $y=2x+c$	
	Sub. (-1, 4) into equation	
	$4=2(-1)+c$	
	$c=6$	
	Equation of AB is $y=2x+6$.	M1
	Gradient of $BC=-\frac{1}{2}$	
	Equation of BC is $y=-\frac{1}{2}x+d$	
	Sub. (31, -2) into equation	
	$-2=-\frac{1}{2}(31)+d$	
	$d=\frac{27}{2}$	
	Equation of BC is $y=-\frac{1}{2}x+\frac{27}{2}$	M1
	B is the point of intersection between the lines AB and BC, so solve	
	the 2 equations.	
	$2x+6=-\frac{1}{2}x+\frac{27}{2}$	
	$x=3$	
	$y=12$	
	Hence, B is (3, 12)	A1
	Equation of AE is $y=-\frac{1}{2}x+e$	
	Sub. (-1, 4) into equation	
	$4=-\frac{1}{2}(-1)+e$	
	$e=\frac{7}{2}$	
	Equation of AE is $y=-\frac{1}{2}x+\frac{7}{2}$	M1
	Sub $E(k,\ 0)$ into equation of AE.	
	$0=-\frac{1}{2}k+\frac{7}{2}$	
	$k=7$	
	E is (7, 0).	A1
9(ii)		



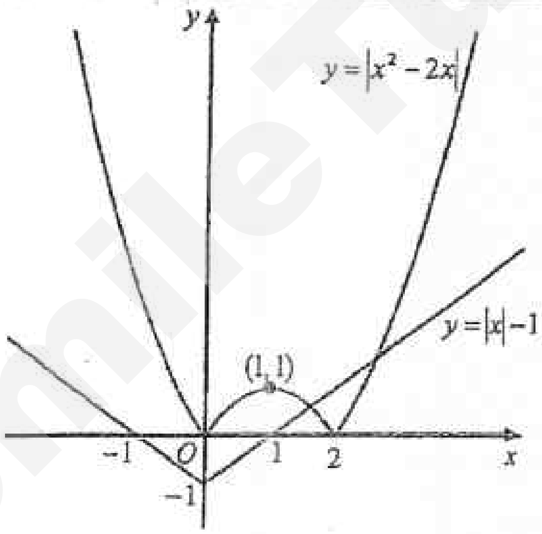
		
	Let D be (g, h)	
	$\frac{4-0}{0-h}=\frac{2}{3}$	
	$h=-6$	M1 for both parts
	$\frac{7-(-1)}{g-(-1)}=\frac{2}{5}$	
	$g=19$	
	Hence, D is (19, -6)	A1
9(iii)	Area of trapezium $ABCD=\frac{1}{2}\begin{vmatrix}-1 & 19 & 31 & 3 & -1\\ 4 & -6 & -2 & 12 & 4\end{vmatrix}$	M1
	$=\frac{1}{2}[(352)-(-128)]$	
	$=240$ units^2.	A1
9(iv)	Midpoint of $BE=\left(\frac{3+7}{2},\frac{12+0}{2}\right)$	
	$=(5, 6)$	M1
	Let F be (x, y)	
	Midpoint of $AF=(5, 6)$	
	$\left(\frac{-1+x}{2},\frac{4+y}{2}\right)=(5, 6)$	M1
	$x=11$, $y=8$	
	Hence, F is (11, 8)	A1
	Total for Q9	12m

10(i)	$y=\sin\left(2x+\frac{\pi}{2}\right)$	
	$\sin\left(2x+\frac{\pi}{2}\right)=0$	M1
	$2x+\frac{\pi}{2}=0,\pi,2\pi$	
	$x=-\frac{\pi}{4}$ (reject), $x=\frac{\pi}{4}$ or $x=\frac{3\pi}{4}$	
	P is $\left(\frac{\pi}{4},0\right)$ and Q is $\left(\frac{3\pi}{4},0\right)$	A1, A1
10(ii)	$\frac{dy}{dx}=2\cos\left(2x+\frac{\pi}{2}\right)$	M1
	At $x=\frac{\pi}{8}$, $\frac{dy}{dx}=2\cos\left(2\left(\frac{\pi}{8}\right)+\frac{\pi}{2}\right)$	
	$\frac{dy}{dx}=-\sqrt{2}$	
	At $x=\frac{\pi}{8}$, $y=\sin\left(2\left(\frac{\pi}{8}\right)+\frac{\pi}{2}\right)$	M1 for both parts.
	$y=\frac{\sqrt{2}}{2}$	
	The equation of l is $y=-\sqrt{2}x+c$	
	$\frac{\sqrt{2}}{2}=-\sqrt{2}\left(\frac{\pi}{8}\right)+c$	M1
	$c=\frac{\sqrt{2}(4+\pi)}{8}$	
	The equation of l is $y=-\sqrt{2}x+\frac{\sqrt{2}(4+\pi)}{8}$	A1
10(iii)	$0=-\sqrt{2}x+\frac{\sqrt{2}(4+\pi)}{8}$	
	$x=\frac{4+\pi}{8}$	
	Sub $x=0$, $y=\frac{\sqrt{2}(4+\pi)}{8}$	
	Area of triangle $=\frac{1}{2}\times\left(\frac{4+\pi}{8}\right)\times\left(\frac{\sqrt{2}(4+\pi)}{8}\right)$	M1

	Area under curve $= \int_0^{\frac{\pi}{4}} \left(\sin\left(2x+\frac{\pi}{2}\right)\right) dx$	M1
	$= \left[-\frac{1}{2}\cos(2x+\frac{\pi}{2})\right]_0^{\frac{\pi}{4}}$	M1
	$= \left[-\frac{1}{2}\cos(\frac{\pi}{2}+\frac{\pi}{2})\right] - \left[-\frac{1}{2}\cos(\frac{\pi}{2})\right]$	
	$= \frac{1}{2} - 0$	
	$= \frac{1}{2}$ units^2	M1
	Sum of areas of shaded regions $= \frac{1}{2} \times \left(\frac{4+\pi}{8}\right) \times \left(\frac{\sqrt{2}(4+\pi)}{8}\right) - \frac{1}{2}$	
	$= 0.063502$ units^2	
	$= 0.0635$ units^2	A1
	Total for Q10	**12m**

--END OF PAPER--

Sec 4E Add Math Prelims P1 Suggested Mark Scheme

Qn	Solution	Mark Scheme
1	$y = kx - 2$ --- ①	
	$y^2 = 4x - x^2$ --- ②	
	Substitute ① into ②:	
	$(kx-2)^2 = 4x - x^2$	M1
	$k^2x^2 - 4kx + 4 - 4x + x^2 = 0$	
	$(k^2+1)x^2 + (-4k-4)x + 4 = 0$	
	For the line to meet the curve, $D \geq 0$	
	$(-4k-4)^2 - 4(k^2+1)(4) \geq 0$	M1
	$16k^2 + 32k + 16 - 16k^2 - 16 \geq 0$	
	$32k \geq 0$	
	$k \geq 0$	A1
	If $k = 1$, the line intersects the curve at two distinct points.	B1
		Total for Q1: 4
2(i)		G1: Shape of $y = \lvert x^2 - 2x \rvert$ G1: Shape of $y = \lvert x \rvert - 1$ G1: Maximum point G1: All intercepts
(ii)	$\lvert x \rvert - \lvert x^2 - 2x \rvert = a + 1$	
	$\lvert x \rvert - 1 = \lvert x^2 - 2x \rvert + a$	
	For exactly one solution, $a = 1$.	B1
		Total for Q2: 5

1

3(a)	(i) Principal value of $\sin^{-1} x$: $-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$ OR $-90° \le \sin^{-1} x \le 90°$ (ii) Principal value of $\tan^{-1} x$: $-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$ OR $-90° < \tan^{-1} x < 90°$	B1 B1
(b)	(i) $y = a\tan(bx)$ Period $= 2\pi \Rightarrow b = 1$ $\left(\frac{\pi}{4}, 2\right)$: $2 = a\tan\left(\frac{\pi}{4}\right) \Rightarrow a = 2$	B1 B1
	(ii) $y = 2\tan(x) \Rightarrow \frac{dy}{dx} = 2\sec^2 x$ At $x = \frac{\pi}{4}, \frac{dy}{dx} = \frac{2}{\cos^2\left(\frac{\pi}{4}\right)} = \frac{2}{\left(\frac{1}{\sqrt{2}}\right)^2} = 4$	M1 A1
		Total for Q3: 6
4	$x^2 + mx + 2m = 0$ $\alpha + \beta = -m$ $\alpha\beta = 2m$ $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$ $= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$ $= \frac{m^2 - 4m}{2m} = \frac{m-4}{2}$ $\left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right) = 1$ Required equation: $x^2 - \frac{m-4}{2}x + 1 = 0$ $2x^2 - (m-4)x + 2 = 0$	B1 for both M1 M1 A1
		Total for Q4: 4
5(i)	$e^y - 1 = \frac{1.6 - 1}{0.5 - 0.2}(x^2 - 0.2)$ $e^y - 1 = 2(x^2 - 0.2)$ $e^y = 2x^2 + 0.6$ When $x = 0, e^y = 0.6$	B1 B1
(ii)	$\ln e^y = \ln(2x^2 + 0.6)$ $y = \ln(2x^2 + 0.6)$	B1
		Total for Q5: 3

6(a) (i)	$\log_x \sqrt{(x+1)(2-x)}$ is defined when $x>0, x\neq 1$ and $(x+1)(2-x)>0 \Rightarrow -1<x<2$ Thus $0<x<2, x\neq 1$	M1 A1
(ii)	Let $y=\ln\sqrt{(x+1)(2-x)}=\frac{1}{2}[\ln(x+1)+\ln(2-x)]$ $\frac{dy}{dx}=\frac{1}{2}\left(\frac{1}{x+1}-\frac{1}{2-x}\right)$	M1 A1
(b)	$9^y+5(3^y-10)=0$ $(3^y)^2+5(3^y-10)=0$ Let $u=3^y$ $u^2+5u-50=0$ $(u+10)(u-5)=0$ $u=-10$ or $u=5$ $3^y=-10$ (no soln) or $3^y=5$ $y\lg 3=\lg 5$ $y=\frac{\lg 5}{\lg 3}=1.46$	 M1 M1 A1
(c)	$x^2+y^2=11xy$ $x^2+y^2-2xy=9xy$ $(x-y)^2=9xy$ $\lg(x-y)^2=\lg(9xy)$ $2\lg(x-y)=\lg 9+\lg x+\lg y$ $\lg(x-y)=\frac{1}{2}\lg x+\frac{1}{2}\lg y+\frac{1}{2}\lg 9$ $=\frac{1}{2}\lg x+\frac{1}{2}\lg y+\lg 3$ Thus $a=\frac{1}{2}, b=\frac{1}{2}, c=3$	 M1 M1 M1 A1 for a and b, A1 for c
		Total for Q6: 12

7(i)	$x^2+y^2-4x-8y=25$ $(x-2)^2-4+(y-4)^2-16=25$ $(x-2)^2+(y-4)^2=45$ Centre: $(2, 4)$, Radius $=\sqrt{45}=\sqrt{(9)(5)}=3\sqrt{5}$ units (shown)	 M1 M1 A1, A1
(ii)	Distance between the centre and the point $(8, 8)$ $=\sqrt{(8-2)^2+(8-4)^2}$ $=\sqrt{52}$ Since $\sqrt{52}>\sqrt{45}$, $(8, 8)$ lies outside the circle.	 M1 A1
(iii)	$y+2x=8$ $x^2+y^2-4x-8y=25$ $x^2+(8-2x)^2-4x-8(8-2x)=25$ $x^2+64-32x+4x^2-4x-64+16x-25=0$ $5x^2-20x-25=0$ $x^2-4x-5=0$ $(x-5)(x+1)=0$ $x=-1$ or $x=5$ $y=10$ $\quad y=-2$ Thus $C(-1, 10)$ and $D(5, -2)$. Distance between C and D $=\sqrt{(5-(-1))^2+(-2-10)^2}$ $=\sqrt{6^2+(-12)^2}$ $=\sqrt{180}$ $=2\sqrt{45}$ Since $CD=2\sqrt{45}$, CD is the diameter of the circle (shown).	 M1 A1, A1 B1
		Total for Q7: 10

8(i)	$\sin^2\theta\cos^2\theta$ $=\left(\frac{1}{2}(2\sin\theta\cos\theta)\right)^2$	M1
	$=\frac{1}{4}(\sin 2\theta)^2$ $=\frac{1}{4}\left(\frac{1}{2}(1-\cos 4\theta)\right)$ since $\cos 4\theta = 1-2\sin^2 2\theta$	M1
	$=\frac{1}{8}(1-\cos 4\theta)$ (shown)	AG1
(ii)	(a) $\int_0^{\frac{\pi}{3}}\sin^2\theta\cos^2\theta\,\mathrm{d}\theta=\int_0^{\frac{\pi}{3}}\frac{1}{8}(1-\cos 4\theta)\,\mathrm{d}\theta$ $=\frac{1}{8}\int_0^{\frac{\pi}{3}}(1-\cos 4\theta)\,\mathrm{d}\theta$ $=\frac{1}{8}\left[\theta-\frac{\sin 4\theta}{4}\right]_0^{\frac{\pi}{3}}$	M1
	$=\frac{1}{8}\left[\frac{\pi}{3}-\frac{\sin\left(\frac{4\pi}{3}\right)}{4}\right]-0$	M1
	$=\frac{1}{8}\left[\frac{\pi}{3}-\frac{1}{4}\left(-\frac{\sqrt{3}}{2}\right)\right]$ $=\frac{1}{8}\left(\frac{\pi}{3}+\frac{\sqrt{3}}{8}\right)$ (shown)	AG1
	(b) $\sin^2\theta\cos^2\theta=\frac{1}{10}$ $\frac{1}{8}(1-\cos 4\theta)=\frac{1}{10}$ $1-\cos 4\theta=\frac{8}{10}$ $\cos 4\theta=\frac{1}{5}$	M1
	Basic angle $= 78.463°$	M1
	$0°\le\theta\le 180° \Rightarrow 0°\le 4\theta\le 720°$ $4\theta = 78.463°, 360°-78.463°,$ $78.463°+360°, 360°-78.463°+360°,$ $\theta = 19.6°, 70.4°, 109.6°, 160.4°$ (1 d.p.)	A2
		Total for Q8: 10

9(i)	$v = pt^2 + qt + 24$ $a = \frac{dv}{dt} = 2pt + q$ $t = 1, a = -4: \quad 2p + q = -4 \quad ---(1)$ $t = 4, v = 0: \quad 16p + 4q = -24 \quad ---(2)$ $(2) \div 4: \quad 4p + q = -6 \quad ---(3)$ $(3) - (1): \quad 2p = -2 \Rightarrow p = -1, q = -2$	 M1 M1 A2
(ii)	$v = -t^2 - 2t + 24$ $s = \int v \, dt$ $= \int -t^2 - 2t + 24 \, dt$ $= -\frac{t^3}{3} - t^2 + 24t + c$ When $t = 0, s = 0, c = 0$ $s = -\frac{t^3}{3} - t^2 + 24t$ Distance $AB = -\frac{(4)^3}{3} - (4)^2 + 24(4) = 58\frac{2}{3}$ m	 M1 M1 A1
		Total for Q9: 7

10(i)	$f(x)=\dfrac{3x+1}{(x+2)(x-3)}=\dfrac{A}{x+2}+\dfrac{B}{x-3}$ $3x+1=A(x-3)+B(x+2)$ Let $x=-2$: $-5=-5A\Rightarrow A=1$ Let $x=3$: $10=5B\Rightarrow B=2$ $\dfrac{3x+1}{(x+2)(x-3)}=\dfrac{1}{x+2}+\dfrac{2}{x-3}$	 B1 B1
(ii)	$f'(x)=\dfrac{d}{dx}\left(\dfrac{1}{x+2}+\dfrac{2}{x-3}\right)$ $=-\dfrac{1}{(x+2)^2}-\dfrac{2}{(x-3)^2}$ Since $(x+2)^2>0$ and $(x-3)^2>0$, $-\dfrac{1}{(x+2)^2}<0$ and $-\dfrac{2}{(x-3)^2}<0$ Thus $f'(x)<0$ and f is a decreasing curve.	 B1 M1 A1
(iii)	$\int_4^6 \dfrac{3x+1}{(x+2)(x-3)}\,dx$ $=\int_4^6 \dfrac{1}{x+2}+\dfrac{2}{x-3}\,dx$ $=\left[\ln(x+2)+2\ln(x-3)\right]_4^6$ $=\left(\ln(6+2)+2\ln(6-3)\right)-\left(\ln 6+2\ln 1\right)$ $=\ln 8+\ln 9-\ln 6$ $=\ln 12$	 M1 M1 A1
		Total for Q10: 8

7

11(i)		$\frac{d}{d\theta}(\sec\theta)$ $=\frac{d}{d\theta}\left(\frac{1}{\cos\theta}\right)$ $=\frac{d}{d\theta}(\cos\theta)^{-1}$ $=-(\cos\theta)^{-2}(-\sin\theta)$ $=\frac{\sin\theta}{\cos^2\theta}$ (shown)	M1 AG1
(ii)	(a)	$\cos\theta=\frac{2}{AX}\Rightarrow AX=\frac{2}{\cos\theta}$ Time taken to travel along $AX=\frac{AX}{10}=\frac{2}{\cos\theta}\times\frac{1}{10}=\frac{\sec\theta}{5}$ $\tan\theta=\frac{PX}{2}\Rightarrow PX=2\tan\theta$ $XQ=10-2\tan\theta$ Time taken to travel along XQ $=\frac{XQ}{60}=\frac{10-2\tan\theta}{60}=\frac{1}{6}-\frac{\tan\theta}{30}$ Thus total time taken T $=\frac{\sec\theta}{5}+\frac{1}{6}-\frac{\tan\theta}{30}$	M1 M1 M1 AG1
	(b)	$T=\frac{\sec\theta}{5}+\frac{1}{6}-\frac{\tan\theta}{30}$ $\frac{dT}{d\theta}=\frac{1}{5}\left(\frac{\sin\theta}{\cos^2\theta}\right)-\frac{1}{30}\sec^2\theta$ For stationary T, $\frac{dT}{d\theta}=0$ $\frac{1}{5}\left(\frac{\sin\theta}{\cos^2\theta}\right)-\frac{1}{30}\sec^2\theta=0$ $6\sin\theta(\sec^2\theta)-\sec^2\theta=0$ $\sec^2\theta[6\sin\theta-1]=0$ $\sec^2\theta=0$ or $\sin\theta=\frac{1}{6}$ $\cos\theta=0$ or basic angle $\theta=9.5941°$ $\theta=\frac{\pi}{2}$ (rejected) or $\theta=9.6°$ Thus $PX=2\tan\theta=2\tan 9.5941°=0.33806\text{ km}$ OR $\frac{1}{5}\left(\frac{\sin\theta}{\cos^2\theta}\right)-\frac{1}{30}\sec^2\theta=0$ $\frac{6\sin\theta-1}{30\cos^2\theta}=0$ $\sin\theta=\frac{1}{6}$ basic angle $\theta=9.5941°$ $\theta=9.6°$	M1 M1 M1 M1, A1
			Total for Q11: 11

8

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2017
Secondary 4

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS 4047/01

Paper 1

Monday 21 August 2017

2 hours

Additional Materials: Writing Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

of 6 printed pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The function f is defined, for all values of x, by

$$f(x) = -\frac{2}{3}x^3 - 3x^2 + 8x + 1.$$

Find the range of values of x such that f is a decreasing function. [3]

2 The graph of $y = -\log_2(x-a)$ has coordinates (7, –2) and (4, b).

(i) Determine the value of a and of b. [2]

(ii) Sketch the graph of $y = -\log_2(x-a)$. [2]

3 A fossil containing radioactive carbon–14 isotopes was recently found. The amount of carbon–14, N, can be expressed as $N = N_0e^{kt}$, where N_0 and k are constants and t refers to the time in years.

The time taken for half of the carbon–14 to decay is 5700 years.

(i) Show that $k = -0.0001216$. [2]

(ii) If the fossil now contains only 39% of its original amount of carbon–14, calculate to the nearest year, the age of the fossil. [3]

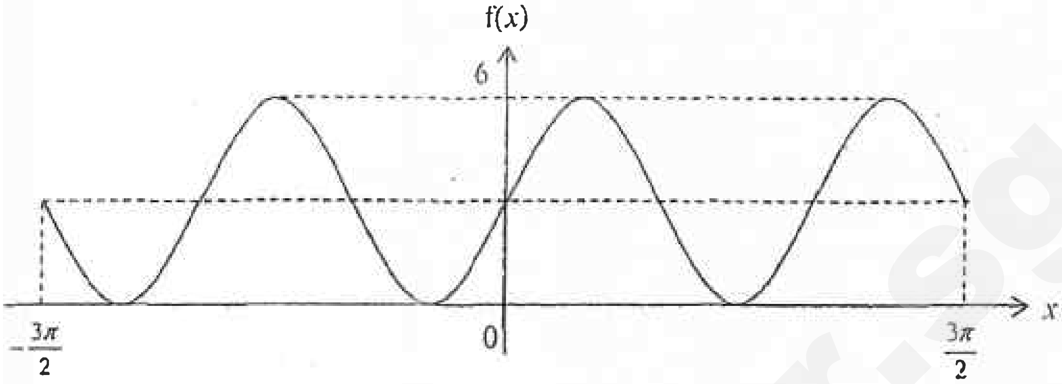
4 (i) Sketch the graph of $y = x^{\frac{2}{3}}$. [2]

(ii) The line $y = \frac{1}{2}x$ intersects the graph of $y = x^{\frac{2}{3}}$ at points A and B.
Find the equation of the perpendicular bisector of AB. [5]

5 (i) Given that $px^2 + qx + 2q$ is always negative, what conditions must apply to the constants p and q? [4]

(ii) Give an example of values of p and q which satisfy the conditions found in part (i). [1]

6



The figure shows part of the graph of $f(x) = a\sin bx + c$, where a, b and c are constants.

(i) State the period and amplitude of $f(x)$. [2]

(ii) State the value c. [1]

(iii) Show that $f(x)$ can be expressed as $3(\cos x + \sin x)^2$. [3]

7 (i) Prove that $\dfrac{1 + \sec A}{\tan A + \sin A} = \operatorname{cosec} A$. [3]

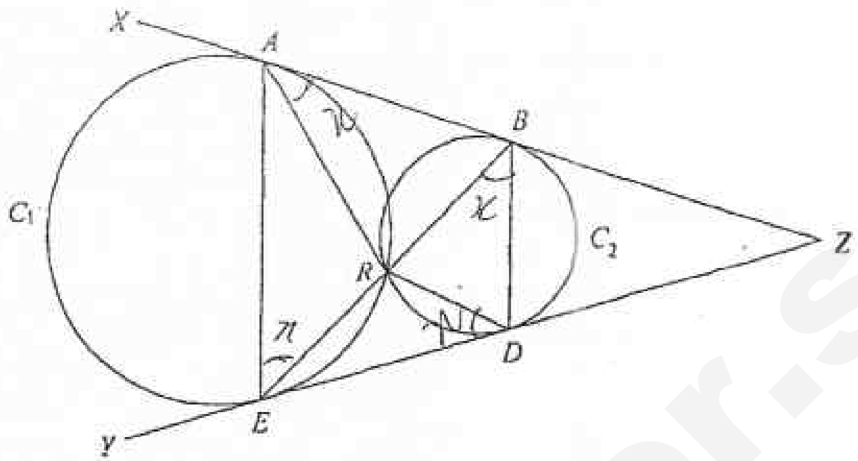
(ii) Hence solve the equation $1 + \sec 2x = 5(\tan 2x + \sin 2x)$ for $-100° \le x \le 100°$. [4]

8 (i) Sketch the graph of $y = |5 - 3x| - 1$, showing all intercepts clearly. [3]

(ii) Find the coordinates of the point of intersection between the line $y = 5x$ and the graph of $y = |5 - 3x| - 1$. [2]

(iii) Determine the set of values of m for which the line $y = mx$ intersects the graph of $y = |5 - 3x| - 1$ at two points. [2]

9

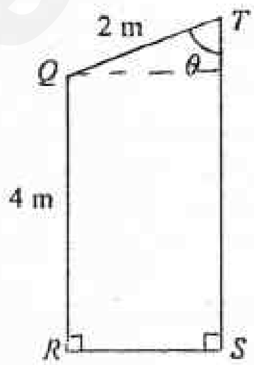


The diagram shows two intersecting circles, C_1 and C_2. The line XZ is tangent to C_1 and C_2 at A and B respectively. The line YZ is tangent to C_1 and C_2 at E and D respectively. Point B is such that $AB = BZ$. Point R lies on both circles and ERB is a straight line.

(i) Prove that AE and BD are parallel. [4]

(ii) Prove that angle BAR = angle RDE. [3]

10



The diagram shows a trapezoidal glass window, $QRST$. The lengths of QR and QT are 4 m and 2 m respectively. Angle QTS is given as θ, where $0 < \theta < \frac{\pi}{2}$.

(i) Show that the area, A m^2, of the window is given by
$$A = 8\sin\theta + \sin 2\theta.$$ [4]

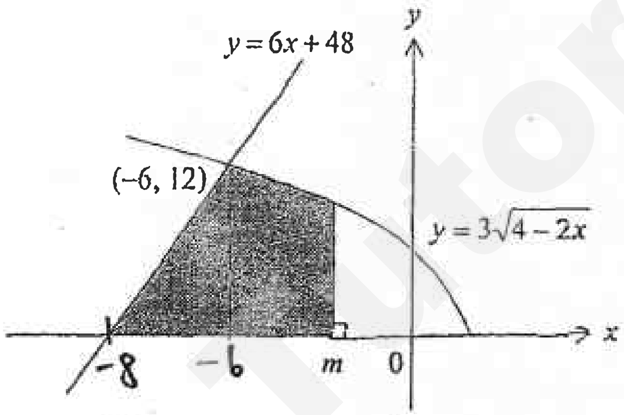
(ii) Given that θ can vary, find the value of θ which gives the greatest area of the window. [5]

11 The equation of a curve is $y = 3\sqrt{4-2x}$.

(i) A particle P moves along the curve in such a way that the y-coordinate of P decreases at a constant rate of 0.12 units per second.

Find the y-coordinate of P at the instant when the x-coordinate of the particle is increasing at 0.16 units per second. [5]

(ii) The diagram shows part of the curve $y = 3\sqrt{4-2x}$ and the line $y = 6x + 48$ which intersects the curve at (–6, 12).



Given that the area of the shaded region is 49 units^2, find the value of the constant m. [6]

12 A car is travelling along a straight road from Junction X to Junction Y. It passes X with a speed of 15 m/s. At the same instant, the driver spots a red traffic light at Y and applies the brakes immediately. During the journey from X to Y, the acceleration, a m/s^2, of the car, t seconds after passing X, is given by $a = -t - 3.5$.

(i) Show that the time taken for the car to come to an instantaneous rest after passing X is 3 seconds. [4]

(ii) Given that the distance between X and Y is 24 m, determine whether the car will be able to stop in time for the red light at Y. Justify your answer. [4]

(iii) State an assumption made. [1]

End of Paper

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2017
Secondary 4

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS
4047/02

Paper 2

Thursday 24 August 2017

2 hours 30 minutes

Additional Materials: Writing Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

This document consists of 5 printed pages and 1 blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

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$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
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$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

Answer all questions.

1 A curve has equation $y = \dfrac{x}{e^{2x}}$.

(i) Find the gradient function for the curve. [3]

(ii) The equation of the normal to the curve at $x = 1$ cuts the axes at $(0, p)$ and $(q, 0)$. Find the value of p and of q, leaving your answers in terms of e. [6]

2 (i) Differentiate $3x \sin 2x$ with respect to x. [3]

(ii) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} x\cos 2x \, dx$. [6]

3 (a) Sketch the graph of the equation $y = e^{-2x}$. [2]

(b) (i) The gradient function of a curve is given by $\dfrac{dy}{dx} = e^{2x} + e^{-2x}$.

Explain why the function is an increasing function. [2]

(ii) Find the equation of the function. [3]

4 (a) (i) Expand and simplify $(1 + a)^8$ in ascending powers of a, up to the term containing a^3. [2]

(ii) Given that $a = x + x^2$, write down the expansion of $(1 + x + x^2)^8$ up to the term containing x^3. [3]

(iii) Using your expansion and a suitable value for x, find the value of 1.0101^8, giving your answer correct to 6 decimal places. [2]

(b) (i) Write down the general term in the binomial expansion of $\left(3x - \dfrac{2}{x^2}\right)^{12}$.

Write down the power of x in this general term. [2]

(ii) Hence, explain why there is no term in x^5 in the binomial expansion of $\left(3x - \dfrac{2}{x^2}\right)^{12}$. [2]

5 The roots of the equation $2x^2 - x + 6 = 0$ are p and q.

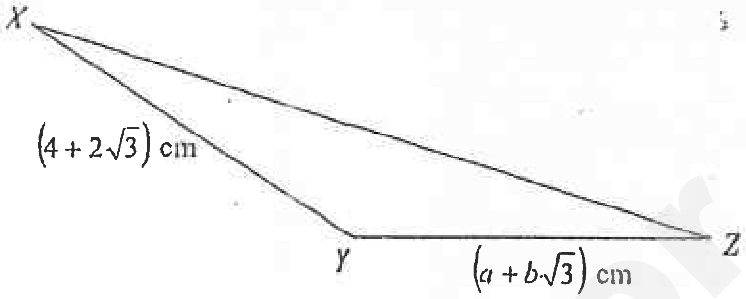
(i) Find the value of $p^2 + q^2$. [3]

(ii) Find the value of $p^3 + q^3$. [2]

(iii) Find a quadratic equation whose roots are $p^2 - q$ and $q^2 - p$. [5]

6 (i) Express $\dfrac{60+36\sqrt{3}}{4+2\sqrt{3}}$ in the form $r+s\sqrt{3}$, where r and s are integers. [3]

(ii) The diagram shows a triangle XYZ.



XY is $(4+2\sqrt{3})$ cm and YZ is $(a+b\sqrt{3})$ cm, where a and b are integers.
The included angle XYZ is 120°.
Given that the area of the triangle is $(15+9\sqrt{3})$ cm^2, find the value of a and of b. [5]

7 A point P has coordinates (8, 0). Another point Q is vertically above P such that $\tan \angle POQ = \dfrac{3}{4}$, where O is the origin.

(i) Find the coordinates of Q. [2]

(ii) Given that OQ is a diameter of a circle, find the equation of the circle. [3]

(iii) A point R is such that angle $ORP = 38°$.
Determine whether R lies within the circle, stating your reasons clearly. [3]

8 (i) Find the remainder when $3x^3 - x^2 + 12x$ is divided by $3x - 1$. [2]

(ii) Hence, state the remainder when $3x^3 - x^2 + 12x + c$ is divided by $3x - 1$. [1]

(iii) Factorise $3x^3 - x^2 + 12x - 4$ completely, showing your workings clearly. [3]

(iv) Express $\dfrac{11x^2+9x+4}{3x^3-x^2+12x-4}$ in partial fractions. [5]

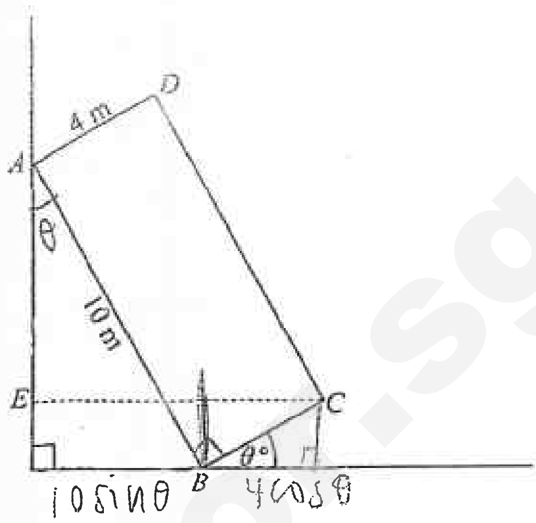
9 A curve has equation $y = (x-1)\ln 2x$, for all values of $x > 0$.

(i) Find $\dfrac{dy}{dx}$. [3]

(ii) Hence, explain why the curve has a stationary point for a value of x between 0.5 and 1. [2]

(iii) Determine the nature of the stationary point. [1]

10 The diagram shows a cuboid structure $ABCD$ that is placed tilted against a vertical wall.



AB is 10 metres, AD is 4 metres and the side BC makes an angle $\theta°$ with the floor. EC is horizontal distance of C from the wall.

(i) Show that EC can be expressed in the form $a \sin \theta + b \cos \theta$, where a and b are constants to be found. [2]

(ii) Express EC in the form $R \sin (\theta + \alpha)$ where $R > 0$ and α is an acute angle. [4]

The structure $ABCD$ will remain tilted against the wall provided EC lies between 7.5 metres and 9.5 metres.

(iii) Find the range of values of θ for the structure $ABCD$ to remain tilted. [4]

11 The amount of active ingredients in a lotion brand L, y units, is dependent on the time, t years after the lotion is manufactured.
The variables y and t are related by the equation $y = ae^{bt}$, where a and b are constants.
Some values of y and t are shown in the table below.

t (years)	1	2	4	5	6
y (units)	950.6	303.2	30.8	9.8	3.1

(i) Plot a graph of $\ln y$ against t. [2]

(ii) Using your graph, find the value of a and of b. [4]

(iii) Find the maximum amount of active ingredients after 3 years. [2]

The amount of active ingredients in another lotion brand M is given by the equation $y = \frac{1000}{e^{\frac{t}{2}}}$.

(iv) By adding a suitable straight line to your graph, find the time after which brand M contains a higher level of active ingredients. [3]

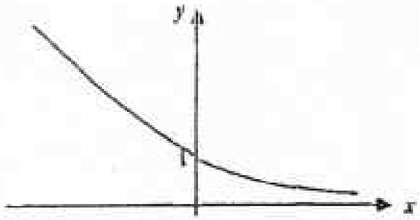
End of Paper

Answers:

1 (i) $\frac{dy}{dx}=\frac{1-2x}{e^{2x}}$ (ii) $p=\frac{1}{e^2}-e^2$ $q=1-\frac{1}{e^4}$

2 (i) $3\sin 2x+6x\cos 2x$ (ii) $\frac{1}{6}\left(\frac{3\pi}{4}-\frac{3}{2}\right)$

3 (a)



(b) (i) $e^{2x}>0$ for all values of x

$\therefore \frac{dy}{dx}=e^{2x}+e-2x>0$

Hence, function is increasing

4 (a) (i) $1+8a+28a^2+56a^3+\ldots$ (ii) $1+8x+36x^2+112x^3+\ldots$

(iii) 1.083712

(b) (i) General term is $\binom{12}{r}(3x)^{12-r}\left(-\frac{2}{x^2}\right)^r$ Power of $x=12-3r$

(ii) $r=\frac{7}{3}$, Since $\frac{7}{3}$ is not an integer, there is no term in x^5.

5 (i) $-\frac{23}{4}$ (ii) $-\frac{35}{8}$ (iii) $8x^2+50x+131=0$

6 (i) $6+6\sqrt{3}$ (ii) $a=6, b=2$

7 (i) $(8, 6)$ (ii) $(x-4)^2+(y-3)^2=25$

(iii) $\angle ORP=38°<53.1$, $\therefore$ R lies outside the circle.

8 (i) 4 (ii) $4+c$ (iii) $(3x-1)(x^2+4)$ (iv) $\frac{2}{(3x-1)}+\frac{3x+4}{(x^2+4)}$

9 (i) $\ln(2x)+1-\frac{1}{x}$ (ii) Since $\frac{dy}{dx}<0$ for $x=0.5$ and $\frac{dy}{dx}>0$ for $x=1$,

$\therefore \frac{dy}{dx}=0$ for $0.5<x<1$

10 $EC=10\sin\theta+4\cos\theta$ (ii) $\sqrt{116}\sin(\theta+21.80°)$ (iii) $22.3°<\theta<40.1°$

11 (ii) $a=2981$ (accept 2836 to 3134), $b=-1.14$ (accept 1.13 to 1.16)

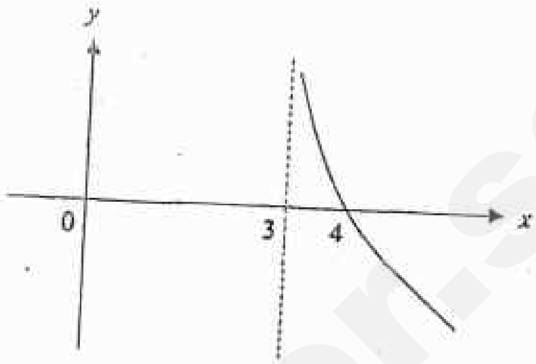
(iii) 94.6 units (accept 94.6 to 99.5) (iv) 1.7 years

Answers:

1 $x < -4$ or $x > 1$

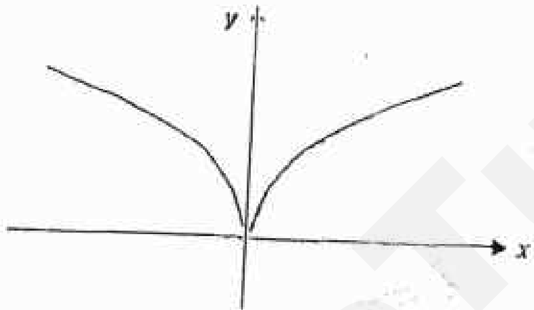
2(i) $a = 3, b = 0$

(ii)



3(ii) 7743 or 7744 years

4(i)



(ii) $y = -2x + 10$

5(i) $p < 0$ & $8p < q < 0$

(ii) Any answer as long as:

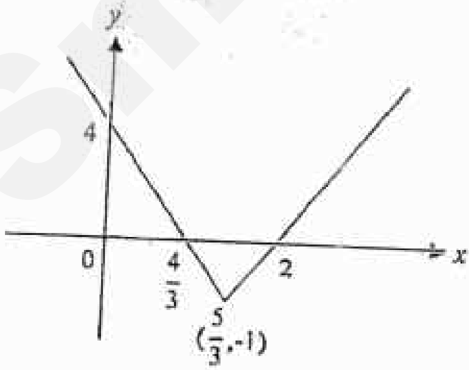
- p and q are negative, and
- $8p < q$

6(i) Period = π, Amplitude = 3

(ii) $c = 3$

7(ii) $x = -95.8°, 5.8°, 84.2°$

8(i)



(ii) Pt of intersection: $(\frac{1}{2}, \frac{5}{2})$

8(iii) $-\frac{3}{5} < m < 3$

10(ii) $\theta = 1.3441$

11(i) $x = -6, y = 12$

(ii) $m = -2.5$

12(ii) $s = 24.75$m, hence the car will not be able to stop in time.

(iii) The driver does not have any reaction time before stepping on the brakes / brakes are not faulty.

1 The function f is defined by $f(x)=\ln\dfrac{\sqrt{x^2+5}}{x}$ for $x>0$.

Show that f is a decreasing function for all values of $x>0$. [3]

2 (i) Sketch, on the same diagram, the graphs of $y=-\dfrac{5}{x^{\frac{1}{2}}}$ and $y^2=4x$. [2]

(ii) State the value of k for which the x-coordinate of the point of intersection of these two graphs satisfies the equation $x^2=k$. [2]

3 You just bought a brand new car. The value, V dollars, of the car depreciates over time. It is given that $V=84000e^{kt}+8500$, where t is the time in years since it was bought and k is a constant.

(i) What is the initial value of the vehicle? [1]

(ii) Calculate the value of k if, after 3 years, the value of the car is halved. [2]

(iii) After having driven the car for 25 years, you decided to change to a new car. A second-hand car dealer offers to buy the old car from you for \$8000. Without using a calculator, justify whether you should accept the offer. [2]

4 (a) Find the values of k for which $3x(x+2)+k^2$ is never negative for all real values of x. [3]

(b) Given that $3x^2+px+84<0$ only when $4<x<k$, find the value of p and of k. [3]

5 (a) Simplify $\log_3 2\times\log_4 3\times\log_5 4\times\ldots\times\log_{n+1} n$. [2]

(b) Using the substitution $u=6^x$, solve the equation $6^{x+1}-6^{1-x}=5$. [4]

[Turn over

6 (i) Prove that $\sec\theta - \dfrac{\cos\theta}{1+\sin\theta} = \tan\theta$. [3]

(ii) Find, in radians, for $0 < \theta < \pi$, the exact values of θ for which

$\sec\theta - \dfrac{\cos\theta}{1+\sin\theta} = \dfrac{1}{3}\cot\theta$. [3]

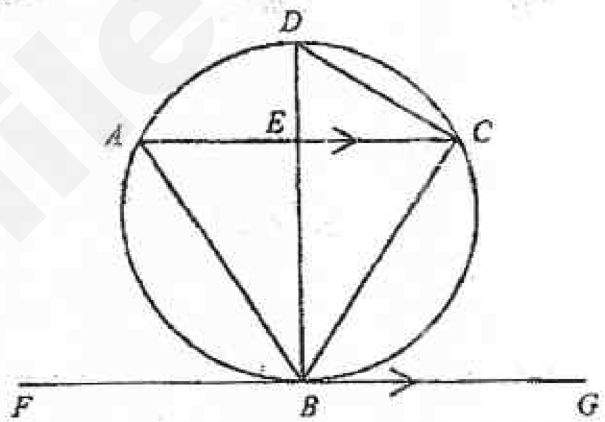
7 The point $P(-1,0)$ is a point on the graph of $y = |kx-2|$.

(i) Show that $k = -2$. [2]

(ii) Sketch the graph of $y = |kx-2|$, indicating the points of intersection with the axes. [3]

(iii) Hence, write down the range(s) of values of x for which $y > 2$. [1]

8 The diagram shows triangles ABC and BCD whose vertices lie on the circumference of a circle. The chords BD and AC intersect at E and AC is parallel to FG. FG is a tangent to the circle at B.



Show that

(i) triangle BCD is similar to triangle BEC, [3]

(ii) $BC^2 = BD \times BE$, [2]

(iii) triangle ABC is an isosceles triangle. [2]

9 A curve has the equation $y=6\sqrt{(1+2x)^3}$

(i) A point P moves along the curve in such a way that the x-coordinate of P is decreasing at a constant rate of 0.04 units per second. Find the x-coordinate of P at the instant the y-coordinate is decreasing at a rate of 0.045 units per second. [4]

(ii) Find the x-coordinate of the curve that splits the area bounded by the curve, the x-axis and the lines $x=2$ and $x=5$, into 2 halves of equal area. [4]

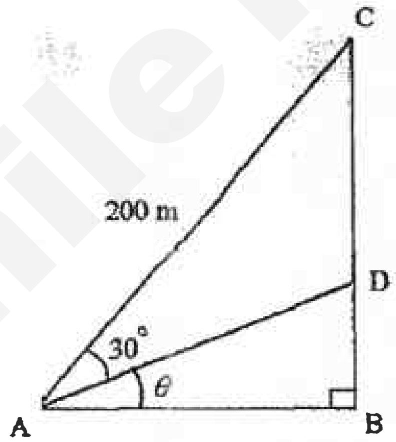
10 The function f is such that $f(x)=2\sin^2 x-\cos^2 x$.

(i) By expressing $f(x)$ in the form of $a+b\cos 2x$, show that $a=\frac{1}{2}$ and $b=-\frac{3}{2}$. [3]

(ii) Sketch the graph of $f(x)$ for $0\le x\le 2\pi$. [3]

(iii) By drawing a suitable line on the same axes, state the number of solutions to the equation $4\pi\sin^2 x-2\pi\cos^2 x-x=2\pi$. [3]

11



The Urban Redevelopment Authority (URA) in Singapore is gazetting a piece of right-angled triangular-shaped land ABC in Hougang Avenue 8. URA plans to build a public skate arena shown in the diagram.
An area ABD is to be built with ramps. $AC=200$ m and $\angle BAD=\theta$, where $0°<\theta<90°$

(i) Show that the area, A m^2, of the triangle ABC is given by $A=5000(\sin 2\theta+\sqrt{3}\cos 2\theta)$. [4]

(ii) Find $\frac{dA}{d\theta}$. [1]

(iii) Find the value of θ for which the area of the triangle ABC is maximum. [4]

[Turn over

12 A particle P travels in a straight line so that its velocity, v m/s, at time t seconds is given by $v = t^2 - 5t + 6$. The particle first crosses the fixed point O at $t = 1.5$ s.

(i) Find the acceleration of the particle at $t = 4$ s. [2]

(ii) Find the time interval during which the particle's velocity is decreasing. [2]

(iii) Find the displacement of the particle from O when it is first instantaneously at rest. [4]

(iv) Find the average speed of the particle for the first three seconds. [3]

End of Paper

1 The area of a right-angled triangle is $(4+6\sqrt{6})\,\text{cm}^2$. The base of the triangle is $(6\sqrt{3}+\sqrt{8})\,\text{cm}$.

(i) Show that the perpendicular height of the triangle, h, can be expressed as $a\sqrt{b}$ cm, where a and b are integers. [4]

(ii) The longest length of the right-angled triangle is H cm. Express H^2 in the form $p+q\sqrt{6}$, where p and q are integers. [3]

2 **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\tan x\sin^2 x\right)=2\sin^2 x+\sec^2 x-1$. [3]

(ii) Hence find $\int_{\frac{\pi}{4}}^{\pi}\sin^2 x\,\mathrm{d}x$, leaving your answer in exact form. [4]

3 The roots of the quadratic equation $2x^2-3x+4=0$ are α and β.

(i) Find the value of $\alpha^2+\beta^2$. [3]

(ii) Show that the value of $\alpha^3+\beta^3$ is $-\frac{45}{8}$. [2]

(iii) Find the quadratic equation, with integer coefficients, whose roots are $\frac{\alpha}{\beta^2+1}$ and $\frac{\beta}{\alpha^2+1}$. [4]

4 The positive y-axis and the line $y=3$ are tangents to a circle C. It is given that the x-coordinate of the centre of C is a, where $a>0$.

(i) Write down the larger possible y-coordinate of the centre of C, in terms of a. [1]

The line L is a tangent to C at the point $(8, 12)$ on the circle. The centre of C lies below and to the left of $(8, 12)$.

(ii) Show that $a=5$ and write down the centre of C. [3]

(iii) Find

(a) the equation of C, [1]

(b) the equation of L, [3]

(c) the equation of the circle which is a reflection of C in the y-axis. [1]

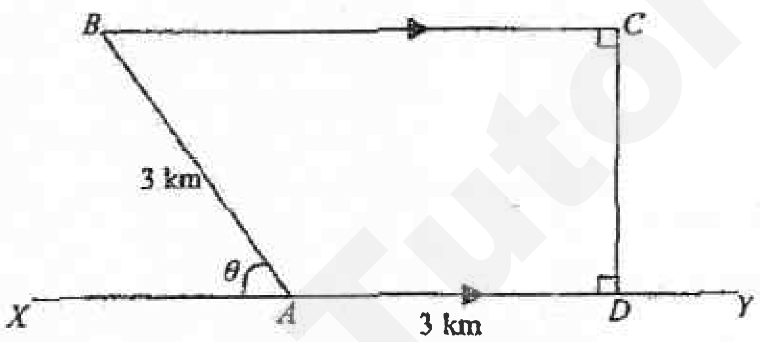
[TURN OVER

5 (a) (i) Write down, in terms of n and y, the first 3 terms in the expansion of $(1+y)^n$. [2]

(ii) Hence or otherwise, find the value of n in the expansion of $(1+x+2x^2)^n$, given that the coefficient of x^2 is 44. [3]

(b) In the expansion of $\left(2x^2-\frac{1}{x}\right)^{12}$, find the ratio of the term independent of x to that of the coefficient of the middle term. [5]

6 Billy signed up for a race and was given a brochure showing the race route.



XY is a straight road. Participants would start running from point A to D, then from D to C, followed by C to B and finally from B back to A. BC is parallel to XY. CD is perpendicular to both BC and XY. $AB = AD = 3$ km and angle XAB is θ °. The total distance of the route is L km.

(i) Show that L can be expressed as $p\cos\theta + q\sin\theta + r$, where p, q and r are constants. [3]

(ii) Express L in the form $R\cos(\theta-\alpha)+r$, where $R > 0$ and α is an acute angle. [2]

(iii) The total length of the route is found to be 13 km. Find the values of θ. [3]

(iv) Billy claims that he can finish the race in under 49 minutes if he maintains his speed of 16 km/h throughout the race regardless of the value of θ. Is Billy's claim true? Explain your answer. [2]

7 **Answer the whole of this question on a sheet of graph paper.**

A particle moving in a certain medium, with speed v m/s, experiences a resistance to motion of R newtons. R and v are related by the equation $R = av^2 + bv$, where a and b are constants.

v	5	10	15	20	25
R	17	44	81	138	185

The table shows the experimental values of the variables v and R, but an error has been made in recording one of the values of R.

(i) Using graph paper, draw the graph of $\frac{R}{v}$ against v. [3]

Use your graph to

(ii) write down the value of v for which its recorded R value was incorrect and find the correct value of R, [2]

(iii) estimate the value of a and of b. [3]

In a different medium, R is directly proportional to v and $R = 30$ when $v = 5$.

(iv) Draw a suitable line on your graph to illustrate the second situation and use it to determine the value of v for which the resistance is the same in both mediums. [3]

8 The function $f(x) = 3x^3 + ax^2 + bx + 2$, where a and b are constants. $x-1$ is a factor of $f(x)$. The remainder when $f(x)$ is divided by $x-2$ is 2.5 times the remainder when $f(x)$ is divided by $x+1$.

(i) Show that $a = 2$ and $b = -7$. [4]

(ii) Without using a calculator, solve $f(x) = 0$. [3]

(iii) Hence solve $3\sin^2 y - 2\sec y - 2\cos y + 4 = 0$ for $0 \le y \le 360°$. [4]

[TURN OVER

9 The equation of the curve is $y = \frac{4x-12}{x+3}$. The point P lies on the curve and has a positive x-coordinate. The normal to the curve at P makes an angle θ with the x-axis such that $\tan\theta = -6$.

(a) Show that the coordinates of P is (9 , 2). [4]

The point Q also lies on the curve and has a positive x-coordinate. The tangent to the curve at Q is parallel to the line $3y - 2x = 6$.

(b) Find the coordinates of Q. [3]

It is given further that the coordinates of R and S are (5 , –4) and (13 , –1) respectively.

(c) Determine whether $PQRS$ is a kite or not. Justify your answer. [2]

(d) Calculate the area of $PQRS$. [2]

10 (a) A curve is such that $\frac{d^2y}{dx^2} = 4e^{-2x+1}$ and the gradient at $(2, e^{-3})$ is $-\frac{2}{e^3} - 4$.

(i) Find $\frac{dy}{dx}$ in terms of x. [3]

(ii) Explain why the curve has no stationary points. [2]

(iii) Find the equation of the curve. [3]

(b) (i) Express $\frac{17+6x-5x^2}{(2x-1)(3-x)^2}$ in partial fractions. [4]

(ii) Hence find $\int \frac{17+6x-5x^2}{(2x-1)(3-x)^2} dx$. [3]

End Of Paper

Paper 1 Answer

Q1

Q. 1 Prelim 2017
AMath P1.

Final

$$f(x) = \ln \frac{\sqrt{x^2+5}}{x}$$

$$= \frac{1}{2}\ln(x^2+5) - \ln x$$

$$f'(x) = \frac{1}{2}\left(\frac{2x}{x^2+5}\right) - \frac{1}{x}$$

$$= \frac{x^2 - x^2 - 5}{x(x^2+5)}$$

$$= -\frac{5}{x(x^2+5)} \quad \text{— M1}$$

if $x > 0$, then $x(x^2+5) > 0$ — M1

$f'(x) < 0$ for $x > 0$

Hence, f is a decreasing function for $x > 0$

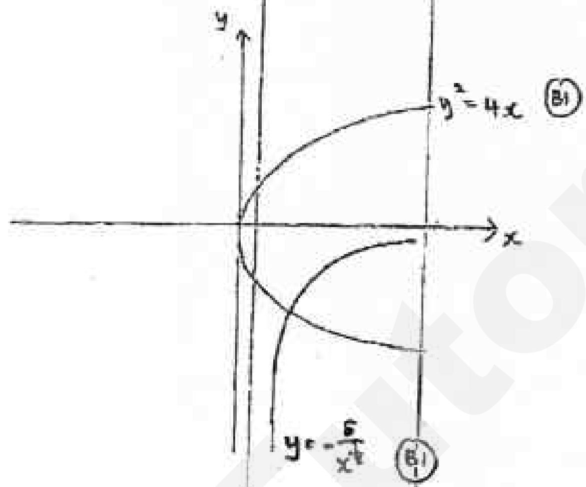
both: A1

3

Q2

Q.2

(i)



(ii) $y = \frac{-5}{x^2}$ — (1)

$y^2 = 4x$ — (2)

sub (1) into (2) :

$\frac{25}{x} = 4x$

$x^2 = \frac{25}{4}$

} either : M1

$\therefore k = \frac{25}{4}$ — A1

4

Q3

Q.3

(i) $V = 84000e^{kt} + 8500$

when $t=0$, $V = 84000 + 8500 = 92,500$ — B1

$\therefore$ initial value of car $= \$92,500$.

(ii) When $t=3$

$V = 84000e^{k(3)} + 8500 = \frac{92,500}{2}$ — M1

$e^{3k} = 0.4494$

$3k = \ln 0.4494$

$\therefore k = -0.2666$

$= -0.267$ (3 sf) — A1

(iii) $V = 84000e^{-0.267t} + 8500$

Since $84000e^{-0.267t} > 0$ OR As $t \to \infty$, $e^{-0.267t} \to 0$

$\therefore V > 8500$ either: M1 $V \to 8500$

Since value of car is at least \$8,500, you should not accept the offer

M1: to mention at least \$8,500 or > \$8,500.

5

Q4

Q.4

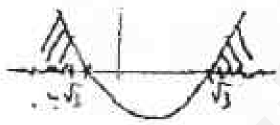
(i) let $y = 3x(x+2) + k^2$

$= 3x^2 + 6x + k^2$

$\therefore \ 6^2 - 4(3)(k^2) \leq 0$ — M1

$12k^2 - 36 \geq 0$

$(k+\sqrt{3})(k-\sqrt{3}) \geq 0$ — M1



$\therefore \ k \leq -\sqrt{3}$ or $k \geq \sqrt{3}$ — A1

(ii)

$(x-4)(x-k) = 0$

$x^2 - 4x - kx + 4k = 0$

$3x^2 + 3(-4-k)x + 12k = 0$

OR $3x^2 - 12x - 3kx + 12k = 0$ } either: M1

OR $3(x-4)(x-k) < 0$

$3x^2 + 3(-4-k)x + 12k < 0$

By comparison,

$12k = 84$

$\therefore \ k = 7$ — A1

and $3(-4-k) = p$

$\therefore \ p = -33$

OR $-12 - 3k = p$

$\therefore \ p = -33$

} — either: A1

6

Q5

Q.5

(i) $\log_3 2 \times \log_4 3 \times \log_5 4 \times \ldots \times \log_{(n+1)} n$

$= \frac{\lg 2}{\lg 3} \times \frac{\lg 3}{\lg 4} \times \frac{\lg 4}{\lg 5} \times \ldots \frac{\lg n}{\lg(n+1)}$ — M1 or using ln

$= \frac{\lg 2}{\lg(n+1)}$ OR $\frac{1}{\log_2 (n+1)}$ OR $\log_{n+1} 2$ — A1

(ii) $6^{x+1} - 6^{1-x} = 5$

$6(6^x) - \frac{6}{6^x} = 5$ — M1

Let $u = 6^x$

$\therefore 6u - \frac{6}{u} - 5 = 0$

$6u^2 - 5u - 6 = 0$ — M1

$(3u+2)(2u-3) = 0$

$\therefore u = \frac{3}{2}$ $\left.(u = -\frac{2}{3} \text{ is rejected})\right\}$ — M1

Hence, $6^x = \frac{3}{2}$

$x = \frac{\lg \frac{3}{2}}{\lg 6}$

$= 0.226$ (3 sf) — A1

6

Q6

Q.6

(i) $\sec\theta - \frac{\cos\theta}{1+\sin\theta} = \tan\theta$

$$\text{LHS} = \frac{1}{\cos\theta} - \frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{1+\sin\theta - \cos^2\theta}{\cos\theta(1+\sin\theta)} \quad \text{— M1}$$

$$= \frac{1+\sin\theta - (1-\sin^2\theta)}{\cos\theta(1+\sin\theta)} \quad \text{OR} \quad \frac{\sin^2\theta+\cos^2\theta+\sin\theta-\cos^2\theta}{\cos\theta(1-\sin\theta)}$$

$$= \frac{\sin^2\theta+\sin\theta}{\cos\theta(1+\sin\theta)} \quad \text{— M1}$$

$$= \frac{\sin\theta(\sin\theta+1)}{\cos\theta(1+\sin\theta)}$$

$$= \tan\theta \ \text{(proven)} \quad \text{— A1}$$

(ii) for $\sec\theta - \frac{\cos\theta}{1+\sin\theta} = \frac{1}{3}\cot\theta$

$$\therefore \tan\theta = \frac{1}{3}\cot\theta$$

$$\tan^2\theta = \frac{1}{3} \quad \text{— M1}$$

$$\tan\theta = \pm\sqrt{\frac{1}{3}}$$

$$\theta = \frac{\pi}{6} \ \text{or} \ \pi - \frac{\pi}{6} = \frac{5\pi}{6} \quad \text{— A1}$$

A1

/6

Q7

Q. 7.

(i) subst. P(−1, 0) into

$y = |kx - 2|$

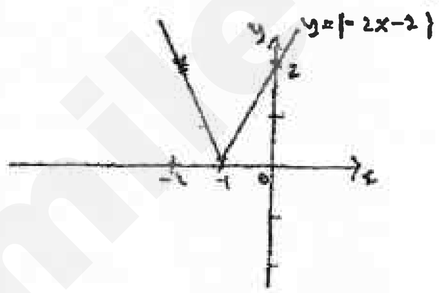
$0 = |-k - 2|$ — M1

$\therefore k = -2$ (shown) — A1

(ii) $y = |-2x - 2|$

when $x = 0$, $y = 2$

when $x = -2$, $y = 2$ — Any pt for $x < -1$ calculated — M1

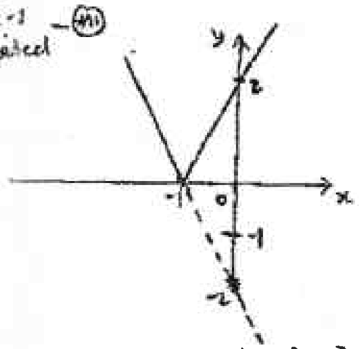


Graph: intercepts shown on axes — M1

correct graph — A1

OR for $y = -2x - 2$

when $x = 0$, $y = -2$



$y = -2x - 2$ drawn — M1

(iii) y not shown above:

when $y = 2$, $-2x - 2 = 2$ or $-2x - 2 = -2$

$x = -2$ or $x = 0$

$\therefore x < -2$ or $x > 0$ — B1

6

Q8

Q.8

(i) $\angle CBG = \angle BDC$ (alt. segment theorem) — M1

$\angle CBG = \angle BCE$ (alt. $\angle$s, $BC \parallel FG$)

$\therefore \angle BDC = \angle BCE$

$\angle CBD = \angle EBC$ (common $\angle$) — M1

$\therefore$ triangle BCD is similar to triangle BEC — A1
(AAA similarity) (shown)

(ii) $\frac{BC}{BD} = \frac{BE}{BC}$ — M1

$\therefore BC^2 = BD \times BE$ (shown) — A1

(iii) $\angle BAC = \angle BDC$ ($\angle$s in same segment) — M1

$\angle BDC = \angle BCE$ (above) — M1

$\therefore \angle BAC = \angle BCE$

Hence $\triangle ABC$ is isosceles (shown)

OR

OR

$\angle CBG = \angle BCA$ (above) — M1

$\angle CBG = \angle CAB$ (alt. segment theorem) — M1

$\therefore \angle BCA = \angle CAB$

Hence, $\triangle ABC$ is isosceles (shown)

7

Q9

Q.9

(i) $y = 6\sqrt{(1+2x)^3} = 6(1+2x)^{\frac{3}{2}}$

when $\frac{dx}{dt} = -0.04$ and $\frac{dy}{dt} = -0.045$ } both: (M1) (or shown below)

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$ — (M1) for $\frac{dy}{dx}$

$-0.045 = 6\left(\frac{3}{2}\right)(1+2x)^{\frac{1}{2}}(2) \cdot (-0.04)$ } (M1) for $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{d}{d}$

$(1+2x)^{\frac{1}{2}} = \frac{0.045}{0.04} \times \frac{1}{18}$

$(1+2x) = 0.003906$

$\therefore x = -0.4980$

$= -0.498$ (3 s.f.) — (A1)

(ii) Let the x-coordinate $= a$

$\therefore \int_2^a 6(1+2x)^{\frac{3}{2}}\,dx = \int_a^5 6(1+2x)^{\frac{3}{2}}\,dx$ —(M1)

$6\left[\frac{2}{5}(1+2x)^{\frac{5}{2}}\left(\frac{1}{2}\right)\right]_2^a = 6\left[\frac{2}{5}(1+2x)^{\frac{5}{2}}\left(\frac{1}{2}\right)\right]_a^5$ —(M1)

$\left[(1+2x)^{\frac{5}{2}}\right]_2^a = \left[(1+2x)^{\frac{5}{2}}\right]_a^5$

$(1+2a)^{\frac{5}{2}} - (1+2(2))^{\frac{5}{2}} = (1+2(5))^{\frac{5}{2}} - (1+2a)^{\frac{5}{2}}$ } either (M1)

$2(1+2a)^{\frac{5}{2}} = (11)^{\frac{5}{2}} + (5)^{\frac{5}{2}}$

$(1+2a)^{\frac{5}{2}} = \frac{457.2}{2}$

$1+2a = 8.782$

$a = 3.891$

$= 3.89$ (3 s.f.) — (A1)

/8

Q10

Q10.

(i) $f(x) = 2\sin^2 x - \cos^2 x$

$= 2(1-\cos^2 x) - \cos^2 x$ — (M1)

$= 2 - 3\cos^2 x$

$= \frac{1}{2} - \frac{3}{2}(2\cos^2 x - 1)$ — (M1)

$= \frac{1}{2} - \frac{3}{2}\cos 2x$ — (A1)

$\therefore a = \frac{1}{2}$ and $b = -\frac{3}{2}$ (shown)

(OR) $f(x) = 2\sin^2 x - \cos^2 x$

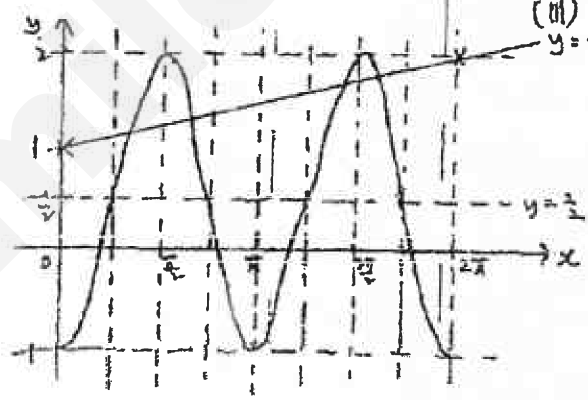
(M1) $= \underline{1 - \cos 2x} - \underline{\frac{\cos 2x + 1}{2}}$ (M1)

$= 1 - \cos 2x - \frac{1}{2}\cos 2x - \frac{1}{2}$

$= \frac{1}{2} - \frac{3}{2}\cos 2x$ — (A1)

$\therefore a = \frac{1}{2}$ and $b = -\frac{3}{2}$

(ii)



(M1) Graph to correctly intersect $y = \frac{1}{2}$

(M1) max = $y = 2$
min = $y = -1$
amplitude = $1\frac{1}{2}$
2 cycles. ($\pi \times 2$)

(A1) correct shape

(iii) $4\pi\sin^2 x - 2\pi\cos^2 x - x = 2\pi$

$2\pi(2\sin^2 x - \cos^2 x) = x + 2\pi$

$2\sin^2 x - \cos^2 x = \frac{1}{2\pi}x + 1$ — (M1)

Draw $y = \frac{1}{2\pi}x + 1$

when $x = 2\pi$, $y = 2$

$\therefore$ no. of solutions = 4 — (A1)

9

Q11

Q.11

(i) $A = \frac{1}{2}\left[200\sin(\theta+30^\circ)\right]\left[200\cos(\theta+30^\circ)\right]$ — M1

$= 20000(\sin\theta\cos 30^\circ + \cos\theta\sin 30^\circ)$
$\times(\cos\theta\cos 30^\circ - \sin\theta\sin 30^\circ)$

$= 20000\left(\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta\right)\left(\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta\right)$ — M1

$= 5000(3\sin\theta\cos\theta - \sqrt{3}\sin^2\theta + \sqrt{3}\cos^2\theta - \sin\theta\cos\theta)$ M1

$= 5000(\sin 2\theta + \sqrt{3}\cos 2\theta)$ — A1

(ii) $\frac{dA}{d\theta} = 5000(2\cos 2\theta - 2\sqrt{3}\sin 2\theta)$
$= 10000(\cos 2\theta - \sqrt{3}\sin 2\theta)$ either: — B1

(iii) Let $\frac{dA}{d\theta} = 10000(\cos 2\theta - \sqrt{3}\sin 2\theta) = 0$

$\therefore \tan 2\theta = \frac{1}{\sqrt{3}}$ — M1

$\therefore 2\theta = 30^\circ$

$\theta = 15^\circ$ — A1

$\frac{d^2A}{d^2\theta} = 10000(-2\sin 2\theta - 2\sqrt{3}\cos 2\theta)$
$= -20000(\sin 2\theta + \sqrt{3}\cos 2\theta)$ either: — M1

when $\theta = 15^\circ$

$\frac{d^2A}{d^2\theta} = -20000(\sin 30^\circ + \sqrt{3}\cos 30^\circ) < 0$

$\therefore$ at $\theta = 15^\circ$, area of triangle is maximum. both: A1

9

Q12

Q.12

(i) $v = t^2 - 5t + 6$

$a = \frac{dv}{dt} = 2t - 5$ — (M1)

at $t = 4$, acceleration $= 3\ m/s^2$ — (A1)

(ii) for velocity to be decreasing;

$a < 0$

$2t - 5 < 0$ — (M1)

$t < \frac{5}{2}$ (or 2.5)

∴ time interval is $0 < t < \frac{5}{2}$ s — (A1)

(iii) At rest, $v = 0$

$t^2 - 5t + 6 = 0$

$(t-2)(t-3) = 0$

$t = 2$ or 3 — (M1)

$s = \int (t^2 - 5t + 6)\, dt$

$= \frac{t^3}{3} - \frac{5t^2}{2} + 6t + c$ — (M1)

when $t = 1.5$, $s = \frac{(1.5)^3}{3} - \frac{5(1.5)^2}{2} + 6(1.5) + c = 0$

$c = -4.5$

$s = \frac{t^3}{3} - \frac{5t^2}{2} + 6t - 4.5$

∴ at $t = 2$, (M1)

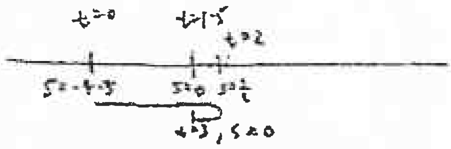
$s = \frac{2^3}{3} - \frac{5(2)^2}{2} + 6(2) - 4.5$

$= \frac{1}{6}$ m (or 0.167 m) — (A1)

Displacement required $= \frac{1}{6}$ m (or 0.167 m)

Q12

(iv)



at $t=0$, $s=-4.5$ m

at $t=3$, $s=\frac{3^3}{3}-\frac{5(3)^2}{2}+6(3)-4.5$ — M1

$= 0$

$\therefore$ average speed $= \dfrac{4.5+\frac{1}{6}+\frac{1}{6}}{3}$ } M1 for distance travelled

$= 1.611$

≈ 1.61 (or $1\frac{11}{18}$) m/s — A1

11

Paper 2 Answer

Q1

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10. $\frac{1}{2} \times h \times (6\sqrt{3} + \sqrt{8}) = 4 + 6\sqrt{6}$.

$h = \frac{8 + 12\sqrt{6}}{6\sqrt{3} + \sqrt{8}} \times \frac{6\sqrt{3} - \sqrt{8}}{6\sqrt{3} - \sqrt{8}}$ [M1]

$= \frac{48\sqrt{3} - 8\sqrt{8} + 72\sqrt{18} - 12\sqrt{48}}{36(3) - 8}$ [M1]

$= \frac{48\sqrt{3} - 16\sqrt{2} + 216\sqrt{2} - 48\sqrt{3}}{100}$ [M1] Simplify: $\sqrt{8}$, $\sqrt{18}$, $\sqrt{48}$

$= \frac{200\sqrt{2}}{100} = 2\sqrt{2}$ cm. [A1]

b. $H^2 = (2\sqrt{2})^2 + (6\sqrt{3} + \sqrt{8})^2$ [M1]

$= 4(2) + 36(3) + 12\sqrt{24} + 8$ [M1]

$= 124 + 12\sqrt{24}$

$= 124 + 24\sqrt{6}$ [A1]

EX 255 (rev 2008)

Q2

2 i. $\frac{d}{dx}(\tan x\sin^2x) = \sin^2x\sec^2x + \tan x(2\sin x)\cos x$ [M1].

$= \sin^2x\left(\frac{1}{\cos^2x}\right) + \frac{\sin x}{\cos x}(2\sin x)\cos x$ [M1] for either

$= \tan^2x + 2\sin^2x$

$= 2\sin^2x + \sec^2x - 1.$ [A1]

ii $\int_{\frac{\pi}{4}}^{\pi}\sin^2x\,dx = \frac{1}{2}\int_{\frac{\pi}{4}}^{\pi}\cancel{2\sin^2x + \sec^2x - 1} - \sec^2x + 1\,dx$ [M1]

$= \frac{1}{2}\int_{\frac{\pi}{4}}^{\pi}2\sin^2x + \sec^2x - 1\,dx + \frac{1}{2}\int_{\frac{\pi}{4}}^{\pi}1 - \sec^2x\,dx$
[M1] [M1]

$= \frac{1}{2}\left[\tan x\sin^2x\right]_{\frac{\pi}{4}}^{\pi} + \frac{1}{2}\left[x - \tan x\right]_{\frac{\pi}{4}}^{\pi}$

$= \frac{1}{2}\left[\tan\pi\sin^2\pi - \tan\frac{\pi}{4}\sin^2\frac{\pi}{4}\right] + \frac{1}{2}\left[\pi - \tan\pi - \frac{\pi}{4} + \tan\frac{\pi}{4}\right]$

$= \frac{1}{2}\left[-\left(\frac{\sqrt{2}}{2}\right)^2\right] + \frac{1}{2}\left[\pi - \frac{\pi}{4} + 1\right]$

$= \frac{1}{2}\left[-\frac{2}{4}\right] + \frac{1}{2}\left[\frac{3\pi}{4} + 1\right]$

$= -\frac{1}{4} + \frac{3\pi}{8} + \frac{1}{2}$

$= \frac{1}{4} + \frac{3\pi}{8}$ [A1]

or

$\int_{\frac{\pi}{4}}^{\pi}2\sin^2x + \sec^2x - 1\,dx = \left[\tan x\sin^2x\right]_{\frac{\pi}{4}}^{\pi}$ [M1]

$2\int_{\frac{\pi}{4}}^{\pi}\sin^2x\,dx + \left[\tan x - x\right]_{\frac{\pi}{4}}^{\pi} = \left[\tan x\sin^2x\right]_{\frac{\pi}{4}}^{\pi}$ [M1]

$2\int_{\frac{\pi}{4}}^{\pi}\sin^2x\,dx = \left[\tan x\sin^2x - \tan x + x\right]_{\frac{\pi}{4}}^{\pi}$ [M1]

$= \pi - \left[\frac{1}{2} - 1 + \frac{\pi}{4}\right]$

$\int_{\frac{\pi}{4}}^{\pi}\sin^2x\,dx = \frac{1}{4} + \frac{3\pi}{8}$ [A1]

Q3

FX 255 (rev 2008)

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3 i. $2x^2 - 3x + 4 = 0$

$\alpha + \beta = \frac{3}{2}$ [M1]

$\alpha\beta = 2$ [M1]

$(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (\frac{3}{2})^2 - 2(2)$

$= -\frac{7}{4}$ [A1]

ii. $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= \frac{3}{2}(-\frac{7}{4} - 2)$ [M1]

$= -\frac{45}{8}$ [A1]

iii $\frac{\alpha}{\beta^2+1} + \frac{\beta}{\alpha^2+1} = \frac{\alpha(\alpha^2+1) + \beta(\beta^2+1)}{(\alpha\beta)^2 + \beta^2 + \alpha^2 + 1}$

$= \frac{\alpha^3 + \alpha + \beta^3 + \beta}{(\alpha\beta)^2 + \alpha^2 + \beta^2 + 1}$

$= \frac{-\frac{45}{8} + \frac{3}{2}}{2^2 - \frac{7}{4} + 1}$ [M1]

$= -\frac{33}{26}$ [M1]

$\left(\frac{\alpha}{\beta^2+1}\right)\left(\frac{\beta}{\alpha^2+1}\right) = \frac{\alpha\beta}{(\alpha\beta)^2 + \beta^2 + \alpha^2 + 1}$

$= \frac{2}{2^2 - \frac{7}{4} + 1}$

$= \frac{8}{13}$ [M1]

$x^2 + \frac{33}{26}x + \frac{8}{13} = 0$

$26x^2 + 33x + 16 = 0$ [A1]

Q4

4i.	y-coordinate $= a+3$ [B1]		
ii	centre $= (a, a+3)$		
	$\sqrt{(a-8)^2+(a+3-12)^2} = a$ [M1]		
	$(a-8)^2 + (a-9)^2 = a^2$		
	$a^2 - 16a + 64 + a^2 - 18a + 81 = a^2$		
	$2a^2 - 34a + 145 - a^2 = 0$		
	$a^2 - 34a + 145 = 0$		
	$a = \dfrac{34 \pm \sqrt{34^2 - 4(1)(145)}}{2(1)}$		
	$= \dfrac{34 \pm 24}{2}$		
	$a = 29$ (rejected) or $a = 5$ [M1]		
	$\therefore$ centre $: (5, 8)$ [A1]		
iii a	Eqn of C : $(x-5)^2 + (y-8)^2 = 25$ [B1]		
b.	$m_{radius} = \dfrac{12-8}{8-5}$		
	$= \dfrac{4}{3}$		
	$m_L \left(\frac{4}{3}\right) = -1$		
	$m_L = -\frac{3}{4}$ [M1]		
	$\therefore$ Eqn of L : $y = -\frac{3}{4}x + c$, $(8, 12)$		
	$12 = -\frac{3}{4}(8) + c$ [M1]		
	$c = 18$		
	$\therefore y = -\frac{3}{4}x + 18$	Either [A1]	
	$4y = -3x + 72$		
iv.	centre $: (-5, 8)$		
	$\therefore$ Eqn of C : $(x+5)^2 + (y-8)^2 = 25$ [B1]		

Q5

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5ai $(1+y)^n = 1+\binom{n}{1}y+\binom{n}{2}y^2+\ldots$ [M1]

$= 1+ny+\frac{n(n-1)}{2}y^2+\ldots$ [M1]

ii $(1+x+2x^2)^n = 1+n(x+2x^2)+\frac{n(n-1)}{2}(x+2x^2)^2+\ldots$ [M1]

$= 1+nx+2nx^2+\frac{n(n-1)}{2}(x^2+\ldots)+\ldots$

$\therefore$ coefficient of $x^2 = 2n+\frac{n(n-1)}{2}$

$2n+\frac{n(n-1)}{2} = 44$ [M1]

$4n+n(n-1) = 88$

$n^2+n(n-1) = 88$

$n^2+3n-88=0$

$(n+11)(n-8)=0$

(rejected) $n=-11$ or $n=8$ [A1]

5b $\left(2x^2-\frac{1}{x}\right)^{12}$

$T_{r+1} = \binom{12}{r}(2x^2)^{12-r}\left(-\frac{1}{x}\right)^r$

$= \binom{12}{r}(2)^{12-r}x^{24-2r}(-1)^r(x)^{-r}$

$= \binom{12}{r}(2)^{12-r}(-1)^r x^{24-3r}$ [M1]

$24-3r=0$

$3r=24$

$r=8$ [M1]

Term independent of $x = \binom{12}{8}(2)^{12-8}(-1)^8$ [M1]

$= 7920$

middle term is the 7^{th} term

when $r=6$, coefficient of $T_7 = \binom{12}{6}(2)^{12-6}(-1)^6$ [M1]

$= 59136$

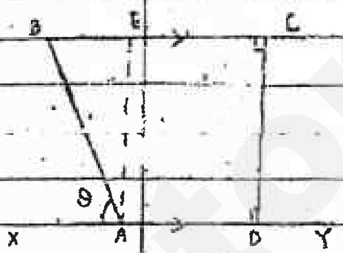
Ratio $= \frac{7920}{59136}$

$= \frac{15}{112}$ OR Ratio $= 15:112$ [A1]

EX 285 (rev 2005)

Q6

6i. $\angle ABC = \theta$ (alt. $\angle$, // lines)



$\therefore \sin\theta = \frac{AE}{3}$

$AE = 3\sin\theta$ [M1]

$\cos\theta = \frac{BE}{3}$

$BE = 3\cos\theta$ [M1]

$L = 3+3+3\cos\theta + 3 + 3\sin\theta$

$= 3\cos\theta + 3\sin\theta + 9$ [A1]

ii $L = R\cos(\theta - \alpha) + r$

$\therefore R = \sqrt{3^2 + 3^2}$

$= \sqrt{18} = 3\sqrt{2}$

[Either [M1]]

$\tan\alpha = 1$

$\alpha = 45°$

$\therefore L = 3\sqrt{2}\cos(\theta - 45°) + 9$ or $L = \sqrt{18}\cos(\theta - 45°) + 9$ [A1].

iii when $L = 13$,

$3\sqrt{2}\cos(\theta - 45) + 9 = 13$ [M1]

$\cos(\theta - 45) = \frac{4}{3\sqrt{2}}$

$\theta - 45 = 19.47, -19.47$ [M1] for B.A

$\theta = 64.47, 25.53$

$\approx 64.5°, 25.5°$ [A1]

iv Distance ran by billy $= \frac{49}{60} \times 16$

$= 13\frac{1}{15}$ km or 13.1 km

max L occur when $\cos(\theta - 45) = 1$, $L = (3\sqrt{2} + 9)$ km. or $= 13.24$ km (*)

No, since the distance covered by Billy is less than the maximum distance, L. [A1]

OR

max time $= \frac{3\sqrt{2}+9}{16} \times 60$

$= 49.6$ min (*) Either (*) [M1]

NO, the maximum time required to finish the race is more than the time taken by Billy. [A1].

EX 255 (rev 2005)

- minus one mark on the Q2 if no label of the y-axis.

[B1]: points
[B1]: Best fitting line
[B1]: Scale of the axes.

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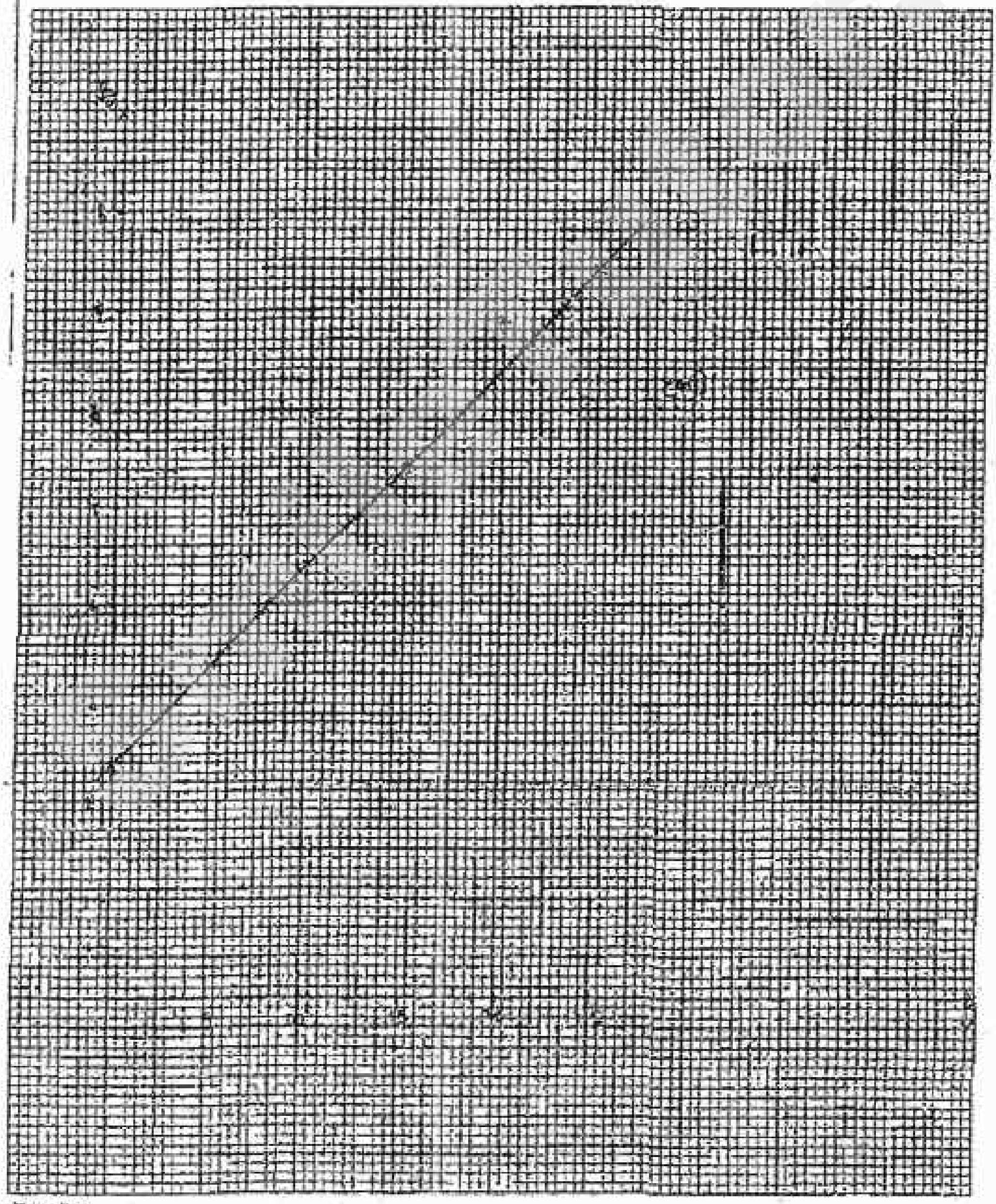

Q7

v	5	10	15	20	25
$\frac{R}{v}$	3.4	4.4	5.4	6.9	7.4

$+bv$

$\frac{R}{v} = av + b$. [M1]

(ii) incorrect $v = 20$. [A1].

when $v = 20$, $\frac{R}{v} = 6.4$.

$\frac{R}{20} = 6.4$

$R = 128$ (± 2) [A1].

(iii) $b = 2.4$ (± 0.1) [A1]

$R = av^2 + bv$

$\frac{R}{v} = av + b$ [M1]

$a = \frac{7-4}{23-8}$

$= \frac{3}{15}$

$= 0.2$ [A1].

$\frac{6.9-4.1}{23.5-7.5} \leq \text{grad} \leq \frac{7.1-3.9}{23.5-8.5}$

$0.175 \leq \text{grad} \leq 0.2185$

(iv). $R = kv$, where k is a constant.

when $R = 30$, $v = 5$,

$30 = k(5)$

$k = 6$. [M1].

$\therefore R = 6v$

$\frac{R}{v} = 6$.

$\therefore v = 18$ (± 0.5) [A1].

Q8

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8i. $f(x) = 3x^3 + ax^2 + bx + 2$

$f(1) = 0$

$3 + a + b + 2 = 0$

$a + b = -5$ — ① (M1)

$f(2) = 2.5\, f(-1)$

$3(2)^3 + 4a + 2b + 2 = [-3 + a - b + 2](2.5)$ (M1)

$26 + 4a + 2b = [-1 + a - b]2.5$

$26 + 4a + 2b = -2.5 + 2.5a - 2.5b$

$1.5a + 4.5b = -28.5$ — ②

from ① $a = -5 - b$ — ③

Sub ③ into ②

$1.5(-5 - b) + 4.5b = -28.5$ (M1)

$-7.5 - 1.5b + 4.5b = -28.5$

$3b - 7.5 = -28.5$

$3b = -21$

$b = -7$; $a = 2$ (A1)

ii $f(x) = 3x^3 + 2x^2 - 7x + 2$

$= (x-1)(3x^2 + 5x - 2)$ (M1)

$= (x-1)(3x-1)(x+2)$ (M1)

$f(x) = 0$

$(x-1)(3x-1)(x+2) = 0$

$x - 1 = 0$ or $3x - 1 = 0$ or $x + 2 = 0$

$x = 1$, $x = \frac{1}{3}$, $x = -2$ (A1)

$$
\begin{array}{r}
3x^2 + 5x - 2 \\
x-1 \,\big)\, 3x^3 + 2x^2 - 7x + 2 \\
-(3x^3 - 3x^2) \\
5x^2 - 7x + 2 \\
-(5x^2 - 5x) \\
-2x + 2 \\
-(-2x + 2) \\
0
\end{array}
$$

EX 255 (rev 2005)

iii	$3\sin^2 y - 2\sec y - 2\cos y + 4 = 0$		
	$3\sin^2 y - \frac{2}{\cos y} - 2\cos y + 4 = 0$		
	$3(1-\cos^2 y)\cos y - 2 - 2\cos^2 y + 4\cos y = 0$ either underlined: [M1]		
	$3\cos y - 3\cos^3 y - 2 - 2\cos^2 y + 4\cos y = 0$		
	$3\cos^3 y + 2\cos^2 y - 7\cos y + 2 = 0$		
	let $Y = \cos y$		
	$\cos y = 1$ or $\cos y = \frac{1}{3}$ or $\cos y = -2$ (N.A) [M1]		
	$\underbrace{y = 0°, 360°}_{[M1]}$ Basic ∡ $= 70.52$		
	$y = 70.52, 360 - 70.52$		
	$= 70.52, 289.48$		
	$\approx 70.5°, 289.5°$ [A1].		
	$y \approx 0°, 70.5°, 289.5°, 360°$		

Q9

EX 255 (rev 2005)

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9a. $y = \frac{4x-12}{x+3}$

$\frac{dy}{dx} = \frac{(x+3)(4)-(4x-12)}{(x+3)^2}$ [M1]

$= \frac{4x+12-4x+12}{(x+3)^2}$

$= \frac{24}{(x+3)^2}$

$m_{normal} = -6$

$m_{tangent} = \frac{1}{6}$

When $\frac{dy}{dx} = \frac{1}{6}$, $\frac{24}{(x+3)^2} = \frac{1}{6}$ [M1]

$(x+3)^2 = 144$

$x+3 = 12$ or $x+3 = -12$

$x = 9$ $\quad$ $x = -15$ (rejected) [M1]

When $x = 9$, $y = 2$ [A1].

$\therefore$ P(9, 2) (shown) [no [A1] if $x=-15$ is not rejected]

b. $3y - 2x = 6$

$y = \frac{2}{3}x + 2$

when $\frac{dy}{dx} = \frac{2}{3}$

$\frac{2}{3} = \frac{24}{(x+3)^2}$ [M1]

$(x+3)^2 = 36$

$x+3 = 6$ or $x+3 = -6$ [M1]

$x = 3$ $\quad$ $x = -9$ (rejected)

$\therefore$ when $x = 3$, $y = 0$.

Q (3, 0) [A1].

c.	$m_{PR} = \frac{2-(-4)}{9-5}$		
	$= \frac{6}{4} = \frac{3}{2}$		
	$m_{QS} = \frac{0-(-1)}{3-13}$ Either [M1]		
	$= -\frac{1}{10}$		
	$m_{PR} \times m_{QS} = -\frac{1}{10}\left(\frac{3}{2}\right)$		
	$= -\frac{3}{20} \neq -1.$ [M1].		
	Since $m_{PR} \times m_{QS} \neq -1$, $\therefore$ PQRS is not a kite.		
d.	Area $= \frac{1}{2}\left\lvert \begin{matrix} 9 & 3 & 5 & 13 & 9 \\ 2 & 0 & -4 & -1 & 2 \end{matrix} \right\rvert$. [M1]		
	$= \frac{1}{2} \lvert (0-12-5+26) - (6+0-52-9) \rvert$		
	$= \frac{1}{2}(64)$		
	$= 32$ units^2 [M1].		
	OR		
9c.	$PQ = \sqrt{(9-3)^2 + (2-0)^2}$		
	$= \sqrt{40} = 6.3245$		
	$QR = \sqrt{(5-3)^2 + (-4-0)^2}$		
	$= \sqrt{20} \approx 4.4721$		
	$PS = \sqrt{(9-13)^2 + (2+1)^2}$ [M1] allow ecf.		
	$= \sqrt{25} = 5$		
	$SR = \sqrt{(13-5)^2 + (-1+4)^2}$		
	$= \sqrt{73} \approx 8.5440$		

Q10

Singapore Examinations and Assessment Board

Name .... Centre/Index No ....

Subject ....

10a. $\frac{d^2y}{dx^2} = 4e^{-2x+1}$

$\frac{dy}{dx} = \frac{4e^{-2x+1}}{-2} + c$ (m1)

$= -2e^{-2x+1} + c$

when $x=2$, $\frac{dy}{dx} = -\frac{2}{e^3} - 4$

$-\frac{2}{e^3} - 4 = -2e^{-3} + c$ (m1)

$c = -4$

$\frac{dy}{dx} = -2e^{-2x+1} - 4$, (A1)

ii. Since $e^{-2x+1} > 0$, (m1) $-2e^{-2x+1} < 0$, $-2e^{-2x+1} - 4 < 0$,

$\frac{dy}{dx} \neq 0$ (A1).

$\therefore$ The curve has no stationary point

OR when $\frac{dy}{dx} = 0$, $e^{-2x+1} = -2$. (m1)

Since $e^{-2x+1} > 0$ for all x, there is no solution for x. (A1)

iii. $y = \int -2e^{-2x+1} - 4\,dx$

$= \frac{-2e^{-2x+1}}{-2} - 4x + c$ (m1).

$= e^{-2x+1} - 4x + c$

$y = e^{-2x+1} - 4x + c$ $(2, e^{-3})$

$e^{-3} = e^{-3} - 8 + c$ (m1)

$c = 8$

$y = e^{-2x+1} - 4x + 8$ (A1).

EX 255 (rev 2005)

bi $\dfrac{17+6x-5x^2}{(2x-1)(3-x)^2} = \dfrac{A}{2x-1} + \dfrac{B}{3-x} + \dfrac{C}{(3-x)^2}$

$= \dfrac{A(3-x)^2 + B(2x-1)(3-x) + C(2x-1)}{(2x-1)(3-x)^2}$

$17+6x-5x^2 = A(3-x)^2 + B(2x-1)(3-x) + C(2x-1)$

let $x=3$, $17+6(3)-5(3)^2 = C(5)$

$5C = -10$

$C = -2$ [M1]

let $x = \frac{1}{2}$, $17+6(\frac{1}{2})-5(\frac{1}{2})^2 = A(3-\frac{1}{2})^2$

$\frac{25}{4}A = 18\frac{3}{4}$

$A = 3$ [M1]

let $x=0$, $17 = 3(3)^2 + B(-1)(3) + (-2)(-1)$

$17 = 27 - 3B + 2$

$-3B = -12$

$B = 4$. [M1]

$\therefore \dfrac{17+6x-5x^2}{(2x-1)(3-x)^2} = \dfrac{3}{2x-1} + \dfrac{4}{3-x} - \dfrac{2}{(3-x)^2}$ [A1]

ii. $\displaystyle\int \frac{17+6x-5x^2}{(2x-1)(6-2x)^2}\,dx$

$= \displaystyle\int \frac{3}{2x-1} + \frac{4}{3-x} - \frac{2}{(3-x)^2}\,dx$ [M1]

$= \frac{3}{2}\ln(2x-1) - 4\ln(3-x) - \left[\frac{2(3-x)^{-1}}{(-1)(-1)}\right] + C$

$= \frac{3}{2}\ln(2x-1) - 4\ln(3-x) - \frac{2}{3-x} + C$

[A1] [A1]

*Penalise 1 mark if no '+c' is seen.

NAME:			CLASS:

YISHUN TOWN SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2017
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC
ADDITIONAL MATHEMATICS PAPER 1 (4047/01)

DATE : 14 AUGUST 2017 | **DAY : MONDAY**

DURATION: 2 h | **MARKS: 80**

ADDITIONAL MATERIALS

Writing Paper x 6
Mathematics Cover Sheet x 1

READ THESE INSTRUCTIONS FIRST

Do not turn over the cover page until you are told to do so.
Write your name, class and class index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/ tape.
Write your answers on the writing papers provided.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This question paper consists of **5** printed pages and **1** blank page

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\text{Area of } \Delta = \frac{1}{2} bc \sin A$$

1 Express $\dfrac{4}{(x^2+4)(x-2)}$ in partial fractions. [5]

2 Solve the equations

(a) $4\log_4 x = 4 + \log_2(x+5)$, [5]

(b) $2(3^{2y+1}) = 6^{y-1}$. [3]

3 The equation of a curve is $y = px^2 - 4x + p$, where p is a constant.

Find the set of values of p for which the curve lies completely above the line $y = 3$. [4]

4 A rectangular block has a height of 3 unit with a square base of area $\left(\dfrac{x}{\sqrt{3}} + \dfrac{\sqrt{12}}{3}\right)$ square units. Given that the volume of the rectangular block is $x\sqrt{45}$ cubic units, without using a calculator, express the value of x in the form $\dfrac{a+\sqrt{b}}{7}$, where a and b are integers. [6]

5 Given that $4x^3 + 16x^2 + 13x + 3 = (x-k)\mathrm{f}(x)$ where $\mathrm{f}(x)$ is a polynomial and k is an integer.

(i) Find the value of k. [2]

(ii) Find $\mathrm{f}(x)$ and show that the x-axis is a tangent to the graph $y = \mathrm{f}(x)$. [4]

(iii) Deduce that there is no real solution for the equation $4x^6 + 16x^4 + 13x^2 + 3 = 0$. [2]

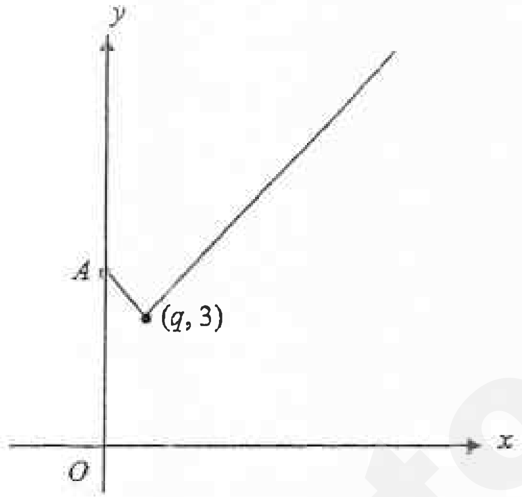
6 (i) Prove that $(\operatorname{cosec}\theta - \cot\theta)(1+\sec\theta) = \tan\theta$. [4]

(ii) Find, in radians, the acute angle θ for which

$$(\operatorname{cosec}\theta - \cot\theta)(1+\sec\theta) = \frac{1}{2}\sec^2\theta.$$ [3]

7 Two obtuse angles A and B are such that $\tan(2A+B) = 4$ and $\sin B = \dfrac{1}{\sqrt{5}}$. Without using a calculator, explain why $135° < A < 180°$. [5]

8 The diagram shows the graph of $y=|1-2x|+p$ for $0 \le x \le 4$.



(i) Find the values of p and of q. [3]

(ii) Find the coordinates of A. [1]

(iii) Find the coordinates of the point(s) of intersection of the line $y=3x$ and the graph of $y=|1-2x|+p$ for $0 \le x \le 4$. [4]

(iv) Determine the set of values of c for which the line $y=c$ intersects the graph of $y=|1-2x|+p$ at exactly one point for $0 \le x \le 4$. [2]

9 In the binomial expansion of $\left(x-\frac{k}{x^2}\right)^7$ in descending powers of x, k is a positive constant.

(i) Write down in terms of k, the first three terms of the expansion. [2]

(ii) Explain why there is no term which is independent of x in this expansion. [3]

(iii) The coefficient of x^4 in the expansion of $(2+x^3)\left(x-\frac{k}{x^2}\right)^7$ is 7, find the value of k. [3]

10 The equation of a curve is $y=2\sqrt{e^x}+\frac{4}{\sqrt{e^x}}+x$.

(i) Find the coordinates of the stationary point of the curve. [5]

(ii) Determine the nature of this stationary point. [3]

11 A triangle ABC is such that point A is (6, 6) and the point C is above point A and lies on the y-axis. Angle $ABC = 90°$ and $AB = BC = \sqrt{20}$ units. The equation of AB is $y + 2x = 18$.

(i) Find the coordinates of C and hence find the equation of BC. [5]

(ii) State the coordinates of M, the midpoint of AC. [1]

(iii) Show that the coordinates of B is (4, 10). [2]

(iv) Calculate the area of quadrilateral $OMBC$. [3]

End of Paper

Paper 1 - Yishun Town 2017 SA2

2a) $4\log_4 x = 4 + \log_2(x+5)$

$$4\left[\frac{\log_2 x}{2}\right] = 4\log_2 2 + \log_2(x+5)$$

$$\log_2 x^2 = \log_2 2^4(x+5)$$

$$x^2 = 16(x+5)$$

$$x^2 - 16x - 80 = 0$$

$$(x+4)(x-20) = 0$$

$x = -4$ or $x = 20$
(NA)

2b) $2(3^{2y+1}) = 6^{y-1}$

$$2(3^{2y})(3) = (6^y)\left(\frac{1}{6}\right)$$

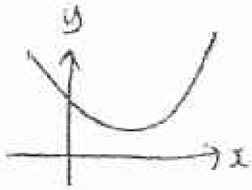
$$\frac{9^y}{6^y} = \frac{1}{36}$$

$$y\lg\left(\frac{9}{6}\right) = \lg\left(\frac{1}{36}\right)$$

$$y \approx -8.84$$

3) $y = px^2 - 4x + p$

$y = 3$



$$px^2 - 4x + p > 3$$

$$px^2 - 4x + p - 3 > 0$$

$$b^2 - 4ac < 0$$

$$(-4)^2 - 4(p)(p-3) < 0$$

$$16 - 4p^2 + 12p < 0$$

$$-4p^2 + 12p + 16 < 0$$

$$p^2 - 3p - 4 > 0$$

$$(p-4)(p+1) > 0$$

$p < -1$ or $p > 4$
(NA)

4)

$$\text{Volume} = \left(\frac{x}{\sqrt{3}} + \frac{\sqrt{12}}{3}\right) \times 3$$

$$x\sqrt{45} = \frac{3x}{\sqrt{3}} + \sqrt{12}$$

$$3x\sqrt{5} - \frac{3x}{\sqrt{3}} = 2\sqrt{3}$$

$$x\left[3\sqrt{5} - \sqrt{3}\right] = 2\sqrt{3}$$

$$x = \frac{2\sqrt{3}}{(3\sqrt{5}-\sqrt{3})}\frac{(3\sqrt{5}+\sqrt{3})}{(3\sqrt{5}+\sqrt{3})}$$

$$= \frac{6\sqrt{15} + 6}{45 - 3}$$

$$= \frac{6\sqrt{15} + 6}{42}$$

$$= \frac{1}{7}\sqrt{15} + \frac{1}{7}$$

$\therefore a = 1,\ b = 15$ //

5i) $4x^3 + 16x^2 + 13x + 3 = (x-k)f(x)$

By observation,

$x = -3$

$x = -1/2$

$\therefore$ Since k is an integer,

$k = -3$

ii) $4x^3 + 16x^2 + 13x + 3 = (x+3)f(x)$

By synthetic division,

	4	16	13	3
$x=-3$		-12	-12	-3
	4	4	1	0

$f(x) = 4x^2 + 4x + 1$ }
At x-axis, $f(x) = 0$

$4x^2 + 4x + 1 = 0$

$b^2 - 4ac$

Since ... x-axis is tangent to curve

5iii) $4x^3+16x^2+13x+3=(x+3)f(x)$

$=(x+3)(4x^2+4x+1)$

$=(x+3)(2x+1)^2$

$4x^6+16x^4+13x^2+3=0$

$4(x^2)^3+16(x^2)^2+13x^2+3=0$

Replace x by x^2,

$x^2+3=0$ $\qquad 2(x^2)+1=0$

$x^2=-3$ $\qquad x^2=-1/2$

$\therefore$ There is no real soln.

6i) $(\operatorname{cosec}\theta-\cot\theta)(1+\sec\theta)$

$=\left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)\left(1+\frac{1}{\cos\theta}\right)$

$=\frac{(1-\cos\theta)(\cos\theta+1)}{(\sin\theta)(\cos\theta)}$

$=\frac{1-\cos^2\theta}{(\sin\theta)(\cos\theta)}$

$=\frac{\sin^2\theta}{(\sin\theta)(\cos\theta)}$

$=\tan\theta$ (shown).

6ii) $\underbrace{(\operatorname{cosec}\theta-\cot\theta)(1+\sec\theta)}=\frac{1}{2}\sec^2\theta$

$\tan\theta=\frac{1}{2}\sec^2\theta$

$\tan\theta=\frac{1}{2}(1+\tan^2\theta)$

Let $y=\tan\theta$,

$y=\frac{1}{2}(1+y^2)$

$2y=1+y^2$

$y^2-2y+1=0$

$(y-1)^2=0$

$y=1$

$\tan^2\theta=1$

$\tan\theta=\pm1$

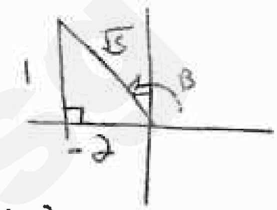
$\alpha=\tan^{-1}(1)$

$=\frac{\pi}{4}$

$\theta=\frac{\pi}{4}$ #

7) $\tan(2A+B)=4$

$\frac{\tan2A+\tan B}{1-\tan2A\tan B}=4$



$\frac{\tan2A+\left(-\frac{1}{2}\right)}{1+\frac{\tan2A}{2}}=4$

$\tan2A-\frac{1}{2}=4+2\tan2A$

$-4\frac{1}{2}=\tan2A$

$270\leq2A\leq360$ since.

$135°\leq A\leq180$ # $\qquad 0\leq B\leq90$

8i) By observation,

$p=3$.

$|1-2x|=0$

$x=1/2$

$q=1/2$.

iv) $c=3$ OR $c>4$

ii) $y=|1-2x|+3$

At A, $x=0$,

$y=|1-2(0)|+3$

$y=4$ $\qquad A(0,4)$ #.

iii) $y=|1-2x|+3$ —①

$y=3x$ —②

① = ②

$|1-2x|+3=3x$

$|1-2x|=3x-3$

$1-2x=3x-3$ OR $1-2x=3-3x$

$4=5x$ $\qquad -2=-x$

$x=4/5$ (NA). $\qquad y=6$ $(2,6)$

10) $y = 2\sqrt{e^x} + \dfrac{4}{\sqrt{e^x}} + x$

$y = 2e^{\frac{x}{2}} + \dfrac{4}{e^{x/2}} + x$

$y = 2e^{\frac{x}{2}} + 4e^{-\frac{x}{2}} + x$

$\dfrac{dy}{dx} = e^{\frac{x}{2}} - 2e^{-x/2} + 1$

At stationary point, $\dfrac{dy}{dx} = 0$

$e^{\frac{x}{2}} - 2e^{-\frac{x}{2}} + 1 = 0$

Let $y = e^{\frac{x}{2}}$,

$y - \dfrac{2}{y} + 1 = 0$

$y^2 - 2 + y = 0$

$y^2 + y - 2 = 0$

$(y+2)(y-1) = 0$

$y = -2 \quad y = 1$

$e^{\frac{x}{2}} = -2 \quad e^{\frac{x}{2}} = 1$

(NA). $\quad \frac{x}{2} = 0$

$x = 0$

$y = 2\sqrt{e^0} + \dfrac{4}{\sqrt{e^0}} + 0$

$= 2 + 4$

$y = 6$

$(0, 6)$

ii) $\dfrac{d^2y}{dx^2} = \frac{1}{2}e^{\frac{x}{2}} + e^{-\frac{x}{2}}$

At $x = 0$,

$\dfrac{d^2y}{dx^2} > 0$ (Minimum)

11) Point C $(0, y)$; $A(6,6)$. $B, (x, y)$

$AB = BC$

$\sqrt{(6-x)^2 + (6-y)^2} = \sqrt{20}$

$(6-x)^2 + (6-y)^2 = 20$ —— ①

$y + 2x = 18$

$y = 18 - 2x$ —— ②

Sub ② into ①,

$(6-x)^2 + (6-18+2x)^2 = 20$

$(6-x)^2 + (2x-12)^2 = 20$

$36 - 12x + x^2 + 4x^2 - 48x + 144 = 20$

$5x^2 - 60x + 160 = 0$

$(x-8)(x-4) = 0$

$x = 8$ OR $x = 4$

$y = 2 \qquad y = 10$

$(8, 2) \qquad (4, 10)$

(NA)

$\therefore B(4, 10)$

$C(0, 10)$

$A(6, 6)$

$m_{BC} = \dfrac{10-10}{4-0} = 0$.

l_{BC}. $y = 10$ #

ii) $M: \left(\dfrac{6+0}{2}, \dfrac{10+6}{2}\right)$

$= (3, 8)$

iii) $B(4, 10)$ #.

iv) Area of quadrilateral

$= \dfrac{1}{2}\begin{vmatrix} 0 & 3 & 4 & 0 & 0 \\ 0 & 8 & 10 & 10 & 0 \end{vmatrix}$

$= \frac{1}{2}|\ldots(32)| = \frac{1}{2}|38| = 19 \text{ units}^2$ #.

9) $\left(x-\frac{k}{x^2}\right)^7 = \binom{7}{0}(x)^7 + \binom{7}{1}(x)^6\left(\frac{-k}{x^2}\right) + \binom{7}{2}(x)^5\left(\frac{-k}{x^2}\right)^2 + \ldots$

$= x^7 + -7kx^4 + 21k^2x + \ldots$

ii)

$$T_{r+1} = \binom{7}{r}(x)^{7-r}\left(\frac{-k}{x^2}\right)^r$$

$$= \binom{7}{r}(-k)^r\left(\frac{x^{7-r}}{x^{2r}}\right)$$

$$= \binom{7}{r}(-k)^r\left(x^{7-3r}\right)$$

$x^{7-3r} = x^0$

$r = 7/3$

Since $r = 7/3 \neq$ integer, there is no term which is independent of x.

iii) $(2+x^3)\left(x-\frac{k}{x^2}\right)^7 = \ldots\ 7x^4 + \ldots$

$$T_{r+1} = \binom{7}{r}(x)^{7-r}\left(\frac{-k}{x^2}\right)^r$$

$$= \binom{7}{r}(-k)^r\left(\frac{x^{7-r}}{x^{2r}}\right)$$

$$T_{r+1} = \binom{7}{r}(-k)^r\left(x^{7-3r}\right)$$

$x^{7-3r} = x^0$

$7-3r = 0$

$r = 7/3$ (NA)

$x^{7-3r} = x^1$

$7-3r = 1$

$3r = 6$

$r = 2$

when $r = 2$,

$T_3 = \binom{7}{2}(-k)^2x^1$

$T_3 = (21k^2)x$

$(2+x^3)(\ldots 21k^2x + \ldots)$

$= 21k^2x^4$

By comparing coefficient of x^4:

$7 = 21k^2$

$\Rightarrow k = \sqrt{3}$ $k = -\sqrt{3}$ (NA).