2017

JC2 H1 Math

1.	SA2 Anderson Junior College	
2.	SA2 Anglo Chinese Junior College	
3.	SA2 Dunman High School	
4.	SA2 Hwa Chong Institution	
5.	SA2 Innova Junior College	
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ANDERSON JUNIOR COLLEGE
JC2 Preliminary Examination 2017

MATHEMATICS 8865/01

Higher 1 **11 September 2017**

Paper 1 **3 hours**

Additional Materials:
Graph Paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagram or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions
Give non-exact numerical answers correct to **3 significant figures**, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
When unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in [ ] at the end of each question or part question.

Name:______________________________________ PDG:_____________

1	2	3	4	5	6

7	8	9	10	11	TOTAL

This Question paper consists of 6 printed pages

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Section A: Pure Mathematics [40 marks]

1	Find the values of k for which $3(k-2)x^2-6x+k>0$ for all values of x. [4] Hence deduce the values of k for which the function $y=(k-2)x^3-3x^2+kx+5$ is strictly increasing for all real values of x. [2]
2	The curve C has equation $y=\frac{1}{2}e^{1-3x^2}$. (i) Without using a calculator, find the equation of the tangent to C at the point P where $x=1$, giving your answer in the form where $y=mx+c$, where m and c are constants in exact terms to be found. [3] The tangent to C at P cuts x-axis at the point A and the y-axis at the point B. (ii) Find the exact coordinates of the midpoint of AB. [2] (iii) Find the length of AB, giving your answer to 3 significant figures. [2]
3	(a) Show that $\frac{d}{dx}\ln\left(\frac{x^3}{1+x^2}\right)=\frac{x^2+3}{x\left(1+x^2\right)}$. [2] Hence deduce the exact value of $\int_1^2 \frac{x^2+3}{2x(x^2+1)}dx$, simplifying your answer to a single term. [3] (b) State the numerical value of $\int_1^2 \ln\left(\frac{x^3}{1+x^2}\right)dx$. [1]
4	Sketch the graph of the curve C with equation $y=2(k-x)x$, where k is a positive constant, showing clearly the coordinates of the points where C cuts the axes. [1] (i) Show that the line $y=\frac{k}{2}x$ always intersects C at two distinct points. [2] The line $y=\frac{k}{2}x$ intersects C at the origin O and another point A where $x=\frac{3k}{4}$. (ii) Find the area of the region between C and the line $y=\frac{k}{2}x$. [3] (iii) State the values of x for which $2kx-2x^2\le\frac{k}{2}x$. [1] Consider the case where $k=2$. (iv) Use your answer in (iii) to deduce the exact values of x for which $4\ln x-2(\ln x)^2\le\ln x$ [2]
5	A new company manufactures souvenirs. The cost, C thousand dollars for producing x hundred souvenirs, is modelled by the equation $C=\frac{169}{2x+1}+2x$, $0\le x\le 20$. (i) Use differentiation to find the number of souvenirs that must be produced to minimise the cost. State the minimum cost, justifying that this cost is a minimum. [5] (ii) Sketch the graph of C against x, showing clearly the coordinates of any

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	turning points and any intersections with the axes [1] The daily revenue collected R thousand dollars, varies with the time t days. The CEO believes that the connection between the rate of change of the daily revenue, $\frac{dR}{dt}$, and the time t days, can be modelled by the equation $\frac{dR}{dt} = 3 - e^{-2t}$, $t \geq 0$. (iii) Sketch the graph of $\frac{dR}{dt}$ against t, showing clearly the coordinates of the point(s) where the curve cuts the vertical axis and the equation of any asymptote(s). Give a practical interpretation of the asymptote(s). [2] (iv) The daily revenue collected when $t = 0$ is \$1000. Find, in terms of t, the daily revenue collected, R thousand dollars, on day t. [3] (v) Hence state the value of t when the daily revenue collected first reaches \$21500. [1] (vi) The daily revenue collected when $t = 0$ is \$1000. Find, in terms of t, the daily revenue collected, R thousand dollars, on day t. [3] (vii) Hence state the value of t when the daily revenue collected first reaches \$21500. [1]
	Section B: Probability and Statistics [60 marks]
6	1. Independent events A and B are such that $P(A) = 0.45$ and $P(B) = 0.4$. (i) Find $P(A \cup B)$. [2] Event C is such that $P(C) = 0.4$, $P(B \mid C) = 0.4$, $P(A \cap C) = 0.18$ and $P(A \cap B \cap C) = 0.1$. (ii) Find $P(B \cap C)$ and hence deduce $P(A' \cap B \cap C)$. [2] (iii) Show that $P(A \cup B \cup C) = 0.83$ and hence find $P(A' \cap B' \cap C')$ [3]
7	A salad bar in a restaurant has 7 types of greens, 3 types of proteins and 6 types of toppings. There are also 2 types of soup and 2 types of yogurt for selection. A promotional set meal consists of a salad plate, plus either a soup or a yoghurt. For the salad plate, a customer needs to choose 3 different types of greens, 1 type of protein, and 2 different types of toppings. (i) Find the number of ways the customers may customise his set meal. [2] Each morning, the employee has to key a password to access the company accounts. The password consists of 3 digits from 1 to 9, followed by 2 letters

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	of the alphabet. Each digit or letter may be used any number of times. Find the number of possible passwords if (ii) there is no other restriction, [1] (iii) the password has exactly one even digit and at least one vowel. [3] One morning, the employee forgot the password. However, he is certain that the digits are all different, but the alphabets are identical. He makes an attempt to type in the password. (iv) Find the probability that the employee gets the password correct in his first attempt. [2]
8	A nursery sells a large number of rose seeds. 25% of the seeds are red rose seeds, and the rest are either yellow or pink rose seeds. The nursery sells the seeds in packs of 12, and each pack contains a random selection of rose seeds. For these packs, the mean number of yellow rose seeds is 3.6. A pack of rose seeds is chosen at random. (i) Show that the probability that the pack contains at most three yellow rose seeds is 0.4925. [2] (ii) Find the probability that more than half of the seeds in the pack are either red or yellow rose seeds. [2] A box contains 200 packs of seeds. (iii) Find the probability that at least 30%, but less than 60% of the packs contain at most three yellow rose seeds. [2] John buys a pack of rose seeds. His pack of seeds contains three red rose seeds, four yellow rose seeds and five pink rose seeds. His child randomly picks three seeds from the pack to plant them in a row. Find the probability that (iv) there are at least two pink rose seeds planted, [3] (v) the third seed planted is a pink rose seed if it is known that at least two pink rose seeds are planted. [3]
9	A college has a large number of students taking mathematics and chemistry. In the block test, the scores of the mathematics test, X marks, is normally distributed with mean 50 marks and standard deviation 8 marks. 3 students are chosen at random. Find the probability that (i) each of the three students score more than 40 marks, [2] (ii) the total marks of the first two students differ from twice the marks of the third student by more than 15 marks. [3] The mathematics marks are moderated to Y marks, using the formula $Y = aX + b$, where a and b are positive constants. 2.04 % of the students have

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	a moderated score of less than 42 marks, while 2.04% of students have a moderated score of more than 78 marks. (iii) Find the value of $\mathrm{E}(Y)$ and show that $\mathrm{Var}(Y) = 77.432$. [3] (iv) Find the values of *a* and *b*. [3] The chemistry marks of the college block test, *C* marks, has a mean of 52 marks and standard deviation 10 marks. A group of 40 chemistry students are randomly selected to attend a feedback session. (v) Find the probability that the average chemistry mark of the group is within 1 mark of the college mean chemistry mark. [2]
10	A baker claims that the mean mass of his 'Xtra' loaf of bread is 800 g. The mass of the loaves is known to have a standard deviation of 10.1 g. A random sample of 50 loaves was taken, and found to have a mean mass of 797.7 grams. (i) Test the baker's claim at the 5% level of significance. [4] (ii) Meanwhile, a group of consumers used the same sample to carry out a different test. They conclude that the baker is overstating the mean mass at the *k*% significance level. Find the smallest value of *k* to three significant figures. [3] The bakery also claims that the average mass of a certain compound in each loaf of healthy bread is 150 mg. The mass of the compound in the loaves is normally distributed and the standard deviation is σ mg. A random sample of 60 loaves of healthy bread is taken, and the mass of compound in each loaf *y* mg is observed. The results are summarised as $\sum(y-150)=60$. A test at 6% shows that the baker is understating the average mass of compound. (iii) Find the possible values that σ can take. [4]
12	A company is selling a particular make of cars. The age of the car *x*, in months, and the advertised selling price *P*, in hundreds of dollars, for 8 cars are given below.

x	5	10	20	50	70	80	100	120
P	546	500	433	329	278	249	187	100

(i) Give a sketch of the scatter diagram for the data, as shown on your GC. [1]

(ii) Find the product moment correlation coefficient and comment on its value in the context of the question. [2]

(iii) Find the equation of the regression line of *P* on *x*, and sketch this line on your diagram. [2]

(iv) Estimate the age of a car that can be bought from the company with a budget of

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	\$28 000. Give reasons why you expect this estimate to be reliable. The number of remaining months the Certificate of Entitlement COE of a car is valid is denoted by y months. It is known that $y = 120 - x$. (v)Find the equation of the regression line of P on y. [2]

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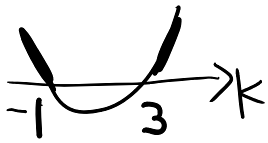
1	$(k-2)x^2-6x+k>0$ for all values of x $3(k-2)>0$ ----(1) and $(-6)^2-4(3)(k-2)(k)<0$ ------(2) From (1): $k>2----(1)$ and From (2): $36-12k(k-2)<0$  $\Rightarrow 36-12k^2+24k<0$ $\Rightarrow -k^2+2k-3<0$ $\Rightarrow k^2-2k+3>0$ $\Rightarrow (k+1)(k-3)>0$ $\Rightarrow k<-1 \quad \text{or} \quad k>3------(2)$ From (1) and (2) : solution is $k>3$ $y=(k-2)x^3-3x^2+kx+5 \Rightarrow \frac{dy}{dx}=3(k-2)x^2-6x+k$ If function is strictly increasing, $\frac{dy}{dx}>0$ for all values of x So $(k-2)x^2-6x+k>0$ From above, solution is $k>3$
2	(i) $y=\frac{1}{2}e^{1-3x^2} \Rightarrow \frac{dy}{dx}=\frac{1}{2}e^{1-3x^2}\left(-6x\right)=-3xe^{1-3x^2}$ At P, $x=1$, $y=\frac{1}{2}e^{1-3}=\frac{1}{2}e^{-2}$, $\frac{dy}{dx}=-3e^{1-3}=-3e^{-2}$ Equation of tangent is $y-\left(\frac{1}{2}e^{-2}\right)=(-3e^{-2})(x-1)$ $y-\left(\frac{1}{2}e^{-2}\right)=(-3e^{-2})(x-1)$ $y=-3e^{-2}x+3e^{-2}+\frac{1}{2}e^{-2}=-3e^{-2}x+\frac{7}{2}e^{-2}$ (ii) At B, $x=0$, $y=\frac{7}{2}e^{-2}$ At A, $y=0$, $-3e^{-2}x+\frac{7}{2}e^{-2}=0 \Rightarrow x=\frac{\left(-\frac{7}{2}e^{-2}\right)}{-3e^{-2}}=\frac{7}{6}$ $A(\frac{7}{6},0)$ $B(0,\frac{7}{2}e^{-2})$ Midpoint of AB is $\left(\frac{\frac{7}{6}+0}{2},\frac{0+\frac{7}{2}e^{-2}}{2}\right)=\left(\frac{7}{12},\frac{7}{4}e^{-2}\right)$ (iii) $AB=\sqrt{\left(\frac{7}{6}\right)^2+\left(\frac{7}{2}e^{-2}\right)^2}$ =1.26

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3

a(i)

$$\ln\left(\frac{x^3}{1+x^2}\right)=\ln x^3-\ln(1+x^2)=3\ln x-\ln(1+x^2)$$

$$\frac{d}{dx}\ln\left(\frac{x^3}{1+x^2}\right)=\frac{3}{x}-\frac{2x}{1+x^2}$$

$$=\frac{3(1+x^2)-2x(x)}{x(1+x^2)}$$

$$=\frac{3+3x^2-2x^2}{x(1+x^2)}=\frac{x^2+3}{x(1+x^2)}$$

a(ii)

$$\int_1^2\frac{x^2+3}{2x(1+x^2)}dx=\frac{1}{2}\int_1^2\frac{x^2+3}{x(1+x^2)}dx$$

$$=\frac{1}{2}\left[\ln\frac{x^3}{1+x^2}\right]_1^2=\frac{1}{2}\left[\ln\frac{2^3}{1+2^2}-\ln\frac{1}{2}\right]$$

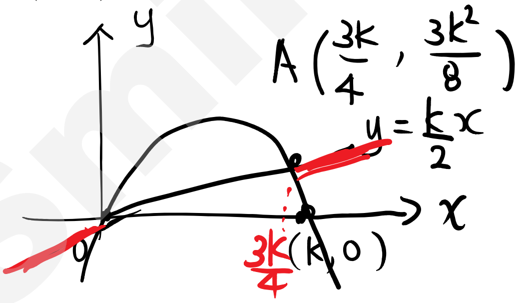
$$=\frac{1}{2}(\ln\frac{8}{5}-\ln\ \frac{1}{2})=\frac{1}{2}(\ln\frac{\left(\frac{8}{5}\right)}{\left(\frac{1}{2}\right)})$$

$$=\frac{1}{2}\ln\frac{16}{5}$$

3b) $\int_1^2\ln\left(\frac{x^3}{1+x^2}\right)dx=-0.0103$

4

$y=2(k-x)x$



(i) At point of intersection of $y=\frac{k}{2}x$ and $y=2(k-x)x$

$$\frac{k}{2}x=2(k-x)x\Rightarrow\frac{k}{2}x=2kx-2x^2\Rightarrow 2x^2-2kx+\frac{k}{2}x=0$$

$$\Rightarrow 2x^2-\frac{3k}{2}x=0---------(1)$$

<u>Method 1:</u>

Observe that Discriminant is $D=\left(-\frac{3k}{2}\right)^2-4(2)(0)=\frac{9k^2}{4}>0$ (since

k>0 $\Rightarrow k^2>0\Rightarrow\frac{9}{4}k^2>0$ for all positive values of k.

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Hence, the quadratic equation (1) will have 2 distinct roots.
So the line intersects the curve at two distinct points.

<u>Alternative Method:</u>

From (1) $x\left(2x-\frac{3k}{2}\right)=0 \Rightarrow x=0 \quad \text{or} \quad x=\frac{3k}{4}\neq 0$

Hence, the quadratic equation (1) will have 2 distinct roots.
So the line intersects the curve at two distinct points.

At A, When $x=\frac{3k}{4}, y=\frac{k}{2}\left(\frac{3k}{4}\right)=\frac{3k^2}{8}$ & A $\left(\frac{3k}{4},\frac{3k^2}{8}\right)$

(ii) Area= $\int_0^{\frac{3k}{4}} \left(2(k-x)x-\frac{k}{2}x\right)\, dx$ =

$$\int_0^{\frac{3k}{4}} -2x^2+\frac{3k}{2}x \; dx=\left[-\frac{2x^3}{3}+\frac{3kx^2}{4}\right]_0^{\frac{3k}{4}}$$

$$=\left(-\frac{2}{3}\left(\frac{3k}{4}\right)^3+\frac{3k}{4}\left(\frac{3k}{4}\right)^2\right)-0=-\frac{2}{3}\left(\frac{27k^3}{64}\right)+\frac{3k}{4}\left(\frac{9k^2}{16}\right)$$

$$=-\frac{9k^3}{32}+\frac{27k^3}{64}=\left(-\frac{9}{32}+\frac{27}{64}\right)k^3=\frac{9}{64}k^3$$

<u>Alternative Method</u>:

Area= $\int_0^{\frac{3k}{4}} 2(k-x)x \; dx-\frac{1}{2}\left(\frac{3k}{4}\right)\left(\frac{3k^2}{8}\right)$

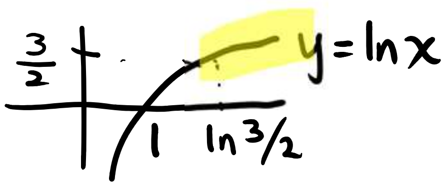
(iii) $2kx-2x^2\leq\frac{k}{2}x$ means $2(k-x)x\leq\frac{k}{2}x\Rightarrow x\leq 0 \quad \text{or} \quad x\geq\frac{3k}{4}$

(iv)Replace x by $\ln x$ and k by 2 in the solution above:

$4\ln x-2(\ln x)^2\leq\ln x$

$\Rightarrow \ln x\leq 0$ or $\ln x\geq\frac{3}{2}$

$\Rightarrow 0<x\leq 1$ or $x\geq e^{\frac{3}{2}}$



5

(i) $C=\frac{169}{2x+1}+2x=169(2x+1)^{-1}+2x$

$\frac{dC}{dx}=169(-1)(2x+1)^{-2}(2)+2=\frac{-338}{(2x+1)^2}+2$

Min C: $\frac{dC}{dx}=0\Rightarrow\frac{-338}{(2x+1)^2}+2=0$

$$2=\frac{338}{(2x+1)^2}\Rightarrow(2x+1)^2=\frac{338}{2}=169$$

$$2x+1=13 \quad \text{or} \quad 2x+1=-13$$

$$x=6 \text{ or } x=\text{-}2 \;\; (\text{rejected}, x\geq 0)$$

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Method 1: $\dfrac{d^2C}{dx^2} = \dfrac{676}{(2x+1)^3}$

At $x = 6, \dfrac{d^2C}{dx^2} > 0$; so C is minimum when x=6.

Method 2:

x	6^-	6	6^+
$\dfrac{dC}{dx}$	-	0	+
Outline			

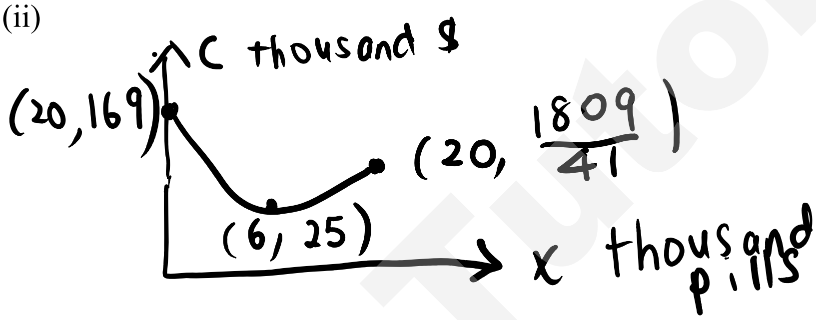
C is minimum.

$C = \dfrac{169}{(2\times6+1)^2} + 2(6) = 25$

6000 pills must be produced.

Minimum production cost is \$25000.

(ii)



(iii)

$\dfrac{dR}{dt} = 3 - e^{-2t}$

$\dfrac{dR}{dt}$ increases and approaches 3 when t is very large.

The daily revenue collected increases at a rate of approximately **3 thousand dollars per day** in the **long run**

(iv) $R = \int 3 - e^{-2t} dt = 3t - \dfrac{e^{-2t}}{-2} + C = 3t + \dfrac{e^{-2t}}{2} + C$

t = 0, R=1: $3(0) + \dfrac{e^0}{2} + C = 1 \Rightarrow \dfrac{1}{2} + C = 1 \Rightarrow C = \dfrac{1}{2}$

$R = 3t + \dfrac{e^{-2t}}{2} + \dfrac{1}{2}$

(v)The revenue first reaches \$21500 when $t = 7$

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6

(i) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=P(A)+P(B)-P(A)\times P(B)$ $\quad\because$ A & B independent

$=0.45+0.4-(0.45)(0.4)=0.67$

(ii) $P(B|C)=0.4\Rightarrow\dfrac{P(B\cap C)}{P(C)}=0.4$

$P(B\cap C)=0.4P(C)=0.4(0.4)=0.16$

$P(A'\cap B\cap C)=P(B\cap C)-P(A\cap B\cap C)=0.16-0.1=0.06$

(iii) <u>Method 1</u>(Formula)

$P(A\cup B\cup C)$

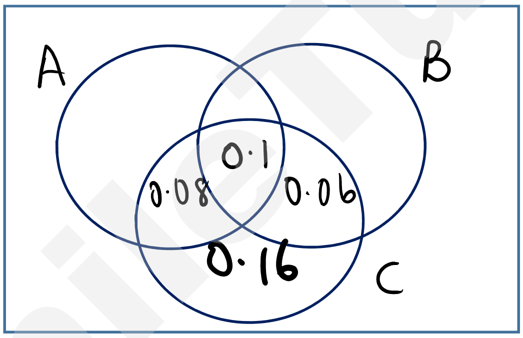
$=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$

=0.45+0.4+0.4 -0.18-0.16-0.18+0.1=0.83

<u>Alternative method (From Venn diagram)</u>

$P(A\cap B'\cap C)=P(A\cap C)-P(A\cap B\cap C)=0.18-0.1=0.08$

$P(A'\cap B'\cap C)=P(C)-0.1-0.08-0.06=0.16$



$P(A\cup B\cup C)=P(A\cup B)+0.16=0.67+0.16=0.83$

$P(A'\cap B'\cap C')=1-P(A\cup B\cup C)=1-0.83=0.17$

7

(i) No of ways= ${}^7C_3\times{}^3C_1\times{}^6C_2\times{}^4C_1$ =6300

(ii) No of codes that can be formed = $9\times9\times9\times26\times26=492804$

(iii) Case 1: one even digit & 2 odd digits, one vowel & one consonant

Case 2: one even digit & 2 odd digits, 2 vowels.

No of codewords= $3(4\times5\times5)\times2(5\times21)+3(4\times5\times5)\times(5\times5)$

=63000+7500=70500

(iv) No of passwords with all different digits, & identical letters

$=9\times8\times7\times26\times1=13104$

Probability $=\dfrac{1}{13104}$

8

Let X be the number of yellow rose seeds out of 12. $X\sim B(12,0.3)$

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	Since $E(X)=12p=3.6 \Rightarrow p=\dfrac{3.6}{12}=0.3$ $P(X \le 3)=0.4925158 \approx 0.4925$ (ii) Let Y be the number of seeds that are either red or yellow rose seeds $Y \sim B(12,0.55)$ Since P(yellow or red)=0.3+0.25=0.55 $P(Y>6)=P(Y\ge 7)=1-P(Y\le 6)=0.527$ (iii) Let W be the number of packs that contain at most three yellow rose seeds, out of 200 packs . $W \sim B(200,0.4925)$ $P(30\% \text{ of } 200 \le W < 60\% \text{ of } 200)=P(60\le W<120)$ $=P(60\le W\le 119)=P(W\le 119)-P(W\le 59)$ $=0.998545\approx 0.999$ (iv)P(at least 2 pink) $=\left(\dfrac{5}{12}\times\dfrac{4}{11}\times\dfrac{3}{10}\right)+3\left(\dfrac{5}{12}\times\dfrac{4}{11}\times\dfrac{7}{10}\right)=\dfrac{4}{11}$ (v)P(third seed is pink|at least 2 pink)= $\dfrac{P(PPP \text{ or } P\overline{P}P \text{ or } \overline{P}PP)}{P(\text{at least 2 pink})}$ $=\dfrac{\left(\dfrac{5}{12}\times\dfrac{4}{11}\times\dfrac{3}{10}\right)+2\left(\dfrac{5}{12}\times\dfrac{4}{11}\times\dfrac{7}{10}\right)}{\dfrac{4}{11}}=\dfrac{\left(\dfrac{17}{66}\right)}{\left(\dfrac{4}{11}\right)}=\dfrac{17}{24}$
9	$X \sim N(50,8^2)$ (i)Prob $=\left(P(X>40)\right)^3$ $=(0.894351)^3=0.715$(ii) $X_1+X_2-2X_3 \sim N(50+50-2(50),8^2+8^2+4(8^2))$ i.e $N(0,384)$ $P(X_1+X_2-2X_3<-15 \text{ or } X_1+X_2-2X_3>15)$ $=P(X_1+X_2-2X_3<-15)+P(X_1+X_2-2X_3>15)$ $=0.221997+0.221997=0.444$ <u>Alternative:</u> $1-P(-15<X_1+X_2-2X_3<15)=1-0.556006=0.444$ (iii) Let $Y \sim N(\mu,\sigma^2)$ $P(Y<42)=P(Y>78)\Rightarrow E(Y)=\dfrac{42+78}{2}=60$ (by symmetry) $P(Y<42)=0.0204 \Rightarrow P(Z<\dfrac{42-60}{\sigma})=0.0204 \Rightarrow P(Z<\dfrac{-18}{\sigma})=0.0204$ $\dfrac{-18}{\sigma}=-2.0455567 \Rightarrow \sigma=\dfrac{-18}{-2.0455567}=8.79956$ Var(Y)=8.79956^2=77.4322655=77.432
	$Y=aX+b$ $E(Y)=aE(X)+b=a(50)+b=50a+b$ $50a+b=60$ --------(1) $Var(Y)=a^2Var(X)=64a^2$ $64a^2$ =77.4333---------(2)

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$$a^2 = \frac{77.4322655}{64} = 1.209879149$$

$a = 1.099995 \approx 1.10$

50(1.099995)+b=60   b=5.0025 $\approx$ 5

(v) $\overline{C} = \dfrac{C_1 + C_2 + \ldots C_{40}}{40}$

Since sample size=40>30 is large, By **CLT**, $\overline{C} \sim N(52, \dfrac{10^2}{40})$

$P(52-1 < \overline{X} < 52+1) = P(51 < \overline{C} < 53) = 0.473$

10 (i) Let X be the mass of a randomly chosen 'Xtra' loaf of bread, and μ the population mean. *X* has a unknown distribution

Test $H_0: \mu = 800$ (baker's claim)

vs $H_1: \mu \neq 800$

Test statistic: Under Ho and since sample size n=50≥30is large, **by Central Limit Theorem,**

$$\overline{X} \sim N\left(800, \frac{10.1^2}{50}\right) \text{ approximately, } Z = \frac{\overline{X} - 800}{\sqrt{\frac{10.1^2}{50}}} \sim N(0,1)$$

Two **tailed** test at the 5% level of significance.

From sample, $\overline{x} = 797.7$,z = - 1.61, p=0 .107

Since p = 0.107>0.05 , do **not reject H₀** .

There is **insufficient evidence** at the **5%** level to conclude that the average mass is not 800 g. We do **not** reject the baker's claim.

OR: There is **insufficient evidence** at the **5%** level to conclude that the baker's claim is not valid.

(ii) If Test $H_0: \mu = 800$ (baker's claim)

vs $H_1: \mu < 800$ **(baker is overstating)**

Then p=0.05367

If bakery is overstating , reject Ho at k%,

p=0.05367< $\dfrac{k}{100}$ $\Rightarrow$ k>5.367

smallest k is 5.37

(iii)

Let Y be the mass of compound in a randomly chosen healthy loaf and μ the population mean. *Y* has a normal distribution

Test $H_0: \mu = 150$ (bakery's claim)

vs $H_1: \mu > 150$ (understating **)**

Test statistic: Under Ho

$$\overline{Y} \sim N\left(150, \frac{\sigma^2}{60}\right) \text{ and } Z = \frac{\overline{Y} - 150}{\frac{\sigma}{\sqrt{60}}} \sim N(0,1)$$

One-**tailed** test at the 6% level of significance.

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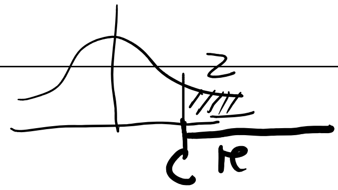
Critical Value:

$P(Z \le C) = 0.94 \Rightarrow C = 1.554774$

Reject Ho if $z > 1.554774$



Since our sample mean $\bar{y} = \dfrac{\sum(y-150)}{60} + 150 = \dfrac{60}{60} + 150 = 151$ Bakery is understating (Reject Ho)

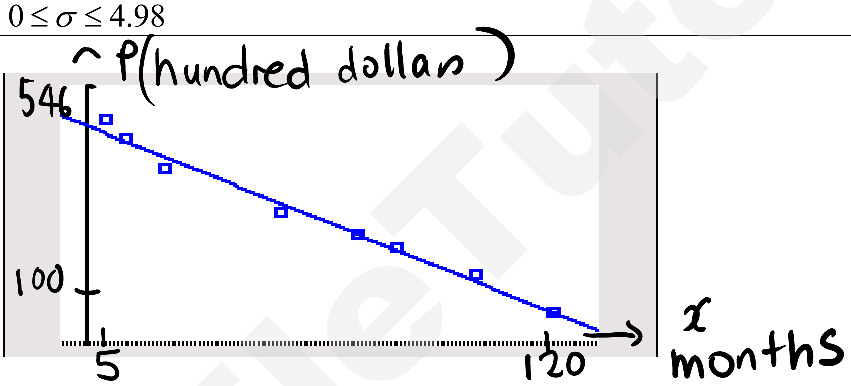
$$\frac{151-150}{\frac{\sigma}{\sqrt{60}}} > 1.554774 \quad \Rightarrow \frac{1}{\left(\frac{\sigma}{\sqrt{60}}\right)} > 1.554774 \quad \Rightarrow \frac{\sqrt{60}}{\sigma} > 1.554774$$

$$\Rightarrow \sqrt{60} > 1.554774\sigma \Rightarrow \sigma < \frac{\sqrt{60}}{1.554774} = 4.98205$$

$$\Rightarrow \sigma < \frac{\sqrt{60}}{1.554774} = 4.98205$$

$0 \le \sigma \le 4.98$

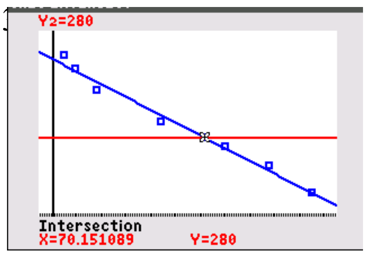
11



(ii) r= -0.992

Since r is close to -1, there is a strong negative linear correlation between the age of the car (x) and the advertised selling price (P). As the age of the car increases, the advertised selling price tends to decrease.

(iii)Regression line is $P = -3.60x + 5$



(iv)Using P on x:

$280 = -3.59669x + 532.3118$

$x = 70.2$

The estimated age of the car is 64.4 months.

The estimate is reliable because r= -0.992 is close to -1, and $P = 280$ is within the sample data range of $130 < P < 546$. Interpolation for2 strongly linearly correlated variables is reliable.

(v)

$y = 120 - x \Rightarrow x = 120 - y$. Replace x by $120 - y$:

$P = -3.5966918(120 - y) + 532.3118$

$P = 3.60y + 101$

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ANGLO-CHINESE JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

Higher 1

MATHEMATICS 8865/01

Paper 1 **16 August 2017**

3 hours

Additional Materials: Cover Sheet
Answer Paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your index number, class and name on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages.

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ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT
JC2 Preliminary Examination 2017

MATHEMATICS 8865
Higher 1
Paper 1

/ 100

Index No: ☐☐☐☐ Form Class: ____________

Name: ______________________________

Calculator model: ____________________

Arrange your answers in the same numerical order.

Place this cover sheet on top of them and tie them together with the string provided.

Question No.	Marks
1	/4
2	/4
3	/4
4	/7
5	/8
6	/13
7	/5
8	/6
9	/6
10	/9
11	/10
12	/12
13	/12

Summary of Areas for Improvement			
Knowledge (K)	Careless Mistakes (C)	Read/Interpret Qn wrongly (R)	Presentation (P)

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Section A: Pure Mathematics [40 marks]

1 A company with businesses in Canada, the United States and Mexico makes calls to these countries on a regular basis. Calls made to Canada, the United States and Mexico through the telco Singcall are charged at 28, 30 and 84 cents per minute respectively.
With Singcall, the bill for all the calls made to these three countries during a particular week was \$90. It was also recorded that the calls made to Mexico was twice as long as those to the United States, and that the total duration of calls made to Canada and Mexico was 120 minutes.
For calls made to each of the three countries, another telco, Sunhub, offers the first 10 minutes of talktime free. Subsequently, calls made to Canada, the United States and Mexico are charged at 38, 40 and 94 cents per minute respectively. For the same call duration made to each country in that particular week, determine if Sunhub's rates are better for the company. [4]

2 Use a non-calculator method to find $\int_{-1}^{1}\left(\mathrm{e}-\mathrm{e}^{-x}\right)^2 \mathrm{d}x$ in the form $a+b(\mathrm{e}^2-\mathrm{e}^{-2})$ where a and b are constants. [4]

3 A bank pays interest on a fixed deposit at a rate of r% per annum. With a principal amount of \$$P$ placed in the bank, if interest is compounded m times per year, after t years, the amount in the bank will be \$$A$ where

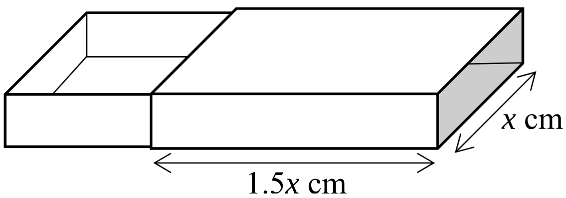
$$A = P(1+\frac{r}{100m})^{tm}.$$

Mr Tan deposits \$7 800 in a fixed deposit that pays 3.2% interest per annum.

(i) Mr Tan intends to use \$10 000 to make the down-payment of a new car after 80 months. If interest is compounded monthly, how much more money does he need in order to do so? Leave your answer to the nearest cent. [2]

(ii) If interest is compounded yearly, find the minimum number of years that it will take him to save \$10 000. [2]

4



A matchbox, shown in the diagram above, consists of an outer cover which is open at opposite ends and an inner box with an open top which slides into the outer cover. The length of the outer cover is 1.5x cm, where x cm is the breadth of the cover. Assume that the entire matchbox is made of cardboard with negligible thickness, and that the inner box has the same dimensions as the outer cover. If the volume of the matchbox is 30 cm^3, show that the area A cm^2 of the cardboard used to make both the inner box and outer cover is given by

$$A = 4.5x^2 + \frac{160}{x}.$$

If the amount of cardboard used is minimum, find the exact dimensions of the matchbox, justifying that they give the minimum amount of cardboard used. [7]

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5 The curve C has equation $y = x - \ln(x^2+1)$.

(i) Use a non-calculator method to find the coordinates of the stationary point on C and determine its nature. [4]

(ii) Find the area bounded by C, the positive x-axis, and the tangent to C at the point where $x = 5$. [4]

6 **(a)** The petrol consumption of a car, in millilitres per kilometre, is advertised to be $P(x) = \dfrac{2500}{x} + \dfrac{2x}{3}$ where x is the speed of the car in km/h.

(i) Find the exact speed of the car when petrol consumption is minimum. [3]

(ii) Sketch the graph of $y = P(x)$ for $x > 0$. [1]

Hence find the range of values of x for which the petrol consumption is at most 90 millilitres per kilometre. [1]

(b) At any time t seconds, a tank is being filled with fuel at a rate given by

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 0.15\pi\sqrt{\pi t + 1},$$

where V is the volume of fuel in cm^3. Given that the tank is empty initially, find the amount of fuel in the tank after 1 minute, correct to 3 decimal places. [3]

The tank is shaped such that when the petrol in it is at a height of h cm, the volume of petrol, V, is given by

$$V = \pi h^3.$$

Find the rate of change of h after 1 minute. [5]

Section B: Statistics [60 marks]

7 Durians are sold by weight at a fruit stall. The popular variety, Black Pearl, sells at \$21 per kg. It is found that the probability that a randomly chosen Black Pearl durian is heavier than 2 kg is 0.322, and that the probability that a randomly chosen Black Pearl durian costs less than \$20 is 0.215. Assuming that the weights of Black Pearl durians follow a normal distribution, find the mean and standard deviation of the weight of a randomly chosen Black Pearl durian. [5]

8 In a game of penalty kicks, a player is given three attempts at scoring. Once the player scores, he wins and the game ends. Henry, who has a 0.7 chance of scoring on any penalty kick, plays the game.

Draw a probability tree diagram to illustrate one such game. [1]

Find the probability that in one game, Henry

(a) scores on the second attempt, [1]

(b) made two attempts, given that he wins. [2]

In 3 such games, find the probability that Henry scores on the first attempt in exactly one game, and on the second attempt in exactly one game. [2]

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9 A school is asked to send a delegation of 7 students to attend the opening ceremony of the Asian Youth Games. They are chosen from 8 swimmers, 5 basketball players and 5 tennis players where no student plays more than one game.
How many different delegations can be formed? [1]
One of the swimmers is the brother of a basketball player. How many different delegations can be formed which include exactly one of the two brothers? [2]
Find the probability that the delegation consists of at least 2 students from each sport. [3]

10 In a wet market, tuna steak and leather jacket steak are sold by mass. The mass, in kg, of tuna steak and leather jacket steak follow independent normal distributions with means and standard deviations as shown in the table below.

	Mean mass	Standard deviation
Tuna steak	1.2	0.3
Leather jacket steak	1.6	0.2

(i) Find the probability that the average mass of one randomly chosen leather jacket steak and two randomly chosen tuna steaks is less than 1.5 kg. [3]

Tuna steaks cost \$17 per kg and leather jacket steaks \$10 per kg.

(ii) Find the probability that a randomly chosen tuna steak costs more than \$22.50 and a randomly chosen leather jacket steak costs more than \$16.50. [2]

(iii) Find the probability that the total cost of a randomly chosen tuna steak and a randomly chosen leather jacket steak is more than \$39. [3]

(iv) Explain why your answer to **(ii)** is smaller than your answer to **(iii)**. [1]

11 A biased cube with exactly one face painted red is thrown n times. Denoting the number of times the red face appears by X, it is found that $\mathrm{E}(X)=\frac{40}{7}$ and $\mathrm{Var}(X)=\frac{200}{49}$.
Find the value of n and hence find the probability that more than one quarter of the throws showed the red face. [5]
Each person in a large group of N people is asked to throw the same cube n times. Using a suitable approximation, determine the least value of N so that there is a probability of more than 0.9 that the mean number of red faces obtained per person is less than 6. [5]

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12 Seven primary school boys took the standing board jump test. The weight, w kg, of each boy and the distance he jumped on the test, x cm, are given in the table below.

w (kg)	42	30	34	37	40	55	45
x (cm)	138	157	158	152	148	126	136

(i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]

(ii) Find the product moment correlation coefficient and comment on its value in the context of the question. [2]

(iii) Find the equation of the least squares regression line of x on w in the form $x = a + bw$, leaving the values of a and b correct to 5 significant figures.
Give an interpretation of the value of b in the context of the question. [2]

(iv) Use the equation of your least squares regression line to calculate an estimate for the standing board jump distance of a boy who weighs

(a) 35 kg,

(b) 15 kg.

Comment on the reliability of your answers. [3]

(v) Aaron also took the test, but it was found that his standing broad jump result was not recorded. After including his weight and the distance he jumped on the test, a new least squares regression line of x on w is calculated to be $x = 202.98 - 1.4249w$.
Given that Aaron weighs 39 kg, find the distance he jumped on the test. [3]

13 **(a)** A manager claims that a cup of coffee brewed by a particular barista contains at least 35 ml of coffee on average. A random sample of 80 cups of coffee brewed by the barista is examined and the quantity x ml of espresso coffee in each cup is measured. The results are summarized by $\sum(x-35) = -40$ and $\sum(x-35)^2 = 950$.

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Suggest a reason why, in this context, the given data is summarised in terms of $(x-35)$ rather than x. [1]

(iii) Test at the 10% significance level whether the manager's claim is valid. [5]

(b) A product designer claims that a new coffee machine brews coffee that contains 35 ml of coffee on average. The variance of the quantity of coffee in each cup is known to be 10.1 ml^2. A random sample of 80 cups of coffee made by the machine is measured. A test at the 10% significance level revealed that the product designer's claim that each cup of coffee is 35 ml on average is valid.
Find the range of values of the mean quantity of coffee in this sample, giving your answer correct to 3 decimal places. [4]

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2017 H1 Prelim Solutions

1	Let the time he spend making calls within Canada, the Unites States and Mexico be x, y and z (in min). $0.28x+0.30y+0.84z=\$90$ $2y=z$ $x+z=120$ $x=40.56,\ \ y=39.71,\ \ z=79.44$ Total bill by Sunhub $=0.38(x-10)+0.4(y-10)+0.94(z-10)$ $=88.77$ Sunhub is cheaper.
2	$\int_{-1}^{1}\left(e-e^{-x}\right)^2\mathrm{d}x=\int_{-1}^{1}e^2-2e^{1-x}+e^{-2x}\mathrm{d}x$ $=\left[e^2x+2e^{1-x}-\frac{1}{2}e^{-2x}\right]_{-1}^{1}$ $=\left(e^2+2-\frac{1}{2}e^{-2}\right)-\left(-e^2+2e^2-\frac{1}{2}e^2\right)$ $=2+\frac{1}{2}\left(e^2-e^{-2}\right)$
3(i)	$A=7800\left(1+\frac{3.2}{100(12)}\right)^{80}=9652.077\approx 9652.08$ 10 000 – 9652.08 = 347.92
(ii)	$10000=7800(1+\frac{3.2}{100})^t$ $\frac{50}{39}=(\frac{129}{125})^t$ $t=\frac{\ln 1.282}{\ln 1.032}=7.888\approx 8$ It will take 8 years.
4	Let h be height of matchbox. $V=x(1.5x)h=30\Rightarrow h=\frac{30}{1.5x^2}=\frac{20}{x^2}$ $A=3xy+2xh+4yh$ $=3x(1.5x)+2xh+4(1.5x)h$ $=4.5x^2+8xh$ $=4.5x^2+8x(\frac{20}{x^2})$ $=4.5x^2+\frac{160}{x}$ $\frac{\mathrm{d}A}{\mathrm{d}x}=9x-\frac{160}{x^2}=0$ $x^3=\frac{160}{9}\Rightarrow x=\sqrt[3]{\frac{160}{9}}\approx 2.6099$ $y=1.5\times\sqrt[3]{\frac{160}{9}}=\sqrt[3]{\frac{27}{8}}\times\sqrt[3]{\frac{160}{9}}=\sqrt[3]{60}$ 

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$$h=\frac{20}{\left(\sqrt[3]{\frac{160}{9}}\right)^2}=20\left(\frac{9}{160}\right)^{\frac{2}{3}}$$

$\frac{\mathrm{d}^2A}{\mathrm{d}x^2}=9+\frac{320}{x^3}>0$ for all x $\therefore$ Min A

x	2.4	2.61	2.8
$\frac{\mathrm{d}A}{\mathrm{d}x}$	-6.1778	0	4.7918

5(i)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[x-\ln(x^2+1)\right]=1-\frac{2x}{x^2+1}$$

$$y=x-\ln(x^2+1)$$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow 1-\frac{2x}{x^2+1}=0$$

$$\frac{2x}{x^2+1}=1$$

$$x^2-2x+1=0$$

$$(x-1)^2=0$$

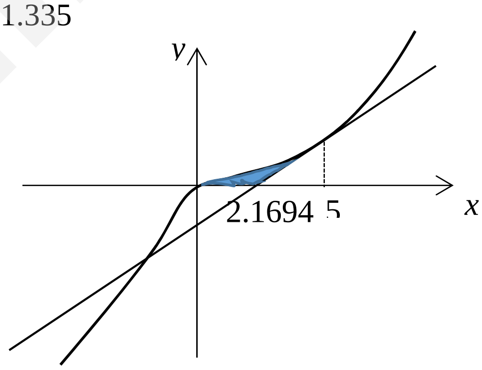
$$\therefore x=1,\ y=1-\ln 2$$

x	1^- 0.5(0.9)	1	1^+ 1.5(1.1)
$\frac{\mathrm{d}y}{\mathrm{d}x}$	0.2(0.0055)	0	0.0769(0.0045)

$\therefore$(1, 1 – ln2) is a stationary point of inflexion.

(ii)

Equation of tangent at x = 5 is
$y=0.61538x-1.335$



when y = 0, x = 2.1694
when x = 5, y = 1.7419

$\int_0^5 y\ \mathrm{d}x$ – Area of triangle

$=\int_0^5[x-\ln(x^2+1)]\ \mathrm{d}x-0.5(5-2.1694)(1.7419)$
$=3.4627-2.4653$
$=0.99739\approx 0.997$ (3 s.f.)

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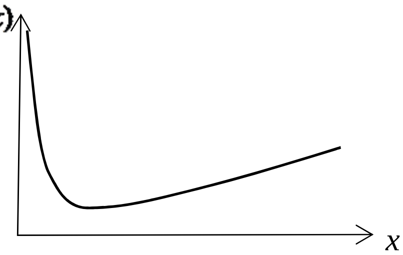
6(a)	$P'(x) = -\dfrac{2500}{x^2} + \dfrac{2}{3} = 0$ $x^2 = 3750$ $\therefore x = 25\sqrt{6}$  Min point (61.237, 81.65) Optimal speed = 81.7 km/h When $P(x) \le 90$, $39.1 \le x \le 95.9$.
6(b)	$\dfrac{dV}{dt} = 0.15\pi\sqrt{\pi t + 1}$ $V = \int 0.15\pi\sqrt{\pi t+1}\ dt = \dfrac{0.15\pi}{1.5\pi}(\pi t+1)^{1.5} + C = 0.1(\pi t+1)^{1.5} + C$ When t = 0, V = 0 $\therefore C = -0.1$ $\therefore V = 0.1(60\pi+1)^{1.5} - 0.1 = 260.755$ OR $V = \int_0^{60} 0.15\pi\sqrt{\pi t+1}\ dt = 260.755$ fr GC $\therefore$ After 1 min, amount of fuel in the tank is 260.755 litres. Vol of fuel = 260.755 $\Rightarrow \pi h^3 = 260.755$ $\therefore h = 4.36208$ $V = \pi(h)^2 h = \pi h^3$ $\dfrac{dV}{dh} = 3\pi h^2$ $\dfrac{dh}{dt} = \dfrac{dh}{dV} \times \dfrac{dV}{dt}$ $= \dfrac{1}{3\pi h^2} \times 0.15\pi\sqrt{\pi(60)+1}$ when $t = 60$ $= 0.036173 \approx 0.0362$ cm/s
7	Let M be the mass of a randomly chosen Black Pearl durian, and S be the cost of a randomly chosen Black Pearl durian. $S = 21M$ $M \sim N(\mu,\ \sigma^2)$ $S \sim N\left(21\mu,\ (21\sigma)^2\right)$ $P(M > 2) = 0.322$ $P(M < 2) = 0.678$ $P(Z < \dfrac{2-\mu}{\sigma}) = 0.678$ $\dfrac{2-\mu}{\sigma} = 0.46211$

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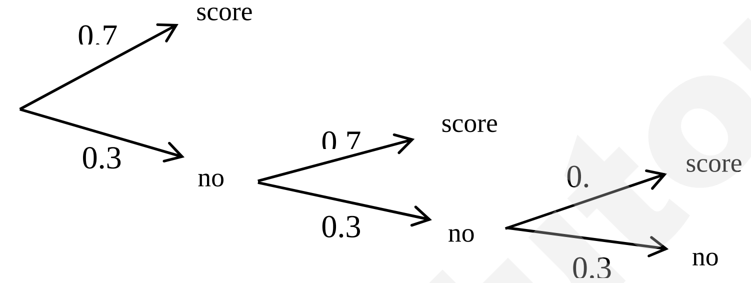
	$P(S<20)=0.215$ $P(Z<\frac{20-21\mu}{21\sigma})=0.215$ $\frac{20-21\mu}{21\sigma}=-0.78919$ Therefore we have two simultaneous equations: $\mu+0.46211\sigma=2$ ----(1) $21\mu-16.573\sigma=20$ -----(2) Solving (1) and (2) $\mu=1.6131=1.61$ (3sf) $\sigma=0.83722=0.837$ (3sf)
8	
(a)	0.3×0.7
(b)	$\frac{0.3\times0.7}{1-\text{P(not scoring)}}=\frac{0.3\times0.7}{1-0.3^3}$ or $\frac{0.3\times0.7}{0.3+0.3\times0.7+0.3^2\times0.7}=\frac{0.21}{0.973}=\frac{30}{139}$
(c)	$3!\times0.7\times0.21\times(1-0.7-0.21)=0.07938\approx0.0794$
9	${}^{18}C_7=31\ 824$ ${}^{2}C_1\,{}^{16}C_6=2\times8\ 008=16\ 016$ Probability = P(3 S + 2 B + 2 T) + P(2 S + 3 B + 2 T) + P(2 S + 2 B + 3 T) $=\frac{{}^8C_3\,{}^5C_2\,{}^5C_2+{}^8C_2\,{}^5C_3\,{}^5C_2+{}^8C_2\,{}^5C_2\,{}^5C_3}{{}^{18}C_7}=\frac{11200}{31\ 824}=\frac{700}{1989}\approx0.352$ (3 s.f.)
10(i)	Let L be the mass of a randomly chosen leather jacket steak. Let T be the mass of a randomly chosen tuna steak. Let A be the average mass of 1 randomly chosen leather jacket and 2 randomly chosen tuna steak. $A=\frac{L+T_1+T_2}{3}$ $\mathrm{E}(A)=E\left(\frac{L+T_1+T_2}{3}\right)$ $=\frac{1}{3}[\mathrm{E}(L)+2\mathrm{E}(T)]=\frac{1}{3}(1.6+2\times1.2)=\frac{4}{3}$

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	$\text{Var}(A) = \text{Var}\left(\frac{L_1 + T_1 + T_2}{3}\right)$ $= \frac{1}{3^2}\left[\text{Var}(L) + 2\text{Var}(T)\right]$ $= \frac{1}{9}\left(0.2^2 + 2\times 0.3^2\right) = \frac{11}{450}$ $A \sim N\left(\frac{4}{3},\ \frac{11}{450}\right)$ $P(A<1.5) = 0.857$ (3sf)
(ii)	Let X be the cost of a randomly chosen tuna steak, and Y be the cost of a randomly chosen leather jacket steak. $X = 17T$ $Y = 10L$ $X \sim N\left(17\times 1.2,\ (17\times 0.3)^2\right) = \left(20.4,\ 26.01\right)$ $Y \sim N\left(10\times 1.6,\ (10\times 0.2)^2\right) = \left(16, 4\right)$ $P(X > 22.50) \times P(Y > 16.50)$ = (0.34026)(0.40130) = 0.13654 = 0.137 (3sf)
(iii)	$X + Y \sim N\left(20.4+16,\ 26.01+4\right) = \left(36.4,\ 30.01\right)$ $P(X + Y > 39) = 0.31753 = 0.318$ (3sf)
(iv)	because the event in (ii) is a subset of the event in (iii), <u>i.e., a tuna steak cost \$24 and a leather jacket steak cost \$16 is not included in (ii) but is included in (iii).</u>
11	Let p be the probability of obtaining a red in a throw. Then $X \sim \text{B}(n, p)$ $\text{E}(X) = np = \frac{40}{7}$ and $\text{Var}(X) = np(1-p) = \frac{200}{49}$ $\Rightarrow \frac{40}{7}(1-p) = \frac{200}{49}$ $1-p = \frac{5}{7} \Rightarrow p = \frac{2}{7}$ $\therefore np = \frac{40}{7}$ $n = \frac{40}{7}\times\frac{7}{2} = 20$ $P(X > 5) = 1 - P(X \le 5) \approx 0.528$ (3 s.f.)
	Let $\overline{X}$ denotes the mean number of reds obtained per person. $\overline{X} = \frac{X_1 + X_2 + ... + X_N}{N}$

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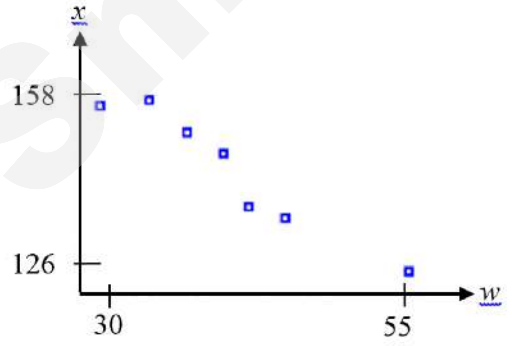
	Since N is large, $\overline{X} \sim N\left(\frac{40}{7}, \frac{200}{49N}\right)$ approximately by Central limit Theorem. Let $Z = \dfrac{\overline{X} - \frac{40}{7}}{\sqrt{\frac{200}{49N}}} \sim N(0, 1)$ $\mathrm{P}(\overline{X} < 6) > 0.9$ $\mathrm{P}\left(Z \le \dfrac{6 - \frac{40}{7}}{\sqrt{\frac{200}{49N}}} \right) > 0.9$ <u>Method 1:</u> $\dfrac{6 - \frac{40}{7}}{\sqrt{\frac{200}{49N}}} > 1.28155$ $N > \dfrac{200}{49}\left(\dfrac{1.28155}{6 - \frac{40}{7}}\right)^2$ $N > 82.11$ The least value of N is 83. **OR**
	Method 2: From GC: $N = 82, \quad \mathrm{P}(\overline{X} < 6) = 0.8998$ $N = 83, \quad \mathrm{P}(\overline{X} < 6) = 0.9012$ The least value of N is 83
12(i)	
(ii)	$r = -0.962$ (3s.f.) There is a <u>strong negative linear relationship</u> between the weight of a boy and the distance of the standing board jump that he can make, i.e., as the weight of a boy increases, the distance of his standing board jump decreases.
(iii)	From G.C., $x = 202.23 - 1.4156w$ (5 s.f.) Meaning of b:

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	$b = -1.4156$ means that <u>1 unit (i.e., kg) increase</u> in the weight of a boy (w) will mean a <u>decrease of 1.4156 units (i.e., cm)</u> in the distance of his standing board jump (x).
(iv)	When $w = 35$ kg, $x = 153$cm (3sf) Since the value of <u>r is close to 1</u> and <u>w is within the range of data</u>, i.e., $30 \le w \le 55$, the estimate is <u>reliable.</u> When $w = 15$ kg, $x = 181$cm (3sf) Since <u>w is outside the given range of data</u>, i.e., $30 \le w \le 55$, the linear relation may no longer hold, therefore the estimate is <u>unreliable</u>.
(v)	Let the distance of Aaron's standing board jump be d. $\bar{w} = \frac{322}{8} = 40.25$ Since $(\bar{w}, \bar{x})$ lies on the regression line, sub $\bar{w}$ to get $\bar{x}$ $\bar{x} = 202.98 - 1.4249(40.25)$ $\bar{x} = 145.6$ $\bar{x} = \frac{1015 + d}{8}$ $145.6 = \frac{1015 + d}{8}$ $d = 150$ cm (3s.f.)
13 (a) (i)	Unbiased estimate of the population mean, $\bar{x} = 35 + \frac{\sum(x-35)}{80} = 35 + \frac{-40}{80} = 34.5$ Unbiased estimate of the population variance, $s^2 = \frac{1}{80-1}\left[\sum(x-35)^2 - \frac{\left(\sum(x-35)\right)^2}{80}\right]$ $= \frac{1}{79}\left[950 - \frac{(-40)^2}{80}\right] = 11.772 = 11.8$ (3sf)
(ii)	"Keeping the recorded values small since they are around 35 ml" or "Giving an indication of the variations around the hypothesized mean of 35 ml".
(iii)	To test H_0: $\mu = 35$ against H_1: $\mu < 35$ 1-tail Z-test at the 10% significance level. Since $n = 80$ is large, by Central Limit Theorem, Under Ho, $\bar{X} \sim N\left(35, \frac{11.772}{80}\right)$ approximately. From GC, p-value = 0.096213

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	Since p-value < 0.10, reject H_0 We conclude that there is <u>sufficient evidence at the 10% level of significance</u> that average cup of single-shot expresso coffee is less than 35 ml, the manager's claim is not valid.
(b)	To test H_0: $\mu = 35$ against H_1: $\mu \neq 35$ 2-tail Z-test at the 10% significance level. Since $n = 80$ is large, by Central Limit Theorem, Under Ho, $\bar{X} \sim N\left(35, \frac{10.1}{80}\right)$ approximately. Since the product designer's claim is valid, H_0 is not rejected, $-1.6449 < \dfrac{\bar{x}-35}{\sqrt{\frac{10.1}{80}}} < 1.6449$ Range of sample mean $34.416 \le \bar{x} \le 35.584$ (3 dp)

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Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

MATHEMATICS (Higher 1) — 8865/01

Paper 1

September 2017

3 hours

Additional Materials: Answer Paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your Name, Index Number and Class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

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For teachers' use:

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Total
Score													
Max Score	**4**	**5**	**8**	**10**	**13**	**3**	**6**	**7**	**10**	**9**	**10**	**15**	**100**

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Section A : Pure Mathematics [40 marks]

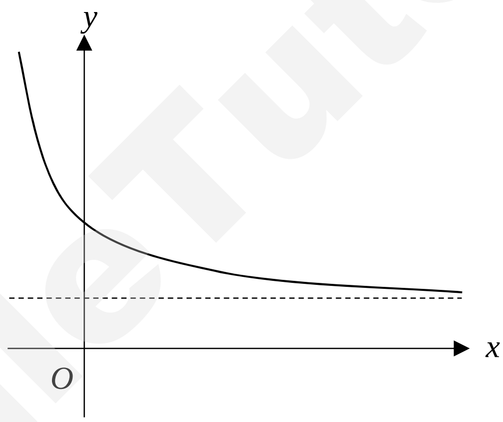
1 Abel, Ben and Caleb travelled from Raffles Place to Bishan using the same route. They hired private transport from different car companies, Car X, Y and Z, respectively. Ben and Caleb had cash vouchers which could be used to offset their final fares. Ben's cash voucher was twice as much in value as Caleb's.

The fare comprises 3 components: A fixed base fare, the distance travelled and the time taken for the journey. The rates of the different companies are shown in the following table.

	Car X	Car Y	Car Z
Base fare ($)	3.20	3.00	3.00
Per kilometre ($)	0.55	0.80	0.45
Per minute ($)	0.29	0	0.20

Abel, Ben and Caleb paid $15.60, $6.60 and $9.40 respectively. Find the distance travelled and the time taken from Raffles Place to Bishan. Assume that the traffic is the same for all 3 journeys. [4]

2



The diagram shows the curve C with equation $y = \ln 2 + 2^{-x}$.

(i) State the exact equation of the asymptote and the exact coordinates of the point of intersection with the y-axis. [2]

(ii) Find the x-coordinate of the point of intersection of C and the line $y = 1$, giving your answer correct to 4 decimal places. [1]

(iii) Write down as an integral an expression for the area of the region bounded by C, $y = 1$ and the y-axis and evaluate this integral. [2]

3 **(a)** Differentiate $\dfrac{\pi^2}{\sqrt{(3-\pi x)}}$. [2]

(b) Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{-x+2\ln x}\right) = x\mathrm{e}^{-x}(2-x)$. [3]

Hence find the exact solution of $\displaystyle\int_1^{\mathrm{e}} x\mathrm{e}^{-x+a}(x-2)\ \mathrm{d}x$ in terms of the constant a. [3]

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4 [It is given that the volume of a cone is $\frac{1}{3}\pi r^2 h$ where h is the vertical height and r the radius of the circular base of the cone.]

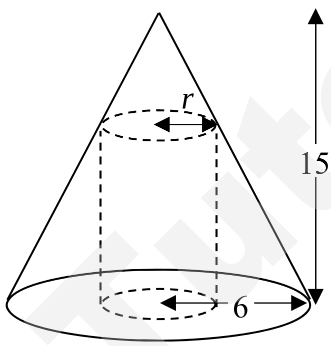
A hollow cone has a base radius 6 cm and height 15 cm. It is made of material with negligible thickness.

(a) The cone is inverted. Initially, the cone is empty and water is poured into it at a rate of 8 cm^3s^{-1}. The depth of water in the cone is x cm at time t seconds.

(i) Show that the volume V cm^3 of the water in the cone is given by $V = \frac{4}{75}\pi x^3$. [2]

(ii) Find the exact rate of increase of the depth of water at the instant when the depth is 5 cm. [3]

(b)



The same cone is now placed in an upright position and a solid cylinder is to be inscribed in the cone (see diagram). Show that the total surface area A cm^2 of the cylinder of radius r cm is given by $A = 30\pi r - 3\pi r^2$, and hence find the exact value of the maximum A. [5]

5 At time 0000, a physicist observes the behaviour of 2 particles E_1 and E_2 for a period of 20 minutes. E_2 is stationary for a few minutes before it starts moving. The speed v, in m/min after t min, of E_1 and E_2 satisfies the equation $v = \frac{1}{3}t$ and $v = \sqrt{(2t-5)}$ respectively.

(i) On the same diagram, sketch the speed-time graphs of E_1 and E_2 during the period of observation of 20 minutes, stating the exact coordinates of any points of intersection with the axes and points of intersection of the two graphs. [2]

(ii) State the duration for which E_2 moves faster than E_1. [1]

(iii) The distance travelled is represented by the area under a speed-time graph. Determine which particle travels a longer distance for the period of observation of 20 minutes. [4]

(iv) The derivative of speed with respect to time is known as acceleration. Without using a calculator, find the time at which E_1 and E_2 have the same acceleration. [3]

The speed of a third particle E_3 satisfies the equation $v = \sqrt{\left(a-(t-5)^2\right)}$, where a is a positive constant. Find the set of values of a, given that E_1 and E_3 will not travel at the same speed at any time t. Show your working clearly. [3]

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Section B : Statistics [60 marks]

6 A sample of 5 people is chosen from a village of large population.

(i) The number of people in the sample who are underweight is denoted by X. State, in context, the assumption required for X to be well modelled by a binomial distribution. [1]

(ii) On average, the proportion of people in the village who are underweight is p. A total of 1000 samples of 5 people are chosen at random from the village and the results are shown in the table below.

x	0	1	2	3	4	5
Number of samples	93	252	349	220	75	11

Using the above results, find $\bar{x}$. Hence estimate the value of p. [2]

7 The Tan family has 2 children while the Wong family has 3 children. The children, together with both their parents, catch a movie at the cinema. At the ticket counter, they realise only 2 rows of consecutive empty seats are left, where Row L has 5 seats and Row M has 4 seats.

Find the number of different possible arrangements if

(i) there are no restrictions, [1]

(ii) the Tan siblings must not sit together. [3]

The 9 of them are randomly seated. Find the probability that Tan siblings sit between their parents given that the Wong family takes Row L. [2]

8 The insurance company Adiva classifies 10% of their car policy holders as 'low risk', 60% as 'average risk' and 30% as 'high risk'. Its statistical database has shown that of those classified as 'low risk', 'average risk' and 'high risk', 1%, 15% and 25% are involved in at least one accident respectively.

Find the probability that a randomly chosen policy holder

(i) is not involved in any accident if the policy holder is classified as "average risk", [1]

(ii) is not involved in any accident, [2]

(iii) is classified as "low risk" if the policy holder is involved in at least one accident. [2]

Two policy holders are chosen at random.

(iv) Find the probability that one is not involved in any accident while the other is involved in at least one accident. [2]

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9 Students pursuing a particular university course are required to take m modules in each semester. At the end of each module, the students have to take an examination which comprises n questions. It may be assumed that for each examination, the number of questions answered correctly by a randomly chosen student follows a binomial distribution $B(n, 0.6)$ with variance 24.

(i) Verify that $n = 100$. [1]

To pass a module, a student must answer at least 50 questions correctly in the examination.

(ii) Find the most probable number of questions that a randomly chosen student answers correctly in an examination. [2]

(iii) Show that the probability that a randomly chosen student passes a module is 0.983. [1]

(iv) Given that a randomly chosen student is at most 90.4% confident of passing all his modules in a semester, find the least value of m. [3]

Forty students in this course are randomly selected and their marks for a particular examination are recorded. Use a suitable approximation to find the probability that on average at most 58 questions are answered correctly. [3]

10 The students in a college are separated into two groups of comparable sizes, Group X and Group Y. The marks for their Mathematics examination are normally distributed with means and variances as shown in the following table.

	Mean	Variance
Group X	55	20
Group Y	34	25

(i) Find, to 1 decimal place, the maximum passing mark if at least 60% of students from Group Y are to pass the examination. [2]

(ii) Find the probability that the total marks of 4 students from Group Y is less than three times the mark of a student from Group X. State clearly the mean and variance of the distribution you use in your calculation. [3]

(iii) The marks of 40 students, with 20 each randomly selected from Group X and Group Y, are used to compute a new mean score, $\overline{M}$. Given that $P(-k < \overline{M} - 44.5 < k) = 0.9545$, find the value of k. [3]

State a necessary assumption for your calculations to hold in parts **(ii)** and **(iii)**. [1]

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11 A retail manager of a large electrical appliances store wants to study the relationship between advertising expenditure, x hundred dollars, and the sales of their refrigerators, y thousand dollars, on a monthly basis. The data is shown in the following table.

x	5	8	12	16	18	20	23
y	12.5	12.9	14.6	15.8	17.0	19.3	20.8

(i) Draw a scatter diagram to illustrate the above data and calculate the product moment correlation coefficient between x and y. [3]

(ii) Find the equation of the regression line of y on x, in the form $y = a + bx$. Sketch this line on your scatter diagram. [2]

(iii) Use a suitable regression line to estimate the advertising expenditure of a particular month if \$15000 was made from the sale of refrigerators. Comment on the reliability of this estimate. [3]

(iv) Explain the meaning of b in the context of this question. [1]

(v) There is an additional expenditure of \$200 for every month of advertising. Without any further calculations, state any change you would expect in the values of the constant b found in part (ii). [1]

12 (a) The centre thickness, X micrometres, of soft contact lenses from a certain company is a normally distributed random variable with mean μ. The company claims that the centre thickness of their lenses is at most 30 micrometres. A random sample of 60 contact lenses is measured. The results are summarised as follows.

$$\Sigma(x-30) = 24 \qquad \Sigma(x-30)^2 = 144$$

Test, at the $2\frac{1}{2}\%$ significance level, whether the claim is justified. [6]

Explain, in the context of the question, the meaning of "at the $2\frac{1}{2}\%$ significance level". [1]

(b) The company Snatch provides a ride-hailing service comprising taxis and private cars in Singapore. Snatch claims that the mean waiting time for a passenger from the booking time to the time of the vehicle's arrival is 7 minutes. A random sample of 30 passengers' waiting times is obtained and the standard deviation of the sample is 2 minutes. A hypothesis test conducted concludes that there is sufficient evidence at the 1% significance level to reject the claim.

(i) State appropriate hypotheses and the distribution of the test statistic used. [3]

(ii) Find the range of values of the sample mean waiting time, $\overline{t}$. [3]

(iii) A hypothesis test is conducted at the 1% significance level whether the mean waiting time of passengers is more than 7 minutes. Using the existing sample, deduce the conclusion of this test if the sample mean waiting time is more than 7 minutes. [2]

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Qn	Suggested Solution
1	Let a be the distance in km, b the time taken in minutes, c be the value of promo discount that caleb had. $\begin{cases} 3.2+0.55a+0.29b=15.6 \\ 3+0.8a=6.6+2c \\ 3+0.45a+0.2b=9.4+c \end{cases}$ $\begin{cases} 0.55a+0.29b=12.4 \\ 0.8a-2c=3.6 \\ 0.45a+0.2b-c=6.4 \end{cases}$ Using GC, $a=12, b=20, c=3$ Hence the time taken was <u>20 minutes</u> and the distance travelled was <u>12 km</u> .
2i	Equation of asymptote: $y = \ln 2$ Coordinates of point of intersection with y-axis: (0, 1 + ln 2)
ii	Using GC, x-coordinate = 1.70438 = 1.7044 (4 d.p.)
iii	Area $=\int_0^{1.70438}\left(\ln 2+2^{-x}-1\right)\mathrm{d}x$ $=0.477006$ $=0.477 \text{ unit}^2$
3(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\pi^2}{\sqrt{(3-\pi x)}}\right)$ $=\pi^2\left(-\frac{1}{2}\right)(3-\pi x)^{-\frac{3}{2}}(-\pi)$ $=\frac{\pi^3}{2}(3-\pi x)^{-\frac{3}{2}}$ $=\frac{\pi^3}{2\sqrt{(3-\pi x)^3}}$
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{-x+2\ln x}\right)$ $=(\mathrm{e}^{-x+2\ln x})(-1+\frac{2}{x})$ $=(\mathrm{e}^{-x})(\mathrm{e}^{\ln x^2})(-1+\frac{2}{x})$ $=x^2\mathrm{e}^{-x}(-1+\frac{2}{x})$ $=x\mathrm{e}^{-x}(2-x)$ (shown) Alternatively (use of product rule) $\mathrm{e}^{-x+2\ln x}=\mathrm{e}^{-x}.\,\mathrm{e}^{2\ln x}=\mathrm{e}^{-x}.\,\mathrm{e}^{\ln x^2}=x$ $\therefore \frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{-x+2\ln x}\right)$ $=\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2\mathrm{e}^{-x}\right)$ $=\mathrm{e}^{-x}(2x)+x^2\left(-\mathrm{e}^{-x}\right)$ $=x\mathrm{e}^{-x}(2-x)$

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	$\int_1^{e} x\mathrm{e}^{-x+a}(x-2)\,\mathrm{d}x$ $=-\mathrm{e}^{a}\int_1^{e} x\mathrm{e}^{-x}(2-x)\,\mathrm{d}x$ $=-\mathrm{e}^{a}\left[\mathrm{e}^{-x+2\ln x}\right]_1^{e}$ $=-\mathrm{e}^{a}\left[\mathrm{e}^{-e+2\ln e}-\mathrm{e}^{-1+2\ln 1}\right]$ $=-\mathrm{e}^{a}\left[\mathrm{e}^{-e+2}-\mathrm{e}^{-1}\right]$ $=\mathrm{e}^{a-1}-\mathrm{e}^{a-e+2}$
4(i)	Let r be the radius of water surface area Using similar triangles, $\frac{r}{6}=\frac{x}{15}\Rightarrow r=\frac{2}{5}x$ Volume of water, $V=\frac{1}{3}\pi\left(\frac{2x}{5}\right)^2 x$ $=\frac{4}{75}\pi x^3$ (shown)
(ii)	Given $\frac{\mathrm{d}V}{\mathrm{d}t}=8$ From part (i), $\frac{\mathrm{d}V}{\mathrm{d}x}=\frac{4}{75}(3)\pi x^2=\frac{4\pi x^2}{25}$ Using Chain Rule, $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{\mathrm{d}x}{\mathrm{d}V}\cdot\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{25}{4\pi x^2}\times 8=\frac{50}{\pi x^2}$ When $x=5$, $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{50}{\pi(5)^2}=\frac{2}{\pi}$ cm/s The rate of increase of the depth of water is $\frac{2}{\pi}$ cm/s when x is 5 cm.
b	Let the height of the cylinder be h. By similar triangles, $\frac{r}{6}=\frac{15-h}{15}\Rightarrow h=15-\frac{5}{2}r$

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Total surface area of the cylinder, $A = 2\pi r^2 + 2\pi rh$

$$= 2\pi r^2 + 2\pi r\left(15 - \frac{5}{2}r\right)$$

$$= 30\pi r - 3\pi r^2 \text{ (shown)}$$

$$\frac{dA}{dr} = 30\pi - 6\pi r$$

$$\frac{dA}{dr} = 0$$

$$30\pi - 6\pi r = 0 \Rightarrow r = 5$$

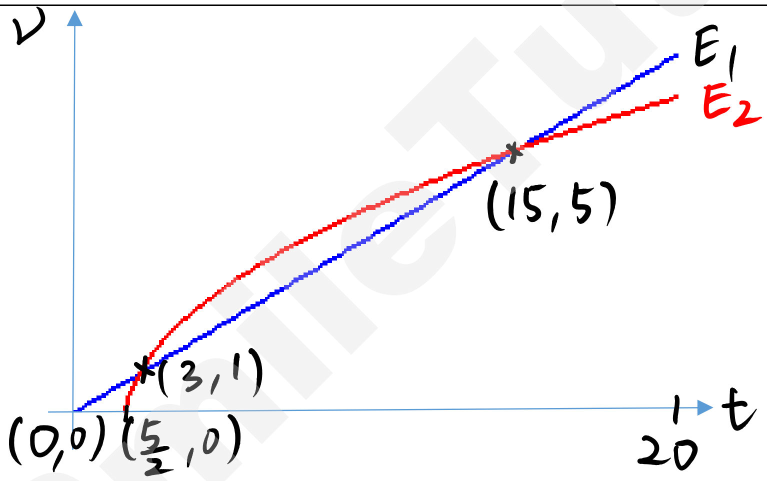
$$\frac{d^2A}{dr^2} = -6\pi < 0$$

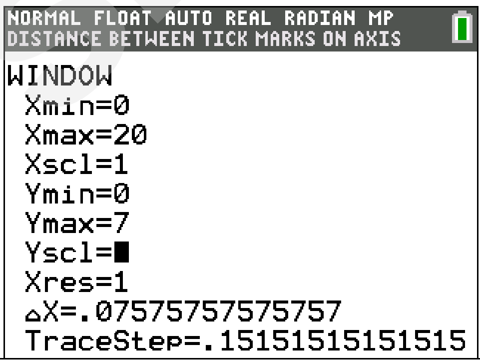
Total surface area is a maximum when $r = 5$.

$\therefore$ maximum value of the total surface area of the cylinder

$= 30\pi(5) - 3\pi(25) = 75\pi \text{ cm}^2$

5i





ii Points of intersection are at $t = 3$ and $t = 15$

Hence duration = $15 - 3 =$ <u>12 minutes</u>

iii Let d_1 and d_2 be the distance travelled by E_1 and E_2 respectively.

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	$\mathrm{d}_1 = \frac{1}{2}(20)(\frac{1}{3}(20)) = \frac{200}{3}\,\mathrm{m}$ (or 66.7 m) $\mathrm{d}_2 = \int_{\frac{5}{2}}^{20} \sqrt{(2t-5)}\,\mathrm{d}t$ $= \left[\frac{2}{3}\frac{(2t-5)^{\frac{3}{2}}}{2} \right]_{\frac{5}{2}}^{20}$ $= \frac{1}{3}\sqrt{35^3}\,\mathrm{m}$ (or 69.0 m) Since $\mathrm{d}_2 > \mathrm{d}_1$, E_2 travelled a longer distance.
iv	Let v_1 and v_2 denote the speeds of E_1 and E_2. To have the same acceleration, $\frac{\mathrm{d}v_1}{\mathrm{d}t} = \frac{\mathrm{d}v_2}{\mathrm{d}t}$ $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{3}t\right) = \frac{\mathrm{d}}{\mathrm{d}t}\left(\sqrt{(2t-5)}\right)$ $\frac{1}{3} = \frac{1}{2}(2t-5)^{-\frac{1}{2}}(2)$ $\frac{1}{3} = \frac{1}{\sqrt{(2t-5)}}$ $2t-5=9$ $t=7$ Hence the time at which they have the same acceleration is <u>00 07</u>
	Suppose the speeds of both particles is the same, ie. $v_1 = v_3$ $\frac{1}{3}t = \sqrt{(a-(t-5)^2)}$ $\frac{1}{9}t^2 = a-(t-5)^2$ $\frac{1}{9}t^2 = a - t^2 + 10t - 25$ $\frac{10}{9}t^2 - 10t + (25-a) = 0$ For the velocities to be always different, $10^2 - 4(\frac{10}{9})(25-a) < 0$ $100 - \frac{1000}{9} + \frac{40}{9}a < 0$ $a < \frac{5}{2}$ since a is positive, set of values of $a = \{a \in \mathbb{R}^+ : a < \frac{5}{2}\}$

6i	Weights of people in the village are independent of each other.
(ii)	$\bar{x} = 1.965$ (from GC) Since $n=5$, $np \approx 1.965 \Rightarrow p \approx 0.393$

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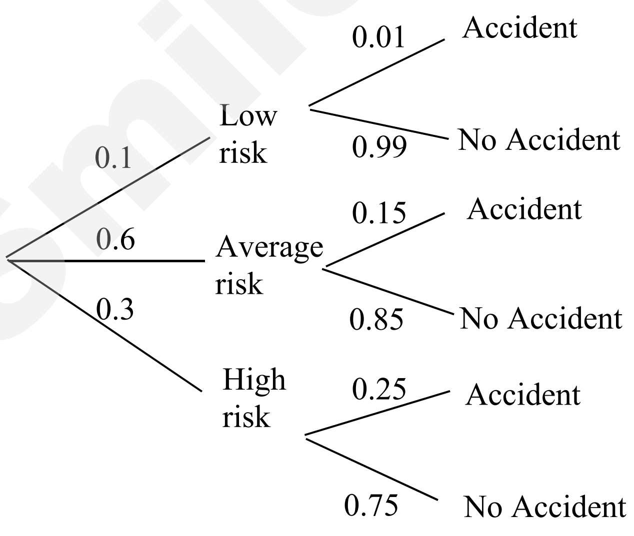
7(i)	No. of ways = 9! = 362880
ii)	No. of ways $=9!-7\times2\times7!$ $=9!-70560$ $=292\ 320$
	Probability $=\dfrac{2\times2}{4!}$ $=\dfrac{1}{6}$ Probability $=\text{P(Tan siblings sit between parents}\mid\text{Wong family takes Row L)}$ $=\dfrac{\text{P(Tan siblings sit between parents and Wong family takes Row L)}}{\text{P(Wong family takes Row L)}}$ $=\dfrac{\frac{5!\times2\times2}{9!}}{\frac{5!4!}{9!}}$ $=\frac{1}{6}$

8 **(i)**	P(holder is not involved in any accident | the holder is classified as “average” risk) = 0.85
(ii)	 Probability of a randomly chosen policy holder not involved in any car accident = (0.1)(0.99) + (0.6)(0.85) + (0.3)(0.75) = 0.834
(iii)	P(policy holder is “low risk” | has met at least one car accident)

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	$= \dfrac{\text{P(holder is classified as "low" risk and met with at least 1 accident)}}{\text{P(holder meets with at least 1 accident)}}$ $= \dfrac{0.1(0.01)}{1-0.834}$ $= \dfrac{1}{166} = 0.00602$ (3 s.f.)
(iv)	Probability = 2(0.834)(1-0.834) =0.276888 (exact)

9 (i)

Let X denote the no. of questions he can answer correctly out of n.

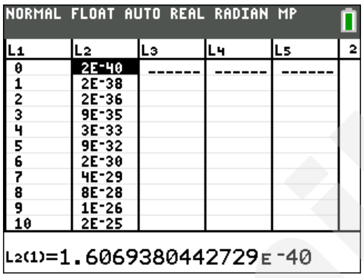
$X \sim \mathrm{B}(n, 0.6)$

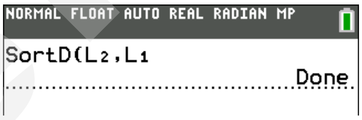
if $n = 100$,

Variance $= npq = 100(0.6)(0.4) = 24$ (verified)

ii

$X \sim \mathrm{B}(100, 0.6)$

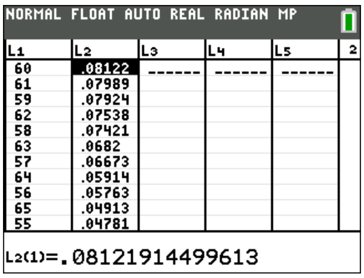




$\mathrm{P}(X = 59) = 0.07924$

$\mathrm{P}(X = 60) = 0.08122$

$\mathrm{P}(X = 61) = 0.07989$



The most probable number of questions answered correctly is 60.

(iii)

Required probability

$= \mathrm{P}(X \geq 50)$

$= 1 - \mathrm{P}(X \leq 49)$

$= 0.98324$ (to 5sf)

$= 0.983$ (to 3sf) (shown)

iv

Let Y denote the no. of exams out of m that he passed.

$Y \sim \mathrm{B}(m, 0.983)$

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	$\mathrm{P}(Y=m)\le 0.904$ $\binom{m}{m}0.983^m(1-0.983)^0\le 0.904$ $0.983^m\le 0.904$ $m\lg 0.983\le \lg 0.904$ $m\ge 5.88621$ least $m=6$
	$X\sim \mathrm{B}(100,0.6)$ $\mathrm{E}(X)=60$ $\mathrm{Var}(X)=24$ By CLT, since $n=40$ is large, $\overline{X}\sim \mathrm{N}\left(60,\frac{24}{40}\right)$ approximately $\mathrm{P}(\overline{X}\le 58)=0.0049117=0.00491$ (3s.f.)

10 (i)	Let Y be the score of Group Y students. $\mathrm{P}(Y\ge a)\ge 0.6$ $\mathrm{P}(Y<a)<0.6$ Thus $a<32.733$ The maximum mark is 32.7
(ii)	$\mathrm{E}\left(Y_1+Y_2+Y_3+Y_4-3X\right)=4\mathrm{E}\left(Y\right)-3\mathrm{E}\left(X\right)=-29$ $\mathrm{Var}\left(Y_1+Y_2+Y_3+Y_4-3X\right)=4\mathrm{Var}\left(Y\right)+9\mathrm{Var}\left(X\right)=280$ $\therefore Y_1+Y_2+Y_3+Y_4-3X\sim \mathrm{N}\left(-29,280\right)$ $\mathrm{P}(Y_1+Y_2+Y_3+Y_4<3X)=\mathrm{P}(Y_1+Y_2+Y_3+Y_4-3X<0)=0.958$
(iii)	$\overline{M}=\frac{X_1+...+X_{20}+Y_1+...+Y_{20}}{40}$ $\mathrm{E}\left(\overline{M}\right)=\frac{20\mathrm{E}(X)+20\mathrm{E}(Y)}{40}=\frac{1}{2}\left(\mathrm{E}(X)+\mathrm{E}(Y)\right)=44.5$ Let $\sigma^2=\mathrm{Var}\left(\overline{M}\right)$ $=\frac{1}{1600}\left(20\mathrm{Var}(X)+20\mathrm{Var}(Y)\right)$ $=\frac{1}{80}\left(\mathrm{Var}(X)+\mathrm{Var}(Y)\right)=0.5625$ $\overline{M}\sim N\left(44.5,0.5625\right)$ Since $\mathrm{P}(-k<\overline{M}-44.5<k)=0.9545$ $\therefore 44.5-k=43.000$ $\Rightarrow k=1.50$ (3 s.f)  **Alternative**

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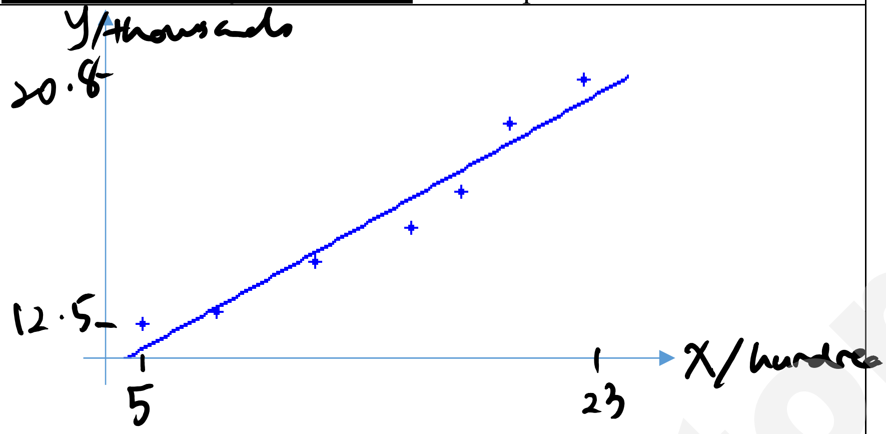
	$\overline{M} \sim N\left(44.5, \sigma^2\right)$ Since $\mathrm{P}(\lvert\overline{M}-44.5\rvert < 2\sigma) = 0.9545$ $\therefore k = 2\sigma = 2\sqrt{0.5625} = 1.50$ (3sf)
	Marks obtained by the students are independent of one another.
11 **i** **ii**	 r = 0.97139 = 0.971 (3 s.f.) The equation of y on x : y = 9.3484 + 0.46531x y = 9.35 + 0.465x (3 s.f.)
iii	Since x is the independent variable, y on x should be used for the estimation. For $y = 15$, $x = 12.146 = 12$ The advertising expenditure is $12,000. This estimate is reliable because : - r is close to 1 which indicates a strong positive linear correlation between x and y. - $y = 15$ is within the given data range (interpolation), $12.5 < y < 20.8$.
iv	b is the gradient of the regression line which indicates that with every $100 spent on advertising in a month, there is an increase of $465 in the sale of refrigerators.
v	There would be no change to b.

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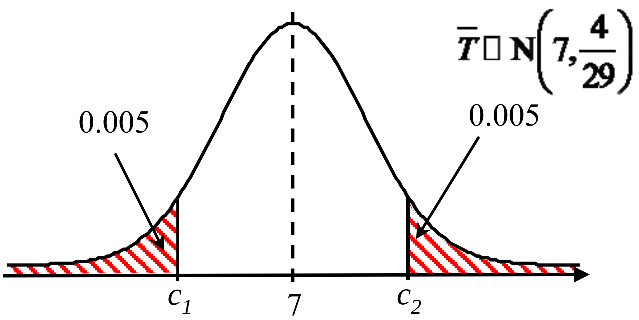
12 **a**	$\bar{x} = \dfrac{\sum(x-30)}{60} + 30 = 30.4$ $s^2 = \dfrac{1}{59}\left[\sum(x-30)^2 - \dfrac{\left(\sum(x-30)\right)^2}{60}\right] = 2.2780$ (5 s.f.) $H_0 : \mu = 30$ $H_1 : \mu > 30$ Conduct a 1-tail test at $2\frac{1}{2}\%$ significance level. Under H_0, $\bar{X} \sim N(30, \dfrac{2.2780}{60})$ approximately. Using a z-test, p-value = $P(\bar{X} > 30.4) = 0.020043 = 0.0200$ (3 s.f.) Since p-value < 0.025, we reject Ho and conclude that there is sufficient evidence at $2\frac{1}{2}\%$ significance level that the mean centre thickness of the soft contact lenses are more than 30 um. Ie. The claim is not justified.
	It means that there is a probability of 0.025 of wrongly rejecting the claim that the mean centre thickness of the soft contact lenses is at most 30 um.
b **(i)**	Let μ be the mean of *X*. $H_0 : \mu = 7$ $H_1 : \mu \neq 7$ $s^2 = \dfrac{30}{29}(\text{sample variance}) = \dfrac{30}{29}(4) = \dfrac{120}{29}$ Under H0, since the sample size is large, the test statistic is $\bar{T} \sim N\left(7, \dfrac{4}{29}\right)$ approximately by Central Limit Theorem.
(ii)	Since the claim is rejected i.e. to reject H0 at 1% significance level.  From GC, $c_1 = 6.04$ and $c_2 = 7.96$. $\bar{t} \leq 6.04$ or $\bar{t} \geq 7.96$ (3 .s.f)

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(iii)	From the two tail test, we know that $\text{p-value (two tail)} \le 0.01$. For a one-tail test, $\text{p-value(one tail)} = \frac{\text{p-value (two tail)}}{2} \le 0.005 < 0.01$, therefore we reject H_0 and conclude that there is sufficient evidence at 1% significance level to say that mean waiting time is more than 7 minutes.

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H1 Mathematics 2017 Preliminary Exam Paper Question

Section A : Pure Mathematics [40 Marks]

1. Whole Food Grocer was having sales and some food items were on offer. Organic feed eggs were having a 15% discount. There was also a $1 discount for every 2 packets of chia seeds purchased. There was no promotion for organic quinoa. The table below shows the total bills and the number of packets of organic quinoa, organic feed eggs and chia seeds Stephanie, Weiwei and Leo bought from Whole Food Grocer. Calculate the original selling price for one packet of each of the 3 food items, giving your answers correct to the nearest cent.

	Quinoa	Eggs	Chia seeds	Total Bill ($)
Stephanie	3	1	2	71.28
Weiwei	2	2	5	91.85
Leo	6	3	3	144.43

[4]

2. Differentiate the following with respect to x.

(a) $\left(x+\ln x\right)^2$, [2]

(b) $\mathrm{e}^{\left(\frac{1}{\sqrt{2-x}}\right)}$. [2]

3. On the same diagram, sketch the graphs of $y=\dfrac{x+1}{2x+3}$ and $y=x^2+x-3$, indicating clearly the equation(s) of any asymptote(s) and any point(s) of intersection. Hence solve the inequality $\dfrac{x+1}{2x+3}\geq x^2+x-3$. [5]

4. **(i)** Express $\dfrac{2x^2+1}{x-4}$ in the form $Ax+B+\dfrac{C}{x-4}$, where A, B and C are constants to be determined. [2]

(ii) Hence, without the use of a calculator, find $\displaystyle\int_5^6 \frac{2x^2+1}{x-4}\,\mathrm{d}x$ in exact form. [3]

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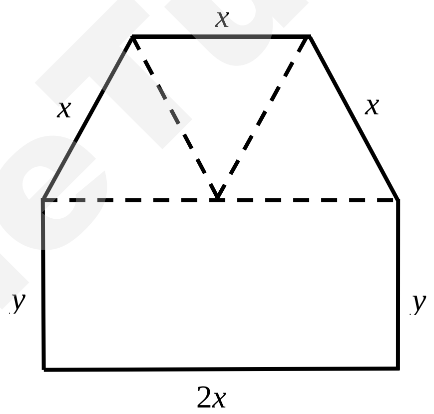
5. **(i)** Sketch the curve C with equation $y=(x+3)(1-x)$, stating clearly the coordinates of the turning point and the axial intercepts. [2]

(ii) The line $y=x+k$ intersects C twice. Find the set of values of k. [3]

(iii) Without the use of a calculator, find the area of the region bounded by C and the line $y=x+5$. [5]

6. **(i)** Kim wants to fence up a vegetable plot in his backyard. The vegetable plot to be fenced up will occupy a rectangle of $2x$ m by y m together with half of a regular hexagon with sides of x m each, as shown in the diagram below. It is given that the area of the vegetable plot is 15 m^2.



Show that the perimeter $P=5x+\dfrac{15}{x}-\dfrac{3\sqrt{3}}{4}x$.

Find, using differentiation, the values of x and y such that P is minimum.

[7]

Two companies provide the cost for the fencing.

Company A	\$90 per metre or part thereof *
Company B	\$95 per metre for the first 10 metre \$84 for the subsequent metre or part thereof

* For example, it costs \$180 to build a fence of 1.2m using Company A

(ii) Find the range of the length of fencing to be built such that it is cheaper to engage Company B. [3]

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(iii) Hence conclude which company Kim should engage to fence his backyard when P is minimum. [2]

Section B: Statistics [60 marks]

7. In an IT department, a staff is tasked to form 7-letter codes (need not be valid words) using the given word 'SPECIAL'. Find the number of codes that can be formed if

(a) there are no restrictions except the code 'SPECIAL' cannot be formed, [2]

(b) all the 3 vowels cannot be together, [2]

(c) the first and the last letters are consonants. [2]

8. A company uses 2 production lines, A and B, to produce lunch boxes. If the lunch box cannot be closed tightly, it will be considered as faulty. Of all the lunch boxes produced, 5% are faulty and 3% of the lunch boxes produced by B are faulty. Among the lunch boxes that are faulty, 60% of them are produced by line A.

One lunch box is selected at random.

(i) Show that the probability that it is produced by line B is $\frac{2}{3}$. [2]

(ii) Find the probability that it is faulty given that it is produced by A. [2]

Two lunch boxes are chosen at random.

(iii) Find the probability that both lunch boxes are produced by B given that exactly one is faulty. [3]

9. The probability of a diner choosing a burger during his visit to Cheeky Chick Café is 0.05. Among the diners who visited Cheeky Chick Café, 20 diners are randomly chosen.

(i) Find the probability that more than 3 diners choose a burger. [2]

(ii) Find the smallest value of n such that there is more than 90% chance of less than n diners choosing a burger. [3]

(iii) The probability of a diner buying a drink in the cafe is p, where $p > 0.5$. Given that the variance of a diner buying a drink is 4.55, find the value of p. [2]

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10. The accountant of a company monitors the number of items produced per month by the company, x (in thousands), together with the total cost of production, $\$y$ (in thousands). The following table shows 12 sets of data collected for a random sample of 12 months.

Number of items produced (x)	21	39	48	24	72	75	15	35	62	81	12	56
Production cost ($\$y$)	40	58	67	45	89	96	37	53	83	102	35	75

(i) Draw a scatter diagram to illustrate the data. [2]

(ii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iii) Find $\bar{x}$ and $\bar{y}$, and mark the point $(\bar{x}, \bar{y})$ on your scatter diagram drawn in part **(i)**. [2]

(iv) Find an equation for the regression line of y on x in the form $y = mx + c$, giving the values of m and c, correct to 2 decimal places. Sketch this line on your scatter diagram. Interpret the meanings of m and c in this context. [3]

(v) Use the equation of your regression line to calculate an estimate for the production cost of 70 thousand items. Comment on the reliability of your estimate. [2]

(vi) The selling price of each item produced is \$2.20. Find the minimum number of items to be produced per month at which the company does not suffer a loss. [2]

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11. The weight of a packet of Calhwa potato chips is known to have a mean of 84 grams and standard deviation 5 grams. The manufacturer claims that the average weight of a packet of potato chips is at least 84 grams. To test this claim, a random sample of 100 such packets of potato chips are selected and tested. The average weight of the 100 packets of potato chips in the sample is 82.9 grams.

(i) State appropriate hypotheses for the test, defining any symbols you use. [2]

(ii) Test, at the 1% significance level, whether the manufacturer's claim is valid. [3]

(iii) State what you understand by the expression 'at the 1% significance level' in part **(ii)**. [1]

(iv) State, giving a reason, whether it is necessary for the weight of the packets of potato chips produced by a manufacturer to follow a normal distribution for the test in part **(ii)** to be valid. [1]

(v) Another random sample of 100 packets of potato chips from another batch gives an average weight of t grams. Find the range of values of t such that there is enough evidence to conclude that the average weight of the packets of potato chips has changed at the 5% level of significance. [5]

12. **(a)** The continuous random variable X has the distribution $\mathrm{N}(\mu, \sigma^2)$. It is given that $\mathrm{P}(X < 15) = 0.841$ and $\mathrm{P}(9 < X < 15) = 0.682$, find the values of μ and σ. [3]

(b) In a city, the minimum temperature in June, denoted by S, is assumed to be normally distributed with mean μ °C and standard deviation 3 °C.

(i) Find the probability that the minimum temperature in June differs from the mean μ by more than 2.5 °C. [3]

(ii) Find the value of μ such that there is a 75% chance that the minimum temperature in June is higher than 11 °C. [2]

In this city, the maximum temperature in June, denoted by T, is also assumed to be normally distributed with mean 20 °C and standard deviation 2.2 °C.

(iii) Find the probability that on a randomly chosen day in June, the maximum temperature is between 17.5 °C and 23 °C. [1]

(iv) Let $\mu = 12$ °C. Find the probability that the maximum temperature on a randomly chosen day in June is more than the average minimum temperature on 2 randomly chosen days in June by less than 10 °C. [4]

(v) State one assumption needed for your calculation in part **(iv)**. Give a reason why the assumption may be unrealistic. [2]

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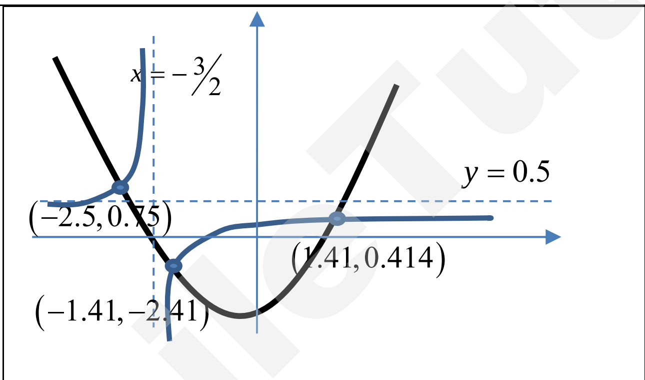
2017 C2 H1 Prelim

	Solutions
1	Let x, y and z be the original selling price per pack of organic quinoa, organic feed eggs and chia seeds in dollars. $3x+0.85y+2z=72.28$ --- (1) $2x+2(0.85)y+5z=93.85$ --- (2) $6x+3(0.85)y+3z=145.43$ --- (3) $x=\$14.90, y=\$11.49, z=\$8.90$
2(a)	$\frac{\mathrm{d}}{\mathrm{d}x}(x+\ln x)^2=2(x+\ln x)\left(1+\frac{1}{x}\right)$ $=\frac{2}{x}(x+\ln x)(x+1)$
2(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^{\left(\frac{1}{\sqrt{2-x}}\right)}=\frac{1}{2(2-x)^{\frac{3}{2}}}\mathrm{e}^{\left(\frac{1}{\sqrt{2-x}}\right)}$
3	 $-2.5\le x<-1.41$ or $-1.41\le x\le 1.41$
4(i)	Using long division $\frac{2x^2+1}{x-4}=2x+8+\frac{33}{x-4}$ OR $\frac{2x^2+1}{x-4}=\frac{(Ax+B)(x-4)+C}{x-4}$ $2x^2+1=(Ax+B)(x-4)+C$ Compare coefficient: $2=A, B=8, C=33$
4(ii)	$\int_5^6 \frac{2x^2+1}{x-4}\,\mathrm{d}x=\int_5^6 2x+8+\frac{33}{x-4}\,\mathrm{d}x$ $=\left[x^2+8x+33\ln\lvert x-4\rvert\right]_5^6$ $=84+33\ln 2-65$ $=19+33\ln 2$

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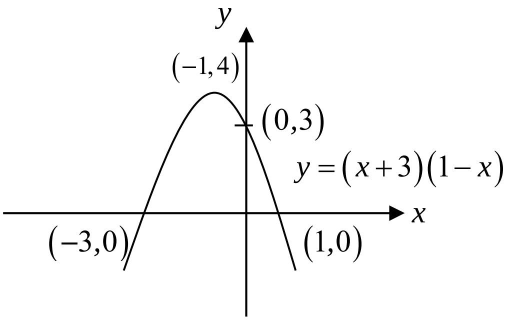
5(i)	
5(ii)	$y=(x+3)(1-x)$ $=x+3-x^2-3x$ $=-x^2-2x+3$ $x+k=-x^2-2x+3$ $x^2+3x+k-3=0$ $b^2-4ac>0$ $3^2-4(k-3)>0$ $9-4k+12>0$ $4k<21$ $k<5.25$ $\{k \in \mathbb{R} : k<5.25\}$
5(iii)	$y=(x+3)(1-x)=x+5$ $-x^2-2x+3=x+5$ $x^2+3x+2=0$ $(x+2)(x+1)=0$ $x=-2, x=-1$ area $=\int_{-2}^{-1}\left(-x^2-2x+3-x-5\right)\mathrm{d}x$ $=\int_{-2}^{-1}\left(-x^2-3x-2\right)\mathrm{d}x$ $=\left[-\frac{x^3}{3}-\frac{3x^2}{2}-2x\right]_{-2}^{-1}$ $=\left(\frac{1}{3}-\frac{3}{2}+2\right)-\left(\frac{8}{3}-\frac{12}{2}+4\right)$ $=\frac{1}{6}\text{units}^2$ Or

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	$\text{area} = \int_{-2}^{-1} (-x^2 - 2x + 3)\mathrm{d}x - \frac{1}{2} \times (3+4) \times 1$ $= \left[-\frac{x^3}{3} - \frac{2x^2}{2} + 3x \right]_{-2}^{-1} - \frac{7}{2}$ $= \frac{1}{6} \text{units}^2$
6(i)	$\text{Area} = 3\left(\frac{1}{2}\right)x^2 \sin\left(\frac{\pi}{3}\right) + 2xy = \frac{3\sqrt{3}}{4}x^2 + 2xy$ $\frac{3\sqrt{3}}{4}x^2 + 2xy = 15$ $2xy = 15 - \frac{3\sqrt{3}}{4}x^2$ $2y = \frac{15}{x} - \frac{3\sqrt{3}}{4}x$ Let P be the perimeter. $P = 5x + 2y$ $= 5x + \frac{15}{x} - \frac{3\sqrt{3}}{4}x$ $\frac{\mathrm{d}P}{\mathrm{d}x} = 5 - \frac{3\sqrt{3}}{4} - \frac{15}{x^2}$ For cost to be minimum, perimeter has to be minimum. $\frac{\mathrm{d}P}{\mathrm{d}x} = 0 \Rightarrow 5 - \frac{3\sqrt{3}}{4} - \frac{15}{x^2} = 0$ $\frac{15}{x^2} = 3.700962 \Rightarrow x^2 = 4.053$ $x = 2.0132$ $\therefore x = 2.01, \quad y = 2.42$ $\frac{\mathrm{d}^2P}{\mathrm{d}x^2} = \frac{30}{x^3} > 0$ Therefore P is a minimum.

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6(ii)	Let N be the perimeter of the fence in integral value Cost from Company $A = 90N$ Cost from Company $B = 10(95) + 84(N-10)$ $= 110 + 84N$ $110 + 84N < 90N$ $6N > 110$ $N > 18.3 \Rightarrow N > 18$
6(iii)	When $x = 2.0132$, $y = 2.4177$ $P = 14.902$ Since $14.902 < 18$, therefore it is cheaper to choose Company *A*.
7(a)	No. of words that can be formed $= 7! - 1$ $= 5039$
(b)	No. of words if 3 vowels are altogether $= 3! \times 5!$ $= 720$ No. of words $= 5040 - 720$ $= 4320$
(c)	No. of words $= {}^4C_2 \times 2! \times 5!$ $= 1440$
8(i)	Let B be the event that the lunch box is produced by production line B. Let F be the event that the lunch box is faulty. $\mathrm{P}(F \cap B) = \mathrm{P}(B) \times \mathrm{P}(F \mid B)$ $= \mathrm{P}(F) \times \mathrm{P}(B \mid F)$ (*) $\mathrm{P}(B)(0.03) = 0.05(0.4)$ $\mathrm{P}(B) = \frac{2}{3}$
8(ii)	Let A be the event that the lunch box is produced by production line A. $\mathrm{P}(A \cap F) = 0.05 \times 0.6 = 0.03$ $\mathrm{P}(F \mid A) = \frac{0.03}{\frac{1}{3}} = 0.09$
8(iii)	$\mathrm{P}(B \cap F') = \frac{2}{3} - 0.02 = 0.64667$ $\mathrm{P}(B \mid \text{only 1 faulty}) = \frac{0.64667 \times 0.02 \times 2}{0.95 \times 0.05 \times 2}$ $= 0.272$
9(i)	Let X denote the number of diners, out of 20, who choose a burger. $X \sim \mathrm{B}(20, 0.05)$ $\mathrm{P}(X > 3) = 1 - \mathrm{P}(X \le 3) = 0.0159$

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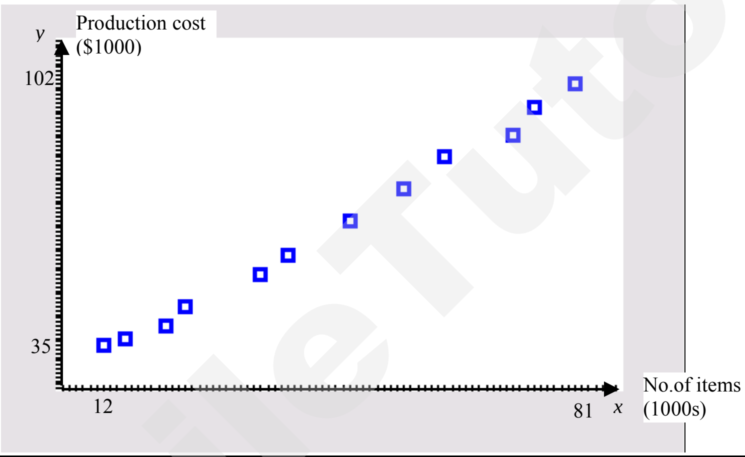
9(ii)	$\mathrm{P}(X < n) > 0.9$ --- (1) $\mathrm{P}(X \le n-1) > 0.9$ --- (2) Using GC, $\mathrm{P}(X \le 1) > 0.736$ $\mathrm{P}(X \le 2) > 0.925$ $\therefore$ smallest value of n is 3.
9(iii)	Let Y denote the number of diners, out of 20, buying a drink in the cafe. $Y \sim \mathrm{B}(20, p)$ $20p(1-p) = 4.55$ --- (1) $p^2 - p + 0.2275 = 0$ $p = 0.35$ or 0.65 Since $p > 0.5$, $p = 0.65$
10(i)	
10(ii)	Product moment correlation coefficient $r = 0.998$ which indicates a strong positive linear correlation between the number of items produced per month by the company together with the total cost of production
10(iii)	$\bar{x} = 45, \quad \bar{y} = 65$

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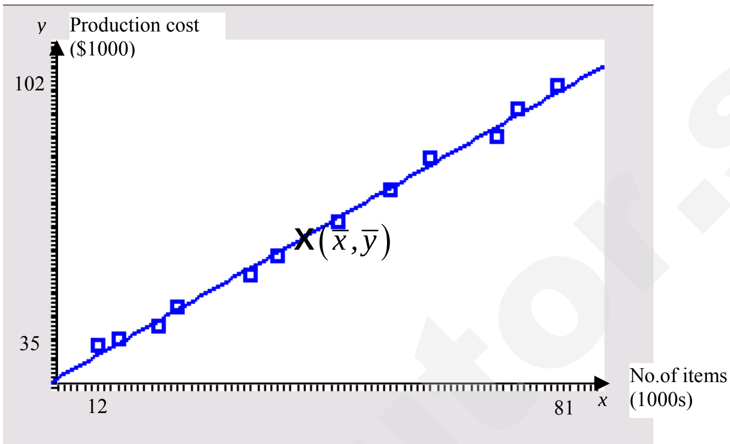
10(iv)	$y = 0.98x + 20.99$ - 0.98 gives the rate at which the production costs are increasing i.e. for every additional item produced, the production cost increases by \$0.98 - OR For every increase in 1000 items produced, there is an increase in the total production cost by 980 dollars. - \$20,990 is the fixed cost of production. 
10(v)	$y = 0.9781x + 20.985$ When $x = 70$, $y = 0.9781 \times 70 + 20.985 = 89.452$, an estimate for the production cost of 70 thousand items is \$89452. Since $x = 70$ lies within the data range $12 \le x \le 81$ and $\lvert r \rvert$ is close to 1, therefore this estimate is reliable.
10(vi)	If x items(1000s) are produced, Total income = $\$2.20 \times x$ The total cost for producing x items is $y = 0.9781x + 20.985$ If there is no loss, $2.20x \ge 0.9781x + 20.985$ $1.2219x \ge 20.985$ $x \ge 17.17407$ Therefore the min number of items to be produced per month is 17175 items.
11(i)	Let X be the weight of a packet of Calhwa potato chips and μ denotes the population mean weight of a packet of potato chips in grams H_0: $\mu = 84$ H_1: $\mu < 84$

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11(ii)	At 1% level of significance, under H_0, since $n = 100$ is large, by Central limit theorem, $\overline{X} \sim N\left(84, \frac{5^2}{100}\right)$ approximately Test statistic $Z = \frac{\overline{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}} \sim N(0,1)$ $p = 0.0139 > 0.01$, we do not reject H_0 and conclude that at the 1% level of significance, there is insufficient evidence to say that the average weight of a packet of potato chips is less than 84 grams
11(iii)	When the level of significance is set at 1%, there is 1% chance that we wrongly conclude the mean weight of a packet of potato chips is less than 84 grams when in fact the mean weight of a packet of potato chips is at least 84 grams.
11(iv)	Since the sample size $n = 100$ is sufficiently large, the **sample mean** weight of the packets of potato chips will be normally distributed by the Central Limit Theorem. Therefore it is not necessary to assume the weight of packets of potato chips follow a normal distribution.
11(v)	$H_0: \mu = 84$ $H_1: \mu \neq 84$ Level of significance: 5% Under H_0, since n =100 is large, by Central limit theorem, $\overline{X} \sim N\left(84, \frac{5^2}{100}\right)$ approximately. Test statistic $\lvert Z \rvert = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$ Rejection region: Reject H_0 if $z \leq -1.95996$ or $z \geq 1.95996$ Since there is sufficient evidence, at 5% level significance to conclude that the average weight of the potato chip has changed, $\frac{t-84}{\frac{5}{\sqrt{100}}} \leq -1.95996$ or $\frac{t-84}{\frac{5}{\sqrt{100}}} \geq 1.95996$ $\Rightarrow 2t - 84 \leq -1.95996$ or $2t - 84 \geq 1.95996$ $t \leq 83.020$ or $t \geq 84.979$ Range of t : $t \leq 83.0$ or $t \geq 85.0$

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12(a)	$\mathrm{P}(X<15)=0.841 \Rightarrow \mathrm{P}\left(Z<\frac{15-\mu}{\sigma}\right)=0.841$ --(1) $\Rightarrow \frac{15-\mu}{\sigma}=0.99858$ $\mathrm{P}(9<X<15)=0.682 \Rightarrow \mathrm{P}\left(\frac{9-\mu}{\sigma}<Z<\frac{15-\mu}{\sigma}\right)=0.682$ $\Rightarrow \mathrm{P}\left(Z<\frac{9-\mu}{\sigma}\right)=0.841-0.682=0.159$ ---(2) $\Rightarrow \frac{9-\mu}{\sigma}=-0.99858$ --- (2) Solving (1) & (2), $\mu=12$ and $\sigma=3.00$ **Alternatively** By observation , $\mu=12$. $\mathrm{P}(X<15)=0.841$ Using GC, $\sigma=3.00$
12(bi)	$S \sim \mathrm{N}\left(\mu,3^2\right)$ $\mathrm{P}(\lvert S-\mu\rvert>2.5)$ --- (1) $=\mathrm{P}\left(\left\lvert\frac{S-\mu}{3}\right\rvert>\frac{2.5}{3}\right)=\mathrm{P}\left(\lvert Z\rvert>\frac{2.5}{3}\right)=2\times\mathrm{P}\left(Z>\frac{2.5}{3}\right)$ --- (2) $=0.405$
b(ii)	$\mathrm{P}(S>11)=0.75$ --- (1) $\mathrm{P}\left(Z\le\frac{11-\mu}{3}\right)=0.25$ $\frac{11-\mu}{3}=-0.67449$, $\mu=13.0\ °\mathrm{C}$
b(iii)	$\mathrm{P}(17.5<T<23)=0.786$
b(iv)	Find $\mathrm{P}\left(0\le T-\frac{S_1+S_2}{2}<10\right)$ Let $W=T-\frac{S_1+S_2}{2}$ $\mathrm{E}(W)=8$ $\mathrm{Var}(W)=9.34$ $W \sim \mathrm{N}(8,9.34)$ $\mathrm{P}\left(0\le T-\frac{S_1+S_2}{2}<10\right)=0.73915=0.739(3\mathrm{s.f})$
(v)	Assume that the minimum and maximum temperatures are independent of each other. It is unrealistic because the weather, e.g. wind direction, rainy weather, etc, will affect both the minimum and maximum temperature of the city.

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INNOVA JUNIOR COLLEGE
JC 2 PRELIMINARY EXAMINATION
in preparation for General Certificate of Education Advanced Level
Higher 1

CANDIDATE NAME			
CLASS		INDEX NUMBER	

MATHEMATICS 8864/01

Paper 1

28 August 2017

3 hours

Additional Materials: Answer Paper
Cover Page
Graph Paper
List of Formulae (MF 15)

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen on both sides of the paper. You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages.

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Section A: Pure Mathematics [35 marks]

1 **(i)** Differentiate $\dfrac{3}{\sqrt{(2x-7)^3}}$. [2]

(ii) Find $\displaystyle\int \frac{1}{e^{4t-3}}+\frac{1}{2t-1}\,dt$. [3]

2 By considering $u = e^{2x}$ or otherwise, solve the equation

$$3-10e^{2x}-8e^{4x}=0,$$

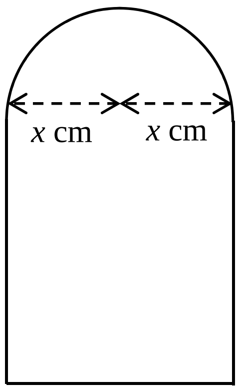
leaving your answer in the form $a\ln b$, where a and b are integers to be determined. [4]

Hence solve the inequality

$$3-10e^{2x}-8e^{4x}\le 0,$$

leaving your answer in exact form. [2]

3 A window frame is in the shape of a rectangle joined to a semicircle of radius x cm.



(i) If the window frame is made using 300 cm of framework with negligible thickness, show that the total area of the window is given by $A = 300x - 2x^2 - \dfrac{1}{2}\pi x^2$. [2]

(ii) Using differentiation, find the maximum area of the window. Leave your answer correct to the nearest integer. [4]

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4 A curve C has equation $y = \ln(3 - x)$.

(i) Sketch C, indicating clearly the exact coordinates of any points of intersection with the axes and the equation of asymptote, if any. [2]

(ii) Find the equation of the tangent to C at the point P where $x = 1$, giving your answer in the form $y = mx + c$, where m and c are exact constants to be determined. [3]

(iii) R is the region bounded by the tangent at P, the curve C and the x-axis. By sketching the equation of the tangent at P on the diagram in part **(i)**, indicate the region R. Hence find the numerical value of the area of region R. [4]

5 The curve C has equation $y = x^4 - 4x^3 + \frac{9}{2}x^2 - 2x + 2$.

(i) Find $\frac{dy}{dx}$. Hence find the coordinates of the stationary points on the curve. [3]

(ii) Use a non-calculator method to determine the nature of each of the stationary points. [2]

(iii) Sketch the graph of C, stating the coordinates of the stationary points and any points where the curve crosses the axes. [2]

(iv) By adding a suitable graph in your diagram in part **(iii)**, solve the inequality $x^4 - 4x^3 + \frac{9}{2}x^2 + x - 9 < 0$, giving your answer correct to 4 decimal places. [2]

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Section B: Statistics [60 marks]

6 The Physical Education (PE) Department of Sunflower Secondary School intends to carry out a survey to investigate the number of hours of exercise each student spends per week. The school has a total of 1200 students studying in four different levels. On one particular school day, a PE teacher selects a random sample of 80 students from those who enter the school via the main gate by

- choosing at random one of the first 15 students who enter the main gate,
- then choosing every 15th student after the first student is chosen.

(i) State the sampling method used by the PE teacher. [1]

(ii) State one disadvantage of the sampling method used in this context. [1]

(iii) Describe briefly how, in this case, the PE teacher might choose a more appropriate random sample. [3]

7 At a lucky draw booth, each contestant will roll an unbiased die. If the die shows a "6", the contestant will pick a counter at random from Box *A*. Otherwise, he will pick a counter at random from Box *B*. Box *A* contains 3 red counters, 2 green counters and 3 yellow counters. Box *B* contains 5 red counters, 3 green counters and 2 yellow counters.

(a) A contestant will win a prize if a yellow counter is picked.

(i) Draw a tree diagram to represent this situation. [2]

(ii) Find the probability that a contestant wins a prize. [2]

(iii) Given that the contestant wins a prize, find the probability that it came from Box *A*. [2]

(b) The rule of winning a prize has now changed. Each contestant needs to pick two counters, without replacement, instead of one. A contestant will win a prize if both counters picked are yellow. Find the probability that a contestant wins a prize. [2]

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8 In a neighbourhood, it is known that 9% of the residents use the bicycle-sharing platform, ShareBike. A sample of n residents is selected at random and the number of residents who use ShareBike in the sample is denoted by the random variable X.

(i) State, in context, an assumption needed for X to be well modelled by a binomial distribution. [1]

(ii) Explain why the assumption stated in part **(i)** may not hold in this context. [1]

Assume now the assumption stated in part **(i)** does in fact hold.

(iii) Find the greatest value of n such that the probability that there is at least 1 resident using ShareBike is less than 0.99. [3]

(iv) Given that $n = 40$, find the probability that more than 3 but at most 5 residents use ShareBike. [2]

(v) Given instead that $n = 70$, using a suitable approximation, find the probability that less than 7 residents use ShareBike. [4]

9 Two electrical components, Type A and Type B, have lifespans of A weeks and B weeks respectively. It is given that A and B are independent random variables with distributions $\mathrm{N}\left(43, 8^2\right)$ and $\mathrm{N}\left(40, 6^2\right)$ respectively.

(i) Find the probability that the total lifespan of 3 randomly chosen Type A components is shorter than thrice the lifespan of a randomly chosen Type B component. [4]

(ii) Find the probability of the event that both the lifespan of a randomly chosen Type A component exceeds 38 weeks and the total lifespan of 2 randomly chosen Type B components exceeds 82 weeks. [4]

(iii) Find the probability that the total lifespan of a randomly chosen Type A component and 2 randomly chosen Type B components exceeds 120 weeks. [3]

(iv) Explain why the answer in part **(ii)** is smaller than the answer in part **(iii)**. [1]

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10 Mr Lee recorded the length of time, t minutes, taken to travel to work when leaving home x minutes after 7 am on 10 mornings over two weeks. The results are as follows.

x	0	5	10	15	20	25	30	35	40	45
t	15	19	30	28	32	39	30	48	53	62

(i) Plot a scatter diagram on graph paper for this data, labelling the axes, using a scale of 2 cm to represent 10 minutes on the t-axis and an appropriate scale for the x-axis. [2]

(ii) Suggest a reason why one of the data points does not seem to follow the trend and indicate the corresponding point on your diagram by labelling it P. [2]

Omit the point P.

(iii) Calculate the product moment correlation coefficient and comment on this value. [2]

(iv) Find the equation of the least squares regression line of t on x, writing your answer in the form $t = ax + b$. [1]

(v) Sketch the regression line on your scatter diagram and interpret the meaning of the value of a in the context of the question. [2]

(vi) Mr Lee needs to arrive at work no later than 8.30 am. Estimate, to the nearest minute, the latest time that he has to leave home without arriving late at work. [3]

11 A large group of Health and Fitness Club members is known to have a mean mass of 85 kg. The trainer claims that the mean mass of his members has decreased under his strict routine. To investigate his claim, the mass, x kg, of 30 randomly chosen members are collated and the results are summarised below.

$$\sum x = 2526, \qquad \sum \left(x - \bar{x}\right)^2 = 544$$

(i) Find unbiased estimates of the population mean and variance. [3]

(ii) Test at the 10% level of significance whether the trainer's claim is valid. [5]

(iii) State the meaning of the p-value obtained in part **(ii)**. [1]

(iv) The trainer makes some adjustments to his training routine and the new population standard deviation is known to be 5 kg. A new sample of 30 members is randomly chosen and the mean mass of this sample is m kg. At the 10% level of significance, find the range of values of m for the trainer's claim to be valid, giving your answer correct to 2 decimal places. [3]

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1

(i)

Let $y=\dfrac{3}{\sqrt{(2x-7)^3}}=3(2x-7)^{-\frac{3}{2}}$

$$\frac{dy}{dx}=3(2x-7)^{-\frac{5}{2}}\cdot\left(\frac{-3}{2}\right)\cdot(2)$$

$$=-9(2x-7)^{-\frac{5}{2}} \qquad \left(\text{or } -\frac{9}{(2x-7)^{\frac{5}{2}}}\right)$$

(ii)

$$\int \frac{1}{e^{4t-3}}+\frac{1}{2t-1}\,dt$$

$$=\int e^{-4t+3}+\frac{1}{2t-1}\,dt$$

$$=-\frac{1}{4}e^{-4t+3}+\frac{1}{2}\ln(2t-1)+c$$

2

(i)

$3-10e^{2x}-8e^{4x}=0$

Let $u=e^{2x}$. Then

$\Rightarrow \quad 3-10u-8u^2=0$

$\Rightarrow \quad (2u+3)(1-4u)=0$

$\Rightarrow \quad u=-\dfrac{3}{2} \quad$ or $\quad u=\dfrac{1}{4}$

$\Rightarrow \quad e^{2x}=-\dfrac{3}{2} \quad$ or $\quad e^{2x}=\dfrac{1}{4}$

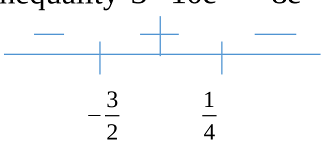
(rej $\because e^{2x}>0$)

$\therefore \quad 2x=\ln\dfrac{1}{4} \Rightarrow x=\dfrac{1}{2}\ln\dfrac{1}{4}=-\ln 2$

$a=-1$ and $b=2$

(ii)

For the inequality $3-10e^{2x}-8e^{4x}\le 0$,



$e^{2x}\le -\dfrac{3}{2} \quad$ or $\quad e^{2x}\ge \dfrac{1}{4}$

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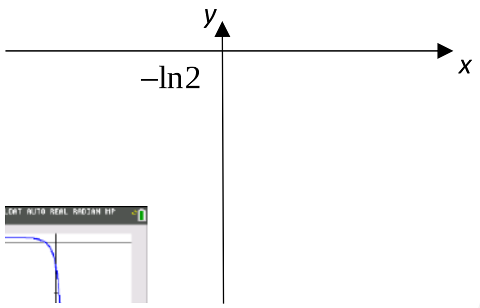
(rej $\because e^{2x} > 0$)

$\therefore \quad x \geq -\ln 2$

Or use graphical method



Therefore,

$\therefore \quad x \geq -\ln 2$

3

(i)

Let A cm^2 be the area of the window and l cm be the length of the rectangle.

$\pi x + 2l + 2x = 300$

$\Rightarrow \; l = 150 - x - \dfrac{\pi x}{2}$

$$A = \frac{1}{2}\pi x^2 + 2xl$$

$$= \frac{1}{2}\pi x^2 + 2x\left(150 - x - \frac{\pi x}{2}\right)$$

$$= \frac{1}{2}\pi x^2 + 300x - 2x^2 - \pi x^2$$

$$A = 300x - 2x^2 - \frac{1}{2}\pi x^2$$

(ii)

For maximum area, $\dfrac{dA}{dx} = 0$

$300 - 4x - \pi x = 0$

$x = \dfrac{300}{4+\pi}$ or 42.007

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Using 1st derivative test,

x	$\left(\frac{300}{4+\pi}\right)^-$	$\frac{300}{4+\pi}$	$\left(\frac{300}{4+\pi}\right)^+$
$\frac{dA}{dx}$	+ve	0	–ve

A is maximum when $x=\frac{300}{4+\pi}$ or 42.007.

Maximum A

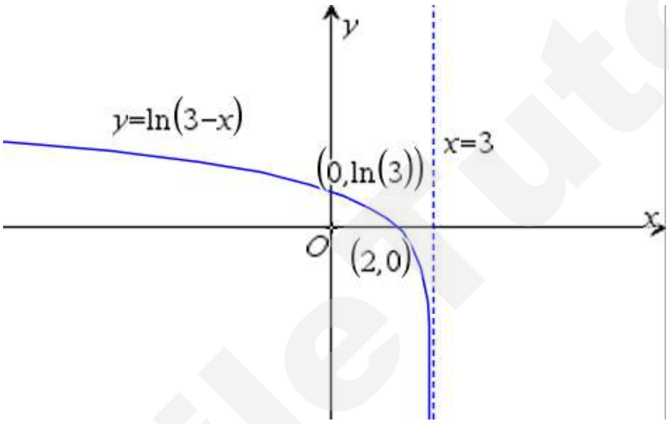
$=300\left(\frac{300}{4+\pi}\right)-2\left(\frac{300}{4+\pi}\right)^2-\frac{1}{2}\pi\left(\frac{300}{4+\pi}\right)^2$

$=6301.115$

$=6301\,\text{cm}^2$ (nearest whole number)

4

(i) $y=\ln(3-x)$



(ii)

$\frac{dy}{dx}=-\frac{1}{3-x}$

At point P, $x=1$; $\frac{dy}{dx}=-\frac{1}{2}$; $y=\ln(2)$

Eqn of tangent at P: $y-\ln(2)=-\frac{1}{2}(x-1)$

$y=\ln(2)-\frac{1}{2}(x-1)$

$y=-\frac{1}{2}x+\ln 2+\frac{1}{2}$

(iii)

$y=-\frac{1}{2}x+\ln 2+\frac{1}{2}$ $y=\ln 2-\frac{1}{2}(x-1)$

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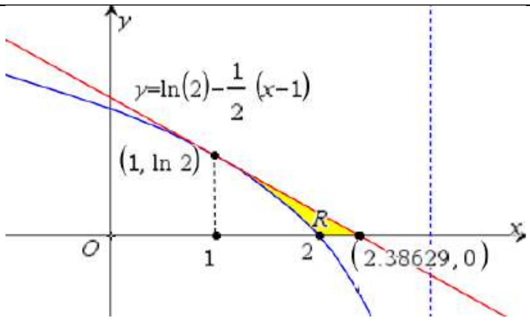

The area of R

= Area of triangle – area under the curve

$$= \frac{1}{2}(2.38629-1)(\ln 2) - \int_1^2 \ln(3-x)\,\mathrm{d}x$$

$$= 0.0941586528$$

$$\approx 0.0942 \text{ units}^2$$

5 (i)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 4x^3 - 12x^2 + 9x - 2$$

Let $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

$$\Rightarrow 4x^3 - 12x^2 + 9x - 2 = 0$$

Using GC, $x = 2$ or $\frac{1}{2}$

When $x = 2$, $y = (2)^4 - 4(2)^3 + 4.5(4) - 2(2) + 2$

$= 0$

When $x = \frac{1}{2}$, $y = \left(\frac{1}{2}\right)^4 - 4\left(\frac{1}{2}\right)^3 + 4.5\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 2$

$= \frac{27}{16}$ or 1.6875

The coordinates of the stationary points are $(2,0)$ and $\left(\frac{1}{2}, \frac{27}{16}\right)$.

(ii)

When $x = 2$,

x	$x = 2^-$	$x = 2$	$x = 2^+$
$\frac{\mathrm{d}y}{\mathrm{d}x}$	$-$	0	$+$
slope			

Thus $(2,0)$ is a minimum point.

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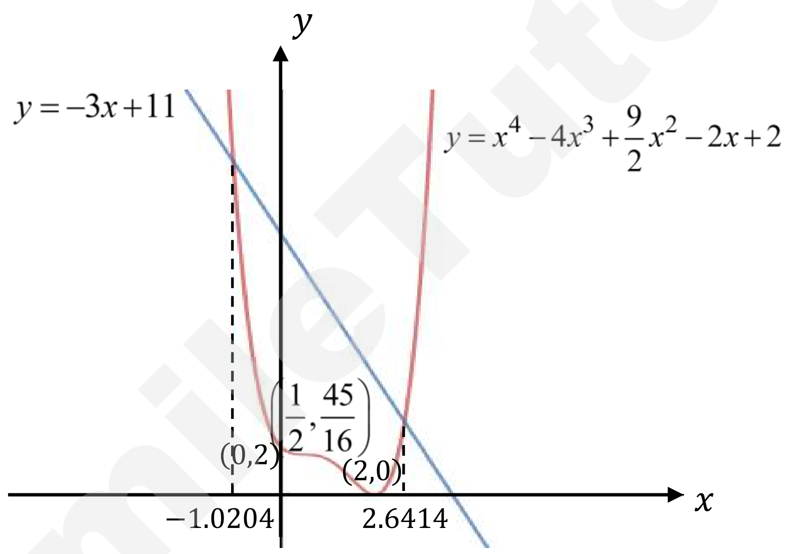
When $x = \frac{1}{2}$,

x	$x = \frac{1}{2}^{-}$	$x = \frac{1}{2}$	$x = \frac{1}{2}^{+}$
$\frac{\mathrm{d}y}{\mathrm{d}x}$	$-$	0	$-$
slope	\	—	\

Thus $\left(\frac{1}{2}, \frac{27}{16}\right)$ is a stationary point of inflexion.

(iii)



(iv)

$$x^4 - 4x^3 + \frac{9}{2}x^2 + x - 9 < 0$$

$$x^4 - 4x^3 + \frac{9}{2}x^2 - 2x + 3x + 2 - 11 < 0$$

$$x^4 - 4x^3 + \frac{9}{2}x^2 - 2x + 2 < -3x + 11$$

Using GC, the x-coordinates of intersection are $x = -1.0204$ or 2.6414

From the graph, $-1.0204 < x < 2.6414$.

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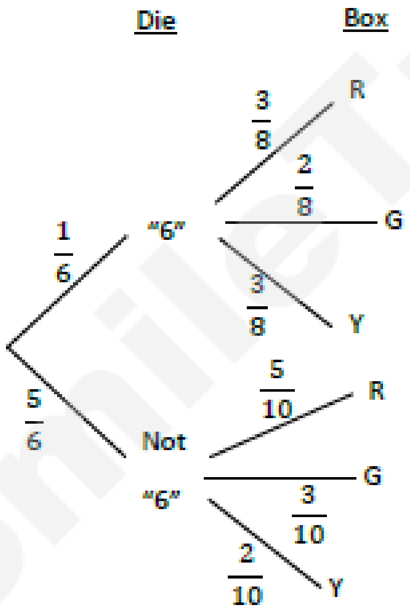
6	(i) Systematic Sampling (ii) One disadvantage is that not all students get an equal chance of being selected as the students who are absent from school will not get a chance to be interviewed. (iii) Stratified Sampling Split the school population into **mutually exclusive subgroups/strata based on the four levels, Sec 1 to 4**. $\frac{80}{1200} \times 100\% \approx 6.67\%$ Choose **random samples** from each subgroup that is **proportional to the size of the subgroup/stratum (6.67%) to make up the sample of 80 students.** **or** Choose **randomly 6.67% of students from each stratum to form a sample of 80 students.**
7	 (a)(ii) $\text{P(wins a prize)} = \left(\frac{1}{6} \times \frac{3}{8}\right) + \left(\frac{5}{6} \times \frac{2}{10}\right)$ $= \frac{11}{48}$

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	(a)(iii) $$P\left(\text{from box A} \mid \text{wins the prize}\right)=\frac{\frac{1}{6}\times\frac{3}{8}}{\frac{11}{48}}$$ $$=\frac{3}{11}$$ (b) $$P\left(\text{wins the grand prize}\right)=\left(\frac{1}{6}\times\frac{3}{8}\times\frac{2}{7}\right)+\left(\frac{5}{6}\times\frac{2}{10}\times\frac{1}{9}\right)$$ $$=\frac{55}{1512}$$
8	(i) The assumption is that the event of a resident using ShareBike or not is independent of any other residents in the neighbourhood. (ii) The assumption may not hold as usually families may use ShareBike together as they are going for the activity together. (iii) $X \sim B(n, 0.09)$ $P(X \geq 1) < 0.99$ $P(X = 0) > 0.01$ $\binom{n}{0}(0.09)^0(1-0.09)^n > 0.01$ $(0.91)^n > 0.01$ $n < \frac{\ln 0.01}{\ln 0.91}$ $n < 48.830$ $\therefore$ Greatest value of n is 48. **<u>Alternative method</u>** Using GC, When $n = 47$, $P(X = 0) = 0.0119$ (>0.01) When $n = 48$, $P(X = 0) = 0.0108$ (>0.01) When $n = 49$, $P(X = 0) = 0.0098$ (<0.01) $\therefore$ Greatest value of n is 48. (iv) $X \sim B(40, 0.09)$ $P(3 < X \leq 5) = P(X \leq 5) - P(X \leq 3)$ $= 0.344$ (3 s.f.)

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	(v) $X \sim \mathrm{B}(70, 0.09)$ Since $n = 70$ is large, $np = 70 \times 0.09 = 6.3 \ \ (>5)$ $nq = 70 \times (1-0.09) = 63.7 \ \ (>5)$ $X \sim \mathrm{N}(6.3, 5.733)$ approximately $\mathrm{P}(X < 7) = \mathrm{P}(X < 6.5)$ (continuity correction) $= 0.533$ (3 s.f.)
9	(i) $A \sim \mathrm{N}\left(43, 8^2\right) \quad B \sim \mathrm{N}\left(40, 6^2\right)$ $\mathrm{P}\left(A_1 + A_2 + A_3 < 3B\right) = \mathrm{P}\left(A_1 + A_2 + A_3 - 3B < 0\right)$ Let $S = A_1 + A_2 + A_5 - 3B$ $\mathrm{E}(S) = 3(43) - 3(40) = 9$ $\mathrm{Var}(S) = 3\left(8^2\right) + 3^2\left(6^2\right) = 516$ $\mathrm{P}\left(S < 0\right) = 0.346$ (3 s.f.) (ii) $B \sim \mathrm{N}\left(40, 6^2\right)$ $\mathrm{E}\left(B_1 + B_2\right) = 2(40) = 80$ $\mathrm{Var}\left(B_1 + B_2\right) = 2\left(6^2\right) = 72$ $B_1 + B_2 \sim \mathrm{N}(80, 72)$ Required probability $= \mathrm{P}\left(A > 38\right) \times \mathrm{P}\left(B_1 + B_2 > 82\right)$ $= 0.299$ (3 s.f.) (iii) $\mathrm{E}\left(A + B_1 + B_2\right) = 43 + 2(40) = 123$ $\mathrm{Var}\left(A + B_1 + B_2\right) = 8^2 + 2\left(6^2\right) = 136$ $A + B_1 + B_2 \sim \mathrm{N}(123, 136)$ Required probability $= \mathrm{P}\left(A + B_1 + B_2 > 120\right)$ $= 0.602$ (3 s.f.) (iv) Because the case in (ii) is a proper subset of the case in (iii). For eg, Part iii contains cases whereby the lifespan of component A may not exceed 38 weeks (eg. 36 weeks) but total lifespan of 2 components of B exceeds 82 weeks (eg. 84 weeks), and yet the total lifespan is more than 120 weeks.

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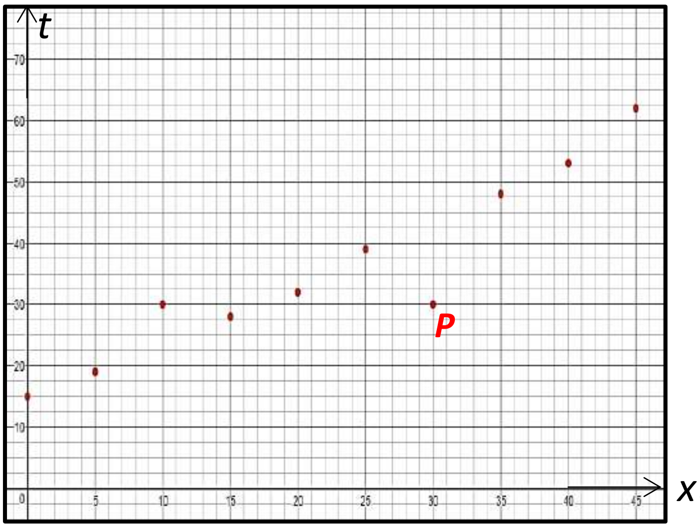
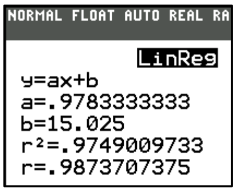
10

(i)



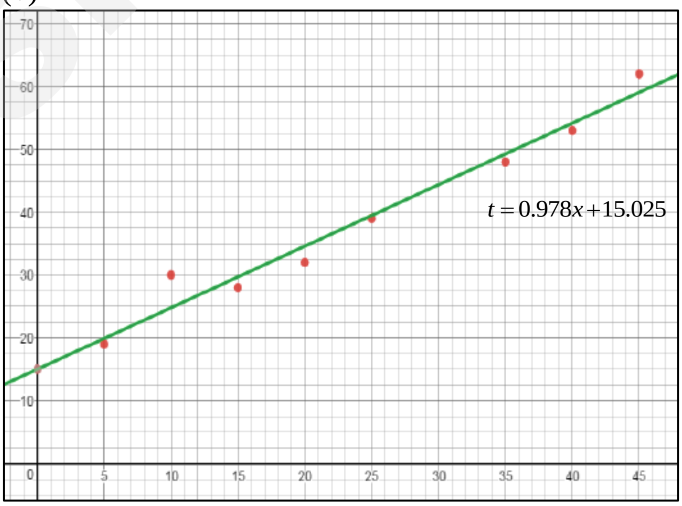
(ii)
Acceptable reasons:
The traffic condition on the road was good (Lesser cars on the road, no traffic jam) and thus he required much shorter travelling time though he left home only at 7.30am.
It was a public holiday/school holiday/Sunday and yet Mr Lee has to work.

(iii)
$r \approx 0.987$
The pmcc is close to 1, indicating a strong positive linear correlation between x and t. I.e. the later Mr Lee leaves home after 7 am, the longer the travelling time would take.



(iv) $t = 0.978x + 15.025$

(v)



$a = 0.978$ means that for every additional minute that Mr Lee delays in leaving home after 7am, his travelling time will increase by 0.978 minutes.

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(vi)

Method 1:

There are 90 minutes from 7 am to 8.30 am.

$$x+t\le 90$$

$$x+(0.97833x+15.025)\le 90$$

$$1.97833x \le 74.975$$

$$x\le 37.898$$

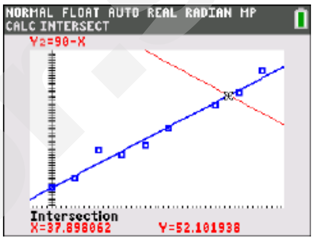
The largest possible value of x is 37 (correct to the nearest minute)

The latest time Mr Lee could leave home without being late for work is 7.37 am.

Method 2:

Sketch the line $x+t=90$ and find x-coordinate of the point of intersection with the regression line.

If x = 38, Mr Lee will arrive late for work. Thus the latest time he needs to leave home is 7.37am.



Method 3:

By Trial & Error, using GC

From part (vi),

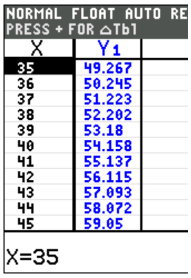
if $x=40, t=54.158, x+t>90$

$x=39, t=53.18,\ \ x+t>90$

$x=38, t=52.202,\ x+t>90$

$x=37, t=51.223, x+t<90$

Thus the latest time he needs to leave home is 7.37am.



NORMAL FLOAT AUTO RE
PRESS + FOR ΔTbl

X	Y1
35	49.267
36	50.245
37	51.223
38	52.202
39	53.18
40	54.158
41	55.137
42	56.115
43	57.093
44	58.072
45	59.05

X=35

11

(i) $\sum x=2526$, $\qquad \sum(x-\bar{x})^2=544$

Unbiased estimates of the population mean μ is

$$\bar{x}=\frac{2526}{30}=84.2$$

Unbiased estimates of the population variance σ^2 is

$$s^2=\frac{30}{29}\left[\frac{544}{30}\right]$$

$$=18.75862$$

$$=18.8\ (3\text{ s.f})$$

(ii)

$H_0:\ \mu=85$

$H_1:\ \mu<85$

Test at 10% significance level

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	Assuming that H_0 is true, Since $n = 30$ is sufficiently large, by the Central Limit Theorem, $\bar{X} \sim \mathrm{N}\left(85, \frac{s^2}{30}\right)$ approximately. Test statistic: $Z = \dfrac{\bar{X} - 85}{\sqrt{\dfrac{18.75862}{30}}} \sim \mathrm{N}(0,1)$ approximately. Using GC, p-value = 0.15584 = 0.156 (3 s.f) $\left(\text{or } z = \dfrac{84.2 - 85}{\sqrt{\dfrac{18.75862}{30}}} = -1.0117 \right)$ Since p-value = 0.15584 > 0.1 (or $z = -1.0117 > -1.28155$), we do not reject H_0 and conclude that there is insufficient evidence at 10% level, that the mean mass of the Health and Fitness Club members has decreased. (or that the trainer's claim is invalid.) (iii) There is 0.15584 probability of drawing **<u>a random sample of 30</u>** Health and Fitness Club members with **<u>sample mean less than 84.2 kg</u>**, assuming that the population mean weight is 85 kg. (iv) $H_0: \mu = 85$ $H_1: \mu < 85$ Test at 10% level significance level. Assuming that H_0 is true, Since n is large, by the Central Limit Theorem, $\bar{X} \sim \mathrm{N}\left(85, \frac{5^2}{30}\right)$ approximately. Test statistic: $Z = \dfrac{\bar{X} - 85}{5/\sqrt{30}} \sim \mathrm{N}(0,1)$ approximately. Since the null hypothesis is rejected, $\Rightarrow$ z-value falls inside critical region $\Rightarrow$ $z\text{-value} < -1.28155$

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	$\Rightarrow \dfrac{m-85}{5/\sqrt{30}} < -1.28155$ $m-85 < -1.1699$ $m < 83.83$ $\therefore \quad 0 < m < 83.83$ **Alternate method** Using $\bar{X} \sim \mathrm{N}\left(85, \dfrac{5^2}{30}\right)$ $\mathrm{P}\left(\bar{X} \le m\right) < 0.1$ $\Rightarrow m < 83.83$ $\therefore \quad 0 < m < 83.83$

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CANDIDATE NAME

CLASS

INDEX NUMBER

MATHEMATICS

8865/01

Paper 1

28 August 2017

3 hours

Additional Materials: Answer Paper
Cover Page
Graph Paper
List of Formulae (MF 26)

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen on both sides of the paper. You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages.

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Section A: Pure Mathematics [40 marks]

1 At the grand opening of a new indoor playground in town, visitors are given discount on entrance fees based on different promotions. The entrance fees to the playground are divided into three categories, namely Toddler (below 3 year old), Child (3 to 12 years old) and Adult (13 years old and above). Three families visit the playground and the number of family members are shown in the table below.

Members	Toddler	Child	Adult
Tan Family	0	2	2
Ng Family	1	1	2
Lim Family	1	3	3

The Tan family paid \$22.50 for their entrance fees after a 25% discount. The Ng family paid \$20 for their entrance fees after a 20% discount. The Lim family bought the membership priced at \$15 nett to enjoy a 50% discount on entrance fees and paid \$41.25 in total. By forming a system of linear equations, find the original entrance fee for each of the categories. [4]

2 **(a)** Differentiate $\dfrac{3}{\sqrt{(2x-7)^3}}$. [2]

(b) Use a non-calculator method to find the exact value of $\displaystyle\int_1^3 \frac{1}{e^{4t-3}}+\frac{1}{2t-1}\,dt$. [4]

3 By considering $u=e^{2x}$ or otherwise, solve the equation

$$3-10e^{2x}-8e^{4x}=0,$$

leaving your answer in the form $a\ln b$, where a and b are integers to be determined. [4]

Hence solve the inequality

$$3-10e^{2x}-8e^{4x}\le 0,$$

leaving your answer in exact form. [2]

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4 A curve C has equation $y = \ln(3 - x)$.

(i) Sketch C, indicating clearly the exact coordinates of any points of intersection with the axes and the equation of the asymptote. [2]

(ii) Find the equation of the tangent to C at the point P where $x = 1$, giving your answer in the form $y = mx + c$, where m and c are exact constants to be determined. [3]

(iii) R is the region bounded by the tangent at P, the curve C and the x-axis. By sketching the equation of the tangent at P on the diagram in part **(i)**, indicate the region R. Hence find the numerical value of the area of region R. [4]

5 An electronic company manufactures smart phones and the manager of this company monitors how the rate of the total manufacturing costs, x million dollars per month, of their new smart phone model changes over a period of t months. The company's financial analyst believes that the relationship between x and t can be modelled by the equation

$$x = t^3 - 13t^2 + 40t + 35, \text{ for } 0 \le t \le 12.$$

(i) Using differentiation, find the minimum value of x, justifying that this value is a minimum. [6]

(ii) Sketch the graph of x against t, giving the coordinates of any intersections with the axes. [2]

(iii) Find the area of the region bounded by the curve, the line t =12 and both t- and x-axes. Give an interpretation of the area you have found, in the context of the question. [3]

The company's accountant believes that the connection between the profit per month, \$$P$ million, is related to x, by the equation

$$P = 45 + 20\ln(3x + 4)$$

(iv) Find the exact value of $\frac{\mathrm{d}P}{\mathrm{d}x}$ for which $t = 8$. [2]

(v) Hence find the rate of increase in profit per month when $t = 8$. [2]

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Section B: Statistics [60 marks]

6 A 4-digit number is chosen at random from the digits $\{1,2,3,4,5\}$ where repetition of digits is not allowed.

Find the probability that the 4-digit number chosen

(i) is an even number, [3]

(ii) is greater than 3000 given that the number is an even number. [3]

7 At a lucky draw booth, each contestant will roll an unbiased die. If the die shows a "6", the contestant will pick a counter at random from Box *A*. Otherwise, he will pick a counter at random from Box *B*. Box *A* contains 3 red counters, 2 green counters and 3 yellow counters. Box *B* contains 5 red counters, 3 green counters and 2 yellow counters.

(a) A contestant will win a prize if a yellow counter is picked.

(i) Draw a tree diagram to represent this situation. [2]

(ii) Find the probability that a contestant wins a prize. [2]

(iii) Given that the contestant wins a prize, find the probability that it came from Box *A*. [2]

(b) The rule of winning a prize has now changed. Each contestant needs to pick two counters, without replacement, instead of one. A contestant will win a prize if both counters picked are yellow. Find the probability that a contestant wins a prize. [2]

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8 In a neighbourhood, it is known that 9% of the residents use the bicycle-sharing platform, ShareBike. A sample of n residents is selected at random and the number of residents who use ShareBike in the sample is denoted by the random variable X.

(i) State, in context, an assumption needed for X to be well modelled by a binomial distribution. [1]

(ii) Explain why the assumption stated in part **(i)** may not hold in this context. [1]

Assume now the assumption stated in part **(i)** does in fact hold.

(iii) Find the greatest value of n such that the probability that there is at least 1 resident using ShareBike is less than 0.99. [3]

It is now given that $n = 20$.

(iv) Find the probability that more than 2 but at most 5 residents use ShareBike. [2]

(v) 40 such random samples are taken and the number of residents using ShareBike is being observed in each sample. Find the probability that the mean number of residents using ShareBike of these observations exceeds 2. [3]

9 Two electrical components, Type A and Type B, have lifespans of A weeks and B weeks respectively. It is given that A and B are independent random variables with distributions $\mathrm{N}\left(43, 8^2\right)$ and $\mathrm{N}\left(40, 6^2\right)$ respectively.

(i) Find the probability that the total lifespan of 3 randomly chosen Type A components is shorter than thrice the lifespan of a randomly chosen Type B component. [4]

(ii) Find the probability of the event that both the lifespan of a randomly chosen Type A component exceeds 38 weeks and the total lifespan of 2 randomly chosen Type B components exceeds 82 weeks. [4]

(iii) Find the probability that the total lifespan of a randomly chosen Type A component and 2 randomly chosen Type B components exceeds 120 weeks. [3]

(iv) Explain why the answer in part **(ii)** is smaller than the answer in part **(iii)**. [1]

[Turn over

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10 Mr Lee recorded the length of time, t minutes, taken to travel to work when leaving home x minutes after 7 am on 10 mornings over two weeks. The results are as follows.

x	0	5	10	15	20	25	30	35	40	45
t	15	19	30	28	32	39	30	48	53	62

(i) Plot a scatter diagram on graph paper for this data, labelling the axes, using a scale of 2 cm to represent 10 minutes on the t-axis and an appropriate scale for the x-axis. [2]

(ii) Suggest a reason why one of the data points does not seem to follow the trend and indicate the corresponding point on your diagram by labelling it P. [2]

Omit the point P.

(iii) Calculate the product moment correlation coefficient and comment on this value. [2]

(iv) Find the equation of the least squares regression line of t on x, writing your answer in the form $t = ax + b$. [1]

(v) Sketch the regression line on your scatter diagram and interpret the meaning of the value of a in the context of the question. [2]

(vi) Mr Lee needs to arrive at work no later than 8.30 am. Estimate, to the nearest minute, the latest time that he has to leave home without arriving late at work. [3]

11 A large group of Health and Fitness Club members is known to have a mean mass of 85 kg. The trainer claims that the mean mass of his members has decreased under his strict routine. To investigate his claim, the mass, x kg, of 30 randomly chosen members are collated and the results are summarised below.

$$\sum x = 2526, \qquad \sum (x - \bar{x})^2 = 544$$

(i) Find unbiased estimates of the population mean and variance. [3]

(ii) Test at the 10% level of significance whether the trainer's claim is valid. [5]

(iii) State the meaning of the p-value obtained in part **(ii)**. [1]

(iv) The trainer makes some adjustments to his training routine and the new population standard deviation is known to be 5 kg. A new sample of 30 members is randomly chosen and the mean mass of this sample is m kg. At the 10% level of significance, find the range of values of m for the trainer's claim to be valid, giving your answer correct to 2 decimal places. [3]

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1	Let \$$x$ be the entry rates for toddlers. Let \$$y$ be the entry rates for children. Let \$$z$ be the entry rates for adults. $0.75(2y+2z)=22.50$ $2y+2z=30 \quad ----(1)$ $0.8(x+y+2z)=20$ $x+y+2z=25 \quad ----(2)$ $0.5(x+3y+3z)+15=41.25$ $x+3y+3z=52.50 \quad ----(3)$ Using GC to solve eq (1), (2) and (3) $x=7.5$ $y=12.5$ $z=2.5$ $\therefore$ the entry rates are \$7.50 (Toddler), \$12.50 (children) and \$2.50 (adult) respectively.
2(i)	(a) Let $y=\dfrac{3}{\sqrt{(2x-7)^3}}=3(2x-7)^{-\frac{3}{2}}$ $\dfrac{dy}{dx}=3(2x-7)^{-\frac{5}{2}}\cdot\left(\dfrac{-3}{2}\right)\cdot(2)$ $=-9(2x-7)^{-\frac{5}{2}} \qquad \left(\text{or } -\dfrac{9}{(2x-7)^{\frac{5}{2}}}\right)$ (b) $\displaystyle\int_1^3 \frac{1}{e^{4t-3}}+\frac{1}{2t-1}\,dt$ $\displaystyle=\int_1^3 e^{-4t+3}+\frac{1}{2t-1}\,dt$ $=\left[-\dfrac{1}{4}e^{-4t+3}+\dfrac{1}{2}\ln(2t-1)\right]_1^3$ $=-\dfrac{1}{4}e^{-9}+\dfrac{1}{2}\ln(5)-\left(-\dfrac{1}{4}e^{-1}+\dfrac{1}{2}\ln(1)\right)$ $=-\dfrac{1}{4}\left(e^{-9}-e^{-1}\right)+\dfrac{1}{2}\ln 5$

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3

(i)

$3-10e^{2x}-8e^{4x}=0$

Let $u=e^{2x}$. Then

$\Rightarrow \quad 3-10u-8u^2=0$

$\Rightarrow \quad (2u+3)(1-4u)=0$

$\Rightarrow \quad u=-\dfrac{3}{2} \quad \text{or} \quad u=\dfrac{1}{4}$

$\Rightarrow \quad e^{2x}=-\dfrac{3}{2} \quad \text{or} \quad e^{2x}=\dfrac{1}{4}$

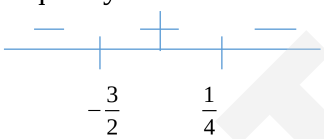
(rej $\because e^{2x}>0$)

$\therefore \quad 2x=\ln\dfrac{1}{4} \Rightarrow x=\dfrac{1}{2}\ln\dfrac{1}{4}=-\ln 2$

$a=-1$ and $b=2$

(ii)

For the inequality $3-10e^{2x}-8e^{4x}\le 0$,

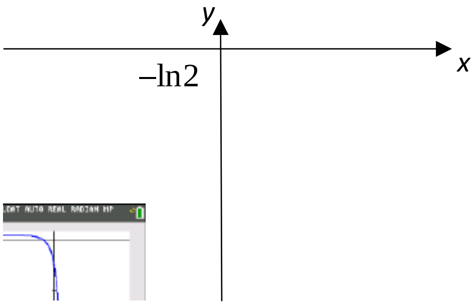


$e^{2x}\le -\dfrac{3}{2} \quad \text{or} \quad e^{2x}\ge \dfrac{1}{4}$

(rej $\because e^{2x}>0$)

$\therefore \quad x\ge -\ln 2$

Or use graphical method



Therefore,

$\therefore \quad x\ge -\ln 2$

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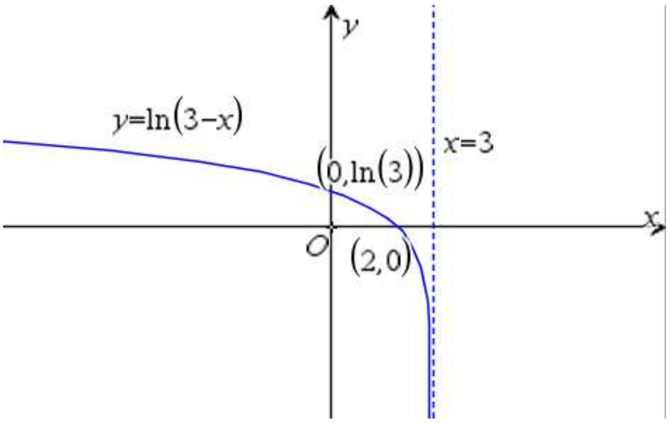
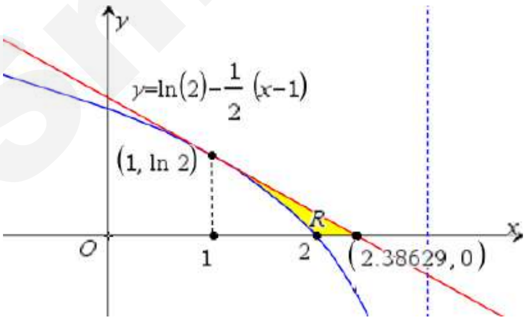
4	(i) $y=\ln(3-x)$  (ii) $\frac{dy}{dx}=-\frac{1}{3-x}$ At point P, $x=1$; $\frac{dy}{dx}=-\frac{1}{2}$; $y=\ln(2)$ Eqn of tangent at P: $y-\ln(2)=-\frac{1}{2}(x-1)$ $y=\ln(2)-\frac{1}{2}(x-1)$ $y=-\frac{1}{2}x+\ln 2+\frac{1}{2}$ (iii) $y=-\frac{1}{2}x+\ln 2+\frac{1}{2}$ $y=\ln 2-\frac{1}{2}(x-1)$  The area of R = Area of triangle – area under the curve $=\frac{1}{2}(2.38629-1)(\ln 2)-\int_1^2 \ln(3-x)\,dx$ $=0.0941586528$ ≈ 0.0942 units^2

5

(i)

$$x = t^3 - 13t^2 + 40t + 35$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 3t^2 - 26t + 40$$

For min value of x, $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$

$$3t^2 - 26t + 40 = 0$$

$$(3t - 20)(t - 2) = 0$$

$$\Rightarrow t = 2 \quad or \quad t = \frac{20}{3}$$

t	2^-	2	2^+
Sign of $\frac{dx}{dt}$	$+$	0	$-$
slope	/	—	\

t	$\left(\frac{20}{3}\right)^-$	$\frac{20}{3}$	$\left(\frac{20}{3}\right)^+$
Sign of $\frac{dx}{dt}$	$-$	0	$+$
slope	\	—	/

Therefore, x is a minimum when $t = \frac{20}{3}$.

Or

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 6t - 26$$

When $t = 2$, $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 6(2) - 26 = -14 < 0$

When $t = \frac{20}{3}$, $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 6\left(\frac{20}{3}\right) - 26 = 14 > 0$

Therefore, x is a minimum when $t = \frac{20}{3}$.

$$x = \left(\frac{20}{3}\right)^3 - 13\left(\frac{20}{3}\right)^2 + 40\left(\frac{20}{3}\right) + 35$$

$$= 20.185$$

$$x = 20.2 \quad \text{or} \quad \frac{545}{27}$$

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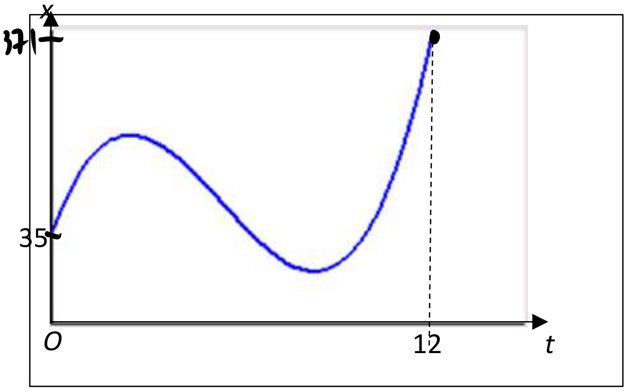
(ii)



(iii)

Area of the region

$$= \int_0^{12} t^3 - 13t^2 + 40t + 35 \ \mathrm{d}t$$

$$= \left[\frac{t^4}{4} - 13\frac{t^3}{3} + 20t^2 + 35t \right]_0^{12}$$

$$= \frac{1}{4}(12)^4 - \frac{13}{3}(12)^3 + 20(12)^2 + 35(20) - 0$$

$$= 996$$

(or using GC to solve)

The total manufacturing cost to manufacture the smart phones for a period of 12 months is \$996 million.

(iv)

$$P = 45 + 20\ln(3x+4)$$

$$\frac{dP}{dx} = \frac{60}{3x+4}$$

When $t = 8$, $x = 35$

$$\frac{\mathrm{d}P}{\mathrm{d}x} = \frac{60}{3(35)+4} = \frac{60}{109}$$

(v)

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\mathrm{d}P}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$$

When $x = 8$,

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{60}{109} \times [3(8)^2 - 26(8) + 40]$$

$$= \frac{1440}{109} \quad \text{or } 13.2$$

The rate of increase in profit when $t = 8$ is \$13.2 million per month.

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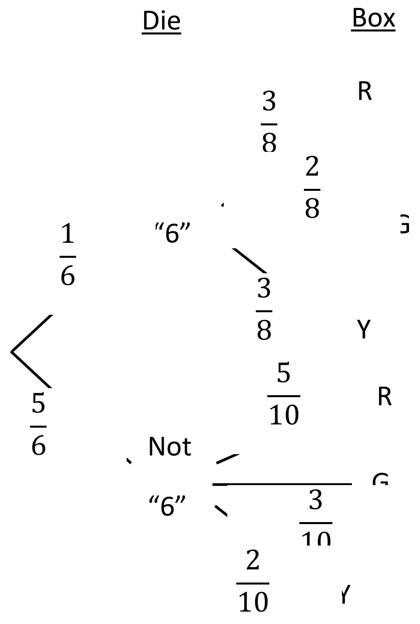
6	(i) Number of ways $= 4\times 3\times 2\times 2$ $= 48$ Required probability $= \dfrac{48}{^5P_4}$ $= \dfrac{48}{120}$ $= 0.4$ (ii) Case 1 : 1st digit is '4' Number of ways $= 1\times 3\times 1\times 2$ $= 6$ Case 2 : 1st digit is '3' or '5' Number of ways $= 2\times 3\times 2\times 2$ $= 24$ Total number of ways $= 24+6=30$ Required probability $= \dfrac{30}{48} = \dfrac{5}{8}$ **<u>Alternative method</u>** $\mathrm{P}(\text{greater than 3000} \mid \text{number is even})$ $= \dfrac{\mathrm{P}(\text{greater than 3000} \cap \text{number is even})}{\mathrm{P}(\text{number is even})}$ $= \dfrac{\frac{24+6}{120}}{0.4}$ $= \dfrac{5}{8}$

7	 (a)(ii) $\mathrm{P}\left(\text{wins a prize}\right)=\left(\frac{1}{6}\times\frac{3}{8}\right)+\left(\frac{5}{6}\times\frac{2}{10}\right)$ $=\frac{11}{48}$ (a)(iii) $\mathrm{P}\left(\text{from box A} \mid \text{wins the prize}\right)=\frac{\frac{1}{6}\times\frac{3}{8}}{\frac{11}{48}}$ $=\frac{3}{11}$ (b) $\mathrm{P}\left(\text{wins the grand prize}\right)=\left(\frac{1}{6}\times\frac{3}{8}\times\frac{2}{7}\right)+\left(\frac{5}{6}\times\frac{2}{10}\times\frac{1}{9}\right)$ $=\frac{55}{1512}$
8	(i) The assumption is that the event of a resident using ShareBike or not is independent of any other residents in the neighbourhood. (ii) The assumption may not hold as usually families may use ShareBike together as they are going for the activity together.

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	(iii) $X \sim \mathrm{B}(n, 0.09)$ $\mathrm{P}(X \geq 1) < 0.99$ $\mathrm{P}(X = 0) > 0.01$ $\binom{n}{0}(0.09)^0(1-0.09)^n > 0.01$ $(0.91)^n > 0.01$ $n < \frac{\ln 0.01}{\ln 0.91}$ $n < 48.830$ $\therefore$ Greatest value of n is 48. **<u>Alternative method</u>** Using GC, When $n = 47$, $\mathrm{P}(X = 0) = 0.0119$ (> 0.01) When $n = 48$, $\mathrm{P}(X = 0) = 0.0108$ (> 0.01) When $n = 49$, $\mathrm{P}(X = 0) = 0.0098$ (<0.01) $\therefore$ Greatest value of n is 48. (iv) $X \sim \mathrm{B}(20, 0.09)$ $\mathrm{P}(2 < X \leq 5) = \mathrm{P}(X \leq 5) - \mathrm{P}(X \leq 2)$ $= 0.260$ (3 s.f.) (v) $\mathrm{E}(X) = 20 \times 0.09 = 1.8$ $\mathrm{Var}(X) = 20 \times 0.09 \times (1-0.09) = 1.638$ Sample mean $\bar{X} = \frac{X_1 + X_2 + ...X_{40}}{40}$ Since $n = 40$ is sufficiently large, by Central Limit Theorem, $\bar{X} \sim \mathrm{N}\left(1.8, \frac{1.638}{40}\right)$ approximately. $\mathrm{P}\left(\bar{X} > 2\right) = 0.161$ (3 s.f.)
9	(i) $A \sim \mathrm{N}\left(43, 8^2\right)$ $B \sim \mathrm{N}\left(40, 6^2\right)$ $\mathrm{P}\left(A_1 + A_2 + A_3 < 3B\right) = \mathrm{P}\left(A_1 + A_2 + A_3 - 3B < 0\right)$ Let $S = A_1 + A_2 + A_5 - 3B$ $\mathrm{E}(S) = 3(43) - 3(40) = 9$ $\mathrm{Var}(S) = 3\left(8^2\right) + 3^2\left(6^2\right) = 516$ $\mathrm{P}\left(S < 0\right) = 0.346$ (3 s.f.)

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(ii)

$B \sim \mathrm{N}\left(40, 6^2\right)$

$\mathrm{E}\left(B_1 + B_2\right) = 2(40) = 80$

$\mathrm{Var}\left(B_1 + B_2\right) = 2\left(6^2\right) = 72$

$B_1 + B_2 \sim \mathrm{N}(80, 72)$

$$\text{Required probability} = \mathrm{P}\left(A > 38\right) \times \mathrm{P}\left(B_1 + B_2 > 82\right)$$
$$= 0.299 \quad (3 \text{ s.f.})$$

(iii)

$\mathrm{E}\left(A + B_1 + B_2\right) = 43 + 2(40) = 123$

$\mathrm{Var}\left(A + B_1 + B_2\right) = 8^2 + 2\left(6^2\right) = 136$

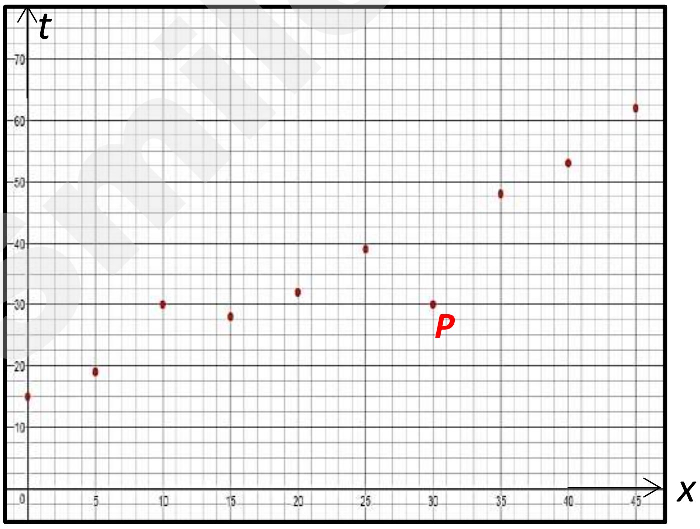
$A + B_1 + B_2 \sim \mathrm{N}(123, 136)$

$$\text{Required probability} = \mathrm{P}\left(A + B_1 + B_2 > 120\right)$$
$$= 0.602 \quad (3 \text{ s.f.})$$

(iv)

Because the case in (ii) is a proper subset of the case in (iii). For eg, Part iii contains cases whereby the lifespan of component A may not exceed 38 weeks (eg. 36 weeks) but total lifespan of 2 components of B exceeds 82 weeks (eg. 84 weeks), and yet the total lifespan is more than 120 weeks.

10 (i)



(ii)

Acceptable reasons:

The traffic condition on the road was good (Lesser cars on the road, no traffic jam) and thus he required much shorter travelling time though he left home only at 7.30am.

It was a public holiday/school holiday/Sunday and yet Mr Lee has to work.

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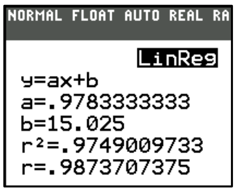
(iii)

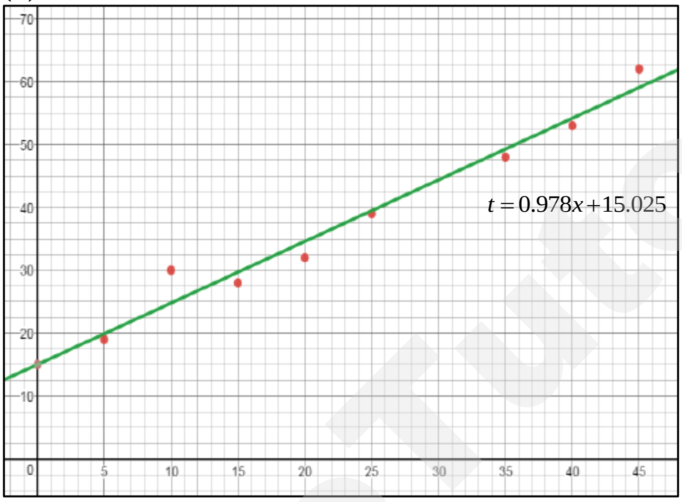
$r \approx 0.987$

The pmcc is close to 1, indicating a strong positive linear correlation between x and t. I.e. the later Mr Lee leaves home after 7 am, the longer the travelling time would take.



(iv) $t = 0.978x + 15.025$

(v)



$a = 0.978$ means that for every additional minute that Mr Lee delays in leaving home after 7am, his travelling time will increase by 0.978 minutes.

(vi)

Method 1:

There are 90 minutes from 7 am to 8.30 am.

$$x + t \leq 90$$

$$x + (0.97833x + 15.025) \leq 90$$

$$1.97833x \leq 74.975$$

$$x \leq 37.898$$

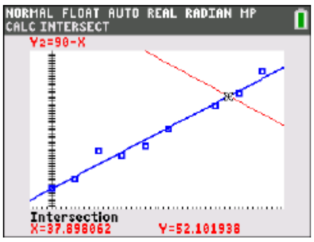
The largest possible value of x is 37 (correct to the nearest minute)

The latest time Mr Lee could leave home without being late for work is 7.37 am.

Method 2:

Sketch the line $x + t = 90$ and find x-coordinate of the point of intersection with the regression line.

If x = 38, Mr Lee will arrive late for work. Thus the latest time he needs to leave home is 7.37am.



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Method 3:

By Trial & Error, using GC

From part (vi),

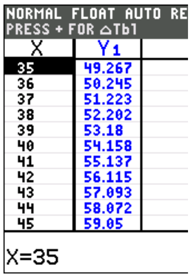
if $x = 40, t = 54.158, x + t > 90$

$x = 39, t = 53.18, \quad x + t > 90$

$x = 38, t = 52.202, \; x + t > 90$

$x = 37, t = 51.223, x + t < 90$

Thus the latest time he needs to leave home is 7.37am.



NORMAL FLOAT AUTO RE
PRESS + FOR △Tbl

X	Y1
35	49.267
36	50.245
37	51.223
38	52.202
39	53.18
40	54.158
41	55.137
42	56.115
43	57.093
44	58.072
45	59.05

X=35

11

(i) $\sum x = 2526, \qquad \sum (x-\bar{x})^2 = 544$

Unbiased estimates of the population mean μ is

$$\bar{x} = \frac{2526}{30} = 84.2$$

Unbiased estimates of the population variance σ^2 is

$$s^2 = \frac{30}{29}\left[\frac{544}{30}\right] = 18.75862 = 18.8 \; (3 \text{ s.f})$$

(ii)

$H_0: \; \mu = 85$

$H_1: \; \mu < 85$

Test at 10% significance level

Assuming that H_0 is true,

Since $n = 30$ is sufficiently large, by the Central Limit Theorem, $\bar{X} \sim N\left(85, \frac{s^2}{30}\right)$ approximately.

Test statistic: $Z = \dfrac{\bar{X} - 85}{\sqrt{\dfrac{18.75862}{30}}} \sim N(0,1)$ approximately.

Using GC, p-value = 0.15584 = 0.156 (3 s.f)

$$\left(\text{or } z = \frac{84.2 - 85}{\sqrt{\dfrac{18.75862}{30}}} = -1.0117 \right)$$

Since p-value = 0.15584 > 0.1 (or $z = -1.0117 > -1.28155$), we do not reject H_0 and conclude that there is insufficient evidence at 10% level, that the mean mass of the Health and Fitness Club members has decreased. (or that the trainer's claim is invalid.)

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(iii)
There is 0.15584 probability of drawing **a random sample of 30** Health and Fitness Club members with **sample mean less than 84.2 kg**, assuming that the population mean weight is 85 kg.

(iv)
$H_0: \mu = 85$

$H_1: \mu < 85$

Test at 10% level significance level.

Assuming that H_0 is true,

Since n is large, by the Central Limit Theorem, $\bar{X} \sim \mathrm{N}\left(85, \frac{5^2}{30}\right)$ approximately.

Test statistic: $Z = \dfrac{\bar{X} - 85}{5/\sqrt{30}} \sim \mathrm{N}(0,1)$ approximately.

Since the null hypothesis is rejected,

$\Rightarrow$ z-value falls inside critical region

$\Rightarrow$ z-value < -1.28155

$\Rightarrow \dfrac{m - 85}{5/\sqrt{30}} < -1.28155$

$m - 85 < -1.1699$

$m < 83.83$

$\therefore \quad 0 < m < 83.83$

Alternate method

Using $\bar{X} \sim \mathrm{N}\left(85, \frac{5^2}{30}\right)$

$\mathrm{P}\left(\bar{X} \le m\right) < 0.1$

$\Rightarrow m < 83.83$

$\therefore \quad 0 < m < 83.83$

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Name: ______________________________________ Class: ______________________

JURONG JUNIOR COLLEGE

Preliminary Examination 2017

MATHEMATICS
Higher 1

8865/01

28 August 2017

Paper 1

3 hours

Additional materials: Answer Paper
Cover Page
List of Formulae (MF 26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work together securely, with the cover page in front.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages.

[Turn over

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Section A: Pure Mathematics [40 marks]

1 John has a total of \$100 000 to be invested in stocks, bonds and gold. The rate of return for stocks, bonds and gold are 12%, 8% and 4% per year respectively. The income generated from stocks is the same as the income generated from bonds and gold combined. John has stipulated that the amount invested in stock should exceed twice the amount invested in bonds by \$4000. Find the total income from his investments at the end of the first year. [4]

2 The curve C has equation $x^2+(y-1)^2=4$.

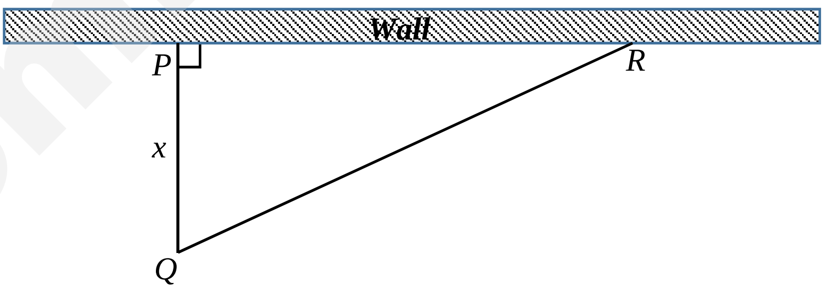
(i) Sketch C and write down the range of values of k such that $x^2+(k-1)^2=4$ has no real roots. [2]

(ii) Find the range of values of k such that the line $y=x+k+1$ intersects C exactly twice. [3]

3 A farmer wishes to make an animal enclosure using the wall as one side of the enclosure. The farmer intends to use 100 m of fencing. If PQ is x,

(i) show that area of the enclosure is $A=5\sqrt{100x^2-2x^3}$, [3]

(ii) find the value of x that gives the maximum possible area of the enclosure. [3]



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4 (i) Sketch on the same diagram, the graphs of C_1: $y = \ln(4x+2)$ and C_2: $y = 2 + \dfrac{3}{2x-7}$, stating the equation of any asymptotes and the exact coordinates of the points of intersection with the axes. [4]

(ii) Solve $\ln(4x+2) > 2 + \dfrac{3}{2x-7}$. [2]

(iii) Using differentiation, find the equation of the tangent to C_1 at $x = 2$. [3]

(iv) Find $\int 2 + \dfrac{3}{2x-7}\, \mathrm{d}x$. [1]

(v) Using your answer to part (iv), find the exact area of the region bounded by C_2, the lines $y = x$, $x = 6$ and $x = 8$. [3]

5 (a) The number of Type A bacteria (in millions) N after t days is modelled by

$$N = \frac{K}{1 + 0.5e^{-0.6t}}.$$

(i) Find the initial number of Type A bacteria (in millions) in terms of K. [1]

(ii) Find the long term population size of Type A bacteria in terms of K. [1]

(iii) After 2 days, the number of Type A bacteria is 320 millions. Find K. [2]

(iv) Sketch the graph of $N = \dfrac{K}{1 + 0.5e^{-0.6t}}$. [2]

(v) Find $\dfrac{\mathrm{d}N}{\mathrm{d}t}$. [2]

(vi) Find the rate of increase of the number of Type A bacteria after the third day. [2]

(b) The growth rate of Type B bacteria is given by $\dfrac{\mathrm{d}P}{\mathrm{d}t} = \dfrac{60e^{-1.1t}}{\left(1 + e^{-1.1t}\right)^2}$ millions per day.

(i) Evaluate $P = \int_0^5 \dfrac{60e^{-1.1t}}{\left(1 + e^{-1.1t}\right)^2}\, \mathrm{d}t$. [1]

(ii) What does P represent in the context of this question? [1]

[Turn Over

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Section B: Statistics [60 marks]

6 A fair six-sided die is tossed once. If the score on the die is 1 or 2, a ball is picked from bag *A*. If the score on the die is 3, 4, 5 or 6, a ball is picked from bag *B*. Bag *A* contains 6 red and 4 blue balls. Bag *B* contains 5 red, 3 blue and 2 green balls. Events *A* and *R* are defined as follows:

Event A = {A ball is picked from bag *A*}

Event R = {A red ball is picked}

Find

(i) $\mathrm{P}(R)$, [2]

(ii) $\mathrm{P}(A'|R)$. [3]

State with a reason whether events *A* and *R* are independent. [1]

7 On average, every 3 out of 8 appointments of a hospital consultant will start late. The number of these appointments which start late is the random variable *L*.

(i) State, in context, two assumptions needed for *L* to be well modelled by a binomial distribution. [2]

The consultant has seven appointments daily. Assume now that *L* follows a binomial distribution.

(ii) Find the most likely numbers of appointments that start late. [2]

(iii) Find the probability that at least half the appointments start late. [2]

(iv) For a particular week, the consultant works 5 days a week. Find the probability that, for no more than 2 of the days, at least half of the appointments start late. [2]

8 Members of the choir are in one these four vocal ranges: soprano, alto, tenor and bass. The sopranos and altos are women while the tenors and basses are men. A choir has four sopranos, three altos, three tenors and two basses.

(a) Find the number of ways to arrange the members of the choir in a row

(i) without restrictions, [1]

(ii) such that those of the same vocal range are together. [2]

(b) Five people are randomly selected from the choir. Find the probability that

(i) all the tenors are chosen, [2]

(ii) at least one women is chosen given that all the tenors are chosen. [3]

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9 The sleep pattern of 250 babies were tracked over 24 months. The age, x months, and the average daily sleep time, t hours, are given in the following table.

X	1	3	6	9	12	18	24
T	15.1	14.5	14.2	13.9	13.5	13.5	13

(i) Give a sketch of the scatter diagram for the data, ass shown on your calculator. [2]

(ii) Find $\overline{x}$ and $\overline{t}$ and mark the point $(\overline{x}, \overline{t})$ on your scatter diagram. [2]

(iii) Find the product moment correlation coefficient. [1]

(iv) Find the equation of the regression line of t on x and draw this line on your scatter diagram. [2]

(v) Calculate an estimate of the average daily sleep time of a 4–month old baby. Comment on the reliability of the estimate. [2]

(vi) Explain why it is not appropriate to estimate the average daily sleep time of a 32–month baby using the equation found in part **(iv)**. [1]

10 A company claims that the mean weight of bags of cashew nuts packed, in grams, is 500. A random sample of 60 bags is selected and the weight, x grams, of each bag is taken. The results obtained are summarised as follows:

$$\sum(x-500) = 318 \text{ and } \sum(x-500)^2 = 25548.4$$

(i) Find unbiased estimates of the population mean and the variance. [3]

(ii) Test at 3% level of significance whether the company's claim is valid. [4]

(iii) State, giving a reason, whether it is necessary to assume a normal distribution in order for the test to be valid. [1]

(iv) Find an inequality satisfied by the level of significance in order for the null hypothesis to be rejected. What conclusion can be drawn if the test is conducted at the 5% level of significance? [2]

[Turn Over

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The company now decides to test the claim that the mean weight of bags of cashew nuts packed, in grams, is at least 500. A second random sample of 70 bags is selected and the sample variance is 23.4^2 g^2 . Using a 5% level of significance, the company finds that the mean weight of bags of cashew nuts is not at least 500.

(v) Find the set of values within which the mean weight of bags of cashew nuts of this sample must lie to 2 decimal places. You may assume that the weight of bags of cashew nuts follows a normal distribution. [4]

11 Cheese tarts of a certain brand are sold in boxes containing 6 tarts. The masses, in grams, of the cheese tarts and of the empty boxes have independent normal distributions with means and standard deviations as shown in the following table.

	Mean (g)	Standard deviation (g)
Cheese tart	60	3.5
Empty box	52	0.8

(i) Find the probability that the mass of a cheese tart is less than 58 grams. [1]

(ii) Find the probability that a randomly chosen box of cheese tarts contains exactly 2 cheese tarts with mass less than 58 grams. [2]

(iii) Find the probability that the total mass of a box containing 6 cheese tarts is more than 415 grams. [3]

The cost of producing cheese tarts is 2.1 cents per gram and the cost of producing empty boxes is 0.3 cents per gram.

(iv) Find the probability that the total cost of producing a box containing 6 cheese tarts is between 747 cents and 774 cents. State the mean and variance of the distribution that you use. [5]

A rival brand of mini cheese tarts have masses with mean 35 grams and standard deviation 3.5 grams. A random sample of 30 mini cheese tarts is taken.

(v) Find the mean mass exceeded by 20% of these mini cheese tarts. [3]

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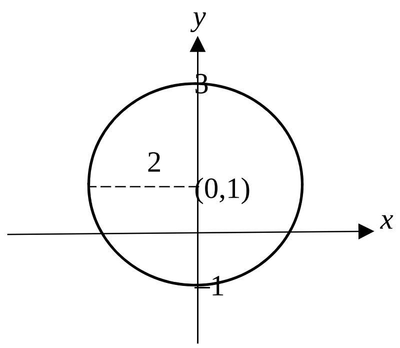
Qn	Soln
1	Let \$$x$, \$$y$, \$$z$ be the amount invested in stocks, bonds and gold respectively. $x+y+z=100\ 000$ $0.12x=0.08y+0.04z \Rightarrow \quad 0.12x-0.08y-0.04z=0$ $x-2y=4000$ Using GC, $x=28\ 000,\ y=12\ 000,\ z=60\ 000$ Total income $=0.12(28000)+0.08(12000)+0.04(60000)=6720$
2(a)	$x^2+(y-1)^2=4$ Curve C is a circle with centre (0,1) and radius 2 units.  $x^2+(k-1)^2=4$ has no real roots $\Rightarrow$ $y=k$ does not intersect the circle $\Rightarrow$ $k<-1$ or $k>3$
(b)	Sub $y=x+k+1$ into $x^2+(y-1)^2=4$, $x^2+(x+k)^2=4$ $x^2+x^2+2kx+k^2=4$ $2x^2+2kx+k^2-4=0$ Since line cuts C twice, Discriminant > 0 $(2k)^2-4(2)(k^2-4)>0$ $4k^2-8k^2+32>0$ $-4k^2+32>0$ $k^2-8<0$ $(k+\sqrt{8})(k-\sqrt{8})<0$ $-\sqrt{8}<k<\sqrt{8}$
3(i)	$QR=100-x$ $PR=\sqrt{(100-x)^2-x^2}$ $=\sqrt{10000-200x+x^2-x^2}$ $=\sqrt{10000-200x}$

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$$A=\frac{1}{2}x\sqrt{10000-200x}$$
$$=\frac{1}{2}\sqrt{x^2\left(10000-200x\right)}$$
$$=\frac{1}{2}\sqrt{10000x^2-200x^3}$$
$$=\frac{10}{2}\sqrt{100x^2-2x^3}$$
$$=5\sqrt{100x^2-2x^3}\quad\text{(shown)}$$

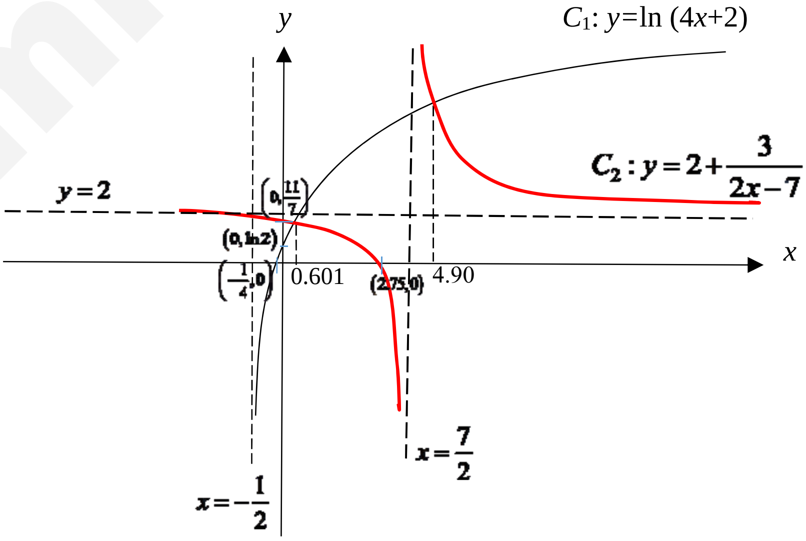
(ii)

$$\frac{\mathrm{d}A}{\mathrm{d}x}=5\left(\frac{1}{2}\right)\left(100x^2-2x^3\right)^{-\frac{1}{2}}\left(200x-6x^2\right)=\frac{5\left(100x-3x^2\right)}{\sqrt{100x^2-2x^3}}$$
$$\frac{5\left(100x-3x^2\right)}{\sqrt{100x^2-2x^3}}=0$$
$$100x-3x^2=0$$
$$x\left(100-3x\right)=0$$
$$x=0\ \text{(N.A.)}\quad or\quad x=\frac{100}{3}$$

x	33.2	$\frac{100}{3}=33.3$	33.4
$\frac{\mathrm{d}A}{\mathrm{d}x}$	0.345	0	−0.174
Slope	/	-	\

A is maximum when $x=\frac{100}{3}$

4 (i)



C_1: y=ln (4x+2): $\left(0,\ln 2\right)$ and $\left(-\frac{1}{4},0\right)$. Asymptote: $x=-\frac{1}{2}$

C_2: $y=2+\frac{3}{2x-7}$: $\left(0,\frac{11}{7}\right)$ and $\left(2.75,0\right)$. Asymptotes: $x=\frac{7}{2}$ and $y=2$.

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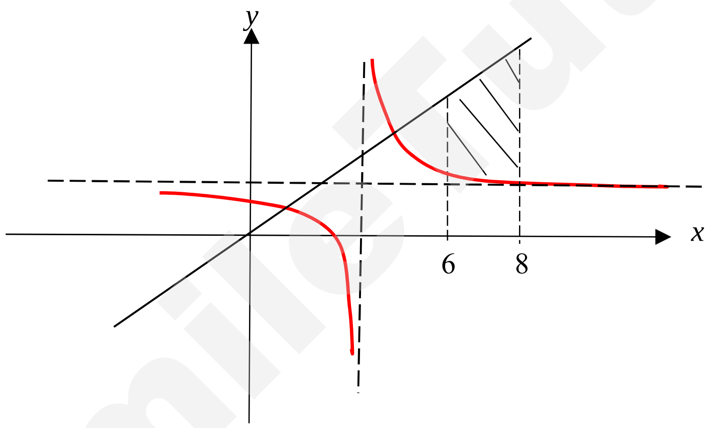
(ii)	For $\ln(4x+2) > 2+\dfrac{3}{2x-7}$, $0.601 < x < \dfrac{7}{2}$ or $x > 4.90$
(iii)	$y=\ln(4x+2)$ $\dfrac{dy}{dx}=\dfrac{4}{4x+2}=\dfrac{2}{2x+1}$ When $x=2$, $y=\ln 10$, $\dfrac{dy}{dx}=\dfrac{2}{5}$ Equation of the tangent to the curve: $y-\ln 10=\dfrac{2}{5}(x-2)$ $y=\dfrac{2}{5}x-\dfrac{4}{5}+\ln 10$
(iv)	$\int 2+\dfrac{3}{2x-7}\ dx = 2x+\dfrac{3}{2}\ln(2x-7)+c$
(v)	 Area $=\int_6^8 x\ dx-\int_6^8 2+\dfrac{3}{2x-7}\ dx$ $=\left[\dfrac{x^2}{2}\right]_6^8-\left[2x+\dfrac{3}{2}\ln(2x-7)\right]_6^8$ $=\left[\dfrac{8^2}{2}-\dfrac{6^2}{2}\right]-\left[\left[2(8)+\dfrac{3}{2}\ln(2(8)-7)\right]-\left[2(6)+\dfrac{3}{2}\ln(2(6)-7)\right]\right]$ $=14-\left[4+\dfrac{3}{2}\ln 9-\dfrac{3}{2}\ln 5\right]$ $=10-\dfrac{3}{2}\ln\dfrac{9}{5}$
	<u>Alternative</u> Area = Area of trapezium $-\int_6^8 2+\dfrac{3}{2x-7}\ dx$

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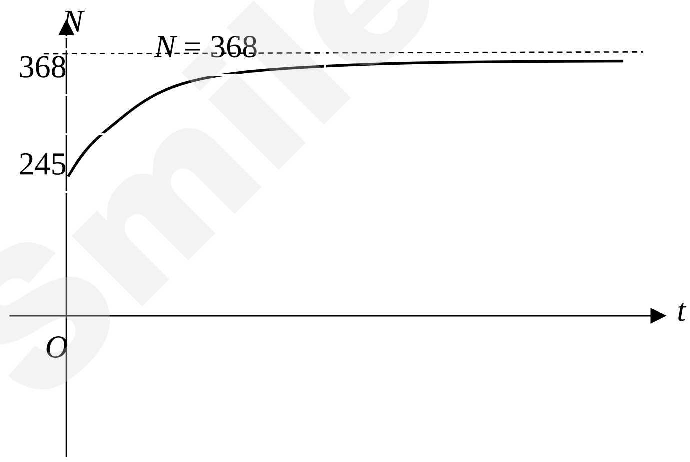
	$=\frac{1}{2}(6+8)(2)-\int_6^8 2+\frac{3}{2x-7}\,dx$ $=14-\left[2x+\frac{3}{2}\ln(2x-7)\right]_6^8$ $=14-\left[\left[2(8)+\frac{3}{2}\ln(2(8)-7)\right]-\left[2(6)+\frac{3}{2}\ln(2(6)-7)\right]\right]$ $=14-\left[4+\frac{3}{2}\ln 9-\frac{3}{2}\ln 5\right]$ $=10-\frac{3}{2}\ln\frac{9}{5}$
5 (a) (i)	$N=\frac{K}{1+0.5e^{-0.6t}}$ When $t=0$, $N=\frac{K}{1+0.5e^{0}}=\frac{2}{3}K$
(ii)	As $t\to\infty, e^{-0.6t}\to 0, N\to K$ The long term population size is K millions.
(iii)	$t=2$, N=320 $320=\frac{K}{1+0.5e^{-0.6(2)}}$ $K=320\left(1+0.5e^{-1.2}\right)=368.191\approx 368$
(iv)	
(v)	$N=\frac{368.19}{1+0.5e^{-0.6t}}=368.19(1+0.5e^{-0.6t})^{-1}$ $\frac{dN}{dt}=-368.29(1+0.5e^{-0.6t})^{-2}\left[(-0.6)0.5e^{-0.6t}\right]$ $=\frac{368.19\times 0.5\times 0.6e^{-0.6t}}{\left(1+0.5e^{-0.6t}\right)^2}$ $\approx\frac{110e^{-0.6t}}{\left(1+0.5e^{-0.6t}\right)^2}$

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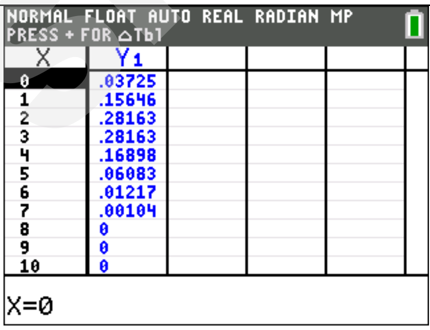
(vi)	After 3 days, the rate of increase is: $\frac{dN}{dt}=\frac{110e^{-0.6(3)}}{\left(1+0.5e^{-0.6(3)}\right)^2}=15.5127\approx 15.5$ millions per day.
(b) (i)	$P=\int_0^5 \frac{60e^{-1.1t}}{\left(1+e^{-1.1t}\right)^2}\,dt=27.051\approx 27.1$
(ii)	P represents the growth of number of Type B bacteria in millions during the first 5 days.
6(i)	$P(R)=\left(\frac{2}{6}\times\frac{6}{10}\right)+\left(\frac{4}{6}\times\frac{5}{10}\right)=\frac{8}{15}$
(ii)	$P(A'\mid R)=\frac{P(A'\cap R)}{P(R)}=\frac{\frac{4}{6}\times\frac{5}{10}}{\frac{8}{15}}=\frac{5}{8}$
	A and R are not independent Any of the following reasons: (1) $P(R\mid A)=\frac{6}{10}\neq P(R)=\frac{8}{15}$ OR (2) $P(A)\times P(R)=\frac{2}{6}\times\frac{8}{15}=\frac{8}{45}$ & $P(A\cap R)=\frac{2}{6}\times\frac{6}{10}=\frac{1}{5}$ $P(A\cap R)\neq P(A)\times P(R)$ OR (3) $P(A'\mid R)=\frac{5}{8}\neq P(A')=\frac{4}{6}$, A' and R are not independent, thus A and R are not independent.
7(i)	(1) The probability that any one appointment will start late remains constant throughout the sample. (2) The punctuality of each appointment is independent of the punctuality of any other appointments. OR Whether an appointment start late is independent of any other appointments that start late.
(ii)	 Using GC, [GC keystrokes: Y_1= binompdf (7, $\frac{3}{8}$,x)], the most likely numbers of appointments that start late = 2 and 3 [Note: most likely number is referring to the **MODE**, not $E(X)$]
(iii)	$L \sim B\left(7,\frac{3}{8}\right)$ $P(L\geq 3.5)=P(L\geq 4)$ $=1-P(L\leq 3)$ $=0.24302$ ≈ 0.243 (3sf)

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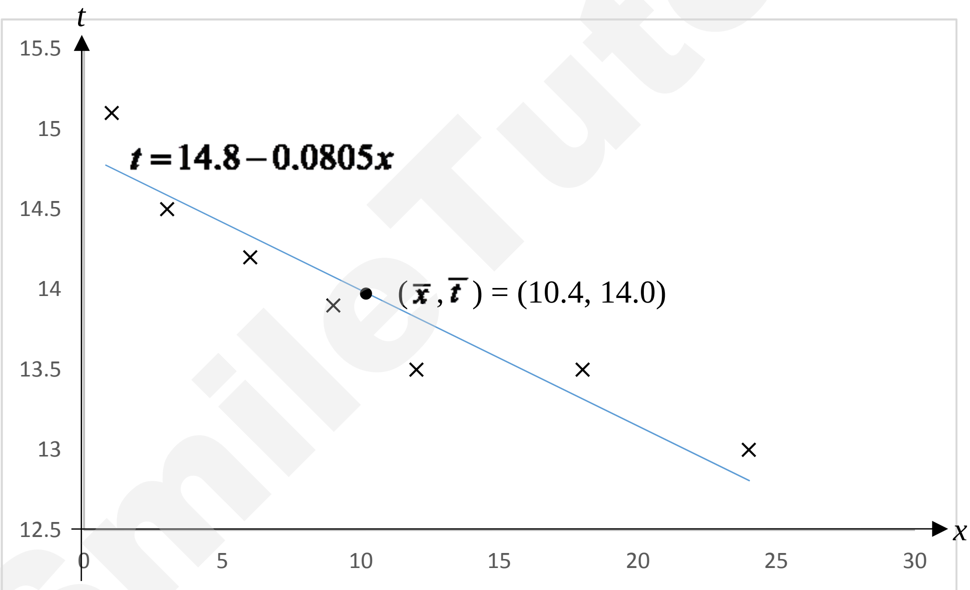
(iv)	Let X denote the number of days out of 5, with at least half of the appointments starting late. $X \sim B(5, 0.24302)$ $\mathrm{P}(X \le 2) = 0.90371 \approx 0.904$ (3sf)
8(a)(i)	No. of ways $= 12! = 479\,001\,600$
(a)(ii)	No. of ways $= (4! \times 3! \times 3! \times 2!) \times 4! = 41472$
(b)(i)	P(all the tenors are chosen)$= \dfrac{{}^3C_3 \times {}^9C_2}{{}^{12}C_5} = \dfrac{36}{792} = \dfrac{1}{22}$ (or 0.0455 3sf)
(b)(ii)	P(at least one woman is chosen | all the tenors are chosen) $= \dfrac{\text{n(1 woman, 3 tenors, 1 bass) + n (2 woman, 3 tenors)}}{\text{n(all tenors are chosen)}}$ $= \dfrac{({}^7C_1 \times {}^3C_3 \times {}^2C_1) + ({}^7C_2 \times {}^3C_3)}{{}^3C_3\,{}^9C_2}$ $= \dfrac{35}{36}$ or 0.972 (3sf)
9(i)	
(ii)	$(\bar{x}, \bar{t}) = (10.4, 14.0)$
(iii)	$r = -0.941$
(iv)	$t = 14.796 - 0.080474x$ $t = 14.8 - 0.0805x$ (3 s.f.)
(v)	When $x = 4$ $t = 14.796 - 0.080474(4) = 14.474 \approx 14.5$ hours (3 s.f.) Since $r = -0.941$ is <u>close to – 1</u>, indicating a strong negative linear correlation between the age and average total sleep time of babies and <u>$x = 4$ is within the data range</u>, this is an interpolation. Hence, the estimate is reliable.
(vi)	$x = 32$ is outside the data range, we are doing extrapolation. The estimate will not be reliable.

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10(i)	$\bar{x}=\frac{\sum(x-500)}{60}+500=\frac{318}{60}+500=505.3$ $s^2=\frac{1}{n-1}\left[\sum(x-500)^2-\frac{\left(\sum(x-500)\right)^2}{n}\right]$ $=\frac{1}{59}\left[25548.4-\frac{(318)^2}{60}\right]$ $=\frac{23863}{59}$ or $404.46\approx 404$ (3sf)
(ii)	$\mathrm{H_0}: \mu=500$ $\mathrm{H_1}: \mu\neq 500$ Under $\mathrm{H_0}$, $\bar{X}\sim \mathrm{N}\left(500, \frac{23863/59}{60}\right)$ approximately by CLT since n = 60 is large. Test statistic $Z=\frac{\bar{X}-500}{\sqrt{\frac{23863/59}{60}}}\sim \mathrm{N}(0,\ 1)$ approximately. α = 0.03 From GC, p-value = 0.0412 Since p-value = 0.0412 > α = 0.03, we do not reject $\mathrm{H_0}$ at the 3% level of significance and conclude there is insufficient evidence that the population mean weight of bags of cashew nuts is not 500g. ie, there is insufficient evidence at the 3% level of significance that the claim is invalid.
(iii)	It is not necessary to assume a normal distribution as the sample size, n = 60, is large, by Central Limit Theorem, $\bar{X}$ has a normal distribution approximately.
(iv)	For $\mathrm{H_0}$ to be rejected, $\alpha\geq 0.0412$. If the test is conducted at the 5% level of significance, since $\alpha=0.05>0.0412=p$-value, we reject $\mathrm{H_0}$ at the 5% level of significance and conclude there is sufficient evidence that the population mean weight of bags of cashew nuts is not 500g. ie, there is sufficient evidence at the 5% level of significance that the claim is invalid.
(v)	$s^2=\frac{70}{69}(23.4^2)=\frac{63882}{115}=555.50\approx 556$ $\mathrm{H_0}: \mu=500$ $\mathrm{H_1}: \mu<500$ Under $\mathrm{H_0}$, $\bar{X}\sim \mathrm{N}\left(500, \frac{555.50}{70}\right)$ Test statistic $Z=\frac{\bar{X}-500}{\sqrt{\frac{555.50}{70}}}\sim \mathrm{N}(0,\ 1)$

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	$\alpha = 0.05$ mean weight of bags of cashew nuts is not at least 500 g $\Rightarrow$ H_0 is rejected at 5% level of significance $\Rightarrow \dfrac{\bar{x}-500}{\sqrt{\frac{555.50}{70}}} \le -1.6449$ $\Rightarrow \bar{x} \le 500-1.6449\sqrt{\frac{555.50}{70}}$ $\Rightarrow \bar{x} \le 495.37(2\text{dp})$
11	Let X and Y denote the mass of a cheese tart and an empty box respectively. $X \sim \mathrm{N}(60, 3.5^2)$ and $Y \sim \mathrm{N}(52, 0.8^2)$
(i)	P(X<58) = 0.28385 $\approx$ 0.284 (3sf)
(ii)	Let N be the number of tarts in a box with mass less than 58g. $N \sim \mathrm{B}(6, 0.28385)$ $\mathrm{P}(N=2) = 0.31790 \approx 0.318$ (3sf)
	Alternatively, Probability $= {}^6C_2(0.28385)^2(1-0.28385)^4$ $= 0.31790 \approx 0.318$ (3sf)
(iii)	Let $T = X_1 + X_2 + \ldots + X_6 + Y$ $\mathrm{E}(T) = 6\mathrm{E}(X) + \mathrm{E}(Y) = 6(60) + 52 = 412$ $\mathrm{Var}(T) = 6\mathrm{Var}(X) + \mathrm{Var}(Y) = 6(3.5^2) + 0.8^2 = 74.14$ $T \sim \mathrm{N}(412, 74.14)$
	P(T > 415) = 0.36376 $\approx$ 0.364 (3sf)
(iv)	$C = 2.1(X_1 + X_2 + \ldots + X_6) + 0.3Y$ $\mathrm{E}(C) = 2.1 \times 6\mathrm{E}(X) + 0.3\mathrm{E}(Y)$ $= (2.1)(6)(60) + 0.3(52)$ $= 771.6$ $\mathrm{Var}(C) = 2.1^2 \times 6\mathrm{Var}(X) + 0.3^2\mathrm{Var}(Y)$ $= 2.1^2(6)(3.5^2) + 0.3^2(0.8^2)$ $= 324.1926$ $C \sim \mathrm{N}(771.6\ ,\ 324.1926)$
	P(747 <C < 774) = 0.467 (3sf)
(v)	Let M denote the mass of a mini cheese tart. $\mathrm{E}(M) = 35$, $\mathrm{Var}(M) = 3.5^2$ $\bar{M} \sim \mathrm{N}(35, \frac{3.5^2}{30})$ approximately by CLT since n = 30 is large. $\mathrm{P}(\bar{M} > a) = 0.2$ $\mathrm{P}(\bar{M} \le a) = 1 - 0.2 = 0.8$ $a = 35.5$ (3sf)

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MERIDIAN JUNIOR COLLEGE
JC2 Preliminary Examination
Higher 1

H1 Mathematics 8865/01

Paper 1 **14 September 2017**

3 Hours

Additional Materials: Writing paper
Graph Paper
List of Formulae (MF 26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics group on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

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Section A: Pure Mathematics [40 marks]

1 Azel, Brenda, Cathy and Dillion went to the wet market to buy three different kinds of fish. As there were no receipts provided, they did not know how much they paid for the individual prices of fish per kilogram. However, Azel, Brenda and Cathy can remember the total amount that they each paid. The weights of the different kinds of fish and the total amounts paid are shown in the following table.

	Azel	Brenda	Cathy	Dillon
Black pomfret (kg)	0.55	0.60	0.40	0.70
Sea Bass (kg)	0.45	0.58	0.75	0.34
Golden snapper (kg)	1.45	1.60	1.70	1.42
Total amounts paid in dollars	38.77	44.18	45.81	[redacted]

Assuming that, for each kind of fish, the price per kilogram paid by them is the same, calculate the total amount that Dillon paid. [5]

2 Differentiate $\ln\left(2x^2+1\right)$ with respect to x. Hence find the exact value of $\int_1^3 \frac{x}{2x^2+1}\,dx$, leaving your answer in the form $a\ln b$ where a and b are constants to be determined. [5]

3 Find the exact equation of the tangent to the curve $\ln y=(2-x)^2$ at the point where $x=3$. [5]

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4 A curve C has equation $y = kx^2 + k$, where k is a positive constant.

(i) Sketch C. [2]

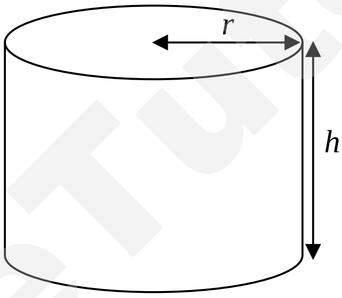
(ii) Find the range of values of k for which the line $y = 4x + 3$ intersects the curve C at 2 distinct points. [4]

It is given that $k = 3$.

(iii) Find the exact x-coordinates of the points of intersection between the line $y = 4x + 3$ and the curve C. [2]

(iv) Hence find the exact area bounded by the line $y = 4x + 3$ and the curve C. [4]

5



A factory decides to design a closed cylindrical water tank of radius r cm and height h cm to hold a maximum of 16000π cm^3 of water. The outer surface area (including the base and lid) of the tank is to be coated with a layer of paint. It is assumed that the thickness of the cylindrical water tank is negligible.

(i) Show that the outer surface area of the tank, S cm^2 is given by $S = \dfrac{32000\pi}{r} + 2\pi r^2$. [3]

(ii) In order to reduce the amount of paint used, the factory wishes to minimize the value of S. Using differentiation, find the value of r for which S is minimized. Hence find the minimum value of S. [5]

(iii) Sketch, in this context, the graph of S against r. [2]

(iv) The tank with minimum value of S is being manufactured. Water is being poured into this cylindrical tank at a constant rate of 1000 cm^3 per minute. Find the rate of change of the depth of water. [3]

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Section B: Statistics [60 marks]

6 Five numbers 1, 3, 6, 7 and 8 are used to form a five-digit number. If each number can only be used once, find the number of ways such that the five-digit number is

(i) formed without further restrictions, [1]

(ii) odd and between 30000 and 80000. [3]

It is now given that all five numbers can be used with repetitions. Find the number of ways to form the five-digit number if it must be even. [2]

7 A manufacturer produces a large number of mugs everyday and the mugs are sold in batches of 50. On average, a proportion p of the mugs are defective. The random variable X is the number of defective mugs in a randomly chosen batch of 50. It is assumed that X has the distribution $\mathrm{B}(50, p)$.

(i) Given that $\mathrm{P}(X = 0 \text{ or } 1) = 0.15$, formulate an equation in p and find the value of p. [3]

For the rest of the question, use $p = 0.06$.

In order to ensure the highest quality in their product, the manufacturer decides to carry out a quality control test using one of the following methods:

Method A: Select 10 mugs from the batch at random and accept the batch if there are fewer than 3 defective mugs, otherwise reject the batch.

Method B: Select 5 mugs from the batch at random and accept the batch if there is no defective mug, reject if there are two or more defective mugs, otherwise select another 5 mugs at random from the batch. If the second sample is drawn, accept the batch if there are fewer than 2 defective mugs, otherwise reject the batch.

(ii) By considering the probability of accepting the batch in each method, justify which method should the manufacturer adopt to carry out the quality control test. [6]

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8 **(a)** A and B are two events such that $\mathrm{P}\left(A \mid B'\right)=\frac{4}{17}$, $\mathrm{P}(B)=\frac{23}{40}$ and $\mathrm{P}(A \cap B)=\frac{3}{8}$.

By using a Venn diagram or otherwise, find $\mathrm{P}\left(A' \cap B'\right)$. [3]

Determine if A and B are independent events. [2]

(b) A basket contains 35 durians, of which 15 are MSW durians and 20 are D24 durians. Of the MSW durians, 4 are infested with maggots and of the D24 durians, 3 are infested with maggots. Two durians are chosen at random from the basket.

(i) Show that the probability that both are MSW durians and at least one durian is infested with maggots is $\frac{10}{119}$. [2]

(ii) Given that at least one durian is infested with maggots, find the probability that both are MSW durians. [3]

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9 Peter is interested to find out how the sale of his books varies with the selling price. Over a period of eight weeks, he sells the books at a different selling price, x (in dollars), which is fixed for each week. He also records the number of books, y, sold in that week. The results are summarised in the following table.

Week	1	2	3	4	5	6	7	8
Selling Price (x dollars)	70	60	50	35	26	17	12	10
Number of Books Sold (y)	3	80	8	20	30	50	75	100

(i) Draw a scatter diagram to illustrate the above information, labelling the axes clearly. [2]

(ii) On the scatter diagram, circle the data point which is an outlier and label it P. [1]

(iii) Omitting the point P, find the product moment correlation coefficient and the least square regression line $y = a + bx$. Sketch this line on your scatter diagram. You are not required to find the axial intercepts. [3]

(iv) Use the least square regression line to estimate the number of books sold when the selling price of the book for that week is \$75. Comment on the reliability of this estimate. [2]

(v) Comment on whether a linear model would be appropriate, referring to both the scatter diagram (omitting the point P) and the context of the question. [2]

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10 **(a)** A chocolate company claims that, on average, the consumption of their dark chocolate over time can decrease one's cholesterol level by at least 20 mg/dL. Over a period of time, a random sample of 80 volunteers who consume this dark chocolate daily have their cholesterol level measured. The table below shows the decrease in the cholesterol level, measured in mg/dL for the 80 volunteers.

Decrease in the cholesterol level (in mg/dL)	17	18	19	20	21	22	23	24
Number of volunteers	8	15	13	18	14	6	4	2

Test the chocolate company's claim at 5% significance level. [6]

(b) The principal of Meridian Childcare Centre claims that a child at the centre spends an average of μ_0 hours on afternoon nap. A survey was conducted and the time, x hours, spent by 60 randomly chosen children on afternoon naps is as follows:

$$\sum x = 185, \quad \sum x^2 = 626$$

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Find the range of values of μ_0 such that the principal is confident, at 4% level of significance, that he did not indicate wrongly the mean time that the children spend on afternoon naps. [4]

[Question 11 is printed on the next page.]

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11 In a particular supermarket, fishes are priced according to their weight. The weight of a randomly chosen Black Tilapia fish has a normal distribution with mean μ grams and standard deviation 32 grams. It is found that 10% of the Black Tilapia fish weigh heavier than 541 grams.

(i) Show that the mean weight of a Black Tilapia fish is 500 grams, correct to the nearest grams. [3]

(ii) Find the probability that out of three randomly chosen Black Tilapia fish, two weigh between 440 grams and 550 grams and one weighs more than 550 grams. [3]

(iii) The Black Tilapia fish is priced at \$ a per kg. It is given that the probability of the total price of 2 randomly chosen Black Tilapia fish cost less than \$6.90 is less than 0.84241, correct to 5 significant figures. Find the range of values of a. [4]

The supermarket also sells Grey Mullet fish. The weight of a randomly chosen Grey Mullet fish follows an independent normal distribution with mean weight 800 grams and standard deviation 50 grams.

(iv) Find the probability that the average weight of two randomly chosen Black Tilapia fish and three randomly chosen Grey Mullet fish exceeds 690 grams. [3]

End of Paper

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2017 H1 MATH (8865/01) JC 2 PRELIM – SOLUTIONS

Qn	Solution
1	**System of Linear Equations**
	Let x = price of black pomfret per kilogram (in dollars) y = price of sea bass per kilogram (in dollars) z = price of golden snapper per kilogram (in dollars) For Azel: $0.55x+0.45y+1.45z=38.77$ For Brenda: $0.60x+0.58y+1.6z=44.18$ For Cathy: $0.4x+0.75y+1.7z=45.81$ Using GC, $x=14.90$, $y=18.00$, $z=15.50$ Total price paid by Dillon = $\$((0.7\times14.90)+(0.34\times18.00)+(1.42\times15.50))$ = \$ 38.56

Qn	Solution
2	**Techniques of Differentiation and Integration**
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(2x^2+1\right)\right)=\frac{4x}{2x^2+1}$ $\int_1^3 \frac{x}{2x^2+1}\,\mathrm{d}x=\frac{1}{4}\int_1^3\frac{4x}{2x^2+1}\,\mathrm{d}x$ $=\frac{1}{4}\left[\ln\left(2x^2+1\right)\right]_1^3$ $=\frac{1}{4}\left(\ln 19-\ln 3\right)$ $=\frac{1}{4}\ln\frac{19}{3}$ $\therefore a=\frac{1}{4},\ b=\frac{19}{3}$

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Qn	Solution
3	**Exponential and Logarithm (Equation of Tangent)**
	$\ln y=(2-x)^2$ $y=\mathrm{e}^{(2-x)^2}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-2(2-x)\mathrm{e}^{(2-x)^2}$ When $x=3$, $\frac{\mathrm{d}y}{\mathrm{d}x}=2\mathrm{e}$, $y=\mathrm{e}$. Equation of tangent is $y-\mathrm{e}=2\mathrm{e}(x-3)$ $\Rightarrow y=2\mathrm{e}x-6\mathrm{e}+\mathrm{e}$ $\Rightarrow y=2\mathrm{e}x-5\mathrm{e}$

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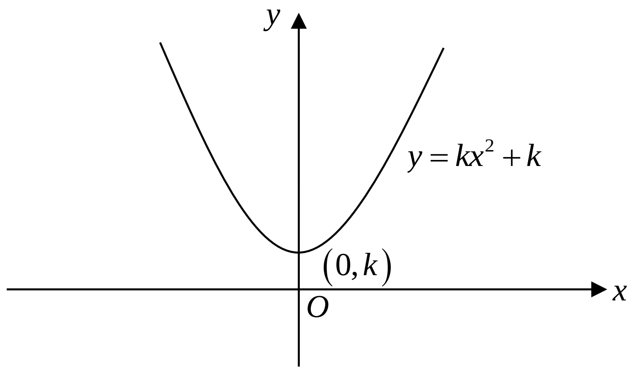
Qn	Solution
4	**Curve Sketching and Application of Integration**
(i)	
(ii)	$kx^2 + k = 4x + 3$ $kx^2 - 4x + k - 3 = 0$ Discriminant > 0 $(-4)^2 - 4(k)(k-3) > 0$ $16 - 4k^2 + 12k > 0$ $k^2 - 3k - 4 < 0$ $(k-4)(k+1) < 0$ $-1 < k < 4$ Since $k > 0$, $0 < k < 4$
(iii)	When $k = 3$ $3x^2 - 4x + 3 - 3 = 0$ $x(3x-4) = 0$ $x = 0$ or $x = \frac{4}{3}$
(iv)	$\int_0^{\frac{4}{3}} 4x + 3 - (3x^2 + 3)\ \mathrm{d}x$ $= \int_0^{\frac{4}{3}} 4x - 3x^2\ \mathrm{d}x$ $= \left[2x^2 - x^3\right]_0^{\frac{4}{3}}$ $= \left[2\left(\frac{4}{3}\right)^2 - \left(\frac{4}{3}\right)^3\right]$ $= \frac{32}{27}$

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Qn	Solution
5	**Maxima/Minima and Connected Rate of Change**

(i)

$16000\pi = \pi r^2 h$

$h = \dfrac{16000}{r^2}$ --- (1)

$S = 2\pi rh + 2\pi r^2$ --- (2)

Substitute (1) into (2),

$S = 2\pi r\left(\dfrac{16000}{r^2}\right) + 2\pi r^2$

$S = \dfrac{32000\pi}{r} + 2\pi r^2$ (shown)

(ii)

For minimum S, $\dfrac{dS}{dr} = 0$.

$\therefore \dfrac{dS}{dr} = -\dfrac{32000\pi}{r^2} + 4\pi r = 0$

$4\pi r = \dfrac{32000\pi}{r^2}$

$r^3 = 8000$

$r = \sqrt[3]{8000}$

$r = 20$

For $r = 20$,

r	20^-	20	20^+
$\dfrac{dS}{dr}$	$-$	0	$+$
Slope of curve	╲	—	╱

$\therefore S$ is minimum when $r = 20$.

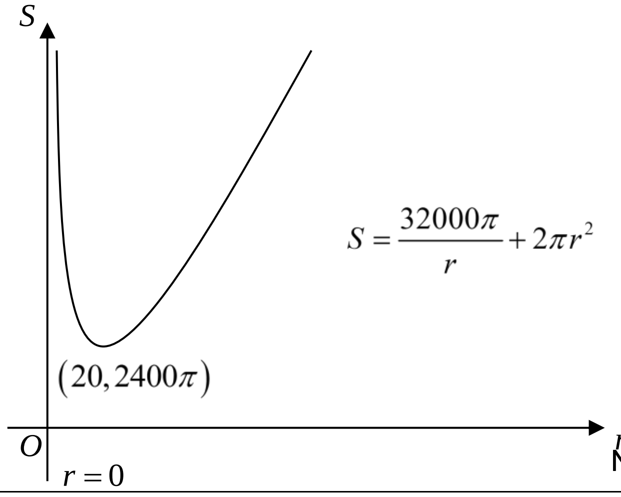
When $r = 20$,

$S = \dfrac{32000\pi}{20} + 2\pi(20)^2$

$= 1600\pi + 800\pi$

$= 2400\pi \text{ cm}^2$ or 7540 cm^2 (3s.f)

(iii)



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(iv)	Given: $\dfrac{\mathrm{d}V}{\mathrm{d}t} = 1000\ \text{cm}^3/\text{min}$ $V = \pi r^2 h$ $V = \pi(20)^2 h$ $V = 400\pi h$ $\dfrac{\mathrm{d}V}{\mathrm{d}h} = 400\pi$ Chain rule: $\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{\mathrm{d}V}{\mathrm{d}t} \times \dfrac{\mathrm{d}h}{\mathrm{d}V}$ $= (1000) \times \dfrac{1}{400\pi}$ $= 0.796\ \text{cm/min}$ (3 s.f)

Qn	Solution
6	**Permutations and Combinations**
(i)	Number of ways = $5! = 120$
(ii)	<u>Case 1 (first digit 3 or 7)</u> Number of ways $= 2 \times 2 \times 3! = 24$ <u>Case 2 (first digit 6)</u> Number of ways $= 3 \times 3! = 18$ Total number of ways $= 24 + 18 = 42$
	Number of ways = $5^4 \times 2 = 1250$

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Qn	Solution
7	**Binomial Distribution**
(i)	Let X be the number of mugs, out of 50, that are defective. $X \sim \mathrm{B}(50, p)$ $\mathrm{P}(X = 0 \text{ or } 1) = 0.15$ $\mathrm{P}(X = 0) + \mathrm{P}(X = 1) = 0.15$ $\binom{50}{0} p^0 (1-p)^{50} + \binom{50}{1} p^1 (1-p)^{49} = 0.15$ $(1-p)^{50} + 50p(1-p)^{49} = 0.15$ Using GC, $p = 0.0659$ (3 s.f.)
(ii)	**Method A:** Let Y be the number of mugs, out of 10, that are defective. $Y \sim \mathrm{B}(10, 0.06)$ $\mathrm{P}(\text{accept the batch})$ $= \mathrm{P}(Y < 3)$ $= \mathrm{P}(Y \le 2)$ $= 0.981$ (3 s.f) **Method B:** Let W be the number of mugs, out of 5, that are defective. $W \sim \mathrm{B}(5, 0.06)$ $\mathrm{P}(\text{accept the batch})$ $= \mathrm{P}(W = 0) + \mathrm{P}(W = 1)\mathrm{P}(W < 2)$ $= \mathrm{P}(W = 0) + \mathrm{P}(W = 1)\mathrm{P}(W \le 1)$ $= 0.73390 + (0.23422)(0.96813)$ $= 0.961$ (3 s.f) Since the **probability** of accepting the batch is **higher for Method A**, the manufacturer should adopt <u>**Method A**</u> to carry out the quality control test as it will shorten the process of the quality control test. Since the **probability** of accepting the batch is **lower for Method B**, the manufacturer should adopt <u>**Method B**</u> to carry out the quality control test as it ensures a more stringent quality control test.

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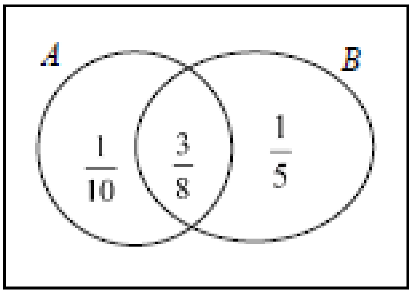
Qn	Solution
8	**Probability**
(a)	$\mathrm{P}(A\|B')=\frac{4}{17}$ $\frac{\mathrm{P}(A\cap B')}{\mathrm{P}(B')}=\frac{4}{17}$ $\mathrm{P}(A\cap B')=\frac{4}{17}\left(1-\frac{23}{40}\right)$ $=\frac{1}{10}$  $\mathrm{P}(A'\cap B')=1-\mathrm{P}(A\cup B)$ $=1-\left(\frac{1}{10}+\frac{23}{40}\right)$ $=\frac{13}{40}$
	$\mathrm{P}(A\cap B)=\frac{3}{8}$ $\mathrm{P}(A)=\frac{1}{10}+\frac{3}{8}=\frac{19}{40}$ $\mathrm{P}(A)\mathrm{P}(B)=\left(\frac{19}{40}\right)\left(\frac{23}{40}\right)=\frac{437}{1600}$ Since $\mathrm{P}(A\cap B)\neq\mathrm{P}(A)\mathrm{P}(B)$, A and B are not independent. (Alternative method) $\mathrm{P}(A)=\frac{1}{10}+\frac{3}{8}=\frac{19}{40}$ $\mathrm{P}(A\|B')=\frac{4}{17}$ Since $\mathrm{P}(A\|B')\neq\mathrm{P}(A)$, A and B' are not independent events, therefore A and B are not independent events.

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(b) (i)	Let X be the event that a randomly chosen durian is MSW and it is infested with maggots. Let Y be the event that a randomly chosen durian is MSW and it is not infested with maggots. P(both are MSW durians and at least one durian is infested with maggots) $=\mathrm{P}(XY)+\mathrm{P}(YX)+\mathrm{P}(XX)$ $=\left(\frac{4}{35}\right)\left(\frac{11}{34}\right)\times 2+\left(\frac{4}{35}\right)\left(\frac{3}{34}\right)$ $=\frac{10}{119}$
(ii)	$\mathrm{P}(\text{at least one durian is infested with maggots})$ $=1-\mathrm{P}(\text{both durians are not infested with maggots})$ $=1-\left(\frac{28}{35}\right)\left(\frac{27}{34}\right)$ $=\frac{31}{85}$ Required probability $=\frac{\frac{10}{119}}{\frac{31}{85}}$ $=\frac{50}{217}$

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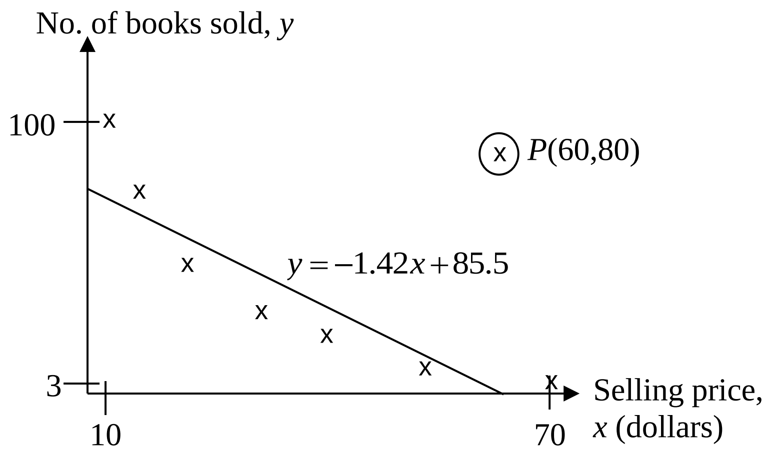
Qn	Solution
9	**Correlation and Regression**
(i), (ii), (iii)	
(iii)	$r = -0.869$ $y = -1.4209x + 85.513$ $= -1.42x + 85.5$ (3 s.f.)
(iv)	When $x = 75$, $y = -1.4209(75) + 85.513$ $= -21.1$(3 s.f.) Since $x = 75$ lies outside the data range of *x*, the estimated value of *y* is not reliable since the linear relationship between *x* and *y* may not longer holds. Or Since the value of *y* (the number of books sold) cannot be negative, the estimated value of *y* is not reliable.
(v)	For a linear model, the number of books sold might fall below zero, hence a linear model might not be appropriate. From the scatter diagram, as the selling price of each book (*x*) increases, the number of books sold (*y*) decreases at a decreasing rate. Thus, a linear model might not be appropriate.

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Qn	Solution
10	**Hypothesis Testing**
(a)	Let X be the decrease in cholesterol level for a randomly chosen volunteer (in mg/dL) Let μ denote the population mean decrease in cholesterol level in volunteers (in mg/dL) Since n = 80 is large, by Central Limit Theorem, $\bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$ approximately. Using GC, $\bar{x} = 19.7375 \ (\text{exact})$, $s^2 = 1.7628^2 \ (5\text{s.f})$ $\mathrm{H_0}$: $\mu = 20$ $\mathrm{H_1}$: $\mu < 20$ Test statistic: $Z = \dfrac{\bar{X} - \mu}{S/\sqrt{n}}$ Level of significance: 5% Reject $\mathrm{H_0}$ if $p-\text{value} < 0.05$ Under $\mathrm{H_0}$, using GC, $p-\text{value} = 0.0914$ (3 s.f) Conclusion: Since $p-\text{value} = 0.0914 > 0.05$, we **do not reject $\mathrm{H_0}$** and conclude that there is **insufficient evidence**, at the 5% significance level, that the mean decrease in cholesterol level is less than 20 mg/dL. Thus, the chocolate company's claim is valid at 5% level of significance.
(b)(i)	Unbiased estimate of μ is $\bar{x} = \dfrac{185}{60} = 3.08$ (3 s.f) Unbiased estimate of σ^2 is $s^2 = \dfrac{1}{59}\left[626 - \dfrac{185^2}{60}\right] = \dfrac{667}{708} = 0.942$ (3 s.f)
(ii)	Since n = 60 is large, by Central Limit Theorem, $\bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$ approximately $\mathrm{H_0}$: $\mu = \mu_0$ $\mathrm{H_1}$: $\mu \neq \mu_0$ Test statistic: $Z = \dfrac{\bar{X} - \mu}{S/\sqrt{n}}$ Level of significance: 4% Reject $\mathrm{H_0}$ if $z-\text{value} < -2.0537$ or $z-\text{value} > 2.0537$

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	Under H_0, $z-\text{value} = \dfrac{3.0833-\mu_0}{\sqrt{0.94209/60}}$ Since the principal is confident that he did not indicate wrongly the mean time that the children took for afternoon naps at 4% level of significance, H_0 is not rejected. $-2.0537 < z-\text{value} < 2.0537$ $-2.0537 < \dfrac{3.0833-\mu_0}{\sqrt{0.94209/60}} < 2.0537$. $2.83 < \mu_0 < 3.34 \quad (3\text{s.f})$

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Qn	Solution
11	**Normal and Sampling Distribution**
(i)	Let X be the weight of a randomly chosen Black Tilapia fish, in grams. $X \sim \mathrm{N}\left(\mu, 32^2\right)$ Given $\mathrm{P}\left(X > 541\right) = 0.10$ Standardizing, $Z \sim \mathrm{N}(0,1)$ $\mathrm{P}\left(X > 541\right) = 0.10$ $1 - \mathrm{P}\left(X \le 541\right) = 0.10$ $\mathrm{P}\left(X \le 541\right) = 0.9$ $\dfrac{541-\mu}{32} = 1.28155$ $\therefore \mu = 500$ (nearest gram)
(ii)	$X \sim \mathrm{N}\left(500, 32^2\right)$ $\mathrm{P}(440 < X < 550) \times \mathrm{P}(440 < X < 550) \times \mathrm{P}(X > 550) \times \dfrac{3!}{2!}$ $= \left(0.91052\right)^2 \times \left(0.059085\right) \times 3$ $= 0.147$ (3 s.f)
(iii)	$X \sim \mathrm{N}\left(500, 32^2\right)$ $\dfrac{a}{1000}\left(X_1 + X_2\right) \sim \mathrm{N}\left(\dfrac{a}{1000}(2)(500), \left(\dfrac{a}{1000}\right)^2 (2)(32^2)\right)$ $\dfrac{a}{1000}\left(X_1 + X_2\right) \sim \mathrm{N}\left(\mathrm{a}, 0.002048a^2\right)$ $\mathrm{P}\left(\dfrac{a}{1000}\left(X_1 + X_2\right) < 6.9\right) < 0.84241$ $\mathrm{P}\left(Z < \dfrac{6.9-a}{\sqrt{0.002048a^2}}\right) < 0.84241$ $\dfrac{6.9-a}{\sqrt{0.002048a^2}} < 1.00441$ $6.9 - a < 0.045455a$ $1.045455a > 6.9$ $a > 6.60$ (3 s.f) Alternatively: $\mathrm{P}\left(\dfrac{a}{1000}\left(X_1 + X_2\right) < 6.9\right) < 0.84241$ Using GC, $\therefore a > 6.60$ (3 s.f)
(iv)	Let Y be the weight of a randomly chosen Grey Mullet fish, in grams. $Y \sim \mathrm{N}\left(800, 50^2\right)$ Let $T = \dfrac{X_1 + X_2 + Y_1 + Y_2 + Y_3}{5} \sim \mathrm{N}\left(\dfrac{1}{5}\left(2(500) + 3(800)\right), \dfrac{1}{5^2}\left(2(32^2) + 3(50^2)\right)\right)$

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	$T \sim \mathrm{N}\left(680,\ 381.92\right)$ $\mathrm{P}(T > 690) = 0.304$ (3 s.f) **Alternatively:** $T = X_1 + X_2 + Y_1 + Y_2 + Y_3 \sim \mathrm{N}\left(2(500) + 3(800),\ 2(32^2) + 3(50^2)\right)$ $T \sim \mathrm{N}\left(3400,\ 9548\right)$ $\mathrm{P}(T > 690 \times 5) = \mathrm{P}(T > 3450) = 0.304$ (3 s.f)

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Class | Adm No

Candidate Name: ____________________

H

2017 Promotional Examination II
Pre-University 2

MATHEMATICS **8865/01**

Paper 1 **12 September 2017**

3 hours

Additional Materials: Answer Paper
List of Formulae (MF 26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, arrange your answers in NUMERICAL ORDER and fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

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Section A: Pure Mathematics [40 marks]

1 Find the range of values of k for which the equation $x^2 + 2kx + 3k + 4 = 0$ has real roots. [3]

2 Find $\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(\frac{\sqrt{2x^2+1}}{2-x}\right)\right]$. [3]

3 The graph of $y = ax^2 + bx + c$ passes through the points (1, 6), (7, 234) and (13, 822). By forming a system of linear equations, find the values of a, b and c. [3]

Hence find the range of values of x for which the graph is decreasing. [2]

4 The curve C has equation $y = 1 - \mathrm{e}^{-2x}$.

(i) Sketch the graph of C, stating the equation(s) of any asymptote(s). [2]

(ii) Find the equation of the tangent to the curve C at $x = 1$, giving your answer in the form $y = mx + c$, where m and c are exact constants to be found. [3]

(iii) Find the exact area bounded by the curve C, the line $x = 1$ and the x-axis. Deduce the exact area bounded by the curve C, $y = 1 - \mathrm{e}^{-2}$ and the y-axis. [3]

5 **(i)** Sketch, on the same diagram, the graphs of $y = \ln(x+3)$ and $y = \frac{2x^2-1}{x^2}$, labelling clearly the equations of any asymptotes. There is no need to find the coordinates of any points where the graphs cross the axes. [4]

(ii) Find the x-coordinate(s) of the point(s) of intersection of the graphs of $y = \ln(x+3)$ and $y = \frac{2x^2-1}{x^2}$. Hence solve the inequality $1 + x^2\ln(x+3) < 2x^2$. [3]

(iii) Find the area bounded by the two curves $y = \ln(x+3)$ and $y = \frac{2x^2-1}{x^2}$, giving your answer to 3 decimal places. [2]

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6 The managing director of a company tracked the rate of output, x units per month, of its product regularly over t months. His analyst believes that x and t can be modelled by the equation

$$x = a + 30t^2 - 2t^3,$$

where $0 \le t \le 12$ and a is a positive constant.

(i) Using differentiation, find the maximum rate of output in the year in terms of a, justifying that this is a maximum. [4]

(ii) Sketch the graph of x against t for $a = 25$ and give an interpretation of the value of a. [3]

(iii) Find the exact area of the region bounded by the graph, the axes and the line $t = 12$ for $a = 25$. Give an interpretation of the value of this area. [3]

The analyst also believes that the profit per month, \$$y$ million, can be modelled by the equation

$$y = \ln[(x-a)^2 + (x-a)], \text{ where } x > a.$$

(iv) By expressing y in terms of t, find the rate at which the profit per month is increasing when $t = 1$. [2]

Section B: Probability and Statistics [60 marks]

7 Consider arranging all the letters of the word **FORMULAE**.

(i) Find the number of different arrangements if there are no restrictions. [1]

(ii) Find the probability that the arrangement starts and ends with a consonant and the vowels are together. [3]

Codewords are formed by arranging 3 letters from the letters of the word **FORMULAE**.

(iii) Find the number of different codewords that can be formed. [2]

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8 In a game, there are three boxes A, B and C. Box A contains 1 red and 9 white balls. Box B contains 2 red and 8 white balls. Box C contains 3 red and 7 white balls. All the red and white balls in the three boxes are indistinguishable other than the colours.

The player selects one of the three boxes by tossing a fair die. The player selects a ball from Box A if the die shows 1, 2 or 3. The player selects a ball from Box B if the die shows 4 or 5. The player selects a ball from Box C if the die shows a 6.

If the player selects a white ball, the game is over. If the player selects a red ball, the person wins \$100 and is allowed to draw another ball from the same box containing the remaining balls. If the second ball is white, the game is over. If the second ball is red, the player wins another \$200 and is allowed to draw another ball from the same box containing the remaining balls. If the third ball is white, the game is over. If the third ball is red, the player wins another \$400.

(i) Draw a tree diagram showing the different outcomes of the game. [2]

(ii) Find the probability that the player wins nothing. Deduce the probability that the player draws at least one red ball. [3]

(iii) Find the probability that the player selects from Box B, given that the player wins \$300. [3]

9 It is known that the masses, in kilograms, of oranges and pears sold at a supermarket are normally distributed. The means and standard deviations of these distributions, and the selling prices, in \$ per kilogram, are shown in the table below.

	Mean (kg)	Standard deviation (kg)	Selling price (\$ per kg)
Oranges	0.27	c	\$3
Pears	0.39	0.05	\$6

(i) Given that 4% of the oranges has a mass less than 0.2 kg, show that $c = 0.04$, correct to 2 decimal places. [2]

(ii) Find the probability that a randomly chosen orange has mass greater than 340g. [1]

(iii) Find the probability that the mass of 3 randomly chosen oranges is within 0.01 kg of the mass of 2 randomly chosen pears, stating clearly the mean and variance of the distribution that you use. [3]

(iv) Find the probability that the cost of 3 randomly chosen oranges and 2 randomly chosen pears exceeds \$7, stating clearly the mean and variance of the distribution that you use. [3]

(v) State an assumption needed for the calculations in part **(iii)** and **(iv)** to be valid. [1]

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10 In an egg farm, eggs are packed in cartons containing 30 eggs each. On average, 5% of the eggs are cracked during the transportation process from the egg farm to a market. At the market, every carton of 30 eggs is checked for cracked eggs. The number of cracked eggs in a randomly chosen carton is denoted by the random variable X.

(i) State, in the context of this question, two assumptions needed to model X using a binomial distribution. [2]

(ii) Explain why one of the assumptions stated in part **(i)** may not hold in this context. [1]

Assume now that these assumptions do in fact hold.

(iii) A carton is rejected if there is more than one cracked egg. Find the probability that a randomly chosen carton is rejected. [2]

(iv) 10 randomly chosen cartons of eggs are checked for cracked eggs. Find the probability that the last carton is the third rejected carton. [4]

(v) The eggs are also packed in trays of n eggs. Find the least value of n such that the probability of obtaining at most two cracked eggs in a randomly chosen tray is less than 0.99. [2]

11 An electric heater was switched on in a cold room and the temperature of the room was noted at five-minute intervals.

Time from switching on electric heater, x (min)	0	5	10	15	20	25	30	35	40
Temperature of room, y (^{o}C)	0.4	1.5	3.4	5.5	7.7	9.7	11.7	13.5	15.4

(i) Draw a sketch of the scatter diagram for the data, as shown on your calculator. [2]

(ii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iii) Find the equation of the regression line of y on x in the form $y = mx + c$, giving the values of m and c correct to 5 decimal places. Draw the line on the scatter diagram in part **(i)** and give an interpretation of m in the context of the question. [3]

(iv) Predict the temperature 2 hours from switching on the electric heater. Give a reason why should this prediction be treated with caution in the context of the question. [2]

(v) It was later found that the temperature was in fact k ^{o}C after the electric heater was switched on for 30 minutes, and the equation of the correct regression line of y on x should be $y = 0.4x$. Find the value of k. [2]

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12 A parent claims that the average speed of vehicles along the road outside a particular school is greater than the speed limit of 40 km per hour. The Traffic Police recorded the speed, x km per hour, of 50 randomly selected vehicles along the road outside the school to obtain unbiased estimates of the population mean and variance of the speed. The data collected are summarised as follows.

$$\sum(x-40)=41, \ \sum(x-40)^2=5173.$$

(i) Suggest why, in this context, the data is summarised in terms of $(x-40)$ rather than x? [1]

(ii) Find unbiased estimates of the population mean and population variance. [3]

(iii) Test, at the 5% level of significance, whether there is sufficient evidence to support the parent's claim. [4]

(iv) State, with a reason, whether it is necessary to assume a normal distribution for the test in part **(iii)** to be valid. [1]

From past records, it is known that the speed along the road outside the school follows a normal distribution with standard deviation of 10 km per hour. To further investigate the parent's claim, the Traffic Police recorded the speed of another 20 randomly selected vehicles along the road outside the school and the mean speed for the second sample is c km per hour.

(v) Show that the unbiased estimate of the population mean speed based on the combined sample of 70 readings is given by $\dfrac{2041+20c}{70}$. [2]

(vi) Find the range of values of c such that there is sufficient evidence to support the parent's claim at the 5% level of significance, based on the combined sample. [3]

End of Paper

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Millennia Institute
H1 Mathematics 2017 Prelim Exam Solution

1	Find the range of values of k for which the equation $x^2+2kx+3k+4=0$ has real roots. [3]
	Solution: $x^2+2kx+3k+4=0$ has real roots $\Rightarrow (2k)^2-4(1)(3k+4)\geq 0$ $\Rightarrow 4k^2-12k-16\geq 0$ $\Rightarrow k^2-3k-4\geq 0$ $\Rightarrow (k-4)(k+1)\geq 0$ $\Rightarrow k\leq -1$ or $k\geq 4$
2	Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(\dfrac{\sqrt{2x^2+1}}{2-x}\right)\right]$. [3]
	Solution: $\dfrac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(\dfrac{\sqrt{2x^2+1}}{2-x}\right)\right]=\dfrac{\mathrm{d}}{\mathrm{d}x}\left[\dfrac{1}{2}\ln\left(2x^2+1\right)-\ln\left(2-x\right)\right]$ $=\dfrac{1}{2}\dfrac{1}{2x^2+1}\left(4x\right)-\dfrac{1}{2-x}(-1)$ $=\dfrac{2x}{2x^2+1}+\dfrac{1}{2-x}$.
3	The graph of $y=ax^2+bx+c$ passes through the points (1, 6), (7, 234) and (13, 822). By forming a system of linear equations, find the values of a, b and c. Hence find the range of values of x for which the graph is decreasing. [2]
	Solution: $y=ax^2+bx+c$ At (1, 6), $6=a+b+c$ … (1) At (7, 234), $234=49a+7b+c$ … (2) At (13, 822), $822=169a+13b+c$ … (3) From graphing calculator, a = 5, b = –2 and c = 3. **Method 1**: Differentiation $y=5x^2-2x+3\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=10x-2$ For y to be decreasing, $\dfrac{\mathrm{d}y}{\mathrm{d}x}<0\Rightarrow 10x-2<0\Rightarrow x<\dfrac{1}{5}$.

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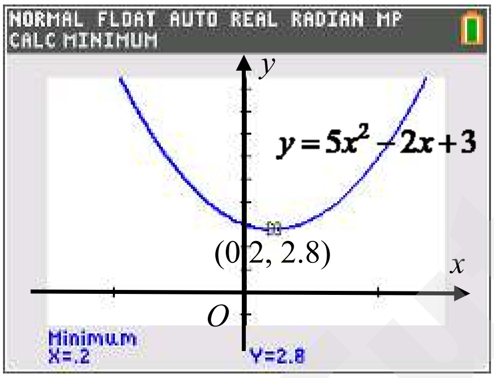
Method 2: Complete the square

$$y=5\left(x^2-\frac{2}{5}x+\frac{3}{5}\right)=5\left[\left(x-\frac{1}{5}\right)^2-\left(-\frac{1}{5}\right)^2+\frac{3}{5}\right]$$

$y=5\left[\left(x-\frac{1}{5}\right)^2+\frac{14}{25}\right]$ has a minimum point at $\left(\frac{1}{5},\frac{14}{25}\right)$ since a = 5 > 0. Hence, graph is decreasing when $x<\frac{1}{5}$.

Method 3: Draw graph



Hence, graph is decreasing when $x<\frac{1}{5}$.

4 The curve C has equation $y=1-\mathrm{e}^{-2x}$.

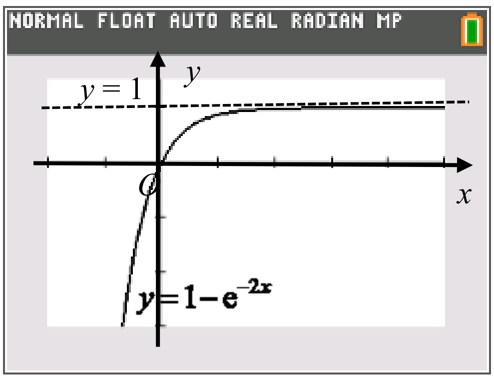
(i) Sketch the graph of C, stating the equation(s) of any asymptote(s).

(ii) Find the equation of the tangent to the curve C at $x=1$, giving your answer in the form $y=mx+c$, where m and c are exact constants to be found.

(iii) Find the exact area bounded by the curve C, the line x = 1 and the x-axis. Deduce the exact area bounded by the curve C, the line y = $1-\mathrm{e}^{-2}$ and the y-axis.

Solution:

(i)



(ii)

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	$y=1-e^{-2x} \Rightarrow \frac{dy}{dx}=-e^{-2x}(-2)=2e^{-2x}$. When $x=1$, $y=1-e^{-2}$ and $\frac{dy}{dx}=2e^{-2}$. **Method 1**: Use $y = mx + c$ $(1-e^{-2})=(2e^{-2})(1)+c \Rightarrow c=1-3e^{-2}$. Required equation is $y=2e^{-2}x+1-3e^{-2}$. **Method 2**: Use $y - b = m(x - a)$ Required equation is $y-\left(1-e^{-2}\right)=\left(2e^{-2}\right)\left(x-1\right)$ i.e. $y=2e^{-2}x-2e^{-2}+1-e^{-2}$ i.e. $y=2e^{-2}x+1-3e^{-2}$, i.e. $m=2e^{-2}$ and $c=1-3e^{-2}$. (iii) Required area = $\int_0^1\left(1-e^{-2x}\right)dx=\left[x-\frac{e^{-2x}}{-2}\right]_0^1$ $=\left(1+\frac{1}{2}e^{-2}\right)-\left(0+\frac{1}{2}\right)=\frac{1}{2}\left(1+e^{-2}\right)$. Required area = $\left(1-e^{-2}\right)(1)-\frac{1}{2}\left(1+e^{-2}\right)$ $=\frac{1}{2}-\frac{3}{2}e^{-2}=\frac{1}{2}\left(1-3e^{-2}\right)$. (deduced)
5	**(i)** Sketch, on the same diagram, the graphs of $y=\ln(x+3)$ and $y=\frac{2x^2-1}{x^2}$, labelling clearly the equations of any asymptotes. There is no need to find the coordinates of any points where the graphs cross the axes. **(ii)** Find the x-coordinate(s) of the point(s) of intersection of the graphs of $y=\ln(x+3)$ and $y=\frac{2x^2-1}{x^2}$. Hence solve the inequality $1+x^2\ln(x+3)<2x^2$. **(iii)** Find the area bounded by the two curves $y=\ln(x+3)$ and $y=\frac{2x^2-1}{x^2}$, giving your answer to 3 decimal places.
	Solution: (i)

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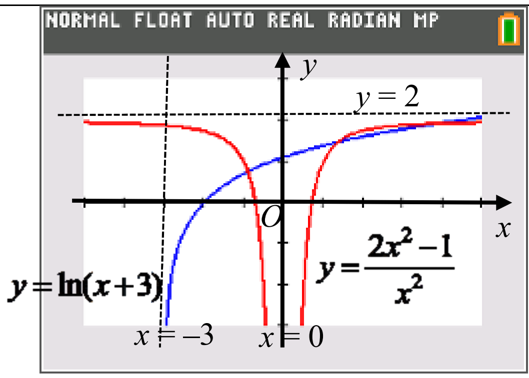


(ii)
From graphing calculator,
Required coordinates are –0.893, 1.38, 3.92. (3 s.f.)

$1+x^2\ln(x+3)<2x^2$

$\Rightarrow \ln(x+3)<\dfrac{2x^2-1}{x^2}$ (since $x\neq 0$)

From graph, $-3<x<-0.893$ or $1.38<x<3.92$. (3 s.f.)

(iii)

Required area $=\displaystyle\int_{1.3841}^{3.9246}\left(\frac{2x^2-1}{x^2}-\ln(x+3)\right)\mathrm{d}x$ [from (ii)]

$= 0.234$ units^2. (3 d.p.) (from graphing calculator)

6 The managing director of a company tracked the rate of output, x units per month, of its product regularly over t months. His analyst believes that x and t can be modelled by the equation

$$x=a+30t^2-2t^3,\text{ where } 0\le t\le 12 \text{ and } a \text{ is a positive constant.}$$

(i) Using differentiation, find the maximum rate of output in the year in terms of a, justifying that this is a maximum.

(ii) Sketch the graph of x against t for a = 25 and give an interpretation of the value of a. [3]

(iii) Find the exact area of the region bounded by the graph, the axes and the line t = 12 for a = 25. Give an interpretation of the value of this area.

The analyst also believes that the profit per month, \$$y$ million, can be modelled by the equation

$$y=\ln[(x-a)^2+(x-a)],\text{ where } x>a.$$

(iv) By expressing y in terms of t, find the rate at which the profit per month is increasing when t = 1.

Solution:

(i) $x=a+30t^2-2t^3$

At stationary point, $\dfrac{\mathrm{d}x}{\mathrm{d}t}=60t-6t^2=0$

$\Rightarrow 6t(10-t)=0 \Rightarrow t=0$ or $t=10$.

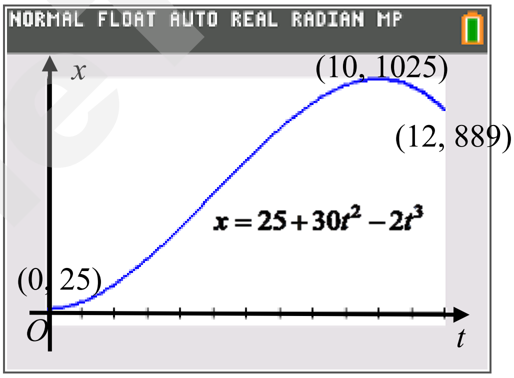
	Method 1 Second derivative test $\dfrac{d^2x}{dt^2}=60-12t.$ When $t=0$, $\dfrac{d^2x}{dt^2}=60>0\Rightarrow$ Rate is a minimum. When $t=10$, $\dfrac{d^2x}{dt^2}=-60<0\Rightarrow$ Rate is a maximum. **Method 2 First derivative test** $\dfrac{dx}{dt}=60t-6t^2=6t(10-t).$

t	0^-	0	0^+	t	10^-	10	10^+
$\dfrac{dx}{dt}$	<0	0	>0	$\dfrac{dx}{dt}$	>0	0	<0
Slope	\	—	/	Slope	/	—	\

Rate is a minimum at $t=0$ and maximum at $t=10$.

Required rate $= a+30(10)^2-2(10)^3=a+1000.$

(ii)



The value of a represents the initial rate of output at the start of the year.

(iii) Required area

$$=\int_0^{12}\left(25+30t^2-2t^3\right)dt=\left[25t+\frac{30t^3}{3}-\frac{2t^4}{4}\right]_0^{12}$$

$$=25(12)+10(12)^3-\frac{1}{2}(12)^4-0=7212.$$

It represents the yearly output, i.e. the company produces 7212 units in the year.

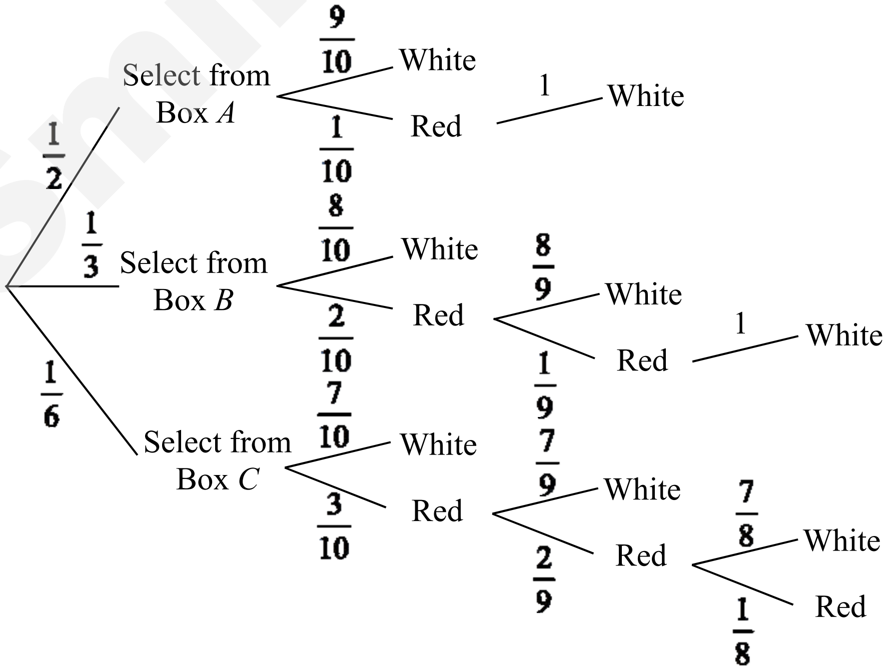
(iv)

$y=\ln[(x-a)^2+(x-a)]$

$y=\ln[(30t^2-2t^3)^2+(30t^2-2t^3)]$

From graphing calculator,

When $t=1$, $\dfrac{dy}{dt}\approx 3.7906=3.79$. (3 s.f.)

	Required rate = 3.79. (3 s.f.) .
7	Consider arranging all the letters of the word **FORMULAE**. **(i)** Find the number of different arrangements if there are no restrictions. **(ii)** Find the probability that the arrangement starts and ends with a consonant and the vowels are together. Codewords are formed by arranging 3 letters from the letters of the word **FORMULAE**. **(iii)** Find the number of different codewords that can be formed.
	(i) Number of ways = 8! = 40320. (ii) Required probability = $\frac{4\times3\times3!\times4!}{40320}=\frac{1728}{40320}=\frac{3}{70}$. (iii) Number of codewords = ${}^{8}C_{3}\times3!$ = 336.
8	In a game, there are three boxes *A*, *B* and *C*. Box *A* contains 1 red and 9 white balls. Box *B* contains 2 red and 8 white balls. Box *C* contains 3 red and 7 white balls. All the red and white balls in the three boxes are indistinguishable other than the colours. The player selects one of the three boxes by tossing a fair die. The player selects a ball fr If the player selects a white ball, the game is over. If the player selects a red ball, the pers the same box containing the remaining balls. If the third ball is white, the game is over. If the third ball is red, the player wins another $400. **(i)** Draw a tree diagram showing the different outcomes of the game. **(ii)** Find the probability that the player wins nothing. Deduce the probability that the player draws at least one red ball. **(iii)** Find the probability that the player selects from Box *B*, given that the player wins $300. [3]
	(i)  (ii)

$$\text{P(wins nothing)} = \frac{1}{2}\times\frac{9}{10}+\frac{1}{3}\times\frac{8}{10}+\frac{1}{6}\times\frac{7}{10}=\frac{50}{60}=\frac{5}{6}.$$

P(draws ≥ 1 red ball) = P(wins something)

$$= 1 - \text{P(wins nothing)} = 1 - \frac{5}{6} = \frac{1}{6}.$$

(iii)

P(selects from Box B | wins \$300)

$$= \frac{\text{P(selects from Box } B \text{ and wins \$300)}}{\text{P(wins \$300)}}$$

$$= \frac{\frac{1}{3}\times\frac{2}{10}\times\frac{1}{9}}{\frac{1}{3}\times\frac{2}{10}\times\frac{1}{9}+\frac{1}{6}\times\frac{3}{10}\times\frac{2}{9}\times\frac{7}{8}} = \frac{\frac{2}{3}}{\frac{2}{3}+\frac{7}{8}} = \frac{16}{37}.$$

9 It is known that the masses, in kilograms, of oranges and pears sold at a supermarket are normally distributed. The means and standard deviations of these distributions, and the selling prices, in \$ per kilogram, are shown in the table below.

	Mean (kg)	Standard deviation (kg)	Selling price (\$ per kg)
Oranges	0.27	c	\$3
Pears	0.39	0.05	\$6

(i) Given that 4% of the oranges has a mass less than 0.2 kg, show that c = 0.04, correct to decimal places. [2]

(ii) Find the probability that a randomly chosen orange has mass greater than 340g.[1]

(iii) Find the probability that the mass of 3 randomly chosen oranges is within 0.01 kg of the mass of 2 randomly chosen pears, stating clearly the mean and variance of the distribution that you use. [3]

(iv) Find the probability that the cost of 3 randomly chosen oranges and 2 randomly chosen pears exceeds \$7, stating clearly the mean and variance of the distribution that you use. [3]

(v) State an assumption needed for the calculations in part **(iii)** and **(iv)** to be valid. [1]

Solution:

(i)

Let X be the mass of a randomly chosen orange.

Then $X \sim \text{N}(0.27, c^2)$.

$\text{P}(X < 0.2) = 0.04$.

$$\Rightarrow \text{P}\left(Z < \frac{0.2-0.27}{c}\right) = 0.04, \text{ where } Z \sim \text{N}(0, 1)$$

From graphing calculator, $\frac{-0.07}{c} \approx -1.7507$

$\Rightarrow c \approx 0.03998 = 0.04.$ (2 d.p.) (shown)

(ii)

From graphing calculator,

$\text{P}(X > 0.34) \approx 0.040059 = 0.0401.$ (3 s.f.) OR

Required probability = $\text{P}(X > 0.34) = \text{P}(X < 0.2) = 0.04.$ (by symmetry)

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	(iii) Let Y and W be the mass, in kg, of 3 randomly chosen oranges and 2 randomly chosen pears respectively. Then $Y \sim N(3\times0.27, 3\times0.04^2)$, i.e. $N(0.81, 0.0048)$ and $W \sim N(2\times0.39, 2\times0.05^2)$, i.e. $N(0.78, 0.005)$. $Y - W \sim N(0.81 - 0.78, 0.0048 + 0.005)$, i.e. $N(0.03, 0.0098)$. Required probability = $P(W - 0.01 < Y < W + 0.01)$ $= P(-0.01 < Y - W < 0.01)$ $\approx 0.076862 = 0.0769$. (3 s.f.) (iv) Let S and T be the cost, in \$, of 3 randomly chosen oranges and 2 randomly chosen pears respectively. Then $S = 3Y \sim N(3\times0.81, 3^2\times0.0048)$, i.e. $N(2.43, 0.0432)$ and $T = 6W \sim N(6\times0.78, 6^2\times0.005)$, i.e. $N(4.68, 0.18)$. $S + T \sim N(2.43 + 4.68, 0.0432 + 0.18)$, i.e. $N(7.11, 0.2232)$. Required probability = $P(S + T > 7)$ $\approx 0.59205 = 0.592$. (3 s.f.) (v) We need to assume that the masses of oranges and pears are independent.
10	In an egg farm, eggs are packed in cartons containing 30 eggs each. On average, 5% of the eggs are cracked during the transportation process from the egg farm to a market. At the market, every carton of 30 eggs is checked for cracked eggs. The number of cracked eggs in a randomly chosen carton is denoted by the random variable X. **(i)** State, in the context of this question, two assumptions needed to model X using a binomial distribution. **(ii)** Explain why one of the assumptions stated in part **(i)** may not hold in this context. [1] Assume now that these assumptions do in fact hold. **(iii)** A carton is rejected if there is more than one cracked egg. Find the probability that a randomly chosen carton is rejected. **(iv)** 10 randomly chosen cartons of eggs are checked for cracked eggs. Find the probability that the last carton is the third rejected carton. **(v)** The eggs are also packed in trays of n eggs. Find the least value of n such that the probability of obtaining at most two cracked eggs in a randomly chosen tray is less than 0.99.
	Solution: (i) We need to assume that: 1. the event that an egg is cracked is independent of that of other eggs. 2. the probability that an egg is cracked is a constant. (ii)

	In this context, the event that an egg is cracked (due to transportation) may affect neighbouring eggs in the same carton to crack and thus the events may not be independent. (iii) P(reject carton) = $P(X > 1)$ where $X \sim B(30, 0.05)$ $= 1 - P(X \le 1)$ $\approx 0.44646 = 0.446$. (3 s.f.) (iv) Let Y be the number of rejected cartons, out of 9. Then $Y \sim B(9, 0.44646)$. Required probability = $P(Y = 2) \times 0.44646$ $\approx 0.051015 = 0.0510$. (3 s.f.) (v) Let W be the number of cracked eggs in a tray, out of n. Then $W \sim B(n, 0.05)$. We want to find least n such that $P(W \le 2) < 0.99$. <table><tr><td>n</td><td>$P(W \le 2)$</td></tr><tr><td>9</td><td>0.99164 > 0.99</td></tr><tr><td>10</td><td>0.9885 < 0.99</td></tr><tr><td>11</td><td>0.98476 < 0.99</td></tr></table>From graphing calculator, least value of n is 10.
11	An electric heater was switched on in a cold room and the temperature of the room was noted at five-minute intervals. <table><tr><td>Time from switching on electric heater, x (min)</td><td>0</td><td>5</td><td>10</td><td>15</td><td>20</td><td>25</td><td>30</td><td>35</td><td>40</td></tr><tr><td>Temperature of room, y (°C)</td><td>0.4</td><td>1.5</td><td>3.4</td><td>5.5</td><td>7.7</td><td>9.7</td><td>11.7</td><td>13.5</td><td>15.4</td></tr></table>**(i)** Draw a sketch of the scatter diagram for the data, as shown on your calculator. **(ii)** Find the product moment correlation coefficient and comment on its value in the context of the data. **(iii)** Find the equation of the regression line of y on x in the form $y = mx + c$, giving the values of m and c correct to 5 decimal places. Draw the line on the scatter diagram in part **(i)** and give an interpretation of m in the context of the question. **(iv)** Predict the temperature 2 hours from switching on the electric heater. Give a reason why should this prediction be treated with caution in the context of the question. **(v)** It was later found that the temperature was in fact k °C after the electric heater was switched on for 30 minutes, and the correct regression line should be $y = 0.4x$. Find the value of k.
	(i)

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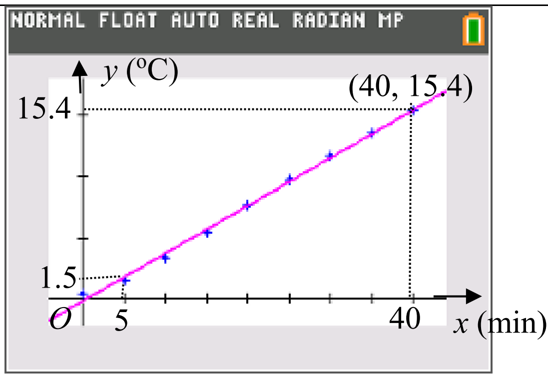

(ii)
From graphing calculator,
required coefficient, $r \approx 0.99870 = 0.999$. (3 s.f)

Since $r > 0$ and $|r| = 0.999$ is very close to 1, the value of r suggests that there is a strong positive linear correlation between the temperature of the room and the time from switching on the electric heater.

(iii)
From graphing calculator,
required equation is $y = 0.38933x - 0.14222$. (5 d.p)
The temperature of the room is expected to increase by 0.3893^{O}C for every one minute increase in time from switching on the electric heater.

(iv)
When $x = 120$,
$y = 0.38933(120) - 0.14222 \approx 46.577 = 46.6$. (3 s.f.)

Required temperature is 46.6 °C.

Possible reasons why prediction should be treated with caution:

1. Extrapolation
This prediction of 46.6^{O}C is rather high and should be treated with caution since $x = 60$ is far outside the range of values of x in the data, i.e. $y = 46.6$ is an extrapolation.

2. Relationship not linear outside data range.
The relationship between the temperature of the room and the time from switching on the electric heater may not be linear any more beyond 40 minutes.

3. Actual temperature too high and can cause a fire.
The actual temperature 120 minutes after switching on the electric heater may be high enough to cause a fire which can be a disaster.

(v)
For the new data, $\sum x = 180,\ \sum y = 57.1 + k$.

$y = 0.4x \Rightarrow \bar{y} = 0.4\bar{x} \Rightarrow \sum y = 0.4\sum x$

$\Rightarrow 57.1 + k = 0.4(180) \Rightarrow k = 14.9$.

12 A parent claims that the average speed of vehicles along the road outside a particular school is greater than the speed limit of 40 km per hour. The Traffic Police recorded the speed, x km per hour, of 50 randomly selected vehicles along the road outside the school to obtain unbiased estimates of the population mean and variance of the speed. The data collected are summarised as follows.

$$\sum(x-40) = 41,\ \sum(x-40)^2 = 5173.$$

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	(i) Suggest why, in this context, the data is summarised in terms of $(x-40)$ rather than x? **(ii)** Find unbiased estimates of the population mean and population va **(iii)** Test, at the 5% level of significance, whether there is sufficient evidence to support t **(iv)** State, with a reason, whether it is necessary to assume a normal distribution for the te From past records, it is known that the speed along the road outside the school follows a normal distribution with standard deviation of 10 km per hour. To further investigate the parent's claim, the Traffic Police recorded the speed of another 20 randomly selected vehicles along the road outside the school and the mean speed for the second sample is c km per hour. **(v)** Show that the unbiased estimate of the population mean speed based on the combined sample of 70 readings is given by $\dfrac{2041+20c}{70}$. **(vi)** Find the range of values of c such that there is sufficient evidence to support the parent's claim at the 5% level of significance, based on the combined sample.
	Solution: (i) This is to keep the recorded speed values small or to give an indication of the variations around the hypothesised mean speed of 40km/h. (ii) Let $y = x - 40$. Then $\sum y = 41$, $\sum y^2 = 5173$. Unbiased estimate of population mean, $\bar{x} = \bar{y} + 40 = \dfrac{\sum y}{50} + 40 = \dfrac{41}{50} + 40 = 40.82.$ Unbiased estimate of population variance, $s_x^2 = s_y^2 = \dfrac{1}{50-1}\left(\sum y^2 - \dfrac{1}{50}\left(\sum y\right)^2\right)$ $= \dfrac{1}{49}\left(5173 - \dfrac{41^2}{50}\right) = \dfrac{5139.38}{49} \approx 104.89 = 105.$ (3 s.f.) (iii) Let X be the speed of a randomly chosen vehicle along the road outside the particular school. Test $H_0 : \mu = 40$ against $H_1 : \mu > 40$ (claim) Under H_0, since $n = 50$ is large, by Central Limit Theorem, $\bar{X} \sim N(40, \dfrac{104.89}{50})$ approximately. Using a one-tail z-test, p-value ≈ 0.28564. Since p-value $\approx 0.28564 > 0.05$, we do not reject H_0 at the 5% level of significance and conclude that there is not enough evidence to support the parent's claim at the 5% level of significance. (iv) It is not necessary to assume a normal distribution for the test in part (iii) to be valid

	since $n = 50$ is large, by Central Limit Theorem, $\overline{X}$ is normally distributed approximately. (v) Required estimate = $\dfrac{\sum x + 20c}{50+20} = \dfrac{50\overline{x} + 20c}{70} = \dfrac{2041+20c}{70}$. (shown) (vi) Test $H_0 : \mu = 40$ against $H_1 : \mu > 40$ (claim) Under H_0, $\overline{X} \sim N\left(40, \dfrac{10^2}{70}\right)$, i.e. $N\left(40, \dfrac{10}{7}\right)$. Using a one-tail z-test at $\alpha = 0.05$, critical value = 41.966. (from graphing calculator) To have sufficient evidence to support the claim at 5% level of significance, we reject H_0 at $\alpha = 0.05$. $\dfrac{2041+20c}{70} \geq 41.966$ i.e. $c \geq 44.8$. (3 s.f.)

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NANYANG JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

Higher 1

MATHEMATICS **8865/01**

Paper 1 **13 September 2017**

3 hours

Additional Materials: Cover Page
Answer Paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagram or graph.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of 6 printed pages.

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Internal Examinations

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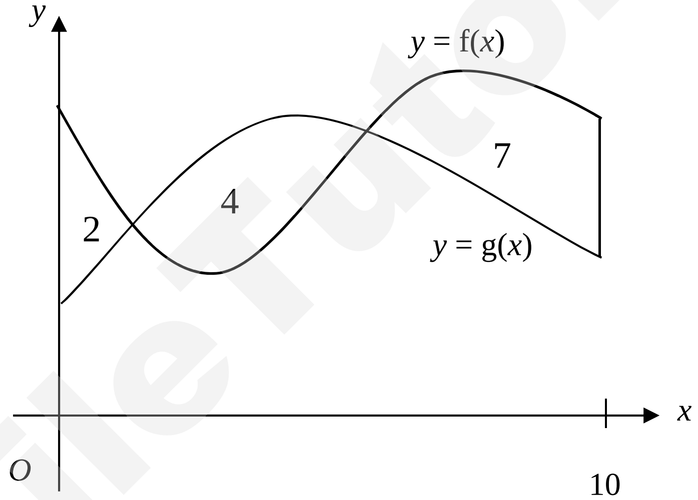
Pure Mathematics [40 marks]

1 Find the range of values of a such that $ax^2 + 4ax + 7a > 2x + 2$ for all real values of x. [4]

2 The graphs of two functions are shown with the areas of the regions between the curves indicated.

(i) What is the total area between the curves for $0 \le x \le 10$? [1]

(ii) What is the value of $\int_0^{10} f(x) - g(x)\,\mathrm{d}x$? [3]



3 Given that u_n is a quadratic polynomial in n and the first three terms are as follows:

$u_1 = 3$, $u_2 = 1$ and $u_3 = -5$.

(i) Find u_n in terms of n. [4]

(ii) Using an algebraic method, find the value of n for which u_n is greater than 2. [2]

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4 The gradient of a curve $y=\mathrm{f}(x)$ at any point (x,y) is given by $\mathrm{f}'(x)=4\mathrm{e}^{1-x}-2\mathrm{e}^{-3-2x}$.

(i) Find the exact x-coordinate of the stationary point on the curve, leaving your answer in the form of $a\ln 2+b$ where a and b are constants to be determined. [2]

(ii) Given that the curve intersects the x-axis at the point where $x=-4$ find the equation of the curve, giving your answer in exact form. [3]

(iii) Sketch the graph of $y=\mathrm{f}(x)$, giving the equations of any asymptotes and the coordinates of the points of intersection with the x- and y-axes. [3]

(iv) Find the equation of the tangent at the point $(-4,0)$, leaving answer in terms of e. Hence find the exact area of the region bounded by the curve, the tangent at the point $(-4,0)$ and the y-axis. [5]

5 NY Press Holdings is an English-language online news media based in Singapore. It is known that the online news will be taken down after 24 hours. Suppose that the number of viewers (in thousands) at t th hour after the release of the news is given by

$$S(t)=50\mathrm{e}^{-0.04t} \quad , \quad 0\le t\le 24$$

(i) Show that the number of viewers (in thousands) in the hour just before the news was taken down is 19.1. [1]

(ii) What is the total number of viewers for the period of 24 hours? [3]

(iii) State an assumption needed for the model to be valid. [1]

Assume that on the average, the number of viewers per hour is 32,000 and the company receives 50 cents for every viewing. In order to attract more viewings, the company uses an advertising site. It is known that for each cent spent on advertising, the number of viewings per hour will increase by 1000.

(iv) Given that the company targets to achieve 50,000 viewings per hour, how much must the company spent on advertising in order to achieve its target? Hence state the amount of revenue (in thousand dollars) per hour. [2]

It is given that the hourly profit achieved by the company after investing x cents on advertising is $\$(16000+180x-10x^2)$.

(v) Use differentiation to find the maximum hourly profit (in thousand dollars), proving that it is maximum. [6]

[Turn over

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Statistics [60 marks]

6 A group of 10 students from various sports CCA are gathered for a photoshoot. The group consists of 3 basketball players, 4 tennis players and 3 players from other sport. The 10 students are arranged randomly in a line.

(i) In how many different ways can the 10 people be arranged in a line? [1]

(ii) Find the probability that no 2 basketball players are next to each other? [2]

(iii) The basketball players are all separated. Find the probability that the 4 tennis players are next to each other. [3]

7 A fixed number, n, of students is observed and the number of those students wearing spectacles is denoted by S.

(i) State, in context, two assumptions needed for S to be well modelled by a binomial distribution. [2]

Assume now that S has the distribution $\mathrm{B}(n, p)$.

(ii) Given that $n = 15$ and $\mathrm{P}(S = 0 \text{ or } S = 1) = 0.3$, write down an equation for the value of p, and find this value numerically. Hence find $\mathrm{P}(2 \le S < 8)$. [3]

It is now given that $n = 30$ and $p = 0.7$.

(iii) Suppose that there are 50 groups of 30 students being observed, find the probability that the mean number of students wearing spectacles is at most 20. [3]

8 Two badminton players, Derek and Benjamin, met in a match, and the winner of the game is the first player to win two sets. The probability that Derek wins a match is p. From the second set onwards, the probability that he wins a set is

- p times of the previous probability if he wins in the preceding set,
- 0.4 if he did not win in the preceding set.

(i) Construct a tree diagram to represent all the possible outcomes. [2]

(ii) Find p if the probability of Derek winning the game is 0.6528. [3]

(iii) Given that $p = 0.64$, find the probability of Derek winning the second set if he wins the game. [3]

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9 The rate of growth, y, of a particular organism in a laboratory is believed to depend in some way on the controlled temperature, x°C. The table shows the results of 8 experiments.

Experiment	1	2	3	4	5	6	7	8
Temperature, x°C.	6	9	11	14	18	22	25	28
Rate of growth, y	5	13	15	a	20	24	26	30

The regression line of y on x is given by $y = 0.993x + 2.11$.

(i) Find the value of a, giving your answer to the nearest integer. [2]

(ii) Give a sketch of your scatter diagram for the 8 sets of data, as shown on your calculator. [2]

(iii) Calculate the product moment correlation coefficient, giving your answer correct to 4 decimal place and comment on its value in relation to your scatter diagram. [2]

(iv) Estimate the temperature in the laboratory when the growth rate of the organism is 18. Comment on the reliability of this prediction. [2]

(v) Explain why it might be unsuitable to use the equation y on x to estimate how the rate of growth of the organism when the temperature is 30°C. [1]

10 The owner of AAA Tuition Company which has many centres in Singapore claims that the mean H1 Mathematics score of their students is 80. A random sample of 50 students from one of the centres is taken. Their mathematics scores, x, are summarised by

$$\sum(x-80) = -40, \qquad \sum(x-80)^2 = 450.$$

Find unbiased estimates of the population mean and variance. [3]

(i) Test, at a 5% level of significance, whether the owner has overstated the mean H1 Mathematics score of their students. [4]

(ii) Suppose a teacher in one of the tuition centres now claims that the mean H1 Mathematics score of a randomly chosen student is not 80. Without carrying out another test, state, with a reason, whether the conclusion in part **(ii)** would remain the same. [2]

Another large random sample of n students gives a mean H1 Mathematics score of 78.8. Given that the population standard deviation of the H1 Mathematics score is now known to be 5, find the largest value of n to conclude that the owner's claim is valid at the same level of significance. [5]

[Turn over

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11 The masses, in kilograms, of the honeydews and durians, sold by a supermarket have independent normal distributions with means and standard deviations as shown in the following table.

	Mean mass	Standard deviation
Honeydew	1.8	b
Durian	1.5	0.3

(i) The mass of the honeydews is a random variable denoted by H. It is known that $\mathrm{P}(H < 2k) = 0.95$ and $\mathrm{P}(H < k) = 0.25$ where k is a constant. Find the value of b. [3]

For the rest of the question, use b = 0.4.

(ii) One honeydew and one durian are chosen at random. Find the probability that the mass of the honeydew is more than 1.7 kg and the mass of the durian is less than 1.7 kg. [2]

(iii) Three honeydews are chosen at random. Find the probability that one of the honeydews has mass less than 1.6 kg and one honeydew has mass more than 1.7 kg each. [4]

(iv) Find the probability that three times the mass of a randomly chosen durian is within ± 0.8kg of the mass of two randomly chosen honeydews. [3]

(v) Honeydews cost \$3 per kilogram and durians cost \$10 per kilogram. Find the mean and variance of the cost of two honeydews and hence find the probability that the total cost of two honeydews and one durian is greater than \$28.50.

[3]

--- End of Paper ---

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no	Solution
1.	$ax^2+4ax+7a>2x+2 \quad \Rightarrow \quad ax^2+(4a-2)x+\left(7a-2\right)>0$ For $Ax^2+Bx+C>0$ for all real values of x, we must have: $A>0$ and $B^2-4AC<0$ That is, $a>0$ and $(4a-2)^2-4a(7a-2)<0$ $(4a-2)^2-4a(7a-2)<0 \quad \Rightarrow \quad 16a^2-16a+4-28a^2+8a<0$ $\Rightarrow \quad -12a^2-8a+4<0 \quad \Rightarrow \quad 3a^2+2a+1>0$ $\Rightarrow \quad 3a^2+2a-1>0$ $\Rightarrow \quad -12a^2-8a+4<0 \quad \Rightarrow \quad 3a^2+2a-1>0$ $\Rightarrow \quad (3a-1)(a+1)>0$ Thus, $a>\frac{1}{3}$ or $a<-1$ Since $a>0$, we have $a>\frac{1}{3}$
2	(a) Total area = 2 + 4 + 7 = 13 (b) $\int_0^{10} f(x)-g(x)\,\mathrm{d}x$ $=\int_0^a f(x)-g(x)\,\mathrm{d}x+\int_a^b f(x)-g(x)\,\mathrm{d}x+\int_b^{10} f(x)-g(x)\,\mathrm{d}x$ $=\int_0^a f(x)-g(x)\,\mathrm{d}x-\int_a^b g(x)-f(x)\,\mathrm{d}x+\int_b^{10} f(x)-g(x)\,\mathrm{d}x$ $=2-4+7$ $=5$
3	$u_n=an^2+bn+c$ When n = 1, $3=a+b+c$...........(1) n = 2, $1=4a+2b+c$...........(2) n =3, $-5=9a+3b+c$...........(3) Using GC, a = -2, b = 4, c = 1. $u_n=-2n^2+4n+1$

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	$-2n^2+4n+1>2$ $2n^2-4n+1<0$ $2(n^2-2n)+1<0$ $2(n-1)^2-2+1<0$ $2(n-1)^2-1<0$ $1-\frac{1}{\sqrt{2}}<n<1+\frac{1}{\sqrt{2}}$ $\text{Sin}\,ce\ n\in\mathbf{Z}^+, n=1$
4	(i) $f'(x)=4e^{1-x}-2e^{-3-2x}$ At stationary point, $\frac{dy}{dx}=0$ $4e^{1-x+1}-2e^{-3-2x}=0$ $4e^{1-x}=2e^{-3-2x}$ $\frac{e^{-3-2x}}{e^{1-x}}=\frac{4}{2}$ $e^{-3-2x-1+x}=2$ $e^{-x-4}=2$ $-x-4=\ln 2$ $\therefore x=-\ln 2-4$ (ii) $y=\int 4e^{1-x}-2e^{-3-2x}\ dx$ $=4\int e^{1-x}\ dx-2\int e^{-3-2x}\ dx$ $=4\left(\frac{e^{1-x}}{-1}\right)-2\left(\frac{e^{-3-2x}}{-2}\right)+C$ $y\ =-4e^{1-x}+e^{-3-2x}+C$ Given that the curve cut the *x*-axis at $x=-4$, so $0=-4e^{1+4}+e^{-3+8}+C$ $0=-4e^5+e^5+C$ $C=3e^5$ Equation of curve: $y=-4e^{1-x}+e^{-3-2x}+3e^5$

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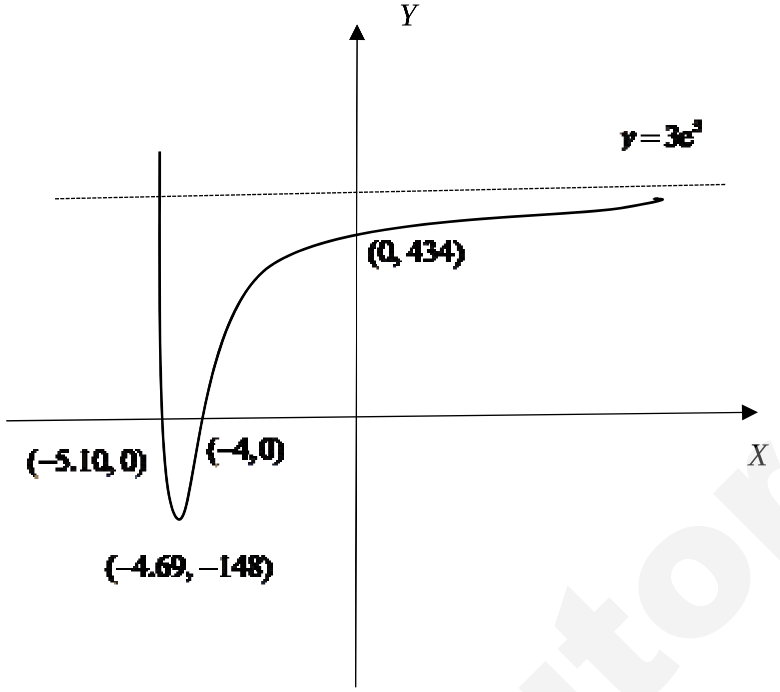
(iii)



(iv)

$$f'(x) = 4e^{-x+1} - 2e^{-2x-3}$$

Gradient of the tangent at the point (-4, 0)

$$= f'(-4) = 4e^{4+1} - 2e^{8-3} = 2e^5$$

Equation of the tangent at the point (-4, 0):

$$y - 0 = 2e^5(x+4)$$
$$y = 2e^5x + 8e^5$$

(v)

Area required

$$= \int_{-4}^{0} \left(2e^5x + 8e^5\right) - \left(-4e^{1-x} + e^{-3-2x} + 3e^5\right)\,dx$$
$$= \int_{-4}^{0} 2e^5x + 5e^5 + 4e^{1-x} - e^{-3-2x}\,dx$$
$$= \left[\frac{2e^5}{2}x^2 + 5e^5x + \frac{4}{-1}e^{1-x} - \frac{1}{-2}e^{-3-2x}\right]_{-4}^{0}$$

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$$= \left[e^5 x^2 + 5e^5 x - 4e^{1-x} + \frac{1}{2} e^{-3-2x} \right]_{-4}^{0}$$

$$= \left(0 + 0 - 4e^1 + \frac{1}{2} e^{-3} \right) - \left(16e^5 - 20e^5 - 4e^5 + \frac{1}{2} e^{-3+8} \right)$$

$$= \frac{1}{2} e^{-3} + \frac{15}{2} e^5 - 4e$$

5

(i) $S(24) = 50e^{-0.04 \times 24} = 19.1$ (thousand papers)

(ii) The total circulation in the 24 hrs

$$\int_0^{24} 50e^{-0.04t}\, dt = \left[\frac{50}{-0.04} e^{-0.041t} \right]_0^{24} = \left[-1250e^{-0.04t} \right]_0^{24}$$

$$= \left(-1250e^{-0.96} \right) - (-1250) = 771.38$$

(iii) Assume the number of viewers at t th hour follows a continuous random variable.

(iv) To increase the circulation from 32 thousand papers to 50 thousand papers, the amoubt spent on advertising will be 50-32 = 18 cents.

Hence total revenue = 50 x (0.5) = 25 (thousand dollars)

(iv) Let R be the hourly profit,

$$R = 16000 + 180x - 10x^2$$

$$\frac{dR}{dx} = 180 - 20x$$

For max R, $\frac{dR}{dx} = 0$

$$x = \frac{180}{20} = 9$$

x	9^-	9	9^+
$\frac{dR}{dx}$	+ve	0	-ve
slope	/	-	\

When x = 9, R is maximum

Maximum $R = 16000 + 180(9) - 10(9)^2 = 16810$

= 1.68 (thousand dollars)

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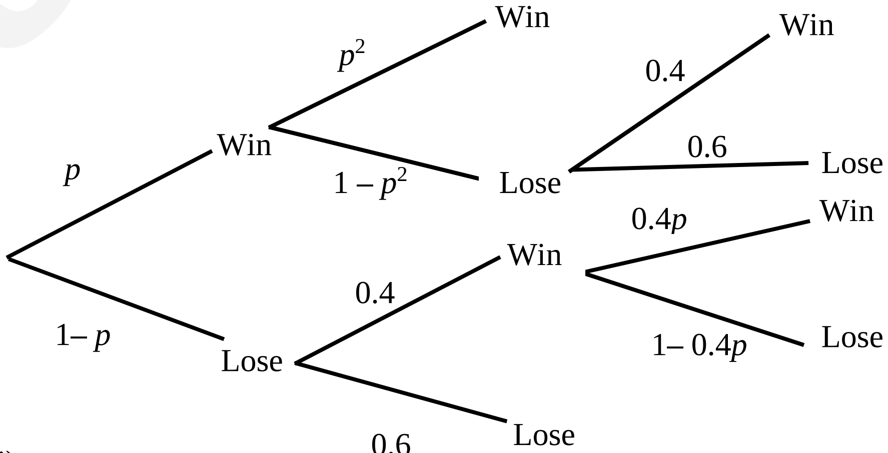
6	(i) No of way = 10! = 3628800 P(no 2 basketball players are next to one another) (ii) $= \frac{7!\binom{8}{3}3!}{3628800} = \frac{7}{15} = 0.467$ P(tennis players are together|basketball players are separated) (iii) $= \frac{\frac{4!4!\binom{5}{3}3!}{3628800}}{\frac{7}{15}} = \frac{1}{49} = 0.0204$
7	2 assumptions 1. Whether the student wear spectacles is independent of one another. 2. The probability of selecting a student wearing spectacle is constant. (ii)S~B(15,*p*) $P(S=0 \text{ or } S=1)=0.3$ $P(S=0)+P(S=1)=0.3$ $(1-p)^{15}+15p(1-p)^{14}=0.3$ $(1-p)^{15}+15p(1-p)^{14}-0.3=0$ Using GC, $p=0.155$ $P(2 \le S < 8)$=0.699 (iii) Let X be no. of the students wearing spectacles out of 30 students. $X \sim B(30, 0.7)$ Since n = 50 is large, by Central Limit theorem, $\overline{X} \sim N(21, \frac{6.3}{50})$ approximately $P(\overline{X} \le 20) = 0.00242$
8(i)	 (ii)

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$\mathrm{P}(\mathrm{WW})+\mathrm{P}(\mathrm{WLW})+\mathrm{P}(\mathrm{LWW})=0.6528$

$p^3+p(1-p^2)(0.4)+(1-p)(0.4)(0.4p)=0.6528$

$0.6p^3-0.16p^2+0.56p-0.6528=0$

Using GC, $p=0.8$

(iii) P(wins second set | wins the game)

$=\dfrac{0.64^3+0.36(0.4)(0.4\times0.64)}{0.64^3+0.64(1-0.64^2)(0.4)+0.36(0.4)(0.4\times0.64)} \quad =\dfrac{0.299008}{0.4501504}$

= 0.664

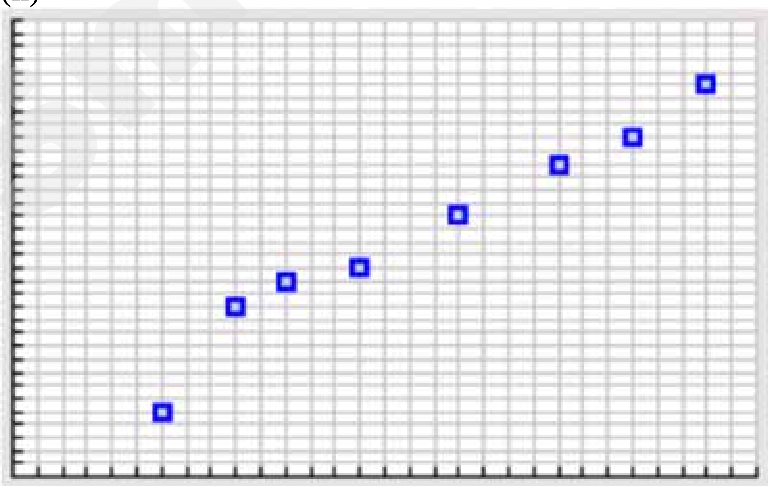
9i $\bar{x}=16.625$

Since the point $(\bar{x},\bar{y})$ will pass through the regression line y on x, $\bar{y}=18.619$.

$\bar{y}=\dfrac{5+13+15+a+20+24+26+30}{8}$

$a=16$

(ii)



(iii) r = 0.9799 (4 dp)

The r value is close to 1, shows that most of the points lie close to the best fit line through the curve. This shows that there is strong positive linear correlation between the growth rate of the organisms and the temperature.

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	(iv) When $y = 18$, sub into $y = 0.993x + 2.11$ $18 = 0.993x + 2.11$ $x = 16.002 = 16.0\,^0$C. The estimate is reliable because the growth rate of the organism is within the data range and r is close to 1. (v) The estimate is unreliable as $x = 30^o c$ is out of the data range of x.
10	Unbiased estimate of population mean, $\bar{x} = \frac{-40}{50} + 80 = 79.2$ Unbiased estimate of population variance, $s^2 = \frac{1}{49}\left[450 - \frac{(-40)^2}{50}\right] = \frac{418}{49} = 8.53061 \approx 8.53$ (i) Let X denote the H1 Mathematics score of a student and μ the population mean H1 Mathematics score. To test $H_o : \mu = 80$ $H_1 : \mu < 80$ Level of significance: 5% Test statistic: Under H_o, $\overline{X} \sim N(80, \frac{8.53061}{50})$ approximately, by Central Limit Theorem since n is large Reject H_o if p-value ≤ 0.05 $\bar{x} = 79.2$, $n = 50$, $s = \sqrt{\frac{418}{49}}$ From GC, p-value = 0.026385 Since p-value $<$ 0.05, there is sufficient evidence to reject H_o and conclude at 5% significant level that there is sufficient evidence that the owner has overstated their mean H1 Mathematics score. (ii) Not the same conclusion Note $H_1 : \mu \neq 80$ (teacher's claim) and p-value = $0.026385 \times 2 = 0.05277 > 0.05$ Do not reject H_0, i.e insufficient evidence to support teacher's claim that the mean H1 Mathematics score is not 80 To test $H_o : \mu = 80$ $H_1 : \mu \neq 80$ at 5% level of significance Under H_o, $\overline{X} \sim N(80, \frac{5^2}{n})$ approximately, by Central Limit Theorem since n is large Reject H_o if p-value ≤ 0.05 owner's claim is valid => do not reject H_o $\therefore p$-value > 0.05

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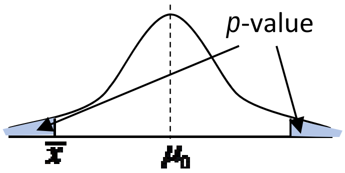
<table>
<tr><td></td><td>

$\mathrm{P}\left(\bar{X} \le \bar{x}\right) > \dfrac{0.05}{2}$, i.e. $\mathrm{P}\left(\bar{X} \le \bar{x}\right) > 0.025$ or



$$\mathrm{P}\left(Z \le \frac{78.8-80}{5/\sqrt{n}} \right) > 0.025$$

$$\frac{78.8-80}{5/\sqrt{n}} > -1.95996$$

$$-0.24\sqrt{n} > -1.95996$$

$$\sqrt{n} < 8.1665$$

$$n < 66.692$$

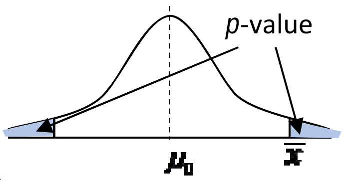
$$n \le 66$$

$\mathrm{P}\left(\bar{X} \ge \bar{x}\right) > \dfrac{0.05}{2}$, i.e. $\mathrm{P}\left(\bar{X} \ge \bar{x}\right) > 0.025$

$$1-\mathrm{P}\left(\bar{X} < \bar{x}\right) > 0.025$$

$$\mathrm{P}\left(\bar{X} < \bar{x}\right) < 0.975$$

$$\mathrm{P}\left(Z < \frac{78.8-80}{5/\sqrt{n}} \right) < 0.975$$



$$\frac{78.8-80}{5/\sqrt{n}} < 1.95996$$

$$-0.24\sqrt{n} < 1.95996$$

$$\sqrt{n} > -8.1665$$

$$\sqrt{n} > 0$$

The largest n is 66

</td></tr>
<tr><td>11i</td><td>$\mathrm{P}(H < 2k) = 0.95$</td></tr>
</table>

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	$P(Z < \frac{2k-1.8}{b}) = 0.95$ $\frac{2k-1.8}{b} = 1.6448$.........(1) $P(H < k) = 0.25$ $P(Z < \frac{k-1.8}{b}) = 0.25$ $\frac{k-1.8}{b} = -0.67449$.........(2) Solving equation (1) and (2), $k = 1.39446$ $b = 0.601$ (ii) Let H be the mass of a randomly chosen honeydew and D be the mass of a randomly chosen durian. H~ N(1.8, 0.4^2) D~ N(1.5, 0.3^2) P(mass of the honeydew is more than 1.7 kg and the mass of the durian is less than 1.7 kg) = P(H > 1.7)$\times$ P(D < 1.7) = 0.448 (iii) Probability = P(H <1.6) $\times$P($1.6 \le H \le 1.7$) $\times$ P(H>1.7) $\times$3! = 0.103 (iv) $H_1 + H_2 \sim N(3.6, 0.32)$ $3D \sim N(4.5, 0.81)$ $3D - (H_1 + H_2) \sim N(0.9, 1.13)$ P($-0.8 \le 3D - (H_1 + H_2) \le 0.8$) =0.408 (v) $C_H = 3(H_1 + H_2) \sim N(10.8, 2.88)$ $C_H + C_D \sim N(25.8, 11.88)$ P($C_H + C_D$ >28.5) =0.217

Section A: Pure Mathematics [40 marks]

1 A farm sells ginger, spear mint, garlic by price per kilogram.
Cheryl bought 1.2 kg of ginger and 1.5 kg of garlic, while her colleague bought 0.9 kg of spear mint and 0.5 kg of garlic. The amount that Cheryl paid is $60.60 and her colleague paid $38.54. Ann, a retailer, bought 36 kg of ginger, 20 kg of spear mint, and 40 kg of garlic. Ann is entitled to have a discount of 25%, and paid $1783.50 after the discount.
Identify the cost and name of the herb with the highest selling price per kg.
State the cost of 2 kg of garlic, without any discount. [4]

2 The curve C has equation $y = 2 - 3e^{-5x}$.

(i) Sketch the graph of C, stating the exact coordinates of any points of intersection with the axes and the equation of the asymptote. [3]

(ii) Without using a calculator, find the exact equation of the tangent to C at the point where $x = 0.2$, expressing y in terms of x. [3]

3 A curve C has equation $y = (x - 2)^2 - 5$.

(i) Find the set of values of k such that the line $y = 2x + k$ intersect C twice. [3]

(ii) Find the exact area of the region bounded by C and the line $y = 2x - 6$. [4]

4 **(a)** Differentiate the following with respect to x.

(i) $\ln\left(\dfrac{4}{\sqrt{12+3x^2}}\right)$, [2]

(ii) $\dfrac{1}{\sqrt{2-3x}}$. [2]

(b) Use a non-calculator method to find $\displaystyle\int_0^6 \frac{\sqrt{e^x}+e^2}{e^{3x}}\,dx$. [4]

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5 A ship builder manufactures yachts. The rate, C thousand dollars per year, at which the total manufacturing costs change is to be monitored regularly over a period of 5 years. The Chief Financial Officer proposed that the relationship between C and the time, t years, can be modelled by the equation

$$C = 25 - 12t + e^{0.8t}, \text{ for } 0 \leq t \leq 5.$$

(i) Use differentiation and this model to find the minimum value of C. Justifying that the value obtained is a minimum. [6]

(ii) Sketch the graph of C against t, giving the coordinates of any intersections with the axes. [2]

(iii) Find the area of the region bounded by C, the t-axis and the lines $t = 0$ and $t = 3$ to 3 decimal places. Give an interpretation of the area that you found, in the context of the question. [2]

The annual profit from the sale of these yachts is P thousands dollars per year. The Chief Financial Officer believes that the relationship between P and t is given by

$$P = 200 + 60\ln(t + 2), \text{ for } 0 \leq t \leq 5.$$

(iv) Find the exact value of t for which $P = 280$. [2]

(v) Sketch the graph of P against t, giving the coordinates of any intersections with the axes. [1]

(vi) Find the exact rate at which the annual profit is increasing when $t = 3$. [2]

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Section B : Probability and Statistics [60 marks]

6 A group of nine people consists of father, mother, their three sons and four daughters. The group arrange themselves in a line for a game. Find the number of different possible arrangements if

(i) the parents are at the two ends and the three sons are all together, [2]

(ii) the four daughters are all separated. [2]

The parents has four tickets for a children ride. If each child is equally likely to be selected, find the probability that there will be more girls selected for the ride. [3]

7 **(a)** In a travel fair, a survey on two destination packages was conducted on a large number of participants. The participants can select at most one destination package. The survey showed that 20% would select destination package *A*, 35% would select destination package *B* and 45% would select neither.

Twelve participants who took part in the survey were randomly selected.

Find the probability that

(i) exactly 4 participants would select destination package *A*. [1]

(ii) at least 4 participants would select destination package *B*. [2]

(iii) fewer than 10 participants but more than 2 would not select destination package *A*. [2]

(iv) Explain the significance of the phrase 'large number' in the first sentence of this question. [1]

(b) A random variable X has a binomial distribution with mean 4 and variance $\frac{4}{3}$. The mean and standard deviation of X are denoted by μ and σ respectively. Find $P(\mu - \sigma < X < \mu + \sigma)$, correct to 4 decimal places. [4]

8 For a camping task, the Task score, t %, and the mean amount of sleep during the camp, s minutes were recorded for a random sample of 9 students. The results are given in the following table.

t	20	30	28	52	76	44	48	60	64
s	450	525	193	530	640	483	554	505	554

(i) Draw a scatter diagram for the data. [2]

(ii) Identify the pair of data which should be regarded as an outlier. Give a possible reason for the occurrence of this pair of data. [2]

Remove the outlier you have identified in part (ii).

(iii) Calculate the new product moment correlation coefficient and find the regression line of s on t for the revised set of data. [2]

(iv) Estimate the mean amount of sleep of the student if his Task score is 80. Comment on the reliability of this estimate. [2]

(v) Using the appropriate regression line, estimate a student's Task score if his mean amount of sleep is 8.8 hours. [2]

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9 A researcher is testing the mileage of a particular type of electric car which the manufacturer claims to have a range of 380 km. A random sample of 50 cars of this type is tested and the range is measured. The results are summarised by

$$\sum(x-380) = -15 \text{ and } \sum(x-380)^2 = 81.$$

(i) Find the exact unbiased estimates of the population mean and variance. [2]

(ii) The researcher suspects that the manufacturer has overstated the mean range of the cars. Test at the 5% level of significance whether there is sufficient evidence to support the researcher's suspicion. [4]

(iii) Explain, whether it is necessary to assume that the range of the cars follow a normal distribution in order for the test to be valid. [1]

Another manufacturer claims that the mean range for their similar powered cars is more than 380 km. Another researcher takes a random sample of 80 cars and records their mileage. A test at the 5 % level of significance shows there is sufficient evidence to support the manufacturer's claim. Assuming the population standard deviation range for these cars is 1.8 km, find the set of values of the mean range of the mileage, correct to 2 decimal places. [3]

10 A restaurant sells cooked crabs and lobsters.

The masses, in kg, of the crabs and lobsters have independent normal distributions with means, standard deviations and selling prices as shown in the following table.

	Mean Mass	Standard deviation of mass	Selling price ($ per kg)
Crabs	1.9	0.2	45
Lobsters	1.6	0.15	85

(i) Find the probability that the mass of a randomly chosen crab is within ±0.1 kg of the mean mass of crabs. [2]

(ii) Find the probability that 4 randomly chosen crabs each has a mass of more than 1.8 kg. [2]

Mr Tan goes to the restaurant and randomly chooses 4 crabs and 6 lobsters.

(iii) Find the probability that the total mass of the 6 lobsters is at least 1.5kg more than the total mass of the 4 crabs. [3]

(iv) Find the probability that Mr Tan pays between $1150 and $1170 for the 4 crabs and 6 lobsters. State the mean and variance of the distribution that you use. [4]

11 A box containing 4 black balls and 6 red balls. Mary draws a ball from the box at random without replacement. One white ball is added to the box, then Jane draws a ball from the box at random.

(i) Draw a probability tree diagram to represent all the possible outcomes. [2]

(ii) Find the probability that Jane draws a black ball. [2]

(iii) Find the probability that Mary draws a black ball, given that Jane draws a ball that is not black. [3]

(iv) Find the probability that *either* Mary draws a black ball *or* Jane draws a ball that is not black or *both*. [2]

Suppose Jane wins a prize if she draws the same coloured ball as Mary. Find the probability that Jane wins a prize, given that Jane does not draw a white ball. [3]

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Section A: Pure Mathematics [40 marks]

1 A farm sells ginger, spear mint, garlic by price per kilogram.
Cheryl bought 1.2 kg of ginger and 1.5 kg of garlic, while her colleague bought 0.9 kg of spear mint and 0.5 kg of garlic. The amount that Cheryl paid is \$60.60 and her colleague paid \$38.54. Ann, a retailer, bought 36 kg of ginger, 20 kg of spear mint, and 40 kg of garlic. Ann is entitled to have a discount of 25%, and paid \$1783.50 after the discount.
Identify the cost and name of the herb with the highest selling price per kg.
State the cost of 2 kg of garlic, without any discount. [4]

	Let \$x, \$y and \$z be the selling price of a kg of ginger, spear mint and garlic respectively. $1.2x + 1.5z = 60.6$ ----(1) $0.9y + 0.5z = 38.54$ ----(2) $0.75(36x + 20y + 40z) = 1783.50$ $36x + 20y + 40z = 2378$ ----(3)	
	Using GC, $x = 27.50$, $y = 32.60$, $z = 18.40$	
	The most expensive herb is spear mint which cost \$32.60 per kg. Cost of 2 kg of garlic is \$36.80	

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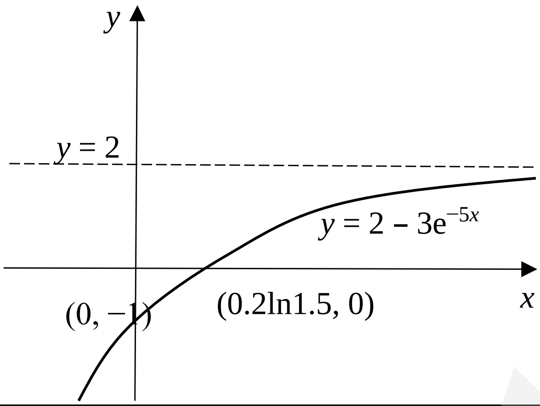
2 The curve C has equation $y = 2 - 3\mathrm{e}^{-5x}$.

(i) Sketch the graph of C, stating the exact coordinates of any points of intersection with the axes and the equation of the asymptote. [3]

(ii) Without using a calculator, find the exact equation of the tangent to C at the point where $x = 0.2$, expressing y in terms of x. [3]

(i)	When $x = 0$, $y = 2 - 3\mathrm{e}^{0} = -1$ When $y = 0$, $2 = 3\mathrm{e}^{-5x}$ $-5x = \ln(2/3)$ $x = -0.2\ln(2/3) = 0.2\ln(3/2) = 0.2\ln 1.5$  	
(ii)	$y = 2 - 3\mathrm{e}^{-5x}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 15\mathrm{e}^{-5x}$	
	At $x = 0.2$, $y = 2 - 3\mathrm{e}^{-1}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 15\mathrm{e}^{-1}$	
	Equation of tangent at $(0.2,\ 2 - 3\mathrm{e}^{-1})$ is $y - (2 - 3\mathrm{e}^{-1}) = 15\mathrm{e}^{-1}(x - 0.2)$ $y = 15\mathrm{e}^{-1}x + (2 - 6\mathrm{e}^{-1})$	
OR	Substitute $x = 0.2$, $y = 2 - 3\mathrm{e}^{-1}$, $m = 15\mathrm{e}^{-1}$ into $y = mx + c$. $c = 2 - 3\mathrm{e}^{-1} - 15\mathrm{e}^{-1}(0.2) = 2 - 6\mathrm{e}^{-1}$ $\therefore$ Equation of tangent at $(0.2,\ 2 - 3\mathrm{e}^{-1})$ is $y = 15\mathrm{e}^{-1}x + (2 - 6\mathrm{e}^{-1})$	

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3 A curve C has equation $y = (x - 2)^2 - 5$.

(i) Find the set of values of k such that the line $y = 2x + k$ intersect C twice. [3]

(ii) Find the exact area of the region bounded by C and the line $y = 2x - 6$. [4]

(i)	$(x-2)^2 - 5 = 2x + k$ $(x-2)^2 - 5 - 2x - k = 0$ $x^2 - 4x + 4 - 5 - 2x - k = 0$ $x^2 - 6x - 1 - k = 0$ As the roots are real and different, $(-6)^2 - 4(-1-k) > 0$ $36 + 4 + 4k > 0$ $4k > -40$ $k > -10$	
(ii)	To find x–coordinates of points of intersection $(x-2)^2 - 5 = 2x - 6$ $x^2 - 6x + 5 = 0$ $(x-5)(x-1) = 0$ $x = 1$ or 5 $\int_1^5 2x - 6 - \left((x-2)^2 - 5\right) \mathrm{d}x$ $= \int_1^5 -5 + 6x - x^2 \mathrm{d}x$ $= \left[-5x + 3x^2 - \frac{x^3}{3}\right]_1^5$ $= \left[-5(5) + 3(5)^2 - \frac{(5)^3}{3}\right] - \left[-5(1) + 3(1)^2 - \frac{(1)^3}{3}\right]$ $= \frac{25}{3} - \left(-\frac{7}{3}\right)$ $= 10\frac{2}{3}$	

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4 **(a)** Differentiate the followings with respect to *x*.

(i) $\ln\left(\dfrac{4}{\sqrt{12+3x^2}}\right)$, [2]

(ii) $\dfrac{1}{\sqrt{2-3x}}$. [2]

(b) Use a non-calculator method to find $\displaystyle\int_0^6 \frac{\sqrt{e^x}+e^2}{e^{3x}}\,dx$. [4]

(a) (i)	Let $y = \ln\left(\dfrac{4}{\sqrt{12+3x^2}}\right) = \ln 4 - \dfrac{1}{2}\ln(12+3x^2)$ $\dfrac{dy}{dx} = -\dfrac{1}{2}\left(\dfrac{6x}{12+3x^2}\right) = -\dfrac{x}{4+x^2}$	
(ii)	$\dfrac{d}{dx}\left(\dfrac{1}{\sqrt{2-3x}}\right) = \dfrac{d}{dx}\left((2-3x)^{-\frac{1}{2}}\right)$ $= -\dfrac{1}{2}(2-3x)^{-\frac{3}{2}}(-3)$ $= \dfrac{3}{2}(2-3x)^{-\frac{3}{2}}$ or $\dfrac{3}{2\left(\sqrt{2-3x}\right)^3}$	
(b)	$\displaystyle\int_0^6 \frac{\sqrt{e^x}+e^2}{e^{3x}}\,dx$ $= \displaystyle\int_0^6 e^{-2.5x}+e^{2-3x}\,dx$ $= \left[\dfrac{e^{-2.5x}}{-2.5}+\dfrac{e^{2-3x}}{-3}\right]_0^6$ $= \left(\dfrac{e^{-15}}{-2.5}+\dfrac{e^{-16}}{-3}\right)-\left(\dfrac{e^0}{-2.5}+\dfrac{e^2}{-3}\right)$ $= \dfrac{2}{5}+\dfrac{e^2}{3}-\dfrac{2}{5e^{15}}-\dfrac{1}{3e^{16}}$	

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$$C = 25 - 12t + e^{0.8t}, \text{ for } 0 \leq t \leq 5.$$

(i) Use differentiation and this model to find the minimum value of C. Justifying that the value obtained is a minimum. [6]

(ii) Sketch the graph of C against t, giving the coordinates of any intersections with the axes. [2]

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The annual profit from the sale of these yachts is P thousands dollars per year. The Chief Financial Officer believes that the relationship between P and t is given by

$$P = 200 + 60\ln(t + 2), \text{ for } 0 \leq t \leq 5.$$

(iv) Find the exact value of t for which $P = 280$. [2]

(v) Sketch the graph of P against t, giving the coordinates of any intersections with the axes. [1]

(vi) Find the exact rate at which the annual profit is increasing when $t = 3$. [2]

(i)	$C = 25 - 12t + e^{0.8t}$, $\frac{dC}{dt} = -12 + 0.8e^{0.8t}$ For turning points, $\frac{dC}{dt} = 0 \Rightarrow 0.8e^{0.8t} = 12$	
	$0.8e^{0.8t} = 12$ $e^{0.8t} = 15$ $0.8t = \ln 15$ $t = \frac{5}{4}\ln 15$ (or 3.39)	
	The minimum $C = 25 - 15\ln 15 + 15 = 40 - 15\ln 15$ (or –0.621)	
	$\frac{d^2C}{dt^2} = 0.64e^{0.8t}$ When $t = \frac{5}{4}\ln 15$, (ie $e^{0.8t} = 15$) $\frac{d^2C}{dt^2} = 0.64(15) = 9.6 > 0$, which shows C is minimum when $t = \frac{5}{4}\ln 15$	
	Or First derivative test.	

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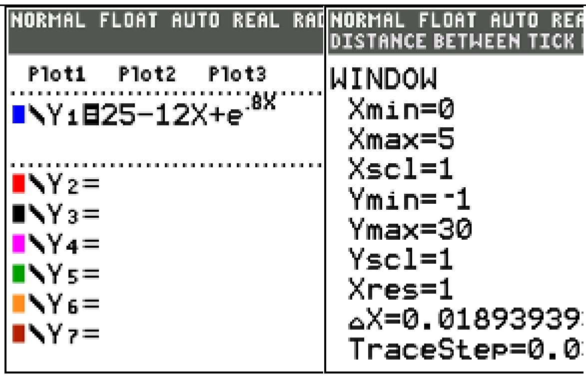
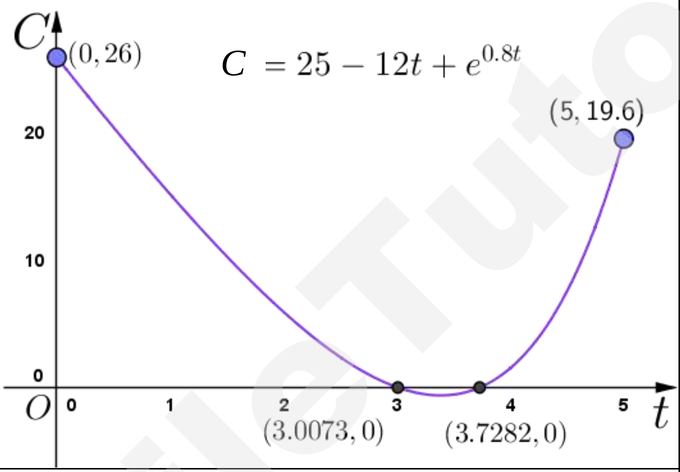
t	3.3	$\frac{5}{4}\ln 15$	3.4
$\frac{dC}{dt}$	−0.789 \	0 —	0.144 /

which shows C is minimum when $t = \frac{5}{4}\ln 15$

(ii)





(iii)

$\int_0^3 25 - 12t + e^{0.8t}\,dt$

$= 33.529$ (3 d.p.)

Represented the total cost during the three years.
or Difference between total cost at the end of the third year and the initial year

(iv)

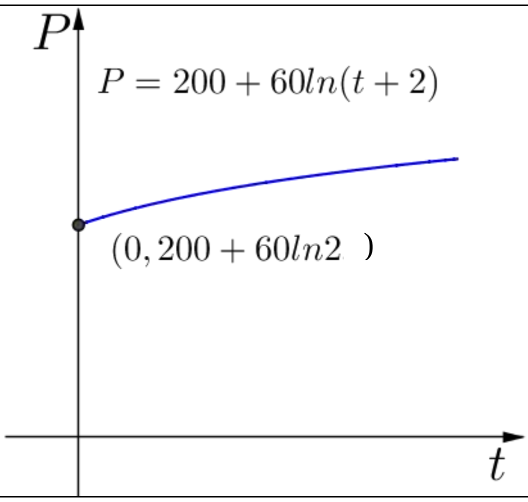
$P = 280$, $280 = 200 + 60\ln(t+2)$

$\ln(t+2) = 8/6$

$t + 2 = e^{4/3}$

$t = e^{4/3} - 2$

(v)



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(vi)	$\dfrac{\mathrm{d}P}{\mathrm{d}t}=\dfrac{60}{t+2}$. When t = 3, $\dfrac{\mathrm{d}P}{\mathrm{d}t}=\dfrac{60}{3+2}$ = 12 The rate at which the annual profit is increasing when t = 3 is $12 thousands dollars per year.	

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Section B : Probability and Statistics [60 marks]

6 A group of nine people consists of father, mother, their three sons and four daughters. The group arrange themselves in a line for a game. Find the number of different possible arrangements if

(i) the parents are at the two ends and the three sons are all together, [2]

(ii) the four daughters are all separated. [2]

The parents has four tickets for a children ride. If each child is equally likely to be selected, find the probability that there will be more girls selected for the ride. [3]

6(i)	2 ways to arrange the two parents at the ends. Group the sons as one unit before arrange with the 4 daughters. 3!(4+1)! Number of ways = 2×3!5! = 1440	
(ii)	First arrange the parents and sons in 5! ways. $\downarrow X \downarrow X \downarrow X \downarrow X \downarrow X \downarrow$ where X could parents or sons As the daughters are all separated, chose 4 of 6 possible slot made by the parents and sons and arrange the daughters in $\binom{6}{4}$ 4! ways. Number of ways where the four daughters are separated $=5!\binom{6}{4}4! = 43200$	
	Case 1: 4 girls, 0 boys Case 2: 3 girls, 1 boy Numbers of ways with more girls $= 1 + \binom{4}{3}\binom{3}{1} = 13$ P(there will have more girls) = $\dfrac{13}{\binom{7}{4}} = \dfrac{13}{35}$	

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7 **(a)** In a travel fair, a survey on two destination packages was conducted on a large number of participants. The participants can select at most one destination package. The survey showed that 20% would select destination package *A*, 35% would select destination package *B* and 45% would select neither.

Twelve participants who took part in the survey were randomly selected.

Find the probability that

(i) exactly 4 participants would select destination package *A*. [1]

(ii) at least 4 participants would select destination package *B*. [2]

(iii) fewer than 10 participants but more than 2 would not select destination package *A*. [2]

(iv) Explain the significance of the phrase 'large number' in the first sentence of this question. [1]

(b) A random variable *X* has a binomial distribution with mean 4 and variance $\frac{4}{3}$. The mean and standard deviation of *X* are denoted by μ and σ respectively. Find $P(\mu - \sigma < X < \mu + \sigma)$, correct to 4 decimal places. [4]

(a)(i)	Let *X* be the random variable "number of participants that would select destination package *A* out of 12." Then $X \sim B(12,0.2)$ $P(X = 4) = 0.132875551 = 0.133$ (3sf)	
(ii)	Let *Y* be the random variable "number of participants that would select destination package *B* out of 12." Then $Y \sim B(12,0.35)$ $P(Y \geq 4) = 1 - P(Y \leq 3) = 0.653347304 = 0.6533$ (3sf)	
	~~$P(Y > 4) = 1 - P(Y \leq 4) = 0.087361546 = 0.0874$ (3sf)~~	
(iii)	Let *W* be the random variable "number of participants that would not select destination package *A* out of 12." Then $W \sim B(12,0.8)$ $P(2 < W < 10) = P(W \leq 9) - P(W \leq 2)$ $= 0.441649725 = 0.442$ (3sf) OR $1 - P(X \leq 2) - P(X \geq 10)$	
(iv)	'Large number' in the first sentence of this question is included so that changes in the percentages of different groups (select Package A, select Package B, select neither) would be negligible.	
(b)	$np = 4$, $np(1-p) = \frac{4}{3} \Rightarrow (1-p) = \frac{1}{3}$, $p = \frac{2}{3}$ and $n = 6$. So $X \sim B(6, \frac{2}{3})$ $P(\mu - \sigma < X < \mu + \sigma)$ $= P(4 - \sqrt{[4/3]} < X < 4 + \sqrt{[4/3]})$ since $\mu = 4$ and $\sigma^2 = \frac{4}{3}$ $= P(2.845299462 < X < 5.154700538)$ $= P(3 \leq X \leq 5)$ since *X* takes 0, 1, 2, …, 6. $= P(X = 3) + P(X = 4) + P(X = 5)$ or $P(X \leq 5) - P(X \leq 2)$ $= 0.812071331 = 0.8121$ (4 dp)	

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8 For a camping task, the Task score, t %, and the mean amount of sleep during the camp, s minutes were recorded for a random sample of 9 students. The results are given in the following table.

t	20	30	28	52	76	44	48	60	64
s	450	525	193	530	640	483	554	505	554

(i) Draw a scatter diagram for the data. [2]

(ii) Identify the pair of data which should be regarded as an outlier. Give a possible reason for the occurrence of this pair of data. [2]

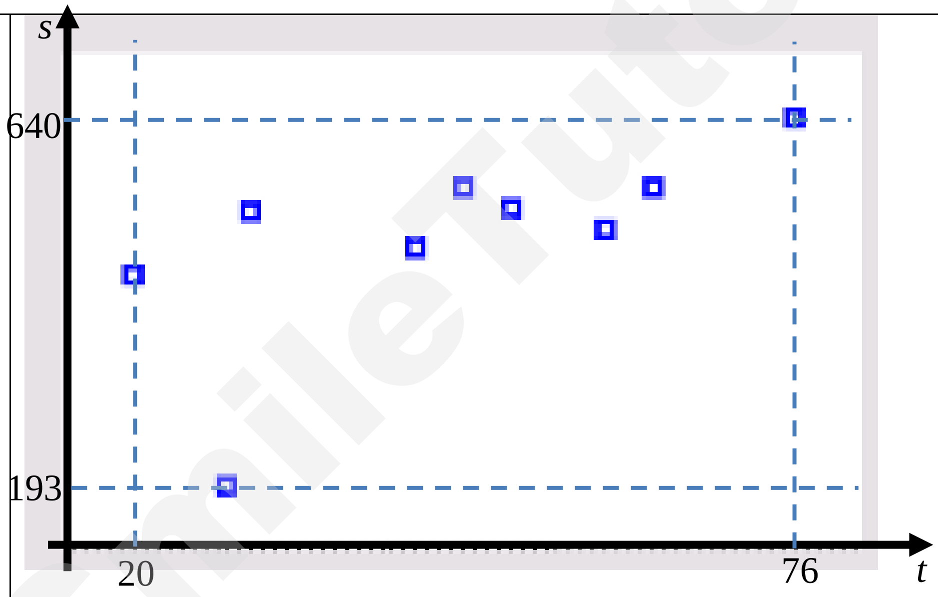
Remove the outlier you have identified in part (ii).

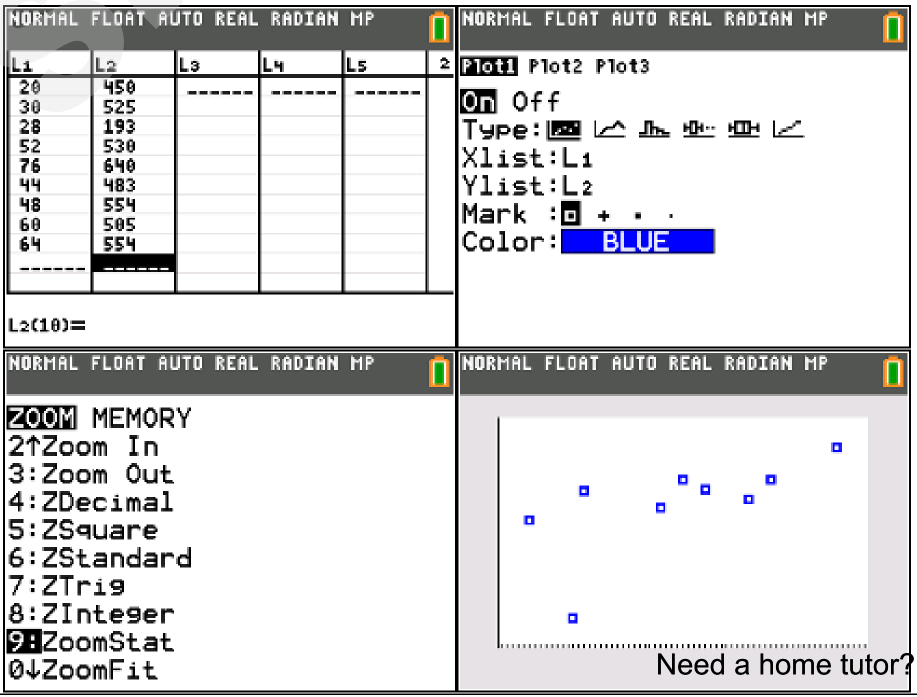
(iii) Calculate the new product moment correlation coefficient and find the regression line of s on t for the revised set of data. [2]

(iv) Estimate the mean amount of sleep of the student if his Task score is 80. Comment on the reliability of this estimate. [2]

(v) Using the appropriate regression line, estimate a student's Task score if his mean amount of sleep is 8.8 hours. [2]

(i)





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	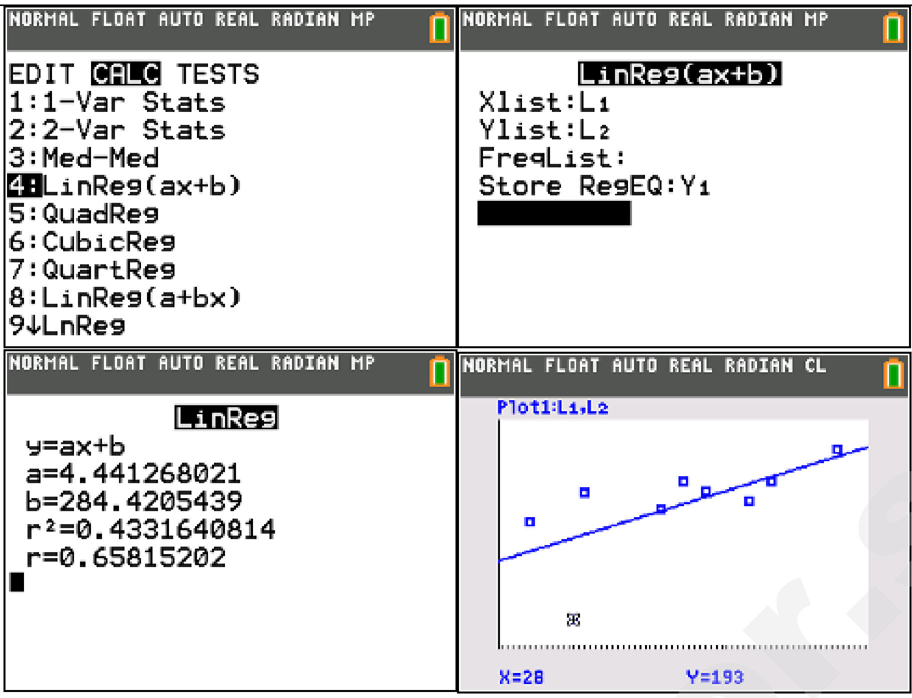	
(ii)	(28, 193) is an outlier, inconsistent with the trend of the other points. Possibly Error recording the amount of sleep or task score.	
(iii)	r = 0.78753 = 0.788 (3sf). $s = 2.4576t + 409.09$	
(iv)	Could t depend on s?	
(iv)	$s = 2.4576t + 409.09$ $s = 2.4576(80) + 409.09 = 606$(3sf) Unreliable, as $x = 80$ is outside the data range [20, 76], ~~through r value and scatter diagram show strong linear correlation~~.	
(v)	Use the line of regression of t on s. $t = 0.252355806s - 84.53012155$ First convert the 8.8 hrs to 528 mins $t = 0.252355806\ (528) - 84.53012155 = 48.71374402 = 48.7$ (3sf)	

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9 A researcher is testing the mileage of a particular type of electric car which the manufacturer claims to have a range of 380 km. A random sample of 50 cars of this type is tested and the range is measured. The results are summarised by

$$\sum(x-380) = -15 \text{ and } \sum(x-380)^2 = 81.$$

(i) Find the exact unbiased estimates of the population mean and variance. [2]

(ii) The researcher suspects that the manufacturer has overstated the mean range of the cars. Test at the 5% level of significance whether there is sufficient evidence to support the researcher's suspicion. [4]

(iii) Explain, whether it is necessary to assume that the range of the cars follow a normal distribution in order for the test to be valid. [1]

Another manufacturer claims that the mean range for their similar powered cars is more than 380 km. Another researcher takes a random sample of 80 cars and records their mileage. A test at the 5 % level of significance shows there is sufficient evidence to support the manufacturer's claim. Assuming the population standard deviation range for these cars is 1.8 km, find the set of values of the mean range of the mileage, correct to 2 decimal places. [3]

Solutions*

(i) $\bar{x} = -\frac{15}{50} + 380 = 379.7$ or $379\frac{7}{10}$ (exact)	
$s^2 = \frac{1}{49}\left(81 - \frac{(-15)^2}{50}\right) = \frac{153}{98}$ or $1\frac{55}{98}$ (exact) ~~= 1.56122449 ≈ 1.56~~	
(ii) We perform a one-tail test,	
$H_0 : \mu = 380$ against $H_1 : \mu < 380$ at 5% level of significance. Reject H_0 if p-value < 0.05	
Under H_0, as σ^2 is unknown and $n = 50$ is large, estimate σ^2 with s^2. Then $\bar{X} \sim N\left(380, \frac{153}{98(50)}\right)$ approximately.	
Carry out z test, p–value = 0.0447775078 Since p-value < 0.05, we reject H_0 and	
conclude that there is sufficient evidence at the 5% level of significance to support the researcher's suspicion that the manufacturer has overstated the mean range of the cars.	
(iii) No, as sample size of 50 is large, by Central Limit Theorem, the sample mean of the cars is approximately normally distributed.	
We perform a one-tail test,	
H_0: $\mu = 380$ against H_1: $\mu > 380$ (manufacturer's claim) at 5% level of significance.	
Under H_0, $\bar{X} \sim N\left(380, \frac{1.8^2}{80}\right)$ approximately.	

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$z = \dfrac{\overline{x} - 380}{\sqrt{\dfrac{1.8^2}{80}}}$ is in the critical region. The critical value is 1.644853627.	
We reject H_o, if $\dfrac{\overline{x} - 380}{\sqrt{\dfrac{1.8^2}{80}}} > 1.644853627$	
Thus the set of values of $\overline{x} > 380.3310204$ $\overline{x} > 380.33$ (2dp)	

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10 A restaurant sells cooked crabs and lobsters.

The masses, in kg, of the crabs and lobsters have independent normal distributions with means, standard deviations and selling prices as shown in the following table.

	Mean Mass	Standard deviation of mass	Selling price ($ per kg)
Crabs	1.9	0.2	45
Lobsters	1.6	0.15	85

(i) Find the probability that the mass of a randomly chosen crab is within ±0.1 kg of the mean mass of crabs. [2]

(ii) Find the probability that 4 randomly chosen crabs each has a mass of more than 1.8 kg. [2]

Mr Tan goes to the restaurant and randomly chooses 4 crabs and 6 lobsters.

(iii) Find the probability that the total mass of the 6 lobsters is at least 1.5kg more than the total mass of the 4 crabs. [3]

(iv) Find the probability that Mr Tan pays between $1150 and $1170 for the 4 crabs and 6 lobsters. State the mean and variance of the distribution that you use. [4]

(i)	Let X be random variable "the mass of a randomly chosen crab."	
	Then $X \sim N(1.9, 0.2^2)$.	
	$P(1.9 - 0.1 < X < 1.9 + 0.1)$ $= P(1.8 < X < 2)$ $= 0.382924922548026$	
(ii)	$P(X > 1.8) = 0.691462461274013$ P(4 randomly chosen crabs each have a mass of more than 2 kg) $= (0.691462461274013)^4 = 0.22859905507626300 = 0.229$ (3SF)	
(iii)	Let Y be random variable "the mass of a randomly chosen lobster." Then $Y \sim N(1.6, 0.15^2)$.	
	Let $A = (Y_1 + Y_2 + \dots + Y_6) - (X_1 + X_2 + X_3 + X_4)$ $\sim N(6\times1.6 - 4\times1.9,\ 6\times0.15^2 + 4\times0.2^2)$	
	$A \sim N(2, 0.295)$	
	$P(X_1 + X_2 + X_3 + X_4 + 1.5 < Y_1 + Y_2 + \dots + Y_6)$ $P(A > 1.5)$ $= 0.821363720484063 = 0.821$ (3sf)	
(iv)	Let $B = 85(Y_1 + Y_2 + \dots + Y_6)$ $+ 45(X_1 + X_2 + X_3 + X_4)$ $\sim N(85\times6\times1.6 + 45\times4\times1.9,\ 85^2\times6\times0.15^2 + 45^2\times4\times0.2^2)$ $B \sim N(1158, 1299.375)$	
	$P(\$1150 < B < \$1170)$	
	$= 0.218212321094298$ $= 0.218$ (3sf)	

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11 A box containing 4 black balls and 6 red balls. Mary draws a ball from the box at random without replacement. One white ball is added to the box, then Jane draws a ball from the box at random.

(i) Draw a probability tree diagram to represent all the possible outcomes. [2]

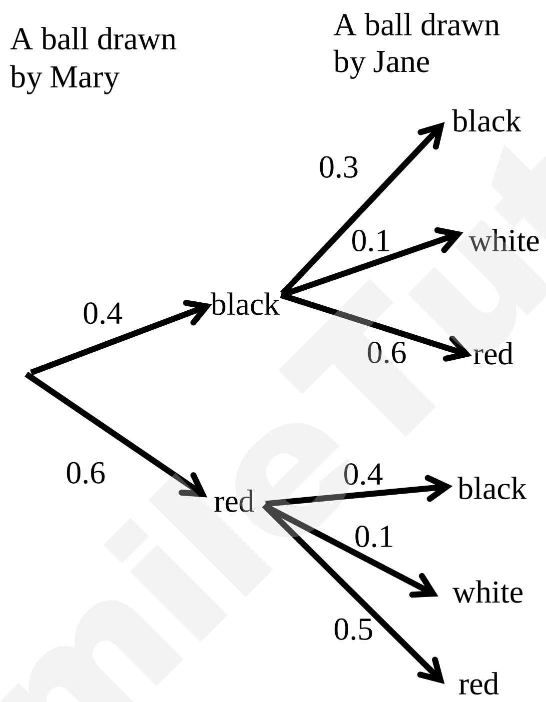
(ii) Find the probability that Jane draws a black ball. [2]

(iii) Find the probability that Mary draws a black ball, given that Jane draws a ball that is not black. [3]

(iv) Find the probability that *either* Mary draws a black ball *or* Jane draws a ball that is not black or *both*. [2]

Suppose Jane wins a prize if she draws the same coloured ball as Mary. Find the probability that Jane wins a prize, given that Jane does not draw a white ball. [3]

Solutions*

(i) 	
Let M be the event "Mary draws a black ball" and J be the event "Jane draws a black ball."	
(ii) P(Jane draws a black ball) = P(J)	
$= 0.4\times0.3 + 0.6\times0.4$	
$= 0.36$ or $9/25$	
(iii) P(Mary draws a black ball given that Jane does not draw a black ball) $= \text{P}(M \mid J')$	
= P(Mary draws a black ball $\cap$ Jane does not draw a black ball) $\div$ P(Jane does not draw a black ball)	
$= 0.4\times (1 - 0.3) \div (1 - 0.36)$ $= 0.4375$ or $7/16$	
(iv) P(Mary draws a black ball or Jane does not draw a black ball or both)	
$= \text{P}(M \cup J')$	
$= \text{P}(M) + \text{P}(J') - \text{P}(M \cap J')$	
$= 0.4 + 0.64 - 0.28$	

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= 0.76 or 19/25	
P(Jane wins a prize | Jane does not draw a white ball) = P(Jane draws the same colour ball as Mary ∩ Jane does not draw a white ball) ÷ P(Jane does not draw a white ball) = P(Jane draws the same colour ball as Mary) ÷ P(Jane does not draw a white ball) = P(Jane and Mary both draws a black ball) + P(Jane and Mary both draws a red ball) ÷ P(Jane does not draw a white ball) = (0.4×0.3 + 0.6×0.5) ÷ 0.9 or (0.4×0.3 + 0.6×0.5) ÷ (0.4×0.9 + 0.6×0.9) = 0.467 or 7/15 **Note:** P(Jane does not draw a white ball) = 0.9 since there are 9 non-white out of 10 balls before Jane draws.	

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MATHEMATICS **8865/01**
Paper 1
13 September 2017
3 hours

Additional Materials: Cover page
Answer papers
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your full name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **7** printed pages.

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Section A: Pure Mathematics [40 marks]

1. A researcher prepares three types of food samples X, Y and Z for his experiment. Each food sample weighs 20 grams and contains three types of ingredients, namely, fibre, wheat and sweetener. The amount of fibre, wheat and sweetener in each sample is given below.

Sample Type	Amount in grams for each 20 grams of sample		
	Fibre	Wheat	Sweetener
X	12	6	2
Y	8	6	6
Z	6	14	0

The researcher wants to prepare a new food sample type T by mixing different amounts of sample types X, Y and Z such that in 20 grams of sample type T, there are 8.8 grams of fibre, 7.6 grams of wheat and 3.6 grams of sweetener.

Determine the amount, in grams, of samples types X, Y and Z in 20 grams of sample type T. [4]

2. Mr Woo purchased x kg of cherries from fruit stall A for \$$a$. He bought 1 kg less cherries from fruit stall B for \$$a$. He realised that fruit stall B charged him more by \$5 per kg. By considering the difference in the unit price of the cherries bought from the two fruit stalls or otherwise, show that $5x^2 - 5x - a = 0$. [2]

Find the maximum weight of cherries that Mr Woo can buy from fruit stall A if he does not want to spend more than \$50 on cherries. Give your answer to the nearest integer. [3]

3. **(a)** Differentiate $e^{x^2} + \dfrac{1}{px^2+1}$ with respect to x, where p is a constant. [2]

(b) Find $\displaystyle\int \frac{2x-1}{x+3}\,dx$. [3]

4. The curve C has equation $y = qx - \ln(2x^2+1)$, where q is a positive constant.

(i) Find, in terms of q, the equation of the tangent to C at the point where $x = 1$. Give your answer in the form $y = ax + b$, with a and b in exact forms. [3]

(ii) Find the exact value of q such that C has 1 stationary point. [3]

(iii) Using the value of q found in **(ii)**, find the equation of the tangent which is parallel to the x-axis. [2]

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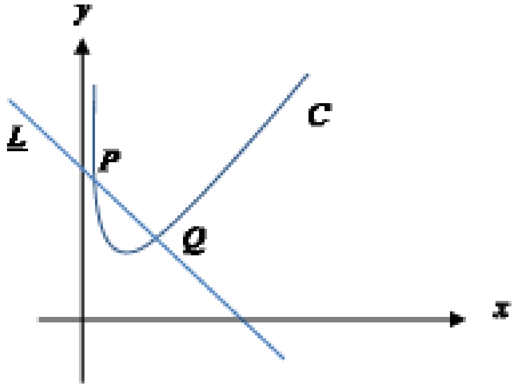
5.



The diagram shows the curve C with equation $y=\left(\sqrt{x}+\dfrac{2k}{\sqrt{x}}\right)^2$ and the line L with equation $y=13k-x$, where k is a positive constant. The graphs intersect at P and Q as shown. Show that the x-coordinates of P and Q are $\frac{1}{2}k$ and $4k$ respectively. [2]

Hence find, in terms of k, the area of the region bounded by C, L, the x-axis and the line $x=k$. [4]

6. The number of customers (in thousands), C, of a new company is believed to be modelled by the equation

$$C=8(1-\mathrm{e}^{-kt}),$$

where t is the number of years from the time the company starts its operation and k is a positive constant.

(i) Given that the company has 7 thousand customers at the end of the 3rd year of operation, determine the exact value of k, giving your answer in the simplest form. [3]

Using the value of k found in (i),

(ii) sketch the graph of C against t, stating the equations of any asymptotes, [2]

(iii) find the exact value of $\dfrac{\mathrm{d}C}{\mathrm{d}t}$ when $t=2$, simplifying your answer. Give an interpretation of the value you have found, in the context of the question. [3]

At the end of the 6th year of operation, the number of customers of the company is now believed to be modelled by the equation

$$C=-0.05t^2+0.7t+5.475,$$

where $t\geq 6$.

(iv) Use differentiation to find the value of t, where $t\geq 6$, which gives the maximum value of C. Hence, find the maximum value of C. [4]

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Section B: Statistics [60 marks]

7. A group of 10 students consisting of 6 females and 4 males bought tickets to attend a concert. If the tickets were for a particular row of 10 adjacent seats, find the number of possible seating arrangements when

(i) the first and last seats were occupied by students of the same gender, [3]

(ii) one particular student did not turn up for the concert. [1]

8. At a stall in a fun-fair, games of chance are played, where at most 1 prize is won per game. The probability that a prize is won in each game is 0.1. For each day, 80 games are played. The random variable X is the number of prizes being won on a particular day.

(i) Find $\mathrm{P}(X > 5)$. [1]

The stall is opened for n days and on each day, 80 games are played.

(ii) If n = 10, find the probability that there are 4 days with at most 5 prizes being won each day. [2]

(iii) If n is large, using a suitable approximation, find the minimum value of n such that the probability that the average number of prizes being won each day exceeds 8.5 is less than 0.1. [4]

9. In a box containing a large number of apples, 15% of the apples are rotten. A random sample of 20 apples is drawn to inspect.

(i) Explain the significance of the phrase 'large number' in the first sentence of the question. [1]

(ii) Find the probability that there is at least 1 but less than 4 rotten apples in the random sample. [3]

A box containing large number of apples is chosen for export if there is no rotten apple from the random sample. If there is at least 1 but less than 4 rotten apples in the random sample, another random sample of 10 apples is drawn from the box to inspect. If there is no rotten apple in the second random sample, the box will be chosen for export. Otherwise, the box will not be chosen for export.

(iii) Find the probability that a randomly chosen box is chosen for export. [2]

(iv) Four boxes each containing a large number of apples are chosen for inspection. If it is known that the first box is chosen for export, find the probability that exactly two out of the four boxes are chosen for export. [2]

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10. A magazine claims that the average time a child spends outdoors is no longer than 14 hours a week. To verify this, Henry randomly selects 50 children and the amount of time that each child spends outdoors in a particular week, x hours, is recorded. The results are summarised as follows.

$$\sum(x-14)=3.9 \qquad \sum(x-14)^2=2.7$$

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Suggest a reason why, in this context, the given data is summarised in terms of $(x-14)$ rather than x. [1]

(iii) Test at the 1% significance level whether the claim made in the magazine is valid. [5]

11. (a) Seven pairs of values of variables x and y are measured where x and y are positive values. Draw a sketch of a possible scatter diagram for each of the following cases:

(i) the product moment correlation coefficient is approximately zero, [1]

(ii) the product moment correlation coefficient is approximately 0.8. [1]

(b) A study on how the trade-in value p, in thousand dollars, of a particular make of used car depreciates with the age of the car t, in years, is conducted. The data for 7 cars is collected and shown in the following table.

Age, t	2	3	4	5	6	7	8
Trade-in value, p	54.0	50.1	45.3	38.6	35.1	33.5	30.4

(i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]

(ii) Find the product moment correlation coefficient and comment on its value in the context of this question. [2]

(iii) Find the equation of the regression line of p on t in the form $p = mt + c$, giving the values of m and c correct to 2 decimal places. [1]

(iv) The data for a second sample of another 6 cars is obtained. The regression lines of p on t and of t on p for the second sample are given respectively as:

$$p = 61.45 - 4.19t \quad \text{and} \quad t = 14.39 - 0.23p\,.$$

Calculate the mean trade-in value and mean age for the combined sample of 13 cars. [3]

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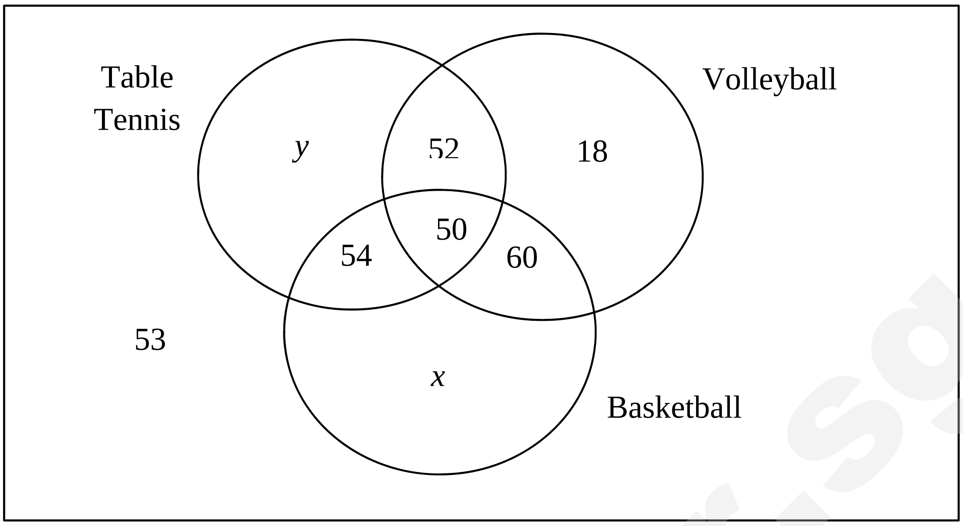
12.



A group of students are surveyed on the types of sport(s) they can play out of the three sports namely table tennis, volleyball and basketball. The numbers of students who can play the different sport(s) are shown in the Venn diagram. The number of students who can play table tennis only is y and the number of students who can play basketball only is x. One of the students is chosen at random.

T is the event that the student can play table tennis.
B is the event that the student can play basketball.
V is the event that the student can play volleyball.

(i) Write down the expressions for P(T) and P(V) in terms of x and y. Given that T and V are independent, show that $13y - 17x = 199$. [3]

(ii) Given that $\mathrm{P}(T \cup B) = \dfrac{379}{450}$, find the values of x and y. [3]

Using the values of x and y found in **(ii)**, find

(iii) $\mathrm{P}(B \cap (T \cup V))$, [1]

(iv) $\mathrm{P}(T \mid V)$. [1]

Three students from the whole group are chosen at random.

(v) Find the probability that among the three students, one can play exactly two sports out of the three sports (table tennis, volleyball and basketball), the other one can play table tennis only and the remaining one cannot play any of the sports. [3]

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13. The masses, in kilograms, of cod fish and salmon fish sold by a fishmonger are normally distributed. The means and standard deviations of these distributions, and the selling prices, in $ per kilogram, of cod fish and salmon fish are shown in the following table.

	Mean (kg)	Standard deviation (kg)	Selling price ($ per kg)
Cod Fish	a	0.1	68
Salmon Fish	0.6	0.15	30

(i) Find the probability that a randomly chosen cod fish has mass less than $(0.2 + a)$ kg. [2]

(ii) It is known that 20% of the cod fish sold by the fishmonger have a mass of at least 0.5 kg. Find the value of a. [3]

Use $a = 0.4$ for the rest of the question.

(iii) Find the probability that the total mass of 4 randomly chosen cod fish is within $\pm$ 0.1 kg of twice the mass of a randomly chosen salmon fish. [4]

(iv) Find the probability that a randomly chosen cod fish has a selling price exceeding $25 and a randomly chosen salmon fish has a selling price exceeding $15. [2]

(v) State an assumption needed for your calculations in **(iii)** and **(iv)**. [1]

- **End of paper** -

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J2 H1 Math Prelim Exam (Solutions)

1. Let the amount of samples types X, Y and Z in 20 grams of sample type T be x, y and z.

Fibre:

$$\left(\frac{12}{20}\right)x+\left(\frac{8}{20}\right)y+\left(\frac{6}{20}\right)z=8.8$$

$$0.6x+0.4y+0.3z=8.8 \qquad ---(1)$$

Wheat :

$$\left(\frac{6}{20}\right)x+\left(\frac{6}{20}\right)y+\left(\frac{14}{20}\right)z=7.6$$

$$0.3x+0.3y+0.7z=7.6 \qquad ---(2)$$

Sweetener :

$$\left(\frac{2}{20}\right)x+\left(\frac{6}{20}\right)y+\left(\frac{0}{20}\right)z=3.6$$

$$0.1x+0.3y+0z=3.6 \quad ---(3)$$

From GC

$x=6,\quad y=10,\ z=4$

There are 6 g of sample type X, 10 g of sample type Y and 4 g of sample type Z in 20 g of sample type T.

2.

$$\frac{a}{x-1}=\frac{a}{x}+5$$

$$\frac{a}{x-1}=\frac{a+5x}{x}$$

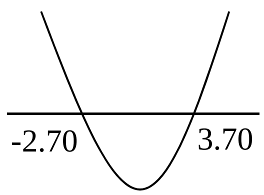
$$ax=ax-a+5x^2-5x$$

$5x^2-5x-a=0$ (shown)

To spend not more than \$50 for x kg of cherries $\Rightarrow a\le 50$

$$5x^2-5x\le 50$$

$$x^2-x-10\le 0$$



Note:
The graphical method to solve the inequality must be shown.

$-2.70\le x\le 3.70$

The maximum amount of cherries Mr Woo can buy is 3 kg.

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3(a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{x^2}+\dfrac{1}{px^2+1}\right)=2x\mathrm{e}^{x^2}-\dfrac{2px}{\left(px^2+1\right)^2}$

(b) $\displaystyle\int \frac{2x-1}{x+3}\,\mathrm{d}x=\int 2-\frac{7}{x+3}\,\mathrm{d}x$
$=2x-7\ln(x+3)+C$

4(i) $\dfrac{\mathrm{d}y}{\mathrm{d}x}=q-\dfrac{4x}{2x^2+1}$

When $x=1$,

$y=q-\ln 3$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x}=q-\dfrac{4}{3}$

Equation of tangent:

$y=\left(q-\dfrac{4}{3}\right)x+c$

$q-\ln 3=\left(q-\dfrac{4}{3}\right)(1)+c\Rightarrow c=\dfrac{4}{3}-\ln 3$

$\therefore y=\left(q-\dfrac{4}{3}\right)x+\dfrac{4}{3}-\ln 3$

Note:
The bracket in $y=\left(q-\dfrac{4}{3}\right)x+\dfrac{4}{3}-\ln 3$ must be clearly shown.

(ii) $\dfrac{\mathrm{d}y}{\mathrm{d}x}=q-\dfrac{4x}{2x^2+1}$

For stationary points,

$\dfrac{\mathrm{d}y}{\mathrm{d}x}=q-\dfrac{4x}{2x^2+1}=0$

$2qx^2+q-4x=0$

Since C has 1 stationary point, $b^2-4ac=0$

$(-4)^2-4(2q)(q)=0$

$16-8q^2=0$

$q^2=2$

$q=-\sqrt{2}$ (NA since $q>0$) or $q=\sqrt{2}$

(iii) $y=\sqrt{2}x-\ln(2x^2+1)$

$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\sqrt{2}-\dfrac{4x}{2x^2+1}$

Given that the tangent is parallel to the x-axis,

$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\sqrt{2}-\dfrac{4x}{2x^2+1}=0$

$2\sqrt{2}x^2+\sqrt{2}-4x=0$

Using GC, $x=0.70711$ or $\dfrac{1}{\sqrt{2}}$.

Hence, the equation of the tangent is $y=1-\ln 2$ or $y=0.307$.

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5.

$$\left(\sqrt{x}+\frac{2k}{\sqrt{x}}\right)^2=13k-x$$

$$x+\frac{4k^2}{x}+4k=13k-x$$

$$x^2+4k^2+4kx=13kx-x^2$$

$$2x^2-9kx+4k^2=0$$

$$(2x-k)(x-4k)=0$$

$$x=\frac{1}{2}k \quad \text{or} \quad x=4k \text{ (shown)}$$

Note:
Factorisation must be shown

Alternative:

$$x=\frac{-(-9k)\pm\sqrt{(-9k)^2-4(2)(4k^2)}}{2(2)}$$

$$=\frac{9k\pm\sqrt{49k^2}}{4}$$

$$=\frac{9k\pm 7k}{4}$$

$$=\frac{16k}{4} \quad \text{or} \quad \frac{2k}{4}$$

$$=4k \quad \text{or} \quad \frac{k}{2} \quad \text{(shown)}$$

$$\text{Required area } =\int_k^{4k}\left(\sqrt{x}+\frac{2k}{\sqrt{x}}\right)^2 \mathrm{d}x+\int_{4k}^{13k}(13k-x)\,\mathrm{d}x$$

$$=\int_k^{4k}\left(x+\frac{4k^2}{x}+4k\right)\mathrm{d}x+\text{Area of triangle}$$

$$=\left[\frac{x^2}{2}+4k^2\ln x+4kx\right]_k^{4k}+\frac{1}{2}(9k)(9k)$$

$$=\left(8k^2+4k^2\ln 4k+16k^2\right)-\left(\frac{1}{2}k^2+4k^2\ln k+4k^2\right)+\frac{81}{2}k^2$$

$$=60k^2+4k^2\ln 4$$

$$=60k^2+8k^2\ln 2$$

6(i)

$$7=8(1-\mathrm{e}^{-3k})$$

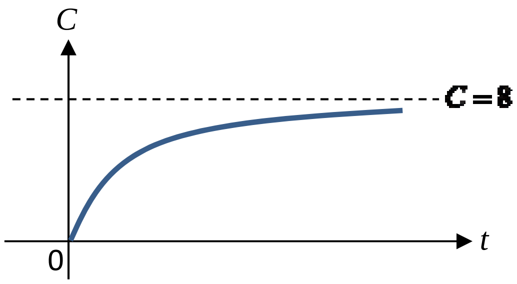
$$\frac{7}{8}=1-\mathrm{e}^{-3k}$$

$$\mathrm{e}^{-3k}=\frac{1}{8}$$

$$-3k=\ln\frac{1}{8}=-3\ln 2$$

$$k=\ln 2$$

(ii)



Note:
Curve is in the first quadrant only

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(iii) $C = 8(1-e^{-kt}) = 8 - 8e^{-t\ln 2}$

$\frac{dC}{dt} = 8\ln 2e^{-t\ln 2}$

$\left.\frac{dC}{dt}\right|_{t=2} = 8\ln 2e^{-2\ln 2}$

$= 8\ln 2e^{\ln\frac{1}{4}}$

$= 8\ln 2\left(\frac{1}{4}\right)$

$= 2\ln 2$

This value indicates that the number of customers is **increasing** at a rate of 2ln2 **thousands per year at the end of the second year of operation.**

(iv) $\frac{dC}{dt} = -0.1t + 0.7$

$-0.1t + 0.7 = 0$

$t = 7$

$C = -0.05(7)^2 + 0.7(7) + 5.475 = 7.925$

Hence, the maximum number of customers is 7.925 **thousands** customers or 7925 when $t = 7$.

7(i) Case 1: First and last seats occupied by males
Number of ways = $4 \times 3 \times 8!$ = 483 840 ways
OR Number of ways = ${}^4C_2 \times 2! \times 8!$ = 483 840 ways

Case 2: First and last seats occupied by females
Number of ways = $6 \times 5 \times 8!$ = 1209 600 ways
OR Number of ways = ${}^6C_2 \times 2! \times 8!$ = 1209 600 ways

$\therefore$ Total number of ways $= 483\ 840 + 1209\ 600 = 1693\ 440$ ways

(ii) Number of ways = $10! =$ 36 288 00 ways

8(i) Given X denotes the number of prizes being won out of 80 games on a particular day.
Then $X \sim B(80, 0.1)$

$P(X > 5) = 1 - P(X \le 5) \approx 0.82308 \approx 0.823$

(ii) $P(X \le 5) \approx 0.17692$
Let Y denotes the number of days with at most 5 prizes being won each day out of 10 days. Then $Y \sim B(10,\ 0.17692)$

Note: For p to be in 5 sf

$P(Y = 4) \approx 0.063970 \approx 0.0640$

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(iii) $\mathrm{E}(X) = 80\times 0.1 = 8$
$\mathrm{Var}(X) = 80\times 0.1\times 0.9 = 7.2$

Since n is large, by Central Limit Theorem, $\overline{X} \sim \mathrm{N}(8, \frac{7.2}{n})$ approximately.

$\mathrm{P}(\overline{X} > 8.5) < 0.1$

$$\Rightarrow \mathrm{P}\left(Z > \frac{8.5-8}{\sqrt{\frac{7.2}{n}}}\right) < 0.1$$

$$\Rightarrow 1-\mathrm{P}\left(Z \le \frac{0.5}{\sqrt{\frac{7.2}{n}}}\right) < 0.1$$

$$\Rightarrow \mathrm{P}\left(Z \le \frac{0.5}{\sqrt{\frac{7.2}{n}}}\right) > 0.9$$

Take note on the change in the inequality

$$\Rightarrow \frac{0.5}{\sqrt{\frac{7.2}{n}}} > 1.2816$$

Solving, $n > 47.300$ (5 s.f.)

The minimum value of n is 48.

<u>Alternative (Using table)</u>
Using GC,
When $n = 47$, $\mathrm{P}(\overline{X} > 8.5) \approx 0.10072 > 0.1$
When $n = 48$, $\mathrm{P}(\overline{X} > 8.5) \approx 0.09835 < 0.1$
When $n = 49$, $\mathrm{P}(\overline{X} > 8.5) \approx 0.09605 < 0.1$

The minimum value of n is 48.

9(i) The phrase 'large number' in the first sentence is required in order to assume that the probability of a rotten apple is approximately constant at 0.15.

(ii) Let X denote the number of rotten apples in a random sample of 20 apples.
Then $X \sim \mathrm{B}(20, 0.15)$
$\mathrm{P}(1 \le X \le 3) = \mathrm{P}(X \le 3) - \mathrm{P}(X = 0) \approx 0.60897 \approx 0.609$
Or
$\mathrm{P}(1 \le X \le 3) = \mathrm{P}(X = 1) + \mathrm{P}(X = 2) + \mathrm{P}(X = 3) \approx 0.60897 \approx 0.609$

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(iii) Let Y denote the number of rotten apples in a random sample of 10 apples.
Then $Y \sim B(10, 0.15)$
P(A randomly chosen box is chosen for export)
$= P(X = 0) + P(1 \le X \le 3) \times P(Y = 0)$
$\approx 0.15865 \approx 0.159$

(iv) Required probability =
$$\frac{\text{P(First box is chosen for export)} \times \text{P(One out of the 3 remaining boxes chosen for export)} \times 3}{\text{P(First box chosen for export)}}$$
$\approx 0.15865 \times (1 - 0.15865)^2 \times 3$
$\approx 0.33691 \approx 0.337$

Alternative solution:
Let W denote the number of boxes chosen for export out of 3.
Then $W \sim B(3, 0.15865)$
$P(W = 1) \approx 0.337$

10(i) Unbiased estimate of the population mean, $\bar{x} = \frac{3.9}{50} + 14 = 14.078$
Unbiased estimate of the population variance,
$$s^2 = \frac{1}{49}\left[2.7 - \frac{(3.9)^2}{50}\right] \approx 0.048894 \approx 0.0489$$

(ii) Possible reasons:
- Keep the recorded values small since they are around 14 hours.
- Give an indication of the variations around the hypothesised mean of 14 hours.

(iii) $H_0: \mu = 14$ vs $H_1: \mu > 14$
Since $n = 50$ is large, by Central Limit Theorem, $\bar{X} \sim N(14, \frac{0.048894}{50})$ approximately.
Level of significance: 1%
Critical region: $z \ge 2.3263$

Note: State the distribution of $\bar{X}$

Standardised test statistic $z = \dfrac{14.078 - 14}{\sqrt{\dfrac{0.048894}{50}}} \approx 2.4943 > 2.3263$
Using GC, p-value $\approx 0.0063100 < 0.01$
Since the p-value is smaller than the level of significance, we reject H_0. There is sufficient evidence, at 1% level of significance, to conclude that Henry's claim is invalid.

Alternative conclusion:
Since the standardised test statistic falls inside the critical region, we reject H_0. There is sufficient evidence, at 1% level of significance, to conclude that Henry's claim is invalid.

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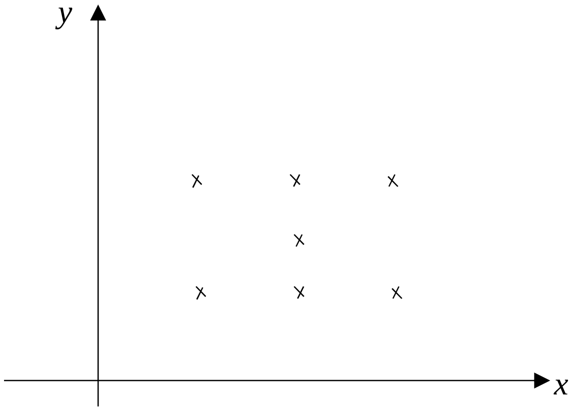
11(a)(i) One possible scatter diagram

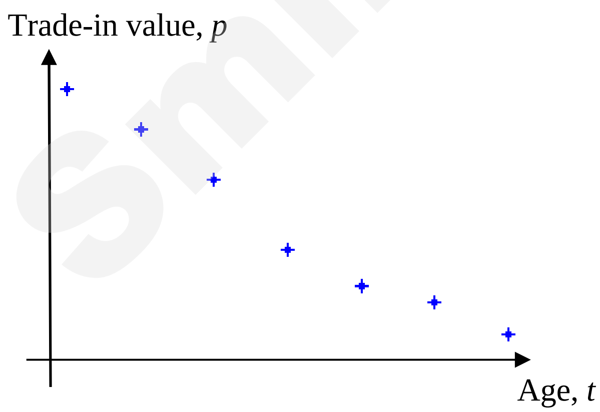


or

Note:
Curve must be symmetrical

(ii) One possible diagram

(b)(i)



Note:
Label the axes correctly

(ii) $r = -0.986$
There is a strong negative **linear** correlation between the age of the car and the average trade-in value of the car.

(iii) $p = -4.0786t + 61.3929$
$p \approx -4.08t + 61.39$ (to 2 d.p.)

(iv) Let the mean trade-in value and mean age of the 6 cars be a and b respectively.

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$p = 61.45 - 4.19t \Rightarrow p + 4.19t = 61.45$
$t = 14.39 - 0.23p \Rightarrow 0.23p + t = 14.39$
Solving using GC, $a = 31.843$ and $b = 7.0661$

Mean age for 13 cars
$= \dfrac{2+3+4+5+6+7+8+6(7.0661)}{13}$
≈ 5.95 (3 s.f)

Mean age for 13 cars is 5.95

Mean trade-in value for 13 cars
$= \dfrac{54+50.1+45.3+38.6+35.1+33.5+30.4+6(31.843)}{13}$
$= 36.8$
Mean trade-in value for 13 cars is 36.8 thousand dollars (or $36800).

12(i) P(T) = $\dfrac{156+y}{287+x+y}$, P(V) = $\dfrac{180}{287+x+y}$, P($T \cap V$) = $\dfrac{102}{287+x+y}$
Given that T and V are independent, this means that
P($T \cap V$) = P(T) × P(V)
$\Rightarrow \dfrac{102}{287+x+y} = \dfrac{156+y}{287+x+y} \times \dfrac{180}{287+x+y}$
$\Rightarrow$ 102 (287 + x + y) = 180 (156 + y)
$\Rightarrow 29\ 274 + 102x + 102y = 28\ 080 + 180y$
$\Rightarrow 1194 + 102x - 78y = 0$
$\Rightarrow -39y + 51x + 597 = 0$
$\Rightarrow 39y - 51x = 597$
$\therefore 13y - 17x = 199$ (Shown) ----- (1)

(ii) P($T \cup B$) = $\dfrac{379}{450}$
$\Rightarrow \dfrac{216+x+y}{287+x+y} = \dfrac{379}{450}$
$\Rightarrow 450(216+x+y) = 379(287+x+y)$
$\Rightarrow 97\ 200 + 450x + 450y = 108\ 773 + 379x + 379y$
$\Rightarrow 71x + 71y = 11\ 573$
$\therefore x + y = 163$ -----(2)

Using GC, solving (1) and (2) simultaneously, $x = 64$ and $y = 99$

(iii) P($B \cap (T \cup V)$) = $\dfrac{164}{450} = \dfrac{82}{225}$

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(iv) $\text{P}(T \mid V) = \dfrac{\text{n}(T \cap V)}{\text{n}(V)} = \dfrac{102}{180} = \dfrac{17}{30}$

(v) Required probability = $\dfrac{166}{450} \times \dfrac{99}{449} \times \dfrac{53}{448} \times 3! \approx 0.0577$

13(i) Let C denote the mass of cod fish. Then $C \sim \text{N}(a,\ 0.1^2)$.

$\text{P}(C < 0.2 + a) = \text{P}\left(Z < \dfrac{(0.2+a)-a}{0.1}\right) = \text{P}(Z < 2) \approx 0.97725 \approx 0.977$

(ii) Given that $\text{P}(C \geq 0.5) = 0.2$

$\Rightarrow \text{P}(C < 0.5) = 0.8$

$\Rightarrow \text{P}(\left(Z < \dfrac{0.5-a}{0.1}\right) = 0.8$

$\Rightarrow \dfrac{0.5-a}{0.1} \approx 0.84162$

$\therefore\ a \approx 0.416$

(iii) Using $a = 0.4$,

Consider $T = C_1 + C_2 + C_3 + C_4 - 2S \sim \text{N}(0.4,\ 0.13)$

$\text{P}(-0.1 < T < 0.1) \approx 0.11993 \approx 0.120$

(iv) Using $a = 0.4$,

Let $68C$ be the selling price of cod fish. Then $68C \sim \text{N}(27.2,\ 46.24)$

Let $30S$ be the selling price of salmon fish. Then $30S \sim \text{N}(18,\ 20.25)$

$\text{P}(68C > 25) \times \text{P}(30S > 15) \approx 0.46858 \approx 0.469$

<u>Alternative</u>

$\text{P}(C > \dfrac{25}{68}) \times \text{P}(S > \dfrac{15}{30}) \approx 0.46858 \approx 0.469$

(v) The mass of all the fish are independent of one another.

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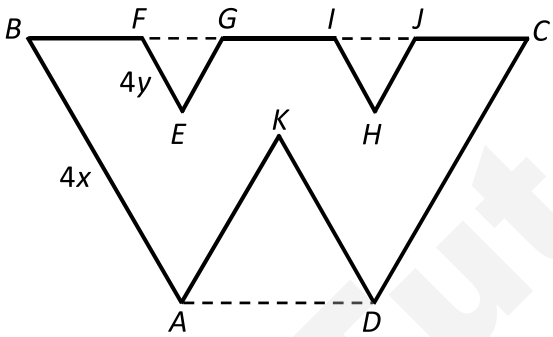
RI H1 Mathematics 2017 Prelim Exam Paper Question

<table>
<tr><td>1</td><td>Differentiate

(a) $\ln\{7\sqrt{(1+6x^2)}\}$, [2]

(b) $\dfrac{1}{6(1-7x)^2}$. [2]</td></tr>
<tr><td>2</td><td>A designer wishes to make a logo for her client. Her design for the logo is shown in the diagram below, which is formed by removing three triangles EFG, HIJ and AKD from a trapezium ABCD.


For aesthetic purpose, her design has the following features:
(1) Angles ABC and BCD are both 60°.
(2) Triangles EFG and HIJ are identical and equilateral.
(3) The points F, G, I and J are on BC such that BF = GI = JC.
(4) Triangle AKD is also equilateral, and the point K lies on the intersection of the line segments AI and GD.

It is given that $AB = CD = 4x$ mm and $EF = 4y$ mm.
(i) Show that $BF = 2(x-y)$ mm. [1]
(ii) Show that $AD = 2(x+y)$ mm. [1]

The perimeter and area of ABFEGIHJCDKA is 566 mm and $4655\sqrt{3}$ mm^2 respectively.

(iii) Find the values of x and y. [4]</td></tr>
<tr><td>3</td><td>Do not use a calculator in answering this question.

(a) Solve the inequality $\dfrac{(x-4)(x^2-4x+4)}{x+4} \geq 0$. [4]

(b) Find the range of values of k for which $(k-1)x^2 - 2x + k + 2 < 0$ for all real values</td></tr>
</table>

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	of x. [4]
4	A jug containing liquid is taken from a refrigerator and placed in a room with a constant temperature of 25 °C. The temperature of the liquid θ °C after time t minutes is given by $$\theta = 25 - Ae^{kt},$$ where A and k are real constants. Initially the temperature of the liquid is 9 °C. After 20 minutes, the temperature of the liquid increases to 17 °C. **(i)** Find the value of A and show that $k = \frac{1}{20}\ln\frac{1}{2}$. [4] **(ii)** Find the temperature of the liquid after 25 minutes. [1] **(iii)** Find the exact duration it takes for the temperature of the liquid to increase from 17 °C to 23 °C. [2] **(iv)** State what happens to θ for large values of t. [1] **(v)** Sketch a graph of θ against t. [2]
5	The curve C_1 has equation $y = \ln(1 + x)$ and the curve C_2 has equation $y = \ln 2 + 1 - x$. **(i)** Sketch the graphs of C_1 and C_2 on the same diagram, stating the equations of any asymptotes and the exact coordinates of any points where the curves crosses the axes. [3] **(ii)** Verify that C_1 and C_2 intersect at $x = 1$. [1] **(iii)** Find, correct to 2 decimal places, the area of the finite region bounded by C_1, C_2 and the x-axis. [3] **(iv)** Use integration to find the exact area of the finite region bounded by C_1, C_2 and the y-axis. Leave your answer in the form $A + B\ln 2$, where A and B are constants to be determined. [5]
6	It is given that $X \sim N(\mu, 7)$ and $P(X < 7) = 0.7$. **(i)** Find the value of μ correct to 3 decimal places. [2] **(ii)** Find $P(X_1 < X_2 + 1)$, where X_1 and X_2 are independent observations of X. [2]
7	A traditional bakery produces two types of biscuits – one with sweet fillings and one

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	with salted fillings. The biscuits are sold in packs of 8, and each pack has a random selection of the two types of biscuits. The mean number of biscuits with sweet fillings in each pack is 3.2. **(i)** Find the probability that a randomly chosen pack contains no biscuits with sweet fillings. [2] **(ii)** Show that the probability that a randomly chosen pack contains at least four biscuits with sweet fillings is 0.406 correct to 3 significant figures. [2] A customer buys 18 packs of biscuits for a wedding. **(iii)** Find the probability that at most 9 of these packs contains at least four biscuits with sweet fillings. [2]
8	Box A contains 2 green marbles and 6 red marbles. Box B contains 4 green marbles and 2 red marbles. Two fair dice are tossed at the same time. Box A is selected if at least one '6' is shown. Otherwise, box B is selected. One marble is then chosen from the selected box and its colour noted. Draw a tree diagram to represent this situation. [4] Find the probability that **(i)** the marble chosen is green, [2] **(ii)** the marble chosen is from box A, given that its colour is red. [2] Give your answers as a fraction in its lowest term.
9	Caffeine is said to affect our sleep at night. In a student research study, different amounts of caffeine, x grams, were given to a test subject for 8 consecutive evenings and the times, t minutes, for the test subject to fall asleep at night were recorded. The results are given in the table below. (table below) **(i)** Draw a scatter diagram to illustrate the data. [2] **(ii)** Calculate the equation of the regression line of t on x, and draw this line on your scatter diagram. [1] **(iii)** Find $\bar{x}$ and $\bar{t}$, and mark the point $(\bar{x}, \bar{t})$ on your scatter diagram. [1]

Day	1	2	3	4	5	6	7	8
x	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
t	15	16	20	23	25	24	30	35

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	(iv) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]
	(v) Use the regression line in part **(ii)** to predict the time taken for the test subject to fall asleep when 1.00 grams of caffeine was given. Comment on the validity of this prediction. [2]
10	A farmer grows watermelons. He claims that the average mass of watermelons in his farm is at least 10 kg. To test this claim, a random sample of 70 watermelons is checked and the masses of watermelons, x kg, are summarised as follows: $$\sum(x-10)=-28,\ \sum(x-10)^2=267.$$ **(i)** Find unbiased estimates of the population mean and variance. [3] **(ii)** Test at the 5 % significance level whether the farmer's claim is valid. [4] The farming process is improved and the new population variance is known to be 3.31 kg^2. A new random sample of 70 watermelons is checked and the total mass of this sample is m kg. A test at the 5 % significance level shows that there is sufficient evidence to suggest that the population mean mass of watermelons is more than 10 kg. **(iii)** Find the range of possible values of m. [3]
11	A pet shop owner carries African bullfrogs. The masses, in kg, of the male and female frogs are independent and are normally distributed with means and standard deviations as shown in the following table:

	Mean mass	Standard deviation
Male	1.4	0.28
Female	0.7	0.14

(i) A male and a female bullfrog are chosen at random. Find the probability that the mass of the female frog is greater than the mass of the male frog, stating clearly the mean and variance of any distribution that you use. [3]

The owner wishes to build a tank for up to four frogs. If he uses material X, the total mass of the frogs must not exceed the maximum safety limit of 4.5 kg.

(ii) Two male and two female bullfrogs are chosen at random. Find the probability that their total mass do not exceed the maximum safety limit of 4.5 kg, stating clearly the mean and variance of any distribution that you use.

If the owner uses material Y, the maximum safety limit of the tank is improved to L kg.

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	(iii) It is 95 % certain that four male bullfrogs chosen at random have a total mass not exceeding the maximum safety limit of L kg. Find, correct to 1 decimal place, the least value of L. [4]
12	Find the number of ways in which the letters of the word SECTION can be arranged if **(i)** the letters are not in alphabetical order, [1] **(ii)** the consonants (S, C, T, N) and vowels (E, I, O) must alternate, [2] **(iii)** all the vowels are together, [2] **(iv)** all the vowels are separated, [2] **(v)** there must be exactly two letters between the two letters E and O. [2] Find, as a fraction in its lowest term, the probability that after arranging the letters of the word SECTION, there is at least one consonant and at least one vowel between the two letters E and O. [4]

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Name	( )	Class	

RIVER VALLEY HIGH SCHOOL
2017 Year 6 Preliminary Examination II
Higher 1

MATHEMATICS 8865/01

Paper 1 **18 Sept 2017**

3 hours

Additional Materials: Answer Paper
List of Formulae (MF26)
Cover Page

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number in the space at the top of this page.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, place the cover page on top of your answer paper and fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

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Section A: Pure Mathematics [40 marks]

1 The curve C has equation $y = 4 + e^{x^2}$. Without using a calculator, find the equation of the tangent to C at the point where $x = 1$, giving your answer in the form $y = mx + c$, where m and c are constants. [4]

Find also the area of the region bounded by the curve C, the tangent line at the point $x = 1$ and the y-axis. [2]

2 **(i)** Differentiate $\ln(x^2 + 1)$ with respect to x. [2]

(ii) Express $\frac{(x+1)^2}{x^2+1}$ in the form $a + \frac{bx}{x^2+1}$, where a, b are real constants. [3]

(iii) Hence find $\int \frac{(x+1)^2}{x^2+1}\,dx$. [2]

3 The line $y = x$ and curve C with equation $y = \frac{1}{2}\left(e^x - 1\right)$ intersect at the point $P(a, a)$, where $a > 0$.

(i) Show that $e^a = 1 + 2a$ [2]

(ii) Given that the area of the region enclosed by the curve C, the x-axis and the line $x = k$ is $\frac{a}{2}$. Find the value of k in terms of a. [3]

(iii) Find the range of values of x such that $e^x - 1 > 2x$. [2]

4 Elman invested \$2000, \$1500 and \$1000 into funds A, B and C respectively. The total yearly interest he earned from these three funds is \$309. Furthermore, the interest earned from fund A was half the sum of the interest earned from B and C.

At the start of the following year, Elman reviewed his portfolio and withdrew all his investment from the three funds. He then invested \$2500 and \$2000 in funds A and B respectively. He managed to earn a yearly interest of \$320. The interest rates from the three funds remained unchanged. Find which fund gave Elman the highest interest rate, stating its value. [6]

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5 In order to improve his performance, Johnson who is a professional sprinter decides to monitor his running speed, S metres per second. His personal trainer believes that the relationship between S and the time, t seconds, can be modelled by the equation $S = t^3 - 10t^2 + 28t$, for $0 \le t \le 5$.

(i) Use differentiation and this model to find the maximum value of S, justifying that this value is a maximum. [5]

(ii) Sketch the graph of S against t, giving the coordinates of any points of intersection with the axes and the stationary points. [2]

(iii) Find the area of the region bounded by the curve, the t-axis and the line $t = 5$. Give an interpretation of the area that you have found, in the context of the question. [2]

Johnson's trainer also records his oxygen intake, σ measured in cm^3 per second during the run. The relationship between σ and t is given by $\sigma = 5\ln(t+4) - 2$, for $0 \le t \le 5$.

(iv) Find the exact value of t for which $\sigma = 23$. [2]

(v) Sketch the graph of σ against t, indicating the range of values of σ. [2]

(vi) Find the rate at which the oxygen intake is increasing when $t = 1$. [1]

Section B: Probability and Statistics [60 Marks]

6 The school ICT assistant needs to schedule a group of 4 Science, 3 Mathematics and 2 Humanities Department teachers for upgrade of their laptops. He decides to arrange the 9 teachers in random order. On a particular day, find the probability that

(i) Miss Tan from the Science Department is first and Mr Ng from the Humanities Department is the last teacher to have their laptops upgraded, [2]

(ii) teachers from the same department are randomly arranged to have their laptops upgraded before teachers from another department to have their laptops upgraded in random order, [2]

(iii) given that the 5 teachers have their laptops upgraded by noon time, there are exactly 2 from the Science Department and exactly 2 from the Mathematics Department. [2]

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7 A game is played with a fair die and two bags of red and blue marbles, labelled A and B. Bag A contains 3 red marbles and 9 blue marbles while bag B contains 7 red marbles and 5 blue marbles. The die is thrown, and if it shows a number more than 4, a marble is drawn at random from bag A. Otherwise, a marble is drawn from bag B. Events X and Y are defined as follows:

X : A marble is drawn from bag A,

Y : A blue marble is drawn.

(i) Find $\mathrm{P}(X)$. [1]

(ii) Draw a tree diagram to illustrate the events X and Y. [2]

(iii) Given that a blue marble is drawn, find the probability that the marble is chosen from bag A. [2]

(iv) Hence or otherwise, determine if event X and Y are independent. [2]

8 Ah Hao food manufacturing company is famous for its high quality curry puff produced for distribution to market places and supermarkets. On average, in a batch of 100 curry puffs produced, 8 of them are identified as too salty by laboratory test. In a monthly routine check, Ministry of Health (MOH) officials randomly pick a sample of n curry puffs from the company for laboratory test.

(i) Denoting the number of curry puff that are too salty in the sample picked by MOH officials by X , state in context, two assumptions needed for X to be well modelled by a binomial distribution. Explain why one of these assumptions may not be true. [3]

(ii) Assuming that X follows a binomial distribution, find the least value of n if the probability that only one of the curry puff checked by the MOH officials is too salty is less than 0.1. [3]

(iii) Using the value of n determined in part **(ii)**, find the probability of having at least 3 but less than 10 curry puffs, being too salty in the random sample chosen by the MOH officials. [2]

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9 **(a)** Eight pairs of values of variables x and y are measured. Draw a sketch of a possible scatter diagram of the data for each of the following cases:

(i) The product moment correlation coefficient is approximately zero. [1]

(ii) The product moment correlation coefficient is approximately -0.95. [1]

(b) Scientists are investigating the relationship between the amount of a chemical compound (KH_2) added in x mg and the production of a cough syrup in y ml in a newly invented medicine production process. The following table shows a series of research results collected.

x/mg	5	10	15	20	25	30	35	40	45	50	55	60
y/ml	1.2	1.5	1.7	2.0	2.4	2.7	3.0	3.4	4.2	4.5	4.9	5.4

(i) The equation of the estimated least squares regression line of y on x for a set of bivariate data is $y = a + bx$. Explain what do you understand by the least square regression line of y on x. Hence find the regression line based on the data in the above table. [3]

(ii) Interpret the values for a and b found part **(b)(i)** in the context of the question. [2]

(iii) It is required to estimate the amount of the chemical compound (KH_2) added when the amount of cough syrup produced is 4.0 ml. By using an appropriate regression line, find the amount of the chemical compound added. State the reason for the use of the regression line. [2]

(iv) Comment on the reliability of the estimate found in part **(b)(iii)**. [2]

10 A group of young physicists is conducting an experiment involving collisions between protons and anti-protons. The amount of energy, x MeV, released in each collision is recorded for 50 collisions. The results are summarized by

$$\sum(x-100) = 720 \quad \text{and} \quad \sum(x-100)^2 = 30500.$$

(i) Find the unbiased estimates for the population mean and variance of the amount of energy released in each collision. [3]

(ii) The group of young physicists claims that the mean amount of energy released in each collision is 108 MeV. Test, at the 5% level of significance, if their claim is correct. State with a reason, whether it is necessary to assume that the amount of energy released in each collision follows a normal distribution. [4]

[Question 10 continues on the next page]

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(iii) A Nobel Prize winner physicist then predicts that the mean amount of energy released in each collision is μ_0 MeV. However, the group of young physicists proposed it should be higher. Find the range of value of μ_0 such that the proposal by the group of young physicists is rejected at the 3% level of significance based on the data in part **(i)**. [4]

11 Each month the amount of electricity, measured in kilowatt-hours (kWh), used by a household in Singapore has mean 500 and standard deviation 80. Assuming normal distribution, find the probability that

(i) in a randomly chosen month, less than 600 kWh is used for a household, [1]

(ii) out of 10 households, there are at most 8 which use less than 600 kWh given that there are at least 4 households use less than 600 kWh, [3]

(iii) there are least 5 months with more than 600 kWh usage per month in a year for a household, [2]

(iv) the amount of electricity used in two randomly chosen months for a household differ by less than 100 kWh. [3]

(v) State any assumption(s) made in the calculation for part **(iv)** and explain briefly why this assumption may not be valid in the real-world. [2]

The charges for electricity used is \$0.22 per kWh.

(vi) Write down the distribution of the total charges of electricity used in any one month. Hence find the probability that, in a randomly chosen month, the total charge is more than \$150. [2]

(vii) Each household receives a bill every three months. Find the least value of m such that the probability that a randomly chosen bill is less than \$$m$ is at least 0.96. [3]

(viii) A campaign was launched to raise awareness of the importance of saving electricity. At the end of the campaign, it was found that the mean monthly electricity usage for a household dropped drastically to 185 kWh with standard deviation 80 kWh. Explain whether a normal distribution is appropriate here. [1]

End of Paper

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Solutions to 2017 Y6 H1 Maths Preliminary Exam II

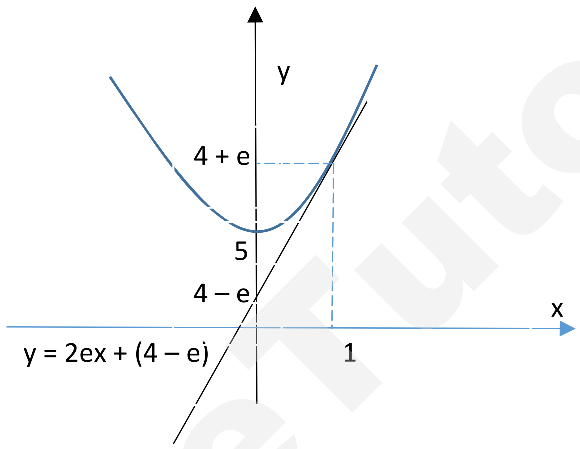
<table>
<tr><th>1</th><th>Working</th><th></th></tr>
<tr><td></td><td>

$y = 4 + e^{x^2} \Rightarrow \dfrac{dy}{dx} = 2xe^{x^2}$

when $x = 1, y = 4 + e, \dfrac{dy}{dx} = 2e$

$y - (4 + e) = 2e(x - 1)$

$y = 2ex + (4 - e)$



$\displaystyle\int_0^1 4 + e^{x^2} - (2ex + 4 - e)\, dx = 1.46$

</td><td></td></tr>
</table>

<table>
<tr><th>2</th><th>Working</th><th></th></tr>
<tr><td>i)</td><td>

$\dfrac{d\ln(x^2+1)}{dx} = \dfrac{1}{x^2+1}.2x = \dfrac{2x}{x^2+1}$

</td><td></td></tr>
<tr><td>ii)</td><td>

$\dfrac{(x+1)^2}{x^2+1} = \dfrac{x^2+2x+1}{x^2+1}$

$= \dfrac{x^2+1}{x^2+1} + \dfrac{2x}{x^2+1}$

$= 1 + \dfrac{2x}{x^2+1}$

</td><td></td></tr>
<tr><td>iii)</td><td>

$\displaystyle\int \frac{(x+1)^2}{x^2+1}\, \mathrm{d}x = \int 1 + \frac{2x}{x^2+1}\, \mathrm{d}x = x + \ln(x^2+1) + c$

</td><td></td></tr>
</table>

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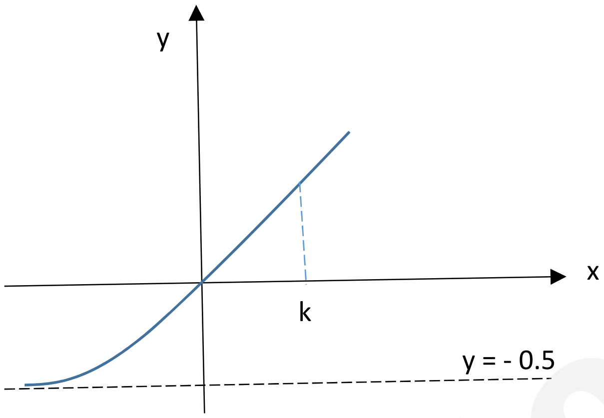
3	**Working**	
i)	Let $x = a$ $\frac{1}{2}\left(e^a - 1\right) = a$ $e^a - 1 = 2a \Rightarrow e^a = 1 + 2a$	
ii)	 $\int_0^k \frac{1}{2}\left(e^x - 1\right) dx = \frac{1}{2}\left[e^x - x\right]_0^k$ $= \frac{1}{2}\left(e^k - k - 1\right)$ $\frac{1}{2}\left(e^k - k - 1\right) = \frac{a}{2}$ $e^k - k = 1 + a = e^a - a$ (from (i)) $\therefore k = a$	
iii)	Using GC, the points of intersection between y = x and the curve is $x = 0$ and 1.256 The range of values of x: $x < 0$ or $x > 1.26$	

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4	**Working**	
	Let a, b and c be the interest rates of each fund in per cent. $\frac{a}{100}(2000)+\frac{b}{100}(1500)+\frac{c}{100}(1000)=309$ $20a+15b+10c=309$ $2.\frac{a}{100}(2000)=\frac{b}{100}(1500)+\frac{c}{100}(1000)$ $40a-15b-10c=0$ $\frac{a}{100}(2500)+\frac{b}{100}(2000)=320$ $25a+20b=320$ Solving the 3 equations, a = 5.15, b = 9.56 and c = 6.26 Fund B gave the highest rate = 9.56%	

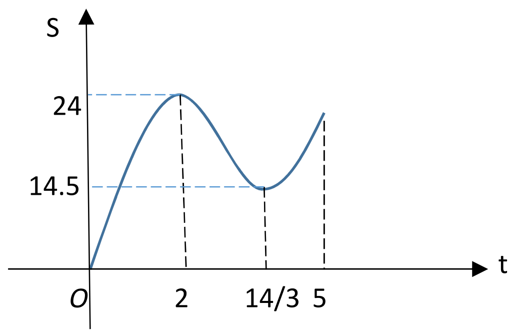
5	**Working**	
i)	$\frac{dS}{dt}=3t^2-20t+28=0$ $t=\frac{20\pm\sqrt{64}}{6}=2$ or $\frac{14}{3}$	

	1.995	**2**	**2.005**
gradient	+	**0**	-
shape			

	4.665	**14/3**	**4.675**
gradient	-	**0**	+
shape			

When $t=2, S=24$ which is maximum.

ii)



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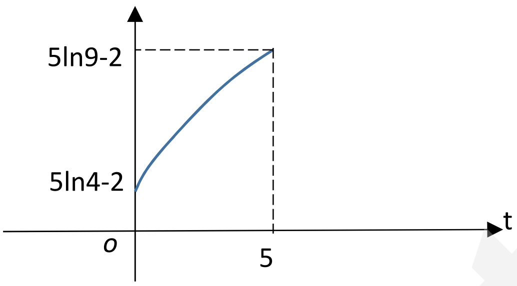
iii)	$\int_0^5 t^3 - 10t^2 + 28t\, dt \approx 89.6$ **89.6 metres** refers to the total distance covered by Johnson during 5 seconds.	
iv)	$5\ln(t+4) - 2 = 23$ $\ln(t+4) = 5$ $t + 4 = e^5$ $t = e^5 - 4$ seconds	
v)		
vi)	When t = 1, $\frac{d\sigma}{dt} = \frac{5}{1+4} = 1$ cm^3/s	

6	Working	
6(i)	The probability = $\frac{1! \times 7! \times 1!}{9!} = \frac{1}{8 \times 9} = \frac{1}{72}$	
6(ii)	The probability = $\frac{(4! \times 3! \times 2!) \times 3!}{9!}$ $= \frac{1728}{362880} = \frac{1}{210}$	
6(iii)	The probability = $\frac{\binom{4}{2}\binom{3}{2}\binom{2}{1} \times 5!}{\binom{9}{5} \times 5!}$ $= \frac{36}{126} = \frac{2}{7}$	

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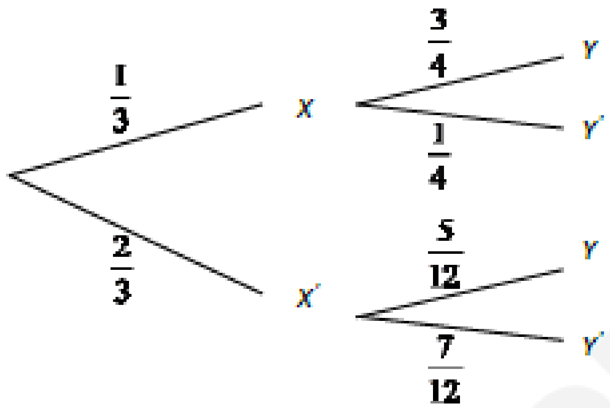
	Working	Marks
7(i)	$\mathrm{P}(X)=\frac{2}{6}=\frac{1}{3}$	
7(ii)	The tree diagram for the events X and Y:  	
7(iii)	$\mathrm{P}(X\|Y)=\frac{\mathrm{P}(X\cap Y)}{P(Y)}$ $=\frac{\frac{1}{3}\times\frac{3}{4}}{\frac{1}{3}\times\frac{3}{4}+\frac{2}{3}\times\frac{5}{12}}$ $=\frac{9}{19}$	
7(iv)	Since $\mathrm{P}(X\|Y)=\frac{9}{19}$, $\mathrm{P}(X)=\frac{1}{3}$ and thus, $\mathrm{P}(X\|Y)\neq\mathrm{P}(X)$ we can conclude that event A and B are not independent events. Alternative: $\mathrm{P}(Y)=\frac{1}{3}\times\frac{3}{4}+\frac{2}{3}\times\frac{5}{12}=\frac{19}{36}$ Then $\mathrm{P}(X\cap Y)=\frac{1}{4}\neq\mathrm{P}(X)\times\mathrm{P}(Y)$ Hence, event A and B are not independent events.	

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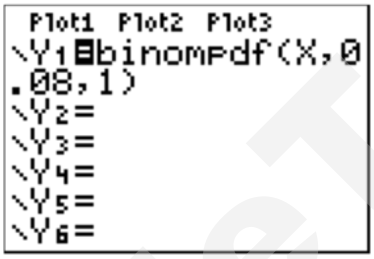
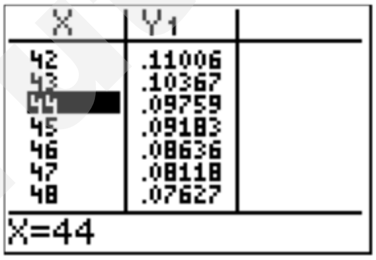
8	Working	Marks
8(i)	The two assumptions are (i) The event of a particular chosen curry puff being too salty is independent of other curry puffs chosen by the MOH officials. (ii) The probability of choosing a curry puff that is too salty is constant for all curry puffs chosen by the MOH officials. (i) may not be true in general as curry puffs are manufactured in batches, thus if 1 curry puff is found to be too salty, curry puffs from the same batch will be salty too. (or any other similar reasoning)	
8(ii)	Assume $X \sim B(n, 0.08)$, Then we need to have $P(X=1) < 0.01$. Using binomial pdf in GC, we can set the necessary and obtain the following probability table for checking:  Hence, from the above table, we deduce that the least value of n should be 44.	
8(iii)	Let n = 44. Then $X \sim B(44, 0.08)$, Then we have $P(3 \le X < 10)$ $= P(X \le 9) - P(X \le 2)$. $= 0.692$	

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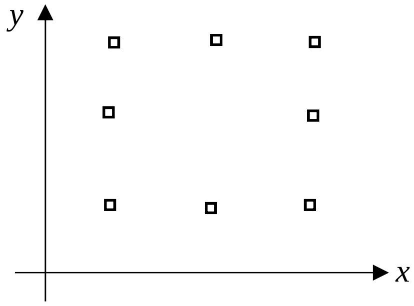
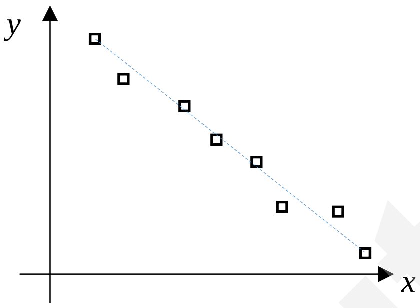
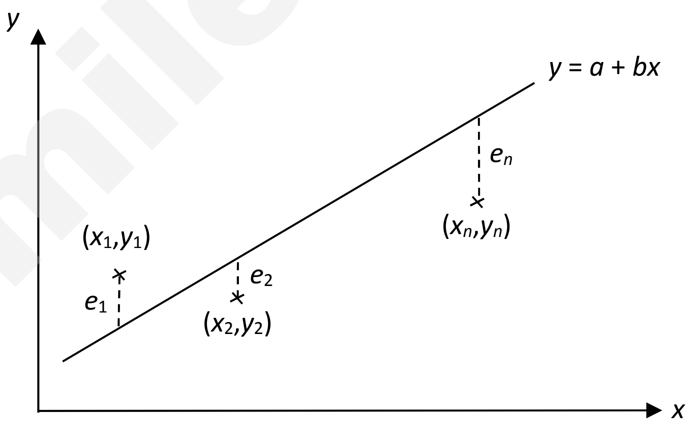
9	Working	Marks
9 (i)	The required scatter diagram are as follows: (i)  (ii)  	
9b(i)	Let the sample of bivariate data be (x_i, y_i) where $i = 1, 2, \ldots n$. Let $e_i = y_i - (a + bx_i)$ be the vertical deviation between the point (x_i, y_i) and the line $y = a + bx$.  The line $y = a + bx$ is the least square regression line for the sample of bivariate data if the sum of the squares of the vertical deviations, i.e. $\sum_{i=1}^{n}(e_i)^2$, is the minimum.	

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	Using GC and the table of values, LinReg y=a+bx a=.5590909091 b=.0774125874 r²=.9835387842 r=.991735239 the regression line of *y* on *x* is $y = 0.559 + 0.0774x$	
9b(ii)	*a* is the amount of cough syrup in ml produced with no amount of the chemical compound KH_2 added in the production process. *b* is the amount of cough syrup in ml produced with the addition of 1 mg of the chemical compound KH_2 added in the production processs.	
9b(iii)	As *x* is the independent variable and *y* is the dependent variable in the data set, we will need to use the regression line of *y* on *x* to estimate the value of *x* given *y* value. Thus, when $y = 4.0$, $4.0 = 0.55909 + 0.077413x$ $\Rightarrow x = 44.448736 \approx 44.4$ mg (3 sig fig)	
9b(iv)	The estimate for *x* value in part 9b(iii) is reliable for the following reasons: i. the correlation coefficient $r = 0.992$ is very close to 1, ii. the given *y* value of 4.0 is within the data range of the *y* value iii. the correct line *y* on *x* is used as *y* is the dependent variable.	

10	**Working**	**Marks**
10(i)	The unbiased estimate for population mean = $\bar{x}$ $= \frac{720}{50} + 100 = 114.4$ The unbiased estimate for population variance = s^2 $= \frac{1}{50-1}\left(\sum(x-100)^2 - \frac{\left(\sum(x-100)\right)^2}{50}\right)$ $= \frac{1}{49}\left(30500 - \frac{720^2}{50}\right) = \frac{2876}{7} = 410.8571429 \approx 411$ (3 sig. fig.)	
10(ii)	Let μ be the mean of the amount of energy released in each collision. For testing of the new claim by the young physicists, Let $H_0: \mu = 108$ $H_1: \mu \neq 108$ We then perform a one tail test at 5% level of significance i.e. $\alpha = 0.05$	

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	Under H0, $\overline{X} \sim N\left(108, \frac{2876/7}{50}\right)$ Then test statistics is $Z = \frac{\overline{X}-108}{\sqrt{\frac{2876/7}{50}}} \sim N(0, 1)$ Using GC, the p-value = 0.0255723191. Since p-value < α, we reject H_0 and conclude that there is sufficient evidence at 5% level of significance that the population mean of the energy released in each collision is not 108 MeV. As the sample size *n* is large (50), it is not necessary to assume that *X*, the amount of energy released in each collision follows a normal distribution as by the Central Limit Theorem, $\overline{X}$ can be approximated by a normal distribution for the test to be valid.	
10(iii)	For testing of the new claim by the young physicists, Let $H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$ We then need to perform a one tailed test at 3% level of significance i.e. $\alpha = 0.03$ Under H0, $\overline{X} \sim N\left(\mu_0, \frac{2876/7}{50}\right)$ Then test statistics is $Z = \frac{\overline{X}-\mu_0}{\sqrt{\frac{2876/7}{50}}} \sim N(0, 1)$. For H0 not to be rejected, $z_{\text{calculated}}$ must not be in the critical region, hence, $\frac{114.4-\mu_0}{\sqrt{\frac{2876/7}{50}}} \le 1.88079361$ $\Rightarrow 114.4-\mu_0 \le 1.88079361\times\sqrt{\frac{2876/7}{50}}$ $\Rightarrow \mu_0 \ge 114.4-1.88079361\times\sqrt{\frac{2876/7}{50}}$ $\Rightarrow \mu_0 \ge 109.0085999$ $\Rightarrow \mu_0 \ge 109$ (3 sig fig)	

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11	Working	Marks
i)	Let W be the r.v. of the amount of electricity used by a household in a month. $W \sim N(500, 80^2)$ $P(W < 600) = 0.89435 \approx 0.894$	
ii)	Let H be the number of households out of 10 which use less than 960 kWh, H ~B(10, 0.89435) $P(H \leq 8 \mid H \geq 4) = \dfrac{P(4 \leq H \leq 8)}{P(H \geq 4)} = \dfrac{P(H \leq 8) - P(H \leq 3)}{1 - P(H \leq 3)}$ $= \dfrac{0.28583}{0.9999868} = 0.28583 \approx 0.286$	
iii)	Let X be the r.v. of number of months that use more than 600 kWh, $X \sim B(12, 0.10565)$. $P(X \geq 5) = 1 - P(X \leq 4) \approx 0.00550$	
iv)	$W_1 - W_2 \sim N(0, 12800)$ $P(\lvert W_1 - W_2 \rvert < 100) = P(-100 < W_1 - W_2 < 100) = 0.62324 \approx 0.623$	
v)	Assume that the amount of electricity used by a household of the two months is independent. This assumption may not be true if the family knows that they have over-used energy in the first month, they may use less in the later months. (or any other reasonable explanations)	
vi)	Let C = 0.22W be the r.v. of the cost of electricity for a household in each month. $C \sim N(110, 309.76)$ $P(C > 150) = 0.011521 \approx 0.0115$	
vii)	$T = C_1 + C_2 + C_3 \sim N(330, 929.28)$ $P(T < m) \geq 0.96$ $m \geq 383.368 \approx 383$ (3 sig. fig)	
viii)	Let V be the r.v of the amount of electricity used by a household per month at the end of the campaign. Supposed $V \sim N(185, 80^2)$ Consider: a) $P(V < 0) = 0.0104$ Since the probability that a household uses negative amount of electricity is 0.0104 (1.04%) which is quite significant, Normal distribution is deemed as unsuitable. OR b) $P(185 - 3(80) < W < 185 + 3(80)) = 0.997$ Since probability of 0.997 of amount of electricity used is within 3 standard deviation and there is quite significant probability that the amount of electricity used is negative, Normal distribution is deemed unsuitable.	

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SERANGOON JUNIOR COLLEGE
2017 JC2 PRELIMINARY EXAMINATION
MATHEMATICS

Higher 1

8865/01

Tuesday **12 Sep 2017**
3 Hours

Additional materials: Writing paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on the cover page and on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets **[ ]** at the end of each question or part question.

The total marks for this paper is **100**.

At the end of the examination, fasten all your work securely together.

This document consists of **8** printed pages and **0** blank page.

[**Turn Over**

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Section A: Pure Mathematics [40 marks]

1 **(a)** Differentiate each of the following functions with respect to x, simplifying your answers:

(i) $\left(\frac{2}{3}x+1\right)^{-6}$ [1]

(ii) $5\mathrm{e}^{1-2x}+\frac{1}{8x^3}$ [2]

(b) Find $\int \frac{1}{\sqrt{2-kt}}\,\mathrm{d}t$, where k is a constant. [2]

2 **(a)** Given that the quadratic graph with equation $y=ax^2+bx+c$ passes through the points with coordinates $(-2,\ 18)$, $(-1, 9)$ and $(1.5,\ 4)$, find the equation of this graph. [3]

(b) Find the exact range of values of k for which the line $y=-x-2$ does not meet the curve $y=(k-1)x^2+kx-3$, where $k \neq 1$. [4]

3 **(i)** On a single diagram, sketch the graphs of $y=\frac{1-3x}{x-2}$ and $y=\ln(x+3)$, stating clearly the equations of any asymptotes and the axial intercepts. [5]

(ii) Find the x-coordinates of the points of intersection, leaving your answer correct to 3 significant figures. [1]

(iii) Hence, find the set of values of x for which $\frac{1-3x}{x-2} \leq \ln(x+3)$. [1]

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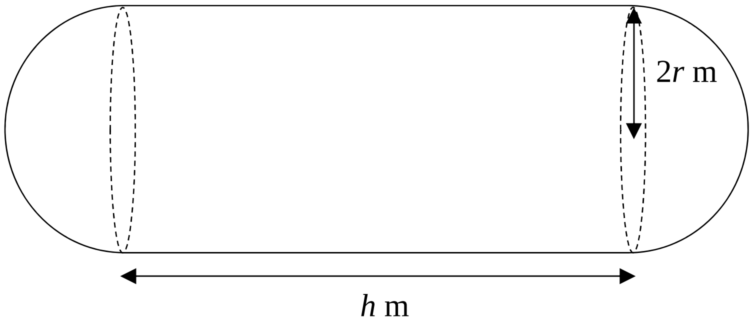
4



An engineer has to design an oil tank with a capacity of 120 m^3. The oil tank consists of a cylindrical body of length h m and two hemispherical ends of radius $2r$ m each, as shown in the diagram.

(i) Show that $h = \left(\dfrac{30}{\pi r^2} - \dfrac{8}{3}r\right)$ m. [2]

(ii) Show that the total surface area of the tank is $\left(\dfrac{120}{r} + \dfrac{16}{3}\pi r^2\right)$ m^2. [2]

(iii) Use a non-calculator method to find the value of r which gives a minimum total surface area of the tank. Hence, find the value of the minimum total surface area of the tank, leaving your answer correct to 2 decimal places. [5]

[It is given that a sphere of radius r has surface area $4\pi r^2$ and volume $\dfrac{4}{3}\pi r^3$]

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5 The management of HoLi, a chain of bubble tea outlets, is selling a 5-year franchise to operate its newest outlet in Kovan. Past experience in similar franchises suggests that the revenue (R) and the operating costs (C), in hundred thousand dollars per year, at time t years, can be modelled by the graphs of

$$R:\ y=\frac{1}{3}t^3-2t^2+4t \quad \text{and}$$

$$C:\ y=e^{2-t}+1 \quad \text{respectively,} \quad \text{for } 0\le t\le 5.$$

(i) Using the axes of y (hundred thousands of dollars per year) against t, sketch, on the same diagram, the graphs of R and C, indicating the coordinates of any stationary point(s) and equation of asymptote(s), showing any necessary working clearly. [5]

(ii) State the coordinates of the point of intersection between the two graphs. [1]

The area under the curve of R and C, from t = 0 to t = T, gives the total revenue and total operating costs at t = T respectively.

(iii) Find the value of t for which the franchise is expected to break even i.e. where the total revenue just covers the total operating costs for the period of t years. [3]

(iv) Compute the approximate total profit, in dollars, expected to be generated over the 5-year period, correcting your answer to 3 significant figures. [2]

(v) Explain, in context, a possible meaning of the horizontal asymptote of curve C. [1]

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Section B: Statistics [60 marks]

6 **(a)** The manager of a bookstore wishes to conduct a survey to seek the customers' opinions on its opening hours. If the manager decides to survey a sample of the first 80 customers who leave the bookstore, give a reason why this sample may not be appropriate. [1]

(b) A surveyor decides to obtain a random sample of 20 residents from the apartment block. He randomly selects 20 units from the apartment block and chooses one resident from each unit.

(i) In the context of the question, explain what is meant by the term 'random sample'. [1]

(ii) Explain why this method may not be appropriate. [1]

(iii) Describe an alternative method so that the surveyor will choose a sample of 20 residents at random from the apartment block of 100 residents. [1]

7 In a large batch of t-shirts printed, a t-shirt printing company wishes to conduct quality checks for the t-shirts. The random variable X is the number of t-shirts which fail the quality check. A random sample of 10 t-shirts are taken. It is assumed that X follows a binomial distribution with unknown constant p being the probability of a t-shirt failing the quality check, where $0.2 < p < 1$.

(i) Find the number of t-shirts expected to fail the quality check in terms of p. [1]

(ii) Given that $\mathrm{P}(X = 1) = 0.141$, formulate an equation in terms of p. Hence, find the value of p. [2]

Taking $p = 0.3$,

(iii) find the largest value of r such that the probability of at least r t-shirts failing the quality check is more than 0.1. [4]

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8 Find how many different arrangements can be made using all letters of the word *PRELIMS* if

(i) there are no restrictions, [1]

(ii) the first and last letters must both be vowels, [2]

(iii) the letters R, L, and M must be together, [2]

(iv) the letters R, L, and M must be separated. [3]

9 The new private car hire company, Snatch, is expanding into the Southeast Asian market. Snatch is sourcing for suitable candidates to fill up their Marketing Manager positions. A candidate has cleared his interview if he passes the first four rounds of interviews.

The probability that a candidate passes the first round of interview is 0.8. If the candidate passes a round of interview, the probability that the candidate will pass the next round of the interview is half the probability of passing the preceding interview. If the candidate fails a round of interview, the candidate will not be allowed to go for the next round of interview. Mr Cheu is shortlisted to go through the interview process.

(i) Illustrate the possible outcomes for the interview process for a candidate on a tree diagram. [2]

(ii) Find the probability that Mr Cheu

(a) clears his interview, [1]

(b) fails to clear his interview, given that he passes the second round of interview. [3]

(iii) 25 candidates were shortlisted to go through the interview. Find the probability that fewer than 3 candidates clear the interview, leaving your answer correct to 5 significant figures. [3]

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10 A manufacturer claimed that the metal rods produced by their machine has a desired length of 50 cm. 100 metal rods were randomly chosen. The masses, x kg, are summarised by

$$\sum(x-55)=-355 \text{ and } \sum(x-55)^2=5622.$$

(i) Find the unbiased estimates of the population mean and variance. [2]

(ii) Test at 1% level of significance, whether the manufacturer has underestimated the mean length of the metal rods. [4]

A new random sample of 100 metal rods is chosen and the mean of this sample is m cm. The population standard deviation is assumed to be 7 cm. A test at 10% level of significance indicates that the manufacturer's claim is valid.

(iii) Find the range of values of m, giving your answer correct to 2 decimal places. [4]

11 Mandy sells her homemade matcha-flavoured macarons. The numbers, x, sold in the first seven months of the year 2017, together with the profits, y dollars, on the sale of these macarons are given in the following table.

	Jan	Feb	Mar	Apr	May	Jun	Jul
x	430	580	320	240	680	160	500
y	850	1240	600	400	1420	300	1050

(i) Give a sketch of the scatter diagram for the data as shown on your calculator. [2]

(ii) Find $\bar{x}$ and $\bar{y}$ and mark the point ($\bar{x}$, $\bar{y}$) on your scatter diagram. [2]

(iii) Calculate the equation of the regression line of y on x, and draw this line on your scatter diagram. [2]

(iv) Calculate the product moment correlation coefficient, and comment on its value in relation to your scatter diagram. [2]

(v) Mandy expects to sell 600 matcha-flavoured macarons for the month of August in 2017. Using an appropriate regression line, estimate her profit for this month. [1]

(vi) Due to an error in her document, Mandy recorded her profits incorrectly. There is a shortfall of 100 dollars for all the profits recorded from January 2017 to July 2017 (as shown in the table above). Comment on whether this recording error will affect the value of the product moment correlation coefficient found in **(iv)**. [1]

[Turn Over

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12 The masses, in kilograms, of watermelons and papayas sold by a supermarket have normal distributions with means and standard deviations as shown in the following table.

	Mean	Standard Deviation
Watermelon	5.42	0.51
Papaya	2.18	0.35

(i) Find the probability that the mass of a watermelon chosen at random is between 5.2 kg and 6.5 kg. [1]

(ii) Find the probability that the total mass of four randomly chosen watermelons is less than 22.8 kg, stating clearly the mean and variance of the distribution that you use. [3]

(iii) Find the probability that the total mass of eight randomly chosen papayas is more than the total mass of four randomly chosen watermelons, stating clearly the mean and variance of the distribution that you use. [3]

(iv) Watermelons cost \$2.80 per kilogram and papayas cost \$1.80 per kilogram. Find the mean and the variance of the total cost of four randomly chosen watermelons and eight randomly chosen papayas and hence, find the probability that the total cost is between \$80 and \$100. [4]

(v) State an assumption for the calculations in parts **(iii)** and **(iv)** to be valid. [1]

End of Paper

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SERANGOON JUNIOR COLLEGE
2017 JC2 PRELIMINARY EXAMINATION
MATHEMATICS (SOLUTIONS)

Higher 1

8865/01

Tuesday **12 Sep 2017**
3 Hours

Additional materials: Writing paper
List of Formulae (MF26)

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1	**(a)** Differentiate each of the following functions with respect to x, simplifying your answers: **(i)** $\left(\frac{2}{3}x+1\right)^{-6}$	[1]
	(ii) $5e^{1-2x}+\frac{1}{8x^3}$	[2]
	(b) Find $\int \frac{1}{\sqrt{2-kt}}\,dt$, where k is a constant.	[2]
	Suggested Solution	
	(a) (i) Let $y=\left(\frac{2}{3}x+1\right)^{-6}$ $\frac{dy}{dx}=(-6)\left(\frac{2}{3}x+1\right)^{-7}\left(\frac{2}{3}\right)$ $=-\frac{4}{\left(\frac{2}{3}x+1\right)^{7}}$	
	(ii) Let $y=5e^{1-2x}+\frac{1}{8x^3}$ $\frac{dy}{dx}=5(-2)e^{1-2x}+\frac{1}{8}(-3)x^{-4}$ $=-10e^{1-2x}-\frac{3}{8x^4}$	
	$\int \frac{1}{\sqrt{2-kt}}\,dt=\int (2-kt)^{-\frac{1}{2}}\,dt$ $=\frac{(2-kt)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)(-k)}+c$ $=\frac{-2\sqrt{2-kt}}{k}+c$	

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2 **(a)** Given that the quadratic graph with equation $y = ax^2 + bx + c$ passes through the points with coordinates $(-2,\ 18)$, $(-1, 9)$ and $(1.5,\ 4)$, find the equation of this graph. [3]

(b) Find the exact range of values of k for which the line $y = -x - 2$ does not meet the curve $y = (k-1)x^2 + kx - 3$, where $k \neq 1$. [4]

Suggested Solution

(a)

$y = ax^2 + bx + c$

At $(-2,18)$: $4a - 2b + c = 18$ --------- (1)

At $(-1,9)$: $a - b + c = 9$ --------- (2)

At $(1.5,4)$: $2.25a + 1.5b + c = 4$ --------- (3)

Using GC, we have $a = 2,\ \ b = -3,\ \ c = 4$.

Hence, the equation of this graph is $y = 2x^2 - 3x + 4$.

(b)

$(k-1)x^2 + kx - 3 = -x - 2$

$(k-1)x^2 + (k+1)x - 1 = 0$

Since the curve does not meet the line,

Discriminant < 0

$(k+1)^2 - 4(k-1)(-1) < 0$

$k^2 + 2k + 1 + 4k - 4 < 0$

$k^2 + 6k - 3 < 0$

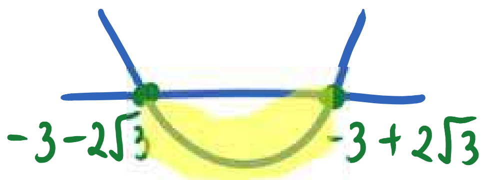
Consider $k^2 + 6k - 3 = 0$

$$k = \frac{-6 \pm \sqrt{6^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{48}}{2}$$

$$= \frac{-6 \pm 4\sqrt{3}}{2}$$

$$= -3 \pm 2\sqrt{3}$$



Hence, $-3 - 2\sqrt{3} < k < -3 + 2\sqrt{3}$

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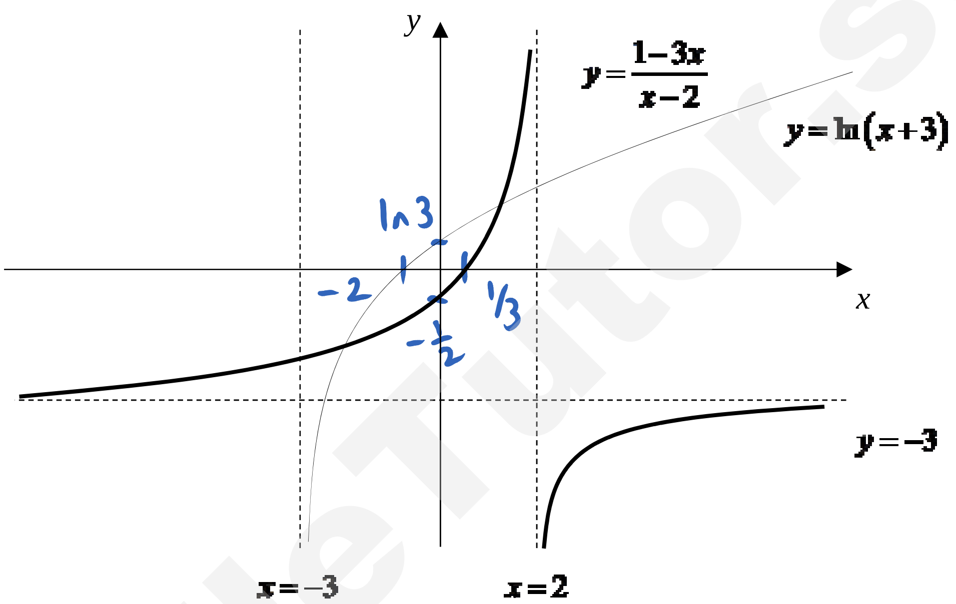
3	**(i)** On a single diagram, sketch the graphs of $y=\dfrac{1-3x}{x-2}$ and $y=\ln(x+3)$, stating clearly the equations of any asymptotes and the axial intercepts.	[5]
	(ii) Find the x-coordinates of the points of intersection, leaving your answer correct to 3 significant figures.	[1]
	(iii) Hence, find the set of values of x for which $\dfrac{1-3x}{x-2} \le \ln(x+3)$.	[1]
	Suggested Solution	
	(i) 	
	(ii) The x-coordinates of the points of intersection are -2.86 and 0.850.	
	(iii) Solution set = $\{x \in \text{P}:\ -2.86 \le x \le 0.850 \text{ or } x > 2\}$	

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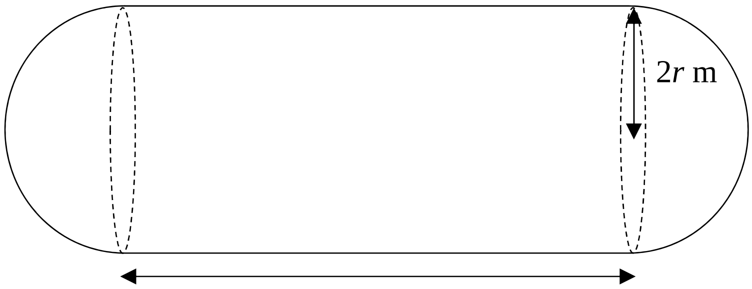
4	 An engineer has to design an oil tank with a capacity of 120 m^3. The oil tank consists of a cylindrical body of length h m and two hemispherical ends of radius $2r$ m each, as shown in the diagram. **(i)** Show that $h=\left(\frac{30}{\pi r^2}-\frac{8}{3}r\right)$ m.	[2]
	(ii) Show that the total surface area of the tank is $\left(\frac{120}{r}+\frac{16}{3}\pi r^2\right)$ m^2.	[2]
	(iii) Use a non-calculator method to find the value of r which gives a minimum total surface area of the tank. Hence, find the value of the minimum total surface area of the tank, leaving your answer correct to 2 decimal places. [It is given that a sphere of radius r has surface area $4\pi r^2$ and volume $\frac{4}{3}\pi r^3$]	[5]
	Suggested Solution	
	(i) Volume of tank, $\pi(2r)^2 h+\frac{4}{3}\pi(2r)^3=120$ $4\pi r^2 h+\frac{32\pi r^3}{3}=120$ $3\pi r^2 h+8\pi r^3=90$ $h=\frac{1}{3\pi r^2}\left(90-8\pi r^3\right)$ $h=\left(\frac{30}{\pi r^2}-\frac{8}{3}r\right)$ m (Shown)	
	(ii) T.S.A of tank, $A=2\pi(2r)h+4\pi(2r)^2$ $=4\pi rh+16\pi r^2$ $=4\pi r\left(\frac{30}{\pi r^2}-\frac{8}{3}r\right)+16\pi r^2$ $=\frac{120}{r}-\frac{32}{3}\pi r^2+16\pi r^2$ $=\frac{120}{r}+\frac{16}{3}\pi r^2$ (Shown)	

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(iii) $A = \dfrac{120}{r} + \dfrac{16}{3}\pi r^2$

$\dfrac{dA}{dr} = -\dfrac{120}{r^2} + \dfrac{32}{3}\pi r$

For minimum value of A, $\dfrac{dA}{dr} = 0$

$-\dfrac{120}{r^2} + \dfrac{32}{3}\pi r = 0$

$-120 + \dfrac{32}{3}\pi r^3 = 0$

$r = \left(\dfrac{45}{4\pi}\right)^{\frac{1}{3}}$ or $\sqrt[3]{\dfrac{45}{4\pi}}$

To show A is minimum,

Method I (2nd derivative)

$\dfrac{d^2A}{dr^2} = \dfrac{240}{r^3} + \dfrac{32}{3}\pi$ (positive, hence minimum)

Method II (sign test)

R	$\left(\sqrt[3]{\dfrac{45}{4\pi}}\right)^-$	$\sqrt[3]{\dfrac{45}{4\pi}}$	$\left(\sqrt[3]{\dfrac{45}{4\pi}}\right)^+$
$\dfrac{dA}{dr}$	–	**0**	+
Slope	$\searrow$	$\rightarrow$	$\nearrow$

$A = \dfrac{120}{\sqrt[3]{\dfrac{45}{4\pi}}} + \dfrac{16}{3}\pi\left(\sqrt[3]{\dfrac{45}{4\pi}}\right)^2$

$= 117.65 \text{ m}^2$ (2 dec places).

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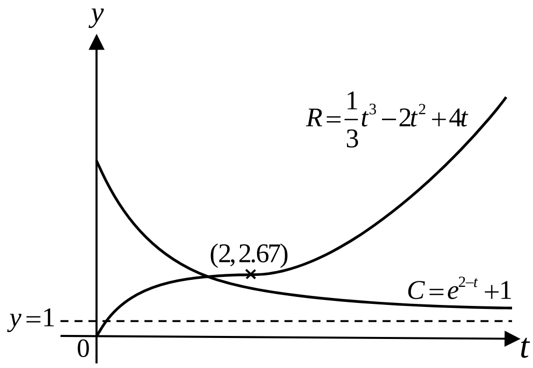
5	The management of HoLi, a chain of bubble tea outlets, is selling a 5-year franchise to operate its newest outlet in Kovan. Past experience in similar franchises suggests that the revenue (R) and the operating costs (C), in hundred thousand dollars per year, at time t years, can be modelled by the graphs of R: $y = \frac{1}{3}t^3 - 2t^2 + 4t$ and C: $y = e^{2-t} + 1$ respectively, for $0 \le t \le 5$.	
	(i) Using the axes of y (hundred thousands of dollars per year) against t, sketch, on the same diagram, the graphs of R and C, indicating the coordinates of any stationary point(s) and equation of asymptote(s), showing any necessary working clearly.	[5]
	(ii) State the coordinates of the point of intersection between the two graphs.	[1]
	The area under the curve of R and C, from t = 0 to t = T, gives the total revenue and total operating costs at t = T respectively.	
	(iii) Find the value of t for which the franchise is expected to break even i.e. where the total revenue just covers the total operating costs for the period of t years.	[3]
	(iv) Compute the approximate total profit, in dollars, expected to be generated over the 5-year period, correcting your answer to 3 significant figures.	[2]
	(v) Explain, in context, a possible meaning of the horizontal asymptote of curve C.	[1]
	Suggested Solution	
	(i) 	

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	$\frac{dR}{dt}=0:\quad t^2-4t+4=0$ $(t-2)^2=0 \quad \Rightarrow \quad t=2$ When $t=2$, $y=\frac{8}{3}$ (or 2.67, rounded off to 3 s.f.)	
	(ii) Coordinates of point of intersection is: $(1.51, 2.63)$	
	(iii) Let T be the no. of years for the franchise to break even. $\int_0^T \frac{1}{3}t^3-2t^2+4t\,dt=\int_0^T e^{2-t}+1\,dt$ $\int_0^T \frac{1}{3}t^3-2t^2+4t-e^{2-t}-1\,dt=0$ $\left[\frac{1}{12}t^4-\frac{2}{3}t^3+2t^2+e^{2-t}-t\right]_0^T=0$ $\frac{1}{12}T^4-\frac{2}{3}T^3+2T^2+e^{2-T}-T-e^2=0$ Using GC, $T\approx 4.13$	
	(iv) $\int_0^5 \frac{1}{3}t^3-2t^2+4t\,dt-\int_0^5 e^{2-t}+1\,dt$ $=18.75-12.339$ (using GC) ≈ 6.41 $\therefore$ Total profit over 5 years is \$641,000.	
	(v) **Possible meaning:** The horizontal asymptote $(y=1)$ of curve C means that the operating costs will stabilize at \$100,000 per year in the long run.	

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6	**(a)** The manager of a bookstore wishes to conduct a survey to seek the customers' opinions on its opening hours. If the manager decides to survey a sample of the first 80 customers who leave the bookstore, give a reason why this sample may not be appropriate.	[1]
	(b) A surveyor decides to obtain a random sample of 20 residents from the apartment block. He randomly selects 20 units from the apartment block and chooses one resident from each unit. **(i)** In the context of the question, explain what is meant by the term 'random sample'. **(ii)** Explain why this method may not be appropriate. **(iii)** Describe an alternative method so that the surveyor will choose a sample of 20 residents at random from the apartment block of 100 residents.	[1] [1] [1]
	Suggested Solution	
	(a) The first 80 customers may not be representative of all bookstore customers. The mid-day and late night shoppers will be unrepresented. (b)(i) In the context of the question, the term "random sample" means that every resident of the same apartment block has the same probability of being selected. The selection of residents is independent. (b)(ii) Since each unit has different number of people, so the probability of being chosen is not equally likely. (b)(iii) Obtain the name list of the 100 residents living at the apartment block and assign a number from 1 to 100 to all the residents. Use a computer program to generate 20 random numbers from 1 to 100. The person who is assigned the chosen number will be selected.	

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7	In a large batch of t-shirts printed, a t-shirt printing company wishes to conduct quality checks for the t-shirts. The random variable X is the number of t-shirts which fail the quality check. A random sample of 10 t-shirts are taken. It is assumed that X follows a binomial distribution with unknown constant p being the probability of a t-shirt failing the quality check and such that $0.2 < p < 1$.	
	(i) Find the number of t-shirts expected to fail the quality check in terms of p.	[1]
	(ii) Given that $\mathrm{P}(X=1)=0.141$, formulate an equation in terms of p. Hence, find the value of p.	[2]
	Taking p = 0.3,	
	(iii) find the largest value of r such that the probability of at least r t-shirts failing the quality check is more than 0.1.	[4]
	Suggested Solution	
	(i) $X \sim \mathrm{B}(10, p)$ $\mathrm{E}(X)=10p$	
	(ii) Given $P(X=1)=0.141$, $\binom{10}{1} p(1-p)^9=0.141$ $10p(1-p)^9=0.141$ Using GC, $p=0.28357=0.284$ (3s.f.) or $p=0.016356$ (rej. since $0.2 < p < 1$)	
	(iii) $\mathrm{P}(X \geq r)>0.1$ $1-\mathrm{P}(X \leq r-1)>0.1$ $\mathrm{P}(X \leq r-1)<0.9$ From GC, When $r=4$, $\mathrm{P}(X \leq 3)=0.64961(<0.9)$ When $r=5$, $\mathrm{P}(X \leq 4)=0.84973(<0.9)$ When $r=6$, $\mathrm{P}(X \leq 5)=0.95265(>0.9)$ $\therefore$ The largest value of r is 5.	

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8	Find how many different arrangements can be made using all letters of the word *PRELIMS* if	
	(i) there are no restrictions,	[1]
	(ii) the first and last letters must both be vowels,	[2]
	(iii) the letters *R*, *L*, and *M* must be together,	[2]
	(iv) the letters *R*, *L*, and *M* must be separated.	[3]
	Suggested Solution	
	(i) No. of arrangements $= 7! = 5040$	
	(ii) No. of arrangements $= 5! \times 2$ $= 240$	
	(iii) No. of arrangements $= 5! \times 3!$ (or ${}^5C_1 \times 4! \times 3!$) $= 720$	
	(iv) No. of arrangements $= {}^5C_3 \times 4! \times 3!$ $= 1440$ *<u>Note</u>: Award M1 for identifying use of slotting method if student writes* ${}^5C_3 \times 4!$ *or* ${}^5C_3 \times 3!$	

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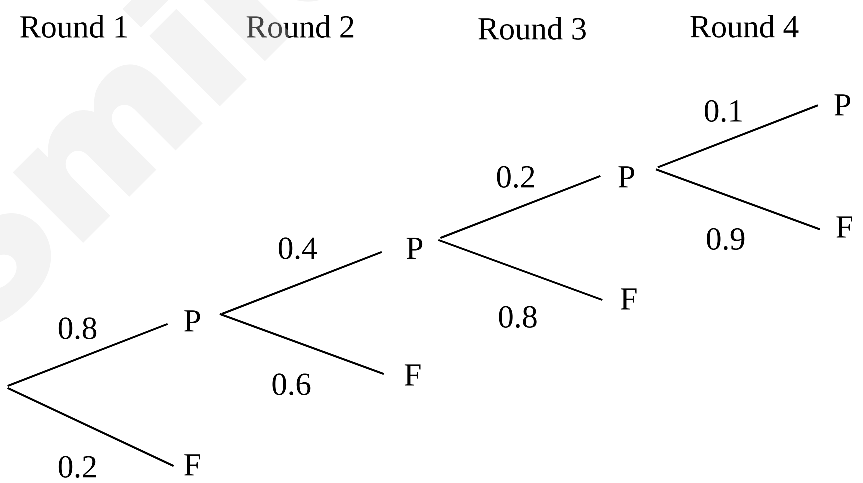
9	The new private car hire company, Snatch, is expanding into the Southeast Asian market. Snatch is sourcing for suitable candidates to fill up their Marketing Manager positions. A candidate has cleared his interview if he passes the first four rounds of interviews. The probability that a candidate passes the first round of interview is 0.8. If the candidate passes a round of interview, the probability that the candidate will pass the next round of the interview is half the probability of passing the preceding interview. If the candidate fails a round of interview, the candidate will not be allowed to go for the next round of interview. Mr Cheu is shortlisted to go through the interview.	
	(i) Illustrate the possible outcomes for the interview process for a candidate on a tree diagram.	[2]
	(ii) Find the probability that Mr Cheu **(a)** clears his interview, **(b)** fails to clear his interview, given that he passes the second round of interview.	[1] [3]
	(iii) 25 candidates were shortlisted to go through the interview. Find the probability that fewer than 3 candidates clear the interview, leaving your answer correct to 5 significant figures.	[3]
	Suggested Solution	
	(i)  P: Passing the round of interview F: Failing the round of interview	
	(ii)(a) Required probability = $0.8 \times 0.4 \times 0.2 \times 0.1 = 0.0064$	

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	(ii)(b) P(fails to clear interview | passes 2nd round of interview) $= \dfrac{\text{P(fails to clear interview \& passes 2nd round of interview)}}{\text{P(passes 2nd round of interview)}}$ $= \dfrac{0.8\times0.4\times0.8+0.8\times0.4\times0.2\times0.9}{0.8\times0.4}$ = 0.98	
	(iii) Let X be the random variable denoting "the number of candidates, out of 25, who cleared the interview." Then $X \sim \text{B}(25, 0.0064)$ Required probability = $\text{P}(X < 3)$ $= \text{P}(X \le 2)$ $= 0.99946$ (5 significant figures)	

10	A manufacturer claimed that the metal rods produced by their machine has a desired length of 50 cm. 100 metal rods were randomly chosen. The lengths, x cm, are summarised by $$\sum(x-55) = -355 \text{ and } \sum(x-55)^2 = 5622.$$ **(i)** Find the unbiased estimates of the population mean and variance.	[2]
	(ii) Test at 1% level of significance, whether the manufacturer has underestimated the mean length of the metal rods.	[4]
	A new random sample of 100 metal rods is chosen and the mean of this sample is m cm. The population standard deviation is assumed to be 7 cm. A test at 10% level of significance indicates that the manufacturer's claim is valid. **(iii)** Find the range of values of m, giving your answer correct to 2 decimal places.	[4]

	Suggested Solution	
	(i) Unbiased estimate of population mean, $\mu = \frac{-355}{100} + 55 = 51.45$ Unbiased estimate of population variance, $s^2 = \frac{1}{99}\left[5622 - \frac{(-355)^2}{100}\right]$ $\approx 44.05808081 = 44.1$	
	(ii) To test $H_o: \mu = 50$ Against $H_1: \mu > 50$ Use a right-tailed test at 1% significance level. Under H_0 and by Central Limit Theorem (since $n = 100$ is large), $\overline{X} \sim N\left(50, \frac{44.05808}{100}\right)$ approximately Test statistic, $Z = \frac{\overline{X} - \mu_o}{\sqrt{\sigma^2 / n}} \sim N(0, 1)$ $\mu_0 = 50$, $s = \sqrt{44.05808}$, $\overline{x} = 51.45$, $n = 100$ Using GC, p-value = 0.0145 (to 3s.f.) Since ***p*-value = 0.0145** $>$ **0.01**, we **do not reject** H_0 and conclude that there is **insufficient** evidence that the manufacturer has underestimated the mean length of the metal rods at the 1% significance level.	
	(iii) To test H_o: $\mu = 50$ Against H_1: $\mu \neq 50$ Using a two-tailed test at 10% level of significance Under H_o and by Central Limit Theorem (since n = 100 is large), $\overline{X} \sim N\left(50, \frac{7^2}{100}\right)$ approximately Test statistic, $Z = \frac{\overline{X} - \mu}{\sigma} \sim N(0, 1)$ Since the manufacturer's claim is valid for this improved experiment at 10% level of significance, the conclusion is not in favour of H_1, we do not reject H_o and z_{test} must lie outside the critical region. $-1.6449 < z_{test} < 1.6449$ $-1.6449 < \frac{m - 50}{\frac{7}{\sqrt{100}}} < 1.6449$ 48.85 < m < 51.15 (2 decimal places)	

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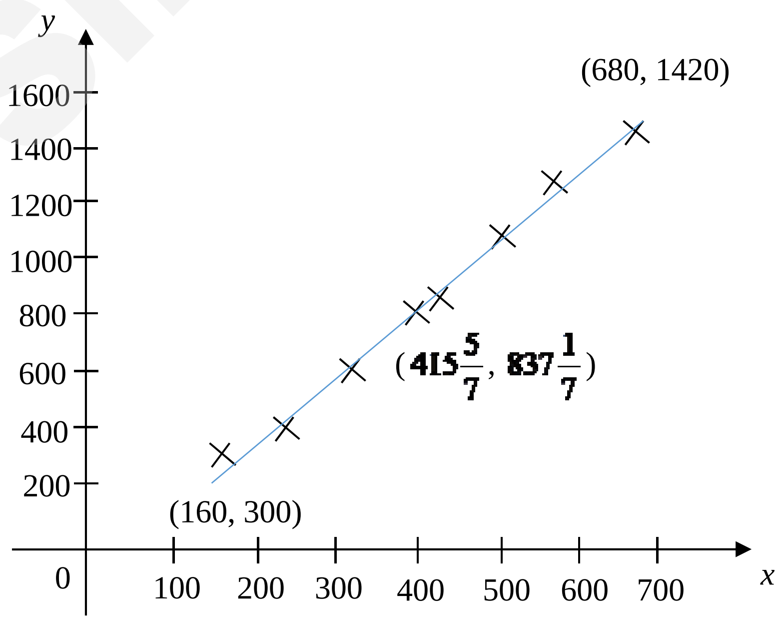
11	Mandy sells her homemade matcha-flavoured macarons. The numbers, x, sold in the first seven months of the year 2017, together with the profits, y dollars, on the sale of these macarons are given in the following table.	

	Jan	Feb	Mar	Apr	May	Jun	Jul
x	430	580	320	240	680	160	500
y	850	1240	600	400	1420	300	1050

	(i) Give a sketch of the scatter diagram for the data as shown on your calculator.	[2]
	(ii) Find $\bar{x}$ and $\bar{y}$ and mark the point ($\bar{x}$, $\bar{y}$) on your scatter diagram.	[2]
	(iii) Calculate the equation of the regression line of y on x, and draw this line on your scatter diagram.	[2]
	(iv) Calculate the product moment correlation coefficient, and comment on its value in relation to your scatter diagram.	[2]
	(v) Mandy expects to sell 600 matcha-flavoured macarons for the month of August in 2017. Using an appropriate regression line, estimate her profit for this month.	[1]
	(vi) Due to an error in her document, Mandy recorded her profits incorrectly. There is a shortfall of 100 dollars for all the profits recorded from January 2017 to July 2017 (as shown in the table above). Comment on whether this recording error will affect the value of the product moment correlation coefficient found in **(iv)**.	[1]
	Suggested Solution	

(i)



	(i) **G1** – Axes labelled with evenly-spaced scale **G1** – Correct data points plotted (end points labelled) (ii) **G1** – Correct ($\overline{x}$, $\overline{y}$) plotted (iii) **G1** – Correct line sketched	
	(ii) $\overline{x} = 415\frac{5}{7}$ (exact) (or 416) $\overline{y} = 837\frac{1}{7}$ (exact) (or 837)	
	(iii) $y = 2.2676x - 105.51$ $\approx 2.27x - 106$ (to 3 sig. fig.)	
	(iv) $r = 0.997$. There is a strong positive linearly correlation between the number of macarons sold and the profit earned from the sale of macarons. As the number of macarons sold increases, the profit from the sale of macarons increases. This explains why the data points show an upward trend and the data points are close to the regression line in the scatter diagram.	
	(v) $y = 2.2676(600) - 105.51$ ≈ 1255.05 ≈ 1260 Mandy's estimated profit for the month of August is \$1260.	
	(vi) As *r* measures the degree of scatter of the data points, an increment for all the values of the profits (values of *y*) will not change the scatter of the data. Hence, there will be no change in the value of *r*.	

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12 The masses, in kilograms, of watermelons and papayas sold by a supermarket have normal distributions with means and standard deviations as shown in the following table.

	Mean	Standard Deviation
Watermelon	5.42	0.51
Papaya	2.18	0.35

(i) Find the probability that the mass of a watermelon chosen at random is between 5.2 kg and 6.5 kg. [1]

(ii) Find the probability that the total mass of four randomly chosen watermelons is less than 22.8 kg, stating clearly the mean and variance of the distribution that you use. [3]

(iii) Find the probability that the total mass of eight randomly chosen papayas is more than the total mass of four randomly chosen watermelons, stating clearly the mean and variance of the distribution that you use. [3]

(iv) Watermelons cost \$2.80 per kilogram and papayas cost \$1.80 per kilogram. Find the mean and the variance of the total cost of four randomly chosen watermelons and eight randomly chosen papayas and hence, find the probability that the total cost is between \$80 and \$100. [4]

(v) State an assumption for the calculations in parts **(iii)** and **(iv)** to be valid. [1]

Suggested Solution

(i)

Let X denote the random variable of the mass of a randomly chosen watermelon.

$X \sim \mathrm{N}\left(5.42, 0.51^2\right)$

$\mathrm{P}(5.2 < X < 6.5) \approx 0.64980 = 0.650$ (3 sig fig.)

(ii)

Let $T = X_1 + X_2 + X_3 + X_4$

$\mathrm{E}(T) = 4(5.42) = 21.68$

$\mathrm{Var}(T) = 4\left(0.51^2\right) = 1.0404$

$T \sim \mathrm{N}(21.68, 1.0404)$

$\mathrm{P}(T < 22.8) = 0.86391 \approx 0.864$ (3 sig fig)

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	(iii) Let Y denote the random variable of the mass of a randomly chosen papaya. $Y \sim \mathrm{N}\left(2.18, 0.35^2\right)$ Let $S = Y_1 + Y_2 + Y_3 + Y_4 + Y_5 + Y_6 + Y_7 + Y_8$ $S \sim \mathrm{N}\left(8(2.18), 8\left(0.35^2\right)\right)$ $S \sim \mathrm{N}(17.44, 0.98)$ $\mathrm{E}(S - T) = 17.44 - 21.68 = -4.24$ $\mathrm{Var}(S - T) = 0.98 + 1.0404 = 2.0204$ $S - T \sim \mathrm{N}(-4.24, 2.0204)$ $\mathrm{P}(S - T > 0) \approx 0.0014275 = 0.00143$ (3 sig fig)	
	(iv) Let C_1 be 1.8S (cost of 6 papayas) $C_1 \sim \mathrm{N}\left(1.8(17.44), 1.8^2(0.98)\right)$ $C_1 \sim \mathrm{N}(31.392, 3.1752)$ Let C_2 be 2.8T (Cost of 4 water melons) $C_2 \sim \mathrm{N}\left(2.8(21.68), 2.8^2(1.0404)\right)$ $C_2 \sim \mathrm{N}(60.704,\ 8.156736)$ $C_1 + C_2 \sim$ N(31.392+60.704, 3.1752+8.156736) $C_1 + C_2 \sim$ N(92.096, 11.331936) P(80 < C_1 + C_2 < 100) $\approx 0.99040 = 0.990$ (3 sig fig).	
	(v) The assumption is that the masses, in kilograms, of watermelons and papayas sold by a supermarket are independently normally distributed.	

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SAINT ANDREW'S JUNIOR COLLEGE

Preliminary Examination

MATHEMATICS
Higher 1 **8865**

Thursday **24th August 2017** **3 hours**

Additional materials : Answer paper
List of Formulae(MF26)
Cover Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, civics group and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.

Answer **all** the questions. Total marks : **100**
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically state otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematic steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

This document consists of **6** printed pages including this page.
[Turn over]

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Section A: Pure Mathematics [40 marks]

1 Find, algebraically, the set of values of k for which $kx^2 + (k-1)x + \dfrac{9}{k} > 0$ for all real values of x. [4]

2 Solve the simultaneous equations,

$$5^{x+2} = 125\left(5^{y-5}\right),$$

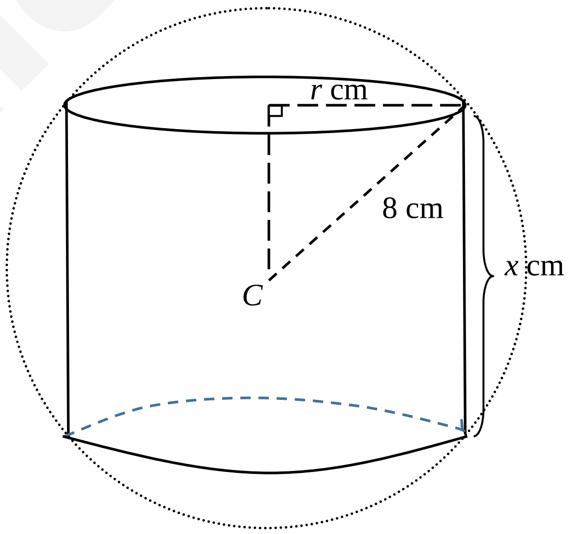
$$\log_{\sqrt{3}}(2x - y) = 4 + 2\log_{\sqrt{3}} 2.$$ [4]

3 (a) Differentiate with respect of x, simplifying your answers,

(i) $\dfrac{1}{\sqrt{x^2 - 3x + 1}}$, [2]

(ii) $\ln\left(\dfrac{x^2 + 3x + 2}{x^2 + 4x + 3}\right)$. [3]

(b) In the diagram below, a solid right cylinder of height x cm and radius r cm is inscribed in a sphere of centre C with radius 8 cm. The circumference of the top and bottom circular surfaces of the cylinder are in contact with the sphere as shown in the figure below.



(i) Show that the volume of the cylinder is

$$V = \frac{\pi}{4}\left(256x - x^3\right) \quad .$$ [2]

(ii) Hence, find the exact maximum value of V as x varies. [4]

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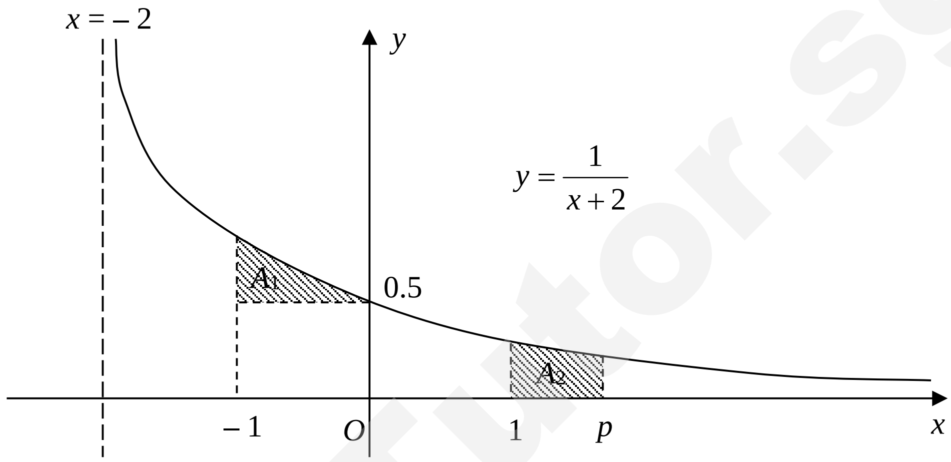
4 (a) Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\left(5\mathrm{e}^{2x}+1\right)^4\right)$. Hence, without the use of a calculator, find $\int_0^2 \mathrm{e}^{2x}\left(5\mathrm{e}^{2x}+1\right)^3 \mathrm{d}x$. [4]

(b) The diagram below, not drawn to scale, shows the curve C with equation $y=\dfrac{1}{x+2}$, $x>-2$. The region A_1 is bounded by C, the lines $x=-1$ and $y=0.5$. The region A_2 is bounded by C, the lines $x=1$, $x=p$ and the x-axis.



Find the exact value of p such that the regions A_1 and A_2 have equal areas. [5]

5 A group of environmentalists conducted a research project to study how the population of the monkeys in a forest could be affected by a newly set-up factory nearby. The population P, of monkeys in the forest, after t months of the opening of the factory, can be modelled by

$$P=500\left(3+\mathrm{e}^{-0.2t}\right).$$

(i) Find the initial population of the monkeys in the forest. [1]

(ii) Find, to the nearest whole number, the population of monkeys after two years. [1]

(iii) **Without the use of a calculator,** determine the number of complete months for the population of monkeys to first fall below 1515. [4]

(iv) Using this model, describe what will happen to the population of monkeys in the forest in the long run. [2]

(v) Sketch, in the context of this question, the graph of P against t. [3]

(vi) Suggest a possible limitation of this model to represent the population of monkeys in the real world. [1]

[Turn Over]

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Section B: Statistics [60 marks]

6 Thomas has six tiles, each with a different letter of his name on it. Thomas randomly arranges these letters in a line. Find the probability that the six tiles are arranged

(a) in the correct order that spells his name. [2]

(b) such that the vowels are separated. [2]

(c) such that the vowels are at the two ends. [2]

7 (a) Eight pairs of values of variables x and y are measured. Draw a sketch of a possible scatter diagram of the data for each of the following cases:

(i) the product moment correlation coefficient is approximately -0.9, [1]

(ii) the product moment correlation coefficient is approximately zero. [1]

(b) A researcher recorded the water temperature T, in ^{o}C, and the depth D, in metres, at noon on a certain day at each of the eight locations in a lake. The results are summarized in the table below.

D (m)	10	50	80	120	200	250	340	400
T (^{o}C)	25.0	23.0	22.2	k	16.4	12.4	10.0	4.0

(i) It is known that the regression line of T on D is given by $T = -0.051424D + 25.908$. Show that the value of k is 19.7, correct to 1 decimal place. [1]

(ii) Give a sketch of the scatter diagram for the data. [2]

(iii) Calculate the product moment correlation coefficient for the revised data and comment on its value in the context of the question. [2]

(iv) Sketch the regression line T on D on your scatter diagram. [1]

(v) Hence, estimate the water temperature when the depth of the water is 550 metres. Comment on the reliability of this estimate. [1]

(vi) Given that 1 kilometre = 1000 metre, rewrite your equation from part **(i)** so that it can used to estimate the temperature of the water when the height is given in kilometres.
State the value of the regression coefficient. [2]

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8 James plays a game by first throwing a biased four-sided die which has faces numbered 1, 2, 3 and 4. Let X denote the number obtained when the die is tossed once. The probability that x is shown on the die is given by $\mathrm{P}(X = x) = \frac{x}{10}$, where x = 1, 2, 3 and 4.

If the number obtained is 3 or 4, he records the number shown as his score.
If the number obtained is 1 or 2, he throws a fair six-sided die and records the sum of the two numbers from his two throws as his score.

(i) Draw a tree diagram to represent all possible outcomes. [2]

Events A and B are defined as follows:

Event A: James' score is at least a 4,
Event B: James' score is an odd number,

(ii) Show that P(A) = $\frac{19}{30}$. [2]

(iii) Find $\mathrm{P}(A|B')$. [4]

(iv) State with a reason, whether the events A and B are mutually exclusive. [2]

9 A wholesaler sells two types of fruit, Type A and Type B. The masses of each type of fruit follow independent normal distributions with the following means and standard deviations:

Type	Mean / kg	Standard Deviation / kg
Type A	1.5	0.1
Type B	1.8	0.2

The wholesale price of Type A and Type B fruit are \$30 per kg and \$35 per kg respectively.

(i) Find the probability that the total mass of three Type A fruits exceeds 5 kg. [3]

(ii) Find the probability that the difference between the total mass of three Type A fruits and twice the mass of a Type B fruit is at least 0.5 kg. [3]

(iii) The total cost of buying two Type A fruits and one Type B fruit exceeds \$$k$ with a probability of 0.1. Determine the value of k. [3]

(iv) Two Type A fruits are selected at random. Determine the probability that only one of them weighs more than 1.3 kg. [2]

[Turn Over]

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10 A pottery manufacturer makes teapots in batches of 30. On average, 8% of teapots are faulty.

(i) State, in context of this question, one assumption needed to model the number of teapots that are faulty by a binomial distribution. [1]

(ii) Find the probability that in a batch of 30 there is more than two faulty teapots. [2]

(iii) The manufacturer produces 240 batches of 30 teapots in one month. Find the expected number of batches which contain exactly one faulty teapot. [3]

(iv) 50 batches of 30 teapots each are randomly chosen. Find the probability that the mean number of faulty teapots in a batch is more than 2.5. [3]

11 A company packs and supplies salt in small packets. The mass of salt in one packet is denoted by x grams. The company claims that the mean mass of salt is at least 10 grams. To test this claim, a sample of 100 packets of salt is randomly chosen. Their masses are summarised by

$$\sum x = 970 \quad \sum x^2 = 9800$$

(i) Find the unbiased estimates of the population mean and variance. [2]

(ii) State, with a reason, whether it is necessary to assume a normal distribution for the test to be valid. [1]

(iii) Carry out a test at the 10% significance level whether the company's claim is valid. [4]

(iv) Explain what is meant by the phrase '10% level of significance' in the context of this question. [1]

(v) Explain the meaning of p-value in this context. [1]

The company introduces a new packaging system and the new population variance is known to be 0.9^2 grams^2. A new random sample of 30 packets of salt is chosen and the mean of this sample is m grams. A test at the 5% significance level indicates that the company's initial claim is valid for this improved process.

(iv) Find the least possible value of m, giving your answer correct to 2 decimal places. [4]

End of Paper

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2017 SAJC Preliminary Examination Solutions

1

$$kx^2+(k-1)x+\frac{9}{k}>0$$

Note that $k \neq 0$.

For the quadratic expression to be positive, the following 3 conditions must be met:

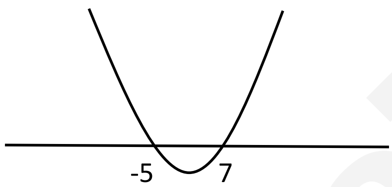
(1) Discriminant < 0 and (2) $k>0$

$$(k-1)^2-4(k)\left(\frac{9}{k}\right)<0$$

$$k^2-2k+1-36<0$$

$$k^2-2k-35<0$$

$$(k+5)(k-7)<0$$



$$-5<k<7$$

Combining (1) and (2), the set of values of k is

$$\left\{k : k \in \mathbb{R}, 0<k<7\right\}$$

2

$$5^{x+2}=5^3(5^{y-5})$$

$$x+2=3+y-5$$

$$x-y=-4$$

$$x=y-4 \text{ --------------------- } (1)$$

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	$\log_{\sqrt{3}}(2x-y)-2\log_{\sqrt{3}}2=4$ $\log_{\sqrt{3}}(2x-y)-\log_{\sqrt{3}}4=4$ $\log_{\sqrt{3}}\frac{(2x-y)}{4}=4$ $\frac{(2x-y)}{4}=3^{\frac{4}{2}}$ $2x-y=4\times 9$ $2x-y=36$ ------------(2) Sub. (1) into (2) $2(y-4)-y=36$ $y=44$ $x=40$
3	(a) (i) Let $y=\frac{1}{\sqrt{x^2-3x+1}}=\left(x^2-3x+1\right)^{-\frac{1}{2}}$ $\frac{dy}{dx}=-\frac{1}{2}\left(x^2-3x+1\right)^{-\frac{3}{2}}(2x-3)$ $=-\frac{2x-3}{2\sqrt{\left(x^2-3x+1\right)^3}}$ (ii) Let $y=\ln\left(\frac{x^2+3x+2}{x^2+4x+3}\right)=\ln\frac{(x+1)(x+2)}{(x+1)(x+3)}=\ln\frac{x+2}{x+3}$ $=\ln(x+2)-\ln(x+3)$ $\frac{dy}{dx}=\frac{1}{x+2}-\frac{1}{x+3}$ $=\frac{1}{(x+2)(x+3)}$ Alternatively,

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$y=\ln\left(\frac{x^2+3x+2}{x^2+4x+3}\right)=\ln(x^2+3x+2)-\ln(x^2+4x+3)$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2x+3}{x^2+3x+2}-\frac{2x+4}{x^2+4x+3}$$

$$=\frac{2x+3}{(x+1)(x+2)}-\frac{2x+4}{(x+1)(x+3)}$$

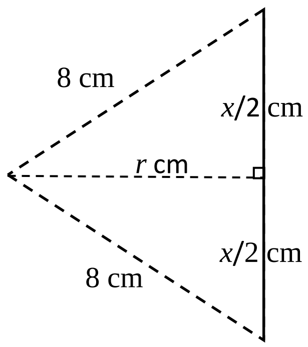
$$=\frac{(2x+3)(x+3)-(2x+4)(x+2)}{(x+1)(x+2)(x+3)}$$

$$=\frac{x+1}{(x+1)(x+2)(x+3)}$$

$$=\frac{1}{(x+2)(x+3)}$$

(b)(i) By Pythagoras's Theorem,

the radius of the cylinder is $r=\sqrt{8^2-\left(\frac{x}{2}\right)^2}$



$$=\sqrt{64-\frac{x^2}{4}}=\sqrt{\frac{256-x^2}{4}}=\frac{\sqrt{256-x^2}}{2}$$

Volume of cylinder is

$$V=\pi\left(\sqrt{\frac{256-x^2}{4}}\right)^2(x)$$

$$=\frac{\pi x\left(256-x^2\right)}{4}$$

$$=\frac{\pi}{4}\left(256x-x^3\right)$$

(ii) $V=\frac{\pi}{4}\left(256x-x^3\right)$

$$\frac{dV}{dx}=\frac{\pi}{4}\left(256-3x^2\right)$$

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When $\dfrac{dV}{dx} = 0$, $x^2 = \dfrac{256}{3}$

Since the height of the cylinder, x, is positive,

$x = \sqrt{\dfrac{256}{3}}$ (= 9.2376)

$\dfrac{d^2V}{dx^2} = \dfrac{\pi}{4}(-6x) = -43.5 < 0$ for positive value of x.

Alternative Method to Check for Maximum

x	9.2	$x = \sqrt{\frac{256}{3}}$	9.3
$\frac{dV}{dx}$	+1.63	0	–5.67
GI	/	—	\

Hence the volume V is maximum when $x = \sqrt{\dfrac{256}{3}}$ cm

At Max $V = \dfrac{\pi}{4}\left(256 - x^2\right)x$

$$= \frac{\pi}{4}\left(256 - \frac{256}{3}\right)\sqrt{\frac{256}{3}}$$

$$= \frac{\pi}{4}\left(\frac{512}{3}\right)\sqrt{\frac{256}{3}}$$

$$= \frac{\pi}{4}\left(\frac{512}{3}\right)\left(\frac{16\sqrt{3}}{3}\right)$$

$$= \frac{2048\sqrt{3}}{9}\pi \text{ cm}^3$$

4 (a)

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	$\frac{d}{dx}\left(5e^{2x}+1\right)^4$ $=4\left(5e^{2x}+1\right)^3\left(10e^{2x}\right)$ $=40e^{2x}\left(5e^{2x}+1\right)^3$ $\int_0^2 e^{2x}\left(5e^{2x}+1\right)^3 dx=\frac{1}{40}\int_0^2 40e^{2x}\left(5e^{2x}+1\right)^3 dx$ From part (i) $\int_0^2 e^{2x}\left(5e^{2x}+1\right)^3 dx=\frac{1}{40}\left[\left(5e^{2x}+1\right)^4\right]_0^2$ $=\frac{1}{40}\left[\left(5e^4+1\right)^4-\left(5e^0+1\right)^4\right]$ $=\frac{1}{40}\left[\left(5e^4+1\right)^4-\left(5e^0+1\right)^4\right]$ $=\frac{1}{40}\left[\left(5e^4+1\right)^4-1296\right]$ (b) $\int_{-1}^{0}\frac{1}{x+2}\,dx-(0.5)=\int_1^p\frac{1}{x+2}\,dx$ $\left[\ln\lvert x+2\rvert\right]_{-1}^{0}-(0.5)=\left[\ln\lvert x+2\rvert\right]_1^p$ $(\ln 2-\ln 1)-0.5=\ln\lvert p+2\rvert-\ln 3$ $\ln(p+2)=\ln 2+\ln 3-0.5$ since $p+2>0$ $(p+2)=e^{\ln 6-0.5}=6e^{-0.5}$ $p=6e^{-0.5}-2$
5	(i) $P=500\left(3+e^{-0.2t}\right)$ When $t=0$, $P=500(3+1)$ $P=2000$ (ii) $P=500\left(3+e^{-0.2(24)}\right)$ $P=1504$ monkeys

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(iii)

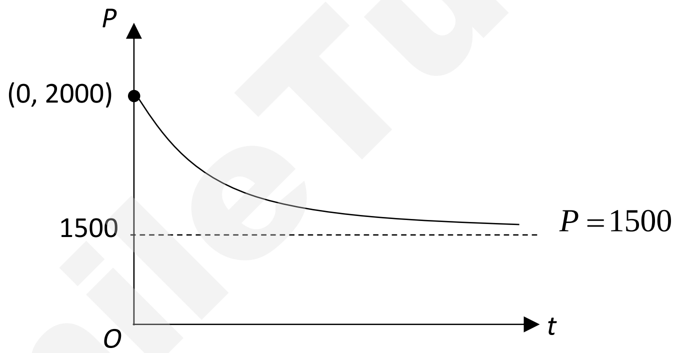
$$500\left(3+e^{-0.2t}\right)<1515$$
$$3+e^{-0.2t}<3.03$$
$$e^{-0.2t}<0.03$$
$$-0.2t<\ln(0.03)$$
$$t>\frac{\ln(0.03)}{-0.2}$$
$$t>17.533$$
$$t\approx 18$$

The population of the monkeys will first drop below 1515 after 18 complete months.

(iv) As $t\to\infty$, $P\to 500(3+0)=1500$.

The population of the monkeys will decrease and approach/stabilise at 1500.

(v)



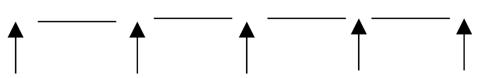
(vi) No. The model may not be accurate in the real world.

In the real world, population of the monkeys may not decrease due to the opening of the factory alone and the monkeys may not stabilise at 1500 due to human interference. (or any possible logical solutions)

6

(i) Probability (tiles are arranged in correct order to spell his name) = $\frac{1}{6!}=\frac{1}{720}$

(ii)



4! ways to arrange consonants.

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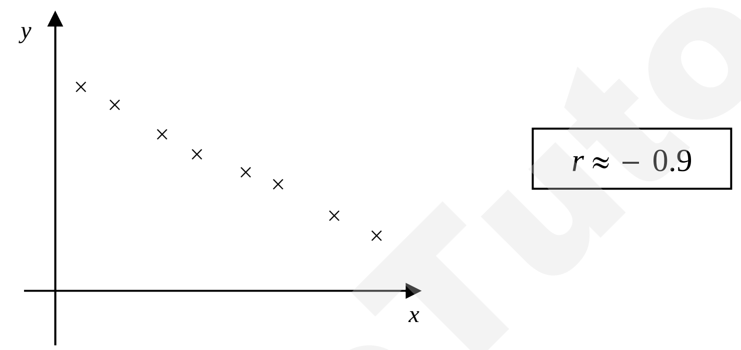
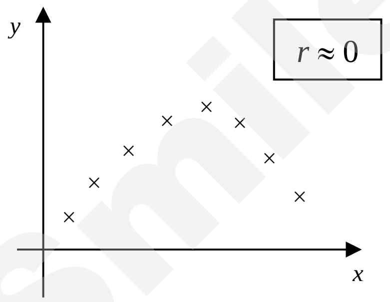
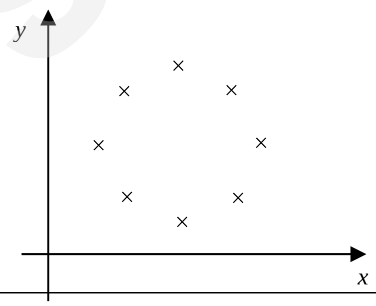
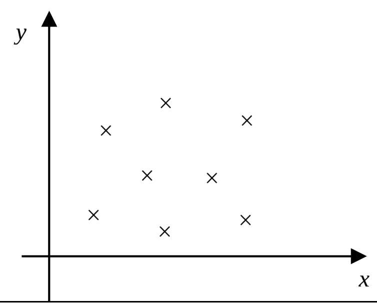
	5 possible slots to insert A & O P(the vowels are separated) = $\frac{4!\,^5P_2}{6!} = \frac{2}{3}$ (iii) 4! Ways to arrange consonants. 2! Ways to arrange A&O $\boxed{A}\ \boxed{T, H, M, S}\ \boxed{O}$ P(the vowels are at the two ends) = $\frac{4!2!}{6!} = \frac{1}{15}$
7	   

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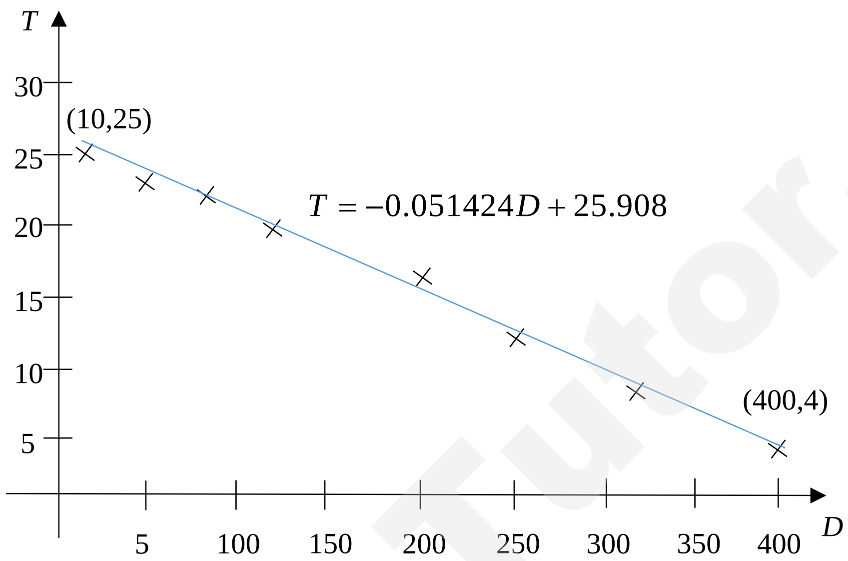
Using $\overline{T} = -0.051424\overline{D} + 25.908$,

substitute $\overline{D} = \frac{1450}{8} = 181.25$, $\overline{T} = \frac{k+113}{8}$

$\Rightarrow \frac{k+113}{8} = -0.051424(181.25) + 25.908$

$\Rightarrow k = 19.6992 = 19.7$ (shown)



$r = -0.992$. It indicates a strong negative linear correlation between the depth of water and the temperature of water. As the depth of water increases, the temperature of water decreases.

Regression line T on D:

$T = -0.051424D + 25.908$

$T = -0.0514D + 25.9$ (3 s.f.)

When D = 550,

$T = -0.051424(550) + 25.908$

$T = -2.38$

Although $r = -0.992$ (3 s.f.) is close to -1, but the estimate may not be reliable as D = 550 falls outside the data range [10, 400]. Hence, the estimate is unreliable.

Regression line T on D where height is given in kilometres:

$T = -51.424D + 25.908$

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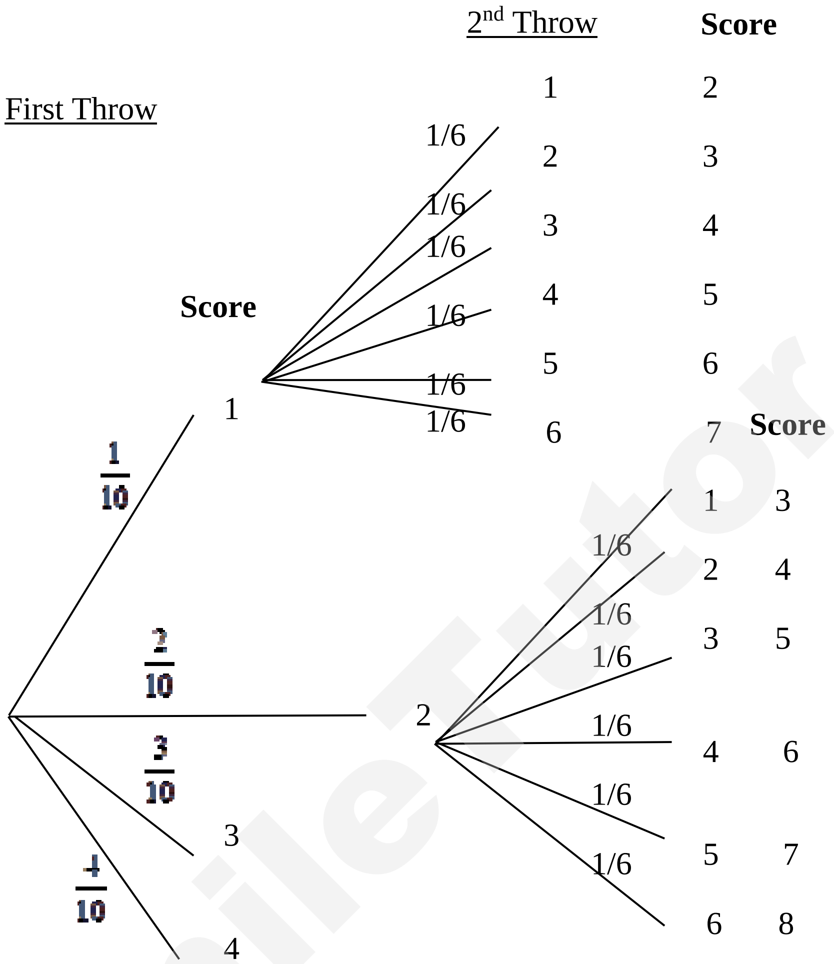
	$T=-51.4D+25.9$ (3 s.f.) The value of the regression coefficient is -0.000051424.
8	 Let X be the score of the first throw and Y be the score of the second throw $\mathrm{P}(A)=1-\mathrm{P}(A')=1-\mathrm{P}(\text{James' score is at most a 3})$ $=1-\mathrm{P}(\text{score is 3 on the 1st throw})$ $-\mathrm{P}(X=1\cap Y=1\text{ or }2)-\mathrm{P}(X=2\cap Y=1)$ $=1-\frac{3}{10}-\left(\frac{1}{10}\times\frac{1}{6}\times 2\right)-\left(\frac{2}{10}\times\frac{1}{6}\right)=\frac{19}{30}$ **<u>Alternative Method</u>** $\mathrm{P}(A)=\mathrm{P}(X=4)+\mathrm{P}(X=1\cap 3\leq Y\leq 6)$

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	$+ \mathrm{P}(X = 2 \cap 2 \le Y \le 6)$ $= \frac{4}{10} + \left(\frac{1}{10} \times \frac{1}{6} \times 4\right) + \left(\frac{2}{10} \times \frac{1}{6} \times 5\right)$ $= \frac{19}{30}$ $\mathrm{P}(A \mid B') = \frac{\mathrm{P}(A \cap B')}{\mathrm{P}(B')} = \frac{\text{P(score is at least 4 and even)}}{\text{P(score is even)}}$ $= \frac{\mathrm{P}(X=4) + \mathrm{P}(X=1 \cap Y = 3 \text{ or } 5) + \mathrm{P}(X=2 \cap Y = 4 \text{ or } 6)}{\mathrm{P}(X=4) + \mathrm{P}(X=1 \cap Y = 1 \text{ or } 3 \text{ or } 5) + \mathrm{P}(X=2 \cap Y = 2 \text{ or } 4 \text{ or } 6)}$ $= \frac{\frac{4}{10} + \left(\frac{1}{10} \times \frac{1}{6} \times 2\right) + \left(\frac{2}{10} \times \frac{1}{6} \times 3\right)}{\frac{4}{10} + \left(\frac{1}{10} \times \frac{1}{6} \times 3\right) + \left(\frac{2}{10} \times \frac{1}{6} \times 3\right)}$ $= \frac{8/15}{11/20}$ $= \frac{32}{33}$ $\mathrm{P}(A \cap B)$ $\mathrm{P}(X = 1 \cap Y = 4 \text{ or } 6) + \mathrm{P}(X = 2 \cap Y = 3 \text{ or } 5)$ $= \left(\frac{1}{10} \times \frac{1}{6} \times 2\right) + \left(\frac{2}{10} \times \frac{1}{6} \times 2\right)$ $= \frac{1}{10} \neq 0$ Events A and B are not mutually exclusive.
9	(i) Let X and Y be the r.v. denoting the mass of the type A and type B fruits respectively. $X \sim N(1.5, 0.1^2)$

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	$Y \sim N(1.8, 0.2^2)$ $X_1 + X_2 + X_3 \sim N(4.5, 0.03)$ $P(X_1 + X_2 + X_3 > 5) = 0.00195$ (to 3 sf) (ii) Let $T = X_1 + X_2 + X_3 - 2Y$ $E(T) = 4.5 - 3.6 = 0.9$ $Var(T) = 0.03 + 4 \times 0.2^2 = 0.19$ $T \sim N(0.9, 0.19)$ $P(\lvert T \rvert \geq 0.5)$ $= P(T \leq -0.5) + P(T \geq 0.5)$ $= 0.000659548 + 0.8206023$ = 0.821 (3 sf) (iii) Let $C = 30(X_1 + X_2) + 35(Y)$ $E(C) = 30(1.5 + 1.5) + 35(1.8) = 153$ $Var(C) = 30^2(0.01 + 0.01) + 35^2(0.04) = 67$ $C \sim N(153, 67)$ $P(C > k) = 0.1$ $P(C \leq k) = 0.9$ $k = 163.49 \approx 163$ (3 sig. fig.) (iv) Required Probability $= P(X > 1.3)P(X \leq 1.3) \times 2!$ $= 0.97725 \times 0.02275 \times 2$ = 0.0445 (3 sf)

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10	(i) The probability of making a faulty teapot is a constant. The event that a teapot is found faulty is independent of other another teapot. (ii) Let X be the number of faulty teapots in a batch of 30. $X \sim \mathrm{B}(30.0.08)$ $P(X>2)=1-P(X\le 2)=0.435$ (iii) Let Y be the number of batches which contain exactly one faulty teapot each. $P(X=1)=0.21382$ $Y \sim \mathrm{B}(240, 0.21382)$ $\mathrm{E}(Y)=240\times 0.21382=51.3$ (iv) Let $\overline{W}$ be the mean number of faulty teapots per batch. Since n = 50 is large, by Central Limit Theorem $\overline{W} \sim \mathrm{N}\left(30\times 0.08, \frac{30\times 0.08\times 0.92}{50}\right)$ approximately $\overline{W} \sim \mathrm{N}\left(2.4, 0.04416\right)$ $P(\overline{W}>2.5)=0.317$

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11	Unbiased estimate of the population mean, $\bar{x} = \frac{970}{100} = 9.7$ Unbiased estimate of the population variance, $s^2 = \frac{1}{99}\left[9800 - \frac{970^2}{100}\right] = \frac{391}{99} = 3.9495$ (ii) It is not necessary to assume a normal distribution for the test to be valid. Since n is large, by Central Limit Theorem, the mean mass of salt is normally distributed approximately. (iii) Let X be the mass of one packet of salt in grams and μ be the population mean of the mass of one packet of salt in grams. $H_o : \mu = 10$ $H_1 : \mu < 10$ Under H_o, Since n = 100 is large, by Central Limit Theorem, $\overline{X} \sim N\left(10, \frac{391}{9900}\right)$ approximately, Use a left tailed z-test at 10 % level of significance, Test Statistic: $Z = \frac{\overline{X} - 10}{s/10} \sim N(0,1)$ Using GC, the p-value = $0.0656 \leq 0.10$ Reject the null hypothesis and conclude that there is sufficient evidence at 10% significance level to reject the company's claim that the mass is at least 10g. (iv) There is a probability of 0.10 that we concluded that the mean mass of the packets of salt is less than 10 grams when it is actually at least 10 grams. The p-value is the lowest significance level at which the sample mean mass of salt is at least 10 grams. (v) Let Y be the random variable of the mass of each packet of salt in the new packaging system.

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	$H_o : \mu = 10$ $H_1 : \mu < 10$ Since n = 30 is large, by Central Limit Theorem Under H_o, $Z = \dfrac{\overline{Y} - 10}{\frac{0.9}{\sqrt{30}}} \sim N(0,1)$ approximately Since company's claim is valid, H_o is not rejected at 5% level of significance, $\dfrac{m-10}{\frac{0.9}{\sqrt{30}}} > -1.6449$ $m > 9.7297$ Least possible value of m is 9.73.

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TEMASEK JUNIOR COLLEGE, SINGAPORE

JC 2
Preliminary Examination 2017

MATHEMATICS

8865/01

Higher 1

29 August 2017

Additional Materials: Answer paper

List of Formulae (MF26)

3 hours

READ THESE INSTRUCTIONS FIRST

Write your Civics group and name on all the work that you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

At the end of the examination, fasten all your work securely together.

This document consists of **5** printed pages.

[Turn over

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Section A: Pure Mathematics [40 marks]

1 The gradient of a curve C is given by $\dfrac{dy}{dx} = 2k + 3 - \dfrac{k+1}{(x-1)^2}$, where $k \in \mathbb{R}$. Find the set of values of k for which C has 2 distinct turning points. [4]

2 (a) Differentiate $\dfrac{e^x - 1}{e^{2x}}$. [2]

(b) Find $\int (2x\sqrt{2x+1} + \sqrt{2x+1})\, dx$. [3]

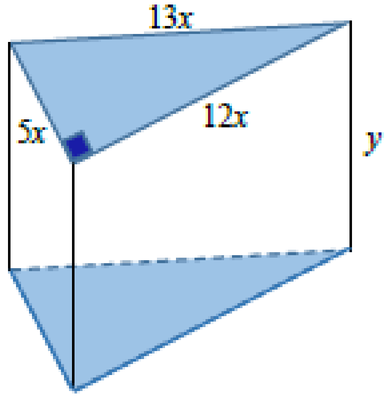
3 The curve C has equation $y = \ln x$, $x > 0$.

(i) Find the equation of the tangent to C at the point where $x = e^2$, leaving your answer in terms of e. [3]

(ii) Sketch the graph of C and the tangent to C at the point where $x = e^2$ on the same diagram, stating the coordinates of intersection with the axes and the equation of any asymptote(s). [2]

(iii) By using the result $\int \ln x \, dx = x \ln x - x + c$, where c is an arbitrary constant, find the area of the region bounded by C, the tangent to C at the point where $x = e^2$, the x-axis and the y-axis. Give your answer in terms of e. [3]

4 A prism with a cross-section in the shape of a right-angled triangle has dimensions (in cm) as shown in the diagram below.



The volume of the prism is 7200 cm^3. Show that the surface area of the prism is given by

$$S = \frac{7200}{x} + 60x^2 \text{ cm}^2.$$ [3]

Without using a calculator, find in surd form the value of x that gives a stationary value of S. Hence state, with a reason, whether S is a maximum or minimum. [4]

It is also given that x is decreasing at 0.5 cm/s, find the rate at which the surface area is decreasing when $x = 5$ cm. [2]

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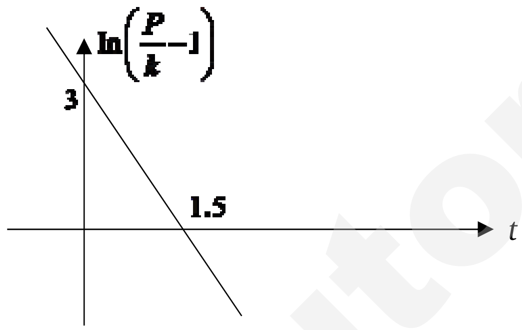
5 The profit P (in thousands dollars) of a company after the start of a promotion "Clearance Sale" can be modelled by the equation

$$P = k(1 + be^{-rt}),$$

where t is the number of days elapsed since the start of the promotion and b, r and k are positive constants.

(i) Express $\ln\left(\frac{P}{k} - 1\right)$ in terms of b, r and t [1]

(ii) The graph of $\ln\left(\frac{P}{k} - 1\right)$ against t is given below.



By using the graph above, show that $b = e^3$ and $r = 2$. [3]

(iii) Using differentiation, show that $\frac{dP}{dt} < 0$ for $t \geq 0$. Hence explain why the maximum profit occurs at $t = 0$. Given that the maximum profit is \$42171, find the value of k, correct to nearest integer value. [5]

The "Clearance Sale" ends after a week and another promotion "Happy Sale" takes place immediately after. The "Happy Sale" lasts for 3 weeks and the profit during "Happy Sale" can be modelled by the equation

$$P = a - \frac{1}{6}(t - 14)^2, \quad \text{for } 7 \leq t \leq 28.$$

(iv) Given that the company's maximum profit during "Happy Sale" is \$18000, find the value of a. [2]

(v) Find the total profit of the company in 4 weeks, correct to the nearest dollars. [3]

Section B: Statistics [60 marks]

6 Find the number of 6-letter passwords that can be formed using the letters from the word SINGAPORE if

(i) repetitions of letters are not allowed. [1]

(ii) at least two vowels must be chosen and repetition of letters are not allowed. [2]

(iii) three distinct vowels and three distinct consonants are chosen, and vowels and consonants must alternate? [3]

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7 A fixed number, n, of taxis entering VICOM is observed and the number of those taxis that fails inspection is denoted by X.

(i) State, in context, two assumptions needed for X to be well modelled by a binomial distribution. [2]

Assume now that X has the distribution $\mathrm{B}(n, p)$.

(ii) Given that $n = 20$ and $p = 0.15$, find $\mathrm{P}(X = 2 \text{ or } 3)$. [2]

(iii) Given that $n = 10$ and $p = 0.3$, find $\mathrm{P}(3 \le X < 6)$. [2]

(iv) Given that $n = 3$ and $\mathrm{P}(X \le 1) = 0.5$, show that $4p^3 - 6p^2 + 1 = 0$. Hence find the value of p. [3]

8 The number of hours, x, spent daily on revision for mathematics and the marks, y, obtained for the mathematics year-end examination are recorded for 10 randomly selected students. The results are given in the following table.

x	1.3	2.1	1.1	2.3	2.7	1.2	3.2	3.4	3.0	2.5
y	68	74	64	76	75	66	85	81	86	75

(i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]

(ii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iii) Find the equation of the regression line of y on x, in the form $y = mx + c$, giving the values of m and c correct to 4 significant figures. Sketch this line on your scatter diagram. [2]

(iv) Use the equation of your regression line to estimate the marks obtained by a student who spends 1.5 hours a day on revision for mathematics. Comment on the reliability of your estimate. [3]

9 A bag contains 3 black balls and 5 white balls. Paul draws a ball at random from the bag and notes the colour. If a black ball is selected, Paul returns it to the bag and adds an additional black ball into the bag. If a white ball is selected, Paul does not return it to the bag but adds 2 black balls into the bag. Paul then draws a ball at random from the bag again and notes the colour.

Draw a tree diagram to represent the information of the two draws. [3]

(i) Show that the probability that Paul selects a black ball on both draws is $\frac{1}{6}$. [1]

(ii) Find the probability that Paul selects a white ball either on his first or second draw, or both. [2]

(iii) Find the probability that Paul selects a white ball on his first draw, given that he selects a black ball on his second draw. [3]

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10 A company claims that their electric-powered V1 cars is designed to travel a mean distance of 500 km on one full charge. To test this claim, a random sample of 80 V1 cars is taken and the distance, *x* km, travelled on one full charge are summarised by

$$\sum(x-500)=46 \quad \text{and} \quad \sum(x-500)^2=460\,.$$

(i) Find the unbiased estimates of the population mean and variance. [3]

(ii) Suggest a reason why, in this context, the given data is summarised in terms of $(x-500)$ instead of *x*. [1]

(iii) Test, at the 5% level of significance, whether the company's claim is valid. [4]

(iv) State, with a reason, whether it is necessary to assume a normal distribution for the test to be valid. [1]

The company introduces a new electric-powered V2 car and claims that the V2 can travel more than the mean distance of the V1 on one full charge. The population variance of the V2 cars is known to be 13 km^2. A random sample of 50 V2 cars is taken.

(v) Find the set of values within which the mean distance of this sample must lie, such that there is not enough evidence from the sample to support the company's claim at the 1% level of significance. [4]

11 Three friends Anand, Beng and Charlie goes racing regularly at the Temasek Circuit, which offers a standard route on Track 1 or a more challenging route on Track 2. The time taken, in minutes, taken by them to complete a round on Tracks 1 and 2 have independent normal distributions with means and standard deviations as shown in the following table.

	Track	Mean	Standard deviation
Anand	1	3.15	0.21
Beng	1	3.18	0.10
Charlie	2	4.22	0.15

(i) Find the probability that Beng takes less than 3.15 minutes to complete a randomly chosen round on Track 1. [1]

(ii) Find the probability that Anand and Beng each takes less than 3.15 minutes to complete a randomly chosen round on Track 1. [2]

(iii) Find the probability that Beng is faster than Anand in completing a randomly chosen round on Track 1. [3]

(iv) Find the probability that out of 8 complete rounds on Track 2, there are more than 4 rounds in which Charlie takes less than 4.20 minutes to complete. [3]

Temasek Circuit charges customers \$18 per minute on Track 1 and \$22 per minute on Track 2. On a particular day, Beng completes 10 rounds on Track 1 and Charlie completes 8 rounds on Track 2.

(v) Find the probability that Beng and Charlie pay a total of less than \$1300. [5]

End of Paper

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Section A: Pure Mathematics [40 marks]

1 The gradient of a curve C is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = 2k + 3 - \frac{k+1}{(x-1)^2}$, where $k \in$ □ . Find the set of values of k for which C has 2 distinct turning points. [4]

1	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2k + 3 - \frac{k+1}{(x-1)^2}$ $= \frac{(2k+3)(x-1)^2 - k - 1}{(x-1)^2}$ $= \frac{(2k+3)x^2 - 2(2k+3)x + k + 2}{(x-1)^2}$ At turning points: $(2k+3)x^2 - 2(2k+3)x + (k+2) = 0$ Since curve has 2 distinct turning points, the equation $(2k+3)x^2 - 2(2k+3)x + (k+2) = 0$ has 2 distinct real roots $\Rightarrow \quad 4(2k+3)^2 - 4(2k+3)(k+2) > 0$ $\Rightarrow \quad 2k^2 + 5k + 3 > 0$ $\Rightarrow \quad (2k+3)(k+1) > 0$ $\therefore k < -\frac{3}{2}$ or $k > -1$	[B1] [M1] correct inequality [A1] correct coefficients [A1]

2 **(a)** Differentiate $\frac{e^x - 1}{e^{2x}}$. [2]

(b) Find $\int (2x\sqrt{2x+1} + \sqrt{2x+1})\, \mathrm{d}x$. [3]

2(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\frac{e^x - 1}{e^{2x}} = \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{e^x}{e^{2x}} - \frac{1}{e^{2x}}\right]$ $= \frac{\mathrm{d}}{\mathrm{d}x}\left[e^{-x} - e^{-2x}\right]$ $= -e^{-x} + 2e^{-2x}$ $= 2e^{-2x} - e^{-x}$	[B1] [B1]
2(b)	$\int 2x\sqrt{2x+1} + \sqrt{2x+1}\, \mathrm{d}x = \int (2x+1)\sqrt{2x+1}\, \mathrm{d}x$ $= \int (2x+1)^{\frac{3}{2}}\, \mathrm{d}x$ $= \frac{2}{5} \cdot \frac{(2x+1)^{\frac{5}{2}}}{2} + c$ $= \frac{(2x+1)^{\frac{5}{2}}}{5} + c$	[B1] [M1] [A1]

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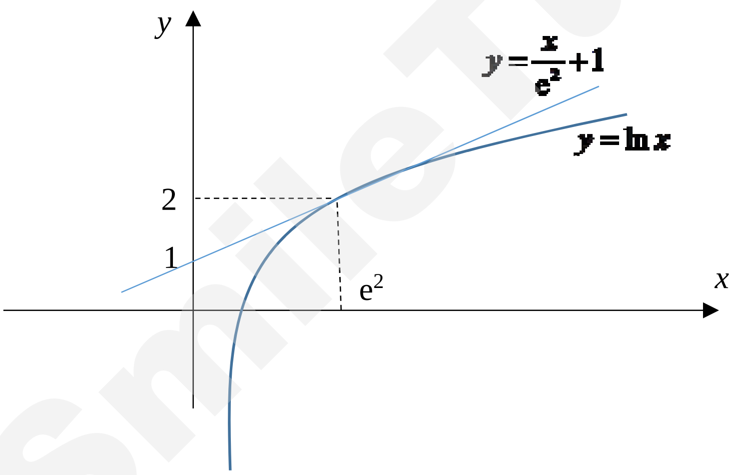
3 The curve C has equation $y = \ln x$, $x > 0$.

(i) Find the equation of the tangent to C at the point where $x = \mathrm{e}^2$, leaving your answer in terms of e. [3]

(ii) Sketch the graph of C and the tangent to C at the point where $x = \mathrm{e}^2$ on the same diagram, stating the coordinates of intersection with the axes and the equation of any asymptote(s). [2]

(iii) By using the result $\int \ln x \, \mathrm{d}x = x\ln x - x + c$, where c is an arbitrary constant, find the area of the region bounded by C, the tangent to C at the point where $x = \mathrm{e}^2$, the x-axis and the y-axis. Give your answer in terms of e. [3]

3(i)	$y = \ln x \quad \Rightarrow \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x}$ When $x = \mathrm{e}^2$, $y = \ln \mathrm{e}^2 = 2$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\mathrm{e}^2}$ Equation of tangent at the point where $x = \mathrm{e}^2$: $y - 2 = \frac{1}{\mathrm{e}^2}(x - \mathrm{e}^2)$ i.e. $y = \frac{x}{\mathrm{e}^2} + 1$	[B1] [M1] [A1]
3(ii)		[B1] Graph of $y = \ln x$, including x-intercept and vertical asymptote $x = 0$ [B1] Graph of $y = \frac{x}{\mathrm{e}^2} + 1$, including y-intercept and intersection with the graph of $y = \ln x$
3(iii)	Required area $= \frac{1}{2}(1+2)\mathrm{e}^2 - \int_1^{\mathrm{e}^2} \ln x \, \mathrm{d}x$ $= \frac{1}{2}(1+2)\mathrm{e}^2 - \left[x\ln x - x\right]_1^{\mathrm{e}^2}$ $= \frac{1}{2}(1+2)\mathrm{e}^2 - \left[\mathrm{e}^2 \ln \mathrm{e}^2 - \mathrm{e}^2 + 1\right]$ $= 1 + \frac{1}{2}\mathrm{e}^2$	[B1] $\int_1^{\mathrm{e}^2} \ln x \, \mathrm{d}x$ [M1] formulate area [A1]

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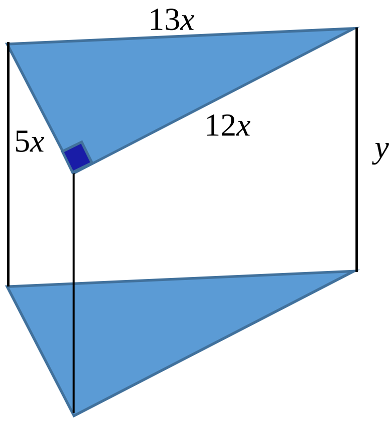
4 A prism with a cross-section in the shape of a right-angled triangle has dimensions (in cm) as shown in the diagram below.



The volume of the prism is 7200 cm^3. Show that the surface area of the prism is given by

$$S = \frac{7200}{x} + 60x^2 \text{ cm}^2.$$ [3]

Without using a calculator, find in surd form the value of x that gives a stationary value of S. Hence state, with a reason, whether S is a maximum or minimum. [4]

It is also given that x is decreasing at 0.5 cm/s, find the rate at which the surface area is decreasing when $x = 5$ cm. [2]

4

Given volume of prism = 7200

We have $\frac{1}{2}(5x)(12x)y = 7200 \quad \Rightarrow \quad y = \frac{240}{x^2}$ [B1]

Surface area of prism

$$S = \frac{1}{2}(5x)(12x) \times 2 + 12xy + 13xy + 5xy$$ [M1][A1]

$$= 60x^2 + 30xy$$

$$= 60x^2 + 30x\left(\frac{240}{x^2}\right)$$

$$= 60x^2 + \frac{7200}{x}$$

$$\frac{dS}{dx} = \frac{-7200}{x^2} + 120x = \frac{120x^3 - 7200}{x^2}$$ [B1]

At minimum S, $\frac{dS}{dx} = 0 \quad \Rightarrow \quad \frac{120x^3 - 7200}{x^2} = 0$ [M1]

$$\Rightarrow \quad 120x^3 - 7200 = 0$$

$$\Rightarrow \quad x^3 = 60$$

$\therefore \quad x = \sqrt[3]{60}$ [A1] for surd form only

Using First Derivative Test:

	$\sqrt[3]{60}^-$	$\sqrt[3]{60}$	$\sqrt[3]{60}^+$
$\frac{dS}{dx}$	-ve	0	+ve

S is minimum at $x = \sqrt[3]{60}$ [B1]

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	Given that x is decreasing at 0.5 cm/s, when $x=5$ $\frac{dx}{dt}=-0.5$ $\frac{dS}{dt}=\frac{dS}{dx}\times\frac{dx}{dx}=\frac{120(5)^3-7200}{5^2}\times -0.5=-156$	[M1]
	Therefore, surface area is decreasing at $156\text{ cm}^2/\text{s}$	[A1]

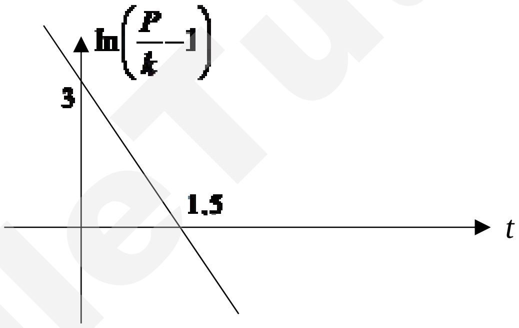
5 The profit P (in thousands dollars) of a company after the start of a promotion "Clearance Sale" can be modelled by the equation

$$P=k(1+be^{-rt}),$$

where t is the number of days elapsed since the start of the promotion and b, r and k are positive constants.

(i) Express $\ln\left(\frac{P}{k}-1\right)$ in terms of b, r and t [1]

(ii) The graph of $\ln\left(\frac{P}{k}-1\right)$ against t is given below.



By using the graph above, show that $b=e^3$ and $r=2$. [3]

(iii) Using differentiation, show that $\frac{dP}{dt}<0$ for $t\geq 0$. Hence explain why the maximum profit occurs at t = 0. Given that the maximum profit is $42171, find the value of k, correct to nearest integer value. [5]

The "Clearance Sale" ends after a week and another promotion "Happy Sale" takes place immediately after. The "Happy Sale" lasts for 3 weeks and the profit during "Happy Sale" can be modelled by the equation

$$P=a-\frac{1}{6}(t-14)^2,\quad \text{for } 7\leq t\leq 28.$$

(iv) Given that the company's maximum profit during "Happy Sale" is $18000, find the value of a. [2]

(v) Find the total profit of the company in 4 weeks, correct to the nearest dollars. [3]

5(i)	$P=k(1+be^{-rt}) \;\Rightarrow\; \frac{P}{k}-1=be^{-rt}$	

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	Then, $\ln\left(\frac{P}{k}-1\right)=\ln(be^{-rt})=\ln b-rt$	[B1]
5(ii)	From graph, y-intercept = 3 $\Rightarrow \ln b=3 \therefore b=\mathrm{e}^3$ Gradient of line $=\frac{-3}{1.5}=-2 \Rightarrow -r=-2 \therefore r=2$	[M1][A1] [M1][A1]
5(iii)	$\frac{\mathrm{d}}{\mathrm{d}t}P=\frac{\mathrm{d}}{\mathrm{d}t}k(1+\mathrm{e}^3\mathrm{e}^{-2t})=\frac{\mathrm{d}}{\mathrm{d}t}k(1+\mathrm{e}^3\mathrm{e}^{-2t})=k(-2\mathrm{e}^{3-2t})$ Since $\mathrm{e}^{3-2t}>0$ for $t\geq 0$, then $-2k(\mathrm{e}^{3-2t})<0$ where k is a constant i.e. $\frac{\mathrm{d}P}{\mathrm{d}t}<0$ for $t\geq 0$ P is decreasing for $t\geq 0$ Hence, P is maximum at $t=0$. Maximum profit occurs at $t=0$. Maximum profit = 42171 $\Rightarrow$ $P=\frac{42171}{1000}$ $k(1+\mathrm{e}^{3-2(0)})=42.171 \Rightarrow k\approx 1.999996498$ $\therefore k=2$	[B1] [B1] for $\frac{\mathrm{d}P}{\mathrm{d}t}<0$ [M1] [A1]
5(iv)	Maximum profit occurs at $t=14$, $\frac{18000}{1000}=a-\frac{1}{6}(14-14)^2$ $a=18$	[M1] [A1]
5(v)	Total profit $=\int_0^7 2(1+\mathrm{e}^{3-2t})\,\mathrm{d}t+\int_7^{28}\left(18-\frac{1}{6}(t-14)^2\right)\mathrm{d}t$ ≈ 240.5855202 Total profit of company in 4 weeks is \$240586	[B1] [B1] [B1]

Section B: Statistics [60 marks]

6 Find the number of 6-letter passwords that can be formed using the letters from the word SINGAPORE if

(i) repetitions of letters are not allowed. [1]

(ii) at least two vowels must be chosen and repetition of letters are not allowed. [2]

(iii) three distinct vowels and three distinct consonants are chosen, and vowels and consonants must alternate? [3]

6(i)	No. of ways $=\binom{9}{6}\times 6!=60480$	[B1] for answer
6(ii)	No. of ways $=60480-\binom{4}{1}\times\binom{5}{5}\times 6!=57600$	[M1] for complement method [A1]
6(iii)	No. of ways $=\binom{4}{3}\times\binom{5}{3}\times 3!\times 3!\times 2=2880$	[B1] for choosing 3 vowels and 3 consonants [B1] for 2 cases of alternating [B1] for answer

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7 A fixed number, n, of taxis entering VICOM is observed and the number of those taxis that fails inspection is denoted by X.

(i) State, in context, two assumptions needed for X to be well modelled by a binomial distribution. [2]

Assume now that X has the distribution $\mathrm{B}(n,\ p)$.

(ii) Given that $n = 20$ and $p = 0.15$, find $\mathrm{P}(X = 2 \text{ or } 3)$. [2]

(iii) Given that $n = 10$ and $p = 0.3$, find $\mathrm{P}(3 \le X < 6)$. [2]

(iv) Given that $n = 3$ and $\mathrm{P}(X \le 1) = 0.5$, show that $4p^3 - 6p^2 + 1 = 0$. Hence find the value of p. [3]

7(i)	• Probability that a taxi fails inspection is constant for all the taxis • Whether a taxi fails inspection or not is independent of another taxi	[B1] [B1]
7(ii)	$X \sim \mathrm{B}(20,\ 0.15)$ $\mathrm{P}(X = 2 \text{ or } 3) = \mathrm{P}(X = 2) + \mathrm{P}(X = 3) = 0.47216 = 0.472$	[M1][A1]
7(iii)	$X \sim \mathrm{B}(10,\ 0.3)$ $\mathrm{P}(3 \le X < 6) = \mathrm{P}(X \le 5) - \mathrm{P}(X \le 2) = 0.56986 = 0.570$	[B1] [B1]
7(iv)	$X \sim \mathrm{B}(3,\ p)$ $\mathrm{P}(X \le 1) = 0.5$ $\Rightarrow \quad (1-p)^3 + 3p(1-p)^2 = 0.5$ $\Rightarrow \quad 1 - 3p + 3p^2 - p^3 + 3p - 6p^2 + 3p^3 = 0.5$ $\Rightarrow \quad 2p^3 - 3p^2 + 1 = 0.5$ $\Rightarrow \quad 4p^3 - 6p^2 + 1 = 0$ (shown) Using GC, $p = 0.5$ or 1.366 (rejected $\because p < 1$) or -0.366 (rejected $\because p > 0$)	[M1] for use of formula [A1] [A1]

8 The number of hours, x, spent daily on revision for mathematics and the marks, y, obtained for the mathematics year-end examination are recorded for 10 randomly selected students. The results are given in the following table.

x	1.3	2.1	1.1	2.3	2.7	1.2	3.2	3.4	3.0	2.5
y	68	74	64	76	75	66	85	81	86	75

(i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]

(ii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iii) Find the equation of the regression line of y on x, in the form $y = mx + c$, giving the values of m and c correct to 4 significant figures. Sketch this line on your scatter diagram. [2]

(iv) Use the equation of your regression line to estimate the marks spent by a student who spends 1.5 hours a day on revision for mathematics. Comment on the reliability of your estimate. [3]

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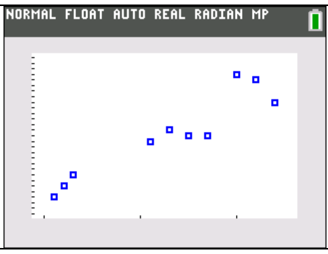
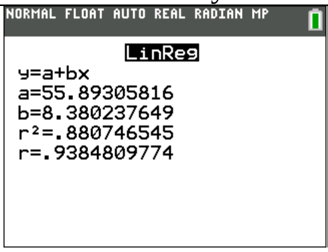
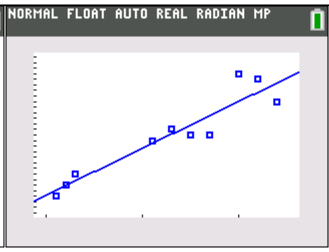
8(i)		[B1] for shape [B1] for axes correctly labelled
8(ii)	$r = 0.938$ (3 s.f.) This indicates a strong positive linear correlation between revision hours and the marks obtained, i.e. as the hours for revision increases, the marks obtained increases linearly.	[B1] [B1]
8(iii)	  $y = 8.3802x + 55.893$ $m = 8.380$ (4 s.f.) and $c = 55.89$ (4 s.f.)	[B1] for regression line [B1] for m and c
8(iv)	When $x = 1.5$, $y = 68.5$ (3 s.f.) The estimate is reliable since r-value is close to 1 and $x = 1.5$ is within the data range	[B1] [B1] [B1]

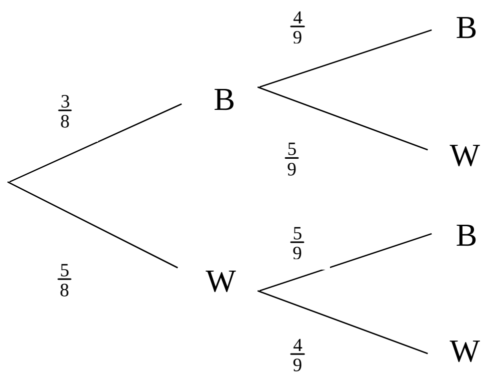
9 A bag contains 3 black balls and 5 white balls. Paul draws a ball at random from the bag and notes the colour. If a black ball is selected, Paul replaces it in the bag and adds an additional black ball into the bag. If a white ball is selected, Paul does not replace it in the bag but adds 2 black balls into the bag. Paul then draws a ball at random from the bag again and notes the colour.

Draw a tree diagram to represent the information of the two draws. [3]

(i) Show that the probability that Paul selects a black ball on both draws is $\frac{1}{6}$. [1]

(ii) Find the probability that Paul selects a white ball either on his first or second draw, or both. [2]

(iii) Find the probability that Paul selects a white ball on his first draw, given that he selects a black ball on his second draw. [3]

9		

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9(i)	P(selects B on both draws) $=\frac{3}{8}\times\frac{4}{9}=\frac{1}{6}$ (shown)	[B1]
9(ii)	P(selects W on first or second draw) $=\frac{5}{8}+\left(\frac{3}{8}\times\frac{5}{9}\right)=\frac{5}{6}$ or $1-\frac{3}{8}\times\frac{4}{9}=\frac{5}{6}$	[M1][A1]
9(iii)	P(selects W on first draw | selects B on second draw) $=\dfrac{\frac{5}{8}\times\frac{5}{9}}{\frac{3}{8}\times\frac{4}{9}+\frac{5}{8}\times\frac{5}{9}}$ $=\frac{25}{37}$	[M1][A1] [A1]

10 A company claims that their electric-powered V1 cars is designed to travel a mean distance of 500 km on one full charge. To test this claim, a random sample of 80 V1 cars is taken and the distance, x km, travelled on one full charge are summarised by

$$\sum(x-500)=46 \quad \text{and} \quad \sum(x-500)^2=460.$$

(i) Find the unbiased estimates of the population mean and variance. [3]

(ii) Suggest a reason why, in this context, the given data is summarised in terms of $(x-500)$ instead of x. [1]

(iii) Test, at the 5% level of significance, whether the company's claim is valid. [4]

(iv) State, with a reason, whether it is necessary to assume a normal distribution for the test to be valid. [1]

The company introduces a new electric-powered V2 car and claims that the V2 can travel more than the mean distance of the V1 on one full charge. The new population variance of the V2 cars is known to be 13 km^2. A random sample of 50 V2 cars is taken.

(v) Find the set of values within which the mean distance of this sample must lie, such that there is not enough evidence from the sample to support the company's claim at the 1% level of significance. [4]

10(i)	$\bar{x}=\frac{46}{80}+500=500.575$ $s^2=\frac{1}{79}\left[460-\frac{46^2}{80}\right]=5.4879$	[B1] [B2]
10(ii)	Values may be too large	[B1]
10(iii)	$H_0: \mu=500$ $H_1: \mu\neq 500$ Level of significance: 5% Test statistic: $Z=\dfrac{\bar{X}-\mu}{\frac{s}{\sqrt{n}}}\sim N(0,1)$ approx. by CLT Under H_0 and using GC, we have $p=0.0281$ (3 s.f.)	[B1] for hypotheses [B1] for application of CLT and s instead of σ [B1] for p-value

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	Since $p < 0.05$, we reject H_0. There is sufficient evidence at the 5% level of significance to conclude that the company's claim is invalid.	[B1] for conclusion
10(iv)	Not necessary since n is large and $\overline{X}$ is still approximately normal by CLT	[B1]
10(v)	$H_0: \mu = 500$ $H_1: \mu > 500$ Level of significance: 1% Test statistic: $Z = \dfrac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$ approx. by CLT Since company's claim is not accepted. H_0 is not rejected $\Rightarrow \dfrac{\overline{x} - 500}{\sqrt{\frac{13}{50}}} < 2.3263$ $\Rightarrow \overline{x} < 501.18$	[B1] for hypotheses [B1] for test statistic with σ and CLT seen or implied [M1] [A1]

11 Three friends Anand, Beng and Charlie goes racing regularly at the Temasek Circuit, which offers a standard route on Track 1 or a more challenging route on Track 2. The time taken, in minutes, taken by them to complete a round on Tracks 1 and 2 have independent normal distributions with means and standard deviations as shown in the following table.

	Track	Mean	Standard deviation
Anand	1	3.15	0.21
Beng	1	3.18	0.10
Charlie	2	4.22	0.15

(i) Find the probability that Beng takes less than 3.15 minutes to complete a randomly chosen round on Track 1. [1]

(ii) Find the probability that Anand and Beng each takes less than 3.15 minutes to complete a randomly chosen round on Track 1. [2]

(iii) Find the probability that Beng is faster than Anand in completing a randomly chosen round on Track 1. [3]

(iv) Find the probability that out of 8 complete rounds on Track 2, there are more than 4 rounds in which Charlie takes less than 4.20 minutes to complete. [3]

Temasek Circuit charges customers \$18 per minute on Track 1 and \$22 per minute on Track 2. On a particular day, Beng completes 10 rounds on Track 1 and Charlie completes 8 rounds on Track 2.

(v) Find the probability that Beng and Charlie pay a total of less than \$1300. [5]

11(i)	Let X and Y be the time taken by Anand and Beng to complete a round on Track 1 $X \sim N(3.15,\ 0.21^2)$ and $Y \sim N(3.18,\ 0.10^2)$ $P(Y < 3.15) = 0.38208 = 0.382$ (3 s.f.)	[B1]
11(ii)	Required probability $= 0.5 \times 0.38208 = 0.19104 = 0.191$	[M1][A1]

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11(iii)	$Y-X \sim \mathrm{N}(3.18-3.15,\ 0.10^2+0.21^2)$ i.e. $Y-X \sim \mathrm{N}(0.03,\ 0.0541)$ $\mathrm{P}(Y-X<0)=0.44868=0.449$ (3 s.f.)	[B2] for $\mathrm{E}(Y-X)$ and $\mathrm{Var}(Y-X)$ or equivalent [B1]
11(iv)	Let W be the time taken by Charlie to complete a round on Track 2 $W \sim \mathrm{N}(4.22,\ 0.15^2)$ $\mathrm{P}(W<4.2)=0.44696$ Let S be the number of rounds which Charlie takes less than 4.20 minutes to complete $S \sim \mathrm{B}(8,\ 0.44696)$ $\mathrm{P}(S>4)=1-\mathrm{P}(S\le 4)=0.25460=0.255$ (3 s.f.)	[B1] for 5 s.f. [M1][A1]
11(v)	Let C be the total cost $C=18(Y_1+Y_2+\ldots+Y_{10})+22(W_1+W_2+\ldots+W_8)$ $\mathrm{E}(C)=(18\times10\times3.18)+(22\times8\times4.22)=1315.12$ $\mathrm{Var}(C)=(18^2\times10\times0.10^2)+(22^2\times8\times0.15^2)=119.52$ $\therefore\quad C \sim \mathrm{N}(1315.12,\ 119.52)$ $\mathrm{P}(C<1300)=0.083327=0.0833$ (3 s.f.)	[B1] for mean and variance of time [B1] for $a^2\mathrm{Var}(Y)$ [B2] for $\mathrm{E}(C)$ and $\mathrm{Var}(C)$ [B1]

End of Paper

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1) $x =$ Number of 6-blade packages sold $y =$ Number of 12-blade packages sold $z =$ Number of 24-blade packages sold $x+y+z=12$ ------ (1) $6x+12y+24z=162$ ------ (2) $2x+3y+4z=35$ ------ (3) $x=5 \quad y=3 \quad z=4$	
2) $y=\dfrac{2}{1+x^2}-1$ $\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{-4x}{(1+x^2)^2}$ The coordinates of points A, B and C are $A(0,1)$, $B(1,0)$ and $C(-1,0)$ At B, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-1$ Thus equation of tangent at B is $y=-x+1$. When $x=0$, $y=-0+1$ = 1. Thus the tangent at B passes through $A(0,1)$. At C, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=1$ Thus equation of tangent at C is $y=x+1$. When $x=0$, $y=0+1$ = 1. Thus the tangent at C passes through $A(0,1)$.	
3(a)(i) $\dfrac{\mathrm{d}}{\mathrm{d}x}\ln\left(\dfrac{5x+2}{4x^2}\right)$ $=\dfrac{\mathrm{d}}{\mathrm{d}x}\left[\ln(5x+2)-\ln 4x^2\right]$ $=\dfrac{5}{5x+2}-\dfrac{2}{x}$	
(ii) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left[\mathrm{e}^{\sqrt{x}}\right]=\dfrac{\mathrm{e}^{\sqrt{x}}}{2\sqrt{x}}$	

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(b)

$$\int_1^4 \frac{e^{\sqrt{x}+\ln 3}}{\sqrt{x}}\,dx = \int_1^4 \frac{e^{\sqrt{x}}e^{\ln 3}}{\sqrt{x}}\,dx$$

$$= \int_1^4 \frac{3e^{\sqrt{x}}}{\sqrt{x}}\,dx$$

$$= 6\int_1^4 \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx$$

$$= 6\left[e^{\sqrt{x}}\right]_1^4$$

$$= 6e^{\sqrt{4}} - 6e$$

$$= 6e(e-1)$$

4(i)

$$\frac{dV}{dt} = 0.0012e^{0.24t} - 0.0594e^{-0.12t}$$

$$0.0012e^{0.24t} - 0.0594e^{-0.12t} = 0$$

$$0.0012e^{0.24t} = 0.0594e^{-0.12t}$$

$$\frac{e^{0.24t}}{e^{-0.12t}} = \frac{0.0594}{0.0012}$$

$$e^{0.36t} = 49.5$$

$$0.36t = \ln 49.5$$

$$t = 10.839$$

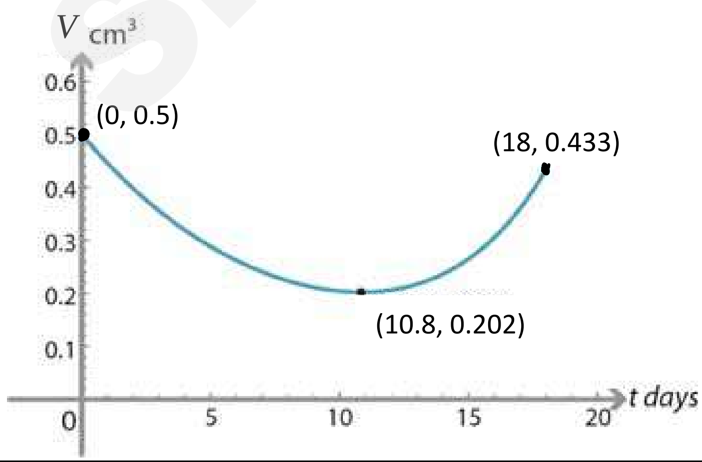
$$V = 0.202$$

sign of $\frac{dV}{dt}$

$-$ $+$

10.839

(ii)



(iii) From GC, $\frac{dV}{dt} = -0.0286$

Rate of decrease = 0.0286

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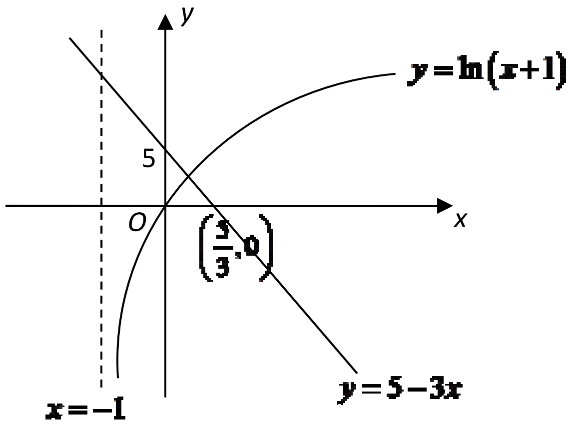

5(a)(i)

$6x+\ln\left(x+1\right)^2<10$

$6x+2\ln\left(x+1\right)<10$

$3x+\ln\left(x+1\right)<5 \Rightarrow \ln(x+1)<5-3x$

Point of intersection is at $x=1.3779$ (3sf).
Hence, from graph, solution of inequality: $-1<x<1.38$

(ii)
$\int_0^{1.3779}\left(5-3x-\ln\left(x+1\right)\right)\,\mathrm{d}x+\int_{1.3779}^{3}\left(\ln\left(x+1\right)-\left(5-3x\right)\right)\,\mathrm{d}x$

≈ 7.76459

≈ 7.76

(b)

$4x^2+\left(5-3x\right)^2=4k^2$

$13x^2-30x+25-4k^2=0$

$b^2-4ac>0$

$30^2-4(13)\left(25-4k^2\right)>0$

$208k^2-400>0$

$13k^2-25>0$

$\left(\sqrt{13}k+5\right)\left(\sqrt{13}k-5\right)>0$

$k<-\frac{5}{\sqrt{13}}$ or $k>\frac{5}{\sqrt{13}}$

$\left\{k\in\mathbb{R}:k<-\frac{5}{\sqrt{13}} \text{ or } k>\frac{5}{\sqrt{13}}\right\}$

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6(i)

Total number of ways of selecting 4 chocolates
$= {}^{14}C_4 = 1001$

Number of ways of selecting 2 soft centres (and 2 hard)
$= {}^{8}C_2 \times {}^{6}C_2 = 420$

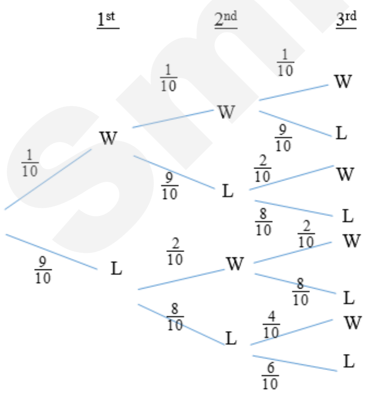
$$\text{P}(\text{ 2 soft}) = \frac{420}{1001}$$
$$= \frac{60}{143} \quad \text{or} \quad 0.420$$

(ii)

Number of ways of selecting at most 3 soft centres
$= 1001 - {}^{8}C_4 \times {}^{6}C_0 = 420$
$= 931$

$$\text{P}(\text{at most 3 soft}) = \frac{931}{1001}$$
$$= \frac{133}{143} \quad \text{or} \quad 0.930$$

7)



$\text{P}(A' \cap B)$ is the probability of the player winning the first 2 stages and lose the 3rd stage.

$$\text{P}(A' \cap B) = \frac{1}{10} \times \frac{1}{10} \times \frac{9}{10} = \frac{9}{1000}$$

$$\text{P}(A') = \frac{9 + 72 + 144 + 432}{1000} = \frac{657}{1000}$$

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P(B) = $\dfrac{9+18+36}{1000} = \dfrac{63}{1000} = 0.063$

P(A')× P(B) = 0.0414 ≠ P($A' \cap B$)

$\therefore$ A' and B are not independent

$\Rightarrow$ A and B are not independent.

8) Let X g be the mass of a cabbage.

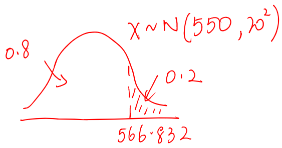
$X \sim N(550, 20^2)$

(i) $P(X > 575) = 0.10565 \approx 0.106$

(ii) $P(X > m) \leq 0.2$

$m \geq 566.832$

Smallest mass is 566.8 g.



(iii) Let \$$C$ be the cost of a cabbage. $C = \dfrac{0.60}{100} X$

$$E(C) = \frac{0.60}{100} \times 550 = 3.3$$

$$\text{Var}(C) = \left(\frac{0.60}{100}\right)^2 \times 20^2 = 0.0144$$

$C \sim N(3.3, 0.0144)$

$P(C < 3.20) = 0.20233 \approx 0.202$

(iv) Let Y g be the mass of half a cabbage. $Y = \dfrac{1}{2} X$

$$Y \sim N\left(\frac{550}{2}, \frac{20^2}{4}\right)$$

$$Y_1 + Y_2 - X \sim N\left(\frac{550}{2} + \frac{550}{2} - 550, \frac{20^2}{4} + \frac{20^2}{4} + 20^2\right)$$

$$Y_1 + Y_2 - X \sim N(0, 600)$$

$P(0 < Y_1 + Y_2 - X < 50) = 0.47939 \approx 0.479$

9) Not every student will have equal chance of being selected as those who do not go to the canteen during lunch break will have no chance of being interviewed.

(i) The probability of a student supporting candidate A is constant for all the students. A student supporting candidate A is independent of whether other students will support candidate A.

(ii) $X \sim B\left(30, \dfrac{4}{9}\right)$

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x	$P(X = x)$
12	0.13058
13	0.14465
14	0.14051

$\therefore$ The most likely number of students who support candidate A is 13.

(iii) mean $= 30 \times \frac{4}{9} = \frac{40}{3}$

standard deviation $= \sqrt{30 \times \frac{4}{9} \times \left(1 - \frac{4}{9}\right)} = 2.7217$

$$P\left(\frac{40}{3} - 2.7217 < X < \frac{40}{3} + 2.7217\right)$$

$= P\left(11 \le X \le 16\right) = P(X \le 16) - P(X \le 10) = 0.72867 \approx 0.729$

Let Y be number of students out of 30 who support candidate B.

$Y \sim B\left(30, \frac{p}{100}\right)$

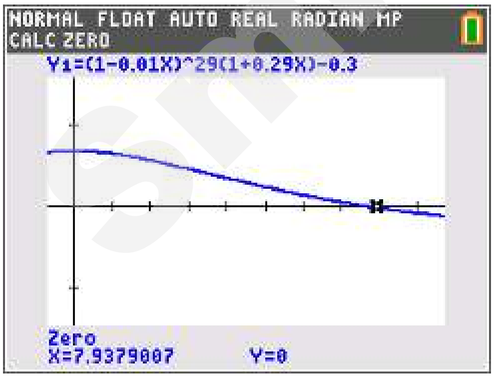
$P\left(Y \le 1\right) = P\left(Y = 0\right) + P\left(Y = 1\right)$

$= \left(1 - 0.01p\right)^{30} + 30 \times 0.01p \times (1 - 0.01p)^{29}$

$= (1 - 0.01p)^{29}(1 - 0.01p + 0.3p)$

$= (1 - 0.01p)^{29}(1 + 0.29p)$

$= 0.3$



$p \approx 7.94$ (correct to 2 d.p.)

10) Let μ kg be the population mean yield of an apple tree.

(i) $H_0 : \mu = 98.5$

$H_1 : \mu > 98.5$

Level of significance : 5%

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Test Statistic: When H_0 is true, $Z = \dfrac{\overline{X} - 98.5}{\dfrac{S}{\sqrt{25}}} \sim N(0, 1)$ approximately

Computation : $\overline{x} = 99.7$, $\quad s^2 = \dfrac{n}{n-1}\left(\dfrac{\sum (x-\overline{x})^2}{n}\right) = \dfrac{178}{24}$

p – value = 0.01379154

Conclusion : Since p – value = 0.0138 < 0.05, H_0 is rejected at 5% level of significance. There is sufficient evidence to conclude that the farmer's claim should not be rejected.

(ii) It is not necessary as the sample size is 25 which is sufficiently large. Central Limit Theorem can be applied and $\overline{X}$ is approximately normal.

(iii) 5% level of significance means that there is a 0.05 probability that the test will conclude that there is an increase in the mean yield of the apple trees when the mean yield is actually 98.5 kg.

(iv) $H_0 : \mu = 102$

$H_1 : \mu \neq 102$

Level of significance : 5%

Rejection region: $z \le -1.95996$ or $z \ge 1.95996$

Conclusion : H_0 is not rejected.

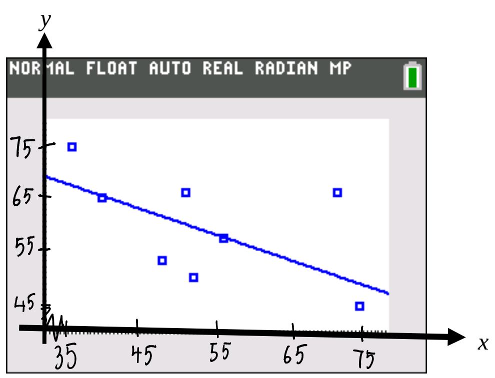
$z = \dfrac{\overline{x} - 102}{\sqrt{\dfrac{7.49}{20}}} \Rightarrow -1.95996 < \dfrac{\overline{x} - 102}{\sqrt{\dfrac{7.49}{20}}} < 1.95996$

$\therefore \{ \overline{x} \in \mathbb{R} : 101 < \overline{x} < 103 \}$

(v) We need to assume that the population variance remained unchanged.

(vi) Since H_0 is not rejected, p – value > 0.05 $\Rightarrow$ p – value > 0.01. Hence H_0 will also not be rejected at 1% level of significance.

11)(i)



(i) $r = -0.53382$

(ii)	The estimate is unreliable as from the scatter diagram, the points do not seem to lie close to straight line and r is not close to – 1.	
(iii)	Let the score be p. $\bar{x}=\dfrac{436}{8}$, $\bar{y}=\dfrac{407+p}{8}$ $\dfrac{407+p}{8}=88.722-0.57976\left(\dfrac{436}{8}\right)$ $\therefore$ p = 50 (nearest integer)	
(iv)	From GC, x = 121.07 – 1.1653y When y = 75, x = 34 (nearest integer) The estimate is not reliable as y = 75 is outside the given data range $45 \le y \le 73$.	

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YISHUN JUNIOR COLLEGE
2017 JC2 Preliminary Examination

MATHEMATICS **8864/01**
HIGHER 1

28 AUGUST 2017
MONDAY 0800h – 1100h

Additional materials :
Answer paper
List of Formulae (MF15)

TIME 3 hours

READ THESE INSTRUCTIONS FIRST

Write your CTG and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise. Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, write down the question number of the questions attempted, model of calculator used on the spaces provided on the cover page. Tie your cover page on top of the answer scripts before submission.
The number of marks is given in brackets [ ] at the end of each question or part question.

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Section A: Pure Mathematics [35 marks]

1 **(i)** Differentiate with respect to *x*,

(a) $5\ln\left(1-3x^2\right)$, [2]

(b) $\dfrac{1}{(2x+3)^2}$. [2]

(ii) Use a non-calculator method to find $\displaystyle\int_1^3 x^3\left(\frac{1}{x}-1\right)^2 \,\mathrm{d}x$. [4]

2 Find the range of values of *k* for which $kx^2+4k-kx-2x$ is always negative. [4]

3 Research has found that the concentration *R* of a drug in the bloodstream, in micrograms per litre, decreases according to the function $R=366e^{-0.0998t}$, $t\geq 0$, where *t* is measured in minutes after the drug is administered.

(i) Sketch the graph of $R=366e^{-0.0998t}$ for $t\geq 0$. [2]

(ii) Find the rate of decrease of *R* at the instant when $t = 20$. [2]

(iii) How long, to the nearest minute, will the concentration of the drug in a patient be 40 micrograms per litre? [2]

The company that manufactures the drug estimates that the profit, in millions of dollars, from the sale of the drug over the years can be modelled by the equation

$$P=-0.03x^3+0.1x^2+x-0.1,\quad 0\leq x\leq 5,$$

where *x* is the number of bottles of the drug, in 10 thousands, produced.

Use differentiation to find the number of bottles that corresponds to the maximum profit, giving your answer correct to 5 significant figures. [5]

4 Sketch the graph of the curve *C* with equation $y=\dfrac{1}{x-2}+1$, stating the coordinates of any points of intersection with the axes and the equations of any asymptotes. [2]

Without the use of a calculator, find the area of the region bounded by *C*, the line $y=x+3$ and the *y*-axis. [6]

5 The curve *C* has equation $y=x^3-2\mathrm{e}^{-x}$. Without using a calculator, find the equation of the tangent to *C* at the point where $x=1$, giving your answer in the form $y=ax+b$, where *a* and *b* are constants. [4]

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Section B: Statistics [60 marks]

6 **(a)** A game is played with a deck of n cards, each distinctly numbered from 1 to n, where n is an even number. A player randomly picks a card from the deck. Events A and B are defined as follows:

A: The card shows an even number.

B: The card shows a multiple of 17.

It is given that $\mathrm{P}(A' \cap B') = \frac{8}{17}$ and $\mathrm{P}(A \cap B) = \frac{1}{34}$.

(i) Show that $\mathrm{P}(B) = \frac{1}{17}$ and hence state the smallest value of n. [5]

(ii) Find $\mathrm{P}(A \mid B)$. [2]

(b) The probability that a train service breaks down is $\frac{1}{15}$. When the train service functions normally, 5% of the people travelling to work by train are late. It is also found that 90% of the people travelling to work by train are punctual.

(i) Draw a tree diagram to represent this situation, showing all possible outcomes and the associated probabilities. [3]

(ii) Given that a randomly chosen person travelling to work by train is late, find the probability that the train service functions normally. [2]

7 In a survey on usage of internet security software conducted with a large number of smartphone users, it was found that 37% of them had anti-virus software A installed on their smartphones, 56% had anti-virus software B installed and 7% did not have any of them installed.

(i) A random sample of twenty smartphone users was chosen. Find the probability that at least eight of the users had anti-virus software A installed on their smartphones. [2]

(ii) Fifty such samples of twenty smartphone users was randomly chosen. Using a suitable approximation, find the probability that there were fewer than thirty samples with at least eight users with anti-virus software A installed on their smartphones. [4]

(iii) Another random sample of n smartphone users was chosen. Given that the probability that at most one user did not have any anti-virus software installed was less than 0.5, show that n would satisfy the inequality $0.93^{n-1}(0.93 + 0.07n) < 0.5$. Hence find the least value of n. [3]

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8 An online blog shop owner has compiled a list of 3000 customers. A sample of 60 customers is chosen to take part in a survey. Describe how the sample could be chosen using systematic sampling. [2]

The purpose of the survey is to find out the customers' opinions about the products sold at the blog shop. Give a reason why a stratified sample might be preferable in this context. [1]

9 An electric power service company keeps records of the installation time of its electricity meters in the houses in a new estate. The time taken, X minutes, to install an electricity meter may be assumed to be normally distributed with mean 50 minutes and standard deviation 10 minutes.

(i) Two randomly chosen houses have their electricity meters installed by the company. Find the probability that each installation takes more than an hour. [2]

(ii) The company wishes to improve its processes such that at least 95% of the installations will take less than 60 minutes. Suppose the population standard deviation is still 10 minutes, find the maximum population mean time taken to install an electricity meter. [3]

Each month, the amount of electricity, measured in kilowatt-hours (kWh), used by a particular household in the estate is normally distributed with mean 522 kWh and standard deviation 26 kWh. The company charges households for electricity used at $0.21 per kWh.

(iii) Find the probability that, in a randomly chosen month, the electricity charge for the household is between $100 and $120. [2]

(iv) The household is billed every two months. Find the largest integral value of d such that the probability that a randomly chosen bill is at least $\$d$ is more than 0.90. State an assumption that is needed in your calculation. [4]

The company also installs gas meters in the houses in the estate. The time taken, Y minutes, to install a gas meter has mean 47 minutes and standard deviation 25 minutes.

(v) Explain why Y cannot be well modelled by a normal distribution. [1]

(vi) A random sample of 55 gas meter installations has been completed in the estate. Find the probability that the mean time taken to install a gas meter is more than 45 minutes. [2]

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10 The following table gives the median monthly household income from work, $\$h$, from each year, t, between 2011 to 2016 (inclusive).

Year, t	2011	2012	2013	2014	2015	2016
Median monthly household income from work, $\$h$	7037	7566	7872	8292	8666	8846

Source: http://www.singstat.gov.sg/statistics/browse-by-theme/household-income-tables

(i) Give a sketch of the scatter diagram for the data. [2]

(ii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iii) Find the values of $\bar{t}$ and $\bar{h}$ and mark the point $(\bar{t}, \bar{h})$ on your scatter diagram in part **(i)**. [2]

(iv) Find the equation of the regression line of h on t, in the form $h = mt + c$, giving the values of m and c correct to 2 decimal places. Sketch this line on your scatter diagram in part **(i)**. [2]

(v) Use a suitable regression line to estimate the year when the median monthly household income from work is \$9700. Comment on the reliability of the estimate obtained. [2]

11 A departmental store manager claims that the mean amount of time that customers spent in the shopping mall is 41 minutes. A random sample of 150 customers is taken and the time, x minutes, spent by each customer is noted. The results are summarized by

$$\sum x = 6386, \quad \sum x^2 = 277270.$$

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Test, at the 5% level of significance, whether the manager's claim is valid. [4]

(iii) State, with a reason, whether it is necessary to assume a normal distribution for the population for the test to be valid. [1]

(iv) Explain, in the context of the question, the meaning of 'at 5% level of significance'. [1]

After several rounds of publicity for the shopping mall, the publicity manager claims that the mean time that customers spent in the shopping mall has increased. The new population variance of the time spent may be assumed to be 49.3 minutes^2. A new random sample of 40 customers is chosen and the mean of this sample is k minutes.

(v) Find the set of values within which k must lie, such that there is not enough evidence from the sample to support the publicity manager's claim at the 10% level of significance. [4]

~ ~ End of paper ~ ~

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YISHUN JUNIOR COLLEG5E

Mathematics Department

PRELIM SOLUTION

Subject : JC2 H1 MATHEMATICS 8864 **Date** :

Qn	Solution
1(i)	**(a)** $\frac{\mathrm{d}}{\mathrm{d}x}\left(5\ln\left(1-3x^2\right)\right)=5\left(\frac{1}{1-3x^2}\right)\left(-6x\right)$ $=-\frac{30x}{1-3x^2}$ **(b)** $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{\left(2x+3\right)^2}\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left(2x+3\right)^{-2}$ $=-2(2x+3)^{-3}(2)$ $=-4(2x+3)^{-3}$
(ii)	$\int_1^3 x^3\left(\frac{1}{x}-1\right)^2\ \mathrm{d}x=\int_1^3\left(x-2x^2+x^3\right)\mathrm{d}x$ $=\left[\frac{x^2}{2}-\frac{2x^3}{3}+\frac{x^4}{4}\right]_1^3$ $=\left(\frac{3^2}{2}-\frac{2(3)^3}{3}+\frac{(3)^4}{4}\right)-\left(\frac{1^2}{2}-\frac{2(1)^3}{3}+\frac{(1)^4}{4}\right)$ $=\frac{27}{4}-\frac{1}{12}$ $=\frac{20}{3}$
2	$\left(-k-2\right)^2-4(k)(4k)<0$ and $k<0$ $k^2+4k+4-16k^2<0$ $-15k^2+4k+4<0$ $(5k+2)(3k-2)>0$ $k<-\frac{2}{5}$ or $k>\frac{2}{3}$ Since $k<0$, $\therefore\ k<-\frac{2}{5}$

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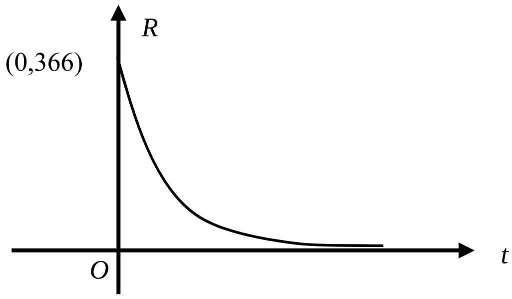
Qn	Solution

3 (i)



(ii)

From GC, when $t = 20$,

$$\frac{dR}{dt} = -4.96$$

The rate of decrease is 4.96 micrograms/litre per min

(iii)

$$40 = 366e^{-0.0998t}$$

$$e^{-0.0998t} = \frac{40}{366}$$

$$-0.0998t = \ln\left(\frac{40}{366}\right)$$

$$t = 22.18$$

$$t \approx 22\text{mins}$$

Alternative solution:
Draw graph of y = 40 and find intersection points.

$$P = -0.03x^3 + 0.1x^2 + x - 0.1$$

$$\frac{dP}{dx} = -0.09x^2 + 0.2x + 1$$

For maximum P, $\frac{dP}{dx}=0$

$$-0.09x^2 + 0.2x + 1 = 0$$

$$x = 4.624753 \quad (x > 0)$$

x	4.624753^-	4.624753	4.624753^+
$\frac{dP}{dx}$	+ve	0	−ve
slope			

Thus, P is maximum when the number of bottles is 46248.

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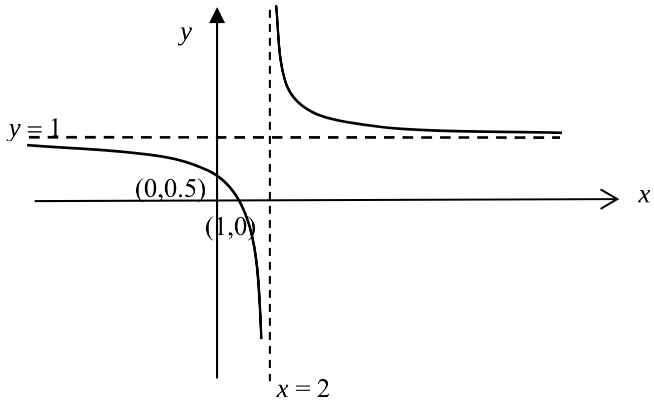
Qn	Solution
4	 $\frac{1}{x-2}+1=x+3$ $1+(x-2)=x^2+x-6$ $x^2=5$ $x=\pm\sqrt{5}$ Area of the region = $\int_{-\sqrt{5}}^{0}\left(x+3-\left(\frac{1}{x-2}+1\right)\right)\mathrm{d}x$ $=\left[\frac{x^2}{2}+2x-\ln\lvert x-2\rvert\right]_{-\sqrt{5}}^{0}$ $=-\ln 2-\left(\frac{5}{2}-2\sqrt{5}-\ln\lvert -\sqrt{5}-2\rvert\right)$ $=-\ln 2-\left(\frac{5}{2}-2\sqrt{5}-\ln\left(2+\sqrt{5}\right)\right)$ $=-\ln 2-\frac{5}{2}+2\sqrt{5}+\ln\left(2+\sqrt{5}\right)$
5	$y=x^3-2e^{-x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2+2e^{-x}$ When $x=1$, $\frac{\mathrm{d}y}{\mathrm{d}x}=3(1)^2+2e^{-1}$ and $y=1-2e^{-1}$ Equation of tangent: $y-\left(1-\frac{2}{e}\right)=\left(3+\frac{2}{e}\right)(x-1)$ $y=\left(3+\frac{2}{e}\right)x-2-\frac{4}{e}$

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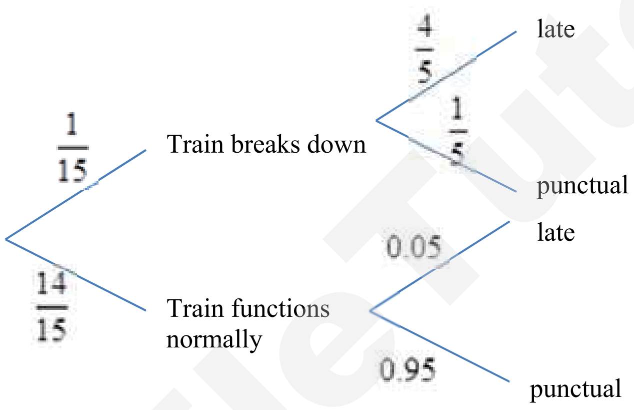
Qn	Solution
6(a)(i)	$P(A\cup B)=1-P(A'\cap B')$ $=\dfrac{9}{17}$ $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ $\dfrac{9}{17}=\dfrac{1}{2}+P(B)-\dfrac{1}{34}$ $\Rightarrow P(B)=\dfrac{1}{17}$ (Shown) Smallest value of n is 34.
(ii)	$P(A\mid B)=\dfrac{\frac{1}{34}}{\frac{1}{17}}=\dfrac{1}{2}$
(b)(i) (ii)	 P (train functions normally | late) $=\dfrac{\frac{14}{15}(0.05)}{0.1}$ ≈ 0.467
7(i)	Let X be the random variable ‘number of smartphone users with anti-virus software A installed on their smartphones out of 20 users’ $X\sim B(20,0.37)$ $P(X\geq 8)=1-P(X\leq 7)$ ≈ 0.47346 $=0.473$ (3 sig fig)
(ii)	Let W be the ‘number of samples with at least eight users with anti-virus software A installed on their smartphone out of 50 samples’ $W\sim B(50, 0.47346)$ Since n is large, $np=23.673>5$, $n(1-p)=26.327>5$ $W\sim N(23.673, 12.465)$ approx $P(W<30)\to P(W<29.5)$ using c.c. ≈ 0.95057 $=0.951$ (3 sig fig)

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Qn	Solution
(iii)	Let Y be the 'number of smartphone users who did not have any anti-virus software installed, out of n' $Y \sim \mathrm{B}(n, 0.07)$ $\mathrm{P}(Y \le 1) < 0.5$ $\mathrm{P}(Y = 0) + \mathrm{P}(Y = 1) < 0.5$ ${}^{n}C_0(0.07)^0(0.93)^n + {}^{n}C_1(0.07)(0.93)^{n-1} < 0.5$ $(0.93)^n + n(0.07)(0.93)^{n-1} < 0.5$ $(0.93)^{n-1}(0.93 + 0.07n) < 0.5$ (shown) Using GC, When $n = 23$, $(0.93)^{n-1}(0.93 + 0.07n) = 0.5146 > 0.5$ When $n = 24$, $(0.93)^{n-1}(0.93 + 0.07n) = 0.4918 < 0.5$ Therefore, least $n = 24$
8	Number the customers from 1 to 3000. $k = \dfrac{3000}{60} = 50$ Randomly choose the first customer from the first 50 customers. Thereafter, select every 50th customer until 60 customers are chosen. Using a stratified sample will take into consideration the different opinions from all the different strata (for example, age group), hence resulting in a sample which is more representative of the population.
9(i)	$X \sim N(50, 10^2)$ Required Prob = $\left[\mathrm{P}(X > 60)\right]^2 \approx (0.158655)^2$ $= 0.02517$ ≈ 0.0252 (3 s.f)
(ii)	Let μ be the population mean time taken (min) the company has to achieve $X \sim N(\mu, 10^2)$ $\mathrm{P}(X < 60) \ge 0.95$ $\mathrm{P}(Z < \dfrac{60-\mu}{10}) \ge 0.95$ $\dfrac{60-\mu}{10} \ge 1.64485$ $\mu \le 43.552$ Maximum $\mu = 43.5$
(iii)	Let W be the amount of electricity (kWh) used in a month by a household $W \sim N(522, 26^2)$ Total charge per month, $B = 0.21W \sim N(109.62, 29.8116)$ $P(100 < B < 120) = 0.932$ (3 s.f)
(iv)	$T = B_1 + B_2 \sim N(219.24, 59.6232)$ $P(T \ge d) > 0.9$ $1 - P(T < d) > 0.9$ $P(T < d) < 0.1$ $d < 209.344$

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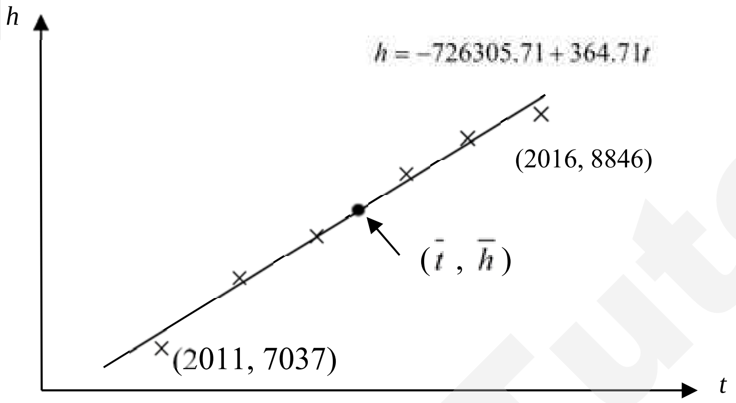
Qn	Solution
	Largest integral value of d is 209. Assume that the electricity used in each month is independent for a particular household.
(v)	Since $\mu - 3\sigma = 47 - 3(25) = -28 < 0$, Time taken to install a gas meter is impossible to be negative, Y is not well modelled by a normal distribution.
(vi)	Since sample size = 55 is large, $\bar{Y} = \frac{Y_1 + Y_2 + ... + Y_{55}}{55} \sim N\left(47, \frac{25^2}{55}\right)$ approx by CLT $P(\bar{Y} > 45) = 0.724$ (3 s.f.)
10(i)	
(ii)	$R \approx 0.992$ (3 s.f) There is a strong positive linear correlation between the median monthly household income from work and the year. As the year increases, the median monthly household income from work increases.
(iii)	$\bar{t} = 2013.5$, $\bar{h} = 8046.5$
(iv)	$h = -726305.71 + 364.71t$ (2 d.p.)
(v)	When $h = 9700$, $9700 = -726305.71 + 364.71t$ $t = 2018.057$ Year: 2018 The estimate is not reliable since the estimate is obtained via extrapolation.
11(i)	Unbiased estimate of the population mean, $\bar{x} = \frac{6386}{150} = 42.573 \approx 42.6$ (3s.f.) Unbiased estimate of the population variance, $s^2 = \frac{1}{149}\left[277270 - \frac{6386^2}{150}\right] = 36.219 \approx 36.2$ (3 s.f.)
(ii)	$H_0 : \mu = 41$ $H_1 : \mu \neq 41$ Test at 5% significance level Under H_0, the test statistic $Z = \frac{\bar{X} - \mu}{S / \sqrt{n}} \sim N(0,1)$ approx. by CLT, where $\mu = 41, s = \sqrt{36.219}, \bar{x} = 42.573, n = 150$.

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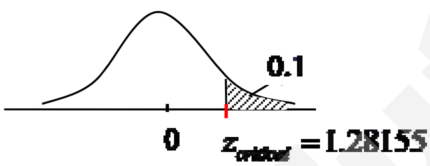
Qn	Solution
	By GC, *p*-value $=0.00137\left(3\text{ s.f.}\right)$. Since *p*-value < 0.05, we reject H_0 and conclude that at 5% level, there is sufficient evidence that the claim is not valid.
(iii)	Since *n* is large, by Central Limit Theorem, the sample mean time spent by 150 customers is approximately normal. Hence it is not necessary to assume a normal distribution for the population for the test to be valid.
(iv)	There is a probability of 0.05 of concluding that the mean time spent by customers is not equal to 41 minutes when it is in fact 41 minutes.
(v)	$H_0 : \mu = 41$ $H_1 : \mu > 41$ (claim) Under H_0, the test statistic $Z = \dfrac{\overline{X}-\mu}{\sigma/\sqrt{n}} \sim \text{N}(0,1)$ approx. by CLT, where $\mu = 41, \sigma = \sqrt{49.3}, \overline{x} = k,\; n = 40$.  Since H_0 is not rejected, $\dfrac{k-41}{\sqrt{49.3}/\sqrt{40}} < 1.28155$ $k < 42.423$ $k < 42.4\left(3\text{ s.f.}\right)$ Required set = $\left\{k \in \square : 0 < k < 42.4\right\}$

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YISHUN JUNIOR COLLEGE
2017 JC2 Preliminary Examination

MATHEMATICS
HIGHER 1

8865/01

28 AUGUST 2017
MONDAY 0800h – 1100h

Additional materials :
Answer paper
List of Formulae (MF26)

TIME 3 hours

READ THESE INSTRUCTIONS FIRST

Write your CTG and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise. Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, write down the question number of the questions attempted, model of calculator used on the spaces provided on the cover page. Tie your cover page on top of the answer scripts before submission.
The number of marks is given in brackets [ ] at the end of each question or part question.

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Section A: Pure Mathematics [40 marks]

1 **(i)** Differentiate with respect to x,

(a) $5\ln\left(1-3x^2\right)$, [2]

(b) $\dfrac{1}{\left(2x+3\right)^2}$. [2]

(ii) Use a non-calculator method to find $\displaystyle\int_1^3 x^3\left(\frac{1}{x}-1\right)^2 \,\mathrm{d}x$. [4]

2 Find the range of values of k for which $kx^2+4k-kx-2x$ is always negative. [4]

3 Research has found that the concentration R of a drug in the bloodstream, in micrograms per litre, decreases according to the function $R=366\mathrm{e}^{-0.0998t}$, $t \geq 0$, where t is measured in minutes after the drug is administered.

(i) Sketch the graph of $R=366\mathrm{e}^{-0.0998t}$ for $t \geq 0$. [2]

(ii) Find the concentration of the drug in the bloodstream of a patient 1 hour after it is administered. [1]

(iii) Find the rate of decrease of R at the instant when t = 20. [2]

(iv) How long, to the nearest minute, will the concentration of the drug in a patient be 40 micrograms per litre? [2]

The company that manufactures the drug estimates that the profit, in millions of dollars, from the sale of the drug over the years can be modelled by the equation

$$P=-0.03x^3+0.1x^2+x-0.1,\quad 0 \leq x \leq 5,$$

where x is the number of bottles of the drug, in 10 thousands, produced.
Use differentiation to find the number of bottles that corresponds to the maximum profit, giving your answer correct to 5 significant figures. [5]

4 Sketch the graph of the curve C with equation $y=\dfrac{1}{x-2}+1$, stating the coordinates of any points of intersection with the axes and the equations of any asymptotes. [2]
Without the use of a calculator, find the area of the region bounded by C, the line $y=x+3$ and the y-axis. [6]

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5 The curve C has equation $y = x^3 - 2e^{-x}$. Without using a calculator, find the equation of the tangent to C at the point where $x = 1$, giving your answer in the form $y = ax + b$, where a and b are constants. [4]

6 Mr Lee wants to buy 6 packets of milo, 3 packets of cereal and 4 packets of coffee. Based on the usual retail price in the supermarket, the total bill will be $93.80.
Mr Lee has a voucher which will entitle him to a 20% discount off each packet of milo, capped at 5 packets. The voucher is not to be used in conjunction with other promotions. If he uses the voucher, his total bill will be $85.30.
The supermarket is currently having a “Buy 2 get 1 free” promotion for milo and for cereal. If he uses the “Buy 2 get 1 free” promotion, his total bill will become $70.40.
Write down and solve equations to find the usual retail price of each packet of milo, cereal and coffee. [4]

Section B: Statistics [60 marks]

7 **(a)** A game is played with a deck of n cards, each distinctly numbered from 1 to n, where n is an even number. A player randomly picks a card from the deck. Events A and B are defined as follows:

A: The card shows an even number.

B: The card shows a multiple of 17.

It is given that $\mathrm{P}(A' \cap B') = \dfrac{8}{17}$ and $\mathrm{P}(A \cap B) = \dfrac{1}{34}$.

(i) Show that $\mathrm{P}(B) = \dfrac{1}{17}$ and hence state the smallest value of n. [5]

(ii) Find $\mathrm{P}(A \mid B)$. [2]

(b) The probability that a train service breaks down is $\dfrac{1}{15}$. When the train service functions normally, 5% of the people travelling to work by train are late. It is also found that 90% of the people travelling to work by train are punctual.

(i) Draw a tree diagram to represent this situation, showing all possible outcomes and the associated probabilities. [3]

(ii) Given that a randomly chosen person travelling to work by train is late, find the probability that the train service functions normally. [2]

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8 A chess team of 5 players is to be selected from 15 boys. In how many ways can the team be chosen if

(i) no more than one of the three best players is to be included, [2]

(ii) at least one of the 4 youngest players is to be included? [2]

9 In a survey on usage of internet security software conducted with a large number of smartphone users, it was found that 37% of them had anti-virus software *A* installed on their smartphones, 56% had anti-virus software *B* installed and 7% did not have any of them installed.

(i) A random sample of twenty smartphone users was chosen. Find the probability that at least eight of the users had anti-virus software *A* installed on their smartphones. [2]

(ii) Fifty such samples of twenty smartphone users was randomly chosen. Find the probability that there were fewer than thirty samples with at least eight users with anti-virus software *A* installed on their smartphones. [3]

(iii) Another random sample of n smartphone users was chosen. Given that the probability that at most one user did not have any anti-virus software installed was less than 0.5, show that n would satisfy the inequality $0.93^{n-1}(0.93+0.07n)<0.5$. Hence find the least value of n. [3]

10 The following table gives the median monthly household income from work, \$$h$, from each year, t, between 2011 to 2016 (inclusive).

Year, t	2011	2012	2013	2014	2015	2016
Median monthly household income from work, \$$h$	7037	7566	7872	8292	8666	8846

Source: http://www.singstat.gov.sg/statistics/browse-by-theme/household-income-tables

(i) Give a sketch of the scatter diagram for the data. [2]

(ii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iii) Find the values of $\bar{t}$ and $\bar{h}$ and mark the point $(\bar{t}, \bar{h})$ on your scatter diagram in part **(i)**. [2]

(iv) Find the equation of the regression line of h on t, in the form $h = mt + c$, giving the values of m and c correct to 2 decimal places. Sketch this line on your scatter diagram in part **(i)**. [2]

(v) Use a suitable regression line to estimate the year when the median monthly household income from work is \$9700. Comment on the reliability of the estimate obtained. [2]

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11 An electric power service company keeps records of the installation time of its electricity meters in the houses in a new estate. The time taken, X minutes, to install an electricity meter may be assumed to be normally distributed with mean 50 minutes and standard deviation 10 minutes.

(i) Two randomly chosen houses have their electricity meters installed by the company. Find the probability that each installation takes more than an hour. [2]

(ii) The company wishes to improve its processes such that at least 95% of the installations will take less than 60 minutes. Suppose the population standard deviation is still 10 minutes, find the maximum population mean time taken to install an electricity meter. [3]

Each month, the amount of electricity, measured in kilowatt-hours (kWh), used by a particular household in the estate is normally distributed with mean 522 kWh and standard deviation 26 kWh. The company charges households for electricity used at \$0.21 per kWh.

(iii) Find the probability that, in a randomly chosen month, the electricity charge for the household is between \$100 and \$120. [2]

(iv) The household is billed every two months. Find the largest integral value of d such that the probability that a randomly chosen bill is at least \$$d$ is more than 0.90. State an assumption that is needed in your calculation. [4]

The company also installs gas meters in the houses in the estate. The time taken, Y minutes, to install a gas meter has mean 47 minutes and standard deviation 25 minutes.

(v) Explain why Y cannot be well modelled by a normal distribution. [1]

(vi) A random sample of 55 gas meter installations has been completed in the estate. Find the probability that the mean time taken to install a gas meter is more than 45 minutes. [2]

[Question 12 is printed on the next page.]

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12 A departmental store manager claims that the mean amount of time that customers spent in the shopping mall is 41 minutes. A random sample of 150 customers is taken and the time, x minutes, spent by each customer is noted. The results are summarized by

$$\sum x = 6386, \qquad \sum x^2 = 277270.$$

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Test, at the 5% level of significance, whether the manager's claim is valid. [4]

(iii) State, with a reason, whether it is necessary to assume a normal distribution for the population for the test to be valid. [1]

(iv) Explain, in the context of the question, the meaning of 'at 5% level of significance'. [1]

After several rounds of publicity for the shopping mall, the publicity manager claims that the mean time that customers spent in the shopping mall has increased. The new population variance of the time spent may be assumed to be 49.3 minutes^2. A new random sample of 40 customers is chosen and the mean of this sample is k minutes.

(v) Find the set of values within which k must lie, such that there is not enough evidence from the sample to support the publicity manager's claim at the 10% level of significance. [4]

~ ~ End of paper ~ ~

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YISHUN JUNIOR COLLEGE
Mathematics Department

PRELIM SOLUTION

Subject : JC2 H1 MATHEMATICS 8865 **Date** :

Qn	Solution
1(i)	(a) $\frac{\mathrm{d}}{\mathrm{d}x}\left(5\ln\left(1-3x^2\right)\right)=5\left(\frac{1}{1-3x^2}\right)\left(-6x\right)$ $=-\frac{30x}{1-3x^2}$ (b) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{\left(2x+3\right)^2}\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left(2x+3\right)^{-2}$ $=-2(2x+3)^{-3}(2)$ $=-4(2x+3)^{-3}$
(ii)	$\int_1^3 x^3\left(\frac{1}{x}-1\right)^2\,\mathrm{d}x=\int_1^3\left(x-2x^2+x^3\right)\mathrm{d}x$ $=\left[\frac{x^2}{2}-\frac{2x^3}{3}+\frac{x^4}{4}\right]_1^3$ $=\left(\frac{3^2}{2}-\frac{2(3)^3}{3}+\frac{(3)^4}{4}\right)-\left(\frac{1^2}{2}-\frac{2(1)^3}{3}+\frac{(1)^4}{4}\right)$ $=\frac{27}{4}-\frac{1}{12}$ $=\frac{20}{3}$
2	$\left(-k-2\right)^2-4(k)(4k)<0$ and $k<0$ $k^2+4k+4-16k^2<0$ $-15k^2+4k+4<0$ $k<0$ and $(5k+2)(3k-2)>0$ $k<-\frac{2}{5}$ or $k>\frac{2}{3}$ Since $k<0$, $\therefore$ $k<-\frac{2}{5}$

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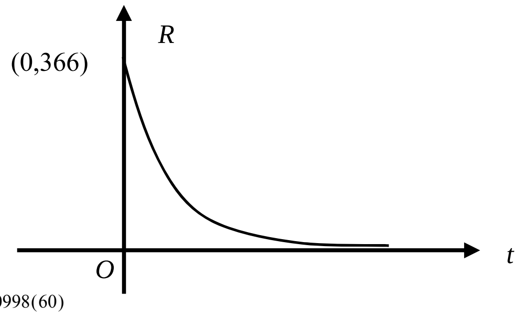
3 (i)



(ii)

$$R = 366e^{-0.0998(60)}$$
$$= 0.918$$

The concentration is 0.918 micrograms/litre

(iii)

From GC, when $t = 20$,

$$\frac{\mathrm{d}R}{\mathrm{d}t} = -4.96$$

The rate of decrease is 4.96 micrograms/litre per min

(iv)

$$40 = 366e^{-0.0998t}$$

$$e^{-0.0998t} = \frac{40}{366}$$

$$-0.0998t = \ln\left(\frac{40}{366}\right)$$

$$t = 22.18$$

$$t \approx 22\text{mins}$$

Alternative solution:
Draw graph of y = 40 and find intersection points.

$$P = -0.03x^3 + 0.1x^2 + x - 0.1$$

$$\frac{\mathrm{d}P}{\mathrm{d}x} = -0.09x^2 + 0.2x + 1$$

For maximum P, $\frac{\mathrm{d}P}{\mathrm{d}x} = 0$

$$-0.09x^2 + 0.2x + 1 = 0$$
$$x = 4.624753 \ (x > 0)$$

x	4.624753^-	4.624753	4.624753^+
$\frac{\mathrm{d}P}{\mathrm{d}x}$	+ve	0	-ve
slope	/	—	\

Thus, P is maximum when the number of bottles is 46248.

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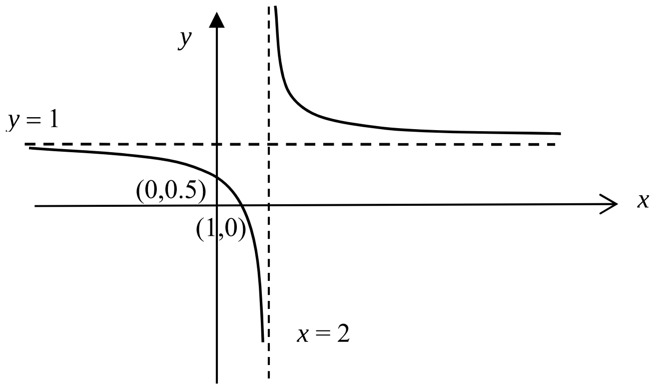
4	 $\frac{1}{x-2}+1=x+3$ $1+(x-2)=x^2+x-6$ $x^2=5$ $x=\pm\sqrt{5}$ Area of the region = $\int_{-\sqrt{5}}^{0}\left(x+3-\left(\frac{1}{x-2}+1\right)\right)\mathrm{d}x$ $=\left[\frac{x^2}{2}+2x-\ln\lvert x-2\rvert\right]_{-\sqrt{5}}^{0}$ $=-\ln 2-\left(\frac{5}{2}-2\sqrt{5}-\ln\lvert -\sqrt{5}-2\rvert\right)$ $=-\ln 2-\left(\frac{5}{2}-2\sqrt{5}-\ln\left(2+\sqrt{5}\right)\right)$ $=-\ln 2-\frac{5}{2}+2\sqrt{5}+\ln\left(2+\sqrt{5}\right)$
5	$y=x^3-2e^{-x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2+2e^{-x}$ When $x=1$, $\frac{\mathrm{d}y}{\mathrm{d}x}=3(1)^2+2e^{-1}$ and $y=1-2e^{-1}$ Equation of tangent: $y-\left(1-\frac{2}{e}\right)=\left(3+\frac{2}{e}\right)(x-1)$ $y=\left(3+\frac{2}{e}\right)x-2-\frac{4}{e}$
6	$6x+3y+4z=93.80$ $5x+3y+4z=85.30$ $4x+2y+4z=70.40$ From GC, x = 8.50, y = 6.40, z = 5.90 The usual retail prices of 1 packet of milo, 1 packet of cereal and 1 packet of coffee are \$8.50, \$6.40 and \$5.90 respectively.

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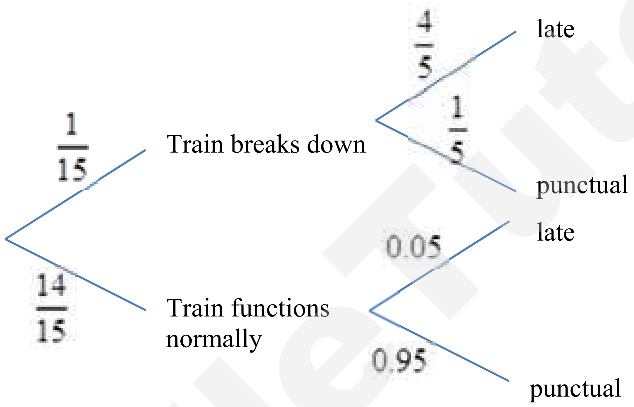
7(a)(i)	$\mathrm{P}(A \cup B) = 1 - \mathrm{P}(A' \cap B')$ $= \dfrac{9}{17}$ $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ $\dfrac{9}{17} = \dfrac{1}{2} + \mathrm{P}(B) - \dfrac{1}{34}$ $\Rightarrow \mathrm{P}(B) = \dfrac{1}{17}$ (Shown) Smallest value of n is 34.
(ii)	$\mathrm{P}(A \mid B) = \dfrac{\frac{1}{34}}{\frac{1}{17}} = \dfrac{1}{2}$
(b)(i) (ii)	 P (train functions normally | late) $= \dfrac{\frac{14}{15}(0.05)}{0.1} = \dfrac{7}{15} \approx 0.467$
8(i)	No of teams = $^{12}C_5 + {^3C_1}\,{^{12}C_4}$ $= 2277$
(ii)	No of teams = $^{15}C_5 - {^{11}C_5}$ $= 2541$
9(i)	Let X be the random variable 'number of smartphone users with anti-virus software A installed on their smartphones out of 20 users' $X \sim \mathrm{B}(20, 0.37)$ $\mathrm{P}(X \geq 8) = 1 - \mathrm{P}(X \leq 7)$ ≈ 0.47346 $= 0.473$ (3 sig fig)
(ii)	Let W be the 'number of samples with at least eight users with anti-virus software A installed on their smartphone out of 50 samples' $W \sim \mathrm{B}(50, 0.47346)$ $\mathrm{P}(W < 30) = \mathrm{P}(W \leq 29)$ ≈ 0.95061 $= 0.951$ (3 sig fig)

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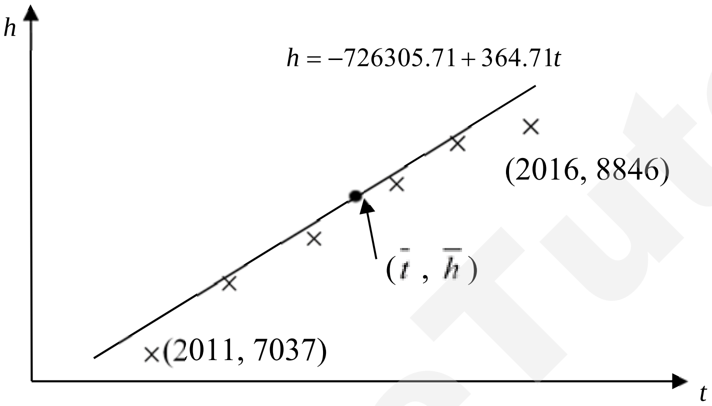
(iii)	Let Y be the 'number of smartphone users who did not have any anti-virus software installed, out of n' $Y \sim \mathrm{B}(n, 0.07)$ $\mathrm{P}(Y \le 1) < 0.5$ $\mathrm{P}(Y = 0) + \mathrm{P}(Y = 1) < 0.5$ ${}^{n}C_{0}(0.07)^{0}(0.93)^{n} + {}^{n}C_{1}(0.07)(0.93)^{n-1} < 0.5$ $(0.93)^{n} + n(0.07)(0.93)^{n-1} < 0.5$ $(0.93)^{n-1}(0.93 + 0.07n) < 0.5$ (shown) Using GC, When $n = 23$, $(0.93)^{n-1}(0.93 + 0.07n) = 0.5146 > 0.5$ When $n = 24$, $(0.93)^{n-1}(0.93 + 0.07n) = 0.4918 < 0.5$ Therefore, least $n = 24$
10(i)	
(ii)	$r \approx 0.992$ (3 s.f.) There is a strong positive linear correlation between the median monthly household income from work and the year. As the year increases, the median monthly household income from work increases.
(iii)	$\bar{t} = 2013.5$, $\bar{h} = 8046.5$
(iv)	$h = -726305.71 + 364.71t$ (2 d.p.)
(v)	When $h = 9700$, $9700 = -726305.71 + 364.71t$ $t = 2018.057$ Year: 2018 Since the estimate is obtained via extrapolation, the estimate is not reliable.
11(i)	$X \sim N(50, 10^2)$ Required Prob = $[\mathrm{P}(X > 60)]^2 \approx (0.158655)^2$ $= 0.02517$ ≈ 0.0252 (3 s.f)

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(ii)	Let μ be the population mean time taken (min) the company has to achieve $X \sim N(\mu, 10^2)$ $P(X < 60) \geq 0.95$ $P(Z < \frac{60-\mu}{10}) \geq 0.95$ $\frac{60-\mu}{10} \geq 1.64485$ $\mu \leq 43.552$ Maximum $\mu = 43.5$
(iii)	Let W be the amount of electricity (kWh) used in a month by a household $W \sim N(522, 26^2)$ Total charge per month, $B = 0.21W \sim N(109.62, 29.8116)$ $P(100 < B < 120) = 0.932$ (3 s.f)
(iv)	$T = B_1 + B_2 \sim N(219.24, 59.6232)$ $P(T \geq d) > 0.9$ $1 - P(T < d) > 0.9$ $P(T < d) < 0.1$ $d < 209.344$ Largest integral value of d is 209. Assume that the electricity used in each month is independent for a particular household
(v)	Since $\mu - 3\sigma = 47 - 3(25) = -28 < 0$, Time taken to install a gas meter is impossible to be negative, Y is unlikely to be normally distributed.
(vi)	Since sample size=55 is large, $\overline{Y} = \frac{Y_1 + Y_2 + ... + Y_{55}}{55} \sim N\left(47, \frac{25^2}{55}\right)$ approx by CLT $P(\overline{Y} > 45) = 0.724$ (3 s.f.)
12(i)	Unbiased estimate of the population mean, $\overline{x} = \frac{6386}{150} = 42.573 \approx 42.6(3\text{s.f.})$ Unbiased estimate of the population variance, $s^2 = \frac{1}{149}\left[277270 - \frac{6386^2}{150}\right] = 36.219 \approx 36.2$ (3 s.f.)
(ii)	$H_0 : \mu = 41$ $H_1 : \mu \neq 41$ Test at 5% significance level Under H_0, the test statistic $Z = \frac{\overline{X} - \mu}{S / \sqrt{n}} \sim N(0,1)$ approx. by CLT, where $\mu = 41, s = \sqrt{36.219}, \overline{x} = 42.573, n = 150$.

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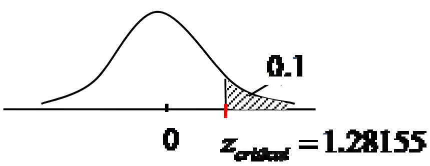
	By GC, p-value $= 0.00137\,(3\text{ s.f.})$. Since p-value < 0.05, we reject H_0 and conclude that at 5% level, there is sufficient evidence that the claim is not valid.
(iii)	Since n is large, by Central Limit Theorem, the sample mean time spent by 150 customers is approximately normal. Hence it is not necessary to assume a normal distribution for the population for the test to be valid.
(iv)	There is a probability of 0.05 of concluding that the mean time spent by customers is not equal to 41 minutes when it is in fact 41 minutes.
(v)	$H_0 : \mu = 41$ $H_1 : \mu > 41$ (claim) Under H_0, the test statistic $Z = \dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0,1)$ approx. by CLT, where $\mu = 41, \sigma = \sqrt{49.3}, \overline{x} = k,\ n = 40$.  Since H_0 is not rejected, $\dfrac{k-41}{\sqrt{49.3}/\sqrt{40}} < 1.28155$ $k < 42.423$ $k < 42.4\,(3\text{ s.f.})$ Required set = $\{k \in \mathbb{R} : 0 < k < 42.4\}$

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