2017

SEC 4 EXP E-MATH

1.	PEI HWA SEC SCH	SA1
2.	ANGLO CHINESE SEC SCH (IN	SA2
3.	BENDEMEER SEC SCH	SA2
4.	CHIJ KATONG CONVENT SEC SCH	SA2
5.	GEYLANG METHODIST SEC SCH	SA2
6.	HOLY INNOCENTS' HIGH SCH	SA2
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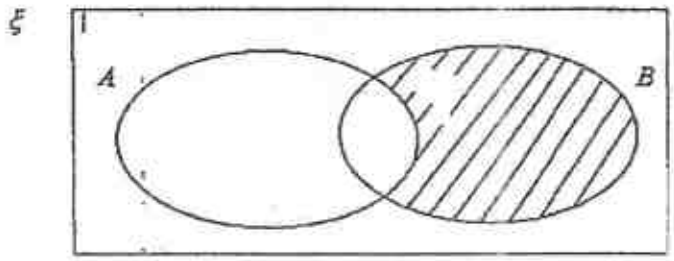
8.	MANJUSRI SEC SCH	SA2
9.	SERANGOON GARDEN SEC SCH (P2)	SA2
10.	TANJONG KATONG SEC SCH	SA2
11.	XINMIN SEC SCH	SA2
12.	YUSOF ISHAK SEC SCH	SA2

ALL WITH ANSWERS

2017 4E EM Pei Hwa Mid Year

Answer all the questions.

1 Express in set notation, the set shaded in the following Venn diagram.



Answer ..... [1]

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2 (a) Simplify $(3+\frac{1}{2}x)(1+x)$.

Answer ..... [1]

(b) Factorise completely $32a^2-18b^2$.

Answer .... [2]

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3 Factorise completely $12bx-6ay+8by-9ax$.

Answer ..... [2]

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4 Write as a single fraction in its simplest form $\frac{5}{2+x}+\frac{6x}{x^2-4}$.

Answer ..... [4]

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5 Show that for all p, where p is a positive integer, $(7p-3)^2-4p(p-3)+6$ is divisible by 15.

Answer

[2]

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6 (a) Express $5-6x-x^2$ in the form $p-(x+q)^2$.

Answer ..... [2]

(b) Hence, sketch the graph of $y=5-6x-x^2$ indicating the y-intercept and the coordinates of the turning point on the graph.

Answer

[2]

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7 A bicycle rental shop uses the formula $C = 5.5 + 3.5h$ to calculate charges for rental of bicycles, where C is the cost of rental and h is the number of hours of rental.

(a) State the basic charge to be paid regardless of the number of hours of rental.

Answer $ ..... [1]

(b) Mathew and Ethan both rented a bicycle each for different number of hours.
The difference in the cost of rental between the two of them is $14.
Find the difference in the number of hours of rental between the two boys.

Answer ..... hours [2]

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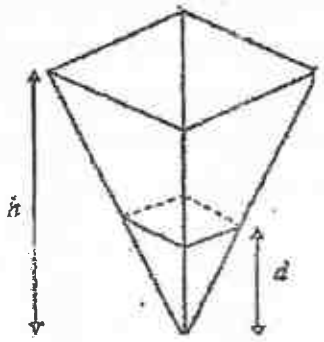
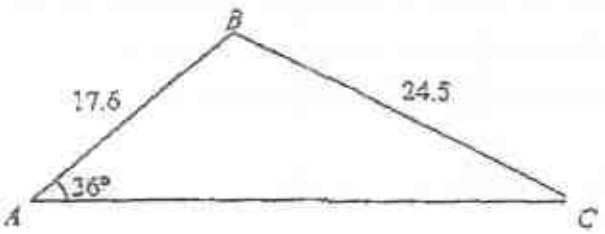
8 The diagram shows an inverted pyramid with a capacity of 800 cm³.



The depth of the liquid in the inverted pyramid, d, is one-third the height, h, of the pyramid.
Calculate the volume of the liquid.

Answer ..... cm³ [2]

9



ABC is a triangle, where $AB = 17.6$ cm, $BC = 24.5$ cm and angle $BAC = 36°$

Find angle ABC.

Answer angle ABC = ... [3]

10 Jane plans to travel back to Singapore from the United States.

In Singapore, the exchange rate is SGD \$1 = USD \$0.71.

In the United States, the exchange rate is USD \$100 = SGD \$153.

Jane wants to change USD \$1426 into Singapore dollars.

Which country should Jane change her money in order to get a better deal?

You must show your calculations.

Answer .....[3]

11 Hector was arranging 315 one-centimetre cubes into a cuboid.

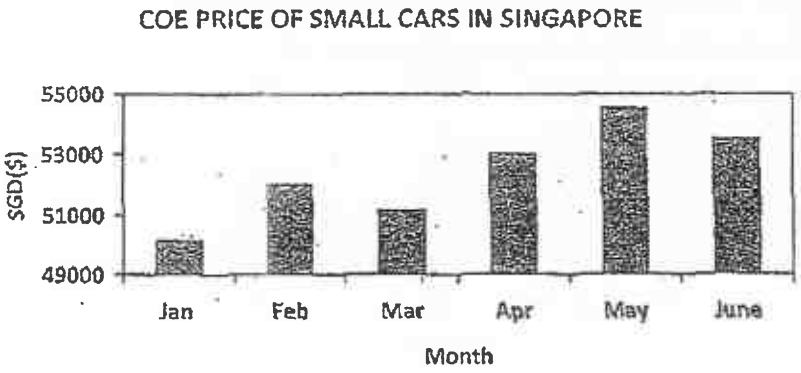
The perimeter of the base of the cuboid is 28 cm.

Each side of the cuboid has a length greater than 3 cm.

Find the height of the cuboid.

Answer .. cm [2]

12 The bar graph shows the COE price of small cars in Singapore over a period of 6 months.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer .....

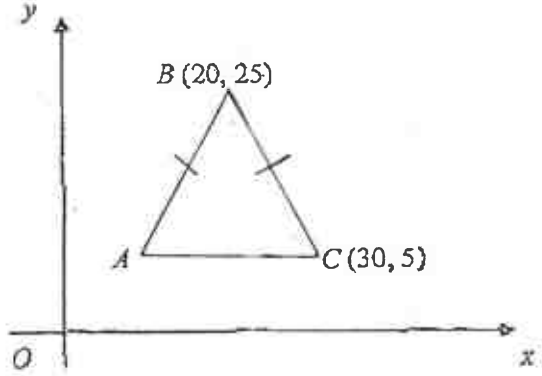
.....

..... [2]

13 The diagram shows an isosceles triangle.

AC is parallel to the x-axis.

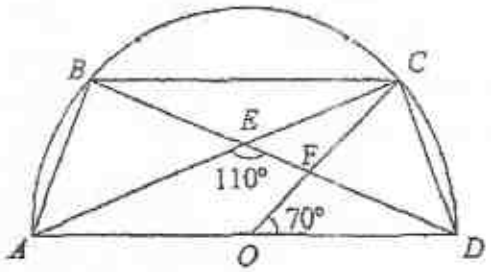
Point B has coordinates (20, 25) and C has coordinates (30, 5)



Find the coordinates of A.

Answer (....., ...) [1]

14



$ABCD$ is a semicircle with centre O.

BED and AEC are straight lines.

Angle $COD = 70°$ and angle $AED = 110°$.

(a) Stating your reasons clearly, calculate

(i) angle ACD,

Answer angle ACD = ...[1]

(ii) angle ADC,

Answer angle ADC = ...[1]

(iii) angle ABC,

Answer angle ABC = ...[1]

(iv) angle BFC.

Answer angle BFC = ...[3]

(b) Explain why BC is parallel to AD.

Answer .....

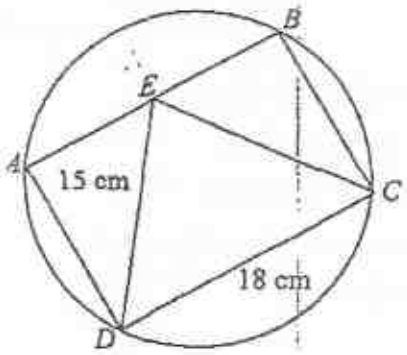
..... [1]

15 The diagram shows a circle $ABCD$.

E is the midpoint of the chord AB.

$ABCD$ is a rectangle.

$DE = 15$ cm and $DC = 18$ cm.



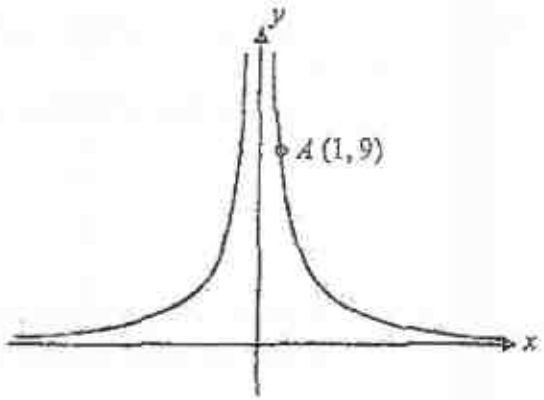
(a) Calculate the area of triangle ADE.

Answer ..... cm² [2]

(b) Calculate the circumference of the circle.

Answer ..... cm [2]

16 The sketch shows the graph of $y = 3^k \times x^{-n}$.
The graph passes through the point A (1, 9).



(a) (i) State a possible value of n.

Answer $n =$ ..... [1]

(ii) Find the value of k.

$k =$ ..... [1]

(b) Given that the coordinates of B is (−2, 2.25), find the length of the line segment AB.

Answer ..... [2]

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17 (a) Express 3780 as the product of its prime factors.

Answer ..... [1]

(b) Using your answer to **part (a)**, explain why 3780 is **not** multiple of 49.

Answer .....

..... [1]

(c) c is a composite number and p is a prime number.
Find the values of p and c such that $3780 \times \frac{c}{p}$ is a perfect square and c has the least value.

Answer $p =$ .....

$c =$ ..... [2]

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18 A map of Singapore is such that 9 cm^2 on the map represents the actual area of 36 km^2 on the land.

(a) Express the scale of the map in the form $1 : n$.

Answer 1 : ..... [2]

(b) The length of Bukit Timah Expressway on the map is 5 cm.
Calculate the actual distance, in kilometres, of the Bukit Timah Expressway.

Answer ..... km [1]

19 The table shows the prices of one litre of petrol and the discounts offered by leading petrol companies

Company	Petrol price per litre	Discount
A	\$1.723	18%
B	\$1.689	15%
C	\$1.702	12% discount plus \$3 off for every \$30 sale after discount

(a) Ronn wants to fill up his car with 55 litres of petrol at Company *C*.
Calculate the total amount Ronn paid for the petrol.

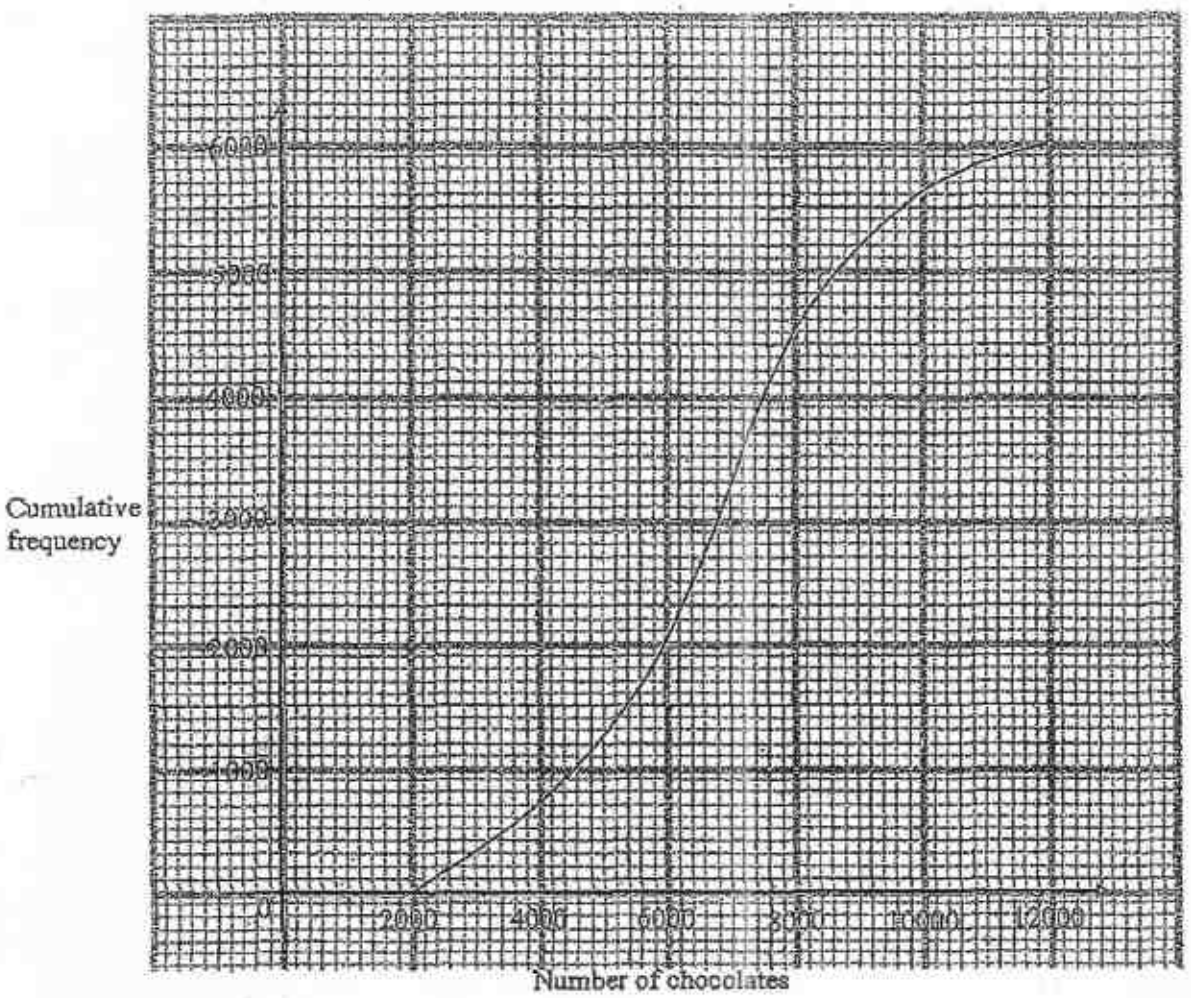
Answer \$ ..... [2]

(b) Comparing Company *A* and *B*, show clearly which company offers a better deal.

Answer ..... [2]

20 6000 customers participated in a contest where they have to guess the number of chocolates in a big glass container.

The cumulative frequency curve below shows the distribution of their guesses.



The actual number of chocolates is 6000.

(a) Find the median.

Answer ..... chocolates [1]

(b) Find the interquartile range.

Answer ..... chocolates [1]

(c) Find the probability that a customer, chosen at random, gave an estimate within 10% of the actual number of chocolates.

Answer ..... [3]

21 Gate B and Gate C are 400 m apart in a park. Gate A is such that angle $ACB = 105°$ and $AB = 550$ m.

(a) Using a scale of 1 cm to 50 m and the line BC is drawn for you, complete the scale drawing of triangle ABC. [1]

(b) A pavilion, inside the park, is located equidistant from the three gates.
By construction, find and label the position of the pavilion P. [2]

(c) Measure and calculate the actual distance between Gate A and the pavilion P.

Answer ..... m [1]

22 The position vectors of A and B are $\begin{pmatrix}3\\4\end{pmatrix}$ and $\begin{pmatrix}-3\\4\end{pmatrix}$ respectively.

(a) Find the length of $\overrightarrow{OB}$.

Answer ..... [1]

(b) C is the point $(0, p)$ where $p > 0$.
$\overrightarrow{OC} = 4\overrightarrow{OA} + 4\overrightarrow{OB}$.
Find the value of p.

Answer $p =$ ..... [2]

(c) What type of quadrilateral is $OACB$?

Answer ..... [1]

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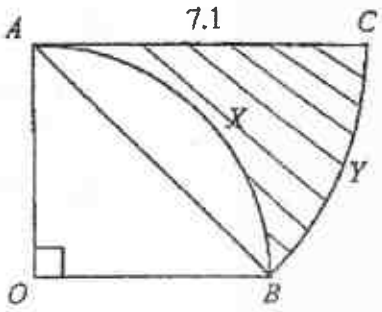
23 In the diagram, angle $AOB = 90°$, AC is parallel to OB and $AC = 7.1$ cm.
AXB is an arc of a circle with centre O and CYB is an arc of a circle with centre A.
Find the area of the shaded region.



Answer ..... cm^2 [5]

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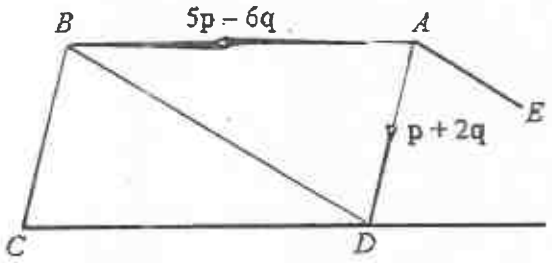
24 In the diagram, $ABCD$ is a parallelogram, $\overrightarrow{AD} = \mathbf{p} + 2\mathbf{q}$ and $\overrightarrow{AB} = 5\mathbf{p} - 6\mathbf{q}$.



(a) Express, as simply as possible, in terms of p and q,

(i) $\overrightarrow{CB}$,

Answer ..... [1]

(ii) $\overrightarrow{DB}$.

Answer ..... [2]

(b) E is a point such that $\overrightarrow{EA} = \mathbf{p} - 2\mathbf{q}$.

(i) Explain why $\overrightarrow{DB}$ is parallel to $\overrightarrow{EA}$.

Answer .....

..... [1]

(ii) Find the ratio of the area of triangle ADE to the area of triangle DBA.

Answer ..... : ..... [2]

End of Paper

MATHEMATICAL FORMULAE

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions.

1 (a) (i) Factorise $-3x^2-2x+5$. [1]

(ii) Simplify $\dfrac{6x+12}{3x^2-15x-42}$. [2]

(b) It is given that $d=\sqrt{\dfrac{5e-f}{ef}}$.

(i) Find d when $e=4$ and $f=2$. [1]

(ii) Express e in terms of d and f. [2]

(c) Solve the equation $\dfrac{3x+2}{5}-\dfrac{1}{2}=\dfrac{x}{2}$. [2]

(d) Solve these simultaneous equations.

$$7x+4y=-37$$
$$x-5y=17$$ [3]

2 In one small packet of gummies, there are both gummy bears and gummy snakes in two colours; red and green. In a large packet, there are 10 small packets.

The information can be represented by the matrix $A = \begin{matrix} \text{Green} & \text{Red} \\ \begin{pmatrix} 5 & 5 \\ 4 & 6 \end{pmatrix} & \begin{matrix} \text{Bear} \\ \text{Snake} \end{matrix} \end{matrix}$

(a) Evaluate the matrix $B = 10A$. [1]

(b) It costs \$0.10 and \$0.12 to produce 1 green and red gummy respectively.

Represent the cost of each colour of gummy in a 2×1 column matrix C in dollars. [1]

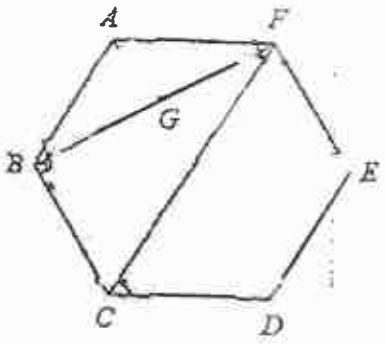
(c) Evaluate the matrix $D = BC$. [1]

(d) State what the elements of D represent. [1]

(e) Another gummy-making company, Company *Y*, packs 6 green gummy bears, 4 red gummy bears, 7 green gummy snakes and 3 red gummy snakes in one small packet. The costs to produce one green gummy and one red gummy remain the same. One large packet is also made up of 10 small packets.

Calculate the total cost for Company *Y* to produce one large packet. [3]

3 (a) The diagram shows a regular hexagon.



(i) Calculate the interior angle of a regular hexagon. [2]

(ii) It is given that $2AG = BC$. Find $\frac{\text{area of triangle } ABF}{\text{area of triangle } BFC}$. [2]

(b) (i) Simplify $\frac{(mn^2)^3}{p^5} \div \frac{n^5}{p^4}$. [2]

(ii) Given that $\frac{2^{q+5}}{4^{2q}} = \frac{1}{16}$, find the value of q. [3]

4 The first five terms in a sequence of numbers are given below.

0, 3, 8, 15, 24...

(a) Find the next two terms. [2]

(b) Find an expression, in terms of n, for the nth term, T_n, of the above sequence. [1]

(c) T_n and T_{n+1} are consecutive terms in the sequence.

Find and simplify an expression, in terms of n, for $T_{n+1} - T_n$. [3]

(d) Explain why two consecutive terms of the sequence cannot have a difference of 8. [2]

5 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation

$$y = x^3 - 4x^2 + \frac{5}{2}$$

Some corresponding values of x and y are given in the table below.

x	−1.5	−1	−0.5	0	0.5	1	1.5	2
y	−9.875	−2.5	1.375	2.5	p	−0.5	−3.125	−5.5

(a) Find the value of p. [1]

(b) Using a scale of 4 cm to represent 1 unit, draw a horizontal x-axis for $-1.5 \le x \le 2$.

Using a scale of 1 cm to represent 1 unit, draw a vertical y-axis for $-12 \le y \le 4$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the coordinates of the maximum point of $y = x^3 - 4x^2 + \frac{5}{2}$, in the range of $-1.5 \le x \le 2$. [1]

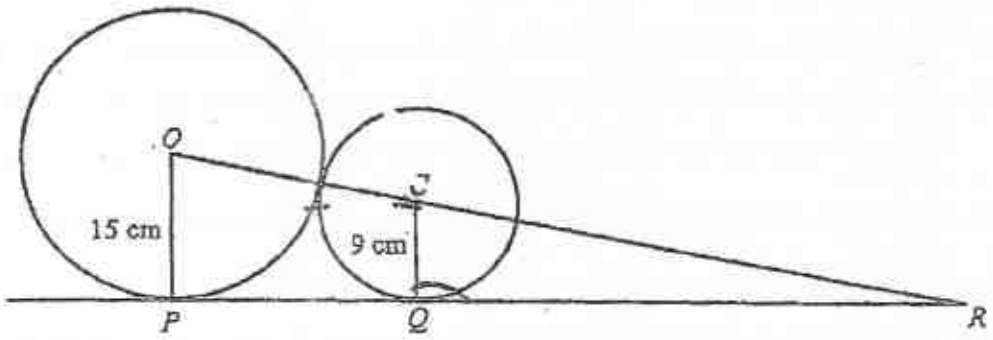
(d) Use your graph to find the solutions to the equation $x^3 - 4x^2 + 6 = 0$, in the range $-1.5 \le x \le 2$. [3]

(e) By drawing a tangent, find the gradient of the curve at $(-1, -2.5)$. [2]

(f) (i) On the same axes, draw the line $y = -3x - 4$ for $-1.5 \le x \le 2$. [1]

(ii) Write down the coordinates of the point where this line intersects the curve. [1]

6 The diagram shows a circle, centre O, with radius 15 cm touching another circle, centre C, with radius 9 cm.
OCR and PQR are straight lines and PQR is a tangent to both the circles at points P and Q.



(a) State the value of angle CQR and explain your answer. [2]

(b) Show that triangles OPR and CQR are similar.
Give a reason for each statement you make. [2]

(c) Find the value of $\dfrac{\text{area of triangle } CQR}{\text{area of trapezium } OCQP}$ [2]

(d) Find the difference in the areas of the two circles.
Leave your answer in terms of π. [2]

7 A company manufactures and sells posters for decoration and display.

(a) The posters manufactured by the company are sold in local shops and department stores. In a particular week, the number of posters available for sale in local shops and department stores are in the ratio 3 : 7.
Given that 160 more posters are available for sale in department stores, find the total number of posters available for sale in that week. [2]

(b) A shop owner bought x posters for \$60 from the company.

(i) Write down an expression, in terms of x, for the cost of each poster in dollars. [1]

The shop owner decides to sell the posters at a profit of \$1 each.

(ii) Write down an expression, in terms of x, for the selling price of each poster in dollars. [1]

The shop owner managed to sell 10 posters at the selling price in (ii).
He decided to sell the rest of the posters at \$5 each.

(iii) Write down an expression, in terms of x, for the total amount of money in dollars, that he collected from the sale of all posters. [1]

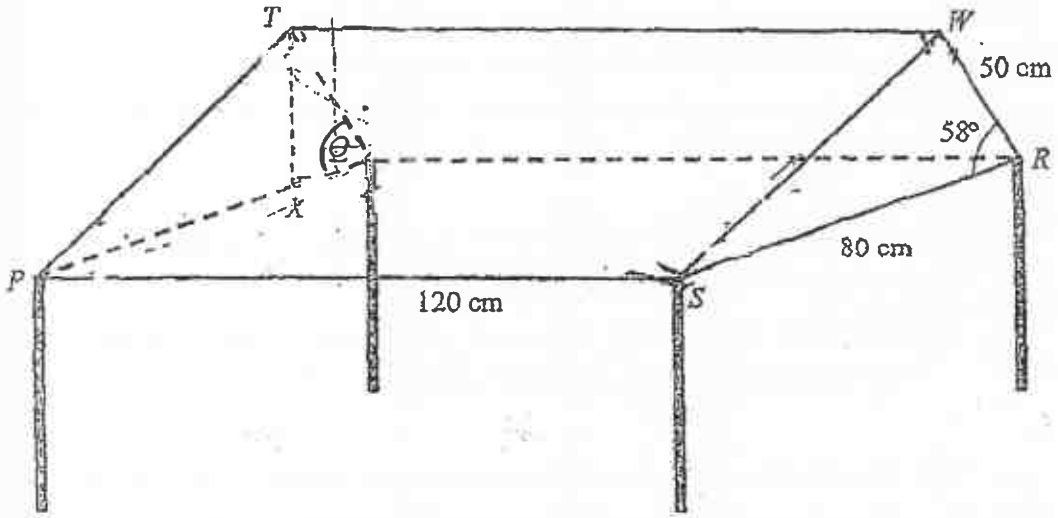
(iv) Given that the shop owner collected a total of \$130 from the sale of all posters, write down an equation in x to represent this information and show that it reduces to

$$x^2 - 34x + 120 = 0$$ [3]

(v) Solve the equation $x^2 - 34x + 120 = 0$. [3]

(vi) Find the cost price of each poster. [1]

8 The diagram shows a table used by an interior designer.
It is made up of a prism and 4 table legs for support.
The rectangle $PQRS$ lies on a horizontal plane.
T is vertically above X.
$PS = 120$ cm, $RS = 80$ cm and $WR = 50$ cm.
Angle $WRS = 58°$.



Calculate

(a) WS, [3]

(b) the volume of the prism, [3]

(c) TX, [2]

(d) XS, [4]

(e) the angle of elevation of T from S. [2]

9 (a) The amount of money, in dollars, spent by a group of 20 students (Group *A*) in the month of May is shown in the stem-and-leaf diagram below.

5	2	5			
6	2	3	7		
7	1	1	5	8	9
8	0	4	5	6	
9	2	3	8	9	
10	5	8			

Key 5|6 means $56

(i) Find the mean amount of money spent by the 20 students. [1]

(ii) Find the standard deviation of the amount of money spent by the 20 students. [1]

(iii) The mean and standard deviation of the amount of money spent by another group of 20 students (Group *B*) in May were $70 and $12 respectively.
Use the information to comment on two differences between the two distributions. [2]

(b) John plays a game at a carnival. In this game, he has to pick 2 coloured balls from two bags, *A* and *B*. He is only allowed to pick one ball from each bag. He has to pick one ball from Bag *A*, followed by another ball from Bag *B*.
Bag *A* contains 2 red balls, 3 blue balls and 6 yellow balls.
Bag *B* contains 4 red balls, 1 blue ball and 4 yellow balls.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

(ii) John will win a large prize if he picks 2 balls that are blue, a small prize if he picks only one ball that is blue and goes home empty-handed otherwise.
Find, as a fraction in the simplest form, the probability that

(a) John will win a large prize, [1]

(b) John will win a small prize, [1]

(c) John will not win anything. [1]

10 A group of students are tasked to design, print and distribute brochures containing tips to save water to students in school, as part of the school's effort to raise awareness of the importance of saving water in school.

The students have been allocated a budget of $1200 to complete this task.

The students are required to print and distribute a copy of the brochure to each student and teacher in the school.

Each brochure is printed on both sides of 2 sheets of A4 size paper.

Students will be given brochures printed in black and white and teachers will be given brochures printed in colour. They will have to purchase the sheets of A4 size paper and toner cartridges from *ABC* bookstore, which will be delivered to school.

In addition, the students are also tasked to design and print 50 copies of A3 size coloured posters containing tips to save water, to be put up in all classrooms and various areas in the school. They have sourced for an external supplier, *XYZ* supplier, to print the posters. The posters will be delivered to school as well.

The information that the students require is found in Annex A, on the opposite page.

The students estimates that they have to distribute the brochures to 1360 students and 90 teachers.

(a) How many sheets of A4 size paper will the students require to purchase to print the brochures for all students and teachers? [1]

(b) How many toner cartridges will the students require to purchase to print the brochures for all students and teachers? [3]

(c) Given that one of the students in the group is a member of *ABC* bookstore and that the students aim to reduce the cost as far as possible, determine if the amount of budget allocated is sufficient to cover all costs.
Justify your answer with relevant mathematical working. [6]

Annex A

1) Cost of purchasing stationaries from *ABC* Bookshop:

Item	Description	Unit Cost (excluding GST)
A4 Paper	White paper	
	1 pack of 100 sheets	$2.00
	1 pack of 500 sheets	$5.00
	5 packs of 500 sheets each	$22.50
	10 packs of 500 sheets each	$42.00
Toner Cartridges	Black printing (each cartridge is able to print 1200 pages)	$136.00
	Colour printing (each cartridge is able to print 900 pages)	$140.00
The above prices are subjected to 7% Goods and Services Tax (GST). Member discount: 10% off total bill, after 7% GST Delivery cost: $30 per trip (not subjected to 7% GST) (Free delivery for minimum purchase of $200 in total bill, inclusive of 7% GST and after member discount.)		

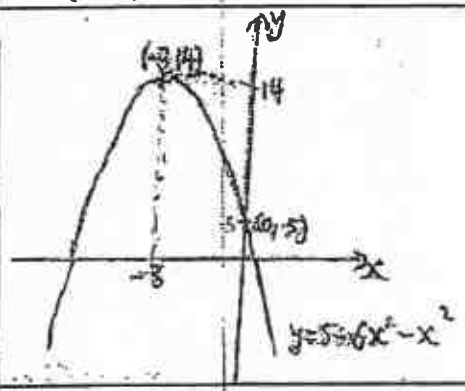
2) Cost of printing A3 size coloured posters

Supplier: *XYZ* Printing

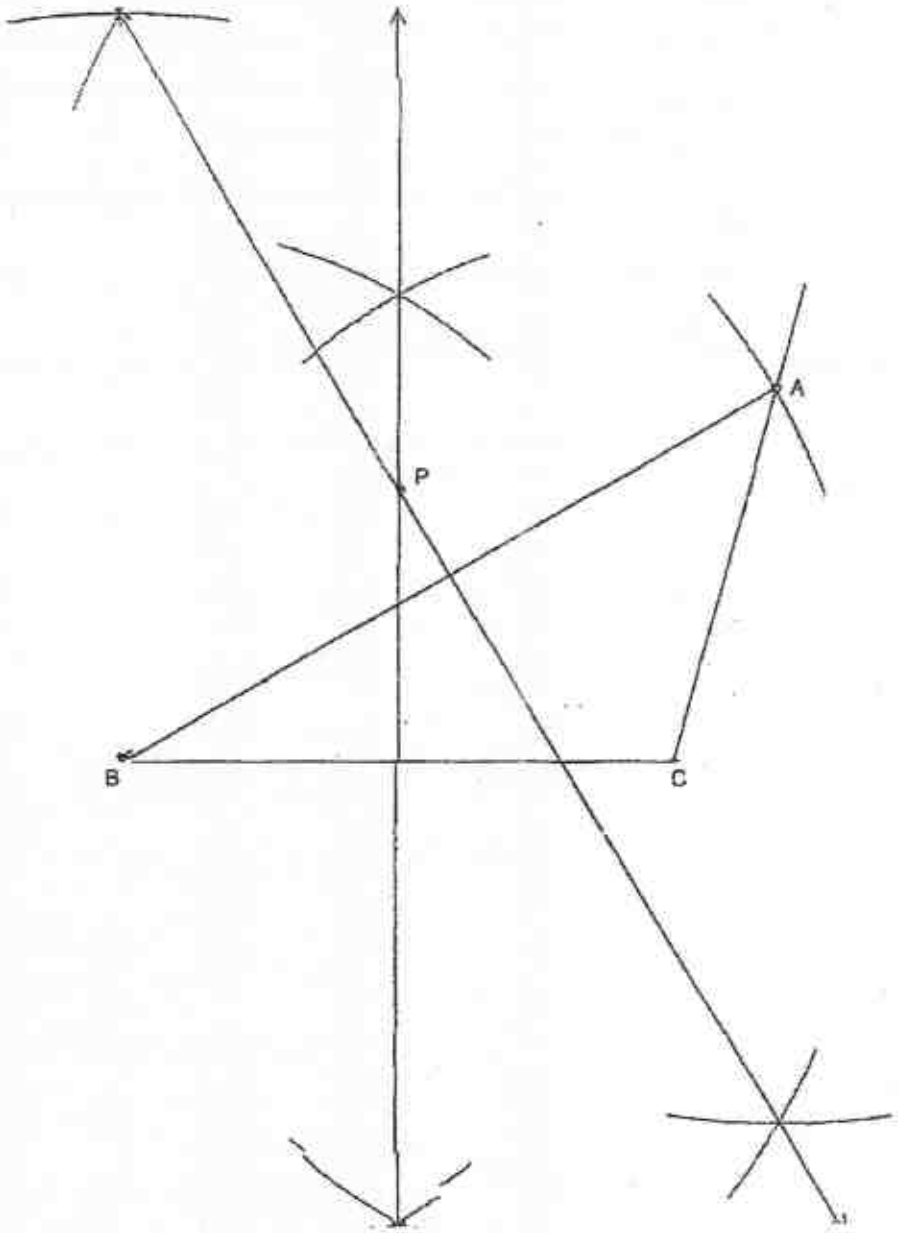
Item	Description	Unit Cost (excluding GST)
Black and White Posters	10 sheets	$25.00
	50 sheets	$120.00
Coloured Posters	10 sheets	$35.00
	50 sheets	$170.00
The above prices are subjected to 7% Goods and Services Tax (GST). Delivery cost: $20 per trip (not subjected to 7% GST) (Free delivery for minimum purchase of $200 in total bill, inclusive of 7% GST.)		

End of Paper

Pei Hwa Secondary School
Mid Year Examination 2017
Sec 4E & 5N Mathematics Paper 1
Answer Key

1(a)	$A' \cap B$
2(a)	$-2x^2+x+3$
2(b)	$2(4a+3b)(4a-3b)$
3	$(4b-3a)(3x+2y)$
4	$\dfrac{11x-10}{(x+2)(x-2)}$
5	$(7p-3)^2-4p(p-3)+6$ $=49p^2-42p+9-4p^2+12p+6$ $=45p^2-30p+15$ $=15(3p^2-2p+1)$ $\therefore$ for all p, $(7p-3)^2-4p(p-3)+6$ is divisible by 15. (Shown)
6(a)	$14-(x+3)^2$
6(b)	
7(a)	\$5.50
7(b)	4 hours
8	$29.6cm^3$ (3s.f.)
9	119.0° (1d.p.)
10	Amount of money Jane will get in Singapore $=\dfrac{1426}{0.71}$ $=SGD\$2008.45$ Amount of money Jane will get in the United States $=\dfrac{153}{100}\times 1426$ $=SGD\$2181.78$

	Jane will change her money in the United States as she will get back more Singapore dollars.
11	7 cm
12	In the graph, the data doesn't start at \$0, but somewhere around \$49000. This makes the differences appear much larger proportionally.
13	(10, 5)
14(a)(i)	90°
14(a)(ii)	55°
14(a)(iii)	125°
14(a)(iv)	75°
14(b)	Angle $BCE = 35°$ (Angles in the same segment) Since angle BCE = angle CAO (by property of alternate angles), BC is parallel to AD.
15(a)	$54cm^2$
15(b)	$68.0cm$
16(a)(i)	$n = -2$
16(a)(ii)	$9 = 3^k \times \left(\frac{1}{3}\right)^{-2}$ $k = 2$
16(b)	7.39 units
17(a)	$2^2 \times 3^3 \times 5 \times 7$
17(b)	Index of 7 is not at least 2
17(c)	$c = 15$ $p = 7$
18(a)	1 : 200000
18(b)	10 km
19(a)	\$76.38
19(b)	Company B offers a better deal.
20(a)	6800
20(b)	2600
20(c)	$\frac{1}{5}$

21a,b	
21(c)	Distance $= 5.7\ (\pm 0.1) \times 50$ $= 285\ (\pm 5)$ m
22(a)	5 units
22(b)	$p = 32$

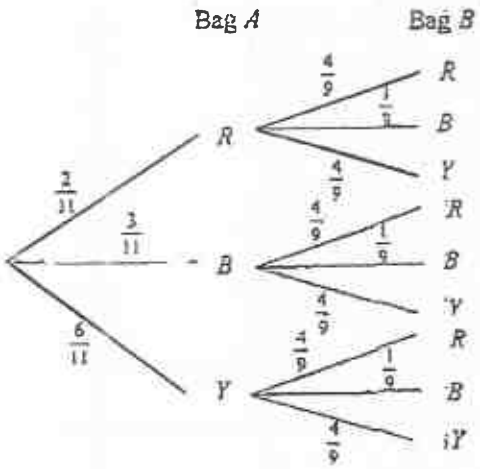
22(c)	Kite
23	12.6 cm^2
24(a)(i)	$-p-2q$
24(a)(ii)	$4p-8q$
24(b)(i)	$\overrightarrow{DB}$ $=4(p-2q)$ $=4\ \overrightarrow{EA}$
24(b)(ii)	$\frac{1}{4}$

PHSS 4E EM MYE Paper 2 2017 Answer Key

No.	Answer
1(a)(i)	$-3x^2-2x+5=(3x+5)(1-x)$
1(a)(ii)	$\frac{2}{x-7}$
1(b)(i)	$d=1.5$ or $d=1\frac{1}{2}$
1(b)(ii)	$e=\frac{f}{5-d^2f}$
1(c)	$x=1$
1(d)	$x=-3,\ y=-4$
2(a)	$\mathbf{B}=\begin{pmatrix}50 & 50\\ 40 & 60\end{pmatrix}$
2(b)	$\mathbf{C}=\begin{pmatrix}0.10\\ 0.12\end{pmatrix}$
2(c)	$\mathbf{D}=\begin{pmatrix}11\\ 11.2\end{pmatrix}$
2(d)	The elements of D represent the cost to produce all the gummy bears and gummy snakes in a large packet respectively.
2(e)	Total cost = \$10.80 + \$10.60 = \$21.40
3(a)(i)	120°
3(a)(ii)	$\frac{1}{2}$
3(b)(i)	$\frac{m^3n}{p}$
3(b)(ii)	$q=3$
4(a)	$T_6=35$ $T_7=48$
4(b)	$T_n=n^2-1$ or $(n+1)(n-1)$
4(c)	$T_{n+1}-T_n=n^2+2n-(n^2-1)$ $=2n+1$

No.	Answer
4(d)	$2n+1=8$ $n=3.5$ Assuming that the difference between two terms is 8, the first consecutive term is 3.5, which does not exist. Therefore, two consecutive terms cannot have a difference of 8. OR The difference $(2n+1)$ is an odd number. Therefore, two consecutive terms cannot have a difference of 8, which is an even number.
5(a)	$p=1.625$
5(b)	If all 8 points plotted correctly, otherwise, at least 6 points plotted correctly. Smooth curve
5(c)	Maximum point = (0, 2.5)
5(d)	From the graph, $x=-1.10\pm0.10$ and $x=1.55\pm0.10$
5(e)	Gradient = 8.67 ± 3
5(f)(i)	Correctly drawn line
5(f)(ii)	$(-0.85, -1.4)$
6(a)	$\angle CQR=90°$ tangent perpendicular to radius
6(b)	$\angle OPR=90°$ (tangent perpendicular to radius) $\angle OPR=\angle CQR$ $\angle PRO=\angle QRC$ (common angle) $\angle POR=\angle QCR$ (corresponding angles, $OP//CQ$) Hence, triangle OPR is similar to triangle CQR. (AA Similarity)
6(c)	$\frac{9}{16}$
6(d)	144π cm^2
7(a)	400
7(b)(i)	$\$\left(\frac{60}{x}\right)$
7(b)(ii)	$\$\left(\frac{60}{x}+1\right)$
7(b)(iii)	$\frac{600}{x}+5x-40$

7(b)(iv)	$\frac{600}{x} + 10 + 5x - 50 = 130$ $\frac{600}{x} + 5x - 170 = 0$ $600 + 5x^2 - 170x = 0$ $5x^2 - 170x + 600 = 0$ $x^2 - 34x + 120 = 0$ (shown)
7(b)(v)	$x = 30$ or $x = 4$
7(b)(vi)	\$2
8(a)	68.3 cm
8(b)	204000 cm^3
8(c)	$TX = 42.4$ cm
8(d)	$XS = 131$ cm
8(e)	$\theta = 17.9°$
9(a)(i)	\$80.15
9(a)(ii)	\$15.60
9(a)(iii)	1. The mean amount of money spent by students in Group *A* is higher than that of Group *B*. On average, students in Group *A* spent more money than students in Group *B*. 2. The standard deviation of the amount of money spent by students in Group *B* is lower than that of Group *A*. There is a smaller spread in the amount of money spent by students in Group *B*./ The amount of money spent by students in Group *B* is more consistent.

9(b)(i)	
9(b)(ii)(a)	$\frac{1}{33}$
9(b)(ii)(b)	$\frac{32}{99}$
9(b)(ii)(c)	$\frac{64}{99}$
10(a)	2900
10(b)	6
10(c)	Cost of purchase from *ABC* Bookstore Total cost with delivery cost, after member discount = \$816.1425 Cost of purchase from *XYZ* Printing Total cost with delivery = \$20 + \$181.90 = \$201.90 Grand total cost = \$816.1425 + \$201.90 = \$1018.04 The amount of budget of \$1200 is sufficient to cover all costs.

Answer **all** the questions

For Examiner's Use

For Examiner's Use

1 Write the following in order of size, smallest first.

$\sqrt{0.81}$ 0.902 $\frac{399}{441}$ $0.86^{\frac{2}{3}}$

Answer ....., ..., ..., ... [2]

smallest *largest*

2 The capacity of a SD card is 256 gigabytes. How many pictures of size 2.5 megabytes each can be stored in this SD card? Give your answer in standard form. (1 gigabyte = 10^9 bytes, 1 megabyte = 10^6 bytes)

Answer ..... [2]

3 Factorise completely $12ac - 14bd + 28bc - 6ad$.

Answer [2]

For Examiner's Use

4 A sum of money was divided equally between Jim, John and Jane.
If Jim gives Jane $20, the ratio would then become 2: 3 :4
What was the total sum of money?

Answer $ ..... [2]

5 Simplify $\frac{7x}{(x-5)^2}+\frac{1}{5-x}$.

Answer ..... [2]

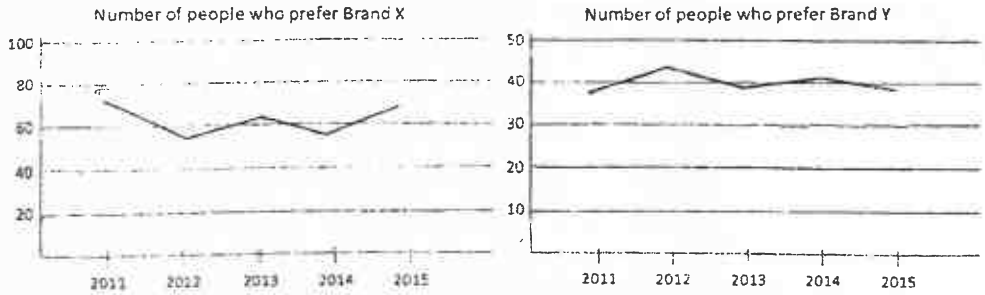
6 Solve the inequalities $-8 \le 2-3x < 8$

Answer ..... [2]

7 12 cooks will take 6 hours to prepare a meal for 180 people.
If 4 of the cooks left the team and the number of people dropped to 150, how many hours would the remaining cooks need to prepare the meal?

Answer ..... hours [2]

8 The diagrams show the result of sales of two competing brands over a few years.



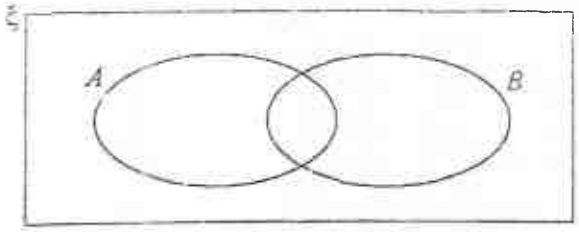
State one aspect of the graph which may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer

..... [2]

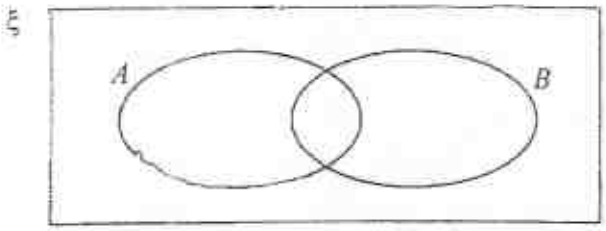
For Examiner's Use

9 (a) On the Venn diagram, shade the region which represents $A \cap B'$.



[1]

(b) $\xi = \{\text{integers } x : 1 \leq x \leq 12\}$
$A = \{\text{prime numbers}\}$
$B = \{\text{multiples of 3}\}$
On the Venn diagram, list down the elements in the appropriate subsets.



[2]

10 Simplify $\left(\frac{x^6}{y^4}\right)^{-\frac{1}{2}} \div \left(\frac{x^{-5}}{y^{-3}}\right)$, giving your answer in positive index.

Answer ..... [3]

For Examiner's Use

For Examiner's Use

11 (a) One day, the rate of exchange between Singapore dollars (S$) and US dollars (US$) was US$1 = S$1.39.

Anthony wanted to bring along US$5000 for a trip to the US. Calculate how much Singapore dollars he would need to exchange.

Answer S$ ..... [1]

(b) There was change of plans at the last minute and Anthony exchanged the US$5000 back into Singapore dollars, at a different exchange rate. If he received S$6850, what was the exchange rate?

Answer US$ ..... = S$ ..... [2]

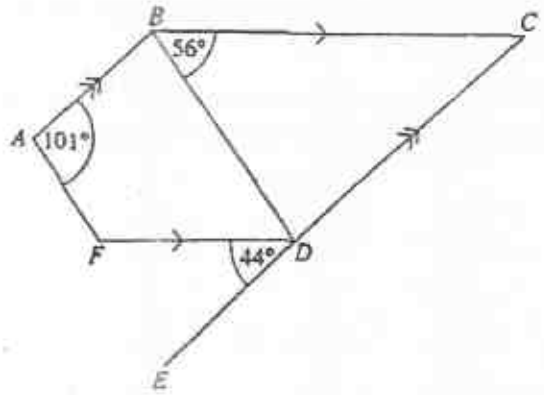
12 A supplier sells watches at $210 each. Jimmy buys the watches from the supplier at a discount of 20%. Jimmy intends to then sell the watches at a profit of 20%.

As a marketing strategy, Jimmy plans to offer a 10% discount on the marked price without affecting his intended 20% profit. Calculate the marked price that Jimmy should sell each watch at.

Answer ..... [3]

For Examiner's Use

13



In the diagram, AB is parallel to EDC and BC is parallel to FD.
Angle $CBD = 56°$, angle $FDE = 44°$ and angle $BAF = 101°$.

State, showing your reasoning, whether AF is or is not parallel to BD.

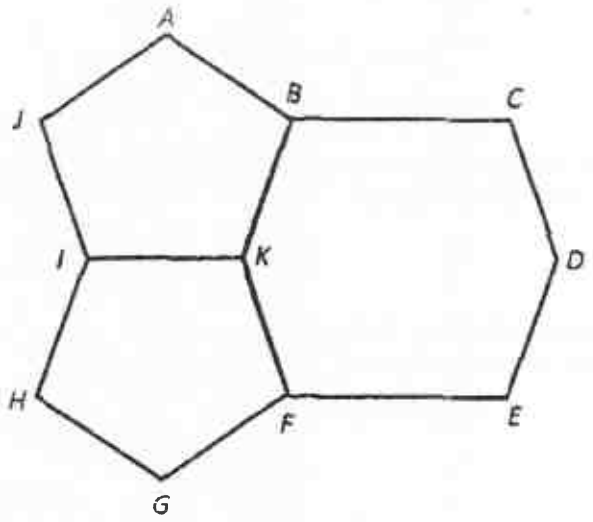
Answer

[3]

For Examiner's Use

For Examiner's Use

14 The diagram shows two pentagons and one hexagon joined together.



(a) Calculate the sum of the interior angles of the hexagon.

Answer ..... ° [1]

(b) Show, by way of calculation, that at least one of the polygons is irregular.

Answer

[2]

For Examiner's Use

For Examiner's Use

15 Written as a product of its prime factors

$$2450 = 2 \times 5^2 \times 7^2$$
$$84 = 2^2 \times 3 \times 7$$

(a) Write down the highest common factor of 2450 and 84, giving your answer as the product of its prime factors.

Answer ..... [1]

(b) The highest common factor of 2450 and $21a$ is 70.
Find the smallest possible value of a, where a is an integer.

Answer $a =$ ..... [1]

(c) The lights on three lighthouses flash at regular intervals. The first light flashes every 84 seconds, the second every 90 seconds and the third every 2450 seconds. The three lights flash together at 0800.
At what time do they next flash together?

Answer ..... [2]

For Examiner's Use

For Examiner's Use

16 William draws at random 2 cards from a stack of 5 cards labelled 5 to 9 without replacement. The sum of the numbers on the two cards is obtained.

(a) Complete the possibility diagram in the answer space below.

+	5	6	7	8	9
5		11			
6	11				
7					
8					
9					

[1]

(b) Calculate the probability that the sum obtained is a multiple of 6.

Answer ..... [1]

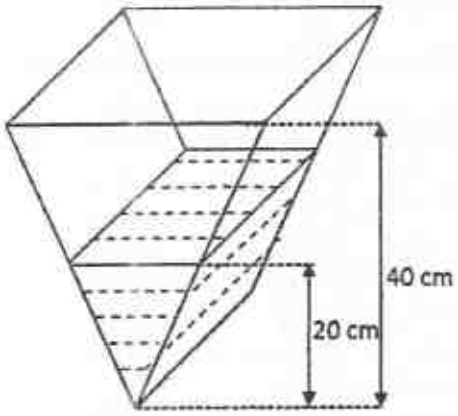
(c) A third card is chosen at random from the stack without replacement. Find the probability that the sum of the numbers on the three cards is 24.

Answer ..... [2]

For Examiner's Use

For Examiner's Use

17



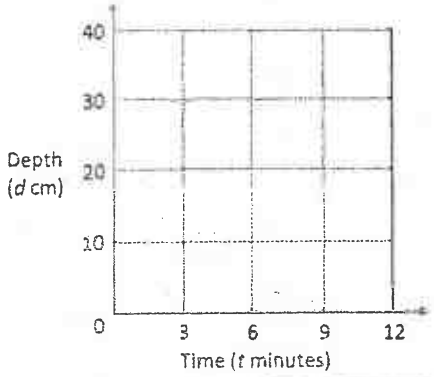
The diagram shows a container in the shape of a prism with a triangular cross-section.
The container has a height of 40 cm.
Water is poured into the empty container at a constant rate.
It takes 12 minutes to fill the container.
After t minutes the depth of the water is d cm.

(a) Find the value of t when $d = 20$.

Answer ..... minutes [2]

(b) On the axes in the answer space, sketch the graph showing how the depth varies during the 12 minutes.

Answer



[2]

For Examiner's Use

For Examiner's Use

18 The table below shows the number of cars and motorcycles passing through an Electronic Road Pricing (ERP) gantry on certain days of the week from 7.30 am to 7.55 am.

	Cars	Motorcycles
Wednesday	320	120
Thursday	380	100
Friday	410	130
Charges per vehicle	\$2	\$0.50

(a) Represent the number of vehicles passing through the gantry in a 3×2 matrix **V**.

Answer ..... [1]

(b) $\mathbf{C} = \begin{pmatrix} 2 \\ 0.5 \end{pmatrix}$. Evaluate $\mathbf{P} = \mathbf{VC}$.

Answer ..... [1]

(c) State what the elements of **P** represent.

Answer

..... [1]

(d) Write down a matrix **D** such that $\mathbf{T} = \mathbf{DP}$ gives you the total charges collected for all vehicles on these three days.

Answer [1]

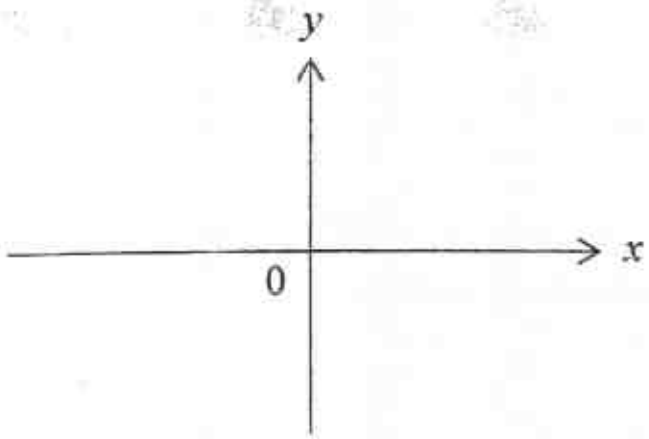
For Examiner's Use

19 (a) Express $x^2 - \frac{1}{4}x$ in the form $(x-b)^2 + c$.

Answer ..... [1]

(b) Sketch the graph of $y = \frac{1}{4}x - x^2$.

Answer



[2]

(c) Find the coordinates of the maximum point of $y = \frac{1}{4}x - x^2$.

Answer (..... , ...) [1]

For Examiner's Use

For Examiner's Use

20

Two bottles of Nescafe Gold Blend Instant Coffee are geometrically similar. The smaller bottle contains 50 g of coffee granules.

(a) The larger bottle is approximately 60% taller than the smaller bottle. Find, in grams, the amount of coffee granules in the larger bottle.

Answer ..... g [2]

(b) The smaller bottle sells for \$5.10 while the larger bottle sells for \$13.25. Which bottle gives the better value for money? You must show your calculations.

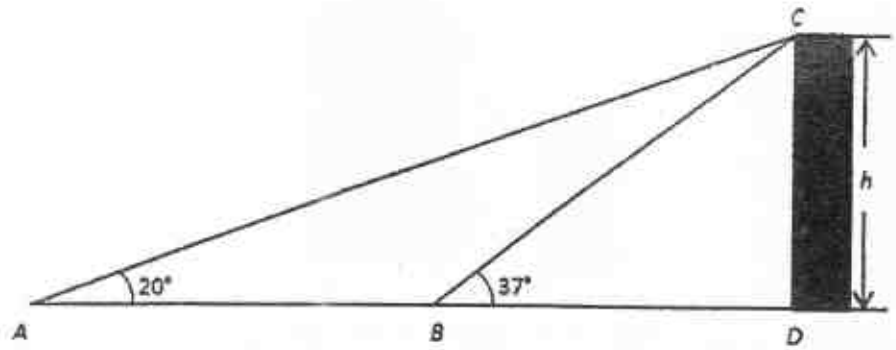
Answer

[2]

For Examiner's Use

For Examiner's Use

21



Joseph walks from point A to point B, which are 400 m apart. A vertical tower of h metres is at point D.
At point A, the angle of elevation to the top of the tower is 20°.
At point B, the angle of elevation to the top of the tower is 37°.

(a) Find AC.

Answer $AC =$ .....m [3]

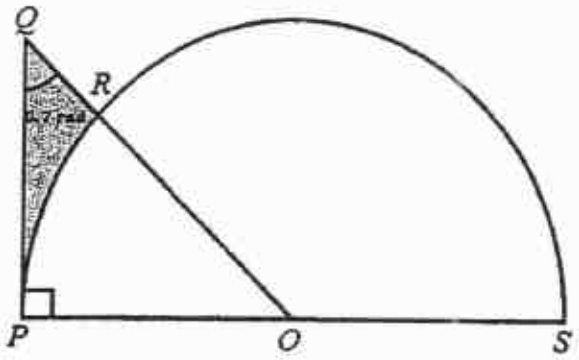
(b) Find h, the height of the tower.

Answer $h =$ ..... m [2]

For Examiner's Use

For Examiner's Use

22 The diagram shows a semi-circle with centre O and radius 8 cm. OP is perpendicular to PQ and angle $PQR = 0.7$ radians.



(a) Find the area of the shaded region.

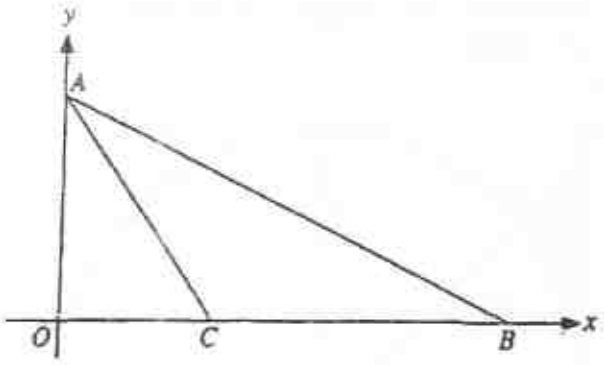
Answer .....cm^2 [4]

(b) Convert 0.7 radians into degrees.

Answer .....° [1]

For Examiner's Use

23



A is the point (0,6) and the gradient of line AB is $-\frac{1}{4}$. C is the point (6,0).

(a) Find the equation of line AB.

Answer [1]

(b) Find the coordinates of B.

Answer (....., ...) [2]

(c) Find the length of AB.

Answer ..... units [1]

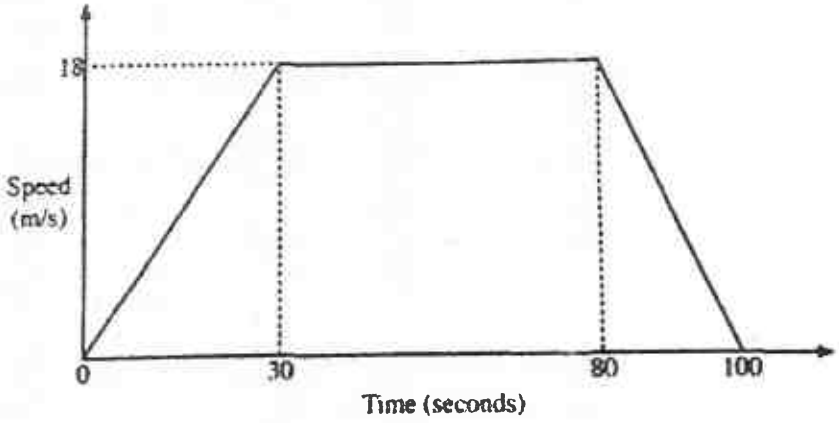
(d) Point D lies on the x-axis and is such that $DC = CB$. Write down the equation of the line that passes through D and is parallel to the y-axis.

Answer ..... [2]

For Examiner's Use

For Examiner's Use

24 The diagram below shows the speed-time graph of an object.



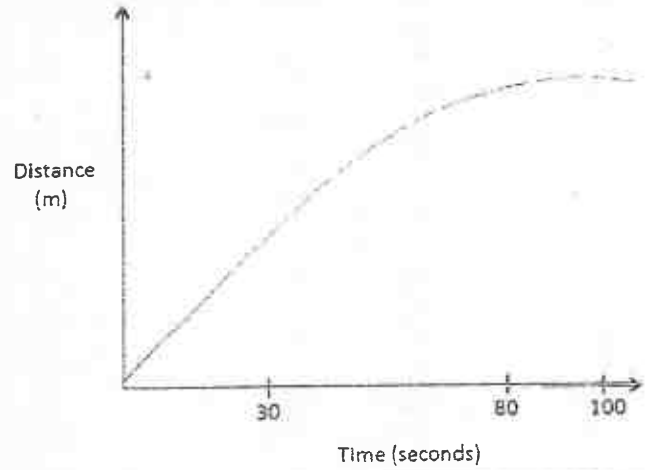
(a) Calculate the speed of the object at 18 seconds. Give your answer in km/h.

Answer .... km/h [2]

(b) Calculate the total distance travelled on the journey.

Answer .....m [2]

(c) Draw the distance-time graph of the object on the grid given below.
You must label the values on the distance-axis clearly.



[2]

End of Paper

Anglo-Chinese School (Barker Road)

PRELIMINARY EXAMINATION 2017

SECONDARY FOUR EXPRESS / FIVE NORMAL ACADEMIC

MATHEMATICS 4048
PAPER TWO

2 HOURS 30 MINS

Additional Materials: Answer Paper (7 sheets)
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your class and candidate number on the cover sheet.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
[redacted]sential working will result in loss of marks.
[redacted]uld be used where appropriate.
[redacted] accuracy is not specified in the question, and if the answer is not exact, give the answer to [redacted] figures. Give answers in degrees to one decimal place.
For π, use either the calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

This paper consists of 11 printed pages inclusive of this page. [Turn over

1 The first three terms in a sequence of numbers, $T_1, T_2, T_3, \ldots$ are given below.

$$T_1 = 1 \times 2 + 10 = 12$$
$$T_2 = 2 \times 3 + 6 = 12$$
$$T_3 = 3 \times 4 + 2 = 14$$

(a) Find T_4. [1]

(b) Show that $T_n = n^2 - 3n + 14$. [2]

(c) Evaluate T_{50}. [1]

(d) Explain why every term in the sequence is even. [2]

---

2 **(a)** It is given that $v^2 = u^2 - 2gh$.

(i) Evaluate v when $u = 30$, $g = 9.8$ and $h = 24$. [2]

(ii) Express u in terms of g, h and v. [2]

(b) Factorise $(x+1)^2 - (y-1)^2$. [2]

(c) Simplify $\dfrac{x^2 - 1}{8 - 3x - 5x^2}$. [3]

(d) Solve the simultaneous equations. [3]

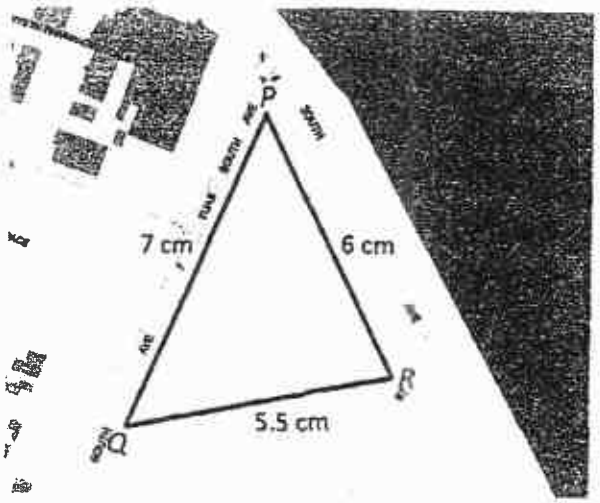
$$1\frac{1}{2}x - 3y = 12$$
$$4y = 3x - 19$$

---

3 (a) The scale of a map is 1 : 7500.

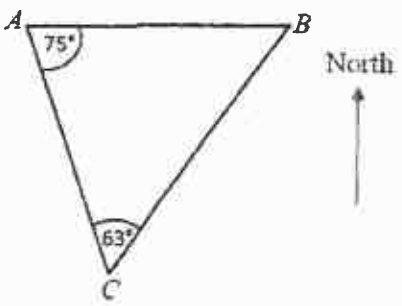
(i) The length of a road on the map is 20.5 cm.
Find the actual length, in kilometres, of the road. [1]

(ii)



On the map, an area formed by a triangle PQR with sides 5.5 cm, 6 cm and 7 cm, is slated for commercial development.
Calculate, in square metres, the actual area. [5]

(b)



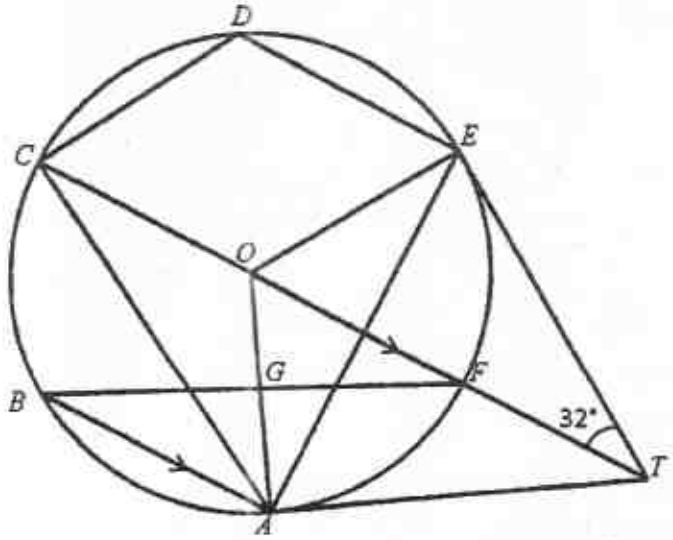
In the diagram, AB is the shoreline. B is due east of A. A boat is at C.
$C = 75°$, angle $ACB = 63°$ and $AB = 35$ m.

(i) ...d the bearing of B from C. [2]

(ii) The area of triangle ABC is 444 m^2. Calculate the shortest distance from the boat to the shore. [1]

(iii) A turtle is crawling along the shoreline. An eagle is at a vertical height of 40 m above C.
It notices the turtle.
Calculate the greatest angle of depression of the turtle as seen from the eagle. [2]

4 In the diagram, O is the centre of the circle.
TA and TE are tangents to the circle. OA and OE are radii of the circle. COT is a straight line.
OA intersects BF at G. CT is parallel to BA.
Angle $OTE = 32°$.



(a) Find

(i) angle AOF, [2]

(ii) angle CDE, [2]

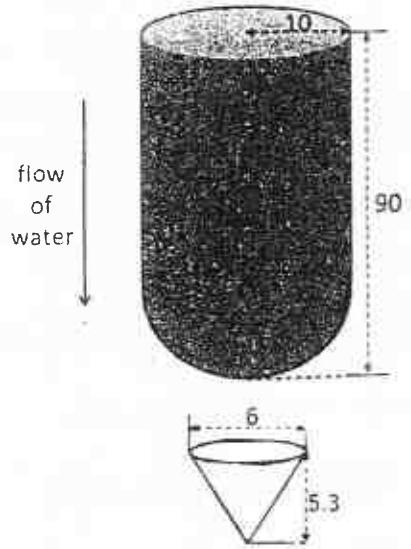
(iii) angle OFG, [2]

(iv) angle AGB. [1]

(b) Explain why points $OETA$ can also be points on the circumference of another circle. [1]

5 The diagram shows a water dispenser that is made up of a cylinder and a hemisphere both of radius 10 cm. The height of the dispenser is 90 cm.

Conical cups of diameter 6 cm and height 5.3 cm are provided to drink the water from the container.



(a) Water is filled to the brim of the dispenser.
Find the amount of water in the dispenser. [2]

(b) Find the capacity of one conical cup.
Give your answer to the nearest cm^3. [2]

(c) Find the external curved surface area of the cup. [2]

(d) Find the height of the water remaining in the dispenser after 250 cups of water has been dispensed. [4]

6 A container can hold 2400 litres of water.

(a) A large tap alone can fill the container in x hours.
Write down an expression, in terms of x, for the amount of water that the large tap can dispense per minute. [1]

(b) A small tap alone will take 1 hour longer than the large tap to fill the container. Write down an expression, in terms of x, for the amount of water that the small tap can dispense per minute. [1]

(c) When both taps are turned on at the same time, they can fill the container in 3 hours.
Form an equation in x and shows that it reduces to $x^2 - 5x - 3 = 0$. [3]

(d) Solve the equation $x^2 - 5x - 3 = 0$, giving your solutions correct to 2 decimal places. [4]

(e) Find the rate of water flow, in litres per minute, of the small tap. [2]

7 **Answer the whole of this question on a single sheet of graph paper.**

A stone is thrown from the top of a cliff next to the sea. The height, h metres, of the stone above sea level t seconds after it is released can be modelled by the equation

$$h = 40 + 8t - \frac{5}{2}t^2$$

Some corresponding values of t and h, correct to 1 decimal place, are given in the table below.

t	0	1	2	3	4	5	6
h	40	45 5	46	41.5	32	17.5	p

(a) Calculate the value of p. [1]

(b) Using a scale of 2 cm to represent 1 second, draw a horizontal t-axis for $0 \le t \le 6$.
Using a scale of 1 cm to represent 5 metres, draw a vertical h-axis for $-10 \le h \le 50$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

raph to estimate

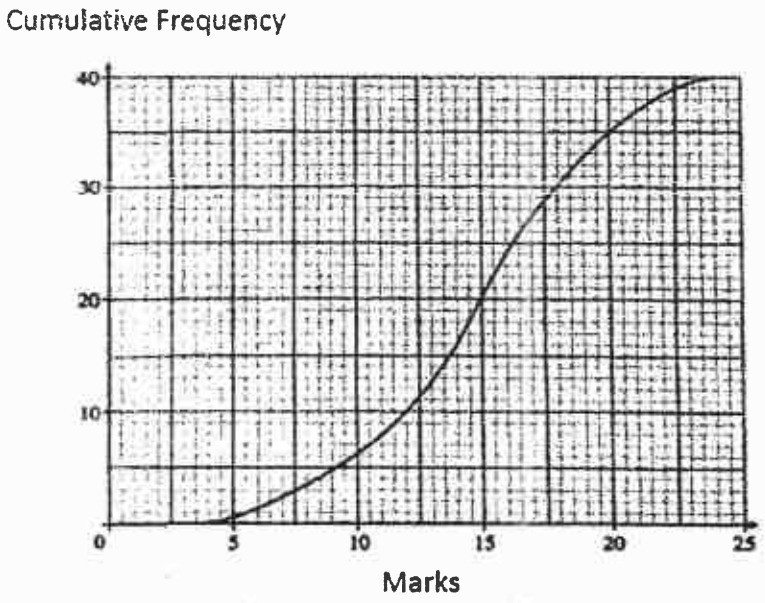
(i) the maximum height of the stone above sea level, [1]

(ii) the length of time that the stone was greater than or equal to 5 m above the top of the cliff, [2]

(iii) the time taken for the stone to hit the water. [1]

(d) By drawing a tangent, find the gradient of the curve at $t = 4$. [2]

8 (a) The marks attained by 40 students in a Mathematics test were recorded.
The cumulative frequency curve shows the distribution of the marks.



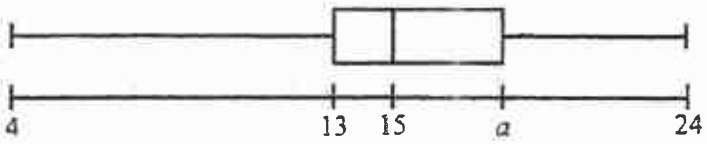
(i) Use the curve to estimate the

(a) the median mark, [1]

(b) the interquartile range. [2]

(ii) 12.5% of students achieved more than x marks in this test. Estimate the value of x. [1]

(iii) The same group of students sat for a Chemistry test. The maximum mark for the test was also 25. The box-and-whisker plot of the distribution of the marks is shown below.



The top 25% of the students for the Chemistry test scored lower than the top 25% in the Mathematics test. Write down the possible range of marks that a can take. [1]

(iv) Describe how the cumulative frequency curve for the marks attained in the Chemistry test may differ from the curve for the Mathematics test. [1]

(b) The weight of 8 students, in kilograms, are listed below:

25, 27, 32, 28, 28, 31, 26, 45

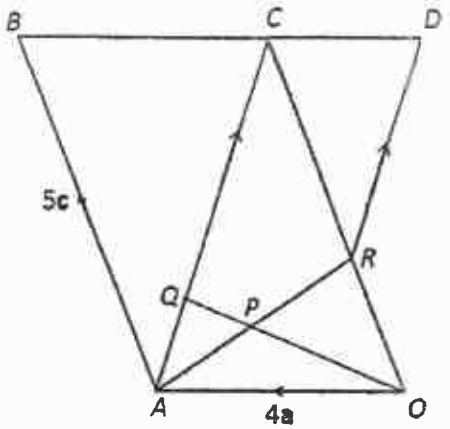
(i) Find the mean weight. [1]

(ii) Explain why the mean may not be an appropriate average to use to summarise the weights of the students. [1]

(iii) Find the standard deviation of the weights. [1]

(iv) Subsequently, it was discovered that the weight of every student was 2 kg less than the actual, due to a faulty weighing scale.
Write down the correct mean and standard deviation of the weights. [2]

9



In the diagram, OA is parallel to DB, AC is parallel to RD and $OABC$ is a parallelogram. $\overrightarrow{OA} = 4\mathbf{a}$ and $\overrightarrow{AB} = 5\mathbf{c}$ respectively. It is given that $OR : RC = 2 : 3$ and $\overrightarrow{AQ} = \frac{1}{3}\overrightarrow{QC}$.

(a) Find, in terms of **a** and **c**, the vectors

(i) $\overrightarrow{OR}$, [1]

(ii) $\overrightarrow{AR}$, [1]

(iii) $\overrightarrow{OQ}$. [2]

(b) P is a point on OQ such that $OP : PQ = 8 : 3$.

(i) Express $\overrightarrow{AP}$ in terms of **a** and **c**. [2]

(ii) Hence write down two facts about A, P and R. [2]

(c) Name a pair of congruent triangles. [1]

(d) Prove that ΔRCD is similar to ΔCOA. [2]

(e) Find

(i) $\dfrac{\text{Area of } \Delta RCD}{\text{Area of } \Delta COA}$, [1]

(ii) $\dfrac{\text{Area of } \Delta OQA}{\text{Area of } \Delta OCA}$. [1]

10 James has gotten a job that pays him a salary of $60 000 annually. He plans to purchase a car but calculates that he can only afford to set aside 30% of his monthly salary for the expenses incurred in owning the car.

(a) Calculate the sum of money that James can afford to set aside monthly for the expenses incurred in owning the car. [1]

He has set his eyes on two cars. He decides to take a loan from a bank for the purchase. He will repay the loan on a monthly basis. The details are given below:

	Brand A (used car)	**Brand B** (new car)
Engine capacity	1600 cc	1400 cc
Cost	$80 000	$90 000
Intended loan amount	50% of cost price	60% of cost price
Intended loan period	5 years	5 years
Type of interest	compound interest at 2.5% per year, compounded yearly	simple interest at 3% per year

The other major expenses in maintaining a car are as follows:

	Brand A (used car)	**Brand B** (new car)
Monthly parking fees	$90	$90
Monthly petrol expenditure	$300	$250
Annual road tax	$744	$626
Annual insurance	$800	$700
Car servicing (twice a year)	$600 each round	$500 each round

(b) Recommend the brand of car that James can purchase, based on the sum of money he can afford to set aside monthly. Justify the decision you make and show your calculations clearly. [7]

End of Paper

Anglo-Chinese School (Barker Road)

Mathematics Paper 1 Marking Scheme
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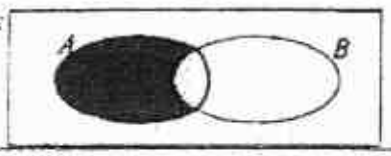
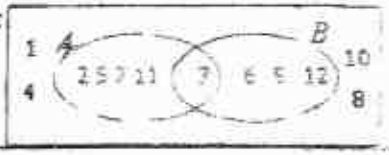
Qn	Steps/Answer
1	$\sqrt{0.81}$ 0.902 0.86^3 $\frac{399}{441}$
2	$(256\times10^9)\div(2.5\times10^6)$ (exact answer) $=1.024\times10^5$
3	$4c(3a+7b)-2d(3a+7b)$ $=(4c-2d)(3a+7b)$ or equivalent $=2(2c-d)(3a+7b)$
4	$\frac{1}{9}=\$20$ Total sum = \$180
5	$=\frac{7x}{(x-5)^2}-\frac{1}{x-5}$ $=\frac{7x-(x-5)}{(x-5)^2}$ $=\frac{6x+5}{(x-5)^2}$
6	$-8\le2-3x$ and $2-3x<8$ $-2<x\le3\frac{1}{3}$
7	12 cooks – 6 hours – 180 people 8 cooks – 9 hours – 180 people 8 cooks – 7.5 hours – 150 people Ans: 7.5 hours
8	Different scale used for the vertical axis may mislead one to think that more people prefer Brand Y to Brand X.
9 (a)	
(b)	
10	$=\left(\frac{x^6}{y^6}\right)^{\frac{1}{2}}\times\left(\frac{y^{-2}}{x^{-2}}\right)$ $=\frac{y^3}{x^3}\times\frac{x^3}{y^3}$ $=\frac{x^3}{y}$
11 (a)	\$6950
(b)	$\frac{\$6850}{5000}$

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Mathematics Paper 1 Marking Scheme
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Qn		Steps/Answer
12	(a)	Cost price of watch for Jimmy $= \frac{80}{100} \times 210$ $= \$168$ Price that Jimmy should sell at $= \frac{120}{100} \times \$168$ $= \$201.60$ Marked price $= \frac{100}{90} \times \$201.60 = \$224$
13		angle FDB = angle CBD = 56° (alternate angles, BC parallel to FD) angle ABD = 180 − (44+56) = 80° angle FAB + angle ABD = 181° **By the property that interior angles of parallel lines are supplementary**, AF is not parallel to BD
14	(a)	720°
	(b)	If the 3 polygons are regular, then angle IKB + angle IKF + angle BKF = 108° + 108° + 120° = 336° By the property that angles at a point add up to 360°, at least one of the polygons must be irregular.
15	(a)	2 x 7
	(b)	10
	(c)	LCM of 84, 90 and 2450 is 44100. Next flash at 2015
16	(a)	

+	5	6	7	8	9
5	-	11	12	13	14
6	11	-	13	14	15
7	12	13	-	15	16
8	13	14	15	-	17
9	14	15	16	17	-

	(b)	$\frac{1}{10}$
	(c)	$\frac{3}{5} - \frac{2}{4} - \frac{1}{3}$ $= \frac{1}{10}$

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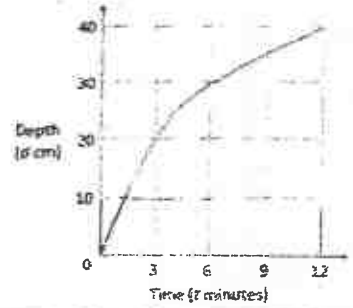
17 (a) $\left(\frac{20}{40}\right)^2 \times 12$

$= 3$ min

(b)



[4]

18 (a) $\mathbf{V} = \begin{pmatrix} 320 & 120 \\ 380 & 100 \\ 410 & 130 \end{pmatrix}$

(b) $\mathbf{P} = \begin{pmatrix} 320 & 120 \\ 380 & 100 \\ 410 & 130 \end{pmatrix} \begin{pmatrix} 2 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 700 \\ 810 \\ 885 \end{pmatrix}$

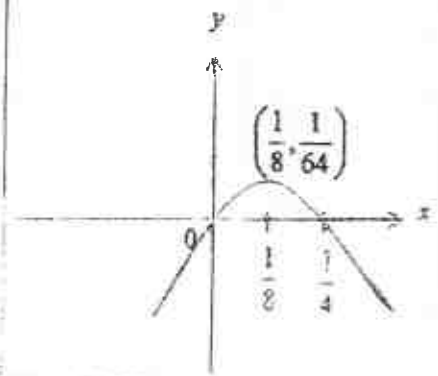
[4]

(c) **P** represents the total charges incurred by all vehicles for each respective day.

(d) $\mathbf{D} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$

19 (a) $\left(x - \frac{1}{8}\right)^2 = \frac{1}{64}$

(b)



[4]

(c) $\left(\frac{1}{8}, \frac{1}{64}\right)$

20 (a) $\frac{V}{50} = \frac{10.6}{1}$

$V_{large} = 204.8$

(b)

Small bottle	Large Bottle
50g - \$5.10	204.8g - \$13.25
1g – 10.2 cents	1g – 6.47

[4]

The large bottle gives more value for money

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21 (a) angle $ACB = 37 - 20 = 17°$

$$\frac{AC}{\sin 143} = \frac{400}{\sin 17}$$

$AC = 823\text{m}$

(b) $h = \sin 20 \times 823.356$

$= 282$ m

22 (a) $$QP = \frac{8}{\tan 0.7 rad} = 9.4979$$

Area of triangle $OPQ = \frac{1}{2}(8)(9.4979) = 37.992$

Area of sector $= \frac{1}{2}(8^2)(0.87079 rad) = 27.865$

Area of shaded region $= 10.1\ \text{cm}^2$

(b) $40.1°$

23 (a) $$\frac{y-6}{x-0} = -\frac{1}{4}$$

$4y = -x + 24$

(b) $x = 24$

$(24, 0)$

(c) 24.7 units

(d) Identify that D has coordinates $(-12, 0)$

$x = -12$

24 (a) $$\text{Speed} = \frac{18}{30} \times 18$$

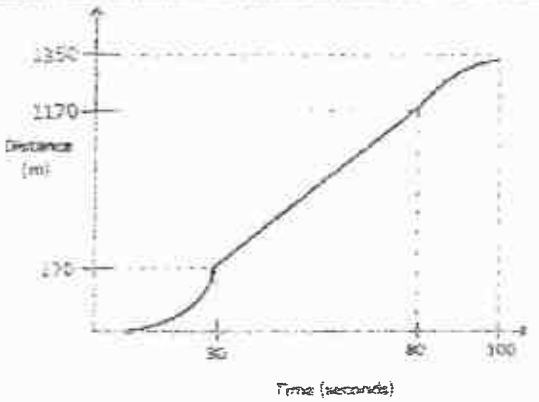
$= 10.8$ m/s

$= 38.88$ km/h

(b) $$\text{Distance} = \left(\frac{1}{2} \times 30 \times 18\right) + (50 \times 18) + \left(\frac{1}{2} \times 20 \times 18\right)$$

$= 1350$ m

(c)



Anglo-Chinese School (Barker Road)

Mathematics Paper 2 Marking Scheme
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Qn		Steps/Answer			
1	(a)	$T_4 = 18$			
	(b)	$T_n = n(n+1)+10-4(n-1)$ $= n^2+n+10-4n+4$ $= n^2-3n+14$			
	(c)	$T_{50} = 2364$			
	(d)	$n^2-3n+14 = n(n-3)+14$ When n is even n(n-3) is (even x odd) = even. When n is odd, n(n-3) is (odd x even) = even. Adding to 14 which is also even, $T_n = n^2-3n+14$ will always be even for all terms.			
2	(ai)	$v^2 = 30^2 - 2(9.8)(24)$ $v = \pm 20.7$			
	(aii)	$v^2 = u^2 - 2gh$ $u^2 = v^2 + 2gh$ $u = \pm\sqrt{v^2+2gh}$			
	(b)	$[(x+1)+(y-1)][(x+1)-(y-1)]$ $= (x+y)(x-y+2)$			
	(c)	$\frac{(x+1)(x-1)}{(1-x)(8+5x)}$			
		$= \frac{(x+1)(x-1)}{-(x-1)(5x+8)}$ or $\frac{-(1-x)(x+1)}{(1-x)(5x+8)}$			
		$= -\frac{(x+1)}{(5x+8)}$ or equivalent			
	(d)	By substitution or elimination method $x = 3$, $y = -2.5$			

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	(a)	1.5375 km	
	(aii)	Conversion from cm to m or cm^2 to m^2 Using cosine rule, $412.5^2 = 525^2 + 450^2 - 2(525)(450)\cos(angle\,BAC)$ $\cos(angle\,BAC) = \frac{-307968.75}{-472500}$ $angle\,BAC = 49.324°$	
		Area of triangle $= \frac{1}{2}(525)(450)\sin 49.324°$ $= 89\,600\ m^2$	
	(bi)	Bearing of C from $B = 63-(90-75)=048°$	
	(bii)	Shortest distance $= \frac{444 \times 2}{35} = 25.4$ m	
	(biii)	Angle of depression $= \tan^{-1}\left(\frac{40}{25.371}\right)$ $=57.6°$	
4	(ai)	angle OTA = angle $OTE = 32°$ (the line joining an external point to the centre of the circle bisects the angle between the tangents) angle $TAO = 90°$ (tangent perpendicular to radius) angle $AOF = (180 - 90 - 32)° = 58°$ (angles sum of triangle AOT)	
	(aii)	angle $AOE = 58 \times 2 = 116°$ angle $AOC = 180 - 58 = 122°$ angle $CDE = \frac{1}{2}(58 \times 2 + 122°) = 119°$	
	(aiii)	angle $GBA = \frac{1}{2}(58°) = 29°$ (angle at centre is twice angle at circumference) angle OFG = angle $GBA = 29°$ (alternate angles, OF parallel to BA)	
	(aiv)	angle $OGF = (180 - 29 - 58)° = 93°$ angle $AGB = 93°$ (vertically opposite angles)	
	(b)	By the property of 'right-angle in a semi-circle', OT is a diameter and points E and A will lie on the circumference $OETA$ are thus four points on the circumference of this circle Or calculate using 'angles in opposite segments are supplementary'	
5	(a)	volume of water $= \pi(10^2)(80) + (\frac{2}{3})(\pi)(10^3)$ $= \ldots\ cm^3$	

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(b) Capacity of one conical cup = $(\frac{1}{3})(\pi)(3^2)(5.3)$

$= 50\ cm^3$

(c) Slant height of cup = $\sqrt{3^2 + 5.3^2}$

$= 6.0902$

Curved surface area of cup = $\pi(3)(6.0902)$

$= 57.4\ cm^2$

(d) Volume of water remaining after dispensing 250 cups

$= 8666\frac{2}{3}\pi - (250 \times \frac{1}{3}\pi(3^2)(5.3)$

$= 4619\frac{2}{3}\pi$

Volume of water in cylinder $= 4619\frac{2}{3}\pi - \frac{2}{3}\pi(10^2) = 4025\pi$

Height of water in cylindrical section $= \frac{4025\pi}{\pi(10^2)} = 40.25$

Height of water remaining in dispenser

$= 40.25 + 10$

$= 50.25$ cm

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6	(a)	$\frac{40}{x}$ litres/minute			
	(b)	$\frac{40}{x+1}$ litres/minute			
	(c)	$180\left(\frac{40}{x+1}+\frac{40}{x}\right)=2400$ $3[40x+40(x+1)]=40x(x+1)$ $40x^2-200x-120=0$ $x^2-5x-3=0$ (shown)			[1]
	(d)	$x=\frac{5\pm\sqrt{(-5)^2-4(1)(-3)}}{2(1)}$ $x=\frac{5\pm\sqrt{37}}{2}$ $x=5.54$ or $x=-0.54$			
	(e)	Rate of water flow for small tap $=\frac{40}{5.541-1}$ $=6.11$ litres per minute			

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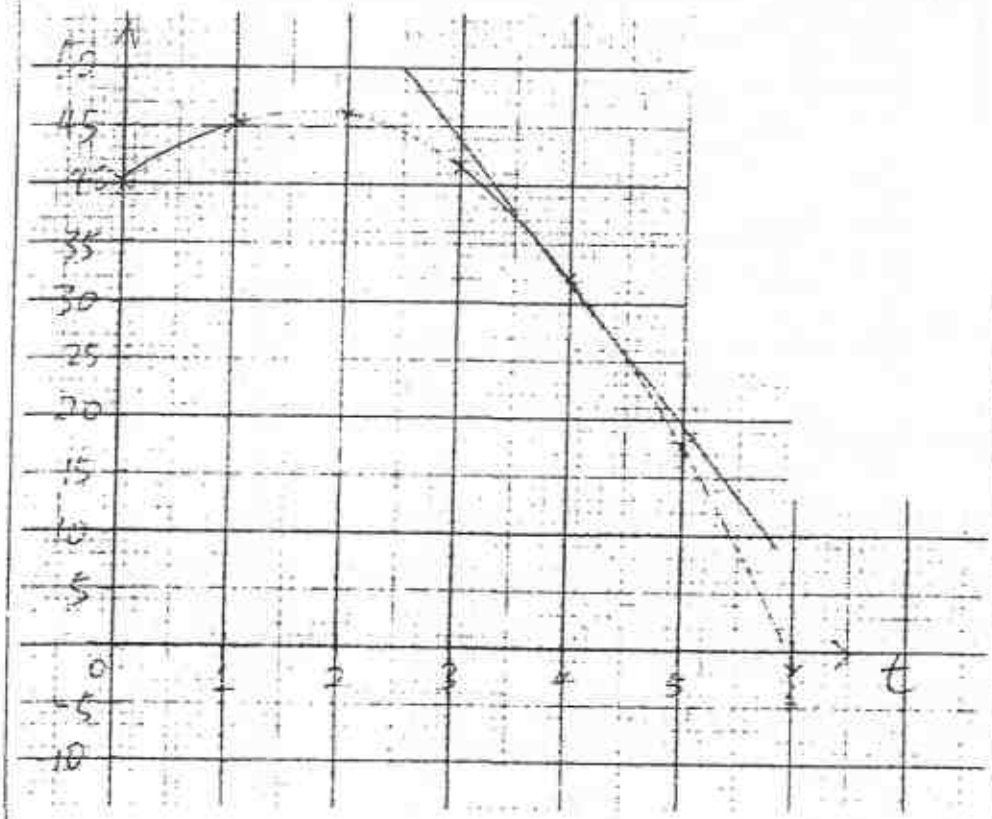
7	(a)	$p = -2$ Please refer to graph  			[10]
	(ci)	Maximum height = 46 m			
	(cii)	Length of time = 2.4 – 0.9 = 1.5 s			
	(ciii)	Time taken to hit water = 5.95s			
	(d)	Tangent drawn correctly. Gradient = $\frac{50-0}{2.5-5.8}$ = −12.1			

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8	(aia)	15 marks			[11]
	(aib)	18 – 12 = 6 marks			
	(aii)	20 marks			
	(aiii)	$15 \leq a < 18$			
	(aiv)	The curve will be steeper before the median mark of 15 and less steep after the median.			
	(bi)	30.25 kg			
	(bii)	There is an outlier 45 kg which would cause the mean to be skewed			
	(biii)	Standard deviation = 5.99			
	(biv)	Correct mean = 32.25 kg Standard deviation remains the same			

Anglo-Chinese School (Barker Road)

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9	(ai)	$\overrightarrow{OR} = \frac{2}{5}\overrightarrow{OC}$ $= 2\mathbf{c}$			[13]
	(aii)	$\overrightarrow{AR} = \overrightarrow{OR} - \overrightarrow{OA}$ $= 2\mathbf{c} - 4\mathbf{a}$			
	(aiii)	$\overrightarrow{OQ} = \overrightarrow{OA} + \overrightarrow{AQ}$ $= 4\mathbf{a} + \frac{1}{4}\overrightarrow{AC}$ $= 4\mathbf{a} + \frac{1}{4}(5\mathbf{c} - 4\mathbf{a})$ $= 3\mathbf{a} + \frac{5}{4}\mathbf{c}$			
	(bi)	$\overrightarrow{AP} = \overrightarrow{AO} + \overrightarrow{OP}$ $= AO + \frac{8}{11}\overrightarrow{OQ}$ $= -4\mathbf{a} + \frac{8}{11}(3\mathbf{a} + \frac{5}{4}\mathbf{c})$ $= -\frac{20}{11}\mathbf{a} + \frac{10}{11}\mathbf{c}$			
	(bii)	$AP = \frac{5}{11}AR$ As point A is common, A, P and R are collinear (i.e. lie on the same straight line).			
	(c)	triangle ABC is congruent to triangle COA			
	(d)	$\angle DCR = \angle AOC$ (alt. $\angle$s, $DC \parallel OA$) $\angle DRC = \angle ACO$ (alt. $\angle$s, $DR \parallel CA$) ΔRCD is similar to ΔCOA. (AA property)			
	(ei)	$\frac{\text{Area of } \Delta RCD}{\text{Area of } \Delta COA} = \left(\frac{RC}{CO}\right)^2$ $= \left(\frac{3}{5}\right)^2$ $= \frac{9}{25}$			
	(eii)	$\frac{\text{Area of } \Delta OQA}{\text{Area of } \Delta OCA} = \frac{QA}{CA} = \frac{1}{4}$			

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(b)

	Brand A		
loan	40000	50% of cost	[8]
Compound amount	$40000(1+\frac{2.5}{100})^5$		
total loan amount	= $45256.33		
monthly instalment	$754.2721419	Divide by 60 months	
Monthly cost of road tax + Insurance + Servicing	$(\frac{744+800+1200}{12})$ =228.67		
Total monthly cost of maintenance	300+90+228.67 =618.67	Adding on monthly petrol and parking costs	
monthly installment + cost of maintenance	1372.94		

	Brand B	
loan	54000	60% of cost
Simple interest	8100	
total loan amount	62100	
monthly instalment	1035	Divide by 60 months
road tax (r)	626	
Insurance (i)	700	
Servicing (s)	1000	
Monthly cost of road tax + Insurance + Servicing	$\frac{626+700+1000}{12}$ =193.83	
	$\frac{250+90+193.83}{12}$	Adding on monthly petrol and parking costs
Total monthly cost of maintenance	$533.83	
monthly installment + cost of maintenance	1568.83	

James can afford Brand A as it is within the sum of money that he can set aside monthly

Register No.	Class

Name : _____

BENDEMEER SECONDARY SCHOOL
2017 PRELIMINARY TWO EXAMINATION
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)
Elementary Mathematics Paper 1
4048/01

DATE : **22 August 2017**
DURATION : **2 hours**
TOTAL : **80 Marks**

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a 2B pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** questions.
Write your answers in the spaces provided on the question paper.
All the diagrams in this paper are **not** drawn to scale.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

FOR EXAMINER'S USE
80

This document consists of **19** printed pages including this cover page.

[Turn over

MATHEMATICAL FORMULAE

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of sphere = $\frac{4}{3}\pi r^3$

Area of triangle ABC = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

For Examiner's use

1 (a) By rounding each number to its nearest ten, calculate $\frac{216.1 + 1083.7}{14.99}$.

(b) Write your answer to part (a) correct to 1 significant figure.

Answer (a) .....[1]

(b) .....[1]

For Examiner's use

2 If the length of a rectangle is 340mm and width is 200mm, both are corrected to the nearest 10mm, calculate the

(a) maximum possible area of this rectangle in cm^2,

(b) lowest possible value of the ratio $\frac{width}{length}$.

Answer (a) ..... [2]

(b) .....[1]

3 James was 82kg and 15% above his ideal weight. He exercised and lost 6% of his initial weight. How many percent of his current weight must James lose in order to reach his ideal weight?

Answer..... [3]

4 (a) Solve $4a(a-3) = 2-(20-6a)$.

(b) Factorise $x^2y^2 + 36 - 4x^2 - 9y^2$ completely.

Answer (a) ..... [2]

(b) .....[3]

5 A flight leaving Singapore to London takes about 13 hours and 15 minutes. If the departure time on a Tuesday from Singapore is 1310 hours and Singapore is 7 hours ahead of London, what day and time, in 24 hour format, does the flight reach London?

Answer .....*hours on* ..... [2]

6 In $\Delta DEF, DF = 10cm, EF = 12cm$ and $\angle EDF = 39°$.

(a) Find $\angle DEF$.

(b) Which is the acceptable answer to part (a)? Explain why the other answer is not applicable.

Answer (a) $\angle DEF$ =.....° , ...° [2]

(b) .....

..... [2]

7 Given that $\frac{a^2}{c^2} - \frac{b^2}{d^2} = 1$, make b the subject.

Answer ..... [2]

8 (a) Evaluate $(2^{-1} - 5^{-2})$ without using a calculator. Show your working clearly.

(b) Simplify $\frac{\sqrt[3]{b^2} \times b^6}{b^{\frac{2}{3}} \times b}$, giving your answer in the form of b^n.

Answer (a) ..... [2]

(b) ..... [2]

9 Siew Teng is x years old and her brother Victor is 2 years older. Their mother is 6 times older than Victor.

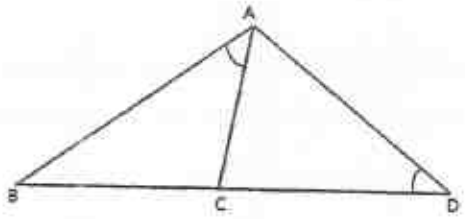
(a) Write down the ratio of Siew Teng's age: Victor's age: Mother's age in terms of x.

(b) Ten years from now, their total ages will be 76. How old was Siew Teng's mother five years ago?

Answer (a) ..... [1]

(b) ..... [2]

10 In the diagram, given that $\angle BAC = \angle BDA$ and C lies on a straight line BD. It is given that AB = 6 cm and BC = 4 cm.



(a) Show that ΔABC and ΔDBA are similar.

.....

.....

.....

.....

..... [2]

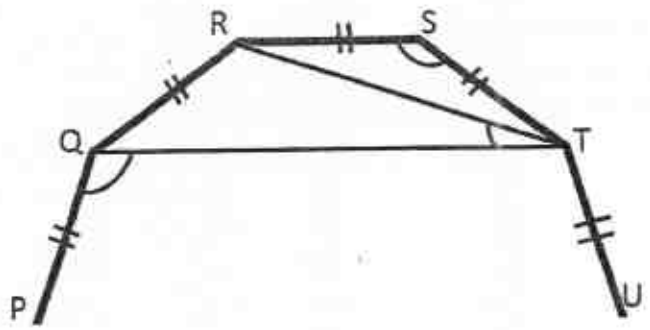
(b) Find BD.

(c) Given the area of ΔABD is 42 cm^2, find the shortest distance from D to AB.

Answer (b) .....*cm* [1]

(c).....*cm*^{2} [2]

11 The below diagram is part of a regular decagon.



Find

(a) $\angle RST$

(b) $\angle RTQ$

(c) $\angle PQT$

Answer (a) .....°[1]

(b) .....° [1]

(c) .....° [2]

12 Two fair six-sided dice are thrown.
Find the probability that

(a) both dice show different numbers,

(b) the sum of the two numbers shown is 12,

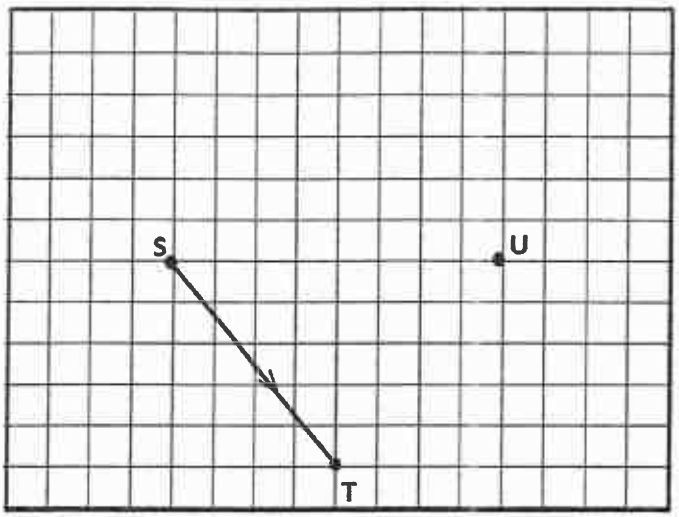
(c) the sum of the two numbers shown is a prime number.

Answer (a)..... [1]

(b)..... [1]

(c)..... [2]

13 The figure below shows the positions of the points S, T and U.



(a) Express $\overrightarrow{ST}$ as a column vector.

(b) V is a point such that STUV is a parallelogram. Draw the parallelogram on the diagram above.

(c) Find the magnitude of $|\overrightarrow{ST}|$ and $|\overrightarrow{TU}|$.

(d) Hence, from your answer in part (c), is $|\overrightarrow{ST}| = |\overrightarrow{TU}|$? What is the specific name of the parallelogram?

Answer (a) ..... [1]

(b) .....See above..... [1]

(c) ..... , ... [2]

(d) .....

.....[2]

14 (a) Hasan invested part of $8000 at 2.4% per annum simple interest and the remaining at 1.8% per annum simple interest. He received a total interest of $348 after two years. How much did he invest at 2.4% per annum simple interest?

(b) Amin bought a car at $70000 and the car depreciated by 25 % at end of first year, 20% at end of second year and 15% at end of third year. What was Amin's car value after 3 years?

Answer (a) $..... [2]

(b) $..... [2]

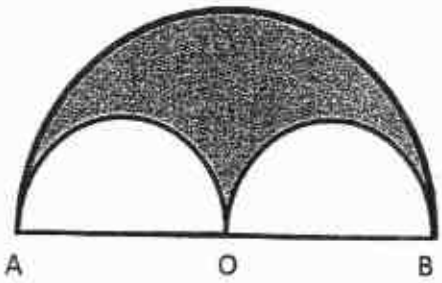
15 The diagram shows 2 small semicircles inside a big semicircle. Given that AB is the diameter of the big semicircle with center O and area of each small semicircle is $\frac{9}{2}\pi\ cm^2$.

Find

(a) the radius of the small semicircle,

(b) the perimeter of the shaded area in terms of π,

(c) the area of the shaded region in terms of π.

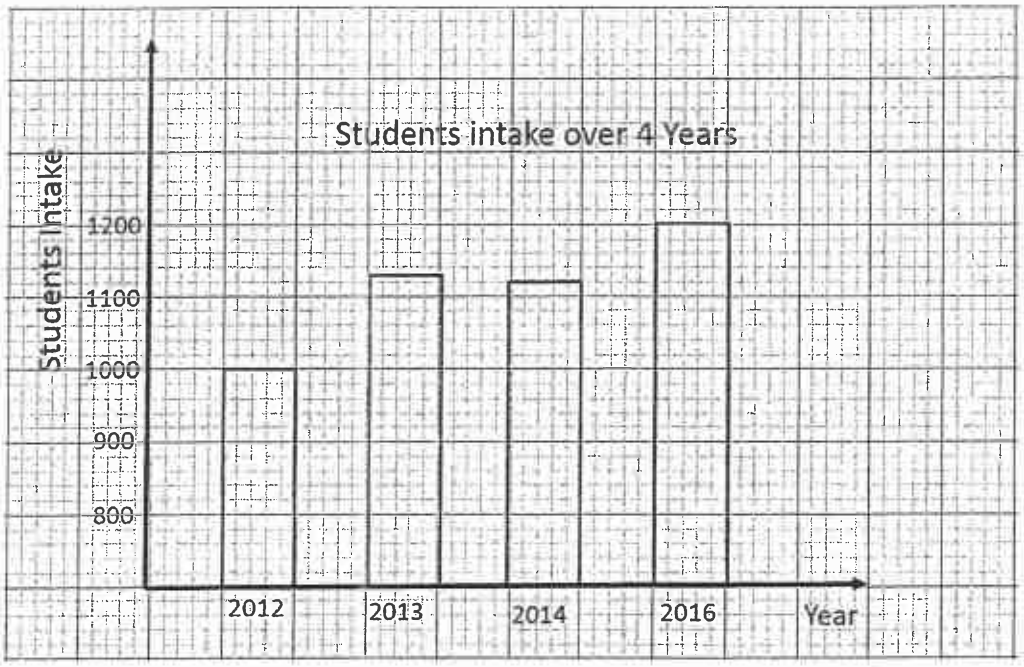


Answer (a) .....*cm* [1]

(b) .....*cm* [1]

(c) .....cm^2 [1]

16 The graph shows the students intake of ABC Secondary school over 4 years.



(a) Express the ratio of the height of the bar representing the students intake in 2012 to that in 2016.

(b) Express the ratio of the student intake in 2012 to the student intake in 2016.

(c) Should both answers you obtain in (a) and (b) be the same?

(d) Explain the similarity or difference in your answers of (a) and (b).

Answer (a) ..... [1]

(b) .....[1]

(c) .....[1]

(d) .....

.....[1]

17 Given the equation of line L_1 is $\frac{1}{2}x - 3y = 9$, find

(a) the coordinates when it cuts the x-axis,

(b) the gradient of the line,

(c) the value of k if the point $(-6, k)$ lies on the line,

(d) the equation of line L_2 that cuts y-axis at 5 and is parallel to L_1.

Answer (a)..... [1]

(b) .....[1]

(c).....[1]

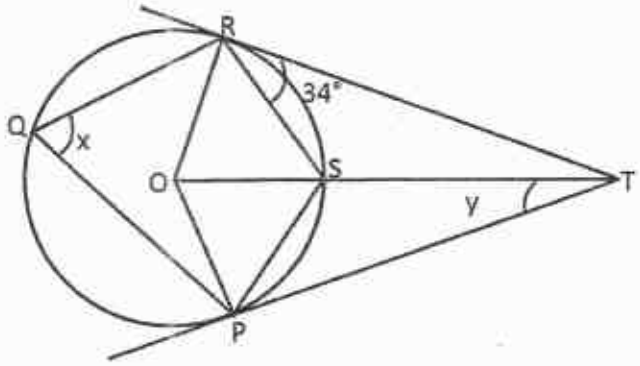
(d).....[1]

18 In the diagram, O is the center of the circle and RT and PT are tangents to the circle at R and P respectively. Find the angles,

(a) x and

(b) y.

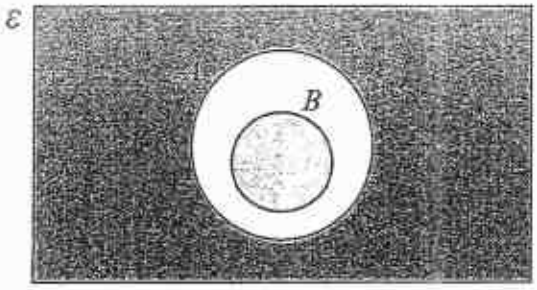
State your reasons clearly.



Answer (a) x =..... [3]

(b) y = ... [1]

19 (a) Use set notation to describe the shaded area in the following Venn diagram.



(b) ε= {numbers from 1 to 10}
A = {even numbers}
B = {prime numbers}
C = {multiples of 2 greater than 6}

(i) List the elements in $A \cap B'$.
(ii) State the relationship between set A and C.

Answer (a) ..... [1]

(b)(i) ..... [1]

(b)(ii) ..... [1]

20 The scale drawing in the answer space below shows the position of towns A and B. Town B is 36 km due South of A. The map scale is given as 1:600 000.

Construct the map of ABCD using the information given below:

(a) Town C which is 54 km from B with a bearing of 085° from B.

(b) Town D is located 18 km from C and on the perpendicular bisector of A and B.

(c) Measure the bearing of Town D from Town A.

Answer (a) .....See above..... [2]

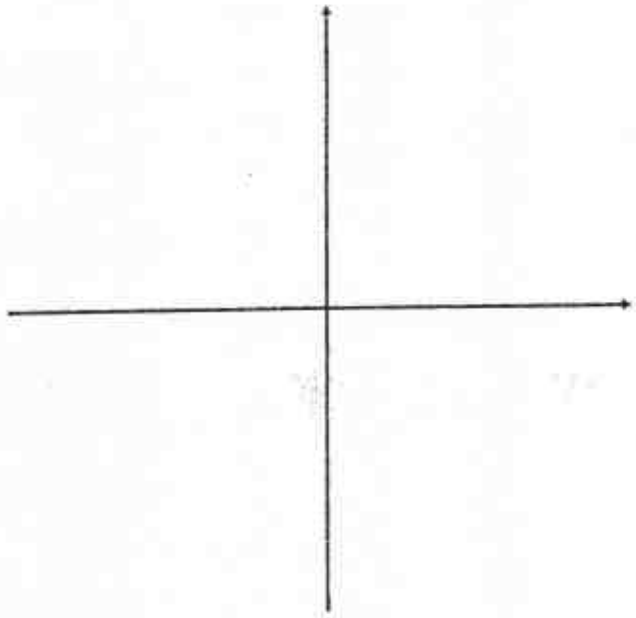
(b) .....See above ..... [2]

(c)..... [1]

21 (a) Express the function $y = -x^2 + 8x - 5$ in the form $y = -(x - h)^2 + k$.

(b) Sketch the graph of the function $y = -x^2 + 8x - 5$. Label the y-intercept and turning point.

(c) Hence, or otherwise, solve the equation $-x^2 + 8x - 5 = -10$



Answer (a) ..... [2]

(b) .....See above ..... [2]

(c) $x =$ ....., ... [2]

--- End of Paper ---

Register No.	Class

Name : _____

BENDEMEER SECONDARY SCHOOL
2017 PRELIMINARY TWO EXAMINATION
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)
Elementary Mathematics Paper 2
4048/02

DATE : **23 August 2017**
DURATION : **2 hours 30 minutes**
TOTAL : **100 marks**

Additional Materials: Cover page
Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST
Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a 2B pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** questions.
All the diagrams in this paper are **not** drawn to scale.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

FOR EXAMINER'S USE
100

This document consists of **11** printed pages including this cover page.

[Turn over

MATHEMATICAL FORMULAE

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 (a) Solve the inequality $\frac{p-2}{4} \leq \frac{1}{2} - \frac{15-2p}{5}$. [3]

(b) (i) Factorise $2q - 18q^3$ completely. [2]

(ii) Hence simplify $\frac{2q-18q^3}{(4q^2-2q)(3q+1)}$. [2]

(c) (i) In January, Joseph's best time to swim 200 metres was 2 minutes 30 seconds.

Calculate his speed in kilometres per hour. [2]

(ii) In December, Joseph's best time is 10% less than his best time in January.

Calculate, in minutes and seconds, his best time in December. [2]

---

2 The first four terms in a sequence of numbers are given below.

$$T_1 = 3 + 2^0 \quad = 4$$
$$T_2 = 5 + 2^1 \quad = 7$$
$$T_3 = 7 + 2^2 \quad = 11$$
$$T_4 = 9 + 2^3 \quad = 17$$

(a) Find T_5. [1]

(b) Find the nth term of the sequence, T_n. [1]

(c) Hence or otherwise, find T_{20}. [1]

(d) Explain why the value of T_n is always odd for all values of n. [1]

(e) T_m and T_{m+1} are consecutive terms in the sequence.

Show that $T_{m+1} - T_m = 2 + 2^{m-1}$. [3]

---

3 A factory produces bottles in both the small and the large size.

(a) It is found that x large bottles can be produced in a minute.
Write down an expression in terms of x, the time taken to produce 1 large bottle, in **seconds**. [1]

(b) 4 more small bottles can be produced in a minute, compared to the large bottles.
Write down an expression in terms of x, the time taken to produce 1 small bottle, in **seconds**. [1]

(c) Given that it takes 2.5 seconds longer to produce a large bottle than a small bottle, form an equation in x and show that it reduces to $x^2 + 4x - 96 = 0$. [3]

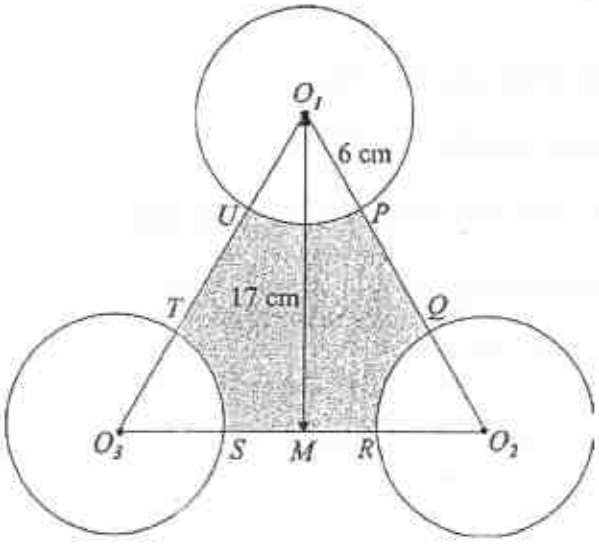
(d) Solve the equation $x^2 + 4x - 96 = 0$. [2]

(e) Hence find the time taken to produce 4000 small bottles, in hours and minutes. [2]

(f) It is known that the factory sells each small bottle at \$0.30 and each large bottle at \$0.50.
Is it more profitable for the factory to produce small or large bottles?
Explain your answer. [3]

---

4 The figure below shows the outline of a spinner toy, which is made up of an equilateral triangle and 3 identical circles with centre O_1, O_2 and O_3 respectively. It is given that the radii of the circles are 6 cm and $O_1M = 17$ cm, where M is the midpoint of SR.



Find (a) PQ, [2]

(b) the perimeter of the shaded region $PQRSTU$ and [3]

(c) the area of the shaded region $PQRSTU$. [3]

---

5 (a) The stem and leaf diagram below shows the marks attained by 15 students in a Mathematics test.

1	3	7				
2	3	6	6			
3	0	4	4	5	7	9
4	1	2	5			
5	0					

Key : 1 | 0 means 10 marks

(i) Using the data given, find the
(a) median mark, [1]
(b) interquartile range and [2]
(c) standard deviation of the marks. [2]

(ii) It was later found that there was a mistake in the marking for the test. As such, every student should get an additional 2 marks.

Describe how the change in marks will affect the median mark and interquartile range. [2]

(b) It is given that a box contains 15 apples and 9 oranges.

Two fruits are then selected from the box at random. If an apple is selected, it is replaced. If an orange is selected, it is not replaced.

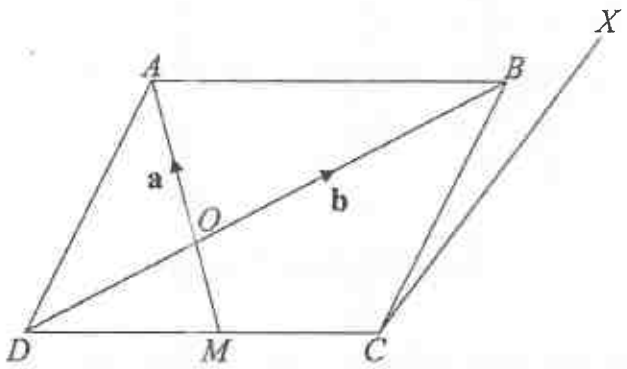
(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

(ii) Find, as a fraction in its simplest form, the probability that

(a) both fruits selected are the same, [2]

(b) at least one of the fruit is an apple. [2]

6 In the following diagram, $ABCD$ is a parallelogram where M is the midpoint of CD and $OD = \frac{1}{3}BD$.



Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$,

(a) express as simply as possible, in terms of **a** and/or **b**,

(i) $\overrightarrow{BD}$, [1]

(ii) $\overrightarrow{AB}$, [1]

(iii) $\overrightarrow{BC}$, [2]

(iv) $\overrightarrow{OM}$. [2]

(b) Given that $\overrightarrow{CX} = \mathbf{a} + \frac{3}{4}\mathbf{b}$, prove that B, D and X are collinear points. [2]

(c) Find the exact value of (i) $\frac{\text{area of } \Delta ODM}{\text{area of } \Delta OAB}$, [2]

(ii) $\frac{\text{area of } \Delta ODM}{\text{area of } ABCD}$. [1]

7 Petrol stations A and B sell two grades of petrol, R92 and P98.

The matrix **L** shows the average amount of petrol sold at the two stations on a day in Week 1.

$$\mathbf{L} = \begin{array}{c} \begin{matrix} \text{R92} & \text{P98} \end{matrix} \\ \begin{pmatrix} 250 & 180 \\ 280 & 180 \end{pmatrix} \end{array} \begin{matrix} \\ \text{Station A} \\ \text{Station B} \end{matrix}$$

(a) Evaluate the matrix $\mathbf{Q} = 7\mathbf{L}$. [1]

(b) It is given that the petrol price (per litre) of grade R92 and P98 are \$2.00 and \$2.40 respectively.

Represent the petrol prices as a column matrix **P**. [1]

(c) Evaluate the matrix $\mathbf{S} = \mathbf{QP}$. [1]

(d) State what the elements of **S** represent. [1]

(e) In Week 2, the average amount of all petrol sold at both petrol stations dropped by 5%. At the same time, the prices of all grades of petrol increased by 5%.

Calculate the earnings made by Station A and Station B respectively in Week 2. [3]

(f) Write down a matrix **X** such that the total earnings of both petrol stations in Week 2 can be calculated using matrix multiplication.

Hence find the total earnings of both petrol stations in Week 2. [2]

8 Figure 1 shows the three-dimensional layout of Roy's living room. The room is shaped like a cuboid with dimensions 4 m by 3.6 m by 3 m, where path *MN* lies across the centre of the room.

- A television is fixed on the wall *QRVU* such that *Y*, the centre of the television, is 1.6 m above *N*.
- Two speakers are fixed at corners *P* and *T* respectively.

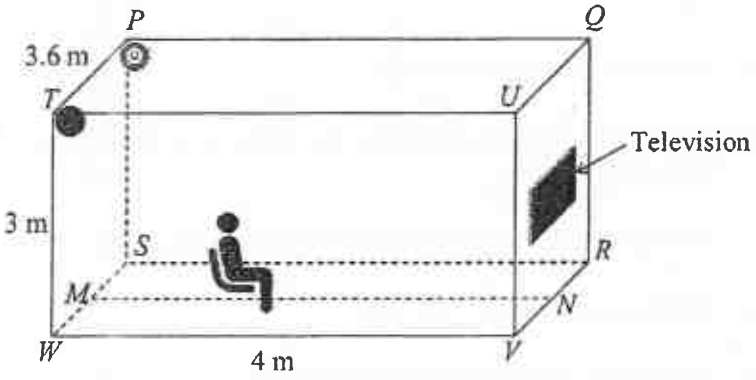


Figure 1

Roy is deciding on the best position to place his armchair along *MN*. The best position will allow him to have an optimal view of the television when seated in the armchair.

Figure 2 shows Roy's eye level at *X*, which is 1.24 m when seated at distance d from the television. It is given that $1.8\,\text{m} \le d \le 3.8\,\text{m}$ for Roy to have an optimal view of the television.

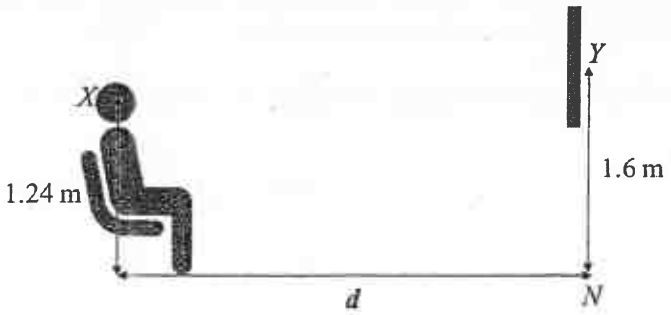


Figure 2

For this question, the dimensions of the television and speakers are negligible.

(a) If Roy chose to place the armchair at the furthest possible optimal distance, find

(i) *TX*, [3]

(ii) $\angle PXT$, [2]

(iii) the angle of elevation of *Y* from *X*. [2]

(b) When the angle of elevation of *Y* from *X* is 12°, will Roy still have an optimal view of the television? Justify your answer. [2]

9 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation

$$y = 5 + \frac{2}{x} - \frac{1}{4}x^2$$

Some corresponding values of x and y are given in the table below.

x	-6	-5	-4	-3	-2	-1.5	-1	-0.5	-0.3
y	-4.33	-1.65	p	2.08	3	3.10	2.75	0.94	-1.69

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-6 \le x \le 0$.
Using a scale of 2 cm to represent 1 unit, draw a vertical y-axis for $-5 \le y \le 4$.

On your axes, plot the points given above and join them with a smooth curve. [3]

(c) By drawing a tangent, find the gradient of the curve at $(-1, 2.75)$. [2]

(d) (i) On the same axes, draw the line L with gradient 0.5 and passes through the point $(-4, -3)$. [1]

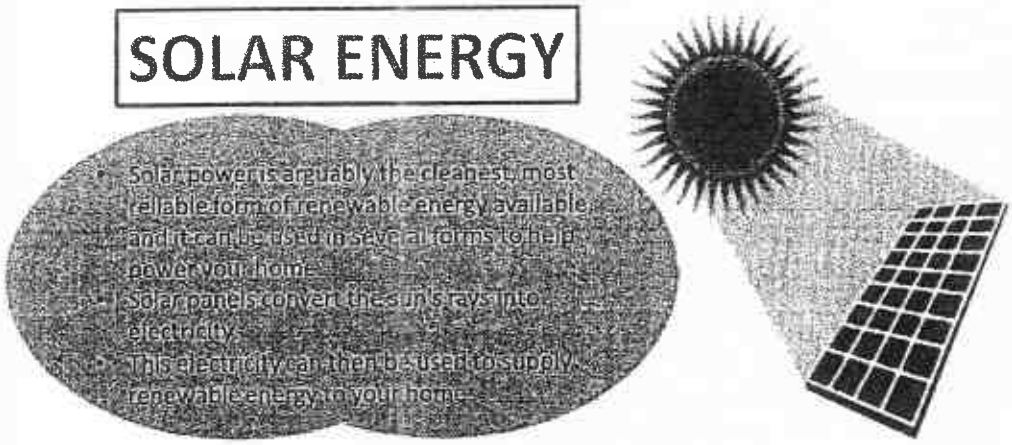
(ii) Write down the equation of the line L. [1]

(iii) The x-coordinate of the point(s) where the line L intersects the curve are the solution(s) to the equation $x^3 + Ax^2 - Bx - 8 = 0$.

Find the values of A and B. [2]

(e) Using the graph, show that $\frac{2}{x} - \frac{1}{4}x^2 + 1 = 0$ has no solution for $x < 0$. [2]

10 Mrs Lim is currently staying at a bungalow with her family. After learning about solar energy from the brochure below, she is thinking of installing solar panels at the bungalow to help reduce the family's electricity bills.



Brochure on Solar Energy

Information that Mrs Lim needs to consider in order to make a decision on the installation can be found under Annex A on the next page.

(a) For the first half of 2017,

(i) calculate the average amount of electricity (in kWh) used by Mrs Lim's family in a month, and [2]

(ii) calculate the average amount (in dollars) paid for electricity usage in a month. [2]

(b) Considering all the information given, should Mrs Lim go ahead with the installation of solar panels for the bungalow?

Justify your answer. [4]

ANNEX A

Table 1: Records of electricity usage by Mrs Lim's family

Electricity Usage for 2017 (in kWh)					
January	February	March	April	May	June
1107.8	**1066.3**	**1123.6**	**1259**	**1249.5**	**1281.6**

Table 2: Charges for electricity usage

Electricity tariff: **21.39 cents per kWh** (Charges subjected to 7% Goods & Services Tax)

Table 3: Details on installing solar panels for Mrs Lim's bungalow

Dimensions of roof area for solar panel installation	**9 m by 4 m**
Dimensions of 1 solar panel	**1.65 m by 1 m**
Cost of installing every 10 solar panels	**$6,250**

Table 4: More about the solar panels

Average amount of electricity produced by 1 solar panel: **19 kWh per month**	Lifespan of solar panels: **20 years**

~END OF PAPER~

Answers:

1a) $p \geq 13\frac{1}{3}$

1b)(i) $2q(1-3q)(1+3q)$ 1b)(ii) $\frac{1-3q}{2q-1}$

1c)(i) 4.8 km/h 1c)(ii) 2 min 15 sec

2a) 27

2b) $2n+1+2^{n-1}$

2c) 524329

3a) $\frac{60}{x}$ s

3b) $\frac{60}{x+4}$ s

3d) $x=-12,\ 8$

3e) 5 h $33\frac{1}{3}$ min or 5 h 34 min

3f) It is more profitable for the factory to produce **large** bottles.

4a) PQ = 7.63 cm

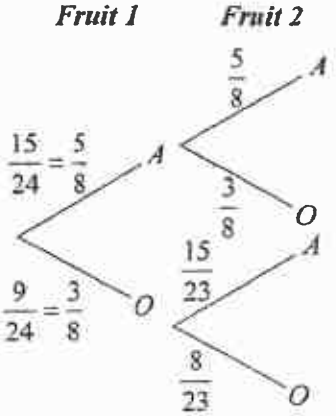
4b) Perimeter = 41.7 cm

4c) Area = 110 cm^2

5a)(i)(a) Median = 34 marks 5a)(i)(b) IQR = 15 marks 5a)(i)(c) SD = 9.99 marks

5a)(ii) The median will increase by 2 and the interquartile range will remain the same.

5b)(i)



5b)(ii)(a) P(both are the same) = $\frac{767}{1472}$ 5b)(ii)(b) P(at least 1 apple) = $\frac{20}{23}$

6a)(i) $\overrightarrow{BD} = -\frac{3}{2}\mathbf{b}$ 6a)(ii) $\overrightarrow{AB} = -\mathbf{a}+\mathbf{b}$

6a)(iii) $\overrightarrow{BC} = -\mathbf{a}-\frac{1}{2}\mathbf{b}$ 6a)(iv) $\overrightarrow{OM} = -\frac{1}{2}\mathbf{a}$

6c)(i) $\dfrac{\text{area of } \Delta ODM}{\text{area of } \Delta OAB} = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$ 6c)(ii) $\dfrac{\text{area of } \Delta ODM}{\text{area of } ABCD} = \dfrac{1}{4} \times \dfrac{2}{3} \times \dfrac{1}{2} = \dfrac{1}{12}$

7a) $\mathbf{Q} = \begin{pmatrix} 1750 & 1260 \\ 1960 & 1260 \end{pmatrix}$ 7b) $\mathbf{P} = \begin{pmatrix} 2.00 \\ 2.40 \end{pmatrix}$ 7c) $\mathbf{S} = \begin{pmatrix} 6524 \\ 6944 \end{pmatrix}$

7d) The earnings of Station A ($6,524) and Station B ($6,944) respectively for Week 1.

7e) The earnings of Station A ($6,507.69) and Station B ($6,926.64) respectively for Week 2.

7f) $\mathbf{X} = \begin{pmatrix} 1 & 1 \end{pmatrix}$

Total earnings $= \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 6507.69 \\ 6926.64 \end{pmatrix}$

$= \begin{pmatrix} 13434.33 \end{pmatrix}$

Total earnings of both stations (Week 2) = $13,434.33

8a)(i) TX = 2.53 m

8a)(ii) $\angle PXT \approx 90.9°$

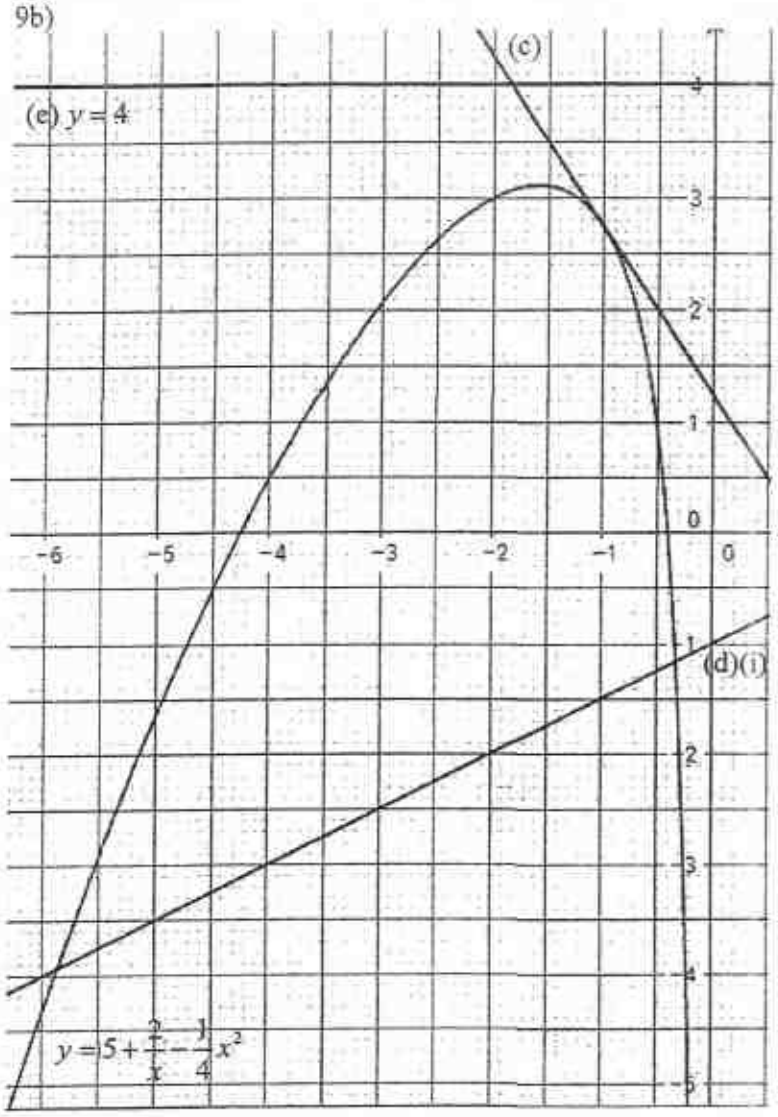
8a)(iii) Angle of elevation = 5.4°

8b) $\tan 12° = \dfrac{0.36}{d}$ $\rightarrow d \approx 1.69$ m

Since 1.69 m is less than the minimum optimal distance 1.8 m, Roy will not have an optimal view of the TV in this case.

9a) $p = 0.5$

9b)



9c) Gradient $= -1.5 \quad (\pm 0.2)$

9d)(ii) $y = \frac{1}{2}x - 1$

9d)(iii) $A = 2$ and $B = 24$

10a)(i) 1181.3 kWh

10a)(ii) \$270.37

10b) Since the average amount paid by Mrs Lim per month will be lesser than what she is currently paying for electricity usage, she should go ahead with the installation.

2017 Sec 4E/5NA Preliminary One Mathematics Marking Scheme

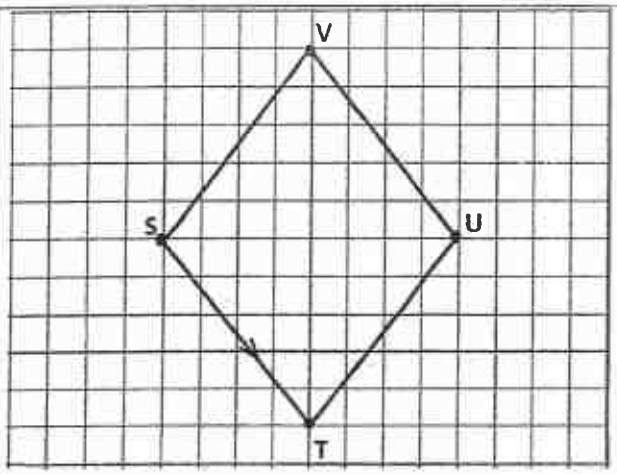
Qn	Answer	Marks
1(a)	130	B1
1(b)	100	B1
2(a)	344 x 204 = 70176 mm^2 1 mm^2 = 0.1^2 cm^2 71196 mm^2 = 0.1 x 0.1 x 70176 cm^2 = 701.76 cm^2 = 702 cm	M1 A1 (accept exact value)
2(b)	$\frac{195}{344}$	B1
3	Ideal weight = $\frac{82}{115} \times 100 = 71.30kg$ Current weight = $\frac{94}{100} \times 82 = 77.08kg$ Per cent = $\frac{77.08-71.30}{77.08} \times 100 = 7.4987 = 7.50\%$	M1 M1, A1
4(a)	$4a^2 - 12a = 2 - 20 + 6a$ $4a^2 - 18a + 18 = 0$ $2a^2 - 9a + 9 = 0$ $(2a - 3)(a - 3) = 0$ $a = \frac{3}{2}, 3$	 M1 A1
4(b)	$x^2y^2 + 36 - 4x^2 - 9y^2$ $= x^2y^2 - 4x^2 - (9y^2 - 36)$ $= x^2(y^2 - 4) - 9(y^2 - 4)$ $= (x^2 - 9)(y^2 - 4)$ $= (x + 3)(x - 3)(y + 2)(y - 2)$	M1 M1 A1
5	Singapore Tuesday 1310 => London Tuesday 0610 Flight 13 hours and 15 minutes => Arrival Tuesday 1925 Or Flight 13 hours and 15 minutes => Arrival 0225 Wednesday Singapore time Singapore 0225 Wednesday => London Tuesday 1925	M1 B1 Or M1 B1 (If no working,

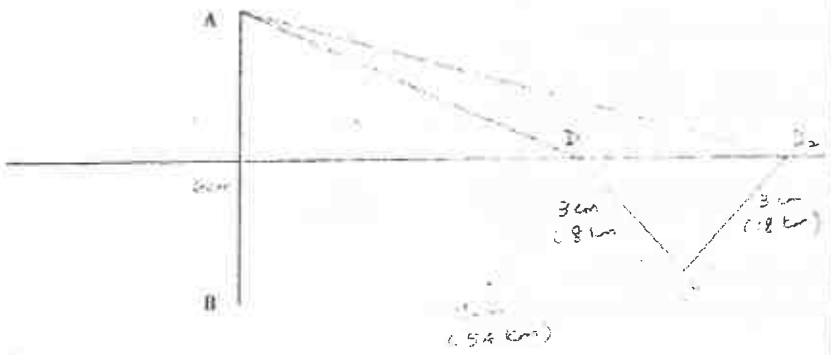
6(a)	$\frac{\sin 39}{12} = \frac{\sin \angle DEF}{10}$ $\angle DEF = \sin^{-1}\frac{10sin39}{12}$ $\angle DEF = 31.63, 180 - 31.63$ $= 31.6, 148.4$	M1 A1
6(b)	Acceptable answer => 31.6°. Reject 148.4° because (148.4 + 39) >180 which is more than angle sum of a triangle.	B1 B1
7	$a^2d^2 - b^2c^2 = c^2d^2$ $b^2c^2 = d^2(a^2 - c^2)$ $b = \pm\frac{d}{c}\sqrt{(a^2 - c^2)}$	M1 A1 No mark if no $\pm$.
8(a)	$\frac{1}{2} - \frac{1}{5^2} = \frac{25-2}{50} = \frac{23}{50}$	M1, A1
8(b)	$b^{\frac{2}{3}+6-\frac{2}{3}-1} = b^5$	M1, A1
9(a)	$x : x + 2 : 6(x + 2)$	B1
9(b)	$(x + 10) + (x + 12) + (6x + 22) = 76$ $x = 4$ $Mother's\ age = 6(4 + 2) - 5 = 31\ years\ old$	M1 A1
10(a)	$In\ \Delta\ ABC\ and\ \Delta DBA$ $\angle BAC = \angle BDA\ (given)$ $\angle ABC = \angle DBA\ (Common\ \angle)$ $\Delta ABC\ is\ similar\ to\ \Delta DBA\ (AA\ Simiarlity)$	}B1(order of vertices must be in corresponding order B1 (statement and reason) No reason no mark
10(b)	$\frac{BC}{BA} = \frac{BA}{BD}$ $\frac{4}{6} = \frac{6}{BD}$	

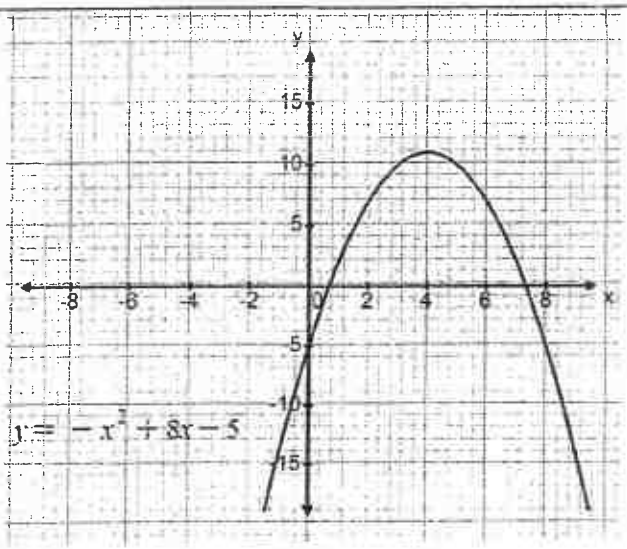
<table>
<tr><td></td><td>$BD = 9$</td><td>B1</td></tr>
<tr><td>10(c)</td><td>Let shortest distance be s.
$\frac{1}{2} \times 6 \times s = 42$
$s = 14cm$</td><td>
M1
A1</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>11(a)</td><td>$\angle RST = \frac{(10-2)180}{10} = 144$</td><td>B1</td></tr>
<tr><td>11(b)</td><td>$\angle SRT = \frac{180-144}{2} = 18° \ (base\ of\ issos.\Delta)$
$\angle RTQ = 18° \ (alt\ \angle)$</td><td>B1</td></tr>
<tr><td>11(c)</td><td>$\angle QRT = \angle QRS - \angle SRT = 144 - 18 = 126°$
$\angle RQT = 180 - 126 - 18 = 36° \ (\angle\ sum\ of\ \Delta)$
$\angle PQT = 144 - 36 = 108°$</td><td>M1

A1</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>12(a)</td><td>$\frac{5}{6}$</td><td>B1</td></tr>
<tr><td>12(b)</td><td>$\frac{1}{36}$</td><td>B1</td></tr>
<tr><td>12(c)</td><td><table>
<tr><th></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th></tr>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td></tr>
<tr><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr>
</table>
$\frac{5}{12}$</td><td>M1

A1</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>13(a)</td><td>$\begin{pmatrix} 4 \\ -5 \end{pmatrix}$</td><td>B1</td></tr>
<tr><td>13(b)</td><td></td><td>B1</td></tr>
</table>

		
13(c)	$\lvert\overrightarrow{ST}\rvert = \sqrt{4^2 + (-5)^2} = 6.40\ unit$ $\lvert\overrightarrow{TU}\rvert = \sqrt{4^2 + 5^2} = 6.40\ unit$	B2
13(d)	Yes. ST=TU. Parallelogram is a rhombus.	B1 B1
14(a)	Let p be the amount invested at 2.4% p.a. $\frac{2.4 \times 2}{100} \times p + \frac{1.8 \times 2}{100} \times (8000 - p) = 348$ $4.8p + 3.6p = 34800 - 28800$ $p = \$5000$	M1 A1
14(b)	$70000 \times 0.75 \times 0.8 \times 0.85 = \$35700$	M1, A1
15(a)	3cm	B1
15(b)	2(Arc length of small semicircle) = $2(\pi \times 3) = 6\pi\ cm$ Radius of big semicircle = 6cm Arc length of big semicircle = $\pi \times 6 = 6\pi\ cm$ Perimeter = $6\pi + 6\pi = 12\pi cm$	 B1
15(c)	Area = $\frac{1}{2}\pi(6^2) - 2(\frac{1}{2}\pi 3^2) = 9\pi\ cm^2$	B1
16(a)	$\frac{3}{5}$	B1
16(b)	$\frac{5}{6}$	B1
16(c)	Yes. Both answers are supposed to be the same.	B1
16(d)	There is a difference in answer because the scale of the vertical	B1

	axis does not start from zero	
17(a)	(18,0)	B1
17(b)	$\frac{1}{6}$	B1
17(c)	k = -4	B1
17(d)	$y = \frac{1}{6}x + 5$	B1
18(a)	$\angle ORS = 90 - 34 = 56\ (radius\ perpendicular\ to\ tangent)$ $\angle ROS = 180 - 2(56) = 68\ (angle\ sum\ of\ issos.triangle)$ $\angle ROP = 2(68) = 136$ $\angle ROP = 2x\ \ (angle\ at\ center = 2\ angles\ at\ circumference)$ $x = 68°$	M1 M1 A1 (If more than 2 reasons not given, deduct 1m overall)
18(b)	$y = 180 - 90 - 68 = 22°\ (angle\ sum\ of\ triangle)$	B1
19(a)	$A' \cup B$	B1
19(b)(i)	{4, 6, 8, 10}	B1
19(b)(ii)	$C \subset A$	B1
20(a) / 20(b)		C1 – Correct angle measurement C1- Correct scale conversion C1 – Perpendicular bisector C1 – Label of Town D (accept either D_1 or D_2)
20(b)	$104° \pm 1°, 114° \pm 1°$	B1
21(a)	$y = -(x^2 - 8x + 5)$ $y = -[(x-4)^2 + 5 - 4^2)]$ $y = -(x-4)^2 + 11$	M1 A1
21(b)		P1- correct shape P1 – correct intercepts and

		turning point.
21(c)	8.58, -0.583	B2

Register No.	Class

Name : _____

BENDEMEER SECONDARY SCHOOL
2017 PRELIMINARY TWO EXAMINATION
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)
Elementary Mathematics
4048 / 02

DATE : **23 August 2017**
DURATION : **2 hours 30 minutes**
TOTAL : **100 marks**

MARK SCHEME

[Turn over

MARK SCHEME

Qn.	Solutions	Remarks
1(a)	$\frac{p-2}{4} \le \frac{1}{2} - \frac{15-2p}{5}$ $\frac{p-2}{4} \le \frac{-25+4p}{10}$ [B1] $10(p-2) \le 4(-25+4p)$ $-6p \le -80$ [B1] $\therefore p \ge 13\frac{1}{3}$ [B1]	
1(b)	(i) $2q-18q^3 = 2q(1-9q^2)$ [B1] $= 2q(1-3q)(1+3q)$ [B1] (ii) $\frac{2q-18q^3}{(4q^2-2q)(3q+1)} = \frac{2q(1-3q)(1+3q)}{(4q^2-2q)(3q+1)}$ $= \frac{2q(1-3q)}{2q(2q-1)}$ [B1] $= \frac{1-3q}{2q-1}$ [B1]	
1(c)	(i) 200 m → 0.2 km, 2 min 30 s → $\frac{1}{24}$ h Speed $= 0.2 / \frac{1}{24}$ [M1] $= 4.8$ km/h [A1] (ii) Best time (Dec) $= 0.9 \times \frac{1}{24}$ [B1] $= \frac{3}{80}$ h = 2 min 15 seconds [B1]	
		Total Marks: 11

MARK SCHEME

2(a)	$T_5 = 11 + 2^4 = 27$ [B1]		
2(b)	n^{th} term $= 2n + 1 + 2^{n-1}$ [B1]		
2(c)	$T_{20} = 2(20) + 1 + 2^{20-1} = 524\,329$ [B1]		
2(d)	Since $2n$ and 2^{n-1} are even, then $T_n = 2n + 1 + 2^{n-1}$ = even + 1 + even = odd [B1]		
2(e)	$T_{m+1} - T_m = 2(m+1) + 1 + 2^{m+1-1} - (2m + 1 + 2^{m-1})$ [B1] $= 2m + 2 + 1 + 2^m - 2m - 1 - 2^{m-1}$ $= 2 + 2^m - 2^{m-1}$ $= 2 + 2^m - \frac{1}{2}(2^m)$ [B1] $= 2 + \frac{1}{2}(2^m)$ $= 2 + 2^{m-1}$ (shown) [B1]		
			Total Marks: 7
3(a)	Time taken to produce 1 large bottle $= \frac{60}{x}$ s [B1]		
3(b)	Time taken to produce 1 small bottle $= \frac{60}{x+4}$ s [B1]		
3(c)	$\frac{60}{x} - \frac{60}{x+4} = 2.5$ [B1] $60(x+4) - 60x = 2.5x(x+4)$ [B1] $240 = 2.5x^2 + 10x$ $x^2 + 4x - 96 = 0$ (shown) [B1]		
3(d)	$x^2 + 4x - 96 = 0$ $(x-8)(x+12) = 0$ [M1] $\therefore x = -12$ (*N.A*), 8 [A1]		
3(e)	Time taken to produce 4000 small bottles $= 4000 \times \frac{60}{8+4}$ [B1] $= 5 \text{ h } 33\frac{1}{3} \text{ min}$ [B1]		Accept: $\approx$ 5 h 33 min
3(f)	In the same duration of time y seconds, Amount earned for selling large bottles $= \$0.50 \times (y/7.5)$ $\approx \$0.067y$ [B1] Amount earned for selling small bottles $= \$0.30 \times (y/5)$ $= \$0.06y$ [B1] $\therefore$ It is more profitable for the factory to produce **large** bottles. [B1]		

MARK SCHEME

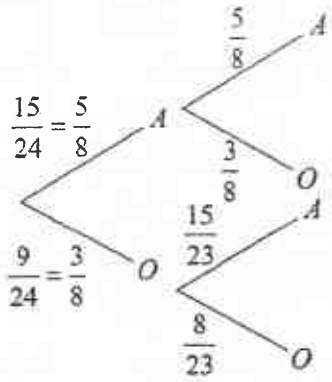
	or Amount earned in 1 min (Large) = 8($0.50) = $4.00 [B1] Amount earned in 1 min (Small) = 12($0.30) = $3.60 [B1] ∴ It is more profitable for the factory to produce **large** bottles. [B1]	
		Total Marks: 12
4(a)	Since M is the midpoint, then O_1M is perpendicular to $O_2\ O_3$. So, $\sin 60^\circ = \dfrac{17}{6+PQ+6}$ [M1] $\therefore PQ = \dfrac{34}{\sqrt{3}} - 12 \approx 7.629909152$ ≈ 7.63 cm [A1] ***or*** Let $O_1\ O_2$ be $2x$. $(2x)^2 = x^2 + 17^2 \rightarrow 3x^2 = 17^2$ $\rightarrow x = \sqrt{96\frac{1}{3}}$ [B1] $\therefore PQ = 2\times\sqrt{96\frac{1}{3}} - 2(6) \approx 7.63$ cm [B1]	
4(b)	Arc length PU $= 6\left(\dfrac{\pi}{3}\right)$ or $\pi\times 2(6)\times\left(\dfrac{60^\circ}{360^\circ}\right)$ ≈ 6.283185307 cm [B1] Perimeter of shaded region $PQRSTU$ $= (6.283185307\times 3) + (7.629909152\times 3)$ [B1] ≈ 41.7 cm [B1] ***or*** Perimeter of shaded region $PQRSTU$ $= 7.629909152 + (3\times\dfrac{\pi}{3}\times 6)$ [B2] ≈ 41.7 cm [B1]	

MARK SCHEME

4(c)	Area of $\Delta O_1O_2O_3$ $= \frac{1}{2}\times 17\times(6+6+\frac{34}{\sqrt{3}}-12)$ ≈ 166.8542278 cm^2 [B1] Area of sector O_1PU $= \frac{1}{2}\times 6^2\times\frac{\pi}{3}$ or or $\pi\times 6^2\times\left(\frac{60^\circ}{360^\circ}\right)$ ≈ 18.84955592 cm^2 [B1] Area of shaded region $PQRSTU$ $= 166.8542278-3(18.84955592)$ ≈ 110 cm^2 [B1] ***or*** Area of shaded region $PQRSTU$ $= \frac{1}{2}\times 17\times 2(\sqrt{96\frac{1}{3}})-\frac{1}{2}\pi(6^2)$ [B2] ≈ 110 cm^2 [B1]	
		Total Marks: 8
5(a)	(i)(a) Median $= 34$ marks [B1] (i)(b) IQR $= 41-26$ [M1] $= 15$ marks [A1] (i)(c) Mean $= \frac{492}{15}$ $= 32.8$ marks S.D. $= \sqrt{\frac{17636}{15}-32.8^2}$ [M1] ≈ 9.99 marks [A1] (ii) The median will increase by 2 and become 36 marks. [B1] The interquartile range will remain the same at 15 marks. [B1]	

MARK SCHEME

5(b) (i)



[B1] Correct branches
[B1] Correct probabilities

(ii)(a) P(both are the same) $=\left(\frac{5}{8}\times\frac{5}{8}\right)+\left(\frac{3}{8}\times\frac{8}{23}\right)$ [B1]

$=\frac{767}{1472}$ [B1]

(ii)(b) P(at least 1 apple) $=1-\left(\frac{3}{8}\times\frac{8}{23}\right)$ [B1]

$=\frac{20}{23}$ [B1]

Total Marks: 13

6(a) (i) $\overrightarrow{BD} = -\frac{3}{2}\mathbf{b}$ [B1]

(ii) $\overrightarrow{AB} = \overrightarrow{AO}+\overrightarrow{OB} = -\mathbf{a}+\mathbf{b}$ [B1]

(ii) $\overrightarrow{BC} = \overrightarrow{BD}+\overrightarrow{DC} = -\frac{3}{2}\mathbf{b}+(-\mathbf{a}+\mathbf{b})$ [B1]

$= -\mathbf{a}-\frac{1}{2}\mathbf{b}$ [B1]

(iv) $\overrightarrow{OM} = \overrightarrow{OD}+\overrightarrow{DM} = -\frac{1}{2}\mathbf{b}+\frac{1}{2}(-\mathbf{a}+\mathbf{b})$ [B1]

$= -\frac{1}{2}\mathbf{a}$ [B1]

6(b) $\overrightarrow{XB} = \overrightarrow{XC}+\overrightarrow{CB} = -\mathbf{a}-\frac{3}{4}\mathbf{b}+\mathbf{a}+\frac{1}{2}\mathbf{b} = -\frac{1}{4}\mathbf{b}$ [B1]

Since $\overrightarrow{BD}=6\overrightarrow{XB}$ → BD // XB and B is a common point, then B, D and X must be collinear points. [B1]

6(c) (i) $\frac{\text{area of } \Delta ODM}{\text{area of } \Delta OAB} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ [B1, B1]

(ii) $\frac{\text{area of } \Delta ODM}{\text{area of } ABCD} = \frac{1}{4}\times\frac{2}{3}\times\frac{1}{2} = \frac{1}{12}$ [B1]

Total Marks: 11

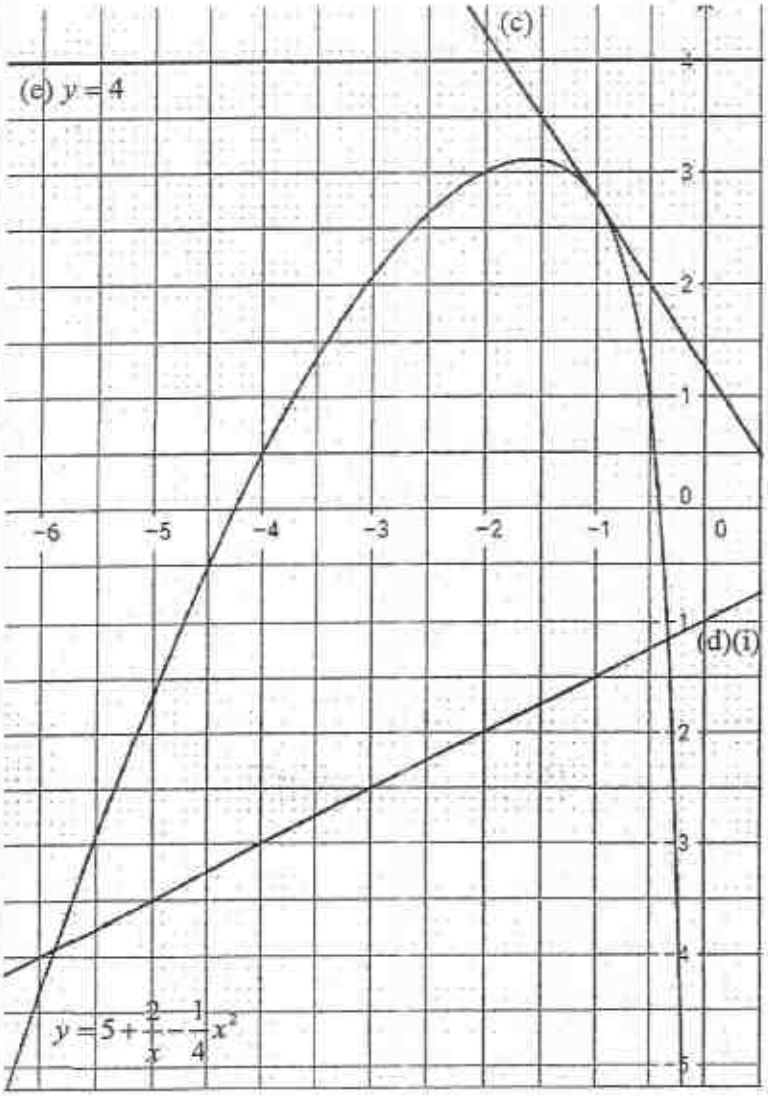
MARK SCHEME

7(a)	$\mathbf{Q} = \begin{pmatrix} 1750 & 1260 \\ 1960 & 1260 \end{pmatrix}$ [B1]	
7(b)	$\mathbf{P} = \begin{pmatrix} 2.00 \\ 2.40 \end{pmatrix}$ [B1]	
7(c)	$\mathbf{S} = \begin{pmatrix} 1750 & 1260 \\ 1960 & 1260 \end{pmatrix}\begin{pmatrix} 2.00 \\ 2.40 \end{pmatrix} = \begin{pmatrix} 6524 \\ 6944 \end{pmatrix}$ [B1]	
7(d)	The earnings of Station A ($6,524) and Station B ($6,944) respectively for Week 1. [B1]	
7(e)	Amount of petrol sold (Week 2) $= 0.95\begin{pmatrix} 1750 & 1260 \\ 1960 & 1260 \end{pmatrix}$ $= \begin{pmatrix} 1662.5 & 1197 \\ 1862 & 1197 \end{pmatrix}$ [B1] Prices of petrol (Week 2) $= 1.05\begin{pmatrix} 2.00 \\ 2.40 \end{pmatrix}$ $= \begin{pmatrix} 2.10 \\ 2.52 \end{pmatrix}$ [B1] Earnings (Week 2) $= \begin{pmatrix} 6507.69 \\ 6926.64 \end{pmatrix}$ The earnings of Station A ($6,507.69) and Station B ($6,926.64) respectively for Week 2. [B1]	
7(f)	$\mathbf{X} = \begin{pmatrix} 1 & 1 \end{pmatrix}$ [B1] Total earnings $= \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 6507.69 \\ 6926.64 \end{pmatrix}$ $= \begin{pmatrix} 13434.33 \end{pmatrix}$ Total earnings of both stations (Week 2) = $13,434.33 [B1]	
		Total Marks: 9

MARK SCHEME

8(a)	(i)	At furthest possible optimal distance, $d = 3.8$ m, $\rightarrow M$ to foot of $X = 4 - 3.8 = 0.2$ m [B1] By Pythagoras' Theorem, W to foot of $X = \sqrt{0.2^2 + (3.6 \div 2)^2} = \sqrt{3.28}$ [B1] So, $TX = \sqrt{3.28 + (3 - 1.24)^2} = \sqrt{6.3776}$ ≈ 2.53 m [B1]	
	(ii)	By Cosine Rule, $3.6^2 = 6.3776 + 6.3776 - 2(6.3776)\cos \angle PXT$ [M1] $\angle PXT \approx 90.9°$ [A1]	
	(iii)	Let the angle of elevation here be θ. $\tan\theta = \dfrac{1.6 - 1.24}{3.8}$ [M1] $\theta \approx 5.4°$ [A1]	
8(b)		$\tan 12° = \dfrac{0.36}{d}$ $\rightarrow d \approx 1.69$ m [B1] Since 1.69 m is less than the minimum optimal distance 1.8 m, Roy will not have an optimal view of the TV in this case. [B1]	
			Total Marks: 9

MARK SCHEME

9(a)	$p = 0.5$	[B1]	
9(b)	 Correct scale + labeling Correct plotting of points Smooth curve	[B1] [B1] [B1]	
9(c)	Drawing of suitable tangent at $x = -1$ Gradient $= \frac{3.5 - 2.75}{-1.5 - (-1)}$ $= -1.5 \quad (\pm 0.2)$	[B1] [B1]	M1 is given if tangent not accurate but correct formula used to find gradient
9(d)	(i) Drawing of correct straight line (ii) $y = \frac{1}{2}x - 1$	[B1] [B1]	

MARK SCHEME

	(iii) For $5+\frac{2}{x}-\frac{1}{4}x^2=\frac{1}{2}x-1$ [B1] $x^3+2x^2-24x-8=0$ So, $A=2$ and $B=24$ [B1]	
9(e)	For $\frac{2}{x}-\frac{1}{4}x^2+1=0 \rightarrow \frac{2}{x}-\frac{1}{4}x^2+5=4$ [B1] For $x<0$, No point of intersection with $y=4$. → No solution (shown) [B1]	
		Total Marks: 12
10(a)	(i) Ave. amount of electricity used per month $=(1107.8+1066.3+1123.6+1259+1249.5+1281.6)/6$ [M1] $=1181.3$ kWh [A1] (ii) Ave. amount paid per month $=1181.3\times\$0.2139\times1.07$ [B1] $\approx\$270.37$ [B1]	
10(b)	Max. no. of solar panels that can be installed = 20 [B1] (Based on calculations $(9\div1.65)\approx5$ [length] and $(4\div)=4$ [width]) ***After installation,*** Ave. amount of electricity saved per month $=19\times20$ $=380$ kWh Ave. amount paid per month $=(1181.3-380)\times\$0.2139\times1.07$ $\approx\$183.40$ [B1] Ave. cost of solar panels per month $=(2\times\$6250)\div(20\times12)$ $\approx\$52.08$ [B1] Total ave. amount paid per month $=\$183.40+\$52.08$ $=\$235.48\ (<\$270.37)$ Since the average amount paid by Mrs Lim per month will be lesser than what she is currently paying for electricity usage, she should go ahead with the installation. [B1]	
		Total Marks: 8

Name _____( ) Class:_____

CHIJ KATONG CONVENT
PRELIMINARY EXAMINATION 2017
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

MATHEMATICS
PAPER 1

4048/01
Duration: 2 hours

Classes: 401, 402, 403, 404, 405, 406, 501, 502

READ THESE INSTRUCTIONS FIRST

Write your name, class and registration number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, hand in separately:
1. **Section A**
2. **Section B**
3. **Section C**

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

FOR EXAMINER'S USE	
Total marks	/80

This question paper consists of 17 printed pages. [Turn over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

CHIJ Katong Convent Preliminary Exam 2017 4048/01 Sec 4E/5NA

Name: _____ ( ) Class: _____

Answer all the questions.

Section A [22 marks]

1 (a) Simplify $\frac{x+1}{x^2-9}-\frac{2}{3-x}$.

Answer ..... [4]

(b) Simplify $\frac{(abc^{-2})^3}{(a^{-4}b^{-1})^{-1}}\times\frac{a^{-6}b^{-7}}{(bc^2)^{-4}}$, leave your answer in positive indices.

Answer ..... [3]

2 Given that $\frac{k}{3}=\sqrt{\frac{A-3b^2}{cA}}$, express A in terms of b, c and k.

Answer $k=$ ..... [3]

[Turn over

3 Factorise the following completely.

(a) $18x^2y+27xy-9xy^3$

Answer ..... [1]

(b) $27a^2-12b^2$

Answer ..... [1]

(c) $3rs-3s-r+1$

Answer ..... [1]

4 Given that $-5 \le x \le 2$ and $-6 \le y \le -1$, find

(a) the largest possible value of $x-y$,

Answer ..... [1]

(b) the smallest possible value of y^2-x^2,

Answer ..... [1]

(c) the smallest possible value of $(x-y)^2$.

Answer ..... [1]

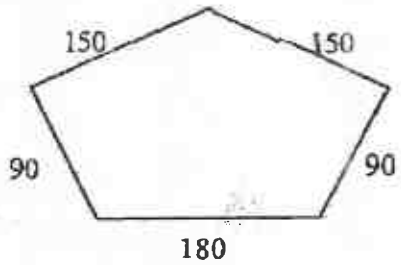
Name: _____ ( ) Class: _____

5 A small bus interchange has 2 feeder buses. Bus number 801 leaves the interchange at 15-minute intervals while number 802 at 25-minutes intervals. If both buses leave together on a particular day, how many times will they leave together in the next 5 hours?

Answer ..... times [3]

---

6 A pond with the shape of a pentagon is shown below (measurements are given in metres and not drawn to scale).



Lamp posts are to be constructed around the pond with the following requirements:

(I) The lamp posts are to be equally spaced from each other.
(II) One lamp post must be constructed at each vertex of the pentagon.
(III) Minimum number of lamp posts are to be constructed to save cost.

Find

(a) the distance between any two lamp posts.

Answer ..... [1]

(b) the number of lamp posts to be constructed.

Answer ..... [2]

Section B [18 marks]

7 When written as the product of their prime factors,

$A = 2^{m+2} \times 3^n$

$B = 2^m \times 3^{n+1} \times 5$, where m and n are positive constants.

Find the lowest common multiple of A and B, giving your answer as a product of its prime factors.

Answer ..... [2]

8 Solve the simultaneous equations.

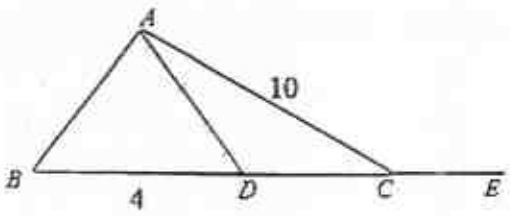
$$\frac{1}{2}x + y = 1,$$

$$\frac{1}{4}x - 3y = 11$$

Answer $x =$ .....

$y =$ ..... [3]

9 In the diagram, $BDCE$ is a straight line, $BD = 4$ cm, $AC = 10$ cm and $AB = AD$.
Given that the area of triangle ABD is 16 cm^2, calculate



(a) the vertical height of triangle ABD. [2]

(b) the value of sin $\angle ACD$. [1]

Answer vertical height = ... cm [2]
sin $\angle ACD$ = ... ° [1]

(c) the value of cos $\angle ACE$.

Answer cos $\angle ACE$ = ... ° [2]

10 During their quest to reach the South Pole on the first day of the new millennium, the Singapore Antarctica 2000 Expedition team experienced temperatures ranging from -35°C to -5°C while their family members in Singapore experienced temperatures ranging from a°C to b°C, where $a < b$.
Find, in terms of a and/or b,

(a) the greatest difference in temperatures between the South Pole and Singapore.

Answer ..... °C [1]

(b) the smallest difference in temperatures between the South Pole and Singapore.

Answer ..... °C [1]

11 Two maps of a new town are drawn. On the first map, a school is represented by an area of 3 cm^2.

The school is represented by an area of 12 cm^2 on the second map.

Given that the scale of the first map is 1 : 80000, find the scale of the second map in the form of 1: n.

Answer 1 : ..... [4]

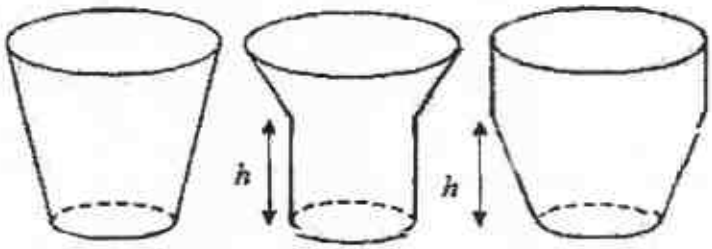
12 Mrs Ang invested \$36 000 in a bank that pays compound interest of 3.2 % per annum, payable every 3 months.

Calculate the amount that Mrs Ang has in the bank after 6 years.

Answer \$..... [2]

Section C [40 marks]

13 Liquid X is poured into three different tanks at a constant rate.
The height of each tank is 2 metres.

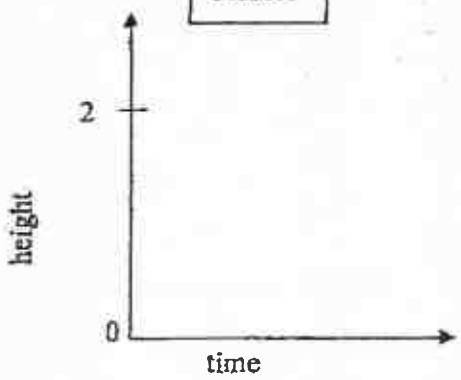


Tank A Tank B Tank C

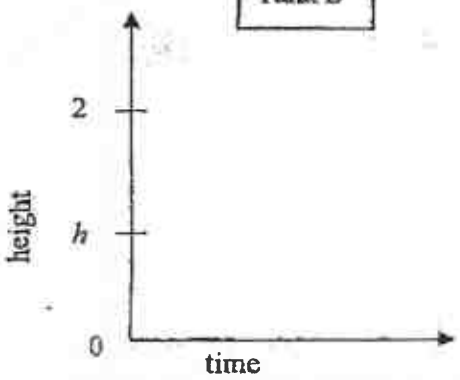
On each of the grids below, sketch the graphs to show how the height of the water changes with time for each tank.

Answer

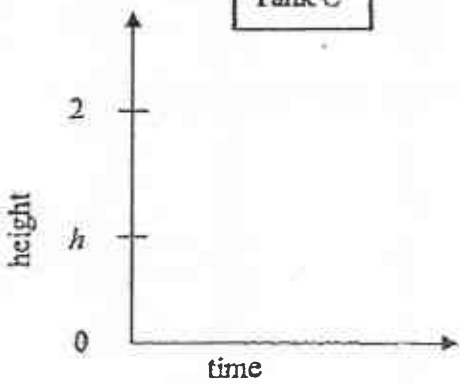
Tank A



Tank B

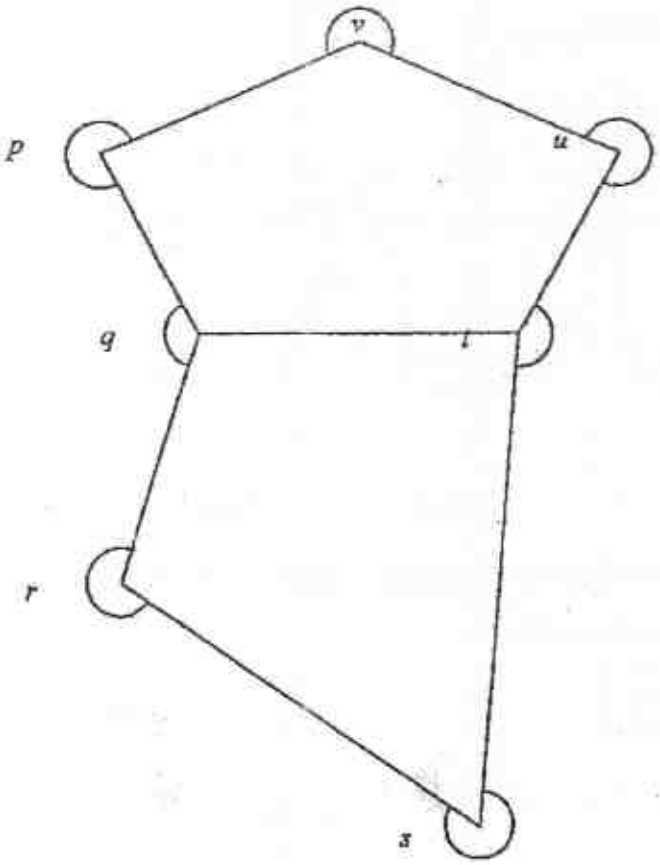


Tank C



[3]

14 (a) Calculate the sum of the angles p, q, r, s, t, u and v shown in the diagram.



Answer ..... ° [2]

(b) A regular polygon has n sides.

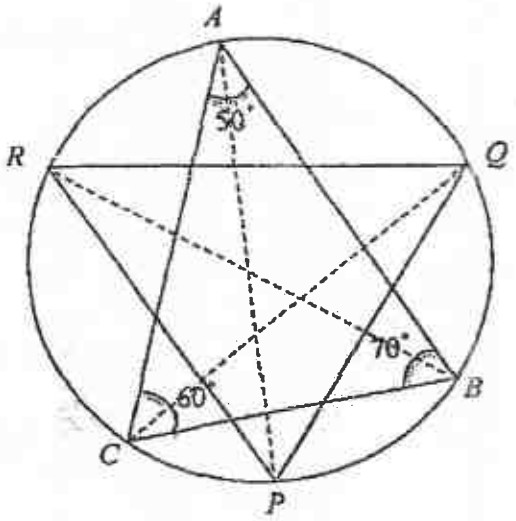
Each exterior angle is $\frac{n}{40}$ degrees.

Find the size of each interior angle in this polygon.

Answer ..... ° [2]

15 In the figure, the vertices of triangle ABC and triangle PQR touch the circumference of the circle.

Given that angle $CAB = 50^{\circ}$, angle $ABC = 70^{\circ}$ and angle $BCA = 60^{\circ}$ and AP, BR and CQ are angle bisectors of angle CAB, angle ABC and angle BCA respectively, find the values of angles RPQ, PQR and PRQ.



Answer angle RPQ = ...° [2]

angle PQR = ...° [1]

angle PRQ = ...° [1]

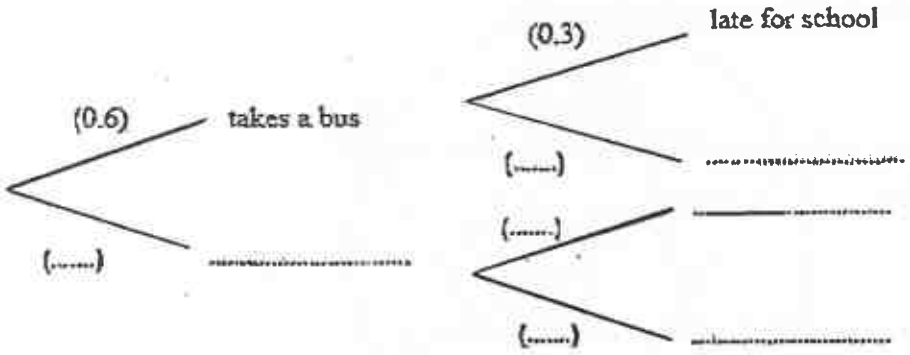
16 The probability that Katie takes a bus is 0.6.

If she takes a bus, the probability that she is late for school is 0.3.

If she does not take a bus, the probability that she is late for school is 0.2.

(a) Complete the probability tree given below

Answer

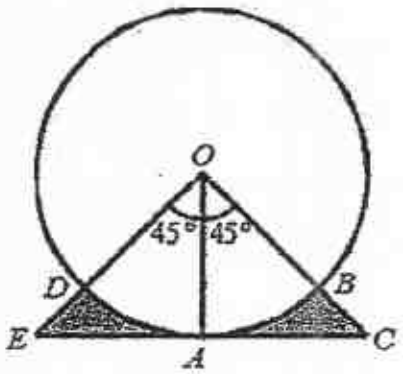


[3]

(b) Calculate the probability that Katie is not late to school.

Answer ..... [2]

17 In the diagram, the circle, centre O, passes through D, A and B.
The tangent at A meets OB produced at C and OD produced at E.
The radius of the circle is 4 cm and angle AOB = angle AOE = 45°.



(a) The area of the shaded region can be expressed as $(a - b\pi)$ cm^2, where a and b are constants.
Find the values of a and b.

Answer $a =$ ..... [2]

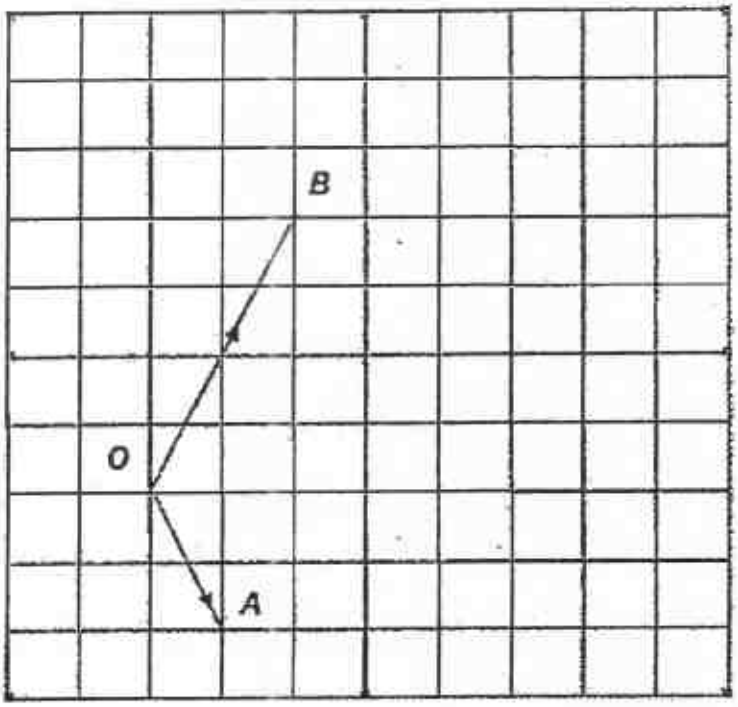
$b =$ ..... [2]

(b) The perimeter of the shaded region can be expressed as $\left(p\pi + 2\sqrt{q}\right)$ cm.
Find the values of p and q.

Answer $p =$ ..... [2]

$q =$ ..... [2]

18 Vectors $\overrightarrow{OB}$ and $\overrightarrow{OA}$ are drawn below.



Given that $\overrightarrow{OP} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$.

(a) (i) locate point P on the grid, mark it with a cross X and label it, [1]

(ii) express $\overrightarrow{OP}$ in terms of $\overrightarrow{OB}$ and/or $\overrightarrow{OA}$.

Answer $\overrightarrow{OP}$ = ... [1]

(b) $OBQA$ is a parallelogram.

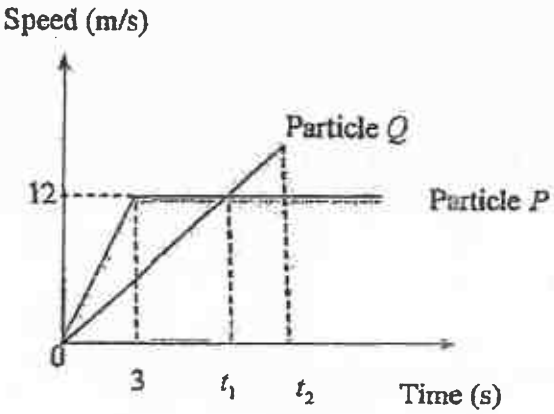
(i) locate point Q on the grid, mark it with a cross X and label it, [1]

(ii) find the column vector representing $\overrightarrow{OQ}$.

Answer $\overrightarrow{OQ}$ = ... [1]

19 The diagram shows the speed-time graphs of two particles P and Q. Both particles

P and Q start from rest. P accelerates uniformly for 3 seconds until it reaches a speed of 12 m/s. It then continues to travel at this constant speed. Q starts from the same point as P but accelerates from rest at a constant rate of 3 m/s^2.



(a) Write down the value of t_1, where the speeds P and Q are the same.

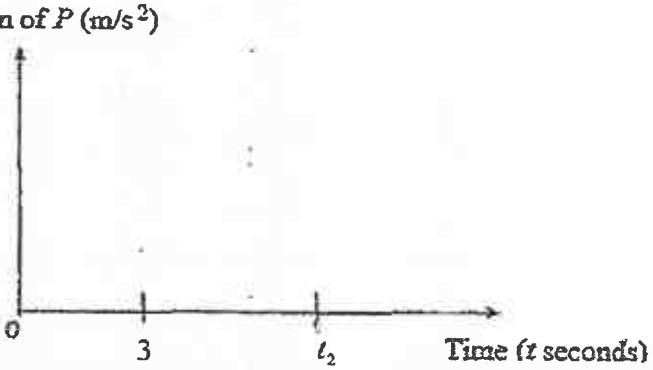
Answer $t_1 =$ ..... [1]

(b) Given that Q overtakes P t_2 seconds after the start of the motion, find the value of t_2

Answer $t_2 =$ ..... [3]

(c) In the answer space below, sketch the acceleration-time graph of P for $0 \le t \le t_2$.

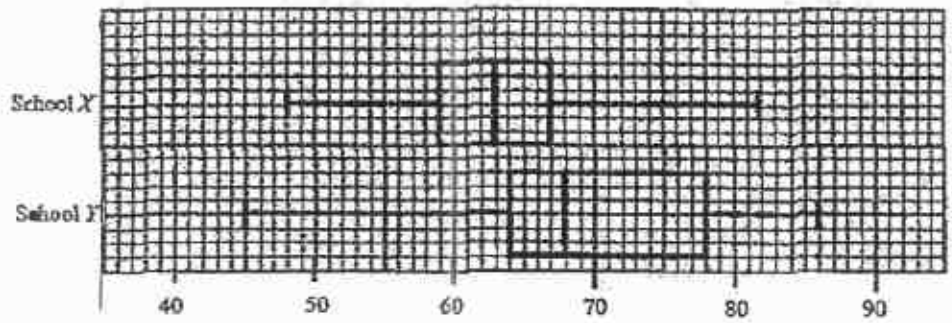
Answer



[1]

20 All the students from 2 schools X and Y took the same examination paper.

The box-and -whisker diagram below shows the results for the two schools.



(a) State, with a reason, which school achieved a better result.

Answer .....

.....

..... [1]

(b) State, with a reason, which school has a more uniformly-distributed mark.

Answer .....

.....

.....[1]

21 The numbers in the Number Triangle are consecutive even numbers.

Name: _____( ) Class: _____

Row	Number Triangle	Sum of row (R)	No. of even numbers (E)	Average of Row (A)
1	2	2	1	2
2	4 6	10	2	5
3	8 10 12	30	3	10
4	14 16 18 20	68	4	p
5	22 24 26 28 30	130	5	26
6	32 34 36 38 40 42	q	6	37

(a) Find the values of p and q.

Answer $p =$ ....., $q =$ ..... [2]

(b) Write down a formula connecting A and E.

Answer ..... [1]

(c) Write down a formula connecting R and E.

Answer ..... [1]

(d) Justify, with reason why the number 6400 could not appear in the column A.

Answer .....

.....

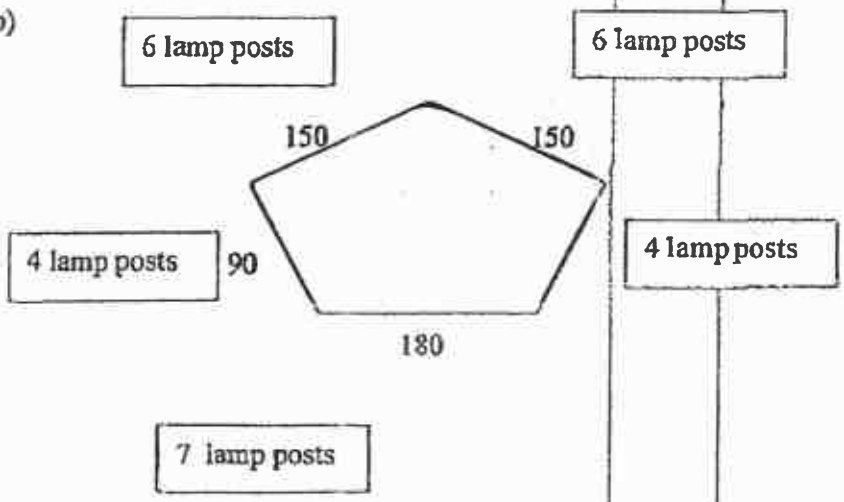
..... [1]

End of Paper

2017 4E/5N P1 E Mathematics Prelim Marking Scheme

Qn	Solution		
1a	Section A $\frac{x+1}{x^2-9}-\frac{2}{3-x}=\frac{x+1}{(x-3)(x+3)}+\frac{2}{x-3}$ $=\frac{x+1+2(x+3)}{(x-3)(x+3)}$ $=\frac{3x+7}{(x+3)(x-3)}$		
1b	$\frac{(abc^{-2})^3}{(a^{-4}b^{-1})^{-1}}\times\frac{a^{-6}b^{-7}}{(bc^2)^{-4}}=\frac{a^3b^3c^{-6}}{a^4b^1}\times\frac{a^{-6}b^{-7}}{b^{-4}c^{-8}}$ $=\frac{a^{-3}b^{-4}c^{-6}}{a^4b^{-3}c^{-8}}$ $=a^{-7}b^{-1}c^2$ $=\frac{c^2}{a^7b}$		
2	$\frac{k}{3}=\sqrt{\frac{A-3b^2}{cA}}$ $\frac{k^2}{9}=\frac{A-3b^2}{cA}$ $k^2cA=9A-27b^2$ $A(k^2c-9)=-27b^2$ $A=\frac{27b^2}{9-k^2c}$ OR $A=\frac{-27b^2}{(ck^2-9)}$		
3	(a) $9xy(2x+3-y^2)$ (b) $3(3a-2b)(3a+2b)$ (c) $(r-1)(3s-1)$		

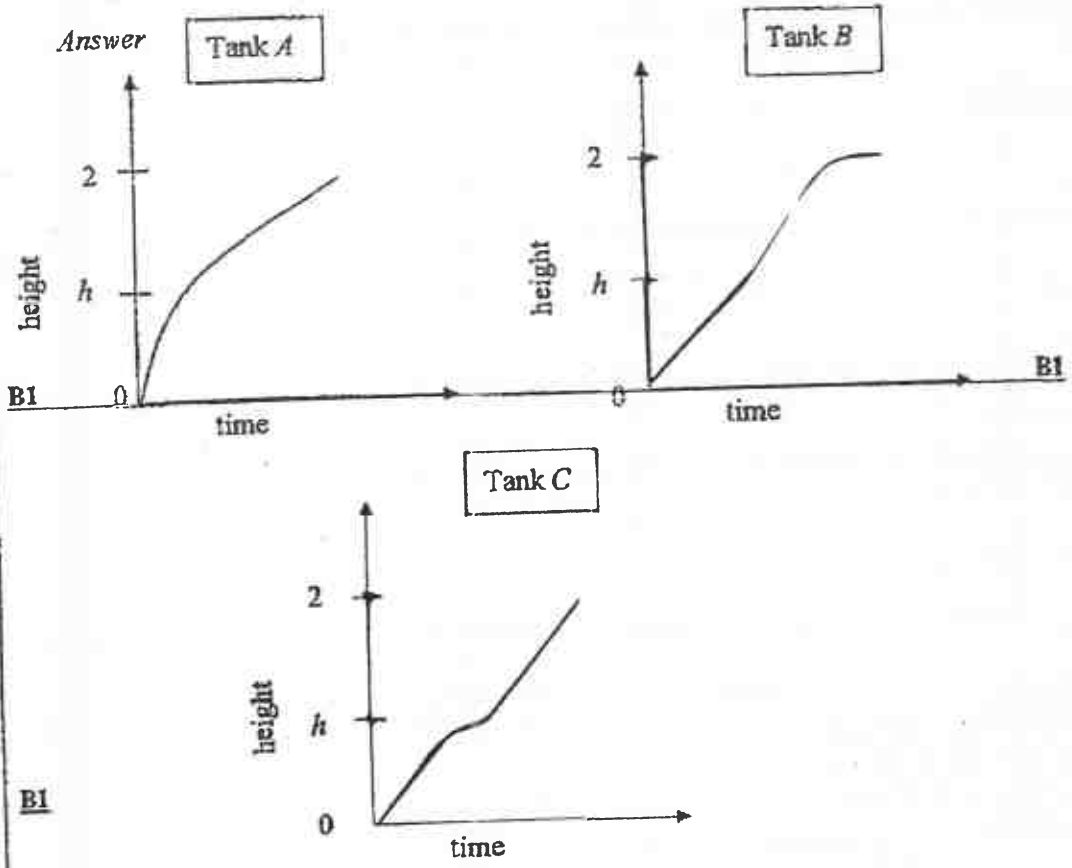
2017 4E/5N P1 E Mathematics Prelim Marking Scheme

Qn	Solution
4	(a) 8 (b) -24 (c) 0
5	Bus S01 Bus S02 3 | 15 25 5 | 5 25 5 | 1 5 1 1 LCM is 75 5 hours = 300 mins $\frac{300}{75}$ = 4 *times*
6	(a) HCF of 150, 90, 180 is 30m (b)  6 + 6 + 4 + 4 + 7 = 27 lamp posts Double counting answer 27 – 5 = 22 lamp posts
	SECTION B [18m]

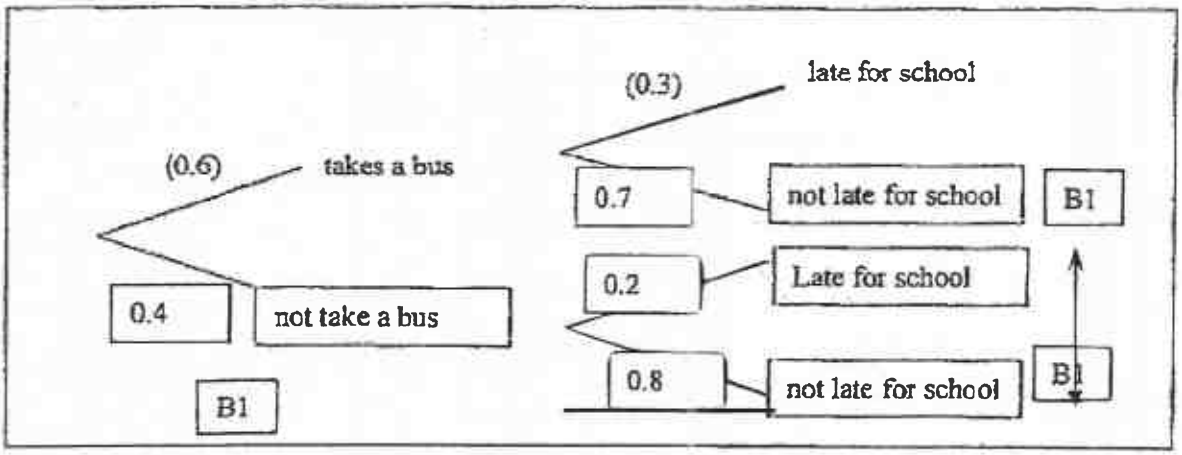
2017 4E/5N P1 E Mathematics Prelim Marking Scheme

Qn	Solution
7	$A = 2^m \times 2^2 \times 3^n$ $B = 2^m \times 3^n \times 3 \times 5$ $LCM = 2^{m+2} \times 3^{n+1} \times 5$
8	$x = 2 - 2y$ $\frac{1}{4}(2-2y) = 11 + 3y$ $y = -3$ $x = 8$
9	(a) $\frac{1}{2} \times 4 \times h = 16$ $h = 8$ (b) $\sin \angle ACD = \frac{8}{10} = \frac{4}{5}$
	(c) $XC = \sqrt{10^2 - 8^2} = 6$ $\cos \angle ACE = -\frac{6}{10} = -\frac{3}{5}$
10	(a) $35 + b$ (b) $5 + a$
11	1 cm^2: 64×10^8 cm^2 Map 1 3 cm^2 : 192 X 10^8 cm^2 Map 2 12 cm^2 : 192 X 10^8 cm^2 1 cm^2 : 16 X 10^8 cm^2 1: 40000
12	Amount = \$36000 $\left(1 + \frac{\frac{3.2}{4}}{100}\right)^{24}$ = \$43586.83

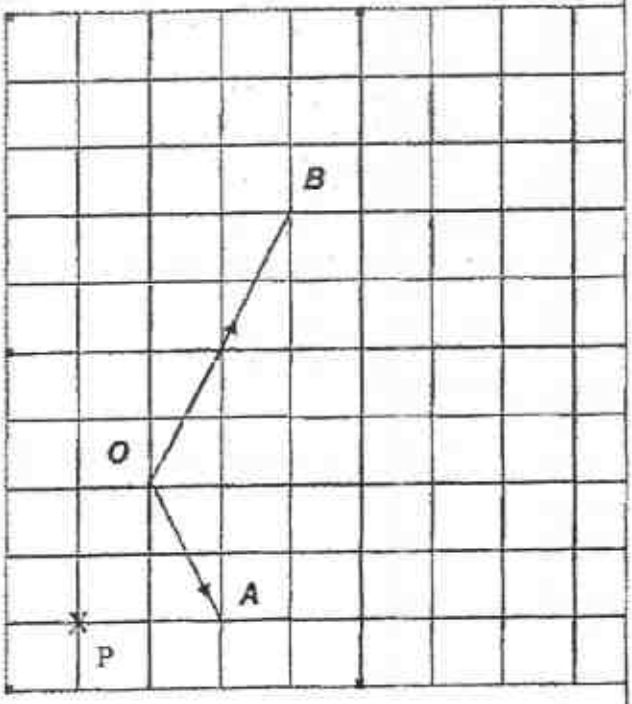
2017 4E/5N P1 E Mathematics Prelim Marking Scheme

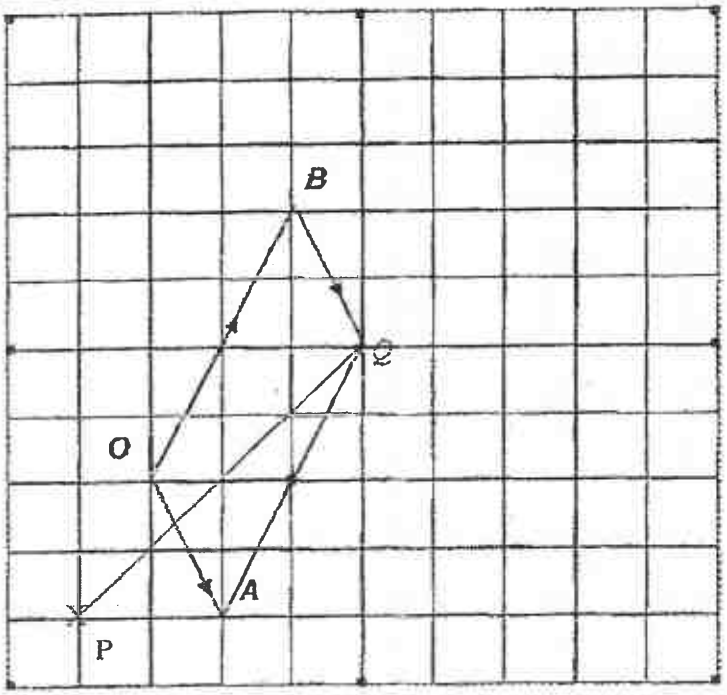
On	Solution		
13	**Section C [40m]** *Answer*  B1 B1 B1		
14	(a) Total angles in the 2 polygons $= 540^0 + 360^0$ $= 900^0$ sum of all required angles $= 7 \times 360^0 - 900^0$ $= 1620^0$		
	$\frac{360^0}{n} = \frac{n}{40}$ (b) $n^2 = 14400$ $n = 120$		
15	angle RPQ = angle RPA + angle APQ = angle RPA + angle ACQ (angles in the same segment) $= 35^0 + 30^0 = 65^0$ angle PQR = angle PQC + angle CQR = angle PAC + angle CBR = 25 + 35 = 60° angle PRQ = 25 + 30 = 55°		

2017 4E/5N P1 E Mathematics Prelim Marking Scheme

Qn	Solution
16 a	
	16b $0.6 \times 0.7 + 0.4 \times 0.8 = 0.74$ **M1** **A1** **(multiplication of probability from the tree)**
17	(a) Angle $OAE = 90^0$ $OA = AE = AC$ Area of shaded region $= \frac{1}{2} \times 8 \times 4 - \frac{1}{2} \times 4^2 \times \pi/2$ $= 16 - 4\pi$ $x = 16$ $y = 4$
17 (b)	$OE = \sqrt{4^2 + 4^2} = \sqrt{32}$ $4\left(\frac{\pi}{2}\right) + 8 + 2\left(\sqrt{32} - 4\right)$ $= 2\pi + 8 + 2\sqrt{32} - 8$ $= 2\pi + 2\sqrt{32}$ $p = 2$ $q = 32$

2017 4E/5N P1 E Mathematics Prelim Marking Scheme

On	Solution		
18 ai			A1

18 aii	$\overrightarrow{OP} = -\frac{1}{2}\overrightarrow{OB}$		A1
18 b			A1

18bii $\overrightarrow{PQ} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ A1

19 (a) 4s

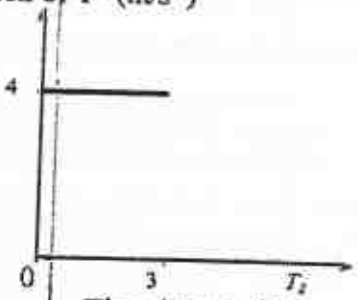
(b) $\frac{1}{2} \times T_2 \times 3T_2 = \frac{1}{2} \times 3 \times 12 + 12(T_2 - 3)$

$T_2^2 - 8T_2 + 12 = 0$

$(T_2 - 6)(T_2 - 2) = 0$

$T_2 = 6$

(c) Acceleration of P (m/s^2)



20 (a)

Sch Y achieved better results because it has a higher median of 68 as compared to 63 for X.

(b)

Sch X is more uniform because of a smaller interquartile range of 8 as compared to 14 for Y.

21 a. p = 17

q = 222

b. $A = E^2 + 1$

c. $R = E^3 + E$

d. $6400 = 80^2$, a perfect square number, but the number in column A are not perfect square numbers.

Answer all the questions.

Section A [30 marks]

1 (a) Expand and simplify $(4x-1)^2-(8x+1)(2x-1)$. [2]

(b) Express $\frac{4x^2-9}{x^2+x-20} \div \frac{4x^2-6x}{16-x^2}$ as a fraction in its lowest term. [3]

(c) Solve the equation $\frac{x}{3}-\frac{2x-1}{x-3}=-2$, leaving your answer correct to 3 decimal places. [3]

(d) y is directly proportional to x^2.
It is known that $y = 144$ for a particular value of x.
Find the percentage change in y when the value of x decreases by 25%. [3]

---

2 During a school's sports day, the number of first, second and third positions won by the different houses are given in the table below.
The number of points won for individual and group events are also given in the table.

Houses	Individual events			Group events		
	First	Second	Third	First	Second	Third
Blue	7	5	4	3	2	0
Green	5	4	6	1	2	1
Red	4	5	5	1	2	2
Yellow	4	6	5	1	0	3
Points	5	3	1	10	6	2

(a) It is given that $\mathbf{A} = \begin{pmatrix} 7 & 5 & 4 \\ 5 & 4 & 6 \\ 4 & 5 & 5 \\ 4 & 6 & 5 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$, evaluate the matrix $\mathbf{P} = \mathbf{AB}$. [2]

(b) Given matrix $\mathbf{C} = \begin{pmatrix} 3 & 2 & 0 \\ 1 & 2 & 1 \\ 1 & 2 & 2 \\ 1 & 0 & 3 \end{pmatrix}$.

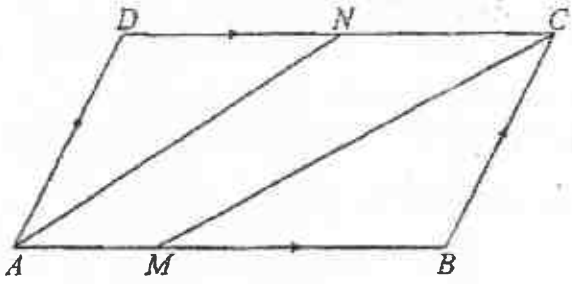
(i) Represent the group event scoring system in a 3×1 matrix $\mathbf{D}$. [1]

(ii) Evaluate the matrix $\mathbf{Q} = \mathbf{CD}$ and explain what do the elements of $\mathbf{Q}$ represent. [2]

(c) The scores of individual events and group events are added for each house.
Using matrix manipulation, determine which house won the overall championship. [2]

---

3 $ABCD$ is a parallelogram.
N is the midpoint of DC and M is the point on AB such that $2AM = MB$.



Given that $\overrightarrow{AB} = 6\mathbf{a}$ and $\overrightarrow{AD} = 4\mathbf{b}$,

(a) Express as simply as possible, in terms of **a** and/or **b**.

(i) $\overrightarrow{AM}$ [1]

(ii) $\overrightarrow{MC}$ [1]

(iii) $\overrightarrow{AN}$ [1]

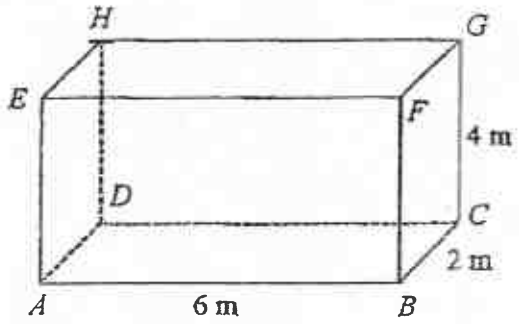
(b) Find the numerical value of

(i) $\dfrac{\text{area of triangle } ADN}{\text{area of parallelogram } ABCD}$, [1]

(ii) $\dfrac{\text{area of triangle } ADN}{\text{area of triangle } AMN}$. [2]

---

4 The diagram shows a rectangular cuboid $ABCDEFGH$.
$AB = 6$ m, $BC = 2$ m and $CG = 4$ m.



(a) Show that angle $HBD = 32.3°$, correct to 1 decimal place. [2]

(b) Calculate angle AFC. [3]

(c) Calculate the greatest angle of elevation of the point H when viewed from the line AB. [1]

Section B [70 marks]
Please begin Question 5 on a NEW sheet of paper

5 (a) Chloe has a total of 126 marks in x tests.
In the next two tests, she scored 9 marks and 8 marks respectively.

Find, in terms of x, her mean mark for the

(i) first x tests, [1]

(ii) $(x+2)$ tests. [1]

Her mean mark for the first x tests was one greater than her mean mark for the $(x+2)$ tests.

(iii) write an equation in x to represent this information and show that it reduces to $x^2+19x-252=0$. [3]

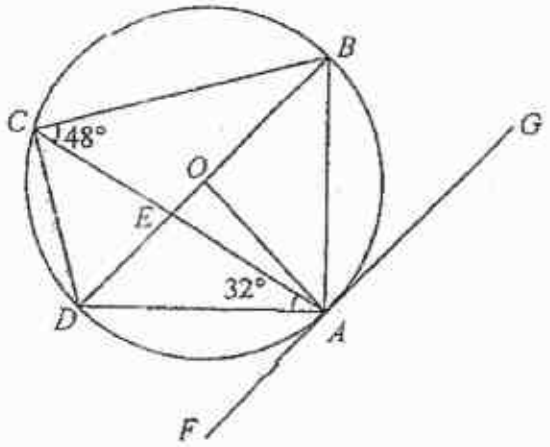
(iv) Solve the equation to find the number of tests Chloe took initially. [3]

(b) Amanda has a mean of 13.5 marks for the first $(x+1)$ tests, but her mark on the last test gave her a mean of 14 marks for the $(x+2)$ tests.

Calculate the number of marks Amanda scored in the last test. [2]

---

6 In the diagram, O is the centre of the circle through A, B, C and D.
FG is the tangent to the circle at A.
AC intersects BD at E.
Angle $ACB = 48°$ and angle $CAD = 32°$.



(a) Calculate the following angles, stating your reasons clearly.

(i) Angle ABO [2]

(ii) Angle CDA [2]

(iii) Angle GAB [2]

(b) Explain why BD is not parallel to GF. [2]

[Turn over

7 **(a)** The frequency table shows the weekly expenditure on food of n students from School X.

Weekly expenditure ($\$x$)	Frequency
$30 < x \leq 40$	8
$40 < x \leq 50$	17
$50 < x \leq 60$	34
$60 < x \leq 70$	p
$70 < x \leq 80$	3

(i) If $\frac{5}{16}$ of the n students have a weekly expenditure of at most \$50, show that the value of p is 18. [2]

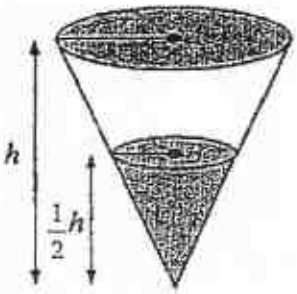
(ii) Calculate an estimate of

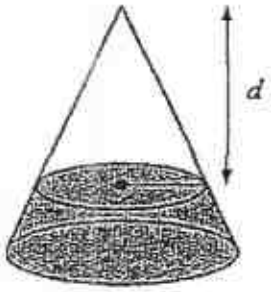
(a) the mean weekly expenditure on food, [1]

(b) the standard deviation. [1]

(iii) The standard deviation of the weekly expenditure on food of students from School Y was \$5.62.
Using this information, comment on one difference between the two distributions. [1]

(b) The diagram shows an inverted cone of height h and radius r.
It contains water to a depth of $\frac{1}{2}h$.



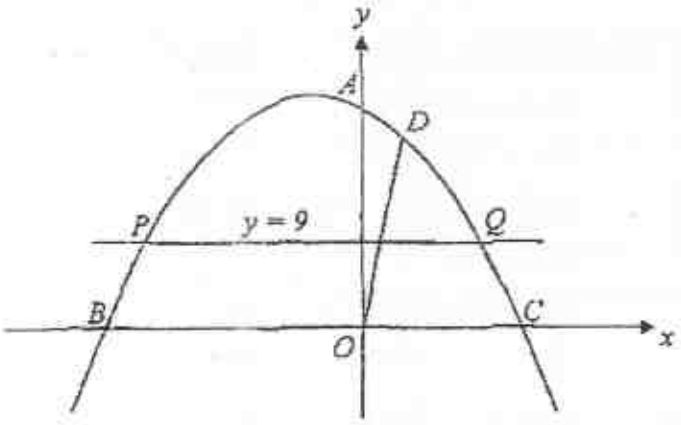


(i) Find the ratio of area of surface B to area of surface A. [1]

(ii) Find the volume of the water if the cone can hold 480 cm^3 of water when full. [2]

(iii) The cone is now inverted such that the liquid rests on the flat circular base of the cone, as shown in the diagram on the right.
Find, in terms of h, an expression for d, the vertical distance of the liquid surface from the tip of the cone. [3]

8 The diagram shows the curve $y = (4 - x)(x + k)$, where k is a constant.
The curve cuts the y-axis at the point $A(0, 24)$, and the x-axis at B and C.



(a) Show that the value of k is 6. [1]

(b) Write down the coordinates of B and C. [2]

(c) Find the coordinates of the maximum point on the curve. [2]

(d) $D(1, m)$ is a point on the given curve.
Find the value of m and the equation of the line OD. [3]

(e) The line $y = 9$ intersects the curve at P and Q. Find the coordinates of P and Q. [3]

9 A student needed to make a circular face mask for a school performing arts event. She took a circular sheet of radius 10 cm and removed two circles, each of radius 2.5 cm for two eyes and an isosceles triangle of base 2 cm and equal sides of 3 cm each for a nose, as shown in **Diagram I**.

The mouth is shown in the **Diagram II**.
It is formed by an arc, AXB, of a circle, centre O and radius 3 cm.
AYB is the arc of another circle with diameter, AB, 3 cm.

She painted the remaining area.

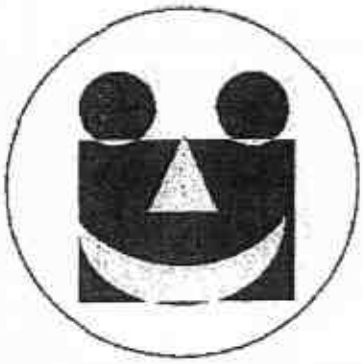


Diagram I

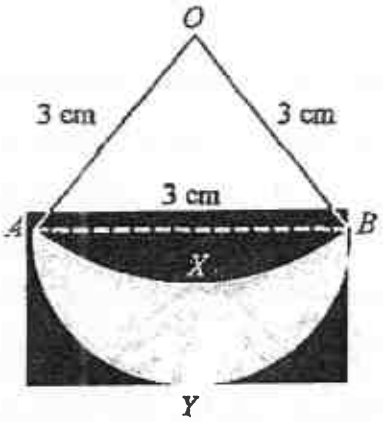


Diagram II

(a) Calculate the area removed. [7]

(b) Calculate the area of mask that was painted. [2]

10 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y=5-\frac{x^2}{10}-\frac{4}{x}$. Some corresponding values are given in the following table.

x	0.5	0.7	1	2	3	4	5	6	7	8
y	−3.0	−0.8	0.9	2.6	2.8	k	1.7	0.7	−0.5	−1.9

(a) Calculate the value of k. [1]

(b) Taking 2 cm to represent 1 unit on each axis, draw a horizontal x-axis for $0\le x\le 8$ and a vertical y-axis for $-3\le y\le 3$, draw the graph of $y=5-\frac{x^2}{10}-\frac{4}{x}$ for the values of x in the range $0.5\le x\le 8$. [3]

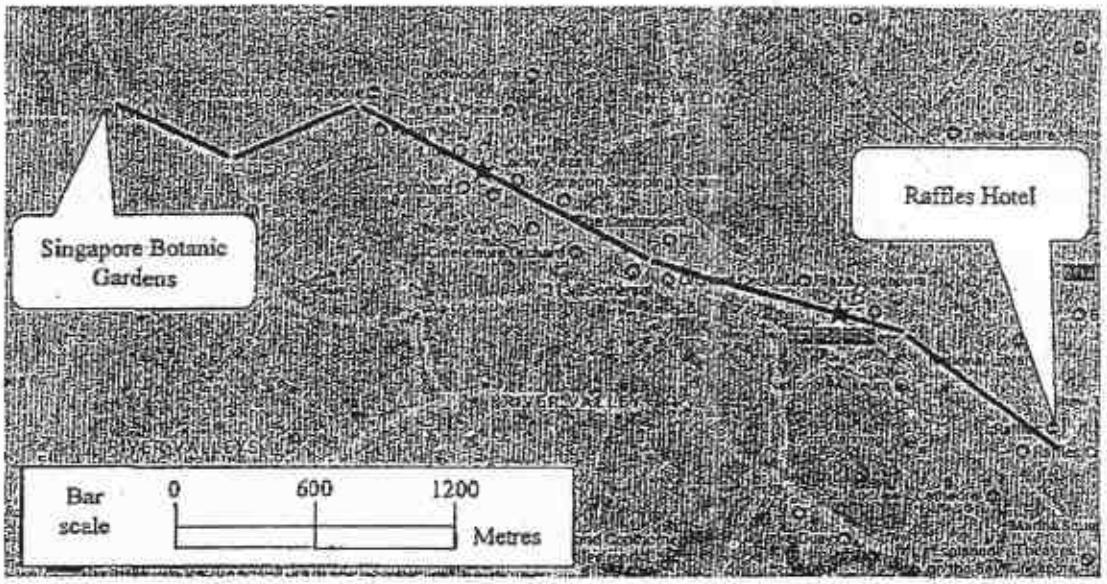
(c) Use your graph to find the greatest value of $5-\frac{x^2}{10}-\frac{4}{x}$ in the interval $0.5\le x\le 8$. [1]

(d) By drawing a tangent, find the gradient of the graph at the point where $x=2$. [2]

(e) Use your graph to solve $5-\frac{x^2}{10}-\frac{4}{x}=2$ in the range $0.5\le x\le 8$. [3]

(f) By drawing a suitable straight line, find the range of values of x in the interval $0.5\le x\le 8$ for which $5-\frac{x^2}{10}-\frac{4}{x}\ge x$. [2]

11 Cheryl works at the Singapore Botanic Gardens.
She needs to rush down to meet a client at Raffles Hotel.
The quickest route from Cheryl's location to Raffles Hotel is indicated on the map with black solid lines.
The bar scale on the lower left corner of the map provides the corresponding actual ground distance.



(a) Calculate the actual travelling distance, in kilometres, between Cheryl's location and Raffles Hotel, giving your answer correct to 2 significant figures. [2]

(b) At 6.14 pm, Cheryl decided to call for a ride from Singapore Botanic Gardens to Raffles Hotel.

Information about FastDel Cab and Aber services and other travelling details are on the opposite page.
Along the way, there are two ERP gantries, indicated by A and B with a star each on the map.

Determine which service Cheryl should choose.
Justify your answer with relevant working. [7]

Travelling time

From	To	Duration
Singapore Botanic Gardens	Orchard ERP (*A*)	6 minutes
Orchard ERP	Handy Road ERP (*B*)	5 minutes
Handy Road ERP	Raffles Hotel	4 minutes

ERP Charges

Orchard (*A*)		**Handy Road Gantry (*B*)**	
12.00 pm – 5.29 pm	$0.50	12.00 pm – 12.04 pm	$0.50
5.30 pm – 5.34 pm	$1.00	12.05 pm – 1.59 pm	$1.00
5.35 pm – 5.59 pm	$1.50	2.00 pm – 2.04 pm	$1.50
6.00 pm – 6.54 pm	$2.00	2.05 pm – 2.54 pm	$2.00
6.55 pm – 6.59 pm	$1.50	2.55 pm – 2.59 pm	$1.50
7.00 pm – 7.59 pm	$1.00	3.00 pm – 5.29 pm	$1.00
		5.30 pm – 5.59 pm	$0.50
		6.00 pm – 7.54 pm	$1.00
		7.55 pm – 7.59 pm	$0.50

FastDel Cab Service

The first 1 km or less		$3.20	
Every 400 m thereafter or less up to 10 km		$0.22	
Every 350 m thereafter or less after 10 km		$0.22	
Current Booking Peak Period ($3.30) Monday to Friday (Except Public Holidays):	6.00 am – 9.29 am	Monday to Sunday & Public Holidays:	6.00 pm – 11.59 pm
Peak Period Surcharge (25% of metered fare) Monday to Friday (Except Public Holidays):	6.00 am – 9.29 am	Monday to Sunday & Public Holidays:	6.00 pm – 11.59 pm
ERP Charge Passengers are required to bear the ERP charge shown on the upper display of the In-vehicle Unit. The ERP charge is deducted each time the taxi passes under the ERP gantry, payable on top of metered fare			

Aber Service

Base Fare	$3.00
Travelling time per minute	$0.20
Travelling distance per km	$0.45
6 pm to 8 pm peak period surge	2.5× of normal fare

End of Paper

4E5N Mathematics Preliminary Exam 2017 (Paper 2)

Section A

1(a)	$(4x-1)^2-(8x+1)(2x-1)$ $=16x^2-8x+1-(16x^2-6x-1)$ $=16x^2-8x+1-16x^2+6x+1$ $=-2x+2$	
1(b)	$\frac{(4x^2-9)}{(x^2+x-20)}\div\frac{(4x^2-6x)}{(16-x^2)}$ $=\frac{(2x-3)(2x+3)}{(x+5)(x-4)}\div\frac{2x(2x-3)}{-(x-4)(x+4)}$ $=\frac{(2x-3)(2x+3)}{(x+5)(x-4)}\times\frac{-(x-4)(x+4)}{2x(2x-3)}$ $=\frac{-(2x+3)(x+4)}{2x(x+5)}$	
1(c)	$\frac{x}{3}-\frac{2x-1}{x-3}=-2$ $\frac{x(x-3)-3(2x-1)}{3(x-3)}=-2$ $x^2-3x-6x+3=-6(x-3)$ $x^2-9x+3=-6x+18$ $x^2-3x-15=0$ $x=\frac{-(3)\pm\sqrt{(-3)^2-4(1)(-15)}}{2(1)}$ $=5.653$ or -2.653	
1(d)	$y=kx^2$ $144=kx^2$ Original value: x New value: $0.75x$ $Y=kX^2$ $Y=k(0.75x)^2$ $=0.5625kx^2$ $=0.5625(144)$ $=81$ Percentage change $=\frac{81-144}{144}\times100$ $=-43.75\%$	

2(a)

$$P = \begin{pmatrix} 7 & 5 & 4 \\ 5 & 4 & 6 \\ 4 & 5 & 5 \\ 4 & 6 & 5 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 54 \\ 43 \\ 40 \\ 43 \end{pmatrix}$$

2(bi)

$$D = \begin{pmatrix} 10 \\ 6 \\ 2 \end{pmatrix}$$

2(bii)

$$Q = \begin{pmatrix} 3 & 2 & 0 \\ 1 & 2 & 1 \\ 1 & 2 & 2 \\ 1 & 0 & 3 \end{pmatrix} \begin{pmatrix} 10 \\ 6 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 42 \\ 24 \\ 26 \\ 16 \end{pmatrix}$$

The elements of **Q** represent the **total score from group events for each house respectively.**

2(c)

$$\text{Total score} = \begin{pmatrix} 54 \\ 43 \\ 40 \\ 43 \end{pmatrix} + \begin{pmatrix} 42 \\ 24 \\ 26 \\ 16 \end{pmatrix}$$

$$= \begin{pmatrix} 96 \\ 67 \\ 66 \\ 59 \end{pmatrix}$$

Blue house won overall championship.

3(ai)	$2\overline{AM} = \overline{MB}$ $\frac{AM}{MB} = \frac{1}{2}$ $\overline{AM} = \frac{1}{3}\overline{AB}$ $= \frac{1}{3}(6\mathbf{a})$ $= 2\mathbf{a}$	
3(aii)	$\overline{MC} = \overline{MB} + \overline{BC}$ $= \frac{2}{3}(6\mathbf{a}) + 4\mathbf{b}$ $= 4\mathbf{a} + 4\mathbf{b}$	
3(aiii)	$\overline{DN} = \frac{1}{2}\overline{DC}$ $= 3\mathbf{a}$ $\overline{AN} = \overline{AD} + \overline{DN}$ $= 3\mathbf{a} + 4\mathbf{b}$	
3(bi)	$\frac{\text{area of triangle } ADN}{\text{area o parallelogram } ABCD} = \frac{\frac{1}{2}(h)(DN)}{(h)(DC)}$ $= \frac{\frac{1}{2}(DN)}{(DC)}$ $= \frac{1}{2} \times \frac{1}{2}$ $= \frac{1}{4}$	
3(bii)	$\frac{\text{area of triangle } ADN}{\text{area of triangle } AMN} = \frac{DN}{AM}$ $= \frac{\frac{1}{2}(DC)}{\frac{1}{3}(DC)}$ $= \frac{3}{2}$	

4(a)

$DB^2 = 6^2 + 2^2$

$= 40$

$DB = \sqrt{40}$

$= 6.3245$

$\tan \angle HBD = \frac{4}{\sqrt{40}}$

$\angle HBD = \tan^{-1}\left(\frac{4}{\sqrt{40}}\right)$

$= 32.311°$

$= 32.3°$ (1 d.p.)

4(b)

$AF^2 = 6^2 + 4^2$ $\quad$ $FC^2 = 2^2 + 4^2$

$= 52$ $\quad$ $= 20$

$AF = \sqrt{52}$ $\quad$ $FC = \sqrt{20}$

$= 7.2111$ $\quad$ $= 4.4721$

$AC = DB$

$= \sqrt{40}$

$= 6.3245$

$AC^2 = AF^2 + FC^2 - 2(AF)(FC)\cos \angle AFC$

$\cos \angle AFC = \frac{AF^2 + FC^2 - AC^2}{2(AF)(FC)}$

$= \frac{52 + 20 - 40}{2\left(\sqrt{52}\right)\left(\sqrt{20}\right)}$

$\angle AFC = \cos^{-1}\left(\frac{32}{2\left(\sqrt{52}\right)\left(\sqrt{20}\right)}\right)$

$= 60.255°$

$= 60.3°$ (1 d.p.)

4(c)

$\tan \angle HAD = \frac{4}{2}$

$\angle HAD = \tan^{-1}(2)$

$= 63.434°$

$= 63.4°$ (1 d.p.)

$\therefore$ greatest angle of elevation is 63.4°

Section B

5(a)	Mean mark for first x tests $= \frac{126}{x}$	
5(b)	Mean mark for first $(x+2)$ tests $= \frac{126+9+8}{x+2}$ $= \frac{143}{x+2}$	
5(c)	$\frac{126}{x} - \frac{143}{x+2} = 1$ $\frac{126(x+2)-143x}{x(x+2)} = 1$ $126x+252-143x = x^2+2x$ $252-17x = x^2+2x$ $x^2+19x-252 = 0$ (shown)	
5(d)	$x^2+19x-252=0$ $(x-9)(x+28)=0$ $x=9$ or -28 (reject) $\therefore$ Chloe took 9 tests initially.	
5(e)	Number of marks Amanda scored in the last test $= 14(x+2)-13.5(x+1)$ $= 14(11)-13.5(10)$ $= 19$	
6(ai)	$\angle BDA = 48°$ (angles in the same segment) $\angle ABO = 90° - 48°$ (right angle triangle in semicircle) $= 42°$ OR $\angle DCE = 90° - 48°$ (right angle triangle in semicircle) $= 42°$ $\angle ABO = 42°$ (angles in the same segment) OR $\angle AOB = 48° \times 2$ $= 96°$ (angle at centre is twice angle at circumference) $42°$ (isosceles triangle AOB)	

6(aii)	$\angle DCE = 42°$ (angles in the same segment) $\angle CDA = 180° - 42° - 32°$ (sum of angles in triangle) $= 106°$ OR $\angle CBD = 32°$ (angles in the same segment) (angles in opposite segement are supplementary) $\angle CDA = 180° - 32° - 42°$ $= 106°$	
6(aiii)	$\angle OAB = 42°$ (base angles of isosceles triangle) $\angle OAG = 90°$ (tangent perpendicular to radius) $\angle GAB = 90° - 42°$ $= 48°$ OR $\angle GAB = 48°$ (alternate segment theorem)	
6(b)	Since $\angle OBA \neq \angle GAB$, it does not satisfy the property of alternate angles with a set of parallel line. Hence, BD is not parallel to GF OR If BD is parallel to GF, $\angle OBA = \angle GAB$, based on alternate angles. Since $\angle OBA \neq \angle GAB$, BD is not parallel to GF.	
7(ai)	$\frac{5}{16}$ ------- $8 + 17 = 25$ students $\therefore 8+17+34+p+3 = \frac{25}{5} \times 16$ $62 + p = 80$ $p = 18$ (shown)	
7(aiia)	$\text{Mean} = \frac{\sum fx}{\sum f}$ $= \$53.875$ $= \$53.88$ (2 d.p.)	
7(aiib)	$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$ $= 9.8734$ $= 9.87$ (3 s.f.)	

7(aiii)	The weekly expenditure on food for School X has a wider spread (less consistent) than that for School Y as the standard deviation for School X is greater than that of School Y.	
7(bi)	$\frac{\text{area of surface B}}{\text{area of surface A}} = \left(\frac{\frac{1}{2}h}{h}\right)^2$ $= \frac{1}{4}$	
7(bii)	$\frac{\text{Volume of water}}{\text{Volume of full cone}} = \left(\frac{1}{2}\right)^3$ $\frac{\text{Volume of water}}{480} = \frac{1}{8}$ $\text{Volume of water} = \frac{1}{8} \times 480$ $= 60 \text{ cm}^3$	
7(biii)	Remainder volume $= 480 - 60 = 420 \text{ cm}^3$ $\frac{\text{Volume of empty part}}{\text{Volume of full cone}} = \left(\frac{d}{h}\right)^3$ $\frac{420}{480} = \left(\frac{d}{h}\right)^3$ $\frac{d}{h} = \sqrt[3]{\frac{7}{8}}$ $d = 0.95647h$ $= 0.956h$ (3 s.f.)	
8(a)	At $A(0, 24)$, $24 = (4-0)(0+k)$ $24 = 4k$ $k = 6$	
8(b)	$B(-6, 0)$ $C(4, 0)$	
8(c)	Line of symmetry: $x = \frac{-6+4}{2} = -1$ At $x = -1$, $-1+6)$ $\therefore$ Coordinate of maximum point $= (-1, 25)$	

8(d)

At $x = 1$,

$m = (4-1)(1+6)$

$= 21$

$\text{gradient} = \frac{21}{1}$

$= 21$

$\therefore$ Equation of line: $y = 21x$

8(e)

Sub. $y = 9$ into equation of graph,

$9 = (4-x)(x+6)$

$9 = -x^2 - 2x + 24$

$x^2 + 2x - 15 = 0$

$(x-3)(x+5) = 0$

$x = 3 \quad \text{or} \quad -5$

P(−5, 9)

Q(3, 9)

9(a)

Area of eyes $= 2 \times \pi r^2$

$= 2 \times (2.5)^2 \pi$

$= 12.5\pi \quad \text{cm}^2$

For isosceles triangle,

$\cos\alpha = \frac{3^2 + 3^2 - 2^2}{2(3)(3)}$

$= \frac{14}{18}$

$\alpha = \cos^{-1}\left(\frac{14}{18}\right)$

$= 38.942°$

Area of nose $= \frac{1}{2}(3)(3)\sin 38.942°$

$= 2.8284 \quad \text{cm}^2$

OR

$h = \sqrt{3^2 - 1^2} = \sqrt{8}$

$\text{Area} = \frac{1}{2} \times 2 \times \sqrt{8}$

$= 2.8284 \text{ cm}^2$

For mouth, $\beta = 60°$

Area of semicircle $= \frac{1}{2}\pi(1.5)^2$

$= \frac{9}{8}\pi \quad cm^2$

Area of sector $= \frac{60}{360}\pi(3)^2$

$= \frac{3}{2}\pi \quad cm^2$

Area of triangle $= \frac{1}{2}(3)(3)\sin 60°$

$= 3.89711 \quad cm^2$

OR

$h = \sqrt{3^2 - 1.5^2} = \sqrt{\frac{27}{4}}$

Area of triangle $= \frac{1}{2} \times 3 \times \sqrt{\frac{27}{4}}$

$= 3.89711 \ cm^2$

Area of mouth $= \frac{9}{8}\pi - \left(\frac{3}{2}\pi - 3.89711\right)$

$= 2.71901 \quad cm^2$

Total area removed $= 12.5\pi + 2.8284 + 2.71901$

$= 44.8173$

$= 44.8 \ cm^2 \quad (3 s.f.)$

9(b)

Area of whole mask $= \pi r^2$

$= 100\pi \quad cm^2$

Area of mask painted $= 100\pi - 44.8173$

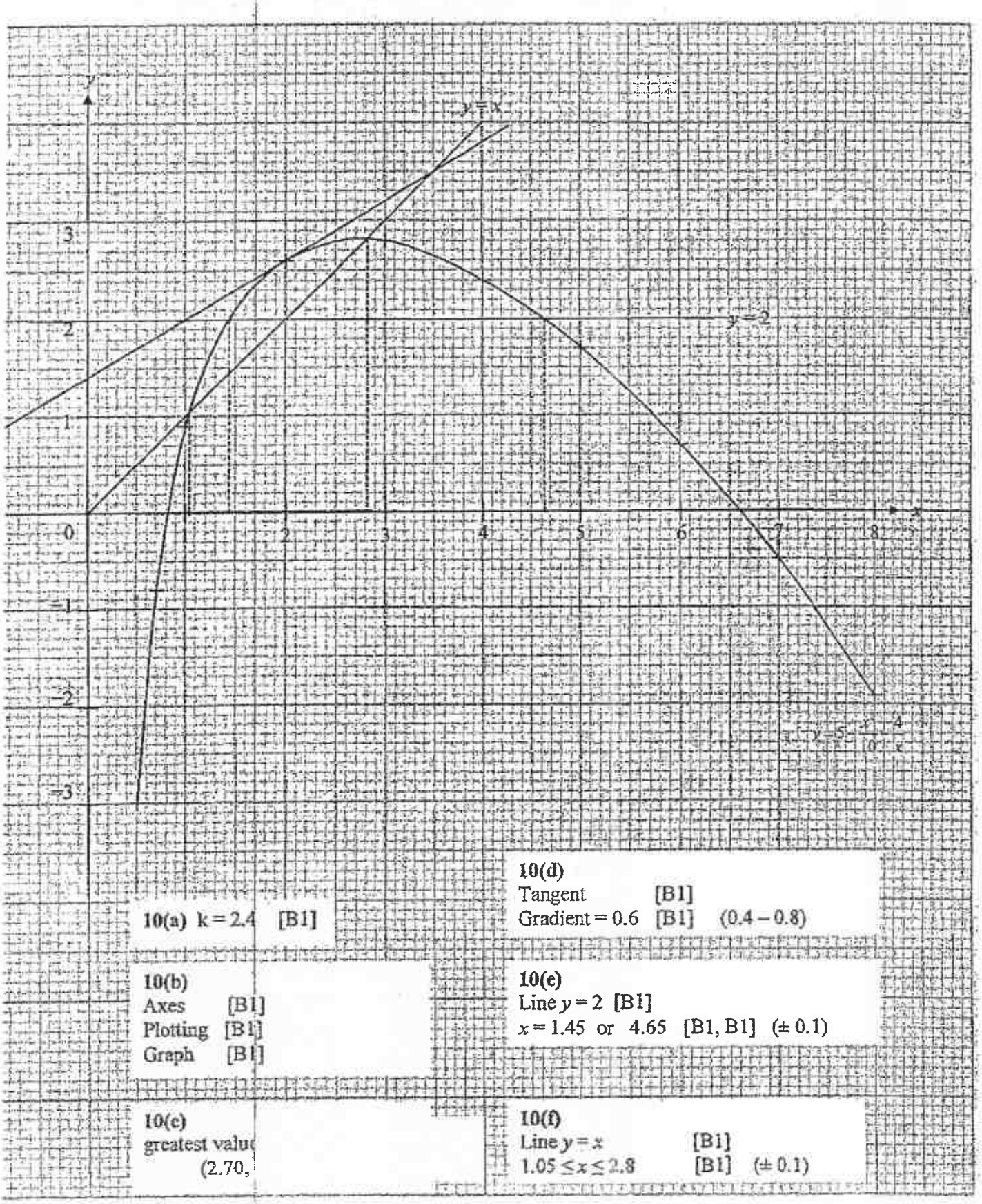
$= 269.341$

$= 269 \quad cm^2 \quad (3 s.f.)$

11(a)	Total distance on map $= 1.8 + 1.9 + 4.7 + 3.8 + 2.8$ $= 15$ cm Actual distance $= \frac{15}{2} \times 600$ $= 4500$ m $= 4.5$ km	
11(b)	**FastDel service** Base fare = \$3.20 400m thereafter or less: $\frac{3500\text{ m}}{400\text{ m}} = 8.75 \approx 9$ Normal fare = $\$3.20 + 9 \times \$0.22$ = \$5.18 **A** Normal fare + peak surcharge = $\$5.18 \times 1.25$ = \$6.475 **B** Total metered fare = \$6.475 + booking + ERP = \$6.475 + \$3.30 + \$3.00 = \$12.775 = \$12.78 (2 d.p.) **Aber service** Base fare = \$3.00 Travelling time fare = $\$0.20 \times 15 = \$3.00$ Distance fare = $\$0.45 \times 4.5 = \$2.025$ Normal fare = \$3 + \$3 + \$2.025 = \$8.025 Total fare = $\$8.025 \times 2.5$ = \$20.0625 = \$20.06 (2 d.p.) Cheryl should choose **FastDel** service .	

Name _____ Index No. _____

Subject _____ Class _____ Date _____



10(a) k = 2.4 [B1]

10(b)
Axes [B1]
Plotting [B1]
Graph [B1]

10(c)
greatest valu
(2.70,

10(d)
Tangent [B1]
Gradient = 0.6 [B1] (0.4 – 0.8)

10(e)
Line $y = 2$ [B1]
$x = 1.45$ or 4.65 [B1, B1] (± 0.1)

10(f)
Line $y = x$ [B1]
$1.05 \le x \le 2.8$ [B1] (± 0.1)

Answer all the questions.

1 (a) Evaluate $\frac{\sqrt{239}-17^2}{34.79^3 \times 13}$, giving your answer correct to 5 significant figures.

Answer _____ [1]

(b) Simplify $5x-2(x+2)$.

Answer _____ [1]

---

2 An estimated number of 36 000 people were present at a concert.

(a) If the estimated number was actually rounded off to 3 significant figures, state the maximum possible number of people at the concert.

Answer _____ [1]

(b) If the estimated number was actually rounded off to 2 significant figures, state the minimum possible number of people at the concert.

Answer _____ [1]

---

3 Factorise completely $6ax-2bx+9ay-3by$.

Answer _____ [2]

---

4 The equation of a curve is $y = x^2 + bx + c$ where b and c are constants.

(a) Given that (2 , 0) is a point on the curve, show that $b = -\frac{4+c}{2}$.

Answer

[2]

(b) If the y-intercept of the curve is 14, find the values of b and c.

Answer $b =$ _____ $c =$ _____ [2]

---

5 Triangle ABC is a right angled triangle. Given that $AB = 13$ cm and $BC = 12$ cm, find two possible lengths for the side AC.

Answer _____ or _____ cm [3]

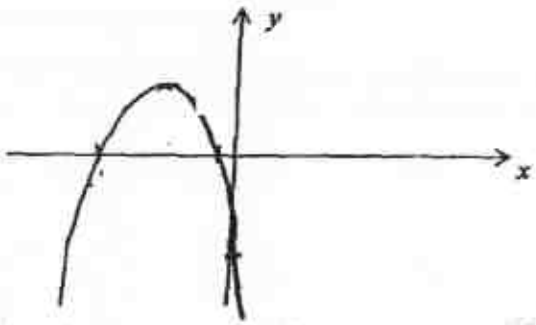
---

6 (a) Express $-x^2 - 5x - 6$ in the form $-(x + a)(x + b)$, where a and b are constants.

Answer _____ [1]

(b) Hence sketch the curve of $y = -x^2 - 5x - 6$, indicating clearly the intercepts and turning point.

Answer



[3]

---

7 Write as a single fraction in its simplest form $\frac{3x}{(x-2)^2} - \frac{2}{2-x}$.

Answer _____ [2]

---

8 The number of apples, oranges and pears at a fruit stall is given by the ratio 2 : 3 : 7.

(a) If there are 126 pears at the fruit stall, find the number of apples at the fruit stall.

Answer _____ [1]

(b) If half the number of oranges at the fruit stall is replaced by papayas, find the fraction of papayas at the fruit stall.

Answer _____ [1]

---

9 Some values of x and y are given in the table below.

x	3	4	6	12
y	8	6	4	2

State whether x and y could be in direct or inverse proportion, and explain why this is so.

Answer _____

_____

_____ [2]

10 Solve the following equations.

(a) $5(x - 4) = 4 - 2(3x + 1)$

Answer $x =$ _____ [2]

(b) $\frac{3x+1}{5} = \frac{1}{x-2} - 4$

Answer $x =$ _____ [3]

11 Factorise the following.

(a) $25x - 30x^2$

Answer _____ [1]

(b) $5x^2 + 13x - 6$

Answer _____ [2]

(c) $12x^2 - 3$

Answer _____ [2]

---

12 A bag costs $3500 in Singapore.
On a trip to the US, Amy manages to find an identical bag that costs US$3000.

1 US dollar = 1.36 Singapore dollars.

Is the bag cheaper in the US or Singapore? You must show your calculations.

Answer _____ [2]

13 The length of a road from one end to the other is 34.1 km.

(a) On a map, the same road measures 5.5 cm. Write down the scale of the map in the form 1 : n.

Answer 1 : _____ [2]

(b) A plot of land of area 88.412 km^2 has been marked out for construction of commercial buildings. What is the area on the map that is marked out for construction of commercial buildings?

Answer _____ cm^2 [3]

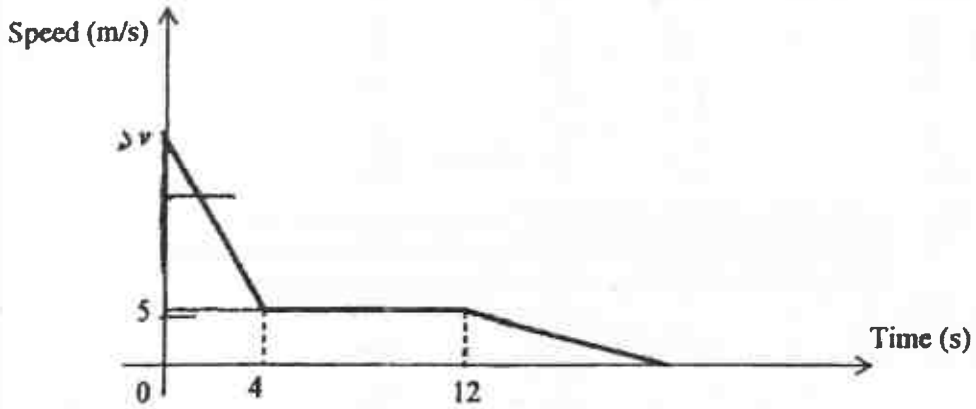
---

14 A computer costs $2300.
During a sale, David buys the computer for $1782.50.
Calculate the percentage discount of the computer during the sale.

Answer _____ % [2]

---

15 A car travelling at an initial speed of v m/s decelerates uniformly for 4 seconds, then travels at a uniform speed of 5 m/s for 8 seconds before decelerating uniformly until it comes to a complete rest. The speed-time graph for the car is shown below.



(a) A van, starting at the same time as the car from the same initial point travels along the same route at a uniform speed of 11 m/s throughout the journey. On the graph above, draw the line representing the speed-time graph of the van, given that $v > 11$. [1]

(b) It is given that deceleration is represented by the gradient of the speed-time graph. The deceleration of the car during the first 4 seconds is 3.75 m/s^2. Show that $v = 20$.

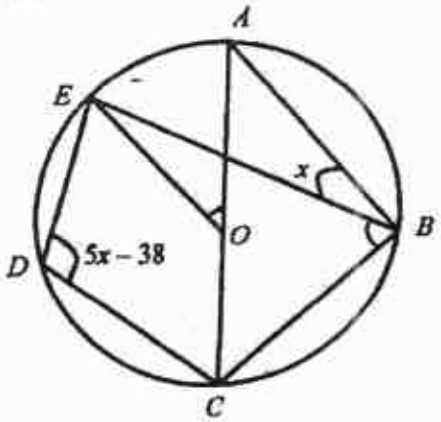
Answer

[2]

(c) It is given that the area under the speed-time graph represents the distance travelled. At how many seconds, after the van and car started from the initial point, will the van overtake the car?

Answer _____ s [4]

16



O is the centre of the circle passing through A, B, C, D and E.
Angle $ABE = x°$, and angle $EDC = (5x - 38)°$.

(a) Find, in terms of x, angle AOE.

Answer _____ ° [1]

(b) Find, in terms of x, angle EBC.

Answer _____ ° [1]

(c) Find x.

Answer $x =$ _____ [2]

17 David's wages, W, varies directly as the square of the number of sales he makes in a month. In January, he makes x number of sales. In February, the number of sales he makes increases by 150% as compared to January. Calculate the percentage change in David's wages in February as compared to January.

Answer _____ % [3]

18 A class of 40 students had their individual weights taken and the mean and standard deviation of the weights were calculated. It was later found out that the weighing machine used was faulty and every student should be heavier by 2 kg. Describe the effect, if any, it would have on the mean and standard deviation that was calculated.

Answer _____

_____

_____ [2]

[Turn over

19 (a) Express 600 as a product of its prime factors, giving your answer in index notation.

Answer _____ [2]

(b) p and q are not prime numbers.

Given that $600 \times pq$ is a perfect square, and that p and q are positive integers smaller than 10, find the smallest possible value of $p - q$.

Answer _____ [2]

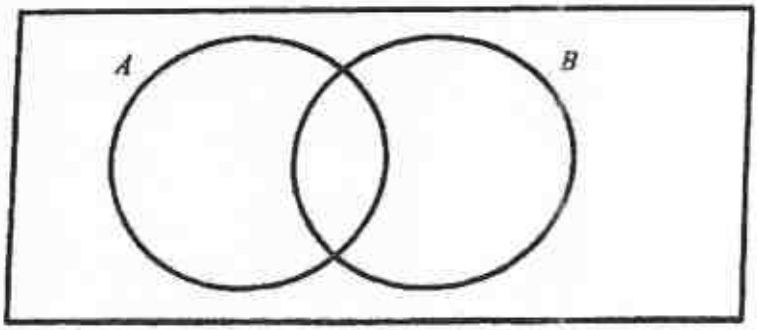
---

20 It is given that

$\xi = \{x : x \text{ is a positive integer smaller than } 10\}$,
$A = \{x : x \text{ is a prime number}\}$,
$B = \{x : x \text{ is an even number}\}$.

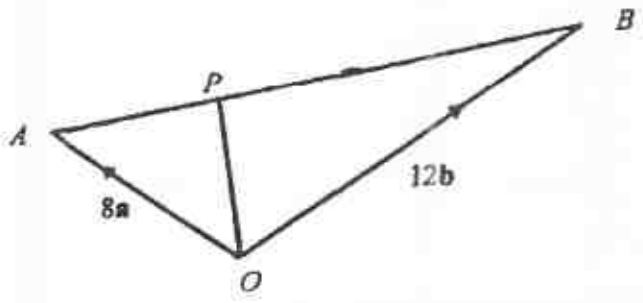
Write down all the numbers in the universal set in the Venn Diagram below.

Answer



[3]

21



OAB is a triangle.
$\overrightarrow{OA} = 8\mathbf{a}$ and $\overrightarrow{OB} = 12\mathbf{b}$.
P is a point on AB such that $AP : PB = 1 : 3$.

(a) Write each of the following in terms of a and b.
Give your answers in their simplest form.

(i) $\overrightarrow{AB}$.

Answer _____ [1]

(ii) $\overrightarrow{AP}$.

swer _____ [1]

[Turn over 14

(b) A line is drawn from O to Q where Q lies on the line AB extended. Given that B is the mid-point of PQ, express $\overrightarrow{OQ}$ in terms of **a** and **b**, giving your answer in its simplest form.

Answer _____ [2]

(c) Find the value of $\frac{\text{Area of triangle } OBQ}{\text{Area of triangle } OAQ}$.

Answer _____ [2]

22 The coordinates of A is $(-3, 5)$ and the coordinates of B is $(7, 10)$.
$\overrightarrow{AC} = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$.

(a) Find $\overrightarrow{AB}$, expressing your answer as a column matrix.

Answer _____ [1]

(b) Find $|\overrightarrow{AC}|$.

Answer _____ [1]

(c) Find the coordinates of C.

Answer _____ [2]

23 An architect designing a walkway draws a scale drawing of the walkway below. The drawing is drawn accurately to a scale of 1 : 10 000.
Point B is directly east of Point A.

Answer

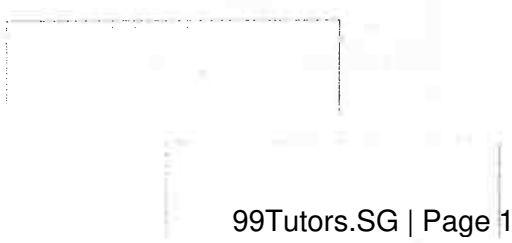
(a) The architect plans to extend the walkway by 0.8 km at a bearing of 145° from point B. Use the scale drawing above to draw the extension of the walkway and label the end of the walkway as Point C. [2]

(b) The walkway is then further extended from Point C back to Point A. By measurement, find the length of the walkway from A to C in kilometres.

Answer _____ km [1]

(c) The architect intends to put a notice board along BC, equidistant from points A and C. By constructing a perpendicular bisector on the scale drawing, indicate and label the position of the notice board with the letter N. [2]

F PAPER



Answer all the questions.

1 (a) Express as a single fraction in its simplest form

$$\frac{1}{p-2}-\frac{2}{4p+3}.$$ [3]

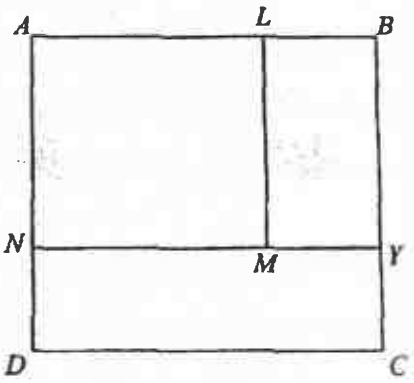
(b) The formula used in an experiment is

$$T=\frac{k(x-a)}{a}.$$

(i) Express x in terms of T, k and a. [2]

(ii) Find, in terms of k, the value of T when $x=3a$. [1]

2 In the given diagram, $ABCD$ and $ALMN$ are squares.
$AB = (3x - 1)$ cm and $AN = (x + 2)$ cm.



(a) Write down the length of LB in terms of x. [1]

(b) The area of the rectangle $LBYM$ is 10 cm^2.
Write down an equation in x and show that it reduces to $2x^2 + x - 16 = 0$. [2]

(c) Solve the equation $2x^2 + x - 16 = 0$, giving your solutions correct to two decimal places. [4]

(d) Which value of x do you have to reject and why? [2]

(e) Hence, calculate the perimeter of $LBYM$, giving your answer to the nearest millimetre. [2]

[Turn over

3 Singapore and Kuala Lumpur are 350.7 km apart.

(a) Ms Wong travelled by car from Singapore to Kuala Lumpur (KL) at an average speed of 90 km/h. How long did the journey take? [1]

(b) Ms Wong left Singapore at 0600. If she had a meeting to attend in KL at 1000, was she early or late for this meeting? [1]

(c) After the 3-hour meeting, Ms Wong took a one-hour lunch-break before making her return journey. She wanted to reach Singapore before the evening peak-hour commenced at 4pm. If the speed limit is 100 km/h, would she be able to reach Singapore by 4pm? [3]

(d) The upcoming Singapore-KL high-speed-rail (HSR) train line boasts a travelling time of 99 minutes in a single direction between the two cities. What is the average speed of the train? [1]

(e) The maximum speed of the train is expected to be 300 km/h. What is the percentage decrease in speed as mentioned in (d), compared to the expected speed? [2]

4 A bag contains 6 tennis-balls comprising of 4 green balls and 2 red balls.

Amy selects a ball at random from the bag and then replaced. She randomly selects another ball from the same bag.

(a) Draw a probability-tree diagram to represent the outcomes. [1]

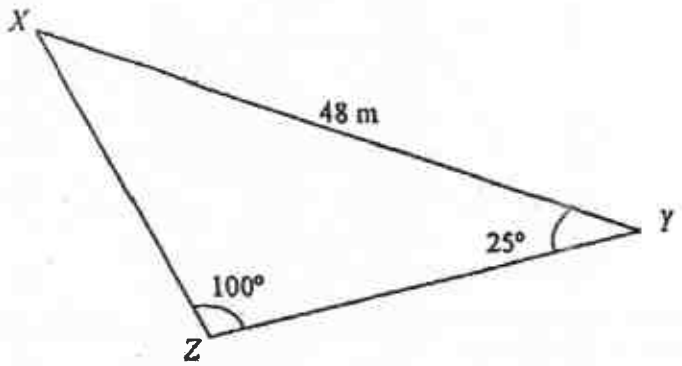
(b) Find, in its simplest form, the probability that the selected balls

(i) are green, [1]

(ii) are of different colours, [2]

(iii) include at least one red ball. [2]

5



X, Y and Z are on level horizontal ground. The bearing of Y from X is 100°.
$XY = 48$ m, angle $XZY = 100°$ and angle $XYZ = 25°$.

(a) Calculate

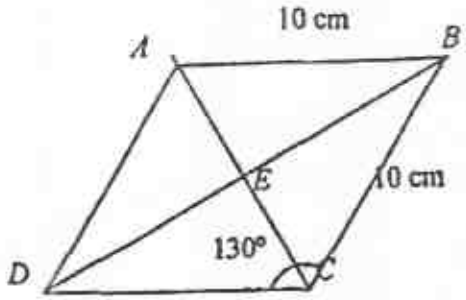
(i) the bearing of X from Y, [1]

(ii) the bearing of Z from X, [2]

(iii) the shortest distance from Z to XY. [3]

(b) If there is a tower of height 10 m at X, calculate the angle of depression of Y from the top of the tower. [2]

6



The diagram shows a cross-section of a rhombus cookie-box, $ABCD$, and E is the intersection-point of AC and BD.
AB // DC and AD // BC, $AB = CD = 10$ cm and angle $BCD = 130°$.

(a) (i) Explain why angle AEB is a right-angle. [1]

(ii) Calculate BD. [2]

(iii) Calculate the length of EC. [1]

(iv) Hence, calculate the area of triangle BCD. [1]

(b) A geometrically similar smaller version of the cookie-box is necessary for smaller quantities of cookies. In the smaller cookie-box, $AB = 8$ cm.

Find the cross-sectional area of the smaller cookie-box. [2]

7 (a) The following table shows the scores of 30 students from Secondary 4 Ace in their Mathematics Examination.

80	88	96	60	59	70	88	97	69	60
39	37	69	74	47	92	72	49	58	66
88	82	100	95	56	77	99	62	79	63

(i) Calculate the mean score for the students in Secondary 4 Ace. [1]

(ii) Calculate the standard deviation for the scores above. [1]

(b) The mean and standard deviation of Secondary 4 Bravo for the same examination are as follow:-

Mean Score	71.75
Standard Deviation	15.6

(i) Which class performed better? Support your claim with evidence. [2]

(ii) Which class had more consistent results? Support your claim with evidence. [2]

8 A funnel is in the form of an inverted right circular cone. Figure 1 shows a vertical cross-section of the funnel. It contains oil and water (which do not mix). The depths of water and oil are all 10 cm, with water at the bottom. It is given that the height of the funnel is 30 cm and the base radius is 9 cm.

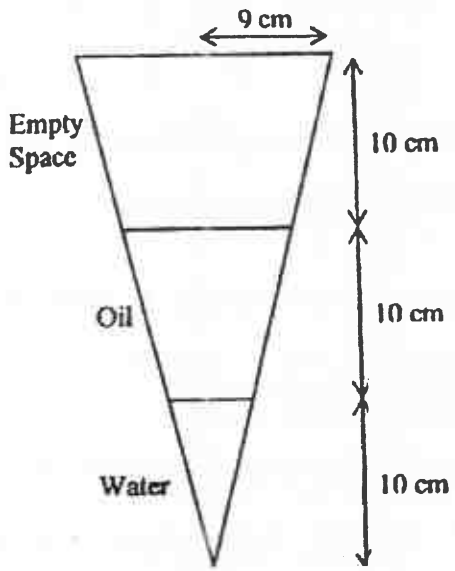


Figure 1

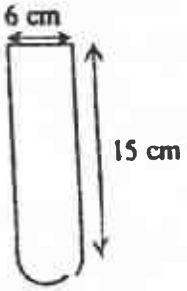


Figure 2

(a) Find the volume of the funnel in terms of π. [1]

(b) Find the fraction of

(i) $\dfrac{\textit{volume of oil}}{\textit{volume of water}}$, [2]

(ii) $\dfrac{\textit{surface area of the funnel in contact with water}}{\textit{total surface area of the interior of the funnel}}$. [2]

(c) All the water in the funnel is then drained through the tap at the vertex of the funnel, into another container formed by a cylinder of diameter 6 cm and surmounted by a hemisphere at the lower part of the cylinder, as shown in Figure 2. The height of the cylindrical part of the container is 15 cm.
Find the depth of water in this container.
(Note: Only the water is drained; the oil remains in the funnel.) [3]

9 Two outlets of a new fast-food chain sell three types of soft drinks, namely Coke, Sprite and Lemon Tea. The tables below show the sales of the soft drinks in the afternoon and evening respectively.

	Afternoon		
	Coke	Sprite	Lemon Tea
Outlet *A*	280	200	150
Outlet *B*	200	300	350

	Evening		
	Coke	Sprite	Lemon Tea
Outlet *A*	420	300	260
Outlet *B*	350	420	540

The sales of the soft drinks in the afternoon are represented by the matrix **A**, where

$$\mathbf{A} = \begin{pmatrix} 280 & 200 & 150 \\ 200 & 300 & 350 \end{pmatrix}.$$

(a) Write down the 2×3 matrix **E** representing the sales in the evening for the two outlets respectively. [1]

The cost price of supplying the soft drinks to the fast-food chain is \$1.20, \$1.00 and \$1.50 for Coke, Sprite and Lemon Tea respectively. The selling price for each soft drink is \$2.00, \$2.00 and \$3.50.
The cost price of supplying the soft drinks is represented by matrix **C**, where

$$\mathbf{C} = \begin{pmatrix} 1.20 \\ 1.00 \\ 1.50 \end{pmatrix}.$$

(b) Write down the column matrix **S** representing the selling price of the soft drinks for the three types of soft drinks respectively. [1]

(c) Calculate $\mathbf{T} = \mathbf{A} + \mathbf{E}$, and describe what matrix **T** represents. [2]

(d) Evaluate **AC** and describe what is represented by the elements of **AC**. [2]

(e) Evaluate $\mathbf{T}(\mathbf{S} - \mathbf{C})$, and explain what the elements of $\mathbf{T}(\mathbf{S} - \mathbf{C})$ represent. [2]

(f) **(i)** If the fast-food chain's general manager would like to evaluate the combined total amount in sales for both outlets for the day, write down the matrix operation he needs to calculate. [1]

(ii) Evaluate the matrix that you have specified in part (i) above. [1]

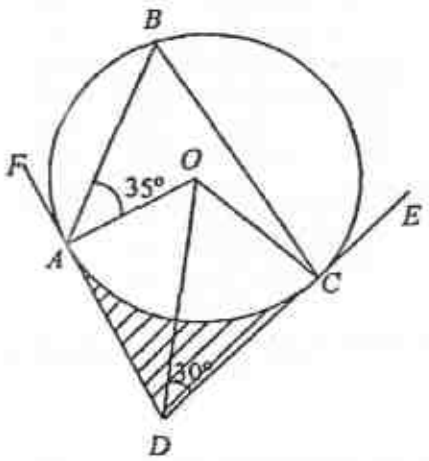
[Turn over

10 (a) (i) Find the value of each interior angle of a regular 15-sided polygon. [2]

(ii) An n-sided polygon has 3 interior angles measuring 140° each.
The remaining interior angles all measure $y°$ each.

Find an expression for y in terms of n. [2]

(b)



The diagram shows a circle ABC, with centre O.
FAD and DCE are tangents to the circle, and $OA = OC = 8$ cm.
Angle $OAB = 35°$ and angle $CDO = 30°$.

(i) Name the pair of congruent triangles. [1]

(ii) Find

(a) angle DOA, [1]

(b) angle CBA, [1]

(c) angle ECB. [1]

(d) the area of the shaded region. [2]

11 **Answer the whole of this question on a sheet of graph paper.**
From the top of a mountain, Barry fires a pellet from an air gun upwards into the air. The height, h metres, of the pellet from Barry t seconds after it is released can be modelled by the equation $h = 1 + 10t - 3t^2$.

Some corresponding values of t and h are given in the table below.

t	0	1	2	3	4	5	6
h	1	8	9	4	m	-24	-47

(a) Calculate the value of m. [1]

(b) Using a scale of 2 cm to represent 1 second, draw a horizontal t-axis for $0 \le t \le 6$.
Using a scale of 1 cm to represent 5 metres, draw a vertical h-axis for $-50 \le h \le 10$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to estimate

(i) the maximum height of the pellet above ground level, [1]

(ii) the length of time that the pellet was more than 2 metres above ground level, [2]

(iii) the time elapsed before the pellet reaches the same level as it was fired from. [1]

(d) By drawing a tangent, find the gradient of the curve at $(5, -24)$.
State the units of your answer. [3]

12 From July 2017 onwards, the price of water to households will be increased in two steps, on 1 July 2017 and on 1 July 2018. At the same time, the Government will be increasing the annual GST Voucher – U-Save rebate for eligible HDB households by between $40 and $120, depending on the flat type. The average change in water bill after the increased U-Save rebates is given in Table A on the next page.

(a) Show that for a 4-room HDB flat, the U-Save Rebate given in July 2017 is $7. [1]

Table B shows how the water tariffs will be increased between 2017 and 2018.
Charlie owns a new 4-room build-to-order (BTO) HDB flat in Woodleigh.
Read and understand the contents of the utility bill dated June 2017 in Table C.

(b) Assuming that the amount of water Charlie used in July 2017 is the same as that for June 2017, calculate the individual charges in July 2017 for

(i) water usage (reading), [1]
(ii) waterborne fee, [1]
(iii) water conservation tax, [1]
(iv) total cost of water services (after deduction of U-Save Rebate). [1]

(c) Assuming that the amount of water Charlie uses for July 2018 is the same as that for June 2017, calculate the total cost of water services in July 2018 (before the U-Save Rebate). [3]

(d) Why do you think that average changes in 2017 and 2018 bills are increasing from 1-room HDB flats to the executive/multi-generation flats? [1]

Table A: Average Change in Water Bill after Increased U-Save Rebates (by HDB Flat Type)
Source: https://www.pub.gov.sg/Documents/WaterPriceRevisionsBrochure.pdf

Water Bill	1-room HDB flat	2-room HDB flat	3-room HDB flat	4-room HDB flat	5-room HDB flat	Executive / Multi-Generation HDB flat
Before price increase	$23	$29	$33	$42	$44	$49
After price increase (2017)	$26	$34	$37	$47	$50	$55
After increased U-Save rebates (2017)	$16	$24	$29	$40	$45	$51
Average change in 2017 Bill	– $7	– $5	– $4	– $2	+ $1	+ $2
Average change in 2018 Bill	– $3	$0	+ $2	+ $5	+ $8	+ $11

Table B: Water Price Revisions
Source: https://www.pub.gov.sg/Documents/WaterPriceRevisionsBrochure.pdf

		Before 1 July 2017		From 1 July 2017		From 1 July 2018	
		Water Price ($/m³)		Water Price ($/m³)		Water Price ($/m³)	
		0 – 40m³	> 40m³	0 – 40m³	> 40m³	0 – 40m³	> 40m³
Potable Water	Tariff	$1.17	$1.40	$1.19	$1.46	$1.21	$1.52
	Water Conservation Tax (% of water tariff)	$0.35 (30% of $1.17)	$0.63 (45% of $1.40)	$0.42 (35% of $1.19)	$0.73 (50% of $1.46)	$0.61 (50% of $1.21)	$0.99 (65% of $1.52)
Used Water	Waterborne Fee	$0.28	$0.28	$0.78	$1.02	$0.92	$1.18
	Sanitary Appliance Fee	$2.80 per fitting*		Combined into Waterborne Fee		Combined into Waterborne Fee	
Total Price		$2.10	$2.61	$2.39	$3.21	$2.74	$3.69

Note: Water is charged per cubic metre (m^3), which is equivalent to 1000 litres.
All figures are before GST.
*For the calculation of total price, the Sanitary Appliance Fee is converted to its volumetric equivalent.

Table C: Utility Bill for June 2017

June 2017 Bill
Account No. ########

Breakdown of Current Charges	Usage	Rate ($)	Amount ($)	Total ($)
Electricity Services				
Reading taken on 28 Jun 2017: 63902	736 kWh	0.2139	157.43	157.43
Water Services by Public Utilities Board				
Reading taken on 28 Jun 2017 : 5094.8	35.8 Cu M	1.1700	41.89	
Waterborne Fee	35.8 Cu M	0.2803	10.03	
Water Conservation Tax	$41.89	30%	12.57	
Sanitary Appliance Fee	2 Fittings	2.8032	5.61	70.10
Refuse Removal by Veolia ES Singapore P L	1 Qty	7.71	7.71	7.71
Subtotal			235.24	235.24
GST	$235.24	7%	16.46	16.46
Current Charges (rounded off)				$251.70

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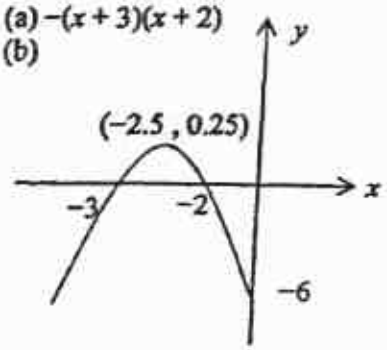
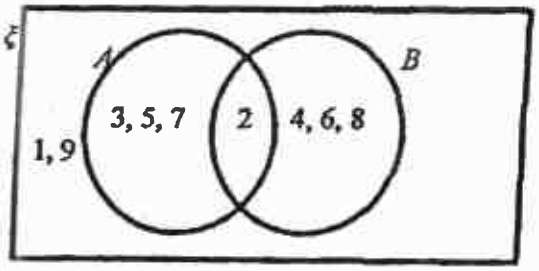
Answer Key

1. (a) −0.00 049 971
   (b) $3x-4$
2. (a) 36 049
   (b) 35 500
3. $(2x+3y)(3a-b)$
4. (a) $b=-\dfrac{4+c}{2}$
   (b) $b=-9$; $c=14$
5. $AC=5$ cm or 17.7 cm
6. (a) $-(x+3)(x+2)$
   (b)



7. $\dfrac{5x-4}{(x-2)^2}$
8. (a) 36
   (b) $\dfrac{1}{8}$
9. –
10. (a) $x=2$
    (b) $x=\dfrac{1}{3}$ or $x=3$
11. (a) $5x(5-6x)$
    (b) $(5x-2)(x+3)$
    (c) $3(2x+1)(2x-1)$
12. cheaper in Singapore
13. (a) 620000
    (b) 2.3 cm²
14. 22.5%
15. (a) -
    (b) –
    (c) $t=5$
16. (a) $2x$
    (b) $90-x$ or $218-5x$
    (c) $x=32$
17. 525%
18. –
19. (a) $600=2^3\times3\times5^2$
    (b) −5
20.



21. (ai) $12b-8a$
    (aii) $3b-2a$
    (b) $21b-6a$
    (c) $\dfrac{3}{7}$
22. (a) $\begin{pmatrix}10\\5\end{pmatrix}$
    (b) 8.06
    (c) (1 , −2)
23. (b) 1.16 km

Answer Key

1. (a) $\frac{2p+7}{(p-2)(4p+3)}$

   (bi) $x=\frac{aT}{k}+a$

   (bii) $T=2k$

2. (a) $(2x-3)$ cm

   (b) –

   (c) $x=2.59$ or -3.09

   (d) –

   (e) 13.5 cm

3. (a) 3.90 h

   (b) She was early for the meeting.

   (c) She would not be able to reach Singapore by 4 pm.

   (d) $212.\dot{5}\dot{4}$ *km / h or* 213 *km / h (to 3 s.f.)*

   (e) $29.\dot{1}\dot{5}\%$ *or* 29.2% *(to 3 s.f.)*

4. (a) –

   (bi) $\frac{4}{9}$

   (bii) $\frac{4}{9}$

   (biii) $\frac{5}{9}$

5. (ai) 280°

   (aii) 165°

   (aiii) 16.9 m

   (b) 11.8°

6. (ai) –

   (aii) 18.1 cm

   (aiii) 4.23 cm

   (aiv) 38.3 cm^2

   (b) 49.0 cm^2

7. (ai) $72.3\dot{6}$ *or* 72.4 *(to 3 s.f.)*

   (aii) 17.6

   (bi) –

   (bii) –

8. (a) $810\pi\ cm^3$

   (bi) $\frac{7}{1}$ *or* 7

   (bii) $\frac{1}{9}$

   (c) 4.33 cm

9. (a) $E=\begin{pmatrix} 420 & 300 & 260 \\ 350 & 420 & 540 \end{pmatrix}$

[Turn over

(b) $S=\begin{pmatrix} 2.00 \\ 2.00 \\ 3.50 \end{pmatrix}$

(c) $T=\begin{pmatrix} 700 & 500 & 410 \\ 550 & 720 & 890 \end{pmatrix}$

Matrix T represents the sales of Coke, Sprite and Lemon Tea in the afternoon and evening at outlets A and B respectively.

(d) $AC=\begin{pmatrix} 761 \\ 1065 \end{pmatrix}$

Matrix AC represents the total cost price of supplying soft drinks to the fast-food chain in the afternoon at outlets A and B respectively.

(e) $T(S-C)=\begin{pmatrix} 1880 \\ 2940 \end{pmatrix}$

Matrix $T(S-C)$ represents the total profits in the afternoon and evening at outlets A and B respectively.

(fi) $\begin{pmatrix} 1 & 1 \end{pmatrix}\left[T\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right]$

(fii) (3770)

10. (ai) 156°

(aii) $y=\dfrac{180n-780}{n-3}$ or $180-\dfrac{240}{n-3}$

(bi) –

(biia) 60°

(biib) 60°

(biic) 65°

(biid) 43.8 cm^2

11. (a) $m=-7$

(b) –

(ci) 9.4 m

(cii) 3.15s

(ciii) 3.35s

(d) -22.64 m/s

12. (a) $7

(bi) $42.60

(bii) $27.92

(biii) $14.91

(biv) $78.44

(c) $97.91

(d) -

[Turn over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Geometry and Measurement

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1. Calculate $\frac{0.85^2 - 5.34}{\sqrt{81.2} + 3.134}$, giving your answer correct to 3 significant figures.

Answer _____ [1]

---

2. A set of numbers is given below.

$$-0.4, \quad \frac{1}{3}, \quad \sqrt[3]{3}, \quad \frac{\pi}{7}, \quad 0.\dot{6}\dot{6}, \quad -\sqrt{4}$$

(a) Write the set of numbers in descending order.

Answer _____ [1]

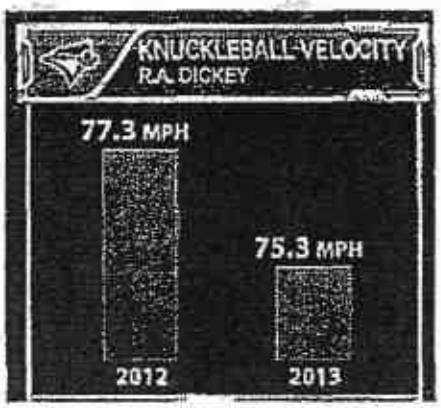
(b) Write down the irrational number(s) from the given set.

Answer _____ [1]

3. Factorise completely $6a^2(a^2-1)-(a^2-1)^2$.

Answer _____ [2]

4. The figure below is extracted from a baseball game broadcast.
It shows the knuckleball velocity statistics of a baseball player.
State one aspect of the data that may be misleading and explain how it might lead to a mis-interpretation of the data by the audience.



Answer _____

_____

_____

_____

_____

_____ [2]

5. Given that $a^2 + 6a = 6$, find the value of $a^3 + 7a^2$.

Answer _____ [2]

6. On Monday, the temperature of a certain location at 12 00 was 34°C.
The temperature dropped to −5°C at 14 00 on Tuesday.
Given that the temperature decreases at a constant rate,
find the temperature at 07 00 on Tuesday.

Answer _____ °C [2]

7. An integer k undergoes a series of operations as shown in the steps below.

Step 1: $\frac{1}{6}$ is added to k.

Step 2: The value from step 1 is multiplied by 24.

Step 3: The value from step 2 is increased by 2.

Step 4: The value from step 3 is divided by 2 to give the resultant value n.

(a) Express n in terms of k.
Give your answer in its simplest form.

Answer _____ [1]

(b) Hence explain why n is an integer and a multiple of 3.

Answer _____

_____

_____

_____ [1]

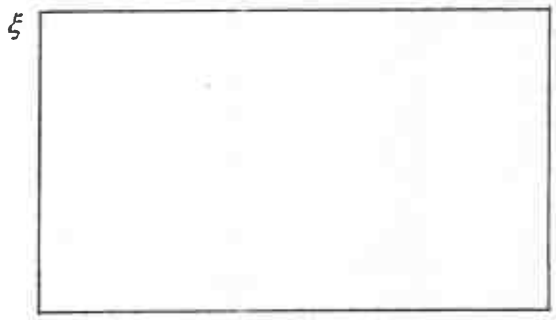
8. V is inversely proportional to the cube of T.
Calculate the percentage change in V, given that T is increased by 300%.

Answer _____ [2]

9. $\xi = \{x : x \text{ is an integer, } 10 < x \leq 23\}$
$A = \{x : x \text{ is an prime number}\}$
$B = \{x : x \text{ is a multiple of } 3\}$

(a) Complete the Venn diagram below to illustrate this information.

Answer [1]



(b) List the elements of $(A \cup B)'$.

Answer _____ [1]

10. It is given that $\cos(180° - A) = -\frac{24}{25}$ and $0° < A < 90°$.

Find, without the use of a calculator, the value of $\sin(180° - A)$.

Answer $\sin(180° - A) =$ _____ [2]

11. Express $-8x-11+x^2$ in the form $(x+p)^2+q$.

Answer _____ [2]

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12. The table below shows the number of books that a group of students has.

Number of books	1	2	3	4
Number of students	5	14	x	7

(a) Write down the largest possible value of x if the mode is 2.

Answer _____ [1]

(b) Find the value of x if the mean is 2.8.

...wer _____ [2]

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13. (a) Express 60 as the product of its prime factors.

Answer 60 = _____ [1]

(b) Find the smallest positive integer value of x for which $60x$ is a multiple of 378.

Answer x = _____ [2]

14. Each term in this sequence is found by adding the same number to the previous term.

a, 13, b, c, 37, ...

(a) Find the values of a, b and c.

Answer $a =$ _____ $b =$ _____ $c =$ _____ [1]

(b) Write down an expression, in terms of n, for the n^{th} term.

Answer _____ [1]

(c) Explain why 121 is not a term in this sequence.

Answer _____

_____

_____

_____

_____

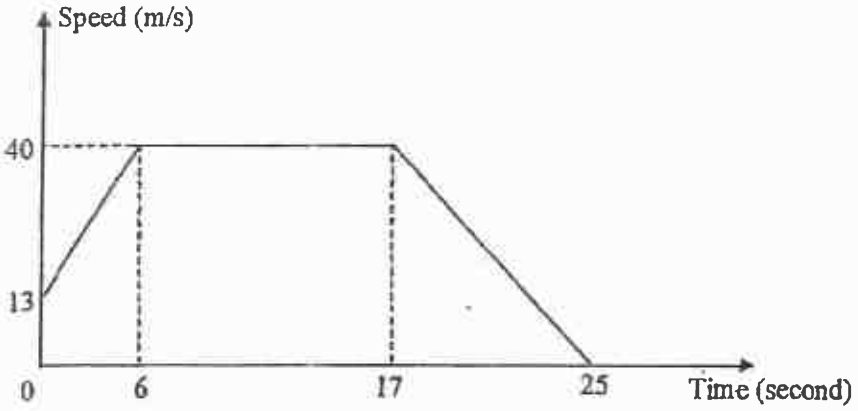
_____

_____

_____

_____ [1]

15. The diagram shows the speed-time graph for the first 25 seconds of a car's journey.



(a) Find the instantaneous speed of the car after travelling for 20 seconds.

Answer _____ m/s [2]

(b) Find the total distance travelled by the car.

Answer _____ m [2]

16. Solve the equation $\frac{8}{3-x} = 5x - 2$.

Answer _____ [3]

17. (a) Simplify $18p^2c^3 \div 4p^5c^{-4}$.

Answer _____ [1]

(b) Given that $9 \times 27^{2n} = 1$, find the value of n.

Answer $n =$ _____ [2]

18. (a) Solve the inequalities $-7 \leq 15-5k < 9$.

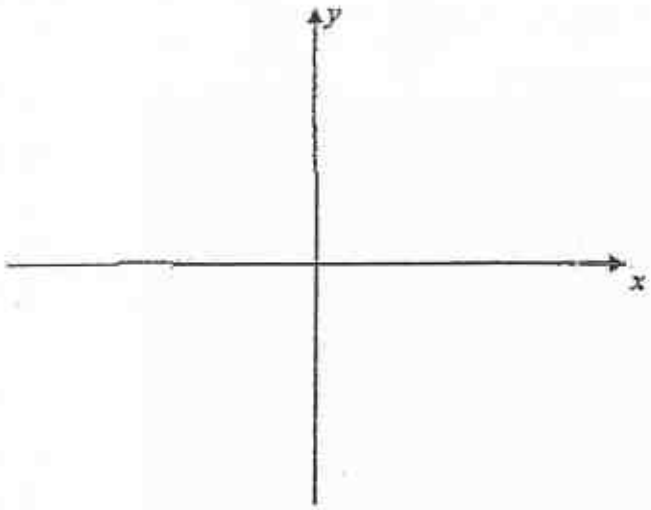
Answer _____ [2]

(b) Write down the integer(s) that satisfy $-7 \leq 15-5k < 9$.

Answer _____ [1]

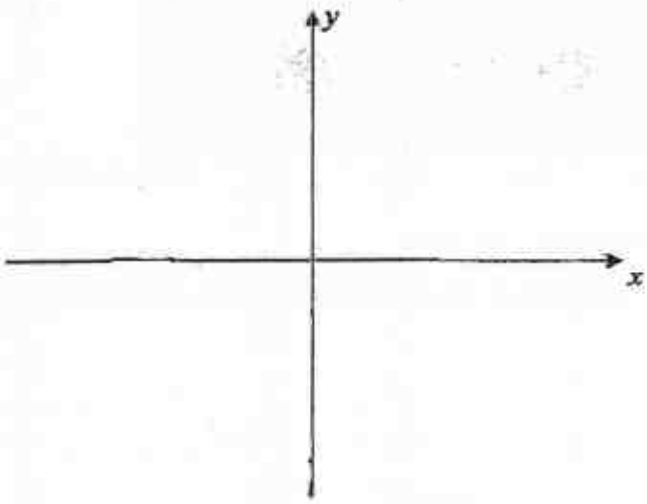
19. (a) (i) Sketch the graph of $y = -\frac{1}{2}x^2$.

Answer [1]



(ii) Sketch the graph of $y = \frac{5}{x^2}$.

Answer [1]



(b) A student claimed that there are roots to the equation $\frac{x^2}{2} + \frac{5}{x^2} = 0$.

Do you agree? Justify your answer.

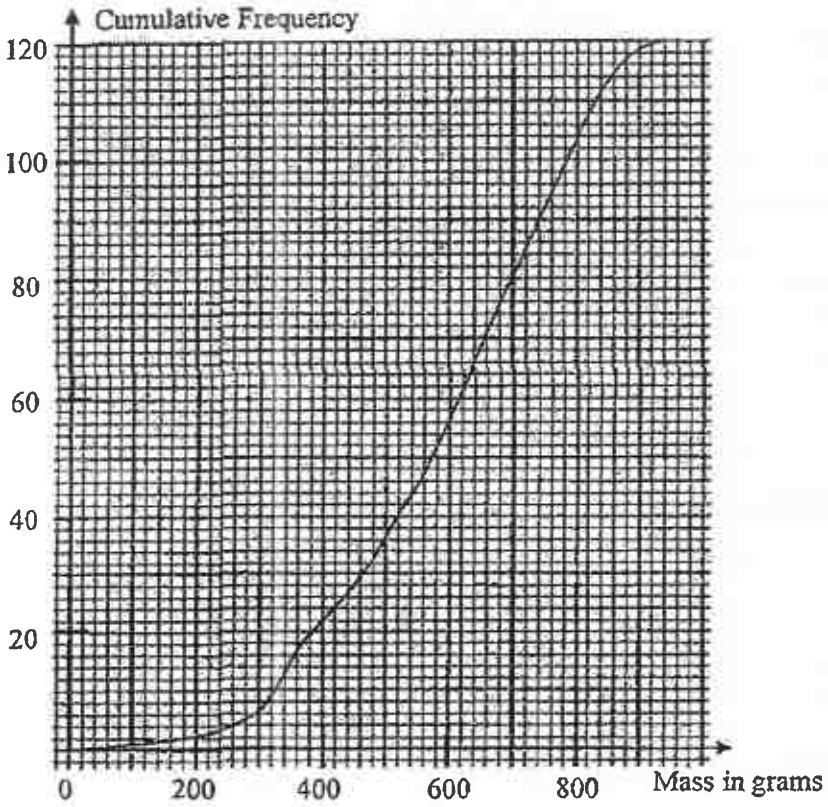
Answer .....

.....

.....

..... [1]

20. The cumulative frequency distribution shows the results of a group of students estimating the mass, in grams, of metal balls in a container.



The actual mass of the metal balls is 500 grams.

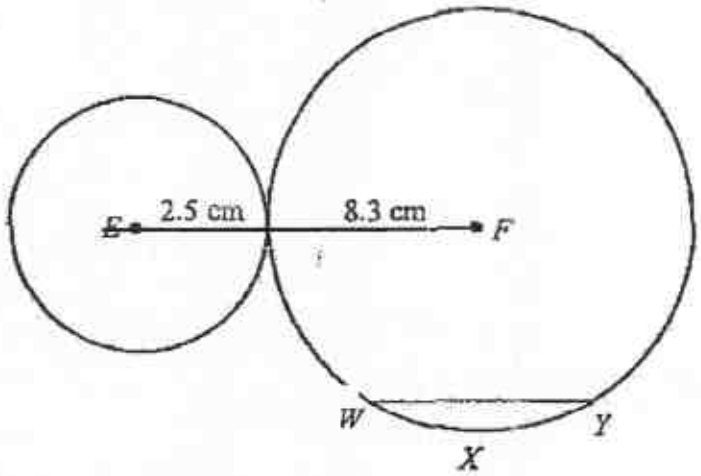
(a) Find the probability that a student, chosen at random, overestimated the mass.

Answer _____ [2]

(b) Find the number of students who gave estimates within 20% of the actual mass.

Answer _____ [2]

21. Two connected discs of radii 2.5 cm and 8.3 cm are shown below.
A clockwise motion in the smaller disc will result in an anti-clockwise motion of the bigger disc.
W, X, Y are points on the circumference of the bigger disc and EF is parallel to WY.
E and F are the center of the smaller and bigger discs respectively.



(a) The smaller disc makes one full complete clockwise rotation.
Find, in terms of π, the angle of rotation made by the larger disc.
Assume that friction is negligible in this question.

Answer _____ radians [2]

(b) Given that $\angle EFW = 1.03$ radians, find the area of the minor segment WXY.

Answer _____ cm^2 [2]

22. A lake has an area of 6.25 km^2.
It is represented by an area of 0.16 cm^2 on map A.

(a) (i) Find the scale of map A in the form $1 : n$.

Answer _____ [2]

(ii) The length of a road on map A is 8.5 cm.
Find the actual length, in kilometres, of the road.

Answer _____ km [1]

(b) The area of the lake is represented on another map B.
The scale of map B is 1 : 450 000.
Find the area, in square centimetres, of the lake represented on map B.

Answer _____ cm^2 [2]

23. The planet Earth can be modelled by a sphere.
The Earth's circumference is estimated to be 40 075 km.

[Take $\pi = 3.142$]

(a) Find the radius, in kilometres, of the Earth.
Give your answer in standard form, correct to 3 significant figures.

Answer _____ km [2]

(b) The speed of light is 3×10^8 m/s.
Express this speed in kilometres per hour.

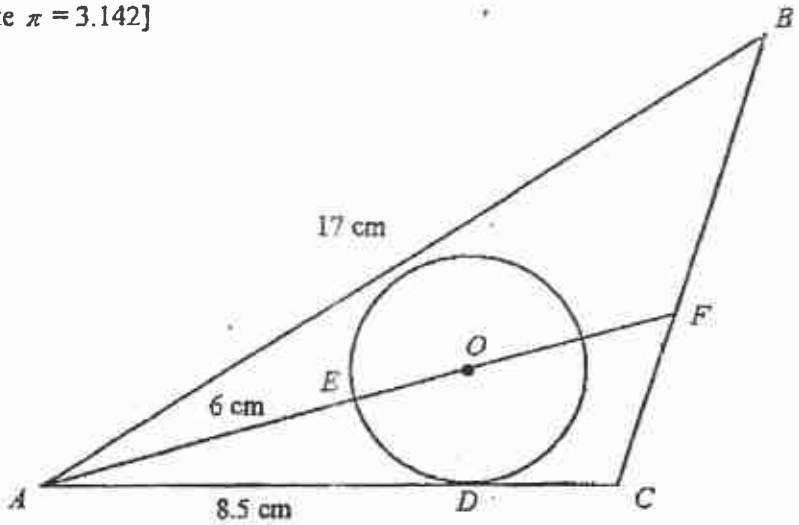
Answer _____ km/h [1]

(c) Find the time taken, in minutes, for a beam of light to travel a distance half the circumference of the Earth.
Give your answer in standard form, correct to 3 significant figures.

Answer _____ minutes [2]

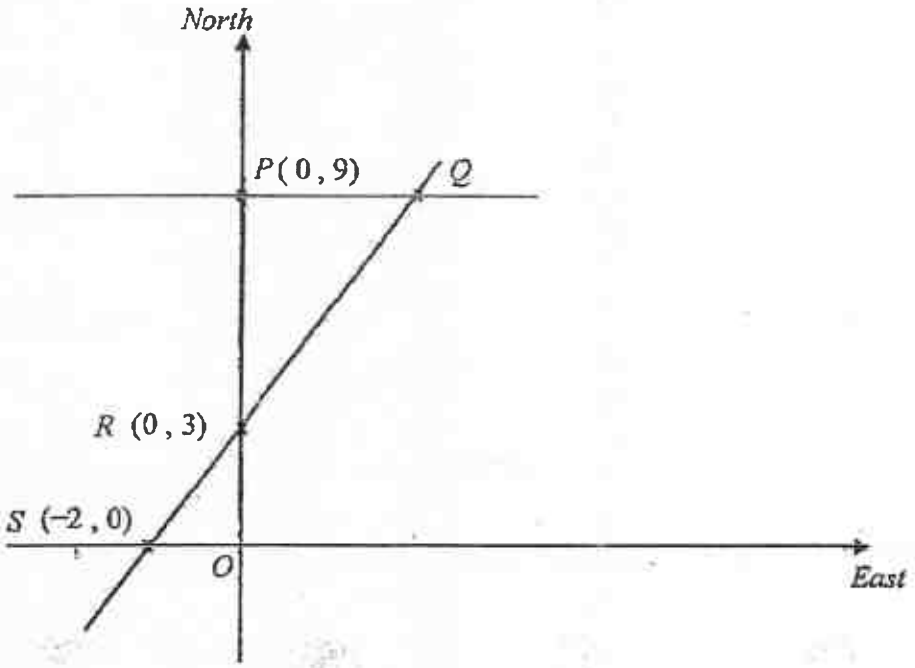
24. In the following figure, a circle with center O is located in triangle ABC.
AC meets the circle at point D and $AD = 8.5$ cm.
E is a point on the circumference of the circle, $AB = 17$ cm and $AE = 6$ cm.
The ratio of the area of triangle ABC to the area of the circle is 5 : 2.
Find the shortest distance from C to AB.

[Take $\pi = 3.142$]



Answer _____ cm [4]

25. In a battleship board game, the position of four ships labelled P, Q, R and S are represented on a Cartesian Plane with the North and East directions given.
Point O is the origin.



(a) Given that line PQ is perpendicular to line OR,
Find the coordinates of the ship at Q.

Answer Q (_____, _____) [3]

(b) Find the distance between Ship P and Ship S.

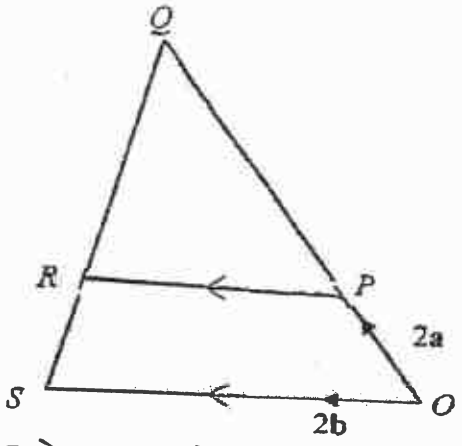
Answer _____ units [1]

(c) Find the bearing of Ship R from Ship Q.

Answer _____ ° [2]

26. In the diagram, $OPRS$ is a trapezium where PR is parallel to OS.

The line OP is produced to the point Q such that $\frac{OP}{OQ} = \frac{1}{3}$.



(a) Given that $\overrightarrow{OP} = 2\mathbf{a}$ and $\overrightarrow{OS} = 2\mathbf{b}$, express in terms of $\mathbf{a}$ and $\mathbf{b}$, as simply as possible,

(i) $\overrightarrow{SQ}$,

Answer ..... [1]

(ii) $\overrightarrow{OR}$

Answer ..... [1]

(b) It is given that $\overrightarrow{OT} = 6\mathbf{a} + 4\mathbf{b}$.

(i) Explain why O, R and T lie on a straight line.

Answer _____

_____

_____

_____ [1]

(ii) State the name of quadrilateral $OQTS$.

Answer _____ [1]

(c) (i) Find, giving your answer as a fraction in its simplest form, $\frac{\text{area of triangle } PQR}{\text{area of triangle } OQS}$

Answer _____ [1]

(ii) Hence write down the ratio of $\frac{\text{area of triangle } PQR}{\text{area of quadrilateral } OPRS}$.

Answer _____ [1]

End of Paper 1

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1. (a) (i) Simplify the expression $\frac{2x^2+7x-4}{x^2-16}$. [2]

(ii) Hence make x the subject of the formula $y=\frac{2x^2+7x-4}{x^2-16}$. [2]

(b) Solve these simultaneous equations. [3]

$$2x = 1-y,$$
$$4x+5y=8.$$

(c) Given that $\frac{1}{x+y}+\frac{2}{x-y}=\frac{2x+5y}{x^2-y^2}$,

(i) show that $\frac{x}{y}=4$. [2]

(ii) Hence find the value of $\left(\frac{3x}{2y}\right)^2$. [2]

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2. Alan bought m water bottles for \$128.

(a) Write down an expression, in terms of m, for the cost, in dollars, of one water bottle. [1]

(b) Alan sold 12 of the water bottles at a profit of \$2 each and the rest at \$7 per water bottle.

Write an expression, in terms of m, for the total amount of money he received from the sale of the water bottles. [1]

(c) Alan found that he made a profit of \$20 from the sale.

Write an equation in m to represent this information and show that it reduces to

$$7m^2-208m+1536=0.$$ [3]

(d) Solve the equation $7m^2-208m+1536=0$. [3]

(e) Find the selling price of each water bottle so that Alan makes a profit of 20%. [1]

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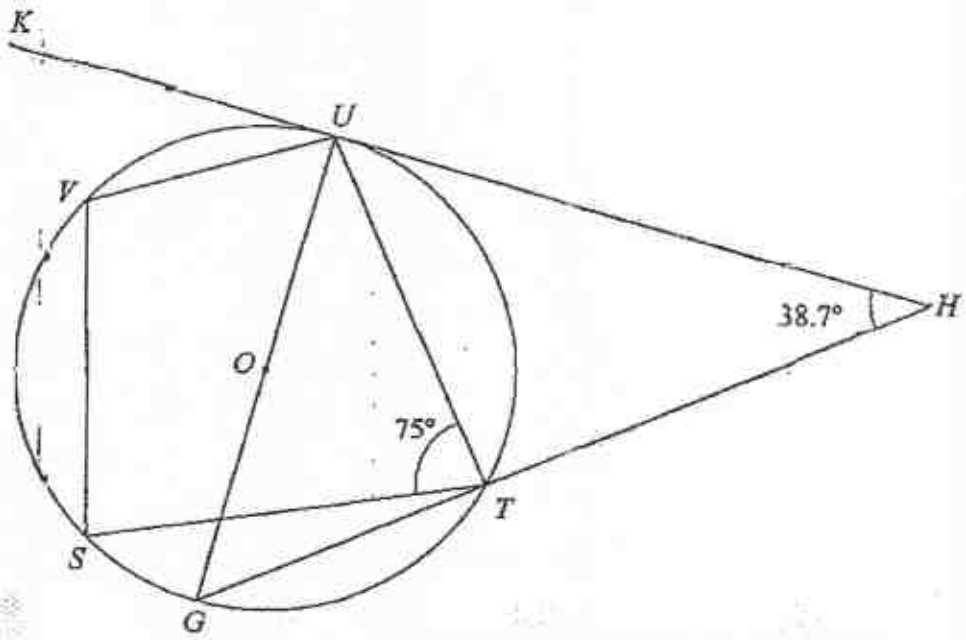
3. In the diagram, the points S, T, U and V lie on a circle with centre O.

G is a point on the circle such that GU is the diameter of the circle.
The tangent KU and the chord GT are extended to meet at point H.
$\angle STU = 75°$ and $\angle GHU = 38.7°$.



(a) Prove that triangle GTU and triangle GUH are similar. [2]

(b) Given that $HU = 8$ cm and $UT : GT = 5 : 4$, find the area of triangle GUH. [3]

(c) Stating your reasons clearly, calculate

(i) $\angle SVU$, [1]

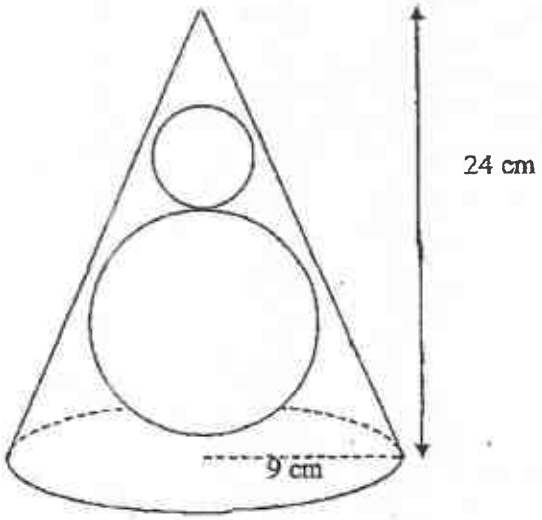
(ii) $\angle GTS$, [1]

(iii) $\angle TGU$, and [1]

(iv) $\angle TOU$. [1]

4. The diagram shows a conical container with radius 9 cm and height 24 cm.

Two balls are placed in the container as shown and 49.5π cm^3 of sand are needed to fill the container completely.



(a) Calculate the total surface area of the container. [2]

(b) If the balls are removed and the container is inverted, find the height of the sand in the container. [4]

(c) The radii of the two balls are in the ratio of 2 : 5.

Calculate the radius of the smaller ball. [4]

5. Answer the whole of this question on a single sheet of graph paper.

The variables x and y are connected by the equation

$$y = x - 2 + \frac{8}{x}.$$

Some corresponding values of x and y are given in the table below.

x	1	1.5	2	3	4	5	6	7	8
y	7.0	4.8	4.0	3.7	4.0	4.6	5.3	h	7.0

(a) Find the value of h. [1]

(b) Using a scale of 2 cm to represent 1 unit on each axis, draw a horizontal x-axis for $0 \le x \le 8$ and a vertical y-axis for $0 \le y \le 8$.

On your axes, plot the points in the given table and join them with a smooth curve. [3]

(c) By drawing a tangent, find the gradient of the curve at (4, 4.0). [2]

(d) Use your graph to solve the equation $x + \frac{8}{x} = 8.5$ for $0 \le x \le 8$. [2]

(e) (i) On the same axes, draw the line $y = 7 - x$ for $0 \le y \le 8$. [2]

(ii) Write down the x-coordinates of the points at which the two graphs intersect. [1]

(iii) Hence state the value of c such that the equation $2x^2 + cx + 8 = 0$ is satisfied by the values of x found in part (e)(ii). [1]

6. Diagram I shows a table with a horizontal plane $ABCD$ such that $AB = 120$ cm and $AD = 70$ cm.
Three vertical planes are erected along three sides of the table such that E and F are vertically above C and D respectively and $CE = DF = 30$ cm.
Q and P are the midpoints of BC and BE respectively.

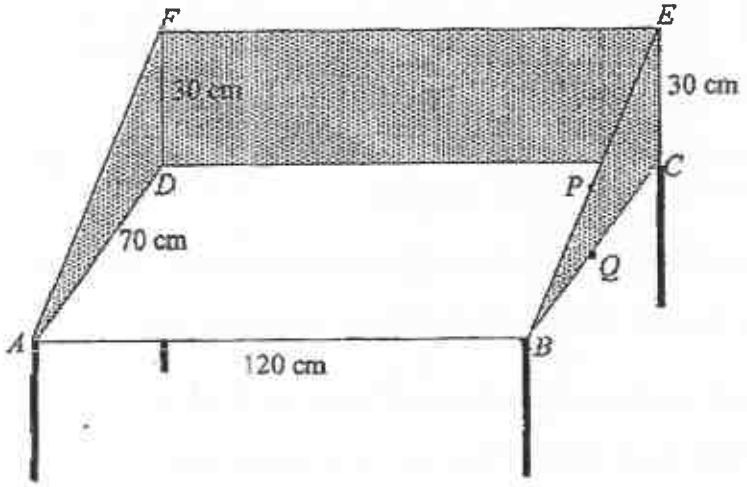


Diagram I

(a) Calculate

(i) AQ, [2]

(ii) angle PAQ. [2]

A wooden board is attached along EF with hinges such that it covers $ABEF$ in Diagram I.
$ABEF$ then becomes a tabletop that can be used by an architect when he draws his designs.
This tabletop can be lifted up and Diagram II shows the side view when this is done.
The new position for B is now B', 60 cm directly above B.

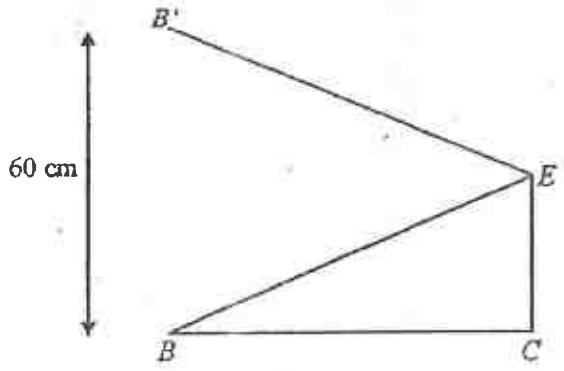


Diagram II

(b) (i) Show that angle BEB' is 46.397°, correct to 3 decimal places. [3]

(ii) Hence find the distance moved by point B, when the tabletop is lifted up to B'. [2]

7. A shop sells two types of cookies, Cranberry and Blueberry.

Each type is sold in packets of three different sizes, small (S), medium (M) and large (L). They are each sold at a different price.

The sales for two consecutive weeks, Week 1 and 2, are given in the following table.

	Week 1			Week 2		
Size	S	M	L	S	M	L
No. of packet of Cranberry cookies sold	15	10	12	7	11	9
No. of packet of Blueberry cookies sold	13	11	14	12	8	17
Cost per packet	\$4	\$5.50	\$6.50	\$4	\$5.50	\$6.50

The matrix **G** shows the sales of the cookies in Week 1.

$$\mathbf{G} = \begin{matrix} \\ \text{Cranberry} \\ \text{Blueberry} \end{matrix} \begin{matrix} \text{S} & \text{M} & \text{L} \end{matrix} \begin{pmatrix} 15 & 10 & 12 \\ 13 & 11 & 14 \end{pmatrix}$$

(a) Write down a matrix **D** to represent the sales of the cookies in Week 2. [1]

(b) Evaluate $\mathbf{M} = (\mathbf{G} + \mathbf{D})$ and state what its elements represent. [2]

(c) The cost of each packet of cookies for each size can be represented by the matrix **C**.

$$\mathbf{C} = \begin{matrix} \text{Cost} \\ \text{S} \\ \text{M} \\ \text{L} \end{matrix} \begin{pmatrix} 4 \\ 5.5 \\ 6.5 \end{pmatrix}$$

Evaluate $\mathbf{L} = \frac{1}{2}(\mathbf{MC})$ and state what its elements represent. [3]

(d) **(i)** Write down a matrix **T** such that **TMC** gives the total sales for the two weeks. [1]

(ii) Hence evaluate **TMC**. [1]

(e) The target sales of the cookies in Week 3, as compared to Week 1 are as follow:

Cranberry: increase by 35%
Blueberry: decrease to 85%

Write down the value of a and of b such that the matrix product

$$\begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} 15 & 10 & 12 \\ 13 & 11 & 14 \end{pmatrix}$$

gives the target sales of the cookies in Week 3. [1]

8. Diagram I shows a regular hexagon $ABCDEF$ and squares $AFPQ$ and $CBRS$.

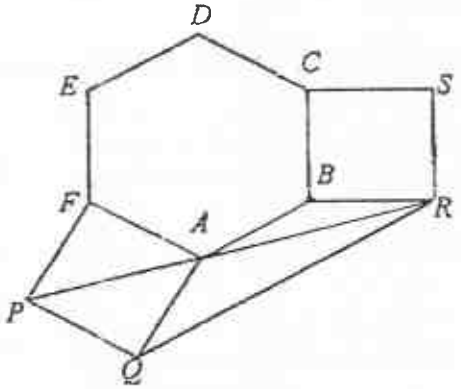


Diagram I

(a) Find

(i) reflex $\angle BAQ$, [2]

(ii) $\angle AQR$. [2]

(b) Show that PAR is a straight line. [2]

(c) Additional squares and hexagons are added to Diagram I to form a regular polygon, $ABR.....Q$, as shown in Diagram II.

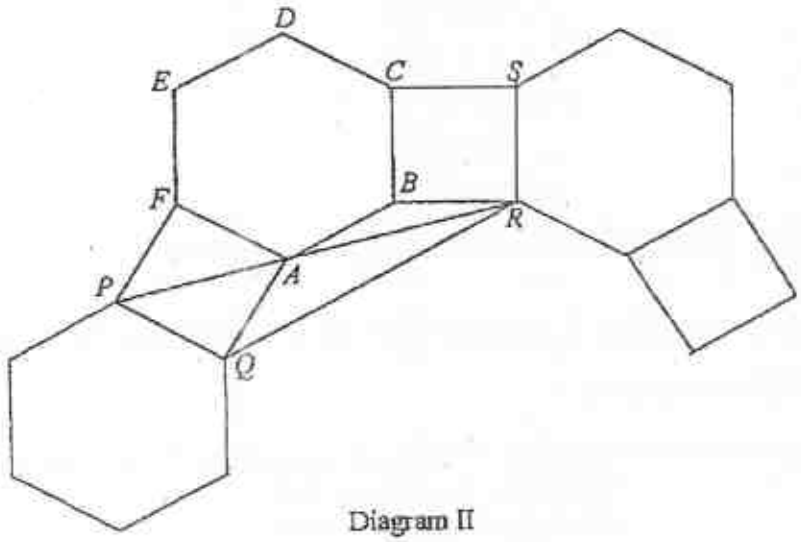


Diagram II

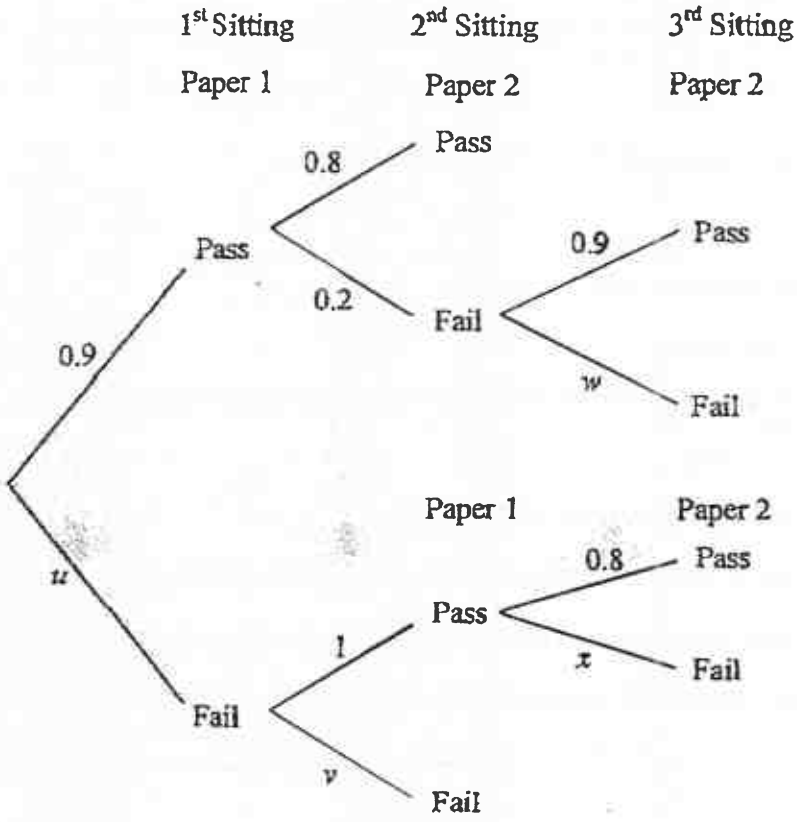
Calculate the number of squares added to form the polygon $ABR....Q$. [3]

9. (a) An entrance examination consists of 2 different papers, Paper 1 and Paper 2.

A candidate must pass Paper 1 before he can proceed to sit for Paper 2.
He must pass both papers in order to pass the examination.
He has a maximum of 3 sittings to pass the examination.

The probability of passing Paper 1 and 2 are 0.9 and 0.8 respectively, and increases by 0.1 for each subsequent attempt of the same paper.

(i) The tree diagram shows the probabilities of the possible outcomes.



Find the respective values of u, v, w, and x. [2]

(ii) Calculate the probability that a candidate

(a) passes the examination at the end of the second sitting, [1]

(b) does not pass the examination. [2]

(iii) If 1000 candidates enrolled for the examination, estimate the number of candidates expected to pass eventually. [1]

9. (b) The stem-and-leaf diagram shows the amount of time, in seconds, a group of boys can hold their breath when under water.

Stem	Leaf
1	5
2	
3	
4	0 0 1 2 3 5 7 7
5	2 4 4 4 5 6 6 6 6 6 7 8
6	1 2 3 3 5 7 8
7	0 0

Key : 4 | 2 means 42

(i) Find the

(a) median time taken, and [1]

(b) mean time taken. [1]

(ii) Is the median or the mean time a better representation, for the time taken by this group of boys?
Explain your answer. [1]

(iii) Calculate the standard deviation. [2]

(iv) Another group of 30 boys measured the time they took to hold their breath underwater.
Their mean time taken was 53.5 seconds and the standard deviation was 7.86.

Compare and comment on the results between these two groups of boys. [1]

10. ERGO is a company that sells ergonomic furniture for homes.
The types of furniture include study table-chair sets, chairs, baby cots and bunk beds.
The table below shows the average time taken by the delivery men to assemble each type of furniture.

Furniture	Average time taken to assemble each piece (minutes)
Study table-chair set	45
Chair	3
Baby cot	12
Bunk bed	105

(a) Find the total average time taken, in hours and minutes, to assemble one set of study table-chair set, one baby cot and one bunk bed. [1]

(b) The Operation Manager in the company is responsible for planning the daily delivery route.
On a particular day, the delivery route is as shown below.

No.	Location	Order	Estimated time of delivery
1	Happy Valley	• 1 study table-chair set • 2 chairs	09 00 to 10 30
2	Joyful Pasture	• 1 baby cot	10 30 to 12 00
3	Dream Cove	• 1 baby cot • 1 bunk bed	10 30 to 12 00
4	Blissful Ave	• 1 study table-chair set • 1 baby cot • 1 bunk bed	13 00 to 15 00
5	Peace Link	• 1 study table-chair set • 1 baby cot	15 00 to 17 00

Additional information needed for the delivery is shown on the opposite page.

The delivery men left the office at 09 15 for the first location at Happy Valley.
After assembling the orders, they proceeded to the second location at Joyful Pasture and arrived at 10 30.

(i) Calculate the average speed, in km/h, of the delivery van, leaving your answer to the nearest whole number.
Do you think the answer is a reasonable estimate of the actual travelling speed of the van? Justify your answer. [3]

(ii) The daily working hours for the delivery men is 08 30 to 18 00, and they are

Determine if the delivery men can leave the office punctually at 18 00 for that day.
Support your answer with appropriate calculations.
State one reasonable assumption you have made in your calculations. [6]

DISTANCE CHART BETWEEN THE VARIOUS LOCATIONS

Distance (in km)	ERGO Office	Happy Valley	Joyful Pasture	Dream Cove	Blissful Ave	Peace Link
ERGO Office	–	13.8	18.1	9.7	7.2	1.9
Happy Valley	13.8	–	4.7	3.8	8	16.3
Joyful Pasture	18.1	4.7	–	6.1	10.6	20
Dream Cove	9.7	3.8	6.1	–	5.4	9.3
Blissful Ave	7.2	8	10.6	5.4	–	8.8
Peace Link	1.9	16.3	20	9.3	8.8	–

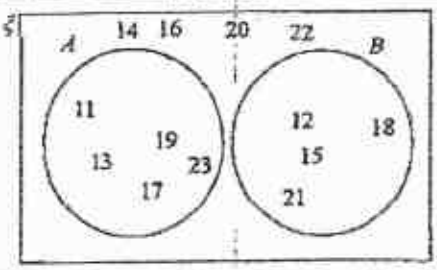
SPEED LIMITS FOR VEHICLES

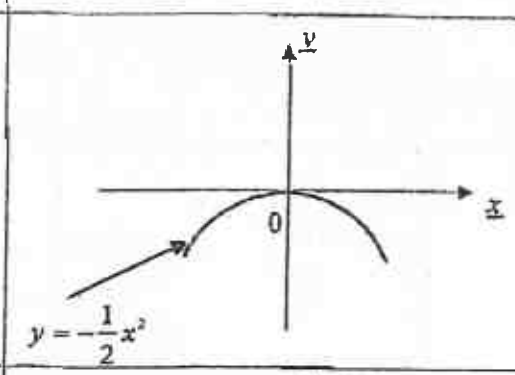
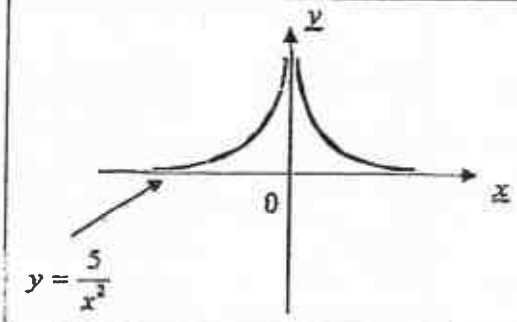
The following speed limits are enforced by LTA to ensure everyone's safety:

Type of Vehicle	Roads	Expressways	Tunnels
Cars & motorcycles	50km/h	70-90km/h	50-80km/h
Buses & coaches	50km/h	60km/h	50-60km/h
Light commercial vehicles (includes Light Goods Vehicles and small buses not exceeding 3.5 tonnes and seating capacity of up to 15 passengers)	50km/h	60-70km/h	50-70km/h
Exceptions: Fire engines, Ambulances, and Government vehicles used by Singapore Police Force or the Singapore Civil Defence Force			

https://www.lta.gov.sg/content/ltaweb/en/roads-and-motoring/road-safety-and-regulations/road-regulations.html

End of Paper 2

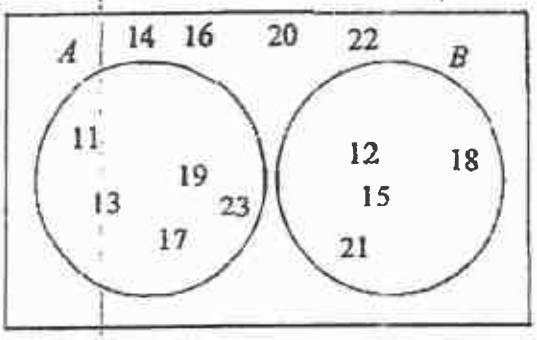
Qn			
1	-0.380	12a	13
2a	$\sqrt[3]{3},\ 0.\dot{6}\dot{6},\ \frac{\pi}{7},\ \frac{1}{3},\ -0.4,\ -\sqrt{4}$	12b	$x=59$
2b	$\sqrt[3]{3},\ \frac{\pi}{7}$	13a	$60 = 2^2 \times 3 \times 5$
3	$(a+1)(a-1)(5a^2+1)$	13b	$x=63$
4	Stating aspect or equivalent – 1 mark Explaining how audience might be mislead or equivalent – 1 mark	14	a = 5 b = 21 and c = 29
5	6	14b	$T_n=8n-3$
6	5.5 °C	14c	When 121 is a term in the sequence, n will have a value of 15.5. A pattern number n must be an integer. The value of 121 is resulted from a value of n = 15.5. This imply that the pattern number of 15.5 doesn't exist and hence 121 is not a term in this sequence.
7a	$n=12k+3$	15a	Speed = 25 m/s
7b	$n=3(4k+1)$ Since k is an integer, $4k+1$ will always be an integer. Therefore, n will be an integer. Based on $n=3(4k+1)$, n can be factorized to give $3(4k+1)$. Hence 3 and $4k+1$ are factors of $n=3(4k+1)$ and n will be a multiple of 3.	15b	759 m
8	-98.4375%	16	$x=1\frac{2}{5}$ or $x=2$
9a		17a	$\frac{9c^7}{2p^3}$
9b	14 , 16 , 20 and 22	17b	$n=-\frac{1}{3}$
10	$\frac{7}{25}$	18a	$1\frac{1}{5} < k \leq 4\frac{2}{5}$
11	$(x-4)^2-27$	18b	2, 3 and 4

Qn			
19ai		23a	6.38×10^3 km
		23b	1.08×10^9 km/h
		23c	1.11×10^{-3} minutes
		24	$x=8.43$ cm
19aii		25a	Coordinates of ship Q is (4, 9)
		25b	9.22 units
		25c	213.7°
19b	No, I do not agree. There are no roots to the equation as there are no common points of intersection between the two curves. These two curves will never meet each other.	26ai	$6a-2b$
20a	$\frac{7}{10}$	26aii	$2a+\frac{4}{3}b$
20b	34	26bi	$\overrightarrow{OT}=6a+4b$ $=3(2a+\frac{4}{3}b)$ $=3\overrightarrow{OR}$ $\overrightarrow{OT}$ is parallel to $\overrightarrow{QR}$ and O is a common point. O, R and T are collinear.
21a	$\frac{50}{83}\pi$	26bii	Trapezium
21b	6.85 cm^2	26ci	$\frac{4}{9}$
22ai	1 : 625000	26cii	4 : 5
22aii 22b	53.125 km 0.309 cm^2		

Sec 4 Express/5 Normal Prelim Paper 1 Marking Scheme

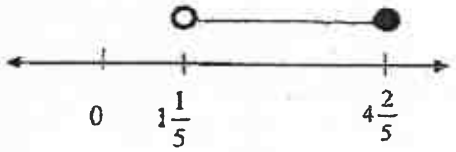
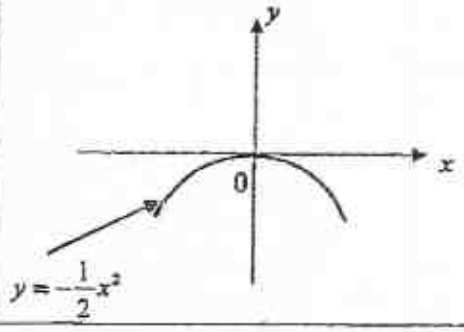
SN	Answer	Mark	Comments
1	$\frac{0.85^2-5.34}{\sqrt{81.2+3.134}}$ $= -0.38019$ $= -0.380$	 B1	 Correct rounding off to 3sf must be shown to be awarded B1
2a	$\sqrt[3]{3} = 1.442249$ $\frac{\pi}{7} = 0.448857$ $0.\dot{6}\dot{6} = \frac{2}{3}$ $\sqrt[3]{3},\ 0.\dot{6}\dot{6},\ \frac{\pi}{7},\ \frac{1}{3},\ -0.4,\ -\sqrt{4}$	 B1	 Correct order
2b	Irrational numbers are $\sqrt[3]{3},\ \frac{\pi}{7}$	B1	
3	$6a^2(a^2-1)-(a^2-1)^2$ $=(a^2-1)[6a^2-(a^2-1)]$ $=(a^2-1)[5a^2+1)]$ $=(a+1)(a-1)(5a^2+1)$	 M1 A1	Accept $5a^4-4a^2-1$ $=(5a^2+1)(a^2-1)$ $=(5a^2+1)(a+1)(a-1)$
4	The chart shown for year 2012 is approximately twice the size of the chart shown in 2013. However, the value of the knuckle velocity in 2012 is not twice the velocity as shown in 2013. Audience might be visually misled into thinking that the baseball player has reduced his knuckle velocity by a great amount.	B1 B1	Stating aspect or equivalent Explaining how audience might be mislead or equivalent

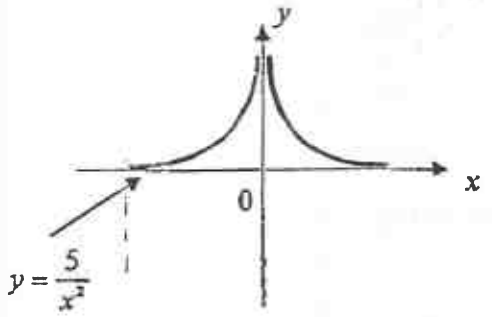
5	$a^2+6a=6$ $a(a^2+6a)=6a$ $a^3+6a^2=6a$ $a^3+6a^2+a^2=6a+a^2$ $a^3+7a^2=6a+a^2$ $=6$	M1 A1	a^3+7a^2 $=a^2(a+7)$ $=a^2[(a+6)+1]$ $=a^2(a+6)+a^2$ $=a(a^2+6a)+a^2$ $=6a+a^2$ $=6$ Or factoise to solve for 2 values of a and perform substitution to find answer.
6	Temperature difference $=34-(-5)$ $=39$ °C Required temperature $=34-\left(\frac{39}{26}\times 19\right)$ $=5.5$ °C	 M1 A1	 Accept workings like $-5+7(1.5)$ $=5.5$
7a	$n=12k+3$	B1	Accept $n=3(4k+1)$ o.e.
7b	$n=3(4k+1)$ **Since k is an integer, $4k+1$ will always be an integer. Therefore, n will be an integer.** Based on $n=3(4k+1)$, n can be factorized to give $3(4k+1)$. **Hence 3 and $4k+1$ are factors of $n=3(4k+1)$ and n will be a multiple of 3.**	 B1	 Only award B1 if student managed to explain both conditions of n.
8	$V=\frac{k}{T^3}$ When T is increased by 300%, New $T=4T$ $V=\frac{k}{(4T)^3}$ $V=\frac{k}{64T^3}$	 M1	

	Percentage decrease $= \dfrac{\frac{k}{64T^3} - \frac{k}{T^3}}{\frac{k}{T^3}} \times 100\%$ $= \dfrac{\frac{1}{64} - 1}{1} \times 100\%$ $= -98.4375\%$	A1	
9a	ξ  	B1	Any missing term will result in zero marks
9b	14 , 16 , 20 and 22	B1	
10	$\cos A = \dfrac{24}{25}$ Let the unknown side of the triangle be x $x^2 = 25^2 - 24^2$ $x^2 = 625 - 576$ $x^2 = 49$ $x = 7$ $\sin(180° - A) = \sin A$ $= \dfrac{7}{25}$	M1 A1	Accept if students show triangles with values of Pythagoras' Theorem applied with writing out the steps.

11	$-8x-11+x^2$ $= x^2-8x+\left(\frac{-8}{2}\right)^2-\left(\frac{-8}{2}\right)^2-11$ $=(x-4)^2-27$	M1 A1	
12a	13	B1	
12b	$\frac{5(1)+14(2)+3x+7(4)}{26+x}=2.8$ $61+3x=2.8(26+x)$ $61+3x=72.8+2.8x$ $0.2x=11.8$ $x=59$	M1 B1	
13a	$60=2^2\times3\times5$	B1	
13b	$378=2\times3^3\times7$ LCM of 60 and 378 $=2^2\times3^3\times5\times7$ $=3780$ $60x=3780$ $x=63$	 M1 B1	 Finding LCM Accept if students have written down workings and could make observations to find the value of x.
14a	$a=5$ $b=21$ and $c=29$	B1	
14b	$T_n=8n-3$	B1	Accept $5+8(n-1)$
14c	$8n-3=121$ $8n=124$ $n=15.5$ **When 121 is a term in the sequence, n will have a value of 15.5. A pattern number n must be an integer.** The value of 121 is resulted from a value of $n=15.5$. This imply that the pattern number of 15.5 doesn't exist and **hence 121 is not a term in this sequence.**	 B1	 Keywords must be seen in students' answer Accept words like **whole number** instead of integer, decimal and fraction accepted too Students must mention that n is not an integer

15a	Let the speed be x m/s $\frac{x-40}{20-17}=\frac{0-40}{25-17}$ $\frac{x-40}{3}=-5$ $x-40=-15$ $x=25$ Speed $=25$ m/s	M1 A1	Deceleration $=\frac{40}{8}$ $=5$ m/s^2 M1 Speed $=40-3(5)$ $=25$ m/s A1
15b	Total distance travelled $=\frac{1}{2}(13+40)(6)+(17-6)(40)+\frac{1}{2}(25-17)(40)$ $=159+440+160$ $=759$ m	M1 A1	
16	$\frac{8}{3-x}=5x-2$ $(3-x)(5x-2)=8$ $15x-6-5x^2+2x=8$ $-5x^2+17x-14=0$ $5x^2-17x+14=0$ $(5x-7)(x-2)=0$ $(5x-7)=0$ or $(x-2)=0$ $x=1\frac{2}{5}$ or $x=2$	M1 M1 A1	Marks awarded if student did not write "x = ____" (i.e some students wrote down 1.4 or 2 as answers) Accept $x=1.4$
17a	$18p^2c^3\div 4p^5c^{-4}$ $=\frac{18p^2c^3}{4p^5c^{-4}}$ $=\frac{9p^{2-5}c^{3+4}}{2}$ $=\frac{9p^{-3}c^7}{2}$ $=\frac{9c^7}{2p^3}$	 B1	Accept $\frac{9}{2}p^{-3}c^7$ Do not accept $4.5p^{-3}c^7$

17b	$9\times27^{2n}=1$ $3^2\times(3^3)^{2n}=3^0$ $3^2\times3^{6n}=3^0$ $2+6n=0$ $n=-\frac{1}{3}$	M1 A1	
18a	$-7\le 15-5k<9$ $-7\le 15-5k$ $-5k\ge -22$ $k\le \frac{22}{5}$ $k\le 4\frac{2}{5}$ and $15-5k<9$ $-5k<9-15$ $-5k<-6$ $k>1\frac{1}{5}$  $1\frac{1}{5}<k\le 4\frac{2}{5}$	 M1 A1	 For both correct inequalities Accept $1.2<k\le 4.4$ [Number line is optional]
18b	2, 3 and 4	B1	
19ai	 	B1	

19aii		B1	
19b	No, I do not agree. There are no roots to the equation as there are no common points of intersection between the two curves. These two curves will never meet each other.	B1	Accept alternative method $x^4+10=0$ $x^4=-10$ $x^4=\sqrt[4]{-10}$ $=$ *no solution* Therefore, there are no roots. Students need to mention that x = no solution and conclude that there are no roots to be given marks.
20a	Number of students who overestimate $=120-36$ $=84$ P(student overestimate the mass) $=\frac{84}{120}$ $=\frac{7}{10}$	M1 A1	 Accept 0.7
20b	120% of actual mass $=\frac{120}{100}\times 500$ $=600$ 80% of actual mass $=\frac{80}{100}\times 500$ $=400$	M1	Working out the respective upper and lower limits of the given range

	Number of students = 56-22 = 34	A1	Readings/Markings must be shown on graph to score M1 if students didn't work out/write down the limits on their answer scripts.
21a	Arc length travelled by smaller disc $= 2.5 \times 2\pi$ $= 5\pi$ cm Let θ be the angle of rotation made by the larger disc $8.3\,\theta = 5\pi$ $\theta = \frac{5\pi}{8.3}$ $= \frac{50\pi}{83}$ or $\frac{50}{83}\pi$ radian	M1 A1	 Accept 0.602π
21b	$\angle FWY = 1.03$ radian (alternate angles) $\angle WFY = \pi - 2(1.03)$ $= 1.08159$ radian Area of segment = $(\frac{1}{2} \times 8.3^2 \times 1.08159) - (\frac{1}{2} \times 8.3^2 \times \sin 1.08159)$ $= 37.25536 - 30.40481$ $= 6.85055$ $= 6.85$ cm^2	 M1 A1	Accept $\angle WFY = 1.081$ *or* 1.082 Area of segment will be 6.84 cm^2 or 6.86 cm^2 respectively. *premature rounding will only be awarded method mark
22ai	Map:Actual 0.16 cm^2 : 6.25 km^2 0.4 cm : 2.5 km 1 cm : 6.25 km Scale of map = 1 : 625000	 M1 A1	
22aii	Map:Actual 1 cm : 6.25 km 8.5 cm : 53.125 km Actual length of road = 53.125 km	 B1	

22b	Map : Actual 1 : 450000 1 cm : 450000 cm 1 cm : 4.5 km 1 cm^2 : 20.25 km^2 Actual : Map 20.25 km^2 : 1 cm^2 1 km^2 : $\frac{1}{20.25}$ cm^2 Area $= \frac{1}{20.25} \times 6.25$ $= 0.308641$ $= 0.309\ cm^2$	M1 A1	Accept $\frac{25}{81}$ cm^2 Students should refrain from giving this answer
23a	$2\pi r = 40075$ Radius $= \frac{40075}{2(3.142)}$ $= 6377.3074$ $= 6.38 \times 10^3$ km	M1 A1	
23b	Speed $= \frac{3 \times 10^8 m}{1s}$ $= \frac{3 \times 10^8 \times 10^{-3}}{\frac{1}{3600}}$ $= 1080000000$ $= 1.08 \times 10^9$ km/h	B1	Accept 1080 000 000 km/h
23c	Time taken $= \frac{\frac{1}{2} \times 40075}{1.08 \times 10^9} \times 60$ $= 1.11319 \times 10^{-3}$ $= 1.11 \times 10^{-3}$ minutes	M1 A1	No marks awarded if speed is wrong.

24	$\angle ODA = 90°$ (tangent perpendicular to radius) Let the radius of the circle be r $(6+r)^2 = r^2 + 8.5^2$ $36 + 12r + r^2 = r^2 + 8.5^2$ $12r = 36.25$ $r = 3.02083$ Area of triangle ABC $= 2.5 \times \pi(3.02083)^2$ $= 71.6802$ Let the shortest distance be x $\frac{1}{2} \times x \times 17 = 71.6802$ $x = 8.4329$ $x = 8.43\text{cm}$	 M1 M1 M1 A1	 Application of pythagoras' Theorem Finding radius Finding area of triangle Accept 3.020 or 3.021 Finding shortest distance
25a	Gradient $= \frac{3-0}{0-(-2)}$ $= \frac{3}{2}$ Equation of line QR is $y = \frac{3}{2}x + 3$ Sub $y = 9$ into $y = \frac{3}{2}x + 3$ $9 = \frac{3}{2}x + 3$ $18 = 3x + 6$ $3x = 12$ $x = 4$ Coordinates of ship Q is (4, 9).	 M1 M1 A1	
25b	Distance between ship P and ship S $= \sqrt{[0-(-2)]^2 + [9-0]^2}$ $= \sqrt{4+81}$ $= \sqrt{85}$ $= 9.2195$ $= 9.22$ units	 B1	Do not accept $\sqrt{85}$

25c	$\tan \angle PQR = \frac{6}{4}$ $\angle PQR = \tan^{-1}(\frac{6}{4})$ $= 56.30993$ Bearing of R from Q $= 360 - 90 - 56.30993$ $= 213.69°$ $= 213.7°$	M1 A1	
26ai	$\overrightarrow{SQ} = \overrightarrow{SO} + \overrightarrow{OQ}$ $= -2b + 6a$ $= 6a - 2b$	B1	Accept $2(3a - b)$
26aii	$\overrightarrow{OR} = \overrightarrow{OQ} + \overrightarrow{QR}$ $= 6a + \frac{2}{3}\overrightarrow{QS}$ $= 6a + \frac{2}{3}(-6a + 2b)$ $= 6a - 4a + \frac{4}{3}b$ $= 2a + \frac{4}{3}b$	B1	$2(a + \frac{2}{3}b)$
26bi	$\overrightarrow{OT} = 6a + 4b$ $= 3(2a + \frac{4}{3}b)$ $= 3\overrightarrow{OR}$ $\overrightarrow{OT}$ is parallel to $\overrightarrow{OR}$ and O is a common point. O, R and T are collinear.	B1	Students must prove that the value of k = 3 and state that there is a common point O to score B1
26bii	Trapezium	B1	
26ci	$\frac{\text{area of } \Delta PQR}{\text{area of } \Delta OQS}$ $= \left(\frac{2}{3}\right)^2$ $= \frac{4}{9}$	B1	

26cii	Ratio of $\frac{\text{area of } \Delta PQR}{\text{area of quadrilateral } OPRS}$ $= \frac{4}{5}$ $= 4 : 5$	B1	Accept $\frac{4}{5}$

Answers to 2017 Preliminary Exam Mathematics Paper 2

On	Answer
1ai	$\frac{2x-1}{x-4}$
aii	$x=\frac{4y-1}{y-2}$
b	$y=2,\ x=-0.5$
cii	36
2a	$\$\frac{128}{m}$
b	$\$\left[12\left(\frac{128}{m}+2\right)+(m-12)7\right]$
d	16 , 13.7 (or $13\frac{5}{7}$)
e	$9.60
3a	$\angle GTU = 90°$ (right angle in semicircle) $\angle GUH = 90°$ (radius perpendicular to tangent) $\Rightarrow \angle GTU = GUH = 90°$ $\angle G$ is a common angle $\therefore$ Triangle GTU and triangle GUH are similar. (All 3 corrsponding angles are equal)
b	$25.6cm^2$
ci	105°
cii	15°
ciii	51.3°
civ	102.6°
4a	979 cm^2
b	10.2 cm
c	3 cm
5a	6.1
c	0.510 (accept 0.4 to 0.6)
d	$x = 1.05, 7.4$ (accept ± 0.05)
eii	$x = 1.2, 3.25$ (accept ± 0.05)
6ai	125 cm
aii	6.8°
bii	61.7 cm
7a	$\begin{pmatrix} 7 & 11 & 9 \\ 12 & 8 & 17 \end{pmatrix}$
b	$\begin{pmatrix} 22 & 21 & 21 \\ 25 & 19 & 31 \end{pmatrix}$ It represents the **total sale or number of cookies** of **each type** and **each size sold** in the two weeks. **(number of cranberry and blueberry** cookies sold in **small, medium and large**

On	Answer
7c	$\begin{pmatrix} 190 \\ 203 \end{pmatrix}$ **Average amount of money collected per week** of **each type** of cookies.
di	$\begin{pmatrix} 1 & 1 \end{pmatrix}$
dii	(746)
e	$a = 1.35,\ b = 0.85$
8ai	210°
aii	30°
b	$\angle BAR = (180° - 150°) \div 2 = 15°$ (base $\angle$ of isos. $\triangle$) $\angle PAR = 45° + 120° + 15° = 180°$ $\therefore$ By the property *Adjacent angles on a straight line is supplementary*, PAR is a straight line
c	4 squares
9ai	$u = 0.1,\ v = 0,\ w = 0.1,\ x = 0.2$
aiia	$0.9 \times 0.8 = 0.72$
aiib	$0.9 \times 0.2 \times 0.1 + 0.1 \times 1 \times 0.2 + 0.1 \times 0 = 0.038$
aiii	$1000 - 1000(0.038) = 962$
9bia	Median time taken = 56 sec
bib	Mean time taken = 53.8 sec
bii	Median, as the extreme value of 15 can lower the mean time taken
biii	Standard deviation = 11.3
biv	The 2 groups of boys have **comparable lung power** since they have **almost the same mean**, but the **second group** of boys are **more consistent** in the amount of time they take to hold their breath under water (or there is a **smaller variation** in the amount of time they take to hold their breath under water) due to the **smaller standard deviation**.

PRELIMINARY EXAM 2017

SECONDARY 4 EXPRESS 5 NORMAL (ACADEMIC)

Mathematics Paper 2

Qn	Solution and Answer	Marks allocation
1ai	$\frac{2x^2+7x-4}{x^2-16}=\frac{(2x-1)(x+4)}{(x-4)(x+4)}=\frac{2x-1}{x-4}$	**M1**: factorization **A1**
aii	$y=\frac{2x^2+7x-4}{x^2-16}$ $y=\frac{2x-1}{x-4}$ $xy-4y=2x-1$ $xy-2x=4y-1$ $x(y-2)=4y-1$ $\therefore x=\frac{4y-1}{y-2}$	**M1** **A1**
b	$2x=1-y$ ------ Eqn 1 $4x+5y=8$ ------ Eqn 2 Subst. Eqn 1 into Eqn 2 $2(1-y)+5y=8$ $3y=6$ $\therefore y=2,\ x=-0.5$	**M1**: method of solving **A1 each**
ci	$\frac{1}{x+y}+\frac{2}{x-y}=\frac{2x+5y}{x^2-y^2}$ $\frac{x-y+2x+2y}{x^2-y^2}=\frac{2x+5y}{x^2-y^2}$ $\Rightarrow 3x+y=2x+5y$ $\Rightarrow x=4y$ $\therefore \frac{x}{y}=4$ (shown)	**M1**: combine LHS as 1 fraction **A1**
cii	$\left(\frac{3x}{2y}\right)^2=\frac{9}{4}\left(\frac{x}{y}\right)^2=\frac{9}{4}(4)^2=36$	**M1**: using (i) **A1**

Qn	Solution and Answer	Marks allocation
2a	$\$\frac{128}{m}$	**B1**: must show unit \$
b	$\$\left[12\left(\frac{128}{m}+2\right)+(m-12)7\right]$	**B1**: o.e.
c	$12\left(\frac{128}{m}+2\right)+(m-12)7-128=20$ $\frac{1536}{m}+24+7m-84-128=20$ $1536+7m^2-208m=0$ $7m^2-208m+1536=0$ (shown)	**M1**: form equation **M1**: simplification **A1**: required equation
d	$7m^2-208m+1536=0$ $\therefore m=\frac{-(-208)\pm\sqrt{(-208)^2-4(7)(1536)}}{2(7)}$ $=\frac{208\pm\sqrt{256}}{14}$ $=16,\ 13.7\ (\text{or}\ 13\frac{5}{7})$	**M1**: method of solving **M1**: simplification **A1**: both answers
e	As no. of water bottles must be a whole number, m = 13.7 is not accepted. Selling price of each bottle for 20% profit $=\$\left[1.2\left(\frac{128}{16}\right)\right]=\$9.60$	(students are STRONGLY ENCOURAGED to explain why one of the values is not accepted) **B1**
3a	$\angle GTU = 90°$ (right angle in semicircle) $\angle GUH = 90°$ (radius perpendicular to tangent) $\Rightarrow \angle GTU = GUH = 90°$ $\angle G$ is a common angle $\therefore$ Triangle GTU and triangle GUH are similar. (All 3 corrsponding angles are equal)	**B1**: 2 statements of evidence **B1**: concluding statement (accept 'By AA similarity')
b	From (a), ΔGTU and ΔGUH are similar $\Rightarrow \frac{TU}{UH}=\frac{GT}{GU}$ $\Rightarrow \frac{TU}{GT}=\frac{UH}{GU}=\frac{5}{4} \quad \Rightarrow \frac{8}{GU}=\frac{5}{4} \quad \Rightarrow GU=\frac{4}{5}\times 8=6.4\text{ cm}$ $\therefore$ Area of triangle $GUH=\frac{1}{2}\times GU\times HU=\frac{1}{2}\times 6.4\times 8=25.6\text{ cm}^2$ **Alternative approach** $\tan 38.7°=\frac{GU}{8} \quad \Rightarrow GU=8\tan 38.7°=6.4092\text{ cm}$ $\therefore$ Area of triangle $GUH=\frac{1}{2}\times GU\times HU=\frac{1}{2}\times 6.4092\times 8=25.6\text{cm}^2$	**M1** **M1, A1**

Qn	Solution and Answer	Marks allocation
3ci	$\angle SVU = 180° - 75° = 105°$ (angles in opposite segment)	**B1**: subtract 1 mark from whole question if no or wrong angle properties
cii	$\angle GTU = 90°$ (right angle in semicircle) $\therefore \angle GTS = 90° - 75° = 15°$	**B1**
ciii	$\angle GUH = 90°$ (radius perpendicular to tangent) $\therefore \angle TGU = 180° - 90° - 38.7° = 51.3°$ (angle sum in triangle)	**B1**
civ	$\angle TOU = 51.3° \times 2 = 102.6°$ (angles at centre is twice angle at circum)	**B1**
4a	Slant height of cone, $l = \sqrt{24^2 + 9^2} = \sqrt{657}$ cm $\therefore$ Total surface area of container $= \pi \times \sqrt{657} \times 9 + \pi \times 9^2 = 979.197... \approx 979 \text{ cm}^2$ (3 s.f.)	**M1** **A1**
b	Volume of container $= \frac{1}{3} \times \pi \times 9^2 \times 24 = 648\pi$ $\left(\frac{\text{Height of sand}}{24}\right)^3 = \frac{49.5\pi}{648\pi} = \frac{11}{144}$ $\therefore$ Height of sand $= \sqrt[3]{\frac{11}{144}} \times 24 = 10.183... \approx 10.2$ cm (3 s.f.)	**M1** (accept method using ratio of radius to find new volume) **M1**: ratios of similar solids **M1, A1**
c	Volume of the 2 balls $= 648\pi - 49.5\pi = 598.5\pi \text{ cm}^3$ $\frac{\text{Volume of small ball}}{\text{Volume of big ball}} = \left(\frac{2}{5}\right)^3 = \frac{8}{125}$ $\Rightarrow$ Volume of small ball $= \frac{8}{133} \times 598.5\pi$ $\frac{4}{3} \times \pi \times r^3 = 36\pi$ $\Rightarrow r^3 = 27$ $\therefore r = 3$ cm	**M1** **M1**: ratios of similar solids (accept method using radius as 2 times and 5 times respectively) **M1**: volume of small ball **A1**
5a	$h = 6.1$	**B1**: c.a.o.
b	See attached graph paper Points Smooth curve	**P2**: all points plotted correctly [**P1**: at least 6 points plotted correctly] **C1**: smooth curve
c	Tangent drawn at (4 , 4.0) Gradient = 0.510 (accept 0.4 to 0.6) (Calculated value = 0.5)	**B1** **B1**
d	Draw $y = 6.5$ $\therefore x = 1.05, 7.4$	**B1** **B1**: $\pm$ 0.05
ei	Draw the line $y = 7 - x$ for $0 \le x \le 8$	**B2**: correct line that span across the required range [**B1**: correct line but not long enough]

Qn	Solution and Answer	Marks allocation
5eii	$x = 1.2, 3.25$	**B1**: both, ± 0.05
eiii	$x - 2 + \frac{8}{x} = 7 - x$ $2x - 9 + \frac{8}{x} = 0$ $2x^2 - 9x + 8 = 0 \qquad \therefore c = -9$	Method using substitution of x values from (eii) is not accepted **B1**
6ai	$BQ = 35$ cm $AQ = \sqrt{35^2 + 120^2} = \sqrt{15625} = 125$ cm	**M1, A1**
aii	$PQ = 15$ cm $\tan P\hat{A}Q = \frac{15}{125}$ $\therefore$ angle $PAQ = \tan^{-1}\left(\frac{15}{125}\right) = 6.84.. \approx 6.8°$ (1 d.p.)	**√M1**: s.o.i, using AQ from (ai) **A1**
bi	$BE = \sqrt{30^2 + 70^2} = \sqrt{5800}$ cm $\therefore \cos BEB' = \frac{5800 + 5800 - 60^2}{2(5800)} = \frac{20}{29}$ $\angle BEB' = \cos^{-1}\left(\frac{20}{29}\right) = 46.3971... \approx 46.397°$ (3 d.p.) [shown] **Alternative approach** $\tan EBC = \frac{30}{70} \Rightarrow \angle EBC = \tan^{-1}\left(\frac{30}{70}\right) = 23.1985...°$ $\angle B'BE = 90° - 23.1985° = 66.8015°$ $\therefore \angle BEB' = 180° - 66.8015° \times 2 = 46.397°$ ($\angle$sum in isos. Δ)	**M1**: find BE, s.o.i. **M1**: applying Cosine Rule **A1** **M1** **M1** **A1**
bii	Distance moved by B is the length of arc on a circle centre E and radius BE, over an angle of BEB'. Distance moved by B $= \frac{46.397}{360} \times 2\pi \times \sqrt{5800} = 61.671... \approx 61.7$cm (3 s.f.)	**M1, A1**
7a	$\mathbf{D} = \begin{pmatrix} 7 & 11 & 9 \\ 12 & 8 & 17 \end{pmatrix}$	**B1**
b	$\mathbf{M} = \begin{pmatrix} 22 & 21 & 21 \\ 25 & 19 & 31 \end{pmatrix}$ It represents the **total sale or number of cookies** of **each type** and **each size** sold in the two weeks. (**number** of **cranberry and blueberry** cookies sold in **small, medium and large size respectively** in 2 weeks.)	**B1** **B1**
c	$\mathbf{L} = \frac{1}{2}\begin{pmatrix} 22 & 21 & 21 \\ 25 & 19 & 31 \end{pmatrix}\begin{pmatrix} 4 \\ 5.5 \\ 6.5 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 88 + 115.5 + 136.5 \\ 100 + 104.5 + 201.5 \end{pmatrix} = \begin{pmatrix} 170 \\ 203 \end{pmatrix}$ **Average amount of money collected per week** of **each type** of cookies.	**√M1**: using **M** from (b), product step, s.o.i **A1** **B1**: interpretation with 'earnings' or 'earned' not accepted.

Qn	Solution and Answer	Marks allocation
7di	$\mathbf{T} = \begin{pmatrix} 1 & 1 \end{pmatrix}$	**B1**
dii	$\mathbf{TMC} = \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 340 \\ 406 \end{pmatrix} = (340 + 406) = (746)$	**B1**: not awarded if not in proper matrix representation
e	$a = 1.35, b = 0.85$	**B1**: both, o.e.
8ai	Int. $\angle$ of hexagon $= 720° \div 6 = 120°$ Reflex $\angle BAQ = 90° + 120° = 210°$	**M1** **A1**
aii	$\angle BAQ = 360° - 210° = 150°$ ($\angle$ sum at a pt.) $\therefore \angle AQR = 180° - 150° = 30°$ (int. $\angle$s, $AB \parallel QR$)	**M1** **A1**
b	$\angle BAR = (180° - 150°) \div 2 = 15°$ (base $\angle$ of isos. Δ) $\angle PAR = 45° + 120° + 15° = 180°$ $\therefore$ By the property *Adjacent angles on a straight line is supplementary*, PAR is a straight line **Alternative approach** $\angle BAR = (180° - 150°) \div 2 = 15°$ (base $\angle$ of isos. Δ) $\angle QAR = 150° - 15° = 135°$ $\angle PAQ + \angle QAR = 45° + 135° = 180°$	**M1** **A1**: showing $\angle PAR$ is 180°, with $\angle$ property and concluding statement
c	Int. $\angle$ of polygon $= \angle BAQ = 150°$ $\Rightarrow$ ext. $\angle$ of polygon $= 30°$ $\Rightarrow$ no. of sides of polygon $= 360° \div 30° = 12$ No. of pairs of square and hexagon $= 6$ Total no. of squares $= 6$ $\therefore$ No. of squares added $= 4$	**M1** **√M1**: using no. of sides **A1**
9ai	$u = 0.1, v = 0, w = 0.1, x = 0.2$	**B2**: all [**B1**: 2 correct]
aiia	$0.9 \times 0.8 = 0.72$	**B1**: o.e.
aiib	$0.9 \times 0.2 \times 0.1 + 0.1 \times 1 \times 0.2 + 0.1 \times 0 = 0.038$	**M1, A1**: o.e.
aiii	$1000 - 1000(0.038) = 962$	**B1**
9bia	Median time taken = 56 sec	**B1**
bib	Mean time taken = 53.8 sec	**B1**
bii	Median, as the extreme value of 15 can lower the mean time taken	**B1**
biii	Standard deviation = 11.3	**B2** [**B1**: correct value but not 3 s.f.]
biv	The 2 groups of boys have comparable lung power since they have almost the same mean, but the second group of boys are **more consistent** in the amount of time they take to hold their breath under water (or there is a **smaller variation** in the amount of time they take to hold their breath under water) due to the **smaller standard deviation.**	**B1**: words in bold and underlined must be seen

Qn	Solution and Answer	Marks allocation
10a	Total time needed to assemble a study table-chair set, 1 baby cot and a bunk bed $= 45 + 12 + 105 = 162$ mins $= 2$ hrs 42 mins	**B1**: working expected
bi	Total distance from ERGO office to Joyful Pasture $= 13.8 + 4.7 = 18.5$ km Total time taken for travelling = Time duration from 09 15 to 10 30 – Total assemble time at Happy Valley $= 75 - (45 + 6) = 24$ mins $\therefore$ Average speed of delivery van $= 18.5 \div \frac{24}{60} = 46.25 \approx 46$ km/h (nearest whole number) This value **may not be a reasonable estimate** of the actual travelling speed of the van, as it **could be higher**, but **due to the road condition** and **time** spent for **stopping** at traffic lights, the **average speed is lower.** **Accept also:** Yes it is a reasonable value as it is within the speed limit by LTA.	**M1**: total distance ÷ total travelling time **A1** **B1**: comment that actual speed could be higher
bii	**Assumption:** • Traffic condition is about the same on the roads to the various locations, such that the average speed of the van is 46 km/h. • Owners are at home when the delivery men reach each location • There is no major traffic delay that day • Delivery van travels on normal road and not using expressway Total travelling time between the various locations from Joyful Pasture to ERGO Office $= \frac{(6.1 + 5.4 + 8.8 + 1.9)\text{ km}}{46\text{ km/h}} \approx 29$ mins (nearest min) Total assemble time at Joyful Pasture to Peace Link $= 12 \times 4 + 105 \times 2 + 45 \times 2 = 348$ mins $\Rightarrow$ Total time needed to complete all delivery and return to ERGO office $= 29 + 348 + 45 = 422$ mins $= 7$ hrs 2 mins Time to reach ERGO office after all delivery = 10 30 + 7 hrs 2 mins = 17 32 $\therefore$ The delivery men **will be able to leave work punctually** at 18 00 that day. * award marks if calculated from the start: ERGO office to all locations and back to ERGO office again	**B1**: any valid assumptions **√M1***: using speed in (bi) **M1*** **M1*** **M1** **B1**: must be supported with appropriate calculation

	Alternative approach: Total time to complete all delivery before lunch = Total travelling time from 10 30 to next location after lunch + total assemble time $= \frac{6.1+5.4}{46} + \left(\frac{12+12+105}{60}\right) = \frac{1}{4} + 2\frac{3}{20} = 2\text{h } 24 \text{ mins}$ $\Rightarrow$ Lunch time at (10 30 + 2 h 24 mins) = 12 54 $\Rightarrow$ Time reach Blissful Ave after lunch = 12 54 + 45 mins = 13 39 Time to reach office after last delivery = 13 39 + Total assemble time after luncgh + Total travelling time after lunch $= 13\,39 + \frac{12\times 2+105+45\times 2}{60} + \frac{8.8+1.9}{46}$ = 13 39 + 3 h 53 mins = 17 32 ∴ The delivery men **will be able to leave work punctually** at 18 00 that day. * award marks if calculated from the start: ERGO office to all locations and back to ERGO office again Accept method using total time to complete delivery and back to office is shorter than total time available from start of delivery at 09 15 to 18 00.	**B1:** valid assumptions as above **M1*** **M1** **M1** **M1*** **B1:** must be supported with appropriate calculation

JUNYUAN SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2017
SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)

CANDIDATE NAME

CLASS | | | | INDEX NUMBER | | |

MATHEMATICS **4048/01**

Paper 1 7 August 2017

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use

This document consists of 19 printed pages (including the Cover Sheet).

[Turn over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 Solve $0.5\left(4-\frac{x}{x+2}\right)=1$.

Answer $x =$ ..... [2]

2 Write as a single fraction $c-d+\frac{1}{c}+\frac{1}{d}-\frac{c^2-d^2}{c+d}$.

Answer ..... [2]

3 Brad invested $4 000 into an account which pays r% per annum interest compounded monthly. His account tripled in value after 320 months.

Find r.

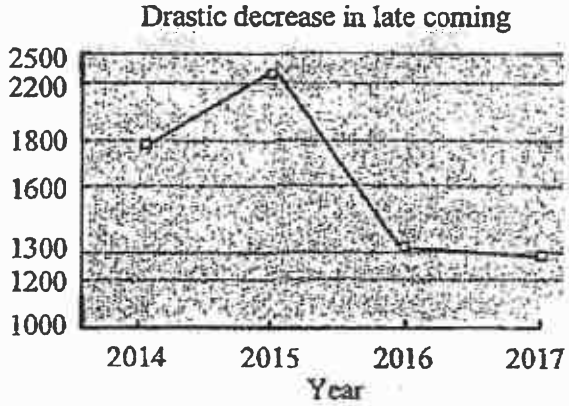
Answer $r =$ ..... [2]

4 An interior angle of a regular polygon is 120° bigger than its exterior angle.

Find the number of sides of the polygon.

Answer ..... [2]

5 The total count of student late coming occurrences in a school is represented by a line graph as shown.



State and explain how the graph can be modified to give a more accurate representation of the late coming occurrences in the school.

Answer .....

.....

..... [2]

6 $\xi = \{\text{even integers } x : 2 < x \leq 14\}$
$A = \{\text{perfect squares}\}$
$B = \{\text{factors of } 12\}$

(a) Draw a Venn diagram to illustrate this information.

Answer

[2]

(b) List the element(s) contained in the set $A \cap B'$.

Answer ..... [1]

---

7 Singapore's tourism hit a record high in 2016, where tourism numbers grew by 7.7% and tourism spending rose by 13.9%.

Some information about the number of tourists and tourism spending are given in the table.

Year	2016	2015
Number of Tourists	1.64×10^7	
Tourism Spending		S$21.4 billion

Estimate, to the nearest dollar, the average tourism spending in 2016.

Answer $ ..... [3]

8 Factorise completely

(a) $(d+e)^2-2(d+e)-8$,

Answer ..... [1]

(b) $1+x-2a-2ax$.

Answer ..... [2]

---

9 Two solid metal cones, which are geometrically similar, have surface areas A_1 and A_2 such that $9A_1=16A_2$.

If the volume of the larger cone is 32 cm^3, find the volume of the smaller cone.

Answer ..... cm^3 [3]

10 **(a)** Express 2 700 as a product of its prime factors.

Answer ..... [1]

(b) Using your answer to part **(a)**, explain why $2\,700b$ is a perfect cube when $b = 10$.

Answer .....

..... [1]

(c) Find the smallest value of p so that $2\,700 \times \sqrt{p}$ is divisible by 14.

Answer $p =$ ..... [1]

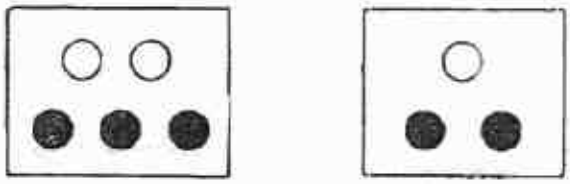
---

11 The length of a rectangle is 3 metres more than its width.
Its perimeter is equal in value to its area.

Find the dimensions of this rectangle.

Answer ..... m by ..... m [3]

12



Box A Box B

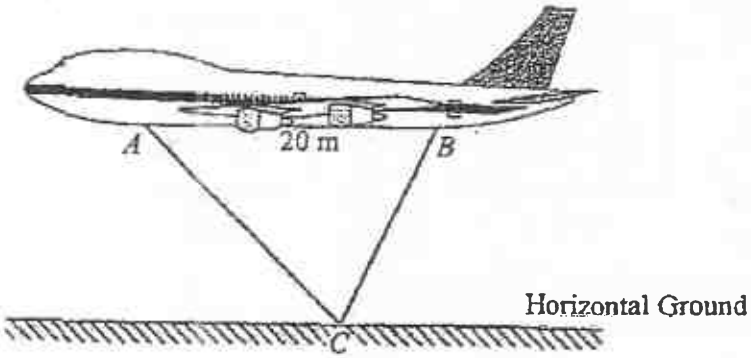
In Box A, there are 3 black balls and 2 white balls.
In Box B, there are 2 black balls and 1 white ball.

Ravi takes at random a ball from Box A and places it in Box B.
He then takes at random a ball from Box B.

Work out the probability that the ball he takes from Box B will be black.

Answer ..... [3]

13



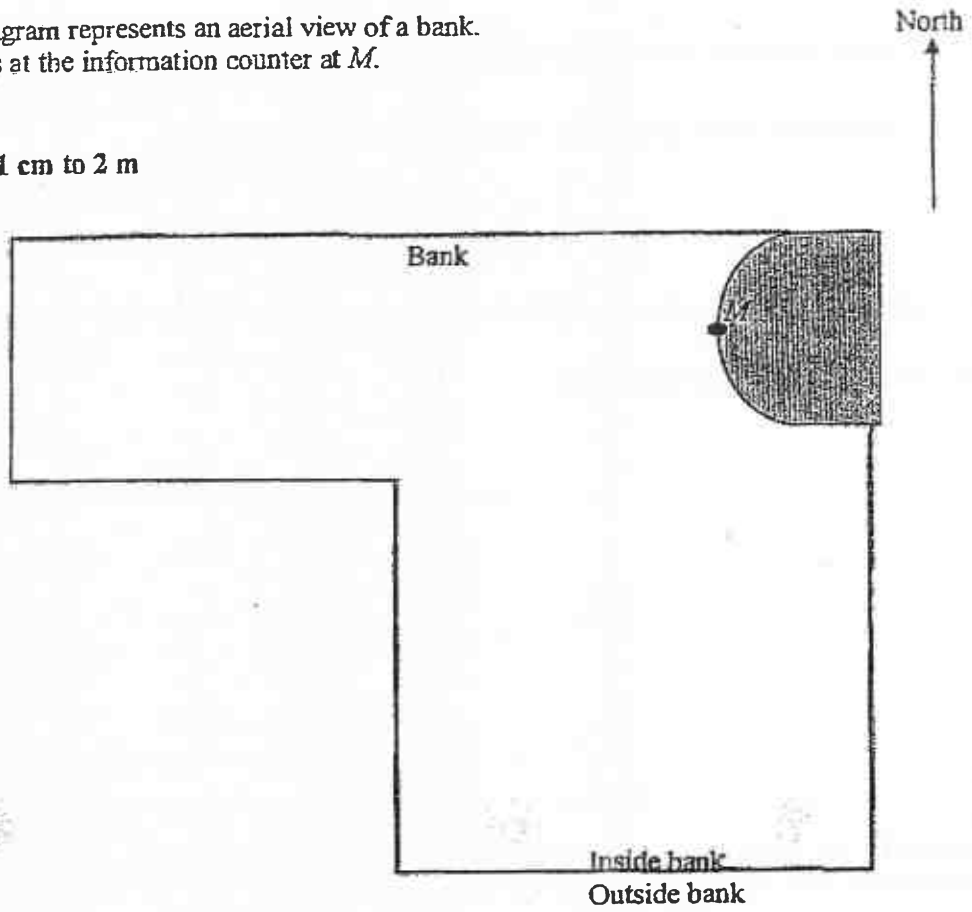
An aeroplane is flying parallel to the ground.
Lights have been fitted at A and B as shown.
When the aeroplane is flying at a certain height, the beams from these lights meet exactly on the ground at C.

The angle of depression of the beam of light from A to C is 50°.
The angle of depression of the beam of light from B to C is 70°.
The distance AB is 20 metres.

Find the height of the aeroplane from the ground when the lights meet at C.

Answer ..... m [3]

14 The diagram represents an aerial view of a bank.
Mark is at the information counter at M.



(a) Mark tethered his dog to a lamp post outside the bank, by means of a leash, at a bearing of 220° and 15 m from M.

On the diagram, mark out the location outside the bank where the dog is tethered to and label this point X.

[1]

(b) Jasmine is keeping a lookout for the dog inside the bank.
She is standing at a point that is equidistant from points M and X.

By showing your working clearly, work out one possible position Jasmine is standing at and label this point J. [1]

(c) The dog is unable to enter the bank.
The leash is 2 m long.

Draw the boundary of the region in which the dog can roam. [1]

15 (a) $(-5, 2)$ is the maximum point of a quadratic curve.

Write the equation of the graph in the form $y = p - (x+q)^2$.

Answer ..... [1]

(b) A straight line on the xy–axes passes through $(-5, 2)$ and cuts the x-axis at $x = 1$.

Find the equation of the straight line.

Answer ..... [2]

16 Ron exchanged 2 000 Singapore Dollars (SGD) for US Dollars (USD) in New York.
The exchange rate was x USD to 1 SGD.

After his trip, he had 8% of his total USD left.
He exchanged the remaining USD for SGD at the rate of 1 USD = 1.46 SGD and received 170 SGD.

Find x.

Answer $x =$ ..... [3]

17 (a) Write down all the integers satisfying the inequalities $-11 < 1 - 3x \leq 2$.

Answer ..... [3]

(b) Given $-6 \leq a \leq -1$ and $2 \leq b \leq 6$, find the range of possible values of $\frac{b}{a}$.

Answer ..... [1]

18 (a) Simplify $x\left(2x^{\frac{1}{4}}\right)^4$.

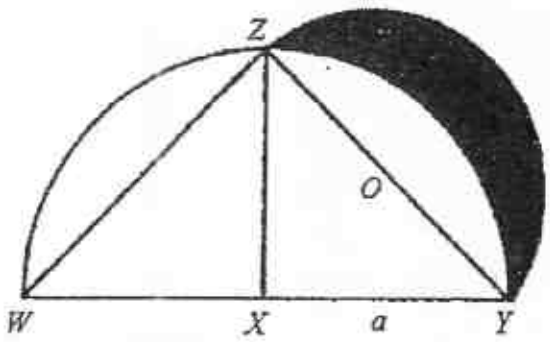
Answer ..... [2]

(b) Evaluate $\frac{3^{n+2}}{5(3^{n-1})}$.

Answer ..... [2]

19 WY is the diameter of a semi-circle with centre X and radius a cm.
Z is on the circumference and angle ZXY is a right angle.
A smaller semi-circle, centred at O, is drawn with ZY as diameter.

Find the area of the shaded region, in terms of a, in its simplest form.



Answer ..... cm^2 [4]

20 The area A of a television screen varies proportionally to the square of its diagonal d. A television set with a diagonal of 30 cm has an area of 440 cm^2.

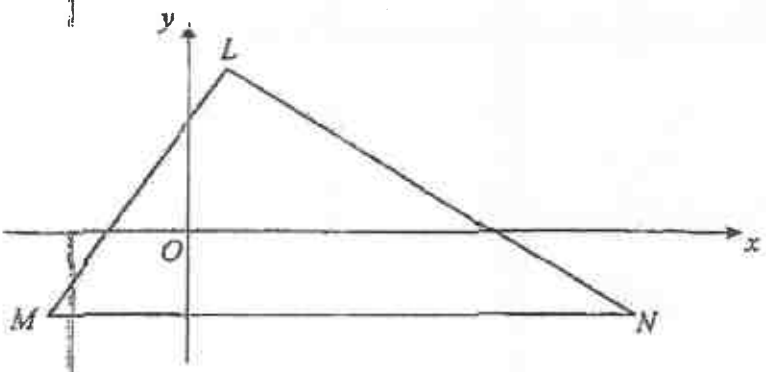
(a) Find the area of a television screen with a diagonal of 75 cm.

Answer ..... cm^2 [3]

(b) State the percentage change in A when d is decreased by 15%.

Answer ..... [1]

21 The diagram below, not drawn to scale, shows triangle LMN.



The equations of the lines LM and LN are $2y = 3x + 5$ and $x + 4y = 24$ respectively.

(a) Find the coordinates of L.

Answer (....., ...) [3]

(b) The coordinates of M are $(-3, -2)$ and MN is parallel to the x-axis.

Write the equation of line MN.

Answer ..... [1]

22 Justin is locked out of his house.
He intends to borrow a ladder.

The only open window is on the second floor, 8 m above the ground.
There is a bush along the edge of the house, 1 m away from the house and 2 m in height.

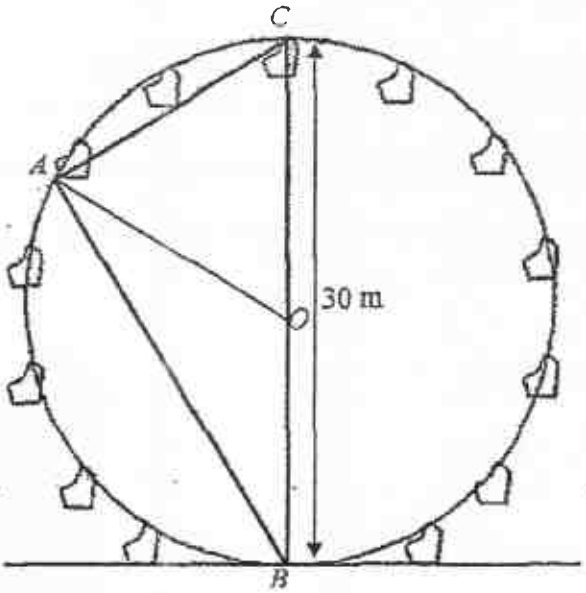
The bush is too thick for Justin to pass through on foot or climb through along the ladder.

What is the minimum length of the ladder Justin needs in order for him to reach the window?

Answer ..... m [4]

23 The diagram shows a circle which represents a ferris wheel with centre O.
The diameter is 30 m.

NOT TO SCALE



(a) A seat starts at B and travels one-third of the circumference to A.

Explain why angle AOB is equal to $\frac{2\pi}{3}$ radian.

Answer ..... [1]

(b) Find the exact value, in radian, of angle ABO.

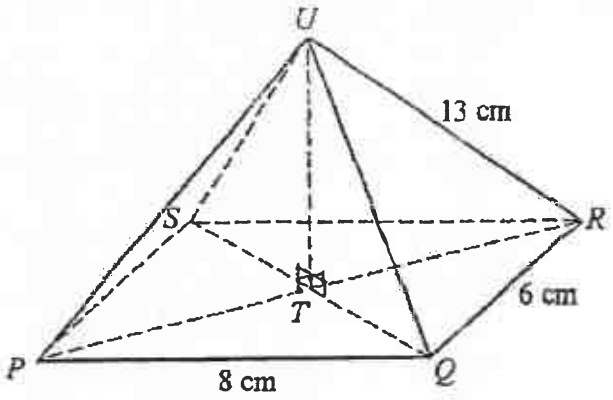
Answer ..... radian [2]

(c) It takes 2.5 minutes for a seat to travel from position B to A.

Find the average speed, in metres per second, of the wheel.

Answer ..... m/s [3]

24 **NOT TO SCALE**



The diagram shows a pyramid on a horizontal rectangular base $PQRS$.
The diagonals of $PQRS$ meet at T.
U is vertically above T.
$PQ = 8$ cm, $QR = 6$ cm and $UR = 13$ cm.

(a) Calculate angle URP.

Answer ..... ° [3]

(b) Find the volume of the pyramid.

Answer ..... cm^3 [2]

(c) Show that triangle PTQ is congruent to triangle RTS.

Answer .....

.....

.....

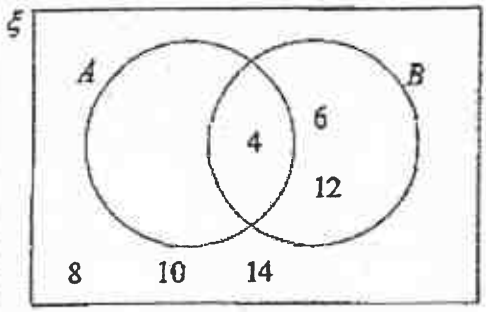
.....

..... [2]

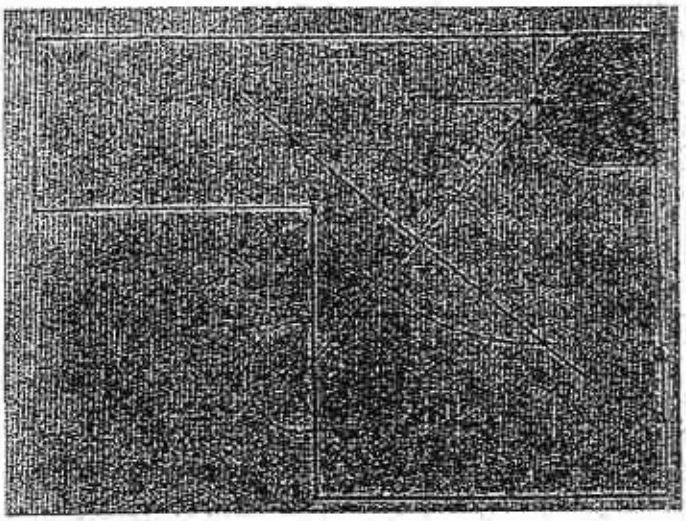
End of Paper

JYSS 4E5N Prelim 2017 Paper 1

No.	Answer	Workings	Marks	*Remarks
1	$x=6$	$0.5(4-\frac{x}{3})=1$ $2-\frac{x}{6}=1$ $\frac{x}{6}=1$ $x=6$	M1 A1	Alternative: $4-\frac{x}{3}=2$ $\frac{x}{3}=2$ $x=6$
2	$\frac{c+d}{cd}$	$c-d+\frac{1}{c}+\frac{1}{d}-\frac{c^2-d^2}{c+d}$ $=c-d+\frac{1}{c}+\frac{1}{d}-\frac{(c+d)(c-d)}{c+d}$ $=\frac{1}{c}+\frac{1}{d}$ $=\frac{c+d}{cd}$	M1 A1	M1 – For any correct method that eliminates $c-d-\frac{c^2-d^2}{c+d}$ to 0.
3	$r=4.13$	$12000=4000(1+\frac{r/12}{100})^{320}$ $3=(1+\frac{r}{1200})^{320}$ $r=4.1268$ $=4.13$	M1 A1	Award M1 for correct substitution of values
4	12	Let x be the size of an exterior angle. $2x+120°=180°$ $x=30°$ $n=\frac{360°}{30°}=12$	M1 A1	
5	The title is biased and does not allow readers to make their own judgement. It should only state "Late coming occurrences in the past 4 years". or The vertical axis has to start from zero so that it does not exaggerate the differences between the number of counts of late-coming. or The scale of the vertical axis have to be consistent and the intervals between the values on the vertical axis have to be equal. This prevents distortion of the graph.		B2	B1 - State the modification B1 – Explain how the modification will make the graph a better representation.

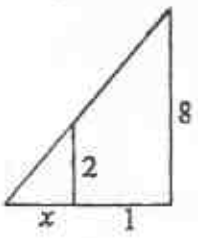
6	(a)	$\xi = \{4, 6, 8, 10, 12, 14\}$ $A = \{4\}$ $B = \{4, 6, 12\}$  		B2	Deduct 1m for each mistake
	(b)	{ } or ϕ		B1	
7		\$1486	Tourism spending in 2016 $= \frac{113.9}{100} \times 21.4 \times 10^9$ $= S\$2.43746 \times 10^{10}$ Average visitor spending $= \frac{2.43746 \times 10^{10}}{1.64 \times 10^7}$ = \$1486.256 = \$1486 (nearest dollar)	M1 M1 A1	No mark is awarded to finding number of visitors in 2015 as this information is not needed.
8	(a)	$(d+e+2)(d+e-4)$	$(d+e)^2 - 2(d+e) - 8$ $= (d+e+2)(d+e-4)$	B1	
	(b)	$(1-2a)(1+x)$	$1 + x - 2a - 2ax$ $= 1 + x - 2a(1+x)$ $= (1-2a)(1+x)$	M1 A1	
9		$13.5 cm^3$	$\frac{A_1}{A_2} = \frac{16}{9}$ $\frac{L_1}{L_2} = \sqrt{\frac{16}{9}} = \frac{4}{3}$ Using $\frac{V_1}{V_2} = (\frac{L_1}{L_2})^3$, $\frac{32}{V_1} = (\frac{4}{3})^3$ $\Rightarrow V_1 = 13.5 \text{ cm}^3$ (3 s.f.)	M1 M1 A1	

No.		Answer	Workings	Marks	*Remarks
10	(a)	$2700 = 2^2 \times 3^3 \times 5^2$		B1	
	(b)	When $b = 10$, $2700 \times 10 = 2^2 \times 3^3 \times 5^2 \times (2 \times 5) = 2^3 \times 3^3 \times 5^3$ Since all the powers of 2700×10 are multiples of 3, $\sqrt[3]{2700 \times 10} = \sqrt[3]{2^3 \times 3^3 \times 5^3} = 2 \times 3 \times 5$, it is a perfect cube.		B1	Accept any correct explanation that $2^3 \times 3^3 \times 5^3$ is a perfect cube.
	(c)	$p = 49$	$\frac{2700 \times \sqrt{p}}{14}$ $= \frac{2^2 \times 3^3 \times 5^2 \times \sqrt{p}}{2 \times 7}$ $\Rightarrow \sqrt{p} = 7$ $\Rightarrow p = 49$	B1	
11		3 m by 6 m	Let width = x m length = $(x+3)$ m Perimeter = Area $2(x+x+3) = x(x+3)$ $4x+6 = x^2+3x$ $x^2 - x - 6 = 0$ $(x+2)(x-3) = 0$ $x = 3$ or $x = -2$ (rej) Width = 3 m Length = 6 m	M1 M1 A1	Correct expression for both area and perimeter Correct factorization Correct values for both width and length
12		$\frac{13}{20}$	Black from Box A then black from Box B: $\frac{3}{5} \times \frac{3}{4} = \frac{9}{20}$ White from Box A then black from Box B: $\frac{2}{5} \times \frac{2}{4} = \frac{1}{5}$ Total probability $= \frac{9}{20} + \frac{1}{5}$ $= \frac{13}{20}$	M1 M1 A1	Accept 0.65

No.		Answer	Workings	Marks	*Remarks
13		16.6 m	$\angle ACB = 60°$ By Sine Rule, $\frac{20}{\sin 60°} = \frac{BC}{\sin 50°}$ $BC = 17.6910\,\text{m}$ $\sin 70° = \frac{h}{17.6910}$ $h = 16.6\,\text{m}$	M1 M1 A1	
14	(a)			B1	
	(b)			B1 No mark for no working	Accept position of J anywhere along the correct perpendicular bisector drawn and inside the bank.
	(c)			B1	Accept boundary as the arc of 1 cm outside the bank. No mark given if arc extends inside the bank.
15	(a)	$y = 2-(x+5)^2$		B1	
	(b)	$y = \frac{1}{3}x - \frac{1}{3}$	Gradient of line passing through $(-5,2)$ and $(1,0)$ $= \frac{2-0}{-5-1} = -\frac{1}{3}$ $y = -\frac{1}{3}x + c$ $0 = -\frac{1}{3}(1) + c$ $c = \frac{1}{3}$ $\Rightarrow \quad y = -\frac{1}{3}x + \frac{1}{3}$	M1 A1	Accept $\frac{1}{3} = 0.333$

No.		Answer	Workings	Marks	*Remarks
16		$x = 0.73$	SGD 1.46 = 1 USD SGD 170 $= \frac{170}{1.46} = 116.438$ USD Total amount of USD $= \frac{116.438}{8} \times 100$ = 1455.475 USD 2000 SGD = 1455.475 USD 1 SGD = 0.7277 USD $x = 0.73$ (2 d.p.)	M1 M1 A1	
17	(a)	$x = 0, 1, 2, 3$	$-11 < 1 - 3x \leq 2$ $-1 \leq 3x < 12$ $-\frac{1}{3} \leq x < 4$ $\Rightarrow x = 0, 1, 2, 3$	M1 M1 A1	Award M2 for any correct method to get $-\frac{1}{3} \leq x < 4$.
	(b)	$-6 \leq \frac{b}{a} \leq -\frac{1}{3}$		B1	Accept $\frac{1}{3} = 0.333$
18	(a)	16	$x(2x^{\frac{1}{4}})^4$ $= x(16x^{-1})$ = 16	 M1 A1	
	(b)	5.4	$\frac{3^{n+2}}{5(3^{n-1})}$ $= \frac{3^{n+2-n+1}}{5}$ $= \frac{3^3}{5}$ = 5.4	 M1 A1	Also accept final answer as $\frac{27}{5}$ or $5\frac{2}{5}$.

No.		Answer	Workings	Marks	*Remarks
19		$\frac{1}{2}a^2$	Area of quadrant $= \frac{1}{4}\pi a^2$ Area of triangle ZXY $= \frac{1}{2}a^2$ Area of segment ZY $= \frac{1}{4}\pi a^2 - \frac{1}{2}a^2$ Diameter ZY $= \sqrt{a^2 + a^2}$ $= \sqrt{2}a$ Area of semi-circle $= \frac{1}{2}\pi\left(\frac{\sqrt{2}a}{2}\right)^2$ $= \frac{1}{4}\pi a^2$ Area of shaded region $= \frac{1}{4}\pi a^2 - \left(\frac{1}{4}\pi a^2 - \frac{1}{2}a^2\right)$ $= \frac{1}{2}a^2$	M1 M1 M1 A1	Accept $0.5a^2$
20	(a)	2750 cm^2	$A = kd^2$ $440 = k(30)^2$ $k = \frac{22}{45}$ $A = \frac{22}{45}(75)^2$ $= 2750$ cm^2	M1 M1 A1	
	(b)	When d decreases by 15%, A becomes $(0.85)^2 = 0.7225$. A decreases by 27.75%.		B1	May use calculator to verify
21	(a)	(2, 5.5)	$2y = 3x + 5$ --- (1) $x + 4y = 24$ --- (2) $x = 24 - 4y$ --- (3) Sub (3) into (1), $2y = 3(24 - 4y) + 5$ $14y = 77$ $y = 5.5$ $x = 2$	M1 M1 A1	1m for correct substitution or elimination method 1m for correct x and y 1m for correct coordinates (accept $5\frac{1}{2}$ or $\frac{11}{2}$)

	(b)	$y=-2$		B1	
22		8.11 m	Let x be the distance from the bush to the ladder. Using similar triangles, $\frac{2}{x}=\frac{8}{x+1}$ $6x=2$ $x=\frac{1}{3}$  Length of ladder $=\sqrt{(1\frac{1}{3})^2+8^2}$ $=8.11$ m	M1 M1 M1 A1	
23	(a)	$\frac{2\pi}{3}$ radian is one third of one revolution (2π).		B1	
	(b)	$\frac{\pi}{6}$ radian	angle ABO $=\frac{180°-120°}{2}$ $=30°$ $=\frac{\pi}{6}$	M1 A1	Accept radian method $\frac{\pi-\frac{2\pi}{3}}{2}=\frac{\pi}{6}$
	(c)	0.209 m/s	Total time $=2.5\times60=150$ s Total distance $=r\theta$ $=\frac{2\pi}{3}(15)$ $=10\pi$ Average speed $=\frac{10\pi}{150}$ $=0.209$ m/s	M1 M1 A1	
24	(a)	67.4°	$TR=\sqrt{3^2+4^2}=5$ $\cos URT=\frac{5}{13}$ Angle $URT=67.4°$	M1 M1 A1	
	(b)	192 cm^3	$UT=\sqrt{13^2-5^2}=12$ Volume $=\frac{1}{\ldots}$	M1 A1	

	(c)	SR = PQ (sides of rectangle) ST = TQ (T is the midpoint of diagonal) TR = TP (T is the midpoint of diagonal) By SSS Test, triangle *PTQ* is congruent to triangle *RTS*.	M1 A1	Accept correct SAS and ASA tests too.

1 (a) Simplify $\frac{5(x-y)^4}{(x+y)^2} \div \frac{(x-y)^3}{6x+6y}$. [2]

(b) Express $\frac{5}{x-1} - \frac{2}{x^2-1}$ as a single fraction in its simplest form. [2]

(c) It is given that $z = \frac{x^2-y^2}{y}$.

(i) Make x the subject of the formula. [2]

(ii) If $x = 2$ and $z = 3$, find the value(s) of y. [3]

(d) Given that $\frac{x+3y}{5x-4y} = \frac{2}{3}$, find the ratio $x : y$. [3]

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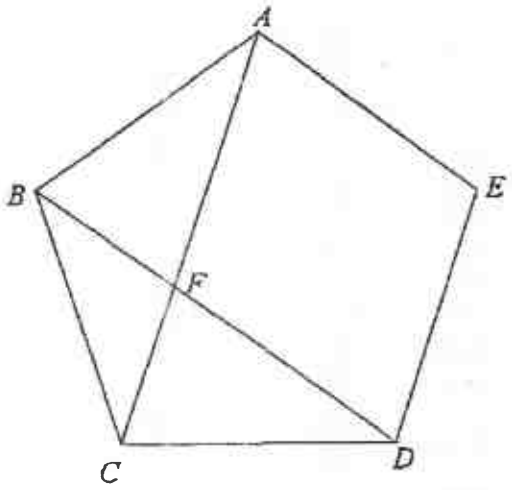
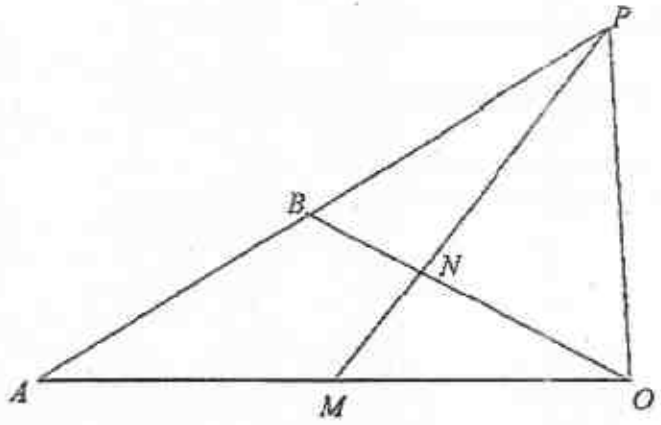
2 (a) The diagram below shows a regular pentagon $ABCDE$.
AC and BD intersect at F.



(i) Find the value of angle CDF, [2]

(ii) Explain why angle $DFA = 108°$. [2]

(b) In the triangle OAB, M is the midpoint of OA.
N is a point on OB such that $ON : NB = 2 : 1$.
MN is produced to P so that $MN : NP = 1 : 2$.



It is given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(i) Express, in terms of **a** and/or **b**,

(a) $\overrightarrow{NB}$, [1]

(b) $\overrightarrow{MN}$, [1]

(c) $\overrightarrow{NP}$. [1]

(ii) Express $\overrightarrow{AB}$ and $\overrightarrow{BP}$ in terms of **a** and **b**. [2]

(iii) Write down two facts about points A, B and P. [2]

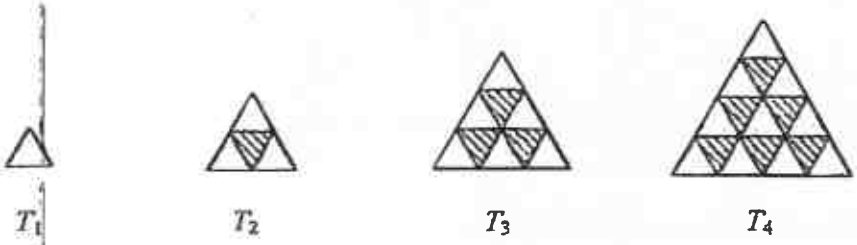
(iv) Find $\dfrac{\text{area of triangle } PMB}{\text{area of triangle } PMA}$. [1]

3 (a) The n^{th} term of a sequence is given by $T_n = \frac{n(n+3)}{2}$.

(i) Which term of the sequence has value 275? [2]

(ii) Explain why each term of the sequence is a whole number. [1]

(b) The diagram shows a sequence of shapes T_1, T_2, T_3, ...
Each shape consists of a number of shaded and unshaded triangles.



T_1 T_2 T_3 T_4

The letter r represents the number of rows of triangles in each shape

The letters S, U and N represent the number of shaded triangles, unshaded triangles and total number of triangles respectively.

The data is recorded in the table below.

Shape		T_1	T_2	T_3	T_4	T_5
Number of rows	r	1	2	3	4	5
Number of shaded triangles	S	0	1	3	6	a
Number of unshaded triangles	U	1	3	6	10	b
Total number of triangles	N	1	4	9	16	c

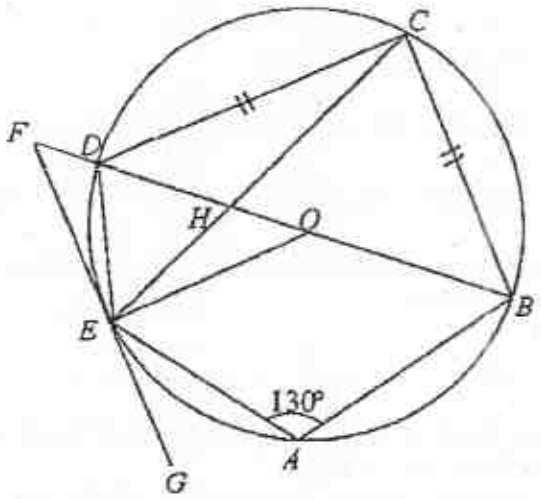
(i) Write down the value of a, of b and of c. [3]

(ii) Write a formula for the total number of triangles in the r^{th} shape, N_r. [1]

(iii) Write a formula for the number of unshaded triangles in the r^{th} shape, U_r. [1]

(iv) Find the number of shaded triangles in shape T_{50}. [2]

4 In the diagram, which is not drawn to scale, O is the centre of the circle.
Points A, B, C, D and E lie on the circumference.



BD is a diameter.
The tangent at E meets BD produced at F.
EC meets BD at H.
$BC = CD$ and angle $EAB = 130°$.

(a) Stating your reasons clearly, find

(i) reflex angle EOB, [1]

(ii) angle ECB, [1]

(iii) angle CBD, [1]

(iv) angle DOE, [1]

(v) angle OFE. [1]

(b) Is the line ED parallel to line BC?
Justify your answer with clear working. [2]

5 The distance between Town P and Town Q is 150 km.
An express bus travels from Town P to Town Q at the average speed of x km/h.
If the average speed of the bus is increased by 15 km/h, the time taken would be 21 minutes less.

(a) Express, in terms of x,

(i) the time taken by the bus at its original speed, [1]

(ii) the time taken by the bus when the speed is increased by 15 km/h. [1]

(b) Form an equation in x and show that it can be reduced to $7x^2 + 105x - 45\,000 = 0$. [3]

(c) Solve the equation in part (b) and hence find the original time taken in hours and minutes, correct to the nearest minute. [4]

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6 At the end of a semester, the final grade of the students is recorded based on their marks obtained from tests, projects, homework and quizzes.
The marks obtained by three students, Aaron, Beatrice and Carly, are given in the following table.

	Tests	Projects	Homework	Quizzes
Aaron	82	95	89	60
Beatrice	72	85	65	57
Carly	88	91	70	64

(a) (i) Write down a 3×4 matrix **M** that represents the information in the table. [1]

(ii) The weightage for each component are as follows:

Tests	50%
Projects	20%
Homework	10%
Quizzes	20%

Represent the weightage as a decimal number in a 4×1 matrix **X**. [1]

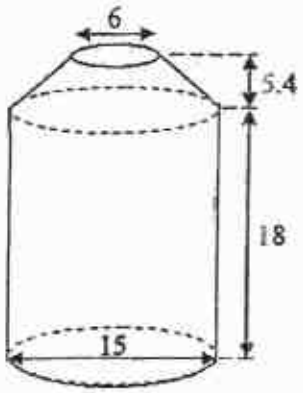
(iii) Evaluate the matric $\mathbf{F} = \mathbf{MX}$. [2]

(iv) State what the elements of **F** represent. [1]

(b) Overall, the cohort did better in projects than in quizzes.

Suggest how the weightage for each component could change so as to improve the final grade of the students. [1]

7 The diagram, not drawn to scale, shows an open container which is made up of a cylinder and a frustum.
A frustum is a cone with part of its top removed.
The cylinder has height 18 cm and diameter of 15 cm.
The conical section has base diameter 15 cm, top diameter 6 cm and height 5.4 cm.



(a) Show that the height of the cone before its top was removed is 9 cm. [2]

(b) The container is filled to its brim with water.

Calculate

(i) the volume of the water in the container, [2]

(ii) the total surface area of the container in contact with water. [3]

(c) All the water in the container is poured into a rectangular tank with a base area of 120 cm^2.

Find the minimum height of the tank so that the water **does not overflow.**
Give your answer as a whole number. [2]

8 The variables x and y are connected by the equation $y = 2x + \frac{50}{x} - 30$, where $x \neq 0$.

Some corresponding values of x and y are given in the following table, corrected to 2 decimal places.

x	1	1.5	3	5	7	9	11	13	15	16
y	22.00	p	−7.33	−10.00	−8.86	−6.44	q	−0.15	3.33	5.13

(a) Find the value of p and of q. [2]

(b) Using a scale of 1 cm to 1 unit on the horizontal x-axis and 2 cm to 5 units on the vertical y-axis, draw the graph of $y = 2x + \frac{50}{x} - 30$ for $1 \leq x \leq 16$. [3]

(c) By drawing a tangent, find the gradient of the curve at the point $x = 10$. [2]

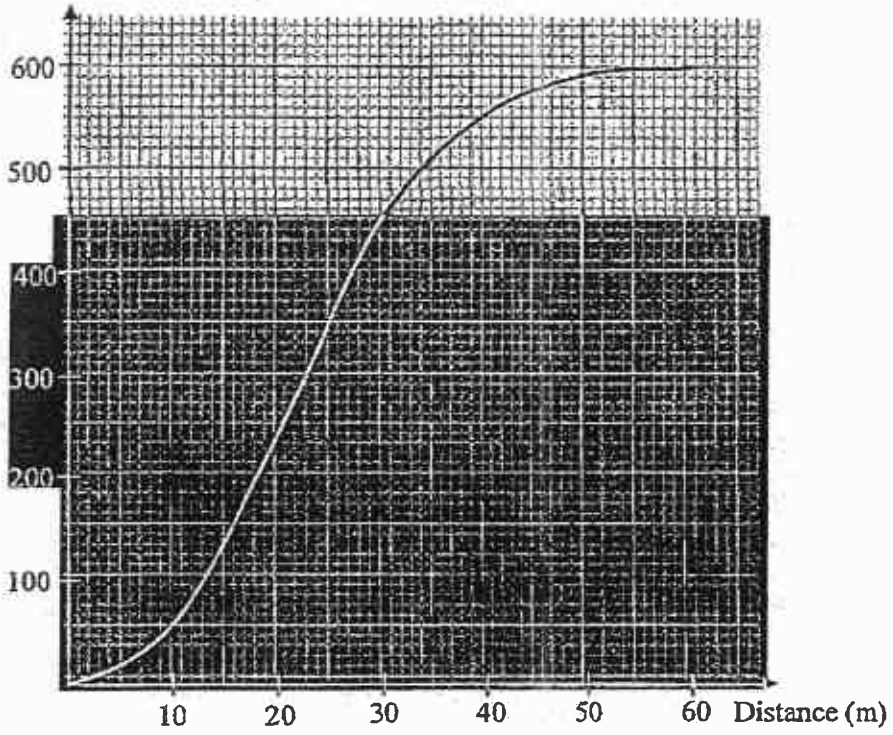
(d) (i) On the same axes, draw the line $y = 3 - 2x$. [1]

(ii) From the graph, state the x-coordinate of the points where this line intersects the curve. [2]

(iii) These values of x are the solutions of the equation $Ax^2 - 33x + B = 0$.

Find the value of A and of B. [2]

9 (a) A group of 600 young children was tested to find the distance that each of them was able to swim in an indoor swimming pool.
The results of the test are shown on the cumulative frequency curve below.



(i) Using the given curve, find for this distribution,

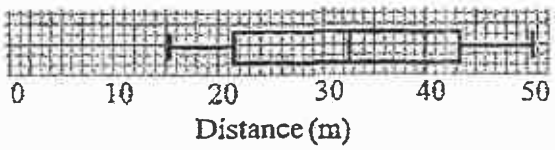
(a) the median, [1]

(b) the interquartile range. [2]

(ii) The distance to pass the test was 35 metres.

Estimate the percentage of children who passed the test. [2]

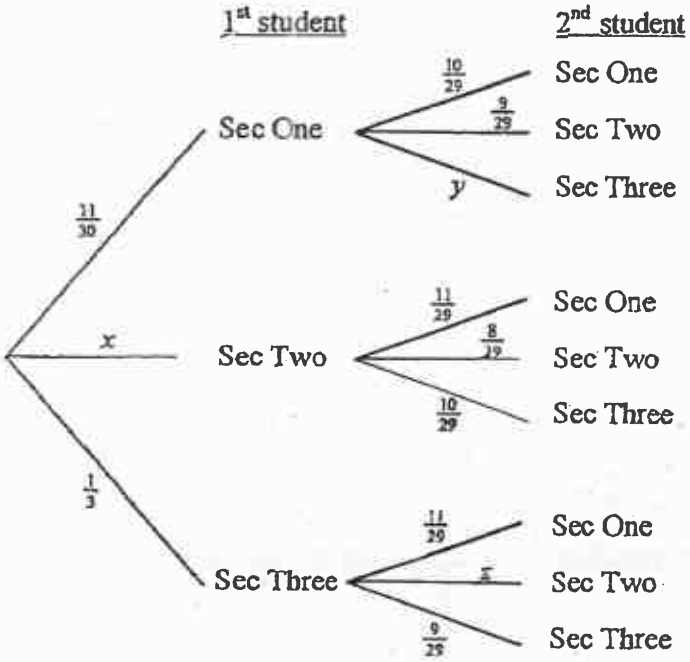
(iii) The same group of children was tested to swim in the outdoor swimming pool.
The box-and-whisker plot shows the distribution of the test.



Make two comparisons between the performances of the children in the two tests. [2]

(b) A room consists of 30 students.
The students are selected from three different levels.
There are 11 students from Sec One, 9 students from Sec Two and 10 students from Sec Three.

Two students are selected from the room to compete with other students from another room.
The tree diagram below shows the possible outcomes and some of their probabilities.



(i) Calculate the value of x, of y and of z as shown on the tree diagram. [3]

(ii) Expressing your answers in fractions in its lowest term, calculate the probability that

(a) both students are from the same level, [1]

(b) both students are of different levels, [1]

(c) one student will be from Sec One and the other from Sec Three. [1]

10 Sarah wants to sell chocolate cupcakes at the next neighbourhood Food Fair.
She intends to bake 180 to 200 cupcakes.
She bakes in batches of 16 cupcakes.
Information that Sarah needs is provided below.

(a) How many times must she bake in order to have a total of 180 to 200 cupcakes? [2]

Sarah needs to decide how much to charge customers for a box of 6 chocolate cupcakes.
She must make sure that she charges enough money to cover all of her costs.

(b) Using your answer from (a), find the number of boxes she will need for the packaging. [1]

(c) Suggest a sensible amount for her to charge for a box of 6 cupcakes.
Justify the decision you make and show your calculations clearly. [7]

Ingredients Recipe makes 16 cupcakes	
114 g	butter
2	eggs
160 g	caster sugar
100 g	plain flour
60 g	cocoa powder
125 ml	evaporated milk
Chocolate cream frosting	

Baking supplies		
Items	**Description**	**Unit cost**
Butter	Pack of 500 g	$4.95
Eggs	Pack of 30 eggs	$3.85
Caster sugar	Pack of 800 g	$2.65
Plain flour	Pack of 1 kg	$1.70
Cocoa powder	Pack of 250 g	$4.10
Evaporated Milk	Can of 350 ml	$1.60
Chocolate Cream Frosting for 50 cupcakes	1 tub	$18
Cupcake liners	Pack of 100 pieces	$4.00
Cupcake boxes	Pack of 5 boxes	$3.00

Booth Rental Fee: $100

END OF PAPER

4E5N Prelim 2017 Paper 2
Answer Key

1a	$\frac{30(x-y)}{x+y}$
1b	$\frac{5x+3}{(x-1)(x+1)}$
1c(i)	$x=\pm\sqrt{y^2+yz}$ or $x=\pm\sqrt{y(y+z)}$
1c(ii)	$y=-4$ or $y=1$
1d	17 : 7
2a(i)	36°
2b(i)	(a) $\frac{1}{3}\mathbf{b}$ (b) $\frac{2}{3}\mathbf{b}-\frac{1}{2}\mathbf{a}$ (c) $\frac{4}{3}\mathbf{b}-\mathbf{a}$
2b(ii)	$\overrightarrow{AB}=\mathbf{b}-\mathbf{a}$ $\overrightarrow{BP}=\mathbf{b}-\mathbf{a}$
2b(iv)	$\frac{1}{2}$
3a(i)	22nd term / T_{22}
3b(i)	$a=10$ $b=15$ $c=25$
3b(ii)	$N_r=r^2$
3b(iii)	$U_r=\frac{r(r+1)}{2}$
3b(iv)	1 225
4a(i)	260°
4a(ii)	50°
4a(iii)	45°
4a(v)	10°
4a(iv)	80°
4b	Not parallel
5a(i)	$\frac{150}{x}$h
5a(ii)	$\frac{150}{x+15}$h

5c	2 hours 3 mins
6a(i)	$\mathbf{M}=\begin{pmatrix}82 & 95 & 89 & 60\\ 72 & 85 & 65 & 57\\ 88 & 91 & 70 & 64\end{pmatrix}$
6a(ii)	$\mathbf{X}=\begin{pmatrix}0.5\\0.2\\0.1\\0.2\end{pmatrix}$
6a(iii)	$\begin{pmatrix}80.9\\70.9\\82\end{pmatrix}$
7b(i)	3 680 cm^3 (3 sf)
7b(ii)	1 260 cm^2 (3 sf)
7c	31 cm
8a	$p=6.33$ $q=-3.45$
8c	1.5 (±1)
8d(ii)	$x=2.2(\pm0.2)$ $x=6.25(\pm0.2)$
8d(iii)	$A=4$ $B=50$
9a(i)	(a) 23 m (b) 14 m
9a(ii)	14.2 %
9b(i)	$x=\frac{3}{10}$, $y=\frac{10}{29}$, $z=\frac{9}{29}$
9b(ii)	(a) $\frac{136}{435}$ (b) $\frac{299}{435}$ (c) $\frac{22}{87}$
10a	12 times
10b	32 boxes
10c	minimum \$7.90 per box so as to cover cost.

Marking Scheme
4E5N Prelim 2017 Paper 2

Qn	Answer Key		Marks
1a	$\frac{30(x-y)}{x+y}$	$\frac{5(x-y)^4}{(x+y)^2} \div \frac{(x-y)^3}{6x+6y}$ $= \frac{5(x-y)^4}{(x+y)^2} \times \frac{6(x+y)}{(x-y)^3}$ $= \frac{30(x-y)}{x+y}$	M1 A1
1b	$\frac{5x+3}{(x-1)(x+1)}$	$\frac{5}{x-1} - \frac{2}{x^2-1}$ $= \frac{5(x+1)}{(x-1)(x+1)} - \frac{2}{(x-1)(x+1)}$ $= \frac{5x+5-2}{(x-1)(x+1)}$ $= \frac{5x+3}{(x-1)(x+1)}$	M1 A1
1c(i)	$x = \pm\sqrt{y^2+yz}$ or $x = \pm\sqrt{y(y+z)}$	$z = \frac{x^2-y^2}{y}$ $yz = x^2 - y^2$ $x^2 = y^2 + yz$ $x = \pm\sqrt{y^2+yz}$	M1 A1
1c(ii)	$y=-4$ or $y=1$	$3 = \frac{(2)^2-y^2}{y}$ $3y = 4 - y^2$ $y^2 + 3y - 4 = 0$ $(y+4)(y-1) = 0$ *factorisation* $y=-4$ or $y=1$	M1 M1 A1
1d	17:7	$\frac{x+3y}{5x-4y} = \frac{2}{3}$ $3(x+3y) = 2(5x-4y)$ $3x+9y = 10x-8y$ $17y = 7x$ $\frac{x}{y} = \frac{17}{7}$ $x:y = 17:7$	M1 M1 A1

2a(i)	$36°$	$\angle BCD$ = Each interior angle $= \frac{(5-2)\times 180}{5} = 108°$ $\angle CDF$ $= \frac{180° - 108°}{2}$ $= 36°$ (base $\angle$ of isos. Δ)	M1 A1
2a(ii)		$\angle ACB = 36°$ (symmetry / congruent triangles) $\angle BFC = 180° - 36° - 36° = 108°$ (isos. Δ) $\angle DFA = 108°$ (vert. opp $\angle$s)	B1 B1
2b(i)	(a) $\frac{1}{3}\mathbf{b}$ (b) $\frac{2}{3}\mathbf{b} - \frac{1}{2}\mathbf{a}$ (c) $\frac{4}{3}\mathbf{b} - \mathbf{a}$	(a) $\overrightarrow{NB} = \frac{1}{3}\overrightarrow{OB} = \frac{1}{3}\mathbf{b}$ (b) $\overrightarrow{MN} = \overrightarrow{ON} - \overrightarrow{OM}$ $= \frac{2}{3}\mathbf{b} - \frac{1}{2}\mathbf{a}$ *(preferred answer)* $= \frac{1}{6}(4\mathbf{b} - 3\mathbf{a})$ (c) $\overrightarrow{NP} = 2\overrightarrow{MN}$ $= \frac{4}{3}\mathbf{b} - \mathbf{a}$ *(preferred answer)* $= \frac{1}{3}(4\mathbf{b} - 3\mathbf{a})$	B1 B1 B1
2b(ii)	$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ $\overrightarrow{BP} = \mathbf{b} - \mathbf{a}$	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \mathbf{b} - \mathbf{a}$ $\overrightarrow{BP} = \overrightarrow{NP} - \overrightarrow{NB}$ $= \frac{4}{3}\mathbf{b} - \mathbf{a} - \frac{1}{3}\mathbf{b}$ $= \mathbf{b} - \mathbf{a}$	B1 B1
2b(iii)		1. B is the midpoint of line ABP. 2. A, B and P are collinear/ABP lies on a straight line	B2
2b(iv)	$\frac{1}{2}$	$\frac{\text{area of triangle } PMB}{\text{area of triangle } PMA}$ $= \frac{\frac{1}{2}(PM)(PB)\sin P}{\frac{1}{2}(PM)(PA)\sin P} = \frac{PB}{PA} = \frac{1}{2}$ *Alternative Mtd* *Use common height*	B1
3a(i)	22^{nd} term / T_{22}	$\frac{n(n+3)}{2} = 275$ $n^2 + 3n - 550 = 0$ $(n-22)(n+25) = 0$ $n = 22$ or $n = -25$ (reject $\because n > 0$) *Alternative Mtd* *Quadratic Formula*	M1 A1
3a(ii)		For any integer value of n, either n is even or $(n+3)$ is even. Hence, $n(n+3)$ is always divisible by 2.	B1 *Accept equivalent reasoning*

3b(i)	$a=10$ $b=15$ $c=25$		B1 B1 B1
3b(ii)	$N_r=r^2$		B1
3b(iii)	$U_r=\frac{r(r+1)}{2}$		B1
3b(iv)	1 225	$N_{50}=50^2=2500$ triangles in total $U_{50}=\frac{50(50+1)}{2}=1275$ triangles unshaded Number of shaded triangles $=2500-1275=1225$	M1 A1
4a(i)	260°	reflex $\angle EOB$ $=130^\circ\times 2$ ($\angle$ at centre $=2\angle$ at circumference) $=260^\circ$	B1
4a(ii)	50°	$\angle ECB=180^\circ-130^\circ$ (angles in opposite segment) $=50^\circ$	B1
4a(iii)	45°	$\angle CBD=(180^\circ-90^\circ)\div 2$ (isosceles triangle, $BC=CD$) $=45^\circ$	B1
4a(iv)	80°	$\angle DOE=260^\circ-180^\circ$ $=80^\circ$	B1
4a(v)	10°	$\angle OFE=180^\circ-90^\circ-80^\circ$ (tangent $\perp$ radius) $=10^\circ$	B1√
4b	Not parallel	$\angle BDE=50^\circ$ (angles in same segment) $\angle CBD=45^\circ$ (from aiii) $\angle CBD\neq\angle BDE$ $\therefore$ Line ED is not parallel to line BC [accept any other mathematically logical method]	M1 A1
5a(i)	$\frac{150}{x}$ h		B1
5a(ii)	$\frac{150}{x+15}$ h		B1
5b		$\frac{150}{x}-\frac{150}{x+15}=\frac{21}{60}$ $\frac{150(x+15)-150x}{x(x+15)}=\frac{7}{20}$ $\frac{2250}{x(x+15)}=\frac{7}{20}$ $45000=7x(x+15)$ $7x^2+105x-45000=0$	M1 M1 A1

5c	2 hours 3 mins	$7x^2 + 105x - 45000 = 0$ $x = \frac{-105 \pm \sqrt{105^2 - 4(7)(-45000)}}{2(7)}$ $x = 73.028$ or $x = -88.028$ Original time taken $= \frac{150}{73.028} = 2.054$ hours $= 2$ hours 3 minutes	M1 A1 M1 A1
6a(i)	$\mathbf{M} = \begin{pmatrix} 82 & 95 & 89 & 60 \\ 72 & 85 & 65 & 57 \\ 88 & 91 & 70 & 64 \end{pmatrix}$		B1
6a(ii)	$\mathbf{X} = \begin{pmatrix} 0.5 \\ 0.2 \\ 0.1 \\ 0.2 \end{pmatrix}$		B1
6a(iii)	$\begin{pmatrix} 80.9 \\ 70.9 \\ 82 \end{pmatrix}$	$\mathbf{F} = \mathbf{MX}$ $= \begin{pmatrix} 82 & 95 & 89 & 60 \\ 72 & 85 & 65 & 57 \\ 88 & 91 & 70 & 64 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.2 \\ 0.1 \\ 0.2 \end{pmatrix}$ $= \begin{pmatrix} 80.9 \\ 70.9 \\ 82 \end{pmatrix}$	B2 All correct B1 1 mistake 0 More than 1 mistake
6a(iv)		The elements in F represent the respective final combined score/grade of Aaron, Beatrice and Carly at the end of the semester.	B1
6b		The teacher could increase the weightage of projects and decrease the weightage of quizzes. *(Any other suitable suggestions)*	B1
7a		Let the height of the original cone be x cm. Using similarity, $\frac{x - 5.4}{x} = \frac{6}{15}$ $15x - 81 = 6x$ $x = 9$ (shown)	M1 A1
7b(i)	3 680 cm^3 (3 sf)	Height of cone that has been removed $9 - 5.4 = 3.6$ cm Volume of water in cylinder $= \pi(7.5)^2(18)$ 3181 cm^3 $= 1012.5\pi$ cm^3	M1 for

		Volume in cone (with top removed) $= \frac{1}{3}\pi(7.5)^2(9) - \frac{1}{3}\pi(3)^2(3.6)$ 496.2 cm^3 $= 157.95\pi$ cm^3 Volume of water in container $= 1012.5\pi + 157.95\pi$ $= 3\,677.1$ $= 3\,680$ cm^3	either of the two A1
7b(ii)	1 260 cm^2 (3 sf)	Slant height, $L = \sqrt{9^2 + 7.5^2} = 11.72$ cm (4 sf) Slant height $l = \sqrt{3.6^2 + 3^2} = 4.686$ cm (4 sf) Surface area of cylinder in contact with water $= 2\pi(7.5)(18) + \pi(7.5)^2$ $= 326.25\pi$ cm^2 1025 cm^2 Surface area of cone in contact with water $= \pi(7.5)(11.72) - \pi(3)(4.686)$ $= 231.98$ cm^2 Required surface area = $326.25\pi + 231.98$ $= 1256.92$ $= 1260$ cm^2	M1 (either one) M1 for either of the two A1
7c	31 cm	Height = $\frac{3677.1}{120} = 30.64$ cm Minimum height = 31 cm (whole number)	M1 A1
8a	$p = 6.33$ $q = -3.45$		B1 B1
8b		See attached: Correct Scale Plotted points Smooth Curve	S1 P1 C1
8c	1.5 (±1)	Draw a tangent at $x = 10$. Gradient = 1.5 (±0.1)	1 1
8d(i)		Draw the line $y = 3 - 2x$.	1
8d(ii)	$x = 2.2(\pm 0.2)$ $x = 6.25(\pm 0.2)$		B1 B1

8d(iii)	$A=4$ $B=50$	$2x+\frac{50}{x}-30=3-2x$ $2x^2+50-30x=3x-2x^2$ $4x^2-33x+50=0$ $\therefore A=4$ and $B=50$	M1 A1
9a(i)	(a) 23 m (b) 14 m	Median = 23 m Interquartile Range $= Q_3 - Q_1$ $= 30-16$ $= 14$ m	B1 M1 A1
9a(ii)	14.2 %	Number of students who passed the test $= 600 - 515^*$ $= 85$ Percentage of students $= \frac{85}{600}\times 100\%$ $= 14.2\ \%$ (3 s.f)	M1 (*accept 80 or 90) *Possible answers: 13.3% or 15% A1
9a(iii)		Children on average swam **further** at the outdoor swimming pool as can be seen from the **bigger median** of 32 m. The **maximum distance** that was covered in the outdoor swimming pool is 50 m, which is lower than the maximum distance covered in the indoor swimming pool which is 60 m. The **interquartile range** for the test in the outdoor swimming pool is 22 m, which is **more than** the test in the indoor swimming pool, 14 m indicating that the distance covered at the indoor swimming pool is **more consistent**.	B2 Any two
9b(i)		$x=\frac{9}{30}=\frac{3}{10}$, $y=\frac{10}{29}$, $z=\frac{9}{29}$	B1 B1 B1
9b(ii)	(a) $\frac{136}{435}$ (b) $\frac{299}{435}$ (c) $\frac{22}{87}$	(a) $\left(\frac{11}{30}\right)\left(\frac{10}{29}\right)+\left(\frac{3}{10}\right)\left(\frac{8}{29}\right)+\left(\frac{1}{3}\right)\left(\frac{9}{29}\right)=\frac{136}{435}$ (b) $1-\frac{136}{435}=\frac{299}{435}$ (c) $\left(\frac{11}{30}\right)\left(\frac{10}{29}\right)+\left(\frac{1}{3}\right)\left(\frac{11}{29}\right)=\frac{22}{87}$	B1 B1 B1
10a	12 times	Number of batches $= \frac{200}{16}=12.5\approx 12$ If she bakes 12 times, she will get $12\times 16=192$ cupcakes	M1 A1 Equivalent working acceptable

10b — 32 boxes — B1

With 192 cupcakes, she needs

$$\frac{192}{6} = 32 \text{ boxes}$$

10c — 7

To follow through from their (a) answer

B2
Quantity of ingredients for 192 cupcakes (shaded box)
- 1 mistake (–1m)
- > 1 mistake (–2m)

B2√
Cost of ingredients for 192 cupcakes (expenses)
- 1 mistake (–1m)
- > 1 mistake (–2m)

B1
Add Booth Rental Fee

B1√
Find cost price of 6 cupcakes ($7.85)

B1
Sensible amount with justification. Profit, transport costs etc are additional consideration.

So long as it makes sense and covers the basic cost price. The minimum amount should be $7.90 per box.

How much of each ingredient is needed:

Item	1 batch	For 12 batches	Sold as:	Need to get:
Butter	114 g	1368 g	500 g	[redacted]
Eggs	2	24	30	[redacted]
Caster sugar	160 g	1920 g	800 g	[redacted]
Plain flour	100 g	1200 g	1 kg	[redacted]
Cocoa powder	60 g	720 g	250 g	[redacted]
Evaporated Milk	125 ml	1500 ml	350 ml	[redacted]
Chocolate Cream Frosting	16 cupcakes	192 cupcakes	50 cupcakes	[redacted]
Cupcake liners	16 cupcakes	192 liners	100 liners	[redacted]
Cupcake boxes		32 boxes	5 boxes	[redacted]

Cost breakdown

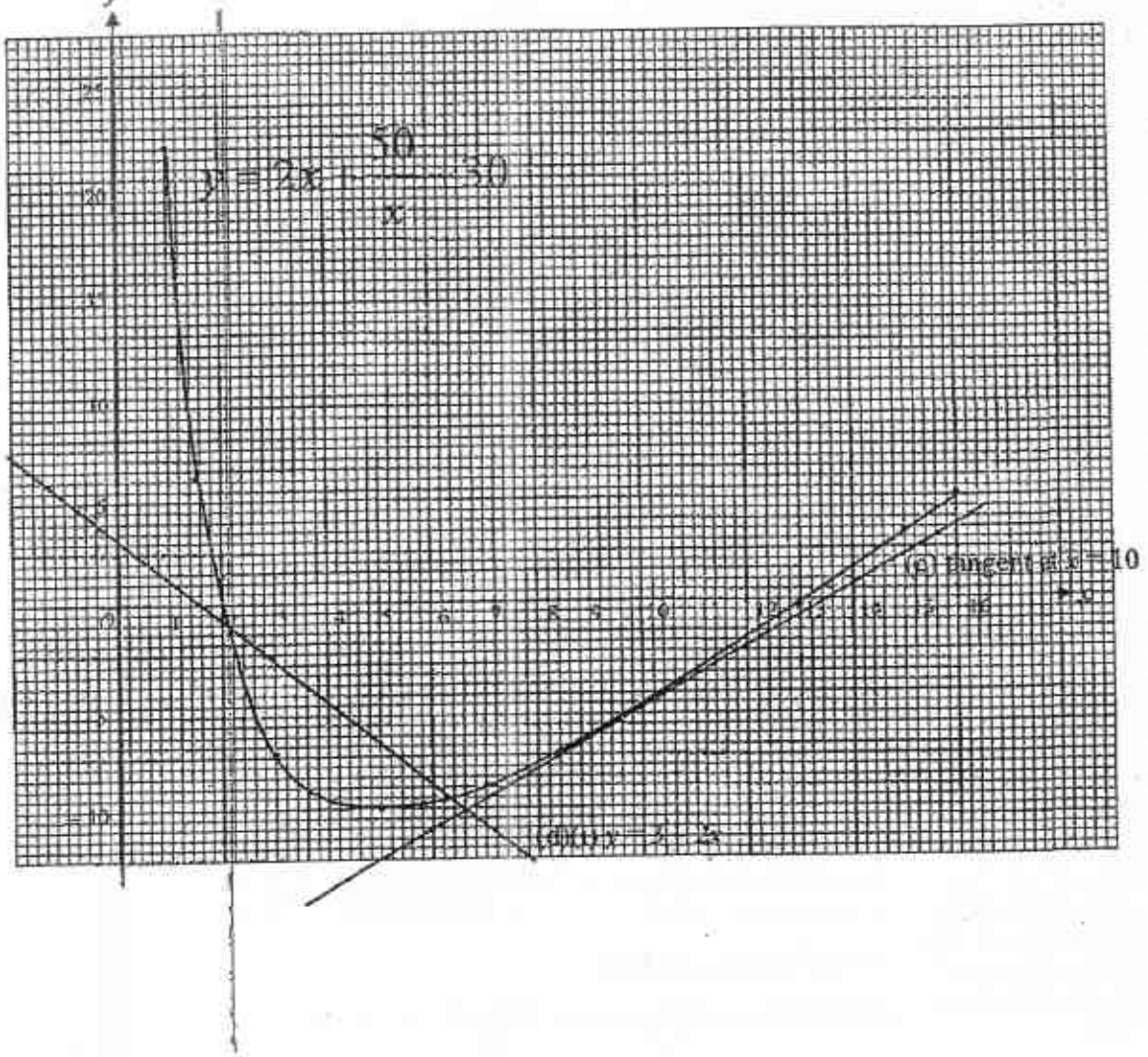
Item	Unit Price	Qty	Expenses
Butter	$4.95	[redacted]	$14.85
Eggs	$3.85	[redacted]	$3.85
Caster sugar	$2.65	[redacted]	$7.95
Plain flour	$1.70	[redacted]	$3.40
Cocoa powder	$4.10	[redacted]	$12.30
Evaporated Milk	$1.60	[redacted]	$8.00
Chocolate Cream Frosting	$18.00	[redacted]	$72.00
Cupcake liners	$4.00	[redacted]	$8.00
Cupcake boxes	$3.00	[redacted]	$21.00
		Sub Total	$151.35
		Booth rental	$100.00

Total expenses ≈ $250.

$$\text{Cost price for 6 cupcakes} = \frac{\$251.35}{192} \times 6 = \$7.85$$

Consider:
- transport costs,
- electricity costs,
- profit to be made

Question 8



Name	Register Number	Class	Calculator Model

智行菩願

MANJUSRI SECONDARY SCHOOL
文 殊 中 學

PRELIMINARY EXAMINATION 2017

Subject:	Mathematics
Paper:	4048/01
Level:	Secondary 4 Express / 5 Normal (Academic)
Date:	7 August 2017
Duration:	2 hours
Setter:	Mr Lee Beng Huat

Candidates answer on the Question Paper
Additional materials: Geometrical Instruments

READ THESE INSTRUCTIONS FIRST

Write your Name, Register Number and Class on all the work you hand in.
Write in dark blue or black pen in the spaces provided on the Question Paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

Marks Obtained
80

This paper consists of **15** printed pages including this cover page.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions.

1 (a) Estimate, correct to the nearest whole number, the value of $\frac{4.97^2 - \sqrt{15}}{\sqrt[3]{30}}$ without the use of a calculator.

Answer ..... [1]

(b) Write down the following in order of size, smallest first.

$\sqrt{0.35}$ 35% 3.5 $\frac{35}{53}$

Answer ....., ..., ..., ... [2]

2 (a) Solve $\frac{1}{3}x + 15 = 9$.

Answer $x =$ ..... [1]

(b) Simplify $15(x-13) + 14(13-x)$.

Answer ..... [2]

3 During a sale there is a discount of 15% on all items selling in a shop. If the discounted price of a watch is \$182.75, find the original price of the watch before the discount.

Answer \$ ..... [2]

4 (a) Simplify $18a^3b \div 6ab^{-3}$.

Answer ..... [1]

(b) Given that $\sqrt{2} \times 4^n = 1$, find the value of n.

Answer $n =$ ..... [2]

---

5 $\xi = \{\text{integers } x : 11 \leq x < 19\}$
$A = \{\text{multiples of 3}\}$
$B = \{\text{prime numbers}\}$

List the elements in

(a) A',

Answer ..... [1]

(b) $A' \cap B$,

Answer ..... [1]

(c) $(A \cup B)'$.

Answer ..... [1]

6 Factorise completely $3ap + 8bq - 12aq - 2bp$.

Answer ..... [2]

7 The plan of a museum is drawn to a scale of 1 : 500.

(a) Find the length, in metres, of a corridor which is represented by a line 10.5 cm long on the plan.

Answer ..... m [1]

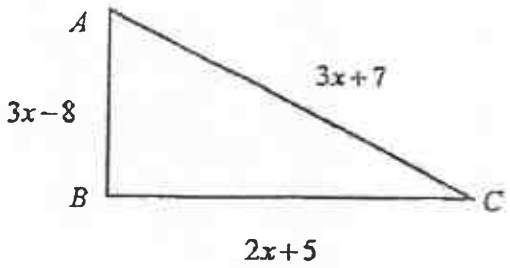
(b) The area of the floor of a bookshop is 500 m^2. Find, in square centimeters, its area on the plan.

Answer ..... cm^2 [2]

8 After Pluto is no longer considered a planet, Mercury is now the smallest planet while Jupiter is still the biggest planet in our solar system.
Planet Mercury has a mass of 3.3×10^{23} kg and Jupitar has a mass of 1.898×10^{27} kg.
How many times is the mass of Jupiter compare to the mass of Mercury?
Give your answer in standard form, correct to 3 significant figures.

Answer ..... [2]

9 The diagram shows a triangle ABC.



(a) One property of a triangle is that the length of the longest side must be less than the sum of the lengths of the two shorter sides.
Form an inequality in x and solve it.

Answer ..... [2]

(b) Given also that the perimeter of the triangle is no more than 85 cm.
Find the largest possible length of the longest side, given x is a prime number.

Answer ..... cm [3]

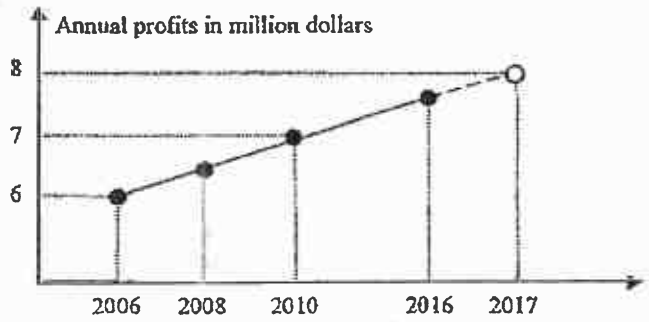
10 Write as a single fraction in its simplest form $\frac{7}{x^2-4}-\frac{2}{2-x}$.

Answer ..... [2]

11 Given that n is a positive integer and $n-\frac{1}{n}=5$. Find the value of $n^2+\frac{1}{n^2}$.

Answer ..... [2]

12 The CEO used the following line graph to show the annual profits made by the company over a number years.



State one aspect of the graph that may be misleading and explain how the annual profits in 2017 can be projected wrongly.

Answer .....

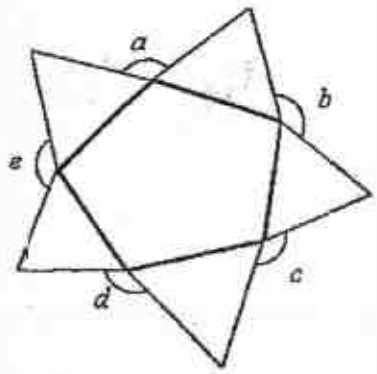
.....

..... [2]

13 Given that $x \quad y = 0.2 : 0.5$ and $y : z = \frac{1}{3} : \frac{1}{2}$, find $x : y : z$.

Answer ..... [3]

14 The diagram shows a pentagon and five equilateral triangles.
Calculate the sum of the angles a, b, c, d and e.

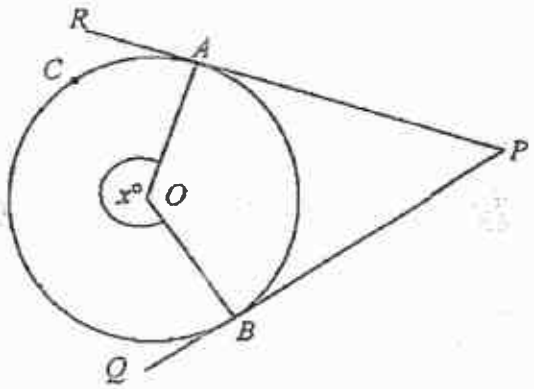


Answer ..... [3]

15 Jane can make 8 dresses in 7 hours. Judy can make 7 dresses in 6 hours.
If Jane and Judy continue to make dresses at the same rate, how long will it take them to make 20 dresses? Give your answer in hours and minutes, to the nearest minutes.

Answer ..... hours ..... minutes [3]

16 A, B and C are points on the circle centre O. PBQ and PAR are tangents to the circle.
Reflex $\angle AOB = x°$.



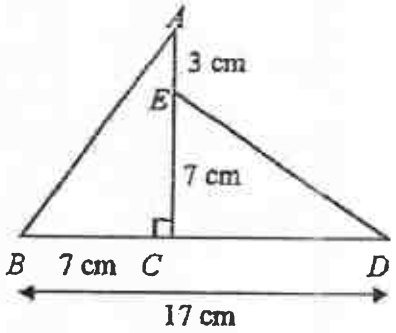
(a) Given C is a point along the major arc AB, express $\angle ACB$ in terms of x.

Answer $\angle ACB =$ ..... [2]

(b) Express $\angle APB$ in terms of x.

Answer $\angle APB =$ ..... [2]

17 In the diagram, $AE = 3$ cm, $EC = 7$ cm, $BC = 7$ cm and $BD = 17$ cm.
Name a pair of congruent triangles, stating your case of congruency.



Answer .....

.....

.....

..... [3]

18 (a) Express 168 as a product of its prime factors.

Answer 168 = ... [2]

(b) Find the smallest positive integer m such that $\frac{168}{\sqrt{m}}$ is a perfect cube.

Answer m = ... [2]

(c) Alice uses all 168 cubes of side 1 unit to make a cuboid. Each of the sides of the cuboid is made up of more than 3 cubes. Find the number of cubes on each side of the cuboid.

Answer ..... by ..... by ..... [2]

19

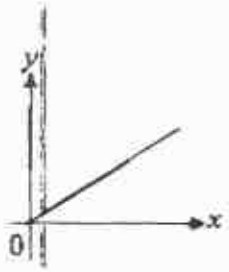


Figure 1

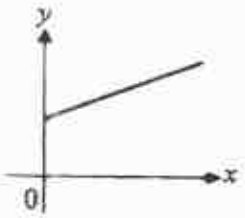


Figure 2

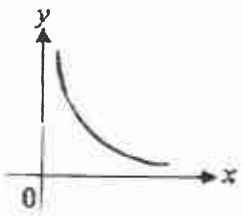


Figure 3

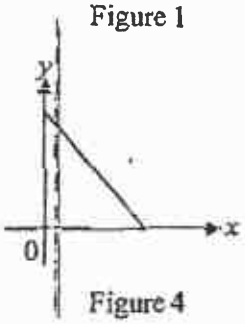


Figure 4

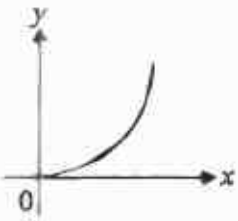


Figure 5

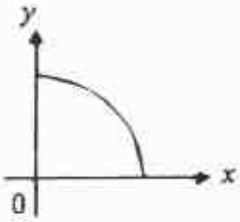


Figure 6

From the graphs above, select one which illustrates each of the following statements.

(a) The amount of pressure y, exerted is inversely proportional to the surface area of a cube, of sides x cm.

Answer Figure ..... [1]

(b) The surface area y, of a sphere is proportional to the square of the radius, x cm.

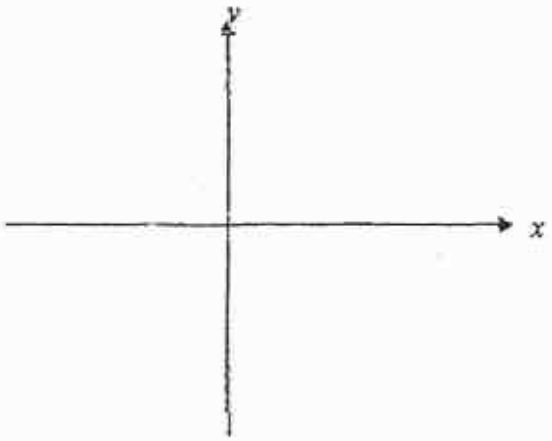
Answer Figure ..... [1]

(c) The total taxi fare \$$y$, of a fixed flag down fees plus x metres of distance travelled, given 1 cent is charged for every metre travelled.

Answer Figure ..... [1]

---

20 Sketch the graph of $y = (x+3)(5-x)$ on the axes below, indicating its turning point and all the intercepts on the axes clearly.



[3]

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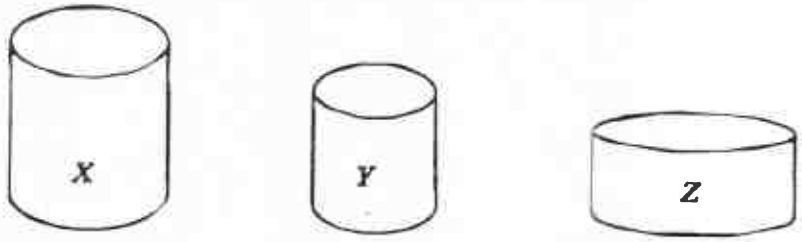
21 There are three mugs X, Y and Z. Mugs X and Y are geometrically similar. The volume of X and Y are 512 cm^3 and 216 cm^3 respectively.



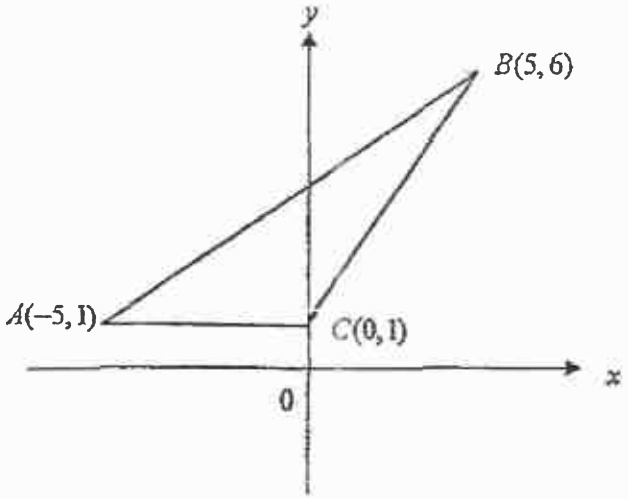
(a) Find the ratio of the surface area of X to Y.

Answer ..... : ..... [2]

(b) The volume of Y is given by the formula $V = \pi r^2 h$ where h is the height of the mug and r the radius of the circular base. Find the volume of Z which has $\frac{2}{3}$ the height of Y and twice the radius of the circular base of Y.

Answer ..... cm^3 [2]

22 In the diagram, the vertices of a triangle A, B and C are $(-5, 1)$, $(5, 6)$ and $(0, 1)$ respectively.



Find

(a) the equation of line BC,

Answer ..... [2]

(b) the equation of the line which passes through A and is parallel to $3x+6y=5$,

Answer ..... [2]

(c) the area of the triangle ABC.

Answer ..... units^2 [2]

23 A frustum and a cone were obtained by slicing a conical container, height $2h$, as shown in Diagram I at the midway of the height. These figures were then attached to a cylinder, height h, to form a new container as shown in Diagram II. Water was poured into the empty container in Diagram II at a constant rate from the top and it took 33 seconds to fill to the brim.

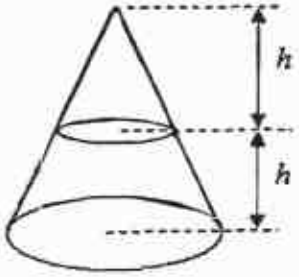


Diagram I

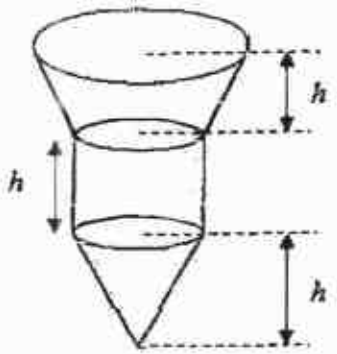


Diagram II

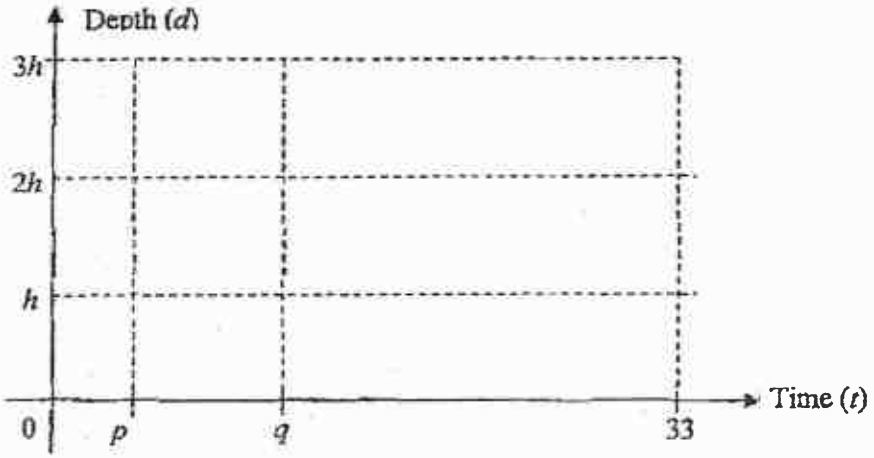
Given that it took p seconds for the water to reach the container to a height of h and q seconds to reach the height $2h$.

(a) Find the value of p and of q.

Answer $p =$ ....., $q =$ ..... [3]

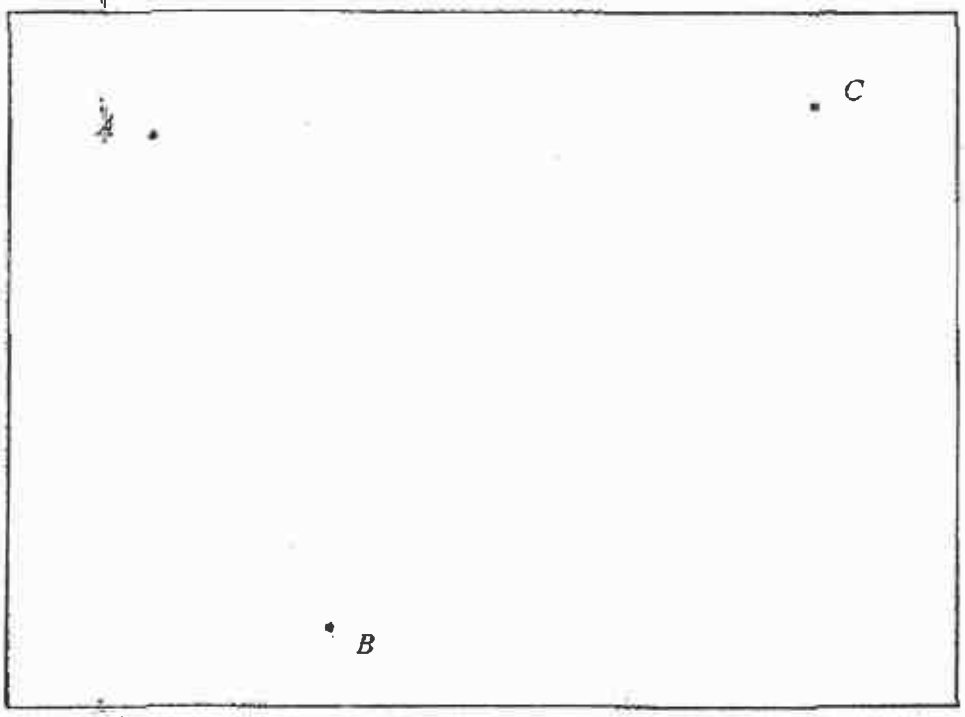
(b) On the grid in the answer space, sketch the graph of the depth of water (d) against the time (t).

Answer



[2]

24 The diagram below is part of the scale drawing of a rectangular field showing the position of 3 soccer players, A, B and C. In the drawing, 1 cm represents 5 m.



(a) The ball is placed in the field equidistant from A and B and 30 m from C.
By constructing suitable lines and arcs in the answer space above, mark and label clearly the position of the ball X.

[2]

(b) Measure and state the distance between player A and the ball X.

Answer ..... m [1]

(c) Both players A and C are to run for the ball.
Player A can run at a speed of 6 m/s while player C's top speed is 7 m/s.
Who will get the ball first? Show your working clearly.

Answer Player ..... [2]

------ *End of Paper* ----

Answer all the questions.

1 (a) It is given that $H = \dfrac{k}{\sqrt{m-n}}$.

(i) Find H when $k = 12$, $m = 6$ and $n = -3$. [1]

(ii) Express n in terms of H, k and m. [2]

(b) Simplify $\dfrac{9a^2b}{(2a)^2} \div \dfrac{12ab^3}{8b^5}$, leaving your answer in positive indices. [2]

(c) Solve the equation $\dfrac{5}{x+7} + \dfrac{4}{11-x} = 1$. [3]

(d) Solve the following simultaneous equations:

$$5x - 3y = 22$$
$$y - 4x + 12 = 0$$ [3]

---

2 (a) Alex needs a loan of \$45 000 to buy a new car.
Bank *ABC* charges an interest rate of 2.45% per annum compounded monthly.
Bank *XYZ* charges a simple interest rate of 2.65% per annum.

If Alex plans to take a five year loan, which bank should he loan from?
Justify your answer. [4]

(b) Alex buys the new car on hire purchase. He uses the \$45 000 loan to pay the 30% down payment and then makes monthly payments of \$1950 for 5 years.

(i) Calculate the cash price of the new car. [1]

(ii) Calculate the interest Alex has to pay in this hire purchase scheme. [2]

(iii) Calculate the rate of simple interest charged for hire purchase.
Leave your answer in 3 decimal places. [1]

(c) Alex took his new car for a road trip from Singapore to Bangkok.
Before the trip. Alex paid S\$109 for 50 litres of petrol to fill up the tank.
In Bangkok, Alex paid a total of 9 408 Thai bahts for 320 litres of petrol he pumped into his car.

Given S\$1 = 24.5 Thai bahts.

Alex said that the petrol price in Bangkok is less than half the petrol price in Singapore.
Do you agree? Justify your answer. [3]

---

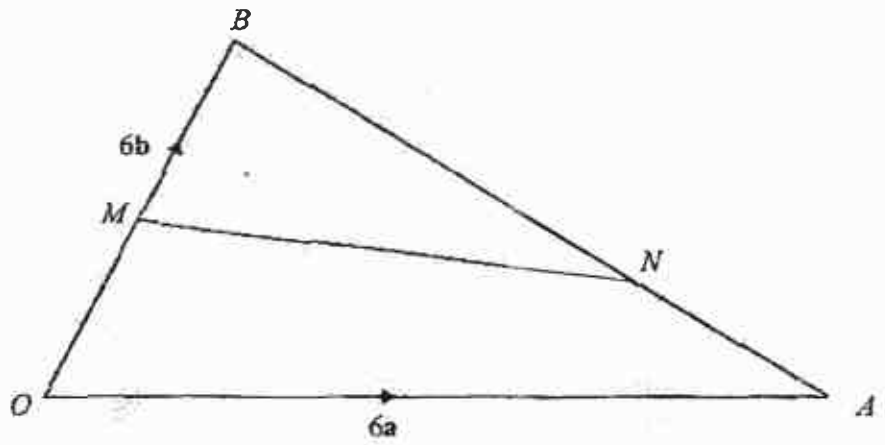
3 (a) Given $\overrightarrow{PQ} = \begin{pmatrix} -7 \\ 24 \end{pmatrix}$ and $\overrightarrow{PS} = \begin{pmatrix} k \\ 12 \end{pmatrix}$.

(i) Find $\left|\overrightarrow{PQ}\right|$. [1]

(ii) Find the value of k such that P, Q and S are collinear. [2]

(iii) Find the coordinates of Q if P is the point $(10, -15)$ [1]

(b) In the diagram, $\overrightarrow{OA} = 6\mathbf{a}$, $\overrightarrow{OB} = 6\mathbf{b}$ and $3\overrightarrow{AN} = \overrightarrow{AB}$. M is the mid-point of OB.



Express, as simply as possible, in terms of **a** and/or **b**,

(i) $\overrightarrow{AN}$, [1]

(ii) $\overrightarrow{ON}$, [1]

(iii) $\overrightarrow{NM}$. [1]

P is a point not shown in the diagram such that $\overrightarrow{MP} = 3\overrightarrow{MN}$.

(iv) Find the position vector of P. [1]

(v) Make two statements about the points O, A and P. [2]

Calculate the value of

(vi) $\dfrac{\text{area of } \Delta AMN}{\text{area of } \Delta BMN}$, [1]

(vii) $\dfrac{\text{area of } \Delta BMN}{\text{area of } \Delta BOA}$. [1]

4 A photocopier prints pages in either 'black and white' or in 'colour'.

(a) In one minute, this photocopier prints x pages in black and white. Write down an expression in terms of x, for the number of seconds it takes to print one page in black and white. [1]

(b) In one minute, this photocopier prints 2 more copies in black and white than it does in colour. Write down an expression, in terms of x, for the number of seconds it takes to print one page in colour. [1]

(c) It takes 1.2 seconds longer to print one page in colour than it takes to print one page in black and white. Form an equation in terms of x and show that it reduces to

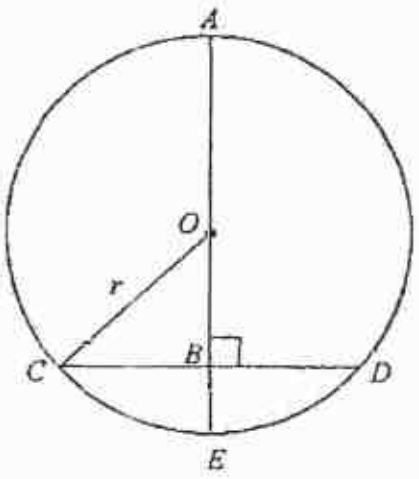
$$x^2 - 2x - 100 = 0.$$ [3]

(d) Solve the equation $x^2 - 2x - 100 = 0$, leaving your answers in 2 decimal places. [2]

(e) Hence, find the time taken in minutes and seconds to print 85 pages in colour. Give your answer corrected to the nearest second. [2]

---

5 The diagram shows a circle, centre O and radius r cm. AB is perpendicular to the CD. Given that $AB = 9$ cm and $CD = 6$ cm.



(a) Express OB in terms of r. [1]

(b) Show that the radius of the circle = 5 cm. [3]

(c) Calculate the area of the minor segment CDE. [4]

6 (a) The first four terms in a sequence of numbers, $u_1, u_2, u_3, u_4, \ldots$, are given below

$$u_1 = 1^2 + 1 = 2$$
$$u_2 = 2^2 + 3 = 7$$
$$u_3 = 3^2 + 5 = 14$$
$$u_4 = 4^2 + 7 = 23$$

(i) Write down an expression for u_5 and show that $u_5 = 34$. [1]

(ii) Find an expression, in terms of n, for u_n. [2]

(iii) Evaluate u_{30}. [1]

(b) A toy manufacturing company makes toy boats and toy cars.
The following table is used in calculating the cost of manufacturing each toy boat and toy car.

	Labour (hours)	Wood (blocks)	Paint (tins)
Boat	6	4	5
Car	4	2	3

This information can be represented by the matrix $T = \begin{pmatrix} 6 & 4 & 5 \\ 4 & 2 & 3 \end{pmatrix}$.

(i) Labour cost \$8 per hour, wood cost \$5 per block and paint costs \$3 per tin.
Represent the cost by a 3×1 column matrix C. [1]

(ii) Evaluate the matrix $V = TC$. [2]

(iii) State what the elements of V represent. [1]

(iv) Given that $W = (80 \quad 50)$,
evaluate WV and explain what the answer represents. [2]

7 The stem and leaf diagram below shows the mass of 21 students.

Stem	Leaf
4	7 7 8
5	0 3 3 4 6 6 6 8 9
6	1 2 4 7 7 8 8
7	0 1

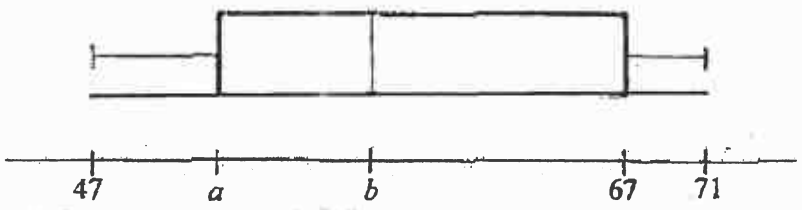
Key : 5|2 means 52 kg

(a) Find

(i) the modal mass, [1]

(ii) the percentage of students more than 62 kg. [1]

(b) The box-and-whisker plot for the above distribution is shown below.



(i) Write down the value of a and of b. [2]

(ii) Find the interquartile range. [1]

(c) Two students are selected from the group.
Calculate the probability that only one student is at least 50 kg. [2]

8 The diagram shows three markers A, B and C placed on a horizontal ground.
The marker A is 250 m from C and the marker B is 400 m due West from A.
Angle $BAC = 65°$

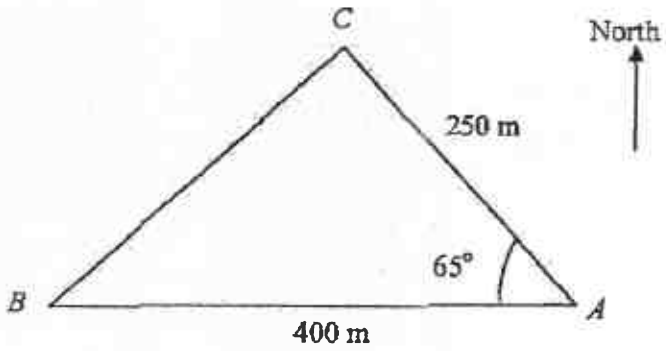


Diagram is not drawn to scale

(a) Calculate

(i) the length BC, [3]

(ii) the area of the triangle ABC, [2]

(iii) the angle ABC and [2]

(iv) the bearing of C from B. [1]

(b) An eagle is hovering vertically above A.
The angle of elevation of the eagle from B is 18°.

Find the angle of depression of C from the eagle. [3]

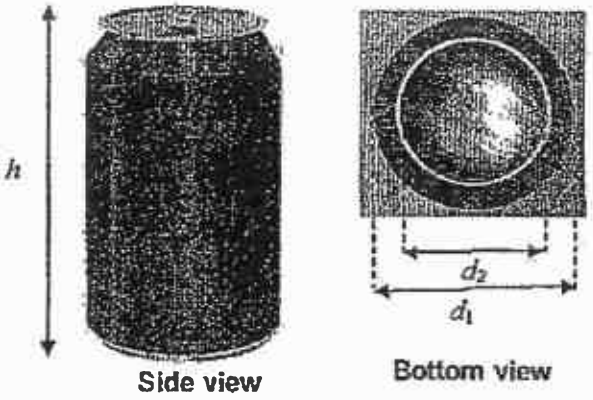
9 Some information about a soda can is shown below.

Soda Can

Height (h): 11.4 cm

Outer Diameter (d_1): 6.4 cm

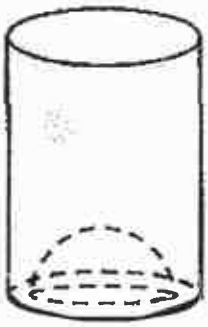
Inner Diameter (d_2): 5.0 cm



Density of soda: 1.2 g/cm^3

Safety Information: the soda can is to be filled to a maximum of 95% of its total volume.

In this question, the soda can (above) can be modelled as a cylinder with an inner hemisphere that is hollowed inwards (concave) at the base of the can.



(a) Calculate

(i) the base area, in square centimetres, of the soda can and [2]

(ii) the total volume, in cubic centimetres, of the soda can. [2]

(b) The material used to make the wall of the soda can must be carefully chosen such that the total mass of each filled soda can is below 620 g.
The manager of the soda manager proposed to use an alloy which has a mass of 0.8 g for every 1 cm^2 to make the can.
If the thickness of the soda can is negligible, will you accept his proposal?
Justify your answer with suitable calculation.

[6]

10 **Answer the whole of this question on a sheet of graph paper.**

The table below gives the values of x and y connected by the equation $y = \frac{x^2}{6} + \frac{12}{x} - 6$.

The table below shows some corresponding values of x and y.

x	1	1.5	2	3	4	5	6	7
y	6.2	2.4	0.7	–0.5	–0.3	0.6	k	3.9

(a) Calculate the value of k. [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $0 \le x \le 8$.
Using a scale of 2 cm to 1 unit, draw a vertical y-axis for $-1 \le y \le 7$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) By drawing a tangent, find the gradient of the curve at $x = 1.5$. [2]

(d) (i) On the same axes, draw the line $y = \frac{x}{6}$. [1]

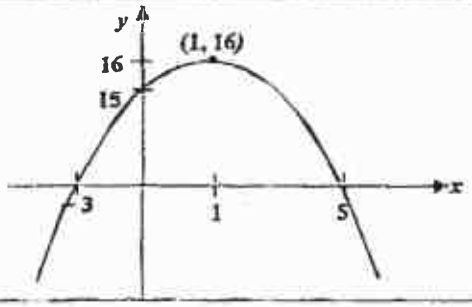
(ii) Write down the x-coordinate of the points where the line intersects the curve. [2]

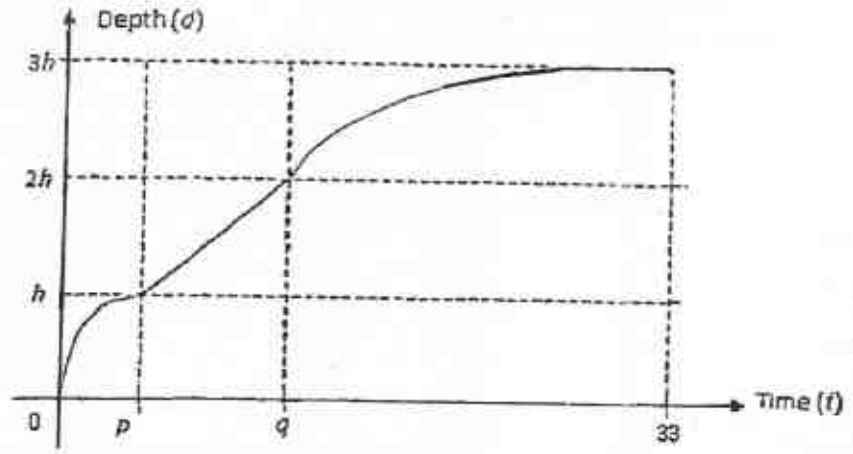
(iii) These values of x is a solution of the equation $x^3 - x^2 + Ax + B = 0$.
Find the value of A and value of B. [2]

... End of Paper 2 ...

Manjusri Secondary School
Preliminary Examination 2017
Elementary Mathematics 4048 Paper 1
Answer key

1(a)	7
1(b)	$35\%, \sqrt{0.35}, \frac{35}{53}, 3.5$
2(a)	-18
2(b)	$x-13$
3	\$215
4(a)	$3a^2b^4$
4(b)	$\frac{1}{4}$
5(a)	11, 13, 14, 16, 17
5(b)	11, 13, 17
5(c)	14, 16
6	$(3a-2b)(p-4q)$
7(a)	52.5 cm
7(b)	3.6cm
	$20\ cm^2$
8	5.75×10^3
9(a)	$x>5$
9(b)	28 cm
10	$\frac{3x+4}{(x+2)(x-2)}$ or $\frac{3x+4}{x^2-4}$
11	27
12	Data from Year 2007, 2009, 2011 to 2015 are missing. The scale in horizontal axis is not consistent. The line graph may not be sloping upward as it seem to be.
13	4 : 10 : 15
14	660°

15	8 hours 40 minutes
16(a)	$\frac{1}{2}(360^\circ - x)$ or $180^\circ - \frac{1}{2}x$
16(b)	$x - 180^\circ$
17	$BC = EC = 7$ cm $CD = CA = 10$ cm $\angle ACB = \angle DCE = 90^\circ$ $\therefore \Delta ABC \equiv \Delta DEC$ (SAS)
18(a)	$2^3 \times 3 \times 7$
18(b)	441
18(c)	$4 \times 6 \times 7$
19(a)	Figure 3
19(b)	Figure 5
19(c)	Figure 2
20	
21(a)	16 : 9
21(b)	576 cm^3
22(a)	$y = x + 1$
22(b)	$y = -\frac{1}{2}x - \frac{3}{2}$ or $2y = -x - 3$
22(c)	12.5 units^2
23(a)	$p = 3$, $q = 12$

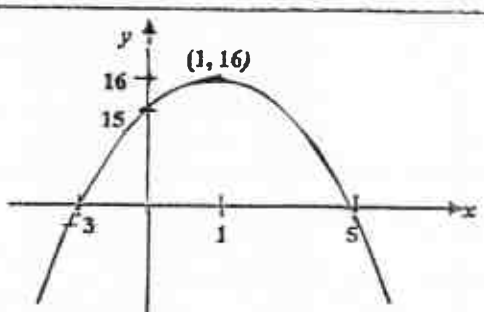
23(b)	
24(b)	25 ± 0.5 m
24(c)	Player *A*

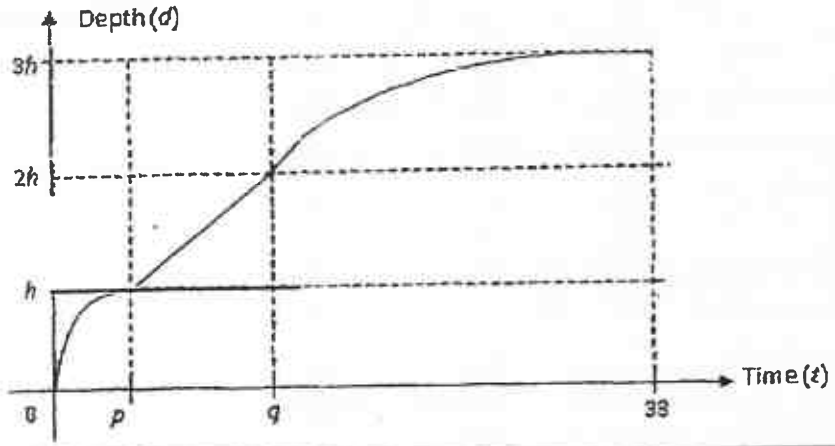
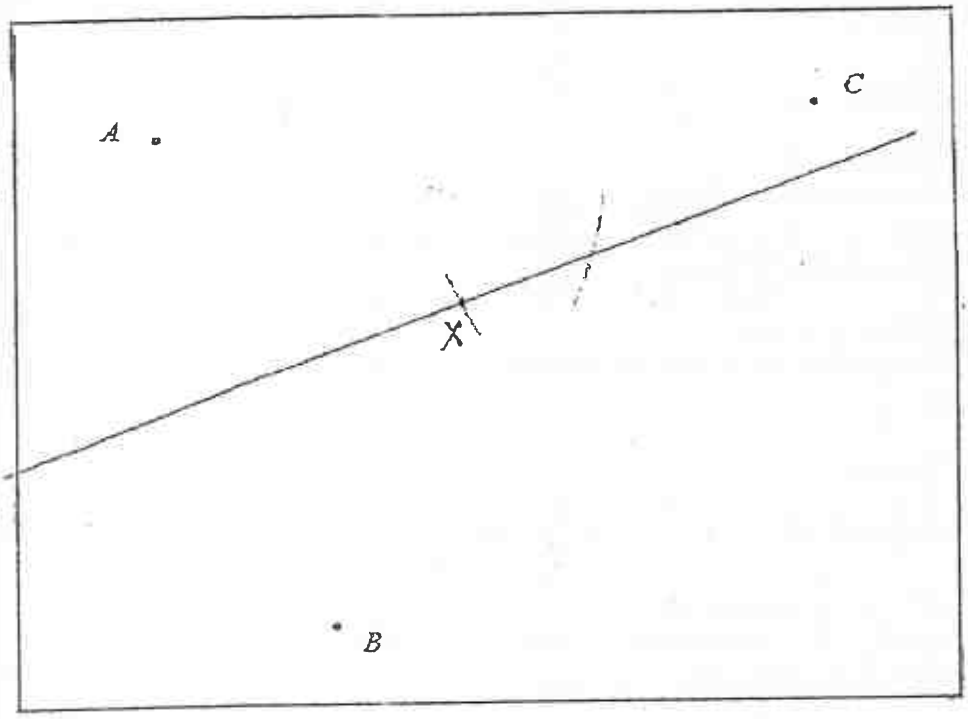
Preliminary Examination 2017
4 Express/ 5 Normal Academic
Elementary Mathematics 4048 Paper 1
Marking Scheme

1 (a)	$\frac{5^2-\sqrt{16}}{\sqrt[3]{27}}=7$	B1
1 (b)	$\sqrt{0.35}\approx 0.59$ $35\%=0.35$ $\frac{35}{53}\approx 0.66$ $35\%,\ \sqrt{0.35},\ \frac{35}{53},\ 3.5$	 M1 A1
2 (a)	$x+45=27$ $x=-18$	B1
2 (b)	$15(x-13)-14(x-13)$ $=x-13$	M1 A1
3	$\frac{100}{85}\times 182.75$ $=\$215$	M1 A1
4 (a)	$3a^2b^4$	B1
4 (b)	$2^{\frac{1}{2}}\times 2^{2n}=2^0$ $\frac{1}{2}+2n=0$ $n=-\frac{1}{4}$	 M1 A1
5 (a)	11, 13, 14, 16, 17	B1
5 (b)	11, 13, 17	B1
5 (c)	14, 16	B1
6	$3ap-12aq+8bq-2bp$ $=3a(p-4q)+2b(4q-p)$ $=(3a-2b)(p-4q)$	 M1 A1

7 (a)	1 cm : 500 cm 1 cm : 5 m 10.5 cm : $10.5 \times 5 = 52.5$ m	B1
7 (b)	$1\ cm^2 : 25\ m^2$ $500\ m^2 : \frac{500}{25} = 20\ cm^2$	M1 A1
8	$\frac{1.898 \times 10^{27}}{3.3 \times 10^{23}} \approx 5751$ $= 5.75 \times 10^3$	M1 A1
9 (a)	$3x - 8 + 2x + 5 > 3x + 7$ $x > 5$	M1 A1
9 (b)	$(3x-8) + (2x+5) + (3x+7) \le 85$ $x \le 10\frac{1}{8}$ Largest possible length = 3×7+7 = 28 cm	M1 B1 A1
10	$\frac{x}{x^2-4} + \frac{2}{x-2}$ $\frac{x+2(x+2)}{(x+2)(x-2)}$ $= \frac{3x+4}{(x+2)(x-2)}$ or $\frac{3x+4}{x^2-4}$	M1 A1
11	$(n - \frac{1}{n})^2 = n^2 - 2 + \frac{1}{n^2}$ $n^2 + \frac{1}{n^2} = 5^2 + 2$ $= 27$	M1 A1
12	Data from Year 2007, 2009, 2011 to 2015 are missing. The scale in horizontal axis is not consistent. The line graph may not be sloping upward as it seem to be. (Do not accept: the vertical axis does not start from 0)	B1 B1
13	$x : y = 2 : 5$ $y : z = 2 : 3$ $x : y : z = 4 : 10 : 15$	B1 B1 B1

14	Sum of interior angles in pentagon $= (5-2)\times 180°$ $= 540°$	M1
	Sum of angles a, b, c, d and $e = 5(360°) - 540° - 10(60°)$ $= 660°$ (deduct one mark if student assumed regular pentagon)	M1 A1
15	In 1 hour, Jane made $\frac{8}{7}$ dresses. Judy made $\frac{7}{6}$ dresses. Both made $(\frac{8}{7}+\frac{7}{6})=\frac{97}{42}$ dresses. Time to make 20 dresses $= 20 \div \frac{97}{42}$ $= 8.659$ hour $= 8$ hours 40 minutes	M1 M1 A1
16 (a)	$\angle AOB = 360° - x$ $\angle ACB = \frac{1}{2}(360° - x)$ or $180° - \frac{1}{2}x$	M1 A1
16(b)	$\angle OAP = \angle OBP = 90°$ $\angle APB = 180° - (360° - x) = x - 180°$	M1, A1
17	$BC = EC = 7$ cm $CD = CA = 10$ cm $\angle ACB = \angle DCE = 90°$ $\therefore \Delta ABC \equiv \Delta DEC$ (SAS)	M1 M1 A1
18 (a)	2 | 168 2 | 84 2 | 42 3 | 21 7 | 7 1 $168 = 2^3 \times 3 \times 7$	M1 A1
18 (b)	$\frac{168}{3\times 7} = 2^3$ $\sqrt{m} = 21$ $m = 441$	M1 A1
18 (c)	$168 = 2^2 \times (2\times 3)\times 7$ $= 4\times 6\times 7$	M1 A1

19 (a)	Figure 3	B1
19 (b)	Figure 5	B1
19 (c)	Figure 2	B1
20		B1 – correct shape B1– indicating turning point B1 – x and y-intercepts
21(a)	$\sqrt[3]{\frac{512}{216}} = \frac{4}{3}$ $\frac{\text{Surface area of } x}{\text{Surface area of } y} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$ Ratio = 16 : 9	M1 A1
21(b)	Volume of Z $= \pi(2r)^2 \frac{2}{3}h$ $= \frac{8}{3} \times \pi r^2 h$ $= \frac{8}{3} \times 216$ $= 576\ \text{cm}^3$	M1 A1
22(a)	Gradient $BC = \frac{6-1}{5-0} = 1$ Equation: $y = x + 1$	B1 B1
22(b)	$m = -\frac{1}{2}$ $y = mx + c$ $1 = -\frac{1}{2}(-5) + c \quad \Rightarrow \quad c = -\frac{3}{2}$ $y = -\frac{1}{2}x - \frac{3}{2}$ or $2y = -x - 3$	M1 A1
22(c)	Area $= \frac{1}{2}(6-1)(0+5)$ $= 12.5\ \text{units}^2$	M1 A1

23(a)	$\frac{\text{Vol of small cone}}{\text{Vol of big cone}} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$ $\frac{\text{Vol of small cone}}{\text{Vol of frustum}} = \frac{1}{7}$ $\frac{\text{Vol of small cone}}{\text{Vol of cylinder}} = \frac{1}{3}$ $p = 33 \div 11 = 3$ $q = 3 \times 4 = 12$	M1 A1 A1
23(b)		B1- correct shape for $0-p$ or $q-33$s B1 - correct shapes for $p-q$ s
		

24(a)	Construct the perpendicular bisector of AB	B1
	Mark the point X 6 cm from C.	B1
24(b)	$5 \times 5 = 25 \pm 0.5$ m	B1
24(c)	Time taken to reach the ball A: $\frac{25}{6} = 4.17$ sec C: $\frac{30}{7} = 4.28$ Player A will get the ball first.	M1 A1

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Elementary Mathematics 4048 Paper 2
Answer key

1	(a)(i)	4		
	(a)(ii)	$n = m - \left(\frac{k}{H}\right)^2$		
1	(b)	$\frac{3b^3}{2a}$		
1	(c)	2 or 3		
1	(d)	$x = 2$ and $y = -4$		
2	(a)	Bank *ABC*.		
2	(b)(i)	\$150 000		
2	(b)(ii)	\$12 000		
2	(b)(iii)	2.286%		
2	(c)	No		
3	(a)(i)	25 units		
3	(a)(ii)	−3.5		
3	(a)(iii)	(3, 9)		
3	(b)(i)	2b – 2a		
3	(b)(ii)	4a + 2b		
3	(b)(iii)	b – 4a		
3	(b)(iv)	12a		
3	(b)(v)	Points *O*, *A* and *P* are collinear points/ form a straight line . *A* is a mid-point of OP / $OA = \frac{1}{2}OP$.		
3	(b)(vi)	$\frac{1}{2}$		

3	(b)(vii)	$\frac{1}{3}$		
4	(a)	$\frac{60}{x}$		
4	(b)	$\frac{60}{x-2}$		
4	(c)	$\frac{60}{x-2}-\frac{60}{x}=1.2$		
4	(d)	$x=-9.05$ or 11.05		
4	(e)	9 min 24 sec		
5	(a)	$9-r$		
	(b)	$(9-r)^2+3^2=r^2$		
	(c)	4.09 cm^2		
6	(a)(i)	$u_5=5^2+9=34$		
6	(a)(ii)	$u_n=n^2+2n-1$		
6	(a)(iii)	959		
6	(b)(i)	$\begin{pmatrix}8\\5\\3\end{pmatrix}$		
6	(b)(ii)	$\begin{pmatrix}83\\51\end{pmatrix}$		
6	(b)(iii)	Elements of V represent the cost of manufacturing each toy boat and toy car respectively.		
6	(b)(iv)	(9190) The answer represents the total cost of manufacturing 80 toy boats and 50 toy cars.		
7	(a)(i)	56 kg		
7	(a)(ii)	$33\frac{1}{3}$% or 33.3%		
7	(b)(i)	$a=53$, $b=58$		
7	(b)(ii)	14 kg		

7	(c)	$\frac{9}{35}$		
8	(a)(i)	371 m		
8	(a)(ii)	45 300 m^2		
8	(a)(iii)	37.6°		
8	(a)(iv)	052.4°		
8	(b)	27.5°		
9	(a)(i)	51.8 cm^2		
9	(a)(ii)	334 cm^3		
9	(b)	Total mass of each filled soda can = 631.308 g Will NOT accept the proposal,		
10	(a)	$k = 2$		
10	(c)	Gradient = -4.8 ± 0.5 (Range accepted from -5.1 to -4.3)		
10	(d)(i)	Draw the line $y = \frac{x}{6}$		
10	(d)(ii)	$x = 2.1 \pm 0.1$ or $x = 5.2 \pm 0.1$		
10	(d)(iii)	$A = -36$, $B = 72$		

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Elementary Mathematics 4048 Paper 2
Marking Scheme

1	(a)(i)	$H = \frac{12}{\sqrt{6-(-3)}}$ $= 4$	B1	
	(a)(ii)	$H\sqrt{m-n} = k$ $m-n = \left(\frac{k}{H}\right)^2$ $n = m-\left(\frac{k}{H}\right)^2$	M1 A1	
	(b)	$\frac{9a^2b}{4a^2} \times \frac{8b^5}{12ab^3} = \frac{9\times 8}{4\times 12} a^{2-3} b^{1+5-3}$ $= \frac{3b^3}{2a}$	M1 A1	
	(c)	$5(11-x)+4(x+7) = (x+7)(11-x)$ $x^2 - 5x + 6 = 0$ $(x-2)(x-3) = 0$ $x = 2$ or $x = 3$	M1 M1 A1	Factorise
	(d)	Substitute $y = 4x-12$ into $5x-3y = 22$ $5x-3(4x-12) = 22$ $x = 2$ and $y = -4$	M1 A1 A1	Elimination method can be used
				11 Marks
2	(a)	Bank *ABC*: Amount = $45\,000\left[1+\frac{2.45}{12(100)}\right]^{5\times 12}$ $= \$50\,858$ Bank *XYZ*: Interest = $45\,000\times\frac{2.65}{100}\times 5 = \$5\,962.50$ Amount = \$50 962 Alex should loan from Bank *ABC*.	M1 M1 M1 A1	

	(b)(i)	Cash Price: $\frac{100}{30}\times 450\,000$ $= \$150\,000$	B1	
	(b)(ii)	Hire Purchase Price $45\,000+(1950\times 5\times 12)=\$162\,000$ Interest = \$12 000	M1 A1	
	(b)(iii)	Rate $= \frac{12\,000\times 100}{105\,000\times 5}$ $= 2.286\%$ (3 d.p)	B1	
	(c)	Price of 1 litre of petrol in Singapore: $\frac{109}{50} = S\$2.18$ Bangkok: $\frac{9408}{320} = 29.4$ Thai bahts $= \frac{29.4}{24.5} = S\$1.20$ Half of Singapore price $= \frac{1}{2}\times 2.18 = \$1.09$ Since 1.20 > 1.09, I do not agree.	M1 M1 A1	
				11 Marks
3	(a)(i)	$\sqrt{(-7)^2+24^2} = 25$ units	B1	
	(a)(ii)	$\overrightarrow{PQ} = n\overrightarrow{PS}$ $\begin{pmatrix}-7\\24\end{pmatrix} = n\begin{pmatrix}k\\12\end{pmatrix}$ $n=2$ $k=-3.5$	M1 A1	Accept $\frac{12}{k}=\frac{24}{-7}$ but not $\frac{k}{12}=\frac{-7}{24}$
	(a)(iii)	$\overrightarrow{OQ} = \overrightarrow{OP}+\overrightarrow{PQ}$ $= \begin{pmatrix}10\\-15\end{pmatrix}+\begin{pmatrix}-7\\24\end{pmatrix}=\begin{pmatrix}3\\9\end{pmatrix}$ Coordinates of $Q = (3, 9)$	B1	
	(b)(i)	$\overrightarrow{AN} = \frac{1}{3}\overrightarrow{AB}$ $= 2\mathbf{b}-2\mathbf{a}$	B1	

	(b)(ii)	$\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN}$ $= 4\mathbf{a} + 2\mathbf{b}$	B1	
	(b)(iii)	$\overrightarrow{NM} = \overrightarrow{OM} - \overrightarrow{ON}$ $= \mathbf{b} - 4\mathbf{a}$	B1	
	(b)(iv)	$\overrightarrow{MP} = 3\overrightarrow{MN}$ $\overrightarrow{OP} = \overrightarrow{OM} - 3\overrightarrow{NM}$ $= 3\mathbf{b} - 3(\mathbf{b} - 4\mathbf{a})$ $= 12\mathbf{a}$	B1	
	(b)(v)	Points O, A and P are collinear points/ form a straight line . A is a mid-point of OP / $OA = \frac{1}{2}OP$.	B1 B1	
	(b)(vi)	$\frac{\text{Area of } \Delta AMN}{\text{Area of } \Delta BMN} = \frac{1}{2}$	B1	
	(b)(vii)	$\frac{\text{Area of } \Delta BMN}{\text{Area of } \Delta BOA} = \frac{1}{3}$	B1	
				12 Marks
4	(a)	$\frac{60}{x}$	B1	
	(b)	$\frac{60}{x-2}$	B1	
	(c)	$\frac{60}{x-2} - \frac{60}{x} = 1.2$ $60x - 60(x-2) = 1.2x(x-2)$ $x^2 - 2x - 100 = 0$ (shown)	M1 M1 A1	Form equation Attempt to simplify
	(d)	$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-100)}}{2(1)}$ $x = -9.05$ or 11.05 (2 d.p)	M1 A1	
	(e)	Time taken $= 85\left(\frac{60}{11.05-2}\right)$ $= 564$ seconds $= 9$ min 24 sec	M1 B1	
				9 Marks

5	(a)	$OB = 9 - r$	B1	
	(b)	$(9-r)^2 + 3^2 = r^2$ $81 - 18r + r^2 + 9 = r^2$ $r = 5$ cm (Shown)	B1 M1 A1	
	(c)	$\sin \angle BOC = \frac{3}{5}$ $\angle BOC = 36.869°$ or 0.6435 rad $\angle COD = 73.739°$ or 1.287 rad Area of sector $= \frac{73.739}{360} \times \pi \times 5^2$ or $\frac{1}{2} \times 5^2 \times 1.287$ $= 16.0875$ cm^2 Area of $\Delta OCD = \frac{1}{2} \times 4 \times 6 = 12$ cm^2 Area of req. segment $= 4.09$ cm^2. (3 s.f.)	B1 M1 M1 A1	
				8 Marks
6	(a)(i)	$u_5 = 5^2 + 9 = 34$	B1	
	(a)(ii)	$u_n = n^2 + 2n - 1$	B1 B1	B1 for n^2 B1 for $2n - 1$
	(a)(iii)	$U_{30} = 30^2 + 2(30) - 1$ $= 959$	B1	
6	(b)(i)	$\begin{pmatrix} 8 \\ 5 \\ 3 \end{pmatrix}$	B1	
	(b)(ii)	$V = \begin{pmatrix} 6 & 4 & 5 \\ 4 & 2 & 3 \end{pmatrix} \begin{pmatrix} 8 \\ 5 \\ 3 \end{pmatrix}$ $= \begin{pmatrix} 83 \\ 51 \end{pmatrix}$	B1 B1	
	(b)(iii)	Elements of V represent the cost of manufacturing each toy boat and toy car respectively.	B1	
	(b)(iv)	$WV = \begin{pmatrix} 80 & 50 \end{pmatrix} \begin{pmatrix} 83 \\ 51 \end{pmatrix} = (9190)$ The answer represents the total cost of manufacturing 80 toy boats and 50 toy cars.	B1 B1	

				10 Marks
7	(a)(i)	Modal mass = 56 kg	B1	
	(a)(ii)	$\frac{7}{21}\times 100\% = 33\frac{1}{3}\%$ or 33.3%	B1	
	(b)(i)	$a = 53$ $b = 58$	B1 B1	
	(b)(ii)	Interquartile range $= 67 - 53$ $= 14$ kg	B1	
	(c)	$\left(\frac{18}{21}\times\frac{3}{20}\right) + \left(\frac{3}{21}\times\frac{18}{20}\right)$ $= \frac{9}{35}$	M1 A1	
				7 Marks
8	(a)(i)	$BC^2 = 250^2 + 400^2 - 2(250)(400)\cos 65°$ $BC = 371.45$ $= 371$ m (3 s.f.)	B1 B1 A1	
	(a)(ii)	Area $= \frac{1}{2}(250)(400)\sin 65°$ $= 45\,315.38$ $= 45\,300$ m^2 (3 s.f.)	M1 A1	
	(a)(iii)	$\frac{\sin \angle ABC}{250} = \frac{\sin 65°}{371.45}$ $\angle ABC = 37.588$ $\approx 37.6°$ (1 d.p.)	M1 A1	
	(a)(iv)	Bearing $= 90° - 37.6°$ $= 052.4°$	B1	
	(b)	Let h be the height of eagle above the ground $\frac{h}{400} = \tan 18°$ $h = 129.967$ m $\tan \angle ACE = \frac{129.967}{250}$ $\angle ACE = 27.46°$ Angle of depression = 27.5° (to 1 d.p.)	 B1 M1 A1	
				11 Marks

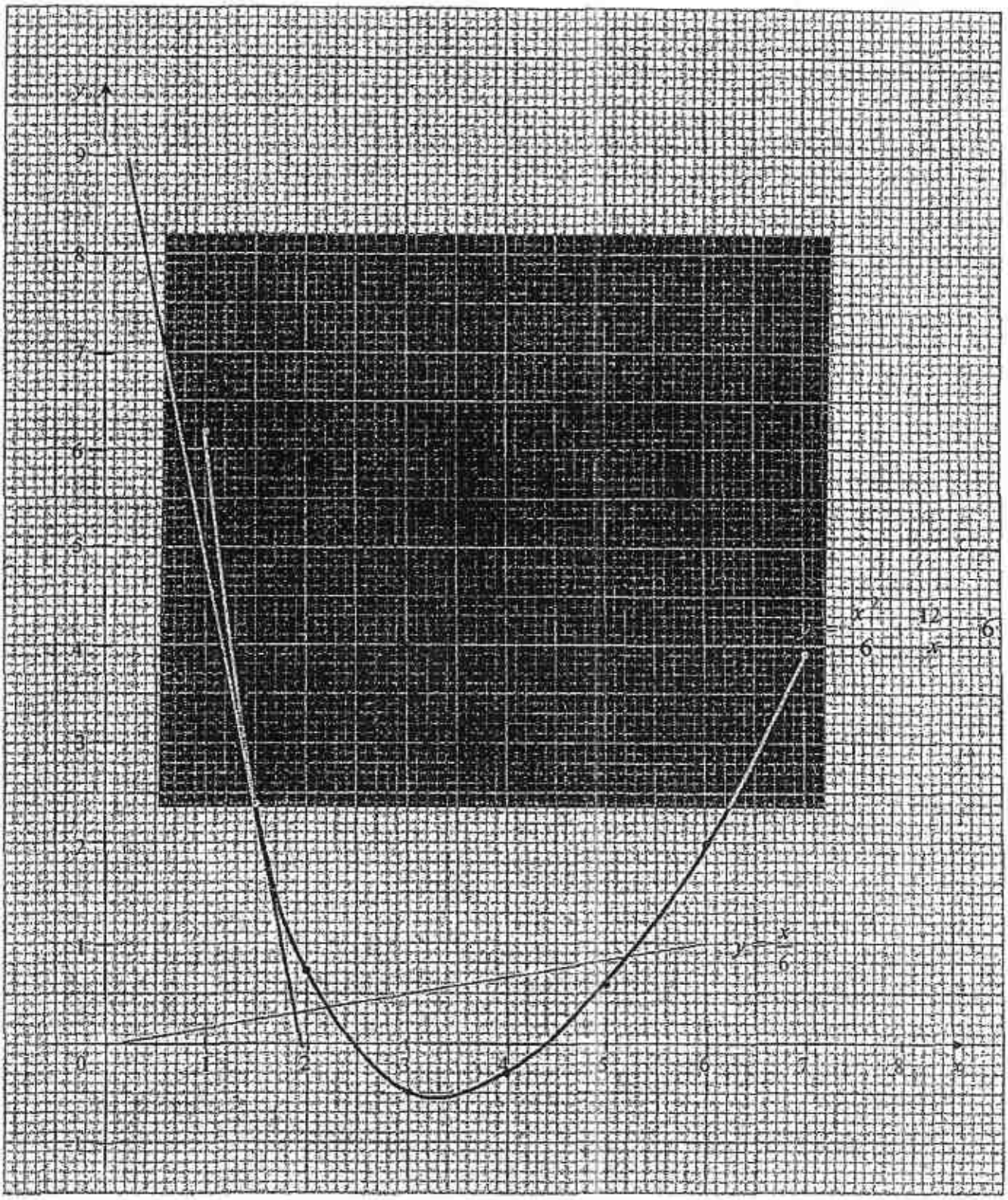
9	(a)(i)	Area of hemisphere $= 2\pi(2.5)^2$ $= 39.2699\ cm^2$ Area of ring $= \pi(3.2^2 - 2.5^2)$ $= 12.534\ cm^2$ Area of the base $= 51.8048$ $= 51.8\ cm^2$	M1 A1	Any one part of working shown
	(a)(ii)	Volume of hemisphere $= \frac{1}{2} \times \frac{4}{3} \times \pi(2.5)^3$ $= 32.7249\ cm^3$ Volume of cylinder $= \pi \times 3.2^2 \times 11.4)$ $= 366.73696\ cm^3$ Volume of the soda can $= 334.01$ $= 334\ cm^3$ (3 s.f.)	M1 A1	Any one part of working shown
	(b)	Surface area of the can $= 2\pi(3.2) \times 11.4 + \pi(3.2)^2 + 51.8048$ $= 313.185$ $= 313\ cm^2$ Mass of the empty can using the proposed material $= 313.185 \times 0.8$ $= 250.548$ g Mass of soda in each can $= 95\% \times 334 \times 1.2$ $= 380.76$ g Total mass of each filled soda can $= 250.548 + 380.76$ $= 631.308$ g Since $631.308 > 620$ g, $\therefore$ I will NOT accept the proposal.	M1 B1 B1 M1 M1 A1	Allow error from part (a) to carry forward in this whole part of question.
				10 Marks
10	(a)	$k = 2$	B1	
	(b)	Refer to attached graph.	B1 – Axes drawn to scale B1 – Points are plotted correctly B1 – Smooth curve plotted	

10	(c)	Tangent is drawn at the point $x = 1.5$ Refer to attached graph Gradient $= -4.8 \pm 0.5$ (Range accepted from -5.3 to -4.3)	B1 B1	
	(d)(i)	Draw the line $y = \frac{x}{6}$ Refer to attached graph.	B1	
	(d)(ii)	$x = 2.2 \pm 0.1$ or $x = 5.2 \pm 0.1$	B1 B1	
	(d)(iii)	$\frac{x^2}{6} + \frac{12}{x} - 6 = \frac{x}{6}$ $x^3 - x^2 - 36x + 72 = 0$ $A = -36$, $B = 72$	M1 A1	 Both correct
				11 Marks

8

Name : _____( )

lass: _____ Date: _____



SERANGOON GARDEN SECONDARY SCHOOL

Vision: Critical Thinkers, Thoughtful Leaders
Mission: Love to Learn, Learn to Lead

PRELIMINARY EXAMINATION 2017

CANDIDATE NAME			
CLASS		REGISTER NUMBER	

MATHEMATICS 4048/02
Paper 2 22 August 2017
Secondary 4 Express/ 5 Normal Academic 2 hours 30 minutes
0800 – 1030

Additional Materials: Writing Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and class register number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

Areas for Improvement		
Error	Penalty	Qn. No.(s)
Accuracy of non-exact answers	– 1	
Missing/ wrong units (for Paper 2 only)	– 1	
Presentation/ Not using ink	– 1	

Name/Signature of Parent/Guardian Date

FOR MARKER'S USE
100

This question paper consists of 13 printed pages and 1 blank page.

Setter: Mr Ng HJ Vetter: Mr Ko TH

MATHEMATICAL FORMULAE

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions.

1 (a) n is a positive integer. Show that $n^2 + n$ is always even. [2]

(b) Solve the equation $p^2 - 7p + 12 = 0$. [2]

Hence solve the equation $q^4 - 7q^2 + 12 = 0$. [2]

(c) A 2.5 km^2 lake has an area of 40 cm^2 on a map.

(i) If the scale of the map is such that 1 cm represents n km, find the value of n. [2]

(ii) The distance between the hospital and the village town on the map is 30 cm. Find the actual distance, in kilometres, between the hospital and the village town. [1]

---

2 Mr Kia is going on a business trip to a province in the same country. There are two options for him to go to the province: by domestic flight or by car.

- If he decides to drive, he would cover a distance of 400 km at a speed of x km/h.
- If he decides to take a domestic flight, he would cover a distance of 300 km at a speed of $(x + 250)$ km/h.

(i) Find an expression, in terms of x, for the time taken to travel from home to the province if Mr Kia decides to drive. [1]

(ii) Find an expression, in terms of x, for the time taken to travel from home to the province if Mr Kia decides to take a domestic flight. [1]

(iii) If the flight time is 210 minutes less than the driving time, form an equation in x and show that it reduces to $7x^2 + 1550x - 200000 = 0$. [3]

(iv) Solve the equation $7x^2 + 1550x - 200000 = 0$, giving your answers correct to 1 decimal place. [3]

(v) If Mr Kia needs to meet his client punctually at 1400, find the latest time that he needs to leave home if he decides to drive. Assume that time has been factored in for the usual traffic conditions. [2]

[Turn over

3 (a) A set of 10 cards is made as shown.

S	T	A	T	I	S	T	I	C	S

The cards are shuffled and placed face down on a desk. A card is drawn at random from the set of cards. It is then replaced and shuffled again before another card is being drawn again.

Calculate the probability that

(i) both cards show the letter T, [2]

(ii) exactly one of the cards shows the letter T. [2]

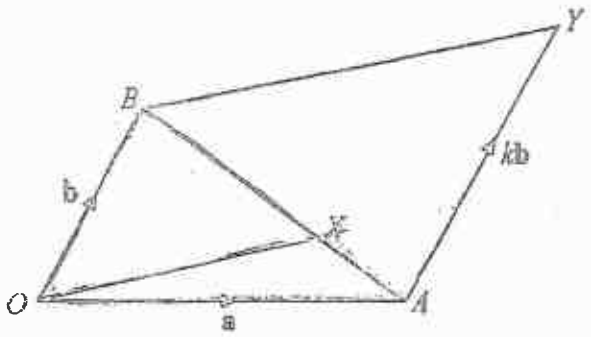
(b) The table shows the ages of 1100 people who entered a 10-km run

Age (x years)		$20 \le x < 30$	$30 \le x < 40$	$40 \le x < 50$
Frequency	Men	375	186	99
	Women	250	122	68

(i) One person is chosen at random. Find, as a fraction in its lowest term, the probability that the person is a man aged less than 40 years old. [1]

(ii) Two persons are chosen at random. Find the probability that both of them are women aged 30 or more. [2]

4 In the diagram, $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{AY} = k\mathbf{b}$. X lies on the line AB such that $\overrightarrow{AX} = \frac{1}{3}\overrightarrow{AB}$.



(i) Express $\overrightarrow{AX}$ and $\overrightarrow{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

(ii) Express $\overrightarrow{BY}$ in terms of k, $\mathbf{a}$ and $\mathbf{b}$. [1]

(iii) Given that OX is parallel to BY, find the value of k. [2]

(iv) The line OX when produced, meets AY at Z. Express $\overrightarrow{AZ}$ in terms of $\mathbf{b}$. [1]

(v) Find the value of

(a) $\dfrac{\text{area of } \Delta OAX}{\text{area of } \Delta OBX}$, [1]

(b) $\dfrac{\text{area of } \Delta AXZ}{\text{area of quadrilateral } XBYZ}$. [2]

[Turn over

5 The following shows the work done by a student in calculating the sum of the first n natural numbers.

n	Series	Sum	Formula
1	1	1	$\frac{1}{2}(1)(1+1)$
2	$1+2$	3	$\frac{1}{2}(2)(2+1)$
3	$1+2+3$	6	$\frac{1}{2}(3)(3+1)$
4	$1+2+3+4$	10	$\frac{1}{2}(4)(4+1)$
⋮	⋮	⋮	⋮
6	$1+2+3+4+5+6$	a	b
⋮	⋮	⋮	⋮
n	$1+2+3+\cdots+n$	c	

(i) Study the pattern and write down the values of a and b. [2]

(ii) Find in terms of n, the value of c. [1]

After doing some additional calculations, the student realised that

$$1^3+2^3+3^3=36=6^2,$$
$$1^3+2^3+3^3+4^3=100=10^2.$$

(iii) Determine the sum of the series

(a) $1^3+2^3+3^3+4^3+5^3+6^3$, [1]

(b) $1^3+2^3+3^3+\cdots+n^3$ in terms of n. [1]

(iv) Hence, using (iii)(b), determine the exact value of the sum of the series

$$3^3+6^3+9^3+12^3+\cdots+300^3.$$ [2]

6 (a) State the order and name of each matrix. [2]

	Matrix	Order	Name of matrix
(i)	$\begin{pmatrix} 2 \\ 5 \\ 12 \end{pmatrix}$		
(ii)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$		

(b) The Tan family owns two cars. Every week (Monday to Friday) on average, Mr Tan spends \$150, \$70 and \$10 on petrol, carpark charges and road pricing (ERP) respectively. Every week (Monday to Friday) on average, Mrs Tan spends \$80, \$45 and \$30 on petrol, carpark charges and road pricing (ERP) respectively.

The information can be represented by the matrix

$$\mathbb{P} = \begin{matrix} \text{Mr Tan} & \text{Mrs Tan} \\ \begin{pmatrix} 150 & 80 \\ 70 & 45 \\ 10 & 30 \end{pmatrix} & \begin{matrix} \text{Petrol} \\ \text{Carpark charges} \\ \text{Road pricing (ERP)} \end{matrix} \end{matrix}.$$

During weekends, the Tan family drives the weekend car and spends on average \$20, \$10 and \$2 on petrol, carpark charges and ERP respectively.

In a year, on average, both Mr Tan and Mrs Tan work for 48 weeks.

(i) Represent the average weekend car expenses of the Tan family by a matrix $\mathbb{R}$. [1]

(ii) Evaluate $\mathbb{Q} = \mathbb{P}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\mathbb{S} = 48\mathbb{Q} + 52\mathbb{R}$. [3]

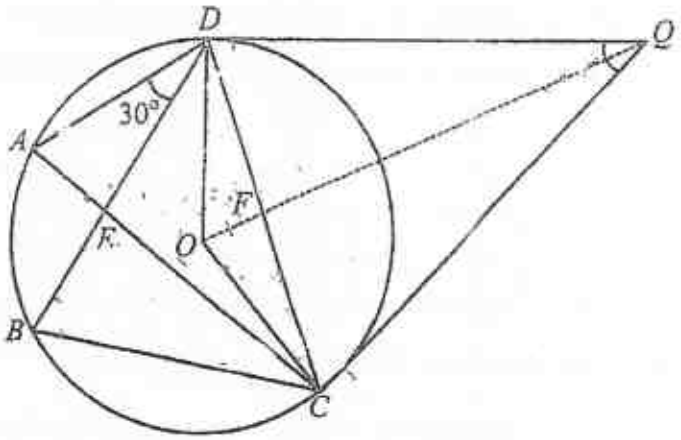
(iii) State what the elements of $\mathbb{S}$ represent. [1]

(iv) The matrix $\mathbb{T}$ is given by $\mathbb{T} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\mathbb{S}$. Evaluate matrix $\mathbb{T}$ and describe in a sentence what the element(s) of the matrix $\mathbb{T}$ represent. [2]

(v) A recent credit card promotion entitles Mr and Mrs Tan 12.5% savings on petrol every time they pump petrol.

Calculate the new expenses for petrol, carpark charges and ERP for the Tan family in a year. [2]

7



In the diagram above, AEC and BED are chords of the circle with centre O. $\angle ADE = 30^\circ$ and $\angle CQD = 50^\circ$. CQ and DQ are tangents to the circle and F is the midpoint of chord CD.

(i) Explain why ΔADE is similar to ΔBCE. [2]

(ii) Name a pair of congruent triangles. [1]

(iii) Find, stating your reasons clearly,

(a) $\angle DAC$, [2]

(b) $\angle BEC$. [1]

(iv) Is it possible to draw a circle that passes through C, O, D and Q? Explain your answer clearly. [1]

8 **Answer the whole of this question on a single sheet of graph paper.**

The table below gives the values of x- and y-coordinates of some points on the graph of $y = \frac{ax}{x+b}$.

x	-0.5	0	1	2	3	4	5
y	-2	0	2	3	3.6	4	4.3

(a) By formulating two equations, find the values of a and b. [3]

(b) Using a scale of 2 cm to represent 1 unit on both the x-axis and y-axis, plot the points given in the table and join them with a smooth curve for $-0.5 \le x \le 5$. [3]

(c) By drawing a suitable tangent, find the gradient of the curve at the point $x = 1.5$. [2]

Using the values of a and b found in (a),

(d) find the solution(s) of the equation

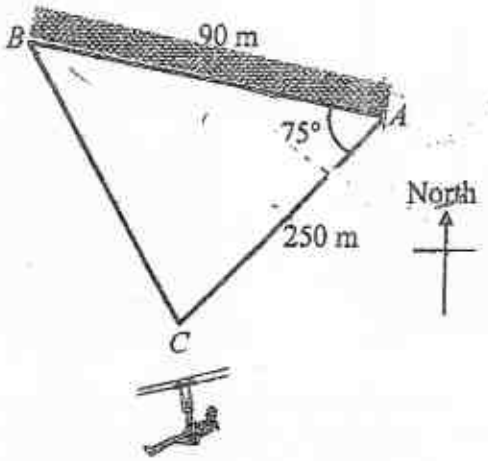
$$\frac{ax}{x+b} = -\frac{1}{3}x + 1,$$

by drawing a suitable straight line on the same axes, [2]

(e) find the range of values of x such that $\frac{ax}{x+b} < 2.5$. [2]

[Turn over

9 Points A and B are points at the bottom a cliff 50 metres tall in height. Point C on a flat ground is 250 metres away from A with AB making an angle of $75°$ with the line AC. The bearing of C from A is $217°$ and A and B are 90 m apart.



Calculate the

(a) bearing of B from A, [1]

(b) area of the land formed by the points A, B and C, [2]

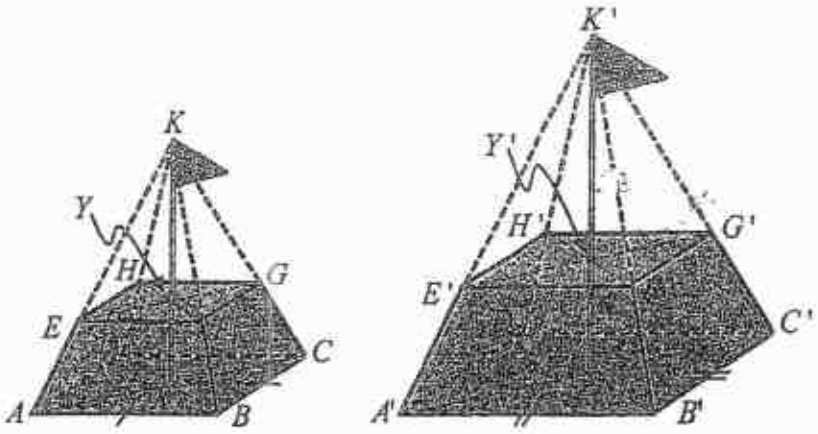
(c) shortest distance from C to the bottom of the cliff. [2]

An outdoor adventure company wants to build a flying fox using a metal cable with the starting point X on the cliff and the landing point at C.

(d) Find the distance away from B vertically below X such that the slope is the greatest. [2]

(e) Find the angle that the metal cable makes with the ground at point C. [2]

10 A company manufactures geometrically similar flagpole bases of two different sizes as shown below.



The bases are made of cement and are in the shape of truncated right pyramids. If each pyramid could be completed, its vertex would be the top of the flagpole at K and K' respectively. The height of the flagpole for the bigger-sized base is 2.5 metres and the ratio of the side length of the bottom surfaces $ABCD$ and $A'B'C'D'$ is $3:5$.

(a) The area of the bottom surface $A'B'C'D'$ is $2500\,\text{cm}^2$. What is the area of the bottom surface $ABCD$? [2]

(b) Given that $E'F' = F'G' = 40\,\text{cm}$, find the length $K'Y'$ and the volume of the base **(as represented by the shaded part)** for the bigger-sized flagpole base. [3]

(c) Hence, find the volume of the base for the smaller-sized flagpole base. [2]

(d) If it costs \$15 to buy a smaller-sized flagpole base and \$25 to buy a bigger-sized flagpole base, which flagpole base is more value for money? Explain with clear working. [2]

[Turn over

11 The concert band of a school intends to rent a concert venue for their annual performance as their school hall is undergoing a renovation.

Information that the chairperson Peter and his committee need is on the opposite page.

As shown in **Figure 1**, seats in the concert hall are arranged along arcs of concentric circles of equal spacing. There are three rows of seats in front and one row of limited seats behind the stage.

(i) Show that angle COD = 1.55 radians and find the area taken up by the stage. [3]

(ii) Each normal concert chair takes up 80 cm of the arc. Show that row 1 can fit a maximum of 47 normal concert chairs. [2]

Peter and his committee decide that they will have a total of 3 rehearsals (including the rehearsal on the actual performance day) and a total of 30 VIP guests. They need to decide whether they should take up Package A or Package B of the concert hall rental offered by the venue management.

(iii) Assuming that Peter and his committee decide to charge \$20, \$15, \$12 and \$25 for Row 1, 2, 3 and 4 respectively, help Peter to decide which package he should take up. Justify the decision with clear calculations and assumption(s) so that Peter can present the proposal to his teacher-in-charge. [5]

Details of the stage

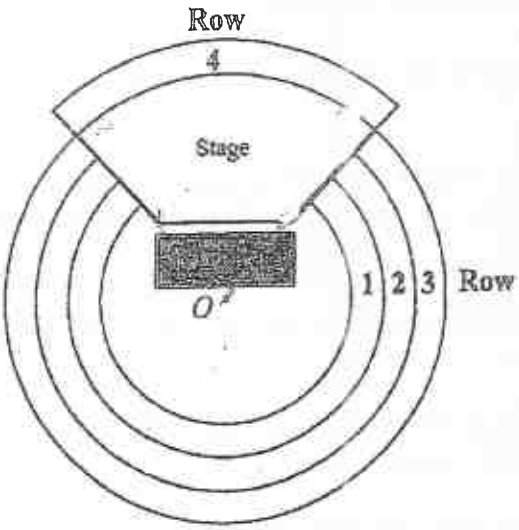


Figure 1

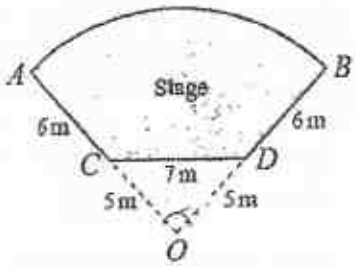


Figure 2

Cost of rental of items

Package	Details (All prices in this column are nett prices)	Cost of renting one normal concert chair (excluding 7% GST)	Cost of renting one VIP concert chair (excluding 7% GST)

A	▪ Basic rental cost: $2800
	▪ Freebies:
	  ◦ Free 150 normal concert chairs
	  ◦ Free 25 VIP chairs
	  ◦ 1st rehearsal (unlimited time usage on day of event): $100
	  ◦ 2nd rehearsal: 20% off normal rehearsal price
	  ◦ 3rd rehearsal and beyond: 10% off normal rehearsal price
		$8	$18

B	▪ Basic rental cost: $1500
	▪ Freebies:
	  ◦ Free 100 normal concert chairs
	  ◦ Free 10 VIP chairs
	▪ All rehearsals cost $120 each with unlimited time usage
	▪ Terms and Condition: Row 4 cannot be opened for selling of tickets.
		$12	$20

END OF PAPER

Sec 4E/5NA Prelims P2 Suggested Mark Scheme

Qn	Solution	
Algebra		
1(a)	$n^2+n=n(n+1)$ If n is odd, then $(n+1)$ is even. If n is even, then $(n+1)$ is odd. Product of an odd number and an even number is even. Thus $n(n+1)$ is even. **Alternative:** If n is odd, then n^2 is odd. Then sum of two odd numbers n and n^2 is even. If n is even, then n^2 is even. Then sum of two even numbers n and n^2 is even.	
(b)	$p^2-7p+12=0$ $(p-3)(p-4)=0$ $p=3$ or $p=4$ $q^4-7q^2+12=0$ Let $p=q^2$. $q^2=3\Rightarrow q=\pm\sqrt{3}$ or $q^2=4\Rightarrow q=\pm2$	
(c) (i)	$40\,\text{cm}^2: 2.5\,\text{km}^2$ $1\,\text{cm}^2: 0.0625\,\text{km}^2$ $1\,\text{cm}: 0.25\,\text{km}$ $n=0.25$	
(ii)	Actual distance between the hospital and the village town $=30\times0.25\,\text{km}$ $=7.5\,\text{km}$	
		Total for Q1: 9

Word problem and quadratic equations		
2(i)	Time taken to travel from home to the province if Mr Kia decides to drive = $\frac{400}{x}$ h.	
(ii)	Time taken to travel from home to the province if Mr Kia decides to take a domestic flight = $\frac{300}{x+250}$ h.	

(iii)	$\frac{400}{x}-\frac{300}{x+250}=\frac{7}{2}$ $400(x+250)-300(x)=\frac{7}{2}(x)(x+250)$ $400x+100000-300x=\frac{7}{2}x^2+875x$ $\frac{7}{2}x^2+775x-100000=0$ $7x^2+1550x-200000=0$ (shown)	
(iv)	$7x^2+1550x-200000=0$ $x=\frac{-1550\pm\sqrt{(1550)^2-4(7)(-200000)}}{14}$ $=\frac{-1550\pm\sqrt{8002500}}{14}$ $=94.919$ or -312.776 $=94.9$ or -312.8 (1d.p.)	
(v)	x must be positive, thus $x=94.919$ If Mr Kia drives, time taken $=\frac{400}{94.919}\text{h}=4.2141\text{h}$ 0947 hrs $\xrightarrow{13\text{ minutes}}$ 1000 hrs $\xrightarrow{4\text{ hr}}$ 1400 hrs He must leave home latest by 0947.	
		Total for Q2: 10

Probability

S	T	A	T	I	S	T	I	C	S

3S, 3T, A, 2I, C

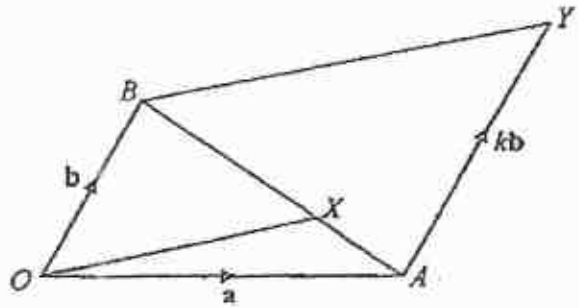
3(a)(i)	P(both cards show the letter T) $=\frac{3}{10}\times\frac{3}{10}$ $=\frac{9}{100}$	
(ii)	P(exactly one of the cards shows the letter T) $=\frac{3}{10}\times\frac{7}{10}+\frac{7}{10}\times\frac{3}{10}$ $=\frac{42}{100}=\frac{21}{50}$	
(b)(i)		

Age (x years)		$20\le x<30$	$30\le x<40$	$40\le x<50$
Frequency	Men	375	186	99
	Women	250	122	68

Probability that the person is a man aged less than 40 years old

	$=\frac{375+186}{1100}$ $=\frac{561}{1100}$ $=\frac{51}{100}$	
(ii)	Probability that both of them are women aged 30 or more $=\frac{122+68}{1100}\times\frac{122+68-1}{1099}$ $=\frac{190}{1100}\times\frac{189}{1099}$ $=\frac{513}{17270}=0.0297$	
		Total for Q3: 7

Vectors



4(i)	$\overrightarrow{AX}=\frac{1}{3}\overrightarrow{AB}=\frac{1}{3}(\mathbf{b}-\mathbf{a})$ $\overrightarrow{OX}-\overrightarrow{OA}=\frac{1}{3}(\mathbf{b}-\mathbf{a})$ $\overrightarrow{OX}=\frac{1}{3}(\mathbf{b}-\mathbf{a})+\mathbf{a}=\frac{1}{3}(2\mathbf{a}+\mathbf{b})$	
(ii)	$\overrightarrow{BY}=-\mathbf{b}+\mathbf{a}+k\mathbf{b}=\mathbf{a}+(k-1)\mathbf{b}$	
(iii)	OX is parallel to $BY \Rightarrow m\overrightarrow{OX}=\overrightarrow{BY}$ $\frac{1}{3}m(2\mathbf{a}+\mathbf{b})=\mathbf{a}+(k-1)\mathbf{b}$ $\begin{cases}\frac{2}{3}m=1\Rightarrow m=\frac{3}{2}\\ \frac{1}{3}m=k-1\Rightarrow k=1+\frac{1}{3}(\frac{3}{2})=\frac{3}{2}\end{cases}$	
(iv)	$\overrightarrow{AZ}=\frac{1}{2}\mathbf{b}$ since $\overrightarrow{OZ}=\overrightarrow{BY}$ and $\overrightarrow{OB}=\overrightarrow{ZY}$.	
(v)(a)	$\frac{\text{area of }\Delta OAX}{\text{area of }\Delta OBX}=\frac{AX}{BX}=\frac{(\frac{1}{3})}{(\frac{2}{3})}=\frac{1}{2}$	
(b)	$\frac{\text{area of }\Delta AXZ}{\text{area of }\Delta ABY}=\left(\frac{1}{3}\right)^2=\frac{1}{9}$ $\frac{\text{area of }\Delta AXZ}{\text{area of quadrilateral }XBYZ}=\frac{1}{8}$	
		Total for Q4: 9

Number patterns

n	Series	Sum	Formula
1	1	1	$\frac{1}{2}(1)(1+1)$
2	$1+2$	3	$\frac{1}{2}(2)(2+1)$
3	$1+2+3$	6	$\frac{1}{2}(3)(3+1)$
4	$1+2+3+4$	10	$\frac{1}{2}(4)(4+1)$
⋮	⋮	⋮	⋮
6	$1+2+3+4+5+6$	a	b
⋮	⋮	⋮	⋮
n	$1+2+3+\cdots+n$	c	

5(i) $b=\frac{1}{2}(6)(6+1)=21$

$a=21$

(ii) $c=\frac{1}{2}(n)(n+1)$

(iii) (a) $1^3+2^3+3^3+4^3+5^3+6^3=21^2=441$

(b) $1^3+2^3+3^3+\cdots+n^3=\left[\frac{1}{2}n(n+1)\right]^2$

(iv)
$$3^3+6^3+9^3+12^3+\cdots+300^3$$
$$=(3\times1)^3+(3\times2)^3+(3\times3)^3+(3\times4)^3+\cdots+(3\times100)^3$$
$$=3^3\left[1^3+2^3+3^3+4^3+\cdots+100^3\right]$$
$$=27\left[\frac{1}{2}(100)(101)\right]^2$$
$$=688567500$$

Total for Q5: 7

Matrices

6(a)

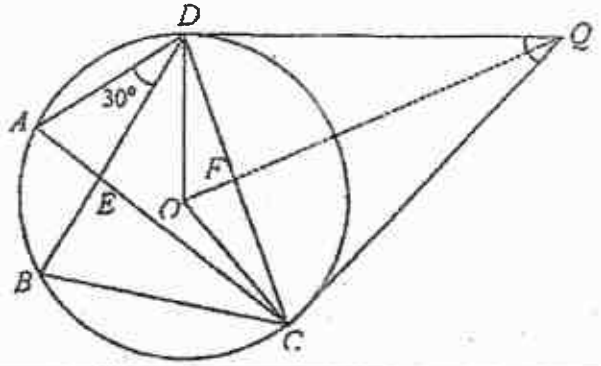
	Matrix	Order	Name of matrix
(i)	$\begin{pmatrix}2\\5\\12\end{pmatrix}$	3×1 <u>OR</u> 3 by 1	Column matrix
(ii)	$\begin{pmatrix}0&0\\0&0\end{pmatrix}$	2×2 <u>OR</u> 2 by 2	Square matrix <u>OR</u> Null matrix <u>OR</u> Zero matrix

(b) (i) $\mathbf{R}=\begin{pmatrix}20\\10\\2\end{pmatrix}$

(ii) Given $\mathbf{P}=\begin{pmatrix}150&80\\70&45\\10&30\end{pmatrix}$

	$Q = P\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}150 & 80\\70 & 45\\10 & 30\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}230\\115\\40\end{pmatrix}$ $S = 48Q + 52R = 48\begin{pmatrix}230\\115\\40\end{pmatrix} + 52\begin{pmatrix}20\\10\\2\end{pmatrix} = \begin{pmatrix}12080\\6040\\2040\end{pmatrix}$	
(iii)	The elements 12080, 6040 and 2040 represent the Tan family's yearly car expenses on petrol, carpark charges and ERP respectively.	
(iv)	$T = \begin{pmatrix}1 & 1 & 1\end{pmatrix}S$ $= \begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}12080\\6040\\2040\end{pmatrix} = (12080+6040+2040)$ $= (20160)_{1\times1}$ It represents the Tan family's total car expenses in a year.	
(v)	<u>Method 1:</u> New yearly expenses for petrol = $0.875\times12080 = \$10570$ carpark charges = \$6040 ERP = \$2040 <u>Method 2:</u> Given $P_{new} = \begin{pmatrix}131.25 & 70\\70 & 45\\10 & 30\end{pmatrix}$ $Q_{new} = P_{new}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}131.25 & 70\\70 & 45\\10 & 30\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}201.25\\115\\40\end{pmatrix}$ $S_{new} = 48Q_{new} + 52R_{new} = 48\begin{pmatrix}201.25\\115\\40\end{pmatrix} + 52\begin{pmatrix}17.5\\10\\2\end{pmatrix} = \begin{pmatrix}10570\\6040\\2040\end{pmatrix}$	
		Total for Q6: 11

Circle Properties (& similarity, congruency)



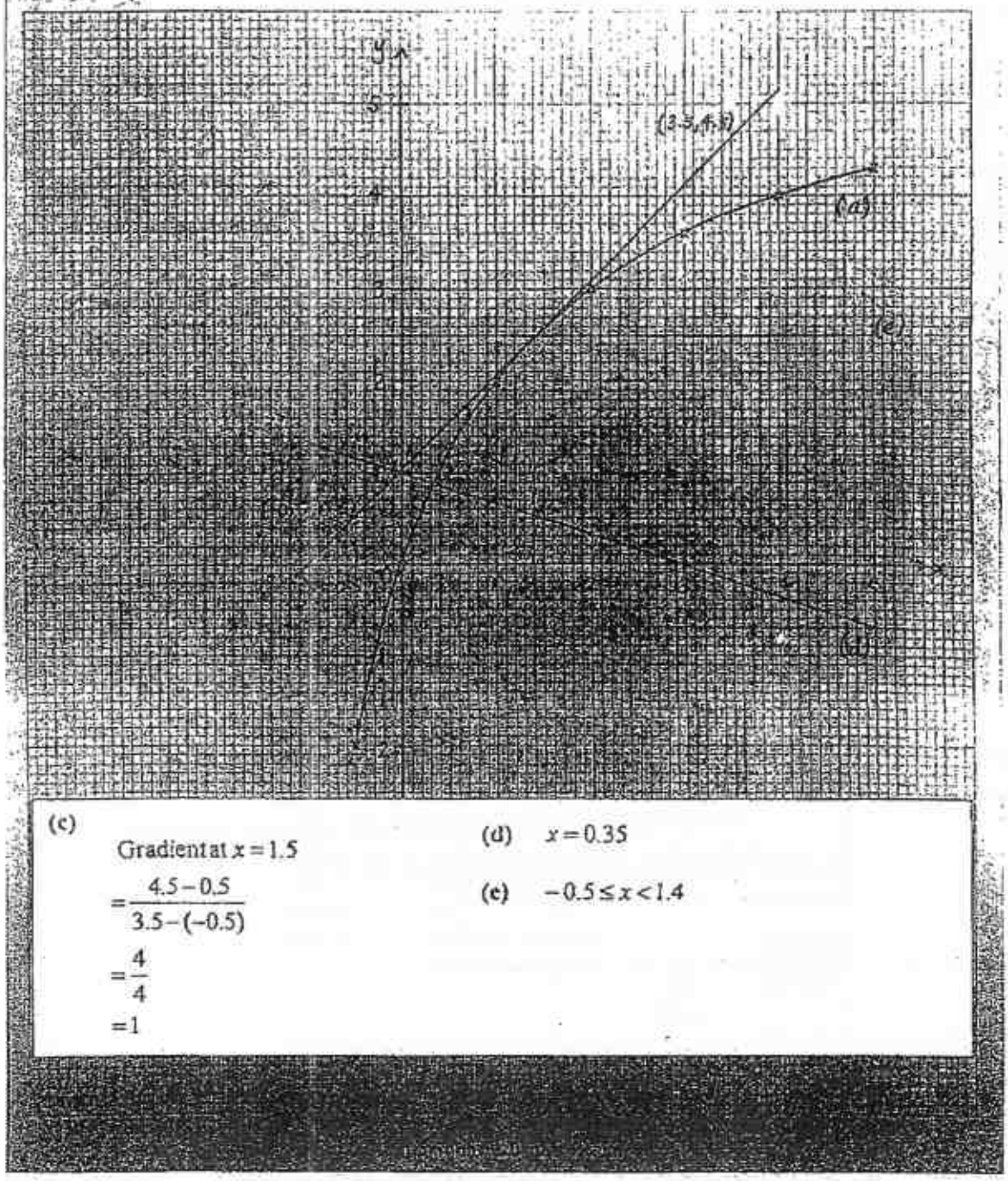
7(i)	$\angle ADE = \angle BCE$ (angle in the same segment) $\angle DAE = \angle CBE$ (angle in the same segment) By the **AA similarity test**, ΔADE is similar to ΔBCE.	
(ii)	Any of the following answers: ▫ ΔDOF is congruent to ΔCOF **OR** ▫ ΔDOQ is congruent to ΔCOQ **OR** ▫ ΔDFQ is congruent to ΔCFQ	
(iii) (a)	$\angle ODQ = \angle OCQ = 90°$ **(tan ⊥ rad)** $\angle DOQ = 360° - 90° - 90° - 50° = 130°$ $\angle DAC = \frac{1}{2}\angle DOC$ $= 65°$ **(angle at centre = twice angle at circumference)**	
(b)	$\angle BEC = \angle AED$ (vertically opposite angles) $= 180° - 30° - 65°$ ($\angle$ sum of triangle) $= 85°$	
(iv)	It is possible to draw a circle that passes through C, O, D and Q since $\angle ODQ = \angle OCQ = 90°$ **(tan ⊥ rad) and angle in a semicircle.** In this case, OQ is the diameter of the circle.	
		Total for Q7: 7

Graph

8(a)	$y = \frac{ax}{x+b}$ (1,2): $2 = \frac{a}{1+b} \Rightarrow a - 2b = 2$ (2,3): $3 = \frac{2a}{2+b} \Rightarrow 2a - 3b = 6$ Solving the two equations simultaneously, $a = 6, b = 2$	
(b)	Refer to graph on page 7.	
(c)		
(d)		
(e)		
		Total for Q8: 12

Name _____

Subject _____



(c) Gradient at $x = 1.5$

$$= \frac{4.5 - 0.5}{3.5 - (-0.5)}$$

$$= \frac{4}{4}$$

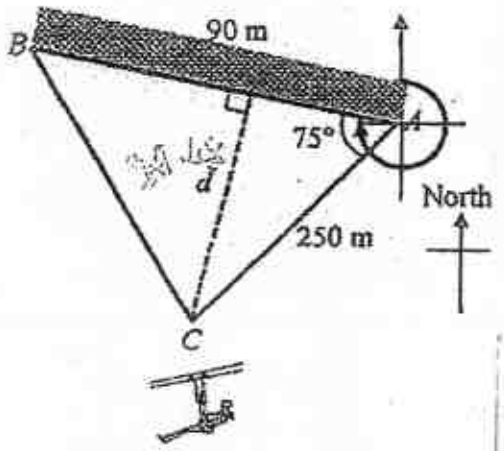
$$= 1$$

(d) $x = 0.35$

(e) $-0.5 \leq x < 1.4$

Trigonometry

9(a) Bearing of B from $A = 217° + 75° = 292°$



(b) Area of the land formed by the points A, B and C

$= \frac{1}{2}(90)(250)\sin 75°$

$= 10866.67$

$= 10900\ \text{m}^2$ (3 s.f.)

(c) Shortest distance from C to the bottom of the cliff

$= d$

$= 250\sin 75°$

$= 241.48$

$= 241\ \text{m}$ (3 s.f.)

(d) Slope is greatest when angle of elevation is the greatest from C.

Distance away from B

$= 90 - 250\cos 75°$

$= 25.295$

$= 25.3\ \text{m}$ (3 s.f.)

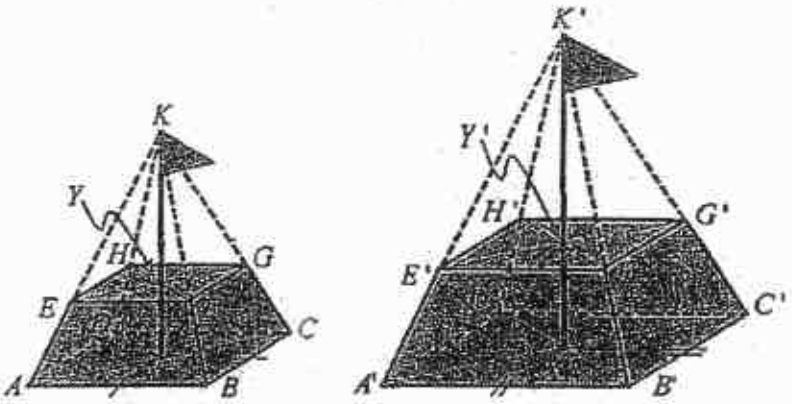
(e) Required angle

$= \tan^{-1}\left(\frac{50}{241.48}\right)$

$= 11.7°$ (1 d.p.)

Total for Q9: 9

Mensuration and similarity involving areas and volumes



10 (a)	$\frac{\text{area of } ABCD}{\text{area of } A'B'C'D'} = \left(\frac{3}{5}\right)^2$ $\frac{\text{area of } ABCD}{2500} = \left(\frac{3}{5}\right)^2$ $\Rightarrow \text{area of } ABCD = \frac{9}{25} \times 2500 = 900\,\text{cm}^2$	
(b)	$\frac{K'Y'}{K'X'} = \frac{40}{50} \Rightarrow K'Y' = \frac{4}{5} \times 2.5 = 2\,\text{m}$ $\text{Volume of the base} = \frac{1}{3} \times 2500 \times 250 - \frac{1}{3} \times 1600 \times 200$ $= 101666\frac{2}{3}\,\text{cm}^3$	
(c)	$\frac{\text{volume of smaller base}}{\text{volume of bigger base}} = \left(\frac{3}{5}\right)^3$ $\frac{\text{volume of smaller base}}{101666\frac{2}{3}} = \left(\frac{3}{5}\right)^3$ $\Rightarrow \text{volume of smaller base} = 21960\,\text{cm}^3$	
(d)	1 cm^3 of the smaller base costs $\$\frac{15}{21960} \approx \$0.000683$ 1 cm^3 of the bigger base costs $\$\frac{25}{101666\frac{2}{3}} \approx \$0.000246$ Since 1 cm^3 of the bigger base costs cheaper, the bigger-sized flagpole base is more value for money.	
		Total for Q10: 9

PRWC – Arc length and area of sector, segment

Details of the stage

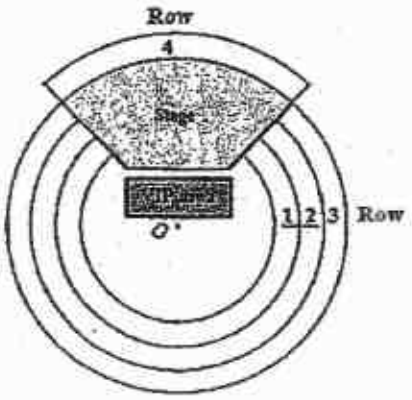


Figure 1

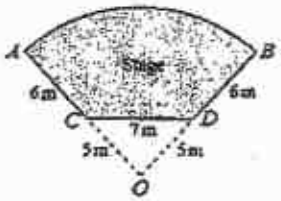


Figure 2

11 (i)

$$\angle COD = \cos^{-1}\left(\frac{5^2+5^2-7^2}{2(5)(5)}\right)$$

$$= \cos^{-1}\left(\frac{1}{50}\right)$$

$$= 1.5508 \text{ rad}$$

$$= 1.55 \text{ rad (3 s.f.)} \quad \text{(shown)}$$

$$\text{Area of stage} = \frac{1}{2}(11)^2(1.5508) - \frac{1}{2}(5)(5)\sin 1.5508$$

$$= 81.326 \text{ m}^2$$

$$= 81.3 \text{ m}^2$$

(ii)

Length of first row

$$= (5+3)(2\pi - 1.5508)$$

$$= 37.859 \text{ m}$$

Number of normal concert seats that can be accommodated in the first row $= \frac{37.859 \text{ m}}{0.80 \text{ m}} = 47.324 \approx 47$ (shown)

(iii)

Row	Length of row	No. of normal concert chairs	Cost of earnings
1	37.859 m	47	47×20=940
2	44.958 m	56	56×15=840
3	52.056 m	65	65×12=780
4	12.5(1.5 =19.38		24×25=600
Total earnings if package A taken up			$3160
Total earnings if package B taken up			$2560

Total number of normal concert chairs needed
= 47 + 56 + 65 + 24 = 192

Package	Cost of renting VIP chairs	Cost of renting normal concert chairs	Cost of rehearsals
A	5×18×1.07 = $96.30	(192−150)×8 ×1.07 = $359.52	100+80+90 = $270
	Total cost for using package A **= $2800 + 96.30 + 359.52 + 270** **= $3525.82**		
B	20×20×1.07 = $428	(192−100)×12 ×1.07 = $1181.28	120×3 = $360
	Total cost for using package B **= $1500 + 428 + 1181.28 + 360** **= $3469.28**		

Profit after taking up package A
= $3160 − 3525.82
= −$365.82

Profit after taking up package B
= $2560 − 3469.28
= −$909.28

Although package B seems cheaper than package A, taking into consideration the earnings, package A has a smaller loss than package B. Thus Peter and his committee should take up package A.

Assumptions:
- Other factors are not taken into consideration. The decision is made purely based on the profit made.

Total for Q11: 10

TANJONG KATONG SECONDARY SCHOOL

Preliminary Examination 2017
Secondary 4

CANDIDATE NAME

CLASS

INDEX NUMBER

MATHEMATICS

4048/01

Paper 1

Friday 18 August 2017

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use

This document consists of 16 printed pages.

[Turn over

Mathematical Formulae

Compound Interest

$$\text{Total Amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r\ell$

Curved surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

For Examiner's Use

For Examiner's Use

Answer all the questions.

1 Calculate $\sqrt[3]{(-3.01)^2 + 2.8}$.

(a) Write down the first five digits on your calculator display.

Answer (a) _____ [1]

(b) Write your answer to part **(a)** correct to 3 decimal places.

Answer (b) _____ [1]

2 These are the first four terms of a sequence.

42 34 26 18

(a) Write down the eighth term in the sequence.

Answer (a) _____ [1]

(b) Write down an expression, in terms of n, for the nth term in the sequence.

Answer (b) _____ [1]

3 Given that $81 \div 27^{\frac{n}{3}} = 9$, find n.

Answer _____ [2]

For Examiner's Use

For Examiner's Use

4 (a) Two integers, 12 and x, are related such that their highest common factor is 6 and their lowest common multiple is 60.
Find the value of integer x.

Answer (a) $x =$ _____ [1]

(b) Andy bought an external hard drive with storage of 1×10^{12} bytes.
A 5-minute-long high definition video takes up 7.2×10^{9} bytes.

Assuming he continues to record all his videos in high definition, what would be the total duration that can be stored in the external hard drive?
Give your answer to the nearest minute.

Answer (b) _____ minutes

[1]

For Examiner's Use

5 The angle, in degrees, of a quadrilateral $EFGH$ are represented by these expressions:
Angle $E = 40 + 2x$, angle $F = 100 - x$, angle $G = 60 + 6x$ and angle $H = 70 + 2x$.

(a) Calculate the value of x.

Answer (a) _____ [2]

(b) What is the name of the quadrilateral?

Answer (b) _____ [1]

6 The value of 200 homes at Mount Ace estate is shown below.

Value of homes ($\$x$)	Number of homes
$200\,000 < x \leq 300\,000$	24
$300\,000 < x \leq 400\,000$	16
$400\,000 < x \leq 500\,000$	85
$500\,000 < x \leq 600\,000$	67
$600\,000 < x \leq 3\,000\,000$	8

The mean value for the homes at Mount Ace estate is \$505 500.
Explain if the mean value is a fair representation for the value of homes at Mount Ace estate. Give your reason.

Answer _____

_____

_____

_____ [2]

[Turn over

For Examiner's Use

For Examiner's Use

7 (a) Factorise completely $8y^2z - 18z + 4x^2y^2 - 9x^2$.

Answer (a) _____ [2]

(b) Simplify $(-ab^{-1})^3 \div \frac{1}{2}a^3b^{-2}$, expressing your answer in positive index form.

Answer (b) _____ [3]

8 $\xi = \{\text{integers } x : 1 \le x \le 20\}$
$P = \{x : \text{prime numbers}\}$
$Q = \{x : 1 + 3x < 18\}$

(a) List the elements in

(i) Q,

Answer(a)(i) _____ [1]

(ii) $P \cap Q$.

Answer(a)(ii) _____ [1]

(b) Show that $P' \cap Q \neq \phi$.

Answer (b) _____

_____ [1]

For Examiner's Use

For Examiner's Use

9 Two geometrically similar bottles A and B have base areas of 27 cm^2 and 75 cm^2 respectively.
Given that the capacity of bottle A is 0.21 litres, find the capacity of bottle B.

Answer _____ *l* [3]

10 A group of 15 students took a Science test and their results are represented in the stem-and-leaf diagram below.

Stem	Leaf
5	3 4 6 7
6	2 2 4 9 9
7	1 3 7
8	0 2 x

5 | 3 represents 53 marks

(a) Given that the range of the Science test results is 32, find the value of x.

Answer (*a*) x = _____ [1]

(b) The passing mark for the Science test is 55. A student from this group is chosen at random. Find the probability that this student failed the test.

Answer (*b*) _____ [1]

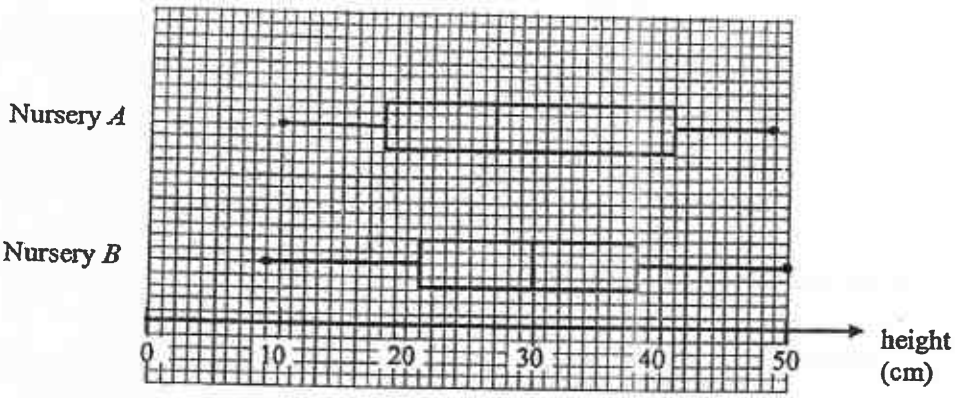
(c) Find the percentage of students who scored more than 75 marks.

Answer (*c*) _____ % [1]

For Examiner's Use

For Examiner's Use

11 The box plots below show the distribution of plants grown in two nurseries, *A* and *B*.



(a) Find the interquartile range for Nursery *A*.

Answer (a) _____ [1]

(b) For each of the statements below, write whether you agree or disagree. Give a reason for each answer, stating clearly which statistics you use to make your decision.

(i) On average, the plants in Nursery *A* grows taller than in Nursery *B*.

Answer _____ because _____

_____ [1]

(ii) A greater proportion of the plants grow above the height of 40 cm in Nursery *B* than do in Nursery *A*.

Answer _____ because _____

_____ [1]

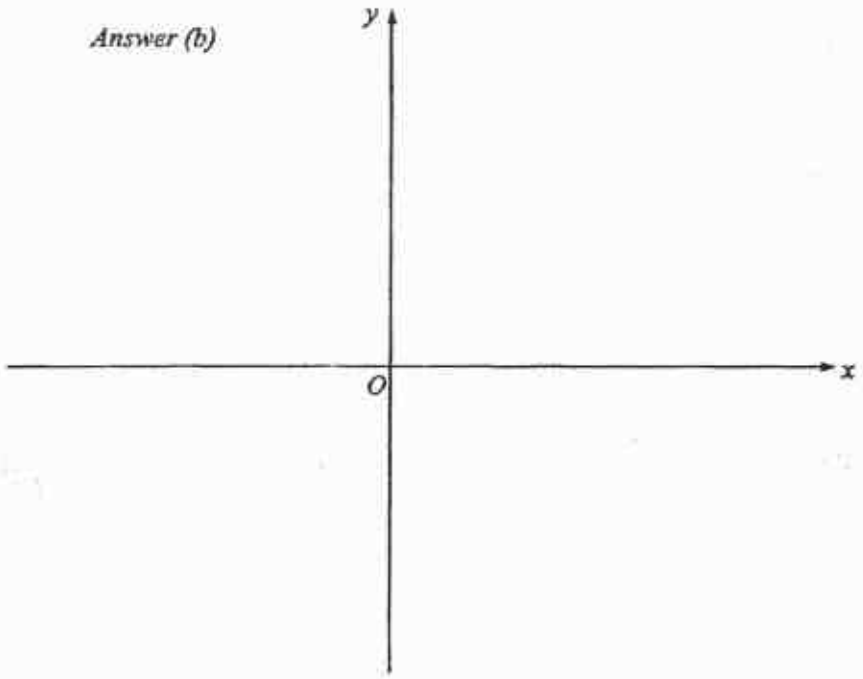
For Examiner's Use

12 (a) Express $x^2 - 6x + 4$ in the form $(x - a)^2 + b$.

Answer (a) _____ [1]

(b) Sketch the graph of $y = x^2 - 6x + 4$.

Answer (b) [2]



(c) The graph of $y = x^2 - 6x + 4$ is reflected in the y-axis. Write down the equation of the line of symmetry for the new graph.

Answer (c) _____ [1]

For Examiner's Use

For Examiner's Use

13 Mr Toh needs to tile his office floor which has an area of 60 square metres (sqm). Which company will offer a cheaper deal for Mr Toh? Justify your answers with calculations.

TIMBRE WORKS	TILE KING
$35 per sqm (for first 40 sqm) 30% discount thereafter	FLAT RATE $25 per sqm

Answer _____ [3]

14 Water is pumped into a cylindrical container at a constant rate such that x litres is pumped in t minutes. 144 litres of water is collected in the cylindrical container after 3 hours.

Find

(a) an equation for x in terms of t,

Answer (a) _____ [2]

(b) the time taken, in hours and minutes to fill a volume of 400 litres.

___ hours _____ minutes [2]

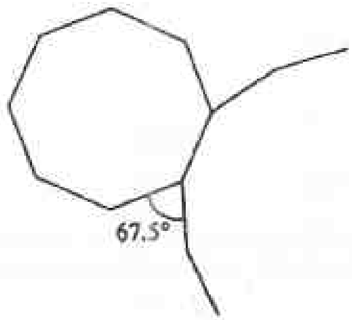
For Examiner's Use

15 **(a)** Explain whether it is possible to form a regular polygon with an interior angle of $125°$.

Answer (a) ____

____ [2]

(b) The diagram shows a sketch of a n-sided regular polygon and a regular octagon. Calculate n.



Answer (b) n = ____________________ [3]

For Examiner's Use

For Examiner's Use

16 Bag A contains three balls numbered 2, 3 and 4 respectively.
Bag B contains four balls numbered 1, 3, 5 and 7 respectively.
A ball is taken at random from each bag and their respective numbers f and g are recorded.

(a) Complete the table to show the possible outcomes for the sum of the two numbers f and g, on the balls selected.

		f, number on ball from Bag A		
		2	3	4
g, number on ball from Bag B	1			
	3			
	5			
	7			

[1]

(b) Find the probability that

(i) $f+g<7$,

Answer (b)(i) _____ [1]

(ii) $f+g$ is an odd number,

Answer (b)(ii) _____ [1]

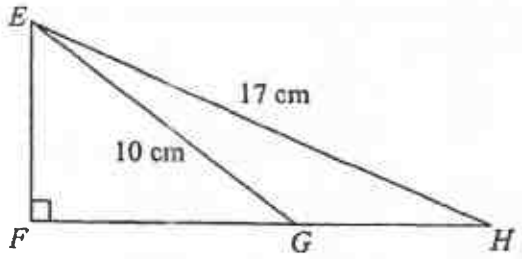
(iii) $f>g$.

Answer (b)(iii) _____ [1]

For Examiner's Use

For Examiner's Use

17 The figure shows triangle EFH where $EH = 17$ cm and $\angle EFH = 90°$
G is a point on FH such that $EG = 10$ cm.



(a) Given that $\sin \angle EGH = \frac{3}{5}$, find

(i) EF,

*Answer (a)(i)*_____ cm [1]

(ii) $\tan \angle EGH$.

*Answer (a)(ii)*_____ [2]

(b) Find the shortest distance from F to EH.

*Answer (b)*_____ cm [3]

(c) A circle C_1 is drawn passing through E, F and G.
A second circle C_2 is drawn passing through E, F and H.
Find the ratio of the circumference of C_1 to circumference of C_2,

Answer (c) _____ : _____ [1]

For Examiner's Use

For Examiner's Use

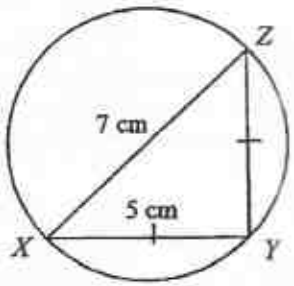
18 The mean, median and mode of the distribution of heights for 9 athletes are all equal to 165 cm.

Three of the athletes have a height of 165 cm and the tallest athlete is 170 cm.

Given that the heights of the athletes are integers, find the least possible height of the shortest athlete.

Answer _____ cm [3]

19 The diagram shows an isosceles triangle inscribed in a circle where $XZ = 7$ cm and $XY = YZ = 5$ cm. Determine whether XZ is a diameter of the circle.
Explain your answer.



Answer _____

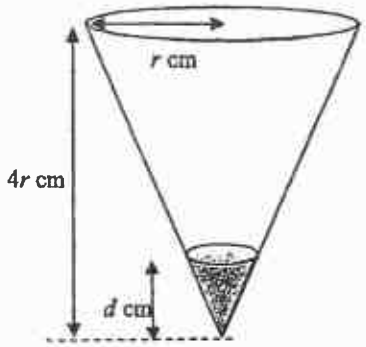
_____

_____ [2]

For Examiner's Use

For Examiner's Use

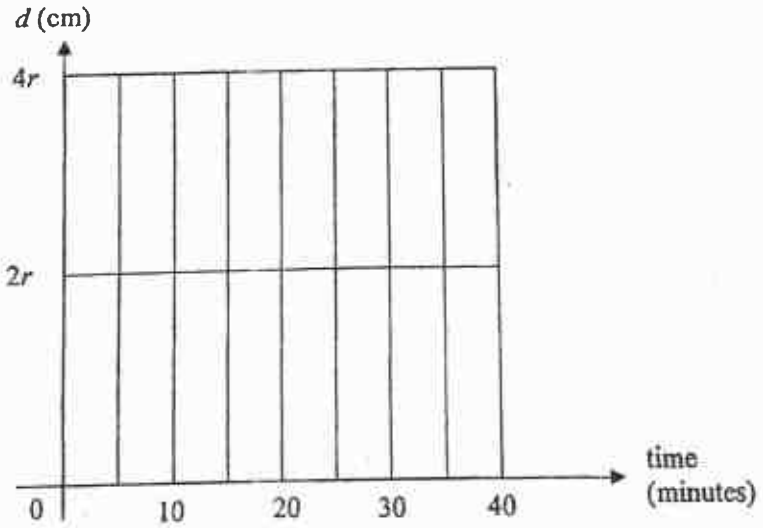
20 A container in the shape of an inverted cone has a top radius of r cm and a height of $4r$ cm. Water is poured into the container at a constant rate. It takes 40 minutes to fill the container completely with water.



(a) Calculate the time taken to fill the container to a height of $2r$ cm.

*Answer (a)*_____ minutes [2]

(b) A graph is drawn to show the relationship between the depth of the water, d cm, and the time taken, t minutes, as the container is filled. Complete the graph to represent how the depth of water changes with time.



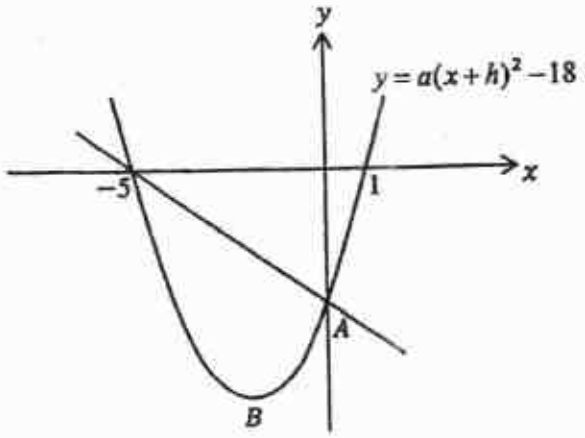
[2]

[Turn over

For Examiner's Use

For Examiner's Use

21 The diagram below shows a curve of $y = a(x+h)^2 - 18$.
The curve cuts the x-axis at -5 and 1 and the y-axis at A.
B is the minimum point on the curve.



(a) Express the equation of the curve in the form of $y = a(x+h)^2 - 18$, where a and h are constants.

Answer (a) $y =$ _____ [3]

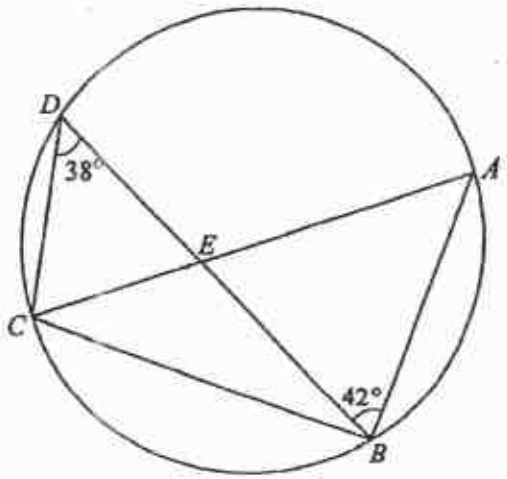
(b) A straight line cuts the curve at $x = -5$ and point A.
Find the equation of the straight line.

_____ [2]

For Examiner's Use

For Examiner's Use

22 The points A, B, C, and D lie on the circumference of a circle such that $\angle BDC = 38°$, $\angle ABD = 42°$ and $\angle ABC = 90°$. Chords AC and BD intersect at E.



(a) (i) Giving your reason, find angle ACD.

Answer (a)(i) _____ [1]

(ii) State whether EC is longer than ED. Give your reason clearly.

Answer (a)(ii) _____ [1]

(b) Describe where the centre of the circle is.

Answer (b) _____ [1]

For Examiner's Use

For Examiner's Use

23 The scale drawing shows the positions of two train stations, P and Q.
The scale is 1 cm to 10 km.

A third train station, R is 80 km from P on a bearing of 150°.

(a) Mark and label on the diagram the position of train station R. [1]

A train, T travels along a path which is equidistant from PR and RQ.

(b) Using ruler and compasses only, mark and label the path in which train T moves. [1]

(c) At a particular instant, the position of train T is such that it is equidistant from train stations P and Q. Using ruler and compasses only, mark and label the position of train T at that instant. [2]

(d) Train T approaches train station R at an average speed of 95 km/h. Calculate the time taken from its position in **(c)** to arrive at R. Give your answer in minutes.

Answer (d) _____ minutes [2]

END OF PAPER

TANJONG KATONG SECONDARY SCHOOL

Preliminary Examination 2017
Secondary 4

CANDIDATE NAME			
CLASS		INDEX NUMBER	

MATHEMATICS 4048/02

Paper 2 **Wednesday 23 August 2017**
2 hours 30 minutes

Additional Materials: Writing Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

[Turn over

This document consists of **11** printed pages and 1 blank page.

1 A soccer club offers annual memberships for both adults and juniors.
The adult annual membership fee is $150.
Junior members need to pay 80% of the adult annual membership fee.

(a) Calculate the discount each junior member receives. [1]

If an adult member does not pay the membership fee by the due date, the club will charge a penalty of 5% per month until the fee is paid.
Simon paid the $150 membership fee exactly two months after the due date.

(b) Calculate the penalty that Simon will be charged. [1]

The soccer club received a statement of the transactions in its saving account for the month of January 2017.

Date	Details	Deposit	Withdrawal	Balance
01 Jan 2017	Brought Forward			$63950.00
09 Jan 2017	Match Fees	$750.00		$64700.00
15 Jan 2017	Withdrawal		[redacted]	$42700.00
23 Jan 2017	Membership Fees	$3800.00		$46500.00
31 Jan 2017	Interest	$124.54		$46624.54

(c) (i) Calculate the withdrawal amount on 15 Jan 2017. [1]

(ii) Interest on the account is calculated on the minimum balance for the month and added to the account on the last day of the month.

What is the annual rate of interest for this account?
Write your answer, correct to one decimal place. [2]

(d) The soccer club plans to invest $120 000 in an account which pays compound interest at the rate of 2% per annum, compounded monthly.
Find the total amount that can be withdrawn at the end of 4 years. [2]

2 A toothpaste firm supplies tubes of toothpaste to 2 different stores.
The number of tubes of toothpaste supplied per delivery to each store, the sizes of the tubes and the number of deliveries made to each store over a year are shown below. [Turn over

		Number of tubes per delivery			Number of deliveries over a year
Size of tube		50 ml	75 ml	100 ml	
Name of store	Econ	400	300	400	2
	Prime	-	200	600	4

(i) Given that $\mathbf{T} = \begin{pmatrix} 400 & 300 & 400 \\ 0 & 200 & 600 \end{pmatrix}$, find the matrix product $\mathbf{S} = \mathbf{T}\begin{pmatrix} 50 \\ 75 \\ 100 \end{pmatrix}$. [1]

(ii) Describe what the elements in S represent. [1]

(iii) Write down two matrices such that the elements of their product under matrix multiplication would give the total number of tubes of toothpaste of each size supplied by the firm over a year. Find this product. [2]

---

3 (a) Solve the inequality $\frac{2p-1}{5} \leq \frac{3+p}{2}$. [2]

(b) Simplify $\frac{12x^2}{4y} \div \frac{6x^3}{y^4}$. [2]

(c) Simplify the expression $\frac{4w^2-36}{2w^2+7w+3}$. [3]

(d) (i) Express as a single fraction in its simplest form

$$\frac{2}{y+3} - \frac{3}{y-1}.$$ [2]

(ii) Solve the equation

$$\frac{2}{y+3} - \frac{3}{y-1} = 5.$$ [3]

---

4 (a) (i) Express 4536 as the product of its prime factors. [1]

(ii) Given that $\frac{4536}{k} = p^3$, where k and p are integers and p is as large as possible, find the values of k and of p. [1]

(iii) The lowest common multiple of two numbers is 4536.
The highest common factor of these two numbers is 126.
Both numbers are greater than 126.

Find the two numbers. [2]

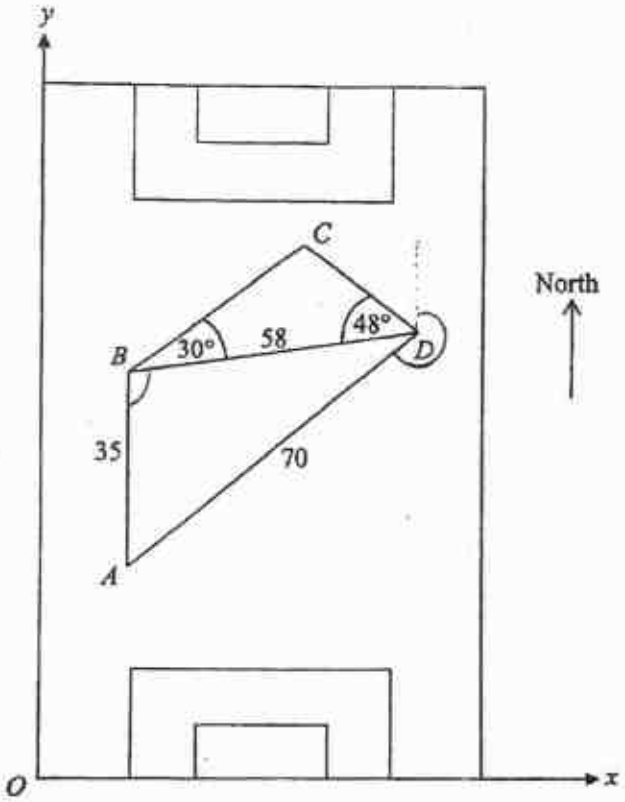
(b) When n is a positive integer, $2n+3$ is an odd number.

(i) Write down an expression for the next odd number greater than $2n + 3$. [1]

(ii) Find and simplify an expression for the difference between the squares of these two odd numbers. [2]

(iii) Hence explain why the difference between the squares of two consecutive odd numbers is always a multiple of 8. [1]

5



(a) During a soccer match a ball is passed from A to B and then from B to D as shown in the diagram. B is due north of A.
$AB = 35$ m, $BD = 58$ m and $AD = 70$ m.

(i) Show that angle $DAB = 55.7°$. [1]

(ii) Find the bearing of A from D. [1]

(iii) Calculate the area of triangle DAB. [2]

(b) Another player is standing at C.
Angle $CBD = 30°$ and angle $BDC = 48°$.
Calculate the length CD. [2]

(c) The x- and y- axes are shown in the diagram.
$\overrightarrow{AD} = \begin{pmatrix} p \\ q \end{pmatrix}$, where p and q are measured in metres.

(i) Show that $p = 57.8$. [1]

(ii) Find the value of q. [2]

6 (a) A has coordinates $(-3, 5)$ and $\overrightarrow{AB}$ is given by $\begin{pmatrix} -7 \\ -4 \end{pmatrix}$.

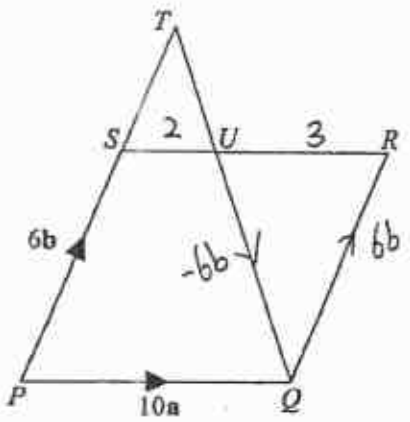
Find

(i) $|\overrightarrow{AB}|$, [1]

(ii) the position vector of B. [1]

(iii) Given that $\overrightarrow{CD}$ is parallel to $\overrightarrow{AB}$, and $\overrightarrow{CD} = \begin{pmatrix} k \\ 16 \end{pmatrix}$, find the value of k. [2]

(b)



$PQRS$ is a parallelogram.
$\overrightarrow{PS} = 6\mathbf{b}$ and $\overrightarrow{PQ} = 10\mathbf{a}$.

U is the point on SR such that $SU : SR = 2 : 5$.
When produced, PS and QU meet at T.

(i) Express each of the following, as simply as possible, in terms of **a** and/or **b**,

(a) $\overrightarrow{PR}$, [1]

(b) $\overrightarrow{SU}$, [1]

(c) $\overrightarrow{TU}$. [2]

(ii) Calculate the value of

(a) $\dfrac{\text{area of triangle } QRU}{\text{area of triangle } QUS}$, [1]

(b) $\dfrac{\text{area of triangle } SUT}{\text{area of triangle } PQT}$. [1]

7 **Answer the whole of this question on a sheet of graph paper.**

An **open** rectangular tank has a square base of side x metres.
The volume of the tank is 9 m^3.

(a) **(i)** Find an expression, in terms of x, for the height of the tank. [1]

(ii) Hence show that the total external surface area of the tank, A square metres, is given by

$$A = x^2 + \frac{36}{x}.$$ [1]

(b) The table below shows some values of x and the corresponding values of A.

x	2	2.5	3	4	5	6	7	8
A	22	20.7	21	25	32.2	42	54.1	p

(i) Find the value of p. [1]

(ii) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $2 \le x \le 8$.
Using a scale of 2 cm to represent 10 m^2, draw a vertical A-axis for $20 \le A \le 80$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

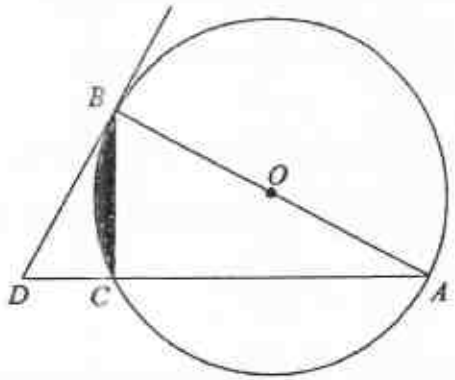
(iii) By drawing a tangent, find the gradient of the curve at the point where $x = 4$. [2]

(iv) Use your graph to find

(a) the value of x for which the surface area is 50 m^2. [1]

(b) the dimensions of the tank which has the least possible surface area. [2]

8 The diagram shows a circle, ABC, centre O.
BD is a tangent to the circle and it meets AC produced at D.



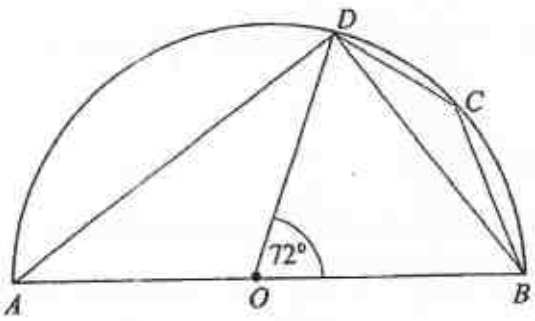
(a) Show that triangles ABD and BCD are similar. [2]

(b) Given that ratio area of triangle ABD : area of triangle $BCD = 4 : 1$ and the radius of the circle is 7.5 cm,

(i) show that angle $BAC = \frac{\pi}{6}$ radian, [2]

(ii) find the perimeter of the shaded region. [3]

(c) In the diagram, A, B, C and D are points on the circumference of a semi-circle, centre O.



(a) Calculate, stating your reasons clearly,

(i) angle DAB, [1]

(ii) angle ABD, [1]

(iii) reflex angle BCD. [2]

(b) Given that $OB = 3.5$ cm, find the area of the segment BCD. [3]

9 (a) The table shows the sizes of 50 pairs of ladies' shoes sold one day in a shoe shop.

Shoe sizes	5	6	7	7.5	8	8.5	9
Number of pairs of shoes sold	4	18	3	5	8	7	5

(i) Find the median shoe size. [1]

(ii) Find the modal shoe size. [1]

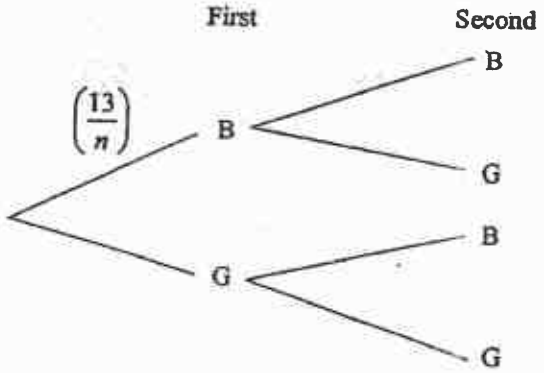
(iii) Explain which central measure would be the most appropriate and useful to the manager when she is ordering stock. [2]

(iv) Find the standard deviation of the shoe sizes. [1]

(v) The standard deviation of the shoe sizes of mens' shoes sold on the same day was 1.52.

Use this information to comment on one difference between the two distributions. [1]

(b) In a class of n students, 13 of them are boys and the rest are girls.
Two students are selected at random to represent the class at a conference.
The tree diagram shows the possible outcomes and their probabilities.



(i) Copy and complete the tree diagram. [2]

(ii) Find, as a single fraction in terms of n, the probability that

(a) the first student selected is a girl, [1]

(b) two boys are selected. [2]

(iii) The probability that two girls selected is $\frac{5}{18}$.

Find the total number of students in the class. [4]

10 Amos makes cookies.

The amount of dough needed to make one cookie is 8 grammes.

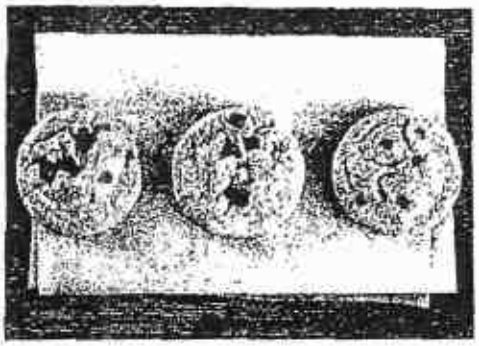
The density of the dough is 0.5333 g/cm^3.

(i) Find the volume of dough needed for each cookie. [1]

The dough is rolled into a sphere before baking.

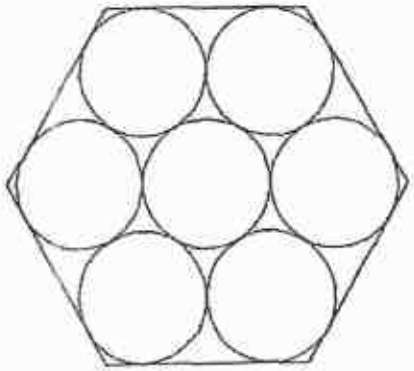
(ii) Calculate the radius of the sphere. [2]

When each cookie is baked, it forms a shape as shown.
The cookie can be modelled as a cylinder of radius 3 cm and a height of 0.7 cm.
The increase in volume is due to air trapped in the cookie.



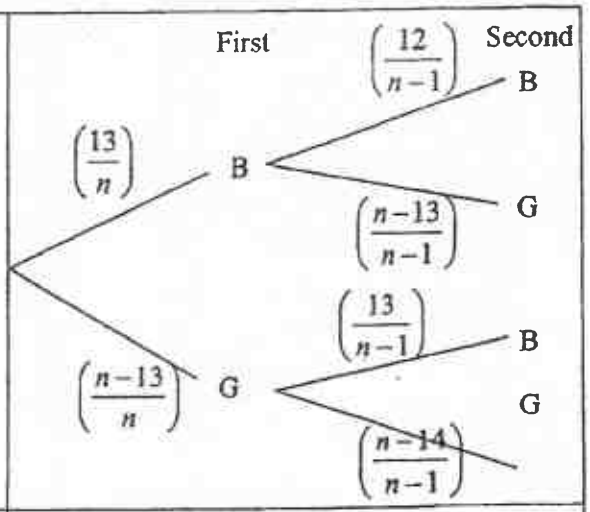
(iii) Calculate the volume of air trapped in the cookie. [2]

A regular hexagonal box is designed to hold 7 such cookies per layer, as shown.



(iv) Find the volume of the box if it is to hold five layers of cookies. [5]

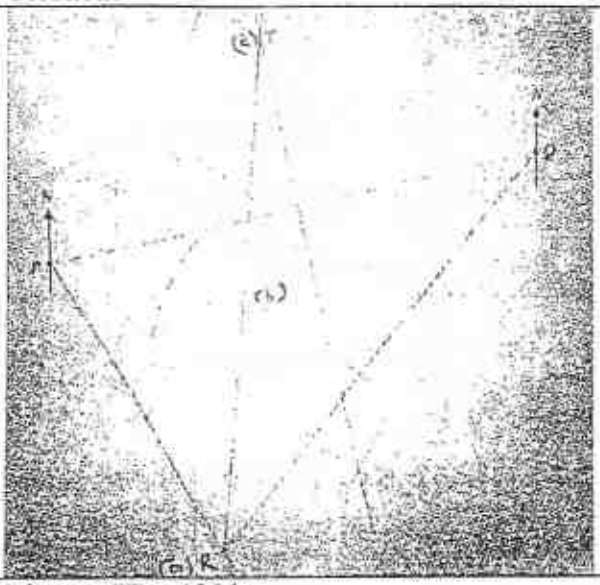
End of Paper

5	ai	$\cos D\hat{A}B = \dfrac{35^2 + 70^2 - 58^2}{2(35)(70)}$		bi	
	aii	235.7°		biia	$\dfrac{n-13}{n}$
	aiii	1011.97 m²		biib	$\dfrac{156}{n(n-1)}$
	b	$CD = 29.6$		biii	$n = 28$ or $n = 9$ (rej)
	ci	$\cos(90° - 55.7°) = \dfrac{p}{70}$	10	i	15.0 cm³
	cii	$q = 39.4$		ii	$r = 1.53$ cm
6	ai	8.06		iii	4.80 cm³
	aii	$\begin{pmatrix} -10 \\ 1 \end{pmatrix}$		iv	814.4745 cm³
	aiii	$k = 28$			
	bia	$\overrightarrow{PR} = 10\mathbf{a} + 6\mathbf{b}$			
	bib	$\overrightarrow{SU} = 4\mathbf{a}$			
	bic	$\overrightarrow{TU} = -4\mathbf{b} + 4\mathbf{a}$			
	biia	$\dfrac{3}{2}$			
	biib	$\dfrac{4}{25}$			

1	a	\$30	7	ai	$\frac{9}{x^2}$
	b	\$15		aii	$4x\left(\frac{9}{x^2}\right)$
	ci	\$22 000		bi	$p = 68.5$
	cii	3.5%		bii	All points correctly plotted Smooth curve drawn
	d	129985.79		biii	Draw tangent at $x = 4$ Grad = 6.38
2	i	$\begin{pmatrix} 82500 \\ 75000 \end{pmatrix}$		biva	$x = 6.8$
	ii	The element in S represent the total volume of toothpaste (in ml) supplied to Econ and Prime respectively.		bivb	Dimensions= 2.5 m × 2.5 m × 1.44 m
	iii	$\begin{pmatrix} 2 & 4 \end{pmatrix}\begin{pmatrix} 400 & 300 & 400 \\ 0 & 200 & 600 \end{pmatrix}$ $\begin{pmatrix} 800 & 1400 & 3200 \end{pmatrix}$	8	a	$\angle BCD = 90°$ (angles in semi-circle) $\angle ABD = 90°$ (tangent perpen. radius) $\therefore \angle ABC = \angle BCD$ $\angle BDC$ is common angle $\therefore \Delta ABD$ and ΔBCD are similar
3	a	$p \geq -17$		bi	$\frac{BD}{CD} = \frac{2}{1} \Rightarrow \frac{AB}{BC} = \frac{2}{1}$ Since radius = 7.5 cm AB =15 and BC = 7.5 cm $\sin B\hat{A}C = \frac{1}{2}$ $B\hat{A}C = \frac{\pi}{6}$ (shown)
	b	$\frac{y^3}{2x}$		bii	15.4 cm
	c	$\frac{4(w-3)}{2w+1}$		cai	$\angle DAB = 36°$ ($\angle$ at centre = 2 $\angle$ at circumference)
	di	$\frac{-y-11}{(y+3)(y-1)}$		caii	$\angle ABD = \frac{180-72}{2}$ (base $\angle$ of isos. Δ) $= 54°$
	dii	$y = 0.318$ or -2.52		caiii	216°
4	ai	$2^3 \times 3^4 \times 7$		cb	1.87 cm²
	aii	$k = 21$ $p = 6$	9	a i	7.25
	aiii	504 and 1134		ii	6
	4bi	$2n + 5$		iii	Mode will be the most appropriate and useful as the manager can stock up more shoes of size 6.
	4bii	$8n + 16$		iv	1.25
	4biii	$8(n + 2)$ is a multiple of 8 for n is a positive integer		v	The shoe sizes of ladies are more consistent than the men's shoe sizes.

For Examiner's Use

For Examiner's Use

Qn No.		Solutions
23	a	
	b	
	c	
	d	Distance TR = 125 km Time = 125 ÷ 95 = 78.9 minutes

For Examiner's Use

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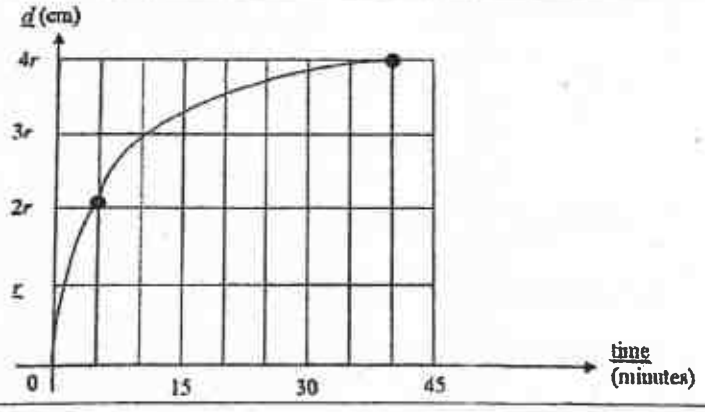
Qn No.		Solutions
	b(i)	$\frac{5}{12}$
	b(ii)	$\frac{2}{3}$
	b(iii)	$\frac{1}{3}$
17	a(i)	EF = 6 cm
	a(ii)	$\tan \angle EGH = -\frac{6}{8} = -\frac{3}{4}$
	b	5.61*cm*
	c	10 : 17
18		Least possible height = 150 cm
19		$XY^2 + YZ^2 = 5^2 + 5^2 = 50$ $XZ^2 = 7^2 = 49$ Since $XY^2 + YZ^2 \neq XZ^2$, XZY is not a right-angled triangle. Hence, XZ is not a diameter (Angle in semicircle).
20	a	Time taken = $40 \div 2^3$ = 5 mins
	b	
21	a	$y = 2(x+2)^2 - 18$
	b	Eqn: $y = -2x - 10$
22	a(i)	$\angle ACD = 42°$ (angles in same segment)
	a(ii)	$\frac{EC}{\sin 38°} = \frac{ED}{\sin 42°}$ Since $42° > 38°$, ED is longer than EC
	b	Given angle ABC = 90°, AC is a diameter of the circle (angle in semicircle) Centre is at midpoint of AC.

[Turn over

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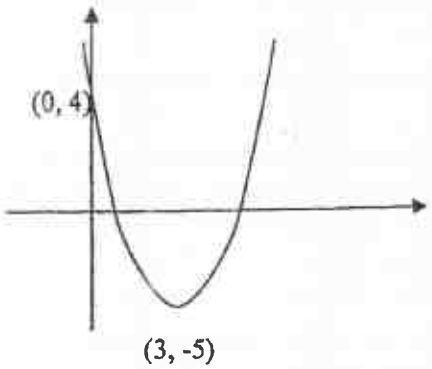
Qn No.		Solutions
	b	
	c	$x = -3$
13		Price for Timbre works $= (40)(35) + 0.7(35)(20)$ $= \$1890$ Price for Tile King $= 25(60)$ $= \$1500$ Tile king is cheaper
14	a	$x = \frac{144}{180}t$ $x = \frac{4}{5}t$
	b	$t = 8h\ 20\text{min}$
15	a	Ext angle of polygon = $180^\circ - 125^\circ = 55^\circ$ No of sides of polygon = $\frac{360}{55} = 6.545$ Since number of sides is not an integer, it is not possible to form a polygon with interior angle 125°.
	b	16 sides
16	a	

		f, number on ball from Bag A		
		2	3	4
g, number on ball from Bag B	1	3	4	5
	3	5	6	7
	5	7	8	9
	7	9	10	11

[Turn over

Answer Key:

Qn No.		Solutions
1	a	2.2804
	b	2.280
2	a	-14
	b	$-8n + 50$
3		$3^{4-n} = 3^2$ $n = 2$
4	a	$x = 2 \times 3 \times 5 = 30$
	b	694 min
5	a	$x = 10°$
	b	Kite
6		It is not a fair representation as - only 37.5% of the homes are valued above \$500,000 (majority of homes are valued less than \$505500) - the mean value is skewed by extreme values in the \$600,000 < x < \$3,000,000 group.
7	a	$(2z + x^2)(2y + 3)(2y - 3)$
	b	$\frac{-2}{b}$
8	a(i)	1, 2, 3, 4, 5
	a(ii)	2, 3, 5
	b	P' are not prime numbers. Since Q contains elements that are not prime, $P' \cap Q$ is not a null set. OR $P' \cap Q = \{1,4\}$ Hence, $P' \cap Q \neq \phi$
9		$V_{big} = 0.972l$
10	a	5
	b	$\frac{2}{15}$
	c	26.7%
11	a	23
	b(i)	Disagree because the median height in A is lesser than in B.
	b(ii)	Disagree because more than 25% of the plants in A grow to height greater than 40cm.
12	a	$(x-3)^2 - 5$

XINMIN SECONDARY SCHOOL
新民中学
SEKOLAH MENENGAH XINMIN
Mid-Year Examination 2017

CANDIDATE NAME

CLASS

INDEX NUMBER

MATHEMATICS 4048/1

Paper 1 **9 May 2017**

Secondary 4 Express / 5 Normal (Academic) **2 hours**

Setter : Ms Pang Hui Chin
Vetter : Mrs Vivien Tay
Moderator: Mrs Sabrina Phang

Additional Materials: Nil

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80.**

Errors	Qn No.	Errors	Qn No.
Accuracy		Simplification	
Brackets		Units	
Geometry		Marks Awarded	
Presentation		Marks Penalised	

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80

Parent's/Guardian's Signature:

This document consists of 22 printed pages. [Turn over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 (a) Factorise completely $3ac - 7c - 18ab + 42b$.

Answer (a) ..... [1]

(b) If $9x^2 + 30x + k$ is a perfect square, state the value of k.

Answer (b) $k =$ ..... [1]

2 Solve the inequality $-2 \leq 2x - 7 < 19$.

er ..... [2]

[Turn over

3 Evaluate, giving your answer in standard form,

(a) $\dfrac{17.31+13.13}{4.041\times\sqrt{898.9}}$,

Answer (a) ..... [1]

(b) $2\left(7.8\times10^{-1}\right)+\left(3.9\times10^{2}\right)$.

Answer (b) ..... [1]

4 Given that x is an integer such that $-4 \le x \le 3$ and y is a prime number such that $0 < y \le 7$, find

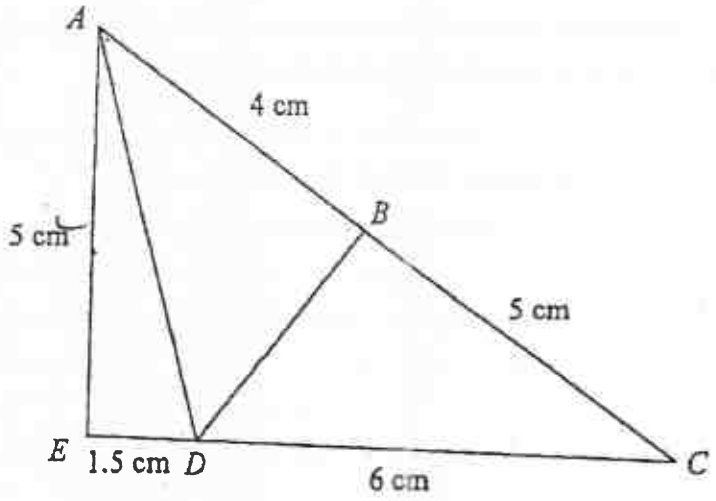
(a) the largest possible value of $\dfrac{x^2}{y}$,

Answer (a) ..... [1]

(b) the least possible value of $x^2 - y^2$.

Answer (b) ..... [1]

5 In the diagram, $AB = 4$ cm, $BC = 5$ cm, $CD = 6$ cm, $DE = 1.5$ cm and $AE = 5$ cm.



Show that triangles ACE and DCB are similar.

Answer In triangles ACE and DCB, ...

.....

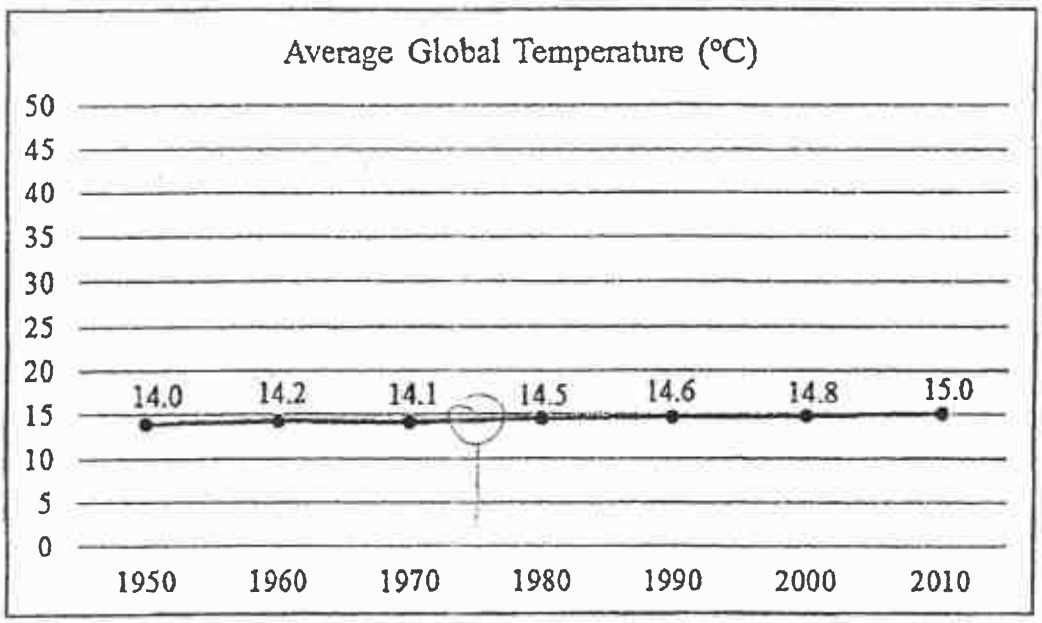
.....

..... [2]

6 Given that $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$, express v in terms of u and f.

..... [2]

7 An article in a newspaper reported the trend in the average global temperature from 1950 to 2010. The article contained the line graph shown below.



Can we determine the average global temperature in 1975 from the line graph? Explain your answer.

Answer .

[2]

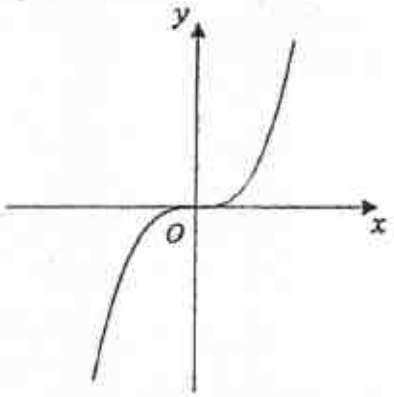
8 Solve $8^{3x-1} = 16$.

Answer $x =$ ..... [2]

9 The equations of the 2 graphs are in the form $y = x^n$.

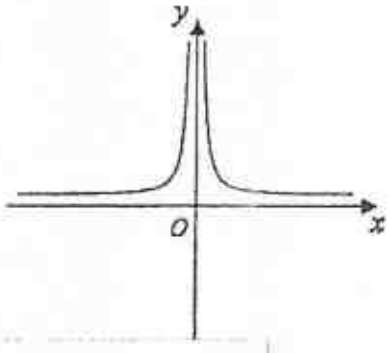
For each of the following, state a possible value of n.

(a)



Answer (a) $n =$ ..... [1]

(b)



Answer (b) $n =$ ..... [1]

10 Written as the product of its prime factors,

$$2160 = 2^4 \times 3^3 \times 5,$$
$$252 = 2^2 \times 3^2 \times 7.$$

(a) Find the smallest positive integer k such that $\frac{2160}{k}$ is a perfect cube.

Answer (a) $k =$ ..... [1]

(b) Write down the HCF of 252 and 2160 in index notation.

Answer (b) ..... [1]

11 The scale of a map is 2 cm : 0.4 km.

(a) Write this scale in the form 1 : n.

Answer (a) ..... : ..... [1]

(b) The actual area of a park is 4 km^2. Find the area, in square centimetres, of the park on the map.

..... cm^2 [2]

12 Solve the following simultaneous equations.

$$3x - 4y = 25$$
$$4x - 5y = 32$$

Answer $x =$ .....

$y =$ ..... [3]

13 In Singapore, Charlie pays $1.45 for 500 ml of bottled water.
When Charlie visited Japan, he paid ¥220 for 32 ounces of bottled water.

1 Singapore dollars = 77.96 Japanese Yen (¥)
1 ounce = 29.57 ml

Is bottled water cheaper in Singapore or in Japan?
You must show your calculations.

..... [3]

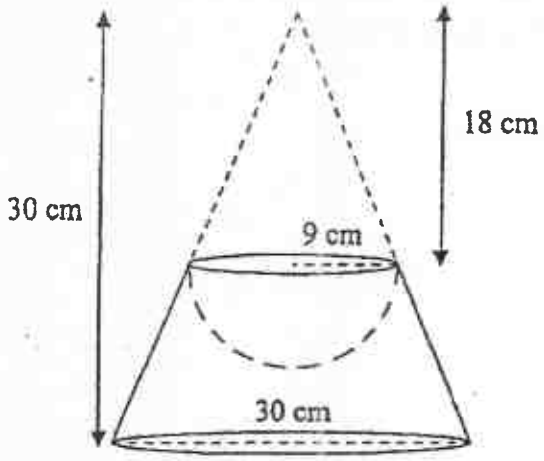
14 Simplify $\frac{1}{3-x}+\frac{3-x}{x^2-9}$.

Answer ..... [3]

15 Simplify $\left(\frac{25x^2y^0}{3x^0y^7}\right)^0 \times \left(\frac{3a}{2}\right)^{-3}$.

..... [3]

16 The diagram below shows a solid pet feeding bowl made from a truncated right circular cone with a hemispherical depression.



The truncated right circular cone is made by removing a cone with base radius 9 cm and and vertical height of 18 cm from a larger solid cone with a base diameter of 30 cm and a vertical height of 30 cm. The hemispherical depression has a radius of 9 cm.

The feeding bowl is to be made out of metal.
Calculate the volume of metal needed to make 10 of such feeding bowls, leaving your answer to the nearest whole number.

er ..... cm^3 [4]

17 Given that P is inversely proportional to $Q^2 + 1$ and that $P = 13$ when $Q = 1$,

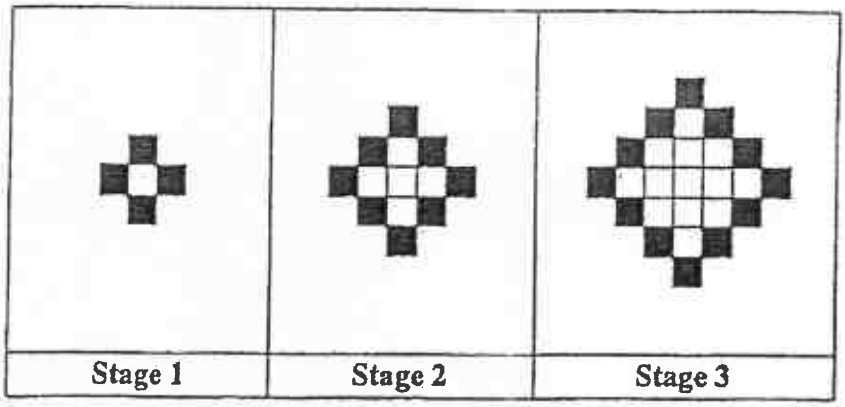
(a) express P in terms of Q,

Answer (a) ..... [2]

(b) find the values of Q when $P = 1$.

) Q = ... or ..... [2]

18 The diagram below shows a sequence of patterns made of squares of sides 1 unit each.



(a) Study the pattern and find the values of x and y.

Stage, n	Shaded area, S	Perimeter, P
1	4	12
2	8	20
3	12	28
4	x	y

Answer (a) $x =$ .....

$y =$ ..... [2]

(b) Express P in terms of n.

Answer (b) ..... [1]

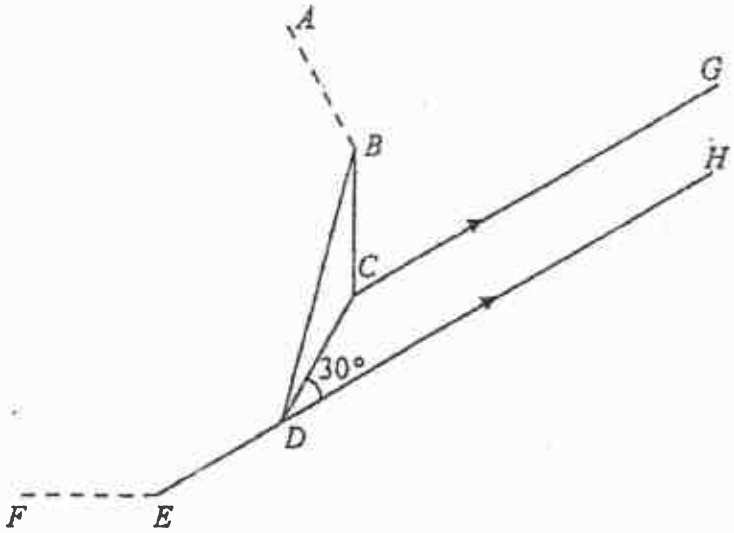
(c) Determine if the number 166 would appear in the P column, stating your reasons clearly.

Answer (c) .....

.....

.... [1]

19 14



In the diagram, $ABCDEF$ is an n-sided regular polygon with exterior angle $CDH = 30°$.
The lines CG and DH are parallel to each other.
Find

(a) the value of n,

Answer (a) $n =$ ..... [1]

(b) obtuse $\angle DCG$,

Answer (b) $\angle DCG =$ .....° [1]

(c) $\angle CBD$.

$\angle CBD =$ .....° [2]

20 $\xi = \{x : x \text{ is an integer such that } 40 \leq x \leq 50\}$

$A = \{x : x \text{ is a multiple of } 3\}$

$B = \{x : 2x + 5 < 99\}$

(a) Draw a Venn diagram to illustrate this information.

Answer (a)

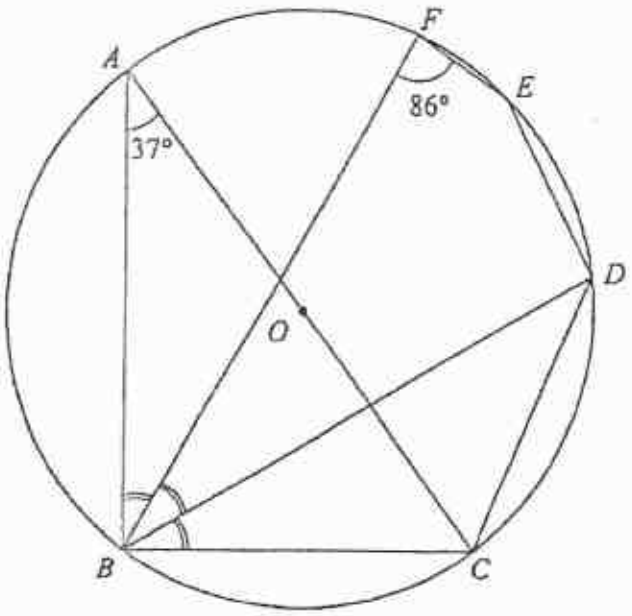
[2]

(b) List the elements of $A' \cap B'$ in set notation.

Answer (b) ..... [1]

(c) On your Venn diagram, shade the region which represents $A \cup B'$. [1]

21 In the diagram, A, B, C, D, E and F lie on a circle with centre O. AC is the diameter of the circle. $\angle ABF = \angle DBF = \angle CBD$.



If $\angle BAC = 37°$ and $\angle BFE = 86°$, find, giving reasons for each answer,

(a) $\angle ACB$,

Answer (a) $\angle ACB =$ ..... ° [2]

(b) $\angle DCA$,

Answer (b) $\angle DCA =$ ..... ° [1]

(c) $\angle FED$.

$\angle FED =$ ..... ° [1]

22 The staff of a company were asked about their monthly salary. The results are shown in the stem-and-leaf diagram.

1	010	050		
2	055	055	980	985
3	010	010	050	050
4	485	800	800	800
5	600	800	800	
6	750	750		
7				
8				
9				
10	999			

Key 3 | 010 means $3010

(a) Find the mean salary of the staff.

Answer (a) $ ..... [1]

(b) Find the median salary of the staff.

Answer (b) $ ..... [1]

(c) Does the mean or the median give a better representation of the salary of the staff in the company? Explain your answer.

Answer (c) .....

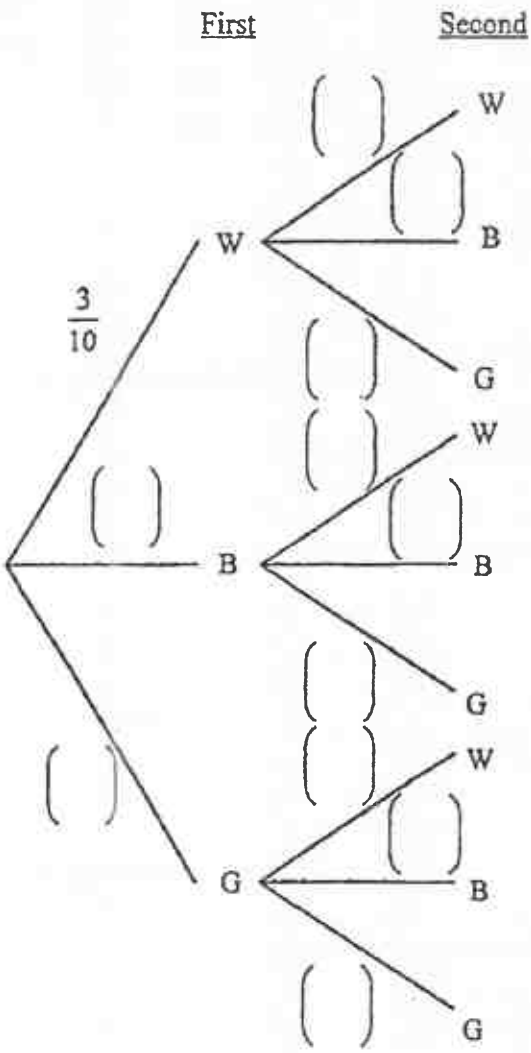
.....

.... [2]

[Turn over

23 3 pairs of white socks, 2 pairs of black socks and 5 pairs of grey socks are mixed and placed in a drawer. On a particular day, Yan Xin woke up late. He randomly snatched two socks from the drawer, put them on and rushed to school.

(a) Complete the following tree diagram to show this information.



[2]

(b) Find, in its simplest form, the probability that Yan Xin has taken

(i) a pair of socks of the same colour,

..... [2]

23 (b) (ii) a pair of socks of different colours,

Answer (b)(ii) ..... [1]

Please turn over for Question 24

24 (a) By completing the square, express $x^2 - 6x + 5$ in the form $(x-a)^2 - b$.

Answer (a) ..... [2]

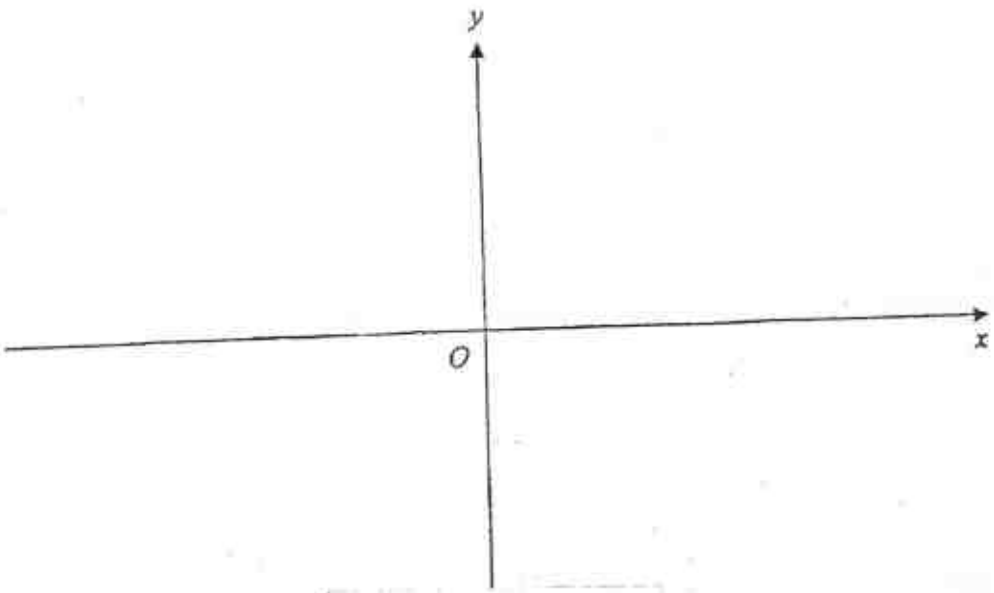
(b) Hence,

(i) solve the equation $x^2 - 6x + 5 = 0$,

Answer (b)(i) $x =$ ..... or ..... [2]

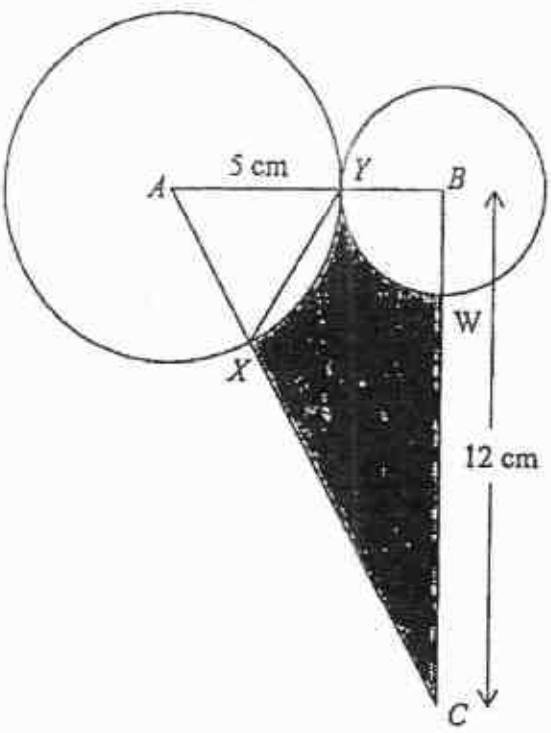
(ii) sketch the graph of $y = x^2 - 6x + 5$.

Answer (b)(ii)



[2]

25



In the diagram, ABC is a right-angled triangle such that two of its vertices A and B are the centres of two circles.

The minor arc length $WY = \frac{3\pi}{2}$ cm, $AY = 5$ cm and $BC = 12$ cm.

(a) Show that the length of BY is 3 cm.

Answer (a)

[1]

(b) Find the size of the angle XAY in radians.

Answer (b) $\angle XAY =$ ..... [2]

25 (c) Hence, find the area of the shaded region.

Answer (c) ..... cm^2 [3]

END OF PAPER

XINMIN SECONDARY SCHOOL
新民中学
SEKOLAH MENENGAH XINMIN
Mid-Year Examination 2017

CANDIDATE NAME

CLASS

INDEX NUMBER

MATHEMATICS 4048/2

Paper 2 **2 May 2017**

Secondary 4 Express / 5 Normal (Academic) **2 hours and 30 minutes**

Setter : Mr Bennett Lim
Vetter : Mrs Vivien Tay
Moderator: Mrs Sabrina Phang

Additional Materials: Writing Paper; Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100.**

Errors	Qn No.	Errors	Qn No.
Accuracy		Simplification	
Brackets		Units	
Geometry		Marks Awarded	
Presentation		Marks Penalised	

For Examiner's Use
100

Parent's/Guardian's Signature:

This document consists of 10 printed pages and 0 blank page. [Turn over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1. Solve the equation $\frac{3}{x-5}-5=\frac{2x}{3-x}$. [4]

---

2. The Hangzhou-Changsa High-speed Railway runs at a speed of 350 km/h and covers a distance of 933 km between the two cities.

   (a) Find the speed of the train in m/s. [2]

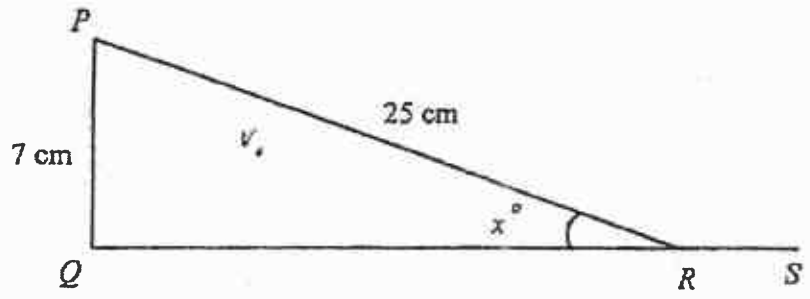
   (b) Calculate the time taken for the train ride, giving your answer in hours and minutes, correct to the nearest minute. [2]

---

3. (a) On 12 September 2013, Tyler invested some money in a bank that pays simple interest at a rate of 3% per annum. He received \$573.75 in total interest on 12 December 2015. How much money did Tyler invest in the bank? [2]

   (b) Tyler also invested \$12 000 in another bank that pays compound interest at a rate of 2.25% per annum compounded half-yearly. How much money will Tyler get back at the end of 5 years? [2]

4.



PQR is a right-angled triangle in which $\angle PRQ = x°$, $PQ = 7$ cm and $PR = 25$ cm. The point S lies on QR produced. Write down, as a fraction, the value of

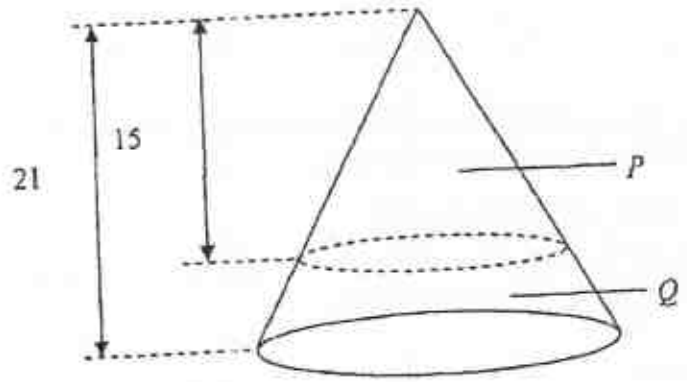
(a) $\cos\angle PRS$, [2]

(b) $\tan(90-x)°$, [1]

(c) $\sin(180-x)°$. [1]

[Turn over

5. The following diagram shows an inverted solid cone that is cut up into 2 sections, P and Q, such that section P is a cone similar to the original cone. The height of cone P is 15 cm and the height of the original cone is 21 cm.



(a) If the curved surface area of cone P is 250 cm^2, calculate the curved surface area of the original cone. [2]

(b) Calculate the ratio of the volume of the original cone to the volume of cone P. [1]

(c) If the volume of section Q is v cm^3, calculate the volume of cone P in terms of v. [2]

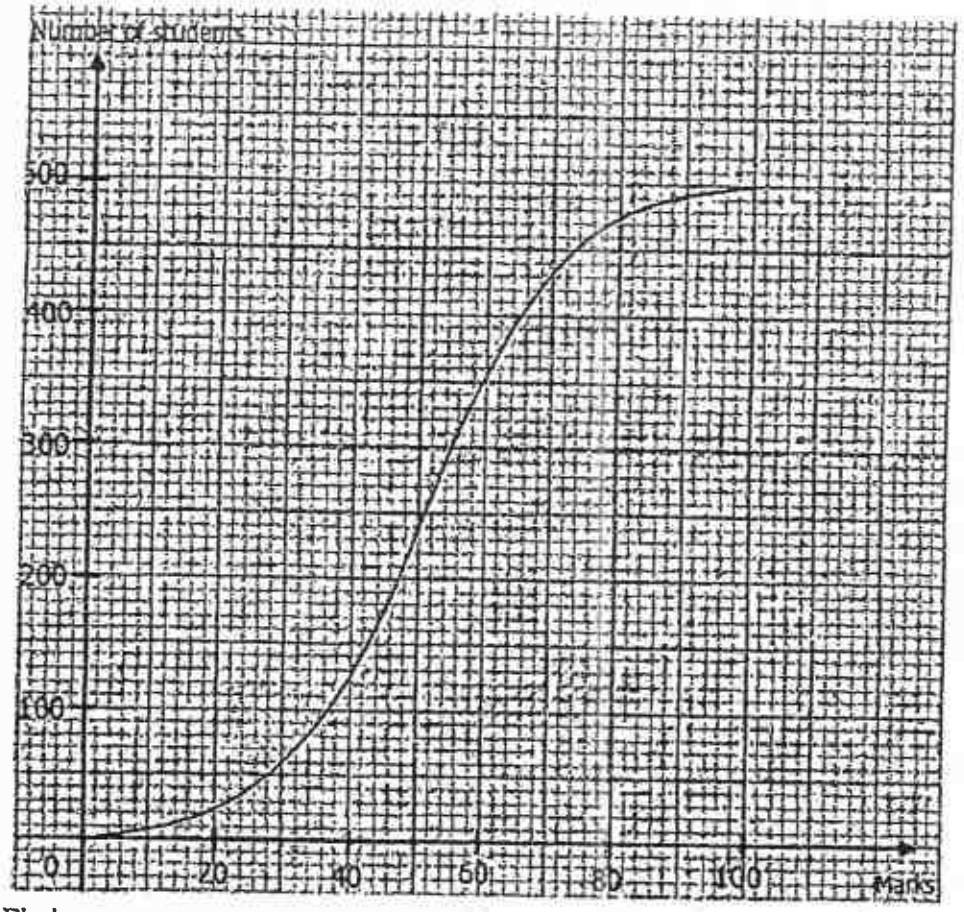
6. The position vector of point A is $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$.

(a) Find $\left|\overrightarrow{AB}\right|$. [2]

(b) Find the coordinates of B. [2]

(c) Given that $\overrightarrow{CD}$ is parallel to $\overrightarrow{BA}$ and $\overrightarrow{CD} = \begin{pmatrix} k \\ 13.6 \end{pmatrix}$, find the value of k. [3]

7. The cumulative frequency curve below illustrates the marks obtained, out of 100, by 500 students in XMSS Mid-Year Examination.



(a) Find

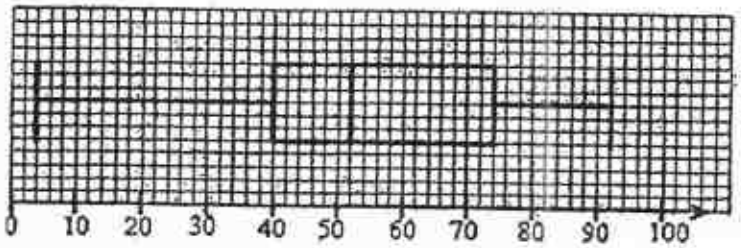
(i) the median mark, [1]

(ii) the interquartile range, [2]

(iii) the percentage of students who scored less than 50 marks. [2]

(b) Given that 15% of students scored a distinction, find the minimum marks students must score to get a distinction. [1]

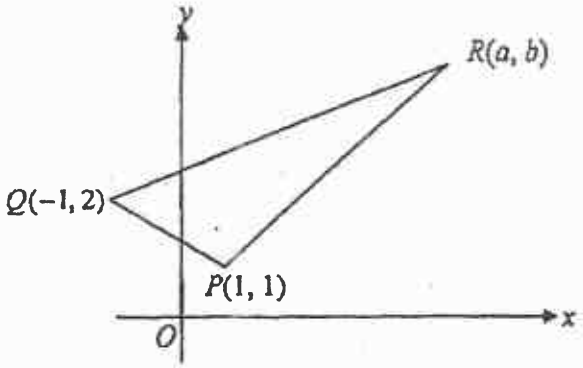
(c) The same 500 students sat for their Preliminary Examination. The box and whiskers diagram below illustrates the marks obtained.



(i) Which examination was more difficult? Give a reason for your answer. [1]

(ii) Which examination had more students scoring more than 70 marks? Explain your answer. [1]

8. The figure shows a triangle PQR with $P(1,1), Q(-1,2)$ and $R(a,b)$. The gradient of PQ, PR and QR are $-2n, 2n$ and n respectively.



Find

(a) the length of PQ, [2]

(b) the value of n, [2]

(c) the coordinates of R, [3]

(d) the equation of line QR. [2]

---

9. (a) It is given that $A = \begin{pmatrix} 2 & 2 \\ -4 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 5 \\ 0 & -1 \end{pmatrix}$.

Find

(i) matrix P if $P = B^2$, [1]

(ii) matrix Q if $A + 2Q = 2B$. [3]

(b) A tour agency records the weekly average number of tour packages to Japan and Korea sold in the months of May and June in 2016.
In May 2016, 25 Japan tour packages and 32 Korea tour packages were sold weekly. In June 2016, 30 Japan tour packages and 40 Korea tour packages were sold weekly. This information can be represented by the matrix

$$L = \begin{array}{c} \begin{matrix} \text{Japan} & \text{Korea} \end{matrix} \\ \begin{pmatrix} 25 & 32 \\ 30 & 40 \end{pmatrix} \end{array} \begin{matrix} \\ \text{May} \\ \text{June} \end{matrix}$$

It is assumed that there are 4 weeks in each month.

(i) The prices of the Japan and Korea tour packages in 2016 were \$690 and \$900 respectively. Represent the prices of the tour packages by a 2×1 column matrix N. [1]

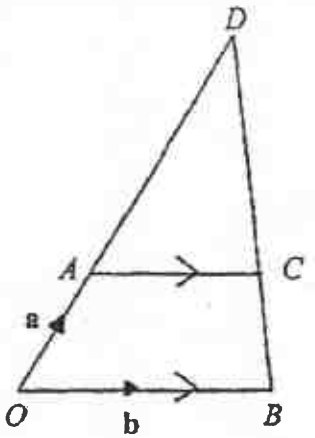
(ii) Evaluate the matrix $R = 4LN$. [2]

(iii) State what the elements of R represent. [1]

(iv) The tour agency decides to offer a discount on the tour packages bought in May and June 2017. The agency estimated a 30% increase and 60% increase in the sales of the Japan tour packages and Korea tour packages respectively compared to 2016.
By using matrix multiplication involving L, calculate the total estimated number of Japan and Korea tour packages sold weekly in May 2017 and June 2017 respec [2]

---

10. In the diagram, $OACB$ is a trapezium where AC is parallel to OB. The lines OA and BC are produced to the point D such that $\frac{OA}{AD} = \frac{1}{2}$.



(a) Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, express, as simply as possible, in terms of $\mathbf{a}$ and/or $\mathbf{b}$,

(i) $\overrightarrow{BD}$, [1]

(ii) $\overrightarrow{OC}$. [2]

(b) Given that $\overrightarrow{OE} = 3\mathbf{a} + 2\mathbf{b}$,

(i) state the name of the quadrilateral $ODEB$, [1]

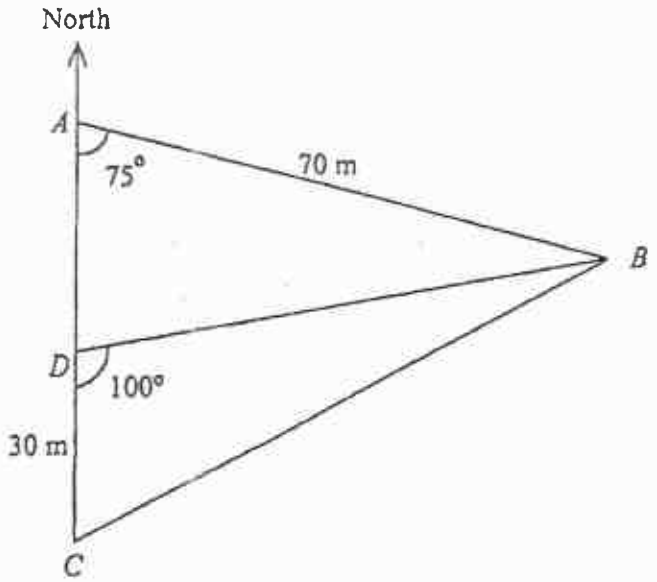
(ii) explain why O, C and E lie in a straight line. [2]

(c) Find

(i) $\dfrac{\text{area of } \Delta ADC}{\text{area of } \Delta ODB}$, [2]

(ii) $\dfrac{\text{area of } \Delta ODB}{\text{area of quadrilateral } ODEB}$. [3]

11. In a laser tag enclosure, A, B, C and D are points on level ground, with A due north of C and D. $\angle BAD = 75°$, $\angle BDC = 100°$, $AB = 70$ m and $CD = 30$ m.



(a) Show that the length of $BD = 68.66$ cm, correct to 2 decimal places. [2]

(b) Calculate

(i) the bearing of D from B, [1]

(ii) the length of CB, [2]

(iii) the area of ΔABD. [2]

In a game, Mario at point B ran along the path BA towards point A at a speed of 8 m/s. Sonic at the top of a 20-metre high guard tower at point D spotted Mario at point B.
He fired a shot that hit Mario when he was closest to the guard tower.
Assume that the time taken by the shot to hit the target from the time it was fired was negligible.

(c) Find

(i) the angle of depression of Mario from Sonic when the shot was fired, [3]

(ii) the time that elapsed from the instant Sonic spotted Mario at point B to the instant Sonic fired the shot. [2]

12. Answer the whole of this question on a sheet of graph paper.

The speed, v, in metres per second of a toy car on a race track propelled by a spring launcher is given by $v = 5 + 4t - t^2$, where t is the time in seconds. The table below shows the corresponding values of t and v.

t	0	1	1.5	2.5	4	5
v	5	8	8.75	8.75	5	0

(a) Draw the graph of $v = 5 + 4t - t^2$ for $0 \le t \le 5$. Use a scale of 2 cm to 1 s on the horizontal t-axis and 2 cm to 1 m/s on the vertical v-axis. [3]

(b) Use your graph to find the maximum speed reached by the car. [1]

(c) (i) By drawing a tangent, find the gradient of the graph at the point when $t = 3.5$ s. [2]

(ii) Use your answer to **c(i)** to explain what was happening to the car at $t = 3.5$ s. [1]

(d) (i) By adding a suitable line to your graph, solve $4t - t^2 - 2 = 0$. [4]

(ii) What do the solutions represent? [1]

---

13. Mr Mah is a motorcycle shop owner in Singapore who sells brand new motorcycles. He is interested in importing the brand new Kawasaki Z100SX motorcycle from Japan. The total costs to be incurred for importing the motorcycles to Singapore, include the amount payable to the manufacturer, shipping costs, government taxes and duty.

Information that Mr Mah needs is on the following page.
Mr Mah is interested in importing 20 motorcycles to sell.

(a) Calculate

(i) the cost of each motorcycle payable to the manufacturer, [1]

(ii) the shipping and insurance cost of each motorcycle. [2]

Mr Mah targets a profit of 15% of his total costs incurred.
Mr Mah needs to decide how much he should sell each motorcycle.

(b) Suggest a sensible selling price for each motorcycle.
Justify your proposed selling price with a **concluding statement.** [7]

[Turn over

Motorcycle Specifications	
Motorcycle Model	Kawasaki Z1000 SX
Year	2017
Weight	228 kg

Cost Payable to Manufacturer	
Price per Unit (S$)	S$18,250
Discount for purchases:	
> 9 units	2.5%
> 19 units	5.0%
> 29 units	7.5%

Shipping and Insurance Cost:	
Net weight (kg)	Cost (S$)
< 2,000	3,250
2,000 – 3,000	4,000
3,001 – 4,000	4,750
4,001 – 5,000	5,500
5,001 – 10,000	6,000
> 10,000	6,500

The following is extracted from the Singapore Land and Transport Authority (LTA[1]) website.

TAX STRUCTURE FOR MOTORCYCLES & SCOOTERS

Registration Fee[2] (RF)	S$140
Additional Registration Fee[3] (ARF)	Tiered Rate:

Vehicle OMV[4]	ARF Rate
First S$5,000	15%
Next S$5,000 (i.e. S$5,001 to S$10,000)	50%
Above S$10,000	100%

Excise Duty[5]	12% of OMV

[1] LTA is responsible for planning, operating, and maintaining Singapore's land transport infrastructure and systems.

[2,3] The RF and ARF are government taxes to be paid by the importer for the registration of the motorcycles for sale in Singapore.

[4] OMV (Open Market Value) – Refer [illegible] acturer of the motorcycle.

[5] It is a tax on the cost paid to the ma[illegible]

10) a) $k = 10$

b) $2^2 \times 3^2$

11 a) 1 : 20000

b) 100

12) $x = 3$, $y = -4$

13) Singapore

14) $\frac{2x}{(3-x)(x+3)}$

15) $\frac{8}{2a^3}$

16) 40150

17 a) $P = \frac{26}{Q^2+1}$

b) 5 or -5

18 a) $x = 16$, $y = 36$

b) $P = 8n + 4$

19) a) $n = 12$

b) 150°

c) 15°

20) b) 47, 49, 50

21) a) 53°

b) 60°

c) 150

22 a) $ 4241.95

b) 3767.50

23 bi) $\frac{33}{95}$

b) $\frac{56}{95}$

24 a) $(x-3)^2 - 4$

bi) 1 or 5

25 b) 0.983

c) 28.6

1a) $(3a-7)(c-6b)$

b) $k=25$

2) $2\frac{1}{2} \leq x < 13$

3a) 2.57×10^{-1}

b) 3.9156×10^{2}

4a) ~~44~~ 8

b) -49

6) $v = \frac{uf}{u-f}$

7) NO because the line segments between the dots have no meaning

8) $x = \frac{7}{9}$

9a) $n = 3, 5 \ldots$
any odd positive integer > 1

b) $n = -2, -4 \ldots$
any even negative integer

Q8a) $x > 4$ b) $n = \frac{1}{4}$

c) R(7, 4) d) $4y = x + 9$

9a) i) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ii) $\begin{pmatrix} 0 & 4 \\ 2 & -4 \end{pmatrix}$

bi) $N = \begin{pmatrix} 690 \\ 900 \end{pmatrix}$ ii) $\begin{pmatrix} 184 & 200 \\ 226 & 800 \end{pmatrix}$

iii) $\begin{pmatrix} 83.7 \\ 103 \end{pmatrix}$

10a i) $3a - b$

ii) $a + \frac{2}{3}b$

bi) Trapezium

ii) $\overrightarrow{OE} = 3\overrightarrow{OC}$

O, C, E collinear.

ci) $\frac{4}{9}$

ii) $\frac{1}{3}$

11a) 68.66 b) 260° ii) 79.6 iii) 1020 m^2

e) 34.6° ii) 7.78 s

Q1) $x = 4$ or 7

2) a) 97.2 m/s b) 2h 40min

Q3 a) $8500 b) $13 420.43

4a) $-\frac{24}{25}$, b) $\tan(90 - x) = \frac{24}{7}$

c) $\sin(180 - x) = \frac{7}{25}$

5a) $440\,cm^2$

b) $\frac{343}{125}$

c) $\frac{125}{216} V$

6a) 5 units b) $(-1, 9)$ c) $m = -3.4$
$k = -10.2$

7a i) 51 (ii) 22 iii) 48%

b) 69

ci) mid-year is more difficult as median mark is lower

ii) Prelim exam. It has more than 25% of students scoring 74m or more higher than mid year. & 25% scoring 62m or more

YUSOF ISHAK SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2017

CANDIDATE NAME

CLASS | INDEX NUMBER

MATHEMATICS 4048 / 01
4 Express / 5 Normal (Academic)

Paper 1

16th August 2017

Candidates answer on the Question Paper

2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

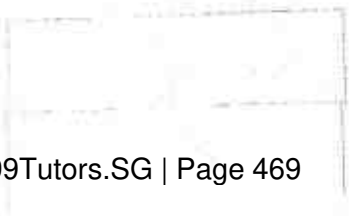
The total number of marks for this paper is 80.

For Examiner's Use

This document consists of 17 printed pages.

Setter: Mr Eric Koh

[Turn over



Answer **all** the questions.

1. Evaluate the following, leaving your answer correct to four significant figures.

$$\frac{-3.3^2 \times \sqrt{2^3}}{\left[1-8\left(7+7^{-1}\right)\right]^2} \times \sin\frac{\pi}{3}$$

Answer ..... [1]

2. The value of a house decreased by 14.3% between 2000 and 2016.
In 2000 the house was valued at $850 000.
Find its value in 2016.

Answer $..... [2]

3. A container is unloaded by 6 men in 24 minutes.
Given that all the men work at the same rate, find how long it would take 9 men to unload the same container.

Answer ..... minutes [2]

4. A car manufacturer states that a particular car
   - Uses 5 litres of fuel in travelling 100 km
   - produces 115 grams of CO_2 for each kilometer travelled.

   Use this information to calculate the mass of CO_2 produced by 1 litre of fuel.
   Give your answer in kilograms.

Answer ..... kg [2]

5. **(a)** Factorise completely $50p^2 - 72q^2$.

   (b) Solve the equation $\frac{x-2}{4} - \frac{x+1}{3} = 1$.

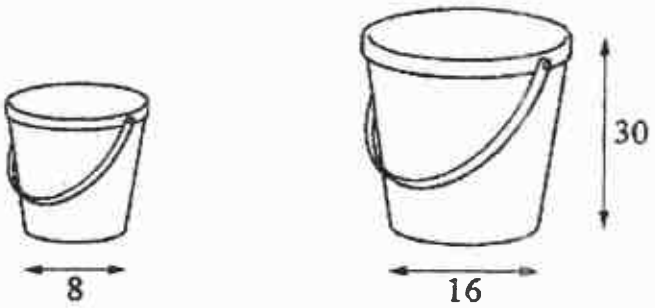
   (c) $T = 2\pi\sqrt{\frac{h}{g}}$. Make h the subject of the formula.

Answer (a) ..... [2]

Answer (b) ..... [2]

Answer (c) ..... [2]

6. Similar buckets are available in two sizes.
The larger bucket has height 30 cm and base diameter 16 cm.
The small bucket has base diameter 8 cm.



(a) Find the height of the small bucket.

(b) Given that the small bucket has volume 850 cm^3, find the volume of the large bucket.

Answer (a) ..... cm [1]

Answer (b) ..... cm^3 [2]

7. The temperature inside a greenhouse is p°C, and outside it is $-q$°C, where p and q are positive integers.
Write down an expression for
(a) the difference between the two temperatures,

(b) the mean of the two temperatures.

Answer (a) .....°C [1]

Answer (b) .....°C [1]

8. Green Line trains run every 10 minutes.
Red Line trains run every 20 minutes.
Purple Line trains run every 35 minutes.
One train from each Line leaves the city centre at 09 00.
After how many minutes will trains from all three Lines next leave the city centre in the same time?

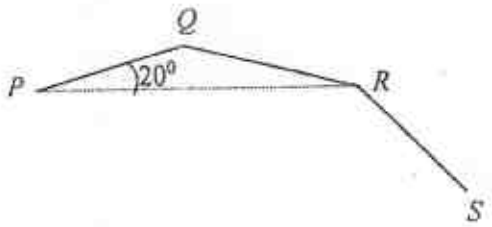
Answer ..... minutes [2]

9. PQ, QR and RS are adjacent sides of a regular polygon. Given that $\angle RPQ = 20°$, calculate

(a) the exterior angle of the polygon,

(b) the number of sides of the polygon,

(c) $\angle PRS$.



Answer (a) ..... [1]

(b) ..... [1]

(c) $\angle PRS$ = ... [1]

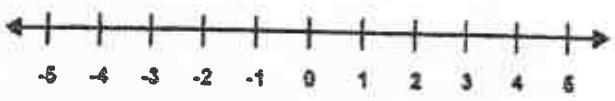
10. P is directly proportional to Q^2.

If Q is increased by 200%, find the percentage increase of P.

Answer .....% [2]

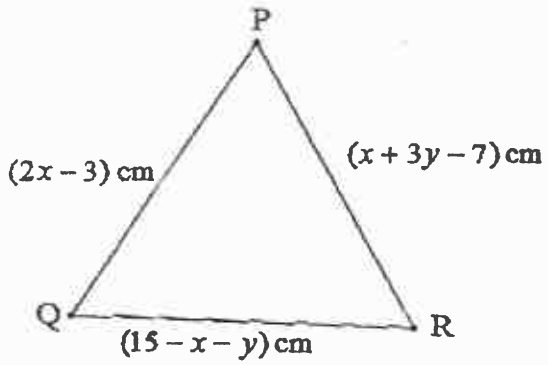
11. Solve the inequalities $\frac{10x+8}{3}+2<5+4x<8$.

Show your solution on the number line below.



[3]

12. The diagram shows an equilateral triangle PQR with $PQ = (2x - 3)$ cm, $QR = (15 - x - y)$ cm and $PR = (x + 3y - 7)$ cm.



(a) Using the information shown in the diagram, write down and simplify two simultaneous equations in x and y.

(b) Solve these equations to find the value of x and the value of y.

Answer (a) .....

.....[2]

(b) x = ... y = ... [2]

13. The information shows the common injuries children suffer in the United States of America (USA) in 2013.

(a) Explain **one way** in which the information is misleading.

Answer .....

.....

..... [2]

(b) Suggest **one recommendation** to overcome the misleading information provided.

Answer .....

.....

..... [1]

14. A map is drawn to a scale of 1 : 50 000.

(a) An airport runway is represented by a line of length 5.8 cm on the map. Calculate, in km, the actual length of the runway.

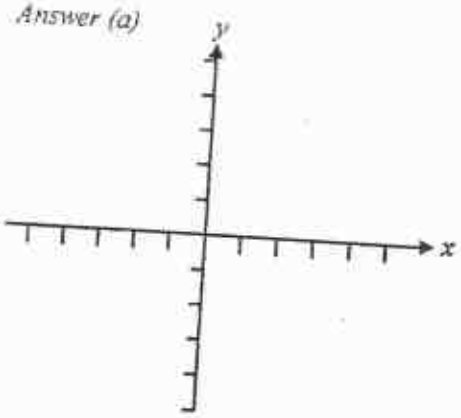
(b) The actual area of the airport is 6.5 km^2. Calculate, in square centimetres, the area on the map which represents the airport.

Answer (a) ..... km [1]

(b) ..... cm^2 [2]

15. (a) Sketch the graph of $y = (1 - x)(x - 3)$.

Answer (a)



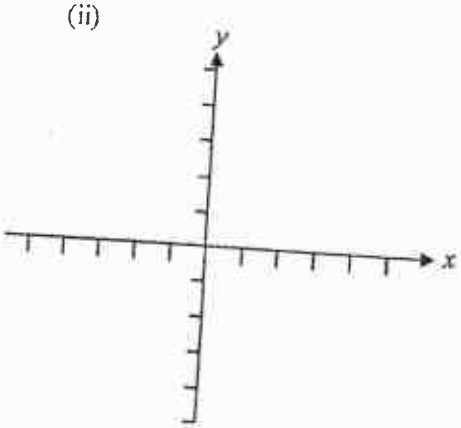
[2]

(b) (i) Express $x^2 - 4x + 5$ in the form $(x - a)^2 + b$.

(ii) Sketch the graph of $y = x^2 - 4x + 5$.

Answer (b)(i) $x^2 - 4x + 5 =$ ..... [1]

(ii)



[2]

16. A company produces three types of soft drinks in 2 different sizes.

The following matrices shows the weekly production, in thousands of litres and the cost per litre in cents, for producing soft drinks of any flavour in 2 different sizes.

$$\begin{array}{c} \\ \text{Regular} \\ \text{Large} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Raspberry} & \text{Orange} & \text{Lemon} \end{array} \\ \begin{pmatrix} 15 & 26 & 18 \\ 14 & 24 & 16 \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ \text{Cost} \end{array} \begin{array}{c} \begin{array}{cc} \text{Regular} & \text{Large} \end{array} \\ \begin{pmatrix} 45 & 60 \end{pmatrix} \end{array}$$

(a) Find $(45 \quad 60)\begin{pmatrix} 15 & 26 & 18 \\ 14 & 24 & 16 \end{pmatrix}$.

Answer (a) ..... [2]

(b) Explain what your answer to **(a)** represents.

Answer (b) .....

..... [1]

17.

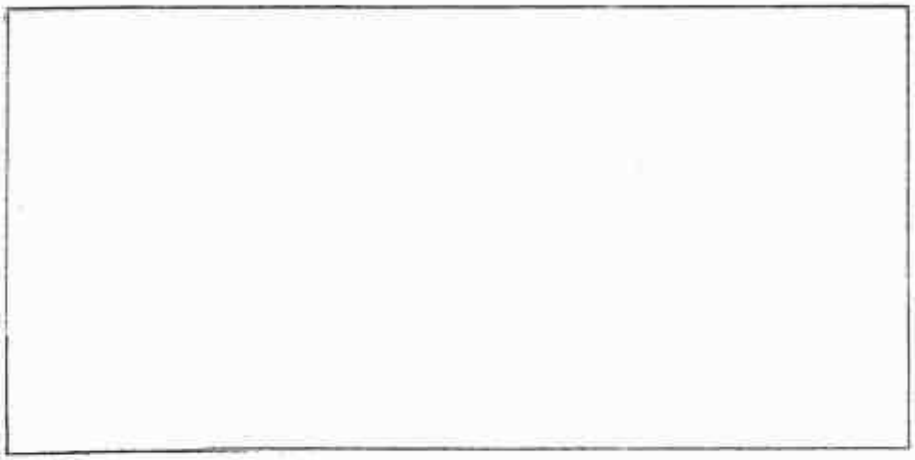
$\varepsilon = \{x : x \text{ is an integer and } 0 < x \leq 15\}$

$A = \{x : x \text{ is a prime number}\}$

$B = \{x : x \text{ is an integer divisible by } 3\}$

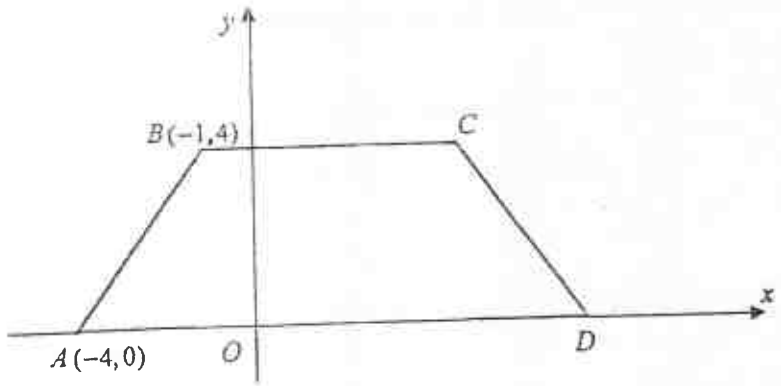
Draw a Venn diagram to illustrate this information, showing elements in each set clearly.

Answer ε



[2]

18. $ABCD$ is a trapezium in which $BC = 8$ units. A is the point $(-4, 0)$ and B is the point $(-1, 4)$. The area of the trapezium is 50 square units.



(a) Calculate the length of AB.

Answer (a)..... [1]

(b) Find the coordinates of C.

Answer (b) (....., ...) [1]

(c) Find the coordinates of D.

Answer (c) (....., ...) [2]

(d) Write down the value of $\cos \angle ABC$.

Answer (d) $\cos \angle ABC =$..... [1]

19. A production line produces loaves of bread with a mass of 500 grams each.
Two separate production lines, P and Q, were operated and 10 loaves were taken as samples from each line which had the following masses:

Line P 502, 487, 488, 490, 507, 500, 498, 491, 505, 490

Line Q 510, 501, 482, 489, 496, 506, 478, 489, 503, 492

(a) Find the mean mass of the products from both lines.

Answer (a) Line P..... [1]

Line Q ..... [1]

(b) Find the standard deviation of the product mass from both lines.

Answer (b) Line P..... [1]

Line Q ..... [1]

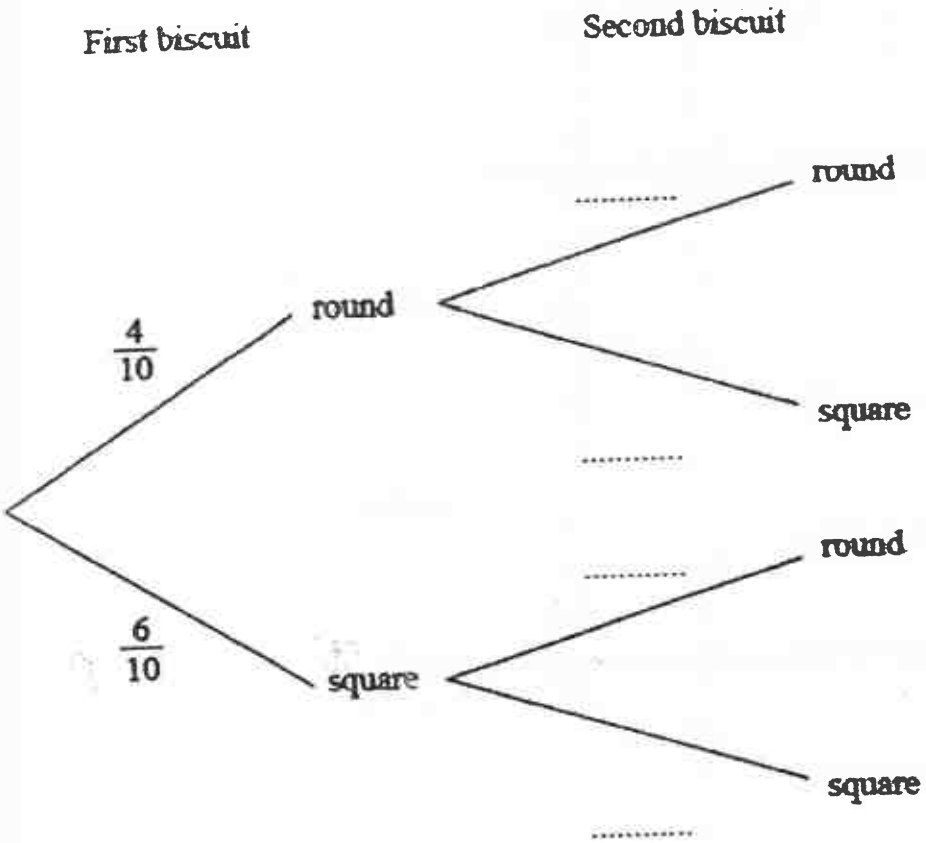
(c) If a loaf from each line is picked at random and each weighs 480 grams and 485 grams respectively, which line did the lighter loaf likely to come from?
Justify your decision with explanation.

Answer .....

.....

..... [2]

20. On a plate there are ten biscuits.
Four of the biscuits are round and six of the biscuits are square.
Joe chooses a biscuit at random from the plate and eats it.
He then chooses another biscuit at random from the plate.
The tree diagram shows the possible outcomes and some of the probabilities.



(a) Complete the tree diagram. [2]

(b) Calculate the probability that Joe chooses

(i) two round biscuits,

(ii) one round biscuit and one square biscuit.

Answer (b)(i) ..... [1]

(b)(ii) ..... [2]

21. (a) Simplify the expression $(3x^2y)^3 \times (5x^{-3}y^4)^{-1}$, giving your answer in positive index notation.

Answer (a) ..... [2]

(b) Solve $\left(\frac{1}{8}\right)^{-\frac{2}{3}} \times 32^{\frac{3}{5}} = 2^{p-2} \div 2^2$.

Answer (b) p = ... [2]

(c) Express the number 0.0040589 in standard form.

Answer (c) ..... [1]

22.

(a) Given that $p = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $q = \begin{pmatrix} m \\ 2 \end{pmatrix}$, find

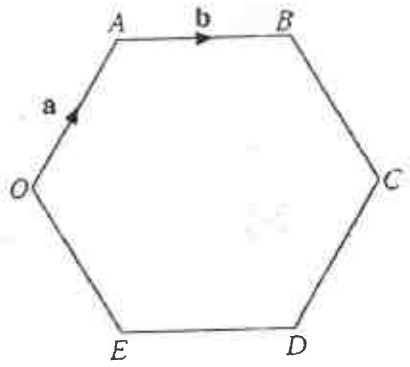
(i) $|p|$,

(ii) the value of m such that $p + q$ is parallel to the y-axis.

Answer (a)(i) .....units [1]

Answer (a)(ii)..... [1]

(b) In the diagram, $OABCDE$ is a regular hexagon. $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{AB} = \mathbf{b}$.



(I) Express the following vectors, as simply as possible, in terms of **a** and **b**.

(i) $\vec{OC}$,

(ii) $\vec{BC}$,

(iii) $\vec{AD}$.

(II) What type of quadrilateral is $ABCD$? Justify your answer using vectors.

Answer (b)(I)(i) ..... [1]

(ii) ..... [1]

(iii) ..... [1]

Answer (II)..... [1]

23. All construction lines must be clearly shown.

(a) Construct, and label clearly, the quadrilateral $ABCD$ in which $AB = BC = CD$, $\angle ABC = 70°$ and $\angle BAD = 100°$.
The line AB has been drawn for you. [2]

(b) On the quadrilateral, construct
(i) the bisector of angle ABC, [1]

(ii) the perpendicular bisector of the line BC. [1]

(c) The two bisectors in **(b)** intersect at the point P. Measure and write down the length of BP, in cm, correct to 1 decimal place.

Answer (c) ..... [1]

End of Paper

YUSOF ISHAK SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2017

CANDIDATE NAME

CLASS | | | | INDEX NUMBER | | |

Mathematics
4 Express / 5 Normal Academic

4048/02

Paper 2

18 August 2017

2 hours 30 minutes

Additional Materials: Answer paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of 12 printed pages.

Setter: Mr Eric Koh

[Turn over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where is } \theta \text{ in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

[3]

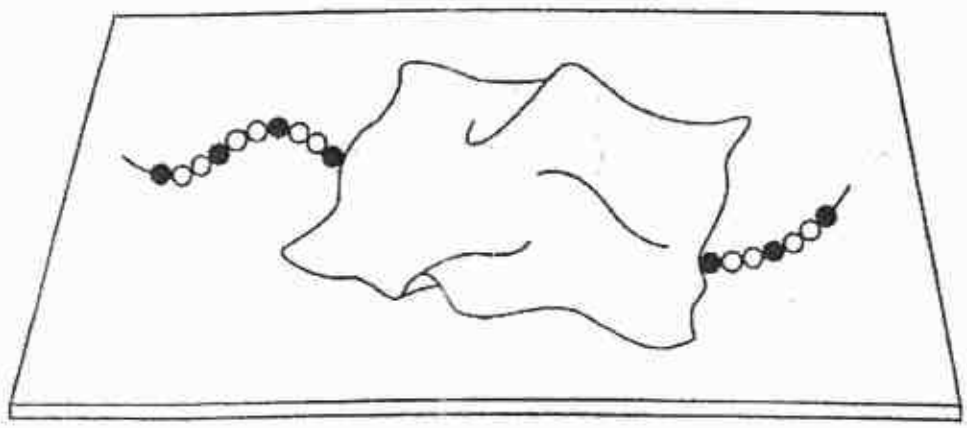
1. (a) Solve the equation $(1+4x)^2 = 81$. [2]

(b) Express as a single fraction in its simplest form $\frac{1}{2x+3} + \frac{3}{2x-1}$. [2]

(c) Find the integers x such that $2x+1 < 9 < 3x+1$. [2]

(d) Factorise completely $a^2 + 9b^2 - 6ab - 2a + 6b$. [2]

2. (a) A string of beads on a table is partly covered by a piece of cloth as shown.
There are 2 white beads between every 2 black beads.
Altogether, there are 14 black beads.
John guessed that the number of white beads was 28.
Do you agree? Justify your decision with calculations. [3]



(b) It is given that $3b = 4a$ and $2c = 5a$.

(i) Find $a:b:c$. [2]

(ii) If $a+b+c = 10$, find b. [3]

[4]

3. John bought x light bulbs for \$25.

(a) Write down an expression in terms of x for the price, in dollars, he had paid for each light bulb.

(b) He wanted to sell each light bulb at a profit of 50 cents. [1]
Show that his selling price for each light bulb was $\$\frac{50+x}{2x}$. [1]

(c) John managed to sell 8 light bulbs at this price. Write down an expression, in terms of x, for
(i) the total amount of money, in dollars, he had received for selling the 8 light bulbs. [1]
(ii) the number of light bulbs that remained unsold. [1]

(d) John sold the remaining light bulbs at \$2 each.
Write down an expression in terms of x for the total amount of money, in dollars, he had received from selling these light bulbs. [1]

(e) John received \$46 altogether.
Form an equation in x and show that it reduces to $x^2 - 29x + 100 = 0$. [3]

(f) Hence or otherwise, find the number of light bulbs John had bought. [3]

[5]

4.

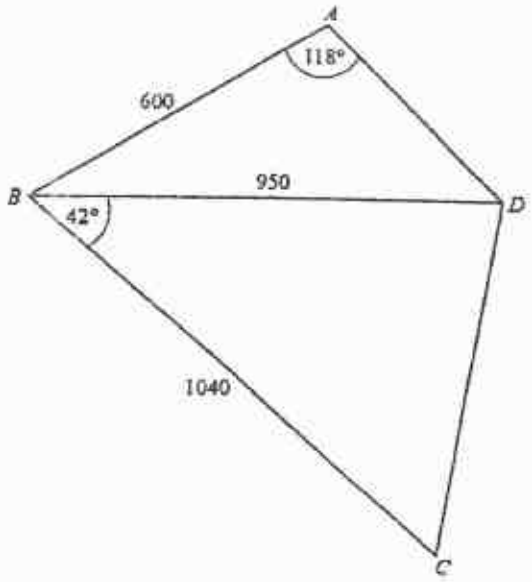


Figure 1

Figure 1 shows the quadrilateral $ABCD$. Quadrilateral $ABCD$ represent a level enclosed area for the rabbits with a path BD.
$AB = 600$ m, $BC = 1040$ m, $BD = 950$ m and $\angle CBD = 42°$ and $\angle BAD = 118°$.

(a) Calculate

(i) $\angle ABD$, [4]

(ii) the length of CD, [4]

(iii) the shortest distance from C to BD. [2]

(b) An eagle is flying directly above the path BD at a height of 500 m.
Calculate the greatest angle of depression of the point C as seen by the eagle. [2]

[6]

5. *P*, *Q*, *R*, *S* and *T* are the different shaped blocks of ice stored in the refrigerated enclosed room.

(a) At 10 p.m. on Monday the cooling system failed, and the blocks started to melt.
At the end of each 24 hour period, the volume of each block was 14% less than its volume at the start of that period.

(i) Block *P* has a volume of 7500 cm^3 at 10 p.m. on Monday.
Calculate its volume at 10 p.m. on Wednesday. [2]

(ii) Block *Q* had a volume of 6490 cm^3 at 10 p.m. on Tuesday.
Calculate the volume at 10 p.m. on the previous day. [2]

(iii) **Showing your working clearly,** find on which day the volume of *R* was half its volume at 10 p.m. on Monday. [2]

(b) At 10 p.m. on Monday, Block *S* was a hemisphere with radius 18 cm.
Calculate

(i) its volume, [2]

(ii) its **total** surface area. [2]

(c) As block *T* melted, its shape was always **geometrically similar** to its original shape.
It had a volume of 5000 cm^3 when its height was 12 cm.
Calculate its height when its volume was 1080 cm^3. [2]

[7]

6. Figure 2A shows the cross-section of an underground train tunnel.

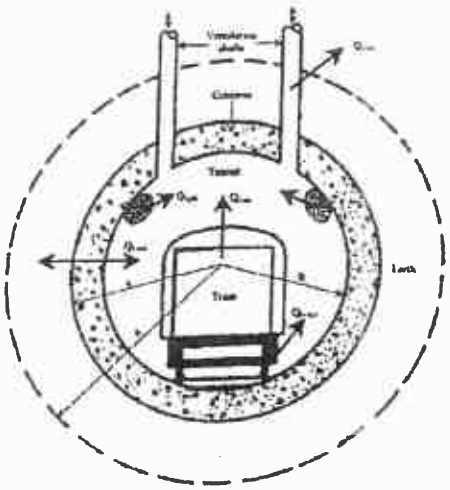


Figure 2A

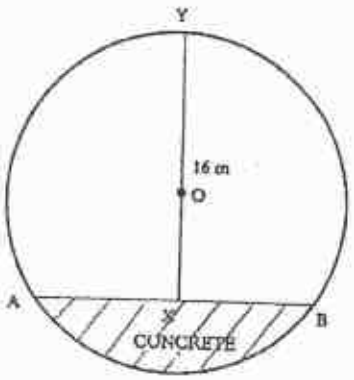


Figure 2B

With reference to Figure 2B.
AB represents the horizontal track surface, where the shaded region beneath it is covered with concrete.
Arc AYB represents the metal ceiling of the tunnel.
O is the centre of the circle with radius r metres.
X is the midpoint of AB and its vertically below Y.
Given that $AB = XY = 16$m.

(a) Calculate

(i) the value of r. [3]

(ii) $\angle AOX$, [1]

(iii) the volume of concrete used for the tunnel, given the tunnel is 900 m long. [3]

(b) A similar model of the tunnel is made. The radius of the model's cross-section is 5 cm.
Calculate the curved surface area of the model's ceiling. [3]

(c) A 130 metre long train travelling at a speed of 50 km/h entered the tunnel.
Calculate the time, in minutes and seconds, needed for the train to completely travel out of the tunnel. [2]

[8]

7.

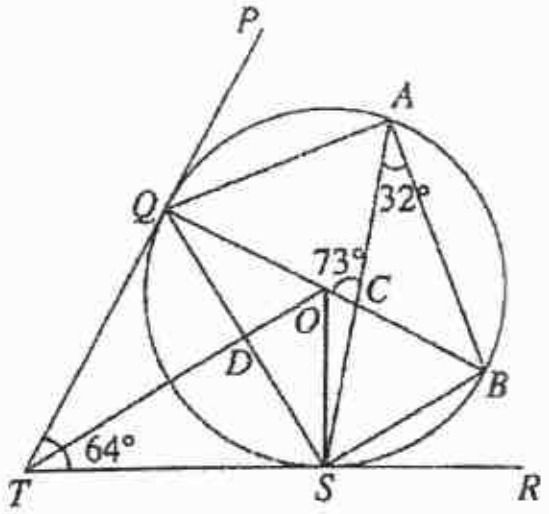


Figure 3

Figure 3 shows the circle $ABSQ$.
$ABSQ$ has centre O. TQP and TSR are tangents to the circle.
$\angle QTS = 64°$, $\angle SAB = 32°$ and $\angle ACQ = 73°$.

(a) Joseph commented that there are **at least** three right angles in Figure 3.
Justify his comment with workings and reasons. [3]

(b) Calculate

(i) $\angle SQB$, [1]

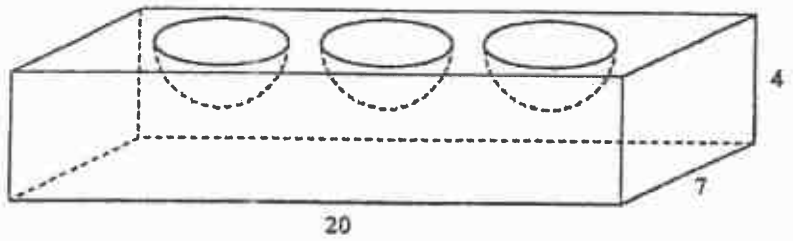
(ii) $\angle TOQ$, [2]

(iii) $\angle ABQ$, [2]

(iv) $\angle BSR$. [2]

[9]

8. A wooden cuboid has length 20 cm, width 7 cm and height 4 cm.
Three **hemisphere**, each of radius 1.5 cm, are hollowed out of the top of the cuboid, to leave the block as shown in the diagram.



(a) Calculate the volume of wood in the block. [2]

(b) The four vertical sides are painted pink.
Calculate the total area that is painted pink. [1]

(c) The inside of each **hemispherical** hollow is painted white.
The flat part of the top of the block is painted green.
Calculate the total area that is painted

(i) white, [1]

(ii) green. [1]

[10]

9. Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = 4x + \frac{60}{x} - 30$.

Some corresponding values of x and y are given in the following table.

x	1.5	2	2.5	3	4	5	7	8
y	16	a	4	2	1	b	6.6	9.5

(a) Calculate the values of a and b. [1]

(b) Using the scales of 2 cm to represent 1 unit of x and 1 cm to represent 1 unit of y, draw the graph of $y = 4x + \frac{60}{x} - 30$ for the range $1.5 \le x \le 8$. [3]

(c) From your graph, find

(i) the least value of y, [1]

(ii) the range of values of x for which $y = 4x + \frac{60}{x} - 30 < 8$. [2]

(d) Find, by drawing a tangent, the gradient of the curve when $x = 5$. [2]

(e) By drawing a suitable straight line on the same axes, find the solutions of the equation $3x^2 + 60 - 30x = 0$. [3]

10. All employees in Singapore have a compulsory savings known as the Central Provident Fund (CPF).
Each worker is required to save a certain percentage of what he earns each month with the CPF and the employer contributes another percentage of his salary to his CPF account.
The total CPF contribution is then kept into 3 accounts in the proportion as shown in the table below.

Contribution rates from 1 January 2016 for private sector and public sector non-pensionable employees being:

- Singapore Citizen
- SPR* from the third year of obtaining SPR status
- SPR during the first two years of obtaining SPR status but who has jointly applied with employer to contribute at full employer-full employee rates

*SPR (Permanent Resident)

Employee's age (years)	Contribution Rates from 1 Jan 2016 (for monthly wages ≥ $750)		
	By Employer (% of wage)	By Employee (% of wage)	Total (% of wage)
55 and below	17	20	37
Above 55 to 60	13	13	26
Above 60 to 65	9	7.5	16.5
Above 65	7.5	5	12.5

Figure 4A

Allocation rates from 1 January 2016 for private sector and public sector non-pensionable employees

Employee's age (years)	Allocation Rates from 1 Jan 2016 (for monthly wages ≥ $750)		
	Ordinary Account (% of wage)	Special Account (% of wage)	Medisave Account (% of wage)
35 and below	23	6	8
Above 35 to 45	21	7	9
Above 45 to 50	19	8	10
Above 50 to 55	15	11.5	10.5
Above 55 to 60	12	3.5	10.5
Above 60 to 65	3.5	2.5	10.5
Above 65	1	1	10.5

Figure 4B

In October 2016, Mr Ong who is 38 years old, earns $3000 a month, while his wife, who is 34 years old, earns $2000 a month.

(a) Calculate Mr Ong's contribution and his employer's contribution to his CPF account monthly. [2]

Both Mr Ong and his wife have just paid the 10% downpayment for their HDB flat which costs $400 000. They intend to pay the rest over a period of 20 years.

(b) Calculate how much they will have to pay per month for the 20 years. [2]

For a part of the amount they have to pay, the Ongs will use the money from both their Ordinary Accounts, and they will borrow the balance from a bank.

(c) Show that the amount from both their Ordinary Accounts to be used for the monthly payment of the flat is $1090. [2]

(d) Calculate the amount of money they have to borrow from the bank over the period of 20 years. [1]

The Ongs have to pay a simple interest rate of 1.48% for Year 1 and 1.58% thereafter.

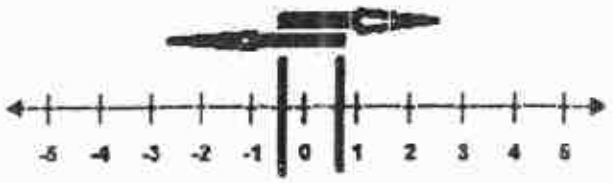
(e) Calculate the total amount they have to pay the bank after 20 years. [3]

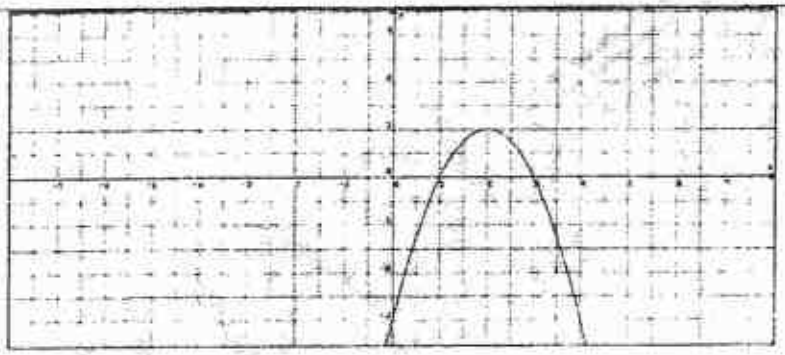
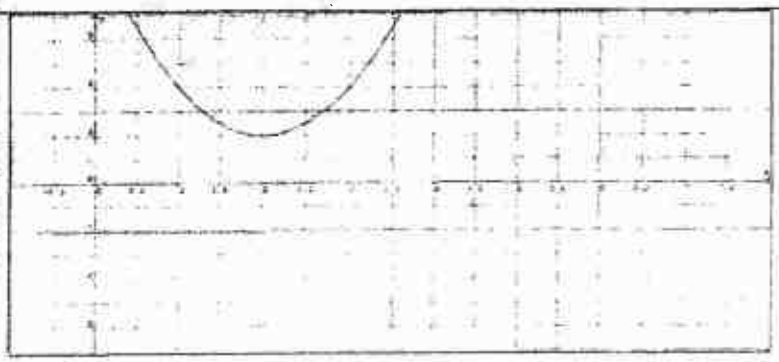
-End of Paper-

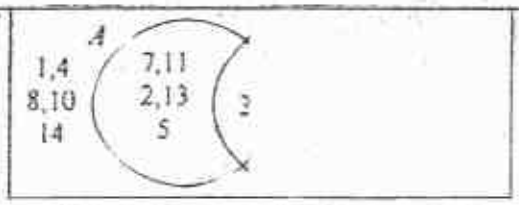
YUSOF ISHAK SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2017
MATHEMATICS PAPER 1
SEC 4E/5N

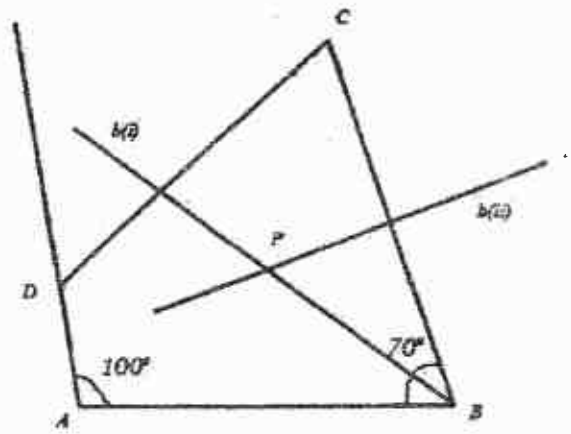
MARKING SCHEME

1	$\frac{-3.3^2 \times \sqrt{2^3}}{\left[1-8\left(7+7^{-1}\right)\right]^2} \times \sin\frac{\pi}{3} = -0.0084628$ [$\sin\frac{\pi}{3}$, radian mode] $= -0.008463$ (4 sig. figures)	Do not accept -0.0001785935 (Degree mode) B1 [1]
2	$\$850000 \times (100-14.3)\%$ $= \$728450$	M1 A1 [2]
3	24×6 16 minutes	M1 A1 [2]
4	1 litre = 20 km 20 km will emit $115 \times 20 = 2300$ grams of CO_2 2.3 kg	M1 A1 [2]
5(a)	$50p^2 - 72q^2$ $2(25p^2 - 36q^2)$ $2(5p-6q)(5p+6q)$	M1 A1 [2]
5(b)	$\frac{x-2}{4} - \frac{x+1}{3} = 1$ $\frac{3(x-2)}{12} - \frac{4(x+1)}{12} = 1$ $\frac{3x-6-4x-4}{12} = 1$ $-x-10 = 12$ $x = -22$	M1 A1 [2]
5(c)	$T = 2\pi\sqrt{\frac{h}{g}}$ $\left(\frac{T}{2\pi}\right)^2 = \frac{h}{g}$ $h = g\left(\frac{T}{2\pi}\right)^2$ or $h = \frac{gT^2}{4\pi^2}$	M1 A1 [2]
6(a)	As the two buckets are similar $\therefore \frac{\text{Height of small bucket}}{\text{Height of large bucket}} = \frac{8}{16}$ $\frac{\text{Height of small bucket}}{30} = \frac{8}{16}$ Height of small bucket $= \frac{8}{16} \times 30 = 15$ cm	B1 [1]
6(b)	$\frac{\text{Volume of large bucket}}{\text{Volum of small bucket}} = \left(\frac{16}{8}\right)^3$ $\frac{\text{Volume of large bucket}}{850} = (2)^3$ Volume of large bucket $= 8 \times 850 = 6800\ \text{cm}^3$	M1 A1 [2]

7(a)	$p+q$	B1 [1]
7(b)	$\frac{1}{2}(p-q)$	B1 [1]
8	LCM of 10, 20, 35 $= 5\times2\times2\times7$ $= 140$ After 140 minutes	M1 A1 [2]
9(a)	40°	B1 [1]
9(b)	9	B1 [1]
9(c)	120°	B1 [1]
10	$P \alpha Q^2$ $P = kQ^2$ where k is a constant New $P_{NEW} = k(3Q)^2$ Percentage increase $= \frac{k(9Q^2 - Q^2)}{kQ^2} \times 100\% = 800\%$	 M1 A1 [2]
11	$\frac{10x+8}{3} + 2 < 5 + 4x < 8$ $\frac{10x+8}{3} + 2 < 5 + 4x$ and $5 + 4x < 8$ $10x + 8 + 6 < 15 + 12x$ and $4x < 3$ $2x > -1$ and $x < \frac{3}{4}$ $x > -\frac{1}{2}$ $\therefore -\frac{1}{2} < x < \frac{3}{4}$  	M1 A1 A1 [3]
12(a)	$2x - 3 = x + 3y - 7$ or $2x - 3 = 15 - x - y$ or $x - 3y = -4$ $3x + y = 18$ $x + 3y - 7 = 15 - x - y$ $2x + 4y = 22$ (Any two of the equations) $x + 2y = 11$	M1 A1 [2]
12(b)	$x = 5$ $y = 3$	M1 A1 [2]

13(a)	The information did not specify the total of number of children surveyed/population. OR The information did not specify the information was obtained in one hositpal/all hositpals in the USA. OR BIG HEADLINE makes you think that 5.3% of children get spinal cord injuries...a pretty scary statistic for parents:	Any 1 with explanation B2 [2] To explain why is this important to mention the population of the children surveyed.
13(b)	For the record, the real figure should be based on the number of injuries per year out of a population of certain number in that country	B1 [1]
14(a)	1 : 50 000 1 cm represent 0.5 km 5.8 cm represent $0.5 \times 5.8 = 2.9$ km	B1 [1]
14(b)	1 cm^2 represent 0.5×0.5 km^2 $\frac{6.5}{0.25} = 26\ cm^2$	M1 A1 [2]
15(a)		
15(b)(i)	$x^2 - 4x + 5 = (x-2)^2 + 1$	B1 [1]
15(b)(ii)		B1 – turning point B1 – y intercept [2]
16(a)	(1515 2610 1770)	M1 A1 [2]
16(b)	The total weekly costs for Raspberry, Orange and Lemon drinks are \$15.15, \$26.10 and \$17.70 respectively	B1 [1]

17		B2 B1 (one number wrong)
18(a)	Length of $AB = \sqrt{4^2+3^2} = 5 units$	B1 [1]
18(b)	$C(7,4)$	B1 [1]
18(c)	$50 = \frac{1}{2}(8+x)\times 4 \Rightarrow x = 17$ $D(13, 0)$	M1 A1 [2]
18(d)	$\cos \angle ABC = -\frac{3}{5}$	B1 [1]
19(a)	Mean mass of Line P = 495.8 g Mean mass of Line Q = 494.6 g	B1 B1 [2]
19(b)	Standard deviation of Line P = 7.07 g Standard deviation of Line Q = 9.92 g	B1 B1 [2]
19(c)	The lighter loaf is likely to come from Q where **the mean is lower**. The mass of line Q's products **are also more varied from their mean** value and hence, **a higher chance of being lighter**.	B1 B1 [2]
20(a)	$\frac{3}{9}, \frac{6}{9}, \frac{4}{9}, \frac{5}{9}$ oe	B1 for all three correct [1]
20(b)(i)	$\frac{12}{90}$	FT from their tree diagram 1FT [1]
20(b)(ii)	$\frac{48}{90}$	FT from their tree diagram. B1 for $\frac{24}{90}$ oe FT seen Or M1 for $\left(\frac{4}{10}\times\frac{6}{9}\right)+\left(\frac{6}{10}\times\frac{4}{9}\right)$ oe FT 2FT [2]

21(a)	$(3x^2y)^3 \times (5x^{-3}y^4)^{-1}$ $= 27x^6y^3 \times \frac{1}{5}x^3y^{-4}$ $= \frac{27x^9}{5y}$	M1 A1 [2]
21(b)	$\left(\frac{1}{8}\right)^{-\frac{2}{3}} \times 32^{\frac{3}{5}} = 2^{p-2} \div 2^2$ $4 \times 8 = 2^{p-4}$ $2^5 = 2^{p-4}$ $p = 9$	M1 A1 [2]
21 (c)	$0.0040589 = 4.0589 \times 10^{-3}$	B1 [1]
22(a)(i)	$\lvert p \rvert = \sqrt{(3)^2 + (4)^2}$ $\lvert p \rvert = 5$ units	B1 [1]
22(a)(ii)	$m = -3$	B1 [1]
22(I)(b)(i)	$\overrightarrow{OC} = 2\overrightarrow{AB} = 2\mathbf{b}$	B1 [1]
22(b)(ii)	$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AO} + \overrightarrow{OC}$ $= -\mathbf{b}-\mathbf{a}+2\mathbf{b}$ $= \mathbf{b}-\mathbf{a}$	B1 [1]
22(b)(iii)	$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$ $= \mathbf{b} + \mathbf{b} - \mathbf{a} - \mathbf{a}$ $= 2\mathbf{b}-2\mathbf{a}$	B1[1]
22(b)(II)	Since $\overrightarrow{AD} = 2\overrightarrow{BC}$ $AD \parallel BC$ $ABCD$ is a trapezium	B1 [1]
23(a) (b)(i) (b)(ii)		(a)[2] (b)(i) [1] (b)(ii) [1] 2 possible location of point D. But no effect on the answer
23(c)	$BP = 3.9\text{ cm} \pm 0.1\text{cm}$	B1 [1]

Yusof Ishak Secondary School
Preliminary Examination 2017
Mathematics Paper 2

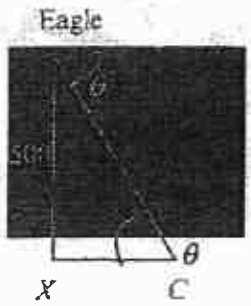
Marking Scheme

1 (a)	$(1+4x)^2 = 81$ $1+4x = \pm\sqrt{81}$ $1+4x = 9$ or $1+4x = -9$ $4x = 8$ or $4x = -10$ $x = 2$ or $x = -2.5$	M1 A1 [2]
1 (b)	$\frac{1}{2x+3} + \frac{3}{2x-1}$ $= \frac{1(2x-1)+3(2x+3)}{(2x+3)(2x-1)}$ $= \frac{2x-1+6x+9}{(2x+3)(2x-1)}$ $= \frac{8x+8}{(2x+3)(2x-1)}$	M1 A1 [2]
1 (c)	$2x+1<9<3x+1$ $2x+1<9$ and $9<3x+1$ $\Rightarrow 2x<8$ and $3x>8$ $\Rightarrow x<4$ and $x>\frac{8}{3}$ $x=3$	M1 A1 [2]
1 (d)	$a^2+9b^2-6ab-2a+6b$ $=(a^2+9b^2-6ab)-2a+6b$ $=(a-3b)^2-2(a-3b)$ $=(a-3b)(a-3b-2)$	M1 A1 [2]

2(a)	Number of sets of 2 white beads and 1 black bead $14-1=13$ Total number of white beads $13\times 2=26$  Disagree Students must be able to explain and show how they obtained the answer	B3 [3]
2(b)(i)	$3b=4a \Rightarrow \frac{a}{b}=\frac{3}{4} \Rightarrow a:b=3:4$ $2c=5a \Rightarrow \frac{a}{c}=\frac{2}{5} \Rightarrow a:c=2:5$ $\therefore a:b:c=6:8:15$	B1 B1 [2]
2(b)(ii)	Let $a=6k, b=8k, c=15k$ $6k+8k+15k=10$ $k=\frac{10}{29}$ $\therefore b=\frac{80}{29}$	M1 A1 A1 [3]

3 (a)	x bulbs cost \$25 1 bulb cost $\$\frac{25}{x}$	B1 [1]
3 (b)	Selling price for each light bulb = $\$\frac{25}{x} + \$0.50$ $= \$\frac{25 + \$0.50x}{x}$ $= \$\frac{2(25 + 0.5x)}{2x}$ $= \$\frac{50 + x}{2x}$	B1 [1]
3 (c)(i)	Total amount = $\$\frac{50 + x}{2x} \times 8$ $= \$\frac{4(50 + x)}{x}$	B1 [1]
3(c)(ii)	Number of unsold light bulbs = $x - 8$	B1 [1]
3(d)	Total amount = $\$2 \times (x - 8)$ $= \$2(x - 8)$	B1 [1]
3(e)	$\frac{4(50 + x)}{x} + 2(x - 8) = 46$ $\frac{200 + 4x}{x} + 2x - 16 = 46$ $\frac{200 + 4x + 2x^2 - 16x}{x} = 46$ $2x^2 - 12x + 200 = 46x$ $2x^2 - 58x + 200 = 0$ $x^2 - 29x + 100 = 0$ (Shown)	M1 A1 A1 [3]
3(f)	$x = \frac{-(-29) \pm \sqrt{(-29)^2 - 4(1)(100)}}{2(1)}$ $x = \frac{29 \pm \sqrt{441}}{2}$ $x = \frac{29 \pm 21}{2}$ $x = 25$ or $x = 4$ The number of light bulbs cannot be less than 8. $\therefore x = 4$ is not applicable The number of light bulbs, $x = 25$.	M1 A1 A1 [3]

[16]

4(a)(i)	In ΔABD, Using Sine Rule, $\frac{\sin 118}{950} = \frac{\sin \angle ADB}{600}$ $\Rightarrow \sin \angle ADB = \frac{600 \times \sin 118}{950}$ $\angle ADB = 33.89°$ $\angle ADB = 33.9°$ (1 decimal place) $\angle ABD = 180° - 118° - 33.9°$ $= 28.1°$	M1 A1 A1 A1 [4]
4(a)(ii)	In ΔBCD, Using Cosine Rule, $CD = \sqrt{950^2 + 1040^2 - 2(950)(1040)\cos 42°}$ $CD = 718.1$ $CD = 718$ m (3 sig. figures)	M2 A1 A1 [4]
4(a)(iii)	Let the required distance be h. Area of $\Delta BCD = \frac{1}{2} \times 950 \times 1040 \times \sin 42°$ $\frac{1}{2} \times 950 \times 1040 \times \sin 42° = \frac{1}{2} \times 950 \times h$ $h = 1040 \times \sin 42°$ $h = 695.9$ $h = 696$ m (3 sig. figures)	M1 A1 [2]
4(b)	The greatest angle of depression occurs when the eagle is directly above the point on BD such that it is nearest to C  $\tan\theta = \frac{500}{695.9}$ $\theta = 35.7°$ Greatest angle of depression is 35.7° (1 decimal place)	M1 A1 [2]

5(a)(i)	On Monday, volume = 7500 cm^3. On Tuesday, volume = 86% of 7500 $= \frac{86}{100} \times 7500 = 6450$ On Wednesday, volume = 86% of 6450 $= \frac{86}{100} \times 6450 = 5547$ $= 5547\ cm^3$ (3 sig. figures)	M1 A1 [2]
5(a)(ii)	Let x be the actual volume of Block Q, the volume of Block Q has been reduced as 86% of its actual volume on Tuesday. $\Rightarrow 86\%\ of\ x = 6450$ $x = 6450 \times \frac{100}{86} = 7500$ Actual volume of Block Q on Monday = $7500 cm^3$ (3 sig. figures)	M1 A1 [2]
5(a)(iii)	Let v be the volume of Block R on Monday On Tuesday, volume = $\frac{86}{100} v = 0.86v$ On Wednesday, volume = $\frac{86}{100}(0.86v) = 0.7396v$ On Thursday, volume = $\frac{86}{100}(0.7396v) = 0.6361v$ On Friday, volume = $\frac{86}{100}(0.6361v) = 0.547v$ On Saturday, volume = $\frac{86}{100}(0.547v) = 0.470v$ Volume reduces to half on Saturday.	M1 A1 [2]
5 (b)(i)	Volume of hemisphere = $\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$ Volume of $S = \frac{1}{2}\left(\frac{4}{3}\pi(18)^3\right)$ $= \frac{2}{3} \times 3.142 \times 5832$ $= 12216.1$ $= 12200\ cm^3$ (3 sig. figures)	M1 A1 [2]
5(b)(ii)	Total surface area of solid hemisphere S $= \frac{1}{2}\left(4\pi r^2\right) + \pi r^2$ $= \frac{1}{2}\left(4\pi(18)^2\right) + \pi(18)^2$ $= 2036016 + 1018008$ $= 3054.024$ $= 3050\ cm^2$ (3 sig. figures)	M1 A1 [2]
5(c)	$\frac{\text{Volume before}}{\text{Volume after}} = \left(\frac{\text{height before}}{\text{height after}}\right)^3$ $\left(\frac{5000}{1080}\right) = \left(\frac{12}{h}\right)^3$	

[18]

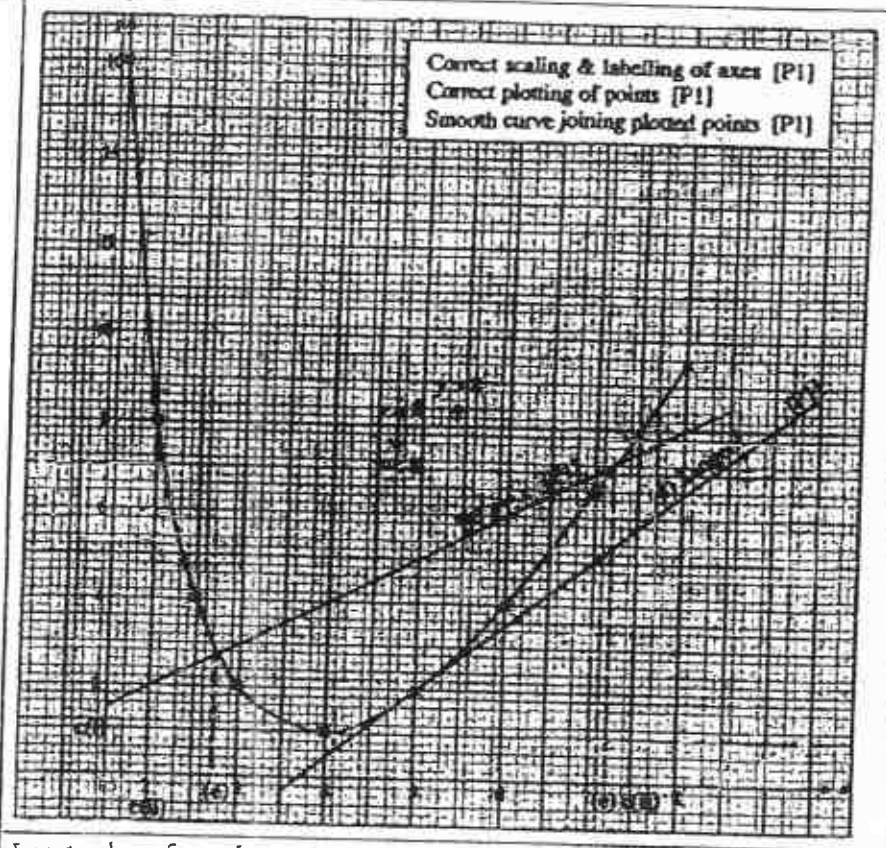
	$\frac{125}{27}=\left(\frac{12}{h}\right)^3$ $\left(\frac{5}{3}\right)^3=\left(\frac{12}{h}\right)^3$ $\frac{5}{3}=\frac{12}{h}$ $5h=36$ $h=7.2$ cm	M1 A1 [2]

6(a)(i)	$OA^2 = OX^2 + AX^2$ $r^2 = (16-r)^2 + 8^2$ $r^2 = 256 - 32r + r^2 + 8^2$ $32r = 320$ $r = 10$	M1 A1 A1 [3]
6(a)(ii)	$\sin \angle AOX = \frac{AX}{OA}$ $= \frac{8}{10}$ $\angle AOX = \sin^{-1}\left(\frac{8}{10}\right)$ $\angle AOX = 53.1°$ (1 decimal place)	 B1 [1]
6(a)(iii)	$\angle AOB = 2(53.1°)$ Shaded region $= \frac{2(53.1°)}{360°} \times \pi \times 10^2 - \frac{1}{2} \times 10 \times 10 \sin 2(53.1°)$ $= 44.74$ m^2 ∴ Volume of concrete used $= 44.74 \times 900$ $= 40\,266$ m^3 $= 40\,300$ m^3 (correct to 3 sig. figures)	M1 A1 A1 [3]
6 (b)	Length of the model tunnel $= \frac{900}{10} \times 5$ $= 450$ m Reflex $\angle AOB = 360° - 2(53.1)° = 253.74°$ Curved surface area $= \frac{253.74°}{360°} \times 2\pi \times 5 \times 450$ $= 9965.6$ cm^2 $= 9970$ cm^2 (3 sig. figures)	 B1 M1 A1 [3]
6 (c)	Total distance the train has to travel $= 900 + 130 = 1030$m. Time taken $= \frac{1030}{50000} \times 60 = 1.236$ minutes 1 minute 14 seconds	M1 A1 [2]

[20]

7(a)	$\angle BSQ = 90°$ (rt. $\angle$ in a semicircle) $\angle BAQ = 90°$ (rt. $\angle$ in a semicircle) $\angle OST$ or $\angle OQT = 90°$ (tangent perp. radius at point of contact)	B1 B1 B1 [3]
7(b)(i)	$\angle SQB = 32°$ ($\angle$s in the same segment)	B1 [1]
7(b)(ii)	$\angle OTQ = \frac{64°}{2} = 32°$ (OT bisects $\angle QTS$) $\angle TOQ = 180 - 32 - 90 = 58°$ ($\angle$ sum of Δ)	M1 A1 [2]
7(b)(iii)	$\angle ACB = 180° - 73° = 107°$ (adj. $\angle$s on a str. line) $\angle ABQ = 180 - 32 - 107 = 41°$ ($\angle$ sum of Δ)	M1 A1 [2]
7(b)(iv)	$QT = ST$ (tangents drawn to circle form ext. point are equal) $\angle DST = \frac{180° - 64°}{2}$ (base $\angle$s of isosceles Δ) $= 58°$ $\angle BSR = 180° - 90° - 58°$ (adj. $\angle$s on a str. line) $= 32°$ OR $\angle BSR = 32°$ (alternate segment theorem)	M1 A1 [2] B1, B1 [2]

8(a)	Volume of wood in the block = $(20 \times 7 \times 4) - 3\left(\frac{1}{2} \times \frac{4}{3} \times \pi \times 1.5^3\right)$ $= 560 - 21.2085$ $= 538.7915$ $= 539\ cm^3$ (3 significant figures)	M1 A1 [2]
8(b)	Total area that is painted pink = $2(7 \times 4) + 2(20 \times 4) = 56 + 160$ $= 216\ cm^2$	B1 [1]
8(c)(i)	Total area that is painted white = $2 \times \pi \times (1.5)^2 \times 3$ $= 42.417$ $= 42.4\ cm^2$ (3 significant figures)	B1 [1]
8(c)(ii)	Total area that is painted green = $(20 \times 7) - 3(\pi \times 1.5^2)$ $= 118.7915$ $= 119\ cm^2$	B1 [1]

9(a)	$a = y = 4(2) + \frac{60}{2} - 30$ $= 8 + 30 - 30$ $= 8$ $b = y = 4(5) + \frac{60}{5} - 30$ $= 20 + 12 - 30$ $= 2$	1 wrong B0 B1 [1]
8(b)		P1 P1 P1 [3]
8(c)(i)	Least value of $y = 1$	[B1]
8(c)(ii)	For $4x + \frac{60}{x} - 30 < 8$ $2 < x < 7.5$	M1 A1 [2]
8(d)	At $x = 5$, using 2 points on the tangent, (7, 5) and (5, 2) Gradient of tangent $= \frac{5-2}{7-5} = \frac{3}{2}$ or 1.5	M1 A1 [2]
8(e)	$3x^2 + 60 - 30x = 0$ $3x + \frac{60}{x} - 30 = 0$ $4x + \frac{60}{x} - 30 = x$ The solution is the intersection of the graphs	M1

	$y = 4x + \frac{60}{x} - 30$ and $y = x$ *i.e.* when $x = 2.75$ or $x = 7.2$	P1 A1 [3]

10(a)	Mr Ong's monthly contribution $= \frac{20}{100} \times \$3000 = \$600$ His employer's monthly contribution $= \frac{17}{100} \times \$3000 = \$510$	B1 B1 [2]
10(b)	They have to pay $\frac{90}{100} \times \$400000 = \$360000$ over 20 years Each month, they have to pay $\frac{\$360000}{20 \times 12} = \$1500$	M1 A1 [2]
10(c)	Amount to be used for monthly payment $= \left(\frac{21}{100} \times \$3000\right) + \left(\frac{23}{100} \times \$2000\right) = \$1090$ (Shown)	M1 A1 [2]
10(d)	They have to borrow $(\$1500 - \$1090) \times 20 \times 12 = \$98400$	B1 [1]
10(e)	They have to pay $\left(\frac{1.48}{100} \times 98400 \times 1\right) = \$1456.32$ Year 1 Interest $\left(\frac{1.58}{100} \times 98400 \times 19\right)$ Year 2 onwards $\$29539.68$ $\$98400 + \$145632 + \$2953968$ $= \$129396$	M1 M1 [A1]