2018

JC2 H1 Math

1.	Anglo Chinese Junior College	
2.	Catholic Junior College	
3.	Hwa Chong Institution	
4.	Meridian Junior College	
5.	Nanyang Junior College	
6.	National Junior College	
7.	Raffles Institution	
8.	River Valley High School	
9.	Saint Andrew's Junior College	
10.	Serangoon Junior College	
11.	Tampines Junior College	
12.	Temasek Junior College	
13.	Victoria Junior College	
14.	Yishun Junior College	

1 Find, algebraically, the set of exact values of m for which $3mx^2-24x+7m>0$ for all real values of x. [4]

2 Find

(i) $\displaystyle\int \frac{5}{\sqrt{3x-1}}\,\mathrm{d}x$, [2]

(ii) $\displaystyle\int \left(\frac{1}{\sqrt{x}}-x\right)^2 \mathrm{d}x$. [3]

3 A culvert is a tunnel structure constructed under roadways to provide cross drainage. One particular type is the low-profile arch culvert as shown in diagram 1. An engineer was tasked to build a low-profile arch culvert drain to improve the drainage system beneath a certain roadway.

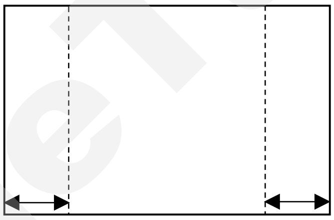


Diagram 1:
Low-profile arch culvert

Diagram 2:
Dimensions of cardboard

Diagram 3:
Cardboard model of the culvert

The engineer decided to use a rectangular piece of cardboard to build a model of the culvert. The rectangular piece of cardboard used is shown in diagram 2 with length $AD=10x$ cm and breadth $DE = y$ cm . The perimeter of this rectangular cardboard *ADEH* is 54 cm. You may assume that the cardboard is of negligible thickness.

To build the culvert model, the engineer needs to fold the cardboard along the dotted lines *BG* and *CF* to create the model as shown in diagram 3. The arch *BC* and *GF* are the semicircles with radius r cm.

(i) Using a non-calculator method, show that the radius of the culvert model in diagram 3 can be expressed as $r=\dfrac{8x}{\pi}$. [1]

(ii) Hence show that the volume of the space below the arch of the model is given by $V=\dfrac{32x^2}{\pi}(27-10x)$. [3]

(iii) Find the value of x for which V is a maximum. Justify that V is maximum for this value of x. [4]

4	**(i)** Find $\int_0^k 3x - e^{-x} + 2 \, dx$ in terms of k, where $k > 0$. [3] The curve C has equation $y = 3x - e^{-x} + 2$. **(ii)** Sketch the graph of C, stating the coordinates of any points of intersection with the axes. [1] **(iii)** Show that the equation of the tangent to the curve at $x = 1$ is $y = \left(3 + \frac{1}{e}\right)x - \frac{2}{e} + 2$. [4] **(iv)** Hence find the exact area enclosed by curve C, the y-axis and the tangent to C at the point where $x = 1$. [4]
5	The top-selling bread from a baker are the Pretzel, the Baguette and the Ciabatta loaf. The cost of making 7 pretzels is equal to the cost of making 6 ciabatta loaves. The total cost to make 50 baguette is \$16 less than the total cost to make 40 ciabatta loaves. The total cost to make 30 pretzels, 30 baguettes and 45 ciabatta loaves is \$123. **(i)** By writing down three linear equations, find the cost price of making each type of bread, correct to the nearest cent. [5] A nearby café decided to make a daily order for baguette from the baker. The baker decided that he will only accept the order if there are at least 16 baguettes ordered. In a simple model, the total manufacturing cost for x baguettes is given by this equation $$C = \frac{x}{3} - \ln(2x - 30) + 20,$$ where C is the manufacturing cost in dollars and x is the number of baguette ordered daily. **(ii)** Sketch the graph of C against x for $x > 15$. Estimate the minimum cost C and state the number of baguettes for which this minimum value occurs. [3] Suppose the baker wants to sell each baguette for \$1, and we let \$$P$ be the profit per baguette that the baker will earn. **(iii)** Formulate an equation relating P and x. [1] **(iv)** Using your formula in **(iii)**, would you advise the baker to accept an order of 20 baguettes? Justify your answer. [2]

6	Over a long period of time, a slimming centre found that at the end of a slimming programme, 20% of their clients lose more than 20 kg and 5% of their clients lose less than 10 kg. By modelling the weight loss of the slimming centre's clients to be a normal distribution, find the mean and variance of the distribution. [4]
7	A candy factory manufactured a large amount of pastilles daily and their candies are randomly packed in boxes of 20. The probability of selecting an orange-flavoured pastille to be packed into a box is 0.25. The random variable X is the number of orange-flavoured pastilles in a box of 20 pastilles. **(i)** State an assumption that is needed for X to be modelled by a binomial distribution. [1] **(ii)** Find the probability that a box of pastilles contains at most 8 orange-flavoured pastilles. [1] Jimmy likes the pastilles very much and would buy a box of pastilles each day from Monday to Friday. For a randomly selected week from Monday to Friday, find the probability that **(iii)** John gets exactly 3 boxes of pastilles that contain at most 8 orange-flavoured pastilles; [2] **(iv)** the box John gets on Friday is the third box of pastilles that contain at most 8 orange-flavoured pastilles. [2]
8	**(a)** There are 8 girls and 10 boys in a class. Find the number of ways to form a class committee consisting of exactly 2 girls and 3 boys. [2] **(b)** The digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 are arranged randomly to form 5-digit numbers. No digit is repeated. Find the number of 5-digit numbers **(i)** greater than 50000; [2] **(ii)** greater than 50000 and are odd. [3]
9	The masses, in grammes, of oranges and tangerines are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean mass	**Standard deviation**
Oranges	181 g	4.77 g

Tangerines	165.8 g	23.07 g

Let X represent the mass (in grammes) of a randomly selected orange.

Let Y represent the mass (in grammes) of a randomly selected tangerine.

(i) Find the probability that a randomly selected orange weighs more than 183 g. [1]

(ii) Juices obtained from squeezing an orange is only 40% of the weight of the orange squeezed while juices obtained from squeezing a tangerine is only 35% of the weight of the tangerine squeezed.

Find $\mathrm{P}\left(0.4\left[X_1+X_2+X_3+X_4\right]>0.35\left[Y_1+Y_2+Y_3+Y_4+Y_5\right]\right)$, stating the mean and variance of the distribution. Explain, in the context of this question, what your answer represents. [5]

(iii) There are many benefits to orange skins. Mass of orange skins peeled, in grammes, have a mean of 90 g and standard deviation of 4 g. In order to collect a large amount of orange skins, a person peeled 55 oranges. Find the probability that the total mass of orange skins he gathered exceeds 5000 g. [4]

10 The head of department for Mathematics believed that the students have done very well for a recent examination and claimed that the mean score for Mathematics for the entire Year 2 cohort is higher than 70. The mathematics score, x, of a random sample of 250 students are summarised as follows.

$$\sum(x-70)=305 \qquad \sum(x-70)^2=35565$$

(i) Calculate the unbiased estimates of the population mean and variance to the nearest 2 decimal places. [3]

(ii) Test at the 5% significant level whether the claim is justified. [4]

(iii) Another random sample of 300 students' scripts for Mathematics were marked. The sample mean score is 71.4 and the sample variance is k. Find the set of values of k for which the same claim is justified at 5% significant level, giving your answer correct to 2 decimal places. [4]

11 A marketing company wishes to investigate the relationship between the amount of time a person goes online while commuting back and forth between home and work. The marketing staff surveyed 10 people and the results are given in the following table.

Amount of commuting time in a week, x (hours)	5	7	9	10	9	12	10	11	8	13
Amount of time spent online in a week, y (hours)	2.5	4.5	6	6.5	7	9.5	9.5	7	6.5	10

(i) Draw a sketch of the scatter diagram for the data, as shown on your calculator. [1]

(ii) Find the product moment correlation coefficient and comment on its value in the context of this question. [2]

(iii) Find the equation of the regression line of y on x, in the form $y = mx + c$, giving the values of m and c correct to 4 decimal places. [1]

(iv) Explain the meaning of the value of m in the context of this question. [1]

(v) Using the equation of the regression line of y on x found in **(iii)**, to estimate the number of hours per week for a person to be online if he/she has 8.5 hours of commuting time. Comment on the reliability of your answer. [3]

The staff decided to survey one more person who has commuted 25 hours in a week and spent 8 hours online while commuting.

(vi) Calculate the new product moment correlation coefficient when this person is included. [1]

(vii) State, with a reason, which of your answers to parts **(ii)** and **(vi)** is a better representation of the correlation between the amount of time spent commuting back and forth between home and work in a week and the amount of time spend online in a week. [1]

12 In a particular year's O-level English results in Singapore, the candidates comprised Singaporeans, Malaysians and other foreigners. One of these candidates is chosen at random.

E is the event that the candidate scored distinction for English.

S is the event that the candidate is a Singaporean.

M is the event that the candidate is a Malaysian.

Given that $P(E) = 0.3$, $P(S) = 0.8$ and $P(E|S) = 0.2$.

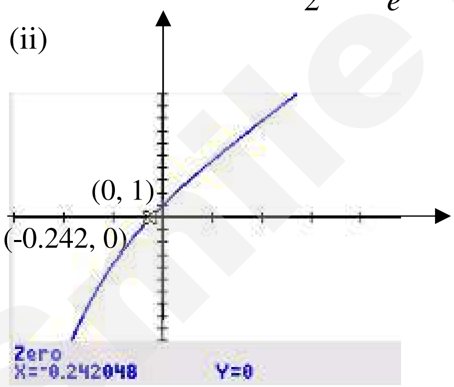
(i) Show that the events E and S are not independent. [1]

(ii) Find $P(E \cap S)$. [1]

	(iii) Find $P(E' \cap S')$.	[2]
	(iv) If $P(M) = 0.15$ and $P(E' \mid M) = 0.2$, find $P(E \cap M)$.	[2]
	(v) It is given that no candidate can hold multiple citizenship. By drawing a Venn diagram, find $P(E' \cap M' \cap S')$. Explain in the context of this question, what your answer represents.	[4]
	(vi) Two candidates from that year were randomly chosen to be interviewed on their experience back in school. Find the probability that one candidate is a Singaporean and the other is a Malaysian.	[2]

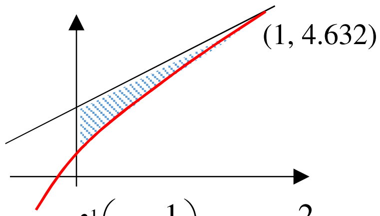
Anglo-Chinese Junior College
2018 H1 Mathematics Prelim Solution

Qn	Solution
1	Since $3mx^2-24x+7m>0$, $\therefore \quad m>0$ and $b^2-4ac<0$ Consider $b^2-4ac<0$ $(-24)^2-4(3m)(7m)<0$ $84m^2>576$ $84m^2-576>0$ $m^2-\frac{48}{7}>0$ $\left(m-\sqrt{\frac{48}{7}}\right)\left(m+\sqrt{\frac{48}{7}}\right)>0$ $m<-4\sqrt{\frac{3}{7}}$ or $m>4\sqrt{\frac{3}{7}}$ Since m>0, $\therefore\ m>4\sqrt{\frac{3}{7}}$
2	(i) $\int\frac{5}{\sqrt{3\text{x}-1}}\,dx=\frac{5\sqrt{3x-1}}{3\left(\frac{1}{2}\right)}+c=\frac{10\sqrt{3x-1}}{3}+c$ (ii) $\int\left(\frac{1}{\sqrt{x}}-x\right)^2dx=\int\frac{1}{x}-2\sqrt{x}+x^2\ dx$ $=\ln x-\frac{4x^{\frac{3}{2}}}{3}+\frac{x^3}{3}+c$
3	(i) With arch GF = 10x – 2x = 8x as the semicircle arch $8x=\left(\frac{1}{2}\right)2(\pi r)$ $\therefore r=\frac{8x}{\pi}$ (Shown) (ii) Since cardboard perimeter =54, $20x+2y=54$ $\therefore\ y=27-10x$ $V=\frac{1}{2}\pi r^2y$ $=\frac{1}{2}\pi\left(\frac{8x}{\pi}\right)^2(27-10x)$ $=\frac{32x^2}{\pi}(27-10x)$ shown

	(iii) $V=\frac{32x^2}{\pi}(27-10x)=\frac{864x^2-320x^3}{\pi}$ $\frac{dV}{dx}=\frac{1728x-960x^2}{\pi}$ $0=\frac{1728x-960x^2}{\pi}=\frac{192x(9-5x)}{\pi}$ x = 1.8 or 0 (rejected since x>0) <table><tr><td>x</td><td>x = 1.75</td><td>x = 1.8</td><td>x = 1.85</td></tr><tr><td>$\frac{dV}{dx}$</td><td>26.7</td><td>0</td><td>-28.3</td></tr></table> Or $\frac{d^2V}{dx^2}=\frac{1728-1920(1.8)}{\pi}=-550\ (3s.f)<0$ Therefore, volume is maximum when x = 1.8 cm.
4	(i) $\int_0^k 3x-e^{-x}+2\ dx=\left[\frac{3}{2}x^2+\frac{1}{e^x}+2x\right]_0^k$ $=\left(\frac{3}{2}k^2+\frac{1}{e^k}+2k\right)-\left(0+\frac{1}{e^0}\right)$ $=\frac{3}{2}k^2+\frac{1}{e^k}-1+2k$ (ii)  (iii) When $x=1$, $y=3-\frac{1}{e}+2=5-\frac{1}{e}$ $\frac{dy}{dx}=3+e^{-x}$. When $x=1$, $\frac{dy}{dx}=3+\frac{1}{e}$ Equation of tangent is $y-\left(5-\frac{1}{e}\right)=\left(3+\frac{1}{e}\right)(x-1)$

$$y=5-\frac{1}{e}+\left(3+\frac{1}{e}\right)x-3-\frac{1}{e}$$

$$y=\left(3+\frac{1}{e}\right)x+2-\frac{2}{e}$$



(iv) $$\int_0^1\left(3+\frac{1}{e}\right)x+2-\frac{2}{e}-\left(3x-e^{-x}+2\right)\,dx$$

$$=\int_0^1\left(3+\frac{1}{e}\right)x+2-\frac{2}{e}\,dx-\left[\frac{3}{2}x^2+\frac{1}{e^x}+2x\right]_0^1$$

$$=\left[\left(3+\frac{1}{e}\right)\frac{x^2}{2}+2x-\frac{2}{e}x\right]_0^1-\left(\frac{5}{2}+\frac{1}{e}\right)$$

$$=\left(\frac{7}{2}-\frac{3}{2e}\right)-\left(\frac{5}{2}+\frac{1}{e}\right)$$

$$=1-\frac{5}{2e}$$

5 (i) Let B represent cost price of a Pretzel
Let T represent cost price of a Baguette
Let C represent cost price of a Ciabatta loaf

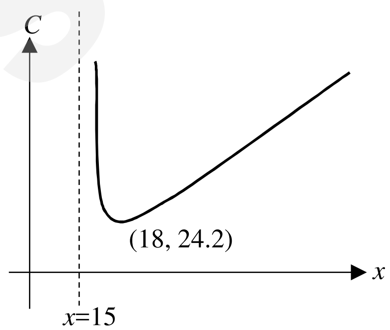
$7P=6C$ ....(1)

$50T-40C=16$ ....(2)

$30P+30T+45C=123$ ....(3)

By GC,
Cost price for one Pretzel is \$1.2
Cost price for one Baguette is \$0.80
Cost price for one Ciabatta is \$1.40

(ii)



Estimated minimum cost is \$24.21 and that occurs when there 18 baguettes.

(iii) $P=1-\dfrac{C}{x}=1-\dfrac{1}{x}\left(\dfrac{x}{3}-\ln(2x-30)+20\right)$

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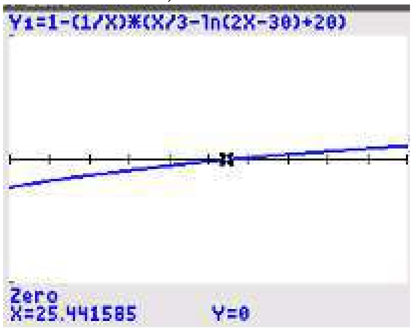
$P=\frac{2}{3}+\frac{1}{x}\ln(2x-30)-\frac{20}{x}$

(iv) From the GC,



As we can see from the graph, the profit is positive only when $x \geq 25.4$. Hence the baker will suffer a loss for an order of 20 baguettes, therefore not advisable to take the order.

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<tr><td>6</td><td>

Let random variable X be the amount of weight loss (in kg) of a customer at the end of a slimming programme.

Given $X \sim N(\mu,\sigma^2)$.

Given $P(X>20)=0.2$ and $P(X<10)=0.05$

$P(X>20)=0.2$

$\Leftrightarrow P(Z>\frac{20-\mu}{\sigma})=0.2$

$\Leftrightarrow \frac{20-\mu}{\sigma}=0.8416$

$\Leftrightarrow 20-\mu=0.8416\sigma$ (1)

$P(X<10)=0.05$

$\Leftrightarrow P(Z>\frac{10-\mu}{\sigma})=0.05$

$\Leftrightarrow \frac{10-\mu}{\sigma}=-1.645$

$\Leftrightarrow 10-\mu=-1.645\sigma$ (2)

(1) – (2), we have

$10=2.486\sigma$

$\sigma=4.02$ (3 sig. fig.)

Substitute $\sigma=4.02$ into (2), we have

$\mu=10+1.645\sigma=16.6$ (3 sig. fig.)

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<tr><td>7</td><td>

(i) The event that each pastilles packed into a box is orange-flavoured or not is independent of the flavour of any other pastille.

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	(ii) Let random variable X be the number of orange-flavoured pastilles in a box of 20 pastilles. $X \sim B(20, 0.25)$ $P(X \le 8) = 0.959$ (iii) Let random variable Y be the number of boxes of pastilles that contain at most 8 orange-flavoured pastilles out of 5 boxes. $Y \sim B(5, 0.959)$ $P(Y = 3) = 0.0148$ (iv) Let random variable W be the number of boxes of pastilles that contain at most 8 orange-flavoured pastilles out of 4 boxes. $W \sim B(4, 0.959)$ A = {2 boxes out of 4 boxes John gets from Monday to Thursday contain at most 8 orange-flavoured pastilles} B = {the box John gets on Friday contains at most 8 orange-flavoured pastilles} $P(A \cap B)$ $= P(A).P(B)$ $= P(W = 2)(0.959)$ $= 0.00887$ (3 sig. fig.)
8	(a) Step 1: Choose 2 girls from 8 girls: ${}^{8}C_{2}$ Step 2: Choose 3 boys from 10 boys: ${}^{10}C_{3}$ Ans: ${}^{8}C_{2} \times {}^{10}C_{3} = 3360$ (b)(i) Step 1: Put one of the digits 5, 6, 7, 8, 9 as the leftmost digit: 5 Step 2: Arrange 4 digits from the remaining 8 digits to the right of the leftmost digit: ${}^{8}P_{4}$ Ans: $5 \times {}^{8}P_{4} = 8400$ (ii) Case 1: Leftmost digit is 5, 7 or 9 Step 1: Put one of the 3 digits 5, 7 or 9 as the leftmost digit: 3 Step 2: Put one of the remaining 4 odd digits as the rightmost digit: 4 Step 3: Arrange 3 digits from the remaining 7 digits between the leftmost and the rightmost digits: ${}^{7}P_{3}$

Case 2: Leftmost digit is 6 or 8

Step 1: Put one of the 2 digits 6 or 8 as the leftmost digit: 2
Step 2: Put one of the 5 odd digits (1, 3, 5, 7 or 9) as the rightmost digit: 5
Step 3: Arrange 3 digits from the remaining 7 digits between the leftmost and the rightmost digits: ${}^{7}P_{3}$

Ans: $3\times4\times{}^{7}P_{3}+2\times5\times{}^{7}P_{3}=4620$

9

Let random variable X be the mass (in grammes) of an orange.
Let random variable Y be the mass (in grammes) of an orange.
Given $X\sim(181,77^2)$ and $Y\sim(165.8,23.07^2)$

(i) $P(X>183)=0.338$ (3 sig. fig.)

(ii)

$$P(0.4(X_1+\ldots+X_4)>0.35(Y_1+\ldots+Y_5))$$
$$=P(0.4(X_1+\ldots+X_4)-0.35(Y_1+\ldots+Y_5)>0)$$
$$E(0.4(X_1+\ldots+X_4)-0.35(Y_1+\ldots+Y_5))$$
$$=(0.4)(4)E(X)-(0.35)(5)E(Y)$$
$$=(0.4)(4)(181)-(0.35)(5)(165.8)$$
$$=-0.55$$

$$Var(0.4(X_1+\ldots+X_4)-0.35(Y_1+\ldots+Y_5))$$
$$=(0.4^2)(4)Var(X)-(0.35^2)(5)Var(Y)$$
$$=(0.4^2)(4)(4.77^2)-(0.35^2)(5)(23.07^2)$$
$$=340.5496073$$
$$0.4(X_1+\ldots+X_4)-0.35(Y_1+\ldots+Y_5)\sim N(-0.55,340.55)$$
$$P(0.4(X_1+\ldots+X_4)-0.35(Y_1+\ldots+Y_5)>0)=0.488$$

The answer is the probability that the total weight of orange juice squeezed from 4 oranges is more than the total weight of tangerine juice squeezed from 5 tangerines.

(iii) Let random variable W be the mass of orange skins peeled (in grammes).
Given $E(W)=90$ and $Var(W)=4^2$.
Let $T=W_1+\ldots+W_5$
Since n = 55 is large, by Central Limit Theorem, T is approximately normally distributed.

$$E(T)=55E(W)=55(90)=4950$$
$$Var(T)=55Var(W)=55(4^2)=880$$
$$T\sim N(4950,880)$$
$$P(T>5000)\approx0.0459$$

10

Let random variable X be the Mathematics scores for Year 2 cohort.

Given n = 250, $\sum(x-70)=305$, $\sum(x-70)^2=35565$.

(i) $\bar{x} = \frac{\sum(x-70)}{n} + 70 = \frac{305}{250} + 70 = 71.22$ (2 dec. places)

Unbiased estimate of population variance:

$$s^2 = \frac{n}{n-1}\left(\frac{\sum(x-70)^2}{n} - \left(\frac{\sum(x-70)}{n}\right)^2\right)$$

$$= \frac{250}{249}\left(\frac{35565}{250} - \left(\frac{305}{250}\right)^2\right)$$

$= 141.3369478$

≈ 141.34 (correct to the nearest 2 decimal places)

(ii) Let $\bar{X} = \frac{X_1 + \ldots + X_n}{n}$

Test H_0: $\mu = 70$

against H_1: $\mu > 70$

at 5% level of significance.

Critical Region: $z > 1.645$

Under H_0, $\bar{X} \sim N(70, \frac{141.34}{250})$ approximately by Central Limit Theorem, since $n = 250$ is large.

Test Statistics: $Z = \frac{\bar{X} - 70}{\sqrt{\frac{141.34}{250}}}$

where $Z \sim N(0,1)$ approximately.

Test Value: $z = \frac{\bar{x} - 70}{\sqrt{\frac{141.34}{250}}} = \frac{71.22 - 70}{\sqrt{\frac{141.34}{250}}} = 1.62 < 1.645$

(z is not in critical region).

p-value: $p = 0.0523 > 0.05$

Do not reject H_0.

Conclusion: There is insufficient evidence at 5% level of significance to conclude that the mean scores of the Year 2 cohort (population) for Mathematics is more than 70.

(iii) Given $\bar{x} = 71.4$

Test H_0: $\mu = 70$

against H_1: $\mu > 70$

at 5% level of significance.

Critical Region: $z > 1.645$

Under H_0, $\bar{X} \sim N(70, \frac{k}{299})$ approximately by Central Limit Theorem, since $n = 250$ is large.

Test Statistics: $Z = \frac{\bar{X} - 70}{\sqrt{\frac{k}{299}}}$

where $Z \sim N(0,1)$ approximately.

Test Value: $z = \dfrac{\bar{x}-70}{\sqrt{\dfrac{k}{299}}} = \dfrac{71.4-70}{\sqrt{\dfrac{k}{299}}} = (1.4)\sqrt{\dfrac{299}{k}}$

To reject H_0 , z must be in critical region.

$z > 1.645$

$(1.4)\sqrt{\dfrac{299}{k}} > 1.645$

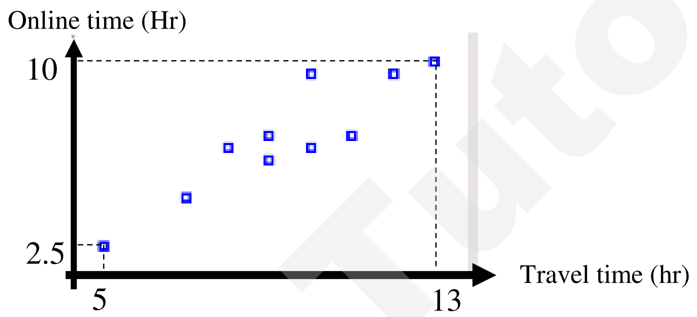
$\sqrt{k} < \dfrac{1.4\sqrt{299}}{1.645}$

$k < \left(\dfrac{1.4}{1.645}\right)^2 (299)$

$k < 216.6071293$

$k < 216.61$

11 (i)



(ii) $r = 0.909865 = 0.910$ (3s.f)

There is a strong, postive linear correlation between the commuting time and the time spent online. As the amount of time spend commuting increases, the amount of time spent online increases

(iii) Least square regression line is

$y = -1.5675 + 0.9008x$ (4d.p)

(iv) $m = 0.9008$ means that for a one hour increase in the travelling time there is an estimated 0.9008 hour increase in time spent online.

(v) $y = -1.5675 + 0.9008\ (8.5) = 6.09$ hr of online time. (3s.f)

Reliable because $x = 8.5$ is within the data range of x (interpolation) and also $r = 0.910$ indicates that there is a strong and positive linear correlation between x and y.

(vi) New r = 0.52082 = 0.521(3s.f)

(vii) The answer in (ii) is more likely to represent the amount of online time while commuting back and forth to work in a week because the r = 0.910 is closer to 1 and shows a stronger positive linear correlation between commuting time and online time.

12

Let the events E, S and M be defined as follow:

E = {a randomly selected candidate scored distinction for English}

S = {a randomly selected candidate is a Singaporean}

M = {a randomly selected candidate is a Malaysian}

Given $P(E) = 0.3$, $P(S) = 0.8$, $P(E|S) = 0.2$.

(i) $P(E|S) = 0.2$ and $P(E) = 0.3$

$P(E|S) \neq P(E)$

Hence events E and S are not independent.

OR

$P(E \cap S) = P(E|S)P(S) = (0.2)(0.8) = 0.16$

$P(E)P(S) = (0.3)(0.8) = 0.24 \neq 0.16$

$\therefore P(E \cap S) \neq P(E)P(S)$

Hence events E and S are not independent.

(ii) $P(E \cap S) = P(E|S)P(S) = (0.2)(0.8) = 0.16$

(iii) $P(E' \cap S') = P[(E \cup S)'] = 1 - P(E \cup S)$

$P(E \cup S)$

$= P(E) + P(S) - P(E \cap S)$

$= 0.3 + 0.8 - 0.16$

$= 0.94$

$P(E' \cap S') = 1 - 0.94 = 0.06$

(iv) $P(M) - 0.15$

$P(E'|M) = 0.2$

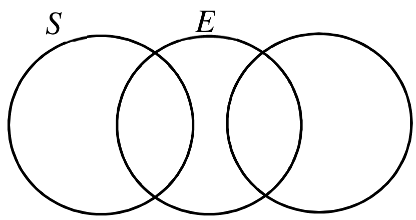
$P(E' \cap M) = P(E'|M)P(M) = 0.15 \times 0.2 = 0.03$

$P(E \cap M) = P(M) - P(E' \cap M)$

$= 0.15 - 0.03$

$= 0.12$

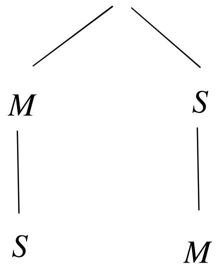
(v)



$P(E' \cap M' \cap S')$
$= 1 - P(E \cup M \cup S)$
$= 1 - 0.97$
$= 0.03$

The answer represents the probability that a randomly selected candidate is a non-Malaysian foreigner (alternatively not a Singaporean nor a Malaysian) and did not score distinction for English.

(vi) $P(S) = 0.8$ and $P(M) = 0.15$



P(one randomly selected candidate is a Singaporean and the other randomly selected candidate is a Malaysian)
= (2)(0.8)(0.15)
= 0.24

Section A: Pure Mathematics [40 marks]

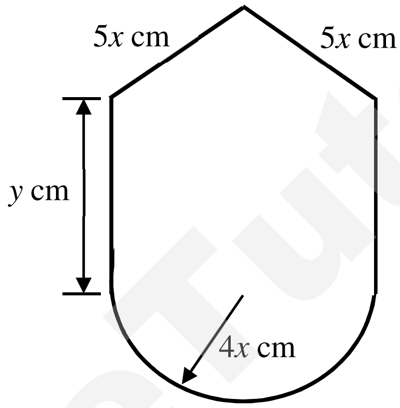
1 **(a)** Find the range of values of c for which the expression x^2+4x+c is always positive for all real values of x. [2]

(b) Show that the roots of the equation $x^2+(1-k)x-k=0$ are real for all real values of k. [2]

2 **(a)** Differentiate $\dfrac{1}{\sqrt{(1-3x)^3}}$ with respect to x. [2]

(b) Use a non-calculator method to find the exact value of $\int_1^9 \dfrac{\left(3-2\sqrt{x}\right)^2}{x^2}\,dx$. [4]

3



A piece of wire of length 80 cm is bent to form the shape shown in the diagram. This shape consists of a semi-circular arc, radius $4x$ cm, two sides of length y cm of a rectangle and two equal sides of an isosceles triangle.

(i) Express y in terms of x. [2]

(ii) Hence, show that the total area enclosed, A cm^2, is such that

$$A=320x-8\pi x^2-28x^2.$$ [3]

(iii) Use a non-calculator method to find the value of x for which A has a stationary value, giving your answer in terms of π. [2]

(iv) Determine whether this value of x makes A a maximum or a minimum. [1]

4 **(a)** The owner of Nutt's Nuts regularly sells cashews for a profit of \$1.50 per kg, pecans for \$1.80 per kg and peanuts for \$0.80 per kg. How many kg of cashews, pecans and pecans should be mixed to obtain a mixture of 100 kg that will sell at \$1.25 per kg so that profit remains the same, given that it is known that the mixture contains five times as many cashews as pecans? [4]

(b) The owner of Nutt's Nuts is also into the construction business. He has a factory capable of producing up to 60 bedroom suites per month. The total cost of materials and labour needed to make the suites is $\left(50x+2x\sqrt{x}\right)$ dollars, where $x>0$.

In addition, there are fixed costs of \$1200 per month.

If x suites are made per month and each suite is sold for $\left(71+\frac{1800}{x}\right)$ dollars,

(i) Determine the manufacturer's monthly profit, $P(x)$. [2]

$[\text{Profit} = \text{selling price} - \text{manufacturing cost}]$

(ii) How many suites should be made and sold each month to maximise profits? [1]

(iii) What is the maximum profit? [1]

5 **(a)** Find the gradient of the curve $y = 3\ln x - \ln 2$ at the point where $y = \ln 32$. [3]

(b) The curve C with equation $y = 3 - e^{2x}$ meets the x-axis at A and the y-axis at B.

(i) Find the exact coordinates of A and B. [2]

(ii) Sketch the curve, stating the equation of any asymptotes. [2]

(iii) Find the equation of the tangent to the curve at the point where $x = \ln 2$, giving your answer in the form $y = ax + b\ln 2 + c$, where a, b and c are integers. [4]

(iv) Find the area bounded by C and the curve $y = 1 - \ln(x+2)$, giving your answer to 1 decimal place. [3]

Section B: Probability and Statistics [60 marks]

6 A random variable X is normally distributed with a mean of μ and a variance of σ^2. Given that $\mathrm{P}(X > 2) = 0.1$ and that $\mathrm{P}(X < 1.75) = 0.18$, find $\mathrm{P}(X > 1.90)$. [4]

7 On average 7% of a certain brand of kitchen lights are faulty. The lights are sold in boxes of 12.

(i) State, in context, two assumptions needed for the number of faulty lights in a box to be well modelled by a binomial distribution. [2]

Assume now that the number of faulty lights in a box has a binomial distribution.

(ii) Find the probability that a box of 12 of these kitchen lights contains at least 1 faulty light. [1]

The boxes are packed into cartons. Each carton contains 20 boxes.

(iii) Find the probability that each box in one randomly selected carton contains at least one faulty light. [1]

(iv) Find the probability that there are at least 20 faulty lights in a randomly selected carton. [2]

(v) Explain why the answer to part **(iv)** is greater than the answer to part **(iii)**. [1]

8 Three-figure numbers are to be formed from the digits 4, 5, 6, 7, 8, 9.

(i) How many different three-figure numbers can be formed
(a) if repetitions are not allowed? [1]
(b) if repetitions are allowed? [1]

Now suppose that repetitions of the digits are allowed.

(ii) Find the probability that a three-digit number chosen at random

(a) contains digits that appear no more than once [2]
(b) has at least one repeated digit but no digit appears more than twice in the number. [2]

9 A rubber seal is fitted to the bottom of a flood barrier. When no pressure is applied, the depth of the seal is 15 cm. When pressure is applied, a watertight seal is created between the flood barrier and the ground.

The table shows the pressure, x kilopascals (kPa), applied to the seal and the resultant depth, y centimetres (cm), of the seal.

x	25	50	75	100	125	150	175	200	250	300
y	14.7	13.4	12.8	11.9	11.0	10.3	9.7	9.0	7.5	6.7

(i) Draw a sketch of the scatter diagram for the data, as shown on your calculator. [2]

(ii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iii) Find the equation of the regression line of y on x, in the form $y = a + bx$. Sketch this line on your scatter diagram. [2]

(iv) Use the equation of your regression line to calculate an estimate of the depth of the seal when it is subjected to a pressure of 225 kPa, giving your answer to one decimal place. [1]

(v) Give two reasons based on the context of this question as to why your equation will not give a realistic estimate of the depth of the seal if it were to be subjected to a pressure of 525 kPa. [2]

10 A golfer practices her driving every morning of her holiday. The lengths (in metres) of a random sample of 40 of her drives at the start of her holiday are measured. The unbiased estimate for the population mean and variance are 182.4 and 729. The population mean length of her drives is denoted by μ. The null hypothesis is $\mu = 190$ is to be tested against the alternative hypothesis $\mu < 190$.

(i) Find the p-value of the test and state the meaning of this p-value in context. [2]

(ii) If the level of significance is $\alpha\%$, find the set of values of α for which the null hypothesis is rejected. [1]

The golfer hopes that her continued practice will increase the mean length of her drives. The lengths (in metres) of a random sample of 60 of her drives at the end of her holiday are summarised by

$$\sum y = 11631 \quad \text{and} \quad \sum \left(y - \bar{y}\right)^2 = 32819.$$

(iii) Find unbiased estimates of the population mean and variance. [3]

(iv) Test whether there is significant evidence, at the 5% level, that the golfer's hope is justified. [4]

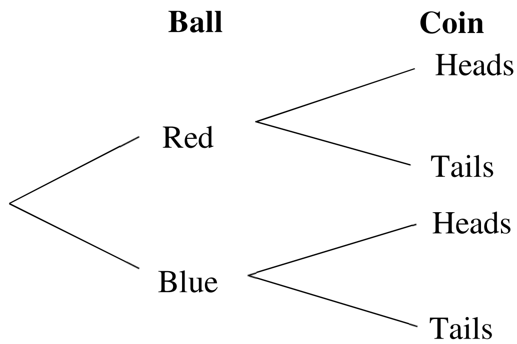
11 **(a)** State in words the relationship between two events R and S when $\mathrm{P}\left(R \cap S\right) = 0$. [1]

(b) The events A and B are independent with $\mathrm{P}\left(A\right) = \frac{1}{4}$ and $\mathrm{P}\left(A \cup B\right) = \frac{2}{3}$. Find $\mathrm{P}\left(B\right)$. [3]

(c) An experiment consists of selecting from a bag and spinning a coin. The bag contains 5 red balls and 7 blue balls. A ball is selected at random from the bag, its colour is noted and then the ball is returned to the bag.

When a red ball is selected, a biased coin with probability $\frac{2}{3}$ of landing heads is spun. When a blue ball is selected a fair coin is spun.

(i) Copy and complete the tree diagram below to show the possible outcomes and associated probabilities. [2]



John selects a ball and spins the appropriate coin.

(ii) Find the probability that he obtains a head. [2]

Given that Mary selected a ball at random and obtained a head when she spun the appropriate coin,

(iii) find the probability that Mary selected a red ball. [3]

John and Mary each repeat this experiment.

(iv) Find the probability that the colour of the ball John selects is the same as the colour of the ball Mary selects. [3]

12 The weights of the oranges sold on a market stall are normally distributed with mean 0.4 kg and standard deviation 0.06 kg. The weights of the lemons sold on the market stall are normally distributed with mean weight 0.1 kg and standard deviation 0.05 kg.

State clearly the mean and variance of the distribution that you use in all the questions below.

(i) Find the value that is exceeded by 75% of the weights of the lemons. [1]

(ii) Ann buys 1 orange and 1 lemon. Calculate the probability that the weight of her orange is more than three times the weight of her lemon. [3]

The selling price of the oranges and lemons are \$2.40 per kilogram and \$1.50 per kilogram respectively.

Candy buys 7 oranges and 20 lemons.

(iii) Find the probability that the total weight of the oranges differ from the total weight of the lemons by less than 1 kg. [3]

(iv) Find the probability that Candy has to pay more than \$10 for her purchase. [3]

End of Paper

AJC 2018 H1 Math JC2 Prelim Solutions

<table>
<tr><td colspan="3">Section A: Pure Mathematics [40 marks]</td></tr>
<tr><td>1</td><td>(a) $x^2+4x+c>0$ for all real values of x
Discriminant $=4^2-4c<0$
$\Rightarrow 4-c<0$
$\Rightarrow c>4$

(b) $x^2+(1-k)x-k=0$
Discriminant $=(1-k)^2-4(-k)$
$=1-2k+k^2+4k$
$=k^2+2k+1$
$=(k+1)^2$
$(k+1)^2\geq 0$ for all real values of k; hence the roots are real for all real values of k</td><td></td></tr>
<tr><td>2</td><td>(a) Let $y=\dfrac{1}{\sqrt{(1-3x)^3}}=(1-3x)^{-\frac{3}{2}}$
$\dfrac{\mathrm{dy}}{\mathrm{d}x}=-\frac{3}{2}(1-3x)^{-\frac{5}{2}}(-3)=\frac{9}{2}(1-3x)^{-\frac{5}{2}}$

(b) $\displaystyle\int_1^9\frac{\left(3-2\sqrt{x}\right)^2}{x^2}\,\mathrm{d}x$
$\displaystyle=\int_1^9\frac{9-12\sqrt{x}+4x}{x^2}\,\mathrm{d}x$
$\displaystyle=\int_1^9 9x^{-2}-12x^{-\frac{3}{2}}+\frac{4}{x}\,\mathrm{d}x$
$\displaystyle=\left[-\frac{9}{x}+\frac{24}{\sqrt{x}}+4\ln x\right]_1^9$
$\displaystyle=\left(-\frac{9}{9}+\frac{24}{3}+4\ln 9\right)-\left(-\frac{9}{1}+\frac{24}{1}+4\ln 1\right)$
$=(7+4\ln 9)-(15)$
$=4\ln 9-8$</td><td></td></tr>
<tr><td>3</td><td>(i) $2y+10x+4\pi x=80\Rightarrow y+5x+2\pi x=40$
$\therefore y=40-5x-2\pi x$

(ii) $A=(\text{area of isosceles }\Delta)+(\text{area of }\cdots)+(\text{area of }\cdots)$
$=\frac{1}{2}(8x)(3x)+8xy+\frac{1}{2}\pi(4x)^2$
$=12x^2+8xy+8\pi x^2$</td><td></td></tr>
</table>

$= 12x^2 + 8x(40 - 5x - 2\pi x) + 8\pi x^2$

$= 12x^2 + 320x - 40x^2 - 16\pi x^2 + 8\pi x^2$

$= 320x - 28x^2 - 8\pi x^2 \quad$ (shown)

(c) $\dfrac{\mathrm{d}A}{\mathrm{d}x} = 320 - 16\pi x - 56x = 0 \Rightarrow x = \dfrac{320}{16\pi + 56} = \dfrac{40}{2\pi + 7}$

(d)

x	2.9	$\dfrac{40}{2\pi+7}$	3.1
$\dfrac{\mathrm{d}A}{\mathrm{d}x}$	+11.8	0	−9.42

$\therefore$ A is a maximum.

4 **(a)** Let $x =$ no. of kg of cashews in the mixture
$y =$ no. of kg of pecans in the mixture
$z =$ no. of kg of peanuts in the mixture

$x + y + z = 100 \quad \cdots\cdots(1)$

$1.5x + 1.8y + 0.8z = 125 \quad \cdots\cdots(2)$

$x = 5y \Rightarrow x - 5y = 0 \quad \cdots\cdots(3)$

Solving system of linear equations by GC, $x = 50,\ y = 10,\ z = 40$.

(b) **(i)** $P(x) = x\left(71 + \dfrac{1800}{x}\right) - \left(50x + 2x\sqrt{x}\right) - 1200$

$= 71x + 1800 - 50x - 2x\sqrt{x} - 1200$

$= 21x - 2x\sqrt{x} + 600$

(ii) $x = 49$

(iii) Maximum profit = 943 dollars

5 **(a)** At the point where $y = \ln 32$,

$\ln 32 = 3\ln x - \ln 2 \Rightarrow 3\ln x = \ln 32 + \ln 2 = \ln 64$

$\ln x^3 = \ln 64$

$x^3 = 64 \Rightarrow x = 4$

$y = 3\ln x - \ln 2 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3}{x}$

At the point where $y = \ln 32$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3}{4}$

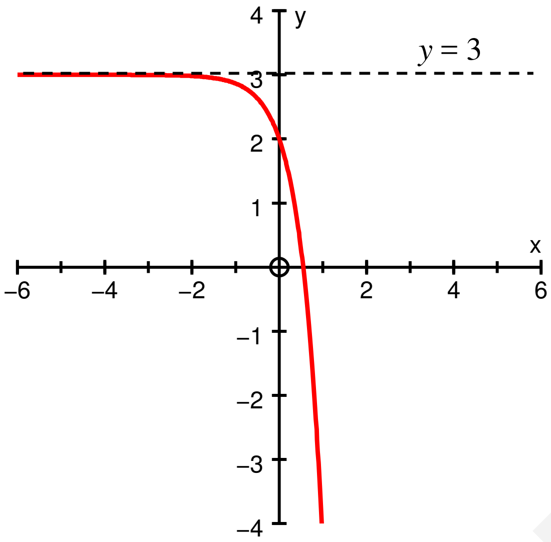
(b) **(i)** At point A, $y = 0$, $e^{2x} = 3 \Rightarrow x = \frac{1}{2}\ln 3$

$\therefore A = \left(\frac{1}{2}\ln 3, 0\right)$

At point B, $x = 0$, $y = 3 - e^0 = 2$

$\therefore B = (0, 2)$

(ii)



(iii) $y = 3 - e^{2x} \Rightarrow \dfrac{dy}{dx} = -2e^{2x}$.

At $x = \ln 2$, $\dfrac{dy}{dx} = -2e^{2\ln 2} = -2e^{\ln 4} = -2(4) = -8$

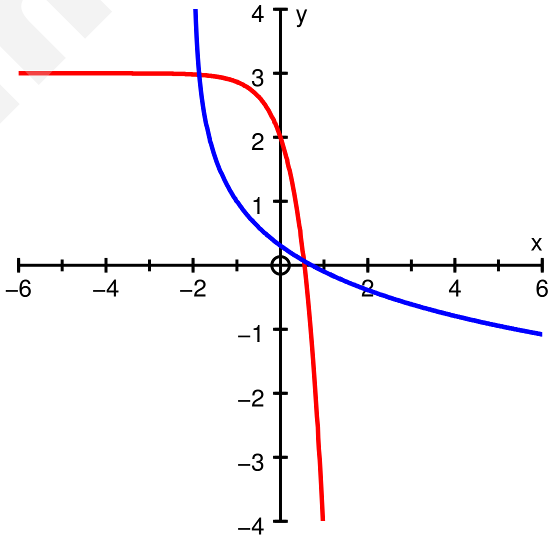
At $x = \ln 2$, $y = 3 - e^{2\ln 2} = 3 - e^{\ln 4} = 3 - 4 = -1$

Equation of tangent at $(\ln 2, -1)$ is:

$y - (-1) = -8(x - \ln 2)$

$y = -8x + 8\ln 2 - 1$

(iv)

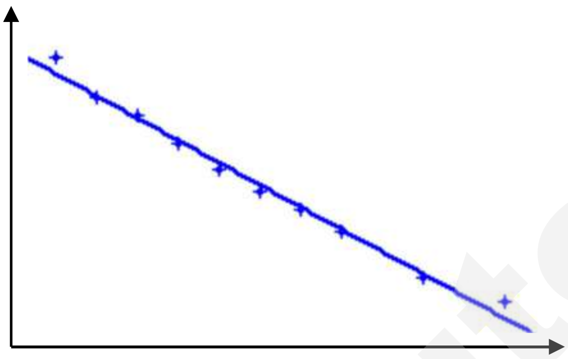


The two graphs intersect at $x = -1.8614$ and $x = 0.53772$

Area bounded $= \int_{-1.8614}^{0.53772} (3 - e^{2x}) - (1 - \ln(x+2))\, dx = 3.6$

Section B: Probability and Statistics [60 marks]

6	$X \sim \mathrm{N}\left(\mu, \sigma^2\right)$ Given $\mathrm{P}(X > 2) = 0.1$ Standardising, $\mathrm{P}\left(Z > \dfrac{2-\mu}{\sigma}\right) = 0.1$ $\dfrac{2-\mu}{\sigma} = 1.2816 \Rightarrow \mu + 1.2816\sigma = 2 \quad (1)$ $\mathrm{P}(X < 1.75) = 0.18$ Standardising, $\mathrm{P}\left(Z < \dfrac{1.75-\mu}{\sigma}\right) = 0.18$ $\dfrac{1.75-\mu}{\sigma} = -0.91537 \Rightarrow \mu - 0.91537\sigma = 1.75 \quad (2)$ Solving (1) & (2): $\mu = 1.8542$ and $\sigma = 0.11379$ $\mathrm{P}(X > 1.90) = 0.344$	
7	**(i)** (1) The outcome of each kitchen light being faulty or not must be independent of any other lights; (2) the probability of any one light is faulty is the same throughout the 12 lights in a box. **(ii)** Let X = no. of kitchen lights that are faulty in a box of 12. $X \sim \mathrm{B}(12, 0.07)$ $\mathrm{P}(X \geq 1) = 1 - \mathrm{P}(X = 0) = 0.58140 = 0.581$ **(iii)** Let Y = no. of boxes in a carton with at least 1 faulty light $Y \sim \mathrm{B}(20, 0.58140)$ $\mathrm{P}(Y = 20) = 0.0000195$ **(iv)** Let T = no. of faulty lights in a carton $T \sim \mathrm{B}(240, 0.07)$ $\mathrm{P}(T \geq 20) = 1 - \mathrm{P}(T \leq 19) = 0.241$ **(v)** The event that there are at least 20 faulty lights in a carton can happen if one box contains at least 1 faulty light or it can happen in many other ways, e.g. 18 boxes with at least 1 faulty light each and one box with at least 2 faulty lights and one box with none. Hence, this explains why the answer to part **(iv)** is greater than the answer to part **(iii)**.	
8	**(i) (a)** No. of different three-figure numbers if repetitions are not allowed $= 6 \times 5 \times 4 = 120$ **(b)** No. of different three-figure numbers if repetitions are allowed $= 6^3 = 216$ **(ii)(a)** P(three-digit numbers which contain digits that appear no more than once)	

	$=\frac{^6P_3}{216}=\frac{120}{216}=\frac{5}{9}$ **(b)** P(three-digit numbers has at least one repeated digit but no digit appears more than twice in the number) $=\frac{^6C_2\times{}^2C_1\times 3}{216}$ or $\frac{6\times5\times3}{216}=\frac{5}{12}$	
9	**(i)**  **(ii)** $r=-0.994$. As r is close to -1 , this suggests that there is a strong negative and linear relationship between the pressure, x kilopascals (kPa), applied to the seal and the resultant depth, y centimetres, of the seal. **(iii)** $y=14.9097-0.0290323x \Rightarrow y=14.9-0.0290x$ **(iv)** When $x=225,\ \ y=14.9097-0.0290323(225)=8.377...=8.4$ **(v)** The estimate of the depth of the seal if it were to be subjected to a pressure of 525 kPa is not reliable because *525 kPa is outside the data range used to form the regression line* (or *it is an extrapolation*). Another reason is because $y=14.9097-0.0290323(525)=-0.332$ when $x=525$ is not possible at all.	
10	**(i)** p-value $=0.0375$. The ***p*-value** is the probability of obtaining a sample mean length less than the observed sample value of 182.4 m given the null hypothesis is true. [or $\mathrm{P}\left(\bar{X}<182.4 \mid H_0 \text{ is true}\right)$ where X is the random variable length of a drive] **(ii)** For null hypothesis to be rejected, p-value $=0.0375<\frac{\alpha}{100}$, i.e. $\alpha>3.75$. **(iii)** Unbiased estimate of the population mean $=\frac{11631}{60}=193.85$ Unbiased estimate of the population variance	

$= \dfrac{1}{59}(32819) = 556.25 = 556$ (3 s.f.)

(iv) $\boldsymbol{Y}$ = length of a drive at the end of golfer's holiday

$H_0 : \mu = 190$ and $H_1 : \mu > 190$

One-tail test at 5% level of significance.

If H_0 is true, then $\bar{Y} \sim N\left(190, \dfrac{\sigma^2}{20}\right)$

Since σ^2 is unknown, we use the unbiased estimate 556.19.

Therefore, $\bar{Y} \sim N\left(190, \dfrac{556.25}{60}\right)$ approximately

Given the observed sample mean $\bar{y} = 193.85$, $p-\text{value} = 0.103$.

Since p-value > 0.05, H_0 is not rejected.

There is no significant evidence at 5% significance level to say that the golfer' hope is justified.

11

(a) R and S are mutually exclusive.

(b) Events A and B are independent, so $P(A \cap B) = P(A) \times P(B) = \dfrac{1}{4} \times P(B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

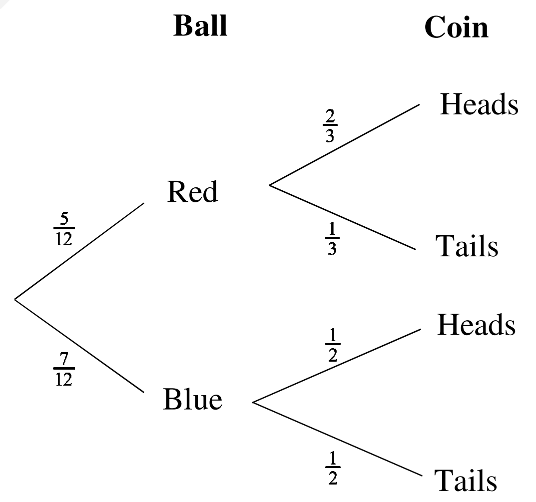
$$\frac{2}{3} = \frac{1}{4} + P(B) - \frac{1}{4} \times P(B)$$

$$\frac{2}{3} = \frac{1}{4} + \frac{3}{4} \times P(B)$$

$$\frac{3}{4} \times P(B) = \frac{5}{12}$$

$$P(B) = \frac{5}{9}$$

(c) **(i)**



(ii) $P(\text{John obtains a head}) = \dfrac{5}{12} \times \dfrac{2}{3} + \dfrac{7}{12} \times \dfrac{1}{2} = \dfrac{41}{72}$ or 0.569

	(iii) $P(\text{Mary selected a red ball} \mid \text{Mary obtained a head})$ $= \dfrac{P(\text{Mary selected a red ball and obtained a head})}{P(\text{Mary obtained a head})}$ $= \dfrac{\frac{5}{12} \times \frac{2}{3}}{\frac{41}{72}} = \dfrac{20}{41}$ or 0.488 **(iv)** $P(\text{John and Mary selected the same coloured ball})$ $= \left(\dfrac{5}{12} \times \dfrac{2}{3}\right)^2 + \left(\dfrac{5}{12} \times \dfrac{1}{3}\right)^2 + 2\left(\dfrac{5}{12} \times \dfrac{2}{3}\right)\left(\dfrac{5}{12} \times \dfrac{1}{3}\right) + 4\left(\dfrac{7}{12} \times \dfrac{1}{2}\right)^2$ $= \dfrac{25}{324} + \dfrac{25}{1296} + \dfrac{25}{324} + \dfrac{49}{144}$ $= \dfrac{37}{72}$ or 0.514 OR $= \left(\dfrac{5}{12}\right)^2 + \left(\dfrac{7}{12}\right)^2 = \dfrac{37}{72}$	
12	Let G = weight, in kg, of an orange $\sim N(0.4, 0.06^2)$ L = weight, in kg, of a lemon $\sim N(0.1, 0.05^2)$ **(i)** $P(L > a) = 0.75 \Rightarrow a = 0.0663$ **(ii)** $3L \sim N(3 \times 0.1,\ 3^2 \times 0.05^2) = N(0.3,\ 0.0225)$ $P(G > 3L) = P(G - 3L > 0) = 0.732$ where $G - 3L \sim N(0.1,\ 0.0261)$ **(iii)** Let $X = \sum_{i=1}^{7} G_i \sim N(7 \times 0.4,\ 7 \times 0.06^2) = N(2.8,\ 0.0252)$ $Y = \sum_{i=1}^{20} L_i \sim N(20 \times 0.1,\ 20 \times 0.05^2) = N(2,\ 0.05)$ $P(-1 < X - Y < 1) = 0.767$ where $X - Y \sim N(0.8,\ 0.0752)$ **(iv)** Let A = price of 7 oranges $= 2.4X$ $\sim N(2.4 \times 2.8,\ 2.4^2 \times 0.0252) = N(6.72,\ 0.145152)$ Let B = price of 20 lemons	

	$= 1.5Y$ $\sim \mathrm{N}\left(1.5\times 2,\ 1.5^2\times 0.05\right)$ i.e. $\mathrm{N}\left(3,\ 0.1125\right)$ Therefore, $A+B \sim \mathrm{N}\left(9.72,\ 0.257652\right)$ $\mathrm{P}\left(A+B>10\right)=0.291$	

Section A: Pure Mathematics [40 marks]

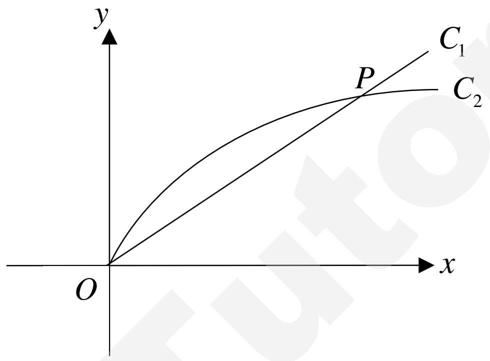
1. Find, algebraically, the range of values of k for which

$$kx^2 - 4x + k < 0$$

for all real values of x. [4]

2. Given that $2x^4 + x^2 - 1 = 0$, use the substitution $u = x^2$ to find the exact values of x. [4]

3.



The diagram shows the graphs of $C_1 : y = \dfrac{2x}{e-1}$ and $C_2 : y = \ln(2x+1)$ for $x \geq 0$. The graphs intersect at the origin O and the point P with coordinates $\left(\dfrac{e-1}{2}, k\right)$.

(i) Find the value of k. [1]

(ii) Find the exact area of the region bounded by C_2, the $y-$axis and the line $y = k$. [5]

(iii) Hence or otherwise, find the exact area of the region bounded by C_1 and C_2. [2]

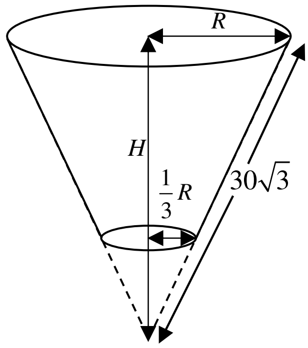
4. The curve C has equation $y = \dfrac{2}{3x+1} - e^{3x-1}$.

(i) Sketch the graph of C, stating the coordinates of any points of intersection with the axes and the equation of the asymptotes. [3]

(ii) Without using a calculator, find the equation of the tangent to C at the point where $x = \dfrac{1}{3}$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be determined. [4]

(iii) The tangent found in part **(ii)** meets the $y-$axis at A and $x-$axis at B. Find the length of AB. [4]

5. [It is given that a right circular cone of base radius r, perpendicular height h and slant height l has volume $\frac{1}{3}\pi r^2 h$ and curved surface area $\pi r l$.]



The diagram shows an open-top waste paper basket created by truncating an inverted right circular cone with negligible thickness. The larger inverted right circular cone has base radius R cm, perpendicular height H cm, and slant height $30\sqrt{3}$ cm. The bottom of the waste paper basket, where the smaller inverted right circular cone is truncated, has radius $\frac{1}{3}R$ cm.

(i) Show that the volume V cm^3, of the waste paper basket is given by

$$V = \frac{26}{81}\pi\left(2700H - H^3\right).$$ [3]

(ii) Without using a calculator, find the maximum value of V as H varies, justifying that this value is a maximum. [5]

An entrepreneur decides to manufacture the waste paper baskets using economy, standard and premium materials. Waste paper baskets manufactured from each material have different unit costs. The table below shows the number of waste paper baskets manufactured from each of the materials and the total costs based on records of 2015, 2016 and 2017.

	2015	**2016**	**2017**
Economy	200	300	350
Standard	170	200	300
Premium	50	60	70
Total cost in $	540.00	687.00	901.50

(iii) Assuming that the unit costs of the waste paper baskets for each material did not change from 2015 to 2018, find the total cost of manufacturing 100 economy, 70 standard and 50 premium waste paper baskets in 2018. [5]

Section B: Statistics [60 marks]

6. As part of Singapore's aim to be a Smart Nation in 10 years, hawker centres are encouraged to go cashless. An initial trial of cashless payment in hawker centres shows that 1 in 5 customers uses cashless payment. A random sample of 12 customers is selected and the number of customers who use cashless payment is denoted by the random variable C.

(i) Explain what is meant in this context by the term 'a random sample'. [1]

(ii) State, in context, an assumption needed for C to be well modelled by a binomial distribution. [1]

Assume now that C has a binomial distribution.

(iii) Find the probability that at least 3 customers use cashless payment. [2]

Eight groups of 12 customers are randomly selected.

(iv) Find the probability that there are exactly 5 groups with at least 3 customers who use cashless payment. [2]

7. A group of 11 people, consisting of 6 men and 5 women, stand in a line for photo-taking. There are 3 married couples in this group, with each married couple consisting of a husband and a wife. Find the number of different possible arrangements if

(i) there is no restriction, [1]

(ii) each married couple stand together, [2]

(iii) men and women alternate, [2]

(iv) a man stands on the extreme left or a woman stands on the extreme right or both. [3]

8. The number of international visitor arrivals in Singapore, x, in thousands, and the Gross Domestic Product (GDP) per capita, y, in thousands of dollars, were recorded for a sample of 8 countries. The results are given in the following table.

Country	Australia	Iran	Japan	Russian Federation	South Korea	Sri Lanka	Taiwan	United Kingdom
x	1,082	25	793	80	631	108	396	519
y	55.7	5.3	38.4	10.6	29.9	4.1	24.6	37.2

(i) Draw a sketch of the scatter diagram for the data, as shown on your calculator. [2]

(ii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iii) Find the regression line of x on y and sketch this line on your scatter diagram. [3]

(iv) Use an appropriate regression line to calculate an estimate of the number of international visitor arrivals from Canada whose GDP per capita is 45100 dollars. Comment on the reliability of your estimate. [2]

9. Kickers chocolates are sold in tins of 5 chocolates. The masses, in grams, of the individual Kickers chocolates and the empty tins have independent normal distributions with means and standard deviations as shown in the following table.

	Mean	Standard Deviation
Individual Kickers Chocolate	53	2.8
Empty Tin	15	0.4

(i) Find the probability that two randomly chosen Kickers chocolate each weigh more than 50 grams. [1]

(ii) Find the probability that the total mass of a tin containing 5 Kickers chocolates is less than 275 grams. State the mean and variance of the distribution that you use. [3]

The masses, in grams, of the individual Venus chocolates have a normal distribution with mean 35 grams and standard deviation σ grams. It is given that 85% of Venus chocolates weigh more than 34 grams.

(iii) Find σ, giving your answer correct to 4 decimal places. [3]

The cost of producing Kickers and Venus chocolates is 2 cents per gram and 3 cents per gram respectively.

(iv) Find the probability that the cost of producing a Kickers chocolate is within 10 cents of the cost of producing a Venus chocolate. State an assumption needed for your calculation. [4]

10. In a game, Mr Lim and Mr Tan take turns to pick a ball, without replacement, from a box which contains 3 red balls, 2 green balls and 1 yellow ball. The game continues until the first player picks a red ball and wins the game. Mr Lim starts the game.

(i) Draw a tree diagram to represent the possible outcomes. [3]

(ii) Find the probability that Mr Tan wins the game. [2]

(iii) If Mr Lim wins the game, find the probability that he wins on his second turn. [3]

(iv) Determine if the events 'Mr Lim has 2 turns' and 'Mr Lim wins the game' are independent. [2]

Mr Lim and Mr Tan play the game 5 times.

(v) Find the probability that Mr Tan wins the game at most two times. [2]

11. A government introduces the carbon tax to encourage companies to reduce carbon dioxide emission and lessen the effect of global warming. A random sample of 60 companies is taken and the amount of annual carbon dioxide emission (in tonnes) are summarised by

$$\sum(x-20000)=286\ 800\ ,\ \sum(x-20000)^2=1\ 429\ 904\ 000\,.$$

(i) Find unbiased estimates of the population mean and variance. [3]

(ii) Test at the 5% significance level, whether there is evidence that the population mean amount of annual carbon dioxide emission is less than 25000 tonnes. [4]

(iii) What do you understand by the terms 'unbiased estimate' and '5% significance level' in the context of this question? [2]

A researcher takes another sample of 60 companies and the mean amount of annual carbon dioxide emission recorded is k tonnes. It is given that the population standard deviation is 2000 tonnes. A test at the 1% significance level indicates that the population mean amount of annual carbon dioxide emission does not differ from 25000 tonnes.

(iv) Find the set of values of k, giving your answer correct to the nearest integer. [5]

End of Paper

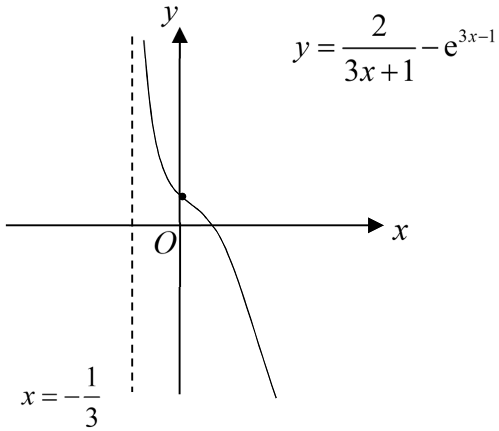
1	
	Since $kx^2-4x+k<0$, graph is "n-shape" and there are no real roots. Hence, $b^2-4ac<0$ and $k<0$ $(-4)^2-4(k)(k)<0$ $16-4k^2<0$ $4-k^2<0$ $(2+k)(2-k)<0$ $k<-2$ or $k>2$ $(\text{reject } \because k<0)$ Therefore, $k<-2$.

2	
	Substituting $u = x^2$, $2u^2 + u - 1 = 0$ $(u+1)(2u-1) = 0$ $u = -1$ or $u = \dfrac{1}{2}$ $\Rightarrow x^2 = -1 \ (\text{reject} \because x^2 \geq 0)$ or $x^2 = \dfrac{1}{2}$ $x^2 = \dfrac{1}{2}$ $x = \pm\dfrac{1}{\sqrt{2}}$

3		
	(i)	When $x=\frac{\mathrm{e}-1}{2}$, $k=\frac{2\left(\frac{\mathrm{e}-1}{2}\right)}{\mathrm{e}-1}=1$
	(ii)	$y=\ln(2x+1)$ $\mathrm{e}^y=2x+1$ $x=\frac{\mathrm{e}^y-1}{2}$ $\int_0^1 \frac{\mathrm{e}^y-1}{2}\,\mathrm{d}y$ $=\frac{1}{2}\int_0^1 \mathrm{e}^y-1\,\mathrm{d}y$ $=\frac{1}{2}\left[\mathrm{e}^y-y\right]_0^1$ $=\frac{1}{2}(\mathrm{e}-1-1)$ $=\frac{1}{2}(\mathrm{e}-2)$
	(iii)	Area $=\frac{1}{2}(1)\left(\frac{\mathrm{e}-1}{2}\right)-\frac{1}{2}(\mathrm{e}-2)$ $=\frac{1}{4}(3-\mathrm{e})$

4

(i)



(ii)

$$\frac{dy}{dx} = -2(3)(3x+1)^{-2} - 3e^{3x-1}$$

When $x = \frac{1}{3}$, $\frac{dy}{dx} = -6(2)^{-2} - 3e^{\frac{3}{3}-1} = -\frac{9}{2}$ and $y = 1 - e^{\frac{3}{3}-1} = 0$

Equation of tangent:

$$y - 0 = -\frac{9}{2}\left(x - \frac{1}{3}\right)$$

$$\frac{9}{2}x + y - \frac{3}{2} = 0$$

$$9x + 2y - 3 = 0$$

(iii)

At $y-$axis, $x = 0$, $y = \frac{3}{2}$.

$\therefore A\left(0, \frac{3}{2}\right)$

At $x-$axis, $y = 0$, $x = \frac{1}{3}$.

$\therefore B\left(\frac{1}{3}, 0\right)$

		Hence, $AB = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{3}{2}\right)^2}$ $= \sqrt{\frac{85}{36}}$ $= 1.5366$ ≈ 1.54 (to 3 s.f.)

5

(i) By Pythagoras' theorem,

$$\left(30\sqrt{3}\right)^2 = R^2 + H^2$$

$$R^2 = 2700 - H^2$$

By similar triangles, truncated part of the cone has dimensions $\frac{1}{3}R$ and $\frac{1}{3}H$.

$$V = \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi\left(\frac{1}{3}R\right)^2\left(\frac{1}{3}H\right)$$

$$= \frac{1}{3}\pi R^2 H - \frac{1}{81}\pi R^2 H$$

$$= \frac{26}{81}\pi R^2 H$$

$$= \frac{26}{81}\pi\left(2700 - H^2\right)H$$

$$= \frac{26}{81}\pi\left(2700H - H^3\right)$$

(ii) At maximum, $\frac{\mathrm{d}V}{\mathrm{d}H} = 0$

$$\frac{26}{81}\pi\left(2700 - 3H^2\right) = 0$$

$$2700 - 3H^2 = 0$$

$$H^2 = 900$$

$$H = 30 \quad \because H > 0$$

H	30^-	30	30^+
$\frac{\mathrm{d}V}{\mathrm{d}H}$	>0	0	<0
	/	—	\

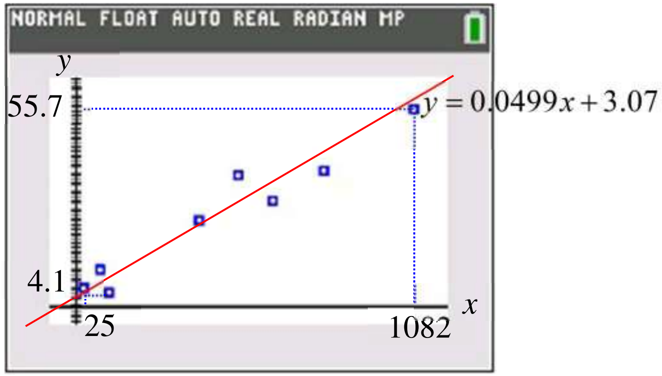
Alternative:

		$\frac{\mathrm{d}^2V}{\mathrm{d}h^2} = \frac{26}{81}\pi(-6H)$ < 0 when $H = 30$ Therefore, V is a maximum when $H = 30$. $V = \frac{26}{81}\pi\left(2700(30)-(30)^3\right)$ $= \frac{52000\pi}{3}$
	(iii)	Let x, y and z represent the unit costs of manufacturing waster paper baskets of economy, standard and premium materials in dollars. $200x+170y+50z=540$ $300x+200y+60z=687$ $350x+300y+70z=901.5$ Using G.C. $x=0.75$ $y=1.5$ $z=2.7$ Cost in 2018: $100(\$0.75)+70(\$1.50)+50(\$2.70)=\$315$

6		
	(i)	A random sample is a sample where all customers have equal chance of being selected and the selection is done independently.
	(ii)	Whether or not a customer uses cashless payment is independent of other customers using cashless payment. <u>OR</u> The probability of customers using cashless payment remains constant at 0.2.
	(iii)	Let C be the random variable denoting the number of customers who use cashless payment, out of 12 customers. $\therefore C \sim \mathrm{B}(12,\ 0.2)$ $\mathrm{P}(C \geq 3) = 1 - \mathrm{P}(C \leq 2)$ $= 0.4416542515$ $\approx 0.442 \quad (\text{to 3 s.f.})$
	(iv)	Let Y be the random variable denoting the number of groups with at least 3 customers who uses cashless payment, out of 8 groups. $\therefore Y \sim \mathrm{B}(8,\ \mathrm{P}(C \geq 3))$ $\Rightarrow Y \sim \mathrm{B}(8,\ 0.4416542515)$ $\mathrm{P}(Y = 5) = 0.1637984338$ $\approx 0.164 \quad (\text{to 3 s.f.})$

7		
	(i)	No. of arrangements $= 11! = 39916800$
	(ii)	No. of arrangements $= 8! \times (2!)^3$ $= 322560$
	(iii)	No. of arrangements $= 6! \times 5!$ $= 86400$
	(iv)	<u>*Man on extreme left*</u> No. of arrangements $= 6 \times 10! = 21772800$ <u>*Woman on extreme right*</u> No. of arrangements $= 5 \times 10! = 18144000$ <u>*Man on extreme left AND woman on extreme right*</u> No. of arrangements $= 6 \times 5 \times 9! = 10886400$ Total no. of arrangements $= 21772800 + 18144000 - 10886400$ $= 29030400$ <u>**Alternative:**</u> *Man on extreme left and woman NOT on extreme right* No. of arrangements $= 6 \times 5 \times 9! = 10886400$ *Woman on extreme right and man NOT on extreme left* No. of arrangements $= 5 \times 4 \times 9! = 7257600$ *Man on extreme left AND woman on extreme right* No. of arrangements $= 6 \times 5 \times 9! = 10886400$

		Total no. of arrangements $=10886400+7257600+10886400$ $=29030400$

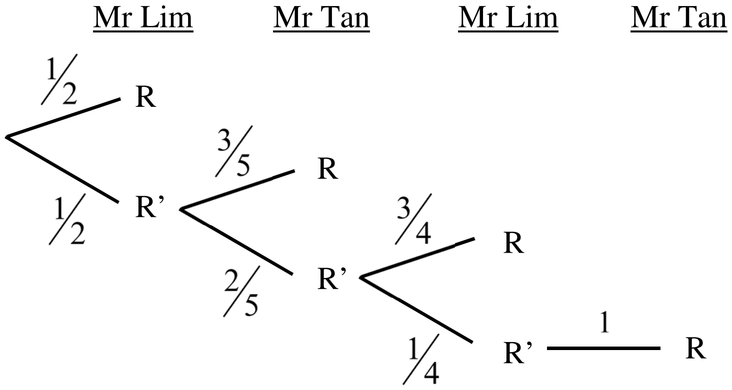
8		
	(i)	
	(ii)	Using GC, $r = 0.970220531 \approx 0.970 \;(3 \text{ s.f.})$. As the value of r is close to 1, this suggests that there is a strong, positive, linear relationship between the number of international visitor arrivals and GDP per capita. As the number of international visitor arrivals increases, GDP per capita increases proportionally.
	(iii)	Using GC, regression line of x on y is $x = 20.04745783y - 61.47085271 \approx 20.0 - 61.5 \quad (\text{to 3 s.f.})$ Making y the subject, $y = \dfrac{x + 61.47085271}{20.04745783} = 0.0498816363x + 3.066266717$ $\approx 0.0499x + 3.07\,(3 \text{ s.f.})$
	(iv)	$y = \dfrac{45100}{1000} = 45.1$ When $y = 45.1$, $x = 20.04745783(45.1) - 61.47085271$ $= 842.6694954$ $= 843\,(\text{to 3 s.f.})$
		The estimated number of visitor arrivals from Canada is 843000. This estimate is reliable as $r = 0.970$ is close to 1 and $y = 45.1$ is within the data range of $4.1 \le y \le 55.7$.

9		
	(i)	Let K be the random variable denoting the mass, in grams, of a randomly selected Kickers chocolate. $\therefore K \sim \mathrm{N}\left(53,\ 2.8^2\right)$ $\mathrm{P}\left(K_1 > 50\right) \times \mathrm{P}\left(K_2 > 50\right) = 0.7361838758$ $\approx 0.736 \quad (\text{to 3 s.f.})$
	(ii)	Let T be the random variable denoting the mass, in grams, of a randomly selected empty tin. $\therefore T \sim \mathrm{N}\left(15,\ 0.4^2\right)$ $K_1 + K_2 + K_3 + K_4 + K_5 + T \sim \mathrm{N}\left(5\times 53 + 15,\ 5\times 2.8^2 + 0.4^2\right)$ $K_1 + K_2 + K_3 + K_4 + K_5 + T \sim \mathrm{N}\left(280,\ 39.36\right)$ $\mathrm{P}\left(K_1 + K_2 + K_3 + K_4 + K_5 + T < 275\right) = 0.2127339029$ $\approx 0.213 \quad (\text{to 3 s.f.})$
	(iii)	Let V be the random variable denoting the mass, in grams, of a randomly selected Venus chocolate. $\therefore V \sim \mathrm{N}\left(35,\ \sigma^2\right)$ $\mathrm{P}\left(V > 34\right) = 0.85$ $\mathrm{P}\left(V < 34\right) = 0.15$ $\mathrm{P}\left(Z < \dfrac{34-35}{\sigma}\right) = 0.15$ $\mathrm{P}\left(Z < \dfrac{-1}{\sigma}\right) = 0.15$ $\dfrac{-1}{\sigma} = -1.03643338$ $\sigma = 0.9648473501 \approx 0.9648 \quad (\text{to 4 d.p.})$
	(iv)	$2K \sim \mathrm{N}\left(2\times 53,\ 2^2\times 2.8^2\right)$ $2K \sim \mathrm{N}\left(106,\ 31.36\right)$ $3V \sim \mathrm{N}\left(3\times 35,\ 3^2\times 0.9648473501^2\right)$ $3V \sim \mathrm{N}\left(105,\ 8.378373681\right)$

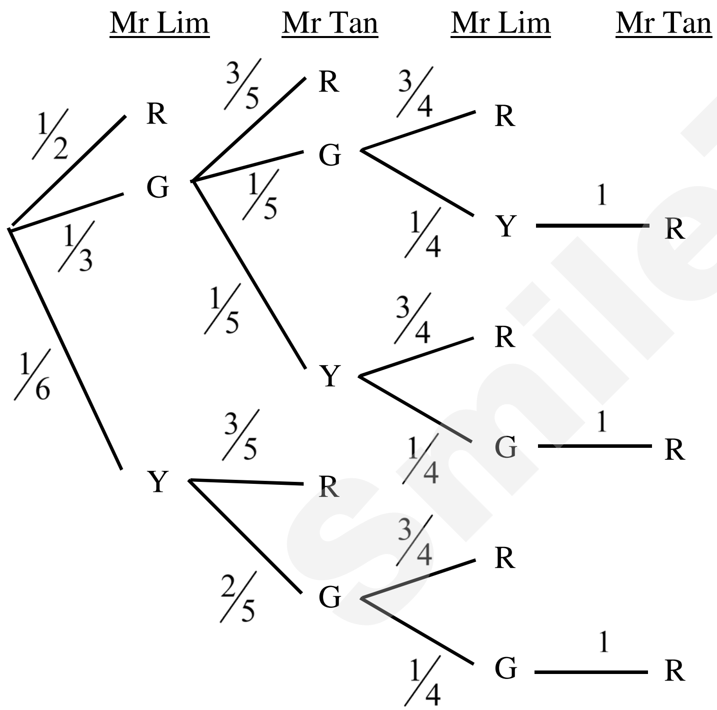
		$2K-3V \sim \mathrm{N}(106-105,\ 31.36+8.378373681)$ $2K-3V \sim \mathrm{N}(1,\ 39.73837368)$ $\mathrm{P}(-10<2K-3V<10)=0.8828159632 \approx 0.883 \quad (\text{to 3 s.f.})$
		The mass of the Kickers chocolates is independent of the mass of the Venus chocolates.

10

(i)



Alternative:



	(ii)	$P(\text{Mr Tan wins the game})$ $=\frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{2}{5}\times\frac{1}{4}\times1$ $=0.35$ **Alternative:** $P(\text{Mr Tan wins the game})$ $=\frac{1}{3}\times\frac{3}{5}+\frac{1}{6}\times\frac{3}{5}+\frac{1}{3}\times\frac{1}{5}\times\frac{1}{4}\times1+\frac{1}{3}\times\frac{1}{5}\times\frac{1}{4}\times1+\frac{1}{6}\times\frac{2}{5}\times\frac{1}{4}\times1$ $=0.35$
	(iii)	$P(\text{Mr Lim wins game on his second turn} \mid \text{Mr Lim wins the game})$ $=\frac{P(\text{Mr Lim wins the game on his second turn})}{P(\text{Mr Lim wins the game})}$ $=\frac{\frac{1}{2}\times\frac{2}{5}\times\frac{3}{4}}{1-0.35}$ $=\frac{0.15}{0.65}=\frac{3}{13}$ **Alternative:** $P(\text{Mr Lim wins game on his second turn} \mid \text{Mr Lim wins the game})$ $=\frac{P(\text{Mr Lim wins the game on his second turn})}{P(\text{Mr Lim wins the game})}$ $=\frac{\frac{1}{3}\times\frac{1}{5}\times\frac{3}{4}+\frac{1}{3}\times\frac{1}{5}\times\frac{3}{4}+\frac{1}{6}\times\frac{2}{5}\times\frac{3}{4}}{1-0.35}$ $=\frac{0.15}{0.65}=\frac{3}{13}$
	(iv)	$P(\text{Mr Lim has two turns})=\frac{1}{2}\times\frac{2}{5}=0.2$

		$\text{P}(\text{Mr Lim wins the game}) = 0.65$ $\text{P}(\text{Mr Lim has two turns and wins the game}) = 0.15$ Since $0.2 \times 0.65 = 0.13 \neq 0.15$, the two events are not independent.
	(v)	Let X be the random variable denoting the number of games Mr Tan wins, out of 5 games. $\therefore X \sim \text{B}(5,\ 0.35)$ $\text{P}(X \leq 2) = 0.764830625 \approx 0.765 \quad (\text{to 3 s.f.})$

11		
	(i)	$\hat{\mu} = \dfrac{286800}{60} + 20000 = 24780$ $s^2 = \dfrac{1}{59}\left[1429904000 - \dfrac{286800^2}{60}\right] = 1000000$
	(ii)	Let X be the random variable denoting the amount of annual carbon dioxide emission, in tonnes, of a randomly selected company. $H_0 : \mu = 25000$ tonnes $H_1 : \mu < 25000$ tonnes Under H_0, $\mu = 25000$ tonnes, and since $n = 60$ is large enough, by Central Limit Theorem, $\bar{X} \sim N\left(25000, \dfrac{1000000}{60}\right)$ approximately. Test statistic, $z = \dfrac{24780 - 25000}{\sqrt{\dfrac{1000000}{60}}} = -1.704112672$ $p\text{-value} = P(Z < -1.704112672)$ $= 0.0441799877$ **<u>Method 1:</u>** At 5% significance level, reject H_0 if $p\text{-value} < 0.05$. Since $p\text{-value} = 0.0441799877 < 0.05$, reject H_0. **<u>Method 2:</u>** At 5% significance level, reject H_0 if $z < -1.644853626$. Since $z = -1.704112672 < -1.644853626$, reject H_0. Conclude that there is sufficient evidence at the 5% significance level that the population mean amount of annual carbon dioxide emission is less than 25000 tonnes.
	(iii)	An unbiased estimate is the value of an estimator for the population mean annual carbon dioxide emission where the expected value of the estimator is equal to the population mean annual carbon dioxide emission, i.e. $E(\theta) = \mu$ where θ is the estimator. 5% significance level means that there is a probability of 0.05 of claiming that the population mean amount of annual carbon dioxide emission is less than 25000 tonnes, when it is actually 25000 tonnes.
	(iv)	$H_0 : \mu = 25000$ tonnes $H_1 : \mu \neq 25000$ tonnes

Under $\mathrm{H_0}$, $\mu = 25000$ tonnes, and since $n = 60$ is large enough, by Central Limit Theorem, $\bar{X} \sim \mathrm{N}\left(25000, \dfrac{2000^2}{60}\right)$ approximately.

Test statistic, $z = \dfrac{k-25000}{\dfrac{2000}{\sqrt{60}}}$

At 1% significance level, reject $\mathrm{H_0}$ if $z < -2.575829303$ or $z > 2.575829303$.

Since $\mathrm{H_0}$ is not rejected,

$$-2.575829303 < \frac{k-25000}{\frac{2000}{\sqrt{60}}} < 2.575829303$$

$$-2.575829303\left(\frac{2000}{\sqrt{60}}\right) < k - 25000 < 2.575829303\left(\frac{2000}{\sqrt{60}}\right)$$

$$25000 - 2.575829303\left(\frac{2000}{\sqrt{60}}\right) < k < 25000 + 2.575829303\left(\frac{2000}{\sqrt{60}}\right)$$

$$24334.92373 < k < 25665.07627$$

$$\Rightarrow \{k \in \because : 24335 \le k \le 25665\}$$

Section A : Pure Mathematics [40 marks]

1 In a computer game, a player is given a plot of land to grow tomatoes, pumpkins and cherries. The player can only plant one type of fruit on the plot of land and the land can only produce 1 kg of fruits each time. The time taken from planting to harvesting the fruits, production cost and selling price for 1 kg of each type of fruit are shown in the table below.

	Time required (Hours)	**Production cost ($)**	**Selling price ($)**
Tomatoes	3	15	35
Pumpkins	5	k	50
Cherries	16	45	100

The total time taken for Jimmy to plant and harvest 23 kg of fruits was 154 hours. The total production cost was $545 and he made a profit of $735 from selling the harvested fruits. Find the value of k. [4]

[You are to assume that Jimmy harvested the fruits once they were ready and there was no time lapse between harvesting and planting new fruits.]

2 **(i)** Show that $x^2 - 2x + 2$ is always positive for all real values of x. Hence, or otherwise, solve the inequality $\frac{x}{x-4} - \frac{1}{2x-3} \geq 0$. [4]

(ii) Hence, solve $\frac{x^2}{x^2-4} - \frac{1}{2x^2-3} < 0$. [3]

3 **(a)** Differentiate $\left(\ln\sqrt{2x}\right)^3$ with respect to x. [3]

(b) The equation of a curve is $y = \frac{a}{2x-a}$, where a is a positive constant.

The point A on the curve has coordinates $(a, 1)$.

(i) Find the equation of the tangent at A. [2]

(ii) If the tangent to the curve at A meets the x-axis at P and the y-axis at Q, and the length of PQ is 7 units, find the exact value of a. [3]

[Turn over

4 **(a)** Find the exact value of k such that $\int_1^2 \frac{1}{2x-1}\,dx = \int_1^5 \frac{k}{\sqrt{2x-1}}\,dx$. [3]

(b) **(i)** On a single diagram, sketch the graphs of $y = 4e^{-x^2} - 1$ and $y = \frac{2x+1}{1-x}$,

stating clearly the equations of any asymptotes and the coordinates of any points of intersection with the $y-$axis. [3]

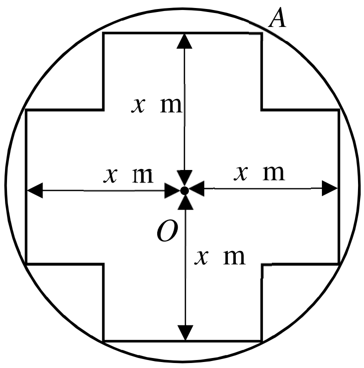
(ii) Find the $x-$coordinate of the point of intersection of $y = 4e^{-x^2} - 1$ and $y = \frac{2x+1}{1-x}$, giving your answer correct to 5 decimal places. [1]

(iii) Write down as an integral an expression for the area of the region bounded by $y = 4e^{-x^2} - 1$, $y = \frac{2x+1}{1-x}$ and the $y-$axis. Evaluate this integral, giving your answer correct to 4 decimal places. [2]

5 **(a)** By expressing $x\sqrt{r^2 - x^2}$ where r is a constant, in the form of $\sqrt{f(x)}$, prove that

$$\frac{d}{dx}\left(x\sqrt{r^2 - x^2}\right) = \frac{r^2 - 2x^2}{\sqrt{r^2 - x^2}}, \text{ where } 0 < x < r.$$ [3]

(b) The diagram shows a flower-bed in the shape of a symmetric cross inscribed in a circle of fixed radius r m with centre O.



(i) By expressing AB in terms of r and x, prove that the area of the cross, A m^2, is given by

$$A = 8x\sqrt{r^2 - x^2} - 4r^2 + 4x^2$$ [3]

(ii) Prove by differentiation, that the value of x for which the area A is maximum will satisfy the equation $5x^4 - 5r^2x^2 + r^4 = 0$. [3]

Let the radius of the circle be 5 m for the rest of the question.

(iii) Find the value of x for which the area A is maximum. [3]

Section B : Probability and Statistics [60 marks]

6 A machine is used to generate codes consisting of 4 characters. A character is either one of the 26 letters of the alphabet A to Z or one of the 10 digits 0 to 9. Each of the 36 characters are equally likely to be generated and each character can be generated more than once.

Find the probability that a randomly generated code has

(i) no digits, [1]

(ii) three different letters and one digit. [3]

7 A factory manufactures paperweights consisting of glass mounted on a wooden base. The volume of glass, in cm^3, in a randomly chosen paperweight has a normal distribution with mean 56.5 and standard deviation 2.9 and the volume of wood, in cm^3, has an independent normal distribution with mean 38.4 and standard deviation σ.

The probability that the total volume of a randomly chosen paperweight exceeds 100 cm^3 is 0.05. Find the value of σ. [4]

8 A shop sells mangoes packed in boxes of 12. It is given that p % of the mangoes are rotten.

(i) Given that the mean number of rotten mangoes in a box is 2.4, show that the value of p is 20. [1]

Use $p = 20$ for the rest of the question.

(ii) A buyer inspects a randomly chosen box of mangoes. Find the probability that

(a) at least 2 mangoes are rotten, [1]

(b) the 10^{th} mango inspected is the second mango that is rotten. [2]

Explain why the answer to part **(b)** is smaller than the answer to part **(a).** [1]

(iii) The shop sells 60 boxes of mangoes. Find the probability that the mean number of rotten mangoes per box is less than 2. [3]

[Turn over

9 A car manufacturer is testing the braking distance for a new model of car. The table shows the braking distance, y metres, for different speeds, x kmh^{-1}, when the brakes are applied.

x	30	50	70	90	110	130
y	25	50	85	155	235	350

(i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]

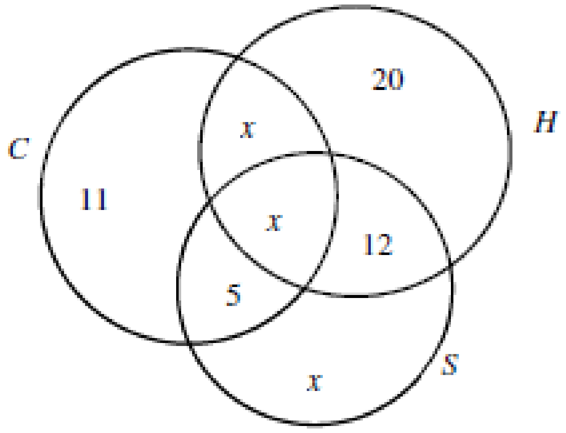
(ii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iii) Find the equation of the regression line of y on x in the form $y = ax + b$, giving the values of a and b correct to 3 significant figures. Sketch this line on your scatter diagram. [2]

(iv) Use the equation of your regression line to calculate estimates of braking distances when the speeds of cars are 100 kmh^{-1} and 150 kmh^{-1}. Comment on the reliability of your estimates. [2]

(v) A seventh set of data (u, v) is added and the equation of the regression line of y on x remains unchanged. State the values of u and v. [1]

10



A college organised Overseas Community Involvement Programme trips to Chiang Mai, Hainan and Surabaya. During the interview, a group of selected students was asked if they had ever visited Chiang Mai, Hainan and Surabaya. The number of students who had visited the 3 different places are shown in the Venn diagram. It was found that every student had visited at least one place. One student is selected at random, and the events C, H and S are defined as follows.

- C is the event that the student visited Chiang Mai.
- H is the event that the student visited Hainan.
- S is the event that the student visited Surabaya.

(i) Find $\mathrm{P}(C)$ and $\mathrm{P}(C \mid H)$, giving your answers in terms of x. [2]

(ii) Given that $\mathrm{P}(C) = \mathrm{P}(C \mid H)$, find the value of x. [2]

(iii) Explain, in the context of this question, what is meant by $\mathrm{P}(C' \mid S')$, and find its value. [4]

(iv) Hence, state with a reason, whether C and S are independent. [2]

11 A glass manufacturer produces large batches of jars. The weights of jars have a standard deviation 10.5 g. The manufacturer claims that the mean weight of a jar is at least 502 g. A random sample of 50 jars is taken and the mean weight is 499 g.

(i) State appropriate hypotheses for the test, defining any symbols you use. [2]

(ii) Test, at the 1% significance level, whether the manufacturer's claim is valid. [3]

(iii) Explain the meaning of "at the 1% significance level " in the context of the question. [1]

A second sample of 60 jars is taken and the weights of the jars, x grams are summarised by

$$\sum(x-502)=-222, \qquad \sum(x-502)^2=8401.13.$$

(iv) Find unbiased estimates for the population mean and variance using this second sample. [3]

A test, at the $\alpha\%$ significance level, shows that there is sufficient evidence to suggest that the population mean weight of jars from this second sample differs from 502 g.

(v) Find the set of possible values of α. [4]

12 A supermarket sells two types of strawberries, A and B. Each type of strawberry comes in different sized packaging. The masses, in kg, of a packet of type A strawberries and a packet of type B strawberries are modelled as having independent normal distributions with means and standard deviations as shown in the table.

Strawberries	**Mean**	**Standard deviation**
Type *A*	1.2	0.2
Type *B*	1.1	0.1

Type A strawberries are sold at \$20 per kg and type B strawberries at \$26 per kg.

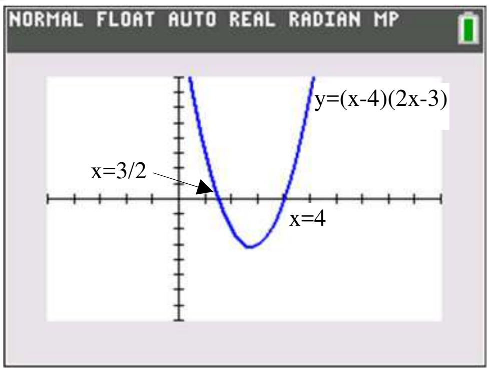
Audrey picks one packet of type A strawberries and one packet of type B strawberries.

(i) Find the probability that none of the two packets picked by Audrey weighs less than 1.2 kg. [2]

(ii) The probability that the total weight of the two packets chosen by Audrey lies between 2.2 kg and m kg is 0.45, where $m > 2.2$. Find the value of m. [3]

(iii) Find the probability that the total cost of the two packets picked by Audrey will not exceed \$50. [3]

(iv) Jackson buys one packet of type A strawberries and three packets of type B strawberries. Find the probability that the packet of type A strawberries weighs more than the average weight of the three packets of type B strawberries by less than 0.2 kg. [4]

Qtn	Solution
1(a) (i)	Let t, p and c be the weights (in kg) of tomatoes, pumpkins and cherries harvested respectively. $t+p+c=23$ ------- (1) $3t+5p+16c=154$ ------- (2) $15t+kp+45c=545$ ------- (3) $35t+50p+100c=735+545=1280$ ------- (4) Solving (1), (2) and (4) using GC, $t=8,\ p=10$ and $c=5$ Substituting into (3), $15(8)+10k+45(5)=545 \Rightarrow k=20$
2(i)	Method 1 $y=x^2-2x+2 \Rightarrow y=(x-1)^2+1$ Since $(x-1)^2 \geq 0$ for all values of x, $\therefore y=x^2-2x+2$ is always positive for all real values of x. Method 2 For $y=x^2-2x+2$, $D=(-2)^2-4(1)(2)=-4<0$ Since coefficient of $x^2>0$, $y=x^2-2x+2$ is always positive for all real values of x. $\frac{x}{x-4}-\frac{1}{2x-3}=\frac{x(2x-3)-(x-4)}{(x-4)(2x-3)}$ $=\frac{2(x^2-2x+2)}{(x-4)(2x-3)} \geq 0$ Since x^2-2x+2 is always positive, we thus solve $(x-4)(2x-3)>0$  $\Rightarrow \quad x<\frac{3}{2}$ or $x>4$

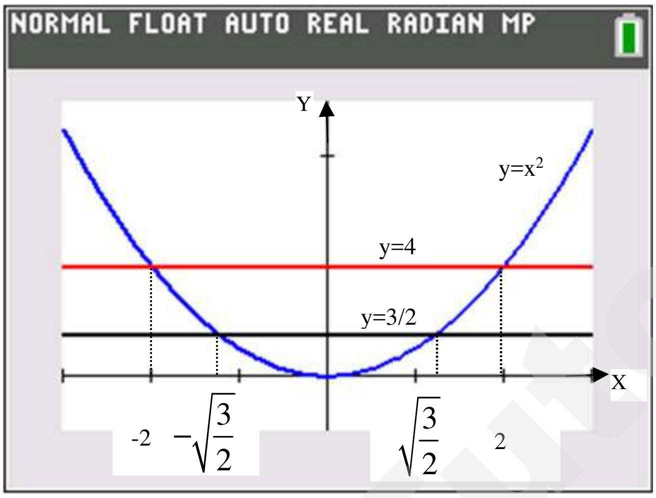
2(ii)

From part (i) and replacing x with x^2,

we have if $\dfrac{x^2}{x^2-4}-\dfrac{1}{2x^2-3}<0$, then

$$\frac{3}{2}<x^2<4 \quad . \; . \; . \; . \; . \; . \; (1)$$

Solving (1), we have:



$$x^2=\frac{3}{2}\Rightarrow x=\pm\sqrt{\frac{3}{2}}$$

$$x^2=4\Rightarrow x=\pm 2$$

Thus, the solution is

$$-2<x<-\sqrt{\frac{3}{2}} \quad \text{or} \quad \sqrt{\frac{3}{2}}<x<2$$

or

$$-2<x<-1.22 \quad \text{or} \quad 1.22<x<2$$

3(a)

<u>Method 1</u>

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\sqrt{2x}\right)^3=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}\ln(2x)\right)^3$$

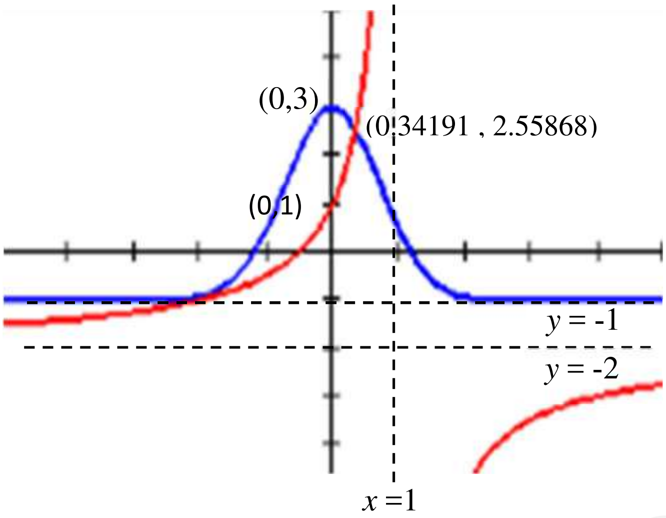
$$=\frac{1}{8}\left(\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln(2x)\right)^3\right)$$

$$=\frac{3}{8}\left(\frac{2}{2x}\right)\left(\ln(2x)\right)^2$$

$$=\frac{3}{8x}\left(\ln(2x)\right)^2$$

<u>Method 2</u>

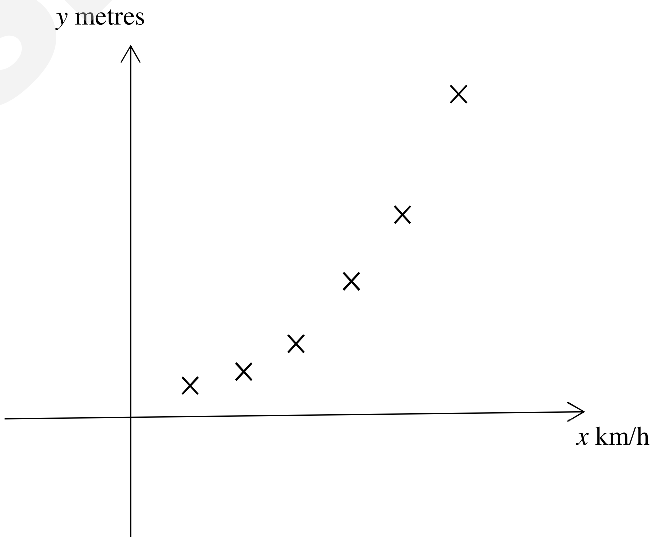
	$\frac{d}{dx}\left(\ln\sqrt{2x}\right)^3$ $=3\left(\ln\sqrt{2x}\right)^2\left(\frac{1}{\sqrt{2x}}\right)\left(\frac{1}{2}(2x)^{-\frac{1}{2}}\times 2\right)$ $=\frac{3}{2x}\left(\ln\sqrt{2x}\right)^2$
3b(i)	$y=\frac{a}{2x-a}$ $\frac{dy}{dx}=-\frac{2a}{(2x-a)^2}$ At point A, $\frac{dy}{dx}=-\frac{2}{a}$ Equation of tangent at A is $y-1=-\frac{2}{a}(x-a)$ $y=-\frac{2}{a}x+3$
b(ii)	When $x = 0$, $y = 3$. Coord of Q is (0 , 3) When $y = 0$, $x = \frac{3}{2}a$. Coord of P is $\left(\frac{3a}{2}, 0\right)$ Length of $PQ = 7$ $3^2+\left(\frac{3a}{2}\right)^2=7^2$ $a=\pm\frac{4}{3}\sqrt{10}$ Since $a > 0$, $\therefore a=\frac{4}{3}\sqrt{10}$
4(a)	$\int_1^2 \frac{1}{2x-1}\,dx=\int_1^5 \frac{k}{\sqrt{2x-1}}\,dx$ $\frac{1}{2}\left[\ln\lvert 2x-1\rvert\right]_1^2=k\left[\sqrt{2x-1}\right]_1^5$ $\frac{1}{2}\ln 3=2k$ $k=\frac{1}{4}\ln 3$

4(b) (i)	
b(ii)	$x-$coordinate of pt of intersection is 0.34191 (5 d.p.)
b(iii)	Area required = $\int_0^{0.34191}\left(4e^{-x^2}-1-\frac{2x+1}{1-x}\right)\mathrm{d}x$ $=0.4028$ (4 d.p.)
5(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x\sqrt{r^2-x^2}\right)$ $=\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{r^2x^2-x^4}\right)=\frac{1}{2}\left(r^2x^2-x^4\right)^{-\frac{1}{2}}\left(2r^2x-4x^3\right)$ $=\frac{r^2x-2x^3}{\sqrt{r^2x^2-x^4}}=\frac{x\left(r^2-2x^2\right)}{x\sqrt{r^2-x^2}}$ $=\frac{r^2-2x^2}{\sqrt{r^2-x^2}}$
5(b) (i)	By Pythagoras' $AB=x-\sqrt{r^2-x^2}$ $A=2x\times 2\sqrt{r^2-x^2}+\left(2x-2\sqrt{r^2-x^2}\right)\times 2\sqrt{r^2-x^2}-(1)$ $=4x\sqrt{r^2-x^2}+4x\sqrt{r^2-x^2}-4\left(r^2-x^2\right)$ $=8x\sqrt{r^2-x^2}-4r^2+4x^2$
(ii)	$A=8x\sqrt{r^2-x^2}-4r^2+4x^2$ $\frac{\mathrm{d}A}{\mathrm{d}x}=8\times\frac{r^2-2x^2}{\sqrt{r^2-x^2}}+8x$ When $\frac{\mathrm{d}A}{\mathrm{d}x}=0$,

	$r^2-2x^2+x\sqrt{r^2-x^2}=0$ $x\sqrt{r^2-x^2}=2x^2-r^2---(2)$ $x^2\left(r^2-x^2\right)=4x^4-4r^2x^2+r^4$ $5x^4-5r^2x^2+r^4=0$
(iii)	$5x^4-125x^2+625=0$ $x=4.25$ or $x=2.63$ In equation (2), $2x^2-r^2\geq 0 \Rightarrow x^2\geq\frac{1}{2}r^2$. That is $x^2\geq\frac{1}{2}(5^2)=12.5$. Hence, $x=2.63$ is rejected. $x=4.25$ [x: 4.25^-, 4.25, 4.25^+; $\frac{\mathrm{d}A}{\mathrm{d}x}$: +ve, 0, -ve; Slope: /, -, \] Hence, when $x=4.25$, A is max
6(i)	Required probility $=\frac{26\times26\times26\times26}{36\times36\times36\times36}=\left(\frac{26}{36}\right)^4=0.272$
6(ii)	Required probability $=\frac{{}^{26}C_3\times{}^{10}C_1\times4!}{36^4}=\frac{1625}{4374}$ or 0.372
7	Let X cm^3 and Y cm^3 be the volumes of glass and wood in a paperweight respectively. $X\sim\mathrm{N}(56.5,\ 2.9^2)$ $Y\sim\mathrm{N}(38.4,\ \sigma^2)$. $\therefore X+Y\sim\mathrm{N}(94.9, 2.9^2+\sigma^2)$ $\mathrm{P}(X+Y>100)=0.05$ $\Rightarrow \mathrm{P}(X+Y\leq100)=0.95$ $\Rightarrow \mathrm{P}\left(Z\leq\frac{100-94.9}{\sqrt{2.9^2+\sigma^2}}\right)=0.95$ $\Rightarrow \frac{5.1}{\sqrt{2.9^2+\sigma^2}}=1.64485$ $\Rightarrow 2.9^2+\sigma^2=\left(\frac{5.1}{1.64485}\right)^2$

Table in (iii):

x	4.25^-	4.25	4.25^+
$\frac{\mathrm{d}A}{\mathrm{d}x}$	+ve	0	-ve
Slope	/	-	\

	$\Rightarrow \sigma = \sqrt{1.20363} = 1.09710 = 1.10$ (to 3 sf)
8(i)	Using mean = np $12\left(\dfrac{p}{100}\right) = 2.4$ $\Rightarrow p = 20$
(ii) (a)	Let X be the no. of rotten mangoes out of 12 mangoes. $X \sim \mathrm{B}(12, 0.2)$ $\mathrm{P}(X \geq 2) = 1 - \mathrm{P}(X \leq 1) = 0.725$
(b)	Let Y be the no. of rotten mangoes out of 11 mangoes $Y \sim \mathrm{B}(11, 0.2)$ Required probability $= 0.2 \times \mathrm{P}(Y = 1)$ $= 0.04724$ $= 0.0472$(to 3 s.f)
	Answer to part **(b)** is smaller than answer to **(a)** because event in **(b)** is a proper subset of event in **(a)**. Event in **(a)** includes cases where 2 rotten mango can be any 2 of the 12 whereas for event (b), the second rotten mango must be the 10^{th} mango.
(iii)	Let $\overline{X}$ be the mean no. of rotten mangoes in a box. Since n is large, by the Central Limit Theorem, $\overline{X} \sim \mathrm{N}\left(2.4\,,\ \dfrac{2.4 \text{ x } 0.8}{60}\right)$ approximately. $\mathrm{P}(\overline{X} < 2) = 0.0127$
9	

(ii)	Product moment correlation coefficient $r = 0.969$ which indicates a strong positive linear correlation between the braking distance, y metres, and different speeds, $x\,\text{kmh}^{-1}$, when the brakes are applied.
(iii)	From GC, regression line is $y = 3.2143x - 107.143 \Rightarrow y = 3.21x - 107$
(iv)	When $x = 100$, $y = 3.2143\times100 - 107.143 = 214.287 = 214$ (3sf) $x = 100$ is within the data range $[30, 130]$ and $r = 0.969$ is close to 1, suggesting strong positive linear correlation between x and y. Therefore the estimated value of $y = 214$ obtained should be reliable. When $x = 150$, $y = 3.2142\times150 - 107.142 = 374.988 = 375$ (3sf) $x = 150$ is outside the data range $[30, 130]$ which means the estimated braking distance of 375 m obtained from the equation of regression line may be unreliable.
(v)	$u = 80, v = 150$
10(i)	$\text{P}(C) = \dfrac{16+2x}{48+3x}$ $\text{P}(C \mid H) = \dfrac{2x}{32+2x} = \dfrac{x}{16+x}$
(ii)	$\dfrac{16+2x}{48+3x} = \dfrac{x}{16+x}$ $(16+2x)(16+x) = x(48+3x)$ $256 + 48x + 2x^2 = 48x + 3x^2$ $x^2 = 256$ $x = 16$
(iii)	$\text{P}(C' \mid S')$ is the probability that a randomly selected student has not visited Chiang Mai given that he/she has not visited Surabaya. $\text{P}(C' \mid S') = \dfrac{\text{P}(C' \cap S')}{\text{P}(S')} = \dfrac{20/96}{47/96}$ $= \dfrac{20}{47}$ or 0.426
(iv)	Since $\text{P}(C' \mid S') = \dfrac{20}{47} \neq \text{P}(C') = \dfrac{1}{2}$ $\therefore C$ and S are not independent.
11(i)	Let X be the weight of a jar and μ denotes the population mean weight of a jar in grams $\text{H}_0 : \mu = 502$ $\text{H}_1 : \mu < 502$

(ii)	Under H_0, Test statistic $Z=\dfrac{\overline{X}-502}{\sqrt{\dfrac{10.5^2}{50}}} \sim N(0,1)$ At 1% level of significance, under H_0, since n = 50 is large, by Central Limit Theorem $\overline{X} \sim N\left(502, \dfrac{10.5^2}{50}\right)$ approximately $p=0.0217>0.01$, we do not reject H0 and conclude that at the 1% level of significance, there is insufficient evidence to claim that the mean weight of a jar is less than 502 g.
(iii)	There is a probability of 0.01 that we wrongly conclude the mean weight of a jar is less than 502 g when in fact the mean weight of a jar is 502 g.
(iv)	Unbiased estimate for the population mean $=\dfrac{\sum(x-502)}{60}+502=498.3$ Unbiased estimate for the population variance $=\dfrac{1}{59}\left(\sum(x-502)^2-\dfrac{\left(\sum(x-502)\right)^2}{60}\right)$ $=128.47$
(v)	$\mathrm{H}_0: \mu=502$ $\mathrm{H}_1: \mu \neq 502$ Under H_0, since n = 60 is large, by Central Limit Theorem $\overline{X} \sim N\left(502, \dfrac{128.47}{60}\right)$ approximately Under H_o, Test statistic $Z=\dfrac{\overline{X}-502}{\sqrt{\dfrac{128.47}{60}}} \sim N(0,1)$ approximately By GC, p-value $=0.011453$ Since H_0 is rejected, $\dfrac{\alpha}{100} \geq 0.011453$ Set of values = $\{\alpha \in \mathbb{R} : 1.15 \leq \alpha \leq 100\}$
12	Let X and Y be the weight (in kg) of a packet of type A and type B strawberries respectively. $X \sim \mathrm{N}\left(1.2, 0.2^2\right)$ $Y \sim \mathrm{N}\left(1.1, 0.1^2\right)$ (i)

$\mathrm{P}(X \geq 1.2)\mathrm{P}(Y \geq 1.2) = 0.0793$

(ii)

Let $T = X + Y \sim \mathrm{N}\left(1.2 + 1.1, 0.2^2 + 0.1^2\right)$

$T \sim \mathrm{N}(2.3, 0.05)$

$\mathrm{P}(T < 2.2) = 0.32736$

$\mathrm{P}(2.2 < T < m) = 0.45$

$\mathrm{P}(T < m) - \mathrm{P}(T < 2.2) = 0.45$

$\mathrm{P}(T < m) = 0.32736 + 0.45 = 0.77736$

$\therefore m = 2.47$

(iii)

Let A and B be the cost of one packet of type A and type B strawberries respectively.

$A = 20X \sim \mathrm{N}\left(20 \times 1.2, 20^2 \times 0.2^2\right) = \mathrm{N}(24, 16)$

$B = 26Y \sim \mathrm{N}\left(26 \times 1.1, 26^2 \times 0.1^2\right) = \mathrm{N}(28.6, 6.76)$

Let C be the total cost for 2 packets of strawberries picked by Audrey.

$C = A + B \sim \mathrm{N}(24 + 28.6, 16 + 6.76) = \mathrm{N}(52.6, 22.76)$

$\mathrm{P}(C \leq 50) = 0.29288 = 0.293$(to 3 s.f)

(iv)

Let M be the average weight of the three packets of type B strawberries bought by Jackson.

$$M = \frac{Y_1 + Y_2 + Y_3}{3}$$

$$M \sim \mathrm{N}\left(\frac{3(1.1)}{3}, \frac{3\left(0.1^2\right)}{3^2}\right) = \mathrm{N}\left(1.1, \frac{0.1^2}{3}\right)$$

$$X - M \sim \mathrm{N}\left(1.2 - 1.1, 0.2^2 + \frac{0.1^2}{3}\right) = \mathrm{N}\left(0.1, \frac{0.13}{3}\right)$$

$\mathrm{P}(0 \leq X - M < 0.2) = 0.36905 = 0.369$(to 3 s.f)

MERIDIAN JUNIOR COLLEGE
JC2 Preliminary Examination
Higher 1

H1 Mathematics 8865/01

Paper 1 **14 September 2018**

3 Hours

Additional Materials: Writing paper
Graph Paper
List of Formulae (MF 26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics group on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1 **(a)** C_1 and C_2 have equations $y = e^{3-2x}$ and $y = 5e^{2x}$ respectively. Determine the exact coordinates of the intersection point. [5]

(b) The graph of $y = ax^2 + bx + c$ has turning point $(1, 4)$ and y-intercept $(0, -1)$.

Determine the values of a, b and c. [4]

2 **(a)** Find the range of values of p for which the equation $x^2 + px + 2 = 0$ has no real roots. [2]

(b) Find the range of values of k for the line $y = 2x + 3$ to intersect the curve $y = kx^2 + (2k+1)x - 3$ at least once. [4]

3 **(a)** The gradient at any point on a particular curve is given by $\frac{dy}{dx} = (2-x)^2$. Given that the curve passes through the point $(0, 2)$, find the equation of the curve. [3]

Hence, find the area of the region bounded by the curve, the x- and y-axes and the line $x = 5$. [1]

(b) Evaluate, in terms of k, the integral $\int_2^k \frac{1}{x^2} - \frac{2}{1-x}\,dx$ where $k > 2$. [3]

4 [It is given that the volume of a cone of radius r and height h is $\frac{1}{3}\pi r^2 h$.]

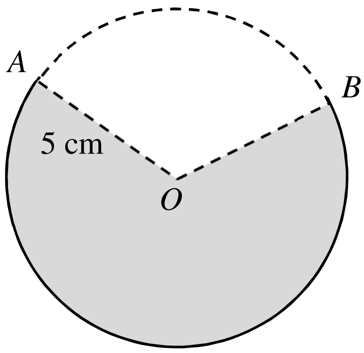


Figure 1

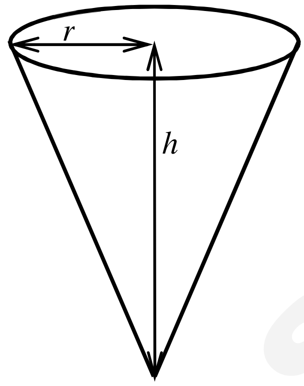


Figure 2

Figure 1 shows a circular piece of paper of radius 5 cm after removing the minor sector *OAB*. The edges *OA* and *OB* are joined to form a cone-shaped drinking cup with radius r and height h as shown in Figure 2.

(i) Show that the volume, V, of the conical cup can be expressed as $\frac{1}{3}\pi\left(25h-h^3\right)$. [2]

(ii) Using differentiation, find the maximum volume of the cup. [5]

(iii) After the cup is fully filled, water leaks at a rate of 3 cm^3 per minute from the cup. Find the exact rate of change of the height of the water in the cup when the height of water is 2 cm. [3]

5 **(i)** Sketch the graph of C with equation $y=\ln\left(2x+3\right)$, stating the equation of any asymptotes and the exact coordinates of intersections with the axes. [2]

The point P on C has coordinates $\left(1, \ln 5\right)$. The tangent to C at P meets the x-axis at T.

(ii) Show that the exact x-coordinate of T is $1-\frac{5}{2}\ln 5$. [4]

(iii) Hence find the area bounded by the tangent to C at P, the x-axis and the curve C, leaving your answers to 4 decimal places. [2]

Section B: Statistics [60 marks]

6 The seven letters in the word ELEMENT are to be arranged to form different 7-letter code words. Find the probability that a code word chosen at random

(i) has all the three E's next to each other, [2]

(ii) has all the three E's separated. [3]

7 An egg wholesaler packs their chicken eggs according to their weights. The chicken eggs are weighed and classified according to the table below.

Criteria	Classification of egg
Weight of chicken egg more than 65 grams	Premium
Weight of chicken egg less than 48 grams	Small

A large batch of randomly chosen chicken eggs are weighed and classified accordingly and it is found that 12% are premium and 6% are small. Assuming a normal distribution, find the mean weight and standard deviation of a randomly chosen chicken egg. [5]

The wholesaler also distributes ostrich eggs for sale.

Explain whether or not a normal model is likely to be appropriate for the weight of an egg chosen at random from the combined group of chicken eggs and ostrich eggs. [1]

8 The events A and B are such that $\mathrm{P}(A') = 0.14$, $\mathrm{P}(B) = 0.34$ and $\mathrm{P}(A' \cap B') = 0.08$.

(i) Draw a Venn Diagram to represent this situation, showing the probability in each of the four regions. [3]

(ii) Determine if A and B are independent events. [2]

(iii) Find the probability that exactly one of A and B occurs. [1]

(iv) Given that $\mathrm{P}(C \mid A) = p$ and events B and C are mutually exclusive, find the largest possible value of p. [3]

9 A machine produces n toys a day. Over a long time, it is found that $100p\%$ of the toys produced by the machine is defective. The number of defective toys produced in a day by the machine is denoted by X.

(i) State, in the context of this question, two conditions needed for X to be well modelled by a binomial distribution. [2]

Assume now that X indeed follows a binomial distribution.

(ii) The mean number and variance of defective toys produced by the machine in a day is 10 and 9.75 respectively. Find the value of n and p. [3]

(iii) Given that there are at most five defective toys produced by the machine on a particular day, find the probability that the last toy produced by the machine is defective. [4]

10 Siti runs a full marathon every year at the annual Scanchart marathon since 2010 and she records the time she takes to complete the marathon, in minutes above 3 hours 30 minutes, starting from her first year of participation. The results are shown in the table below.

Year (x)	2010	2011	2012	2013	2014	2015	2016	2017
time (t)	80.2	56.6	48.2	42.5	37.3	34.9	29.7	25.0

(i) Draw a scatter diagram for the above data, clearly labelling the axes. [2]

(ii) Find the product moment correlation coefficient and comment on its value in the context of the question. [2]

(iii) Find the equation of the regression line t on x. Interpret the gradient of the regression line in the context of the question. [2]

(iv) Using the regression line you have found in **(iii)**, predict Siti's timing for the marathon she will run in 2018. Comment on the reliability of your prediction. [2]

(v) Explain why a linear model may not be appropriate in this context. [1]

11 A manufacturer claims that his company produces cans of coffee that have an average volume of 230 millilitres each, with a standard deviation of 10 millilitres. A random sample of 80 such cans is taken. A test, is carried out at the 5% significance level, on whether the manufacturer has overstated the mean volume.

(i) Write down appropriate hypotheses to test the manufacturer's claim, defining any symbols you use. [2]

(ii) State what you understand by the term "at 5% level of significance" in the context of the question. [1]

(iii) Use an algebraic method to calculate the set of possible values of the average volume of beverage dispensed for which the null hypothesis would not be rejected. [5]

(iv) Explain why there is no need to know anything about the population distribution of the volumes of the beverages. [2]

12 Brandon can choose to take the subway or taxi to and fro between his home and office. The one-way journey times, in minutes, by taxi and by subway have independent normal distributions. The means and standard deviations of these distributions are shown in the following table.

	Mean	Standard Deviation
Taxi	59	2
Subway	61	3

(i) Find the probability that a randomly chosen taxi journey takes less than an hour. [1]

(ii) Find the probability that two randomly chosen taxi journeys take more than an hour each. [2]

(iii) The probability that the total journey time taken for a randomly chosen trip to and fro between home and office by taxi is more than two hours in total is denoted by p. Without calculating its value, explain why p will be greater than your answer in part **(ii)**. [1]

(iv) Brandon takes a subway from his home to his office to pick up a bulky item. He then takes a taxi back to his home to store the item. Finally, he takes a taxi back to his office. Find the probability that Brandon's total journey takes more than 3 hours. [3]

Journeys are charged by the time taken. For the taxi journey, the charge is \$0.69 per minute and for the subway journey, the charge is \$0.13 per minute.

Let A represent the cost of the taxi journey from Brandon's home to work.

Let B represent the cost of the subway journey from Brandon's home to work.

(v) Find $\mathrm{P}\left(2A-\left(B_1+B_2\right)<68\right)$ and explain, in the context of the question, what your answer represents. [5]

End of Paper

2018 H1 MATH (8865/01) JC 2 PRELIM – SUGGESTED SOLUTIONS

Qn	Solutions
1	**Exponential & Logarithm**
(a)	$e^{3-2x} = 5e^{2x}$ $\ln\left(e^{3-2x}\right) = \ln 5 + \ln\left(e^{2x}\right)$ $\Rightarrow 3-2x = \ln 5 + 2x$ $\Rightarrow x = \frac{3-\ln 5}{4} = \frac{3}{4} - \frac{1}{4}\ln 5$ $\Rightarrow y = 5e^{2\left(\frac{3-\ln 5}{4}\right)}$ $\Rightarrow y = 5e^{\left(\frac{3-\ln 5}{2}\right)}$ Hence coordinates are $\left(\frac{3}{4} - \frac{1}{4}\ln 5,\ \ 5e^{\left(\frac{3-\ln 5}{2}\right)}\right)$.
(b)	$y = ax^2 + bx + c$ $\frac{dy}{dx} = 2ax + b$ At $(1,4)$, $4 = a + b + c$ $0 = 2a + b$ At $(0,-1)$, $-1 = c$ Using GC to solve, $a = -5,\ b = 10$.

Qn	Solution
2	**Equations & Inequalities**
(a)	$x^2 + px + 2 = 0$ No real roots $\Rightarrow$ Discriminant < 0 $p^2 - 4(1)(2) < 0$ $\left(p-\sqrt{8}\right)\left(p+\sqrt{8}\right) < 0$ $-\sqrt{8} < p < \sqrt{8}$ $-2\sqrt{2} < p < 2\sqrt{2}$
(b)	$kx^2 + \left(2k+1\right)x - 3 = 2x + 3$ $kx^2 + \left(2k+1\right)x - 3 - 2x - 3 = 0$ $kx^2 + \left(2k-1\right)x - 6 = 0$ Intersect at least once $\Rightarrow$ Discriminant ≥ 0 $\left(2k-1\right)^2 - 4k\left(-6\right) \geq 0$ $4k^2 - 4k + 1 + 24k \geq 0$ $4k^2 + 20k + 1 \geq 0$  Using GC, $k \leq -4.95$ or $k \geq -0.0505$ $\left(3 \text{ s.f.}\right)$

Qn	Solution
3	**Techniques of Integration**
(a)	$\dfrac{dy}{dx}=(2-x)^2$ Equation of curve is $y=\int(2-x)^2 dx=\int x^2-4x+4\ dx$ $y=\dfrac{x^3}{3}-2x^2+4x+C$ Since (0,2) is on the curve, $C=2$. $\therefore\ y=\dfrac{x^3}{3}-2x^2+4x+2$ Using GC, required area = $\int_0^5 \dfrac{x^3}{3}-2x^2+4x+2\ dx=28.75\,\text{units}^2$
(b)	$\int_2^k \dfrac{1}{x^2}-\dfrac{2}{1-x}\ dx$ $=\left[-\dfrac{1}{x}+2\ln\lvert 1-x\rvert\right]_2^k$ $=\left(-\dfrac{1}{k}+2\ln\lvert 1-k\rvert\right)-\left(-\dfrac{1}{2}+2\ln\lvert 1-2\rvert\right)$ $=-\dfrac{1}{k}+2\ln\lvert 1-k\rvert+\dfrac{1}{2}$

Qn	Solution
4	**Application of Differentiation**
(i)	Using Pythagoras' Theorem, $r^2+h^2=5^2$ $\Rightarrow r^2=25-h^2$ $V=\frac{1}{3}\pi r^2 h$ $=\frac{1}{3}\pi\left(25-h^2\right)h$ $=\frac{1}{3}\pi\left(25h-h^3\right)$
(ii)	$V=\frac{1}{3}\pi\left(25h-h^3\right)$ $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{1}{3}\pi\left(25-3h^2\right)$ For max vol, $\frac{\mathrm{d}V}{\mathrm{d}h}=0$ $\frac{1}{3}\pi\left(25-3h^2\right)=0$ $25-3h^2=0$ $h^2=\frac{25}{3}$ since $h>0, h=\frac{5}{\sqrt{3}}=\frac{5\sqrt{3}}{3}$ 1st derivative test

h	$\frac{5\sqrt{3}}{3}^-$	$\frac{5\sqrt{3}}{3}$	$\frac{5\sqrt{3}}{3}^+$
$\frac{\mathrm{dV}}{\mathrm{dh}}$	+	0	-
Slope of curve	/	—	\

$\therefore h=\frac{5\sqrt{3}}{3}$ gives max vol.

Alternatively: Using 2nd derivative test

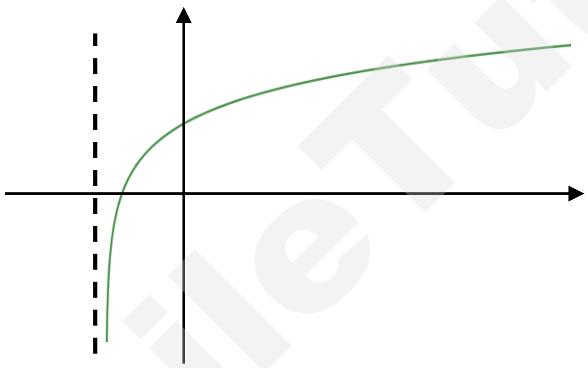
$\frac{\mathrm{d}^2V}{\mathrm{d}h^2}=\frac{1}{3}\pi(-6h)$

$=-2\pi h$

When $h=\frac{5\sqrt{3}}{3}, \frac{\mathrm{d}^2V}{\mathrm{d}h^2}=-\frac{10\sqrt{3}\pi}{3}<0$

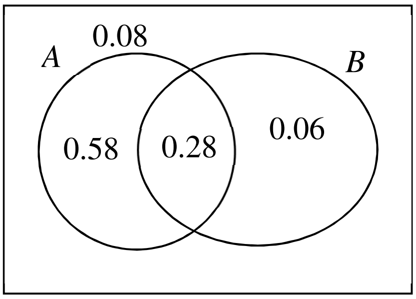
$\therefore h=\frac{5\sqrt{3}}{3}$ gives max vol.

	$V=\frac{1}{3}\pi\left(25\left(\frac{5\sqrt{3}}{3}\right)-\left(\frac{5\sqrt{3}}{3}\right)^3\right)$ $=50.3833$ $=50.4$ (to 3 s.f.)
(iii)	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}$ when $h=2, \frac{\mathrm{d}V}{\mathrm{d}h}=\frac{1}{3}\pi\left(25-3\left(2^2\right)\right)=\frac{13\pi}{3}$ Given $\frac{\mathrm{d}V}{\mathrm{d}t}=-3$, $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{3}{13\pi}\times(-3)$ $=\frac{-9}{13\pi}$

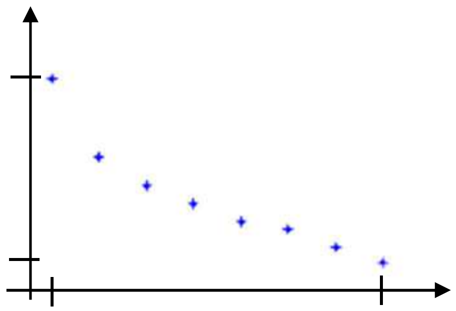
Qn	Solution
5	**Applications of Differentiation & Integration & Curve Sketching**
(i)	
(ii)	$y=\ln(2x+3)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{2x+3}$ At $x=1$, gradient of tangent = $\frac{2}{5}$ Equation of tangent : $y-\ln 5=\frac{2}{5}(x-1)$ At T, $y=0$, $0-\ln 5=\frac{2}{5}(x-1)$ $-\frac{5}{2}\ln 5=x-1$ $x=1-\frac{5}{2}\ln 5$ (Shown)
(iii)	Area = $\int_{1-\frac{5}{2}\ln 5}^{1}\left(\frac{2}{5}(x-1)+\ln 5\right)\mathrm{d}x-\int_{-1}^{1}\ln(2x+3)\,\mathrm{d}x=1.2143$(4 d.p.) **<u>OR</u>** Area = $\frac{1}{2}\left[1-\left(1-\frac{5}{2}\ln 5\right)\right](\ln 5)-\int_{-1}^{1}\ln(2x+3)\,\mathrm{d}x=1.2143$(4 d.p.)

Qn	Solution
6	**Probability (Using P&C)**
(i)	Total number of arrangements = $\frac{7!}{3!} = 840$ Number of arrangements where all the three E's are next to each other = 5! Required probability = $\frac{5!}{840} = \frac{1}{7}$
(ii)	Number of ways to arrange L, M, N T = 4! = 24 Number of ways to slot in the three E's = $\binom{5}{3} = 10$ Required probability = $\frac{24 \times 10}{840} = \frac{2}{7}$

Qn	Solutions
7	**Normal Distribution**
(i)	Let X kg be the weight of a randomly chosen egg. $X \sim \mathrm{N}\left(\mu, \sigma^2\right)$ $\mathrm{P}(X > 65) = 0.12 \Rightarrow \mathrm{P}\left(Z > \dfrac{65-\mu}{\sigma}\right) = 0.12$ $\Rightarrow \dfrac{65-\mu}{\sigma} = 1.174987$ $65 - \mu = 1.174987\sigma$ -------- (1) $\mathrm{P}(X < 48) = 0.06 \Rightarrow \mathrm{P}\left(Z < \dfrac{48-\mu}{\sigma}\right) = 0.06$ $\Rightarrow \dfrac{48-\mu}{\sigma} = -1.554774$ $48 - \mu = -1.554774\sigma$ -------- (2) Solving using GC $\sigma = 6.2277 \approx 6.23$ $\mu = 57.683 \approx 57.7$
	A normal model is not appropriate because when we combine both groups with different distributions, it may result in a bi-modal distribution (i.e. with two modes) which contrasts with a normal distribution that has only one mode.

Qn	Solution
8	**Probability (Using Venn Diagrams)**
(i)	
(ii)	$\mathrm{P}(A)\mathrm{P}(B)=(0.86)(0.34)=0.2924\neq 0.28$ Since $\mathrm{P}(A\cap B)\neq \mathrm{P}(A)\mathrm{P}(B)$, A and B are not independent.
(iii)	Required probability = 0.58 + 0.06=0.64
(iv)	$\mathrm{P}(C\mid A)=p$ $\dfrac{\mathrm{P}(C\cap A)}{\mathrm{P}(A)}=p$ $\mathrm{P}(C\cap A)=0.86p$ Largest p occurs when $\mathrm{P}(C\cap A)=0.58$ $0.58 = 0.86p$ $p=\dfrac{29}{43}=0.674$ (3 s.f.)

Qn	Solution
9	**Binomial Distribution**
(i)	The probability that a randomly chosen toy is defective is constant for all toys. Whether a randomly chosen toy is defective is independent from any other toys.
(ii)	$X \sim \mathrm{B}(n, p)$ $\mathrm{E}(X) = 10 \quad \Rightarrow \quad np = 10$ $\mathrm{Var}(X) = 9.75 \quad \Rightarrow \quad np(1-p) = 9.75$ $1 - p = \frac{9.75}{10} \quad \Rightarrow \quad p = 0.025$ $\therefore n = \frac{10}{0.025} = 400$
(iii)	Let Y be the number of defective toys produced out of 399 toys $Y \sim \mathrm{B}(399, 0.025)$ Required probability $= \frac{\mathrm{P}(Y \le 4) \times 0.025}{\mathrm{P}(X \le 5)}$ = 0.0109 (3 s.f.)

Qn	Solution
10	**Correlation and Regression**
(i)	
(ii)	Using GC, $r = -0.93492 = -0.935$ (3s.f.) Since r is close to -1, there is a strong negative linear correlation between the year of participation, x and the time above 3 hours and 30 minutes, she takes to complete the marathon, t.
(iii)	Using GC, $t = 13611 - 6.7381x$ $\Rightarrow t = 13600 - 6.74x$ (to 3 s.f.) The gradient of -6.74 means that the time taken for Siti to complete the marathon decreases by 6.74 minutes with the passing of every year.
(iv)	When $x = 2018$, $t = 13611 - 6.7381(2018)$ $t = 13.978 = 14.0$ min (to 3 s.f) The predicted time for Siti to complete the marathon in 2018 is 3 hours 44 min. Since $x = 2018$ falls outside the data range of x, the linear relation between t and x may no longer hold. Thus the prediction is not reliable.
(v)	A linear model would not be appropriate in the long run since Siti will be limited by her human capacity and her timing at some point in time is likely to reach a plateau. (OR, a linear model would imply an indefinite decrease, which would imply a negative time in the long run which is not possible.)

Qn	Solution
11	**Hypothesis Testing**
(i)	Let X be the volume of beverage dispensed by the vending machine (in millilitres). Let μ denote the population mean volume of beverage dispensed by the vending machine (in millilitres). $\mathrm{H_0} : \mu = 230$ $\mathrm{H_1} : \mu < 230$
(ii)	There is a probability of 0.05 of incorrectly concluding that the volume of the can of coffee is less than 230 millilitres of beverage, when in fact, it is 230 millilitres.
(iii)	Under H_0, since n = 80 is large, by Central Limit Theorem, $\overline{X} \sim \mathrm{N}\left(230, \frac{10^2}{80}\right)$ approximately. Test Statistic: $Z = \dfrac{\overline{X} - 230}{\sqrt{\frac{10^2}{80}}}$ Level of significance: 5% Reject H_0 if z-value < -1.6449 Under H_0, $z - \text{value} = \dfrac{\overline{x} - 230}{\sqrt{10^2/80}}$ Since the null hypothesis is not rejected at 5% significance level, $\mathrm{H_0}$ should not be rejected. z-value > -1.6449 $\dfrac{\overline{x} - 230}{\sqrt{10^2/80}} > -1.6449$ $\overline{x} > 228.16$ $\therefore \ \{\overline{x} \in \mathbb{R}^+ : \overline{x} > 228\}$
(iv)	There is no need to know anything about the population distribution as the sample size, 80, is sufficiently large and thus by Central Limit Theorem, $\overline{X} \sim \mathrm{N}\left(230, \frac{10^2}{80}\right)$ approximately.

Qn	Solution
12	**Sampling**
(i)	Let X and Y be the journey times by a randomly chosen taxi and a randomly chosen subway respectively (in minutes). $X \sim \mathrm{N}\left(59, 2^2\right)$ $Y \sim \mathrm{N}\left(61, 3^2\right)$ $\mathrm{P}\left(X < 60\right) = 0.69146 \approx 0.691$ (3s.f.)
(ii)	$\left[\mathrm{P}\left(X > 60\right)\right]^2 = \left(1 - 0.69146\right)^2 = 0.095195 \approx 0.0952$ (3s.f.)
(iii)	The event in **(ii)** is a proper subset of the event in **(iii)**, thus p is greater than the answer in part **(ii)**.
(iv)	$X_1 + X_2 + Y \sim \mathrm{N}\left(2\times 59 + 61, 2\times 2^2 + 3^2\right)$ $X_1 + X_2 + Y \sim \mathrm{N}\left(179, 17\right)$ $\mathrm{P}\left(X_1 + X_2 + Y_3 > 180\right) = 0.40418 \approx 0.404$ (3s.f.)
(v)	$A = 0.69X \sim \mathrm{N}\left(40.71, 1.9044\right)$ $B = 0.13Y \sim \mathrm{N}\left(7.93, 0.1521\right)$ $2A - \left(B_1 + B_2\right) \sim \mathrm{N}\left(65.56, 7.9218\right)$ $\mathrm{P}\left(2A - \left(B_1 + B_2\right) < 68\right) = 0.80701 \approx 0.807$ The answer represents the probability that 2 times the cost of a randomly chosen taxi journey exceeds the cost of the sum of 2 randomly chosen subway journeys by less than \$68.

NANYANG JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

Higher 1

MATHEMATICS **8865/01**

Paper 1 **10 September 2018**

3 hours

Additional Materials: Cover Page
Answer Paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagram or graph.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1 David went to a seafood restaurant on three different days to eat lobster, fish and crab. He observed that the price per kilogram of lobster and fish remained constant for all his three visits and the price per kilogram of crab was the same for his first two visits but increased by 20% on his third visit. In addition, the restaurant gave a fifty dollars discount for any bill exceeding \$400. The mass of lobster, fish and crab that he ordered as well as the bill before discount for each visit are shown in the table below.

	First visit	Second visit	Third visit
Lobster (kg)	3.20	4.50	5.60
Fish (kg)	1.50	1.20	2.00
Crab (kg)	6.00	5.20	4.80
Bill before discount (\$)	289.39	309.43	422.76

Find the price per kilogram of lobster, fish and crab during his first visit to the restaurant. [4]

2 The volume of a solid sphere is decreasing at a constant rate of $2\,\text{cm}^3\ \text{s}^{-1}$. Find the rate of change of its total surface area when its radius is 2 cm. [4]

[It is given that the surface area of a sphere is $4\pi r^2$ and the volume of a sphere is $\frac{4}{3}\pi r^3$, where r is the radius of the sphere.]

3 **(a)** Differentiate $\dfrac{\left(\mathrm{e}-\mathrm{e}^{-x}\right)^2}{\mathrm{e}^x}$ with respect to x. [3]

(b) Find $\displaystyle\int \frac{2}{\sqrt{5x-2}}\,\mathrm{d}x$, simplifying your answer. [2]

4 The curve C has equation $y=\mathrm{e}^{-2x+3}$.

(i) Sketch the graph of C, stating the exact coordinates of any points of intersection with the axes and the equations of any asymptotes. [2]

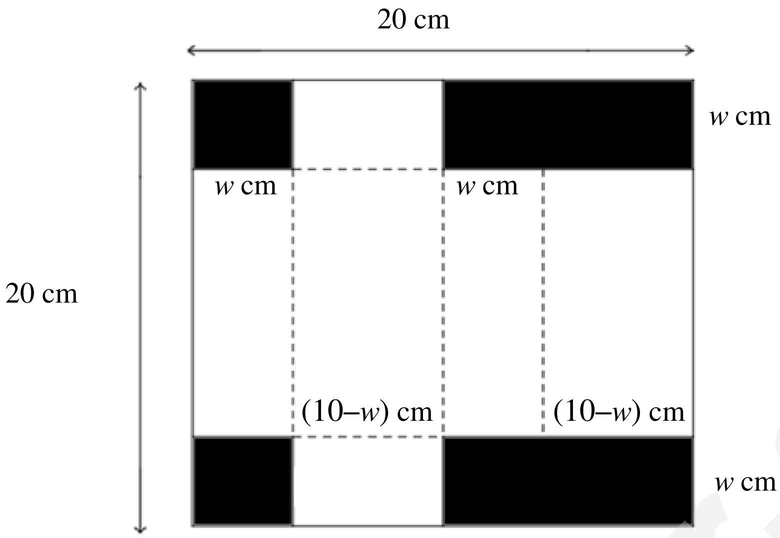
(ii) Find the equation of the tangent to C at the point $x=1$, giving your answer in the form $y=mx+c$, where m and c are exact constants to be determined. [3]

(iii) This tangent meets the x-axis at $x=p$. Find the exact area of the region bounded by C, the tangent, the line $x=2$ and the x-axis. [4]

(iv) Find the range of values of k for which C and $y=\dfrac{1}{k}\mathrm{e}^{-x+3}-\mathrm{e}^3$ do not intersect. [4]

Question 5 is printed on the next page

5



A cardboard, with negligible thickness, is in the shape of a square with side 20 cm. The shaded portions are to be cut off the cardboard and the remaining cardboard will be folded into a box with a top as shown in the diagram above. The volume of the box is V cm^3.

(i) Show that $V = 2w^3 - 40w^2 + 200w$. [2]

(ii) Given that w can vary, using differentiation, find the exact length of w when the volume of the box is a maximum. [5]

A company manufactures the box for sandwiches and sells x (in thousands) of them per month. The monthly revenue \$$R$ is given by the equation $R = 10x - \dfrac{x^2}{10}$.

(iii) Sketch the graph of R against x, stating the coordinates of the intersections with the axes. [2]

(iv) State the maximum monthly revenue of the company and the number of sandwich boxes they must sell to achieve it. [2]

In addition, the monthly cost \$$C$ in producing x sandwich boxes (in thousands) is given by the equation $C = 50 + 2x$.

(v) Denoting the monthly profit receives by the company monthly be P, find an equation relating P and x. [1]

(vi) Justify whether maximum revenue and profit can be achieved at the same time by producing the same number of sandwich boxes. [2]

Section B: Statistics [60 marks]

6 Find the number of different arrangements of the eleven letters in the word 'PERSONALITY' if the arrangements are such that

(i) P, E and R are together, [2]

(ii) S, O and N are separated, [2]

(iii) P, E and R are together or S, O and N are separated. [3]

7 A manufacturer produces balloons of which 40% are oval and 60% are round. 20 balloons are randomly selected and packed into a packet.

(i) In a randomly selected packet of balloons, find the probability that

(a) 14 of them are round, [1]

(b) at least half of them are round. [2]

(ii) 6 packets of balloons are randomly selected. Find the probability that less than 4 of them have at least half of the balloons that are round. [2]

(iii) Instead of packing 20 balloons into one packet, the manufacturer decides to pack 80 balloons into one packet. 60 packets of balloons are randomly selected. Find the probability that on average, at most 49 balloons are round. [3]

Question 8 is printed on the next page

8 A recent study done on the graduates from *SSS* University aims to explore the relationship between their final grade point average (GPA) and their starting salaries. The starting salaries, y thousand dollars, of a random sample of 8 graduates from the university with GPA x are given in the following table.

x	3.2	4.8	2.3	3.6	1.8	4.5	2.7	3.4
y	4.1	5.2	3.5	4.3	3.2	5.8	3.4	4.7

(i) Give a sketch of the scatter diagram of the data. [2]

(ii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iii) Find the equation of the regression line of y on x in the form $y = ax + b$. Sketch this line on your scatter diagram. [2]

(iv) Use the equation of your regression line to calculate an estimate of the starting salary for a graduate who have a GPA of 4.2. State two reasons why you would expect this to be a reliable estimate. [3]

9 A bag contains 3 red balls and 7 blue balls. Whenever a red ball is drawn, it will be replaced in the bag and whenever a blue ball is drawn, it is not replaced. 3 balls are drawn one after another. Construct a probability tree showing this information. [2]

Find the probability that

(i) all the balls are blue, [1]

(ii) at least one of the balls is blue, [2]

(iii) exactly two of the balls are blue. [2]

Given that exactly 2 of the 3 balls drawn are blue, find

(iv) the probability that the first ball drawn is blue. [3]

10 The mean mass of cereal in a packet is printed as 475 grams on its packaging. The manager suspects that the mean mass may not be 475 grams. He took 30 randomly chosen packets and measured their mass and the data is summarized as follows.

$$\sum x = 14\,127, \qquad \sum x^2 = 6\,655\,913$$

(i) Find the unbiased estimates of the population mean and variance. [2]

(ii) Test at the 5% significance level whether the manager's suspicion is correct. [5]

(iii) State, with a reason, whether it is necessary to assume the mass of cereal in a packet has a normal distribution. [1]

(iv) The manager now wants to change the mean mass printed on the packaging to m grams. Based on the sample above and using 5% significant level, find the maximum mean mass in grams (to the nearest whole number) that should be printed so that it will not overstate the actual mass of the cereal. [6]

11 Every morning, a student needs to reach the bus stop at 7:30am to catch a bus to school. If he reaches school after 8:00am, he will be considered late. Assume that the waiting times for a bus is normally distributed with mean 8 minutes and variance 5 minutes^2, and the duration of the bus journey is normally distributed with mean 20 minutes and variance 4 minutes^2.

(i) On a randomly chosen day, find the probability that he will be late for school. [3]

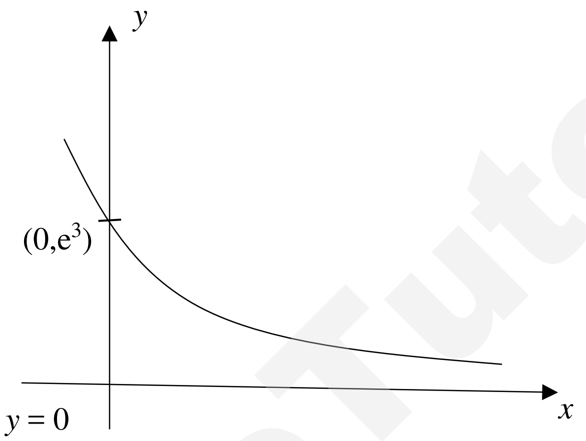
(ii) In 20 days, what is the expected number of days he will be late for school? [1]

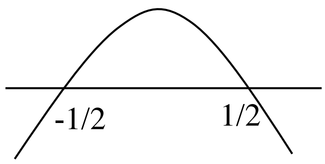
(iii) In order to reduce the probability of him being late for school, he has to reach the bus stop earlier than 7:30am. Find the latest time he needs to reach the bus stop for this probability to be less than 0.01. [3]

(iv) Find the probability that the mean time taken to travel from the bus stop (including waiting for the bus) to school in 40 days is between 28 and 29 minutes. [2]

(v) Bus fare is charged at \$0.085 per minute. Find the probability he has to pay more than \$8.60 for 5 days. [3]

1	Let x, y and z be the price per kilogram of the lobsters, fish and crabs respectively during his first visit. $3.20x+1.50y+6.00z=289.39$ $4.50x+1.20y+5.20z=309.43$ $5.60x+2.00y+4.80(1.2z)=422.76-50$ Using GC, $x=34.70, y=12.90, z=26.50$	
2	Let V, A, r be the volume, total surface area, radius of the hemisphere respectively. Given that $\frac{\mathrm{d}V}{\mathrm{d}t}=-2$, find $\frac{\mathrm{d}A}{\mathrm{d}t}$ when $r = 2$. $V=\frac{4}{3}\pi r^3 \quad \Rightarrow \quad \frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$ Since, $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}r}\frac{\mathrm{d}r}{\mathrm{d}t}$ $-2=4\pi(2)^2\frac{\mathrm{d}r}{\mathrm{d}t}$ $\frac{\mathrm{d}r}{\mathrm{d}t}=-\frac{1}{8\pi}$ To find $\frac{\mathrm{d}A}{\mathrm{d}t}$: $A=4\pi r^2$ $\frac{\mathrm{d}A}{\mathrm{d}r}=8\pi r$ From $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}r}\frac{\mathrm{d}r}{\mathrm{d}t}$ $\frac{\mathrm{d}A}{\mathrm{d}t}=8\pi(2)\left(-\frac{1}{8\pi}\right)=-2 \quad \text{cm}^2\text{s}^{-1}$ Hence, the surface area is decreasing at the rate of 2 cm^2s^{-1} when $r = 2$ cm.	
3	(a) $\frac{\mathrm{d}}{\mathrm{d}x}\frac{\left(\mathrm{e}-\mathrm{e}^{-x}\right)^2}{\mathrm{e}^x}$ $=\frac{\mathrm{d}}{\mathrm{d}x}\frac{\mathrm{e}^2-2\mathrm{e}^{1-x}+\mathrm{e}^{-2x}}{\mathrm{e}^x}$ $=\frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{e}^{2-x}-2\mathrm{e}^{1-2x}+\mathrm{e}^{-3x})$ $=-\mathrm{e}^{2-x}+4\mathrm{e}^{1-2x}-3\mathrm{e}^{-3x}$	

	(b) $\int \frac{2}{\sqrt{5x-2}}\,dx$ $= 2\int (5x-2)^{-\frac{1}{2}}\,dx$ $= 2\frac{(5x-2)^{\frac{1}{2}}}{\frac{1}{2}(5)} + C$ $= \frac{4}{5}\sqrt{5x-2} + C$	
4	(a)(i)  	
	(ii) $y = e^{-2x+3}$ $\frac{dy}{dx} = -2e^{-2x+3}$ When $x = 1$, $y = e$ $\frac{dy}{dx} = -2e$ $y - e = -2e(x-1)$ $y = e - 2e(x-1)$ $= -2ex + 3e$	
	(iii) $y = -2ex + 3e$ When $y = 0$, $x = p$ $2ep = 3e$ $p = \frac{3}{2}$	

	$\int_1^2 \mathrm{e}^{-2x+3}dx - \frac{1}{2}\left(\frac{3}{2}-1\right)\left(-2\mathrm{e}+3\mathrm{e}\right)$ $= \left[\frac{\mathrm{e}^{-2x+3}}{-2}\right]_1^2 - \frac{1}{2}\left(\frac{1}{2}\right)(\mathrm{e})$ $= -\frac{\mathrm{e}^{-1}}{2} + \frac{\mathrm{e}}{2} - \frac{\mathrm{e}}{4}$ $= -\frac{1}{2\mathrm{e}} + \frac{\mathrm{e}}{4}$	
	(b) For the 2 curve to intersect $\mathrm{e}^{-2x+3} = \frac{1}{k}\mathrm{e}^{-x+3} - \mathrm{e}^3$ $k\mathrm{e}^{-2x}\mathrm{e}^3 = \mathrm{e}^{-x}\mathrm{e}^3 - k\mathrm{e}^3$ $k\mathrm{e}^{-2x} = \mathrm{e}^{-x} - k$ Let $y = \mathrm{e}^{-x}$ $ky^2 - y + k = 0$ For y to have no solution so that they don't intersect $1 - 4(k)(k) < 0$ $(1-2k)(1+2k) < 0$ $k < -\frac{1}{2}$ *or* $k > \frac{1}{2}$  	
5	(i) $V = (20-2w)(10-w)w$ $= 2(10-w)^2 w$ $= 2(100-20w+w^2)w$ $= 200w - 40w^2 + 2w^3$	
	(ii) $\frac{\mathrm{d}V}{\mathrm{d}w} = 6w^2 - 80w + 200$ $\frac{\mathrm{d}V}{\mathrm{d}w} = 0$ $6w^2 - 80w + 200 = 0$ $3w^2 - 40w + 100 = 0$ $(3w-10)(w-10) = 0$ $w = \frac{10}{3}$ or $w = 10$ When $w = \frac{10}{3}$,	

w	$\left(\frac{10}{3}\right)^-$	$\frac{10}{3}$	$\left(\frac{10}{3}\right)^+$
Gradient	+ve	0	-ve

Hence, when $w=\frac{10}{3}$, it will give a maximum volume.

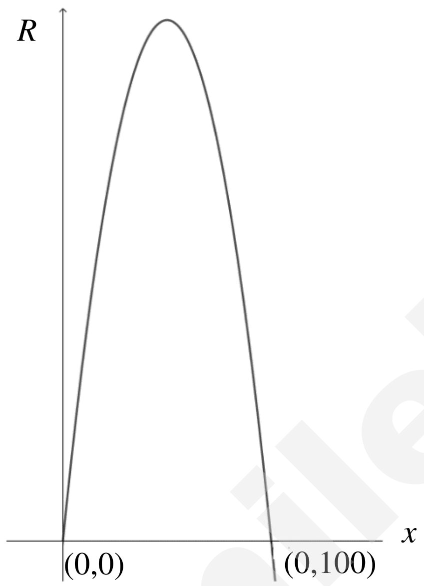
When $w=10$,

w	$(10)^-$	10	$(10)^+$
Gradient	-ve	0	+ve

Hence, when $w=10$, it will give a minimum volume.

Therefore, $w=\frac{10}{3}$.

(iii)



(iv)
number of sandwich boxes = 50 000
maximum revenue = \$250

(v)

$$P=10x-\frac{x^2}{10}-(50+2x)$$
$$=8x-\frac{x^2}{10}-50$$

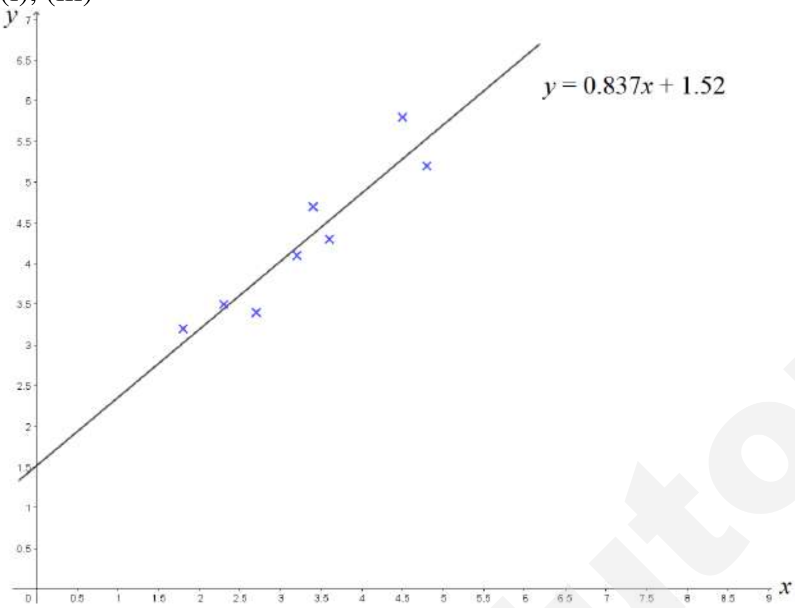
(vi)

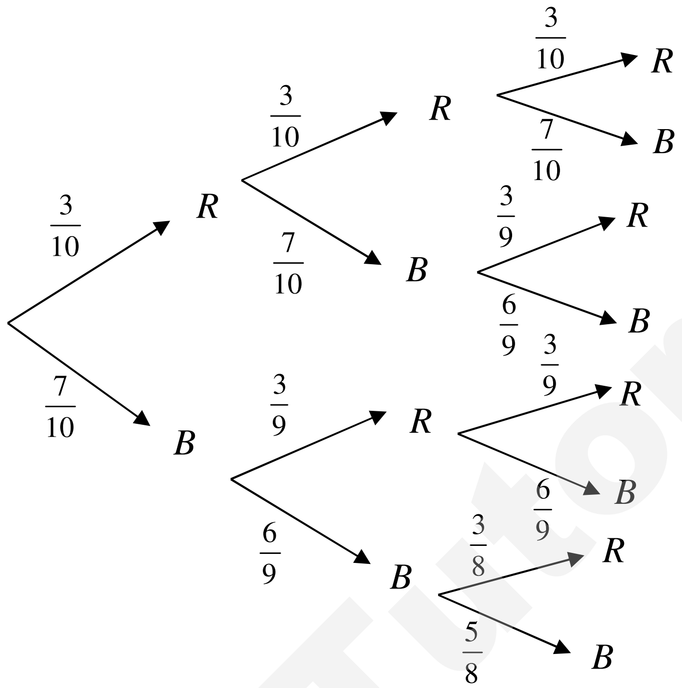
$$P=8x-\frac{x^2}{10}-50$$
$$\frac{dP}{dx}=8-\frac{x}{5}$$

For maximum profit, $\frac{dP}{dx}=0$

$$8-\frac{x}{5}=0$$

	$x=40$ Maximum profit happens when the company has to produce 40 000 boxes. Since the company has to produce 50 000 sandwich boxes to attain maximum revenue and produce 40 000 sandwich boxes to attain maximum profit, the level of production is not the same to produce maximum revenue and maximum profit.	
6	(i) No. of ways = 3! 9! = 2 177 280 (ii) No. of ways = 8! ${}^{9}C_{3}$ 3! = 20 321 280 (iii) No. of ways where P, E and R are together and S, O and N are separated = 3! 6! ${}^{7}C_{3}$ 3! = 907 200 $\therefore$ No. of ways P, E and R are together or S, O and N are separated = No. of ways P, E and R are together + No of ways S, O and N are separated – No. of ways where P, E and R are together and S, O and N are separated = 2 177 280 + 20 321 280 – 907 200 = 2 1591 360	
7	(i)(a) Let X be the random variable denoting the number of balloons out of 20 that are round. $X \sim \mathrm{B}(20, 0.6)$ $\mathrm{P}(X=14)=0.12441=0.124$	
	(i)(b) $\mathrm{P}(X \geq 10)$ $=1-\mathrm{P}(X \leq 9)$ $=0.87249$ $=0.872$	
	(ii) Let Y be the random variable denoting the number of packets out of 6 with at least half of the balloons round. $Y \sim \mathrm{B}(6, 0.87249)$ $\mathrm{P}(Y<4)$ $=\mathrm{P}(Y \leq 3)$ $=0.030738$ $=0.0307$	
	(iii) Let W be the random variable denoting the number of balloons (in a packet) out of 80 that are round. $W \sim \mathrm{B}(80, 0.6)$ $\overline{W}=\dfrac{W_1+W_2+\ldots+W_{60}}{60}$ Since n = 60 is large, by Central Limit Theorem, $\overline{W} \sim \mathrm{N}(80(0.6), \dfrac{80(0.6)(0.4)}{60})$ approximately.	

	$\mathrm{P}\left(\bar{W} \le 49\right)$ $= 0.96145$ $= 0.961$	
8	(i), (iii) 	
	(ii) $r = 0.936$ As the GPA increases, the starting salary increases in a strong linear correlation	
	(iii) $y = 0.837x + 1.52$	
	(iv) $y = 0.837(4.2) + 1.52 = 5.0354$ Starting salary = \$5035.40 The r value of 0.936 is close to 1. $x = 4.2$ is within the data range.	

9	Let R denote the event "a red ball is drawn" and B denote the event "a blue ball is drawn". 	
	(i) P(all the balls are blue) = $\frac{7}{10}\times\frac{6}{9}\times\frac{5}{8}=\frac{7}{24}$ or 0.291 (to 3 sig fig)	
	(ii) P(at least one of the balls is blue) = 1 – P(all balls are red) $= 1 - \frac{3}{10}\times\frac{3}{10}\times\frac{3}{10}$ $= \frac{973}{1000}$ or 0.973	
	(iii) P(exactly two of the balls are blue) = P(RBB) + P(BRB) + P(BBR) $= \left(\frac{3}{10}\times\frac{7}{10}\times\frac{6}{9}\right)+\left(\frac{7}{10}\times\frac{3}{9}\times\frac{6}{9}\right)+\left(\frac{7}{10}\times\frac{6}{9}\times\frac{3}{8}\right)$ $= \frac{847}{1800}$ or 0.471 (to 3 sig fig)	

	(iv) P(the first ball drawn is blue | exactly 2 of the 3 balls drawn are blue) $= \dfrac{\text{P(the first ball drawn is blue AND exactly 2 of the 3 balls drawn are blue)}}{\text{P(exactly 2 of the 3 balls drawn are blue)}}$ $= \dfrac{\text{P(BRB)} + \text{P(BBR)}}{\text{P(exactly 2 of the 3 balls drawn are blue)}}$ $= \dfrac{\left(\frac{7}{10}\times\frac{3}{9}\times\frac{6}{9}\right)+\left(\frac{7}{10}\times\frac{6}{9}\times\frac{3}{8}\right)}{\frac{847}{1800}}$ $= \dfrac{85}{121}$ or 0.702	
10	(i) $n = 30$, $\overline{x} = \dfrac{\sum x}{n} = \dfrac{14127}{30} = 470.9 = 471$ (to 3 sig. fig) $s^2 = \dfrac{1}{n-1}\left(\sum x^2 - \dfrac{\left(\sum x\right)^2}{n}\right) = \dfrac{1}{29}\left(6655913 - \dfrac{(14127)^2}{30}\right) = 120.99 = 121$ (to 3 sig fig)	
	(b) Let μ denote the population mean mass of packets of cereals H_0: $\mu = 475$ H_1: $\mu \neq 475$ Under H_0, $\overline{X} \sim \mathrm{N}\left(475, \dfrac{120.99}{30}\right)$ by Central Limit Theorem. $Z = \dfrac{\overline{X} - 475}{\sqrt{\dfrac{120.99}{30}}} \sim \mathrm{N}(0,1)$ At 5% significant level, reject H_0 if p-value ≤ 0.05	

	Using GC, p-value $= 0.041192 \leq 0.05$ Reject H_o and conclude that 5% significance level, there is sufficient evidence to suggest that the manager's suspicion is correct, i.e. a packet of cereal may not be 475 grams.	
	(c) It is not necessary to assume that the weight of packets of cereals have a normal distribution because the sample size is large, the distribution of the sample mean is approximately normal by the Central Limit Theorem.	
	(d) H_0: $\mu = m$ H_1: $\mu < m$ Under H_0, $\overline{X} \sim \mathrm{N}\left(m, \frac{120.99}{30}\right)$ by Central Limit Theorem. $Z = \frac{\overline{X} - m}{\sqrt{\frac{120.99}{30}}} \sim \mathrm{N}(0,1)$ At 1% significant level, reject H_0 if p-value ≤ 0.05 Since we do not want to reject H_0 $P(\overline{X} \leq 470.9) \geq 0.05$ $P(Z \leq \frac{470.9 - m}{\sqrt{\frac{120.99}{30}}}) \geq 0.05$ $\frac{470.9 - m}{\sqrt{\frac{120.99}{30}}} \geq -1.64485$ $m \leq 474.2$ The packaging should indicate 474 grams.	
11	(i) Let W and J denote the average waiting time and journey time $W \sim \mathrm{N}(8,5)$ $J \sim \mathrm{N}(20,4)$ Let T denote the sum of the waiting and journey time $T \sim \mathrm{N}(28,9)$ $\mathrm{P}(T > 30) = 0.25249 = 0.252$ (to 3 sig fig)	
	(ii) Expected number of days late $= 20 \times 0.25249 = 5.05$ (to 3 sig fig)	

	(iii) Let t be the time that is exceeded by less than 1% of his waiting and journey time $\mathrm{P}(T > t) < 0.01$ $\mathrm{P}(T < t) > 0.99$ $t > 34.98$ To nearest minutes t = 35 minutes So the latest time he can be at the bus stop is 7:25am if he is to have less than 1% chance of being late.	
	(iv) Let $\overline{T}$ be the mean traveling time for 40 days $\overline{T} \sim \mathrm{N}(28, \frac{9}{40})$ $\mathrm{P}(28 < \overline{T} < 29) = 0.48249 = 0.482$ (to 3 sig fig)	
	(v) Let F denote the bus fare per day so $F = 0.085J$ $F \sim \mathrm{N}(0.085 \times 20, 0.085^2 \times 4)$ $F \sim \mathrm{N}(1.7, 0.0289)$ For 5 days $F_1 + ... + F_5 \sim \mathrm{N}(8.5, 0.1445)$ $\mathrm{P}(F_1 + ... + F_5 > 8.6) = 0.39625 = 0.396$ (to 3 sig fig)	

NATIONAL JUNIOR COLLEGE
SENIOR HIGH 2 PRELIMINARY EXAMINATION
Higher 1

MATHEMATICS 8865/01

Paper 1 **11 September 2018**

3 hours

Additional Materials: Answer Paper
List of Formulae (MF26)
Cover Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, registration number, subject tutorial group, on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in the brackets [ ] at the end of each question or part question.

This document consists of **7** printed pages.

National Junior College

Section A: Pure Mathematics [40 marks]

1 Find the exact value of $\displaystyle\int_{-1}^{0}\left(\sqrt{e^x}-\frac{3}{\sqrt{e^x}}\right)^2 \mathrm{d}x$. [3]

2 The curve C has equation $y=ax^2+bx+c$.

(i) Given that $b=2$ and $c=-1$, find the range of values of a such that C has 2 real distinct roots for all real values of x. [2]

(ii) Given that $a>0$ and $b=0$, find the range of values of c such that C does not intersect the line $y=2$. [2]

3 **(i)** Differentiate $\ln\left(x^2+9\right)$. [1]

(ii) Express $\dfrac{2x^2-x+9}{(1-x)\left(x^2+9\right)}$ in the form $\dfrac{A}{1-x}+\dfrac{Bx}{x^2+9}$ where A and B where are integers to be determined. [2]

(iii) Hence find $\displaystyle\int \frac{2x^2-x+9}{(1-x)\left(x^2+9\right)}\,\mathrm{d}x$. [3]

4 A curve C has equation $y=e^{(x+1)^3}$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and explain why $\dfrac{\mathrm{d}y}{\mathrm{d}x}\geq 0$ for all real values of x. [3]

(ii) Hence find the exact coordinates of the stationary point of C. [2]

(iii) Sketch C, stating the exact coordinates of any points of intersection with the axes and the equations of any asymptotes. Mark the point of inflexion with a cross. [3]

(iv) Find the equation of the tangent to C at the point where $x=-1.5$, giving your answer in the form of $y=mx+c$ where m and c are exact constants. [2]

(v) Using the values of m and c found in **(iv)**, find the range of values of x such that $e^{(x+1)^3}\leq mx+c$. [1]

5 A company produces three flavours of chocolate bars: Milk, Dark and White. Each chocolate bar weighs 100 grams. The manufacturing cost of 500 grams of Milk chocolate bars is the same as the manufacturing cost of a Dark Chocolate bar. The total manufacturing cost of 10 Dark chocolate bars is \$16.70 more than the manufacturing cost of 8 White chocolate bars. The total manufacturing cost of 6 Milk, 5 Dark Chocolate and 3 White chocolate bars is \$25.08.

(i) By writing down three linear equations, find the manufacturing cost of each flavour of chocolate bars. [3]

The company is trialling a new flavour of chocolate bar, Caramel. A financial consultant for the company predicts that the profit \$$P$, generated by selling a Caramel chocolate bar, will be related to the manufacturing cost (\$$x$) by the equation

$$P = 6(3x)^{0.4} - 0.01x^4 - 8,\ \ x \geq 0\,.$$

(ii) Sketch the graph of P against x, stating the coordinates of the points where the graph crosses the x- and P-axes. Find the maximum value of P. [3]

(iii) Find the value of $\dfrac{\mathrm{d}P}{\mathrm{d}x}$ at $x = 2.3$ and give an interpretation of the value found in the context of the question. [2]

(iv) Given that the manufacturing cost of a Caramel chocolate bar is \$2.30, state the selling price. $[\text{Profit } = \text{ selling price} - \text{manufacturing cost}]$ [1]

(v) If the manufacturing cost of a Caramel chocolate bar is increased to \$6, would you advise the company to produce chocolate bar of this flavour? Justify your answer. [1]

The financial consultant predicts that the marginal cost of manufacturing a Caramel chocolate bar, C (in thousands) is related to the quantity manufactured, q (in thousands) by the equation

$$C = \frac{1}{8}q^2(q-4) + 3,\ \ q > 0\,.$$

[Marginal cost = additional cost incurred in the production of one more unit of a good or service]

(vi) Use differentiation to find the minimum value of C, justifying that the value is minimum. Give an interpretation of the value found in the context of the question. [4]

(vii) State the area bounded by the curve C, the line $q = 1$ and the axes. Give an interpretation of the value found in the context of the question. [2]

Section B: Statistics [60 marks]

6 The digits 1, 2, 3, 4 and 5 are arranged randomly to form a five-digit number. No digit is repeated. Find the probability that

(i) the last two digits are both odd, [2]

(ii) not all odd digits are together, [3]

(iii) the number is greater than 30 000 and odd. [3]

7 5.2% of all insurance agents from a large insurance company, Prodential, obtained an Advanced Diploma in Insurance (ADI). Each agent can only receive one ADI each. A sample that contains 30 randomly chosen agents from Prodential is obtained.

(i) Find the probability that at least three insurance agents obtained an ADI each. [2]

(ii) Explain the importance of choosing a sample from a 'large' company. [1]

$100p$% of all insurance agents from another large insurance company, Avila, obtained an Advanced Diploma in Insurance (ADI), where $p < 0.5$. A sample that contains 10 randomly chosen agents from Avila is obtained. It is given that the number of insurance agents with ADI in this sample can be modelled by a binomial distribution.

(iii) Given the probability that 5 agents from Avila obtained an ADI each is 0.12294, show that p satisfies an equation of the form $p(1-p) = k$ where k is a constant to be determined. Hence find the value of p correct to 2 decimal places. [3]

Given instead that $p = 0.24$.

(iv) Based on the same sample, state the mean and variance of the number of agents from Avila who have obtained an ADI. [1]

(v) Forty samples, where each sample consists of 10 randomly chosen agents from Avila, are chosen. Find the probability that the mean number of insurance agents with ADI is between 2.3 and 2.5. [2]

8 A school's concert band comprises 24 woodwind players, n brass players and 10 percussion players. $\frac{1}{3}$ of all woodwind players, $\frac{2}{5}$ of all brass players and $\frac{4}{5}$ of all percussion players are Senior High students, while the rest are Junior High students.

One student from the concert band is selected at random.

(i) Given that he or she is a Senior High student, show that the probability of selecting a brass player is $\frac{n}{n+40}$. [2]

(ii) Find, in terms of n, the probability that the student is neither a percussion player nor a Senior High student. [2]

(iii) The probability of selecting a Senior High student or a percussion player but not both is $\frac{1}{3}$. Find the value of n. [3]

(iv) Using the value of n found in **(iii)**, explain whether the event of selecting a Junior High student is independent of the event of selecting a brass player. [2]

9 A ball manufacturer claims that average diameter of the balls is 15 cm. To test this claim, a random sample of 100 balls is checked and the diameters, x cm, are summarised by

$$\sum(x-15)=23 \qquad \sum(x-15)^2=113.26.$$

(i) Find the unbiased estimates of the population mean and variance. [2]

(ii) Test, at the 5% significance level whether the manufacturer's claim is valid. [4]

(iii) Explain, in the context of the question, the meaning of "at the 5% significance level". [1]

(iv) State, giving a reason, whether any assumption is needed in order for the test to be valid. [1]

A new sample of a 100 balls is collected and the mean diameter of this sample is 15.25 cm. A test, at the $\alpha\%$ significance level, shows that the manufacturer's claim is justified. Find the set of values of α. [2]

10 **(i)** Sketch a scatter diagram that might be expected when x and y are related approximately as given in each of the cases (A) and (B) below. In each case your diagram should include 6 points, approximately equally spaced with respect to x, and with all x- and y- values positive.

(A) $y = a + bx^2$, where a and b are positive constants.

(B) $y = c + \dfrac{d}{x}$, where c is a positive constant and d is a negative constant. [2]

Obesity is becoming increasingly prevalent across the globe. To investigate the effects of obesity on one's health, a study was conducted to determine if the blood pressures of adults aged between 40 and 50 years old are dependent on their Body Mass Index (BMI). Data from six patients in this age-group from a hospital was collected. Their BMI, m, kg m^{-2} and systolic blood pressure, s, in mmHg, are as follows.

m	22	27	31	36	40	44
s	120	150	168	172	179	183

(ii) Sketch the scatter diagram for these values, labelling the axes clearly. [2]

(iii) Using your answer to part **(ii)**, explain why model (B) is more appropriate for modelling these values and calculate the product moment correlation coefficient for this case. [3]

(iv) Find the equation of the regression lines of s on $\dfrac{1}{m}$ and $\dfrac{1}{m}$ on s. [2]

(v) Choose an appropriate line found in part **(iv)** and use it to estimate the BMI of another patient (of a similar age profile) whose systolic blood pressure is 110 mmHg. Comment on the reliability of your estimate. [2]

(vi) State, in context, a limitation of using the regression equation in part **(v)** to estimate the systolic blood pressure of *other* people with known BMI in the interval $22 \le m \le 44$. [1]

11 In this question you should state clearly the values of the parameters of any normal distribution you use.

A supermarket sells two types of durians, Red Prawn and Black Gold. The masses, in kilograms, of the durians each have independent normal distributions. The means and standard deviations of these distributions, and the selling prices, in $ per kilogram, are shown in the following table.

	Mean mass (kg)	Standard deviation (kg)	Selling price ($ per kg)
Red Prawn	0.25	k	1.50
Black Gold	0.35	0.03	2.40

(i) Over a long period of time, it is found that 20% of Red Prawn durians have mass equal to or less than 0.15 kg. Find the value of k. [2]

Suppose that the true value of k is 0.02.

(ii) Three Red Prawn durians are randomly selected. Find the probability that exactly one of the durians has mass less than 0.24 kg and exactly one of the durians has mass more than 0.26 kg. [2]

(iii) Find the probability that the total mass of six randomly chosen Red Prawns durians is within 0.2 kg of the total mass of five randomly chosen Black Gold durians. [4]

Mr Phang buys three Red Prawn durians and three Black Gold durians. Mr Fong buys ten Red Prawn durians.

(iv) Find the probability that Mr Phang pays more than Mr Fong. [4]

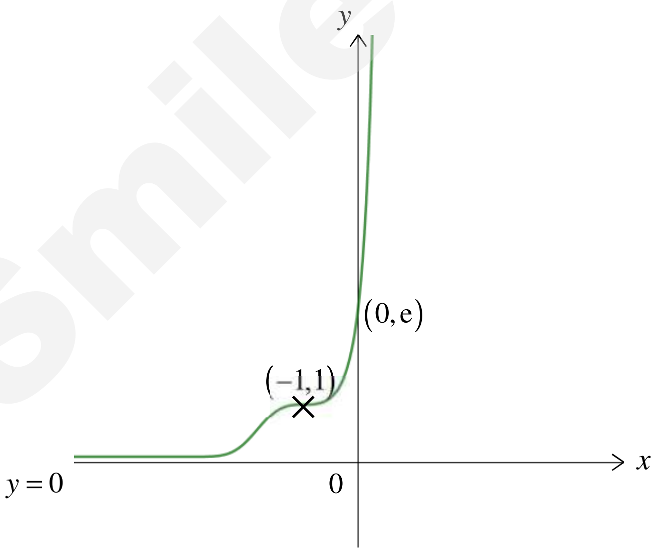
--- END OF PAPER ---

Qn	Solution
1	$\int_{-1}^{0}\left(\sqrt{e^{x}}-\frac{3}{\sqrt{e^{x}}}\right)^{2}dx=\int_{-1}^{0}\left(\left(e^{x}\right)^{\frac{1}{2}}-3\left(e^{x}\right)^{-\frac{1}{2}}\right)^{2}dx$ $=\int_{-1}^{0}\left(\left(e^{x}\right)-6+9\left(e^{-x}\right)\right)dx$ $=\left[e^{x}-6x-9e^{-x}\right]_{-1}^{0}$ $=\left[e^{0}-0-9e^{0}\right]-\left[e^{-1}-6(-1)-9e^{-1(-1)}\right]$ $=(1-0-9)-\left(e^{-1}+6-9e\right)$ $=-14-e^{-1}+9e$

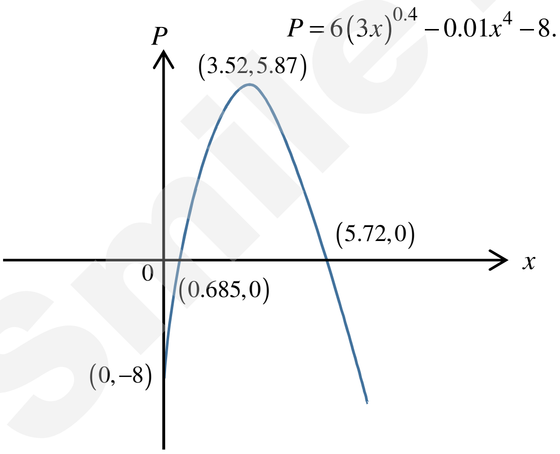
Qn	Solution
2(i)	$y = ax^2 + 2x - 1$ Since the curve has 2 real distinct roots, $b^2 - 4ac > 0,\ a \neq 0$ $4 - 4(a)(-1) > 0$ $a > -1,\ a \neq 0$
(ii)	$y = ax^2 + c$ **Method 1** Min Point of curve: $(0, c)$ Since the curve does not intersect the line $y = 2$ and $a > 0$, $c > 2$ **Method 2** $ax^2 + c = 2$ $ax^2 + (c-2) = 0$ Since the curve does not intersect the line, $b^2 - 4ac < 0$ $-4(a)(c-2) < 0$ $4(a)(c-2) > 0$ Since $a > 0,\ c - 2 > 0 \Rightarrow c > 2$

Qn	Solution
3(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln(x^2+9)\right] = \frac{2x}{x^2+9}$
(ii)	$\frac{A}{1-x} + \frac{Bx}{x^2+9} = \frac{A(x^2+9)+(1-x)(Bx)}{(1-x)(x^2+9)}$ $= \frac{Ax^2+9A+Bx-Bx^2}{(1-x)(x^2+9)}$ $= \frac{(A-B)x^2+Bx+9A}{(1-x)(x^2+9)}$

Qn	Solution
	Comparing coefficients with $\dfrac{2x^2-x+9}{(1-x)(x^2+9)}$: $x^2: A-B=2$ $x: B=-1$ $\text{constant}: 9A=9 \Rightarrow A=1$ Therefore, $A=1,\ B=-1$ Hence $\dfrac{2x^2-x+9}{(1-x)(x^2+9)}=\dfrac{1}{1-x}-\dfrac{x}{x^2+9}$ (shown)
(iii)	$\displaystyle\int \frac{2x^2-x+9}{(1-x)(x^2+9)}\,\mathrm{d}x=\int \frac{1}{1-x}-\frac{x}{x^2+9}\,\mathrm{d}x$ $\displaystyle=\int \frac{1}{1-x}\mathrm{d}x-\frac{1}{2}\int \frac{2x}{x^2+9}\,\mathrm{d}x$ $\displaystyle=-\ln\lvert 1-x\rvert-\frac{1}{2}\ln(x^2+9)+C$

Qn	Solution
4(i)	$y=\mathrm{e}^{(x+1)^3}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{(x+1)^3}\times\dfrac{\mathrm{d}}{\mathrm{d}x}\left[(x+1)^3\right]$ $=3(x+1)^2\,\mathrm{e}^{(x+1)^3}$ Since $(x+1)^2\ge 0$ and $\mathrm{e}^{(x+1)^3}>0$ therefore $\dfrac{\mathrm{d}y}{\mathrm{d}x}\ge 0$.
(ii)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$ $3(x+1)^2\,\mathrm{e}^{(x+1)^3}=0$ Since $\mathrm{e}^{(x+1)^3}>0$ for all x, $(x+1)^2=0$ $x=-1$ When $x=-1$, $y=1$ so the coordinates of the stationary point is $(-1,1)$.
(iii)	

(iv)	$\frac{dy}{dx}=3(x+1)^2 e^{(x+1)^3}$ when $x=-\frac{3}{2}$, $y=e^{\left(-\frac{3}{2}+1\right)^3}=e^{-\frac{1}{8}}$, $\frac{dy}{dx}=3\left(\frac{1}{4}\right)e^{-\frac{1}{8}}=\frac{3}{4}e^{-\frac{1}{8}}$ Equation of tangent at $x=-\frac{3}{2}$: $y-e^{-\frac{1}{8}}=\frac{3}{4}e^{-\frac{1}{8}}\left[x-\left(-\frac{3}{2}\right)\right]$ $y=\frac{3}{4}e^{-\frac{1}{8}}\left(x+\frac{3}{2}\right)+e^{-\frac{1}{8}}=\frac{3}{4}e^{-\frac{1}{8}}x+\frac{9}{8}e^{-\frac{1}{8}}+e^{-\frac{1}{8}}$ $=\frac{3}{4}e^{-\frac{1}{8}}x+\frac{17}{8}e^{-\frac{1}{8}}$ Therefore, $m=\frac{3}{4}e^{-\frac{1}{8}}$, $c=\frac{17}{8}e^{-\frac{1}{8}}$
(v)	$e^{(x+1)^3}\leq\frac{3}{4}e^{-\frac{1}{8}}x+\frac{17}{8}e^{-\frac{1}{8}}$ By GC $-2.83\leq x\leq -0.173$

Qn	Solution
5(i)	Let M, D and W be the manufacturing cost (in \$) of a 100 gram Milk, Dark and White chocolate bar respectively. 500 grams of Milk chocolate bars is equivalent to 5 Milk chocolate bars. $5M = D$ $10D - 8W = 16.7$ $6M + 5D + 3W = 25.08$ or $5M - D + 0W = 0$ $0M + 10D - 8W = 16.7$ $6M + 5D + 3W = 25.08$ By GC: $M = 0.63$ $D = 3.15$ $W = 1.85$
(ii)	 Maximum value of P is 5.87 (2.d.p.).
(iii)	By GC, $\frac{dP}{dx} = 1.77\,(2.d.p)$ at $x = 2.3$. When manufacturing cost to produce a Caramel chocolate bar is \$2.30, every dollar spent on manufacturing cost will generate a profit of \$1.77.
(iv)	Profit $= 6\left[3(2.3)\right]^{0.4} - 0.01(2.3)^4 - 8 = \$4.7126$ Selling price $= 4.7126 + 2.3 = \$7.01\,(2.d.p.)$

<table>
<tr>
<td>(v)</td>
<td>
When $x = 6$, $P = -1.89397$. This suggests that when there is no manufacturing cost, the company will make a loss at $1.89 dollars per Caramel chocolate bar, which is not advisable.

Or

$\frac{\mathrm{d}P}{\mathrm{d}x}$ at $x = 6$ is $-7.37\,(2\text{d.p.})$ which suggest that the company will make a loss of $7.37 for every dollar spent when the manufacturing cost to produce a Caramel chocolate bar is at $6.
</td>
</tr>
<tr>
<td>(vi)</td>
<td>
$C = \frac{1}{8}q^2(q-4)+3 = \frac{1}{8}q^3 - \frac{1}{2}q^2 + 3$
$\frac{\mathrm{d}C}{\mathrm{d}q} = \frac{3}{8}q^2 - q = q\left(\frac{3}{8}q - 1\right)$

To find the minimum value of C as q varies,
$\frac{\mathrm{d}C}{\mathrm{d}q} = q\left(\frac{3}{8}q - 1\right) = 0$
$q = 0$ or $C = \frac{3}{8}q - 1 = 0 \Rightarrow q = \frac{8}{3}$
Reject $q = 0$ since $q > 0$

To calculate minimum value of C,
$C = \frac{1}{8}\left(\frac{8}{3}\right)^2\left(\frac{8}{3} - 4\right) + 3 = \frac{49}{27} = 1.81\,(2\text{d.p.})$

To show that it is minimum value,
<table>
<tr><td>$q = \frac{8}{3} - 0.01$
$= 2.6567$</td><td>$q = \frac{8}{3}$</td><td>$q = \frac{8}{3} + 0.01$
$= 2.6767$</td></tr>
<tr><td>$\frac{\mathrm{d}C}{\mathrm{d}q} = -0.00996$</td><td>$\frac{\mathrm{d}C}{\mathrm{d}q} = 0$</td><td>$\frac{\mathrm{d}C}{\mathrm{d}q} = 0.0101$</td></tr>
</table>

OR
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	$\frac{\mathrm{d}^2C}{\mathrm{d}q^2}=\frac{3}{4}q-1$ when $q=\frac{8}{3},\frac{\mathrm{d}^2C}{\mathrm{d}q^2}=\frac{3}{4}\left(\frac{8}{3}\right)-1=1>0$ Hence $C=1.8148=1.81(3.\text{s.f.})$ is a minimum value at $q=\frac{8}{3}$ The lowest marginal cost of \$1814.81 is achieved when the quantity of chocolate bar manufactured is about 2666.
(vii)	By GC $\int_0^1\left[\frac{1}{8}q^2(q-4)+3\right]\mathrm{d}t=2.864583=2.86(3.\text{s.f.})$ The total cost of manufacture for the first thousand of Caramel chocolate bars is \$2864.58.

Qn	Solution
6(i)	Required probability $= \frac{3!\times 3\times 2}{5!} = \frac{36}{120} = \frac{3}{10}$ Or $\frac{3}{5}\times\frac{2}{4} = \frac{3}{10}$
(ii)	Number of ways such that all odd digits are together $= 3!\times 3! = 36$ Number of ways such that not all odd digits are together $= 5! - 36 = 84$ Required probability $= \frac{84}{5!} = \frac{7}{10}$
(iii)	Case 1: 3 or 5 as the first digit Probability $= \frac{2\times 2\times 3!}{5!} = \frac{1}{5}$ Case 2: 4 as the first digit Probability $= \frac{1\times 3\times 3!}{5!} = \frac{3}{20}$ Required probability $= \frac{1}{5} + \frac{3}{20} = \frac{7}{20}$

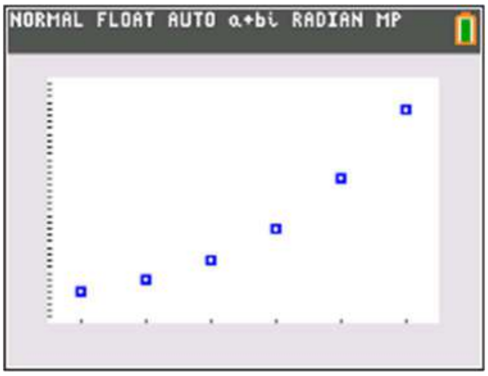
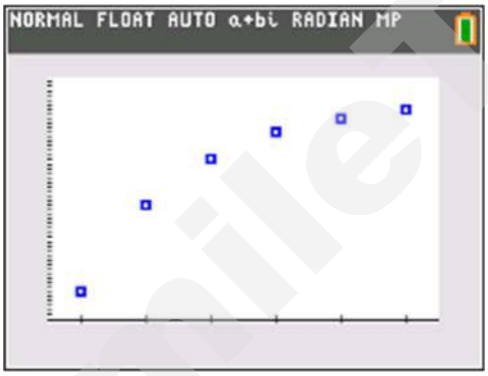
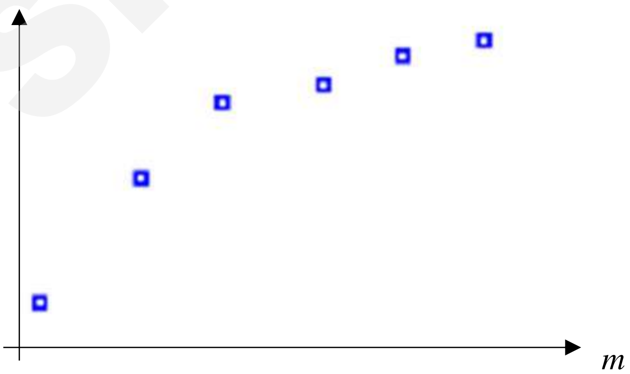
Qn	Solution
7(i)	Let Y denote the number of insurance agents with ADI out of 30 randomly chosen Prodential insurance agents. Then $$Y \sim \mathrm{B}(30,\ 0.052).$$ $\mathrm{P}(Y \geq 3) = 1 - \mathrm{P}(Y \leq 2)$ $= 0.2032366271$ $= 0.203 (3\text{s.f.})$
(ii)	In order for the number of insurance agents to follow a binomial distribution, the **probability of an agent chosen to have an ADI must be constant**. This can be assumed true only when the company is large.
(iii)	Let W denote the number of insurance agents with ADI out of 10 randomly chosen Avila insurance agents. Then $$W \sim \mathrm{B}(10,\ p).$$ $\mathrm{P}(W = 5) = 0.12294$ $\binom{10}{5} p^5 (1-p)^5 = 0.12294$ $p(1-p) = \left(\frac{0.12294}{252}\right)^{\frac{1}{5}} \approx 0.21760,$ i.e., $k = 0.21760 (5\text{s.f.})$ $p^2 - p + 0.21760 = 0$ $p = 0.68 \text{ or } 0.32$ Since $p < 0.5$, $p = 0.32$
(iv)	$\mathrm{E}(W) = 10 \times 0.24 = 2.4$ $\mathrm{Var}(W) = 10 \times 0.24 \times (1 - 0.24) = 1.824$
(v)	Since sample size, 40, is large, by Central Limit Theorem, $$\overline{W} \sim \mathrm{N}\left(2.4, \frac{1.824}{40}\right) \text{ approximately.}$$ Therefore, $\mathrm{P}(2.3 < \overline{W} < 2.5) \approx 0.36042 = 0.360$ (to 3 s.f.).

Qn	Solution
8	<table><tr><td></td><td>Senior High</td><td>Junior High</td><td>**Total**</td></tr><tr><td>Woodwind</td><td>8</td><td>16</td><td>**24**</td></tr><tr><td>Brass</td><td>$\frac{2}{5}n$</td><td>$\frac{3}{5}n$</td><td>$\boldsymbol{n}$</td></tr><tr><td>Percussion</td><td>8</td><td>2</td><td>**10**</td></tr><tr><td>**Total**</td><td>$\frac{2}{5}n+16$</td><td>$\frac{3}{5}n+18$</td><td>$\boldsymbol{n+34}$</td></tr></table>
8(i)	P(brass player | SH student) $=\frac{\mathrm{P}(\text{brass player} \cap \text{SH student})}{\mathrm{P}(\text{SH student})}$ $=\frac{\text{No. of Senior High brass players}}{\text{No. of Senior High concert band students}}$ $=\frac{\frac{2}{5}n}{\frac{1}{3}\times 24+\frac{2}{5}n+\frac{4}{5}\times 10}$ $=\frac{\frac{2}{5}n}{\frac{2}{5}n+16}$ $=\frac{n}{n+40}$
(ii)	P(neither SH student nor percussion player) = P(either JH woodwind player or JH brass player) $=\frac{\text{No. of JH woodwind players}+\text{No. of JH brass players}}{\text{Total no. of concert band students}}$ $=\frac{16+\frac{3}{5}n}{24+n+10}$ $=\frac{\frac{3}{5}n+16}{n+34}$ $=\frac{3n+80}{5n+170}$

(iii)	$\mathrm{P}(\text{SH student or percussion player but not both})$ $= \mathrm{P}(\text{SH student or percussion player})$ $-\mathrm{P}(\text{SH student and percussion player})$ $= \dfrac{\left(\frac{2}{5}n+8+8+2\right)-8}{n+34}$ $= \dfrac{\frac{2}{5}n+10}{n+34}$ $\dfrac{\frac{2}{5}n+10}{n+34} = \dfrac{1}{3}$ $\dfrac{6}{5}n+30 = n+34$ $6n+150 = 5n+170$ $n = 20$
(iv)	$\mathrm{P}(\text{JH student and brass player}) = \dfrac{12}{54} = \dfrac{2}{9}$ $\mathrm{P}(\text{JH student}) = \dfrac{30}{54} = \dfrac{5}{9}$ $\mathrm{P}(\text{brass player}) = \dfrac{20}{54} = \dfrac{10}{27}$ $\mathrm{P}(\text{JH student}) \times \mathrm{P}(\text{brass player}) = \dfrac{50}{243}$ $\mathrm{P}(\text{JH student}) \times \mathrm{P}(\text{brass player})$ $\neq \mathrm{P}(\text{JH student and brass player})$ Hence the two events are not independent.

Qn	Solution
9(i)	$\bar{x}=\frac{\sum x}{n}=\frac{23}{100}+15=15.23$ $s^2=\frac{1}{99}[113.26-\frac{(23)^2}{100}]\approx 1.09061=1.09(3\text{s.f.})$
(ii)	Test H_0 : $\mu=15$ against H_1 : $\mu\neq 15$ Level of significance = 5% (2-tailed) Under H_0, $\bar{X}\sim N\left(15,\frac{1.09061}{100}\right)$ approximately by Central Limit Theorem since sample size $n=100$ is large. Hence $Z=\frac{\bar{X}-15}{\sqrt{\frac{S^2}{100}}}:\ N(0,1)$ approximately. From GC, Critical region: $z\leq -1.95996$ or $z\geq 1.95996$ Test statistic, $z=\frac{15.23-15}{\sqrt{\frac{1.09061}{100}}}=2.202>1.95996$ Or p-value = $0.0276<0.05$ H_0 is rejected. Hence we conclude that there is sufficient evidence at the 5 % significance level to show that the manufacturer's claim is not valid.
(iii)	It means that there is a probability of 0.05 of wrongly rejecting the claim that the mean diameter of ball is 15 cm.
(iv)	No assumption about the diameter of the balls is required. Given that the sample size 100 (>20) is large, the mean diameter of the balls will follow a Normal Distribution approximately by Central Limit Theorem.
Last part	Do not reject H_0 if p-value $>\frac{\alpha}{100}$ From GC,

Qn	Solution
	p-value = 0.016670 $\therefore \frac{\alpha}{100} < 0.016670 \Rightarrow \alpha < 1.67$ (3.s.f.)

Qn	Solution
10(i)	$y = a + bx^2$  $y = c + \frac{d}{x}$ 
(ii)	 End points: (22, 120) & (44, 183)
(iii)	• The equation $y = c + \frac{d}{x}$, is more appropriate.

Qn	Solution
	• From the scatter diagram, as m increases, s increases at an decreasing rate. $r = -0.982$
(iv)	Regression line of s on $\frac{1}{m}$: $s = 248.61 - 2731.4\left(\frac{1}{m}\right)$ $= 249 - \frac{2730}{m}$ (3 s.f.) Regression line of $\frac{1}{m}$ on s: $\frac{1}{m} = 0.088937 - 0.00035326\, s$ $= 0.0889 - 0.000353s$ (3 s.f.)
(v)	Since systolic blood pressure depends on weight, we should use the regression line of s on $\frac{1}{m}$: $110 = 248.61 - 2731.4\left(\frac{1}{m}\right)$ $m \approx 19.706$ Since $s = 110$ lies outside the range of values of s, **extrapolation** is required which gives an **unreliable** estimate.
(vi)	• Too few patients were selected for the equation of the regression line to be reliable in estimation. • The data is only valid for estimating the blood pressure for people of a similar age profile. • The data is only valid for estimating the blood pressure of patients with similar medical conditions. • A person's blood pressure is not fixed and is influenced by other factors at time of measurement, such as physical activity and/or varying emotional states like anxiety.

Qn	Solution
11(**i**)	Let A be the random variable denoting the mass of a Red Prawn durian. $A \sim \mathrm{N}(0.25, k^2)$ $\mathrm{P}(A \le 0.15) = 0.2$ $\mathrm{P}\left(Z \le \frac{0.15-0.25}{k}\right) = 0.2$ $\mathrm{P}\left(Z \le \frac{-0.10}{k}\right) = 0.2$ $-\frac{0.10}{k} = -0.84162$ $k = 0.119 \, (3\text{s.f.})$
(ii)	$\mathrm{P}(A<0.24)\mathrm{P}(0.24 \le A \le 0.26)\mathrm{P}(A \ge 0.26) \times 3! = 0.219$
(iii)	Let B be the random variable denoting the mass of a Black Gold durian. Let $T = (A_1 + A_2 + \ldots + A_6) - (B_1 + B_2 + \ldots + B_5)$ $\mathrm{E}(T) = 6\mathrm{E}(A) - 5\mathrm{E}(B) = 6(0.25) - 5(0.35) = -0.25$ $\mathrm{Var}(T) = 6\mathrm{Var}(A) + 5\mathrm{Var}(B) = 6(0.02)^2 + 5(0.03)^2 = 0.0069$ $\mathrm{P}(-0.2 < T < 0.2) = 0.274$
(iv)	Let \$$V$ and \$$W$ be the amount that Mr Phang and Mr Fong pay respectively.

$V = 1.5\left(A_{V1} + A_{V2} + A_{V3}\right) + 2.4\left(B_1 + B_2 + B_3\right)$

$W = 1.5\left(A_{W1} + A_{W2} + \ldots + A_{W10}\right)$

Need to compute $\mathrm{P}(V > W)$ or $\mathrm{P}(V - W > 0)$

$\mathrm{E}(V - W) = \mathrm{E}(V) - \mathrm{E}(W)$

$= (1.5)(3)\mathrm{E}(A) + (2.4)(3)\mathrm{E}(B) - \left[(1.5)(10)\mathrm{E}(A)\right]$

$= 3.645 - 3.75$

$= -0.105$

$\mathrm{Var}(V - W) = \mathrm{Var}(V) + \mathrm{Var}(W)$

$= (1.5)^2(3)\mathrm{Var}(A) + (2.4)^2(3)\mathrm{Var}(B) + (1.5)^2(10)\mathrm{Var}(A)$

$= 0.018252 + 0.009$

$= 0.027252$

$V - W \sim \mathrm{N}(-0.105, 0.027252)$

$\mathrm{P}(V - W > 0) = 0.262$

Name	( )	Class	

RIVER VALLEY HIGH SCHOOL
2018 Year 6 Preliminary Examination
Higher 1

MATHEMATICS

8865/01

Paper 1

17 Sep 2018

3 hours

Additional Materials: Answer Paper
List of Formulae (MF26)
Cover Page

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number in the space at the top of this page.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, place the cover page on top of your answer paper and fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1 Find the values of α for which the expression $\alpha x^2+\alpha x-2$ is negative. [4]

2 To create a password, Ah Boy decided to have a password format such that each of its first 3 characters are chosen from the letters A to Z. This is followed by a 6-digit number which is derived from the product of the 3 numbers associated with the first 3 letters as seen in the ASCII table below. An example of a password is "ABC287430" where $287430=65\times 66\times 67$.

Alphabet	Number	Alphabet	Number	Alphabet	Number
A	65	J	74	S	83
B	66	K	75	T	84
C	67	L	76	U	85
D	68	M	77	V	86
E	69	N	78	W	87
F	70	O	79	X	88
G	71	P	80	Y	89
H	72	Q	81	Z	90
I	73	R	82		

He left 3 hints for himself to reveal a forgotten password:

(i) The sum of the number associated with the first letter, twice the number associated with the second letter and thrice the number associated with the last letter is 422.

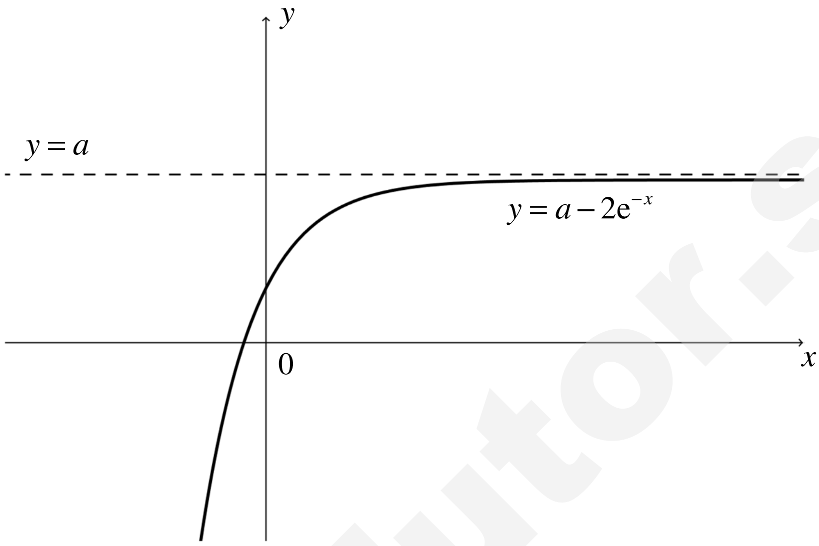
(ii) The sum of twice the number associated with the first letter and three times the number associated with the second letter is 264 more than the number associated with the last letter.

(iii) Thrice the number associated with the first letter is 20 less than the sum of the number associated with the second letter and twice the number associated with the last letter.

Construct a system of linear equations and solve it to find his password. [4]

3 **(a)** Differentiate $\ln\left(\dfrac{x-4}{x+3}\right)$ with respect to x, leaving your answer as a single fraction. [3]

(b) The diagram shows the graph of the curve C with equation $y = a - 2e^{-x}$, where $a > 0$.



(i) Find the equation of the tangent to C at the point where the graph crosses the y-axis. [3]

(ii) For the case where $a = 3$, find the exact area of the region bounded by C, the tangent found in part **(i)** and the line $x = a$. [3]

4 **(i)** Sketch the graph of $y = \dfrac{4+x}{3-x}$, indicating clearly any asymptotes and axial intercepts. [2]

(ii) By adding another suitable graph on the same diagram in part **(i)**, find the range of values of x for which $\dfrac{4+x}{3-x} > -2x+3$. Hence, solve $\dfrac{x-4}{3+x} < -2x-3$. [5]

(iii) Find the exact area of the region bounded by the graph of $y = \dfrac{4+x}{3-x}$, the x and y axes and the line $x = -2$. [3]

[Turn over]

5 **(a)** SHVR Pte Ltd's human resource department estimated that the labour cost is related to the size of the company by the function $C = \mathrm{e}^{\frac{m}{100}} + 1$, where \$$C$ (in thousands) is the total labour cost and m is the size of the company in terms of number of employees. The company has 300 employees currently and is expanding its operation so that the labour cost is increasing at a rate of \$1000 per year. Find the rate at which SHVR Pte Ltd is increasing in size. [4]

(b) Preparing for a product launch, the marketing department of SHVR Pte Ltd estimated that their sales \$$S$ (in thousands) over time t (in years) from product launch is modelled by the equation $S = 2\mathrm{e}^{-t^2+2t} + 10$.

(i) Find the exact maximum sales in the sales projection by the marketing department in thousands. [5]

(ii) Sketch the graph of S against t, indicating the exact coordinates of the axial intercept and turning point. Describe the long term sales of the product. [2]

(iii) Find the cumulative total sales over the first 5 years. [2]

Section B: Probability and Statistics [60 Marks]

6 **(a)** Find the number of ways to arrange the word 'PIKACHU' such that no two vowels are next to each other. [2]

(b) A cafeteria has a menu as follows:

ABC Cafeteria	
Mains Pork Chicken Beef Mutton	
Sides	**Drinks**
Fries Mashed Potatoes Pasta Salad Coleslaw Baked Beans Potato Salad Rice	Iced Lemon Tea Barley Coffee Milk Tea Plain Water

A paying customer is entitled to choose at least 1 main, any number of sides and only 1 drink.

(i) Find the number of possible combinations of food items that is offered by the cafeteria to a paying customer. [3]

(ii) Simone does not eat beef, fries, mashed potatoes and potato salad. Her classmate ordered food for her randomly with choice of 1 main, 3 sides and 1 drink. Find the probability that the food chosen contains at least one item that she does not eat. [3]

[Turn over]

7 VR Secondary School Mathematics department wants to investigate the relationship between the Elementary Mathematics and Additional Mathematics scores obtained by their students. A random sample of 7 students are chosen and their respective scores are given below:

Elementary Mathematics (x)	83	83	80	51	64	74	65
Additional Mathematics (y)	69	67	67	35	50	27	47

(i) Sketch the scatter diagram for the given data and find the product moment correlation coefficient. [2]

(ii) On the scatter diagram, **circle** the point that is most likely an outlier of the data. Find the product moment correlation coefficient when the outlier is omitted from the data. [2]

(iii) Explain why it is not appropriate to comment on the relationship between x and y based on just the r-value without reference to the scatter diagram [1]

For the following part, exclude the outlier identified in part **(ii)** in all calculations.

(iv) Using the regression line of y on x, estimate a student's Elementary Mathematics score when his Additional Mathematics score is 30, rounding it to the nearest whole number. Comment on the reliability of the estimate. [3]

8 Events A and B are such that $\mathrm{P}(A'\cap B)=0.5$, $\mathrm{P}(A)=0.48$ and $\mathrm{P}(B)=0.66$. Find

(i) $\mathrm{P}(A\cup B)'$; [3]

(ii) $\mathrm{P}(A'|B')$; [3]

(iii) $\mathrm{P}(A\cap B)$ and determine if events A and B are independent. [3]

9 A receptionist considers her day to be a "good" day when she gets less than 10 rude calls for the particular day. The number of "good" days in a work week of 5 days, denoted by X, is observed over a long period of time. The probability of having a "good" day is denoted by p.

(i) State two assumptions made to model X as a binomial distribution. [2]

(ii) Given that the probability of a work week with less than two "good" days is 0.888, form an equation in terms of p and solve for p, giving your answer correct to 3 decimal places. [3]

(iii) Given that the receptionist has less than two "good" days in a particular work week, find the probability that the receptionist has a "good" day on the last day of the week. [3]

(iv) A calendar year has 52 weeks and the receptionist is assumed to work 5 days per week regardless of any public holiday. Find the probability that the mean number of "good" days for a week over a calendar year is not more than 2. [3]

10 A newspaper report claims that the mean starting monthly salary of a fresh university graduate is \$3500. However a human resource manager believes that this claim is incorrect. A random sample of 60 fresh university graduates is surveyed and their starting monthly salaries, \$$x$, are summarized by

$$\sum x = 207000, \quad \sum (x - 3400)^2 = 5450000.$$

(i) Calculate the unbiased estimates of the population mean and variance. [2]

(ii) Test, at the 5% significance level, whether the human resource manager's belief is correct. [4]

(iii) State, in the context of the question, the meaning of the p-value found in part **(ii)**. [1]

A second sample of 50 fresh university graduates is surveyed and the sample mean and standard deviation of their starting salaries are found to be \$$\bar{y}$ and \$342 respectively.

(iv) Find the range of values of $\bar{y}$ such that this second sample would result in accepting the manager's belief at the 5% significance level. [3]

(v) Suppose the claim by the newspaper is correct and the standard deviation of the starting salaries of fresh university graduates is \$543, find the probability that a random sample of 50 fresh university graduates have its mean salary below \$3350. [2]

[Turn over]

11 Gary likes to take part in duathlons where there is a total running distance of 15 km and a total cycling distance of 36 km. His timings, in minutes, for running and cycling are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean	Standard Deviation
Timing for running (min)	100	15
Timing for cycling (min)	μ	σ

(i) The probability that he completes his cycling in less than an hour is equal to the probability that he completes his cycling after 2 hours. Write down the value of μ. [1]

(ii) The probability that he completes his cycling under 80 minutes is 0.158655. Show that $\sigma = 10$. [2]

(iii) Find the probability that thrice the time needed for him to complete cycling is more than an hour from twice the time needed for him to complete the running. [2]

(iv) Find the probability that his timing for cycling is less than 80 minutes and his timing for running is less than 90 minutes. [2]

(v) Show that the probability of Gary finishing the duathlon under 170 minutes is 0.134. Give a reason why this probability is greater than the probability calculated in part **(iv)**. [2]

Gary's target timing for completing a duathlon is under 2 hours 50 minutes. Over the past two years, he has already completed 9 duathlons.

(vi) Find the probability that he achieves his target timing in at least 3 but fewer than 6 of the duathlons over the last two years. [3]

End of Paper

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Y6 H1 Prelim 2018 Paper Solutions

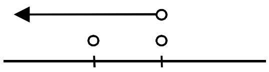
1	Solution [4]	

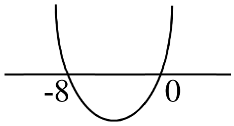
For $\alpha x^2 + \alpha x - 2$ to be always negative,

$\alpha^2 - 4\alpha(-2) < 0$ and $\alpha < 0$

$\alpha(\alpha + 8) < 0$

$-8 < \alpha < 0$





$\therefore -8 < \alpha < 0$

2	Solution [4]	

Let α, β and γ be the values associated with the first, second and third alphabets respectively.

$\alpha + 2\beta + 3\gamma = 422$

$2\alpha + 3\beta - \gamma = 264$

$3\alpha - \beta - 2\gamma = -20$

Using GC, $\alpha = 65, \quad \beta = 69, \quad \gamma = 73$

The following 6 numbers are $65 \times 69 \times 73 = 327405$

Password is AEI327405.

3	Solution [9]	

(a)

Let $y = \ln\left(\frac{x-4}{x+3}\right) = \ln(x-4) - \ln(x+3)$

$$\frac{dy}{dx} = \frac{1}{x-4} - \frac{1}{x+3}$$

$$= \frac{x+3-x+4}{(x-4)(x+3)}$$

$$= \frac{7}{(x-4)(x+3)}$$

(b) (i)

$y = a - 2e^{-x}$

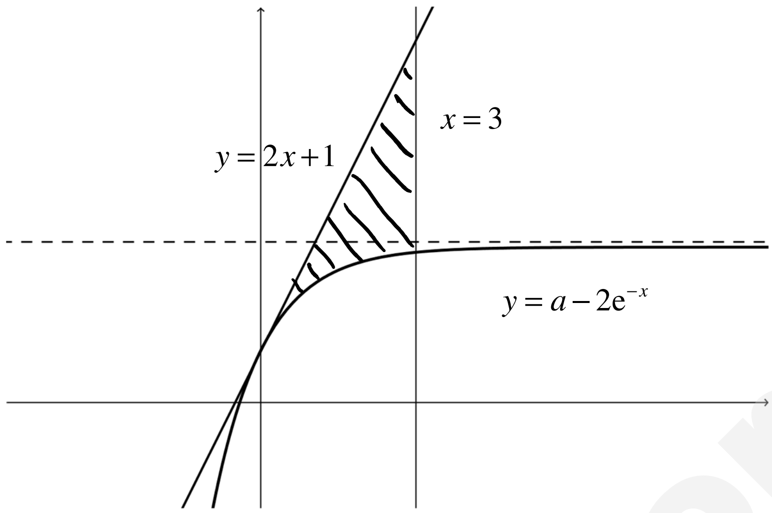
$$\frac{dy}{dx} = 2e^{-x}$$

When $x = 0$, $y = a - 2$, $\frac{dy}{dx} = 2$

$y - (a-2) = 2x$

$y = 2x + (a-2)$

(ii)



Area

$=\int_0^3 2x+1-\left(3-2e^{-x}\right)\,dx$

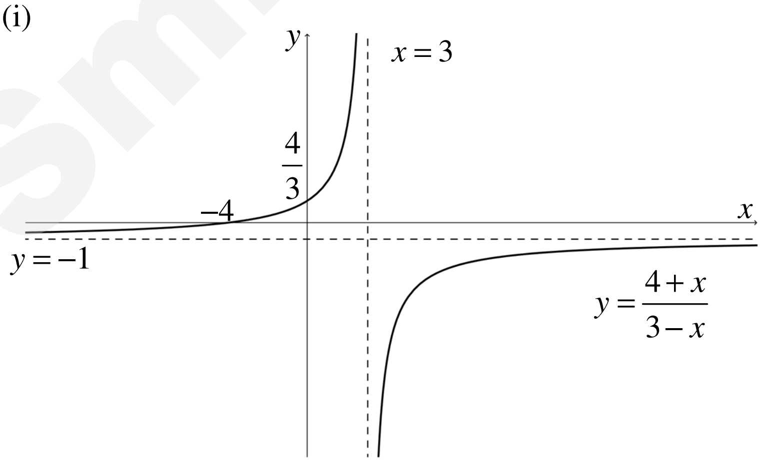
$=\int_0^3 2x-2+2e^{-x}\,dx$

$=\left[x^2-2x-2e^{-x}\right]_0^3$

$=\left(9-6-2e^{-3}\right)-(-2)$

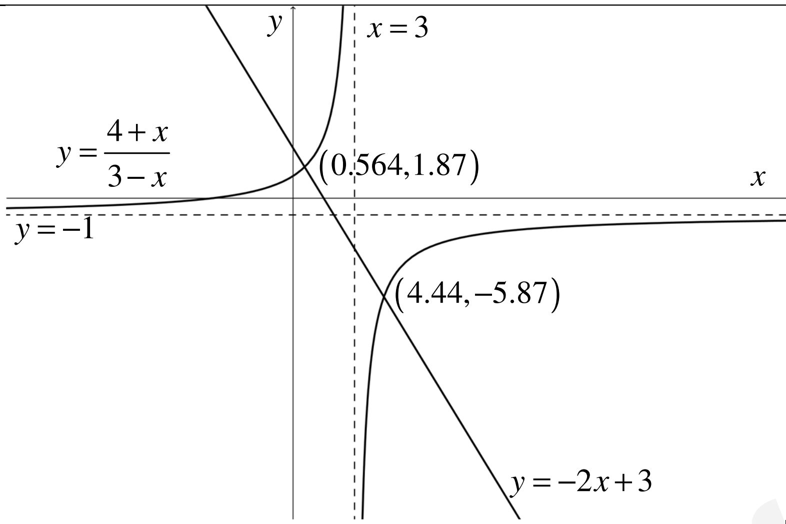
$=5-2e^{-3}$ units^2

4	Solution [10]	
	(i)	



(ii)

Add in the graph of $y=-2x+3$,



Using GC, the 2 points of intersections are $(0.564, 1.87)$ and $(4.44, -5.87)$. Hence $0.564 < x < 3$ or $x > 4.44$

$\frac{x-4}{3+x} < -2x-3$

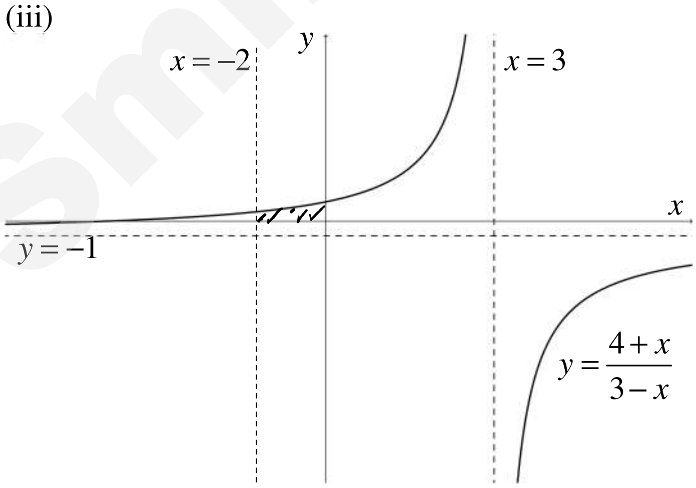
$\Rightarrow \frac{4-x}{3+x} > 2x+3$

Replace x by $-x$

$0.564 < -x < 3$ or $-x > 4.44$

$\therefore x < -4.44$ or $-3 < x < -0.564$

(iii)



Area required

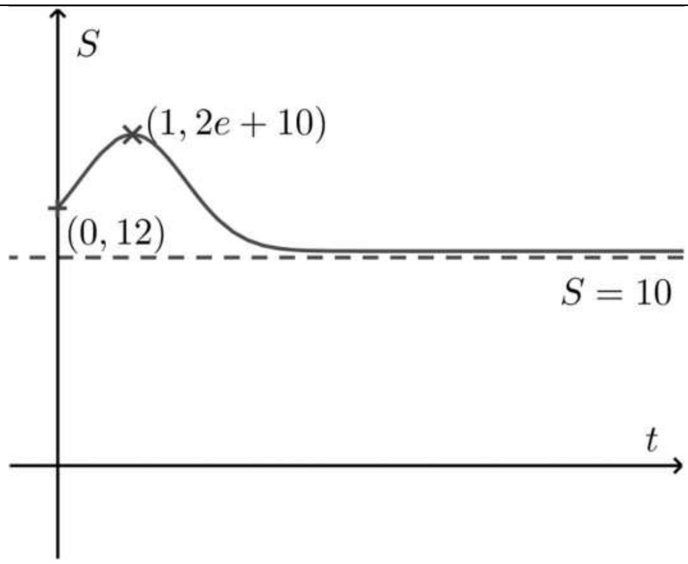
$= \int_{-2}^{0} \frac{4+x}{3-x}\, dx$

$= \int_{-2}^{0} -1 + \frac{7}{3-x}\, dx$

	$=\left[-x-7\ln\lvert 3-x\rvert\right]_{-2}^{0}$ $=(-7\ln 3)-(2-7\ln 5)$ $=7\ln\left(\frac{5}{3}\right)-2$ units^2	

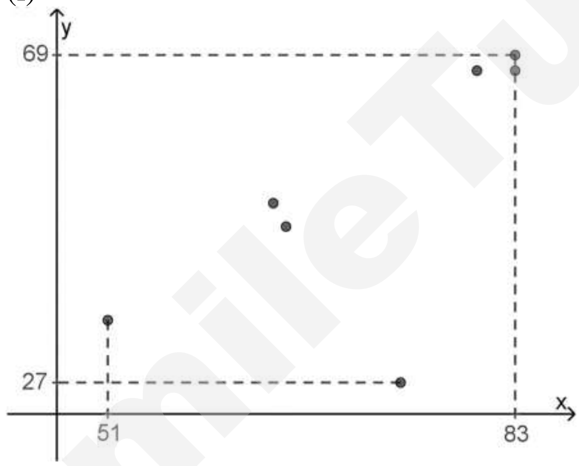
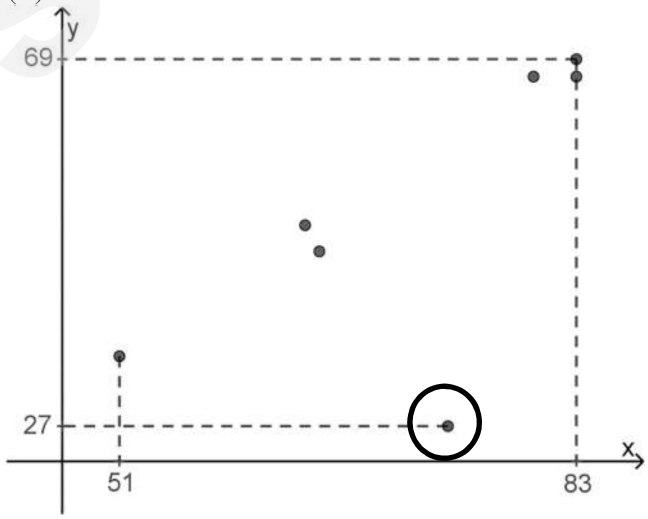
5	Solution [13]	
	(a) Find $\frac{dm}{dt}$ when $\frac{dC}{dt}=1$ and $m=300$. $C=e^{\frac{m}{100}}+1$ $\frac{dC}{dm}=\frac{1}{100}e^{\frac{m}{100}}$ Given $\frac{dC}{dt}=1$ at $m=300$, $\frac{dC}{dt}=\frac{dC}{dm}\times\frac{dm}{dt}$ $1=\frac{1}{100}e^{\frac{300}{100}}\times\frac{dm}{dt}$ $\frac{dm}{dt}=\frac{100}{e^3}=4.98$ employees per year(3 s.f.)	
	(b)(i) $S=2e^{-t^2+2t}+10$ $\frac{dS}{dt}=2(-2t+2)e^{-t^2+2t}$ when $\frac{dS}{dt}=0$, $0=2(-2t+2)e^{-t^2+2t}$ $-2t+2=0$ $t=1$ $\left.\frac{d^2S}{dt^2}\right\rvert_{t=10}=-10.8731055<0\Rightarrow$ maximum at $t=1$ OR [table below] Therefore S is maximum at $t=1$ $\therefore$ max $S=2e^{-(1)^2+2(1)}+10=\$\ (2e+10)$ thousands	
	(b)(ii)	

t	0.995	1	1.005
$\frac{dS}{dt}$	0.054364223	0	-0.054364223

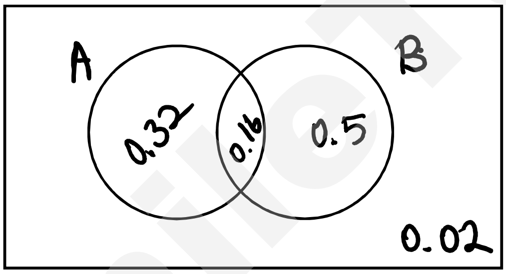
	 The long term sales of the product stabilizes at \$10 thousand.	
	(b)(iii) Total sales $= \int_0^5 2\mathrm{e}^{-t^2+2t} + 10 \, \mathrm{d}t$ $=$ \$58.9 thousand (3 s.f.)	

6	Solution [8]	
	(a) Arrange the consonants first then slot in the vowels into the 5 slots which are between the consonants and at both ends of the 4 letters. No. of ways $= 4! \times \binom{5}{3} \times 3!$ $= 1440$	
	(b)(i) 2 choices for every main, minus 1 where all mains are not added i.e. $(2^4 - 1)$ ways (Alternatively, this can be found by $\binom{4}{1} + \binom{4}{2} + \binom{4}{3} + 1$) Similarly, 2 choices for each side i.e. 2^7 ways (Alternatively, this can be found by $\binom{7}{1} + \binom{7}{2} + \binom{7}{3} + \binom{7}{4} + \binom{7}{5} + \binom{7}{6} + 1$) Lastly choose 1 drink out of 5 choices i.e. $\binom{5}{1}$ ways	

	No. of combinations $=\left(2^4-1\right)\times\left(2^7\right)\times\begin{pmatrix}5\\1\end{pmatrix}$ $=9600$	
	(b)(ii) Number of combinations where the food chosen does not contain any food which Simone does not eat $=\begin{pmatrix}3\\1\end{pmatrix}\begin{pmatrix}4\\3\end{pmatrix}\begin{pmatrix}5\\1\end{pmatrix}=60$ Total number of combinations $=\begin{pmatrix}4\\1\end{pmatrix}\begin{pmatrix}7\\3\end{pmatrix}\begin{pmatrix}5\\1\end{pmatrix}=700$ Probability that the chosen food contains at least 1 item that Simone does not eat $=1-\dfrac{60}{700}=\dfrac{32}{35}$ or 0.914 (3 sig fig)	

7	Solution [8]	
	(i)  $r=0.689$ (3 sig fig)	
	(ii)  $r=0.993$ (3 sig fig)	

	(iii) The r value of 0.689 which is not close to 1 suggests that the 2 scores do not have a strong linear correlation but the scatter diagram shows that other than an outlier point, the rest of the points do exhibit a strong linear correlation.	
	(iv) Regression line of y on x is $y = -19.24043716 + 1.057377049x$ When $y = 30$, $30 = -19.24043716 + 1.057377049x$ $x = 46.56847546$ ≈ 47 The estimate is not reliable as the given value of y is outside the range of the data for y after excluding the outlier.	

8	Solution [9]	
	(i)  $\mathrm{P}(A \cup B) = \mathrm{P}(A' \cap B) + \mathrm{P}(A)$ $= 0.5 + 0.48$ $= 0.98$ $\mathrm{P}(A \cup B)' = 1 - 0.98$ $= 0.02$	
	(ii) $\mathrm{P}(A' \mid B') = \dfrac{\mathrm{P}(A' \cap B')}{\mathrm{P}(B')}$ $= \dfrac{0.02}{1 - 0.66}$ $= 0.0588235294$ ≈ 0.0588	
	(iii)	

	$\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)=\mathrm{P}(A\cup B)$ $0.48+0.66-\mathrm{P}(A\cap B)=0.98$ $\mathrm{P}(A\cap B)=0.16$ OR $P(A\cap B)=P(B)-P(A'\cap B)$ $=0.66-0.5$ $=0.16$ $\mathrm{P}(A)\times\mathrm{P}(B)=0.48\times0.66=0.3168\neq\mathrm{P}(A\cap B)$ A and B are not independent events	

9	Solution [11]	
	(i) The probability of a day being "good" is assumed to be constant. The event of a day being "good" is independent of another day being "good" OR "Good" days occur independently	
	(ii) Let X be the r.v. "no of good days in a week". $X \sim \mathrm{B}(5,p)$ $\mathrm{P}(X<2)=0.888$ $\mathrm{P}(X=0)+\mathrm{P}(X=1)=0.888$ $\binom{5}{0}(1-p)^5+\binom{5}{1}p(1-p)^4=0.888$ $(1-p)^5+5p(1-p)^4=0.888$ Solving, $p=0.36980863\approx0.370$	
	(iii) Let Y be the r.v. number of good days for the first 4 days of a week $Y\sim\mathrm{B}(4,p)$ P("good" day on last day | less than 2 "good" days in a week) $=\dfrac{P(Y=0)\times p}{P(X<2)}$ $=\dfrac{P(Y=0)\times0.36980863}{0.888}$ $=0.0656831364$ ≈0.0657	

(iv)

Since $n = 52$ is large, by CLT

$$\overline{X} \sim \mathrm{N}\left(1.849043, \frac{1.16525104}{52}\right) \text{ approx.}$$

$$\mathrm{P}\left(\overline{X} \le 2\right)$$

$$= 0.8427979243 \approx 0.843$$

10	Solution [12]	

(i)

$$\bar{x} = \frac{207000}{60} = 3450$$

$$\Sigma(x - 3400) = 3000$$

$$s^2 = \frac{1}{59}\left[5450000 - \frac{3000^2}{60}\right] = 89830.50847 \approx 89800$$

(ii)

Let μ be the mean starting salaries of fresh graduates.

Test H_0: $\mu = 3500$ (i.e. manager's belief is incorrect)

Against H_1: $\mu \ne 3500$ (i.e. manager's belief is correct)

Perform a 2-tailed test at 5% level of significance.

Test Statistics:

Under H_0, $\overline{X} \sim \mathrm{N}\left(3500, \frac{s^2}{60}\right)$ approximately

$$Z = \frac{\overline{X} - 3500}{s/\sqrt{60}} \sim \mathrm{N}(0,1) \text{ approximately}$$

Using GC, p-value = 0.196284 (>0.05)

Do not reject H_0. There is insufficient evidence at 5% level of significance that the human resource manager's belief is correct.

(iii)

p-value of 0.196284 is the smallest value of significance level for which the claim that the mean starting monthly salary of a fresh university graduate is \$3500 would be rejected.

Or

p-value of 0.196284 is the probability of obtaining a sample mean as extreme as the one obtained, assuming the claim

	that the mean starting monthly salary of a fresh university graduate is \$3500 is true.	
	(iv) Test $H_0:\ \mu = 3500$ Against $H_1:\ \mu \neq 3500$ Perform a 2-tailed test at 5% level of significance. $s^2 = \frac{50}{49} \times 342^2 = 119351.0204$ Test Statistics: Under H_0, $\overline{Y} \sim \mathrm{N}\left(3500, \frac{119351.0204}{50}\right)$ approximately $Z = \frac{\overline{Y} - 3500}{\sqrt{119024.4898/50}} \sim \mathrm{N}(0,1)$ approximately Since H_0 is rejected, $\frac{\overline{y} - 3500}{\sqrt{119351.0204/50}} < -1.95996$ $\overline{y} < 3404.241954$ <u>Or</u> $\frac{\overline{y} - 3500}{\sqrt{119351.0204/50}} > 1.95996$ $\overline{y} > 3595.758046$ Answer: $\overline{y} < 3400$ or $\overline{y} > 3600$ (to 3sf)	
	(v) $\overline{X} \sim \mathrm{N}\left(3500, \frac{543^2}{50}\right)$ approximately by CLT since sample size (50) is large. $\mathrm{P}(\overline{X} < 3350) = 0.0254$	

11	Solution [12]	
	(i) $\mu = \frac{60+120}{2} = 90$ minutes	
	(ii) Let C be Gary's timing for cycling.	

	$C \sim N(90, \sigma^2)$ $Z = \dfrac{C-90}{\sigma} \sim N(0,1)$ $\mathrm{P}(C<80) = 0.158655$ $\mathrm{P}\left(Z < \dfrac{80-90}{\sigma}\right) = 0.158655$ $\dfrac{80-90}{\sigma} = -1.000001057$ $\sigma = \dfrac{-10}{-1.000001057} \approx 10$	
	(iii) Let R be Gary's timing for running. $R \sim N(100, 15^2)$ $3C - 2R \sim N(70, 150)$ $\mathrm{P}(3C-2R>60) = 0.7928919719 \approx 0.793$ (3 sig fig)	
	(iv) Required probability $= \mathrm{P}(C<80) \times \mathrm{P}(R<90)$ $= 0.0400592579$ ≈ 0.0401 (3 sig fig)	
	(v) $C + R \sim N(190, 325)$ $\mathrm{P}(C+R<170) = 0.1336287896 \approx 0.134$ (3 sig fig)(shown) The event in part (iv) is but a proper subset of that in this part, hence the probability is here is greater than that in (iv).	
	(vi) Let X denote the number of duathlons which Gary manages to achieve his target timing, out of 9 duathlons. $X \sim B(9, 0.134)$ $\mathrm{P}(3 \le X < 6) = \mathrm{P}(X \le 5) - \mathrm{P}(X \le 2)$ $= 0.1080999944$ ≈ 0.108 (3 sig fig)	

SAINT ANDREW'S JUNIOR COLLEGE

Preliminary Examination

MATHEMATICS

Higher 1 **8865/01**

Wednesday **29th August 2018** **3 hours**

Additional materials : Answer paper
List of Formulae (MF26)
Cover Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, civics group and index number on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Answer **all** the questions. Total marks : **100**

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically state otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematic steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

At the end of the examination, fasten all your work securely together.

This document consists of **7** printed pages including this page.

Section A: Pure Mathematics [40 marks]

1. A company manufactures three different types of candy. Each candy is filled with either fruit, cream or nut. The candies are packed into three types of boxes and sold as follows:

Filling / Type of box	Fruit	Cream	Nut	Selling Price
Square	4	4	12	\$9.40
Heart	12	4	4	\$7.60
Round	8	8	8	\$11

The company manufactures 4800 fruit-filled candies, 4000 cream-filled candies and 5600 nut-filled candies weekly.

(i) Find the number of boxes of each type of candies the company manufactures each week. **[3]**

The manufacturing cost per candy is \$0.20 for fruit-filled candy, \$0.25 for cream-filled candy and \$0.30 for nut-filled candy.

(ii) Assuming that the cost of boxes are negligible, find the profit the company makes if all the boxes are sold each week. **[2]**

2. Two curves are given by the equation $\mathrm{f}(x) = ax^2 + 2x - 3$ and $\mathrm{g}(x) = -ax - 4$ respectively where $a \in \mathbb{R}$. Show algebraically that these two curves will intersect each other at 2 distinct points for all real values of *a*. **[3]**

3. The curve S_1 has equation $y = \frac{1}{16}x^3 - \frac{3}{16}x^2 + 1$.

(i) Use differentiation to find the *x*-coordinates of its stationary points and determine the nature of its stationary points. **[4]**

(ii) Find the equation of the tangent at $x = -2$ in exact form. **[2]**

Another curve S_2 has equation $y = \dfrac{3x+4}{x+4}$.

(iii) Sketch the graph of S_1 and S_2 on the same diagram, stating clearly any intersections with each other and equations of any asymptotes. **[4]**

(iv) Form an integral that represent the area bounded by the curves S_1 and S_2 for the region $x \geq 0$. Hence find this area. **[2]**

4. A microbiologist measures the population of a certain type of bacteria. He starts the experiment at time $t=0$. After t hours, the population, n, is given by the formula:

$$n = A\left(1-\mathrm{e}^{-Bt}\right)$$

where A and B are positive constants.

(i) Sketch the graph of n against t, stating the equation of the asymptote. **[2]**

When $t=2, n=8000$ and when $t=4, n=12000$.

(ii) Show that $2\mathrm{e}^{-4B}-3\mathrm{e}^{-2B}+1=0$. **[2]**

(iii) Use the substitution $y=\mathrm{e}^{-2B}$ to solve the equation in part **(ii)** and hence find the value of B in the form $k\ln 2$, where k is a constant. **[2]**

(iv) Determine the population size in the long run. **[2]**

5. In a free market, the price of a product is determined by the relationship between its supply and demand. A manufacturer of headphones is formulating plans to increase production.

(a) The marketing research department determined that the selling price of headphones is given by the equation: $p=-0.5e^{0.0005x}+0.003x+250$, where x is the monthly demand and p is the selling price per headphone (in dollars).

(i) Use differentiation to find the monthly demand that gives maximum selling price of the headphone. [You do not need to prove that it is a maximum value.] **[3]**

(ii) Sketch the graph p against x, stating the stationary point and coordinates it makes with the axes. **[2]**

(b) The production of headphones is dependent on the supply of one of its components, copper.

(i) The supply of copper (in grams), at time t weeks in any year can be modelled by

$$r(t)=\frac{10}{t+1}+12.5, \quad 0\le t\le 46.$$

Find $\int_0^{46}\left(\frac{10}{t+1}+12.5\right)\mathrm{d}t$ in exact form and interpret the value you found in the context of the question. **[3]**

(ii) Due to a sudden shortage of copper at the start of a particular production year, the model for the supply of copper is changed to a quadratic function f(t). The initial supply of copper is 10.5 grams. The supply decreases up to week 20 and then increases for the rest of the year. Suggest a suitable function, f(t), to model the supply of copper at week t. given that the minimum supply is 5.5 grams at week 20. **[3]**

(iii) Would the supply of copper modelled by f(t), be able to match the intended supply of copper modelled by r(t) within that particular year? Justify your answer. **[1]**

[Turn over

Statistics [60 marks]

6. A company is selling a type of Multi-Purpose Vehicle (MPV). The number of MPV sold in a day, *X*, over a period of 60 randomly chosen days are recorded and the data obtained is summarised by

$$\sum(x-10)=-399 \qquad \sum(x-10)^2=2845$$

(i) Find the unbiased estimates of the population mean and variance of the number of MPV sold. **[2]**

(ii) Find the probability that the mean number of MPV sold per day is at least 3 over a period of 100 days. **[2]**

7. **(a)** The events *A* and *B* are such that $\mathrm{P}(A'|B')=\frac{9}{16}$, $\mathrm{P}(A)=\frac{9}{20}$ and $\mathrm{P}(A\cup B)=\frac{16}{25}$.

Find $\mathrm{P}(B)$ and $\mathrm{P}(A\cap B)$. **[5]**

Determine if *A* and *B* are independent events. **[2]**

(b) There are 100 people working in the SAINTS company. Their ages range from 20 years old to 55 years old. The age and gender distribution of the staff are summarised in the following table:

	Number of staff	
Age group	Female	Male
20 years old to 25 years old	25	28
26 years old to 39 years old	17	19
40 years old to 55 years old	3	8

To celebrate the 40th anniversary of the SAINTS company, two staff members are randomly selected to be the emcees of the 40th anniversary dinner.

Find the probability that the emcees are

(i) of different gender, **[1]**

(ii) from different age groups. **[2]**

8. Tom owns a cheese tart specialty shop. On a daily basis, he prepares enough ingredients to bake exactly 500 cheese tarts a day. On average, Tom sells 80% of his cheese tarts per day. Let the random variable A be the number of cheese tarts that Tom sells in any randomly chosen day.

(i) State, in context, two assumptions needed for A to be well modelled by a binomial distribution. **[2]**

Assume now that A has a binomial distribution.

(ii) Find the probability that Tom sells between 300 and 420 cheese tarts inclusive given that he sells at least 380 cheese tarts in any randomly chosen day. **[3]**

(iii) The cost price of the ingredients to make 500 cheese tarts is \$1000. Find the probability that Tom can make a profit if he sells the cheese tarts at \$2.50 each on any randomly chosen day. **[2]**

(iv) Find the probability that the mean number of cheese tarts sold per day is at most 395 over a period of 30 days. **[3]**

9. Ms Lee, a teacher from Shine College, claims that students spend an average of 4 hours studying in the college per week. Another teacher, Mr Ng, suspects that the claim is inaccurate. A survey was conducted and the time, x hours, spent by 50 randomly chosen students were recorded and found that their studying time in the college in hours per week had a mean of 3.8 hours and a standard deviation of 0.46 hours.

Carry out a test at 5% significance level to confirm the other teacher's suspicion. **[5]**

The same survey on another random sample of 50 students was conducted and it is found that there is significant evidence that the population mean studying time per week is not 4 hours at 5% level of significance. Using only this information, and giving a reason in each case, state whether each of the following statements is (i) necessarily true, (ii) necessarily false, (iii) neither necessarily true nor necessarily false.

(a) There is significant evidence at the 10% significance level that the population mean studying time in the college is not 4 hours. **[2]**

(b) There is significant evidence at the 2.5% significance level that the population mean studying time in the college is less than 4 hours. **[2]**

A publicity campaign on the night study program is held. Ms Lee claims that as a result of the publicity campaign, the average time that students spent studying in the college is now μ_0 hours. Another survey on 8 students was conducted after the publicity campaign, and the time, in hours, spent studying in the Shine College were recorded as follows:

4.3 4.5 3.8 4.4 4.9 3.5 4.6 3.7

[Turn over

It is now known that the time that students studying in Shine College follows a normal distribution with a standard deviation of 0.463 hours.

Using the data from the sample of 8 students after the publicity campaign on night study program was held, find the largest value of μ_0, to the nearest minute, so that Ms Lee can justify at 5% level of significance that she has not overstated the average time that the students spent studying in Shine College. **[4]**

10. A physical instructor conducted a test to assess the physical fitness level of his student, Mary. To do this, he timed Mary when she carried out a predetermined number of sit-ups on different occasions. The table below shows the number of sit-ups, x, and the time taken, t seconds, Mary took to complete the sit-ups.

x	20	30	40	50	55	60	65	70
t	53	77	120	175	215	254	298	350

(i) Calculate the product moment correlation coefficient between x and t. Draw a scatter diagram for the above data. **[3]**

(ii) Calculate the equation for the regression line of t on x. Give, in context, an interpretation for the gradient of the regression line of t on x. Sketch also the line t on x on your scatter diagram. **[3]**

(iii) Estimate the time that Mary would take to complete 67 sit-ups. Comment also on the reliability of your answer. **[3]**

(iv) State the value of product moment correlation coefficient if the physical instructor records the time in minutes. Justify your answer. **[2]**

(v) Another physical instructor suggests that, instead of carrying out a predetermined number of sit-ups, Mary should complete as many sit-ups as possible in predetermined periods of time. How, if at all, would your method of the regression equation for data generated in this way differ from your method of calculation in part **(ii)**? Explain your reason clearly. **[2]**

11. A factory manufactures a large number of screws and screw caps. The diameter of the screws in millimeters has the distribution $\mathrm{N}\left(10, 0.5^2\right)$. In order to protect the screws from rusting, they are given a coating which increases the diameter of each screw by 2 %.

(i) Show that the mean diameter of a randomly chosen screw after coating is 10.2 mm. **[1]**

(ii) A worker chooses three coated screw for testing randomly. Find the probability that one of the screw has diameter between 9 mm and 11 mm, while the other two have diameter more than 11 mm each. **[3]**

The diameter of a screw cap in millimeters has the distribution $\mathrm{N}\left(\mu, \sigma^2\right)$. It is known that 20 % of the screw caps have diameter greater than 13 mm and 5 % have diameter of less than 10 mm.

(iii) Find the values of μ and σ, giving your answers correct to 3 significant figures. **[3]**

A coated screw and a screw cap are a good fit if

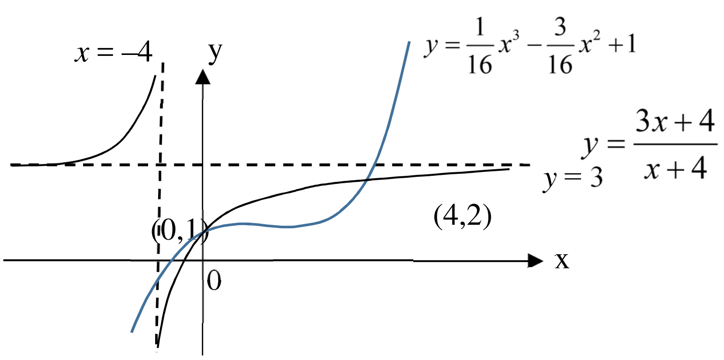
- the diameter of the screw cap is greater than the diameter of the coated screw and,
- the diameter of the screw cap is larger than the diameter of the coated screw by not more than 0.3 cm.

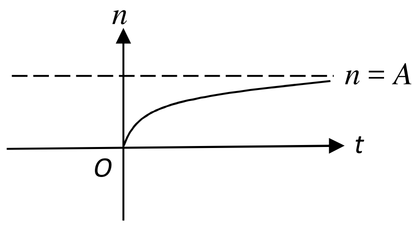
(iv) A worker randomly chooses a coated screw and a screw cap for a fit test. Find the probability that the coated screw and the screw cap are a good fit. **[3]**

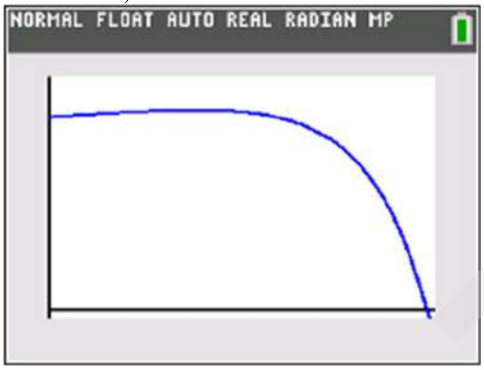
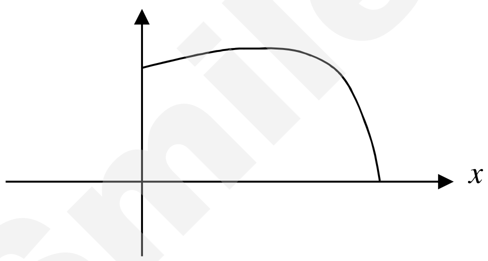
~End of Paper~

2018 SAJC JC2 H1 Math Prelim Exams: Solution

Qn	Solution
1 (i)	Let x, y, z be the number of square, heart and round type boxes manufactured weekly respectively. $4x+12y+8z=4800.....(1)$ $4x+4y+8z=4000.....(2)$ $12x+4y+8z=5600.....(3)$ Using GC, $x=200,\ y=100,\ z=350$ The company manufactures 200 square boxes, 100 heart boxes and 350 round boxes.
(ii)	Total Cost = $(4800\times0.20)+(4000\times0.25)+(5600\times0.30)=3640$ Total revenue $=(200\times9.4)+(100\times7.6)+(350\times11)=6490$ Profit = \$6490 – \$3640 = \$2850
2	At intersection point(s): Equate the 2 equations: $ax^2+2x-3=-ax-4$ Rearrange: $ax^2+(a+2)x+1=0\ldots(1)$ Discriminant : $=(a+2)^2-4(a)(1)$ $=a^2+4a+4-4a$ $=a^2+4>0$ for all real values of a The 2 curves will intersect each other at 2 distinct points.
3(i)	Differentiate: $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{16}x^2-\frac{3}{8}x$ For Stationary points, $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $\frac{3}{16}x^2-\frac{3}{8}x=0$ $x(\frac{3}{16}x-\frac{3}{8})=0$ $x=0$ or $x=2$ To determine nature of stationary points, recommend second derivative test. $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{3}{8}x-\frac{3}{8}$ At $x=0$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{3}{8}<0$ $\therefore$ max point. At $x=2$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{3}{8}(2)-\frac{3}{8}=\frac{3}{8}>0$ $\therefore$ min point.

	Alternative Solution: x | 0^- Eg - 0.1 | 0 | 0^+ Eg 0.1 $\frac{dA}{dx}$ | 0.0394 | 0 | -0.0356 Shape | / | — | \\ $x=0$ is a max point. x | 2^- Eg 1.9 | 2 | 2^+ Eg 2.1 $\frac{dA}{dx}$ | -0.0356 | 0 | 0.0394 Shape | \\ | — | / $x=2$ is a max point.
(ii)	At $x=-2$, $y=\frac{1}{16}(-2)^3-\frac{3}{16}(-2)^2+1=-\frac{1}{4}$, $\frac{dy}{dx}=\frac{3}{16}(-2)^2-\frac{3}{8}(-2)=\frac{3}{2}$, Equation of tangent at $x=-2$: $y+\frac{1}{4}=\frac{3}{2}(x+2)$ or $\left(y=\frac{3}{2}x+\frac{11}{4}\right)$
(iii)	$y=\frac{3x+4}{x+4}=3-\frac{8}{x+4}$ Vertical Asymptote: $x=-4$ Horizontal Asymptote: $y=3$ 

(iv)	Area bounded by the 2 curves $=\int_0^4 \left(\frac{3x+4}{x+4}\right)-\left(\frac{1}{16}x^3-\frac{3}{16}x^2+1\right)\,\mathrm{d}x$ Using GC, required area = 2.45 units^2
4 (i)	
(ii)	$t=2,\quad n=8000$ $8000=A\left(1-\mathrm{e}^{-2B}\right)----(1)$ $t=4,\quad n=12000$ $12000=A\left(1-\mathrm{e}^{-4B}\right)---(2)$ $\frac{(2)}{(1)}\Rightarrow\frac{12}{8}=\frac{3}{2}=\frac{1-\mathrm{e}^{-4B}}{1-\mathrm{e}^{-2B}}$ $2-2\mathrm{e}^{-4B}=3-3\mathrm{e}^{-2B}$ $2\mathrm{e}^{-4B}-3\mathrm{e}^{-2B}+1=0---(3)$
(iii)	$2\left(\mathrm{e}^{-2B}\right)^2-3\mathrm{e}^{-2B}+1=0---(4)$ $y=\mathrm{e}^{-2B}$, equation (4) $\Rightarrow$ $2y^2-3y+1=0$ $(2y-1)(y-1)=0$ $y=\frac{1}{2}$ or $y=1$ $\mathrm{e}^{-2B}=1\Rightarrow B=0$ (reject since $B>0$) $\mathrm{e}^{-2B}=\frac{1}{2}\Rightarrow B=\frac{1}{2}\ln 2$
(iv)	$8000=A\left(1-e^{-2B}\right)----(1)$ From (iii), $\mathrm{e}^{-2B}=\frac{1}{2}$. $8000=A\left(1-\frac{1}{2}\right)$ Solving, $A=16000$ $\therefore n=16000\left(1-\mathrm{e}^{-0.347t}\right)$ $t\to\infty,\quad n\to 16000$

	The maximum population size is 16000.
5 a(i)	$\frac{dp}{dx} = (-0.5)(0.0005)e^{0.0005x} + 0.003$ $= -0.00025e^{0.0005x} + 0.003$ $\frac{dp}{dx} = 0 \Rightarrow -0.00025e^{0.0005x} + 0.003 = 0$ $-0.00025e^{0.0005x} + 0.003 = 0$ $e^{0.0005x} = 12$ ln both sides: $x = \frac{\ln 12}{0.0005}$ =2000ln12 (or 4970)
aii	From GC,  Window setting : $0 < x \le 15000$; $-10 < p \le 500$ 
bi	Given $r(t) = \frac{10}{t+1} + 12.5$, $\int_0^{46} \left(\frac{10}{t+1} + 12.5 \right) dt = \left[10\ln\lvert t+1 \rvert + 12.5t \right]_0^{46}$ =10ln47+575 It represents the **total** supply of of copper in 46 weeks of production.
bii	Given minimum point: $(20, 5.5)$ Suggested model is $f(t) = a(t-20)^2 + 5.5$

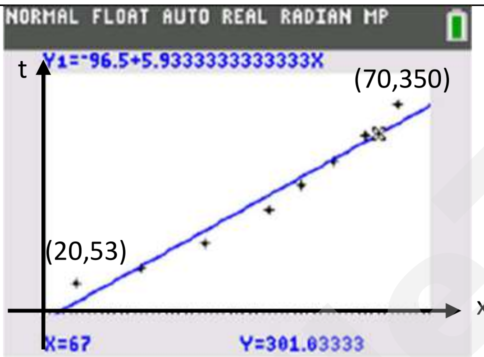
	Given $(0,10.5)$: $10.5 = a(-20)^2 + 5.5$ $a = \dfrac{1}{80}$ $\mathrm{f}(t) = \dfrac{1}{80}(t-20)^2 + 5.5$
biii	$\dfrac{10}{t+1} + 12.5 = \dfrac{1}{80}(t-20)^2 + 5.5$ Using GC, intersection at week 44. Yes, f(t) will reach r(t)
6(i)	Let X be the number of MPVs sold. Unbiased estimate of the population mean of X, $\bar{x} = \left(\dfrac{-399}{60}\right) + 10 = 3.35$ Unbiased estimates of the population variance of X, $s^2 = \dfrac{1}{59}\left[2845 - \dfrac{(-399)^2}{60}\right] = 3.2483 \approx 3.25$ (to 3 s.f.)
(ii)	Since n is large, by Central Limit Theorem, $\bar{X} \sim N(3.35, \dfrac{3.2483}{100})$ approximately $P(\bar{X} \geq 3) = 0.97393 \approx 0.974$
7(a)	$\mathrm{P}(A' \mid B') = \dfrac{9}{16}$ $\dfrac{\mathrm{P}(A' \cap B')}{\mathrm{P}(B')} = \dfrac{9}{16}$ $\dfrac{1 - \mathrm{P}(A \cup B)}{1 - \mathrm{P}(B)} = \dfrac{9}{16}$ $\dfrac{1 - \frac{16}{25}}{1 - \mathrm{P}(B)} = \dfrac{9}{16}$ $1 - \mathrm{P}(B) = \dfrac{16}{25}$ $\mathrm{P}(B) = \dfrac{9}{25}$ Since $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ $\dfrac{16}{25} = \dfrac{9}{20} + \dfrac{9}{25} - \mathrm{P}(A \cap B)$ $\mathrm{P}(A \cap B) = \dfrac{17}{100}$ $\mathrm{P}(A) \times \mathrm{P}(B) = \left(\dfrac{9}{20}\right) \times \left(\dfrac{9}{25}\right) = \dfrac{81}{500}$ Since $\mathrm{P}(A \cap B) \neq \mathrm{P}(A)\mathrm{P}(B)$, A and B are not independent.

bi	Required Probability $=\dfrac{{}^{55}C_1\times{}^{45}C_1}{{}^{100}C_2}=\dfrac{1}{2}$ (or $=2\left(\dfrac{55}{100}\right)\left(\dfrac{45}{99}\right)=\dfrac{1}{2}$)
bii	(see table below) Required Probability from different age groups P(1 from Grp A, 1 from Grp B) + P(1 from Grp A, 1 from Grp C) + P(1 from Grp B, 1 from Grp C) $=\dfrac{{}^{53}C_1\times{}^{36}C_1+{}^{53}C_1\times{}^{11}C_1+{}^{36}C_1\times{}^{11}C_1}{{}^{100}C_2}=\dfrac{2887}{4950}$ (or $=2\left(\dfrac{53}{100}\right)\left(\dfrac{36}{99}\right)+2\left(\dfrac{53}{100}\right)\left(\dfrac{11}{99}\right)+2\left(\dfrac{36}{100}\right)\left(\dfrac{11}{99}\right)=\dfrac{2887}{4950}$)
8i)	Each cheese tart is equally likely to be sold. Or The probability of selling a cheese tart remains a constant **at 0.8** for every cheese tart. The sale of a cheese tart is independent of the sale of another cheese tart.
(ii)	Let A be the number of cheese tarts that Tom sells out of 500. $A \sim B(500, 0.8)$ Required Probability $=\text{P}(300\le A\le 420 \mid A\ge 380)$ $=\dfrac{\text{P}(300\le A\le 420\cap A\ge 380)}{\text{P}(A\ge 380)}$ $=\dfrac{\text{P}(380\le A\le 420)}{\text{P}(A\ge 380)}$ $=\dfrac{\text{P}(A\le 420)-\text{P}(A\le 379)}{1-\text{P}(A\le 379)}$ $=\dfrac{0.99049-0.012256}{1-0.012256}$ $=0.990$
(iii)	To make a profit, he must earn more than \$1000, which means he must sell more than \$1000/\$2.50 = 400 cheese tarts. Required Probability $=\text{P}(A>400)$ $=1-\text{P}(A\le 400)$ $=0.482$

Table in bii:

Group	Age Group	No of staff
A	20-25 years	53
B	26-39	36
C	40-55	11

(iv)	$A \sim B(500, 0.8)$ $E(A) = 500(0.8) = 400$ $Var(A) = 500(0.8)(0.2) = 80$ $\overline{A} = \dfrac{A_1 + A_2 + \cdots + A_{30}}{30}$ Since $n = 30$ is large, by Central Limit Theorem, $\overline{A} = \dfrac{A_1 + A_2 + \cdots + A_{30}}{30} \sim N\left(400, \dfrac{80}{30}\right)$ approximately $P(\overline{A} \le 395)$ $= 0.00110$
9	Let *X* be the time, in hours, that <u>a randomly chosen student</u> spends studying in Shine College, and μ be the population mean. Unbiased estimate of the population variance $s^2 = \dfrac{50}{49}(0.46)^2 = 0.21592 \approx 0.216$ (3 sig. fig.) Test $H_o : \mu = 4$ against $H_1 : \mu \neq 4$ Under H_o, since sample size = 50 is large, by Central Limit Theorem, $\overline{X} \cdots N\left(4, \dfrac{0.21592}{50}\right)$ approximately Use a two-tailed test at 5% level of significance. Using GC, with $\overline{x} = 3.8$, $s = \sqrt{0.21592}$, n =50, z value $= -3.0435$ p value $= 0.0023387 \le 0.05$ We **reject** H_o and conclude that there is **sufficient evidence** at 5% significance level to conclude that students **do not spend an average of 4 hours** studying in college (or to confirm Mr Ng's suspicion that the students do not spend an average of 4 hours studying in college). (a)Necessarily true. Since p-value $= k \le 0.05 \Rightarrow k < 0.1$ for any real number of *k*. Therefore, there is significant evidence at the 10% significance level that the population mean studying time in the Shine college is not 4 hours. (b) Necessarily true also, For a two-tailed test: Since p-value = $k \le 0.05$, then for a one-tailed test, p-value = $\dfrac{k}{2} \le 0.025$.
	From the GC, $\overline{x} = 4.2125$ Test $H_o : \mu = \mu_0$ against $H_1 : \mu < \mu_0$

	Under H_o, $\overline{X} \sim N\left(\mu_0, \frac{0.463^2}{8}\right)$ $Z = \frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$ For Ms Lee to justify that he has not overstated the average time, do not reject H_0 at 5% (z-calc does not lie in critical region) $z_{calc} = \frac{4.2125-\mu_0}{0.463/\sqrt{8}} > -1.644853626$ $4.2125-\mu_0 > -1.644853626\left(\frac{0.463}{\sqrt{8}}\right)$ $\mu_0 < 4.4818$ hrs $\mu_0 < 4$ hrs 28.91 mins Largest possible value of μ_0 is 4 hours 28 mins (268 mins)
10 (i)	 The product moment correlation coefficient, r, is 0.979 (3sf).
(ii)	From GC, equation for the regression line of t on x is $t = 5.9333x - 96.5$ $t = 5.93x - 96.5$ There is an increase of 5.93 seconds for every situp Mary takes. (see graph for the regression line $t = 5.93x - 96.5$)
(iii)	When $x = 67$, $t = 5.9333(67) - 96.5$ $= 301s$ Since $x = 67$, is within data range and the product moment correlation coefficient is close to 1 suggesting a strong positive linear correlation, the predicted value is reliable.
(iv)	The product moment correlation coefficient remains at 0.979 as it is **not affected** by linear transformation of the variables.
(v)	An appropriate regression equation would be $x = c + dt$ since the time is now the independent variable.
11i	Let X be the random variable "diameter of a screw in mm" $X \sim \mathrm{N}(10, 0.5^2)$ $Y = 1.02X$ where Y is the new diameter after the coating. $\mathrm{E}(Y) = 1.02\mathrm{E}(X)$ New mean diameter = $1.02 \times 10 = 10.2$

ii	$\text{Var}(Y)=\text{Var}(1.02X)=1.02^2(0.5)^2$ $Y=1.02X \sim \text{N}\left(1.02(10),1.02^2(0.5)^2\right)$ $Y\sim N(10.2,0.2601)$ Required Probability $=3\text{P}(9<Y<11)\left[\text{P}(Y>11)\right]^2$ $=0.00953$
(iii)	Let C be the random variable "diameter of a screw cap in mm" $C\sim \text{N}\left(\mu,\sigma^2\right)$ $P(C<10)=0.05$ $P\left(Z<\frac{10-\mu}{\sigma}\right)=0.05$ where $Z\sim N(0,1)$ $\frac{10-\mu}{\sigma}=-1.64485$ $\mu-1.64485\sigma=10\cdots(1)$ $P(C>13)=0.20$ $P\left(Z>\frac{13-\mu}{\sigma}\right)=0.20$ $\frac{13-\mu}{\sigma}=0.84162$ $\mu+0.84162\sigma=13\cdots(2)$ Solving (1) and (2) using GC, $\mu=12.0,\ \sigma=1.21$
(iv)	$C-Y\sim \text{N}\left(12-10.2,1.2065^2+0.2601\right)$ $C-Y\sim \text{N}(1.8,1.7158)$ Required Probability $=\text{P}(0<C-Y\le 3)$ $=0.735$

SERANGOON JUNIOR COLLEGE
2018 JC2 PRELIMINARY EXAMINATION
MATHEMATICS

Higher 1

8865/01

Tuesday **11 September 2018**
3 Hours

Additional materials: Writing paper

List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on the cover page and on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets **[ ]** at the end of each question or part question.

The total marks for this paper is **100**.

At the end of the examination, fasten all your work securely together.

This document consists of **7** printed pages and **1** blank page.

[**Turn Over**

Section A: Pure Mathematics [40 marks]

1 ♥ represents one ticket to a ride in the amusement park.

Kim's Purchases

Amusement Park	
Transformer	♥ ♥ ♥
Hurricane	♥ ♥ ♥ ♥ ♥
Battlestar	♥ ♥ ♥ ♥
Total Cost: $53.00	

Don's Purchases

Amusement Park	
Transformer	♥ ♥
Hurricane	♥ ♥ ♥ ♥
Battlestar	♥ ♥ ♥
Total Cost: $39.00	

It is known that the cost of a ride on the Battlestar is 25% less than the total cost of a ride on the Transformer and a ride on the Hurricane.

(i) Find the cost for each of the rides. [3]

To enjoy discounts on the rides in the amusement park, Kim applies to be a Gold member which entitles him to a 40% discount on all Battlestar rides and Don applies to be a Silver member which entitles him to a 20% discount on all Transformer rides.

Assuming that Kim and Don took the number of rides as shown above,

(ii) what is the total amount that both Kim and Don have to pay altogether after the discounts were given to both of them? [1]

2 **(a)** Find the exact range of values of k for which $y = x^2 + (k-1)x + 3$ is always positive. [3]

(b) **(i)** On the same diagram, sketch the graphs of $y = -\frac{1}{2} - \left(\frac{1}{5}\right)^x$ and $y = \frac{11-3x}{2x-1}$, indicating the axial intercepts and the coordinates of the intersection points between the two graphs and the equations of asymptotes, if any. [5]

(ii) Find the range of values of x for which

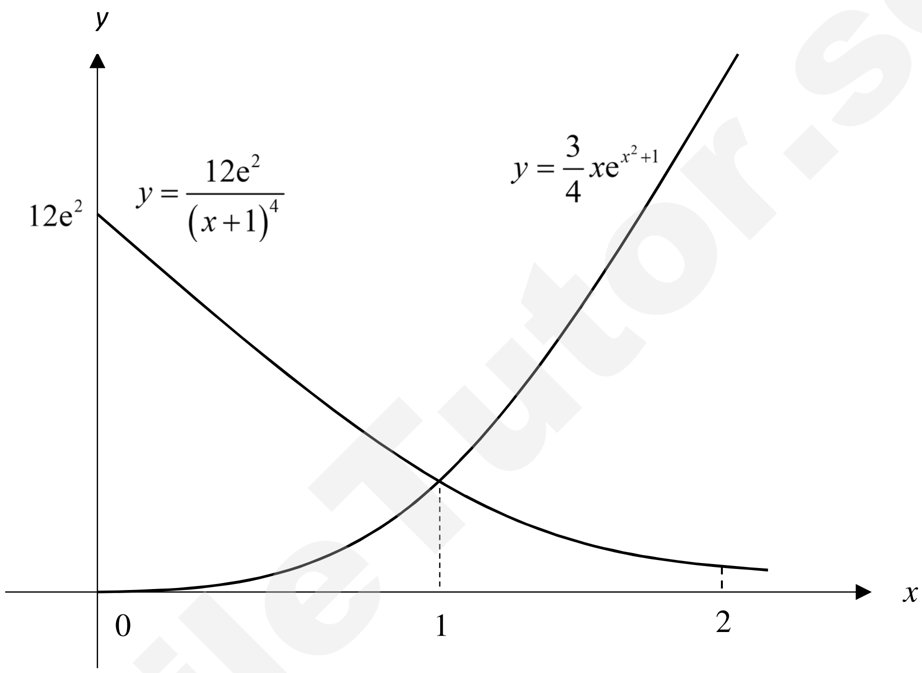
$$\frac{11-3x}{2x-1} + \frac{1}{2} + \left(\frac{1}{5}\right)^x \geq 0.$$ [1]

(iii) Hence, solve $\frac{11+3x}{-2x-1} + \frac{1}{2} + 5^x \geq 0$. [2]

3 **(i)** Differentiate e^{x^2+1} with respect to x. [1]

(ii) Hence, find $\int \frac{3}{4}xe^{x^2+1}\ dx$ [2]

(iii) The diagram shows parts of the graphs of $y=\frac{3}{4}xe^{x^2+1}$ and $y=\frac{12e^2}{(1+x)^4}$. The graph of $y=\frac{3}{4}xe^{x^2+1}$ cuts the origin and the two graphs intersect at $x=1$.



Find, in terms of e, the exact area bounded by the graphs of $y=\frac{3}{4}xe^{x^2+1}$, $y=\frac{12e^2}{(1+x)^4}$, the line $x=2$ and the $x-$axis. [4]

4 The curve C has equation $y=\ln\left(4-x^2\right)$.

(i) Show, algebraically, that x can only take values between -2 and 2. [1]

(ii) Find the x-coordinate of the point A, for which the gradient of the tangent at A is parallel to the line $3y=2x-5$. [2]

(iii) Hence, find the equation of the tangent at the point A in the form $y=mx+c$, where m and c are exact constants to be determined. [1]

The tangent to the curve at A cuts the y-axis at B.

(iv) Find the exact value of the length of AB. [2]

5 A company manufactures bicycles. The rate at which the total manufacturing cost changes, C thousand dollars per year, is to be monitored regularly over a period of 6 years. The company's economist proposes that the relationship between C and the time, t years, can be modelled by the equation

$$C = 25 - 6t + \mathrm{e}^{0.7t}, \quad \text{for } 0 \leq t \leq 6.$$

(i) Using differentiation and this model, find the minimum value of C, leaving your answer correct to two decimal places. [4]

(ii) Sketch the graph of C against t, stating the coordinates of any intersections with the axes and the end points. [2]

(iii) Find the area of the region bounded by the curve, the t-axis and the lines $t = 0$ and $t = 6$, leaving your answer to two decimal places. Give an interpretation of the area that you have found, in the context of the question. [2]

The company also manufactures bicycle helmets. The economist proposes that the relationship between the profit \$$P$ and the manufacturing cost \$$x$, can be modelled by the equation

$$P = 7\sqrt{x} - 0.8x\,.$$

(iv) Sketch the graph of P against x, stating the coordinates of the intersections with the x-axis. [2]

(v) Given that the manufacturing cost of a bicycle helmet is \$40, find the selling price of a bicycle helmet. [1]
[You may assume that profit = selling price – manufacturing cost].

(vi) If the manufacturing cost of a bicycle helmet is \$80, would you advise the company to produce the bicycle helmet? Justify your answer. [1]

Section B: Probability and Statistics [60 marks]

6 In a particular population, the probability that an individual chosen at random will have a certain infection is p. Every individual will take a medical test for the infection. If an individual has the infection, there is a probability of 0.95 that the test will prove positive. If an individual does not have the infection, there is a probability of 0.85 that the test will prove negative.

(i) Draw a probability tree diagram to illustrate the above–mentioned information. [2]

(ii) The conditional probability that a randomly chosen individual whose test is negative does not have the infection is $\frac{17}{21}$. Find the value of p. [2]

7 **(a)** A group of fifteen people consists of one pair of brothers, one set of three sisters and five married couples. The fifteen people are seated in a row.

(i) Find the number of arrangements if men and women alternate. [1]

(ii) Find the number of arrangements in which each married couple is seated together but no two sisters sit next to one another. [2]

(iii) Find the probability that either the brothers are next to each other or the sisters are all next to one another or both. [2]

(b) There are 15 male students and 10 female students in a class. Seven students are to be selected from the class, and the order of selection does not matter.
Find the number of selections in which there are at least three students of each gender being selected. [3]

8 **(a)** The random variable X has distribution $B(7, p)$ and $P(X = 1 \text{ or } 4) = 0.25$. Write down an equation in terms of p and find the possible values of p. [2]

(b) A company manufactures a large number of teapots. On average, p% of the teapots are defective. The teapots are randomly packed into boxes of 15. Given that the mean number of defective teapots in a box is 0.3, show that [1]
$p = 2$.

(i) State, in context of this question, one assumption needed to model the number of defective teapots by a binomial distribution. [1]

(ii) Find the probability that at least 90% of the teapots in a randomly chosen box are not defective. [2]

(iii) n boxes are randomly chosen. The boxes which have more than 10% of the teapots that are defective are rejected. Find the greatest value of n such that the probability of rejecting at least 3 boxes is at most 0.01. [3]

9 The total distance run per week, x kilometres, and amount of weight loss, y kilograms, of 8 men undergoing a particular special training programme after a period of time are given in the following table.

x	0.6	1.2	1.5	2.4	2.5	3.2	3.6	3.4
y	0.5	2.5	2.9	5.5	5	8.1	9	10

(i) Give a sketch of the scatter diagram of the data, as shown on your calculator. [2]

(ii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iii) Find the equation of the regression line of y on x in the form $y = ax + b$, giving the values of a and b correct to 4 significant figures. Explain the meaning of value of a in the context of the question. Sketch this line on your scatter diagram. [3]

(iv) Use a suitable regression line to calculate an estimate of the weight loss for a man who runs 700 metres daily. Comment on the reliability of this estimate. [2]

(v) It is decided to record the distance run per week for person in metres instead of kilometres.Without any further calculations, state any change you would expect in the value of the product moment correlation coefficient. [1]

10 **In this question you should state clearly the values of the parameters of any normal distributions that you use.**

The masses, in grams, of oranges have the distribution $N(250, 15^2)$ and the masses, in grams, of pears have the distribution $N(200, 10^2)$.

(i) It is 95% certain that 6 oranges chosen at random have a total mass exceeding L grams. Find the value of L, correct to 1 decimal place. [2]

(ii) Find the probability that the total mass of two randomly chosen oranges is within 50 grams of twice the mean mass of the pears. [2]

(iii) Find the probability that the average mass of 5 randomly chosen oranges is more than the average mass of 3 randomly chosen pears by more than 55 grams. [3]

A certain recipe requires 6 oranges and 10 pears. The recipe requires the oranges and pears to be prepared by peeling them and removing the cores. This process reduces the mass of each orange by 10% and the mass of each pear by 15%.

(iv) Find the probability that the total mass, after preparation, of 6 randomly chosen oranges and 10 randomly chosen pears is at least 3 kilograms. [3]

(v) Oranges are sold at 60 cents per 100 grams and pears are sold at 70 cents per 100 grams in a particular store. Let W represents the mass of a randomly chosen orange and V represents the mass of a randomly chosen pear.

Find $\mathrm{P}(3(0.7V) - 0.6(W_1 + W_2) > 130)$ and explain, in the context of this question, what your answer represents. [4]

11 **(a)** A particular junior college has 30 classes with different class sizes. The Principal of the college wishes to take a sample of 60 students to meet up with a ministry official for a short conversation. He chooses 2 students at random from each of the classes. State, with a reason, why this method will not give a random sample of 60 students from the college. Describe how you would obtain a random sample of 60 students from the college. [2]

(b) A manager of a particular departmental store claims that the average amount of time that customers spent shopping in the store is not greater than 45 minutes. A random sample of 80 customers is chosen and the time, t minutes, spent by each customer is recorded. The results are summarised by

$$\sum(t-50) = -342 \quad \text{and} \quad \sum(t-50)^2 = 2112.$$

(i) Find the unbiased estimates of the population mean and variance. [3]

(ii) Test the manager's claim at the 1% level of significance. [4]

(iii) Explain the meaning of "p – value" in the context of the question. [1]

(iv) Without carrying out any further hypothesis tests, what would you conclude if a two-tailed test is carried out instead at 5% level of significance? [1]

The department store introduces free Wi-Fi so that the customers can spend more time in the store. The same manager claims that the average amount of time that customers spent shopping in the store is now more than 45 minutes. To test his claim, a random sample of 20 customers is chosen and it is found that their mean average time is m minutes.

(v) Given the population standard deviation is 12 minutes, find the set of values of m for which the manager's claim is valid at the 1% level of significance. [3]

(vi) State a necessary assumption for the test in **(v)** to be valid. [1]

SERANGOON JUNIOR COLLEGE
2018 JC2 PRELIMINARY EXAMINATION
MATHEMATICS

Higher 1

8865/01

Tuesday **11 September 2018**
3 Hours

Additional materials: Writing paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on the cover page and on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets **[ ]** at the end of each question or part question.

The total marks for this paper is **100**.

At the end of the examination, fasten all your work securely together.

This document consists of **7** printed pages and **1** blank page.

[**Turn Over**

Section A: Pure Mathematics [40 marks]

1

Amusement Park	
Transformer	♥ ♥ ♥
Hurricane	♥ ♥ ♥ ♥ ♥
Battlestar	♥ ♥ ♥ ♥
Total Cost: $53.00	

Kim's Ticket

Amusement Park	
Transformer	♥ ♥
Hurricane	♥ ♥ ♥ ♥
Battlestar	♥ ♥ ♥
Total Cost: $39.00	

Don's Ticket

It is known that the cost of a ride on the Battlestar is 25% less than the total cost of a ride on the Transformer and a ride on the Hurricane.

(i) find the cost for each of the rides. [3]

Kim is a Gold member which entitles him to a 40% discount on all Battlestar rides and Don is a Silver member which entitles him to a 20% discount on all Transformer rides.

Assuming that Kim and Don took the number of rides as shown in their tickets above,

(ii) what is the total amount that both Kim and Don have to pay altogether after the discounts were given to both of them? [1]

Suggested Solution

(i)

Let x, y and z be the cost of each ride on the Transformer, Hurricane and Battlestar respectively.

$3x+5y+4z=53$ ------- (1)

$2x+4y+3z=39$ ------- (2) for writing equations (1) and (2)

$z=0.75(x+y)$ for writing equation (3)

$0.75x+0.75y-z=0$ ------- (3)

Using the GC, $x=5.5$, $y=2.5$, $z=6$ for all correct answers

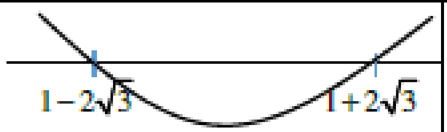
Hence, the cost of a ride on the Transformer, Hurricane and Battlestar are $5.50, $2.50 and $6 respectively.

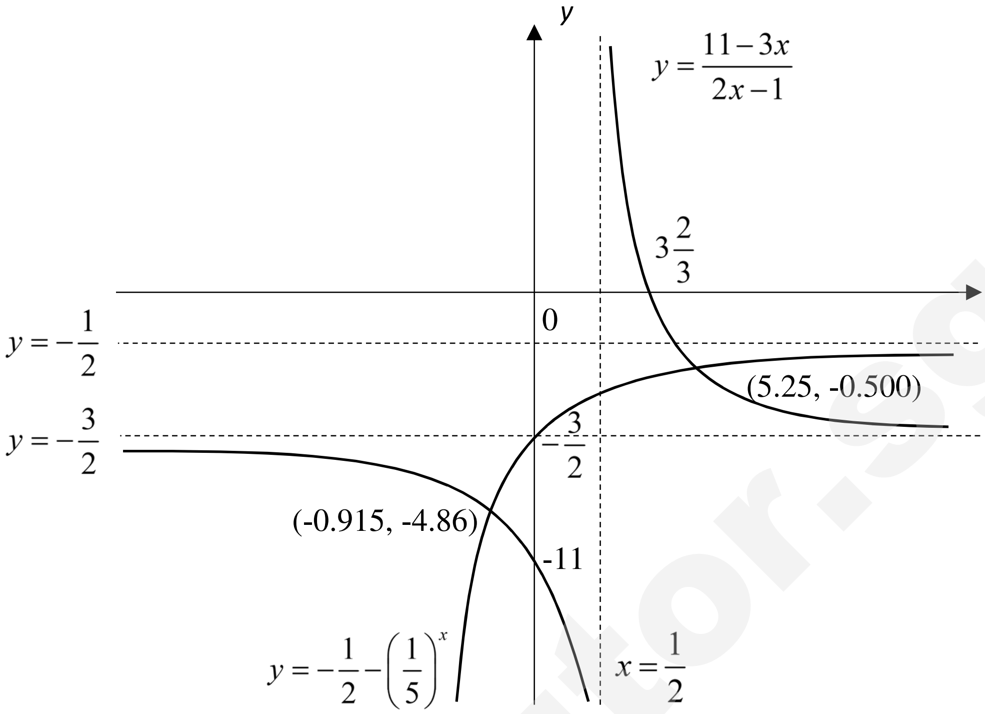
(ii)

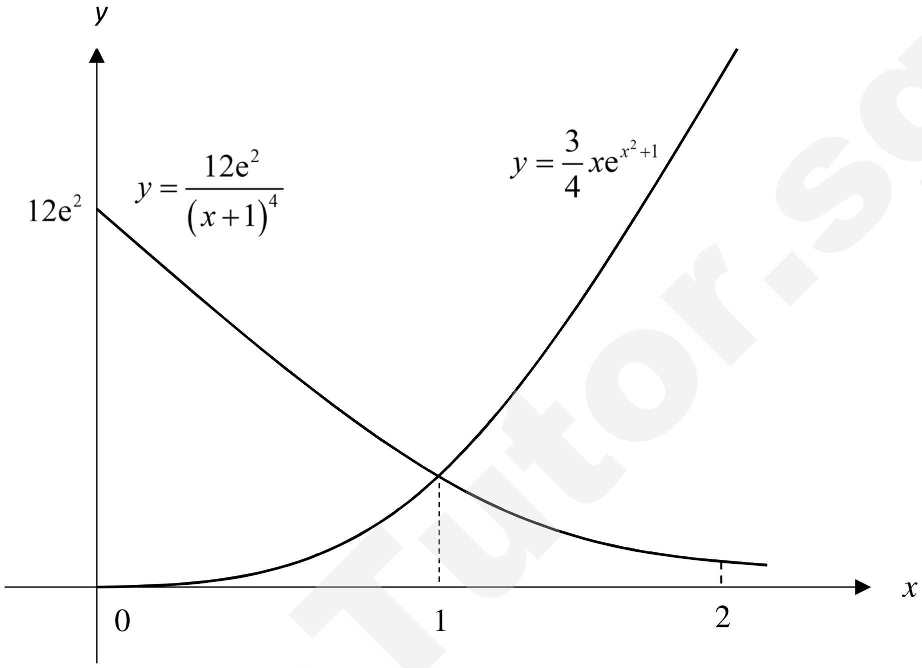
Total amount $=3(5.5)+5(2.5)+4(0.6\times6)+2(0.8\times5.5)+4(2.5)+3(6)$

$= \$80.20$

After the discounts were given, Kim and Don will have to pay a total of $80.20.

2	**(a)** Find the exact range of values of k for which $y=x^2+(k-1)x+3$ is always positive.	[3]
	(b)(i) On the same diagram, sketch the graphs of $y=-\frac{1}{2}-\left(\frac{1}{5}\right)^x$ and $y=\frac{11-3x}{2x-1}$, indicating the axial intercepts and the coordinates of the intersection points between the two graphs and the equations of asymptotes, if any.	[5]
	(ii) Find the range of values of x for which $$\frac{11-3x}{2x-1}+\frac{1}{2}+\left(\frac{1}{5}\right)^x \geq 0.$$	[1]
	(iii) Hence, solve $\frac{11+3x}{-2x-1}+\frac{1}{2}+5^x \geq 0$.	[2]
	Suggested Solution	
	(a) For $y=x^2+(k-1)x+3$ is always positive, the coefficient of $x^2 = 1 > 0$, we just need to satisfy the condition: Discriminant < 0 $(k-1)^2-4(1)(3)<0$ **<u>Method 1</u>** $k^2-2k-11<0$	
	Consider $k^2-2k-11=0$.	
	$k=\frac{2\pm\sqrt{(-2)^2-4(1)(-11)}}{2(1)}$ $k=1+2\sqrt{3}$ or $k=1-2\sqrt{3}$  Hence, $1-2\sqrt{3}<k<1+2\sqrt{3}$	
	<u>Method 2</u> $(k-1)^2-\left(\sqrt{12}\right)^2<0$	
	$\left(k-1-\sqrt{12}\right)\left(k-1+\sqrt{12}\right)<0$	
	$\left[k-\left(1+2\sqrt{3}\right)\right]\left[k-\left(1-2\sqrt{3}\right)\right]<0$  Hence, $1-2\sqrt{3}<k<1+2\sqrt{3}$	
	(b)(i)	

		
	(b)(ii) $\frac{11-3x}{2x-1}+\frac{1}{2}+\left(\frac{1}{5}\right)^x\geq 0$ $\frac{11-3x}{2x-1}\geq -\frac{1}{2}-\left(\frac{1}{5}\right)^x$ From the graph, $x\leq -0.915$ or $\frac{1}{2}<x\leq 5.25$	
	(b)(iii) $\frac{11+3x}{-2x-1}+\frac{1}{2}+5^x\geq 0$ $\frac{11-3(-x)}{2(-x)-1}+\frac{1}{2}+\left(\frac{1}{5}\right)^{(-x)}\geq 0$	
	Replacing the 'x' in (b)(ii) by '$-x$', $-x\leq -0.915$ or $\frac{1}{2}<-x\leq 5.25$ $x\geq 0.915$ or $-5.25\leq x<-\frac{1}{2}$	
3	**(i)** Differentiate e^{x^2+1} with respect to x.	[1]
	(ii) Hence, find $\int \frac{3}{4}xe^{x^2+1}\,dx$	[2]

	(iii) The diagram shows parts of the graphs of $y=\frac{3}{4}x\mathrm{e}^{x^2+1}$ and $y=\frac{12\mathrm{e}^2}{(1+x)^4}$. The graph of $y=\frac{3}{4}x\mathrm{e}^{x^2+1}$ cuts the origin and the two graphs intersect at $x=1$.	
		
	Find, in terms of e, the exact area bounded by the graphs of $y=\frac{3}{4}x\mathrm{e}^{x^2+1}$, $y=\frac{12\mathrm{e}^2}{(1+x)^4}$, the line $x=2$ and the $x-$axis.	[4]
	Suggested Solutions	
	(i) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{x^2+1}\right)=2x\mathrm{e}^{x^2+1}$	
	(ii) $\int \frac{3}{4}x\mathrm{e}^{x^2+1}\ \mathrm{d}x = \frac{3}{8}\int 2x\mathrm{e}^{x^2+1}\ \mathrm{d}x$	
	$= \frac{3}{8}\mathrm{e}^{x^2+1}+c$	
	(iii) Area of bounded region	
	$=\int_0^1 \frac{3}{4}x\mathrm{e}^{x^2+1}\ \mathrm{d}x+\int_1^2 \frac{12\mathrm{e}^2}{(x+1)^4}\ \mathrm{d}x$	

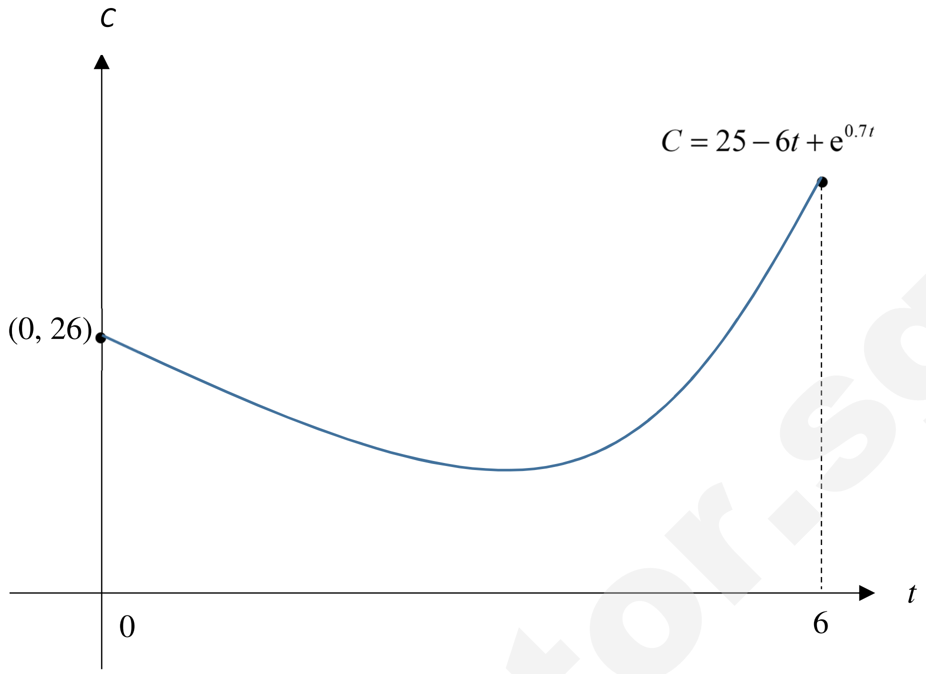
	$= \int_0^1 \frac{3}{4} x e^{x^2+1} \, dx + 12e^2 \int_1^2 (x+1)^{-4} \, dx$	
	$= \left[\frac{3}{8} e^{x^2+1} \right]_0^1 + 12e^2 \left[\frac{(x+1)^{-3}}{-3} \right]_1^2$	
	$= \frac{3}{8}e^2 - \frac{3}{8}e - 4e^2 \left(\frac{1}{27} - \frac{1}{8} \right)$	
	$= \frac{3}{8}e^2 - \frac{3}{8}e + \frac{19}{54}e^2$	
	$= \frac{157}{216}e^2 - \frac{3}{8}e$	
4	The curve C has equation $y = \ln(4 - x^2)$.	
	(i) Show, algebraically, that x can only take values between -2 and 2.	[1]
	(ii) Find the x-coordinate of the point A, for which the gradient of the tangent at A is parallel to the line $3y = 2x - 5$.	[2]
	(iii) Hence, find the equation of the tangent at the point A in the form $y = mx + c$, where m and c are exact constants to be determined.	[1]
	The tangent to the curve at A cuts the y-axis at B.	
	(iv) Find the exact value of the length of AB.	[2]
	Suggested Solution	
	(i) The curve $y = \ln(4 - x^2)$ is only defined when $4 - x^2 > 0$. Hence, $x^2 - 4 < 0$ $(x-2)(x+2) < 0$ Therefore $-2 < x < 2$. Hence, x can only take values between -2 and 2.	
	(ii) $y = \ln(4 - x^2)$ $\frac{dy}{dx} = \frac{-2x}{4 - x^2} = \frac{2x}{x^2 - 4}$ When $\frac{dy}{dx} = \frac{2}{3}$,	
	$\frac{2x}{x^2 - 4} = \frac{2}{3}$	
	$2x^2 - 6x - 8 = 0$	
	$x^2 - 3x - 4 = 0$	
	$(x+1)(x-4) = 0$	
	$x = -1$ or $x = 4$ (rejected since the graph is undefined at $x = 4$)	
	(iii)	

	When $x=-1, \quad y=\ln 3$	
	Equation of tangent at A, $y-\ln 3=\frac{2}{3}(x+1)$	
	$y=\frac{2}{3}x+\frac{2}{3}+\ln 3$	
	(iv) At B, $y=\frac{2}{3}(0)+\frac{2}{3}+\ln 3$	
	$y=\frac{2}{3}+\ln 3$	
	The coordinates of A is $(-1, \ \ln 3)$ and B is $\left(0, \ \frac{2}{3}+\ln 3\right)$	
	Length AB = $\sqrt{(-1-0)^2+\left[\ln 3-\left(\frac{2}{3}+\ln 3\right)\right]^2} = \frac{\sqrt{13}}{3}$ units	
5	A company manufactures bicycles. The rate at which the total manufacturing cost changes, C thousand dollars per year, is to be monitored regularly over a period of 6 years. The company's economist proposes that the relationship between C and the time, t years, can be modelled by the equation	
	$C = 25 - 6t + e^{0.7t}, \quad \text{for } 0 \le t \le 6.$	
	(i) Using differentiation and this model, find the minimum value of C, leaving your answer correct to two decimal places.	[4]
	(ii) Sketch the graph of C against t, stating the coordinates of any intersections with the axes.	[2]
	(iii) Find the area of the region bounded by the curve, the t-axis and the lines $t = 0$ and $t = 6$, leaving your answer to two decimal places. Give an interpretation of the area that you have found, in the context of the question.	[2]
	The company also manufactures bicycle helmets. The economist proposes that the relationship between the profit \$$P$ and the manufacturing cost \$$x$, can be modelled by the equation	
	$P=7\sqrt{x}-0.8x$.	
	(iv) Sketch the graph of P against x, stating the coordinates of the intersections with the x-axis.	[2]

	(v) Given that the manufacturing cost of a bicycle helmet is \$40, find the selling price of a bicycle helmet. [You may assume that profit = selling price – manufacturing cost].	[1]
	(vi) If the manufacturing cost of a bicycle helmet is \$80, would you advise the company to produce the bicycle helmet? Justify your answer.	[1]
	Suggested Solution	
	(i) $C = 25 - 6t + e^{0.7t}$	
	$\frac{dC}{dt} = -6 + 0.7e^{0.7t}$	
	Setting $\frac{dC}{dt} = 0$,	
	$-6 + 0.7e^{0.7t} = 0$	
	$e^{0.7t} = \frac{60}{7}$	
	$0.7t = \ln\frac{60}{7}$	
	$t = 3.0692$	

t	$(3.0692)^-$	3.0692	$(3.0692)^+$
$\frac{dC}{dt}$	-ve	0	+ve
Slope	\	___	/

	Hence, $t = 3.0692$ gives a minimum value of C.	
	$C = 25 - 6(3.0692) + e^{0.7(3.0692)}$ $= 15.1563$ $= 15.16$	
	(ii)	



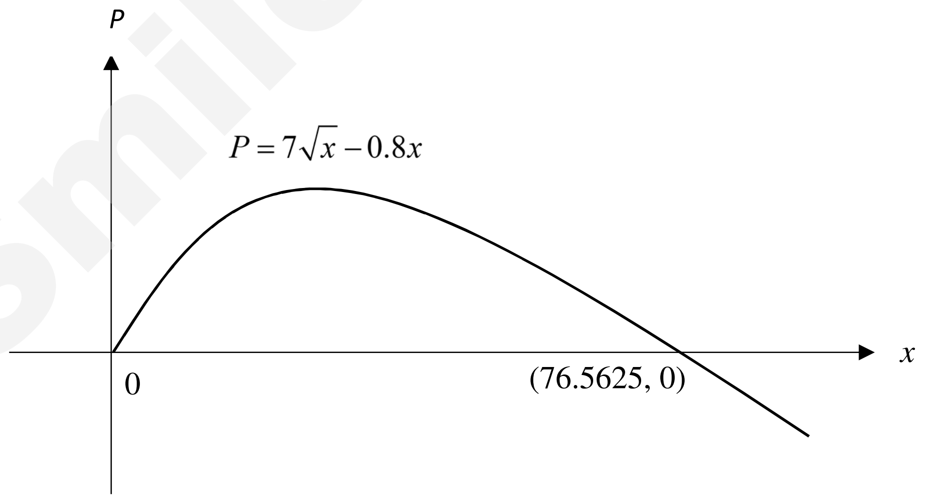
(iii)

Area under curve = $\int_0^6 25 - 6t + e^{0.7t} \, dt$

= 135.84 (From GC)

The total manufacturing cost for the first 6 years is $135,840.

(iv)



G1 – Shape of graph
G1 – Coordinates of x-intercept

(v)

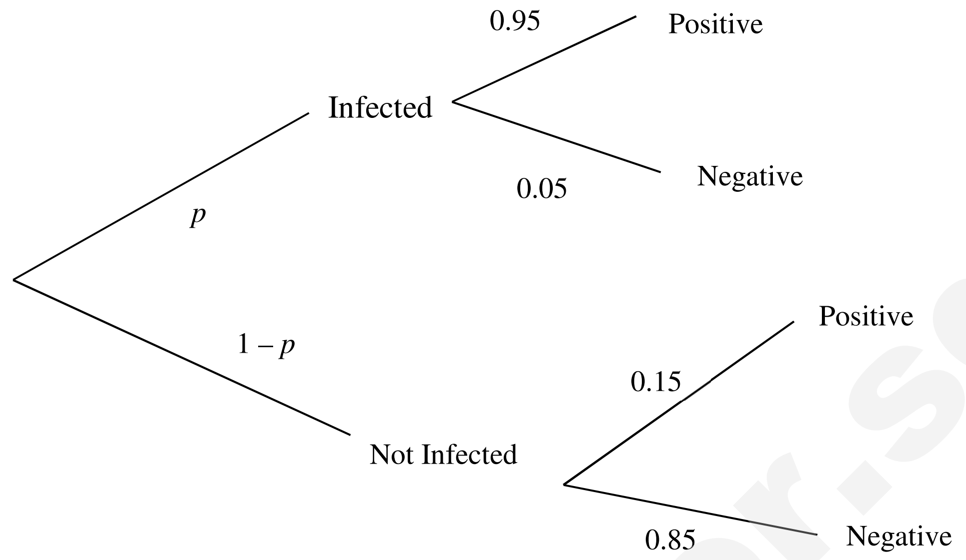
$P = 7\sqrt{x} - 0.8x$

When $x = 40$,

$P = 7\sqrt{40} - 0.8(40)$

$= 12.27$ (2 d.p.)

	Selling price = \$40 + \$12.27 = \$52.27 The selling price of a bicycle helmet is \$52.27.	
	(vi) When $x = 80$, $P = 7\sqrt{80} - 0.8(80)$ $= -1.39$ (3 sig. fig.) Since the profit is negative, the company will make a loss. Hence the company should not manufacture the bicycle helmet if the manufacturing cost is \$80. OR From the graph in (iv), if $x > 76.5625$, the profit is negative. This means that the company will make a loss. Hence, the company should not manufacture the bicycle helmet if the manufacturing cost is \$80.	
	Section B: Probability and Statistics [60 marks]	
6	In a particular population, the probability that an individual chosen at random will have a certain infection is *p*. Every individual will take a medical test for the infection. If an individual has the infection, there is a probability of 0.95 that the test will prove positive. If an individual does not have the infection, there is a probability of 0.85 that the test will prove negative.	
	(i) Draw a probability tree diagram to illustrate the above–mentioned information.	[2]
	(ii) The conditional probability that a randomly chosen individual whose test is negative does not have the infection is $\frac{17}{21}$. Find the value of *p*.	[2]
	Suggested Solution	



1 mark per level

$$\text{P(does not have infection | test is negative)} = \frac{17}{21}$$

$$\frac{\text{P(does not have infection AND test is negative)}}{\text{P(test is negative)}} = \frac{17}{21}$$

$$\frac{0.85(1-p)}{0.05p+0.85(1-p)} = \frac{17}{21}$$

$$\frac{0.85-0.85p}{0.85-0.8p} = \frac{17}{21}$$

$17.85 - 17.85p = 14.45 - 13.6p$

$4.25p = 3.4$

$p = 0.8$

7	**(a)** A group of fifteen people consists of one pair of brothers, one set of three sisters and five married couples. The fifteen people are seated in a row.	
	(i) Find the number of arrangements if men and women alternate.	[1]
	(ii) Find the number of arrangements in which each married couple is seated together but no two sisters sit next to one another.	[2]
	(iii) Find the probability that either the brothers are next to each other or the sisters are all next to one another or both.	[2]
	(b) There are 15 male students and 10 female students in a class. Seven students are to be selected from the class, and the order of selection does not matter.	
	Find the number of selections in which there are at least three students of each gender being selected.	[3]

Suggested Solution

(a)

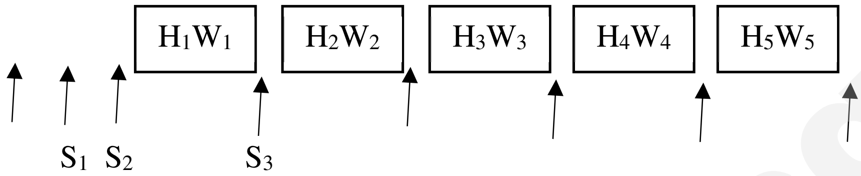
W W W W W W W W

M M M M M M M

(i) Number of arrangements = $8!7! = 203\ 212\ 800$

(ii) $[{}^8C_3\ 3! -]$

Number of arrangements = $7!\ (2!)^5\ {}^8C_3\ 3! = 54\ 190\ 080$



(iii) Let X be the event denoting "the brothers are next to each other"
Let Y be the event denoting "the sisters are next to each other"

$$\begin{aligned}&\mathrm{P}(X \cup Y)\\ &= \mathrm{P}(X) + \mathrm{P}(Y) - \mathrm{P}(X \cap Y)\\ &= \frac{14!2!}{15!} + \frac{13!3!}{15!} - \frac{12!2!3!}{15!}\\ &= \frac{43}{273} \text{ or } 0.157 \text{ (to 3 sig fig)}\end{aligned}$$

(b)

	Number of selections
Case 1: 3 male and 4 female students	${}^{15}C_3{}^{10}C_4 = 95\ 550$
Case 2: 4 male and 3 female students	${}^{15}C_4{}^{10}C_3 = 163800$
Total number of selections	${}^{15}C_3{}^{10}C_4 + {}^{15}C_4{}^{10}C_3 = 259\ 350$

8 **(a)** The random variable X has distribution $\mathrm{B}(7, p)$ and $\mathrm{P}(X = 1 \text{ or } 4) = 0.25$. Write down an equation in terms of p and find the possible values of p. [2]

(b) A company manufactures a large number of teapots. On average, p% of the teapots are defective. The teapots are randomly packed into boxes of 15. Given that the mean number of defective teapots in a box is 0.3, show that $p = 2$. [1]

(i) State, in context of this question, one assumption needed to model the number of defective teapots by a binomial distribution. [1]

(ii) Find the probability that at least 90% of the teapots in a randomly chosen box are not defective. [2]

(iii) n boxes are randomly chosen. The boxes which have more than 10% of the teapots that are defective are rejected. Find the greatest value of [3]

	n such that the probability of rejecting at least 3 boxes is at most 0.01.	
	Suggested Solution	
	(a) $P(X = 1) + P(X = 4) = 0.25$ ${}^7C_1 p(1-p)^6 + {}^7C_4 p^4(1-p)^3 = 0.25$ $7p(1-p)^6 + 35p^4(1-p)^3 = 0.25$ Using GC, $p = 0.0479$ or $p = 0.680$	
	(b) Let X be the random variable denoting "the number of defective teapots in a box of 15" $X \sim B(15, p)$ $E(X) = 15\left(\frac{p}{100}\right)$ $0.3 = 15\left(\frac{p}{100}\right)$ $p = 2$ (Shown) **(i)** The event that a teapot is defective is independent of the event that another teapot being defective. The probability of selecting a defective teapot is 0.02 for each teapot. **(ii)** Let Y be the random variable denoting "the number of non-defective teapots out of 15" $Y \sim B(15, 1 - 0.02)$ $Y \sim B(15, 0.98)$ $P(Y \geq 0.9 \times 15)$ $= P(Y \geq 13.5)$ $= P(Y \geq 14)$ $= 1 - P(Y \leq 13)$ $= 0.96466$ $= 0.965$ **Alternative Method** $P(15 - X \geq 0.9 \times 15)$ $= P(15 - X \geq 13.5)$ $= P(X \leq 1.5)$ $= P(X \leq 1)$ $= 0.96466$ $= 0.965$ **(iii)** $P(X > 0.10 \times 15)$ $= P(X > 1.5)$ $= P(X \geq 2)$	

	$= 1 - P(X \le 1)$ $= 1 - 0.96466$ $= 0.03534$ Let W be random variable denoting "the number of rejected boxes out of n boxes" $W \sim B(n, 0.03534)$ $P(W \ge 3)$ $= 1 - P(W \le 2) \le 0.01$ When $n = 12$, $1 - P(W \le 2) = 0.0076385$ When $n = 13$, $1 - P(W \le 2) = 0.0096713$ When $n = 14$, $1 - P(W \le 2) = 0.011989$ Greatest value of $n = 13$	
9	The total distance run per week, x kilometres, and amount of weight loss, y kilograms, of 8 men undergoing a particular special training programme after a period of time are given in the following table.	

x	0.6	1.2	1.5	2.4	2.5	3.2	3.6	3.4
y	0.5	2.5	2.9	5.5	5	8.1	9	10

(i) Give a sketch of the scatter diagram of the data, as shown on your calculator. [2]

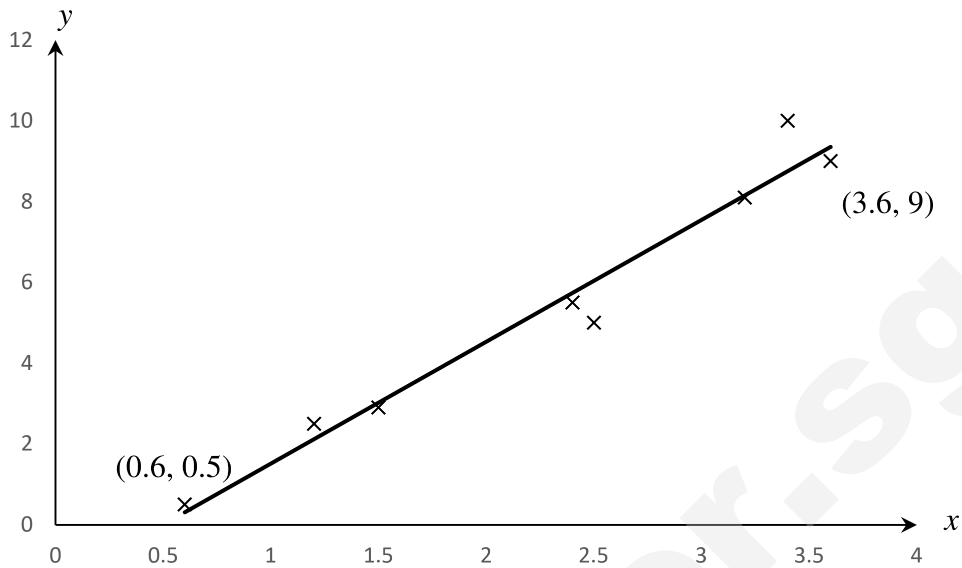
(ii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iii) Find the equation of the regression line of y on x in the form $y = ax + b$, giving the values of a and b correct to 4 significant figures. Explain the meaning of value of a in the context of the question. Sketch this line on your scatter diagram. [3]

(iv) Use a suitable regression line to estimate calculate an estimate of the weight loss for a man who runs 700 metres daily. Comment on the reliability of this estimate. [2]

(v) It is decided to record the distance run per week for person in metres instead of kilometres.Without any further calculations, state any change you would expect in the value of the product moment correlation coefficient. [1]

	Suggested Solution	

	(i)  1 mark – the scale is evenly spaced and the axes are clearly labelled 1 mark – the correct number of points labelled and the endpoints indicated 1 mark – correct regression line drawn within the data range. **(ii)** $r = 0.98102 = 0.981$. Since r is close to 1, there is a strong positive linear correlation between the total distance run per week and the amount of weight loss. As the total distance run per week increases, the amount of weight loss increases. **(iii)** $y = 3.0141x - 1.49497$ $a = 3.014$, $b = -1.495$ <u>The meaning of a</u> For every 1 km increase in the total distance run per week, there is a weight of loss of 3.014 kg. **(iv)** When $x = 0.7(7) = 4.9$ km per week, $y = 3.0141(4.9) - 1.49497 = 13.3$ kg. There is an estimate of 13.3 kg of weight loss. Although r is close to 1, $x = 4.9$ km is outside the data range [0.6 – 3.6], the linear correlation may not hold, the estimate obtained by extrapolation is not reliable. **(v)** There is no change in the value of product moment correlation coefficient.	
10	**In this question you should state clearly the values of the parameters of any normal distributions that you use.** The masses, in grams, of oranges have the distribution $N(250, 15^2)$ and the masses, in grams, of pears have the distribution $N(200, 10^2)$.	

	(i) It is 95% certain that 6 oranges chosen at random have a total mass exceeding L grams. Find the value of L, correct to 1 decimal place.	[2]
	(ii) Find the probability that the total mass of two randomly chosen oranges is within 50 grams of twice the mean mass of the pears.	[2]
	(iii) Find the probability that the average mass of 5 randomly chosen oranges is more than the average mass of 3 randomly chosen pears by more than 55 grams.	[3]
	A certain recipe requires 6 oranges and 10 pears. The recipe requires the oranges and pears to be prepared by peeling them and removing the cores. This process reduces the mass of each orange by 10% and the mass of each pear by 15%.	
	(iv) Find the probability that the total mass, after preparation, of 6 randomly chosen oranges and 10 randomly chosen pears is at least 3 kilograms.	[3]
	(v) Oranges are sold at 60 cents per 100 grams and pears are sold at 70 cents per 100 grams in a particular store. Let W represents the mass of a randomly chosen orange and V represents the mass of a randomly chosen pear. Find $\mathrm{P}(3(0.7V) - 0.6(W_1 + W_2) > 130)$ and explain, in the context of this question, what your answer represents.	[4]
	Suggested Solution	
	Let X and Y be the random variables denoting the mass of a randomly chosen orange in grams and the mass of a randomly chosen pear in grams respectively. $X \sim \mathrm{N}(250, 15^2)$ and $Y \sim \mathrm{N}(200, 10^2)$ **(i)** $X_1 + X_2 + \ldots + X_6 \sim \mathrm{N}(6 \times 250, 6 \times 15^2)$ $X_1 + X_2 + \ldots + X_6 \sim \mathrm{N}(1500, 1350)$ $\mathrm{P}(X_1 + X_2 + \ldots + X_6 > L) = 0.95$ $\mathrm{P}(X_1 + X_2 + \ldots + X_6 \leq L) = 0.05$ $L = 1439.564219$ $= 1439.6$ grams (to 1 decimal place) **(ii)** $X_1 + X_2 - 400 \sim \mathrm{N}(2 \times 250 - 400, 2 \times 15^2 + 0)$ $X_1 + X_2 - 400 \sim \mathrm{N}(100, 450)$ $\mathrm{P}(-50 < X_1 + X_2 - 2(200) < 50)$ $= \mathrm{P}(-50 < X_1 + X_2 - 400 < 50)$ $= 0.0092110$ $= 0.00921$ (to 3 sig fig)	

(iii) $\bar{X} = \dfrac{X_1 + X_2 + ... + X_5}{5} \sim N(250, \dfrac{15^2}{5})$

$\bar{X} \sim N(250, 45)$

$\bar{Y} = \dfrac{Y_1 + Y_2 + Y_3}{3} \sim N(200, \dfrac{10^2}{3})$

$\bar{X} - \bar{Y} \sim N(250 - 200, 45 + \dfrac{10^2}{3})$

$\bar{X} - \bar{Y} \sim N(50, \dfrac{235}{3})$

$P(\bar{X} - \bar{Y} > 55)$
$= 0.28606$
$= 0.286$ (to 3 sig fig)

(iv) $T = 0.9(X_1 + X_2 + \ldots + X_6) + 0.85(Y_1 + Y_2 + \ldots + Y_{10})$
$T \sim N(0.9\times6\times250 + 0.85\times10\times200,\ 0.9^2\times6\times15^2 + 0.85^2\times10\times10^2)$
$T \sim N(3050, 1816)$
$P(T \geq 3000)$
$= 0.87966$
$= 0.880$ (to 3 sig fig)

(v) $W \sim N(250, 15^2)$ and $V \sim N(200, 10^2)$
$A = 3(0.7V) - 0.6(W_1 + W_2)$
$A \sim N(0.7\times3\times200 - 0.6\times2\times250,\ 0.7^2\times3^2\times(10^2) + 0.6^2\times2\times15^2)$
$A \sim N(120, 603)$
$P(A > 130)$
$= 0.341919$
$= 0.342$ (to 3 sig fig)

$P(3(0.7V) - 0.6(W_1 + W_2) > 130)$ refers to the probability that thrice the cost of a randomly chosen pear exceeds the total cost of two randomly chosen oranges by more than \$1.30.

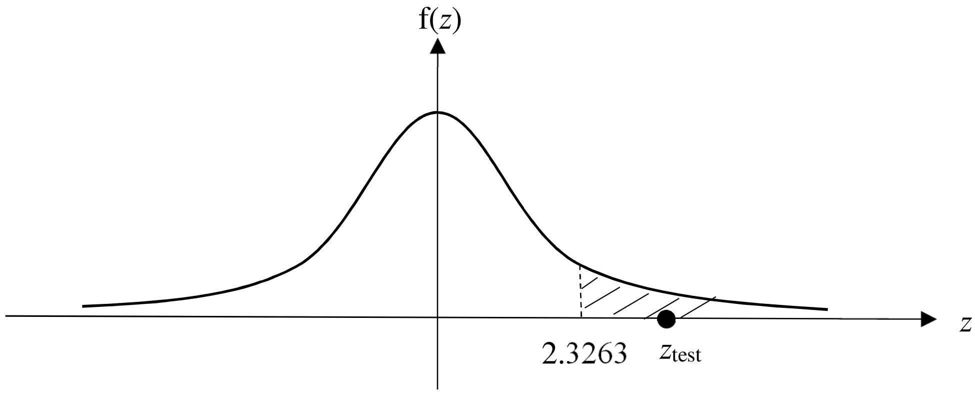
11 **(a)** A particular junior college has 30 classes with different class sizes. The Principal of the college wishes to take a sample of 60 students to meet up with a ministry official for a short conversation. He chooses 2 students at random from each of the classes. State, with a reason, why this method will not give a random sample of 60 students from the college. Describe how you would obtain a random sample of 60 students from the college. [2]

(b) A manager of a particular departmental store claims that the average amount of time that customers spent shopping in the store is not greater than 45 minutes. A random sample of 80 customers is chosen and the time, t minutes, spent by each customer is recorded. The results are summarised by

$$\sum(t - 50) = -342 \quad \text{and} \quad \sum(t - 50)^2 = 2112.$$

(i) Find the unbiased estimates of the population mean and variance. [3]

	(ii) Test the manager's claim at the 1% level of significance.	[4]
	(iii) Explain the meaning of "p – value" in the context of the question.	[1]
	(iv) Without carrying out any further hypothesis tests, what would you conclude if a two-tailed test is carried out instead at 5% level of significance?	[1]
	The department store introduces free Wi-Fi so that the customers can spend more time in the store. The same manager claims that the average amount of time that customers spent shopping in the store is now more than 45 minutes. To test his claims, a random sample of 20 customers is chosen and it is found that their mean average time is *m* minutes.	
	(v) Given the population standard deviation is 12 minutes, find the set of values of *m* for which the manager's claim is valid at the 1% level of significance.	[3]
	(vi) State a necessary assumption for the test in **(v)** to be valid.	[1]
	Suggested Solution	
11	**(a)** This does not give a random sample because the students will have different probabilities of being selected due to the different class sizes. Obtain a list of students studying in the college (sampling frame). Randomly select 60 students from the list to participate in the short conversation with the ministry official.	
	(b) **(i)** Unbiased estimate of population mean $= \frac{\sum(t-50)}{80}+50$ $= \frac{-342}{80}+50$ $= 45.725$ Unbiased estimate of population variance $= \frac{1}{79}\left[\sum(t-50)^2 - \frac{[\sum(t-50)]^2}{80}\right]$ $= \frac{1}{79}\left[2112 - \frac{(-342)^2}{80}\right]$ $= 8.22722$ $= 8.23$ (to 3 sig fig)	
	Let X be the random variable denoting "the amount of time a randomly chosen customer spent in the departmental store" and μ be "the average amount of time the customers spent in the departmental store" **(ii)** To test $H_0: \mu = 45$ minutes (manager's claim)	

	$H_1: \mu > 45$ minutes Right–tailed test at 1% level of significance Under H_0, $\bar{X} \sim N(45, \frac{8.22722}{80})$ approximately by Central Limit Theorem since $n = 80$ is large Using GC, p – value = 0.0118867 = 0.0119 (to 3 sig fig) Since p – value > 0.01, we do not reject H_0 and there is insufficient evidence to conclude that the average amount of time the customers spent in the department store is more than 45 minutes at 1% level of significance. The manager's claim is valid.	
	(iii) p – value is the lowest level of significance at which the evidence is sufficient to conclude the average amount of time the customers spent in the department store is more than 45 minutes.	
	(iv) p – value of the two tailed test = $2 \times$ p – value$_{(1-\text{ tailed test})}$ $= 2 \times 0.0118867$ $= 0.0237734 = 0.0238$ Since the p – value < 0.05, we reject H_o and conclude there is sufficient evidence to conclude that the average amount of time the customers spent in the department store differs from 45 minutes at 5% level of significance.	
	(v) Let Y be the random variable denoting "the amount of time a randomly chosen customer spent in the departmental store after the introduction of Wi-Fi in the store" To test $H_0: \mu = 45$ minutes $H_1: \mu > 45$ minutes (manager's claim) Right–tailed test at 1% level of significance Under H_0, $\bar{Y} \sim N(45, \frac{12^2}{20})$ For the manager's claim to be valid, we reject H_0, z_{test} lies within the critical region.  $z_{test} \geq 2.3263$	

		$\dfrac{m-45}{\sqrt{\dfrac{12^2}{20}}} \geq 2.3263$ $m \geq 45 + 2.3263\sqrt{\dfrac{12^2}{20}}$ $m \geq 51.242$ The solution set = $\{m \in \square : m \geq 51.3\}$	
	(vi)	**Assumption for part (v) to be valid** The time spent by the customers in the department store follows a normal distribution.	

H1

TAMPINES JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

MATHEMATICS 8865/01

Paper 1

Monday, 10 September 2018

3 hours

Additional Materials: Answer Paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics group on all the work you hand in, including the Cover Page.

Write in dark blue or black pen on both sides of the paper.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1 Find algebraically the range of values of k for which

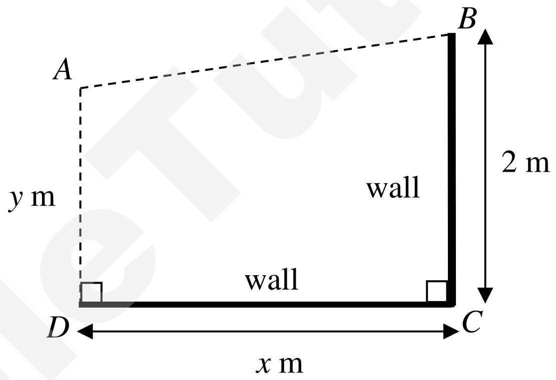
$$3kx^2 - 6x + k > 0$$

for all real values of x. [4]

2 **(i)** Differentiate $\dfrac{1}{3x^2+4}$ with respect to x. [2]

(ii) Find $\int \left(3-\sqrt{x}\right)^2 \, dx$. [3]

3



The diagram shows a garden $ABCD$ in the shape of a trapezium next to the walls BC and CD. The wall BC is 2 m long and the wall CD is x m long. The broken lines AB and AD represent fences. The fence AD is y m long and is parallel to BC. The total length of the fences is 5 m.

(i) Show that the area, A m^2, of the garden $ABCD$ is given by

$$A = \frac{1}{12}\left(33x - x^3\right).$$ [3]

(ii) Use differentiation to find the maximum value of A. Justify that this is the maximum value. [4]

4 The curve C has equation $y = e^{2x-1} - 3$.

(i) Sketch the graph of C, stating the exact coordinates of any points of intersection with the axes and the equations of any asymptotes. [3]

(ii) Without using a calculator, find the equation of the tangent to C at the point where $x = \frac{1}{2}$, giving your answer in the form $y = mx + c$, where m and c are constants. [4]

(iii) Find the exact area of the region bounded by C, the x-axis and the line $x = 2$. [4]

5 A student decided to track his daily expenditure, \$$S$, for t days, $1 \le t \le 20$. He spent \$12 on Day 1. Every day, he spent more money than the previous day up till Day 10. On Day 10, he spent \$30 and realised that he had used up too much of his budget. He decided to cut his expenditure and spent less money than the previous day for the remaining days. He noticed that his expenditure can be modelled using a quadratic equation.

(i) Show that the expenditure is modelled by

$$S = -\frac{2}{9}t^2 + \frac{40}{9}t + \frac{70}{9}.$$ [4]

(ii) Find the student's expenditure on Day 20. [1]

(iii) Sketch the graph of S against t, for $1 \le t \le 20$, stating the coordinates of the end points and turning point. [2]

(iv) Would this be a good equation to use to estimate his daily expenditure in the long run? Justify your answer. [1]

The student's mother recommended that he could set a daily budget of \$20 instead.

(v) By adding a suitable line to your graph in **(iii)**, find the range of Days for which the student would have exceeded this daily budget. [3]

To improve his financial planning, the student should set a new daily budget which is the minimum amount he does not exceed 75% of the time based on his previous record.

(vi) How much should this new daily budget be? [2]

Section B: Probability and Statistics [60 marks]

6 A bakery produces two types of cookie. One type of cookie contains nuts, and the other type contains no nuts. There is a constant probability that a cookie contains nuts. The cookies are sold in packs of 6. Each pack has a random selection of cookies. For these packs, the mean number of cookies containing nuts is 0.8.

(i) Find the probability that a pack chosen at random has at least 2 cookies containing nuts. [3]

A customer buys 10 packs of cookies for a party.

(ii) Find the probability that less than 4 of these packs have at least 2 cookies containing nuts. [2]

7 A group of 7 boys and 5 girls are standing in a queue. Find the number of different possible arrangements if

(i) the girls must stand together, [2]

(ii) all the girls must be separated. [2]

Find the probability that there are at most 2 boys in the front half of the queue. [3]

8 The year, x, and the mean amount spent on credit cards per household in Singapore in the 1st Quarter, y thousand dollars, are given in the following table.

x	2000	2003	2006	2009	2011	2013	2015	2017
y	2.01	3.00	3.63	5.23	6.73	8.73	9.95	10.3

(i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]

(ii) Find the product moment correlation coefficient. [1]

(iii) Find the equation of the regression line of y on x, in the form $y = mx + c$, giving the values of m and c correct to 3 significant figures. Sketch this line on your scatter diagram. [2]

(iv) Calculate an estimate of the mean amount spent on credit cards per household in Singapore in the 1st Quarter of 1998. Comment on the reliability of your estimate. [2]

(v) Without calculating the estimate, state two reasons why you would expect the estimate of the mean amount spent on credit cards per household in Singapore in the 1st Quarter of 2016 to be reliable. [2]

9 A riding qualification involves two separate parts, theory and practical. To succeed in the riding qualification, a student must first pass theory followed by practical. Students who fail theory at the first attempt always make a second attempt, while students who pass theory at the first attempt cannot make a second attempt. Students are allowed at most two attempts at theory but only one attempt at practical.

A is the event that the student passes theory at the first attempt,
B is the event that the student passes practical,
C is the event that the student passes theory at the second attempt.

It is given that $\mathrm{P}(A) = 0.2$, $\mathrm{P}(B) = 0.4$, $\mathrm{P}(C \mid A') = 0.7$ and $\mathrm{P}(A \cup B) = 0.52$.

(i) Determine whether the events A and B are independent. [2]

(ii) Explain, in the context of the question what is meant by $\mathrm{P}(C \mid A)$, and find its value. [2]

(iii) Draw a tree diagram to represent the information above. [2]

(iv) Find the probability that a student chosen at random will succeed in the riding qualification. [2]

There are n students who take the riding qualification.

(v) Find the least value of n given that the probability that none of the students will succeed in the riding qualification is less than 0.1. [4]

10 The masses, in kg, of two types of oranges, A and B, sold by a supermarket have independent normal distributions with means and standard deviations as shown in the following table.

	Mean	Standard deviation
Type A	0.26	σ
Type B	0.16	0.02

It is found that 40% of oranges of type A have a mass less than 0.25 kg.

(i) Show that the standard deviation of the distribution of the mass of oranges of type A is 0.04 kg, correct to 2 decimal places. [2]

(ii) Find the probability that two randomly chosen oranges of type A each have a mass of more than 0.25 kg, giving your answer correct to 2 decimal places. [2]

(iii) Without any calculation, explain why the probability that the total mass of two randomly chosen oranges of type A is more than 0.5 kg is greater than your answer to part **(ii)**. [1]

4 oranges of type A and 3 oranges of type B are chosen at random.

(iv) Find the probability that the total mass of 4 oranges of type A is at least 0.6 kg more than the total mass of 3 oranges of type B. [4]

Oranges of type A cost \$4.50 per kg and oranges of type B cost \$5 per kg.

(v) Find the probability that the total cost of 4 oranges of type A and 3 oranges of type B is between \$6.50 and \$7.50. State the mean and variance of the distribution that you use.

[4]

11 Intensity of light is measured in lumens. A light bulb manufacturing company claims that the mean intensity of light from its standard 60 watt light bulbs is at least 800 lumens. A random sample of 50 standard 60 watt light bulbs is checked and the intensity of light from the light bulbs, x lumens, are summarised by

$$\sum(x-800)=-300, \qquad \sum(x-800)^2=34924.$$

(i) Calculate unbiased estimates of the population mean and variance. [3]

(ii) Test, at the 5% significance level, whether the company's claim is valid. [5]

(iii) State the meaning of the p-value in context. [1]

(iv) State, giving a reason, whether it is necessary to assume a normal distribution for this test to be valid. [1]

The manufacturing company claims that the mean intensity of light from its energy-efficient 15 watt light bulbs is 850 lumens. It is known that the standard deviation of the intensity of light from energy-efficient 15 watt light bulbs is 10 lumens. A consumer organisation decides to check the manufacturer's claim and measures the intensities of a random sample of 20 energy-efficient 15 watt light bulbs. Using a 5% significance level, the consumer organisation finds that the manufacturer overestimated the intensity of light from its energy-efficient 15 watt light bulbs.

(v) Find the set of values within which the mean intensity of light from the random sample of 20 energy-efficient light bulbs must lie. [4]

End of Paper

2018 JC2 Preliminary Examination
H1 Mathematics
Solution

1 **[4m]**	$3kx^2 - 6x + k > 0$ $3k > 0$ and Discriminant < 0 Discriminant $= (-6)^2 - 4(3k)(k)$ $= 36 - 12k^2$ $36 - 12k^2 < 0$ $3 - k^2 < 0$ $(\sqrt{3} - k)(\sqrt{3} + k) < 0$ $k < -\sqrt{3}$ or $k > \sqrt{3}$ Since $3k > 0$, $k > \sqrt{3}$	

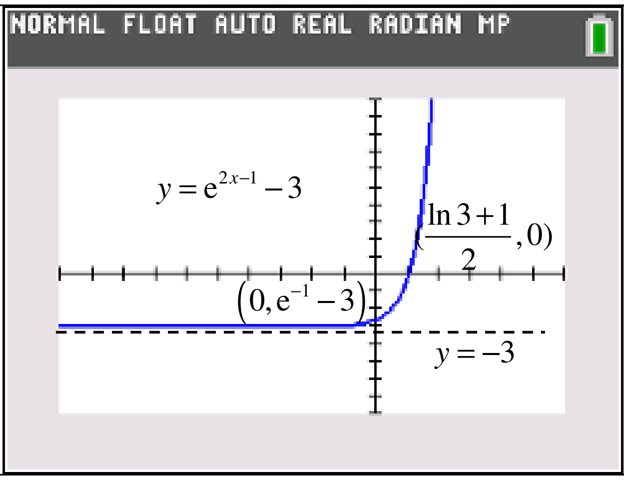
2(i) **[5m]**	$y = \frac{1}{3x^2+4} = \left(3x^2+4\right)^{-1}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -\left(3x^2+4\right)^{-2}(6x)$ $= -\frac{6x}{\left(3x^2+4\right)^2}$	
(ii)	$\int\left(3-\sqrt{x}\right)^2 \mathrm{d}x$ $= \int\left(9 - 6\sqrt{x} + x\right)\mathrm{d}x$ $= 9x - \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^2}{2} + c$ $= 9x - 4x^{\frac{3}{2}} + \frac{x^2}{2} + c$	

3(i) **[7m]**	$A = \frac{1}{2}(y+2)x \quad ----(1)$ Using pythagoras theorem, $x^2 = (5-y)^2 - (2-y)^2$ $x^2 = \left(25 - 10y + y^2\right) - \left(4 - 4y + y^2\right)$ $x^2 = 21 - 6y$ $y = \frac{21-x^2}{6} \quad ----(2)$	

	Substitute (2) into (1) $A=\frac{1}{2}\left(\frac{21-x^2}{6}+2\right)x$ $=\frac{1}{2}\left(\frac{33-x^2}{6}\right)x$ $=\frac{1}{12}\left(33x-x^3\right)$ (shown)	
(ii)	$\frac{dA}{dx}=\frac{1}{12}\left(33-3x^2\right)$ When $\frac{dA}{dx}=0$, $\frac{1}{12}\left(33-3x^2\right)=0$ $33-3x^2=0$ $x^2=11$ $x=\pm\sqrt{11}$ Since $x>0$, $x=\sqrt{11}=3.3166=3.32$ m (to 3 s.f) To show A is maximum: Using first derivative test, (see table below) Or Using second derivative test, $\frac{d^2A}{dx^2}=\frac{1}{12}(-6x)=-\frac{x}{2}$ When $x=\sqrt{11}$, $\frac{d^2A}{dx^2}=-\frac{\sqrt{11}}{2}<0$ Therefore A is maximum when $x=\sqrt{11}$. Maximum area $=\frac{1}{12}\left(33\left(\sqrt{11}\right)-\left(\sqrt{11}\right)^3\right)=6.08\text{ m}^2$	

x	$\sqrt{11}^-$	$\sqrt{11}$	$\sqrt{11}^+$
Sign of $\frac{dA}{dx}$	+	0	-
	/	–	\

4(i)
[11m]



(ii)

$$\frac{dy}{dx} = 2e^{2x-1}$$

When $x = \frac{1}{2}$,

$$\frac{dy}{dx} = 2e^{2\left(\frac{1}{2}\right)-1} = 2, \quad y = e^{2\left(\frac{1}{2}\right)-1} - 3 = -2$$

Equation of tangent at P: $y - (-2) = 2(x - \frac{1}{2})$

$$y = 2x - 3$$

(iii)

$$\int_{\frac{\ln 3+1}{2}}^{2} e^{2x-1} - 3 \, dx$$

$$= \left[\frac{e^{2x-1}}{2} - 3x\right]_{\frac{\ln 3+1}{2}}^{2}$$

$$= \left[\frac{e^{2(2)-1}}{2} - 3(2)\right] - \left[\frac{e^{2\left(\frac{\ln 3+1}{2}\right)-1}}{2} - 3\left(\frac{\ln 3+1}{2}\right)\right]$$

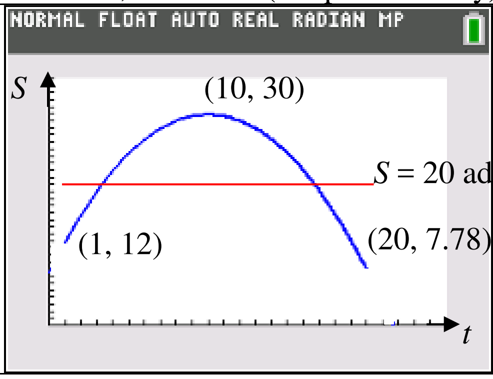
$$= \frac{e^3}{2} - 6 - \frac{3}{2} + \frac{3\ln 3}{2} + \frac{3}{2}$$

$$= \frac{e^3}{2} + \frac{3\ln 3}{2} - 6$$

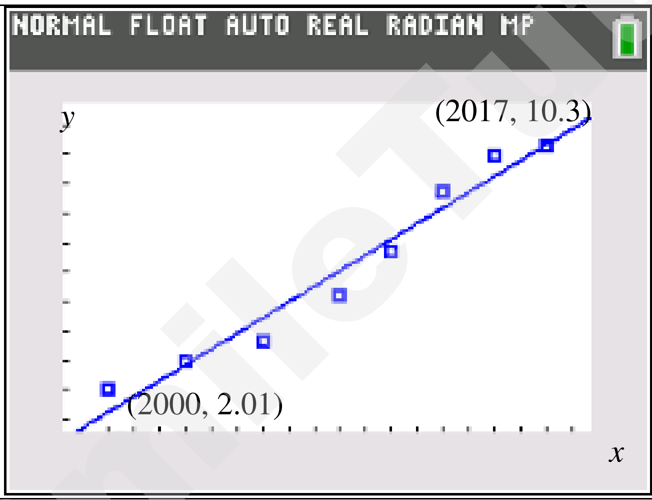
5(i)
[13m]

Let $S = at^2 + bt + c$

\$12 on Day 1: $12 = a(1)^2 + b(1) + c$

	\$30 on Day 10 is maximum: $30 = a(10)^2 + b(10) + c$ $\frac{\mathrm{d}S}{\mathrm{d}t} = 2at + b$ $0 = 2a(10) + b$ Using GC, $a = -\frac{2}{9}, b = \frac{40}{9}, c = \frac{70}{9}$	
(ii)	At $t = 20$, S = \$7.78 (2 d.p. for money)	
(iii)		
(iv)	No, as his expenditure would eventually become negative which is impossible.	
(v)	Add horizontal line $S = 20$ Intersection points at 3.29 and 16.7 He exceeded \$20 from Day 4 to Day 16.	
(v)	75% of 20 days = 15 days At t = 7, S = 28. The new daily budget should be \$28.	

6(i) **[5m]**	Let X be the number of cookies containing nuts out of 6. $X \sim \mathrm{B}(6, p)$ Since $\mathrm{E}(X) = 0.8$ $6p = 0.8$ $p = \frac{2}{15}$ $\mathrm{P}(X \geq 2)$ $= 1 - \mathrm{P}(X \leq 1)$ $= 0.18509$ (to 5 s.f) $= 0.185$ (to 3 s.f)	
(ii)	Let Y be the number of packs with at least 2 cookies containing nuts out of 10. $Y \sim \mathrm{B}(10, 0.18509)$	

	$\mathrm{P}(Y<4)=\mathrm{P}(Y\le 3)$ $=0.904$ (3sf)	
7(i) **[7m]**	Number of arrangements = $8!\times 5!$ = 4838400	
(ii)	Number of arrangements = $7!\times\binom{8}{5}\times 5!$ = 33868800	
	Probability = $\dfrac{\text{No. ways 1 boy + No. ways 2 boys}}{\text{Total no. ways}}$ $=\dfrac{\binom{7}{1}\times 6!+\binom{7}{2}\times\binom{5}{4}\times 6!}{\binom{12}{6}\times 6!}$ $=\dfrac{4}{33}$	
8(i) **[9m]**	 	
(ii)	By GC, product moment correlation coefficient r = 0.980 (3sf)	
(iii)	By GC, $y=0.53403x-1066.8$ (5sf) $y=0.534x-1070$ (3sf)	
(iv)	$y=0.53403(1998)-1066.8$ (5sf) $=0.192$ thousand dollars (3sf) The estimate is not reliable as 1998 lies outside the data range. Extrapolation is not a good practice.	
(v)	The correlation coefficient is very close to 1, indicating a strong positive linear relationship. 2016 also lies within the data range and interpolation is a good practice. Hence, the estimate is reliable.	

9 (i)

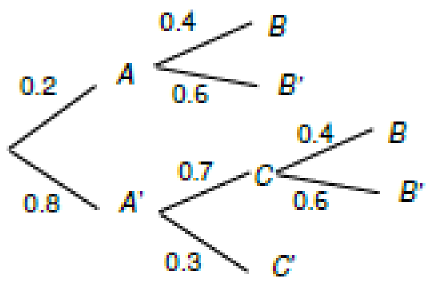
$$\begin{aligned}P(A\cap B) &= 0.2+0.4-0.52\\ &= 0.08\\ &= 0.2\times 0.4\\ &= P(A)\times P(B)\end{aligned}$$

Thus A and B are independent.

(ii)

$P(C\mid A)$ is the probability of a student passing theory at second attempt if he passed theory at first attempt. Since this is not possible as C and A are mutually exclusive, $P(C\mid A)=0$

(iii)



(iv)

$$\begin{aligned}\text{Probability} &= (0.2)(0.4)+(0.8)(0.7)(0.4)\\ &= 0.304\end{aligned}$$

(v)

Let Y be the number of students who succeed out of n.

$Y \sim B(n, 0.304)$

$P(Y=0)<0.1$

Using GC,

n	$P(Y=0)$
6	0.1137
7	0.0791

Least $n=7$

10(i)
[13m]

Let A be the mass of oranges of type A, in kg.

$A \sim N(0.26, \sigma^2)$

$P(A<0.25)=0.4$

$P\left(Z<\dfrac{0.25-0.26}{\sigma}\right)=0.4$

$P\left(Z<\dfrac{-0.01}{\sigma}\right)=0.4$

$\dfrac{-0.01}{\sigma}=-0.25335$

$\sigma=0.0395=0.04$ (shown)

(ii)

Required probability

	$= \mathrm{P}(A > 0.25) \times \mathrm{P}(A > 0.25)$ $= (0.6)^2$ $= 0.36$ (to 2 d.p)	
(iii)	Part **(iii)** includes more cases in addition to the case in part **(ii)**. For example, the mass of one orange of type A is less than 0.25 kg but the mass of the other orange of type A is more than 0.25 kg such that the total mass of two randomly chosen oranges of type A is more than 0.5 kg.	
(iv)	Let A and B be the mass of oranges of type A and type B, in kg, respectively $A \sim \mathrm{N}(0.26, 0.04^2)$ $B \sim \mathrm{N}(0.16, 0.02^2)$ $(A_1 + A_2 + A_3 + A_4) - (B_1 + B_2 + B_3) \sim \mathrm{N}(4(0.26) - 3(0.16), 4(0.04^2) + 3(0.02^2))$ $(A_1 + A_2 + A_3 + A_4) - (B_1 + B_2 + B_3) \sim \mathrm{N}(0.56, 0.0076)$ $\mathrm{P}((A_1 + A_2 + A_3 + A_4) \geq (B_1 + B_2 + B_3) + 0.6)$ $= \mathrm{P}((A_1 + A_2 + A_3 + A_4) - (B_1 + B_2 + B_3) \geq 0.6)$ = 0.323 (to 3 s.f)	
(v)	$4.5(A_1 + A_2 + A_3 + A_4) + 5(B_1 + B_2 + B_3) \sim \mathrm{N}(7.08, 0.1596)$ $\mathrm{P}(6.50 < 4.5(A_1 + A_2 + A_3 + A_4) + 5(B_1 + B_2 + B_3) < 7.50)$ = 0.780 (to 3 s.f)	

11(i) **[14m]**	Unbiased estimate of population mean $\bar{x} = \frac{-300}{50} + 800 = 794$ Unbiased estimate of population variance $s^2 = \frac{1}{49}\left(34924 - \frac{(-300)^2}{50}\right)$ $= 676$	
(ii)	Let μ be the population mean intensity of light from the standard 60 watt light bulb, in lumens Let X be the intensity of light from the standard 60 watt light bulb, in lumens To test $\mathrm{H}_0 : \mu = 800$ against $\mathrm{H}_1 : \mu < 800$ at 5% level of significance. Since n = 50 is large, by Central Limit Theorem, $\overline{X} \sim \mathrm{N}\left(800, \frac{676}{50}\right)$ approximately under H_0.	

	Test statistic: $Z=\dfrac{\overline{X}-800}{\sqrt{\dfrac{676}{50}}}\sim \mathrm{N}(0,1)$ approximately under H_0 $z_{test}=\dfrac{794-800}{\sqrt{\dfrac{676}{50}}}=-1.63$ (to 3 s.f) Using G.C., $p=0.051362=0.0514$, $z_{test}=-1.63$, $\bar{x}=794$, $n=50$ Since $p=0.0514>0.05$, we do not reject H_0 and conclude that there is insufficient evidence, at the 5% level of significance, that the mean intensity of light from the standard 60 watt light bulb is less than 800 lumens. Hence the company's claim is valid.	
(iii)	The smallest level of significance such that the company's claim that the mean intensity of light from the standard 60 watt light bulbs is at least 800 lumens is rejected is 5.14%.	
(iv)	It is not necessary to assume normal distribution. Since n = 50 is large, by Central Limit Theorem, the mean intensity of light from the standard 60 watt light bulbs is approximately normally distributed.	
(v)	Let μ be the population mean intensity of light from the energy-efficient 15 watt light bulbs Let Y be the intensity of light from the energy-efficient 15 watt light bulbs , in lumens To test $\mathrm{H}_0: \mu=850$ against $\mathrm{H}_1: \mu<850$ at 5% level of significance. Since n = 20 is large, by Central Limit Theorem, $\overline{Y}\sim \mathrm{N}\left(850, \dfrac{10^2}{20}\right)$ approximately under H_0. Test statistic: $Z=\dfrac{\overline{Y}-850}{\sqrt{\dfrac{10^2}{20}}}\sim \mathrm{N}(0,1)$ under H_0 $z_{\text{test}}=\dfrac{\bar{y}-850}{\sqrt{\dfrac{10^2}{20}}}=\dfrac{\bar{y}-850}{\sqrt{5}}$ Since the manufacturer overestimated the intensity of light from its energy-efficient 15 watt light bulbs, we reject H_0 To reject H_0, $z_{\text{test}}<z_{\text{crit}}$	

	$\dfrac{\overline{y}-850}{\sqrt{5}}<-1.6449$ $\overline{y}-850<-3.6780$ $\overline{y}<846.322$ $\overline{y}\leq 846$ (to 3 s.f) Set of values = $\left\{\overline{y}\in \square : \overline{y}\leq 846\right\}$	

TEMASEK JUNIOR COLLEGE, SINGAPORE

JC 2
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Higher 1

TEMASEK JUNIOR COLLEGE

MATHEMATICS

8865/01

Paper 1

12 Sep 2018

Additional Materials: Answer paper

List of Formulae (MF26)

3 hours

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Section A: Pure Mathematics [40 marks]

1 **(i)** Differentiate $\ln(e+e^x)^2$. [2]

(ii) Given that $\int_0^a \frac{e^x}{e+e^x}\,dx = 1$, find the value of a. [4]

2 **(a)** By means of the substitution $u=\sqrt{x}$, and without the use of a graphing calculator, find the value of x which satisfies the equation $5\sqrt{x}-\frac{8}{\sqrt{x}}=6$. [3]

(b) Find, algebraically, the set of values of k for which

$$x^2-2x+kx+k^2+1>0$$

for all real values of x. [3]

3 **(i)** On the same diagram, sketch the graphs of $y=\ln(x+4)$ and $y=\frac{1}{(x-2)^2}$, showing the equations of the asymptotes and any axial intercept(s). [5]

(ii) Hence find the range of values of x for which $\ln(x+4)\geq\frac{1}{(x-2)^2}$. [3]

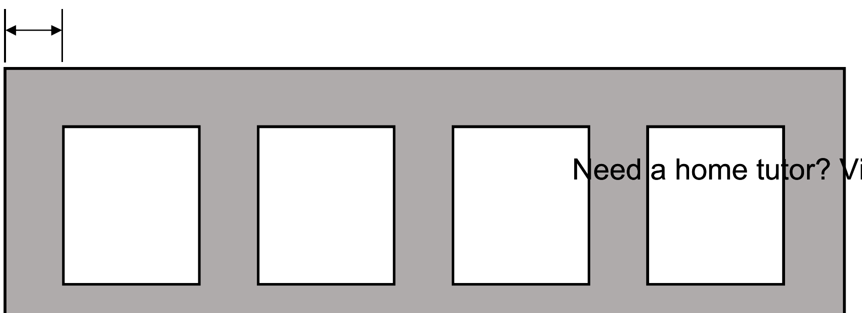
4 A particular industrial machine generates annual revenue at the rate $R'(t)=5000-20t^2$ dollars where t is the age of the machine in years. The annual operating and servicing costs for the machine is given by $C'(t)=2000+10t^2$ dollars.

(i) Sketch $y=R'(t)$ and $y=C'(t)$ on the same diagram, indicating clearly the axial intercepts. [3]

(ii) Find the range of values of t for which the machine generates a profit, that is the useful life of the machine. [2]

(iii) Use integration to find the total profit generated by the machine over the period of useful life. [3]

5



The diagram shows a photo frame design with 8 identical rectangular holes, each x cm by y cm, cut out for the display of photographs. The holes are spaced 4 cm from one another and 4 cm from the edges of the photo frame.

It is given that x and y can vary but the total area of the 8 holes must be 576 cm^2.

(i) Show that the shaded area, A cm^2, the portion of the photo frame not covered by the photos is given by

$$A = 48x + \frac{2880}{x} + 240.$$ [3]

(ii) Find the values of x and y for which A is a minimum. Hence find the minimum value of A in the form $a\sqrt{15} + b$ where a and b are integers to be found. [6]

(iii) Find the rate of change of A when x is changing at the rate of 0.2 cm s^{-1} at the instant when the value of y is twice the value of x. [3]

Section B : Statistics [60 marks]

6 A supermarket sells a particular type of durians. The masses of these durians are normally distributed with mean μ and standard deviation σ in kilograms. As part of quality control, the supermarket would discard durians that weigh less than 0.6 kg and reserve those that weigh more than 2 kg for their regular customers. Based on past data, the supermarket usually discards and reserves 15% and 1% of the durians respectively. Find the values of μ and σ. [4]

7

The diagram shows an observation wheel with 24 capsules. Each capsule can carry passengers up to a maximum load of 3000 kg. The weights of male passengers have mean 70 kg and standard deviation 8.9 kg.

(i) The operator of the observation wheel allows n randomly chosen male passengers to enter a capsule. Find the greatest value of n such that the probability that the total weight of the n male passengers exceed the maximum load is less than 0.01. [4]

(ii) Explain whether it is necessary to assume that the weights of male passengers are normally distributed. [1]

8 A manufacturer sells a new wifi router that is designed to have a mean signal range of 100 m. A quality control manager suspects that there is a flaw in the manufacturing process and the routers produced have a mean signal range that differs from 100 m. A random sample of 53 wifi routers is tested and found to have a mean signal range of 95.7 m and standard deviation of 11.7 m.

(i) Find an unbiased estimate of the population variance.
Explain what is meant by "unbiased estimate" in this context. [2]

(ii) Test at the 2.5% level of significance whether the quality control manager's suspicion is justified. [4]

A second sample of 53 wifi routers is tested and the unbiased estimates for the population mean and standard deviation calculated using this second sample are 97.8 m and s m respectively. A test at the 2.5% significance level does not indicate that the routers have a mean signal range of less than 100 m. Find the range of values of s that would result in such a conclusion. [4]

9 Eight students signed up for a weekly private tuition at eight different centres. They were surveyed on the monthly fees (\$$x$) they paid and their subsequent test scores (y %) after 6 months. The results are given in the following table.

Student	A	B	C	D	E	F	G	H
x	200	300	180	340	220	280	400	500
y	44	51	62	56	48	50	59	65

(i) Plot a scatter diagram for the data. Giving a reason, identify a data pair which should be regarded as suspect. [2]

The suspect data pair is subsequently removed from the data set.

(ii) Calculate the correlation coefficient for the revised data set. Comment on the value obtained. [2]

(iii) Find the equation of the regression line of y on x, and use it to predict the test score of a student who is paying \$350 for tuition, correct to the nearest integer value. Comment on the reliability of your prediction. [3]

(iv) A new equation of the regression line of y on x, $y = 32.0555 + 0.066149x$ is obtained when a new data pair was added. If the value of x of this data pair is 550, find the corresponding value of y. [3]

10 A machine is used to generate codes consisting of two integers followed by four letters. Each of the two integers generated is equally likely to be any of the nine integers 1 – 9. The integer 0 is not used. Each of the four letters generated is equally likely to be any of the seven letters of the alphabets $\{A, B, C, D, E, F, G\}$.

(i) Find the number of codes that can be formed, if no letter or integer is repeated in the code. [2]

From (ii) onwards, letters and integers can be repeated in the codes.

Find the number of codes that can be formed

(ii) with two same integers, [2]

(iii) with exactly one vowel and three consonants. [2]

Hence find the **probability** that the last letter of a randomly chosen code is a vowel given that there are exactly one vowel and three consonants. [3]

[Turn over

11 A confectionary produces a large number of sweets every day. On average, 20% of the sweets are wasabi-flavoured and the rest are caramel-flavoured.

(i) A random sample of n sweets is chosen. If the probability that there are at least three wasabi-flavoured sweets in the sample is at least 0.7, find the least possible value of n. [3]

The manufacturer decides to put the sweets randomly into packets of 20.

(ii) Find the probability that such a packet contains less than 3 wasabi-flavoured sweets. [2]

(iii) A customer selects packets of 20 sweets at random from a large consignment until she finds a packet with exactly 12 caramel-flavoured sweets. Give a reason why a Binomial Distribution is not an appropriate model for the number of packets she selects in the context of the question. [1]

The packets are then packed into boxes. Each box contains 10 packets.

(iv) Find the probability that all the packets in a randomly chosen box contain at least 3 wasabi-flavoured sweets. [2]

(v) Find the probability that there are at least 30 wasabi-flavoured sweets in a randomly chosen box. [1]

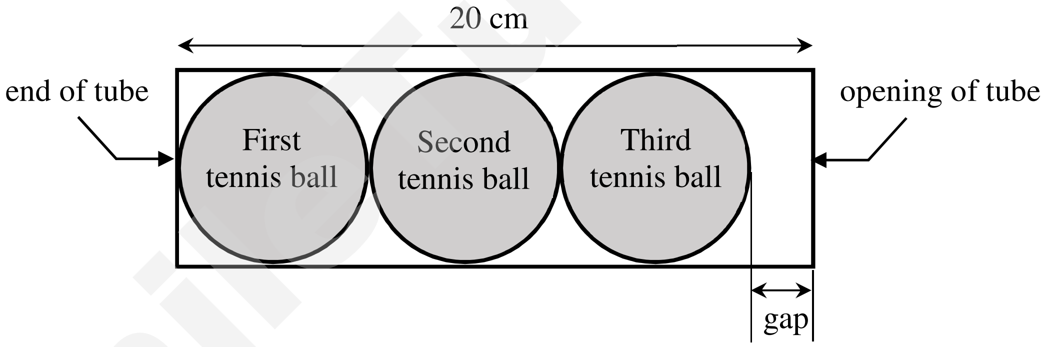
(vi) Explain why the answer to **(v)** is greater than the answer to **(iv)**. [1]

12 A company manufactures tennis balls and packs them into cylindrical tubes for sale. The tennis balls have radii that are normally distributed with mean 3.3 cm and standard deviation 0.2 cm.

(i) Find the probability that the radius of a randomly selected tennis ball lies between 3.135 cm and 3.465 cm. Without any further calculation, explain, with the aid of a diagram, how the answer obtained would compare with the probability that the radius lies between 3.465 cm and 3.795 cm. [3]

(ii) 3 tennis balls are randomly selected. Find the probability that exactly one of them has a radius less than 3.4 cm and two of them have radii greater than 3.4 cm each. [2]

The cylindrical tubes are 20 cm long. 3 tennis balls are randomly selected and packed into a cylindrical tube such that the first tennis ball is in contact with the end of the tube and each subsequent ball is in contact with its neighbouring ball as shown in the diagram below. Assume that the centres of all the tennis balls are horizontally aligned.



(iii) Find the probability that a gap exists between the third tennis ball and the opening of the tube. [3]

(iv) Find the range of values of k such that the probability that the gap between the third tennis ball and the opening of the tube is more than k cm is at most 0.15. [3]

State an assumption used in your calculations. [1]

--- End of paper ---

H

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(ii) Given that $\int_0^a \frac{\mathrm{e}^x}{\mathrm{e}+\mathrm{e}^x}\ \mathrm{d}x = 1$, find the value of a. [4]

[Solution]

(i) $$\frac{\mathrm{d}}{\mathrm{d}x}\ln(\mathrm{e}+\mathrm{e}^x)^2 = 2\frac{\mathrm{d}}{\mathrm{d}x}\ln(\mathrm{e}+\mathrm{e}^x)$$
$$= \frac{2\mathrm{e}^x}{\mathrm{e}+\mathrm{e}^x}$$

(ii) From (i), $\int \frac{2\mathrm{e}^x}{\mathrm{e}+\mathrm{e}^x}\ \mathrm{d}x = \ln(\mathrm{e}+\mathrm{e}^x)^2$

$$\int_0^a \frac{2\mathrm{e}^x}{\mathrm{e}+\mathrm{e}^x}\ \mathrm{d}x = \left[\ln(\mathrm{e}+\mathrm{e}^x)^2\right]_0^a$$

$$2\int_0^a \frac{\mathrm{e}^x}{\mathrm{e}+\mathrm{e}^x}\ \mathrm{d}x = 2\ln(\mathrm{e}+\mathrm{e}^a) - 2\ln(\mathrm{e}+1)$$

$$2\int_0^a \frac{\mathrm{e}^x}{\mathrm{e}+\mathrm{e}^x}\ \mathrm{d}x = 2\ln\left(\frac{\mathrm{e}+\mathrm{e}^a}{\mathrm{e}+1}\right)$$

Given $\int_0^a \frac{\mathrm{e}^x}{\mathrm{e}+\mathrm{e}^x}\ \mathrm{d}x = 1$

$$1 = \ln\left(\frac{\mathrm{e}+\mathrm{e}^a}{\mathrm{e}+1}\right)$$

$$\frac{\mathrm{e}+\mathrm{e}^a}{\mathrm{e}+1} = \mathrm{e}$$

$$\mathrm{e}+\mathrm{e}^a = \mathrm{e}^2+\mathrm{e}$$

$$a = 2$$

2 **(a)** By means of the substitution $u=\sqrt{x}$, and without the use of a graphing calculator, find the value of x which satisfies the equation $5\sqrt{x}-\dfrac{8}{\sqrt{x}}=6.$ [3]

(b) Find, algebraically, the set of values of k for which

$$x^2-2x+kx+k^2+1>0$$

for all real values of x. [3]

[Solution]

(a)

$5\sqrt{x}-\dfrac{8}{\sqrt{x}}=6 \Rightarrow 5u-\dfrac{8}{u}=6$

$5u^2-6u-8=0$

$(5u+4)(u-2)=0$

$u=-\dfrac{4}{5}$ or $u=2$

$\sqrt{x}=-\dfrac{4}{5}$ (Reject) or $\sqrt{x}=2$

$x=4$

(b) Coefficient of $x^2>0$ and Discriminant <0 since the graph is above x-axis.

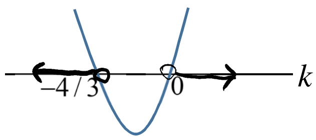
$(-2+k)^2-4(k^2+1)<0$

$4-4k+k^2-4k^2-4<0$

$-3k^2-4k<0$

$3k^2+4k>0$

$k(3k+4)>0$



$k<-\dfrac{4}{3}$ or $k>0$

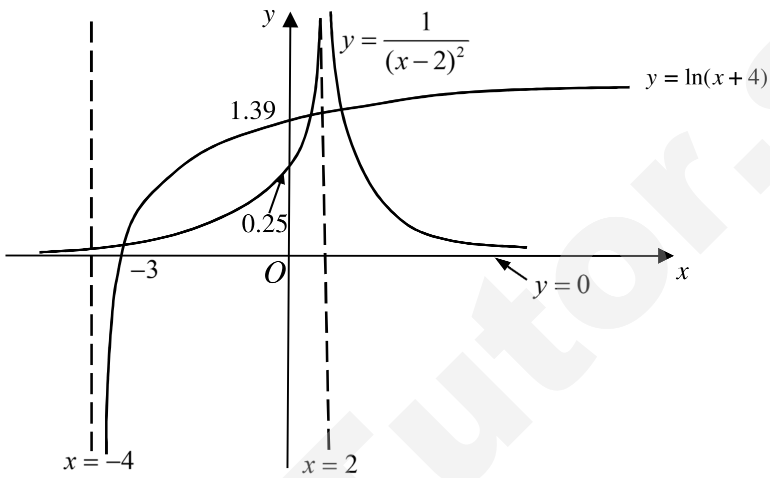
The set of values of k is $\{k\in\mathbb{R} : k<-\dfrac{4}{3} \quad \text{or} \quad k>0\}$

3 **(i)** On the same diagram, sketch the graphs of $y=\ln(x+4)$ and $y=\dfrac{1}{(x-2)^2}$, showing the equations of the asymptotes and any axial intercept(s). [5]

(ii) Hence find the range of values of x for which $\ln(x+4)\geq\dfrac{1}{(x-2)^2}$. [3]

[Solution]

(i)



(ii) The intersections are $x=-2.96$, 1.22 and 2.72

From the graph, $-2.96\leq x\leq 1.22$ or $x\geq 2.72$

4 A particular industrial machine generates annual revenue at the rate $R'(t)=5000-20t^2$ dollars where t is the age of the machine in years. The annual operating and servicing costs for the machine is given by $C'(t)=2000+10t^2$ dollars.

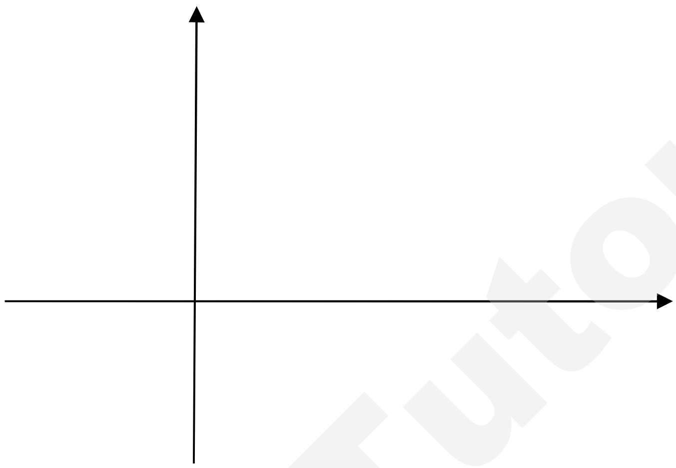
(i) Sketch $y = R'(t)$ and $y = C'(t)$ on the same diagram, indicating clearly the axial intercepts. [3]

(ii) Find the range of values of t for which the machine generates a profit, that is the useful life of the machine. [2]

(iii) Use integration to find the total profit generated by the machine over the period of useful life. [3]

[Solution]

(i)



(ii) From GC, $t = 10$

For $R'(t) > C'(t)$, the range of values of t is $0 \le t < 10$.

(iii) Total profit generated $= \int_0^{10} \left[5000 - 20t^2 - \left(2000 + 10t^2\right) \right] \mathrm{d}t$

$$= \int_0^{10} \left[3000 - 30t^2 \right] \mathrm{d}t$$

$$= \left[3000t - \frac{30}{3} t^3 \right]_0^{10}$$

$$= 20000$$

5

The diagram shows a photo frame design with 8 identical rectangular holes, each x cm by y cm, cut out for the display of photographs. The holes are spaced 4 cm from one another and 4 cm from the edges of the photo frame.

It is given that x and y can vary but the total area of the 8 holes must be 576 cm^2.

(i) Show that the shaded area, A cm^2, the portion of the photo frame not covered by the photos is given by

$$A = 48x + \frac{2880}{x} + 240.$$ [3]

(ii) Find the values of x and y for which A is a minimum. Hence find the minimum value of A in the form $a\sqrt{15} + b$ where a and b are integers to be found. [6]

(iii) Find the rate of change of A when x is changing at the rate of 0.2 cm s^{-1} at the instant when the value of y is twice the value of x. [3]

[Solution]

(i) Total area of 8 holes, $8xy = 576 \Rightarrow y = \frac{72}{x}$

Area of shaded region,

$$A = (4x + 5\times4)(2y + 3\times4) - 8xy$$
$$= 8xy + 48x + 40y + 240 - 8xy$$
$$= 48x + \frac{2880}{x} + 240 \quad \text{(shown)}$$

(ii) $\frac{dA}{dx} = 48 - \frac{2880}{x^2}$

$\frac{dA}{dx} = 0 \quad \Rightarrow \quad 48 - \frac{2880}{x^2} = 0$

$\Rightarrow \quad x = \sqrt{60}$ or $-\sqrt{60}$ (reject as $x > 0$)

$\Rightarrow \quad y = \frac{72}{\sqrt{60}}$

x	$\sqrt{60}^-$	$\sqrt{60}$	$\sqrt{60}^+$
$\frac{\mathrm{d}A}{\mathrm{d}x}$	− ve	0	+ ve

$\therefore$ A is minimum at $x=\sqrt{60}$.

<u>Alternative:</u> Using the second derivative test,

$\frac{\mathrm{d}^2 A}{\mathrm{d}x^2}=\frac{5600}{x^3}>0$ for $x=\sqrt{60}$

$\therefore$ A is minimum at $x=\sqrt{60}$.

$$A=48\sqrt{60}+\frac{2880}{\sqrt{60}}\times\frac{\sqrt{60}}{\sqrt{60}}+240=48\times2\sqrt{15}+48\sqrt{60}+240=96\sqrt{15}+96\sqrt{15}+240=192\sqrt{15}+240$$

Minimum A is $192\sqrt{15}+240$ where a = 192 and b = 240

(iii) $y=2x \quad \Rightarrow \quad 2x=\frac{72}{x}$

$\Rightarrow \quad x=6$

$\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}$

$\frac{\mathrm{d}A}{\mathrm{d}t}=\left(48-\frac{2800}{x^2}\right)\times0.2$

$\frac{\mathrm{d}A}{\mathrm{d}t}=\left(48-\frac{2880}{6^2}\right)\times0.2$

$=-\frac{32}{5}$ (or -6.4)

A is decreasing at rate of $+\frac{32}{5}$ $\mathrm{cm}^2\ \mathrm{s}^{-1}$

Section B : Statistics [60 marks]

6 A supermarket sells a particular type of durians. The masses of these durians are normally distributed with mean μ and standard deviation σ in kilograms. As part of quality control, the supermarket would discard durians that weigh less than 0.6 kg and reserve those that weigh more than 2 kg for their regular customers. Based on past data, the supermarket usually discards and reserves 15% and 1% of the durians respectively. Find the values of μ and σ. [4]

[Solution]

Let X denote the weight of a particular type of durian.

$X \sim N(\mu, \sigma^2)$

$\mathrm{P}(X < 0.6) = 0.15$

$\mathrm{P}\left(Z < \dfrac{0.6-\mu}{\sigma}\right) = 0.15$

From GC, $\dfrac{0.6-\mu}{\sigma} = -1.0364 ------(1)$

$\mathrm{P}(X > 2) = 0.01$

$\mathrm{P}\left(Z > \dfrac{2-\mu}{\sigma}\right) = 0.01$

From GC, $\dfrac{2-\mu}{\sigma} = 2.3263 ------(2)$

Solving, $\sigma = 0.416$, $\mu = 1.03$ (shown)

7

The diagram shows an observation wheel with 24 capsules. Each capsule can carry passengers up to a maximum load of 3000 kg. The weights of male passengers have mean 70 kg and standard deviation 8.9 kg.

(i) The operator of the observation wheel allows n randomly chosen male passengers to enter a capsule. Find the greatest value of n such that the probability that the total weight of the n male passengers exceed the maximum load is less than 0.01. [4]

(ii) Explain whether it is necessary to assume that the weights of male passengers are normally distributed. [1]

[Solution]

Let M be the weight of a male passenger
$\mathrm{E}(M) = 70$, $\mathrm{Var}(M) = 8.9^2$

Let $T = M_1 + M_2 + M_3 + \ldots + M_n$
Assuming that n is large, by Central Limit Theorem,
$T \sim \mathrm{N}(70n, 8.9^2 n)$ approximately

(i) $\mathrm{P}(T > 3000) < 0.01$

From GC

n	$\mathrm{P}(T > 3000)$
40	$0.00019 < 0.01$
41	$0.0113 > 0.01$

Greatest value of n is 40

(ii) It is not necessary to assume that the weights of male passengers are normally distributed as the total weight of n male passengers is approximately normally distributed by the Central Limit Theorem since n is large.

8 A manufacturer sells a new wifi router that is designed to have a mean signal range of 100 m. A quality control manager suspects that there is a flaw in the manufacturing process and the routers produced have a mean signal range that differs from 100 m. A random sample of 53 wifi routers is tested and found to have a mean signal range of 95.7 m and standard deviation of 11.7 m.

(i) Find an unbiased estimate of the population variance.
Explain what is meant by “unbiased estimate” in this context. [2]

(ii) Test at the 2.5% level of significance whether the quality control manager’s suspicion is justified. [4]

A second sample of 53 wifi routers is tested and the unbiased estimates for the population mean and standard deviation calculated using this second sample are 97.8 m and *s* m respectively. A test at the 2.5% significance level does not indicate that the routers have a mean signal range of less than 100 m. Find the range of values of *s* that would result in such a conclusion. [4]

[Solution]

(i) Let X be the signal range of a router and μ be the population mean signal range.

Unbiased estimate of the population variance is $s^2 = \frac{53}{52}\left(11.7^2\right) = 139.5225$

s^2 is an unbiased estimate of the population variance means that the mean of the sampling distribution of S^2, i.e. $E(S^2)$ is equal to the actual population variance.

(ii) $H_0 : \mu = 100$

$H_1 : \mu \neq 100$

Level of significance: 2.5%

Under H_0, test statistic: $Z = \dfrac{\overline{X} - 100}{\frac{\sqrt{139.5225}}{\sqrt{53}}} \sim N(0,1)$ approximately

by Central Limit Theorem since n =53 is large

Given $\overline{x} = 95.7$

From GC, p-value $= 0.00804 < 0.025$

Since p-value < level of significance, we reject H_0.

There is sufficient evidence at 2.5% significance level that the quality control manager’s suspicion that the routers produced have a mean signal range that differs from 100 m is justified.

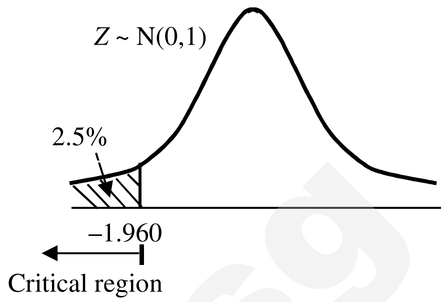
(iii) $H_0 : \mu = 100$

$H_1 : \mu < 100$

Level of significance: 2.5%

test statistic: $Z = \dfrac{\overline{X} - 100}{\frac{s}{\sqrt{53}}} \sim N(0,1)$ approximately

by Central Limit Theorem since n = 53 is large



Since H_0 is not rejected,

z_{cal} lies outside the critical region

$$\frac{97.8 - 100}{\frac{s}{\sqrt{53}}} > -1.95996$$

$\Rightarrow \quad s > 8.17 \quad$ (3 sig fig)

9 Eight students signed up for a weekly private tuition at eight different centres. They were surveyed on the monthly fees (\$$x$) they paid and their subsequent test scores (y %) after 6 months. The results are given in the following table.

Student	A	B	C	D	E	F	G	H
x	200	300	180	340	220	280	400	500
y	44	51	62	56	48	50	59	65

(i) Plot a scatter diagram for the data. Giving a reason, identify a data pair which should be regarded as suspect. [2]

The suspect data pair is subsequently removed from the data set.

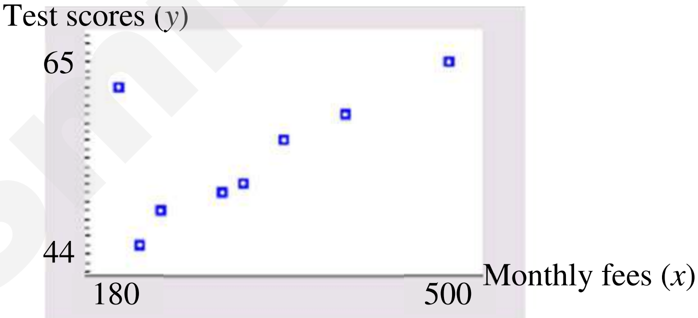
(ii) Calculate the correlation coefficient for the revised data set. Comment on the value obtained. [2]

(iii) Find the equation of the regression line of y on x, and use it to predict the test score of a student who is paying \$350 for tuition, correct to the nearest integer value. Comment on the reliability of your prediction. [3]

(iv) A new equation of the regression line of y on x, $y = 32.0555 + 0.066149x$ is obtained when a new data pair was added. If the value of x of this data pair is 550, find the corresponding value of y. [3]

[Solution]

(i)



The data pair (180, 62) should be regarded as suspect because it does not follow the trend that as x increases, y increases.

(ii) From GC, $r = 0.989$. There is a strong positive linear correlation between x and y.

(iii) $y = 31.6 + 0.0677x$ (3 s.f.)

When $x = 350$, $y = 31.627 + 0.067683(350) = 55.3 \approx 55$

Since $x = 350$ is within the data range [180, 500] and $r = 0.989$ is close to 1, the prediction is reliable.

(iv) Let the unknown y value be k.

$\bar{x} = 348.75$

$\bar{y} = \dfrac{373+k}{8}$

Since ($\bar{x}$, $\bar{y}$) lies on the regression line,

$\dfrac{373+k}{8} = 32.0555 + 0.066149(348.75)$

$\Rightarrow \quad k = 68$

10 A machine is used to generate codes consisting of two integers followed by four letters. Each of the two integers generated is equally likely to be any of the nine integers 1 – 9. The integer 0 is not used. Each of the four letters generated is equally likely to be any of the seven letters of the alphabets $\{A, B, C, D, E, F, G\}$.

(i) Find the number of codes that can be formed, if no letter or integer is repeated in the code. [2]

From (ii) onwards, letters and integers can be repeated in the codes.

Find the number of codes that can be formed

(ii) with two same integers, [2]

(iii) with exactly one vowel and three consonants. [2]

Hence find the **probability** that the last letter of a randomly chosen code is a vowel given that there are exactly one vowel and three consonants. [3]

[Solution]

(i) Number of codes required = ${}^{9}C_{2} \times 2! \times {}^{7}C_{4} \times 4! = 60480$

Alternative

- ${}^{9}P_{2} \times {}^{7}P_{4}$
- $(9\times8) \times (7\times6\times5\times4)$

(ii) Number of codes required = $9\times7^{4} = 21609$

$$\underbrace{(9\times1)}_{2^{\text{nd}}\text{ integer same}} \times \underbrace{(7\times7\times7\times7)}_{\text{letters can repeat}}$$

(iii) No. of codes = $(9^{2})\times(5^{3}\times2)\times4 = 81000$

Notes: 9^{2}: integers can repeat
5^{3}: 3 consonant letters can repeat
2^{1}: 1 vowel chosen can be letter A or E
4 : 1 vowel chosen can be in any of the 4 positions CCCV, CCVC, CVCC, VCCC

$$\text{P}(\text{last letter is a vowel} \mid \text{exactly one vowel and three consonants})$$

$$= \frac{\text{P}(\text{last letter is a vowel and there are exactly one vowel and three consonants})}{\text{P}(\text{exactly one vowel and three consonants})}$$

$$= \frac{\dfrac{9^{2}\times5^{3}\times2}{9^{2}\times7^{4}}}{\dfrac{81000}{9^{2}\times7^{4}}} = 0.25$$

1 vowel chosen can only be in last position

sample space for both probabilities: total no. of codes = $9^{2}\times7^{4}$

[M1, A1]

11 A confectionary produces a large number of sweets every day. On average, 20% of the sweets are wasabi-flavoured and the rest are caramel-flavoured.

(i) A random sample of n sweets is chosen. If the probability that there are at least three wasabi-flavoured sweets in the sample is at least 0.7, find the least possible value of n. [3]

The manufacturer decides to put the sweets randomly into packets of 20.

(ii) Find the probability that such a packet contains less than 3 wasabi-flavoured sweets. [2]

(iii) A customer selects packets of 20 sweets at random from a large consignment until she finds a packet with exactly 12 caramel-flavoured sweets. Give a reason why a Binomial Distribution is not an appropriate model for the number of packets she selects in the context of the question. [1]

The packets are then packed into boxes. Each box contains 10 packets.

(iv) Find the probability that all the packets in a randomly chosen box contain at least 3 wasabi-flavoured sweets. [2]

(v) Find the probability that there are at least 30 wasabi-flavoured sweets in a randomly chosen box. [1]

(vi) Explain why the answer to **(v)** is greater than the answer to **(iv)**. [1]

[Solution]

(i) Let X be the number of wasabi-flavoured sweets out of n.

$X \sim \mathrm{B}(n,\ 0.2)$

$\mathrm{P}(X \geq 3) \geq 0.7$

$1 - \mathrm{P}(X \leq 2) \geq 0.7$

n	$1-\mathrm{P}(X \leq 2)$
17	$0.6904 < 0.7$
18	$0.7287 > 0.7$

Least n is 18.

(ii) Let Y be the number of wasabi-flavoured sweets out of 20.

$Y \sim \mathrm{B}(20,\ 0.2)$

$\mathrm{P}(Y<3)=\mathrm{P}(Y\le 2)$

$= 0.206$

(iii) The number of packets selected (i.e, the number of trials) is not fixed.

(iv) Let W be the number of packets which contains at least 3 wasabi-flavoured sweets out of 10.

$W \sim \mathrm{B}(10,\ 1-0.20608)$

$W \sim \mathrm{B}(10,\ 0.79392)$

$\mathrm{P}(W=10)=0.0995$

Alternative method:

$(1-0.20608)^{10}=0.0995$

(v) Let V be the number of wasabi-flavoured sweets out of 200.

$V \sim \mathrm{B}(200,\ 0.2)$

$\mathrm{P}(V\ge 30)=1-\mathrm{P}(V\le 29)=0.972$

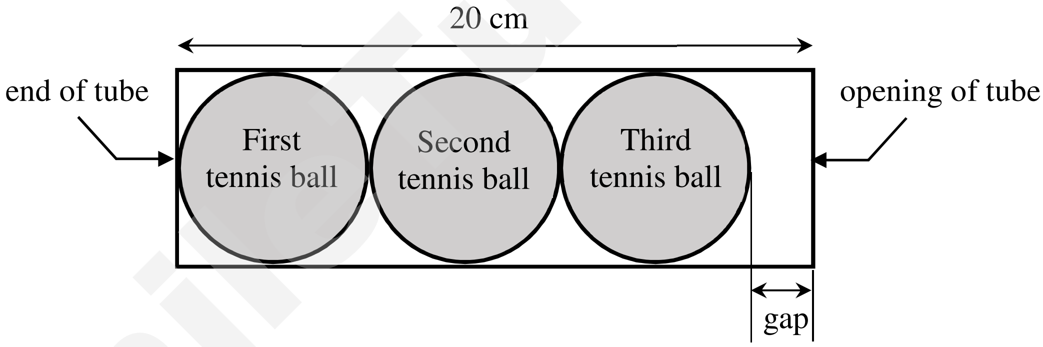
(vi) Part **(iv)** is a subset of part **(v)**, for example part (v) include cases where some packets have less than 3 wasabi sweets but overall the 10 packets have at least 30 wasabi sweets.

12 A company manufactures tennis balls and packs them into cylindrical tubes for sale. The tennis balls have radii that are normally distributed with mean 3.3 cm and standard deviation 0.2 cm.

(i) Find the probability that the radius of a randomly selected tennis ball lies between 3.135 cm and 3.465 cm. Without any further calculation, explain, with the aid of a diagram, how the answer obtained would compare with the probability that the radius lies between 3.465 cm and 3.795 cm. [3]

(ii) 3 tennis balls are randomly selected. Find the probability that exactly one of them has a radius less than 3.4 cm and two of them have radii greater than 3.4 cm each. [2]

The cylindrical tubes are 20 cm long. 3 tennis balls are randomly selected and packed into a cylindrical tube such that the first tennis ball is in contact with the end of the tube and each subsequent ball is in contact with its neighbouring ball as shown in the diagram below. Assume that the centres of all the tennis balls are horizontally aligned.



(iii) Find the probability that a gap exists between the third tennis ball and the opening of the tube. [3]

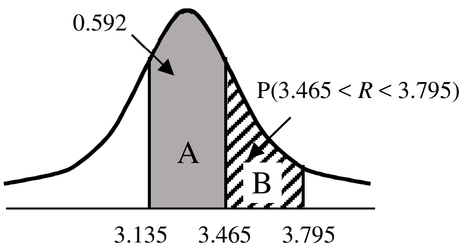
(iv) Find the range of values of k such that the probability that the gap between the third tennis ball and the opening of the tube is more than k cm is at most 0.15. [3]

State an assumption used in your calculations. [1]

[Solution]

Let R be the radius of a randomly chosen tennis ball, $R \sim \mathrm{N}\left(3.3,\ 0.2^2\right)$

(i) $\mathrm{P}\left(3.135 < R < 3.465\right)$
$\approx 0.59063 = 0.591$ (3 sf)



From diagram, $\mathrm{P}\left(3.135 < R < 3.465\right) > \mathrm{P}\left(3.465 < R < 3.795\right)$

since area A is larger than area B given that the widths of the two intervals $3.135 < R < 3.465$ and $3.465 < R < 3.795$ are the same.

(ii) Required probability
$= \mathrm{P}\left(R < 3.4\right) \times \mathrm{P}\left(R > 3.4\right) \times \mathrm{P}\left(R > 3.4\right) \times 3$
$= (0.69146) \times (1 - 0.69146)^2 \times 3$
$= 0.197$ (3 sf)

(iii) Let G be the gap between the third tennis ball and the opening of the tube

$G = 20 - 2(R_1 + R_2 + R_3)$

$G \sim \mathrm{N}(\ 20 - 2(3.3 \times 3),\ 2^2(0.2^2 \times 3)\ )$
i.e. $G \sim \mathrm{N}(\ 0.2,\ \sqrt{0.48}^2\ )$

$\mathrm{P}(G > 0) = 0.614$ (3 s.f.)

Alternative

Let D be the diameters of 3 tennis balls

$D = 2(R_1 + R_2 + R_3)$

$D \sim \mathrm{N}(\ 2(3.3 \times 3),\ 2^2(0.2^2 \times 3)\ )$
i.e. $D \sim \mathrm{N}(\ 19.8,\ \sqrt{0.48}^2\ )$

$\mathrm{P}(D < 20) = 0.614$ (3 s.f.)

(iv) $\mathrm{P}(G > k) \le 0.15$

$1 - \mathrm{P}(G \le k) \le 0.15$

$\mathrm{P}(G \le k) \ge 0.85$

From GC,

$k \ge 0.918$ (3 sf)

Using invNorm function

Alternative

$\mathrm{P}(20 - D > k) \le 0.15$

$\mathrm{P}(D < 20 - k) \le 0.15$

From GC,

$20 - k \le 19.082$

$k \ge 0.918$ (3 sf)

Using invNorm function

The radii of the tennis balls are independent of one another.

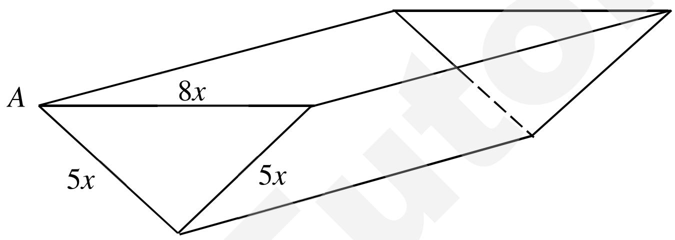
1 The curve $y=(k-6)x^2-5x$ has a minimum point. Find algebraically the set of values of k for which the curve intersects the line $y=3x-k$ at two distinct points for all real values of x. [4]

2 The curve C_1 has equation $y=\ln(x+2)$ and the curve C_2 has equation $y=\dfrac{x+2}{x-1}$.

(i) On the same diagram, sketch C_1 and C_2, stating the exact coordinates of any points of intersection with the axes and the equations of any asymptotes. [4]

(ii) Find the area of the finite region bounded by C_1, C_2 and the line $x=2$. [2]

3



The diagram shows an open container with a rectangular top *ABEF*, constructed from 4 sheets of metal of negligible thickness. The ends *ABC* and *FED* are isosceles triangles with sides $AC = BC = ED = FD = 5x$ cm and $AB = FE = 8x$ cm. The sides *BEDC* and *AFDC* are rectangles with width $5x$ cm and length y cm. The total area of the metal sheets is 500 cm^2.

(a) **(i)** Show that the volume, V cm^3, of the container is given by $V=600x-\dfrac{144}{5}x^3$. [3]

(ii) Use differentiation to find the maximum value of V as x varies, justifying that this value is a maximum. [4]

(b)

It is given that $x = 2$.

Water is poured into the empty container at a rate of 10 cm^3/s. The water level reaches a depth of h cm after t seconds (see diagram above). Find the rate of increase of the depth of water when the volume of water in the container is 242.4 cm^3. [4]

4 **(a)** Express $\dfrac{5x+1}{(x+1)(2x+1)}$ in the form $\dfrac{A}{x+1}-\dfrac{B}{2x+1}$, where A and B are integers to be determined. Hence, differentiate $\dfrac{5x+1}{(x+1)(2x+1)}$ with respect to x. [5]

(b) Find $\displaystyle\int\left(3x+\frac{2}{x^2}\right)^2 \mathrm{d}x$. [3]

5 A stall in a food bazaar sells three types of cupcakes: vanilla, red velvet and white chocolate. The price of a vanilla cupcake, red velvet cupcake and white chocolate cupcake is \$1.50, \$3.50 and \$3.00 respectively.

On Monday, 400 cupcakes were baked. At the end of the day, $\dfrac{5}{9}$ of the vanilla cupcakes baked were sold and $\dfrac{3}{4}$ of the red velvet cupcakes baked were sold. The number of vanilla cupcakes sold was 70 more than the number of white chocolate cupcakes sold. The amount collected from selling these cupcakes was \$450.

If all the cupcakes were sold at the end of Monday, the stall would have collected \$970.

(i) By writing down three linear equations, find the number of each type of cupcake baked on Monday. [4]

(ii) Given that the production cost of a vanilla cupcake, red velvet cupcake and white chocolate cupcake is \$0.60, \$ 2.00 and \$1.80 respectively, find the profit earned by the stall on Monday and interpret the numerical value obtained in the context of the question. [2]

In order to attract more customers, the stall is trialling a new product – cookies. The cookies are sold by weight. It is predicted that the total profit \$$P$ will be related to the weight of cookies produced (x kg) by the equation

$$P=-10x^2+140x-400.$$

You may assume that all the cookies produced are sold.

(iii) Sketch the graph of P against x, stating the coordinates of the intersections with the axes. [2]

(iv) State the weight of cookies produced when the profit is a maximum. Give this value of P. [2]

(v) Give an interpretation, in context, of the value of P when $x=0$. [1]

Section B: Statistics (60 marks)

6 Events A and B are such that $\mathrm{P}(B)=\frac{7}{18}$, $\mathrm{P}(A\mid B)=\frac{4}{7}$ and $\mathrm{P}(A'\cap B')=\frac{1}{3}$. Find

(i) $\mathrm{P}(A\cap B)$, [1]

(ii) $\mathrm{P}(A)$. [3]

7 A username for an online portal consists of six characters. It is stipulated that the username can only contain characters chosen from the twenty-six letters of the alphabet A - Z and ten digits 0 - 9.

Kaykay is creating a username on this portal. She decides that the first four characters of her username will consist of only letters and the last two characters of the username will consist of only digits.

Suppose that repetitions are allowed, find the probability that she forms a username containing

(i) the letter K exactly once and the digits are different, [3]

(ii) the letter K as its first character or 3 as its fifth character, but not both. [3]

8 On average $100p\%$ of a certain company's pea seeds germinate. The pea seeds are sold in trays of 24.

(i) State, in context, two assumptions needed for the number of pea seeds that germinated in a tray to be well modelled by a binomial distribution. [2]

Assume now that the number of pea seeds that germinated in a tray has a binomial distribution.

(ii) The probability that 15 or 16 pea seeds germinate in a tray is 0.086550 correct to 6 decimal places. Find the value of p to a suitable degree of accuracy, given that $p>0.5$. [3]

The trays are packed into cartons. Each carton contains 8 trays.

(iii) Find the probability that each tray in one randomly selected carton contains at least twenty pea seeds that germinated. [3]

(iv) Find the probability that there are at least 160 pea seeds that germinated in a randomly selected carton. [2]

(v) Explain why the answer to part **(iv)** is greater than the answer to part **(iii)**. [1]

9 A shop at a beach sells ice cream. For each of the nine days in July, the hours of sunshine, x, and the number of ice cream cones sold, y, are given in the table below.

x	4.3	6.9	0.0	10.4	5.2	1.8	8.0	9.2	2.1
y	74	78	13	156	80	44	134	130	55

(i) Give a sketch of the scatter diagram for the data. [2]

(ii) Find the product moment correlation coefficient. [1]

(iii) Find the equation of the regression line of y on x in the form $y = mx + c$, giving the values of m and c correct to 3 significant figures. Sketch this line on your scatter diagram. [2]

(iv) The shop closed early on one of the days as the owner wanted to attend a birthday party. Suggest, giving a reason, which day this was. [2]

(v) The shop sells 190 ice cream cones on a particular day with 9.5 hours of sunshine.

Use the equation of your regression line in part **(iii)** to calculate an estimate of the number of ice cream cones sold in a day with 9.5 hours of sunshine.

Give a reason why this estimated number is much lower than the actual number of ice cream cones sold. [2]

(vi) Use the appropriate regression line to estimate the number of hours of sunshine during a day when 100 ice cream cones are sold. Comment on the reliability of your estimate. [3]

10 A factory produces packets of a particular brand of coffee powder. Each packet is supposed to contain a mass of 800 g of coffee powder. The supervisor suspects that the machine packing the coffee powder is not operating properly. A random sample of 50 packets is taken and the masses of coffee powder in the packets are measured. The masses, x g, are summarised by

$$\sum(x-800) = -75.6, \qquad \sum(x-800)^2 = 1020.2.$$

(i) Find the unbiased estimates of the population mean and variance. [3]

(ii) Test, at the 5% significance level, whether the supervisor's suspicion is valid. [4]

(iii) State, giving a reason, whether it is necessary to assume a normal distribution for this test to be valid. [1]

After alterations to the machine, the supervisor observes that the machine seems to deliver more than 800 g of coffee in a packet. A random sample of 20 packets is selected. The sample mean is m g. A test at the 5% significance level is carried out on this sample, and the supervisor's observation is justified. Assuming that the mass of coffee powder in a packet produced after alterations to the machine is normally distributed with standard deviation 18 g, find the set of possible values of m. [5]

11 In a high school, the times taken, in minutes, for boys and girls to complete their 2.4 km test run have independent normal distributions with means and standard deviations as shown in the following table.

	Mean	Standard deviation
Boys	11.51	0.72
Girls	13.17	0.99

(i) Find the probability that a randomly chosen boy takes less than 10 minutes to complete the run. [1]

A boy who takes less than 10 minutes to complete the run is considered a fast runner.

(ii) A boy is chosen at random. Given that he is a fast runner, find the probability that he takes less than 9.50 minutes to complete the run. [3]

(iii) Less than 40% of the boys take more than t minutes to complete the run. Find the set of values of t. [2]

Two boys and two girls are randomly chosen. Find the probability that

(iv) the total time taken by the two boys differs from the total time taken by the two girls by at least 1 minute, [4]

(v) the average time taken by the two boys and two girls is less than the mean time taken by the boys in the high school. [4]

End of paper

1)

Minimum turning point: $k-6>0$

$k>6$

$(k-6)x^2-5x=3x-k$

$(k-6)x^2-8x+k=0$

Discriminant > 0: $b^2-4ac>0$

$(-8)^2-4(k-6)k>0$

$64-4k^2+24k>0$

$k^2-6k-16<0$

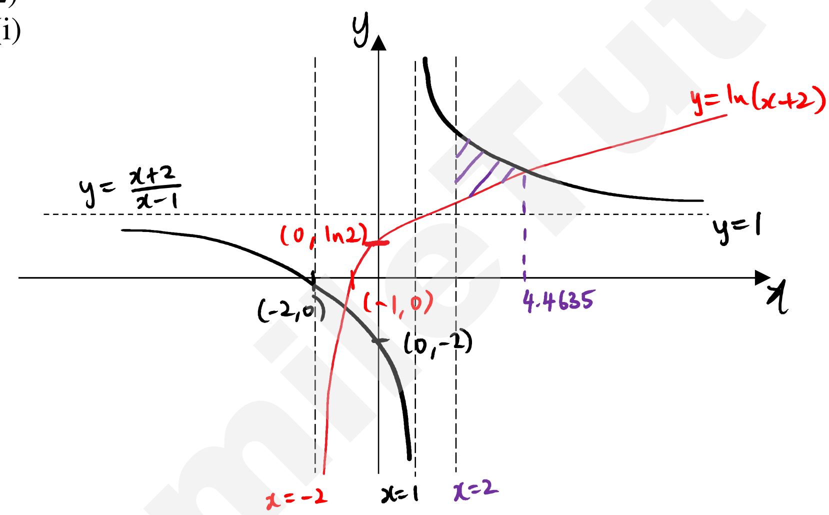
$(k-8)(k+2)<0$

$-2<k<8$

Since $k>6$ and $-2<k<8 \;\Rightarrow\; 6<k<8$

$\{k \in \because : 6<k<8\}$

2)
(i)



(ii) From the GC, the point of intersection is (4.4635, 1.8662).

By GC, Area of the finite region $= \displaystyle\int_{2}^{4.4635}\left[\frac{x+2}{x-1}-\ln(x+2)\right]\mathrm{d}x = 2.14$

3)

(i) Height of ABC $=\sqrt{(5x)^2-(4x)^2}=3x$

$5xy\times 2+\frac{1}{2}(8x)(3x)\times 2=500$

$10xy+24x^2=500$

$y=\dfrac{500-24x^2}{10x}$

$$V = \frac{1}{2}(8x)(3x)\,y$$
$$= 12x^2\left(\frac{500-24x^2}{10x}\right)$$
$$= \frac{6000x^2-288x^4}{10x}$$
$$= 600x - \frac{144}{5}x^3$$

(ii)

$$\frac{\mathrm{d}V}{\mathrm{d}x} = 600 - 86.4x^2$$

$$x^2 = \frac{600}{86.4} = \frac{125}{18}$$

Since $x > 0,\ x = 2.6352$

	2.6352^-	0	2.6352^+
Sign of $\frac{\mathrm{d}V}{\mathrm{d}x}$	+	0	-
Slope of Tangent	/	-	\

By sign test, V is maximum when $x = 2.6352$.

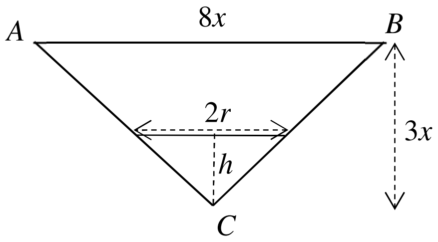
When $x = 2.6352,\ V = 1054.09 \approx 1050$

(b) When $x = 2,\ y = \dfrac{500-24(2)^2}{10(2)} = 20.2$

$$V_w = \frac{1}{2}\times 2r\times h\times 20.2 = 20.2rh$$

$3x = 6,\ \ 8x = 16$

By similar triangles:



$$\frac{h}{6} = \frac{2r}{16}$$

$$r = \frac{8}{6}h$$

$$V_w = 20.2\left(\frac{8}{6}h\right)h = \frac{404}{15}h^2$$

When $V_w = 242.4,\ \dfrac{404}{15}h^2 = 242.4 \Rightarrow h = 3$

$$\frac{\mathrm{d}V_w}{\mathrm{d}h} = \frac{808}{15}h \quad , \qquad \frac{\mathrm{d}V_w}{\mathrm{d}t} = 10$$

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}h}{\mathrm{d}V_w}\times\frac{\mathrm{d}V_w}{\mathrm{d}t}$$
$$= \frac{15}{808(3)}\times 10 = 0.0619$$

4)

(a)

$$\frac{5x+1}{(x+1)(2x+1)} = = \frac{A}{x+1} - \frac{B}{2x+1}$$
$$= \frac{A(2x+1) - B(x+1)}{(x+1)(2x-1)}$$
$$= \frac{(2A-B)x + A - B}{(x+1)(2x-1)}$$

$2A - B = 5$

$A - B = 1$

Solving simultaneously, $A = 4, B = 3$

$$\frac{5x+1}{(x+1)(2x+1)} = \frac{4}{x+1} - \frac{3}{2x+1}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{5x+1}{(x+1)(2x+1)}\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{4}{x+1} - \frac{3}{2x+1}\right)$$
$$= -\frac{4}{(x+1)^2} + \frac{6}{(2x+1)^2}$$

(b)

$$\int\left(3x + \frac{2}{x^2}\right)^2 \mathrm{d}x = \int\left(9x^2 + \frac{12}{x} + \frac{4}{x^4}\right)\mathrm{d}x$$
$$= 3x^3 + 12\ln x - \frac{4}{3x^3} + C$$

5)

(i) Let v, r and w represent the number of vanilla cupcakes, red velvet cupcakes and white chocolate cupcakes **baked** respectively.

$v + r + w = 400 - (1)$

$1.50v + 3.50r + 3.00w = 970 - (2)$

$\frac{5}{9}v(1.50) + \frac{3}{4}r(3.50) + \left(\frac{5}{9}v - 70\right)(3.00) = 450$

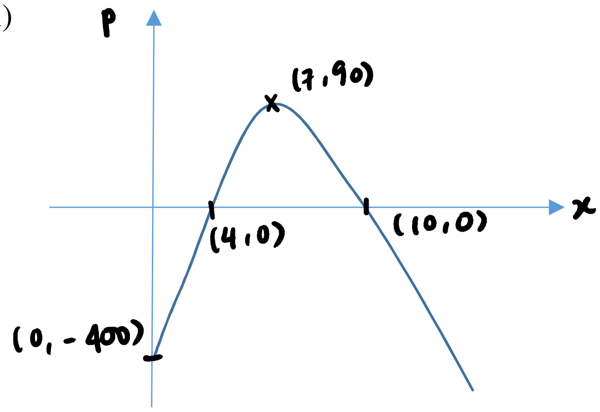
$\frac{5}{2}v + 2.625r = 660 - (3)$

By GC, $v = 180, r = 80, w = 140$

(ii) Profit = $450 - 180(0.60) - 80(2) - 140(1.80) = -70$

The store made a loss of \$70 on Monday.

(iii)



(iv) The weight of cookies produced is 7 kg. The maximum value of P is \$90.

(v) It is the fixed cost of the stall.

6)

(i)
$$\frac{P(A \cap B)}{P(B)} = \frac{4}{7}$$
$$P(A \cap B) = \frac{4}{7}P(B)$$
$$= \frac{4}{7}\left(\frac{7}{18}\right) = \frac{2}{9}$$

(ii)
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$1 - P(A' \cap B') = P(A) + P(B) - P(A \cap B)$$
$$1 - \frac{1}{3} = P(A) + \frac{7}{18} - \frac{2}{9}$$
$$\frac{2}{3} - \frac{3}{18} = P(A)$$
$$P(A) = \frac{1}{2}$$

7)

(a) __ __ __ __ __ __

Number of usernames formed when repetitions are allowed $= 26^4 \times 10^2 = 45697600$

Number of codes with letter k exactly once and 2 different digits

$= (4 \times 1 \times 25 \times 25 \times 25) \times (10 \times 9) = 5625000$

Required probability= $= \frac{5625000}{45697600} \approx 0.123$

(b) Case 1: k __ __ __ __ __

Number of usernames formed with k as its first character but not 3 as its fifth character $= 1 \times 26 \times 26 \times 26 \times 9 \times 10 = 1581840$

Case 2: __ __ __ __ 3 __

Number of usernames formed with 3 as its fifth character but not k as its first character $= 25\times26\times26\times26\times1\times10 = 4394000$

Required probability $= \dfrac{1581840+4394000}{45697600} = \dfrac{17}{130} \approx 0.131$

8)

(i)

1) The probability that a pea seed germinates is the same for all 24 seeds.

2) A pea seed germinates independently of all other pea seeds.

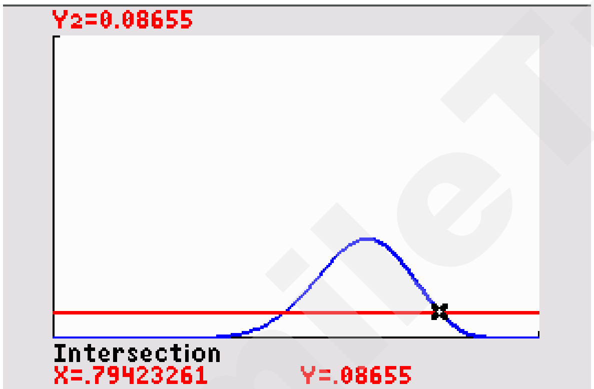
(ii)

Let X be the number of seeds that germinate out of a tray of 24.

$$X \sim \mathrm{B}(24, p)$$

$\mathrm{P}(X=15)+\mathrm{P}(X=16)=0.086550$

$\left({}^{24}C_{15}\right)(p)^{15}(1-p)^{9}+\left({}^{24}C_{16}\right)(p)^{16}(1-p)^{8}=0.086550$



By GC, $p=0.794233=0.7942$ or $p=0.476512$ (rej $\because p>0.5$)

(iii)

$\left[\mathrm{P}(X\geq20)\right]^{8}=\left[1-\mathrm{P}(X\leq19)\right]^{8}$

$=0.00121$

(iv) Let Y be the number of pea seeds that germinate out of 192 pea seeds in a carton.

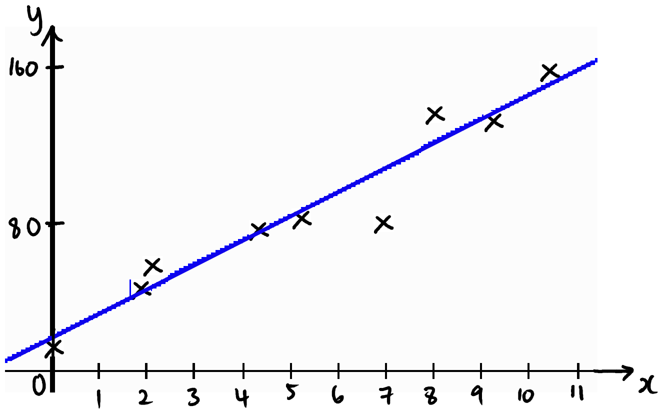
$$Y \sim \mathrm{B}(192, 0.794233)$$

$\mathrm{P}(Y\geq160)=1-\mathrm{P}(Y\leq159)=0.103$

(v) Answer in part (iv) is greater than the answer in (iii) because the event ‘each of the 8 trays in a carton contains at least 20 pea seeds that germinated’ is a subset of the event ‘a carton contains at least 160 pea seeds that germinated’.

9)

(i), (iii)



(ii) $r = 0.965$

(iii) $y = 12.579x + 17.940$

$y = 12.6x + 17.9$

(iv) The day with 6.9 hours of sunshine. The diagram shows that point (6.9, 78) is the furthest below the regression line as compared to the other points.

(v) $y = 12.579(9.5) + 17.940 = 137$

The particular day with 9.5 hours of sunshine where 190 ice cream cones were sold might not be in July. It might be a day in December which is the holiday season and more people visit the beach. Hence, it is not appropriate to use the regression line for estimation.

OR

The owner could have ran a sale on ice cream cones.

(vi) Since x and y are measured and we need to estimate x given y, regression line of x on y is used.

$x = 0.074099y - 0.96799$

$x = 0.074099(100) - 0.96799 = 6.44$

The estimate is reliable as $r = 0.965$ is close to 1, a linear model is appropriate and $y = 100$ is within the data range of $13 \le y \le 156$.

10)

(i) Let $y = x - 800$, so

$\sum y = -75.6$, $\quad \sum y^2 = 1020.2$

$$\bar{x} = \bar{y} + 800 = \frac{-75.6}{50} + 800 = 798.488$$

$$s_x^{\ 2} = s_y^{\ 2} = \frac{1}{49}\left[1020.2 - \frac{(-75.6)^2}{50}\right] = 18.488 = 18.5$$

(ii) Let μ g be the population mean mass.

$H_0 : \mu = 800$

$H_1 : \mu \neq 800$

Level of significance: 5%

Test statistics: Since $n = 50$ is sufficiently large, by Central Limit Theorem, $\overline{X}$ is approximately normal.

When H_0 is true, $Z = \dfrac{\overline{X} - 800}{S/\sqrt{50}} \sim \mathrm{N}\left(0,1\right)$ approximately

Computation: $\bar{x} = 798.488$, $s^2 = 18.488$

$p - \text{value} = 0.0129$

Since $p - \text{value} = 0.0129 < 0.05$, H_0 is rejected at 5% level of significance. Hence, there is sufficient evidence that the supervisor's suspicion is valid.

(iii) It is not necessary to assume the mass of a packet of coffee powder follows a normal distribution. Since sample size = 50 is sufficiently large, by Central Limit Theorem, the sample mean mass of coffee powder ($\overline{X}$) is approximately normally distributed.

$\mathrm{H}_0 : \mu = 800$

$\mathrm{H}_1 : \mu > 800$

Level of significance: 5%

Test statistics: When H_0 is true, $Z = \dfrac{\overline{X} - 800}{18/\sqrt{20}} \sim \mathrm{N}\left(0,1\right)$

Rejection region: $z \geq 1.6449$

Computation: $z = \dfrac{m - 800}{18/\sqrt{20}}$

Owner's claim accepted

$\Rightarrow$ H_0 is rejected

$\dfrac{m - 800}{18/\sqrt{20}} \geq 1.6449$

$m \geq 807$

$\therefore \{m \in \because : m \geq 807\}$

11)

Let X mins be the time taken by a boy and Y mins be the time taken by a girl.

$X \sim \mathrm{N}\left(11.51, 0.72^2\right)$

$Y \sim \mathrm{N}\left(13.17, 0.99^2\right)$

(i) $\mathrm{P}\left(X < 10\right) = 0.017987 = 0.0180$

(ii)

$\mathrm{P}\left(X < 9.5 \mid X < 10\right)$

$= \dfrac{\mathrm{P}\left(X < 9.5\right)}{\mathrm{P}\left(X < 10\right)}$

$= 0.146$

(iii)

$\mathrm{P}(X > t) < 0.4$

$1 - \mathrm{P}(X \le t) < 0.4$

$\mathrm{P}(X \le t) > 0.6$

$t > 11.7$

$\therefore \{t \in \because : t > 11.7\}$

(iv)

$\mathrm{E}\left(X_1 + X_2 - (Y_1 + Y_2)\right) = 2\mathrm{E}(X) - 2\mathrm{E}(Y) = -3.32$

$\mathrm{Var}\left(X_1 + X_2 - (Y_1 + Y_2)\right) = 2\mathrm{Var}(X) + 2\mathrm{Var}(Y) = 2.997$

$X_1 + X_2 - (Y_1 + Y_2) \sim \mathrm{N}(-3.32, 2.997)$

$\mathrm{P}(X_1 + X_2 \ge Y_1 + Y_2 + 1) + \mathrm{P}(X_1 + X_2 \le Y_1 + Y_2 - 1)$

$= \mathrm{P}\left(X_1 + X_2 - (Y_1 + Y_2) \ge 1\right) + \mathrm{P}\left(X_1 + X_2 - (Y_1 + Y_2) \le -1\right)$

$= 0.916$

(v)

$$\bar{W} = \frac{X_1 + X_2 + Y_1 + Y_2}{4}$$

$$\mathrm{E}(\bar{W}) = \frac{1}{4}\left[2\mathrm{E}(X) + 2\mathrm{E}(Y)\right] = 12.34$$

$$\mathrm{Var}(\bar{W}) = \frac{1}{16}\left[2\mathrm{Var}(X) + 2\mathrm{Var}(Y)\right] = 0.1873125$$

$\bar{W} \sim \mathrm{N}(12.34, 0.1873125)$

$\mathrm{P}(\bar{W} < 11.51) = 0.0276$

Section A: Pure Mathematics [40 marks]

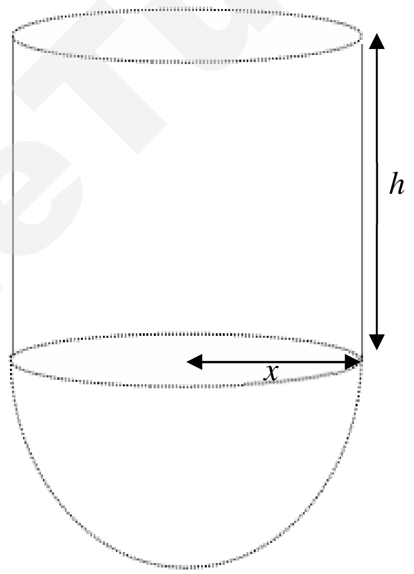
1 **(a)** **(i)** Sketch the graph of $y = 2 + \dfrac{2}{x-1}$, stating clearly the coordinates of all the points of intersection with the axes and the equations of any asymptotes. [2]

(ii) By adding a suitable graph, state the range of values of x that satisfy the inequality $\dfrac{2}{x-1} \le x$. [3]

(b) Find the set of values of p for which $x^2 - 2x + 2 = p$ has no real roots. [2]

2 [It is given that a sphere of radius r has surface area $4\pi r^2$ and volume $\dfrac{4}{3}\pi r^3$.]

A structure consists of a cylindrical top with two open ends and a hemispherical bottom, as shown in the diagram below. The cylinder has base radius x cm and height h cm. The external surface area of the structure is 40 cm^2.



(i) Show that the volume of the structure, V cm^3, is given by $V = 20x - \dfrac{1}{3}\pi x^3$. [3]

(ii) Use a non-calculator method to find the maximum value of this volume, giving your answer in the form $\dfrac{p}{q}\sqrt{\dfrac{r}{\pi}}$, where p, q and r are integers. Justify that this is the maximum value. [4]

3 **(a)** Differentiate $\dfrac{2}{e^{-x}+1}$. [2]

(b) Find $\displaystyle\int \left(1 - \frac{3}{x}\right)^2 \mathrm{d}x$. [3]

4 The curve C has equation $y = \ln(3 - 2x)$.

(i) Without using a calculator, find the equation of the tangent to C at the point where $x = \frac{1}{2}$, giving your answer in the form $y = mx + c$, where m and c are constants. [3]

(ii) This tangent meets the x-axis at A and the y-axis at B. Find the exact area of the triangle OAB. [3]

(iii) Find the exact area of the region bounded by C, the x-axis and the y-axis. [3]

5 One day, Kiera chanced upon an online shop which sells boys' light-weight pants. The weight of the pants varies according to the waist length of the pants as shown in the table below.

Waist length (inches)	Weight (grams)
25	170
29	200
32	232

After reading many positive online reviews about these pants, Kiera decided to buy 100 pairs of pants and try to sell them at a local flea market. As she predicted that most of the teenage boys would not be plump, she purchased four times as many smallest sized pants as the biggest sized pants. When all the pants were checked out, the total weight of the purchase was 19.12 kg and the price was \$1000.

(i) Find the number of the different sized pants that Kiera has purchased. [4]

(ii) Assuming that all the pants are sold at the same price, state the minimum price that Kiera has to sell for a pair of pants so that she would not make a loss. [1]

After the purchase of 100 pants, Kiera started on her advertising plan. She estimated that the revenue from the pants is given by $R = -0.02x^3 + 0.01x^2 + 1.2x - 1.1$, where R is the revenue in thousands of dollars and x is the amount, in hundreds of dollars, spent on advertising. Revenue is referred to as the income from the sale of goods to customers while profit is the difference between the revenue and the amount spent on buying, operating or producing something.

(iii) Sketch the graph of $R = -0.02x^3 + 0.01x^2 + 1.2x - 1.1$ for $2 \le x \le 6$. [2]

(iv) Find the minimum amount, to the nearest dollar, that Kiera needs to spend on advertising to generate a revenue of \$2150. [1]

(v) Use your calculator to estimate the maximum revenue. State the value of x for which this maximum revenue occurs. [2]

(vi) In order for Kiera to generate this maximum revenue and assuming all the pants are sold, state the price that Kiera has to sell for a pair of pants. Hence, find the profit that Kiera will make. [2]

Section B: Probability and Statistics [60 marks]

6 A school is required to send a delegation of 10 teachers to attend the Teachers' Conference 2018. This group of 10 teachers are to be selected from a pool of 7 Mathematics teachers, 5 Humanities teachers and 4 Science teachers.

(i) How many different delegations can be formed? [1]

One of the Mathematics teachers is the brother of a particular Humanities teacher.

(ii) How many different delegations can be formed such that the siblings cannot be in the same delegation? [2]

(iii) Find the probability that the delegation consists of at least 3 teachers from each department. (Mathematics, Humanities and Science are considered as 3 different departments.) [3]

7 A government agency publishes a report on smoking. In a study, 30% of smokers are classified as 'light smoker', 50% as 'moderate smoker' and 20% as 'heavy smoker'. Of those classified as 'light smoker', 'moderate smoker' and 'heavy smoker', 10%, 18% and 25% develop lung cancer in the next 10 years respectively.

(i) Draw a tree diagram to represent the above information. [2]

Find the probability that a randomly chosen smoker in the study

(ii) develops lung cancer in the next 10 years, [2]

(iii) is either a 'moderate smoker' or develops lung cancer in the next 10 years or both. [2]

A randomly chosen smoker in the study is found to have lung cancer.

(iv) Find the probability that he is a 'heavy smoker'. [2]

Two smokers in the study are randomly chosen.

(v) Find the probability that one develops lung cancer while the other does not develop lung cancer in the next 10 years. [2]

8 Hugedelay produces delay lines for use in communications. The delay times for a delay line is measured in nanoseconds (ns). It is found that 10% of the delay times are less than 274.6 ns and 7.5% are more than 288.2 ns. Assuming that the delay times for Hugedelay are normally distributed, find the mean and variance of this distribution. [4]

9 Eggs produced at a chicken farm are packaged in boxes of six. For any egg, the probability that it is broken when it reaches the retail outlet is 0.1. A box is said to be sub-standard if it contains at least two broken eggs. The number of broken eggs in a box is the random variable X.

(i) State, in context, the assumption needed for X to be well modelled by a binomial distribution. [1]

Assume now that X has a binomial distribution.

(ii) Find the probability that a randomly selected box is sub-standard. [2]

(iii) A random sample of n boxes is taken. Find the greatest value of n such that the probability that there are more than three sub-standard boxes is less than 0.01. [3]

(iv) Ten boxes are chosen at random. Find the most likely number of boxes that are sub-standard. [2]

(v) Give a possible reason in context why the assumption made in part **(i)** may not be valid. [1]

10 Gift baskets are individually wrapped to customers' requirements. The supervisor recorded the time taken by a new staff to complete wrapping his first 11 gift baskets. The data are given based on the order of wrapping the gift baskets.

Gift basket	1	2	3	4	5	6	7	8	9	10	11
No of items in gift basket, x	40	20	60	50	20	30	10	58	49	19	38
Time in min to complete wrapping, t	54.5	36.8	52.3	44.5	31.3	28.2	22.1	41.2	36.0	28.3	32.1

(i) Give a sketch of the scatter diagram for the data. Label the points from 1 to 11 according to the order of wrapping the gift basket. [2]

(ii) Find the regression line of t on x in the form of $t = ax + b$, giving the values of a and b correct to 2 decimal places. Sketch this line on your scatter diagram. Suggest a possible reason, in context, why the first five data points should be excluded. [3]

The supervisor decided to use only the last 6 data points.

(iii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iv) The equation of the regression line is $t = 19.1 + 0.361x$. Estimate the time taken to wrap a gift basket with 44 items. Give two reasons why you would expect this estimate to be reliable. [2]

11 Scientists believe that the mean duration of a certain viral infection in adults is 3.6 days. From the records of a random sample of 100 infected adults, the average duration of infection is 3.42 days and the standard deviation is 1.005 days.

(i) Test, at the 5% significance level, whether the scientists' claim should be rejected. [4]

(ii) State, giving a reason, whether it is necessary to assume a normal distribution for this test to be valid. [1]

A young scientist claims that treating people with vitamin Q will reduce the average duration of infection. A second random sample of 50 adults having the infection is treated with vitamin Q. The duration of infection, y, are summarised by

$$\sum y = 166 \qquad \sum (y - \bar{y})^2 = 55$$

(iii) Find unbiased estimates of the population mean and variance using this second sample. [2]

The population mean duration of infection after being treated with vitamin Q is μ days. Using the sample data, a significance test of the null hypothesis $\mu = 3.6$ against the alternative hypothesis $\mu < 3.6$ is carried out by the young scientist at the $\alpha\%$ significance level.

(iv) Find the range of values of α for which the null hypothesis is rejected. [3]

12 Leon Fish Farm breeds only salmon and scallops. The salmon are packed in boxes of half a dozen each while the scallops are packed in boxes of 100 each. The masses, in kilograms, of salmon and scallops sold by the fish farm have independent normal distributions. The means and standard deviations of these distributions, and the selling prices, in $ per kilogram, are shown in the following table.

	Mean (kg)	Standard deviation (kg)	Selling price ($ per kg)
Salmon	5	1.5	30
Scallops	0.2	0.05	16

(i) Find the probability that a randomly chosen salmon weighs more than 5.5 kg. [1]

(ii) A customer buys 60 randomly chosen boxes of salmon. Find the probability that the average number of salmon weighing more than 5.5 kg per box is at most 2. [3]

(iii) The probability that the total mass of 2 boxes of scallops exceeding m kg is more than 0.95. Find the range of values of m. [3]

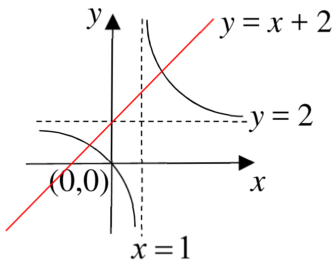
Let V be the selling price of a box of salmon.
Let W be the selling price of a box of scallops.

(iv) Find $\mathrm{P}\left(-150 < V_1 + V_2 - 6W < 150\right)$ and explain, in the context of this question, what your answer represents. [5]

− End Of Paper −

Yishun Junior College
2018 JC2 H1 Math Preliminary Examination
Solutions

No	**Solutions**
1	(i) $y = 2 + \frac{2}{x-1}$



(ii)

$\frac{2}{x-1} \le x$

$2 + \frac{2}{x-1} \le x + 2$

Additional graph to add in: $y = x + 2$

Intersection points between the 2 graphs:

$(-1,1)$ and $(2,4)$

Hence,

$-1 \le x < 1$ or $x \ge 2$

(b)

$x^2 - 2x + 2 - p = 0$

Since there are no real roots,

Discriminant < 0

$(-2)^2 - 4(1)(2-p) < 0$

$4 - 8 + 4p < 0$

$4p < 4$

$\{p \in \cdots : p < 1\}$

2	

2

(i) $2\pi x^2 + 2\pi xh = 40$

$h = \dfrac{40 - 2\pi x^2}{2\pi x}$

Volume of the structure

$= \dfrac{1}{2}.\dfrac{4}{3}\pi x^3 + \pi x^2 h$

$= \dfrac{2}{3}\pi x^3 + \pi x^2\left(\dfrac{40 - 2\pi x^2}{2\pi x}\right)$

$= \dfrac{2}{3}\pi x^3 + x\left(20 - \pi x^2\right)$

$= 20x - \dfrac{1}{3}\pi x^3$ (shown)

(ii) $\dfrac{dV}{dx} = 20 - \pi x^2$

At stationary value, $\dfrac{dV}{dx} = 0$.

$20 - \pi x^2 = 0$

$\therefore x = \sqrt{\dfrac{20}{\pi}}$ as $x > 0$

When $x = \sqrt{\dfrac{20}{\pi}}^{-}$, $V = 20\sqrt{\dfrac{20}{\pi}} - \dfrac{1}{3}\pi\left(\sqrt{\dfrac{20}{\pi}}\right)^3$

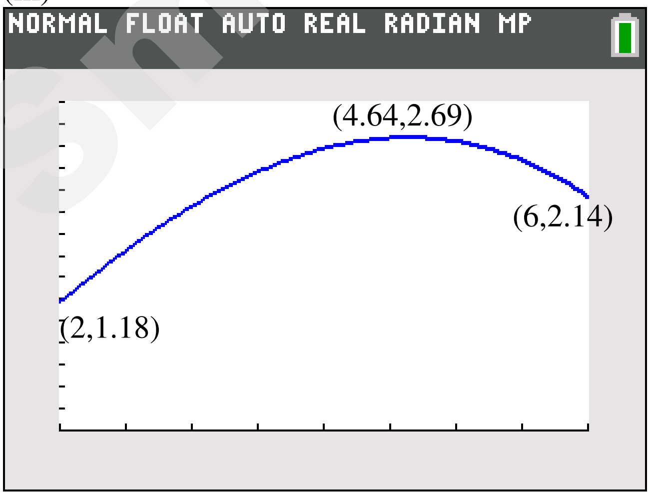
$= \dfrac{40}{3}\sqrt{\dfrac{20}{\pi}}$

$= \dfrac{80}{3}\sqrt{\dfrac{5}{\pi}}$

x	$\sqrt{\dfrac{20}{\pi}}^{-}$	$\sqrt{\dfrac{20}{\pi}}$	$\sqrt{\dfrac{20}{\pi}}^{+}$
$\dfrac{dV}{dx}$	+ve	0	-ve
slope	/	-	\

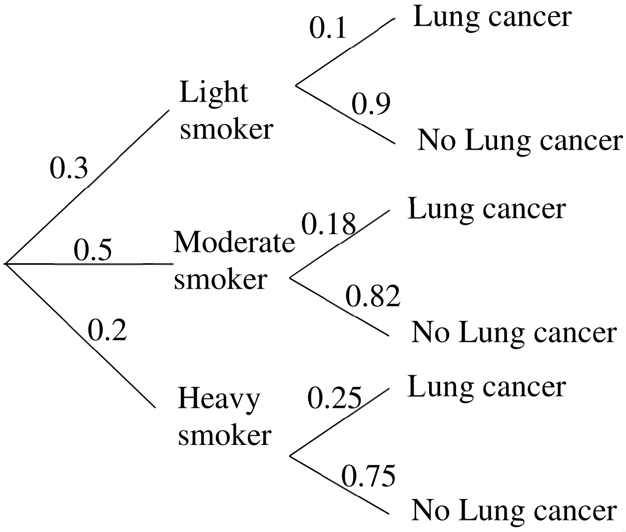
Hence, volume is maximum.

3	(a) $\frac{2}{e^{-x}+1}$ $\frac{d\left[2\left(e^{-x}+1\right)^{-1}\right]}{dx}$ $= -2\left(e^{-x}+1\right)^{-2}\left(-e^{-x}\right)$ $= 2e^{-x}\left(e^{-x}+1\right)^{-2}$ (b) $\int\left(1-\frac{3}{x}\right)^2 dx$ $= \int\left(1-\frac{6}{x}+\frac{9}{x^2}\right)dx$ $= x-6\ln\lvert x\rvert-\frac{9}{x}+C$
4	(i) $y=\ln(3-2x)$ $\frac{dy}{dx}=\frac{-2}{3-2x}$ When $x=\frac{1}{2}$, $y=\ln\left[3-2\left(\frac{1}{2}\right)\right]=\ln 2$, $\frac{dy}{dx}=\frac{-2}{3-2\left(\frac{1}{2}\right)}=-1$ Equation of tangent: $y-\ln 2=-1\left(x-\frac{1}{2}\right)$ $y=-x+\frac{1}{2}+\ln 2$ (ii) When $y=0$, $x=\frac{1}{2}+\ln 2$ $A\left(\frac{1}{2}+\ln 2,0\right)$ When $x=0$, $y=\frac{1}{2}+\ln 2$ $B\left(0,\frac{1}{2}+\ln 2\right)$

	area of triangle $OAB = \frac{1}{2}\left(\frac{1}{2}+\ln 2\right)\left(\frac{1}{2}+\ln 2\right)$ $= \frac{1}{2}\left(\frac{1}{2}+\ln 2\right)^2$ units^2 (iii) Required area $= \int_0^{\ln 3} \frac{1}{2}\left(3-\mathrm{e}^{y}\right)\,\mathrm{d}y$ $= \frac{1}{2}\left[3y-\mathrm{e}^{y}\right]_0^{\ln 3}$ $= \frac{1}{2}\left[3\ln 3-\mathrm{e}^{\ln 3}-0+1\right]$ $=\frac{1}{2}\left[3\ln 3-2\right]$ units^2
5	(i) Let x, y, and z be the number of 25-inch, 29-inch and 32-inch pants respectively. $x+y+z=100$ $x-4z=0$ $170x+200y+232z=19120$ Solving, $x = 40$, $y = 50$, $z = 10$ (ii) Minimum price to sell per pair = \$10 (iii)  (iv) When $R = 2.15$, $x = 3.1438579$ Minimum amount spent = \$315

	(v) Maximum point (4.6419, 2.68535) Maximum revenue = \$2685.35 when x = 4.64 (vi) Price per pair = $\frac{2685.35}{100} = 26.85$ At maximum revenue, \$464 is spent on advertising. Profit made $= 2685.35 - 1000 - 464.19$ = \$1221.16
6	(i) No of different delegations $= {}^{16}C_{10} = 8008$ (ii) <u>Method 1</u> No of different delegations $= {}^{14}C_9 \times 2 + {}^{14}C_{10}$ $= 5005$ <u>Method 2</u> No of different delegations $= {}^{16}C_{10} - {}^{14}C_8$ $= 5005$ (iii) Required probability $= \frac{{}^7C_4 \times {}^5C_3 \times {}^4C_3 + {}^7C_3 \times {}^5C_4 \times {}^4C_3 + {}^7C_3 \times {}^5C_3 \times {}^4C_4}{{}^{16}C_{10}}$ $= \frac{2450}{8008} = \frac{175}{572}$ Or 0.306 (3 s.f)

7 (i)



(ii) Required probability

$= 0.3\times0.1+0.5\times0.18+0.2\times0.25$

$= 0.17$

(iii) Required probability

$= 0.5+0.17-0.5\times0.18$

$= 0.58$

(iv)

$\text{P(Heavy smoker} \mid \text{Develops lung cancer)}$

$= \dfrac{0.2\times0.25}{0.17}$

$= \dfrac{5}{17}$ or 0.294

Required probability

$= 0.17\times(1-0.17)\times2$

$= \dfrac{1411}{5000}$ or 0.2822

8 Let X be the delay times (in ns) for a delay line.
and $X \sim \mathrm{N}(\mu, \sigma^2)$

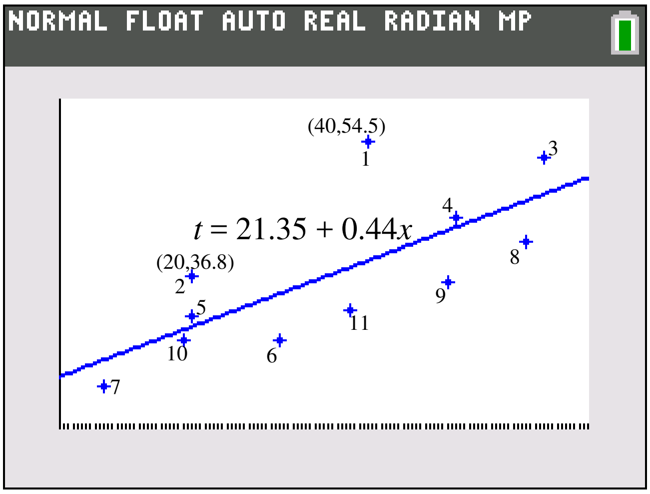
Given that $\mathrm{P}(X < 274.6) = 0.1$

$$P\left(Z<\frac{274.6-\mu}{\sigma}\right)=0.1$$

$\dfrac{274.6-\mu}{\sigma}=-1.28155167$

$\mu-1.28155167\sigma=274.6$ ------ (1)

	$P(X > 288.2) = 0.075$ $P\left(Z > \frac{288.2-\mu}{\sigma}\right) = 0.075$ $\frac{288.2-\mu}{\sigma} = 1.439531471$ $\mu + 1.439531471\sigma = 288.2$ ------- (2) From GC, $\mu = 281.0051152$ $\sigma = 4.998074$ i.e. Mean =281 (3 sf) Variance = σ^2 = 25.0 (3 sf)
9	(i) Assumption: Whether or not an egg is broken is independent of another egg. (ii) $X \sim B(6, 0.1)$ P(a box of eggs is sub-standard) = $P(X \geq 2)$ $= 1 - P(X \leq 1)$ $= 0.114$ (3 sf) (iii) Let W be the number of boxes that are sub-standard out of n. W~N(n, 0.114265) Given $P(W > 3) < 0.01$ $1 - P(W \leq 3) < 0.01$ From GC, When $n = 8$, $P(W > 3) = 0.00815 < 0.01$ When $n = 9$, $P(W > 3) = 0.01336 > 0.01$ Therefore, greatest value of number of boxes chosen is 8. (iv) Let Y be the number of boxes that are sub-standard out of 10. $Y \sim B(10, 0.114265)$ $P(Y = 0) = 0.2972$ $P(Y = 1) = 0.3834$ $P(Y = 2) = 0.2226$ i.e. Most likely number = 1 (v) Breakages are not independent of each other. (if one egg in a box is broken, it is more likely the others will be).

10	(i) Scatter diagram  (ii) From GC the required regression line is $t = 21.3521 + 0.43763x$ $t = 21.35 + 0.44x$ (2dp) This is likely because the new staff was not familiar with the wrapping process initially for the first 5 gift baskets. (iii) Considering only the last 6 points, $r = 0.974$. This shows that there is a strong positive linear correlation between x and t. As the number of items in the gift basket increases, the timing to complete wrapping increases. (iv) When $x = 44$, $t = 19.1 + 0.361(44) = 34.984$. i.e. The estimated wrapping time for 44 items is 35.0 mins. The estimate is reliable because it is obtained by interpolation and $r = 0.974$ is close to +1.
11	Let X be the duration of a certain viral infection in an adult. (i) H_0: $\mu = 3.6$ H_1: $\mu \neq 3.6$ Under H_0, test statistic is, $Z = \dfrac{\overline{X} - \mu}{S/\sqrt{n}} \sim N(0,1)$ approximately (by CLT) where μ=3.6, $s^2 = \dfrac{100}{99}(1.005)^2$, $\overline{x} = 3.42$, n=100 From GC, p-value = 0.0747382

	Since p-value = 0.0747382 > 0.05, we do not reject H_0, and conclude that, at 5% level there is no significant evidence that the average duration of infection is different from the scientist's claim. (ii) Not necessary. Since n = 100 is large, by CLT, the sample mean duration of a certain viral infection in an adult follows a normal distribution approximately. (iii) unbiased estimate of population mean, $\bar{y} = \frac{\sum y}{50} = \frac{166}{50} = 3.32$ unbiased estimate of population variance, $s^2 = \frac{\sum (y-\bar{y})^2}{49} = \frac{55}{49}$ (iv) H_0: $\mu = 3.6$ H_1: $\mu < 3.6$ Under H_0, test statistic is, $Z = \frac{\bar{X}-\mu}{S/\sqrt{n}}$ ~ N(0,1) approximately (by CLT) where μ=3.6, $s^2 = \frac{55}{49}$, $\bar{x} = 3.32$, n = 50 From GC, p-value = 0.0308262 Given null hypothesis is rejected, p-value = $0.0308262 < \frac{\alpha}{100}$ i.e. α>3.08262 Ans: $\alpha \geq 3.09$
12	Let S be the mass (in kg) of a salmon, Let A be the mass (in kg) of a scallop. $S \sim N(5, 1.5^2)$, $A \sim N(0.2, 0.05^2)$ (i) P(S > 5.5) = 0.369 (ii) Let X be the number of salmon that weigh more than 5.5 kg in a box of 6. $X \sim B(6, 0.3694414)$ E(X) = 6(0.3694414) = 2.2166484 Var(X) = 6(0.3694414)(0.6305586) = 1.3977267 Since n = 60 is large, by CLT, Let $\bar{X} = \frac{X_1 + X_2 + \cdots + X_{60}}{60}$ $\bar{X} \sim N\left(2.2166484, 1.3977267/60\right)$ approximately $P(\bar{X} \leq 2) = 0.0779$

	(iii) Let $G = A_1 + A_2 + \ldots + A_{200}$ $G \sim \mathrm{N}(200(0.2), 200(0.05)^2)$ i.e. $G \sim \mathrm{N}(40, 0.5)$ $\mathrm{P}(G > m) > 0.95$ $1 - \mathrm{P}(G \le m) > 0.95$ $\mathrm{P}(G \le m) < 0.05$ $m < 38.8369$ $m < 38.8$ (iv) $V = 30(S_1 + S_2 + \ldots + S_6) \sim \mathrm{N}(30(6\times5), 30^2(1.5)^2\times6)$ i.e. $V \sim \mathrm{N}(900, 12150)$ $W = 16(A_1 + A_2 + \ldots + A_{100}) \sim \mathrm{N}(16(100\times0.2), 16^2(0.05)^2\times100)$ i.e. $W \sim \mathrm{N}(320, 64)$ $V_1 + V_2 - 6W \sim \mathrm{N}(2(900) - 6\times320, 2(12150)+6^2(64))$ i.e. $V_1 + V_2 - 6W \sim \mathrm{N}(-120, 26604)$ $\mathrm{P}\left(-150 < V_1 + V_2 - 6W < 150\right) = 0.524$ It is the probability that the selling price of 2 boxes of salmon differs from six times the selling price of a box of scallops by less than \$150. Or It is the probability of the ‘the difference between the selling price of 2 boxes of salmon and six times the selling price of a box of scallops is within ± 150.