## 2019 Secondary 4 EMath

<table>
<thead>
<tr>
<th></th>
<th>School Name</th>
<th>Grade</th>
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<tbody>
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<td>2.</td>
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<td>3.</td>
<td>Nan Chiau High</td>
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<td>4.</td>
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<td>5.</td>
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<td>6.</td>
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<td>7.</td>
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<td>11.</td>
<td>West Spring Secondary</td>
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</tr>
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<td>12.</td>
<td>Yuying Secondary</td>
<td>SA2</td>
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ANG MO KIO SECONDARY SCHOOL
MID-YEAR EXAMINATION 2019
SECONDARY FOUR EXPRESS / FIVE NORMAL ACADEMIC

MATHEMATICS
Paper 1

Setter: Mrs Seah Kwan Chaet

Wednesday 15 May 2019 2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact,
give the answer to three significant figures. Give answers in degrees to one decimal place.
For \pi, use either your calculator value or 3.142, unless the question requires the answer in terms of \pi.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner’s Use

80

This document consists of 17 printed pages and 1 blank page.
Mathematical Formulae

Compound interest

Total amount = \( P \left(1 + \frac{r}{100}\right)^n \)

Mensuration

Curve surface area of a cone = \( \pi rl \)

Surface area of a sphere = \( 4\pi r^2 \)

Volume of a cone = \( \frac{1}{3} \pi r^2 h \)

Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

Area of triangle \( ABC = \frac{1}{2} ab \sin C \)

Arc length = \( r\theta \), where \( \theta \) is in radians

Sector Area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

Trigonometry

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\( a^2 = b^2 + c^2 - 2bc \cos A \)

Statistics

Mean = \[ \frac{\sum fx}{\sum f} \]

Standard deviation = \[ \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2} \]
1 (a) Simplify $x + 7 - x(2 - 3x)$.

**Answer**

(b) Factorise completely $4a + 12a - x - 3$.

**Answer**

2 The pie chart shows the sales for 4 different flavours of cookies.

State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

**Answer**
3 (a) Write down the set notation for the set represented by the shaded region in the Venn diagram below.

![Venn diagram]

Answer

(b) \( \mathcal{E} = \{ x : x \text{ is an integer, } 1 \leq x \leq 9 \} \)
\( A = \{ x : x \text{ is an odd number} \} \)
\( B = \{ x : x \text{ is a factor of } 6 \} \)

(i) Draw a Venn diagram in the box below to illustrate the above information.

Answer

(ii) List the elements in the set \( (A \cup B)' \).

Answer
4 (a) Express $x^2 - 2x - 2$ in the form $p + (x + q)^2$.

Answer

(b) Sketch the graph of $y = x^2 - 2x - 2$ on the axes below.
Indicate clearly the values where the graph crosses the $x$– and $y$– axes.

Answer

5 The exchange rate between Singapore dollars ($) and Japanese Yen (¥) was $1 = ¥81.7339$. Mr Toshi bought a watch priced at $1550. The shop charged 1.8% commission as he chose to pay in Japanese Yen using his credit card. What was the total cost of the watch in Japanese Yen?

Answer ¥
6 Write as a single fraction in its simplest form \( \frac{4}{3-x} + \frac{1}{x^2-3x} \).

Answer

---

7 A map is drawn to a scale of 1 : 500 000.

(a) The length of a runway at an airport is 0.6 cm on the map. Calculate the actual length of the runway in kilometres.

Answer \( \text{km} \) [1]

(b) The airport has an area of 5 km\(^2\). Calculate the area, in square centimetres, of the airport on the map.

Answer \( \text{cm}^2 \) [2]
The monthly telephone charges offered by a telecommunications company is obtained by adding a fixed charge of $28 and the total usage for the month as shown below.

<table>
<thead>
<tr>
<th>Usage Per Month</th>
<th>Rate (cents per minute)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First 5 hours</td>
<td>Free</td>
</tr>
<tr>
<td>Next 20 hours</td>
<td>3.5</td>
</tr>
<tr>
<td>Exceed 20 hours</td>
<td>10</td>
</tr>
</tbody>
</table>

If the total usage for the month of February was 31 hours, calculate the telephone charges for that month.

Answer: $ \text{[2]}$

9 Two pails, $X$ and $Y$, are geometrically similar. The volume of pail $X$ is $512 \text{ cm}^3$. Given that the base area of pail $Y$ is four times the base area of pail $X$, find the volume of pail $Y$.

![Diagram of two pails](image)

Answer: $ \text{[3]}$
10 The diagram below shows a triangle $PQR$ with coordinates $P(0, 5)$ and $Q(7, 0)$. The area of the triangle is $28 \text{ cm}^2$.

Find
(a) the coordinates of $R$,

Answer $R(\text{ } \text{ } \text{ })$ [2]

(b) the equation of the line $QR$.

Answer $..$ [2]

VI If the length of a rectangle is increased by $30\%$ and its breadth is decreased by $20\%$, find the percentage change in its area.

Answer $\%$ [2]
12 Written as the product of its prime factors, \(126 = 2 \times 3^2 \times 7\).
(a) Express 720 as a product of its prime factors.

\[\text{Answer}\]

(b) Find the largest integer which is a factor of both 126 and 720.

\[\text{Answer}\]

(c) Find the smallest possible value of \(m\) if \(720m\) is both a perfect square and a perfect cube.

\[\text{Answer} \quad m = \]

13 In the diagram below, \(AB = 8\) cm, \(BC = 9\) cm and \(\angle BAC = 42^\circ\). Find \(\angle ACB\).

\[\text{Answer}\]

\[\text{[Turn Over} \]
14 The diagram below shows part of a circle $ABC$ with centre $O$ and radius 10 cm. The radius $OC$ makes an angle of 1.2 radians with the radius $OB$. $ACD$ is a sector with centre $A$ and radius 16 cm.

Find the area of the shaded region $BCD$.

Answer $\text{cm}^2$ [5]
The cumulative frequency curve shows the marks obtained by 200 pupils in a particular Science examination. The maximum mark is 80.

Cumulative Frequency

Find the
(a) (i) median,

Answer

(ii) interquartile range.

Answer

(b) Find the passing mark if 36% of the pupils passed the examination.

Answer
The sum of a series of numbers is given below

$$S_n = 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}.$$  

(a) Find the value of $S_{35}$.

Answer [1]

(b) Find the value of $n$ given that $S_n = 1378$.

Answer [2]

(c) If $T = 101 + 102 + 103 + \ldots + 199$, find the value of $T$.

Answer [2]

(d) Given the sum of even integers is $P = 2 + 4 + 6 + \ldots + 100$, find the value of $P$.

Answer [2]

(e) Using your answer in part (d), find the sum of all the odd integers between 0 and 100.

Answer [1]
A drinks vending machine takes 50 cent coins and $1 coins. A drink costs $1.50. The probability that the machine will accept a particular 50 cent coin is 0.9 and that it will accept a particular $1 coin is 0.85.

(a) What is the probability that the machine will not accept a particular 50 cent coin?

Answer

(b) John put one 50 cent coin and one $1 coin into the machine. Calculate the probability that
(i) the machine will not accept both coins,

Answer

(ii) John will get a drink only when he inserts another $1 coin.

Answer

(c) Peter only has three 50 cent coins. Calculate the probability that
(i) the machine will accept all three coins,

Answer

(ii) Peter will not get a drink.

Answer
18 \hspace{1cm} \overrightarrow{AB} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} -5 \\ 2 \end{pmatrix} \text{ and } B \text{ is the point } (7, 0). \text{ Find }

(a) \hspace{1cm} |\overrightarrow{AB}|.

\text{Answer} \hspace{1cm} [1]

(b) \hspace{1cm} \text{the coordinates of the point } C.

\text{Answer} \hspace{1cm} C ( \quad , \quad ) \hspace{1cm} [2]

19 \hspace{1cm} \text{The marks scored by 12 students from a Sec 4 class in a test are listed below.}

\begin{array}{cccccccc}
72, 72, 27, 38, 85, 54, 32, 76, 46, 68, 56, 64, 92.
\end{array}

\text{The diagram below shows a box-and-whisker plot representing their results.}

\hspace{1cm}

\hspace{1cm}

(a) \hspace{1cm} \text{Find the value of } x \text{ and of } y.

\text{Answer} \hspace{1cm} x = \quad , \quad y = \hspace{1cm} [2]

(b) \hspace{1cm} \text{A new student later joined the group and sat for the same test. If the median mark remains unchanged, find the mark scored by the new student.}

\text{Answer} \hspace{1cm} [1]
The diagram below shows a cone fitted inside a sphere. \( V \) is the vertex of the cone and \( O \) is the centre of the sphere. Given that the cone has a height of 10 cm and a base radius of 5 cm, find the radius of the sphere, \( R \).

Answer cm [3]
21 (a) Simplify each of the following, giving your answer as a positive index.

\((x^{-3}y)^{-5}\)

**Answer**

(ii) \(\frac{1}{18x^3} \div \frac{4\sqrt{x}}{4\sqrt{x}}\)

**Answer**

(b) Solve the equation \(2^{5x-1} = 3^{5x-1}\).

**Answer** \(x = \)
The diagram shows part of a regular \(n\)-sided polygon, \(PQRST\ldots\).

(a) Explain why triangles \(QRS\) and \(TSR\) are congruent.

\textit{Answer}

(b) It is given that angle \(AQR = 30^\circ\). Find

(i) the value of \(n\),

\textit{Answer} \hspace{1cm} [1]

(ii) angle \(RTS\),

\textit{Answer} \hspace{1cm} [2]

(iii) angle \(OPT\).

\textit{Answer} \hspace{1cm} [2]
ANG MO KIO SECONDARY SCHOOL
MID-YEAR EXAMINATION 2019
SECONDARY FOUR EXPRESS / FIVE NORMAL ACADEMIC

MATHMATICS
Paper 2

Setter: Mdm Kwa Leng Leng

Monday 13 May 2019 2 hours 30 minutes

Candidates answer on the Question Paper.
Additional Materials: Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact,
give the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer
in terms of \( \pi \).

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.
Mathematical Formulae

**Compound interest**

Total amount = \( P \left(1 + \frac{r}{100}\right)^n \)

**Mensuration**

Curve surface area of a cone = \( \pi rl \)

Surface area of a sphere = \( 4\pi r^2 \)

Volume of a cone = \( \frac{1}{3} \pi r^2 h \)

Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

Area of triangle \( ABC = \frac{1}{2} ab \sin C \)

Arc length = \( r\theta \), where \( \theta \) is in radians

Sector Area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

**Trigonometry**

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

**Statistics**

\[
\text{Mean} = \frac{\sum fx}{\sum f}
\]

\[
\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}
\]
1 (a) It is given that \( y + z = \frac{4y - 6z}{7x} \).

(i) Find \( x \) when \( y = 2 \) and \( z = -1 \). \( [1] \)

(ii) Express \( z \) in terms of \( x \) and \( y \). \( [2] \)

(b) Solve the equation \( \frac{2a + 1}{3} + \frac{a - 2}{4} = 2 \). \( [2] \)
(e) Solve these simultaneous equations.

\[ 4p + 3q = 2 \]
\[ 5p - 4q = 49 \]

[3]

(d) Simplify \( \frac{4h^2 - 36}{4h^2 - 5h - 21} \).

[3]
(a) In \( \triangle OAB \), the point \( X \) on \( AB \) is such that \( AX : XB = 1 : 3 \). \( OXY \) is a straight line and \( XY = \frac{1}{4} Y \).

Given that \( \overrightarrow{OA} = 2a \), \( \overrightarrow{OX} = 6b \), write each of the following in terms of \( a \) and \( b \). Give your answers in their simplest form.

(i) \( \overrightarrow{AX} \), \[1\]

(ii) \( \overrightarrow{OB} \), \[2\]

(iii) \( \overrightarrow{AY} \). \[2\]

(b) State 2 facts about \( \overrightarrow{AY} \) and \( \overrightarrow{OB} \). \[2\]
3 Famous Factory produces pastries and delivers them to two outlets every day that each outlet is open. There are 3 deliveries for each day. The number of pastries supplied in a single delivery is given by the matrix \( P \).

\[
\begin{pmatrix}
60 & 80 \\
30 & 50 \\
80 & 100
\end{pmatrix}
\]

(a) Evaluate the matrix \( D = 3P \). [1]

(b) A chicken pie is sold at $2. A tuna puff is sold at $1.80. A cream puff is sold at $1.40. Represent these prices in a \( 1 \times 3 \) matrix \( C \). [1]

(c) Evaluate the matrix \( Q = CD \). [2]

(d) State what the elements of \( Q \) represent. [1]
(e) In a particular month, Outlet 1 was opened for 22 days and Outlet 2 was opened for 30 days. Write down a $2 \times 2$ matrix $E$ such that the matrix $F = QE$, where the elements of $F$ represent the amount of money collected from selling all the pastries in each outlet in that month. Hence evaluate $F$. [2]

(f) There are two promotion schemes proposed. Scheme A is price reduction of 10% on all pastries and Scheme B is price reduction of 20% on chicken pies only. Supposing all pastries at each outlet are sold at the end of a day, calculate the daily amount of money collected at each outlet from the sale of pastries under each promotion schemes. Hence, propose the suitable scheme for each outlet. [2]
4. \(ABCD\) is a square with sides 12cm and \(BCE\) is a semi-circle with centre \(O\). \(AF\) is a tangent to the semi-circle at point \(E\) and \(BOCF\) is a straight line.

(a) Name a triangle similar to \(\Delta ADG\). [1]

(b) Show that \(\Delta OEF\) is similar to \(\Delta ABF\). [2]
(e) Find the length of $EF$. [2]

(d) Find the ratio

(i) $\frac{\text{Area of } \triangle EFC}{\text{Area of } \triangle ECB}$ [1]

(ii) $\frac{\text{Area of } \triangle OEF}{\text{Area of quadrilateral } ABOE}$ [2]
Xavier and Yves were running on a 400m track in Bishan Stadium. Both of them started from the same point. Xavier started running at a speed of $x$ m/s. At the same time, Yves also started running in the opposite direction at speed of $(x+3)$ m/s and he met Xavier on the track again after $t$ s.

(a) Write down two expressions that each represents the distance travelled by Xavier and Yves after $t$ s. [2]

(b) Using the two expressions of distances in part (a), show that $t = \frac{400}{2x+3}$. [1]

(c) Zed started off together with the 2 boys and he ran in the same direction as Yves. However, he ran at a speed of $(x - 1)$ m/s. Find, in terms of $x$, the time taken when Xavier and Zed met each other on the track again. [1]
(d) Given that Xavier met Zed 20 seconds after passing Yves, form an equation in terms of $x$ and show that it simplifies to \( 4x^2 + 4x - 83 = 0 \). [3]

(e) Solve the equation \( 4x^2 + 4x - 83 = 0 \). [3]

(f) Find the time taken by Xavier to complete running one round around the track. [1]
6. The diagram shows a solid trapezoidal prism. The base $ABCD$ is on flat ground and $ABGH$, $BCFG$, $CDEF$ and $ADEH$ are vertical rectangular planes. $AB = 35\text{m}$, $BC = 23\text{m}$, $CD = 17\text{m}$, $AH = 8\text{m}$, $AD = 20\text{m}$, $\angle ABC = 42^\circ$ and $AB$ is parallel to $DC$.

![Diagram of a trapezoidal prism]

Calculate
(a) the length of $AC$, [3]

(b) the angle of depression of $A$ from $F$, [2]

(c) the area of triangle $ABC$, [2]
(d) the perpendicular distance from point $D$ to the line $AB$,

(e) the surface area of the prism.
7 (a) In the figure below, $AB$ is a diameter of the circle with centre $O$. $AZY$ and $XBY$ are straight lines. $\angle BAZ = 47^\circ$ and $\angle BYZ = 23^\circ$.
Stating your reasons clearly, find $\angle BZX$.

(b) The above diagram shows two rhombuses $ABCD$ and $EFGC$. $\angle BFG = 114^\circ$ and $\angle DAE = 109^\circ$. Stating your reasons clearly, calculate

(i) $\angle FBC$.

(ii) $\angle FGE$.

(iii) $\angle EMB$. 

[3]

[1]

[2]
8 (a) Dice $A$ has the numbers 1, 2, 3 and 4 engraved on it while dice $B$ has the numbers 2, 3, 5 and 7 engraved on it. They are rolled one after another and the sum of the two rolls is then recorded on a possibility diagram.

(i) Complete the possibility diagram.

<table>
<thead>
<tr>
<th>Dice $B$</th>
<th>Dice $A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

(ii) Find the probability that both dice show odd numbers.

(iii) Find the probability that the sum of the two numbers is prime.

(iv) Find the probability that the sum of the two numbers is at most 9.
8 (b) A group of 23 basketball players were asked how many points they scored in a season of matches. The results are presented in the following stem-and-leaf diagram.

```
 4 | 3 5 6 7 7 9
 5 | 1 2 4 6 8
 6 | 0 3 4 6 7 8 9
 7 | 1 1 2 3
 8 | 9
```

Key: 4 | 3 means 43 points

(i) Find the range. [1]

(ii) Find the median score. [1]

(iii) A special award is given to players who scored more than 75 points in a season. Find the percentage of players who attained this award. [1]
(iv) A moderation has to be done and 2 points are to be added across all the scores. 
   Explain how the median score is affected by this moderation. [1]

(v) Find the standard deviation of the scores. [2]

(vi) The standard deviation of the scores of another group of players was 7.96 points. 
   Use this information to comment on one difference between the two groups. [1]
The variables $x$ and $y$ are connected by the equation

$$y = 2x + \frac{18}{x^2}.$$  

Some corresponding values of $x$ and $y$ are given in the following table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>20</td>
<td>11</td>
<td>$a$</td>
<td>8</td>
<td>9.1</td>
<td>10.7</td>
<td>12.5</td>
</tr>
</tbody>
</table>

(a) Find the value of $a$.  

(b) Draw the graph on the graph paper provided and staple it at the end of this question paper.

Using a scale of 2 cm to 1 unit, draw a horizontal $x$-axis for $0 \leq x \leq 6$.

Using a scale of 1 cm to 1 unit, draw a vertical $y$-axis for $0 \leq y \leq 22$.

On your axes, plot the points given in the table and join them with a smooth curve.

(c) Use your graph to solve $2x + \frac{18}{x^2} = 12$ in the range $0 \leq x \leq 6$.

(d) By drawing a tangent, find the gradient of the graph at the point where $x = 1.5$.  


(e) On the same axes, draw the graph of \( y = x + 10 \) for \( 0 \leq x \leq 6 \).

(i) Write down the \( x \)-coordinate of the point where the two graphs intersect.

(ii) Given that this value of \( x \) is a solution to the equation
\[
x^2 + Ax^2 + Bx + 18 = 0,
\]
find the value of \( A \) and the value of \( B \).
The Open Electricity Market is an initiative by the Energy Market Authority (EMA) that allows households to buy electricity from a retailer of their choice to benefit from their competitive pricing or continue to buy from SP Group at the regulated tariff rate, which is reviewed every quarter. There are two types of standard price plans:

- **Fixed Price Plans** are for consumers who prefer a constant electricity rate throughout the contract duration. The rate may be higher or lower than the regulated tariff rate during the contract duration.

- **Discount off the Regulated Tariff Plans** provide a fixed discount off the regulated tariff rate, but the electricity rates will change when the regulated tariff rates changes every quarter.

Jimmy is reviewing some price plans of two retailers. He stays in a private condominium and a part of his utilities bills is given below.

![Bar graph showing electricity usage for six months](image)

(a) Calculate the mean value of the electricity usage over the 6 months. Explain why this mean may not be an appropriate average to assist Jimmy in reviewing his options. [2]
(b) The fact sheets of two of the retailers are shown in the INSERT.

(i) The electricity retailers charge a fee if you terminate the contract before its expiry date. If Jimmy signs a **12-Month Fixed Price Plan** contract with **Sembcorp Power Pte Ltd** starting on 1 January 2019 but considers to switch over to **iSwitch Pte Ltd** from 1 March 2019 onwards, calculate the early termination charge that he as to pay. [2]

(ii) Jimmy forgets to pay his bill for the electricity usage in the month of February 2019 before the payment due date in March. Considering a **12-Month Fixed Price Plan** contract, which company imposes a higher late payment charge? Justify your answer and show your calculations clearly. [2]
(iii) By considering the period of 1 January to 28 February 2019, suggest the plan in iSwitch Pte Ltd that Jimmy should choose. Justify your suggestion and show your calculations clearly.
The fact sheets of two of the retailers are shown below. All fees and charges stated in these fact sheets are inclusive of Goods and Services Tax (GST).

<table>
<thead>
<tr>
<th>Name of retailer</th>
<th>iSwitch Pte Ltd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Plan</td>
<td>'Chope' the Rate (12 Months)</td>
</tr>
<tr>
<td>Electricity Rate</td>
<td>17.62 cents/kWh</td>
</tr>
<tr>
<td>Late Payment Charge</td>
<td>1% of the outstanding amount per month</td>
</tr>
<tr>
<td>Early Termination Charge</td>
<td>$100 if terminated within 3 days from sign-up</td>
</tr>
<tr>
<td></td>
<td>30% of months left × average of latest 2 months bill if terminated after 3 days from sign-up</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Name of retailer</th>
<th>Semcor Power Pte Ltd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Plan</td>
<td>12-Month Fixed Price Plan</td>
</tr>
<tr>
<td>Electricity Rate</td>
<td>18.65 cents/kWh</td>
</tr>
<tr>
<td>Late Payment Charge</td>
<td>$5.35 per bill</td>
</tr>
<tr>
<td>Early Termination Charge</td>
<td>Termination Rate × Unexpired Months</td>
</tr>
<tr>
<td></td>
<td>Termination Rate per month:</td>
</tr>
<tr>
<td></td>
<td>HDB 1-Room $10.70 / HDB 2-Room $16.05 /</td>
</tr>
<tr>
<td></td>
<td>HDB 3-Room $21.40 / HDB 4-Room $32.10 /</td>
</tr>
<tr>
<td></td>
<td>HDB 5-Room $37.45 /</td>
</tr>
<tr>
<td></td>
<td>HDB Executive &amp; Private Condominium $42.80 /</td>
</tr>
<tr>
<td></td>
<td>Terrace $74.90 / Semi-Detached $101.65 / Bungalow $214</td>
</tr>
</tbody>
</table>

(Adapted from source: https://www.ema.gov.sg/openelectricitymarket.aspx)

The historical electricity tariff rate of SP Group is shown below.

<table>
<thead>
<tr>
<th>SP Group Electricity Tariff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rates (cents/kWh) are exclusive of 7% Goods and Services Tax (GST)</td>
</tr>
<tr>
<td>January 2018</td>
</tr>
<tr>
<td>21.56</td>
</tr>
</tbody>
</table>

(Adapted from source: https://www.spgroup.com.sg)
1. \( x^2 - 3x = 0 \)

\( x(x - 3) = 0 \)

\( x = 0 \) or \( x = 3 \)

B1

2. \( 2x + 3y = 5 \)

Accept \( 2(2) + 3(-1) = 1 \)

B1

3. \( \text{HCF} = 2 \times 3 = 6 \)

B1

4. \( n = 2^1 \times 3^1 \times 5 = 10 \times 15 = 150 \)

M1

5. \( \text{Area} \) of sector \( ABC = \frac{1}{2} \times 10^2 \times 0.6 \times \pi \)

M1

6. \( n^2 = 2766 \times 0.6 \times \pi \)

M1

7. \( F = \sqrt{99(99 + 1)} = 9900 \)

M1

8. \( P = 2(1 + 2 + 3 + \ldots + n) \)

M1

9. \( \text{Gradient} = \frac{3}{7} \)

M1

10. \( \text{Gradient} = \frac{3}{7} \)

M1

11. \( 1 + 0.8 + 1.1 = 2.9 \)

M1

12. \( a^2 + b^2 = c^2 \)

M1

13. \( a^2 - 4 = 0 \)

M1

14. \( a^2 + 5 = 6 \)

M1

15. \( \text{Area} \) of sector \( ABC = \frac{1}{2} \times 10^2 \times 0.6 \times \pi \)

M1

16. \( \text{Area} \) of sector \( ABC = \frac{1}{2} \times 10^2 \times 0.6 \times \pi \)

M1

17. \( \text{Area} \) of sector \( ABC = \frac{1}{2} \times 10^2 \times 0.6 \times \pi \)

M1

18. \( \text{Area} \) of sector \( ABC = \frac{1}{2} \times 10^2 \times 0.6 \times \pi \)

M1
25a
\[ \frac{3x}{2} = \frac{1}{2x} \]

M1 for rearranging as fractional index.

25b
\[ y = 2 \]

M1

25c
\[ x^2 + y^2 = 1 \]

M1

25d
\[ y = \frac{1}{x} \]

M1

25e
\[ x = 1, y = 0 \]

A1

25f
\[ \frac{3x}{2} = \frac{1}{2x} \]

M1

25g
\[ y = 2 \]

A1

25h
\[ OR = 15, QR = 15, QS = 15 \]

M1

25i
\[ ES = \frac{180 - 150}{2} = 15^\circ \] (base \( \angle C \) of \( \triangle ABC \))

M1

25j
\[ \text{Sum of interior \( \angle \)s of \( 
\triangle \)PORST} = (3 - 2) \times 180 = 360^\circ \]

M1

25k
\[ \angleOPT = \frac{360 - 150}{2} = 45^\circ \]

M1

\[ \angleQTS = 15 + 30 = 45^\circ \]

M1

\[ \angleERT = 45^\circ \]

M1

\( \boxed{a} \) must state corresponding \( \angle \)

26a
\[ 3a = 5b \]

B1

26b
\[ OB = 15, QR = 15, QS = 15 \]

A1

26c
\[ AF = 60, \quad OF = 60 - 20 = 40 \]

A1

26d
\[ \triangle Q = \triangle P = \triangle R = \triangle T \]

M1 for triangle law.

26e
\[ \angle A = 110^\circ, \quad \angle B = 110^\circ, \quad \angle C = 110^\circ \]

M1 for triangle law.

26f
\[ \frac{300}{200} = 1.5 \]

B1

26g
\[ \frac{10}{300} = 0.333^\circ \]

B1

26h
\[ \frac{300}{200} = 1.5 \]

B1
56. **Scheme A:** $\frac{C^2}{2}$

\[
\begin{align*}
\text{Area} &= \frac{C^2}{2} \\
&= \frac{(120)(120)}{2} \\
&= 7200 \\
\end{align*}
\]

**Scheme B:**

\[
\begin{align*}
\text{Area} &= \frac{C^2}{2} \\
&= \frac{(120)(120)}{2} \\
&= 7200 \\
\end{align*}
\]

Both schemes have the same area of 7200 m².

57. **Scheme A:**

\[
\begin{align*}
\text{Area} &= \frac{C^2}{2} \\
&= \frac{(120)(120)}{2} \\
&= 7200 \\
\end{align*}
\]

**Scheme B:**

\[
\begin{align*}
\text{Area} &= \frac{C^2}{2} \\
&= \frac{(120)(120)}{2} \\
&= 7200 \\
\end{align*}
\]

Both schemes have the same area of 7200 m².

58. **Scheme A:**

\[
\begin{align*}
\text{Area} &= \frac{C^2}{2} \\
&= \frac{(120)(120)}{2} \\
&= 7200 \\
\end{align*}
\]

**Scheme B:**

\[
\begin{align*}
\text{Area} &= \frac{C^2}{2} \\
&= \frac{(120)(120)}{2} \\
&= 7200 \\
\end{align*}
\]

Both schemes have the same area of 7200 m².
COMMONWEALTH SECONDARY SCHOOL
MID-YEAR EXAMINATION 2019

MATHEMATICS
PAPER 1

Name: ____________________________ ( ) Class: ____________

SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC

Friday 03 May 2019
10 30 – 12 30
2 h

4048/1

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give
the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in
terms of \( \pi \).

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Name of setter: Mr Toinh Long Teng

<table>
<thead>
<tr>
<th>For Examiner's Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Presentation</td>
</tr>
<tr>
<td>Accuracy</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

Parent's Signature: __________________________

This paper consists of 17 printed pages including the cover page.

[Turn over
Mathematical Formulae

Compound Interest

Total amount = \( P \left(1 + \frac{r}{100}\right)^n \)

Mensuration

Curved surface area of a cone = \( \pi rl \)
Surface area of a sphere = \( 4\pi r^2 \)
Volume of a cone = \( \frac{1}{3} \pi r^2 h \)
Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

Area of triangle \( ABC = \frac{1}{2} ab \sin C \)

Arc length = \( r\theta \), where \( \theta \) is in radians
Sector area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

Trigonometry

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\( a^2 = b^2 + c^2 - 2bc \cos A \)

Statistics

Mean = \( \frac{\sum fx}{\sum f} \)

Standard deviation = \( \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2} \)
1 Write the following in order of size, in descending order.
\[ \pi, \ 3.142, \ \frac{22}{7}, \ 3.142 \]

*Answer.*

2 Factorise completely \(16p^4 - 81q^2\).

*Answer.*

3 Given that \(\frac{1}{27} = 9^k\), find \(k\).

*Answer* \(k = \ldots\)
4 The stem-and-leaf diagram shows the times, in minutes, taken by some students to complete a task.

\[
\begin{array}{c|cccc}
3 & 2 & 4 & 5 & 6 \\
4 & 0 & 1 & 1 & 9 & 9 \\
\end{array}
\]

Key: 34 represents 34 minutes

For these times, find

(a) the lower quartile,

\[\text{Answer} \text{ minutes } [1]\]

(b) the mean,

\[\text{Answer} \text{ minutes } [1]\]

(c) the standard deviation,

\[\sqrt{\frac{x^2 - \overline{x}^2}{n}} = 5.81\]

\[\text{Answer} \text{ minutes } [1]\]

5 Sketch the graph of \( y = -(2-x)^2 + 1 \) on the axes below.

Indicate clearly the values where the graph crosses the x- and y-axes and the coordinates of any turning points.
6 The diagram shows the speed-time graph for part of a car's journey between two sets of traffic lights.

\[ v \quad \text{Speed (m/s)} \]
\[ 0 \quad 4 \quad \text{Time (seconds)} \]

The distance travelled in the first 4 seconds is 20 metres.
(a) Calculate the value of \( v \).

*Answer \( v = \ldots \) [1]*

The car then decelerated at twice the rate of the acceleration.
(b) Calculate the average speed of the car for the entire journey.

*Answer \ldots \text{m/s} [2]*

7 A club is made up of members that are either a child, an adult or a senior citizen. There are 19 children and 14 senior citizens. The club wishes to maintain that at most three-fifths of the members are adults.

By forming an inequality, find the maximum number of adults possible.

*Answer \ldots* [3]
8. The Venn diagram illustrates the relationship between two different types of quadrilaterals.

Quadrilaterals

Parallelograms

Kites

(a) What special shape is represented by the intersection of the sets representing Parallelograms and Kites?

Answer

(b) Using an appropriate symbol, complete the statement:

\{Parallelograms\} \ldots \{Trapeziums\}

Answer

9. A sum of money was divided between A, B and C in the ratio 2 : 3 : 4.
If the money had been divided equally between them, A would have received an extra $40. What was the total sum of money?

Answer

10. The probability that a phone will ring in a thirty-minute interval is \(\frac{5}{6}\). What is the probability that the phone will not ring in a one-hour interval?

Answer
11 Adam can lay 70 bricks in 30 minutes. Charlie can lay 80 bricks in 45 minutes. Adam and Charlie work together to lay a total of 1000 bricks. If they continue to lay bricks at the same rate, how long will it take them to lay the 1000 bricks? Give your answer in hours and minutes, to the nearest minute.

Answer

12 The diagram shows three of the sides, $AB$, $BC$ and $CD$ of a regular polygon. $AB$ produced and $DC$ produced meets at $E$. Angle $BEC = 108^\circ$

$A$ $B$ $C$ $D$

Find the number of sides of the regular polygon.

Answer
13 Solve \( \frac{5x}{14} - \frac{2x - 3}{21} = 1 \).

Answer \( x = \) \( [2] \)

14 Solve \( x^2 + 4x - 11 = 0 \) by using completing the square.

Answer \( x = \) or \( x = \) \( [3] \)

15 Given that the value of \( \frac{1}{x^2} + \frac{1}{y^2} = 3 \) and \( xy = 4 \), find the value of \( (x + y)^3 \).

Answer \( .. \) \( [3] \)
The attractive force, \( F \) newtons, between two stars, is inversely proportional to the square of the distance between the centres of the two stars, \( r \) km.

(a) Sketch the graph of \( F \) against \( r \).

(b) The distance between the centres of star \( A \) and star \( B \) is 150% larger than the distance between the centres of star \( C \) and star \( D \). Find the ratio of the attractive force between star \( A \) and star \( B \) to the attractive force between star \( C \) and star \( D \).
17 (a) Write down a possible equation for each of the sketch graphs below. In each case select one of the equations from the box below.

\[
\begin{align*}
y &= x^2 - 3 \\
y &= -x^2 + 3 \\
y &= 3^x \\
y &= -x^3 + 3 \\
y &= 3^{-x} \\
y &= x^3 + 3
\end{align*}
\]

(i) Answer

(ii) Answer

(iii) Answer
(h) The diagram shows three lines \( l_1, l_2 \) and \( l_3 \).

The equation of \( l_1 \) is \( y = 2x + 1 \).

(i) \( l_1 \) and \( l_2 \) have the same \( y \)-intercept. State a possible value of the gradient of \( l_2 \).

Answer \( \ldots \) [1]

(ii) \( l_1 \) and \( l_3 \) have the same gradient. State a possible equation of \( l_3 \).

Answer \( \ldots \) [1]

18 A solid cylinder has radius \( r \) cm and height \( h \) cm. A solid hemisphere has radius \( r \) cm. The total surface area of the cylinder and hemisphere are equal.

Work out, in terms of \( r \), the volume of the cylinder.

Answer \( \ldots \) cm\(^3\) [3]
19 Daniel invested a sum of money in an account paying compound interest at \( r \% \) per year. After
3 years, the money had earned total interest of 20\% of the principal.

Find the value of \( r \), correct to one decimal place.

\[ \text{Answer} \]

20 Two glasses are geometrically similar. The height of the small glass is 9 cm and the height of
the large glass is 15 cm.

If a big tank of fruit juice can fill up 125 small glasses, how many large glasses could be filled
from the same big tank?
In the diagram, $A$, $B$ and $C$ are points on a circle. $AB = 7$ cm, $BC = 10$ cm and $AC = 12$ cm. Explain, with reasons, whether $AC$ is a diameter of the circle.

*Answer*

22 It is given that $T = 2\pi \sqrt{\frac{L}{g}}$.

(a) Calculate the value of $T$ when $L = 1.2$ and $g = 9.81$. Write your answer correct to three decimal places.

*Answer* ..

(b) Rearrange the formula to make $g$ the subject.

*Answer* $g =$ ..
23 The Singapore River is 3.2 kilometres long. It is represented on a map with a distance of 2 cm.

(a) Express the scale of the map in the form $1:n$.

Answer

(b) The length of the Ayer Rajah Expressway is 27 km, corrected to the nearest kilometre. Find the greatest possible distance of the expressway on the map.

Answer cm [1]

(c) The Jurong Lake District is represented by an area of 1.4 square centimetres on the map. Find the actual area, in square kilometres, of the district.

Answer km$^2$ [2]
24 (a) Express 84 as a product of its prime factors, leaving your answer in index notation.

\[ 84 = 2^2 \times 3 \times 7 \]

Answer \[ 2^2 \times 3 \times 7 \] [1]

(b) Find the highest common factor of 84 and \( 2 \times 3^3 \times 5 \).

\[ \text{HCF} = 2 \times 3 \]

Answer [1]

(c) The lowest common multiple of 84 and \( x \) is \( 2^3 \times 3 \times 7^2 \). Find the smallest possible value of \( x \).

\[ \text{LCM} = 2^3 \times 3 \times 7^2 \]

Answer \[
\]

[2]
25 The \( n \)th term of a sequence is given by \( n(n + 3) \).

(a) Find the fifth term.

Answer

(b) One term in the sequence is 154. Find the value of \( n \) for this term.

Answer \( n = \)

(c) Explain why every term in the sequence is an even number.

Answer
In the diagram, $A, B, C$ and $D$ are points on a circle, centre $O$. $BE$ and $DE$ are tangents to the circle at points $B$ and $D$ respectively. Angle $BCD = 5x^\circ$ and angle $BED = 2x^\circ$.

Find $x$.

Showing your working clearly and give reasons.

Answer $x =$ [4]
COMMONWEALTH SECONDARY SCHOOL
MID YEAR EXAMINATION 2019

MATHEMATICS
PAPER 2

Name: _______________________________ ( ) Class: ________

SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC
4048/2

Monday 6 May 2019
08 00 – 10 30
2h 30 min

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
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the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in
terms of \( \pi \).

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Name of setter: Mrs Ang YM
Mr Koh HY

For Examiner's Use

| Total | 100 |

Parent's Signature: _______________________________

This paper consists of 13 printed pages including the cover page.

[Turn over
Mathematical Formulae

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Mensuration

Curved surface area of a cone = \( \pi rl \)
Surface area of a sphere = \( 4\pi r^2 \)
Volume of a cone = \( \frac{1}{3} \pi r^2 h \)
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Sector area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

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\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\( a^2 = b^2 + c^2 - 2bc \cos A \)

Statistics

Mean = \( \frac{\sum fx}{\sum f} \)

Standard deviation = \( \sqrt{\frac{\sum f x^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2} \)
1. (a) Solve the inequality \( \frac{3-x}{5} < 1 + \frac{2x+1}{4} \). [2]

(b) Express as a single fraction in its simplest form \( \frac{4y}{3-2y} - \frac{y}{(2y-3)^2} \). [2]

(c) Simplify \( \frac{(-3h)^2}{8h^3j^5} + \frac{27h^4j^2}{4j^5} \). [2]

(d) Simplify \( \left( \frac{256p^{10}}{q^{20}r^x} \right)^{-\frac{1}{4}} \). [2]

(e) Solve the equation \( \frac{10}{x^2 - 9} - \frac{3}{x + 3} = 1 \). [3]

---

2. One astronomical unit (1 au) is a unit of length defined as 149597870700 metres, which is roughly the average distance between the earth and the sun.

(a) Express one astronomical unit in metres, correct to 3 significant figures in standard form. [1]

(b) The speed of sound is 343 m/s. How long, in seconds, does sound take to travel a distance of 1 au? Give your answer in standard form, correct to 3 significant figures. [2]

(c) The average distance between the earth and the moon is 384400 km. Express this distance as a percentage of 1 au. [2]

(d) A rocket travels a distance of one metre in 8000 nanoseconds (ns) and \( 1 \text{ns} = 10^{-9} \text{ s} \). How long, in seconds, does the rocket take to travel 1 au. Give your answer in standard form, correct to 3 significant figures. [2]
3. Answer the whole of this question on a sheet of graph paper.

A stone was projected directly up a slope. Its distance, \( y \) metres, from the bottom of the slope, \( t \) seconds after it was projected, is given in the table below.

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>1.80</td>
<td>3.10</td>
<td>5.50</td>
<td>6.85</td>
<td>7.75</td>
<td>8.30</td>
<td>8.40</td>
</tr>
</tbody>
</table>

(a) Using a scale of 2 cm to represent 1 second, draw a horizontal \( t \)-axis for \( 0 \leq t \leq 6 \). Using a scale of 2 cm to represent 1 metre, draw a vertical \( y \)-axis for \( 0 \leq y \leq 9 \).
On your axes, draw the graph of \( y \) against \( t \). [3]

(b) Use your graph to find the distance of the stone from the bottom of the slope when \( t = 2.5 \). [1]

(c) (i) By drawing a tangent, find the gradient of the curve at \( t = 3 \). [2]

(ii) State briefly what this gradient represents. [1]

(d) At the instance the stone was projected, an object was released down the slope from a point 8 metres from the bottom. This object moved directly down the slope at a constant speed of 2 m/s.

(i) On the same axes, draw the graph representing the motion of this object. [1]

(ii) Use your graphs to find when the stone and the object passed each other. [1]

4. \( A \) is the point \((-2, 7)\) and \( B \) is the point \((6, -4)\).

(a) Find the length of the line \( AB \). [2]

(b) Find the equation of the line \( AB \). [2]

(c) The equation of another line \( l \) is \( 8y + 11x = -16 \).
Show how you can tell that the line \( l \) is parallel to the line \( AB \). [2]

(d) The coordinates of \( C \) is \((2, k)\). Given that the points \( A \), \( B \) and \( C \) are collinear, find the value of \( k \). [2]
Three buoys, $A$, $B$ and $C$, are positioned in a lake to provide a course for a water sports event. $AB = 800$ m and $A$ is due south of $C$. $\angle ABC = 34^\circ$ and the bearing of $B$ from $A$ is $026^\circ$. $E$ is a point on $AB$ which is $300$ m from $B$.

(a) Calculate $AC$. [2]

(b) Find the area of triangle $ABC$. [2]

(c) Calculate the bearing of $A$ from $B$. [2]

(d) A helicopter, $H$, is hovering at a point vertically above $E$.

(i) The angle of elevation of the helicopter from $B$ is $11^\circ$. Calculate the vertical height of the helicopter above $E$. [2]

(ii) $F$ is a point on $BC$ which is nearest to the helicopter. Calculate the angle of depression of $F$ from the helicopter. [3]
6. (a) Given that \( P = \begin{pmatrix} w & 1 \\ 0 & 2 \end{pmatrix} \) and \( P^2 = \begin{pmatrix} 6w - 9 & w + 2 \\ 0 & 4 \end{pmatrix} \), where \( w \) is a constant.

Find the value(s) of \( w \). \[3\]

(b) A waffle maker produces three different types of waffles: Red bean, Chocolate and Peanut, for distribution to its outlets at various locations.

The table below shows the quantity delivered to each location each time.

<table>
<thead>
<tr>
<th></th>
<th>Chocolate</th>
<th>Red bean</th>
<th>Peanut</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outlet 1</td>
<td>225</td>
<td>140</td>
<td>125</td>
</tr>
<tr>
<td>Outlet 2</td>
<td>265</td>
<td>115</td>
<td>245</td>
</tr>
<tr>
<td>Outlet 3</td>
<td>245</td>
<td>125</td>
<td>175</td>
</tr>
</tbody>
</table>

(i) Represent the data in the above table by a \( 3 \times 3 \) matrix \( A \). \[1\]

(ii) Hence, find, by matrix multiplication, the total number of waffles delivered to each outlet. \[2\]

(c) The following table shows the selling price and the cost price of 1 unit of each type of waffle.

<table>
<thead>
<tr>
<th></th>
<th>Chocolate</th>
<th>Red Bean</th>
<th>Peanut</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling Price ($)</td>
<td>1.20</td>
<td>0.80</td>
<td>1.00</td>
</tr>
<tr>
<td>Cost Price ($)</td>
<td>0.60</td>
<td>0.50</td>
<td>0.40</td>
</tr>
</tbody>
</table>

(i) Represent the data in the above table by a matrix \( C \) such that \( AC \) gives the total selling price and total cost price of each outlet. Hence, evaluate \( AC \). \[3\]

(ii) Find the profit earned by outlet 2. \[1\]
7. A rectangular wall is 208 cm by 169 cm. The border around the wall is to be covered with tiles. The tiles measure \(x\) cm by \((x - 5)\) cm. Each tile is placed so that its longer side is vertical. Some of the tiles are shown in the diagram below.

(a) Express, in terms of \(x\), the number of tiles that will fit
   (i) across the top row of the border,
   (ii) along one vertical side of the border. \([1] \quad [1]\)

(b) Given that 74 tiles are required to fill the whole border around the wall, form an equation and show that it reduces to \(3x^2 - 44x + 65 = 0\). \([3]\)

(c) Solve the equation \(3x^2 - 44x + 65 = 0\). \([2]\)

(d) Explain why one of the solutions in part (c) must be rejected as the length of the tile. \([1]\)

(e) Find the area of the wall that will be filled with tiles. \([2]\)
8. The diagram shows the cross-section design of the medal for a sports event. It is made up of two identical shapes.

\[ \overset{\text{A}}{\text{YB}} \text{ is an arc of a circle with centre } C. \]
\[ \text{XA and } XB \text{ are tangents to the circle.} \]
\[ XC = 4 \text{ cm and } \angle XCB = 60^\circ. \]

(a) Calculate the length of \( BC \). [2]  

(b) Find the total cross-sectional area of the medal. [3]  

(c) If the medal has a thickness of 0.7 cm, find the volume of this piece of medal. [2]
9. In the diagram, \( AB \) is parallel to \( QP \).

(a) Prove that triangle \( PQR \) is similar to triangle \( ABR \). \([3]\)

(b) State another pair of similar triangles. \([1]\)

(c) Given that \( PQ = 5 \) cm, \( PR = 4 \) cm and \( AB = 12 \) cm, calculate the length of \( AR \) using similar triangles. \([2]\)

(d) Find the numerical value of the ratio of \( \frac{\text{Area of triangle } PQR}{\text{Area of triangle } ABR} \). \([1]\)

(e) Given that the area of triangle \( CPQ \) is 15 cm\(^2\), find the area of \( ABPQ \). \([2]\)
10. (a) In the afternoon, on a certain stretch of road, the speeds of 100 vehicles were recorded. The cumulative frequency curve shows the distribution of the speeds of the vehicles.

(i) Use the curve to estimate

(a) the median speed,

(b) the interquartile range of the speeds,

(c) the 80th percentile,

(d) the percentage of vehicles that travels within the speed limit if the speed limit on the road is 60 km/h.

(ii) From the curve, find the probability that at least one of the two vehicles chosen at random travels at a speed more than 55 km/h.
(b) The speeds of 100 vehicles passing the same stretch of road at night were also recorded. The box-and-whisker plot shows the distribution of the speeds.

(i) Make two comments comparing the speeds of the vehicles in the afternoon and at night. [2]

(ii) Find the range of the speeds of the vehicles at night. [1]
11. Jewel Changi Airport is set to become an iconic landmark of Singapore and a key tourist attraction. The building occupies an area of about 134 000 m² on the ground.

(a) A map has a scale of 1 : 7500. Calculate the area of the building on the map. [2]

(b) The diagram below shows the view of the building from the top. A Skytrain Bridge runs through the building at about 18 m above the ground of Forest Valley.

Diagram 1 [Source: Jewel Changi Development & Straits Times Graphic]

(i) The actual circumference of the whole circular Forest Valley is about 105 m. Find the actual radius of the whole circular Forest Valley in terms of $\pi$, in metres. [1]
(ii) Mr Yap, a lighting designer, plans to hang string lights from point $X$ on the skytrain bridge, down to point $Y$ on the ground, which is on the circumference of the Forest Valley as illustrated in diagram 1. Assuming $XY$ is taut, he claims that he will need at least 34 m of string lights.

Given that $Z$ is another point on the ground, which is also on the circumference of the Forest Valley, vertically below point $X$, and the minor arc length $ZY$ is 35 m, verify if his claim is true or false, by showing your working clearly. [6]

END OF PAPER
1. \( \frac{22}{7} \), \(3.142\), \(3.142\), \(\pi\)

2. 
\[
16p^4 - 81q^4 = (4p^2)^2 - (9q^2)^2 \\
= (4p^2 - 9q^2)(4p^2 + 9q^2) \\
= (2p - 3q)(2p + 3q)(4p^2 + 9q^2)
\]

3. 
\[
\frac{1}{27} = 9^k \\
3^{2k} = 3^{-3} \\
k = -1.5
\]

4a. 
\[
\frac{34 + 35}{2} = 34.5
\]

4b. 
\[
39\frac{2}{3} \text{ or } 39.7
\]

4c. 
\[
\sqrt{\frac{14465}{9} - \left(39\frac{2}{3}\right)^2} = 5.81 \text{ (3 s.f.)}
\]

5.

\[
\frac{1}{2}(4)(v) = 20 \Rightarrow v = 10
\]

6a. 
\[
\frac{1}{2}(4)(v) = 20 \Rightarrow v = 10
\]

6b. 
\( t = 6 \)

- total distance = \(\frac{1}{2}(6)(10) = 30\)
- avg speed = \(\frac{30}{6} = 5 \text{ m/s}\)
Let \( x \) be the number of adults.

\[
\frac{x}{x+19+14} \leq \frac{3}{5}
\]

\[
5x \leq 3x + 99
\]

\[
x \leq 49.5
\]

Therefore, the maximum number of adults is 49.

8a Rhombus

8b \( \subseteq \) or \( \subseteq \)

9 1 unit - $40

Total sum of money = 40 \times 9 = $360

10 \[
\frac{1}{x} \cdot \frac{1}{6} = \frac{1}{36}
\]

11 Adam's rate = \[
\frac{70}{30}
\]

Charlie's rate = \[
\frac{80}{45}
\]

Total rate = \[
\frac{70}{30} + \frac{80}{45} = \frac{37}{9}
\]

Time = \[
\frac{1000}{37} = 243.24 \text{ min (5 s.f.)} = 4 \text{ hours 3.24 min}
\]

Time taken is 4 hours 4 minutes.

12 \[
\angle EBC = \frac{180^\circ - 108^\circ}{2} = 36^\circ \Rightarrow \text{no. of sides} = \frac{360^\circ}{36^\circ} = 10
\]

13 \[
5x - 2x - 3 = 1,
\]

\[
15x - 2(2x - 3) = 42
\]

\[
15x - 4x + 6 = 42
\]

\[
x = 16 \Rightarrow x = \frac{36}{11} = 3.27
\]

14 \[
(x+2)^2 - 4 = 10
\]

\[
(x+2)^2 = 15
\]

\[
x + 2 = \pm\sqrt{15}
\]

\[
x = -2 \pm \sqrt{15} \Rightarrow x = 1.87 \text{ or } -5.87 \text{ (3 s.f.)}
\]

15 \[
\frac{1}{x^2} + \frac{1}{y^2} = 3
\]

\[
x^2 + y^2 = 3x^2 y^2
\]

\[
(x+y)^2 = 3x^2 y^2 + 2xy
\]

\[
= 3(4)^2 + 2(4)
\]

\[
= 56
\]
16b
\[ r_{AB} = \frac{5}{2} r_{CD} \]
\[ r_{AB}^2 = \frac{25}{4} r_{CD}^2 \]
\[ F_{AB} = \frac{r_{AB}^2}{r_{CD}^2} = \frac{4}{25} \]
Answer: 4 : 25

17ai
\[ y = -x^3 + 3 \]
aii
\[ y = x^2 - 3 \]
aiii
\[ y = 3 - x \]
bi
Any value greater than 2
bii
\[ y = 2x + c, \text{ where } c \neq 0 \]

18
\[ 2\pi r^2 + 2\pi rh = 2\pi r^2 + \pi r^2 \]
\[ h = \frac{r}{2} \]
vol. of cylinder = \( \pi r^2 h \)

19
\[ \rho \left(1 + \frac{r}{100}\right)^3 = 1.2 \rho \]
\[ 1 + \frac{r}{100} = \sqrt[3]{1.2} \]
\[ r = 6.3 \text{ (1 d.p.)} \]
\[ l_1 = \frac{9}{15} = \frac{3}{5} \]
\[ V_1 = \left( \frac{3}{5} \right)^3 = \frac{27}{125} \]
\[ V_2 = \frac{27}{125} \]
\[ 125V_1 = 27V_2 \]

Therefore, 27 large glasses could be filled.

\[ 7^2 + 10^2 \neq 12^2 \]
\[ \therefore \ AB^2 + BC^2 \neq AC^2 \]

By Pythagoras Theorem, \( \angle ABC \neq 90^\circ \)

By Right-Angle in Semicircle, \( AC \) is not a diameter.

\[ \frac{T}{2\pi} = \sqrt{\frac{L}{g}} \Rightarrow \frac{T^2}{4\pi^2} = \frac{L}{g} \Rightarrow g = \frac{4\pi^2L}{T^2} \]

\[ 2 : 3.2 \times 1000 \times 100 \]
\[ 1 : 160000 \]

\[ 27.5 \times 2 = 17.1875 \text{ cm} \]

\[ \frac{4 \text{ cm}^2 \text{ represents } 10.24 \text{ km}^2}{1.4 \text{ cm}^2 \text{ represents } 3.584 \text{ km}^2} \]

\[ 2^2 \times 3 \times 7 \]
\[ 2 \times 3 = 6 \]

\[ 2^3 \times 1 \times 7^2 = 392 \]

\[ 5(5 + 3) = 40 \]

\[ n(n + 3) = 154 \]
\[ n^2 + 3n = 154 \]
\[ (n - 11)(n + 14) = 0 \]

\[ n = 11 \text{ or } n = -14 \text{ (rejected)} \]

When \( n \) is odd, \( n + 3 \) is even. When \( n \) is even, \( n + 3 \) is odd.

Product of odd and even is always even.

\[ \angle BAD = 180^\circ - 5x^\circ \text{ (angles in opposite segments)} \]
\[ \angle BOD = 2 \times \angle BAD \text{ (angle at centre} = 2 \times \text{angle at circumference)} \]
\[ = 360^\circ - 10x^\circ \]
\[ 360^\circ - 10x^\circ + 2x^\circ + 90^\circ \times 2 = 360^\circ \text{ (tangent} \perp \text{radius, total interior angle of quadrilateral)} \]

\[ x = 22.5 \]
<table>
<thead>
<tr>
<th>Qn No</th>
<th>Solutions</th>
</tr>
</thead>
</table>
| 1a    | \[
\frac{3-x}{5} < 1 + \frac{2x+1}{4} \\
\frac{3-x}{5} < \frac{4+2x+1}{4} \\
4(3-x) < 5(5+2x) \\
12-4x < 25+10x \\
-14x < 13 \\
x > \frac{13}{14}
\] |
| 1b    | \[
\frac{4y}{3-2y} - \frac{y}{(2y-3)^2} \\
= \frac{-4y}{2y-3} \cdot \frac{y}{(2y-3)^2} \\
= \frac{-4y(2y-3)-y}{(2y-3)^2} \\
= \frac{11y-8y^2}{(2y-3)^2} \\
= \frac{y(11-8y)}{(2y-3)^2} \\
= \frac{y(11-8y)}{(3-2y)^2}
\] or \[
\frac{4y}{3-2y} - \frac{y}{(2y-3)^2} \\
= \frac{4y}{3-2y} \cdot \frac{y}{(2y-3)^2} \\
= \frac{4y(3-2y)-y}{(2y-3)^2} \\
= \frac{11y-8y^2}{(2y-3)^2} \\
= \frac{y(11-8y)}{(2y-3)^2} \\
= \frac{y(11-8y)}{(3-2y)^2}
\] |
| 1c    | \[
\frac{(3h)^2 + 27h^2}{8h^2y^2 + 4f^4} \\
= \frac{9h^2}{8h^2y^2} \cdot \frac{27h^4}{4f^4} \\
= \frac{1}{6h^2f^4}
\] |
| 1d    | \[
\left( \frac{256p^{16}}{q^{20}r^{-4}} \right)^{\frac{1}{4}} \\
= \left( \frac{q^{20}r^{-4}}{256p^{16}} \right)^{\frac{1}{4}} \\
= \frac{q^{5}r^{-1}}{4p^{4}} \\
= \frac{q^{5}r^{-1}}{4p^{4}} \\
= \frac{q^{5}r^{-1}}{4p^{4}} \\
= \frac{1}{4p^{4}q^{3}r^{-3}}
\]
\[ \frac{10}{x^2 - 9} - \frac{3}{x + 3} = 1 \]
\[10 - 3(x - 3) = x^2 - 9\]
\[10 - 3x + 9 = x^2 - 9\]
\[x^2 + 3x - 28 = 0\]
\[(x + 7)(x - 4) = 0\]
\[x = -7 \quad \text{or} \quad x = 4\]

2a \quad 1.50 \times 10^{11} \text{ m (3 sig. fig.)}

2b \quad \text{Time taken for sound to travel 1 au} = \frac{1.4959 \times 10^{11}}{343}
\quad = 4.36 \times 10^4 \text{ s (3 sig. fig.)}

2c \quad \frac{384400 \times 10^8}{1.4959 \times 10^{11}} \times 100
\quad = 0.257 \% \text{ (3 sig. fig.)}

2d \quad \text{Time taken} = 1.4959 \times 10^{11} \times 8000 \times 10^{-6}
\quad = 1.20 \times 10^6 \text{ s (3 sig. fig.)}

3 \quad \text{Answers on the last page}

4a \quad AB = \sqrt{(-2 - 6)^2 + (7 - (-4))^2}
\quad = \sqrt{64 + 121}
\quad = \sqrt{185} = 13.6 \text{ units}

4b \quad \text{Gradient of } AB = \frac{7 - (-4)}{-2 - 6} = \frac{11}{-8}
\quad \text{Equation of } AB:
\quad y - 7 = -\frac{11}{8}(x + 2) \quad \text{or} \quad 7 = -\frac{11}{8}(-2) + c
\quad c = \frac{17}{4}
\quad y = -\frac{11}{8}x + \frac{17}{4}
\quad 8y = -11x + 34

4c \quad 8y + 11x = -16
\quad 8y = -11x - 16
\quad y = -\frac{11}{8}x - 2

\text{Since they have the same gradient, they are parallel.}
4d \[ 8k = -11(2) + 34 \quad \text{or} \quad \frac{7-k}{-2-2} = -\frac{11}{8} \]
\[ k = 1 \frac{1}{2} \quad \quad k = 1 \frac{1}{2} \]

5a Using Sine Rule,
\[ \frac{\sin 120^\circ}{800} = \frac{\sin 34^\circ}{AC} \]
\[ AC = \frac{800 \times \sin 34^\circ}{\sin 120^\circ} \]
\[ AC = 516.560 \quad \quad \text{= 517 m (3 sig. fig.)} \]

5b Area of triangle \( ABC = \frac{1}{2} (800)(516.560) \sin 26^\circ \]
\[ = 90577.9 \quad \quad \text{= 90600 m}^2 \text{ (3 sig. fig.)} \]

5c Bearing of \( A \) from \( B = 360^\circ - 120^\circ - 34^\circ \) or \( 180^\circ + 26^\circ \)
\[ = 206^\circ \]

5d (i) Let the vertical height of the helicopter above \( E \) be \( h \)
\[ \tan 11^\circ = \frac{h}{300} \]
\[ h = 300 \tan 11^\circ \]
\[ h = 58.314 \]
\[ h = 58.3 \text{ m (3 sig. fig.)} \]

(ii) \[ \sin 34^\circ = \frac{EF}{300} \]
\[ EF = 300 \sin 34^\circ \]
Let the angle of depression of \( F \) from the helicopter be \( \theta \)
\[ \tan \theta = \frac{58.314}{300 \sin 34^\circ} \]
\[ \theta = 19.16^\circ \quad (1 \text{ d.p.}) \]

6a \[
\begin{pmatrix}
w & 1 \\
0 & 2
\end{pmatrix}
\begin{pmatrix}
w & 1 \\
0 & 2
\end{pmatrix}
= \begin{pmatrix}
w+2 & w+2 \\
0 & 4
\end{pmatrix}
\]
\[
\begin{pmatrix}
w^2 & w+2 \\
0 & 4
\end{pmatrix}
= \begin{pmatrix}
w+2 & w+2 \\
0 & 4
\end{pmatrix}
\]
\[ w^2 = 6w - 9 \]
\[ w^2 - 6w + 9 = 0 \]
\[ (w-3)^2 = 0 \]
\[ w = 3 \]
6b (i) \[ A = \begin{pmatrix} 225 & 140 & 125 \\ 265 & 115 & 245 \\ 245 & 125 & 175 \end{pmatrix} \]

(ii) \[
\begin{pmatrix}
225 & 140 & 125 \\
265 & 115 & 245 \\
245 & 125 & 175 \\
\end{pmatrix} \begin{pmatrix}
1 \\
1 \\
1 \\
\end{pmatrix} 
= \begin{pmatrix}
490 \\
625 \\
545 \\
\end{pmatrix}
\]

Total number of waffles delivered to outlet 1 is 490, outlet 2 is 625 and outlet 3 is 545.

6c (i) \[
C = \begin{pmatrix} 1.20 & 0.60 \\ 0.80 & 0.50 \\ 1.00 & 0.40 \end{pmatrix} \]

or \[
C = \begin{pmatrix} 0.60 & 1.20 \\ 0.50 & 0.80 \\ 0.40 & 1.00 \end{pmatrix}
\]

\[
AC = \begin{pmatrix} 225 & 140 & 125 \\ 265 & 115 & 245 \\ 245 & 125 & 175 \end{pmatrix} \begin{pmatrix} 1.20 & 0.60 \\ 0.80 & 0.50 \\ 1.00 & 0.40 \end{pmatrix} = \begin{pmatrix} 507 & 255 \\ 655 & 314.50 \\ 569 & 270.50 \end{pmatrix}
\]

or \[
AC = \begin{pmatrix} 225 & 140 & 125 \\ 265 & 115 & 245 \\ 245 & 125 & 175 \end{pmatrix} \begin{pmatrix} 0.60 & 1.20 \\ 0.50 & 0.80 \\ 0.40 & 1.00 \end{pmatrix} = \begin{pmatrix} 255 & 350 \\ 314.50 & 655 \\ 279.50 & 569 \end{pmatrix}
\]

(ii) Profit earned by outlet 2 = $340.50

7a (i) \[
\frac{208}{x-5} \text{ tiles}
\]

(ii) \[
\frac{169}{x} \text{ tiles}
\]

7b \[
2 \left( \frac{208}{x-5} \right) + 2 \left( \frac{169}{x} \right) - 4 = 74
\]

\[
\frac{416}{x-5} + \frac{338}{x} = 78
\]

\[
416x + 338(x-5) = 78x(x-5)
\]

\[
416x + 338x - 1690 = 78x^2 - 390x
\]

\[
78x^2 - 1144x + 1690 = 0
\]

\[
3x^2 - 44x + 65 = 0 \text{ (shown)}
\]
(3x - 5)(x - 13) = 0

\[ x = \frac{5}{3} = \frac{1}{2} \text{ or } x = 13 \]

7d If \( x = \frac{1}{2} \), the width of the tile will be negative.

7e Taking \( x = 13 \),
area of the wall that will be filled with tiles = \( 13(13 - 5) \times 74 \)
= 7696 cm²

8a \( \cos 60° = \frac{BC}{4} \)

\( BC = 2 \) cm

8b Total cross-sectional area of medal

\[ = 2 \left( \frac{1}{2} \right) (2^2) \left( \frac{4\pi}{3} \right) + 2 \left( \frac{1}{2} \right) (4)(2) \sin 60° \]

\[ = 30.6115 \]

\[ = 30.6 \text{ cm}^2 \text{ (3 sig. fig)} \]

8c Volume of the medal = 30.6115 \times 0.7

\[ = 21.428 \]

\[ = 21.4 \text{ cm}^3 \text{ (3 sig. fig)} \]

9a \( \angle BAR = \angle QPR \) (alternate angles)

\( \angle ABR = \angle PQR \) (alternate angles)

\( \angle ARB = \angle PRO \) (vertically opposite angles)

Since all corresponding angles are equal, triangle \( PQR \) is similar to triangle \( ABR \).

9b Triangle \( CPO \) and Triangle \( CBA \)

9c Using similar triangles,

\[ \frac{AR}{PR} = \frac{AB}{PQ} \]

\[ \frac{AR}{4} = \frac{12}{5} \]

\[ PB = 9.6 \text{ cm or } 9\frac{3}{5} \text{ cm} \]

9d Area of triangle \( PQR \) = \( \left( \frac{5}{12} \right)^2 = \frac{25}{144} \)

Area of triangle \( ABR \) = \( \left( \frac{5}{12} \right)^2 = \frac{25}{144} \)

9e \[ \left( \frac{5}{12} \right)^2 = \frac{15}{Area \text{ of triangle } CBA} \]

Area of triangle \( CBA \) = 86.4 cm²
Area of $ABPQ = 71.4 \text{ cm}^2$

10a  
(i)  
(a) median = 56.5 km/h

(b) Interquartile range $= 62.5 - 50 = 12.5$ km/h

(c) $80^{th}$ percentile $= 64$ km/h

(d) 65%

(ii) Probability $= 1 - \left( \frac{43 \times 42}{100 \times 99} \right)
= \frac{1349}{1650}$

10b  
(i) The median speed in the afternoon is 56.5 km/h while the median speed at night is 60 km/h. Vehicles at night travel faster. The interquartile range in the afternoon is 12.5 km/h while the interquartile range at night is 20 km/h. The speeds of the vehicles in the afternoon are less widespread (or more consistent) than the speeds at night due to smaller interquartile range in the afternoon.

(ii) Range $= 82 - 30 = 52$ km/h

11a  
1 cm : 75 m  
1 cm$^2$ : 5625 m$^2$

Area of building on the map $= \frac{134000}{5625}$  
$= 23.8$ cm$^2$ (3 sig. fig)

11b  
(i) actual circumference $= 105$ m

$2\pi r = 105$

Actual radius $= \frac{105}{2\pi}$ m

(ii) Let the centre of forest valley be $O$ and angle $ZOX$ be $\theta$ radian

$\frac{105}{2\pi} \theta = 35$

$\theta = \frac{2\pi}{3}$

Using Cosine Rule,

$YZ^2 = \left( \frac{105}{2\pi} \right)^2 + \left( \frac{105}{2\pi} \right)^2 - 2 \left( \frac{105}{2\pi} \right) \left( \frac{105}{2\pi} \right) \cos \frac{2\pi}{3}$

$YZ = 28.9447$ m
By Pythagoras Theorem,
\[ X Y^2 = 18^2 + Y Z^2 \]
\[ XY = 34.1 \text{ m (3 sig. fig)} \]

His claim is false.

3b \( y = 6.2 \)

3c

(i) gradient = \( \frac{8.5 - 5}{4.5 - 1.3} = 1.09375 \)

(ii) The gradient represents the speed of the stone at \( t = 3 \).

3d

(ii) At \( t = 1.65 \)

3(a) correct labelled axes & scale, points plotted correctly and smooth curve
3(c) correct tangent
3(d) graph of object
READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a 2B pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 80.
Mathematical Formulae

Compound interest

Total amount = $P \left(1 + \frac{r}{100}\right)^n$

Mensuration

Curved surface area of a cone = $\pi r l$

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3} \pi r^2 h$

Volume of a sphere = $\frac{4}{3} \pi r^3$

Area of triangle $ABC = \frac{1}{2} ab \sin C$

Arc length = $r \theta$, where $\theta$ is in radians

Sector area = $\frac{1}{2} r^2 \theta$, where $\theta$ is in radians

Trigonometry

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

Statistics

Mean = $\frac{\Sigma fx}{\Sigma f}$

Standard deviation = $\sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$
Answer all the questions.

1 Given that $64^k \times 16 = 1$, find the value of $k$.

\[ \text{Answer } k = \ldots \ldots \ldots \ldots \ [1] \]

2 The stem-and-leaf diagram shows the Science quiz marks scored by a group of 15 students.

\[
\begin{array}{c|c c c c}
0 & 7 \\
1 & 2 \\
2 & 4 & X \\
3 & 0 & 4 & 5 & 6 \\
4 & 3 & 5 & 6 & 6 & 6 & 7 & 9 \\
\end{array}
\]

Key: 0 | 7 represents 7 marks

Given that the median mark is twice the interquartile range, find the value of $X$.

\[ \text{Answer } X = \ldots \ldots \ldots \ldots \ [2] \]

3 The LCM and HCF of $p$ and $q$ are 60 and 6 respectively. Given that both $p$ and $q$ are between 6 and 60 and $p < q$, find the value of $p$ and of $q$.

\[ \text{Answer } p = \ldots \ldots, q = \ldots \ldots \ [2] \]
4. A, B and C are three non-empty sets satisfying the following conditions:
   \[ A \subset B, \ A \cap C \neq \emptyset, \ A \notin C \text{ and } C \notin B. \]
   Draw a clearly labelled Venn diagram to illustrate the above information.

   *Answer*

5. Sketch the graph of \( y = (x + 2)(4 - x) \) on the axes below.
   Indicate clearly the x-intercepts, y-intercept and the coordinates of the turning point.

   *Answer*
6 Tide detergent is having its monthly promotion at MAMA Supermarket.

Show that the cost of the detergent is directly proportional to its volume.

Answer

7 The employees of a company are offered a wage increment which is calculated according to one of the following schemes:

**Scheme A:** An increment of 5% of their present wages.

**Scheme B:** An increment of 3% of their present wages plus additional $8 per week.

Employees earning $x per week at present will receive higher increment if they have chosen **Scheme A**. Find the range of values of x.

Answer
8 Solve \( \frac{1}{x} - \frac{3}{2x+1} = 1 \).

Answer \( x = \ldots \ldots \ldots \ldots \) [3]

9 Rearrange \( y = \sqrt{\frac{2x+y}{3x-5}} \) to express \( x \) in terms of \( y \).

Answer \( x = \ldots \ldots \ldots \ldots \) [3]
10 Mr Lim invested $10 000 in a fund that pays a compound interest of 2.75% per annum, compounded quarterly. Find the interest earned, correct to the nearest dollar, at the end of 3 years.

Answer $620 [3]

11 The diagram shows a container in the shape of a prism with a triangular cross-section. Water is poured into the container until the depth of water is \( \frac{3}{5} \) the height of the container.

If the volume of the container is 200 ml, find the volume of water in the container.

Answer \( \frac{180}{5} \) ml [3]
12 (a) Convert 80 km/h to m/s.

\[ \text{Answer} \quad \text{m/s} \quad [1] \]

(b) In Marina Coastal Expressway (MCE), the speed limit of all vehicles is 80 km/h. A car is travelling at a constant speed of 30 m/s and passes a stationary traffic police motorcycle. The traffic police immediately gives chase, accelerating uniformly to reach a maximum speed of 40 m/s and continues with this speed until it overtakes the car.

(i) Calculate the acceleration of the police motorcycle.

\[ \text{Answer} \quad \text{m/s}^2 \quad [1] \]

(ii) In the axes below, draw the distance–time graph for the police motorcycle for the first 30 seconds. The distance–time graph for the car has been drawn for you.

\[ \text{Answer} \]

\[ \begin{array}{c}
\text{Distance (m)} \\
\hline
1000 \\
900 \\
800 \\
700 \\
600 \\
500 \\
400 \\
300 \\
200 \\
100 \\
\hline
\end{array} \]

\[ \begin{array}{c}
\text{Time (s)} \\
\hline
0 \\
15 \\
30 \\
\hline
\end{array} \]
13 \( PQRS \) is a parallelogram.
\[ \overrightarrow{PQ} = \left( -\frac{3}{4} \right), \overrightarrow{PS} = \left( \frac{12}{5} \right). \]
Calculate \( |\overrightarrow{PR}| \).

---

Answer \( \ldots \ldots \ldots \ldots \) units [3]

---

14 By observing the pattern in the last digit of \( 3^n \) and of \( 8^n \), where \( n > 0 \), find the last digit in the subtraction \( 3^{31} - 8^{16} \).

---

Answer \( \ldots \ldots \ldots \ldots \) [3]

The diagram below shows part of a site map of the newly launched Build-To-Order (BTO) flats at Fernvale Road.

Andy and his parents applied for this BTO project under the Multi-Generation Priority Scheme which allows married children to make a joint application with their parents for two units in the same project.

Their application has gotten them a good queue number in the selection of units. Andy has chosen the unit 580, marked as A.

State the unit which Andy's parents should choose if they wish to satisfy the following criteria:
- equidistant from the multi storey carpark, C, and the LRT station, L, and
- equidistant from the line LC and AC.

Answer ........................................ [3]
17 In the diagram, $O$ is the centre of the circle with radius 13 cm. $PQ$ and $RS$ are perpendicular equal chords of length 24 cm and intersecting at $K$.

Calculate the length of $OK$.
Show your workings and give reasons.

Answer .................. cm [3]
18 For a $n$-sided polygon, the largest interior angle is $100^\circ$ and the smallest interior angle is $20^\circ$. Find the value(s) of $n$.

Answer $n =$ ........................... [4]

19 (a) Factorise $(2x - 7)(x - 2) - 9$ completely.

Answer .................................. [2]

(b) Hence, solve $2(y - 1)^2 - 11y + 16 = 0$.

Answer .................................. [2]
20 When \( x + 8 \) is divided by \( y \), the result is 4. When \( x \) is divided by \( y \), its quotient is 2 and the remainder is 6. By forming two equations in terms of \( x \) and of \( y \), solve for \( x \) and \( y \).

\[
\text{Answer } x = \ldots..., \ y = \ldots...
\]

21 In the diagram, \( A, B, C \) and \( D \) lie on the circumference of the circle. \( ADP, ABQ, BCP \) and \( DCQ \) are straight lines. \( \angle BAD = 54^\circ \) and \( \angle CPD = 27^\circ \).

\[
\text{Find } \angle AQD.
\]

Show your working and give reasons.

\[
\text{Answer } \angle AQD = \ldots\ldots\ldots\ldots\ldots\ldots^\circ
\]
A carpenter wants to create a wooden toy for his son. He removed a right-angled cone and a hemisphere from a uniform cylindrical wood. The radius of the base of the cone and hemisphere are \( r \) cm. The distance between the top of the hemisphere and the vertex of the cone is also \( r \) cm.

Given that the curved surface area of the cone is equal to the curved surface area of the hemisphere, work out the volume of the toy, in terms of \( r \).

\[ \text{Answer} \quad \text{cm}^3 \quad [4] \]
23 The histogram illustrates the length of stay (in days) in Australia for a group of Singaporean tourists last year.

(a) Complete the following table.

<table>
<thead>
<tr>
<th>Answer</th>
<th>Length of stay (days)</th>
<th>Numbers of tourists</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 &lt; x ≤ 5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5 &lt; x ≤ 10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10 &lt; x ≤ 15</td>
<td></td>
</tr>
<tr>
<td></td>
<td>15 &lt; x ≤ 20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20 &lt; x ≤ 25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>25 &lt; x ≤ 30</td>
<td></td>
</tr>
<tr>
<td></td>
<td>30 &lt; x ≤ 35</td>
<td></td>
</tr>
<tr>
<td></td>
<td>35 &lt; x ≤ 40</td>
<td></td>
</tr>
</tbody>
</table>

(b) Calculate the fraction of the Singaporean tourists who stayed in Australia longer than 15 days.

Answer .................................. [1]

(c) Calculate an estimate for the mean and standard deviation of the length of stay.

Answer .................................. 

Mean = ................ days

Standard deviation = .......... days  [2]
24 (a) Explain why \((x - 2)^4 (x^2 - 4x + 4)\) is both a perfect square and a perfect cube when \(x\) is an integer.

**Answer**

........................................................................................................................... [2]

(b) The figure is made up of 2 big squares, 2 small squares and a rectangle \(WXYZ\). The perimeter of rectangle \(WXYZ\) is 20 cm. The total area of all the four squares is 140 cm\(^2\).

![Diagram of WXYZ]

Find the area of \(WXYZ\).

**Answer** ...................................... cm\(^2\) [4]
25 Mrs Tan sells three different types of cakes. The table below shows the number of cakes bought by Childcare Centre A and B for Children’s Day Celebration.

<table>
<thead>
<tr>
<th></th>
<th>Sponge</th>
<th>Chocolate</th>
<th>Cheese</th>
</tr>
</thead>
<tbody>
<tr>
<td>Childcare Centre A</td>
<td>32</td>
<td>27</td>
<td>20</td>
</tr>
<tr>
<td>Childcare Centre B</td>
<td>44</td>
<td>45</td>
<td>38</td>
</tr>
</tbody>
</table>

(a) Represent this information in a $2 \times 3$ matrix, $P$.

\[
\text{Answer } P = \begin{pmatrix} \hline \end{pmatrix} \quad [1]
\]

(b) The selling price for each sponge cake, chocolate cake and cheese cake is $3.20, $4.50 and $4.80 respectively.

The information can be represented as $Q = \begin{pmatrix} 3.2 \\ 4.5 \\ 4.8 \end{pmatrix}$.

Evaluate the matrix $PQ$.

\[
\text{Answer } PQ = \begin{pmatrix} \hline \end{pmatrix} \quad [1]
\]

(c) Mrs Tan has a special promotion for cakes order for Children’s Day celebration.

- Get the second sponge cake at half price.
- Buy three chocolate cakes for the price of two.
- Cheese cakes 1-for-1!

Write down a $3 \times 3$ matrix, $R$, such that $RQ$ will give the discounted price for each type of cakes sold to Childcare Centre A and B.

\[
\text{Answer } R = \begin{pmatrix} \hline \end{pmatrix} \quad [2]
\]
25 (d) Evaluate matrix \( S = PRQ \). 

\[
Answer \quad S = \begin{pmatrix} \end{pmatrix} \quad [2]
\]

(e) Describe what the elements in matrix \( S \) represents.

\[
Answer
\]

......................................................................................................................................................... [1]

- End -
READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a 2B pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give
the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in
terms of \( \pi \).

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 100.
Mathematical Formulae

Compound interest

Total amount = \( P \left(1 + \frac{r}{100}\right)^n \)

Mensuration

Curved surface area of a cone = \( \pi rl \)
Surface area of a sphere = \( 4\pi r^2 \)
Volume of a cone = \( \frac{1}{3}\pi r^2h \)
Volume of a sphere = \( \frac{4}{3}\pi r^3 \)

Area of triangle \( ABC = \frac{1}{2} ab \sin C \)

Arc length = \( r\theta \), where \( \theta \) is in radians

Sector area = \( \frac{1}{2} r^2\theta \), where \( \theta \) is in radians

Trigonometry

\( \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \)

\( a^2 = b^2 + c^2 - 2bc \cos A \)

Statistics

Mean = \( \frac{\Sigma fx}{\Sigma f} \)

Standard deviation = \( \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2} \)
1. (a) Simplify \( \frac{4b^3}{3a} + \frac{(-4ab^3)^2}{2a^{-4}b^2} \). Give your answer in positive indices.

(b) Factorise \( 14xb + 3ay - 2xy - 21ab \) completely.

(c) Solve the inequality \( \frac{x}{5} < \frac{x+2}{3} \leq \frac{3-2x}{4} \).
(d) Simplify \( \frac{2x(1-6x)}{4x^2-1} - \frac{2}{2x+1} \)  

(e) Express \( 2x^2 + 6x - 15 \) in the form \( a(x + b)^2 + c \), where \( a \), \( b \) and \( c \) are constants.

Hence, solve the equation \( 2x^2 + 6x - 15 = 0 \), giving your answers correct to four decimal places.
2. \( ABCDE \) is a pentagon.

(a) Calculate the area of the pentagon \( ABCDE \).
(b) The pentagon $ABCDE$ is a cross-section of a prism which is a model of a house as shown below.

![Diagram of a prism]

(i) Calculate the total surface area of the model, including the base. [2]

(ii) Calculate the angle of depression of $H$ from $E$. [3]
3. There was a newspaper article on Singapore’s population published in 2018.

**Singapore’s population grows slightly to 5.638 million, with non-resident numbers stable.**

Singapore’s population rose 0.5 percent to 5.638 million from 2017 to 2018.

The slight increase over the past year was due to stable growth in citizen population, which rose 1 percent to 3.472 million citizens as of June 2018.

The 1 percent growth in citizens was due to citizen births and immigration.

![Population Chart]


(a) Calculate the Singapore’s total population in 2017.
(b) Express the number of Foreign Domestic Workers as a percentage of the Singapore's total population in 2018.

(c) Based on the information shown in this article, do you agree that Singapore's total population will reach 6.3 million by 2030? Support your answer with appropriate workings and state an assumption that you have made in your calculation.

(d) State one aspect of the diagram that may be misleading and explain how this may lead to a misinterpretation of information.
4. The diagram shows a rectangular table top.

```
   x m
```

The area of the table top is 5 square metres.
The length of the table top is \( x \) metres.
The perimeter of the table top is \( y \) metres.

(a) Show that \( y = 2x + \frac{10}{x} \). [2]

(b) The variables \( x \) and \( y \) are connected by the equation \( y = 2x + \frac{10}{x} \).
Some corresponding values of \( x \) and \( y \), correct to one decimal place, are given in the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>( p )</td>
<td>12</td>
<td>( q )</td>
<td>9</td>
<td>10.5</td>
<td>13.7</td>
<td>17.3</td>
</tr>
</tbody>
</table>

(i) Find the value of \( p \) and of \( q \). [1]

(ii) Using a scale of 1 cm to represent 1 m, draw a horizontal \( x \)-axis for \( 0 \leq x \leq 8 \).
Using a scale of 1 cm to represent 1 m, draw a vertical \( y \)-axis for \( 0 \leq y \leq 22 \).
On your axes, plot the points given in the table and join them with a smooth curve. [3]
(iii) Use your graph to find the length of the table top if the perimeter of the table top is 16 m.

[2]

(iv) By drawing a suitable straight line, find the x coordinate of the point at which the gradient of the curve is \( \frac{1}{3} \).

[2]
5. Given equation of a line \( L \) is \( 3x - 2y = 8 \).

   (a) State the gradient of line \( L \). \[1\]

   (b) If \( P(k, -2) \) is a point on the line \( L \), find the value of \( k \). \[1\]

   (c) Find the equation of another line that is parallel to the x-axis and passes through \( P \). \[1\]

   (d) Calculate the perpendicular distance from the origin, \( O \), to the line \( L \). \[4\]
6.

In the diagram, \( \overrightarrow{OA} = a, \overrightarrow{OB} = 6b \) and \( \overrightarrow{OA} = \frac{1}{3} \overrightarrow{OC} \).

\( D \) is a point on \( AB \) such that \( 3\overrightarrow{AD} = \overrightarrow{DB} \) and \( E \) is a point on \( BC \) such that \( \overrightarrow{CE} : \overrightarrow{EB} = 4 : 5 \).

(a) Express, as simply as possible, in terms of \( a \) and \( b \),

(i) \( \overrightarrow{BA} \),

(ii) \( \overrightarrow{DB} \),

(iii) \( \overrightarrow{CB} \),

(iv) \( \overrightarrow{AE} \).
(b) Write down the relationship between \(OD\) and \(AE\). Explain your answer. \([2]\)

(e) Find the ratio of

(i) area of triangle \(CAE\) : area of triangle \(AOD\), \([2]\)

(ii) area of triangle \(CAE\) : area of triangle \(AOB\). \([2]\)
7. (a) There are two boxes of sweets containing toffees and chocolates.
Box A contains 8 toffees and 4 chocolates, whereas box B contains 7 toffees and 3 chocolates.
Jolin loves chocolates.
One of the boxes is chosen at random and a sweet is taken out.
If she gets a chocolate, she will consume it.
If she did not get a chocolate from the first selection, she will place the sweet into the other box and she will select again from the other box.
Jolin limits herself to two selections.

Find, as a fraction in its simplest form, the probability that
(i) Jolin will have her favourite chocolate from the first selection, \[3\]
(ii) Jolin will have her favourite chocolate. \[3\]
Your friend, Kenton gives you a chance to win $1000 by playing a game of "Guess the number". There are two options of the game that he allows you to choose.

**Option A**
He uses a random number generator to choose a number from 1 to 8.
If you guess it correctly, you win.

**Option B**
You flip a fair coin.
If the coin lands on head, Kenton will roll a fair 6-sided die. If you manage to guess what it rolled, you win.
If the coin lands on tail, Kenton will use a random number generator to choose a random number from 1 to 8. If you guess the number correctly, you win.

Which option will you choose? Explain your answer.
Figure 1 shows a dreamcatcher-inspired ornament which is made up of wire. The ornament consists of an inner circle $ABCDEF$ and an outer circle $GHIJKL$, both with centre $O$, and 6 identical "petal" designs such as $AGB$ and $BHC$.

Team Hōʻola will be making these ornaments to raise funds for their Voluntary Welfare Organisation.

To estimate the amount of wire to be purchased, Head of Fundraising team, Janice modelled the following:

- The arc $AG$ is an arc of another circle with centre $X$, radius 18 cm and
  \[ \angle AXG = \frac{\pi}{6} \text{ radians}. \]

  This information is illustrated in Figure 2.

Figure 2
A regular hexagon forms within the inner circle \( ABCDEF \) with \( OB = 5 \text{ cm} \).
This information is illustrated in Figure 3.

![Figure 3](image)

(a) Show that the radius of the outer circle is 13.31 cm, correct to 2 decimal places.

[6]
(b) Team Hō'ola decided to make 50 dreamcatcher-inspired ornaments and Janice chanced upon the following promotion.

10 m silver craft bendable DIY wire (2mm thick) $9.28

What is the estimated cost to make these ornaments?
(c) Find the area of the shaded region as shown in Figure 1. [5]
9. (a) The cumulative frequency curve below shows the time spent in minutes by a group of 80 teenagers on Instagram (a social media platform) on a particular day.

Use the curve to estimate

(i)  the median,  

(ii) the interquartile range.
(b) Through a market research, it was found out that the time spent on Facebook (another social media platform) is less popular and less consistent among the same group of 80 teenagers. A second cumulative frequency curve for the same group of 80 teenagers spending their time on Facebook is drawn. Describe how the second cumulative frequency curve may differ from the curve for Instagram. [2]

(c) The box-and-whisker plot represents the distribution of the time spent for the same group of 80 teenagers on SnapChat (another social media platform).

For this group of 80 teenagers, which of the social media platforms - Instagram, Facebook or Snapchat, is the most popular? Support your answer with an appropriate statistical measure. [2]
10. Ms Tan got her new home recently and the layout of her house is shown in the diagram below. The layout is drawn to scale.

(a) Express the scale of the map in the form of $1:n$. [1]
Ms Tan decides to get a television set for her living room.

She found the following infographic online.

Viewing distance guide: linking screen sizes to how far you sit from the TV

<table>
<thead>
<tr>
<th>Viewing distance</th>
<th>Screen Size Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 1.5m</td>
<td>Up to 32 inch</td>
</tr>
<tr>
<td>1.5 to 2m</td>
<td>32 to 39 inch</td>
</tr>
<tr>
<td>2 to 2.5m</td>
<td>40 to 45 inch</td>
</tr>
<tr>
<td>2.5 to 3m</td>
<td>46 to 56 inch</td>
</tr>
<tr>
<td>Over 3m</td>
<td>Over 56 inch</td>
</tr>
</tbody>
</table>

What is the range of television size which Ms Tan should get for her living room? [2]
Ms Tan decided to shop for her television set online and she saw the following options:

![LG OLED 55" and 65" 4K SMART TV](image)

Ms Tan pays a downpayment of $2,000 for her television set as suggested in (b). She pays the remaining amount over 3 months, with a simple interest rate of 7% per annum.

Calculate her monthly instalment. [3]
Answers

1 \[4^{3k} \times 4^2 = 4^6\]
\[3k + 2 = 0\]
\[k = -\frac{2}{3}\text{ exact only!}\]

2 Median = 36
Interquartile Range = 18
\[Q_1 = 46 - 18 = 28\]
\[X = 8 \rightarrow \text{digit in the ones place.}\]

3 HCF = 6 = 2 \times 3
LCM = 60 = 2^2 \times 3 \times 5
\[p = 2^2 \times 3 = 12\]
\[q = 2 \times 3 \times 5 = 30\]

4

5

6 \[\frac{1.79}{10} = 179\]
\[\frac{17.9}{2} \times 50 = 179\]
\[\frac{26.85}{150} = 179\]
\[\text{Must show for all three!}\]

Since \[\frac{\text{cost}}{\text{volume}} = \text{constant},\] the cost is directly proportional to its volume.

7 \[0.05x > 0.03x + 8 \text{ or } 1.05x > 1.03x + 8\]
\[0.02x > 8\]
\[x > 400\]

8 \[
\frac{2x+1-3x}{x(2x+1)} = 1
\]
\[1 - x = 2x^2 + x\]
\[2x^2 + 2x - 1 = 0\]
\[x = \frac{-2 \pm \sqrt{(2)^2 - 4(2)(-1)}}{2(2)} = 0.366 \text{ or } -1.37\]

9 \[y^2 = \frac{2x+3}{3x-5}\]
\[3xy^2 - 5y^2 = 2x + y\]
\[3xy^2 - 2x = 5y - y\]
\[x(3y^2 + 1) = 3y^2 - y\]
\[x = \frac{y(3y^2 - 2)}{3y - 2} \text{ or } \frac{3y - 2}{3y^2 + 2}\]

10 \[10000 \left(1 + \frac{0.25}{100}\right)^4 - 10000 = 10856.92 - 10000 = 856.92 = \$857\]

11 Not cube because one of the component is constant!!

12a \[\text{Solution: } \frac{80 \times 1000 \text{ m}}{3600 \text{ s}} = \frac{200}{9} = \frac{22}{9}\]

12b \[\frac{40}{15} = 2\frac{2}{3}\]

12c Distance (m)

[Graph showing smooth curve only!]

Quarterly is 4 times!

Mixed number or 3sf only!!
13 \[
\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = \left(\frac{-3}{4}\right) + \left(\frac{12}{5}\right) = \left(\frac{9}{9}\right)
\]

\[|\overrightarrow{PR}| = \sqrt{9^2 + 9^2} = \sqrt{162} = 12.7 \text{ units}\]

PQ is not perpendicular to PSI

Length PR ≠ length PQ + length QRI

14
\[
3^1 = 3 \\
3^2 = 9 \\
3^3 = 27 \\
3^4 = 81 \\
3^5 = 243
\]
\[
8^1 = 8 \\
8^2 = 64 \\
8^3 = 512 \\
8^4 = 4096 \\
8^5 = 32768
\]

Last digit for \(3^n\) is 3, 9, 7, 1

Last digit for \(8^n\) is 8, 4, 2, 6

\[\frac{3^1}{4} = 7\frac{3}{4} \rightarrow \text{Remainder} = 3 \rightarrow \text{Last digit} = 7\]

\[\frac{16}{4} = 4 \rightarrow \text{Remainder} = 0 \rightarrow \text{Last digit} = 6\]

Last digit of \(3^{11} - 8^{16} = 7 - 6 = 1\)

We must see that you had chosen the digit "7" and "6".

Equation of line must be seen in this page. Or else, we will take it as you use calculator mode 3, 4.

\[x = -3 \text{ or } 0 \text{ or } 4 \rightarrow \text{only accept exact answers.}\]
16. Draw perpendicular bisector of LC.
   Draw angle bisector of AC.
   Unit 518.

17. Let M be the midpoint of RS.
   \[ OM = \sqrt{13^2 - 12^2} = 5 \] (Perpendicular bisector of chord)
   \[ OK = \sqrt{5^2 + 5^2} = 7.07 \] (Equal chords)
   Leave answer in 3sf as you are finding length!

18. \[ 20 \leq \frac{(n-2) \times 180 - 100 - 20}{n-2} \leq 100 \]
   \[ 20(n-2) \leq (n-2) \times 180 - 120 \leq 100(n-2) \]
   \[ 20n - 40 \leq 180n - 360 - 120 \leq 100n - 200 \]
   \[ 20n - 40 \leq 180n - 480 \text{ or } 180n - 480 \leq 100n - 200 \]
   \[ 440 \leq 160n \text{ or } 80n \leq 280 \]
   \[ 2.75 \leq n \text{ or } n \leq 3.5 \]
   \[ 2.75 \leq n \leq 3.5 \]
   \[ n = 3 \]
   Guess & check allowed for this qn.

19a. \[ 2x^2 - 4x - 7x + 14 - 9 = 2x^2 - 11x + 5 = (2x - 1)(x - 5) \]
   Factorise, not completing the square! Read carefully.

19b. \[ 2(y - 1)^2 - 11(y - 1) + 5 = 0 \]
   \[ x = y - 1 \rightarrow \text{must be seen! Read question. It says "Hence".} \]
   \[ y - 1 = \frac{1}{2} \text{ or } 5 \]
   \[ y = 1.5 \text{ or } 6 \]

20. \[ \frac{x+8}{x} = 4 \]
   \[ x+8 = 4x \]
   \[ x = 2y + 6 \]
   \[ x = 4y \]
   \[ y = 2y + 6 \]
   \[ 2y + 6 + 8 = 4y \]
   \[ 2y = 14 \]
   \[ y = 7 \]
   \[ x = 20 \]
   Do not write mixed number for algebra!

21. \[ \angle BCD = 180 - 54 = 126 \text{ (} \angle \text{ in opp segment) } \]
   \[ \angle DCP = 180 - 126 = 54 \text{ (} \angle \text{ on a straight line) } \]
   \[ \angle ADC = 27 + 54 = 81 \text{ (ext } \angle \text{ of } \Delta) \]
   \[ \angle AQD = 180 - 54 - 81 = 45 \text{ (} \angle \text{ sum of } \Delta) \]
   \[ \angle ABP = 180 - 54 - 27 = 99 \text{ (} \angle \text{ sum of } \Delta) \]
   \[ \angle ADQ = 180 - 99 = 81 \text{ (} \angle \text{ in opposite segment) } \]
   \[ \angle AQD = 180 - 54 - 81 = 45 \text{ (} \angle \text{ sum of } \Delta) \]
   Easier method:

22. \[ \pi rl = 2\pi r^2 \]
   \[ l = 2r \]
   \[ h = \sqrt{(2r)^2 - r^2} = \sqrt{3}r \]
   \[ \text{Vertical height and slanted length are different!} \]
   \[ \text{Volume} = \pi r^2(\sqrt{3}r + 2r) - \frac{1}{3} \pi r^2(\sqrt{3}r) - \frac{2}{3} \pi r^3 \]
   \[ = \pi r^3 \left(\sqrt{3} + 2 - \frac{\sqrt{3}}{3} - \frac{2}{3}\right) \]
   \[ = 7.82r^3 \]
   Refer to cover page if you had left your answers in \( \pi \).
23a

<table>
<thead>
<tr>
<th>Length of stay (days)</th>
<th>Numbers of tourists</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; x ≤ 5</td>
<td>10</td>
</tr>
<tr>
<td>5 &lt; x ≤ 10</td>
<td>10</td>
</tr>
<tr>
<td>10 &lt; x ≤ 15</td>
<td>10</td>
</tr>
<tr>
<td>15 &lt; x ≤ 20</td>
<td>2</td>
</tr>
<tr>
<td>20 &lt; x ≤ 25</td>
<td>2</td>
</tr>
<tr>
<td>25 &lt; x ≤ 30</td>
<td>3</td>
</tr>
<tr>
<td>30 &lt; x ≤ 35</td>
<td>3</td>
</tr>
<tr>
<td>35 &lt; x ≤ 40</td>
<td>3</td>
</tr>
</tbody>
</table>

23b

\[ \frac{13}{43} \]

23c

Mean = \( \frac{1327}{96} \) or 13.9

SD = 11.0

3sf only! Do not round to nearest integer!

24a

\[(x - 2)^4 (x^2 - 4x + 4) = (x - 2)^4 (x - 2)^2 = (x - 2)^6 \rightarrow \text{ must be seen!} \]

Since the power 6 is both a multiple of 2 and of 3, it is both a perfect cube and a perfect square.

24b

\[2x + 2y = 20\]
\[x + y = 10\]

\[2x^2 + 2y^2 = 140\]
\[x^2 + y^2 = 70\]

\[(x + y)^2 = x^2 + y^2 + 2xy\]
\[2xy = 100 - 70\]
\[xy = 15\]

Alternative method is to solve of x and y to get 8.16 or 1.84 and find \( xy = 15.0 \text{ km}^2 \)

25a

\[ P = \begin{pmatrix} 32 & 27 & 20 \\ 44 & 45 & 38 \end{pmatrix} \]

25b

\[ PQ = \begin{pmatrix} 319.9 \\ 525.7 \end{pmatrix} \]

25c

\[ R = \begin{pmatrix} \frac{3}{4} & 0 & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \]

Work with exact! This is percentage, not money hence, it should not be 2dp!

Only accept exact or 3sf.

25d

\[ S = \begin{pmatrix} 32 & 27 & 20 \\ 44 & 45 & 38 \end{pmatrix} \begin{pmatrix} \frac{3}{4} & 0 & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 24 & 18 & 10 \\ 33 & 30 & 19 \end{pmatrix} \begin{pmatrix} 3.2 \\ 4.5 \\ 4.8 \end{pmatrix} = \begin{pmatrix} 205.8 \\ 331.8 \end{pmatrix} \]

25e

They represent the total money collected from Childcare Centre A and B respectively for the sales of cakes after discount.

Money collected from sales is not money earned! The word "earned/earnings" relates to profit.
NAN CHIAU HIGH SCHOOL
MID-YEAR EXAMINATION 2019
SECONDARY FOUR EXPRESS

MARKING SCHEME

MATHMATICS
Paper 2
Candidates answer on the Question Paper

May 2019, Thursday
2 hours 30 minutes
1. (a) Simplify \( \frac{4b^3}{3a} + \frac{(-4ab^3)^2}{2a^{-4}b^2} \). Give your answer in positive indices. [2]

\[
\frac{4b^3}{3a} + \frac{(-4ab^3)^2}{2a^{-4}b^2} = \frac{4b^3}{3a} \times \frac{2a^4b^2}{16a^7b^8} \quad \text{[M1] convert expression to multiplication & use of power}
\]

\[
= \frac{b^2}{3a} \times \frac{1}{2a^6b^6} \quad \text{[A1]}
\]

(b) Factorise \( 14xb + 3ay - 2xy - 21ab \) completely. [2]

\[
14xb + 3ay - 2xy - 21ab = 14xb - 2xy - 21ab + 3ay
\]

\[
= 2x(7b - y) - 3a(7b - y) \quad \text{[M1] factorise common term}
\]

\[
= (7b - y)(2x - 3a) \quad \text{[A1]}
\]

(c) Solve the inequality \( \frac{x}{5} \leq \frac{x+2}{3} \leq \frac{3-2x}{4} \). [3]

\[
\frac{3-2x}{4} \geq \frac{x+2}{3} \quad \text{and} \quad \frac{x+2}{3} > \frac{x}{5}
\]

\[
3(3 - 2x) \leq 4(x + 2) \quad \text{and} \quad 5(x + 2) > 3x
\]

\[
9 - 6x \geq 4x + 8 \quad \text{and} \quad 5x + 10 > 3x
\]

\[
x \geq 10x \quad \text{and} \quad 2x > -10
\]

\[
x \leq \frac{1}{10} \quad \text{[M1] and} \quad x > -5 \quad \text{[M1]}
\]

\[-5 < x \leq \frac{1}{10} \quad \text{[A1]}
\]
(d) Simplify \( \frac{2x(1-6x)}{4x^2-1} - \frac{2}{2x+1} \).

\[
\begin{align*}
\frac{2x(1-6x)}{4x^2-1} & - \frac{2}{2x+1} \\
= & \frac{2x(1-6x)}{2x+1} - \frac{2}{2x+1} \\
= & \frac{2x(2x-1)}{(2x+1)(2x-1)} - \frac{2}{2x+1} \\
= & \frac{2x-12x^2+4x+2}{(2x+1)(2x-1)} \\
= & \frac{-12x^2-2x+2}{(2x+1)(2x-1)} \\
= & -\frac{2(6x^2-x-1)}{(2x+1)(2x-1)} \\
= & -\frac{2(3x-1)(2x+1)}{(2x+1)(2x-1)} \\
= & -\frac{2(3x-1)}{2x-1} \\
\end{align*}
\]

\[M1\] common denominator

\[M1\] simplify numerator

\[M1\] factorize numerator

\[A1\]

(e) Express \( 2x^2 + 6x - 15 \) in the form \( a(x + b)^2 + c \), where \( a \), \( b \), and \( c \) are constants.

Hence, solve the equation \( 2x^2 + 6x - 15 = 0 \), giving your answers correct to four decimal places.

\[
\begin{align*}
2x^2 + 6x - 15 & = 2(x^2 + 3x - \frac{15}{2}) \\
= & 2 \left( x + \frac{3}{2} \right)^2 - \frac{9}{4} - \frac{15}{2} \\
= & 2 \left( x + \frac{3}{2} \right)^2 - \frac{39}{4} \\
= & 2 \left( x + \frac{3}{2} \right)^2 - \frac{39}{2} \\
\end{align*}
\]

\[M1\] which to do completing the square

\[A1\]

\[
\begin{align*}
2x^2 + 6x - 15 & = 0 \\
2 \left( x + \frac{3}{2} \right)^2 - \frac{39}{2} & = 0 \\
\left( x + \frac{3}{2} \right)^2 & = \frac{39}{4} \\
\end{align*}
\]

\[M1\] solving using completing the square

\[
\begin{align*}
x + \frac{3}{2} & = \pm \sqrt{\frac{39}{4}} \\
\end{align*}
\]

\[
\begin{align*}
x & = 1.6225 \text{ or } x = -4.6225 \\
\end{align*}
\]

\[A1\]
2. ABCDE is a pentagon.

(a) Calculate the area of the pentagon ABCDE.

\[ 10^2 = 8.5^2 + 3.5^2 - 2(8.5)(3.5)\cos B\hat{A}E \] \[ \text{[M1] use of cosine rule} \]

\[ \cos B\hat{A}E = \frac{8.5^2 + 3.5^2 - 10^2}{2(8.5)(3.5)} \]

\[ \cos B\hat{A}E = -0.260504 \]

\[ \alpha = 74.90^\circ \]

\[ B\hat{A}E = 105.1^\circ \text{ or } 295.1^\circ \text{ (ref)} \] \[ \text{[M1] ability to find angle } B\hat{A}E \]

Or

\[ \hat{A}BE = 19.75^\circ \text{ or } 340.25^\circ \text{ (ref)} \]

\[ \hat{B}\hat{E}A = 55.15^\circ \text{ or } 304.85^\circ \text{ (ref)} \]

Area of \( \triangle ABE = \frac{1}{2}(8.5)(3.5)\sin 105.1^\circ \] \[ \text{[M1] finding area of triangle} \]

\[ = 14.36 \text{ cm}^2 \]

\[ BE = \sqrt{10^2 - 1^2} \]

\[ = \sqrt{98} \text{ cm} \]

Area of \( BCDE = \frac{1}{2}(5 + 6)(\sqrt{98}) \] \[ \text{[M1] finding area of trapezium} \]

\[ = 54.72 \text{ cm}^2 \]

Area of \( ABCDE = 14.36 + 54.72 \]

\[ = 69.1 \text{ cm}^2 \] \[ \text{[A1]} \]
(b) The pentagon ABCDE is a cross-section of a prism which is a model of a house as shown below.

(i) Calculate the total surface area of the model, including the base.

\[
\text{total surface area} = 2(69.08) + 10(5 + 8.5 + 3.5 + 6 + \sqrt{99})\]

\[
= 468 \text{ cm}^2 \quad [\text{AI}]
\]

(ii) Calculate the angle of depression of H from E.

Let the angle of depression be \( \theta \)

Student use
(i) angle of depression = angle of elevation or
(ii) angle of depression = 90° − 57.5°

\[
HD = \sqrt{10^2 + 99} \quad [\text{M1}] \quad \text{finding length HD}
\]

\[
= \sqrt{199} \text{ cm}
\]

\[
tan\theta = \frac{6}{\sqrt{199}} \quad [\text{M1}] \quad \text{finding relevant angle (either angle EHD or HED)}
\]

\[
\theta = 23.0° \quad [\text{A1}]
\]
3. There was a newspaper article on Singapore’s population published in 2018.

**Singapore’s population grows slightly to 5.638 million, with non-resident numbers stable.**

Singapore’s population rose 0.5 percent to 5.638 million from 2017 to 2018.

The slight increase over the past year was due to stable growth in citizen population, which rose 1 percent to 3.472 million citizens as of June 2018.

The 1 percent growth in citizens was due to citizen births and immigration.

(a) Calculate the Singapore’s total population in 2017.

\[
100.5\% \rightarrow 5.638 \text{ million} \\
100\% \rightarrow \frac{100}{100.5} \times 5.638 \text{ million} \quad [M1] \quad \text{showing} \quad \frac{100}{100.5} \times 5.638 \\
= 5.61 \times 10^6 \quad [A1] \quad \text{or} \quad 5.61 \text{ million}
\]
(b) Express the number of Foreign Domestic Workers as a percentage of the Singapore’s total population in 2018.

Number of foreign domestic workers

\[
\frac{15}{100} \times 1.644 \times 10^6
\]

\[
= 0.2466 \times 10^6 \quad \text{------- [M1] must in 10^6 or million}
\]

\[
\frac{0.2466 \times 10^6}{5.838 \times 10^6} \times 100\% = 4.37\% \quad \text{------- [A1]}
\]

(c) Based on the information shown in this article, do you agree that “Singapore’s total population will reach 6.3 million by 2030?” Support your answer with appropriate workings and state an assumption that you have made in your calculation.

\[2030 - 2018 = 12 \text{ years}\]

Population of citizen by 2030

\[= (1.01)^{12} \times 3.472 \times 10^6 \quad \text{------- [M1] finding the increase of citizen only}\]

\[= 3.912 \times 10^6 \]

Total population by 2030

\[= (3.912 + 0.522 + 1.644) \times 10^6 \]

\[= 6.08 \times 10^6 \quad \text{------- [A1]}
\]

Assumption:

There is no increase for PRs and Non-Residents or
There is a constant growth of citizens at 1% per year \[\text{------- [M1]}\]

(d) State one aspect of the diagram that may be misleading and explain how this may lead to a misinterpretation.

Accept any logical answer

(i) Pictogram used alternate of a male and female figure \[\text{--- [B1]}\]

[Reason] misleading readers that there is an equal number of male and female population. \[\text{--- [B1]}\]

(ii) Inaccurate use of \(\frac{1}{4}\) pictogram “figure” for “non-resident” & \(\frac{3}{4}\) in “citizen” \[\text{--- [B1]}\]

[Reason]

in PRs : if 1 figure \(\rightarrow 0.261 \text{ M}\)

in citizen : 3.472 M should be represented by 13.3 figures

not 13 + \(\frac{3}{4}\) figure \[\text{--- [B1]}\]
4. The diagram shows a rectangular table top.

The area of the table top is 5 square metres.
The length of the table top is \( x \) metres.
The perimeter of the table top is \( y \) metres.

(a) Show that \( y = 2x + \frac{10}{x} \). 

Width of table top = \( \frac{5}{x} \)  

Perimeter, \( y = x + \frac{5}{x} + x + \frac{5}{x} \)  

\[ y = 2x + \frac{10}{x} \]  

(b) The variables \( x \) and \( y \) are connected by the equation \( y = 2x + \frac{10}{x} \). 

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.5</th>
<th>1.5</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>12</td>
<td>9</td>
<td>10.5</td>
<td>13.7</td>
<td>17.3</td>
<td></td>
</tr>
</tbody>
</table>

Find the value of \( p \) and of \( q \). 

\( p = 21 \) 

\( q = 9.7 \) (accept also 9.67)  

(c) Using a scale of 1 cm to represent 1 unit, draw a horizontal \( x \)-axis for \( 0 \leq x \leq 8 \).

Using a scale of 1 cm to represent 1 unit, draw a vertical \( y \)-axis for \( 0 \leq y \leq 22 \).

On your axes, plot the points given in the table and join them with a smooth curve.
(d) Use your graph to find the possible width of the table top if the perimeter of the

table top is 16 m. [2]

When \( P = 16 \),
\[ x = 0.7 \text{ (±0.1)} \quad [B1] \text{ answer must be rejected as length is longer} \]

or \[ x = 7.3 \text{ (±0.1)} \quad [B1] \]

(e) By drawing a suitable straight line, find the \( x \)-coordinate of the point at which

gradient of the curve is \( \frac{1}{3} \). [2]

Draw a line with \( \text{grad} = \frac{1}{3} \) on the graph [M1]

\( x \)-coordinate = 2.3 (±0.1) [A1]
5. Given equation of a line $L$ is $3x - 2y = 8$.

(a) State the gradient of line $L$.

\[
2y = 3x - 8
\]
\[
y = \frac{3}{2}x - 4
\]
Gradient $= \frac{3}{2}$ \[B1\]

(b) If $P \ (k, -2)$ is a point on the line $L$, find the value of $k$.

\[
3k - 2(-2) = 8
\]
\[
k = \frac{4}{3} \quad \text{[B1]}
\]

(c) Find the equation of another line that is parallel to the $x$-axis and passes through $P$.

\[
y = -2 \quad \text{[B1]}
\]

(d) Calculate the perpendicular distance from the origin, $O$, to the line $L$.

\[
3x - 2y = 8
\]
When $x = 0, \quad y = -4 \quad (0, -4)$
When $y = 0, \quad x = \frac{8}{3} \quad \frac{8}{3}, 0$ \[M1\] finding relevant coordinates

Distance between 2 points
\[
= \sqrt{\left(\frac{8}{3}\right)^2 + (-4)^2} \quad \text{[M1]}
\]
\[
= 4.8074 \text{ units}
\]

Let the perpendicular distance be $h$.
\[
\frac{1}{2} (4) \left(\frac{8}{3}\right) = \frac{1}{2} (h)(4.8074) \quad \text{[M1] suitable method to find $h$}
\]

\[
h = 2.22 \text{ units} \quad \text{[A1] accept $h = 2.21$ units}
\]
In the diagram, $\overrightarrow{OA} = a$, $\overrightarrow{OB} = 6b$ and $\overrightarrow{OC} = \frac{1}{3} \overrightarrow{OC}$.

$D$ is a point on $AB$ such that $3\overrightarrow{AD} = 2\overrightarrow{DB}$ and $E$ is a point on $BC$ such that $\overrightarrow{CE} : \overrightarrow{EB} = 4 : 5$.

(a) Express, as simply as possible, in terms of $a$ and $b$,

(i) $\overrightarrow{BA}$,

\[
\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = a - 6b
\]

(ii) $\overrightarrow{OD}$,

\[
\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OA} - \frac{2}{5} \overrightarrow{BA} = a - \frac{2}{5} (a - 6b) = \frac{3}{5} (a + 4b)
\]
(iii) \( \overline{CB} \),

\[
\overline{CB} = \overline{OB} - \overline{OC} \\
= 6b - 3a \\
= 3(2b - a)
\]

(iv) \( \overline{AE} \),

\[
\overline{AE} = \overline{AC} + \overline{CE} \\
= 2a + \frac{4}{9} \overline{CB} \\
= 2a + \frac{4}{9}(6b - 3a) \\
= \frac{2}{3}(a + 4b)
\]

(b) Write down the relationship between \( OD \) and \( AE \). Explain your answer.

\[
\overline{OD} = \frac{3}{5}(a + 4b) \\
\overline{AE} = \frac{2}{3}(a + 4b)
\]

Since \( \overline{OD} \) and \( \overline{AE} \) are parallel lines, \( M1 \) – Relationship between \( OD \) and \( AE \)

\( OD \) and \( AE \) are parallel lines. \( A1 \) – Parallel lines
(c) Find the ratio of

(i) \[ \frac{\text{area of triangle CAE}}{\text{area of triangle AOD}} = \frac{\frac{1}{2} (CA)(AE) \sin \theta}{\frac{1}{2} (OA)(OD) \sin \theta} = \frac{AE}{OD} \times \frac{CA}{OA} = \frac{10}{9} \times \frac{2}{1} = \frac{20}{9} \]

\[20:9\] \\
\[\text{AI}\]

(ii) \[ \frac{\text{area of triangle CAE}}{\text{area of triangle AOB}} = \frac{\frac{1}{2} (h)(AD)}{\frac{1}{2} (h)(AB)} \times \frac{\text{area of triangle AOD}}{\text{area of triangle AOB}} = \frac{20}{9} \times \frac{2}{5} = \frac{8}{9} \]

\[8:9\] \\
\[\text{AI}\]
7. (a) There are two boxes of sweets containing toffees and chocolates. Box A contains 8 toffees and 4 chocolates, whereas box B contains 7 toffees and 3 chocolates.

Jolin loves chocolates.
One of the boxes is chosen at random and a sweet is taken out.
If she gets a chocolate, she will consume it.
If she did not get a chocolate from the first selection, she will place the sweet into the other box and she will select again from the other box.
Jolin limits herself to two selections.

Find, as a fraction in its simplest form, the probability that
(i) Jolin will have her favourite chocolate from the first selection,

\[
P(\text{first selection}) = P(\text{Box A}) + P(\text{Box B})
\]
\[
= \left(\frac{1}{2}\right)\left(\frac{4}{12}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{10}\right)
\]
\[
= \frac{19}{60}
\]

(ii) Jolin will have her favourite chocolate.

\[
P(\text{favourite}) = P(\text{at first}) + P(\text{at second})
\]
\[
= \frac{19}{60} + P(\text{no A, yes B}) + P(\text{yes A, no B})
\]
\[
= \frac{19}{60} + \left(\frac{1}{2}\right)\left(\frac{8}{12}\right)\left(\frac{3}{11}\right) + \left(\frac{1}{2}\right)\left(\frac{7}{10}\right)\left(\frac{4}{13}\right)
\]
\[
= \frac{4421}{8580}
\]
(b) Your friend, Kenton gives you a chance to win $1000 by playing a game of “Guess the number”. There are two options of the game that he allows you to choose.

**Option A**
He uses a random number generator to choose a number from 1 to 8.
If you guess it correctly, you win.

**Option B**
You flip a fair coin.
If the coin lands on head, Kenton will roll a fair 6-sided die. If you manage to guess what it rolled, you win.
If the coin lands on tail, Kenton will use a random number generator to choose a random number from 1 to 8. If you guess the number correctly, you win.

Which option will you choose? Explain your answer.

<table>
<thead>
<tr>
<th>Option A</th>
<th>Option B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(\text{win}) = \frac{1}{8}$</td>
<td>$P(\text{win}) = P(\text{H, die}) + P(\text{T, No, Gen})$</td>
</tr>
<tr>
<td></td>
<td>$= \frac{1}{2} \cdot \frac{1}{6} + \frac{1}{2} \cdot \frac{1}{8}$</td>
</tr>
<tr>
<td></td>
<td>$= \frac{7}{48}$</td>
</tr>
</tbody>
</table>

Since **Option B** has a higher probability, **Option B** should be chosen.
A regular hexagon forms within the inner circle $ABCDEF$ with $OB = 5$ cm. This information is illustrated in Figure 3.

![Figure 3](image)

(a) Show that the radius of the outer circle is 13.31 cm, correct to 2 decimal places.

Let $M$ be the midpoint of $AB$.
Consider $\triangle AGX$,
Using Cosine Rule,

$$AG^2 = 18^2 + 18^2 - 2(18)(18) \cos \frac{\pi}{6}$$

$$AG = 9.31749 \text{ cm}$$

Consider $\triangle AMO$,

$$\angle AMO = 360^\circ / 12 = 30^\circ$$

$$AM = 5 \sin 30^\circ$$

$$MO = 5 \cos 30^\circ$$

Consider $\triangle AGM$,

By Pythagoras theorem,

$$GM = \sqrt{AG^2 - AM^2}$$

$$= \sqrt{9.31749^2 - (5 \sin 30^\circ)^2}$$

$$= 8.9758 \text{ cm}$$

Radius $= GM + MG$
(b) Team H8's team decided to make 50 dreamcatcher-inspired ornaments and Janice chanced upon the following promotion.

```
10 m silver craft bendable DIY wire (2mm thick)
$9.28
```

What is the estimated cost to make these ornaments?

\[
\text{arc } AG = (18) \left(\frac{\pi}{4}\right) \\
= 3\pi \text{ cm} \\
\text{Petals} = 12 \times 3\pi \\
= 36\pi \text{ cm} \\
\text{Circumference of small circle} = 2\pi(5) \\
= 10\pi \\
\text{Circumference of big circle} = 2\pi(13.30597) \\
= 26.619\pi \\
\text{Amount of wire for 50 dreamcatchers} \\
= (26.619\pi + 10\pi + 36\pi) \times 50 \\
= 3630.59\pi \text{ cm} \\
= 36.3059\pi \text{ m} \\
\text{Rolls of wire needed} \\
= \frac{36.3059\pi}{10} \\
= 11.4058 \\
= 12 \text{ (rounded up)} \\
\text{Cost} = 12 \times \$9.28
\]

\[= \$111.36 \text{ (to 2dp)}\]
Find the area of the shaded region as shown in Figure 1.

\[
\text{area of } \Delta AGX = \frac{1}{2} (8.9758)(5 \sin 30^\circ) = 11.21975 \text{ cm}^2
\]

\[
\text{area of } \Delta OXA = \frac{1}{2} (5 \cos 30^\circ)(5 \sin 30^\circ) = 5.41266 \text{ cm}^2
\]

\[
\text{area of shaded region} = \pi (13.30597)^2 - \frac{\pi}{6} (18)(18) \sin \frac{\pi}{6} - 3.823 \text{ cm}^2
\]

\[
= 310.7504 \text{ cm}^2
\]

\[
= 311 \text{ cm}^2
\]
9. (a) The cumulative frequency curve below shows the time spent in minutes by a group of 80 teenagers on Instagram (a social media platform) on a particular day.

Use the curve to estimate

(i) the median.

\[
\frac{N}{2} = 40 \\
\text{median} = 162.5 \text{ min}
\]

(ii) the interquartile range.

\[
\frac{N}{4} = 20 \quad \frac{3N}{4} = 60
\]

Q1 = 125 min \hspace{1cm} Q3 = 200 min

\[
\text{Interquartile Range} = Q3 - Q1
\]

\[
= 200 - 125 \\
= 75 \text{ min}
\]
(b) Through a market research, it was found out that the time spent on Facebook (another social media platform) is less popular and less consistent among the same group of 80 teenagers. A second cumulative frequency curve for the same group of 80 teenagers spending their time on Facebook is drawn. Describe how the second cumulative frequency curve may differ from the curve for Instagram.

The curve will shift to the left
and has a gentler slope
as compared to the given curve.

(c) The box-and-whisker plot represents the distribution of the time spent for the same group of 80 teenagers on Snapchat (another social media platform).

For this group of 80 teenagers, which of the social media platforms - Instagram, Facebook or Snapchat, is the most popular? Support your answer with an appropriate statistical measure.

<table>
<thead>
<tr>
<th>Median (Instagram)</th>
<th>Median (Facebook)</th>
<th>Median (SnapChat)</th>
</tr>
</thead>
<tbody>
<tr>
<td>162.5 min</td>
<td>&lt; 162.5 min</td>
<td>225 min</td>
</tr>
</tbody>
</table>

Since Snapchat has the highest median, it is the most popular.

*If students show no data evidence but mentioned about highest median, award 1 mark.
10. Ms Tan got her new home recently and the layout of her house is shown in the diagram below. The layout is drawn to scale.

(a) Express the scale of the map in the form of 1:n

1 cm = 1 m
1 cm = 100 cm

1:100
(b) Ms Tan decides to get a television set for her living room.

She found the following infographic online.

![Viewing distance guide](image)

Viewing distance guide: linking screen sizes to how far you sit from the TV

<table>
<thead>
<tr>
<th>Viewing Distance</th>
<th>Screen Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 1.5m</td>
<td>Up to 32 inch</td>
</tr>
<tr>
<td>1.5 to 2m</td>
<td>32 to 39 inch</td>
</tr>
<tr>
<td>2 to 2.5m</td>
<td>40 to 45 inch</td>
</tr>
<tr>
<td>2.5 to 3m</td>
<td>46 to 56 inch</td>
</tr>
<tr>
<td>Over 3m</td>
<td>Over 56 inch</td>
</tr>
</tbody>
</table>

What is the range of television size which Ms Tan should get for her living room?

Distance of sofa from TV = 3 cm × 100 cm
= 3 m

Range = 46 to 56 inch
Ms Tan decided to shop for her television set online and she saw the following options:

Ms Tan pays a downpayment of $2,000 for her television set as suggested in (b). She pays the remaining amount over 3 months, with a simple interest rate of 7% per annum.

Calculate her monthly instalment.

Ms Tan to purchase 55 inch TV.

Remaining amount = $2,988 – $2,000

= $988

\[
\text{Interest} = \frac{988 \times 7 \times \frac{3}{12}}{100}
\]

= $17.29

Monthly instalment = \(\frac{988 + 17.29}{3}\)

= $335.10 (to 2dp)
ST. MARGARET’S SECONDARY SCHOOL
Mid-Year Examinations 2019

MATHEMATICS
4048/01

Paper 1
13 May 2019
Secondary 4 Express
2 hours

Additional Materials: Nil

READ THESE INSTRUCTIONS FIRST

Write your name, registration number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.
Mathematical Formulae

Compound Interest

Total amount = \( P \left(1 + \frac{r}{100}\right)^n \)

Mensuration

Curved surface area of cone = \( \pi rl \)

Surface area of a sphere = \( 4\pi r^2 \)

Volume of cone = \( \frac{1}{3} \pi r^2 h \)

Volume of sphere = \( \frac{4}{3} \pi r^3 \)

Area of triangle \( ABC = \frac{1}{2} ab \sin C \)

Arc length = \( r\theta \), where \( \theta \) is in radians

Sector area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

Trigonometry

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

Statistics

\[ \text{Mean} = \frac{\sum fx}{\sum f} \]

\[ \text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} \]
1  (a) Expand and simplify $(4x - 1)^2 - (8x + 1)(2x - 1)$.

Answer (a)___________________ [2]

(b) Simplify $\frac{(2x^2)^3}{5\sqrt{x}} \times 4x^{-2}$, giving your answer in the form $ax^n$, where $a$ and $n$ are rational numbers.

Answer (b)___________________ [2]

2  (a) The area of Singapore is about 710 km$^2$. Express the area in square metres, giving your answer in standard form.

Answer (a) __________________ [2]

(b) $2019.04 = 2 \times 10^3 + 1 \times 10 + 9 \times 10^m + 4 \times 10^n$, where $m$ and $n$ are integers.

Write down the value of $m$ and $n$.

Answer (b) $m = ___________, n = ___________ [2]
3  (a) Solve \( \frac{2}{m^2} - 3 = \frac{1}{m} \). 

Answer (a) ____________________ [2] 

(b) Hence solve \( \frac{2}{(3y-1)^2} - 3 = \frac{1}{(3y-1)} \). 

Answer (b) ____________________ [2] 

4  An interior angle of a regular hexagon is three times the size of the exterior angle of another \( n \)-sided regular polygon, Find the value of \( n \). 

Answer ______________________ [2]
The number 2200, written as the product of their prime factors, are $2^3 \times 5^2 \times 11$.

(a) Express 5880 as the product of its prime factors.

(b) Hence write down the greatest integer that will divide both 2200 and 5880 exactly.

(c) Write down the smallest integer $k$, such that $\sqrt{\frac{2200}{k}}$ will give a whole number.

Answer (a)____________________[2]

Answer (b)____________________[1]

Answer (c)____________________[1]
Four points lie on a line segment such that \( AB : BC = 1 : 2 \) and \( BC : CD = 8 : 5 \).

(a) Find \( AB : BD \).

Answer (a)____________________[1]

(b) If \( A \) represents the number \(-7\) and \( D \) represents the number 98.4, find the number that is represented by \( C \).

Answer (c)____________________[2]

7 (a) Factorise \( 4x^2 - 28xy + 49y^2 \) completely.

Answer (a)____________________[2]

(b) Given that \( 4x^2 - 28xy + 49y^2 = 0 \), express \( x \) in terms of \( y \).

Answer (b)____________________[1]
The number of days some students were late for school was recorded in the table as follows.

<table>
<thead>
<tr>
<th>Number of days late</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students</td>
<td>13</td>
<td>9</td>
<td>7</td>
<td>x</td>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

(a) Write down the largest possible value of $x$ if the mode is 0 days.

Answer (a)____________________[1]

(b) Find the smallest possible value of $x$ if the median is 3 days.

Answer (b)____________________[1]

(c) Find the value of $x$ if mean is 2.

Answer (c)____________________[2]

An open field has an area of 112.5 km². It is represented by an area of 18 cm² on map $X$.

(a) Find the scale of the map in the form $1 : n$.

Answer (a)____________________[2]

(b) Map $Y$ has a scale of $1 : 400 000$. A road is measured 2.4 cm on Map $X$.

Find, in centimetres, the length representing this road on Map $Y$.

Answer (b)____________________[2]
10  The volume of cylinder $A$ of radius $r$ cm and height $h$ cm is 360 cm$^3$.

(a) Find the volume of cylinder $B$ of radius $2r$ and height $\frac{1}{3}h$ cm.

Answer (a) ______________________ [2]

(b) Cylinder $C$ is similar to cylinder $A$. If the radius of cylinder $C$ is $0.5r$ cm, find the volume of cylinder $C$.

Answer (b) ______________________ [2]

11  (a) The sketch represents the graph of $y = x^n$. Write down a possible value of $n$.

Answer (a) ______________________ [1]
(b) Write down a possible equation for this graph.

\[ y = x^2 \]

Answer (b) __________________ [1]

12 (a) On the Venn diagram, shade the set \( A \cap B' \).

\( \mathbb{H} = \{x : 0 < x \leq 30\} \)
\( M = \{x : x \text{ is a perfect cube}\} \)
\( N = \{x : x = 2k + 1, k \text{ integer}\} \)

Find \( \mathbb{H} \cap M \).

Answer ______________________ [1]

(b) \( \mathbb{H} = \{x : 0 < x \leq 30\} \)
\( M = \{x : x \text{ is a perfect cube}\} \)
\( N = \{x : x = 2k + 1, k \text{ integer}\} \)

Find \( M \cap N \).

Answer ______________________ [2]
13 A bookshelf contains 20 fiction and 5 non-fiction books.

(a) Write down the probability that a book drawn at random from the bookshelf will be non-fiction.

Answer (a)____________________[1]

(b) Given that $p$ non-fiction books are added to the bookshelf such that the probability of drawing a non-fiction book is 0.5, find the value of $p$.

Answer (b)____________________[2]

14 Regina exercises on Tuesdays and Fridays.

On Tuesdays, she jogs for 40 minutes and sprints for 15 minutes.
On Fridays, she jogs 10 more minutes and sprints 5 minutes less.

This information can be represented by the matrix $R = \begin{pmatrix} 40 & 15 \\ 10 & -5 \end{pmatrix}$.

(a) Regina’s jogging speed is 8 km/h and her sprinting speed is 10 km/h.
Represent these speeds in a 2×1 column matrix $S$.

Answer (a) $S = \underline{\hphantom{100}}$[1]

(b) Evaluate the matrix $P = \frac{1}{60} RS$.

Answer (b) $P = \underline{\hphantom{100}}$[2]

(c) What is the distance covered on Fridays?

Answer (c)____________________[2]
15 Theresa weighed 7 oranges in a supermarket.
   The mean mass of the oranges was 138 grams.
   The standard deviation of the masses of the oranges was 4.29 grams.
   The scales in the supermarket was faulty.
   The correct mass of each orange was 15 grams more than what Theresa has recorded.
   Write down the correct values for the mean and standard deviation of each orange.

Answer
   mean = _______________ [1]
   Standard Deviation = _______________ [1]

16 The number of goals scored by 2 soccer players, Fati and Bala, in 6 matches are listed below.

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>1</th>
<th>3</th>
<th>2</th>
<th>3</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fati</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Bala</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

One of them has to be selected for the national team. Who should be selected?
Justify your answer with clear working.

Answer
   ____________________________________________ [2]
17 Given that \( \mathbf{u} = \begin{pmatrix} 8 \\ -2 \end{pmatrix} \), \( \mathbf{v} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} \) and \( \mathbf{w} = \begin{pmatrix} 16 \\ p \end{pmatrix} \). Find

(a) \( |\mathbf{u} - \mathbf{v}| \).

Answer (a) ______________________ [2]

(b) \( 2\mathbf{v} + \mathbf{u} \).

Answer (b)________________________ [1]

18 The force (F units) between two particles is inversely proportional to the square of the distance (x units) between them.

When the distance between two particles is \( x \), the force is \( F \). When the distance is reduced to 0.5\( x \), what is the ratio of the force to the original force?

Answer __________________________ [2]

SMSS 2019
The point $R$ is such that $\overrightarrow{QR} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$. It is given that $\overrightarrow{RS} = \begin{pmatrix} -12 \\ h \end{pmatrix}$.

Find the two possible values of $h$ which will make $PQRS$ a trapezium.

$Answer$ ____________________ [2]
20 (a) Solve the linear inequalities \( \frac{4}{3}x - 14 \leq 3(x - 4) < 7.5 \) and represent their solution on the number line provided. [3]

(b) Hence, write down the smallest integer which satisfies \( \frac{4}{3}x - 14 \leq 3(x - 4) < 7.5 \).

Answer (b) __________________ [1]

21 A man invested $7600 in a fund that pays 2.8% compound interest per annum, compounded every half-yearly. Calculate the total amount of money he has at the end of 4 years, giving your answer correct to the nearest 10 cents.

Answer ____________________ [2]
The letters of the word ‘RESILIENCE PACKAGE’ are pasted onto 17 plastic balls, one letter on each ball and they are then put into a box. A ball is drawn at random. If it is a ball with a consonant, it is put back into the box and a new ball is drawn. If it is a ball with a vowel, it is NOT put back into the box and a second ball is then drawn.

(a) Complete the tree diagram.
(b) Calculate the probability that two vowels are drawn.

\[ \text{Answer (b)} \] 

23 Ada draws this graph to show the happiness index of her country for the last 4 years.

\[ \text{Annual Happiness Index} \]

State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

\[ \text{Answer} \]

\[ \text{____________________________________________________________________} \]

\[ \text{____________________________________________________________________} \]

\[ \text{____________________________________________________________________} \]

\[ \text{____________________________________________________________________} \]

\[ \text{____________________________________________________________________} \] [2]
24 In the triangle $ABC$, $D$ and $E$ are points on $AB$ and $AC$ respectively such that 

\[ \angle ADE = \angle ACB. \]

(a) Show that triangle $AED$ and triangle $ABC$ are similar.

___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________

[2]

Given that $AD = 8$ cm, $DB = 4$ cm, $AE = 6$ cm and $EC = x$ cm, find

(b) the value of $x$,

Answer (b) ______________________[1]

(c) the area of triangle $ADE : area$ of quadrilateral $BCED$.

Answer (c) ______________________ [2]
25 (a) Express \( y = x^2 - 6x + 4 \) in the form \( y = (x - a)^2 + b \).

Answer (a) ___________________ [2]

(b) Hence sketch the curve \( y = x^2 - 6x + 4 \), indicating clearly, the points of intersection with the \( y \) axis and the turning point. [2]
ST. MARGARET’S SECONDARY SCHOOL
Mid-Year Examinations 2019

CANDIDATE NAME

CLASS

REGISTER NUMBER

MATHEMATICS 4048/02

Paper 2 6 May 2019

Secondary 4 Express 2 hours 30 minutes

Additional Materials: Writing papers
Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, registration number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give
the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in
terms of \( \pi \).

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

This document consists of 22 printed pages
Mathematical Formulae

Compound Interest

Total amount = \( P \left( 1 + \frac{r}{100} \right)^n \)

Mensuration

Curved surface area of a cone = \( \pi rl \)

Surface area of a sphere = \( 4\pi r^2 \)

Volume of a cone = \( \frac{1}{3} \pi r^2 h \)

Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

Area of triangle \( ABC = \frac{1}{2} ab \sin C \)

Arc length = \( r \theta \), where \( \theta \) is in radians

Sector area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

Trigonometry

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\( a^2 = b^2 + c^2 - 2bc \cos A \)

Statistics

Mean = \( \frac{\sum fx}{\sum f} \)

Standard deviation = \( \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2} \)
1 The graph \( y = (x - 1)(x + 3) \) cuts the \( x \)-axis at \( A \) and \( B \) and the \( y \)-axis at \( C \).

(a) Find the coordinates of \( C \). \([1]\)

(b) Find the coordinates of \( M \), the minimum point of the curve. \([2]\)

Hence,

(c) (i) find the equation of the line \( MC \), and \([2]\)

(ii) the length of the line joining \( M \) to \( C \). \([2]\)
2 (a) Factorise completely

[2]

(b) Express \( \frac{y-1}{y^2-1} - \frac{2}{1-y} \) as a single fraction in its simplest form. [3]
(c) Given that $b = a + \frac{c}{x}$ and $d = \frac{c}{b}$, express $x$ in terms of $a$, $c$ and $d$. [3]
3. In the diagram, the circle of radius \( x \) cm with centre \( O \) is touching the sides of the right angled triangle \( ABC \) at \( D, E, F \), where angle \( ACB \) is 90°.

\[ \text{[Diagram]} \]

(a) Find the length of \( CE \) in terms of \( x \). State the circle properties used in your working. \([2] \]

(b) Given also that \( AD = 7 \) cm and \( BD = 5 \) cm, form an equation in \( x \), and show that it simplifies to \( x^2 + 12x - 35 = 0 \). \([3] \]
(c) Solve the equation \( x^2 + 12x - 35 = 0 \), giving both answers correct to 2 decimal places. [3]

(d) Hence find the shaded area. [2]
4 toothpicks are arranged to form a square. The diagram shows the first three of a sequence of figures that are formed. All the squares are of the same size.

![Figures 1 to 3 showing arranged toothpicks forming squares](image)

<table>
<thead>
<tr>
<th>Figure Number (n)</th>
<th>Number of vertical toothpicks (V)</th>
<th>Total number of toothpicks (S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>18</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>7</td>
<td>v</td>
<td>s</td>
</tr>
</tbody>
</table>

The number of vertical sides (V) and the total number of sides (S) are recorded in the table above.

(a) Find the value of v and of s. [2]

(b) Write down a formula that shows the relationship between S and n. [1]
(c) If the total number of sides is 700, find the value of \( n \). [2]

(d) Give a reason why the value of \( S \) cannot be a prime number. [1]
5 In the diagram, \( DB \) is a diameter of the circle and \( ST \) is a tangent to the circle at \( A \). \( \angle BDA = 2x \), \( \angle BDC = x \), \( \angle CBD = 7y \) and \( \angle DBA = 5y \).

(a) Explain why angle \( BCD = 90^\circ \). [1]

(b) Find the value of \( x \) and of \( y \). [4]
(c) Given that $M$ is the mid-point of $BD$,

(i) determine if $\angle MAT$ is a right angle. Explain your answer. \hspace{1cm} [1]

(ii) Find $\angle DMA$. \hspace{1cm} [2]
6 (a) Find the values of the unknown in each of the following.

(b) If \( \mathbf{P} = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \) and \( \mathbf{Q} = \begin{pmatrix} -2 & 1 \\ 5 & 9 \end{pmatrix} \), find the matrix \( \mathbf{P}^2 + 3\mathbf{Q} \).
(c) The following table shows the working hours of 3 clerks in a week.

<table>
<thead>
<tr>
<th></th>
<th>Regular Working Hours</th>
<th>Overtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fatimah</td>
<td>40</td>
<td>4</td>
</tr>
<tr>
<td>Vani</td>
<td>45</td>
<td>3</td>
</tr>
<tr>
<td>June</td>
<td>38</td>
<td>7</td>
</tr>
</tbody>
</table>

The hourly wages of these clerks are $20 for regular working hours and $30 for overtime.

(i) Represent the working hours of the clerks by a matrix A and the hourly wage rates by a matrix B such that AB exists. [2]

(ii) Find AB. [2]

(iii) Explain what your answer in (ii) represents. [1]

The elements in AB represent the wages earned by Fatimah, Vani and June respectively.

(iv) The hourly wage for regular working hours increased by 20% and overtime increased by 15% respectively, but the working hours remain the same. Calculate the increase in the company’s expense in wages paid to the three clerks using matrix multiplication. [3]
In the above diagram, $C$ represents the foot of a vertical tower $CT$. The points $A$, $B$ and $C$ are on horizontal ground, where angle $CAB = 38^\circ$, $AC = 60$ m and $BC = 45$ m. Given that the angle of elevation of $T$ from $A$ is $25^\circ$, calculate

(i) the height of the tower, [2]

(ii) angle $ABC$, [2]

(iii) the angle of depression of $B$ from $T$. [2]
A ship sails 7 km from $P$ to $Q$. It then sails 5 km from $Q$ to $R$ on a bearing of 070°. Given that angle $PQR = 145°$, calculate

(i) the bearing of $Q$ from $P$, [2]

(ii) how far $Q$ is east of $P$, [2]

(iii) the distance $PR$. [2]
8 (a) The box-and-whisker plot below shows the distribution of the ages (in years) of 50 members from the Rainbow Country Club.

(i) State the median and find the interquartile range. [3]

(ii) The table below shows the ages of 50 members from Sunshine Country Club.

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Members</td>
<td>8</td>
<td>7</td>
<td>15</td>
<td>8</td>
<td>6</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>
(iii) Sunshine Club claims that their members are generally younger than those from Rainbow Club. Do you agree? Give a reason for your answer. [1]

(iv) Which country club has a wider spread of ages among its members? Give a reason for your answer. [1]
8 (b) A school has two gardens, Famous Garden and Eco-Garden. The heights of 120 plants from the Eco-Garden were measured. The cumulative frequency diagram below shows the distribution of these heights.

Use the graph to estimate the

(i) 40th percentile,
(ii) percentage of the plants whose height is at least 45 cm. [2]

The heights of 120 plants from the Famous Garden were also measured. It was found that the height measured has the same median but a smaller interquartile range compared to that of the Eco-Garden.

(iii) Describe how the cumulative frequency curve of the Famous Garden may differ from that of the Eco-Garden given above. [2]
There are 120 commercial buildings in the Sunflower City. A commercial building generates solid waste at a rate of 2.75 m$^3$ per day.

The solid waste of commercial buildings to that of the entire city is in the ratio 4 : 10.

The Seashell landfill has a capacity of $1.2 \times 10^7$ m$^3$. Currently, 50% of the landfill is filled.

(a) Calculate the volume of solid waste the city generates per week. (Assume 5 working days in a week) \[2\]

(b) Burning the solid waste in incinerators reduces its volume by 90%. However, only 60% of the solid waste can be burned. Determine how long more, in years, will the Seashell landfill be completely filled. \[6\]
(c) Norman commented that incineration is a good way of getting rid of rubbish. Do you agree? Give a reason for your answer. [2]
10 Answer the whole of this question on a sheet of graph paper.

The following is a table of values for the graph of $y = x + \frac{6}{x}$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>7</td>
<td>5.5</td>
<td>5</td>
<td>4.9</td>
<td>5</td>
<td>$p$</td>
<td>6.2</td>
<td>7</td>
<td>8.8</td>
</tr>
</tbody>
</table>

(a) Calculate the value of $p$. \[1\]

(b) Using a scale of 2 cm to 1 unit on each axis, draw the graph of $y = x + \frac{6}{x}$ for $1 \leq x \leq 8$. \[3\]

(c) Use your graph to find the values of $x$ for which $y = 6.5$. \[2\]

(d) By drawing a suitable tangent to your curve, find the coordinates of the point at which the gradient of the tangent is equal to $-2$. \[2\]

(e) By drawing another suitable line on the same axes, use your graph to find the solutions of the equation $\frac{3x}{2} + \frac{6}{x} = 7$. \[3\]
Answer:

1(a) \(-2x + 2\)  
(b) \(\frac{2}{5}x^{\frac{15}{2}}\)

2(a) \(7.1 \times 10^8 \text{ m}^2\)  
(b) \(m = 0, n = -2\)

3(a) \(\frac{2}{3}, -1\)  
(b) \(\frac{5}{9}, 0\)

4 9

5(a) \(2^3 \times 3 \times 5 \times 7^2\)  
(b) 40  
(c) 22

6(a) \(4 : 13\)  
(b) 67.4

7(a) \((2x - 7y)^2\)  
(b) \(x = \frac{7y}{2}\)

8(a) 12  
(b) 18  
(c) 7

9(a) \(1 : 250\ 000\)  
(b) 1.5 cm

10(a) 480 cm\(^3\)  
(b) 45 cm\(^3\)

11(a) any negative odd integer  
(b) any exponential equation

12(a)

(b) \{1, 27\}

13(a) \(\frac{1}{5}\)  
(b) 15

14(a) \(\begin{pmatrix} 8 \\ 10 \end{pmatrix}\)  
(b) \(\begin{pmatrix} 47 \\ 6 \\ 1 \\ 2 \end{pmatrix}\)  
(c) \(8 \frac{1}{3} \text{ km}\)

15 Mean = 153 g, SD = 4.29 g

16 SD of Fati = 0.816, SD of Bala = 1.73, Mean of Fati = Mean of Bala = 2

Select Fati as he is more consistent in his score.
17(a) 10 units  (b) \[ \binom{12}{10} \]

18  4 : 1

19  10, –4

20(a)

\[ \begin{array}{c}
\text{Point} \\
\text{1.2} \\
\text{6.5}
\end{array} \]

(b)  –1

21  $8494.10$

22(a) \[ \frac{9}{17}, \frac{8}{17}, \frac{9}{17}, \frac{8}{17}, \frac{9}{16}, \frac{7}{16} \]  (b) \[ \frac{7}{34} \]

23  Scale was not stated in the question.

Height of simile face or area of simile face could be used to represent the readings from each year which may lead to misinterpretation of data.

24(a) \( \angle EAD = \angle BAC \) (common angle)

\( \angle ADE = \angle ACB \) (given)

Triangle \( ADE \) is similar to triangle \( ACB \) (\( AA \) similarity test)

(b) 10  (c) 1 : 3

25(a) \((x - 3)^2 \)  5  (b)

\( (3, -5) \)
ST. MARGARET’S SECONDARY SCHOOL
Mid-Year Examinations 2019

CANDIDATE NAME

CLASS

REGISTER NUMBER

MATHMATICS

4048/02

Paper 2

6 May 2019

Secondary 4 Express

2 hours 30 minutes

Additional Materials: Writing papers
Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, registration number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give
the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in
terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.
Mathematical Formulae

Compound Interest

Total amount = \( P \left(1 + \frac{r}{100}\right)^n \)

Mensuration

Curved surface area of a cone = \( \pi rl \)

Surface area of a sphere = \( 4\pi r^2 \)

Volume of a cone = \( \frac{1}{3} \pi r^2 h \)

Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

Area of triangle \( ABC = \frac{1}{2} ab \sin C \)

Arc length = \( r \theta \), where \( \theta \) is in radians

Sector area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

Trigonometry

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

Statistics

Mean = \( \frac{\sum fx}{\sum f} \)

Standard deviation = \( \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} \)
The graph \( y = (x - 1)(x + 3) \) cuts the x-axis at \( A \) and \( B \) and the y-axis at \( C \).

(a) Find the coordinates of \( C \). \[1\]

\[ y = (x - 1)(x + 3) \]

At \( C \), \( x = 0 \),

\[ y = (0 - 1)(0 + 3) \]

\[ = -3 \]

\[ \therefore \text{Coordinates of } C \text{ are } (0, -3). \]

(b) Find the coordinates of \( M \), the minimum point of the curve. \[2\]

When \( y = 0 \), \( x = -3, 1 \)

\( x \)-coordinate of \( M = \frac{-3+1}{2} \)

\[ = -1 \]

\( y \)-coordinate of \( M = (-1 - 1)(-1 + 3) \)

\[ = -4 \]

\[ \therefore \text{Coordinates of } M \text{ are } (-1, -4) \]

Hence,

(c) (i) find the equation of the line \( MC \), and \[2\]

\[ M(-1, -4), \quad C(0, -3) \]

Gradient \( MC = \frac{-4+3}{-1-0} \)

\[ = 1 \]

Equation of \( MC : y = x - 3 \)

(ii) the length of the line joining \( M \) to \( C \). \[2\]

\[ M(-1, -4), \quad C(0, -3) \]

Length of \( MC = \sqrt{(0 + 1)^2 + (-3 + 4)^2} \)

\[ = \sqrt{2} \]

\[ = 1.41 \text{ units} \]
2 (a) Factorise completely

(i) \(2p^2 + 6p - pq - 3q\),

\[
2p^2 + 6p - pq - 3q \\
= 2p(p + 3) - q(p + 3) \\
= (p + 3)(2p - q)
\]

(ii) \(24 - 54x^2\).

\[
24 - 54x^2 \\
= 6(4 - 9x^2) \\
= 6(2 + 3x)(2 - 3x)
\]

(b) Express \(\frac{y-1}{y^2-1} - \frac{2}{1-y}\) as a single fraction in its simplest form.

\[
\frac{y - 1}{y^2 - 1} - \frac{2}{1-y} \\
= \frac{y - 1}{(y+1)(y-1)} + \frac{2}{y-1} \\
= \frac{y - 1 + 2(y + 1)}{(y+1)(y-1)} \\
= \frac{3y + 1}{(y+1)(y-1)}
\]
(c) Given that \( b = a + \frac{c}{x} \) and \( d = \frac{c}{b} \), express \( x \) in terms of \( a, c \) and \( d \).

\[
\begin{align*}
\frac{c}{d} &= a + \frac{c}{x} \\
\frac{c}{x} &= \frac{c}{d} - a \\
\frac{c}{x} &= \frac{c-ad}{d} \\
x &= \frac{cd}{c-ad}
\end{align*}
\]
3 In the diagram, the circle of radius $x$ cm with centre $O$ is touching the sides of the right angled triangle $ABC$ at $D$, $E$, $F$, where angle $ACB$ is $90^\circ$.

(a) Find the length of $CE$ in terms of $x$. State the circle properties used in your working.  

\[ \angle ACB = 90^\circ \text{ (given)} \]
\[ \Rightarrow \angle CEO = 45^\circ \text{ (tangents from external point)} \]
\[ \angle CEO = 90^\circ \text{ (tangent is perpendicular to radius)} \]
\[ \Rightarrow \angle EOC = 45^\circ \text{ (angle sum of triangle)} \]

\[ \angle CEO = \angle EOC \Rightarrow \text{triangle } CEO \text{ is isosceles} \]

Hence $CE = OE$
\[ = x \]

(b) Given also that $AD = 7$ cm and $BD = 5$ cm, form an equation in $x$, and show that it simplifies to $x^2 + 12x - 35 = 0$.  

$CF = CE$ (tangents from external point)
\[ = x \]

By Pythagoras' Theorem
\[ (5 + x)^2 + (7 + x)^2 = (5 + 7)^2 \]
\[ 25 + 10x + x^2 + 49 + 14x + x^2 = 144 \]
\[ 2x^2 + 24x - 70 = 0 \]
\[ x^2 + 12x - 35 = 0 \text{ (shown)} \]
(c) Solve the equation \(x^2 + 12x - 35 = 0\), giving both answers correct to 2 decimal places. 

\[
x^2 + 12x - 35 = 0 \\
x = \frac{-12 \pm \sqrt{12^2 - 4(1)(-35)}}{2(1)} \\
= \frac{-12 \pm \sqrt{284}}{2} \\
= 2.426 \text{ or } -14.426 \\
= 2.43 \text{ or } -14.43 \text{ (to 2 d.p.)}
\]

(d) Hence find the shaded area.

Since \(x > 0\), \(x = 2.426\) 

Shaded area = \(\frac{(5 + 2.426)(7 + 2.426)}{2} - \pi (2.426)^2\) 

= 16.5 cm\(^2\) (to 3 s.f.)
4 toothpicks are arranged to form a square. The diagram shows the first three of a sequence of figures that are formed. All the squares are of the same size.

![Image of square figures](image)

Figure 1  
Figure 2  
Figure 3

<table>
<thead>
<tr>
<th>Figure Number (n)</th>
<th>Number of vertical toothpicks (V)</th>
<th>Total number of toothpicks (S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>18</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>7</td>
<td>v</td>
<td>s</td>
</tr>
</tbody>
</table>

The number of vertical sides (V) and the total number of sides (S) are recorded in the table above.

(a) Find the value of v and of s.  \[2\]

\[
\begin{align*}
2 & \quad 5 & \quad 9 & \quad 14 & \quad 20 & \quad 27 & \quad 35 \\
3 & \quad 4 & \quad 5 & \quad 6 & \quad 7 & \quad 8 \\
4 & \quad 10 & \quad 18 & \quad 28 & \quad 40 & \quad 54 & \quad 70 \\
6 & \quad 8 & \quad 10 & \quad 12 & \quad 14 & \quad 16
\end{align*}
\]

\[v = 35, \quad s = 70\]

(b) Write down a formula that shows the relationship between S and n.  \[1\]

\[n = 1, \quad s = (1)(1 + 3) = 4; \quad n = 2, \quad s = (2)(2 + 3) = 10\]

\[s = n(n + 3)\]
(c) If the total number of sides is 700, find the value of $n$.  

\[
\begin{align*}
n(n + 3) &= 700 \\
n^2 + 3n - 700 &= 0 \\
(n + 28)(n - 25) &= 0
\end{align*}
\]

\[ n = -28 \text{ is rejected.} \quad \therefore \quad n = 25 \]

(d) Give a reason why the value of $S$ cannot be a prime number.  

Values of $S$ start from 4 and are even numbers, hence it cannot be a prime number.
In the diagram, $DB$ is a diameter of the circle and $ST$ is a tangent to the circle at $A$. 
$\angle BDA = 2x$, $\angle BDC = x$, $\angle CBD = 7y$ and $\angle DBA = 5y$.

(a) Explain why angle $BCD = 90^\circ$.  
Since $BD$ is a diameter, by “right angle in a semi-circle” property, $\angle BCD = 90^\circ$.

(b) Find the value of $x$ and of $y$.

\[ x + 2x + 7y + 5y = 180^\circ \text{ (angles in opposite segments)} \]
\[ \Rightarrow x + 4y = 60^\circ \quad (1) \]
\[ x + 7y + 90^\circ = 180^\circ \text{ (angle sum of triangle)} \]
\[ \Rightarrow x + 7y = 90^\circ \quad (2) \]

\[ (2) - (1) \quad 3y = 30^\circ \]
\[ y = 10^\circ \]

Substitute into (1) \[ x = 60^\circ - 4(10^\circ) \]
\[ = 20^\circ \]
(c) Given that \( M \) is the mid-point of \( BD \),

(i) determine if \( \angle MAT \) is a right angle. Explain your answer.  

\[ M \text{ is the mid-point of } BD \]
\[ \Rightarrow M \text{ is the centre of the circle} \]
\[ \Rightarrow MA \text{ is a radius of the circle} \]
Since \( ST \) is a tangent, by “tangent is perpendicular to radius” property
\[ \angle MAT \text{ is a right angle.} \]

(ii) Find \( \angle DMA \).  

\[ \Delta DMA \text{ is isosceles, hence} \]
\[ \angle MAD = \angle MDA \]
\[ = 2(20^\circ) \]
\[ = 40^\circ \]
\[ \angle DMA = 180^\circ - 2 \times 40^\circ \text{ (angle sum of triangle)} \]
\[ = 100^\circ \]
6 (a) Find the values of the unknown in each of the following.

\[
\begin{pmatrix} 1 & 0 & 4 \\ 5 & 2 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ a \end{pmatrix} = \begin{pmatrix} 8 \\ b \end{pmatrix}.
\]

\[
\begin{pmatrix} 1 + 0 + 4a \\ 5 + 4 - 3a \end{pmatrix} = \begin{pmatrix} 8 \\ b \end{pmatrix}
\]

1 + 4a = 8
a = 1.75

b = 5 + 4 - 3(1.75)
b = 3.75

(b) If \( P = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \) and \( Q = \begin{pmatrix} -2 & 1 \\ 5 & 9 \end{pmatrix} \), find the matrix \( P^2 + 3Q \).

\[
P^2 + 3Q
= \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} + 3 \begin{pmatrix} -2 & 1 \\ 5 & 9 \end{pmatrix}
= \begin{pmatrix} 9 & 8 \\ 16 & 17 \end{pmatrix} + \begin{pmatrix} -6 & 3 \\ 15 & 27 \end{pmatrix}
= \begin{pmatrix} 3 & 11 \\ 31 & 44 \end{pmatrix}
\]
(c) The following table shows the working hours of 3 clerks in a week.

<table>
<thead>
<tr>
<th></th>
<th>Regular Working Hours</th>
<th>Overtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fatimah</td>
<td>40</td>
<td>4</td>
</tr>
<tr>
<td>Vani</td>
<td>45</td>
<td>3</td>
</tr>
<tr>
<td>June</td>
<td>38</td>
<td>7</td>
</tr>
</tbody>
</table>

The hourly wages of these clerks are $20 for regular working hours and $30 for overtime.

(i) Represent the working hours of the clerks by a matrix $A$ and the hourly wage rates by a matrix $B$ such that $AB$ exists.  

$$A = \begin{pmatrix} 40 & 4 \\ 45 & 3 \\ 38 & 7 \end{pmatrix}, \quad B = \begin{pmatrix} 20 \\ 30 \end{pmatrix}$$

(ii) Find $AB$.  

$$AB = \begin{pmatrix} 40 & 4 \\ 45 & 3 \\ 38 & 7 \end{pmatrix} \begin{pmatrix} 20 \\ 30 \end{pmatrix} = \begin{pmatrix} 800 + 120 \\ 900 + 90 \\ 760 + 210 \end{pmatrix} = \begin{pmatrix} 920 \\ 990 \\ 970 \end{pmatrix}$$

(iii) Explain what your answer in (ii) represents.  

The elements in $AB$ represent the wages earned by Fatimah, Vani and June respectively.

(iv) The hourly wage for regular working hours increased by 20% and overtime increased by 15% respectively, but the working hours remain the same. Calculate the increase in the company’s expense in wages paid to the three clerks using matrix multiplication.  

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 40 & 4 \\ 45 & 3 \\ 38 & 7 \end{pmatrix} \begin{pmatrix} 4 \\ 4.5 \end{pmatrix} = \begin{pmatrix} 123 & 14 \end{pmatrix} \begin{pmatrix} 4 \\ 4.5 \end{pmatrix} = (555)$$

The increase in wages paid is $555.
In the above diagram, $C$ represents the foot of a vertical tower $CT$. The points $A$, $B$ and $C$ are on horizontal ground, where angle $CAB = 38^\circ$, $AC = 60$ m and $BC = 45$ m. Given that the angle of elevation of $T$ from $A$ is $25^\circ$, calculate:

(i) the height of the tower,

\[
tan 25^\circ = \frac{CT}{60}
\]

\[
CT = 60 \tan 25^\circ = 27.978 = 28.0 \text{ (to 3 s.f.)}
\]

Height of tower is 28.0 m

(ii) angle $ABC$,

\[
\frac{\sin \angle ABC}{60} = \frac{\sin 38^\circ}{45}
\]

\[
\angle ABC = \sin^{-1} \left( \frac{60 \sin 38^\circ}{45} \right) = 55.173^\circ = 55.2^\circ \text{ (to 1 d.p.)}
\]

(iii) the angle of depression of $B$ from $T$.

Let $\theta$ be the angle of depression

\[
tan \theta = \frac{27.978}{45}
\]

\[
\theta = tan^{-1} \left( \frac{27.978}{45} \right) = 31.870^\circ = 31.9^\circ \text{ (to 1 d.p.)}
\]
A ship sails 7 km from $P$ to $Q$. It then sails 5 km from $Q$ to $R$ on a bearing of $070^\circ$. Given that angle $PQR = 145^\circ$, calculate

(i) the bearing of $Q$ from $P$, \[\begin{align*}
x & = 360^\circ - 70^\circ - 145^\circ \\
& = 145^\circ
\end{align*}\]

Bearing of $Q$ from $P$
\[\begin{align*}
& = 180^\circ - 145^\circ \\
& = 035^\circ
\end{align*}\]

(ii) how far $Q$ is east of $P$, \[\begin{align*}
sin 35^\circ &= \frac{d}{7} \\
d &= 7 \sin 35^\circ \\
& = 4.01503 \\
& = 4.02 \text{ km}
\end{align*}\]

Ans: $Q$ is 4.20 km east of $P$.

(iii) the distance $PR$. \[\begin{align*}
PR^2 &= 7^2 + 5^2 - 2(7)(5) \cos 145^\circ \\
PR &= \sqrt{74 - 70 \cos 145^\circ} \\
& = 11.5 \text{ km (to 3 s.f.)}
\end{align*}\]
8 (a)  The box-and-whisker plot below shows the distribution of the ages (in years) of 50 members from the Rainbow Country Club.

![Box-and-Whisker Plot]

(i)  State the median and find the interquartile range.  

Median = 42 years  
Interquartile range = 58 – 31  
= 27 years

(ii)  The table below shows the ages of 50 members from Sunshine Country Club.

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Members</td>
<td>8</td>
<td>7</td>
<td>15</td>
<td>8</td>
<td>6</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

Find the median and the interquartile range.

Median = 40 years  
Interquartile range = 45 – 35  
= 10 years
(iii) Sunshine Club claims that their members are generally younger than those from Rainbow Club. Do you agree? Give a reason for your answer.

I agree that members of Sunshine Club are generally younger than those from Rainbow Club because the median age of Sunshine Club (40 years) is lower that the median age of Rainbow Club (42 years).

(iv) Which country club has a wider spread of ages among its members? Give a reason for your answer.

Rainbow Club has a wider spread of ages among its members because its interquartile range (27 years) is bigger/larger than that of Sunshine Club (10 years).
8 (b) A school has two gardens, Famous Garden and Eco-Garden. The heights of 120 plants from the Eco-Garden were measured. The cumulative frequency diagram below shows the distribution of these heights.

Use the graph to estimate the

(i) 40th percentile, \[1\]

\[120 \times 0.4 = 48\]

From the graph, 40th percentile = 33 cm.
(ii) percentage of the plants whose height is at least 45 cm.  

No. of plants with length at least 45 cm = 120 – 92  
= 28

Percentage of plants whose height is at least 45 cm  

\[
= \frac{28}{120} \times 100\%  
= 23\frac{1}{3}\% 
\]

The heights of 120 plants from the Famous Garden were also measured. It was found that the height measured has the same median but a smaller interquartile range compared to that of the Eco-Garden.

(iii) Describe how the cumulative frequency curve of the Famous Garden may differ from that of the Eco-Garden given above.

The cumulative frequency curve of the Famous Garden will be steeper and it will intersect the graph of the Eco-Garden at the median height.
There are 120 commercial buildings in the Sunflower City. A commercial building generates solid waste at a rate of 2.75 m$^3$ per day.

The solid waste of commercial buildings to that of the entire city is in the ratio 4 : 10.

The Seashell landfill has a capacity of $1.2 \times 10^7$ m$^3$. Currently, 50% of the landfill is filled.

(a) Calculate the volume of solid waste the city generates per week. (Assume 5 working days in a week) [2]

Volume of waste generated by a commercial building per week
$= 120 \times 5 \times 2.75$
$= 1650$ m$^3$

Volume of solid waste generated by the city per week
$= \frac{10}{4} \times 1650$
$= 4125$ m$^3$

(b) Burning the solid waste in incinerators reduces its volume by 90%. However, only 60% of the solid waste can be burned. Determine how long more, in years, will the Seashell landfill be completely filled. [6]

60% of solid waste that can be burned
$= 0.6 \times 4125$
$= 2475$ m$^3$

Remain of solid waste after incineration
$= 0.1 \times 2475$
$= 247.5$ m$^3$

40% of waste that cannot be burned
$= 0.4 \times 4125$
$= 1650$ m$^3$

50% of landfill that is not used
$= 0.5 \times 1.2 \times 10^7$
$= 6 \times 10^6$ m$^3$

No. of years left where the landfill will be completely filled
$= 6 \times 10^6 \div (1650 + 247.5) \div 52$
$= 3162.055 \div 52$
$= 60.8$ years

The landfill will be completely filled in another 60.8 years.
(c) Norman commented that incineration is a good way of getting rid of rubbish. Do you agree? Give a reason for your answer. [2]

Agree.
Reason: It is a more efficient use of space.
OR
The process of incineration can produce electricity that can be used.

Disagree.
Reason: Incineration facilities are costly.
OR
The process of incineration emits / produces hazardous pollutants that are harmful.
10 Answer the whole of this question on a sheet of graph paper.

The following is a table of values for the graph of \( y = x + \frac{6}{x} \).

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>7</td>
<td>5.5</td>
<td>5</td>
<td>4.9</td>
<td>5</td>
<td>p</td>
<td>6.2</td>
<td>7</td>
<td>8.8</td>
</tr>
</tbody>
</table>

(a) Calculate the value of \( p \). \[1\]

(b) Using a scale of 2 cm to 1 unit on each axis, draw the graph of \( y = x + \frac{6}{x} \) for \( 1 \leq x \leq 8 \). \[3\]

(c) Use your graph to find the values of \( x \) for which \( y = 6.5 \). \[2\]

(d) By drawing a suitable tangent to your curve, find the coordinates of the point at which the gradient of the tangent is equal to \(-2\). \[2\]

(e) By drawing another suitable line on the same axes, use your graph to find the solutions of the equation \( \frac{3x}{2} + \frac{6}{x} = 7 \). \[3\]
READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
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Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the
answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This paper consists of 18 printed pages (including this cover page)
Mathematical Formulae

Compound Interest

Total amount = \( P \left(1 + \frac{r}{100}\right)^n \)

Mensuration

Curved surface area of a cone = \( \pi rl \)

Surface area of a sphere = \( 4\pi r^2 \)

Volume of a cone = \( \frac{1}{3} \pi r^2 h \)

Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

Area of triangle \( ABC = \frac{1}{2} ab \sin C \)

Arc length = \( r \theta \), where \( \theta \) is in radians

Sector area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

Trigonometry

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\( a^2 = b^2 + c^2 - 2bc \cos A \)

Statistics

Mean = \( \frac{\sum fx}{\sum f} \)

Standard Deviation = \( \sqrt{\frac{\sum f x^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} \)
1. The first 4 terms of a sequence of numbers $T_1$, $T_2$, $T_3$ and $T_4$ are given below:
   $T_1 = 1^2 + 2^2 = 5$
   $T_2 = 2^2 + 3^2 = 13$
   $T_3 = 3^2 + 4^2 = 25$
   $T_4 = 4^2 + 5^2 = 41$

   (a) Write down an expression in terms of $n$, for the $n^{th}$ term $T_n$ of the sequence.

   Answer: ............................................. [1]

   (b) Explain why $T_n$ is always odd.

   Answer:
   ..........................................................
   ..........................................................
   ..........................................................
   ..........................................................

2. Simplify $\frac{a^2b}{2} \times (2a^0b^{-2})^3$, leaving your answer in positive index form.

   Answer: ............................................. [2]

   [Turn Over]
3. \( x = \frac{4}{3} \) is a solution to the equation \( 18x^2 - kx - 20 = 0 \), where \( k \) is a constant.

(a) Find the value of \( k \).

\[ \text{Answer} \quad k = \quad \text{[1]} \]

(b) Hence, find the other solution to the equation \( 18x^2 - kx - 20 = 0 \).

\[ \text{Answer} \quad \text{[1]} \]

4. Given that the size of an exterior angle of a regular polygon is one-fifth its interior angle, find the number of sides of the polygon.

\[ \text{Answer} \quad \text{[2]} \]
5. Water is poured at a constant rate into each of the containers shown below.

![Containers](image)

The graphs show the depth of water in the containers as they are being filled. Match the graphs with the appropriate container and complete the table below.

```
<table>
<thead>
<tr>
<th>Depth</th>
<th>Depth</th>
<th>Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph 1</td>
<td>Time</td>
<td>Graph 2</td>
</tr>
</tbody>
</table>
```

**Answer:**

<table>
<thead>
<tr>
<th>Graph</th>
<th>Container</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
</tr>
</tbody>
</table>

6. (a) The graph below can be represented by the equation, \( y = ax^n + b \).

State the value of \( b \), of \( n \) and the range of \( a \).

![Graph](image)

**Answer**

\( b = \ldots, n = \ldots \)  

[Turn Over]
(b) On the axes in the answer space, sketch the graph of \( y = ax^n \), where \( a < 0 \) and \( n = 2 \). [1]

7. Find the largest prime number that satisfy the inequality \( 3 + y \leq 15 - 2y < y + 10 \). [3]
8. Solve the simultaneous equations
\[
8x + 3y = 4 \\
4x = y + 4
\]

Answer \( x = \ldots \) \( y = \ldots \) [3]

9. It is given that \( p = \begin{pmatrix} 6 \\ 5 \end{pmatrix}, \ q = \begin{pmatrix} -4 \\ 3 \end{pmatrix} \) and \( r = \begin{pmatrix} 8 \\ -6 \end{pmatrix} \).

(a) Find \( |q - 2p| \).

Answer \ldots \ldots \ldots [2]

(b) Use vectors to explain whether \( r \) is parallel to \( q \).

Answer \ldots \ldots \ldots \ldots \ldots \ldots \ldots [1]

[Turn Over]
10. Simplify and express \( \frac{9m^2 - 12mn + 4n^2}{m^2 - n^2} + \frac{3m - 2n}{m - n} \) as a single fraction.

**Answer** ................................................................. [3]

11. The ground area of a scale model is 1 : 100 of the actual ground area.

(a) Given that the length of the model is 3 m, calculate the length of the house in metre.

**Answer** .................................................................m [1]

(b) Given that the actual volume of a storeroom in the house is 15 000 000 cm\(^3\), calculate the volume of the storeroom in the model in cm\(^3\).

**Answer** .................................................................cm\(^3\) [2]
12. A bag contains a total of 80 green, red and blue balls. There are 18 red balls in the bag.

(a) If a ball is picked randomly, the probability of picking a green ball is $\frac{3}{8}$. How many blue balls are there?

\[ \text{Answer} \]

(b) Two balls are picked randomly from the bag. Find the probability of

(i) picking a green ball and a red ball,

\[ \text{Answer} \]

(ii) picking at least one red ball.

\[ \text{Answer} \]
13. Find the equation of the straight line
   (a) passing through the point \((-2, 5)\) and parallel to the \(y\)-axis,

   \[ \text{Answer} \]

   \[ \text{[1]} \]

   (b) which passes through the point \((2, -5)\) and its gradient is the same as the line \(y - 3x - 5 = 0\).

   \[ \text{Answer} \]

   \[ \text{[2]} \]

14. In the diagram triangle \(ACD\) is right-angled, and \(B\) is on \(AC\) such that \(\sin \angle ABD = \frac{x}{y}\).

   Find in terms of \(x\) and \(y\).
   (a) \(\tan \angle BDC\),

   \[ \text{Answer} \]

   \[ \text{[2]} \]

   (b) \(\cos \angle ABD\).

   \[ \text{Answer} \]

   \[ \text{[1]} \]
15. (a) An insurance agent is paid a basic monthly salary of $1200 and a commission of 8% on all the sales if the monthly sales made by the insurance agent exceed $7500. The insurance agent’s salary for a particular month was $3694. Calculate the total sales made by the insurance agent.

Answer .................................................. [2]

(b) Two taps $X$ and $Y$ are turned on at the same time to fill up an empty tank. The rate at which Tap $X$ fills up the tank per minute is twice as fast as that of Tap $Y$. When used together, both taps can fill the tank with 300 cm$^3$ of water in 5 minutes. Find the rate of flow of Tap $Y$ in cm$^3$/minute.

Answer .................................................. cm$^3$/minute [2]

[Turn Over]
16. The table below shows the frequency of visits to the library by some students in Betta School in a week.

<table>
<thead>
<tr>
<th>Number of visits</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>x</td>
<td>2</td>
</tr>
</tbody>
</table>

(a) If the mode is 3, write down the smallest possible value of x.

Answer $x =$ ............................................. [1]

(b) If the median is 2, write down the largest possible value of x.

Answer $x =$ ............................................. [1]

(c) Given that the mean is $\frac{1}{22}$, calculate the value of x.

Answer $x =$ ............................................. [2]

17. The diagram shows a speed-time graph of a car. The total distance travelled in 35 s is 540 m.

(a) Calculate the maximum speed $V$ m/s.

Answer $V =$ ............................................. m/s [2]
(b) Calculate the speed of the car at 28 seconds.

Answer \[ \frac{16}{6} \text{ m/s} \] \[ \text{[2]} \]

18. The base areas of two geometrically similar pyramids are 98 cm\(^2\) and 242 cm\(^2\).

(a) Find the ratio of the height of the larger pyramid to the height of the smaller pyramid.

Answer \[ \text{[1]} \]

(b) Given the total surface area of the smaller pyramid is 343 cm\(^2\). Find the total surface area of the larger pyramid.

Answer \[ \text{[1]} \]
(c) Find the percentage difference in volumes of the two pyramids in terms of the larger pyramid.

19. (a) $s$ is directly proportional to the square of $v$. It is known that $s = 36$ for a particular value of $v$.

Find the value of $s$ when this value of $v$ is halved.

Answer $s = \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldOTS
20. Written as a product of its prime factors, 
\[ p = 3^x \times 5^y \times 7^3 \] and \[ q = 5 \times 7^2 \times 11 \]

(a) Find the smallest value of \( x \) and of \( y \) for which \( p \) is a multiple of 35.

\[ \text{Answer: } x = \ldots \ldots \ldots, \ y = \ldots \ldots \ldots \] [2]

(b) Explain why \( 55q \) is a perfect square.

\[ \text{Answer: } \]

(c) Show your workings and state, with reason(s), if the product of \( p \) and \( q \) is an odd or even number.

\[ \text{Answer: } \] [1]

[Turn Over]
21. (a) Describe the shaded region in set notation.

\[ \mathcal{E} \]

(b) Given that

\[ \xi = \{ x \text{ is a positive integer and } 0 < x < 10 \} \]

\[ A = \{ x : x \text{ is a prime number} \} \]

\[ B = \{ x : x \text{ is a factor of } 12 \} \]

\[ C = \{ x : \frac{50}{x} < 8 \} \]

(i) Find \( B \cap C \).

Answer ...................................................................... [1]

(ii) List the elements of \( (A \cup B)' \).

Answer ...................................................................... [1]

(ii) Draw a Venn diagram to illustrate the given information. [2]
22. In triangle PQR, X is a point on QR such that QX:XR is 3:2 and RS is parallel to XY.

(a) Show that triangle QXY is similar to triangle QRS.

Answer:
..............................................................
..............................................................
..............................................................
..............................................................
..............................................................

(b) Given that the area of triangles QXY and PXR are 27 cm² and 46 cm² respectively. Find the area of
(i) triangle QRS,

Answer ..............................................cm² [2]

(ii) triangle PQR.

Answer ..............................................cm² [2]

[Turn Over]
The sector $CEF$ with centre at $C$ is inscribed in the square $ABCD$ of side $4$ cm.

Given that $AF = FB$, $DE = EA$ and angle $ECF = 0.64$ radian.

(a) Calculate the perimeter of the sector $CEF$.

(b) Find the area of the shaded region.

Answer .................................................. cm $^2$ [4]
BEATTY SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2019

SUBJECT : Mathematics
PAPER : 4048 / 02
SETTER : Mr Teo CK
          Miss Chong HY

LEVEL : Sec 4 Express
        Sec 5 Normal (Academic)
DURATION : 2 hours 30 minutes
DATE : 29 August 2019

CLASS : NAME : REG NO :

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the
answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of
π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use

100

This paper consists of 23 printed pages (including this cover page)

[Turn over
Mathematical Formulae

Compound Interest

Total amount = \( P(1 + \frac{r}{100})^n \)

Mensuration

Curved surface area of a cone = \( \pi rl \)

Surface area of a sphere = \( 4\pi r^2 \)

Volume of a cone = \( \frac{1}{3}\pi r^2 h \)

Volume of a sphere = \( \frac{4}{3}\pi r^3 \)

Area of triangle \( ABC = \frac{1}{2}ab \sin C \)

Arc length = \( r\theta \), where \( \theta \) is in radians

Sector area = \( \frac{1}{2}r^2\theta \), where \( \theta \) is in radians

Trigonometry

\( \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \)

\( a^2 = b^2 + c^2 - 2bc \cos A \)

Statistics

Mean = \( \frac{\Sigma fx}{\Sigma f} \)

Standard Deviation = \( \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2} \)
Answer all the questions.

1 (a) Simplify \( \left( \frac{64y^9}{1^{12}} \right)^{\frac{1}{3}} \).

(b) Express as a single fraction in its simplest form.

(i) \( \frac{20x^2}{9} + \frac{4xy^3}{15y} \).

(ii) \( \frac{4}{3x-1} - \frac{5}{2+x} \).
(c) (i) Express \( x^2 - 5x - 8 \) in the form \( (x+a)^2 + b \). \[1\]

(ii) Hence, solve the equation \( x^2 - 5x - 8 = 0 \), giving your answers correct to two decimal places. \[3\]
2. An ice-cream shop sells 4 different flavours of ice-cream. The table below shows the number of cups of ice-cream sold by the shop over 3 days.

<table>
<thead>
<tr>
<th></th>
<th>Chocolate</th>
<th>Vanilla</th>
<th>Strawberry</th>
<th>Durian</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friday</td>
<td>252</td>
<td>168</td>
<td>84</td>
<td>105</td>
</tr>
<tr>
<td>Saturday</td>
<td>305</td>
<td>158</td>
<td>115</td>
<td>152</td>
</tr>
<tr>
<td>Sunday</td>
<td>316</td>
<td>191</td>
<td>134</td>
<td>167</td>
</tr>
</tbody>
</table>

(a) Represent the above information using a $3 \times 4$ matrix $A$.  

(b) The selling price of each cup of ice-cream is shown in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Chocolate</th>
<th>Vanilla</th>
<th>Strawberry</th>
<th>Durian</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling Price</td>
<td>$2.10</td>
<td>$1.80</td>
<td>$2.05</td>
<td>$2.55</td>
</tr>
</tbody>
</table>

Write down a matrix $P$ such that the product $AP$ represents the total sales of the ice-cream for each day respectively. Evaluate $AP$.  

[1] [2]
(c) Evaluate \( (1 \ 1 \ 1)A \) and state what the elements of the product represents. [2]

(d) The cost price of each cup of ice-cream is shown in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Chocolate</th>
<th>Vanilla</th>
<th>Strawberry</th>
<th>Durian</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cost Price</strong></td>
<td>$0.40</td>
<td>$0.45</td>
<td>$0.55</td>
<td>$0.80</td>
</tr>
</tbody>
</table>

Write down two matrices such that the elements of their product represent the total profit received by the shop over the 3 days for the sale of each flavor of ice-cream respectively.

Evaluate this product. [2]
3 (a) The diagram shows a toy which is made up of a cylindrical part and a hemispherical part.

The height of the cylindrical part is 4 cm and the radius of the hemispherical part is 6 cm.

(i) Find the radius of the cylindrical part, given that the volume of the cylindrical part is 150 cm$^3$.

(ii) Calculate total surface area of the toy.
(b) In the diagram, \(A, B, C\) and \(D\) lie on a circle with centre \(O\). Angle \(AOC = 4x^\circ\).

(i) Find angle \(ADC\), giving your answer in terms of \(x^\circ\). [2]

State a reason for your answer.

(ii) Given further that angle \(ABC = 5x^\circ + 12^\circ\), find the value of \(x\). [2]
4 (a) Construct a triangle $ABC$ such that $AB = 13\, \text{cm}$, $AC = 12\, \text{cm}$, angle $ABC = 55^\circ$, and angle $ACB$ is acute. The line $AB$ has been drawn for you. [2]

(b) Construct the angle bisector of angle $BAC$ such that it intersects the line $BC$, and label the point of intersection $D$. [2]

(c) Construct the perpendicular bisector of $AD$ such that it intersects the line $AC$, and label the point of intersection $E$. Measure and write down the value of $DE$. [2]
In the diagram, $\overline{OA} = 2a$, $\overline{OB} = b$. $BC$ is parallel to $OA$ and $BC = \frac{3}{2} \overline{OA}$. $X$ is a point on $OC$ such that $OX = \frac{2}{3} XC$. $Y$ is the midpoint of $BC$.

(a) Express in terms of $a$ and/or $b$, as simply as possible,

(i) $\overline{AB}$,

(ii) $\overline{OC}$,

(iii) $\overline{OX}$,

(iv) $\overline{AX}$.

(b) What can you deduce about the points $A$, $X$ and $B$? Justify your answer.
(c) (i) \( AY \) produced meets \( OB \) produced at a point \( Z \). Given that \( 
abla Z = h \nabla Y \), express \( \nabla Z \) in terms of \( a, b \) and \( h \). [3]

(ii) Given also that \( OZ = kOB \), express \( \nabla Z \) in terms of \( a, b \) and \( k \). [1]

(iii) Hence, show that \( h = 4 \) and \( k = 4 \). [2]

(d) Find the value of

(i) \( \frac{\text{area of } \triangle OAX}{\text{area of } \triangle OAC} \), [1]

(ii) \( \frac{\text{area of } \triangle OBX}{\text{area of } \triangle ABC} \). [1]
6. In the diagram, \( P \) and \( Q \) are the bases of two lighthouses such that \( P \) is located 40 km due north of \( Q \).

\( A \) is a boat 30 km from \( P \) and on a bearing of 055° from \( P \).

(a) Find the distance \( AQ \). [3]

(b) Find the bearing of \( A \) from \( Q \). [2]
Light from $P$ can be seen within 20 km radius of $P$, beyond which the light becomes too faint to be seen. When the boat moves from $A$ to $Q$, $X$ and $Y$ are the positions on the boat’s journey which are 20 km from $P$.

(c) Calculate

(i) the shortest distance of the boat from $P$.  

(ii) the smallest angle of elevation of the lighthouse $P$ from the boat as it travels from $X$ to $Y$, given that the height of lighthouse $P$ is 600m.
The diagram shows two roads $XY$ and $YZ$ meeting at point $Y$. The roads are perpendicular to each other. $XY = 45$ km and $YZ = 25$ km.

Cyclist $A$ is travelling from point $X$ towards point $Y$ at a constant speed of $20$ km/h. Cyclist $B$ is travelling from point $Z$ towards point $Y$ at a constant speed of $10$ km/h.

(a) Write down an expression, in terms of $t$, for the distance in kilometres

(i) between cyclist $A$ and point $Y$, after $t$ hours, [1]

(ii) between cyclist $B$ and point $Y$, after $t$ hours. [1]

(b) Form an expression, in terms of $t$, for the shortest distance, $d$, between the two cyclists and show that it reduces to $\sqrt{500t^2 - 2300t + 2650}$. Hence find the shortest distance between the two cyclists after 15 minutes. [4]
(c) The two cyclists are 10 km apart at a certain instant, $t$ hours. Form a quadratic equation in terms of $t$ and show that it reduces to $10t^2 - 46t + 51 = 0$. [2]

(d) Given that $t < 2$, find the time, in minutes, when the two cyclists are 10 km apart. Correct your answer to 3 significant figures. [3]
A scientist wanted to test the effect of different music on the growth of plants. 80 *Rosa chinensis* plants were exposed to Beethoven’s Ninth Symphony, and their growth was observed over a period of 10 days.
The cumulative frequency curve below shows the increase in the height of the plants at the end of 10 days.

**Increase in heights of plants**
(a) Find
(i) the median increase in the height of the plants,
(ii) the interquartile range,
(iii) the 90th percentile.

(b) Another group of 80 *Rosa chinensis* plants were exposed to Bach’s Goldberg Variations, and their growth were also observed over a period of 10 days. The box-and-whisker plot below shows the increase in the heights of the plants after 10 days.

![Box-and-whisker plot showing increase in height (cm)]

(i) Describe how the cumulative frequency graph of the growth of the 80 plants exposed to Bach’s Goldberg Variations will differ from that of those exposed to Beethoven’s Ninth Symphony.

(ii) Make two comparisons between the growths of the plants under the two conditions.
9 The variables $x$ and $y$ are connected by the equation $y = \frac{1}{2}x^2 + 0.1x^2 - 3$. The table below shows some values of $x$ and the corresponding values of $y$, correct to 2 decimal places.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-0.98</td>
<td>-2.40</td>
<td>-2.55</td>
<td>-2.48</td>
<td>-2.04</td>
<td>-1.37</td>
<td>$p$</td>
<td>0.61</td>
</tr>
</tbody>
</table>

(a) Find the value of $p$.

\[-0.98 + 0.5\]

(b) In the space provided on the next page, using a scale of 2 cm to represent 1 unit on the horizontal axis and 2 cm to represent 0.5 units on the vertical axis, draw the graph of

\[y = \frac{1}{2}x^2 + 0.1x^2 - 3\] for $0.5 \leq x \leq 6$.

(c) Using your graph, write down the solution(s) to the equation \[\frac{1}{2}x^2 + 0.1x^2 - 1.5 = 0\].

(d) By drawing a tangent, find the gradient of the curve at $x = 4$.

(e) By drawing a suitable straight line on the same axes, solve the equation \[\frac{1}{2}x^2 + 0.1x^2 - 0.5x = 0\].
10 A right conical container of capacity $24\pi$ cm$^3$ and vertical angle 60° is completely filled with water. The height of the container is $h$ cm and the base radius is $r$ cm.

(a) Find the value of $r$ and of $h$. [5]
The water in the container is poured into another identical container $B$ so that the depth of water in container $A$ is $\frac{1}{2}h$ cm.

(b) Find the volume of water in container $B$ in terms of $\pi$. [2]
A class of students plans to sell breakfast sets during the National Day carnival in order to raise funds for a charity.

Each breakfast set consists of 2 scrambled eggs, 2 slices of toast, 2 sausages, and a cup of coffee.

The students estimate that they will sell 250 breakfast sets.

<table>
<thead>
<tr>
<th>Item</th>
<th>Description</th>
<th>Unit cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eggs</td>
<td>Pasar Fresh Eggs (10 per pack)</td>
<td>$1.80</td>
</tr>
<tr>
<td></td>
<td>Pasar Fresh Eggs (30 per pack)</td>
<td>$4.35</td>
</tr>
<tr>
<td>Bread</td>
<td>Sunshine Enriched Soft White Bread (14 slices)</td>
<td>$1.80</td>
</tr>
<tr>
<td></td>
<td>FairPrice Wholemeal Bread (12 slices)</td>
<td>$1.55</td>
</tr>
<tr>
<td>Sausages</td>
<td>Tierney’s Chicken Hot Dog (10 per pack)</td>
<td>$5.25</td>
</tr>
<tr>
<td></td>
<td>FairPrice Sausages (6 per pack) ($0.35 off per 2 packs)</td>
<td>$3.20</td>
</tr>
<tr>
<td>Coffee</td>
<td>Nescafe 3 in 1 Instant Coffee (35 per pack) (Buy 5 get 1 free)</td>
<td>$6.15</td>
</tr>
<tr>
<td></td>
<td>Gold Roast 3 in 1 Coffee Mix (25 per pack)</td>
<td>$3.95</td>
</tr>
</tbody>
</table>

(a) Find the lowest possible total cost of the ingredients for the breakfast sets. [5]
(b) The school provides $200 in funding for the students, and up to 30% of the sales can be used to cover for their expenses, while the remaining goes to the charity. The students also wish to raise at least $600 for charity. Find the minimum they must charge for each breakfast set (to the nearest ten cents). Justify your answer, showing all necessary workings clearly. State an assumption you have made in your calculations. [4]
Beatty Secondary School

4ESN Preliminary Examination 2019

1a \( a^2 + (a + 1)^2 \) or \( 2n^2 + 2n + 1 \)

1b For any two consecutive numbers, one would be even and the other one odd. Since the square of an even number is even and the square of an odd number is odd, one of the squares of the two consecutive numbers will be even, and the other will be odd. Hence, their sum will be odd.

2 \( \frac{a^2b}{2} \times \left( \frac{2a^2b - 1}{2} \right)^2 = \frac{a^2b}{2} \times 8b^2 \) \( = \frac{4a^2}{b^3} \)

3a \( 18 \left( \frac{4}{3} \right)^2 - \left( \frac{4}{3} \right) - 20 = 0 \)

\( k = 9 \)

3b \( 18x^2 - 9x - 20 = 0 \)

\( (3x - 4)(6x + 5) = 0 \)

\( x = \frac{4}{3} \) or \( x = -\frac{5}{6} \) (the other solution)

4 \( x + 5x = 180 \)

\( x = 30 \)

\( \frac{360}{30} = 12 \)

Number of sides = 12

5 Graph Container

<table>
<thead>
<tr>
<th>1</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>C</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
</tr>
</tbody>
</table>

6a \( b = 2 \)

\( n = 1 \)

Range of \( a: a > 0 \)

7

<table>
<thead>
<tr>
<th>7</th>
<th>3 + ( y \leq 15 - 2y ) and 15 - 2y &lt; ( y + 10 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>5 \leq 3y</td>
</tr>
<tr>
<td>7</td>
<td>3y \leq 12</td>
</tr>
<tr>
<td>7</td>
<td>5 \leq y</td>
</tr>
<tr>
<td>7</td>
<td>y \geq \frac{5}{3}</td>
</tr>
</tbody>
</table>

M1 for each correct answer

8 Largest prime number is 3.

9a \( y = \frac{4x - 4}{5} \) (1)

\( \frac{4x}{5} \) (2)

From (2), \( y = 4x - 4 \) (3)

Sub (3) in (1),

\( 8x = 4(4x - 4) = 4 \)

\( 8x = 16 \)

\( x = 2 \)

\( y = \frac{4}{5} \)

M1

9b \[ \sqrt{(-16)^2 + (-7)^2} \]

\[ = \sqrt{365} \]

\[ = 17.5 \text{ units (3 s.f.)} \]

A1
9b If \( r \) is parallel to \( q \),
\[ r = kq \]
Now,
\[ \left( \begin{array}{c} 8 \\ -6 \end{array} \right) = -2 \left( \begin{array}{c} -4 \\ 3 \end{array} \right) \]
i.e. \( r = -2q \) or \( q = -\frac{1}{2} r \)
Hence, \( r \) is parallel to \( q \).

10
\[
\frac{9m^2 - 12mn + 4n^2}{m^2 - n^2} + \frac{3m - 2n}{m - n}
= \frac{(3m - 2n)(3m - 2n)}{m - n} + \frac{3m - 2n}{m - n}
= \frac{(3m - 2n)(3m - 2n)}{m - n} \times \frac{m - n}{m - n}
= \frac{3m^2 - 2n}{m - n}
\]
M1 – correct factorisation
M1 – for changing divide to multiplication and interchange the numerator with the denominator of the second fraction.

11a Area Scale
1 : 100

Linear Scale
1 : 10

Length of the house = 3 x 10
= 30 m

11b Volume of the model = \( \left( \frac{1}{10} \right)^3 \times 15000000 \)
= 15000 cm³

12a \[
\begin{array}{c}
3 & 30 \\
8 & 80 \\
\end{array}
\]
\[
80 - 30 = 50 \quad 30 - 18 = 12 \\
\]
M1

12b \[
\begin{array}{c}
30 & 18 & 18 & 18 \\
80 & 79 & 80 & 79 \\
\end{array}
\]
\[
= \frac{27}{158}
\]
M1

12bi \( P(\text{at least 1 red ball}) \)
\[
1 - P(\text{no red ball})
= \frac{32}{80} \times \frac{61}{79}
= \frac{1269}{3160}
\]
B1

OR

13a \[ x = \frac{-2}{3} \]

13b \[ y = 2x + 5 \]

Sub \( (2, -5) \),
\[-5 = 3C + c \]
\[ c = 11 \]
M1

14a \[ \tan \angle BDC = \frac{\sqrt{y^2 - x^2}}{x} \]

14b \[ \cos \angle ABD = -\cos \angle BDC = -\frac{\sqrt{y^2 - x^2}}{y} \]

B1

15a Total sales = (3694 - 1200) \times \frac{100}{8}
= $31175
M1

15b Tap X: 2r cm²/min
Tap Y: r cm³/min

In 5 min,
\[ S(2r) + 5r = 300 \]
\[ r = 20 \]

Rate of Tap Y = 20 cm³/min
M1

OR
<table>
<thead>
<tr>
<th>Qn</th>
<th>Expression</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>16a</td>
<td>$V_x: V_T = 2:1$</td>
<td>$V_x = \frac{300}{3} = 100 \text{ cm}^3$</td>
</tr>
<tr>
<td></td>
<td>Rate = $\frac{100}{5} = 20 \text{ cm}^3/\text{min}$</td>
<td>M1</td>
</tr>
<tr>
<td>16b</td>
<td>$3^2 + 5^2 + 4^2 + 1 = x^2 + 2$</td>
<td>$x = 9$</td>
</tr>
<tr>
<td>16c</td>
<td>$\frac{3(0) + 5(0) + 4(2) + 3x + 2(4)}{14 + x} = \frac{1}{2}$</td>
<td>$21 + 3x = 45$</td>
</tr>
<tr>
<td></td>
<td>$14 + x = 22$</td>
<td>$462 + 66x = 630 + 45x$</td>
</tr>
<tr>
<td></td>
<td>$21x = 168$</td>
<td>$x = 8$</td>
</tr>
<tr>
<td>17a</td>
<td>Total dist = $\frac{1}{2} v(10 + 35)$</td>
<td>$v = 22.5 \text{ m/s}$</td>
</tr>
<tr>
<td></td>
<td>$540 = 22.5v$</td>
<td>A1</td>
</tr>
<tr>
<td>17b</td>
<td>$a = \frac{24}{10} = 2.4 \text{ m/s}^2$</td>
<td>$v = 24 - 2.4(2.4)$</td>
</tr>
<tr>
<td></td>
<td>Speed = $24 - 3(2.4)$</td>
<td>M1</td>
</tr>
<tr>
<td>18a</td>
<td>$H_k = \frac{242}{98} = \frac{11}{7}$</td>
<td>B1</td>
</tr>
<tr>
<td>18b</td>
<td>$A_k = \frac{7}{11}$</td>
<td>$A_k = 121 \times 343$</td>
</tr>
<tr>
<td></td>
<td>$\frac{A_k}{A_l} = \frac{7}{11}$</td>
<td>$A_l = 49$</td>
</tr>
<tr>
<td></td>
<td>$A_k = \frac{49}{243}$</td>
<td>$A_l = 121 \times 49$</td>
</tr>
<tr>
<td></td>
<td>$A_k = 847 \text{ cm}^2$</td>
<td>BI</td>
</tr>
</tbody>
</table>
19a  \[ s = kr^2 \]
   \[ 36 = kr^2 \]
   \[ s = k\left(\frac{r}{2}\right)^2 \]
   \[ s = 9 \]
   MI

19b  \[ y = \frac{k}{x} \]
   \[ k = 24 \]
   \[ y = \frac{24}{9} \]
   \[ y = \frac{8}{3} \]
   MI, A1

20a  \[ 35 = 5 \times 7 \]
     \[ 3^2 \times 5^2 \times 7^1 \] needs to have a factor of 5 and 7.
     Therefore, \( x = 0, y = 1 \)
     B1 - if 1 correct
     B2 - If 2 correct

20b  \[ 55q = 5 \times 11 \times q \]
     \[ = 5 \times 11 \times 5 \times 7 \times 11 \]
     \[ = 5^2 \times 7^2 \times 11^2 \]
     Since the indices of the prime factors are multiples of 2,
     \[ 55q \] is a perfect square.
     B1

20c  \[ p \times q = 3^2 \times 5^2 \times 7^3 \times 11^1 \]
     \[ = 3^2 \times 5^2 \times 7^3 \times 11 \]
     Since \( p \times q \) is a product of prime factors that are odd,
     \( p \times q \) is an odd number.
     B1

21a  \[ (P \cap Q) \cup (P \cap Q) \]
     or
     \[ (P \cap Q) \cup (P \cap Q) \]
     B1

21bi  \[ B \cap C = \emptyset \] or \{\}
     B1

21bii  \[ (A \cup B) = \{8, 9\} \]
     B1

21biii  \[ \text{B1} - 5 \text{ out of 9 correct} \]
         \[ \text{B2} - \text{All correct} \]

22bi  Since \( \triangle OXY \) is similar to \( \triangle YXZ \),
      \[ \text{Area of } \triangle OQR \]
      \[ \frac{1}{2} \times QR \times h \]
      \[ = \frac{25}{9} \times \frac{3}{2} \]
      \[ = 75 \text{ cm}^2 \]
      \[ \text{Area of } \triangle OXZ \]
      \[ \frac{1}{2} \times XZ \times h \]
      \[ = \frac{25}{9} \times \frac{5}{2} \]
      \[ = 115 \text{ cm}^2 \]
      B1 - Either 2 of these reasons

22bii  Consider \( \triangle APQR \) and \( \triangle APXR \):
      \[ \text{Area of } \triangle APQR \]
      \[ \frac{1}{2} \times QR \times h \]
      \[ = \frac{1}{2} \times 5 \times h \]
      \[ QR = 5 \]
      \[ \text{Area of } \triangle APXR \]
      \[ \frac{1}{2} \times XR \times h \]
      \[ = \frac{1}{2} \times 2 \times h \]
      \[ XR = 2 \]
      \[ \text{Area of } \triangle APQR \]
      \[ = \frac{5}{2} \times 46 \]
      \[ = 115 \text{ cm}^2 \]
      MI, A1
### Question 23a

EC = \sqrt{4^2 + 2^2}
= \sqrt{20}

Perimeter of sector CEF
= 2\sqrt{20} + \sqrt{20} \times 0.64
= 11.8\text{ cm}

### Question 23b

Area of sector CEF
= \frac{1}{2} (\sqrt{20})^2 \times 0.64
= 6.4\text{ cm}^2

Area of shaded region
= 4^2 - \left(\frac{1}{2} \times 4 \times 2\right) - 6.4
= 1.6\text{ cm}^2
Answer Key

1(a) \( \frac{t^4}{4v^3} \)

1(b)(i) \( \frac{25x}{3y^2} \)

1(b)(ii) \( \frac{13-11x}{(3x-1)(2+x)} \)

1(c)(i) \( \left( \frac{x-5}{2} \right)^2 - \frac{57}{4} \)

1(c)(ii) \( x = 6.27 \) or \( -1.27 \)

2(a) \[
A = \begin{bmatrix}
252 & 168 & 84 & 105 \\
305 & 158 & 115 & 152 \\
316 & 191 & 134 & 167
\end{bmatrix}
\]

2(b) \[
P = \begin{bmatrix}
2.10 \\
1.80 \\
2.05 \\
2.55
\end{bmatrix}
\]

\[
AP = \begin{bmatrix}
1271.55 \\
1548.25 \\
1707.95
\end{bmatrix}
\]

2(c) The elements represents the total number of cups of ice cream sold over the 3 days of each flavor respectively.

2(d) \( (1484.10 \quad 697.95 \quad 499.50 \quad 742) \)

3(a)(i) \( r = 3.45 \text{ cm} \)

3(a)(ii) \( 426 \text{ cm}^2 \)

3(b)(i) \( 2x^2 \)

3(b)(ii) \( x^0 = 24 \)

4(c) \( 6.3 \text{ cm} \)

5(a)(i) \( b - 2a \)

5(a)(ii) \( b + 3a \)

5(a)(iii) \( \frac{2}{5}(b + 3a) \)

5(a)(iv) \( -\frac{4}{5}a + \frac{2}{5}b \)

5(b) \[
\frac{AB}{2} = \frac{5}{2}AX
\]

\( A, X, \text{ and } B \) are collinear.

5(c)(i) \( \frac{AZ}{h} = b - \frac{1}{2}a \)

5(c)(ii) \( \frac{AZ}{k} = -2a + kb \)

5(c)(iii) \( \overline{AG} \)
5(d)(i) \[ \frac{2}{5} \]
5(d)(ii) \[ \frac{2}{5} \]
6(a) 62.3 km
6(b) 023.2°
6(c) 15.8 km
6(d) 1.7°
7(a)(i) \((45 - 20t)\) km
7(a)(ii) \((25 - 10t)\) km
7(b) 45.9 km
7(c) \((AG)\)
7(d) 112 minutes
8(a)(i) 2.25 cm
8(a)(ii) 0.75 cm
8(a)(iii) 2.85 cm
8(b)(i) The cumulative frequency graph of the growth of plants exposed to Bach will be **less steep** in the middle/steeper at the upper quartile/wider range/bigger range compared to that of plants exposed to Beethoven.

Or

The middle of the cumulative frequency graph of the growth of plants exposed to Bach is **shifted to the right** compared to that of plants exposed to Beethoven.

8(b)(ii) The plants grow **better** when exposed to Bach on average compared to when exposed to Beethoven as the **median increase in heights of the plants when exposed to Bach is higher than those exposed to Beethoven.**

The increase in heights of the plants are more spread when exposed to Bach than when exposed to Beethoven as the interquartile range is larger when exposed to Bach than when exposed to Beethoven.

9(a) -0.48
9(b) (graph)
9(c) 0.6 or 3.8
9(d) 0.792
9(e) 1.1 or 4.95
10(a) \( h = 6 \text{ cm} \)
\( r = 3.46 \text{ cm} \)
10(b) \( 2\pi \)
11(a) \$428.65
11(b) Any value above $3.3146

Assume no cost incurred for cooking. (or any reasonable)
READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets \([\ ]\) at the end of each question or part question.
The total number of marks for this paper is 100.

This paper consists of 23 printed pages (including this cover page)

| Turn over |
Answer all the questions.

1 (a) Simplify \( \left( \frac{64y^9}{t^{12}} \right)^{\frac{1}{3}} \).

\[
\left( \frac{64y^9}{t^{12}} \right)^{\frac{1}{3}} = \left( \frac{t^{12}}{64y^9} \right)^{\frac{1}{3}} = \frac{t^4}{4y^3} \quad \text{- M1}
\]

(b) Express as a single fraction in its simplest form.

(i) \( \frac{20x^2}{9} + \frac{4xy^3}{15y} \).

\[
\frac{20x^2}{9} + \frac{4xy^3}{15y} = \frac{20x^2 \cdot 15y + 4xy^3 \cdot 9}{9 \cdot 15y} = \frac{25x}{3} \quad \text{- B1}
\]

(ii) \( \frac{4}{3x-1} - \frac{5}{2+x} \).

\[
\frac{4}{3x-1} - \frac{5}{2+x} = \frac{4(2+x) - 5(3x-1)}{(3x-1)(2+x)} = \frac{13-11x}{(3x-1)(2+x)} \quad \text{- A1}
\]

(c) (i) Express \( x^2 - 5x - 8 \) in the form \((x + a)^2 + b\).

\[
x^2 - 5x - 8 = \left( x - \frac{5}{2} \right)^2 - \left( \frac{5}{2} \right)^2 - 8 = \left( x - \frac{5}{2} \right)^2 - \frac{57}{4} \quad \text{- B1}
\]

(ii) Hence, solve the equation \( x^2 - 5x - 8 = 0 \), giving your answers correct to two decimal places.

\[
x^2 - 5x - 8 = 0
\]

\[
x - \frac{5}{2} = \pm \sqrt{\frac{57}{4}}
\]

\[
x = \frac{5}{2} \pm \frac{\sqrt{57}}{2}
\]

\[
x \approx 6.27 \text{ or } -1.27 \quad \text{- A2}
\]
An ice-cream shop sells 4 different flavours of ice-cream. The table below shows the number of cups of ice-cream sold by the shop over 3 days.

<table>
<thead>
<tr>
<th></th>
<th>Chocolate</th>
<th>Vanilla</th>
<th>Strawberry</th>
<th>Durian</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friday</td>
<td>252</td>
<td>168</td>
<td>84</td>
<td>105</td>
</tr>
<tr>
<td>Saturday</td>
<td>305</td>
<td>158</td>
<td>115</td>
<td>152</td>
</tr>
<tr>
<td>Sunday</td>
<td>316</td>
<td>191</td>
<td>134</td>
<td>167</td>
</tr>
</tbody>
</table>

(a) Represent the above information using a $3 \times 4$ matrix $A$.

$$
A = \begin{pmatrix}
252 & 168 & 84 & 105 \\
305 & 158 & 115 & 152 \\
316 & 191 & 134 & 167
\end{pmatrix} \quad \text{--- B1}
$$

(b) The selling price of each cup of ice-cream is shown in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Chocolate</th>
<th>Vanilla</th>
<th>Strawberry</th>
<th>Durian</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling Price</td>
<td>$2.10</td>
<td>$1.80</td>
<td>$2.05</td>
<td>$2.55</td>
</tr>
</tbody>
</table>

Write down a matrix $P$ such that the product $AP$ represents the total sales of all the ice-cream for each day respectively. Evaluate $AP$.

$$
P = \begin{pmatrix}
2.10 \\
1.80 \\
2.05 \\
2.55
\end{pmatrix} \quad \text{--- B1}
$$

$$
A = \begin{pmatrix}
252 & 168 & 84 & 105 \\
305 & 158 & 115 & 152 \\
316 & 191 & 134 & 167
\end{pmatrix}
$$

$$
AP = \begin{pmatrix}
1271.55 \\
1548.25 \\
1707.95
\end{pmatrix} \quad \text{--- B0}
$$

Evaluate $(1 \ 1 \ 1)A$ and state what the elements of the product represents.

$$
(1 \ 1 \ 1)A = \begin{pmatrix}
252 & 168 & 84 & 105 \\
305 & 128 & 115 & 152 \\
316 & 191 & 134 & 167
\end{pmatrix}
$$

The elements represent the total number of cups of ice-cream sold over the 3 days of each flavor respectively. --- B1

(d) The cost price of each cup of ice-cream is shown in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Chocolate</th>
<th>Vanilla</th>
<th>Strawberry</th>
<th>Durian</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost Price</td>
<td>$0.40</td>
<td>$0.45</td>
<td>$0.55</td>
<td>$0.80</td>
</tr>
</tbody>
</table>

Write down two matrices such that the elements of their product represent the total profit received by the shop over the 3 days for the sale of each flavor of ice-cream respectively. Evaluate the product.

$$
\begin{pmatrix}
1.70 & 0 & 0 & 0 \\
0 & 1.35 & 0 & 0 \\
0 & 0 & 1.50 & 0 \\
0 & 0 & 0 & 1.75
\end{pmatrix}
$$

$$
= \begin{pmatrix}
148.410 & 697.95 & 499.50 & 742
\end{pmatrix} \quad \text{--- A1}
$$
(a) The diagram shows a toy which is made up of a cylindrical part and a hemispherical part.
The height of the cylindrical part is 4 cm and the radius of the hemispherical part is 6 cm.

(i) Find the radius of the cylindrical part, given that the volume of the cylindrical part is 150 cm³.

\[ \pi r^2 h = 150 \]
\[ r^2 = \frac{150}{\pi \times 4} \]
\[ r = 3.4549 \]
\[ r = 3.45 \text{ cm (3sf)} \]

(ii) Calculate total surface area of the toy.

\[ S_d \text{ of hemisphere} = 2 \pi r^2 = 108 \pi \]
Total \( SA = 108 \pi + 2 \pi r^2 = 2 \pi (3.4549)^2 = 59.575 \pi \]
\[ = 426.178 \text{ cm}^2 \]

OR

\[ S_d \text{ of hemisphere} = 3 \pi (6)^2 = 108 \pi \]
\[ S_A \text{ of cylinder} = \pi (3.4549)^2 + 2 \pi (3.4549)h = 59.575 \pi \]
\[ TSA = 108 \pi + 59.575 \pi = 426 \text{ cm}^2 \]

(b) In the diagram, \( A, B, C \) and \( D \) lie on a circle with centre \( O \). Angle \( \angle OAC = 4x^\circ \).

(i) Find angle \( \angle ALC \), giving your answer in terms of \( x^\circ \).

\[ \angle AOC = 2 \angle A' \text{ (angle at centre = 2 angle at circumference)} \]

(ii) Given further that angle \( \angle ABC = 5x^\circ + 12^\circ \), find the value of \( x \).

\[ 5x^\circ + 12^\circ = 180^\circ - 2x^\circ \]
\[ 7x^\circ = 168^\circ \]
\[ x^\circ = 24^\circ \]

OR

\[ 2(5x + 12) = 360 - 4x \]
\[ 10x + 24 = 360 - 4x \]
\[ 14x = 336 \]
\[ x = 24 \]
4 (a) Construct a triangle $ABC$ such that $AB = 13\text{ cm}$, $AC = 12\text{ cm}$, angle $ABC = 55^\circ$, and angle $ACB$ is acute. The line $AB$ has been drawn for you.

(b) Construct the angle bisector of angle $BAC$ such that it intersects the line $BC$, and label the point of intersection $D$.

(c) Construct the perpendicular bisector of $AD$ such that it intersects the line $AC$, and label the point of intersection $E$. Measure and write down the value of $DE$.

---

5

In the diagram, $\overline{OA} = 2a$, $\overline{OB} = b$. $BC$ is parallel to $OA$ and $BC = \frac{3}{2} OA$. $X$ is a point on $BC$ such that $O\overline{X} = \frac{2}{5} \overline{XC}$. $Y$ is the midpoint of $BC$.

(a) Express in terms of $a$ and/or $b$, as simply as possible,

(i) $\overline{AC}$

(ii) $\overline{OC}$

(iii) $\overline{OX}$

(iv) $\overline{AX}$

(b) Write down two facts about the points $A$, $X$, and $B$.

Write down two facts about the points $A$, $X$, and $B$.

No question labels = minus 1 overall
(c) \(AY\) produced meets \(OB\) produced at a point \(Z\). Given that \(\overline{AZ} = k\overline{AY}\), express \(\overline{AZ}\) in terms of \(a, b\) and \(h\).

\[
\overline{AZ} = h\overline{AY} = h\overline{AB} + h\overline{BY} = M1
\]

\[
= b - \frac{1}{2} a = M1
\]

\[
\overline{AZ} = bh - \frac{1}{2} ha = A1
\]

(ii) Given also that \(\overline{OZ} = k\overline{OB}\), express \(\overline{AZ}\) in terms of \(a, b\), and \(k\).

\[
\overline{AZ} = -2a + kb = B1
\]

(iii) Hence, show that \(k = 4\) and \(k = 4\).

\[
bh - \frac{1}{2} ha = 2a + kb
\]

Comparing

\[
k = 4, k = 4 = A1, A1
\]

(d) Find the value of

\[\text{area of } \triangle OAX \text{ area of } \triangle OAC\]

(i) \[
\text{area of } \triangle OAX = \frac{2}{3} \text{ area of } \triangle OAC = B1
\]

(ii) \[
\text{area of } \triangle OBY = \frac{3}{5} \text{ area of } \triangle ABC = A1
\]

6 In the diagram, \(P\) and \(Q\) are the bases of two lighthouses such that \(P\) is located 40 km due north of \(Q\).

\(A\) is a boat 30 km from \(P\) and on a bearing of \(055^\circ\) from \(P\).

(a) Find the distance \(AQ\).

\[
AQ^2 = 30^2 + 40^2 - 2(30)(40)\cos 125^\circ = M1M1
\]

\[
AQ = 62.2621525...
\]

\[
AQ = 62.3 \text{ km (3 sf)} = A1
\]

(b) Find the bearing of \(A\) from \(Q\).

\[
\sin \angle PQA = \frac{30}{62.2621525}
\]

\[
\sin \angle PAQ = \frac{30 \times \sin 125^\circ}{62.2621525}
\]

\[
\angle PQA = 23.2469296^\circ
\]

Bearing of \(A\) from \(Q = 023.2^\circ\).
Light from \( P \) can be seen within 20 km radius of \( P \), beyond which the light becomes too faint to be seen. When the boat moves from \( A \) to \( Q \), \( X \) and \( Y \) are the positions on the boat’s journey which are 20 km from \( P \).

(c) Calculate

(i) the shortest distance of the boat from \( P \).

\[
\angle PQX = \angle PQA = 23.246926^\circ
\]

\[
\sin \angle PQX = \frac{h}{40} \\
\frac{h}{15.78778476} \text{ km} \\
\frac{h}{15.8} \text{ km} 
\]

(ii) the smallest angle of elevation of the lighthouse \( P \) from the boat as it travels from \( X \) to \( Y \), given that the height of lighthouse \( P \) is 600 m.

\[
\tan^{-1}\left(\frac{600}{30000}\right) = \frac{600}{30000} \\
= 1.7183 \\
= 1.7^\circ 
\]

7. The diagram shows two roads \( XY \) and \( YZ \) meeting at point \( Y \). The roads are perpendicular to each other. \( YX = 45 \) km and \( YZ = 25 \) km.

Cyclist \( A \) is travelling from point \( X \) towards point \( Y \) at a constant speed of 20 km/h.

Cyclist \( B \) is travelling from point \( Y \) towards point \( Z \) at a constant speed of 10 km/h.

(i) Write down an expression, in terms of \( t \), for the distance in kilometres

\( 45 - 20t \) km

(ii) Between cyclists \( A \) and point \( Y \), after \( t \) hours.

\( 25 - 10t \) km

(iii) Between cyclist \( B \) and point \( Y \), after \( t \) hours.

\( 25 - 10t \) km

(iv) Term an expression, in terms of \( t \), for the shortest distance, \( d \), between the two cyclists and show that it reduces to \( \sqrt{10t^2 - 46t + 53} \).

Hence find the shortest distance between the two cyclists after 15 minutes.

\[
d^2 = (45 - 20t)^2 + (25 - 10t)^2 \\
d = \sqrt{2025 - 1800t + 400t^2 + 625 - 500t + 100t^2} \\
= \sqrt{500t^2 - 2300t + 2650} \\
At t = 0.25 \text{ hour,} \\
d = \sqrt{500(0.25)^2 - 2300(0.25) + 2650} \\
= 45.893... \\
= 45.9 \text{ km} 
\]
(c) The two cyclists are 10 km apart at a certain instant, \( r \) hours.
Form a quadratic equation in terms of \( r \) and show that it reduces to
\[ 10r^2 - 46r + 51 = 0. \]

\[
\begin{align*}
10 &= \sqrt{500r^2 - 2300r + 2650} \quad \text{\(-M1\)} \\
100 &= 500r^2 - 2300r + 2650 \\
500r^2 - 2300r + 2550 &= 0 \\
20r^2 - 92r + 102 &= 0 \\
10r^2 - 46r + 51 &= 0 \quad \text{\(-B1\)}
\end{align*}
\]

(d) Given that \( r < 2 \), find the time, in minutes, when the two cyclists are 10 km apart.
Correct your answer to 3 significant figures.

\[
\begin{align*}
10r^2 - 46r + 51 &= 0 \\
\quad t &= \frac{(-46) \pm \sqrt{(-46)^2 - 4(10)(51)}}{2(10)} \quad \text{\(-M1\)} \\
\quad &= \frac{46 \pm \sqrt{76}}{20} \\
\quad &= \frac{2.73588 \text{ or } 1.8641}{\text{\(-A1\)}} \\
\text{Hence } t &= 1.8641 \quad \text{\(-112 \text{ minutes (3 s.f.)-} \text{\(-A1\)}}
\end{align*}
\]
The variables \( x \) and \( y \) are connected by the equation \( y = \frac{1}{2x^3} + 0.1x^2 - 3 \). The table below shows some values of \( x \) and the corresponding values of \( y \), correct to 2 decimal places.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-0.98</td>
<td>-2.40</td>
<td>-2.55</td>
<td>-2.48</td>
<td>-2.04</td>
<td>-1.37</td>
<td>-0.61</td>
<td></td>
</tr>
</tbody>
</table>

(a) Find the value of \( p \).

\[ p = -0.48 \]  -- B1

(b) In the space provided on the next page, using a scale of 2 cm to represent 1 unit on the horizontal axis and 2 cm to represent 0.5 units on the vertical axis, draw the graph of \( \frac{1}{2x^3} + 0.1x^2 - 1.5 = 0 \). Refer to attached.

(c) Using your graph, write down the solution(s) to the equation \( \frac{1}{2x^3} + 0.1x^2 - 1.5 = 0 \).

\( x = 0.5 \) or \( x = 3.6 \) -- A1

(d) By drawing a tangent, find the gradient of the curve at \( x = 4 \).

Tangent drawn. -- M1

\[ \text{gradient} = \frac{-0.25}{-2.15} = \frac{0.792}{0.54} \]  (actual = 0.784375) -- A1

(e) By drawing a suitable straight line on the same axes, solve the equation \( \frac{1}{2x^3} + 0.1x^2 - 0.5x = 0 \).

\[ \frac{1}{2x^3} + 0.1x^2 - 0.5x = 0 \]

\[ \frac{1}{2x^3} + 0.1x^2 - 3 = 0.5x - 3 \]

\( \therefore \) Draw \( y = 0.5x - 3 \) -- M1

Correctly drawn graph -- B1

\( x = 1.1 \) or \( 4.95 \pm 0.1 \) -- B1

(a) Find
(i) the median increase in the height of the plants,
\[ 2.25 \text{ cm} \quad -- \text{B1} \]
(ii) the interquartile range,
\[ 2.5 - 1.75 = 0.75 \text{ cm} \quad -- \text{M1, A1} \]
(iii) the 90\(^{th}\) percentile.
\[ \frac{90\% \times 80}{100} = 72 \]
\[ \text{90\(^{th}\) percentile} = 2.85 \text{ cm} \quad -- \text{A1} \]

(all answers \( \pm 0.025 \))

(b) Another group of 80 *Rosa chinensis* plants were exposed to Bach's Goldberg Variations, and their growth was also observed over a period of 10 days. The box-and-whisker plot below shows the increase in the heights of the plants after 10 days.

(I) Describe how the cumulative frequency graph of the growth of the 80 plants exposed to Bach's Goldberg Variations will differ from that of those exposed to Beethoven's Ninth Symphony.

The cumulative frequency graph of the growth of plants exposed to Bach will be less steep in the middle/steeper at the upper quartile/wider range/broader range compared to that of plants exposed to Beethoven. -- B1

The middle of the cumulative frequency graph of the growth of plants exposed to Bach is shifted to the right compared to that of plants exposed to Beethoven. -- B1

(ii) Make two comparisons between the growth of the plants under the two conditions.

The plants grow better when exposed to Bach on average compared to when exposed to Beethoven as the median increase in heights of the plants when exposed to Bach is higher than those exposed to Beethoven. -- B1

The increase in heights of the plants are more spread when exposed to Bach than when exposed to Beethoven as the interquartile range is larger when exposed to Bach than when exposed to Beethoven. -- B1
A right conical container of capacity $24\pi$ cm$^3$ and vertical angle $60^\circ$ is completely filled with water. The height of the container is $h$ cm and the base radius is $r$ cm.

(a) Find the value of $r$ and of $h$.

\[
\frac{1}{3} \pi r^2 h = 24\pi \quad \text{--- M1}
\]
\[
h = 72 \quad \text{--- M1}
\]
\[
\frac{r}{\tan 30^\circ} = 6 \quad \text{--- M1}
\]
Hence, $r = \frac{72 \tan 30^\circ}{3} = 3.4610... = 3.46$ cm --- M1
\[
h = 6 \text{ cm} \quad \text{--- A1}
\]

OR

\[
r^2 + h^2 = (2r)^2 \]
\[
r^2 = \frac{h^2}{3} \quad \text{--- (1)}
\]
\[
\frac{1}{3} \pi r^2 h = 24\pi \quad \text{--- (2)}
\]
sub (1) into (2):

\[
\frac{1}{3} \pi \left( \frac{h^2}{3} \right) h = 24\pi
\]
\[
h^3 = 216
\]
\[
h = 6 \text{ cm}
\]
\[
r = \sqrt[3]{\frac{216}{3}}
\]
\[
r = 3.46 \text{ cm}
\]
The water in the container is poured into another identical container $B$ so that the depth of water in container $A$ is $\frac{1}{2} h$ cm.

(b) Find the volume of water in container $B$ in terms of $\pi$. [2]

\[
V_B = \frac{7}{8} \times 24\pi = 21\pi \text{ cm}^3
\]

OR

Volume remaining = \[
\frac{1}{3} \pi \left( \frac{\sqrt{12}}{2} \right)^2 \left( \frac{5}{2} \right) = 3\pi
\]

Volume in $B = 24\pi - 3\pi = 21\pi \text{ cm}^3$

11 A class of students plan to sell breakfast sets during the National Day carnival in order to raise funds for Food Bank Singapore.

Each breakfast set consists of 2 scrambled eggs, 2 slices of toast, 2 sausages, and a cup of coffee.

The students estimate that they will sell 250 breakfast sets.

<table>
<thead>
<tr>
<th>Item</th>
<th>Description</th>
<th>Unit cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eggs</td>
<td>Pasar Fresh Eggs (10 per pack)</td>
<td>$1.80</td>
</tr>
<tr>
<td></td>
<td>Pasar Fresh Eggs (30 per pack)</td>
<td>$4.55</td>
</tr>
<tr>
<td>Bread</td>
<td>Sunshine Enriched Soft White Bread (14 slices)</td>
<td>$1.80</td>
</tr>
<tr>
<td></td>
<td>Fair Price Wholemeal Bread (12 slices)</td>
<td>$1.55</td>
</tr>
<tr>
<td>Sausages</td>
<td>Tienney’s Chicken Hot Dog (10 per pack)</td>
<td>$3.25</td>
</tr>
<tr>
<td></td>
<td>Fair Price Sausages (per pack)</td>
<td>$2.50</td>
</tr>
<tr>
<td></td>
<td>(Buy 5 get 1 free)</td>
<td>$3.70</td>
</tr>
<tr>
<td></td>
<td>Gold Roast 3 in 1 Coffeemix (25 per pack)</td>
<td>$3.95</td>
</tr>
</tbody>
</table>

(5) Find the lowest possible total cost of the ingredients for the breakfast sets.

<table>
<thead>
<tr>
<th>Item</th>
<th>10 per pack</th>
<th>20 per pack</th>
<th>Mix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eggs</td>
<td>(50)(1.80)</td>
<td>(12)(2.55)</td>
<td>(16)(4.35) + (2)(1.80) = 73.20</td>
</tr>
<tr>
<td></td>
<td>(90)</td>
<td>(20)</td>
<td></td>
</tr>
<tr>
<td>Bread</td>
<td>(30)(1.80)</td>
<td>(42)(1.55)</td>
<td>(35)(1.80) + (1)(1.55) = 64.55</td>
</tr>
<tr>
<td></td>
<td>(40)</td>
<td>(42)</td>
<td>(28)(1.80) + (9)(1.55) = 64.55</td>
</tr>
<tr>
<td>Sausages</td>
<td>(50)(5.25)</td>
<td>(84)(3.20)</td>
<td>(82)(3.20) - (41)(0.35) + (1)(5.25) = 253.30</td>
</tr>
<tr>
<td></td>
<td>252,50</td>
<td>254,10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(80)(3.20)</td>
<td>(40)(0.35)</td>
<td>(80)(3.20) - (40)(0.35) + (2)(5.25) = 252.5</td>
</tr>
<tr>
<td>Coffee</td>
<td>(7)(6.15)</td>
<td>(16)(3.95)</td>
<td>(5)(6.15) + (2)(3.95) = 38.65</td>
</tr>
<tr>
<td></td>
<td>43.05</td>
<td>39.50</td>
<td></td>
</tr>
</tbody>
</table>

B1 for each correct choice for each item.

Total cost = $73.20 + 64.55 + 253.30 + 38.65 = $428.65 -- A1

(not lowest cost per item used: M1 for each correct item, no A1)

(insufficient quantities purchased: M0)
(b) The school provides $200 in funding for the students, and up to 30% of the sales can be used to cover their expenses, while the remaining goes to the charity. The students also wish to raise at least $600 for charity. Find the minimum they must charge for each breakfast set (to the nearest ten cents). Justify your answer, showing all necessary workings clearly. State an assumption you have made in your calculations.

\[ \text{min charge to raise } \$600 = \frac{(428.65 - 200) + 600}{250} = 3.3146 \quad \text{-- M1 (ECF their (a))} \]

\[ \left( \frac{30}{100} \right) (3.3146)(250) = 248.595 > 431.60 - 200 \quad \text{-- M1 (ECF their (a))} \]

\[ \therefore \text{min charge } = 3.40 \quad \text{-- A1} \]

Assume no cost incurred for cooking. (or any reasonable) \text{ -- B1}

**OR**

\[ \text{min charge to raise } \$600 = \frac{(428.65 - 200) + 600}{250} = 3.3146 \quad \text{-- M1 (ECF their (a))} \]

\[ \text{min charge to cover expenses } = \frac{\left[ \frac{(428.65 - 200) x 100}{30} \right] + 250 = 3.048}{250} \quad \text{-- M1 (ECF their (a))} \]

\[ \therefore \text{min charge } = 3.40 \quad \text{-- A1} \]

Assume no cost incurred for cooking. (or any reasonable) \text{ -- B1}

**Alternative assumptions:**
1) No accidents that result in wastage during the event
2) No free samples were given out
3) All 250 sets were sold
## Class Full Name Index Number

<table>
<thead>
<tr>
<th>Class</th>
<th>Full Name</th>
<th>Index Number</th>
</tr>
</thead>
</table>

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**Preliminary Examination 2019**

**Mathematics**  
**Paper 1**

Secondary 4 Express/ 4A1/ 5 Normal Academic  
30 August 2019  
2 hours

Additional Materials: Nil

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**Read These Instructions First**

Write your name, index number and class on all the work you hand in.  
Write in dark blue or black pen on both sides of the paper.  
You may use a pencil for any diagrams or graphs.  
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.  
If working is needed for any questions it must be shown with the answer.  
Omission of essential working will result in loss of marks.  
Calculators should be used where appropriate.  
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.  
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).

At the end of the examination, fasten all your work securely together.  
The number of marks is given in brackets [ ] at the end of each question or part question.  
The total marks for this paper is 80.

**Do Not Open This Paper Until You Are Told To Do So**

Setter: Mr Alvis Mazon Tan

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This document consists of **20** printed pages, including this cover page.
Mathematical Formulae

Compound interest

Total amount = \( P \left( 1 + \frac{r}{100} \right)^n \)

Mensuration

Curve surface area of a cone = \( \pi rl \)

Surface area of a sphere = \( 4\pi r^2 \)

Volume of a cone = \( \frac{1}{3} \pi r^3 h \)

Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

Area of triangle \( ABC = \frac{1}{2} ab \sin C \)

Arc length = \( r\theta \), where \( \theta \) is in radians

Sector Area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

Trigonometry

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\( a^2 = b^2 + c^2 - 2bc \cos A \)

Statistics

Mean = \( \frac{\sum fx}{\sum f} \)

Standard deviation = \( \sqrt{\frac{\sum f x^2}{\sum f} - \left( \frac{\sum f x}{\sum f} \right)^2} \)
Answer all the questions.

1 Write down the following in ascending order.

\[
\frac{25}{38}, \sqrt{0.49}, 0.60^2, 0.701
\]

Answer \(\ldots\), \(\ldots\), \(\ldots\), \(\ldots\) [1]

2 (a) Expand and simplify \((2x - 1)(2 - 3x) - 3x(2x - 5)\).

Answer (a) \(\ldots\) [2]

(b) Factorise completely \(24ab - 4ac + pc - 6pb\).

Answer (b) \(\ldots\) [2]

3 Calculate \(\frac{13.5^3}{6.48 - 2.57}\), giving your answers corrected to 2 significant figures.

Answer \(\ldots\) [1]
4 If the radius of a sphere increases by 10%, find the percentage increase in its volume.

Answer ………………………..% [2]

5 On a certain day the exchange rate between the pounds (£) and the Singapore dollars was S$1.684 = £1.

(a) Calculate the amount of pounds that Renee can buy with S$1263.

Answer (a) £= ……………………….. [1]

(b) After four weeks, she realized she has too much pounds and she now wants to change £200 back to Singapore dollars. If the loss by this transaction is S$6, what is the current exchange rate? Leave your answers corrected to 4 decimal places.

Answer (b) £1 = S$……………….. [2]
Integers $P$ and $Q$, written as products of their prime factors, are

\[ P = 2^2 \times 3 \times k^2 \quad \text{and} \quad Q = 2^3 \times 7 \times k, \]

where $k$ is a prime number.

(a) Express, in terms of $k$ and as a product of its prime factors, the smallest possible integer which is exactly divisible by both $P$ and $Q$.

\[ \text{Answer (a)} \quad \text{…………………….. \[1\]} \]

(b) Find the smallest integer, $n$, such that $27 kn$ is a multiple of $P$. Give your answer in terms of $k$ if necessary.

\[ \text{Answer (b) } n = \text{…………………….. \[2\]} \]
Kai Xuan has written down seven numbers. The mean of these numbers is 8, the median is 7 and the mode is 11. The smallest number is an even prime number and the largest number is eight times the smallest number. The second and third numbers are consecutive numbers.

Find the seven numbers.

\[ \text{Answer} = \ldots \ldots , \ldots \ldots , \ldots \ldots , \ldots \ldots , \ldots \ldots , \ldots \ldots \]  \[ \text{[2]} \]

Rearrange the formula \[ v = \frac{-u^2 + 5}{u^2 - a} \] and make \( u \) the subject of the formula.

\[ \text{Answer} u = \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \]  \[ \text{[3]} \]
The graph shows the total revenue, in billion dollars, of three different fast food chain.

![Graph showing total global revenue for three fast food chains: McDonald's (35.8 billion), Burger King (9.5 billion), and KFC (4.3 billion).]

State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer
10 (a) Solve the inequalities $8 + x < 10 + \frac{3}{2}x \leq 15.5 - 2x$.

Answer (a) $x = \ldots$…………………………………………………………………………………………… [3]

(b) Hence, write down the largest rational number that satisfies

$8 + x < 10 + \frac{3}{2}x \leq 15.5 - 2x$

Answer (b) $x = \ldots$…………………………………………………………………………………………… [1]
The first four terms in a sequence of numbers $T_1, T_2, T_3, T_4$ are as follow

\[
T_1 = \frac{2}{1} - \frac{3}{2}
\]

\[
T_2 = \frac{3}{2} - \frac{4}{2^2}
\]

\[
T_3 = \frac{4}{2^2} - \frac{5}{2^3}
\]

\[
T_4 = \frac{5}{2^3} - \frac{6}{2^4}
\]

(a) Write down the $n^{th}$ line and show that it can be expressed as $T_n = \frac{n}{2^n}$.

Answer (a) ..................... [3]

(b) Hence or otherwise, evaluate the following sum and leave your answer as a fraction.

\[
T_1 + T_2 + T_3 + \ldots \ldots \ldots \ldots . T_{11}
\]

Answer (b) ..................... [2]
A = \{\text{points lying on the line } 2x + y = 8\}
B = \{\text{points lying on the line } 3x - 4y = 12\}
C = \{\text{points lying on the line } mx - 4y = c\}

(a) Is (-1,6) \in A? Explain your answer clearly.

(b) Find the element \( p \) such that \( p \in (A \cap B) \).

\text{Answer (b)} \ p = \ldots \ldots \ldots \ldots [2]

(c) Write down a possible value of \( m \) and of \( c \) such that \( B \cap C = \emptyset \).

\text{Answer (c)} \ m = \ldots \ldots \ldots \ldots [1]
\quad c = \ldots \ldots \ldots \ldots [1]
The diagram below shows an irregular hexagon. Calculate the value of $a + b + c + d + e + f$.

\[ \text{Answer} \quad \ldots \ldots \ldots \ldots \quad [2] \]

Jia Lung invested some money in the savings account for 4 years. The rate of compound interest was fixed at 4% per annum compounded annually. At the end of 4 years, there was $8436.48 in her account.

How much did Jia Lung invest in the account?

\[ \text{Answer} \quad \ldots \ldots \ldots \ldots \quad [3] \]
Akshay jogs at a speed of 10 km/h.
One evening he jogged around his neighborhood for 1 hour 30 minutes.

(a) Calculate the distance that Akshay covered.

\[ \text{Distance} = \text{Speed} \times \text{Time} \]

\[ \text{Answer (a)} = \text{_________ km} \quad [1] \]

(b) Given that the scale of the neighborhood is 1: 25000, find in cm, the map distance that he covered.

\[ \text{Map Distance} = \text{Actual Distance} \times \frac{1}{\text{Scale}} \]

\[ \text{Answer (b)} = \text{_________ cm} \quad [2] \]

(c) A reservoir located in his neighborhood occupies a total area of 1.70 cm\(^2\) on the map. What is the actual area, in m\(^2\), of the reservoir?

\[ \text{Actual Area} = \text{Map Area} \times (\text{Scale})^2 \]

\[ \text{Answer (c)} = \text{_________ m}^2 \quad [2] \]
16

$PQR$ is a right-angled triangle. $QRT$ is a straight line.

$PR = 12\, \text{cm}$ and $QR = 19\, \text{cm}$.

Find the values of the following, giving your answer to two decimal places where necessary.

(a) $\tan \angle PQR$

(b) $\cos \angle TRP$

Answer (a) …………… [2]

Answer (b) …………… [2]
In the diagram, $O$ is the centre of the circle $BCD$ with radius 20 cm and $CD$ is the diameter of the circle. The ratio of the length $AB$ to the length $AD$ is 0.5.

$A$ is a point on $BC$ produced such that $AD$ is a tangent to the circle at $D$.

Calculate the area of the shaded region.

\[ \text{Answer (b) \ .................. cm}^2 \]
The table below shows the ages of 16 employees who work part-time at a cafe.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>21</td>
<td>16</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>17</td>
<td>19</td>
<td>65</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>22</td>
<td>17</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>19</td>
<td>19</td>
<td>18</td>
<td></td>
</tr>
</tbody>
</table>

(a) Complete the dot diagram to show the distribution of the ages of the employees.

(answer a)

```
16 17 18 19 20 21 22 23 65
```

Ages of Employees (years)

(b) Find the median of the distribution of ages.

Answer (b) Median = …………………… years

(c) Calculate the mean age of the employees.

Answer (c) Mean = …………………… years

(d) Pranav made the following statement:

“The mean is the most accurate way to determine the average age of the employees.”

Validate if Pranav statement is true.
Allyson participates in a game show. In order to win a prize, she has to navigate through a maze. The prize is located at checkpoint $X$. There are no prizes awarded at checkpoint $Y$ and $Z$. The diagram below shows four junctions $A$, $B$, $C$ and $D$ in the maze. Once Allyson runs pass a junction, she is not able to make a turnaround.

The probability that Allyson goes straight, without changing direction, at every junction is $\frac{3}{7}$.

(a) Find the probability that Allyson hits the dead end.

(b) Find the probability that Allyson wins a prize.

---

Answer (a) ...................... [2]

Answer (b) ...................... [3]
Two similar jugs have base area of 45 cm$^2$ and 125 cm$^2$.

(a) Find the ratio of the height of the smaller jug to the ratio of the height of the larger jug.

\[ \text{Answer (a)} \quad \ldots : \ldots \quad [1] \]

(b) The curved surface area of the smaller jug is 63 cm$^2$. Find the curved surface area of the larger jug.

\[ \text{Answer (b)} \quad \ldots \ldots \ldots \ldots \text{cm}^2 \quad [2] \]

(c) The capacity of the larger jug is 2.5 litres. Find the capacity of the smaller jug. Give your answer in cubic centimetres.

\[ \text{Answer (c)} \quad \ldots \ldots \ldots \ldots \text{cm}^3 \quad [2] \]
$OPQR$ is a parallelogram such that $\overrightarrow{PQ} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ and $P$ is the point $(3,2)$.

(a) Express $\overrightarrow{RP}$ as a column vector.

(b) The point $J$ lies on $\overrightarrow{RP}$ produced such that $\overrightarrow{PJ} = m\overrightarrow{RP}$

Show that $\overrightarrow{OJ} = \begin{pmatrix} 3+m \\ 2-2m \end{pmatrix}$
The diagram shows the speed time graph of a car. The car starts from rest and accelerates uniformly to a speed of 15 m/s in 10 seconds. The car then travels at a constant speed for some time before it decelerates uniformly at 0.75 m/s$^2$ until it comes to rest. The whole journey takes one minute.

(a) Given that the speed of the car after $x$ seconds is $v$ m/s, express $v$ in terms of $x$.

Answer $v =$……………………… [1]

(b) For how long does the car travel at the maximum speed?

Answer ………………………… [2]

(c) Calculate the total distance travelled by the car during this 1 minute journey.

Answer ………………………… m [2]
(d) Hence, sketch the distance time graph for the whole journey, indicating all relevant values in your sketch.
READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
You may use a pencil for any diagrams or graphs.
Do not use staples, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any questions it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact,
give the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 100.

DO NOT OPEN THIS PAPER UNTIL YOU ARE TOLD TO DO SO

Setter: Mrs Jane Cheng

For Examiner’s use

100
Mathematical Formulae

Compound interest

Total amount = \( P \left( 1 + \frac{r}{100} \right)^n \)

Mensuration

Curve surface area of a cone = \( \pi rl \)

Surface area of a sphere = \( 4\pi r^2 \)

Volume of a cone = \( \frac{1}{3} \pi r^2 h \)

Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

Area of triangle \( ABC = \frac{1}{2} ab \sin C \)

Arc length = \( r\theta \), where \( \theta \) is in radians

Sector Area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

Trigonometry

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\( a^2 = b^2 + c^2 - 2bc \cos A \)

Statistics

Mean = \( \frac{\sum fx}{\sum f} \)

Standard deviation = \( \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2} \)
Answer all the questions.

1. (a) Solve the inequality \( \frac{x+2}{3} \geq \frac{4-x}{7} \). [2]

   (b) Express as a single fraction in its simplest form \( \frac{2x}{(3x-5)^2} + \frac{x}{5-3x} \). [2]

   (c) Simplify \( \left( \frac{27a^6}{b^{12}} \right)^{-\frac{1}{3}} \). [2]
(d) Simplify \( \frac{24p^3q^2}{5r^3} \div \frac{8p^4r}{15q^3} \). [2]

(e) Solve the equation \( \frac{15}{x+2} = 2x + 3 \). [3]
A series of diagrams of shaded and unshaded small squares is shown below. The shaded squares are those which lie on the diagonals of the diagram.

(a) Copy and complete the table below.

| Diagram, $n$ | 1 | 2 | 3 | 4 | 5 | [2]  
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of shaded squares, $S$</td>
<td>1</td>
<td>5</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of unshaded squares, $U$</td>
<td>0</td>
<td>4</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total number of squares, $T$</td>
<td>1</td>
<td>9</td>
<td>25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) By observing the number patterns, without drawing further diagrams,

(i) write down the total number of squares in diagram 12, [1]

(ii) find an expression, in terms of $n$, for the total number of squares, $T$. [1]

(c) (i) Find an expression, in terms of $n$, for the number of shaded squares, $S$. [1]

(ii) Write down the number of the diagram that has 41 shaded squares. [1]

(d) Hence, or otherwise, find an expression, in terms of $n$, for the number of unshaded squares, $U$. [2]
3 \hspace{1cm} P is the point \((-5, 12)\) and \(Q\) is the point \((5, -4)\)

(a) Find the length of \(PQ\). 

(b) Find the equation of the line \(PQ\).
(c) The equation of the line $l_1$ is $8x + 5y + 10 = 0$.
(i) Show how you can decide whether the line $l_1$ does or does not intersect the line $PQ$.

(ii) The equation of line $l_2$ is $3y = 4x - 39$.
Find the coordinates of the point of intersection of the line $l_1$ and the line $l_2$. [3]
Mrs Tan is a Korean Language teacher. She conducts classes for basic and advanced students on weekdays and weekends. Each student has a 15-week block of lessons with one lesson per week. The matrix K shows the number of students she teaches each week in one 15-week block.

<table>
<thead>
<tr>
<th>Basic</th>
<th>Advanced</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
</tr>
</tbody>
</table>

Weekday
Weekend

(a) Evaluate the matrix $P = 15K$. [1]

(b) Mrs Tan charges $20 for each basic lesson and $32 for each advanced lesson. Represent the lesson charges in a $2 \times 1$ matrix $L$. [1]

(c) Evaluate the matrix $T = PL$. [2]
(d) State what the elements of $T$ represent. [1]

(e) Mrs Tan wants to attract more students, so in the next 15-week block she reduces her prices by 10%.
For this block of lessons, on weekdays she has 15 basic students and 5 advanced students.
On weekends she has 7 basic students and 6 advanced students.

Calculate the total amount of money she earns for this 15-week block of lessons. [3]
5 The cumulative frequency graph shows the distribution of the age groups of the *Fitness First* club.

(a) Complete the grouped frequency table for the ages of the members. [1]

<table>
<thead>
<tr>
<th>Age (x)</th>
<th>20 ≤ x &lt; 30</th>
<th>30 ≤ x &lt; 40</th>
<th>40 ≤ x &lt; 50</th>
<th>50 ≤ x &lt; 60</th>
<th>60 ≤ x &lt; 70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td>15</td>
</tr>
</tbody>
</table>

(ii) Calculate the mean age of each member. [1]

(iii) Calculate the standard deviation. [2]
(iv) Find the percentage of members whose age is 30 years old and above but less than 60 years old.

(v) A magazine article stated that citizens aged 50 and above are less active than those aged below 40.
Comment on whether the data from the Fitness First club supports this claim.

(b) The table below gives information about the ages of the members in the Any Time Fitness club.

<table>
<thead>
<tr>
<th></th>
<th>Members aged under 50</th>
<th>Members aged 50 or over</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>50</td>
<td>34</td>
</tr>
<tr>
<td>Female</td>
<td>36</td>
<td>30</td>
</tr>
</tbody>
</table>

(i) One of these members is selected at random.
Find, as a fraction in its lowest terms, the probability that he or she is under 50.

(ii) Two of the members are selected at random.
Find the probability that
(a) both members are female,
(b) they are both aged 50 or over, but only one is a male member.
A litre of 95-octane unleaded petrol cost \( \$x \) in January 2019.

(a) Mr Ang paid \( \$85.50 \) for his petrol. Write down in terms of \( x \), the amount of petrol bought. [1]

Mr Bala paid \( \$100 \) for his 98-octane unleaded petrol which cost 25 cents more per litre.

(b) Write down in terms of \( x \), the amount of petrol bought by Mr Bala. [1]

(c) If Mr Ang received 2 litres less petrol than Mr Bala, write down an equation to represent this information and show that it can reduce to

\[ 16x^2 - 112x + 171 = 0. \] [3]
(d) Solve the equation $16x^2 - 112x + 171 = 0$. 

(e) The price of the 98-octane unleaded petrol in January 2019 was a reduction of 7% on the price in December 2018. Find the price of the 98-octane unleaded petrol in December 2018 if it cost less than $3 for a litre of 95-octane unleaded petrol in January 2019.
The diagram shows a container consisting of a square bottom with rectangular sides, each 20 cm by 4 cm, and a regular pyramid on top with perpendicular height given by \( VX \). Water is poured into the container till the brim of the cuboid.

(a) Find the height \( VX \) of the pyramid. [2]

(b) Calculate the total surface area of the container. [2]

(c) Find the volume of water in the cuboid. [1]
The container is now inverted as shown in the diagram below.

(d) Calculate the depth of the water in the pyramid when inverted. [3]

(e) Another smaller container, which is geometrically similar, has a square base of 225 cm². Both containers are made of the same material. Find the mass of the smaller container in grams, given that the mass of the empty larger container is 1.28 kg. [2]
In the diagram, $OABC$ is a parallelogram and $D$ is the midpoint of $BC$. $BE$ and $OC$ produced intersect at the point $F$. $BE : BF = 1 : 3$ and $OC : OF = 1 : 2$.

Let $OA = a$ and $OC = c$.

(a) Express and simply the following vectors in terms of $a$ and $c$.

(i) $\vec{AC}$

(ii) $\vec{BF}$

(iii) $\vec{OD}$
(iv) $\vec{OE}$

(b) State two facts about the vectors $\vec{OD}$ and $\vec{OE}$ from the results in (a).

(c) Find the ratio of the areas of

(i) $\triangle ODF$ and $\triangle OEF$,

(ii) $\triangle OCD$ and $OABC$,

(iii) $\triangle OCD$ and $OABF$. 

Two school teams, Novotel and Temasek, are participating in an Amazing Race in Bishan Park. The diagram shows the paths in the park.

The teams assemble at $P$ before heading to $Q$ to start the race.

$P$ is due north of $R$.

The bearing of $R$ from $Q$ is $241^\circ$.

The distance $PR$ is 72 metres and the distance $RQ$ is 85 metres.

(a) Find the distance $PQ$. [3]
(b) The final station of the race is at R, each team is required to find a clue that is hidden at point S before completing the race at R. The bearing of S from R is 099° and QS is 54 metres. Given that there are two possible locations for S, find the two possible values of angle RSQ.
(c) Both teams manage to find the clue at the same time and team Novotel is closer to R than team Temasak.

Team Novotel claims that they are the winner.

Given that the speed of team Novotel is 30% less than the speed of team Temasak when they travel from S to R.

Do you agree with team Novotel that they will win the race?

Justify your answer with clear working in your calculations.
10 Answer the whole of this question on a sheet of graph paper.

The table below gives some values of \( x \) and the corresponding values of \( y \) for 
\[ y = x(1 + x)(5 - x). \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>–2</th>
<th>–1</th>
<th>–0.5</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>14</td>
<td>0</td>
<td>–1.375</td>
<td>8</td>
<td>18</td>
<td>( p )</td>
<td>20</td>
<td>0</td>
</tr>
</tbody>
</table>

(a) Find the value of \( p \). [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal \( x \)-axis for \( -2 \leq x \leq 5 \).
Using a scale of 2 cm to represent 5 units, draw a vertical \( y \)-axis for
\( -5 \leq y \leq 25 \).
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) By drawing a tangent, find the gradient of the curve where \( x = 4 \). [2]

(d) (i) On the same axes, draw the line \( 2x + y = 12 \) for \( -2 \leq x \leq 5 \). [1]

(ii) Write down the \( x \)-coordinates of the points where this line intersects the curve. [2]

(iii) The \( x \)-coordinates of the points where the two graphs intersect are solutions of the equation \( x^3 + Ax^2 + Bx + 12 = 0 \). Find the value of \( A \) and the value of \( B \). [2]
PRELIMINARY EXAMINATION 2019

4048/01

MATHEMATICS
Paper 1

Secondary 4 Express/ 4A1/ 5 Normal Academic
30 August 2019

Additional Materials: Nil

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

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For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 80.

DO NOT OPEN THIS PAPER UNTIL YOU ARE TOLD TO DO SO

For Examiner's use

Setter: Mr Alvis Mazon Tan

This document consists of 20 printed pages, including this cover page.
Mathematical Formulae

Compound interest

\[ \text{Total amount} = P \left(1 + \frac{r}{100}\right)^t \]

Mensuration

Curve surface area of a cone = \( \pi rl \)

Surface area of a sphere = \( 4\pi r^2 \)

Volume of a cone = \( \frac{1}{3} \pi r^2 h \)

Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

Area of triangle \( ABC = \frac{1}{2} ab \sin C \)

Are length = \( r\theta \), where \( \theta \) is in radians

Sector Area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

Trigonometry

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

Statistics

Mean = \[ \frac{\sum fx}{\sum f} \]

Standard deviation = \[ \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} \]
Answer all the questions.

1. Write down the following in ascending order.

\[
\frac{25}{38} \quad \sqrt{0.49} \quad 0.60^2 \quad 0.701
\]

Answer \( \frac{25}{38}, \sqrt{0.49}, 0.701, 0.60^2 \) [1]

2. (a) Expand and simplify \((2x - 1)(2 - 3x) - 3x(2x - 5)\).

\[
(2x - 1)(2 - 3x) - 3x(2x - 5) = 4x - 6x^2 - 2 + 3x - 6x^2 + 15x \quad \text{[MI]}
\]

\[
= -12x^2 + 22x - 2 \quad \text{[CA]}
\]

Answer (a) \(-2(6x^2 - 11x + 1)\) [2]

(b) Factorise completely \(24ab - 4ac + pc - 3ab\).

\[
24ab - 4ac + pc - 3ab = 4a(6b - c) + p(c - 6b) \quad \text{[MI]}
\]

\[
= 4a(c - 6b) - p(b - c) \quad \text{[CA]}
\]

Answer (b) \( (4a - p)(6b - c) \) [2]

3. Calculate \( \frac{13.5^3}{6.48 - 2.57} \), giving your answers corrected to 2 significant figures.

Answer \( 630 \) [1]
If the radius of a sphere increases by 10%, find the percentage increase in its volume.

\[
\frac{\frac{4}{3} \pi r^3 (1.1)^3 - \frac{4}{3} \pi r^3}{\frac{4}{3} \pi r^3} \times 100\% \quad [\text{M1}]
\]

\[
= 33.1\% \quad [\text{A1}]
\]

Answer \(33.1\%\) \[2\]

5

On a certain day, the exchange rate between the pound (£) and the Singapore dollar was S$1.684 = £1.

(a) Calculate the amount of pounds that Renee can buy with S$1263.

\[
\frac{1263}{1.684} = 750 \quad [\text{M1}]
\]

Answer (a) £ 750 \[1\]

(b) After four weeks, she realized she has too much pounds and she now wants to change £200 back to Singapore dollars.

If the loss by this transaction is S$6, what is the current exchange rate? Leave your answers corrected to 4 decimal places.

With reference to the original exchange rate

\[
\begin{align*}
£200 &= S$1.684 \times 200 \\
&= S$336.80
\end{align*}
\]

\[
S$336.80 - S$6 = S$330.8
\]

New exchange rate:

\[
\frac{S$330.8}{£200} = S$1.654 \quad [\text{A1}]
\]

Answer (b) £1 = S$1.654 \[2\]
Integers $P$ and $Q$, written as products of their prime factors, are

$P = 2^2 \times 3 \times k^2$ and $Q = 2^3 \times 7 \times k$, where $k$ is a prime number.

(a) Express, in terms of $k$ and as a product of its prime factors, the smallest possible integer which is exactly divisible by both $P$ and $Q$.

$L.C.M. = 2^3 \times 3 \times 7 \times k^2 \boxed{[B1]}$

Answer (a) $2^3 \times 3 \times 7 \times k^2 \boxed{[1]}$

(b) Find the smallest integer, $n$, such that $27kn$ is a multiple of $P$. Give your answer in terms of $k$ if necessary.

$p = 2^2 \times 3 \times k$

$27kn = 3^3 kn$

$= 3^3 (3k)(2^2 k) \boxed{[M1]}$

Answer (b) $n = \boxed{[2]}$
Kai Xuan has written down seven numbers.
The mean of these numbers is 8, the median is 7 and the mode is 11.
The smallest number is an even prime number and the largest number is eight times the smallest number.
The second and third numbers are consecutive numbers.

Find the seven numbers.

\[ m_1 \] \[ x_1, x_2, x_3, x_4, x_5, x_6, x_7 \]
\[ 2 \quad 7 \quad 11 \quad 11 \quad 16 \]
\[ 2 + 7 + 11 + 11 + 16 + x_2 + x_3 = 7 \]
\[ : x_2 + x_3 = 9 \Rightarrow x_2 = 4, x_3 = 5. \]

[ A1 ] Answer = 2, 4, 5, 7, 11, 11, 16

---

8

Rearrange the formula \( P = \frac{u^2 + 5}{u^2 - a} \) and make \( u \) the subject of the formula.

\[ \frac{u^2 - a}{u^2} \cdot u^2 - a = -u^2 + 5 \]
\[ u + v = -u^2 + 5 \] [ M1 ]
\[ u + v = ar + 5 \]
\[ u^2(v + 1) = ar + 5 \] [ M1 ]
\[ u = \pm \sqrt{\frac{ar + 5}{v + 1}} \] [ A1 ]

\[ \pm \sqrt{\frac{ar + 5}{v + 1}} \]

Answer \( u = \quad \) \[ u = \quad \]
The graph shows the total revenue, in billion dollars, of three different fast food chains.

State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer:

1. The size of the fries and hence the area was used to compare the global revenue.

2. Total area for McDonald's appears to be twice that of Burger King, but in fact, the revenue is more than 3 times of the latter.
10 (a) Solve the inequalities $8 + x < 10 + \frac{3}{2}x \leq 15.5 - 2x$.

$8 + x < 10 + \frac{3}{2}x \quad \text{and} \quad 10 + \frac{3}{2}x \leq 15.5 - 2x$ [M1]

$-\frac{1}{2}x < 2 \quad \text{and} \quad x \leq \frac{11}{7}$ [M1]

$x > 4 \quad \text{and} \quad x \leq \frac{11}{7}$ [M1]

The solution is $-4 < x \leq \frac{11}{7}$ [A1]

Answer (a) $x = \frac{-4 < x \leq \frac{11}{7}}{7}$ [3]

(b) Hence, write down the largest rational number that satisfies

$8 + x < 10 + \frac{3}{2}x \leq 15.5 - 2x$

Answer (b) $x = \frac{\frac{11}{7}}{7}$ [1]
The first four terms in a sequence of numbers $T_1, T_2, T_3, T_4$ are as follow

$$T_1 = \frac{2}{1} - \frac{3}{2}$$

$$T_2 = \frac{3}{2} - \frac{4}{2^2}$$

$$T_3 = \frac{4}{2^2} - \frac{5}{2^3}$$

$$T_4 = \frac{5}{2^3} - \frac{6}{2^4}$$

\vdots

(a) Write down the $n$th line and show that it can be expressed as $T_n = \frac{n}{2^n}$.

Answer (a) ................................ [3]

(b) Hence or otherwise, evaluate the following sum and leave your answer as a fraction.

$$T_1 + T_2 + T_3 + \cdots + T_{11}$$

\[
\begin{align*}
T_1 + T_2 + \cdots + T_{11} &= \left(\frac{2}{1} - \frac{3}{2}\right) + \left(\frac{3}{2} - \frac{4}{2^2}\right) + \left(\frac{4}{2^2} - \frac{5}{2^3}\right) + \cdots + \left(\frac{11}{2^9} - \frac{12}{2^{10}}\right) + \left(\frac{12}{2^{10}} - \frac{13}{2^{11}}\right) \\
&= \frac{2}{1} - \frac{13}{2^{11}} \\
&= \frac{4083}{2^{11}}
\end{align*}
\]

Answer (b) ................................ [2]
\[ A = \{ \text{points lying on the line } 2x + y = 8 \} \]
\[ B = \{ \text{points lying on the line } 3x - 4y = 12 \} \]
\[ C = \{ \text{points lying on the line } mx - 4y = c \} \]

(a) Is \((-1, 6) \in A\)? Explain your answer clearly.

\[
\text{Sub} (-1, 6) \text{ into } A
\]
\[
\text{L.H.S.} = 2(-1) + 6 = 4
\]
\[
\text{R.H.S.} = 8, \text{ since L.H.S } \neq \text{ R.H.S.}, (-1, 6) \notin A \] \[\{B1\} \]  \[\{\] \[\]

(b) Find the element \(p\) such that \(p \in (A \cap B)\).

\[
2x + y = 8 \quad \text{(1)}
\]
\[
3x - 4y = 12 \quad \text{(2)}
\]

\[
\text{Sub (1) into (2)}
\]
\[
3x - 4(8 - 2x) = 12 \]  \[\{M1\} \]
\[
11x = 44
\]
\[
x = 4
\]
\[
y = 0
\]
\[
p = (4, 0) \]  \[\{A1\} \]

Answer (b) \(p = (4, 0)\) \[\{2\} \]

(c) Write down a possible value of \(m\) and of \(c\) such that \(B \cap C = \emptyset\).

\[
B \cap C = \emptyset \implies \text{No Solution.}
\]

The line \(y = \frac{3}{4}x - 3\) and \(mx - 4y = c\) must be parallel but cannot have the same \(y\)-intercept.

\[
m = 3 \]  \[\{B1\} \]
\[
c \neq 12 \]  \[\{B1\} \]

Answer (c) \(m = 3\) \[\{1\} \]
\[
c \neq 12 \]  \[\{1\} \]

Any Value Except 12.
The diagram below shows an irregular hexagon. Calculate the value of $a + b + c + d + e + f$.

Sum of interior $\angle$s in a hexagon: $(6-2) \times 180^\circ = 720^\circ$

$360^\circ \times 6 = 2160^\circ$

Sum of reflex $\angle$s: $a + c + e + f = 2160^\circ - 720^\circ = 1440^\circ$ [AI]

Answer $1440^\circ$ [2]

Jia Lung invested some money in her savings account for 4 years. The rate of compound interest was fixed at 4% per annum compounded annually. At the end of 4 years, there was $8436.48 in her account.

How much did Jia Lung invest in the account?

$A = P \left(1 + \frac{r}{100}\right)^n$

$8436.48 = P \left(1 + \frac{4}{100}\right)^4$ [M1]

$8436.48 = P(1.04)^4$

$P = \frac{8436.48}{(1.04)^4} = \$7211.54$ [A1]

Answer $\$7211.54$ [3]
Akshay jogs at a speed of 10 km/h.
One evening he jogged around his neighborhood for 1 hour 30 minutes.

(a) Calculate the distance that Akshay covered.

\[
\begin{align*}
10 \text{ km/h} \times 1.5 \text{ h} &= 15 \text{ km} \\
\text{Answer (a)} &= 15 \text{ km} \\
\end{align*}
\]

(b) Given that the scale of the neighborhood is 1: 25000, find in cm, the map distance that he covered.

\[
\begin{align*}
1 \text{ cm} : 25000 \text{ cm} \\
1 \text{ cm} : 0.25 \text{ km} \\
15 \text{ km} \Rightarrow 15 \text{ cm} \\
0.25 \text{ km} = 60 \text{ cm} \quad \text{Answer (b)} = 60 \text{ cm} \\
\end{align*}
\]

(c) A reservoir located in his neighborhood occupies a total area of 1.70 cm² on the map. What is the actual area, in m², of the reservoir?

\[
\begin{align*}
1 \text{ cm}^2 : 625000000 \text{ cm}^2 \\
1 \text{ cm}^2 : 625.00 \text{ m}^2 \\
1.70 \text{ cm}^2 : 1.70 \times 625.00 \text{ m}^2 \\
&= 10625 \text{ m}^2 \\
\text{Answer (c)} &= 10625 \text{ m}^2 \\
\end{align*}
\]
PQR is a right-angled triangle. QRT is a straight line. 
PR = 12 cm and QR = 19 cm.

Find the values of the following, giving your answer to two decimal places where necessary.

(a) \( \tan \angle PQR \)

\[
\tan \angle PQR = \frac{QP}{PQ} = \frac{\sqrt{19^2 - 12^2}}{12} = \frac{\sqrt{361 - 144}}{12} = \frac{\sqrt{217}}{12} \\
= 0.81
\]

Answer (a) \( 0.81 \) [2]

(b) \( \cos \angle TRP \)

\[
\cos \angle TRP = \cos (180^\circ - \angle TRP) \\
= -\cos (\angle TRP) \\
= -\frac{12}{19}
\]

Answer (b) \( -\frac{12}{19} \) [2]
In the diagram, $O$ is the centre of the circle $BCD$ with radius 20 cm and $CD$ is the diameter of the circle. The ratio of the length $AB$ to the length $AD$ is 0.5.

$A$ is a point on $BC$ produced such that $AD$ is a tangent to the circle at $D$.

Calculate the area of the shaded region.

$$
\times ADB = \sin^{-1} \left( \frac{1}{2} \right) = \frac{\pi}{6} \text{ rad}
$$

$$
\times BDC = \frac{\pi}{2} \text{ rad} - \frac{\pi}{6} \text{ rad} = \frac{\pi}{3} \text{ rad} \left( \text{tangent at point} \right)
$$

$$
\times BCD = \pi \text{ rad} - \frac{\pi}{2} \text{ rad} - \frac{\pi}{3} \text{ rad} = \frac{\pi}{6} \text{ rad} \left[ \text{AIJ} \right]
$$

$$
\times BOD = 2 \times \times BCD \left( \text{At circumference} \right)
$$

$$
\times BOD = 2 \times \frac{\pi}{3} \text{ rad} = \frac{\pi}{3} \text{ rad} \left[ \text{MIJ} \right]
$$

Area of shaded region = Area of sector $OBD$ - Area of $\triangle OBD$

$$
= \frac{1}{2} (20)^2 \left( \frac{\pi}{3} \right) - \frac{1}{2} (20)^2 \sin \left( \frac{\pi}{3} \right) \left[ \text{MIJ} \right]
$$

$$
= 56.2 \text{ cm}^2
$$

Answer (b) .........................cm$^2$ [5]
The table below shows the ages of 16 employees who work part-time at a cafe.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>21</td>
<td>16</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>17</td>
<td>19</td>
<td>65</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>22</td>
<td>17</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>19</td>
<td>19</td>
<td>18</td>
<td></td>
</tr>
</tbody>
</table>

(a) Complete the dot diagram to show the distribution of the ages of the employees.

![Dot diagram](image)

**Answer (a)**

(b) Find the median of the distribution of ages.

\[
\text{Median} = \frac{20 + 21}{2} = 20.5 \quad [B1]
\]

**Answer (b) Median = 20.5 years**

(c) Calculate the mean age of the employees.

\[
\text{Mean} = \frac{\sum fx}{\sum f} = \frac{366}{16} = 22.9 \quad [B1]
\]

**Answer (c) Mean = 22.9 years**

(d) Pranav made the following statement:

"The mean is the most accurate way to determine the average age of the employees" Validate if Pranav statement is true.

1. **Not Valid** [B1]
2. Mean is affected by extremal points in the data set. [B1]
Allyson participates in a game show. In order to win a prize, she has to navigate through a maze. The prize is located at checkpoint X. There are no prizes awarded at checkpoint Y and Z. The diagram below shows four junctions A, B, C and D in the maze. Once Allyson runs past a junction, she is not able to make a turnaround.

The probability that Allyson goes straight, without changing direction, at every junction is \( \frac{3}{7} \).

(a) Find the probability that Allyson hits the dead end.

\[
P(\text{Allyson hits the dead end}) = \left( \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} \right) + \left( \frac{4}{7} \times \frac{3}{7} \times \frac{4}{7} \right) \quad \text{[M1]}
\]

\[
= \frac{64}{343} + \frac{48}{343} \quad \text{[M2]}
\]

\[
= \frac{112}{343} \quad \text{[A1]}
\]

Answer (a) \( \frac{112}{343} \) \[2\]

(b) Find the probability that Allyson wins a prize.

\[
P(\text{Allyson wins a prize}) = \left( \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} \right) + \left( \frac{4}{7} \times \frac{3}{7} \times \frac{4}{7} \right) \quad \text{[M1]}
\]

\[
= \frac{64}{343} + \frac{48}{343} \quad \text{[M2]}
\]

\[
= \frac{112}{343} \quad \text{[A1]}
\]

Answer (b) \( \frac{112}{343} \) \[3\]
Two similar jugs have base area of 45 cm$^2$ and 125 cm$^2$.

(a) Find the ratio of the height of the smaller jug to the ratio of the height of the larger jug.

\[
\left( \frac{h_{\text{small}}}{h_{\text{large}}} \right)^2 = \frac{45}{125} \Rightarrow \frac{h_{\text{small}}}{h_{\text{large}}} = \sqrt{\frac{45}{125}} = \frac{3}{5} \quad \text{[BJ]}
\]

(b) The surface area of the bottom of the smaller jug is 65 cm$^2$.
Find the surface area of the bottom of the larger jug.

Answer (a) \( \frac{3}{5} \) \quad [1]

(c) The capacity of the larger jug is 2.5 litres. Find the capacity of the smaller jug. Give your answer in cubic centimetres.

\[
\left( \frac{3}{5} \right)^3 = \frac{V_{\text{small}}}{2.5 \text{ L}} \quad \text{[MJ]}
\]

\[
V_{\text{small}} = \frac{2.5}{125} \times 2.5 \text{ L}
\]

\[
= 0.54 \text{ L}
\]

\[
= 540 \text{ cm}^3 \quad \text{[AI]}
\]

Answer (b) \( 175 \) cm$^2$ [2]

Answer (c) \( 540 \) cm$^3$ [2]
OPQR is a parallelogram such that \( PQ = (2, 4) \) and \( P \) is the point \((3, 2)\).

(a) Express \( \overrightarrow{RP} \) as a column vector.

\[
\overrightarrow{RP} = \overrightarrow{OQ} + \overrightarrow{QP} \\
= \overrightarrow{QP} + \overrightarrow{OP} \\
= \overrightarrow{-PQ} + \overrightarrow{OP} \quad [M1] \\
= \left(\begin{array}{c}
2 \\
4
\end{array}\right) + \left(\begin{array}{c}
3 \\
2
\end{array}\right) \\
= \left(\begin{array}{c}
-2 \\
2
\end{array}\right) \quad [A1]
\]

Answer (a) .................................................. [2]

(b) The point \( J \) lies on \( \overrightarrow{RP} \) produced such that \( PJ = 2\overrightarrow{RP} \).

Show that \( \overrightarrow{PJ} = \left(\begin{array}{c}
3 + m \\
2 - 2m
\end{array}\right) \).

\[
\overrightarrow{PJ} - \overrightarrow{OP} = m(\overrightarrow{RP}) \\
\overrightarrow{PJ} = m(\overrightarrow{RP}) + \overrightarrow{OP} \\
= m\left(\begin{array}{c}
-2 \\
2
\end{array}\right) + \left(\begin{array}{c}
3 \\
2
\end{array}\right) \\
= \left(\begin{array}{c}
3 + m \\
2 - 2m
\end{array}\right) \quad [A1]
\]

[2]
The diagram shows the speed time graph of a car. The car starts from rest and accelerates uniformly to a speed of 15 m/s in 10 seconds. The car then travels at a constant speed for some time before it decelerates uniformly at 0.75 m/s² until it comes to rest. The whole journey takes one minute.

(a) Given that the speed of the car after \( x \) seconds is \( v \) m/s, express \( v \) in terms of \( x \).

By similar \( \triangle s \):

\[
\frac{15}{10} = \frac{v}{x}
\]

\[ v = 1.5x \]

Answer \( v = 1.5x \) \[1\]

(b) For how long does the car travel at the maximum speed?

Let the time where he ends at constant speed be \( t \).

\[
\frac{(0-15) \text{ m/s}}{(60-t) \text{ s}} = -0.75 \text{ m/s}^2 \]

\[ t = 40 \text{ s} \]

Answer \( 40 \) \[2\]

Duration: \( 40 \text{ s} - 10 \text{ s} = 30 \text{ s} \) \[A1\]

(c) Calculate the total distance travelled by the car during this 1 minute journey.

\[
\frac{1}{2} (60 + 30) \times 15 \text{ m/s} \]

\[ = 675 \text{ m} \]

Answer \( 675 \text{ m} \) \[2\]
(d) Hence, sketch the distance time graph for the whole journey, indicating all relevant values in your sketch.

Distance (m)

\[ 0 \quad 10 \quad 20 \quad 30 \quad 40 \quad 50 \quad 60 \]

\[ 75 \quad 125 \quad 175 \quad 225 \]

[BI]: Correct Shape
[BI]: Correct Values
Mathematics
Paper 2
Secondary 4 Express / Secondary 5 Normal (Academic)
3 September 2019
2 hours 30 minutes
Candidates answer on the Question Paper.
Additional Materials: Graph Paper

Read These Instructions First
Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is required for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give your
answer to three significant figures. Give answer in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms
of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 100.

Do Not Open This Paper Until You Are Told To Do So

Setter: Mrs Jane Cheng

This document consists of 21 printed pages, including this cover page.
Answer all the questions.

1 (a) Solve the inequality \( \frac{x+2}{3} \geq \frac{4-x}{7} \).

\[ 7(x+2) \geq 3(4-x) \]
\[ 7x + 14 \geq 12 - 3x \]
\[ 10x \geq -2 \]
\[ x = -\frac{1}{5} \]

(b) Express as a single fraction in its simplest form \( \frac{2x}{(3x-5)^2} - \frac{x}{3x-5} \)

\[ = \frac{2x-x(3x-5)}{(3x-5)^2} \]
\[ = \frac{2x-x}{3x-5} \]

(c) Simplify \( \left( \frac{27a^6}{b^{12}} \right)^{-\frac{1}{2}} \)

\[ = \left( \frac{b^{12}}{27a^6} \right)^{\frac{1}{3}} \]
\[ = \frac{b^2}{3a^2} \]
(d) Simplify \( \frac{24p^{2}q^{2}}{5r^{3}} \div \frac{8p^{4}r^{3}}{15q^{3}} \):

\[
= \frac{24p^{2}q^{2}}{5r^{3}} \times \frac{15q^{3}}{8p^{4}r^{3}} \tag{m1}
\]

\[
= \frac{9q^{2+3}p^{3-4}}{r^{3+1}} \tag{m1}
\]

\[
= \frac{9q^{5}}{pr^{4}} \tag{A1}
\]

(e) Solve the equation \( \frac{15}{2x+2} = 2x + 8 \):

\[
\begin{align*}
\text{Islandwide Beltway} & \quad x + 6 - 15 = 0 \\
2x^2 + 7x - 9 &= 0 \\
(2x + 9)(x - 1) &= 0 \\
2x + 9 &= 0 \quad \text{or} \quad x - 1 &= 0 \\
x &= -\frac{9}{2} \quad \text{or} \quad x = 1
\end{align*}
\]

\[
\boxed{\begin{align*}
\text{Islandwide Beltway} & \quad x + 6 - 15 = 0 \\
2x^2 + 7x - 9 &= 0 \\
(2x + 9)(x - 1) &= 0 \\
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x &= -\frac{9}{2} \quad \text{or} \quad x = 1
\end{align*}}
\]

\[
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2x + 9 &= 0 \quad \text{or} \quad x - 1 &= 0 \\
x &= -\frac{9}{2} \quad \text{or} \quad x = 1
\end{align*}}
\]
2 A series of diagrams of shaded and unshaded small squares is shown below. The shaded squares are those which lie on the diagonals of the diagram.

(a) Copy and complete the table below.

<table>
<thead>
<tr>
<th>Diagram, ( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>( (2n-1)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of shaded squares, ( S )</td>
<td>1</td>
<td>5</td>
<td>9</td>
<td>13</td>
<td>17</td>
<td>( (2n-1)^2 )</td>
</tr>
<tr>
<td>Number of unshaded squares, ( U )</td>
<td>0</td>
<td>4</td>
<td>16</td>
<td>26</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>Total number of squares, ( T )</td>
<td>1</td>
<td>9</td>
<td>25</td>
<td>49</td>
<td>81</td>
<td>( (2n-1)^2 )</td>
</tr>
</tbody>
</table>

(b) By observing the number patterns, without drawing further diagrams,
(i) write down the total number of squares in diagram \( 1 \),
\[ n = 1, \quad (2n-1)^2 = (2\times1-1)^2 = 1^2 = 1 \]  
(ii) find an expression, in terms of \( n \), for the total number of squares, \( T \).
\[ T = (2n-1)^2 - 4n^3 - 4n + 1 \]  

(c) (i) Find an expression, in terms of \( n \), for the number of shaded squares, \( S \).
\[ S = 4n - 3 \]  
(ii) Write down the number of the diagram that has 41 shaded squares.
\[ 4n - 3 = 41 \]
\[ 4n = 44 \]
\[ n = 11 \]  

(d) Hence, or otherwise, find an expression, in terms of \( n \), for the number of unshaded squares, \( U \).
\[ U = T - S \]
\[ = (2n-1)^2 - (4n^3 - 3) \]  
\[ = 4n^2 - 4n + 1 - 4n + 2 \]  
\[ = 4n^2 - 8n + 4 \]  
\[ = 4(n^2 - 2n + 1) \]
3. \( P \) is the point \((-5, 12)\) and \( Q \) is the point \((5, -4)\).

(a) Find the length of \( PQ \).

\[
\ell = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}
\]
\[
= \sqrt{(12 + 4)^2 + (-5 - 5)^2}
\]
\[
= \sqrt{3 \times 6}
\]
\[
= 18.67
\]
\[
= 18.9 \text{ units (3 s.f.)}
\]

(b) Find the equation of the line \( PQ \).

\[
y = \frac{y_2 - y_1}{x_2 - x_1} \cdot x + c
\]
\[
= \frac{-8}{5} \cdot 5 + 4
\]
\[
y = -8x + 4 \quad \text{(A1)}
\]

OR

\[
5y = -8x + 20
\]

\[
y = \frac{-8}{5}x + 4
\]

\[
-4 = \frac{-8}{5} \cdot 5 + c
\]
\[
c = 4 \quad \text{(m)}
\]
(e) The equation of the line $l_1$ is $8x + 5y + 10 = 0$.

(i) Show how you can decide whether the line $l_1$ does or does not intersect the line $PQ$?

\[ l_1 \quad 5y = -8x - 10 \quad m = -\frac{8}{5} \]

\[ y = -\frac{8}{5}x - 2 \]

Both equations have the same gradient, they are parallel to each other.

Hence, the line $l_1$ does not intersect with $PQ$.

(ii) The equation of line $l_2$ is $3y = 4x - 30$.

Find the coordinates of the point of intersection of the line $l_1$ and the line $l_2$.

\[ 8x + 5y = -10 \]

\[ 4x - 3y = 30 \]

Sub $y = -8$ into eqn (1)

\[ 8x + 5(-8) = -10 \]

\[ 8x = -10 + 40 \]

\[ 8x = 30 \]

\[ x = \frac{30}{8} \]

\[ x = 3.75 \]
Mrs Tan is a Korean Language teacher. She conducts classes for basic and advanced students on weekdays and weekends. Each student has a 15-week block of lessons with one lesson per week. The matrix \( K \) shows the number of students she teaches each week in one 15-week block.

\[
K = \begin{pmatrix}
12 & 3 \\
5 & 8 \\
\end{pmatrix}
\]

(a) Evaluate the matrix \( P = 15K \).

\[
P = 15 \begin{pmatrix}
12 & 3 \\
5 & 8 \\
\end{pmatrix}
= \begin{pmatrix}
180 & 45 \\
75 & 120 \\
\end{pmatrix}
\]

(b) Mrs Tan charges $20 for each basic lesson and $32 for each advanced lesson. Represent the lesson charges in a \( 2 \times 1 \) matrix \( L \).

\[
L = \begin{pmatrix}
20 \\
32 \\
\end{pmatrix}
\]

(c) Evaluate the matrix \( T = \begin{pmatrix}
120 & 45 \\
75 & 120 \\
\end{pmatrix} \begin{pmatrix}
20 \\
32 \\
\end{pmatrix} \).

\[
T = \begin{pmatrix}
120 & 45 \\
75 & 120 \\
\end{pmatrix} \begin{pmatrix}
20 \\
32 \\
\end{pmatrix}
= \begin{pmatrix}
5040 \\
5340 \\
\end{pmatrix}
\]
(d) State what the elements of \( T \) represent.

The elements of \( T \) represent the total amount of money Mrs Tan collects for a 15-week block of lessons on weekdays and weekends respectively.

(e) Mrs Tan wants to attract more students, so in the next 15-week block she reduces her prices by 10%.
For this block of lessons, on weekdays she has 15 basic students and 5 advanced students.
On weekends she has 7 basic students and 6 advanced students.

Calculate the total amount of money she earns for this 15-week block of lessons.

\[
0.9 \times \left( 15 \times 20 + (5+6) \times 32 \right) \times 15
\]

\[
= \$ 10,692
\]
The cumulative frequency graph shows the distribution of the age groups of the Fitness First club.

(a) Cumulative Frequency

(i) Complete the grouped frequency table for the ages of the members.

<table>
<thead>
<tr>
<th>Age (x)</th>
<th>20 ≤ x &lt; 30</th>
<th>30 ≤ x &lt; 40</th>
<th>40 ≤ x &lt; 50</th>
<th>50 ≤ x &lt; 60</th>
<th>60 ≤ x &lt; 70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>10</td>
<td>25</td>
<td>90</td>
<td>60</td>
<td>15</td>
</tr>
</tbody>
</table>

(ii) Calculate the mean age of each member.

\[
\bar{x} = \frac{(10 \times 25 + 25 \times 35 + 90 \times 45 + 60 \times 55 + 15 \times 65)}{200}
\]

\[
= \frac{4725}{200} = 23.625 \text{ years old}
\]

(iii) Calculate the standard deviation.

\[
SD = \sqrt{\frac{\sum (x^2)}{N} - \left(\frac{\sum x}{N}\right)^2} = \sqrt{\frac{464000}{200} - (47.25)^2}
\]

\[
= 9.3508
\]

\[
= 9.3 \times (3 \text{ s.f.)}
\]
(iv) Find the percentage of members whose age is 30 years old and above but less than 60 years old.

\[
\text{No. of members} = 25 + 90 + 60 = 175
\]

\[
\frac{175}{200} \times 100\% = 87.5\% \quad \text{(4)}
\]

(v) A magazine article stated that citizens aged 50 and above are less active than those aged below 40.

Comment on whether the data from the Fitness First club supports this claim.

\[
\begin{align*}
\text{percentage of members} & \geq 50 \quad \frac{75}{100} \times 100\% = 75\% \quad \text{(4)} \\
\text{percentage of members} < 40 \quad \frac{30}{100} \times 100\% = 30\%
\end{align*}
\]

It is not true that citizens \( \geq 50 \) yrs old are less active \( \text{\textcolor{red}{(4)}} \)

(b) The table below gives information about the ages of the members in the Any Time Fitness club.

<table>
<thead>
<tr>
<th></th>
<th>Members aged under 50</th>
<th>Members aged 50 or over</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>35</td>
<td>34</td>
</tr>
<tr>
<td>Female</td>
<td>36</td>
<td>30</td>
</tr>
</tbody>
</table>

(i) One of these members is selected at random. Find, as a fraction in its lowest terms, the probability that he or she is under 50.

\[
\frac{86}{150} \times \frac{150}{149} = \frac{43}{749} \quad \text{(4)}
\]

(ii) Two of the members are selected at random. Find the probability that

(a) both members are female,

\[
p(\text{both are female}) = \frac{66}{150} \times \frac{65}{149} = \frac{143}{745} \quad \text{(4)}
\]

(b) they are both aged 50 or over, but only one is a male member.

\[
p(\text{both \( \geq 50 \) yrs old, only one is a male member}) = \frac{34}{150} \times \frac{30}{149} + \frac{30}{150} \times \frac{34}{149} = \frac{69}{745} \quad \text{(4)}
\]
A litre of 95-octane unleaded petrol cost $x in January 2019.

(a) Mr Ang paid $85.50 for his petrol. Write down in terms of $x$, the amount of petrol bought. 

\[ \text{Mr Ang} = \frac{85.50}{x} \text{ litre} \]  

Mr Bala paid $100 for his 98-octane unleaded petrol which cost 25 cents more per litre.

(b) Write down in terms of $x$, the amount of petrol bought by Mr Bala.

\[ \text{Mr Bala} = \frac{100}{x + 0.25} \text{ litre} \]

(c) If Mr Ang received 2 litres less petrol than Mr Bala, write down an equation to represent this information and show that it can reduce to $16x^2 - 112x + 171 = 0$. 

\[ \frac{100}{x + 0.25} = 2 \times \frac{x}{x + 0.25} \]

\[ 2x - \frac{29}{2} = 2x + \frac{1}{2}x \]

\[ 2x^2 + \left( \frac{1}{2} - \frac{29}{2} \right) + 21 \frac{3}{8} = 0 \]

\[ 2x^2 - \frac{29}{2} x + \frac{171}{8} = 0 \]

\[ 16x^2 - 112x + 171 = 0 \text{ shown} \]
(d) Solve the equation \(16x^2 - 112x + 171 = 0\). [2]

\[
\begin{align*}
  a &= 16 & b &= -112 & c &= 171 \\
  x &= \frac{-(-112) \pm \sqrt{(-112)^2 - 4(16)(171)}}{2 \times 16} \\
  x &= \frac{47 \pm \sqrt{2201 - 1104}}{32} \\
  x &= \frac{47 \pm 43}{32} \\
  x &= \frac{90}{32} \text{ or } x = \frac{2}{32} \\
  x &= 2.8125 \text{ (2 d.p.)} \text{ or } x = 0.625
\end{align*}
\]

(e) The price of the 98-octane unleaded petrol in January 2019 was a reduction of 7\% on the price in December 2018. Find the price of the 98-octane unleaded petrol in December 2018 if it cost less than $3 for a litre of 95-octane unleaded petrol in January 2019.

95-octane:
\[
x = \$2.25 \text{ (Jan 2019)}
\]

98-octane:
\[
2.25 + 0.25 = \$2.50 \text{ per litre in Jan 2019}
\]

\[
\frac{2.50}{0.93} \times 100\% = 2.68817
\]

\[
= \$2.69 \text{ (2 d.p.)}
\]

Price of 98-octane in Dec 2018 was $2.69
7. The diagram shows a container consisting of a square bottom with rectangular sides, each 20 cm by 4 cm, and a regular pyramid on top with perpendicular height given by $VX$. Water is poured into the container till the brim of the cuboid.

![Diagram of container]

(a) Find the height $VX$ of the pyramid.

$$\text{Slant height } l = \sqrt{24^2 - 10^2} = \sqrt{476} \text{ cm}$$

$$VX^2 = (\sqrt{476})^2 - 10^2$$

$$VX = \sqrt{376} = 19.407 \text{ cm}$$

(b) Calculate the total surface area of the container.

$$\text{Total surface area} = 4 \times (20 \times 4) + 8 \times 20 + 4 \times \frac{1}{2} \times 20 \times \sqrt{476}$$

$$= 320 + 872.697$$

$$= 1592.697$$

$$= 1590 \text{ cm}^2$$

(c) Find the volume of water in the cuboid.

$$\text{Volume of water in the cuboid} = 20 \times 20 \times 14$$

$$= 1600 \text{ cm}^3$$
The container is now inverted as shown in the diagram below.

\[ \frac{V_1}{V_2} = \left( \frac{h_1}{h_2} \right)^3 \]

Let \( \frac{h_1}{h_2} = x \)

\[ h_1 = 19.39x \]

(d) Calculate the depth of the water in the pyramid when inverted.

Let \( x \) be the ratio of the height of water to the height of the pyramid (x).

\[ \frac{1}{3} (20x)(20x)(19.39x) = 160 \]  
\[ 19.39x^2 = 1 \]  
\[ x^2 = \frac{1}{19.39} \]

(x) Height of water = \( 0.852 \text{ cm} \)

(e) Another smaller container, which is geometrically similar, has a square base of 225 cm². Both containers are made of the same material. Find the mass of the smaller container in grams, given that the mass of the empty larger container is 1.28 kg.

\[ \frac{A_1}{A_2} = \left( \frac{l_1}{l_2} \right)^2 \]

\[ \frac{l_1}{l_2} = \sqrt{\frac{225}{400}} \]

\[ = \frac{3}{4} \]

\[ \frac{V_1}{V_2} = \left( \frac{3}{4} \right)^3 \]

\[ = \frac{27}{64} \]
8 In the diagram, $OABC$ is a parallelogram and $D$ is the midpoint of $BC$. $BE$ and $OC$ produced intersect at the point $F$. $BE:BF = 1:3$ and $OC:OF = 1:2$.

Let $\overrightarrow{OA} = a$ and $\overrightarrow{OC} = c$.

(a) Express and simplify the following vectors in terms of $a$ and $c$.

(i) $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = -a + c$

(ii) $\overrightarrow{BF} = \overrightarrow{BO} + \overrightarrow{OF} = -\overrightarrow{AO} + \overrightarrow{OF} = -a + \frac{2}{3}c$

(iii) $\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD} = \frac{1}{2}a$
(iv) \( \overrightarrow{OE} \)

\[ \overrightarrow{OE} = \overrightarrow{OF} + \overrightarrow{FE} \]

\[ = 2 \varepsilon + \frac{3}{2} \overrightarrow{FE} \quad (\text{M1}) \]

\[ = 2 \varepsilon - \frac{3}{2} (-\varepsilon + \varepsilon) \]

\[ = \frac{4}{3} \varepsilon + \frac{3}{2} \varepsilon \]

\[ = \frac{5}{3} (\varepsilon + \frac{3}{2} \varepsilon) \quad (\text{M1}) \]

(b) State two facts about the vectors \( \overrightarrow{OD} \) and \( \overrightarrow{OE} \) from the results in (a).

\[ \overrightarrow{OE} = \frac{5}{3} \overrightarrow{OB} \]

\[ \frac{\overrightarrow{OE}}{\overrightarrow{OB}} = 5:3 \]

1 \( \overrightarrow{OE} \) is parallel to \( \overrightarrow{OB} \), \( O, D, E \) and \( E \) are collinear. (\text{M1})

2 \( \overrightarrow{OE} : \overrightarrow{OD} = 5:3 \) (\text{M1})

(c) Find the ratio of the areas of

(i) \( \triangle ODF \) and \( \triangle OFE \),

\[ \text{Area of } \triangle ODF : \text{Area of } \triangle OFE = 3 \] (\text{M1})

(ii) \( \triangle OCD \) and \( \triangle ABC \),

\[ \text{Area of } \triangle OCD : \text{Area of } \triangle ABC = 1:4 \]

(iii) \( \triangle OCD \) and \( \triangle ABE \).

\[ \begin{array}{ccc}
\triangle OCD & \triangle ABE & \triangle BCF \\
1 & 4 & 1 \\
2 & 1 & \text{M1} \\
1 & 4 & 2 \\
\end{array} \]

\[ \text{Area of } \triangle OCD : \text{Area of } \triangle ABE = 1:6 \] (\text{M1})
Two school teams, *Novotel* and *Temasek*, are participating in an Amazing Race in Bishan Park. The diagram shows the paths in the park.

The teams assemble at *P* before heading to *Q* to start the race.

*P* is due north of *R*.

The bearing of *R* from *Q* is 241°.

The distance *PR* is 72 metres and the distance *RQ* is 85 metres.

(a) Find the distance *PQ*. [3]

\[
PQ = \sqrt{72^2 + 85^2 - 2 \times 72 \times 85 \times \cos 61°}
\]

\[
PQ = 80.467
\]

\[
= 80.5 \text{ m (3 s.f.)}
\]
(b) The final station of the race is at $R$, each team is required to find a **clue** that is hidden at point $S$ before completing the race at $R$. The bearing of $S$ from $R$ is $099^\circ$ and $QS$ is 54 metres. Given that there are **two** possible locations for $S$, find the two possible values of angle $RSQ$.

\[
\angle QRS = 99^\circ - 61^\circ = 38^\circ
\]

In $\triangle RQS$

\[
\frac{\sin \angle RQS}{54} = \frac{\sin 38^\circ}{8x}
\]

\[
\sin \angle RQS = \frac{8x \sin 38^\circ}{54}
\]

\[
\angle RQS = \sin^{-1} \left( \frac{8x \sin 38^\circ}{54} \right)
\]

\[
\angle RQS = 72.7188^\circ
\]

\[
\angle RQS = 180^\circ - 72.7188^\circ
\]

\[
= 104.2812^\circ
\]

\[
= 104.3^\circ \quad \text{(1 d.p.)}
\]
(e) Both teams manage to find the clue at the same time and team Novotel is closer to R than team Temasak. Team Novotel claims that they are the winner.

Given that the speed of team Novotel is 30% less than the speed of team Temasak when they travel from S to R.

Do you agree with team Novotel that they will win the race?

Justify your answer with clear working in your calculations.

\[ \text{Novotel} - S_2 - \text{Temasak} - S_1 \]

\[
\begin{align*}
\angle RAS_2 &= 180' - 38' - 104.2812' \\
&= 37.7188' \\
&= 37.7^\circ \\
\angle RAS_1 &= 180' - 38' - 74.868 \\
&= 66.2812 \\
&= 66.3^\circ
\end{align*}
\]

Let the speed of team Temasak be \( x \)

\[
\begin{align*}
\text{Time}_{\text{Novotel}} &= \frac{53.66}{0.7x} = \frac{76.657}{x} \\
\text{Time}_{\text{Temasak}} &= \frac{80.3017}{x}
\end{align*}
\]

Time taken by Team Novotel is less than time taken by Team Temasak.

Yes, I agree that Team Novotel will win the race.
10 Answer the whole of this question on a sheet of graph paper.

The table below gives some values of \( x \) and the corresponding values of \( y \) for \( y = x(1 + x)(5 - x) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>-0.5</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>14</td>
<td>0</td>
<td>-1.375</td>
<td>8</td>
<td>18</td>
<td>( p )</td>
<td>20</td>
<td>0</td>
</tr>
</tbody>
</table>

(a) Find the value of \( p \).

(b) Using a scale of 2 cm to 1 unit, draw a horizontal \( x \)-axis for \(-2 \leq x \leq 5\). Using a scale of 2 cm to represent 5 units, draw a vertical \( y \)-axis for \(-5 \leq y \leq 25\). On your axes, plot the points given in the table and join them with a smooth curve.

(c) By drawing a tangent, find the gradient of the curve where \( x = 4 \).

(d) (i) On the same axes, draw the line \( 2x + y = 12 \) for \(-2 \leq x \leq 5\).

(ii) Write down the \( x \)-coordinates of the points where this line intersects the curve.

(iii) The \( x \)-coordinates of the points where the two graphs intersect are solutions of the equation \( x^3 - 4x^2 + 7x + 12 = 0 \). Find the value of \( A \) and the value of \( B \).

\[
\begin{align*}
y &= x(5 + 4x - x^2) \\
   &= -x^3 + 4x^2 + 5x \\
-2x + 12 &= -x^3 + 4x^2 + 5x \\
-2x + 12 &= -2x + 12 \\
-2x + 12 &= -2x + 12 \\
-2x + 12 &= -2x + 12 \\
A &= -4 \\
B &= -7
\end{align*}
\]

~~~ End of Paper ~~~
Q 7:01

(a) \[ p = \boxed{A1} \]

(b) plotting of pts \[ (1) \]

Smooth curve \[ (1) \]

Label and scale \[ (1) \]

(c) \[ m = \frac{9 - 20}{5 - 4} = -11 \pm \]

(d1) \[ y = -2x + 12 \]

d(11) \[ x = 1.2 \pm 0.1 \] \[ (A2) \]

and \[ x = 4.9 \pm 0.1 \] \[ (A2) \]

(d11) \[ A = -4 \]

(b) \[ B = -7 \] \[ (A1) \]
READ THESE INSTRUCTIONS FIRST

Write your index number, and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142, unless the question requires the answers in terms of $\pi$.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

**ANSWERS TO QUESTIONS 1 TO 15 MUST BE HANDED IN SEPARATELY FROM ANSWERS TO QUESTIONS 16 TO 24.**
Compound interest

Total amount = \( P\left(1 + \frac{r}{100}\right)^n \)

Mensuration

Curved surface area of a cone = \( \pi rl \)

Surface area of a sphere = \( 4\pi r^2 \)

Volume of a cone = \( \frac{1}{3}\pi r^2h \)

Volume of a sphere = \( \frac{4}{3}\pi r^3 \)

Area of triangle \( ABC = \frac{1}{2}ab \sin C \)

Arc length = \( rl \), where \( l \) is in radians

Sector area = \( \frac{1}{2}r^2l \), where \( l \) is in radians

Trigonometry

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

Statistics

Mean = \( \frac{\sum fx}{\sum f} \)

Standard deviation = \( \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} \)
3

**Answer** all the questions.

1. Solve \( \frac{x}{4} + 13 = 6 \).

   \[ \text{Answer } \frac{x}{4} + 13 = 6 \] \[ x = \text{.................} \quad [1] \]

2. Peter boards a bus at bus stop A at 6.50 am. The bus travels to bus stop B at an average speed of 60 km/h to school. He reaches bus stop B at 7.15 am.

   What is the distance between the 2 bus stops?

   \[ \text{Answer } \text{.................} \quad [2] \]

3. John wrote down five numbers.
   The mean of these numbers is 8, the median is 6 and the mode is 5.
   The largest number is three times the smallest number.

   Find the five numbers.

   \[ \text{Answer } \text{.................} \quad [2] \]

4. A field which has an area of 1800 m\(^2\), is used to plant sunflowers.
   It is known that an acre of land, which is about 4047 m\(^2\), can grow an average of 20 000 sunflower plants.
   Each sunflower plant has an average seeding rate of 1500 seeds.

   Calculate an estimate of the total number of seeds that can be harvested from the field, leaving your answer in standard form, correct to 3 significant figures.

   \[ \text{Answer } \text{.................} \quad [2] \]
5 Simplify \(\frac{3x}{5} - \frac{4(2-3x)}{7}\).

\[\text{Answer} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad [2]\]

6 \(\xi = \{x : x \text{ is a positive integer and } x < 25\}\)
\(A = \{x : x \text{ is a perfect square}\}\)
\(B = \{x : x \text{ is an odd number}\}\)

(a) Find \(n(A \cap B)\).

\[\text{Answer} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad [1]\]

(b) On the Venn diagram, shade the region which represents \((A \cap B)'.\)

7 Factorise fully \(6ac + 9ad - 12bd - 8bc\).

\[\text{Answer} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad [2]\]
8 \( A \) is the point \((-2, 5)\) and \( \overrightarrow{BA} = \begin{pmatrix} -4 \\ 8 \end{pmatrix} \).

(a) Find the coordinates of point \( B \).

\[ \text{Answer } B (\ldots \ldots \ldots, \ldots \ldots \ldots) \text{ [1]} \]

(b) Calculate \( |\overrightarrow{BA}| \).

\[ \text{Answer } \ldots \ldots \ldots \ldots \ldots \ldots \ldots \text{ [1]} \]

9 Andrew weighed ten large watermelons from his farm.

The mean mass of the watermelons was 9070 grams.
The standard deviation of the masses of the watermelons was 362.8 grams.

The scales used by Andrew were found to be inaccurate.
The correct mass of each watermelon turns out to be 1650 grams more than Andrew recorded.

Write down the correct values for the mean and standard deviation (SD).

\[ \text{Answer } \text{Mean} = \ldots \ldots \ldots \ldots \ldots \ldots \ldots \text{ g \ [2]} \]
\[ \text{SD} = \ldots \ldots \ldots \ldots \ldots \ldots \ldots \text{ g} \]
10 A man purchased a painting in 2016.  
The value of the painting increased by 8% in 2017.  
In 2018, the value of the painting was $73288.80, an increase of 20% as compared to 2017.  
Express the cost of the painting in 2016 as a percentage of the cost of the painting in 2018.

Answer …………………………. % [3]

11 (a) David makes a mocktail drink for his kids.  
He uses apple juice, lime juice and sparkling water in the ratio 3 : 2 : 7 respectively.  
He uses 2.1 litres of sparkling water.

(i) How much apple juice does he use?

Answer ………………………. litres [1]

(ii) How much mocktail drink does he make altogether?

Answer ………………………. litres [1]

(b) Wen Ni makes a mocktail drink using lemon juice, syrup and coconut water.  
The ratio lemon juice : syrup is \( \frac{1}{2} : \frac{1}{3} \).  
The ratio syrup : coconut water is 1 : 8.

Find the ratio lemon juice : syrup : coconut water.

Answer ……………. : …………. : …………. [2]
12 Jane draws a triangle.
The first angle is 20º bigger than the second angle
The third angle is three times the size of the first angle.

Form an equation and solve it to find the angles of the triangle.

\[ \text{Answer} \quad \theta, \quad \theta, \quad \theta \] [3]

13 It is given that \( x \) and \( y \) are in direct proportion.
The difference in the value of \( y \), when \( x \) is 5 and when \( x \) is 17, is 54.

(a) Find an equation connecting \( x \) and \( y \).

\[ \text{Answer} \quad \] [3]

(b) Find the value of \( x \) when \( y = 84 \).

\[ \text{Answer} \quad \] [1]
14 When written as the product of their prime factors,

\[ m = 2^2 \times 5^2 \times 11 \]
\[ n = 2^3 \times 3 \times 5 \times 7 \]

Find

(a) the largest integer which is a factor of \( m \) and \( n \),

\[ \text{Answer} \]  

(b) the smallest integer \( p \) such that \( mp \) is a perfect cube,

\[ \text{Answer} \]  

(c) the smallest positive integer value of \( q \) such that \( \sqrt[3]{\frac{mn}{q}} \) is an integer.

\[ \text{Answer} \]  

15 (a) Solve the inequalities \(-15 \leq 8 - 3x < 2\).

\[ \text{Answer} \]  

(b) Write down all the integers that satisfy \(-15 \leq 8 - 3x < 2\).

\[ \text{Answer} \]
The diagram shows a regular hexagon and a triangle.

(a) Calculate the sum of the interior angles of the regular hexagon.

Answer \[  \text{ } \circ \] [1]

(b) Calculate the sum of the angles \(a, b, c, d, e, f\) and \(g\).

Answer \[  \text{ } \circ \] [2]
17 The diagram shows a right circular cone which is divided into 3 parts $A$, $B$ and $C$ by planes parallel to the base as shown. $B$ and $C$ are frustums of a cone. The height of each part is $h$ cm.

The curved surface area of $A$ is 200 cm$^2$

(a) Show that $A_A : A_{A+B}$ is 1 : 4, where $A_A$ is the curved surface area of $A$ and $A_{A+B}$ is the combined curved surface areas of $A$ and $B$.

(b) Find $A_B$, the curved surface area of $B$.

Answer .................................. cm$^2$ [1]

(c) Find the ratio of the volume of $B$ to the volume of $C$.

Answer .................................. [3]
18  The diagram shows the positions of three towns $P$, $Q$ and $R$.  
$PQ = 35$ km and $QR = 45$ km.  
The bearing of $P$ from $Q$ is $235^\circ$ and the bearing of $R$ from $Q$ is $205^\circ$.  

Calculate the distance between towns $P$ and $R$.  

\[ \text{Answer} \quad \ldots \ldots \ldots \ldots \quad \text{km} \quad [3] \]
The diagram shows six points $A$, $B$, $C$, $D$, $E$ and $F$ on the circumference of a circle with centre at $O$.

It is given that the diameter $BE$ of the circle is 14 cm, $\angle BED = 35^\circ$ and $AB = ED$.

Calculate the area of the shaded region.

\[
\text{Answer} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \text{cm}^2 \quad [5]
\]
In the diagram, the equation of the line $L_1$ is $2y = 3x - 5$.

The line $L_2$ has the same gradient as the line $L_1$. $A$ and $D$ lie on the $y$-axis. $BC$ is parallel to the $y$-axis and $BC = 8$ units.

(a) Find the equation of the line $L_2$.

Answer ……………………………… [2]

(b) What is the area of quadrilateral $ABCD$?

Answer ……………………………… units$^2$ [1]

(c) Find the value of $\tan \angle ABO$.

Answer ……………………………… [1]
21  (a)  The period of oscillation of a pendulum is given by \( T = 2\pi \sqrt{\frac{l}{g}} \).

Rearrange the formula to make \( l \) the subject.

Answer …………………………. [2]

(b)  A solid is made from a cylinder and a cone.
Both the cylinder and cone have radius 2.5 cm and height 5 cm.

![Diagram of a cylinder and a cone]

Calculate the total surface area of the solid.

Answer ……………………. cm\(^2\) [3]
22 A gift company sells three hamper packages containing packets of biscuits, bars of chocolates and bottles of wine. The cost of one packet of biscuit, one bar of chocolate and one bottle of wine are $7.20, $10.80 and $32.00 respectively.

<table>
<thead>
<tr>
<th></th>
<th>Biscuit</th>
<th>Chocolate</th>
<th>Wine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamper A</td>
<td>6 packets</td>
<td>5 bars</td>
<td>3 bottles</td>
</tr>
<tr>
<td>Hamper B</td>
<td>9 packets</td>
<td>7 bars</td>
<td>4 bottles</td>
</tr>
<tr>
<td>Hamper C</td>
<td>10 packets</td>
<td>8 bars</td>
<td>2 bottles</td>
</tr>
</tbody>
</table>

The quantity of the items packed into each of the three types of hamper packages is shown in the table above.

These information can be represented by the matrices

\[
H = \begin{pmatrix}
6 & 5 & 3 \\
9 & 7 & 4 \\
10 & 8 & 2
\end{pmatrix}
\quad \text{and} \quad
P = \begin{pmatrix}
7.20 \\
10.80 \\
32.00
\end{pmatrix}
\]

(a) Evaluate the matrix \( R = HP \).

Answer \( R = \) ……………………… [2]

(b) State what the elements of \( R \) represent.

………………………………………………………………………………………….… [1]

(c) A total of 20 Hamper A, 25 Hamper B and 30 Hamper C are sold. Represent the total number of hamper packages sold in a \( 1 \times 3 \) matrix \( T \).

Answer \( T = \) …………………… [1]

(d) Using matrix multiplication, find the total amount of money obtained by the gift company from the sales of the hamper packages \( A, B \) and \( C \).

Answer \$ …………………… [2]
23 Some boys are placed into five groups, A, B, C, D and E, based on their heights. The pie chart shows the proportion of boys in each group.
Group A comprises \( \frac{1}{4} \) of the boys, Group B 30% of the boys and Group C has 18 boys.
Group D is represented by a 60° sector.

(a) Find the percentage of the boys who are in Group D.

Answer ……………………….. % [1]

(b) Given that the number of boys in group B is 36, find the total number of boys.

Answer ……………………….. boys [2]

(c) Calculate the number of boys in group E.

Answer ……………………….. boys [2]
24 A bag contains 20 marbles, \( n \) of which are red and the rest are yellow. A marble is chosen at random and not replaced.

(a) Write down, in terms of \( n \), the probability that the marble is yellow.

\[ \text{Answer} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad [1] \]

A second marble is chosen at random.

(b) Find, in terms of \( n \), the probability that both marbles are yellow.

\[ \text{Answer} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad [1] \]

(c) (i) The probability that both marbles are yellow is \( \frac{39}{95} \).

Show that \( n^2 - 39n + 224 = 0 \)

\[ \text{[2]} \]

(ii) Solve the equation \( n^2 - 39n + 224 = 0 \) to find the number of yellow marbles in the bag.

\[ \text{Answer} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad [3] \]
READ THESE INSTRUCTIONS FIRST
Write your index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.
Answers to Questions 1 to 6 must be handed in separately from answers to Questions 7 to 10.
Mathematical Formulae

Compound interest

Total amount \( P \left(1 + \frac{r}{100}\right)^n \)

Mensuration

Curved surface area of a cone = \( \pi rl \)

Surface area of a sphere = \( 4 \pi r^2 \)

Volume of a cone = \( \frac{1}{3} \pi r^2 h \)

Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

Area of triangle \( ABC = \frac{1}{2} ab \sin C \)

Arc length = \( r \theta \), where \( \theta \) is in radians

Sector area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

Trigonometry

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\( a^2 = b^2 + c^2 - 2bc \cos A \)

Statistics

Mean = \( \frac{\sum fx}{\sum f} \)

Standard deviation = \( \sqrt{\frac{\sum f x^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} \)
1 (a) Factorise $27a^4 - 3$.

Answer ............................................. [2]

(b) Express as a single fraction in its simplest form

(i) $\frac{2(x-1)^2}{4y^3} + \frac{6y(x-1)}{8y^2}$,

Answer ............................................. [1]

(ii) $\frac{3}{m-2} - \frac{2}{3m-1}$.

Answer ............................................. [2]

(c) Solve the equation $2^{2-x} = \frac{1}{\sqrt{2^{5x+1}}}$.

Answer ............................................. [2]
(d) (i) Express \( x^2 - 8x - 6 \) in the form \( (x - b)^2 - c \).

*Answer* ………………………………………… [1]

(ii) **Hence** solve the equation \( x^2 - 8x - 6 = 0 \), giving your answers correct to one decimal place.

*Answer* \( x = \ldots \) or \( \ldots \) [3]
In the diagram below, \( OB = BC \) and \( AF = FB \). It is given that \( OA : AE = 2 : 1 \) and \( ED : DC = 4 : 3 \). \( O\overrightarrow{A} = 2\mathbf{a} \) and \( O\overrightarrow{B} = \mathbf{b} \).

(a) Express, as simply as possible, in terms of \( \mathbf{a} \) and/or \( \mathbf{b} \),

(i) \( \overrightarrow{CE} \),

Answer ………………………………………………….. [1]

(ii) \( \overrightarrow{CD} \),

Answer ………………………………………………….. [1]

(iii) \( \overrightarrow{BA} \),

Answer ………………………………………………….. [1]
(iv) \( \overline{OF} \) \\
Answer \………………………………………………………… [2]

(v) \( \overline{FD} \) \\
Answer \………………………………………………………… [2]

(b) Find

(i) \( \frac{\text{Area of } \triangle OBA}{\text{Area of } \triangle OBE} \) \\
Answer \………………………………………………………… [1]

(ii) \( \frac{\text{Area of } \triangle OBA}{\text{Area of } \triangle OCE} \) \\
Answer \………………………………………………………… [2]
3 (a) Celine wishes to buy a new car. The price of the car is $98 000.

(i) The car dealer requires Celine to make a downpayment of $19 600. Express this amount as a percentage of the price of the car.

Answer ..................................% [1]

(ii) After making the downpayment, Celine decides to take a bank loan for the rest of the money to be paid to the car dealer.
Bank OCC charges an interest rate of 2.78% per annum compounded half yearly.
Bank DBB charges a simple interest rate of 2.99% per annum.
If Celine decides to take a seven year loan, which bank should she loan from? Justify your answer.

Answer ..................................................................................................................
..................................................................................................................... [4]

(b) Celine visits the petrol station weekly to refill petrol, refilling 51 litres of petrol each and every time. In order to save on petrol costs, she decides to drive to Johor Bahru, Malaysia, weekly to refill the petrol.

(i) How much does she need to pay weekly to refill petrol in Singapore if the price of petrol is $2.25 per litre?

Answer S$.......................................... [1]
(ii) How much does she need to pay to refill petrol weekly in Johor Bahru if the price of petrol is RM 2.08 per litre?

Answer RM......................... [1]

(iii) (a) How much is Celine able to save weekly if she refills the petrol in Malaysia? Give your answer in terms of Singapore dollars. (Given the exchange rate is S$1 = RM 3.05)

Answer S$............................ [2]

(b) Express the amount of savings as a percentage of the cost of refilling petrol weekly in Singapore.

Answer .........................% [1]
The diagram below shows a circle $ABCD$ with centre $O$. $AT$ and $CT$ are tangents to the circle and angle $ATO = 40^\circ$.

(a) Find, giving reasons for each answer,

(i) angle $AOD$,

$Answer \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots [2]$

(ii) angle $ABC$,

$Answer \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots [2]$

(iii) angle $ADC$,

$Answer \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots [1]$

(iv) angle $OCD$.

$Answer \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots [1]$
(b) Calculate the area of the shaded region, given that the radius of the circle is 5 cm.

Answer ..................cm² [4]
The diagram above shows a prism $ABCDEF$ whose cross section is an isosceles triangle.

It is given that $AB = 8$ cm, $BC = 12$ cm, and $AF = 3$ cm.

(a) Show that $\angle ABF = 21.6^\circ$, correct to 1 decimal place. \[3\]

(b) Find the angle of elevation of $E$ from $B$.

Answer ………………….° \[3\]
(c) (i) Calculate the volume of the prism $ABCDEF$.

Answer …………………… $\text{cm}^3$ [2]

(ii) The prism is melted and moulded into a hemisphere. Calculate the radius of the hemisphere.

Answer …………………… $\text{cm}$ [2]
6 Answer the whole of this question on the sheet of graph paper.

The variables \( x \) and \( y \) are connected by the equation

\[
y = \frac{x^2}{3} + \frac{2}{x} - 3.
\]

Some corresponding values of \( x \) and \( y \), correct to two decimal places, are given in the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>( p )</td>
<td>(-0.67)</td>
<td>(-0.92)</td>
<td>(-0.67)</td>
<td>(-0.12)</td>
<td>(0.67)</td>
<td>(2.83)</td>
<td>( q )</td>
</tr>
</tbody>
</table>

(a) Find the values of \( p \) and \( q \).

Answer \( p = \ldots \ldots \ldots \)

\( q = \ldots \ldots \ldots \) [2]

(b) Using a scale of 2 cm to represent 1 unit on each axis, draw a horizontal \( x \)-axis for \( 0 \leq x \leq 5 \) and a vertical \( y \)-axis for \( -2 \leq y \leq 6 \).

[3]

On your axes, plot the points given in the table and join them with a smooth curve.

(c) Use your graph to find the solutions to the equation \( \frac{x^2}{3} + \frac{2}{x} = 3 \).

Answer \( x = \ldots \ldots \) or \( \ldots \ldots \) [2]
(d) By drawing a tangent, find the gradient of the curve at \((3, 0.67)\).

Answer .................................. [2]

(e) By drawing a suitable line on the graph, solve the equation

\[
\frac{x^2}{3} + \frac{2}{x} + 2x - 6 = 0
\]

Answer Equation of line \(y = \ldots\) [1]

\(x = \ldots\) [1]
In the lucky draw, there are 3 types of tickets to be drawn from a box: $5 tickets, $20 tickets and “N” tickets. If a customer draws a $5 or a $20 ticket, the ticket will be exchanged for a $5 or $20 shopping voucher respectively. However, if the customer draws an “N” ticket in the first draw, the “N” ticket will be placed back into the box and the customer will draw a ticket a second time. The second draw is the final one.

Given \( \frac{3}{5} \) of the tickets are $5 tickets, \( \frac{1}{10} \) of the tickets are $20 tickets and the rest are “N” tickets, complete the tree diagram below.

(i) Find the probability that a customer will draw an “N” ticket.

\[ \text{Answer} \] \[ \text{[1]} \]

(ii) Find the probability that a customer will win a cash voucher.

\[ \text{Answer} \] \[ \text{[2]} \]
(iii) If both Mary and Peter take part in the lucky draw, what is the probability that at least one of them will win a cash voucher?

Answer ........................................ [2]

(b) The cumulative frequency curve below shows the distribution of the weekly wages of 80 employees.
Use the curve to estimate

(i) the median wage,

\[ \text{Answer } \$ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots [1] \]

(ii) the interquartile range of the wages,

\[ \text{Answer } \$ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots [1] \]

(iii) the percentage of the employees who earned at least $500.

\[ \text{Answer } \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots [2] \]
8  (a) The first four terms in a sequence are $\frac{1}{3}, -\frac{1}{6}, \frac{1}{12}, -\frac{1}{24}$

Find the 5th term.

\[ \text{Answer} \] ............................... [1]

(b) The \( n \)th term of a sequence is given by \( T_n = \frac{1}{n^3+1} \). Find the sum of the 4th and 5th terms.

\[ \text{Answer} \] ............................... [3]

(c) The first four terms in a sequence of numbers are given below

\[ T_1 = 1^3 + 5 = 6 \]
\[ T_2 = 2^3 + 7 = 15 \]
\[ T_3 = 3^3 + 9 = 36 \]
\[ T_4 = 4^3 + 11 = 75 \]
(i) Find $T_5$.

Answer .................................. [1]

(ii) Express $T_n$ in the form of $an^3 + bn^2 + cn + d$, where $a$, $b$, $c$ and $d$ are integers to be determined.

Answer $a =$ .................

$b =$ .........................

c = .........................

d = .........................[2]
\( A\) represents a triangular plot of land on horizontal ground. \( AB = 8 \text{ km}, \ BC = 5 \text{ km}, \ AC = 11 \text{ km} \) and \( B \) is due west of \( C \).

(a) Calculate

(i) the bearing of \( A \) from \( C \),

Answer \( ..................................^{\circ} \) [3]

(ii) the bearing of \( C \) from \( A \),

Answer \( ..................................^{\circ} \) [2]
(iii) the reflex angle $ABC$.

$Answer$ ........................................ $^\circ$ [2]

(iv) the area of the triangular plot of land $ABC$.

$Answer$ ........................................ km$^2$ [2]

(b) A vertical lamp post is erected at point $A$. State with a reason, whether the angle of depression of $B$ or of $C$ is larger from the top of the lamp post.

$Answer$ ........................................................................................................

......................................................................................................................... [2]
Mr Wong is thinking of applying for a credit card that gives the most savings in terms of dining, grocery and petrol. His gross monthly expenses (before any discounts) are listed in the table below:

<table>
<thead>
<tr>
<th>Type of Expenses</th>
<th>Amount ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Petrol</td>
<td>350</td>
</tr>
<tr>
<td>Grocery</td>
<td>100</td>
</tr>
<tr>
<td>Dining</td>
<td>400</td>
</tr>
</tbody>
</table>

After doing some research, he decided to narrow his options to the three cards listed below:

<table>
<thead>
<tr>
<th>Credit Card</th>
<th>Savings on Petrol</th>
<th>Savings on Dining</th>
<th>Savings on Grocery</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBSH Card</td>
<td>• 14% upfront discount plus 5% cash rebate (on monthly petrol expenses) with monthly minimum spending of $600 on the CBSH card</td>
<td>• 5% cash rebate on dining expenses with monthly minimum spending of $600 on the CBSH card</td>
<td>• 5% cash rebate on grocery expenses with monthly minimum spending of $600 on the CBSH card</td>
</tr>
<tr>
<td>BSOP Card</td>
<td>• 15% upfront discount plus 6% cash rebate (on monthly petrol expenses) with minimum spending of $1000 on BSOP Card</td>
<td>• 5% cash rebate on all dining expenses with minimum monthly spending of $1000 on the BSOP Card</td>
<td>• 5% cash rebate on grocery expenses with minimum spending of $600 on the BSOP Card</td>
</tr>
<tr>
<td>CBCO Card</td>
<td>• 14% upfront discount plus 4.3% cash rebate (on monthly petrol expenses) with monthly minimum spending of $800 on the CBCO Card plus 2.1% cash rebate (on monthly petrol expenses) with monthly minimum spending of $400 on petrol</td>
<td>• 5% cash rebate on all dining expenses with monthly minimum spending of $800 on the CBCO Card</td>
<td>• 5% cash rebate on grocery expenses with monthly minimum spending of $800 on the CBCO Card</td>
</tr>
</tbody>
</table>
Which credit card should Mr Wong apply for so as to maximise his savings, given that he can only apply for one card and this card is to be used only for these three types of expenses? Show your working clearly. [9]
### Marking Scheme for Sec 4E/5NA EMaP P1

<table>
<thead>
<tr>
<th>Qn</th>
<th>Solutions</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 1  | \[
\frac{x}{4} + 13 = 6 \\
\frac{x}{4} = -7 \\
x = -28
\] | B1 |
| 2  | \[
\begin{align*}
25 \text{ min} &= \frac{25}{60} = \frac{5}{12} \text{ hrs} \\
D &= S \times T \\
D &= 60 \times \frac{5}{12} = 25 \text{ km}
\end{align*}
\] | A1 |
| 3  | 5, 5, 6, 9, 15 | B1 (1st 3 numbers), B1 (last 2 numbers) |
| 4  | 4047 m² = 1 Acre grows 20 000 **sunflower plants** 
\[
= 20000 \times 1500 \text{ seeds} \\
= 3 \times 10^7 \text{ seeds}
\] | M1 |
|    | \[
1 \text{ m}^2 = \frac{3 \times 10^7}{4047} \text{ seeds} \\
1800 \text{ m}^2 = \frac{3 \times 10^7}{4047} \times 1800 \text{ seeds} \\
= 1.33 \times 10^7
\] | A1 |
| 5  | \[
\begin{align*}
\frac{3x - 4(2 - 3x)}{5} &= \frac{21x - 20(2 - 3x)}{35} \\
&= \frac{21x - 40 + 60x}{35} \\
&= \frac{81x - 40}{35}
\end{align*}
\] | M1 |

6a | 2 | B1 |

6b | ![Venn Diagram](image) | B1 |
<table>
<thead>
<tr>
<th>Question</th>
<th>Solution</th>
</tr>
</thead>
</table>
| 7 | \[
6ac + 9ad - 12bd - 8bc = 3a(2c + 3d) - 4b(3d + 2c) = (3a - 4b)(2c + 3d)\] |
| 8a | \[
\overline{OB} = \overline{OA} - \overline{BA} = \left(\begin{array}{c}
-2 \\
5
\end{array}\right) - \left(\begin{array}{c}
-4 \\
8
\end{array}\right) = \left(\begin{array}{c}
2 \\
-3
\end{array}\right)
\]
\[
B = (2, -3)
\] |
| 8b | \[
|\overline{AB}| = \sqrt{(-4)^2 + (8)^2} = 8.94
\] |
| 9 | Mean = 10,720 g  
SD = 362.8 g |
| 10 | Cost of painting in 2017 = \[
\frac{73288.8}{1.2} = \$61074
\]
Cost of painting in 2016 = \[
\frac{61704}{1.08} = \$56550
\]
Percentage = \[
\frac{56550}{73288.8} \times 100\% = 77.2\%
\] |
| 11a | 7 units represent 2.1l  
1 unit represents 0.3l  
3 units represent 0.9l |
| 11b | 1 unit represents 0.3l  
12 units represent 3.6l |
| 11c | Lemon : Simply Syrup : Coconut water  
\[
\frac{1}{2} : \frac{1}{3} : 8
\]  
\[
\frac{1}{2} \times 6 : \frac{1}{3} \times 6 : 8 \times 2
\]  
\[
3 : 2 : 16
\] |
| 12 | \[
x + (x - 20) + 3x = 180 \quad (x + 20) + x + 3(x + 20) = 180
\]
\[
5x - 20 = 180 \quad \text{or} \quad 5x + 80 = 180
\]
\[
5x = 200 \quad 5x = 100
\]
\[
x = 40^\circ \quad x = 20^\circ
\]
\[
40^\circ, 20^\circ, 120^\circ \quad 40^\circ, 20^\circ, 120^\circ
\] |
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| 13a | $y = kx$ | or $x = ky$
|   | $y = 5k$  \[\text{--------- (1)}\] | $5 = ky$  \[\text{--------- (1)}\]
|   | $y = 17k$  \[\text{--------- (2)}\] | $17 = ky$  \[\text{--------- (2)}\]
|   | $(2) - (1); \quad 17k - 5k = 54$ | $\frac{17}{k} - \frac{5}{k} = 54$ |
|   | $12k = 54$ | $17 - 5 = 54k$ |
|   | $k = 4.5$ | $k = \frac{2}{9}$ |
|   | $y = 4.5x$ | $x = \frac{2}{9}y$ |
| 13b | $y = 4.5x$ | $84 = 4.5x$ |
|   | $x = \frac{2}{3}$ |
| 14a | 20 |   |
| 14b | 1210 |   |
| 14c | 924 |   |
| 15a | $15 \leq 8 - 3x$  \[\text{--------- (1)}\] | $8 - 3x < 2$ |
|   | $3x \leq 23$  \[\text{--------- (2)}\] | $6 < 3x$ |
|   | $x \leq \frac{23}{3}$ | $2 < x$ |
|   | $2 < x \leq \frac{23}{3}$ |   |
| 15b | 3, 4, 5, 6, 7 |   |
| 16a | Sum of interior angles = $(6 - 2) \times 180 = 720^\circ$ |   |
| 16b | Sum of angles = $7 \times$ angles at a point \[\text{--- Sum of interior angle of hexagon + sum of interior angle of triangle}\] = $7 \times 360 - [720 + 180]$ \[= 1620^\circ\] |   |
|   | $\angle a + \angle g + \angle f + \angle b = 4 \times (360 - 120) = 960^\circ$ | $\angle d + \angle c + \angle e = (3 \times 360) - 2(120) - 180 = 660^\circ$ |
|   | Sum of angles = $960^\circ + 660^\circ = 1620^\circ$ |   |
17a \[
\frac{A_A}{A_{A+B}} = \left(\frac{l_A}{l_{A+B}}\right)^2
= \left(\frac{h}{2h}\right)^2
= \left(\frac{1}{4}\right)
\]

17b \[
\frac{200}{200 + B} = \left(\frac{1}{4}\right)
B = 600
\]

17c \[
\frac{V_A}{V_{A+B+C}} = \left(\frac{l_A}{l_{A+B+C}}\right)^3
= \left(\frac{h}{3h}\right)^3
= \left(\frac{1}{27}\right)
\]
\[
\frac{V_A}{V_{A+B}} = \left(\frac{l_A}{l_{A+B}}\right)^3
= \left(\frac{h}{2h}\right)^3
= \left(\frac{1}{8}\right)
\]

Vol of B : Vol of C = 7 : 19

18 \[
\angle PQR = 235 \div 205 = 30^\circ
\]
\[
PR^2 = 35^2 + 45^2 - 2(35)(45)\cos 30^\circ
= 522.0199781
\]
\[
PR = \sqrt{522.0199781}
= 22.8 \text{ km}
\]
<table>
<thead>
<tr>
<th>19</th>
<th>( \angle DOE = 180 - 35 - 35 = 110^\circ )</th>
<th>M1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Area of segment = area of sector ( DOC ) – area of triangle ( DOC )</td>
<td>[ \text{M1, M1} ]</td>
</tr>
<tr>
<td></td>
<td>( = \frac{110}{360} \times \pi \times (7)^2 - \frac{1}{2} (7)(7) \sin 110^\circ )</td>
<td>[ = 24.01415413 ]</td>
</tr>
<tr>
<td></td>
<td>Area of section of circle</td>
<td>[ \text{M1} ]</td>
</tr>
<tr>
<td></td>
<td>= area of circle – 2 x area of segment</td>
<td>[ \text{A1} ]</td>
</tr>
<tr>
<td></td>
<td>= ( \pi (7)^2 - 2(24.01415413) )</td>
<td>[ = 106cm^2 ]</td>
</tr>
<tr>
<td></td>
<td>OR ( \angle DOE = 180 - 35 - 35 = 110^\circ )</td>
<td>[ \text{M1} ]</td>
</tr>
<tr>
<td></td>
<td>2 x ( Area of sector BOD + Area of triangle DOE )</td>
<td>[ \text{M1} ]</td>
</tr>
<tr>
<td></td>
<td>= 2 x ( \left( \frac{180-110}{360} \times \pi \times (7)^2 + \frac{1}{2} (7)(7) \sin 110^\circ \right) )</td>
<td>[ \text{M1, M1} ]</td>
</tr>
<tr>
<td></td>
<td>= 2 ( (29.932396 + 23.224692) )</td>
<td>[ = 106cm^2 ]</td>
</tr>
<tr>
<td>20a</td>
<td>Line ( L_1 ): ( 2y = 3x - 5 )</td>
<td>[ \text{B1} ]</td>
</tr>
<tr>
<td></td>
<td>Gradient = 1.5</td>
<td>[ \text{B1} ]</td>
</tr>
<tr>
<td></td>
<td>Line ( L_2 ): ( y = 1.5x + 5.5 ) or ( 2y = 3x + 11 )</td>
<td>[ \text{B1} ]</td>
</tr>
<tr>
<td>20b</td>
<td>( \text{area} = \frac{5}{3} \times 8 = 13\frac{1}{3} \text{ units}^2 )</td>
<td>[ \text{B1} ]</td>
</tr>
<tr>
<td>20c</td>
<td>( \tan \angle ABO = \frac{5}{\sqrt{3}} = 1.5 )</td>
<td>[ \text{B1} ]</td>
</tr>
<tr>
<td>21a</td>
<td>( T = \frac{2\pi}{\sqrt{\frac{l}{g}}} )</td>
<td>[ \text{M1} ]</td>
</tr>
<tr>
<td></td>
<td>( \frac{T}{2\pi} = \sqrt{\frac{l}{g}} )</td>
<td>[ \text{A1} ]</td>
</tr>
<tr>
<td></td>
<td>( \left( \frac{T}{2\pi} \right)^2 = \frac{l}{g} )</td>
<td></td>
</tr>
<tr>
<td>Question</td>
<td>Solution</td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>----------</td>
<td></td>
</tr>
</tbody>
</table>
| 21b | \[ l = \sqrt{5^2 + 2.5^2} = 5.59016994 \]  
Total surface area = curved SA of cone + curved surface area of cylinder + base area  
\[ = \pi(2.5)(\sqrt{31.25}) + 2\pi(2.5)(5) + \pi(2.5)^2 \]  
\[ = 142 \text{ cm}^2 \text{ (3sf)} \]  
| M1, M1 (any 2) | A1 |
| 22a | \[ R = \begin{pmatrix} 6 & 5 & 3 \\ 9 & 7 & 4 \\ 10 & 8 & 2 \end{pmatrix} \begin{pmatrix} 7.20 \\ 10.80 \\ 32 \end{pmatrix} \]  
\[ = \begin{pmatrix} 193.20 \\ 268.40 \\ 222.40 \end{pmatrix} \]  
| M1 | A1 |
| 22b | \( R \) represent the cost of each hamper.  
| B1 |
| 22c | \( T = (20 \ 25 \ 30) \)  
| BJ |
| 22d | \( \begin{pmatrix} 20 & 25 & 30 \end{pmatrix} \begin{pmatrix} 193.20 \\ 268.40 \\ 222.40 \end{pmatrix} \)  
\[ = \$17246 \]  
| M1 | A1 |
| 23a | \[ \frac{60}{360} \times 100\% = 16.7\% \]  
| B1 |
| 23b | 30% rep 108° represent 36 boys  
1° represent \( \frac{36}{108} \) boys  
360° represent \( \frac{36}{108} \times 360 \) = 120 boys  
| M1 | A1 |
| 23c | No of boys in group E = Total boys – boys in \((A + B + C + D)\)  
\[ = 120 \) – 36 – 18 – 20 \]  
\[ = 16 \text{ boys} \]  
| M1 | A1 |
| 24a | \[ \frac{20-n}{20} \]  
| B1 |
| 24b | \[ \frac{20-n}{20} \times \frac{19-n}{19} \]  
\[ = \frac{(20-n)(19-n)}{380} \]  
or  
\[ = \frac{380-39n+n^2}{380} \]  
| B1 |
\[
\begin{array}{|c|c|}
\hline
24c & \frac{(20-n)(19-n)}{380} = 39 \\
& \frac{380}{95} \\
& (20-n)(19-n) = 156 \\
& 380 - 20n - 19n + n^2 = 156 \\
& n^2 - 39n + 224 = 0 \text{ (Shown)} \\
& \text{M1} \\
& \text{A1} \\
\hline
24d & n^2 - 39n + 224 = 0 \\
& (n - 32)(n - 7) = 0 \\
& \text{Either } n = 32 \text{ or } n = 7 \\
& \text{No of yellow marbles} = 20 - 7 \\
& = 13 \\
& \text{M1} \\
& \text{M1} \\
& \text{A1} \\
\hline
\end{array}
\]
<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 1a | $27a^4 - 3 = 3(9a^4 - 1)$  
     | $= 3[(3a^2)^2 - 1]$  
     | $= 3(3a^2 - 1)(3a^2 + 1)$ | M1 |
|    | Comments: Students did not factorise $(9a^4 - 1)$ | A1 |
| 1b | (i) $\frac{2(x-1)^2}{4y^3} \div \frac{6y(x-1)}{8y^2} = \frac{2(x-1)^2}{4y^3} \times \frac{8y^2}{6y(x-1)}$  
     | $= \frac{2(x-1)}{3y^2}$ | B1 |
|    | Comments: Students made careless mistake as they cancelled the powers instead of applying the indices rules | |
|    | (ii) $\frac{3}{m-2} \div \frac{2}{3m-1} = \frac{3(3m-1) - 2(m-2)}{(m-2)(3m-1)}$  
     | $= \frac{7m+1}{(m-2)(3m-1)}$ | M1 |
|    | Comments: Students made mistake when they expand $-2(m - 2)$. | A1 |
1c \[ 2^{2-x} = \frac{1}{\sqrt{2^{2x+1}}} \]
\[ 2^{2-x} = 2^{-1(3x+1)} \]
\[ 2 - x = -\frac{5}{3}x + \frac{1}{3} \]
\[ \frac{2}{3}x = -\frac{7}{3} \]
\[ x = -\frac{7}{2} \]

Comments:
Students did not apply the indices rules \( \frac{1}{a^m} = a^{-m} \), \( \sqrt{a} = a^{\frac{1}{2}} \) and \( 1 = a^0 \).

1d
(i) \[ x^2 - 8x - 6 = (x - 4)^2 - 16 - 6 \]
\[ = (x - 4)^2 - 22 \]

Comments:
Majority of the students did it correctly.

(ii) \[ (x - 4)^2 - 22 = 0 \]
\[ (x - 4)^2 = 22 \]
\[ x = 4 \pm \sqrt{22} \]
\[ x = -0.7 \text{ or } x = 8.7 \text{ (to 1 dp)} \]

Comments:
Many students did not follow the instruction and use the requested method. Some of them did not correct the answers to one decimal place.

2
(a)(i) \( 3a - 2b \)
(ii) \( \frac{3}{7} (3a - 2b) \)
(iii) \( 2a - b \)
(iv) \( a + \frac{1}{2}b \)
(v) \[ \frac{FD}{FB + BC + CD} = -\frac{1}{2}(2a - b) + \frac{1}{2}(2b) + \frac{3}{7}(3a - 2b) \]
\[ = \frac{2}{7}a + \frac{9}{14}b \]

(b)(i) \( \frac{2}{3} \)

(ii) \[ \frac{\text{Area of } \Delta OBA}{\text{Area of } \Delta OCE} = \frac{\frac{1}{2} \times OB \times OA \times \sin \angle BOA}{\frac{1}{2} \times OC \times OE \times \sin \angle BOA} \]
\[ = \frac{\frac{1}{2} \times 1 \times 2 \times \sin \angle BOA}{\frac{1}{2} \times 2 \times 3 \times \sin \angle BOA} \]
\[ = \frac{1}{3} \]

Comments:
Badly done. Students did not consider the direction of the vectors, answers without vector notation. Could not find the ratio of areas, answers given with units.

3a

(i) \[ \frac{19600}{98000} \times 100\% = 20\% \]

(ii) Bank OCC
\[ A = 78400 \left( 1 + \frac{2}{100} \right)^{14} = \$95114.73 \]

Interest paid = 95114.73 - 78400 = $16714.73

Bank DBB
Choose Bank DBB as lesser interest charged.

Comments:
Students thought that the bank with more interest is to be chosen. Forgot that this is a loan.

3b
(i)  
$2.25 \times 51 = $114.75$

(ii)  
$2.08 \times 51 = \text{RM}106.08$

(iii)(a)  
Converting to Singapore dollars, Celine paid $\frac{106.08}{3.00} = $35.3478
She saves $79.97 weekly

(iii)(b)  
$\frac{79.97}{114.75} \times 100\% = 69.691\% \approx 69.7\%$

Comments:  
Students did not give answers correct to 2 decimal places.

4
(a)  
(i) Angle DAT = 90° (tangent perpendicular to radius)  
Angle AOD = 90° - 40° = 50° (sum of angles in a triangle)

(ii) Angle AOC = 50° \times 2 = 100°  
Angle ABC = 100° \div 2 = 50° (angle at centre = 2 times angle at Circumference)

(iii) Angle ADC = 180° - 50° = 130° (angles in opp segments)

(iv) Angle OCD = \frac{180° - 50°}{2} = 65° (Base angles of isosceles triangle)

Comments:  
Students did not write the angle properties properly.
\[
I = 78400 \times \frac{2.99}{100} \times 7 = 16409.12
\]

Choose Bank DBB as lesser interest charged.

**Comments:**
Students thought that the bank with more interest is to be chosen. Forgot that this is a loan.

### 3b

- **(i)**
  \[2.25 \times 51 = $114.75\]

- **(ii)**
  \[2.08 \times 51 = RM106.08\]

- **(iii)(a)**
  Converting to Singapore dollars, Celine paid \[\frac{166.08}{3.08} = 54.38\]
  She saves S$79.97 weekly

- **(iii)(b)**
  \[\frac{79.97}{114.75} \times 100\% = 69.69\% = 69.7\%\]

**Comments:**
Students did not give answers correct to 2 decimal places.

### 4

- **(i)** Angle DAT = 90° (tangent perpendicular to radius)
  Angle AOD = 90° − 40° = 50° (sum of angles in a triangle)

- **(ii)** Angle AOC = 50° × 2 = 100°
  Angle ABC = 100° ÷ 2 = 50° (angle at centre = 2 times angle at Circumference)

- **(iii)** Angle ADC = 180° − 50° = 130° (angles in opp segments)

- **(iv)** Angle OCD = \[\frac{180° - 50°}{2} = 65°\] (Base angles of isosceles triangle)

**Comments:**
Students did not write the angle properties properly.
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| 1a | \(27a^4 - 3 = 3(9a^4 - 1)\)  
    |  
    | \(= 3\left((3a^2)^2 - 1\right)\)  
    |  
    | \(= 3(3a^2 - 1)(3a^2 + 1)\)  
    | M1  
    | A1 | Comments:  
    | Students did not factorise \((9a^4 - 1)\) |
| 1b | \(\frac{2(x-1)^2}{4y^3} + \frac{6y(x-1)}{8y^2} = \frac{2(x-1)^2}{4y^3} \times \frac{8y^2}{6y(x-1)}\)  
    | \(= \frac{2(x-1)}{3y^2}\)  
    | B1 | Comments:  
    | Students made careless mistake as they cancelled the powers instead of applying the indices rules |
|     | (ii)  
    | \(\frac{3}{m-2} + \frac{2}{3m-1} = \frac{3(3m-1) - 2(m-2)}{(m-2)(3m-1)}\)  
    |  
    | \(= \frac{7m+1}{(m-2)(3m-1)}\)  
    | M1  
    | A1 | Comments:  
    | Students made mistake when they expand \(-2(m-2)\). |
1c \[2^{2-x} = \frac{1}{\sqrt[3]{2^{5x+1}}}\]
\[2^{2-x} = 2^{\frac{1}{3(5x+1)}}\]
\[2 - x = \frac{5x - 1}{3}\]
\[\frac{2}{3}x = \frac{7}{3}\]
\[x = \frac{7}{2}\]

Comments:
Students did not apply the indices rules \(\frac{1}{a^{-n}} = a^{n}\), \(\sqrt[3]{a} = a^{\frac{1}{3}}\) and \(1 = a^0\).

1d (i) \[x^2 - 8x - 6 = (x-4)^2 - 16 - 6 = (x-4)^2 - 22\]
Comments:
Majority of the students did it correctly.

(ii) \[(x-4)^2 - 22 = 0\]
\[(x-4)^2 = 22\]
\[x = 4 \pm \sqrt{22}\]
\[x = -0.7 \text{ or } x = 8.7 \text{ (to 1 dp)}\]
Comments:
Many students did not follow the instruction and use the requested method. Some of them did not correct the answers to one decimal place.

2. (a)(i) \(3a - 2b\)
(ii) \(\frac{3}{7}(3a - 2b)\)
(iii) \(2a - b\)
(iv) \(a + \frac{1}{2}b\)

Comments:

(b)

\[ \tan 40^\circ = \frac{5}{TC} \]

\[ TC = 5.9588\text{ cm} \]

Area of \( \triangle OTC = \frac{1}{2} \times 5.9588 \times 5 = 14.897\text{ cm}^2 \)

Area of sector \( ODC = \frac{1}{2} \times (5)^2 \times \frac{50\pi}{180} = 10.908\text{ cm}^2 \)

Area of shaded region = 14.897 - 10.908 = 3.989 \approx 3.99\text{ cm}^2

Comments:
Some students could not find the area of sector correctly. Did not convert the angle from degrees to radians correctly or choose the right formula for area of sector.

(a)

\[ 3^2 = 8^2 + 8^2 - 2(8)(8) \cos ABF \]

\[ \angle ABF = \cos^{-1}\left(\frac{119}{128}\right) \]

\[ = 21.61384575^\circ \]

\[ = 21.6^\circ \text{ (shown)} \]

(b)

Length of \( BE = \sqrt{12^2 + 8^2} = 14.422\text{ cm} \)

Area of triangle \( DCE = \frac{1}{2} \times 8 \times 8 \times \sin 21.614^\circ = 11.787\text{ cm}^2 \)

Perpendicular height from \( E \) to \( CD = \frac{2 \times 11.787}{8} = 2.94675\text{ cm} \)

Angle of elevation = \( \sin^{-1}\left(\frac{2.94675}{14.422}\right) = 11.8^\circ \)

Comments:
Students used the wrong triangle \( EDB \) to find angle of elevation of \( E \) from \( B \) and assumed that \( \angle EDB = 90^\circ \).

(c)(i)

Volume of prism = 11.787 \times 12 = 141.447\text{ cm}^3 \approx 141\text{ cm}^3
(c)(ii) Since volume of prism equals to volume of hemisphere,

\[
\text{Radius} = \sqrt[3]{\frac{141.447 \times 3}{2\pi}} = 4.0723 \text{ cm} \approx 4.07 \text{ cm}
\]

Comments:
Students did not apply the formula to find volume of hemisphere.

6

(a) \( p = 1.08, \ q = 5.73 \)

(b)

(c) \( x = 0.7 \pm 0.1 \) or \( x = 2.6 \pm 0.1 \)

(d) Gradient at \((3, 0.67)\) = \(1.78 \pm 0.2\)

B1, B1
B1: correct plot
B1: correct shape
B1: correct scale and axes

B1 B1
B1: draw the tangent line on graph
B1 for the answer
\[ y = -2x + 3 \]

\[ x = 1.88 \quad \text{(accept } x = 1.85 \text{ to } 1.9) \]

Comments:
Many students did not answer the part: equation of the line.
(a)\(\frac{3}{10}\)

(b)(i) $400$

(ii) \(470 - 330 = 140\)

(iii) \(\frac{15}{80} \times 100\% = 18.75\%\)

Comments:
1. For part (a), many students drew the tree diagram for the 2nd draw for the cash vouchers side. This would not have happened if they understood the question that once the customer drew a cash voucher, they will not be given a chance to draw again.

2. For part (aiii), many students did not know that they should use the answer to part (a(ii)) to help them find the answer. They went on to use other ways to find the answer which was wrong.
3. For part (biii), many left their answers as 18.8 (correct to 3 sf), which is not right as the answer is 18.75 which is an exact answer, hence had to penalize students who rounded their answers to 3 sf.

\[
T_5 = \frac{1}{48}
\]

(b)

\[
T_4 = \frac{1}{17}
\]

\[
T_5 = \frac{1}{26}
\]

\[
\text{Sum} = \frac{1}{17} + \frac{1}{26} = \frac{43}{442}
\]

(c)(i) \( T_2 = 5^2 + 13 = 138 \)

(ii)

\[
T_n = n^3 + 2n + 3
\]

\[
a = 1, \ b = 0, \ c = 2, \ d = 3
\]

Comments:
This question is ok and most students are able to get full marks. Those who did not make mistakes/gave up the last part which they should not have as it was just an expansion of algebraic expression.

9

(a)(i)

Bearing = 270° + \cos^{-1}\left(\frac{11 + 3^2 - 8^2}{2 \times 11 \times 5}\right) = 270° + 41.801° \approx 311.8°

(ii)

Bearing = 180° - (90° - 41.8°) = 131.8°

(iii)

\[
\frac{8}{\sin 41.8°} = \frac{11}{\sin ABC}
\]

\( \angle ABC \) (acute) = 66.4°

However, angle ABC is obtuse (seen from the diagram), hence actual angle \( ABC = 180° - 66.4° = 113.6° \)

Reflex \( \angle ABC = 360° - 113.6° = 246.4° \)

OR
\[
\cos ABC = \frac{8^2 + 5^2 - 11^2}{2 \times 5 \times 8}
\]

Angle \(ABC = 113.5782^\circ\)

Reflex \(\angle ABC = 360^\circ - 113.5782^\circ = 246.4^\circ\) (to 1 dp)

(iv)

Area = \(\frac{1}{2} \times 11 \times 5 \times \sin 41.8 = 18.3 \text{ km}^2\)

(b) Point B. Point B is nearer to point A than point C

Comments:
1. Students lost marks in part (iii), especially those who used sine rule to get the angle \(ABC\). Many did not find the obtuse angle \(ABC\) and used the acute angle \(ABC\) instead as they have forgotten that \(\sin \theta = \sin (180^\circ - \theta)\).

2. Many students leave their answers to 3 sf for angles which is incorrect as it should be to 1 dp. Pls take note of this small but important detail.

<table>
<thead>
<tr>
<th>10</th>
<th>CBSH Card</th>
</tr>
</thead>
<tbody>
<tr>
<td>Petrol savings = 0.14 \times 350 + 0.05 \times 350 = $66.50</td>
<td></td>
</tr>
<tr>
<td>Dining Savings = 0.05 \times 400 = 20</td>
<td></td>
</tr>
<tr>
<td>Grocery savings = 0.05 \times 100 = 5</td>
<td></td>
</tr>
<tr>
<td>Total savings = $91.50</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B1</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th></th>
<th>BSOP Card</th>
</tr>
</thead>
<tbody>
<tr>
<td>Petrol savings = 0.15 \times 350 = $52.50</td>
<td></td>
</tr>
<tr>
<td>Dining Savings = 0 (as minimum monthly spending on the card is less than $1000)</td>
<td></td>
</tr>
<tr>
<td>Grocery savings = 0.05 \times 100 = 5</td>
<td></td>
</tr>
<tr>
<td>Total savings = $57.50</td>
<td></td>
</tr>
</tbody>
</table>

| B1 |

<table>
<thead>
<tr>
<th></th>
<th>CBCO Card</th>
</tr>
</thead>
<tbody>
<tr>
<td>Petrol savings = 0.14 \times 350 + 0.043 \times 350 = $64.05</td>
<td></td>
</tr>
<tr>
<td>Dining Savings = 0.05 \times 400 = 20</td>
<td></td>
</tr>
<tr>
<td>Grocery savings = 0.05 \times 100 = 5</td>
<td></td>
</tr>
<tr>
<td>Total savings = $89.05</td>
<td></td>
</tr>
</tbody>
</table>

| B1 |

He should apply for the CBSH card
Comments:
1. Some students use the amount that was listed in the criteria for the rebates/discount to be used: example, students used $1000 for calculation of savings for BSOP card rather than the expenses of Mr Wong which was given in the question. This is a result of misinterpreting the question.

2. Students cannot understand the term ‘upfront discount’ which means regardless of the amount spent, the discount will be given the moment the customer presents the card. Quite a number of students lost marks here.
FAIRFIELD METHODIST SCHOOL (SECONDARY)
PRELIMINARY EXAMINATION 2019
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

MATHEMATICS

Paper 1

Date: 27 August 2019
Duration: 2 hours

Candidates answer on the Question Paper.

--- READ THESE INSTRUCTIONS FIRST ---

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact,
give the answer to 3 significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

<table>
<thead>
<tr>
<th>For Examiner's Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper 1</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Paper 2</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

Setter: Miss Shamsiah Zainalabidin

This question paper consists of 19 printed pages including the cover page.
Mathematical Formulae

Compound interest

Total amount \( = P \left( 1 + \frac{r}{100} \right)^n \)

Mensuration

Curved surface area of a cone = \( \pi rl \)

Surface area of a sphere = \( 4\pi r^2 \)

Volume of a cone = \( \frac{1}{3} \pi r^2 h \)

Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

Area of a triangle \( ABC = \frac{1}{2} ab \sin C \)

Arc length = \( r\theta \), where \( \theta \) is in radians

Sector area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

Trigonometry

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

Statistics

Mean = \[ \frac{\sum fx}{\sum f} \]

Standard deviation = \[ \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2} \]
1. (a) Express 3780 as a product of its prime factors.

Answer (a) ........................................... [1]

(b) Hence, find the smallest integer by which 3780 must be multiplied to obtain a perfect square.

Answer (b) ........................................... [1]

2. Given that $A = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$, $B = (-3, 9)$ and $C = \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}$, find

(a) $\frac{1}{2} BA$,

Answer (a) ........................................... [1]

(b) $C^2$.

Answer (b) ........................................... [1]
3 The curve $y = (x - 1)(x + k)$ has a minimum point $Q$ as shown.

(a) Write down the equation of the line of symmetry of this curve.

Answer (a) .................................. [1]

(b) Write down the value of $k$.

Answer (b) $k =$ ....................... [1]

4 The volume, $V \text{ cm}^3$, of a cylinder is directly proportional to $r^2h$, where $r$ is the radius and $h$ is the height of the cylinder. Find the percentage change in the volume when the radius is increased by 50% and the height is decreased by 20%.

Answer ........................................ % [2]
Charlene wanted to impress her parents by showing the rapid increase in her marks in 5 tests. Suggest, with reason, which graph she should use to impress her parents.

**Answer:**

6. The estimated atomic mass of 12 billion nitrogen atoms is $2.80 \times 10^{-9}$ grams.

(a) Express the mass of 1 nitrogen atom in picograms, leaving your answer in standard form.

$[1 \text{ pico} = 10^{-12}]$

**Answer (a) ....................... picograms[1]**

(b) The atomic mass of a helium atom is $6.684 \times 10^{-24}$ g. Express the ratio of the mass of a helium atom to a nitrogen atom in the form of $n : 1$, leaving your answer in standard form.

**Answer (b) ....................... : 1 [2]**
7 The height of Mount Kiki is 4.2 km and the temperature at the foot of the mountain is 31°C. The temperature decreases constantly at a rate of 8 °C for every 700 m.

(a) Calculate the temperature at the peak of the mountain.

Answer (a) ..................°C [1]

(b) Calculate the height from the foot of the mountain at which the temperature is 7°C.

Answer (b) ...................... m [1]

(c) A man took 6 hours 43 minutes to climb from the foot of the mountain to the peak. Given that he reached the peak at 12:25, at what time did he begin his climb?

Answer (c) ...................... [1]
8. The pressure, $P$, of a fixed mass of gas at a constant temperature is inversely proportional to the volume, $V$, of the gas.

(a) Sketch the graph of $P$ against $V$ on the axis provided.

(b) When the pressure of the gas is 4 Nm$^{-2}$, the volume is 8 m$^3$. Find $P$ when $V = 12$ m$^3$.

Answer (a) ................. Nm$^{-2}$ [1]

Answer (b) ................. Nm$^{-2}$ [1]

9. A map is drawn to a scale of 1 : 120 000.

(a) Calculate the actual distance, in km, represented by 6.3 cm on the map.

(b) A lake has an actual area of 3.9 km$^2$. Find the area of the lake on the map, in square centimetres.

Answer (a) ................. km [1]

Answer (b) ................. cm$^2$ [2]
10. Consider the number pattern:
   \[1 + 3 = 4 = 2 \times 2\]
   \[1 + 3 + 5 = 9 = 3 \times 3\]
   \[1 + 3 + 5 + 7 = 16 = 4 \times 4\]
   \[1 + 3 + 5 + 7 + 9 = 25 = 5 \times 5\]

   (a) Write down the sixth line in the pattern.

   Answer (a) .................................................. [1]

   (b) Using the above number pattern, find the sum \(1 + 3 + 5 + 7 + 9 + \ldots + 81\)

   Answer (b) .................................................. [1]

11. John deposited $10 000 in a bank paying an interest of 10% per annum, compounded half yearly. Calculate the amount of interest he would receive after 2 years.

   Answer $ ............................................. [3]
12 Match the correct graphs A to E, found below to represent each of the following statements.

(a) The cost, \( y \), of taxi fare which consists of a fixed charge plus an amount proportional to the distance travelled, \( x \).

(b) The volume, \( y \), of a sphere is proportional to the cube of the radius \( x \).

(c) The distance travelled by an object, \( y \), varies directly with the time taken, \( x \).

\[ \begin{align*}
A & \quad B & \quad C & \quad D & \quad E \\
& & & & \\
& & & & \\
\end{align*} \]

Answer (a) ................................... [1]

(b) ................................... [1]

(c) ................................... [1]

13 (a) On the Venn diagram in the answer space, shade the region which represents \((P \cup Q)'\).

Answer

\[ \begin{align*}
& \quad E \\
& \quad Q \\
& \quad P \\
\end{align*} \]

(b) \( E = \{ x: x \text{ is an integer between 0 and 21} \} \)

\( A = \{ x: x \text{ is a multiple of 5} \} \)

\( B = \{ x: x \text{ is not a prime number} \} \)

(i) List the elements contained in the set \((A \cup B)'\).

Answer (b)(i) ................................... [1]

(ii) Find \( n(A \cap B) \).

Answer (b)(ii) ................................... [1]
14 (a) Solve the inequality \( 4 - 3x \leq \frac{2-x}{3} < \frac{4+x}{5} \).

Answer (a) .................................................. [2]

(b) Hence, represent the solution on the number line below. [1]

Answer

15 (a) Express \( x^2 - 9x + 45 \) in the form \( (x-p)^2 + q \).

Answer(a) .................................................. [1]

(b) Hence, solve the equation \( x^2 - 9x + 45 = 50 \), giving your answers correct to two decimal places.

Answer(b) \( x = \ldots \text{ or } \ldots \) [2]
16 In the diagram, \( \sin A = \frac{5}{13} \) and angle \( A \) is an obtuse angle.

Leaving your answer, as a fraction, find the value of

(a) \( \sin A - \cos A \),

(b) \( \cos(180^\circ - A) + \tan(A - 90^\circ) \).

**Answer (a) ......................... [2]**

**Answer (b) ......................... [2]**
17 (a) Factorise completely $4a^2 + 2ab - 14xb - 28ax$.

Answer (a) ........................................... [2]

(b) Solve the equation $\frac{3x+1}{7} = -\frac{3-x}{4}$.

Answer (b) $x =$ ...........................................[2]
18. The graph below shows the speed-time graph of a moving object.

(a) Describe the motion of the object for the first 4 seconds.

Answer(a) ................................................................. [1]

(b) Given that after 4 seconds, the object started to decelerate at a rate of 5 m/s², find the value of T.

Answer(b) ...................... [1]

(c) Sketch the distance-time graph of the object for the first 16 seconds.

Answer(c) ................................................................. [2]
19 Construct a quadrilateral \( PQRS \) such that \( PQ = 10 \text{ cm}, QS = QR = 9 \text{ cm}, RS = 5 \text{ cm} \) and \( \angle PQR = 120^\circ \). \( PQ \) has already been drawn. [2]

(a) Construct the perpendicular bisector of \( PQ \). [1]

(b) The perpendicular bisector meets \( PS \) at \( T \). Hence, measure and write down the length \( RT \).

\[ \text{Answer (a)} \]

\[ \text{Answer(b) } RT = \ldots \ldots \ldots \ldots \ldots \ldots \text{ cm} \] [1]
The table below shows the drug testing results of 36 athletes for 2018 Olympic Games.

<table>
<thead>
<tr>
<th>Suspected of drugs consumption</th>
<th>Yes</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td>No</td>
<td>3</td>
<td>21</td>
</tr>
</tbody>
</table>

(a) Present the results in the probability tree diagram below.

Answer(a) [2]

(b) Find the probability that an athlete

(i) is suspected of taking drugs and tested positive,

Answer(b)(i) [1]

(ii) receives a negative test result.

Answer(b)(ii) [2]
21. (a) Simplify \( \left( \frac{a^2 - a^3}{a^3} \right) - 3(a)^6 \).

(b) Given that \( \frac{1}{9^{1-3x}} = 2 \times 3^{x-1} \), find the value of \( x \).

Answer (a) ......................... [2]

Answer (b) \( x = \) ......................... [3]
The numbers 4, 6, 7, 9, 2, 5, 9, 12, 2, x and y have a mean of 7 and a mode of 9. Find:
(a) the values of the two numbers x and y, given that x < y,

Answer (a) x = .........., y = ........... [2]

(b) the median,

Answer (b) .................................... [1]

(c) the standard deviation of this set of eleven numbers.

Answer (c) .................................... [2]
23. The figure below consists of a pentagon and five identical equilateral triangles.

(a) Calculate angle $EGH$.

Answer $(a)$: $\theta^\circ$ [2]

(b) Explain why $AI = AB$.

Answer: 

[Explanation]

[1]

(c) Calculate angle $AEI$.

Answer $(c)$: $\theta^\circ$ [2]
24  (a) Expand and simplify $y(y - 2) + 12y^2 - 6y$.

(b) Express $\frac{6}{3y+7} - \frac{1}{49-9y^2}$ as a single fraction in its simplest form.

$\text{Answer(a)}$ .................................. [2]

$\text{Answer(b)}$ .................................. [3]

End of paper
READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place
in the case of angles in degrees, unless a different level of accuracy is specified in the
question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part
question.

The total number of marks for this paper is 100.
Mathematical Formulae

Compound interest

Total amount = \( P \left( 1 + \frac{r}{100} \right)^n \)

Mensuration

Curved surface area of a cone = \( \pi rl \)

Surface area of a sphere = \( 4\pi r^2 \)

Volume of a cone = \( \frac{1}{3} \pi r^2 h \)

Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

Area of a triangle \( ABC = \frac{1}{2} ab \sin C \)

Arc length = \( r\theta \), where \( \theta \) is in radians

Sector area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

Trigonometry

\( \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \)

\( a^2 = b^2 + c^2 - 2bc \cos A \)

Statistics

Mean = \( \frac{\sum fx}{\sum f} \)

Standard deviation = \( \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2} \)
1 (a) Express $\frac{3x}{x-3} + \frac{2}{x+4}$ as a single fraction. [2]

(b) Using factorisation, simplify fully $(x^2 + 5)^2 - (x^2 - 3)^2$. [2]
1 (c) Solve $2x^3 - 13x^2 - 24x = 0$. [3]

(d) $n$ is an integer. Showing your working clearly, explain why the sum of $\frac{1}{2}n(n+1)$ and $\frac{1}{2}(n+1)(n+2)$ is always a square number. [2]
2. The diagram shows two circles with equal radii.
P, T and R are points on the circle with centre Q.
Q, T, S and U are points on the circle with centre R.
PQRS is a straight line.

(i) Show that the triangles PTR and UQT are congruent. [3]
2 (ii) Name another triangle that is congruent to \(PTR\). [1]

(iii) Explain why \(TQ\) is parallel to \(SU\). [1]

(iv) Stating the reasons clearly, find the value of angle \(UQR\). [1]
$AEC$ and $BDC$ are straight lines. $AE = 12$ cm and $BD = 4$ cm.

$CE = x$ cm and $CD = (2x - 5)$ cm. Angle $ACB = \theta^\circ$.

(a) Show that $\frac{\text{Area of triangle } CDE}{\text{Area of triangle } ABC} = \frac{CE \times CD}{AC \times BC}$. [2]

(b) It is given that $\frac{\text{Area of triangle } CDE}{\text{Area of triangle } ABC} = \frac{1}{3}$.

Using the result from part (a), form an equation in $x$ and show that it simplifies to $2x^2 - 19x + 6 = 0$. [3]
3  (c)  (i) Solve the equation $2x^2 - 19x + 6 = 0$, giving your answers correct to 2 decimal places.  [3]

(ii) State, with a reason, which of these solutions does not apply to triangle $CDE$.  [1]

(d) Given that $\theta = 25$, calculate $DE$.  [3]
The diagram below shows an open cylindrical container with a diameter of 12 cm and height of 4 cm.

(a) Assuming the thickness of the container is negligible, calculate the area of material needed to make one container. Give your answer correct to the nearest square centimetre. [3]

(b) A hemispherical pan is completely filled with 13 litres of soup. As many containers as possible are completely filled with the soup from the pan.

(i) Calculate the number of containers which are filled. [3]
4 (b) (II) Calculate the volume of soup which is left in the pan, giving your answer in cubic centimetres. [2]

(iii) Calculate the radius of the hemispherical pan, giving your answer correct to the nearest millimetre. [2]

(c) Peter has two different containers, which are geometrically similar to each other. The heights of the containers are in the ratio of 2 : 3. Write down the ratio of the volumes of soup these containers hold when full. [1]
In the diagram, the rectangle $PQRS$ represents a vertical cliff face. The foot of the cliff, $PQ$, runs from East to West, and is at sea level. A ship is in the sea at $T$.
Angle $OPT = 75^\circ$, angle $PTQ = 63^\circ$ and $PQ = 45$ m.
(a) Find the bearing of $T$ from $Q$. [2]
5 (b) Show that $QT = 48.8$ m, correct to three significant figures. [2]

(c) Calculate the shortest distance from the ship to the cliff. [2]
5 (d) The angle of depression of the ship when viewed from \( R \) is 16°.

(i) Find the height of the cliff.  

(ii) Calculate the greatest possible value of the angle of elevation of the top of the cliff when viewed from the ship.  

[2]
6. When \( x \) copies of a book are printed, the cost \( \$C \) of each copy is given by the formula
\[
C = 10 + \frac{2400}{x}
\]
(a) The table gives some values of \( x \) and the corresponding values of \( C \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>600</th>
<th>800</th>
<th>1200</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C )</td>
<td>34</td>
<td>22</td>
<td>18</td>
<td>16</td>
<td>14</td>
<td>13</td>
<td>( p )</td>
</tr>
</tbody>
</table>

(i) Find the value of \( p \). [1]

(ii) On the grid, plot the points given in the table and join them with a smooth curve. [3]
6 (b) Use your graph to estimate the number of books to be printed if the cost of printing each book is $15. [1]

(c) (i) By drawing a tangent, find the gradient of the curve at the point where $x = 300$. [2]

(ii) Describe briefly what this gradient represents. [1]

(d) In order to sell $x$ books, the selling price of each book must be $\$ \left( 25 - \frac{x}{60} \right)$. [1]

(i) On the axes, used in part (a), draw the graph of $C = 25 - \frac{x}{60}$ for the values of $x$ from 0 to 1200. [2]

(ii) Use your graphs to find the range of the number of books that should be printed if no loss to be incurred. [1]
7 The diagram shows a tangent $APQ$ to two circles with centre $C$ and $E$. The points $A$, $B$, $C$, $D$, $E$ and $F$ lie on the same straight line. It is given that $BF = 16$ cm and $DF = 12$ cm.

(a) (i) Show that the triangles $APC$ and $AQE$ are similar. [2]

(ii) Hence, find the length of $AB$. [2]
7 (b) Show that angle $E4Q$ is $\frac{\pi}{6}$. [1]

(c) Calculate the perimeter of the shaded region. [4]
8 (a) In the diagram, $WXYZ$ is a parallelogram. $M$ is a point on $XY$ such that $XM : MX = 3 : 2$, $WX = 6p + 3q$ and $WZ = 10p - 5q$.

![Diagram of parallelogram WXYZ with points M and X, Y, Z labeled.](image)

(i) Find, in terms of $p$ and/or $q$,

(a) $WM$.

(b) $ZM$.

[1]
8

(a) (ii) (a) Find area of triangle $\triangle WMX$ : area of $\triangle WXYZ$. [1]

(b) The area of triangle $\triangle WMX$ is 8 units$^2$.
Hence, calculate the area of $\triangle WXYZ$. [1]

(iii) Given that $N$ is on $WX$ produced such that $ZMN$ is a straight line.
Express $WN$ in terms of $p$ and $q$. [1]
8 (b) Coordinates of $A$ and $B$ are $(-3, 3)$ and $(7, -13)$ respectively.

(i) Write $\overrightarrow{AB}$ as a column vector.  

(ii) Find the acute angle formed by the line $AB$ with the horizontal axis.  

(iii) If the gradient of $AB = \frac{2m}{n}$, express $\overrightarrow{AB}$ in terms of $m$ and $n$.  

(iv) Another vector $\overrightarrow{CD}$ is parallel to $\overrightarrow{AB}$ and has the magnitude thrice that of $\overrightarrow{AB}$. Write down the possible vectors of $\overrightarrow{CD}$.  

The cumulative frequency curve below shows the travelling time of 120 working adults travelling to work daily by train.

(a) Use the graph to estimate
(i) the median of travelling time, [1]

(ii) the 20th percentile of travelling time, [1]
9 (a) (iii) the interquartile range of travelling time, \[2\]

(iv) the percentage of the total number of adults who spend more than 45 minutes travelling to work every day. \[1\]

(b) Another 120 working adults travelled to work by bus. The travelling time is illustrated in the box and whisker diagram below.

Find the median travelling time and the interquartile range. Hence, compare and comment on the travelling times by train and bus in two different ways. \[3\]
One working adult is chosen at random. Assume that the travelling times between the train and bus are independent. The working adult makes the first trip on Monday by train and the second trip on Tuesday by bus.
Expressing each answer as a fraction in its lowest terms, calculate the probability that the working adult took
(i) more than 55 minutes on both trips, [1]
(ii) more than 55 minutes on one trip, but not the other. [2]
The diagram below shows a race in the Olympic Games. For certain races, the athletes do not all start from the same part of the track. This is called “staggered start”.

Figure 1

The grass field comprises two semi-circular ends of radius 36.5 m and two straight lengths of 84.39 m each. The field is surrounded by a running track of 8 lanes, each of width 1220 mm.

The route along which the running distance is measured for each lane is as below:

<table>
<thead>
<tr>
<th>Lane</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lane 1</td>
<td>300 mm from inner edge of the lane</td>
</tr>
<tr>
<td>Lanes 2 to 8</td>
<td>200 mm from inner edge of the lane</td>
</tr>
</tbody>
</table>
10 (a) (i) Show that the total distance that an athlete in Lane 1 would have to run to complete one lap of the track is 400 m. [2]

(ii) Show that the staggered start line for Lane 8 is 53.03 m from the start line for Lane 1 (distance of i) as seen in Figure 1. [3]

(iii) Explain why a “staggered start” is needed for each runner in Lane 1 to Lane 8 to complete one lap of the track. [1]
An athlete wants to incorporate in his training a minimum of 150 minutes of brisk walking weekly, at the an average speed of 6.8 km/h. He claims that he needs to walk briskly around the track in Lane 8 five rounds daily to hit his target.

Justify whether his claims is true or false. Show your working clearly.
<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| **2019 Sec 4Exp/5NA Preliminary Examination**<br>**Mathematics Paper 1**<br>**Answer Key** | **1a** $2^2 \times 3^3 \times 5 \times 7$ | **1b** 105 | **2a** (28.5) | **2b**
|   |   |   |   | $\begin{pmatrix} 7 & 14 \\ -7 & -14 \end{pmatrix}$ |
| **4** | 80% | **5a** Charlene should use Graph 2 as the scale for the vertical axis is bigger and does not start from zero, making the difference in marks between each test looks bigger. | **6a** $2.325 \times 10^{-7}$ picograms | **5b**
| **6b** | $2.86 \times 10^{-3}$ : 1 | **7a** $-17 \, ^\circ C$ | **8a** | **8b** $P = \frac{32}{12} = 2.67 \, Nm^{-2}or\frac{2}{3} \, Nm^{-2}$
| **7c** | 05 42 | **8a** |   | **9a** 7.56 km |
| **9b** | $2.71 \, cm^2 (3 \, s.f.)$ | **9b** |   | **10a** $1 + 3 + 5 + 7 + 9 + ... + 81 = 41 \times 41 = 1681$
| **10b** | $1 + 3 + 5 + 7 + 9 + ... + 81 = 41 \times 41 = 1681$ | **11** $S2155.06 (2 \, d.p.)$ | **11a** $\text{Total} = 49 \, \frac{13}{7}$ |
| **12a** | D | **12c** |   | **12a** |
| **13a** | {2,3,7,11,13,17,19} | **13bii** 3 | **14a** $x \geq \frac{1}{4}$ | **13b**
| **14b** |   |   |   | **14b** |
| **15a** | $(x - 4.5)^2 = 24.75$ | **15b** 25.28 or 15.23 (2 d.p.) | **16a** $\frac{17}{13}$ | **16b** $2(\alpha - 7x)(2a + b)$
| **16b** | $\frac{215}{75}$ |   |   | **17a** 2 |
| **17b** | $x = -5$ | **17b** 7 sec | **18a** | **18b** 7 sec |
| **19a** | $10.0 \pm 0.1 \, cm$ | **19b** 11 | **20bii** 7 | **20a** $x = \frac{6}{7}$ |
| **20b** | $\frac{36}{36}$ | **20bii** 7 | **20b** | **22a** $y = 12$
| **21a** | a $- 4$ | **21b** $x = \frac{6}{7}$ | **22a** | **22c** 3.55 |
| **22a** | 7 | **22c** 3.55 | **23a** 168° | **23a** 168° |
| **23c** | $36^\circ$ | **24a** $13y^2 - 8y$ | **24b** $\frac{41 - 18y}{(7 + 3y)(7 - 3y)}$ | **24b**

FMS(S) Sec 4 Exp / 5 N(A) Preliminary Examination 2019
Mathematics Paper 1
Marking Scheme for Sec 4 Exp/ SNA Mathematics P1 2019

<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Marking Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>[ 3780 = 2^3 \times 3^2 \times 5 \times 7 ]</td>
<td>B1</td>
</tr>
<tr>
<td>1b</td>
<td>[ 2^3 \times 3^2 \times 5 \times 7 = \text{perfect sq.} ] Therefore, ( 3 \times 5 \times 7 = 105 )</td>
<td>B1</td>
</tr>
<tr>
<td>2a</td>
<td>[ \frac{\frac{1}{2} \times 81}{2} ] [ = \frac{1}{2} \left( \frac{9}{2} \right)^2 ] [ = \frac{1}{2} \left( \frac{81}{4} \right) ] [ = \frac{81}{8} ]</td>
<td>B1</td>
</tr>
<tr>
<td>3b</td>
<td>( x = -4 )</td>
<td>B1</td>
</tr>
<tr>
<td>3b</td>
<td>( \frac{1}{2} + 4k = -4 ) [ \frac{1}{2} - 4k = -8 ] [ -4k = -9 ] [ k = \frac{9}{4} ]</td>
<td>B1</td>
</tr>
<tr>
<td>4</td>
<td>New radius, ( r_1 = 1.5r ), new height, ( h_1 = 0.8h ) [ V_1 = \frac{4}{3} \pi r_1^2 h_1 ] [ V_1 = \frac{4}{3} \pi (1.5r)^2 (0.8h) ] [ V_1 = \frac{4}{3} \pi (1.5r)^2 (0.8h) ] [ \therefore V_1 = 1.8V ] % change in ( V ) [ \frac{1.8 - 1}{1} \times 100% ] [ = 80% ]</td>
<td>B1 or B2</td>
</tr>
</tbody>
</table>

Name: ______________________ ( ) Class: ____________

6a Mass of 12 billion nitrogen atoms \[ = 2.80 \times 10^9 \text{ g} = 2.80 \times 10^9 \text{ picograms} \] Mass of 1 nitrogen atom \[ = \frac{2.80 \times 10^9}{12 \times 10^6} \] \[ = 0.233 \times 10^3 \text{ picograms} \] \[ = 2.33 \times 10^2 \text{ picograms} \] | B1 |

6b Helium : : Nitrogen \[ 6.694 \times 10^{-2} \] \[ 4.2 \times 10^{-3} \] \[ 0.2333 \times 10^{-2} \] \[ 8.3 \times 10^{-4} \] \[ : 1 \] \[ : 1 \] \[ : 1 \] \[ : 6 \] | A1 |

7b Amt. of increase in temp with decrease in alt. \[ 31 \times 17 \] \[ = 529 \times 48 \] \[ = 12,432 \] \[ = 24 \] | B1 |

8a Time he has to climb \[ 12 \text{ km} \] \[ 43 \text{ m} \] | B1 |

8b \[ P = \frac{F}{A} \] \[ k = P/V \] \[ = 4(3) \times 2 \] \[ = 12 \] \[ = 2.67 \text{ nm}^2 \text{ or } 2.67 \text{ Nm}^{-2} \] \[ \therefore P = \frac{12}{3} \text{ Nm}^2 \text{ or } \frac{2.67 \text{ Nm}^2}{3} \] | B1 |

FMS(S/S) Sec 4 Exp / 5 N(A) Preliminary Examination 2019
Mathematics Paper 1
| Name: | 9a | 1 cm : 120 000 cm  
1 cm = \frac{120000}{1000} = 120 km  
6.3 cm = 6.3 \times 1.2 km = 7.56 km |
|-----|----|---|
| 9b | \( (1 \text{ cm})^2 = (1.2 \text{ km})^2 \)  
\( 1 \text{ cm}^2 = 1.44 \text{ km}^2 \)  
\( 2.71 \text{ cm}^2 = 3.9 \text{ km}^2 \)  
\( 3.9 \)  
\( 1.44 \)  
\( 3.9 \)  
\( 2.71 \) |
| 10a | \( 1 + 3 + 5 + 7 + 9 + 11 + 13 = 49 = 7 \times 7 \) |
| 10b | \( 1 + 3 + 5 + 7 + 9 + ... + 81 = 41 \times 41 = 1681 \) |
| 11 | \( t = P\left(1 + \frac{r}{100}\right)^n \)  
\( = 10000\left(1 + \frac{2}{100}\right)^{10} \)  
\( = \$12155.0625 \)  
\( \text{Interest} = \$12155.0625 - \$10000 \)  
\( = \$2155.0625 \) |
| 12a | B |
| 12b | D |
| 12c | C |
| 13a | Answer 6 |
| 13b1 | \( \{2, 3, 7, 11, 13, 17, 19\} \) |
| 13bii | 3 |

---

| Name: | 14a | \( 4 - 3x < \frac{2 - x}{3} \)  
\( \text{or} \quad \frac{2 - x}{3} < \frac{4 + x}{5} \)  
\( -9x + x \leq 2 - 12 \)  
\( -5x - 3x < 12 - 10 \)  
\( -8x \leq -10 \)  
\( -8x < 2 \)  
\( x > \frac{-1}{4} \)  
\( x < \frac{1}{4} \) |
| 14b | \( x = 1 \)  
| 15a | \( x > \frac{-5}{6} \)  
\( \text{or} \quad 9x + (2) (2) = 45 \)  
\( (x - 2) = 26.25 \) |
| 15b | B1 |
| 15c | B1 |
| 16a | \( \text{Length of base of triangle} \)  
\( = \sqrt{13^2 - 3^2} = 12 \text{ units} \)  
\( \sin \angle A = \cos \angle A \)  
\( = \sin \angle A = -\cos(180^\circ - \angle A) \)  
\( = \frac{5}{13} \)  
\( \frac{12}{13} \)  
\( \frac{13}{17} \)  
\( \frac{17}{13} \)  
\( \frac{13}{5} \)  
\( \frac{216}{75} \) |
| 16b | \( \cos (180^\circ - \angle A) + \tan (\angle A - 90^\circ) \)  
\( = \frac{12}{13} \)  
\( = \frac{12}{13} \)  
\( \frac{13}{5} \)  
\( \frac{216}{75} \) |

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FSM(S) Sec 4 Exp / S N(A) Preliminary Examination 2019  
Mathematics Paper 1
17a
\[ 4a^3 + 2ab - 14ab - 28a + \frac{20a^2 b}{5} = 2(2a + b)(2a + 2b) \]
\[ = (2a + b)(4a + 2a + 2b) \]
\[ = 2(a + 7b)(2a + b) \]

17b
\[ \frac{x + 1}{4} = \frac{3 - x}{4} \]
\[ 4(3x + 1) = -7(3 - x) \]
\[ 12x + 4 = -21 + 7x \]
\[ 12x - 7x = -21 - 4 \]
\[ 5x = -25 \]
\[ x = -5 \]

18a
The object is moving at constant speed of \( 5 \text{ ms}^{-1} \) with zero acceleration for the first 4 sec.

18b
Gradient = 5
\[ \frac{15}{5} = \frac{3}{T} \]
\[ T = 4 + 3 = 7 \text{ sec} \]

18c
All 3 parts of the correct graph shape = B2
1 wrong shape = B2
If no graph = B2
2 or more wrong shape = 0 marks

19a, b
Refer to last page for the details

19e
10.0 ± 0.1 cm

20a
Suspected of drugs consumption
Yes
Positive
\[ \frac{11}{12} \]
No
Negative
\[ \frac{1}{12} \]
Any 2 correct ans
\[ = \frac{11}{12} \times 2 \]
\[ = \frac{3}{24} \]

20bii
\[ P(\text{receiving negative result}) = \frac{12}{36} + \frac{24}{36} = \frac{36}{36} \]
\[ = \frac{21}{36} = \frac{21}{24} \]

21
\[ (a - 1)^2 - 3(a) = a^2 (a - 1)^2 - 3 \]
\[ = a^2 (a - 1) - 3 \]
\[ = a - 4 \]
21b \( \frac{1}{(3^2 + 3^2)^{\frac{1}{2}}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{3} \)
\( 6x - 2 = \frac{5x}{2} \)
\( 6x - 5x = -2 \)
\( x = \frac{5}{2} \)
\( x = 2.5 \)

22a Let \( x = 9 \) as mode is 9
mean = \( \frac{5+6+7+9+9+12+12+9+y}{11} \)
\( 7 = \frac{65 + y}{11} \)
\( 77 = 65 + y \)
\( y = 12 \)

22b Median position = \( \frac{11 + 1}{2} = 6 \)
Arranging the values in ascending order,
2, 2, 4, 5, 6, 7, 9, 9, 9, 12, 12
Hence the median at 6th position is 7

22c Standard deviation
= 3.55

23a Int angle \( \angle IGH \)
= \( \frac{180^\circ}{3} \) (angle in equilateral triangle)
= 60°
Angle in pentagon
= \( \frac{(5-2) \times 180^\circ}{5} \)
= 108°
Angle \( \angle EDH = 60^\circ + 108^\circ \)
= 168°

23b Since the triangle is equilateral and all triangles are identical,
\( AB = BC = AC = AI \). Hence \( AI = AB \)

23c Angle \( \angle GEI = \angle CEA \)
\( = \frac{180^\circ - 108^\circ}{2} \) (base angles in isos. triangle)
\( = 36^\circ \)
\( \therefore \angle EAI = 108^\circ - (36^\circ \times 2) = 36^\circ \)

24a \( y(2-2) + 12y^3 + 6y \)
\( = y^2 - 2y + 12y^3 + 6y \)
\( = 13y^3 - 8y \)

24b \( \frac{1}{7} - \frac{40 + 2y^2}{7} \)
\( = \frac{7 - (3y)}{7} \)
\( = \frac{7 + 3y}{7} - \frac{21}{7} \)
\( = \frac{42 - 12y}{7} - \frac{12y}{7} \)
\( = \frac{7 + 3y}{7} - 3y \)

FMS(S) Sec 4 Exp / 5 (A) Preliminary Examination 2019
Mathematics Paper 1
<table>
<thead>
<tr>
<th>Answer Key</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1(a)</strong></td>
<td>(3x^2 + 14x - 6) (\div (x - 3)(x + 4))</td>
</tr>
<tr>
<td><strong>1(c)</strong></td>
<td>(x = 0) or (x = -1.5) or (x = 8)</td>
</tr>
<tr>
<td><strong>1(d)</strong></td>
<td>Since ((n + 1)^2 = (n + 1)(n + 1)), therefore, it is a square number. Or ((n + 1)^2) has a repeated same factor, therefore it is a square number.</td>
</tr>
<tr>
<td><strong>2(i)</strong></td>
<td>Statement with reasons for SAA/RHS/SAS/SSS</td>
</tr>
<tr>
<td><strong>2(ii)</strong></td>
<td>Triangle (QUS)</td>
</tr>
<tr>
<td><strong>2(iii)</strong></td>
<td>Angle (TQS = Angle SUT) (Angle in same segment) = (60^\circ) (mentioned in (i) that triangle (QTR) is equilateral triangle)</td>
</tr>
<tr>
<td><strong>2(iv)</strong></td>
<td>(30^\circ)</td>
</tr>
<tr>
<td><strong>3(c)(ii)</strong></td>
<td>(x = 0.327, DC = 2x - 5 = 4.356 &lt; 0). Therefore, (x) cannot be (0.33).</td>
</tr>
<tr>
<td><strong>4(a)</strong></td>
<td>(264 \text{ cm}^2) (nearest (\text{cm}^2))</td>
</tr>
<tr>
<td><strong>4(b)(ii)</strong></td>
<td>(333 \text{ cm}^2) (nearest (\text{cm}^2))</td>
</tr>
<tr>
<td><strong>4(c)</strong></td>
<td>(8.27)</td>
</tr>
<tr>
<td><strong>5(b)</strong></td>
<td>(48.8 \text{ m} (3 \text{s.f.}))</td>
</tr>
<tr>
<td><strong>5(d)(i)</strong></td>
<td>(14.0 \text{ m} (3 \text{s.f.}))</td>
</tr>
<tr>
<td><strong>6(a)</strong></td>
<td>(x = 12)</td>
</tr>
<tr>
<td><strong>6(c)(ii)</strong></td>
<td>The gradient represent the rate of decrease of the cost of printing each book when number of books is 300. Or The gradient represent the rate at which the cost of printing each book is decreasing.</td>
</tr>
<tr>
<td><strong>7(a)(i)</strong></td>
<td>(AB = 2)</td>
</tr>
<tr>
<td><strong>7(a)(ii)</strong></td>
<td>(210 \leq x \leq 700) (accept 180 - 220; 680 - 730, with interval of 10)</td>
</tr>
<tr>
<td><strong>7(b)</strong></td>
<td>(\angle CAP = \sin^{-1} \left( \frac{1}{2} \right) = 30^\circ = \frac{\pi}{6})</td>
</tr>
<tr>
<td><strong>7(c)</strong></td>
<td>(17.4 \text{ cm} (3 \text{s.f.}))</td>
</tr>
</tbody>
</table>

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**Fairfield Methodist School (Secondary)**

**2019 Sec 4Exp 5NA Preliminary Examination**

**Answer Key for Mathematics Paper 1**
<table>
<thead>
<tr>
<th>Answer Key</th>
</tr>
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<tbody>
<tr>
<td>8(a)(i)(b) [ 2p + 5q ]</td>
</tr>
<tr>
<td>8(a)(ii)(b) [ \frac{90}{3} \text{ units}^2 \text{ or } 26\frac{2}{3} \text{ or } 26.7 (3 \text{ s.f.)} ]</td>
</tr>
<tr>
<td>8(b)(i) [ \overrightarrow{AB} = \left( \frac{10}{16} \right) ]</td>
</tr>
<tr>
<td>8(b)(iii) [ \overrightarrow{AB} = k \left( \frac{n}{-2m} \right) \text{ or } \overrightarrow{AB} = k \left( \frac{-n}{2m} \right) ]</td>
</tr>
<tr>
<td>9(a)(i) 41 minutes</td>
</tr>
<tr>
<td>9(a)(iii) IQR = 48 - 29 = 19 mins</td>
</tr>
</tbody>
</table>

9(b) The travelling time using train is much shorter (faster) than using a bus as the median time travelling with a train < median time travelling with a bus. The travelling time using train is less spread out (more consistent) than using travelling using a bus as the interquartile range of travelling time of a train < interquartile range of travelling time of a bus.

9(c)(i) \[ \frac{7}{240} \]

9(c)(ii) \[ \frac{37}{120} \]

10(a)(iii) Using staggered start, each runner runs exactly 400 meters or same distance.

10(b) His claim is false.
### Preliminary Examination 2019
Mathematics Paper 2
Marking Scheme

<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Marks</th>
<th>Total</th>
</tr>
</thead>
</table>
| 1(a) | \[
\begin{align*}
3x + 2 &= \frac{2}{x - 3} + \frac{4}{x - 4} \\
3(x^2 + 9) + 2(x - 3) &= (x - 3)(x + 4) \\
3x^2 + 12x + 2x - 6 &= (2x - 3)(x + 4) \\
3x^2 + 14x - 6 &= (x - 3)(x + 4)
\end{align*}
\] | M1 | 2 |

| 1(b) | \[
\begin{align*}
(x^2 + 5)^2 - (x^2 - 3)^2 &= (x^2 + 5 - x^2 + 3)(x^2 + 5 + x^2 - 3) \\
&= (8)(2x^2 + 2) \\
&= 16x^2 + 16 \\
&= 16\left(x^2 + 1\right) \text{ or } 16x^2 + 16
\end{align*}
\] | M1 | 2 |

| 1(c) | \[
\begin{align*}
x^2 - 13x - 24 &= 0 \\
x(2x - 24x - 8) &= 0 \\
x &= 0 \text{ or } x = -1.5 \text{ or } x = 8
\end{align*}
\] | A1 (two correct), A1 | 2 |

| 1(d) | \[
\begin{align*}
\frac{1}{2}(n + 1) + \frac{1}{2}(n + 1)(n + 2) &= \frac{1}{2}(1) + \frac{1}{2}(2)(3) \\
&= \frac{1}{2}(n + 1)(2n + 2) \\
&= \frac{1}{2}(n + 1)(2n + 1) \\
&= (n + 1)^2
\end{align*}
\] Since \((n + 1)^2 = (n + 1)(n + 1)\), therefore, it is a square number. \(n + 1\) has a repeated term factor, therefore it is a square number. | B1 | 3 |

| 2(i) | \[
\begin{align*}
QR &= QT = RT \text{ (radius of circles centred at } Q \text{ and } R) \\
\text{Therefore, triangle } QRT \text{ is an equilateral triangle.} \\
\text{Angle } Q &= \text{Angle } R = \text{Angle } T = 60^\circ
\end{align*}
\] | B1 | 3 |

| 2(ii) | \[
\begin{align*}
\text{Angle } TQ &= \text{Angle } SUT \text{ (Angle in same segment) = } 60^\circ \\
\text{Angle } TQ &= \text{Angle } TQR = \text{Angle } TQR = \text{Angle } QRT = \text{Angle } SUT \text{ (all are } 60^\circ) \text{ (triangle } QTR \text{ is equilateral triangle).} \\
\text{Since angle } TQR = \text{Angle } SUT, \text{ the form alternate angle, therefore } TQ \text{ is parallel to } SU. \\
\text{Since angle } TQR = \text{Angle } SUT, \text{ by converse property of alternate angle, } TQ \text{ is parallel to } SU.
\end{align*}
\] | B1 | 3 |

| 2(iii) | \[
\begin{align*}
\text{Angle } UQR &= \text{Angle } UQ = 90^\circ \text{ (right angle in a semi-circle)} \\
\text{Since they are the same and they form interior angle, therefore, } TQ \text{ is parallel to } SU. \\
\text{Since angle } TQU = \text{Angle } UQ = 90^\circ, \text{ by converse property of interior angle, } TQ \text{ is parallel to } SU.
\end{align*}
\] | B1 | 3 |
3(a) Area of triangle CDE
Area of triangle ABC
\[ \frac{1}{2} \times CD \times CE \times \sin\theta \]
\[ \frac{1}{2} \times AC \times BC \times \sin\theta \]
\[ \frac{CD \times CE}{AC \times BC} \]
M1
AG1
2

3(b) \[ \frac{CD \times CE}{AC \times BC} = \frac{1}{3} \]
\[ \frac{(x + 2)(x - 5)}{x} = \frac{3}{x} \]
\[ x = 2 \text{ or } x = 5 \text{ (acceptable)} \]
M1
M1 (Expand and simplify)

3(c)(i) \[ 2x^2 - 19x + 6 = 0 \]
\[ x = \frac{19 \pm \sqrt{(-19)^2 - 4\cdot2\cdot6}}{2\cdot2} \]
\[ x = \frac{19 \pm \sqrt{337}}{4} \]
\[ x = 9.1729 \text{ or } x = 0.3270484 \]
\[ x = 9.17 \text{ or } x = 0.33 \text{ (2 d.p.)} \]
A1, A1
3

3(c)(ii) When \( x = 0.3271 \), \( DC = 2x - 5 = -4.356 < 0 \).
Therefore, \( x \) cannot be 0.33.
B1

3(d) \[ DE = \sqrt{13.3458^2 + 9.1729^2 - 2(13.3458)(9.1729)\cos25^\circ} \]
= 36.35327
= 36.35 (3 s.f.)
M2
A1
3

4(a) Total surface area of bowl
\[ = \pi x^2 + 2\pi x \cdot 6 \]
\[ = 263.839 \]
\[ = 264 \text{ cm}^2 \text{ (nearest cm}^2\text{)} \]
M1
M1
M1
12

4(b)(i) Number of bowls completely filled
\[ = \frac{13}{(x+6)(x+4)} \]
\[ = \frac{13}{12 \times 100 \text{cm}^2} \]
\[ = \frac{13}{720 \text{ cm}^2} \]
\[ = 0.01741 \]
M1
A1

4(b)(ii) Volume of soup left
\[ = 13 \cdot 28 \times (\pi \cdot 6^2 \cdot 4) \]
\[ = 333.9984 \]
M1
A1

4(b)(iii) Volume of scoop left
\[ = 0.7363 \times (\pi \times 6^2 \times 4) \]
\[ = 333.094 \]
M1

4(b)(iv) Volume of sphere pan = 13 litres
\[ = 13000 \text{ cm}^3 \]
\[ \frac{1}{3} \cdot \pi \cdot r^2 \cdot h \]
\[ = 13000 \text{ cm}^3 \]
\[ r = 25 \text{ cm} \]
\[ h = 180.37 \text{ cm} \]
\[ h = 184 \text{ mm (nearest mm)} \text{ or } 18.4 \text{ cm (nearest cm)} \]
M1
A1
2

4(c) \[ \frac{3}{5} = \frac{8}{x} \]
\[ x = \frac{8 \cdot 15}{27} \]
\[ x = 11.11 \text{ (nearest cm)} \]
B1

5(a) \[ \sin 25^\circ = \sin 35^\circ \cdot \sin 75^\circ \]
\[ = \frac{5}{8} \cdot \frac{\sqrt{5}}{4} \]
\[ = 0.282775 \text{ or } 0.283 \text{ (4 s.f.)} \]
M1

5(b) \[ \sin 25^\circ = \sin 35^\circ \cdot \sin 75^\circ \]
\[ = \frac{5}{8} \cdot \frac{\sqrt{5}}{4} \]
\[ = 0.282775 \text{ or } 0.283 \text{ (4 s.f.)} \]
M1

5(c) \[ \sin 42^\circ = \frac{h}{x} \]
\[ h = \sin 42^\circ \cdot x \]
\[ = 0.207918 \cdot x \]
\[ h = 12.61 \text{ in } \text{ (3 s.f.)} \]
A1
2

5(d)(i) Let the height of the cliff be \( H \).
\[ \tan 16^\circ = \frac{H}{13.9983} \]
\[ H = 13.9983 \cdot \tan 16^\circ \]
\[ = 13.593 \text{ or } 14.0 \text{ m (3 s.f.)} \]
M1
A1
2

5(d)(ii) Angle of elevation = \[ \tan^{-1} \left( \frac{13.9983}{32.642715} \right) \]
\[ = 23.196^\circ \]
M1
Note: if note can be FTI
A1
2

6(a)(i) \( p = 12 \)
B1
1

6(a)(ii) Plot the points
Draw a smooth curve
P2
C1
3

6(b) 400 \text{ (accept answer: } 460 - 500) \]
B1
1
<table>
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<tbody>
<tr>
<td>6</td>
<td>(c)(i) Draw a tangent line&lt;br&gt;Gradient of tangent = -20.1 / 500 = -0.04025&lt;br&gt;Accept answers: (0.0230 to 0.030)&lt;br&gt;Actual answer: -0.02666 = -0.02567</td>
<td>B1</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>(c)(ii) The gradient represent the rate of decrease of the cost of printing each book when number of books is 300. Or&lt;br&gt;The gradient represent the rate at which the cost of printing each book is decreasing</td>
<td>B1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>(d)(i) Plot points (2, 25), (600, 15) and (1200, 5) Draw a line</td>
<td>P1</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>(d)(ii) 210 ≤ x ≤ 790 (accept 180 to 220; 680 to 730, with interval of 10)</td>
<td>L1</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>(a)(i) Angle APC = 90° (tangent perpendicular radius)&lt;br&gt;Angle AQE = 90° (tangent perpendicular radius)&lt;br&gt;Therefore, angle APC = angle AQE&lt;br&gt;Angle APC = angle QAE (common angle)&lt;br&gt;Since there are two pairs of corresponding angles are equal, triangle APC and triangle AQE are similar.</td>
<td>B1</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>(a)(ii) ( \frac{AC}{PC} = \frac{AE}{QE} )&lt;br&gt;( AB = 2 )&lt;br&gt;( AB + 4 = 6 )&lt;br&gt;( AB + 2 = 8 )&lt;br&gt;( AB + 10 = 10 )&lt;br&gt;( 3AB + 2 = AB + 10 )&lt;br&gt;( 3AB + 6 = AB + 10 )&lt;br&gt;( 2AB = 4 )&lt;br&gt;( AB = 2 )</td>
<td>M1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>(b) Angle EQ = Angle CAP&lt;br&gt;( \sin \angle CAP = \frac{AC}{PC} = \frac{2}{1} = 2 )&lt;br&gt;( \angle CAP = \sin^{-1} \left( \frac{1}{2} \right) = 30° = \frac{\pi}{6} )</td>
<td>A1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>(c) Perimeter of the shaded region = Arc PD + PQ + Arc QD&lt;br&gt;( = \frac{120 \times 2 \pi x 2 \pi}{360} + \frac{60 \times 2 \pi x 6}{360} + 4.18879 + 6.92820 + 6.28318 )&lt;br&gt;( = 17.40017 )&lt;br&gt;( = 17.4 \text{ cm (3 s.f.) or 17.5 cm (3 s.f.) (if rounded off to 5 s.f. working)} )</td>
<td>M1</td>
<td>4</td>
</tr>
</tbody>
</table>

8 | (a)(i) \( WM = \frac{1}{2} (W + X + 3M) = 6p + 3q + \frac{5}{3} (-10p + 5q) \)<br>\( WM = 2p + 5q \) | B1 | 1 |
| 8 | (a)(b) Note: If notation is wrong, minus 1 marks for overall (a)(b) | | |
| 8 | (b) Area of triangle WXY Area of triangle WXX<br>Area of triangle WXY<br>Area of triangle WXY<br>Area of triangle WXYZ<br>\( = \frac{1}{2} (\frac{1}{2} (120 - 06) \times 3) + \frac{1}{2} (120 - 06) \times 1) \)<br>\( = 120 - 06 \times 3 \times 1 \)<br>\( = 120 \times 06 \times 1 \)<br>\( = 120 \times 06 \times 1 \)<br>\( = 120 \times 06 \times 1 \) | M1 | 2 |

8 | (b)(i) Area of triangle WXY = 8x = 2x units squared or 28.4 or 26.7 (3 s.f.) | B1 | 1 |

9 | (a)(i) 41 minutes | B1 | 1 |
<p>| 9 | (a)(ii) 25 minutes | B1 | 1 |
| 9 | (b) Q = 29 mins, R = 48 mins&lt;br&gt;IQR = 48 - 29 = 19 mins | M1, A1 or B1 | 1 |
| 9 | (iv) Percentage of adults who spend more than 40 minutes travelling to work everyday&lt;br&gt;( \frac{120 - 06}{120} \times 100% = \frac{44}{120} \times 100% = \frac{37}{3} \times 100% = 37.77% = 37.8% ) | M1 | 1 |
| 9 | (b) Median for bus = 43 mins&lt;br&gt;IQR for bus = 55 - 27 = 28 mins&lt;br&gt;The travelling time using train is much shorter (faster) than using a bus as the median time travelling with a train &lt; median time travelling with a bus.&lt;br&gt;The travelling time using train is less spread out (more consistent) than using travelling using a bus as the interquartile range of travelling time of a train &lt; interquartile range of travelling time of a bus. | B1 | 3 |
| 9 | (c)(i) Probability of finishing work in time&lt;br&gt;( P(\text{more than 55 minutes}) = \frac{120 - 06}{120} x 4 = \frac{14}{120} x 4 = 0.14 \times 4 = 0.56 ) | B1 | 1 |
| 9 | (c)(ii) P(\text{more than 55 minutes on one trip})&lt;br&gt;( = \frac{120 - 06}{120} x 3 \times 106 = 0.14 \times 3 \times 106 = 55 &lt; 55 \times 4 = 220 ) | M1 A1 | 2 |</p>
<table>
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<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>10(a)(i)</td>
<td>Total distance for Lane 1&lt;br&gt;$= 2 \times x \times (36.5 + 0.3) - (84.39 \times 2)$&lt;br&gt;$= 400.001 \text{ m}$&lt;br&gt;$= 400 \text{ m (S.F.)}$</td>
<td>M1 for 36.5 + 0.3&lt;br&gt;AG1</td>
<td>2</td>
</tr>
<tr>
<td>10(a)(ii)</td>
<td>Total distance for Lane 8&lt;br&gt;$= 2 \times x \times (36.5 + 0.2 + 1.22 \times 7) - (84.39 \times 2)$&lt;br&gt;$= 453.031$&lt;br&gt;Staggered start = 453.031 - 400&lt;br&gt;$= 53.03 \text{ m (2 d.p.)}$</td>
<td>M1 for 1.22 \times 7&lt;br&gt;M1 for the formula&lt;br&gt;AG1</td>
<td>3</td>
</tr>
<tr>
<td>10(a)(iii)</td>
<td>Using staggered start, each runner runs exactly 400 meters or run the same distance.</td>
<td>B1</td>
<td>1</td>
</tr>
</tbody>
</table>

**10(b)**<br>Speed = 6.8 km/h = $6.8 \times 1000 = 6.8 \times 1000 / 60 = 113.33 \text{ m/min}$<br>Time taken to complete one round in Lane 8<br>$453.031 \text{ m} / 113.33 \text{ m/min} = 3.99733 \text{ mins}$<br>Time taken to complete five rounds in Lane 8<br>$= 3.99733 \times 5 = 19.9865 \text{ mins}$<br>Time taken to complete five rounds in Lane 8 in 1 week<br>$= 19.9865 \times 7 = 139.905 \text{ mins (less than 150 minutes)}$<br>His claim is false.<br>Or 10b<br>Total distance for average speed of 6.8 km/h<br>$= 6.8 \times 1000 \times 150 \text{ mins}$<br>$= 6.8 \times 1.5 \text{ km/h}$<br>$= 17 \text{ km} = 17000 \text{ m}$<br>Total distance covered for 5 rounds in 1 week<br>$= 453.03 \times 5 \times 7 = 15856.05 \text{ m}$
Since $15856.05 \text{ m} < 17000 \text{ m}$, his claim is false.<br>

**Or 10b**<br>Total distance for average speed of 6.8 km/h<br>$= 6.8 \times 1000 \times 150 \text{ mins}$<br>$= 6.8 \times 1.5 \text{ km/h}$<br>$= 17 \text{ km} = 17000 \text{ m}$
No. of rounds covered in 1 week = $17000 / 453.03 = 37.525$<br>Since 35 rounds < 37.525 rounds, his claim is false.
HILLGROVE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2019
SECONDARY 4 (EXPRESS) / 5 (NORMAL ACADEMIC)

CANDIDATE NAME ( )

CLASS

CENTRE NUMBER S

INDEX NUMBER

MATHMATICS 4048/02

Paper 2

Candidates answer on the Question Paper.

No Additional Materials are required.

26 August 2019

2 hours 30 minutes

8.00 a.m. to 10.30 a.m.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer all questions

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if answer is not exact, give

the answer to three significant figures. Give answers in degrees to one decimal place.

For \( \pi \), use either your calculator value or 3.142, unless the question requires the

answer in terms of \( \pi \).

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part

question.

The total number of marks for this paper is 100.

Parent’s/ Guardian’s Signature:

________________________

Setter: Miss Li Zhiyi

This document consists of 25 printed pages
Mathematical Formulae

**Compound interest**

Total amount = \[ P \left( 1 + \frac{r}{100} \right)^n \]

**Mensuration**

Curved surface area of a cone = \( \pi rl \)

Surface area of a sphere = \( 4\pi r^2 \)

Volume of a cone = \( \frac{1}{3} \pi r^2 h \)

Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

Area of triangle \( ABC = \frac{1}{2} ab \sin C \)

Arc length = \( r\theta \), where \( \theta \) is in radians

Sector area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

**Trigonometry**

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

**Statistics**

Mean = \[ \frac{\sum f x}{\sum f} \]

Standard deviation = \[ \sqrt{\frac{\sum f x^2}{\sum f} - \left( \frac{\sum f x}{\sum f} \right)^2} \]
Answer all the questions.

1 (a) Solve the inequality $\frac{2+3x}{5} \leq \frac{3-x}{2}$.

(b) Simplify $\frac{16x^2 - y^2}{4x^2 + 9xy + 2y^2}$.
(c) Given that \( x = -\frac{1}{2} \) is a solution of \((m-1)x^2 + (m-2)x = 2m-15\), find the value of \( m \) and the second possible value of \( x \). [3]

(d) Solve the equation \( \frac{x+5}{3x-1} = \frac{15x-1}{3x-1} \). [3]
2. $\overline{AB} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\overline{OB} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$, where $O$ is the origin.

(a) Find the length of $AB$. [1]

(b) Find the equation of line $AB$. [2]

(c) $C$ is a point on $x$–axis such that angle $ACB$ is a right angle. Find the tangent of angle $ABC$. [2]
(d) The equation of the line $m$ is $3y + 2x = 6$. Line $m$ intersects the $x$-axis at $D$ and $y$-axis at $E$.

(i) Show how you can tell that the line $m$ does not intersect the line $AB$. [2]

(ii) Find $ED$ [2]

(iii) What type of quadrilateral is $ABDE$? Explain your answer. [3]
Rashidi wants to make a closed rectangular box to contain his toys. The box has a rectangular base of sides $3x$ cm and $x$ cm and its height is $h$ cm.

(a) (i) The volume of the box is $90$ cm$^3$. Express $h$ in terms of $x$. 

(ii) Show that the total surface area of the box $A$ cm$^2$, is given by $A = \frac{240}{x} + 6x^2$. 

The table shows some corresponding values of $x$ and $A$, correct to the nearest integer.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>246</td>
<td>144</td>
<td>134</td>
<td>156</td>
<td>$b$</td>
<td>256</td>
<td>328</td>
<td>414</td>
</tr>
</tbody>
</table>

(a) Find the value of $b$. 

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal $x$-axis for $0 \leq x \leq 8$. Using a scale of 2 cm to represent 50 units, draw a vertical $A$-axis for $0 \leq A \leq 500$. On your axes, plot the points given in the table and join them with a smooth curve on the grid in page 9.
(c) By drawing a tangent, find the gradient of the curve at (2, 144).

(d) Rashidi wants to make a box with a total surface area of 100 cm² without changing the volume of the box. Is it possible to do so? Explain your answer.
Replace this page with graph paper.
4. \( A, B \) and \( C \) represent three islands.

Both boats \( X \) and \( Y \) left from island \( A \).

The bearings of \( B \) and \( C \) from \( A \) is \( 160^\circ \) and \( 100^\circ \) respectively.

Boat \( X \) sailed towards \( B \) at a speed of \( x \) km/h. Boat \( Y \) sailed towards \( C \) at a speed of \((x + 32)\) km/h.

Boat \( X \) and \( Y \) took 2 hours to reach \( B \) and \( C \) respectively.

\[
\begin{array}{c}
\text{(a) Write an expression, in terms of } x, \text{ for the distance travelled by Boat } X. \\
\text{(b) } B \text{ and } C \text{ are 150 km apart.} \\
\end{array}
\]

\[
\begin{align*}
\text{(b) } B \text{ and } C \text{ are 150 km apart.} \\
\text{Write down an equation in } x \text{ to represent this information and show that it reduces to } \\
x^2 + 32x - 4601 = 0. \\
\end{align*}
\]
(b) Solve the equation \( x^2 + 32x - 4601 = 0 \).

(c) Boat \( X \) returned from \( B \) to \( A \) and took 30 more minutes than the onward journey.

Find the speed of Boat \( X \) for the return journey.
$O$ is the centre of the circle passing through $A$, $B$, $C$, $D$ and $E$. $UBX$ is the tangent to the circle at $B$. $ACX$ is a straight line which passes through $O$. $\angle CBX = \angle CXB = 30^\circ$ and $\angle EDC = 140^\circ$.

(a) Stating your reasons clearly, find,

(i) $\angle BAC$,
(ii) \( \angle CEB \)  

(iii) \( \angle CBE \)  

(iv) \( \angle OBY \)  

(b) \( T \) is a point such that angle \( ATE \) is 60° and on the same side as point \( B \).  

State whether point \( T \) lies in the circle.  

Explain your answer.
Angela and Ruth went on a free-and-easy trip to Sydney.

(a) They both exchanged S$1600 at a money changer in Singapore. The money changer used an exchange rate between Singapore dollars (S$) and Australian dollars (A$) of A$ 1 = S$0.95.

Calculate the amount of Australian dollars they received.

(b) They booked their accommodation through the Waterbnb application.

Each night at the accommodation costs A$110.25. However, there was a 10% service charge. An additional 5% daily cleaning fee was imposed only on the cost of the accommodation.

They booked a total of 8 nights at the accommodation.

Calculate the total cost of their accommodation in Australian dollars.
(e) Ruth also purchased a watch in Sydney. To pay for the watch, she borrowed S$ 2340 for 2 years at an interest of 5.6% per annum compounded half-yearly.

Calculate the amount of interest she paid for the watch.

(e) Sydney has a population of 4.627 million and Darwin has a population of 132 000.

Calculate how many more people live in Sydney than in Darwin, giving your answer in standard form.
Hillgrove organised an annual school carnival to raise funds for a children’s home.

Two different classes sold three different flavours of Bubble Tea to raise funds.

The table below shows the number of Bubble Tea sold by each class and the price of the Bubble Tea.

<table>
<thead>
<tr>
<th>Flavours</th>
<th>Class 4-10</th>
<th>Class 4-11</th>
<th>Price per cup</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milk Oolong</td>
<td>30</td>
<td>20</td>
<td>$1.90</td>
</tr>
<tr>
<td>Cheese Peach</td>
<td>20</td>
<td>40</td>
<td>$3.20</td>
</tr>
<tr>
<td>Brown Sugar</td>
<td>15</td>
<td>10</td>
<td>$2.50</td>
</tr>
</tbody>
</table>

(a) Represent the number of different flavours of Bubble Tea sold by each class in a $3 \times 2$ matrix, $F$.  

(b) Represent the prices of Bubble Tea in a $1 \times 3$ matrix, $P$.  

(c) Evaluate the matrix $R = PF$.  

[1] [1] [2]
(d) Explain what each element in matrix $R$ means.

(e) $M$ is \[ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \]. Evaluate $A = RM$ and explain what the element in $A$ means.
The diagram represents the roof of a house. The ridge AB is horizontal and is built centrally above the rectangular plane WXYZ. AM is the height of the ridge above WXYZ. It is given that WX = 15 m, XY = 8 m, AB = 7 m and AM = 2.5 m.

(a) Show that WM is 5.657 m, correct to 4 significant figures. [2]

(b) Find

(i) angle AWM, [2]
(ii) $AW$.

(iii) $AX$.

(c) Find the smallest angle of elevation of $A$ from a point along $WX$. [2]
$ABCD$ is a square of length 12 cm. $DQP$ and $BRS$ are two identical quadrants and arcs $QP$ and $RS$ meet at point $X$. $DCX$ is a straight line.

(a) Find

(i) the perimeter of the shaded region, \[4\]
(ii) the area of \(QRX\). [Note: \(QRX\) is not a triangle]

(b) Show that triangle \(DAB\) is similar to triangle \(RCQ\).
There was a hit and run car accident along the Pan-Island Expressway (PIE). It was raining the day before and the road was wet. PIE has a speed limit of 80 km/h. The police needed to find out who the driver was and there were some items found at the scene.

Exhibit A: slip of paper with driver’s telephone number

Exhibit B: driver’s shoes print - 25 cm in length

Exhibit C: driver's vehicle skid marks - 350 ft. in length
It is given that the velocity of the car is directly proportional to the square root of the length of skid marks.

The police also uses graphs below to help them to solve crimes.

\[
\text{height of a person (cm)}
\]

\[
\begin{array}{c}
180 \\
27
\end{array}
\]

\[
\text{length of foot (cm)}
\]
Note that the graphs only give predicted estimated values.

(a) Given that there are 8-digits in the driver's telephone number, find the total number of possible telephone numbers.
The police then narrowed to three suspects with their details below.

<table>
<thead>
<tr>
<th></th>
<th>Daniel Lee</th>
<th>Jacob Yap</th>
<th>Samuel Wong</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td>1.78 m</td>
<td>1.66 m</td>
<td>1.82 m</td>
</tr>
<tr>
<td>Brand of vehicle they own</td>
<td>Kia</td>
<td>Toyota</td>
<td>Volkswagen</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Speed limit</th>
<th>Composition Fine</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Light Vehicle</td>
</tr>
<tr>
<td></td>
<td>(Examples: motor cars, motorcycles, light goods van, etc.)</td>
</tr>
<tr>
<td>Exceeding road speed limit by 1 – 20 km/h</td>
<td>$150</td>
</tr>
<tr>
<td>Exceeding road speed limit by 21 – 30 km/h</td>
<td>$200</td>
</tr>
<tr>
<td>Exceeding road speed limit by 31 – 40 km/h</td>
<td>$300</td>
</tr>
<tr>
<td>Exceeding 41 km/h</td>
<td>prosecution in court</td>
</tr>
</tbody>
</table>
(b) Find the driver who was involved in the accident and determine the appropriate fine for speeding ticket. Justify your decision with calculations.
HILLGROVE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2019
SECONDARY 4 (EXPRESS) / 5 (NORMAL ACADEMIC)

CANDIDATE NAME ( ) CLASS
CENTRE NUMBER S INDEX NUMBER

MATHMATICS 4048/01
Paper 1 22 August 2019

Candidates answer on the Question Paper. 2 hours
No Additional Materials are required. 8.00 a.m. to 10.00 a.m.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Parent's/ Guardian's Signature:

Setter: Miss Li Ziyi

This document consists of 21 printed pages.
Mathematical Formulae

**Compound interest**

Total amount = \( P \left( 1 + \frac{r}{100} \right)^n \)

**Mensuration**

Curved surface area of a cone = \( \pi rl \)

Surface area of a sphere = \( 4\pi r^2 \)

Volume of a cone = \( \frac{1}{3} \pi r^2 h \)

Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

Area of triangle \( ABC = \frac{1}{2}ab \sin C \)

Arc length = \( r\theta \), where \( \theta \) is in radians

Sector area = \( \frac{1}{2}r^2\theta \), where \( \theta \) is in radians

**Trigonometry**

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

**Statistics**

Mean = \( \frac{\Sigma fx}{\Sigma f} \)

Standard deviation = \( \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \left( \frac{\Sigma fx}{\Sigma f} \right)^2} \)
Answer all the questions.

1. Write the following numbers in order of size, starting with the largest.

\[
\frac{37}{100}, \quad 0.37, \quad 0.\overline{3}7, \quad \left(\frac{37}{100}\right)^2
\]

Answer: \_, \_, \_, \_ \quad [1]

2. Write down the sets represented by the following shaded regions.

(a)

\[\begin{aligned}
\text{\textcolor{gray}{A}} & \quad \text{\textcolor{gray}{B}} \\
\text{\textcolor{gray}{A}} & \quad \text{\textcolor{gray}{B}}
\end{aligned}\]

Answer: \_ \_ \_ \_ \quad [1]

(b)

\[\begin{aligned}
\text{\textcolor{gray}{A}} & \quad \text{\textcolor{gray}{B}} \\
\text{\textcolor{gray}{A}} & \quad \text{\textcolor{gray}{B}}
\end{aligned}\]

Answer: \_ \_ \_ \_ \quad [1]
3. Show that \((1+5n)^2 - 6\) is a multiple of 5 for all integer values of \(n\).

Answer

4. (a) Express \(x^2 - 6x + 5\) in the form of \((x-a)^2 + b\).

Answer

(b) Hence, solve \(x^2 - 6x + 5 = 0\).

Answer \(\quad\) or \(\quad\) [1]
5 Nithya runs a photography club.

Her club currently has 62 boys and 39 girls.

Her aim is to have at least 45% of members to be girls.

Find the smallest number of girls she needs to recruit to achieve this.

Answer: ______________________ [2]

6 A pizza is divided in the ratio of 3 : 7 : 5 for Adrian, Ben and Carl respectively. Ben then divides his share in the ratio of 9 : 7 and give it to Adrian and Carl respectively.

Carl says that both he and Adrian have the same amount of pizza now.

Do you agree with Carl? Explain your answer.

Answer: ______________________ [2]
7 It is given that \( y = \frac{3x + 5b}{x - 2c} \).

(a) Find \( y \) when \( x = 1, \ b = 2, \ c = 4 \).

\[ \text{Answer} \quad [1] \]

(b) Express \( x \) in terms of \( y, \ b \) and \( c \).

\[ \text{Answer} \quad [2] \]

8 Given that \( \cos x = \frac{21}{29} \), where \( x \) is an acute angle, find the value of \( \sin x + \cos(180^\circ - x) - \tan x \) without the use of calculator.

\[ \text{Answer} \quad [3] \]
9 (a) The product of three prime numbers $a$, $b$ and $c$ is $m$, where $m$ is an even number.

The smallest and the largest number out of the three prime numbers are $a$ and $c$ respectively.

State the value of $a$ and explain your answer.

Answer

(b) Given that $b = 23$ and $m$ is less than 670, find the largest possible value of $c$.

Answer

[2]
The diagram shows a plot of land labelled $WXYZ$.

(a) Construct the bisector of angle $WZY$.  

(b) Construct the perpendicular bisector of $WZ$.  

(c) (i) The government is planning to build a playground that is equidistant from $W$ and $Z$ and $WZ$ and $YZ$. Mark and label the location of the playground as $P$.  

(ii) Hence, find the bearing of $P$ from $Y$.  

Answer
11 (a) Given that $4(x + 2y) = 5x - 3y$, find the ratio of $x : y$.

(b) Solve $\frac{27^b}{9} = 81^{b+1}$.
12 Winnie is drawing an $n$-sided regular polygon.

The total sum of the interior angles is $h^\circ$.

Given that $h + n = 1631$, find the values of $h$ and $n$.

\[ \text{Answer} \quad h = \underline{\phantom{1000}} \]

\[ \quad n = \underline{\phantom{1000}} \quad [4] \]
In the diagram, $A$ and $B$ are points on a circle, centre $O$. The radius of the circle is 5 cm and length of $AB$ is 9.4 cm.

Find the area of the minor segment $AB$.

Answer  ________________ [4]
$ABCD$ is a square and $AY$ is perpendicular to $XZ$. $XYZ$ and $ADZ$ are straight lines. $Y$ is the midpoint of $CD$.

(a) Show that triangle $DYZ$ is congruent to triangle $CYX$.

(b) Show that $AZ = AX$. 

[2]
Serene has two boxes of cards.  
One box contains cards that has either a circle or a square.  
The other box contain cards that are green or blue in colour.

Serene picks a card from each box at random.  
The probability that she picks a circle card is \( c \).  
The probability that she picks a green card is \( g \).

(a)  Complete the table for the card that Serene picks, writing each probability in terms of \( c \) and \( g \).

<table>
<thead>
<tr>
<th>Event</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle and Green</td>
<td></td>
</tr>
<tr>
<td>Square and Green</td>
<td></td>
</tr>
<tr>
<td>Circle and Blue</td>
<td></td>
</tr>
<tr>
<td>Square and Blue</td>
<td></td>
</tr>
</tbody>
</table>

(b)  The probability that she picks a circle card is \( \frac{2}{7} \).

The probability that she picks a circle card and green card is \( \frac{1}{5} \).

Hence, or otherwise, find the total number of green cards given that the total number of blue and green cards is 30.

Answer
The diagram shows the speed – time graph for a truck’s journey.

The distance travelled for the first $t$ seconds is 756 m.

(a) Find $t$.

\[ \text{Answer} \] [2]

(b) Calculate the deceleration at the 15th second of the journey.

\[ \text{Answer} \] [2]

(c) Find the average speed in km/h.

\[ \text{Answer} \] [2]

17 The diagram shows a parallelogram $OXYZ$. 

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HGV Sec 4E/5N E-Mathematics Prelim Paper 1
$\overrightarrow{OA} = a$ and $\overrightarrow{OB} = b$. $BED$ and $AEC$ are lines parallel to the sides of the parallelogram. $\overrightarrow{OX} = 3\overrightarrow{OA}$ and $\overrightarrow{OZ} = 2\overrightarrow{OB}$.

(a) Express $\overrightarrow{EY}$ in terms of $a$ and $b$, as simply as possible.

Answer [2]

(b) Write down two vectors that can be written as $b - 3a$.

Answer [2]

(c) Find the ratio of the area of triangle $BCE$ to the area of parallelogram $OXYZ$.

Answer [2]
The diagram shows a toy that combines a cone and a hemisphere. The slant height of the cone is 17 cm and the perpendicular height of the cone is 15 cm.

(a) Show that the radius of the hemisphere is 8 cm.

(b) Calculate the surface area of the toy.

Answer

[2]

[3]
(c) The company wants to make a miniature version of the toy such that the volume of the miniature toy is $\frac{1}{6}$ of the volume of the actual toy.

Calculate the radius of the miniature toy.

Answer [2]
19 The travelling times taken by 50 students from home to school are recorded. The cumulative frequency curve below shows the distribution of their times.

(a) Use your graph to estimate

(i) the median

Answer

(ii) the interquartile range

Answer
(b) Find the probability that two students, chosen at random, take less than 70 minutes to travel to school.

\[ \text{Answer } \] [2]

(c) A student states that 32% of the students took at least 36 minutes to travel to school.

Comment on whether the data from the school supports this claim.

\[ \text{Answer } \] [2]
The diagram shows two triangles $ABC$ and $BDC$.

$AC = 3 \text{ cm}, AB = 4 \text{ cm}, BC = 5 \text{ cm}, BD = 10 \text{ cm}$ and $\angle CBD = 113^\circ$.

(a) Show that $\angle CAB$ is a right angle.

(b) Find $\angle BDC$.

Answer

[2]

HGV Sec 4E/5N E-Mathematics Prelim Paper 1
(c) Find the area of $ABDC$.

\textit{Answer} \hspace{1cm} [2]

(d) A circle is drawn through the points $A$, $B$ and $C$. State the length of the diameter of the circle. Explain your answer.

\hspace{1cm}

\hspace{1cm}

\hspace{1cm}

\hspace{1cm}

\hspace{1cm}

\hspace{1cm}

\hspace{1cm}

\hspace{1cm}

\hspace{1cm} [2]
**HILLGROVE SECONDARY SCHOOL**  
**PRELIMINARY EXAMINATION 2019**  
**SECONDARY 4 (EXPRESS) / 5 (NORMAL ACADEMIC)**

**MATHMATICS**  
4048/02

Paper 2  
26 August 2019

Candidates answer on the Question Paper.  
No Additional Materials are required.  
2 hours 30 minutes  
8.00 a.m. to 10.30 a.m.

**READ THESE INSTRUCTIONS FIRST**

Write your class, index number and name on the work you hand in.  
Write in dark blue or black pen on both sides of the paper.  
You may use an HB pencil for any diagrams or graphs.  
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.

If working is needed for any question it must be shown with the answer.  
Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.  
If the degree of accuracy is not specified in the question, and if answer is not exact, give the answer to three significant figures.  
Give answers in degrees to one decimal place.

For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).

At the end of the examination, fasten all your work securely together.  
The number of marks is given in brackets [ ] at the end of each question or part question.  
The total number of marks for this paper is 100.

**For Examiner’s Use**

**TOTAL**

100

**Mathematical Formulas**

**Compound interest**

Total amount = \( P \left(1 + \frac{r}{100}\right)^n \)

**Mansuration**

- Curved surface area of a cone = \( \pi rh \)
- Surface area of a sphere = \( 4\pi r^2 \)
- Volume of a cone = \( \frac{1}{3} \pi r^2 h \)
- Volume of a sphere = \( \frac{4}{3} \pi r^3 \)
- Area of triangle \( ABC \) = \( \frac{1}{2} \cdot \text{base} \cdot \text{height} \)
- Area of sector = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

**Trigonometry**

- \( \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \)
- \( a^2 = b^2 + c^2 - 2bc \cos A \)

**Statistics**

- Mean = \( \frac{\sum fx}{\sum f} \)
- Standard deviation = \( \sqrt{\frac{\sum f x^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} \)
1 (a) Solve the inequality \( \frac{2+3x}{5} \leq \frac{3-x}{2} \).

\[
\frac{2+3x}{5} \leq \frac{3-x}{2} \\
4+6x \leq 15-5x \quad \text{[M1]} \\
11x \leq 11 \\
x \leq 1 \quad \text{[CAI]}
\]

(b) Simplify \( \frac{16x^2-y^2}{4x^2+9xy+2y^2} \).

\[
\frac{(4x+y)(4x-y)}{4x+y+2y} \\
\frac{4x-y}{4x+y} \quad \text{[M1]}
\]

(c) Given that \( x = -\frac{1}{2} \) is a solution of \((m-1)x^3+(m-2)x = 2m-15\), find the value of \( m \) and the second possible value of \( x \).

\[
\text{Since } x = -\frac{1}{2} \text{ is a solution,} \\
\frac{1}{8}(m-1)(-\frac{1}{2})^3 + (-\frac{1}{2}) \left(\frac{3}{4}ight) = 2\left(-\frac{1}{2}\right) - 15 \\
-\frac{1}{16} - \frac{1}{4}m + \frac{3}{4} = 2\left(-\frac{1}{2}\right) - 15 \\
\frac{6}{4} = \frac{0}{4}m \\
m = 7. \quad \text{[M1]}
\]

\(6x^2 + 5x = -1 \)
\(x^2 + 5x + 1 = 0 \)
\(3x+1)(2x+1) = 0 \)

\(x = \frac{-1}{3} \) is the second possible value. \([CA\text{I}]\)

(d) Solve the equation \( x + \frac{1}{x} = \frac{15x-1}{3x-1} \).

\[
(x-1)(3x-1) = 15x-1 \quad \text{[M1]} \\
3x^2 - x + 15x - 5 = 15x - 1 \\
3x^2 - x - 4 = 0 \\
(3x-4)(x+1) = 0 \quad \text{[CA\text{I}]} \\
x = -1 \text{ or } 1\frac{1}{3} \quad \text{[CAI]}
\]
2. \( \vec{AB} = \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \) and \( \vec{OB} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \), where \( O \) is the origin.

(a) Find the length of \( AB \).

\[
AB = \sqrt{3^2 + 2^2 + (-2)^2} = \sqrt{13}
\]

(b) Find the equation of line \( AB \).

Gradient \( \vec{m} = \frac{4}{3} \) and \( \vec{n} = (4, 0, 0) \), so \( \Delta y = \Delta x \).

\[
\theta = \frac{4}{3} \quad \text{and} \quad c = 2\frac{2}{3}
\]

(c) \( C \) is a point on \( xy \)-axis such that angle \( ABC \) is a right angle. Find the tangent of angle \( ABC \).

\[ \tan \angle ABC = \frac{2}{3} \]

(d) The equation of the line \( m \) is \( 3y + 2x = 6 \). Line \( m \) intersects the \( x \)-axis at \( D \) and the \( y \)-axis at \( E \).

(i) Show how you can tell that the line \( m \) does not intersect the line \( AB \).

(ii) Find \( E \) \( \text{if} \quad y = 0 \quad \text{and} \quad x = 0 \)...

(iii) What type of quadrilateral is \( ABDE \)? Explain your answer.

\[ \vec{AE} \text{ is parallel to } \vec{AD} \text{ because } \vec{AE} = \vec{AD} \]

\[ \vec{AE} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \]

\[ \vec{AE} \text{ is also parallel to } \vec{AD} \text{ because } \vec{AE} = \vec{AD} \]

\[ \therefore \text{ABDE is a parallelogram because it has 2 pairs of parallel sides.} \]
3 Rashidi wants to make a closed rectangular box to contain his toys. The box has a rectangular base of sides $3x$ cm and $x$ cm and its height is $h$ cm.

(a) (i) The volume of the box is $90$ cm$^3$.

Express $h$ in terms of $x$.

$$V_0 = (3x)(x)(h)$$

$$h = \frac{V_0}{3ax}$$

$$h = \frac{90}{3x} \quad \text{[cm]}$$

(ii) Show that the total surface area of the box $A$ cm$^2$, is given by $A = \frac{240}{x} + 6x^2$. [2]

$$A = 2(3x)(x) + 2(3x)(h) \quad \text{[cm]}$$

$$= 6x^2 + 6xh$$

$$= 6x^2 + 6x \cdot \frac{90}{3x} \quad \text{[cm]}$$

$$= 240 + 6x^2 \quad \text{[cm]}$$

The table shows some corresponding values of $x$ and $A$, correct to the nearest integer:

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>246</td>
<td>144</td>
<td>134</td>
<td>86</td>
<td>56</td>
<td>38</td>
<td>27</td>
</tr>
</tbody>
</table>

(a) Find the value of $b$.

$$b = \frac{240}{5} + \frac{(38)}{5}$$

$$= \frac{148}{5} \quad \text{[cm]}$$

(b) Using a scale of $2$ cm to represent $1$ unit, draw a horizontal $x$-axis for $0 \leq x \leq 8$.
Using a scale of $2$ cm to represent $50$ units, draw a vertical $A$-axis for $0 \leq A \leq 500$.

On your axes, plot the points given in the table and join them with a smooth curve on the grid in page 9. [3]
4. A, B and C represent three islands.

Both boats X and Y left from island A.

The bearings of B and C from A is 160° and 100° respectively.

Boat X sailed towards B at a speed of \(x\) km/h. Boat Y sailed towards C at a speed of \((x+32)\) km/h.

Boat X and Y took 2 hours to reach B and C respectively.

(a) Write an expression, in terms of \(x\), for the distance travelled by Boat X.

\[
\text{Dist.} = 2x \times \text{time}
\]

\[
\text{Dist.} = 2x
\]

(b) B and C are 150 km apart.

Write down an equation in \(x\) to represent this information and show that it reduces to

\[x^3 + 32x - 4601 = 0\]
(b) Solve the equation \( x^2 + 32x - 4601 = 0 \).

\[
\begin{align*}
x &= -\frac{32 \pm \sqrt{32^2 - 4 \times 1 \times (-4601)}}{2} \\
&= -32 \pm \sqrt{5344} \\
&= -32 \pm 73.118 \text{ cm} \\
&= -5.57 \text{ cm}, 53.57 \text{ cm}
\end{align*}
\]

(c) Boat X returned from B to A and took 30 more minutes than the onward journey.

Find the speed of Boat X for the return journey.

\[
\text{Speed} = \frac{2 \times 1.3 \times 691.18034}{2.5} \\
= \frac{4 \times 9.3 \times 84.727}{3.0} \\
= 53.0 \text{ cm/s}, 7 \text{ cm/s}
\]

5. \( O \) is the centre of the circle passing through \( A, B, C, D \) and \( E \). \( UBX \) is the tangent to the circle at \( B \). \( ACX \) is a straight line which passes through \( O \). \( \angle CBX = \angle CBX = 30^\circ \) and \( \angle EDC = 140^\circ \).

(a) Stating your reasons clearly, find,

(i) \( \angle BAC \),

\[
\text{Note:}
\begin{align*}
\angle BAC &= 30^\circ + 30^\circ \quad \text{(ext } x \text{ in a triangle)} \\
\angle ABC &= 90^\circ \quad \text{(alt. segment)} \\
\angle BAC &= 120^\circ - 90^\circ \quad \text{(sum } z \text{ in a triangle)} \\
&= 30^\circ \quad \text{(M)}
\end{align*}
\]
(ii) \( \angle CEB \)

\[
\angle CEB = 30^\circ \text{ (iii) } \angle CBE,
\]

\[
\angle CBE = 190^\circ - 140^\circ = 50^\circ \text{ (iv) } \angle DOY.
\]

\[
\angle DOY = 180^\circ - 120^\circ = 60^\circ
\]

(b) \( T \) is a point such that angle \( \angle ATE = 60^\circ \).

State whether point \( T \) lies in the circle.

Explain your answer.

6. Angela and Ruth went on a free-and-easy trip to Sydney.

(a) They both exchanged S$1600 at a money changer in Singapore. The money changer used an exchange rate between Singapore dollars (S$) and Australian dollars (A$) of \( \text{S} 1 = \text{A} 0.95 \).

Calculate the amount of Australian dollars they received.

(b) They booked their accommodation through the Waterbnb application.

Each night at the accommodation costs A$110.25. However, there was a 10% service charge. An additional 5% daily cleaning fee was imposed only on the cost of the accommodation.

They booked a total of 8 nights at the accommodation.

Calculate the total cost of their accommodation in Australian dollars.
(c) Ruth also purchased a watch in Sydney.

To pay for the watch, she borrowed S$2340 for 2 years at an interest of 5.6% per annum compounded half-yearly.

Calculate the amount of interest she paid for the watch.

\[
\text{Amount} = 2340 \left( 1 + \frac{0.056}{2} \right)^2 = 2640 \\
= 2.12 \times 2340.269 \\
= 5.7 \times 137.269 \quad [\text{CAI}]
\]

(d) Sydney has a population of 4,627 million and Darwin has a population of 132,000.

(e) Calculate how many more people live in Sydney than in Darwin, giving your answer in standard form.

\[
4.627 \times 10^8 - 1.32 \times 10^5 = 4.497 \times 10^8 \quad [\text{CAI}]
\]

7 Hillgrove organised an annual school carnival to raise funds for a children's home.

Two different classes sold three different flavours of Bubble Tea to raise funds.

The table below shows the number of Bubble Tea sold by each class and the price of the Bubble Tea.

<table>
<thead>
<tr>
<th>Flavours</th>
<th>Class 4-10</th>
<th>Class 4-11</th>
<th>Price per cup</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milk Oolong</td>
<td>30</td>
<td>20</td>
<td>$1.90</td>
</tr>
<tr>
<td>Cheese Peach</td>
<td>20</td>
<td>40</td>
<td>$3.20</td>
</tr>
<tr>
<td>Brown Sugar</td>
<td>10</td>
<td>10</td>
<td>$2.50</td>
</tr>
</tbody>
</table>

(a) Represent the number of different flavours of Bubble Tea sold by each class in a 3 \times 2 matrix, \( F \).

\[
F = \begin{pmatrix}
20 & 10 \\
30 & 20 \\
40 & 15 \\
\end{pmatrix} \quad [\text{B1}]
\]

(b) Represent the prices of Bubble Tea in a 1 \times 3 matrix, \( P \).

\[
P = \begin{pmatrix}
1.90 & 3.20 & 2.50 \\
\end{pmatrix} \quad [\text{B1}]
\]

(c) Evaluate the matrix \( R = PF \).

\[
R = PF = \begin{pmatrix}
1 & 2 & 3 \\
3 & 2 & -5 \\
2 & 1 & 4 \\
\end{pmatrix} \begin{pmatrix}
20 & 20 & 20 \\
30 & 40 & 15 \\
\end{pmatrix} = \begin{pmatrix}
151.5 & 191 \\
\end{pmatrix} \quad [\text{CAI}]
\]
(d) Explain what each element in matrix \( R \) means.

The elements in \( R \) represent the total amount of money collected from houses 4-10 and 4-11.

\[
\begin{bmatrix}
54.5 & 141 \\
344.5 & 1
\end{bmatrix}
\]

(a) Evaluate \( A - RM \) and explain what the element in \( A \) means.

\[
A = \begin{bmatrix}
150.5 & 141 \\
344.5 & 1
\end{bmatrix}
\]

\[
m = \begin{bmatrix}
15 & 141 \\
344.5 & 1
\end{bmatrix}
\]

The element represents the total amount of money collected from house 4-10 and 4-11.

(b) Find angle \( \angle AM \).

\[
\tan \angle AM = \frac{10}{15}
\]

\[
\angle AM = \tan^{-1} \left( \frac{10}{15} \right) = 33.66^\circ
\]

The diagram represents the roof of a house. The ridge \( AB \) is horizontal and is built centrally above the roof. The slope of the ridge \( XYZ \), \( AM \) is the height of the ridge above \( WXYZ \). It is given that \( WX = 15 \text{ m}, XY = 8 \text{ m}, AB = 12 \text{ m} \) and \( AM = 2.5 \text{ m} \).

(a) Show that \( PM \) is 5.68 m, correct to 3 significant figures.

\[
PM = \sqrt{4^2 + 4.5^2}
\]

\[
= 5.68 \text{ m}
\]

(b) Find the angle \( \angle AM \).

\[
\tan \angle AM = \frac{10}{15}
\]

\[
\angle AM = \tan^{-1} \left( \frac{10}{15} \right) = 33.66^\circ
\]
(ii) \[ NW = \sqrt{2.5^2 + 6.456254149^2} \] 
\[ = 6.9264 \text{ m} \] 

(iii) \[ AX : \] 
\[ \sin \theta = \frac{4}{6.9264} \] 
\[ \theta = 11.98^\circ \] 
\[ YAX = 90^\circ - 11.98^\circ = 78.02^\circ \] 
\[ YAX = \theta + 90^\circ - 11.98^\circ \] 
\[ YAX = 78.02^\circ - 11.98^\circ = 66.04^\circ \] 
\[ XY = 130.246 \text{ cm} \] 
\[ XY = 11.962 \text{ cm} \] 
\[ XI = 11.962 \text{ cm} \] 

(c) Find the smallest angle of elevation of \( A \) from a point along \( MW \). 
\[ \sin \theta = \frac{4}{1.5} \] 
\[ \theta = 23.56^\circ \text{ (approx)} \] 

(c) Find the perimeter of the shaded region, 
\[ p = \sqrt{12^2 + 12^2} \] 
\[ = \sqrt{288} \text{ cm} \] 
\[ \text{Area of circle} = \frac{1}{4} \cdot \pi \cdot 14^2 \text{ cm}^2 \] 

Arc length \( L = r \theta \) 
\[ = 12 \cdot \frac{360}{180} \] 
\[ = 12 \pi \text{ cm} \] 

Perimeter 
\[ = \pi \frac{12\pi}{4} + \left( 12 - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \right) + 12 + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \] 
\[ = 37.31 \text{ cm} \]
(ii) \( \text{the area of } \triangle OXY \) \( \text{is not a triangle} \). [3] 

Area \( \triangle OXY \) 

\[
\frac{1}{2} \times \left( \frac{\text{base}}{2} \right) \times \text{height} \times \text{length of side} \]

\[
= \frac{1}{2} \times \left( \frac{920}{2} \right) \times \text{height} \times \text{length of side} \]

\[
= 4513.37 \text{ cm}^2 \]

Area \( \triangle OXX \) = \( 15.45133 \times 24 \) \( \text{cm}^2 \) 

\[
= 15.4797 \text{ cm}^2 \]

\[
= 9.47 \text{ cm}^2 \] [2m]

(b) Show that \( \triangle BAB \) is similar to \( \triangle RCD \). [3]

\[
\angle BAB = \angle RCD = 90^\circ \text{ (vertical angles)}
\]

\[
\angle BAA = \angle ACD \text{ (corresponding angles)}
\]

\[
\angle ABB = \angle CBB = 90^\circ \text{ (base of \( \triangle ABB \))}
\]

\[
\angle CBB = \angle ABB \text{ (base of \( \triangle CBB \))}
\]

\[
\triangle BAB \sim \triangle RCD 
\]

The driver was also found to have a mobile phone in his pocket with the number dialled to the victim's phone. The times of the calls were also found to be consistent with the times of the accident. [2m]

There was a hit and run car accident along the Pan-Island Expressway (PIE). It was raining the day before and the road was wet. PIE has a speed limit of 80 km/h. The police needed to find out who the driver was and there were some items found at the scene.

Exhibit A: slip of paper with driver's telephone number

Exhibit B: driver's shoes print - 25 cm in length

Exhibit C: driver's vehicle skid marks - 350 ft. in length

It is given that the velocity of the car is directly proportional to the square root of the length of skid marks.

The police also uses graphs below to help them to solve crimes.

\[
\text{height of a person (cm)}
\]

\[
180 \quad 27
\]

\[
\text{length of foot (cm)}
\]
The police then narrowed to three suspects with their details below.

<table>
<thead>
<tr>
<th></th>
<th>Daniel Lee</th>
<th>Jacob Yap</th>
<th>Samuel Wong</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td>1.78 m</td>
<td>1.66 m</td>
<td>1.82 m</td>
</tr>
<tr>
<td>Brand of vehicle they own</td>
<td>Kit</td>
<td>Toyota</td>
<td>Volkswagen</td>
</tr>
</tbody>
</table>

The Composition Fine

**Light Vehicle**
(Examples: motor cars, motorcycles, light goods vans, etc.)

<table>
<thead>
<tr>
<th>Speed limit</th>
<th>Fine (Light Vehicle)</th>
<th>Fine (Heavy Vehicle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exceeding road speed limit by 1 - 20 km/h</td>
<td>$150</td>
<td>$200</td>
</tr>
<tr>
<td>Exceeding road speed limit by 21 - 30 km/h</td>
<td>$200</td>
<td>$250</td>
</tr>
<tr>
<td>Exceeding road speed limit by 31 - 40 km/h</td>
<td>$300</td>
<td>$400</td>
</tr>
<tr>
<td>Exceeding 41 km/h</td>
<td>prosecution in court</td>
<td></td>
</tr>
</tbody>
</table>

**Heavy Vehicle**
(Examples: tractors, buses, tractors, etc.)

---

(a) Given that there are 8-digits in the driver's telephone number, find the total number of possible telephone numbers.

Note that the graphs only give predicted estimated values.

(b) Find the driver who was involved in the accident and determine the appropriate fine for speeding ticket.

Justify your decision with calculations.
(b) let the height be $h$.

\[
\frac{h}{25} = \frac{180}{17}
\]

$\therefore h = 166.33 \text{ cm}$.

Since velocity of car is directly proportional to square of height, we have:

\[V = k \sqrt{h} \]

where $k$ is a constant.

\[25 = k \sqrt{25} \]

\[k = \frac{25}{5} \]

\[V = \frac{25}{5} \sqrt{h} \]

To find velocity at that instant,

\[V = \frac{25}{5} \sqrt{25} = 66.14378 \text{ m/s} \]

\[= 66.14378 \times 3600 \text{ mph} \]

\[\approx 234.5 \text{ mph} \]

\[\text{Exceed} = 90 - 234.5 = -144.5 \text{ mph} \]

\[\approx 25.06 \text{ mph} \]
HILLGROVE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2019
SECONDARY 4 (EXPRESS) / 5 (NORMAL
ACADEMIC)

CANDIDATE
NAME

CENTRE
NUMBER
S

INDEX
NUMBER

MATHEMATICS

Paper 1

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and, if answer is not exact, give
the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142 unless the question requires the
answer in terms of \( \pi \).

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Parent's/ Guardian's Signature:

TOTAL

This document consists of 21 printed pages.
1. Write the following numbers in order of size, starting with the largest:

\[
\frac{37}{100}, \quad 0.37, \quad 0.3\bar{7}, \quad \left(\frac{37}{100}\right)^2
\]

\[
\frac{27}{100} = 0.27, \quad \left(\frac{37}{100}\right) = 0.1369
\]

Answer: 0.3\bar{7}, 0.37, \frac{37}{100}, \left(\frac{27}{100}\right)^2 [1]

2. Write down the sets represented by the following shaded regions.

(a)

(b)

Answer: (a) $A \cup B$, (b) $A \cap B$ [1]

3. Show that $(1+5n)^2 - 6$ is a multiple of 5 for all integer values of $n$.

Answer:

\[
(1+5n)^2 - 6 = 1 + 10n + 25n^2 - 6
\]

\[
= 25n^2 + 10n - 5 \quad \text{[Clm]}
\]

\[
= 5(5n^2 + 2n - 1)
\]

Since $5(5n^2 + 2n - 1)$ is a factor of 5, [Al]

$\therefore (1+5n)^2 - 6$ is a multiple of 6. [2]

4. (a) Express $x^2 - 6x + 5$ in the form $(x-a)^2 + b$.

\[
x^2 - 6x + 5 = x^2 - 6x + \left(-\frac{3}{2}\right)^2 - \left(-\frac{3}{2}\right)^2 + 5
\]

\[
= (x-3)^2 - 4
\]

Answer: $(x-3)^2 - 4$ [Al] [1]

(b) Hence, solve $x^2 - 6x + 5 = 0$.

\[
(x-3)^2 - 4 = 0
\]

\[
(x-3)^2 = 4
\]

\[
x-3 = \pm 2
\]

\[
x = 5 \quad \text{or} \quad 1
\]

Answer: 5 or 1 [Al] [1]
5. Nithya runs a photography club.
   Her club currently has 62 boys and 39 girls.
   Her aim is to have at least 45% of members to be girls.
   Find the smallest number of girls required to join her club.

   \[ \frac{39 - x}{62 + 39 + x} \geq 0.45 \]
   \[ x \geq \frac{110}{1} \]
   \[ x \geq \frac{110}{110} \]
   \[ x = 10 \text{ girls} \] [Ch]

6. A pizza is divided in the ratio of 3:7:5 for Adrian, Ben, and Carl respectively. Ben then divides his share in the ratio of 9:7 and give it to Adrian and Carl respectively.
   Carl says that both he and Adrian have the same amount of pizza now. Do you agree with Carl? Explain your answer.

   \[ \frac{39 - x}{62 + 39 + x} \geq 0.45 \]
   \[ x \geq \frac{110}{110} \]
   \[ x = \frac{110}{110} \]
   \[ \text{Answer} \]

7. It is given that \( y = \frac{3x + 5b}{x - 2c} \).
   (a) Find \( y \) when \( x = 1 \), \( b = 2 \), \( c = 4 \).
   \[ y = \frac{3(1) + 5(2)}{1 - 2(4)} \]
   \[ y = \frac{3 + 10}{1 - 8} \]
   \[ y = \frac{13}{-7} \]
   \[ y = -\frac{13}{7} \] [Ch]
   \[ \text{Answer} \]

8. Given that \( \cos x = \frac{21}{29} \), where \( x \) is an acute angle, find the value of \( \sin x + \cos(180° - x) - \tan x \) without the use of calculator.

   \[ \alpha = \sqrt{21^2 - 20^2} \text{ [Id]} \]
   \[ \sin x = \frac{20}{29} \]
   \[ \cos(180° - x) = -\cos x = -\frac{21}{29} \]
   \[ \tan x = \frac{20}{21} \text{ [Id]} \]
   \[ \sin x + \cos(180° - x) - \tan x = \frac{20}{29} - \frac{21}{29} - \frac{20}{21} \text{ [Id]} \]
   \[ = \frac{20}{29} - \frac{21}{29} - \frac{20}{21} \text{ [Id]} \]
   \[ = -\frac{601}{588} \text{ [Ch]} \]
   \[ \text{Answer} \]
9 (a) The product of three prime numbers $a$, $b$ and $c$ is $m$, where $m$ is an even number.

The smallest and the largest number out of the three prime numbers are $a$ and $c$—respectively—$c > a$.

State the value of $a$ and explain your answer.

*Answer* $a$ must be $1$. [1] For $m$ to be an even number, one of the three prime numbers must be even. 2 is the only even prime number. [1]

(b) Given that $b = 23$ and $m$ is less than 670, find the largest possible value of $c$.

\[ 2 \times 23 \times c < 670 \text{ (1)} \]

\[ c < \frac{670}{50} \]

\[ c < 13 \frac{10}{23} \]

\[ \therefore \text{largest } c = 13. \text{ [1] } \]

10 The diagram shows a plot of land labelled $WXYZ$.

(i) Construct the bisector of angle $WZ$.

(ii) Construct the perpendicular bisector of $WZ$.

(iii) The government is planning to build a playground that is equidistant from $W$ and $Z$ and $WZ$ and $YZ$. Mark and label the location of the playground as $P$.

(ii) Hence, find the bearing of $P$ from $Y$.

*Answer* $217.5^\circ \pm 1^\circ$ [1]
11 (a) Given that $4(x + 2y) = 5x - 3y$, find the ratio of $x : y$.

\[
4x + 8y = 5x - 3y \\
4y = x \\
\frac{x}{4y} = \frac{1}{4} \quad [\text{Eqn}]
\]

\[
\therefore x : y \Rightarrow 4 : 1
\]

Answer \( x : y = 4 : 1 \) \[2\]

(b) Solve \( \frac{27^b}{9} = 81^{b+1} \).

\[
\frac{27^b}{9} = 81^{b+1} \\
\frac{27^b}{3^2} = (3^4)^{b+1} \\
27^b \cdot 3^{-2} = 3^{4b+4} \\
3^{3b} \cdot 3^{-2} = 3^{4b+4} \\
3^{3b-2} = 3^{4b+4} \\
3b - 2 = 4b + 4 \\
-2 = b + 4 \\
-6 = b \\
\]

Answer \( b = -6 \) \[2\]

12 Winnie is drawing an \( n \)-sided regular polygon.

The total sum of the interior angles is \( 180^\circ \times n \).

Given that \( h + n = 1631 \), find the values of \( h \) and \( n \).

\[
(\text{Eqn}) \times 180^\circ = h, \\
180n = 1631 - n, \\
180n + n = 1631 \\
181n = 1631 \\
181n = 1631 \\
181n = 1631 \\
181n = 1631 \\
181n = 1631 \\
181n = 1631
\]

Answer \( h = 1501, n = 11 \) \[6\]
In the diagram, A and B are points on a circle, centre O. The radius of the circle is 5 cm and length of AB is 9.4 cm.

Find the area of the minor segment AB.

\[
\sin \beta = \frac{w - \frac{s}{2}}{s} \quad \text{(CMD: find } \beta \text{ area)}
\]

\[
\beta = 2 \times \sin^{-1} \left( \frac{w - \frac{s}{2}}{s} \right)
\]

\[
= 140.10 \text{ or } 124.9^\circ
\]

Area \( \beta \) minor segment AB = \( 140.10 \times 0.33423 \times \frac{1}{2} \times \frac{1}{2} \times 0.75 \times \pi(103.125^\circ)
\]

\[
= 22.5 \text{ cm}^2 \quad \text{(CMD: sector + triangle)}
\]

Answer: 22.5 cm²
15 Serene has two boxes of cards.
One box contains cards that have either a circle or a square.
The other box contains cards that are green or blue in colour.

Serene picks a card from each box at random.
The probability that she picks a circle card is \( c \).
The probability that she picks a green card is \( g \).

(a) Complete the table for the card that Serene picks, writing each probability in terms of \( c \) and \( g \).

<table>
<thead>
<tr>
<th>Event</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle and Green</td>
<td>( \frac{c}{g} )</td>
</tr>
<tr>
<td>Square and Green</td>
<td>( \frac{1-c}{g} )</td>
</tr>
<tr>
<td>Circle and Blue</td>
<td>( \frac{c}{1-g} )</td>
</tr>
<tr>
<td>Square and Blue</td>
<td>( \frac{1-c}{1-g} )</td>
</tr>
</tbody>
</table>

(b) The probability that she picks a circle card is \( \frac{2}{7} \).
The probability that she picks a circle card and green card is \( \frac{1}{3} \).

Hence, or otherwise, find the total number of green cards given that the total number of blue and green cards is 30.

\[
\text{Answer: } 21 \text{ cards}
\]

16 The diagram shows the speed – time graph for a truck’s journey.
The distance travelled for the first 10 seconds is 756 m.

(b) Calculate the deceleration at the 15th second of the journey.

\[
\text{Deceleration} = \frac{C(t) - 0}{(10 - t)^2}
\]

Answer: \( A = 18 \) m/s²

(c) Find the average speed in km/h.

\[
\text{Average speed} = \frac{15t + 2000}{15t - 3200}
\]

Answer: \( 30 \) km/h
The diagram shows a parallelogram $OXYZ$.

(a) Express $\overline{BY}$ in terms of $a$ and $b$, as simply as possible.

$$\frac{\overline{OZ}}{\overline{OA}} = \frac{2}{1}, \quad \frac{\overline{OC}}{\overline{OD}} = \frac{2}{1}, \quad \frac{\overline{BY}}{\overline{AB}} = \frac{2}{1}$$

$$\overline{BY} = \frac{\overline{AB}}{2}$$

Answer: $2a + b$ [2]

(b) Write down two vectors that can be written as $\mathbf{b} - 3\mathbf{a}$.

(c) Find the ratio of the area of triangle $\triangle BCE$ to the area of parallelogram $OXYZ$.

$$\frac{\text{Area } \triangle BCE}{\text{Area } OXYZ} = \frac{1}{2}$$

Answer: $1:2$ [2]

The diagram shows a toy that contains a cone and a hemisphere.

(a) Show that the radius of the hemisphere is 8 cm.

$$r = \sqrt{17^2 - 13^2}$$

$$= \sqrt{289 - 169}$$

$$= 8 \text{ cm}$$

$$\therefore \text{ radius } = 8 \text{ cm}$$ [CA]

(b) Calculate the surface area of the toy.

$$\text{Surface area} = \pi rl + 2\pi r^2$$

$$= \pi (8)(17) + 2\pi (8)^2$$

$$= 224.32 + 200.96$$

$$= 425.28 \text{ cm}^2$$ [M]

Answer: $425.28 \text{ cm}^2$ [3]
(c) The company wants to make a miniature version of the toy such that the volume of the miniature toy is \( \frac{1}{6} \) of the volume of the actual toy. Calculate the radius of the miniature toy.

\[
\left( \frac{r_2}{8} \right)^3 = \frac{1}{6} \\
\Rightarrow r_2^3 = \frac{8}{3} \\
\Rightarrow r_2 = \sqrt[3]{\frac{8}{3}} \\
\Rightarrow r_2 = \sqrt[3]{\frac{8}{3}} \text{ cm} \\
\text{Answer: } r_2 \approx 1.00 \text{ cm} \quad [2]
\]

19 The travelling times taken by 50 students from home to school are recorded. The cumulative frequency curve below shows the distribution of their times.

Use your graph to estimate:

(i) the median

\[ Q_2 = \text{value at } 50 \text{th point} \]

Answer: \( \ldots \) \text{ min} \quad [1]

(ii) the interquartile range

\[ Q_1 = \text{value at } 25 \text{th point}, \quad Q_3 = \text{value at } 75 \text{th point} \]

\[ IQR = Q_3 - Q_1 \]

Answer: \( \ldots \) \text{ min} \quad [2]
(b) Find the probability that two students, chosen at random, take less than 70 minutes to travel to school.

\[
P(2 \text{ students take less than 70 min}) = \frac{91}{100} \times \frac{99}{100} = \frac{91}{100} \quad \text{(approx)}
\]

\[
= \frac{241}{100} \quad \text{(approx)} [\text{M}]
\]

Answer: \(\frac{241}{100}\) [2]

(c) A student states that 32% of the students took at least 36 minutes to travel to school.

Comment on whether the data from the school supports this claim.

\[
\text{Answer: No. of students who took at least 36 min} = 50 - 16 = 34
\]

\[
\frac{241}{100} \times 100\% = 24.1\% \text{ took at least 36 min, not 32\%.} [\text{CA}]
\]

20 The diagram shows two triangles \(ABC\) and \(BDC\).

\[
\text{Given: } AB = 3 \text{ cm, } BC = 5 \text{ cm, } BD = 10 \text{ cm and } \angle CBD = 113^\circ.
\]

(a) Show that \(\angle CBA\) is a right angle.

\[
\text{In } \triangle ABC, \quad AB = 3 \text{ cm, } BC = 5 \text{ cm, } BD = 10 \text{ cm and } \angle CBD = 113^\circ.
\]

\[
\text{By tangent: } \quad \frac{\sin A}{\sin B} = \frac{AC}{BC} \quad \text{and } \quad \sin \angle A = \frac{AC}{BC} = \frac{3}{5}
\]

\[
\angle CBA = 90^\circ \quad \text{(verified)} [\text{CA}]
\]

(b) Find \(\angle BDC\).

\[
\angle BDC = 180^\circ - 46^\circ - 113^\circ = 21^\circ \quad \text{(verified)} [\text{CA}]
\]

Answer: \(21^\circ\) [2]
(c) Find the area of \(ABDC\):

\[
\text{Area } \frac{1}{2} \times 2 \times 3 = \left( \frac{1}{2} \times 5 \times 10 \times \sin 113^\circ \right) \quad \text{[M1]}
\]

\[
= 25 \cdot 0.126 \\
= 3.0 \text{ cm}^2 \quad \text{[A1]}
\]

Answer \(2\lambda \cdot 0 \text{ cm}^2\) \([2]\)

(d) A circle is drawn through the points \(A, B\) and \(C\). State the length of the diameter of the circle. Explain your answer.

\[
\text{Diameter } = 5 \text{ cm} \quad \text{[E1]}
\]

Since \( \angle ACH = 90^\circ\), \(BC\) must be the diameter due to the property of a right angle in a semicircle. \([B1]\)
ADDITIONAL MATHEMATICS 4047/01

[80 marks] SEMESTER ONE EXAMINATION 13 May 2019

2 hours

Additional material: Writing paper

INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.
Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer ALL questions.
Write your answers on the writing paper provided.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Write the brand and model of your calculator in the space provided below.

INFORMATION FOR CANDIDATES

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For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Brand / Model of Calculator:

For Examiner’s Use

This question paper consists of 7 printed pages, including the cover page.

Setter: Ms Shen Sirui  Vetter: Mr Nara
1. ALGEBRA

**Quadratic Equation**
For the equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

**Binomial expansion**
\[(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,
\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)...(n-r+1)}{r!} \)

2. TRIGONOMETRY

**Identities**
\[
\sin^2 A + \cos^2 A = 1
\]
\[
\sec^2 A = 1 + \tan^2 A
\]
\[
\cosec^2 A = 1 + \cot^2 A
\]
\[
\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
\sin 2A = 2 \sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]
\[
\tan 2A = \frac{2\tan A}{1 - \tan^2 A}
\]

**Formulae for \( \triangle ABC \)**
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\text{Area of } \triangle = \frac{1}{2}ab \sin C
\]
1  (i) On the same diagram sketch the curve \( y^2 = 8x \) and \( y = 6x^{-2} \). [2]

(ii) Find the coordinates of the point of intersection of the two curves. [3]
2 A particle moves along the curve $y = e^{2x}$ in such a way that the $y$-coordinate of the particle is increasing at a constant rate of 0.3 units per second. Find the $y$-coordinate of the particle at the instant when the $x$-coordinate of the particle is increasing at 0.01 units per second. [4]

3 The equation of a curve is $y = 3x^2 - kx + 2k - 4$, where $k$ is a constant. Show that the line $y = 2x + 5$ intersects the curve for all real values of $k$. [5]
4 (a) Given that $(3^{x+2})(2^{x-2}) = 6^{2x}$, find the value of $6^x$. [3]

(b) The side of an equilateral triangle is $6(\sqrt{3} - 1)$ cm. **Without using a calculator,** find the exact value of the area of the equilateral triangle in the form $(a + b\sqrt{c})$ cm$^2$, where $a$, $b$ and $c$ are integers. [4]

5 Find the range of values of $x$ for which the gradient of the graph $y = x^4 - 3x^3 - 6x^2 + 6$ is increasing. [5]
A curve has the equation \( y = (2x - 3)^2 - 1 \).

(i) Find the coordinates of the points at which the curve intersects the x-axis. [2]

(ii) Sketch the graph of \( y = |(2x - 3)^2 - 1| \). [3]

(iii) Using your graph, state the range of values of \( k \) for which \( |(2x - 3)^2 - 1| = k \) has 4 solutions. [1]
7 It is given that $f'(x) = x + \sin 4x$ and $f(0) = \frac{3}{4}$.
Show that $f''(x) + 16f(x) = 8x^2 + 17$. [5]

8 Solve the equation $6\sin^2 x + 5 \cos x = 5$ for $0^\circ < x < 360^\circ$. [5]
9  (a) Given that the first two non-zero terms in the expansion, in ascending powers of \( x \), of \((1 + bx)(1 + ax)^6\) are 1 and \(-\frac{21}{4} x^2\) and that \( a > 0 \), find the value of \( a \) and of \( b \). [5]

(b) Find the term independent of \( x \) in the expansion of \((2x + \frac{1}{x^2})^9\). [3]
The equation of a curve is \( y = \frac{x^2}{2x-1} \).

(i) Find the coordinates of the stationary points of the curve. [4]

(ii) Determine the nature of each of the stationary points of the curve. [4]
The diagram shows part of the curve \( y = \frac{12}{(3x+2)^2} \) meeting the y-axis at point A. The tangent to the curve at A intersects the x-axis at point B. Point C lies on the curve such that BC is parallel to the y-axis. Find

(i) the equation of AB, \[4\]

(ii) the area of the shaded region. \[5\]
12  (a) State the values between which the principal value of $\tan^{-1} x$ must lie. Give your answer in terms of $\pi$. 

(b) The diagram below shows triangle $ABC$ where $AD = 2 \text{ m}$, $DC = 3 \text{ m}$ and $BD = h \text{ m}$. $BD$ is perpendicular to $AC$ and $\theta_1 + \theta_2 = 45^\circ$.

![Diagram of triangle ABC with points A, D, B, C and angles $\theta_1$ and $\theta_2$]

By using a suitable formula for $\tan(\theta_1 + \theta_2)$, find the value of $h$. 

[Turn over]
The Ultraviolet Index describes the level of solar radiation on the earth’s surface. The Ultraviolet Index, $U$, measured from the top of a building is given by $U = 6 - 5 \cos qt$, where $t$ is the time in hours, $0 \leq t \leq 20$, from the lowest value of Ultraviolet Index and $q$ is a constant. It takes 10 hours for the Ultraviolet Index to reach its lowest value again.

(i) Explain why it is impossible to measure a Ultraviolet Index of 12. [1]

(ii) Show that $q = \frac{\pi}{5}$. [1]

(iii) The top of the building is equipped with solar panels that supply power to the building when the Ultraviolet Index is at least 3.5. Find the duration, in hours and minutes, that the building is supplied with power by the solar panels. [5]
ADDITIONAL MATHEMATICS
Paper 2 [ 100 marks ]

Candidates answer on the question paper.

INSTRUCTIONS TO CANDIDATES
Do not open this booklet until you are told to do so.
Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
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Answer ALL questions.
Write your answers in the spaces provided on the question paper.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Write the brand and model of your calculator in the space provided below.

INFORMATION FOR CANDIDATES
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

<table>
<thead>
<tr>
<th>Brand / Model of Calculator</th>
<th>For Examiner's Use</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total 100</td>
</tr>
</tbody>
</table>

This question paper consists of 15 printed pages.
Setter: Mr. Gabriel Cheow
Vetter: Mr. Narayanan
1. ALGEBRA

Quadratic Equation
For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion

\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n
\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. TRIGONOMETRY

Identities

\[
\sin^2 A + \cos^2 A = 1
\]

\[
\sec^2 A = 1 + \tan^2 A
\]

\[
\cosec^2 A = 1 + \cot^2 A
\]

\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
\sin 2A = 2 \sin A \cos A
\]

\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]

\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \triangle ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2} bc \sin A
\]
1. The roots of the quadratic equation $2x^2 - 8x + 9 = 0$ are $\alpha$ and $\beta$.

(i) Show that the value of $\alpha^3 + \beta^3$ is 10. [3]

(ii) Find a quadratic equation whose roots are $\frac{1}{\alpha^2 + \beta}$ and $\frac{1}{\alpha + \beta^2}$. [4]
2 The function \( f(x) = 6x^3 + ax^2 + bx - 12 \), where \( a \) and \( b \) are constants, is exactly divisible by \( x + 2 \) and leaves a remainder of 5 when divided by \( x + 1 \).

(i) Find the value of \( a \) and of \( b \). [4]

(ii) By showing your working clearly, factorise \( f(x) \). [3]

(iii) Hence, solve the equation \( 6(2^{3y}) + 2^{2y+3} - 2^{y+4} - 12 = 2^{2y} \) [4]
(i) Express \( \frac{2x+16}{(x^2+4)(2x-1)} \) in partial fractions. \([5]\)

(ii) Differentiate \( \ln(x^2 + 4) \) with respect to \( x \). \([2]\)

(iii) Hence, using your results in (i) and (ii), find \( \int \frac{x+8}{(x^2 + 4)(2x-1)} \, dx \). \([4]\)
4 Prove the following identities.

(a) \((\sec x - \tan x)(\csc x + 1) = \cot x\)

\[\text{LHS} = (\sec x - \tan x)(\csc x + 1)\]

(b) \(\frac{1 - \cos 2x + \sin x}{\sin 2x + \cos x} = \tan x\)
The lines \( y = 8 \) and \( 4x + 3y = 30 \) are tangent to a circle \( C \) at the points \((-1, 8)\) and \((3, 6)\) respectively.

(i) Show that the equation of \( C \) is \( x^2 + y^2 + 2x - 6y - 15 = 0 \). [5]

(ii) Explain whether or not the x-axis is tangent to \( C \). [3]

(iii) The points \( Q \) and \( R \) also lie on the circle, and the length of the chord \( QR \) is 2 units. Calculate the shortest distance from the center of \( C \) to the chord \( QR \). [2]
The table shows experimental values of two variables $x$ and $y$, which are known to be connected by the equation $yx^n = A$, where $n$ and $A$ are constants.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>22.0</td>
<td>13.0</td>
<td>8.9</td>
<td>6.9</td>
<td>5.3</td>
</tr>
</tbody>
</table>

(i) Plot $\log y$ against $\log x$ and draw a straight line graph.  

[3]
(ii) Use your graph to estimate the value of $A$ and of $n$. [4]

(iii) On the same diagram, draw the line representing the equation $y = x^2$ and hence find the value of $x$ which satisfies the equation $x^{n+2} = A$. [2]
7 The diagram shows a trapezium with vertices $A(-2.5, 0), B(0, p), C(1, 3)$ and $D$. The sides $AB$ and $DC$ are parallel and the angle $DAB$ is $90^\circ$. Angle $ABO$ is equal to angle $CBO$.

(i) Express the gradients of the lines $AB$ and $CB$ in terms of $p$ and hence, or otherwise, show that $p = 5$. [3]
(ii) Find the coordinates of $D$. [4]

(iii) Find the area of the trapezium $ABCD$. [2]
8 (a) Solve the equation \(3 \log_3 x = 8 - 4 \log_3 x\). [5]

(b) It is given that \(\log_a x = p\) and \(\log_a y = q\).

Express \(\log_a ax^2 y^3\) in terms of \(p\) and \(q\). [3]
The figure shows a stage prop $ABC$ used by a member of the theatre, leaning against a vertical wall $OP$. It is given that $AB = 30$ cm, $BC = 100$ cm, $\angle ABC = \angle AOC = 90^\circ$ and $\angle BCO = \theta$.

(i) Show that $OC = (100 \cos \theta + 30 \sin \theta)$ cm. Let $D$ be foot of $B$ on $OC$, let $E$ be foot of $A$ on $BD$.

(ii) Express $OC$ in terms of $R \cos(\theta - \alpha)$, where $R$ is a positive constant and $\alpha$ is an acute angle.

(iii) State the maximum value of $OC$ and the corresponding value of $\theta$.

(iv) Find the value of $\theta$ for which $OC = 80$ cm.
10 Given that \( y = a + b \cos 4x \), where \( a \) and \( b \) are integers, and \( x \) is in radians,

(i) state the period of \( y \). [1]

Given that the maximum and minimum values of \( y \) are 3 and \(-5\) respectively, find

(ii) the amplitude of \( y \). [1]

Using the values of \( a \) and \( b \) found in part (iii),

(iv) sketch the graph of \( y = a + b \cos 4x \) for \( 0 \leq x \leq \pi \). [3]

(v) On the same set of axes, sketch the graph of \( y = |4 \sin 3x| \), and hence state the
number of solutions of \( a + b \cos 4x = |4 \sin 3x| \). [3]
11 The dimensions of a cuboid are 3x cm by 2x cm by h cm and its total surface area is 312 cm$^2$. The volume of the cuboid is $V$ cm$^3$.

(i) Express $h$ in terms of $x$. [2]

(ii) Show that $V = \frac{36}{5} x(26 - x^2)$. [2]

(iii) Find the maximum volume of the cuboid as $x$ varies, giving your answer to the nearest cm$^3$. [5]
ADDITIONAL MATHEMATICS 4047/01
[80 marks] SEMESTER ONE EXAMINATION 13 May 2019
2 hours

Additional material: Writing paper

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For Examiner’s Use

Brand / Model of Calculator

This question paper consists of 7 printed pages, including the cover page.

Setter: Ms Shen Sirui  Vetter: Mr Nara
1. ALGEBRA

**Quadratic Equation**
For the equation \( ax^2 + bx + c = 0 \),
\[
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**Binomial expansion**
\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n-1}ab^{n-1} + b^n,
\]
where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. TRIGONOMETRY

**Identities**

\[
\begin{align*}
\sin^2 A + \cos^2 A &= 1 \\
\sec^2 A &= 1 + \tan^2 A \\
\csc^2 A &= 1 + \cot^2 A \\
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\tan(A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A \\
\tan 2A &= \frac{2\tan A}{1 - \tan^2 A}
\end{align*}
\]

**Formulae for \( \triangle ABC \)**

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

Area of \( \triangle = \frac{1}{2}ab \sin C \)
1 (i) On the same diagram sketch the curve $y^2 = 8x$ and $y = 6x^{-2}$. [2]

(ii) Find the coordinates of the point of intersection of the two curves. [3]

<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td><img src="image1.jpg" alt="Image" /></td>
</tr>
</tbody>
</table>
| ii | $y^2 = 8x - - - (1)$
    | $y = 6x^{-2} - - - (2)$
    | Sub (2) into (1): $(6x^{-2})^2 = 8x$
    | $36$
    | $x^4 = 8x$
    | $x^5 = 4.5$
    | $x = 1.3509$
    | $y = 3.2877$
    | Intersection: $(1.35, 3.29)$ |

<table>
<thead>
<tr>
<th>Mark</th>
</tr>
</thead>
</table>
| B1 for $y^2 = 8x$
| B1 for $y = 6x^{-2}$
| M1 for substitution
| M1 for value of $x$ or $y$
| A1 |
2 A particle moves along the curve $y = e^{2x}$ in such a way that the $y$-coordinate of the particle is increasing at a constant rate of 0.3 units per second. Find the $y$-coordinate of the particle at the instant when the $x$-coordinate of the particle is increasing at 0.01 units per second.

[4]

<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$y = e^{2x}$</td>
<td>M1 for $\frac{dy}{dx}$</td>
</tr>
<tr>
<td></td>
<td>$\frac{dy}{dx} = 2e^{2x}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{dy}{dx} = \frac{dy}{dt} + \frac{dx}{dt}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$2e^{2x} = 0.3 \div 0.01$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$e^{2x} = 15$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = \frac{\ln 15}{2}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sub $x = \frac{2}{\ln 15}$, $y = e^{2(\frac{\ln 15}{2})} = 15$</td>
<td>A1</td>
</tr>
</tbody>
</table>

3 The equation of a curve is $y = 3x^2 - kx + 2k - 4$, where $k$ is a constant. Show that the line $y = 2x + 5$ intersects the curve for all real values of $k$.

[5]

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<th>Solution</th>
<th>Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$y = 3x^2 - kx + 2k - 4$</td>
<td>M1 for combining equations</td>
</tr>
<tr>
<td></td>
<td>$y = 2x + 5$</td>
<td>M1 for $ax^2 + bx + c = 0$</td>
</tr>
<tr>
<td></td>
<td>(1) = (2): $3x^2 - kx + 2k - 4 = 2x + 5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$3x^2 - kx - 2x + 2k - 9 = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$3x^2 - (k + 2)x + 2k - 9 = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$b^2 - 4ac = [-k + 2]^2 - 4(3)(2k - 9)$</td>
<td>M1 for $(k - 10)^2 + 12$</td>
</tr>
<tr>
<td></td>
<td>$= k^2 + 4k + 4 - 24k + 108$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= k^2 - 20k + 111$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= (k - 10)^2 - 10^2 + 112$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= (k - 10)^2 + 12$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Since $(k - 10)^2 + 12 &gt; 0$, $b^2 - 4ac &gt; 0$ and line intersects the curve for all real values of $k$.</td>
<td>A1 for conclusion</td>
</tr>
</tbody>
</table>
4 (a) Given that \((3^{x+2})(2^{x-2}) = 6^{2x}\), find the value of \(6^x\). \([3]\)

(b) The side of an equilateral triangle is \(6(\sqrt{3} - 1)\) cm. **Without using a calculator**, find the exact value of the area of the equilateral triangle in the form \((a + b\sqrt{c})\) cm\(^2\), where \(a, b\) and \(c\) are integers. \([4]\)

<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Mark</th>
</tr>
</thead>
</table>
| a  | \( (3^{x+2})(2^{x-2}) = 6^{2x} \)  
\( 3^x(3^2)(2^x)(2^{-2}) = 6^{2x} \)  
\( 6^x \frac{9}{4} = 6^{2x} \)  
\( 6^x = \frac{9}{4} \) | M1 for \(3^x(3^2)\) or \((2^x)(2^{-2})\)  
M1 for \(6^x \left(\frac{9}{4}\right)\)  
A1 |
| b  | Area = \(\frac{1}{2} [6(\sqrt{3} - 1)]^2 \sin 60\)  
\(= \frac{1}{2} (36)(3 - 2\sqrt{3} + 1) \left(\frac{\sqrt{3}}{2}\right) \)  
\(= 9\sqrt{3}(4 - 2\sqrt{3}) \)  
\(= 36\sqrt{3} - 54 \)  
\(= -54 + 36\sqrt{3} \) | M1  
M1 for \(3 - 2\sqrt{3} + 1\)  
M1 for \(\frac{\sqrt{3}}{2}\)  
A1 |

5 Find the range of values of \(x\) for which the gradient of the graph \(y = x^4 - 3x^3 - 6x^2 + 6\) is increasing. \([5]\)

<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Mark</th>
</tr>
</thead>
</table>
| \(y = x^4 - 3x^3 - 6x^2 + 6\)  
\(\frac{dy}{dx} = 4x^3 - 9x^2 - 12x\)  
\(\frac{d^2y}{dx^2} = 12x^2 - 18x - 12\)  
\(12x^2 - 18x - 12 > 0\)  
\(2x^2 - 3x - 2 > 0\)  
\((2x + 1)(x - 2) > 0\)  
\(x < -\frac{1}{2}, x > 2\) | M1 for \(\frac{dy}{dx}\)  
M1 for \(\frac{d^2y}{dx^2}\)  
M1 for \(\frac{d^2y}{dx^2} > 0\)  
M1 for factorised form  
A1 |
A curve has the equation \( y = (2x - 3)^2 - 1 \).

(i) Find the coordinates of the points at which the curve intersects the \( x \)-axis. [2]

(ii) Sketch the graph of \( y = |(2x - 3)^2 - 1| \). [3]

(iii) Using your graph, state the range of values of \( k \) for which \( |(2x - 3)^2 - 1| = k \) has 4 solutions. [1]

<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Mark</th>
</tr>
</thead>
</table>
| i  | \((2x - 3)^2 - 1 = 0\)  
\(2x - 3 = \pm 1\)  
\(x = 1, \quad x = 2\)  
\((1, 0) \quad (2, 0)\) | M1  
A1 or B2 |
| ii |  | T1 for turning point (1.5, 1)  
P1 for (1, 0) and (2, 0)  
C1 for shape of graph |
| iii| \(0 < k < 1\) | B1 (no mark if students got part ii wrong) |
It is given that \( f'(x) = x + \sin 4x \) and \( f(0) = \frac{3}{4} \).

Show that \( f''(x) + 16f(x) = 8x^2 + 17 \). [5]

<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x) = ( \frac{x^2}{2} - \frac{\cos 4x}{4} + c )</td>
<td>M1 for ( \frac{x^2}{2} - \frac{\cos 4x}{4} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{3}{4} = 0 - \frac{1}{4} + c )</td>
<td>M1 for ( f(x) = \frac{x^2}{2} - \frac{\cos 4x}{4} + 1 )</td>
<td></td>
</tr>
<tr>
<td>( c = 1 )</td>
<td>M1 for ( 1 + 4 \cos 4x )</td>
<td></td>
</tr>
<tr>
<td>( f(x) = \frac{x^2}{2} - \frac{\cos 4x}{4} + 1 )</td>
<td>M1 for sub into ( f''(x) + 16f(x) )</td>
<td></td>
</tr>
<tr>
<td>( f''(x) = 1 + 4 \cos 4x )</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>( f''(x) + 16f(x) = 1 + 4 \cos 4x + 16(\frac{x^2}{2} - \frac{\cos 4x}{4} + 1) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( = 1 + 4 \cos 4x + 8x^2 - 4 \cos 4x + 16 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( = 8x^2 + 17 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8 Solve the equation \( 6 \sin^2 x + 5 \cos x = 5 \) for \( 0^\circ < x < 360^\circ \). [5]

<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 6(1 - \cos^2 x) + 5 \cos x = 5 )</td>
<td>M1 for ( 1 - \cos^2 x )</td>
<td></td>
</tr>
<tr>
<td>( 6 - 6 \cos^2 x + 5 \cos x - 5 = 0 )</td>
<td>M1 for equation</td>
<td></td>
</tr>
<tr>
<td>( 6 \cos^2 x - 5 \cos x - 1 = 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( (6 \cos x + 1)(\cos x - 1) = 0 )</td>
<td>M1 for ( \cos x = -\frac{1}{6} )</td>
<td></td>
</tr>
<tr>
<td>( \cos x = -\frac{1}{6} ), ( \cos x = 1 )</td>
<td>M1 for basic angle</td>
<td></td>
</tr>
<tr>
<td>( \alpha = 80.405 ) (Rej)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x = 180 - \alpha, 180 + \alpha )</td>
<td>A1 for both answers</td>
<td></td>
</tr>
<tr>
<td>( x = 99.6^\circ, 260.4^\circ )</td>
<td>Ignore if students do not reject ( \cos x = 1 )</td>
<td></td>
</tr>
</tbody>
</table>
9 (a) Given that the first two non-zero terms in the expansion, in ascending powers of \( x \), of \((1 + bx)(1 + ax)^6\) are 1 and \(-\frac{21}{4} x^2\) and that \( a > 0 \), find the value of \( a \) and of \( b \). [5]

(b) Find the term independent of \( x \) in the expansion of \( \left(2x + \frac{1}{x^2}\right)^9\). [3]

<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>((1 + ax)^6 = 1 + \binom{6}{1}(ax)^1 + \binom{6}{2}(ax)^2 + \cdots)\n(= 1 + 6ax + 15a^2x^2 + \cdots)\n((1 + bx)(1 + ax)^6 = (1 + bx)(1 + 6ax + 15a^2x^2 + \cdots))\n(= 1 + 6ax + bx + 15a^2x^2 + 6abx^2 + \cdots)\n(6a + b = 0)\n(b = -6a - - - (1))\n(15a^2 + 6ab = -\frac{21}{4} - - - (2))\n sub (1) into (2): (15a^2 + 6a(-6a) = -\frac{21}{4})\n(21a^2 = \frac{21}{4})\n(a^2 = \frac{1}{4})\n(a = \frac{1}{2})\n(b = -3)</td>
<td>M1 for (1 + 6ax + 15a^2x^2)\nM1 for (6a + b = 0)\nM1 for (15a^2 + 6ab = -\frac{21}{4})\nA1\nA1</td>
</tr>
<tr>
<td>b</td>
<td>(T_{r+1} = \binom{9}{r} (2x)^{9-r} \left(\frac{1}{x^2}\right)^r)\nFor (x^0), (x^{9-r}(x)^{-2r} = x^0)\nr = 3\n(T_{3+1} = \binom{9}{3} (2x)^{9-3} \left(\frac{1}{x^2}\right)^3)\n(= 84(2x)^6(x)^{-6})\n(= 5376)</td>
<td>M1 for (\binom{9}{r} (2x)^{9-r} \left(\frac{1}{x^2}\right)^r)\nM1 for (r = 3)\nA1</td>
</tr>
</tbody>
</table>
10 The equation of a curve is \( y = \frac{x^2}{2x-1} \).

(i) Find the coordinates of the stationary points of the curve. [4]

(ii) Determine the nature of each of the stationary points of the curve. [4]

<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Mark</th>
</tr>
</thead>
</table>
| i  | \( y = \frac{x^2}{2x-1} \)  
\( \frac{dy}{dx} = \frac{2x(2x-1) - 2x^2}{(2x-1)^2} \)  
\( = \frac{2x^2 - 2x}{(2x-1)^2} \)  
when \( \frac{dy}{dx} = 0, \frac{2x^2 - 2x}{(2x-1)^2} = 0 \)  
\( 2x(x-1) = 0 \)  
\( x = 0, x = 1 \)  
\( y = 0, y = 1 \)  
Stationary points: \([0, 0]\) and \([1, 1]\) | M1 for quotient or product rule |
| ii | \( \frac{d^2y}{dx^2} = \frac{(4x-2)(2x-1)^2 - 4(2x-1)(2x^2 - 2x)}{(2x-1)^4} \)  
when \( x = 0, \frac{d^2y}{dx^2} = -2 < 0 \)  
\( (0, 0) \) is maximum point | M1 for \( \frac{d^2y}{dx^2} \) |
|    | when \( x = 1, \frac{d^2y}{dx^2} = 2 > 0 \)  
\( (1, 1) \) is minimum point | M1 for sub either \( x = 0 \) or \( x = 1 \) into \( \frac{d^2y}{dx^2} \) |
|    | OR | A1 for both coordinates |
|    | \( \begin{array}{|c|c|c|c|} 
 x & -0.1 & 0 & 0.1 \\
 \frac{dy}{dx} & > 0 & 0 & < 0 \\
\end{array} \) | |
|    | \( (0, 0) \) is maximum point. | |
|    | \( \begin{array}{|c|c|c|c|} 
 x & 0.9 & 1 & 1.1 \\
 \frac{dy}{dx} & < 0 & 0 & > 0 \\
\end{array} \) | |
|    | \( (1, 1) \) is minimum point. | |

[Turn over]
The diagram shows part of the curve \( y = \frac{12}{(3x+2)^2} \) meeting the \( y \)-axis at point \( A \). The tangent to the curve at \( A \) intersects the \( x \)-axis at point \( B \). Point \( C \) lies on the curve such that \( BC \) is parallel to the \( y \)-axis. Find

(i) the equation of \( AB \),

(ii) the area of the shaded region.

<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Mark</th>
</tr>
</thead>
</table>
| i  | \( y = \frac{12}{(3x+2)^2} \)  
  \( \frac{dy}{dx} = -24(3x + 2)^{-3} (3) \)  
  \( = -\frac{72}{(3x + 2)^3} \)  
  when \( x = 0, \frac{dy}{dx} = -9 \)  
  when \( x = 0, y = 3 \)  
  Line \( AB \): \( y = -9x + 3 \) | M1 for dy/dx  
  M1 for dy/dx at \( A \)  
  M1 for \( y = 3 \)  
  A1 |
| ii | sub \( y = 0, 0 = -9x + 3 \)  
  \( x = \frac{1}{3} \)  
  \( B = \left( \frac{1}{3}, 0 \right) \) | M1 for \( x \)-coordinate of \( B \) |
\[
\text{Area of } OACB = \int_{0}^{\frac{1}{3}} 12(3x + 2)^{-2} \, dx \\
= \left[ \frac{12(3x + 2)^{-1}}{-1(3)} \right]_{0}^{\frac{1}{3}} \\
= \left[ -\frac{4}{3x + 2} \right]_{0}^{\frac{1}{3}} \\
= -\frac{4}{3 \left( \frac{1}{3} \right)} + 2 - \left( -\frac{4}{3(0) + 2} \right) \\
= \frac{2}{3}
\]

\[
\text{Area of } \Delta OAB = \frac{1}{2} \left( \frac{1}{3} \right) (3) = \frac{1}{2}
\]

\[
\text{Area of shaded region} = \frac{2}{3} - \frac{1}{2} = \frac{1}{6} \text{ unit}^2
\]

M1 for $-\frac{4}{3x+2}$ (independent of limits)

M1 for area of $OACB$

M1 for area of tri $OAB$

A1
12 (a) State the values between which the principal value of $\tan^{-1} x$ must lie. Give your answer in terms of $\pi$. [1]

(b) The diagram below shows triangle $ABC$ where $AD = 2$ m, $DC = 3$ m and $BD = h$ m. $BD$ is perpendicular to $AC$ and $\theta_1 + \theta_2 = 45^\circ$.

By using a suitable formula for $\tan(\theta_1 + \theta_2)$, find the value of $h$. [5]

<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>$\frac{\pi}{2} &lt; \tan^{-1} x &lt; \frac{\pi}{2}$</td>
<td>B1</td>
</tr>
<tr>
<td>b</td>
<td>$\tan (\theta_1 + \theta_2) = \frac{\tan \theta_1 + \tan \theta_2}{1 - \tan \theta_1 \tan \theta_2}$</td>
<td>M1 for tan addition formula</td>
</tr>
<tr>
<td></td>
<td>$\tan 45 = \frac{2}{h + h}$</td>
<td>M1 for either $\tan \theta_1 = \frac{2}{h}$ or $\tan \theta_2 = \frac{3}{h}$</td>
</tr>
<tr>
<td></td>
<td>$2 - \left(\frac{2}{h}\right) \left(\frac{3}{h}\right)$</td>
<td>M1 for $\tan 45 = 1$</td>
</tr>
<tr>
<td></td>
<td>$\frac{2}{h} + \frac{3}{h}$</td>
<td>M1 for $h^2 - 5h - 6 = 0$</td>
</tr>
<tr>
<td></td>
<td>$1 = \frac{2}{h} + \frac{3}{h}$</td>
<td>$h = 6$</td>
</tr>
<tr>
<td></td>
<td>$1 - \left(\frac{2}{h}\right) \left(\frac{3}{h}\right) = \frac{5}{h}$</td>
<td>$h = -1$ (rej)</td>
</tr>
<tr>
<td></td>
<td>$h^2 - 5h - 6 = 0$</td>
<td>A1</td>
</tr>
</tbody>
</table>

[Turn over]
The Ultraviolet Index describes the level of solar radiation on the earth’s surface. The Ultraviolet Index, $U$, measured from the top of a building is given by $U = 6 - 5 \cos qt$, where $t$ is the time in hours, $0 \leq t \leq 20$, from the lowest value of Ultraviolet Index and $q$ is a constant. It takes 10 hours for the Ultraviolet Index to reach its lowest value again.

(i) Explain why it is impossible to measure a Ultraviolet Index of 12. \[1\]

(ii) Show that $q = \frac{\pi}{5}$. \[1\]

(iii) The top of the building is equipped with solar panels that supply power to the building when the Ultraviolet Index is at least 3.5. Find the duration, in hours and minutes, that the building is supplied with power by the solar panels. \[5\]

<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
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</tr>
</thead>
</table>
| i  | Max $U = 6 + 5 = 11$  
Since max value of $U = 11$, we cannot measure a Ultraviolet Index of 12. | B1 for stating max value of $U$ |
| ii | $10 = \frac{2\pi}{q}$  
$q = \frac{2\pi}{10} = \frac{\pi}{5}$ | B1 for $q = \frac{2\pi}{10}$ |
| iii | $6 - 5 \cos \frac{\pi}{5} t = 3.5$  
$\cos \frac{\pi}{5} t = \frac{1}{2}$  
$\alpha = \frac{\pi}{3}$  
$\frac{\pi}{5} t = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}, \frac{\pi}{3}, 2\pi - \frac{\pi}{3} + 2\pi$  
$t = 1.6666, 8.3333, 11.66, 18.33$  
Duration = $(8.3333 - 1.6666) + (18.33 - 11.66)$  
= 13.3367  
= 13 hours 20 mins | M1 for forming equation  
M1 for basic angle  
M1 for $\frac{\pi}{3}, 2\pi - \frac{\pi}{3}$  
M1 for all 4 values  
A1 |
INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.
Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer ALL questions.
Write your answers in the spaces provided on the question paper.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Write the brand and model of your calculator in the space provided below.

INFORMATION FOR CANDIDATES

You are expected to use a scientific calculator to evaluate explicit numerical expressions.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

<table>
<thead>
<tr>
<th>Brand / Model of Calculator</th>
<th>For Examiner’s Use</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
</tr>
<tr>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

This question paper consists of 15 printed pages.

Setter: Mr. Gabriel Cheow
Vetter: Mr. Narayanan
1. ALGEBRA

Quadratic Equation
For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where $n$ is a positive integer and \(\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}\)

2. TRIGONOMETRY

Identities

\[
\sin^2 A + \cos^2 A = 1 \\
\sec^2 A = 1 + \tan^2 A \\
\csc^2 A = 1 + \cot^2 A \\
\sin(A + B) = \sin A \cos B \pm \cos A \sin B \\
\cos(A + B) = \cos A \cos B \mp \sin A \sin B \\
\tan(A + B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \\
\sin 2A = 2 \sin A \cos A \\
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A \\
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for $\triangle ABC$

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \\
a^2 = b^2 + c^2 - 2bc \cos A \\
\Delta = \frac{1}{2} bc \sin A
\]
1 The roots of the quadratic equation \(2x^2 - 8x + 9 = 0\) are \(\alpha\) and \(\beta\).

(i) Show that the value of \(\alpha^3 + \beta^3\) is 10.

\[
\alpha + \beta = 4, \alpha\beta = \frac{9}{2}
\]

M1 – sum & pdt

\[
\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta
= 4^2 - 9
= 7
\]

M1

\[
\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)
= (4)\left(7 - \frac{9}{2}\right)
= 10\text{ (shown)}
\]

A1

(ii) Find a quadratic equation whose roots are \(\frac{1}{\alpha^2 + \beta}\) and \(\frac{1}{\alpha + \beta}\).

New sum: \[
\frac{1}{\alpha^2 + \beta} + \frac{1}{\alpha + \beta} = \frac{\alpha + \beta^2 + \alpha^2 + \beta}{(\alpha^2 + \beta)(\alpha + \beta)}
\]

\[
= \frac{\alpha + \beta^2 + \alpha^2 + \beta}{\alpha^3 + \beta^2 + \alpha^2 + \beta^2}
\]

\[
= \frac{4}{139}
\]

M1

Allow for ECF

New pdt: \[
\frac{1}{\alpha^2 + \beta} \times \frac{1}{\alpha + \beta} = \frac{1}{(\alpha^2 + \beta)(\alpha + \beta)}
\]

\[
= \frac{1}{10 + \frac{9}{2} + \left(\frac{9}{2}\right)^2}
\]

\[
= \frac{4}{139}
\]

M1

New eqn: \[
x^2 - \frac{44}{139}x + \frac{4}{139} = 0
\]

A1

\[
139x^2 - 44x + 4 = 0
\]
2 The function \( f(x) = 6x^3 + ax^2 + bx - 12 \), where \( a \) and \( b \) are constants, is exactly divisible by \( x + 2 \) and leaves a remainder of 5 when divided by \( x + 1 \).

(i) Find the value of \( a \) and of \( b \). [4]

\[
\begin{align*}
\text{for } x = 2 &\quad f(2) = 0 \\
-48 + 4a - 2b - 12 &\quad 2a - b = 30 \quad \text{Eqn 1} \\
\text{for } x = -1 &\quad f(-1) = 5 \\
-6 + a - b - 12 &\quad a - b = 23 \quad \text{Eqn 2} \\
\text{Eqn 1} - \text{Eqn 2: } a = 7 \\
\text{Sub into Eqn 1: } b = -16
\end{align*}
\]

(ii) By showing your working clearly, factorise \( f(x) \). [3]

\[
\begin{align*}
6x^3 + 7x^2 - 16x - 12 &\quad = (x + 2)(Ax^2 + Bx + C) \\
\text{By observation: } A = 6, C = 6 \\
\Rightarrow 6x^3 + 7x^2 - 16x - 12 &\quad = (x + 2)(6x^2 + Bx - 6) \\
\text{Let } x = 1 \\
6 + 7 - 16 - 12 &\quad = (3)(6 + B - 6) \\
-15 &\quad = 3B \\
B &\quad = -5 \\
6x^3 + 7x^2 - 16x - 12 &\quad = (x + 2)(6x^2 - 5x - 6) \\
&\quad = (x + 2)(3x + 2)(2x - 3)
\end{align*}
\]

(iii) Hence, solve the equation \( 6(2^{3y}) + 2^{2y+3} - 2^{y+4} - 12 = 2^{2y} \). [4]

\[
\begin{align*}
6(2^{3y}) + 2^{2y+3} - 2^{y+4} - 12 &\quad = 2^{2y} \\
6(2^{3y}) + 8(2^{2y}) - 16(2^y) - 12 &\quad = 2^{2y} \\
6(2^y)^3 + 7(2^y)^2 - 16(2^y) - 12 &\quad = 0 \\
\text{Let } x = 2^y \\
\Rightarrow (x + 2)(3x + 2)(2x - 3) &\quad = 0 \\
x &\quad = -2, -\frac{3}{2}, \frac{3}{2} \\
2^y &\quad = -2 \ (\text{rej.}), -\frac{2}{3} (\text{rej.}), \frac{3}{2} \\
y &\quad = 3 \ln 2, \ln \frac{3}{2}, \ln 1.5 \\
y &\quad = 3 \ln 2 = 0.585 \ (3sf)
\end{align*}
\]
(i) Express \( \frac{2x + 16}{(x^2 + 4)(2x-1)} \) in partial fractions.

\[
\frac{2x + 16}{(x^2 + 4)(2x-1)} = \frac{Ax + B}{x^2 + 4} + \frac{C}{2x-1}
\]

\[
2x + 16 = (Ax + B)(2x - 1) + C(x^2 + 4)
\]

Let \( x = 0.5 \):
\[
17 = C \left( \frac{17}{4} \right)
\]

\[ C = 4 \quad \text{M1} \]

Let \( x = 0 \):
\[
16 = B(-1) + 4(4)
\]

\[ B = 0 \quad \text{M1} \]

Let \( x = -1 \):
\[
14 = -A(-3) + 20
\]

\[ 3A = -6 \quad A = -2 \quad \text{M1} \]

\[
\therefore \quad \frac{2x + 16}{(x^2 + 4)(2x-1)} = \frac{-2x}{x^2 + 4} + \frac{4}{2x - 1} \quad \text{A1}
\]

(ii) Differentiate \( \ln(x^2 + 4) \) with respect to \( x \).

\[
\frac{d}{dx} [\ln(x^2 + 4)] = \frac{2x}{x^2 + 4} \quad \text{B2}
\]

(iii) Hence, using your results in (i) and (ii), find \( \int \frac{x + 8}{(x^2 + 4)(2x-1)} \, dx \).

\[
\int \frac{x + 8}{(x^2 + 4)(2x-1)} \, dx = \frac{1}{2} \int \frac{2x + 16}{(x^2 + 4)(2x-1)} \, dx
\]

\[
= \frac{1}{2} \int \left( \frac{-2x}{x^2 + 4} + \frac{4}{2x - 1} \right) \, dx \quad \text{M1 partial frac}
\]

\[
= -\frac{1}{2} \int \frac{2x}{x^2 + 4} \, dx + \frac{1}{2} \left( 2 \ln(2x - 1) + c_1 \right) \quad \text{M1}
\]

\[
= -\frac{1}{2} \ln(x^2 + 4) + c_2 + \ln(2x - 1) + \frac{1}{2} c_1 \quad \text{M1}
\]

\[
= \ln(2x - 1) - \frac{1}{2} \ln(x^2 + 4) + c \quad \text{A1}
\]
Prove the following identities.

(a) \((\sec x - \tan x)(\cosec x + 1) = \cot x\)

\[LHS = (\sec x - \tan x)(\cosec x + 1)\]

\[= \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x} + \frac{\sin x}{\sin x}\right)\]

\[= \frac{(1 - \sin x)(1 + \sin x)}{\cos x \sin x}\]

\[= \frac{\cos^2 x}{\cos x \sin x}\]

\[= \frac{\cos x}{\sin x}\]

\[= \cot x\]

\[= RHS \text{ (proven)}\]

(b) \(\frac{1 - \cos 2x + \sin x}{\sin 2x + \cos x} = \tan x\)

\[LHS = \frac{1 - \cos 2x + \sin x}{\sin 2x + \cos x}\]

\[= \frac{1 - (1 - 2\sin^2 x) + \sin x}{2\sin x \cos x + \cos x}\]

\[= \frac{2\sin^2 x + \sin x}{\cos x(2\sin x + 1)}\]

\[= \frac{\sin x(2\sin x + 1)}{\cos x(2\sin x + 1)}\]

\[= \frac{\sin x}{\cos x}\]

\[= \tan x\]

\[= RHS \text{ (proven)}\]
The lines $y = 8$ and $4x + 3y = 30$ are tangent to a circle $C$ at the points $(-1,8)$ and $(3,6)$ respectively.

(i) Show that the equation of $C$ is $x^2 + y^2 + 2x - 6y - 15 = 0$.  

Let centre of circle be $O$.  
Horizontal tangent at $(-1,8)$ means that $O$ is on the line $x = -1$.  

To find normal of circle at $(3,6)$:  
$4x + 3y = 30$  
$y = -\frac{4}{3}x + 10$  
$\therefore m_{\text{normal}} = -\frac{3}{4}$  
eqn of normal: $y - 6 = -\frac{3}{4}(x - 3)$  

When $x = -1, y = 3 \Rightarrow O(-1,3)$  

Horizontal tangent is $y = 8$. Hence radius is $5$.  

(ii) Explain whether or not the $x$ axis is tangent to $C$.  

$C$ has centre $(-1,3)$ and radius $5$.  
Hence its horizontal tangents are $y = 3 \pm 5 \Rightarrow y = 8$ or $y = -2$  
x-axis is $y = 0$, which is between the two horizontal tangents.  
Hence the $x$ axis will cut through $C$ at two points.  
Hence the $x$ axis is not tangent to $C$.  

Alternative solution: Sub $y = 0$ into eqn of $C$, show that $b^2 - 4ac \neq 0$.

(iii) The points $Q$ and $R$ also lie on the circle, and the length of the chord $QR$ is $2$ units. Calculate the shortest distance from the center of $C$ to the chord $QR$.  

Let $M$ be midpoint of $QR$. Hence $OM$ perpendicular to $QR$. 
Hence, $OM$ is shortest distance from $C$ to chord $QR$. 
Consider right-angled triangle $OMR$. 
By Pythagoras Theorem,  
$OM = \sqrt{5^2 - \left(\frac{2\cdot3}{2}\right)^2}$  
$= \sqrt{24} = 2\sqrt{6}$  
$= 4.90$ (3sf)
The table shows experimental values of two variables \( x \) and \( y \), which are known to be connected by the equation \( yx^n = A \), where \( n \) and \( A \) are constants.

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
x & 1.0 & 1.5 & 2.0 & 2.5 & 3.0 \\
\hline
y & 22.0 & 13.0 & 8.9 & 6.9 & 5.3 \\
\hline
\end{array}
\]

(i) Plot \( \log y \) against \( \log x \) and draw a straight line graph.

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\log x & 0 & 0.176 & 0.301 & 0.398 & 0.477 \\
\hline
\log y & 1.34 & 1.11 & 0.949 & 0.839 & 0.724 \\
\hline
\end{array}
\]

Scale: 4 cm to 0.1 units on X-axis, 2 cm to 0.1 units on Y-axis.
Scale used must be appropriate in order to award L1.
(ii) Use your graph to estimate the value of $A$ and of $n$. [4]

\[ yx^n = A \]

\[ \log y + n \log x = \log A \] \quad \text{(M1)}

\[ \log y = -n \log x + \log A \]

\[ Y = mX + c \]

\[ \Rightarrow m = -n, c = \log A \]

\[ m = \frac{0.7 - 1.34}{0.5 - 0} \] \quad \text{(M1)}

\[ = -1.28 \]

\[ n = 1.28 \] \quad \text{(A1)}

\[ c = 1.34 \]

\[ \log A = 1.34 \]

\[ A = 10^{1.34} \]

\[ = 21.9 \] \quad \text{(A1)}

(iii) On the same diagram, draw the line representing the equation $y = x^2$ and hence find the value of $x$ which satisfies the equation $x^{n+2} = A$. [2]

\[ Y = x^2 \]

\[ \log y = 2 \log x \]

\[ \text{Draw: } Y = 2x \] \quad \text{(M1 for drawing line)}

\[ x^{n+2} = A \]

\[ (n + 2) \log x = \log A \]

\[ 2 \log x = -n \log x + \log A \]

Let graph 1 be $\log y = 2 \log x$, and

Let graph 2 be $\log y = -n \log x + \log A$

From graph, let intersection be $(X, Y)$.

\[ (X, Y) = (0.41, 0.82) \]

\[ \log x = 0.41 \]

\[ x = 10^{0.41} \]

\[ = 2.57 \] \quad \text{(A1)}
The diagram shows a trapezium with vertices $A(-2.5, 0), B(0, p), C(1, 3)$ and $D$. The sides $AB$ and $DC$ are parallel and the angle $DAB$ is $90^\circ$. Angle $ABO$ is equal to angle $CBO$.

(i) Express the gradients of the lines $AB$ and $CB$ in terms of $p$ and hence, or otherwise, show that $p = 5$.

\[ m_{AB} = \frac{p - 0}{2.5 - 0} = \frac{2p}{5} \]

\[ m_{BC} = \frac{3 - p}{1} \]

\[ m_{AB} = -m_{BC} \]

\[ \frac{2p}{5} = \frac{p - 3}{1} \]

\[ 2p = 5p - 15 \]

\[ 3p = 15 \]

\[ p = 5 \text{ (shown)} \]
(ii) Find the coordinates of \( D \).

\[
m_{CD} = m_{AB} = 2
\]

\[
m_{AD} = -\frac{1}{2} \because AD \perp CD
\]

Let \( D(k, h) \)

\[
m_{CD} = \frac{3 - h}{1 - k} = \frac{2}{1 - k}
\]

\[
3 - h = 2 - 2k
\]

\[
h = 2k + 1 \quad \cdots \text{Eqn 1}
\]

\[
m_{AD} = \frac{h - 0}{k + 2.5} = -\frac{1}{2}
\]

\[
2h = -k - 2.5 \quad \cdots \text{Eqn 2}
\]

Eqn 1 in Eqn 2: \( 2(2k + 1) = -k - 2.5 \)

\[
5k = -2 - 2.5
\]

\[
k = -0.9
\]

\[
h = -9.3
\]

\[
\therefore D(-0.9, -0.8) \quad \text{A1}
\]

Alternative method: finding eqn of line \( AD \) and eqn of line \( CD \).

(iii) Find the area of the trapezium \( ABCD \).

\[
\text{Area} = \frac{1}{2} \begin{vmatrix} 1 & 0 & -2.5 & -0.9 & 1 \\ 3 & 5 & 0 & -0.8 & 3 \\ \end{vmatrix}
\]

\[
= \frac{1}{2} \left[ (5 + 2) - \left( -12.5 - 0.8 \right) \right]
\]

\[
= 21 \text{ units}^2 \quad \text{A1}
\]
8 (a) Solve the equation \(3 \log_x 3 = 8 - 4 \log_3 x\).

\[
3 \log_x 3 = 8 - 4 \log_3 x
\]

\[
\frac{3}{\log_3 x} = 8 - 4 \log_3 x \quad \text{M1 common log base 3 eqn}
\]

Let \(y = \log_3 x\)

\[
\frac{3}{y} = 8 - 4y
\]

\[
3 = 8y - 4y^2
\]

\[
4y^2 - 8y + 3 = 0
\]

\( (2y - 3)(2y - 1) = 0 \)

\( y = 1.5 \text{ or } 0.5 \quad \text{M1} \)

\( x = 3^{1.5} \text{ or } 3^{0.5} \)

\( = \sqrt{27} \text{ or } \sqrt{3} \quad \text{A1, A1} \)

(b) It is given that \( \log_a x = p \) and \( \log_a y = q \).

Express \( \log_a (ax^2y^3) \) in terms of \( p \) and \( q \).

\[
\log_y ax^2y^3 = \log_y a + 2 \log_y x + 3 \log_y y
\]

\[
= \frac{1}{\log_a y} + 2 \frac{\log_a x}{\log_a y} + 3
\]

\[
= \frac{1}{q} + 2pq + 3
\]

\( \text{M1 splitting of logs} \quad \text{A1, A1} \)
The figure shows a stage prop $ABC$ used by a member of the theatre, leaning against a vertical wall $OP$. It is given that $AB = 30$ cm, $BC = 100$ cm, $\angle ABC = \angle AOC = 90^\circ$ and $\angle BCO = \theta$.

(i) Show that $OC = (100\cos \theta + 30\sin \theta)$ cm. [2]

Let $D$ be foot of $B$ on $OC$, let $E$ be foot of $A$ on $BD$.

\[
\cos \theta = \frac{CD}{100} \Rightarrow CD = 100 \cos \theta \quad \text{(M1)}
\]

\[
\sin \theta = \frac{AE}{30} \Rightarrow AE = 30 \sin \theta \quad \text{(M1)}
\]

$OC = CD + AE = 100 \cos \theta + 30 \sin \theta$

(ii) Express $OC$ in terms of $R \cos(\theta - \alpha)$, where $R$ is a positive constant and $\alpha$ is an acute angle. [3]

\[
R = \sqrt{100^2 + 30^2} = \frac{10\sqrt{109}}{30} = \frac{10\sqrt{109}}{10} = \frac{10}{\sqrt{109}} \quad \text{(M1 for R)}
\]

\[
\alpha = \tan^{-1}\left(\frac{30}{100}\right) = 16.7^\circ (1dp)
\]

$\therefore OC = 10\sqrt{109} \cos(\theta - 16.7^\circ)$ [A1]

(iii) State the maximum value of $OC$ and the corresponding value of $\theta$. [2]

$OC_{\text{max}} = 10\sqrt{109}$

\[\theta = 16.7^\circ \quad \text{B1, B1} \]

(iv) Find the value of $\theta$ for which $OC = 80$ cm. [3]

\[80 = 10\sqrt{109} \cos(\theta - 16.7^\circ) \]

\[\cos(\theta - 16.7^\circ) = \frac{8}{\sqrt{109}} \quad \text{(M1)} \]

\[\theta - 16.7^\circ = 39.98^\circ \quad (\theta \text{ is acute}) \quad \text{M1} \]

\[\theta = 56.7^\circ \quad \text{A1} \]
**MARK SCHEME**

10 Given that \( y = a + b \cos 4x \), where \( a \) and \( b \) are integers, and \( x \) is in radians,
   (i) state the period of \( y \). [1]

\[ \frac{\pi}{2} \quad \text{B1} \]

Given that the maximum and minimum values of \( y \) are 3 and \(-5\) respectively, find
(ii) the amplitude of \( y \), [1]

\[ \text{amplitude} = \frac{3 - (-5)}{2} = 4 \quad \text{B1} \]

(iii) the value of \( a \) and of \( b \). [2]

\[ b = 4 \quad \text{B1} \]
\[ a = -1 \quad \text{B1} \]

Using the values of \( a \) and \( b \) found in part (iii),
(iv) sketch the graph of \( y = a + b \cos 4x \) for \( 0 \leq x \leq \pi \). [3]

(v) On the same set of axes, sketch the graph of \( y = |4 \sin 3x| \), and hence state the number of solutions of \( a + b \cos 4x = |4 \sin 3x| \). [3]

Number of solutions = 2 \quad \text{A1}
11 The dimensions of a cuboid are $3x$ cm by $2x$ cm by $h$ cm and its total surface area is $312$ cm$^2$. The volume of the cuboid is $V$ cm$^3$.

(i) Express $h$ in terms of $x$. 

$$2[3x(2x) + 3xh + 2xh] = 312$$

$$6x^2 + 5xh = 156$$

$$h = \frac{156 - 6x^2}{5x}$$

(ii) Show that $V = \frac{36}{5} x(26 - x^2)$.

$$V = (3x)(2x)\left(\frac{156 - 6x^2}{5x}\right)$$

$$= 6x\left(\frac{156 - 6x^2}{5}\right)$$

$$= \frac{36}{5} x(26 - x^2)$$

(iii) Find the maximum volume of the cuboid as $x$ varies, giving your answer to the nearest cm$^3$.

$$\frac{dV}{dx} = \frac{36}{5}[(26 - x^2) + x(-2x)]$$

$$- \frac{36}{5}[-3x^2 + 26]$$

$$\frac{dV}{dx} = -9$$

$$3x^2 - 26 = 0$$

$$x^2 = \frac{26}{3}$$

$$x = \pm \sqrt{\frac{26}{3}} (\text{req. -ve} : x > 0)$$

$$\frac{d^2V}{dx^2} = \frac{36}{5}(-6x)$$

$$\left.\frac{d^2V}{dx^2}\right|_{x=\sqrt{\frac{26}{3}}} = \frac{36}{5}(-6)\left(\frac{\sqrt{26}}{3}\right) < 0 \Rightarrow \text{max}$$

$$V = \frac{36}{5} \left(\frac{26}{3}\right)\left(26 - \frac{26}{3}\right)$$

$$= 367.4 \ldots$$

$$= 367 \text{ cm}^3$$
West Spring Secondary School
PRELIMINARY EXAMINATION 2019

Mathematics Paper 1
Secondary 4 Express/4 Normal Academic (O)/5 Normal Academic

Name ___________________________ ( ) Date 2 September 2019
Class _______________ Duration _____________ 2 hours

Candidates answer on the question paper
READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the
answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
The number of marks is given in [] at the end of each question or part question.
The total number of marks for this paper is 80.

FOR EXAMINER’S USE

/80

This document consists of 21 printed pages including this cover page.
Setter Mr Kok Yeong Haur [Turn over
Mathematical Formulae

**Compound interest**

Total amount = \( P \left(1 + \frac{r}{100}\right)^n \)

**Mensuration**

Curved surface area of a cone = \( \pi rl \)

Surface area of a sphere = \( 4 \pi r^2 \)

Volume of a cone = \( \frac{1}{3} \pi r^2 h \)

Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

Area of triangle \( ABC = \frac{1}{2} ab \sin C \)

Arc length = \( r \theta \), where \( \theta \) is in radians

Sector area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

**Trigonometry**

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

**Statistics**

\[
\text{Mean} = \frac{\sum fx}{\sum f}
\]

\[
\text{Standard deviation} = \sqrt{\frac{\sum f x^2 - \left(\frac{\sum fx}{\sum f}\right)^2}{\sum f}}
\]
Answer all the questions.

1. Solve \( \frac{x}{3} + 11 = 7 \)

   \[ \text{Answer } x = \ldots \ldots \] \[ \text{[1]} \]

2. Given that \( \sqrt[3]{2^{18}} = \left( \frac{1}{k} \right)^6 \), find \( k \).

   \[ \ldots \ldots \]

   \[ \text{Answer } k = \ldots \ldots \] \[ \text{[2]} \]

3. A set of five numbers is shown below.
   
   \[ 7 \quad 5 \quad 18 \quad 2 \quad 7 \]

   (a) Write down the median.

   \[ \text{Answer } \ldots \ldots \] \[ \text{[1]} \]

   (b) When one of the number is removed from the set, the median and the range do not change. Which number was removed?

   \[ \text{Answer } \ldots \ldots \] \[ \text{[1]} \]
4  \[ n \text{ is a positive integer.} \]

Show that \( (5n+2)^2 - (5n-2)^2 \) is a multiple of 8.

*Answer*

5  Factorise completely \( 4ax + 15by - 20ay - 3bx \).

*Answer*  

6  The frequency, \( f \text{ Hz} \), of a note produced by a string is proportional to the square root of the tension, \( T \text{ newtons} \), of the string.

For two identical strings, the ratio of the frequencies of the notes produced is \( 3 : 1 \).

Find the ratio of the tensions in the strings.

*Answer*  

[2]
A village of 120 people has two newspapers, the Arirang and the Busan. 35% of the villagers read the Arirang, 60% read the Busan, and 15% read neither.

\[ \mathcal{E} = \{\text{all people in the village}\} \]
\[ A = \{\text{people who read the Arirang}\} \]
\[ B = \{\text{people who read the Busan}\} \]

(a) State which Venn Diagram represents the village.

<table>
<thead>
<tr>
<th>Diagram 1</th>
<th>Diagram 2</th>
<th>Diagram 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram 1" /></td>
<td><img src="image2.png" alt="Diagram 2" /></td>
<td><img src="image3.png" alt="Diagram 3" /></td>
</tr>
</tbody>
</table>

Answer: Diagram 1

(b) In the diagram you have selected in (a), shade the region that represents the people in the village who read Arirang but not Busan.

Answer: Diagram 1

(c) Find the percentage of the villagers who read both newspapers.

Answer: 5%
8 A model of an auditorium is built using a scale of 1 : 250. The interior volume of the model is 125 000 cm³.

Find the actual interior volume, in m³, of the auditorium. Give your answer to 3 significant figures in standard form.

Answer .......... m³ [3]

9 John can either buy or rent a particular laptop. The cost of buying is $1288. The cost of renting is 25% of the price for the first year, and a monthly rental fee of $16.50 after the first year.

If \( x \) is the number of months after the first year, use the information to form an inequality in \( x \) and calculate, in years and months, when it becomes more expensive to rent than to buy the laptop.

Answer .......... years .......... months [3]
The diagram shows the fuel gauge of Kumar’s car. The fuel gauge indicates the amount of petrol in the car.

(a) Find the fraction of the car tank that is not filled with petrol.

Answer ......................... [1]

The fuel tank can store a maximum of 50 litres of petrol.

For cars travelling into Johor Bahru, their fuel tanks must be at least \( \frac{3}{4} \) full.

(b) Calculate how much fuel must Kumar top up in Singapore before he can enter Johor Bahru.

Answer ...................... litres [2]
11 (a) Express \( x^2 - 8x + 19 \) in the form \((x - a)^2 + b\).

Answer

(b) Hence, sketch the graph of \( y = x^2 - 8x + 19 \) on the axes below, labelling clearly the turning point and intercept(s).

Answer
ABC is a straight line.
AB = AM and CB = CN.
Angle MAB = \( p \)° and angle MBN = 90°.

(a) Find angle CBN in terms of \( p \).

Answer

(b) Explain with workings if AM is parallel to CN.
The diagram represents a park $ABC$.

(a) Construct the perpendicular bisector of $BC$. \hspace{1cm} [1]

(b) Construct the bisector of angle $ABC$. \hspace{1cm} [1]

(c) A café is to be built in the park, nearer to $B$ than to $C$ and nearer to $AB$ than to $BC$. Shade the region where the café is to be built. \hspace{1cm} [1]
Sam draws this graph to show the percentages of his students that passed Chemistry exam for the last four years.

(a) State and explain one aspect of the graph that may be misleading.

(b) Based on the statistic, explain if Sam can also claim that the number of students passing Chemistry has increased?
$ABCD$ is a square.
Point $Z$ lies on $CD$ such that $A, X, Y$ and $Z$ form a straight line.
Angle $AXB = \text{angle } DYA = 90^\circ$.

By considering angle $DAY = \theta$, prove that triangles $ABX$ and $DAY$ are congruent.

Answer
The table shows the times taken by 140 girls to complete the West Spring Cross Country 2019.

<table>
<thead>
<tr>
<th>Time (in minutes)</th>
<th>10 ≤ x &lt; 20</th>
<th>20 ≤ x &lt; 30</th>
<th>30 ≤ x &lt; 40</th>
<th>40 ≤ x &lt; 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of girls</td>
<td>25</td>
<td>39</td>
<td>62</td>
<td>14</td>
</tr>
</tbody>
</table>

(a) Calculate an estimate of
   (i) the mean time,

   Answer \[ \text{minutes} \] \[1\]

(ii) the standard deviation.

Answer \[ \text{minutes} \] \[1\]

(b) The mean time for the boys to complete the run was 23.8 minutes and the standard deviation was 10.4 minutes.

Make two comments comparing the times of the girls and the boys.

1. \[ \text{Comment 1} \]

2. \[ \text{Comment 2} \] \[2\]

\[ \text{Answer} \] \[3\]
(a) Calculate the sum of interior angles of a pentagon.

(b) Calculate the sum of the angles $a, b, c, d, e, f$ and $g$ in the diagram.
The table shows the travel times in minutes between some stations on an MRT route.

<table>
<thead>
<tr>
<th>Admiralty</th>
<th>Sembawang</th>
<th>Yishun</th>
<th>Khatib</th>
<th>Yio Chu Kang</th>
<th>Ang Mo Kio</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>9</td>
<td>6</td>
<td>4</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>13</td>
<td>10</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>a</td>
<td>15</td>
<td>9</td>
<td>b</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>17</td>
<td>6</td>
<td>7</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

(a) Find the values of $a$ and $b$.

Answer $a = \ldots \ldots \ldots$, $b = \ldots \ldots \ldots$ [2]

(b) A train leaves Sembawang MRT station and reaches Ang Mo Kio MRT station at 20:08.

(i) Calculate the time when the train leaves Sembawang MRT station.

Answer \ldots \ldots \ldots [1]

(ii) Given that the distance between Sembawang MRT and Ang Mo Kio MRT stations is 12.5 km, find the average speed of the train in km/h, between these two stations.

Answer \ldots \ldots \ldots km/h [1]
(a) Express 600 as the product of its prime factors.

(b) The number $600k$ is a perfect cube.

Find the smallest positive integer value of $k$.

Answer $k = \ldots$ [1]

(c) $x$ is a number between 950 and 1000.

The highest common factor of $x$ and 600 is 20.

Find the smallest possible value of $x$.

Answer $x = \ldots$ [2]
A factory makes wooden tables and chairs.

A table requires 8 hours of labour (L), 9 planks of wood (W) and 3 tins of paint (P). A chair requires x hours of labour (L), 2 planks of wood (W) and 1 tin of paint (P).

(a) Represent this information in a 2 x 3 matrix, P.

\[
P = \begin{pmatrix}
L & W & P \\
\end{pmatrix}
\]

Table
Chair

[1]

(b) The cost of labour is $10 per hour, the cost of wood is $20 per plank and the cost of paint is $4 per tin.

Find, in terms of x, the matrix \( R = P \begin{pmatrix} 10 \\ 20 \\ 4 \end{pmatrix} \).

Answer \( R = \begin{pmatrix} \end{pmatrix} \) [2]

(c) Explain what each element in matrix \( R \) represents.

[1]

(d) The cost of a table is four times the cost of a chair. Calculate x.

Answer \( x = \) [1]
The diagram shows part of the speed-time graph of an object over a period of 50 seconds. The object accelerates uniformly from 10 m/s to \( v \) m/s in 20 seconds. It then decelerates uniformly for the next 15 seconds. Thereafter it maintains a constant speed of 10 m/s. The object travelled 450 m in the first 20 seconds.

(a) Calculate the value of \( v \).

Answer: \( v = \ldots \ldots \ldots \ldots \ldots \text{m/s} \) [2]

(b) Find the acceleration of the object after 7 seconds.

Answer: \( \ldots \ldots \ldots \ldots \text{m/s}^2 \) [1]

(c) Complete the speed-time graph.

Answer
$ABC$ is an isosceles triangle with vertices on a circle with centre $O$ and radius 15 cm. The height of the triangle $ABC$ is 25 cm.

Calculate the area of the shaded region.
A company sells two sizes of the same brand of drink.

1.5 litre $3.40
6.5 litres $16.80

(a) Show that the cost of the drink is not directly proportional to the volume of the drink.

Answer

(b) The bottles are all geometrically similar.
The height of the 1.5 litre is 18 cm.

Calculate the height of the 6.5 litres bottle.

Answer cm
24. \( A \) is the point \((1, 1)\).
\[
\overrightarrow{AB} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}.
\]

\( D \) divides \( BC \) such that \( BD : DC = 1 : 1 \).
(a) Find \( \overrightarrow{BC} \).

\[\text{Answer} \quad \overrightarrow{BC} = \begin{pmatrix} \quad \end{pmatrix}\]

[2]

(b) Find \( |\overrightarrow{AD}| \).

\[\text{Answer} \quad |\overrightarrow{AD}| = \quad \]

[2]

(c) \( P \) is the point \((3, 9)\).

Use vectors to show whether or not \( ABPC \) is a parallelogram.

\[\text{Answer}\]

[2]

End of Paper 1
Compound interest

Total amount \( P\left(1 + \frac{r}{100}\right)^n \)

Mensuration

Curved surface area of a cone = \( \pi rl \)
Surface area of a sphere = \( 4\pi r^2 \)

Volume of a cone = \( \frac{1}{3} \pi r^2 h \)

Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

Area of triangle \( ABC = \frac{1}{2} ab \sin C \)

Arc length = \( r \theta \), where \( \theta \) is in radians

Sector area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

Trigonometry

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\( a^2 = b^2 + c^2 - 2bc \cos A \)

Statistics

Mean = \( \frac{\sum fx}{\sum f} \)

Standard deviation = \( \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} \)
1
(a) Simplify \( \frac{9a^2}{4b^3} - \frac{15a^4}{12ab^3} \) \[1\]
(b) Express as a single fraction in its simplest form \( \frac{5}{p+2} - \frac{2}{2p-3} \) \[2\]
(c) Solve the inequality \( 7 - 2x \geq 3x - 8 \) \[2\]
(d) It is given that \( a = \frac{1}{2} \sqrt{\frac{\pi (b^2 - 6)}{c}} \).
Express \( b \) in terms of \( a \) and \( c \). \[2\]

2
The diagram show patterns of grey and white squares.

Diagram 1  Diagram 2  Diagram 3  Diagram 4

<table>
<thead>
<tr>
<th>Diagram</th>
<th>Number of grey squares (G)</th>
<th>Number of white squares (W)</th>
<th>Total number of squares (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>17</td>
<td>8</td>
<td>25</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
<td>32</td>
<td>49</td>
</tr>
<tr>
<td>5</td>
<td>49</td>
<td>x</td>
<td>y</td>
</tr>
</tbody>
</table>

(a) Find the values of \( x \) and \( y \). \[2\]
(b) 
(i) Write down an expression, in terms of \( n \), for the total number of squares \( T \) for Diagram \( n \). \[1\]
(ii) Explain why it is not possible to have a diagram with a total of 226 squares. \[1\]
(c) Write an equation connecting \( G \), \( W \) and \( T \). \[1\]
(d) If there are 161 grey squares in Diagram 10, calculate the number of white squares. \[2\]
3 (a) One litre of petrol costs $2.25 in a particular petrol station. During National Day, there was a promotion of 7% fuel discount. Members are entitled to an additional 10% discount on their fuel purchase. Karen, who is a member, paid $61.58 for her petrol on National Day. Calculate to 1 decimal place, the number of litres she pumped. [2]

(b) The Singapore Savings Bonds (SSB) for the month of Aug pays the following interest rate for the first three years. The interest is paid out twice a year for every year.

<table>
<thead>
<tr>
<th>Rate</th>
<th>1.95%</th>
<th>2.00%</th>
<th>2.10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
<td>1st</td>
<td>2nd</td>
<td>3rd</td>
</tr>
</tbody>
</table>

Sam bought $20 000 of the SSB for Aug.

(i) Calculate the interest Sam will receive in the first half of the first year. [1]

(ii) Calculate the total interest earned after three years. [1]

Sam also invested $20 000 in a savings account of a bank with a rate of compound interest of 1.98% per year. He leaves the money in the account for 3 years.

(iii) Calculate the total amount of interest he will earn after 3 years. Give your answer to the nearest cent. [2]

(iv) Based on the given information, give one possible reason why Sam would prefer to invest more in SSB than in the bank. [1]

c) Din booked a hotel in China using his credit card. The hotel costs RMB 390 per night. Din booked the hotel for 4 nights and has two payment options:

Option A: The hotel charge in Singapore dollars using the hotel's exchange rate. Option B: The hotel charge in RMB, after which Din's credit card company will convert to Singapore dollars using the company's exchange rate. There is a fee of 0.5% charged by the credit card company for the currency conversion.

Din found the currency exchange rates for the hotel and the credit card company

<table>
<thead>
<tr>
<th>Hotel:</th>
<th>S$1 = RMB 4.97</th>
</tr>
</thead>
<tbody>
<tr>
<td>Credit Card:</td>
<td>S$1 = RMB 5.06</td>
</tr>
</tbody>
</table>

Explain with workings, which payment option Din should choose. [2]
4 (a) The equation of line $p$ is $2x + 3y = 12$.
The line cuts the $x$-axis at point $A$ and the $y$-axis at point $B$.

(i) Find the length of $AB$,  

A point $C$ lies on the line $p$ such that it is equidistant from the coordinate axes.  

(ii) Show that the coordinates of point $C$ is $(2.4, 2.4)$.  

(iii) Write down the equation of the line which passes through $C$ and is parallel to the $y$-axis.

(b) The diagram shows a parallelogram $ABCD$.
$AC$ and $BD$ intersects at $E$.
$G$ is a point on $AB$ such that $2AG = GB$.
$AF : AE = 1 : 2$.
$\overline{AG} = a$ and $\overline{AF} = b$.

(i) Use vectors to determine if $D, F$ and $G$ lie on a straight line.  

(ii) Find the ratio of

(a) $\frac{\text{the area of } \triangle AFG}{\text{the area of } \triangle DFC}$  

(b) $\frac{\text{the area of } \triangle AFG}{\text{the area of } \triangle DEC}$.
The diagram shows a circle, centre $O$.
$AF$ is a tangent to the circle.
The line $CD$ is parallel to the diameter of the circle $BE$.
$DEF$ is a straight line.
The lines $AC$ and $BE$ intersect at $G$.
Angle $OAC = 45^\circ$ and angle $ACB = 30^\circ$

(i) Find, giving reasons for each answer,
(a) angle $BEG$,
(b) angle $CAB$,
(c) angle $EFA$. 

(ii) Determine with workings if triangles $GAB$ and $DCE$ are similar

(b) A sector has radius 8 cm and angle 0.873 radians.
It is then formed into a cone by joining the two radii together.
Calculate the perpendicular height of the cone.
Points $P$, $S$, $Q$ and $R$ are at ground level.
$Q$ is on a bearing of $240^\circ$ from $R$.
$R$ is $60$ m due East of $P$.
$S$ is due North of $Q$.
$PQ = 70$ m.

(a) Find angle $PQR$.  
(b) Calculate the bearing of $P$ from $Q$.  
(c) Calculate $QR$.  
(d) An engineer, $X$, walks along a straight line from $S$ to $Q$.
Calculate the shortest distance of $X$ from $P$ during this journey.  
(e) $S$ is the base of a vertical tower.
$T$ is the point on top of the tower vertically above $S$.
The angle of depression of $R$ from $T$ is $27^\circ$.
Calculate the height of the tower.
The variables $x$ and $y$ are connected by the equation

$$y = x + \frac{4}{x} - 5.2.$$ 

Some corresponding values of $x$ and $y$, correct to 2 decimal places, are given in the table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0.7</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
<th>7.0</th>
<th>7.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>1.21</td>
<td>-0.20</td>
<td>-1.03</td>
<td>-1.2</td>
<td>$p$</td>
<td>-0.20</td>
<td>0.60</td>
<td>2.37</td>
<td>2.83</td>
</tr>
</tbody>
</table>

(a) Find the value of $p$.  

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal $x$-axis for $0 < x \leq 8$. 
Using a scale of 4 cm to represent 1 unit, draw a vertical $y$-axis for $-2 \leq y \leq 3$. 
On your axes, plot the points given in the table and join them with a smooth curve.

(c) Use your graph to solve the equation $x^2 - 4x + 4 = 0$.  

(d) By drawing a tangent, find the value of $x$ where the gradient of the curve is 0.75.

(e) (i) On the same axes, draw the line $y = \frac{3}{2}x - 2$, for $0 < x \leq 8$.  
(ii) Write down the $x$-coordinate of the point where this line intersects the curve.

(iii) This value of $x$ is a solution of the equation $x^2 + Ax + B = 0$. 
Find the value of $A$ and the value of $B$.  

The diagram shows a conical bottle of height $h$ and radius $R$ that is filled with water. When rests on its base, the water in the bottle is 8 cm from its vertex.

(a) Express $r_1$ in terms of $R$ and $h$.

Hence show that the volume of the water can be expressed as
\[ V = \frac{1}{3} \pi R^2 \left( h - \frac{512}{h^2} \right) \]  
[3]

When the same conical bottle is turned upside down, the water level is 2 cm from its base.

(b) Show that the volume of water
\[ V = \frac{1}{3} \pi R^2 \left( h - 2 \right)^3 \]  
[2]

(c) Using your answers from part (a) and (b), or otherwise, write down an equation in $h$ and show that it reduces to
\[ h^2 - 2h - 84 = 0. \]  
[2]

(d) Solve the equation $h^2 - 2h - 84 = 0$, giving your solutions correct to one decimal place.  
[3]

(e) Calculate the volume of water in the conical bottle if $R = 7$ cm.  
[2]
9 (a) The marks of 120 students in a Physics test were recorded. The cumulative frequency curve below shows the distribution of the marks.

(i) Use the curve to estimate
   (a) the median mark,
   (b) the interquartile range of the marks.

(ii) The criteria for distinction is 45 marks.
    Estimate the percentage of students who scored distinction.
(iii) The marks of the same 120 students in a Chemistry test were also recorded.
The box-and-whisker plot shows the distribution of the marks.

Make two comments comparing the marks of the students for Physics and for Chemistry. [2]

(h) The table summarises the number of practice papers each student did before taking the Physics test.

<table>
<thead>
<tr>
<th>Number of practice papers</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students</td>
<td>23</td>
<td>40</td>
<td>19</td>
<td>26</td>
<td>12</td>
</tr>
</tbody>
</table>

(i) One student is selected at random.
Find the probability that the student did not do any practice papers. [1]

(ii) Two students are selected at random.
Find, as a fraction in its simplest form, the probability that
(a) they both did three practice papers,
(b) one had done more than two practice papers and the other had done fewer than two practice papers. [2]
Zander runs a restaurant in a shopping mall for the month of June.

His full-service, non-corner, 24-hour restaurant is made up of the following sections:

<table>
<thead>
<tr>
<th>Percentage of Restaurant Floor Area</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>60%</td>
<td>Dining Area</td>
</tr>
<tr>
<td>30%</td>
<td>Kitchen</td>
</tr>
<tr>
<td>10%</td>
<td>Others (e.g. cashier, dish washing, receiving, storage etc)</td>
</tr>
</tbody>
</table>

The restaurant has 70 seats in the dining area.
Full-service restaurants typically have about one seat per 12 square feet (sqf).

(a) Estimate the floor area of the restaurant in sqf.

(b) The monthly rental of the restaurant is calculated in dollars based on the table.

<table>
<thead>
<tr>
<th>Location in mall</th>
<th>Monthly Rent in Dollars per square feet ($/sqft)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&lt; 1000</td>
</tr>
<tr>
<td>Corner</td>
<td>50</td>
</tr>
<tr>
<td>Non-Corner</td>
<td>40</td>
</tr>
</tbody>
</table>

The mall management also charge a “Maintenance & Advertising” cost every month. This cost is based on the floor area of the tenant, and is $200 per 100 sqf.

Use the table and information provided to calculate the monthly rental cost, inclusive of the “Maintenance & Advertising” cost.

(c) In addition to the monthly rental cost calculated in part (b), Zander estimates that he will have these costs each month:

- Food Raw Materials & Ingredients $14,400*
- Utilities $21,600**
- Other administrative cost (fixed) $2,000

* Dependent on hours of operation; based on 24-hour and $20 per hour
** Dependent on hours of operation; based on 24-hour and $30 per hour

The shopping mall requires all restaurant tenants to open at least 12 hours each day, and at least till 1 am.

Zander requires 20 workers who are paid according to their working hours (Table 1).

To help understand and improve his business, Zander also collected information on his restaurant revenue at different times of the day (Table 2).
Zander needs a monthly profit of at least $7000 for his repayment for the loan he took for his restaurant business.

(i) Determine with workings, if Zander is able to repay his loan in June. [4]

(ii) Suggest a sensible opening hours for Zander’s restaurant in July that will allow him to pay his loans. Justify your decision and show all calculations clearly. [2]

End of Paper ☺
<table>
<thead>
<tr>
<th>Q</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x = -12$</td>
</tr>
<tr>
<td>2</td>
<td>$k = \frac{1}{2}$</td>
</tr>
<tr>
<td>3a</td>
<td>7</td>
</tr>
<tr>
<td>3b</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>$(4a - 3b)(x - 5y)$</td>
</tr>
<tr>
<td>6</td>
<td>9 : 1</td>
</tr>
<tr>
<td>7a</td>
<td>Diagram 1</td>
</tr>
<tr>
<td>7b</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>7c</td>
<td>10%</td>
</tr>
<tr>
<td>8</td>
<td>$1.95 \times 10^4$</td>
</tr>
<tr>
<td>9</td>
<td>It is more expensive to rent after 5 years 11 months</td>
</tr>
<tr>
<td>10a</td>
<td>7</td>
</tr>
<tr>
<td>10b</td>
<td>16</td>
</tr>
<tr>
<td>11a</td>
<td>$\frac{3}{8}$ litres</td>
</tr>
<tr>
<td>12a</td>
<td>$P$</td>
</tr>
<tr>
<td>13</td>
<td>Construction</td>
</tr>
<tr>
<td>14a</td>
<td>Misleading feature: the percentage is not proportional to the number of books drawn. How it is misleading: gives the impression there is huge increase in the percentage of students passing Chemistry.</td>
</tr>
<tr>
<td>14b</td>
<td>No, Sam cannot claim that the number of students passing has increased. This is because the number of students taking the subject may not be the same over the years</td>
</tr>
</tbody>
</table>
| 15 | Method 1 (AAS): Let $\angle DAY = \theta$. \[
\angle AXB = \angle DYA = 90^\circ \text{ (given)} \\
\angle BAX = 90^\circ - \theta \text{ (complementary angles)} \\
= 90^\circ - (180^\circ - 90^\circ - \angle ADY) \text{ (sum of } \Delta) \\
= \angle ADY \\
AB = DA \text{ (sides of a square are equal)} \\
\triangle ABX and \triangle DAY are congruent (AAS) |
| 16ai | Mean = 29.6 min (3sf) |
| 16a(ii) | SD = 8.98 min (3 sf) |
| 16b | Two comments: |
2) Boys' recorded time is shorter. 2.58 vs 2.64 in 2.5 km, girls' time is shorter.
3) Girls' timing are more consistent as their SD is smaller.

17a 54°
17b 1620°
18a \( a = 18 \)
\( b = 11 \)
18bii Speed 44.1 km/h
19a \( 2^3 \times 3 \times 5^2 \)
19b 45
19c 980

20a \( P = \begin{pmatrix} 8 & 9 & 3 \\ x & 2 & 1 \end{pmatrix} \)

20b \( P \begin{pmatrix} 10 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 & 9 & 3 \\ 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 10 \\ 20 \end{pmatrix} \)
\( = \begin{pmatrix} 272 \\ 10x + 44 \end{pmatrix} \)

20c The cost of making a table and a chair respectively.
20d \( x = 2.4 \)
21a \( v = 35 \)
21b Acceleration \( = \frac{35 - 10}{20} = 1.25 \text{ m/s}^2 \)

21c Diagram

22 350 cm³

23a Since the ratio of cost/volume are not the same, the cost is not directly proportional to the volume.

23b \( h_1 = 29.3 \text{ cm} \)

24a \( \begin{pmatrix} 6 \\ 2 \end{pmatrix} \)

24b 4.12
24c Parallelogram
Qns  Solution  
1a  \( \frac{9}{5ab} \)  
1b  \( \frac{8p-19}{(p+2)(2p-3)} \)  
1c  \( x \leq 3 \)  
1d  \( b = \pm \sqrt{\frac{4a^2c + 6\pi}{\pi}} \)  
2a  \( y = 121, x = 72 \)  
2b  \( T = (2n-1)^2 \)  
2bii  Column (T) consists of perfect squares only but 226 is not a perfect square.  
2c  \( T = G + W \)  
2d  200  
3a  \( N = 32.7 \) litres  
3b  Interest = $1.95  
3bii  $1210  
3biii  $1211.68  
3biv  One possible reason - interest earned is comparable but SSB pays interest twice every year compared to the bank, which pays the interest after 3 years.  
3c  Din should choose Option B.  
4ai  7.21  
4a  \( \begin{bmatrix} 12 & 12 \\ 5 & 5 \end{bmatrix} \)  
4a  \( \begin{bmatrix} \frac{12}{5} & \frac{12}{5} \end{bmatrix} \)  
4a  \( x = \frac{12}{5} \)  
4b  \( D, F \) and \( G \) lie on a straight line  
4bii  1 \( \frac{1}{9} \)  
4biii  1 \( \frac{1}{12} \)  
5aia  60°  
5aib  15°  
5acic  45°  
5b  7.92 cm  
6a  25.4°  
6b  0.36°  
6c  115m  
6d  39.8 m  
6e  50.8 m  
7a  \( p = 3 \frac{4}{3} - 5.2 = -0.87 \)  
7b  (see graph below)  

All points plotted correctly
7c  \( x = 2 \)

7d  Draw a tangent with gradient = 0.75 on the graph

\( x = 4 \)

7ei  See graph

7eii  \( x = 1.1 \) \( (1.05 \leq x \leq 1.15) \)

7eiii  \( \therefore A = 5.4 \) and \( B = -8 \)

8a  \( \frac{1}{3} \pi R^2 \left( h - \frac{512}{h^2} \right) \)
8b \[ \frac{1}{3} \pi R^3 \left( \frac{h-2}{h^2} \right) \]

8d \( h = 10.21954446 = 10.2 \) or \( h = -8.22 \)

8e \( 2790 \text{ cm}^3 \)

9aia Median = 34 marks

9aib IQR = 39 - 29 = 10 marks

9aii Percentage with distinction = 11.7%

9aiii 2 comments:

1) On average, students scored higher in Physics than in Chemistry, as Physics median of 34 is higher than Chemistry median of 30.

2) Students' marks in Physics more consistent than Chemistry as Physics IQR of 10 is lower than Chemistry IQR of 15.

9bi \( \frac{23}{120} \)

9biia \( \frac{65}{1428} \)

9biib \( \frac{57}{170} \)

10a 1400 sq ft

10b 44800

10ci As amount is less than $7000, Zander will not be able to meet his repayment.

10cii Any sensible operating hours with relevant workings to prove that profit earned is more than $7000

Operating hours must be at least 12 hours
Operating hours must be till 1 am
Calculations for Food Raw Materials and Utilities must be based on hours of operation

One possible solution of operating hours: 8 am to 1 am
YUYING SECONDARY SCHOOL
PRELIMINARY EXAMINATION
Secondary 4 Express/ 5 Normal (Academic) / 4N1 'O'

NAME

CLASS

REG. NO

MATHEMATICS

4048/01

Paper 1

26 August 2019
2 hours

Candidates answer on the Question Paper.

Setter: Mr Tai Kay Seng

READ THESE INSTRUCTIONS FIRST

Write your name, class and class register number on the work that you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use

<table>
<thead>
<tr>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
</tr>
</tbody>
</table>

This document consists of 16 printed pages. [Turn Over
Compound interest

\[ \text{Total amount} = P \left(1 + \frac{r}{100}\right)^n \]

Mensuration

Curved surface area of a cone = \( \pi rl \)
Surface area of a sphere = \( 4\pi r^2 \)
Volume of a cone = \( \frac{1}{3}\pi r^2 h \)
Volume of a sphere = \( \frac{4}{3}\pi r^3 \)
Area of triangle \( ABC = \frac{1}{2} ab \sin C \)
Arc length = \( r\theta \), where \( \theta \) is in radians
Sector area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

Trigonometry

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]
\[ a^2 = b^2 + c^2 - 2bc \cos A \]

Statistics

\[ \text{Mean} = \frac{\sum fx}{\sum f} \]
\[ \text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} \]
3

Answer all the questions

1 Write the following numbers in order of size, starting with the smallest.

\[ 0.85, \quad \frac{8}{13}, \quad (0.75)^2, \quad \sqrt{0.49} \]

Answer

2 Leo is travelling to Johor Bahru from Singapore. He wants to change 400 Singapore Dollars into Malaysian Ringgit.
In Singapore, the exchange rate is 1 Singapore Dollar = 3.02 Malaysian Ringgit.
In Johor Bahru, the exchange rate is 1 Malaysian Ringgit = 0.328 Singapore Dollars.

Where should he change his money to get more Malaysian Ringgit and by how much more?

Answer

Ringgit [2]

3 Show that \((5n-1)^2 + 4\) is a multiple of 5.

Answer
4 Given that $5 \times 5^{2n-2} = 125$, find the value of $n$.

Answer $n = \text{[2]}$

5 The first five terms of a sequence are as follows:

$78, \ a, \ b, \ 69, \ c, \ \ldots$  

(a) Find the value of $a$, $b$, and $c$.

Answer (a) $a = \text{[2]}$

$b = \text{[2]}$

$c = \text{[2]}$

(b) Write down an expression, in terms of $n$, for the $n$th term of the sequence.

Answer (b) \[ \text{[1]} \]

(c) Explain why 32 is not a term of this sequence.

Answer (c) \[ \text{[1]} \]
6 The angles, in degrees, of a quadrilateral $ABCD$ are represented by these expressions:
Angle $A = (80 - 4x)^\circ$, angle $B = (10 + 3x)^\circ$, angle $C = (5x + 90)^\circ$ and angle $D = (15x - 10)^\circ$.
(a) Calculate the value of $x$.

(b) What is the name of the quadrilateral? 

Answer (a) [1]

Answer (b) [1]

7 Two boxes are geometrically similar. The base area of the smaller box is 64 cm$^2$ while the base area of the larger box is 81 cm$^2$.

Calculate the volume of the smaller box as a percentage of the volume of the bigger box.

Answer % [2]

8 (a) Express 324 as a product of its prime factors.

(b) A number has exactly 8 factors. Two of the factors are 4 and 18.
List all the factors of the number.

Answer (a) [1]

Answer (b) [2]
9 The length of a rectangle exceeds its width by 2 cm. 
(a) If the diagonal is 10 cm, find the width of the rectangle.

\[ \text{Answer (a)} \quad \text{cm} \quad [2] \]

(b) How many squares of side 3 cm can be cut out from this rectangle?

\[ \text{Answer (b)} \quad [1] \]

10 Kenneth invests $2000 for 3 years at a fixed rate of compound interest. 
At the end of the first year there is $2100 in his account. 
(a) What is the rate of compound interest?

\[ \text{Answer (a)} \quad \% \quad [2] \]

(b) How much does Kenneth have in his account at the end of 3 years?

\[ \text{Answer (b)} \quad \$ \quad [2] \]
11 The diagram shows the box-and-whisker plot for the power, in kilowatts (kW), supplied to an electrical circuit.

(a) Find the median of the power supplied.

\[ \text{Answer (a)} \quad \text{kW} \quad [1] \]

(b) Find the interquartile range.

\[ \text{Answer (b)} \quad \text{kW} \quad [1] \]

12 \( A \) is the point \((-1, 4)\), \( B \) is the point \((2, -5)\) and \( O \) is the origin.
(a) Find the gradient of the line \( OA \).

\[ \text{Answer (a)} \quad [1] \]

(b) Find the equation of the line through \( B \) parallel to \( OA \).

\[ \text{Answer (b)} \quad [2] \]
The Venn Diagram represents the sets $A$ and $B$.

(a) List the elements of $(A \cup B)'$.

(b) Find $n(A' \cap B)$.

(c) Set $C$ is defined as the first two multiples of 32. Insert set $C$ in the Venn diagram above.

14 Simplify

(a) $27x^3y^{-2} + 18xy^3$

(b) $\frac{2}{(x-3)^2} - \frac{1}{3-x}$
15 A bicycle accelerates from rest at a constant rate to a certain speed in 10 seconds. It maintains at this speed for the next 10 seconds. The total distance travelled by the bicycle in the 20 seconds is 240 metres.

(a) Calculate the speed of the bicycle in the tenth second and

\[ \text{Answer (a)} \quad \quad \text{m/s} \quad [2] \]

(b) draw the speed-time graph of the bicycle journey for the first 20 seconds.

\[ \text{Speed (m/s)} \]

\[ \begin{array}{c|c|c|c}
0 & 10 & 20 \\
\hline
0 & 10 & 20 \\
\hline
10 & 20 & 30 \\
\hline
\end{array} \]

Time (s)

[2]

16 Factorise completely

(a) \(4x^2 - 10x - 6\),

(b) \(10x^3y - 5xy + 2x - 1\).

\[ \text{Answer (a)} \quad [2] \]

\[ \text{Answer (b)} \quad [2] \]
17 A two digit number is formed using the digits 4, 8 and 9. Repetition of the digit is allowed.
(a) List the sample space.

Answer (a) [1]

(b) Find the probability that a number selected at random is
(i) a prime number,

Answer (b)(i) [1]

(ii) divisible by 5.

Answer (b)(ii) [1]

18

The diagram above is made up of a square, a regular pentagon and an incomplete regular polygon ABCDE of $n$ sides. Find the value of $n$.

Answer $n =$ [3]
The volume, $V$, of a given mass of gas, is inversely proportional to the pressure, $P$.

(a) Sketch a volume-pressure graph for the mass of gas.

Answer (a) \[ V \]

\[ \]

When the volume is 3 m$^3$, the pressure of the gas is 200 N/m$^2$.

(b) Find the equation for $V$ in terms of $P$.

Answer (b) $V = \quad [2]$

(c) Calculate the pressure when the volume is 5 m$^3$.

Answer (c) \[ \quad \text{N/m}^2 \quad [1] \]
A rubber cone of diameter 6 cm and height 10 cm is cut in half to make two rubber door stoppers. Find

(a) the volume of a rubber stopper,

Answer (a) \( \text{cm}^3 [1] \)

(b) the total surface area of a rubber stopper.

Answer (b) \( \text{cm}^2 [3] \)
21 The following shows the Formula 1 track where the turns along the track are numbered 01 to 23.

(a) Estimate the actual length of the track from turn 04 to turn 07.

\[ \text{Answer (a)} \quad \text{m} [1] \]

(b) A racer finished the 309.316 km race in 1 h 45.599 min. Calculate the average speed in km/h.

\[ \text{Answer (b)} \quad \text{km/h} [1] \]

(c) Suggest and explain a possible speed when the racer
(i) went past the grandstand,

\[ \text{Answer (i)} \]

(ii) was at turn 05.

\[ \text{Answer (ii)} \]

................................. [1]
$ABCD$ is a quadrilateral.

(a) Write each of the following in terms of $p$ and $q$.

(i) $\overline{AC}$

(ii) $\overline{BX}$

(iii) $\overline{XD}$

(iv) Explain why $B, X$ and $D$ lie in a straight line.

\[ \text{Answer} \]
23. A particular restaurant offers 3 different dinner set menu Deluxe, Superior and Economy Set Package. The following table shows the orders for the three set packages on three days of a particular week.

<table>
<thead>
<tr>
<th></th>
<th>Deluxe</th>
<th>Superior</th>
<th>Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friday</td>
<td>35</td>
<td>45</td>
<td>60</td>
</tr>
<tr>
<td>Saturday</td>
<td>70</td>
<td>85</td>
<td>150</td>
</tr>
<tr>
<td>Sunday</td>
<td>90</td>
<td>130</td>
<td>180</td>
</tr>
</tbody>
</table>

(a) Represent the number of orders for each type of set package on the three days by a $3 \times 3$ matrix $A$.

Answer (a) $A = \begin{pmatrix} 35 & 45 & 60 \\ 70 & 85 & 150 \\ 90 & 130 & 180 \end{pmatrix}$ [1]

(b) Given that each Deluxe, Superior and Economy Set Package costs $188, $88 and $38 respectively, write down a $3 \times 1$ matrix $B$ showing the price for each type of the set packages.

Answer (b) $B = \begin{pmatrix} 188 \\ 88 \\ 38 \end{pmatrix}$ [1]

(c) Evaluate the matrix $C = AB$.

Answer (c) $C = \begin{pmatrix} 35 \cdot 188 + 45 \cdot 88 + 60 \cdot 38 \\ 70 \cdot 188 + 85 \cdot 88 + 150 \cdot 38 \\ 90 \cdot 188 + 130 \cdot 88 + 180 \cdot 38 \end{pmatrix}$ [1]

(d) State what the elements of $C$ represent.

Answer (d)
(e) For Mothers’ Day, the restaurant gives a discount of 20% for Deluxe Set, 15% for Superior Set and 10% for Economy Set.

Matrix \( N \) is a \( 3 \times 1 \) matrix that represents the price for each type of the set packages after the respective discount.

\[ M = \begin{pmatrix} 20 & 30 & 45 \end{pmatrix} \]

represents the order for these set packages on Mothers’ Day. Evaluate \( Q = MN \) and state what the element of \( Q \) represents.

\[ Answer \ (e) \ Q = \]

\[ Answer \ (e) \]

24 Density, \( d \) kg/m\(^3\), of a material is the mass, \( m \) kg, per unit volume, \( v \) m\(^3\), in which

\[ d = \frac{m}{v}. \]

An alloy is a mixture of metals.

If 0.0002 m\(^3\) of copper is mixed with 0.0008 m\(^3\) of tin, 7.62 kg of the alloy is formed.

If 0.0005 m\(^3\) of copper is mixed with 0.0005 m\(^3\) of tin, 8.1 kg of the alloy is formed.

Calculate the density of each of the two metals.

\[ Answer \ d_{\text{copper}} = \quad \text{kg/m}^3, \ d_{\text{tin}} = \quad \text{kg/m}^3 \] [5]
YUYING SECONDARY SCHOOL
PRELIMINARY EXAMINATION
Secondary 4 Express / 5 Normal (Academic) / 4N1

NAME
CLASS
REG. NO

MATHEMATICS

4048/02
28 Aug 2019
2 hours 30 minutes
Setters: Mr Lee Mun Tat
Ms Wee Li Hui

Paper 2

Candidates answer on the Question Paper.
Additional Materials: Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use

Total

100

This document consists of 21 printed pages.
Mathematical Formulae

**Compound interest**

Total amount = \( P \left( 1 + \frac{r}{100} \right)^n \)

**Mensuration**

Curved surface area of a cone = \( \pi rl \)

Surface area of a sphere = \( 4 \pi r^2 \)

Volume of a cone = \( \frac{1}{3} \pi r^2 h \)

Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

Area of triangle \( ABC = \frac{1}{2} ab \sin C \)

Arc length = \( r \theta \), where \( \theta \) is in radians

Sector area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

**Trigonometry**

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\( a^2 = b^2 + c^2 - 2bc \cos A \)

**Statistics**

Mean = \( \frac{\sum fx}{\sum f} \)

Standard deviation = \( \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2} \)
Answer all the questions.

1 (a) It is given that \( V = \frac{1}{3} ax^3 + by. \)

(i) Find \( V \) when \( x = 2, \ y = -1, \ a = 3 \) and \( b = 5. \) [1]

(ii) Express \( x \) in terms of \( V, \ a, \ b \) and \( y. \) [2]

(b) Simplify \( \frac{6ab + 15b}{4a^2 - 25}. \) [2]
(c) Solve the equation \( 2 = \frac{3}{x+1} + \frac{1}{x(x+1)} \).

(d) Solve the inequality \( \frac{x}{3} + \frac{5}{6} < \frac{5x}{3} \) and state the smallest possible prime number which satisfies the inequality \( \frac{x}{3} + \frac{5}{6} < \frac{5x}{3} \).
The table shows the number of lightning flashes, in billions, on Earth, from year 2016 to 2018.

<table>
<thead>
<tr>
<th>Year 2016</th>
<th>Year 2017</th>
<th>Year 2018</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.21</td>
<td>2.98</td>
<td>3.11</td>
</tr>
</tbody>
</table>

(a) The number of lightning flashes in 2016 can be expressed as $k$ millions. Find $k$.  

(b) Assuming there are 365 days in a year, find the average number of lightning flashes that can be seen in a day in 2018. Give your answer in standard form, correct to 3 significant figures. 

(c) Express the lightning flashes in 2016 as a percentage of the total lightning flashes from 2016 to 2018.
(d) The number of lightning flashes in 2015 is 3.65 billions. Calculate the percentage decrease in the number of lightning flashes from 2015 to 2018. [2]

(e) It is predicted that the number of lightning flashes will increase by 25% from 2018 to 2019. Calculate the predicted number of lightning flashes in 2019, giving your answer to the nearest billion. [2]
3 Answer the whole of this question on a sheet of graph paper.

The variables \( x \) and \( y \) are connected by the equation \( y = \frac{x}{10}(15 - x^2) \).

Some corresponding values of \( x \) and \( y \) are given in the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-1.8</td>
<td>-2.2</td>
<td>-1.4</td>
<td>0</td>
<td>1.4</td>
<td>2.2</td>
<td>2.2</td>
<td>( q )</td>
<td>0.4</td>
<td>-5</td>
</tr>
</tbody>
</table>

(a) Calculate the value of \( q \). [1]

(b) Using a scale of 2 cm to represent 1 unit on each axis, draw a horizontal \( x \)-axis for \(-3 \leq x \leq 5\) and a vertical \( y \)-axis for \(-5 \leq y \leq 3\).

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the solutions of \( \frac{x}{10}(15 - x^2) = -1 \) for \(-3 \leq x \leq 5\). [2]

(d) By drawing a tangent, find the gradient of the curve at \( x = 3.5 \). [2]

(e) (i) On the same axes, draw the line with gradient \( \frac{1}{2} \) that passes through the coordinates \((-3, -1)\). [1]

(ii) Write down the equation of the line. [1]

(iii) Write down the coordinates of the points where the line intersects the curve. [2]
In the diagram above, $AZ$ is parallel to $XY$ and $WX$ is parallel to $ZY$.

(a) State a triangle that is similar to triangle $DBC$. [1]

(b) Show that triangle $WAC$ is similar to triangle $YZC$. [2]
(c) It is given that $AB = 6\text{ cm}, BC = 8\text{ cm}, CZ = 10\text{ cm} \text{ and } ZY = 12\text{ cm}$.

(i) If the area of triangle $DBC$ is $48\text{ cm}^2$, find the area of triangle $DXY$. \[1\]

(ii) Find the ratio $WX : ZY$. \[2\]

(iii) Find $\frac{\text{area of trapezium } ACYX}{\text{area of triangle } CZY}$. \[2\]
5 (a) The position vector of point $A$ is $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$ and the position vector of point $B$ is $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

(i) Find the column vector $\overrightarrow{AB}$.

(ii) Find $|\overrightarrow{AB}|$.

(iii) Given that $\overrightarrow{AC} = 3\overrightarrow{AB}$, find the coordinates of $C$. 

[1] [2] [2]
(b) The point $P$ has coordinates $(4, -2)$ and $\overrightarrow{PQ} = \begin{pmatrix} -8 \\ 12 \end{pmatrix}$.

(i) Find the equation of the line $PQ$. [2]

(ii) The equation of another line is $3x + 2y = 11$. Show how you can tell that this line does not intersect the line $PQ$. [2]
The diagram below shows a cross-sectional view of a river.

The profile of the river is modelled by the arc $ABC$ as shown in the diagram below. The arc $ABC$ is part of a sector with centre $O$.

Given $AC = 10$ m and angle $ABC = 2.3\text{ rad}$,

(a) (i) show that $AO = 6.703$ m, correct to 4 significant figures. [4]
(ii) Find that the length of arc $ABC$. 

(b) The cross-section of the river with the arc $ABC$ superimposed on it is shown in the diagram below. The volume of rain flowing down a 100 m stretch of the river is 1700 m$^3$. Determine if the rain will overflow from the river and cause flooding to this stretch of the river.
The diagram shows a circle with centre $O$ and passing through the points $A, B, C, D$ and $E$. $AD$ is a diameter of the circle and $\angle COD = 68^\circ$. $OA$ is parallel to $BC$.

(a) Find, giving reasons for each answer,

(i) angle $OAC$, 

(ii) angle $ODC$, 

(iii) angle $ABC$,  

[1] [1] [2]
(iv) angle $CAB$. \[2\]

(b) $X$ is a point on $AD$ such that $AX = \frac{1}{3} AD$.

Given that the area of $\triangle AED = 30 \text{ cm}^2$, find the area of $\triangle AXE$. \[2\]

(c) Given that the shaded area of the segments is $36.375 \text{ cm}^2$,

find the radius of the circle. \[2\]
A water tank has a capacity of 1500 litres.

(a) The water is pumped into the tank at $x$ litres per minute.

Write down an expression, in terms of $x$, for the time taken, in minutes, for an empty water tank to be completely full. [$1$]

(b) If the rate of pumping water is increased by 3 litres per minute, write down an expression, in terms of $x$, for the time taken, in minutes, for an empty water tank to be completely full. [$1$]

(c) Given that the difference in the time taken is 28 minutes, form an equation in $x$ and show that it simplifies to $7x^2 + 21x - 1125 = 0$. [$3$]
(d) Solve the equation \(7x^2 + 21x - 1125 = 0\), giving your solutions correct to 3 decimal places. [3]

(e) Hence, find the time taken for the tank to be completely full when the water is pumped in at a rate of \(x\) litres per minute. Give your answer in hours and minutes, correct to the nearest minute. [2]
In the diagram, \( ABC \) is a horizontal triangular field in which \( AB = 68 \text{ m} \), \( AC = 45 \text{ m} \) and \( \angle BAC = 118^\circ \).

(a) Calculate

(i) the length of \( BC \),

(ii) the area of triangle \( ABC \),
(b) Find the shortest distance from $A$ to $BC$. [2]

(c) A vertical tower $XA$ stands at $A$. The angle of elevation of the top of the tower from $C$ is $16^\circ$. Calculate

(i) the height of the tower. [2]

(ii) the greatest angle of elevation of the top of the tower when viewed from any point along $BC$. [2]
10 The table below shows the weight of 40 students from class 4A.

<table>
<thead>
<tr>
<th>Weight (kg)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 &lt; x ≤ 50</td>
<td>13</td>
</tr>
<tr>
<td>50 &lt; x ≤ 60</td>
<td>16</td>
</tr>
<tr>
<td>60 &lt; x ≤ 70</td>
<td>8</td>
</tr>
<tr>
<td>70 &lt; x ≤ 80</td>
<td>3</td>
</tr>
</tbody>
</table>

(a) Calculate an estimate of
(i) the mean weight, [2]

(ii) the standard deviation. [2]

(b) The mean and standard deviation weight of class 4B are shown below.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>60.5 kg</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>12.5 kg</td>
</tr>
</tbody>
</table>

Make two comparisons between the weight of the two classes. [2]
(c) A student has seven 50 cents coins and five 20 cents coins in his wallet. He takes \( \frac{2}{3} \) coins out of the wallet, at random, one after the other. The coins taken out are not replaced.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

(ii) Find the probability that the total value of the two coins taken out is

(a) 40 cents, [1]

(b) 70 cents. [2]

~~~ End of Paper ~~~
3 Answer all the questions

1 Write the following numbers in order of size, starting with the smallest.

\[
0.85, \quad \frac{8}{13}, \quad (0.75)^2, \quad \sqrt{0.49}
\]

Answer \( (0.75)^2, \frac{8}{13}, \sqrt{0.49}, 0.85 \) [1]

2 Leo is travelling to Johor Bahru from Singapore. He wants to change 400 Singapore Dollars into Malaysian Ringgit.

In Singapore, the exchange rate is 1 Singapore Dollar = 3.02 Malaysian Ringgit.
In Johor Bahru, the exchange rate is 1 Malaysian Ringgit = 0.328 Singapore Dollars.

Where should he change his money to get more Malaysian Ringgit and by how much more?

Singapore:
- \( 400 \times 3.02 = 1208 \) Ringgit
- Difference = 1219.51 - 1208 = 11.51 Ringgit

Johor Bahru:
- \( 400 \div 0.328 = 1219.51 \) Ringgit

Answer Johor Bahru

3 Show that \((5n-1)^2 + 4\) is a multiple of 3.

Answer

\[
(5n-1)^2 + 4 = 25n^2 - 10n + 5
\]

\[ M1 \]

\[ 5(5n^2 - 2n + 1) \] [A1]

4 Given that \(5 \times 5^{n-2} = 125\), find the value of \(n\).

\[
\begin{align*}
5 \times 5^{n-2} &= 125 \\
5^{1+2n-2} &= 5^3 \\
1+2n-2 &= 3 \\
2n &= 4 \\
n &= 2 \quad \text{[M1]}
\end{align*}
\]

Answer \(n = 2\) [2]

5 The first five terms of a sequence are as follows:

\(8, \quad a, \quad b, \quad 69, \quad c, \quad \ldots\)

(a) Find the value of \(a, b,\) and \(c\).

\[
\begin{align*}
as &= \frac{78 - 69}{3} = 75 \\
b &= \frac{78 - 69}{3} = 72 \\
c &= \frac{78 - 69}{3} = 66
\end{align*}
\]

Answer (a) \( a = 75, \quad b = 72, \quad c = 66 \) [2]

(b) Write down an expression, in terms of \(n\), for the \(n\)th term of the sequence.

Answer (b) \(81 - 3n\) [1]

(e) Explain why 32 is not a term of this expression.

Answer (e) \(81 - 3n = 32, \quad n = 16.3\). \(n\) is not an integer. [1]

Hence 32 is not a term of this expression. [1]

6 The angles, in degrees, of a quadrilateral \(ABCD\) are represented by these expressions:

Angle \(A = (80 - 45)^\circ\), angle \(B = (10 + 3x)^\circ\), angle \(C = 2(5x + 90)^\circ\) and angle \(D = (3x - 10)^\circ\).

(a) Calculate the value of \(x\).

\[
\begin{align*}
80 - 4x + 10 + 3x + 5x + 90 + 15x - 10 &= 360 \\
170 + 12x &= 360 \\
x &= 10
\end{align*}
\]

Answer (a) \(x = 10\) [1]

(b) What is the name of the quadrilateral?

Answer (b) Trapezium [1]
7 Two boxes are geometrically similar. The base area of the smaller box is 64 cm² while the base area of the larger box is 81 cm².

Calculate the volume of the smaller box as a percentage of the volume of the bigger box.

\[ \frac{A_2}{A_1} = \left( \frac{1}{\sqrt{2}} \right)^3 = 64 \]
\[ \frac{V_2}{V_1} = \left( \frac{1}{\sqrt{2}} \right)^3 = \frac{512}{729} \]
\[ \frac{V_2}{V_1} \times 100\% = \frac{512}{729} \times 100\% = 70.2\% \]

Answer: 70.2% [41]

8 (a) Express 324 as a product of its prime factors.

Answer (a) \(2^2 \times 3^4\) [11]

(b) A number has exactly 9 factors. Two of the factors are 4 and 18.
List all the factors of the number.

\[4 = 2^2\]
\[18 = 2 \times 3^2\]

Factors: \(2^2, 2^1, 3^2, 3^1, 6, 9, 12, 18, 36\) [41]

Answer (b) [2]

9 The length of a rectangle exceeds its breadth by 2 cm.
(a) If the diagonal is 10 cm, find the width of the rectangle.
\[x^2 + (x+2)^2 = 10^2\] [41]
\[2x^2 + 4x - 96 = 0\]
\[x^2 + 2x - 48 = 0\]
\[(x + 8)(x - 6) = 0\]
x = -8 (rej) or 6 [41]

Answer (a) 6 cm [2]

(b) How many squares of side 3 cm can be cut out from this rectangle?

Answer (b) 4 [11]

10 Kenneth invests $2000 for 3 years at a fixed rate of compound interest.
At the end of the first year there is $2100 in his account.
(a) What is the rate of compound interest?
\[2100 = 2000 \left(1 + \frac{r}{100}\right)^1\] [M1]
\[r = \left(\frac{2100}{2000}\right) \times 100 = 5\]

Answer (a) 5% [2]

(b) How much does Kenneth have in his account at the end of 3 years?
\[2331.525\] [M1]

Answer (b) $2331.525 [2]

11 The diagram shows the box-and-whisker plot for the power, in kilowatts (kW), supplied to an electrical circuit.

(a) Find the median of the power supplied.

Answer (a) 30.0 kW [1]

(b) Find the interquartile range.

Answer (b) 2.5 kW [1]

12 A is the point \((-1, 4)\), B is the point \((2, -5)\) and O is the origin.
(a) Find the gradient of the line OA.
\[\frac{4 - 0}{-1 - 0} = -4\]

Answer (a) -4 [11]

(b) Find the equation of the line through B parallel to OA.
\[y = -4x + c\] [M1]
Sub \((2, -5)\) into eqn
\[c = 3\]

Answer (b) \(y = -4x + 3\) [41] [2]
13. The Venn Diagram represents the sets $A$ and $B$.

(a) List the elements of $(A \cup B)^c$.

Answer (b) \(\{1, 6, 10, 16\}\) \(\text{[1]}\)

(b) Find $n(A \cap B)$.

Answer (b) \(2\) \(\text{[1]}\)

(c) Set $C$ is defined as the first two multiples of 32. Insert set $C$ in the Venn diagram above.

Answer (c) \(\text{[1]}\)

14. Simplify

(a) \(\frac{27x^2y^2 + 18xy^3}{18xy^2} - \frac{3}{2}x^2y^{-2} \cdot 3^{-2}y^{-2} \cdot 2^{-3}y^{-3}\)

Answer (a) \(\frac{3}{2}x^2y^{-5}\) \(\text{[1]}\)

(b) \(\frac{2}{x-3} + \frac{1}{3-x} \cdot \frac{2}{x-3} + \frac{1}{3-x} \cdot \frac{2}{x-3}\)

Answer (b) \(\frac{x-1}{(x-3)^2}\) \(\text{[3]}\)

15. A bicycle accelerates from rest at a constant rate to a certain speed in 10 seconds. It maintains at this speed for the next 10 seconds. The total distance travelled by the bicycle in the 20 seconds is 240 metres.

(a) Calculate the speed of the bicycle in the tenth second and

\[
\frac{1}{2}(v)(10) + (x)(10) = 240 \quad [M1]
\]

\[x = 16 \quad [A1]\]

Answer (a) \(\text{[2]}\)

(b) Draw the speed-time graph of the car's journey for the first 20 seconds.

Answer (b) \(\text{[2]}\)

16. Factorise

(a) \(4x^2 - 16x - 6\), \(= 2(2x^2 - 5x - 3)\) \(\text{[M1]}\)

\(= 2(2x + 1)(x - 3)\) \(\text{[A1]}\)

Answer (a) \(\text{[2]}\)

(b) \(10x^2y - 5xy + 2x - 1\), \(= 5xy(2x - 1) + 1(2x - 1)\) \(\text{[M1]}\)

\(= (5xy + 1)(2x - 1)\) \(\text{[A1]}\)

Answer (b) \(\text{[2]}\)
A two digit number is formed using the digits 4, 8 and 9. Repetition of the digit is allowed.

(a) List the sample space.

\[ \text{Answer (a)} = \{44, 48, 49, 84, 88, 89, 94, 98, 99\} \] [1]

(b) Find the probability that a number selected at random is

(i) a prime number.

\[ \text{Answer (b)(i)} = \frac{1}{9} \] [1]

(ii) divisible by 5.

\[ \text{Answer (b)(ii)} = 0 \] [1]

---

19 The volume, \( V \), of a given mass of gas, is inversely proportional to the pressure, \( P \).

(a) Sketch a volume-pressure graph for the mass of gas.

\[ \text{Answer (a)} \]

When the volume is 3 m\(^3\), the pressure of the gas is 200 N/m\(^2\).

(b) Find the equation for \( V \) in terms of \( P \).

\[ V = \frac{k}{P} \] [M1]

\[ 3 = \frac{k}{200} \Rightarrow k = 600 \] [M1]

\[ V = \frac{600}{P} \] [A1]

\[ \text{Answer (b)(i)} \]

(c) Calculate the pressure when the volume is 5 m\(^3\).

\[ \text{Answer (c)} \]

---

\[ \text{Turn Over} \]

---

\[ \text{Sec 4E/SNA/4N1'O' EM_P1} \]

\[ \text{YYSS_PRELIM_2019} \]
A rubber cone of diameter 6 cm and height 10 cm is cut in half to make two rubber door stoppers. Find

(a) the volume of a rubber stopper,

\[ V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (3)^2 (10) = 47.1 \text{ cm}^3 \]  

Answer (a) \(47.1 \text{ cm}^3\) [4]

(b) the total surface area of a rubber stopper.

Slant height \(s = \sqrt{10^2 + 3^2} = \sqrt{89} \text{ cm} \)

\[ A = \frac{1}{2} \pi r (s + h) = \frac{1}{2} \pi \times 3 \times \sqrt{89} + \frac{1}{2} \pi \times 3 \times 10 = 93.3 \text{ cm}^2 \]  

Answer (b) \(93.3 \text{ cm}^2\) [8]

21. The following shows the Formula 1 track where the turns along the track are numbered 01 to 23.

(a) Estimate the actual length of the track from turn 04 to turn 07.

Answer (a) \(1200 \text{ to } 1400\) m [1]

(b) A racer finished the 3850 km race in 1 h 45.599 min. Calculate the average speed in km/h.

\[ 1 \text{ h } 45.599 \text{ min } = 1.759983 \text{ h} \]

Av. Speed = \(\frac{3850}{1.759983} = 216\) km/h

Answer (b) \(216\) km/h [4]

(c) Suggest and explain a possible speed when the racer
(i) went past the grandstand,

Answer (i) When the racer passed the grandstand, the speed should be higher than the average speed as the course is relatively straight.
Speed should be around 200 km/h. ...................... [1]

(ii) was at turn 05.

Answer (ii) The racer should slow down at turn 05 and the speed is lower than the average speed. Speed should be around 130 km/h. .......... [1]
23. A particular restaurant offers 3 different dinner set menu Deluxe, Superior and Economy Set Package. The following table shows the orders for the three set packages on three days of a particular week.

<table>
<thead>
<tr>
<th></th>
<th>Deluxe</th>
<th>Superior</th>
<th>Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friday</td>
<td>35</td>
<td>45</td>
<td>60</td>
</tr>
<tr>
<td>Saturday</td>
<td>70</td>
<td>85</td>
<td>150</td>
</tr>
<tr>
<td>Sunday</td>
<td>80</td>
<td>130</td>
<td>180</td>
</tr>
</tbody>
</table>

(a) Represent the number of orders for each type of set package on the three days by a 3 x 3 matrix.

\[
\begin{pmatrix}
35 & 45 & 60 \\
70 & 85 & 150 \\
90 & 130 & 180
\end{pmatrix}
\]

Answer (a) A = \[\begin{pmatrix}
35 & 45 & 60 \\
70 & 85 & 150 \\
90 & 130 & 180
\end{pmatrix}\] [1]

(b) Given that each Deluxe, Superior and Economy Set Package costs $188, $88 and $38 respectively, write down a 3 x 3 matrix B showing the price for each type of the set packages.

\[
B = \begin{pmatrix}
188 \\
88 \\
38
\end{pmatrix}
\]

Answer (b) B = \[
\begin{pmatrix}
188 \\
88 \\
38
\end{pmatrix}
\] [1]

(c) Evaluate the matrix \( C = AB \).

\[
C = AB = \begin{pmatrix}
35 & 45 & 60 \\
70 & 85 & 150 \\
90 & 130 & 180
\end{pmatrix} \begin{pmatrix}
188 \\
88 \\
38
\end{pmatrix}
\]

\[
C = \begin{pmatrix}
35 \\
70 \\
90
\end{pmatrix} \begin{pmatrix}
188 \\
88 \\
38
\end{pmatrix}
\]

\[
C = \begin{pmatrix}
6600 \\
5540 \\
35200
\end{pmatrix}
\]

Answer (c) \( C = \begin{pmatrix}
6600 \\
5540 \\
35200
\end{pmatrix}\) [1]

(d) State what the elements of \( C \) represents.

Answer (d) ...It represents the restaurant takings from the three set packages on each day. [1]
(e) For Mothers’ Day, the restaurant gives a discount of 20% for Deluxe Set, 15% for Superior Set, and 10% for Economy Set.

Matrix $N$ is a $3 \times 1$ matrix that represents the price for each type of the set packages after the respective discounts.

$N = \begin{bmatrix} 150.4 \\ 74.8 \\ 34.2 \end{bmatrix}$

$M = \begin{bmatrix} 20 & 30 & 45 \end{bmatrix}$ represents the order for these set packages on Mothers’ Day.

Evaluate $Q = MN$ and state what the element of $Q$ represents.

$Q = \begin{bmatrix} 150.4 \\ 74.8 \\ 34.2 \end{bmatrix} \begin{bmatrix} 20 \\ 30 \\ 45 \end{bmatrix} = \begin{bmatrix} 6791 \end{bmatrix}$

Answer (e) $Q = \begin{bmatrix} 6791 \end{bmatrix}$

Answer (e) …The restaurant total takings from the three set packages on Mothers’ Day.

Density, $d$, $\text{kg/m}^3$, of a material is the mass, $m$, $\text{kg}$, per unit volume, $v$, $\text{m}^3$, in which

$d = \frac{m}{v}$

An alloy is a mixture of metals.

If 0.002 $\text{m}^3$ of copper is mixed with 0.0008 $\text{m}^3$ of tin, 7.62 kg of the alloy is formed.

If 0.0005 $\text{m}^3$ of copper is mixed with 0.0005 $\text{m}^3$ of tin, 8.1 kg of the alloy is formed.

Calculate the density of each of the two metals.

$7.62 = d_c \times 0.002 + d_t \times 0.0008 \quad (1)$

$8.1 = d_c \times 0.0005 + d_t \times 0.0005 \quad (2)$

Sub $d_t = 7300$ into (2)

$d_c = 8900$

Answer $d_{copper} = \quad \text{kg/m}^3$, $d_{tin} = \quad \text{kg/m}^3$
Marking Scheme
Sec 4E5N/4N1-Maths P2-Prelim-2019

1 (a) (i)
\[ \nu = \frac{1}{3} (2^2)(0) + (-1)(5) \]
= -1

1 (a) (ii)
\[ \nu = \frac{1}{3} ax^2 + by \]
\[ V - by = \frac{1}{3} ax^2 \]
\[ 3(V - by) = ax^2 \]
\[ \frac{a}{x} = \frac{3(V - by)}{a} \]
\[ x = \sqrt[3]{\frac{3(V - by)}{a}} \]

1 (b)
\[ \frac{6ab + 15b}{4a^2 - 25} = \frac{3b(2a + 5)}{(2a + 5)(2a - 5)} \]
= \frac{3b}{2a - 5}

1 (c)
\[ x + 1 \quad x(x+1) \]
\[ 2 = \frac{3x + 1}{x(x+1)} \]
\[ 2 = \frac{3x + 1}{x^2 + x} \]
\[ 2x^2 + 2x = 3x + 1 \]
\[ 2x^2 - x - 1 = 0 \]
\[ (2x + 1)(x - 1) = 0 \]
\[ x = -\frac{1}{2} \text{ or } x = 1 \]

1 (d)
\[ x, \frac{5}{3}, \frac{5x}{3} \]
\[ 2 + 5, \frac{10x}{6} \]
\[ 2x + 5 < 10x \]
\[ 5 < 8x \]
\[ 8x > 5 \]
\[ x > \frac{5}{8} \]

2 (a) 3210

2 (b) \[ \frac{3.11 \times 10^7}{365} = 8.52 \times 10^5 \text{ (a.s.f.)} \]

2 (c) \[ 2.14 \times 10^8 \]

2 (d) \[ 3.65 \times 10^5 \]

2 (e) \[ \frac{125}{100} = 4 \text{ billion (nearest billion)} \]

2 (f) Refer to graph paper.
### Triangle DXY

4 (a) \[ \angle ACW = \angle ZCY \text{ (vert. opp. angles)} \]
\[ \angle WAC = \angle ZYC \text{ (WAIZY, alt. angles)} \]

By AA similarity, triangle WAC is similar to triangle ZYC.

4 (c) (i)

<table>
<thead>
<tr>
<th>Area of triangle DBC</th>
<th>Area of triangle DXY</th>
</tr>
</thead>
</table>
| \( \frac{8}{24} \)   | \( \frac{1}{9} \) \( \text{Area of triangle DXY} = 48 \times 9 \)

\[ = 432 \text{ cm}^2 \]

4 (c) (ii)

By similarity,
\[ \frac{WA}{YZ} = \frac{AC}{ZC} \]
\[ \frac{WA}{14} = \frac{10}{12} \]
\[ = \frac{12}{14} = \frac{6}{7} \]
\[ WX \cdot ZY = 12 \times 5 \]

4 (c) (iii)

<table>
<thead>
<tr>
<th>Area of ( ACXY )</th>
<th>Area of triangle ( CZY )</th>
</tr>
</thead>
</table>
| \( \frac{6 + 8}{24} \text{ (in)} \) | \( \frac{19}{5} \text{ (in)} \) \[ \text{(shown)} \]

5 (a) (i)

\[ AB = AO + OB \]
\[ = \left( \frac{3}{5}, -8 \right) \]

(ii)

\[ \sqrt{3^2 + 8^2} = 8.54 \text{ units} \ (3 \text{ s.f.)} \]

(iii) \[ OC = \overrightarrow{OA} + \overrightarrow{AC} \]
\[ = \left( \frac{-1}{5}, 3 \right) + 3 \left( \frac{-8}{5} \right) \]
\[ = \left( \frac{8}{5}, -\frac{19}{5} \right) \]

(b) (i) \( \text{Gradient of } PQ = \frac{-2}{-8} = \frac{3}{4} \)

(ii) \[ 3x + 2y = 11 \]
\[ = 3 \left( \frac{3}{4} \right) + c \]
\[ c = 4 \]
\[ y = \frac{3}{4} x + 4 \]

Both lines have the same gradient so they are parallel and will not intersect.

6 (a) (i) \[ \text{Reflex } \angle AOC = 4.6 \text{ rad (angle at centre = } 2 \times \text{ angle at circum)} \]
\[ \angle AOC = 2 \pi - 4.6 \approx 1.684 \]
\[ AO = \frac{5}{\sin 0.842} = 6.703 \text{ m} \]

(ii) \[ \text{Arc } ABC = 6.703 \times 1.684 \approx 11.3 \text{ m} \]

(b) \[ \text{Cross sectional area} = \frac{1}{2} \times (6.703^2 \times 1.684 - \frac{1}{2} (6.703)^2 \sin 1.684) \]
\[ = 15.5099 \]
\[ = 15.5 \text{ m}^2 \]
(d) \[ x = \frac{-21 \pm \sqrt{21^2 - 4(-1125)(7)}}{2(7)} \]
\[ = 11.266 \text{ or } -14.266 \]

(c) \[ t = 1200 \]
\[ = 31.266 \text{ mins} \]
\[ = 2 \text{ h 13 mins} \]

9(o)(ii) \[ BC^2 = 65^2 + 45^2 - 2(65)(45) \cos 118 \]
\[ = 97.582 \text{ cm} \]
\[ = 97.6 \text{ cm} \]

(i) \[ \frac{-65(45) \sin 118}{2} \]
\[ = 138.0 \text{ cm} \]

(b) \[ h = \frac{97.582}{2} \text{ h} = 1350.0 \text{ cm} \]
\[ = 12.77 \text{ m} \]

(e)(i) \[ h = 45 \tan 16 \]
\[ = 12.9 \text{ m} \]

8(a) \[ x = 1500 \]

(b) \[ x = 1500 \]
\[ x + 3 \]

(c) \[ \frac{1500}{x} = \frac{1500}{x+3} = 28 \]
\[ 4500 = 28x(x+3) \]
\[ 28x^2 + 84x - 4500 = 0 \]
\[ 7x^2 + 21x - 1125 = 0 \]
(shown)

(b) Class 4B is heavier than 4A as the mean is higher.
Class 4A weights more consistent as the S.D. is smaller.

(c)(i) \[ 45 \times 13 = 595 \times 16 \]
\[ = 55.25 \text{ kg} \]

(i) \[ \sqrt{\frac{125400}{49}} \]
\[ = 9.06 \text{ kg} \]

3 s.f.)
### Question (ii(a))

<table>
<thead>
<tr>
<th>Event</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{5}{12} \times \frac{4}{11} )</td>
<td>( \frac{5}{33} )</td>
</tr>
</tbody>
</table>

### Question (b)

\[
\frac{5}{12} \times \frac{7}{11} + \frac{5}{11} \times \frac{7}{12} = \frac{35}{66}
\]