CAMBRIDGE
International Examinations

Cambridge IGCSE

Grade thresholds – June 2017

Cambridge IGCSE Additional Mathematics (0606)

Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the June 2017 examination.

		minimum raw mark required for grade:				
	maximum raw mark available	A	B	C	D	E
Component 11	80	56	41	26	21	15
Component 12	80	58	43	29	24	18
Component 13	80	58	44	29	24	18
Component 21	80	62	47	33	27	22
Component 22	80	55	43	30	25	21
Component 23	80	62	47	33	27	22

Grade A* does not exist at the level of an individual component.

The maximum total mark for this syllabus, after weighting has been applied, is **160**.

The overall thresholds for the different grades were set as follows.

Option	Combination of Components	A*	A	B	C	D	E
AX	11, 21	148	118	88	59	48	37
AY	12, 22	140	113	86	59	49	39
AZ	13, 23	149	120	91	62	51	40

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

ADDITIONAL MATHEMATICS **0606/11**

Paper 1 **May/June 2017**

MARK SCHEME

Maximum Mark: 80

Published

This document consists of 8 printed pages.

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MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method marks, awarded for a valid method applied to the problem.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
B	Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘**dep**’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

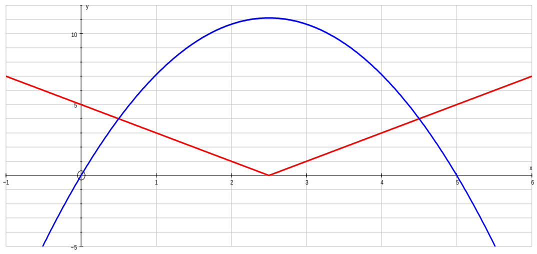
Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(i)	$kx-5=x^2+4x$ $x^2+(4-k)x+5=0$	**M1**	equating line and curve equation and collecting terms to form an equation of the form $ax^2+bx+c=0$ x terms must be gathered together, maybe implied by later work
	For a tangent $(4-k)^2=20$	**DM1**	correct use of discriminant
	$k=4+2\sqrt{5}$	**A1**	Accept $k=4+\sqrt{20}$
	Alternative Gradient of line = k Gradient of curve = $\frac{\mathrm{d}y}{\mathrm{d}x}=2x+4$ Equating: $k=2x+4$	**M1**	
	substitution of $k=2x+4$ or $x=\frac{k-4}{2}$ in $kx-5=x^2+4$ and simplify to a quadratic equation in k or x	**DM1**	
	$k=4+2\sqrt{5}$	**A1**	Accept $k=4+\sqrt{20}$
1(ii)	Normal gradient = $-\frac{1}{4+2\sqrt{5}}\times\frac{4-2\sqrt{5}}{4-2\sqrt{5}}$	**M1**	use of negative reciprocal and attempt to rationalise using a form of $a-b\sqrt{5}$ or $a-\sqrt{20}$ or *their* equivalent from (i)
	$=-\frac{4-2\sqrt{5}}{-4}$ oe $=1-\frac{\sqrt{5}}{2}$	**A1**	$-\frac{4-2\sqrt{5}}{-4}$ oe leading to $1-\frac{\sqrt{5}}{2}$
2	$\mathrm{p}(3)=27+9a+3b-48$	**M1**	attempt to find $\mathrm{p}(3)$
	$3a+b=9$ oe	**A1**	
	$\mathrm{p}'(x)=3x^2+2ax+b$ $\mathrm{p}'(1)=3+2a+b$	**M1**	attempt to differentiate and find $\mathrm{p}'(1)$ must have 2 terms correct
	$2a+b=-3$ oe	**A1**	
	$a=12,\ b=-27$	**A1**	for both
3(a)	x^3y^7	**B2**	**B1** for each term

Question	Answer	Marks	Guidance
3(b)(i)	for $(t-2)^{\frac{3}{2}}=(t-2)^{\frac{1}{2}}(t-2)$ soi	**M1**	
	$(t-2)^{\frac{1}{2}}(4+5(t-2))$	**A1**	
	$(t-2)^{\frac{1}{2}}(5t-6)$	**A1**	
3(b)(ii)	2 and $\frac{6}{5}$	**B1**	**FT** on *their* $(t-2)^{\frac{1}{2}}(5t-6)$, must have 2
4(a)(i)	$f>5$, $f(x)>5$	**B1**	
4(a)(ii)	$\frac{y-5}{3}=e^{-4x}$ or $\frac{x-5}{3}=e^{-4y}$	**B1**	
	$-4x=\ln\left(\frac{y-5}{3}\right)$ or $-4y=\ln\left(\frac{x-5}{3}\right)$	**B1**	
	leading to $f^{-1}(x)=-\frac{1}{4}\ln\left(\frac{x-5}{3}\right)$ or $f^{-1}(x)=\frac{1}{4}\ln\left(\frac{3}{x-5}\right)$ or $f^{-1}(x)=\frac{1}{4}(\ln 3-\ln(x-5))$ or $f^{-1}(x)=-\frac{1}{4}(\ln(x-5)-\ln 3)$	**B1**	
	Domain $x>5$	**B1**	
4(b)	$\ln(x^2+5)=2$	**B1**	
	$x^2+5=e^2$	**B1**	
	$x=1.55$ or better or $\sqrt{e^2-5}$	**B1**	
5(a)(i)	$\overrightarrow{OM}=\overrightarrow{OC}+\frac{1}{2}\left(\overrightarrow{OA}-\overrightarrow{OC}\right)$ oe	**M1**	may be implied by correct answer.
	$\frac{1}{2}(\mathbf{a}+\mathbf{c})$	**A1**	

Question	Answer	Marks	Guidance
5(a)(ii)	$\mathbf{b} = \frac{5}{2}\overrightarrow{OM}$ oe , $\frac{5}{2}$(*their* (i)) or $\overrightarrow{OM} = \frac{2}{3}\left(\mathbf{b} - \overrightarrow{OM}\right)$	**M1**	dealing with ratio correctly to relate **b** or $\overrightarrow{OB}$ to $\overrightarrow{OM}$
	$= \frac{5}{4}(\mathbf{a} + \mathbf{c})$	**A1**	
5(b)(i)	$\lvert -10\mathbf{i} + 24\mathbf{j} \rvert = 26$ $\mathbf{p} = \frac{39}{26}(-10\mathbf{i} + 24\mathbf{j})$	**M1**	magnitude of $-10\mathbf{i} + 24\mathbf{j}$ and use with 39
	$\mathbf{p} = -15\mathbf{i} + 36\mathbf{j}$	**A1**	
5(b)(ii)	If parallel to the y-axis, **i** component is zero	**M1**	realising **i** component is zero
	so $2\mathbf{p} + \mathbf{q} = 12\mathbf{j}$	**DM1**	use of 12
	$\mathbf{q} = 30\mathbf{i} - 60\mathbf{j}$	**A1**	
5(b)(iii)	$\lvert\mathbf{q}\rvert = 30\sqrt{1^2 + (-2)^2}$ or $\sqrt{900} \times \sqrt{5}$	**M1**	attempt at magnitude of *their* **q**
	$\lvert\mathbf{q}\rvert = 30\sqrt{5}$	**A1**	Answer Given: must have full and correct working
6(i)	$\frac{1}{2} \times 12^2 \times \theta = 150$	**M1**	use of sector area
	$\theta = 2.083$, so $\theta = 2.08$ to 2dp	**A1**	
6(ii)	Area of triangle AOB = $\frac{1}{2} \times 12^2 \sin 2.08$	**M1**	correct method for area of triangle
	Area of segment = $150 - \frac{1}{2} \times 12^2 \times \sin 2.08$	**A1**	allow unsimplified, using $\theta = 2.08,\ 2.083$ or $\frac{150}{72}$
	$\sin 1.04 = \frac{\frac{AB}{2}}{12}$	**M1**	correct trigonometric statement using $\theta = 2.08,\ 2.083$ or $\frac{150}{72}$ with attempt to obtain AB
	AB = awrt 20.7	**A1**	
	Shaded area = *their* $AB \times 8 -$ *their* segment area	**M1**	execution of a correct 'plan'(rectangle – segment)
	awrt 78.4 or 78.5	**A1**	

Question	Answer	Marks	Guidance
6(iii)	Arc *AB* = 25 or 24.96	**B1**	
	Perimeter= 25 + *their AB* + 16	**M1**	correct ‘plan’ (arc + *their AB* + 2×8)
	awrt 61.7	**A1**	
7	differentiation to obtain answer in the form $p\left(3x^2+8\right)^{\frac{2}{3}}$ or $qx\left(3x^2+8\right)^{\frac{2}{3}}$	**M1**	
	$6x\left(3x^2+8\right)^{\frac{2}{3}}$	**B1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{5}{3}\times6x\left(3x^2+8\right)^{\frac{2}{3}}$	**A1**	all correct
	When $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ only solution is $x=0$	**DM1**	$qx\left(3x^2+8\right)^{\frac{2}{3}}=0$ and attempt to solve
	$x=0$ and $3x^2+8$ = 0 has no solutions	**A1**	
	Stationary point at $(0,32)$	**A1**	
	correct gradient method with substitution of *x* values either side of zero or equivalent valid method	**M1**	
	correct conclusion from correct work using a correct $\frac{\mathrm{d}y}{\mathrm{d}x}$	**A1**	
8(i)		**B5**	**B1** for shape of modulus function **B1** for *y* intercept = 5 (for modulus graph only) **B1** for *x* intercept = 2.5 at the V of a modulus graph **B1** for shape of quadratic function for $-1 \leqslant x \leqslant 6$ **B1** for intercepts at *x* = 0 and *x* = 5 for a quadratic graph
8(ii)	$2x-5=\pm4$	**B1**	one correct answer
	$x=\frac{9}{2}$	**M1**	solution of two different correct linear equations or solution of an equation obtained from squaring both sides or use of symmetry from first solution.
	$x=\frac{1}{2}$	**A1**	second correct solution

Question	Answer	Marks	Guidance
8(iii)	$16\left(\frac{1}{2}\right)^2-80\left(\frac{1}{2}\right)+36=4$ and $16\left(\frac{9}{2}\right)^2-80\left(\frac{9}{2}\right)+36=4$	**B1**	verification using both x values or for forming and solving $16x^2-80x+36=0$
8(iv)	using *their* values from (ii) in an equality of the form $a\leqslant x\leqslant b$ or $a<x<b$	**M1**	
	$\frac{1}{2}\leqslant x\leqslant\frac{9}{2}$ cao	**A1**	
9(i)	$5+4\left(\sec^2\left(\frac{x}{3}\right)-1\right)$ leading to given answer	**B1**	use of correct identity
9(ii)	$3\tan\left(\frac{x}{3}\right)\quad(+c)$	**B1**	
9(iii)	attempt to integrate using (i) and/or (ii)	**M1**	
	Area = $\int_{\frac{\pi}{2}}^{\pi}4\sec^2\left(\frac{x}{3}\right)+1\,\mathrm{d}x$	**A1**	all correct
	$\left[12\tan\left(\frac{x}{3}\right)+x\right]_{\frac{\pi}{2}}^{\pi}$	**DM1**	correct method for evaluation using limits in correct order
	$=\left(12\tan\frac{\pi}{3}+\pi\right)-\left(12\tan\frac{\pi}{6}+\frac{\pi}{2}\right)$	**A1**	
	$=8\sqrt{3}+\frac{\pi}{2}$	**A1**	
10(a)	differentiation of a quotient or equivalent product	**M1**	
	correct differentiation of e^{3x}	**B1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3\mathrm{e}^{3x}\left(4x^2+1\right)-8x\mathrm{e}^{3x}}{\left(4x^2+1\right)^2}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3\mathrm{e}^{3x}}{4x^2+1}-\frac{8x\mathrm{e}^{3x}}{\left(4x^2+1\right)^2}$	**A1**	everything else correct including brackets where needed, allow unsimplified

Question	Answer	Marks	Guidance
10(b)(i)	one term differentiated correctly	**M1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -4\sin\left(x+\frac{\pi}{3}\right) + 2\sqrt{3}\cos\left(x+\frac{\pi}{3}\right)$	**A1**	all correct
	When $x = \frac{\pi}{2}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -5$	**A1**	
10(b)(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}t}$ $-5 \times \frac{\mathrm{d}x}{\mathrm{d}t} = 10$ oe	**M1**	correct use of rates of change
	$\frac{\mathrm{d}y}{\mathrm{d}t} = -2$	**A1**	**FT** answer to (i)

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ADDITIONAL MATHEMATICS **0606/12**

Paper 1 **May/June 2017**

MARK SCHEME

Maximum Mark: 80

Published

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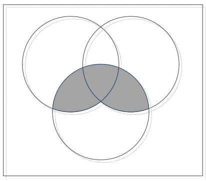
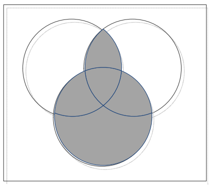
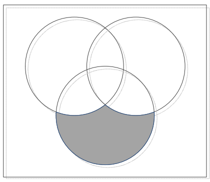
Question	Answer	Marks	Partial Marks
1	 $(A \cup B) \cap C$  $(A \cap B) \cup C$  $(A' \cap B') \cap C$	**B3**	**B1** for each
2	attempt at differentiating a quotient, must have minus sign and $(x+1)^2$ in the denominator	**M1**	
	for $\left(5x^2+4\right)^{-\frac{1}{2}}$	**B1**	
	for $\frac{1}{2}(10x)\left(5x^2+4\right)^{-\frac{1}{2}}$	**DB1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x+1)\frac{1}{2}(10x)\left(5x^2+4\right)^{-\frac{1}{2}} - \left(5x^2+4\right)^{\frac{1}{2}}}{(x+1)^2}$	**A1**	all else correct
	When $x = 3$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{11}{112}$	**A1**	must be exact
	Alternative $y = \left(5x^2+4\right)^{\frac{1}{2}}(x+1)^{-1}$	**M1**	attempt to differentiate a product
	for $\left(5x^2+4\right)^{-\frac{1}{2}}$	**B1**	
	for $\frac{1}{2}(10x)\left(5x^2+4\right)^{-\frac{1}{2}}$	**DB1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}10x\left(5x^2+4\right)^{-\frac{1}{2}}(x+1)^{-1} + \left(5x^2+4\right)^{\frac{1}{2}}\left(-(x+1)^{-2}\right)$	**A1**	all else correct
	When $x = 3$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{11}{112}$	**A1**	A1 must be exact

Question	Answer	Marks	Partial Marks
3(a)	$\mathbf{v} = 3\sqrt{5}\times\frac{1}{\sqrt{5}}(\mathbf{i}-2\mathbf{j})$	**M1**	attempt to find the magnitude of $(\mathbf{i}-2\mathbf{j})$ and use
	$= 3\mathbf{i}-6\mathbf{j}$	**A1**	for $3\mathbf{i}-6\mathbf{j}$ only
3(b)	$\mathbf{w} = 2\cos 30^{\circ}\,\mathbf{i} + 2\sin 30^{\circ}\,\mathbf{j}$	**M1**	attempt to use trigonometry correctly to obtain components
	$= \sqrt{3}\mathbf{i}+\mathbf{j}$	**A1**	
4	$3^n - n3^{n-1}\left(\frac{x}{6}\right) + n(n-1)3^{n-2}\left(\frac{x}{6}\right)^2$ $3^n = 81$ **,** so $n = 4$	**B1**	
	$4\times 3^3 \times -\frac{1}{6} = a$	**M1**	for $-n3^{n-1}\left(\frac{x}{6}\right)$**,** ${}^{n}C_1 3^{n-1}\left(-\frac{x}{6}\right)$ or $\binom{n}{1}3^{n-1}\left(-\frac{x}{6}\right)$**,** with/without *their n*
	$a = -18$	**A1**	using *their n* and equating to a to obtain $a = -18$
	$\frac{4\times 3}{2}\times 3^2 \times \frac{1}{36} = b$	**M1**	for $n(n-1)3^{n-2}\left(\frac{x}{6}\right)^2$**,** ${}^{n}C_2 3^{n-2}\left(\frac{x}{6}\right)^2$ or $\binom{n}{2}3^{n-2}\left(\frac{x}{6}\right)^2$**,** with/without *their n*
	$b = \frac{3}{2}$	**A1**	using *their n* and equating to b to obtain $b = \frac{3}{2}$
5(i)	$v = -12\sin 3t$	**B1**	
5(ii)	12	**B1**	**FT** on *their* (i) of the form $k\sin 3t$, must be $\lvert k\rvert$
5(iii)	$a = -36\cos 3t$	**B1**	allow unsimplified
	$3t = \frac{\pi}{2}$, 1.57 or better	**B1**	
	$t = \frac{\pi}{6}$ or 0.524	**B1**	
5(iv)	4 cao	**B1**	may be obtained from knowledge of cosine curve

Question	Answer	Marks	Partial Marks
6(i)	$\frac{1}{\sin\theta}\times\frac{1}{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}}$	**M1**	for $\cot\theta=\frac{\cos\theta}{\sin\theta}$, $\tan\theta=\frac{\sin\theta}{\cos\theta}$, $\text{cosec}\,\theta=\frac{1}{\sin\theta}$
	dealing with the fractions correctly	**M1**	
	$\frac{1}{\sin\theta}\times\frac{\sin\theta\cos\theta}{\cos^2\theta+\sin^2\theta}$	**M1**	use of identity
	$=\cos\theta$	**A1**	correct simplification, with all correct
	Alternative 1 $\frac{\text{cosec}\,\theta}{\frac{1}{\tan\theta}\left(1+\tan^2\theta\right)}$	**M1**	dealing with fractions
	$=\frac{\tan\theta\,\text{cosec}\,\theta}{\sec^2\theta}$	**M1**	use of appropriate identity
	$=\frac{\sin\theta}{\cos\theta}\times\frac{1}{\sin\theta}\times\cos^2\theta$	**M1**	for $\cot\theta=\frac{1}{\tan\theta}$, $\tan\theta=\frac{\sin\theta}{\cos\theta}$, $\sec\theta=\frac{1}{\cos\theta}$, $\text{cosec}\,\theta=\frac{1}{\sin\theta}$
	$=\cos\theta$	**A1**	correct simplification, with all correct
	Alternative 2 $\frac{\text{cosec}\,\theta}{\frac{1}{\cot\theta}\left(\cot^2\theta+1\right)}$	**M1**	dealing with fractions
	$=\frac{\cot\theta\,\text{cosec}\,\theta}{\text{cosec}^2\theta}$	**M1**	use of appropriate identity
	$=\frac{\cot\theta}{\text{cosec}\,\theta}$ $=\frac{\cos\theta}{\sin\theta}\times\sin\theta$	**M1**	for $\cot\theta=\frac{\cos\theta}{\sin\theta}$, $\tan\theta=\frac{\sin\theta}{\cos\theta}$, $\text{cosec}\,\theta=\frac{1}{\sin\theta}$
	$=\cos\theta$	**A1**	correct simplification, with all correct

Question	Answer	Marks	Partial Marks
6(ii)	$\int_0^a \cos 2\theta \, \mathrm{d}\theta = \left[\frac{1}{2}\sin 2\theta\right]_0^a$	**B1**	
	$\frac{1}{2}\sin 2a = \frac{\sqrt{3}}{4}$	**M1**	use of $\left[k\sin 2\theta\right]_0^a = \frac{\sqrt{3}}{4}$ to obtain $k\sin 2a = \frac{\sqrt{3}}{4}$
	$2a = \frac{\pi}{3}$	**DM1**	attempt to solve equation of the form $k\sin 2a = \frac{\sqrt{3}}{4}$, with $-1 \leqslant \frac{\sqrt{3}}{4k} \leqslant 1$, must have a correct order of operations dealing with the double angle
	$a = \frac{\pi}{6}$, 0.167π or better	**A1**	
7(i)	$\lg y = \lg A + bx$	**B1**	straight line form, may be implied by correct values of both *A* and *b* later
	Gradient = *b*,	**M1**	equating gradient to *b*
	$b = 3$	**A1**	
	Use of substitution into one of the following $2.2 = \lg A + 0.5b$ $3.7 = \lg A + b$ $158.489 = A \times 10^{0.5b}$ $5011.872 = A \times 10^{b}$ or equivalent valid method leads to $\lg A = 0.7$	**M1**	
	$A = 5,\ 5.01$ or $10^{0.7}$	**A1**	
	Alternative 1 $\lg y = \lg A + bx$	**B1**	straight line form, may be implied by correct work later
	$2.2 = \lg A + 0.5b$	**M1**	one correct equation
	$3.7 = \lg A + b$	**A1**	both equations correct
	attempt to solve 2 correct equations	**M1**	
	leading to $b = 3$ **and** $A = 5,\ 5.01$ or $10^{0.7}$	**A1**	

Question	Answer	Marks	Partial Marks
7(i)	**Alternative 2** $y = A\left(10^{bx}\right)$ $158.489 = A \times 10^{0.5b}$	**M1**	one correct equation
	$5011.872 = A \times 10^{b}$	**A1**	both correct
	$\frac{5011.872}{158.489} = 10^{0.5b}$	**M1**	attempt to solve 2 correct equations
	leading to $b = 3$	**A1**	correct *b*
	Use of substitution leads to $A = 5,\ 5.01$ or $10^{0.7}$	**A1**	correct *A*
7(ii)	Substitute *A* and *b* correctly into either $y = A\left(10^{0.6b}\right)$, $\lg y = \lg A + 0.6b$ or $\lg y = \lg A + 0.6 \lg 10^{b}$ or using $\lg y = 1.8 + 0.7$	**M1**	correct statement using *their A* and *b* **correctly** in either equation or using $\lg y = 3x + 0.7$
	$y = 316$, 315 or $10^{2.5}$	**A1**	
7(iii)	Substitute *A* and *b* correctly into either $600 = A\left(10^{bx}\right)$, $\lg 600 = \lg A + bx$ or $\lg 600 = \lg A + x \lg 10^{b}$ or using $\lg 600 = 3x + 0.7$	**M1**	correct statement using *their A* and *b* **correctly** in either equation or using $\lg y = 3x + 0.7$
	$x = 0.693$	**A1**	
8(a)(i)	2520	**B1**	
8(a)(ii)	360	**B1**	
8(a)(iii)	1080	**B1**	
8(a)(iv)	6 or 8 to start with No of ways = $2 \times 5 \times 4 \times 3 \times 2$ = 240	**B1**	
	9 to start with No of ways = $1 \times 5 \times 4 \times 3 \times 3$ = 180	**B1**	
	Total number of ways = 420	**DB1**	Dependent on both previous B marks

Question	Answer	Marks	Partial Marks
8(a)(iv)	**Alternative 1** All numbers > 6000 – all odd numbers > 6000	**B1**	plan and attempt to use, must be using 1080
	$1080-180-480$	**B1**	for 180 and 480
	Total number of ways = 420	**DB1**	Dependent on both previous B marks
	Alternative 2 Even numbers > 60000 : Odd numbers > 60000 7 : 11	**B1**	correct ratio
	Total number of ways = $\frac{7}{18}\times 1080$	**B1**	
	= 420	**DB1**	Dependent on both previous B marks
8(b)(i)	480700	**B1**	
8(b)(ii)	26460	**B1**	
8(b)(iii)	With brother and sister $^{23}C_5 = 33649$	**B1**	for $^{23}C_5$ or $^{23}C_5 \times {}^kC_k$
	Without brother and sister $^{23}C_7 = 245157$	**B1**	for $^{23}C_7$ or $^{23}C_7 \times {}^kC_k$
	Total number of ways = 278806	**B1**	for $^{23}C_5 + {}^{23}C_7$ and evaluation
9(a)(i)	3×2	**B1**	
9(a)(ii)	correct attempt to multiply the 2 matrices	**M1**	
	$\mathbf{C} = \begin{pmatrix} 6 & -6 \\ 5 & 2 \\ 19 & -8 \end{pmatrix}$	**A2**	–1 for each incorrect element
9(b)(i)	$\mathbf{X}^{-1} = \frac{1}{13}\begin{pmatrix} -7 & 12 \\ -4 & 5 \end{pmatrix}$	**B2**	**B1** for correct use of determinant **B1** for correct matrix
9(b)(ii)	$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{13}\begin{pmatrix} -7 & 12 \\ -4 & 5 \end{pmatrix}\begin{pmatrix} 26 \\ 52 \end{pmatrix}$	**B1**	
	attempt to evaluate using inverse from (i) together with pre-multiplication to obtain a 2×1 matrix	**M1**	
	$x = 34,\ y = 12$	**A2**	**A1** for each
10(i)	0.5	**B1**	for 0.5 from correct work only

Question	Answer	Marks	Partial Marks
10(ii)	$15^2 = 8^2 + 8^2 - (2\times8\times8\times\cos AOB)$ $AOB = 2.43075$ rads	**M1**	use of cosine rule (or equivalent) to obtain angle *AOB*.
	$DOC = AOB - 2(their\ AOD)$	**M1**	use of angle *AOD* and symmetry
	$DOC = 1.43$ to 2 dp	**A1**	Answer Given: need to have seen either 2.431 or better, or 1.431 or better in previous calculations
	Alternative 1 $15 = 2\times8\times\sin\left(\frac{1+DOC}{2}\right)$	**M1**	use of basic trigonometry
	use of $\frac{1+0.5DOC}{2}$	**M1**	may be implied
	$DOC = 1.43$ to 2 dp	**A1**	Answer Given: need to have seen either 2.431 or better, or 1.431 or better or 1.215 or better in previous calculations
	Alternative 2		
	$15^2 = 8^2 + 8^2 - (2\times8\times8\times\cos AOB)$ $AOB = 2.43075$ rads $\angle AOB \times 8 =$ arc AB	**M1**	use of cosine rule (or equivalent) to obtain angle AOB.
	$\frac{\text{arc}AB - 8}{8} = \angle DOC$	**M1**	attempt at *DOC*, must be a complete method with *AOB* found
	$DOC = 1.43$ to 2 dp	**A1**	Answer Given: need to have seen either 2.431 or better, or 1.431 or better or 1.215 or better in previous calculations
	Alternative 3 Equating 2 different forms for the area of triangle *AOB* $\frac{15\sqrt{31}}{4} = \frac{1}{2}\times8^2\sin AOB$, $AOB = 2.43075$ rads	**M1**	using both different forms of the area of triangle *AOB*
	$DOC = AOB - 2(\text{their } AOD)$	**M1**	use of angle *AOD* and symmetry
	$DOC = 1.43$ to 2 dp	**A1**	Answer Given: need to have seen either 2.431 or better, or 1.431 or better in previous calculations

Question	Answer	Marks	Partial Marks
10(iii)	$\sin\left(\frac{1.43}{2}\right)=\frac{\frac{DC}{2}}{8}$ or $DC^2=8^2+8^2-(2\times8\times8\times\cos1.43)$	**M1**	use of cosine rule or basic trigomoetry to obtain *DC*
	$DC=10.49$	**A1**	awrt 10.5, may be implied
	Perimeter = $10.49+4+4+15$ = 33.5	**A1**	awrt 33.5
10(iv)	$\frac{1}{2}\times8^2(2.43-\sin2.43)-\frac{1}{2}\times8^2(1.431-\sin1.431)$	**B1**	area of one appropriate sector; allow unsimplified; may be implied by a correct segment
	area of one appropriate triangle, allow unsimplified	**B1**	
	an appropriate segment, allow unsimplified	**B1**	
	= 42.8 (allow awrt 42.8)	**B1**	final answer
	Alternative 1 Area of a trapezium + 2 small segments	**B1**	one appropriate small sector, allow unsimpified (could be doubled)
	Each small segment = $\frac{1}{2}\times8^2(0.5-\sin0.5)$	**B1**	an appropriate triangle, allow unsimplfied (could be doubled)
	Area of trapezium = $\frac{1}{2}(15+10.5)\times(6.041-2.784)$	**B1**	attempt at trapezium, must have a correct attempt at finding the distance between the parallel sides – allow unsimplified
	Total area = 42.8 (allow awrt 42.8)	**B1**	final answer
	Alternative 2 Area of 2 small sectors + area of triangle *ODC* – the area of triangle *OAB* Area of a small sector = $\frac{1}{2}\times8^2\times\frac{1}{2}$	**B1**	area of small sector, allow unsimplified, (could be doubled)
	Area of triangle *ODC* = $\frac{1}{2}\times8^2\times\sin1.43$	**B1**	area of triangle *ODC*, allow unsimplified
	Area of triangle *OAB* = $\frac{1}{2}\times8^2\times\sin2.43$	**B1**	area of triangle *OAB*, allow unsimplified
	Total area = 42.8 (allow awrt 42.8)	**B1**	final answer

Question	Answer	Marks	Partial Marks
10(iv)	**Alternative 3** Area of rectangle + 2 small triangles + 2 small segments Each small segment = $\frac{1}{2}\times 8^2\left(0.5-\sin 0.5\right)$	**B1**	area of a small segment, allow unsimplified, could be doubled
	$\frac{1}{2}\times\frac{\left(15-10.49\right)}{2}\left(6.041-2.784\right)$	**B1**	area of a small triangle, allow unsimplified, could be doubled
	Area of rectangle = $10.49\times\left(6.041-2.784\right)$	**B1**	allow unsimplified, could be doubled
	Total area = 42.8 (allow awrt 42.8)	**B1**	final answer
	Alternative 4 Sector *AOB* – sector *AOD* – sector *COB* – triangle *DOC*	**B1**	area of one appropriate sector; allow unsimplified; may be implied by a correct segment
	$\left(\frac{1}{2}\times 8^2\times 2.43\right)-2\left(\frac{1}{2}\times 8^2\times 0.5\right)-\left(\frac{1}{2}\times 8^2\sin 1.43\right)$ Area = sector *AOB* – segment *DC* – triangle *AOB*	**B1**	area of one appropriate triangle, allow unsimplified
	$\left(\frac{1}{2}\times 8^2\times 2.43\right)-\text{(their segment)}-\left(\frac{1}{2}\times 8^2\sin 2.43\right)$	**B1**	an appropriate segment, allow unsimplified
	Total area = 42.8 (allow awrt 42.8)	**B1**	final answer
11(i)	me^{2x-1} where m is numeric constant	**M1**	
	$\mathrm{f}\left(x\right)=\frac{1}{2}\mathrm{e}^{2x-1}\quad\left(+c\right)$	**A1**	condone omission of +*c*
	$\frac{7}{2}=\frac{1}{2}+c$	**DM1**	correct attempt to find arbitrary constant
	$\mathrm{f}\left(x\right)=\frac{1}{2}\mathrm{e}^{2x-1}+3$	**A1**	must be an equation
11(ii)	$k\mathrm{e}^{2x-1}$ where *k* is a numeric constant	**M1**	
	$\mathrm{f}''\left(x\right)=2\mathrm{e}^{2x-1}$	**A1**	
	$2x-1=\ln\left(\frac{4}{k}\right)$	**DM1**	attempt to equate to 4 and use logarithms
	$x=\frac{1}{2}+\ln\sqrt{2}$	**A1**	

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ADDITIONAL MATHEMATICS **0606/13**
Paper 1 **May/June 2017**
MARK SCHEME
Maximum Mark: 80

Published

This document consists of **8** printed pages.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

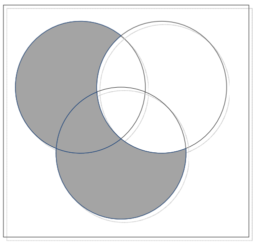
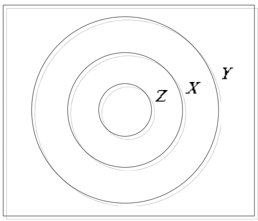
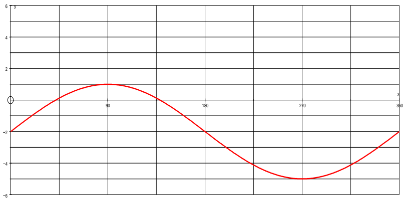
Types of mark

M	Method marks, awarded for a valid method applied to the problem.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
B	Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘**dep**’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1(a)		**1**	
1(b)		**1**	
2(i)	4	**1**	
2(ii)	40° or $\frac{2\pi}{9}$ or 0.698 rad	**1**	
3(i)		**3**	**B1** for a complete cycle starting and ending at –2 **B1** for max at $y = 1$ and min at $y = –5$ **B1** for a completely correct graph
3(ii)	5	**1**	**FT** *their* min value for y
4(i)	Area = $\frac{1}{2}(3+2\sqrt{5})(4+6\sqrt{5})$ $=\frac{1}{2}(12+26\sqrt{5}+60)$	**M1**	use of correct formula and attempt to expand out the brackets
	$=36+13\sqrt{5}$	**A1**	
4(ii)	$\frac{3+2\sqrt{5}}{2+3\sqrt{5}}$	**B1**	
	$=\frac{3+2\sqrt{5}}{2+3\sqrt{5}}\times\frac{2-3\sqrt{5}}{2-3\sqrt{5}}$	**M1**	
	$=\frac{6-5\sqrt{5}-30}{4-45}$ $=\frac{24+5\sqrt{5}}{41}$	**A1**	for answer

Question	Answer	Marks	Partial Marks
5	When $x = 4,\ y = 5$	**B1**	for *y*
	$\frac{dy}{dx} = \frac{1}{2} \times 4(4x+9)^{-\frac{1}{2}}$	**B1**	for $2(4x+9)^{-\frac{1}{2}}$, allow unsimplified
	When $x = 4,\ \frac{dy}{dx} = \frac{2}{5}$, so perp grad = $-\frac{5}{2}$	**M1**	obtaining numerical gradient for normal
	Equation of normal $y - 5 = -\frac{5}{2}(x-4)$ $(2y = 30 - 5x)$	**M1**	for equation of normal
	$A(6,\ 0),\ B(0,\ 15)$	**A2**	**A1** for each
	Midpoint $\left(3,\ \frac{15}{2}\right)$	**B1**	**FT** on *their x*/*y* intercepts
6(a)(i)	dealing with multiplication and addition	**M1**	implied by 2 correct elements
	A + 3**C** = $\begin{pmatrix} -12 & 7 \\ 11 & 7 \end{pmatrix}$	**A1**	
6(a)(ii)	correct attempt to multiply	**M1**	implied by 2 correct elements
	BA = $\begin{pmatrix} 17 & 9 \\ 14 & 18 \\ -3 & -1 \end{pmatrix}$	**A1**	
6(b)(i)	$\mathbf{X}^{-1} = \frac{1}{10}\begin{pmatrix} -2 & 3 \\ -4 & 1 \end{pmatrix}$	**B2**	**B1** for $\frac{1}{10}$, **B1** for $\begin{pmatrix} -2 & 3 \\ -4 & 1 \end{pmatrix}$
6(b)(ii)	$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{1}{10}\begin{pmatrix} -2 & 3 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} 5 & -10 \\ 15 & 20 \end{pmatrix}$	**M1**	pre-multiplication using matrix from (b)(i)
	$= \begin{pmatrix} 3.5 & 8 \\ -0.5 & 6 \end{pmatrix}$	**A2**	–1 for each incorrect element

Question	Answer	Marks	Partial Marks
7(a)	LHS = $\dfrac{\dfrac{\sin^2\theta}{\cos^2\theta}+\sin^2\theta}{\cos\theta+\dfrac{1}{\cos\theta}}$	**M1**	for obtaining all in terms of $\sin\theta$ and $\cos\theta$
	$=\dfrac{\dfrac{\sin^2\theta+\sin^2\theta\cos^2\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta+1}{\cos\theta}}$	**M1**	for simplification using addition of fractions
	$=\dfrac{\sin^2\theta\left(1+\cos^2\theta\right)}{\cos\theta\left(\cos^2\theta+1\right)}$ $=\dfrac{\sin^2\theta}{\cos\theta}$	**M1**	for factorisation and subsequent cancelling of common term
	$\tan\theta\sin\theta$ = RHS	**A1**	correct final simplification
	Alternative $\dfrac{\sec^2\theta-1-\cos^2\theta+1}{\cos\theta+\sec\theta}$	**M1**	use of correct identities
	$=\dfrac{\left(\sec\theta-\cos\theta\right)\left(\sec\theta+\cos\theta\right)}{\left(\sec\theta-\cos\theta\right)}$ $=\sec\theta-\cos\theta$	**M1**	attempt to factorise and simplify
	$=\dfrac{1-\cos^2\theta}{\cos\theta}$	**M1**	simplification to obtain terms in $\sin\theta$ and $\cos\theta$ only
	$=\dfrac{\sin^2\theta}{\cos\theta}$ $=\tan\theta\sin\theta$	**A1**	for final simplification
7(b)	$\sin\phi=\dfrac{x}{3},\ \cos\phi=\dfrac{3}{y}$	**M1**	for obtaining $\sin\phi$ and $\cos\phi$ in terms of x and y and attempt to use correct identity
	Using $\sin^2\phi+\cos^2\phi=1$ leads to $\dfrac{x^2}{9}+\dfrac{9}{y^2}=1$ and hence $x^2y^2+81=9y^2$	**M1**	attempt at simplification
	81	**A1**	

Question	Answer	Marks	Partial Marks
	Alternative method using substitution $\left(9\times\frac{9}{\cos^2\phi}\right)-\left(\frac{9}{\cos^2\phi}\times 9\sin^2\phi\right)$	**M1**	attempt to substitute in for *x* and *y*
	$=\left(\frac{81}{\cos^2\phi}\right)-\left(\frac{81\sin^2\phi}{\cos^2\phi}\right)$	**M1**	simplification of fractions
	$=\frac{81\left(1-\sin^2\phi\right)}{\cos^2\phi}$ or $81\left(\sec^2\phi-\tan^2\phi\right)$ leading to 81	**A1**	use of correct identity to obtain 81
8(i)	$\mathrm{p}\left(-\frac{1}{2}\right)=-\frac{2}{8}+\frac{a}{4}-2+b$	**M1**	for attempt at $\mathrm{p}\left(-\frac{1}{2}\right)$
	leading to $a+4b=9$ oe	**A1**	
	$\mathrm{p}(1)=2+a+4+b$ leading to $a+b=-18$ oe	**B1**	
	solution of simultaneous equations	**M1**	
	$a=-27,\ b=9$	**A1**	for both
8(ii)	attempt at factorisation using either long division or observation	**M1**	
	$(2x+1)\left(x^2-14x+9\right)$	**A1**	
8(iii)	attempt to solve q(*x*) = 0	**M1**	
	$x=7\pm2\sqrt{10},\ -\frac{1}{2}$	**A1**	for all 3 solutions
9(i)	$\left[3\mathrm{e}^{5x}+\mathrm{e}^{-5x}\right]_{-k}^{k}=6$	**B2**	**B1** for each term integrated correctly
	$\left(3\mathrm{e}^{5k}+\mathrm{e}^{-5k}\right)-\left(3\mathrm{e}^{-5k}+\mathrm{e}^{5k}\right)=6$	**M1**	for use of limits with $a\mathrm{e}^{5x}+b\mathrm{e}^{-5x}$
	$2\mathrm{e}^{5k}-2\mathrm{e}^{-5k}=6$	**A1**	correct unsimplified
	$\mathrm{e}^{5k}-\mathrm{e}^{-5k}=3$	**A1**	correct simplification to obtain given answer

Question	Answer	Marks	Partial Marks
9(ii)	$y^2 - 3y - 1 = 0$	**M1**	for correct attempt to obtain a quadratic equation in terms of y or e^{5x}
	$y = \frac{3 \pm \sqrt{9+4}}{2}$, $y = e^{5k} = 3.303$ only	**DM1**	for attempt to solve quadratic equation and solve for k
	$k = 0.239$	**A1**	**A0** if more than one solution is given
10(i)	for attempt to differentiate a product	**M1**	
	$\frac{5}{5x+1}$	**B2**	**B1** for $\frac{1}{5x+1}$
	$\frac{\mathrm{d}y}{\mathrm{d}x} = (10x+2) \times \frac{5}{5x+1} + 10\ln(5x+1)$	**A1**	all else correct
10(ii)	$(10x+2) \times \frac{5}{5x+1} = 10$	**B1**	simplification to obtain 10, allow if seen in (i)
	$10\int \ln(5x+1)\,\mathrm{d}x$ $= (10x+2)\ln(5x+1) - 10x$	**M1**	use of result from part (i)
	$\int \ln(5x+1)\,\mathrm{d}x$ $= \frac{(5x+1)}{5}\ln(5x+1) - x$	**A1**	
10(iii)	$\left[(x+0.2)\ln(5x+1) - x\right]_0^{\frac{1}{5}}$	**M1**	use of limits in result from (ii)
	$= -\frac{1}{5} + \frac{2}{5}\ln 2 = \frac{-1+\ln 4}{5}$ cao	**A1**	
11(i)	attempt to differentiate	**M1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 6 - \frac{3}{2}x^{\frac{1}{2}}$	**A1**	
	When $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	**M1**	equating to zero and attempt to solve
	$x = 16,\ y = 32$	**A1**	both correct

Question	Answer	Marks	Partial Marks
11(ii)	$\frac{d^2y}{dx^2}=-\frac{3}{4}x^{-\frac{1}{2}}$	**B1**	correct differentiation
	This is negative so a maximum point	**DB1**	correct conclusion
11(iii)	When $x = 4$, $\frac{dy}{dx}=3$	**B1**	
	$\partial y \approx \frac{dy}{dx}\times h$	**M1**	use of small increases
	$\approx 3h$	**A1**	**FT** *their* (iii)
12(i)	attempt to differentiate	**M1**	
	$6\cos 2t+6$	**A1**	
12(ii)	$\cos 2t=-1$	**M1**	attempt to equate (i) to zero and solve
	$t=\frac{\pi}{2}$	**A1**	
12(iii)	attempt to integrate	**M1**	
	$x=-\frac{3}{2}\cos 2t+3t^2+2t \quad (+c)$	**A2**	–1 for each error
	When $t=0,\ x=0$, so $c=\frac{3}{2}$	**M1**	attempt to find c
	$x=\frac{3}{2}-\frac{3}{2}\cos 2t+3t^2+2t$	**A1**	

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ADDITIONAL MATHEMATICS **0606/21**

Paper 2 **May/June 2017**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **7** printed pages.

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MARK SCHEME NOTES

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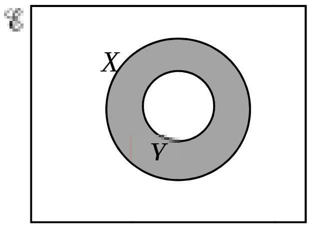
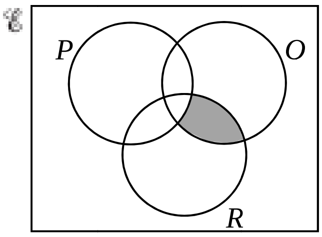
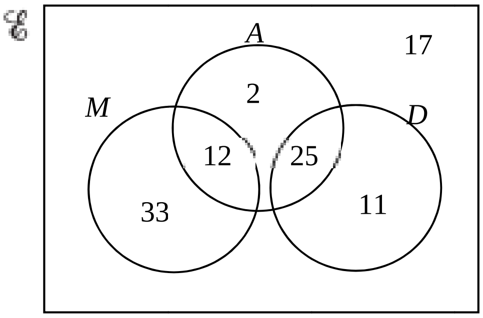
Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	Integrates	**M1**	must be clear attempt to integrate at least one term
	$[y=]x^4+x\left(+c\right)$	**A1**	Both terms correct
	$17=2^4+2+c$	**DM1**	Substitution of $x = 2$, $y = 17$ to find c
	$y=x^4+x-1$ cao	**A1**	must have $y =$
2(a)	$2\sqrt{6}\times3\sqrt{3}=6\sqrt{18}$ oe	**M1**	method must be shown – simplifies and combines product
	$18\sqrt{2}$	**A1**	If all over common denominator then consider the product for M1A1
	$9\sqrt{2}$ oe soi leading to final answer of $27\sqrt{2}$	**B1**	
2(b)	$[x=]\dfrac{6+\sqrt{3}}{2-\sqrt{3}}$	**M1**	Expanding and making x subject – condone slips but must be of equivalent difficulty
	$[x=]\dfrac{6+\sqrt{3}}{2-\sqrt{3}}\times\dfrac{2+\sqrt{3}}{2+\sqrt{3}}$ oe and multiplies out numerator and denominator	**M1**	numerator at least 3 terms; $12+2\sqrt{3}+6\sqrt{3}+3$
	$15+8\sqrt{3}$	**A1**	
3(i)	$\dfrac{2x}{x^2+1}$ final answer	**B2**	**B1** for $\dfrac{1}{x^2+1}\times\left(ax+b\right)$, a or b must be non-zero
3(ii)	$\delta y=their\left(\dfrac{2(3)}{(3)^2+1}\right)\times h$ or better	**M1**	Substitutes $x = 3$ into *their* $\dfrac{dy}{dx}$ **and** multiplies by h
	$\dfrac{6}{10}h$ oe	**A1**	
4(a)(i)	36	**B1**	
4(a)(ii)	7	**B1**	
4(b)	$[y=]5\sin4x+7$	**B4**	**B1** for each of 5, 4 and 7 and **B1** for sine Accept $a = 5$, $b = 4$, $c = 7$ for **B3**

Question	Answer	Marks	Guidance
5(i)	$16+32ax+24a^2x^2+8a^3x^3+a^4x^4$	**B2**	**B1** for at most 2 terms incorrect or missing or for correct but unsimplified form **SC1** for $16+32ax+24ax^2+8ax^3+ax^4$ or all terms correct listed
5(ii)	$24a^2=8a^3$ and solves to given answer	**B1**	or verifies that $a = 3$ leads to coeff of 216 for both terms must be from correct terms in (i)
5(iii)	$x = –0.01$ or $ax = –0.03$ soi	**M1**	
	$16 + 32(3)(–0.01) +24(9)(–0.01)^2$ leading to 16 – 0.96 + 0.0216 or 15.06…isw	**A1**	Must show clear substitution into their expansion for A1 and reach a value which rounds to 15.1
6(i)	$(\mathbf{M}=)\begin{pmatrix}90&10&30\\0&45&0\\25&0&15\\10&0&100\end{pmatrix}$	**B1**	columns and/or rows may be interchanged but must be consistent
6(ii)	$(\mathbf{LM}=)\begin{pmatrix}1&1&1&1\end{pmatrix}\begin{pmatrix}90&10&30\\0&45&0\\25&0&15\\10&0&100\end{pmatrix}=\begin{pmatrix}125&55&145\end{pmatrix}$	**B1**	Answer must be of correct order and must be consistent with a correct **M**
6(iii)	The total numbers of each type of ticket sold by all 4 cinemas oe	**B1**	
6(iv)	$(\mathbf{N}=)\begin{pmatrix}5\\4\\3\end{pmatrix}$	**B1**	Calculation not required
	The **total** income of **all** (4) cinemas or other valid comment e.g. **total** income from **all** ticket sales	**B1**	**Total** cost/value of tickets etc.
7(a)	 	**B2**	**B1** for each
7(b)(i)	$\mathrm{n}(M\cap D)=0$ or $M\cap D=\varnothing$	**B1**	No additional brackets e.g. $M\cap D=\{\varnothing\}$ is **B0**

Question	Answer	Marks	Guidance
7(b)(ii)		**B3**	**B1** correct intersection of circles with 12 and 25 correct **B1** 33, 2, 11 correctly placed **B1FT** 17; must be on the Venn diagram **and** identified as the required answer **FT** on 100– (sum of *their* 5 correctly positioned values)
8(a)	$\left[{}^{30}P_2 =\right]870$	**B1**	
8(b)(i)	${}^2C_1 \times {}^{14}C_{10}$ oe (2 × 1001)	**M1**	Condone $\begin{pmatrix}14\\4\end{pmatrix}$ for $\begin{pmatrix}14\\10\end{pmatrix}$
	2002	**A1**	implies **M1**
8(b)(ii)	$\left({}^2C_1 \times {}^5C_4 \times {}^9C_6\right) + \left({}^2C_1 \times {}^5C_5 \times {}^9C_5\right)$ oe (840 + 252) or ${}^2C_1 \times {}^{14}C_{10} -$ $\left({}^2C_1 \times {}^5C_1 \times {}^9C_9 + {}^2C_1 \times {}^5C_2 \times {}^9C_8 + {}^2C_1 \times {}^5C_3 \times {}^9C_7\right)$ {2002 – (10 + 80 + 720)}	**M3**	**M3** for fully correct method soi **M2** for all necessary products but not summed with no extra products seen soi **M1** for one correct three term product soi
	1092	**A1**	implies **M3**
9(i)	Substitution of $y = 2(1-x)$	**M1**	Must be attempt at full substitution. Condone one sign error in substitution. Condone omission of = 0 or incorrect rhs
	$-3x^2+2x+1=0$ oe $\left(3x^2-2x-1=0\right)$	**A1**	Terms collected
	Solving *their* quadratic found from eliminating one variable $(3x+1)(1-x)$ or $(3x+1)(x-1)$	**M1**	can be implied by a correct pair of *x* values
	$\left(-\frac{1}{3}, \frac{8}{3}\right)$ oe and $(1, 0)$ oe isw nfww	**A2**	**A1** for each or **A1** for a correct pair of *x*-coordinates or a correct pair of *y*-coordinates

Question	Answer	Marks	Guidance
9(ii)	$[m=]\frac{1}{2}$ cao	**B1**	
	$\left(\frac{1}{3},\frac{4}{3}\right)$	**B1**	**FT**
	$y-their\frac{4}{3}=their\frac{1}{2}\left(x-their\frac{1}{3}\right)$	**M1**	or $y=their\frac{1}{2}x+c$ and substitutes their midpoint and reaches $c=\ldots$
	$6y-3x=7$	**A1**	allow any equivalent form with integer coeffs/constant
10(i)	$\begin{array}{lllll} t & 1 & 1.5 & 2 & 2.5 \\ \ln P & 1.48 & 2.12 & 2.76 & 3.4(0) \end{array}$	**M1**	allow ln$P$ values to 1 dp rounded or truncated (1.5, 2.1, 2.8, 3.4)
	single ruled line drawn within tolerance at least for t between 1 and 2.5	**A1**	All points within 1 square of line / must **not** pass through origin
10(ii)	e^{their3}	**M1**	
	18 to 22.2	**A1**	
10(iii)	(0, c) with $0.1 \leqslant c \leqslant 0.3$ (0.2)	**B1**	allow $y = c$ condone $c =\ldots$
	m in the range $1.25 \leqslant m \leqslant 1.34$ (1.28)	**B1**	
10(iv)	$\ln P=(their1.28)t+their0.2$	**M1**	or $\ln P=(\ln b)t+\ln a$
	$P=e^{(their1.28)t+their0.2}$	**M1**	or $\ln b=m=their1.28$ and $\ln a=c=their0.2$
	$P=e^{their0.2}e^{(their1.28)t}$	**A1**	or $1.10 \leqslant a \leqslant 1.35$ $3.49 \leqslant b \leqslant 3.82$
10(v)	$1000 * e^{their0.2}\times e^{their1.28t}$ or $1000 * their\ a\times their\ b^t$	**M1**	A correct relationship e.g. $1.3t * \ln(1000)-0.2$ where * is = or an inequality sign
	5.3	**A1**	5.2 to 5.5 must be to 1dp

Question	Answer	Marks	Guidance
11(i)	$\frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}=\frac{\cos^2 x+\sin^2 x}{\sin x\cos x}$ oe	**B2**	**B1**for either $\cot x=\frac{\cos x}{\sin x}$ or $\tan x=\frac{\sin x}{\cos x}$ used **B1**for correctly placing over a common denominator or for splitting into 3 correct terms **not** just for stating or working from both sides
	Valid use of Pythagorean identity e.g. $\cos^2 x+\sin^2 x=1$	**B1**	
	Simplification to sec$x$ (correct solution only)	**B1**	**not** if working from both sides
11(ii)	$\cos x=\frac{1}{2}$ soi	**M1**	
	60, 300	**A1**	Correct pair
	$\cos x=-\frac{1}{2}$ soi	**M1**	
	120, 240	**A1**	Correct pair
12(i)	$\left[v=\frac{\mathrm{d}\left(3t-\cos 5t+1\right)}{\mathrm{d}t}=\right]3+5\sin 5t$	**B2**	**B1** for either with no other terms or for both with 1 extra
	their $\left(3+5\sin 5t\right)=0$	**M1**	Must be from an attempt to differentiate
	awrt 0.76	**A1**	0.7570187525
	awrt 1.13	**A1**	1.12793684
	substitutes *their* t values into s (4.07…, 3.58…)	**DM1**	must be two values
	0.48 to 0.49 [m]	**A1**	Final **A1** may imply earlier **A1**s
12(ii)	25cos 5t	**M1**	Differentiating *their* v correctly providing at least 2 terms with one trig function
	–25	**A1**	Ignore +25 following –25

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

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ADDITIONAL MATHEMATICS **0606/22**

Paper 2 **May/June 2017**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **14** printed pages.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘dep’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1	5x + 3 = 3x – 1 oe or 5x + 3 = 1 – 3x oe	**M1**	
	x = –2 and x = –0.25 only mark final answer	**A2**	nfww **A1** for x = –2 ignoring extras implies M1 if no extras seen If M0 then **SC1** for any correct value with at most one extra value
	Alternative method $(5x+3)^2 = (1-3x)^2$ oe soi	**M1**	
	$16x^2 + 36x + 8 = 0$ oe	**A1**	
	x = –0.25, x = –2 only; mark final answer	**A1**	
2	**Without using a calculator…** Sufficient evidence must be seen to be convinced that a calculator has not been used. Withhold the mark for any step that is unsupported.		
	deals with the negative index soi	**B1**	e.g. $\left(\frac{3-\sqrt{5}}{1+\sqrt{5}}\right)^2$
	rationalises $\frac{3-\sqrt{5}}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}}$ oe	**M1**	allow for $\frac{1+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$
	multiplies out correctly $\frac{3-4\sqrt{5}+5}{1-5}$ oe	**A1**	allow for $\frac{3+4\sqrt{5}+5}{9-5}$
	squares correct binomial $\left(-2+\sqrt{5}\right)^2 = \left(4-4\sqrt{5}+5\right)$ oe	**A1**	allow for $\left(2+\sqrt{5}\right)^2 = \left(4+4\sqrt{5}+5\right)$
	$9-4\sqrt{5}$ cao	**A1**	dep on all previous marks awarded

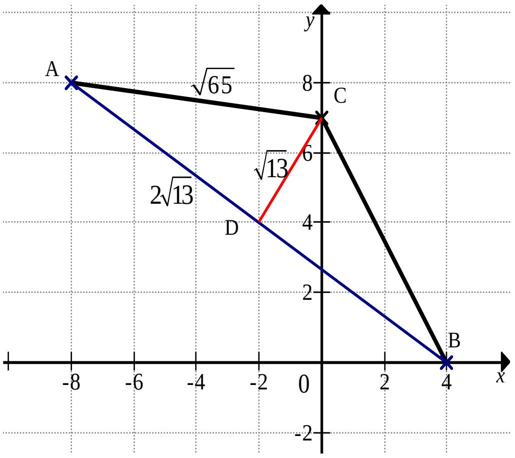
Question	Answer	Marks	Partial Marks
2	**Alternative method 1:** dealing with the negative index soi	**B1**	
	correctly squaring with at least 3 terms in the numerator and denominator $\frac{3-\sqrt{5}}{1+\sqrt{5}} \times \frac{3-\sqrt{5}}{1+\sqrt{5}} = \frac{9-6\sqrt{5}+5}{1+2\sqrt{5}+5}$ oe	**B1**	
	rationalising *their* $\left(\frac{14-6\sqrt{5}}{6+2\sqrt{5}} \times \frac{6-2\sqrt{5}}{6-2\sqrt{5}}\right)$ oe	**M1**	
	multiplying out correctly; at least 3 terms in the numerator but condone a single value for the denominator $\frac{84-64\sqrt{5}+60}{36-20}$ oe	**A1**	
	$9-4\sqrt{5}$ cao	**A1**	
	Alternative method 2 dealing with the negative index soi	**B1**	
	$9-6\sqrt{5}+5=\left(a+b\sqrt{5}\right)\left(1+2\sqrt{5}+5\right)$	**M1**	
	$14=6a+10b$ $-6=2a+6b$ oe	**A1**	
	$a=9$ cao	**A1**	
	$b=-4$ cao	**A1**	
	Alternative method 3 for dealing with the negative index soi	**B1**	
	$[3-\sqrt{5}=\left(c+d\sqrt{5}\right)\left(1+\sqrt{5}\right)$ leading to] $c+5d=3$ $c+d=-1$	**M1**	
	$c=-2$ and $d=1$	**A1**	
	$\left(-2+\sqrt{5}\right)^2=4-4\sqrt{5}+5$	**A1**	
	$9-4\sqrt{5}$ cao	**A1**	

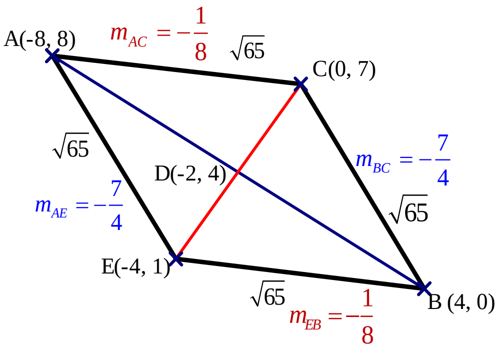
Question	Answer	Marks	Partial Marks
3	Correctly finding a correct linear factor or root	**B1**	from a valid method, e.g. factor theorem used or long division or synthetic division: $\mathrm{f}(2)=10(2^3)-21(2^2)+4=0$ or $\begin{array}{rl} & 10x^2 - x - 2 \\ x-2 & \overline{)\,10x^3 - 21x^2 \quad + 4} \\ & \underline{10x^3 - 20x^2} \\ & \qquad -x^2 \\ & \qquad \underline{-x^2 + 2x} \\ & \qquad\qquad -2x + 4 \\ & \qquad\qquad \underline{-2x + 4} \\ & \qquad\qquad\qquad 0 \end{array}$ or $\begin{array}{c\|cccc} 2 & 10 & -21 & 0 & 4 \\ & \downarrow & 20 & -2 & -4 \\ \hline & 10 & -1 & -2 & 0 \end{array}$
	correct linear factor stated or implied by, e.g. $(x-2)(10x^2-x-2)$	**B1**	$(x-2)$ or $(2x-1)$ or $(5x+2)$ do not allow $\left(x-\frac{1}{2}\right)$ or $\left(x+\frac{2}{5}\right)$
	Correct quadratic factor $(10x^2-x-2)$ or $(5x^2-8x-4)$ or $(2x^2-5x+2)$	**B2**	found using any valid method; **B1** for any 2 terms correct
	$(x-2)(2x-1)(5x+2)$ mark final answer	**B1**	must be written as a correct product of all 3 linear factors; only award the final **B1** if **all** previous marks have been awarded
			If quadratic factor is not found but correct remaining linear factors are found using e.g. the factor theorem or long division or synthetic division etc. with correct, sufficient, complete working to justify that no calculator has been used allow: **B1** for correctly finding a correct linear factor or root **B1** for a correct linear factor stated or implied **SC3** for the full, complete and correct working to find the remaining two linear factors and arrive at the correct product of 3 linear factors

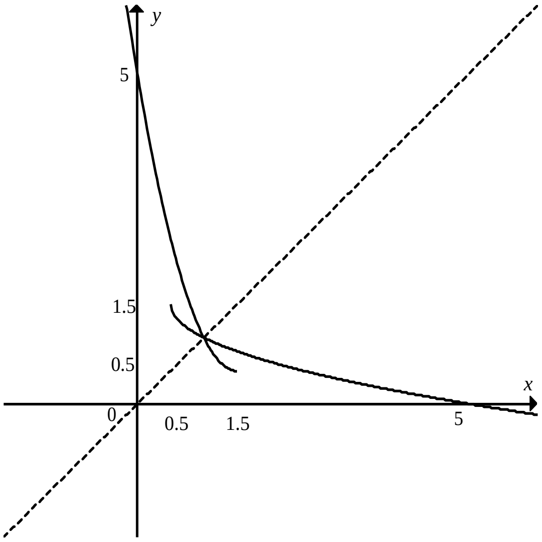
Question	Answer	Marks	Partial Marks
4	$\frac{dy}{dx} = 6x - 7$ soi	**B1**	
	$m_{\text{normal}} = -\frac{1}{5}$ soi	**B1**	finds or uses correct gradient of normal
	$m_{\text{tangent}} = 5$ soi or $(6x-7)\left(-\frac{1}{5}\right) = -1$ oe	**M1**	uses $m_1 m_2 = -1$ with numerical gradients
	$6x - 7 = 5$ oe $\Rightarrow x = 2$	**A1**	
	$y = 9$	**A1**	
	$k = 47$	**A1**	
	Alternative method $m_{\text{normal}} = -\frac{1}{5}$	**B1**	
	$m_{\text{tangent}} = 5$	**M1**	
	$3x^2 - 12x + 11 - c = 0$ oe	**A1**	
	solving $3x^2 - 12x + 12 = 0$ oe to find $x = 2$	**A1**	
	$y = 9$	**A1**	
	$k = 47$	**A1**	
5(i)	$\left(\textit{their}\, 2x^4\right)(0.2 - \ln 5x) + 0.4x^5\left(\textit{their}\,\frac{-5}{5x}\right)$ oe or $\textit{their}\, 0.4x^4 - \left(\left(\textit{their}\, 2x^4\right)\ln 5x + 0.4x^5\left(\textit{their}\,\frac{5}{5x}\right)\right)$ oe	**M1**	clearly applies correct form of product rule
	$-2x^4 \ln 5x$ isw	**A1**	nfww
5(ii)	$3\ln 5x$ or $\ln 5x + \ln 5x + \ln 5x$	**B1**	

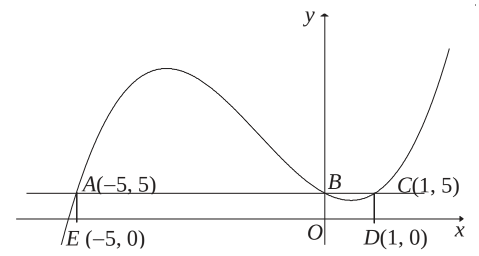
Question	Answer	Marks	Partial Marks
5(iii)	$\frac{3}{-2}\int\left(-2x^4\ln 5x\right)\mathrm{d}x$ oe	**M1**	**FT** $k = 2$ from (i) allow for $\frac{3}{2}\int\left(2x^4\ln 5x\right)\mathrm{d}x$ or, when $k = -2$, for $\int\left(x^4\ln 5x\right)\mathrm{d}x = -0.2x^5(0.2 - \ln 5x)$ or $-\frac{2}{3}\int\left(3x^4\ln 5x\right)\mathrm{d}x = 0.4x^5(0.2 - \ln 5x)$ oe or, when FT $k = 2$, for $\int\left(x^4\ln 5x\right)\mathrm{d}x = 0.2x^5(0.2 - \ln 5x)$ or $\frac{2}{3}\int\left(3x^4\ln 5x\right)\mathrm{d}x = 0.4x^5(0.2 - \ln 5x)$ oe
	$-\frac{3}{2}\left(0.4x^5(0.2 - \ln 5x)\right)[+c]$ oe isw cao	**A1**	nfww; implies M1 An answer of $0.6x^5(0.2 - \ln 5x)$ following $k = 2$ from (i) implies M1 A0
6	Uses $b^2 - 4ac$	**M1**	
	$(p-q)^2 - 4(p)(-q)$	**A1**	implies **M1**
	$p^2 + 2pq + q^2$	**M1**	correctly simplifies
	$(p+q)^2 \geqslant 0$ oe cao isw	**A1**	
	Alternative method $(px-q)(x+1)\quad[=0]$ or $\frac{-(p-q)\pm\sqrt{(p+q)^2}}{2p}$	**M2**	or **M1** for $(px+q)(x-1)\quad[=0]$ or $\frac{-(p-q)\pm\sqrt{(p-q)^2-4(p)(-q)}}{2p}$
	$x = \frac{q}{p},\quad x = -1$	**A1**	
	for conclusion, e.g. p and q are real therefore $\frac{q}{p}$ is real [and –1 is real]	**A1**	
7(a)(i)	7	**B1**	
7(a)(ii)	$\frac{1}{7}$ or $\frac{1}{their7}$	**B1**	**FT** *their* 7 must not be 1 if following through

Question	Answer	Marks	Partial Marks
7(b)	$y=81^{-\frac{1}{4}}$ or $y=3^{-1}$ or $y=9^{-\frac{1}{2}}$ oe	**M1**	Anti-logs
	$y=\frac{1}{3}$ only or 0.333[3….] only	**A1**	nfww; implies the M1; y = …. must be seen at least once If M0 then **SC1** for e.g. $81^{-\frac{1}{4}}=\frac{1}{3}$ as final answer
7(c)	$\frac{2^{5(x^2-1)}}{(2^2)^{x^2}}$ oe or $\frac{4^{\frac{5}{2}(x^2-1)}}{4^{x^2}}$ oe or $\frac{32^{x^2}\times 32^{-1}}{4^{x^2}}$ or $\log 32^{x^2-1}-\log 4^{x^2}=\log 16$ oe	**B1**	converts the terms given left hand side to powers of 2 or 4; may have cross-multiplied or separates the power in the numerator correctly or applies a correct log law
	$2^{3x^2-5}=16$ oe $\Rightarrow 3x^2-5=4$ oe or $4^{\frac{3}{2}x^2-\frac{5}{2}}=16$ oe $\Rightarrow \frac{3}{2}x^2-\frac{5}{2}=2$ oe or $\frac{8^{x^2}}{32}=16$ oe $\Rightarrow x^2\log 8=\log 512$ oe or $(x^2-1)\log 32-x^2\log 4=\log 16$ oe	**M1**	combines powers and takes logs or equates powers; or brings down all powers for an equation already in logs condone omission of necessary brackets for M1; condone one slip
	$[x=]\pm\sqrt{3}$ isw cao or ± 1.732050... rot to 3 or more figs. isw	**A1**	
8(i)	$y-8=-\frac{8}{12}(x-(-8))$ oe isw or $y[-0]=-\frac{8}{12}(x-4)$ oe isw or $3y=-2x+8$ oe isw	**B2**	**B1** for $m_{AB}=-\frac{8}{12}$ oe or **M1** for $\frac{8-0}{-8-4}$ oe
8(ii)	$(-8-4)^2+(8[-0])^2$ oe	**M1**	any valid method
	$\sqrt{208}$ isw or $4\sqrt{13}$ isw or 14.4222051… rot to 3 or more sf	**A1**	implies **M1** provided nfww

Question	Answer	Marks	Partial Marks
8(iii)	[coordinates of *D* =] (–2, 4) soi	**B1**	If coordinates of *D* not stated then a calculation for m_{CD} or a relevant length with the coordinates clearly embedded must be shown to imply **B1**
	Gradient methods: $\left[m_{CD} = \frac{7 - \mathit{their}4}{0 - \mathit{their}(-2)} = \right] \quad \mathit{their}\left(\frac{3}{2}\right)$  	**M1**	or **Length of sides methods:** finds or states $AC^2 = 65$ or $AC = \sqrt{65}$ or $AC^2 = (-8-0)^2 + (8-7)^2$ oe or $AC = \sqrt{(-8-0)^2 + (8-7)^2}$ oe **and** $CD^2 = \mathit{their}13$ or $CD = \mathit{their}\sqrt{13}$ or $CD^2 = (0 - \mathit{their}(-2))^2 + (7 - \mathit{their}4)^2$ oe or $CD = \sqrt{(0 - \mathit{their}(-2))^2 + (7 - \mathit{their}4)^2}$ oe **and** $AD^2 = \mathit{their}52$ or $AD = \mathit{their}2\sqrt{13}$ or $AD^2 = (-8 - \mathit{their}(-2))^2 + (8 - \mathit{their}4)^2$ or $AD = \sqrt{(-8 - \mathit{their}(-2))^2 + (8 - \mathit{their}4)^2}$ or uses a valid method with *their* coordinates of *D* to find the exact area of the triangle and equates to $\frac{1}{2}(AD)(CD)\sin(ADC)$
	states $\frac{3}{2} \times \left(-\frac{8}{12}\right) = -1$ oe or $\frac{3}{2}$ is the negative reciprocal of $-\frac{2}{3}$ oe or finds the equation of the perpendicular bisector of *AB* as $y = \frac{3}{2}x + 7$ independently of *C* and states that *C* lies on this line.	**A1**	applies Pythagoras to confirm, using integer values, that 65 = 13 + 52 or finds e.g. $AC = \sqrt{65}$ using $\sqrt{(2\sqrt{13})^2 + (\sqrt{13})^2}$ or solves $\frac{1}{2}\left(2\sqrt{13}\right)\left(\sqrt{13}\right)\sin ADC = 13$ or $\left(\sqrt{65}\right)^2 = (2\sqrt{13})^2 + (\sqrt{13})^2 - 2(2\sqrt{13})(\sqrt{13})\cos ADC$ to show *ADC* is a right angle
8(iv)	$\begin{pmatrix} -4 \\ 1 \end{pmatrix}$ or $-4\mathbf{i} + \mathbf{j}$	**B1**	condone coordinates

Question	Answer	Marks	Partial Marks
8(v)	Full valid method e.g. for **showing** that e.g. $\overrightarrow{CB} = \begin{pmatrix}4\\0\end{pmatrix} - \begin{pmatrix}0\\7\end{pmatrix} = \begin{pmatrix}4\\-7\end{pmatrix}$ or **showing** that e.g. $\overrightarrow{AC} = \begin{pmatrix}0\\7\end{pmatrix} - \begin{pmatrix}-8\\8\end{pmatrix} = \begin{pmatrix}8\\-1\end{pmatrix}$ oe **and** $\overrightarrow{EB} = \begin{pmatrix}4\\0\end{pmatrix} - \begin{pmatrix}-4\\-1\end{pmatrix} = \begin{pmatrix}8\\-1\end{pmatrix}$ oe or comparing gradients of both pairs of opposite sides and showing they are pairwise the same or comparing the lengths of both pairs of opposite sides and showing that they are pairwise the same or showing that length *AC* = length *AE* or that the length *BC* = length *BE* or comparing the gradients and lengths of a pair of opposite sides or showing that *D* is the midpoint of *CE* or showing that length *DC* = length *DE* and that *C*, *D* and *E* are collinear	**B2**	**B1** for incomplete method e.g. for **stating** that $\overrightarrow{CB} = \begin{pmatrix}4\\-7\end{pmatrix}$ or $\overrightarrow{AC} = \begin{pmatrix}8\\-1\end{pmatrix} = \overrightarrow{EB}$ or just showing that one pair of opposite sides is parallel or has the same length or just showing that length *DC* = length *DE* or just showing that *C*, *D* and *E* are collinear 
9(i)	$2(x-1.5)^2 + 0.5$ isw	**B3**	or **B3** for *a* = 2 and *b* = 1.5 and *c* = 0.5 provided not from wrong format isw or **B2** for $2(x-1.5)^2 + c$ where $c \neq 0.5$ or *a* = 2 and *b* = 1.5 or **SC2** for $2(x-1.5) + 0.5$ or $2\left((x-1.5)^2 + \frac{1}{4}\right)$ seen or **B1** for $(x-1.5)^2$ seen or for *b* = 1.5 or for *c* = 0.5 or **SC1** for 3 correct values seen in incorrect format e.g. $2(x-1.5x)+0.5$ or $2(x^2-1.5)+0.5$

Question	Answer	Marks	Partial Marks
9(ii)		**B3**	**B1** for correct graph for f over correct domain or correct graph for f – 1 over correct domain **B1** for vertex marked for f or f – 1 and intercept marked for f or f – 1 **B1** for idea of symmetry – either symmetrical by eye, ignoring any scale or line $y = x$ drawn and labelled Maximum of 2 marks if not fully correct
9(iii)	$\frac{x-0.5}{2}=\left(y-1.5\right)^2$	**M1**	**FT** *their a,b,c*, provided *their a* $\neq$1 and *a,b,c* are all non-zero constants or $\frac{y-0.5}{2}=\left(x-1.5\right)^2$ and reverses variables at some point
	$\mathrm{f}^{-1}(x)=1.5-\sqrt{\frac{x-0.5}{2}}$ oe	**A1**	must have selected negative square root only; condone y = ... etc.; must be in terms of x
			If M0 then **SC2** for $\mathrm{f}^{-1}(x)=\frac{6-\sqrt{8x-4}}{4}$ oe or **SC1** for $\mathrm{f}^{-1}(x)=\frac{-(-6)\pm\sqrt{36-4(2)(5-x)}}{2(2)}$ oe
	$x \geqslant \frac{1}{2}$ oe	**B1**	
10(i)	$\sin^{-1}\left(\frac{3}{4}\right)$ soi	**M1**	implied by 0.848[06…]
	0.848[06…] rot to 3 or more figs or 2.29[35…] rot to 3 or more figs	**M1**	implied by a correct answer of acceptable accuracy
	0.544 486... rot to 3 or more figs isw	**A1**	
	1.03 or 1.02630... rot to 4 or more figs isw	**A1**	Maximum 3 marks if extra angles in range; no penalty for extra values outside range $0 \leqslant x \leqslant \frac{\pi}{2}$

Question	Answer	Marks	Partial Marks
10(ii)	Correctly uses $\tan^2 y = \sec^2 y - 1$ and/or $\frac{\sin y}{\cos y}$ **and** $\sin^2 y = 1 - \cos^2 y$	**M1**	for using correct relationship(s) to find an equation in terms of a single trigonometric ratio
	$3\sec^2 y - 14\sec y - 5 = 0$ $\Rightarrow (3\sec y + 1)(\sec y - 5)$ or $5\cos^2 y + 14\cos y - 3 = 0$ $\Rightarrow (5\cos y - 1)(\cos y + 3)$	**DM1**	for factorising or solving their 3-term quadratic dependent on the first M1 being awarded
	$[\cos y = -3]\ \cos y = \frac{1}{5}$	**A1**	
	78.5 or 78.4630... rot to 2 or more decimal places isw	**A1**	
	281.5 or 281.536.... rot to 2 or more decimal places isw	**A1**	Maximum 4 marks if extra angles in range; no penalty for extra values outside range $0 \leqslant x \leqslant 360$
11(i)	$\frac{x^4}{4} + \frac{4x^3}{3} - \frac{5x^2}{2} + 5x\,[+c]$ isw	**B2**	**B1** for any 3 correct terms
11(ii)	$x^3 + 4x^2 - 5x + 5 = 5$ and rearrange to $x\left(x^2 + 4x - 5\right) = 0$ oe soi	**B1**	
	Solves *their* $x^2 + 4x - 5\,[= 0]$ soi	**M1**	
	$x = -5,\ x = 1$ soi	**A1**	
	$OEAB = 25,\ OBCD = 5$	**A1**	

Question	Answer	Marks	Partial Marks
11(iii)	Correct or correct **FT** substitution of 0, *their* –5 seen in $\left[\frac{x^4}{4}+\frac{4x^3}{3}-\frac{5x^2}{2}+5x\right]_{their-5}^{0}$	**M1**	dependent on at least **B1** in (i)
	Correct or correct **FT** substitution of *their* 1, 0 seen in $\left[\frac{x^4}{4}+\frac{4x^3}{3}-\frac{5x^2}{2}+5x\right]_{0}^{their1}$	**M1**	dependent on at least **B1** in (i)
	$their\frac{1175}{12} - theirOEAB + theirOBCD - their\frac{49}{12}$ oe	**M1**	for the strategy needed to combine the areas; may be in steps; $97.91\dot{6} - 25 + 5 - 4.08\dot{3}$
	$\frac{886}{12}$ oe or $73\frac{5}{6}$ oe or $73.8\dot{3}$ rot to 3 or more sig figs	**A1**	all method steps must be seen; not from wrong working If M0 then allow **SC3** for $\int_{-5}^{0}(x^3+4x^2-5x)\,dx - \int_{0}^{1}(x^3+4x^2-5x)\,dx$ oe $=\left[\frac{x^4}{4}+\frac{4x^3}{3}-\frac{5x^2}{2}\right]_{-5}^{0} - \left[\frac{x^4}{4}+\frac{4x^3}{3}-\frac{5x^2}{2}\right]_{0}^{1}$ $=\left[0-\left(\frac{625}{4}-\frac{500}{3}-\frac{125}{2}\right)\right]-\left[\left(\frac{1}{4}+\frac{4}{3}-\frac{5}{2}\right)-0\right]$ $=\frac{443}{6}$ oe or **SC2** for $\int_{their(-5)}^{0}(x^3+4x^2-5x)\,dx - \int_{0}^{their1}(x^3+4x^2-5x)\,dx$ oe $=\left[\frac{x^4}{4}+\frac{4x^3}{3}-\frac{5x^2}{2}\right]_{their(-5)}^{0} - \left[\frac{x^4}{4}+\frac{4x^3}{3}-\frac{5x^2}{2}\right]_{0}^{their1}$ $=\left[F(0)-F(their(-5))\right]-\left[F(their1)-F(0)\right]$
12(i)	$-6(2x+1)^{-2}$ or $\frac{-6}{(2x+1)^2}$ oe isw	**B1**	Allow $-3(2x+1)^{-2}\times 2$ or $\frac{-3\times 2}{(2x+1)^2}$ oe
	Denominator or $(2x+1)^2$ is positive [and numerator negative therefore $g'(x)$ is always negative] oe	**B1**	**FT** *their* $g'(x)$ of the form $\frac{-k}{(2x+1)^2}$ oe where $k > 0$; Allow $(2x+1)^{-2}$ is always positive
12(ii)	$g > 0$	**B1**	
12(iii)	$\frac{3k}{2x+1}+3$ oe isw	**B1**	

Question	Answer	Marks	Partial Marks
12(iv)	$\frac{3k}{2(0)+1}+3=5$	**B1**	
	$k=\frac{2}{3}$ isw	**B1**	implies the first **B1**
12(v)	$x>-\frac{1}{2}$	**B1**	

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

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ADDITIONAL MATHEMATICS **0606/23**

Paper 2 **May/June 2017**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **7** printed pages.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method marks, awarded for a valid method applied to the problem.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
B	Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(a)	$\log_7 2.5 = 2x+5$ or $\log_7\left(\frac{2.5}{7^5}\right) = 2x$ or $(2x+5)\log 7 = \log 2.5$	**M1**	correct first anti-logging step
	$[x=]\frac{\log_7 2.5 - 5}{2}$ or $\frac{1}{2}\log_7\left(\frac{2.5}{7^5}\right) = x$ or $x = \frac{1}{2}\left(\frac{\log 2.5}{\log 7} - 5\right)$	**M1**	isolates *x*
	–2.26(4...)	**A1**	
1(b)	$5^2 p^{-3} q^{\frac{5}{4}}$ oe	**B3**	**B1** for each term If B0 then allow **M1** for numerator of $125q^{\frac{3}{2}}$ or denominator of $5p^3q^{\frac{1}{4}}$
2(i)	*B* and *C* with valid reason	**B2**	**B1** for one graph and valid reason or both graphs and no reason
2(ii)	*B* only with valid reason	**B2**	**B1** for graph *B* or valid reason
3	$[m=]\frac{13-5}{1-0.2}$ or 10 soi	**M1**	or $13 = m + c$ and $5 = 0.2m + c$ and subtracting/substituting to solve for *m* or *c*, condone one error
	$Y - 13 = \textit{their } 10(X - 1)$ or $Y - 5 = \textit{their } 10(X - 0.2)$ or $13 = \textit{their } 10 + c$ or $5 = \textit{their } 10 \times 0.2 + c$	**M1**	or using *their m* or *their c* to find *their c* or *their m*, without further error
	$\sqrt[3]{y} = (\textit{their } m)\frac{1}{x} + (\textit{their } c)$ or $\sqrt[3]{y} = (\textit{their } m)\left(\frac{1}{x} - 1\right) + 13$ or $\sqrt[3]{y} = (\textit{their } m)\left(\frac{1}{x} - 0.2\right) + 5$	**M1**	*their m* and *c* must be validly obtained
	$y = \left(\frac{10}{x} + 3\right)^3$ or $y = \left(10\left(\frac{1}{x} - 1\right) + 13\right)^3$ or $y = \left(10\left(\frac{1}{x} - 0.2\right) + 5\right)^3$ cao, isw	**A1**	

Question	Answer	Marks	Guidance
4(a)(i)	$\begin{pmatrix}-4\\3\end{pmatrix}$	**B1**	
4(a)(ii)	$\sqrt{11^2+(-15)^2}$ or better	**M1**	
	$\frac{1}{\sqrt{346}}\begin{pmatrix}11\\-15\end{pmatrix}$	**A1**	
4(b)	$\overrightarrow{OR}=\overrightarrow{OP}+\frac{3}{4}\overrightarrow{PQ}$ soi	**M1**	or $\overrightarrow{OR}=\overrightarrow{OQ}-\frac{1}{4}\overrightarrow{PQ}$ soi
	$\left[\overrightarrow{OR}=\right]\mathbf{p}+\frac{3}{4}(\mathbf{q}-\mathbf{p})$	**M1**	or $\left[\overrightarrow{OR}=\right]\mathbf{q}-\frac{1}{4}(\mathbf{q}-\mathbf{p})$
	$\left[\overrightarrow{OR}=\right]\frac{1}{4}\mathbf{p}+\frac{3}{4}\mathbf{q}$ oe	**A1**	
5(a)	$(9\times8\times7\times6\times1)+(8\times8\times7\times6\times1)$ soi	**M2**	**M1** for one correct product of the sum
	5712	**A1**	
5(b)	${}^9C_4\times{}^5C_4+{}^9C_3\times{}^5C_5$ oe	**M2**	**M1** for one correct product of the sum
	[630 + 84 =] 714	**A1**	
6	$64=2^n$	**M1**	
	$n = 6$	**A1**	
	$their\,6(2)^{their\,(6-1)}\times(-a)=-16b$ oe	**M1**	
	$their\,\frac{6\times(6-1)}{2}(2)^{their\,(6-2)}\times(-a)^2=100b$ oe	**M1**	
	attempts to solve	**DM1**	dep on both M1 marks being awarded; must have correctly or correct FT eliminated one unknown
	$a = 5$	**A1**	
	$b = 60$	**A1**	

Question	Answer	Marks	Guidance
7(i)	$k\left(1+4x\right)^9$	**M1**	
	$4\times 10\left(1+4x\right)^9$ or better	**A1**	
	$(1+4x)^{10}(\mathit{their}-\sin x)+$ $\cos x\left(\mathit{their}\left(4\times 10\times\left(1+4x\right)^9\right)\right)$	**M1**	clearly applies product rule
	$(1+4x)^{10}(-\sin x)+\cos x\left(4\times 10\times\left(1+4x\right)^9\right)$	**A1**	all correct
7(ii)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{4x-5}\right)=4\mathrm{e}^{4x-5}$ soi	**B1**	
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\tan x\right)=\sec^2 x$ soi	**B1**	
	clearly applies correct form of quotient rule $\frac{\tan x\left(\mathit{their}\ 4\mathrm{e}^{4x-5}\right)-\mathrm{e}^{4x-5}\left(\mathit{their}\sec^2 x\right)}{\left(\tan x\right)^2}$	**M1**	or correct form of product rule to $\mathrm{e}^{4x-5}(\tan x)^{-1}$ $4\mathrm{e}^{4x-5}(\tan x)^{-1}+\mathrm{e}^{4x-5}(\tan x)^{-2}\times\sec^2 x$
	$\frac{\tan x\left(4\mathrm{e}^{4x-5}\right)-\mathrm{e}^{4x-5}\left(\sec^2 x\right)}{\left(\tan x\right)^2}$ isw	**A1**	all correct
8(i)	$\frac{\pi}{3}$	**B1**	
	6 [cm]	**B1**	
8(ii)	[major arc =]$\left(2\pi-\mathit{their}\frac{\pi}{3}\right)\mathit{their}\ r$	**M1**	
	10π + 6 cao	**A1**	
8(iii)	$\frac{1}{2}(\mathit{their}6)^2\left(2\pi-\mathit{their}\frac{\pi}{3}\right)$	**M1**	$\frac{1}{2}(\mathit{their}6)^2\left(\mathit{their}\frac{\pi}{3}\right)$
	$\frac{1}{2}(\mathit{their}6)^2\sin\left(\mathit{their}\frac{\pi}{3}\right)$	**M1**	$\frac{1}{2}(\mathit{their}6)^2\sin\left(\mathit{their}\frac{\pi}{3}\right)$
	Sector + triangle	**M1**	$\pi\times\mathit{their}6^2$ – (Sector – triangle)
	$30\pi+9\sqrt{3}$	**A1**	

Question	Answer	Marks	Guidance
9(i)	$\frac{y}{9}=\sqrt{x-1}$ with attempt to swop x and y at some point or $\frac{x}{9}=\sqrt{y-1}$	**M1**	attempt to swop; may be in later work that contains an error
	$\left[\mathrm{f}^{-1}(x)=\right]\left(\frac{x}{9}\right)^2+1$ oe	**A1**	condone $y=$ ... etc; must be a function of x
	$x > 0$	**B1**	
9(ii)	f(51)	**M1**	or $\mathrm{fg}(x)=9\sqrt{x^2+1}$
	$9\sqrt{50}$ oe	**A1**	
9(iii)	$[\mathrm{gf}(x)=]\left(9\sqrt{x-1}\right)^2+2$	**M1**	
	$[\mathrm{gf}(x)=]\,81(x-1)+2$ or better	**A1**	
	$\mathit{their}\left(81x-79\right)=5x^2+83x-95\rightarrow$ $\mathit{their}\left(5x^2+2x-16\,[=0]\right)$	**M1**	provided $\mathit{their}(81x-79)$ of the form $ax+b$ for non-zero a and b
	1.6 oe only	**A1**	must disregard other solution
10(a)	$\sin x=0.5$, $\sin x=-0.5$	**M1**	
	$\frac{\pi}{6},\ -\frac{\pi}{6},\ \frac{5\pi}{6},\ -\frac{5\pi}{6}$ oe	**A2**	**A1** for any correct pair of angles if M0 then **SC1** for a correct pair of angles
10(b)	$2y+15=\tan^{-1}\left(\frac{1}{3}\right)$ soi	**M1**	
	18.43(49...) and 198.43(49...)	**M1**	
	1.7, 91.7	**A2**	**A1** for each

Question	Answer	Marks	Guidance
10(c)	Uses $\cot^2 z = \operatorname{cosec}^2 z - 1$ oe	M1	for using correct identity or identities to obtain an equation in terms of a single trigonometric ratio
	$2\operatorname{cosec}^2 z + 7\operatorname{cosec} z - 4 = 0 \Rightarrow$ $(2\operatorname{cosec} z - 1)(\operatorname{cosec} z + 4)$	DM1	for dealing with quadratic
	$[\sin z = 2\ ]\ \sin z = -\frac{1}{4}$	M1	
	194.5, 345.5	A2	A1 for each
11(i)	$5+\sqrt{10x} = \frac{5x+20}{4} \to \cancel{20} + 4\sqrt{10x} = 5x + \cancel{20}$	M1	or better; equates and solves as far as clearing the fraction
	$\left[\frac{x}{\sqrt{x}} = \right]\sqrt{x} = \frac{4\sqrt{10}}{5}$ oe	M1	Simplifies as far as $\sqrt{x} = \cdots$
	$x = 6.4$ cao	A1	squares and simplifies to 6.4
	$[y=]13$	B1	
11(ii)	(area of trapezium $=$) *their* 57.6	B1	FT $x =$ *their* 6.4, $y =$ *their* 13 using any valid method
	$\int_0^{6.4}\left(5+\sqrt{10x}\right)\mathrm{d}x$	M1	
	$\int (10x)^{\frac{1}{2}}\,\mathrm{d}x = k\,(10x)^{\frac{3}{2}}$ or	M1	or $\int \sqrt{10}x^{\frac{1}{2}}\,\mathrm{d}x = k\,\sqrt{10}\,(x)^{\frac{3}{2}}$
	$\left[5x + \frac{2(10x)^{\frac{3}{2}}}{3\times 10}\right]$	A1	or $\left[5x + \frac{2(10)^{\frac{1}{2}}(x)^{\frac{3}{2}}}{3}\right]$
	their $\left[5(6.4) + \frac{2(10\times 6.4)^{\frac{3}{2}}}{3\times 10}\right]$ − *their* 57.6 oe	M1	limits used correctly or correct FT and subtraction of trapezium; *their* $\frac{992}{15}$ − *their* 57.6
	$\frac{128}{15}$ or 8.53 oe	A1	allow 8.5333333.... rot to 4 or more sf

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

ADDITIONAL MATHEMATICS	**0606/11**
Paper 1	**May/June 2017**
	2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **12** printed pages.

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[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 The line $y = kx - 5$, where k is a positive constant, is a tangent to the curve $y = x^2 + 4x$ at the point A.

(i) Find the exact value of k. [3]

(ii) Find the gradient of the normal to the curve at the point A, giving your answer in the form $a + b\sqrt{5}$, where a and b are constants. [2]

2 It is given that $p(x) = x^3 + ax^2 + bx - 48$. When $p(x)$ is divided by $x - 3$ the remainder is 6. Given that $p'(1) = 0$, find the value of a and of b. [5]

3 **(a)** Simplify $\sqrt{x^8y^{10}} \div \sqrt[3]{x^3y^{-6}}$, giving your answer in the form x^ay^b, where a and b are integers. [2]

(b) **(i)** Show that $4(t-2)^{\frac{1}{2}} + 5(t-2)^{\frac{3}{2}}$ can be written in the form $(t-2)^p(qt+r)$, where p, q and r are constants to be found. [3]

(ii) Hence solve the equation $4(t-2)^{\frac{1}{2}} + 5(t-2)^{\frac{3}{2}} = 0$. [1]

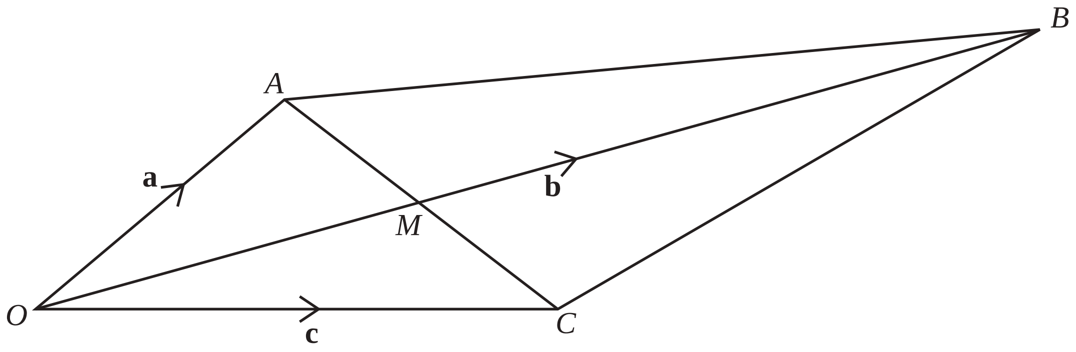
4 **(a)** It is given that $f(x) = 3e^{-4x} + 5$ for $x \in \mathbb{R}$.

(i) State the range of f. [1]

(ii) Find f^{-1} and state its domain. [4]

(b) It is given that $g(x) = x^2 + 5$ and $h(x) = \ln x$ for $x > 0$. Solve $hg(x) = 2$. [3]

5 **(a)**



The diagram shows a figure *OABC*, where $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$. The lines *AC* and *OB* intersect at the point *M* where *M* is the midpoint of the line *AC*.

(i) Find, in terms of **a** and **c**, the vector $\overrightarrow{OM}$. [2]

(ii) Given that $OM : MB = 2:3$, find **b** in terms of **a** and **c**. [2]

(b) Vectors **i** and **j** are unit vectors parallel to the x-axis and y-axis respectively.

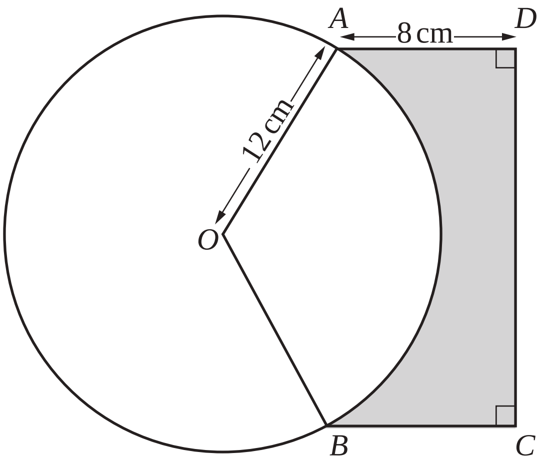
The vector **p** has a magnitude of 39 units and has the same direction as $-10\mathbf{i} + 24\mathbf{j}$.

(i) Find **p** in terms of **i** and **j**. [2]

(ii) Find the vector **q** such that $2\mathbf{p} + \mathbf{q}$ is parallel to the positive y-axis and has a magnitude of 12 units. [3]

(iii) Hence show that $|\mathbf{q}| = k\sqrt{5}$, where k is an integer to be found. [2]

6



The diagram shows a circle, centre *O*, radius 12 cm. The points *A* and *B* lie on the circumference of the circle and form a rectangle with the points *C* and *D*. The length of *AD* is 8 cm and the area of the minor sector *AOB* is 150 cm^2.

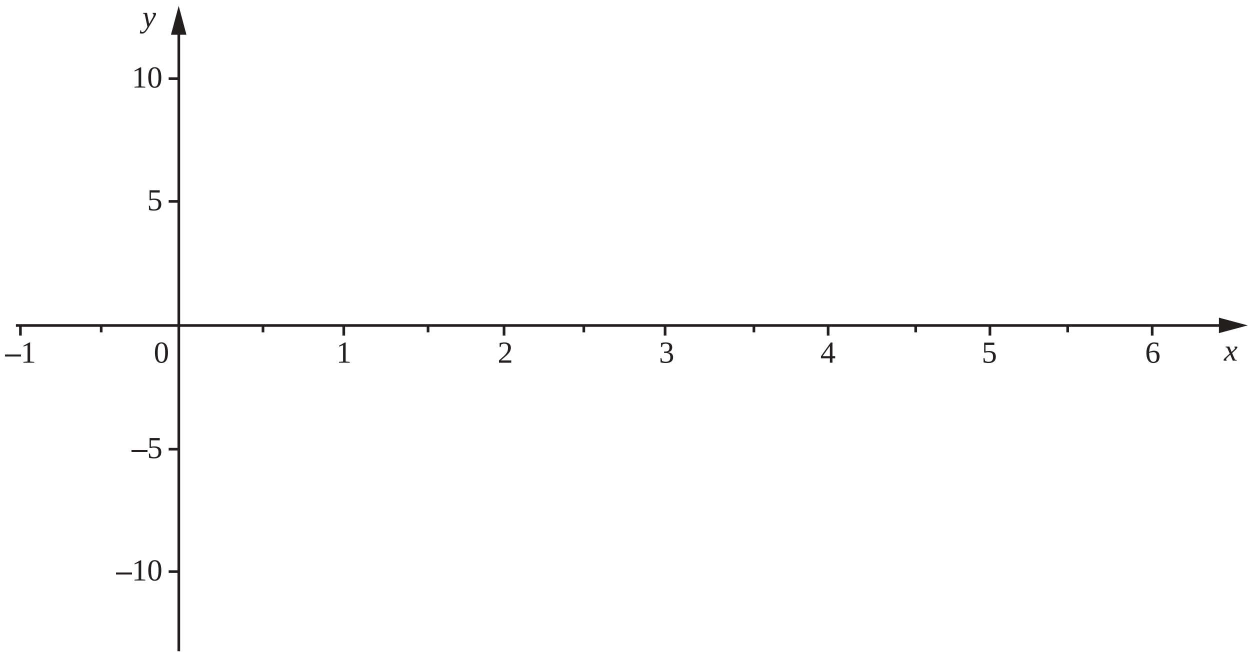
(i) Show that angle *AOB* is 2.08 radians, correct to 2 decimal places. [2]

(ii) Find the area of the shaded region *ADCB*. [6]

(iii) Find the perimeter of the shaded region *ADCB*. [3]

7 Show that the curve $y = (3x^2 + 8)^{\frac{5}{3}}$ has only one stationary point. Find the coordinates of this stationary point and determine its nature. [8]

8 **(i)** On the axes below sketch the graphs of $y = |2x - 5|$ and $9y = 80x - 16x^2$. [5]



(ii) Solve $|2x - 5| = 4$. [3]

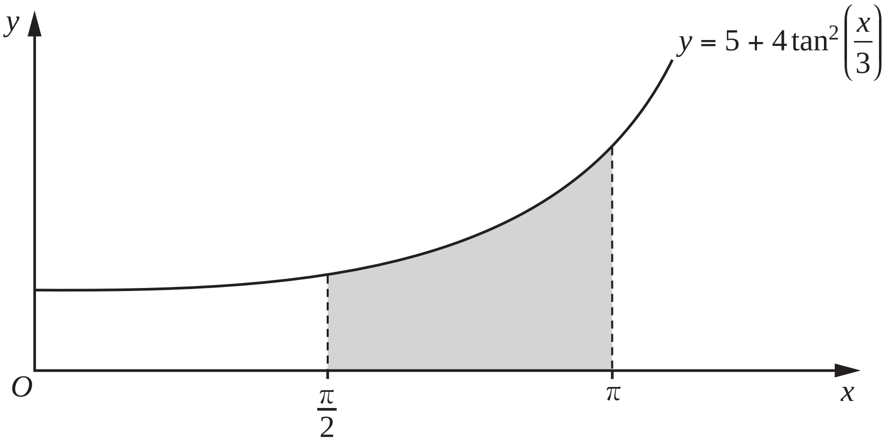
(iii) Hence show that the graphs of $y = |2x - 5|$ and $9y = 80x - 16x^2$ intersect at the points where $y = 4$. [1]

(iv) Hence find the values of x for which $9|2x - 5| \leqslant 80x - 16x^2$. [2]

9 **(i)** Show that $5 + 4\tan^2\left(\frac{x}{3}\right) = 4\sec^2\left(\frac{x}{3}\right) + 1$. [1]

(ii) Given that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\tan\left(\frac{x}{3}\right)\right) = \frac{1}{3}\sec^2\left(\frac{x}{3}\right)$, find $\int \sec^2\left(\frac{x}{3}\right)\mathrm{d}x$. [1]

(iii)



The diagram shows part of the curve $y = 5 + 4\tan^2\left(\frac{x}{3}\right)$. Using the results from parts **(i)** and **(ii)**, find the exact area of the shaded region enclosed by the curve, the x-axis and the lines $x = \frac{\pi}{2}$ and $x = \pi$. [5]

Question 10 is printed on the next page.

10 **(a)** Given that $y = \frac{e^{3x}}{4x^2 + 1}$, find $\frac{dy}{dx}$. [3]

(b) Variables *x*, *y* and *t* are such that $y = 4\cos\left(x + \frac{\pi}{3}\right) + 2\sqrt{3}\sin\left(x + \frac{\pi}{3}\right)$ and $\frac{dy}{dt} = 10$.

(i) Find the value of $\frac{dy}{dx}$ when $x = \frac{\pi}{2}$. [3]

(ii) Find the value of $\frac{dx}{dt}$ when $x = \frac{\pi}{2}$. [2]

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

0147311657

ADDITIONAL MATHEMATICS **0606/12**

Paper 1 **May/June 2017**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

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Write your Centre number, candidate number and name on all the work you hand in.
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[Turn over

Mathematical Formulae

1. ALGEBRA

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For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

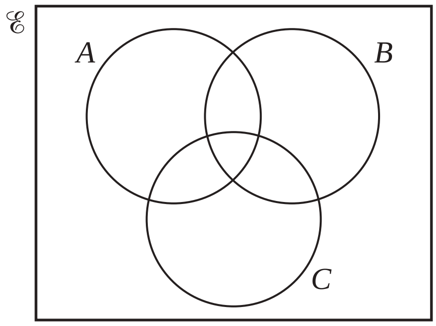
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

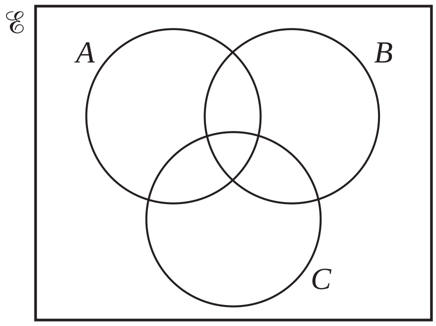
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

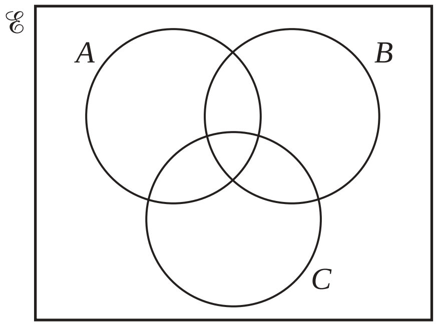
1 On each of the Venn diagrams below, shade the region which represents the given set.



$(A \cup B) \cap C$



$(A \cap B) \cup C$



$(A' \cap B') \cap C$

[3]

2 It is given that $y = \dfrac{(5x^2 + 4)^{\frac{1}{2}}}{x + 1}$. Showing all your working, find the exact value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 3$. [5]

3 Vectors $\mathbf{i}$ and $\mathbf{j}$ are unit vectors parallel to the x-axis and y-axis respectively.

(a) The vector $\mathbf{v}$ has a magnitude of $3\sqrt{5}$ units and has the same direction as $\mathbf{i} - 2\mathbf{j}$. Find $\mathbf{v}$ giving your answer in the form $a\mathbf{i} + b\mathbf{j}$, where a and b are integers. [2]

(b) The velocity vector $\mathbf{w}$ makes an angle of 30° with the positive x-axis and is such that $|\mathbf{w}| = 2$. Find $\mathbf{w}$ giving your answer in the form $\sqrt{c}\,\mathbf{i} + d\mathbf{j}$, where c and d are integers. [2]

4 The first 3 terms in the expansion of $\left(3 - \frac{x}{6}\right)^n$ are $81 + ax + bx^2$. Find the value of each of the constants n, a and b. [5]

5 A particle P moves in a straight line, such that its displacement, x m, from a fixed point O, t s after passing O, is given by $x = 4\cos(3t) - 4$.

(i) Find the velocity of P at time t. [1]

(ii) Hence write down the maximum speed of P. [1]

(iii) Find the smallest value of t for which the acceleration of P is zero. [3]

(iv) For the value of t found in part **(iii)**, find the distance of P from O. [1]

6 **(i)** Show that $\dfrac{\operatorname{cosec}\theta}{\cot\theta + \tan\theta} = \cos\theta$. [4]

It is given that $\displaystyle\int_0^a \frac{\operatorname{cosec}2\theta}{\cot 2\theta + \tan 2\theta}\,\mathrm{d}\theta = \frac{\sqrt{3}}{4}$, where $0 < a < \dfrac{\pi}{4}$.

(ii) Using your answer to part **(i)** find the value of a, giving your answer in terms of π. [4]

7 It is given that $y = A(10^{bx})$, where A and b are constants. The straight line graph obtained when $\lg y$ is plotted against x passes through the points $(0.5, 2.2)$ and $(1.0, 3.7)$.

(i) Find the value of A and of b. [5]

Using your values of A and b, find

(ii) the value of y when $x = 0.6$, [2]

(iii) the value of x when $y = 600$. [2]

8 **(a)** A 5-digit number is to be formed from the seven digits 1, 2, 3, 5, 6, 8 and 9. Each digit can only be used once in any 5-digit number. Find the number of different 5-digit numbers that can be formed if

(i) there are no restrictions, [1]

(ii) the number is divisible by 5, [1]

(iii) the number is greater than 60 000, [1]

(iv) the number is greater than 60 000 and even. [3]

(b) Ranjit has 25 friends of whom 15 are boys and 10 are girls. Ranjit wishes to hold a birthday party but can only invite 7 friends. Find the number of different ways these 7 friends can be selected if

(i) there are no restrictions, [1]

(ii) only 2 of the 7 friends are boys, [1]

(iii) the 25 friends include a boy and his sister who cannot be separated. [3]

9 **(a)** Given that $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ -1 & 2 \\ 4 & 5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$ and $\mathbf{C} = \mathbf{AB}$,

(i) state the order of **A**, [1]

(ii) find **C**. [3]

(b) The matrix $\mathbf{X} = \begin{pmatrix} 5 & -12 \\ 4 & -7 \end{pmatrix}$.

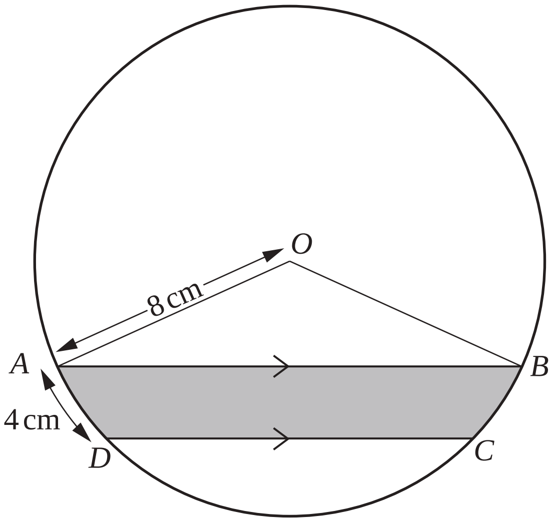
(i) Find $\mathbf{X}^{-1}$. [2]

(ii) Using $\mathbf{X}^{-1}$, find the coordinates of the point of intersection of the lines

$$12y = 5x - 26,$$
$$7y = 4x - 52.$$

[4]

10



The diagram shows a circle, centre *O*, radius 8 cm. The points *A*, *B*, *C* and *D* lie on the circumference of the circle such that *AB* is parallel to *DC*. The length of the arc *AD* is 4 cm and the length of the chord *AB* is 15 cm.

(i) Find, in radians, angle *AOD*. [1]

(ii) Hence show that angle $DOC = 1.43$ radians, correct to 2 decimal places. [3]

(iii) Find the perimeter of the shaded region. [3]

(iv) Find the area of the shaded region. [4]

Question 11 is printed on the next page.

11 The curve $y = \mathrm{f}(x)$ passes through the point $\left(\frac{1}{2}, \frac{7}{2}\right)$ and is such that $\mathrm{f}'(x) = \mathrm{e}^{2x-1}$.

(i) Find the equation of the curve. [4]

(ii) Find the value of x for which $\mathrm{f}''(x) = 4$, giving your answer in the form $a + b\ln\sqrt{2}$, where a and b are constants. [4]

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8262841617

ADDITIONAL MATHEMATICS **0606/13**

Paper 1 **May/June 2017**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

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The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **15** printed pages and **1** blank page.

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CAMBRIDGE
International Examinations

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

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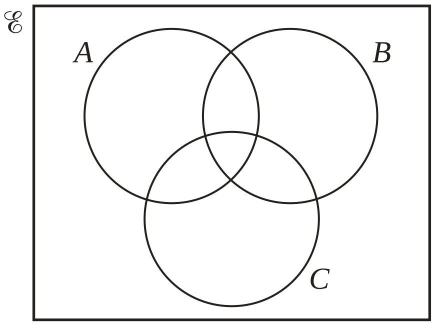
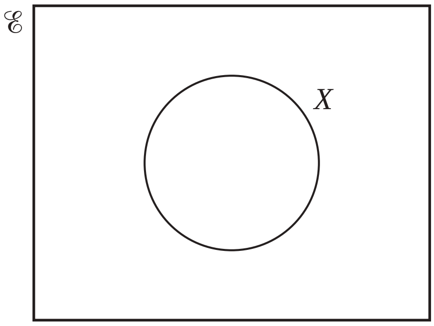
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** On the Venn diagram below, shade the region which represents $(A \cap B') \cup (C \cap B')$. [1]



(b) Complete the Venn diagram below to show the sets Y and Z such that $Z \subset X \subset Y$. [1]

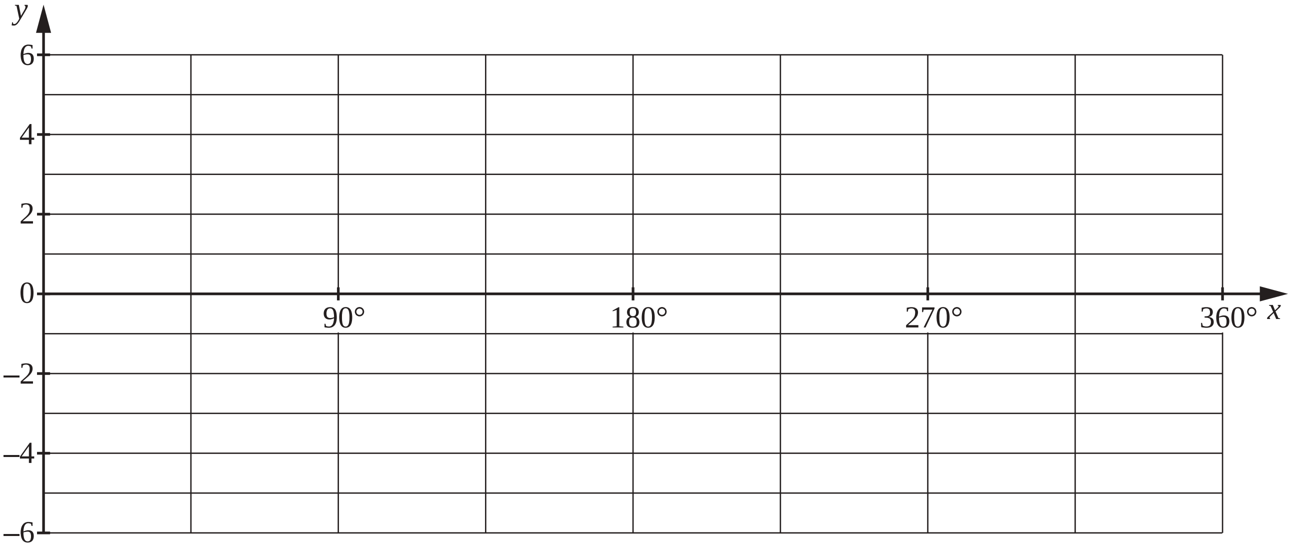


2 Given that $y = 3 + 4\cos 9x$, write down

(i) the amplitude of y, [1]

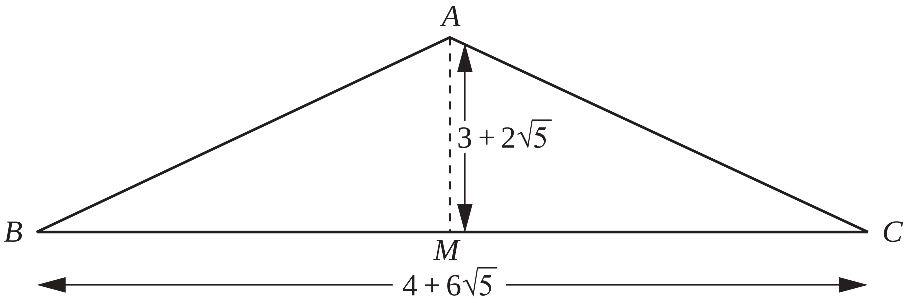
(ii) the period of y. [1]

3 **(i)** On the axes below, sketch the graph of $y = 3\sin x - 2$ for $0° \leqslant \theta \leqslant 360°$. [3]



(ii) Given that $0 \leqslant |3\sin x - 2| \leqslant k$ for $0° \leqslant x \leqslant 360°$, write down the value of k. [1]

4 In this question, all dimensions are in centimetres.



The diagram shows an isosceles triangle *ABC*, where $AB = AC$. The point *M* is the mid-point of *BC*. Given that $AM = 3 + 2\sqrt{5}$ and $BC = 4 + 6\sqrt{5}$, find, **without using a calculator**,

(i) the area of triangle *ABC*, [2]

(ii) tan *ABC*, giving your answer in the form $\dfrac{a + b\sqrt{5}}{c}$ where *a*, *b* and *c* are positive integers. [3]

5 The normal to the curve $y = \sqrt{4x+9}$, at the point where $x = 4$, meets the x- and y-axes at the points A and B. Find the coordinates of the mid-point of the line AB. [7]

6 **(a)** Given that $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 5 & 1 \\ 2 & 4 \\ -1 & 0 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} -5 & 2 \\ 3 & 1 \end{pmatrix}$, find

(i) $\mathbf{A} + 3\mathbf{C}$, [2]

(ii) $\mathbf{BA}$. [2]

(b) **(i)** Given that $\mathbf{X} = \begin{pmatrix} 1 & -3 \\ 4 & -2 \end{pmatrix}$, find $\mathbf{X}^{-1}$. [2]

(ii) Hence find $\mathbf{Y}$, such that $\mathbf{XY} = \begin{pmatrix} 5 & -10 \\ 15 & 20 \end{pmatrix}$. [3]

7 **(a)** Show that $\dfrac{\tan^2\theta + \sin^2\theta}{\cos\theta + \sec\theta} = \tan\theta\sin\theta$. [4]

(b) Given that $x = 3\sin\phi$ and $y = \dfrac{3}{\cos\phi}$, find the numerical value of $9y^2 - x^2y^2$. [3]

8 It is given that $p(x) = 2x^3 + ax^2 + 4x + b$, where a and b are constants. It is given also that $2x + 1$ is a factor of $p(x)$ and that when $p(x)$ is divided by $x - 1$ there is a remainder of -12.

(i) Find the value of a and of b. [5]

(ii) Using your values of a and b, write $p(x)$ in the form $(2x + 1)q(x)$, where $q(x)$ is a quadratic expression. [2]

(iii) Hence find the exact solutions of the equation $p(x) = 0$. [2]

9 It is given that $\int_{-k}^{k} (15e^{5x} - 5e^{-5x})dx = 6$.

(i) Show that $e^{5k} - e^{-5k} = 3$. [5]

(ii) Hence, using the substitution $y = e^{5k}$, or otherwise, find the value of k. [3]

10 It is given that $y = (10x + 2)\ln(5x + 1)$.

(i) Find $\frac{dy}{dx}$. [4]

(ii) Hence show that $\int \ln(5x + 1)\,dx = \frac{(ax + b)}{5}\ln(5x + 1) - x + c$, where a and b are integers and c is a constant of integration. [3]

(iii) Hence find $\int_0^{\frac{1}{5}} \ln(5x+1)\,\mathrm{d}x$, giving your answer in the form $\dfrac{d+\ln f}{5}$, where d and f are integers. [2]

11 A curve has equation $y = 6x - x\sqrt{x}$.

(i) Find the coordinates of the stationary point of the curve. [4]

(ii) Determine the nature of this stationary point. [2]

(iii) Find the approximate change in y when x increases from 4 to $4 + h$, where h is small. [3]

12 A particle moves in a straight line, such that its velocity, v ms^{-1}, t s after passing a fixed point O, is given by $v = 2 + 6t + 3\sin 2t$.

(i) Find the acceleration of the particle at time t. [2]

(ii) Hence find the smallest value of t for which the acceleration of the particle is zero. [2]

(iii) Find the displacement, x m from O, of the particle at time t. [5]

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Cambridge International Examinations
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CANDIDATE NAME

CENTRE NUMBER ☐☐☐☐☐

CANDIDATE NUMBER ☐☐☐☐

6595404132

ADDITIONAL MATHEMATICS **0606/21**

Paper 2 **May/June 2017**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **12** printed pages.

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Mathematical Formulae

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$$\Delta = \frac{1}{2} bc \sin A$$

1 Find the equation of the curve which passes through the point (2, 17) and for which $\frac{\mathrm{d}y}{\mathrm{d}x} = 4x^3 + 1$. [4]

2 **Do not use a calculator in this question.**

(a) Show that $\sqrt{24} \times \sqrt{27} + \frac{9\sqrt{30}}{\sqrt{15}}$ can be written in the form $a\sqrt{2}$, where a is an integer. [3]

(b) Solve the equation $\sqrt{3}(1+x) = 2(x-3)$, giving your answer in the form $b + c\sqrt{3}$, where b and c are integers. [3]

3 The variables x and y are such that $y = \ln(x^2 + 1)$.

(i) Find an expression for $\frac{dy}{dx}$. [2]

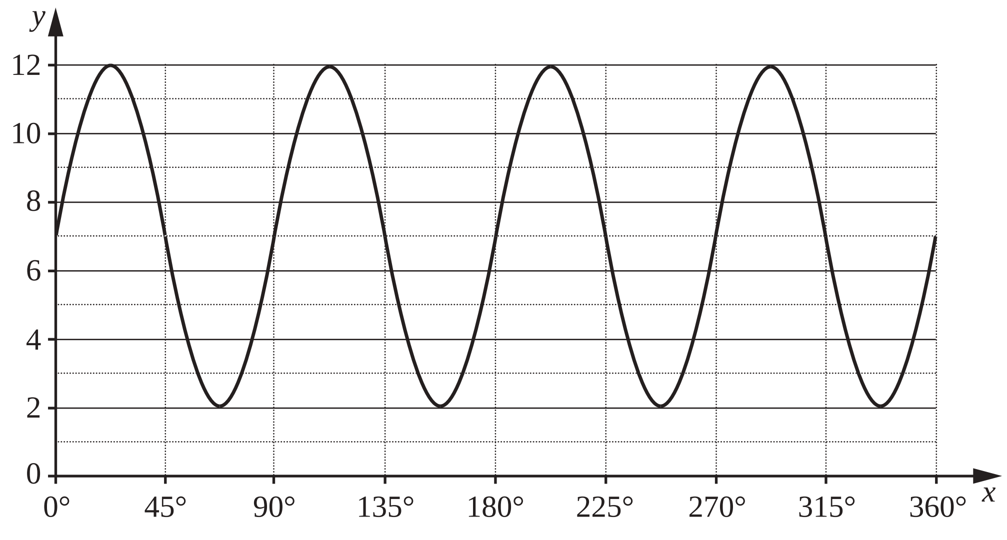
(ii) Hence, find the approximate change in y when x increases from 3 to $3 + h$, where h is small. [2]

4 **(a)** Given that $y = 7\cos 10x - 3$, where the angle x is measured in degrees, state

(i) the period of y, [1]

(ii) the amplitude of y. [1]

(b)



Find the equation of the curve shown, in the form $y = a\mathrm{g}(bx) + c$, where $\mathrm{g}(x)$ is a trigonometric function and a, b and c are integers to be found. [4]

5 **(i)** Given that a is a constant, expand $(2 + ax)^4$, in ascending powers of x, simplifying each term of your expansion. [2]

Given also that the coefficient of x^2 is equal to the coefficient of x^3,

(ii) show that $a = 3$, [1]

(iii) use your expansion to show that the value of 1.97^4 is 15.1 to 1 decimal place. [2]

6 Four cinemas, *P*, *Q*, *R* and *S* each sell adult, student and child tickets. The number of tickets sold by each cinema on one weekday were

P: 90 adult, 10 student, 30 child
Q: 45 student
R: 25 adult, 15 child
S: 10 adult, 100 child.

(i) Given that $\mathbf{L} = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}$, construct a matrix, **M**, of the number of tickets sold, such that the matrix product **LM** can be found. [1]

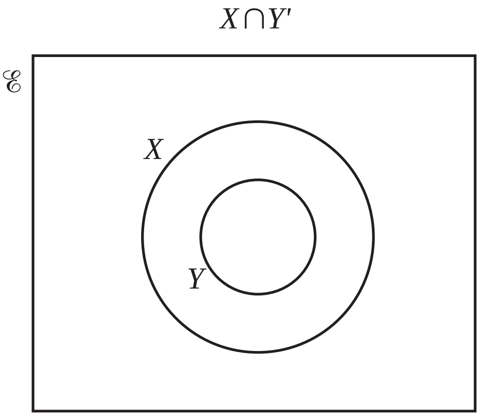
(ii) Find the matrix product **LM**. [1]

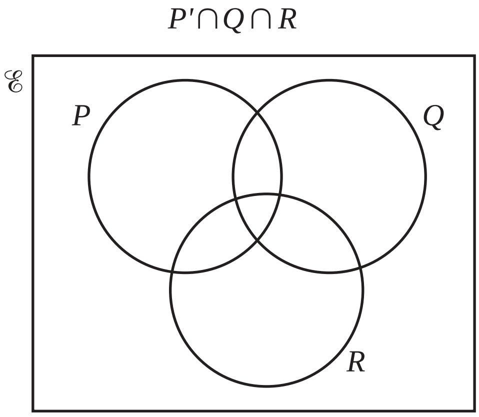
(iii) State what information is represented by the matrix product **LM**. [1]

An adult ticket costs \$5, a student ticket costs \$4 and a child ticket costs \$3.

(iv) Construct a matrix, **N**, of the ticket costs, such that the matrix product **LMN** can be found and state what information is represented by the matrix product **LMN**. [2]

7 **(a)** On each of the Venn diagrams below shade the region which represents the given set.





[2]

(b) In a group of students, each student studies at most two of art, music and design. No student studies both music and design.

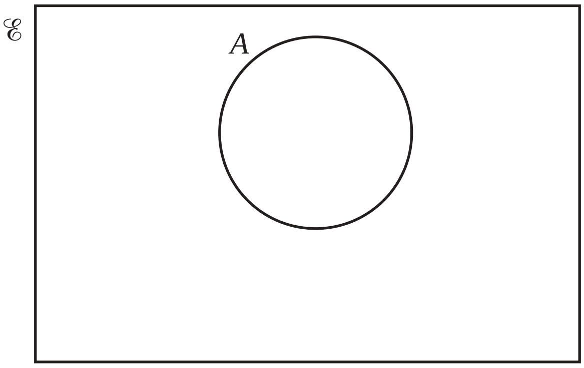
A denotes the set of students who study art,
M denotes the set of students who study music,
D denotes the set of students who study design.

(i) Write the following using set notation.

No student studies both music and design. [1]

There are 100 students in the group. 39 students study art, 45 study music and 36 study design. 12 students study both art and music. 25 students study both art and design.

(ii) Complete the Venn diagram below to represent this information and hence find the number of students in the group who do not study any of these subjects.



[3]

8 **(a)** A football club has 30 players. In how many different ways can a captain and a vice-captain be selected at random from these players? [1]

(b) A team of 11 teachers is to be chosen from 2 mathematics teachers, 5 computing teachers and 9 science teachers. Find the number of different teams that can be chosen if

(i) the team must have exactly 1 mathematics teacher, [2]

(ii) the team must have exactly 1 mathematics teacher and at least 4 computing teachers. [4]

9 The curve $3x^2 + xy - y^2 + 4y - 3 = 0$ and the line $y = 2(1 - x)$ intersect at the points A and B.

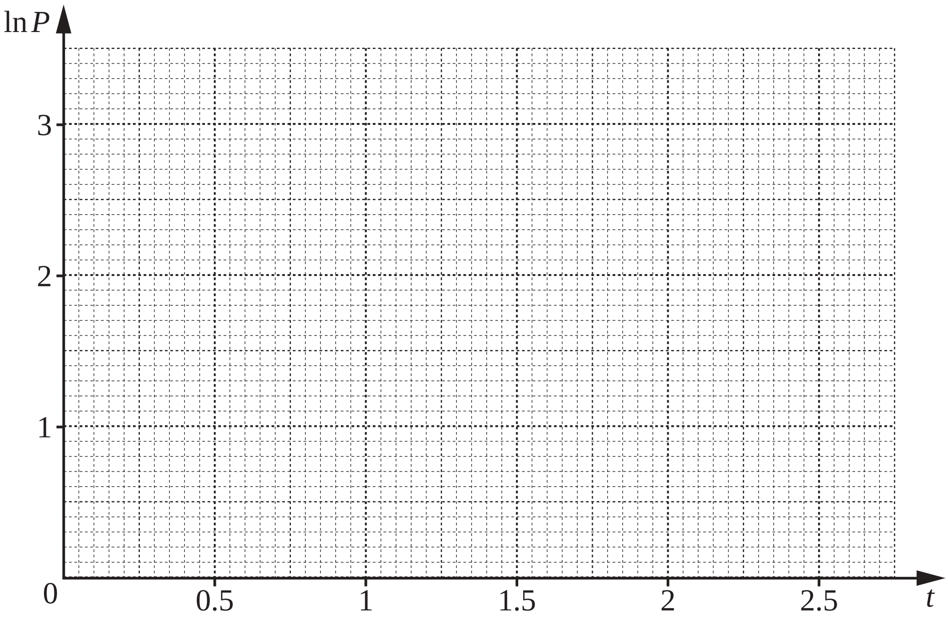
(i) Find the coordinates of A and of B. [5]

(ii) Find the equation of the perpendicular bisector of the line AB, giving your answer in the form $ax + by = c$, where a, b and c are integers. [4]

10 The table shows values of the variables t and P.

t	1	1.5	2	2.5
P	4.39	8.33	15.8	30.0

(i) Draw the graph of $\ln P$ against t on the grid below. [2]



(ii) Use the graph to estimate the value of P when $t = 2.2$. [2]

(iii) Find the gradient of the graph and state the coordinates of the point where the graph meets the vertical axis. [2]

(iv) Using your answers to part **(iii)**, show that $P = ab^t$, where a and b are constants to be found. [3]

(v) Given that your equation in part **(iv)** is valid for values of t up to 10, find the smallest value of t, correct to 1 decimal place, for which P is at least 1000. [2]

11 **(i)** Prove that $\sin x(\cot x + \tan x) = \sec x$. [4]

(ii) Hence solve the equation $|\sin x(\cot x + \tan x)| = 2$ for $0^\circ \leqslant x \leqslant 360^\circ$. [4]

Question 12 is printed on the next page.

12 A particle moves in a straight line so that, t seconds after passing a fixed point O, its displacement, s m, from O is given by

$$s = 1 + 3t - \cos 5t.$$

(i) Find the distance between the particle's first two positions of instantaneous rest. [7]

(ii) Find the acceleration when $t = \pi$. [2]

Cambridge International Examinations
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CANDIDATE NAME	

CENTRE NUMBER		CANDIDATE NUMBER

* 8 7 1 5 8 8 0 4 4 *

ADDITIONAL MATHEMATICS **0606/22**

Paper 2 **May/June 2017**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

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Mathematical Formulae

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$$a^2 = b^2 + c^2 - 2bc \cos A$$

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1 Solve $|5x+3| = |1-3x|$. [3]

2 **Without using a calculator**, express $\left(\dfrac{1+\sqrt{5}}{3-\sqrt{5}}\right)^{-2}$ in the form $a + b\sqrt{5}$, where a and b are integers. [5]

3 **Without using a calculator**, factorise the expression $10x^3 - 21x^2 + 4$. [5]

4 The point P lies on the curve $y = 3x^2 - 7x + 11$. The normal to the curve at P has equation $5y + x = k$. Find the coordinates of P and the value of k. [6]

5 **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}[0.4x^5(0.2 - \ln 5x)] = kx^4 \ln 5x$, where k is an integer to be found. [2]

(ii) Express $\ln 125x^3$ in terms of $\ln 5x$. [1]

(iii) Hence find $\int (x^4 \ln 125x^3)\, \mathrm{d}x$. [2]

6 Show that the roots of $px^2 + (p - q)x - q = 0$ are real for all real values of p and q. [4]

7 **(a)** Given that $a^7 = b$, where a and b are positive constants, find,

(i) $\log_a b$, [1]

(ii) $\log_b a$. [1]

(b) Solve the equation $\log_{81} y = -\frac{1}{4}$. [2]

(c) Solve the equation $\frac{32^{x^2-1}}{4^{x^2}} = 16$. [3]

8 **Solutions to this question by accurate drawing will not be accepted.**

The points A and B are $(-8, 8)$ and $(4, 0)$ respectively.

(i) Find the equation of the line AB. [2]

(ii) Calculate the length of AB. [2]

The point C is $(0, 7)$ and D is the mid-point of AB.

(iii) Show that angle ADC is a right angle. [3]

The point E is such that $\overrightarrow{AE} = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$.

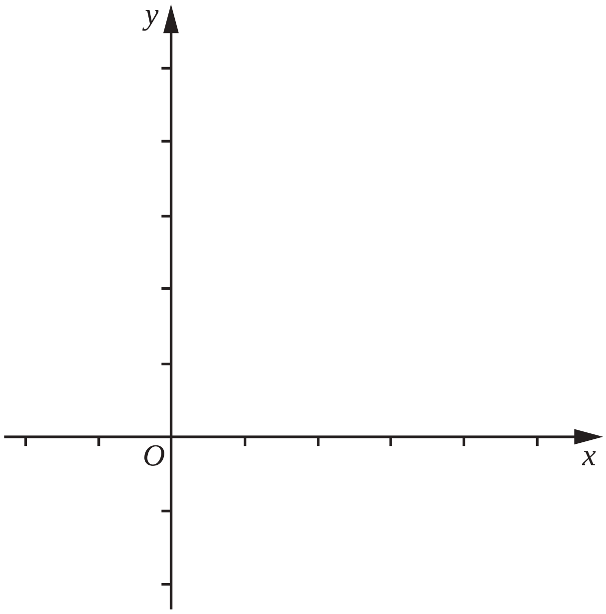
(iv) Write down the position vector of the point E. [1]

(v) Show that $ACBE$ is a parallelogram. [2]

9 A function f is defined, for $x \leqslant \frac{3}{2}$, by $\mathrm{f}(x) = 2x^2 - 6x + 5$.

(i) Express $\mathrm{f}(x)$ in the form $a(x-b)^2 + c$, where a, b and c are constants. [3]

(ii) On the same axes, sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, showing the geometrical relationship between them. [3]



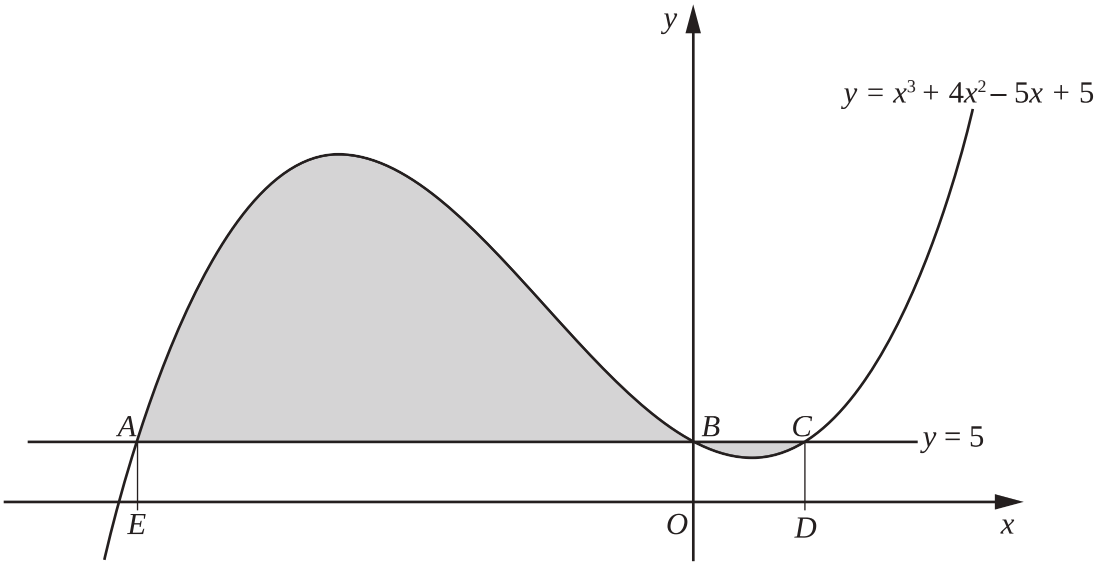
(iii) Using your answer from part **(i)**, find an expression for $\mathrm{f}^{-1}(x)$, stating its domain. [3]

10 Solve the equation

(i) $4\sin\left(3x - \frac{\pi}{4}\right) = 3$ for $0 \leqslant x \leqslant \frac{\pi}{2}$ radians, [4]

(ii) $2\tan^2 y + \sec^2 y = 14\sec y + 3$ for $0° \leqslant y \leqslant 360°$. [5]

11



The diagram shows part of the curve $y = x^3 + 4x^2 - 5x + 5$ and the line $y = 5$. The curve and the line intersect at the points A, B and C. The points D and E are on the x-axis and the lines AE and CD are parallel to the y-axis.

(i) Find $\int (x^3 + 4x^2 - 5x + 5)\,\mathrm{d}x$. [2]

(ii) Find the area of each of the rectangles $OEAB$ and $OBCD$. [4]

(iii) Hence calculate the total area of the shaded regions enclosed between the line and the curve. You must show all your working. [4]

Question 12 is printed on the next page.

12 The function g is defined, for $x > -\frac{1}{2}$, by $g(x) = \frac{3}{2x+1}$.

(i) Show that $g'(x)$ is always negative. [2]

(ii) Write down the range of g. [1]

The function h is defined, for all real x, by $h(x) = kx + 3$, where k is a constant.

(iii) Find an expression for $hg(x)$. [1]

(iv) Given that $hg(0) = 5$, find the value of k. [2]

(v) State the domain of hg. [1]

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

ADDITIONAL MATHEMATICS **0606/23**

Paper 2 **May/June 2017**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **15** printed pages and **1** blank page.

DC (ST/FC) 129465/2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

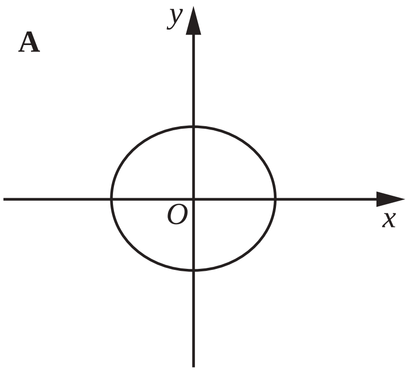
$$a^2 = b^2 + c^2 - 2bc\cos A$$

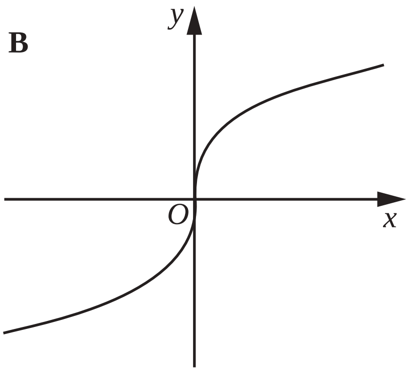
$$\Delta = \frac{1}{2}bc\sin A$$

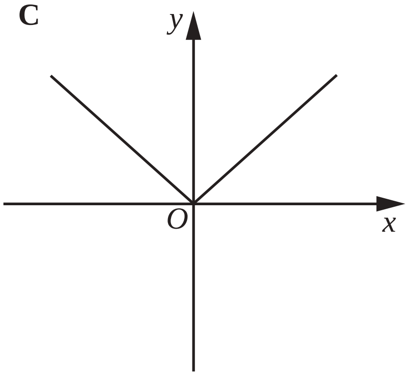
1 **(a)** Solve the equation $7^{2x+5} = 2.5$, giving your answer correct to 2 decimal places. [3]

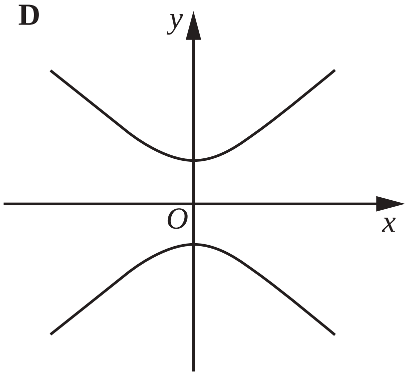
(b) Express $\dfrac{(5\sqrt{q})^3}{(625p^{12}q)^{\frac{1}{4}}}$ in the form $5^a p^b q^c$, where a, b and c are constants. [3]

2







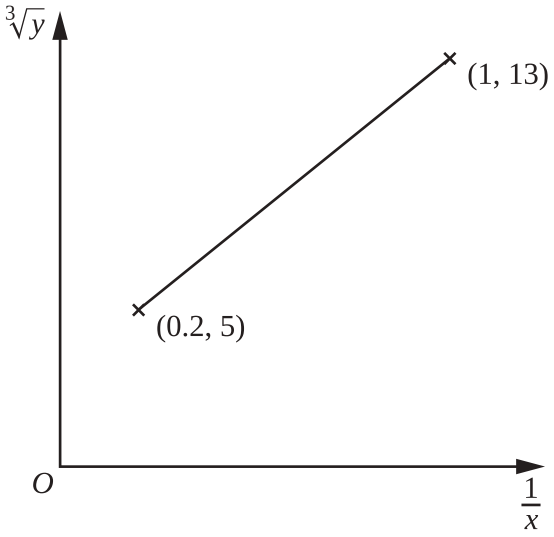


The four graphs above are labelled **A**, **B**, **C** and **D**.

(i) Write down the letter of each graph that represents a function, giving a reason for your choice. [2]

(ii) Write down the letter of each graph that represents a function which has an inverse, giving a reason for your choice. [2]

3



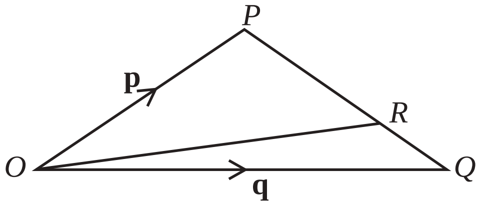
Variables x and y are such that when $\sqrt[3]{y}$ is plotted against $\frac{1}{x}$, a straight line graph passing through the points (0.2, 5) and (1, 13) is obtained. Express y in terms of x. [4]

4 **(a)** Vectors **a**, **b** and **c** are such that $\mathbf{a} = \begin{pmatrix} 5 \\ -6 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 11 \\ -15 \end{pmatrix}$ and $3\mathbf{a} + \mathbf{c} = \mathbf{b}$.

(i) Find **c**. [1]

(ii) Find the unit vector in the direction of **b**. [2]

(b)



In the diagram, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$. The point R lies on PQ such that $PR = 3RQ$. Find $\overrightarrow{OR}$ in terms of **p** and **q**, simplifying your answer. [3]

5 **(a)** How many 5-digit numbers are there that have 5 different digits and are divisible by 5? [3]

(b) A committee of 8 people is to be selected from 9 men and 5 women. Find the number of different committees that can be selected if the committee must have at least 4 women. [3]

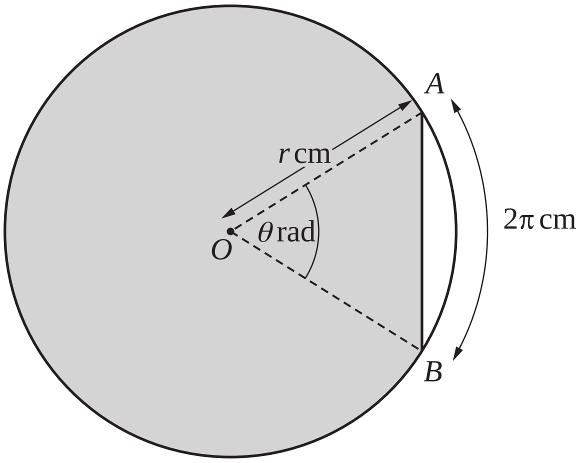
6 The first three terms of the binomial expansion of $(2-ax)^n$ are $64-16bx+100bx^2$. Find the value of each of the integers *n*, *a* and *b*. [7]

7 Differentiate with respect to x,

(i) $(1+4x)^{10}\cos x$, [4]

(ii) $\dfrac{e^{4x-5}}{\tan x}$. [4]

8



The diagram shows a circle, centre O of radius r cm, and a chord AB. Angle $AOB = \theta$ radians. The length of the major arc AB is 5 times the length of the minor arc AB. The minor arc AB has length 2π cm.

(i) Find the value of θ and of r. [2]

(ii) Calculate the exact perimeter of the shaded segment. [2]

(iii) Calculate the exact area of the shaded segment. [4]

9 The functions f and g are defined, for $x > 1$, by

$$\mathrm{f}(x) = 9\sqrt{x-1},$$
$$\mathrm{g}(x) = x^2 + 2.$$

(i) Find an expression for $\mathrm{f}^{-1}(x)$, stating its domain. [3]

(ii) Find the exact value of fg(7). [2]

(iii) Solve $\mathrm{gf}(x) = 5x^2 + 83x - 95$. [4]

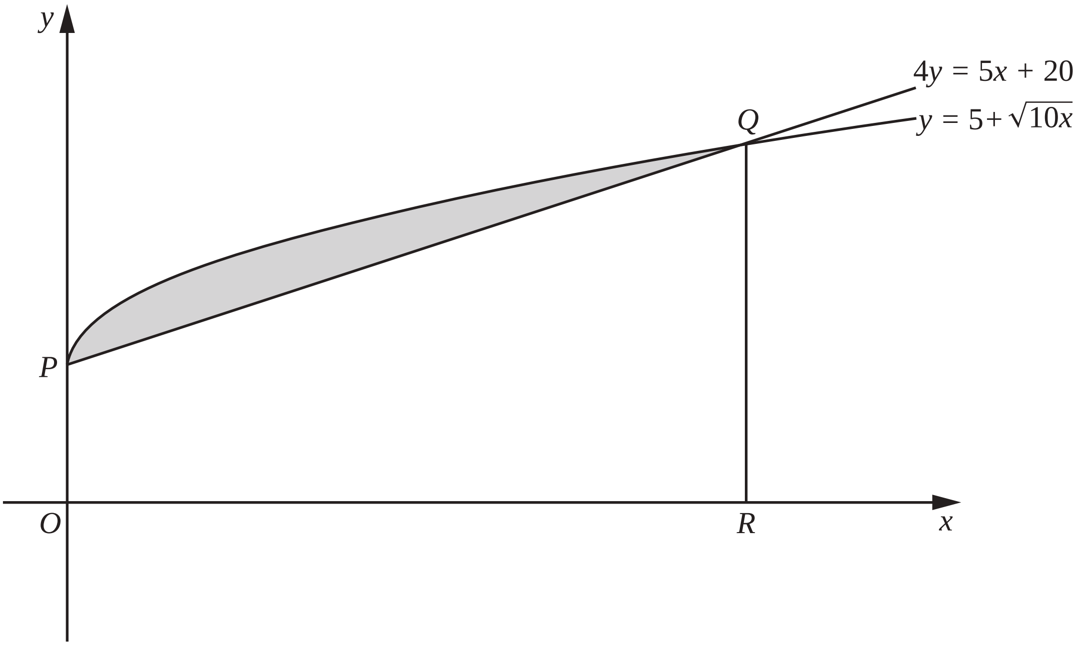
10 Solve the equation

(a) $2|\sin x| = 1$ for $-\pi \leqslant x \leqslant \pi$ radians, [3]

(b) $3\tan(2y + 15^\circ) = 1$ for $0^\circ \leqslant y \leqslant 180^\circ$, [4]

(c) $3\cot^2 z = \operatorname{cosec}^2 z - 7\operatorname{cosec} z + 1$ for $0^\circ \leqslant z \leqslant 360^\circ$. [5]

11



The diagram shows part of the curve $y = 5 + \sqrt{10x}$ and the line $4y = 5x + 20$. The line and curve intersect at the points $P(0, 5)$ and Q. The line QR is parallel to the y-axis.

(i) Find the coordinates of Q. [4]

(ii) Find the area of the shaded region. You must show all your working. [6]

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Cambridge Assessment
International Education

Grade thresholds – November 2017

Cambridge IGCSE Additional Mathematics (0606)

Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the November 2017 examination.

		minimum raw mark required for grade:				
	maximum raw mark available	A	B	C	D	E
Component 11	80	56	42	28	22	17
Component 12	80	56	40	25	20	15
Component 13	80	70	53	35	26	17
Component 21	80	50	38	26	21	16
Component 22	80	59	42	24	19	14
Component 23	80	63	44	26	18	10

Grade A* does not exist at the level of an individual component.

The maximum total mark for this syllabus, after weighting has been applied, is **160**.

The overall thresholds for the different grades were set as follows.

Option	Combination of Components	A*	A	B	C	D	E
AX	11, 21	132	106	80	54	43	33
AY	12, 22	148	115	82	49	39	29
AZ	13, 23	146	133	97	61	44	27

Cambridge Assessment International Education
Cambridge International General Certificate of Secondary Education

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ADDITIONAL MATHEMATICS **0606/11**

Paper 1 **October/November 2017**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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This document consists of **7** printed pages.

MARK SCHEME NOTES

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Types of mark

M	Method marks, awarded for a valid method applied to the problem.
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Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(i)	$A' \cap B$	B1	
1(ii)	$A \cap B \cap C$	B1	
1(iii)	$A \cup B$	B1	
2(i)	$\text{p}\left(\frac{1}{2}\right)=\frac{a}{8}+\frac{b}{4}-\frac{13}{2}+4$	M1	attempt at $p\left(\frac{1}{2}\right)$
	$\text{p}'(x)=3ax^2+2bx-13$ $\text{p}'\left(\frac{1}{2}\right)=\frac{3a}{4}+b-13$	M1	attempt at $p'\left(\frac{1}{2}\right)$
	leading to $a+2b=20$ and $3a+4b-52=0$	A1	at least one correct equation
	solution of simultaneous equations	DM1	
	$a=12,\ b=4$	A1	for both
2(ii)	$p(-1)=-12+4+13+4$	M1	
	9	A1	FT on *their* integer values of a and b
3(a)	$Tg^{\frac{1}{2}}=2\pi l^{\frac{1}{2}}$ $T^2g=4\pi^2 l$	B1	multiplication/dealing with power of $\frac{1}{2}$ or squaring
	$l=\frac{T^2g}{4\pi^2}$ or $\left(\frac{Tg^{\frac{1}{2}}}{2\pi}\right)^2$	B1	for either
3(b)	$y^2-4y+3=0$ leading to $y=1,\ y=3$	M1	reduction to quadratic equation and attempt to solve
	$x^{\frac{1}{3}}=1,\ \ x^{\frac{1}{3}}=3$	DM1	attempt to solve $x^{\frac{1}{3}}=k$ (positive k)
	$x=1,\ x=27$	A2	A1 for each

Question	Answer	Marks	Guidance
4(i)	$\frac{1}{2}$	**B1**	
4(ii)	$\lg y = mx^2 + c$ $\lg y = \frac{1}{2}x^2 + 1$	**B2**	–1 for each error
4(iii)	$y = 10^{\left(\frac{x^2}{2}+1\right)}$	**B1**	dealing with lg on *their* (ii)
	$y = 10\left(10^{\frac{x^2}{2}}\right)$	**B2**	B1 for each, dependent on first B1
5(i)	(0, 20)	**B1**	
5(ii)	31.7	**B1**	
5(iii)	$2\text{e}^{2x} - 8\text{e}^{-2x} \quad (+c)$	**B2**	B1 for each correct term
5(iv)	Area of trapezium = $\frac{1}{2}(20 + 31.7)$ $= 25.86$ or 25.85	**B1**	
	$\left[2\text{e}^{2x} - 8\text{e}^{-2x}\right]_0^1 = \left(2\text{e}^2 - 8\text{e}^{-2}\right) - (-6)$	**M1**	substitution of both limits, must have come from integration of the form $a\text{e}^{2x} + b\text{e}^{-2x}$.
	19.7	**A1**	
	Required area = 6.15, 6.16, 6.17	**A1**	
6(a)(i)	$\text{f} \geqslant 3$	**B1**	must be using a correct notation
6(a)(ii)	$(4x-1)^2 + 3 = 4$	**M1**	correct order
	solution of resulting quadratic equation	**DM1**	
	$x = 0,\ x = \frac{1}{2}$	**A1**	both required

Question	Answer	Marks	Guidance
6(b)(i)	$xy-4y=2x+1$	**M1**	'multiplying out'
	$x(y-2)=4y+1$ $x=\frac{4y+1}{y-2}$	**M1**	collecting together like terms
	$\text{h}^{-1}(x)=\frac{4x+1}{x-2}$	**A1**	correct answer with correct notation
	Range $\text{h}^{-1}\neq 4$	**B1**	must be using a correct notation
6(b)(ii)	$\text{h}^2(x)=\text{h}\left(\frac{2x+1}{x-4}\right)$ $=\frac{2\left(\frac{2x+1}{x-4}\right)+1}{\left(\frac{2x+1}{x-4}\right)-4}$	**M1**	dealing with h^2 correctly
	dealing with fractions within fractions	**M1**	
	$=\frac{5x-2}{17-2x}$ oe	**A1**	
7(i)	$\ln(2x+1)-\ln(2x-1)$	**B1**	
7(ii)	attempt to differentiate	**M1**	
	$\frac{\text{d}y}{\text{d}x}=\frac{2}{2x+1}-\frac{2}{2x-1}+4$	**A1**	all correct
	attempt to obtain in required form	**DM1**	
	$=\frac{16x^2-8}{4x^2-1}$	**A1**	A1 all correct
7(iii)	When $\frac{\text{d}y}{\text{d}x}=0$, $16x^2-8=0$	**M1**	setting $\frac{\text{d}y}{\text{d}x}=0$ and attempt to solve
	$x=\frac{1}{\sqrt{2}}$ only	**A1**	

Question	Answer	Marks	Guidance
7(iv)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{32x\left(4x^2-1\right)-8x\left(16x^2-8\right)}{\left(4x^2-1\right)^2}$	**M1**	attempt at second derivative and conclusion or equivalent method
	When $x=\frac{1}{\sqrt{2}}$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ is + ve, so minimum	**A1**	
8(a)(i)	${}^8C_6\times{}^6C_4$	**B1**	either ${}^8C_6$ or ${}^6C_4$
	420	**B1**	
8(a)(ii)	${}^{12}C_8+{}^{12}C_{10}$	**B2**	B1 for each
	= 561	**B1**	
	Alternate scheme: $1001\ -\left(2\times{}^{12}C_9\right)$	**B1 B1**	
	= 561	**B1**	
8(b)(i)	136 080	**B1**	
8(b)(ii)	No of ways ending with 0 - 15 120	**B1**	
	No of ways ending with 5 - 13 440	**B1**	
	Total 28 560	**B1**	
8(b)(iii)	Starting with 6 or 8 - 13 440	**B1**	
	Starting with 7 or 9 - 16 800	**B1**	
	Total = 30 240	**B1**	
9(i)	$\tan\left(\frac{PAQ}{2}\right)=2.4$	**M1**	valid method
	$PAQ=2.352(01....)$ $PAQ=2.35$ correct to 3 sf	**A1**	must see greater than 3 sf then rounding
9(ii)	$PBQ=0.790$ or 0.792	**B1**	
9(iii)	$\left(2.352\times10\right)+\left(0.790\times24\right)$	**M1,A1**	M1 for correct attempt at an arc length A1 for one correct arc length
	= awrt 42.5	**A1**	

Question	Answer	Marks	Guidance
9(iv)	$\left(\left(\frac{1}{2}\times 24^2\times 0.790\right)-\left(\frac{1}{2}\times 24^2\times\sin 0.790\right)\right)$	B1,B1	B1 for a correct sector area allow, unsimplified B1 for a correct area of a triangle, allow unsimplified
	$+\left(\left(\frac{1}{2}\times 10^2\times 2.352\right)-\left(\frac{1}{2}\times 10^2\times\sin 2.352\right)\right)$	B1	correct plan, dependent on both previous B marks
	$=22.94+82.1$ = 105	B1	
10(a)	$\frac{3}{4}=\sin^2 2x$	B1	dealing correctly with cosec
	$\sin 2x=\pm\frac{\sqrt{3}}{2}$ $2x=60,\ 120,\ 240,\ 300$	M1	correct method of solution including dealing with 2*x* correctly, may be implied by one correct solution.
	$x=30,\ 60,\ 120,\ 150$	A2	A1 for each correct pair
10(b)	$\tan\left(y-\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}}$	M1	dealing with order of operations to obtain a first solution
	$y-\frac{\pi}{4}=\frac{\pi}{6},\ \frac{7\pi}{6}$	M1	M1 for attempt to obtain a second solution
	$y=\frac{5\pi}{12},\ \frac{17\pi}{12}$	A2	A1 for each

Cambridge Assessment International Education
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ADDITIONAL MATHEMATICS **0606/12**

Paper 1 **October/November 2017**

MARK SCHEME

Maximum Mark: 80

Published

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MARK SCHEME NOTES

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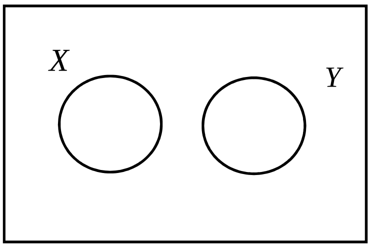
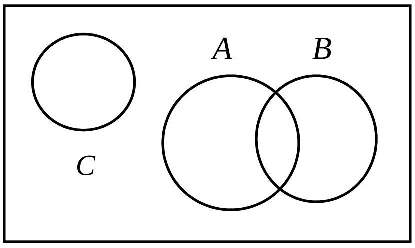
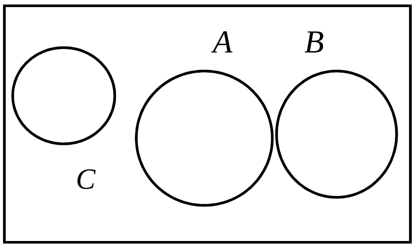
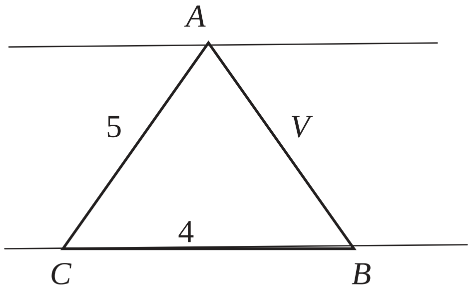
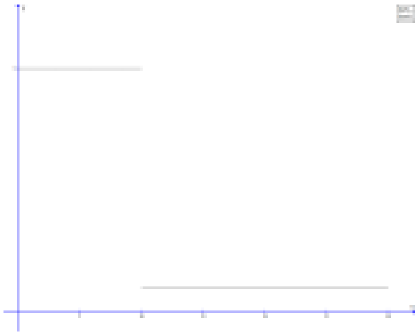
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(i)		**1**	
1(ii)	Either  Or 	**2**	**B1** for *C* with no intersection with either *A* or *B* (allow if *C* is not represented by a circle) **B1** for all correct, *C* must be represented by a circle
2	$a=4$	**B1**	
	$b=6$	**B1**	
	$c=-2$	**M1, A1**	**M1** for use of $\left(\frac{\pi}{12},\ 2\right)$ to obtain *c*, using *their* values of *a* and of *b*
3(i)	$32-20x^2+5x^4$	**B3**	**B1** for each correct term
3(ii)	$\left(32-20x^2+5x^4\right)\left(\frac{1}{x^2}\text{............}+\frac{9}{x^4}\right)$	**B1**	$\frac{1}{x^2}$ and $\frac{9}{x^4}$
	Independent of *x*: $-20+45$	**M1**	attempt to deal with 2 terms independent of *x*, must be looking at terms in x^2 and $\frac{1}{x^2}$ and terms in x^4 and $\frac{1}{x^4}$
	= 25	**A1**	**FT** *their* answers from (i) (*their* -20×1) + (*their* 5×9)

Question	Answer	Marks	Guidance
4	correct differentiation of $\ln\left(3x^2+2\right)$	**B1**	
	attempt to differentiate a quotient or a product	**M1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\left(x^2+1\right)\left(\frac{6x}{3x^2+2}\right)-2x\ln\left(3x^2+2\right)}{\left(x^2+1\right)^2}$	**A1**	all other terms correct.
	When $x=2,\ \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{5\left(\frac{12}{14}\right)-4\ln 14}{25}$	**M1**	**M1dep** for substitution and attempt to simplify
	$=\frac{6}{35}-\frac{4}{25}\ln 14$	**A2**	**A1** for each correct term, must be in simplest form
5(i)	**Either** Gradient $=-0.2$	**B1**	
	$\lg y=-0.2x+c$	**B1**	$\lg y=mx+c$ soi
	correct attempt to find c	**M1**	must have previous B1
	$\lg y=0.42-0.2x$ or $\lg y=\frac{21}{50}-\frac{x}{5}$	**A1**	line in either form, allow equivalent fractions
	Or $0.3=0.6m+c$	**B1**	
	$0.2=1.1m+c$	**B1**	
	attempt to solve for both m and c	**M1**	must have at least one of the previous B marks
	Leading to $\lg y=0.42-0.2x$ or $\lg y=\frac{21}{50}-\frac{x}{5}$	**A1**	line in either form, allow equivalent fractions

Question	Answer	Marks	Guidance
5(ii)	**Either** $y = 10^{(0.42 - 0.2x)}$	**M1**	dealing with the index, using their answer to **(i)**
	$y = 10^{0.42}\left(10^{-0.2x}\right)$ $y = 2.63\left(10^{-0.2x}\right)$	**A2**	**A1** for each
	Or $y = A\left(10^{bx}\right)$ leads to $\lg y = \lg A + bx$ Compare this form with their equation from (i)	**M1**	comparing their answer to **(i)** with $\lg y = \lg A + bx$ may be implied by one correct term from correct work
	$\lg A = 0.42$ so $A = 2.63$	**A1**	
	$b = -0.2$	**A1**	A1 for each
6(i)	$y \in \mathbb{R}$ oe	**B1**	Must have correct notation i.e. no use of x
6(ii)	$y > 3$ oe	**B1**	Must have correct notation i.e. no use of x
6(iii)	$\mathrm{f}^{-1}(x) = \mathrm{e}^{x}$ or $\mathrm{g}(4) = 35$	**B1**	First B1 may be implied by correct answer or by use of 35
	$\mathrm{f}^{-1}\mathrm{g}(4) = \mathrm{e}^{35}$	**B1**	
6(iv)	$\dfrac{y-3}{2} = x^2$ or $\dfrac{x-3}{2} = y^2$	**M1**	valid attempt to obtain the inverse
	$\mathrm{g}^{-1}(x) = \sqrt{\dfrac{x-3}{2}}$	**A1**	correct form, must be $\mathrm{g}^{-1}(x) =$ or $y =$
	Domain $x > 3$	**B1**	Must have correct notation
7(i)	$\mathrm{p}\left(\dfrac{1}{2}\right)$: $\dfrac{a}{8} + 2 + \dfrac{b}{2} + 5 = 0$	**M1**	substitution of $x = \dfrac{1}{2}$ and equating to zero (allow unsimplified)
	$\mathrm{p}(-2)$: $-8a + 32 - 2b + 5 = -25$	**M1**	substitution of $x = -2$ and equating to –25 (allow unsimplified)
	leading to $a + 4b + 56 = 0$ $4a + b - 31 = 0$ oe	**M1**	**M1dep** for solution of simultaneous equations to obtain a and b
	$a = 12,\ b = -17$	**A2**	**A1** for each

Question	Answer	Marks	Guidance
7(ii)	$12x^3+8x^2-17x=0$ $x=0$	**B1**	for $x=0$
	$x=-\frac{1}{3}\pm\frac{\sqrt{55}}{6}$ oe	**B1**	
8			
8(i)	$\angle ABC=67.4^{\circ}$	**B1**	
	$\frac{4}{\sin BAC}=\frac{5}{\sin 67.4^{\circ}}$	**M1**	attempt at the sine rule, using 4 and 5 (or e.g. use of cosine rule followed by sine rule on triangle shown)
	$\angle BAC=47.6^{\circ}$	**A1**	may be implied by later work
	Angle required = $180^{\circ}-47.6^{\circ}-67.4^{\circ}\ =65^{\circ}$	**A1**	Answer Given
8(ii)	$V^2=5^2+4^2-\left(2\times5\times4\times\cos65^{\circ}\right)$	**M1**	attempt at the cosine rule or sine rule to obtain V – allow if seen in (i)
	$V=4.91$ or $\frac{4}{\sin BAC}=\frac{V}{\sin 65}$	**A1**	
	Distance to travel: $\frac{120}{\sin 67.4^{\circ}}$	**M1**	distance to travel – allow if seen in **(i)**
	130 or $\sqrt{120^2+50^2}$	**A1**	
	Time taken: $\frac{130}{4.91}$	**M1**	**M1dep** for correct method to find the time, must have both of the previous M marks
	26.5	**A1**	

Question	Answer	Marks	Guidance
	Alternative method $AC = \frac{120}{\cos 25}$ oe	**M1**	correct attempt at *AC*
	$=132.4$	**A1**	Allow 132
	Speed for this distance = 5	**M1A1**	**M1dep A1** for speed, it must be 5 exactly for A1, must have first M mark
	Time taken $= \frac{132.4}{5}$	**M1**	**M1dep** for a correct method to find the time, must have both of the previous M marks
	= 26.5	**A1**	
9(a)		**B3**	**B1** for line joining $(0,5)$ and $(10,5)$ **B1** for a line joining $(10,0.5)$ and $(30,0.5)$ **B1** all correct with **no solid** line joining $(10,5)$ to $(10,0.5)$
9(b)(i)	3	**B1**	
9(b)(ii)	$\frac{\mathrm{d}v}{\mathrm{d}t} = -15\mathrm{e}^{-5t} + \frac{3}{2}$	**M1**	attempt to differentiate, must be in the form $a\mathrm{e}^{-5t} + b$
	When $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$, $\mathrm{e}^{-5t} = 0.1$	**M1**	**M1dep** for equating to zero and attempt to solve, must be of the form $a\mathrm{e}^{-5t} = b,\ b > 0$ to obtain an equation in the form $-5t = k$ where *k* is a logarithm or < 0
	$t = 0.461$	**A1**	

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Question	Answer	Marks	Guidance
9(b)(iii)	**Either** attempt to integrate, must be in the form $ce^{-5t}+dt^2$	**M1**	
	$s=-\frac{3}{5}e^{-5t}+\frac{3}{4}t^2 \quad (+c)$	**A1**	
	When $t=0,\ s=0 \quad$ so $c=\frac{3}{5}$	**M1**	**M1dep** for attempt to find *c* and substitute $t=0.5$
	$s=0.738$	**A1**	
	Or attempt to integrate, must be in the form $ce^{-5t}+dt^2$	**M1**	
	$\left[-\frac{3}{5}e^{-5t}+\frac{3}{4}t^2\right]_0^{0.5}$	**A1**	
	correct use of limits	**M1**	**M1dep**
	leading to $s=0.738$	**A1**	
10(i)	$5\angle BAC=6.2,\ \angle BAC=1.24$	**B1**	
10(ii)	$\sin 0.62=\frac{BD}{5}$, $BD=2.905,\ 2.91$	**B1**	valid method to find *BD*
	Arc *BFC*: $\pi\times BD \quad (=9.13)$	**M1**	attempt to find arc length *BFC,* using *their BD*
	Perimeter: $9.13+6.2=15.3$	**A1**	
10(iii)	Area: $\left(\frac{1}{2}\times\pi\times2.91^2\right)-\left(\left(\frac{1}{2}\times5^2\times1.24\right)-\left(\frac{1}{2}\times5^2\times\sin1.24\right)\right)$	**B3**	**B1** for area of semi circle (= 13.3) **B1** for area of sector (= 15.5) **B1** for area of triangle (= 11.8)
	$9.58\leqslant$ Area $\leqslant 9.62$	**B1**	final answer

Question	Answer	Marks	Guidance
11(a)	$\tan\left(\phi+35^{\circ}\right)=\frac{2}{5}$	**M1**	dealing correctly with cot and an attempt at solution of $\tan\left(\phi+35\right)=c$, order must be correct, to obtain a value for $\phi+35$
	$\phi+35^{\circ}=21.8^{\circ},\ 201.8^{\circ},\ 381.8^{\circ}$	**M1**	**M1dep** for an attempt at a second solution in the range, (180° + *their* first solution in the range oe)
	$\phi=166.8^{\circ},\ \ 346.8^{\circ}$	**A2**	**A1** for each
11(b)(i)	**Either** $\dfrac{\frac{1}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}}$	**M1**	expressing all terms in terms of $\sin\theta$ and $\cos\theta$ where necessary
	$=\frac{1}{\cos\theta}\left(\frac{\sin\theta\cos\theta}{\cos^{2}\theta+\sin^{2}\theta}\right)$	**M1**	dealing with the fractions correctly to get $\frac{\sin^{2}\theta+\cos^{2}\theta}{\sin\theta\cos\theta}$ in denominator or as in left hand column
	$=\frac{\sin\theta}{(1)}$	**A1**	use of identity, together with a complete and correct solution, withhold A1 for incorrect use of brackets
	Or $\dfrac{\sec\theta}{\frac{1}{\tan\theta}+\tan\theta}$ $=\dfrac{\sec\theta}{\frac{1+\tan^{2}\theta}{\tan\theta}}$	**M1**	dealing with fractions in the denominator correctly to get $\frac{1+\tan^{2}\theta}{\tan\theta}$ in the denominator, allow $\tan\theta$ taken to the numerator
	$=\frac{\sec\theta\tan\theta}{\sec^{2}\theta}$	**M1**	use of the identity to get $\sec^{2}\theta$
	$=\frac{\tan\theta}{\sec\theta}\ \ =\frac{\sin\theta}{\cos\theta}\times\cos\theta\ \ =\sin\theta$	**A1**	expressing all terms in terms of $\sin\theta$ and $\cos\theta$ and simplification to the given answer, withhold A1 for incorrect use of brackets

Question	Answer	Marks	Guidance
11(b)(ii)	$\sin 3\theta = -\frac{\sqrt{3}}{2}$	**M1**	correct attempt to solve for θ, order must be correct, may be implied by one correct solution
	$3\theta = -\frac{2\pi}{3},\ -\frac{\pi}{3},\ \frac{4\pi}{3}$ $\theta = -\frac{2\pi}{9},\ -\frac{\pi}{9},\ \frac{4\pi}{9}$	**A3**	**A1** for each

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ADDITIONAL MATHEMATICS **0606/13**
Paper 1 **October/November 2017**
MARK SCHEME
Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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This document consists of **10** printed pages.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method marks, awarded for a valid method applied to the problem.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
B	Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘**dep**’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	Using $\tan^2\theta+1=\sec^2\theta$ to obtain $y=2(\tan^2\theta+1)$ or $(x+5)^2=\sec^2\theta-1$ $(x+5)^2+1=\frac{y}{2}$	**M1**	use of correct identity
	$y=2((x+5)^2+1)$ oe	**A1**	
2	$\frac{dy}{dx}=10e^{5x}+3$ an attempt at integration in form $ae^{5x}+bx$	**M1**	
	$y=\frac{10}{5}e^{5x}+3x\ \ (+c)$	**A1**	condone omission of c
	attempt to find c using $x=0, y=9$	**M1**	**M1dep**
	$y=2e^{5x}+3x+7$	**A1**	
3	$9<4k(k-4)$ $4k^2-16k-9$	**M1**	use of the discriminant with correct values
	$(2k-9)(2k+1)$	**M1**	**M1dep** for solution of *their* quadratic to obtain critical values
	Critical values $\frac{9}{2},\ \ -\frac{1}{2}$	**A1**	
	$k<-\frac{1}{2},\ \ k>\frac{9}{2}$	**A1**	
4	$a=3$	**B1**	
	$b=8$	**B1**	
	$\frac{5}{2}=3\cos\left(8\times\frac{\pi}{12}\right)+c$	**M1**	substitution of $x=\frac{\pi}{12}$ and $y=\frac{5}{2}$ to find c
	$c=4$	**A1**	
5(i)	$\frac{5}{14}(7x-10)^{\frac{2}{5}}$	**B2**	**B1** for $k(7x-10)^{\frac{2}{5}}$

Question	Answer	Marks	Guidance
5(ii)	$\frac{5}{14}\left[\left(7x-10\right)^{\frac{2}{5}}\right]_6^a=\frac{25}{14}$ $\frac{5}{14}\left(7a-10\right)^{\frac{2}{5}}-\frac{5}{14}\left(7\times 6-10\right)^{\frac{2}{5}}=\frac{25}{14}$ $\left(7a-10\right)^{\frac{2}{5}}-4=5$	**M1**	correct application of limits for $k\left(7x-10\right)^{\frac{2}{5}}$
	$a=\frac{9^{\frac{5}{2}}+10}{7}$	**M1**	**M1dep** for evaluation of $\left(7\times 6-10\right)^{\frac{2}{5}}$ and correct order of operations to find *a*, including dealing with power.
	$a=\frac{253}{7}$ or $36\frac{1}{7}$	**A1**	
6(i)	Gradient = $\frac{2.4-0.9}{0.2-0.8}$ (= –2.5)	**B1**	
	$\ln y=-\frac{5}{2}x^2+c$	**M1**	straight line form and correct substitutions to find *c*
	$\ln y=-\frac{5}{2}x^2+2.9$ oe	**A1**	
	<u>Alternative method</u> $2.4=p(0.2)+q$ $0.9=p(0.8)+q$	**B1**	
	Correct method of solution to find *p* and *q* from two correct equations	**M1**	**M1dep**
	$\ln y=-\frac{5}{2}x^2+2.9$	**A1**	
6(ii)	$y=\mathrm{e}^{\left(-\frac{5}{2}x^2+2.9\right)}$	**M1**	dealing with ln
	$y=\mathrm{e}^{-\frac{5}{2}x^2}\times\mathrm{e}^{2.9}$	**M1**	**M1dep** for dealing with the index
	$y=18.2z^{-\frac{5}{2}}$	**A1**	

Question	Answer	Marks	Guidance
7(i)	$64-48x^2+15x^4$	**B3**	**B1** for each correct term in final line of response
7(ii)	$\left(64-48x^2+15x^4\right)\left(\frac{1}{x^2}+2+x^2\right)$	**B1**	**B1**for $\frac{1}{x^2}+2+x^2$ oe
	at least two correctly obtained products leading to terms in x^2	**M1**	
	Term in x^2 : $64+15-96$	**A1**	**FT** for correct evaluation of *their* 64 + (2 × *their*–48) + *their* 15
	$=-17$	**A1**	
8(i)	attempt to differentiate a product	**M1**	
	$\frac{dy}{dx}=\left((x-4)\times\frac{5}{3}\times3(3x-1)^{\frac{2}{3}}\right)+(3x-1)^{\frac{5}{3}}$	**A2**	**A1** for $(+)\left((x-4)\times\frac{5}{3}\times3(3x-1)^{\frac{2}{3}}\right)$ **A1** for $(+)(3x-1)^{\frac{5}{3}}$
	$=(3x-1)^{\frac{2}{3}}\left((5x-20)+(3x-1)\right)$	**M1**	use of $(3x-1)^{\frac{5}{3}}=(3x-1)^{\frac{2}{3}}(3x-1)$
	$=(3x-1)^{\frac{2}{3}}(8x-21)$	**A1**	
8(ii)	When $x=3$, $\frac{dy}{dx}=8^{\frac{2}{3}}\times3$	**M1**	$(3\times3-1)^{\frac{2}{3}}\times k$ or $(9-1)^{\frac{2}{3}}\times k$ or $4\times k$ (where k is any number)
	$\partial y=8^{\frac{2}{3}}\times3\times h$	**M1**	**M1dep** for *their* $\left((9-1)^{\frac{2}{3}}\times k\right)\times h$
	$\partial y=12h$	**A1**	
9(a)(i)	720	**B1**	
9(a)(ii)	240	**B1**	
9(a)(iii)	$k\times4!\ \times2$ or $240-k\times4!\ \times2$ or correct equivalents with no extra terms added or subtracted	**B1**	
	$4\times4!\ \times p$ or correct equivalents with no extra terms added or subtracted	**B1**	
	192	**B1**	

Question	Answer	Marks	Guidance
9(b)(i)	6435	**B1**	
9(b)(ii)	With twins: ${}^{13}C_6$ or 1716 Without twins: ${}^{13}C_8$ or 1287	**B2**	**B1** for ${}^{13}C_6$ or 1716 or ${}^{13}C_8$ or 1287 **B1** for (${}^{13}C_6$ and ${}^{13}C_8$) or (1716 and 1287) with no multiples and no extra terms
	Total: 1716 + 1287 = 3003	**B1**	3003 from a correct method
10(a)	matrix multiplication, must have at least 2 correct elements	**M1**	
	$\mathbf{AB} = \begin{pmatrix} 13 & 8 \\ 2a-5b & 3a+4b \end{pmatrix}$	**A1**	
	$2a-5b=18$ $3a+4b=4$	**M1**	formation and solution of simultaneous equations
	leading to $a=4,\ b=-2$	**A1**	
	<u>Alternate scheme</u> $\mathbf{AB} = \begin{pmatrix} 13 & 8 \\ 18 & 4 \end{pmatrix}$ $\mathbf{ABB}^{-1} = \begin{pmatrix} 13 & 8 \\ 18 & 4 \end{pmatrix} \mathbf{B}^{-1}$	**M1**	Correct plan
	Correct inverse	**B1**	
	$\mathbf{A} = \begin{pmatrix} 4 & -1 \\ a & b \end{pmatrix} = \frac{1}{23}\begin{pmatrix} 13 & 8 \\ 18 & 4 \end{pmatrix}\begin{pmatrix} 4 & -3 \\ 5 & 2 \end{pmatrix}$	**M1**	Correct order and method of multiplication with at least two correct elements
	leading to $a=4,\ b=-2$	**A1**	
10(b)(i)	$-\frac{1}{17}\begin{pmatrix} 1 & 5 \\ 4 & 3 \end{pmatrix}$ oe	**B2**	**B1** for $-\frac{1}{17}$ **B1** for $\begin{pmatrix} 1 & 5 \\ 4 & 3 \end{pmatrix}$
10(b)(ii)	$\mathbf{Z} = -\frac{1}{17}\begin{pmatrix} 1 & 5 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ 4 & 0 \end{pmatrix}$	**M1**	pre-multiplication with two elements correct
	$= -\frac{1}{17}\begin{pmatrix} 19 & 2 \\ 8 & 8 \end{pmatrix}$ oe	**A2**	**A1** for four correct of $-\frac{1}{17}$, 19, 2, 8, 8

Question	Answer	Marks	Guidance
11(i)	1.48	**B1**	
11(ii)	$\frac{1}{2}\times 10^2 \times \theta = 21.8$	**M1**	correct use of sector area
	$\theta = 0.436$	**A1**	
11(iii)	$\angle BOC = \frac{2\pi - 1.48 - 0.436}{2} \quad (= 2.18(4))$	**B1**	2.18(4) or unsimplified
	$BC = 20\sin\left(\frac{1}{2}\angle BOC\right)$ or $BC = \frac{10 \times \sin BOC}{\sin\left(\frac{\pi - BOC}{2}\right)}$ or $BC = \sqrt{(200 - 200\cos BOC}$ $BC = 17.7(5)$	**M2**	**M1** for a complete correct method to find *BC* using *their* angle *BOC* **M1** for a correct plan using 14.8, *their BC* and 10 × *their* answer to (ii)
	Perimeter = $14.8 + (2 \times 17.7(5)) + 4.36$ $= 54.7$ or 54.6	**A1**	awrt 54.7 or awrt 54.6

Question	Answer	Marks	Guidance
11(iv)	Area = $\left(\frac{1}{2}\times10^2\times1.48\right)+21.8+2\left(\frac{1}{2}\times10^2\sin 2.18(4)\right)$	**B2**	**B1** for $\left(\frac{1}{2}\times10^2\times1.48\right)+21.8$ **B1** for $2\left(\frac{1}{2}\times10^2\sin 2.18(4)\right)$
	= 178	**B1**	awrt 178 from correct working
	Alternative method 1 Segment area = $\frac{1}{2}\left(10^2\left(2.18-\sin 2.18\right)\right)$	**B1**	**B1** for $2\times\frac{1}{2}\left(10^2\left(2.18(4)-\sin 2.18(4)\right)\right)$
	Area required = $100\pi-2\times\frac{1}{2}\left(10^2\left(2.18(4)-\sin 2.18(4)\right)\right)$	**B1**	
	= 178	**B1**	awrt 178 from correct working
	Alternative method 2 Area of trapezium = $\frac{1}{2}\left((13.5+4.33)(17.1)\right)$	**B1**	correct area of trapezium *ABCD* (allow unsimplified)
	Area of segments = $\frac{1}{2}\left(10^2\left(1.48-\sin 1.48\right)\right)$ + $\frac{1}{2}\left(10^2\left(0.436-\sin 0.436\right)\right)$	**B1**	correct area of both segments (allow unsimplified)
	= 178	**B1**	awrt 178 from correct working

Question	Answer	Marks	Guidance
12(i)	$2x^2+5x-12=0$ or $y^2+3y-28=0$	**M1**	attempt to get in terms of one variable
	$(2x-3)(x+4)=0$ or $(y+7)(y-4)=0$	**M1**	**M1dep** for solution of a three term quadratic
	leading to $x=-4,\ y=-7$ and $x=\frac{3}{2},\ y=4$	**A2**	**A1** for each 'pair'
	Midpoint $M\left(\frac{\frac{3}{2}-4}{2},\frac{4+(-7)}{2}\right)\left(=\left(-\frac{5}{4},-\frac{3}{2}\right)\right)$	**A1**	correctly obtained midpoint
	Gradient of *PQ* = 2	**B1**	may be implied
	Perp gradient = $-\frac{1}{2}$	**M1**	$\frac{-1}{\textit{their}\text{ gradient of }PQ}$
	Perp bisector: $y+\frac{3}{2}=-\frac{1}{2}\left(x+\frac{5}{4}\right)$	**M1**	**M1dep** for equation of perp bisector using *their* perp gradient and *their* midpoint. (unsimplified)
	$y=-\frac{1}{2}(-10)-\frac{17}{8}=\frac{23}{8}$ or $\frac{23}{8}=-\frac{1}{2}x-\frac{17}{8}\to x=-10$	**A1**	all correct so far and for verification using a correct equation

Question	Answer	Marks	Guidance
12(ii)	Area = $\frac{1}{2}\times\left(\frac{17}{8}+1\right)\times\frac{5}{4}$	**M1**	finding *R*, *S* and *RS*
	correct method for finding area	**M1**	**M1dep**
	$= \frac{125}{64}$ or 1.95 or $1\frac{61}{64}$	**A1**	
	<u>Alternative method 1</u> Area = $\frac{1}{2}\times\frac{\sqrt{125}}{4}\times\frac{\sqrt{125}}{8}$	**M1**	finding *R*, *S*, *RM* and *MS*
	correct method for finding area	**M1**	**M1dep**
	$= \frac{125}{64}$ or 1.95 or $1\frac{61}{64}$	**A1**	
	<u>Alternative method 2</u> Area = $\frac{1}{2}\begin{vmatrix}0 & 0 & \frac{-5}{4} & 0\\ 1 & \frac{-17}{8} & \frac{-3}{2} & 1\end{vmatrix}$	**M1**	finding *R* and *S* to obtain *their* $\frac{1}{2}\begin{vmatrix}0 & 0 & \frac{-5}{4} & 0\\ 1 & \frac{-17}{8} & \frac{-3}{2} & 1\end{vmatrix}$
	$= \frac{1}{2}\left\|-\frac{5}{4}-\frac{85}{32}\right\|$ oe	**M1**	**M1dep** for correct method of evaluation
	$= \frac{125}{64}$ or 1.95 or $1\frac{61}{64}$	**A1**	

Cambridge Assessment International Education
Cambridge International General Certificate of Secondary Education

ADDITIONAL MATHEMATICS **0606/21**

Paper 2 **October/November 2017**

MARK SCHEME

Maximum Mark: 80

Published

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Question	Answer	Marks	Guidance
1	$x^2-6x-7(>0)$	**B1**	
	$(x-7)(x+1)(>0)$	**M1**	
	Critical values 7 and –1	**A1**	
	$x>7$ or $x<-1$	**A1**	
2	$\dfrac{(1+\sin\theta)-(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$	**M1**	Dealing with fractions
	$=\dfrac{2\sin\theta}{(1-\sin^2\theta)}$	**A1**	Simplification
	$=\dfrac{2\sin\theta}{\cos^2\theta}$	**M1**	Use of identity (seen anywhere)
	$=2\tan\theta\sec\theta$	**M1**	Use of $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ and $\sec\theta=\dfrac{1}{\cos\theta}$ (seen anywhere)
3	$2=\log_5 25$	**B1**	
	$\log_5 25+\log_5(x-7)=\log_5 25(x-7)$ $10x+5=25(x-7)$	**M1**	
	$180=15x$	**M1**	Equate, clear brackets and collect terms.
	$12=x$	**A1**	

Question	Answer	Marks	Guidance
4	$x-2\left(4-\sqrt{3}x\right)=5\sqrt{3}$	**M1**	Eliminate y
	$x=\frac{5\sqrt{3}+8}{2\sqrt{3}+1}$	**A1**	
	$x=\frac{\left(5\sqrt{3}+8\right)\left(2\sqrt{3}-1\right)}{\left(2\sqrt{3}+1\right)\left(2\sqrt{3}-1\right)}$	**M1**	Multiply by $\left(a\sqrt{b}+c\right)$ as appropriate
	$x=2+\sqrt{3}$	**A1**	
	$y=1-2\sqrt{3}$	**A1**	
	<u>Alternative method</u> $\sqrt{3}\left(5\sqrt{3}+2y\right)+y=4$	**M1**	Eliminate x
	$y=\frac{-11}{\left(2\sqrt{3}+1\right)}$	**A1**	
	$y=\frac{-11\left(2\sqrt{3}-1\right)}{\left(2\sqrt{3}+1\right)\left(2\sqrt{3}-1\right)}$	**M1**	Multiply by $\left(a\sqrt{b}+c\right)$ as appropriate
	$y=1-2\sqrt{3}$	**A1**	
	$x=2+\sqrt{3}$	**A1**	
5(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{5}{3x+2}\right)=-5\left(3x+2\right)^{-2}\times 3$	**M1**	$-5\left(3x+2\right)^{-2}$
		A1	$\times 3$
5(ii)	$\int\frac{30}{\left(3x+2\right)^{2}}\,\mathrm{d}x=\left[\frac{-10}{\left(3x+2\right)}\right]$	**M1**	$\frac{1}{\left(3x+2\right)}$
		A1	$\times -10$
5(iii)	$\left[\frac{-10}{\left(3x+2\right)}\right]_{1}^{2}=-\frac{10}{8}+\frac{10}{5}$	**M1**	Insert limits and subtract
	$=\frac{3}{4}$	**A1**	
6(i)	$2q+3p=13$	**B1**	

Question	Answer	Marks	Guidance
6(ii)	Multiply matrices correctly	**M1**	
	$2p+pq=12$	**A1**	
6(iii)	$4p+p(13-3p)=24$	**M1**	Eliminate q
	$3p^2-17p+24=0$	**A1**	
	$(3p-8)(p-3)=0$	**M1**	Solve
	$p=3,\ q=2$	**A1**	
7	$\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2-\frac{1}{x^2}(+C)$	**B2**	**B1** for $3x^2$ **B1** for $-\frac{1}{x^2}$.
	$x=1,\ \frac{\mathrm{d}y}{\mathrm{d}x}=1\rightarrow C=-1$	**B1**	
	$y=x^3+\frac{1}{x}-x+D$ $x=1,\ y=3\rightarrow D=2$	**B2**	**B1** for two correct terms in x
	$y=x^3+\frac{1}{x}-x+2$	**B1**	
8	$z^2=a^2+3(a+3)^2+2a(a+3)\sqrt{3}$ $=79+b\sqrt{3}$	**M1**	
	$a^2+3(a+3)^2=79$ and $2a(a+3)=b$	**A1**	**FT** Equate correctly to obtain both eqns
	$a^2+3a^2+18a+27=79$ $4a^2+18a-52=0$	**M1**	Expand and simplify to obtain 3 term quadratic
	$(a-2)(4a+26)=0$	**M1**	
	$a=2,\quad b=20$	**A2**	**A1** for each
9(i)	$1+4x+6x^2+4x^3+x^4$	**B1**	
9(ii)	$1296-864x+216x^2-24x^3+x^4$	**B2**	Minus 1 each error.
9(iii)	$1295-868x+210x^2-28x^3=175$	**M1**	Subtract and equate to 1
	$28x^3-210x^2+868x-1120=0$	**A1**	

Question	Answer	Marks	Guidance
9(iv)	$28(2)^3-210(2)^2+868(2)-1120$	**M1**	Inserts $x=2$
	$=224-840+1736-1120=0$ $(x-2)$ is a factor	**A1**	
	$(x-2)(28x^2-154x+560)$	**M1A1**	**M1** for 28 and 560 seen oe **A1** for –154
	$b^2-4ac<0$ shown	**B1**	
10(i)	$\mathbf{r}_A=(2\mathbf{i}+4\mathbf{j})+t(\mathbf{i}+\mathbf{j})$	**B1**	
10(ii)	$\mathbf{r}_B=(10\mathbf{i}+14\mathbf{j})+t(-2\mathbf{i}-3\mathbf{j})$	**B1**	
10(iii)	$\mathbf{r}_B-\mathbf{r}_A=(8\mathbf{i}+10\mathbf{j})+t(-3\mathbf{i}-4\mathbf{j})$	**M1**	
	$X^2=(8-3t)^2+(10-4t)^2$	**M1A1**	
10(iv)	Differentiate	**M1**	
	$\frac{dX^2}{dt}=2(8-3t)(-3)+2(10-4t)(-4)$ oe	**A1**	
	$\frac{dX^2}{dt}=0 \rightarrow t=2.56$ $\rightarrow X=0.4$	**B2**	**B1** for value of t **B1** for value of X.
11(i)	$x^2-2x+(kx+3)^2=8$	**M1**	Eliminate y
	$(1+k^2)x^2+(6k-2)x+1=0$	**A1**	
	$b^2-4ac=0 \rightarrow (6k-2)^2-4(1+k^2)=0$	**M1**	
	$k=\frac{3}{4}$	**A1**	Answer given
11(ii)	$x=\frac{-b}{2a} \rightarrow x=\frac{-2.5}{2\times1.5625}$	**M1**	
	$=-0.8$	**A1**	
	$y=0.75\times-0.8+3=2.4$	**A1**	**FT**

Question	Answer	Marks	Guidance
11(iii)	Eqn of PQ $\frac{y-2.4}{x+0.8} = \frac{-4}{3}$	**M1**	
	$\rightarrow 3y = 4-4x$	**A1**	
12(i)	$\frac{\mathrm{d}(\cos x)^{-1}}{\mathrm{d}x} = \frac{1}{\cos^2 x} \times \sin x$	**M1**	$\frac{1}{\cos^2 x}$
		A1	$\times \sin x$
12(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \sec^2 x + \frac{4\sin x}{\cos^2 x}$	**B1**	$\sec^2 x$
		B1	$\frac{4\sin x}{\cos^2 x}$
12(iii)	$\frac{1}{\cos^2 x} + \frac{4}{\cos x} \times \frac{\sin x}{\cos x} = 4$	**M1**	Equate *their* (i) to 4 and multiply by $\cos^2 x$
	$\rightarrow 1 + 4\sin x = 4\cos^2 x$	**M1**	Use of identity and simplify
	$4\sin^2 x + 4\sin x - 3 = 0$	**A1**	
	$(2\sin x - 1)(2\sin x + 3) = 0$	**M1**	Solve
	$x = \frac{\pi}{6}, \frac{5\pi}{6}$	**A2**	**A1** for each

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Question	Answer	Marks	Partial Marks
1	$z^2 = 7 + 4\sqrt{3}$	**B1**	Accept $4 + 3 + 4\sqrt{3}$
	$a\left(7 + 4\sqrt{3}\right) + b\left(2 + \sqrt{3}\right) = 1 + \sqrt{3}$	**M1**	Equate both $\sqrt{3}$ terms and constant terms to obtain two equations in a and b.
	$7a + 2b = 1$ $4a + b = 1$	**A1**	Both correct. Accept equation with a multiple of $\sqrt{3}$
	Attempt to solve a pair of linear simultaneous eqns to $a =$ or $b =$	**M1**	**M1dep**
	$a = 1$and $b = -3$	**A1**	
2	$2x^{1.5} + 6x^{-0.5} = x\left(x^{0.5} + 5x^{-0.5}\right)$	**M1**	Attempt to multiply by $x^{0.5} + 5x^{-0.5}$ or $x^{0.5}$ or divide by $x^{0.5}$
	$2x^{1.5} + 6x^{-0.5} = x^{1.5} + 5x^{0.5}$ or $x^{1.5} - 5x^{0.5} + 6x^{-0.5} = 0$ or $\dfrac{2x^2 + 6}{x + 5} = x$ or $\dfrac{2x + \frac{6}{x}}{1 + \frac{5}{x}} = x$	**A1**	Simplified numerical powers
	$x^2 - 5x + 6 = 0$	**M1**	**M1dep** obtain a three term quadratic. Allow errors in signs and coefficients but not powers
	$(x - 3)(x - 2) = 0$	**M1**	Solve a three term quadratic
	$x = 3$ or 2 only	**A1**	
3	Correctly obtain a value of $x = 2$	**B1**	Inequality not required
	Correctly obtain a value of $x = -\dfrac{1}{2}$	**B1**	Inequality not required
	$x > 2$ and $x < -\dfrac{1}{2}$	**B1**	**B1dep** mark final answer(s). Allow $2 < x < -\dfrac{1}{2}$

Question	Answer	Marks	Partial Marks
4	$x+4=y^2$	**B1**	
	$7y-x=16$ $7y-16+4=y^2$	**B1**	allow 2^4 for16
	$y^2-7y+12\to(y-3)(y-4)(=0)$ or $x^2-17x+60\to(x-5)(x-12)(=0)$	**M1**	**Attempt** to eliminate x or y to obtain a three term quadratic.
	Solve a three term quadratic	**M1**	**M1dep**
	$\to y=3,\ x=5$ or $y=4\ x=12$	**A1**	Allow for values seen even if correct pairs not clear.
5(i)	${}^{10}C_4=210$	**B1**	
5(ii)	2 Mystery 2 others = ${}^5C_2\times{}^5C_2$ =100 3 Mystery 1 other = ${}^5C_3\times{}^5C_1$ = 50 4 Mystery = ${}^5C_4$ =5 Total 155	**B3**	**B1** for one combination, unsimplifiied **B1** for second combination, unsimplifiied **B1** for third combination, unsimplifiied and total
	Alternative Method All – 0 Mystery – 1 Mystery	**B1**	All minus 0 or 1 or both
	$=210-{}^5C_4-{}^5C_1\times{}^5C_3$	**B1**	**B1dep** 1Mystery and 0 mystery unsimplified
	$=210-5-5\times10=155$	**B1**	**B1dep** final answer
5(iii)	2M1C1R = ${}^5C_2\times{}^3C_1\times{}^2C_1=60$ 1M2C1R = ${}^5C_1\times{}^3C_2\times{}^2C_1=30$ 1M1C2R $={}^5C_1\times{}^3C_1\times{}^2C_2$ $=15$ Total 105	**B3**	**B1** for one combination, unsimplifiied **B1** for second combination, unsimplifiied **B1** for third combination, unsimplifiied and total
6(i)	$\pi x^2h=500\to h=\dfrac{500}{\pi x^2}$	**B1**	Ignore units Condone r for x
6(ii)	$A=2\pi x^2+2\pi xh$	**M1**	Correct expression for *A* and insert for *their h.*
	$=2\pi x^2+2\pi x\times\dfrac{500}{\pi x^2}$ = $2\pi x^2+\dfrac{1000}{x}$	**A1**	Answer given Condone r for x.

Question	Answer	Marks	Partial Marks
6(iii)	Differentiate: at least one power reduced by 1	**M1**	
	$\frac{\mathrm{d}A}{\mathrm{d}x}=4\pi x-\frac{1000}{x^2}$	**A1**	
	$\frac{\mathrm{d}A}{\mathrm{d}x}=0\rightarrow x=\sqrt[3]{\frac{1000}{4\pi}}$ isw or $\left(x=4.3(0)\right)$	**A1**	
	$A=2\pi\left(4.3\right)^2+\frac{1000}{4.3}=349\,\text{cm}^2$	**A1**	awrt 349
	$\frac{\mathrm{d}^2A}{\mathrm{d}x^2}=4\pi+\frac{2000}{x^3}$ (>0) or a positive value $(\rightarrow$ min)	**B1**	Correct second differential (need not be evaluated) and conclusion. or Examine correct gradient either side of $x=4.3$ and conclusion
7(i)	(Gradient or $\frac{\mathrm{d}y}{\mathrm{d}x}$) $=\frac{3x-1}{\sqrt{x}}$	**B1**	Gradient = Negative reciprocal. Can be implied.
	$=3x^{\frac{1}{2}}-x^{-\frac{1}{2}}$	**B1**	± One correct term
	$y=2x^{\frac{3}{2}}-2x^{\frac{1}{2}}\left(+C\right)$	**M1**	at least 1 fractional power increased by1.
	$-10=2-2+C\rightarrow C=-10$	**A1**	one term correct with simplified coefficients
	$y=2x^{\frac{3}{2}}-2x^{\frac{1}{2}}-10$	**A1**	For *C* from correct working.
7(ii)	$x=4\rightarrow y=16-4-10=2$	**B1**	
	$\rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=6-\frac{1}{2}=5.5$	**B1**	
	Eqn with *their* grad and point (4, ...)	**M1**	
	Eqn of tangent: $\frac{y-2}{x-4}=5.5\rightarrow y=5.5x-20$ oe	**A1**	Must be in the form $y=mx+c$ but accept $2y=11x-40$
8(i)	$2\mathbf{A}=\begin{pmatrix}4 & 2\\ 8 & 6\end{pmatrix}$	**B1**	
	$\left(2\mathbf{A}\right)^{-1}=\frac{1}{8}\begin{pmatrix}6 & -2\\ -8 & 4\end{pmatrix}$	**B2**	**B1** for $\begin{pmatrix}6 & -2\\ -8 & 4\end{pmatrix}$ **B1** for $\frac{1}{8}$

Question	Answer	Marks	Partial Marks
8(ii)	$4x+2y=-5$ $8x+6y=-9$	**B1**	
	Pre multiply $\begin{pmatrix}-5\\-9\end{pmatrix}$ by a 2×2 matrix.	**M1**	Allow recovery
	$\begin{pmatrix}x\\y\end{pmatrix}=\frac{1}{8}\begin{pmatrix}6&-2\\-8&4\end{pmatrix}\begin{pmatrix}-5\\-9\end{pmatrix}$	**M1**	Pre multiply *their* $\begin{pmatrix}-5\\-9\end{pmatrix}$ by *their* answer to (i)
	$\begin{pmatrix}x\\y\end{pmatrix}=\frac{1}{8}\begin{pmatrix}-12\\4\end{pmatrix}=\begin{pmatrix}-1.5\\0.5\end{pmatrix}$	**A2**	**A1** for x value **A1** for y value oe Allow both unsimplified
9(i)	$\frac{\mathrm{d}}{\mathrm{d}x}(x\ln x)=x\times\frac{1}{x}+\ln x$ isw	**M1A1**	Product rule. One correct term + another term. Allow unsimplified.
9(ii)	$\int 1+\ln x\mathrm{d}x=x\ln x$	**M1**	Correct use of (i) and must be dealing with 2 terms. soi
	$\int \ln x dx=x\ln x-x+(C)$	**A1**	Correct answer with no working is fine.
9(iii)	$\int_k^{2k}\ln x\,\mathrm{d}x=[2k\ln 2k-2k]-[k\ln k-k]$ $=k(2\ln 2k-\ln k-1)$	**M1**	Insert limits and subtract correctly using *their* result from (ii) which must contain an ln function
	$=k\left(\ln(2k)^2-\ln k-1\right)$	**M1**	Uses $n\ln a=\ln a^n$ somewhere oe
	$=k\left(\ln\left(\frac{4k^2}{k}\right)-1\right)$	**M1**	Uses $\ln a-\ln b=\ln\left(\frac{a}{b}\right)$ or $\ln a+\ln b=\ln ab$ somewhere
	$=k(\ln 4k-1)$	**A1**	Answer given Correct completion.

Question	Answer	Marks	Partial Marks
10(i)	$c=1\rightarrow 6\left(1\right)^3-7\left(1\right)^2+1=0\rightarrow\left(c-1\right)$ is a factor.	**B1**	Or correct division. Finding or using one correct factor.
	Attempt to factorise or use long division to obtain $6c^2\ldots\pm 1$ or $6c^2\pm c\ldots$ respectively	**M1**	
	$\left(c-1\right)\left(6c^2-c-1\right)=0$	**A1**	
	$\left(c-1\right)\left(2c-1\right)\left(3c+1\right)=0$	**A1**	
	$c=1,\ \frac{1}{2},-\frac{1}{3}$	**A1**	**FT** From three different linear factors
10(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\sec^2 x+6\cos x$	**B2**	**B1** for each term
10(iii)	$\frac{1}{\cos^2 x}+6\cos x=7$	**B1**	**B1dep** Replaces $\sec^2 x$ by $\frac{1}{\cos^2 x}$
	$\rightarrow 6\cos^3 x-7\cos^2 x+1=0$	**B1**	**B1dep** Answer given so all steps must be correct.
10(iv)	$\cos x=1,\ \frac{1}{2},\ -\frac{1}{3}.$ $\rightarrow x=0,\ 1.05\left(\text{or } \frac{\pi}{3}\right),\ 1.91$	**A2**	**A1** for 2 values awrt **A1** for third value and no others in range. No credit for answers in degrees
11(i)	$y=0\rightarrow\left(x-4\right)\left(x+1\right)=0$	**M1**	Solve
	$\rightarrow A$ is $\left(4,0\right)$ nfww	**A1**	Indication somewhere that $x=4$ when $y=0$
11(ii)	$4+3x-x^2=mx+8$ $x^2+\left(m-3\right)x+4=0$	**M1**	Eliminate y.
	$b^2-4ac\left(=0\right)\rightarrow\left(m-3\right)^2=16$	**M1**	**M1dep** Use of discriminant
	$m=-1$	**A1**	Do not award if $m=7$ is not discarded
11(iii)	Obtain quadratic $x^2+\left(m-3\right)x+4=0$ using *their* m and attempt to solve.	**M1**	Working must be seen for any marks to be awarded. Must not be awarded if m is not obtained correctly
	Point B (2, 6)	**A1**	

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11(iv)	Area under curve $=\int_2^4 \left(4+3x-x^2\right)\mathrm{d}x$ Integrate powers increased in at least 2 terms	**M1**	
	$=\left[4x+\frac{3}{2}x^2-\frac{1}{3}x^3\right]_2^4$	**A1**	
	$=\left[16+24-\frac{64}{3}\right]-\left[8+6-\frac{8}{3}\right]$ $=7\frac{1}{3}$	**M1**	**M1dep** Insert limits of *their* 2 and 4 and subtract in correct order. May be implied by $18\frac{2}{3}-\ldots$
	Intercept is (8,0) so area of triangle = $\frac{6\times 6}{2}=18$	**M1**	Area of triangle using *their B* = $\frac{\left(their 8-x_B\right)}{2}\times y_B$ or Attempt to find other suitable areas to result in a complete method.
	Shaded area $=18-7\frac{1}{3}=10\frac{2}{3}$	**A1**	Accept 10.7. Must not be awarded if point *B* is not obtained correctly.

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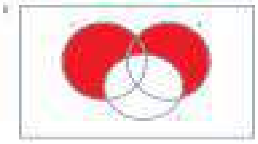
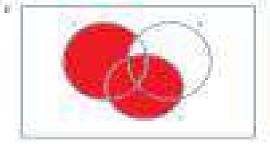
soi	seen or implied

Question	Answer	Marks	Guidance
1(a)	 	**B2**	**B1** for each
1(b)	$n(P')$ = 18	**B1**	
	$n((Q \cup R) \cap P)$ = 11	**B1**	
	$n(Q' \cup P)$ = 29	**B1**	
2	$3x-1=5+x$ $x = 3$	**B1**	
	$3x-1=-5-x$ oe	**M1**	M1 not earned if incorrect equation(s) present
	$x = -1$	**A1**	
3	$\frac{p(\sqrt{3}+1)+(\sqrt{3}-1)}{(\sqrt{3}-1)(\sqrt{3}+1)}=q+3\sqrt{3}$	**M1**	on LHS take common denominator or rationalise each term or multiply throughout
	$p(\sqrt{3}+1)+(\sqrt{3}-1)=2q+6\sqrt{3}$ oe	**A1**	correct eqn with no surds in denominators of LHS
	equate surd/non surd parts	**M1**	equate and solve for p or q ($\neq 0$)
	$p = 5$ and $q = 2$	**A1**	
4	$\log_3 3=1$ or $\log_3 9=2$	**B1**	implied by one correct equation
	$x+1=3y$	**B1**	
	$x-y=9$	**B1**	
	solve correct equations for x or y	**M1**	
	$x=14$ and $y=5$	**A1**	
5(i)	$\overrightarrow{OX}=\lambda(1.5\mathbf{b}+3\mathbf{a})$	**B1**	
5(ii)	$\overrightarrow{AB}=\mathbf{b}-\mathbf{a}$ or $\overrightarrow{BA}=\mathbf{a}-\mathbf{b}$	**B1**	
	$\overrightarrow{OX}=\mathbf{a}+\mu(\mathbf{b}-\mathbf{a})$	**B1**	
5(iii)	$1.5\lambda=\mu$ or $3\lambda=1-\mu$	**M1**	$\overrightarrow{OX}=\overrightarrow{OX}$ and equate for **a** or **b**
	$\mu=\frac{1}{3}$ $\lambda=\frac{2}{9}$	**A2**	**A1** for each

Question	Answer	Marks	Guidance
5(iv)	$\frac{AX}{XB}=\frac{1}{2}$	**B1**	Accept 1 : 2 but not $\frac{1}{2}:1$
5(v)	$\frac{OX}{XD}=\frac{2}{7}$	**B1**	Accept 2 : 7 but not $\frac{2}{7}:1$
6(i)	$f^2 = f(f)$ used algebraic $([(x + 2)^2 + 1] + 2)^2 + 1$	**M1**	numerical or algebraic
	17	**A1**	
6(ii)	$x=\frac{y-2}{2y-1}$	**M1**	change x and y
	$2xy-x=y-2 \rightarrow y(2x-1)=x-2$	**M1**	**M1dep** multiply, collect y terms, factorise
	$y=\frac{x-2}{2x-1}$ $\quad [=g(x)]$	**A1**	correct completion
6(iii)	$gf(x)=\frac{\left[(x+2)^2+1\right]-2}{2\left[(x+2)^2+1\right]-1}$ oe	**B1**	
	$\frac{(x+2)^2-1}{2(x+2)^2+1}=\frac{8}{19}$ $3(x+2)^2=27$ oe $\quad 3x^2 + 12x - 15 = 0$	**M1**	*their* gf = $\frac{8}{19}$ and simplify to quadratic equation
	solve quadratic	**M1**	**M1dep** Must be of equivalent form
	$x=1 \quad x=-5$	**A1**	
7(i)	$v=0 \rightarrow \cos 2t=\frac{1}{3}$	**M1**	set $v=0$ and solve for $\cos 2t$
	$\rightarrow t=0.615$ or 0.616	**A1**	
7(ii)	$s=\frac{3}{2}\sin 2t-t \quad (+c)$	**M1A1**	**M1** for $\sin 2t$ and $\pm t$
	$t=\frac{\pi}{4}\rightarrow \qquad s=1.5-\frac{\pi}{4} \qquad (=0.715)$	**A1**	
7(iii)	$a=-6\sin 2t$	**M1A1**	**M1** for $-\sin 2t$
	$t = 0.615 \rightarrow a = -5.66$ or -5.65 or $-2\sqrt{8}$	**A1**	condone substitution of degrees

Question	Answer	Marks	Guidance
8(i)	$\cos\alpha = \frac{1}{3}$ oe	**M1**	
	$\alpha = 70.5°$	**A1**	
8(ii)	speed $= \sqrt{3^2 - 1^2}$	**M1**	Pythagoras/trig ratio/cosine rule
	$\sqrt{8}$ or $2\sqrt{2}$ or 2.83 m s^{-1}	**A1**	
8(iii)	time = $\frac{50}{their\sqrt{8}}$	**M1**	
	$\frac{25\sqrt{2}}{2}$ or 17.7s	**A1**	
8(iv)	*their* $8(\text{iii})$ seen	**B1**	
	$BC = 10\sqrt{2}$ or 14.1 m or 14.2 m	**B1**	
9(i)	$\frac{\mathrm{d}}{\mathrm{d}x}(\ln x) = \frac{1}{x}$ and $\frac{\mathrm{d}}{\mathrm{d}x}x^3 = 3x^2$ or $\frac{\mathrm{d}}{\mathrm{d}x}x^{-3} = -3x^{-4}$	**B1**	seen
	Substitution of *their* derivatives into quotient rule	**M1**	
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\ln x}{x^3}\right) = \frac{x^3 \times \frac{1}{x} - 3x^2 \ln x}{x^6}$ oe	**A1**	correct completion
9(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \to 1 - 3\ln x = 0$ $\ln x = \frac{1}{3}$	**M1**	equate given $\frac{\mathrm{d}y}{\mathrm{d}x}$ to zero and solve for ln$x$ or x
	$x = \mathrm{e}^{\frac{1}{3}}$	**A1**	seen
	$y = \frac{1}{3\mathrm{e}}$	**A1**	seen
9(iii)	$\frac{\ln x}{x^3} = \int \frac{1 - 3\ln x}{x^4}\mathrm{d}x$ oe	**M1**	use given statement in (i)
	$\int \frac{1}{x^4}\mathrm{d}x = \frac{-1}{3x^3}$	**B1**	seen anywhere
	$\int \frac{\ln x}{x^4}\mathrm{d}x = -\frac{1}{9x^3} - \frac{\ln x}{3x^3}$ (+C) oe	**A2**	**A1** for each term

Question	Answer	Marks	Guidance
10(a)	LHS $=\dfrac{\sin^2 x+(1+\cos x)^2}{\sin x(1+\cos x)}$	**B1**	correct addition of fractions
	$=\dfrac{1+2\cos x+1}{\sin x(1+\cos x)}$	**B1**	expansion and use of identity
	$=\dfrac{2(1+\cos x)}{\sin x(1+\cos x)}=2\mathrm{cosec}x$	**B1**	factorisation and completion
10(b)(i)	$\mathrm{cosec}^2 y-1+\mathrm{cosec}y-5=0$ $\mathrm{cosec}^2 y+\mathrm{cosec}y-6=0$	**M1**	use of identity for $\cot^2 y$ to obtain quadratic in cosec$y$
	$(\mathrm{cosec}y-2)(\mathrm{cosec}y+3)=0$	**M1**	solve 3 term quadratic for cosec$y$
	$\sin y=\dfrac{1}{2}$, $\sin y=-\dfrac{1}{3}$	**M1**	obtain values for sin$y$
	$y=30°, 150°, 199.5°, 340.5°$	**A2**	**A1** for 2 values
10(b)(ii)	$2z+\dfrac{\pi}{4}=\dfrac{5\pi}{6}$ or $\dfrac{7\pi}{6}$ (2.6…, 3.6…)	**M2**	**M1** equate to $\dfrac{5\pi}{6}$ **M1** equate to $\dfrac{7\pi}{6}$
	$z=\dfrac{7\pi}{24}$ or $\dfrac{11\pi}{24}$ (0.916, 1.44)	**A2**	**A1** for 1 value
11(i)	Other root $=4$	**B1**	
	$\mathrm{f}(x)=(x-3)(x-3)(x-4)$ $=x^3-10x^2+33x-36$	**M1**	multiply out $(x-3)(x-3)(x\pm p)$
	$a=-10$ $b=33$	**A2**	**A1** for each Can be implied by correct cubic
11(ii)	$x=6,\ x=6,\ x=1$ $x=2,\ x=2,\ x=9$ $x=1,\ x=1,\ x=36$	**B4**	**B1** for each of first two sets **B2** for third set

Cambridge International Examinations
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CANDIDATE NAME				
CENTRE NUMBER			CANDIDATE NUMBER	

1404301964

ADDITIONAL MATHEMATICS **0606/11**

Paper 1 **October/November 2017**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **16** printed pages.

DC (NH/FC) 135359/3

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1} b + \binom{n}{2}a^{n-2} b^2 + \ldots + \binom{n}{r}a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

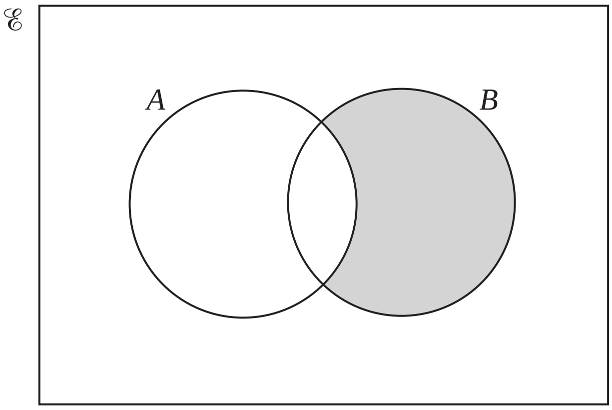
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

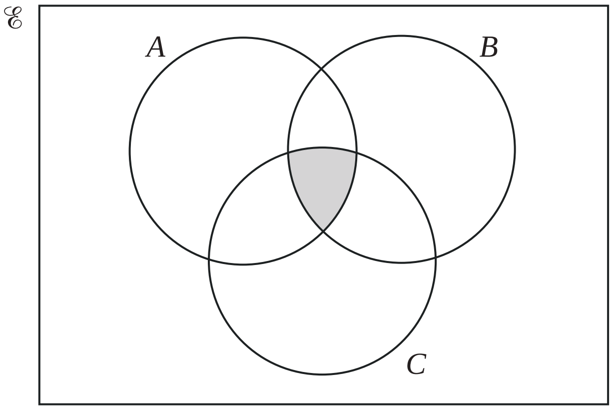
1 Express in set notation the shaded regions shown in the Venn diagrams below.

(i)



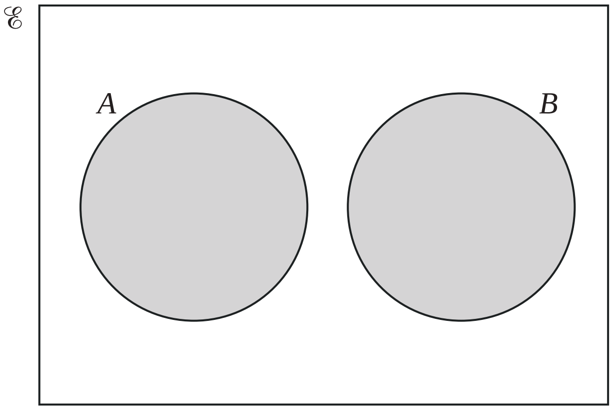
.... [1]

(ii)



.... [1]

(iii)



.... [1]

2 The polynomial $p(x)$ is $ax^3 + bx^2 - 13x + 4$, where a and b are integers. Given that $2x - 1$ is a factor of $p(x)$ and also a factor of $p'(x)$,

(i) find the value of a and of b. [5]

Using your values of a and b,

(ii) find the remainder when $p(x)$ is divided by $x + 1$. [2]

3 **(a)** Given that $T = 2\pi l^{\frac{1}{2}} g^{-\frac{1}{2}}$, express l in terms of T, g and π. [2]

(b) By using the substitution $y = x^{\frac{1}{3}}$, or otherwise, solve $x^{\frac{2}{3}} - 4x^{\frac{1}{3}} + 3 = 0$. [4]

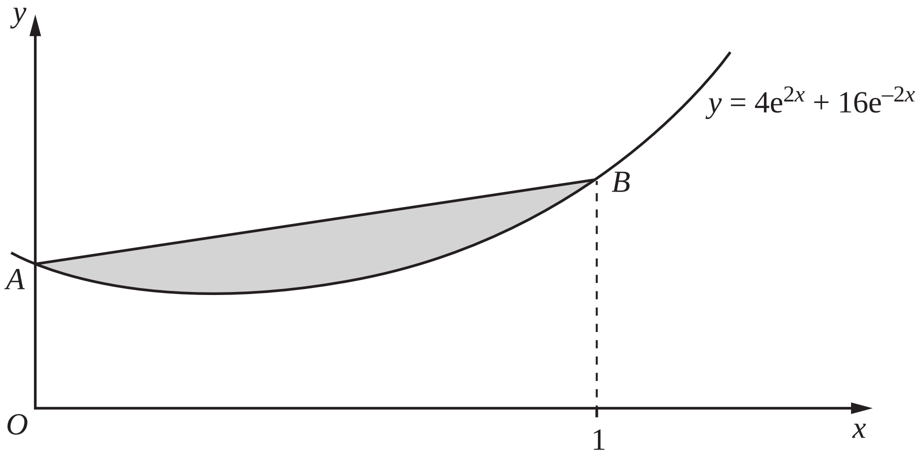
4 When lg$y$ is plotted against x^2 a straight line is obtained which passes through the points (4, 3) and (12, 7).

(i) Find the gradient of the line. [1]

(ii) Use your answer to part **(i)** to express lg$y$ in terms of x. [2]

(iii) Hence express y in terms of x, giving your answer in the form $y = A\left(10^{bx^2}\right)$ where A and b are constants. [3]

5



The diagram shows part of the graph of $y = 4e^{2x} + 16e^{-2x}$ meeting the y-axis at the point A and the line $x = 1$ at the point B.

(i) Find the coordinates of A. [1]

(ii) Find the y-coordinate of B. [1]

(iii) Find $\int\left(4e^{2x} + 16e^{-2x}\right)dx$. [2]

(iv) Hence find the area of the shaded region enclosed by the curve and the line AB. You must show all your working. [4]

6 **(a)** Functions f and g are such that, for $x \in \mathbb{R}$,

$$f(x) = x^2 + 3,$$

$$g(x) = 4x - 1.$$

(i) State the range of f. [1]

(ii) Solve $fg(x) = 4$. [3]

(b) A function h is such that $\mathrm{h}(x)=\dfrac{2x+1}{x-4}$ for $x\in\mathbb{R},\ x\neq 4$.

(i) Find $\mathrm{h}^{-1}(x)$ and state its range. [4]

(ii) Find $\mathrm{h}^{2}(x)$, giving your answer in its simplest form. [3]

7 **(i)** Write $\ln\left(\frac{2x+1}{2x-1}\right)$ as the difference of two logarithms. [1]

A curve has equation $y = \ln\left(\frac{2x+1}{2x-1}\right) + 4x$ for $x > \frac{1}{2}$.

(ii) Using your answer to part **(i)** show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{ax^2 + b}{4x^2 - 1}$, where *a* and *b* are integers. [4]

(iii) Hence find the *x*-coordinate of the stationary point on the curve. [2]

(iv) Determine the nature of this stationary point. [2]

8 **(a)** 10 people are to be chosen, to receive concert tickets, from a group of 8 men and 6 women.

(i) Find the number of different ways the 10 people can be chosen if 6 of them are men and 4 of them are women. [2]

The group of 8 men and 6 women contains a man and his wife.

(ii) Find the number of different ways the 10 people can be chosen if both the man and his wife are chosen or neither of them is chosen. [3]

(b) Freddie has forgotten the 6-digit code that he uses to lock his briefcase. He knows that he did not repeat any digit and that he did not start his code with a zero.

(i) Find the number of different 6-digit numbers he could have chosen. [1]

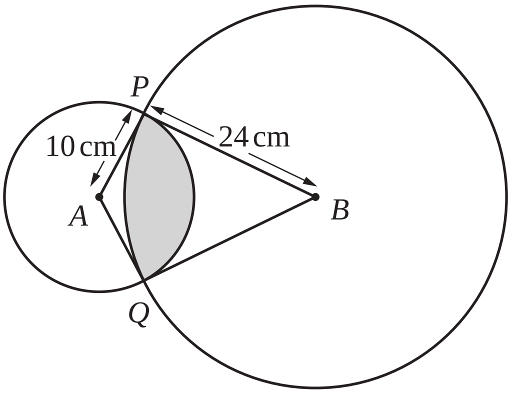
Freddie also remembers that his 6-digit code is divisible by 5.

(ii) Find the number of different 6-digit numbers he could have chosen. [3]

Freddie decides to choose a new 6-digit code for his briefcase once he has opened it. He plans to have the 6-digit number divisible by 2 and greater than 600 000, again with no repetitions of digits.

(iii) Find the number of different 6-digit numbers he can choose. [3]

9



The diagram shows a circle, centre *A*, radius 10 cm, intersecting a circle, centre *B*, radius 24 cm. The two circles intersect at the points *P* and *Q*. The radii *AP* and *AQ* are tangents to the circle with centre *B*. The radii *BP* and *BQ* are tangents to the circle with centre *A*.

(i) Show that angle *PAQ* is 2.35 radians, correct to 3 significant figures. [2]

(ii) Find angle *PBQ* in radians. [1]

(iii) Find the perimeter of the shaded region. [3]

(iv) Find the area of the shaded region. [4]

Question 10 is printed on the next page.

10 **(a)** Solve $3\operatorname{cosec} 2x - 4\sin 2x = 0$ for $0° \leqslant x \leqslant 180°$. [4]

(b) Solve $3\tan\left(y - \frac{\pi}{4}\right) = \sqrt{3}$ for $0 \leqslant y \leqslant 2\pi$ radians, giving your answers in terms of π. [4]

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CANDIDATE NAME	

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ADDITIONAL MATHEMATICS **0606/12**

Paper 1 **October/November 2017**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

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This document consists of **16** printed pages.

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[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

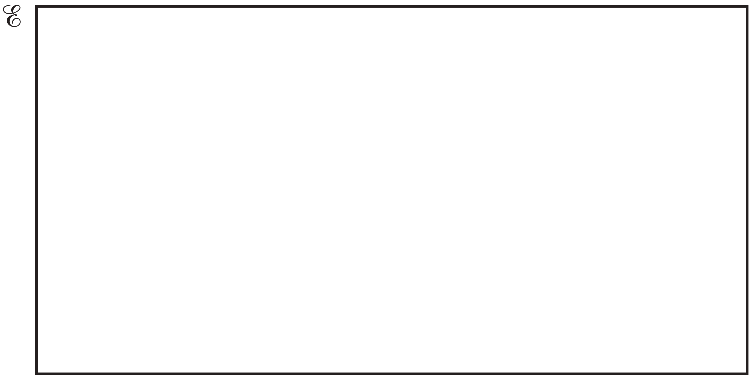
Formulae for ∆ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

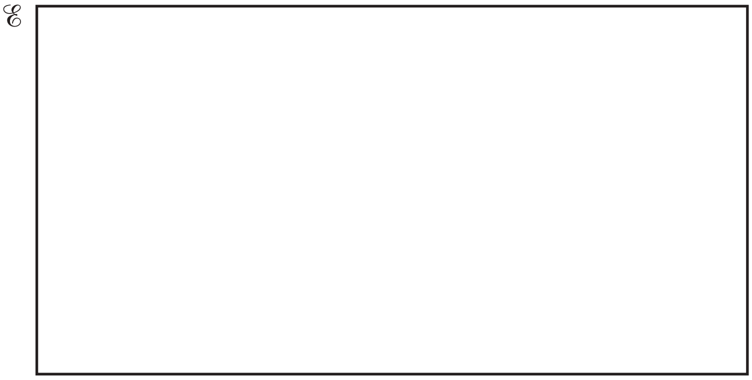
$$\Delta = \frac{1}{2} bc \sin A$$

1 **(i)** On the Venn diagram below, draw sets X and Y such that $\mathrm{n}(X \cap Y) = 0$.



[1]

(ii) On the Venn diagram below, draw sets A, B and C such that $C \subset (A \cup B)'$.



[2]

2 The graph of $y = a\sin(bx) + c$ has an amplitude of 4, a period of $\frac{\pi}{3}$ and passes through the point $\left(\frac{\pi}{12},\ 2\right)$. Find the value of each of the constants a, b and c. [4]

3 **(i)** Find, in ascending powers of x, the first 3 terms in the expansion of $\left(2-\dfrac{x^2}{4}\right)^5$. [3]

(ii) Hence find the term independent of x in the expansion of $\left(2-\dfrac{x^2}{4}\right)^5\left(\dfrac{1}{x}-\dfrac{3}{x^2}\right)^2$. [3]

4 Given that $y = \dfrac{\ln(3x^2+2)}{x^2+1}$, find the value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 2$, giving your answer as $a + b\ln 14$, where a and b are fractions in their simplest form. [6]

5 When $\lg y$ is plotted against x, a straight line is obtained which passes through the points (0.6, 0.3) and (1.1, 0.2).

(i) Find $\lg y$ in terms of x. [4]

(ii) Find y in terms of x, giving your answer in the form $y = A\left(10^{bx}\right)$, where A and b are constants. [3]

6 Functions f and g are defined, for $x > 0$, by

$$\mathrm{f}(x) = \ln x,$$
$$\mathrm{g}(x) = 2x^2 + 3.$$

(i) Write down the range of f. [1]

(ii) Write down the range of g. [1]

(iii) Find the exact value of $\mathrm{f}^{-1}\mathrm{g}(4)$. [2]

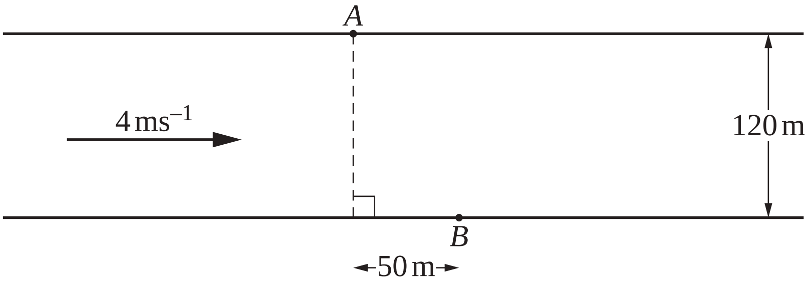
(iv) Find $\mathrm{g}^{-1}(x)$ and state its domain. [3]

7 A polynomial $\mathrm{p}(x)$ is $ax^3 + 8x^2 + bx + 5$, where a and b are integers. It is given that $2x - 1$ is a factor of $\mathrm{p}(x)$ and that a remainder of –25 is obtained when $\mathrm{p}(x)$ is divided by $x + 2$.

(i) Find the value of *a* and of *b*. [5]

(ii) Using your values of *a* and *b*, find the exact solutions of $\mathrm{p}(x) = 5$. [2]

8

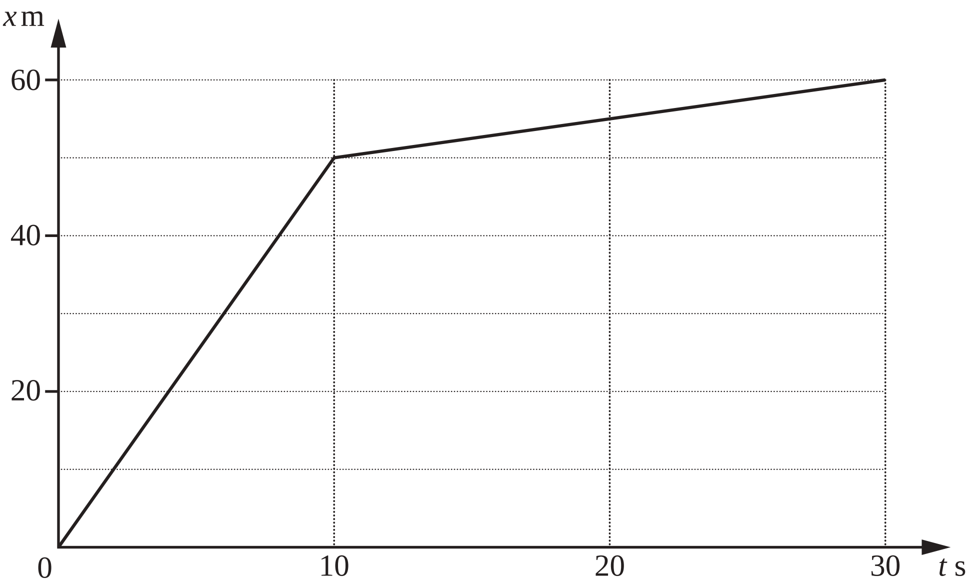


The diagram shows a river which is 120 m wide and is flowing at 4 ms^{-1}. Points *A* and *B* are on opposite sides of the river such that *B* is 50 m downstream from *A*. A man needs to cross the river from *A* to *B* in a boat which can travel at 5 ms^{-1} in still water.

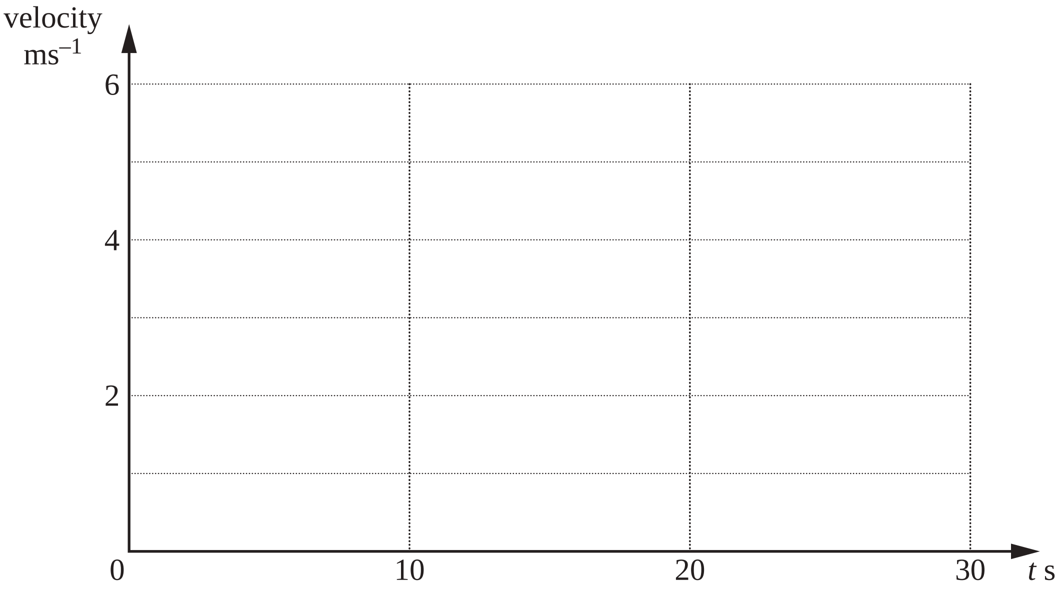
(i) Show that the man must point his boat upstream at an angle of approximately 65° to the bank. [4]

(ii) Find the time the man takes to cross the river from *A* to *B*. [6]

9 **(a)**



The diagram shows the displacement-time graph of a particle P which moves in a straight line such that, t s after leaving a fixed point O, its displacement from O is x m. On the axes below, draw the velocity-time graph of P.



[3]

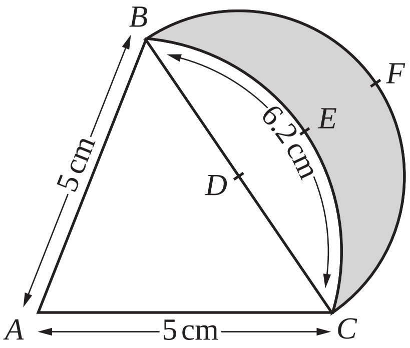
(b) A particle Q moves in a straight line such that its velocity, v ms^{-1}, t s after passing through a fixed point O, is given by $v = 3\mathrm{e}^{-5t} + \frac{3t}{2}$, for $t \geqslant 0$.

(i) Find the velocity of Q when $t = 0$. [1]

(ii) Find the value of t when the acceleration of Q is zero. [3]

(iii) Find the distance of Q from O when $t = 0.5$. [4]

10



The diagram shows an isosceles triangle *ABC*, where $AB = AC = 5$ cm. The arc *BEC* is part of the circle centre *A* and has length 6.2 cm. The point *D* is the midpoint of the line *BC*. The arc *BFC* is a semi-circle centre *D*.

(i) Show that angle *BAC* is 1.24 radians. [1]

(ii) Find the perimeter of the shaded region. [3]

(iii) Find the area of the shaded region. [4]

11 **(a)** Solve $2\cot(\phi + 35^\circ) = 5$ for $0^\circ \leqslant \phi \leqslant 360^\circ$. [4]

Question 11(b) is printed on the next page.

(b) **(i)** Show that $\dfrac{\sec\theta}{\cot\theta+\tan\theta}=\sin\theta$. [3]

(ii) Hence solve $\dfrac{\sec 3\theta}{\cot 3\theta+\tan 3\theta}=-\dfrac{\sqrt{3}}{2}$ for $-\dfrac{\pi}{2}\leqslant\theta\leqslant\dfrac{\pi}{2}$, giving your answers in terms of π. [4]

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CANDIDATE NAME			
CENTRE NUMBER	☐☐☐☐☐	CANDIDATE NUMBER	☐☐☐☐

6581423218

ADDITIONAL MATHEMATICS **0606/13**

Paper 1 **October/November 2017**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

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The total number of marks for this paper is 80.

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International Examinations

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1} b + \binom{n}{2}a^{n-2} b^2 + \ldots + \binom{n}{r}a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Given that $y = 2\sec^2\theta$ and $x = \tan\theta - 5$, express y in terms of x. [2]

2 A curve is such that its gradient at the point (x, y) is given by $10e^{5x} + 3$. Given that the curve passes though the point $(0, 9)$, find the equation of the curve. [4]

3 Find the set of values of k for which the equation $kx^2 + 3x - 4 + k = 0$ has no real roots. [4]

4 The graph of $y = a\cos(bx) + c$ has an amplitude of 3, a period of $\frac{\pi}{4}$ and passes through the point $\left(\frac{\pi}{12}, \frac{5}{2}\right)$. Find the value of each of the constants a, b and c. [4]

5 **(i)** Find $\int (7x-10)^{-\frac{3}{5}}\mathrm{d}x$. [2]

(ii) Given that $\int_{6}^{a} (7x-10)^{-\frac{3}{5}}\mathrm{d}x = \frac{25}{14}$, find the exact value of a. [3]

6 When $\ln y$ is plotted against x^2 a straight line is obtained which passes through the points (0.2, 2.4) and (0.8, 0.9).

(i) Express $\ln y$ in the form $px^2 + q$, where p and q are constants. [3]

(ii) Hence express y in terms of z, where $z = \mathrm{e}^{x^2}$. [3]

7 **(i)** Find, in ascending powers of x, the first 3 terms in the expansion of $\left(2-\frac{x^2}{4}\right)^6$. Give each term in its simplest form. [3]

(ii) Hence find the coefficient of x^2 in the expansion of $\left(2-\frac{x^2}{4}\right)^6\left(\frac{1}{x}+x\right)^2$. [4]

8 It is given that $y = (x-4)(3x-1)^{\frac{5}{3}}$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = (3x-1)^{\frac{2}{3}}(Ax+B)$, where A and B are integers to be found. [5]

(ii) Hence find, in terms of h, where h is small, the approximate change in y when x increases from 3 to 3 + h. [3]

9 **(a)** A 6-digit number is to be formed using the digits 1, 3, 5, 6, 8, 9. Each of these digits may be used only once in any 6-digit number. Find how many different 6-digit numbers can be formed if

(i) there are no restrictions, [1]

(ii) the number formed is even, [1]

(iii) the number formed is even and greater than 300 000. [3]

(b) Ruby wants to have a party for her friends. She can only invite 8 of her 15 friends.

(i) Find the number of different ways she can choose her friends for the party if there are no restrictions. [1]

Two of her 15 friends are twins who cannot be separated.

(ii) Find the number of different ways she can now choose her friends for the party. [3]

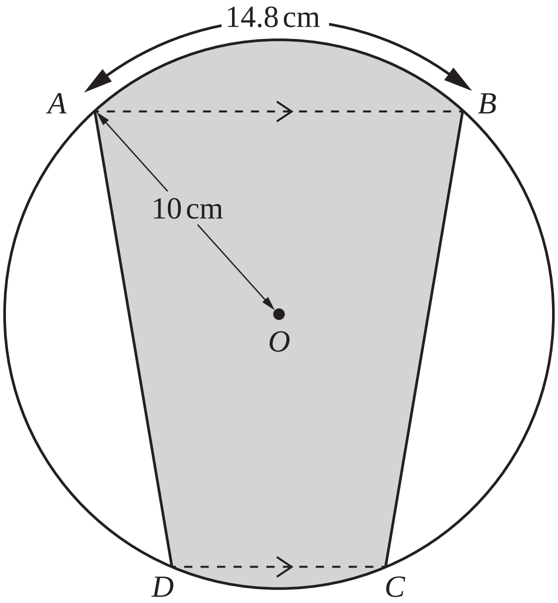
10 **(a)** Given that $\mathbf{A} = \begin{pmatrix} 4 & -1 \\ a & b \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 3 \\ -5 & 4 \end{pmatrix}$ and $\mathbf{AB} = \begin{pmatrix} 13 & 8 \\ 18 & 4 \end{pmatrix}$, find the value of a and of b. [4]

(b) It is given that $\mathbf{X} = \begin{pmatrix} 3 & -5 \\ -4 & 1 \end{pmatrix}$, $\mathbf{Y} = \begin{pmatrix} -1 & 2 \\ 4 & 0 \end{pmatrix}$ and $\mathbf{XZ} = \mathbf{Y}$.

(i) Find $\mathbf{X}^{-1}$. [2]

(ii) Hence find $\mathbf{Z}$. [3]

11



The diagram shows a circle, centre *O*, radius 10 cm. The points *A*, *B*, *C* and *D* lie on the circumference of the circle such that *AB* is parallel to *DC*. The length of the minor arc *AB* is 14.8 cm. The area of the minor sector *ODC* is 21.8 cm^2.

(i) Write down, in radians, angle *AOB*. [1]

(ii) Find, in radians, angle *DOC*. [2]

(iii) Find the perimeter of the shaded region. [4]

(iv) Find the area of the shaded region. [3]

12 The line $y = 2x + 1$ intersects the curve $xy = 14 - 2y$ at the points P and Q. The midpoint of the line PQ is the point M.

(i) Show that the point $\left(-10,\ \frac{23}{8}\right)$ lies on the perpendicular bisector of PQ. [9]

The line PQ intersects the y-axis at the point R. The perpendicular bisector of PQ intersects the y-axis at the point S.

(ii) Find the area of the triangle RSM. [3]

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CANDIDATE NAME

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7690408726

ADDITIONAL MATHEMATICS **0606/21**

Paper 2 **October/November 2017**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
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Mathematical Formulae

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Solve the inequality $(x-1)(x-5) > 12$. [4]

2 Show that $\dfrac{1}{1-\sin\theta} - \dfrac{1}{1+\sin\theta} = 2\tan\theta\sec\theta$. [4]

3 Solve the equation $\log_5(10x+5) = 2 + \log_5(x-7)$. [4]

4 Solve the following simultaneous equations for x and y, giving each answer in its simplest surd form.

$$\sqrt{3}x + y = 4$$

$$x - 2y = 5\sqrt{3}$$

[5]

5 **(i)** Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{5}{3x+2}\right)$. [2]

(ii) Use your answer to part **(i)** to find $\int \frac{30}{(3x+2)^2}\,\mathrm{d}x$. [2]

(iii) Hence evaluate $\int_1^2 \frac{30}{(3x+2)^2}\,\mathrm{d}x$. [2]

6 It is given that $\mathbf{M} = \begin{pmatrix} 2 & p \\ -3 & q \end{pmatrix}$ where p and q are integers.

(i) If det $\mathbf{M} = 13$, find an equation connecting p and q. [1]

(ii) Given also that $\mathbf{M}^2 = \begin{pmatrix} 4-3p & 12 \\ -6-3q & -3p+q^2 \end{pmatrix}$, find a second equation connecting p and q. [2]

(iii) Find the value of p and of q. [4]

7 Find y in terms of x, given that $\dfrac{d^2y}{dx^2} = 6x + \dfrac{2}{x^3}$ and that when $x = 1$, $y = 3$ and $\dfrac{dy}{dx} = 1$. [6]

8 Given that $z = a + (a+3)\sqrt{3}$ and $z^2 = 79 + b\sqrt{3}$, find the value of each of the integers *a* and *b*. [6]

9 **(i)** Expand $(1 + x)^4$, simplifying all coefficients. [1]

(ii) Expand $(6 - x)^4$, simplifying all coefficients. [2]

(iii) Hence express $(6 - x)^4 - (1 + x)^4 = 175$ in the form $ax^3 + bx^2 + cx + d = 0$, where a, b, c and d are integers. [2]

(iv) Show that $x = 2$ is a solution of the equation in part **(iii)** and show that this equation has no other real roots. [5]

10 In this question **i** is a unit vector due east and **j** is a unit vector due north. Units of length and velocity are metres and metres per second respectively.

The initial position vectors of particles A and B, relative to a fixed point O, are $2\mathbf{i} + 4\mathbf{j}$ and $10\mathbf{i} + 14\mathbf{j}$ respectively. Particles A and B start moving at the same time. A moves with constant velocity $\mathbf{i} + \mathbf{j}$ and B moves with constant velocity $-2\mathbf{i} - 3\mathbf{j}$. Find

(i) the position vector of A after t seconds, [1]

(ii) the position vector of B after t seconds. [1]

It is given that X is the distance between A and B after t seconds.

(iii) Show that $X^2 = (8 - 3t)^2 + (10 - 4t)^2$. [3]

(iv) Find the value of t for which $(8 - 3t)^2 + (10 - 4t)^2$ has a stationary value and the corresponding value of X. [4]

11 The line $y = kx + 3$, where k is a positive constant, is a tangent to the curve $x^2 - 2x + y^2 = 8$ at the point P.

(i) Find the value of k. [4]

(ii) Find the coordinates of P. [3]

(iii) Find the equation of the normal to the curve at P. [2]

12 **(i)** Differentiate $(\cos x)^{-1}$ with respect to x. [2]

(ii) Hence find $\frac{\mathrm{d}y}{\mathrm{d}x}$ given that $y = \tan x + 4(\cos x)^{-1}$. [2]

(iii) Using your answer to part **(ii)** find the values of x in the range $0 \leqslant x \leqslant 2\pi$ such that $\frac{\mathrm{d}y}{\mathrm{d}x} = 4$. [6]

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CANDIDATE NAME ☐

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8047669716

ADDITIONAL MATHEMATICS **0606/22**

Paper 2 **October/November 2017**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

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Mathematical Formulae

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$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 If $z = 2 + \sqrt{3}$ find the integers a and b such that $az^2 + bz = 1 + \sqrt{3}$. [5]

2 Solve the equation $\dfrac{2x^{1.5}+6x^{-0.5}}{x^{0.5}+5x^{-0.5}}=x$. [5]

3 Solve the inequality $|3x-1|>3+x$. [3]

4 Solve the simultaneous equations

$$\log_2(x+4) = 2\log_2 y,$$

$$\log_2(7y - x) = 4.$$

[5]

5 Naomi is going on holiday and intends to read 4 books during her time away. She selects these books from 5 mystery, 3 crime and 2 romance books. Find the number of ways in which she can make her selection in each of the following cases.

(i) There are no restrictions. [1]

(ii) She selects at least 2 mystery books. [3]

(iii) She selects at least 1 book of each type. [3]

6 The volume of a closed cylinder of base radius x cm and height h cm is $500\,\text{cm}^3$.

(i) Express h in terms of x. [1]

(ii) Show that the total surface area of the cylinder is given by $A = 2\pi x^2 + \frac{1000}{x}\,\text{cm}^2$. [2]

(iii) Given that x can vary, find the stationary value of A and show that this value is a minimum. [5]

7 The gradient of the normal to a curve at the point with coordinates (x, y) is given by $\frac{\sqrt{x}}{1-3x}$.

(i) Find the equation of the curve, given that the curve passes through the point $(1, -10)$. [5]

(ii) Find, in the form $y = mx + c$, the equation of the tangent to the curve at the point where $x = 4$. [4]

8 The matrix **A** is $\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$.

(i) Find $(2\mathbf{A})^{-1}$. [3]

(ii) Hence solve the simultaneous equations

$$2y + 4x + 5 = 0,$$

$$6y + 8x + 9 = 0.$$

[4]

9 **(i)** Find $\frac{\mathrm{d}}{\mathrm{d}x}(x\ln x)$. [2]

(ii) Hence find $\int \ln x \, \mathrm{d}x$. [2]

(iii) Hence, given that $k > 0$, show that $\int_{k}^{2k} \ln x \, dx = k(\ln 4k - 1)$. [4]

10 **(i)** **Without using a calculator**, solve the equation $6c^3 - 7c^2 + 1 = 0$. [5]

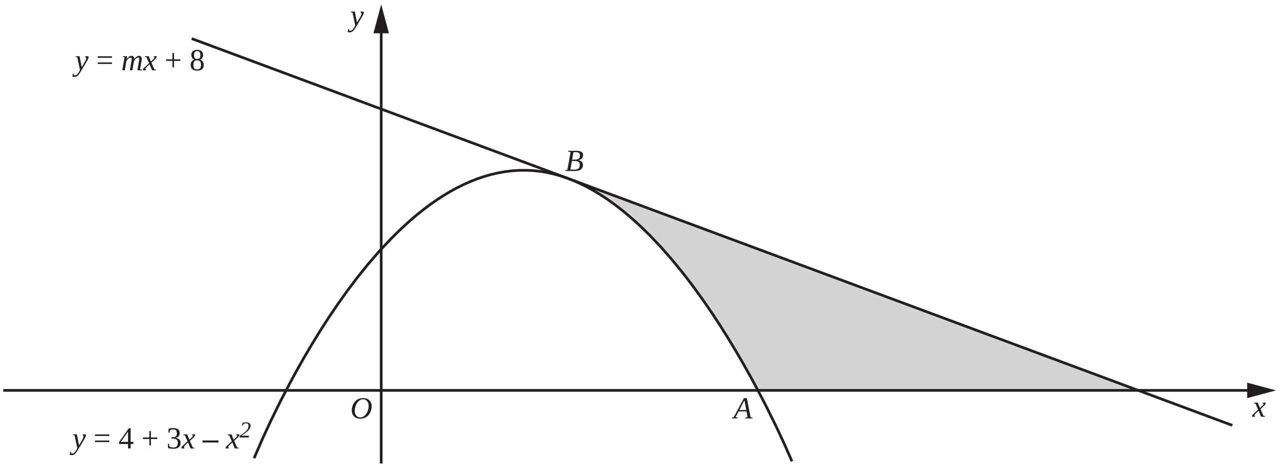
It is given that $y = \tan x + 6\sin x$.

(ii) Find $\frac{dy}{dx}$. [2]

(iii) If $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 7$ show that $6\cos^3 x - 7\cos^2 x + 1 = 0$. [2]

(iv) Hence solve the equation $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 7$ for $0 \leqslant x \leqslant \pi$ radians. [2]

11



The diagram shows the curve $y = 4 + 3x - x^2$ intersecting the positive x-axis at the point A. The line $y = mx + 8$ is a tangent to the curve at the point B. Find

(i) the coordinates of A, [2]

(ii) the value of m, [3]

(iii) the coordinates of *B*, [2]

(iv) the area of the shaded region, showing all your working. [5]

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CANDIDATE NAME	

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ADDITIONAL MATHEMATICS **0606/23**

Paper 2 **October/November 2017**

2 hours

Candidates answer on the Question Paper.

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DC (NF/AR) 135372/4

Mathematical Formulae

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Quadratic Equation

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}\,b + \binom{n}{2}a^{n-2}\,b^2 + \ldots + \binom{n}{r}a^{n-r}\,b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

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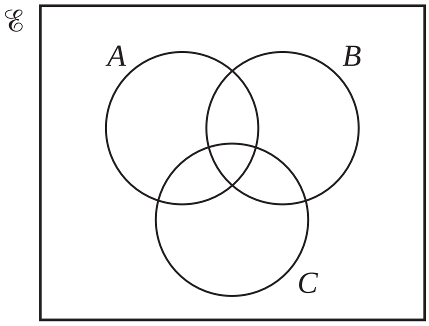
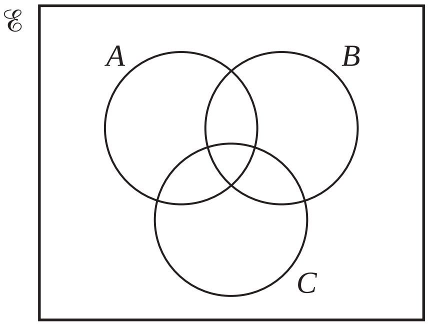
$$\Delta = \frac{1}{2}\,bc \sin A$$

1 **(a)** On each of the diagrams below, shade the region which represents the given set.

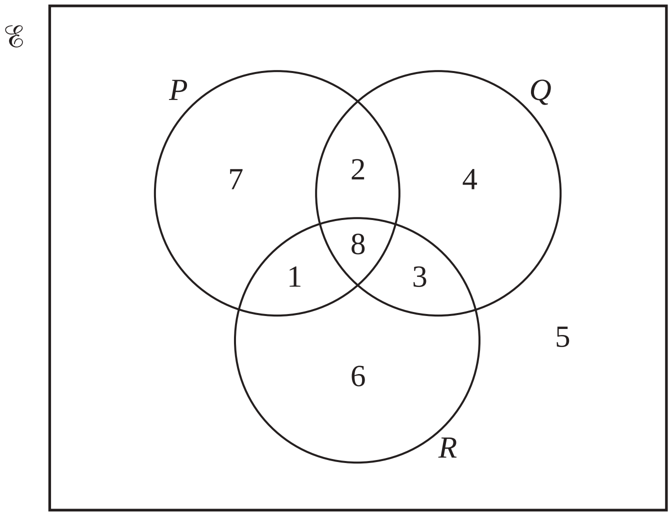


$(A \cup B) \cap C'$



$(A \cap B') \cup C$ [2]

(b)



The Venn diagram shows the number of elements in each of its subsets.

Complete the following.

$n(P') =$ ....

$n((Q \cup R) \cap P) =$ ....

$n(Q' \cup P) =$ .... [3]

2 Solve the equation $|3x-1| = |5+x|$. [3]

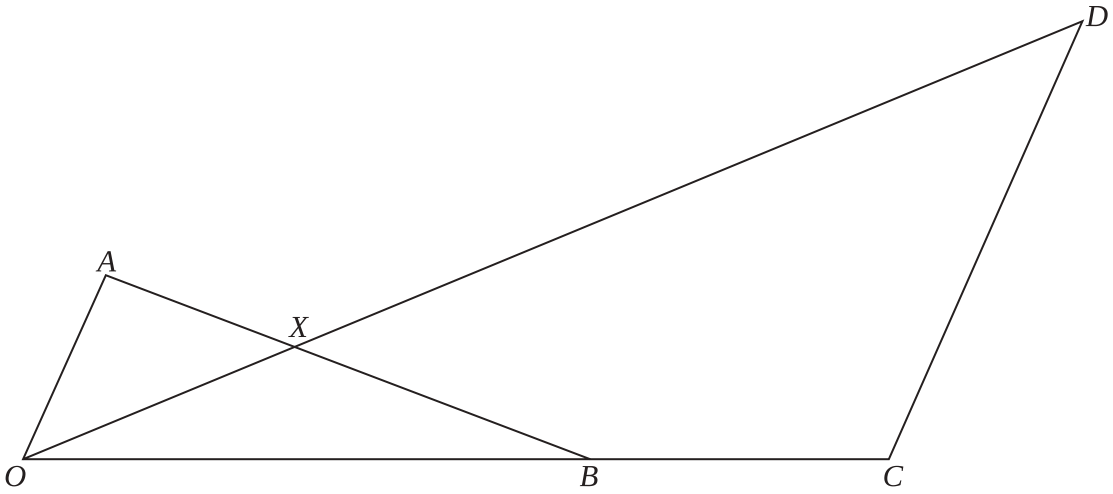
3 Find integers p and q such that $\dfrac{p}{\sqrt{3}-1} + \dfrac{1}{\sqrt{3}+1} = q + 3\sqrt{3}$. [4]

4 Solve the simultaneous equations

$$\log_3(x+1) = 1 + \log_3 y,$$

$$\log_3(x-y) = 2.$$ [5]

5



The diagram shows points O, A, B, C, D and X. The position vectors of A, B and C relative to O are $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \frac{3}{2}\mathbf{b}$. The vector $\overrightarrow{CD} = 3\mathbf{a}$.

(i) If $\overrightarrow{OX} = \lambda\overrightarrow{OD}$ express $\overrightarrow{OX}$ in terms of λ, $\mathbf{a}$ and $\mathbf{b}$. [1]

(ii) If $\overrightarrow{AX} = \mu\overrightarrow{AB}$ express $\overrightarrow{OX}$ in terms of μ, $\mathbf{a}$ and $\mathbf{b}$. [2]

(iii) Use your two expressions for $\overrightarrow{OX}$ to find the value of λ and of μ. [3]

(iv) Find the ratio $\frac{AX}{XB}$. [1]

(v) Find the ratio $\frac{OX}{XD}$. [1]

6 The functions f and g are defined for real values of x by

$$f(x) = (x+2)^2 + 1,$$

$$g(x) = \frac{x-2}{2x-1}, \; x \neq \frac{1}{2}.$$

(i) Find $f^2(-3)$. [2]

(ii) Show that $g^{-1}(x) = g(x)$. [3]

(iii) Solve $gf(x) = \frac{8}{19}$. [4]

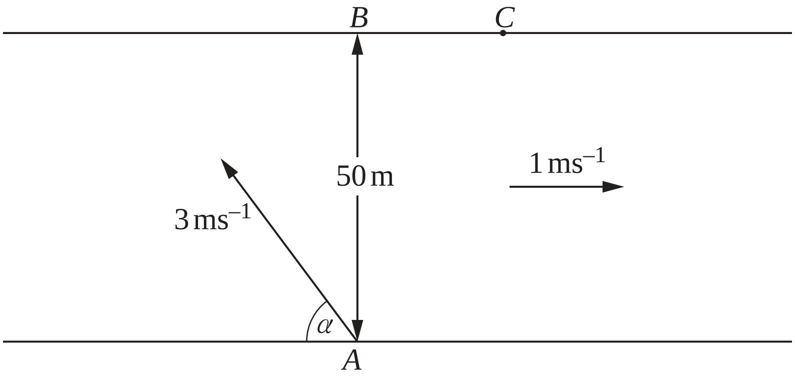
7 A particle moving in a straight line passes through a fixed point O. Its velocity, v ms^{-1}, t s after passing through O, is given by $v = 3\cos 2t - 1$ for $t \geqslant 0$.

(i) Find the value of t when the particle is first at rest. [2]

(ii) Find the displacement from O of the particle when $t = \frac{\pi}{4}$. [3]

(iii) Find the acceleration of the particle when it is first at rest. [3]

8



A man, who can row a boat at 3 ms^{-1} in still water, wants to cross a river from A to B as shown in the diagram. AB is perpendicular to both banks of the river. The river, which is 50 m wide, is flowing at 1 ms^{-1} in the direction shown. The man points his boat at an angle α° to the bank. Find

(i) the angle α, [2]

(ii) the resultant speed of the boat from A to B, [2]

(iii) the time taken for the boat to travel from *A* to *B*. [2]

On another occasion the man points the boat in the same direction but the river speed has increased to 1.8 ms^{-1} and as a result he lands at the point *C*.

(iv) State the time taken for the boat to travel from *A* to *C* and hence find the distance *BC*. [2]

9 **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\ln x}{x^3}\right) = \frac{1 - 3\ln x}{x^4}$. [3]

(ii) Find the exact coordinates of the stationary point of the curve $y = \frac{\ln x}{x^3}$. [3]

(iii) Use the result from part **(i)** to find $\int\left(\frac{\ln x}{x^4}\right)\mathrm{d}x$. [4]

10 **(a)** Show that $\dfrac{\sin x}{1+\cos x}+\dfrac{1+\cos x}{\sin x}=2\operatorname{cosec} x$. [3]

(b) Solve the following equations.

(i) $\cot^2 y+\operatorname{cosec} y-5=0$ for $0° \leqslant y \leqslant 360°$ [5]

(ii) $\cos\left(2z+\frac{\pi}{4}\right)=-\frac{\sqrt{3}}{2}$ for $0 \leqslant z \leqslant \pi$ radians [4]

Question 11 is printed on the next page.

11 The cubic equation $x^3 + ax^2 + bx - 36 = 0$ has a repeated positive integer root.

(i) If the repeated root is $x = 3$ find the other positive root and the value of a and of b. [4]

(ii) There are other possible values of a and b for which the cubic equation has a repeated positive integer root. In each case state all three integer roots of the equation. [4]

Cambridge Assessment
International Education

Grade thresholds – June 2018

Cambridge IGCSE™ Additional Mathematics (0606)

Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the June 2018 examination.

		minimum raw mark required for grade:				
	maximum raw mark available	A	B	C	D	E
Component 11	80	64	47	31	25	19
Component 12	80	57	43	30	24	18
Component 13	80	64	47	31	25	19
Component 21	80	59	45	31	25	20
Component 22	80	62	51	39	33	27
Component 23	80	59	45	31	25	20

Grade A* does not exist at the level of an individual component.

The maximum total mark for this syllabus, after weighting has been applied, is **160**.

The overall thresholds for the different grades were set as follows.

Option	Combination of Components	A*	A	B	C	D	E
AX	11, 21	145	123	92	62	50	39
AY	12, 22	141	119	94	69	57	45
AZ	13, 23	145	123	92	62	50	39

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ADDITIONAL MATHEMATICS **0606/11**

Paper 1 **May/June 2018**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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This document consists of **10** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

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- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

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Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

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Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

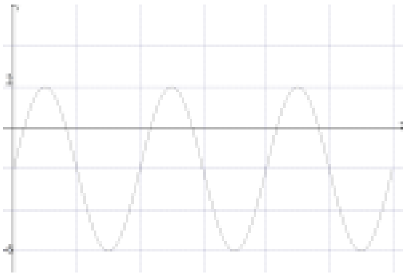
M	Method marks, awarded for a valid method applied to the problem.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
B	Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘**dep**’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Partial Marks</th></tr>
<tr><td>1</td><td>Substitution and simplification to obtain a 3 term quadratic in one variable</td><td>M1</td><td>substitution of $y = x + 4$ or $x = y - 4$ and simplification to 3 terms.</td></tr>
<tr><td></td><td>$x^2 - 2x - 8 = 0$ or $2x^2 - 4x - 16 = 0$
or
$y^2 - 10y + 16 = 0$ or $y^2 - 10y + 16 = 0$</td><td>A1</td><td>correct equation of the form
$ax^2 + bx + c = 0$ or $ay^2 + by + c = 0$</td></tr>
<tr><td></td><td>Solution of quadratic equation</td><td>M1</td><td>M1 dep</td></tr>
<tr><td></td><td>$x = 4,\ y = 8$
$x = -2,\ y = 2$</td><td>A2</td><td>A1 for each pair</td></tr>
<tr><td>2</td><td>Midpoint $\left(\frac{5}{2}, -1\right)$</td><td>B1</td><td></td></tr>
<tr><td></td><td>Gradient of line $= -\frac{8}{3}$</td><td>B1</td><td></td></tr>
<tr><td></td><td>Gradient of perp $= \frac{3}{8}$</td><td>M1</td><td></td></tr>
<tr><td></td><td>Equation of perp bisector:
$y + 1 = \frac{3}{8}\left(x - \frac{5}{2}\right)$</td><td>M1</td><td>M1 dep Using <i>their</i> perpendicular gradient and <i>their</i> midpoint</td></tr>
<tr><td></td><td>$6x - 16y - 31 = 0$ or
$-6x + 16y + 31 = 0$</td><td>A1</td><td></td></tr>
<tr><td>3</td><td>
<table>
<tr><th>A</th><th>B</th><th>C</th><th>D</th></tr>
<tr><td></td><td>✓</td><td></td><td></td></tr>
<tr><td></td><td></td><td>✓</td><td>✓</td></tr>
<tr><td></td><td></td><td>✓</td><td></td></tr>
<tr><td>✓</td><td></td><td></td><td></td></tr>
</table>
</td><td>4</td><td>B1 for either each row correct or each column correct – mark to candidate’s advantage.</td></tr>
<tr><td>4(i)</td><td>$b = 4$</td><td>B1</td><td></td></tr>
<tr><td></td><td>$c = 6$</td><td>B1</td><td></td></tr>
<tr><td></td><td>$2 = a + 4\sin\frac{\pi}{2}$</td><td>M1</td><td>Evaluation of a using <i>their b</i> and <i>their c</i> and the given point.</td></tr>
<tr><td></td><td>$a = -2$</td><td>A1</td><td></td></tr>
</table>

Question	Answer	Marks	Partial Marks
4(ii)		**3**	**B1** for $-6 \leqslant y \leqslant 2$ **B1** for 3 complete cycles **B1** for all correct
5(i)	The number of bacteria at the start of the experiment	**B1**	
5(ii)	$20\,000 = 800e^{kt}$ so $\frac{20\,000}{800} = e^{2k}$ or $\ln 20\,000 = \ln 800 + \ln(e^{2k})$	**M1**	use of given equation and attempt to solve for e^{2k} or use logs correctly
	$2k = \ln 25$	**M1**	correct method to obtain $2k$
	1.61	**A1**	
5(iii)	$P = 800e^{3\ln 5}$	**M1**	Substitution of $t = 3$ in formula using *their k*
	$= 100\,000$	**A1**	answer in range 99800 to 100200
6(a)	$\left(\frac{\log_3 p}{\log_3 2} \times \log_3 2\right) + \log_3 q$ or $\log_3 2^{\log_2 p} + \log_3 q$	**B1**	
	$\log_3 p + \log_3 q$ or $\log_3(2^{\log_2 p} \times q)$	**B1**	**B1 dep**
	$\log_3 pq$	**B1**	**B1 dep**
6(b)	$(\log_a 5 - 1)(\log_a 5 - 3) = 0$	**M1**	solution of quadratic equation
	$\log_a 5 = 1,\ a = 5$ $\log_a 5 = 3,\ a = \sqrt[3]{5}$ or 1.71 or $5^{\frac{1}{3}}$	**A2**	**A1** for $a = 5$ **A1** for $a = \sqrt[3]{5}$ or 1.71or $5^{\frac{1}{3}}$
7(i)	$\frac{1}{2}\begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}$	**2**	**B1** for $\frac{1}{2}$ **B1** for $\begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}$

Question	Answer	Marks	Partial Marks
7(ii)	$4x-2y=\frac{5}{2}$ $-5x+3y=\frac{7}{2}$	**B1**	Relating solution of these equations to matrix in **(i)** B1 for adapted equation or $\begin{pmatrix}2.5\\3.5\end{pmatrix}$ or $2\begin{pmatrix}x\\y\end{pmatrix}$
	$\begin{pmatrix}x\\y\end{pmatrix}=\frac{1}{2}\begin{pmatrix}3&2\\5&4\end{pmatrix}\begin{pmatrix}2.5\\3.5\end{pmatrix}$ or $2\begin{pmatrix}x\\y\end{pmatrix}=\frac{1}{2}\begin{pmatrix}3&2\\5&4\end{pmatrix}\begin{pmatrix}5\\7\end{pmatrix}$	**M1**	Correct method for pre-multiplication by *their* inverse matrix.
	$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}7.25\\13.25\end{pmatrix}$ or $\begin{pmatrix}\frac{29}{4}\\\frac{53}{4}\end{pmatrix}$ $x=7.25,\ y=13.25$	**A2**	**A1** for each. Condone in matrix form.
8(a)	$3(2\mathbf{i}-5\mathbf{j})-4(\mathbf{i}-3\mathbf{j})$	**M1**	For expansion and collection of terms
	$3\mathbf{p}-4\mathbf{q}=2\mathbf{i}-3\mathbf{j}$	**A1**	
	Magnitude of their $2\mathbf{i}-3\mathbf{j}$ $\sqrt{2^2+(-3)^2}$	**M1**	For method to find magnitude
	Unit vector = $\frac{2\mathbf{i}-3\mathbf{j}}{\sqrt{13}}$	**A1**	
8(b)(i)	$v^2=2.73^2+1.25^2$	**B1**	Correct use of Pythagoras
	$v=3.00$	**B1**	
8(b)(ii)	$\tan\theta=\frac{1.25}{2.73}$ oe	**M1**	Use of a trig funtion to obtain a relevant angle
	Angle to *AB*= 24.6° or 0.429 radians	**A1**	
9(i)	$256x^8-64x^6+7x^4$	**3**	**B1** for each term

Question	Answer	Marks	Partial Marks
9(ii)	$\frac{1}{x^4}+\frac{2}{x^2}+1$	**B1**	
	$\left(256x^8-64x^6+7x^4\right)\left(\frac{1}{x^4}+\frac{2}{x^2}+1\right)$ $\left(256\times1-64\times2+7\times1\right)x^4$	**M1**	**M1** for three correctly obtained products leading to terms in x^4 using *their* $256x^8-64x^6+7x^4$ and *their* $\frac{1}{x^4}+\frac{2}{x^2}+1$
	Coefficient of x^4 is $256-128+7$ = 135	**A1**	
10(a)	$\frac{5+6\sqrt{5}}{6+\sqrt{5}}\times\frac{6-\sqrt{5}}{6-\sqrt{5}}$	**M1**	for rationalisation
	$=\frac{30-5\sqrt{5}+36\sqrt{5}-30}{31}$	**M1**	**M1dep** for expanding the numerator to obtain four terms.
	$=\frac{31\sqrt{5}}{31}=\sqrt{5}$	**A1**	**A1** for $\sqrt{5}$ from correct working
10(b)	$\sqrt{3}\times\left(\sqrt{2}\right)^6\times\sqrt{2}=\sqrt{6}\times2^3$		
	$8\sqrt{6}$	**B2**	**B1** for $\sqrt{6}$ from $\sqrt{3}\times\sqrt{2}$ **B1** for 8 from $\left(\sqrt{2}\right)^6$ or 2^3

Question	Answer	Marks	Partial Marks
10(c)	**EITHER**: $x^2+\sqrt{2}x-4=0$	**B1**	3 term quadratic equation equated to zero
	$x=\dfrac{-\sqrt{2}\pm\sqrt{18}}{2}$	**M1**	use of the quadratic formula
	for use of $\sqrt{18}=3\sqrt{2}$	**M1**	**M1 dep**
	$\sqrt{2},\quad -2\sqrt{2}$	**A1**	For both from full working
	OR: $x^2+\sqrt{2}x-4=0$	**B1**	
	$\left(x+\dfrac{\sqrt{2}}{2}\right)^2=4+\dfrac{1}{2}$ $x=\pm\dfrac{3}{\sqrt{2}}-\dfrac{\sqrt{2}}{2}$	**M1**	Correct use of completing the square method
	$x=-\dfrac{4}{\sqrt{2}},\ \dfrac{2}{\sqrt{2}}$	**M1**	**M1dep** for dealing with $\sqrt{2}$ in denominator
	$x=\sqrt{2},\quad -2\sqrt{2}$	**A1**	
11(i)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=16-\dfrac{54}{x^3}$	**M1**	for $\dfrac{\mathrm{d}y}{\mathrm{d}x}=16\pm\dfrac{p}{x^3}$
	Equating to zero and obtaining x^3	**M1**	**M1dep**
	$x=\dfrac{3}{2},\ y=36$	**A2**	**A1** for each

Question	Answer	Marks	Partial Marks
11(ii)	**EITHER**: When $x=1,\ y=43$ When $x=3,\ y=51$	**B1**	**B1** for both
	$\left(\frac{1}{2}(43+51)\times 2\right)-\int_1^3 16x+\frac{27}{x^2}\ \mathrm{d}x$		
	Area of trapezium = $\left(\frac{1}{2}(43+51)\times 2\right)$ oe	**B1**	**FT** from *their P* and *their Q*
	Integration to find area under curve	**M1**	for $\left[px^2+\frac{q}{x}\right]$
	= $\left[8x^2-\frac{27}{x}\right]$	**A1**	Integration correct
	$=\left[8\times 3^2-\frac{27}{3}\right]-\left[8\times 1^2-\frac{27}{1}\right]$	**M1**	**M1dep** for application of limits
	Required area = 94 – 82 =12	**A1**	
	OR: When $x=1,\ y=43$ When $x=3,\ y=51$	**B1**	**B1** for both
	Equation of *PQ:* $y=4x+39$	**B1**	Equation of line **FT** from *their P* and *their Q*
	Integration of their $4x+39-16x-\frac{27}{x^2}$	**M1**	for $\left[px+qx^2+\frac{r}{x}\right]$
	= $\left[39x-6x^2+\frac{27}{x}\right]$	**A1**	All correct
	$=\left[39\times 3-6\times 3^2+\frac{27}{3}\right]$ $-\left[39\times 1-6\times 1^2+\frac{27}{1}\right]$	**M1**	**M1dep** for application of limits
	Required area = 72 – 60 = 12	**A1**	

Question	Answer	Marks	Partial Marks
12	$\frac{\mathrm{d}y}{\mathrm{d}x} = (2x-5)^{\frac{1}{2}} \quad (+c)$	**M1**	for $k(2x-5)^{\frac{1}{2}}$,
	for $(2x-5)^{\frac{1}{2}}$	**A1**	
	Substitution to obtain arbitrary constant	**M1**	**M1 dep** Using $\frac{\mathrm{d}y}{\mathrm{d}x} = 6$ when $x = \frac{9}{2}$
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = (2x-5)^{\frac{1}{2}} + 4$	**A1**	for correct $\frac{\mathrm{d}y}{\mathrm{d}x}$
	Integration of *their* $k(2x-5)^{\frac{1}{2}} + c$	**M1**	**M1 dep** on first M1 for integration of $k(2x-5)^{\frac{1}{2}}$ to obtain $m(2x-5)^{\frac{3}{2}}$
	$y = \frac{1}{3}(2x-5)^{\frac{3}{2}} + 4x \quad (+d)$	**A1**	for $\frac{1}{3}(2x-5)^{\frac{3}{2}} + 4x$ **FT** *their* (non -zero) constant
	Finding constant	**M1**	**M1 dep** for obtaining arbitrary constant for $m(2x-5)^{\frac{3}{2}} + nx + d$ using $x = \frac{9}{2}, \quad y = \frac{2}{3}$
	$y = \frac{1}{3}(2x-5)^{\frac{3}{2}} + 4x - 20$	**A1**	for correct equation

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ADDITIONAL MATHEMATICS **0606/12**
Paper 1 **May/June 2018**
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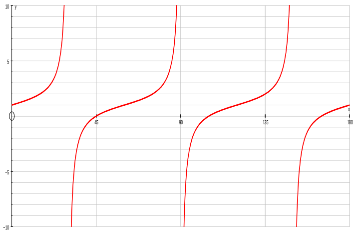
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1(i)	$\frac{\pi}{3}$ or 60°	**B1**	
1(ii)		**3**	**B1** for 3 asymptotes at $x=30^{\circ}$, 90° and 150°; the curve must approach but not cross all 3 of the asymptotes and be in the 1st and 4th quadrants **B1** for starting at $(0,1)$ and finishing at $(180,1)$ **B1** for all correct
2	For an attempt to obtain an equation in *x* only	**M1**	
	$9x^2-(k+1)x+4=0$	**A1**	correct 3 term equation
	$(k+1)^2-(4\times 9\times 4)$	**M1**	**M1dep** for correct use of b^2-4ac oe
	Critical values $k=11$, $k=-13$	**A1**	
	$-13<k<11$	**A1**	For the correct range
3	$e^y=ax^2+b$	**B1**	may be implied, $b\neq 0$
	either $3=5a+b$ $1=3a+b$ or Gradient = 1, so $a=1$	**M1**	correct attempt to find *a* or *b* by use of simultaneous equations or finding the gradient and equating it to *a*
	Coefficient of x^2 is 1	**A1**	
	Intercept is –2	**A1**	
	$y=\ln(x^2-2)$	**A1**	For correct form
4(i)	$3=\ln(5t+3)$ $e^3=5t+3$ or better	**B1**	
	$t=3.42$	**B1**	
4(ii)	$\frac{dx}{dt}=\frac{5}{5t+3}$	**M1**	for $\frac{k_1}{5t+3}$
	When $t=0$, $\frac{dx}{dt}=\frac{5}{3}$, 1.67 or better	**A1**	all correct

Question	Answer	Marks	Partial Marks
4(iii)	If $t > 0$ each term in $\frac{k_1}{5t+3} > 0$ so never negative oe	B1	**dep** on **M1** in **(ii)** **FT** on *their* $\frac{k_1}{5t+3}$, provided $k_1 > 0$
4(iv)	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \frac{k_2}{(5t+3)^2}$	M1	
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{25}{(5t+3)^2}$ When $t = 0$, $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{25}{9}$ or -2.78	A1	all correct
5(i)	$a = 243,\ b = -45,\ c = \frac{10}{3}$	3	**B1** for each coefficient, must be simplified
5(ii)	$\left(243 - \frac{45}{x} + \frac{10}{3x^2}\right)\left(4 + 36x + 81x^2\right)$	B1	For $\left(4 + 36x + 81x^2\right)$
	for having 3 terms independent of x	M1	
	Independent term is $972 - 1620 + 270 = -378$	A1	
6	attempt to differentiate quotient or equivalent product	M1	
	$\frac{\mathrm{d}}{\mathrm{d}x}(2x-1)^{\frac{1}{2}} = (2x-1)^{-\frac{1}{2}}$ for a quotient $\frac{\mathrm{d}}{\mathrm{d}x}(2x-1)^{-\frac{1}{2}} = -(2x-1)^{-\frac{3}{2}}$ for a product	B1	
	either $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{2x-1} - (x+2)\left[(2x-1)^{-\frac{1}{2}}\right]}{\left(\sqrt{2x-1}\right)^2}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = (2x-1)^{-\frac{1}{2}} - (x+2)\left[(2x-1)^{-\frac{3}{2}}\right]$	A1	All other terms correct
	When $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, $2x - 1 = x + 2$	M1	equate to zero and attempt to solve
	$x = 3$	A1	
	$y = \sqrt{5}$, $\frac{5}{\sqrt{5}}$, 2.24	A1	

Question	Answer	Marks	Partial Marks
7(i)	1000	**B1**	
7(ii)	$2000 = 1000e^{\frac{t}{4}}$	**B1**	
	$t = 4\ln 2,\ \ \ln 16$	**M1**	For $4\ln k$ or $\ln k^4,\ k > 0$
	2.77	**A1**	
7(iii)	$B = 1000e^2$ $= 7389,\ 7390$	**B1**	
8(a)	$3\left(1-\sin^2\theta\right)+4\sin\theta = 4$	**M1**	use of correct identity
	$\left(3\sin\theta-1\right)\left(\sin\theta-1\right)=0$ $\sin\theta = \frac{1}{3},\ \ \sin\theta = 1$	**M1**	For attempt to solve a 3 term quadratic equation in $\sin\theta$ to obtain $\sin\theta =$
	$\theta = 19.5^{\circ},\ \ 160.5^{\circ}$	**A1**	
	90°	**A1**	
8(b)	$\tan 2\phi = \sqrt{3}$ $2\phi = \frac{\pi}{3},\ -\frac{2\pi}{3}$	**M1**	obtaining an equation in $\tan 2\phi$ and correct attempt to solve for one solution to reach $2\phi = k$
	for one correct solution $\phi = \frac{\pi}{6},$ or 0.524	**A1**	
	for attempt at a second solution	**M1**	
	$\phi = -\frac{\pi}{3}$, or -1.05	**A1**	for a correct second solution and no other solutions within the range
9(a)(i)	1000	**B1**	
9(a)(ii)	for use of power rule	**M1**	
	for addition or subtraction rule	**M1**	**dep** on previous **M1**
	$\lg\frac{1000a}{b^2}$	**A1**	Allow $\lg\frac{10^3 a}{b^2}$
9(b)(i)	$x^2 - 5x + 6 = 0$	**M1**	For attempt to obtain a quadratic equation and solve
	$x = 3,\ x = 2$	**A1**	for both

Question	Answer	Marks	Partial Marks
9b(ii)	$(\log_4 a)^2 - 5\log_4 a + 6 = 0$	**M1**	For the connection with **(i)** and attempt to deal with at least one logarithm correctly, either $4^{their\,3}$ or $4^{their\,2}$
	$a = 64$	**A1**	
	$a = 16$	**A1**	
10(i)	$AC^2 = \left(4\sqrt{3}-5\right)^2 + \left(4\sqrt{3}+5\right)^2$	**M1**	For attempt to use the cosine rule
	$-2\left(4\sqrt{3}-5\right)\left(4\sqrt{3}+5\right)\cos 60^\circ$	**A1**	For all correct unsimplified
	$AC^2 = 123$	**M1**	**M1 dep** for attempt to evaluate without use of calculator
	$AC = \sqrt{123}$	**A1**	
	ALTERNATIVE METHOD		
	Taking *D* as the foot of the perpendicular from *A*: Find *AD*, *BD*, *DC* $AC^2 = AD^2 + DC^2$	**M1**	For a complete method to get AC^2
	$AC^2 = \left(\frac{12-5\sqrt{3}}{2}\right)^2 + \left(\frac{15+4\sqrt{3}}{2}\right)^2$	**A1**	For all correct unsimplified
	$AC^2 = 123$	**M1**	**M1dep** for attempt to evaluate without use of calculator
	$AC = \sqrt{123}$	**A1**	

Question	Answer	Marks	Partial Marks
10(ii)	$\frac{AC}{\sin 60^{\circ}} = \frac{4\sqrt{3}-5}{\sin ACB}$ or $\sin ACB = \frac{AD}{AC}$	**M1**	For attempt at the sine rule or trigonometry involving right-angled triangles
	For attempt at cosec	**M1**	**dep** on first **M** mark $\text{cosec}ACB = \frac{2\sqrt{123}}{\sqrt{3}\left(4\sqrt{3}-5\right)}$ or $\frac{2\sqrt{41}}{\left(4\sqrt{3}-5\right)}$ oe
	$\text{cosec } ACB = \frac{2}{\sqrt{3}} \frac{\sqrt{123}}{\left(4\sqrt{3}-5\right)} \times \frac{4\sqrt{3}+5}{4\sqrt{3}+5}$	**M1**	**dep** on previous **M** mark for a statement involving rationalisation using $a\sqrt{3}+b$
	$= \frac{2\sqrt{41}}{23}\left(4\sqrt{3}+5\right)$	**A1**	For rationalisation using $\frac{4\sqrt{3}+5}{4\sqrt{3}+5}$ oe and simplification
	ALTERNATIVE METHOD		
	$\frac{1}{2}\left(4\sqrt{3}-5\right)\left(4\sqrt{3}+5\right)\sin 60 \quad = \frac{23\sqrt{3}}{4}$	**M1**	Area of *ABC*
	$\frac{1}{2}\sqrt{123}\left(4\sqrt{3}+5\right)\sin ACB \quad = \frac{23\sqrt{3}}{4}$	**M1**	For attempt at a second area of *ABC* and equating to first area
	For attempt at cosec	**M1**	**dep** on first 2 **M** marks
	$= \frac{2\sqrt{41}}{23}\left(4\sqrt{3}+5\right)$	**A1**	Need to be convinced no calculator is being used in simplification

Question	Answer	Marks	Partial Marks
11	When $x=0,\ y=\frac{1}{2}$	**B1**	For $y=\frac{1}{2}$
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\mathrm{e}^{4x}$	**B1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}$, Gradient of normal = –2	**B1**	**FT** on *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$, must be numeric
	either: Normal $y-\frac{1}{2}=-2x$ or: Gradient of normal = $-\frac{OA}{OB}$	**M1**	For an attempt at a normal equation passing through *their* $\left(0,\frac{1}{2}\right)$ and a substitution of $y=0$
	When $y=0,\ x=\frac{1}{4}$	**A1**	
	EITHER: $\int_0^{\frac{1}{4}}\frac{1}{8}\mathrm{e}^{4x}+\frac{3}{8}\mathrm{d}x$	**M1**	For attempt to integrate to obtain $k_1\mathrm{e}^{4x}+\frac{3}{8}x,\ k_1\neq\frac{1}{8},\ k_1\neq\frac{1}{2}$
	$\left[\frac{1}{32}\mathrm{e}^{4x}+\frac{3x}{8}\right]_0^{\frac{1}{4}}$	**A1**	For correct integration
	Use of limits	**M1**	**M1dep**
	For area of triangle $=\frac{1}{16}$	**B1**	**FT** on *their* $x=\frac{1}{4}$
	$=\frac{\mathrm{e}}{32}$	**A1**	final answer in correct form
	OR: $\int_0^{\frac{1}{4}}\frac{1}{8}\mathrm{e}^{4x}+\frac{3}{8}-\frac{1}{2}+2x\ \mathrm{d}x$	**M1**	For attempt at subtraction and attempt to integrate to obtain $k_1\mathrm{e}^{4x}+\frac{3}{8}x+k_2x+k_3x^2,\ k_1\neq\frac{1}{8}$
	$\left[\frac{1}{32}\mathrm{e}^{4x}-\frac{1}{8}x+x^2\right]_0^{\frac{1}{4}}$	**A2**	–1 for each error for integration
	for use of limits	**M1**	**M1dep**
	$=\frac{\mathrm{e}}{32}$	**A1**	final answer in correct form

Question	Answer	Marks	Partial Marks
12(a)	$p=\frac{1}{4}$	**B1**	
	$p+q-4q+6=4$	**B1**	**FT** on *their p*
	$q=\frac{3}{4}$	**B1**	
12(b)	$\left(x^{\frac{1}{3}}+3\right)\left(x^{\frac{1}{3}}+1\right)=0$	**M1**	For attempt to factorise and solve, or solve using the quadratic formula oe, a quadratic in $x^{\frac{1}{3}}$ or *u*
	$x^{\frac{1}{3}}=-1$ or $u=-1$ $x^{\frac{1}{3}}=-3$ or $u=-3$	**A1**	For both
	$x=-1$	**A1**	
	$x=-27$	**A1**	

Cambridge Assessment International Education
Cambridge International General Certificate of Secondary Education

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ADDITIONAL MATHEMATICS **0606/13**

Paper 1 **May/June 2018**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **10** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

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GENERIC MARKING PRINCIPLE 3:

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- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
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- marks are not deducted for omissions
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MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

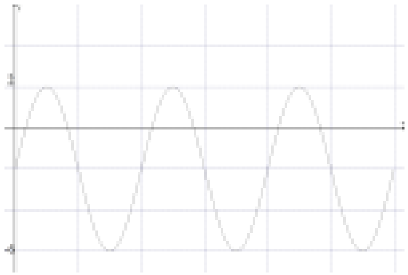
M	Method marks, awarded for a valid method applied to the problem.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
B	Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Partial Marks</th></tr>
<tr><td>1</td><td>Substitution and simplification to obtain a 3 term quadratic in one variable</td><td>M1</td><td>substitution of $y = x + 4$ or $x = y – 4$ and simplification to 3 terms.</td></tr>
<tr><td></td><td>$x^2 - 2x - 8 = 0$ or $2x^2 - 4x - 16 = 0$
or
$y^2 - 10y + 16 = 0$ or $y^2 - 10y + 16 = 0$</td><td>A1</td><td>correct equation of the form
$ax^2 + bx + c = 0$ or $ay^2 + by + c = 0$</td></tr>
<tr><td></td><td>Solution of quadratic equation</td><td>M1</td><td>M1 dep</td></tr>
<tr><td></td><td>$x = 4,\ y = 8$
$x = -2,\ y = 2$</td><td>A2</td><td>A1 for each pair</td></tr>
<tr><td>2</td><td>Midpoint $\left(\frac{5}{2}, -1\right)$</td><td>B1</td><td></td></tr>
<tr><td></td><td>Gradient of line $= -\frac{8}{3}$</td><td>B1</td><td></td></tr>
<tr><td></td><td>Gradient of perp $= \frac{3}{8}$</td><td>M1</td><td></td></tr>
<tr><td></td><td>Equation of perp bisector:
$y + 1 = \frac{3}{8}\left(x - \frac{5}{2}\right)$</td><td>M1</td><td>M1 dep Using <i>their</i> perpendicular gradient and <i>their</i> midpoint</td></tr>
<tr><td></td><td>$6x - 16y - 31 = 0$ or
$-6x + 16y + 31 = 0$</td><td>A1</td><td></td></tr>
<tr><td>3</td><td><table>
<tr><th>A</th><th>B</th><th>C</th><th>D</th></tr>
<tr><td></td><td>✓</td><td></td><td></td></tr>
<tr><td></td><td></td><td>✓</td><td>✓</td></tr>
<tr><td></td><td></td><td>✓</td><td></td></tr>
<tr><td>✓</td><td></td><td></td><td></td></tr>
</table></td><td>4</td><td>B1 for either each row correct or each column correct – mark to candidate’s advantage.</td></tr>
<tr><td>4(i)</td><td>$b = 4$</td><td>B1</td><td></td></tr>
<tr><td></td><td>$c = 6$</td><td>B1</td><td></td></tr>
<tr><td></td><td>$2 = a + 4\sin\frac{\pi}{2}$</td><td>M1</td><td>Evaluation of a using <i>their b</i> and <i>their c</i> and the given point.</td></tr>
<tr><td></td><td>$a = -2$</td><td>A1</td><td></td></tr>
</table>

Question	Answer	Marks	Partial Marks
4(ii)		**3**	**B1** for $-6 \leqslant y \leqslant 2$ **B1** for 3 complete cycles **B1** for all correct
5(i)	The number of bacteria at the start of the experiment	**B1**	
5(ii)	$20\,000 = 800\mathrm{e}^{kt}$ so $\frac{20\,000}{800} = \mathrm{e}^{2k}$ or $\ln 20\,000 = \ln 800 + \ln(\mathrm{e}^{2k})$	**M1**	use of given equation and attempt to solve for e^{2k} or use logs correctly
	$2k = \ln 25$	**M1**	correct method to obtain $2k$
	1.61	**A1**	
5(iii)	$P = 800\mathrm{e}^{3\ln 5}$	**M1**	Substitution of $t = 3$ in formula using *their k*
	$= 100\,000$	**A1**	answer in range 99800 to 100200
6(a)	$\left(\frac{\log_3 p}{\log_3 2} \times \log_3 2\right) + \log_3 q$ or $\log_3 2^{\log_2 p} + \log_3 q$	**B1**	
	$\log_3 p + \log_3 q$ or $\log_3(2^{\log_2 p} \times q)$	**B1**	**B1 dep**
	$\log_3 pq$	**B1**	**B1 dep**
6(b)	$(\log_a 5 - 1)(\log_a 5 - 3) = 0$	**M1**	solution of quadratic equation
	$\log_a 5 = 1,\ a = 5$ $\log_a 5 = 3,\ a = \sqrt[3]{5}$ or 1.71 or $5^{\frac{1}{3}}$	**A2**	**A1** for $a = 5$ **A1** for $a = \sqrt[3]{5}$ or 1.71or $5^{\frac{1}{3}}$
7(i)	$\frac{1}{2}\begin{pmatrix}3 & 2\\5 & 4\end{pmatrix}$	**2**	**B1** for $\frac{1}{2}$ **B1** for $\begin{pmatrix}3 & 2\\5 & 4\end{pmatrix}$

Question	Answer	Marks	Partial Marks
7(ii)	$4x-2y=\frac{5}{2}$ $-5x+3y=\frac{7}{2}$	**B1**	Relating solution of these equations to matrix in **(i)** B1 for adapted equation or $\begin{pmatrix}2.5\\3.5\end{pmatrix}$ or $2\begin{pmatrix}x\\y\end{pmatrix}$
	$\begin{pmatrix}x\\y\end{pmatrix}=\frac{1}{2}\begin{pmatrix}3&2\\5&4\end{pmatrix}\begin{pmatrix}2.5\\3.5\end{pmatrix}$ or $2\begin{pmatrix}x\\y\end{pmatrix}=\frac{1}{2}\begin{pmatrix}3&2\\5&4\end{pmatrix}\begin{pmatrix}5\\7\end{pmatrix}$	**M1**	Correct method for pre-multiplication by *their* inverse matrix.
	$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}7.25\\13.25\end{pmatrix}$ or $\begin{pmatrix}\frac{29}{4}\\\frac{53}{4}\end{pmatrix}$ $x=7.25,\ y=13.25$	**A2**	**A1** for each. Condone in matrix form.
8(a)	$3(2\mathbf{i}-5\mathbf{j})-4(\mathbf{i}-3\mathbf{j})$	**M1**	For expansion and collection of terms
	$3\mathbf{p}-4\mathbf{q}=2\mathbf{i}-3\mathbf{j}$	**A1**	
	Magnitude of their $2\mathbf{i}-3\mathbf{j}$ $\sqrt{2^2+(-3)^2}$	**M1**	For method to find magnitude
	Unit vector = $\frac{2\mathbf{i}-3\mathbf{j}}{\sqrt{13}}$	**A1**	
8(b)(i)	$v^2=2.73^2+1.25^2$	**B1**	Correct use of Pythagoras
	$v=3.00$	**B1**	
8(b)(ii)	$\tan\theta=\frac{1.25}{2.73}$ oe	**M1**	Use of a trig funtion to obtain a relevant angle
	Angle to AB= 24.6° or 0.429 radians	**A1**	
9(i)	$256x^8-64x^6+7x^4$	**3**	**B1** for each term

Question	Answer	Marks	Partial Marks
9(ii)	$\frac{1}{x^4}+\frac{2}{x^2}+1$	**B1**	
	$\left(256x^8-64x^6+7x^4\right)\left(\frac{1}{x^4}+\frac{2}{x^2}+1\right)$ $\left(256\times1-64\times2+7\times1\right)x^4$	**M1**	**M1** for three correctly obtained products leading to terms in x^4 using *their* $256x^8-64x^6+7x^4$ and *their* $\frac{1}{x^4}+\frac{2}{x^2}+1$
	Coefficient of x^4 is $256-128+7$ = 135	**A1**	
10(a)	$\frac{5+6\sqrt{5}}{6+\sqrt{5}}\times\frac{6-\sqrt{5}}{6-\sqrt{5}}$	**M1**	for rationalisation
	$=\frac{30-5\sqrt{5}+36\sqrt{5}-30}{31}$	**M1**	**M1dep** for expanding the numerator to obtain four terms.
	$=\frac{31\sqrt{5}}{31}=\sqrt{5}$	**A1**	**A1** for $\sqrt{5}$ from correct working
10(b)	$\sqrt{3}\times\left(\sqrt{2}\right)^6\times\sqrt{2}=\sqrt{6}\times2^3$		
	$8\sqrt{6}$	**B2**	**B1** for $\sqrt{6}$ from $\sqrt{3}\times\sqrt{2}$ **B1** for 8 from $\left(\sqrt{2}\right)^6$ or 2^3

Question	Answer	Marks	Partial Marks
10(c)	**EITHER**: $x^2+\sqrt{2}x-4=0$	**B1**	3 term quadratic equation equated to zero
	$x=\frac{-\sqrt{2}\pm\sqrt{18}}{2}$	**M1**	use of the quadratic formula
	for use of $\sqrt{18}=3\sqrt{2}$	**M1**	**M1 dep**
	$\sqrt{2},\quad -2\sqrt{2}$	**A1**	For both from full working
	OR: $x^2+\sqrt{2}x-4=0$	**B1**	
	$\left(x+\frac{\sqrt{2}}{2}\right)^2=4+\frac{1}{2}$ $x=\pm\frac{3}{\sqrt{2}}-\frac{\sqrt{2}}{2}$	**M1**	Correct use of completing the square method
	$x=-\frac{4}{\sqrt{2}},\ \frac{2}{\sqrt{2}}$	**M1**	**M1dep** for dealing with $\sqrt{2}$ in denominator
	$x=\sqrt{2},\quad -2\sqrt{2}$	**A1**	
11(i)	$\frac{dy}{dx}=16-\frac{54}{x^3}$	**M1**	for $\frac{dy}{dx}=16\pm\frac{p}{x^3}$
	Equating to zero and obtaining x^3	**M1**	**M1dep**
	$x=\frac{3}{2},\ y=36$	**A2**	**A1** for each

Question	Answer	Marks	Partial Marks
11(ii)	**EITHER**: When $x=1,\ y=43$ When $x=3,\ y=51$	**B1**	**B1** for both
	$\left(\frac{1}{2}(43+51)\times 2\right)-\int_1^3 16x+\frac{27}{x^2}\ \mathrm{d}x$		
	Area of trapezium = $\left(\frac{1}{2}(43+51)\times 2\right)$ oe	**B1**	**FT** from *their P* and *their Q*
	Integration to find area under curve	**M1**	for $\left[px^2+\frac{q}{x}\right]$
	= $\left[8x^2-\frac{27}{x}\right]$	**A1**	Integration correct
	$=\left[8\times 3^2-\frac{27}{3}\right]-\left[8\times 1^2-\frac{27}{1}\right]$	**M1**	**M1dep** for application of limits
	Required area = 94 – 82 =12	**A1**	
	OR: When $x=1,\ y=43$ When $x=3,\ y=51$	**B1**	**B1** for both
	Equation of *PQ:* $y=4x+39$	**B1**	Equation of line **FT** from *their P* and *their Q*
	Integration of their $4x+39-16x-\frac{27}{x^2}$	**M1**	for $\left[px+qx^2+\frac{r}{x}\right]$
	= $\left[39x-6x^2+\frac{27}{x}\right]$	**A1**	All correct
	$=\left[39\times 3-6\times 3^2+\frac{27}{3}\right]$ $-\left[39\times 1-6\times 1^2+\frac{27}{1}\right]$	**M1**	**M1dep** for application of limits
	Required area = 72 – 60 = 12	**A1**	

Question	Answer	Marks	Partial Marks
12	$\frac{\mathrm{d}y}{\mathrm{d}x} = \left(2x-5\right)^{\frac{1}{2}} \quad \left(+c\right)$	**M1**	for $k\left(2x-5\right)^{\frac{1}{2}}$,
	for $\left(2x-5\right)^{\frac{1}{2}}$	**A1**	
	Substitution to obtain arbitrary constant	**M1**	**M1 dep** Using $\frac{\mathrm{d}y}{\mathrm{d}x} = 6$ when $x = \frac{9}{2}$
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = \left(2x-5\right)^{\frac{1}{2}} + 4$	**A1**	for correct $\frac{\mathrm{d}y}{\mathrm{d}x}$
	Integration of *their* $k\left(2x-5\right)^{\frac{1}{2}} + c$	**M1**	**M1 dep** on first M1 for integration of $k\left(2x-5\right)^{\frac{1}{2}}$ to obtain $m\left(2x-5\right)^{\frac{3}{2}}$
	$y = \frac{1}{3}\left(2x-5\right)^{\frac{3}{2}} + 4x \quad \left(+d\right)$	**A1**	for $\frac{1}{3}\left(2x-5\right)^{\frac{3}{2}} + 4x$ **FT** *their* (non -zero) constant
	Finding constant	**M1**	**M1 dep** for obtaining arbitrary constant for $m\left(2x-5\right)^{\frac{3}{2}} + nx + d$ using $x = \frac{9}{2}, \quad y = \frac{2}{3}$
	$y = \frac{1}{3}\left(2x-5\right)^{\frac{3}{2}} + 4x - 20$	**A1**	for correct equation

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ADDITIONAL MATHEMATICS **0606/21**

Paper 2 **May/June 2018**

MARK SCHEME

Maximum Mark: 80

Published

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nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
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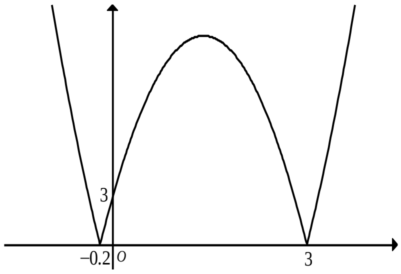
soi	seen or implied

Question	Answer	Marks	Partial Marks
1(i)(a)	A is not a [proper] subset of B oe	**B1**	
1(i)(b)	A and C are mutually exclusive oe or A intersection C is the empty set oe	**B1**	
1(ii)(a)	$\mathrm{n}\left(A \cup B\right) = 3$	**B1**	
1(ii)(b)	$x \in \left(A \cap C'\right)$ oe	**B1**	
2(i)	$k \times \frac{1}{3x-1}$	**M1**	
	$3 \times \frac{1}{3x-1}$	**A1**	
2(ii)	$x = \frac{11}{15}$ soi	**B1**	
	$0.125 \approx \textit{their}\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=\textit{their}\frac{11}{15}} \times \delta x$ oe	**M1**	
	0.05 nfww	**A1**	
3(i)	$\left({}^{12}P_7 =\right) 3\,991\,680$	**B1**	
3(ii)	$(4 \times {}^{11}P_6 =)\ 1\,330\,560$	**B1**	
3(iii)	4! × 4! × 2 oe	**M2**	**M1** for 4! × 4! oe only or ${}^4P_4 \times {}^4P_3$ oe only
	1152	**A1**	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Partial Marks</th></tr>
<tr><td>4(i)</td><td>$2(-4)^3+3(-4)^2-4a-12=0$ with one correct interim step leading to
$a=-23$</td><td>B1</td><td>Note: = 0 must be seen or may be implied by e.g. $-92 = 4a$ or $92 = -4a$
or convincingly showing that
$2(-4)^3+3(-4)^2-4(-23)-12=0$
or correct synthetic division at least as far as
$\begin{array}{r|rrrr} -4 & 2 & 3 & a & -12 \\ & & -8 & 20 & -4a-80 \\ \hline & 2 & -5 & a+20 & 0 \end{array}$
then $a = -23$
or correct long division to, e.g. verify –23, at least as far as
$\begin{array}{rl} & 2x^2-5x\ -3 \\ \hline x+4\,\big) & 2x^3+3x^2-23x-12 \\ & \underline{2x^3+8x^2} \\ & \quad -5x^2-23x \\ & \quad -\underline{5x^2-20x} \\ & \qquad\qquad -3x-12 \\ & \qquad\qquad \underline{-3x-12} \\ & \qquad\qquad\qquad 0 \end{array}$</td></tr>
<tr><td></td><td>$\mathrm{p}(1)=2+3-23\ -12$
$b=-30$</td><td>B1</td><td></td></tr>
<tr><td>4(ii)</td><td>finds a correct quadratic factor
e.g. $(2x^2-5x-3)$</td><td>B2</td><td>B1 for quadratic factor with 2 correct terms
OR
B1for finding $(x-3)$ using factor theorem
B1for convincingly finding $(2x+1)$ as third factor</td></tr>
<tr><td></td><td>Product of three linear factors
$(2x+1)(x-3)(x+4)$</td><td>M1</td><td></td></tr>
<tr><td></td><td>$x=-\dfrac{1}{2},\ \ x=3,\ \ x=-4$ nfww</td><td>A1</td><td>If M0 then SC1 if quadratic factorised correctly but does not show full factorisation but does give all 3 solutions correctly</td></tr>
<tr><td>5(i)</td><td>Putting $y=\mathrm{f}(x)$, changing subject to x and swopping x and y or vice versa</td><td>M1</td><td></td></tr>
<tr><td></td><td>$\mathrm{f}^{-1}(x)=\dfrac{1}{2}\left(\dfrac{1}{x}+5\right)$ or $\dfrac{5x+1}{2x}$ oe isw</td><td>A1</td><td></td></tr>
<tr><td>5(ii)</td><td>$x>0$ oe</td><td>B1</td><td></td></tr>
</table>

Question	Answer	Marks	Partial Marks
5(iii)	$\dfrac{1}{2\left(\dfrac{1}{2x-5}\right)-5}$	**B1**	
	$\dfrac{1}{\dfrac{2-5(2x-5)}{2x-5}}$ oe	**M1**	**FT** if expression of equivalent difficulty e.g. $\dfrac{1}{\left(\dfrac{1}{2x-5}\right)-5}$
	Completes to $\dfrac{2x-5}{-10x+27}$ oe final answer	**A1**	
6(i)	$16x = 40$ oe	**M1**	
	$x = 2.5$ oe (radians)	**A1**	
6(ii)	$\frac{1}{2}(16)^2(2.5)$ oe	**M1**	
	320	**A1**	
6(iii)	$\frac{1}{2}r^2(their2.5) = (their320) - 140$ oe	**M1**	**FT** provided *their* 320 > 140
	correct simplification to $r^2 = \ldots$	**M1**	**dep** on first **M1**
	12	**A1**	
7(i)	$4\tan x + 4x\sec^2 x$ isw	**B2**	Fully correct **B1** for one correct term as part of e.g. a sum of 2 terms
7(ii)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{3x+1}\right) = 3\mathrm{e}^{3x+1}$	**B1**	
	$\dfrac{(x^2-1)(their3\mathrm{e}^{3x+1}) - their(2x)\mathrm{e}^{3x+1}}{(x^2-1)^2}$	**M1**	
	$\dfrac{(x^2-1)(3\mathrm{e}^{3x+1}) - 2x\mathrm{e}^{3x+1}}{(x^2-1)^2}$ oe isw	**A1**	
8(i)	Takes logs of both sides	**M1**	
	$\ln y = \ln a + n\ln x$ or $\lg y = \lg a + n\lg x$	**A1**	

Question	Answer	Marks	Partial Marks
8(ii)	$n =$ –0.2 to –0.3 nfww	**B1**	
	attempts to equate y-intercept to ln a or forms *their* ln equation with *their* gradient and a point on the line or uses two points on the line to form a pair of simultaneous equations	**M1**	
	$a = e^{4.7}$ isw or 110 or 109.9[47…]	**A1**	maximum of 2 marks if no coordinates stated
8(iii)	use of ln(50) and ln$x$ = 3 to 3.2	**M1**	or for $\frac{50}{theira} = x^{their\ n}$ or better or for $\ln 50 = \ln(theira) + (their\ n)\ln x$ oe
	awrt 22 or 23 to 2 significant figures	**A1**	implies **M1**
9(i)	$5\left(x-\frac{7}{5}\right)^2-\frac{64}{5}$	**B3**	**B1** for each of p, q, r correct in correct format; allow correct equivalent values. If **B0**, then **SC2** for $5\left(x-\frac{7}{5}\right)-\frac{64}{5}$ or **SC1** for correct values but incorrect format
9(ii)		**B4**	**B2** for fully correct shape in correct position or **B1** for fully correct shape translated parallel to the x-axis **B1** for y-intercept at (0, 3) marked on graph **B1** for roots marked on graph at –0.2 and 3
9(iii)	$0 < k < \left\|their\left(-\frac{64}{5}\right)\right\|$	**B2**	**FT** *their* **(i)** **B1** for any inequality using *their* $\frac{64}{5}$ or max y value is *their* 12.8soi
10(i)	$v = \frac{ds}{dt} = -3\sin 3t$	**B1**	
	When $v = 0$, $t = \frac{\pi}{3}$	**B1**	

Question	Answer	Marks	Partial Marks
10(ii)	Finding *s* when $t = \frac{\pi}{4}$ and $t = \frac{\pi}{2}$	**M1**	
	Finding *s* when $t = their\frac{\pi}{3}$ and correct plan	**M1**	Using *their* **(i)** correctly
	1.29 nfww	**A1**	
10(iii)	$a = \frac{dv}{dt} = -9\cos 3t$	**B1**	
	9	**B1**	**FT** *their* $k\cos 3t$
11(a)	$10\left(1-\sin^2 x\right)+3\sin x = 9$	**M1**	
	Solves $10\sin^2 x - 3\sin x - 1 = 0$ oe	**M1**	**dep** on first **M1** Solves *their* three term quadratic in sin *x*
	$\sin x = \frac{1}{2}$, $\sin x = -\frac{1}{5}$	**A1**	
	30º, 150º and 191.5º, 348.5º awrt	**A2**	**A1** for any two correct solutions
11(b)	$3\frac{\sin 2y}{\cos 2y} = 4\sin 2y$ oe	**M1**	
	Solves $3\sin 2y - 4\sin 2y\cos 2y\,[=0]$	**M1**	**dep** on first **M1**
	$\sin 2y = 0$ $\cos 2y = \frac{3}{4}$	**A1**	
	Any two of π, 0.72273…, 5.56045… nfww	**A1**	
	$\frac{\pi}{2}$, 0.361, 2.78 awrt nfww	**A1**	**SC**: cancels out sin2*y* after **M1M0** allow **SC1** for 0.72273… and 5.56045... and **SC1**for 0.361 and 2.78
12(i)	$\tan 30 = \frac{h}{x/2}$ oe	**M1**	
	Correct completion to given answer	**A1**	
	$V = 5\sqrt{3}h^2$ isw	**B1**	

Question	Answer	Marks	Partial Marks
12(ii)(a)	$\frac{\mathrm{d}V}{\mathrm{d}h} = their 10\sqrt{3}h$ or $\frac{5\sqrt{3}}{2}$	**B1**	**FT** $their V = kh^2$
	$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}h}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t}$ soi	**M1**	
	$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{their\left(\frac{\mathrm{d}V}{\mathrm{d}h}\right)} \times 0.5$	**M1**	
	0.115 or 0.11547 to 0.1155 oe	**A1**	
12(ii)(b)	$\left(\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}t} =\right) 2\sqrt{3} \times their \frac{1}{5\sqrt{3}}$	**M1**	
	$\frac{2}{5}$	**A1**	

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ADDITIONAL MATHEMATICS **0606/22**

Paper 1 **May/June 2018**

MARK SCHEME

Maximum Mark: 80

Published

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This document consists of **10** printed pages.

Cambridge Assessment International Education

[Turn over

Generic Marking Principles

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GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

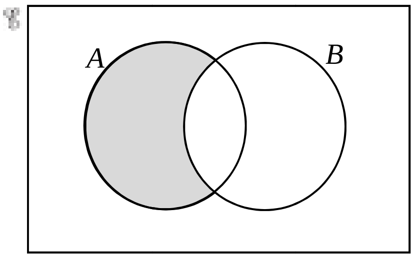
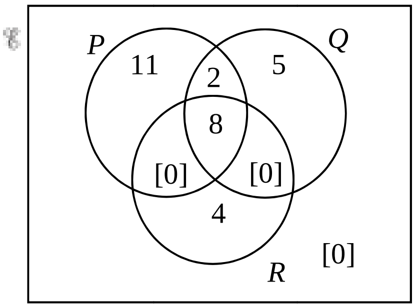
Types of mark

M	Method marks, awarded for a valid method applied to the problem.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
B	Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘**dep**’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

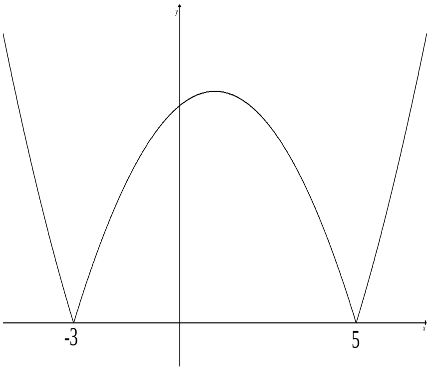
awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1(i)	Uses $\cot\theta = \frac{\cos\theta}{\sin\theta}$ $\frac{\cos^2\theta + \sin^2\theta}{\sin\theta}$ Uses $\cos^2\theta + \sin^2\theta = 1$ Completes to $\frac{1}{\sin\theta} = \operatorname{cosec}\theta$	**B3**	**B1** for using $\cot\theta = \frac{\cos\theta}{\sin\theta}$ oe or $\tan\theta = \frac{\sin\theta}{\cos\theta}$ oe at some stage **B1** for use of $\cos^2\theta + \sin^2\theta = 1$ oe **B1** for common denominator of $\sin\theta$ oe either in a compound fraction or in two partial fractions or for writing $\frac{1-\sin^2\theta}{\sin\theta}$ as $\frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta}$ oe **Maximum of 2 marks if not fully correct or does not complete to cosecθ**
1(ii)	$\sin\theta = \frac{1}{4}$	**M1**	
	14.5° or 14.47[751…] rot to 4 or more figures isw	**A1**	Not from wrong working
2(a)		**B1**	
2(b)		**B3**	**B1** for 8 correctly placed and all the empty regions correct **B1** for 11, 2, 5 correctly placed **B1** for 4 correctly placed **maximum of 2 marks if fully correct but other values such as 30, 21 and/or 15 present within the diagram**
	their 12	**B1**	**STRICT FT** *their* Venn diagram

Question	Answer	Marks	Partial Marks
3	p(–3) = 0 or p(2) = –15 stated or implied	**M1**	
	$-54 + 9a + 72 + b = 0$ or better	**A1**	finds one correct equation; implies **M1**
	$16 + 4a - 48 + b = -15$ or better	**A1**	finds another correct equation; implies **M1**
	Solves a pair of simultaneous equations in *a* and *b*	**M1**	**dep** on first **M1** condone one sign or arithmetic error in *their* solution; as far as finding one unknown
	$a = -7$, $b = 45$	**A1**	
	60 cao	**A1**	
4	Eliminates one of the unknowns	**M1**	
	Simplifies to a correct 3-term quadratic: $2x^2 + 4x - 16$ [= 0] oe or $2y^2 - 6y - 36$ [= 0] oe	**A1**	
	Factorises or solves $(x+4)(x-2)=0$ oe or $(y+3)(y-6)=0$ oe	**M1**	**FT** *their* 3-term quadratic in *x* or *y*;
	(2, 6) and (–4, –3) oe	**A2**	Not from wrong working **A1** for either (2, 6) or (–4, –3) or **A1** for $x = 2$ and $x = -4$ or $y = 6$ and $y = -3$
5(a)	$^7P_4$ or $7 \times 6 \times 5 \times 4$ oe	**M1**	
	840	**A1**	
5(b)(i)	20	**B1**	
5(b)(ii)	$^5C_1 \times {}^4C_1 \times {}^2C_1$ or $5 \times 4 \times 2$ oe	**M1**	
	40	**A1**	
5(b)(iii)	$^5C_3 + {}^4C_3$ oe	**M1**	
	14	**A1**	

Question	Answer	Marks	Partial Marks
6(i)	(Arc length =) 1.5×5 oe soi	**M1**	implied by 7.5
	(*DE* =) 10sin(0.75) oe soi	**M1**	implied by awrt 6.82
	34.3 or answer in range 34.31 to 34.32	**A1**	
6(ii)	(Area sector =) $\frac{1}{2} \times 5^2 \times 1.5$ oe	**M1**	implied by 18.75
	(Area triangle =) $\frac{1}{2} \times 5^2 \times \sin(1.5)$ oe	**M1**	implied by awrt 12.47
	31.2 or answer in range 31.21 to 31.22	**A1**	
7(i)	$\lvert their(\mathbf{a}+\mathbf{c})\rvert = \sqrt{their(5^2+14^2)}$	**M1**	
	$\sqrt{221}$	**A1**	mark final answer
7(ii)	[(2 + *m*)**i** + (3 – 5*m*)**j** therefore] *their* (2 + *m*) = 0	**M1**	for attempting to form **a** + *m***b** and equate the scalar of the **i** component to 0
	m = –2 only	**A1**	implies **M1**
7(iii)	[(2*n* – 1)**i** + (3*n* + 5)**j** = 3**i** + 11**j** or *n*(2**i** + 3**j**) = (3**i** + 11**j**) + (**i** – 5**j**) oe leading to] 2*n* – 1 = 3 or 3*n* + 5 = 11 oe, soi	**M1**	
	n = 2 only	**A1**	implies **M1**

Question	Answer	Marks	Partial Marks
8(a)	$\begin{pmatrix} -2 & 6 \\ 1 & 12 \end{pmatrix}$	**B2**	**B1** for a 2 by 2 matrix with 2 or 3 correct elements
	$their\left[\frac{1}{-30}\begin{pmatrix} 12 & -6 \\ -1 & -2 \end{pmatrix}\right]$ oe isw	**B2**	**FT** *their* non-singular **BA** **B1 FT** for either $\frac{1}{their(-30)}\begin{pmatrix} & \\ & \end{pmatrix}$ or $\ldots\times their\begin{pmatrix} 12 & -6 \\ -1 & -2 \end{pmatrix}$ If *their* **BA** is singular, **B0** then **SC1** for $\ldots\times their\begin{pmatrix} 12 & -6 \\ -1 & -2 \end{pmatrix}$ OR **Alternative method $\mathbf{A}^{-1}\mathbf{B}^{-1}$:** **B2** for $\mathbf{A}^{-1} = \frac{1}{-5}\begin{pmatrix} -3 & 1 \\ -1 & 2 \end{pmatrix}$ isw or $\mathbf{B}^{-1} = \frac{1}{6}\begin{pmatrix} -5 & 2 \\ -3 & 0 \end{pmatrix}$ isw or **B1** for a multiplier of $\frac{1}{-5}$ or for $\begin{pmatrix} -3 & 1 \\ -1 & 2 \end{pmatrix}$ or for a multipler of $\frac{1}{6}$ or for $\begin{pmatrix} -5 & 2 \\ -3 & 0 \end{pmatrix}$ **B2 FT** for $\mathbf{A}^{-1}\,\mathbf{B}^{-1} = their\frac{1}{-30}\times their\begin{pmatrix} 12 & -6 \\ -1 & -2 \end{pmatrix}$ or **B1 FT** for a 2 by 2 matrix with 2 or 3 correct elements **Maximum of 3 marks if not fully correct**
8(b)(i)	2 × 3	**B1**	
8(b)(ii)	$\begin{pmatrix} 2 & -\frac{1}{2} \end{pmatrix}$ oe isw	**B2**	**B1** for each correct element; must be in a 1 by 2 matrix or **M1** for a full method as far as finding values for the two elements

Question	Answer	Marks	Partial Marks
9(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{\sin x}\right)=\frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)$ oe	**B2**	**B1** for $\frac{1}{2}(\sin x)^{-\frac{1}{2}}\times ...$ or for $\frac{1}{2}(\sin x)^{-\frac{1}{2}}$ or for $\frac{1}{2}(...)^{-\frac{1}{2}}\times\cos x$ or for $their\,\frac{1}{2}(\sin x)^{\left(their\frac{1}{2}\right)-1}\times\cos x$
	$their\left(4x^3\right)\sqrt{\sin x}$ $+x^4\left(their\,\frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)\right)$ oe	**M1**	Applies correct form of product rule
	$4x^3\sqrt{\sin x}+x^4\left(\frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)\right)$ oe isw	**A1**	Not from wrong working
9(ii)	$\int\left(4x^3\sqrt{\sin x}\right)\mathrm{d}x$ $+\int\left(x^4\times\frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)\right)\mathrm{d}x$ $=x^4\sqrt{\sin x}$ oe	**M1**	or $\int x\,\mathrm{d}x+2\int\left(\frac{x^4\cos x}{2\sqrt{\sin x}}+4x^3\sqrt{\sin x}\right)\mathrm{d}x$ oe **FT** *their* **(i)**
	$\frac{x^2}{2}+2x^4\sqrt{\sin x}$ $[+c]$	**A2**	**A1** for $\int x\,\mathrm{d}x+2x^4\sqrt{\sin x}$
10(a)(i)		**B2**	**B1** for correct shape **B1** for roots marked on the graph or seen nearby provided graph drawn and one root is negative and one is positive
10(a)(ii)	Any correct domain	**B1**	
10(b)(i)	$\frac{4}{3x-1}$	**B1**	mark final answer

Question	Answer	Marks	Partial Marks
10(b)(ii)	Correct method for finding inverse function e.g. swopping variables **and** changing subject or vice versa; or indicates $(\mathrm{hg})^{-1}(x) = \mathrm{g}^{-1}\mathrm{h}^{-1}(x)$ and finds $\mathrm{g}^{-1}(x) = \frac{x+1}{3}$ and $\mathrm{h}^{-1}(x) = \frac{4}{x}$	**M1**	**FT** only if *their*hg(x) of the form $\frac{a}{bx+c}$ where a, b and c are integers
	$\left[(\mathrm{hg})^{-1}(x) =\right] \frac{1}{3}\left(\frac{4}{x}+1\right)$ oe isw or $\left[(\mathrm{hg})^{-1}(x) =\right] \frac{4+x}{3x}$ oe isw	**A1**	**FT** *their* $(\mathrm{hg})^{-1}(x) = \frac{a-cx}{bx}$ oe If **M0** then **SC1** for *their* hg(x) of the form $y = \frac{a}{x} + b$ oe leading to *their* (hg)$^{-1}$(x) of the form $y = \frac{a}{x-b}$ isw
10(c)	*a* cao	**B1**	
11(a)	$\frac{(2x-1)^{\frac{4}{3}}}{\frac{4}{3}\times 2}$ $[+c]$ oe isw	**B2**	**B1** for $k \times \frac{(2x-1)^{\left(\frac{1}{3}+1\right)}}{\left(\frac{1}{3}+1\right)}$ where $k \neq 0$
11(b)(i)	$k\cos 4x\,[+c]$ where $k < 0$ or $k = \frac{1}{4}$	**M1**	
	$-\frac{1}{4}\cos 4x\,[+c]$	**A1**	
11(b)(ii)	Sight of correct substitution of limits: $-\frac{1}{4}\cos\frac{4\pi}{4} - \left(-\frac{1}{4}\cos\frac{4\pi}{8}\right)$ oe	**M1**	**FT** *their* $k\cos 4x$ from **(b)(i)** **dep** on **M1** awarded in **(b)(i)**
	$\frac{1}{4}$	**A1**	does **not** imply **M1**

Question	Answer	Marks	Partial Marks
11(c)	$\int e^{\frac{x}{3}}\,dx = ke^{\frac{x}{3}}$ [$+c$]	M1	k any non-zero constant
	$k = 3$	A1	
	Sight of correct substitution of limits: *their* $ke^{\frac{\ln 8}{3}} -$ *their* ke^{0} oe	M1	dep on first M1
	Shows how to deal with the power of the first term e.g. $\frac{\ln 8}{3} = \ln 8^{\frac{1}{3}}$ or $\frac{\ln 8}{3} = \ln 2$ or $3\left(\sqrt[3]{8}\right)$ seen	B1	
	$6 - 3 = 3$	A1	Not from wrong working
12(i)	$\tan\frac{\pi}{12} = \frac{r}{h}$ oe	M1	
	$r = h(2-\sqrt{3})$ or $r = h\tan\frac{\pi}{12}$ oe	A1	
	$[V =]\frac{1}{3}\pi\left(2-\sqrt{3}\right)^2 h^2 \times h$ oe	M1	Correctly uses *their* expression for r in terms of h in formula for volume of a cone dependent on finding an expression connecting r and h
	$[V =]\frac{\pi\left(4-4\sqrt{3}+3\right)h^3}{3}$ oe correctly leading to $[V =]\frac{\pi\left(7-4\sqrt{3}\right)h^3}{3}$ **AG**	A1	
12(ii)	Correct derivative of V e.g. $\frac{3\pi\left(7-4\sqrt{3}\right)h^2}{3}$ oe isw	B1	
	$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ soi	B1	
	$\frac{1}{\textit{their}\left(\frac{dV}{dh}\right)\Big\vert_{h=5}} \times 30$	M1	if correct implies B1 B1; if incorrect, a correct FT statement implies the second B1
	5.32	A1	

Cambridge Assessment International Education
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ADDITIONAL MATHEMATICS **0606/23**

Paper 2 **May/June 2018**

MARK SCHEME

Maximum Mark: 80

Published

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- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method marks, awarded for a valid method applied to the problem.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
B	Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘**dep**’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

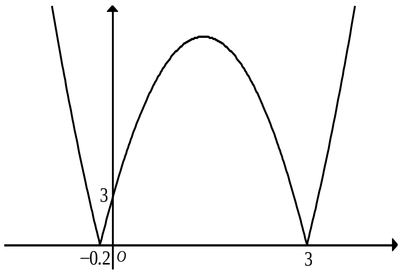
Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1(i)(a)	A is not a [proper] subset of B oe	**B1**	
1(i)(b)	A and C are mutually exclusive oe or A intersection C is the empty set oe	**B1**	
1(ii)(a)	$\mathrm{n}(A \cup B) = 3$	**B1**	
1(ii)(b)	$x \in (A \cap C')$ oe	**B1**	
2(i)	$k \times \dfrac{1}{3x-1}$	**M1**	
	$3 \times \dfrac{1}{3x-1}$	**A1**	
2(ii)	$x = \dfrac{11}{15}$ soi	**B1**	
	$0.125 \approx$ *their* $\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=their\frac{11}{15}} \times \delta x$ oe	**M1**	
	0.05 nfww	**A1**	
3(i)	$\left({}^{12}P_7 =\right) 3\,991\,680$	**B1**	
3(ii)	$(4 \times {}^{11}P_6 =)\ 1\,330\,560$	**B1**	
3(iii)	4! × 4! × 2 oe	**M2**	**M1** for 4! × 4! oe only or ${}^4P_4 \times {}^4P_3$ oe only
	1152	**A1**	

Question	Answer	Marks	Partial Marks
4(i)	$2(-4)^3+3(-4)^2-4a-12=0$ with one correct interim step leading to *a* = –23	**B1**	Note: = 0 must be seen or may be implied by e.g. –92 = 4*a* or 92 = –4*a* or convincingly showing that $2(-4)^3+3(-4)^2-4(-23)-12=0$ or correct synthetic division at least as far as $\begin{array}{rrrrr} -4\;\vert & 2 & 3 & a & -12 \\ \vert & & -8 & 20 & -4a-80 \\ \hline & 2 & -5 & a+20 & 0 \end{array}$ then *a* = –23 or correct long division to, e.g. verify –23, at least as far as $\begin{array}{rl} & 2x^2-5x\ -3 \\ x+4\big) & 2x^3+3x^2-23x-12 \\ & \underline{2x^3+8x^2} \\ & \quad -5x^2-23x \\ & \quad -\underline{5x^2-20x} \\ & \qquad\qquad -3x-12 \\ & \qquad\qquad \underline{-3x-12} \\ & \qquad\qquad\qquad 0 \end{array}$
	$\text{p}(1)=2+3-23\ -12$ *b* = –30	**B1**	
4(ii)	finds a correct quadratic factor e.g. $(2x^2-5x-3)$	**B2**	**B1** for quadratic factor with 2 correct terms OR **B1**for finding (*x* – 3) using factor theorem **B1**for convincingly finding (2*x* + 1) as third factor
	Product of three linear factors (2*x* + 1)(*x* – 3)(*x* + 4)	**M1**	
	$x = -\frac{1}{2}$, *x* = 3, *x* = – 4 nfww	**A1**	If **M0** then **SC1** if quadratic factorised correctly but does not show full factorisation but does give all 3 solutions correctly
5(i)	Putting *y* = f(*x*), changing subject to *x* and swopping *x* and *y* or vice versa	**M1**	
	$\text{f}^{-1}(x)=\frac{1}{2}\left(\frac{1}{x}+5\right)$ or $\frac{5x+1}{2x}$ oe isw	**A1**	
5(ii)	*x* > 0 oe	**B1**	

Question	Answer	Marks	Partial Marks
5(iii)	$\dfrac{1}{2\left(\dfrac{1}{2x-5}\right)-5}$	**B1**	
	$\dfrac{1}{\dfrac{2-5(2x-5)}{2x-5}}$ oe	**M1**	**FT** if expression of equivalent difficulty e.g. $\dfrac{1}{\left(\dfrac{1}{2x-5}\right)-5}$
	Completes to $\dfrac{2x-5}{-10x+27}$ oe final answer	**A1**	
6(i)	$16x = 40$ oe	**M1**	
	$x = 2.5$ oe (radians)	**A1**	
6(ii)	$\dfrac{1}{2}(16)^2(2.5)$ oe	**M1**	
	320	**A1**	
6(iii)	$\dfrac{1}{2}r^2(their2.5)=(their320)-140$ oe	**M1**	**FT** provided *their* 320 > 140
	correct simplification to $r^2 = \ldots$	**M1**	**dep** on first **M1**
	12	**A1**	
7(i)	$4\tan x + 4x\sec^2 x$ isw	**B2**	Fully correct **B1** for one correct term as part of e.g. a sum of 2 terms
7(ii)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{3x+1}\right)=3\mathrm{e}^{3x+1}$	**B1**	
	$\dfrac{(x^2-1)(their3\mathrm{e}^{3x+1})-their(2x)\mathrm{e}^{3x+1}}{(x^2-1)^2}$	**M1**	
	$\dfrac{(x^2-1)(3\mathrm{e}^{3x+1})-2x\mathrm{e}^{3x+1}}{(x^2-1)^2}$ oe isw	**A1**	
8(i)	Takes logs of both sides	**M1**	
	$\ln y = \ln a + n\ln x$ or $\lg y = \lg a + n\lg x$	**A1**	

Question	Answer	Marks	Partial Marks
8(ii)	$n =$ –0.2 to –0.3 nfww	**B1**	
	attempts to equate y-intercept to ln a or forms *their* ln equation with *their* gradient and a point on the line or uses two points on the line to form a pair of simultaneous equations	**M1**	
	$a = e^{4.7}$ isw or 110 or 109.9[47…]	**A1**	maximum of 2 marks if no coordinates stated
8(iii)	use of ln(50) and ln$x$ = 3 to 3.2	**M1**	or for $\frac{50}{their a} = x^{their\ n}$ or better or for $\ln 50 = \ln(their a) + (their\ n)\ln x$ oe
	awrt 22 or 23 to 2 significant figures	**A1**	implies **M1**
9(i)	$5\left(x-\frac{7}{5}\right)^2-\frac{64}{5}$	**B3**	**B1** for each of p, q, r correct in correct format; allow correct equivalent values. If **B0**, then **SC2** for $5\left(x-\frac{7}{5}\right)-\frac{64}{5}$ or **SC1** for correct values but incorrect format
9(ii)		**B4**	**B2** for fully correct shape in correct position or **B1** for fully correct shape translated parallel to the x-axis **B1** for y-intercept at (0, 3) marked on graph **B1** for roots marked on graph at –0.2 and 3
9(iii)	$0 < k < \left\lvert their\left(-\frac{64}{5}\right)\right\rvert$	**B2**	**FT** *their* **(i)** **B1** for any inequality using *their* $\frac{64}{5}$ or max y value is *their* 12.8soi
10(i)	$v=\frac{\mathrm{d}s}{\mathrm{d}t}=-3\sin 3t$	**B1**	
	When $v = 0$, $t = \frac{\pi}{3}$	**B1**	

Question	Answer	Marks	Partial Marks
10(ii)	Finding *s* when $t = \frac{\pi}{4}$ and $t = \frac{\pi}{2}$	**M1**	
	Finding *s* when $t = \textit{their}\frac{\pi}{3}$ and correct plan	**M1**	Using *their* **(i)** correctly
	1.29 nfww	**A1**	
10(iii)	$a = \frac{\mathrm{d}v}{\mathrm{d}t} = -9\cos 3t$	**B1**	
	9	**B1**	**FT** *their* $k\cos 3t$
11(a)	$10(1-\sin^2 x)+3\sin x = 9$	**M1**	
	Solves $10\sin^2 x - 3\sin x - 1 = 0$ oe	**M1**	**dep** on first **M1** Solves *their* three term quadratic in sin *x*
	$\sin x = \frac{1}{2},\ \sin x = -\frac{1}{5}$	**A1**	
	30º, 150º and 191.5º, 348.5º awrt	**A2**	**A1** for any two correct solutions
11(b)	$3\frac{\sin 2y}{\cos 2y} = 4\sin 2y$ oe	**M1**	
	Solves $3\sin 2y - 4\sin 2y\cos 2y\ [= 0]$	**M1**	**dep** on first **M1**
	$\sin 2y = 0\ \ \cos 2y = \frac{3}{4}$	**A1**	
	Any two of π, 0.72273…, 5.56045… nfww	**A1**	
	$\frac{\pi}{2}$, 0.361, 2.78 awrt nfww	**A1**	**SC**: cancels out sin2*y* after **M1M0** allow **SC1** for 0.72273… and 5.56045... and **SC1**for 0.361 and 2.78
12(i)	$\tan 30 = \frac{h}{x/2}$ oe	**M1**	
	Correct completion to given answer	**A1**	
	$V = 5\sqrt{3}h^2$ isw	**B1**	

Question	Answer	Marks	Partial Marks
12(ii)(a)	$\frac{\mathrm{d}V}{\mathrm{d}h} = their 10\sqrt{3}\,h$ or $\frac{5\sqrt{3}}{2}$	**B1**	**FT** $their V = k\,h^2$
	$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}h}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t}$ soi	**M1**	
	$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{their\left(\frac{\mathrm{d}V}{\mathrm{d}h}\right)} \times 0.5$	**M1**	
	0.115 or 0.11547 to 0.1155 oe	**A1**	
12(ii)(b)	$\left(\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}t} =\right) 2\sqrt{3} \times their \frac{1}{5\sqrt{3}}$	**M1**	
	$\frac{2}{5}$	**A1**	

Cambridge International Examinations
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ADDITIONAL MATHEMATICS **0606/11**

Paper 1 **May/June 2018**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **16** printed pages.

DC (SC/SG) 145845/2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Solve the equations

$$y - x = 4,$$

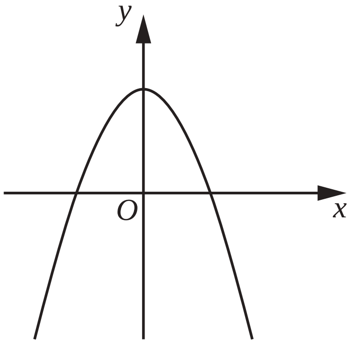
$$x^2 + y^2 - 8x - 4y - 16 = 0.$$

[5]

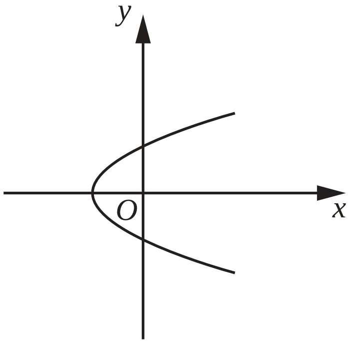
2 Find the equation of the perpendicular bisector of the line joining the points (1, 3) and (4, −5). Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [5]

3 Diagrams **A** to **D** show four different graphs. In each case the whole graph is shown and the scales on the two axes are the same.

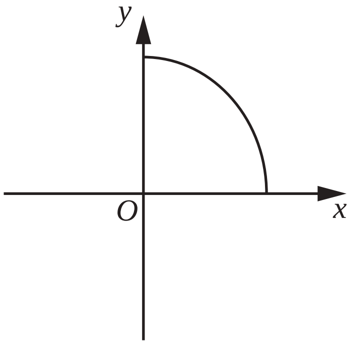
A



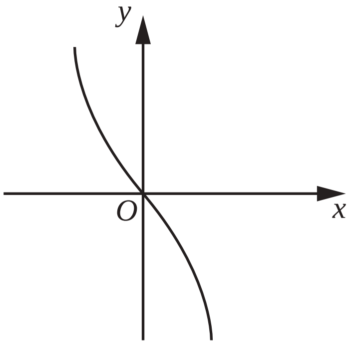
B



C



D

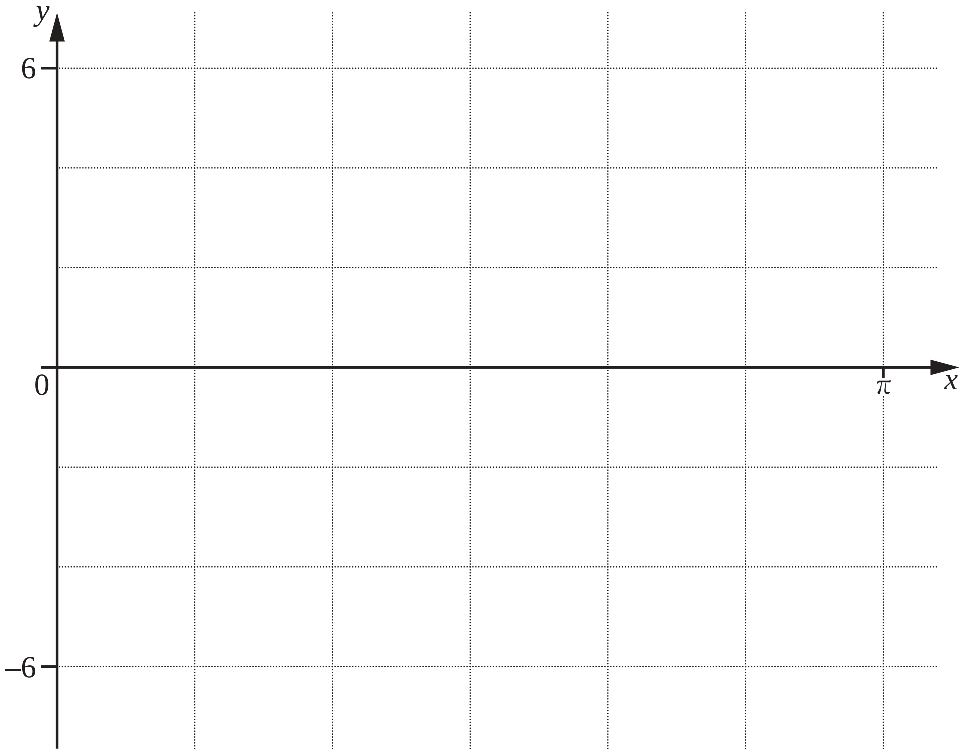


Place ticks in the boxes in the table to indicate which descriptions, if any, apply to each graph. There may be more than one tick in any row or column of the table. [4]

	A	**B**	**C**	**D**
Not a function				
One-one function				
A function that is its own inverse				
A function with no inverse				

4 **(i)** The curve $y = a + b\sin cx$ has an amplitude of 4 and a period of $\frac{\pi}{3}$. Given that the curve passes through the point $\left(\frac{\pi}{12}, 2\right)$, find the value of each of the constants a, b and c. [4]

(ii) Using your values of a, b and c, sketch the graph of $y = a + b\sin cx$ for $0 \leqslant x \leqslant \pi$ radians. [3]



5 The population, P, of a certain bacterium t days after the start of an experiment is modelled by $P = 800\mathrm{e}^{kt}$, where k is a constant.

(i) State what the figure 800 represents in this experiment. [1]

(ii) Given that the population is 20 000 two days after the start of the experiment, calculate the value of k. [3]

(iii) Calculate the population three days after the start of the experiment. [2]

6 **(a)** Write $(\log_2 p)(\log_3 2) + \log_3 q$ as a single logarithm to base 3. [3]

(b) Given that $(\log_a 5)^2 - 4\log_a 5 + 3 = 0$, find the possible values of a. [3]

7 **(i)** Find the inverse of the matrix $\begin{pmatrix} 4 & -2 \\ -5 & 3 \end{pmatrix}$. [2]

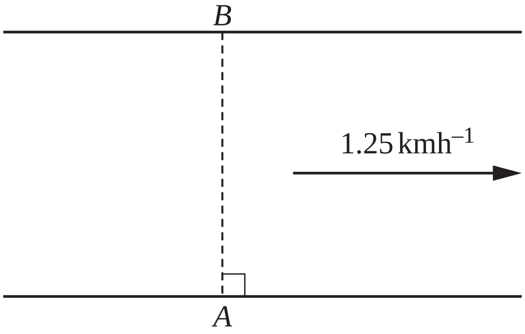
(ii) Hence solve the simultaneous equations

$$\begin{aligned} 8x - 4y - 5 &= 0, \\ -10x + 6y - 7 &= 0. \end{aligned}$$

[4]

8 **(a)** Given that $\mathbf{p} = 2\mathbf{i} - 5\mathbf{j}$ and $\mathbf{q} = \mathbf{i} - 3\mathbf{j}$, find the unit vector in the direction of $3\mathbf{p} - 4\mathbf{q}$. [4]

(b)



A river flows between parallel banks at a speed of 1.25 kmh^{-1}. A boy standing at point A on one bank sends a toy boat across the river to his father standing directly opposite at point B. The toy boat, which can travel at v kmh^{-1} in still water, crosses the river with resultant speed 2.73 kmh^{-1} along the line AB.

(i) Calculate the value of v. [2]

The direction in which the boy points the boat makes an angle θ with the line *AB*.

(ii) Find the value of θ. [2]

9 **(i)** Find the first 3 terms in the expansion of $\left(2x - \frac{1}{16x}\right)^8$ in descending powers of x. [3]

(ii) Hence find the coefficient of x^4 in the expansion of $\left(2x - \frac{1}{16x}\right)^8 \left(\frac{1}{x^2} + 1\right)^2$. [3]

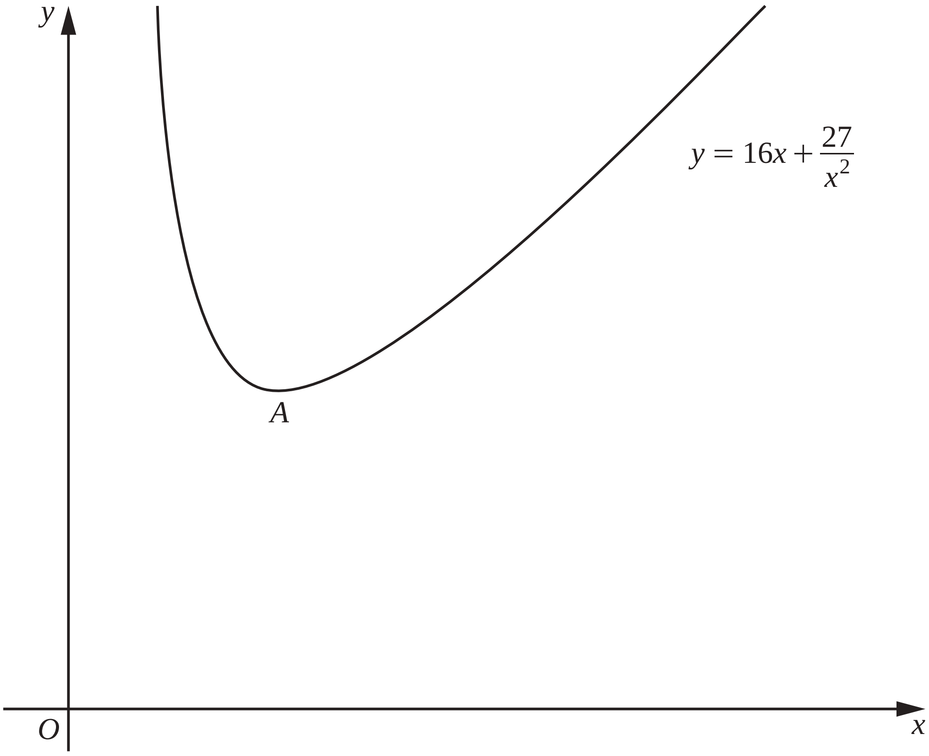
10 Do not use a calculator in this question.

(a) Simplify $\frac{5+6\sqrt{5}}{6+\sqrt{5}}$. [3]

(b) Show that $3^{0.5}\times(\sqrt{2})^7$ can be written in the form $a\sqrt{b}$, where a and b are integers and $a > b$. [2]

(c) Solve the equation $x+\sqrt{2}=\frac{4}{x}$, giving your answers in simplest surd form. [4]

11



The diagram shows part of the graph of $y = 16x + \frac{27}{x^2}$, which has a minimum at A.

(i) Find the coordinates of A. [4]

The points P and Q lie on the curve $y = 16x + \dfrac{27}{x^2}$ and have x-coordinates 1 and 3 respectively.

(ii) Find the area enclosed by the curve and the line PQ. You must show all your working. [6]

Question 12 is printed on the next page.

12 A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = (2x-5)^{-\frac{1}{2}}$. Given that the curve has a gradient of 6 at the point $\left(\frac{9}{2}, \frac{2}{3}\right)$, find the equation of the curve. [8]

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ADDITIONAL MATHEMATICS **0606/12**

Paper 1 **May/June 2018**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

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Mathematical Formulae

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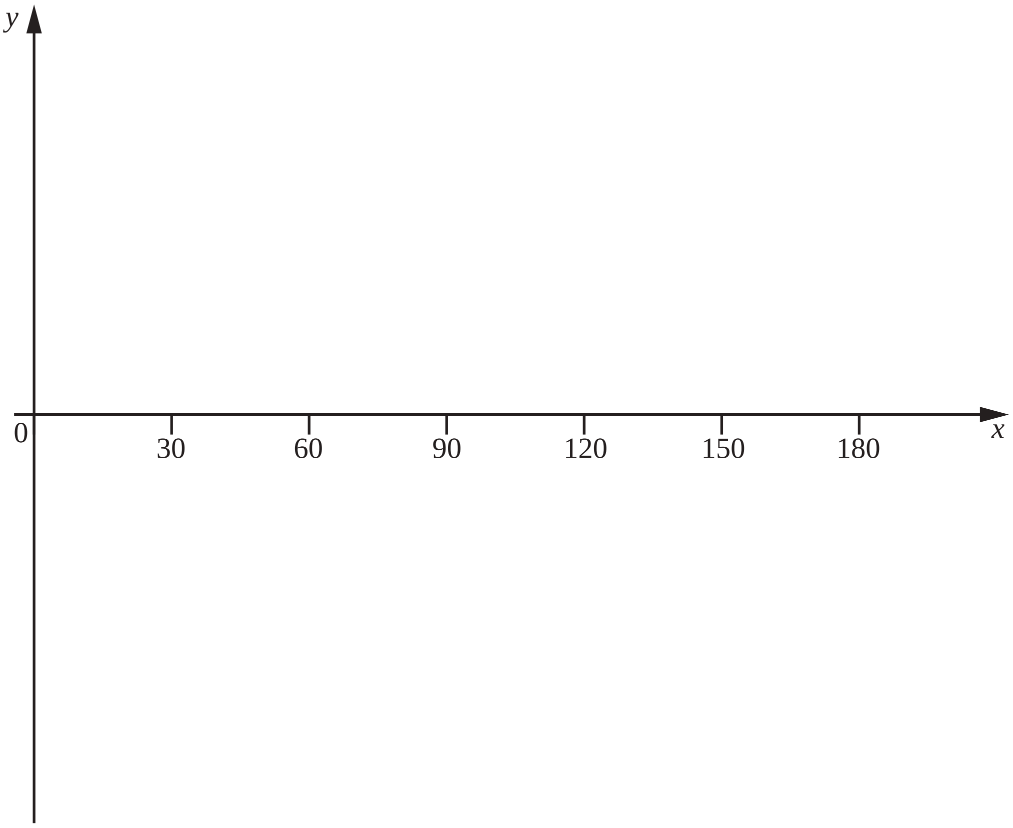
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 It is given that $y = 1 + \tan 3x$.

(i) State the period of y. [1]

(ii) On the axes below, sketch the graph of $y = 1 + \tan 3x$ for $0° \leqslant x° \leqslant 180°$. [3]



2 Find the values of k for which the line $y = 1 - 2kx$ does not meet the curve $y = 9x^2 - (3k + 1)x + 5$. [5]

3 The variables x and y are such that when e^y is plotted against x^2, a straight line graph passing through the points (5, 3) and (3, 1) is obtained. Find y in terms of x. [5]

4 A particle P moves so that its displacement, x metres from a fixed point O, at time t seconds, is given by $x = \ln(5t + 3)$.

(i) Find the value of t when the displacement of P is 3m. [2]

(ii) Find the velocity of P when $t = 0$. [2]

(iii) Explain why, after passing through O, the velocity of P is never negative. [1]

(iv) Find the acceleration of P when $t = 0$. [2]

5 **(i)** The first three terms in the expansion of $\left(3-\frac{1}{9x}\right)^5$ can be written as $a+\frac{b}{x}+\frac{c}{x^2}$. Find the value of each of the constants a, b and c. [3]

(ii) Use your values of a, b and c to find the term independent of x in the expansion of

$$\left(3-\frac{1}{9x}\right)^5(2+9x)^2.$$ [3]

6 Find the coordinates of the stationary point of the curve $y = \dfrac{x+2}{\sqrt{2x-1}}$. [6]

7 A population, B, of a particular bacterium, t hours after measurements began, is given by $B = 1000e^{\frac{t}{4}}$.

(i) Find the value of B when $t = 0$. [1]

(ii) Find the time taken for B to double in size. [3]

(iii) Find the value of B when $t = 8$. [1]

8 **(a)** Solve $3\cos^2\theta + 4\sin\theta = 4$ for $0° \leqslant \theta \leqslant 180°$. [4]

(b) Solve $\sin 2\phi = \sqrt{3}\cos 2\phi$ for $-\frac{\pi}{2} \leqslant \phi \leqslant \frac{\pi}{2}$ radians. [4]

9 **(a)** **(i)** Solve $\lg x = 3$. [1]

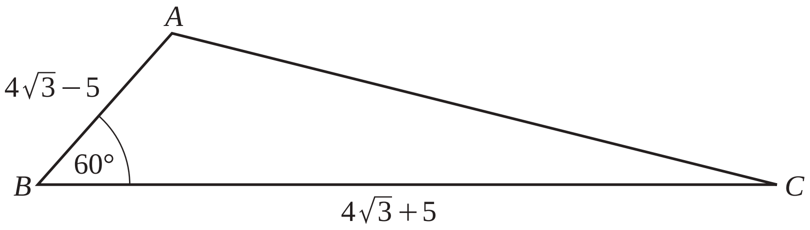
(ii) Write $\lg a - 2\lg b + 3$ as a single logarithm. [3]

(b) **(i)** Solve $x - 5 + \frac{6}{x} = 0$. [2]

(ii) Hence, showing all your working, find the values of *a* such that $\log_4 a - 5 + 6\log_a 4 = 0$. [3]

10 Do not use a calculator in this question.

All lengths in this question are in centimetres.



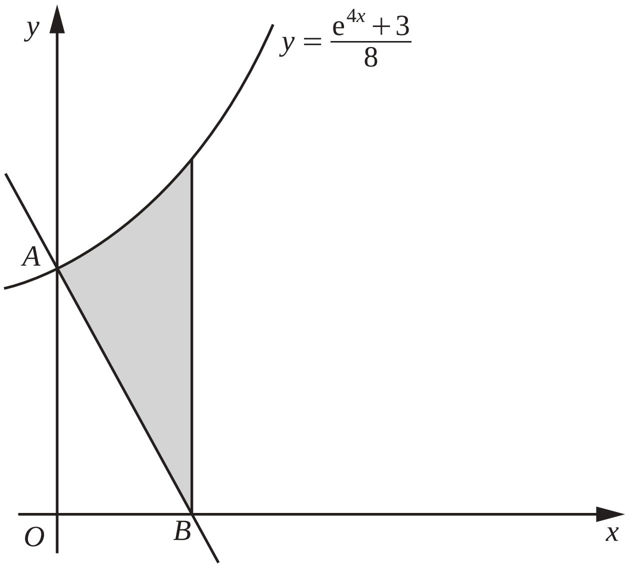
The diagram shows the triangle *ABC*, where $AB = 4\sqrt{3} - 5$, $BC = 4\sqrt{3} + 5$ and angle $ABC = 60°$.

It is known that $\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$, $\tan 60° = \sqrt{3}$.

(i) Find the exact value of *AC*. [4]

(ii) Hence show that $\operatorname{cosec} ACB = \frac{2\sqrt{p}}{q}(4\sqrt{3}+5)$, where p and q are integers. [4]

11



The diagram shows the graph of the curve $y = \frac{e^{4x} + 3}{8}$. The curve meets the y-axis at the point A.

The normal to the curve at A meets the x-axis at the point B. Find the area of the shaded region enclosed by the curve, the line AB and the line through B parallel to the y-axis. Give your answer in the form $\frac{e}{a}$, where a is a constant. You must show all your working.

[10]

Question 12 is printed on the next page.

12 Do not use a calculator in this question.

(a) Given that $\frac{6^p \times 8^{p+2} \times 3^q}{9^{2q-3}}$ is equal to $2^7 \times 3^4$, find the value of each of the constants p and q. [3]

(b) Using the substitution $u = x^{\frac{1}{3}}$, or otherwise, solve $4x^{\frac{1}{3}} + x^{\frac{2}{3}} + 3 = 0$. [4]

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

5006741950

ADDITIONAL MATHEMATICS **0606/13**

Paper 1 **May/June 2018**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
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Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **16** printed pages.

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[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Solve the equations

$$y - x = 4,$$

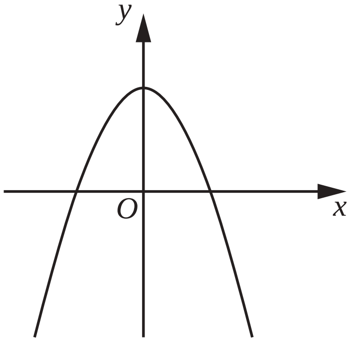
$$x^2 + y^2 - 8x - 4y - 16 = 0.$$

[5]

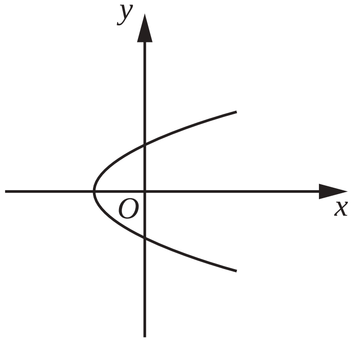
2 Find the equation of the perpendicular bisector of the line joining the points (1, 3) and (4, -5). Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [5]

3 Diagrams **A** to **D** show four different graphs. In each case the whole graph is shown and the scales on the two axes are the same.

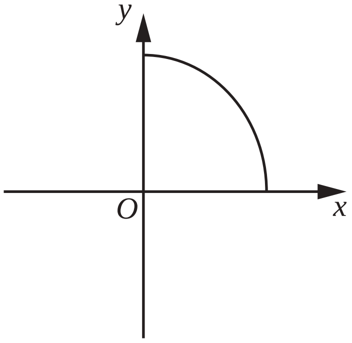
A



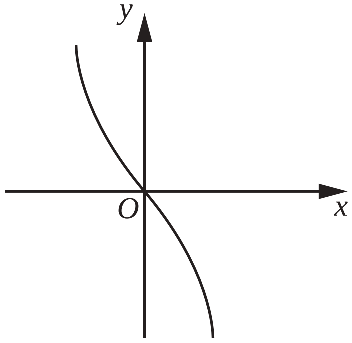
B



C



D

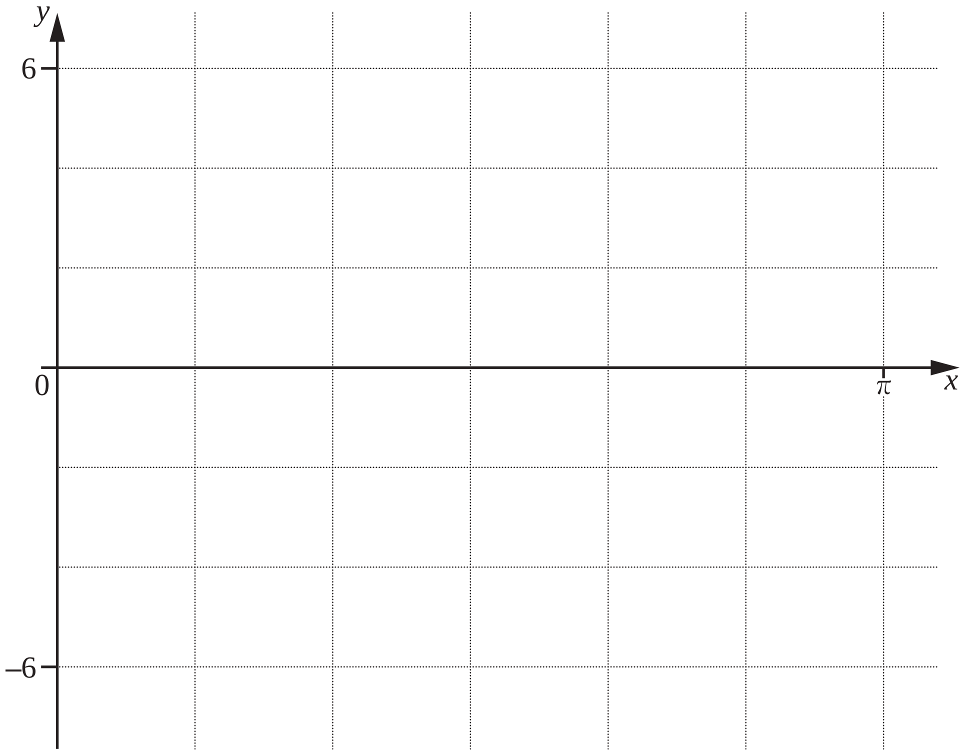


Place ticks in the boxes in the table to indicate which descriptions, if any, apply to each graph. There may be more than one tick in any row or column of the table. [4]

	A	**B**	**C**	**D**
Not a function				
One-one function				
A function that is its own inverse				
A function with no inverse				

4 **(i)** The curve $y = a + b\sin cx$ has an amplitude of 4 and a period of $\frac{\pi}{3}$. Given that the curve passes through the point $\left(\frac{\pi}{12}, 2\right)$, find the value of each of the constants a, b and c. [4]

(ii) Using your values of a, b and c, sketch the graph of $y = a + b\sin cx$ for $0 \leqslant x \leqslant \pi$ radians. [3]



5 The population, P, of a certain bacterium t days after the start of an experiment is modelled by $P = 800e^{kt}$, where k is a constant.

(i) State what the figure 800 represents in this experiment. [1]

(ii) Given that the population is 20 000 two days after the start of the experiment, calculate the value of k. [3]

(iii) Calculate the population three days after the start of the experiment. [2]

6 **(a)** Write $(\log_2 p)(\log_3 2) + \log_3 q$ as a single logarithm to base 3. [3]

(b) Given that $(\log_a 5)^2 - 4\log_a 5 + 3 = 0$, find the possible values of *a*. [3]

7 **(i)** Find the inverse of the matrix $\begin{pmatrix} 4 & -2 \\ -5 & 3 \end{pmatrix}$. [2]

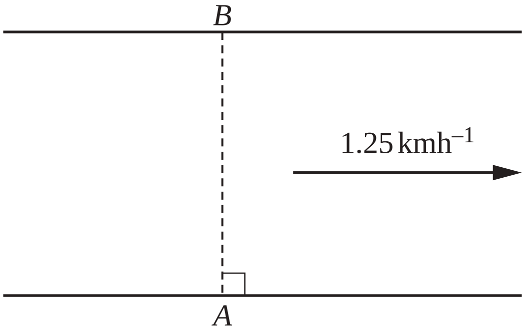
(ii) Hence solve the simultaneous equations

$$8x - 4y - 5 = 0,$$
$$-10x + 6y - 7 = 0.$$

[4]

8 **(a)** Given that $\mathbf{p} = 2\mathbf{i} - 5\mathbf{j}$ and $\mathbf{q} = \mathbf{i} - 3\mathbf{j}$, find the unit vector in the direction of $3\mathbf{p} - 4\mathbf{q}$. [4]

(b)



A river flows between parallel banks at a speed of 1.25 kmh^{-1}. A boy standing at point A on one bank sends a toy boat across the river to his father standing directly opposite at point B. The toy boat, which can travel at v kmh^{-1} in still water, crosses the river with resultant speed 2.73 kmh^{-1} along the line AB.

(i) Calculate the value of v. [2]

The direction in which the boy points the boat makes an angle θ with the line *AB*.

(ii) Find the value of θ. [2]

9 **(i)** Find the first 3 terms in the expansion of $\left(2x - \frac{1}{16x}\right)^8$ in descending powers of x. [3]

(ii) Hence find the coefficient of x^4 in the expansion of $\left(2x - \frac{1}{16x}\right)^8 \left(\frac{1}{x^2} + 1\right)^2$. [3]

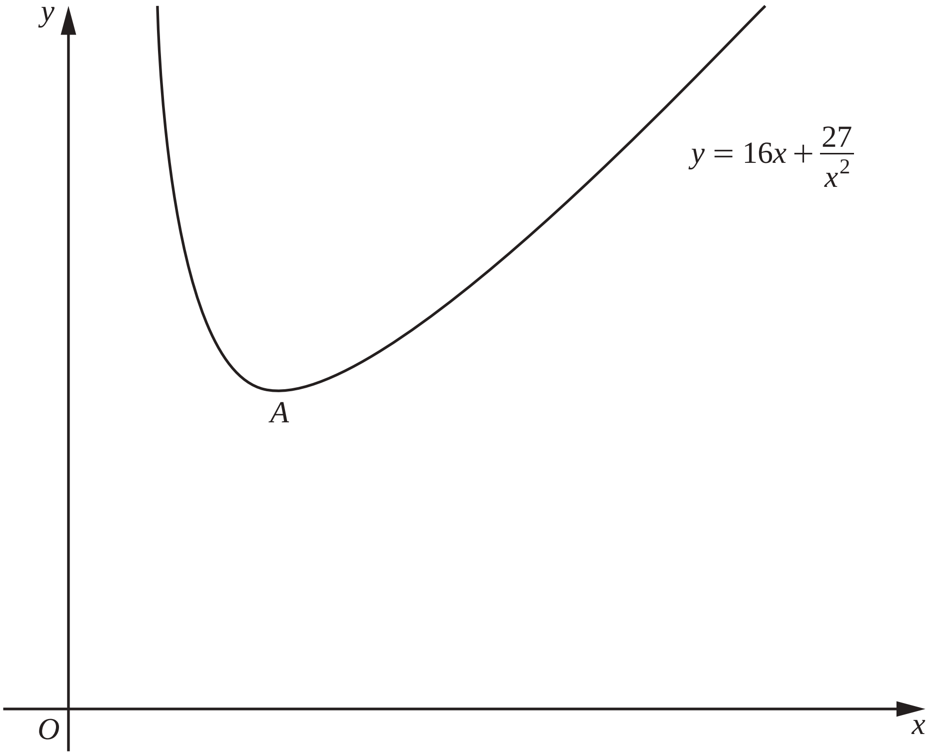
10 Do not use a calculator in this question.

(a) Simplify $\frac{5+6\sqrt{5}}{6+\sqrt{5}}$. [3]

(b) Show that $3^{0.5} \times (\sqrt{2})^7$ can be written in the form $a\sqrt{b}$, where a and b are integers and $a > b$. [2]

(c) Solve the equation $x + \sqrt{2} = \frac{4}{x}$, giving your answers in simplest surd form. [4]

11



The diagram shows part of the graph of $y = 16x + \frac{27}{x^2}$, which has a minimum at A.

(i) Find the coordinates of A. [4]

The points P and Q lie on the curve $y = 16x + \dfrac{27}{x^2}$ and have x-coordinates 1 and 3 respectively.

(ii) Find the area enclosed by the curve and the line PQ. You must show all your working. [6]

Question 12 is printed on the next page.

12 A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = (2x-5)^{-\frac{1}{2}}$. Given that the curve has a gradient of 6 at the point $\left(\frac{9}{2}, \frac{2}{3}\right)$, find the equation of the curve. [8]

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CANDIDATE NAME

CENTRE NUMBER ☐☐☐☐☐

CANDIDATE NUMBER ☐☐☐☐

6234149436

ADDITIONAL MATHEMATICS **0606/21**

Paper 2 **May/June 2018**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

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Write in dark blue or black pen.
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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1} b + \binom{n}{2}a^{n-2} b^2 + \ldots + \binom{n}{r}a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

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Identities

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 A, B and C are subsets of the same universal set.

(i) Write each of the following statements in words.

(a) $A \not\subset B$ [1]

(b) $A \cap C = \varnothing$ [1]

(ii) Write each of the following statements in set notation.

(a) There are 3 elements in set A or B or both. [1]

(b) x is an element of A but it is not an element of C. [1]

2 The variables x and y are such that $y = \ln(3x - 1)$ for $x > \frac{1}{3}$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(ii) Hence find the approximate change in x when y increases from $\ln(1.2)$ to $\ln(1.2) + 0.125$. [3]

3 A 7-character password is to be selected from the 12 characters shown in the table. Each character may be used only once.

	Characters			
Upper-case letters	A	B	C	D
Lower-case letters	e	f	g	h
Digits	1	2	3	4

Find the number of different passwords

(i) if there are no restrictions, [1]

(ii) that start with a digit, [1]

(iii) that contain 4 upper-case letters and 3 lower-case letters such that all the upper-case letters are together and all the lower-case letters are together. [3]

4 **Do not use a calculator in this question.**

It is given that $x+4$ is a factor of $\mathrm{p}(x) = 2x^3 + 3x^2 + ax - 12$. When $\mathrm{p}(x)$ is divided by $x-1$ the remainder is b.

(i) Show that $a = -23$ and find the value of the constant b. [2]

(ii) Factorise $\mathrm{p}(x)$ completely and hence state all the solutions of $\mathrm{p}(x) = 0$. [4]

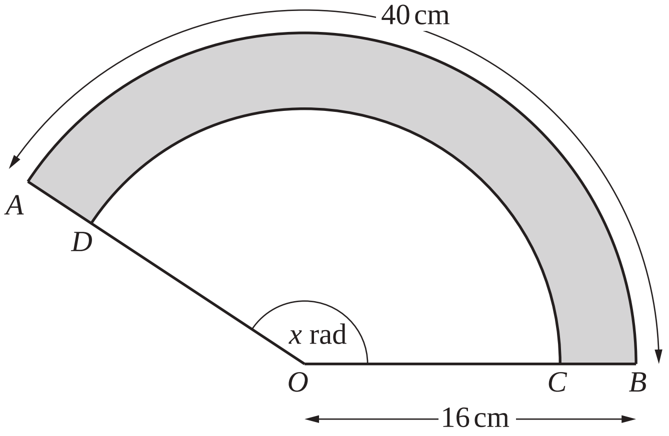
5 The function f is defined by $\mathrm{f}(x) = \dfrac{1}{2x - 5}$ for $x > 2.5$.

(i) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

(ii) State the domain of $\mathrm{f}^{-1}(x)$. [1]

(iii) Find an expression for $\mathrm{f}^{2}(x)$, giving your answer in the form $\dfrac{ax+b}{cx+d}$, where a, b, c and d are integers to be found. [3]

6



In the diagram *AOB* and *DOC* are sectors of a circle centre *O*. The angle *AOB* is x radians. The length of the arc *AB* is 40 cm and the radius *OB* is 16 cm.

(i) Find the value of x. [2]

(ii) Find the area of sector *AOB*. [2]

(iii) Given that the area of the shaded region *ABCD* is 140 cm^2, find the length of *OC*. [3]

7 Differentiate with respect to x

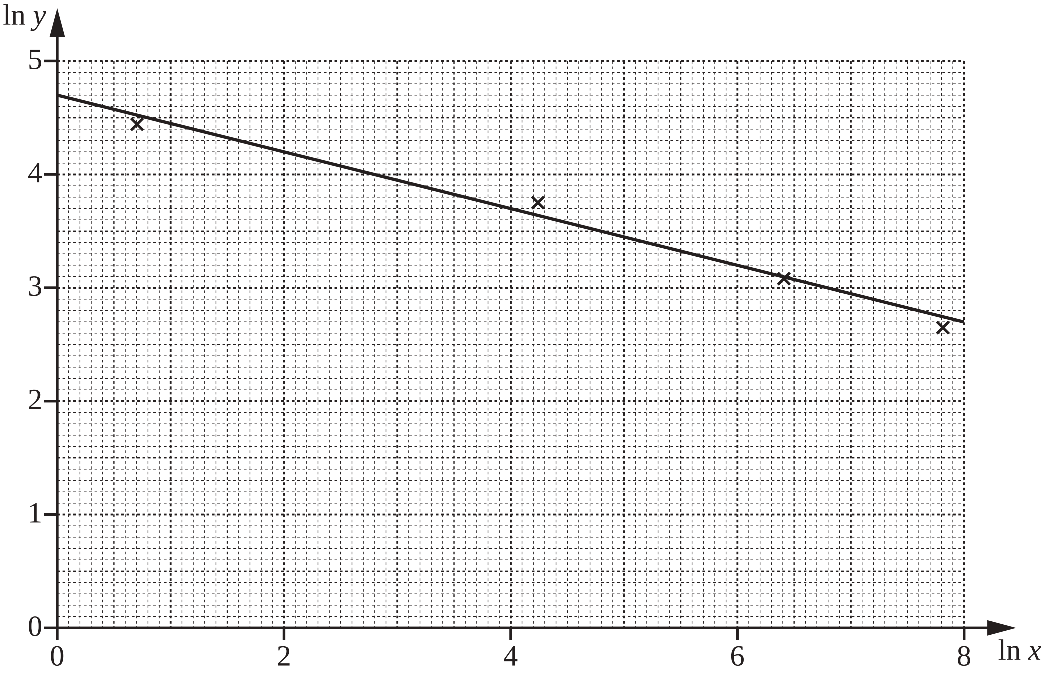
(i) $4x\tan x$, [2]

(ii) $\dfrac{e^{3x+1}}{x^2-1}$. [3]

8 An experiment was carried out recording values of y for certain values of x. The variables x and y are thought to be connected by the relationship $y = ax^n$, where a and n are constants.

(i) Transform the relationship $y = ax^n$ into straight line form. [2]

The values of $\ln y$ and $\ln x$ were plotted and a line of best fit drawn. This is shown in the diagram below.

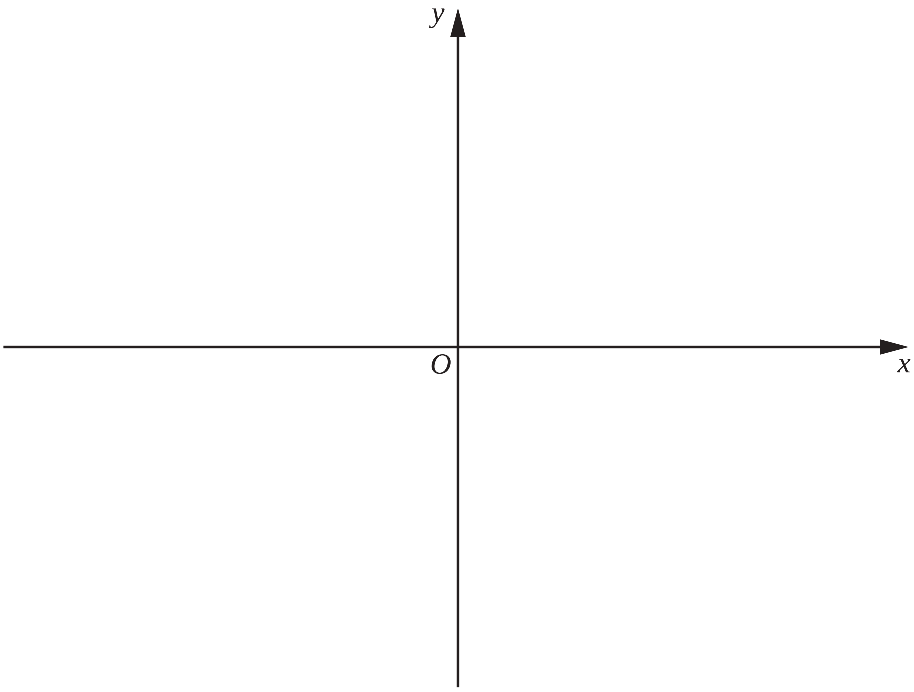


(ii) Use the graph to find the value of a and of n, stating the coordinates of the points that you use. [3]

(iii) Find the value of x when $y = 50$. [2]

9 **(i)** Express $5x^2 - 14x - 3$ in the form $p(x+q)^2 + r$, where p, q and r are constants. [3]

(ii) Sketch the graph of $y = \left|5x^2 - 14x - 3\right|$ on the axes below. Show clearly any points where your graph meets the coordinate axes. [4]



(iii) State the set of values of k for which $\left|5x^2 - 14x - 3\right| = k$ has exactly four solutions. [2]

10 A particle moves in a straight line such that its displacement, s metres, from a fixed point O at time t seconds, is given by $s = 4 + \cos 3t$, where $t \geqslant 0$. The particle is initially at rest.

(i) Find the exact value of t when the particle is next at rest. [2]

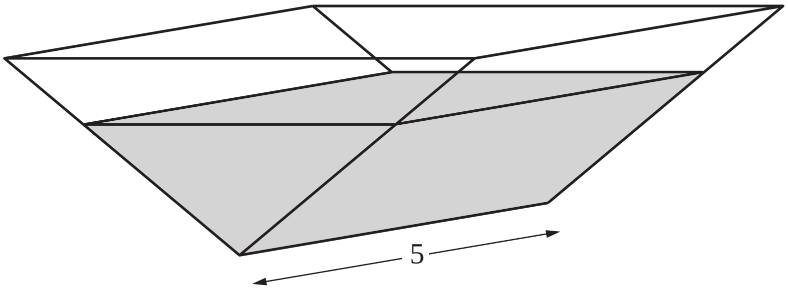
(ii) Find the distance travelled by the particle between $t = \frac{\pi}{4}$ and $t = \frac{\pi}{2}$ seconds. [3]

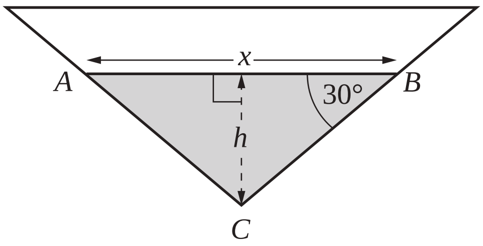
(iii) Find the greatest acceleration of the particle. [2]

11 **(a)** Solve $10\cos^2 x + 3\sin x = 9$ for $0° < x < 360°$. [5]

(b) Solve $3\tan 2y = 4\sin 2y$ for $0 < y < \pi$ radians. [5]

12 In this question all lengths are in metres.





A water container is in the shape of a triangular prism. The diagrams show the container and its cross-section. The cross-section of the water in the container is an isosceles triangle *ABC*, with angle *ABC* = angle *BAC* = 30°. The length of *AB* is *x* and the depth of water is *h*. The length of the container is 5.

(i) Show that $x = 2\sqrt{3}h$ and hence find the volume of water in the container in terms of *h*. [3]

(ii) The container is filled at a rate of $0.5\,\text{m}^3$ per minute. At the instant when h is $0.25\,\text{m}$, find

(a) the rate at which h is increasing, [4]

(b) the rate at which x is increasing. [2]

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Cambridge International General Certificate of Secondary Education

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER					

ADDITIONAL MATHEMATICS **0606/22**

Paper 2 **May/June 2018**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1} b + \binom{n}{2}a^{n-2} b^2 + \ldots + \binom{n}{r}a^{n-r} b^r + \ldots + b^n,$$

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Formulae for ∆ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

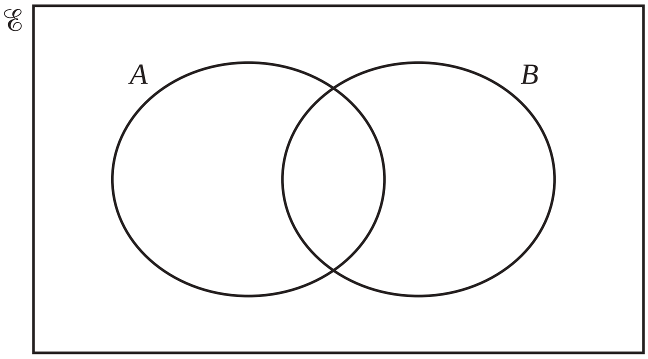
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 (i) Show that $\cos\theta\cot\theta + \sin\theta = \operatorname{cosec}\theta$. [3]

(ii) Hence solve $\cos\theta\cot\theta + \sin\theta = 4$ for $0° \leqslant \theta \leqslant 90°$. [2]

2 **(a)** On the Venn diagram below, shade the region that represents $A \cap B'$.



[1]

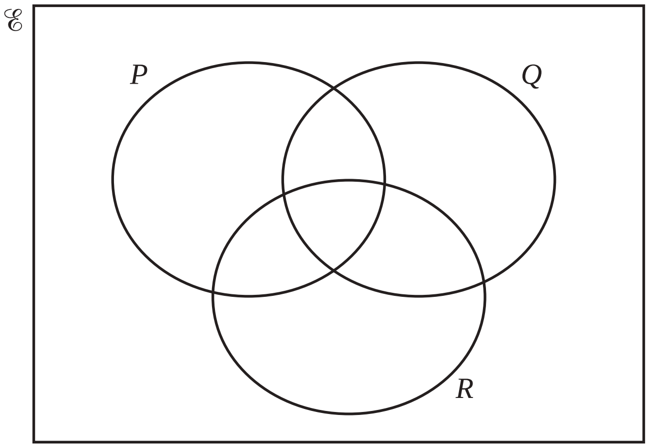
(b) The universal set ℰ and sets P, Q and R are such that

$(P \cup Q \cup R)' = \varnothing$, $P' \cap (Q \cap R) = \varnothing$,

$\mathrm{n}(Q \cap R) = 8$, $\mathrm{n}(P \cap R) = 8$, $\mathrm{n}(P \cap Q) = 10$,

$\mathrm{n}(P) = 21$, $\mathrm{n}(Q) = 15$, $\mathrm{n}(\mathscr{E}) = 30$.

Complete the Venn diagram to show this information and state the value of n(R).



n(R) = ... [4]

3 It is given that $x+3$ is a factor of the polynomial $\mathrm{p}(x)=2x^3+ax^2-24x+b$. The remainder when $\mathrm{p}(x)$ is divided by $x-2$ is -15. Find the remainder when $\mathrm{p}(x)$ is divided by $x+1$. [6]

4 Find the coordinates of the points where the line $2y - 3x = 6$ intersects the curve $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 5$. [5]

5 **(a)** Four parts in a play are to be given to four of the girls chosen from the seven girls in a drama class. Find the number of different ways in which this can be done. [2]

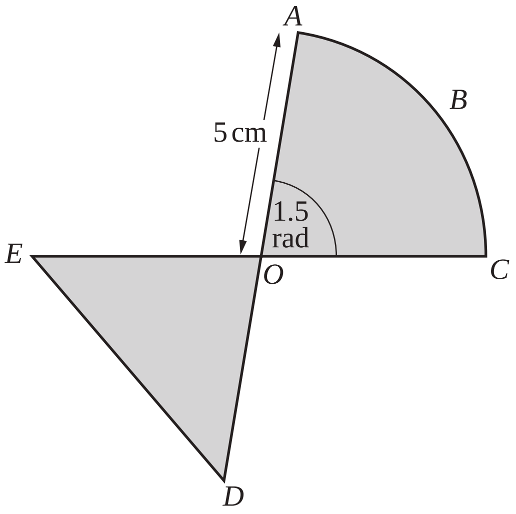
(b) Three singers are chosen at random from a group of 5 Chinese, 4 Indian and 2 British singers. Find the number of different ways in which this can be done if

(i) no Chinese singer is chosen, [1]

(ii) one singer of each nationality is chosen, [2]

(iii) the three singers chosen are all of the same nationality. [2]

6



In the diagram, *ABC* is an arc of the circle centre *O*, radius 5 cm, and angle *AOC* is 1.5 radians. *AD* and *CE* are diameters of the circle and *DE* is a straight line.

(i) Find the total perimeter of the shaded regions. [3]

(ii) Find the total area of the shaded regions. [3]

7 Vectors **i** and **j** are vectors parallel to the x-axis and y-axis respectively.

Given that $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$, $\mathbf{b} = \mathbf{i} - 5\mathbf{j}$ and $\mathbf{c} = 3\mathbf{i} + 11\mathbf{j}$, find

(i) the exact value of $|\mathbf{a} + \mathbf{c}|$, [2]

(ii) the value of the constant m such that $\mathbf{a} + m\mathbf{b}$ is parallel to **j**, [2]

(iii) the value of the constant n such that $n\mathbf{a} - \mathbf{b} = \mathbf{c}$. [2]

8 **(a)** $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 1 & -3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 0 & -2 \\ 3 & -5 \end{pmatrix}$. Find $(\mathbf{BA})^{-1}$. [4]

(b) The matrix **X** is such that **XC** = **D**, where $\mathbf{C} = \begin{pmatrix} -2 & 5 & 3 \\ 0 & 10 & 4 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} -4 & 5 & 4 \end{pmatrix}$.

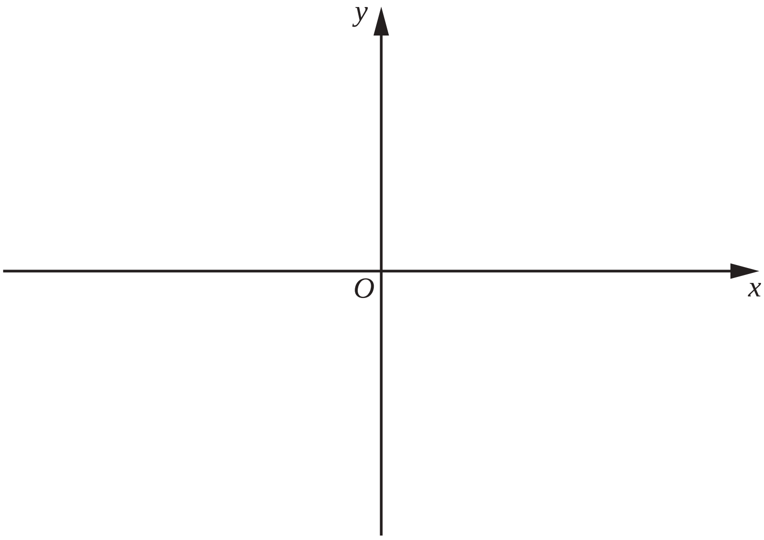
(i) State the order of the matrix **C**. [1]

(ii) Find the matrix **X**. [2]

9 **(i)** Differentiate $x^4(\sqrt{\sin x})$ with respect to x. [4]

(ii) Hence find $\int\left(x+\frac{x^4\cos x}{\sqrt{\sin x}}+8x^3(\sqrt{\sin x})\right)\mathrm{d}x$. [3]

10 **(a)** **(i)** On the axes below, sketch the graph of $y = |(x+3)(x-5)|$ showing the coordinates of the points where the curve meets the x-axis. [2]



(ii) Write down a suitable domain for the function $f(x) = |(x+3)(x-5)|$ such that f has an inverse. [1]

(b) The functions g and h are defined by

$$g(x) = 3x - 1 \qquad \text{for } x > 1,$$
$$h(x) = \frac{4}{x} \qquad \text{for } x \neq 0.$$

(i) Find hg(x). [1]

(ii) Find $(\text{hg})^{-1}(x)$. [2]

(c) Given that $p(a) = b$ and that the function p has an inverse, write down $p^{-1}(b)$. [1]

11 **(a)** Find $\int \sqrt[3]{2x-1}\,\mathrm{d}x$. [2]

(b) **(i)** Find $\int \sin 4x\,\mathrm{d}x$. [2]

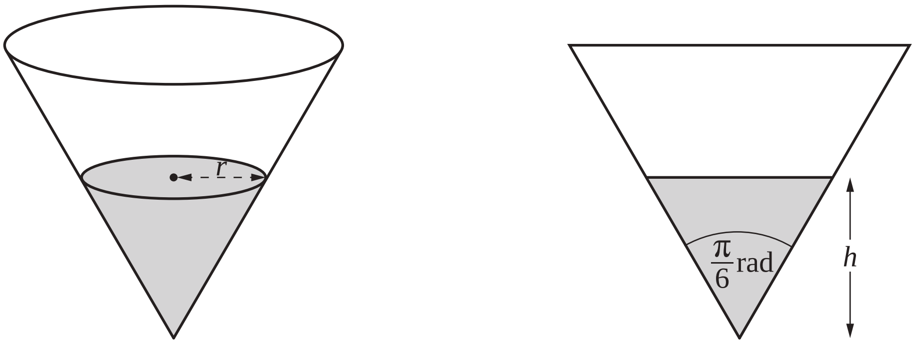
(ii) Hence evaluate $\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \sin 4x\,\mathrm{d}x$. [2]

(c) Show that $\int_{0}^{\ln 8} \mathrm{e}^{\frac{x}{3}}\,\mathrm{d}x = 3$. [5]

12 In this question all lengths are in centimetres.

The volume of a cone of height h and base radius r is given by $V = \frac{1}{3}\pi r^2 h$.

It is known that $\sin\frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}$, $\cos\frac{\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$, $\tan\frac{\pi}{12} = 2-\sqrt{3}$.



A water cup is in the shape of a cone with its axis vertical. The diagrams show the cup and its cross-section. The vertical angle of the cone is $\frac{\pi}{6}$ radians. The depth of water in the cup is h. The surface of the water is a circle of radius r.

(i) Find an expression for r in terms of h and show that the volume of water in the cup is given by $V = \frac{\pi(7-4\sqrt{3})h^3}{3}$. [4]

(ii) Water is poured into the cup at a rate of $30\,\text{cm}^3\,\text{s}^{-1}$. Find, correct to 2 decimal places, the rate at which the depth of water is increasing when $h = 5$. [4]

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Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

ADDITIONAL MATHEMATICS **0606/23**

Paper 2 **May/June 2018**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **15** printed pages and **1** blank page.

DC (JM) 162865

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1} b + \binom{n}{2}a^{n-2} b^2 + \ldots + \binom{n}{r}a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 A, B and C are subsets of the same universal set.

(i) Write each of the following statements in words.

(a) $A \not\subset B$ [1]

(b) $A \cap C = \varnothing$ [1]

(ii) Write each of the following statements in set notation.

(a) There are 3 elements in set A or B or both. [1]

(b) x is an element of A but it is not an element of C. [1]

2 The variables x and y are such that $y = \ln(3x - 1)$ for $x > \frac{1}{3}$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(ii) Hence find the approximate change in x when y increases from $\ln(1.2)$ to $\ln(1.2) + 0.125$. [3]

3 A 7-character password is to be selected from the 12 characters shown in the table. Each character may be used only once.

	Characters			
Upper-case letters	A	B	C	D
Lower-case letters	e	f	g	h
Digits	1	2	3	4

Find the number of different passwords

(i) if there are no restrictions, [1]

(ii) that start with a digit, [1]

(iii) that contain 4 upper-case letters and 3 lower-case letters such that all the upper-case letters are together and all the lower-case letters are together. [3]

4 **Do not use a calculator in this question.**

It is given that $x + 4$ is a factor of $\mathrm{p}(x) = 2x^3 + 3x^2 + ax - 12$. When $\mathrm{p}(x)$ is divided by $x - 1$ the remainder is b.

(i) Show that $a = -23$ and find the value of the constant b. [2]

(ii) Factorise $\mathrm{p}(x)$ completely and hence state all the solutions of $\mathrm{p}(x) = 0$. [4]

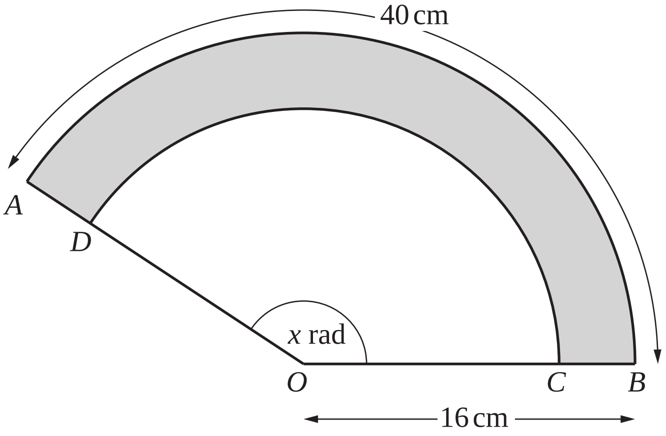
5 The function f is defined by $\mathrm{f}(x) = \dfrac{1}{2x-5}$ for $x > 2.5$.

(i) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

(ii) State the domain of $\mathrm{f}^{-1}(x)$. [1]

(iii) Find an expression for $\mathrm{f}^{2}(x)$, giving your answer in the form $\dfrac{ax+b}{cx+d}$, where a, b, c and d are integers to be found. [3]

6



In the diagram *AOB* and *DOC* are sectors of a circle centre *O*. The angle *AOB* is x radians. The length of the arc *AB* is 40 cm and the radius *OB* is 16 cm.

(i) Find the value of x. [2]

(ii) Find the area of sector *AOB*. [2]

(iii) Given that the area of the shaded region *ABCD* is 140 cm^2, find the length of *OC*. [3]

7 Differentiate with respect to x

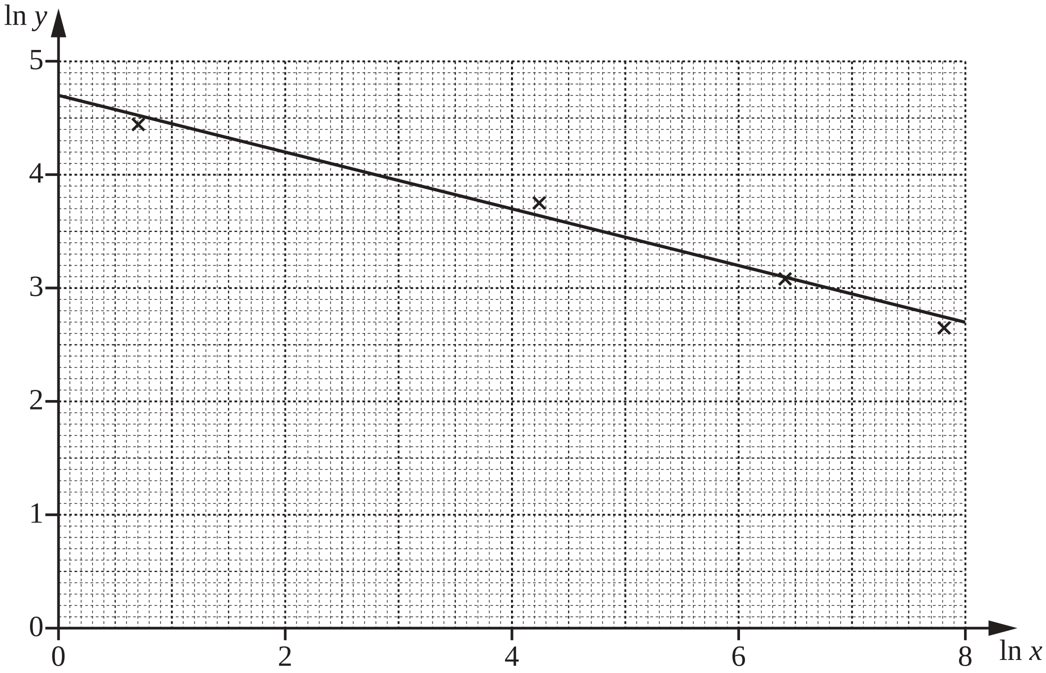
(i) $4x\tan x$, [2]

(ii) $\dfrac{e^{3x+1}}{x^2-1}$. [3]

8 An experiment was carried out recording values of y for certain values of x. The variables x and y are thought to be connected by the relationship $y = ax^n$, where a and n are constants.

(i) Transform the relationship $y = ax^n$ into straight line form. [2]

The values of $\ln y$ and $\ln x$ were plotted and a line of best fit drawn. This is shown in the diagram below.

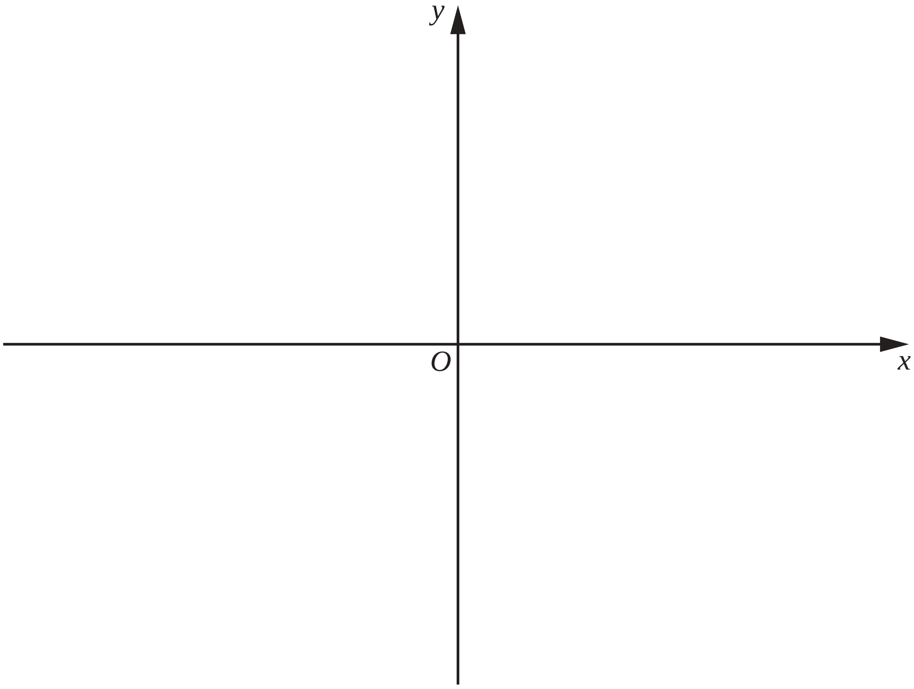


(ii) Use the graph to find the value of a and of n, stating the coordinates of the points that you use. [3]

(iii) Find the value of x when $y = 50$. [2]

9 **(i)** Express $5x^2 - 14x - 3$ in the form $p(x+q)^2 + r$, where p, q and r are constants. [3]

(ii) Sketch the graph of $y = \left|5x^2 - 14x - 3\right|$ on the axes below. Show clearly any points where your graph meets the coordinate axes. [4]



(iii) State the set of values of k for which $\left|5x^2 - 14x - 3\right| = k$ has exactly four solutions. [2]

10 A particle moves in a straight line such that its displacement, s metres, from a fixed point O at time t seconds, is given by $s = 4 + \cos 3t$, where $t \geqslant 0$. The particle is initially at rest.

(i) Find the exact value of t when the particle is next at rest. [2]

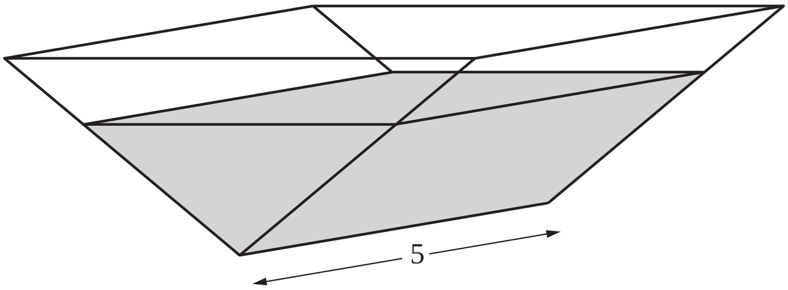
(ii) Find the distance travelled by the particle between $t = \frac{\pi}{4}$ and $t = \frac{\pi}{2}$ seconds. [3]

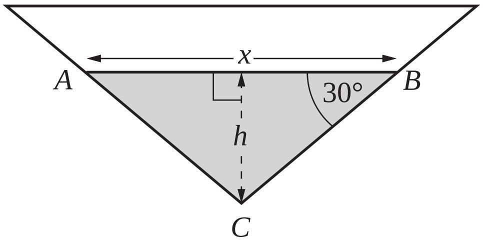
(iii) Find the greatest acceleration of the particle. [2]

11 **(a)** Solve $10\cos^2 x + 3\sin x = 9$ for $0° < x < 360°$. [5]

(b) Solve $3\tan 2y = 4\sin 2y$ for $0 < y < \pi$ radians. [5]

12 In this question all lengths are in metres.





A water container is in the shape of a triangular prism. The diagrams show the container and its cross-section. The cross-section of the water in the container is an isosceles triangle *ABC*, with angle *ABC* = angle *BAC* = 30°. The length of *AB* is *x* and the depth of water is *h*. The length of the container is 5.

(i) Show that $x = 2\sqrt{3}h$ and hence find the volume of water in the container in terms of *h*. [3]

(ii) The container is filled at a rate of $0.5\,\text{m}^3$ per minute. At the instant when h is $0.25\,\text{m}$, find

(a) the rate at which h is increasing, [4]

(b) the rate at which x is increasing. [2]

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ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key messages

Candidates are to be reminded of the importance of working to a suitable level of accuracy throughout a question in order to be able to give their final answer correctly to the required level of accuracy. It is also essential that candidates ensure that they have met the requirements of each question. In questions that require the candidate to show a specific result, it is essential that each step of the solution is shown clearly.

General comments

There were many scripts of a high standard showing a good understanding of the syllabus and the correct applications of the techniques required. Most solutions were set out clearly and, where the need arose, the blank page in the question/answer booklet was utilised as intended, or extra pages were used.

Comments on specific questions

Question 1

(a) In both parts of this question, it was essential that the correct notation was used. A single digit answer was required as the question asked for the number of elements in the given sets. An answer of the form {6} or {1} was therefore considered to be incorrect. However, most candidates answered correctly.

(b) Most candidates were able to produce a correct Venn diagram with set *P* enclosed in set *Q* and sets *Q* and *R* separate.

(c) Any correct answers were acceptable, with most candidates obtaining at least one of the results given below.

Answers: **(a)(i)** 6 **(ii)** 1 **(c)** $S' \cup T'$ or $(S \cup T)'$ and $(X \cap Y) \cup (X \cap Z)$ or $X \cap (Y \cup Z)$

Question 2

The following criteria were needed to gain marks: a maximum point in the first quadrant, intercepts on the axes either labelled or written below the graph, cusps on the *x*-axis and a correct shape of the curve for $x < -\frac{1}{2}$ and $x > 3$. Marks were usually lost when the cusps were drawn as stationary points and the outer parts of the curve were the incorrect shape. It is suggested that the graph of the quadratic equation without the modulus is drawn faintly or with construction (dotted) lines to start with. A reflection in the *x*-axis should then ensure the correct shape throughout. The appearance of such construction lines will not be penalised but seen as an aid to obtaining a correct sketch.

Question 3

(i) Most candidates were able to obtain the first three terms of the required expansion. There was the occasional arithmetic slip or sign error in some cases. It must be noted that a few candidates chose to take out a factor of 3, giving their final answer as $243 - 54x + 5x^2$. These candidates had not answered the question correctly, not understanding that a factor of 3 could only be taken out if the final answer was written as $3\left(243-54x+5x^2\right)$.

(ii) A correct expansion of $\left(x-\frac{2}{x}\right)^2$ was obtained by most candidates and then used correctly with their answer to **Part (i)** to obtain the two terms independent of *x* which would then lead to the final answer.

Answers: **(i)** $729-162x+15x^2$ **(ii)** –2856

Question 4

(i) The majority of candidates recognised the notation $\text{p}'(x)$ and used it correctly with the remainder theorem to obtain the given result.

(ii) The factor theorem was used by most candidates to obtain a second equation in *a* and *b*. This equation, together with the given result from **Part (i)**, were usually solved correctly.

(iii) It was intended that candidates use either algebraic long division or observation to write $\text{p}(x)$ in the required form. Some candidates chose to use synthetic division. This method will only be correct if the resulting extra factor of 2 (the result $(2x-1)\left(2x^2+28x+49\right)$ is obtained) is taken into account. Candidates must be careful when using synthetic division by a factor of the form $ax+b$ where $a\neq\pm1$ or 0.

(iv) This is an example of where some candidates did not take note of the demand of the question. The demand was to factorise $\text{p}(x)$. Some candidates factorised $Q(x)$ only and some chose to write down the solutions to $\text{p}(x)=0$.

Answers: **(ii)** $a=27,\ b=84$, **(iii)** $(2x-1)\left(x^2+14x+49\right)$ **(iv)** $(2x-1)(x+7)^2$

Question 5

(i) The product rule for logarithms was applied appropriately by the majority of candidates to obtain the correct result.

(ii) The product rule and the power rule for logarithms were applied appropriately by the majority of candidates to obtain the correct result.

(iii) Many candidates did not obtain full marks for this part of the question as they did not answer the question completely. Most obtained the correct result of $p=\frac{1}{6}$, but some candidates stopped at this point and gave a final answer of 0.17. Candidates should be guided by the mark allocation. In this case there is a mark allocation of 3 marks which is too generous for the solution of a simple linear equation. Of those candidates that did continue and solve to obtain a value for x, most were successful, giving their final answer to the required level of accuracy.

Answers: **(i)** $2+p$ **(ii)** $7p-4$ **(iii)** 1.26

Question 6

(a) An answer of products of two matrices was expected. Those candidates that gave the extra answers of **CBA** and **AA** were not able to obtain credit for these answers but are to be commended on recognising that these were valid options.

(b) (i) Most candidates obtained a correct inverse matrix.

(ii) A few candidates did not take note of the word 'Hence' in the instruction to find the matrix **Z**. Using a method involving the solution of four simultaneous equations in four unknowns was not an acceptable answer. It was intended that pre-multiplication of the given equation by the inverse matrix obtained in **Part (i)** was used. Many candidates did just this, with very few instances of post-multiplication being seen.

Answers: **(a)** **BA** and **CB** **(b)(i)** $\frac{1}{16}\begin{pmatrix} 3 & 2 \\ -5 & 2 \end{pmatrix}$ **(ii)** $\frac{1}{16}\begin{pmatrix} 16 & 3 \\ -16 & -5 \end{pmatrix}$

Question 7

It was essential that no use of calculators was made in this question. It was therefore necessary that each step in the working of the solution be shown.

(i) Most candidates used either the area of a trapezium or the area of a triangle and a rectangle with sufficient evidence of expansion without a calculator to obtain the correct result.

(ii) Most candidates were able to obtain $\cot\theta = \frac{4}{10-2\sqrt{5}}$ or equivalent. Some errors in the length involving *DC* were made, but this did not preclude the awarding of a method mark for a correct attempt at rationalisation. This was another part of the paper where some candidates did not read the requirements of the question and did not give their final answer in the required form. A check on this should be made at the end of each question.

Answers: **(i)** $10+22\sqrt{5}$ **(ii)** $\frac{1}{2}+\frac{\sqrt{5}}{10}$

Question 8

(a) (i) There were quite a few candidates who did not appreciate the fact that when a particle is travelling at constant velocity, the acceleration is zero. It was not intended that the gradient at the instant $t = 5$ be calculated, although if it was done correctly the mark was awarded.

(ii) Most candidates realised that they needed to find the total area under the graph. There was only the occasional arithmetic slip made by some candidates.

(b) (i) This part of the question was intended to test whether candidates were aware of the difference between velocity and speed. Unfortunately, there were many incorrect answers of –2.5.

(ii) This part of the question was intended to test whether candidates realised that they needed to be working in radians as well as the need for differentiation. Fortunately, there were many completely correct solutions. It should be noted that some candidates, when choosing not to give an exact answer, did not give their final answer to the correct level of accuracy.

Answers: **(a)(i)** 0 **(ii)** 110 m **(b)(i)** 2.5ms^{-1} **(ii)** $\frac{\pi}{4}$ or 0.785

Question 9

(i) Very few incorrect solutions were seen, with most candidates making correct use of the area of the sector to obtain an expression for the angle of the sector first and then making use of the arc length and the radius to obtain the given result for the perimeter. It should be noted that candidates should be working in radians for efficiency, but as this was not a requirement of the question, those that chose to work in degrees were not penalised.

(ii) Most candidates realised the need to differentiate the expression for the perimeter and equate it to zero to find the value of *r* for which the perimeter has a stationary value. This was another example of candidates not reading the requirements of the question as many lost a mark by not finding this value of P. It was also essential that working be shown to determine the nature of the stationary point. Most chose to use the second derivative method and arrive at the correct conclusion appropriately. If the second derivative method is not being used and the method of inspection of the gradient either side of the stationary point is being considered, it is essential that candidates make this clear, usually by using a table which has clear headings.

Answer: **(ii)** 24, minimum

Question 10

(i) There were many completely correct solutions to this part, with candidates integrating correctly and making correct use of arbitrary constants. Candidates must ensure, however, that they give their final answer as an equation as required, not as an expression.

(ii) Apart from those candidates that mistakenly thought that the gradient of the tangent was 10, most applied a correct method. The final accuracy mark was not awarded if the correct level of accuracy was not used. Answers in exact form were acceptable as were unsimplified answers as the form of the final answer was not specified..

Answers: **(i)** $y = e^{2x} + \frac{3x^2}{2} + 8x - 6$ **(ii)** $y + 2.26 = -\frac{1}{12}\left(x - \frac{1}{4}\right)$

Question 11

(a) Most candidates were able to obtain at least one mark in this part of the question, by obtaining an equation in terms of $\sin x$ and $\cos x$ together with one correct solution. Too many candidates divided their equation through by $\sin x$ and thus did not consider the solutions of the equation $\sin x = 0$. Some candidates were also unable to obtain both solutions to $\cos x = \pm\frac{1}{\sqrt{2}}$, not having considered the solution obtained from $\cos x = -\frac{1}{\sqrt{2}}$.

(b)(i) Many correct solutions were seen with most candidates showing enough working to obtain the given result.

(ii) Most candidates realised that they needed to use the result from **Part (i)** and hence attempted to solve the equation $\cos 3\theta = \frac{1}{2}$. It was pleasing to see that many candidates are now a lot more confidant in dealing with negative angles. Most chose to leave their answers in terms of π.

Answers: **(a)** $0^\circ, 45^\circ, 135^\circ, 180^\circ$ **(b)(ii)** $\pm\frac{\pi}{9},\ \pm\frac{5\pi}{9}$

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

To succeed in this examination, candidates need to be able to interpret and use all the information given in a problem. Candidates should read each question carefully and identify key statements. Candidates also need to show sufficient method so that marks can be awarded. Candidates need to be aware of instructions in questions such as '...showing all your working'. Such instructions mean that when a solution is incomplete, often through calculator use, a significant loss of marks will result. Candidates should ensure that their answers are given to at least the accuracy demanded in a question. When no particular accuracy is required, candidates should ensure that they follow the instructions printed on the front page of the examination paper. Candidates need to take care to ensure that their calculator is in the appropriate mode when working with trigonometric expressions.

General comments

Most candidates were well prepared for this examination and many excellent solutions were offered. Candidates were able to recall and use manipulative technique when needed. Most candidates were also able to formulate problems into mathematical terms and select and apply appropriate techniques of solution.

The presentation of work was generally clear and logical. Some candidates made good use of the blank pages at the end of the paper or used additional paper. This ensured that their work was legible and could be marked. Candidates who did this usually added a note in their script to indicate that their answer was written, or continued, elsewhere. This was very helpful.

Many candidates offered complete solutions, with all working shown. Candidates who relied on their calculator to solve equations or evaluate definite integrals, for example, often lost marks. This was because key steps in the method, which were required, were omitted. Showing clear and full method is essential if a question asks candidates to 'Show that…' a result is in a particular form. This instruction indicates that the answer has been given and that the marks will be awarded for the method. Working back from the given answer is rarely successful in this case. The need for this was highlighted in **Questions 2(i)** and **7(i)** in this examination.

When candidates are required to 'Explain why' something is valid or correct, it is important that any explanation is not contradictory or does not contain incorrect statements. This was required in **Questions 7(ii)**, **9(a)(i)** and **9(b)(iii)** in this paper.

In order for final answers to be accurate to three significant figures, working values must be given to a greater accuracy. This avoids a premature approximation error. This was evident in **Question 2(ii)**, **10(ii)** and **11(b)(ii)** in this paper.

Most candidates attempted to answer all questions. Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

Generally, this question was well answered. A few candidates reversed the answers to **Parts (i)** and **(ii)**. The most common error was to find $^{15}C_3$ in **Part (iii)**.

Answers: **(i)** 1 081 575 **(ii)** 40 320 **(iii)** 2730

Question 2

(i) Almost all candidates understood the need to apply the quotient rule, or rearranged correctly and used the product rule. Most earned 3 or 4 marks. The few candidates who were attempting to work back from the given answer often stated the derivative of e^x as xe^x. Some candidates omitted brackets or did not show convincing working to find the given answer. A few candidates omitted to state the given answer as part of their solution. This was penalised.

(ii) A good number of candidates found the value of the derivative when $x = 2$ and evaluated this to at least 3 significant figures, as needed. The solution was usually completed by multiplying this by h. A few candidates rounded the value of the derivative when $x = 2$ to fewer than 3 significant figures and seemed to be applying the idea of approximation to the answer when, in fact, the method being used was the approximation.

Answer: **(ii)** $–0.0261h$

Question 3

(i) Some excellent sketches were seen, with candidates taking care over the axis of symmetry of the curve as well as the y-intercept and amplitude. A few candidates initiated their sketch at the correct point but then clearly used the x-axis as the axis of symmetry. Most candidates attempted a graph of correct period and only a few sketches had incorrect amplitude.

(ii) Almost all candidates answered this part correctly. Those few who were incorrect usually arranged the three values in a different order.

Answer: **(ii)** $a = –1$, $b = 5$, $c = 3$

Question 4

(a) Candidates all understood that the brackets on the left needed to be expanded and the terms collected. Mostly this was managed correctly. Occasional sign slips were made and a few arithmetic errors were seen. The majority of candidates were able to find the critical values for their quadratic expression and most of these were able to give an inequality of the correct form for their answer. When stating critical values, it may be less confusing for some candidates to write, for example,

CV: $x = –1$, $x = \frac{3}{4}$, as many better candidates did. This may have reduced the errors made by a few candidates who wrote, for example, CV: $x \leqslant –1, \frac{3}{4}$, which they then stated as their answer.

(b) The majority of candidates used the given equation, correctly wrote down the values for a, b and c and applied $b^2 – 4ac$. Most of these candidates were able to find the discriminant as –1. The best candidates understood that, as the value was independent of k, there were no real roots *whatever the value of k*. Indication of this was required for full marks to be given. A few candidates multiplied through by 4 and worked with $x^2 + 4kx + 4k^2 + 4 = 0$. This was allowed. Some of these candidates did not multiply through correctly and this was not permitted as it was unnecessary. A few candidates were unable to state the correct a, b and c. These candidates usually included xs in their expressions or incorrectly grouped the x^2 and k^2 terms, for example.

Answer: **(a)** $–1 \leqslant x \leqslant \frac{3}{4}$

Question 5

In this question, candidates needed to apply problem solving skills and work their way through the correct, multi-step solution. Care needed to be taken at all stages to ensure that the information given was used in the correct way. Most candidates were able to correctly find the gradient of *AB*. A few candidates inverted the calculation and usually did this consistently, appearing to have a correct solution as they had made a repeated error. Almost all candidates understood that the gradient of *CD* could be found using the product of the gradients being equal to –1. Many candidates then either formed an equation using the gradient in terms of *k* or formed the equation of the line *CD* and substituted $x = 3$. A good number of candidates found the *x*-coordinate of *D* in this way. At this point in the solution, a few candidates would have benefitted from rereading the question to check which equation they were trying to find. While many candidates went on to state an acceptable form of the correct equation, a few found the equation of *CD* or *AB* rather than the perpendicular bisector. These candidates usually omitted to find the mid-point of *CD*. Some candidates used an incorrect gradient, after finding the mid-point correctly.

Answer: $y = -\frac{3}{2}\left(x - \frac{13}{2}\right)$

Question 6

(i) This part was almost universally correct. A few candidates omitted brackets and stated $\ln y = \ln A + \ln bx$. This was penalised unless there was clear evidence that $\ln y = \ln A + (\ln b)x$ was intended.

(ii) Many candidates would have benefitted from rereading the question as the equation of the line of best fit was often omitted. Some candidates needed to take more care with the form of the line of best fit as, after stating $Y = 1.4X + 2.2$, they often stopped and did not replace *Y* with ln*y*. Many candidates did not round their correct values of *A* and *b* to 1 significant figure, as required. A few candidates used points which were not on the line, commonly (0.8, 3.4), to find the value of *m* and/or *c*. This was not permitted. Some candidates needed to take more care with reading the scale when reading the value of the *y*-intercept as it was often stated as 2 or 2.1. Occasionally the value of the intercept was calculated using their gradient instead of reading it from the graph. This introduced an unnecessary opportunity to make an error. Weaker candidates tended to confuse *A* with ln*A* and *b* with ln*b*. A few weaker candidates anti-logged by incorrectly using the base 10 or worked with lg throughout, instead of ln.

(iii) The simplest method of solution for this part was to use the graph to find $\ln y = 6$ when $x = 2.7$ and then anti-log. This method was not dependent on having the correct values for *A* and *b*. A few candidates did this, although most used the exponential or logarithmic equation they had found.

Answers: **(i)** $\ln y = \ln A + x\ln b$ **(ii)** $\ln y = 1.4x + 2.2$; $A = 9$, $b = 4$ **(iii)** $y = 400$

Question 7

(i) A good number of candidates earned full marks for this part. A few made slips with the 2 or the $\frac{1}{2}$ when applying the chain rule to $\sqrt{x^2+1}$, but this was not common. A few candidates were unable to manipulate their unsimplified answer to the form required. Again, this was not common.

(ii) This was very well answered with a high proportion of candidates earning both marks. A few candidates gave at least a partially correct explanation. Some candidates gave more comment than was required and made an error. Those candidates who kept their solutions simple and stated that $\frac{dy}{dx} = 0$ at a stationary point and that it was not possible for $2x^2$ to be –1, were the most successful.

Answer: **(i)** $\frac{dy}{dx} = \frac{2x^2+1}{(x^2+1)^{\frac{1}{2}}}$

Question 8

(i) This part was almost universally correct.

(ii) A good number of correct solutions were seen. Some candidates misunderstood the ratio and attempted to work with $\overrightarrow{AC} = 3\overrightarrow{CB}$. Candidates who formed a proportion $\frac{AC}{CB} = \frac{1}{3}$ usually avoided this error. A good proportion of candidates were able to form a correct vector route to find $\overrightarrow{OC}$. Some candidates, again, should have reread the question as, having found $\overrightarrow{OC}$ they did not complete the solution. Most candidates who attempted to find the unit vector were successful. A few candidates multiplied by the magnitude of the vector, instead of dividing by it. A few other candidates seemed to think the magnitude was the unit vector.

(iii) Again, a good number of correct answers were seen. Some formed $\overrightarrow{DA}$ and then negated it. Others used the route $\overrightarrow{OD} - \overrightarrow{OA}$ successfully. Weaker candidates often misread their own writing, using $\overrightarrow{OD}$ as $\overrightarrow{AD}$ or omitted to understand that $\overrightarrow{AD}$ was a fractional part of $\overrightarrow{OA}$ as they multiplied by λ only. Candidates who were unable to successfully interpret the ratio in **Part (ii)** generally repeated the error in this part.

Answers: **(i)** $4\mathbf{i} - 16\mathbf{j}$ **(ii)** $\frac{3\mathbf{i} + 8\mathbf{j}}{\sqrt{73}}$ **(iii)** $\frac{-\lambda}{\lambda+1}(2\mathbf{i} + 12\mathbf{j})$

Question 9

(a) (i) The simplest explanations offered were based upon each x is mapped to a unique y, therefore a function, and the function being many-one, therefore no inverse. Most candidates were able to state a satisfactory reason to explain why the function had no inverse but very few candidates justified the mapping being a function. Weaker candidates commented that as it had input and output it was a function or tried to find the inverse function and comment on issues with the domain. This was not accepted as the inverse did not exist in this case. A few candidates were unclear in their comments as to whether they were considering g or its inverse. It was not uncommon to suggest 'it is one-many', for example.

(ii) An excellent number of correct expressions were stated for the composite function and its domain. A few candidates needed to take more care as $6(6x^4 + 5) + 5$ was not an uncommon incorrect answer amongst those seen. Weaker candidates occasionally stated the answer $(6x^4 + 5)^2$.

(iii) This part proved challenging for many. A reasonable number of correct answers were seen, but the most common incorrect answer offered was 5.

(iv) Again, this proved to be challenging. Some candidates misinterpreted the phrase *For this value of k*. Rather than understanding that this required the negative fourth root to be taken, many found the value of their expression for the inverse of h when x was either 5 or 0. Some candidates remembered to take ± the fourth root. Most of these omitted to discard the positive root, however. Most candidates earned 2 marks and this was usually for giving an answer of $\mathrm{h}^{-1}(x) = \sqrt[4]{\frac{x-5}{6}}$.

(b) (i) A good number of correct answers were seen. A few candidates made a slip with the inequality sign and $p \leqslant 2$ was not uncommon from these candidates. Weaker candidates offered $p > 5$ or similar or simply stated p was real.

(ii) Candidates who took care with the graph of $y = \mathrm{p}(x)$, making sure that the y-intercept and the asymptote were correct before reflecting in the line $y = x$ usually earned all 3 marks. Those who drew the asymptotes on the diagram were more successful than those who did not. This was often the feature that was missing, with many curves tending to the x-axis and the y-axis. A good number of candidates sketched a graph of the correct exponential shape and usually indicated that they understood that all that was required to sketch the inverse function was to reflect it in the given line. Some candidates unnecessarily found the rule for the inverse function and this was, on occasion, unhelpful, as the graphs they sketched were not symmetrical. A few candidates ignored the given line and reflected their p(*x*) in the *x*-axis.

(iii) This part of the question was very well answered, with almost all candidates understanding the connection between the graphs and the equation given.

Answers: **(a)(ii)** $6(6x^4+5)^2+5$; all real x **(iii)** 0 **(iv)** $\mathrm{h}^{-1}(x)=-\sqrt[4]{\frac{x-5}{6}}$ **(b)(i)** $p>2$

Question 10

(i) Many fully correct answers were seen to this part of the question. Most candidates used the suggested substitution and found a pair of values for *u*. This usually resulted in a fully correct solution. A few candidates square-rooted their values for *u*, instead of squaring them or simply restated them as the values of *x*. Occasional slips were made in simplifying the initial equation. Many candidates who did this stated no method of solution, solving using their calculator. These candidates were penalised. It is important to show how the solutions to any quadratic equation have been found.

(ii) Candidates used various approaches to answer this part. Most commonly the difference between the area under the curve and the area of the trapezium was attempted. A good number of candidates offered fully correct and complete solutions, showing all key method steps, as required. Some candidates should take care with the accuracy of working values. These candidates made premature approximation errors, rounding their areas to 2 or 3 significant figures, before calculating their final difference of areas. The final answer was often given to 2 significant figures, when at least 3 were required. However, in this case, it was possible to work with exact values and state the exact answer. Those who insist upon rounding should write down a more accurate answer, before attempting to round, to avoid a possible loss of accuracy mark. Only a few candidates stated the integral they were attempting to find without any integration of terms being seen or without a difference of values, such as F(4) – F(0.25), being found. Candidates who worked out the difference of the expressions and then integrated were unlikely to make rounding errors. However, some candidates did not choose the correct values from **Part (i)** for their upper and lower limits. This error was also compounded by candidates who integrated to find the area under the line instead of using $\frac{1}{2}(a+b)\times h$.

Answers: **(i)** *A*(0.25, 3.75), *B*(4, 15) **(ii)** 2.8125

Question 11

(a) A good number of candidates understood the need to simplify the expression before integrating and were able to do this successfully. Some of these candidates thought that $x^2\times x^6=x^{12}$. It was a requirement that candidates stated the constant of integration in their answer. Many candidates omitted it and were penalised in this part of the question. The very weakest candidates attempted to integrate each term in the numerator and the denominator offering answers such as

$$\frac{\frac{x^3}{3}\left(\frac{x^7}{7}+x\right)}{\frac{x^7}{7}}.$$

(b)(i) This was well answered with almost all candidates stating the answer as a multiple of $\sin(4\theta-5)$. A few candidates had clearly differentiated but most candidates were fully correct in their answer. A few candidates earned M1 only for having the negative of the correct answer or for multiplying by 4.

(ii) A reasonable number of fully correct solutions were seen. As candidates were directed to use the previous part of the question to answer this part, they needed to show full method to indicate that they had done so. Many candidates earned the method mark for a correct substitution of limits which was shown. Some candidates needed to take more care with the accuracy of their final answer. These candidates should, perhaps, write down a more accurate answer to the question before attempting to round. This may avoid the loss of an accuracy mark. Other candidates needed to take care over their presentation as they omitted brackets. A few candidates were working in degrees, which was not valid here. These candidates may do better if they checked the mode of their calculator at the start of each question. A few candidates gave a choice of answers in degrees or radians. This was not condoned.

Answers: **(a)** $\frac{x^3}{3} - \frac{1}{3x^3} + c$ **(b)(i)** $\frac{\sin(4\theta - 5)}{4}\,(+c)$ **(ii)** 0.0353

Cambridge Assessment
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Grade thresholds – March 2019

Cambridge IGCSE™ Additional Mathematics (0606)

Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the March 2019 examination.

		minimum raw mark required for grade:				
	maximum raw mark available	A	B	C	D	E
Component 12	80	66	49	32	25	18
Component 22	80	62	49	37	32	27

Grade A* does not exist at the level of an individual component.

The maximum total mark for this syllabus, after weighting has been applied, is **160**.

The overall thresholds for the different grades were set as follows.

Option	Combination of Components	A*	A	B	C	D	E
AY	12, 22	148	128	98	69	57	45

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ADDITIONAL MATHEMATICS **0606/12**

Paper 12 **March 2019**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

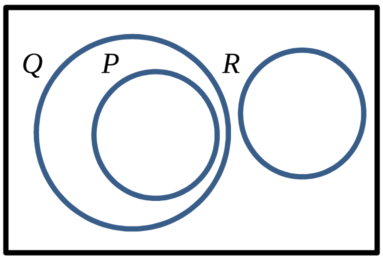
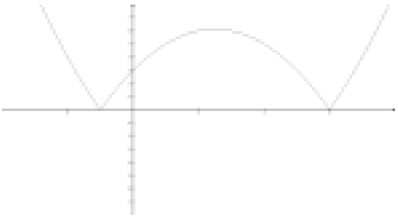
Types of mark

M	Method marks, awarded for a valid method applied to the problem.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
B	Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1(a)(i)	6	**B1**	
1(a)(ii)	1	**B1**	
1(b)		**2**	**B1** for *P* contained within *Q* **B1** for *Q* and *R* separate
1(c)	$S' \cap T'$ or $(S \cup T)'$ oe	**B1**	
	$(X \cap Y) \cup (X \cap Z)$ or $X \cap (Y \cup Z)$ oe	**B1**	
2		**4**	**B1** for general shape with maximum point in 1st quadrant **B1** for $\left(-\frac{1}{2}, 0\right)$ and $(3, 0)$ soi **B1** for $(0, 3)$ soi **B1 dep** on first B1, with cusps and correct shape for $x < -\frac{1}{2}$ and $x > 3$
3(i)	$729 - 162x + 15x^2$	**3**	**B1** for 729 **B1** for $-162x$ **B1** for $15x^2$ Mark final answer
3(ii)	$(729 - 162x + 15x^2)\left(x^2 - 4 + \frac{4}{x^2}\right)$	**B1**	for expansion of $\left(x - \frac{2}{x}\right)^2$
	Term independent of $x = -2916 + 60$	**M1**	for attempt to find independent term, must be considering 2 products using *their* answer to part **(i)**
	$= -2856$	**A1**	
4(i)	$p'(x) = 6x^2 + 2ax + b$	**B1**	for $p'(x) = 6x^2 + 2ax + b$
	$p'(-3) = 54 - 6a + b$, $= -24$ leading to $6a - b = 78$	**B1**	must be convinced of correct substitution and simplification AG

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Question	Answer	Marks	Partial Marks
4(ii)	$\text{p}\left(\frac{1}{2}\right): \frac{2}{8}+\frac{a}{4}+\frac{b}{2}-49=0$	**M1**	for attempt at $\text{p}\left(\frac{1}{2}\right)$ equated to 0
	$6a-b=78$ $a+2b=195$ oe	**M1**	**M Dep** on previous M for attempt to solve both equations
	leading to $a=27$	**A1**	
	$b=84$	**A1**	
4(iii)	$(2x-1)(x^2+14x+49)$	**2**	**M1** for factorisation by observation or by long division
4(iv)	$(2x-1)(x+7)^2$	**B1**	
5(i)	$\log_4 16+\log_4 p$	**M1**	for dealing with product correctly
	$2+p$	**A1**	
5(ii)	$7\log_4 x-\log_4 256$	**M1**	for dealing with power and division correctly
	$7p-4$	**A1**	
5(iii)	$2+p-(7p-4)=5$ leading to $p=\frac{1}{6}$	**M1**	for use of parts **(i)** and **(ii)** to obtain a value for *p*
	so $x=4^{\frac{1}{6}}$	**M1**	for correct attempt to deal with $\log_4$ in order to obtain *x*
	$x=1.26$	**A1**	
6(a)	**BA** and **CB**	**2**	**B1** for one correct product of 2 matrices **B1** for a second correct product of 2 matrices, with no other incorrect products
6(b)(i)	$\frac{1}{16}\begin{pmatrix} 3 & 2 \\ -5 & 2 \end{pmatrix}$ oe	**2**	**B1** for $\frac{1}{16}$ soi **B1** for $\begin{pmatrix} 3 & 2 \\ -5 & 2 \end{pmatrix}$

Question	Answer	Marks	Partial Marks
6(b)(ii)	$\mathbf{X}^{-1}\mathbf{XZ} = \mathbf{X}^{-1}\mathbf{Y}$ $\mathbf{Z} = \frac{1}{16}\begin{pmatrix} 3 & 2 \\ -5 & 2 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ 2 & 0 \end{pmatrix}$	**M1**	for pre-multiplication by *their* inverse matrix
	attempt at matrix multiplication	**M1**	**M1 Dep** on previous M mark, must have at least 2 correct elements
	$\mathbf{Z} = \frac{1}{16}\begin{pmatrix} 16 & 3 \\ -16 & -5 \end{pmatrix}$ oe	**A1**	
7(i)	Area = $\frac{1}{2}\left(8+6\sqrt{5}\right)\left(10-2\sqrt{5}\right)$	**M1**	for a correct method of finding the area of the trapezium
	$=10+22\sqrt{5}$	**A2**	**A1** for 10 with sufficient working seen **A1** for $22\sqrt{5}$ with sufficient working seen
7(ii)	$\cot\theta = \frac{4}{10-2\sqrt{5}}$	**B1**	
	$= \frac{4\left(10+2\sqrt{5}\right)}{\left(10-2\sqrt{5}\right)\left(10+2\sqrt{5}\right)}$	**M1**	for attempt to rationalise an expression for $\cot\theta$, some evidence of expansion must be seen
	$=\frac{1}{2}+\frac{\sqrt{5}}{10}$	**A1**	
8(a)(i)	0	**B1**	
8(a)(ii)	Area under curve = $\frac{1}{2}(2\times10)+(4\times10)+\frac{1}{2}(10+20)\times4$	**M1**	for attempt to find the total area under the graph
	= 110	**A1**	
8(b)(i)	When $t=\frac{7\pi}{12}$, $v=-2.5$	**M1**	for substitution of $t=\frac{7\pi}{12}$ and correct attempt to evaluate
	Speed = 2.5	**A1**	must be positive
8(b)(ii)	$a=6\cos 2t$	**M1**	for differentiation to get acceleration, must be of the form $m\cos 2t$
	When acceleration = 0, $\cos 2t = 0$	**M1**	**M Dep** on previous M mark for equating to zero and correct attempt to solve to get a solution in radians.
	$t=\frac{\pi}{4}$ or 0.785	**A1**	

Question	Answer	Marks	Partial Marks
9(i)	$\frac{1}{2}r^2\theta = 36$ $\theta = \frac{72}{r^2}$	**M1**	for use of the area of the sector
	$P = 2r + r\theta$	**M1**	for attempt to find *P* making use of the area
	$P = 2r + \frac{72}{r}$	**A1**	for attempt to simplify to obtain AG
9(ii)	$\frac{\mathrm{d}P}{\mathrm{d}r} = 2 - \frac{72}{r^2}$	**M1**	for attempt to differentiate to obtain the form $a + \frac{b}{r^2}$ and equate to zero
	When $\frac{\mathrm{d}P}{\mathrm{d}r} = 0,\ r = 6$	**A1**	
	$P = 24$	**A1**	
	$\frac{\mathrm{d}^2P}{\mathrm{d}r^2} = \frac{144}{r^3}$ positive so minimum	**B1**	**FT** on *their* positive *r*, for a correct method to determine the nature of the stationary point leading to a correct conclusion. If the second derivative is evaluated, it must be correct for *their r*.
10(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2\mathrm{e}^{2x} + 3x\ \ (+c)$	**2**	**M1** for attempt to integrate to obtain the form $m\mathrm{e}^{2x} + nx$ **A1** all correct
	$c = 8$	**M1**	**M1 Dep** on previous M mark for attempt to get *c*
	$y = \mathrm{e}^{2x} + \frac{3x^2}{2} + 8x\ (+d)$	**2**	**M1** for attempt to integrate again to obtain the form $pe^{2x} + qx^2\,(+rx)$ **A1** all correct, **FT** on *their* $k\mathrm{e}^{2x}$ and *their c*
	$d = –6$	**M1**	**M1 Dep** on previous M mark for attempt to get *d*
	$y = \mathrm{e}^{2x} + \frac{3x^2}{2} + 8x - 6$	**A1**	

Question	Answer	Marks	Partial Marks
10(ii)	When $x = \frac{1}{4}$, $y = -2.26$ $\frac{dy}{dx} = 12.0$	**M1**	for attempt to obtain both y and $\frac{dy}{dx}$ using *their* work from **(i)**
	$y + 2.26 = -\frac{1}{12}\left(x - \frac{1}{4}\right)$	**2**	**M1 Dep** on previous M mark for attempt to obtain the equation of the normal **A1** allow unsimplified, must be using correct accuracy or exact equivalents.
11(a)	$2\sin x\left(\cos^2 x - 1\right) = 0$	**M1**	for obtaining in terms of sin and cos to obtain one solution correctly
	$\sin x = 0,\ \ x = 0^o,\ 180^o$	**B1**	for $x = 0^o$, 180^o and no other in the given range for the solution of this equation
	$\cos x = \pm\frac{1}{\sqrt{2}},\ x = 45^o,\ 135^o$	**A1**	for $x = 45^o$, 135^o and no other in the given range for the solution of this equation
11(b)(i)	$\frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}$	**M1**	for dealing with cot and sec
	$\frac{\cos^2\theta}{\cos\theta}$	**M1**	for correct use of identity
	$\cos\theta$	**A1**	for all correct working to gain AG
11(b)(ii)	$\cos 3\theta = \frac{1}{2}$ $\theta = \frac{5\pi}{9}$ or $\frac{\pi}{9}$	**M1**	for use of part **(i)** and attempt to solve correctly to obtain a positive angle, may be implied by one correct solution
	$\theta = -\frac{5\pi}{9}$ or $-\frac{\pi}{9}$	**M1**	for use of part **(i)** and attempt to solve correctly to obtain a negative angle, may be implied by one correct solution
	$\theta = \pm\frac{\pi}{9},\ \pm\frac{5\pi}{9}$	**A2**	**A1** for one correct pair of solutions **A1** for a second pair of solutions with no extra solutions within the range

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ADDITIONAL MATHEMATICS **0606/22**

Paper 22 **March 2019**

MARK SCHEME

Maximum Mark: 80

Published

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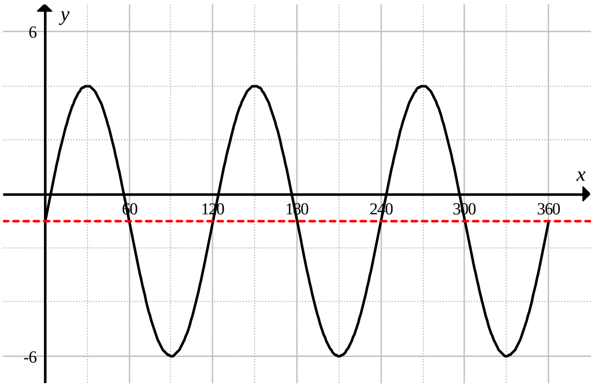
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1(i)	1081575	**B1**	
1(ii)	40320	**B1**	
1(iii)	2730	**B1**	
2(i)	$\frac{\mathrm{d}}{\mathrm{d}x}(\ln x)=\frac{1}{x}$, $\frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{e}^x)=\mathrm{e}^x$ soi	**B2**	**B1** for each
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{e}^x\times \mathit{their}\frac{1}{x}-(\ln x)\times \mathit{their}\,\mathrm{e}^x}{(\mathrm{e}^x)^2}$	**M1**	
	correct completion to given answer, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1-x\ln x}{x\mathrm{e}^x}$	**A1**	
2(ii)	$\delta y=\left(\frac{1-2\ln 2}{2\mathrm{e}^2}\right)\times h$ soi	**M1**	
	–0.0261[…]*h* isw	**A1**	
3(i)	Fully correct curve 	**B3**	**B1** for correct shape for sine with *y*-intercept at –1 **B1** for curve with period 120° **B1** for curve with amplitude 5 Maximum of 2 marks if not fully correct.
3(ii)	$a=-1$ $b=5$ $c=3$	**B2**	**B1** for any 2 correct
4(a)	Expands, rearranges to form a 3-term quadratic on one side $4x^2+x-3[*0]$	**M1**	
	Critical values $\frac{3}{4}$ and –1	**A1**	
	$-1\leqslant x\leqslant\frac{3}{4}$ final answer	**A1**	**FT** *their* critical values

Question	Answer	Marks	Partial Marks
4(b)	$k^2-4\left(\frac{1}{4}\right)\left(k^2+1\right)$	**M1**	
	–1	**A1**	
	discriminant independent of *k* and negative oe	**A1**	**FT** *their* –1
5	$\left[m_{AB}=\right]\frac{2+4}{3-7}$ oe or $-\frac{3}{2}$ soi	**M1**	
	$\left[m_{CD}=\right]$ *their* $\frac{2}{3}$ oe, soi	**M1**	
	their $\frac{2}{3}=\frac{3+3}{k-2}$ oe or 3 + 3 = *their* $\frac{2}{3}(x-2)$ oe	**M1**	
	$k = 11$ nfww	**A1**	
	$\left(\frac{(their\,11)+2}{2},\frac{3+-3}{2}\right)$ oe	**M1**	
	$y=-\frac{3}{2}(x-6.5)$ oe isw	**A1**	**FT** *their* m_{AB} and (*their* 6.5, 0)
6(i)	Takes logs, to any base, of both sides and applies the addition/multiplication law for logs $\ln y=\ln(Ab^x)\Rightarrow\ln y=\ln A+\ln b^x$	**M1**	
	$\Rightarrow\ln y=\ln A+x\ln b$	**A1**	
6(ii)	ln *y* = 1.4*x* + 2.2 oe or ln *y* = *x*ln 4 + ln 9 oe	**B2**	**B1** for either *m* = 1.4 or ln *b* = 1.4 or *c* = 2.2 or ln *A* = 2.2
	$[A=\mathrm{e}^{their\,2.2}=]\,9$ and $[b=\mathrm{e}^{their\,1.4}=]\,4$	**B2**	**FT** *their* 2.2 and *their* 1.4 **B1 FT** for $A=\mathrm{e}^{their\,2.2}$ or $b=\mathrm{e}^{their\,1.4}$ or correct FT decimal rounded to more than 1 sf
6(iii)	$\ln y=6$ or $y=their\,9(their\,4^{2.7})$ or $y=\mathrm{e}^{their\,2.2}(\mathrm{e}^{their\,1.4\times2.7})$ or $\ln y=their\,1.4(2.7)+their\,2.2$ or $\ln y=(2.7)\ln(their\,4)+\ln(their\,9)$	**M1**	
	awrt 400 correct to 1 sf	**A1**	

Question	Answer	Marks	Partial Marks
7(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{x^2+1}\right)=\frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}}\times 2x$	**B2**	**B1** for $\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{x^2+1}\right)=kx\left(x^2+1\right)^{-\frac{1}{2}}$ where $k \neq 1$
	$\sqrt{x^2+1}$ $+x\times \mathit{their}\left(\frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}}\times 2x\right)$	**M1**	
	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]\frac{2x^2+1}{\left(x^2+1\right)^{\frac{1}{2}}}$ or $a = 2, b = 1, p = \frac{1}{2}$ nfww	**A1**	
7(ii)	Complete argument e.g. For stationary points $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and when a and b are positive, ax^2+b cannot be 0 or $2x^2$ cannot be -1	**B2**	**FT** *their* positive a and b **B1 FT** for a partially correct argument e.g. Because $\frac{\mathrm{d}y}{\mathrm{d}x}$ cannot be 0.
8(i)	$6\mathbf{i}-4\mathbf{j}-\left(2\mathbf{i}+12\mathbf{j}\right)$ oe	**M1**	
	$4\mathbf{i}-16\mathbf{j}$ oe, isw	**A1**	
8(ii)	$\left[\overrightarrow{OC}=\right]\overrightarrow{OA}+\frac{1}{4}\overrightarrow{AB}$ oe or $\left[\overrightarrow{OC}=\right]\overrightarrow{OB}-\frac{3}{4}\overrightarrow{AB}$ oe or $\left[\overrightarrow{OC}=\right]\frac{1}{4}\overrightarrow{OB}+\frac{3}{4}\overrightarrow{OA}$ oe or $3(x-2)=6-x$ and $3(y-12)=-4-y$	**M1**	
	$3\mathbf{i}+8\mathbf{j}$ oe	**A1**	
	$\left\lvert\overrightarrow{OC}\right\rvert=\sqrt{\mathit{their}3^2+\mathit{their}8^2}$	**M1**	
	$\mathit{their}\,\frac{3\mathbf{i}+8\mathbf{j}}{\sqrt{73}}$	**A1**	**FT** *their* $3\mathbf{i}+8\mathbf{j}$ and *their* $\sqrt{73}$
8(iii)	$-\frac{\lambda}{1+\lambda}\left(2\mathbf{i}+12\mathbf{j}\right)$ oe, isw	**B2**	**B1** for $\frac{\lambda}{1+\lambda}\left(2\mathbf{i}+12\mathbf{j}\right)$ seen or $\overrightarrow{OD}=\frac{1}{1+\lambda}\left(2\mathbf{i}+12\mathbf{j}\right)$ oe

Question	Answer	Marks	Partial Marks
9(a)(i)	Valid explanation e.g. Each *x* is mapped to a unique value of *y* [and so g is a function] but the inverse does not exist because it is many to one oe	**B2**	**B1** for either each *x* is mapped to a unique value of *y* oe or for inverse does not exist because it is many to one oe
9(a)(ii)	$\left[g^2(x)=\right]\quad 6(6x^4+5)^4+5$ isw for all real *x*	**B2**	**B1** for $\left[g^2(x)=\right]\quad 6(6x^4+5)^4+5$ isw **B1** for correct domain
9(a)(iii)	[*k* =] 0	**B1**	
9(a)(iv)	$x^4=\frac{y-5}{6}$ soi	**M1**	or $y^4=\frac{x-5}{6}$
	$x=\pm\sqrt[4]{\frac{y-5}{6}}$	**A1**	or $y=\pm\sqrt[4]{\frac{x-5}{6}}$
	$h^{-1}(x)=-\sqrt[4]{\frac{x-5}{6}}$	**A1**	If **M1 A0 A0**, allow **SC1** for an answer of $h^{-1}(x)=\sqrt[4]{\frac{x-5}{6}}$ or $y=\sqrt[4]{\frac{x-5}{6}}$
9(b)(i)	p > 2	**B1**	
9(b)(ii)	For p: Correct exponential shape tending to $y=2$ passing through (0, 5)	**B2**	**B1** for each
	For the inverse function: Approximate reflection of p in the dotted line passing through (*their* 5, 0)	**B1**	
9(b)(iii)	Valid explanation e.g. The graphs do not intersect and so there are no solutions oe	**B1**	
10(i)	Eliminates *x* or *y* e.g. $3x+3=x+5\sqrt{x}+1$ or $3+3u^2=u^2+5u+1$	**M1**	
	Rearranges to a 3-term quadratic e.g. $0=2x-5\sqrt{x}+2$ or $0=2u^2-5u+2$	**A1**	
	Factorises or solves $0=2x-5\sqrt{x}+2$ oe or $0=2u^2-5u+2$ oe	**M1**	
	$\sqrt{x}=0.5$, $\sqrt{x}=2$ or $u=0.5$, $u=2$	**A1**	

Question	Answer	Marks	Partial Marks
	$A(0.25, 3.75)$ $B(4, 15)$ oe	**A2**	**A1** for each or for $x = 0.25$ and $x = 4$

Question	Answer	Marks	Partial Marks
10(ii)	**Method 1: Finding the area of the trapezium and subtracting**		
	Valid method to find the area of the trapezium soi	**M1**	
	$\frac{1125}{32}$ or $35\frac{5}{32}$ or 35.2 or 35.15625 rot to 4 or more figs, soi	**A1**	
	Attempts to integrate $\int_{their\,0.25}^{their\,4}(x+5\sqrt{x}+1)\,\mathrm{d}x\ [-their\,35.2]$	**M1**	
	$\left[\frac{x^2}{2}+\frac{5x^{\frac{3}{2}}}{\frac{3}{2}}+x\right]_{their\,0.25}^{their\,4}$ $[-their\,35.2]$ oe	**A1**	
	F(*their* 4) – F(*their* 0.25) $[-their\,35.2]$	**M1**	
	$\frac{45}{16}$ or $2\frac{13}{16}$ or 2.8125 isw or 2.81, or 2.812	**A1**	
	Method 2: Finding the difference of two integrals		
	Attempts to integrate $\int_{their\,0.25}^{their\,4}(x+5\sqrt{x}+1-(3+3x))\,\mathrm{d}x$ or $\int_{their\,0.25}^{their\,4}(-2x+5\sqrt{x}-2)\,\mathrm{d}x$ oe	**M2**	**M1** for an attempt to form the difference with at most one error and attempts to integrate
	$\left[their\left(\frac{-2x^2}{2}+\frac{5x^{\frac{3}{2}}}{\frac{3}{2}}-2x\right)\right]_{their\,0.25}^{their\,4}$ oe	**A1**	**FT** dep on at least M1 already awarded; must be at least 3 terms and, if FT, must be of equivalent difficulty
	F(*their* 4) – F(*their* 0.25)	**M1**	
	$\frac{45}{16}$ or $2\frac{13}{16}$ or 2.81, 2.812 or 2.8125	**A2**	

Question	Answer	Marks	Partial Marks
11(a)	$\frac{x^2(x^6+1)}{x^6}=x^2+\frac{1}{x^4}$ soi	**B1**	
	$\frac{x^3}{3}+\frac{x^{-3}}{-3}+c$ oe, isw	**B2**	**B1** for any two out of three terms correct
11(b)(i)	$k\sin(4\theta-5)$ where $k>0$ or $k=-\frac{1}{4}$	**M1**	
	$\frac{\sin(4\theta-5)}{4}$ $(+c)$	**A1**	
11(b)(ii)	$\frac{\sin(4(2)-5)}{4}-\frac{\sin(4(1.25)-5)}{4}$ or $\frac{\sin(3)}{4}-\frac{\sin(0)}{4}$	**M1**	**FT** *their* **(b)(i)**, **dep** on **M1** awarded in **(b)(i)**
	0.0353 or 0.03528[…] oe, cao	**A1**	

Cambridge Assessment International Education
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

6715046212

ADDITIONAL MATHEMATICS **0606/12**

Paper 1 **February/March 2019**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **15** printed pages and **1** blank page.

DC (JM/CB) 165270/2

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1} b + \binom{n}{2}a^{n-2} b^2 + \ldots + \binom{n}{r}a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** Given that $\mathscr{E} = \{ x : 1 < x < 20\}$,

$A = \{\text{multiples of } 3\}$,

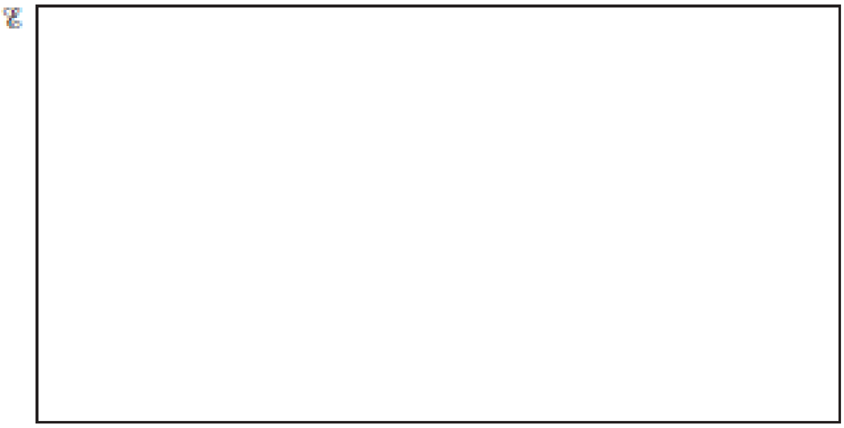
$B = \{\text{multiples of } 4\}$,

find

(i) $\text{n}(A)$, [1]

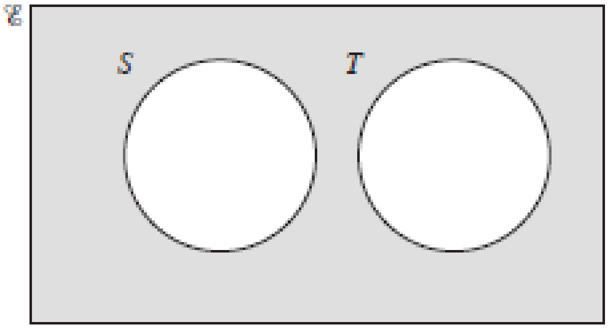
(ii) $\text{n}(A \cap B)$. [1]

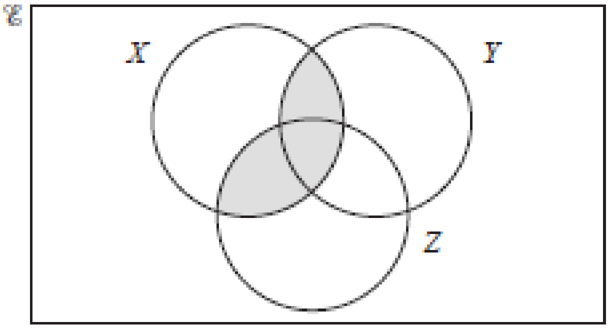
(b) On the Venn diagram below, draw the sets P, Q and R such that $P \subset Q$ and $Q \cap R = \varnothing$.



[2]

(c) Using set notation, describe the shaded areas shown in the Venn diagrams below.





.... [2]

2 On the axes below, sketch the graph of the curve $y = \left|2x^2 - 5x - 3\right|$, stating the coordinates of any points where the curve meets the coordinate axes.

[4]

3 **(i)** Find the first 3 terms in the expansion, in ascending powers of x, of $\left(3-\frac{x}{9}\right)^6$. Give the terms in their simplest form. [3]

(ii) Hence find the term independent of x in the expansion of $\left(3-\frac{x}{9}\right)^6\left(x-\frac{2}{x}\right)^2$. [3]

4 The polynomial $p(x) = 2x^3 + ax^2 + bx - 49$, where a and b are constants. When $p'(x)$ is divided by $x + 3$ there is a remainder of -24.

(i) Show that $6a - b = 78$. [2]

It is given that $2x - 1$ is a factor of $p(x)$.

(ii) Find the value of a and of b. [4]

(iii) Write $p(x)$ in the form $(2x - 1)Q(x)$, where $Q(x)$ is a quadratic factor. [2]

(iv) Hence factorise $p(x)$ completely. [1]

5 It is given that $\log_4 x = p$. Giving your answer in its simplest form, find, in terms of p,

(i) $\log_4(16x)$, [2]

(ii) $\log_4\left(\dfrac{x^7}{256}\right)$. [2]

Using your answers to **parts (i)** and **(ii)**,

(iii) solve $\log_4(16x) - \log_4\left(\dfrac{x^7}{256}\right) = 5$, giving your answer correct to 2 decimal places. [3]

6 **(a)** Given that $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 & -4 \\ 2 & 5 \\ 3 & 1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 & -2 & 0 \end{pmatrix}$, write down the matrix products which are possible. You do not need to evaluate your products. [2]

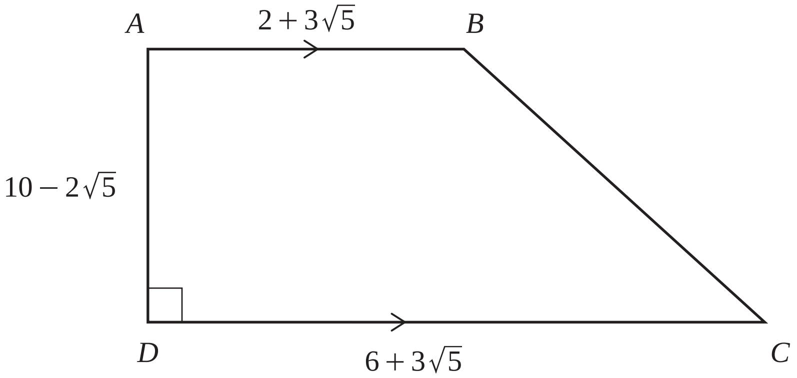
(b) It is given that $\mathbf{X} = \begin{pmatrix} 2 & -2 \\ 5 & 3 \end{pmatrix}$ and $\mathbf{Y} = \begin{pmatrix} 4 & 1 \\ 2 & 0 \end{pmatrix}$.

(i) Find $\mathbf{X}^{-1}$. [2]

(ii) Hence find the matrix **Z** such that $\mathbf{XZ} = \mathbf{Y}$. [3]

7 Do not use a calculator in this question.

All lengths in this question are in centimetres.

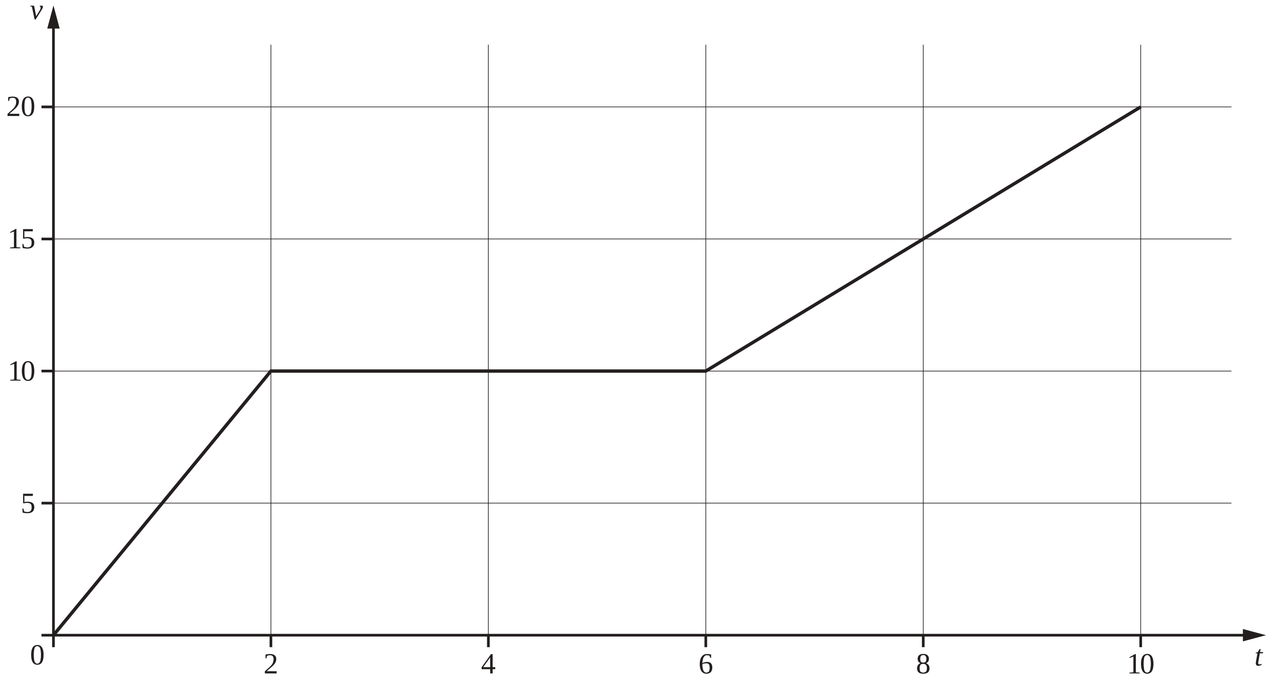


The diagram shows the trapezium *ABCD*, where $AB = 2+3\sqrt{5}$, $DC = 6+3\sqrt{5}$, $AD = 10-2\sqrt{5}$ and angle $ADC = 90°$.

(i) Find the area of *ABCD*, giving your answer in the form $a+b\sqrt{5}$, where a and b are integers. [3]

(ii) Find $\cot BCD$, giving your answer in the form $c+d\sqrt{5}$, where c and d are fractions in their simplest form. [3]

8 **(a)**



The diagram shows the velocity-time graph of a particle P moving in a straight line with velocity $v\,\text{ms}^{-1}$ at time t seconds after leaving a fixed point.

(i) Write down the value of the acceleration of P when $t = 5$. [1]

(ii) Find the distance travelled by the particle P between $t = 0$ and $t = 10$. [2]

(b) A particle Q moves such that its velocity, $v\,\text{ms}^{-1}$, t seconds after leaving a fixed point, is given by $v = 3\sin 2t - 1$.

(i) Find the speed of Q when $t = \frac{7\pi}{12}$. [2]

(ii) Find the least value of t for which the acceleration of Q is zero. [3]

9 The area of a sector of a circle of radius r cm is $36\,\text{cm}^2$.

(i) Show that the perimeter, P cm, of the sector is such that $P = 2r + \frac{72}{r}$. [3]

(ii) Hence, given that r can vary, find the stationary value of P and determine its nature. [4]

10 A curve is such that when $x = 0$, both $y = -5$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 10$. Given that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 4\mathrm{e}^{2x} + 3$, find

(i) the equation of the curve, [7]

(ii) the equation of the normal to the curve at the point where $x = \frac{1}{4}$. [3]

11 **(a)** Solve $\sin x \cos x = \frac{1}{2}\tan x$ for $0° \leqslant x \leqslant 180°$. [3]

(b) **(i)** Show that $\sec\theta - \dfrac{\sin\theta}{\cot\theta} = \cos\theta$. [3]

(ii) Hence solve $\sec 3\theta - \dfrac{\sin 3\theta}{\cot 3\theta} = \dfrac{1}{2}$ for $-\dfrac{2\pi}{3} \leqslant \theta \leqslant \dfrac{2\pi}{3}$, where θ is in radians. [4]

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

ADDITIONAL MATHEMATICS **0606/22**

Paper 2 **February/March 2019**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
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Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
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The total number of marks for this paper is 80.

This document consists of **14** printed pages and **2** blank pages.

DC (KS/TP) 165271/2

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}\,b + \binom{n}{2}a^{n-2}\,b^2 + \ldots + \binom{n}{r}a^{n-r}\,b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

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Identities

$$\sin^2 A + \cos^2 A = 1$$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}\,bc\sin A$$

1 A band can play 25 different pieces of music. From these pieces of music, 8 are to be selected for a concert.

(i) Find the number of different ways this can be done. [1]

The 8 pieces of music are then arranged in order.

(ii) Find the number of different arrangements possible. [1]

The band has 15 members. Three members are chosen at random to be the treasurer, secretary and agent.

(iii) Find the number of ways in which this can be done. [1]

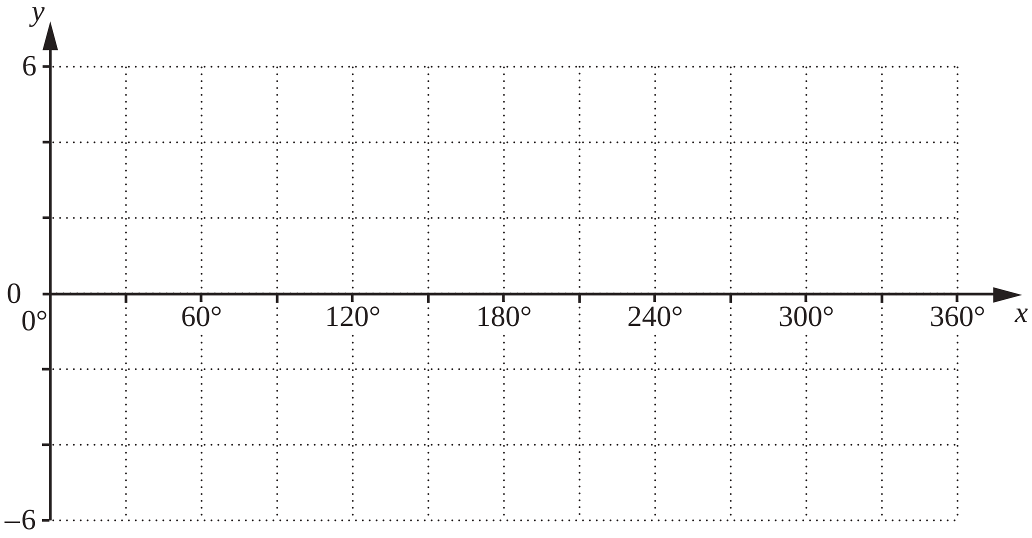
2 Variables x and y are related by the equation $y = \frac{\ln x}{\mathrm{e}^x}$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1 - x\ln x}{x\mathrm{e}^x}$. [4]

(ii) Hence find the approximate change in y as x increases from 2 to $2 + h$, where h is small. [2]

3 The function f is defined, for $0^\circ \leqslant x \leqslant 360^\circ$, by $\mathrm{f}(x) = a + b\sin cx$, where a, b and c are constants with $b > 0$ and $c > 0$. The graph of $y = \mathrm{f}(x)$ meets the y-axis at the point $(0, -1)$, has a period of 120° and an amplitude of 5.

(i) Sketch the graph of $y = \mathrm{f}(x)$ on the axes below. [3]



(ii) Write down the value of each of the constants a, b and c. [2]

a = ... b = ... c = ...

4 **(a)** Find the values of x for which $(2x+1)^2 \leqslant 3x+4$. [3]

(b) Show that, whatever the value of k, the equation $\frac{x^2}{4}+kx+k^2+1=0$ has no real roots. [3]

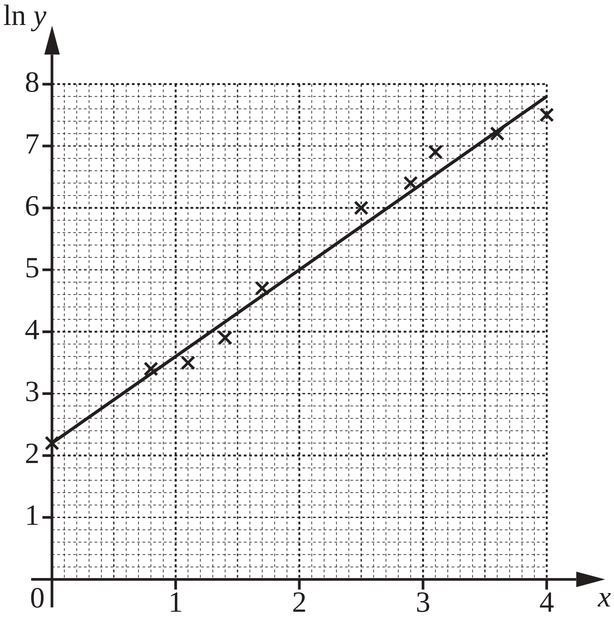
5 Solutions to this question by accurate drawing will not be accepted.

The points $A(3,\ 2)$, $B(7,\ -4)$, $C(2,\ -3)$ and $D(k,\ 3)$ are such that CD is perpendicular to AB. Find the equation of the perpendicular bisector of CD. [6]

6 The relationship between experimental values of two variables, x and y, is given by $y = Ab^x$, where A and b are constants.

(i) Transform the relationship $y = Ab^x$ into straight line form. [2]

The diagram shows $\ln y$ plotted against x for ten different pairs of values of x and y. The line of best fit has been drawn.



(ii) Find the equation of the line of best fit and the value, correct to 1 significant figure, of A and of b. [4]

(iii) Find the value, correct to 1 significant figure, of y when $x = 2.7$. [2]

7 **(i)** Given that $y = x\sqrt{x^2+1}$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{ax^2+b}{(x^2+1)^p}$, where a, b and p are positive constants. [4]

(ii) Explain why the graph of $y = x\sqrt{x^2+1}$ has no stationary points. [2]

8 Relative to an origin O, the position vectors of the points A and B are $2\mathbf{i} + 12\mathbf{j}$ and $6\mathbf{i} - 4\mathbf{j}$ respectively.

(i) Write down and simplify an expression for $\overrightarrow{AB}$. [2]

The point C lies on $\overrightarrow{AB}$ such that $AC : CB$ is $1 : 3$.

(ii) Find the unit vector in the direction of $\overrightarrow{OC}$. [4]

The point D lies on $\overrightarrow{OA}$ such that $OD : DA$ is $1 : \lambda$.

(iii) Find an expression for $\overrightarrow{AD}$ in terms of λ, **i** and **j**. [2]

9 **(a)** It is given that $g(x) = 6x^4 + 5$ for all real x.

(i) Explain why g is a function but does not have an inverse. [2]

(ii) Find $g^2(x)$ and state its domain. [2]

It is given that $h(x) = 6x^4 + 5$ for $x \leqslant k$.

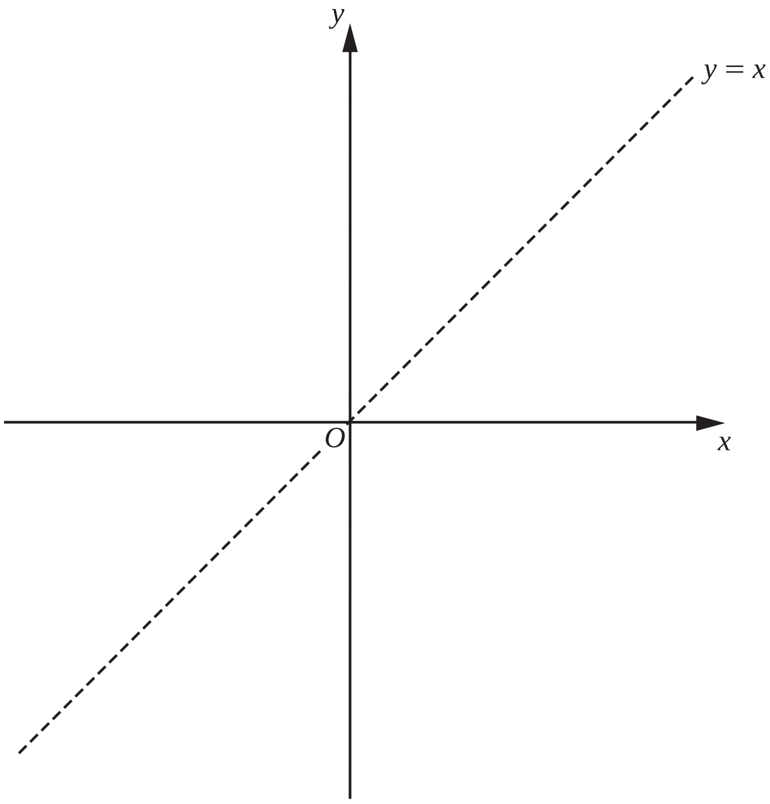
(iii) State the greatest value of k such that h^{-1} exists. [1]

(iv) For this value of k, find $h^{-1}(x)$. [3]

(b) The function p is defined by $p(x) = 3e^x + 2$ for all real x.

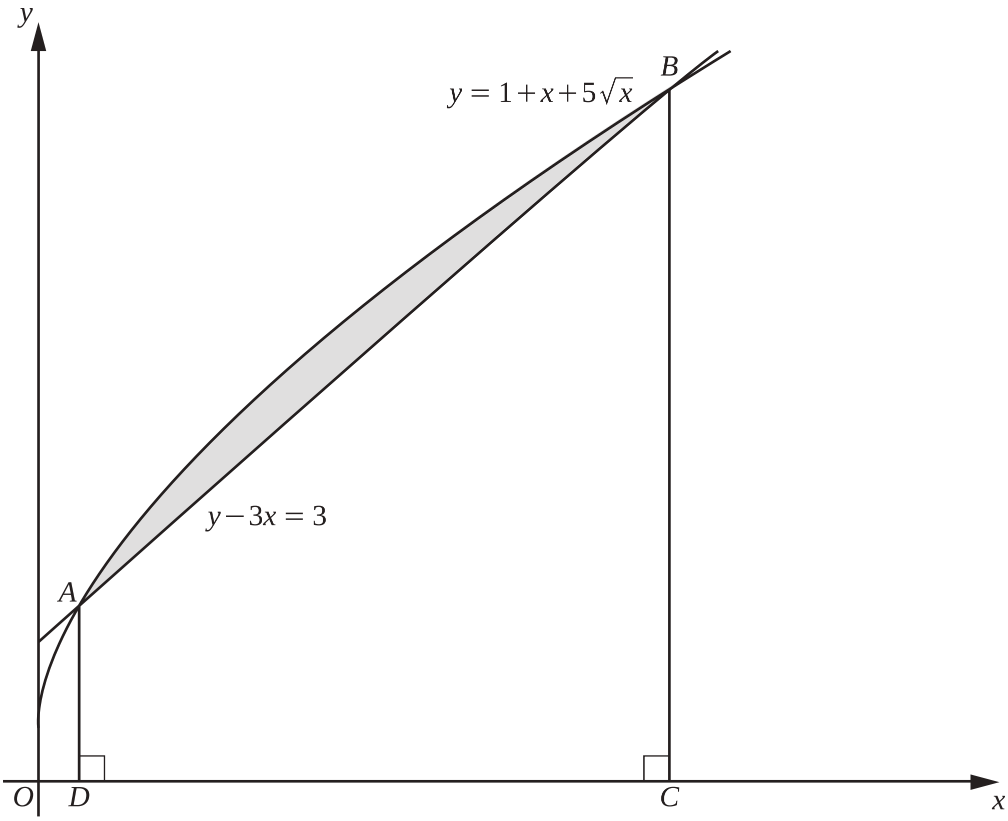
(i) State the range of p. [1]

(ii) On the axes below, sketch and label the graphs of $y = p(x)$ and $y = p^{-1}(x)$. State the coordinates of any points of intersection with the coordinate axes. [3]



(iii) Hence explain why the equation $p(x) = p^{-1}(x)$ has no solutions. [1]

10



The diagram shows the curve $y = 1 + x + 5\sqrt{x}$ and the straight line $y - 3x = 3$. The curve and line intersect at the points A and B. The lines BC and AD are perpendicular to the x-axis.

(i) Using the substitution $u^2 = x$, or otherwise, find the coordinates of A and of B. You must show all your working. [6]

(ii) Find the area of the shaded region, showing all your working. [6]

11 **(a)** Find $\int \frac{x^2(x^6+1)}{x^6}\,\mathrm{d}x$. [3]

(b) **(i)** Find $\int \cos(4\theta - 5)\,\mathrm{d}\theta$. [2]

(ii) Hence evaluate $\int_{1.25}^{2} \cos(4\theta - 5)\,\mathrm{d}\theta$. [2]

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Cambridge Assessment
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Grade thresholds – June 2019

Cambridge IGCSE™ Additional Mathematics (0606)

Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the June 2019 examination.

		minimum raw mark required for grade:				
	maximum raw mark available	A	B	C	D	E
Component 11	80	65	50	34	27	20
Component 12	80	55	43	31	25	19
Component 13	80	64	48	31	25	19
Component 21	80	66	52	39	33	28
Component 22	80	61	49	37	31	25
Component 23	80	58	43	29	23	18

Grade A* does not exist at the level of an individual component.

The maximum total mark for this syllabus, after weighting has been applied, is **160**.

The overall thresholds for the different grades were set as follows.

Option	Combination of Components	A*	A	B	C	D	E
AX	11, 21	145	131	102	73	60	48
AY	12, 22	138	116	92	68	56	44
AZ	13, 23	141	122	91	60	48	37

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ADDITIONAL MATHEMATICS **0606/11**

Paper 1 **May/June 2019**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **12** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

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- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

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- marks are not deducted for omissions
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MARK SCHEME NOTES

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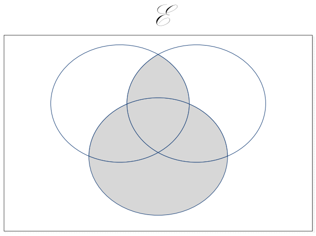
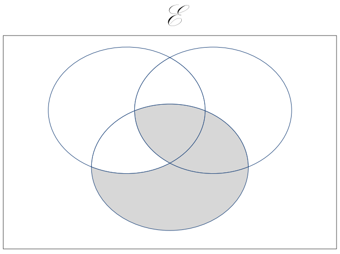
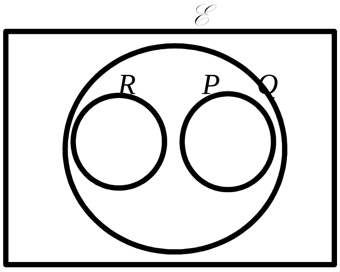
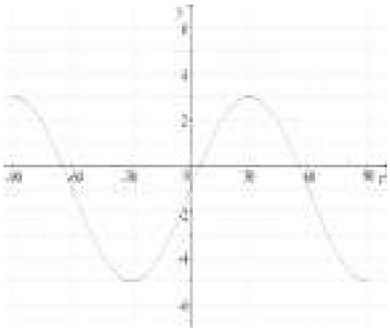
Types of mark

M	Method marks, awarded for a valid method applied to the problem.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
B	Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘**dep**’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(a)		**B1**	
		**B1**	
1(b)		**B2**	**B1** for $P \subset R$ and $Q \subset R$ **B1** for $P \cap Q = \varnothing$
2(i)	4	**B1**	
2(ii)	120° or $\frac{2\pi}{3}$	**B1**	
2(iii)		**B3**	**B1** for a complete curve starting at $\left(-90^{\circ}, 3\right)$ and finishing at $\left(90^{\circ}, -5\right)$ **B1** for $-5 \leqslant y \leqslant 3$ for a complete curve Minimum point(s) at $y = -5$ Maximum point(s) at $y = 3$ **DepB1** for a fully correct sine curve satisfying both the above and passing through $\left(-60^{\circ}, -1\right), \left(0^{\circ}, -1\right)$ and $\left(60^{\circ}, -1\right)$
3(i)	–12	**B1**	
3(ii)	$\left(2\times -3-1\right)\left(k-3\right)-12=23$ oe or $2\left(-3\right)^{2}+\left(2k-1\right)\left(-3\right)-k-12=23$	**M1**	
	$k=-2$	**A1**	

Question	Answer	Marks	Guidance
3(iii)	$(2x-1)(x-2)-12=-25$ $2x^2-5x+15=0$	**M1**	expansion and simplification to a 3 term quadratic equation equated to zero, using *their k*.
	Discriminant: $25-(4\times2\times15)$ $= -95$	**M1**	using discriminant for their three term quadratic equation
	which is < 0 so no real solutions	**A1**	cao for correct discriminant and correct conclusion
4(i)	$a=256$	**B1**	
	$8\times2^7\times bx[=256x]$ oe or $\frac{8\times7\times2^6\times(bx)^2}{2}\left[=cx^2\right]$ oe	**M1**	
	$b=\frac{1}{4}$ oe, $c=112$	**A2**	**A1** for each
4(ii)	$(256+256x+112x^2)\left(4x^2-12+\frac{9}{x^2}\right)$	**B1**	for $\left(4x^2-12+\frac{9}{x^2}\right)$
	Terms independent of x are $(256\times(-12))+(112\times9)$ $=-3072+1008$	**M1**	adding and selecting $(\mathit{their}\,256\times\mathit{their}\,(-12))+(\mathit{their}\,112\times\mathit{their}\,9)$
	$= -2064$	**A1**	
5(i)	$v=20\times\frac{1}{\sqrt{3^2+4^2}}\begin{pmatrix}3\\4\end{pmatrix}$ oe	**M1**	finding and using the magnitude of $\begin{pmatrix}3\\4\end{pmatrix}$
	$v=\begin{pmatrix}12\\16\end{pmatrix}$	**A1**	
5(ii)	$\boldsymbol{r}_p=\begin{pmatrix}1\\2\end{pmatrix}+\begin{pmatrix}12\\16\end{pmatrix}t$	**M1**	correct use of position vector and *their* velocity vector
		A1	

Question	Answer	Marks	Guidance
5(iii)	$\begin{pmatrix}17\\18\end{pmatrix}+\begin{pmatrix}8\\12\end{pmatrix}t=\begin{pmatrix}1\\2\end{pmatrix}+\begin{pmatrix}12\\16\end{pmatrix}t$ Leading to $17+8t=1+12t$ or $18+12t=2+16t$	**M1**	equating position vectors of both particles at time *t* and solve either equation for *t*
	$t=4$	**A1**	
	Position vector of collision $\begin{pmatrix}49\\66\end{pmatrix}$	**A1**	
6	Method 1 $3x^2-2x+1=2x+5$ leading to	**M1**	equating the equations of the line and the curve and rearranging to obtain a three term quadratic equated to zero
	$3x^2-4x-4=0$	**A1**	
	$x=-\frac{2}{3}$ and $x=2$	**A1**	
	$\int_{-\frac{2}{3}}^{2}\left(2x+5-\left(3x^2-2x+1\right)\right)\,\mathrm{d}x$	**M1**	subtraction (either way round)
	$\int_{-\frac{2}{3}}^{2}\left(4+4x-3x^2\right)\,\mathrm{d}x$	**M1**	integration to $Ax+Bx^2+Cx^3$
	$\left[4x+2x^2-x^3\right]_{-\frac{2}{3}}^{2}$	**A1**	for $4x+2x^2-x^3$ oe
	$(8+8-8)-\left(-\frac{8}{3}+\frac{8}{9}+\frac{8}{27}\right)$ $=8--\frac{40}{27}$	**M1**	**Dep** on preceding M1 correct use of limits
	$=\frac{256}{27}$ or 9.48 or $9\frac{13}{27}$	**A1**	

Question	Answer	Marks	Guidance
6	Method 2 $3x^2 - 2x + 1 = 2x + 5$ leading to	**M1**	equating the line and the curve and rearranging to obtain a three term quadratic equated to zero
	$3x^2 - 4x - 4 = 0$	**A1**	
	$x = -\frac{2}{3}$ and $x = 2$	**A1**	
	Area of trapezium = $\frac{1}{2}\left(\frac{11}{3}+9\right)\times\frac{8}{3}$	**B1**	area of the trapezium, allow unsimplified
	Area under curve = $\int_{-\frac{2}{3}}^{2} 3x^2 - 2x + 1\,\mathrm{d}x$	**M1**	integration to $Ax + Bx^2 + Cx^3$
	$=\left[x^3 - x^2 + x\right]_{-\frac{2}{3}}^{2}$	**A1**	for $x^3 - x^2 + x$
	$=\left((8-4+2)-\left(-\frac{8}{27}-\frac{4}{9}-\frac{2}{3}\right)\right)$ $6 - -\frac{38}{27}$	**M1**	**DepM1** for correct use of limits.
	Shaded Area = $\frac{152}{9} - \frac{200}{27}$ $= \frac{256}{27}$ or 9.48 or $9\frac{13}{27}$	**A1**	
7(a)	Method 1 $\log_3 x + \frac{\log_3 x}{\log_3 9} = 12$	**B1**	change to base 3 logarithm
	$\frac{3\log_3 x}{2} = 12$ $x = 3^8$ or $\sqrt[3]{3^{24}}$	**M1**	simplification and dealing with base 3 logarithms to obtain a power of 3
	$x = 6561$	**A1**	

Question	Answer	Marks	Guidance
7(a)	Method 2 $\frac{\log_9 x}{\log_9 3}+\log_9 x=12$	**B1**	change to base 9
	$3\log_9 x=12$ $x=9^4$ or $\sqrt[3]{9^{12}}$	**M1**	simplification and dealing with base 9 logarithms to obtain a power of 9
	$x=6561$	**A1**	
7(b)	Method 1 $\log_4\left(3y^2-10\right)=\log_4\left(y-1\right)^2+\frac{1}{2}$	**B1**	use of power rule
	$\log_4\frac{3y^2-10}{\left(y-1\right)^2}=\frac{1}{2}$	**B1**	**DepB1** for use of division rule
	$\frac{3y^2-10}{\left(y-1\right)^2}=2$	**B1**	for $\frac{1}{2}=\log_4 2$
	$y^2+4y-12=0$	**M1**	**Dep** on first two B marks simplification to a three term quadratic.
	$y=2$ only	**A1**	
7(b)	Method 2 $\log_4\left(3y^2-10\right)=\log_4\left(y-1\right)^2+\frac{1}{2}$	**B1**	use of power rule
	$\log_4\left(3y^2-10\right)=\log_4\left(y-1\right)^2+\log_4 2$	**B1**	for $\log_4 2$
	$3y^2-10=2\left(y-1\right)^2$	**B1**	**Dep** on first B1 use of the multiplication rule
	$y^2+4y-12=0$	**M1**	**Dep** on first and third B marks. simplification to a 3 term quadratic
	$y=2$ only	**A1**	

Question	Answer	Marks	Guidance
8(i)	$\mathrm{f} > -1$	**B1**	or $\mathrm{f}(x) > -1$, $y > -1$, $(-1, \infty)$, $\{y : y > -1\}$
8(ii)	$\mathrm{e}^y = \frac{x+1}{5}$ oe	**M1**	a complete valid method to obtain the inverse function
	$y = \ln\left(\frac{x+1}{5}\right)$ or $\mathrm{f}^{-1}(x) = \ln\left(\frac{x+1}{5}\right)$ oe	**A1**	
	Domain $x > -1$ or $(-1, \infty)$	**B1**	**FT** *their* **(i)** or correct
8(iii)	$\mathrm{g}(1) = 5$ so $\mathrm{fg}(1) = \mathrm{f}(5)$	**M1**	evaluation using correct order of operations
	$5\mathrm{e}^5 - 1 = 741$	**A1**	awrt 741 or $5\mathrm{e}^5 - 1$
8(iv)	$\mathrm{g}^2(x) = (x^2+4)^2 + 4$	**M1**	correct use of g^2
	$x^4 + 8x^2 + 16 + 4 = 40$ $(x^2+4)^2 = 36$ or $x^4 + 8x^2 - 20 = 0$ $(x^2+10)(x^2-2) = 0$	**M1**	**DepM1** for forming and solving a quadratic in x^2
	$x = \pm\sqrt{2}$ only	**A1**	
9(i)	Method 1 $600\pi = 2\pi r^2 + 2\pi rh$	**B1**	
	$h = \frac{600\pi - 2\pi r^2}{2\pi r}$	**M1**	making *h* subject from a two term expression for SA.
	$V = \pi r^2 h$ $V = \pi r^2\left(\frac{600\pi\text{-}2\pi r^2}{2\pi r}\right)$ $V = \pi r^2\left(\frac{300}{r} - r\right)$ $V = 300\pi r - \pi r^3$	**A1**	correct substitution and manipulation to obtain given answer

Question	Answer	Marks	Guidance
9(i)	Method 2 $600\pi = 2\pi r^2 + 2\pi rh$	B1	
	$600\pi\text{r} = 2\pi r^3 + 2\pi r^2 h$	M1	multiplying both sides by r
	$\frac{600\pi\text{r-}2\pi r^3}{2} = \pi r^2 h$ $V = \pi r^2 h$ $V = 300\pi r - \pi r^3$	A1	correct manipulation to obtain $\pi r^2 h$
9(ii)	$\frac{\mathrm{d}V}{\mathrm{d}r} = 300\pi - 3\pi r^2$	M1	differentiation of given formula to $A+Br^2$
	When $\frac{\mathrm{d}V}{\mathrm{d}r} = 300\pi - 3\pi r^2 = 0$	M1	equating to zero and attempt to solve
	$r = 10$	A1	
	$V = 2000\pi$ or 6280 or 6283	A1	
	$\frac{\mathrm{d}^2V}{\mathrm{d}r^2} = -6\pi r$, $\frac{\mathrm{d}^2V}{\mathrm{d}r^2} < 0$ so maximum	B1	cao for $\frac{\mathrm{d}^2V}{\mathrm{d}r^2} = -6\pi r$, $\frac{\mathrm{d}^2V}{\mathrm{d}r^2} = -60\pi$ or other correct method leading to maximum
10(i)	Method 1 $\lg y = A + Bx^2$	B1	statement soi
	$16 = A + 6B$ $4 = A + 2B$	M1	one correct equation
	leading to $A = -2$ and $B = 3$	A2	**A1** for each
10(i)	Method 2 $\lg y = A + Bx^2$	B1	statement soi
	Gradient = B $B = 3$	B1	
	$16 = A + 6B$ or $4 = A + 2B$	M1	a correct equation
	$A = -2$	A1	

Question	Answer	Marks	Guidance
10(i)	<u>Method 3</u> $\lg y - 4 = 3(x^2 - 2)$ or $\lg y - 16 = 3(x^2 - 6)$ OR $4 = 3(2) + c$ or $16 = 3(6) + c$	**M1**	correct equation or for correct method for finding constant.
	$\lg y = A + Bx^2$	**B1**	statement soi by *their A* and *B*
	Hence $y = 10^{3x^2-2}$ $B = 3$	**B1**	
	$A = -2$	**A1**	
10(ii)	$y = 10^{-2+3\left(\frac{1}{\sqrt{3}}\right)^2}$	**M1**	correct use of *their A* and *B*
	$y = 0.1$ oe	**A1**	
10(iii)	$2 = 10^{3x^2-2}$	**M1**	correct use of *their A* and *B*
	$\lg 2 = 3x^2 - 2$ $x = \sqrt{\frac{\lg 2 + 2}{3}}$	**M1**	complete correct method to solve for *x*
	$x = 0.876$	**A1**	

Question	Answer	Marks	Guidance
11(i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\left(x^2+1\right)\left(2x-3\right)^{-\frac{1}{2}}+2x\left(2x-3\right)^{\frac{1}{2}}$	**M1**	differentiation of a product
		B1	for $\frac{\mathrm{d}}{\mathrm{d}x}\left(2x-3\right)^{\frac{1}{2}}=\frac{1}{2}\times 2\left(2x-3\right)^{-\frac{1}{2}}$ oe
		A1	all else correct i.e. $\frac{\mathrm{d}y}{\mathrm{d}x}=\left(x^2+1\right)\mathrm{f}(x)+2x\left(2x-3\right)^{\frac{1}{2}}$
	$=\left(2x-3\right)^{-\frac{1}{2}}\left(x^2+1+2x\left(2x-3\right)\right)$	**M1**	correctly taking out a factor of $\left(2x-3\right)^{-\frac{1}{2}}$ or correctly using $\left(2x-3\right)^{\frac{1}{2}}$ as denominator
	$=\frac{5x^2-6x+1}{\left(2x-3\right)^{\frac{1}{2}}}$	**A1**	
11(ii)	When $x=2$, $y=5$	**B1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=9$, so gradient of normal = $-\frac{1}{9}$	**M1**	substitution to obtain gradient and correct method for gradient of normal
	Equation of normal $y-5=-\frac{1}{9}\left(x-2\right)$	**M1**	**DepM1** for equation of normal
	$x+9y-47=0$ or $-x-9y+47=0$	**A1**	Must be in this form

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ADDITIONAL MATHEMATICS **0606/12**

Paper 1 **May/June 2019**

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Maximum Mark: 80

Published

This document consists of **11** printed pages.

Cambridge Assessment International Education

[Turn over

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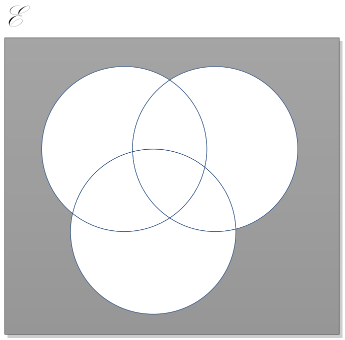
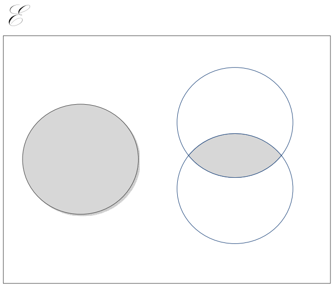
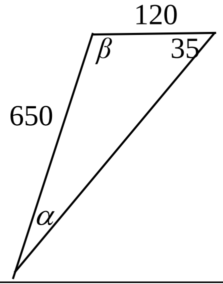
Question	Answer	Marks	Guidance
1(a)		**B1**	
		**B1**	

Question	Answer	Marks	Guidance
1(b)	$P=\{30^{\circ},150^{\circ},210^{\circ},330^{\circ}\}$	**B1**	May be seen or implied in a Venn diagram Allow without set notation
	$Q=\{30^{\circ},150^{\circ}\}$	**B1**	May be seen or implied in a Venn diagram Allow without set notation
	$P\cap Q=\{30^{\circ},150^{\circ}\}$	**B1**	**Dep** on both previous B marks Must be in set notation
2	**Either:** $(2x+3)^2(x-1)=3(2x+3)$ $(2x+3)(2x^2+x-6)(=0)$	**M1**	For attempt to equate line and curve and attempt to simplify to $2x+3$ × a quadratic factor or cancelling $2x+3$ and obtaining a quadratic factor
	$(2x+3)(2x^2+x-6)=0$ $(2x+3)(2x-3)(x+2)=0$	**M1**	**Dep** for attempt at 3 linear factors from a linear term and a quadratic term
	$\left(-\frac{3}{2},0\right)$	**B1**	
	$\left(\frac{3}{2},18\right)$	**A1**	**Dep** on first M mark only
	$(-2,-3)$	**A1**	**Dep** on first M mark only
	Or: $(2x+3)^2(x-1)=3(2x+3)$ $4x^3+8x^2-9x-18(=0)$	**M1**	For attempt to equate line and curve and attempt to simplify to a cubic equation, by collecting like terms
	$(x+2)(4x^2-9)$ $(2x-3)(2x^2+7x+6)$ $(2x+3)(2x^2+x-6)$ $(2x+3)(2x-3)(x+2)(=0)$	**M1**	**Dep** For attempt to find a factor from a 4 term cubic equation (usually $x+2$), do long division oe to obtain a quadratic factor and factorise this quadratic factor
	$\left(-\frac{3}{2},0\right)$	**A1**	
	$\left(\frac{3}{2},18\right)$	**A1**	
	$(-2,-3)$	**A1**	
3(i)	1000	**B1**	

Question	Answer	Marks	Guidance
3(ii)	$\frac{\mathrm{d}B}{\mathrm{d}t} = 400\mathrm{e}^{2t} - 1600\mathrm{e}^{-2t}$	**B1**	
	$3 = \mathrm{e}^{2t} - 4\mathrm{e}^{-2t}$ oe	**M1**	For equating an equation of the form $a\mathrm{e}^{2t} + b\mathrm{e}^{-2t}$ to 1200 and dividing by 400
	$\mathrm{e}^{4t} - 3\mathrm{e}^{2t} - 4 = 0$	**A1**	
3(iii)	$\left(\mathrm{e}^{2t}+1\right)\left(\mathrm{e}^{2t}-4\right)=0$	**M1**	For attempt to factorise and solve, dealing with exponential correctly, to obtain $\mathrm{e}^{2t} = ...$
	$t = \ln 2,\ \frac{1}{2}\ln 4$ or awrt 0.693 only isw if appropriate	**A1**	
4(a)	$a = \frac{5}{2}$	**B1**	
	$b = -\frac{3}{2}$	**B1**	
	$c = \frac{11}{2}$	**B1**	
4(b)	$9x^{\frac{1}{2}} - 3y^{-\frac{1}{2}} = 12$ $4x^{\frac{1}{2}} + 3y^{-\frac{1}{2}} = 14$	**M1**	For attempt to solve simultaneous equations. Must reach $kx^{\frac{1}{2}} = ...$ or $ky^{-\frac{1}{2}} = ...$ oe
	$x = 4$	**A1**	
	$y = \frac{1}{4}$	**A1**	
5(i)	$9.6 = 12\theta$	**M1**	For use of arc length
	$\theta = 0.8$	**A1**	

Question	Answer	Marks	Guidance
5(ii)	Either $\tan\theta = \frac{AB}{12}$, $(AB = 12.36)$ Or $OB = \frac{12}{\cos\theta}$ $(OB = 17.22)$	**M1**	For attempt to find AB or OB using *their* θ May be implied by a correct triangle area Allow if using degrees consistently
	Either Area $\Delta OAB = \frac{1}{2}\times 12\times$ *their* 12.36 Or Area $\Delta OAB = \frac{1}{2}\times 12\times$ *their* $17.22\times\sin\theta$ $(= 74.1 \text{ or } 74.2)$	**M1**	Allow if using degrees consistently For attempt to find area of triangle using *their* θ
	Area of sector $OAC = \frac{1}{2}\times 12^2\times 0.8$ $= 57.6$	**B1**	Allow unsimplified
	Area of shaded region = 16.5 or 16.6	**A1**	
6(a)(i)	40320	**B1**	
6(a)(ii)	No. of ways with maths books as 1 unit = $5!$ or $5\times 4!$ or ${}^5P_5$ or 120	**B1**	
	No. of ways maths books can be arranged amongst themselves = 4! or ${}^4P_4$ or 24	**B1**	
	Total = $(5!\ \times 4!\ \text{oe}) = 2880$	**B1**	
6(a)(iii)	No. of ways with maths books as 1 unit and geography books as 1 unit = $3!$ or ${}^3P_3$ or $3\times 2!$ or 6	**B1**	
	No. of ways maths books can be arranged amongst themselves and geography books can be arranged amongst themselves = $4!\ \times 3!$ or ${}^4P_4\times{}^3P_3$ or 144	**B1**	
	Total = $(3!\ \times 4!\ \times 3!\ \text{oe})$ = 864	**B1**	
6(b)(i)	${}^{12}C_6 = 924$	**B1**	

Question	Answer	Marks	Guidance
6(b)(ii)	**Either:** $924 - {}^8C_6$	**M1**	For *their* **(i)** – the number of teams of just men
	Total = 896	**A1**	
	Or: 5M 1W : ${}^8C_5 \times {}^4C_1$ (= 224) 4M 2W : ${}^8C_4 \times {}^4C_2$ (= 420) 3M 3W : ${}^8C_3 \times {}^4C_3$ (= 224) 2M 4W : ${}^8C_2 \times {}^4C_4$ (= 28)	**M1**	For a complete method
	Total = 896	**A1**	
7(i)		**B1**	For correct triangle, may be implied by a correct sine rule or cosine rule.
	$\frac{120}{\sin\alpha}$ or $\frac{120}{\sin(55-\theta)} = \frac{650}{\sin 35}$ or $\frac{650}{\sin 145}$	**M1**	For use of a correct sine rule to obtain $\alpha = ...$ or $\theta = ...$ Or for a correct cosine rule leading to a value for *v*, followed by a correct sine rule leading to one of the other angles
	$\alpha = 6.08^{\circ}$ or $\beta = 138.9$	**A1**	May be implied by a correct $\theta =$ awrt 49°
	Bearing is 048.9° or 049°	**A1**	
7(ii)	Either $\frac{v_r}{\sin(145 - their\alpha)} = \frac{650}{\sin 35}$ or $\frac{120}{\sin(their\alpha)}$ Or $v_r^2 = 650^2 + 120^2 -$ $(2 \times 650 \times 120)\cos(145 - their\alpha)$	**M1**	For use of sine rule or cosine rule to find resultant velocity Do not allow for a right-angled triangle May be seen in **(i)**
	$v_r = 745$	**A1**	For correct resultant velocity, allow awrt 745
	Time taken = $\frac{1250}{their\ 744.7}$	**M1**	For correct attempt at finding time using *their v* , $\neq$ 650, 120, 770 or 530
	$=1.68$ hours or I hour 41 mins or 101 mins	**A1**	

Question	Answer	Marks	Guidance
8(i)	$e^y = \frac{m}{x} + c$	**B1**	May be implied by subsequent work
	Either $20 = 2m + c$ $8 = 4m + c$	**M1**	For at least 1 correct equation
		M1	**Dep** For attempt to solve *their* 2 equations simultaneously to obtain at least one unknown
	leading to $m = -6,\ c = 32$	**A1**	For both
	$y = \ln\left(32 - \frac{6}{x}\right)$	**A1**	Must have correct brackets Mark the final answer given
	Or: Gradient = $m = (-6)$	**M1**	For attempt to find gradient and equate it to *m*
	$20 = 2m + c$ or $8 = 4m + c$ or $e^y - 8 = m\left(\frac{1}{x} - 4\right)$ or $e^y - 20 = m\left(\frac{1}{x} - 2\right)$	**M1**	For at least 1 correct equation, may be using *their m*
	leading to $c = 32$ and $m = -6$	**A1**	For both $m = -6,\ c = 32$
	$y = \ln\left(32 - \frac{6}{x}\right)$	**A1**	
8(ii)	$x > \frac{3}{16}$ oe	**B1**	
8(iii)	$y = \ln 30$ isw	**B1**	
8(iv)	$2 = \ln\left(32 - \frac{6}{x}\right)$	**M1**	For a correct substitution and attempt to re-arrange using 2, *their* 32 and *their* – 6, keeping exactness to obtain *x* =
	$x = \frac{6}{32 - e^2}$ oe	**A1**	Must be exact

Question	Answer	Marks	Guidance
9(i)	$5 = 4 + 2\cos 3x$	**M1**	For attempt to solve trig equation to obtain one correct solution
	$\frac{\pi}{9}$	**A1**	
	$-\frac{\pi}{9}$	**A1**	
9(ii)	**Either:** $\int_{-\frac{\pi}{9}}^{\frac{\pi}{9}} 4 + 2\cos 3x - 5 \ \mathrm{d}x$	**M1**	For use of subtraction method
	$\left[\frac{2}{3}\sin 3x - x\right]_{-\frac{\pi}{9}}^{\frac{\pi}{9}}$	**M1**	For attempt to integrate to obtain the form $a\sin 3x + bx$
		B1	For $\frac{2}{3}\sin 3x$
		B1	For $-x$, may be implied by $4x - 5x$
	$\left(\frac{\sqrt{3}}{3} - \frac{\pi}{9}\right) - \left(-\frac{\sqrt{3}}{3} + \frac{\pi}{9}\right)$	**M1**	**Dep** on previous M mark for correct application of *their* limits in radians from **(i)** retaining exactness
	Shaded area = $\frac{2\sqrt{3}}{3} - \frac{2\pi}{9}$ oe isw	**A1**	

Question	Answer	Marks	Guidance
9(ii)	**Or:** Area of rectangle = $5\times\frac{2\pi}{9}$	**M1**	$5\times$ the difference of *their* limits in exact radians
	Area under curve = $\left[4x+\frac{2}{3}\sin 3x\right]_{-\frac{\pi}{9}}^{\frac{\pi}{9}}$	**M1**	For attempt to integrate to obtain the form $a\sin 3x+bx$
		B1	For $\frac{2}{3}\sin 3x$
		B1	For $4x$
	$\left(\frac{\sqrt{3}}{3}+\frac{4\pi}{9}\right)-\left(-\frac{\sqrt{3}}{3}-\frac{4\pi}{9}\right)$ $\left(=\frac{2\sqrt{3}}{3}+\frac{8\pi}{9}\right)$	**M1**	**Dep** on previous M mark for correct application of *their* limits in exact radians from **(i)** retaining exactness
	Shaded area = $\frac{2\sqrt{3}}{3}-\frac{2\pi}{9}$ oe isw	**A1**	
10(i)	$800=4x^2h$	**B1**	
	$h=\frac{800}{4x^2}$ oe or $xh=\frac{800}{4x}$ oe	**B1**	
	$(S=)2hx+8xh+4x^2$ oe	**M1**	Allow if h is substituted at this point
	$S=4x^2+\left(\frac{2000}{x}\right)$	**A1**	Leading to AG, must have S = or surface area = at some point and no errors

Question	Answer	Marks	Guidance
10(ii)	$\left(\frac{\mathrm{d}S}{\mathrm{d}x}=\right)8x-\frac{2000}{x^2}$	**B1**	For correct differentiation
	When $\frac{\mathrm{d}S}{\mathrm{d}x}=0,\quad x=\sqrt[3]{250}$ oe (6.30)	**M1**	For equating to zero and attempt to solve, must get as far as $x =$ …, must be using the form $ax+\frac{b}{x^2}$
		A1	For correct positive x
	$S=476$ only	**A1**	
	$\frac{\mathrm{d}^2S}{\mathrm{d}x^2}=8+\frac{4000}{x^3}$ $\frac{\mathrm{d}^2S}{\mathrm{d}x^2}>0$ or 24 so minimum	**B1**	For a correct convincing method, with enough detail to reach a correct conclusion of a minimum. Must be using $x=\sqrt[3]{250}$ oe
11		**M1**	For attempt at differentiating a product
		B1	For $\frac{2}{3}\times3(3x+1)^{-\frac{1}{3}}$
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)(x-2)\times\frac{2}{3}\times3(3x+1)^{-\frac{1}{3}}+(3x+1)^{\frac{2}{3}}$	**A1**	For all other terms correct
	$y=\frac{4}{3}$	**B1**	
	When $x=\frac{7}{3},\ \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{13}{3}$	**M1**	For attempt at normal equation using $-\frac{1}{\textit{their } m}$ and *their y* when $x=\frac{7}{3}$
	Equation of normal: $y-\frac{4}{3}=-\frac{3}{13}\left(x-\frac{7}{3}\right)$	**A1**	For correct normal equation, may be implied by a correct final answer
	At y-axis, $y=\frac{73}{39}$ $\left(0,\frac{73}{39}\right)$ isw	**A1**	

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ADDITIONAL MATHEMATICS **0606/13**

Paper 1 **May/June 2019**

MARK SCHEME

Maximum Mark: 80

Published

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Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **8** printed pages.

[Turn over

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

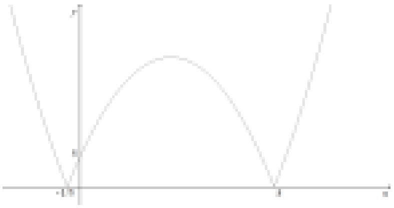
B Mark for a correct result or statement independent of Method marks.

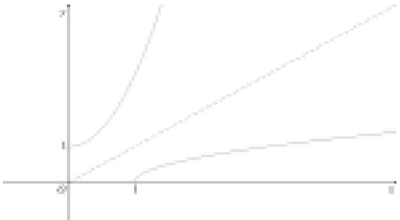
When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

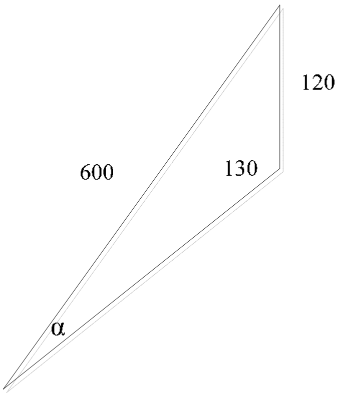
awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$A \cap B = \varnothing$	**B1**	
	$Z \subset (X \cap Y)$	**B2**	**B1** for identifying $X \cap Y$
2	$a = \frac{3}{2}$	**B1**	
	$b = \frac{7}{3}$	**B1**	
	$c = 3$	**B1**	
3	$x^2 + (3 - m)x + m - 4 = 0$	**M1**	For equating line and curve and attempting to obtain a quadratic equation equated to zero
	Discriminant: $(3-m)^2 - 4(m-4)$	**M1**	**Dep** For use of $b^2 - 4ac$, could be implied by use of quadratic formula
	$(m-5)^2$	**A1**	
	Always positive or zero for any *m*, so line and curve will always touch or intersect	**A1**	For a suitable comment/conclusion
4(i)		**B1**	For $\frac{6x^3}{(2x^3+5)}$
		M1	For attempt to differentiate a quotient
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x-1)\frac{6x^2}{(2x^3+5)} - \ln(2x^3+5)}{(x-1)^2}$	**A1**	For all other terms correct
	When $x = 2$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{24}{21} - \ln 21$ or $\frac{8}{7} - \ln 21$, or –1.90	**A1**	
4(ii)	$-1.90p$ oe	**B1**	

Question	Answer	Marks	Guidance
5(i)		**B1**	For shape with maximum in 1st quadrant
		B1	For $\left(-\frac{1}{3}, 0\right)$ and $(5, 0)$
		B1	For $(0, 5)$
		B1	All correct with cusps and correct shape for $x < -\frac{1}{3}$ and $x > 5$
5(ii)		**M1**	For attempt to find maximum point
	Maximum point when $x = \frac{7}{3}$	**A1**	For $x = \frac{7}{3}$
	$y = \frac{64}{3}$ so $k = \frac{64}{3}$	**A1**	
6(i)	$\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} \times \sin\theta$ oe	**M1**	For dealing with sec, tan and cosec in terms of sin and cos
	$\frac{1 - \sin^2\theta}{\cos\theta}$	**M1**	For simplification and use of identity
	$\frac{\cos^2\theta}{\cos\theta}$	**A1**	For simplification to AG
6(ii)	$\cos 2\theta = \frac{\sqrt{3}}{2}$	**M1**	For use of part **(i)** and attempt to solve to get as far as $2\theta = ...$
	$2\theta = 30^{\circ},\ 330^{\circ}$	**M1**	For dealing with double angle correctly, may be implied by one correct solution
	$\theta = 15^{\circ},\ 165^{\circ}$	**A1**	For both
6(iii)	$\sin\left(\phi + \frac{\pi}{3}\right) = \pm\frac{1}{\sqrt{2}}$ $\phi + \frac{\pi}{3} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}$	**M1**	For correct attempt to solve, may be implied by $\phi + \frac{\pi}{3} = \frac{\pi}{4}$
		M1	**Dep** For dealing with compound angle correctly
	$\phi = \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{23\pi}{12}$	**A2**	**A1** for one correct pair, **A1** for a second correct pair with no extra solutions in the range.

Question	Answer	Marks	Guidance
7(i)	$AC^2 = \left(2\sqrt{5}-1\right)^2 + \left(2+\sqrt{5}\right)^2$	**M1**	For use of Pythagoras' theorem and attempt to expand brackets
	$= 20 - 4\sqrt{5} + 1 + 4 + 4\sqrt{5} + 5$	**A1**	For correct unsimplified, must be convinced of non-calculator use
	$AC = \sqrt{30}$	**A1**	
7(ii)	$\tan ACB = \frac{2\sqrt{5}-1}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$	**M1**	For attempt at tan *ACB* and rationalisation
	$= \frac{4\sqrt{5}-2-10+\sqrt{5}}{4-5}$ oe	**M1**	**Dep** For seeing at least 3 terms in the numerator
	$= 12 - 5\sqrt{5}$	**A1**	
7(iii)	$\sec^2 ACB = \tan^2 ACB + 1$ $= 144 - 120\sqrt{5} + 125 + 1$	**M1**	For use of identity using *their* **(ii)**
	$= 270 - 120\sqrt{5}$	**A1**	
8(i)	$g \geqslant 1$	**B1**	Must be using correct notation
8(ii)	$g\left(\sqrt{62}\right) = 125$	**B1**	
	$f^{-1}(x) = \frac{1}{3}\ln x$	**B1**	
	$\frac{1}{3}\ln 125 = \ln 5$	**B1**	For correct order and manipulation to obtain the given answer, need to see $\frac{1}{3}\ln 125$
8(iii)	$3e^{3x} = 24$	**M1**	For dealing with derivatives correctly
	$x = \frac{1}{3}\ln 8$	**A1**	
	$x = \ln 2$	**A1**	
8(iv)		**B3**	**B1** for correct g with intercept **B1** for $y = x$ and/or implication of symmetry **B1** for correct g^{-1} with intercept
9(a)(i)	$7! = 5040$	**B1**	

Question	Answer	Marks	Guidance
9(a)(ii)	Treating the 4 trophies as 1 unit so there are 4! ways	**B1**	Maybe implied by a correct answer
	There are also 4! ways of arranging the football trophies amongst themselves	**B1**	
	Total = $4! \times 4! = 576$	**B1**	
9(a)(iii)	Treating the 4 football trophies as 1 unit and the 2 cricket trophies as 1 unit so there are 3! ways	**B1**	Maybe implied by a correct answer
	There are also 4! ways of arranging the football trophies amongst themselves and 2 ways of arranging the cricket trophies	**B1**	Maybe implied by a correct answer
	Total = $3! \times 4! \times 2 = 288$	**B1**	
9(b)(i)	3003	**B1**	
9(b)(ii)	28	**B1**	
9(b)(iii)	3003 – 1	**M1**	For *their* **(i)** – 1
	3002	**A1**	**FT**
10(i)		**M1**	Attempt to integrate to obtain $k(2x+3)^{\frac{1}{2}}$
	$\frac{dy}{dx} = (2x+3)^{\frac{1}{2}} \quad (+c)$	**A1**	All correct, condone omission of +*c*
	$5 = 3 + c$	**M1**	**Dep** For attempt at *c*
	$\frac{dy}{dx} = (2x+3)^{\frac{1}{2}} + 2$	**M1**	For a further attempt to integrate
	$y = \frac{1}{3}(2x+3)^{\frac{3}{2}} + 2x(+d)$	**A1**	All correct, condone omission of +*d*
	$-\frac{1}{3} = \frac{8}{3} + 1 + d$	**M1**	For attempt at *d*
	$y = \frac{1}{3}(2x+3)^{\frac{3}{2}} + 2x - 4$	**A1**	Must have *y* =

Question	Answer	Marks	Guidance
10(ii)	When $x = 3,\ y = 11$	**M1**	For attempt to find *y* using *their* **(i)**
		M1	**Dep** For attempt at normal
	Normal: $y - 11 = -\frac{1}{5}(x - 3)$	**A1**	Al1 correct unsimplified
	$x + 5y - 58 = 0$	**A1**	For correct form
11(i)		**B1**	For correct triangle, may be implied by subsequent work
	$\frac{120}{\sin\alpha} = \frac{600}{\sin 130}$	**M1**	For use of the correct sine rule
	$\alpha = 8.81^{\circ}$	**A1**	Allow greater accuracy
	Bearing 041.2° or 041°	**A1**	Allow greater accuracy
11(ii)	$\frac{v_r}{\sin 41.19} = \frac{600}{\sin 130} = \frac{120}{\sin\alpha}$	**M1**	For use of sine rule using *their* α or cosine rule
	$v_r = 515.8$ awrt 516	**A1**	
	Time taken $= \frac{2500}{515.8}$	**M1**	For attempt to find time using *their* v_r, not 600, 720 or 480
	$= 4.85$ or 4.84	**A1**	

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ADDITIONAL MATHEMATICS **0606/21**

Paper 2 **May/June 2019**

MARK SCHEME

Maximum Mark: 80

Published

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isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1	$6x^2+7x-20\,[*0]$	M1	where * may be any inequality sign or =
	Critical values $\frac{4}{3}$, $-\frac{5}{2}$	A1	
	$x \leqslant -\frac{5}{2}$ or $x \geqslant \frac{4}{3}$ final answer	A1	**FT** *their* critical values using outside regions
2(i)	$\frac{\mathrm{d}}{\mathrm{d}x}(\ln x)=\frac{1}{x}$ soi	B1	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x^3\left(\frac{1}{x}\right)-3x^2\ln x}{\left(x^3\right)^2}$ or $x^{-3}\left(\frac{1}{x}\right)+\left(-3x^{-4}\right)\ln x$	M1	
	Completion to given answer: $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1-3\ln x}{x^4}$	A1	
2(ii)	$\left(\frac{1-3\ln \mathrm{e}}{\mathrm{e}^4}\right)h$	M1	
	$-\frac{2h}{\mathrm{e}^4}$ oe or –0.0366h awrt	A1	
3(i)	Correct shape 0.6 oe indicated on x-axis 3 indicated on y-axis	3	**B1** correct shape must have cusp on x-axis **B1** for each correct point There must be a sketch to award the marks for the intercepts and sketch should be continuous with one intersection only on each axis
3(ii)	Solves $5x-3=x-2$ oe or $(5x-3)^2=(2-x)^2$	M1	
	$[x=]\frac{1}{4}$ oe	A1	
	$[x=]\frac{5}{6}$ oe	B1	

Question	Answer	Marks	Partial Marks
4	$\left(\sqrt{5}-3\right)^2 = 5+9-2(3)\sqrt{5}$	**M1**	
	$\dfrac{their\left(14-6\sqrt{5}\right)}{\sqrt{5}+1}\times\dfrac{\sqrt{5}-1}{\sqrt{5}-1}$	**M1**	Attempts to rationalise or forms a pair of simultaneous equations e.g. $5p+q=14, \quad p+q=-6$
	$\dfrac{their(14\sqrt{5}-30-14+6\sqrt{5})}{5-1}$	**M1**	multiplies out; numerator must have at least 3 terms; condone one sign error in numerator; denominator may be 4 or $5-\sqrt{5}+\sqrt{5}-1$ or solves *their* simultaneous equations to find one unknown
	$5\sqrt{5}-11$	**A1**	or $p=5, q=-11$
5(i)	$-\dfrac{10}{6}$ oe	**B1**	
5(ii)	27	**B1**	
5(iii)	Attempts to find total area	**M1**	
	$\dfrac{1}{2}(23+their\ k+6)\times 10$ or $\dfrac{1}{2}\times 4\times 10+23\times 10+\dfrac{1}{2}\times 6\times 10$	**M1**	
	280	**A1**	
6(a)	$(x+3)(x-3)-2x(-x)$	**B1**	
	their det $\mathbf{A}=0$	**M1**	Can be implied by later work
	$[x=] \quad \pm\sqrt{3}$ isw	**A1**	
6(b)(i)	3 × 2 or 3 by 2	**B1**	
6(b)(ii)	**BC** is a 3 by 3 matrix and **CB** is a 2 by 2 matrix [so they cannot be the same] oe or [**CB** =] $\begin{pmatrix} 6 & 5 \\ 41 & 15 \end{pmatrix}$ [so not equal] or finding one correct element of **CB** as being different from **BC and** commenting that the elements are different, [the matrices cannot be the same] oe	**B2**	**B1** for a partially correct statement e.g. The orders are not the same **or BC** is a 3 by 3 matrix **or CB** is a 2 by 2 matrix or **B1** for 3 correct elements or **B1** for finding one correct element of **CB** as being different from **BC**, without further comment

Question	Answer	Marks	Partial Marks
7(i)	$\sec^2 u$	**B1**	
7(ii)	Attempts $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$ or $\frac{dy}{dx}=\frac{dy}{du}\div\frac{dx}{du}$	**M1**	
	$\frac{dy}{dx}=\frac{their\sec^2 u}{3u^2}$	**A1**	**FT** *their* **(i)**
	$u=\sqrt[3]{x-1}$ soi	**B1**	
	$\frac{\sec^2(\sqrt[3]{x-1})}{3(\sqrt[3]{x-1})^2}$ cao	**A1**	final answer If **B1** only then **SC1** for $k(x-1)^{-\frac{2}{3}}\sec^2(x-1)^{\frac{1}{3}}$
8(i)	[angle *ECD* =] $\frac{5\pi}{18}$ oe or 0.873 soi	**B1**	
	Attempts to find *AC* and subtract 8	**M1**	e.g. $AC=\frac{8}{\cos\frac{2\pi}{9}}$
	[*DC* =] 2.44	**A1**	
	$\frac{1}{2}\times 8\times theirAC\times\sin\frac{2\pi}{9}$ OR $\frac{1}{2}\times 8\times 8\tan\left(\frac{2\pi}{9}\right)-\frac{1}{2}\times 8^2\times\frac{2\pi}{9}$ $-\frac{1}{2}\times their\ 2.44^2\times their\frac{5\pi}{18}$	**M2**	**M1** for $\frac{1}{2}\times 8^2\times\frac{2\pi}{9}$ or for $\frac{1}{2}\times their\ 2.44^2\times their\frac{5\pi}{18}$ seen
	awrt 1.91	**A1**	
8(ii)	*their*(6.712 – 2.443) + *their* 2.443$\left(\frac{5\pi}{18}\right)$ + 8$\left(\frac{2\pi}{9}\right)$	**M2**	**M1** for either arc seen
	awrt 12.0	**A1**	
9(a)(i)	39 916 800	**B1**	
9(a)(ii)	$5!\times 6!$ oe	**M1**	
	86 400	**A1**	

Question	Answer	Marks	Partial Marks
9(b)(i)	${}^5C_3 \times {}^3C_1$ oe	**M1**	
	30	**A1**	
9(b)(ii)	${}^5C_2 \times {}^3C_2 + {}^5C_1 \times {}^3C_1$ oe	**M1**	
	45	**A1**	
10(i)	$\frac{4-3}{1-p}=\frac{1}{3}$ oe	**M1**	**ALT** uses $y = mx + c$ with *A* and *B* as far as an equation in *p* only
	–2	**A1**	
10(ii)	**Either:** Finds midpoint *AB* $\left(\frac{their\ p+1}{2}, \frac{3+4}{2}\right)$	**B1**	**FT** *their p*
	Verifies $(-0.5, 3.5)$ is on *L*	**B1**	
	$y=-3x+2$ therefore *m* = –3 oe **and** $\frac{1}{3}\times -3=-1$ oe	**B1**	
	Or: finds midpoint *AB* $\left(\frac{their\ p+1}{2},\frac{3+4}{2}\right)$	**B1**	**FT** *their p*
	$\frac{1}{3}\times -3=-1$ oe	**B1**	
	$y-3.5=-3(x+0.5)$ **and** completion to $y=-3x+2$	**B1**	
10(iii)	*q* = 4	**B1**	
10(iv)	22.5 nfww	**B2**	**B1** for correct method to find area using correct values e.g. $\frac{1}{2}\times AB\times MC$ where *M* is the midpoint of *AB*

Question	Answer	Marks	Partial Marks
11(a)(i)	$\frac{1}{\sin\theta}\left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)$	**M2**	**M1** for either $\frac{\operatorname{cosec}\theta-\cot\theta}{\sin\theta}=\frac{1}{\sin\theta}\left(\operatorname{cosec}\theta-\frac{\cos\theta}{\sin\theta}\right)$ or $\frac{\operatorname{cosec}\theta-\cot\theta}{\sin\theta}=\frac{1}{\sin\theta}\left(\frac{1}{\sin\theta}-\cot\theta\right)$
	$\frac{1-\cos\theta}{1-\cos^2\theta}$	**M1**	
	$\frac{1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)}=\frac{1}{1+\cos\theta}$	**A1**	
11(a)(ii)	awrt 233.1	**B2**	with no extras in range **B1** for $\cos\theta=-\frac{3}{5}$ soi
11(b)	$3\phi-4=\tan^{-1}\left(-\frac{1}{2}\right)$ soi	**M1**	
	awrt 0.132, 1.18	**A2**	with no extras in range **A1** for one correct
12(a)	$\frac{e^{2x}}{2}$ seen	**B1**	
	$\frac{e^{2a}}{2}-\frac{1}{2}=50$	**M1**	Uses limits correctly for their integral and sets = 50
	Rearranges and takes logs to base e: $2a=\ln 101$ oe	**M1**	Using *their* integral
	$a=\frac{1}{2}\ln 101$ or $\ln\sqrt{101}$ final answer	**A1**	Allow any exact equivalent
12(b)(i)	$[y=]\,3x-\frac{2}{5}\sin 5x\ [+c]$	**B2**	**B1** for $-k\sin 5x$ where $k>0$
	$\frac{8\pi}{5}=\frac{3\pi}{5}-\frac{2}{5}\sin\left(5\times\frac{\pi}{5}\right)+c$	**M1**	
	$y=3x-\frac{2}{5}\sin 5x+\pi$	**A1**	

Question	Answer	Marks	Partial Marks
12(b)(ii)	$\left[\int y\,\mathrm{d}x = \int\left(3x - \frac{2}{5}\sin 5x + \pi\right)\mathrm{d}x\right]$ $= \frac{3x^2}{2} + \frac{2}{25}\cos 5x + \pi x\,[+c]$	**B3**	**B2** for $\frac{2}{25}\cos 5x$ oe nfww and **B1FT** for $\frac{3x^2}{2} + \ldots + \pi x\,[+c]$
	their F(π) – *their* F$\left(\frac{\pi}{2}\right)$	**M1**	
	16[.0] or 15.95 to 15.96 or $\frac{13\pi^2}{8} - \frac{2}{25}$	**A1**	

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ADDITIONAL MATHEMATICS **0606/22**

Paper 2 **May/June 2019**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **9** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1	**Either:** Quotient rule: $\frac{\mathrm{d}}{\mathrm{d}x}(\sin x) = \cos x$ soi	**B1**	
	$\frac{\mathrm{d}}{\mathrm{d}x}(\ln x^2) = \frac{2}{x}$ oe soi	**B1**	
	$\frac{(\ln x^2)(\textit{their}\ \cos x) - (\sin x)\left(\textit{their}\ \frac{2}{x}\right)}{(\ln x^2)^2}$ oe	**M1**	
	$\frac{(\ln x^2)\cos x - (\sin x)\left(\frac{2}{x}\right)}{(\ln x^2)^2}$ oe isw	**A1**	
	Or: Product rule on $y = (\sin x)(\ln x^2)^{-1}$ $\frac{\mathrm{d}}{\mathrm{d}x}(\sin x) = \cos x$ soi	**B1**	
	$\frac{\mathrm{d}}{\mathrm{d}x}\left((\ln x^2)^{-1}\right) = -(\ln x^2)^{-2} \times \frac{2}{x}$	**B1**	
	$(\sin x) \times \textit{their}\left(-(\ln x^2)^{-2} \times \frac{2}{x}\right)$ $+ (\textit{their}\cos x)(\ln x^2)^{-1}$ oe	**M1**	
	$(\sin x) \times \left(-(\ln x^2)^{-2} \times \frac{2}{x}\right)$ $+ (\cos x)(\ln x^2)^{-1}$ oe isw	**A1**	
2	$k^2 - 4(k-1)(-k)$ oe	**B1**	
	$k(5k-4)$	**M1**	
	Correct critical values 0, 0.8 oe	**A1**	
	$k < 0, k > 0.8$ oe	**A1**	**FT** *their* critical values provided *their* $ak^2 + bk + c > 0$ has positive a and there are 2 values; mark final answer If **B1 M0** allow **SC1** for a final answer of $k > 0.8$ oe

Question	Answer	Marks	Partial Marks
3(i)	Uses $x = 2$ as a root: $a(2^3) - 12(2^2) + 5(2) + 6 = 0$	**M1**	or $\begin{array}{r\|cccc} 2 & a & -12 & 5 & 6 \\ & \downarrow & 2a & -24+4a & -38+8a \\ \hline & a & -12+2a & -19+4a & -32+8a = 0 \end{array}$
	Solves $8a - 48 + 10 + 6 = 0$ to find $a = 4$	**A1**	or solves $-32 + 8a = 0$ to find $a = 4$ If **M0** then **SC1** for $4(2^3) - 12(2^2) + 5(2) + 6 = 0$ or showing that the synthetic division with $a = 4$ results in a remainder of 0
3(ii)	$(x-2)(4x^2 - 4x - 3)$	**B2**	**B1** for any two terms correct in quadratic factor
	Product of three correct linear factors: $(x-2)(2x-3)(2x+1)$	**B1**	
	$x = 2,\ x = 1.5,\ x = -0.5$ oe	**B1**	**dep** on all previous marks having been earned If **B2 B0** then award **SC1** for correct factorisation of correct quadratic factor leading to 3 correct roots
4	**Either:** $A = \frac{1}{4}\pi x^2$ oe, soi	**B1**	
	$\frac{\mathrm{d}A}{\mathrm{d}x} = \frac{2}{4}\pi x$ oe, soi	**B1**	
	$\frac{\mathrm{d}x}{\mathrm{d}t} = 0.01$ soi	**B1**	
	$\frac{2}{4}\pi(6) \times 0.01$	**M1**	**FT** *their* $\frac{\mathrm{d}A}{\mathrm{d}x}$ when $x = 6$
	0.03π or exact equivalent	**A1**	mark final answer
	Or: $A = \pi r^2$ and $r = \frac{x}{2}$ soi	**B1**	
	$\frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r$ oe soi	**B1**	
	$\frac{\mathrm{d}r}{\mathrm{d}t} = 0.005$ oe soi	**B1**	
	$2\pi(3) \times 0.005$	**M1**	
	0.03π or exact equivalent	**A1**	

Question	**Answer**	**Marks**	**Partial Marks**
5(i)	$5(x-1.5)^2-10.25$ isw	**B3**	**B1** for each of *p*, *q*, *r* correct in correctly formatted expression; allow correct equivalent values If **B0** then **SC2** for $5(x-1.5)-10.25$ or **SC1** for correct values but other incorrect format
5(ii)	$\frac{their\ -10.25}{5}$ is least value when $x = their 1.5$	**B2**	**STRICT FT** *their* part (i); **B1 STRICT FT** for each
6(a)	2×4 or 2 by 4	**B1**	
6(b)(i)	$(\mathbf{A}^{-1}=)\ \frac{1}{2}\begin{pmatrix}3 & 4\\1 & 2\end{pmatrix}$ isw	**B2**	**B1** for $\frac{1}{2}\times their\begin{pmatrix} & \\ & \end{pmatrix}$ or for $k\times\begin{pmatrix}3 & 4\\1 & 2\end{pmatrix}$, where *k* is not 0 or 1
6(b)(ii)	**B** = $\frac{1}{4}\begin{pmatrix}13 & 20\\5 & 8\end{pmatrix}$ oe isw	**B3**	**FT** *their* $\mathbf{A}^{-1}$ provided **B1** earned in **(b)(i)** **B1** for the strategy of using $\mathbf{A}^{-1}$: $\mathbf{B}=\mathbf{A}^{-1}\mathbf{A}^{-1}$ soi or $\mathbf{AB}=\mathbf{A}^{-1}$ soi or $\mathbf{BA}=\mathbf{A}^{-1}$ soi or $\mathbf{B}=(\mathbf{A}^{-1})^2$ and **B1** for two or three elements of **B** correct or correct FT
7(a)	$\lg(x^2-3)=\lg 1$ soi	**M1**	
	-2 and 2	**A1**	Implies **M1**
7(b)(i)	**Two separate terms in numerator:** $(\sin(2x+5))\ln a$ or $\log_a a^{\sin(2x+5)}=(\sin(2x+5))\log_a a$	**B1**	**Combines terms in numerator:** or $\ln\left(\frac{a^{\sin(2x+5)}}{a}\right)$
	$-\ln a$ or $\log_a a^{-1}=-\log_a a$	**B1**	or $(\sin(2x+5)-1)\ln a$ or $\frac{\ln\left(\frac{a^{\sin(2x+5)}}{a}\right)}{\ln a}=\log_a\left(a^{\sin(2x+5)-1}\right)$
	$\sin(2x+5)-1$	**B1**	**dep** all previous marks awarded;
7(b)(ii)	$-\frac{1}{2}\cos(2x+5)+(\ their\ -1)\ x\ \ (+\ c)$	**B3**	**FT** *their* numerical *k* **B2** for $-\frac{1}{2}\cos(2x+5)$ seen or **B1** for $a\cos(2x+5)$, $a<0$ or for $\frac{1}{2}\cos(2x+5)$ or for $-\frac{1}{2}\cos 2x+5$ seen

Question	Answer	Marks	Partial Marks
8(a)	$-\frac{20}{8}a^3\left[x^3\right]$ and $-\frac{6}{32}a\left[x^5\right]$ oe soi or $\frac{20}{8}a^3\left[x^3\right]$ and $\frac{6}{32}a\left[x^5\right]$ oe soi	**B2**	**B1** for either $-\frac{20}{8}a^3\left[x^3\right]$ or $-\frac{6}{32}a\left[x^5\right]$ oe or for $-\frac{{}^6C_3}{8}a^3\left[x^3\right]$ **and** $-\frac{{}^6C_5}{32}a\left[x^5\right]$ oe, or for $\frac{{}^6C_3}{8}a^3\left[x^3\right]$ **and** $\frac{{}^6C_5}{32}a\left[x^5\right]$ oe or for $\frac{20}{8}a^3\left[x^3\right]$ **and** $ka\left[x^5\right]$ oe where $k > 0$ or for $ka^3\left[x^3\right]$ **and** $\frac{6}{32}a\left[x^5\right]$ oe where $k > 0$
	$their\frac{20}{8}a^3 = 120 \times their\frac{6}{32}a$ oe soi	**M1**	
	$\left[a =\right] \pm 3$	**A1**	
8(b)(i)	$1 + 40x + 760x^2 + 9120x^3$	**B2**	**B1** for three out of the four terms correct If **B0** then **SC1** for $1, 40x, 760x^2, 9120x^3$ seen but not summed
8(b)(ii)	$1 + 40(-0.01) + 760(-0.01)^2 + 9120(-0.01)^3$ or 1 – 0.4 + 0.076 – 0.00912 oe leading to 0.66688 cao	**B2**	or **M1** for use of $x = -0.01$ oe in *their* expansion seen or implied by e.g. 0.66688 without working or 1 – 0.4 + 0.076 – 0.00912
9(a)	$6(1-\cos^2 x) - 13\cos x = 1$ oe	**B1**	
	Solves or factorises *their* 3-term quadratic	**M1**	
	70.5 and 289.5	**A2**	with no extras in range **A1** for either, ignoring extras in range
9(b)(i)	Numerator: Substitution of $\tan y = \frac{\sin y}{\cos y}$	**M1**	
	Denominator: Substitution of $1 + \tan^2 y = \sec^2 y$ or substitution of $1 + \tan^2 y = 1 + \frac{\sin^2 y}{\cos^2 y}$ **and** correct rearrangement to $\frac{1}{\cos^2 y}$ oe	**M1**	
	Correct completion to $4\sin y$ cao	**A1**	

Question	Answer	Marks	Partial Marks
9(b)(ii)	–0.848[06…] rot to 3 or more figures	**B1**	with no extras in range
10(a)	$\sqrt{5^2+(-15)^2}$ seen	**M1**	
	$\frac{1}{5\sqrt{10}}(5\mathbf{i}-15\mathbf{j})$ oe, isw	**A1**	
10(b)(i)	$\begin{pmatrix}9\\12\end{pmatrix}$ oe, soi	**B1**	
	$\begin{pmatrix}3\\-5\end{pmatrix}+\frac{2}{3}\left(their\begin{pmatrix}9\\12\end{pmatrix}\right)$ oe, soi or $\begin{pmatrix}12\\7\end{pmatrix}-\frac{1}{3}\left(their\begin{pmatrix}9\\12\end{pmatrix}\right)$ oe, soi	**M1**	
	$\begin{pmatrix}9\\3\end{pmatrix}$	**A1**	If B1 M0, award SC1 for a final answer of $\begin{pmatrix}-9\\-3\end{pmatrix}$ oe
10(b)(ii)	Forms a valid vector relationship using $\overrightarrow{DC}$ and e.g. $\overrightarrow{OD}$ or $\overrightarrow{DB}$ e.g. $their\begin{pmatrix}9\\3\end{pmatrix}=\overrightarrow{OD}+\begin{pmatrix}6\\1.25\end{pmatrix}$ oe or $\overrightarrow{DB}=\begin{pmatrix}6\\1.25\end{pmatrix}+\frac{1}{3}\times their\begin{pmatrix}9\\12\end{pmatrix}$ oe	**M1**	or $\overrightarrow{DC}=\overrightarrow{OC}-\overrightarrow{OD}$ $\begin{pmatrix}6\\1.25\end{pmatrix}=their\begin{pmatrix}9\\3\end{pmatrix}-\frac{1}{\lambda}\begin{pmatrix}12\\7\end{pmatrix}$ soi or $\overrightarrow{DC}=\overrightarrow{DB}-\overrightarrow{CB}$ $\begin{pmatrix}6\\1.25\end{pmatrix}=\frac{\lambda-1}{\lambda}\begin{pmatrix}12\\7\end{pmatrix}-\frac{1}{3}\times their\begin{pmatrix}9\\12\end{pmatrix}$ soi
	Finds a correct proportion e.g. $\overrightarrow{OB}=4\overrightarrow{OD}$ oe soi or $3\overrightarrow{OB}=4\overrightarrow{DB}$ oe soi	**A1**	or solves a correct equation in λ e.g. $6=9-\frac{1}{\lambda}\times 12$
	$\lambda=4$	**A1**	from a fully correct method
11(i)	$v\neq 0$ or $v>0$ oe	**B1**	
11(ii)	Differentiates : $4\times -3(t+1)^{-4}$ oe, isw	**B2**	**B1** for $k(t+1)^{-4}$ with $k\neq -12$
	$-\frac{1}{108}$ oe or –0.00926	**B1**	
11(iii)	Integrates: $[s=]-2(t+1)^{-2}+2$ oe, isw	**B3**	**B2** for $-2(t+1)^{-2}$ $[+c]$ or **B1** for $k(t+1)^{-2}$ $[+c]$ with $k\neq -2$

Question	Answer	Marks	Partial Marks
11(iv)	Finds *their s* from **(iii)** when $t = 4$ or when $t = 3$ or finds $\left[their\left(-2(t+1)^{-2}\ [+2]\ \right)\right]_3^4$ $= their\left(-2(4+1)^{-2} - \left(-2(3+1)^{-2}\right)\right)$	**M1**	
	$\frac{9}{200}$ or 0.045	**A1**	
12(a)(i)	g > –9	**B1**	
12(a)(ii)	$x > 1$	**B1**	
12(a)(iii)	$[\text{gf}(x) =]\quad 4(5x-2)^2 - 9$	**B1**	
	$100x^2 - 80x - 38 = 0$ or $(5x-2)^2 = \frac{45+9}{4}$	**M1**	
	$[x =]\quad \frac{-(-80) \pm \sqrt{(-80)^2 - 4(100)(-38)}}{2(100)}$ leading to $\frac{4+3\sqrt{6}}{10}$ oe only or $\frac{1}{5}\left(2 + \sqrt{\frac{54}{4}}\right)$ or better only	**A1**	
12(b)(i)	(They are) **reflection**s (of each other) in (the line) ***y* = *x*** oe	**B1**	
12(b)(ii)	$x^2 = y^2 + 1$ or $y^2 = x^2 + 1$	**M1**	
	$x = [\pm]\sqrt{y^2+1}$ or $y = [\pm]\sqrt{x^2+1}$	**A1**	
	$-\sqrt{x^2+1}$ nfww	**A1**	

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ADDITIONAL MATHEMATICS **0606/23**

Paper 2 **May/June 2019**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **8** printed pages.

Cambridge Assessment International Education

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method marks, awarded for a valid method applied to the problem.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
B	Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1	For attempting to solve $9x^2 + 17x - 2\,[*0]$	**M1**	where * may be any inequality sign or =
	Critical values $\frac{1}{9}, \quad -2$	**A1**	
	$-2 < x < \frac{1}{9}$ final answer	**A1**	**FT** *their* critical values from $ax^2 + bx + c < 0$ with $a > 0$
2	$\frac{\mathrm{d}}{\mathrm{d}x}(\tan 3x) = 3\sec^2 3x$ soi	**B1**	
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\cos\frac{x}{2}\right) = -\frac{1}{2}\sin\frac{x}{2}$ soi	**B1**	
	Applies correct form of product rule	**M1**	
	$\left[\frac{\mathrm{d}y}{\mathrm{d}x} =\right]$ $-\frac{1}{2}\left(\sin\frac{x}{2}\right)\tan 3x + 3\left(\sec^2 3x\right)\cos\frac{x}{2}$ oe isw	**A1**	
3(i)	$\frac{7-9}{4-(-3)}$ oe or $-\frac{2}{7}$ seen	**M1**	
	$y - 4 = their\left(-\frac{2}{7}\right)(x-6)$ or $y = their\left(-\frac{2}{7}\right)x + c$ and *their* $c = \frac{40}{7}$ oe	**M1**	
	$2x + 7y = 40$ oe	**A1**	
3(ii)	$\sqrt{their\left(\frac{40}{2}\right)^2 + their\left(\frac{40}{7}\right)^2}$	**M1**	**FT** *their* equation from part **(i)**
	20.8[00…]	**A1**	
4(i)	Correct graph	**B3**	**B1** Correct sinusoidal shape with midline at $y = 4$ **B1**Two cycles **B1** Correct amplitude **Maximum of 2 marks if not fully correct**
4(ii)	180°	**B1**	

Question	Answer	Marks	Partial Marks
4(iii)	3	**B1**	
5(a)(i)	3 by 2 or 3 × 2	**B1**	
5(a)(ii)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$	**B1**	
5(b)	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and **B** is $\mathbf{C}^{-1}$ oe	**B2**	**B1** for either
5(c)	$\frac{1}{9a}\begin{pmatrix} 5 & -4a \\ 1 & a \end{pmatrix}$	**B2**	**B1** for $\frac{1}{9a}\begin{pmatrix} & \\ & \end{pmatrix}$ or $\ldots\times\begin{pmatrix} 5 & -4a \\ 1 & a \end{pmatrix}$
6(i)	$9(3x-5)^{2}-2$ isw	**B2**	**B1** for $k(3x-5)^{2}$ $k \neq 9$ seen
	$54(3x-5)^{[1]}$ isw	**B2**	**B1** for $k(3x-5)^{[1]}$ $k \neq 54$ seen
6(ii)	Solves *their* $9(3x-5)^{2}-2=0$	**M1**	
	$[x=]$ $\frac{5}{3}\pm\frac{\sqrt{2}}{9}$ or exact equivalent	**A1**	
6(iii)	Substitutes *their* $\frac{5}{3}+\frac{\sqrt{2}}{9}$ or *their* $\frac{5}{3}-\frac{\sqrt{2}}{9}$ into *their* $54(3x-5)^{[1]}$ and considers sign of result	**M1**	
	When $x=\frac{5}{3}+\frac{\sqrt{2}}{9}$ $\frac{d^{2}y}{dx^{2}}>0$ so minimum and when $x=\frac{5}{3}-\frac{\sqrt{2}}{9}$ $\frac{d^{2}y}{dx^{2}}<0$ so maximum	**A1**	

Question	Answer	Marks	Partial Marks
7(i)	$\left[AD = BC =\right] 35$ soi	**B1**	
	Valid method for finding *DC*	**M1**	
	$\left[DC =\right] 19.2836...$	**A1**	
	$50 \times \frac{4\pi}{9}$ oe	**M1**	
	$35 + 35 + 19.2836\ldots + 50 \times \frac{4\pi}{9}$ = 159 or awrt 159 isw	**A1**	
7(ii)	Sector – triangle: $\frac{1}{2} \times 50^2 \times \frac{4\pi}{9}$	**M1**	or Segment + trapezium : $\frac{1}{2} \times 50^2 \left(\frac{4\pi}{9} - \sin\frac{4\pi}{9}\right)$
	$-\left(\frac{1}{2} \times \mathit{their}15^2 \times \sin\left(\frac{4\pi}{9}\right)\right)$ oe	**M1**	$+\left(\frac{1}{2}(64.2787... + 19.2836...) \times 26.81155\right)$
	1630 or 1634.538… rot to 4 or more figs, isw	**A1**	
8(a)(i)	$p = 2, \quad q = 7, \quad r = -7$	**B3**	**B1** for each or **M1** for any two of $8\left(x^2\right)^7\left(-\frac{1}{px}\right), \frac{8 \times 7}{2}\left(x^2\right)^6\left(-\frac{1}{px}\right)^2,$ $\frac{8 \times 7 \times 6}{3 \times 2}\left(x^2\right)^5\left(-\frac{1}{px}\right)^3$ or better
8(a)(ii)	Valid explanation	**B1**	
8(b)	$\frac{n(n-1)}{2}\left(-\frac{1}{2}\right)^2 [x]$ seen or implied	**B1**	
	$\mathit{their}\left(\frac{n(n-1)}{2}\left(-\frac{1}{2}\right)^2\right) = 30$	**M1**	
	$n^2 - n - 240 = 0$	**A1**	
	16	**A1**	

Question	Answer	Marks	Partial Marks
9	Roots of curve: (4, 0) or (–4, 0) oe	**B1**	
	Intersections: (–3, 7) or (3, 7) oe	**B1**	
	Correct strategy for finding area	**B1**	
	$\int(16-x^2)\,\mathrm{d}x=16x-\frac{x^3}{3}$ or $\int(9-x^2)\,\mathrm{d}x=9x-\frac{x^3}{3}$	**M1**	
	F(*b*) – F(*a*)	**M1**	
	$\frac{148}{3}$ or $49\frac{1}{3}$ or 49.3[33…]	**A1**	
10(i)	$\overrightarrow{PQ}=\mathbf{q}-\mathbf{p}$ $\overrightarrow{QA}=2\mathbf{p}-\mathbf{q}$ $\overrightarrow{PB}=4\mathbf{q}-\mathbf{p}$	**B3**	**B2** for any two correct or **B1** for $\overrightarrow{OA}=2\mathbf{p}$ or $\overrightarrow{OB}=4\mathbf{q}$ soi
10(ii)	$\overrightarrow{PQ}=\lambda(4\mathbf{q}-\mathbf{p})-\mu(2\mathbf{p}-\mathbf{q})$ oe isw	**B2**	**B1** for $\overrightarrow{PQ}=\lambda\overrightarrow{PB}-\mu\overrightarrow{QA}$ soi
10(iii)	For equating the coefficients of **p** or **q** in $\mathbf{q}-\mathbf{p}=\lambda(4\mathbf{q}-\mathbf{p})-\mu(2\mathbf{p}-\mathbf{q})$	**M1**	
	$4\lambda+\mu=1$ oe $\lambda+2\mu=1$ oe	**A1**	**FT** *their* **(ii)** provided in terms of λ, μ, **p** and **q**
	Solves *their* equations in λ and μ	**M1**	
	$\lambda=\frac{1}{7}$, $\mu=\frac{3}{7}$	**A1**	
11(i)	$\left[v=\frac{\mathrm{d}x}{\mathrm{d}t}=\right]\quad 5+\cos t$	**B1**	
	$5+\cos t\neq 0$ (and so never at rest) oe	**B1**	
11(ii)	$x=5\left(\frac{\pi}{3}\right)+\sin\left(\frac{\pi}{3}\right)$ or $x=5\left(\frac{\pi}{2}\right)+\sin\left(\frac{\pi}{2}\right)$ seen	**M1**	
	2.75 to 2.752	**A1**	

Question	Answer	Marks	Partial Marks
11(iii)	$\left[a = \frac{\mathrm{d}v}{\mathrm{d}t} = \right] -\sin t$	**M1**	**FT** *their* v provided of the form $k \pm \cos t$
	$[t = 4,\ a = -\sin 4 =]$ 0.757 or 0.7568[024…]	**A1**	
11(iv)	Valid method soi e.g. $their(-\sin t) = 0$ or $\cos t = -1$ sketch of $v = 5 + \cos t$	**M1**	
	$t = \pi$	**A1**	
12	Eliminates y: $10x^3 - 19x^2 - x = 4x - 6$ oe	**M1**	
	Collects terms: $10x^3 - 19x^2 - 5x + 6 = 0$	**A1**	
	$x - 2$ is a factor soi	**B1**	
	Correct quadratic factor: $(10x^2 + x - 3)$	**B2**	**B1** for any two correct elements of quadratic factor
	Factorises *their* 3-term quadratic or solves *their* 3-term quadratic = 0: $(5x + 3)(2x - 1)$ oe	**M1**	
	(–0.6, –8.4), (0.5, –4)	**A2**	**A1** for $x = -0.6$, $x = 0.5$ or $y = -8.4$, $y = -4$ nfww or for one correct (x, y) pair
	$\left(\frac{their(-0.6 + 0.5)}{2}, \frac{their(-8.4 + -4)}{2}\right)$ oe	**M1**	
	$(-0.05, -6.2)$ oe	**A1**	

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CANDIDATE NAME

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2049112309

ADDITIONAL MATHEMATICS **0606/11**

Paper 1 **May/June 2019**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **15** printed pages and **1** blank page.

DC (CE/CB) 165273/2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

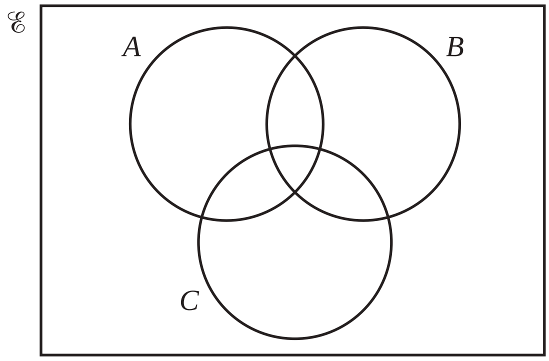
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

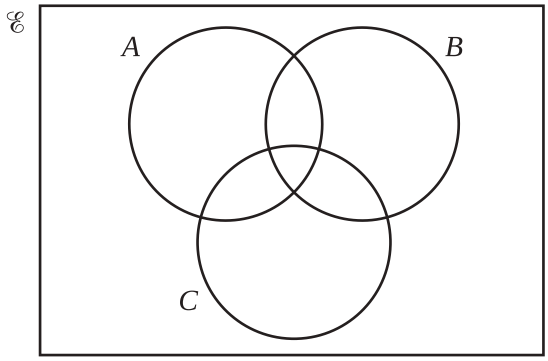
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** On the Venn diagrams below, shade the region indicated.



$(A \cap B) \cup C$



$(A' \cup B) \cap C$

[2]

(b) On the Venn diagram below, draw sets P, Q and R such that

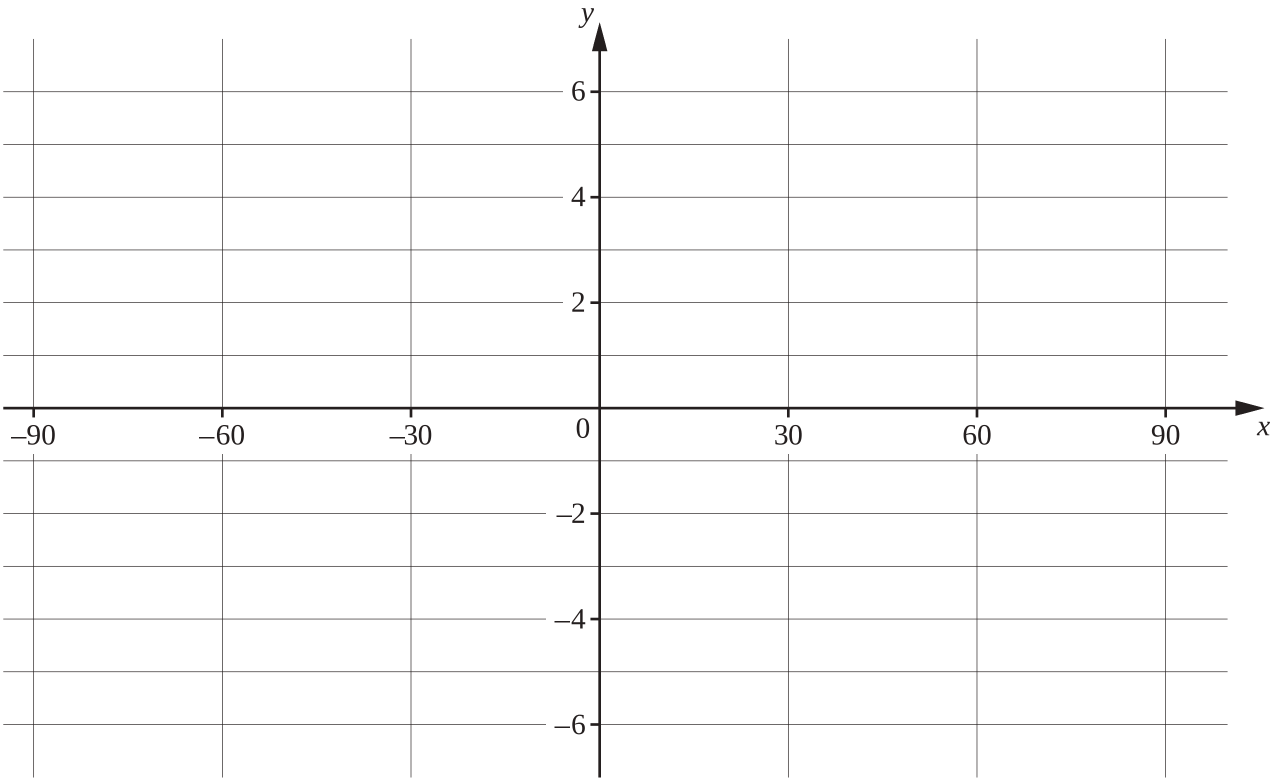
$$P \subset R,\ \ Q \subset R \quad \text{and} \quad P \cap Q = \varnothing.$$

[2]

2 **(i)** Write down the amplitude of $4\sin 3x - 1$. [1]

(ii) Write down the period of $4\sin 3x - 1$. [1]

(iii) On the axes below, sketch the graph of $y = 4\sin 3x - 1$ for $-90° \leqslant x° \leqslant 90°$.



[3]

3 The polynomial $\mathrm{p}(x) = (2x-1)(x+k)-12$, where k is a constant.

(i) Write down the value of $\mathrm{p}(-k)$. [1]

When $\mathrm{p}(x)$ is divided by $x+3$ the remainder is 23.

(ii) Find the value of k. [2]

(iii) Using your value of k, show that the equation $\mathrm{p}(x) = -25$ has no real solutions. [3]

4 **(i)** The first 3 terms, in ascending powers of x, in the expansion of $\left(2+bx\right)^8$ can be written as $a + 256x + cx^2$. Find the value of each of the constants a, b and c. [4]

(ii) Using the values found in **part (i)**, find the term independent of x in the expansion of $\left(2+bx\right)^8\left(2x-\dfrac{3}{x}\right)^2$. [3]

5 A particle P is moving with a velocity of $20\,\text{ms}^{-1}$ in the same direction as $\begin{pmatrix}3\\4\end{pmatrix}$.

(i) Find the velocity vector of P. [2]

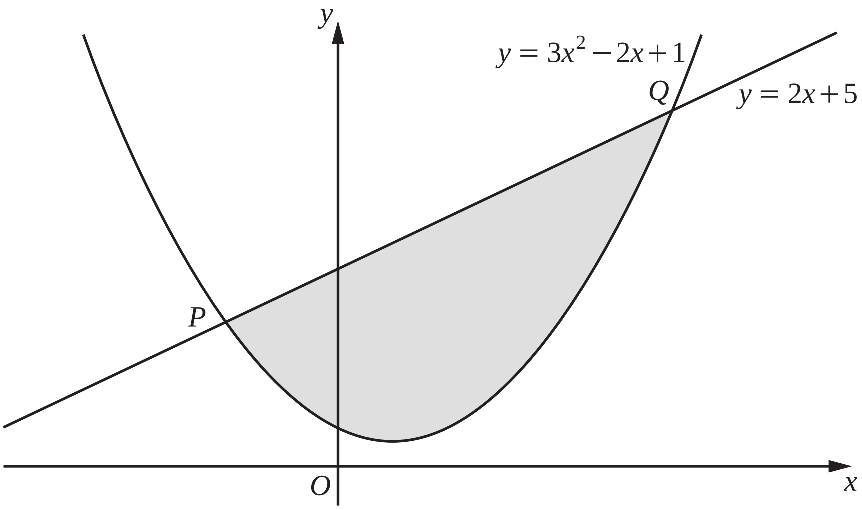
At time $t = 0\,\text{s}$, P has position vector $\begin{pmatrix}1\\2\end{pmatrix}$ relative to a fixed point O.

(ii) Write down the position vector of P after t s. [2]

A particle Q has position vector $\begin{pmatrix}17\\18\end{pmatrix}$ relative to O at time $t = 0\,\text{s}$ and has a velocity vector $\begin{pmatrix}8\\12\end{pmatrix}\,\text{ms}^{-1}$.

(iii) Given that P and Q collide, find the value of t when they collide and the position vector of the point of collision. [3]

6



The diagram shows the curve $y = 3x^2 - 2x + 1$ and the straight line $y = 2x + 5$ intersecting at the points P and Q. Showing all your working, find the area of the shaded region. [8]

7 **(a)** Solve $\log_3 x + \log_9 x = 12$. [3]

(b) Solve $\log_4(3y^2 - 10) = 2\log_4(y-1) + \frac{1}{2}$. [5]

8 It is given that $f(x) = 5e^x - 1$ for $x \in \mathbb{R}$.

(i) Write down the range of f. [1]

(ii) Find f^{-1} and state its domain. [3]

It is given also that $g(x) = x^2 + 4$ for $x \in \mathbb{R}$.

(iii) Find the value of fg(1). [2]

(iv) Find the exact solutions of $\mathrm{g}^2(x) = 40$. [3]

9 In this question all lengths are in centimetres.

A closed cylinder has base radius r, height h and volume V. It is given that the total surface area of the cylinder is 600π and that V, r and h can vary.

(i) Show that $V = 300\pi r - \pi r^3$. [3]

(ii) Find the stationary value of V and determine its nature. [5]

10 When lg y is plotted against x^2 a straight line graph is obtained which passes through the points $(2, 4)$ and $(6, 16)$.

(i) Show that $y = 10^{A+Bx^2}$, where A and B are constants. [4]

(ii) Find y when $x = \dfrac{1}{\sqrt{3}}$. [2]

(iii) Find the positive value of x when $y = 2$. [3]

11 It is given that $y = \left(x^2+1\right)\left(2x-3\right)^{\frac{1}{2}}$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{Px^2+Qx+1}{\left(2x-3\right)^{\frac{1}{2}}}$, where P and Q are integers. [5]

(ii) Hence find the equation of the normal to the curve $y = \left(x^2 + 1\right)\left(2x - 3\right)^{\frac{1}{2}}$ at the point where $x = 2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [4]

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ADDITIONAL MATHEMATICS **0606/12**

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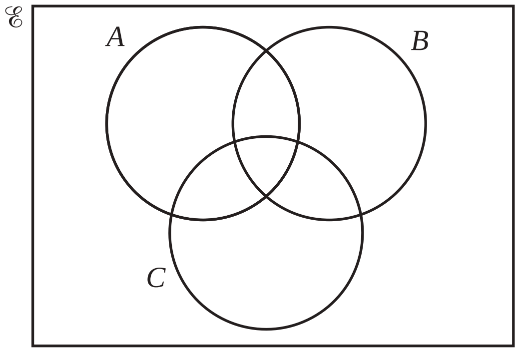
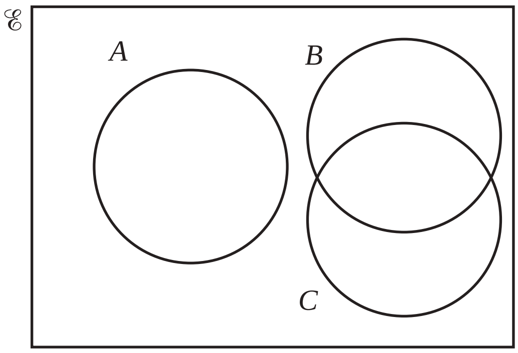
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** On the Venn diagrams below, shade the region indicated.



$A' \cap B' \cap C'$



$A \cup (B \cap C)$

[2]

(b)

$$\mathscr{E} = \{x : 0° \leqslant x \leqslant 360°\}$$

$$P = \{x : \cos 2x = 0.5\}$$

$$Q = \{x : \sin x = 0.5\}$$

Find $P \cap Q$. [3]

2 Do not use a calculator in this question.

Find the coordinates of the points of intersection of the curve $y = (2x+3)^2(x-1)$ and the line $y = 3(2x+3)$. [5]

3 The number, B, of a certain type of bacteria at time t days can be described by $B = 200e^{2t} + 800e^{-2t}$.

(i) Find the value of B when $t = 0$. [1]

(ii) At the instant when $\frac{dB}{dt} = 1200$, show that $e^{4t} - 3e^{2t} - 4 = 0$. [3]

(iii) Using the substitution $u = e^{2t}$, or otherwise, solve $e^{4t} - 3e^{2t} - 4 = 0$. [2]

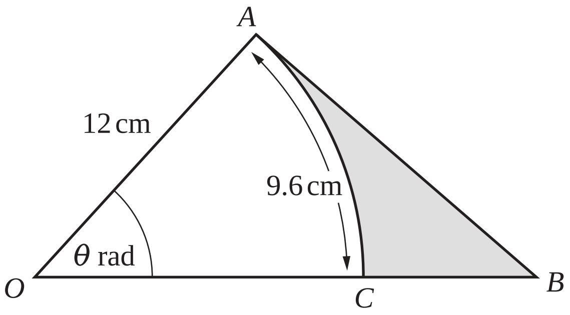
4 **(a)** Given that $\dfrac{\left(pr^2\right)^{\frac{3}{2}}\sqrt{qr}}{q^2\left(pr^2\right)^{-1}}$ can be written in the form $p^aq^br^c$, find the value of each of the constants a, b and c. [3]

(b) Solve

$$3x^{\frac{1}{2}} - y^{-\frac{1}{2}} = 4,$$
$$4x^{\frac{1}{2}} + 3y^{-\frac{1}{2}} = 14.$$

[3]

5



The diagram shows the right-angled triangle *OAB*. The point *C* lies on the line *OB*. Angle $OAB = \frac{\pi}{2}$ radians and angle $AOB = \theta$ radians. *AC* is an arc of the circle, centre *O*, radius 12 cm and *AC* has length 9.6 cm.

(i) Find the value of θ. [2]

(ii) Find the area of the shaded region. [4]

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6 **(a)** Eight books are to be arranged on a shelf. There are 4 mathematics books, 3 geography books and 1 French book.

(i) Find the number of different arrangements of the books if there are no restrictions. [1]

(ii) Find the number of different arrangements if the mathematics books have to be kept together. [3]

(iii) Find the number of different arrangements if the mathematics books have to be kept together and the geography books have to be kept together. [3]

(b) A team of 6 players is to be chosen from 8 men and 4 women. Find the number of different ways this can be done if

(i) there are no restrictions, [1]

(ii) there is at least one woman in the team. [2]

7 A pilot wishes to fly his plane from a point *A* to a point *B* on a bearing of 055°. There is a wind blowing at 120 km h^{-1} from the west. The plane can fly at 650 km h^{-1} in still air.

(i) Find the direction in which the pilot must fly his plane in order to reach *B*. [4]

(ii) Given that the distance between *A* and *B* is 1250 km, find the time it will take the pilot to fly from *A* to *B*. [4]

8 When e^y is plotted against $\frac{1}{x}$, a straight line graph passing through the points $(2, 20)$ and $(4, 8)$ is obtained.

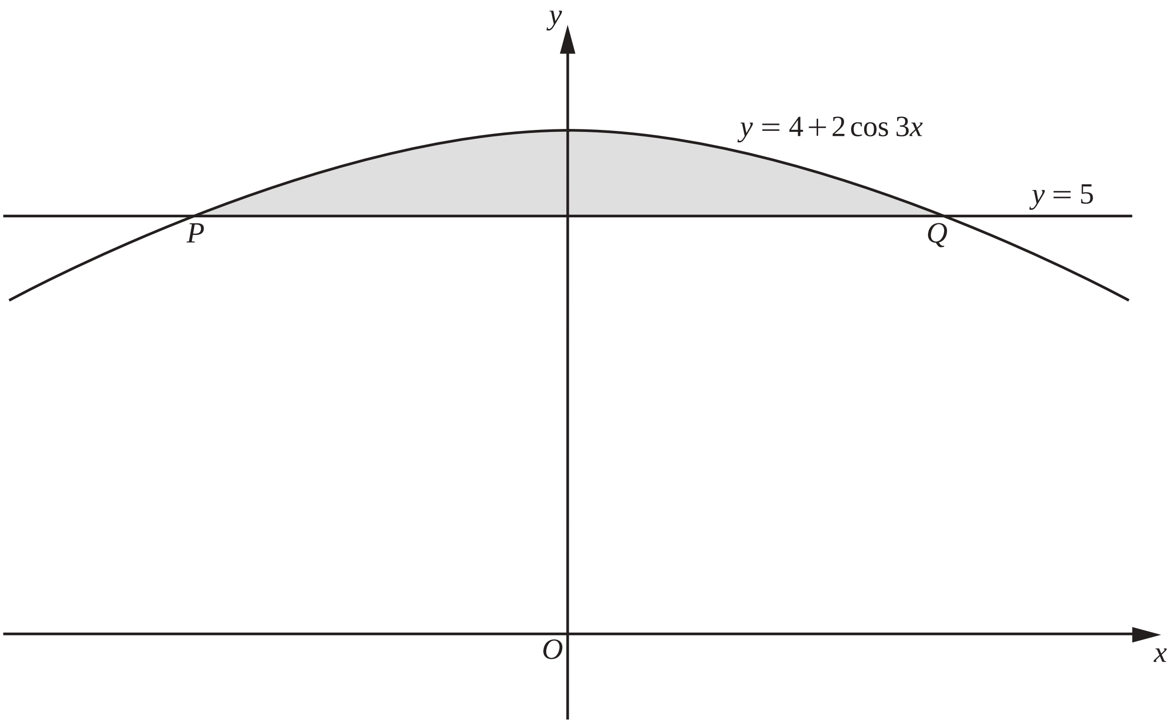
(i) Find y in terms of x. [5]

(ii) Hence find the positive values of x for which y is defined. [1]

(iii) Find the exact value of y when $x = 3$. [1]

(iv) Find the exact value of x when $y = 2$. [2]

9

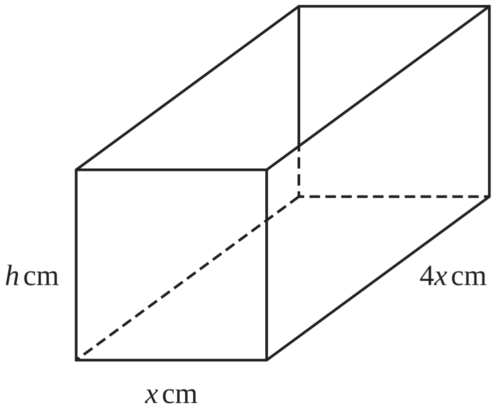


The diagram shows the curve $y = 4 + 2\cos 3x$ intersecting the line $y = 5$ at the points P and Q.

(i) Find, in terms of π, the x-coordinate of P and of Q. [3]

(ii) Find the exact area of the shaded region. You must show all your working. [6]

10



The diagram shows an open container in the shape of a cuboid of width x cm, length $4x$ cm and height h cm. The volume of the container is 800 cm^3.

(i) Show that the external surface area, $S\,\text{cm}^2$, of the open container is such that $S = 4x^2 + \dfrac{2000}{x}$. [4]

(ii) Given that x can vary, find the stationary value of S and determine its nature. [5]

Question 11 is printed on the next page.

11 The normal to the curve $y=(x-2)(3x+1)^{\frac{2}{3}}$ at the point where $x=\frac{7}{3}$, meets the y-axis at the point P. Find the exact coordinates of the point P. [7]

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Cambridge International General Certificate of Secondary Education

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

6938333361

ADDITIONAL MATHEMATICS **0606/13**

Paper 1 **May/June 2019**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
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DO **NOT** WRITE IN ANY BARCODES.

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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **16** printed pages.

DC (KN/TP) 165276/2

Cambridge Assessment International Education

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\mathrm{cosec}^2 A = 1 + \cot^2 A$$

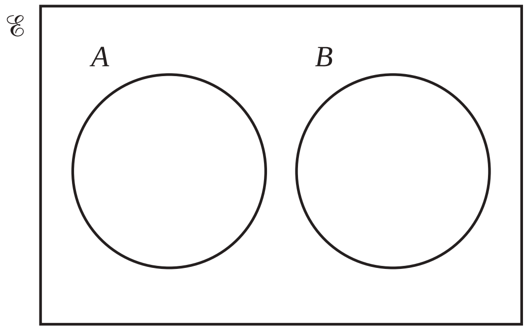
Formulae for ΔABC

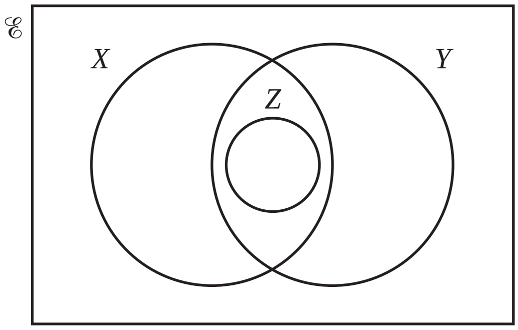
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Describe, using set notation, the relationship between the sets shown in each of the Venn diagrams below.





....

[3]

2 Given that $\frac{\sqrt{p}\left(qr\right)^{-2}}{p^2q^{\frac{1}{3}}r}=\frac{1}{p^aq^br^c}$, find the value of each of the constants a, b and c. [3]

3 Show that the line $y = mx + 4$ will touch or intersect the curve $y = x^2 + 3x + m$ for all values of m. [4]

4 It is given that $y = \dfrac{\ln(2x^3+5)}{x-1}$ for $x > 1$.

(i) Find the value of $\dfrac{dy}{dx}$ when $x = 2$. You must show all your working. [4]

(ii) Find the approximate change in y as x increases from 2 to $2+p$, where p is small. [1]

5 **(i)** On the axes below, sketch the graph of $y = \left|3x^2 - 14x - 5\right|$, showing the coordinates of the points where the graph meets the coordinate axes.

[4]

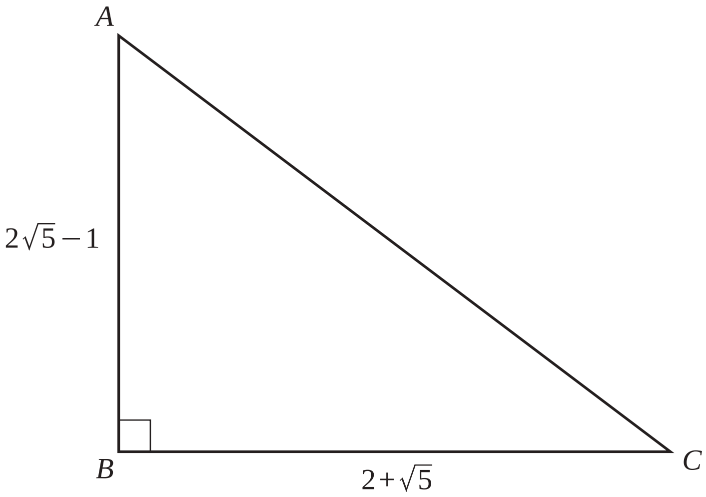
(ii) Find the exact value of k such that $\left|3x^2 - 14x - 5\right| = k$ has 3 solutions only. [3]

6 **(a)** **(i)** Show that $\sec\theta - \dfrac{\tan\theta}{\operatorname{cosec}\theta} = \cos\theta$. [3]

(ii) Solve $\sec 2\theta - \dfrac{\tan 2\theta}{\operatorname{cosec} 2\theta} = \dfrac{\sqrt{3}}{2}$ for $0° \leqslant \theta \leqslant 180°$. [3]

(b) Solve $2\sin^2\left(\phi + \dfrac{\pi}{3}\right) = 1$ for $0 < \phi < 2\pi$ radians. [4]

7 **Do not use a calculator in this question.**
In this question, all lengths are in centimetres.



The diagram shows the triangle *ABC* such that $AB = 2\sqrt{5}-1$, $BC = 2+\sqrt{5}$ and angle $ABC = 90^\circ$.

(i) Find the exact length of *AC*. [3]

(ii) Find $\tan ACB$, giving your answer in the form $p+q\sqrt{r}$, where p, q and r are integers. [3]

(iii) Hence find $\sec^2 ACB$, giving your answer in the form $s+t\sqrt{u}$ where s, t and u are integers. [2]

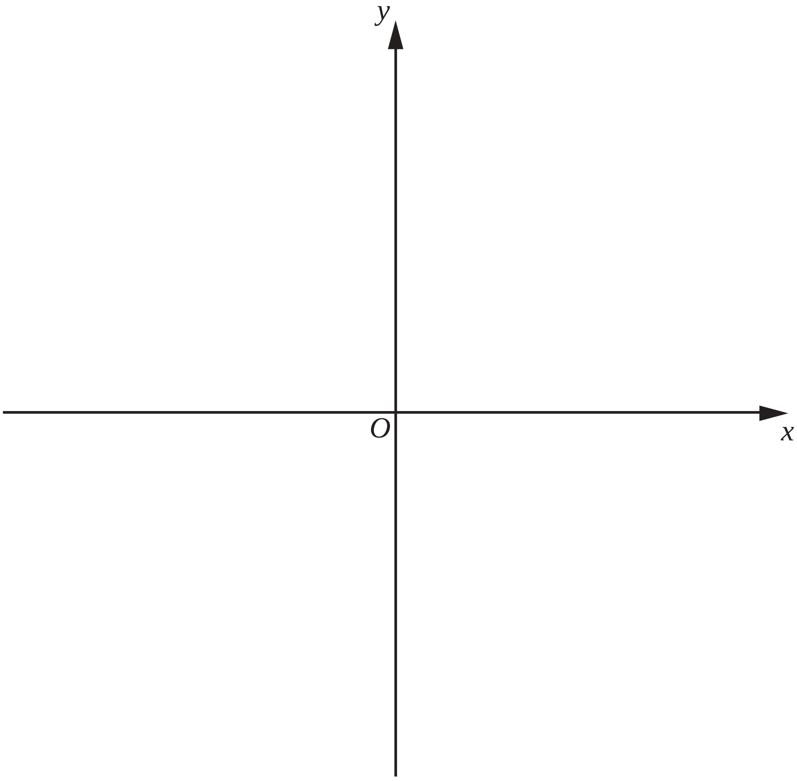
8 $\mathrm{f} : x \mapsto \mathrm{e}^{3x}$ for $x \in \mathbb{R}$
$\mathrm{g} : x \mapsto 2x^2 + 1$ for $x \geqslant 0$

(i) Write down the range of g. [1]

(ii) Show that $\mathrm{f}^{-1}\mathrm{g}(\sqrt{62}) = \ln 5$. [3]

(iii) Solve $\mathrm{f}'(x) = 6\mathrm{g}''(x)$, giving your answer in the form $\ln a$, where a is an integer. [3]

(iv) On the axes below, sketch the graph of $y = g$ and the graph of $y = g^{-1}$, showing the points where the graphs meet the coordinate axes.



[3]

9 **(a)** Jack has won 7 trophies for sport and wants to arrange them on a shelf. He has 2 trophies for cricket, 4 trophies for football and 1 trophy for swimming. Find the number of different arrangements if

(i) there are no restrictions, [1]

(ii) the football trophies are to be kept together, [3]

(iii) the football trophies are to be kept together and the cricket trophies are to be kept together. [3]

(b) A team of 8 players is to be chosen from 6 girls and 8 boys. Find the number of different ways the team may be chosen if

(i) there are no restrictions, [1]

(ii) all the girls are in the team, [1]

(iii) at least 1 girl is in the team. [2]

10 A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = (2x+3)^{-\frac{1}{2}}$. The curve has a gradient of 5 at the point where $x = 3$ and passes through the point $\left(\frac{1}{2}, -\frac{1}{3}\right)$.

(i) Find the equation of the curve. [7]

(ii) Find the equation of the normal to the curve at the point where $x = 3$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [4]

Question 11 is printed on the next page.

11 A pilot wishes to fly his plane from a point *A* to a point *B*. The bearing of *B* from *A* is 050°. A wind is blowing from the north at a speed of $120\,\text{km}\,\text{h}^{-1}$. The plane can fly at $600\,\text{km}\,\text{h}^{-1}$ in still air.

(i) Find the bearing on which the pilot must fly his plane in order to reach *B*. [4]

(ii) Given that the distance from *A* to *B* is 2500 km, find the time taken to fly from *A* to *B*. [4]

Cambridge Assessment International Education
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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8109129254

ADDITIONAL MATHEMATICS **0606/21**

Paper 2 **May/June 2019**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **15** printed pages and **1** blank page.

DC (NF/TP) 165272/1

Cambridge Assessment International Education

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1} b + \binom{n}{2}a^{n-2} b^2 + \ldots + \binom{n}{r}a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

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$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

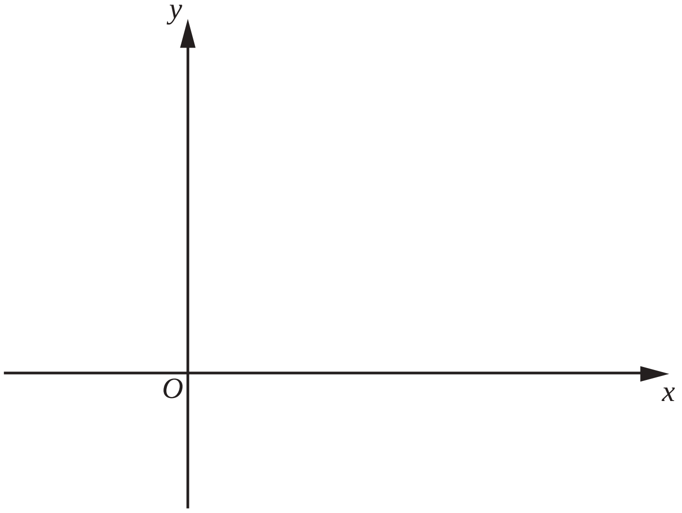
1 Find the values of x for which $x(6x + 7) \geqslant 20$. [3]

2 Two variables x and y are such that $y = \frac{\ln x}{x^3}$ for $x > 0$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1-3\ln x}{x^4}$. [3]

(ii) Hence find the approximate change in y as x increases from e to e + h, where h is small. [2]

3 **(i)** Sketch the graph of $y = |5x - 3|$ on the axes below, showing the coordinates of the points where the graph meets the coordinate axes.

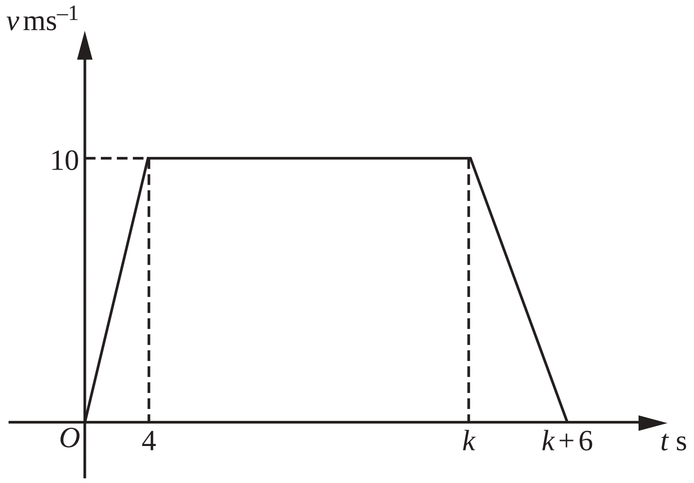


[3]

(ii) Solve the equation $|5x - 3| = 2 - x$. [3]

4 **Without using a calculator**, express $\dfrac{\left(\sqrt{5}-3\right)^2}{\sqrt{5}+1}$ in the form $p\sqrt{5}+q$, where p and q are integers. [4]

5



The velocity-time graph represents the motion of a particle travelling in a straight line.

(i) Find the acceleration during the last 6 seconds of the motion. [1]

(ii) The particle travels with constant velocity for 23 seconds. Find the value of k. [1]

(iii) Using your answer to **part (ii)**, find the total distance travelled by the particle. [3]

6 **(a)** $\mathbf{A} = \begin{pmatrix} x+3 & -x \\ 2x & x-3 \end{pmatrix}$

Given that **A** does not have an inverse, find the exact values of x. [3]

(b) $\mathbf{B} = \begin{pmatrix} 0 & 3 \\ -4 & 1 \\ 5 & 2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 0 & 1 & 2 \\ 3 & -4 & 5 \end{pmatrix}$

(i) Write down the order of matrix **B**. [1]

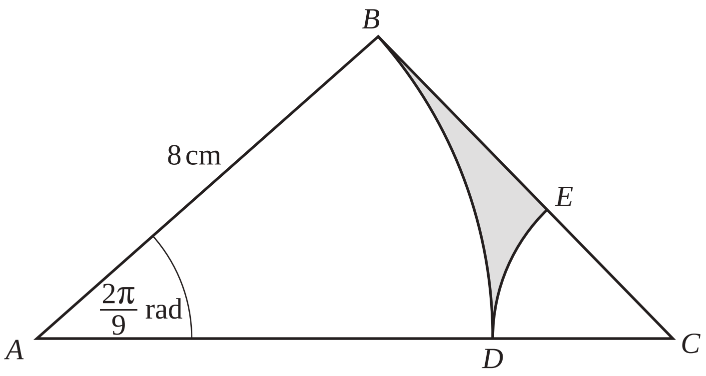
(ii) The matrix $\mathbf{BC} = \begin{pmatrix} 9 & -12 & 15 \\ 3 & -8 & -3 \\ 6 & -3 & 20 \end{pmatrix}$. Explain why $\mathbf{CB} \neq \mathbf{BC}$. [2]

7 The variables x, y and u are such that $y = \tan u$ and $x = u^3 + 1$.

(i) State the rate of change of y with respect to u. [1]

(ii) Hence find the rate of change of y with respect to x, giving your answer in terms of x. [4]

8



The diagram shows a right-angled triangle ABC with $AB = 8\,\text{cm}$ and angle $ABC = \frac{\pi}{2}$ radians. The points D and E lie on AC and BC respectively. BAD and ECD are sectors of the circles with centres A and C respectively. Angle $BAD = \frac{2\pi}{9}$ radians.

(i) Find the area of the shaded region. [6]

(ii) Find the perimeter of the shaded region. [3]

9 (a) Eleven different television sets are to be displayed in a line in a large shop.

(i) Find the number of different ways the televisions can be arranged. [1]

Of these television sets, 6 are made by company *A* and 5 are made by company *B*.

(ii) Find the number of different ways the televisions can be arranged so that no two sets made by company *A* are next to each other. [2]

(b) A group of people is to be selected from 5 women and 3 men.

(i) Calculate the number of different groups of 4 people that have exactly 3 women. [2]

(ii) Calculate the number of different groups of at most 4 people where the number of women is the same as the number of men. [2]

10 Solutions to this question by accurate drawing will not be accepted.

The points A and B have coordinates $(p, 3)$ and $(1, 4)$ respectively and the line L has equation $3x + y = 2$.

(i) Given that the gradient of AB is $\frac{1}{3}$, find the value of p. [2]

(ii) Show that L is the perpendicular bisector of AB. [3]

(iii) Given that $C(q, -10)$ lies on L, find the value of q. [1]

(iv) Find the area of triangle ABC. [2]

11 **(a)** **(i)** Show that $\frac{\operatorname{cosec}\theta - \cot\theta}{\sin\theta} = \frac{1}{1+\cos\theta}$. [4]

(ii) Hence solve $\frac{\operatorname{cosec}\theta - \cot\theta}{\sin\theta} = \frac{5}{2}$ for $180° < \theta < 360°$. [2]

(b) Solve $\tan\left(3\phi - 4\right) = -\frac{1}{2}$ for $0 \leqslant \phi \leqslant \frac{\pi}{2}$ radians. [3]

12 **(a)** Given that $\int_0^a \mathrm{e}^{2x}\,\mathrm{d}x = 50$, find the exact value of a. You must show all your working. [4]

(b) A curve is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = 3 - 2\cos 5x$. The curve passes through the point $\left(\frac{\pi}{5}, \frac{8\pi}{5}\right)$.

(i) Find the equation of the curve. [4]

(ii) Find $\int y\mathrm{d}x$ and hence evaluate $\int_{\frac{\pi}{2}}^{\pi} y\mathrm{d}x$. [5]

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Cambridge Assessment International Education
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME

CENTRE NUMBER ☐☐☐☐☐ CANDIDATE NUMBER ☐☐☐☐

3034680663

ADDITIONAL MATHEMATICS **0606/22**

Paper 2 **May/June 2019**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

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Write in dark blue or black pen.
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DC (LK) 165275/2

Cambridge Assessment International Education

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

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$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Given that $y = \dfrac{\sin x}{\ln x^2}$, find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [4]

2 Find the values of k for which the equation $(k-1)x^2 + kx - k = 0$ has real and distinct roots. [4]

3 **(i)** Given that $x-2$ is a factor of ax^3-12x^2+5x+6, use the factor theorem to show that $a=4$. [2]

(ii) Showing all your working, factorise $4x^3-12x^2+5x+6$ and hence solve $4x^3-12x^2+5x+6=0$. [4]

4 A circle has diameter x which is increasing at a constant rate of $0.01\,\text{cm}\,\text{s}^{-1}$. Find the exact rate of change of the area of the circle when $x = 6\,\text{cm}$. [5]

5 **(i)** Express $5x^2 - 15x + 1$ in the form $p(x+q)^2 + r$, where p, q and r are constants. [3]

(ii) Hence state the least value of $x^2 - 3x + 0.2$ and the value of x at which this occurs. [2]

6 **(a)** State the order of the matrix $\begin{pmatrix} 0 & 1 & 4 & 8 \\ 5 & 8 & 1 & 6 \end{pmatrix}$. [1]

(b) $\mathbf{A} = \begin{pmatrix} 2 & -4 \\ -1 & 3 \end{pmatrix}$

(i) Find $\mathbf{A}^{-1}$. [2]

(ii) Hence, given that $\mathbf{ABA} = \mathbf{I}$, find the matrix $\mathbf{B}$. [3]

7 **(a)** Solve $\lg(x^2-3)=0$. [2]

(b) **(i)** Show that, for $a>0$, $\dfrac{\ln a^{\sin(2x+5)}+\ln\left(\dfrac{1}{a}\right)}{\ln a}$ may be written as $\sin(2x+5)+k$, where k is an integer. [3]

(ii) Hence find $\displaystyle\int \frac{\ln a^{\sin(2x+5)}+\ln\left(\dfrac{1}{a}\right)}{\ln a}\,\mathrm{d}x$. [3]

8 **(a)** In the binomial expansion of $\left(a-\frac{x}{2}\right)^6$, the coefficient of x^3 is 120 times the coefficient of x^5. Find the possible values of the constant a. [4]

(b) **(i)** Expand $(1+2x)^{20}$ in ascending powers of x, as far as the term in x^3. Simplify each term. [2]

(ii) Use your expansion to show that the value of 0.98^{20} is 0.67 to 2 decimal places. [2]

9 **(a)** Solve $6\sin^2 x - 13\cos x = 1$ for $0° \leqslant x \leqslant 360°$. [4]

(b) **(i)** Show that, for $-\frac{\pi}{2} < y < \frac{\pi}{2}$, $\frac{4\tan y}{\sqrt{1+\tan^2 y}}$ can be written in the form $a\sin y$, where a is an integer. [3]

(ii) Hence solve $\frac{4\tan y}{\sqrt{1+\tan^2 y}} + 3 = 0$ for $-\frac{\pi}{2} < y < \frac{\pi}{2}$ radians. [1]

10 **(a)** Find the unit vector in the direction of $5\mathbf{i} - 15\mathbf{j}$. [2]

(b) The position vectors of points A and B relative to an origin O are $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 12 \\ 7 \end{pmatrix}$ respectively. The point C lies on AB such that $AC : CB$ is 2 : 1.

(i) Find the position vector of C relative to O. [3]

The point D lies on OB such that $OD : OB$ is $1 : \lambda$ and $\overrightarrow{DC} = \begin{pmatrix} 6 \\ 1.25 \end{pmatrix}$.

(ii) Find the value of λ. [3]

11 The velocity, v m s^{-1}, of a particle travelling in a straight line, t seconds after passing through a fixed point O, is given by $v = \dfrac{4}{(t+1)^3}$.

(i) Explain why the direction of motion of the particle never changes. [1]

(ii) Showing all your working, find the acceleration of the particle when $t = 5$. [3]

(iii) Find an expression for the displacement of the particle from O after t seconds. [3]

(iv) Find the distance travelled by the particle in the fourth second. [2]

12 **(a)** The functions f and g are defined by

$$\mathrm{f}(x) = 5x-2 \quad \text{for } x > 1,$$
$$\mathrm{g}(x) = 4x^2-9 \quad \text{for } x > 0.$$

(i) State the range of g. [1]

(ii) Find the domain of gf. [1]

(iii) Showing all your working, find the exact solutions of $\mathrm{gf}(x) = 4$. [3]

Question 12(b) is printed on the next page.

(b) The function h is defined by $\mathrm{h}(x)=\sqrt{x^2-1}$ for $x \leqslant -1$.

(i) State the geometrical relationship between the graphs of $y=\mathrm{h}(x)$ and $y=\mathrm{h}^{-1}(x)$. [1]

(ii) Find an expression for $\mathrm{h}^{-1}(x)$. [3]

Cambridge Assessment International Education
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME	

CENTRE NUMBER		CANDIDATE NUMBER	

9164116781

ADDITIONAL MATHEMATICS **0606/23**

Paper 2 **May/June 2019**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

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DC (SC/CB) 165277/2

Cambridge Assessment International Education

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

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$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Find the values of x for which $9x^2 + 18x - 1 < x + 1$. [3]

2 Differentiate $\tan 3x \cos \frac{x}{2}$ with respect to x. [4]

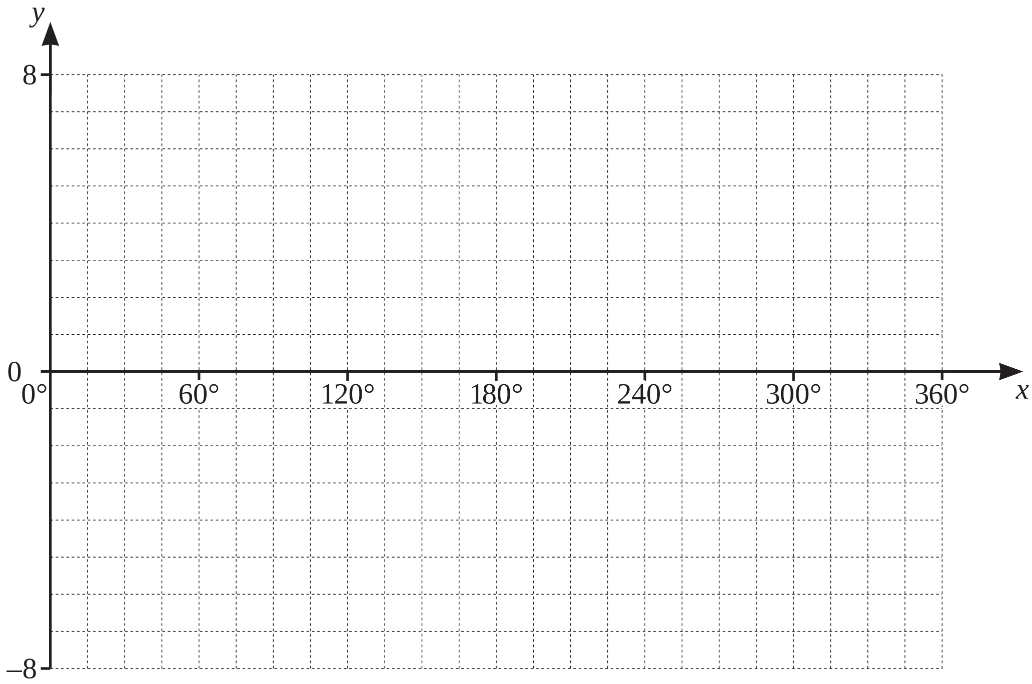
3 The points A, B and C have coordinates $(4, 7)$, $(-3, 9)$ and $(6, 4)$ respectively.

(i) Find the equation of the line, L, that is parallel to the line AB and passes through C. Give your answer in the form $ax + by = c$, where a, b and c are integers. [3]

(ii) The line L meets the x-axis at the point D and the y-axis at the point E. Find the length of DE. [2]

4 The function f is defined, for $0° \leqslant x \leqslant 360°$, by $\mathrm{f}(x) = 4 + 3\sin 2x$.

(i) Sketch the graph of $y = \mathrm{f}(x)$ on the axes below.



[3]

(ii) State the period of f. [1]

(iii) State the amplitude of f. [1]

5 **(a)** Given that $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 0 & -1 \\ 6 & 4 \end{pmatrix}$ and that $\mathbf{A} + \mathbf{O} = \mathbf{A}$,

(i) state the order of the matrix **A**, [1]

(ii) write down the matrix **O**. [1]

(b) $\mathbf{B} = \begin{pmatrix} 1 & -1 \\ 3 & 2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 0.4 & 0.2 \\ -0.6 & 0.2 \end{pmatrix}$.

Find the matrix product **BC** and state a relationship between **B** and **C**. [2]

(c) $\mathbf{D} = \begin{pmatrix} a & 4a \\ -1 & 5 \end{pmatrix}$, where a is a positive integer. Find $\mathbf{D}^{-1}$ in terms of a. [2]

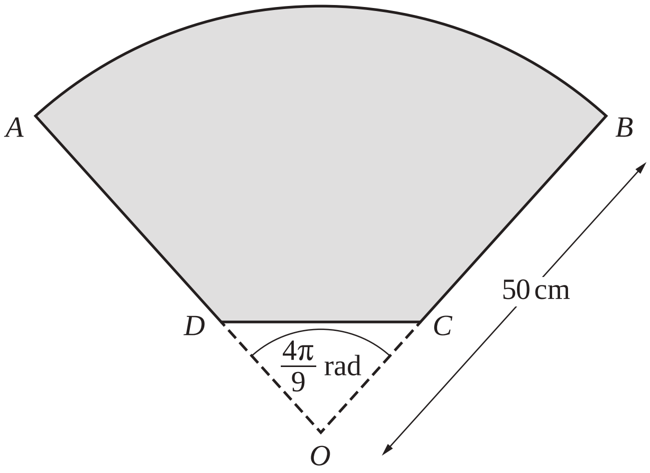
6 A curve has equation $y = (3x-5)^3 - 2x$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [4]

(ii) Find the exact value of the x-coordinate of each of the stationary points of the curve. [2]

(iii) Use the second derivative test to determine the nature of each of the stationary points. [2]

7



The diagram shows a company logo, *ABCD*. The logo is part of a sector, *AOB*, of a circle, centre *O* and radius 50 cm. The points *C* and *D* lie on *OB* and *OA* respectively. The lengths *AD* and *BC* are equal and *AD* : *AO* is 7 : 10. The angle *AOB* is $\frac{4\pi}{9}$ radians.

(i) Find the perimeter of *ABCD*. [5]

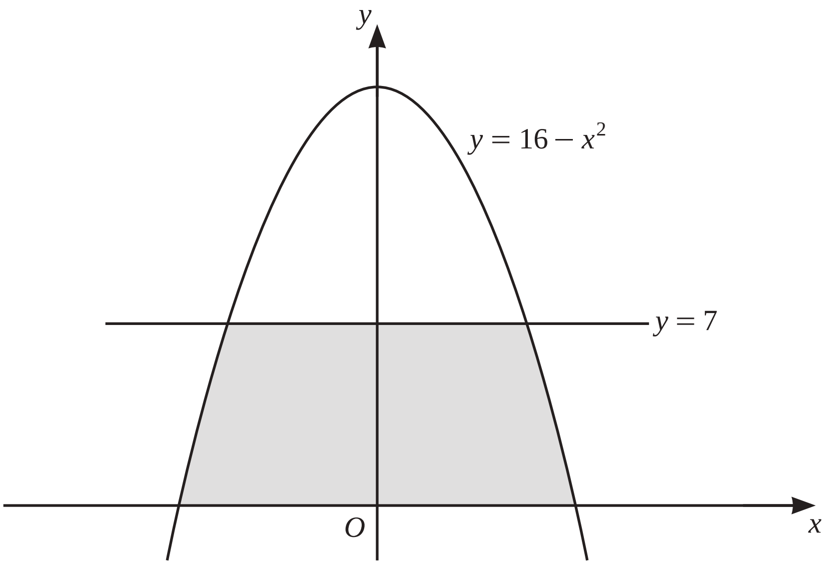
(ii) Find the area of *ABCD*. [3]

8 **(a)** **(i)** Given that $\left(x^2-\frac{1}{px}\right)^8 = x^{16}-4x^{13}+qx^{10}+rx^7+\ldots,$ find the value of each of the constants p, q and r. [3]

(ii) Explain why there is no term independent of x in the binomial expansion of $\left(x^2-\frac{1}{px}\right)^8$. [1]

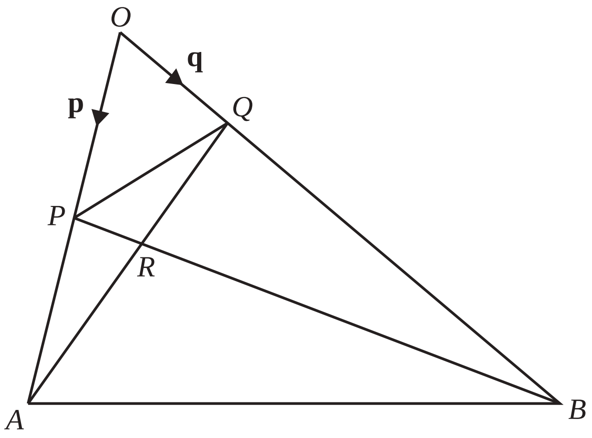
(b) In the binomial expansion of $\left(1-\frac{\sqrt{x}}{2}\right)^n$, where n is a positive integer, the coefficient of x is 30. Form an equation in n and hence find the value of n. [4]

9



The diagram shows the curve $y = 16 - x^2$ and the straight line $y = 7$. Find the area of the shaded region. You must show all your working. [6]

10



The diagram shows a triangle *OAB*. The point *P* is the midpoint of *OA* and the point *Q* lies on *OB* such that $\overrightarrow{OQ} = \frac{1}{4}\overrightarrow{OB}$. The position vectors of *P* and *Q* relative to *O* are **p** and **q** respectively.

(i) Find, in terms of **p** and **q**, an expression for each of the vectors $\overrightarrow{PQ}$, $\overrightarrow{QA}$ and $\overrightarrow{PB}$. [3]

(ii) Given that $\overrightarrow{PR} = \lambda\overrightarrow{PB}$ and that $\overrightarrow{QR} = \mu\overrightarrow{QA}$, find an expression for $\overrightarrow{PQ}$ in terms of λ, μ, **p** and **q**. [2]

(iii) Using your expressions for $\overrightarrow{PQ}$, find the value of λ and of μ. [4]

11 A particle travelling in a straight line passes through a fixed point O. The displacement, x metres, of the particle, t seconds after it passes through O, is given by $x = 5t + \sin t$.

(i) Show that the particle is never at rest. [2]

(ii) Find the distance travelled by the particle between $t = \frac{\pi}{3}$ and $t = \frac{\pi}{2}$. [2]

(iii) Find the acceleration of the particle when $t = 4$. [2]

(iv) Find the value of t when the velocity of the particle is first at its minimum. [2]

Question 12 is printed on the next page.

12 Do not use a calculator in this question.

The line $y = 4x - 6$ intersects the curve $y = 10x^3 - 19x^2 - x$ at the points A, B, and C. Given that C is the point (2, 2), find the coordinates of the midpoint of AB. [10]

Cambridge Assessment
International Education

Grade thresholds – November 2019

Cambridge IGCSE™ Additional Mathematics (0606)

Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the November 2019 examination.

		minimum raw mark required for grade:				
	maximum raw mark available	A	B	C	D	E
Component 11	80	60	44	29	24	18
Component 12	80	60	44	29	24	18
Component 13	80	60	42	25	19	14
Component 21	80	63	46	29	23	18
Component 22	80	63	46	29	23	18
Component 23	80	69	50	30	22	13

Grade A* does not exist at the level of an individual component.

The maximum total mark for this syllabus, after weighting has been applied, is **160**.

The overall thresholds for the different grades were set as follows.

Option	Combination of Components	A*	A	B	C	D	E
AX	11, 21	141	123	90	58	47	36
AY	12, 22	141	123	90	58	47	36
AZ	13, 23	144	129	92	55	41	27

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Cambridge Assessment International Education
Cambridge International General Certificate of Secondary Education

ADDITIONAL MATHEMATICS **0606/11**

Paper 1 **October/November 2019**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **10** printed pages.

[Turn over

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GENERIC MARKING PRINCIPLE 6:

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MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

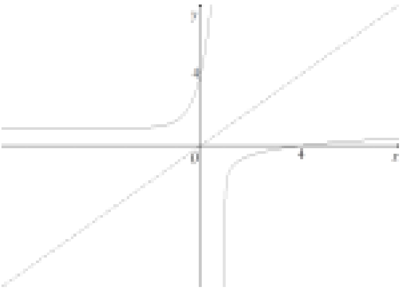
M	Method marks, awarded for a valid method applied to the problem.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
B	Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$A' \cap B$ oe	**B1**	
	$(X \cap Y) \cup (X \cap Z)$ or $X \cap (Y \cup Z)$	**B1**	
2	$2x^2 + 3x + k = kx - 3$	**M1**	For an attempt to equate and simplify to a 3 term quadratic equation, allow an error in one term
	$2x^2 + (3-k)x + (k+3) = 0$	**A1**	
	$(3-k)^2 - 4\times 2\times(k+3)$	**M1**	For attempt to use the discriminant, allow previous error, leading to a quadratic equation in terms of *k*
	$k^2 - 14k - 15 = 0$ giving critical values of -1 and 15	**A1**	For critical values
	$-1 < k < 15$	**A1**	
3	Either $7^x \times 7^{2y}$ or $49^{\frac{x}{2}} \times 49^y$ or $5^{5x} \times 5^{2y}$ or $25^{\frac{5x}{2}} \times 25^y$	**M1**	For expressing the terms on the left hand side of either one of the 2 equations in terms of powers of 7, 49, 5 or 25
	$7^x \times 7^{2y} = 7^0$ or $49^{\frac{x}{2}} \times 49^y = 49^0$	**A1**	
	$5^{5x} \times 5^{2y} = 5^{-2}$ or $25^{\frac{5x}{2}} \times 25^y = 25^{-1}$	**A1**	
	leading to $x + 2y = 0$ and $5x + 2y = -2$	**M1**	For attempt to solve two linear equations, with integer coefficients and constants, in terms of *x* and *y*
	$x = -\frac{1}{2}$, $y = \frac{1}{4}$	**A1**	
4(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(4x^2+1\right)\right) = \frac{8x}{4x^2+1}$	**B1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(2x-3)\frac{8x}{\left(4x^2+1\right)} - 2\ln\left(4x^2+1\right)}{(2x-3)^2}$	**M1**	For attempt to differentiate a quotient
		A1	For all other terms, not including $\frac{8x}{4x^2+1}$, correct
4(ii)	When $x = 2$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{16}{17} - 2\ln 17$ $= -4.73$	**M1**	For attempt to find value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 2$ and multiply by *h*
	Change in $y = -4.73h$	**A1**	

Question	Answer	Marks	Guidance
5(i)	$\text{f} > 1$	**B1**	Must be using correct notation
	$\text{g} \in \mathbb{R}$	**B1**	Must be using correct notation
5(ii)	$\text{g}(0) = 1,\ \text{g}(1) = 2$ and attempt at $\text{f}(2)$	**M1**	For attempt at g^2 and correct order
	$\text{f}(2) = 164.8$ awrt 165	**A1**	
5(iii)		**B3**	**B1** for correct f and $(0, 4)$, must be in first and second quadrant **B1** for correct f^{-1} and $(4, 0)$, must be in first and fourth quadrant **B1** for $y = x$ and/or symmetry implied, by 'matching intercepts'. No intersection.
6	$\frac{\text{d}y}{\text{d}x} = k(8x + 5)^{-\frac{1}{2}}$	**M1**	For attempt to differentiate, must be in the form $k(8x + 5)^{-\frac{1}{2}}$
	$\frac{\text{d}y}{\text{d}x} = 4(8x + 5)^{-\frac{1}{2}}$	**A1**	
	When $x = \frac{1}{2}$, $y = 3$	**B1**	
	Normal: $y - 3 = -\frac{3}{4}\left(x - \frac{1}{2}\right)$	**M1**	For attempt at the normal when $x = \frac{1}{2}$, using correct process for *their* $\frac{\text{d}y}{\text{d}x}$ and *their* y.
	$6x + 8y - 27 = 0$	**A1**	

Question	Answer	Marks	Guidance
7(i)	lg y = lg A + x lg b	**B1**	For statement, may be implied by subsequent work
	Either 6 = lg A + 3.4 lg b or 3.6 = lg A + 2.2 lg b	**M1**	For one correct equation
		M1	For another correct equation and attempt to solve simultaneously
	lg b = 2, b = 100	**A1**	
	lg A = –0.8, A = $10^{-0.8}$ or 0.158	**A1**	
	Or Gradient = lg b = 2	**M1**	equating gradient to lg b and attempt to evaluate
	b = 100	**A1**	Must be identified as b
	6 = lg A + 3.4 lg b or 3.6 = lg A + 2.2 lg b	**M1**	For a correct equation and attempt to find lg A
	lg A = –0.8, A = $10^{-0.8}$ or 0.158	**A1**	Must be identified as A
7(ii)	lg 900 = –0.8 + 2x oe	**M1**	For correct use of y = 900
	x = 1.88	**A1**	
8(i)	$BC^2 = \left(7+\sqrt{5}\right)^2 + \left(7-\sqrt{5}\right)^2$ $= 49 + 14\sqrt{5} + 5 + 49 - 14\sqrt{5} + 5$ = 108	**M1**	For use of Pythagoras' theorem and attempt to expand and simplify
	$BC = 6\sqrt{3}$	**A1**	
	Perimeter = $22 + 6\sqrt{5} + 6\sqrt{3}$	**A1**	

Question	Answer	Marks	Guidance
8(ii)	**Either** $\frac{1}{2}\left(4+3\sqrt{5}+11+2\sqrt{5}\right)\left(7+\sqrt{5}\right)$ $=\frac{1}{2}\left(15+5\sqrt{5}\right)\left(7+\sqrt{5}\right)$ $=\frac{1}{2}\left(105+35\sqrt{5}+15\sqrt{5}+25\right)$	**M1**	**Either** For a valid method and attempt to expand out and simplify
	Or $\left(4+3\sqrt{5}\right)\left(7+\sqrt{5}\right)+\frac{1}{2}\left(7+\sqrt{5}\right)\left(7-\sqrt{5}\right)$ $= 28 + 21\sqrt{5} + 4\sqrt{5} + 15 + \frac{1}{2}(49 - 5)$	**M1**	**Or** For a valid method and attempt to expand out and simplify
	Area = $65+25\sqrt{5}$	**A2**	**A1**for each term
9(i)	**Either** $15^2 = 10^2 + 10^2 - 200\cos AOB$ $\cos AOB = -0.125$	**M1**	For use of cosine rule
	$AOB = 1.696$ so 1.70 to 2 dp	**A1**	Must have justification to 2 dp
	Or $\sin\left(\frac{AOB}{2}\right)=\frac{7.5}{10}$ $\frac{AOB}{2}=0.8481$	**M1**	For use of basic trig
	$AOB = 1.696$ so 1.70 to 2 dp	**A1**	

Question	Answer	Marks	Guidance
9(ii)	Angle $DOC = \frac{\pi}{3}$	**B1**	
	Either $AOD = BOC = 0.5\left(2\pi - \frac{\pi}{3} - 1.696\right)$ $AOD = BOC = 1.77$	**M1**	For attempt to get *AOD* or *BOC*
	Arc lengths = 17.7	**M1**	For attempt at arc length using their previous answer
	Perimeter = $15 + 10 + (2 \times 17.7) = 60.4$	**A1**	
	Or Arc *AB* = 17 or Arc *CD* = $\frac{10\pi}{3}$	**M1**	For either arc length
	(20π – arc *AB* – arc *CD*)	**M1**	
	Perimeter = 60.4	**A1**	
9(iii)	**Either** Area of each sector = $\frac{1}{2}10^2(1.770)$	**M1**	For area of sector using their *BOC*
	Area of triangles = $\left(\frac{1}{2} \times 100 \times \sin\frac{\pi}{3}\right) + \left(\frac{1}{2} \times 100 \sin 1.70\right)$	**M1**	For area of one triangle using the sine rule oe
	Total area = 177 + 43.3 + 49.6	**M1**	For plan
	Area = awrt 270	**A1**	
	Or Area of upper segment = $\frac{1}{2}10^2(1.696 - \sin 1.696)$	**M1**	For area of a sector or area of a triangle using the sine rule oe
	Area of lower segment = $\frac{1}{2}10^2\left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)$	**M1**	For whichever has not been obtained in previous part
	Shaded area = 100π – are of the 2 segments Area = 314.2 – 35.2 – 9.06	**M1**	For plan
	Area = awrt 270	**A1**	

Question	Answer	Marks	Guidance
10	$1.5 = 2 + \cos 3x$ $\cos 3x = -0.5$	**M1**	For correct attempt to find points of intersection
	$3x = \frac{2\pi}{3}, \ \frac{4\pi}{3}$	**M1**	For dealing with $3x$ correctly
	$x = \frac{2\pi}{9}$ or 40°	**A1**	
	$x = \frac{4\pi}{9}$ or 80°	**A1**	
	Either $\int_{\frac{2\pi}{9}}^{\frac{4\pi}{9}} 1.5 - \left(2 + \cos 3x\right)\, \mathrm{d}x$	**M1**	For subtraction method – condone omission of or incorrect limits
	$\left[-0.5x - k\sin 3x\right]_{\frac{2\pi}{9}}^{\frac{4\pi}{9}}$	**M1**	For attempt to integrate – condone omission of or incorrect limits
	$\left[-0.5x - \frac{1}{3}\sin 3x\right]_{\frac{2\pi}{9}}^{\frac{4\pi}{9}}$	**A1**	All correct – condone omission of or incorrect limits
	$\left(-\frac{2\pi}{9} + \frac{\sqrt{3}}{6}\right) - \left(-\frac{\pi}{9} - \frac{\sqrt{3}}{6}\right)$	**M1**	Dep for application of limits, must be in radians
	Area = $\frac{\sqrt{3}}{3} - \frac{\pi}{9}$	**A1**	
	Or $\left(1.5 \times \frac{2\pi}{9}\right)$	**M1**	For attempt at rectangle (must include subtraction subsequently)
	$\left[2x + k\sin 3x\right]_{\frac{2\pi}{9}}^{\frac{4\pi}{9}}$	**M1**	For attempt to integrate – condone omission of or incorrect limits
	$\left[2x + \frac{1}{3}\sin 3x\right]_{\frac{2\pi}{9}}^{\frac{4\pi}{9}}$	**A1**	All correct – condone omission of or incorrect limits
	$\left(\left(\frac{8\pi}{9} - \frac{\sqrt{3}}{6}\right) - \left(\frac{4\pi}{9} + \frac{\sqrt{3}}{6}\right)\right)$	**M1**	Dep for application of limits, must be in radians
	Area = $\frac{\sqrt{3}}{3} - \frac{\pi}{9}$	**A1**	

Question	Answer	Marks	Guidance
11(a)(i)	362 880	**B1**	
11(a)(ii)	$7!\ \times 2$	**B1**	For 7!
	10 080	**B1**	For $7!\ \times 2$ leading to 10080
11(a)(iii)	Total = $4!\ \times 4!\ \times 3! = 3456$	**B3**	**B1** for treating as 4 separate units 4! **B1** for either number of ways of arranging the maths books amongst themselves 4! or the number of ways of arranging the physics books amongst themselves 3!
11(b)(i)	18 564	**B1**	
11(b)(ii)	Total 3738	**B4**	**B1** 4 boys 3150 **B1** 5 boys 560 **B1** 6 boys 28
12	$\frac{\mathrm{d}y}{\mathrm{d}x} = k\cos\left(x+\frac{\pi}{3}\right)\ +c$	**M1**	For attempt to integrate
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -2\cos\left(x+\frac{\pi}{3}\right)\ +c$	**A1**	All correct, condone omission of +*c*
	$5 = -2\cos\frac{2\pi}{3}+c$	**M1**	Dep for attempt to find *c*
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -2\cos\left(x+\frac{\pi}{3}\right)+4$	**A1**	
	$y = p\sin\left(x+\frac{\pi}{3}\right)\quad (+qx+d)$	**M1**	attempt to integrate a second time to obtain $y = p\sin\left(x+\frac{\pi}{3}\right)$
	$y = -2\sin\left(x+\frac{\pi}{3}\right)+4x+d$	**A1**	All correct, condone omission of +*d*
	$\frac{5\pi}{3} = -2\sin\frac{2\pi}{3}+\frac{4\pi}{3}+d$	**M1**	Dep for attempt to find a second arbitrary constant
	$y = -2\sin\left(x+\frac{\pi}{3}\right)+4x+\frac{\pi}{3}+\sqrt{3}$ or $y = -2\sin\left(x+\frac{\pi}{3}\right)+4x+2.78$	**A1**	

Cambridge Assessment International Education
Cambridge International General Certificate of Secondary Education

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ADDITIONAL MATHEMATICS **0606/12**

Paper 1 **October/November 2019**

MARK SCHEME

Maximum Mark: 80

Published

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rot	rounded or truncated
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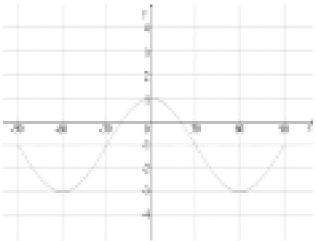
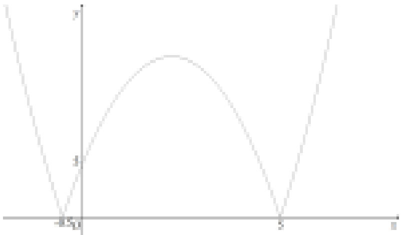
soi	seen or implied

Question	Answer	Marks	Guidance
1(i)		**B3**	**B1** for y intercept $(0,1)$, must have a graph **B1** for starting and finishing at $(\pm 90,-1)$ **B1** for all correct, must be attempt at a curve passing through $(\pm 30,-1)$ and $(\pm 60,-3)$
1(ii)	2	**B1**	
1(iii)	120° or $\frac{2\pi}{3}$	**B1**	
2	$\lg y^2 = mx + c$	**B1**	May be implied by subsequent work
	Gradient = $-4 \quad (=m)$	**B1**	
	$c = 32$	**B1**	
	$y = 10^{their\ \frac{c}{2} + their\ \frac{mx}{2}}$	**M1**	Dep on first B1 Use of $\lg y^2 = 2\lg y$ and $10^{their\ \frac{c}{2} + their\ \frac{mx}{2}}$ Or use of $y^2 = 10^{(their\ c + their\ mx)}$ and $10^{their\ \frac{c}{2} + their\ \frac{mx}{2}}$
	$y = 10^{16-2x}$	**A1**	
3	$\left(1-\frac{x}{7}\right)^{14} = 1 - 2x + \frac{13}{7}x^2$	**B2**	All terms correct or **B1** for 2 correct terms
	$(1-2x)^4 = 1 - 8x + 24x^2 \ldots$	**B2**	First three terms correct or **B1** for one incorrect term
	Product = $1 - 10x + \frac{293}{7}x^2$	**M1**	For attempt to multiply out to obtain $(1) - 10x + mx^2$, $m \neq 16$
	$a = -10,\ b = \frac{293}{7}$	**A1**	For both, need to identify a and b
4(i)		**B4**	**B1** for shape, with max in first quadrant **B1** for $(-0.5,0)$ and $(5,0)$ **B1** for $(0,5)$ **B1** all correct, with cusps and correct curvature for $x < 0.5$ and $x > 5$

Question	Answer	Marks	Guidance
4(ii)	$k = 0$	**B1**	Not from incorrect work
	Stationary point when $y = \pm\frac{121}{8}$ or ± 15.125	**M1**	For attempt to find *y*-coordinate of stationary point, must be a complete method i.e. Use of calculus Use of discriminant, Use of completing the square Use of symmetry Allow if seen in part (i), but must be used in (ii)
	$k > \frac{121}{8}$	**A1**	cao
5a(i)	fg	**B1**	
5a(ii)	g^{-1}	**B1**	
5a(iii)	f^{-1}	**B1**	
5a(iv)	g^2	**B1**	
5(b)(i)	Undefined at $x = 0$ oe	**B1**	
5(b)(ii)	$4 = a + b$ $h'(x) = \frac{p}{x^3}$ and attempt at $h'(1)$	**M1**	For attempt at $h(1)$ and differentiation to obtain $h'(1)$, must have the form $h'(x) = \frac{p}{x^3}$ oe
	$b = -8$ $a = 12$	**A1**	For both
6(a)	$p^{\frac{7}{2}}q^{\frac{5}{3}}r^{-\frac{7}{3}}$	**B3**	**B1** for each term or for each of $a = \frac{7}{2}$, $b = \frac{5}{3}$, $c = -\frac{7}{3}$

Question	Answer	Marks	Guidance
6(b)	**Either** $\log_7 x + \frac{2}{\log_7 x} = 3$	**M1**	For change of base.
	$(\log_7 x)^2 - 3\log_7 x + 2 = 0$ $\log_7 x = 1, \quad \log_7 x = 2$	**M1**	Dep for forming a 3 term quadratic equation in $\log_7 x$ and a correct attempt to solve
	$x = 7, \quad x = 49$	**M1**	Dep on both previous M marks for dealing with a base 7 logarithm correctly
		A1	For both
	Or $\frac{1}{\log_x 7} + 2\log_x 7 = 3$	**M1**	For change of base
	$2(\log_x 7)^2 - 3\log_x 7 + 1 = 0$ $\log_x 7 = 1, \quad \log_x 7 = 0.5$	**M1**	Dep for forming a 3 term quadratic equation in $\log_x 7$ and a correct attempt to solve
	$x = 7, \quad x = 49$	**M1**	Dep on both previous M marks for dealing with a base *x* logarithm correctly
		A1	For both
	Or $\frac{\lg x}{\lg 7} + 2\frac{\lg 7}{\lg x} = 3$ or $\lg 1000$	**M1**	For change of base
	$(\lg x)^2 - 3\lg 7(\lg x) + 2(\lg 7)^2 = 0$ $\lg x = 2\lg 7 \qquad \lg x = \lg 7$	**M1**	Dep for forming a 3 term quadratic equation in $\lg x$ and a correct attempt to solve
	$x = 7, \quad x = 49$	**M1**	Dep on both previous M marks for dealing with a base 10 logarithm correctly
		A1	For both, must be exact
7(i)	$\frac{dy}{dx} = \left(e^{x^2} + 1\right) + 2xe^{x^2}(x+5)$	**B1**	For $2xe^{x^2}$
		M1	For attempt at differentiating a product or expanding brackets and differentiating a product
		A1	For all other terms, apart from $2xe^{x^2}$, correct

Question	Answer	Marks	Guidance
7(ii)	When $x = 0.5$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 9.35$	**M1**	For attempt to find *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 0.5$ and multiplication by p
	Approximate change = $9.35p$	**A1**	
7(iii)	$\frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}t}$ $9.346 \times \frac{\mathrm{d}x}{\mathrm{d}t} = 2$	**M1**	For use of correct rates of change equation using *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 0.5$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 2$
	$\frac{\mathrm{d}x}{\mathrm{d}t} = 0.214$	**A1**	FT on $\frac{2}{\textit{their}\ 9.346}$ Must be correct to at least 3 sf
8(a)(i)	**Either** $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 3 \\ 3 & 0 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 2 & 1 \\ 1 & 3 \\ 1 & 1 \\ 0 & 1 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \end{pmatrix}$	**B2**	For correct matrices in correct order or **B1** if one correct matrix and a slip in one element of the other matrix
	Or $\begin{pmatrix} 4 & 2 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 & 1 & 0 & 3 \\ 1 & 3 & 1 & 1 & 0 \\ 1 & 0 & 2 & 3 & 1 \end{pmatrix}$ or $\begin{pmatrix} 4 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 & 1 & 0 & 3 \\ 1 & 3 & 1 & 1 & 0 \end{pmatrix}$	**B2**	For correct matrices in correct order or **B1** if one correct matrix and a slip in one element of the other matrix
8(a)(ii)	$\begin{pmatrix} 10 \\ 10 \\ 6 \\ 2 \\ 12 \end{pmatrix}$ or $\begin{pmatrix} 10 & 10 & 6 & 2 & 12 \end{pmatrix}$ Team E	**M1**	For matrix multiplication of *their* (i), with at least 2 elements correct, **must be in correct form**, may be unsimplified
		A1	All correct and identifying team E
8(b)(i)	$\frac{1}{6}\begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix}$	**B2**	**B1** for $\frac{1}{6}$ and **B1** for $\begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix}$

Question	Answer	Marks	Guidance
8(b)(ii)	$\mathbf{C} = \mathbf{A}^{-1}\mathbf{B}$	**M1**	For pre-multiplication by *their* inverse from **(i)**
	$\mathbf{C} = \frac{1}{6}\begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 5 & 0 \\ 1 & -2 \end{pmatrix}$	**M1**	Dep for matrix multiplication, using *their* inverse from (i), at least 2 elements correct
	$= \frac{1}{6}\begin{pmatrix} 21 & -2 \\ -9 & -2 \end{pmatrix}$ oe	**A1**	
9(i)	$\pi r^2 h = 1200\pi$	**B1**	
	$h = \frac{1200}{r^2}$ or $\pi rh = \frac{1200\pi}{r}$ and substitution into *their S*	**B1**	Must have attempt to use in an equation for *S*
	$S = 2\pi r^2 + \left(2\pi r \times \frac{1200}{r^2}\right)$ leading to given answer	**B1**	
9(ii)	$\frac{\mathrm{d}S}{\mathrm{d}r} = 4\pi r - \frac{2400\pi}{r^2}$	**M1**	Must obtain the form $Ar + \frac{B}{r^2}$
	When $\frac{\mathrm{d}S}{\mathrm{d}r} = 0$, $r = \sqrt[3]{600}$, 8.43	**M1**	Dep for equating to zero and attempt to solve to obtain $r = \ldots$
		A1	For correct *r*
	$S_{\min} = 1340$ or 1341	**A1**	
	Either $\frac{\mathrm{d}^2S}{\mathrm{d}r^2} = 4\pi + \frac{4800\pi}{r^3}$ $\frac{\mathrm{d}^2S}{\mathrm{d}r^2} > 0$ so minimum	**B1**	For a correct method to reach a correct conclusion If *r* is not calculated, then must state that $r > 0$
	Or Consideration of gradient e.g. $\begin{array}{\|c\|c\|c\|c\|}\hline \boldsymbol{r} & <8.43 & 8.43 & >8.43 \\ \hline \frac{\mathrm{d}S}{\mathrm{d}r} & - & 0 & + \\ \hline \end{array}$ Minimum point	**B1**	Must be making a correct and convincing argument with sufficient detail

Question	Answer	Marks	Guidance
10(i)	**Either** $18^2 = 10^2 + 10^2 - 200\cos AOB$	**M1**	Attempt to use cosine rule
	$\cos AOB = -0.62$	**A1**	Allow unsimplified
	$AOB = 2.2395$ or greater accuracy, so 2.24 (to 2 dp) or $AOB = 2.239...$ so 2.24 (to 2 dp) $AOB = 2.240$ so 2.24 (to 2 dp)	**A1**	Must justify 2 dp
10(i)	**Or** $\sin\frac{AOB}{2} = \frac{9}{10}$ or $\tan\frac{AOB}{2} = \frac{9}{\sqrt{19}}$ or $\cos\frac{AOB}{2} = \frac{\sqrt{19}}{10}$	**M1**	Attempt at trig using a right angled triangle
	$\frac{AOB}{2} =$ awrt 1.12	**A1**	
	$AOB = 2.2395$ or greater accuracy, so 2.24 (to 2 dp) or $AOB = 2.239...$ so 2.24 (to 2 dp) $AOB = 2.240$ so 2.24 (to 2 dp)	**A1**	Must justify 2 dp
10(ii)	$AOC = 2\pi - 2(2.2395)$ or $\frac{AOC}{2}$ or $ABC = \pi - (2.2395)$ oe	**M1**	For attempt to find angle AOC or ABC $AOC = 2\pi - 2(\textit{their } AOB)$ $ABC = \pi - (\textit{their } AOB)$ oe
	$AOC = 1.804$ or 1.803	**A1**	Condone 1.8 or 1.80
	Arc length = 18.04 or 18.03	**M1**	For attempt at arc length using $10 \times \textit{their } AOC$
	$AC = 20\sin\frac{AOC}{2}$ or $36\sin\frac{ABC}{2}$ or $\sqrt{10^2 + 10^2 - 200\cos AOC}$ or $\sqrt{18^2 + 18^2 - 648\cos ABC}$ $= 15.69$ or 15.7	**M1**	For attempt at AC using *their AOC*, or ABC but $AOC \neq 2.24$ or $\frac{2\pi}{3}$
	Perimeter = 33.7	**A1**	Allow awrt 33.7

Question	Answer	Marks	Guidance
10(iii)	Area of sector = 50×1.804 = 90.2 or 90.15	**M1**	For attempt at sector area $\frac{1}{2}\times10^2\times their\ AOC$ *AOC* must be in radians
	Area of triangle = $50\sin1.804$ = 48.6 or 48.66	**M1**	For attempt at area of triangle $\frac{1}{2}\times10^2\times$ sin *their AOC* *AOC* must be in radians
	Shaded area = 41.6 or 41.5	**A1**	Lack of accuracy is penalised here
11	$\frac{\mathrm{d}y}{\mathrm{d}x}=2(3x-1)^{\frac{1}{3}}+c$	**M1**	For $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\ a(3x-1)^{\frac{1}{3}}$, condone omission of + *c*
		A1	All correct, condone omission of *c*
	$6=4+c$	**M1**	Dep for attempt to find *c*
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\ 2(3x-1)^{\frac{1}{3}}+2$	**A1**	All correct, may be implied by $c=2$
	$y=\frac{1}{2}(3x-1)^{\frac{4}{3}}+2x+d$	**M1**	For attempt to integrate *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ to obtain the form $y=b(3x-1)^{\frac{4}{3}}\quad(+mx+d)$
		A1	All correct, condone omission of *d*
	$11=14+d$	**M1**	Dep for attempt to find *d*, a second arbitrary constant, having used an arbitrary constant for $\frac{\mathrm{d}y}{\mathrm{d}x}$
	$y=\frac{1}{2}(3x-1)^{\frac{4}{3}}+2x-3$	**A1**	

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ADDITIONAL MATHEMATICS	**0606/13**
Paper 1	**October/November 2019**
MARK SCHEME	
Maximum Mark: 80	

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **9** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

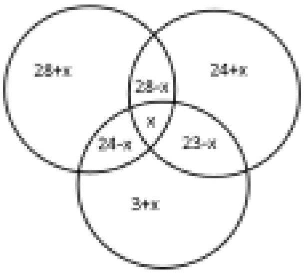
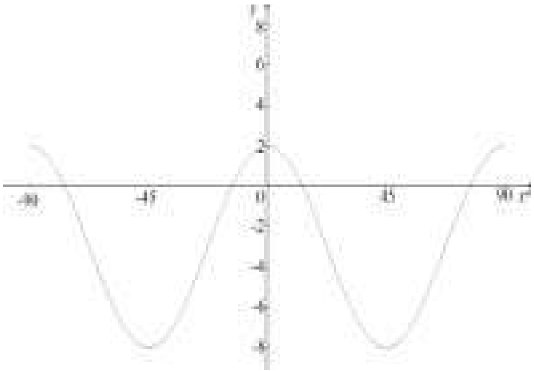
A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

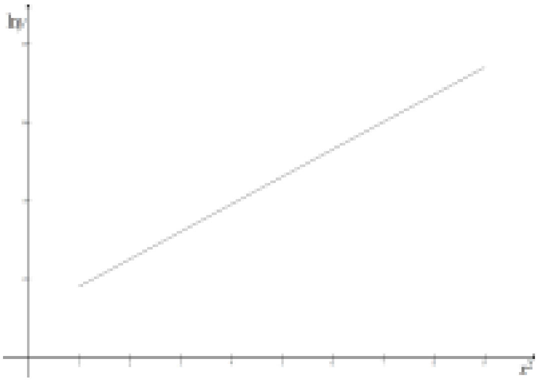
When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(i)		**M1**	for a Venn diagram showing at least 4 correct 'parts' in terms of *x*
		A1	for all 7 'parts' correct in terms of *x* on a Venn diagram or in working. May be implied by a correct equation.
	$80+24+x+23-x+3+x=145$ $50+28+x+28-x+24+x=145$ $75+28+x+24-x+3+x=145$ $50+80+75-(23+28+24)+x=145$ or equivalents	**M1**	for forming an equation in *x* using sum of 'parts' = 145 or $50+80+75-(23+28+24)+x=145$ Equations must be seen
	$x=15$	**A1**	from correct working only
1(ii)	43	**B1ft**	for *their x* plus 28
2(i)		**B4**	**B1** for a maximum at $(0, 2)$ **B1** for minimums at $y=-8$ and no other minimums **B1** for starting at $(-90^{\circ}, 2)$ and finishing at $(90^{\circ}, 2)$ **B1** for a fully correct curve with correct shape, particularly at end points, that has earned all three previous B marks.
2(ii)	5	**B1**	
2(iii)	90°	**B1**	
3(i)	$\frac{dy}{dx}=kx\left(3x^2-1\right)^{-\frac{4}{3}}$	**M1**	
	$\frac{dy}{dx}=-\frac{1}{3}\times 6x\left(3x^2-1\right)^{-\frac{4}{3}}$	**A1**	
3(ii)	When $x=\sqrt{3}$, $\frac{dy}{dx}=-\frac{\sqrt{3}}{8}$ $-\frac{\sqrt{3}}{8}p$ or $-0.217p$	**B1**	**FT** on *their* $\frac{dy}{dx}$ of the form $kx\left(3x^2-1\right)^{-\frac{4}{3}}$

Question	Answer	Marks	Guidance
3(iii)	When $x=\sqrt{3},\quad y=\frac{1}{2}$	**B1**	for $y=\frac{1}{2}$
	Normal: $y-\frac{1}{2}=\frac{8}{\sqrt{3}}\left(x-\sqrt{3}\right)$	**M1**	Dep on M1in part(i). An equation of the normal using *their* normal gradient, $\sqrt{3}$ and *their y*
		A1	allow unsimplified
4(i)	$-\frac{1}{13}\begin{pmatrix}-1 & -2\\ -4 & 5\end{pmatrix}$ oe	**B2**	**B1** for $-\frac{1}{13}$ **B1** for $\begin{pmatrix}-1 & -2\\ -4 & 5\end{pmatrix}$
4(ii)	$\frac{1}{13}\begin{pmatrix}1 & 2\\ 4 & -5\end{pmatrix}\begin{pmatrix}12\\ 7\end{pmatrix}$ $=\frac{1}{13}\begin{pmatrix}26\\ 13\end{pmatrix}$	**M1**	for pre-multiplication by *their* inverse from (i)
		M1	for correct method for matrix multiplication
	$=\begin{pmatrix}2\\ 1\end{pmatrix}$	**A1**	
	$x=1.11$	**B1**	
	$y=\frac{\pi}{4}$ or 0.785	**B1**	
5(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(x^2+3\right)\right) = \frac{2x}{\left(x^2+3\right)}$	**B1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\left(x^2+3\right)\frac{2x}{\left(x^2+3\right)}+2x\ln\left(x^2+3\right)$	**M1**	for product rule
		A1	**FT** *their* $\frac{2x}{\left(x^2+3\right)}$
5(ii)	$\left(x^2+3\right)\ln\left(x^2+3\right)=\int 2x+2x\ln\left(x^2+3\right)\ \mathrm{d}x$	**M1**	for using *their* result from (i) for $2x+kx\ln\left(x^2+3\right)$
	$\int x\ln\left(x^2+3\right)\ \mathrm{d}x$ $=\frac{1}{2}\left(x^2+3\right)\ln\left(x^2+3\right)-\frac{x^2}{2}(+c)$	**A1**	

Question	Answer	Marks	Guidance
6(i)	$\ln y = \ln A + x^2 \ln b$ or $\lg y = \lg A + x^2 \lg b$	**B1**	May be implied by a table of values for x^2 and $\ln y$ or $\lg y$ or axes labelled $\ln y$ or $\lg y$ and x^2
		**M1**	for attempt to plot either $\ln y$ or $\lg y$ against x^2 using an evenly spaced scale on each axis.
		A2	**A2** All points on a correct line (for $1 \leqslant x^2 \leqslant 9$) with axes correctly labelled **A1** One point not on the correct line or a correct line with axes not correctly labelled. **A0** Two or more points not on the correct line or one point not on the line and axes incorrect
6(ii)	Gradient = $\ln b$ or $\lg b$ $\ln b \approx 0.7$ or $\lg b \approx 0.3$ leading to	**M1**	for a complete method using the gradient of *their* straight-line graph of lg$y$ or ln$y$ against x^2 to obtain b
	$b = 2$ (allow 1.6 – 2.4)	**A1**	from correct working
	Intercept = $\ln A$ or $\lg A$ $\ln A \approx 1.1$ $\lg A \approx 0.5$ leading to	**M1**	for a complete method using intercept of *their* straight-line graph of lg$y$ or ln$y$ against x^2 to find A
	$A = 3$ (allow 2.5 – 3.6)	**A1**	from correct working
6(iii)	$100 = 3\left(2^{x^2}\right)$ or $\ln 100 = their 1.1 + their 0.7x^2$ or $\lg 100 = their 0.5 + their 0.3x^2$ or reading from lg$y$ = 2 to obtain x^2 or from ln$y$ = 4.6 to obtain x^2	**M1**	for a valid method to find x^2 Substitution methods should be using values of A and b in range
	leading to $x = 2.25$ (allow 2.0 – 2.7)	**A1**	for an answer in range from correct working
7(a)(i)	15 120	**B1**	
7(a)(ii)	1680	**B1**	
7(a)(iii) Method 1	Total = 2310	**B3**	B1 1st digit is 7 or 9 1680 or 210×8 B1 1st digit is 8 630 or 210×3
7(a)(iii) Method 2	Total = 2310	**B3**	B1 for 5th digit is 2,4 or 6 1890 or 210×9 B1 for 5th digit is 8 420 or 210×2

Question	Answer	Marks	Guidance
7(b)(i)	3003	**B1**	
7(b)(ii)	28	**B1**	
7(b)(iii)	Total 1419	**B3**	B1 Including husband and wife 495 B1 Excluding husband and wife 924
8(a)(i)	$\log_a a + 2\log_a y + \log_a x$	**M1**	for $\log_a a + \log_a x + \log_a y^2$ **and** $\log_a y^2 = 2\log_a y$
	$1+2q+p$	**A1**	
8(a)(ii)	$3\log_a x - \log_a y - \log_a a$	**M1**	for $\log_a x^3 - (\log_a a + \log_a y)$ **and** $\log_a x^3 = 3\log_a x$
	$3p-q-1$ or $3p-(q+1)$	**A1**	
8(a)(iii)	$\frac{1}{p}+\frac{1}{q}$	**B1**	
8(b)	$m-3m^2+4=0$ $m=\frac{4}{3},\quad (-1)$ $x=\frac{\lg\frac{4}{3}}{\lg 3}$, $x=\frac{\ln\frac{4}{3}}{\ln 3}$ or $\lg_3\frac{4}{3}$	**M1**	for obtaining a quadratic in m or 3^x
		M1	Dep for attempt to solve quadratic and deal with 3^x correctly
	$x=0.262$ only	**A1**	
9(i)	$100=2r+2r\theta+3r\theta$	**M1**	for addition of $2r$ and two arc lengths with at least one correct arc length
	$\theta=\frac{100-2r}{5r}$ or $\frac{20}{r}-\frac{2}{5}$ oe	**A1**	
9(ii)	$\frac{1}{2}9r^2\theta-\frac{1}{2}4r^2\theta$	**M1**	for subtraction of two sector areas with at least one sector area correct.
	$\frac{5r^2}{2}\left(\frac{100-2r}{5r}\right)$	**A1**	Must expand and simplify to obtain given answer $50r-r^2$
9(iii)	$\frac{dA}{dr}=50-2r$ $0=50-2r$ leading to $r=25$	**M1**	for differentiation and equating to zero and obtaining r or for using completing the square $-(25-r)^2+25^2$
	Max when $A=625$	**A1**	

Question	Answer	Marks	Guidance
9(iv)	When $r=10,\ \frac{dA}{dr}=30$	**B1**	
	$\frac{dr}{dt}=\frac{3}{30}$	**M1**	for $\frac{dr}{dt}=\frac{3}{their 30}$ where *their* 30 has been obtained from an evaluation of $\frac{dA}{dr}$ at $r=10$
	$\frac{dr}{dt}=0.1$ or $\frac{1}{10}$	**A1**	
9(v)	$\frac{d\theta}{dr}=-\frac{20}{r^2}$ oe	**B1**	
	$\frac{d\theta}{dr}=-\frac{1}{5}$ oe $\frac{d\theta}{dt}=\frac{1}{10}\times-\frac{1}{5}$ oe	**M1**	for $their\frac{dr}{dt}\times their\frac{d\theta}{dr}$ with both evaluated at $r=10$
	$\frac{d\theta}{dt}=-\frac{1}{50}$ or -0.02	**A1**	
10(a)(i)	$\pm\frac{20--20}{5}$	**M1**	for finding the gradient of the relevant part
	8	**A1**	
10(a)(ii)	7.5	**B1**	
10(a)(iii)	$\frac{1}{2}(5+7.5)20+\left(\frac{1}{2}\times2.5\times20\right)$ or $20\times5+\left(\frac{1}{2}\times2.5\times20\right)+\left(\frac{1}{2}\times2.5\times20\right)$ oe	**M1**	for a correct expression for total area using *their* 7.5
	150	**A1**	
10(b)(i)	$x=3e^{2t}+t+c$	**M1**	for $ke^{2t}+t$ Condone omission of c
	$0=3e^0+0+c$ When $t=0,\ x=0$ so $c=-3$	**M1**	Dep for substitution to find c
	$x=3e^{2t}+t-3$	**A1**	

Question	Answer	Marks	Guidance
10(b)(ii)	$\frac{dv}{dt} = 12e^{2t}$ so $12e^{2t} = 24$	**M1**	for ke^{2t} equated to 24
	$2t = \ln 2$	**M1**	Dep for correct order of operations to obtain $2t$
	$t = \frac{1}{2}\ln 2$, $\ln\sqrt{2}$ or 0.347	**A1**	

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ADDITIONAL MATHEMATICS **0606/21**

Paper 2 **October/November 2019**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **7** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

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- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

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- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

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MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method marks, awarded for a valid method applied to the problem.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
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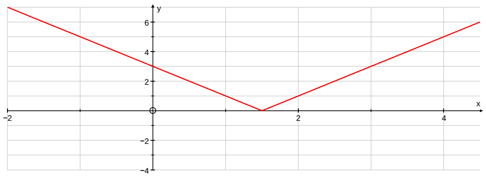
When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘**dep**’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1(i)		**B2**	**B1** shape **B1** Correct intersection with axes.
1(ii)	$7 = 2x - 3 \rightarrow x = 5$	**B1**	
	Uses $7 = 3 - 2x$ oe	**M1**	
	$x = -2$	**A1**	
2	$p = 2$ $q = 4$ $r = 3$	**B3**	**B1** for each

Question	Answer	Marks	Partial Marks
3(a)	obtain $e^{5x-3} = 3$	**M1**	**OR** Take logs → $2x + 1 = \ln 3 + 4 - 3x$
	take logs correctly → $5x - 3 = \ln 3$	**M1**	**OR** Collect like terms → $5x = 3 + \ln 3$
	$x = \frac{3+\ln 3}{5}$ or $x = 0.820$	**A1**	
3(b)`	Use of laws of logs → $\lg(y-6)(y+15) = 2$	**M1**	
	Uses $10^2 = 100$ → $[(y-6)(y+15)] = 100$	**B1**	
	Obtain correct quadratic → $y^2 + 9y - 190 = 0$	**A1**	
	Solve a three term quadratic	**M1**	
	$y = 10$ only	**A1**	
4	Eliminate x or y	**M1**	
	$x = \frac{7+5\sqrt{2}}{3+2\sqrt{2}}$ or $y = \frac{1}{3+2\sqrt{2}}$	**A1**	
	Multiply numerator and denominator by $3-2\sqrt{2}$	**M1**	
	$x = 1+\sqrt{2}$	**A1**	
	$y = 3-2\sqrt{2}$	**A1**	
5(i)	Differentiate	**M1**	Obtain $2\cos 2t$ or $-2\sin 2t$
	$v = 6\cos 2t - 8\sin 2t$	**A1**	
	$a = -12\sin 2t - 16\cos 2t$	**A1**	
5(ii)	Equate v to 0 and attempt to solve	**M1**	
	$\tan 2t = 0.75$	**A1**	or $\sin 2t = 0.6$ or $\cos 2t = 0.8$
	$t = 0.32(2)$	**A1**	Must be in radians
5(iii)	Insert value of t into expression for a	**M1**	Radians or degrees
	$a = -20$	**A1**	Must have used radians

Question	Answer	Marks	Partial Marks
6	Eliminate y	M1	
	$x^2 - x - 5 = 0$	A1	
	Use formula	M1	
	$x = \frac{1 \pm \sqrt{21}}{2}$	A1	
	$y = \frac{21 \pm \sqrt{21}}{2}$	A1	
	Find mid-point	M1	$(0.5, 10.5)$
	Show that mid-point lies on $x + y = 11$	A1	
7(a)(i)	$\mathrm{f}(0.5) = 0.5 + 4.5 - 5 = 0$	B1	
7(a)(ii)	Factorise to obtain $2x^2$ and 5	M1	
	$(2x-1)(2x^2 + x + 5)$	A1	
7(b)(i)	Replace $\tan x$ by $\frac{\sin x}{\cos x}$ and $\sec x$ by $\frac{1}{\cos x}$	M1	$13\frac{\sin x}{\cos^2 x} - 4\sin x - \frac{5}{\cos^2 x} = 0$
	Uses $\cos^2 x = 1 - \sin^2 x$	M1	$13\sin x - 4\sin x(1 - \sin^2 x) - 5 = 0$
	$4\sin^3 x + 9\sin x - 5 = 0$	A1	Completed correctly
7(b)(ii)	$2\sin^2 x + \sin x + 5 = 0$ no real roots	B1	Suitable statement seen
	$2\sin x - 1 = 0$	M1	Attempt to solve
	$x = \frac{\pi}{6}$	A1	
	$x = \frac{5\pi}{6}$	A1	
8(i)	$-2e^{-2x}$ seen	B1	
	Product rule	M1	Clear attempt
	$e^{-2x}(1 - 2x)$	A1	

Question	Answer	Marks	Partial Marks
8(ii)	Set $\frac{dy}{dx}=0$ and attempt to solve	**M1**	Must have two terms
	$\left(\frac{1}{2}, \frac{1}{2e}\right)$	**A1**	
8(iii)	Attempt to find $\frac{dy}{dx}$ at $x=1$	**M1**	
	$y-\frac{1}{e^2}=\frac{-1}{e^2}(x-1)$ or $y=-\frac{1}{e^2}x+\frac{2}{e^2}$	**A1**	
8(iv)	Integrate **part(i)** $xe^{-2x}=\int\left(-2xe^{-2x}+e^{-2x}\right)dx$	**M1**	
	Integrate e^{-2x} and make $\int xe^{-2x}dx$ the subject	**M1**	
	$\frac{-xe^{-2x}}{2}-\frac{e^{-2x}}{4}+c$	**A1**	
9(i)	$\frac{1}{3}$	**B1**	
	$\times\begin{pmatrix}-3 & -2\\ 9 & 5\end{pmatrix}$	**B1**	
9 (ii)	$\mathbf{B}^2=\begin{pmatrix}10 & 7\\ 42 & 31\end{pmatrix}$	**B2**	Minus one each error
9(iii)	$\mathbf{C}=\mathbf{B}^2-\mathbf{BA}$	**M1**	
	$\mathbf{BA}=\begin{pmatrix}1 & 1\\ -15 & -3\end{pmatrix}$	**A1**	
	$\mathbf{C}=\begin{pmatrix}9 & 6\\ 57 & 34\end{pmatrix}$	**A1**	
9(iv)	$\mathbf{D}=\mathbf{B}^2\mathbf{A}^{-1}$	**M1**	
	$\mathbf{D}=\frac{1}{3}\begin{pmatrix}33 & 15\\ 153 & 71\end{pmatrix}$	**A2**	Minus one each error
10(i)	$81+108x+54x^2+12x^3+x^4$	**B3**	**B1** for coefficients **B1** for powers **B1** for all Correct

Question	Answer	Marks	Partial Marks
10(ii)	Identify and select two terms in x and equate to zero	**M1**	$81 - 54p = 0$
	$p = 1.5$	**A1**	
10(iii)	Constant term = $-108p = -162$	**A1**	**FT** using *their p*
10(iv)	Correctly identify two terms in x^2	**M1**	x^2 term = $108 - 12p$
	$108 - 18 = 90$	**A1**	
11(i)	Uses correct triangle with v_w opposite 10° Sides of 300 and 280 include 10°	**M1**	
	Use cosine rule	**M1**	${v_w}^2 = 300^2 + 280^2 - 2 \times 300 \times 280 \cos 10$
	$v_w = 54.3$	**A1**	
11(ii)	Use sine rule	**M1**	$\frac{280}{\sin\alpha} = \frac{54.3}{\sin 10°}$
	$\alpha = 63°$ or $64°$	**A1**	
	Bearing 117° or 116°	**A1**	

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ADDITIONAL MATHEMATICS **0606/22**

Paper 2 **October/November 2019**

MARK SCHEME

Maximum Mark: 80

Published

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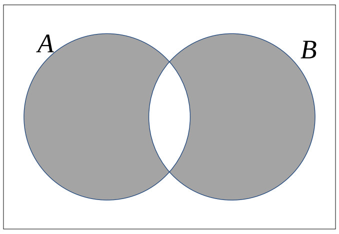
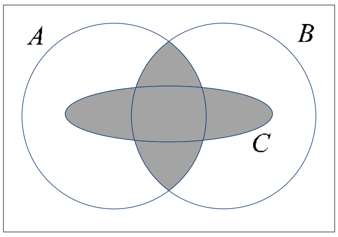
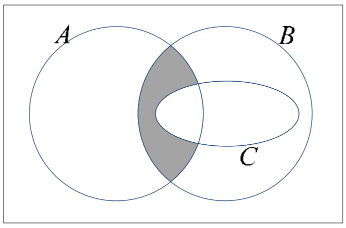
Question	Answer	Marks	Guidance
1		**B1**	
		**B1**	
		**B 1**	
2	$\frac{dy}{dx} = 6\cos 3x$	**B1**	
	$-3\sin 3x$	**B1**	
	$\frac{d^2y}{dx^2} = -18\sin 3x - 9\cos 3x$	**B1**	**FT** Correct derivative of *their* $\frac{dy}{dx}$
	Insert and collect like terms	**M1**	Must insert for y, *their* $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ correctly resulting in 6 terms.
	$k = -15$	**A1**	Allow $-15\sin 3x$ seen nfww
3(i)	${}^{14}P_5$ or $14 \times 13 \times 12 \times 11 \times 10$	**M1**	
	240 240	**A1**	cao
3(ii)	${}^{3}P_1 \times {}^{5}P_2 \times {}^{6}P_2$ or $3 \times (5 \times 4) \times (6 \times 5)$	**M1**	Two of the three elements multiplied by …
	= 1800	**A1**	
3(iii)	${}^{6}P_2 \times {}^{8}P_3$ or $(6 \times 5) \times (8 \times 7 \times 6)$	**M1**	One element multiplied by … Clear intention to multiply
	= 10 080	**A1**	

Question	Answer	Marks	Guidance
4	$kx + 3 = x^2 + 5x + 12$ $\rightarrow x^2 + (5 - k)x + 9(= 0)$	**M1**	Equate and attempt to simplify to all terms on one side.
	Use discriminant of *their* quadratic.	**M1**	**dep**
	$(5 - k)^2 - 36$ oe	**A1**	Unsimplified
	$k = -1$ and 11	**A1**	Both boundary values
	$-1 < k < 11$	**A1**	Must be in terms of k.
	OR $2x + 5 \sim k$	**M1**	Connect gradients of line and curve
	$y = (2x+5)x+3 \rightarrow$ $2x^2 + 5x + 3 = x^2 + 5x + 12$	**M1**	Eliminate k and y.
	$x^2 = 9 \rightarrow x = \pm 3$	**A1**	
	$k = 11$ or $k = -1$	**A1**	
	$-1 < k < 11$	**A1**	
5(i)	$\frac{dy}{dx} = \frac{-2k}{(x+1)^3}$	**B1**	oe Unsimplified
	Gradient of normal = $\frac{(x+1)^3}{2k}$ or Gradient of tangent = -3	**M1**	Gradient of normal = $\frac{-1}{\text{gradient of tangent}}$
	$\frac{8}{2k} = \frac{1}{3}$ or $\frac{2k}{8} = -3$	**M1**	Equate gradient of normal to $\frac{1}{3}$ at $x = 1$ or equate gradient of tangent to -3 at $x = 1$
	$k = 12$	**A1**	
5(ii)	$x = 2 \rightarrow \frac{dy}{dx} = -\frac{8}{9}$ or *their* $\frac{-2k}{27}$	**B1**	**FT**
	$y = \frac{4}{3}$ or *their* $\frac{k}{9}$	**B1**	**FT**
	$\frac{y - \frac{4}{3}}{x - 2} = -\frac{8}{9}$ or $y = -\frac{8}{9}x + \frac{28}{9}$	**B1**	isw

Question	Answer	Marks	Guidance
6(i)	$\dfrac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}+1}+\dfrac{\frac{1}{\cos x}+1}{\frac{\sin x}{\cos x}}$	**M1**	Use $\tan x=\dfrac{\sin x}{\cos x}$ and $\sec x=\dfrac{1}{\cos x}$ throughout
	$\dfrac{\sin x}{1+\cos x}+\dfrac{1+\cos x}{\sin x}$	**M1**	**dep** Multiply by cos$x$
	$\dfrac{\sin^2 x+1+2\cos x+\cos^2 x}{(1+\cos x)\sin x}$	**M1**	**dep** Add *their* fractions correctly and expand (1 + cos$x$)^2 correctly
	$\dfrac{2(1+\cos x)}{(1+\cos x)\sin x}$	**M1**	**dep** Use sin^2x + cos^2x = 1 and take out a factor of 2.
	All correct AG	**A1**	Do not award if brackets missing at any point **or** x missing more than twice **or** x misplaced. Do not credit mixed variables.
	OR $\dfrac{\tan^2 x+(\sec x+1)^2}{\tan x(\sec x+1)}$	**M1**	Add fractions
	$=\dfrac{2\sec^2 x+2\sec x}{\tan x(\sec x+1)}$	**M1**	**dep** Expand brackets correctly and use 1 + tan^2x = sec^2x
	$\dfrac{2\sec x}{\tan x}$	**M1**	**dep** Cancel sec$x$ + 1
	$\dfrac{2}{\cos x}\times\dfrac{\cos x}{\sin x}$	**M1**	**dep** Use $\tan x=\dfrac{\sin x}{\cos x}$ and $\sec x=\dfrac{1}{\cos x}$ oe
	All correct AG	**A1**	Do not award if brackets missing at any point **or** x missing more than twice **or** x misplaced. Do not credit mixed variables.
6(ii)	3sin^2x + sin$x$ – 2 = 0 oe	**B1**	Obtain three term quadratic.
	(3sin$x$ – 2)(sin$x$ + 1) = 0	**M1**	Solve three term quadratic
	41.8° awrt	**A1**	
	138.2° awrt	**A1**	Mark final answers This mark is not awarded if there are more solutions in the range.

Question	Answer	Marks	Guidance
7(a)	$2 \times 4 \times p = 40 \rightarrow p = 5$	**B1**	May be obtained later.
	$(x-2)(x-4)(x-p) = 0$	**M1**	Factorise cubic
	$a = -11$	**A1**	Expand and identify
	$b = 38$	**A1**	
	OR $2 \times 4 \times p = 40 \rightarrow p = 5$	**B1**	May be obtained later.
	Obtain equations $4a + 2b = 32$ $16a + 4b = -24$ and attempt to solve	**M1**	
	$a = -11$	**A1**	
	$b = 38$	**A1**	
7(b)	Find $x = -1$	**M1**	Trial value/s and finds a root or shows that $(x + 1)$ or $(x + 4)$ or $(x - 10)$ divides into $x^3 - 5x^2 - 46x - 40$.
	$(x + 1)(x^2 - 6x - 40)$ (= 0) or $(x + 4)(x^2 - 9x - 10)$(= 0) or $(x - 10)(x^2 + 5x + 4)$(= 0)	**A1**	Factorise to give linear and quadratic factor
	$(x + 1)(x + 4)(x - 10)$ (= 0)	**M1**	Solve the quadratic to give 2 roots
	$x = -1, -4, 10$	**A1**	
	OR Uses factor theorem to find a root $(-1)^3 - 5(-1^2) - 46(-1) - 40$ or $-1 - 5 + 46 - 40 = 0$ $\rightarrow x = -1$	**M1**	This may be awarded for $x = -4$ or $x = 10$.
	Uses factor theorem to attempt to find further roots	**M1**	At least two more trials.
	$(-4)^3 - 5(-4)^2 - 46(-4) - 40$ or $- 64 - 80 + 184 - 40 = 0$ $\rightarrow x = -4$	**A1**	
	$(10)^3 - 5(10)^2 - 46(10) - 40$ or $1000 - 500 - 460 - 40 = 0$ $\rightarrow x = 10$	**A1**	

Question	Answer	Marks	Guidance
8(i)	$\sqrt{5^2+12^2}=13$	**M1**	
	$\boldsymbol{v_A}=-\frac{5}{2}\mathbf{i}-6\mathbf{j}$ or $\frac{1}{2}\left(-5\mathbf{i}-12\mathbf{j}\right)$	**A1**	
8(ii)	$\lvert v_B\rvert=\sqrt{12^{12}+\left(-9\right)^2}$	**M1**	Use Pythagoras
	15	**A1**	Do not allow ± 15. Mark final answer.
8(iii)	$\boldsymbol{r_A}=\begin{pmatrix}20\\-7\end{pmatrix}+t\begin{pmatrix}-2.5\\-6\end{pmatrix}$ or $\boldsymbol{r_A}=\left(20-2.5t\right)\mathbf{i}+\left(-7-6t\right)\mathbf{j}$	**B1**	**FT** on *their* $\boldsymbol{v}_A$ only if of the form $k(-5\mathbf{i}-12\mathbf{j})$ where $k \neq 1$ or 0.
	$\boldsymbol{r_B}=\begin{pmatrix}-67\\11\end{pmatrix}+t\begin{pmatrix}12\\-9\end{pmatrix}$ or $\boldsymbol{r_B}=\left(-67+12t\right)\mathbf{i}+\left(11-9t\right)\mathbf{j}$	**B1**	
8(iv)	$20-2.5t=-67+12t$ or $-7-6t=11-9t$	**M1**	Equate x or y coordinates. Must have two terms in both coordinates.
	$t=6$	**A1**	nfww Ignore other value of t.
	$\boldsymbol{r}=\begin{pmatrix}5\\-43\end{pmatrix}$ only or $\boldsymbol{r}=5\mathbf{i}-43\mathbf{j}$	**A1**	**A0** if further value of $\boldsymbol{r}$ found.
9(i)	Midpoint (1, 2)	**B1**	May be seen on diagram
	Gradient of $AB=-\frac{3}{4}$	**B1**	
	Gradient of PM $=\frac{-1}{\textit{their}\text{ gradient of }AB}=\frac{4}{3}$	**M1**	Use $m_1 \times m_2 = -1$
	Equation PM $\frac{y-2}{x-1}=\frac{4}{3}$	**M1**	**dep** Attempt to find equation of line with *their* midpoint and *their* gradient of PM. If $y = mx + c$ used c must be found.
	$y=\frac{4}{3}x+\frac{2}{3}$	**A1**	
9(ii)	$s=\frac{4}{3}r+\frac{2}{3}$	**B1**	**FT** Insert (r, s) into *their* linear equation to

Question	Answer	Marks	Guidance
			obtain s =…
9(iii)	$(r-1)^2 + (s-2)^2 = 100$ oe	**B1**	**FT** Use Pythagoras with *their* (1, 2)
	Eliminate r or s	**M1**	From one linear and one quadratic expression. Unsimplified
	$25r^2 - 50r - 875 = 0$ oe or $25s^2 - 100s - 1500 = 0$ oe	**A1**	
	$(5r + 25)(5r - 35) = 0$ oe or $(5s - 50)(5s + 30) = 0$ oe	**M1**	Solve three term quadratic Can be implied by correct solution.
	$r = 7$, $s = 10$	**A1**	Do not award if negative values of r and s are also given nfww
	OR Equivalent method such as: $\overrightarrow{MP} = \begin{pmatrix} a \\ b \end{pmatrix} \to a^2 + b^2 = 100$ and $\frac{b}{a} = \frac{4}{3}$	**B1**	Using distance =10 and gradient = $\frac{4}{3}$.
	Eliminate a or b	**M1**	
	$a^2 + \left(\frac{4a}{3}\right)^2 = 100$ or $\left(\frac{3b}{4}\right)^2 + b^2 = 100$	**A1**	
	$\to a = (\pm)6$ and $b = (\pm)8$	**M1**	Solve
	$r = 7$, $s = 10$	**A1**	
10(i)	Quotient rule or product rule	**M1**	
	$\frac{x - 2x\ln x}{x^4}$ or $\frac{x - \ln x.2x}{x^4}$ oe isw	**A2/1/0**	Minus one each error. Allow unsimplified.
10(ii)	$x - 2x\ln x = 0$	**M1**	Set $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and attempt to solve. Must have two terms and obtain $\ln x = k$ only.
	$x = 1.65$ awrt or $\sqrt{\mathrm{e}}$	**A1**	
	$y = 0.184$ awrt or $\frac{1}{2\mathrm{e}}$	**A1**	

Question	Answer	Marks	Guidance
10(iii)	$\frac{\ln x}{x^2} = \int \frac{1}{x^3} - \frac{2\ln x}{x^3} \mathrm{d}x$	**M1**	Integrate *their* derivative from (i) which must have two terms. Condone omission of dx.
	$\frac{-1}{2x^2}$	**A1**	Find $\int \frac{1}{x^3} \mathrm{d}x$
	$\int \frac{\ln x}{x^3} \mathrm{d}x = -\frac{1}{4x^2} - \frac{\ln x}{2x^2} + (C)$	**A1**	oe Rearrange and complete
10(iv)	Insert limits and subtract correctly	**M1**	**dep** Must be inserting into **two** terms in x from (iii). Values explicitly seen if expression is incorrect.
	$\frac{3}{16} - \frac{\ln 2}{8}$ or 0.101 awrt	**A1**	
11	$(\sqrt{5}-3)(\sqrt{5}+3) = -4$	**B1**	Seen anywhere
	Attempt formula	**M1**	
	$x = \frac{-3 \pm 5}{2(\sqrt{5}-3)}$	**A1**	
	Multiply by *their* $(\sqrt{5}+3)$	**M1**	Attempt must be seen with a further line of working. oe
	$x = \sqrt{5}+3$	**A1**	oe Mark final answer
	$x = \frac{-1(\sqrt{5}+3)}{4}$	**A1**	oe Mark final answer

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ADDITIONAL MATHEMATICS **0606/23**

Paper 2 **October/November 2019**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **7** printed pages.

Cambridge Assessment International Education

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method marks, awarded for a valid method applied to the problem.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
B	Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$x = 1$	**B1**	
	$-3x - 2 = x + 4$ oe	**M1**	
	$x = -1.5$ oe	**A1**	
2(i)	$\dfrac{\frac{1}{\sin} - \frac{\cos x}{\sin x}}{1 - \cos x}$	**M1**	express in terms of sin*x* and cos*x*
	$\dfrac{(1-\cos x)}{\sin(1-\cos x)}$	**A1**	rewrite not as a fraction within a fraction
	$\dfrac{1}{\sin x} = \operatorname{cosec} x$	**A1**	correct completion **answer given**
2(ii)	$\left[\sin x = \dfrac{1}{2}\right]$ $x = 30°$	**B1**	
	$x = 150°$ nfww	**B1**	no extra answers
3	$(1 + ax)^5 = 1 + 5ax + 10a^2x^2 + 10a^3x^3$ soi	**B1**	4 terms not ${}^nC_r$ notation
	$[2] + (10a + b)x + (5ab + 20a^2)x^2$	**M1**	obtain expansion with 2 terms in x, 2 terms in x^2
	equate terms in x and x^2 to give two equations in a and b each consisting of three terms	**M1**	
	$10a + b = 32$ $5ab + 20a^2 = 210$	**A1**	correct equations imply previous two M marks
	eliminate b	**M1**	
	obtain $3a^2 - 16a + 21 = 0$ correctly	**A1**	**answer given**
	$a = 3$ and $b = 2$	**B1**	
	$c = 720$ only	**B1**	no additional answers
4(i)	$y = 2(x - 1)^2 - 9$	**B3**	$a = 2$, $b = 1$, $c = -9$ in correct form. **B1** for each
4(ii)	minimum *their* –9	**B1**	**FT** from *their* correct form, with $a > 0$
	when $x =$ *their* 1	**B1**	**FT** from *their* correct form, with $a > 0$

Question	Answer	Marks	Guidance
4(iii)	$x=\sqrt{p}$ or $p=x^2$ soi	**B1**	
	$(x-1)=\sqrt{\frac{9}{2}}$ or $\left(\sqrt{p}-1\right)=\sqrt{\frac{9}{2}}$ oe	**M1**	$(x-b)=\sqrt{\frac{-c}{a}}$ $\left(\sqrt{p}-b\right)=\sqrt{\frac{-c}{a}}$ using *their* values of *a*, *b*, *c* from (i)
	$p = 9.74$	**A1**	completion not involving use of quadratic formula
5(a)	$\tan\left(y-\frac{\pi}{4}\right)=(\pm)\sqrt{3}$	**M1**	± 1.73…
	$y-\frac{\pi}{4}=\frac{\pi}{3}$ or $\frac{2\pi}{3}$	**A1**	1.04(7…) or 2.09(4…)
	$y=\frac{7\pi}{12}$ or 1.83	**A1**	
	$y=\frac{11\pi}{12}$ or 2.88	**A1**	
5(b)	correctly rewrite equation in terms of sinz and cosz	**M1**	
	use $\sin^2 z = 1 - \cos^2 z$	**M1**	appropriate use of Pythagorean identity for forming an equation in one trig ratio
	$6\cos^2 z - 7\cos z + 1 = 0$ **oe**	**A1**	
	$(6\cos z - 1)(\cos z - 1) = 0$	**M1**	solve three term quadratic in cosz
	80.4°	**A1**	
	279.6°	**A1**	
6(i)	$[\tan ACB =]\ \frac{3+\sqrt{3}}{3-\sqrt{3}}$	**B1**	
	rationalise with $3+\sqrt{3}$	**M1**	
	simplify showing at least 3 terms in numerator to $2+\sqrt{3}$	**A1**	
6(ii)	$(AC)^2=\left(3+\sqrt{3}\right)^2+\left(3-\sqrt{3}\right)^2$ oe	**M1**	Pythagoras
	at least 4 terms $12+6\sqrt{3}+12-6\sqrt{3}$	**A1**	

Question	Answer	Marks	Guidance
	$AC = 2\sqrt{6}$	**A1**	
7(i)	evidence of differentiation $(3x + 2)^{-3}$	**M1**	
	$-12(3x + 2)^{-3} \times 3$	**A1**	may use PR or QR on fraction part
	+1	**B1**	
	set *their* $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	**M1**	$1 - 36(3x + 2)^{-3} = 0$
	$x = 0.43$ nfww	**A1**	
	$y = 0.98$ only	**A1**	
7(ii)	$\frac{-2}{3x+2}$ oe	**B1**	
	$\frac{1}{2}x^2$	**B1**	
	$\left[\frac{-2}{6+2}+2\right]-\left[\frac{-2}{2}\right]$	**M1**	insert correct limits into *their* two term integral and subtract two non-zero terms in correct order
	2.75 nfww	**A1**	2.75 following B1 B1implies M1
8(i)	$p = -4$	**B1**	
8(ii)	$(x - 2)(x - 3)(x + 4)$	**M1**	**FT** $(x - 2)(x - 3)(x - p)$
	$(x^2 - 5x + 6)(x + 4)$	**A1**	**FT** $(x^2 - 5x + 6)(x - p)$ multiply out two factors
	correctly obtain $a = -1$ $x^3 - x^2 - 14x + 24$	**A1**	**answer given**
	$b = -14$ stated	**B1**	
8(iii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 2x - 14$	**B1**	**FT** *their* numerical b $3x^2 - 2x + b$
8(iv)	set *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to 2	**M1**	**FT** *their* numerical b
	$x = 2$	**A1**	
	$y = 40$ only	**A1**	no additional answers
8(v)	$y - 40 = 2(x + 2)$ $(y = 2x + 44)$	**B1**	
9(i)	$\overrightarrow{AD} = 2\mathbf{a} + \mathbf{b}$	**B1**	

Question	Answer	Marks	Guidance
	$\overrightarrow{OX} = \mathbf{a} + \lambda(2\mathbf{a} + \mathbf{b})$	**B1**	
9(ii)	$\overrightarrow{BC} = 3\mathbf{a} - 2\mathbf{b}$	**B1**	
	$\overrightarrow{OX} = 2\mathbf{b} + \mu(3\mathbf{a} - 2\mathbf{b})$	**B1**	
9(iii)	$\overrightarrow{OX} = \overrightarrow{OX}$ and equate for **a** or **b**	**M1**	
	$1 + 2\lambda = 3\mu$ and $\lambda = 2 - 2\mu$	**A1**	
	solve correct equations for λ or μ	**M1**	
	$\lambda = \frac{4}{7}$ and $\mu = \frac{5}{7}$	**A1**	
9(iv)	$\frac{4}{3}$ or 4 : 3	**B1**	**FT** $\lambda/(1 - \lambda)$ $0 < \lambda < 1$
10(i)	$\mathrm{gf}(x) = \mathrm{e}^{2(\ln(3x+2))} - 4$	**B1**	
	their gf = 5	**M1**	
	use $\ln a^p = p\ln a$ or $\mathrm{e}^{\ln a} = a$ or $\ln \mathrm{e}^a = a$	**B1**	correct use of log/exponential relationship seen anywhere
	$3x + 2 = 3$ or $(3x + 2)^2 = 9$	**A1**	3 may take the form of $\mathrm{e}^{0.5\ln 9}$ 9 may take the form of $\mathrm{e}^{\ln 9}$
	$x = \frac{1}{3}$ only	**A1**	
10(ii)	$x = \frac{\mathrm{e}^y - 2}{3}$	**M1**	find x in terms of y
	$\frac{\mathrm{e}^x - 2}{3}$ $\left(= \mathrm{f}^{-1}(x) \text{ or } = y\right)$	**A1**	interchange x and y correct completion
10(iii)	$\frac{\mathrm{e}^x - 2}{3} = \mathrm{e}^{2x} - 4$	**M1**	*their* $\mathrm{f}^{-1}(x) = \mathrm{g}(x)$
	$3\mathrm{e}^{2x} - \mathrm{e}^x - 10$ $(= 0)$	**A1**	obtain quadratic in e^x must be arranged as a three term quadratic in order shown
	$(3\mathrm{e}^x + 5)(\mathrm{e}^x - 2)$ $(= 0)$	**M1**	solve for e^x
	$x = \ln 2$ or 0.693 only	**A1**	

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CANDIDATE NAME	

CENTRE NUMBER		CANDIDATE NUMBER	

ADDITIONAL MATHEMATICS **0606/11**

Paper 1 **October/November 2019**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **16** printed pages.

DC (CE/SW) 172295/2

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1} b + \binom{n}{2}a^{n-2} b^2 + \ldots + \binom{n}{r}a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

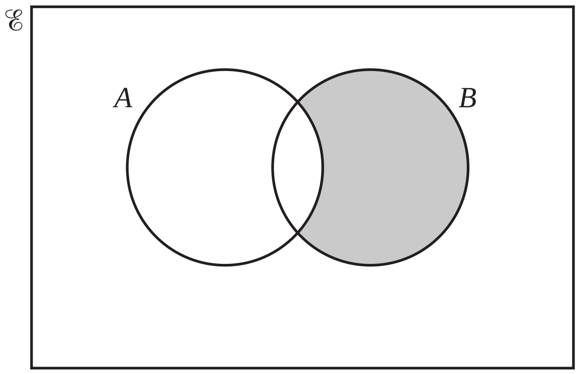
Formulae for ΔABC

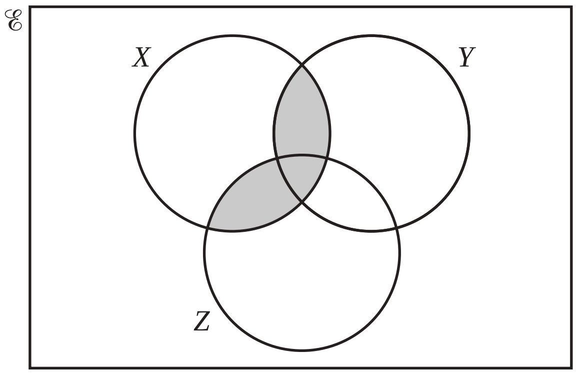
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Using set notation, describe the regions shaded on the Venn diagrams below.





.... [2]

2 Find the values of k for which the line $y = kx - 3$ and the curve $y = 2x^2 + 3x + k$ do not intersect. [5]

3 Given that $7^x \times 49^y = 1$ and $5^{5x} \times 125^{\frac{2y}{3}} = \frac{1}{25}$, calculate the value of x and of y. [5]

4 It is given that $y = \dfrac{\ln(4x^2 + 1)}{2x - 3}$.

(i) Find $\dfrac{dy}{dx}$. [3]

(ii) Find the approximate change in y as x increases from 2 to $2 + h$, where h is small. [2]

5

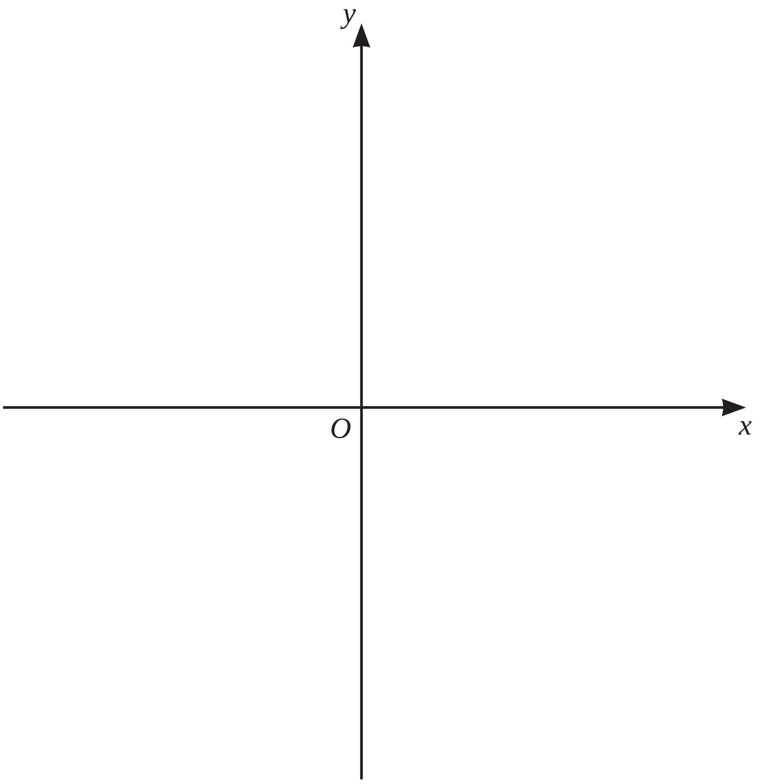
$$f(x) = 3e^{2x} + 1 \quad \text{for } x \in \mathbb{R}$$

$$g(x) = x + 1 \quad \text{for } x \in \mathbb{R}$$

(i) Write down the range of f and of g. [2]

(ii) Evaluate $fg^2(0)$. [2]

(iii) On the axes below, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$, stating the coordinates of the points where the graphs meet the coordinate axes. [3]



6 Find the equation of the normal to the curve $y = \sqrt{8x+5}$ at the point where $x = \frac{1}{2}$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [5]

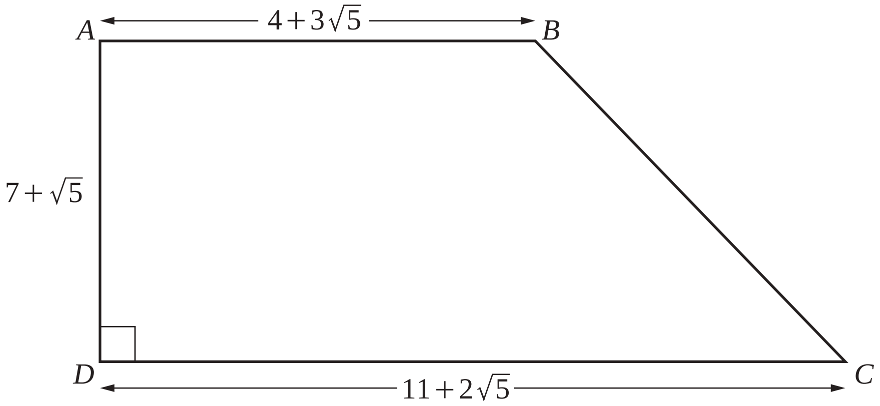
7 When lg y is plotted against x, a straight line graph passing through the points (2.2, 3.6) and (3.4, 6) is obtained.

(i) Given that $y = Ab^x$, find the value of each of the constants A and b. [5]

(ii) Find x when $y = 900$. [2]

8 **Do not use a calculator in this question.**

In this question, all lengths are in centimetres.

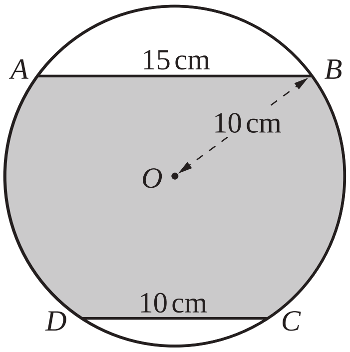


The diagram shows the trapezium *ABCD* in which angle *ADC* is 90° and *AB* is parallel to *DC*. It is given that $AB = 4+3\sqrt{5}$, $DC = 11+2\sqrt{5}$ and $AD = 7+\sqrt{5}$.

(i) Find the perimeter of the trapezium, giving your answer in simplest surd form. [3]

(ii) Find the area of the trapezium, giving your answer in simplest surd form. [3]

9



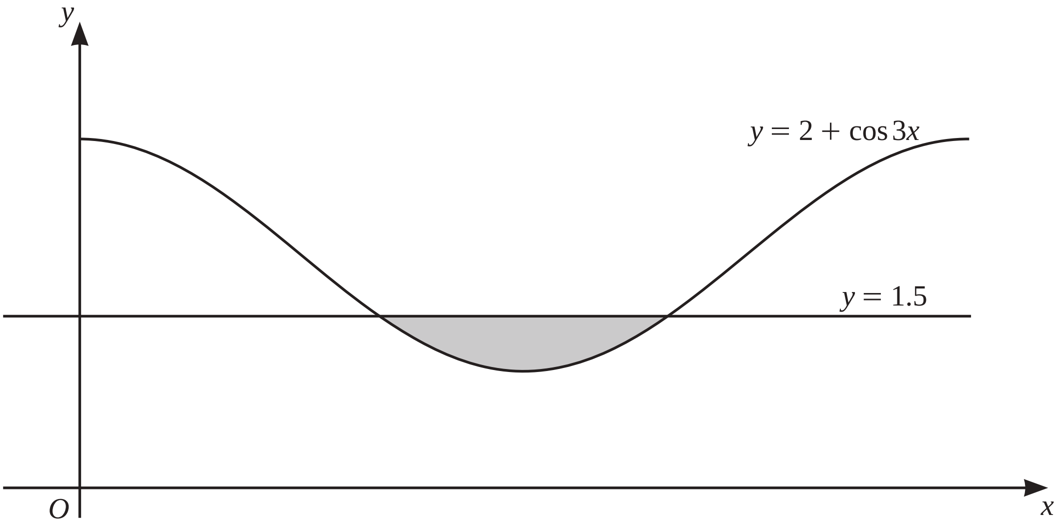
The diagram shows a circle with centre O and radius 10 cm. The points A, B, C and D lie on the circle such that the chord $AB = 15$ cm and the chord $CD = 10$ cm. The chord AB is parallel to the chord DC.

(i) Show that the angle AOB is 1.70 radians correct to 2 decimal places. [2]

(ii) Find the perimeter of the shaded region. [4]

(iii) Find the area of the shaded region. [4]

10



The diagram shows part of the graph of $y = 2 + \cos 3x$ and the straight line $y = 1.5$. Find the exact area of the shaded region bounded by the curve and the straight line. You must show all your working. [9]

Continuation of working space for Question 10

11 **(a)** Jess wants to arrange 9 different books on a shelf. There are 4 mathematics books, 3 physics books and 2 chemistry books. Find the number of different possible arrangements of the books if

(i) there are no restrictions, [1]

(ii) a chemistry book is at each end of the shelf, [2]

(iii) all the mathematics books are kept together and all the physics books are kept together. [3]

(b) A quiz team of 6 children is to be chosen from a class of 8 boys and 10 girls. Find the number of ways of choosing the team if

(i) there are no restrictions, [1]

(ii) there are more boys than girls in the team. [4]

Question 12 is printed on the next page.

12 A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2\sin\left(x + \frac{\pi}{3}\right)$. Given that the curve has a gradient of 5 at the point $\left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$, find the equation of the curve. [8]

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

7218149806

ADDITIONAL MATHEMATICS **0606/12**

Paper 1 **October/November 2019**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

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Mathematical Formulae

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$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

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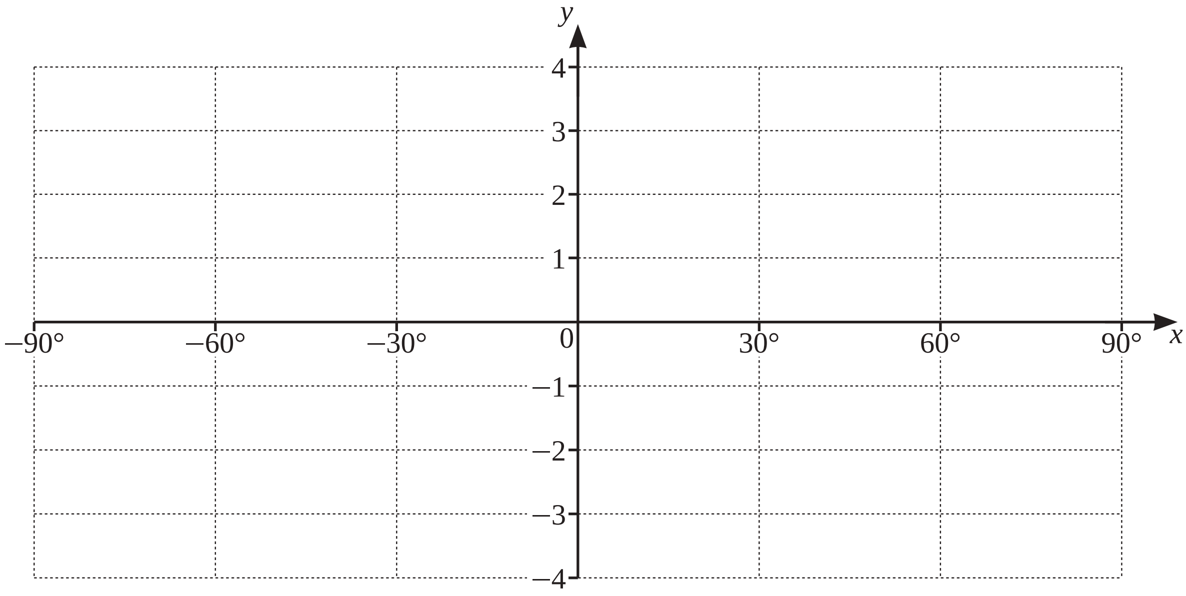
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(i)** On the axes below, sketch the graph of $y = 2\cos 3x - 1$ for $-90° \leqslant x \leqslant 90°$.



[3]

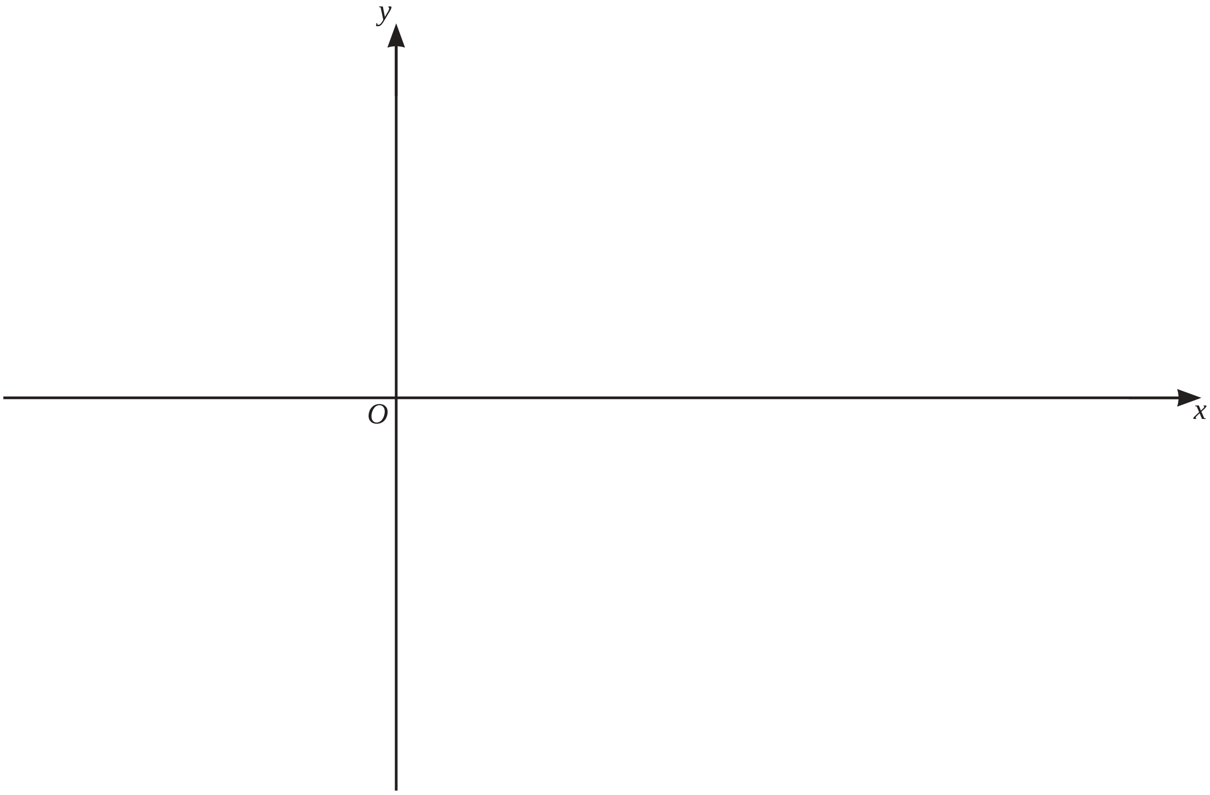
(ii) Write down the amplitude of $2\cos 3x - 1$. [1]

(iii) Write down the period of $2\cos 3x - 1$. [1]

2 When $\lg y^2$ is plotted against x, a straight line is obtained passing through the points $(5, 12)$ and $(3, 20)$. Find y in terms of x, giving your answer in the form $y = 10^{ax+b}$, where a and b are integers. [5]

3 The first three terms in the expansion of $\left(1-\frac{x}{7}\right)^{14}(1-2x)^4$ can be written as $1+ax+bx^2$. Find the value of each of the constants a and b. [6]

4 **(i)** On the axes below, sketch the graph of $y = \left|2x^2 - 9x - 5\right|$ showing the coordinates of the points where the graph meets the axes. [4]



(ii) Find the values of k for which $\left|2x^2 - 9x - 5\right| = k$ has exactly 2 solutions. [3]

5 **(a)** It is given that $\mathrm{f} : x \mapsto \sqrt{x}$ for $x \geqslant 0$,
$\mathrm{g} : x \mapsto x + 5$ for $x \geqslant 0$.

Identify each of the following functions with one of f^{-1}, g^{-1}, fg, gf, f^2, g^2.

(i) $\sqrt{x+5}$ [1]

(ii) $x - 5$ [1]

(iii) x^2 [1]

(iv) $x + 10$ [1]

(b) It is given that $\mathrm{h}(x) = a + \dfrac{b}{x^2}$ where a and b are constants.

(i) Why is $-2 \leqslant x \leqslant 2$ not a suitable domain for $\mathrm{h}(x)$? [1]

(ii) Given that $\mathrm{h}(1) = 4$ and $\mathrm{h}'(1) = 16$, find the value of a and of b. [2]

6 **(a)** Write $\dfrac{\sqrt{p}\left(\dfrac{qp}{r}\right)^2}{p^{-1}\sqrt[3]{qr}}$ in the form $p^a q^b r^c$, where a, b and c are constants. [3]

(b) Solve $\log_7 x + 2\log_x 7 = 3$. [4]

7 It is given that $y = \left(1 + \mathrm{e}^{x^2}\right)(x + 5)$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(ii) Find the approximate change in y as x increases from 0.5 to $0.5 + p$, where p is small. [2]

(iii) Given that y is increasing at a rate of 2 units per second when $x = 0.5$, find the corresponding rate of change in x. [2]

8 **(a)** Five teams took part in a competition in which each team played each of the other 4 teams. The following table represents the results after all the matches had been played.

Team	Won	Drawn	Lost
A	2	1	1
B	1	3	0
C	1	1	2
D	0	1	3
E	3	0	1

Points in the competition were awarded to the teams as follows

4 for each match won, 2 for each match drawn, 0 for each match lost.

(i) Write down two matrices whose product under matrix multiplication will give the total number of points awarded to each team. [2]

(ii) Evaluate the matrix product from **part (i)** and hence state which team was awarded the most points. [2]

(b) It is given that $\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 5 & 0 \\ 1 & -2 \end{pmatrix}$.

(i) Find $\mathbf{A}^{-1}$. [2]

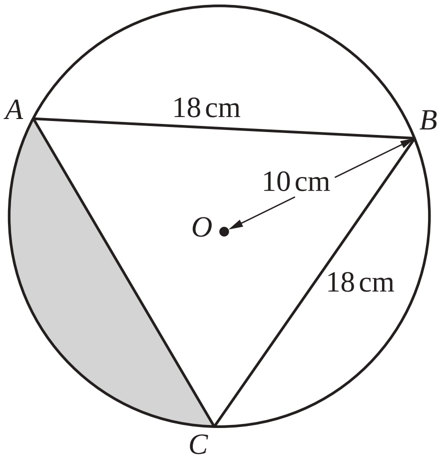
(ii) Hence find the matrix **C** such that **AC** = **B**. [3]

9 A solid circular cylinder has a base radius of r cm and a height of h cm. The cylinder has a volume of $1200\pi\,\text{cm}^3$ and a total surface area of $S\,\text{cm}^2$.

(i) Show that $S = 2\pi r^2 + \dfrac{2400\pi}{r}$. [3]

(ii) Given that h and r can vary, find the stationary value of S and determine its nature. [5]

10



The diagram shows a circle centre O, radius 10 cm. The points A, B and C lie on the circumference of the circle such that $AB = BC = 18$ cm.

(i) Show that angle $AOB = 2.24$ radians correct to 2 decimal places. [3]

(ii) Find the perimeter of the shaded region. [5]

Continuation of working space for Question 10(ii).

(iii) Find the area of the shaded region. [3]

Question 11 is printed on the next page.

11 A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2(3x-1)^{-\frac{2}{3}}$. Given that the curve has a gradient of 6 at the point (3, 11), find the equation of the curve. [8]

Cambridge Assessment International Education
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME			
CENTRE NUMBER	☐☐☐☐☐	CANDIDATE NUMBER	☐☐☐☐

ADDITIONAL MATHEMATICS **0606/13**

Paper 1 **October/November 2019**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **15** printed pages and **1** blank page.

DC (SC/FC) 172297/1

Cambridge Assessment International Education

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 In a group of 145 students, the numbers studying mathematics, physics and chemistry are given below. All students study at least one of the three subjects.

x students study all 3 subjects

24 students study both mathematics and chemistry

23 students study both physics and chemistry

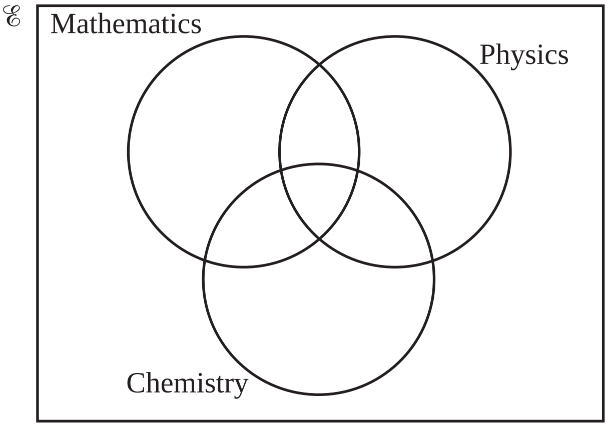
28 students study both mathematics and physics

50 students study chemistry

75 students study physics

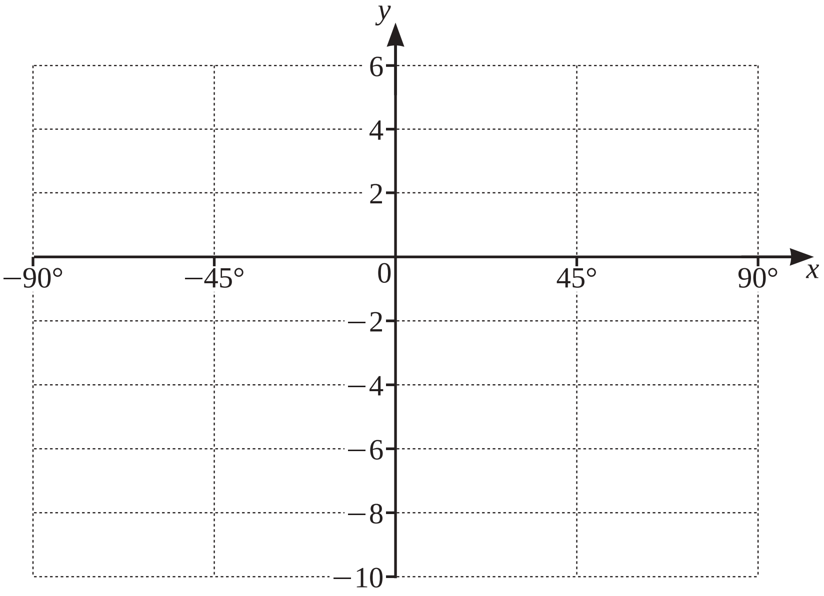
80 students study mathematics

(i) Using the Venn diagram, find the value of x. [4]



(ii) Find the number of students who study mathematics only. [1]

2 **(i)** On the axes below, sketch the graph of $y = 5\cos 4x - 3$ for $-90° \leqslant x \leqslant 90°$.



[4]

(ii) Write down the amplitude of y. [1]

(iii) Write down the period of y. [1]

3 **(i)** Differentiate $y = \left(3x^2 - 1\right)^{-\frac{1}{3}}$ with respect to x. [2]

(ii) Find the approximate change in y as x increases from $\sqrt{3}$ to $\sqrt{3} + p$, where p is small. [1]

(iii) Find the equation of the normal to the curve $y = \left(3x^2 - 1\right)^{-\frac{1}{3}}$ at the point where $x = \sqrt{3}$. [3]

4 It is given that $\mathbf{A} = \begin{pmatrix} 5 & 2 \\ 4 & -1 \end{pmatrix}$.

(i) Find $\mathbf{A}^{-1}$. [2]

(ii) Hence find, in radians, the acute angles x and y such that

$$\begin{aligned} 5\tan x + 2\tan y &= 12, \\ 4\tan x - \tan y &= 7. \end{aligned}$$

[5]

5 **(i)** Differentiate $(x^2+3)\ln(x^2+3)$ with respect to x. [3]

(ii) Hence find $\int x\ln(x^2+3)\,dx$. [2]

6

x	1	1.5	2	2.5	3
y	6	14.3	48	228	1536

The table shows values of the variables x and y such that $y = Ab^{x^2}$, where A and b are constants.

(i) Draw a straight line graph to show that $y = Ab^{x^2}$. [4]

(ii) Use your graph to find the value of A and of b. [4]

(iii) Estimate the value of x when $y = 100$. [2]

7 **(a)** A 5-digit code is to be chosen from the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9. Each digit may be used once only in any 5-digit code. Find the number of different 5-digit codes that may be chosen if

(i) there are no restrictions, [1]

(ii) the code is divisible by 5, [1]

(iii) the code is even and greater than 70 000. [3]

(b) A team of 6 people is to be chosen from 8 men and 6 women. Find the number of different teams that may be chosen if

(i) there are no restrictions, [1]

(ii) there are no women in the team, [1]

(iii) there are a husband and wife who must not be separated. [3]

8 **(a)** Given that $\log_a x = p$ and $\log_a y = q$, find, in terms of p and q,

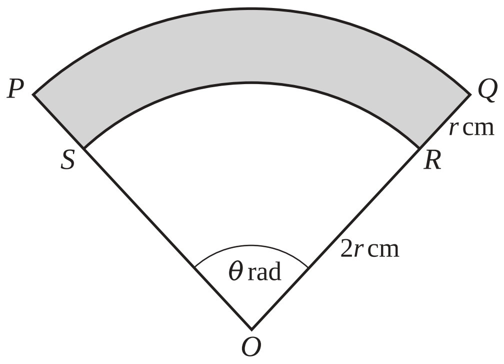
(i) $\log_a axy^2$, [2]

(ii) $\log_a\left(\dfrac{x^3}{ay}\right)$, [2]

(iii) $\log_x a + \log_y a$. [1]

(b) Using the substitution $m = 3^x$, or otherwise, solve $3^x - 3^{1+2x} + 4 = 0$. [3]

9



The diagram shows a sector *OPQ* of the circle centre *O*, radius $3r$ cm. The points *S* and *R* lie on *OP* and *OQ* respectively such that *ORS* is a sector of the circle centre *O*, radius $2r$ cm. The angle $POQ = \theta$ radians. The perimeter of the shaded region *PQRS* is 100 cm.

(i) Find θ in terms of r. [2]

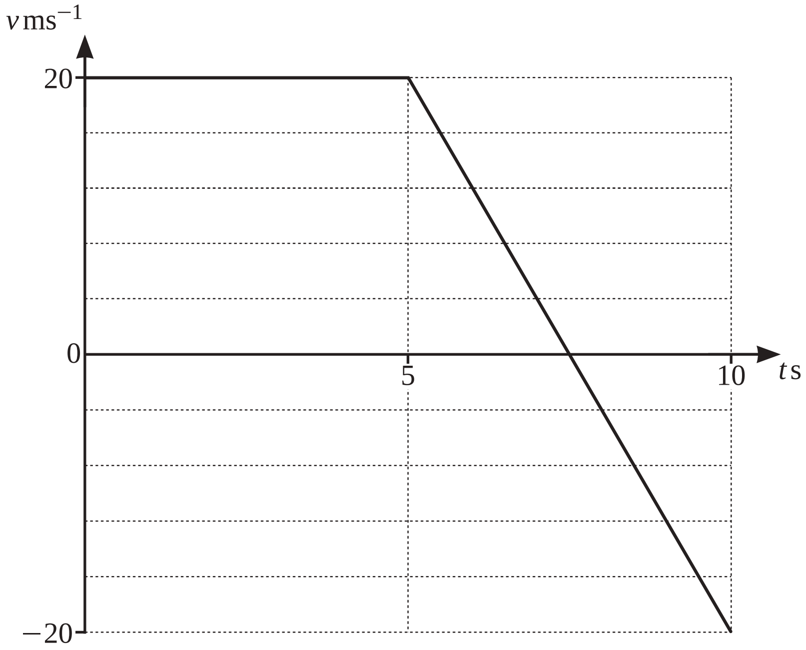
(ii) Hence show that the area, A cm^2, of the shaded region *PQRS* is given by $A = 50r - r^2$. [2]

(iii) Given that r can vary and that A has a maximum value, find this value of A. [2]

(iv) Given that A is increasing at the rate of $3\,\text{cm}^2\,\text{s}^{-1}$ when $r = 10$, find the corresponding rate of change of r. [3]

(v) Find the corresponding rate of change of θ when $r = 10$. [3]

10 (a)



The velocity-time graph for a particle P is shown by the two straight lines in the diagram.

(i) Find the deceleration of P for $5 \leqslant t \leqslant 10$. [2]

(ii) Write down the value of t when the speed of P is zero. [1]

(iii) Find the distance P has travelled for $0 \leqslant t \leqslant 10$. [2]

(b) A particle Q has a displacement of x m from a fixed point O, t s after leaving O. The velocity, $v \text{ ms}^{-1}$, of Q at time t s is given by $v = 6e^{2t} + 1$.

(i) Find an expression for x in terms of t. [3]

(ii) Find the value of t when the acceleration of Q is 24 ms^{-2}. [3]

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

5120051451

ADDITIONAL MATHEMATICS **0606/21**

Paper 2 **October/November 2019**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
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The total number of marks for this paper is 80.

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DC (NF/SG) 172320/3

Cambridge Assessment International Education

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \, .$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

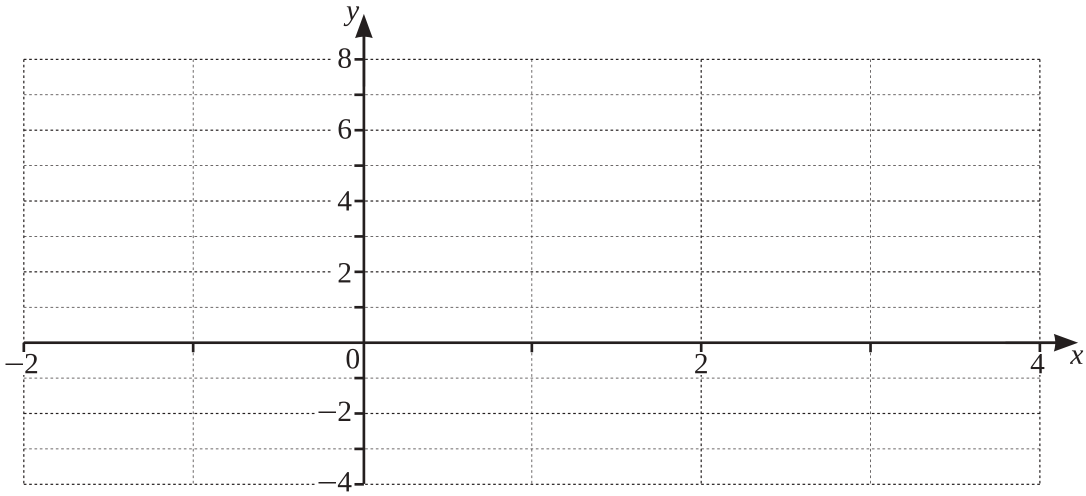
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

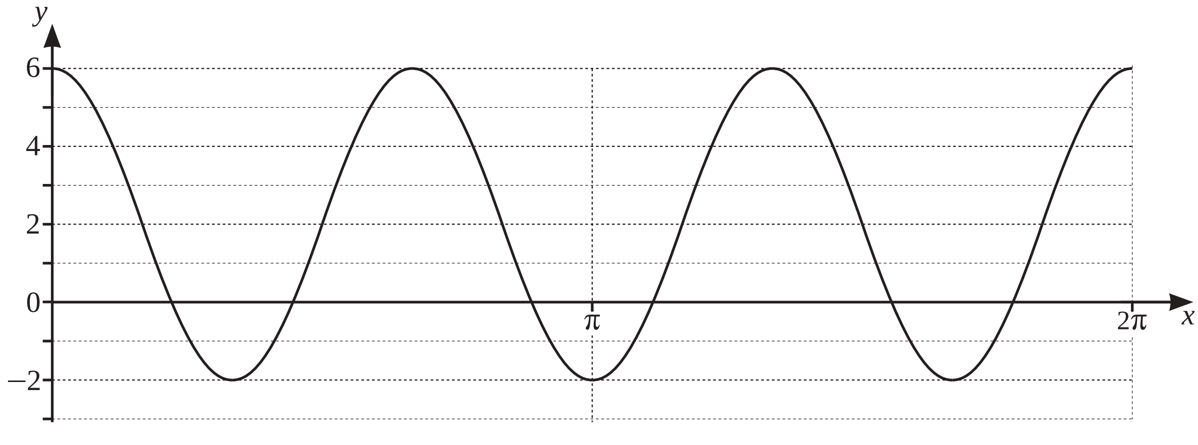
1 **(i)** On the axes below, draw the graph of $y = |2x - 3|$.



[2]

(ii) Solve the equation $7 - |2x - 3| = 0$. [3]

2



The figure shows part of the graph of $y = p + q\cos rx$. Find the value of each of the integers p, q and r.

$p =$

$q =$

$r =$

[3]

3 **(a)** Solve $e^{2x+1} = 3e^{4-3x}$. [3]

(b) Solve $\lg(y-6) + \lg(y+15) = 2$. [5]

4 **Do not use a calculator in this question.**

Solve the following simultaneous equations, giving your answers for both x and y in the form $a+b\sqrt{2}$, where a and b are integers.

$$2x+y=5$$

$$3x-\sqrt{2}y=7$$

[5]

5 A particle is moving in a straight line such that t seconds after passing a fixed point O its displacement, s m, is given by $s = 3\sin 2t + 4\cos 2t - 4$.

(i) Find expressions for the velocity and acceleration of the particle at time t. [3]

(ii) Find the first time when the particle is instantaneously at rest. [3]

(iii) Find the acceleration of the particle at the time found in **part (ii)**. [2]

6 Do not use a calculator in this question.

The curve $xy = 11x + 5$ cuts the line $y = x + 10$ at the points A and B. The mid-point of AB is the point C. Show that the point C lies on the line $x + y = 11$. [7]

7 **(a)** **(i)** Use the factor theorem to show that $2x-1$ is a factor of $\mathrm{p}(x)$, where $\mathrm{p}(x)=4x^3+9x-5$. [1]

(ii) Write $\mathrm{p}(x)$ as a product of linear and quadratic factors. [2]

(b) **(i)** Show that $13\tan x\sec x-4\sin x-5\sec^2 x=0$ can be written as $4\sin^3 x+9\sin x-5=0$. [3]

(ii) Using your answers to **part (a)(ii)** and **part (b)(i)** solve the equation

$$13\tan x\sec x-4\sin x-5\sec^2 x=0 \quad \text{for } 0<x<2\pi \text{ radians.}$$ [4]

8 The equation of a curve is given by $y = xe^{-2x}$.

(i) Find $\frac{dy}{dx}$. [3]

(ii) Find the exact coordinates of the stationary point on the curve $y = xe^{-2x}$. [2]

(iii) Find, in terms of e, the equation of the tangent to the curve $y = xe^{-2x}$ at the point $\left(1, \frac{1}{e^2}\right)$. [2]

(iv) Using your answer to **part (i)**, find $\int xe^{-2x}dx$. [3]

9 Given that $\mathbf{A} = \begin{pmatrix} 5 & 2 \\ -9 & -3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ 6 & 5 \end{pmatrix}$, find

(i) $\mathbf{A}^{-1}$, [2]

(ii) $\mathbf{B}^2$, [2]

(iii) the matrix **C**, where $\mathbf{B}^{-1}\mathbf{C} + \mathbf{A} = \mathbf{B}$, [3]

(iv) the matrix **D**, where $\mathbf{B}^{-2}\mathbf{D}\mathbf{A} = \mathbf{I}$. [3]

10 **(i)** Expand $(3+x)^4$ evaluating each coefficient. [3]

In the expansion of $\left(x-\frac{p}{x}\right)(3+x)^4$ the coefficient of x is zero.

(ii) Find the value of the constant p. [2]

(iii) Hence find the term independent of x. [1]

(iv) Show that the coefficient of x^2 is 90. [2]

11 A plane, which can travel at a speed of $300\,\text{km}\,\text{h}^{-1}$ in still air, heads due north. The plane is blown off course by a wind so that it travels on a bearing of 010° at a speed of $280\,\text{km}\,\text{h}^{-1}$.

(i) Find the speed of the wind. [3]

(ii) Find the direction of the wind as a bearing correct to the nearest degree. [3]

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Cambridge International General Certificate of Secondary Education

CANDIDATE NAME	

CENTRE NUMBER							CANDIDATE NUMBER					

5192885901

ADDITIONAL MATHEMATICS **0606/22**

Paper 2 **October/November 2019**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.
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The total number of marks for this paper is 80.

This document consists of **16** printed pages.

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Cambridge Assessment International Education

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1} b + \binom{n}{2}a^{n-2} b^2 + \ldots + \binom{n}{r}a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

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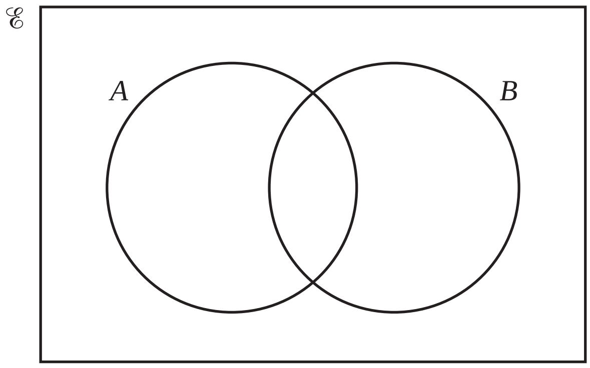
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

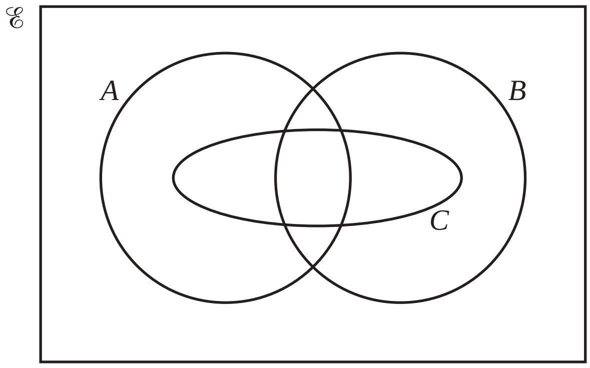
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

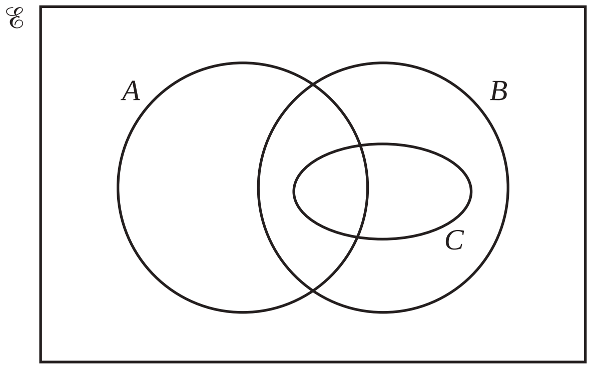
1 On each of the Venn diagrams below, shade the region indicated.



$(A' \cap B) \cup (A \cap B')$



$(A \cap B) \cup C$



$A \cap B \cap C'$

[3]

2 Given that $y = 2\sin 3x + \cos 3x$, show that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} + 3y = k\sin 3x$, where k is a constant to be determined. [5]

3 A 5-digit code is formed using the following characters.

Letters	a	e	i	o	u	
Numbers	1	2	3	4	5	6
Symbols	@	*	#			

No character can be repeated in a code. Find the number of possible codes if

(i) there are no restrictions, [2]

(ii) the code starts with a symbol followed by two letters and then two numbers, [2]

(iii) the first two characters are numbers, and no other numbers appear in the code. [2]

4 Find the values of k for which the line $y = kx + 3$ does not meet the curve $y = x^2 + 5x + 12$. [5]

5 At the point where $x = 1$ on the curve $y = \frac{k}{(x+1)^2}$, the normal has a gradient of $\frac{1}{3}$.

(i) Find the value of the constant k. [4]

(ii) Using your value of k, find the equation of the tangent to the curve at $x = 2$. [3]

6 **(i)** Show that $\frac{\tan x}{1+\sec x}+\frac{1+\sec x}{\tan x}=\frac{2}{\sin x}$. [5]

(ii) Hence solve the equation $\frac{\tan x}{1+\sec x}+\frac{1+\sec x}{\tan x}=1+3\sin x$ for $0° \leqslant x \leqslant 180°$. [4]

7 **(a)** The cubic equation $x^3 + ax^2 + bx - 40 = 0$ has three positive integer roots. Two of the roots are 2 and 4. Find the other root and the value of each of the integers *a* and *b*. [4]

(b) **Do not use a calculator in this question.**

Solve the equation $x^3 - 5x^2 - 46x - 40 = 0$ given that it has three integer roots, only one of which is positive. [4]

8 **(i)** A particle A travels with a speed of $6.5\,\text{ms}^{-1}$ in the direction $-5\mathbf{i}-12\mathbf{j}$. Find the velocity, $\boldsymbol{v}_A$, of A. [2]

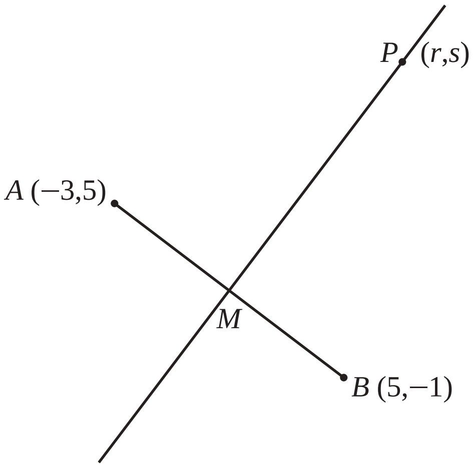
(ii) A particle B travels with velocity $\boldsymbol{v}_B = 12\mathbf{i}-9\mathbf{j}$. Find the speed, in ms^{-1}, of B. [2]

Particle A starts moving from the point with position vector $20\mathbf{i}-7\mathbf{j}$. At the same time particle B starts moving from the point with position vector $-67\mathbf{i}+11\mathbf{j}$.

(iii) Find $\boldsymbol{r}_A$, the position vector of A after t seconds, and $\boldsymbol{r}_B$, the position vector of B after t seconds. [2]

(iv) Find the time when the particles collide and the position vector of the point of collision. [3]

9



The diagram shows the points A (–3, 5) and B (5, –1). The mid-point of AB is M and the line PM is perpendicular to AB. The point P has coordinates (r, s).

(i) Find the equation of the line PM in the form $y = mx + c$, where m and c are exact constants. [5]

(ii) Hence find an expression for s in terms of r. [1]

(iii) Given that the length of *PM* is 10 units, find the value of r and of s. [5]

10 **(i)** Given that $y = \frac{\ln x}{x^2}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(ii) Find the coordinates of the stationary point on the curve $y = \frac{\ln x}{x^2}$. [3]

(iii) Using your answer to **part (i)**, find $\int \frac{\ln x}{x^3} \mathrm{d}x$. [3]

(iv) Hence evaluate $\int_1^2 \frac{\ln x}{x^3} \mathrm{d}x$. [2]

Question 11 is printed on the next page.

11 Do not use a calculator in this question.

Solve the quadratic equation $(\sqrt{5}-3)x^2+3x+(\sqrt{5}+3)=0$, giving your answers in the form $a+b\sqrt{5}$, where a and b are constants. [6]

Cambridge Assessment International Education
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

ADDITIONAL MATHEMATICS **0606/23**

Paper 2 **October/November 2019**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator

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DC (NH/FC) 172299/2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1} b + \binom{n}{2}a^{n-2} b^2 + \ldots + \binom{n}{r}a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

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$$\mathrm{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Solve $|3x+2| = x+4$. [3]

2 **(i)** Show that $\dfrac{\operatorname{cosec}x - \cot x}{1-\cos x} = \operatorname{cosec}x$. [3]

(ii) Hence solve $\dfrac{\operatorname{cosec}x - \cot x}{1-\cos x} = 2$ for $0° < x < 180°$. [2]

3 The first four terms in the expansion of $(1+ax)^5(2+bx)$ are $2+32x+210x^2+cx^3$, where a, b and c are integers. Show that $3a^2-16a+21=0$ and hence find the values of a, b and c. [8]

4 **(i)** Given that $y = 2x^2 - 4x - 7$, write y in the form $a(x-b)^2 + c$, where a, b and c are constants. [3]

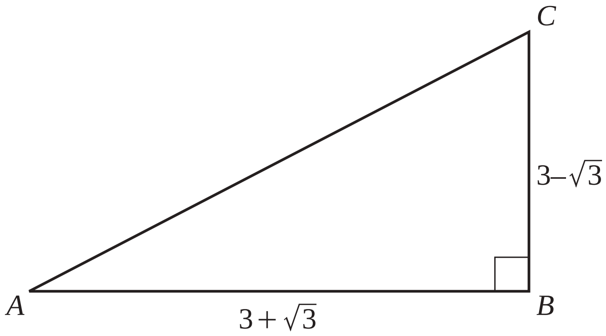
(ii) Hence write down the minimum value of y and the value of x at which it occurs. [2]

(iii) Using your answer to **part (i)**, solve the equation $2p - 4\sqrt{p} - 7 = 0$, giving your answer correct to 2 decimal places. [3]

5 **(a)** Solve $3\cot^2\left(y-\frac{\pi}{4}\right)=1$ for $0 < y < \pi$ radians. [4]

(b) Solve $7\cot z+\tan z=7\operatorname{cosec} z$ for $0° \leqslant z \leqslant 360°$. [6]

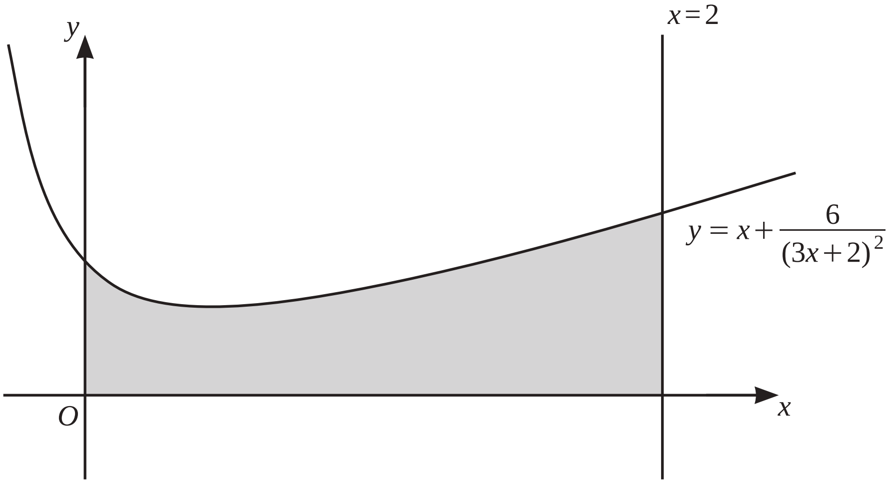
6 **Do not use a calculator in this question.**



(i) Find tan *ACB* in the form $r+s\sqrt{3}$, where *r* and *s* are integers. [3]

(ii) Find *AC* in the form $t\sqrt{u}$, where *t* and *u* are integers and $t \neq 1$. [3]

7



The diagram shows part of the curve $y = x + \frac{6}{(3x+2)^2}$ and the line $x = 2$.

(i) Find, correct to 2 decimal places, the coordinates of the stationary point. [6]

(ii) Find the area of the shaded region, showing all your working. [4]

8 The roots of the equation $x^3 + ax^2 + bx + 24 = 0$ are 2, 3 and p, where p is an integer.

(i) Find the value of p. [1]

(ii) Show that $a = -1$ and find the value of b. [4]

Given that a curve has equation $y = x^3 - x^2 + bx + 24$ find, using your value of b,

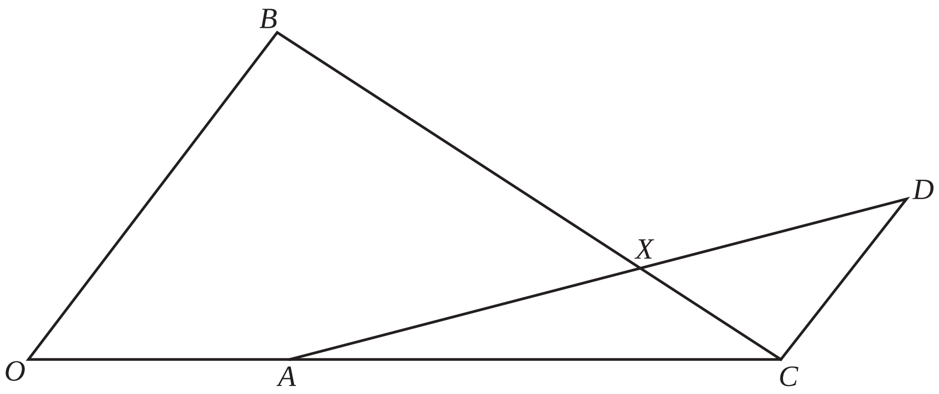
(iii) $\dfrac{dy}{dx}$, [1]

(iv) the integer value of x for which the gradient of the curve is 2 and the corresponding value of y. [3]

The coordinates of the point P on the curve are given by the values of x and y found in **part (iv)**.

(v) Find the equation of the tangent to the curve at P. [1]

9



The diagram shows points O, A, B, C, D and X. The position vectors of A, B, and C relative to O are $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = 2\mathbf{b}$ and $\overrightarrow{OC} = 3\mathbf{a}$. The vector $\overrightarrow{CD} = \mathbf{b}$.

(i) Given that $\overrightarrow{AX} = \lambda\overrightarrow{AD}$, find $\overrightarrow{OX}$ in terms of λ, **a** and **b**. [2]

(ii) Given that $\overrightarrow{BX} = \mu\overrightarrow{BC}$, find $\overrightarrow{OX}$ in terms of μ, **a** and **b**. [2]

(iii) Hence find the value of λ and of μ. [4]

(iv) Find the ratio $\frac{AX}{XD}$. [1]

10 The functions f and g are defined by

$$\mathrm{f}(x) = \ln(3x+2) \quad \text{for } x > -\frac{2}{3},$$
$$\mathrm{g}(x) = \mathrm{e}^{2x} - 4 \quad \text{for } x \in \mathbb{R}.$$

(i) Solve $\mathrm{gf}(x) = 5$. [5]

(ii) Find $f^{-1}(x)$. [2]

(iii) Solve $f^{-1}(x) = g(x)$. [4]

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