2015 Sec 4 E-Maths

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13	Tanjong Katong Sec	SA2
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Index Number	Class	Marks: / 80

Anglo-Chinese School
(Barker Road)

PRELIMINARY EXAMINATION 2015

SECONDARY FOUR EXPRESS/FIVE NORMAL ACADEMIC

MATHEMATICS 4016
PAPER ONE

2 HOURS

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your class and candidate number on the cover sheet.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either the calculator value or 3.142, unless the question requires the answer in terms of π.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for Paper One is 80.

3 s.f.		Simplify fraction	
1 d.p.		Truncation error	

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved Surface area of cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle} = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

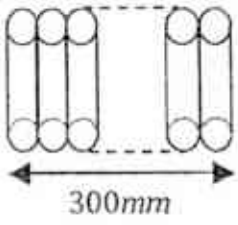
$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions

For Examiner's Use

1 The diameter of polyester fiber is 9.8 micrometres each.
They are woven into sheets of width 300 millimetres.
If each sheet is made by arranging polyester fibre strands side-by-side without overlapping, find the number of fiber strands in each sheet, expressing the answer in standard form.



Answer _____ [2]

For Examiner's Use

2 Charlie invested $25000 into a business which gave a guaranteed return at 4% per annum compounded every quarterly.
Find the total return earned after 3 years, giving your answers to the nearest cents.

Answer _____ [2]

For Examiner's Use

3 At 12 noon, the temperature of the Sahara Desert is $45.4°C$ while the temperature recorded at 12 midnight is $4.9°C$. Assuming that the temperature decreases uniformly as the time passes, find the time when the temperature first reaches $26.5°C$.

Answer _____ a.m./p.m. [2]

For Examiner's Use

4 A factory can assemble 4000 watches in 8 hours by 50 workers.
In their latest project, they are to assemble 13500 watches.
Assuming that all workers work at the same rate, calculate how many more workers are required if they are to complete the task in 18 hours.

Answer _____ workers [2]

3

For Examiner's Use

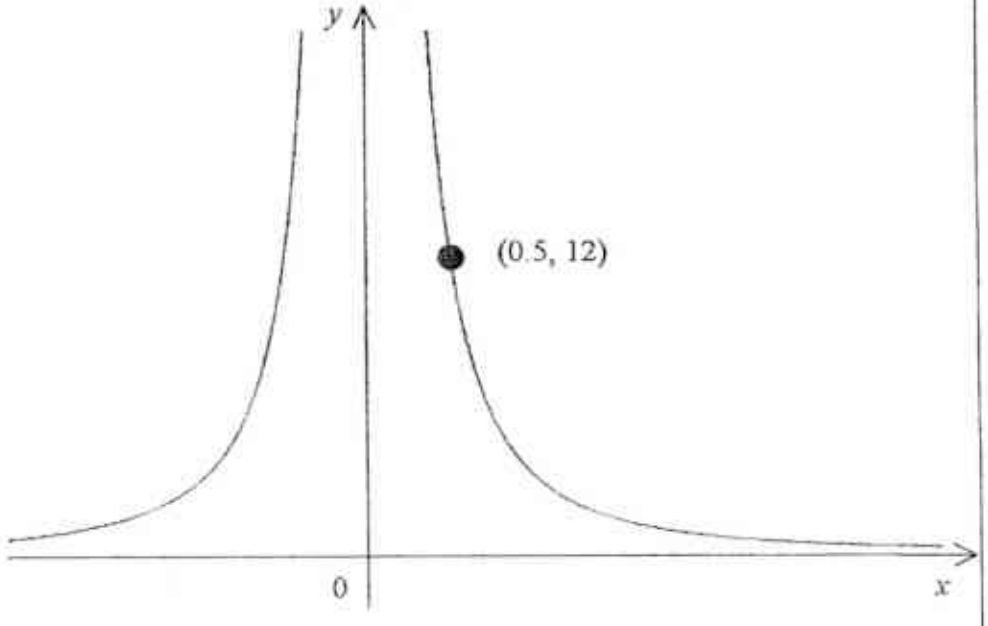
5 When the Goods and Services Tax (GST) increases by 2%, James decreases his expenditure by 2%. James claimed that his expenditure neither increases nor decreases. Explain if he is right or wrong.

Answer: .....

.....

..... [2]

6 The diagram shows the graph of $y = kx^{-2}$.



Given that it passes through (0.5, 12),

(i) find the value of k.

(ii) State the equation of the line of symmetry for this graph.

Answer (i) k = ... [1]

(ii) ..... [1]

For Examiner's Use

7 An area of 1250 km^2 is represented on a map by an area of 50 cm^2.

Find

(a) the scale of the map in the form of 1: n,

Answer _____ [1]

(b) the length of a road on the map with an actual distance of 27 km.

Answer _____ cm [1]

For Examiner's Use

8 Simplify $\dfrac{\left(3a^{-\frac{1}{2}}\right)^2 \times 4a^2b}{12b^{-2}}$, leaving your answers as positive indices.

Answer _____ [2]

4

For Examiner's Use

9 The probability that an amateur dart archer actually hits the bull's eye is $\frac{1}{6}$.

If three consecutive attempts are made, find the probability that the bull's eye will be hit

(a) three times,

Answer (a) _____ [1]

(b) not at all,

Answer (b) _____ [1]

(c) at least twice.

Answer (c) _____ [1]

For Examiner's Use

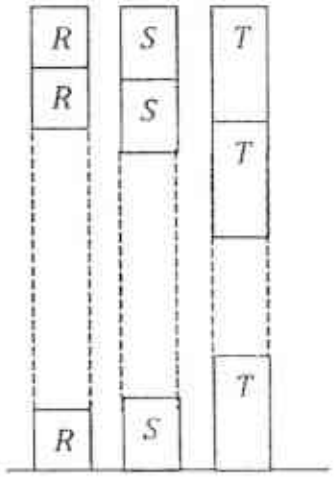
10 **(a)** 495 and a number N have a LCM of 4950 and a HCF of 15.

Given that $4950 = 2 \times 3^2 \times 5^2 \times 11$, find the number N.

Answer (a) $N =$ _____ [1]

(b) Three types of cylindrical cans, R, S and T, of the same radii, have heights 50 cm, 0.6 m and 0.72 m. They are stacked to the **same** height for a game to be played.

Find the minimum number of can T.



Answer (b) _____ cans [2]

11 The volume, V, of a cylinder varies directly to the square of its base radius r cm ~~and height h cm~~. If the base radius decreases by 50% and the height increases by 40%, find the percentage change in the volume of the cylinder.

Answer _____ % [3]

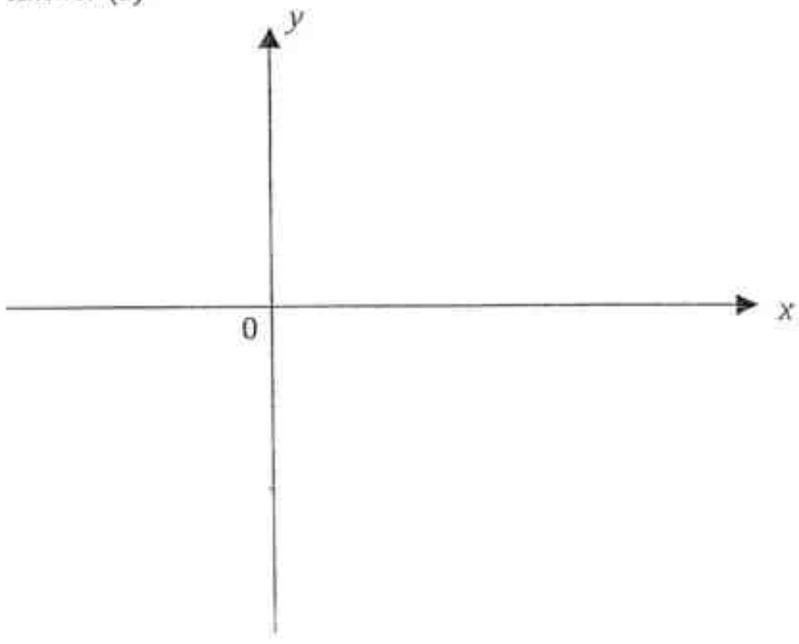
For Examiner's Use

12 **(a)** Express $y = -x^2 + 4x - 9$ in the form $y = -(x-a)^2 + b$.

Answer (a) _____ [1]

(b) Hence, sketch the graph of $y = -x^2 + 4x - 9$ and write down the coordinates of the turning point and y-intercept.

Answer (b) [2]



13 The first four terms of a number sequence are 5, 12, 19 and 26.

(a) If the nth term of the number sequence can be expressed in the form of $pn + q$, find the values of p and q.

Answer (a) $p =$ _____ [1]

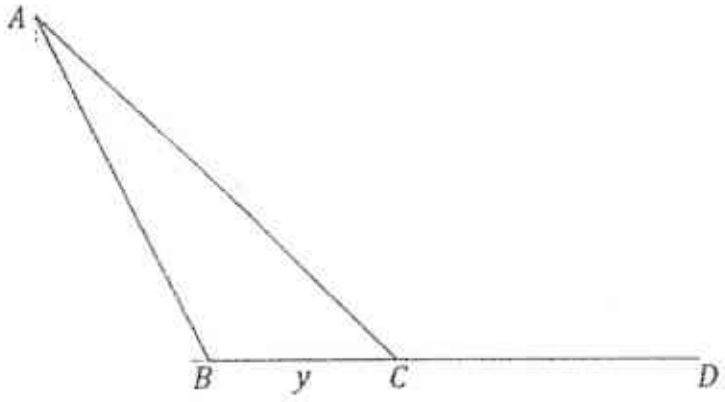
$q =$ _____ [1]

(b) Also, deduce the nth term of a sequence that has –3, 4, 11 and 18 as its first four terms.

Answer (b) _____ [1]

For Examiner's Use

14 In the diagram, BCD is a straight line.



Given that $BC = y$ cm, $AC = 2BC$ and the area of ΔABC is $0.8y^2$ cm^2, find the exact value of

(a) $\sin \angle ACD$,

Answer (a) _____ [2]

(b) $\cos \angle ACD$.

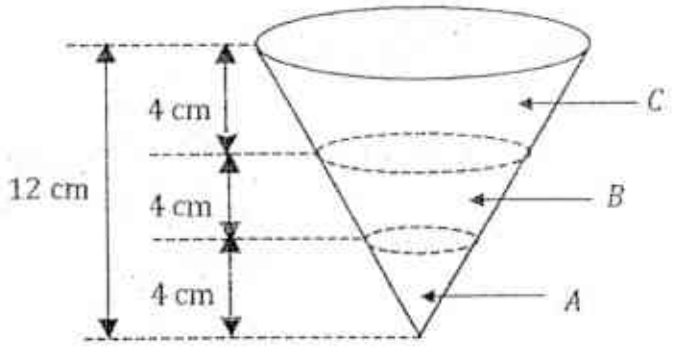
Answer (b) _____ [1]

For Examiner's Use

6

For Examiner's Use

15 A solid cone is cut into three equal parts, A, B and C, by planes parallel to the base, as shown below.
The height of the cone is 12 cm.



Find the ratio of the

(a) diameters of the bases C to B,

Answer (a) _____ [1]

(b) the volumes of A to C.

Answer (b) _____ [2]

For Examiner's Use

16 **(a)** Given that $x + y = 6$ and $xy = 2$, find the value of $\frac{2}{x^2} + \frac{2}{y^2}$.

Answer (a) _____ [2]

(b) Given that $\frac{3x - 2y}{y - 4x} = \frac{3}{2}$, find the value of $\frac{3x}{7y}$.

Answer (b) _____ [2]

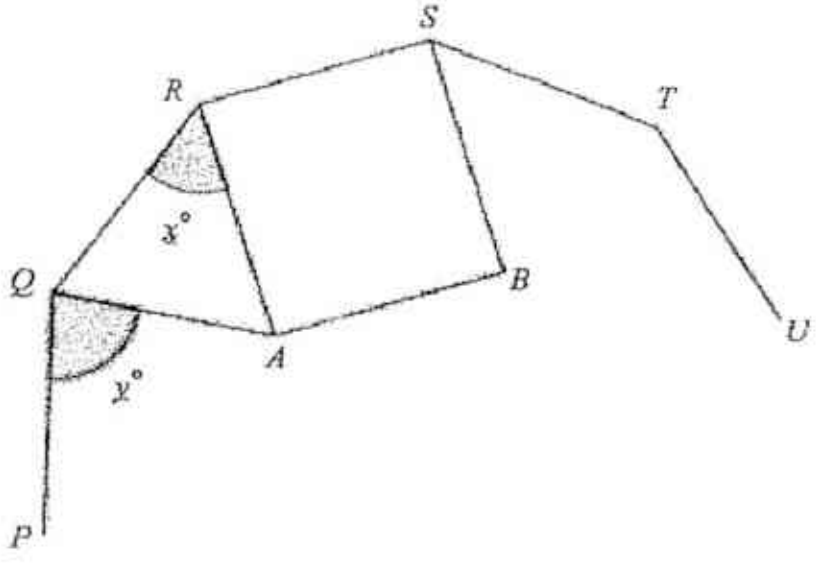
For Examiner's Use

17 (a) Find the size of each exterior angle of a regular decagon.

For Examiner's Use

Answer (a) _____ ° [1]

(b) *PQRSTU*... is part of a regular polygon which has interior angles of 144°. *RABS* is a square.



Find

(i) the value of x,

Answer (b) (i) _____ ° [1]

(ii) the value of y.

Answer (b) (ii) _____ ° [2]

For Examiner's Use

18 (a) Solve the inequalities $\frac{x}{2}+2<\frac{2x+7}{3}\leq 9-2x$.

Answer (a) _____ [2]

(b) Represent your solutions in **part (a)** on the number line in the answer space below.

Answer (b) [1]

(c) With reference to your solutions in **part (a)**, write down the smallest value of x^2.

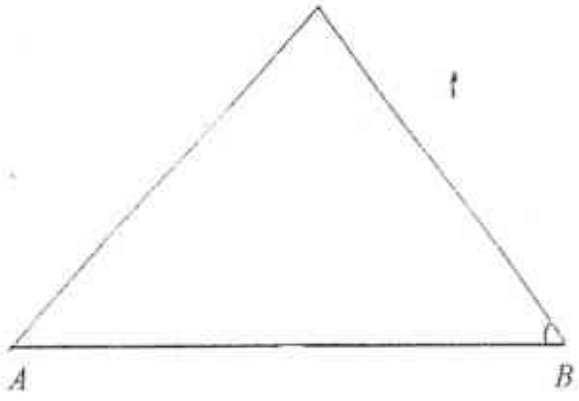
Answer (c) _____ [1]

8

For Examiner's Use

19 (a) In the spaces below, construct a triangle ABC such that $AB = 7.4$ cm, $BC = 5.5$ cm and $\angle ABC = 54°$.

Answer (a)



[2]

(b) A point S is on the diagram.

It is equidistant from the lines AC and AB and points A and B.

With suitable construction lines shown, mark point S. [3]

For Examiner's Use

20 A is a point $(1, 2)$, $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

(a) Find $\left|\overrightarrow{BC}\right|$.

Answer (a) _____ units [1]

(b) Express $\overrightarrow{AC}$ as a column vector.

Answer (b) _____ [2]

(c) Given that $\overrightarrow{CD} = \begin{pmatrix} -2 \\ s \end{pmatrix}$, find the two possible values of s such that $ABCD$ is a trapezium.

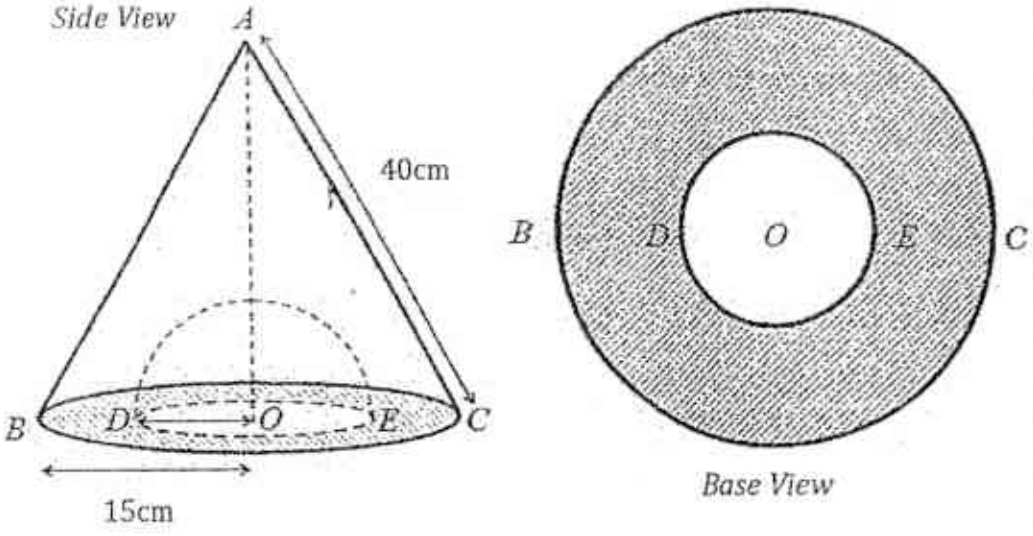
Answer (c) $s =$ _____ or _____ [3]

9

For Examiner's Use

21 The diagram shows the side and base views of a solid party hat.

It is made up of a cone and a hemisphere cut out from the bottom of the cone.



Given that the slant height, AC, and radius, OB, of the cone are 40 cm and 15 cm respectively and the shaded area of the base is 176π cm^2, find

(a) the radius, OD, of the hemisphere,

Answer (a) _____ cm [2]

(b) the volume of the party hat,

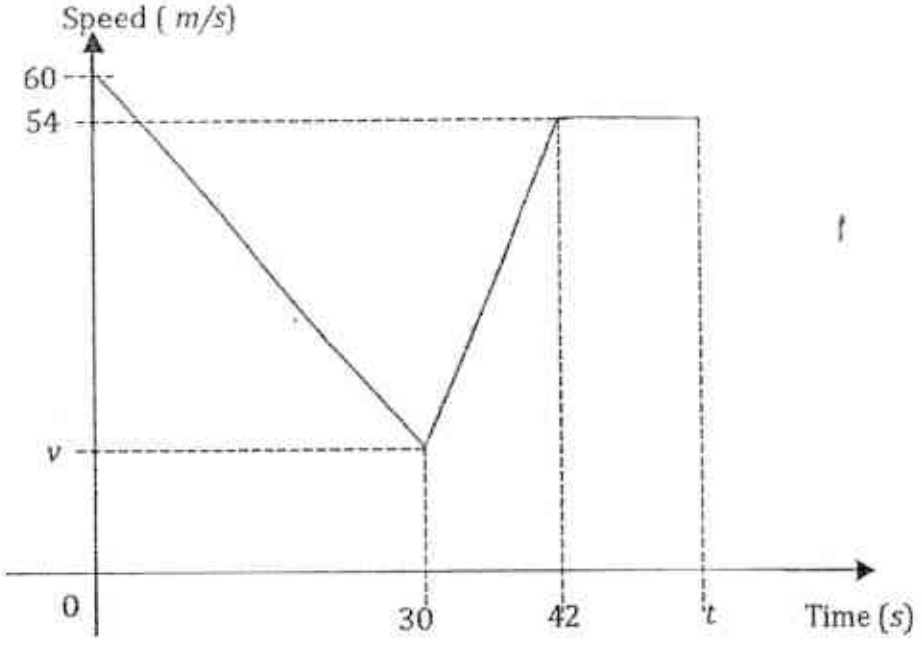
Answer (b) _____ cm^3 [2]

(c) the total surface area of the party hat.

Answer (c) _____ cm^2 [2]

For Examiner's Use

22 The diagram shows the speed-time graph for the first t seconds of the motion of a particle.



(a) Given that the particle undergoes a retardation of 1.2m/s^2 at 20s, in the spaces below, show that the value of v is 24.

Answer (a)

[2]

(b) The particle travelled a distance of 702 metres between the 42 and t second, find the value of t.

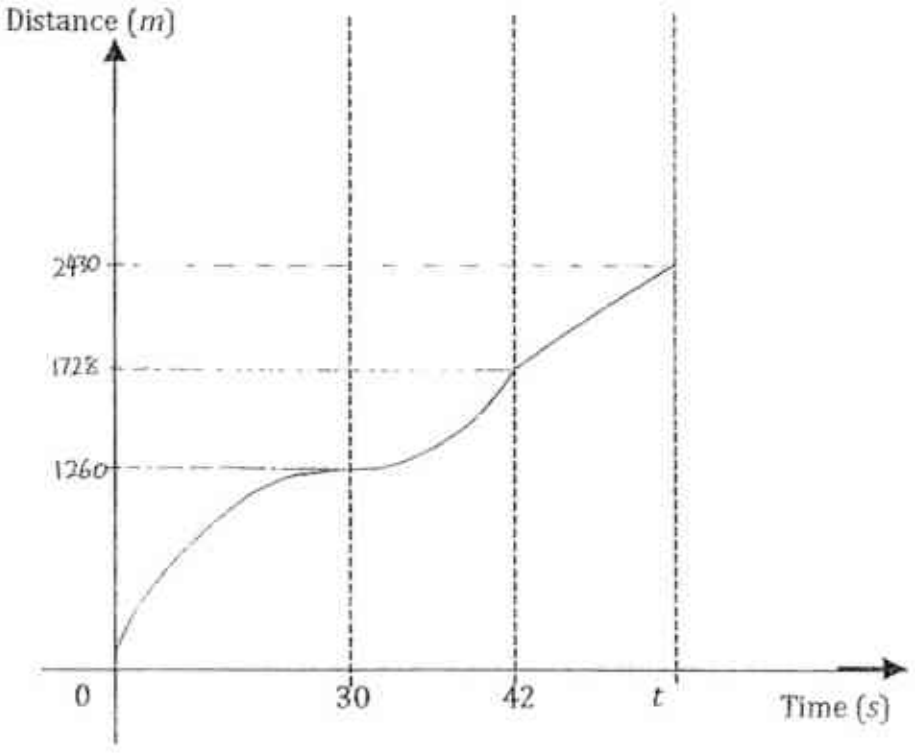
Answer (b) t = _____ [1]

10

For Examiner's Use

(c) In the axes below, sketch the distance-time graph of the particle for the entire duration of t seconds.

Answer (c)



[3]

For Examiner's Use

For Examiner's Use

23 The equation of the line l is $3x + 4y = 24$.

(a) Given that line l crosses the x-axis and y-axis at points A and B respectively, find the coordinates of the points A and B.

Answer (a) A ( ) [1]
B ( ) [1]

(b) Find the shortest distance from the origin to AB.

Answer (b) units [3]

(c) Line l intersects another line h at a point C.

(i) Given that the gradient of line h is $\frac{3}{4}$ and it passes through the origin, write down the equation of line h.

Answer (c) (i) [1]

(ii) Hence, find the coordinates of C.

Answer (c) (ii) C ( ) [2]

11

[END OF PAPER 1]

1. 3.06×10^4
2. $3170.63
3. 5.36pm
4. 25 workers
5. Let the expenditure be $x & original GSt of 5%
   Total expense is $1.05x.
   With the increment of GST by 2%, with 2% drop in expenditure,ie.$0.98x, the GST is now 7% x 0.98x=$0.0686x.
   New total expenses is 0.98x+ 0.0686x=$1.0486x
   So there is actually a decrease in total expenses.
   Or
   If figures is rounded to 2dp, James is correct to say that there is no change.

6.i K=3

6ii. x=0

7a.1:500000

7b.5.4cm

8.$3ab^3$

9a.$\frac{1}{216}$

9b.$\frac{125}{216}$

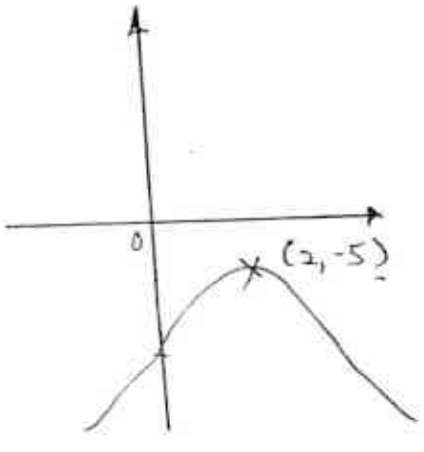
9c.$\frac{2}{27}$

10a.N=150

10b.25 cans

11.65%

12.a-$(x-2)^2-5$



13a.P=7 q=-2

13b.7n-10

14a. $\frac{4}{5}$

14b. $\frac{3}{5}$

15a. 3:2

15b. 1:19

16a. 16

16b. $\frac{1}{6}$

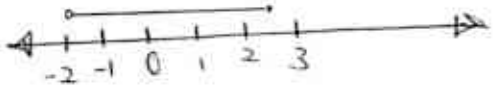
17a. 36^0

17bi. 54^0

17bii. 81^0

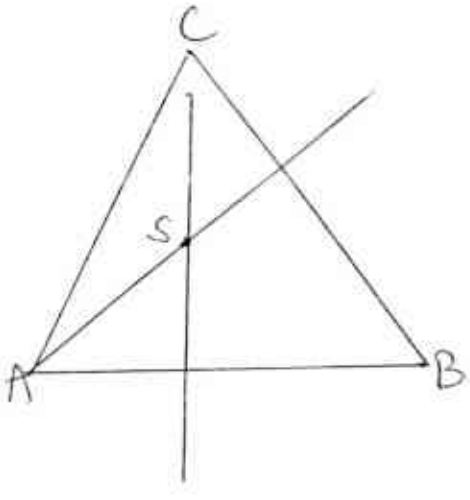
18a. $-2 < x \leq \frac{5}{2}$

18b.



18c. 0

19.



20a.3.61units

20b. $\binom{6}{3}$

20c. s=0 or 3

21a. 7 cm

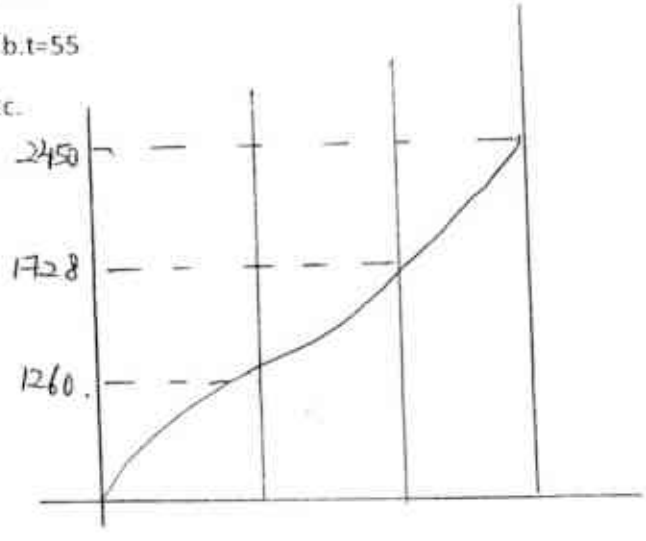
21b. $\frac{1}{3}\pi(15)^2\left(\sqrt{20^2-15^2}\right)-\frac{1}{2}$

21c.2750cm^2

22.a.24

22b.t=55

22c.



23a.A(8,0) B(0,6)

23b. → Let the shortest distance from the origin to AB be d cm

23c.$y=\frac{3}{4}x$

23.ii.c(4,3)

Area of $\triangle = \frac{1}{2} \times 8 \times 6$

$\frac{1}{2} \times AB \times d = 24$

$\sqrt{8^2+6^2} \times d$.

Anglo-Chinese School
(Barker Road)

PRELIMINARY EXAMINATION 2015

SECONDARY FOUR EXPRESS/FIVE NORMAL ACADEMIC

MATHEMATICS 4016
PAPER TWO

2 HOURS 30 MINUTES

Additional Materials: Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your class and candidate number on the cover sheet.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
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Answer all questions.
If working is needed for any question it must be shown with the answer.
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Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either the calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions
[100 marks]

1. Given that a universal set $\xi = \{x : x \text{ is a positive integer and } x \leq 14\}$,
$A = \{x : x \text{ is an odd integer and } 3x + 5 > 13\}$ and $B = \{x : x \text{ is a factor of } 28\}$.

(a) Find the following

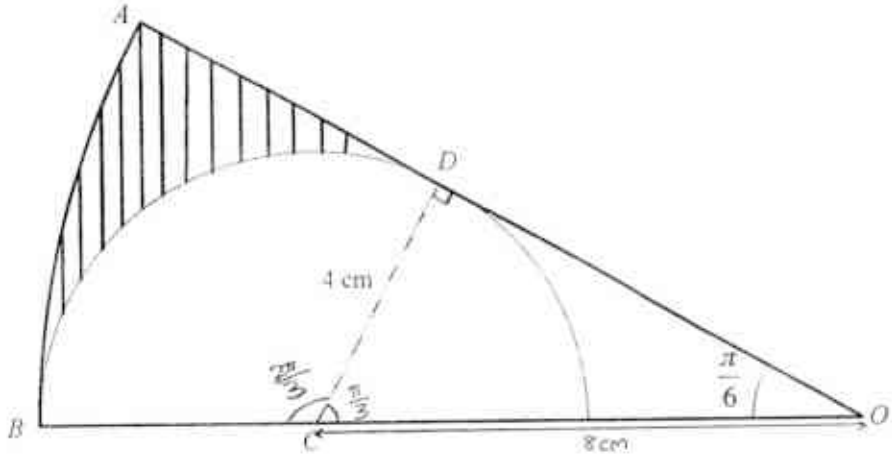
(i) $A \cup B$ [1]

(ii) $n(A \cap B')$ [2]

(b) With the help of a Venn diagram, list out the elements illustrating the sets ξ, A and B. [2]

---

2. The diagram shows the sector OAB of a circle with centre O and the radius OA. A semi-circle with centre C and radius 4 cm is drawn inside the sector such that OA is the tangent to the semi-circle at D.



Given that $\angle AOB = \frac{\pi}{6}$ radians, calculate

(a) the perimeter of the shaded region, [3]

(b) the area of the shaded region, [3]

/5

3. A shop sells three varieties of ice-creams, Vanilla, Chocolate and Strawberry. Each variety is sold in either cone or bowl. The sales of the ice-creams in two consecutive days are shown in the table below.

	DAY ONE		DAY TWO	
	Cone	Bowl	Cone	Bowl
Vanilla ice-cream	20	11	27	16
Chocolate ice-cream	35	19	33	21
Strawberry ice-cream	28	20	19	22

The information for the Day One's sales can be represented by

the matrix $\boldsymbol{R} = \begin{pmatrix} 20 & 11 \\ 35 & 19 \\ 28 & 20 \end{pmatrix}$.

The information for Day Two is represented by a matrix **S**.

(a) Write down the matrix $\boldsymbol{S}$. [1]

(b) Calculate $\boldsymbol{R} + \boldsymbol{S}$. [1]

(c) Describe what is represented by the elements $\boldsymbol{R} + \boldsymbol{S}$. [1]

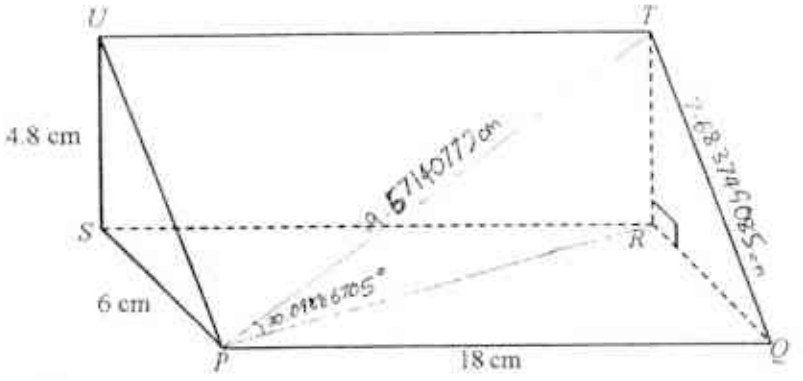
Suppose the price of Vanilla ice-cream is \$1, Chocolate ice-cream is \$2 and Strawberry ice-cream is \$2.

(d) Represent these prices by a 1×3 matrix $\boldsymbol{T}$. [1]

(e) Calculate $\boldsymbol{TS}$. [1]

(f) Describe what is represented by the elements of $\boldsymbol{TS}$. [1]

4.



The diagram shows a floatation device shaped in the form of a triangular prism in which $\angle QRT = 90°$, $US = 4.8$ cm, $SP = 6$ cm and $PQ = 18$ cm.

(a) Calculate

(i) the length of PT, [2]

(ii) $\angle RPT$. [2]

(b) Prove that triangles PRU and QST are congruent. [2]

(c) Styrofoam is needed to make such a floatation device.
Calculate the volume of styrofoam needed to make 10 such floatation devices. [2]

---

5. **(a)** Given that $w = \sqrt{\dfrac{u-v}{1+u}}$, express u in terms of w and v.

State the value(s) of w that does not exist in the equation. [3]

(b) Solve the equation $\dfrac{x+1}{2x-3} - \dfrac{14-3x}{4x^2-9} = 0$. [3]

(c) Solve the equation $4^{x^2+3} = 8^{\frac{5x}{3}+3}$. [2]

16

---

6. A car travels from town A to town B at an average speed of k km/h and returns from town B to town A at an average speed of $k + 15$ km/h. Given that the distance between town A and town B is 360 km and that the average speed of the entire journey is 60 km/h,

(a) Find the total time taken for the entire journey. [2]

(b) **(i)** Find an expression in terms of k for the time taken from town A to town B. [1]

(ii) Find an expression in terms of k for the time taken from town B to town A. [1]

(c) Show that k can be expressed in an equation, and that it simplifies to $k^2 - 45k - 450 = 0$. [3]

(d) Solve $k^2 - 45k - 450 = 0$. [3]

---

7. **Answer the whole of this question on a single sheet of graph paper.**

The variables x and y are connected by the equation $y = \frac{1}{2}x^2(3 - x)$.

Some corresponding values of y, corrected to 1 decimal place where necessary, are given in the following table.

x	-1.5	- 1.3	-1	-0.5	0	0.5	1	1.5	2	2.5	3	3.3	3.5
y	5.1	3.6	2	p	0	0.3	1	1.7	2	1.6	0	-1.6	-3.1

(a) Find the value of p. [1]

(b) Taking 2 cm to represent 1 unit on each axis, draw the graph of $y = \frac{1}{2}x^2(3 - x)$, for values $-1.5 \le x \le 3.5$. [3]

(c) By drawing a tangent, find the gradient of the curve at (2.5, 1.6). [2]

(d) **(i)** By adding a suitable line on the graph, use your graph to find the solution(s) of the equation $\frac{1}{4}x^2(3 - x) = x + 2$. [2]

(ii) This value of x is a solution of the equation $x^3 + Ax^2 + Bx + C = 0$.
Find the value of A, of B and of C. [1]

---

8. (a) 40 students from class **Y** were weighed and the results, recorded to the nearest kilogram were as follows.

Weight (x kilograms)	Frequency
$54 < x \leq 58$	3
$58 < x \leq 62$	p
$62 < x \leq 66$	9
$66 < x \leq 70$	q
$70 < x \leq 74$	14
$74 < x \leq 78$	5

(i) Show that the values of p and q are 3 and 6 respectively given that the mean is 68 kg. [3]

(ii) Find the standard deviation of the weights of the students in the class. [2]

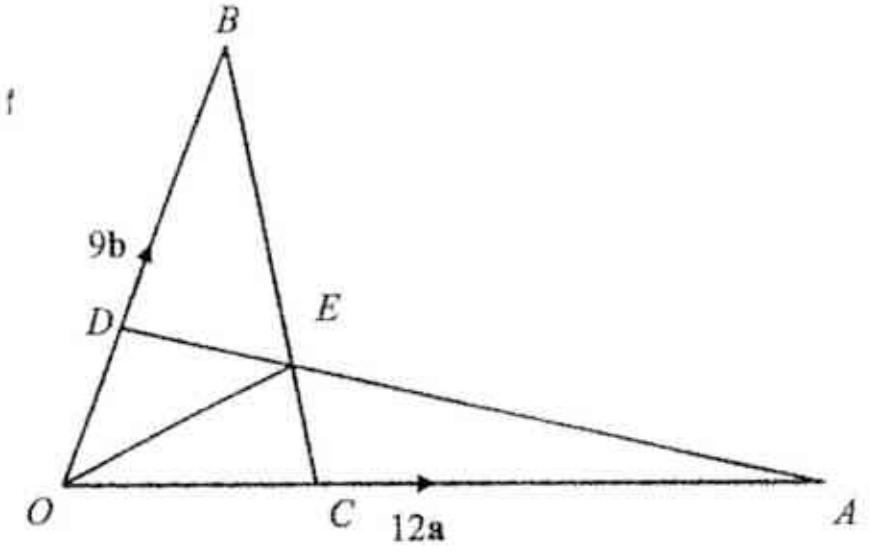
(iii) Given that the mean and standard deviation of another class **Z** is 60 kg and 7 kg respectively, comment on the distribution of the weights of the students in both classes. [2]

(b) To achieve the ideal weight, a student must be 70 kg and below.

(i) Calculate the probability that if two students in class **Y** are chosen at random, one achieved an ideal weight but the other did not. [2]

(ii) Calculate the probability that if three students in class **Y** are chosen at random, at least one student achieved the ideal weight. [2]

9. In the diagram, $\overrightarrow{OA} = 12\mathbf{a}$ and $\overrightarrow{OB} = 9\mathbf{b}$. It is given that $OD = \frac{1}{2}DB$ and $OC = \frac{1}{3}OA$.



(a) Express, as simply as possible, in terms of **a** and **b**,

(i) $\overrightarrow{BC}$, [1]

(ii) $\overrightarrow{DA}$. [1]

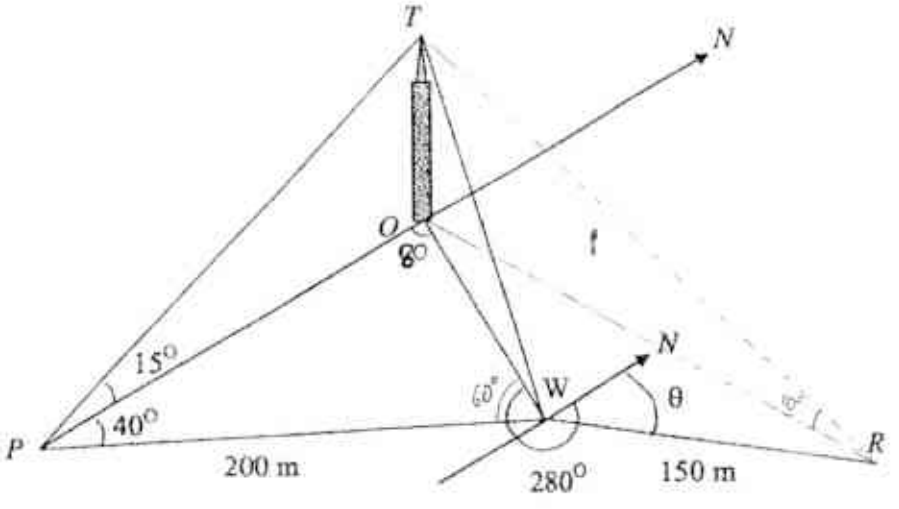
(b) Given that $\frac{\text{area of } \Delta ODE}{\text{area of } \Delta ODA} = \frac{1}{4}$, find the position vector of E in terms of **a** and **b**. [2]

(c) Calculate the numerical value of $\frac{BE}{EC}$. [2]

(d) Show that the areas of ΔOCE and ΔODE are equal. [3]

(e) Find $\frac{\text{area of } \Delta BDE}{\text{area of } \Delta BOC}$. [2]

10.



O, P, W and R are points on a horizontal plane. A vertical memorial tower, OT, is due north of P. The angle of elevation of a man at P to the top of the tower is 15°. He walks a distance of 200 metres to point W. Given that the bearing of W from P is 040° and the bearing of O from W is 280°,

calculate

(a) $\angle PWO$, [2]

(b) OP, [2]

(c) the height of the tower OT, [2]

(d) using the answer from part (c), the angle of depression of W from T. [3]

From W, the man walks 150 metres to a point R on a bearing of θ° on the horizontal plane. At R, the angle of elevation of the man to the top of the tower is 10°.

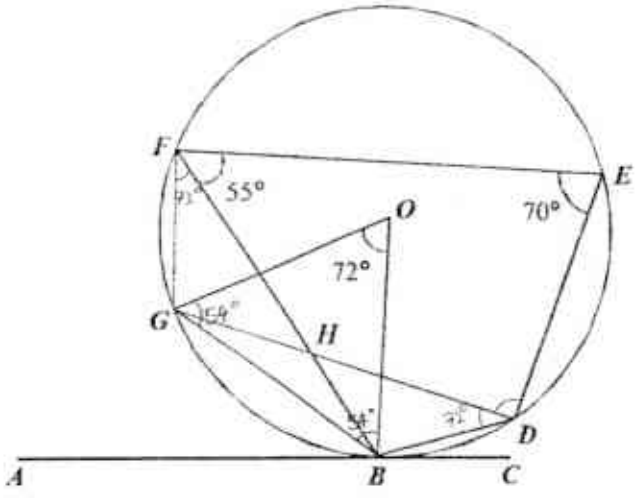
Calculate

(e) angle OWR, [3]

(f) the bearing of θ on the horizontal plane. [1]

/8

11.



(a) In the diagram, ABC is a tangent to the circle centre O. Given that $\angle BFE = 55^\circ$, $\angle FED = 70^\circ$ and $\angle GOB = 72^\circ$, calculate

(i) $\angle OBG$, [1]

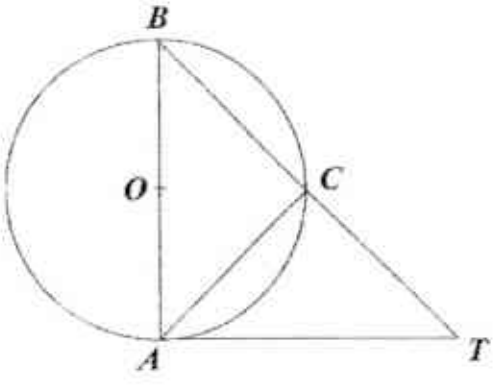
(ii) $\angle GDB$, [1]

(iii) $\angle EDG$, [1]

(iv) $\angle GHB$, [2]

(v) the sum of interior angles of the pentagon $BDEFG$. [2]

(b) In the diagram, AB is the diameter of the circle with centre O. AT is the tangent to the circle and TB cuts the circle at C.



(i) Prove that ΔABT is similar to ΔCAT. [3]

(ii) Given that $BC = 18$ cm and $AT = 12$ cm, calculate the length of CT. [3]

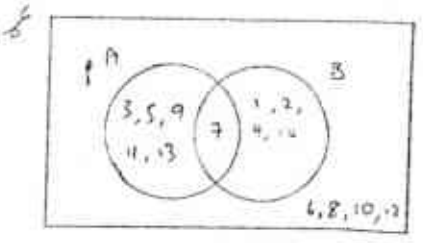
END OF PAPER 2

Answer Prelims 2015 Sec4/5 Paper 2

1. (a)(i) $A \cup B = \{1,2,3,4,5,7,9,11,13,14\}$ [B1]

(ii) $n(A \cap B') = 5$ [B2]

(b) [B2]



2. **(a)**

$$\sin\frac{\pi}{6} = \frac{4}{OC}$$

$$OC = 8cm$$

$$OD = \sqrt{8^2 - 4^2} = \sqrt{48}$$

$$OA = OB = 12cm$$

$$AD = 12 - \sqrt{48} = 5.0718$$ [M1]

$$\text{Arc } AB = r\theta = 12 \times \frac{\pi}{6} = 2\pi$$

$$\text{Angle } BCD = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$$

$$\text{Arc } BD = 4 \times \frac{2\pi}{3} = \frac{8\pi}{3}$$ [M1]

$$\text{Per of the shaded region} = = \frac{8\pi}{3} + 2\pi + 5.0718 = 19.7cm$$ [A1]

(b) $$\text{Area of sector } OAB = \frac{1}{2}(8+4)^2\left(\frac{\pi}{6}\right) = 12\pi$$ [M1]

$$\text{Area of Triangle } OCD = \frac{1}{2}\left(\sqrt{48}\right)(4) = 2\sqrt{48}$$

$$\angle BCD = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$$

$$\text{Area of sector } BCD = \frac{1}{2}(4)^2\left(\frac{2\pi}{3}\right) = \frac{16}{3}\pi$$ [M1]

$$\text{Area of shaded region} = 12\pi - 2\sqrt{48} - \frac{16}{3}\pi = 7.09 \text{ cm}^2$$ [A1]

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3. **(a)** $\begin{pmatrix} 27 & 16 \\ 33 & 21 \\ 19 & 22 \end{pmatrix}$ [B1]

(b) $\begin{pmatrix} 47 & 27 \\ 68 & 40 \\ 47 & 42 \end{pmatrix}$ [B1]

(c) Total sales of each flavour in both bowls n cones respectively. [B1]

(d) $\begin{pmatrix} 1 & 2 & 2 \end{pmatrix}$ [B1]

(e) $\begin{pmatrix} 1 & 2 & 2 \end{pmatrix}\begin{pmatrix} 27 & 16 \\ 33 & 21 \\ 19 & 22 \end{pmatrix} = \begin{pmatrix} 131 & 102 \end{pmatrix}$ [B1]

(f) Total sales for Day 2 sold in cones and bowls respectively [B1]

4 **(a)** **(i)** $RP = \sqrt{6^2 + 18^2} = 18.973665$ [M1]
$\approx 19.0cm$

$PU = \sqrt{4.8^2 + 6^2} = 7.6837$

$PT = \sqrt{18^2 + 7.6837^2} \approx 19.6cm$ [A1]

OR

$PT = \sqrt{18.97366^2 + 4.8^2} \approx 19.6cm$

(ii) $\tan \angle RPT = \dfrac{4.8}{18.9737}$ [M1]

$\angle \text{RPT} \approx 14.2^{\circ}$ [A1]

(b) $PU = QT$ (Prop of Rect), $UR = TS$ (Diagonals of Rect), [B2,1]
$PR = QS$ (Diagonals of Rect)
Therefore by (SSS), triangles PRU and QST are congruent. (proven)

(c) Vol of device $= \dfrac{1}{2} \times 4.8 \times 6 \times 18 = 259.2cm^3$ [M1]

Vol of 10 devices $= 259.2 \times 10 = 2592cm^3$ [A1]

5. (a)

$$w = \sqrt{\frac{u-v}{1+u}}$$

$w^2(1+u) = u - v$ [M1]

$u(w^2 - 1) = -w^2 - v$

$u = \frac{w^2+v}{1-w^2}$ or $\frac{-w^2-v}{w^2-1}$ [A1]

$w \neq 1$ or $w \neq -1$ [B1]

(b)

$$\frac{x+1}{2x-3} - \frac{14-3x}{(2x-3)(2x+3)} = 0$$

$$\frac{(x+1)(2x+3)-(14-3x)}{(2x-3)(2x+3)} = 0$$ [M1]

$2x^2 + 3x + 2x + 3 - 14 + 3x = 0$

$2x^2 + 8x - 11 = 0$

$x = \frac{-8 \pm \sqrt{64 - 4(2)(-11)}}{2(2)}$ [B1]

$= 1.08$ or -5.08 [A1]

(c) $2^{2x^2+6} = 2^{5x+9}$ [M1]

$\therefore 2x^2 + 6 = 5x + 9$

$(2x+1)(x-3) = 0$ [A1]

$\therefore x = -0.5$ or 3

6. (a)

Av Speed $= \frac{720}{T}$ [M1]

$60 = \frac{720}{T}$

$T = 12$ hrs [A1]

(b) $\text{Time}_{A \text{ to } B} = \frac{360}{k}$ [B1]

$\text{Time}_{B \text{ to } A} = \frac{360}{k+15}$ [B1]

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(c)

$$\frac{360}{k}+\frac{360}{k+15}=12$$

$$360(k+15)+360k=12k(k+15)$$ [M1]

$$360k+5400+360k=12k^2+180k$$ [M1]

$$0=12k^2-540k-5400$$

$$k^2-45k-450=0 \text{ (shown)}$$ [M1]

(d)

$$k^2-45k-450=0$$

$$=\frac{-(-45)\pm\sqrt{2025-4(1)(-450)}}{2(1)}$$ [M1]

$$=-8.42 \text{ or } 53.4$$ [A2]

8 **(a)(i)** $31+p+q=40$

$$p+q=9----------(1)$$ [M1]

$$2132+60p+68q=40\times 68$$

$$15p+17q=147----------(2)$$

$$(1)\times 17:\ 17p+17q=153-------(3)$$ [M1]

$$(3)-(2)\ 2p=6$$

$$p=3$$

$$q=6 \text{ (shown)}$$ [A1]

(ii) $\overline{X}=68$ [M1]

$$\text{std dev}=\sqrt{\frac{186272}{40}-(68)^2}$$ [A1]

$$=5.73kg$$

(iii)

Mean of Class Y > Mean of Class Z

∴ Wt of Class Y > Wt of Class Z

Std Dev of Class Y < Std Dev of Class Z

∴ Spread of Class Y < Spread of Class Z [B2]

(b)(i) $$P(\text{ideal weight})=\left(\frac{21}{40}\right)\left(\frac{19}{39}\right)+\left(\frac{19}{40}\right)\left(\frac{21}{39}\right)$$ [M1]

$$P(\text{ideal weight})=\frac{133}{260}$$ [A1]

(ii) $$P(\text{at least 1 student ach ideal wt})=1-\left(\frac{19}{40}\right)\left(\frac{18}{39}\right)\left(\frac{17}{38}\right)$$ [M1]

$$=\frac{469}{520}$$ [A1]

9.

(a) (i) $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = 4\mathbf{a} - 9\mathbf{b}$ [B1]

(ii) $\overrightarrow{DA} = \overrightarrow{OA} - \overrightarrow{OD} = 12\mathbf{a} - 3\mathbf{b}$ [B1]

(b) since

$$\frac{\text{Area of } \Delta ODE}{\text{Area of } \Delta ODA} = \frac{1}{4}$$

$$\frac{DE}{DA} = \frac{1}{4}$$ [M1]

$$\overrightarrow{DE} = \frac{1}{4}(12\mathbf{a} - 3\mathbf{b}) = 3\mathbf{a} - \frac{3}{4}\mathbf{b}$$

$$\overrightarrow{OE} = 3\mathbf{a} - \frac{3}{4}\mathbf{b} + 3\mathbf{b} = 3\mathbf{a} + \frac{9}{4}\mathbf{b}$$ [A1]

(c)

$$\overrightarrow{BE} = 3\mathbf{a} + \frac{9}{4}\mathbf{b} - 9\mathbf{b} = 3\left(\mathbf{a} - \frac{9}{4}\mathbf{b}\right)$$

$$\overrightarrow{EC} = \overrightarrow{OC} - \overrightarrow{OE} = 4\mathbf{a} - 3\mathbf{a} - \frac{9}{4}\mathbf{b} = \mathbf{a} - \frac{9}{4}\mathbf{b}$$ [M1]

$$\therefore \frac{BE}{EC} = \frac{3}{1}$$ [A1]

(d)

$$\frac{\text{Area of } \Delta OEC}{\text{Area of } \Delta OEB} = \frac{\frac{1}{2} \times EC \times h}{\frac{1}{2} \times BE \times h} = \frac{1}{3}$$ [M1]

$$\frac{\text{Area of } \Delta ODE}{\text{Area of } \Delta OEB} = \frac{\frac{1}{2} \times OD \times h}{\frac{1}{2} \times OB \times h} = \frac{1}{3}$$ [M1]

$$\therefore \frac{\text{Area of } \Delta OEC}{\text{Area of } \Delta ODE} = \frac{1}{1}$$ Hence, they are equal. [A1]

(e)

$$\therefore \frac{\text{Area of } \Delta BDE}{\text{Area of } \Delta BOC} = \frac{\text{Area of } \Delta BDE}{\text{Area of } \Delta BDE + \Delta ODE + \Delta OCE}$$ [M1]

$$= \frac{2}{2+1+1} = \frac{1}{4}$$ [A1]

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10 (a) $\angle PWO = 280^\circ - 180^\circ - 40^\circ$ (alt $\angle$s) [M1]

$= 60^\circ$ [A1]

(b) $\frac{OP}{\sin 60} = \frac{200}{\sin 80}$ [M1]

$OP = 175.877$

$= 176$ cm [A1]

(c) $\tan 15 = \frac{OT}{175.877}$ [M1]

$\therefore\ OT = 47.1261 \approx 47.1m$ [A1]

(d) $OW^2 = 175.877^2 + 200^2 - 2(175.877)(200)\cos 40$ [M1]

$= 130.5407$

$\tan \angle OWT = \frac{47.1261}{130.5407}$ [M1]

$\angle$ of depression $= 19.8494$

$\approx 19.8^\circ$ [A1]

(e) $\tan \angle OWT = \frac{47.1261}{OR}$ [M1]

$= 2.67.265$

$\cos \angle OWR = \frac{130.5407^2 + 150^2 - 267.265^2}{2(130.5407)(150)}$ [M1]

$\angle OWR = 144.518 \approx 144.5^\circ$ [A1]

(f)

$\angle OWN = 180 - 100$

≈ 80

$\theta = 144.518 - 80$ [B1]

$\approx 064.5^\circ$

11. (a) (i) $\angle OBG = \frac{180 - 72}{2} = 54^\circ$ ($\angle$ sum of isoc Δ) [B1]

(ii) $\angle GDB = \frac{72}{2} = 36^\circ$ ($\angle$ at ctr $= 2\angle$s at cir.) [B1]

(iii) $\angle GFB = 36^\circ$ ($\angle$ in same seg)

$\angle FDG = 180 - 36 - 55 = 89^\circ$ (opp$\angle$ cyclic quad) [B1]

(iv) $\angle FHD = 360 - 70 - 55 - 89 = 146^\circ$ ($\angle$sum of Quad) [M1]

$\angle GHB = \angle FHD = 146^\circ$ (vert opp $\angle$s) [A1]

(v) sum of int $\angle$s of $BDEFG = (5 - 2)180$ [M1]

$= 540^\circ$ [A1]

(b) **(i)** $\angle BAT = 90^{\circ}$ (tan $\perp$ *rad*) [M1]

$\angle BCA = 90^{\circ}$ ($\angle$ *in semicircle*) [M1]

$\angle ACT = 90^{\circ}$ (*adj*$\angle s$ on a st line)

$\angle BAT = \angle ACT = 90^{\circ}$

$\angle ATB = \angle CTA$ (*common*$\angle$) [M1]

$\therefore \Delta ABT$ is similar to ΔCAT

(ii) $\dfrac{CT}{AT} = \dfrac{AT}{BT}$ [M1]

$\dfrac{CT}{12} = \dfrac{12}{18+CT}$

$CT^2 + 18CT - 144 = 0$ [M1]

$(CT + 24)(CT - 6) = 0$

$CT = -24$ (NA) or 6 cm [A1]

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ANDERSON SECONDARY SCHOOL
Preliminary Examination 2015
Secondary Four Express & Five Normal

CANDIDATE NAME: _____

CLASS: _____ / _____ INDEX NUMBER: _____

MATHEMATICS 4016/01

Paper 1 24 August 2015

2 hours

0800 – 1000h

Additional Materials: Writing paper

READ THESE INSTRUCTIONS FIRST

Write your name, centre number and index number in the spaces at the top of this page and on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
If working is needed for any question it must be **neatly and clearly** shown in the space below the question.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

This document consists of 17 printed pages.

23

Setter: Mdm Wong Lai Fong

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

For Examiner's Use

Answer **ALL** questions.

For Examiner's Use

1 Consider the following numbers:

$$27^{\frac{1}{3}},\ \pi,\ \frac{22}{7},\ 3.1\dot{},\ 3.33,\ \sqrt{10}$$

(a) Write the above numbers in order of size, smaller first.

(b) State which of the above numbers are irrational.

Answer (a) _____ [1]

(b) _____ [1]

2 (a) Evaluate $\sqrt{\frac{1-\sin 12.3}{e^3}}$, giving your answer correct to 3 significant figures.

(b) At room temperature, an oxygen atom in the air travels 500 metres per second. Find the time taken, in minutes, for an oxygen atom to travel 750 megametres in room temperature. Give your answer in standard form.

Answer (a) _____ [1]

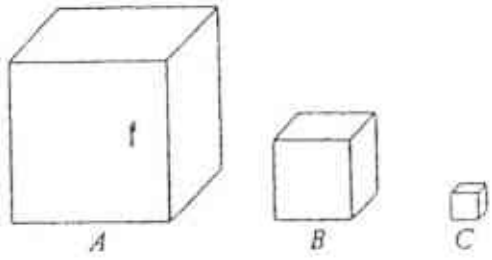
(b) _____ min [1]

For Examiner's Use

3 The weekly sales of three brands of bread A, B and C are shown in the following table:

Bread	A	B	C
Sales (in number of loaves)	600	300	100

Brady draws three *geometrically similar* cubes, labelled A, B and C (as shown below) to represent the above information on the weekly sales of the three brands A, B and C respectively. Explain why this representation is misleading.



Answer _____

_____

_____ [1]

4 Given a sequence

$$2, \frac{3}{2}, \frac{5}{4}, \frac{9}{8}, \frac{17}{16}, \ldots$$

(a) Write down the next term in the sequence.

(b) One of the terms in the sequence is $\frac{x}{y}$.

Write down, in terms of y only, the term that is immediately before $\frac{x}{y}$.

(c) Without finding further terms, explain if $\frac{201}{200}$ is a term in the sequence.

Answer (a) _____ [1]

(b) _____ [1]

(c) _____ [1]

5 (a) Solve $(x-2)^6 = \pi$, leaving your answer(s) correct to 4 significant figures.

(b) Given that $3^{\sqrt{m}} = 4 - e^0$, find the value(s) of m.

Answer (a) $x =$ _____ [2]

(b) $m =$ _____ [2]

6 (a) A piece of string can be cut into exact number of 15-cm pieces.
Find the least number of 12-cm pieces that the original piece of string could be cut into.

(b) Solve the inequalities $\frac{x}{2} + 1 \leq 2x - 5 < 5(x+1) - 22$.

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Answer (a) _____ [1]

(b) _____ [3]

For Examiner's Use

7 (a) A polygon has n sides. Two of its exterior angles are 70° and 80°, and the remaining exterior angles are each 14°. Find the value of n.

(b) A contractor is looking at two different type of tiles, P and H.
Each tile of type P is in the shape of a regular pentagon while that of type H is a regular hexagon.
If the contractor needs to fit the same type of tiles together on a floor without any gaps, explain which type of tiles he should use.

Answer (a) $n =$ _____ [2]

(b) _____

_____

_____

_____

_____ [2]

8 An area of 12.5 m^2 is represented by 8 cm^2 on a map.

(a) Calculate, in m^2, the actual **exact** area of a field which is represented by a circle of area 16π cm^2.

(b) Find the scale of the map in the form $1 : n$

Answer (a) _____ m^2 [1]

(b) _____ [2]

For Examiner's Use

9 (a) Given that $t = 1 \pm \sqrt{\frac{x}{y^3}}$, make x the subject.

(b) Express $\frac{2}{4m^2 - 1} - \frac{3}{1+2m}$ as a single fraction in its simplest form.

Answer (a) _____ [2]

(b) _____ [2]

For Examiner's Use

10 A survey was carried out with a group of drivers to find out the average number of cups of coffee they consume daily. The survey findings are as shown in the table.

Number of cups	0	1	2	3	4
Number of drivers	2	8	7	3	1

(a) Find
(i) the mode,
(ii) the median.

(b) A driver who consumes x cups of coffee daily is later excluded from the survey. If the mode and the median remain unchanged, find the possible value(s) of x.

Answer (a) (i) _____ [1]

(ii) _____ [1]

(b) _____ [2]

11 (a) If Nathan sells his branded watch for \$7500, he will lose 30%.
Find the amount he must sell his watch in order for him to make a profit of 40%.

(b) Nathan borrowed \$12 000 from a money-lender who charged a compound interest of R% per month.
After $2\frac{1}{3}$ years, Nathan found that he owed a total of \$15 000.
Calculate the value of R.

Answer (a) \$ _____ [2]

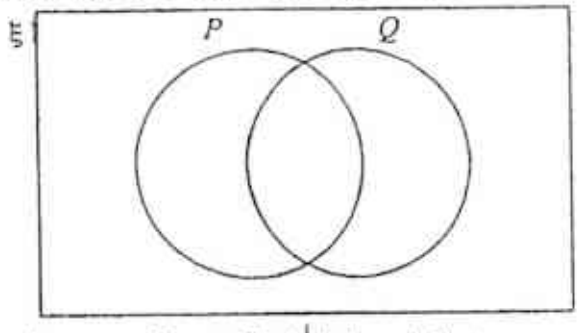
(b) R = _____ [2]

For Examiner's Use

12 (a) Given that ξ = {all points on a plane},
L = {all points on a given straight line},
and C = {all points on the circumference of a given circle}.
Write down the possible value(s) of $n(L \cap C)$.

(b) Given that $A = \{(x, y): y = \sin x\}$ and $B = \{(x, y): y = \cos(k - x)\}$.
Write down a possible value of k, other than $k = 0$, such that $A = B$.

(c) The Venn diagram shows a universal set ξ and the two sets P and Q.



In this Venn diagram, add a set T which is such that
$P \cap T = \varnothing$ and $Q \cap T = T$.

Answer (a) _____ [1]

(b) _____ [1]

(c) *In the diagram above.* [2]

13 Solve the equations

(a) $5x = 10x^2$

(b) $(y^2 + 1)(y^2 - 1) = 0$

Answer (a) $x =$ _____ [2]

(b) $y =$ _____ [2]

For Examiner's Use

14 Given that $\mathbf{P} = \begin{pmatrix} 3 & 1 & -2 \end{pmatrix}$ and $\mathbf{L} = \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix}$.

(a) Calculate **LP**.

(b) M is a $a \times b$ matrix which has elements that are either 1 or 0 such that

$$\mathbf{ML} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}.$$

(i) State the value of a and of b.

(ii) Find M.

Answer (a) _____ [1]

(b) (i) $a =$ _____ ; $b =$ _____ [2]

(ii) _____ [1]

For Examiner's Use

15 The point (0, 1) is marked on each axes in the answer space.
On these axes, sketch the graphs of

(a) $2(x + y) = 1$

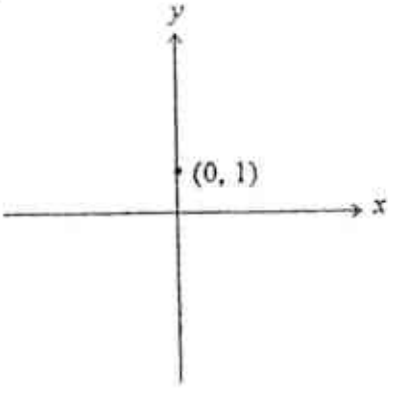
(b) $y = \frac{1}{x} + 1$

(c) $y = 2^{-x}$

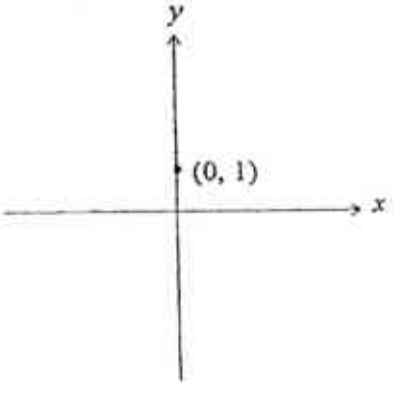
[3]

Answer

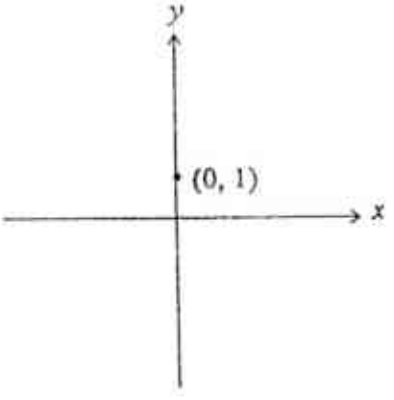
(a)



(b)

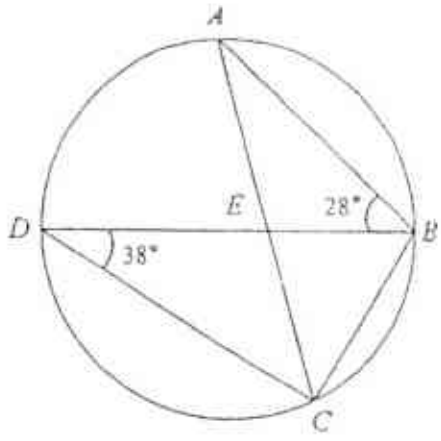


(c)



For Examiner's Use

16 Given A, B, C and D are points on the circumference of the circle such that $\angle BDC = 38°$, $\angle ABD = 28°$ and $\angle BCD = 90°$.
Chords AC and BD intersect at E.
Using appropriate geometrical reasons,
(a) find $\angle ACD$,
(b) explain where the centre of the circle is,
(c) determine, without further calculation, if EC is longer than ED.



Answer (a) $\angle ACD =$ _____ (Reason: _____) [1]

(b) _____
_____
_____ [1]

(c) _____
_____
_____ [1]

17 A mark, M on a cliff is p m below the sea level at high tide.
At low tide, M is above sea level and the sea level is q m lower than at high tide.
(a) Given that p and q are positive integers, find, in terms of p and q, the distance of M above sea level at low tide.
(b) If $p = 1.3$ and $q = 2.8$, calculate how far M is above or below sea level when the sea level is exactly halfway between high tide and low tide.

Answer (a) _____ m [1]

(b) _____ [1]

For Examiner's Use

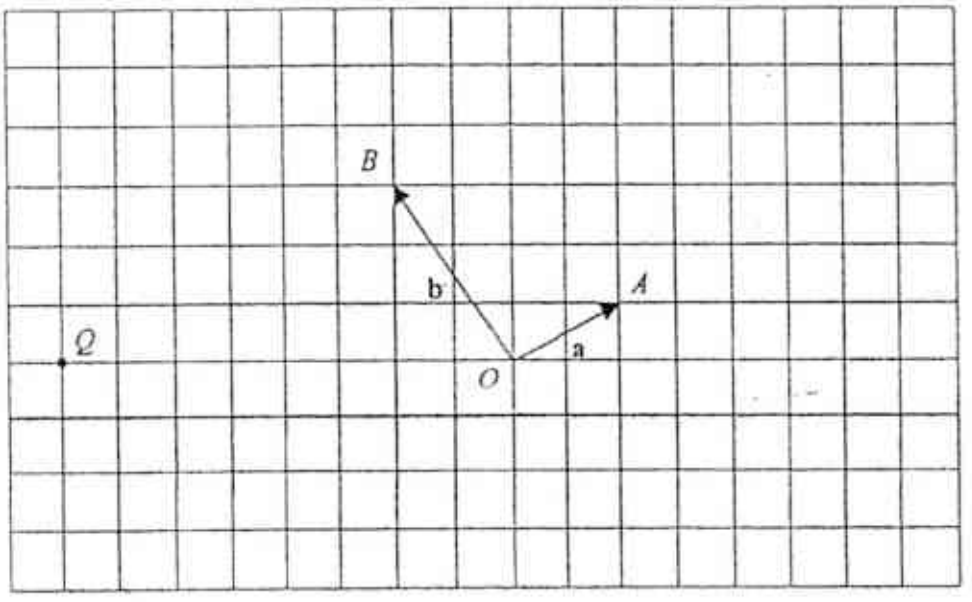
18 In the diagram, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Mark and label clearly in the diagram, the point P such that $\overrightarrow{OP} = 2\mathbf{a} - \mathbf{b}$.

(b) Determine the value of λ such that $\overrightarrow{OQ} = \lambda\mathbf{a} + \mathbf{b}$.

(c) If $\mathbf{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, find $\left|\overrightarrow{AQ}\right|$.

Answer (a)



Answer (a) *In the diagram above.* [1]

(b) $\lambda =$ _____ [1]

(c) _____ [1]

For Examiner's Use

19 (a) A man is deciding which of the following options will give him more water flowing into his flowerbed:

Option A	Option B
A hose with a diameter of 8 cm	Two hoses with diameter of 5 cm each.

The man chooses option B and his reasoning is that the two hoses have a bigger combined diameter of $5 + 5 = 10 > 8$. Is he right? Explain.

(b) (i) Sketch the graph of $y = 9 - (2 - x)^2$ on the axes provided, indicating clearly its turning point and the intercepts on the axes.

(ii) The graph of $y = 9 - (2 - x)^2$ is reflected in the y-axis.
Write down the coordinates of the turning point of the new graph.

For Examiner's Use

Answer (a) _____

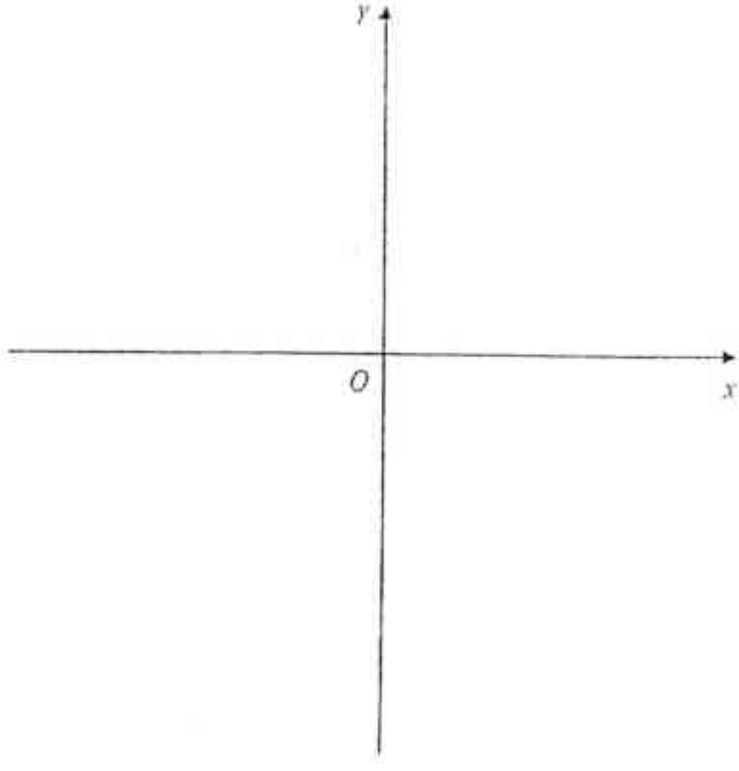
_____

_____

_____

_____ [2]

(b) (i)



[2]

(ii) _____ [1]

For Examiner's Use

20 (a) A researcher claims that the weight loss, W, of a mice in an experiment is directly proportional to the square of the dosage, D, of a herb administered to this mice. If the weight loss is to be multiplied by 10, find the factor the dosage should be multiplied.

(b) It is thought that x is inversely proportional to y^n.
The following table shows some values of x and the corresponding values of y.

x	0.5	2
y	1	0.5

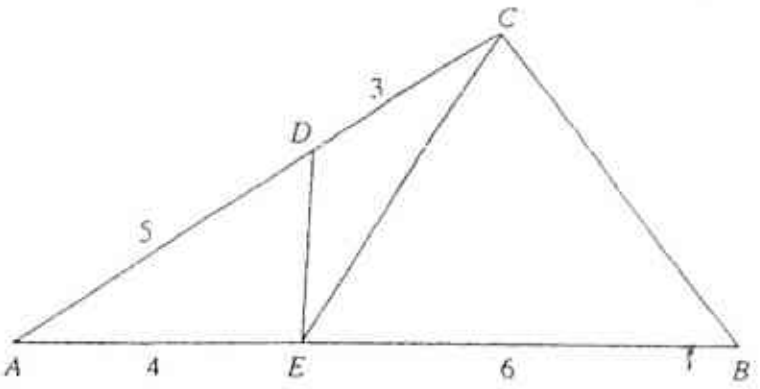
Form an equation relating x and y.

Answer (a) _____ [2]

(b) _____ [2]

For Examiner's Use

21 In triangle ABC, $AE = 4$ cm, $BE = 6$ cm, $AD = 5$ cm and $CD = 3$ cm.



(a) Name a pair of similar triangles and show that they are similar.

(b) Given that area of ΔCDE is x cm^2, find the area of ΔBCE in terms of x.

(c) A point is selected at random from inside ΔABC.
Find the probability that the point lies inside ΔADE.

For Examiner's Use

Answer (a) _____

_____

_____ [3]

(b) _____ cm^2 [3]

(c) _____ [1]

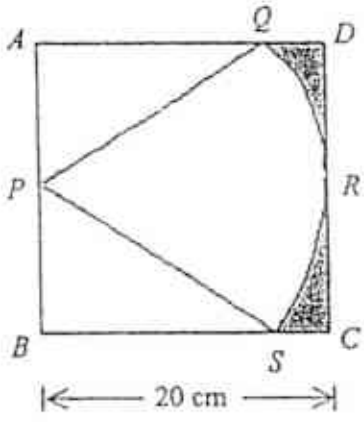
For Examiner's Use

22 In the figure, P is a mid-point of AB. Sector $PQRS$, with centre P, is inscribed in the square $ABCD$ of side 20 cm.

(a) State the exact value of $\cos \angle QPB$.

(b) Find $\angle QPS$ in radians.

(c) Express the shaded region as a percentage of the area of square $ABCD$.



For Examiner's Use

Answer (a) _____ [1]

(b) _____ [2]

(c) _____ [3]

ANDERSON SECONDARY SCHOOL
2015 Preliminary Examination
Secondary Four Express / Five Normal
MATHEMATICS PAPER 1 (4016/01)

Answer Key

1 (a) $27^{\frac{1}{3}}$, 3.1, π, $\frac{22}{7}$, $\sqrt{10}$, 3.33

(b) π, $\sqrt{10}$

2 (a) 0.0923

(b) 2.5×10^{4}

3 Sales represented by the heights of the similar cubes or by the volumes

4 (a) $\frac{33}{32}$

(b) $\frac{y+2}{y}$

(c) denominator is not 2*

5 (a) 3.210 or 0.7898

(b) 1

6 (a) 5

(b) $x>4$

7 (a) 17

(b) Interior $\angle$ of tile $P = 108°$
Interior $\angle$ of tile $H = 120°$
Since 360 is divisible by 120 but not by 108, use type H that fit together without gaps.

8 (a) 25π

(b) 1 : 125

9 (a) $x = y^3(t-1)^2$

(b) $\frac{5-6m}{(2m+1)(2m-1)}$

10 (a) (i) 1

(ii) 2

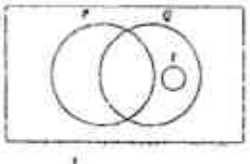
(b) 0

11 (a) 15 000

(b) 0.800

12 (a) 0, 1 or 2

(b) Any odd multiple of $\frac{\pi}{2}$

(c) 

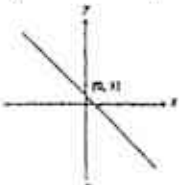
13 (a) 0 or $\frac{1}{2}$

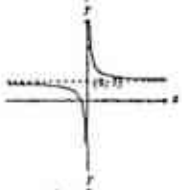
(b) ±1

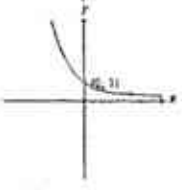
14 (a) $\begin{pmatrix} 6 & 2 & -4 \\ 12 & 4 & -8 \\ 9 & 3 & -6 \end{pmatrix}$

(b) (i) $a = 3$, $b = 3$

(ii) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

15 (a) 

(b) 

(c) 

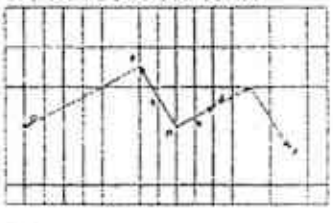
16 (a) 28°, angles in same segment

(b) Since $\angle BCD = 90°$ (rt. $\angle$ in semicircle), DB is a diameter.
Centre is midpoint of DB.

(c) Since $\angle EDC > \angle ECD = 28°$, EC is longer than ED.

17 (a) $q - p$

(b) 0.1 m above sea level

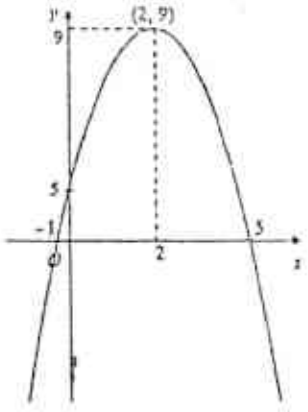
18 (a) 

(b) −3

(c) 10.0

19 (a) Wrong. Cross-sectional area of A > cross-sectional of B

19 (b) (i)



(ii) (−2, 9)

20 (a) 3.16

(b) $x = \frac{1}{2y^2}$

21 (a) $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{2}$ (given)

$\angle DAE = \angle BAC$ (common)

ΔABC is similar to ΔADE (SAS)

(b) $4x$

(c) $\frac{1}{4}$

22 (a) $-\frac{1}{2}$

(b) $\frac{\pi}{3}$

(c) 4.34%

ANDERSON SECONDARY SCHOOL
Preliminary Examination 2015
Secondary Four Express & Five Normal

CANDIDATE NAME:

CLASS: / INDEX NUMBER:

MATHEMATICS 4016/01

Paper 2

25 August 2015

2 hours 30 minutes

0800 – 1030h

Additional Materials: Writing paper (10 sheets)
Graph paper (2 sheets)

READ THESE INSTRUCTIONS FIRST

Write your name, centre number and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
If working is needed for any question it must be **neatly and clearly** shown in the space below the question.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer all the questions.

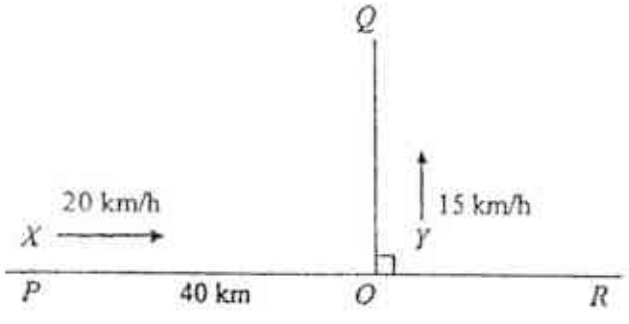
1 (a) A straight line passes through the points $A(8, 4)$ and $B(14, 12)$.

(i) Another line, parallel to the y-axis and passing through the point $(9, 2)$, meets the line segment AB at the point D.
Calculate the coordinates of D. [2]

(ii) The line segment AB undergoes a clockwise rotation R about the point A such that B', the image of B, lies on the negative y-axis.
Find the coordinates of B'. [2]

(iii) Hence, find the equation of the image of the line AB under rotation R. [1]

(b) The diagram shows a road junction at O with PR perpendicular to OQ and $OP = 40$ km. A cyclist X starting from P travels towards R at a constant speed of 20 km/h. At the same time, another cyclist Y starting from O travels towards Q at a constant speed of 15 km/h.



(i) Find the distance between the cyclists 1 hour after the start. [1]

(ii) Given that after t hours, the cyclists are 60 km apart, show that $25t^2 - 64t - 80 = 0$. [3]

(iii) Solve $25t^2 - 64t - 80 = 0$ and hence find the distance cyclist X is from O when the cyclists are 60 km apart. [3]

2 (a) Michelle wants to buy a car which costs $130 000.
She decides to make a 20% down payment and take a loan from Ah Long Finance Company to pay for the remaining cost of the car.

The table below shows the monthly repayment plan (per $1 000 borrowed) based on a simple interest rate of 3.3% per annum offered by Ah Long Finance Company.

Term of loan (in years)	3	5	7	10
Monthly repayment (for every $1000 borrowed)	$30.53	$19.42	$m	$11.08

(i) Find the value of m. [2]

(ii) Using the table above, calculate the amount of interest Michelle will have to pay if she chooses the term of loan of 10 years. [2]

(b) Math Powerhouse supplies electricity to households and each household has to pay a fixed charge of $12 and a further charge of 12 cents for each unit of electricity used in a month.

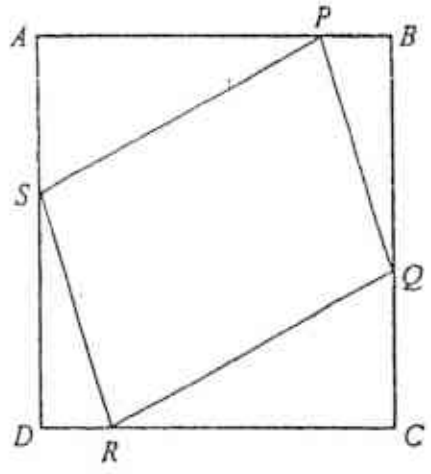
(i) Write down a formula connecting the total cost, C dollars, and the number of units, n, of electricity used. [1]

(ii) Calculate the number of units used when the total cost was $31.92. [1]

(iii) Math Powerhouse offers another method of payment: 17 cents for each unit of electricity used in a month but no fixed charge.
Each household can choose to pay either using the method with fixed charge or the one without.
The Wong household uses N units of electricity each month.

Suggest to the Wong household which method of payment is a better choice. [3]

3



In the diagram, $ABCD$ is a rectangle. Points P, Q, R and S lie on AB, BC, CD and DA such that $AP = BQ = CR = DS$.

(a) Showing all your reasons clearly, prove that

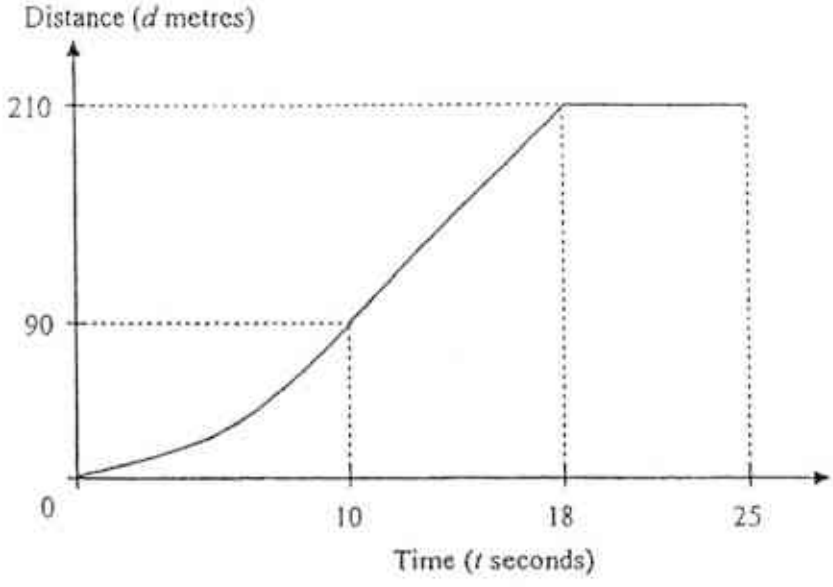
(i) $AS = CQ$. [2]

(ii) triangle APS is congruent to triangle CRQ. [3]

(b) Write down two reasons to prove that $PQRS$ is a parallelogram. [2]

35

4 The diagram below shows the distance-time graph of an object A for the first 25 seconds of its motion along a straight path. The object increases its speed uniformly from 3 m/s for the first 10 seconds of the journey.



(a) Find the average speed of object A for the first 25 seconds of its journey. [1]

(b) Find the speed of object A when $t = 15$. [1]

(c) Find the acceleration of object A when $10 < t < 18$. [1]

(d) On a sheet of graph paper, draw the speed-time graph of object A for the first 25 seconds, using a scale of 2 cm to represent 5 seconds on the horizontal axis and a scale of 2 cm to represent 5 m/s for the vertical axis. [3]

Another object B also starts its motion together with object A along the same straight path. The speed of object B, v m/s, is given by $v = t$.

(e) On the same axes drawn in (d), draw the speed-time graph of object B for the first 25 seconds. [1]

(f) Write down the significance of the point where the two graphs intersect. [1]

(g) Without any calculation, use your graphs to explain which object travels a greater distance during the first 15 seconds. [1]

5 **Answer the whole of this question on a sheet of graph paper.**

Anderson Printing Company makes of a profit of y thousand dollars from the printing of x thousand T-shirts where $y = 8 - x - \frac{10}{x+1}$.

The table below shows some corresponding values of x and y for this equation.

x	0	0.5	1	2	3	4	5	6	7
y	−2	0.83	2	2.67	2.5	2	1.33	0.57	−0.25

(a) Explain what does the value of −2 for y signifies. [1]

(b) Using a scale of 2 cm to represent 1 unit on the x-axis and 2 cm to represent 0.5 unit on the y-axis, draw the graph of $y = 8 - x - \frac{10}{x+1}$ for $0 \le x \le 7$. [3]

(c) Using your graph, find

(i) the solution(s) of the equation $8 - x - \frac{10}{x+1} = 0$, [1]

(ii) the number of T-shirts the company should print in order to obtain the maximum profit. [1]

(d) Explain the significance of the solution(s) in (c)(i). [1]

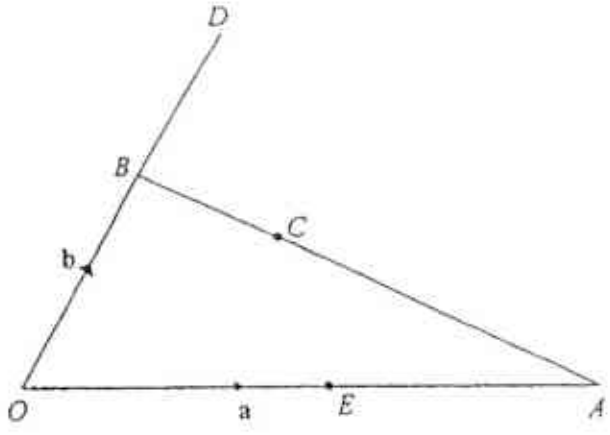
(e) By drawing a suitable straight line, find the number of T-shirts the company would be printing when the profit per shirt is $1. [2]

(f) (i) Determine graphically the x-coordinate of point P on the curve such that the gradient of the curve at P is −0.5. [2]

(ii) State what the value of −0.5 in part (i) represents. [1]

36

6 In the diagram, OAB is a triangle. C is a point on AB such that $AC:CB = 2:1$. The side OB is produced to the point D such that $OB:BD = 3:2$.



It is given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Express, as simply as possible, in terms of a and/or b,

(i) $\overrightarrow{AB}$, [1]

(ii) $\overrightarrow{AC}$, [1]

(iii) $\overrightarrow{OC}$, [1]

(iv) $\overrightarrow{OD}$. [1]

(b) Show that $\overrightarrow{CD} = \mathbf{b} - \frac{1}{3}\mathbf{a}$. [2]

(c) It is given that E is a point on OA such that $\overrightarrow{OE} = \frac{5}{9}\mathbf{a}$.

Express, as simply as possible, in terms of a and b, the vector $\overrightarrow{ED}$. [1]

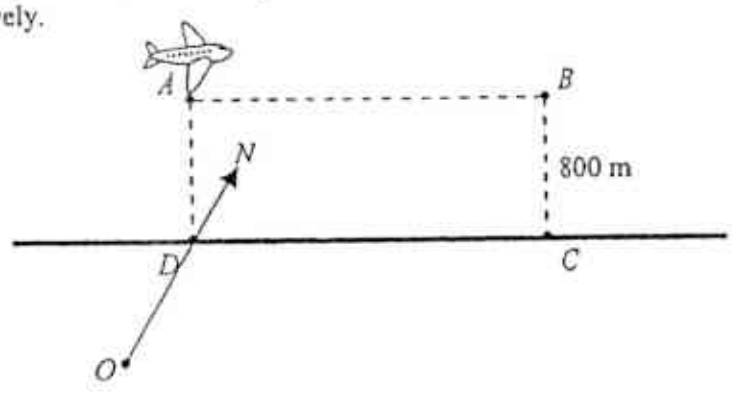
(d) With reasons clearly provided, show that D, C and E are collinear. [2]

(e) Given that $|\mathbf{a}| = 8$, $|\mathbf{b}| = 6$ and $\angle BOA = 60°$, calculate

(i) $\left|\overrightarrow{AB}\right|$, [2]

(ii) the area of triangle ABE. [2]

7 Three points O, C and D lie on the ground such that D is due north of O and C is due east of D. An aeroplane flies eastwards from A, a point 800 m directly above D. After 30 seconds, it reaches a point B, a point 800 m directly above C.
The angles of elevation of the plane from O at A and at B are 45° and 30° respectively.



Calculate

(a) the distance OD and OC, [2]

(b) the bearing of C from O, [2]

(c) the distance AB travelled by the plane, giving your answer to the nearest metre, [2]

(d) the speed of the plane, giving your answer to the nearest km/h. [2]

---

8 Mr Eu is planning to make a pet feeding bowl for his terrier.
He first obtains a solid wooden right circular cone with base diameter 50 cm and vertical height 50 cm as shown in Diagram I.

To make the feeding bowl, a smaller right circular cone of radius r cm is first removed from the original cone to form a frustum of height h cm as shown in Diagram II.

A hemispherical depression of r cm is then removed from the frustum to make a solid wooden feeding bowl as shown in Diagram III.

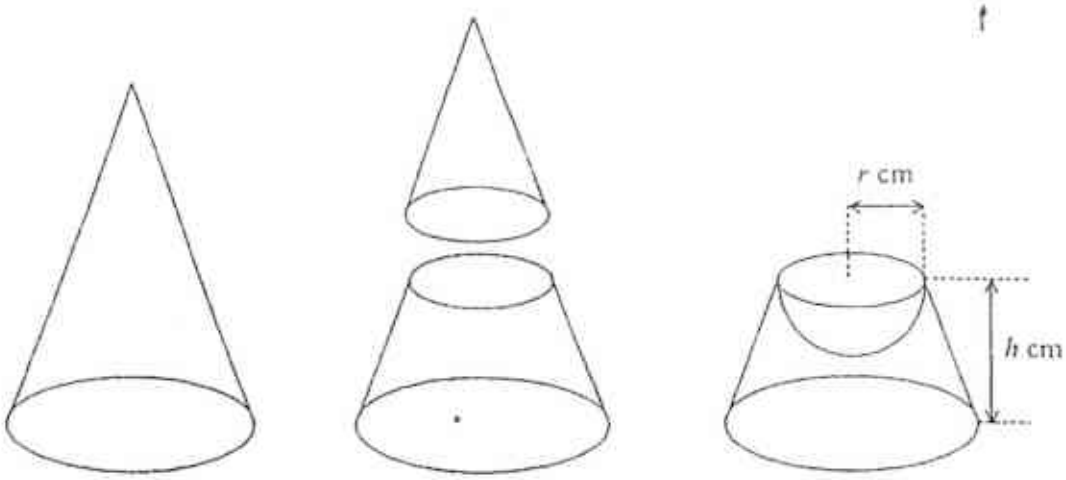


Diagram I Diagram II Diagram III

(a) Find the volume of the original wooden cone.
Leave your answer in terms of π. [1]

(b) Given that the volume of the cone removed is 21.6% of the original cone, show that the height of the frustum is 20 cm. [2]

(c) Find the volume of the hemispherical portion removed. [2]

(d) Mr Eu wants to paint the entire (both inner and outer) feeding bowl red.
Calculate the total surface area to be painted. [3]

(e) After using the feeding bowl for a week, Mr Eu thinks that the hemispherical volume created to fill the dog feeds is insufficient for his greedy terrier.
He says, "I should have created a hemispherical depression of radius 18 cm."
Explain to Mr Eu, with mathematical evidence, why his hypothetical feeding bowl cannot be created. [2]

9 Two bus companies use 3 different types of buses – small, medium and big. The table below shows the number of bus trips run on 3 consecutive days by the two bus companies.

			Bus Company	
			SAS	SNRT
No. of bus trips	Day 1	Small	8	15
		Medium	20	30
		Big	6	5
	Day 2	Small	10	15
		Medium	20	35
		Big	12	6
	Day 3	Small	6	8
		Medium	13	14
		Big	9	5

The information for the number of trips run on Days 1, 2 and 3 can be represented by the matrices $P = \begin{pmatrix} 8 & 15 \\ 20 & 30 \\ 6 & 5 \end{pmatrix}$, $Q = \begin{pmatrix} 10 & 15 \\ 20 & 35 \\ 12 & 6 \end{pmatrix}$ and $R = \begin{pmatrix} 6 & 8 \\ 13 & 14 \\ 9 & 5 \end{pmatrix}$ respectively.

(a) Evaluate $A = \frac{1}{3}(P + Q + R)$ and describe what is represented by the elements of A. [2]

(b) Let $B = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$.

(i) Explain which matrix, 3AB or 3BA, exists. [1]

(ii) Describe what the elements of the matrix that exists in (b)(i) represent. [1]

(c) The number of passengers per trip on a small, medium and big bus is 12, 25 and 50 respectively.

Write down the product of P and two other matrices such that the elements in this product represent the number of passengers the two bus companies carry on Day 1. **You need not** evaluate this product. [2]

(d) Company SAS charges \$22 per passenger while company SNRT charges \$25 per passenger.

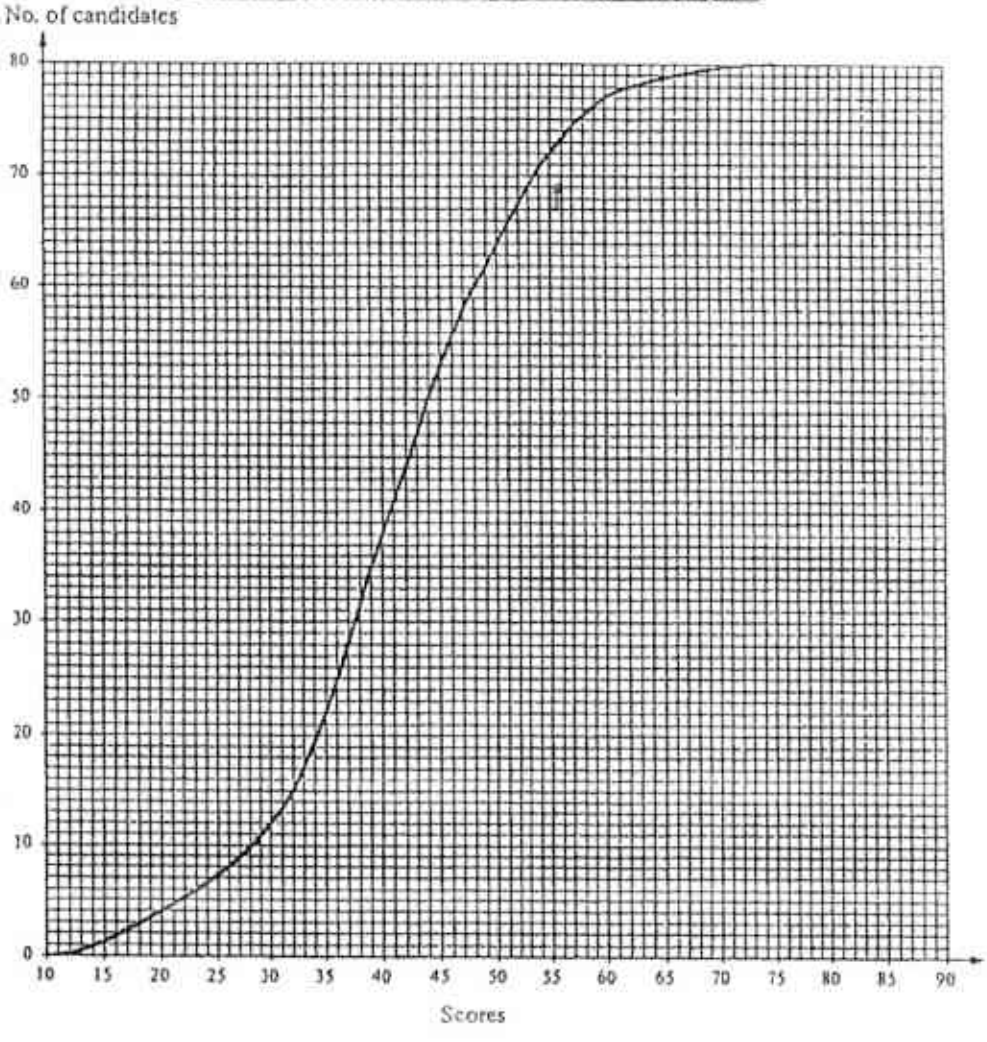
(i) Write down, but **do not** evaluate, the product of 3 matrices that gives the total amount each bus company collects on Day 1. [2]

(ii) Let the matrix in (d)(i) be M.

Explain what the matrix $M\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ represents. [1]

10 The cumulative distribution of scores gained by a group of 80 candidates in the first round of aptitude test for admission to ANDE School is shown below.

Cumulative distribution of scores of 80 candidates



(a) Using the cumulative frequency curve, find

(i) the median score, [1]

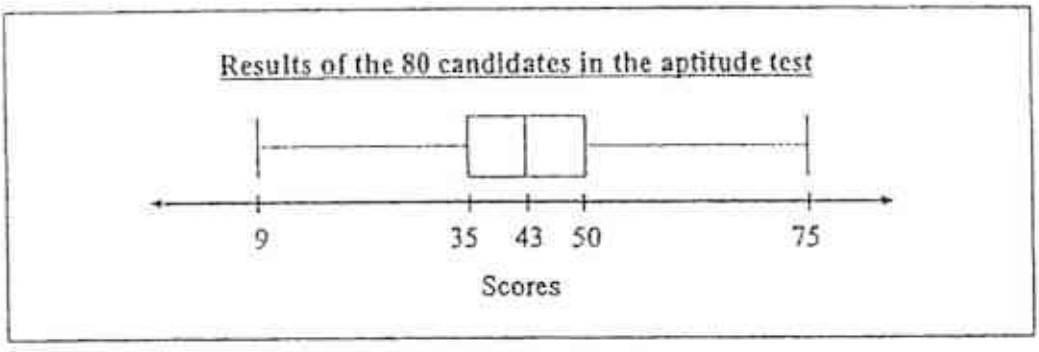
(ii) the interquartile range. [1]

(b) If 20% of the candidates qualified for the direct admission, use your graph to estimate the qualifying score for direct admission to ANDE School. [1]

(c) Candidates who scored at least 35 proceeded to the second round of test. Find the probability that a candidate, selected at random, failed to proceed to the second round of test by a score difference of 2. [2]

(d) Several days later, it was noticed that there was an error in the marking, and that all candidates should get 3 more marks in the first round of test. State what adjustment, if any, should be made to the above cumulative frequency graph to reflect the correct information. [1]

(e) Another group of 80 candidates also took the same aptitude test in the first round. The results of this second group are illustrated in the box-and-whisker plot below.



(i) Write down the median and the range of this distribution. [2]

(ii) If the probability that a candidate chosen random scored between a and b is 0.5, suggest a possible pair of values of a and b. [1]

(iii) Mr Loh compared the scores of the second group of candidates with the corrected scores of the first group and remarked that the performance of the two groups was comparable.
Do you agree with him? Justify your answer. [2]

24

39

ANDERSON SECONDARY SCHOOL
2015 Preliminary Examination
Secondary Four Express / Five Normal
MATHEMATICS PAPER 2 (4016/02)

Answer Key

1 (a) (i) $\left(9, 5\frac{1}{3}\right)$

(ii) $(0, -2)$

(iii) $y = \frac{3}{4}x - 2$

(b) (i) 25 km

(iii) $t = 3.48$; 29.6 km

2 (a) (i) $m = 14.65$

(ii) \$34278.40

(b) (i) $C = 12 + 0.12n$

(ii) $n = 166$

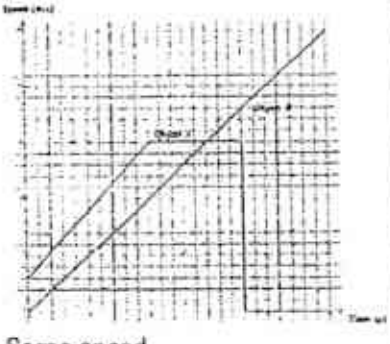
(iii) If uses 240 units each month, either payment; if uses more than 240 units, use fixed charge payment, else use no fixed charge payment.

4 (a) $8\frac{2}{5}$ m/s

(b) 15 m/s

(c) 0 m/s^2

(d), (e)

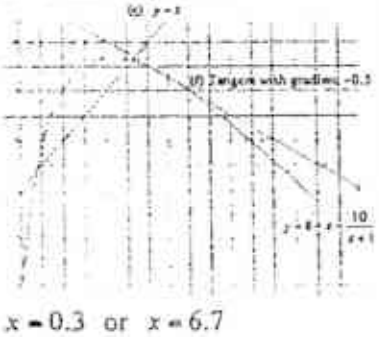


(f) Same speed

(g) Object A

5 (a) Loss of \$2000 incurred when no T-shirt is printed.

(b)



(c) (i) $x = 0.3$ or $x = 6.7$

(ii) 2150 T-shirts

5 (d) The company makes a profit when it prints more than 300 but less than 6700 T-shirts.

(e) 400 or 2600 T-shirts

(f) (i) $x = 3.5$

(ii) The rate of decrease of profit per T-shirt printed.

6 (a) (i) $b - a$

(ii) $\frac{2}{3}(b - a)$

(iii) $\frac{1}{3}(a + 2b)$

(iv) $\frac{5}{3}b$

(c) $\frac{5}{3}(b - \frac{1}{3}a)$

(e) (i) $\sqrt{52}$

(ii) 9.24 units^2

7 (a) $OD = 800$ m; $OC = 800\sqrt{3}$ m

(b) 054.7°

(c) 1131 m

(d) 136 km/h

8 (a) $\frac{31250\pi}{3}$ cm^3

(c) 7070 cm^3

(d) 6190 cm^3

(e) No. If the hemispherical depression is of radius 18 cm, the height of the cone to be removed would be 36 cm. Then the height of the frustum would be only 14 cm, which is insufficient to accommodate a hemispherical depression of 18 cm.

40

9 (a)
$$\begin{pmatrix} 8 & 12\frac{2}{3} \\ 17\frac{2}{3} & 26\frac{1}{3} \\ 9 & 5\frac{1}{3} \end{pmatrix}$$

The average number of trips per day made by each type of buses run by each of the two bus companies.

(b) 3BA exists since the dimension of matrix B is 1×3 and the that of A is 3×2.

(c)
$$\begin{pmatrix} 12 & 25 & 50 \end{pmatrix}\begin{pmatrix} 8 & 15 \\ 20 & 30 \\ 6 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

(d) (i)
$$\begin{pmatrix} 12 & 25 & 50 \end{pmatrix}\begin{pmatrix} 8 & 15 \\ 20 & 30 \\ 6 & 5 \end{pmatrix}\begin{pmatrix} 22 & 0 \\ 0 & 25 \end{pmatrix}$$

(ii) The amount bus company SNRT collects more than SAS does on Day 1.

10 (a) (i) 40.5

(ii) 14

(b) 50

(c) $\frac{1}{16}$

(d) Move the graph 3 marks to the right.

(e) (i) Median = 43; Range = 66

(ii) Possible pairs are:
(1) $a = 9$ and $b = 43$
(2) $a = 35$ and $b = 50$
(3) $a = 43$ and $b = 75$

(iii) I agree with Mr Loh that the performance of the two groups was comparable as their median scores of 43;5 and 43 were close. Furthermore, their scores for interquartile range of 14 and 15 were also approximately the same.

mm

BEATTY SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2015

SUBJECT : Mathematics

LEVEL : Secondary 4E/5N

PAPER : 4016 / 01

DURATION : 2 hours

SETTER : Mr Bernard Lee

DATE : 1 September 2015

CLASS :	NAME :	REG NO :

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

For Examiner's Use
80

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

For Examiner's Use

Answer **all** the questions.

For Examiner Use

1 Write the following in order of size, smallest first.

$$\sqrt[3]{-\frac{8}{27}},\ -0.6,\ -0.7,\ -\frac{19}{30}$$

Answer ....., ..., ..., ... [2]
smallest largest

---

2 **(a)** Calculate $\frac{0.123^2}{\sqrt[3]{52}-3.7}$.

Write down the first five digits of your answer.

Answer (a) ..... [1]

(b) Write your answer to **part (a)** correct to 2 significant figures.

Answer (b) ..... [1]

---

3 Given that $16^k \times 0.5 = 2^{2015}$, find the value of k.

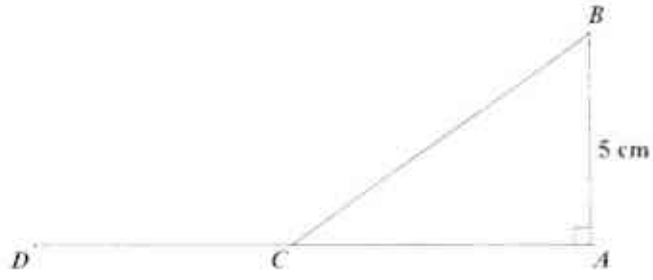
Answer $k =$ ..... [2]

42

For Examiner's Use

4 In ΔABC, $AB = 5$ cm and $\angle CAB = 90°$. AC is produced to D.

For Examiner's Use



Given that $\sin \angle BCD = \frac{3}{5}$,

(a) show that $BC = 8\frac{1}{3}$ cm,

Answer (a)

[1]

(b) calculate the exact length of AC,

Answer (b) ..... cm [1]

(c) calculate $\cos \angle BCD$, giving your answer as a fraction.

Answer (c) ..... [1]

---

5 The length of each edge of a cube is increased by 50%.

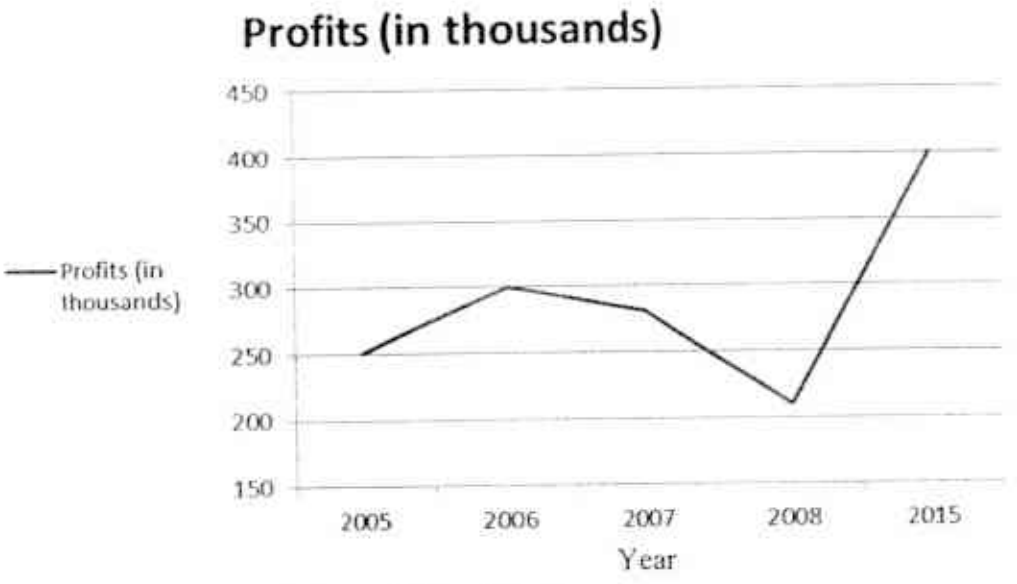
Calculate the percentage increase in its volume.

Answer ..... % [2]

For Examiner's Use

6 (a) The line graph below shows the profits that a company has made over a few years.

For Examiner Use



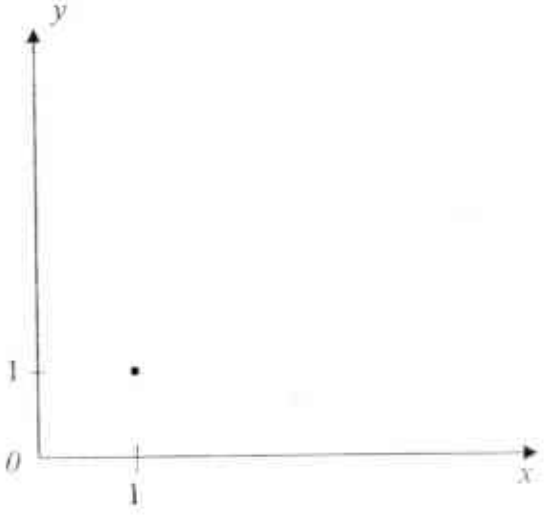
Explain one way in which the graph is misleading.

Answer .....

.....

..... [2]

(b) On the axes below, sketch the graph of $y = 3(2^x)$



[1]

43

For Examiner's Use

7 Loch Ness is a large, deep, freshwater lake in the Scottish Highlands.

(a) Its deepest point is 230 000 cm. Express 230 000 in standard form.

For Examiner Use

Answer (a) ..... [1]

(b) Loch Ness has a volume of 7.5 km^3. Convert 7.5 km^3 to cm^3, expressing your answer in standard form.

Answer (b) ..... cm^3 [1]

(c) Brenda Sherratt was the first person to swim 36.2 km along the length of Loch Ness at an average speed of 1.15 km/h in July 1966. Calculate the number of hours and minutes she took, correct to the nearest minute.

Answer (c) ..... hours ..... minutes [1]

---

8 The volume, V cm^3, of an object is directly proportional to the cube of its diameter, d cm. Given that $V = 32.5$ when $d = 5$,

(a) find a formula connecting V and d,

Answer (a) ..... [2]

(b) calculate the diameter of the object when it has a volume of 89 cm^3.

Answer (b) ..... cm [1]

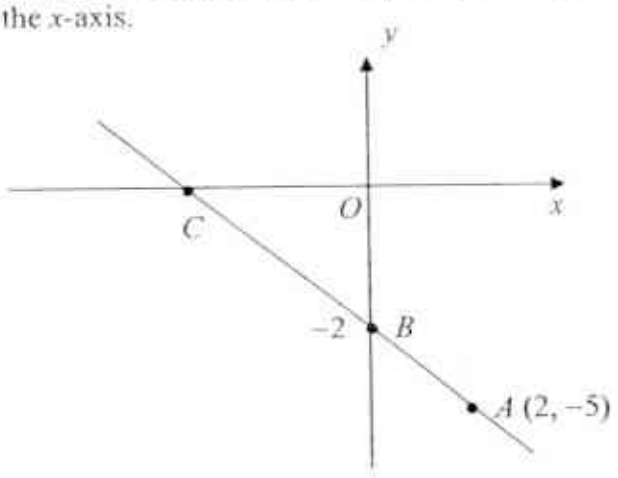
(c) write down a possible shape of the object, giving a reason for your answer.

Answer (c) .....

..... [1]

---

9 The diagram shows a sketch of a straight line passing through the points $A\ (2, -5)$, B on the y-axis and C on the x-axis.



(a) Find the equation of the line.

Answer (a) ..... [2]

(b) Find the coordinates of C.

Answer (b) C (....., ...) [1]

---

10 Each term in a sequence is found by multiplying the same constant to the previous term.

p, 12, q, 48, r, ...

(a) Write down 2 possible values of q.

Answer (a) q = ... or ..... [1]

(b) Write down the ratio $\frac{p}{r}$.

Answer (b) ..... [1]

(c) Explain why 386 is not a term in this sequence.

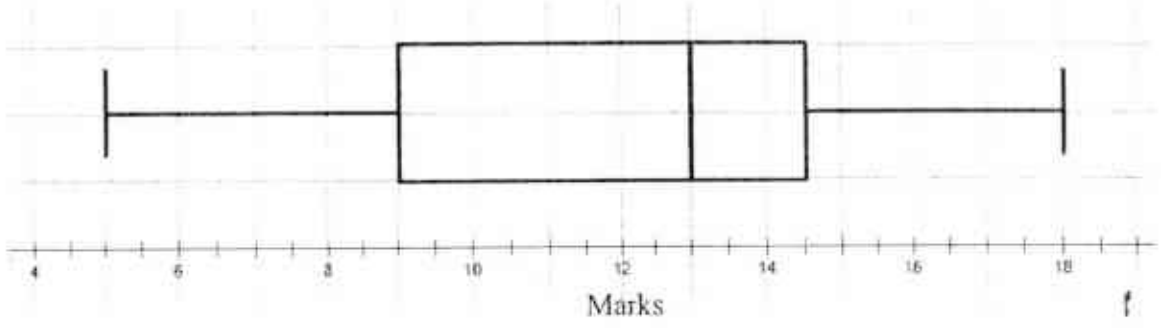
Answer (c) .....

..... [1]

For Examiner's Use

11 The marks obtained by a class of students in a test are represented by the box-and-whisker plot below.

For Examiner's Use



(a) Find the interquartile range of the marks.

Answer (a) ..... [1]

(b) Their marks were arranged in ascending order.
Only 1 student scored 13 marks, and this student was the 15th student in the arranged list.
Calculate the number of students in the class.

Answer (b) ..... [1]

---

12 Alvin wants to change 300 Singapore Dollars (S$) to Thai Baht (THB).
The exchange rate is S$1 = 24.95 THB.
For S$300, the money changer gives Alvin the maximum number of 1000 THB notes that can be bought and returns Alvin the change in S$, correct to the nearest 10 cents.

Calculate the amount of THB and S$ that Alvin will receive.

Answer ..... THB and S$ ..... [3]

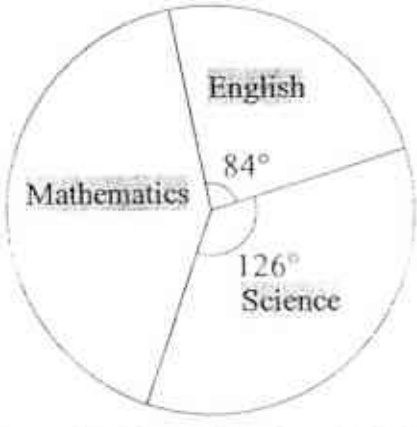
---

13 The cash price of a television set is $1 260.
The hire-purchase scheme requires the buyer to pay a deposit of 20% of the cash price plus 18 equal monthly instalments of $59.36.
Calculate the interest rate per annum of the hire-purchase scheme.

Answer ..... % [3]

---

14 Some students were surveyed and asked to write down their favourite subject.
The results were presented in the pie chart below.



(a) Calculate the ratio of the number of students who chose Mathematics to the number of students who chose English.

Answer (a) ..... : .....[2]

(b) Calculate the smallest possible number of students who were surveyed.

45

Answer (b) ..... [2]

---

15 Ivan invested some money in a savings account for 2 years.
The rate of compound interest was fixed at 10% per annum compounded half-yearly.
At the end of the 2 years there was \$9724.05 in his account.

How much did Ivan invest in the account?

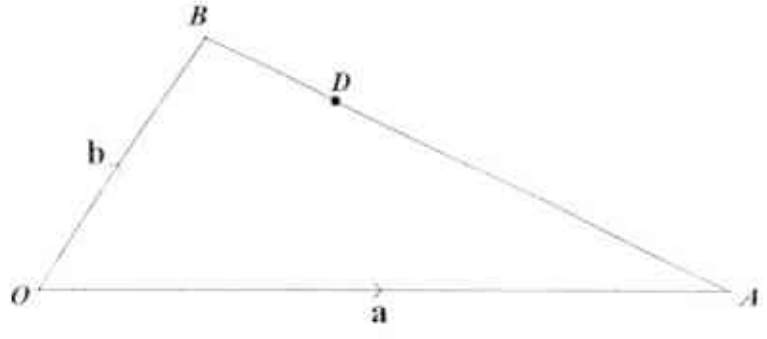
Answer \$ ..... [2]

---

16 OAB is a triangle.

D is the point on BA such that $BD = \frac{2}{7}BA$.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.



(a) Find $\overrightarrow{AD}$ in terms of **a** and **b**.

Answer (a) .....[1]

(b) E is the point on OA such that $5OE = 2EA$.
Using vectors, determine if ED is parallel to OB.

Answer (b)

[2]

For Examiner's Use

17 ξ = {positive integers x: $2x < 22$}
A = {factors of 60}
B = {prime numbers}

(a) Draw a Venn diagram to illustrate the information.

Answer (a)

[2]

(b) List the elements contained in the set $A \cap B'$.

Answer (b) .....[1]

---

18 Simplify

(a) $\dfrac{1}{x^2-1} - \dfrac{1}{x-1}$,

Answer (a) ..... [3]

(b) $3\sqrt{x^{-2}y^4} \div 12x^5y^{-4}$, leaving your answer in positive index.

46

Answer (b) ..... [2]

---

19 (a) Chris travels from Town A to Town B at a constant speed of u km/h.

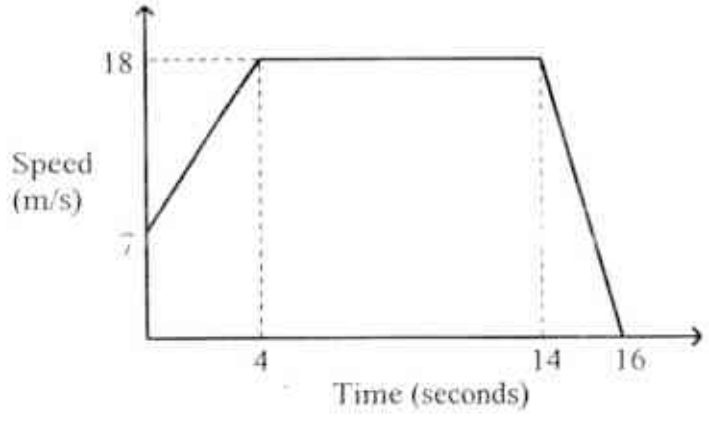
(i) Convert u km/h into m/s, leaving your answer in terms of u.

Answer (i)..... m/s [1]

(ii) Steven sets off from Town B for Town A at the same time as Chris at v km/h. Given that the distance between the two towns is 250 km, express, in terms of u and v, the distance from Town A where they will meet.

Answer (ii)..... km [2]

(b) The diagram shows the speed-time graph for a car journey between two points.



Calculate the average speed of the journey.

Answer (b)..... m/s [2]

For Examiner's Use

20 Most stop signs around the world are in the shapes of regular octagons. Some of these stop signs have diameters of 750 mm, i.e. $AB = 750$ mm, as shown in the figure below.

For Examiner Use

(a) Calculate the area of the stop sign, leaving your answer in square centimetres.

Answer (a)..... cm^2 [2]

(b) Calculate the length of one side of the stop sign, leaving your answer in centimetres.

47

Answer (b)..... cm [2]

21 A bag contains 4 balls, numbered 1, 2, 3 and 4.
Two balls are taken from the bag at random, one after the other, **without replacement.**

(a) Draw a possibility diagram to represent the outcomes.

Answer (a)

[2]

(b) Find, in its simplest form, the probability that

(i) the product of the two numbers is a square number.

Answer (b)(i)..... [1]

(ii) one number is odd and another number is even.

Answer (b)(ii)..... [1]

(iii) the sum of the numbers is more than 5.

Answer (b)(iii)..... [1]

22 **(a)** **(i)** Express 594 as a product of its prime factors.

(ii) Find the smallest positive integer k such that $\frac{594}{\sqrt{k}}$ is a perfect cube.

Answer (a)(i) ..... [1]

(ii) k = ... [1]

(b) Megan is playing with 594 cubes. Megan uses all 594 cubes to make a cuboid. Each of the sides of the cuboid is made up of more than 3 cubes.

Find the number of cubes on each side of the cuboid.

Answer (b) .....cubes by .....cubes by.....cubes [1]

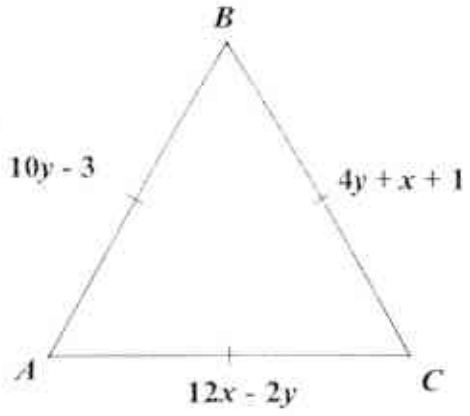
(c) Each cube has a volume of 2 cm^3. Calculate the maximum number of cubes that Megan can fit into a box of dimensions 8 cm by 10 cm by 10 cm.

Answer (c) ..... [2]

For Examiner's Use

23 The diagram shows an equilateral triangle of lengths $(4y + x + 1)$ cm, $(10y - 3)$ cm and $(12x - 2y)$ cm.

For Examiner Use



By solving for x and y, calculate the area of the triangle.

Answer ..... cm^2 [5]

For Office Use

24 **(a)** Construct triangle ABC where $BC = 8$ cm and $AC = 10$ cm.
AB has already been drawn. [1]

For Examiner Use

Answer (a), (b), (c) and *(d)*.

(b) Construct the perpendicular bisector of AB. [1]

(c) Constructor the bisector of angle ABC. [1]

(d) Mark clearly a possible point which is inside the triangle, equidistant from BC and BA, and is nearer to A than B.

Label this point P. [1]

4

For Answer's Use

Answer Key

For Examiner's Use

1 (a) -0.7 , $\sqrt[3]{-\frac{8}{27}}$, $-\frac{19}{30}$, -0.6

2 (a) 0.4653 (b) 0.47

3 504

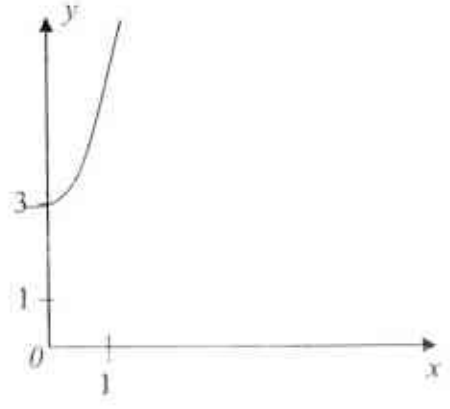
4 (a) Show (b) $6\frac{2}{3}$ (c) $-\frac{4}{5}$

5 237.5%

6 (a) Either one of the misleading feature + effect

Misleading Feature	Effect
Inconsistent scale on horizontal axis	Exaggerates the differences between the years
Not all years are shown	Profits could be very low from 2009 to 2014
Unequal spacing of years	Misrepresents the trend
Vertical axis does not start from 0	Rate of gain in profits appear steeper

6 (b)



7 (a) 2.3×10^{5} (b) 7.5×10^{15} (c) 31 hours 29 minutes

8 (a) $V = 0.26d^3$ (b) 7.00 cm

8 (c) Hemisphere. Volume of hemisphere when d = 5 is $32.7\ cm^3 \approx 32.5\ cm^3$.

9 (a) $y = -\frac{3}{2}x - 2$ (b)(i) $(-\frac{4}{3}, 0)$

10 (a) 24 or –24 (b) $\frac{1}{16}$

10 (c) The next few terms in the sequence are 96, 192, 384, 768, hence 386 is not a term in the sequence

11 (a) 5.5 (b) 29

12 7000 THB and S$19.40

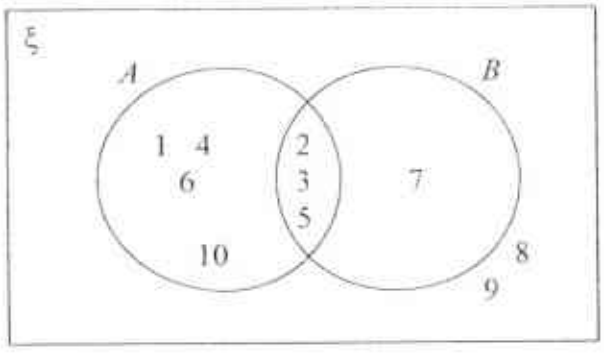
13 4%

14 (a) 25 : 14 (b) 60

15 $8000

16 (a) $\frac{5}{7}\mathbf{b} - \frac{5}{7}\mathbf{a}$ (b) Since $\overrightarrow{ED} = \frac{5}{7}\overrightarrow{OB}$, ED is parallel to OB.

17 (a)



17(b) {1, 4, 6, 10}

18 (a) $\frac{-x}{(x+1)(x-1)}$ (b) $\frac{y^b}{4x^b}$

19 (a)(i) $\frac{5u}{18}$ (a)(ii) $\frac{250u}{u+v}$ (b) 15.5 m/s

20 (a) 3980 cm^2 (b) 28.7 cm

21 (a)

	1	2	3	4
1		(2, 1)	(3, 1)	(4, 1)
2	(1, 2)		(3, 2)	(4, 2)
3	(1, 3)	(2, 3)		(4, 3)
4	(1, 4)	(2, 4)	(3, 4)	

21 (b)(i) $\frac{1}{6}$ (b)(ii) $\frac{2}{3}$ (b)(iii) $\frac{1}{3}$

22 (a)(i) $2 \times 3^3 \times 11$ (a)(ii) 484 (b) 6 by 9 by 11 (c) 294

23 8.77 cm^2

24 Construction

50

BEATTY SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2015

SUBJECT : Mathematics

LEVEL : Sec 4 Express

PAPER : 4016 / 02

DURATION : 2 hours 30 minutes

SETTER : Ms Chai YL

DATE : 2 September 2015

CLASS :	NAME :	REG NO :

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
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Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P(1 + \frac{r}{100})^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Solve the inequality $3x+10 \leq \frac{2}{3}(2x+25)$. [2]

(b) Given that $R = \frac{3}{t} - \frac{2t}{s}$,

(i) find R when $t = -5$ and $s = 3$, [1]

(ii) express s in terms of R and t. [2]

(c) Simplify $\frac{4p^2 - 36}{5p^2 + 8p - 21}$. [2]

(d) Solve the equation $\frac{y+2}{4} = \frac{8}{y-2}$. [3]

---

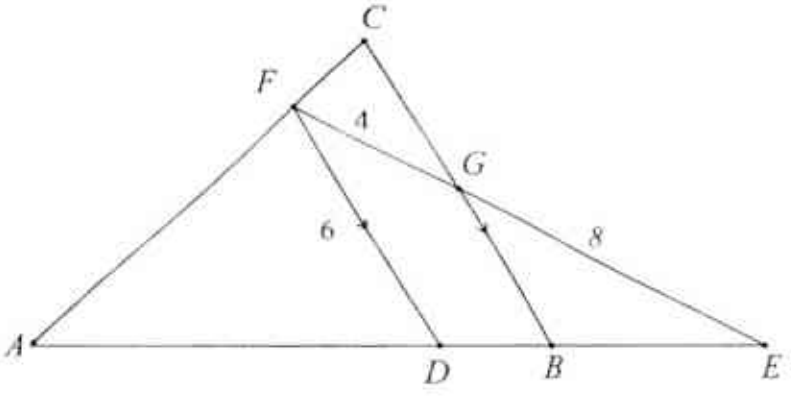
2 In the diagram, BC is parallel to DF, $DF = 6$ cm, $GE = 8$ cm and $FG = 4$ cm.

(a) Prove that triangles BEG and DEF are similar. [2]

(b) Hence calculate the length of BG. [2]

(c) If $AC = 4FC$, find the length of BC. [2]

(d) If the area of ADF is 20 cm², find the area of $BDFG$. [2]



3 **(a)** Part of the utilities bill for the month of July of a household is shown below.

Water Services by Public Utilities Board			
	Usage	Rate	Amount ($)
Water	4.7 Cu M	$ 1.1700	5.50
Waterborne Fee	4.7 Cu M	$ 0.2803	
Water Conservation Tax		30%	
⋮			
Goods & Services Tax (GST)		7%	

Water Conservation Tax is imposed as a percentage of the total water consumption to reinforce the message that water is precious.

(i) Find the Water Conservation Tax for this bill. [1]

(ii) Calculate the total charges for water usage for this household, including GST. [2]

(iii) The value in part **(ii)** is 12 % of the entire utilities bill. Find the total charges for the month of July. [2]

(b) A supermarket has three check-out counters manned by Abby, Brian and Chloe. The following table shows the number of notes of denomination $2, $10 and $50 in their cash registers at the end of a business day.

	Denomination		
	$2	$10	$50
Abby	12	10	25
Brian	20	15	30
Chloe	18	8	24

(i) Given that $P = \begin{pmatrix} 12 & 10 & 25 \\ 20 & 15 & 30 \\ 18 & 8 & 24 \end{pmatrix}$ and $Q = \begin{pmatrix} 2 \\ 10 \\ 50 \end{pmatrix}$, evaluate PQ and explain what the elements in this matrix product represent. [2]

The three cashiers started the day with $150, $100 and $180 in their cash registers respectively.

(ii) Write down a (3×1) matrix S to represent these amounts. [1]

(iii) Write down another matrix T such that a matrix operation involving P, Q, S and T will give the total sales of the supermarket for the day. Hence find this amount. [2]

4 A factory producing bottled drinks mixes two different juices, X and Y, by piping in the required amounts using taps of two sizes, small and large, and the mixture is stored in a tank of capacity 2 m^3.

The small tap dispenses X at the rate of x m^3 per minute while the large tap dispenses Y at the rate of y m^3 per minute.

The tank is filled completely in $3\frac{1}{3}$ minutes when the taps are turned on together.

(a) Show that $y = \frac{3}{5} - x$. [2]

(b) When used alone, the small tap will take 4 minutes longer than the large tap to fill the tank. Form an equation in x and show that it reduces to $10x^2 - 16x + 3 = 0$. [3]

(c) Solve the equation $10x^2 - 16x + 3 = 0$, giving your answers correct to 3 significant figures. [3]

(d) Calculate, to the nearest minute, the time taken by the large tap to fill the tank. [2]

5 **(a)** The position vector of point A is $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

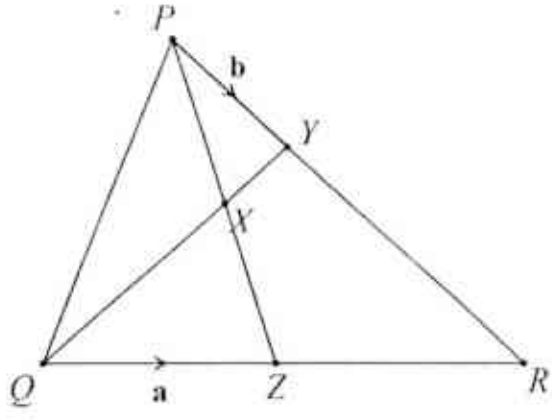
(i) Find $\left|\overrightarrow{AB}\right|$. [1]

(ii) Find the coordinates of B. [2]

(iii) Given that $\overrightarrow{CD}$ is parallel to $\overrightarrow{BA}$, and $\overrightarrow{CD} = \begin{pmatrix} k \\ 13.6 \end{pmatrix}$, find the value of k. [2]

(b) In the diagram, $PR = 3PY$, X is the midpoint of PZ and Z is the midpoint of QR.

$\overrightarrow{QZ} = \mathbf{a}$ and $\overrightarrow{PY} = \mathbf{b}$.



(i) Express in terms of **a** and/or **b**, as simply as possible

(a) $\overrightarrow{PZ}$ [1]

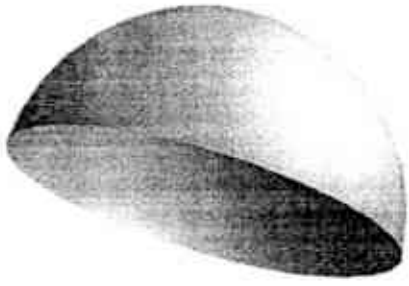
(b) $\overrightarrow{XZ}$ [1]

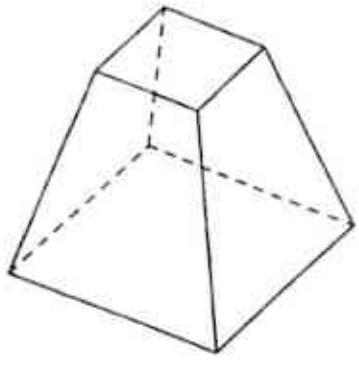
(c) $\overrightarrow{QX}$ [2]

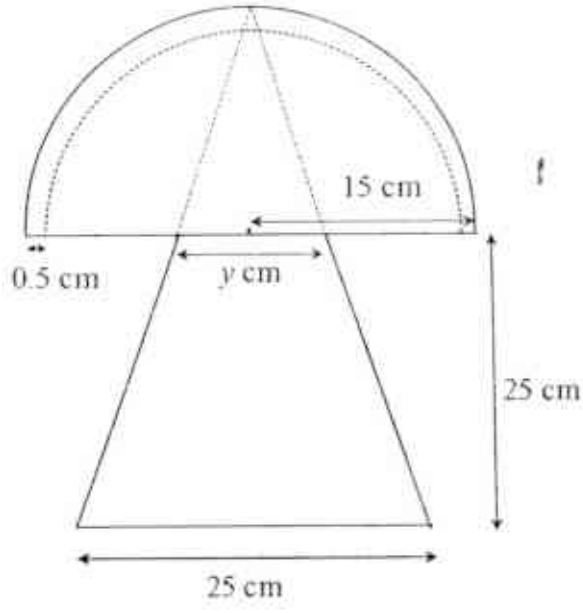
(ii) Find the value of $\dfrac{\text{area of } \Delta PQX}{\text{area of } \Delta PQR}$. [2]

6 A mushroom shape table lamp consists of a solid base in the shape of a frustum of height 25 cm, and the lampshade is a hemispherical shell with **external** radius 15 cm. The base of the frustum is a square of side 25 cm.

The parts of the lamp and its cross-section are shown below.







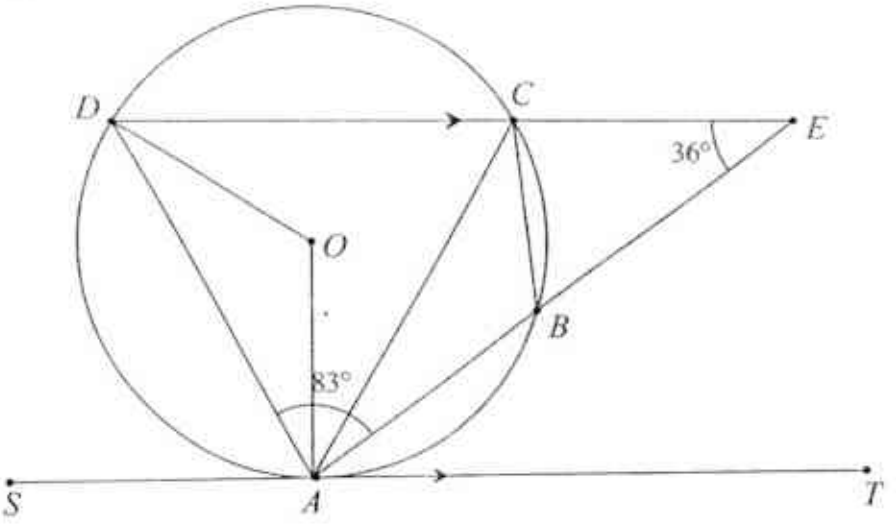
(a) **(i)** Show that $y = 9.375$. [1]

(ii) Calculate the volume of the base. [2]

(b) The lampshade is made of glass of thickness 0.5 cm. Calculate the volume of glass used to make one such lampshade. [3]

(c) If a protective layer is applied on the **entire** lampshade, find the total area which needs to be painted. [3]

7 **(a)** In the diagram, A, B, C and D are points on the circumference of the circle centre O.
ABE and DCE are straight lines and ST is a tangent to the circle at A.
ST is parallel to DE, $\angle CEB = 36°$ and $\angle DAB = 83°$.



(i) Calculate

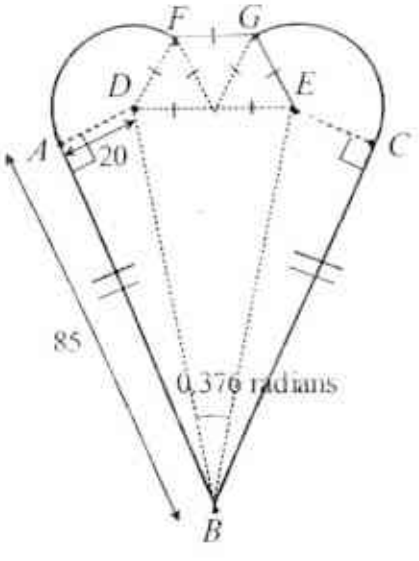
(a) angle OAB, [1]

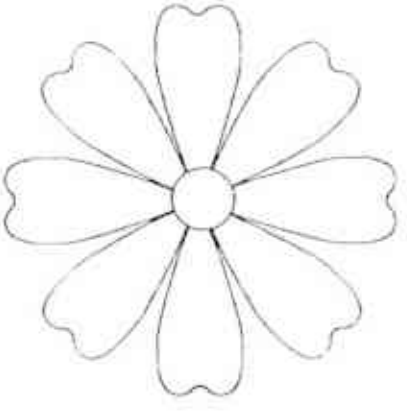
(b) angle ABC, [2]

(c) angle AOD. [2]

(ii) Show that ACD is an isosceles triangle. [1]

(b) The diagram shows a piece of cardboard used to make a daisy for a carnival float. Eight such pieces are used to make one daisy. Each petal is made up of two sectors, ADF and GEC and six triangles.
The radii of the two sectors are 20 cm, $AB = CB = 85$ cm and $\angle DBE = 0.376$ radians.





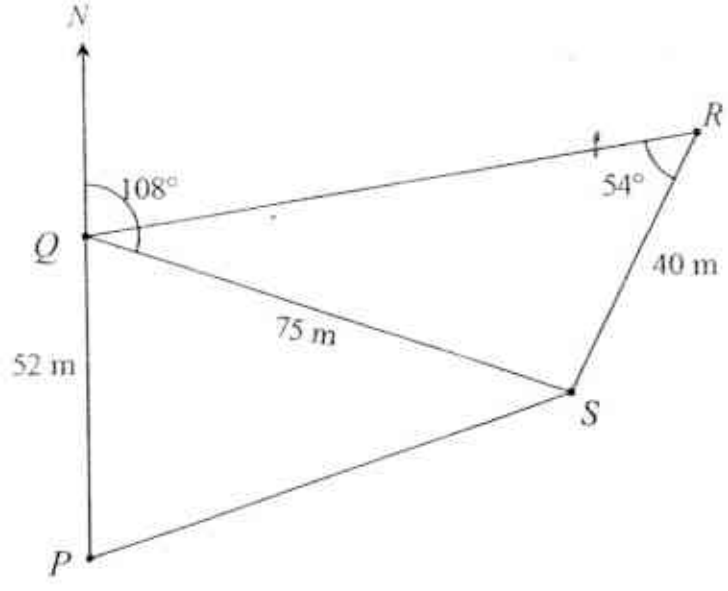
(i) Calculate $\angle ADB$, giving your answer in radians, correct to 2 decimal places. [2]

(ii) Find the area of one sector. [4]

8 P, Q, R and S are four corners of a plot of land gazetted for development into a neighbourhood park.

$PQ = 52$ m, $QS = 75$ m, $RS = 40$ m and $\angle QRS = 54°$.

P is due south of Q and the bearing of S from Q is 108°.



(a) Find the distance PS. [2]

(b) Find the bearing of Q from R. [3]

(c) Calculate the area of $PQRS$. [2]

A flag pole is erected at Q and the angle of elevation of the top of this flag pole from P is 13°.

(d) Calculate the height of the flag pole. [2]

9 **(a)** The lifespan, measured to the nearest hour, of a brand of batteries, *Power Max*, are given in the table below.

Number of hours	35 – 39	40 – 44	45 – 49	50 – 54	55 – 59
Number of batteries	5	10	50	20	3

(i) Calculate the

(a) mean. [1]

(b) standard deviation. [2]

(ii) Another brand, *Durable*, has a mean lifespan of p hours and a standard deviation of q hours. If p is larger than your answer to **(i) (a)** and q is smaller than your answer to **(i) (b)**, decide which brand of batteries you prefer and explain your choice. [2]

(b) On any day, Alice may go to work by bus, MRT or taxi.

The probability that she takes the bus is $\frac{1}{5}$.

The probability that she takes the MRT is $\frac{3}{4}$.

(i) Find the probability that she goes to work by taxi. [1]

If she takes the bus, the probability that she will be late for work is $\frac{1}{8}$.

If she takes the MRT, the probability that she will be late is $\frac{1}{15}$.

She will not be late if she takes a taxi. Find the probability that

(ii) Alice will be late for work on any given day. [2]

(iii) Alice will not be late for two of three consecutive days. [2]

10 ***Answer the whole of this question on a single sheet of graph paper.***

A group of research scientists brought 320 pelicans to an island. The number of pelicans, y, was counted at intervals and recorded in the table below.

x (years)	0	0.5	1.5	2	2.5	3.5	4.5	5.5	6	7
y	320	266	264	n	351	469	556	554	500	250

(a) Using a scale of 2 cm to represent 1 year on the x –axis and 2 cm to represent 50 animals on the y-axis for $100 \leq y \leq 600$, plot the values given and join them with a smooth curve. [3]

(b) Use your curve to estimate

(i) the value of n, [1]

(ii) the maximum number of pelicans on the island within these 7 years. [1]

(c) Give a possible reason why the number of pelicans decreased at the beginning. [1]

(d) The scientists also brought 100 penguins to the same island, and noticed that the number of penguins increased at a steady rate of 90 per year.

(i) Write down an equation connecting the number of penguins, y, and the number of years, x. [1]

(ii) On the same axes, draw the graph of the equation found in **(d) (i).** [1]

(iii) Find the value of x when the number of pelicans is equal to the number of penguins. [1]

(e) Estimate the value of x at the instant of time when the rate of increase of number of pelicans is equal to the rate of increase of the number of penguins, showing clearly how you obtain your answer. [2]

~ ~ End of Paper ~ ~

4E/5N Prelim 2015 Math Paper 2 Ans Key

1 **(a)** $x \le 4$ **(b)** **(i)** $2\frac{11}{15}$ **(ii)** $s = \frac{2t^2}{3 - Rt}$ **(c)** $\frac{4(p-3)}{5p-7}$

(d) $y = 6$ or $y = -6$

2 **(a)** $\angle BEG = \angle DEF$ (common angle)
$\angle GBE = \angle FDE$ (corresponding angles, $BC \parallel DF$)
Hence BEG and DEF are similar (AA)

(b) $BG = 4$ cm **(c)** $BC = 8$ cm **(d)** $11\frac{1}{9}$ cm^2

3 **(a)** **(i)** \$1.65 **(ii)** \$ 9.06 (2 d.p) **(iii)** \$75.50 (2 d.p)

(b) **(i)** $\begin{pmatrix} 1374 \\ 1690 \\ 1316 \end{pmatrix}$ **(ii)** $S = \begin{pmatrix} 150 \\ 100 \\ 180 \end{pmatrix}$ **(iii)** $T = (1 \quad 1 \quad 1)$, $T(PQ - S) = (3950)$

The elements represent the total amount of money in each of the cash registers at the end of the day.

4 **(a)** - **(b)** - **(c)** 1.38 or 0.217 (3 s.f)
(d) $x = 1.38$ is rejected
Time taken by large tap = 5 minutes (nearest minute)

5 **(a)** **(i)** 5 units **(ii)** B is $(-1, 9)$ **(iii)** $n = -3.4$, $k = -10.2$

(b) **(i)** **(a)** $\overrightarrow{PZ} = 3\mathbf{b} - \mathbf{a}$ **(b)** $\overrightarrow{XZ} = \frac{1}{2}(3\mathbf{b} - \mathbf{a})$ **(c)** $\overrightarrow{QX} = \frac{1}{2}(3\mathbf{a} - 3\mathbf{b})$

(ii) $\frac{\text{area of } \Delta PQX}{\text{area of } \Delta PQR} = \frac{1}{4}$

6 **(a)** **(i)** - **(ii)** 7890 cm^3 (3 s.f) **(b)** 684 cm^3 (3 s.f) **(c)** 2780 cm^2 (3 s.f)

7 **(a)** **(i)** **(a)** 54° **(b)** 119° **(c)** 122° **(ii)** -

(b) **(i)** 1.34 radians (2 d.p) **(ii)** 503 cm^2 (3 s.f)

8 **(a)** 76.9 m (3 s.f) **(b)** 262.4° **(c)** 3330 m^2 **(d)** 12.0 m (3 s.f)

9 **(a)** **(i)** **(a)** 47.3 hours (3 s.f) **(b)** 4.18 hours

(ii) *Durable* because the average lifespan is longer since it's mean is larger, and lifespan is also more consistent because the standard deviation is smaller.

(b) **(i)** $\frac{1}{20}$ **(ii)** $\frac{3}{40}$ **(iii)** $\frac{12321}{64000}$

10 **(b)** **(i)** 300 ± 10 **(ii)** 570 ± 10 **(c)** Pelicans were getting used to a new environment
(d) **(i)** $y = 100 + 90x$ **(iii)** $x = 5.2 \pm 0.1$

(e) Either 2 ± 0.1 or 4 ± 0.2

57

4E/5N Prelim 2015 Math Paper 2

1 **(a)** $3x+10 \le \frac{2}{3}(2x+25)$

$9x+30 \le 4x+50$ [M1]

$5x \le 20$

$x \le 4$ [A1]

(b) Given that $R = \frac{3}{t} - \frac{2t}{s}$,

(i) find R when $t = -5$ and $s = 3$,

$$R = \frac{3}{-5} - \frac{2(-5)}{3}$$

$$= 2\frac{11}{15}$$ [B1]

(ii) express s in terms of R and t.

$Rts = 3s - 2t^2$

$3s - Rts = 2t^2$ [M1]

$s(3 - Rt) = 2t^2$

$s = \frac{2t^2}{3 - Rt}$ [A1]

(c) $\frac{4p^2 - 36}{5p^2 + 8p - 21}$

$= \frac{4(p-3)(p+3)}{(5p-7)(p+3)}$ [M1]

$= \frac{4(p-3)}{5p-7}$ [A1]

(d) $\frac{y+2}{4} = \frac{8}{y-2}$

$y^2 - 4 = 32$ [M1]

$y^2 = 36$ [M1]

$y = 6$ or $y = -6$ [A1 for both correct answers]

---

2 **(a)** $\angle BEG = \angle DEF$ (common angle)

$\angle GBE = \angle FDE$ (corresponding angles, $BC \parallel DF$) [M1]

Hence BEG and DEF are similar (AA) [A1] 58

(b) $\frac{BG}{6} = \frac{8}{12}$ [M1]

$BG = 4$ cm [A1]

(c) $\frac{BC}{6} = \frac{4}{3}$ [M1]

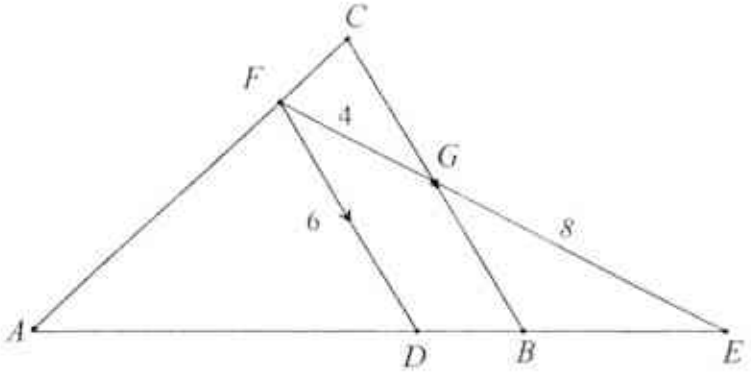
$BC = 8$ cm [A1]

(d) Area of ADF = area of DEF

Area of $BEG = \frac{4}{9}$ (area of DEF) [M1]

Area of $BDFG = \frac{5}{9} \times 20$

$= 11\frac{1}{9}$ cm^2 [A1]



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3 (a) (i) Find the Water Conservation Tax for this bill.

Water Conservation Tax $= \frac{30}{100} \times \$5.50$

$= \$1.65$ [B1]

(ii) Calculate the total charges for water usage for this household, including GST.

Total $= \$ (5.50 + 4.7 \times 0.2803 + 1.65) \times 1.07$ [M1]

$= \$ 9.06$ (2 d.p) [A1]

(iii) The value in part **(iii)** is 12% of the entire utilities bill. Find the total charges for the month of July.

Total charges for July $= \$9.0601 \times \frac{100}{12}$ [M1]

$= \$75.50$ (2 d.p) [A1]

(b) (i) $PQ = \begin{pmatrix} 12 & 10 & 25 \\ 20 & 15 & 30 \\ 18 & 8 & 24 \end{pmatrix} \begin{pmatrix} 2 \\ 10 \\ 50 \end{pmatrix}$

$= \begin{pmatrix} 1374 \\ 1690 \\ 1316 \end{pmatrix}$ [B1]

The elements represent the total amount of money in each of the cash registers at the end of the day. [B1]

(ii) $S = \begin{pmatrix} 150 \\ 100 \\ 180 \end{pmatrix}$

(iii) $T = (1 \quad 1 \quad 1)$ [M1]

$$T(PQ - S) = (1 \quad 1 \quad 1)\left[\begin{pmatrix} 1374 \\ 1690 \\ 1316 \end{pmatrix} - \begin{pmatrix} 150 \\ 100 \\ 180 \end{pmatrix}\right]$$

$= (3950)$ [A1]

---

4 **(a)** Express y in terms of x. [2]

$x + y = \dfrac{2}{3\frac{1}{3}}$ [M1]

$= \dfrac{3}{5}$

Hence $y = \dfrac{3}{5} - x$ [A1]

(b) $\dfrac{2}{x} = \dfrac{2}{y} + 4$ [M1]

$2y = 2x + 4xy$

$2\left(\dfrac{3}{5} - x\right) = 2x + 4x\left(\dfrac{3}{5} - x\right)$

$6 - 10x = 10x + 12x - 20x^2$ [M1]

$20x^2 - 32x + 6 = 0$ [A1]

$10x^2 - 16x + 3 = 0$

(c) $x = \dfrac{-(-16) \pm \sqrt{(-16)^2 - 4(10)(3)}}{2(10)}$ [M1]

$= \dfrac{16 \pm \sqrt{136}}{20}$

$= 1.3830$ or 0.21690

$= 1.38$ or 0.217 (3 s.f) [A1, A1]

(d) $x = 1.38$ is rejected

Time taken by large tap $= \dfrac{2}{0.6 - 0.21690}$ [M1] 5?

$= 5.22$

$= 5$ minutes (nearest minute) [A1]

5 **(a)** **(i)** $\left|\overrightarrow{AB}\right| = \sqrt{(-3)^2 + 4^2}$

$= 5$ units [B1]

(ii) $\overrightarrow{OB} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ [M1]

B is $(-1, 9)$ [B1]

$\overrightarrow{CD} = n\overrightarrow{BA}$

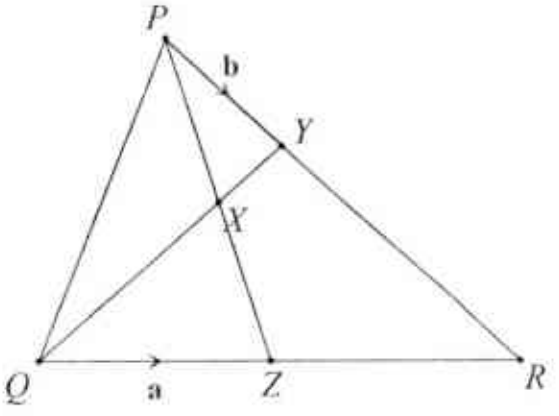
(iii) $\begin{pmatrix} k \\ 13.6 \end{pmatrix} = n\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ [M1]

$n = -3.4$

$k = -10.2$ [A1]

(b) In the diagram, $PR = 3PY$, X is the midpoint of PZ and Z is the midpoint of QR.

$\overrightarrow{QZ} = \mathbf{a}$ and $\overrightarrow{PY} = \mathbf{b}$.



(i) Express in terms of **a** and/or **b**, as simply as possible

(a) $\overrightarrow{PZ} = 3\mathbf{b} - \mathbf{a}$ [B1]

(b) $\overrightarrow{XZ} = \frac{1}{2}(3\mathbf{b} - \mathbf{a})$ [B1]

(c) $\overrightarrow{QX} = \mathbf{a} - \frac{1}{2}(3\mathbf{b} - \mathbf{a})$ [M1]

$= \frac{1}{2}(3\mathbf{a} - 3\mathbf{b})$ [B1]

(ii) Find the value of $\frac{\text{area of } \Delta PQX}{\text{area of } \Delta PQR}$. [2]

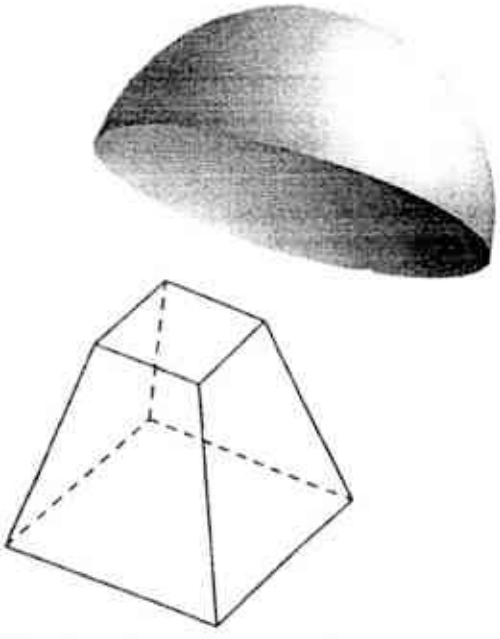
$PQX = \frac{1}{2}(PQZ)$

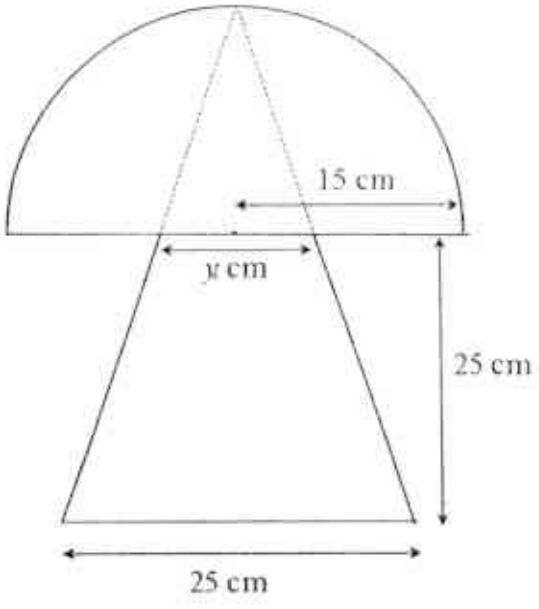
$= \frac{1}{2}(\frac{1}{2}PQR)$ [M1]

$\frac{\text{area of } \Delta PQX}{\text{area of } \Delta PQR} = \frac{1}{4}$ [A1]

6 A mushroom shape table lamp consists of a solid base in the shape of a frustum of height 25 cm, and the lampshade is a hemispherical shell with **external** radius 15 cm. The base of the frustum is a square of side 25 cm.

The parts of the lamp and its cross-section are shown below.





(a) (i) Show that $y = 9.375$. [1]

$$\frac{y}{25} = \frac{15}{40} \qquad \text{[M1]}$$

$$y = 9.375$$

(ii) Calculate the volume of the base. [2]

$$\text{Volume of base} = \frac{1}{3}(25)^2(25+15) - \frac{1}{3}(9.375)^2(15) \qquad \text{[M1]}$$

$$= 7893.88$$

$$= 7890 \text{ cm}^3 \text{ (3 s.f)} \qquad \text{[A1]}$$

(b) The lampshade is made of glass of thickness 0.5 cm. Calculate the volume of glass used to make one such lampshade. [3]

$$\text{Volume of glass} = \frac{2}{3}\pi(15)^3 - \frac{2}{3}\pi(15-0.5)^3 \qquad \text{[M1, M1]}$$

$$= 683.558$$

$$= 684 \text{ cm}^3 \text{ (3 s.f)}$$

(c) If a protective layer is applied on the **entire** lampshade, find the total area which needs to be painted. [3]

$$\text{Total area} = 2\pi(15)^2 + 2\pi(14.5)^2 + \pi(15) - \pi(14.5)^2 \qquad \text{[M1, M1]}$$

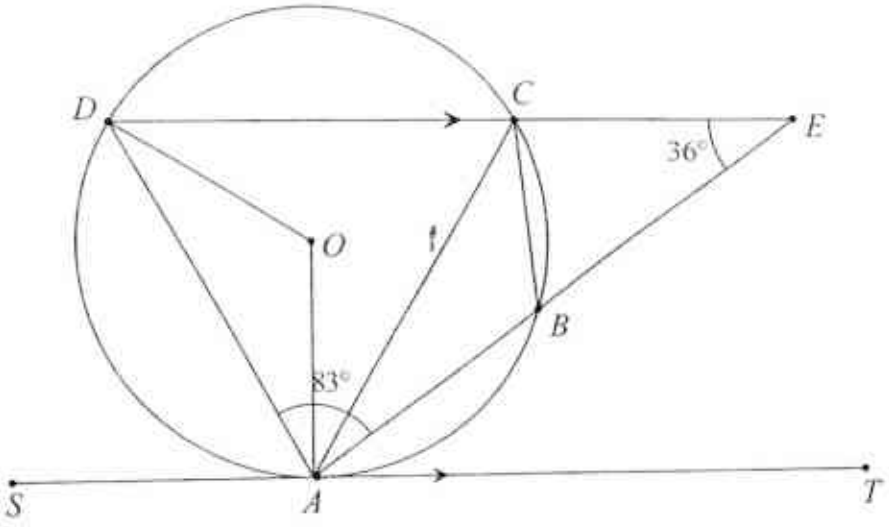
$$= 2781.09$$

$$= 2780 \text{ cm}^2 \text{ (3 s.f)} \qquad \text{[A1]}$$

7 **(a)** In the diagram, A, B, C and D are points on the circumference of the circle centre O.

ABE and DCE are straight lines and ST is a tangent to the circle at A.

ST is parallel to DE, $\angle CEB = 36°$ and $\angle DAB = 83°$.



(i) Calculate

(a) angle OAB, [1]

$\angle EAT = 36°$ (alt angles, $DE \parallel ST$)

$\angle OAB = 90° - 36°$ (tangent perpendicular to radius)

$= 54°$ [A1]

(b) angle ABC, [1]

$\angle ADE = 180° - 36° - 83°$ (angle sum of Δ) [M1]

$= 61°$

$\angle ABC = 180° - 61°$ (angles in opposite segments)

$= 119°$ [A1]

(c) angle AOD. [2]

$\angle OAD = 83° - 54°$

$= 29°$

$\angle AOD = 180° - 2(29°)$ [M1]

$= 122°$ [A1]

(ii) Show that ACD is an isosceles triangle. [1]

$\angle ACD = \frac{1}{2}(122°)$ (angle at centre = twice angle at circumference)

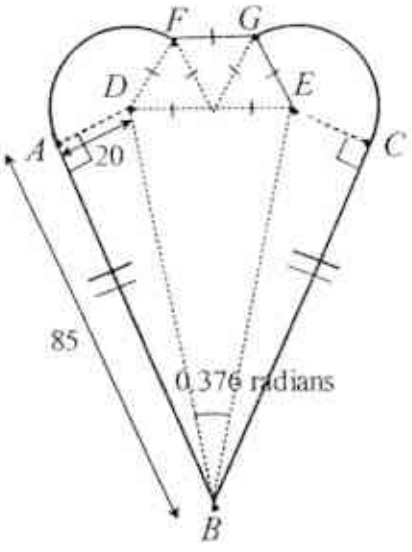
$= 61°$ [B1]

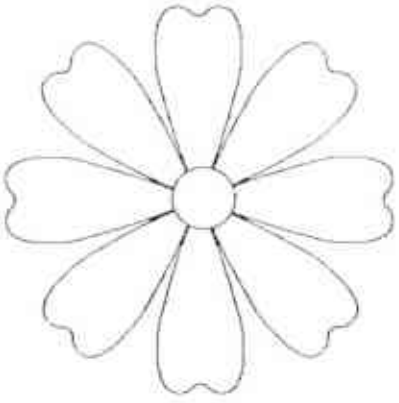
Hence ACD is isosceles.

(b) The diagram shows a piece of cardboard used to make a daisy for a carnival float. Eight such pieces are used to make one daisy. Each petal is made up of two sectors, ADF and

GEC, and six triangles.

The radii of the two sectors are 20 cm, $AB = CB = 85$ cm and $\angle DBE = 0.376$ radians.





(i) Calculate $\angle ADB$, giving your answer in radians, correct to 2 decimal places. [2]

$$\tan \angle ADB = \frac{85}{20} \qquad \text{[M1]}$$

$$\angle ADB = 1.3397$$

$$= 1.34 \text{ radians (2 d.p)} \qquad \text{[A1]}$$

(ii) Find the area of one sector. [4]

$$\angle BDE = \frac{1}{2}(\pi - 0.376)$$

$$= 1.3828 \qquad \text{[M1]}$$

$$\angle ADF = 2\pi - 1.3397 - \frac{\pi}{3} - 1.3828$$

$$= 2.5134 \qquad \text{[M1]}$$

$$\text{Area of one sector} = \frac{1}{2}(20)^2(2.5134) \qquad \text{[M1]}$$

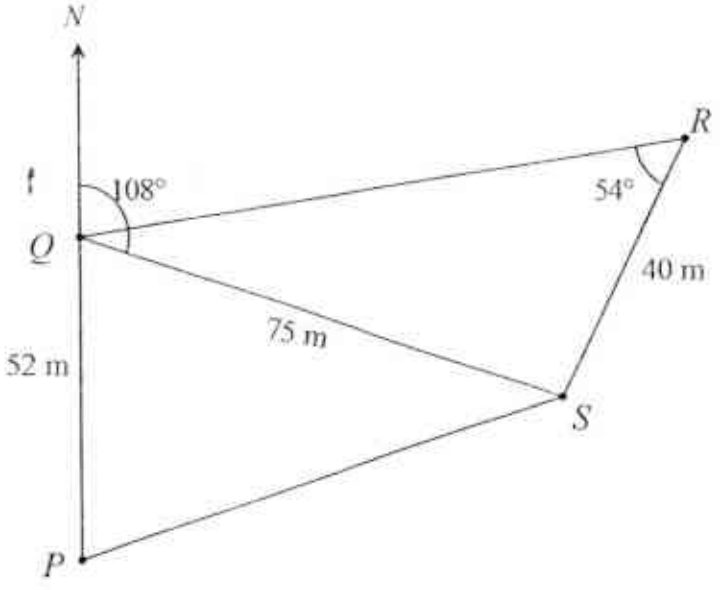
$$= 503 \text{ cm}^2 \text{ (3 s.f)} \qquad \text{[A1]}$$

61

8 P, Q, R and S are four corners of a plot of land gazetted for development into a neighbourhood park.

$PQ = 52$ m, $QS = 75$ m, $RS = 40$ m and $\angle QRS = 54°$.

P is due south of Q and the bearing of S from Q is 108°.



(a) Find the distance PS. [2]

$PS^2 = 52^2 + 75^2 - 2(52)(75)\cos 72°$ [M1]

$PS = 76.932$

$= 76.9$ m (3 s.f) [A1]

(b) Find the bearing of Q from R. [3]

$\frac{\sin \angle RQS}{40} = \frac{\sin 54°}{75}$ [M1]

$\angle RQS = 25.561°$

Bearing of Q from $R = 180° + 108° - 25.561°$ [M1]

$= 262.4°$ [A1]

(c) Calculate the area of this plot of land. [2]

Area of land $= \frac{1}{2}(52)(75)\sin 72° + \frac{1}{2}(40)(75)\sin(180° - 54° - 25.561°)$ [M1]

$= 3329.7$

$= 3330$ m^2 (3 s.f) [A1]

A flag pole is erected at Q and the angle of elevation of the top of flag pole from P is 13°.

(d) Calculate the height of the rock wall. [2]

$\tan 13° = \frac{h}{52}$ [M1]

Height of rock wall $= 52 \times \tan 13°$

$= 12.0$ m (3 s.f)

9 **(a)** **(i)** Calculate, for these batteries, the

(a) Mean $= \frac{4166}{88}$

$= 47.3$ hours (3 s.f) [B1]

(b) S. D. $= \sqrt{\frac{198762}{88} - \left(\frac{4166}{88}\right)^2}$ [M1]

$= 4.18$ hours [A1]

(ii) Another brand, *Durable*, has a mean lifespan of p hours and a standard deviation of q hours. If p is larger than your answer to **(i) (a)** and q is smaller than your answer to **(i) (b)**, decide which brand of batteries you prefer and explain your choice. [2]

Durable because the average lifespan is longer since it's mean is larger, and lifespan is also more consistent because the standard deviation is smaller. [B1 for choice, B1 for explanation]

(b) On any day, Alice may go to work by bus or MRT with probabilities of $\frac{1}{5}$ and $\frac{3}{4}$ respectively. She may also take a taxi to work on some days.

(i) Find the probability that she goes to work by taxi. [1]

P (take taxi) $= 1 - \frac{1}{5} - \frac{3}{4}$

$= \frac{1}{20}$

If she takes the bus, the probability that she will be late for work is $\frac{1}{8}$.

If she takes the MRT, the probability that she will be late is $\frac{1}{15}$.

She will not be late if she takes a taxi. Find the probability that

(ii) Alice will be late for work on any given day. [2]

P (late on any day) $= \frac{1}{5} \times \frac{1}{8} + \frac{3}{4} \times \frac{1}{15}$

$= \frac{3}{40}$

(iii) Alice will be punctual for two of three consecutive days. [2]

P (punctual for 2 out of 3 days) $= \left(\frac{37}{40} \times \frac{37}{40} \times \frac{3}{40}\right) \times 3$ [M1]

$= \frac{12321}{64000}$ 62 [A1]

10 ***Answer the whole of this question on a single sheet of graph paper.***

A group of research scientists brought 320 pelicans to an island. The number of pelicans, y, was counted at intervals and recorded in the table below.

x (years)	0	0.5	1.5	2	2.5	3.5	4.5	5.5	6	7
y	320	266	264	n	351	469	556	554	500	250

(a) Using a scale of 2 cm to represent 1 year on the x –axis and 2 cm to represent 50 animals on the y-axis for $100 \leq y \leq 600$, plot the values given and join them with a smooth curve. [3]

(b) Use your curve to estimate

(i) the value of n, [1]

(ii) the maximum number of pelicans on the island within these 7 years. [1]

(c) Explain briefly why the number of pelicans decreased at the beginning. [1]

(d) The scientists also brought 100 penguins to the same island, and noticed that the number of penguins increased at a steady rate of 90 per year.

(i) Write down an equation connecting the number of penguins, y, and the number of years, x. [1]

(ii) On the same axes, draw the graph of the equation found in **(d) (i).** [1]

(iii) Find the value of x when the number of pelicans is equal to the number of penguins. [1]

(e) Estimate when the rate of increase of number of pelicans is equal to the rate of increase of the number of penguins. [2]

– – End of Paper – –

Calculator Model :	

Class	Full Name	Index Number

bowen
SECONDARY
I believe, therefore I am

PRELIMINARY EXAMINATION 2
2015

O

4016/01

MATHEMATICS
Paper 1

Secondary 4 Express/ 5 Normal Academic
25th August 2015

2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any questions it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 80.

DO NOT OPEN THIS PAPER UNTIL YOU ARE TOLD TO DO SO

For Examiner's use
80

This document consists of **18** printed pages, including this cover page.

Setter: Ms Melissa Chong

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curve surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector Area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

For Examiner's Use

Answer **all** the questions.

For Examiner's Use

1 Write the following in order of size, smallest first.

$$0.55^{\frac{2}{3}} \quad \sqrt{0.21} \quad \frac{23}{41} \quad 0.671$$

Answer ..... [1]

2 The diameter of a spherical atom is 125 picometres.

(a) Express the diameter in metres, giving your answer in standard form.

(b) Find the volume in cm^3 of one billion such atoms. Give your answer in standard form correct to 1 significant figure.

Answer (a) ..... m [1]

(b) ..... cm^3 [2]

3 The sine of an angle is 0.821.
Give two possible values for the angle.

Answer ..... or ..... [2]

For Examiner's Use

4 Simplify the following, leaving your answer in positive index.

(a) $\left(\frac{a^{\frac{3}{2}}}{2}\right)^{-2}$

(b) $\frac{\sqrt{a}}{(-2a)^3 \times 3a^0}$

Answer (a) ..... [1]

(b) ..... [1]

5 Express $\frac{2}{2-t} + \frac{3t-5}{t^2-7t+10}$ as a single fraction in its simplest form.

Answer ..... [3]

For Examiner's Use

6 Solve the inequality $\frac{x-12}{3} \leq \frac{x-15}{6}$.

Answer ..... [2]

7 A man wants to invest $100 000 in an investment scheme for a period of 2 years.
Bank A offers a compound interest of 5% per annum, compounded yearly.
Bank B offers a compound interest of 4.9% per annum, compounded monthly.
Showing your working clearly, indicate the bank that he should invest his money in.

Answer ..... [3]

For Examiner's Use

For Examiner's Use

8 Two bottles are geometrically similar. The smaller bottle has a capacity of 1 litre and the larger bottle has a capacity of 9 litres. Calculate the height of the smaller bottle as a percentage of the height of the larger bottle.

Answer .....% [2]

9 Three interior angles of an n-sided polygon are 160°, 164° and 168°. The size of each of the remaining exterior angles is 13°. Find the value of n.

Answer n = ... [2]

10 The intensity I, of a given light source is inversely proportional to the square of distance, d. For a given distance of d cm, the intensity is 120 units. Find the new value of I when d is halved.

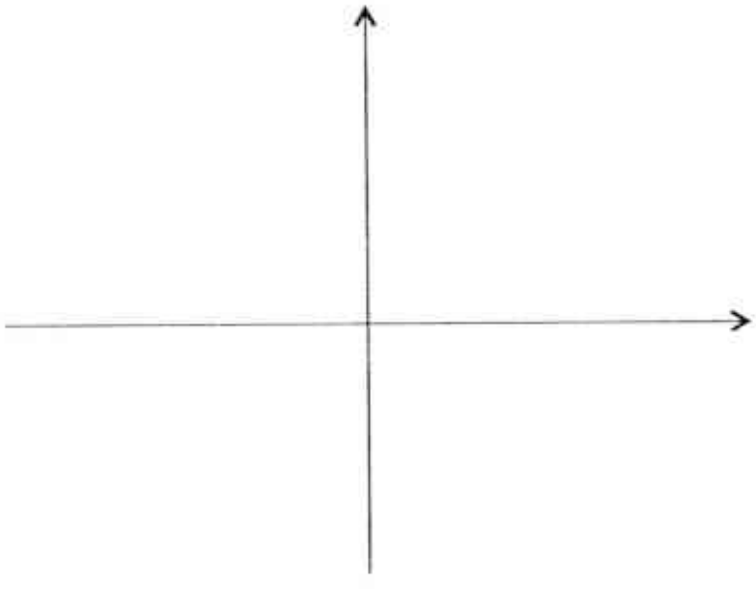
Answer ..... [2]

For Examiner's Use

11 **(a)** Express $x^2 - 6x - 10$ in the form of $(x-a)^2 + b$.

Answer (a) ..... [1]

(b) Sketch the graph of $y = x^2 - 6x - 10$. [2]



For Examiner's Use

For Examiner's Use

12 Written as the product of prime factors, $3500 = 2^2 \times 5^3 \times 7$.

(a) Express 720 as a product of its prime factors.

(b) Hence, write down

(i) the largest integer which is a factor of both 3500 and 720.

(ii) the smallest positive integer k for which $720k$ is a multiple of 3500.

Answer (a) ..... [1]

(b)(i) ..... [1]

(b)(ii) ..... [1]

13 Factorise completely

(a) $\frac{9}{16}x^2 - 1$,

(b) $ab - a - b + 1$.

Answer (a) ..... [1]

(b) ..... [1]

For Examiner's Use

14

$\mathcal{E} = \{\text{integers } x : 1 \leq x \leq 12\}$
$A = \{\text{even numbers}\}$
$B = \{\text{multiples of 4}\}$

(a) Draw a Venn Diagram to illustrate this information.

(b) List the elements contained in the set $(A \cap B)'$.

Answer (a) [2]

(b) ..... [1]

15

(a) **(i)** The nth term of a sequence is given by $(2n - 1)^2$. Write down the first 4 terms.

(ii) Explain clearly why all the terms in the sequence can never be even.

(b) The first four terms of another sequence are 25, 49, 81, 121.....
By comparing this sequence with your answer in **(ai)**, write down the nth term.

Answer (a)(i) ..... [1]

(ii) .....

.....

.....

.....[1]

(b) .....[1]

For Examiner's Use

For Examiner's Use

16 The gradient of the line joining $A(8, p)$ and $B(-2, 7)$ is 4. Find

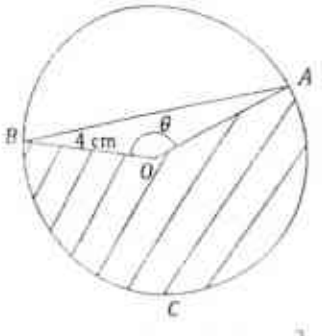
(a) the equation of the line AB,

(b) the length of AB.

Answer (a) ..... [2]

(b) ..... [1]

17 The figure shows a circle, centre O of radius 4 cm.



Given that the area of triangle OAB is 6 cm^2, find

(a) θ in radians,

(b) the area of the shaded region,

(c) the perimeter of the major sector ABC.

Answer (a) ..... radians [2]

(b) ..... cm^2 [1]

(c) ..... cm [1]

For Examiner's Use

18 $\overrightarrow{AB} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$ and $\overrightarrow{DC} = \frac{2}{3}\overrightarrow{AB}$.

(a) Express $\overrightarrow{CD}$ as a column vector.

(b) Given that A is the point $(1, -3)$, find the coordinates of B.

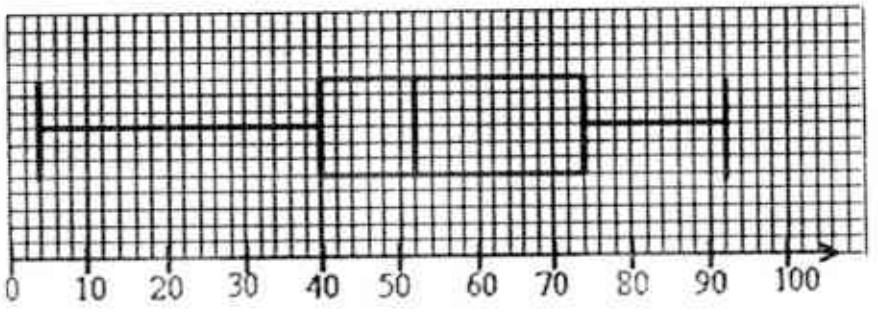
(c) What is the special name given to the quadrilateral $ABCD$?

Answer (a) ..... [1]

(b) ..... [1]

(c) ..... [1]

19 A group of students sat for their Preliminary Examination Maths paper and their results are represented in the box-and-whisker plot.



(a) Find the median of the results.

(b) Find the interquartile range of the results.

Answer (a) ..... [1]

(b) ..... [1]

For Examiner's Use

20

For Examiner's Use

The matrices below shows the number of tour packages to Hong Kong, Korea and Taiwan sold in May and June.

$$\begin{array}{c} \\ \text{Hongkong} \\ \text{Korea} \\ \text{Taiwan} \end{array} \begin{array}{c} \begin{array}{cc} \text{May} & \text{June} \end{array} \\ \begin{pmatrix} 17 & 15 \\ 32 & 36 \\ 11 & 13 \end{pmatrix} \end{array}$$

The price of the Hong Kong, Korea and Taiwan package is \$690, \$1500 and \$1200 respectively.

(a) If $M = (690 \quad 1500 \quad 1200)\begin{pmatrix} 17 & 15 \\ 32 & 36 \\ 11 & 13 \end{pmatrix}$, evaluate M.

(b) Explain what the elements of M represent.

(c) Evaluate $\frac{1}{2}M\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and explain what the answer represents.

Answer (a) ..... [1]

(b) .....

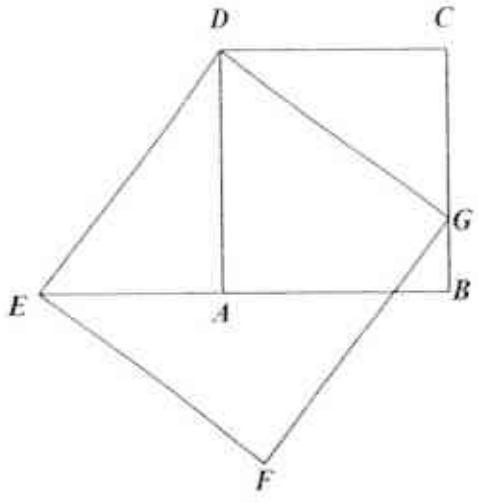
..... [1]

(c) .....

..... [2]

For Examiner's Use

21 In the diagram, $ABCD$ is a square and $DEFG$ is a rectangle. EAB is a straight line.



(a) Show that angle ADE = angle CDG

(b) Prove that triangle ADE is congruent to triangle CDG.

Answer

(a) [2]

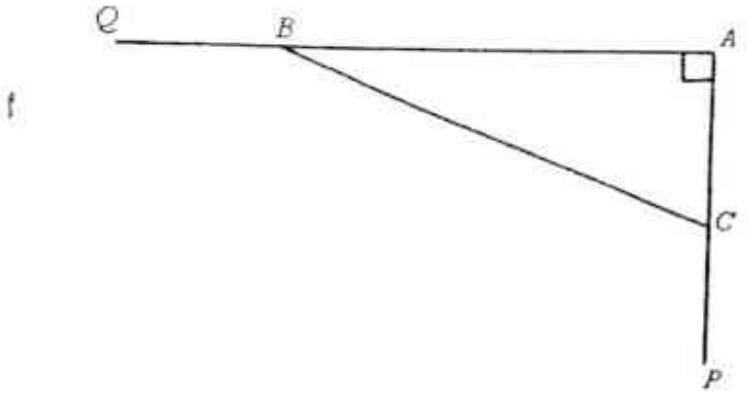
(b) [2]

For Examiner's Use

22

For Examiner's Use

ABC is a right-angled triangle in which $AB = 24$ cm and $BC = 25$ cm. AC is produced to P and AB is produced to Q.



(a) Given that C is the midpoint of AP and the area of ΔAPQ is 3 times the area of ΔACB, find the length of AQ.

(b) Express, as a fraction, the value of

(i) $\tan \angle APQ$,

(ii) $\cos \angle BCP$.

Answer (a) ..... [2]

(b)(i)..... [1]

(ii)..... [1]

For Examiner's Use

23 Jenny is playing with 715 one-centimetre cubes.

(a) She uses all 715 cubes to make a cuboid. All of the sides of the cuboid are longer than 1 cm. Find the dimensions of the cuboid.

(b) Jenny makes the largest cube possible using some of the 715 cubes.
How many cubes does she have left over?

Answer (a) ..... cm by ..... cm by ..... cm [2]

(b) ..... [2]

24 A bag initially contains 6 blue balls, x green balls and y red balls. The probability of drawing a blue ball is $\frac{1}{3}$. If 3 more green balls are added into the bag and 1 red ball is removed from the bag, the possibility of drawing a red ball from the bag is $\frac{1}{4}$. Find the value of y.

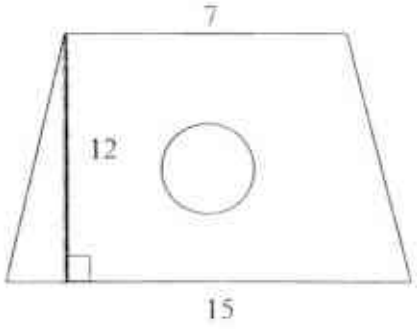
Answer ..... [3]

For Examiner's Use

25

For Examiner's Use

The cross section of a bronze medal is a trapezium with a circular hole, as shown. All measurements are in centimetres.



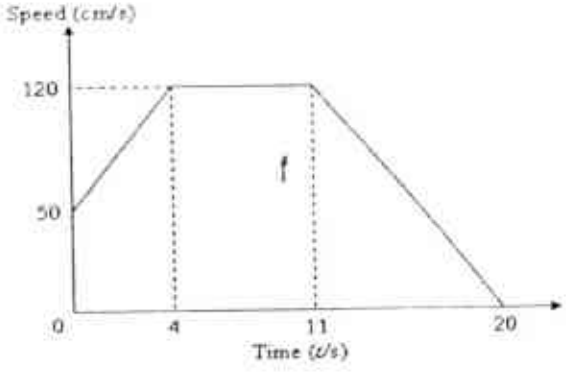
The diameter of the circular hole is 1cm and the uniform thickness of the medal is 0.5cm.
The mass of 1 cubic centimetre of bronze is 8.4g.
The price of 1 gram of the bronze is $2.45.

Calculate the value of the bronze in the medal, giving your answer to the nearest cent.

Answer ..... [4]

For Examiner's Use

26 The diagram below shows the speed-time graph of a particle over a period of 20 seconds.

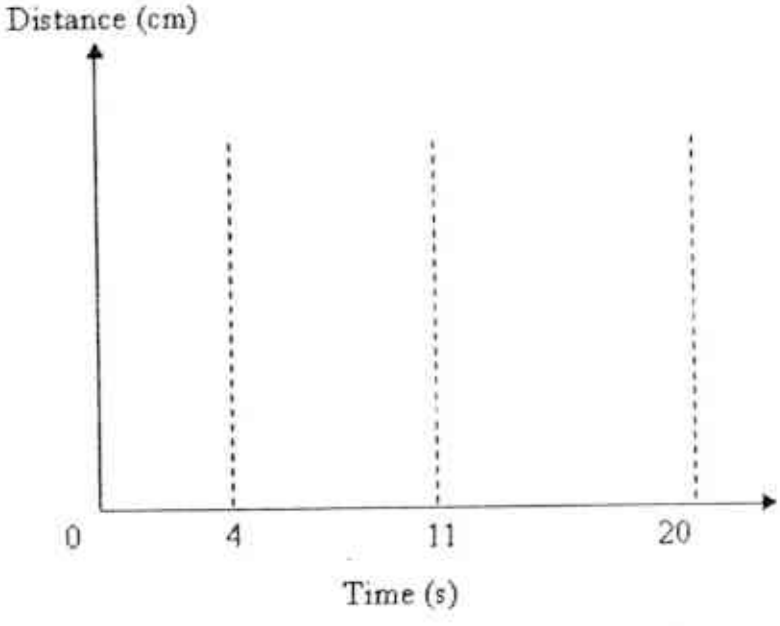


(a) Find,

(i) the acceleration of the particle at time $t = 3$.

(ii) the speed when time $t = 17$.

(b) Sketch the distance-time graph for the same journey. [2]



Answer (a)(i) ..... cm/s^2 [1]

(ii)..... cm/s [2]

71

For Examiner's Use

27

(a) Construct triangle XYZ where $XZ = 8.5$ cm, $\angle ZYX = 52°$ and Z is above the line XY.
XY has already been drawn. [1]

Answer for (a), (b) and (c).

(b) Construct

(i) the perpendicular bisector of the line XY. [1]

(ii) the bisector of angle ZXY [1]

(c) Mark clearly a possible point which is inside the triangle, equidistant from XZ and XY and nearer to Y than to X.

Label this point P. [1]

END OF PAPER

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Answers

1	$\sqrt{0.21} \quad \frac{23}{41} \quad 0.671 \quad 0.55^{\frac{7}{3}}$
2(a)	1.25×10^{-10}
(b)	1×10^{15}
3	55.2° or 124.8° or 0.963 rad or 2.18 rad
4(a)	$4/a^3$
(b)	$-\frac{1}{24a^{\frac{5}{2}}}$
5	$\frac{t+5}{(t-5)(t-2)}$
6	$x \leq 9$
7	Bank A - \$110 250; Bank B - \$110 274.27; Bank B
8	48.1
9	27
10	480
11(a)	$(x - 3)^2 - 19$
12(a)	$2^4 \times 3^2 \times 5$
(b)(i)	20
(b)(ii)	175
13(a)	$(\frac{3}{4}x+1)(\frac{3}{4}x-1)$
(b)	$(a - 1)(b - 1)$
14(b)	{1, 2, 3, 5, 6, 7, 9, 10, 11}
15(a)(i)	1, 9, 25, 49
(ii)	(2n - 1) is always an odd number. When squared, it will always still remain as an odd number.
(b)	$(2n + 3)^2$
16(a)	$y = 4x + 15$
(b)	41.2 units

17(a)	2.29 rad
(b)	31.9 cm^2
(c)	24.0 cm
18(a)	$\begin{pmatrix} \frac{4}{3} \\ -4 \end{pmatrix}$
(b)	(-1, 3)
(c)	trapezium
19(a)	52
(b)	34
20(a)	(72 930 79950)
(b)	The total amount earned from all the tour packages to Hong Kong, Korea and Taiwan in May and June respectively
(c)	(76 440); The average amount of money earned from all the tour packages to Hong Kong, Korean and Taiwan in May and June;
22(a)	36 cm
(b)(i)	18/7
(ii)	- 7/25
23(a)	5 by 11 by 13
(b)	203
24	6
25	$1350.20
26(a)(i)	17.5 cm/s^2
(ii)	40 cm/s

Calculator Model :	

Class	Full Name	Index Number

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I believe, therefore I am

PRELIMINARY EXAMINATION II
2015

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4016/02

MATHEMATICS
Paper 2

Secondary 4 Express / 5 Normal Academic
27th Aug 2015

2 hours 30 minutes

Additional Materials: Graph Paper (1 sheet)
Answer Paper (6 sheets)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HD pencil for any diagrams or graphs.
Do not use staples, paper clips or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of a scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answers to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value of 3.142, unless the question requires the answer in terms of π.

At the end of the test, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

DO NOT OPEN THIS PAPER UNTIL YOU ARE TOLD TO DO SO

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100

This document consists of **11** printed pages, including this cover page.

73

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r l$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Simplify $\sqrt[3]{27m^6n^9}$. [1]

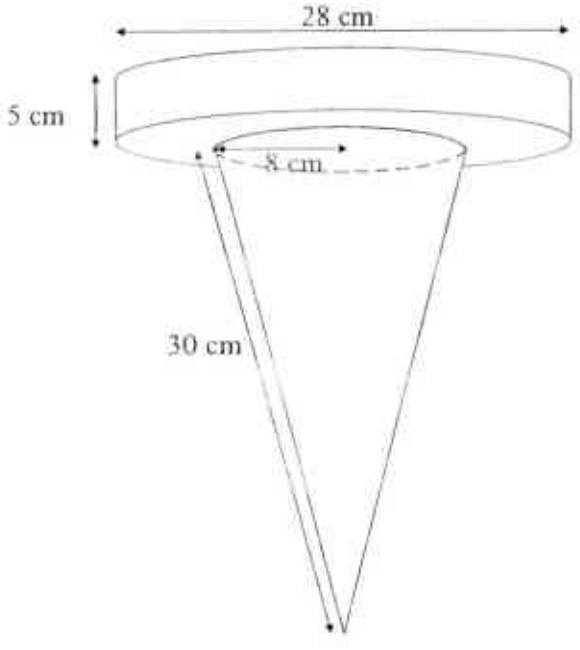
(b) Factorise $9q^2 + 9pq^2 - q - p$ completely. [2]

(c) Simplify $\dfrac{(a+b)^2}{2a^2 + 2b^2 + 4ab}$. [2]

(d) **(i)** Express $\dfrac{5}{x-1} + \dfrac{3}{1-x^2}$ as a single fraction [2]

(ii) Hence, solve $2\left(\dfrac{5}{x-1} + \dfrac{3}{1-x^2}\right) = \dfrac{1}{3}$. [3]

2 The figure shows an Olympic Torch comprising a cylinder attached to the base of a cone.



(a) Show that the height of the cone is $\sqrt{836}$ cm. [1]

(b) Calculate

(i) the volume of the figure, [2]

(ii) the surface area of the figure. [3]

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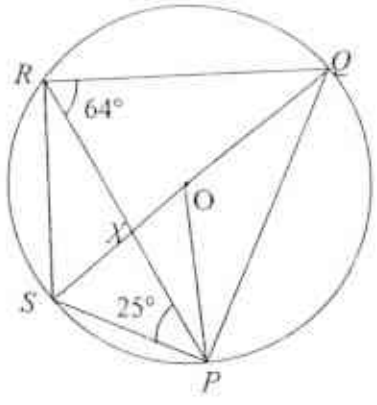
3 In the diagram, P, Q, R and S are points on the circle with centre O. PR intersect QS at X. $\angle QRP = 64°$ and $\angle SPR = 25°$.



(a) Show that ΔRXQ is similar to ΔSXP. [2]

(b) Calculate, stating your reasons clearly,

(i) $\angle SXP$, [1]

(ii) $\angle RPQ$, [1]

(iii) $\angle SRP$, [1]

(iv) $\angle RPO$. [2]

(c) If SX: XQ = 1: 3, calculate the area of ΔRXS if the area of ΔPXQ is 72 cm^2. [2]

4 **(a)** Given ε = {integers x: $2 < x < 23$},

A = {even numbers},

B = {prime numbers},

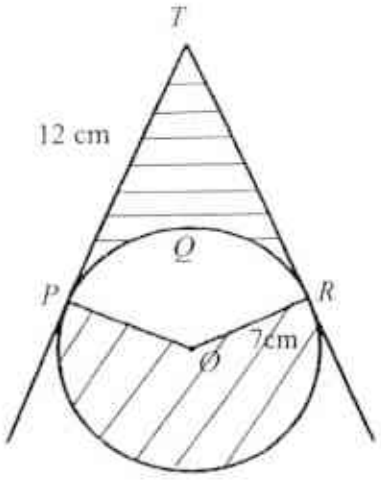
C = {multiples of 4}.

(i) Draw a Venn diagram to illustrate this information. [2]

(ii) List the elements contained in the set $(A \cap C')$. [1]

(iii) Write down $n(A \cup B \cup C)'$. [1]

4 **(b)** Given that the radius of the circle is 7 cm and $TP = TR = 12$ cm, calculate

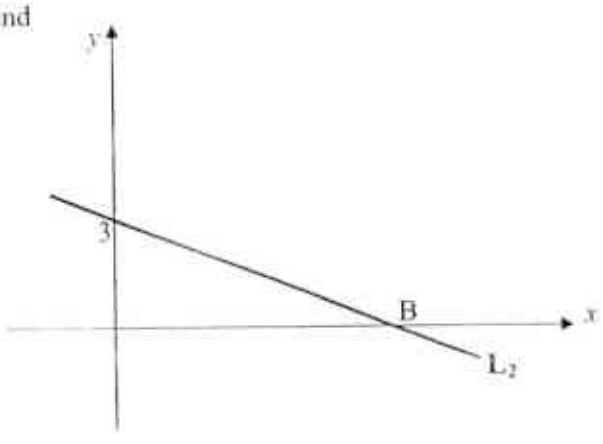


(i) reflex $\angle POR$ in radians, [2]

(ii) arc length PQR, [1]

(iii) area of the shaded region. [3]

5 A line $\mathbf{L_1}$ is parallel to the line $\mathbf{L_2}$: $2y + x - 6 = 0$ and passes through the point A (-2, 5). Find



(a) the equation of $\mathbf{L_1}$, [2]

(b) the coordinates of B where the line $\mathbf{L_2}$ cuts the x-axis, [1]

(c) the value of k if the coordinates C $(k, 1)$ lies on the line $\mathbf{L_1}$, [1]

(d) the length of DE where the coordinates D and E are the points where $\mathbf{L_1}$ cuts the x and y-axis respectively, [2]

(e) the area the trapezium bounded by the lines L_1, L_2 and the x and y-axis. [2]

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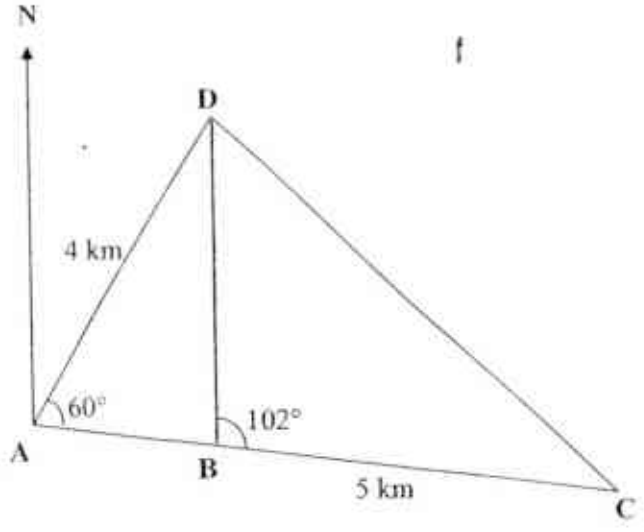
6 The diagram shows a school labeled as A and three other landmarks B, C and D. Given that D is due north of B, AD = 4 km, BC = 5 km, $\angle BAD = 60°$ and $\angle CBD = 102°$, calculate



(a) the bearing of A from D, [1]

(b) distance between B and D, [2]

(c) distance between C and D, [2]

(d) the bearing of D from C, [3]

(e) the shortest distance from B to CD, [1]

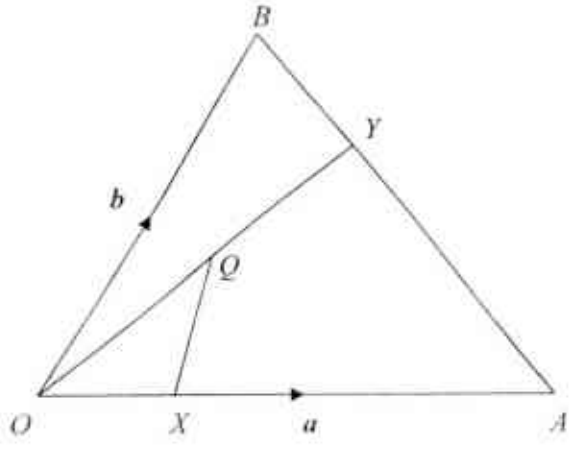
(f) A 9-storey HDB flat of height 950 m is located at B and X is the highest point on the flat. Calculate the greatest angle of elevation of X when viewed from a point on CD. [1]

7 **(a)** The position vectors of A, B, C and D are $\mathbf{a} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$ and $\mathbf{d} = \begin{pmatrix} p \\ 1 \end{pmatrix}$ respectively whereby $AB \parallel CD$.

(i) Find the value of p, [2]

(ii) Hence, find the value of $\left|\overrightarrow{AD}\right|$. [2]

(b) In the diagram, X and Y are points on OA and AB respectively such that $XA = 3OX$ and $AY = 2YB$. Q is the midpoint of OY.



It is given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
Express, as simply as possible, in terms of **a** and/ or **b**,

(i) $\overrightarrow{YO}$, [2]

(ii) $\overrightarrow{XQ}$, [2]

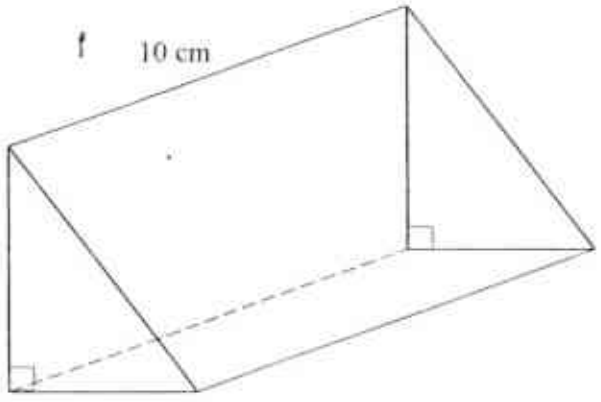
(iii) $\overrightarrow{XB}$, [1]

(iv) Write down 2 facts about vectors $\overrightarrow{XQ}$ and $\overrightarrow{XB}$. [2]

(v) Find the ratio of $\dfrac{\Delta OXQ}{\Delta OQB}$. [1]

76

8 The diagram shows a right-angled triangular prism. The perpendicular height of the right-angled triangle is 7 cm more than the base and is 1 cm less than the hypotenuse.



(a) Given that the base of the triangle is x cm, write down an expression , in terms of x, for

(i) the perpendicular height, [1]

(ii) the hypotenuse. [1]

(b) Write down an expression for the total surface area of the prism. [2]

(c) Given that the total surface area is 360 cm^2, show that it simplifies to

$$x^2 + 37x - 210 = 0.$$ [1]

(d) Solve the equation $x^2 + 37x - 210 = 0$. [2]

(e) Calculate the volume of the prism. [2]

9 ***Answer the whole of this question on a sheet of graph paper.***

The variables x and y are connected by the equation $y = x + \frac{10}{x} - 4$.

Some corresponding values of x and y are given in the table below.

x	1	1.5	2	3	4	5	6	7	8
y	7	***a***	3	2.3	2.5	3	3.6	4.4	5.25

(a) Calculate the value of ***a*** correct to 2 decimal places. [1]

(b) Using a scale of 2 cm to represent 1 unit on both axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the value of x when the gradient of the curve $y = x + \frac{10}{x} - 4$ is equal to zero. [1]

(d) **(i)** On the same axes, draw the graph of the straight line $y = 1 - x$. [1]

(ii) Hence, use your graph to estimate the coordinates of the point Q on the curve where the gradient of the curve is -1. [1]

(e) By drawing a suitable straight line on the same axes, use your graph to find the solutions of the equation $\frac{1}{2}x + \frac{10}{x} - 7 = 0$. [3]

77

10 A container contains 3 red marbles and equal number of blue and green marbles.

(a) Two marbles are drawn without replacement. If the probability of drawing both red marbles is $\frac{1}{7}$, calculate

(i) the total number of blue and green marbles in the container. [2]

(ii) the probability of getting different coloured marbles. [1]

(b) The following stem and leaf diagram shows the weight (kg) distribution of 16 Bowemians.

STEM	LEAF
4	5 7 9
5	1 5 5 7
6	0 1 6 7 8 9
7	1 1 6

Key : 4 | 5 means 45 kg

(i) Find the median and the interquartile range of the distribution. [3]

(ii) Find the mean and standard deviation of the weights. [3]

11 Three brands of rice are sold at Sheng Yong Supermarket in two locations, Bedok and Buangkok

The following tables show the price of each brand at each location.

The prices are given in dollars per carton of 5 kg.

Data is given for the months June and July of 2015.

Brand / Location	June			July	
	Bedok	Buangkok		Bedok	Buangkok
Golden Cockroach	12.30	12.35	Golden Cockroach	12.60	12.75
Loyal Umbrella	13.55	13.50	Loyal Umbrella	13.95	13.80
Crazy Family	14.20	14.55	Crazy Family	14.65	14.95

The information for June can be represented by the matrix $M = \begin{pmatrix} 12.30 & 12.35 \\ 13.55 & 13.50 \\ 14.20 & 14.55 \end{pmatrix}$

The information for July is represented by a matrix N.

(a) Write down the matrix N. [1]

(b) Calculate N – M. [1]

(c) Describe what is represented by the elements of (N – M). [1]

(d) Given that $S = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$, calculate R = SM. [1]

(e) Describe what is represented by the elements of R. [1]

(f) Write down the matrix T such that the product NT will show the total cost, in dollars, of buying 10 kg of each brand at two locations in July. [1]

(g) Calculate NT. [1]

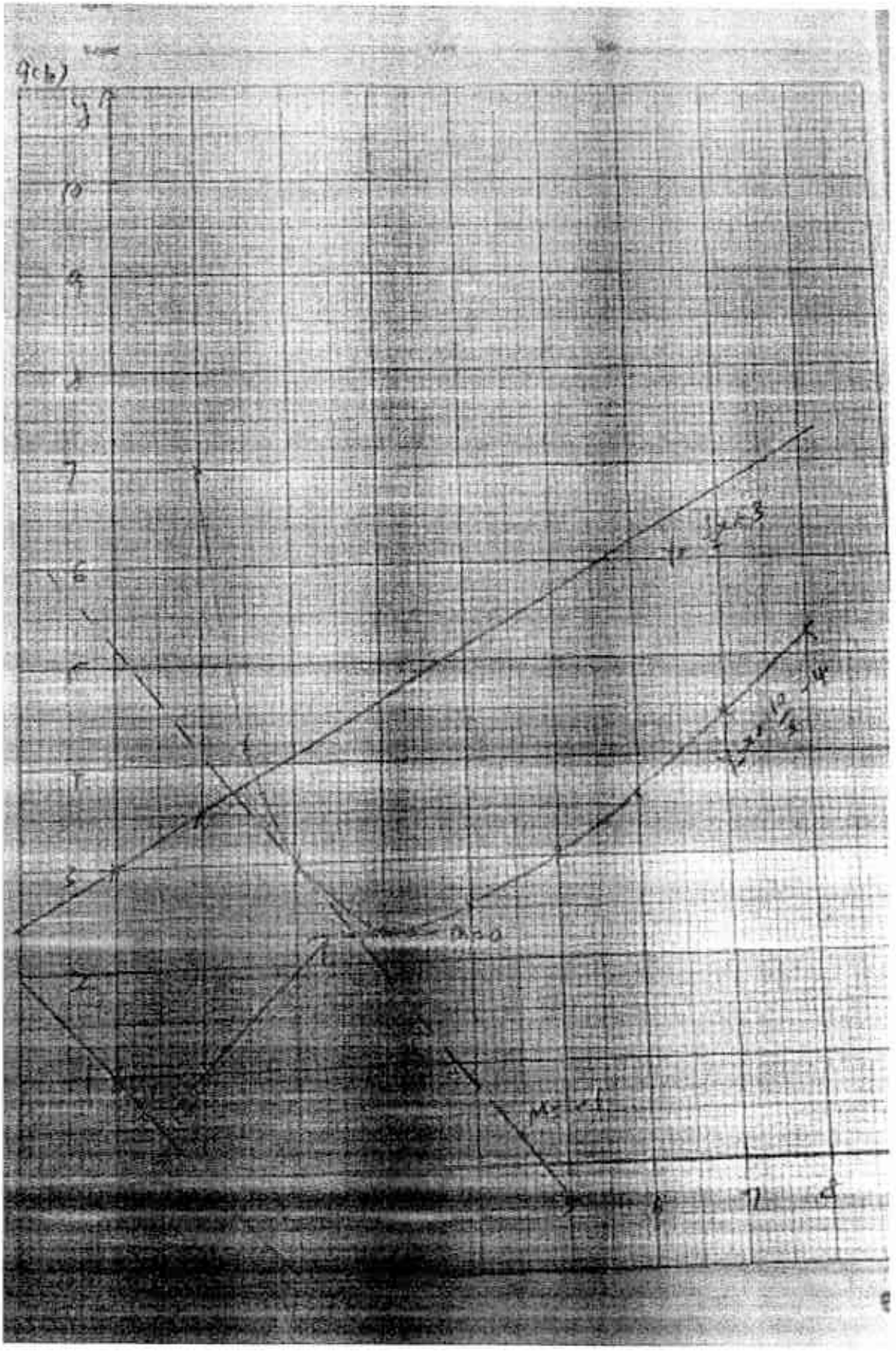
End of Paper

Answer Key

1a	$3m^2n^3$	7ai	p = 7
b	$(3q+1)(3q-1)(q+p)$	ii	9.43
c	1/2	bi	$-\frac{1}{3}(a-2b)$
di	$\frac{5x+2}{x^2-1}$	ii	$\frac{1}{3}(-\frac{1}{4}a+b)$
ii	$x = 30.4 or -0.292$	iii	$-\frac{1}{4}a+b$
		iv	1/3 1) XB = 3XQ 2) X, Q, B lies on a straight line
2a	$h^2+8^2=30^2$ Thus h = $\sqrt{836}$ (Shown)	v	1: 2
b	5014 cm^3	8a	$(x+7)$ cm
c	2223 cm^2	b	$(x+8)$ cm
3a	$\angle PRQ = \angle QSP$ (*Angle* sin *samesegment*) $\angle RQS = \angle RPS$ (*Angle* sin *samesegment*) $\&\angle RXQ = \angle PXS$ (*vertoppangles*) $\therefore \Delta RXQ$ *is similar to* ΔPXS (*AAA*)	c	$(x^2+37x+150)$ am^2
bi	91°	e	300 cm^2
ii	65°	9a	$a = 4.17$
iii	26°	b	See Graph
iv	39°	c	3.16
c	8 cm^2	di	See Graph
4a		e	$y = \frac{1}{2}x+3, x = 1.6$
aii	{6,10,14,18,22}	10ai	4
iii	3	ii	16/21
4bi	4.20 rad	bi	Median = 60.5, IQ = 15.5
ii	14.6 cm	ii	Mean = 60.5, SD = 9.29
iii	143 cm^2		
5a	$y = -\frac{1}{2}x+4$	11a	$\begin{pmatrix} 12.60 & 12.75 \\ 13.95 & 13.20 \\ 14.65 & 14.95 \end{pmatrix}$
b	(6, 0)	b	$\begin{pmatrix} 0.30 & 0.40 \\ 0.40 & 0.30 \\ 0.45 & 0.04 \end{pmatrix}$
c	k = 6	c	The increase in the price of different brand of rice over at the 2 locations in the 2 months
d	(0, 4)	d	(40.05 40.40)
e	7 cm^2	e	Total price of one carton of 3 rice at each location in the month of June

6a	222°	g	$\begin{pmatrix} 50.70 \\ 55.50 \\ 59.20 \end{pmatrix}$
b	3.54 km		
c	6.70 km		
d	313°		
e	2.58 km		
g	20.2°		

79


9(b)

Name:		Index Number:		Class:	

CATHOLIC HIGH SCHOOL
2015 Preliminary Examination 3
Secondary 4

MATHEMATICS

4016/01

Paper 1

1 September 2015
2 hours

Candidates answer in the space provided on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE ON THE MARGINS.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to **three significant figures**. Give answers in **degrees to one decimal place**.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **80**.

For Examiner's Use only
80

This document consists of **20** printed pages. 80

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = $\pi r l$

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Express 864 as the product of its prime factors.

Answer (a)..... [1]

(b) Written as a product of its prime factors, $720 = 2^4 \times 3^2 \times 5$.

Find the smallest possible integer value of n such $720n$ is a multiple of 864.

Answer (b) $n =$..... [2]

---

2 The proton, which is the nucleus of a hydrogen atom, can be pictured as a sphere whose radius is 160 nanometres.

(a) Express 160 nanometres in metres, giving your answer in standard form.

Answer (a).....m [1]

(b) How many protons must be placed in a straight line to make a length of 64 cm?
Give your answer in standard form.

Answer (b)..... [1]

---

3 Factorise the following expressions completely.

(a) $2a^2 + 5ab - 3b^2$

Answer (a)..... [1]

(b) $x^2y - 4y - 12 + 3x^2$

Answer (b)..... [3]

---

4 **(a)** Solve the inequality $x + 5 \leq 3 + 2x < 24 - x$.

Answer (a)..... [2]

(b) Hence, write down all the prime numbers that satisfy the inequality in **(a)**.

Answer (b)..... [1]

5 An open field has an area of 112.5 km^2.
It is represented by an area of 18 cm^2 on map X.

(a) Find the scale of the map in the form 1 : n.

Answer (*a*) 1 : .....[2]

(b) Map Y has a scale of 1 : 400 000 .
A road is measured 2.4 cm on Map X.

Find, in centimetres, the length representing this road on Map Y .

Answer (*b*)..... cm [2]

---

6 Simplify $\left(\frac{x^6}{1000}\right)^{\frac{1}{3}} \div \left(256x^4\right)^{-\frac{1}{4}}$, leaving your answer in positive index.

Answer .....[2]

---

7 Solve the equation $\frac{2x}{2x-3}+1=\frac{1}{2-3x}$.

Answer $x=$ .....[3]

---

8 It is given that $\xi=\{x:0<x<15,\ x \text{ is an integer}\}$.
$A=\{x:3<x\leq 9\}$ and $B=\{x:4\leq x<12,\ x \text{ is a prime number}\}$.

(a) Draw a Venn diagram in the space below to illustrate this information.

Answer (*a*) [2]

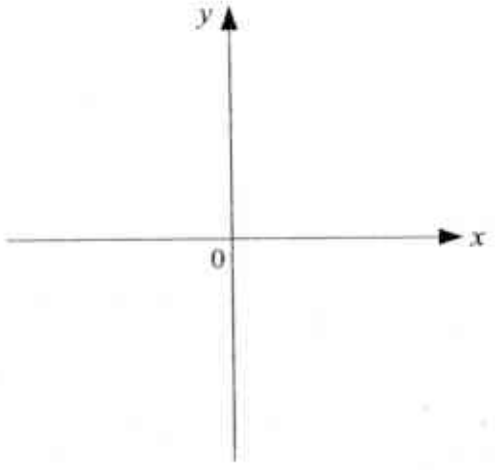
(b) Write down $n(A' \cap B)$.

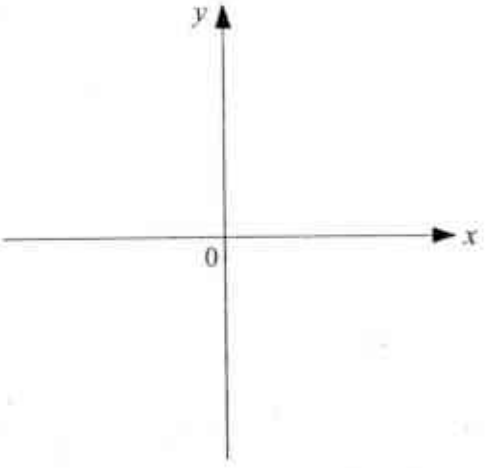
Answer (*b*).....[1]

---

9 **(a)** On the axes given, sketch the following graphs, indicating the x and y intercepts where relevant.

(i) $y = -x^2$ **(ii)** $y = 2^x$ [2]





(b) Hence explain why the equation $2^x + x^2 = 0$ has no solution.

Answer (*b*).....

.....

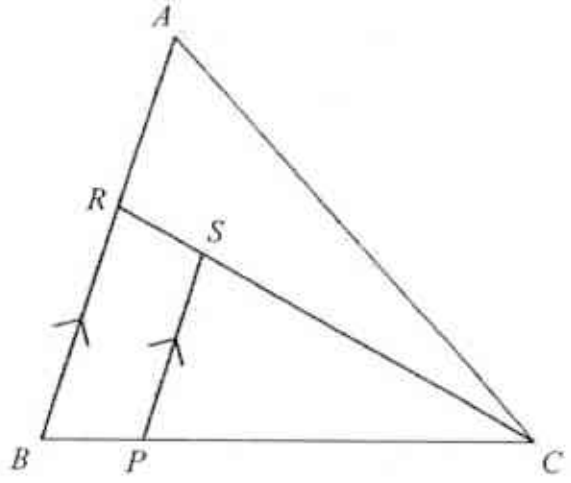
..... [1]

---

10 Simplify $\frac{3pq^3}{2r^2} \times \frac{qr^3}{5p^2} \div \frac{q^2}{10pr}$.

Answer ..... [2]

11 In the figure, AB and SP are parallel lines. P lies on the line BC such that $BP : PC = 1 : 4$ and R lies on the line AB such that $BR : RA = 5 : 3$.



(a) Explain why triangles BCR and PCS are similar.

Answer (*a*) In triangles BCR and PCS, ...

.....

.....

.....

..... [1]

(b) If the area of ΔBCR is 50 cm^2, calculate the area of ΔPCS.

Answer (*b*).....cm^2 [2]

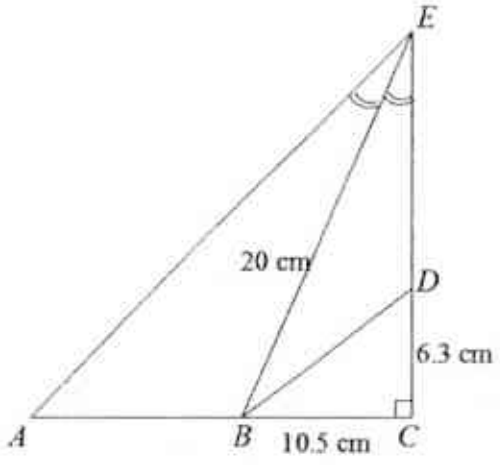
(c) Find $\dfrac{\text{area of } \Delta PCS}{\text{area of } \Delta ABC}$, giving your answer as a fraction in the simplest form.

Answer (*c*).....[1]

12 In the figure, triangle ACE is a right-angle triangle.
B and D are points on AC and CE respectively such that $BC = 10.5$ cm, $BE = 20$ cm and $CD = 6.3$ cm.
The line BE bisects the angle AEC.

Find

(a) angle BEC,



Answer (*a*).....° [1]

(b) the length of AE,

Answer (*b*)..... cm [2]

(c) $\tan \angle BDE$, giving your answer as a fraction in the simplest form.

Answer (*c*) ..... [1]

84

13 Two stores advertise the same LED television set during the Great Singapore Sale.

STORE *A*

$1500 + 7% GST*

STORE *B*

- Deposit of $220

plus 12 monthly instalments of $120

- Price includes 7% GST*

**GST : Goods and Services Tax*

(a) Which store sells the television set at a lower price? Justify your answer.

Answer (*a*) Store ..... offers a lower price because .....

.....

.....

.....

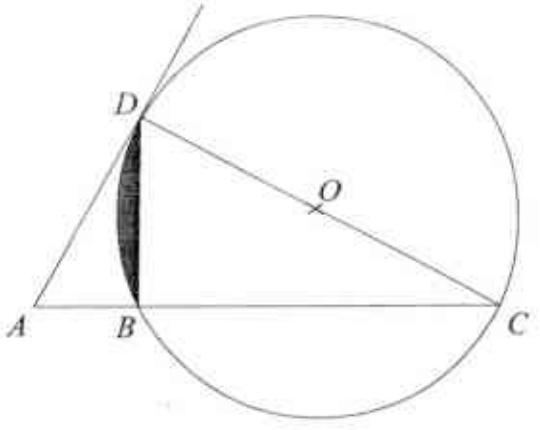
.....

..... [2]

(b) Calculate the amount of GST charged on the television set in Store *B*.

Answer (*b*) $..... [1]

14 In the diagram, O is the centre of circle BCD with radius 20 cm and CD is a diameter of the circle.
A is a point on BC produced such that AD is a tangent to the circle at D.



(a) Given that $\frac{AB}{AD}=\frac{1}{2}$, explain why angle $BCD=\frac{\pi}{6}$ radian.

Answer (a) [2]

(b) Calculate the area of the shaded region.

Answer (b)..... cm^2 [3]

85

15 The illumination, I units, of a bulb varies inversely as the square of the distance, d metres. Given that the illumination is 9 units when the distance is 2 m,

(a) express I in terms of d,

Answer (*a*) $I =$ ..... [1]

(b) find the percentage change in the illumination of the bulb which is required to reduce the distance of the bulb to $\frac{1}{5}$ of its original value.

Answer (*b*)..... % [2]

---

16 Express $\frac{2}{x+2} - \frac{9x+20}{(x+3)(x+2)}$ as a fraction in its simplest form.

Answer ..... [2]

---

17 Han Brothers Travel has tour packages to Australia, South Korea and Japan.

The table below shows the number of people who have signed up for the respective packages from January to June and from July to December.

	Period 1	Period 2
	From January to June	From July to December
Australia	15	29
South Korea	25	24
Japan	20	31

The information for the number of people who signed up for packages to Australia, South Korea and Japan can be represented by the matrix $\mathbf{P} = \begin{pmatrix} 15 & 29 \\ 25 & 24 \\ 20 & 31 \end{pmatrix}$.

(a) Write down a 1×3 matrix $\mathbf{Q}$ such that the matrix multiplication $\mathbf{C} = \mathbf{QP}$ gives the total number of people who signed up for the tour packages in period 1 and period 2 respectively.
Hence find $\mathbf{C}$.

Answer (*a*) $\mathbf{Q} =$ ..... [1]

$\mathbf{C} =$ ..... [1]

(b) Given that $\mathbf{R} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, calculate $\mathbf{D} = \mathbf{CR}$.

Answer (*b*) $\mathbf{D} =$ .....[1]

(c) Describe what is represented by the element(s) of **D**.

Answer (*c*) .....

.....

..... [1]

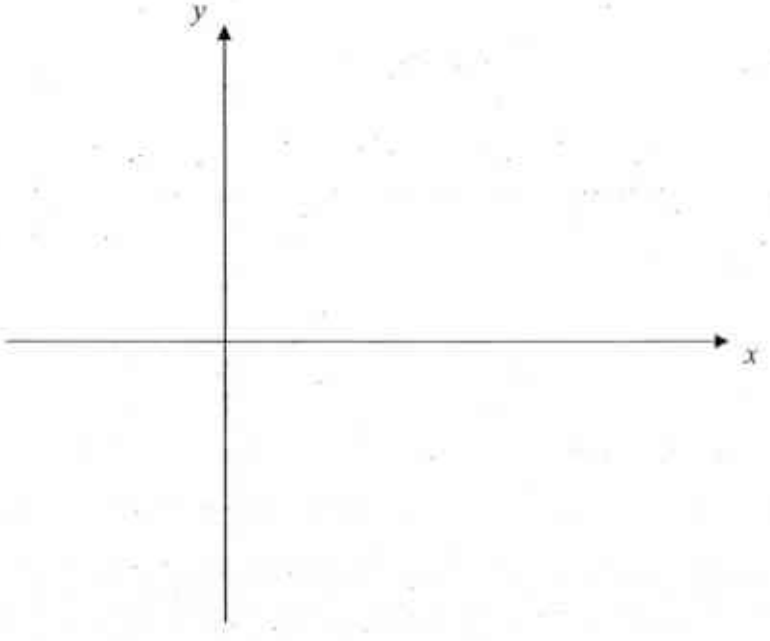
---

18 **(a)** Express $y = 4x - x^2$ in the form $y = a - (x + b)^2$, where a and b are constants.

Answer (*a*) $y =$ ..... [2]

(b) Sketch the graph of $y = 4x - x^2$, showing the axes intercepts and coordinates of the turning point clearly.

Answer (*b*) [2]



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19 Point A and B have coordinates (1, 3) and (1, 15) respectively.
Point P lies on the line $y = 6 + x$ and is equidistant from A and B.

(a) Explain, with working clearly shown, why P is **not** the mid-point of AB.

Answer (*a*) [2]

(b) Find the coordinates of P.

Answer (*b*) P (..... , ...) [2]

---

20 The average daily temperature, in degrees Celsius (°C), of City A measured in 11 days are shown below.

29.5 32.1 33.2 30.1 33.9 32.7 29.9 33.0 30.7 33.6 29.9

(a) Find

(i) the median temperature,

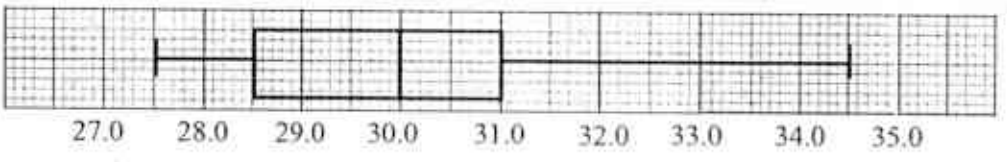
Answer (*a*)(i) ..... °C [1]

87

(ii) the interquartile range.

Answer (*a*)(ii) .....°C [2]

(b) The box-and-whisker diagram shows the average temperature of City *B* in the same period.



A report commented that the average temperature of City *B* during these 11 days is generally higher than that of City *A*.

Do you agree? Give a reason to support your answer.

Answer (*b*) .....

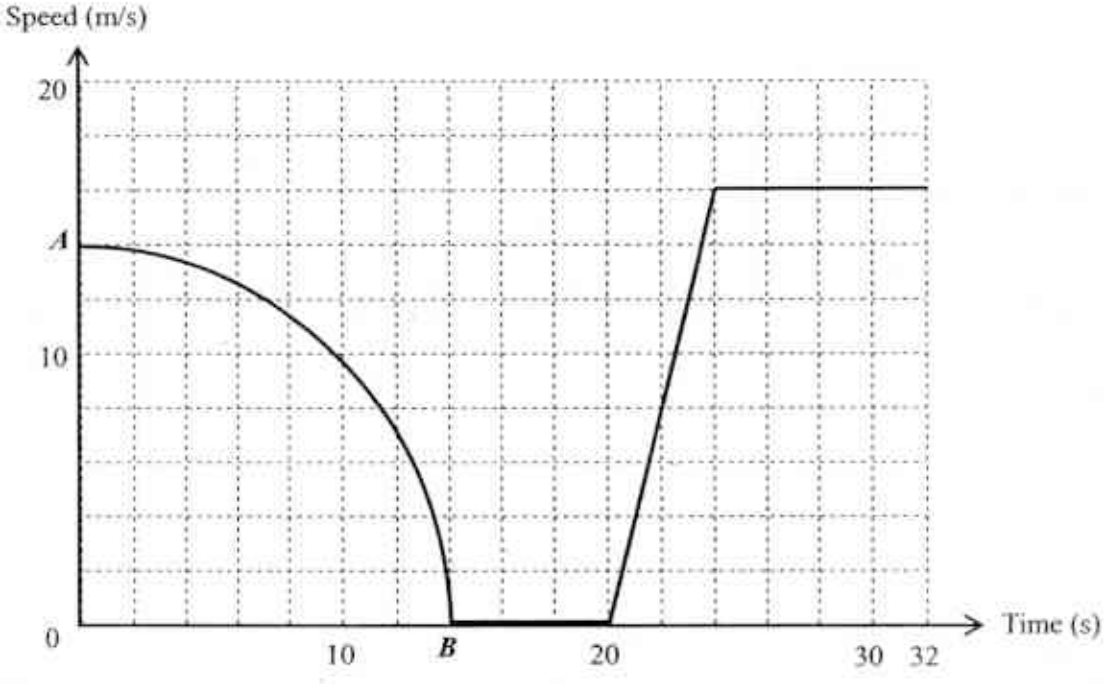
.....

..... [1]

21 The diagram below shows the speed-time graph of a car for a journey.

The car decelerates from an initial speed of 14 m/s for the first 14 seconds and remained at rest for the next 6 seconds. It then accelerates uniformly for the next 4 seconds and remained at a constant speed of 16 m/s thereafter.

AB is an arc of a quadrant.



(a) Find the acceleration at $t = 23$ s.

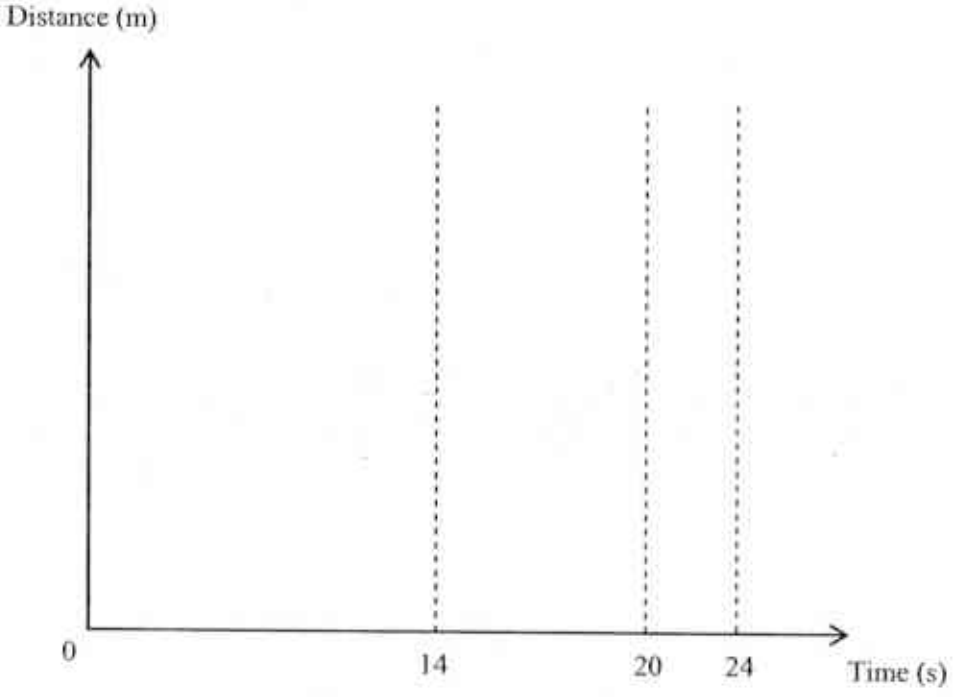
Answer (*a*) ..... m/s^2 [1]

(b) Taking π to be $\frac{22}{7}$, find the total distance travelled for the entire journey.

Answer (*b*)..... m [2]

(c) On the axes provided below, sketch the distance-time graph of the vehicle for the first 24 seconds of the journey, indicating the distances travelled after $t = 14$ s, $t = 20$ s and $t = 24$ s on the vertical axis clearly.

Answer (c) [2]



22 Magician William randomly draws 2 cards from a stack of 5 cards labelled 5 to 9, one after another without replacement.

The sum of the numbers on the two cards is obtained.

(a) Complete the possibility diagram in the answer space below.

Answer (a) [2]

+	5	6	7	8	9
5		11			
6	11				
7					
8					
9					

(b) Calculate the probability that the sum obtained is a multiple of 5.

Answer (b) ..... [1]

(c) A third card is chosen at random from the stack without replacement.
Find the probability that the sum of the numbers on the three cards is 24.

Answer (c) ..... [2]

89

23 In a triangular field ABC, B is due east of A, C is at a bearing of 025° from A and $AC = 460$ m.

AB has already been drawn.

(a) Using a scale of 1 cm to represent 50 m, construct the triangular field ABC.

Answer (*a*) [1]

(b) A statue is located at point Q which is equidistant from the points B and C and equidistant from the lines AC and AB.

By constructing the appropriate perpendicular and angle bisectors, mark the point Q and estimate the actual distance CQ.

Answer (*b*) CQ = ...m [3]

END OF PAPER

Answer **all** the questions.

1 **(a)** Express 864 as the product of its prime factors.

Answer (a) $2^5 \times 3^3$ [1]

(b) Written as a product of its prime factors, $720 = 2^4 \times 3^2 \times 5$.

Find the smallest possible integer value of n such $720n$ is a multiple of 864.

LCM of 720 and $864 = 2^5 \times 3^3 \times 5 = 4320$

$$\therefore \quad n = \frac{4320}{720} = 6$$

Answer (b) $n =$ 6 [2]

---

2 The proton, which is the nucleus of a hydrogen atom, can be pictured as a sphere whose radius is 160 nanometres.

(a) Express 160 nanometres in metres, giving your answer in standard form.

Answer (a) 1.6×10^{-7} m [1]

(b) How many protons must be placed in a straight line to make a length of 64 cm?
Give your answer in standard form.

Answer (b) 2×10^6 [1]

---

3 Factorise the following expressions completely.

(a) $2a^2 + 5ab - 3b^2$

Answer (a) $(2a - b)(a + 3b)$ [1]

(b) $x^2y - 4y - 12 + 3x^2$

$$x^2y - 4y - 12 + 3x^2 = y(x^2 - 4) + 3(x^2 - 4)$$

$$= (x^2 - 4)(y + 3)$$

$$= (x + 2)(x - 2)(y + 3)$$

Answer (b) $(x + 2)(x - 2)(y + 3)$ [3]

---

4 **(a)** Solve the inequality $x + 5 \leq 3 + 2x < 24 - x$.

$$x + 5 \leq 3 + 2x < 24 - x$$

$$\begin{matrix} x + 5 \leq 3 + 2x \\ x \geq 2 \end{matrix} \quad \text{and} \quad \begin{matrix} 3 + 2x < 24 - x \\ 3x < 21 \\ x < 7 \end{matrix}$$

Hence, $2 \leq x < 7$

Answer (a) $2 \leq x < 7$ [2]

(b) Hence, write down all the prime numbers that satisfy the inequality in **(a)**.

Answer (b) 2 , 3 , 5 [1]

---

5 An open field has an area of 112.5 km^2.

It is represented by an area of 18 cm^2 on map X.

(a) Find the scale of the map in the form $1 : n$.

$18cm^2 : 112.5km^2$

$1cm^2 : 6.25km^2$

$1cm : 2.5km$

$1cm : 250000cm$

Answer (*a*) 1 : 250 000 [2]

(b) Map Y has a scale of 1 : 400 000.
A road is measured 2.4 cm on Map X.

Find, in centimetres, the length representing this road on Map Y.

$MapX : 1cm \rightarrow 2.5km$

$\therefore 2.4cm \rightarrow 6km$

$MapY : 1cm \rightarrow 4km$

$\therefore 6km \rightarrow 1.5cm$

Answer (*b*) 1.5 cm [2]

---

6 Simplify $\left(\frac{x^6}{1000}\right)^{\frac{1}{3}} \div \left(256x^4\right)^{-\frac{1}{4}}$, leaving your answer in positive index.

$$\left(\frac{x^6}{1000}\right)^{\frac{1}{3}} \div \left(256x^4\right)^{-\frac{1}{4}} = \frac{x^2}{10} \div \frac{1}{4x}$$

$$= \frac{2x^3}{5}$$

Answer $\frac{2x^3}{5}$ [2]

---

7 Solve the equation $\frac{2x}{2x-3}+1=\frac{1}{2-3x}$.

$$\frac{2x}{2x-3}+1=\frac{1}{2-3x}$$
$$\frac{2x+(2x-3)}{2x-3}=\frac{1}{2-3x}$$
$$\frac{4x-3}{2x-3}=\frac{1}{2-3x}$$
$$(4x-3)(2-3x)=2x-3$$
$$-12x^2+17x-6=2x-3$$
$$12x^2-15x+3=0$$
$$4x^2-5x+1=0$$
$$(4x-1)(x-1)=0$$
$$x=\frac{1}{4} \quad \text{or} \quad x=1$$

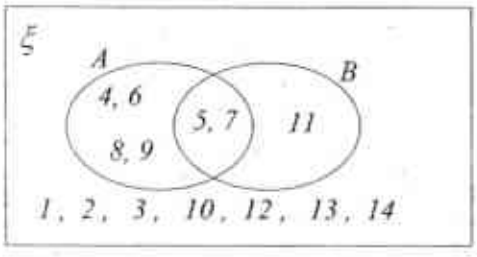
Answer $x=\frac{1}{4}$ or 1 [3]

---

8 It is given that $\xi=\{x:0<x<15,\ x \text{ is an integer}\}$.
$A=\{x:3<x\leq 9\}$ and $B=\{x:4\leq x<12,\ x \text{ is a prime number}\}$.

(a) Draw a Venn diagram in the space below to illustrate this information.

Answer (a) [2]



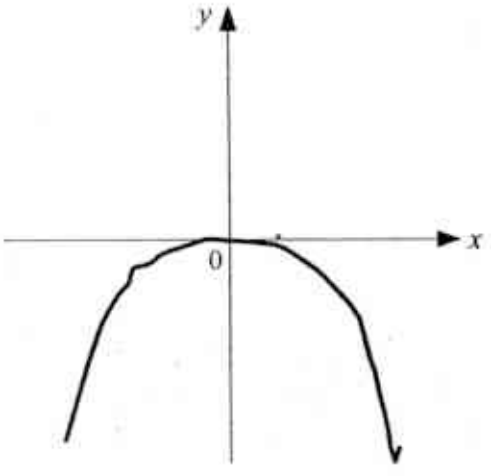
(b) Write down $n(A' \cap B)$.

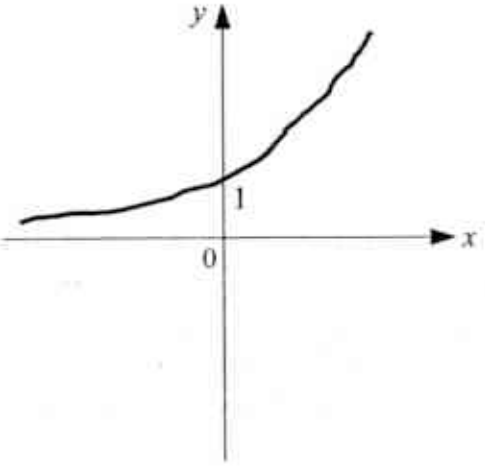
Answer (b) 1 [1]

---

9 **(a)** On the axes given, sketch the following graphs, indicating the x and y intercepts where relevant.

(i) $y = -x^2$ **(ii)** $y = 2^x$ [2]





(b) Hence explain why the equation $2^x + x^2 = 0$ has no solution.

Answer (b) $2^x + x^2 = 0$ has no solution since the graphs of $y = 2^x$ and $y = -x^2$ do not intersect. [1]

---

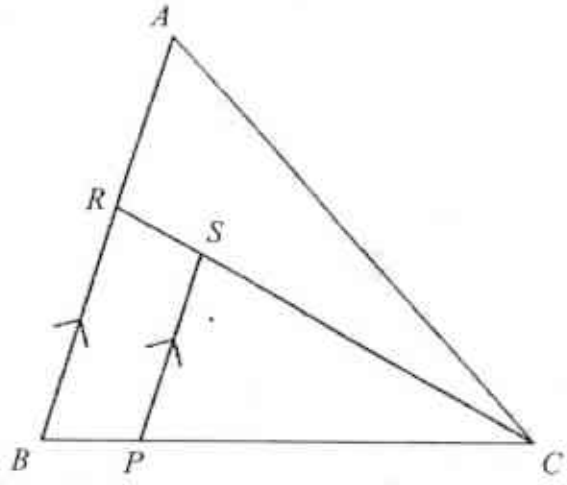
10 Simplify $\frac{3pq^3}{2r^2} \times \frac{qr^3}{5p^2} \div \frac{q^2}{10pr}$.

$$\frac{3pq^3}{2r^2} \times \frac{qr^3}{5p^2} \div \frac{2q^2}{10pr} = \frac{3pq^3}{2r^2} \times \frac{qr^3}{5p^2} \times \frac{10pr}{q^2}$$
$$= 3q^2r^2$$

Answer $3q^2r^2$ [2]

---

11 In the figure, AB and SP are parallel lines. P lies on the line BC such that $BP : PC = 1 : 4$ and R lies on the line AB such that $BR : RA = 5 : 3$.



(a) Explain why triangles BCR and PCS are similar.

In triangles BCR and PCS,

$\angle BCR$ is shared

$\angle CBR = \angle CPS$ (corresponding angles)

$\angle CRB = \angle CSP$ (angle sum of triangle)

(b) If the area of ΔBCR is 50 cm^2, calculate the area of ΔPCS.

$$\frac{\text{Area of } \Delta PCS}{\text{Area of } \Delta BCR} = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$

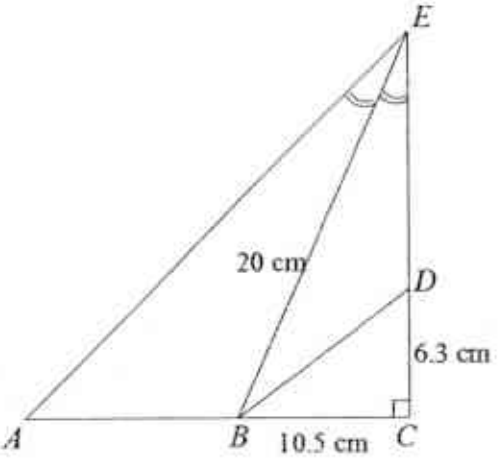
$$\text{Area of } \Delta PCS = \frac{16}{25} \times 50 = 32 \text{ cm}^2$$

Answer (*b*) 32 cm^2 [2]

(c) Find $\dfrac{\text{area of } \Delta PCS}{\text{area of } \Delta ABC}$, giving your answer as a fraction in the simplest form.

Answer (*c*) $\dfrac{2}{5}$ [1]

12 In the figure, triangle ACE is a right-angle triangle.
B and D are points on AC and CE respectively such that $BC = 10.5\,\text{cm}$, $BE = 20\,\text{cm}$ and $CD = 6.3\,\text{cm}$.
The line BE bisects the angle AEC.



Find

(a) angle BEC,

$$\sin \angle BEC = \frac{10.5}{20}$$

$$\angle BEC = 31.66° \approx 31.7°$$

Answer (a) 31.7 ° [1]

(b) the length of AE,

$$CE = \sqrt{20^2 - 10.5^2} = \sqrt{289.75}$$

$$\angle AEC = 2(31.66) = 63.32°$$

$$\cos 63.32 = \frac{\sqrt{289.75}}{AE}$$

$$AE = 37.9$$

Answer (b) 37.9 cm [2]

(c) $\tan \angle BDE$, giving your answer as a fraction in the simplest form.

Answer (c) $-\frac{5}{3}$ [1]

13 Two stores advertise the same LED television set during the Great Singapore Sale.

STORE *A*

$1500 + 7% GST*

STORE *B*

- Deposit of $220

plus 12 monthly instalments of $120

- Price includes 7% GST*

**GST : Goods and Services Tax*

(a) Which store sells the television set at a lower price? Justify your answer.

Answer (*a*) Store **A** offers a lower price because

$$\text{Price of LCD TV (Store A)} = \frac{107}{100} \times \$1500$$
$$= \$1605$$
$$\text{Price of LCD TV (Store B)} = \$220 + 12 \times \$120$$
$$= \$1660$$

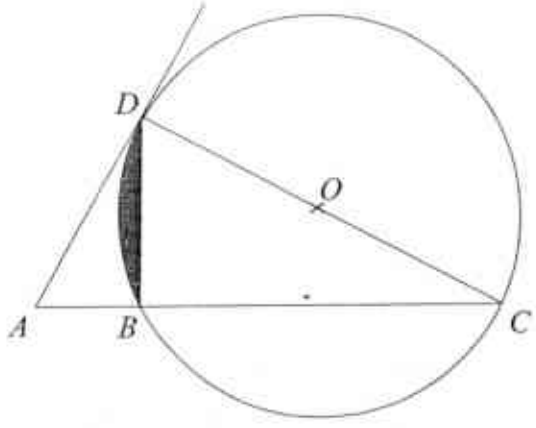
Store A offers better deal since the cost of the TV set is lower at Store A. [2]

(b) Calculate the amount of GST charged on the television set in Store *B*.

$$\text{Amount of GST of the TV set in Store B} = \frac{7}{107} \times \$1650$$
$$= \$107.94$$

Answer (*b*) $107.94 $ [1]

14 In the diagram, O is the centre of circle BCD with radius 20 cm and CD is a diameter of the circle.
A is a point on BC produced such that AD is a tangent to the circle at D.



(a) Given that $\frac{AB}{AD}=\frac{1}{2}$, explain why angle $BCD=\frac{\pi}{6}$ radian.

Answer (a) [2]

$\angle ADB=\sin^{-1}\frac{1}{2}=\frac{\pi}{6}$

$\angle BDC=\frac{\pi}{2}-\frac{\pi}{6}$ (tangent perpendicular radius)

$=\frac{\pi}{3}$

$BCD=\pi-\frac{\pi}{2}-\frac{\pi}{3}=\frac{\pi}{6}$

(b) Calculate the area of the shaded region.

$\angle BOD=2\left(\frac{\pi}{6}\right)=\frac{\pi}{3}$

shaded area $=\frac{1}{2}\left(20^2\right)\left(\frac{\pi}{3}\right)-\frac{1}{2}\left(20^2\right)\sin\frac{\pi}{3}$

$=36.2$

Answer (b) 36.2 cm^2 [3]

15 The illumination, I units, of a bulb varies inversely as the square of the distance, d metres. Given that the illumination is 9 units when the distance is 2 m,

(a) express I in terms of d,

$$I = \frac{k}{d^2}$$
$$k = (9)(2^2)$$
$$= 36$$
$$\therefore I = \frac{36}{d^2}$$

Answer (a) $I = \dfrac{36}{d^2}$ [1]

(b) find the percentage change in the illumination of the bulb which is required to reduce the distance of the bulb to $\frac{1}{5}$ of its original value.

$$I_O = \frac{k}{d^2}$$
$$I_N = \frac{k}{(0.2d)^2} = \frac{25k}{d^2}$$
$$\% \text{ change} = \frac{\frac{25k}{d^2} - \frac{k}{d^2}}{\frac{k}{d^2}} \times 100 = 2400$$

Answer (b) 2400% [2]

---

16 Express $\dfrac{2}{x+2} - \dfrac{9x+20}{(x+3)(x+2)}$ as a fraction in its simplest form.

$$\frac{2}{x+2} - \frac{9x+20}{(x+3)(x+2)} = \frac{2(x+3)-(9x+20)}{(x+3)(x+2)}$$
$$= \frac{2x+6-9x-20}{(x+3)(x+2)}$$
$$= \frac{-7x-14}{(x+2)(x+3)}$$
$$= \frac{-7(x+2)}{(x+2)(x+3)} = \frac{-7}{x+3}$$

Answer $\dfrac{-7}{x+3}$ [2]

---

17 Han Brothers Travel has tour packages to Australia, South Korea and Japan.

The table below shows the number of people who have signed up for the respective packages from January to June and from July to December.

	Period 1	Period 2
	From January to June	From July to December
Australia	15	29
South Korea	25	24
Japan	20	31

The information for the number of people who signed up for packages to Australia, South Korea and Japan can be represented by the matrix $\mathbf{P} = \begin{pmatrix} 15 & 29 \\ 25 & 24 \\ 20 & 31 \end{pmatrix}$.

(a) Write down a 1×3 matrix **Q** such that the matrix multiplication $\mathbf{C} = \mathbf{QP}$ gives the total number of people who signed up for the tour packages in period 1 and period 2 respectively.
Hence find **C**.

Answer (a) $\mathbf{Q} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$ [1]

$\mathbf{C} = \begin{pmatrix} 60 & 84 \end{pmatrix}$ [1]

(b) Given that $\mathbf{R} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, calculate $\mathbf{D} = \mathbf{CR}$.

$D = \begin{pmatrix} 60 & 84 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 144 \end{pmatrix}$

Answer (b) $\mathbf{D} = \begin{pmatrix} 144 \end{pmatrix}$ [1]

95

(c) Describe what is represented by the element(s) of **D**.

Answer (*c*) Total number of people who signed up for the tour packages in the year. [1]

---

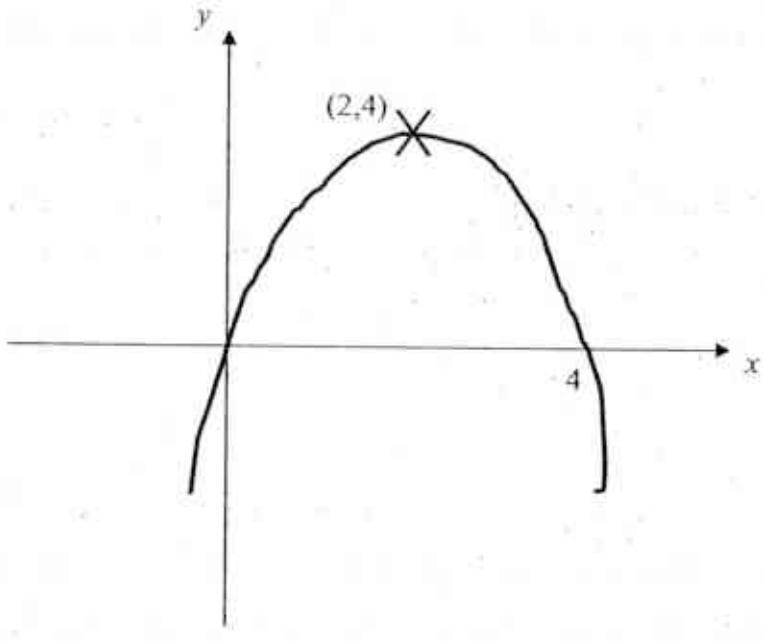
18 **(a)** Express $y = 4x - x^2$ in the form $y = a - (x + b)^2$, where a and b are constants.

$$\begin{aligned} y &= -(x^2 - 4x) \\ &= -[(x-2)^2 - 4] \\ &= -(x-2)^2 + 4 \end{aligned}$$

Answer (*a*) $y = 4 - (x-2)^2$ [2]

(b) Sketch the graph of $y = 4x - x^2$, showing the axes intercepts and coordinates of the turning point clearly.

Answer (*b*) [2]



---

19 Point A and B have coordinates (1, 3) and (1, 15) respectively.
Point P lies on the line $y = 6 + x$ and is equidistant from A and B.

(a) Explain, with working clearly shown, why P is **not** the mid-point of AB.

Answer (a) [2]

Mid-point of $AB = (1, 9)$

Substituting $x = 1$ into $y = 6 + x$, $y = 7 \neq 9$

Thus, the mid-point of AB does not lie on the line $y = 6 + x$ and P cannot be the mid-point.

(b) Find the coordinates of P.

Point P lies along the line $y = 9$ since it is at equidistant from A and from B.

Since P lies on the line $y = 6 + x$, substituting $y = 9$, $x = 3$

Answer (b) $P(3, 9)$ [2]

---

20 The average daily temperature, in degrees Celsius (°C), of City A measured in 11 days are shown below.

29.5 32.1 33.2 30.1 33.9 32.7 29.9 33.0 30.7 33.6 29.9

(a) Find

(i) the median temperature,

Answer (a)(i) 32.1°C [1]

9C

(ii) the interquartile range.

$Q_1 = 29.9$

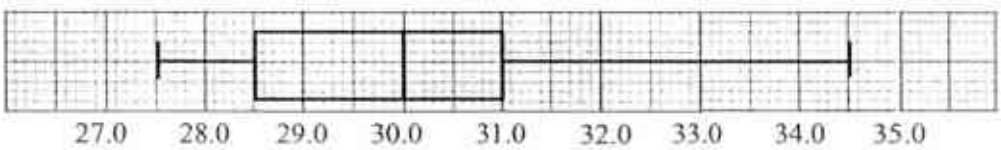
$Q_3 = 33.2$

Interquartile range $= 33.2 - 29.9$

$= 3.3^{\circ}C$

Answer (*a*)(ii) 3.3°C [2]

(b) The box-and-whisker diagram shows the average temperature of City *B* in the same period.



A report commented that the average temperature of City *B* during these 11 days is generally higher than that of City *A*.

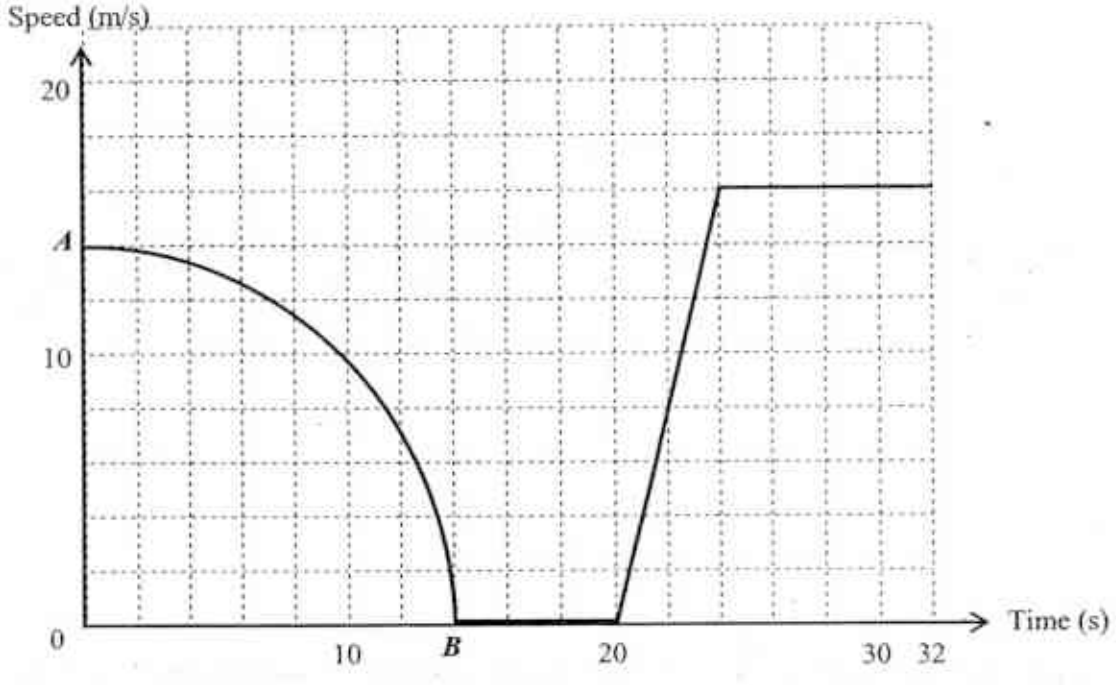
Do you agree? Give a reason to support your answer.

Answer (*b*) No. Median of City *A* is 32.1°C, higher than that of *B*, 30.0°C. [1]

21 The diagram below shows the speed-time graph of a car for a journey.

The car decelerates from an initial speed of 14 m/s for the first 14 seconds and remained at rest for the next 6 seconds. It then accelerates uniformly for the next 4 seconds and remained at a constant speed of 16 m/s thereafter.

AB is an arc of a quadrant.



(a) Find the acceleration at $t = 23$ s.

$$\frac{16-0}{24-20} = 4 \text{ m/s}^2$$

Answer (a) 4 m/s^2 [1]

(b) Taking π to be $\frac{22}{7}$, find the total distance travelled for the entire journey.

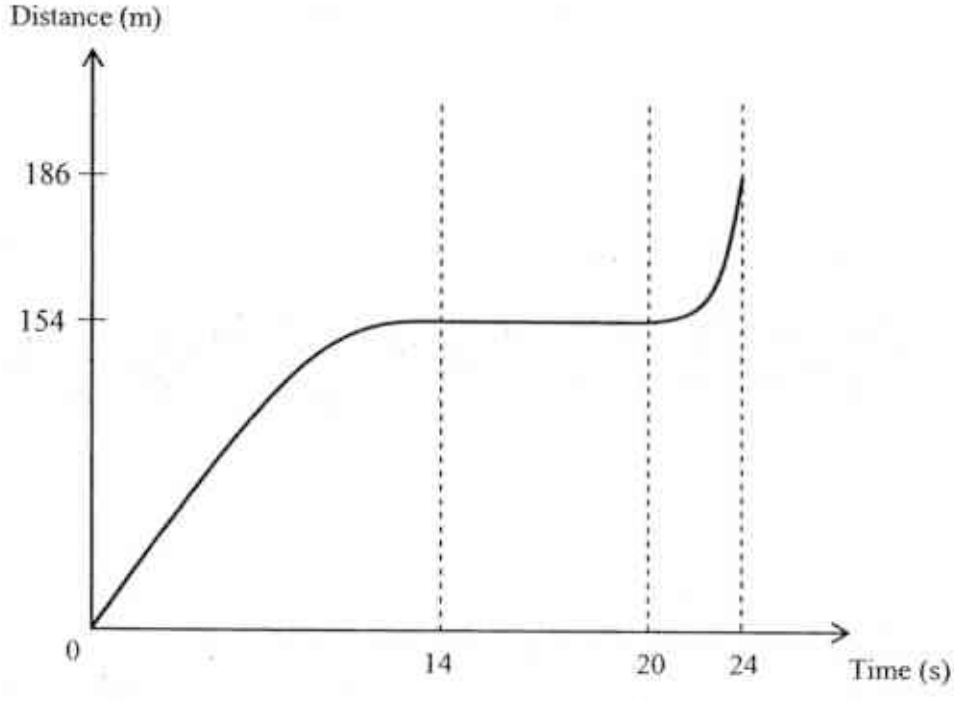
$$\text{Total distance} = \frac{1}{4}\left(\frac{22}{7}\right)(14^2) + \frac{1}{2}(16)(8+12)$$
$$= 154 + 160$$
$$= 314$$

Answer (b) 314 m [2]

97

(c) On the axes provided below, sketch the distance-time graph of the vehicle for the first 24 seconds of the journey, indicating the distances travelled at $t = 14$ s and $t = 24$ s on the vertical axis clearly.

Answer (c) [2]



22 Magician William randomly draws 2 cards from a stack of 5 cards labelled 5 to 9, one after another without replacement.

The sum of the numbers on the two cards is obtained.

(a) Complete the possibility diagram in the answer space below.

Answer (a) [2]

+	5	6	7	8	9
5	X	11	12	13	14
6	11	X	13	14	15
7	12	13	X	15	16
8	13	14	15	X	17
9	14	15	16	17	X

(b) Calculate the probability that the sum obtained is a multiple of 5.

Answer (b) $\frac{1}{5}$ [1]

(c) A third card is chosen at random from the stack without replacement.
Find the probability that the sum of the numbers on the three cards is 24.

$$\text{P(sum is 24)} = 6\left(\frac{1}{5}\times\frac{1}{4}\times\frac{1}{3}\right) = \frac{1}{10}$$

OR $$\text{P(sum is 24)} = \frac{6}{20}\times\frac{1}{3} = \frac{1}{10}$$

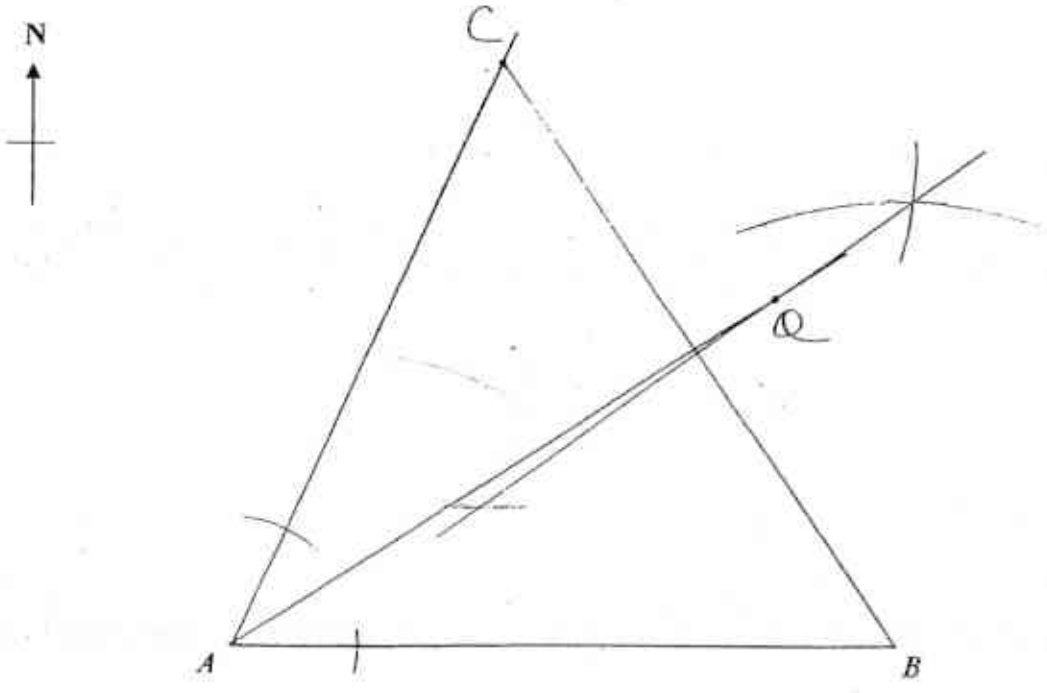
Answer (c) $\frac{1}{10}$ [2]

23 In a triangular field ABC, B is due east of A, C is at a bearing of 025° from A and $AC = 460$ m.

AB has already been drawn.

(a) Using a scale of 1 cm to represent 50 m, construct the triangular field ABC.

Answer (a) [1]



(b) A statue is located at point Q which is equidistant from the points B and C and equidistant from the lines AC and AB.

By constructing the appropriate perpendicular and angle bisectors, mark the point Q and estimate the actual distance CQ.

Answer (b) 250 - 300 m [3]

END OF PAPER

Name:		Index Number:		Class:	

CATHOLIC HIGH SCHOOL
2015 Preliminary Examination 3
Secondary 4

MATHEMATICS **4016/02**

Paper 2 **2 September 2015**

2 hours 30 minutes

Additional materials: Answer booklets **A**, **B** and **C**

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE ON THE MARGINS.

Answer **all** questions.
Attempt **questions 1 to 4** on **Answer booklet A**,
questions 5 to 8 on **Answer booklet B**,
questions 9 to 11 on **Answer booklet C**.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to **three significant figures**. Give answers in **degrees to one decimal place**.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **100**.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{ , where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{ , where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

Attempt **questions 1 to 4** on **Answer Booklet A**.

1 **(a)** Given that $x^2 + y^2 = a$ and $xy = b$, find $(2x - 2y)^2$ in terms of a and b. [2]

(b) Given that $\frac{3a - bc}{2ac - 5b} = \frac{1}{2}$,

(i) find the exact value of c when $a = -3$ and $b = 2$, [1]

(ii) express c in terms of a and b, [2]

(c) Given that $\frac{y}{x} = 2015$ and $\frac{z}{y} = 2015$, where $x \neq 0$ and $y \neq 0$.

Find the value of $\frac{y + z}{x + y}$. [2]

---

2 Teddy, Colin and Azmat each decided to buy a new motorbike that was priced at $8 500.

(a) Teddy was given a discount and he paid $8202.50 for the motorbike in cash.

(i) Calculate the percentage discount he received. [1]

(ii) The value of the new motorbike depreciates by 12% during the first year. In the second year, its value depreciates by 20% of its value at the beginning of the year.

If Teddy sold off his motorbike at the end of the second year, how much money would he lose? [3]

(b) Colin paid a down payment of 20% of the price of the motorbike and the balance to be paid by instalments at a simple interest rate of 5% per annum for a period of 2.5 years.

Calculate

(i) the amount of down payment paid, [1]

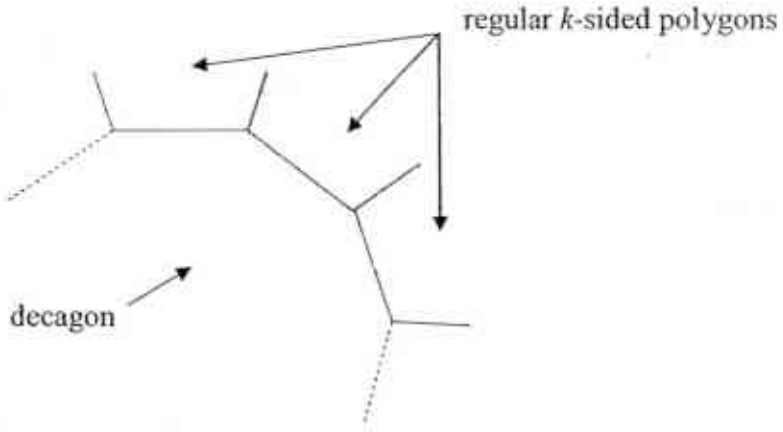
(ii) the amount of instalment payable in a month. [2]

(c) Azmat paid a down payment of $2612 and the balance to be paid at the end of 2 years with compound interest rate of 3.75% per annum.

How much would the motorbike cost him altogether? [2]

100

3 A number of regular k-sided polygons are placed together in a ring to form a regular decagon as shown in the diagram below.



(a) Find

(i) the interior angle of the decagon, [2]

(ii) the value of k. [2]

(b) If n-sided regular polygons that are placed together in a ring to form a N-sided polygon,

(i) show that $N = \dfrac{2n}{n-4}$. [3]

(ii) Hence or otherwise, explain why a regular octagon cannot be formed by placing a number of regular polygons in a ring. [1]

---

4 **(a)** The position vectors of a point A, B and C are $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$, $\begin{pmatrix} 5 \\ 20 \end{pmatrix}$ and $\begin{pmatrix} 10 \\ 0 \end{pmatrix}$ respectively.

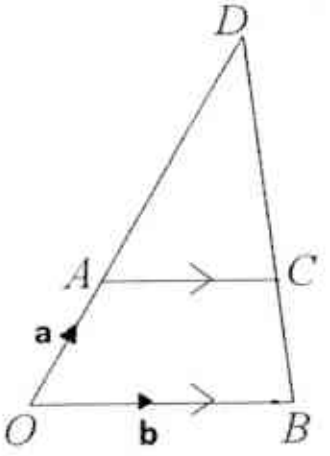
Find

(i) the column vector $\overrightarrow{AB}$, [1]

(ii) the coordinates of M such that M is the result of the translation of point A by $\begin{pmatrix} -6 \\ -8 \end{pmatrix}$, [2]

(iii) the equation of the line AM. [2]

(b) In the diagram, $OACB$ is a trapezium where AC is parallel to OB.
The line OA is produced to the point D such that $\frac{OA}{AD} = \frac{1}{2}$.



(i) Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, express, as simply as possible, in terms of $\mathbf{a}$ and/or $\mathbf{b}$,

(a) $\overrightarrow{BD}$, [1]

(b) $\overrightarrow{OC}$. [2]

(ii) Given that $\overrightarrow{OE} = 3\mathbf{a} + 2\mathbf{b}$,

(a) state the name of the quadrilateral $ODEB$, [1]

(b) explain why O, C and E lie in a straight line. [1]

(iii) Find, giving your answers as fractions in the simplest form,

(a) $\frac{\text{area of } \Delta ADC}{\text{area of } \Delta ODB}$, [1]

(b) $\frac{\text{area of } \Delta ADC}{\text{area of quadrilateral } ODEB}$. [1]

Attempt **questions 5 to 8** on **Answer Booklet B**.

5 The first four terms in a sequence of numbers, T_1, T_2, T_3 and T_4, ... are given below.

$$T_1 = 4-3=1$$
$$T_2 = 9-6=3$$
$$T_3 = 16-9=7$$
$$T_4 = 25-12=13$$

(i) Study the pattern and write down the line for T_5. [1]

(ii) T_n can be expressed in the form an^2+bn+c, where a, b and c are constants.

Find the values of a, b and c. [3]

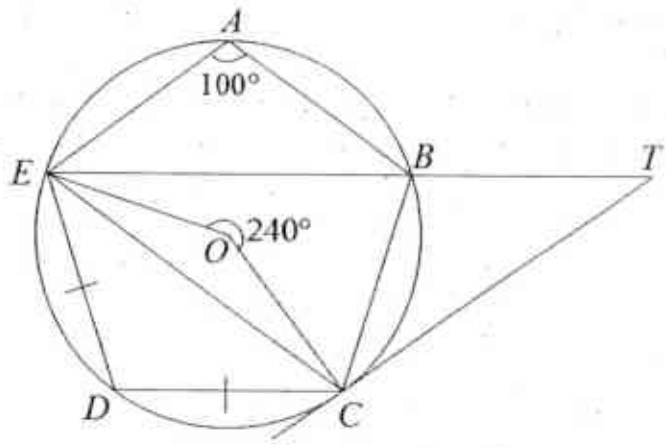
(iii) Find k such that $T_k = 73$. [2]

The first four terms of another sequence are 3, 7, 13, 21.

(iv) By using part **(i)** and **(ii)** or otherwise, write down an expression, in terms of n, for the nth term T_n of this sequence. [1]

---

6 In the diagram, O is the centre of circle $ABCDE$ where $DE = DC$.
Reflex angle $EOC = 240°$ and angle $EAB = 100°$.
(*You must not assume CT is a tangent to the circle at C.*)



(a) Find, giving reasons for each answer,

(i) angle DCE, [2]

(ii) angle CBE, [1]

(iii) angle CEB. [2]

(b) Given that $CTE = 20°$ and EBT is a straight line, show that CT is a tangent to the circle at C. [2]

---

7 The organizing committee of the national day parade is expecting 30 000 spectators at the floating platform.

The committee plans to have two entrances, the East Entrance and the West Entrance. The East Entrance will allow x number of spectators to enter in a minute while the West Entrance will allow y number of spectators to enter in a minute.

(i) Write down an expression, in terms of x, for the time taken in minutes for 30 000 spectators to enter the floating platform via the East Entrance only. [1]

Opening the East Entrance only will take 30 more minutes for all spectators to enter than opening the West Entrance only.

(ii) Show that $y = \dfrac{1000x}{1000 - x}$. [2]

(iii) If both entrances are opened at the same time, 30 000 spectators will take exactly 2.5 hours to enter the venue.

Form an equation in x and show that it reduces to $x^2 - 2200x + 200\ 000 = 0$. [2]

(iv) Solve the equation $x^2 - 2200x + 200\ 000 = 0$, giving your answers correct to the nearest whole number. [2]

(v) Explain why one of the answers in **(iv)** has to be rejected. [1]

---

8 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation $y = x + \frac{5}{x} - 3$, $x > 0$.

Some corresponding values of x and y are given in the table below.

x	0.5	1	1.5	2	2.5	3	4	5
y	7.5	3	1.83	1.5	1.5	1.67	2.25	3

(a) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $0 \le x \le 5$.
Using a scale of 2 cm to represent 1 unit, draw a vertical y-axis for $0 \le y \le 8$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) Use the graph to find the x-coordinate of the minimum point. [1]

(c) By drawing a tangent, find the gradient of the curve at the point where $x = 1.5$. [2]

(d) **(i)** On the same axes in **(a)**, draw the line $y = \frac{1}{3}x + 1$. [1]

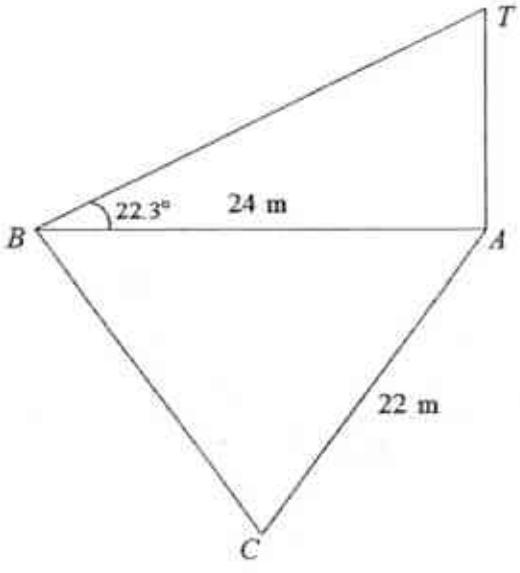
(ii) Write down the x-coordinates of the intersection points between this line and the curve for $0 \le x \le 5$. [1]

(iii) The values of x in **(d)(ii)** are solutions of the equation $2x^2 + Ax + B = 0$, where A and B are integer constants.

Find the values of A and B. [3]

Attempt **questions 9 to 11** on **Answer Booklet C**.

9 In the diagram, A is the foot of a cliff and B and C are yachts in the sea.
A is due east of B and the bearing of C from A is $214°$.
$AB = 24$ m and $AC = 22$ m.



(a) The angle of elevation of the top of the cliff, T, from B is $22.3°$.

Find the height of the cliff, TA. [2]

(b) Find the distance BC and hence, determine the angle BCA. [4]

(c) Calculate

(i) the bearing of B from C, [2]

(ii) the area of triangle ABC. [2]

(d) Determine the shortest distance from A to BC. [2]

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10 **Diagram I** shows a water bottle.
The cover of the water bottle is a hemisphere of radius 6 cm.
The portion of the water bottle that contains water is a cylinder of height 16 cm.

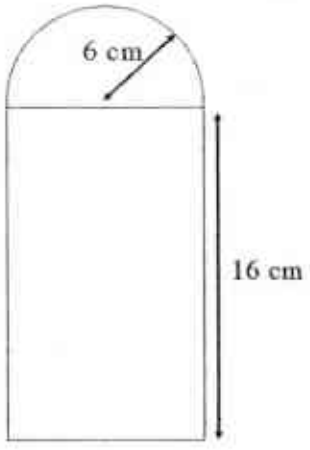


Diagram I

(a) Calculate the total surface area, including the base, of the outside of the water bottle. [2]

Diagram II shows a container which is a prism and whose cross-section is a trapezium.
The lengths of the parallel sides of the trapezium are 40 cm and 60 cm.
The depth of the container is 30 cm.

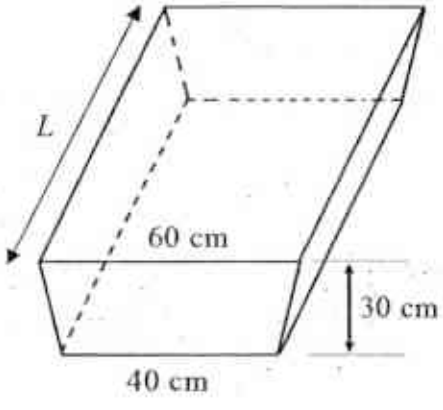


Diagram II

(b) 20 water bottles in **Diagram I** are filled with water to the brim of the cylinder and then poured into an empty trapezoidal container as shown in **Diagram II**.

Given that the trapezoidal container is completely filled with water, calculate the length of the container, L, giving your answer correct to 2 decimal places. [3]

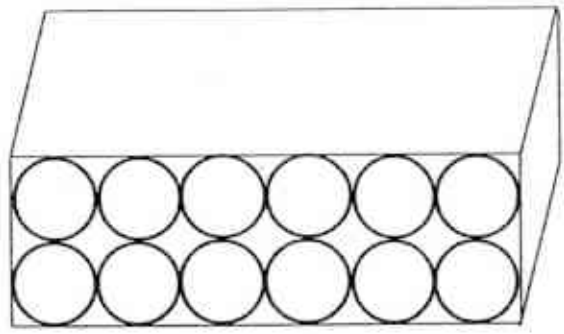


Diagram III

Diagram III shows twelve of these water bottles all facing in the same direction, which just fit into a box.

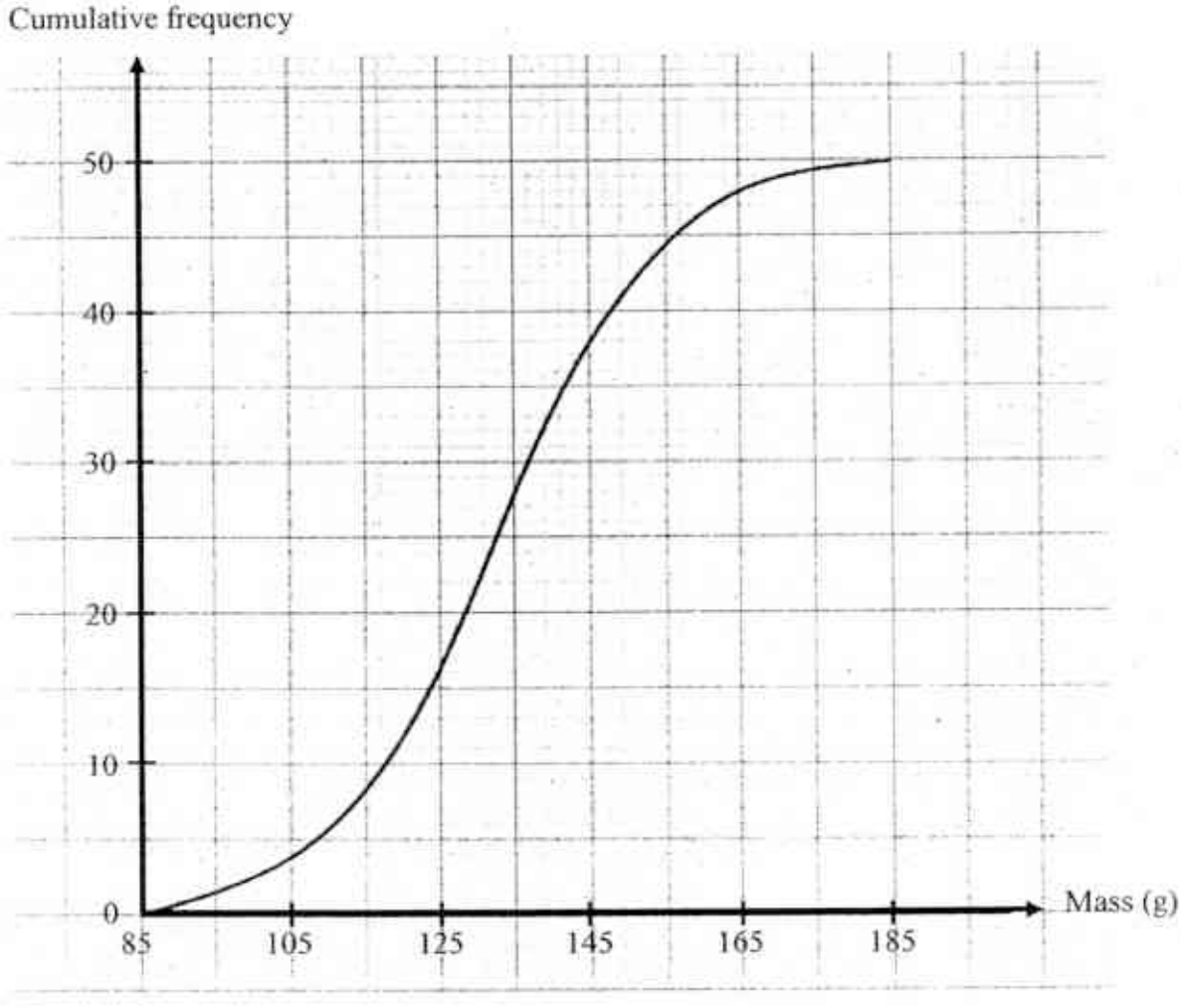
(c) Calculate

(i) the volume of the inside of the box. [2]

(ii) the percentage of volume of the box that is **not** occupied by the water bottles. [2]

---

11 The graph below shows a cumulative frequency curve depicting the masses of 50 oranges.



ok

(a) From the cumulative frequency curve, find the values of p and q in the grouped frequency table shown below. [2]

Mass, x (g)	$x \leq 85$	$85 < x \leq 105$	$105 < x \leq 125$	$125 < x \leq 145$	$145 < x \leq 165$	$165 < x \leq 185$
No. of oranges	0	4	p	q	10	2

(b) Hence, calculate an estimate of the

(i) mean mass, [2]

(ii) standard deviation of the 50 oranges. [2]

(c) Two oranges are chosen at random from the crate of 50 oranges, one after another without replacement.

Find, as a fraction in its simplest form, the probability that

(i) both oranges weigh at most 145 g, [1]

(ii) one orange weighs more than 165 g but the other weighs at most 125 g. [2]

(d) Another crate of 50 oranges have the same median but a smaller interquartile range. Describe how this cumulative frequency curve will differ from the given curve. [1]

END OF PAPER

Answer **all** the questions.

1 **(a)** Given that $x^2 + y^2 = a$ and $xy = b$, find $(2x - 2y)^2$ in terms of a and b. [2]

(b) Given that $\dfrac{3a - bc}{2ac - 5b} = \dfrac{1}{2}$,

(i) find the exact value of c when $a = -3$ and $b = 2$, [1]

(ii) express c in terms of a and b, [2]

(c) Given that $\dfrac{y}{x} = 2015$ and $\dfrac{z}{y} = 2015$, where $x \neq 0$ and $y \neq 0$.

Find the value of $\dfrac{y + z}{x + y}$. [2]

Solution

(a)

$$(2x - 2y)^2 = 4(x - y)^2$$

$$= 4(x^2 + y^2 - 2xy)$$

$$= 4(a - 2b) \quad (\text{or } 4a - 8b)$$

(b)(i)

$$c = \frac{6(-3) + 5(2)}{2(-3 + 2)} = \frac{-8}{-2} = 4$$

(b)(ii)

$$\frac{3a - bc}{2ac - 5b} = \frac{1}{2}$$

$$6a - 2bc = 2ac - 5b$$

$$2ac + 2bc = 6a + 5b$$

$$2c(a + b) = 6a + 5b$$

$$c = \frac{6a + 5b}{2(a + b)}$$

(c)

$\frac{y}{x} = 2015 \Rightarrow y = 2015x$

$\frac{z}{y} = 2015 \Rightarrow z = 2015y$

$y + z = 2015(x + y)$

$\frac{y+z}{x+y} = 2015$

Alternative

$\frac{y}{x} = 2015 \Rightarrow y = 2015x$

$\frac{z}{y} = 2015 \Rightarrow z = 2015y = \left(2015^2\right)x$

$\frac{y+z}{x+y} = \frac{2015x + \left(2015^2\right)x}{x + 2015x}$

$= \frac{4062240x}{2016x} = 2015$

---

2 Teddy, Colin and Azmat each decided to buy a new motorbike that was priced at \$8 500.

(a) Teddy was given a discount and he paid \$8202.50 for the motorbike in cash.

(i) Calculate the percentage discount he received. [1]

(ii) The value of the new motorbike depreciates by 12% during the first year. In the second year, its value depreciates by 20% of its value at the beginning of the year.

If Teddy sold off his motorbike at the end of the second year, how much money would he lose? [3]

(b) Colin paid a down payment of 20% of the price of the motorbike and the balance to be paid by instalments at a simple interest rate of 5% per annum for a period of 2.5 years.

Calculate

(i) the amount of down payment paid, [1]

(ii) the amount of instalment payable in a month. [2]

(c) Azmat paid a down payment of \$2612 and the balance to be paid at the end of 2 years with compound interest rate of 3.75% per annum.

How much would the motorbike cost him altogether? [2]

Solution

(a)(i)

$\%\ \text{discount} = \frac{8500 - 8202.5}{8500} \times 100 = 3.5$

(a)(ii)

Value of car at end of second year $= 0.8 \times 0.88 \times 8500$

$= \$5984$

Total amount Cody will lose $= 8202.50 - 5984$

$= \$2218.50$

(b)(i)

Down payment $= \frac{20}{100} \times \$8500 = \$1700$

(b)(ii)

Balance $= \$8500 - \$1700 = \$6800$

Interest $= \frac{6800 \times 2.5 \times 5}{100} = \$850$

Monthly instalment $= \frac{6800 + 850}{30} = \$255$

(c)

Balance $= \$8500 - \$2612 = \$5888$

Total amount $= \$5888\left(1 + \frac{3.75}{100}\right)^2 = \$6337.88$

Total cost $= \$6337.88 + \$2612 = \$8949.88$

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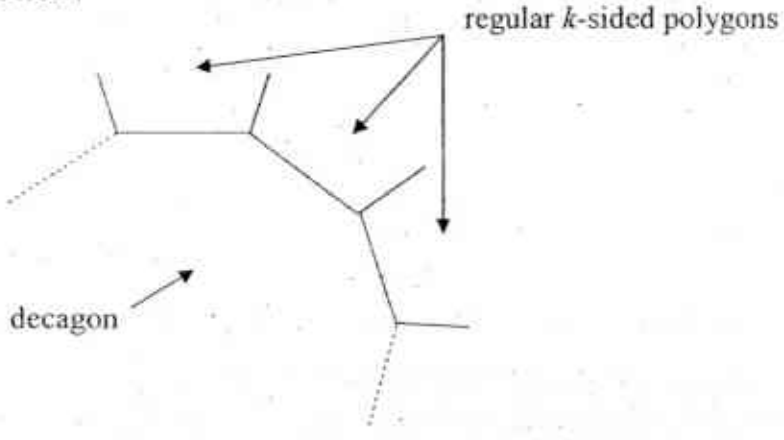
3 A number of regular k-sided polygons are placed together in a ring to form a regular decagon as shown in the diagram below.



(a) Find

(i) the interior angle of the decagon, [2]

(ii) the value of k. [2]

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(b) If n-sided regular polygons that are placed together in a ring to form a N-sided polygon,

(i) show that $N = \dfrac{2n}{n-4}$. [3]

(ii) Hence or otherwise, explain why a regular octagon cannot be formed by placing a number of regular polygons in a ring. [1]

Solution

(a)(i)

$$\text{Size of each interior angle} = \frac{180(10-2)}{10}$$

$$= 144^\circ$$

(a)(ii)

$$\text{interior angle of } k\text{-sided polygon} = \frac{360-144}{2}$$

$$= 108^\circ$$

$$108k = 180(k-2)$$

$$72k = 360$$

$$k = 5$$

(b)(i)

$$\frac{180(N-2)}{N} + 2\left(\frac{180(n-2)}{n}\right) = 360$$

$$\frac{(N-2)}{N} + \frac{2(n-2)}{n} = 2$$

$$\frac{nN - 2n + 2Nn - 4N}{Nn} = 2$$

$$3Nn - 2n - 4N = 2Nn$$

$$Nn - 4N = 2n$$

$$N = \frac{2n}{n-4}$$

(b)(ii)

When $N = 8$,

$$8 = \frac{2n}{n-4}$$

$$8n - 32 = 2n$$

$$6n = 32$$

$$n = 5.33 \quad (\text{N.A. since } n \text{ must be integer})$$

4 **(a)** The position vectors of a point A, B and C are $\begin{pmatrix}-3\\5\end{pmatrix}$, $\begin{pmatrix}5\\20\end{pmatrix}$ and $\begin{pmatrix}10\\0\end{pmatrix}$ respectively.

Find

(i) the column vector $\overrightarrow{AB}$, [1]

(ii) the coordinates of M such that M is the result of the translation of point A by $\begin{pmatrix}-6\\-8\end{pmatrix}$, [2]

(iii) the equation of the line AM. [2]

Solutions

(a)(i)

$$\overrightarrow{AB}=\begin{pmatrix}8\\15\end{pmatrix}$$

(a)(ii)

$$\overrightarrow{OM}=\begin{pmatrix}-3\\5\end{pmatrix}+\begin{pmatrix}-6\\-8\end{pmatrix}$$

$$=\begin{pmatrix}-9\\-3\end{pmatrix}$$

M is $(-9\ ,\ -3\ )$

(b)

$$m_{AM}=\frac{-8}{-6}=\frac{4}{3}$$

Equation of AM is $y=\frac{4}{3}x+c$.

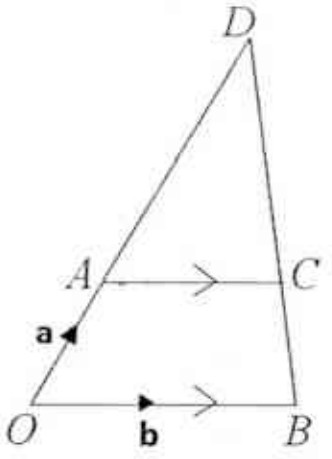
At $A(-3\ ,\ 5)$,

$$5=\frac{4}{3}(-3)+c$$

$$c=9$$

$$y=\frac{4}{3}x+9\ \left(\text{ or } 3y=4x+27\right)$$

(b) In the diagram, $OACB$ is a trapezium where AC is parallel to OB.
The line OA is produced to the point D such that $\frac{OA}{AD} = \frac{1}{2}$.



(i) Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, express, as simply as possible, in terms of $\mathbf{a}$ and/or $\mathbf{b}$,

(a) $\overrightarrow{BD}$, [1]

(b) $\overrightarrow{OC}$. [2]

(ii) Given that $\overrightarrow{OE} = 3\mathbf{a} + 2\mathbf{b}$,

(a) state the name of the quadrilateral $ODEB$, [1]

(b) explain why O, C and E lie in a straight line. [1]

(iii) Find, giving your answers as fractions in the simplest form,

(a) $\dfrac{\text{area of } \Delta ADC}{\text{area of } \Delta ODB}$, [1]

(b) $\dfrac{\text{area of } \Delta ADC}{\text{area of quadrilateral } ODEB}$. [1]

Solutions

(i)(a)

$\overrightarrow{BD} = 3\mathbf{a} - \mathbf{b}$

(i)(b)

$$\overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{DC}$$

$$= 3\mathbf{a} + \frac{2}{3}(\mathbf{b} - 3\mathbf{a})$$

$$= \mathbf{a} + \frac{2}{3}\mathbf{b}$$

(ii)(a)

Trapezium

(ii)(b)

$$\overrightarrow{OE} = 3\mathbf{a} + 2\mathbf{b}$$

$$= 3\left(\mathbf{a} + \frac{2}{3}\mathbf{b}\right)$$

$$= 3\overrightarrow{OC}$$

So, O, C and E are collinear.

(iii)(a)

$$\frac{\text{area of } \Delta ADC}{\text{area of } \Delta ODB} = \frac{4}{9}$$

(iii)(b)

$$\frac{\text{area of } \Delta ADC}{\text{area of quadrilateral } ODEB} = \frac{4}{9} \times \frac{1}{3}$$

$$= \frac{4}{27}$$

.08

5 The first four terms in a sequence of numbers, T_1, T_2, T_3 and T_4, ... are given below.

$$T_1 = 4 - 3 = 1$$
$$T_2 = 9 - 6 = 3$$
$$T_3 = 16 - 9 = 7$$
$$T_4 = 25 - 12 = 13$$

(i) Study the pattern and write down the line for T_5. [1]

(ii) T_n can be expressed in the form $an^2 + bn + c$, where a, b and c are constants.

Find the values of a, b and c. [3]

(iii) Find k such that $T_k = 73$. [2]

The first four terms of another sequence are 3, 7, 13, 21.

(iv) By using part **(i)** and **(ii)** or otherwise, write down an expression, in terms of n, for the nth term T_n of this sequence. [1]

Solution

(i)

$T_5 = 36 - 15 = 21$

(ii)

$T_n = (n+1)^2 - 3(n)$

$= n^2 - n + 1$

$a = 1$ $b = -1$ $c = 1$

(iii)

$n^2 - n + 1 = 73$

$n^2 - n - 72 = 0$

$(n+8)(n-9) = 0$

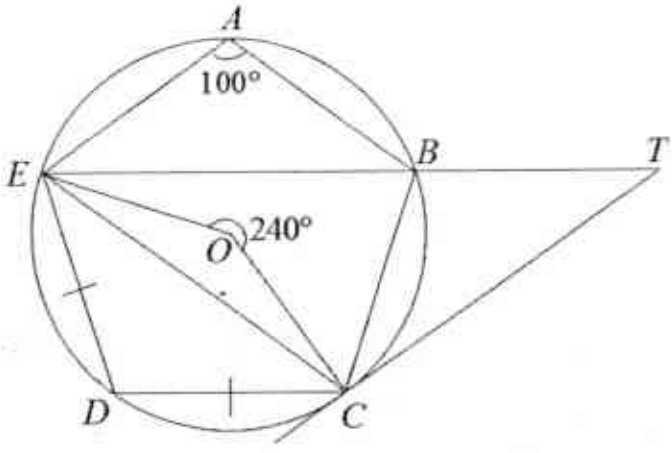
$n = -8$(N.A.) or $n = 9$

(iv)

$T_n = (n+1)^2 - (n+1) + 1$

$= n^2 + n + 1$

6 In the diagram, O is the centre of circle $ABCDE$ where $DE = DC$.
Reflex angle $EOC = 240°$ and angle $EAB = 100°$.
(*You must not assume CT is a tangent to the circle at C.*)



(a) Find, giving reasons for each answer,

(i) angle DCE, [2]

(ii) angle CBE, [1]

(iii) angle CEB. [2]

(b) Given that $CTE = 20°$ and EBT is a straight line, show that CT is a tangent to the circle at C. [2]

Solution

(a)(i)

$\angle CDE = 120°$ (angle at center = twice angle at circumference)

$\angle DCE = \dfrac{180° - 120°}{2}$ (isosceles triangle)

$= 30°$

(a)(ii)

$\angle CBE = 180° - 120°$ (opposite angles of cyclic quadrilateral)

$= 60°$

(a)(iii)

$\angle BCE = 180° - 100°$ (opposite angles of cyclic quadrilateral)

$= 80°$

$\angle CEB = 180° - 80° - 60°$ (angle sum of triangle)

$= 40°$

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(b)

$\angle BCT = 180 - 20 - (180 - 60)$ (angle sum of triangle)

$= 40°$

$\angle BCO = 80 - \frac{1}{2}(180 - 120)$ (isosceles triangle)

$= 50°$

$\angle OCT = 40 + 50$

$= 90°$

Hence by tangent perpendicular radius,
CT is a tangent to the circle at C.

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7 The organizing committee of the national day parade is expecting 30 000 spectators at the floating platform.

The committee plans to have two entrances, the East Entrance and the West Entrance. The East Entrance will allow x number of spectators to enter in a minute while the West Entrance will allow y number of spectators to enter in a minute.

(i) Write down an expression, in terms of x, for the time taken in minutes for 30 000 spectators to enter the floating platform via the East Entrance only. [1]

Opening the East Entrance only will take 30 more minutes for all spectators to enter than opening the West Entrance only.

(ii) Show that $y = \frac{1000x}{1000 - x}$. [2]

(iii) If both entrances are opened at the same time, 30 000 spectators will take exactly 2.5 hours to enter the venue.

Form an equation in x and show that it reduces to $x^2 - 2200x + 200\ 000 = 0$. [2]

(iv) Solve the equation $x^2 - 2200x + 200\ 000 = 0$, giving your answers correct to the nearest whole number. [2]

(v) Explain why one of the answers in **(iv)** has to be rejected. [1]

Solution

(i)

$\frac{30\ 000}{x}$

(ii)

$$\frac{30\ 000}{x}-\frac{30\ 000}{y}=30$$

$$\frac{1000}{x}-\frac{1000}{y}=1$$

$$\frac{1000y-1000x}{xy}=1$$

$$1000y-1000x=xy$$

$$1000y-xy=1000x$$

$$y=\frac{1000x}{1000-x}$$

(iii)

$$x+y=\frac{30000}{150}\ \left(\text{ or } 150x+150y=30000\right)$$

$$x+\frac{1000x}{1000-x}=200$$

$$x(1000-x)+1000x=200(1000-x)$$

$$-x^2+2000x=200000-200x$$

$$x^2-2200x+200000=0$$

(iv)

$$x=\frac{2200\pm\sqrt{(-2200)^2-4(1)(200000)}}{2}$$

$$=\frac{2200\pm\sqrt{4040000}}{2}$$

$$=2104.99 \quad \text{or} \quad x=95.01$$

$$\approx 2105 \qquad \approx 95$$

(v)

$x=2105$ has to be rejected since $y=200-2105<0$.

8 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation $y = x + \frac{5}{x} - 3$, $x > 0$.

Some corresponding values of x and y are given in the table below.

x	0.5	1	1.5	2	2.5	3	4	5
y	7.5	3	1.83	1.5	1.5	1.67	2.25	3

(a) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $0 \leq x \leq 5$.
Using a scale of 2 cm to represent 1 unit, draw a vertical y-axis for $0 \leq y \leq 8$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) Use the graph to find the x-coordinate of the minimum point. [1]

(c) By drawing a tangent, find the gradient of the curve at the point where $x = 1.5$. [2]

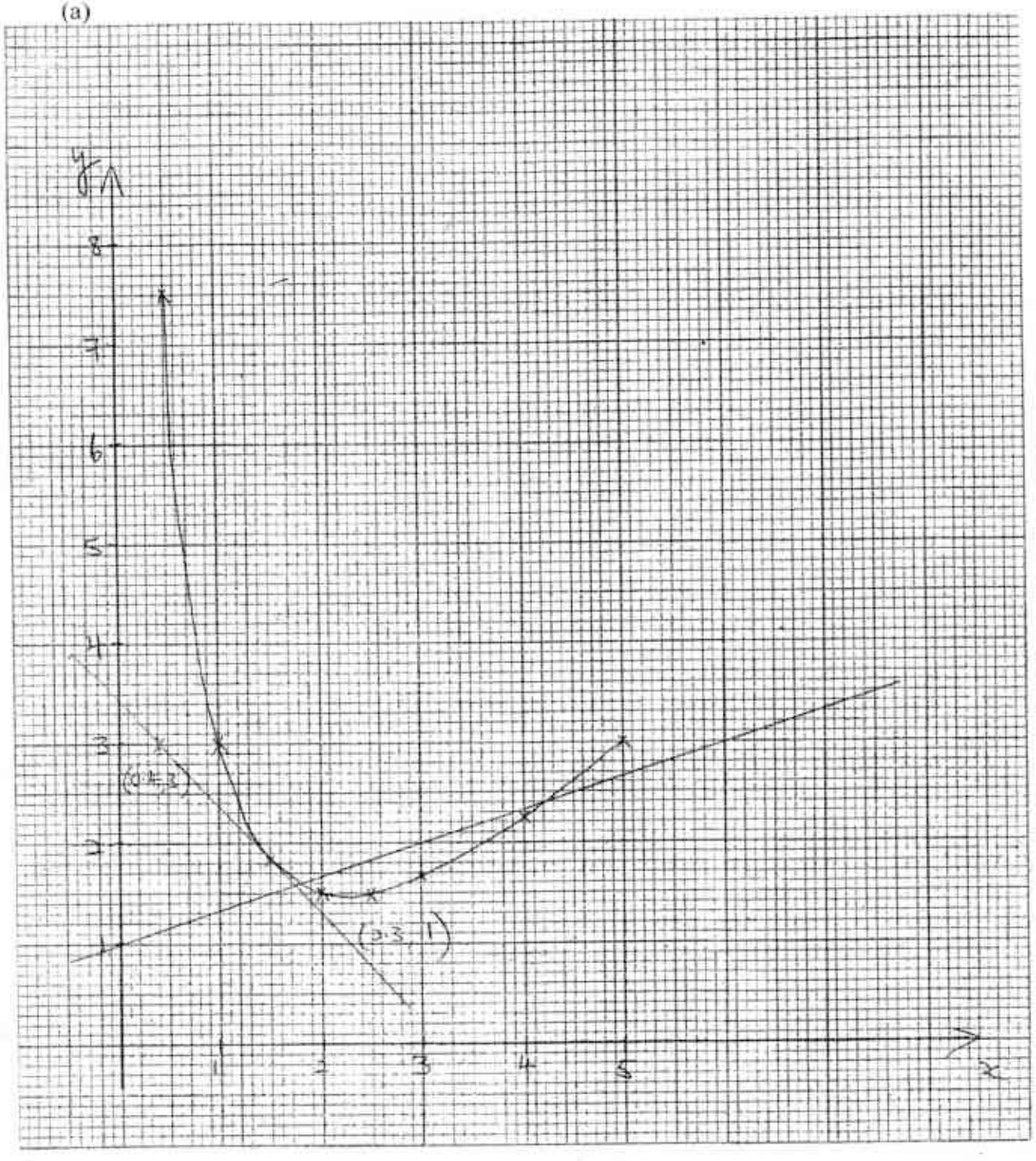
(d) **(i)** On the same axes in **(a)**, draw the line $y = \frac{1}{3}x + 1$. [1]

(ii) Write down the x-coordinates of the intersection points between this line and the curve for $0 \leq x \leq 5$. [1]

(iii) The values of x in **(d)(ii)** are solutions of the equation $2x^2 + Ax + B = 0$, where A and B are integer constants.

Find the values of A and B. [3]

(a)



(a)

On graph paper.

(b)

$x = 2.3$ (accept coordinate ±0.1)

(c)

Gradient $= -1.05$ (accept gradient ± 0.2)

(d)(i)

On graph

(d)(ii)

$x = 1.8$ (accept 1.7 to 1.9), 4.2 (accept 4.1 to 4.3)

(d)(iii)

$$\frac{1}{3}x + 1 = x + \frac{5}{x} - 3$$

$$\frac{2}{3}x + \frac{5}{x} - 4 = 0$$

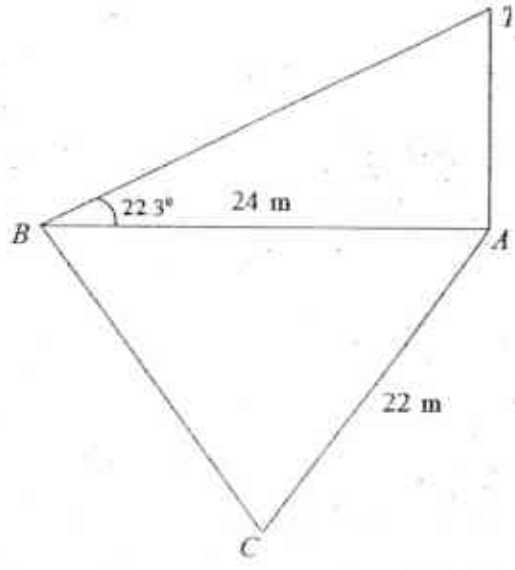
$$2x^2 - 12x + 15 = 0$$

$$A = -12$$

$$B = 15$$

---

9 In the diagram, A is the foot of a cliff and B and C are yachts in the sea.
A is due east of B and the bearing of C from A is $214°$.
$AB = 24$ m and $AC = 22$ m.



(a) Given that the angle of elevation of the top of the cliff, T, from B is $22.3°$.

Find the height of the cliff, TA. [2]

(b) Find the distance BC and hence, determine the angle BCA. [4]

(c) Calculate

(i) the bearing of B from C, [2]

(ii) the area of triangle ABC. [2]

(d) Determine the shortest distance from A to BC. [2]

Solution

(a)

$$\tan 22.3° = \frac{TA}{24}$$

$$TA = 9.843 \approx 9.84 \text{ m}$$

(b)

$$\angle BAC = 56°$$

$$BC = \sqrt{24^2 + 22^2 - 2(24)(22)\cos 56°}$$

$$= 21.667 \approx 21.7 \text{ m}$$

$$\frac{\sin \angle BCA}{24} = \frac{\sin 56°}{21.66}$$

$$\angle BCA = 66.674° \approx 66.7°$$

(c)(i)

$$\angle BCN_C = 66.67 - 34$$
$$= 32.67°$$

$$\text{bearing} = 360 - 32.67$$
$$= 327.325°$$
$$\approx 327.3°$$

(c)(ii)

$$\text{Area of triangle} = \frac{1}{2}(24)(22)\sin 56°$$
$$= 218.86$$
$$\approx 219 \text{ m}^2$$

(d)

$$\sin 66.67° = \frac{AP}{22}$$

$$AP = 20.201 \approx 20.2 \text{ m}$$

10 **Diagram I** shows a water bottle.
The cover of the water bottle is a hemisphere of radius 6 cm.
The portion of the water bottle that contains water is a cylinder of height 16 cm.

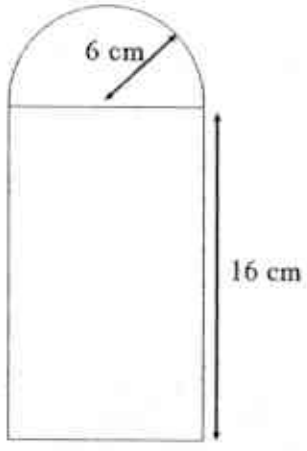


Diagram I

(a) Calculate the total surface area, including the base, of the outside of the water bottle. [2]

Diagram II shows a container which is a prism and whose cross-section is a trapezium.
The lengths of the parallel sides of the trapezium are 40 cm and 60 cm.
The depth of the container is 30 cm.

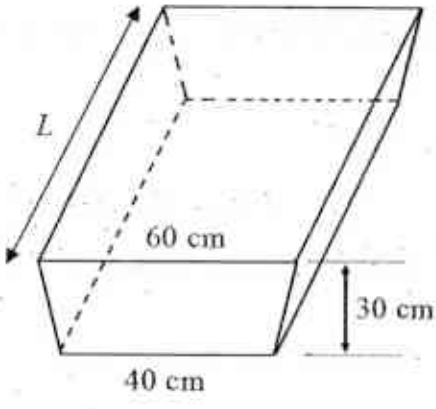


Diagram II

(b) 20 water bottles in **Diagram I** are filled with water to the brim of the cylinder and then poured into an empty trapezoidal container as shown in **Diagram II**.

Given that the trapezoidal container is completely filled with water, calculate the length of the container, L, giving your answer correct to 2 decimal places. [3]

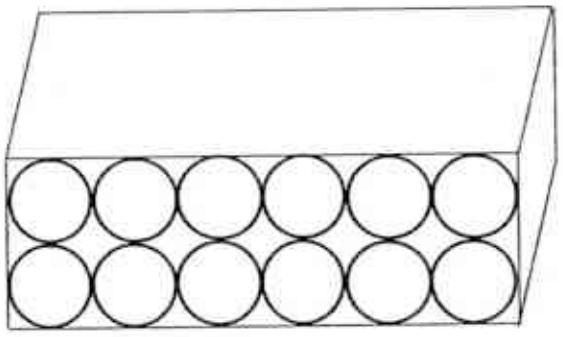


Diagram III

Diagram III shows twelve of these water bottles all facing in the same direction, which just fit into a box.

(c) Calculate

(i) the volume of the inside of the box. [2]

(ii) the percentage of volume of the box that is **not** occupied by the water bottles. [2]

Solution

(a)

$$\text{total surface area} = \frac{1}{2}(4\pi)(6)^2 + (2\pi)(6)(16) + (\pi)(6)^2$$
$$= 300\pi \text{ cm}^2$$
$$= 942.47$$
$$\approx 942 \text{ cm}^2$$

(b)

$$\text{volume of water} = (\pi)(6)^2(16) \times 20$$
$$= 11520\pi = 36190 \text{ cm}^3$$

$$\frac{1}{2}(30)(60+40)L = 36190$$

$$L \approx 24.13$$

(c)(i)

$$\text{volume of box} = 6(2\times 6) \times 2(2\times 6) \times 22$$

$$= 38016 \text{ cm}^3$$

(c)(ii)

$$\text{Volume of 1 water bottle, including the cap} = \frac{1}{2}\left(\frac{4}{3}\pi\right)(6)^3 + 576\pi$$

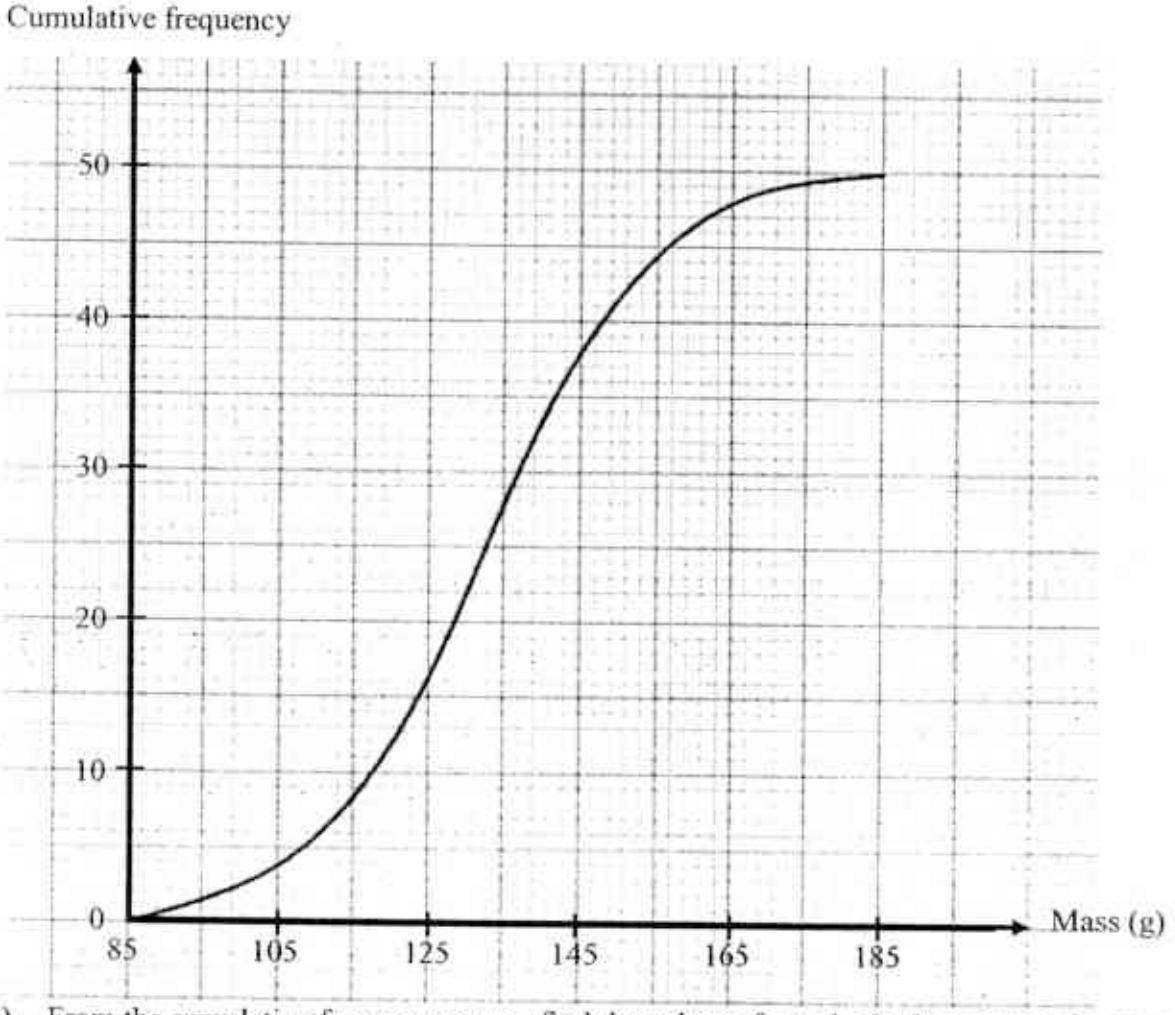
$$= 720\pi = 2261 \text{ cm}^3$$

$$\% \text{ not occupied} = \frac{38016 - 2261 \times 12}{38016} \times 100$$

$$= 28.6\%$$

---

11 The graph below shows a cumulative frequency curve depicting the masses of 50 oranges.



(a) From the cumulative frequency curve, find the values of p and q in the grouped frequency table shown below. [2]

Mass, x (g)	$x \leq 85$	$85 < x \leq 105$	$105 < x \leq 125$	$125 < x \leq 145$	$145 < x \leq 165$	$165 < x \leq 185$
No. of oranges	0	4	p	q	10	2

(b) Hence, calculate an estimate of the

(i) mean mass, [2]

(ii) standard deviation of the 50 oranges. [2]

(c) Two oranges are chosen at random from the crate of 50 oranges, one after another without replacement.

Find, as a fraction in its simplest form, the probability that

(i) both oranges weigh at most 145 g, [1]

(ii) one orange weighs more than 165 g but the other weighs at most 125 g. [2]

(d) Another crate of 50 oranges have the same median but a smaller interquartile range. Describe how this cumulative frequency curve will differ from the given curve. [1]

Solution

(a)

$p = 12$

$q = 22$

(b)(i)

$$\text{Mean mass} = \frac{95\times 4+115\times 12+135\times 22+155\times 10+175\times 2}{50}$$

$$= \frac{6630}{50}$$

$$= 132.6 \text{ g}$$

(b)(ii)

$$\text{Standard deviation} = \sqrt{\frac{4(95)^2+12(115)^2+22(135)^2+10(155)^2+2(175)^2}{50}-(132.6)^2}$$

$$= 19.03$$

$$\approx 19.0 \text{ g}$$

(c)(i)

$$\frac{38}{50}\times\frac{37}{49}=\frac{703}{1225}$$

(c)(ii)

$$\frac{2}{50}\times\frac{16}{49}\times 2=\frac{32}{1225}$$

(d)

The cumulative frequency curve for this crate of 50 oranges will be steeper.

Calculator Model:

Name: Mark Scheme	Class	Class Register Number/ Centre No./Index No.

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

Parent's Signature

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PRELIMINARY EXAMINATION 2015
SECONDARY 4

Mathematics **4016/01**

Paper 1 Friday 21 August 2015

2 hours

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number clearly in the spaces provided at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams, graphs or rough working.
Do not use highlighters or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

/80

This document consists of 18 printed pages

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r l$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions.

1 From the set of numbers $\left\{-\frac{2}{7}, -5, 0, 2.1, \sqrt{0.4}, \sqrt{4}, \sqrt{40}, \sqrt[3]{0.064}\right\}$, write down

(a) all the integers,

Answer (a) $-5, 0, \sqrt{4}$ [1]

(b) all the rational numbers.

Answer (b) $-\frac{2}{7}, -5, 0, 2.1, \sqrt{4}, \sqrt[3]{0.064}$ [1]

---

2 Factorise completely $x^2 + a^2 - 4b^2 - 2ax$.

$$x^2 + a^2 - 4b^2 - 2ax = (x-a)^2 - 4b^2$$
$$= (x - a + 2b)(x - a - 2b)$$

or

$$x^2 + a^2 - 4b^2 - 2ax = (a-x)^2 - 4b^2$$
$$= (a - x + 2b)(a - x - 2b)$$

Answer ..... [2]

---

3 During the Great Singapore Sale, a refrigerator is sold at a 25% discount. A further reduction of 10% is applied on a discounted price if a discount coupon is used.
A customer pays $2430 for a refrigerator using a discount coupon.
Calculate the original price.

90% × 75% × original price = $2430
Original price = $2430 ÷ (0.9 × 0.75)
= $ 3600

Answer $..... [2]

4 (a) Simplify $\left(\frac{2x^{-\frac{1}{2}}y^{3}}{y^{-2}}\right)^{-2}$.

$$\left(\frac{2x^{-\frac{1}{2}}y^{3}}{y^{-2}}\right)^{-2} = \left(2x^{-\frac{1}{2}}y^{3}\right)^{-2}$$

$$= 2^{-2}xy^{-10}$$

$$= \frac{x}{4y^{10}}$$

Answer (a)..... [2]

(b) Given that $\frac{(343)^{x+1}}{7} = 49^{2x}$, find the value of x.

$$\frac{(343)^{x+1}}{7} = 49^{2x}$$

$$\left(7^{3}\right)^{x+1} = 7\left(7^{2}\right)^{2x}$$

$$7^{3x+3} = 7^{4x+1}$$

Comparing powers, $3x+3=4x+1$

$$x = 2$$

Answer (b) x = ... [2]

---

5 When written as a product of their prime factors,

$A = 2^2 \times 3^2 \times 5$,

$B = 2 \times 3 \times 5^2$.

Given that the HCF and LCM of A, B and C is 6 and 6300 respectively, find the smallest possible value of C.

HCF = 6 = 2×3

LCM = 6300 = $2^2 \times 3^2 \times 5^2 \times 7$

C = 2× 3 ×7 = 42

Answer C =..... [2]

6 (a) Solve the equation $1+\frac{3}{4x-13}=\frac{2}{5}$.

$$1+\frac{3}{4x-13}=\frac{2}{5}$$
$$\frac{3}{4x-13}=-\frac{3}{5}$$
$$12x-39=-15$$
$$12x=24$$
$$x=2$$

Answer (a) $x=$ ..... [2]

(b) The relationship between p and q can be expressed as $\frac{2p+3q}{p+q}=5$.

Find the value of $\frac{2p}{q}$.

$$\frac{2p+3q}{p+q}=5$$
$$5p+5q=2p+3q$$
$$3p=-2q$$
$$\frac{p}{q}=-\frac{2}{3}$$
$$\therefore \frac{2p}{q}=2\left(-\frac{2}{3}\right)$$
$$=-1\frac{1}{3}$$

Answer (b) $\frac{2p}{q}=$ ..... [2]

---

7 Find all the integer values of x that satisfy the inequality $-1<\frac{2(4-5x)}{3}<8$.

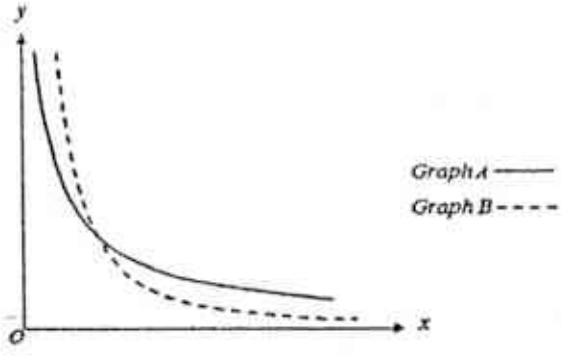
$$-1<\frac{2(4-5x)}{3}<8$$
$$-1<\frac{2(4-5x)}{3} \text{ and } \frac{2(4-5x)}{3}<8$$
$$-3<8-10x \quad \text{and} \quad 8-10x<24$$
$$10x<11 \quad \text{and} \quad -10x<16$$
$$x<1.1 \quad \text{and} \quad x>-1.6$$
$$\therefore -1.6<x<1.1$$

Hence the integer values of x are -1, 0 and 1.

05

Answer $x=$ ..... [3]

8 (a) The graphs of $y=\frac{3}{x}$ and $y=\frac{3}{x^2}$ are shown on the axes below.



(i) Identify whether Graph A or B is the graph for $y=\frac{3}{x^2}$.

Answer (a)(i) Graph...B..... [1]

(ii) Write down the coordinates of the intersection point of the two graphs.

Answer (a)(ii) (__1____ , ___3___) [1]

(b) (i) Express $y=-x^2+5x-2$ in the form $y=-(x-a)^2+b$.

$y=-x^2+5x-2$

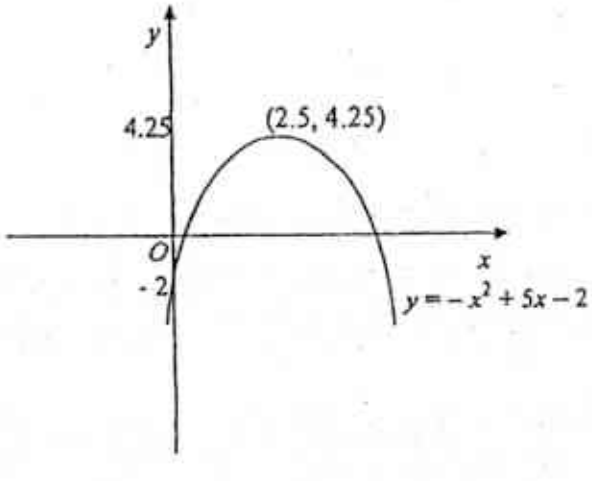
$y=-(x^2-5x+2)$

$y=-\left[x^2-5x+\left(\frac{-5}{2}\right)^2+2-\left(\frac{-5}{2}\right)^2\right]$

$y=-\left(x-2\frac{1}{2}\right)^2+4\frac{1}{4}$

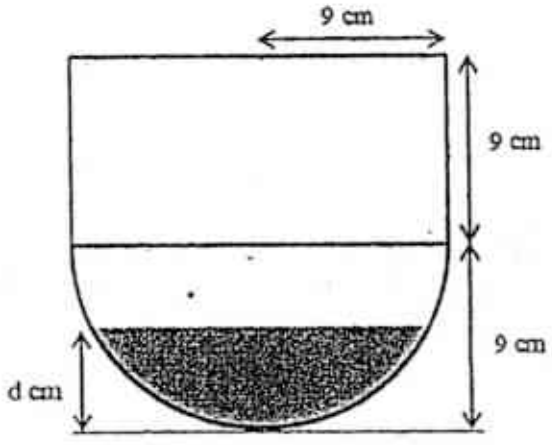
Answer (b)(i) y=..... [2]

(ii) Hence, sketch the graph of $y=-x^2+5x-2$.

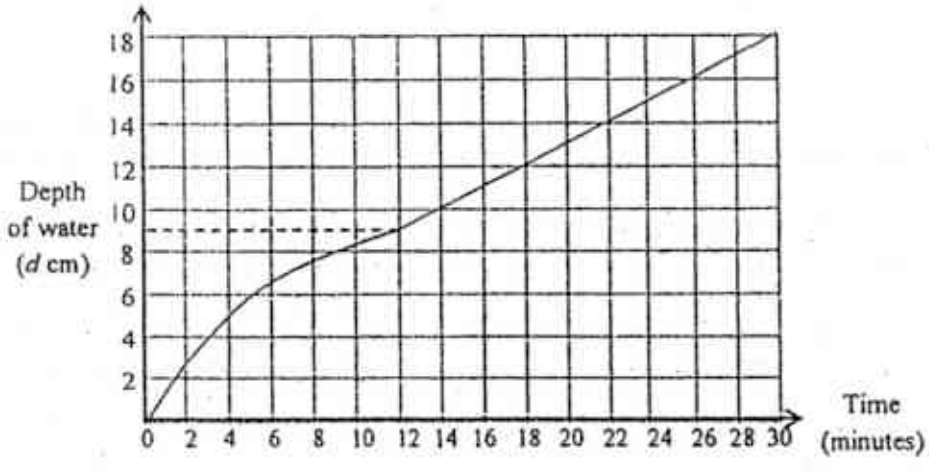


[2]

9 The diagram shows the cross-section of a container, made up of a cylinder and a hemisphere. The cylinder has height 9 cm and a circular base with radius 9 cm. Water is poured into the empty container at a constant rate and fills it in 30 minutes. On the given axes, sketch the graph showing the relationship between the depth of the water, d cm, and the time, t minutes, as the container is being filled up during the 30 minutes.



Answer

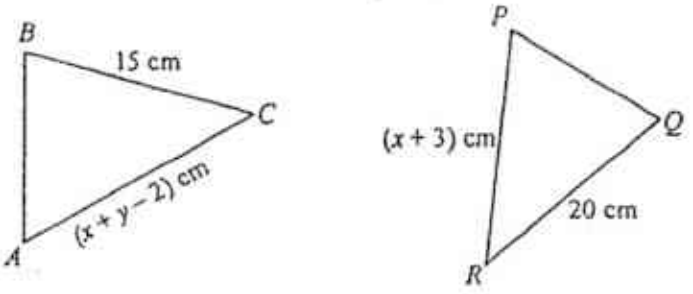


$$\frac{\text{Vol of hemisphere}}{\text{Vol of cylinder}} = \frac{\frac{2}{3}\pi(9)^3}{\pi(9)^2(9)}$$

$$= \frac{2}{3}$$

Time taken to fill up hemisphere $= \frac{2}{5} \times 30\text{mins}$

$= 12$ mins



[1]

he diagram above, triangle ABC is similar to triangle PQR.
Form an equation in x and y.

Since triangle ABC is similar to triangle PQR,

$$\frac{15}{20}=\frac{x+y-2}{x+3}$$

$$20x+20y-40=15x+45$$

$$5x+20y=85$$

$$x+4y=17$$

[2]

Answer (a) ..... [2]

Find the values of x and y if $2x+3y=20$.

$2x+3y=20$ ——————— (1)

$x+4y=17$ ——————— (2)

$(2)\times 2: 2x+8y=34$ ——————— (3)

$(3)-(1)$: $5y=14$

$$y=2.8 \text{ or } 2\frac{4}{5}$$

[1]

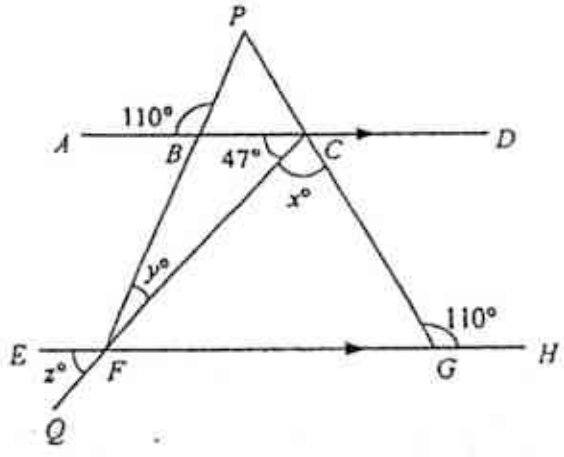
When $y=2.8$ or $2\frac{4}{5}$, $x=5.8$ or $5\frac{4}{5}$

[1]

Answer (b) $x=$....., $y=$ ..... [2]

[1]

13 In the diagram, *ABCD* is parallel to *EFGH*. *QFC*, *PBF* and *PCG* are straight lines.



(a) Find the value of

(i) x,

$x° + 47° = 110°$ (alt. $\angle s$, $AD // EH$)

$x = 63$

Answer (a)(i) x = ... [1]

(ii) y,

$\angle FBC = 110°$ (vert. opp. $\angle s$)

$y° = 180° - 110° - 47°$ ($\angle s$ sum of Δ)

$y = 23$

Answer (a)(ii) y = ... [1]

(iii) z.

$z° = 47°$ (corr. $\angle s$, $AD // EH$)

$z = 47$

Answer (a)(iii) z = ... [1]

(b) Explain, stating the reasons, whether PB is or is not equal to PC.

Answer (b) PB ..... equal to PC because

..... [1]

PB is equal to PC because $\angle PBC = \angle PCB = 70°$, thus they are base angles of an isosceles triangle PBC.

14 The lengths of three square flower beds P, Q and R are in the ratio 2 : 3 : n, where n is an integer. Given that the actual area of P is 64 m^2 and the total area of P, Q and R is 784 m^2, find the value of n.

	P	:	Q	:	R
Length	2	:	3	:	n
Area	4	:	9	:	n^2
Actual Area	64 m^2	:	144 m^2	:	$(784-64-144)\ m^2 = 576\ m^2$

$16n^2 = 576$

$n^2 = 36$

$n = 6$

Answer n=..... [3]

15 80 students took a Physical Fitness test and the number of push-ups that they did in one minute are shown in the frequency table below.

No. of push-ups (x)	Frequency
$0 < x \le 10$	8
$10 < x \le 20$	17
$20 < x \le 30$	21
$30 < x \le 40$	22
$40 < x \le 50$	10
$50 < x \le 60$	2

Find the mean and the standard deviation of the number of push-ups that the students did in one minute.

No. of push-ups (x)	Mid-class value, x_m	Frequency
$0 < x \le 10$	5	8
$10 < x \le 20$	15	17
$20 < x \le 30$	25	21
$30 < x \le 40$	35	22
$40 < x \le 50$	45	10
$50 < x \le 60$	55	2

$$\text{Mean} = \frac{5(8)+(15)(17)+(25)(21)+(35)(22)+(45)(10)+(55)(2)}{80}$$

$$= 26\frac{7}{8}$$

Using calculator, Standard deviation = 12.6 (3 sig.fig.)

Or Standard deviation $= \sqrt{\frac{\sum f x^2}{\sum f} - \left(\frac{\sum f x}{\sum f}\right)^2}$

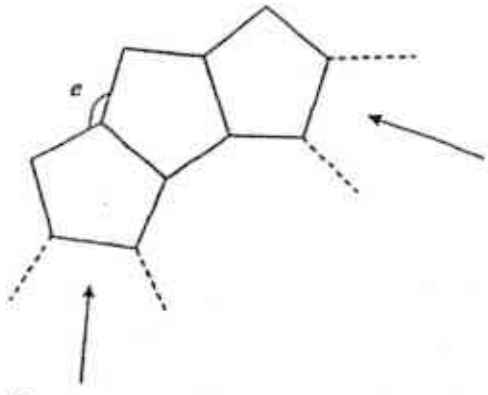
$$= \sqrt{\frac{70400}{80} - (26\frac{7}{8})^2}$$

$$= 12.6$$

Answer Mean = ... [2]

Standard deviation = ... [2]

17 The diagram shows part of a ring formed by a number of regular pentagons placed together.



Find

(a) the value of e,

Each interior angle of pentagon $= 108°$

$e = 360° - 2 \times 108°$

$= 144°$

Answer (a) $e =$ ..... [2]

(b) the number of pentagons in the ring.

$$\text{No of pentagons} = \frac{360°}{180° - 144°} = \frac{360°}{36°} = 10$$

Answer (b) ..... [1]

---

18 One solution of the equation $3x^2 + x + k = 0$, where k is a constant, is $x = -2$.

Find

(a) the value of k,

$3x^2 + x + k = 0$

When $x = -2, 3(-2)^2 + (-2) + k = 0$

$12 - 2 + k = 0$

$\therefore k = -10$

Answer (a) $k =$ ..... [1]

(b) the second solution of the equation.

$3x^2 + x - 10 = 0$

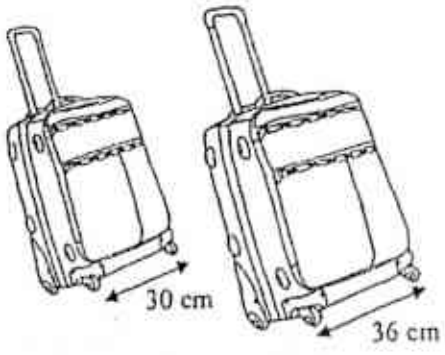
$(3x - 5)(x + 2) = 0$

$x = 1\frac{2}{3}$ or $x = -2$

The second solution of the equation is $x = 1\frac{2}{3}$.

Answer (b) $x =$ ..... [1]

19 The diagram shows two luggage bags which are geometrically similar. The cost of the luggage bag is proportional to its volume. The larger luggage bag has a length of 36 cm and the smaller one has a length of 30 cm. If the smaller luggage bag costs \$175, find the cost of the larger luggage bag.



$$\frac{C_1}{C_2}=\frac{V_1}{V_2}=\left(\frac{36}{30}\right)^3$$

$$\frac{C_1}{175}=\left(\frac{6}{5}\right)^3$$

$$C_1=\left(\frac{6}{5}\right)^3\times 175$$

$$=302.40$$

The larger luggage bag costs \$302.40.

Answer \$..... [2]

---

20 (a) p is inversely proportional to q^2. It is known that $p = 36$ for a particular value of q. Find the value of p when this value of q is trebled.

$p=\frac{k}{q^2}$, k is a non-zero constant

$k=pq^2$

$=36q^2$ for a particular value of q

$p=\frac{36q^2}{(3q)^2}$ when q is trebled

$=4$

Answer (a) p =..... [2]

(b) If Alex and Benji work together, they can complete a job in 12 days. Benji alone takes 5 days to complete $\frac{1}{6}$ of the job. How many days will Alex take to complete the job alone?

Benji takes $6\times5=30$ days to complete the job alone.

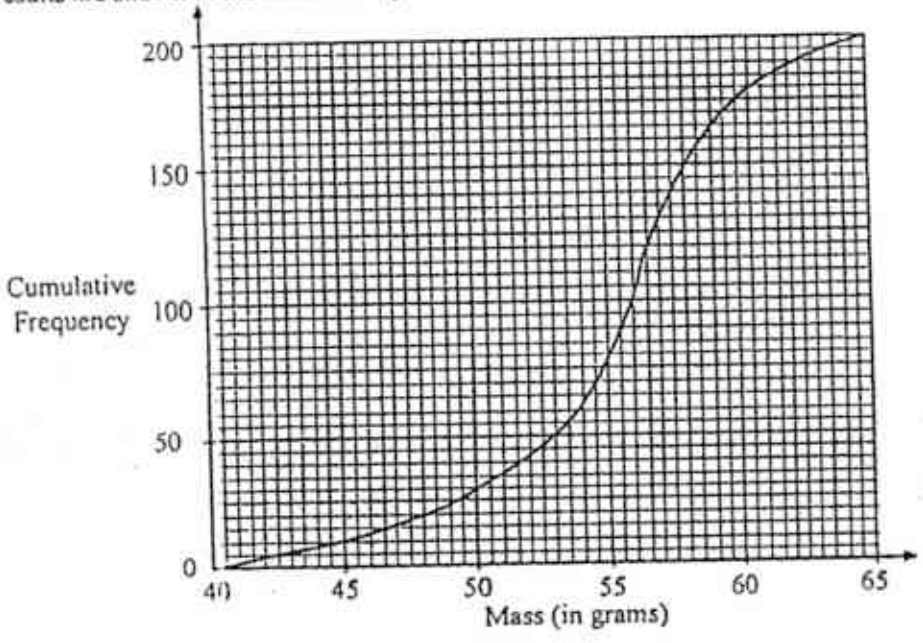
In 12 days, Benji completes $\frac{12}{30}=\frac{2}{5}$ of the job

and Alex completes $\frac{3}{5}$ of the job.

Therefore Alex takes $\frac{5}{3}\times12=20$ days to complete the job alone.

Answer (b) .....days [3]

23 The masses of a sample of 200 eggs from Rainbow Farm were measured and the results are shown in the cumulative curve below.



The heaviest 25% of the eggs are classified as "Grade 1".
The lightest 20% of the eggs are classified as "Grade 3".
The remaining eggs are classified as "Grade 2".

Using the graph,

(a) find an estimate for the least possible difference between the mass of an egg classified as "Grade 3" and the mass of an egg classified as "Grade 1".

Least possible difference $= 58 - 51.5$ g
$= 6.5$ g

Answer (a) .....grams [1]

Two eggs are chosen from the sample group.

(b) Find the probability that both eggs are classified as "Grade 2".

$$P(\text{both eggs are Grade 2}) = \frac{110}{200} \times \frac{109}{199}$$
$$= \frac{1199}{3980}$$

Answer (b) ..... [2]

Name	Class	Class Register Number

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

Parent's Signature

PRELIMINARY EXAMINATION 2015
SECONDARY 4

4016/02

MATHEMATICS
Paper 2

Friday 28 August 2015
2 hours 30 minutes

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

/100

This Question Paper consists of 7 printed pages and 1 blank page.

[Turn Over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r l$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

3 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation

$$y = \frac{1}{2}(8x - x^2 - \frac{12}{x})$$

Some of the corresponding values of x and y, correct to 2 decimal places, are given in the following table.

x	1	2	3	4	5	6	7	8
y	−2.50	p	5.50	6.50	6.30	5.00	2.64	−0.75

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal axis for $0 \le x \le 8$. [3]
Using a scale of 2 cm to 1 unit, draw a vertical axis for $-3 \le y \le 7$.
On your axes, plot the points given in the table and join them with a smooth curve.

(c) By drawing a tangent, find the gradient of the curve at $x = 3.5$. [2]

(d) (i) On the same axes, draw the line $y = \frac{1}{3}x$ for $0 \le x \le 8$. [1]

(ii) Write down the x-coordinate of the points where this line intersects the curve. [1]

(iii) This value of x is a solution of the equation $Ax - 2Bx^2 - \frac{12}{x} = 0$.
Find the values of A and B. [2]

---

4 The vertices of a triangle ABC are $A(2,6)$, $B(16,6)$ and $C(8,12)$.

Find

(a) (i) $\overrightarrow{BC}$, [2]

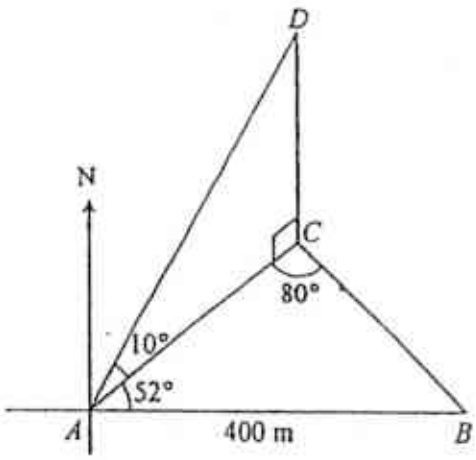
(ii) $\left|\overrightarrow{BC}\right|$. [2]

(b) (i) Find the coordinates of point M on BC where $BC : BM = 4 : 1$. [4]

(ii) State the position vector of point M. [1]

(iii) Express $\overrightarrow{AM}$ as a column vector. [2]

7



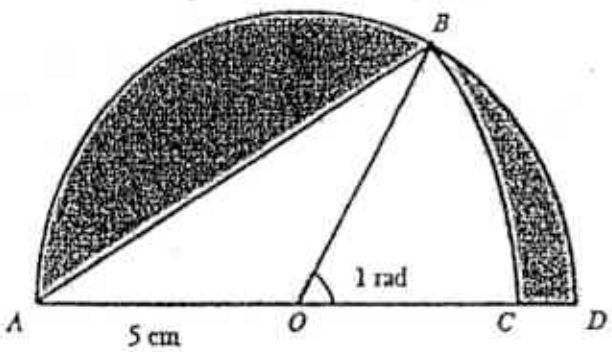
In the diagram, points A, B and C are on level ground and B is due east of A. CD is a vertical building at point C, angle $ACB = 80°$, angle $BAC = 52°$, $AB = 400$ m and the angle of elevation of the top of the building from A is 10°. Find

(a) the distance AC, [3]
(b) the height of the building CD, [2]
(c) the maximum angle of elevation from AB to the top of the building. [3]

The bearing of point E from point A is 112°. If AE is 500 m, find
(d) (i) angle CAE [2]
(ii) the distance from C to E. [2]

---

8 $ABDCO$ is a semi-circle with centre at O, radius 5 cm and diameter AD. Arc BC is part of a circle with centre at A. Given angle $BOC = 1$ rad, find



(a) the length of arc BD, [1]
(b) the length of arc BC, [3]
(c) perimeter of the shaded region, [3]
(d) area of the shaded region. [3]

	Answer Key
	$\frac{2}{(x-3)(x-4)}$
(i)	$x=\pm\sqrt{\frac{2py^2}{k}+5}$
(ii)	1) $\frac{2py^2}{k}+5\geq 0$ 2) $k\neq 0$
(i)	$(4\times 5)-8=12$
(ii)	$m=11$ $p=22$ $t=110$
(iii)	$r=n^2-n$ or $n(n-1)$
	$n=14.5$ or -13.5 $\Rightarrow n\notin\mathbb{Z}^+$ Therefore it's not one of the results of an equation of the sequence.
	$p=3$
	See attached
	Gradient = 1.01
(i)	See attached
(ii)	1.45 and 7.10
(iii)	$A=7\frac{1}{3}, B=\frac{1}{2}$
(i)	$\overline{BC}=\begin{pmatrix}-8\\6\end{pmatrix}$
(ii)	10 units
(i)	$M(14,7\frac{1}{2})$
(ii)	$\overline{OM}=\begin{pmatrix}14\\7\frac{1}{2}\end{pmatrix}$
(iii)	$\overline{AM}=\begin{pmatrix}12\\1\frac{1}{2}\end{pmatrix}$
	(i) $\frac{800}{x}$ km/l (ii) $\frac{800}{x-10}$ km/l
	$x=68.44l$ or $-58.44l$(2d.p)
	937 km
	$A=\begin{pmatrix}8\\30\\20\end{pmatrix}$
	$B=\begin{pmatrix}2 & 1 & 1\end{pmatrix}$ $BA=(66)$

rely.

imeter

Calculator Model :

Name	Class	Class Register Number

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

Parent's Signature

PRELIMINARY EXAMINATION 2015
SECONDARY 4

4016/02

MATHEMATICS
Paper 2

SOLUTIONS

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100

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125

Mathematical Formulae

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$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

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$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 (a) Simplify $\frac{4}{(x-2)(x-4)} - \frac{2}{(x-2)(x-3)}$. [3]

(b) (i) Make x the subject for the following equation. [3]

$$y = \sqrt{\frac{(x^2-5)k}{2p}}$$

(ii) State the 2 conditions such that x is a real value. [2]

Soln:

(a)
$$\frac{4}{(x-2)(x-4)} - \frac{2}{(x-2)(x-3)}$$
$$= \frac{4(x-3)-2(x-4)}{(x-2)(x-3)(x-4)}$$
$$= \frac{4x-12-2x+8}{(x-2)(x-3)(x-4)}$$
$$= \frac{2x-4}{(x-2)(x-3)(x-4)}$$
$$= \frac{2(x-2)}{(x-2)(x-3)(x-4)}$$
$$= \frac{2}{(x-3)(x-4)}$$

(b) (i)
$$y = \sqrt{\frac{(x^2-5)k}{2p}}$$
$$y^2 = \frac{(x^2-5)k}{2p}$$
$$\frac{2py^2}{k} = x^2 - 5$$
$$x^2 = \frac{2py^2}{k} + 5$$
$$x = \pm\sqrt{\frac{2py^2}{k} + 5}$$

(ii)
1) $\frac{2py^2}{k} + 5 \geq 0$

2) $k \neq 0$

2 (a) In the following sequence of equations,

$$\begin{aligned}(1\times 2)-2 &= 0\\ (2\times 3)-4 &= 2\\ (3\times 4)-6 &= 6\\ &\vdots\\ (m\times 12)-p &= t\\ &\vdots\\ (n\times y)-z &= r\end{aligned}$$

(i) State the 4th equation [1]

(ii) Find the values of m, p and t. [3]

(iii) Express r in terms of n. [2]

(b) Determine if $r=196$ can be the result of an equation in the sequence. [2]

Soln:

(a) (i) 4th equation: $(4\times 5)-8=12$

(ii) $m=11$

$p=22$

$t=(11\times 12)-22$

$=110$

(iii) $(n\times y)-z=r$

$(n\times(n+1))-2n=r$

$n^2+n-2n=r$

$n^2-n=r$

$r=n^2-n$ or $n(n-1)$

(b) $r=n^2-n$

$196=n^2-n$

$n^2-n-196=0$

$n=14.5$ or -13.5

$\Rightarrow n\notin\mathbb{Z}^+$

Therefore it's not one of the results of an equation of the sequence.

3 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation

$$y = \frac{1}{2}(8x - x^2 - \frac{12}{x})$$

Some of the corresponding values of x and y, correct to 2 decimal places, are given in the following table.

x	1	2	3	4	5	6	7	8
y	−2.50	p	5.50	6.50	6.30	5.00	2.64	−0.75

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal axis for $0 \leq x \leq 8$. [3]
Using a scale of 2 cm to 1 unit, draw a vertical axis for $-3 \leq y \leq 7$.
On your axes, plot the points given in the table and join them with a smooth curve.

(c) By drawing a tangent, find the gradient of the curve at $x = 3.5$. [2]

(d) (i) On the same axes, draw the line $y = \frac{1}{3}x$ for $0 \leq x \leq 8$. [1]

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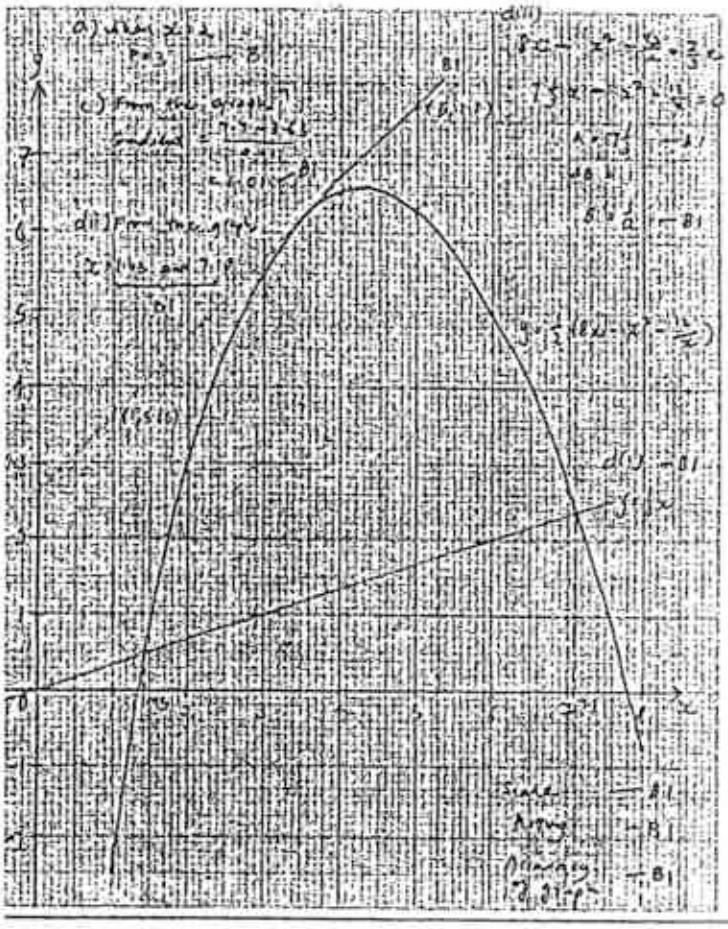
(iii) This value of x is a solution of the equation $Ax - 2Bx^2 - \frac{12}{x} = 0$.
Find the values of A and B. [2]



ces of a triangle ABC are $A(2,6)$, $B(16,6)$ and $C(8,12)$.

$\overrightarrow{BC}$. [2]

$\left|\overrightarrow{BC}\right|$. [2]

Find the coordinates of point M on BC where $BC : BM = 4 : 1$. [4]

State the position vector of point M. [1]

Express $\overrightarrow{AM}$ as a column vector. [2]

Maximum distance

$$= \frac{800}{68.4428 - 10} \times 68.4428$$

$$= 936.9$$

$$= 937\text{km (3s.f)}$$

---

6 The table below shows the price of a ticket for each category for a musical, Swan Lake.

Child (below 12 years old)	$8
Adult	$30
Senior Citizen(above 55 years old)	$20

(a) Write down a column matrix A to represent the above information [1]

(b) Mrs Lim bought four tickets for her 70 year old father, her 9 year old and 11 year old daughters, and herself.
Write down a matrix B such that the product BA gives the total amount of money Mrs Lim paid for the tickets.
Hence, find this product. [3]

(c) The table below shows information about the musical.

Number of tickets sold

	Child	Adult	Senior Citizen
Saturday	37	u	25
Sunday	44	85	v

(i) Form a matrix multiplication if the ticket sales collected on Saturday and Sunday are $2686 and $3522 respectively. [1]

(ii) Find the values of u and v. [2]

(d) (i) Evaluate the matrix $P = (1.2 \quad 2)T$ where T is the matrix representing the revenue collected on Saturday and Sunday. [1]

(ii) Explain what the matrix $(1.2 \quad 2)$ means and what the answer in d(i) represents. [2]

Soln

(a) $A = \begin{pmatrix} 8 \\ 30 \\ 20 \end{pmatrix}$

(b) $B = \begin{pmatrix} 2 & 1 & 1 \end{pmatrix}$

$$BA = \begin{pmatrix} 2 & 1 & 1 \end{pmatrix}\begin{pmatrix} 8 \\ 30 \\ 20 \end{pmatrix}$$

$$= (16 + 30 + 20)$$

$$= (66)$$

(c) (i) $$\begin{pmatrix} 37 & u & 25 \\ 44 & 85 & v \end{pmatrix}\begin{pmatrix} 8 \\ 30 \\ 20 \end{pmatrix} = \begin{pmatrix} 2686 \\ 3522 \end{pmatrix}$$

(ii) $(37 \times 8) + (u \times 30) + (25 \times 20) = 2686$

$u = 63$

$(44 \times 8) + (85 \times 30) + (v \times 20) = 3522$

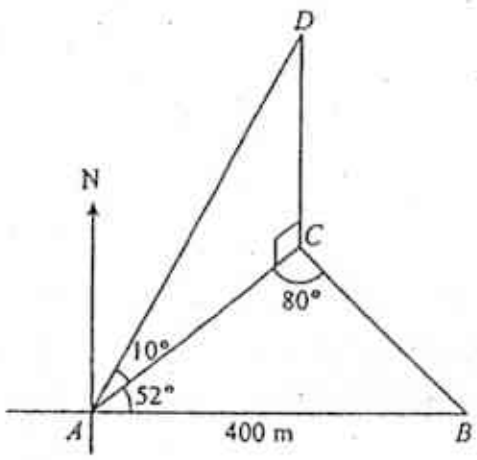
$v = 31$

(d) (i) $$\begin{pmatrix} 1.2 & 2 \end{pmatrix}\begin{pmatrix} 2686 \\ 3522 \end{pmatrix}$$

$$= (10267.2)$$

(ii) Increase in price by 20% and 100% on Saturday and Sunday respectively.
Total sales of tickets for the Saturday and Sunday

7



In the diagram, points A, B and C are on level ground and B is due east of A. CD is a vertical building at point C, angle $ACB = 80°$, angle $BAC = 52°$, $AB = 400$ m and the angle of elevation of the top of the building from A is 10°. Find

(a) the distance AC, [3]
(b) the height of the building, [2]
(c) the maximum angle of elevation from AB to the top of the building. [3]

The bearing of point E from point A is 112°. If AE is 500 m, find
(d) (i) angle CAE [2]
(ii) the distance from C to E. [2]

Soln:

(a) $\angle ABC = 180° - 52° - 80°$ (sum of angles of triangle)
$= 48°$

Using Sine Rule:

$$\frac{AC}{\sin 48°} = \frac{AB}{\sin 80°}$$

$$AC = \frac{400}{\sin 80°} \times \sin 48°$$

$= 301.8436$

$= 302$m(3 sig. fig)

(b) $$\tan \angle DAC = \frac{DC}{AC}$$

$$\tan 10° = \frac{DC}{301.8436}$$

$DC = 53.2231$

$= 53.2$m(3 sig. fig)

(c) Let perpendicular distance from C to AB be h

$$\sin 52° = \frac{h}{AC}$$

$$\sin 52° = \frac{h}{301.8436}$$

$h = 237.856$

Maximum angle of elevation

$$= \tan^{-1}\left(\frac{53.2231}{237.856}\right)$$

$= 12.613°$

$= 12.6°$(1 dec. place)

$$\sin 1 = \frac{BM}{5}$$

$$BM = 4.20735 \text{ cm}$$

$$\sin 0.5 = \frac{BM}{AB}$$

$$= \frac{4.20735}{AB}$$

$$AB = 8.7758 \text{cm}$$

$$\text{Arc } BC = r\theta$$

$$= 8.7758 \times 0.5$$

$$= 4.3879$$

$$= 4.39 \text{cm (3 sig. fig.)}$$

(c) $\text{Arc } AB = 5(\pi - 1)$

$$= 10.70796$$

Perimeter

$$= Arc\ AB + \text{Line } AB + BD + BC + CD$$

$$= Arc\ AB + \text{Line } AB + BD + BC + (OD - OC)$$

$$= Arc\ AB + \text{Line } AB + BD + BC + (OD - (\text{Line } AB - 5)$$

$$= 10.70796 + 8.7758 + 5 + 4.3879 + (5 - (8.7758 - 5))$$

$$= 30.09586$$

$$= 30.1 \text{cm (3sig.fig)}$$

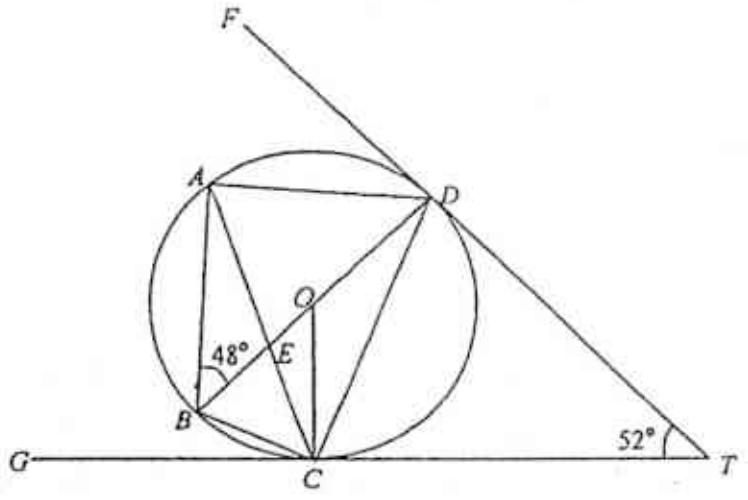
(d) Shaded area

$$= \frac{\pi 5^2}{2} - \frac{1}{2}(8.7758^2)(0.5)$$

$$= 20.0162$$

$$= 20.0 \text{cm}^3 \text{ (3sig.fig)}$$

07



The diagram, shows a circle with centre O and points A, B, C and D lie on its circumference. TDF and TCG are tangents to the circle at D and C respectively. $BEOD$ is a straight line. Given that angle $CTD = 52°$ and angle $ABD = 48°$, calculate

(i) angle COD, [1]
(ii) angle ACG, [2]
(iii) angle DEA. [2]

Given that the radius of the circle is 6 cm, find
(i) CT, [2]
(ii) the area of triangle DCT. [2]

If given two points S and R on the circumference of the circle, Mary commented that tangent lines drawn from these points will always meet. Do you agree? Explain. [2]

Soln:

(i) $\angle OCT = \angle ODT = 90°$(tan $\perp$ radius)

$\angle COD = 360° - 90° - 90° - 52°$(angle sum of quadrilateral)

$= 128°$

(ii)

$\angle ACD = \angle ABD$

$= 48°$(angles in the same segment)

$\angle DCT = \dfrac{180° - 522°}{2}$(base angles in isos. triangle, $DT = CT$)

$= 64°$

$\angle ACG = 180° - 48° - 64°$(sum of angles in triangle)

$= 68°$

130

Name: ______________________________ Register Number: ______

Class: ________

Clementi Town Secondary School
Preliminary Examination 2015
Secondary 4 Express / 5 Normal Academic

MATHEMATICS **4016/01**
Paper 1 **2 September 2015**
2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

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The total number of marks for this paper is 80.

For Examiner's Use
80

This question paper consists of 17 printed pages, including this cover page.

131

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Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

For examiner's Use

For Examiner's Use

3 The line $6x + 3y = 18$ crosses the x-axis at C.

(a) Find the coordinates of point C.

Answer (a) C (....., ...) [1]

(b) Find the gradient of the line.

Answer (b)..... [1]

4 The ratio of Ahmad's expenditure to Bala's expenditure is 4 : 9 and the ratio of Charlie's expenditure to Bala's expenditure is 5 : 3.
The difference between Ahmad's and Charlie's expenditure is \$55.
Calculate the amount of Bala's expenditure.

Answer \$..... [2]

5 **(a)** Given that $8^{3d-1} = 1$, find d.

Answer (a) $d =$..... [1]

(b) Given that $\frac{1}{2^x} = 4$, find x.

Answer (b) $x =$..... [1]

For Examiner's Use

6 The surface of a spherical solid of radius 5 m is to be painted and each tin of paint can paint an area of $6\pi m^2$.

How many tins of paint are to be bought to paint the spherical solid?

Answer ..... [3]

For Examiner's Use

7 **(a)** Express 2016 as a product of its prime factors.

Answer (a) ..... [1]

(b) Given that $\frac{2016}{k}$ is a perfect square, find the smallest possible integer value of k.

Answer (b) $k =$ ..... [2]

For Examiner's Use

8 A cyclist cycles at an average speed of 28 km/h for 45 minutes.
After resting for 25 minutes, he continues to cycle for another 12 km in 30 minutes.

Calculate the average speed of the cyclist in km/h for the entire journey.

Answer ..... km/ h [3]

9 (a) Solve the inequality $x+6<13\leq 5x-2$.

Answer (a) ..... [2]

(b) Hence, write down the largest possible prime value of x that satisfies $x+6<13\leq 5x-2$.

Answer (b) $x=$..... [1]

For Examiner's Use

10 **(a)** Express $x^2 - 6x - 16$ in the form of $(x-a)^2 - b$.

Answer (a) ..... [1]

(b) Hence, write down the minimum value of $y = x^2 - 6x - 16$.

Answer (b) ..... [1]

(c) Write down the equation of the line of symmetry of the graph of $y = x^2 - 6x - 16$.

Answer (c) ..... [1]

11 Ali's height is 150 cm.
Brian's height is 110% of Chandra's height.
Brian's height is 96 % of Ali's height.

What is Chandra's height in metres?

Answer ..... m [3]

For Examiner's Use

12

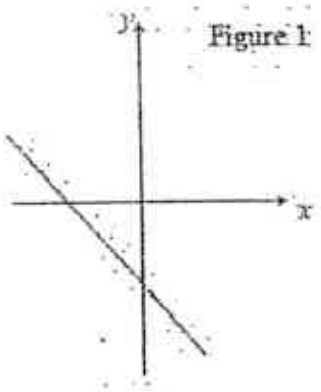


Figure 1

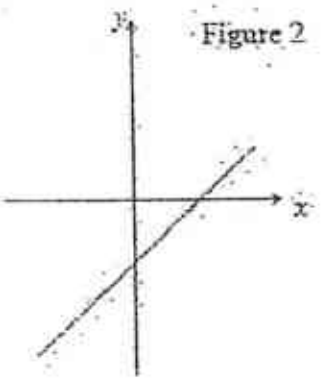


Figure 2

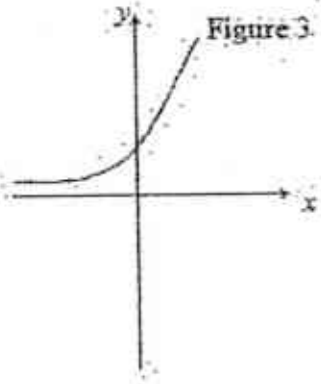


Figure 3

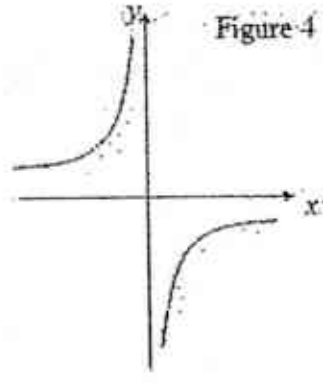


Figure 4

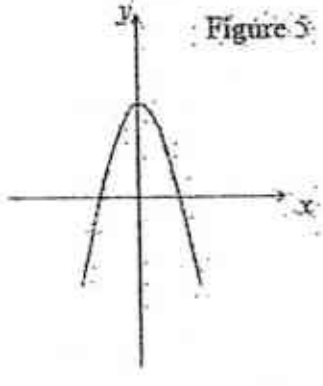


Figure 5

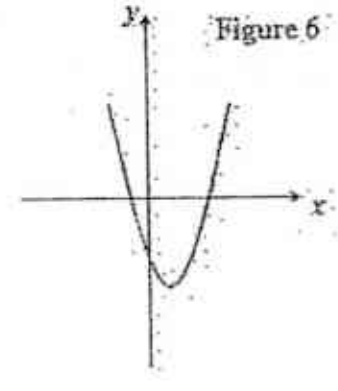


Figure 6

Which of the graphs shown above could be the graph of

(a) $y = -\frac{1}{x}$,

Answer (a) Figure ..... [1]

(b) $y + x = -2$,

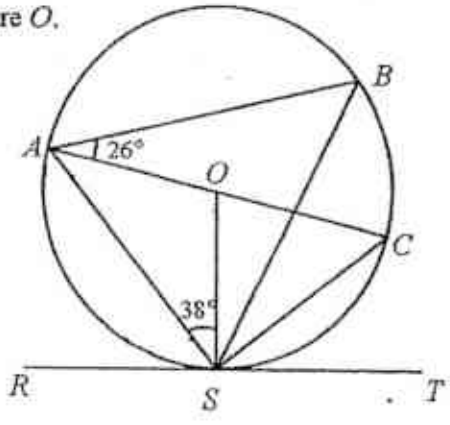
Answer (b) Figure ..... [1]

(c) $y = 1 - x^2$?

Answer (c) Figure ..... [1]

For Examiner's Use

13 In the diagram, AOC is the diameter of the circle, centre O.
RST is the tangent at S.
$\angle ASO = 38°$ and $\angle BAC = 26°$.



(a) Find $\angle ASR$.

Answer (a) $\angle ASR =$ ..... ° [1]

(b) Find $\angle ACS$.

Answer (b) $\angle ACS =$ ..... ° [1]

(c) Find $\angle BSC$.

Answer (c) $\angle BSC =$ ..... ° [1]

14 $\mathcal{E} = \{$ integers x: $2 < x \leq 10\}$
$P = \{$ prime numbers $\}$
$Q = \{$ factors of 12 $\}$

(a) List the elements in Q.

Answer (a) ..... [1]

(b) List the elements in $(P \cup Q)'$.

Answer (b) ..... [1]

(c) Find $n(P \cap Q')$.

Answer (c) ..... [1]

For Examiner's Use

15 A hawker sells green tea and jasmine tea, each available in small, medium and large cups.
The matrices show the number of cups sold during lunch and the price of one cup of each size.

$$\begin{array}{c} \\ \text{Green Tea} \\ \text{Jasmine Tea} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Small} & \text{Medium} & \text{Large} \end{array} \\ \begin{pmatrix} 4 & 8 & 2 \\ 1 & 5 & 0 \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ \text{Small} \\ \text{Medium} \\ \text{Large} \end{array} \begin{array}{c} \begin{array}{c} \text{Price} \end{array} \\ \begin{pmatrix} 1.20 \\ 1.50 \\ 1.80 \end{pmatrix} \end{array}$$

(a) Find $\begin{pmatrix} 4 & 8 & 2 \\ 1 & 5 & 0 \end{pmatrix}\begin{pmatrix} 1.2 \\ 1.5 \\ 1.8 \end{pmatrix}$.

Answer (a) ..... [2]

(b) Explain what your answer to (a) represents.

Answer (b) .....

..... [1]

16 **(a)** Simplify $\dfrac{4x+6}{2xy+3y}$.

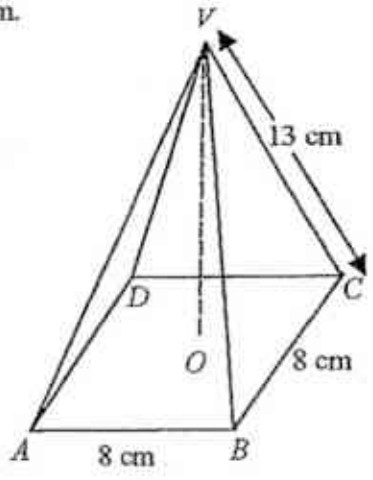
Answer (a)..... [1]

(b) Express as a single fraction in its simplest form $\dfrac{4}{a+3b} - \dfrac{a-21b}{a^2-9b^2}$.

Answer (b)..... [2]

For Examiner's Use

17 $ABCDV$ is a right pyramid with a square base of side 8 cm. VO is vertical and $VC = 13$ cm.



(a) Find the length of OC.

Answer (a) $OC =$ ..... cm [1]

(b) Find the vertical height VO.

Answer (b) $VO =$ .....cm [1]

(c) Calculate the volume of the pyramid.

Answer (c).....cm^3 [1]

For Examiner's Use

18 **(a)** Factorise completely $8ab + 1 - 4a - 2b$.

Answer (a) ..... [2]

(b) Factorise $3x^2 - 15x + 18$.

Answer (b) ..... [2]

19 It is given that x is inversely proportional to the square root of y.
When $x = 6$, $y = 4$.

(a) Find the equation relating x and y.

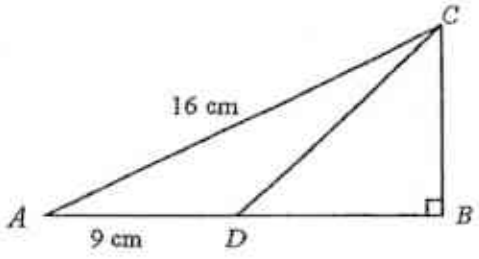
Answer (a) ..... [2]

(b) Hence, find the value of x when $y = 256$.

Answer (b) $x =$ ..... [1]

For Examiner's Use

20 In the diagram, ADB is a straight line, $\angle ABC = 90°$, $AC = 16\,\text{cm}$, $AD = 9\,\text{cm}$ and area of $\Delta ADC = 36\,\text{cm}^2$.



(a) Prove that $\angle CAD = 30°$.

Answer (a)

[2]

(b) Find the shortest distance between point D and the line AC.

Answer (b)..... cm [1]

(c) Find the length of CD.

Answer (c)..... cm [2]

(d) Find $\sin \angle ADC$.

Answer (d)..... [2]

For Examiner's Use

21 (a) Convert 2015 picoseconds to seconds, giving your answer in standard form.

Answer (a).....'seconds [1]

(b) In the Singapore Budget 2015, the Ministry of Education was allocated $10.9 billion from the total budget of $67 820 000 000.

(i) How much was allocated to the other ministries?
Give your answer in billion.

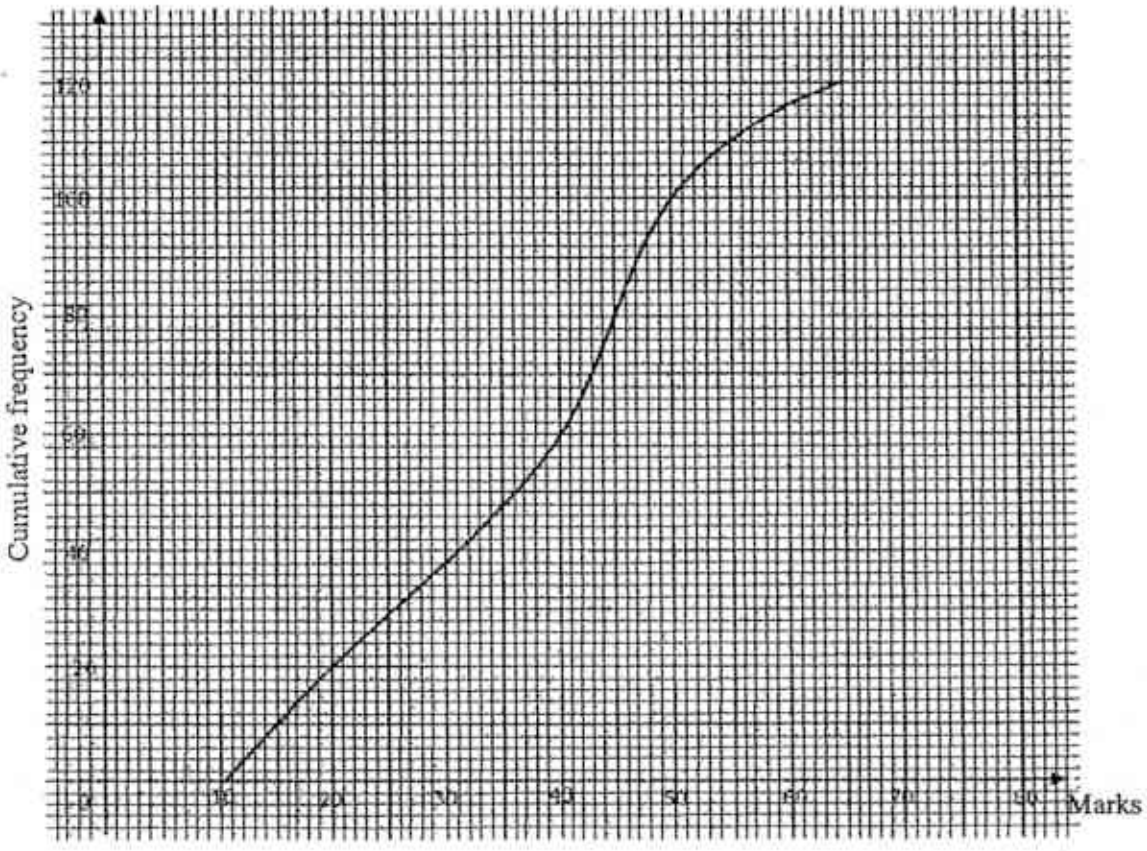
Answer (b)(i) $ ..... billion [2]

(ii) Calculate the percentage of the total budget allocated to the Ministry of Education.

Answer (b)(ii) ..... % [2]

For Examiner's Use

22 The cumulative frequency curve below illustrates the marks obtained, out of 70, by 120 students in test *A*. The passing mark is 35.



(a) Use the graph to find

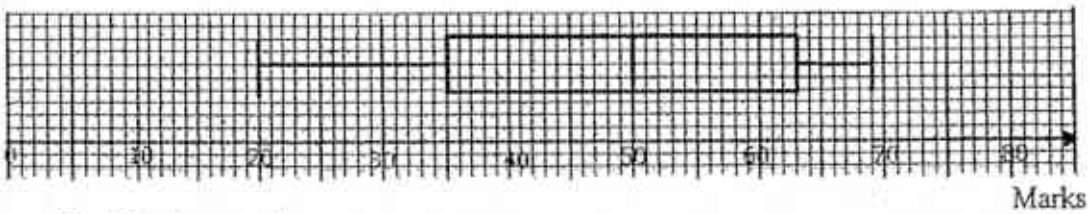
(i) the median mark,

Answer (a)(i) ..... [1]

(ii) the interquartile range.

Answer (a)(ii) ..... [1]

(b) The box-and-whisker plot represents the marks, out of 70, obtained by the same group of 120 students in test *B* two weeks later.



Find the interquartile range.

Answer (b) ..... [1]

For Examiner's Use

(c) In which exam did the students perform better? Justify your answer.

Answer (c) .....

.....

.....

.....

..... [2]

23 The numerator of a fraction is x and its denominator is y when expressed in its simplest form.
The sum of the numerator and the denominator of the fraction is 21.
When 5 is added to the numerator, the fraction becomes 1.

(a) Write down two simultaneous equations, in terms of x and y, to represent this information.

Answer (a).....

..... [2]

(b) Solve the simultaneous equation to find the value of x and y.

Answer (b) x =.....

y = ... [2]

(c) Hence, state the reciprocal of the original fraction.

Answer (c) ..... [1]

For Examiner's Use

24 The diagram shows the speed-time graph of a particle over a period of 15 seconds.
The particle uniformly decelerated from 10 m/s to v m/s in 5 seconds.
It then maintains at this speed for the next 5 seconds and accelerates uniformly at 2 m/s^2 for another 5 seconds.
The distance travelled in the first 5 seconds is 35 m.

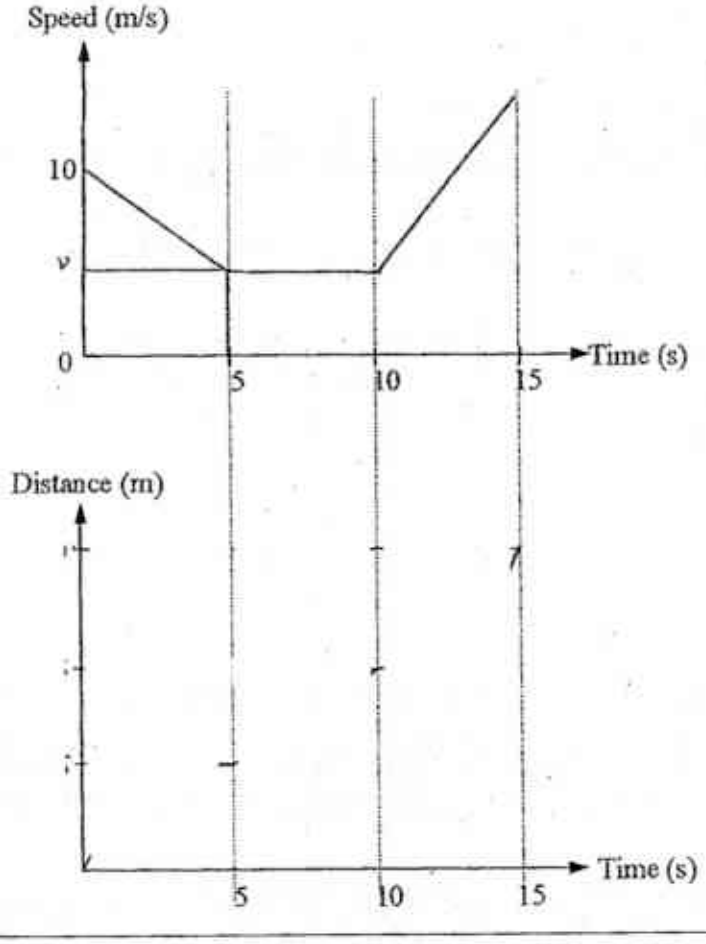
(a) Find the value of v.

Answer (a) $v =$ ..... [1]

(b) Find the speed of the particle after 15 seconds.

Answer (b)..... m/s [1]

(c) Complete the corresponding distance-time graph. [3]



For Examiner Use

~ END OF PAPER ~

Name: ______________________________ Register Number: ______

Class: _______

Clementi Town Secondary School
Preliminary Examination 2015
Secondary 4 Express / 5 Normal Academic

MATHEMATICS
Paper 1

Marking Scheme

4016/01
2 September 2015
2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Do not open the booklets until you are told to do so.
Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.
Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use
/ 80

This question paper consists of 17 printed pages, including this cover page.

139

[Turn over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

For Examiner's Use

Answer **all** the questions.

1 **(a)** Simplify $7(a-15)-8(a-15)$.

$15-a$ B1

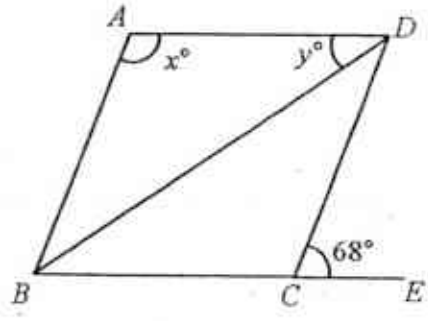
Answer (a)..... [1]

(b) Solve $(3b+1)(b-2)=0$.

$-\frac{1}{3}$ 2 B1

Answer (b) b = ... or ..... [1]

2 The diagram shows a sketch of a rhombus.



(a) Find x.

112 B1

Answer (a) x = ... [1]

(b) Find y.

34 B1

Answer (b) y = ... [1]

140

For Examiner's Use

3 The line $6x + 3y = 18$ crosses the x-axis at C.

(a) Find the coordinates of point C.

B1

Answer (a) C (3 , 0) [1]

(b) Find the gradient of the line.

B1

Answer (b) -2 [1]

For Examiner's Use

4 The ratio of Ahmad's expenditure to Bala's expenditure is 4 : 9 and the ratio of Charlie's expenditure to Bala's expenditure is 5 : 3.
The difference between Ahmad's and Charlie's expenditure is \$55.
Calculate the amount of Bala's expenditure.

A	:	B	C
4	:	9	
		3	: 5
		9	: 15

M1

11 U ---- \$55
1 U ------\$5
9 U ------ \$45

A1

Answer \$ ..... [2]

5 **(a)** Given that $8^{3d-1} = 1$, find d.

$\frac{1}{3}$ B1

Answer (a) $d =$ ..... [1]

(b) Given that $\frac{1}{2^x} = 4$, find x.

B1

Answer (b) $x =$ -2 [1]

For Examiner's Use

6 The surface of a spherical solid of radius 5 m is to be painted and each tin of paint can paint an area of $6\pi\,m^2$.

How many tins of paint are to be bought to paint the spherical solid?

Surface area of sphere $= 4\pi(5)^2$ M1
$= 100\pi$

$\frac{100\pi}{6\pi} = 16.66667$ M1

No of tins = 17 A1

Answer ..... [3]

7 **(a)** Express 2016 as a product of its prime factors.

$2^5 \times 3^2 \times 7$ B1

Answer (a) ..... [1]

(b) Given that $\frac{2016}{k}$ is a perfect square, find the smallest possible integer value of k.

$\frac{2^5 \times 3^2 \times 7}{k} = \frac{(2\times2\times3)\times(2\times2\times3)\times 2\times 7}{k}$ M1

k = 14 A1

Answer (b) k = ... [2]

For Examiner's Use

8 A cyclist cycles at an average speed of 28 km/h for 45 minutes.
After resting for 25 minutes, he continues to cycle for another 12 km in 30 minutes.

Calculate the average speed of the cyclist in km/h for the entire journey.

Total time taken = 45 + 30 + 25 M1
= 100 mins = $1\frac{2}{3}$h

Total Distance travelled = 12 + 28 x $\frac{3}{4}$ M1
= 33 km

Average speed = 33/$1\frac{2}{3}$
= 19.8 km/h A1

Answer ..... km/ h [3]

9 **(a)** Solve the inequality $x+6<13\leq 5x-2$.

$x+6<13\leq 5x-2$
$x+6<13$ or $13\leq 5x-2$
$x<7$ $3\leq x$ M1

$3\leq x<7$ A1

Answer (a) ..... [2]

(b) Hence, write down the largest possible prime value of x that satisfies $x+6<13\leq 5x-2$.

5 B1

Answer (b) x =..... [1]

For Examiner's Use

10 **(a)** Express $x^2 - 6x - 16$ in the form of $(x-a)^2 - b$.

B1

$(x-3)^2 - 25$

Answer (a) ..... [1]

(b) Hence, write down the minimum value of $y = x^2 - 6x - 16$.

B1

$y = -25$

Answer (b) ..... [1]

(c) Write down the equation of the line of symmetry of the graph of $y = x^2 - 6x - 16$.

$x = 3$ B1

Answer (c) ..... [1]

11 Ali's height is 150 cm.
Brian's height is 110% of Chandra's height.
Brian's height is 96 % of Ali's height.

What is Chandra's height in metres?

Brian's height $= 0.96 \times 150$ M1
$= 144$ cm

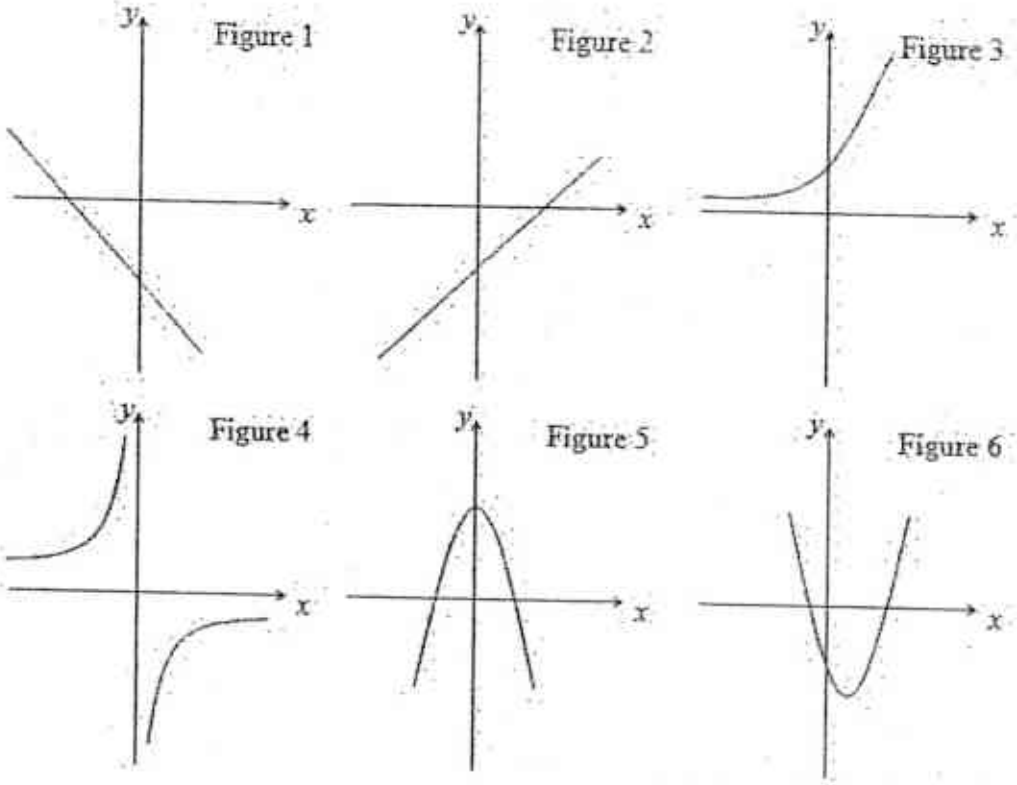
Chandra's height $= \frac{100}{110} \times 144$ M1
≈ 1.31m (3s.f) A1

Answer ..... m [3]

145

For Examiner's Use

12



Which of the graphs shown above could be the graph of

(a) $y = -\frac{1}{x}$,

B1

4

Answer (a) Figure ..... [1]

(b) $y + x = -2$,

B1

1

Answer (b) Figure ..... [1]

(c) $y = 1 - x^2$.

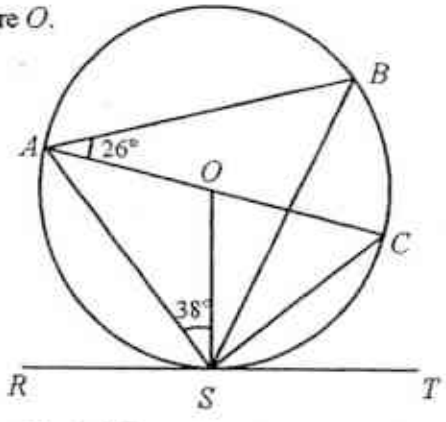
5 B1

Answer (c) Figure ..... [1]

For Examiner's Use

For Examiner's Use

13 In the diagram, AOC is the diameter of the circle, centre O.
RST is the tangent at S.
$\angle ASO = 38°$ and $\angle BAC = 26°$.



(a) Find $\angle ASR$.

$\angle ASR = 90° - 38°$ (Tangent to circle)
$= 52°$
B1

Answer (a) $\angle ASR =$ .....^{o} [1]

(b) Find $\angle ACS$.

$\angle ACS = \angle OSC$ (Base angle of isos. triangle)
$= 90° - 38°$ (angle in a semi circle)
$= 52°$ B1

Answer (b) $\angle ACS =$ .....^{o} [1]

(c) Find $\angle BSC$.

$\angle BSC = \angle BAC$
$= 26°$ (Angle in same segment)
B1

Answer (c) $\angle BSC =$ .....^{o} [1]

---

14 $\mathcal{E} = \{$ integers x: $2 < x \leq 10\}$
$P = \{$ prime numbers$\}$
$Q = \{$ factors of 12$\}$

(a) List the elements in Q.

{3,4,6} B1

Answer (a) ..... [1]

(b) List the elements in $(P \cup Q)'$.

{8,9,10} B1

Answer (b) ..... [1]

(c) Find $n(P \cap Q')$.

2 B1

Answer (c) ..... [1]

For Examiner's Use

15 A hawker sells green tea and jasmine tea, each available in small, medium and large cups.
The matrices show the number of cups sold during lunch and the price of one cup of each size.

$$\begin{array}{c} \\ \text{Green Tea} \\ \text{Jasmine Tea} \end{array}\begin{array}{c} \begin{array}{ccc}\text{Small} & \text{Medium} & \text{Large}\end{array} \\ \begin{pmatrix} 4 & 8 & 2 \\ 1 & 5 & 0 \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ \text{Small} \\ \text{Medium} \\ \text{Large} \end{array}\begin{array}{c} \begin{array}{c}\text{Price}\end{array} \\ \begin{pmatrix} 1.20 \\ 1.50 \\ 1.80 \end{pmatrix} \end{array}$$

(a) Find $\begin{pmatrix} 4 & 8 & 2 \\ 1 & 5 & 0 \end{pmatrix}\begin{pmatrix} 1.2 \\ 1.5 \\ 1.8 \end{pmatrix} = \begin{pmatrix} 4\text{x}1.2 + 8\text{x}1.5 + 2\text{x}1.8 \\ 1\text{x}1.2 + 5\text{x}1.5 + 0\text{x}1.8 \end{pmatrix}$ M1

$= \begin{pmatrix} 20.4 \\ 8.7 \end{pmatrix}$ A1

Answer (a) ..... [2]

(b) Explain what your answer to (a) represents. B1

Answer (b) The elements in (a) represents the total amount earned by the drink stall for selling green tea and jasmine tea respectively..... [1]

16 (a) Simplify $\dfrac{4x+6}{2xy+3y}$.

$\dfrac{2}{y}$ B1

Answer (a)..... [1]

(b) Express as a single fraction in its simplest form $\dfrac{4}{a+3b} - \dfrac{a-21b}{a^2-9b^2}$.

$\dfrac{4}{a+3b} - \dfrac{a-21b}{a^2-9b^2} = \dfrac{4(a-3b)-(a-21b)}{(a+3b)(a-3b)}$ M1

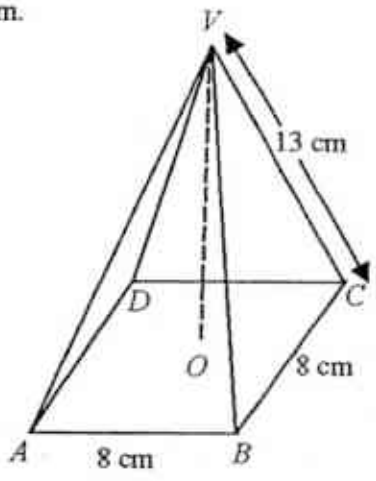
$= \dfrac{3a+9b}{(a+3b)(a-3b)}$

$= \dfrac{3}{a-3b}$ A1

Answer (b)..... [2]

For Examiner's Use

17 $ABCDV$ is a right pyramid with a square base of side 8 cm. VO is vertical and $VC = 13$ cm.



(a) Find the length of OC.

$$OC = \frac{1}{2}AC$$
$$= \frac{1}{2}\sqrt{8^2 + 8^2}$$
$$\approx 5.66 \text{ cm (3 s.f)} \quad \text{B1}$$

Answer (a) $OC =$ ..... cm [1]

(b) Find the vertical height VO.

$$VO = \sqrt{13^2 - OC^2}$$
$$= \sqrt{137}$$
$$\approx 11.7 \text{ cm (3 s.f)} \quad \text{B1}$$

Answer (b) $VO =$ .....cm [1]

(c) Calculate the volume of the pyramid.

$$\text{Volume} = \frac{1}{3}(8^2)\sqrt{137}$$
$$\approx 250 \text{ cm}^3 \text{ (3 s.f)} \quad \text{B1}$$

Answer (c).....cm^3 [1]

144

For Examiner's Use

18 **(a)** Factorise completely $8ab+1-4a-2b$.

$8ab+1-4a-2b = 4a(2b-1)-(-1+2b)$ M1

$= (4a-1)(2b-1)$ A1

Answer (a) ..... [2]

(b) Factorise $3x^2-15x+18$.

$3x^2-15x+18 = 3(x^2-5x+6)$ M1

$= 3(x-3)(x-2)$ A1

Answer (b) ..... [2]

---

19 It is given that x is inversely proportional to the square root of y.

When $x=6$, $y=4$.

(a) Find the equation relating x and y.

$x = \frac{k}{\sqrt{y}}$, where k is a constant M1

$k = 12$

$x = \frac{12}{\sqrt{y}}$ A1

Answer (a) ..... [2]

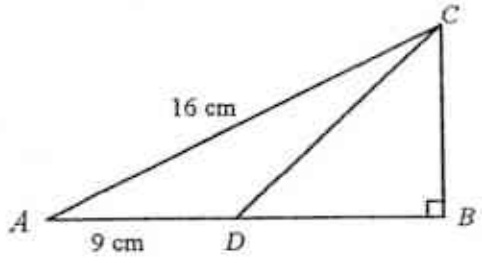
(b) Hence, find the value of x when $y=256$.

$\frac{3}{4}$ B1

Answer (b) $x =$ ..... [1]

For Examiner's Use

20 In the diagram, ADB is a straight line, $\angle ABC = 90°$, $AC = 16$ cm, $AD = 9$ cm and area of $\Delta ADC = 36\text{ cm}^2$.



(a) Prove that $\angle CAD = 30°$.

Answer (a)

$$\frac{1}{2}\times 9\times 16\times \sin\angle CAD = 36 \quad \text{M1}$$

$$\sin\angle CAD = 0.5$$

$$\angle CAD = 30° \quad \text{(Proved)} \quad \text{A1}$$

[2]

(b) Find the shortest distance between point D and the line AC.

$$\frac{1}{2}\times 16\times \text{shortest distance} = 36$$

$$\text{shortest distance} = 36\div 8 = 4.5 \text{ cm} \quad \text{B1}$$

Answer (b)..... cm [1]

(c) Find the length of CD.

$$CD = \sqrt{9^2+16^2-2\times 9\times 16\times \cos 30°} \quad \text{M1}$$

$$\approx 9.36 \text{ cm (3 s.f)} \quad \text{A1}$$

Answer (c)..... cm [2]

(d) Find $\sin\angle ADC$.

$$BC = \frac{36\times 2}{9} \quad \text{M1}$$

$$= 8$$

$$\sin\angle ADC = \sin\angle BDC$$

$$\sin\angle ADC = \frac{BC}{CD}$$

$$\sin\angle BCD \approx 0.855 \text{ (3 s.f)} \quad \text{A1}$$

Answer (d)..... [2]

For Examiner's Use

21 (a) Convert 2015 picoseconds to seconds, giving your answer in standard form.

2.015×10^{-9} B1

Answer (a)..... seconds [1]

(b) In the Singapore Budget 2015, the Ministry of Education was allocated \$10.9 billion from the total budget of \$67 820 000 000.

(i) How much was allocated to the other ministries?
Give your answer in billion.

$6.782\times10^{10}-10.9\times10^{9}$ M1
$=5.692\times10^{10}$

56.92 A1

Answer (b)(i) \$ ..... billion [2]

(ii) Calculate the percentage of the total budget allocated to the Ministry of Education.

$\frac{10.9\times10^{9}}{6.782\times10^{10}}\times100\%$ M1

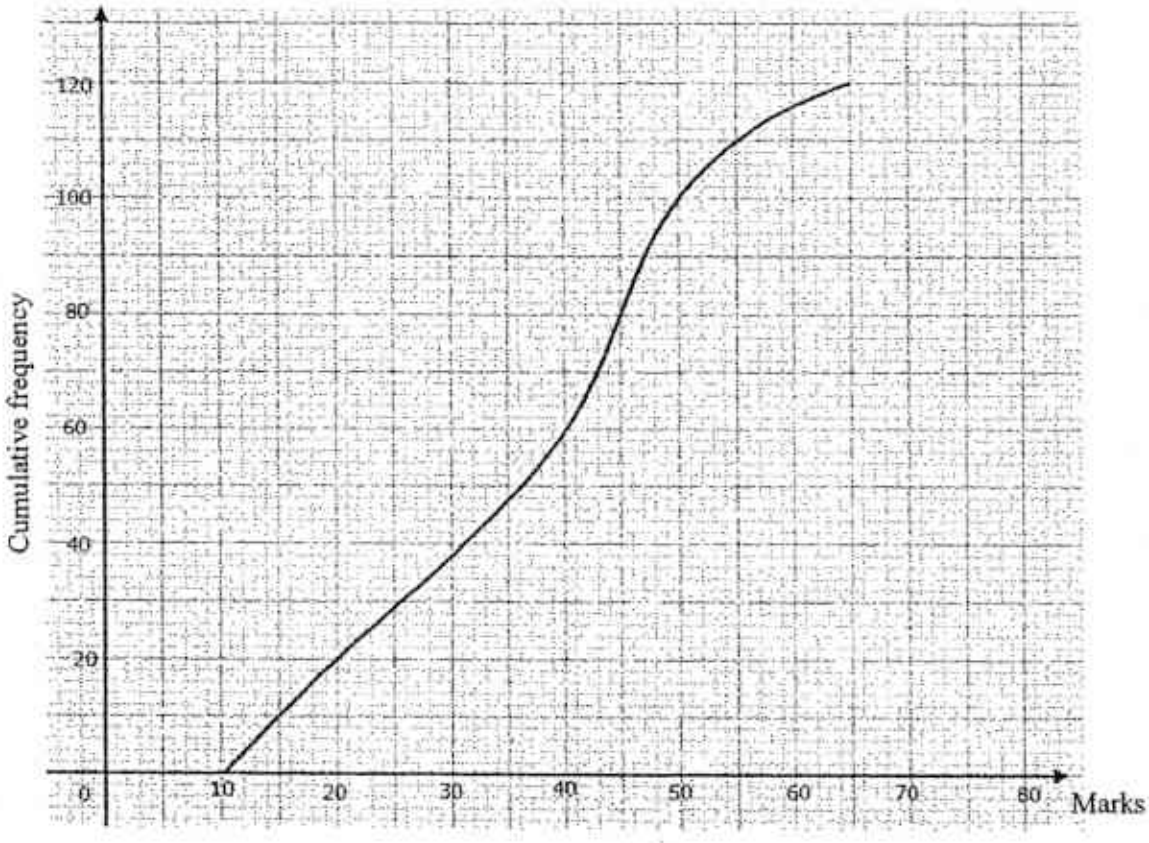
$=16.1\%$ A1

Answer (b)(ii) ..... % [2]

For Examiner's Use

For Examiner's Use

22 The cumulative frequency curve below illustrates the marks obtained, out of 70, by 120 students in test A. The passing mark is 35.



(a) Use the graph to find

(i) the median mark,

40 B1

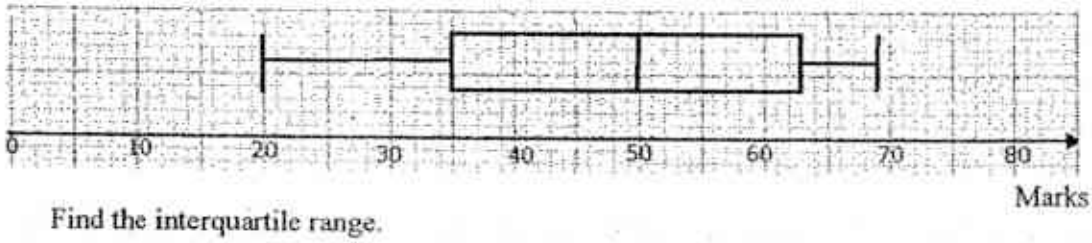
Answer (a)(i) ..... [1]

(ii) the interquartile range.,

Accept Ans: 21 to 22 B1

Answer (a)(ii) ..... [1]

(b) The box-and-whisker plot represents the marks, out of 70, obtained by the same group of 120 students in test B two weeks later.



Find the interquartile range.

28 B1

Answer (b) ..... [1]

14 C

For Examiner's Use

(c) In which exam did the students perform better? Justify your answer.

Answer (c) Test B has a higher mean mark than that of Test A, hence the students perform better for Test B.

.....[2]

For Examiner's Use

23 The numerator of a fraction is x and its denominator is y when expressed in its simplest form.
The sum of the numerator and the denominator of the fraction is 21.
When 5 is added to the numerator, the fraction becomes 1.

(a) Write down two simultaneous equations, in terms of x and y, to represent this information.

Answer (a) $\frac{x+5}{y}=1$ B1

$x+y=21$ B1 [2]

(b) Solve the simultaneous equation to find the value of x and y.

Answer (b) $x=8$ B1

$y=13$ B1 [2]

(c) Hence, state the reciprocal of the original fraction.

Answer (c) $\frac{13}{8}$ B1 [1]

For Examiner's Use

24 The diagram shows the speed-time graph of a particle over a period of 15 seconds.
The particle uniformly decelerated from 10 m/s to v m/s in 5 seconds.
It then maintains at this speed for the next 5 seconds and accelerates uniformly at 2 m/s^2 for another 5 seconds.
The distance travelled in the first 5 seconds is 35 m.

(a) Find the value of v.

4 B1

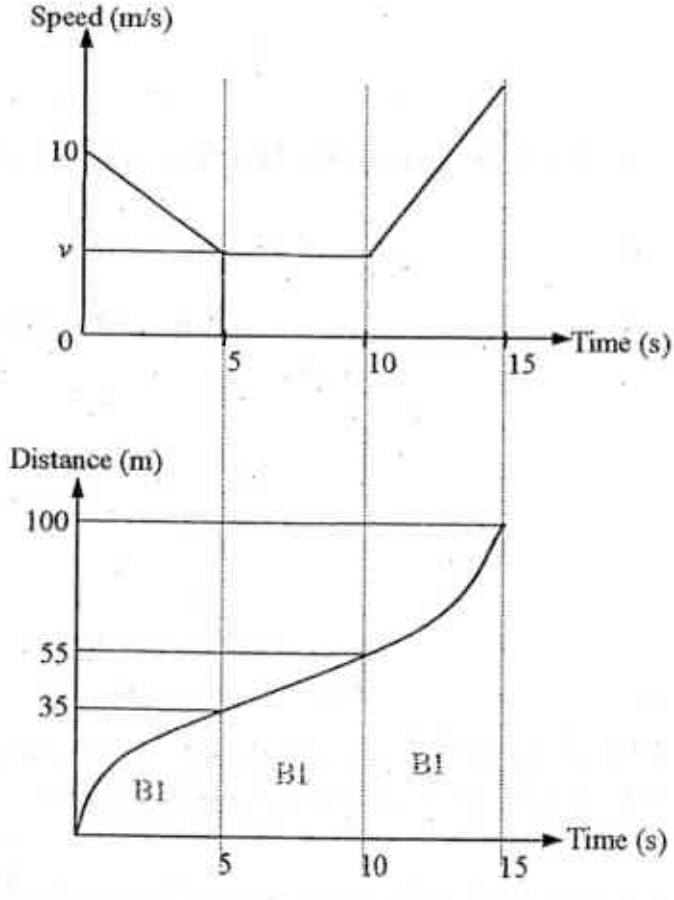
Answer (a) v = ... [1]

(b) Find the speed of the particle after 15 seconds.

14 B1

Answer (b) ..... m/s [1]

(c) Complete the corresponding distance-time graph. [3]



~ *END OF PAPER* ~

147

Name : ______________________ Register Number : ______

Class : ______

Clementi Town Secondary School
Preliminary Examination 2015
Secondary 4 Express / 5 Normal Academic

Mathematics **4016/2**

Paper 2

14 September 2015
2 hours 30 minutes

Additional Materials provided: Answer Paper (7 sheets)
Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Do not open the booklets until you are told to do so.
Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the answer paper.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.
Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Solve $(2x+1)^2 = 25$. [2]

(b) Simplify $\frac{x+2y}{4} + \frac{2x^2-8y^2}{8}$. [2]

(c) It is given that $z = \sqrt{\frac{x}{3x-y}}$.

(i) Find z when $x = 3$ and $y = 1.5$. [1]

(ii) Express x in terms of y and z. [2]

(d) Express $2 - \frac{4f-g}{f+2g}$ as a single fraction in its simplest form. [2]

---

2 **(a)** Lenny bought a brand new car at a selling price of \$109 000.
He paid 40% of the selling price in cash and took a 5-year loan for the rest of the amount.
If the bank charges 2.8% of simple interest per annum for the loan, calculate

(i) the total interest payable, [2]

(ii) the amount Lenny has to pay per month for the loan. [2]

(b) Lenny's new car uses fuel at an average rate of 9.5 litres per 100 km driven.
In an average year, Lenny drives 15 000 km.
The retail price of fuel is \$2.25 per litre and Lenny has a loyalty card that gives him 10% discount off the retail price.
Assuming the price of fuel remains the same, work out the amount Lenny would expect to spend on fuel in one year, correct to the nearest dollar. [2]

(c) The original value of the car is its selling price of \$109 000.
For each of the first five years, the value of the car decreases by 13% of its value at the start of every year.
After three years, Lenny decides to sell the car.
Calculate the overall percentage reduction in the value of the car compared with its original value, giving your answer to 2 decimal places. [3]

149

3

Diagram I

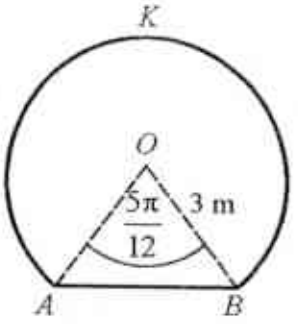


Diagram II

A 20-metre long tunnel can be found along a hiking trail, as shown in Diagram I.
The cross-section of this tunnel is a segment AKB of a circle centre O and radius 3 m, as shown in Diagram II.

It is given that angle $AOB = \frac{5\pi}{12}$.

(a) Calculate

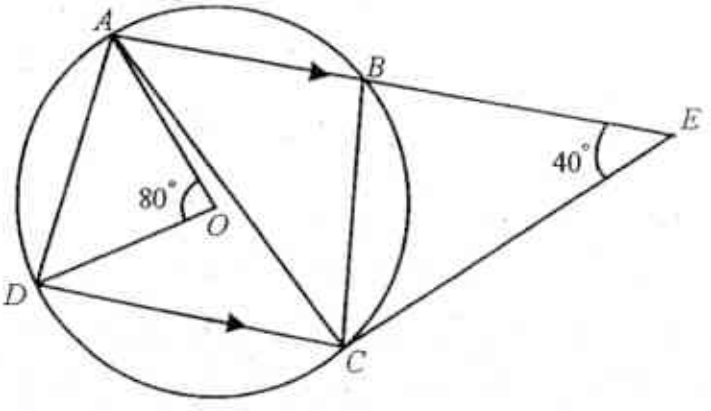
(i) the length of AB, [2]

(ii) the perimeter of the segment AKB, [2]

(iii) the area of the segment AKB. [3]

(b) During a construction project, the tunnel is to be permanently sealed off by filling it with cement.
Work out the total cost of cement required to completely fill the tunnel, assuming that the cost of cement is \$30 per cubic metre. [2]

4



(a) In the diagram above, $ABCD$ is a cyclic quadrilateral in a circle with centre O.
ABE is a straight line which is parallel to DC.
Given that angle $AEC = 40°$ and angle $AOD = 80°$, find

(i) angle DAO, [1]

(ii) angle ACD. [1]

(b) Given further that angle $ODC = 30°$, find angle OAC. [1]

(c) By showing your working and stating all the reasons clearly,

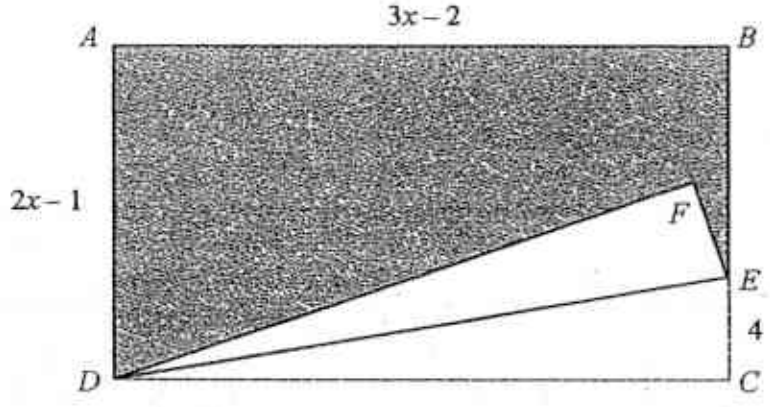
(i) prove that triangle ACE is isosceles, [2]

(ii) show that triangle BCE and triangle DAC are similar. [2]

(d) A point, F, is on the same side of AD as O, such that angle AFD is 30°.

Determine whether the point F lies on the circumference of the circle, inside the circle, or outside the circle.
Give a reason for your answer. [2]

5



A rectangular piece of paper $ABCD$ is folded along the line ED such that C is moved to F. The length of AB is $(3x-2)$ cm, AD is $(2x-1)$ cm and EC is 4 cm.

(a) Write down an expression, in terms of x, for

(i) area of rectangle $ABCD$, [1]

(ii) area of the triangle DEF. [1]

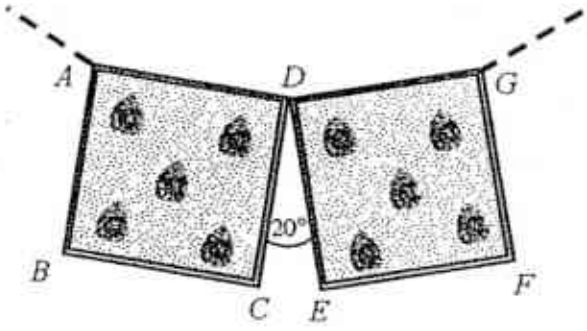
(b) Given that the area of the shaded region $ABEFD$ is 184 cm^2, form an equation in x and show that it reduces to $6x^2 - 19x - 174 = 0$. [2]

(c) Solve the equation $6x^2 - 19x - 174 = 0$, giving the answers correct to three decimal places. [3]

(d) Find the perimeter of the folded piece of paper. [2]

150

6



(a) The diagram above shows part of the design of a necklace.
AD and DG are sides of a regular polygon.
$ABCD$ and $DEFG$ are squares, while angle $CDE = 20°$.

(i) Find the total number of sides of this regular polygon. [3]

(ii) Calculate angle AGF. [2]

(b) The first five terms in a sequence of numbers, $T_1, T_2, T_3, T_4, T_5, \ldots$ are given by $-9, -4, 1, 6, 11, \ldots$.

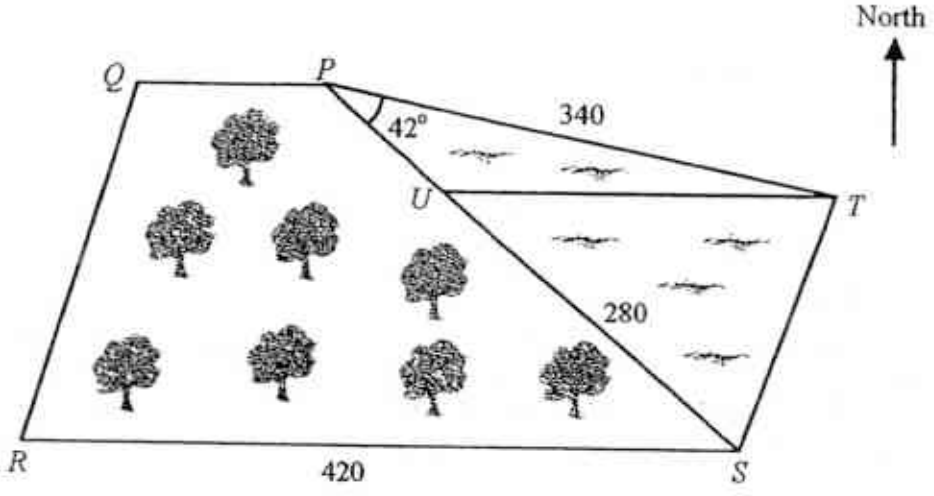
(i) Find T_6 and T_7. [1]

(ii) Is it possible for the terms beyond T_5 to be negative?
Explain your answer. [1]

(iii) Find an expression, in terms of n, for T_n. [2]

(iv) Evaluate T_{150}. [1]

7



The diagram shows a large garden, $PQRS$, which overlooks a triangular pond, PST.
A bridge is built across the pond from U to T, where U is a point on PS and T is due east of U.
Angle $TPU = 42°$ and the bearing of P from T is 290°.
$PT = 340$ m, $SU = 280$ m and $RS = 420$ m.

(a) Calculate

(i) angle PTU, [1]

(ii) the bearing of S from P. [2]

(b) After a stroll in the garden, Joe stops at U.

Suppose the entrance to the garden is at P, how far is Joe away from the entrance? [2]

(c) Calculate the area of the triangular pond, PST. [2]

(d) During a renovation of the garden, the original bridge is replaced by a new bridge that would join PS to T by the shortest distance.
Calculate the length of this new bridge. [2]

(e) When the pond appears on an accurate map, $PT = 17$ mm.
Find the area of the pond on this map in square millimetres. [2]

8 **(a)** P is the point (2, 1).

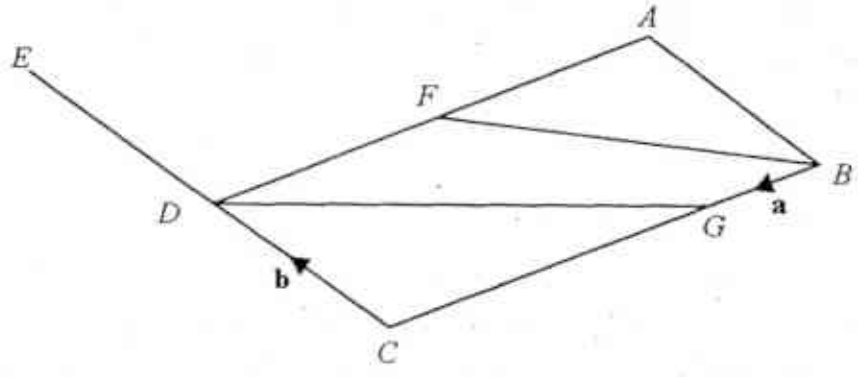
The position vector of Q is $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$.

(i) Write down the column vector $\overrightarrow{PQ}$. [1]

(ii) Find the exact value of $\left|\overrightarrow{PQ}\right|$. [2]

(iii) Find the position vector of R, such that $\overrightarrow{QR} = 3\overrightarrow{QP}$. [1]

(b)



$ABCD$ is a parallelogram and E lies on CD produced such that $CD = DE$.
F is the midpoint of AD and G is a point on BC such that $BG : GC = 1 : 3$.
Given $\overrightarrow{BG} = \mathbf{a}$ and $\overrightarrow{CD} = \mathbf{b}$,

(i) express, as simply as possible, in terms of **a** and/or **b**,

(a) $\overrightarrow{DF}$, [1]

(b) $\overrightarrow{BF}$, [1]

(c) $\overrightarrow{BE}$. [1]

(ii) Make two statements about B, F and E. [2]

(iii) Find the numerical value of

(a) $\dfrac{\text{area of triangle } AFB}{\text{area of triangle } DCG}$, [1]

(b) $\dfrac{\text{area of triangle } AFB}{\text{area of quadilateral } FBGD}$. [1]

9 Answer the whole of this question on a sheet of graph paper.

The table below shows the number of residents (n) in Town S after t years.
The variables t and n are connected by the equation $n = 60(2^t) + 150$.

Year (t)	0	1	2	3	4	5	6	7
Number of residents (n)	210	270	390	630	1110	2070	k	7830

(a) Find the value of k. [1]

(b) Using a scale of 2 cm to represent 1 year, draw a horizontal axis for $0 \le t \le 7$.
Using a scale of 2 cm to represent 1000 residents, draw a vertical axis for $0 \le n \le 8000$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to estimate the number of years it took for the population of the town to reach 1000. [1]

(d) **(i)** By drawing a tangent, find the gradient of the curve at (5, 2070). [2]

(ii) What does this gradient represent? [1]

(e) The number of residents in another town, Town M is given by the equation $n = 7000 - 600t$ for $0 \le t \le 7$.

(i) On the same axes, draw a graph to represent the number of residents in Town M. [2]

(ii) Find the value of t when the two towns have the same number of residents. [1]

153

10 **(a)** A goodie bag contains 5 chocolates and 3 candy bars.
Ann takes an item randomly from the goodie bag and eats it.
Ben then takes an item randomly from the goodie bag.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

Find, as a fraction in its simplest form, the probability that

(ii) both of them take a candy bar, [1]

(iii) Ben takes a chocolate, [1]

(iv) one of the items taken is a chocolate. [1]

(b) Twenty students took part in a Mathematics quiz which has a maximum of 10 marks.
Their marks are shown in the frequency table below.

Mark	2	3	4	5	6	7	8	9	10
Frequency	1	1	0	2	5	3	1	4	3

(i) Write down the median mark. [1]

(ii) Calculate the mean mark. [1]

(iii) What is the passing mark of the quiz if 40% of the students passed? [1]

(iv) Calculate the standard deviation. [2]

(v) The same group of students took part in a Science quiz with the same maximum mark.
The mean mark for the Science quiz was 7.5 and the standard deviation was 2.02.
Make two comparisons between the performances of the students in these two quizzes. [2]

– End of Paper –

Name : ______________________ Register Number : ______

Class : ______

Clementi Town Secondary School
Preliminary Examination 2015
Secondary 4 Express / 5 Normal Academic

Mathematics
Paper 2

MARKING SCHEME

4016/2
14 September 2015
2 hours 30 minutes

Additional Materials provided: Answer Paper (7 sheets)
Graph paper (1 sheet)

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Calculators should be used where appropriate.
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Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Solve $(2x+1)^2 = 25$. [2]

(b) Simplify $\frac{x+2y}{4} + \frac{2x^2-8y^2}{8}$. [2]

(c) It is given that $z = \sqrt{\frac{x}{3x-y}}$.

(i) Find z when $x = 3$ and $y = 1.5$. [1]

(ii) Express x in terms of y and z. [2]

(d) Express $2 - \frac{4f-g}{f+2g}$ as a single fraction in its simplest form. [2]

1 **(a)** $(2x+1) = \pm\sqrt{25}$

$x = \frac{5-1}{2}$ or $\frac{-5-1}{2}$

$x = 2$ or -3 [B2 – 1 mark for each correct answer]

1 **(b)** $\frac{x+2y}{4} + \frac{2(x-2y)(x+2y)}{8}$ [M1 – factorise]

$= \frac{x+2y}{4} \times \frac{8}{2(x-2y)(x+2y)}$

$= \frac{1}{x-2y}$ [A1]

1 **(c)** **(i)** $z = 0.632$ [B1]

(ii) $z^2 = \frac{x}{3x-y}$

$3z^2x - z^2y = x$

$3z^2x - x = z^2y$ [M1 – grouping like terms]

$x(3z^2-1) = z^2y$

$x = \frac{z^2y}{3z^2-1}$ [A1]

1 **(d)** $\frac{2f+4g-(4f-g)}{f+2g}$ [M1 – combine as a single fraction]

$= \frac{5g-2f}{f+2g}$ [A1]

154

2 (a) Lenny bought a brand new car at a selling price of $109 000.
He paid 40% of the selling price in cash and took a 5-year loan for the rest of the amount.
If the bank charges 2.8% of simple interest per annum for the loan, calculate

(i) the total interest payable, [2]

(ii) the amount Lenny has to pay per month for the loan. [2]

(b) Lenny's new car uses fuel at an average rate of 9.5 litres per 100 km driven.
In an average year, Lenny drives 15 000 km.
The retail price of fuel is $2.25 per litre and Lenny has a loyalty card that gives him 10% discount off the retail price.
Assuming the price of fuel remains the same, work out the amount Lenny would expect to spend on fuel in one year, correct to the nearest dollar. [2]

(c) The original value of the car is its selling price of $109 000.
For each of the first five years, the value of the car decreases by 13% of its value at the start of every year.
After three years, Lenny decides to sell the car.
Calculate the overall percentage reduction in the value of the car compared with its original value, giving your answer to 2 decimal places. [3]

2 (a) (i) $\text{Interest} = \$\dfrac{(0.6\times 109\,000)\times 2.8\times 5}{100}$ [M1 – simple interest]

$= \$9156$ [A1]

(ii) $\text{Monthly payment} = \$\dfrac{(0.6\times 109\,000)+9156}{5\times 12}$ [M1 – instalment]

$= \$1242.60$ [A1]

2 (b) Amount of fuel required in a year $= 9.5\times 150 = 1425$ litres
Price of 1 litre of fuel after discount $= \$2.025$ } [M1 – either one]
Amount of money spent on fuel in a year $= \$2\,885.625 \approx \$2\,886$ [A1]

2 (c) $109\,000\times(0.87)^3 = 71776.827$ [M1 – depreciation]

$\dfrac{109\,000-71776.827}{109\,000}\times 100\%$ [M1 – find % change]

$\approx 34.15\%$ (to 2 d.p.) [A1]

3

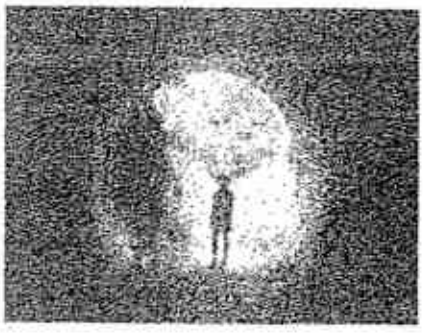


Diagram I

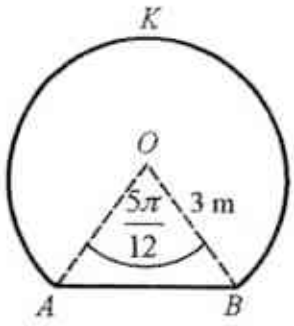


Diagram II

A 20-metre long tunnel can be found along a hiking trail, as shown in Diagram I.
The cross-section of this tunnel is a segment AKB of a circle centre O and radius 3 m, as shown in Diagram II.

It is given that angle $AOB = \frac{5\pi}{12}$.

(a) Calculate

(i) the length of AB, [2]

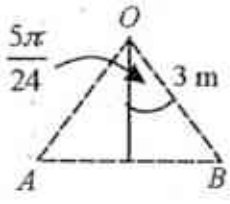
(ii) the perimeter of the segment AKB, [2]

(iii) the area of the segment AKB. [3]

(b) During a construction project, the tunnel is to be permanently sealed off by filling it with cement.
Work out the total cost of cement required to completely fill the tunnel, assuming that the cost of cement is \$30 per cubic metre. [2]

3 (a) (i) $AB = 2\left(3\sin\frac{5\pi}{24}\right)$ [M1 – use trigo ratio]

≈ 3.65 m (3 s.f.) [A1]



OR by cosine rule

3 (a) (ii) Perimeter $= 3\left(2\pi - \frac{5\pi}{12}\right) + AB$ [M1 – find arc length]

≈ 18.6 m (3 s.f.) [A1]

3 (a) (iii) Area of minor segment $= \frac{1}{2}(3)^2\left(\frac{5\pi}{12} - \sin\frac{5\pi}{12}\right)$ [M1]

≈ 1.54382 m^2

Area of major segment $AKB = \pi(3)^2 - 1.54382$ [M1]

≈ 26.7305 m^2

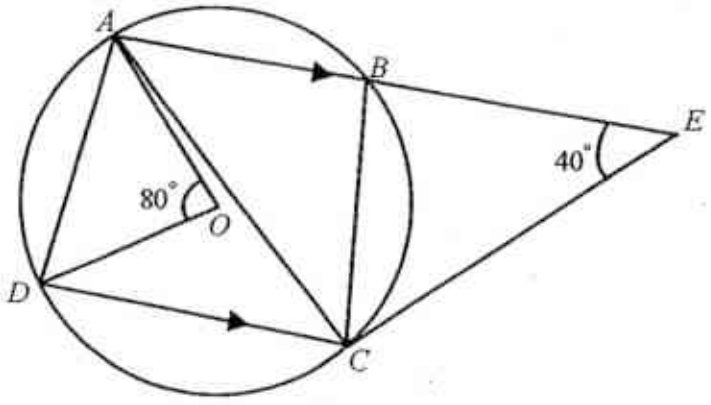
≈ 26.7 m^2 (3 s.f.) [A1]

3 (b) Volume of tunnel $= 26.7305 \times 20 = 534.61$ m^3 [M1]

Total cost of cement $= 30 \times 534.61 \approx$ \$16000 (3 s.f.) or \$16038.30 [A1]

55

4



(a) In the diagram above, $ABCD$ is a cyclic quadrilateral in a circle with centre O.
ABE is a straight line which is parallel to DC.
Given that angle $AEC = 40°$ and angle $AOD = 80°$, find

(i) angle DAO, [1]

(ii) angle ACD. [1]

(b) Given further that angle $ODC = 30°$, find angle OAC. [1]

(c) By showing your working and stating all the reasons clearly,

(i) prove that triangle ACE is isosceles, [2]

(ii) show that triangle BCE and triangle DAC are similar. [2]

(d) A point, F, is on the same side of AD as O, such that angle AFD is 30°.

Determine whether the point F lies on the circumference of the circle, inside the circle, or outside the circle.
Give a reason for your answer. [2]

4 (a) (i) $\angle DAO = \dfrac{180° - 80°}{2} = 50°$ (isos. triangle) [A1 – with reason]

4 (a) (ii) $\angle ACD = \dfrac{1}{2}(80°) = 40°$ ($\angle$ at centre = 2 $\angle$ at circumf) [A1 – with reason]

4 (b) $\angle OAC = 180° - 50° - 50° - 40° - 30° = 10°$ (sum of $\angle$s in a triangle) [A1 – with reason]

4 (c) (i) $\angle EAC = \angle ACD = 40°$ (from (a)(ii); alt. $\angle$s, $AE // DC$)
Hence $\angle EAC = 40°$ [M1]
Since $\angle EAC = \angle AEC$
Hence, triangle ACE is isosceles. [A1]

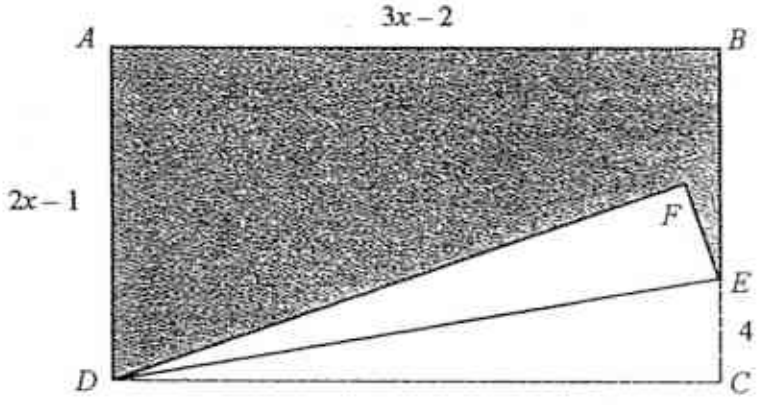
4 (c) (ii) $\angle CBE = \angle ADC$ (ext $\angle$ of cyclic quad)
$\angle BEC = \angle DCA = 40°$ (from (a)) } [A1 – with reason]

By AA similarity test, triangle BCE and triangle DAC are similar. [A1]

4 **(d)** F lies outside the circle. [B1]

If F lies on the circumference, $\angle AFD = 40°$.
If F lies inside the circle, $40° < \angle AFD < 80°$. ($\angle$ at centre = 2 $\angle$ at circumf)
Hence F must lie outside the circle [A1 – with reason]

5



A rectangular piece of paper $ABCD$ is folded along the line ED such that C is moved to F. The length of AB is $(3x - 2)$ cm, AD is $(2x - 1)$ cm and EC is 4 cm.

(a) Write down an expression, in terms of x, for
(i) area of rectangle $ABCD$, [1]
(ii) area of the triangle DEF. [1]

(b) Given that the area of the shaded region $ABEFD$ is 184 cm^2, form an equation in x and show that it reduces to $6x^2 - 19x - 174 = 0$. [2]

(c) Solve the equation $6x^2 - 19x - 174 = 0$, giving the answers correct to three decimal places. [3]

(d) Find the perimeter of the folded piece of paper. [2]

5 **(a)** **(i)** $(3x-2)(2x-1) = 6x^2 - 7x + 2$ [B1]

5 **(a)** **(ii)** $\frac{1}{2}(4)(3x-2) = 6x - 4$ [B1]

5 **(b)** $6x^2 - 7x + 2 - 2(6x - 4) = 184$ [M1]
$6x^2 - 19x - 174 = 0$ [A1]

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5 (c) $x = \dfrac{-(-19) \pm \sqrt{(-19)^2 - (4)(6)(-174)}}{(2)(6)}$ [M1 – quadratic formula]

$x = 7.196$ or -4.030 [A2 – 1 mark for each correct answer]

5 (d) $DC = 3x - 2 = 19.588$

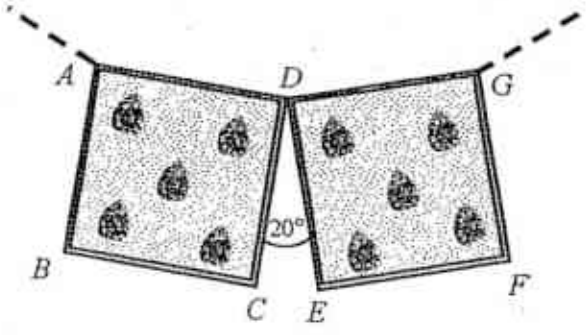
$DE = \sqrt{19.588^2 + 4^2} = 19.992$ cm [M1 – find DE by Pythagoras Theorem]

Perimeter $= 3x - 2 + 2x - 1 + 2x - 5 + 19.992$

$= 7x + 11.992$

≈ 62.4 cm [A1]

6



(a) The diagram above shows part of the design of a necklace.
AD and DG are sides of a regular polygon.
$ABCD$ and $DEFG$ are squares, while angle $CDE = 20°$.

(i) Find the total number of sides of this regular polygon. [3]

(ii) Calculate angle AGF. [2]

6 (b) The first five terms in a sequence of numbers, $T_1, T_2, T_3, T_4, T_5, \ldots$ are given by $-9, -4, 1, 6, 11, \ldots$.

(i) Find T_6 and T_7. [1]

(ii) Is it possible for the terms beyond T_5 to be negative?
Explain your answer. [1]

(iii) Find an expression, in terms of n, for T_n. [2]

(iv) Evaluate T_{150}. [1]

6 (a) (i) Interior angle of polygon ($\angle ADG$) $= 360^\circ - 90^\circ - 90^\circ - 20^\circ$

$= 160^\circ$ [M1 – angles at a point]

$\therefore$ Exterior angle $= 180^\circ - 160^\circ = 20^\circ$

No. of sides $= \dfrac{360}{20}$ [M1 – int or ext $\angle$ of polygon method]

$= 18$ [A1]

6 (a) (ii) $\angle AGD = \dfrac{180^\circ - 160^\circ}{2} = 10^\circ$ [M1 – isos Δ]

$\angle AGF = 10^\circ + 90^\circ = 100^\circ$ [A1]

6 (b) (i) $T_6 = 16, T_7 = 21$ [B1]

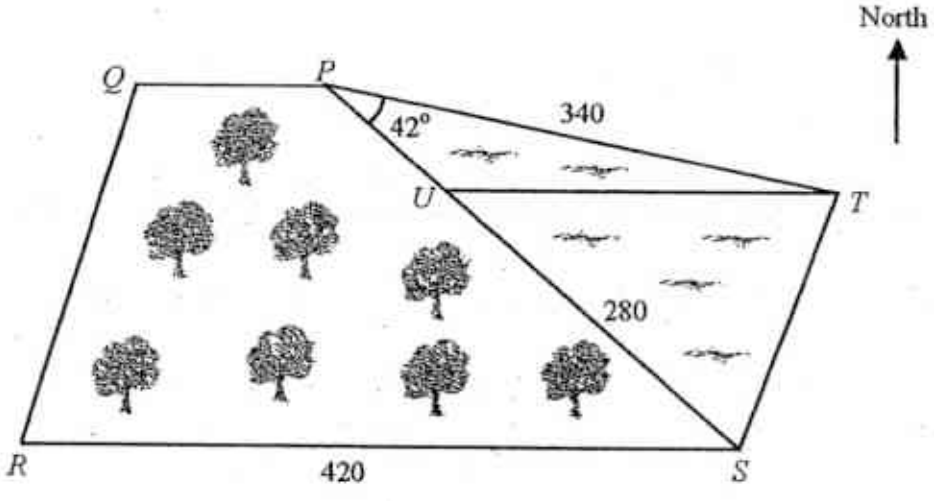
6 (b) (ii) No. Every term in this sequence is larger than the previous by 5. Since the terms after 2nd term is positive, any of the terms after the 5th term can never be negative. [B1]

6 (b) (iii) $T_n = -9, -4, 1, 6, 11, \ldots.$
$T_n + 14 = 5, 10, 15, 20, 25, \ldots.$ } [M1 – recognise the pattern]
$T_n + 14 = 5n$

$T_n = 5n - 14$ [A1]

6 (b) (iv) $T_{150} = 5(150) - 14 = 736$ [B1]

7



The diagram shows a large garden, $PQRS$, which overlooks a triangular pond, PST.
A bridge is built across the pond from U to T, where U is a point on PS and T is due east of U.
Angle $TPU = 42^\circ$ and the bearing of P from T is 290°.
$PT = 340$ m, $SU = 280$ m and $RS = 420$ m.

(a) Calculate

(i) angle PTU, [1]

(ii) the bearing of S from P. [2]

(b) After a stroll in the garden, Joe stops at U.

Suppose the entrance to the garden is at P, how far is Joe away from the entrance? [2]

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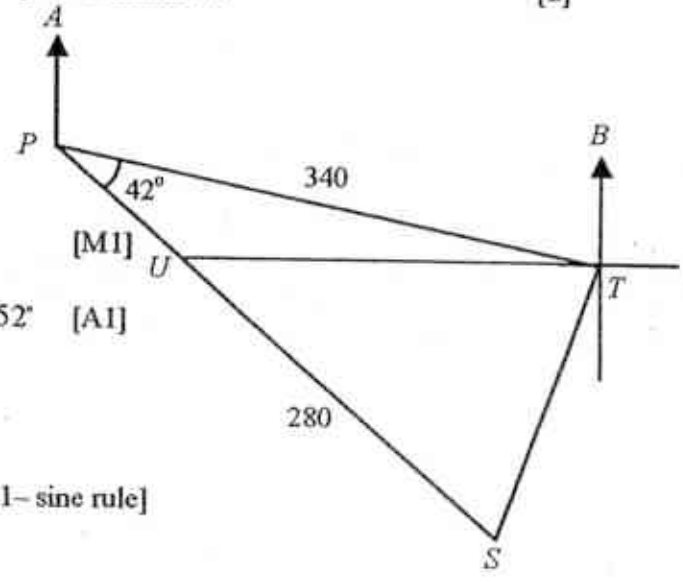
(c) Calculate the area of the triangular pond, PST. [2]

(d) During a renovation of the garden, the original bridge is replaced by a new bridge that would join PS to T by the shortest distance.
Calculate the length of this new bridge. [2]

(e) When the pond appears on an accurate map, $PT = 17$ mm.
Find the area of the pond on this map in square millimetres. [2]



7 **(a)(i)** $\angle PTU = 290^\circ - 270^\circ = 20^\circ$ [B1]

7 **(a)(ii)** $\angle BTP = 360^\circ - 290^\circ = 70^\circ$

$\angle APT = 180^\circ - 70^\circ = 110^\circ$ [M1]

Bearing of S from P $= 110^\circ + 42^\circ = 152^\circ$ [A1]

7 **(b)** $\angle PUT = 180^\circ - 42^\circ - 20^\circ = 118^\circ$

$\dfrac{PU}{\sin 20^\circ} = \dfrac{340}{\sin 118^\circ}$ [M1– sine rule]

$PU = \dfrac{340}{\sin 118^\circ} \times \sin 20^\circ$

$= 131.703$

$= 132$ m (3 s.f.) [A1]

7 **(c)** Area of the pond $= \dfrac{1}{2}(340)(280 + 131.703)\sin 42^\circ$ [M1 – $\dfrac{1}{2}ab\sin c^\circ$]

$= 46832.12$

$= 46800$ m^2 (3 s.f.) [A1]

7 **(d)** Let x be the shortest distance from PS to T.

Area of the pond $= 46832.12$ m^2

$\dfrac{1}{2}(x)(280 + 131.703) = 46832.12$ [M1– area of triangle]

$x = 227.504 \approx 228$ m (3 s.f.) [A1]

7 **(e)** Ratio of length $= 340$ m : 17 mm $= 20$ m : 1 mm
Ratio of area $= 400$ m^2 : 1 mm^2 [M1– area of similar figures]

Area of pond on the map $= \dfrac{46832.12}{400} \approx 117$ mm^2 [A1]

8 **(a)** P is the point (2, 1).

The position vector of Q is $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$.

(i) Write down the column vector $\overrightarrow{PQ}$. [1]

(ii) Find the exact value of $\left|\overrightarrow{PQ}\right|$. [2]

(iii) Find the position vector of R, such that $\overrightarrow{QR} = 3\overrightarrow{QP}$. [1]

(a) **(i)** $\overrightarrow{OP} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \begin{pmatrix} -3 \\ 6 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$ [B1]

(a) **(ii)** $\left|PQ\right| = \sqrt{5^2 + 5^2}$ [M1]

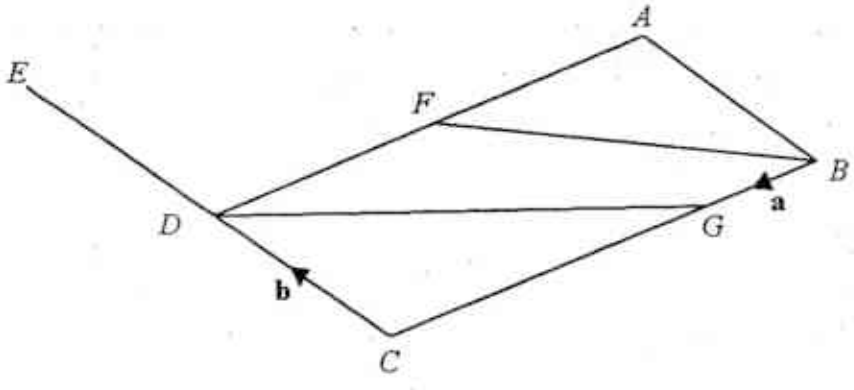
$= \sqrt{50}$ or $5\sqrt{2}$ units [A1]

(a) **(iii)** $\overrightarrow{QR} = 3\overrightarrow{QP}$

$\overrightarrow{OR} - \overrightarrow{OQ} = 3\left(\overrightarrow{OP} - \overrightarrow{OQ}\right)$

$\overrightarrow{OR} = 3\overrightarrow{OP} - 2\overrightarrow{OQ} = 3\begin{pmatrix} 2 \\ 1 \end{pmatrix} - 2\begin{pmatrix} -3 \\ 6 \end{pmatrix} = \begin{pmatrix} 12 \\ -9 \end{pmatrix}$ [B1]

(b)



$ABCD$ is a parallelogram and E lies on CD produced such that $CD = DE$.
F is the midpoint of AD and G is a point on BC such that $BG : GC = 1 : 3$.
Given $\overrightarrow{BG} = \mathbf{a}$ and $\overrightarrow{CD} = \mathbf{b}$,

(i) express, as simply as possible, in terms of **a** and/or **b**,

(a) $\overrightarrow{DF}$, [1]

(b) $\overrightarrow{BF}$, [1]

(c) $\overrightarrow{BE}$. [1]

(ii) Make two statements about B, F and E. [2]

(iii) Find the numerical value of

(a) $\dfrac{\text{area of triangle } AFB}{\text{area of triangle } DCG}$, [1]

(b) $\dfrac{\text{area of triangle } AFB}{\text{area of quadilateral } FBGD}$. [1]

(b) (i) (a) $\overrightarrow{DF} = -\overrightarrow{AF} = -\frac{1}{2}\overrightarrow{BC} = -\frac{1}{2}(4\mathbf{a}) = -2\mathbf{a}$ [B1]

(b) (i) (b) $\overrightarrow{BF} = \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DF} = 4\mathbf{a} + \mathbf{b} - 2\mathbf{a} = 2\mathbf{a} + \mathbf{b}$ [B1]

(b) (i) (c) $\overrightarrow{BE} = \overrightarrow{BC} + \overrightarrow{CE} = 4\mathbf{a} + 2\mathbf{b}$ [B1]

(b) (ii) $\therefore \overrightarrow{BE} = 2\overrightarrow{BF}$

(1) $BE = 2BF$ (2) B, F and E lie on the same straight line. [B2]

(b) (iii) (a) $\dfrac{\text{area of triangle } AFB}{\text{area of triangle } DCG} = \dfrac{2}{3}$ [B1]

(b) (iii) (b) $\dfrac{\text{area of triangle } AFB}{\text{area of parallelogram } ABCD} = \dfrac{1}{4} = \dfrac{2}{8}$

$\dfrac{\text{area of triangle } AFB}{\text{area of triangle } DCG} = \dfrac{2}{3}$ (from (a))

$\therefore \dfrac{\text{area of triangle } AFB}{\text{area of quadilateral } FBGD} = \dfrac{2}{3}$ [B1]

9 Answer the whole of this question on a sheet of graph paper.

The table below shows the number of residents (n) in Town S after t years.

The variables t and n are connected by the equation $n = 60(2^t) + 150$.

Year (t)	0	1	2	3	4	5	6	7
Number of residents (n)	210	270	390	630	1110	2070	k	7830

(a) Find the value of k. [1]

(b) Using a scale of 2 cm to represent 1 year, draw a horizontal axis for $0 \le t \le 7$.
Using a scale of 2 cm to represent 1000 residents, draw a vertical axis for $0 \le n \le 8000$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to estimate the number of years it took for the population of the town to reach 1000. [1]

(d) **(i)** By drawing a tangent, find the gradient of the curve at (5, 2070). [2]

(ii) What does this gradient represent? [1]

(e) The number of residents in another town, Town M is given by the equation $n = 7000 - 600t$ for $0 \le t \le 7$.

(i) On the same axes, draw a graph to represent the number of residents in Town M. [2]

(ii) Find the value of t when the two towns have the same number of residents. [1]

9 (a) $k = 60(2^6) + 150 = 3990$
[B1]

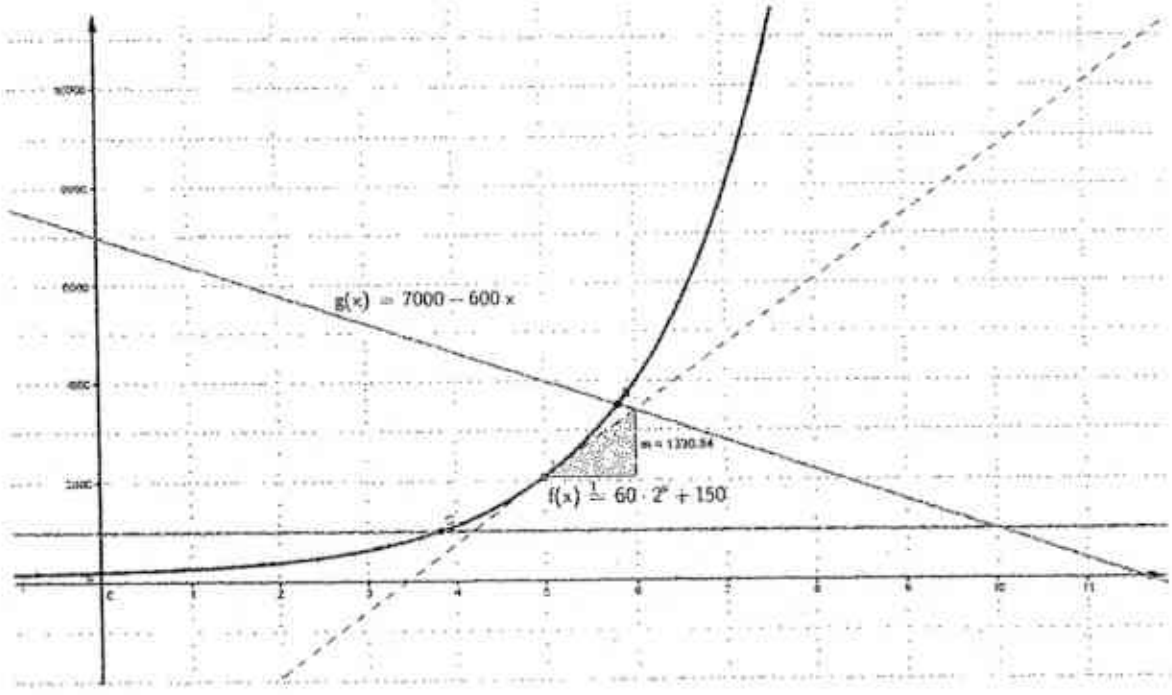
9 (b) Draw the axes with correct scale. [B1]
Plot all given points. [B1]
Draw a smooth curve through all plots. [B1]
(Refer to graph)

9 (c) $t = 3.8$ years (± 0.1) [B1]

9 (d) (i) Draw the tangent at $t = 5$. [M1]
Gradient = 1330 (± 100) [A1]
(ii) The gradient represents the rate at which the number of residents in the town is increasing at the instant when $t = 5$. [B1]

9 (e) (i) Plot at least 2 points from the given equation. [B1]
Draw a straight through all plots. [B1]
(Refer to graph)

9 (e) (ii) From the graph, $t = 5.8$ [B1]



10 (a) A goodie bag contains 5 chocolates and 3 candy bars.
Ann takes an item randomly from the goodie bag and eats it.
Ben then takes an item randomly from the goodie bag.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

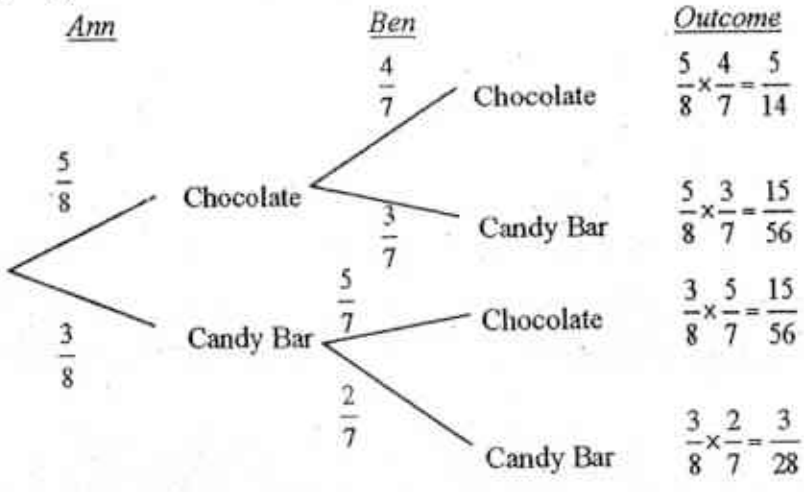
Find, as a fraction in its simplest form, the probability that

(ii) both of them take a candy bar, [1]

(iii) Ben takes a chocolate, [1]

(iv) one of the items taken is a chocolate. [1]

10 (a) (i)



[B2 for no errors, B1 for 1-2 errors, B0 for more than 2 errors]

10 (a) (ii) $\frac{3}{8} \times \frac{2}{7} = \frac{3}{28}$ [B1]

10 (a) (iii) $\frac{5}{8} \times \frac{4}{7} + \frac{3}{8} \times \frac{5}{7} = \frac{5}{8}$ [B1]

10 (a) (iv) $\frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{5}{7} = \frac{15}{28}$ [B1]

10 (b) Twenty students took part in a Mathematics quiz which has a maximum of 10 marks. Their marks are shown in the frequency table below.

Mark	2	3	4	5	6	7	8	9	10
Frequency	1	1	0	2	5	3	1	4	3

(i) Write down the median mark. [1]

(ii) Calculate the mean mark. [1]

(iii) What is the passing mark of the quiz if 40% of the students passed? [1]

(iv) Calculate the standard deviation. [2]

(v) The same group of students took part in a Science quiz with the same maximum mark.
The mean mark for the Science quiz was 7.5 and the standard deviation was 2.02.
Make two comparisons between the performances of the students in these two quizzes. [2]

10 (b) (i) Median = 7 [B1]

10 (b) (ii) Mean $= \frac{2\times1+3\times1+5\times2+6\times5+7\times3+8\times1+9\times4+10\times3}{20} = 7$ [B1]

10 (b) (iii) Passing mark = 8 [B1]

10 (b) (iv) Standard deviation $= \sqrt{\frac{2^2\times1+3^2\times1+5^2\times2+6^2\times5+7^2\times3+8^2\times1+9^2\times4+10^2\times3}{20} - 7^2}$
$= 2.21$ (3 s.f.) [M1] [A1]

10 (b) (v) - The students generally performed **better** for the Science Quiz as the mean mark for the Science Quiz is higher than the mean mark for the Mathematics Quiz.
- The marks for the Science Quiz are more **consistent** than the marks for the Mathematics Quiz. [B2]

– End of Paper –

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Answer all the questions.

1 (a) The selling price of a bedroom set after a 25% discount is $4 800.

(i) Find the original price of the bedroom set. [1]

(ii) John bought the bedroom set at the selling price of $4800 on hire purchase. He paid a 10% deposit and paid the remaining amount in monthly instalments of $210 per month. If the shop charges a simple interest of 25% per annum, find the number of years John will take to repay his loan. [2]

(b) Mrs Lim deposits $20 000 into a bank that pays a compound interest of x% per year. Find the value of x if she receives $6 898 in interest after 12 years. [3]

(c) The exchange rate between Singapore and the United States is US$1 = SG$1.35. Susan bought 5 handbags from the US that cost US$320 each and sold the bags for SG$500 each. Find the percentage of the total profit made. [2]

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2 (a) Make s the subject of the formula $2rs = \frac{\sqrt{3s^4+2r^2}}{7}$. [3]

(b) (i) Expand and simplify $\left(p - \frac{1}{p}\right)^2$. [1]

(ii) It is given that $p^2 + \frac{1}{p^2} = 11$. Hence or otherwise, find the values of $\frac{1}{3}\left(p - \frac{1}{p}\right)$. [3]

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3 A solid metal hemisphere of radius 15 cm is melted to form some solid identical cones and solid identical spheres with radius 2.5 cm

(a) Given that each cone and each sphere have the same volume, find the height of each cone. [2]

(b) Find the number of spheres formed if the number of spheres is twice the number of cones. [2]

(c) Calculate the total surface area of each cone. [2]

(d) The surfaces of the cones are painted red. A tin of 200 ml of paint that costs $3.50 can paint an area of 450 cm^2. Calculate the total cost of paint needed to paint all the cones. [2]

---

4 Diagram I shows a semi-circular drain $ABCD$ of length 5 m. On a rainy day, the drain is filled with water to a height of 0.7 m. Diagram II shows the cross-section of the drain when it is filled with water. The cross-section has centre O and a diameter of 2.5 m.

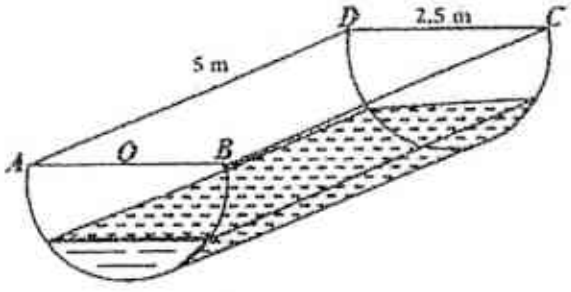


Diagram I

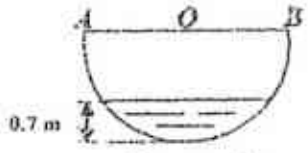


Diagram II

(a) Calculate the curved surface area of the drain that is in contact with water. [4]

(b) Find the total volume of the water in the drain. [3]

---

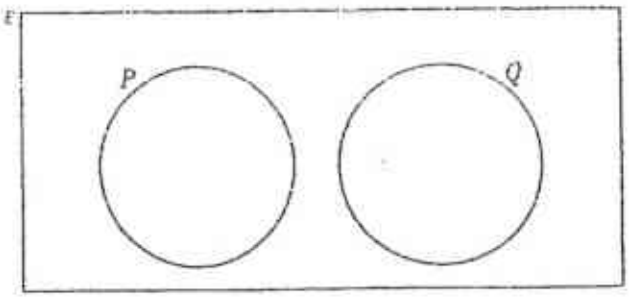
5 (a) A universal set is given as $\varepsilon = \{x : x \text{ is an even integer less than } 20\}$.
It is given that $A = \{x : x \text{ is a multiple of } 8\}$ and $B = \{x : x \text{ is a multiple of } 4\}$.

(i) Using set notation, write down the relationship between set A and set B. [1]

(ii) Hence, represent sets ε, A and B in a Venn Diagram, showing all the elements clearly. [2]

(iii) On your Venn Diagram, shade the region $A' \cap B$. [1]

(b) Refer to the Venn Diagram below and answer the questions that follow.

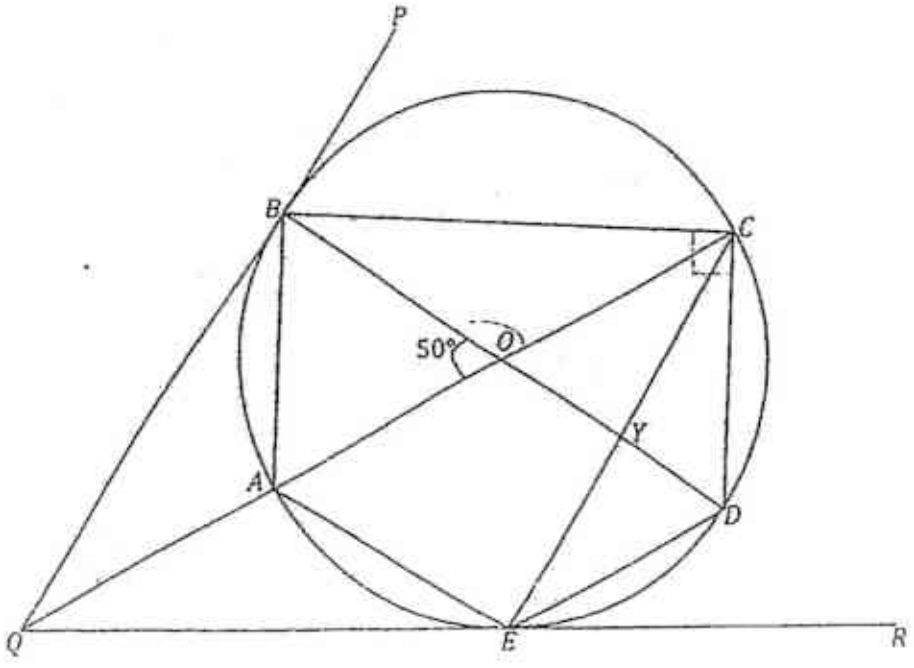


(i) Find $P \cap Q$. [1]

(ii) Write down the set notation for the shaded region. [1]

(iii) If $\varepsilon = \{t, u, v, w, x, y, z\}$, $P = \{u, w, y\}$ and $x \notin Q$, list all the possible elements in Q. [1]

6



In the diagram above, $ABCDE$ are on the circumference of a circle with centre O. QP and QR are tangents to the circle at B and E respectively. AC and CE intersect BD at O and Y respectively and $QAOC$ is a straight line.

(a) It is given that $\angle BOA = 50°$. Showing your reasons clearly, calculate

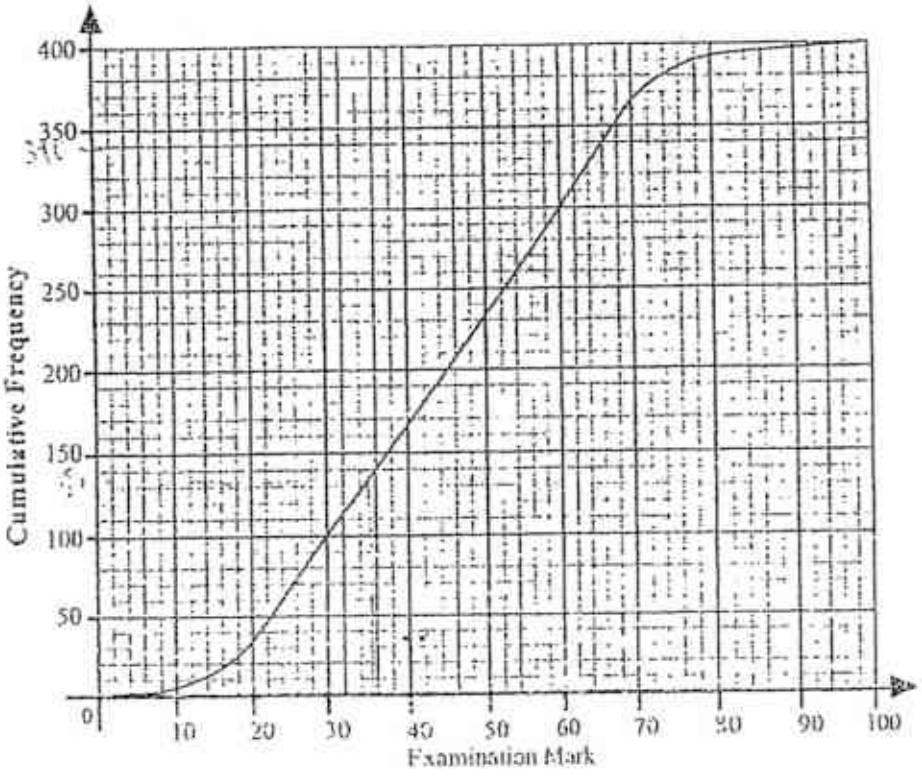
(i) $\angle BCA$, [1]

(ii) $\angle AQE$, [1]

(iii) $\angle CED$. [2]

(b) Given that $\angle BXD = 100°$, explain clearly whether point X is in the circle, on the circumference or outside the circle. [2]

7 The diagram below shows the cumulative frequency curve for the marks of 400 pupils who sat for a Mathematics Examination.



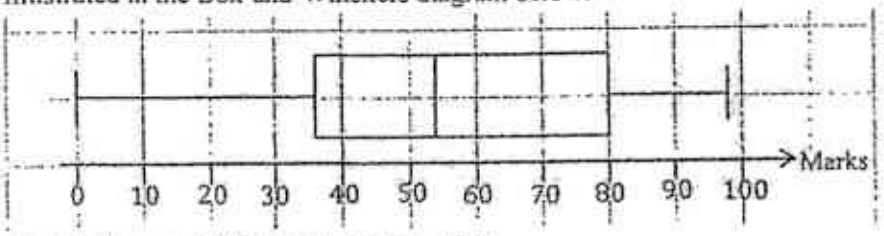
(a) Use your graph to find

(i) the median mark, [1]

(ii) the interquartile range. [1]

(b) Two students were selected at random. Find the probability that one of the students obtained less than 35 marks and the other obtained more than 66 marks. [2]

(c) The same group of students sat for a Chemistry Examination and the results are illustrated in the Box-and-Whiskers diagram below.



Use the Box-and-Whiskers diagram to find

(i) the median mark, [1]

(ii) the interquartile range. [1]

(d) Compare the marks obtained by the 400 students for the Mathematics Examination and Chemistry Examination in two different ways. [2]

---

8 Consider the following sequence

Line 1	$1^3 + 2^3 = 3^2 = \left(\frac{2 \times 3}{2}\right)^2$
Line 2	$1^3 + 2^3 + 3^3 = 6^2 = \left(\frac{3 \times 4}{2}\right)^2$
Line 3	$1^3 + 2^3 + 3^3 + 4^3 = 10^2 = \left(\frac{4 \times 5}{2}\right)^2$
Line 4	... ...

(a) (i) Write down the 4^{th} line of the sequence. [1]

(ii) Write down in terms of n, the sum of $1^3 + 2^3 + 3^3 + \ldots\ldots + n^3$. [1]

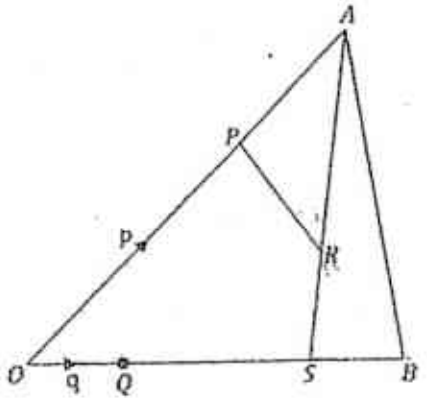
(b) Use your result in (a) to find the sum of

(i) $3^3 + 4^3 + 5^3 + 6^3 + \ldots\ldots + 50^3$ [2]

(ii) $2^3 + 4^3 + 6^3 + 8^3 + \ldots\ldots + 50^3$ [2]

---

9 In the diagram below, $\overrightarrow{OP} = \mathrm{p}$ and $\overrightarrow{OQ} = \mathrm{q}$. It is given that $\overrightarrow{OP} = \frac{2}{3}\overrightarrow{OA}$, $\overrightarrow{OQ} = \frac{1}{3}\overrightarrow{OS}$, $SB = OQ$ and $3SR = SA$.



(a) Express as simply as possible in terms of p and q.

(i) $\overrightarrow{SA}$. [1]

(ii) $\overrightarrow{PB}$. [1]

(iii) $\overrightarrow{PR}$. [2]

(b) State 2 facts about $\overrightarrow{PR}$ and $\overrightarrow{PB}$. [2]

(c) Find the numerical value of

$$\frac{\text{Area of triangle } RSB}{\text{Area of triangle } APR}.$$ [2]

10 Two language schools conduct courses for Korean, Japanese and Chinese Language. The number of students for each language course in May and June are given in the table below.

	May			June		
	Korean	Japanese	Chinese	Korean	Japanese	Chinese
School X	150	220	116	200	250	136
School Y	90	5	10	351	308	457

The number of students for each language course in both schools in May can be represented by the matrix

$$\mathbf{M} = \begin{pmatrix} 150 & 220 & 116 \\ 90 & 5 & 10 \end{pmatrix}$$

Similarly, the number of students for each language course in both schools in June can be represented by the matrix **J**.

(a) Write down the matrix **J**. [1]

(b) Evaluate $\mathbf{T} = \frac{1}{2}(\mathbf{M} + \mathbf{J})$. [1]

(c) State what the elements of **T** represent. [1]

The course fees were \$200, \$180 and \$100 for Korean, Japanese and Chinese language courses respectively.

(d) Write down a matrix **S** such that the elements of **MS** represent the total course fees collected by each school in the month of May
Hence, evaluate **MS**. [2]

---

11 A car travelled from town X to town Y, which were 320 km apart, at an average speed of x km/h. A van travelled on the same road from town Y to town X at an average speed which was 5 km/h slower than the speed of the car.

(a) Write down an expression in terms of x, for the time taken by the car to travel from town X to town Y. [1]

(b) Write down an expression in terms of x, for the time taken by the van to travel from town Y to town X. [1]

(c) Given that the difference in time taken for both vehicles to reach their respective destination is 15 minutes, form an equation in x and show that it reduces to

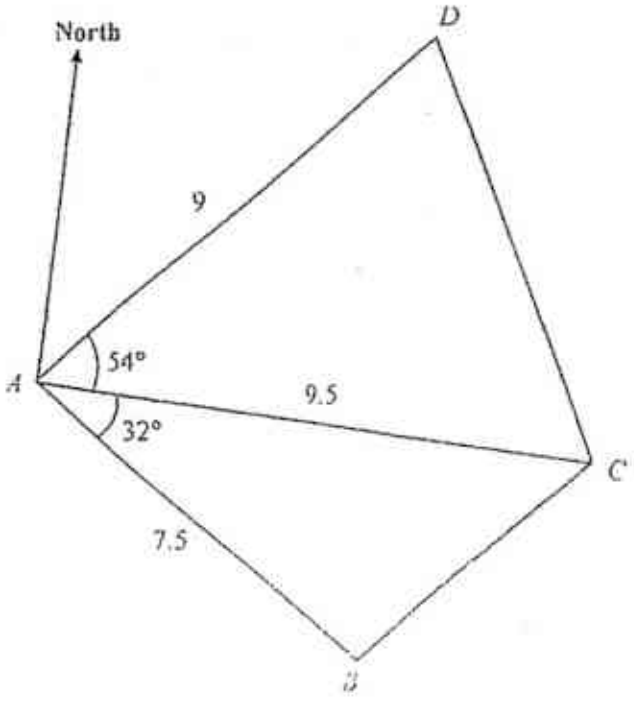
$$x^2 - 5x - 6400 = 0.$$ [3]

(d) Solve the equation $x^2 - 5x - 6400 = 0$, giving your answer correct to 2 decimal places. [2]

(e) Hence, find in hours and minutes, the time when the two vehicles meet. [3]

---

12 A, B, C and D are points on level ground such that C is due east of A. It is given that $AB = 7.5$ m, $AC = 9.5$ m, $AD = 9$ m, $\angle DAC = 54°$ and $\angle CAB = 32°$.



(a) Calculate

(i) angle ABC, given that angle ABC is an obtuse angle. [3]

(ii) the bearing of C from B. [2]

(b) A tree is at D. The angle of depression of C from the top of the tree is 25°.
Calculate the

(i) height of the tree, [3]

(ii) greatest angle of elevation of the top of the tree from a point along the line AB. [2]

13 **Answer the whole of this question on a piece of graph paper.**

The variables x and y are connected by the equation $y = x + \frac{6}{x} - 1$.

The table below shows some values of x and their corresponding values of y.

x	0.75	1	1.5	2	2.5	3	4	5	6	8
y	7.75	6	4.5	4	3.9	4	4.5	5.2	6	7.75

(a) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $0 \leq x \leq 8$.
Using a scale of 2 cm to 1 unit, draw a vertical y-axis for $0 \leq y \leq 8$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) Use your graph to find the minimum value of y. [1]

(c) (i) On the same axes, draw the graph of $x + 3y = 21$. [1]

(ii) Write down the x-coordinates of the points where the two graphs intersect. [1]

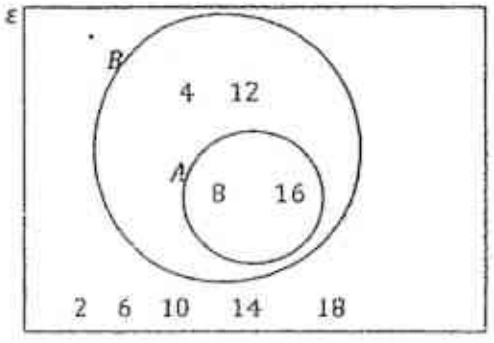
(iii) These values of x is a solution to the equation $ax^2 + bx + c = 0$, where a, b and c are integers. Find the values of a, b and c. [2]

(d) By drawing a suitable tangent to your curve, find the coordinates of the point at which the gradient of the curve is $-\frac{1}{2}$. [2]

END OF PAPER

Answer Key

1. (a)(i) \$6400 (ii) 3 (b) 2.50 (to 3 s.f.) (c) 15.7%
2. (a) $s = \pm\sqrt{\frac{2r^2}{196r^2-3}}$ (b)(i) $p^2 - 2 + \frac{1}{p^2}$ (ii) ± 1
3. (a) 10 cm (b) 72 (c) 101 cm^2 (d) \$31.50
4. 13.9 m^2 (b) 5.63 m^3
5. (a)(i) $A \subset B$ (ii)



(b) (i) $P' \cap Q = \emptyset$ or null set (ii) $(P \cup Q)'$ (iii) $Q = \{t, v, z\}$

6. (a)(i) 25° (ii) 40° (iii) 25° (b) As $\angle BXD > 90°$, by converse of right angle in a semicircle, X is inside the circle.
7. (a)(i) 45 (ii) 30 (b) $\frac{27}{266}$ (c)(i) 54 (ii) 44
   (d) Overall, the students did better in Chemistry as compared to Mathematics as the median for Chemistry is greater than the median for Mathematics. The marks for Chemistry is more spread out as compared to Mathematics as the interquartile range for Chemistry is larger.
8. (a)(i) $1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 15^2 = \left(\frac{5\times6}{2}\right)^2$ (ii) $1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$ (b)(i) 1 625 616 (ii) 845 000
9. (a)(i) $-3q + \frac{3}{2}p$ (ii) $-p + 4q$ (iii) $2q - \frac{1}{2}p$ (b) P, R and B are collinear (c) ½
10. (a) $J = \begin{pmatrix} 200 & 250 & 136 \\ 351 & 303 & 457 \end{pmatrix}$ (b) $\begin{pmatrix} 175 & 235 & 126 \\ 220.5 & 156.5 & 233.5 \end{pmatrix}$
    (c) T represents the average number of students taking each language lessons for the months of May and June in School X and Y respectively. (d) $\begin{pmatrix} 81\,200 \\ 19\,900 \end{pmatrix}$
11. (a) $\frac{320}{x}$ (b) $\frac{320}{x-5}$ (d) $x = 82.54$ or $x = -77.54$ (e) 2 h 0 min
12. (a)(i) 106.7° (ii) 048.7° (b)(i) 3.92 m (ii) 23.6°
13. (b) $y = 3.9$ (c)(ii) 0.9 or 5.15 (iii) $a = 2, b = -12, c = 9$ (d) (2,3)

Answer all the questions.

1 The numbers 756 and 1200 written as the products of their prime factors are:

$756 = 2^2 \times 3^3 \times 7$

$1200 = 2^4 \times 3 \times 5^2$

Find

(a) the smallest integer n if $756n$ is a multiple of 1200.

(b) the smallest integer m if $1200m$ is a perfect cube.

Answer (a) $n =$ ..... [1]

(b) $m =$ ..... [1]

2 Some boys were seated around a circular table. A bag containing 51 sweets was passed around the table, starting from Tim. Each boy took the same number of sweets. When the bag reached Tim again, there was only 3 sweets left. If Tim took one more sweet and had x sweets altogether, express the number of boys seated around the table in terms of x.

Answer ..... [1]

3 A tank was filled with water. Mary poured out $\frac{1}{2}$ of the water from the tank into another container. She then poured out $\frac{1}{3}$ of the remaining water in the tank into another container. She then poured out $\frac{1}{4}$ of the remaining water in the tank into another container and so on. How many times would she need to pour out the water from the tank so that the remaining water in the tank is exactly $\frac{1}{15}$ of the original amount?

Answer ..... [2]

4 A rectangular lake with an actual area of 13.5 km^2 is represented as an area of c cm^2 on a map.

(a) Express the scale of the map in the form 1 : n, where n is an integer.

(b) The lake is represented on another map with a scale of 1 : 100 000. If the breadth of the lake is 2 cm on the second map, find the actual length of the lake in kilometres

Answer (a) 1 : ..... [2]

(b) Actual length = ... km [2]

5 It takes 8 workers to complete a job. The job can be completed 4 days earlier if 2 more workers are added. How many workers will be needed if the job needs to be completed 10 days earlier?

Answer ..... [2]

---

6 The mass of an oxygen atom is 2.66×10^{-20} milligrams and the mass of a hydrogen atom is 1.67×10^{-12} picograms

(a) Find the ratio of the mass of an oxygen atom to the mass of a hydrogen atom is its simplest form.

(b) Find the total mass in kilograms of one oxygen atom and two hydrogen atoms, giving your answer in standard form.

Answer (a) ..... : ..... [1]

(b) ..... kg [2]

---

7 (a) Factorise completely $12a^2x - b^2y + 4a^2y - 3b^2x$.

(b) It is given that $12a^2x - b^2y + 4a^2y - 3b^2x = 0$. Hence, find the values of a if $b = 7, x = a$ and $y = 10$.

Answer (a) ..... [3]

(b) a = ... [2]

---

8 Solve the following pair of simultaneous equations.

$$2^x \times 16^y = 8$$

$$125^y \div 5^{-x} = \frac{1}{625}$$

Answer x = ... and y = ... [4]

---

9 An unbiased coin and a six-sided fair die with numbers 1 to 6 are tossed. Represent the sample space using a possibility diagram. [1]

(a) Find the probability of getting a head on the coin and a number less than 5 on the die.

(b) Sam and Tim decided to play a game. If a head is obtained from the coin, 2 points will be awarded. If a tail is obtained from the coin, 1 point will be deducted. The total points is calculated by adding the number obtained on the die with the points obtained from the tossing of the coin.
Find the probability of
(i) obtaining a total of 4 points,
(ii) obtaining more than 5 points.

Answer (a) ..... [1]

(b) (i) ..... [1]

(ii) ..... [1]

10 (a) Solve the inequality $\frac{1}{5}(5x - 29) \leq \frac{30-2x}{2} < 9$.

(b) Represent the solution on the number line below.

(c) If x is a factor of 98, find the value of x.

[1]

Answer (a) ..... [3]

(c) x = ... [1]

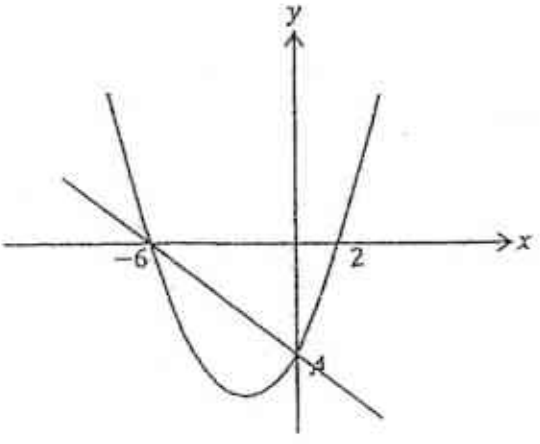
---

11 y is inversely proportional to the square of x and $y = p$ for a particular value of x. Express the value of y in terms of p when this value of x is halved.

Answer ..... [3]

---

12 The diagram below shows a curve of $y = a(x + h)^2 - 5$. The curve cuts the x-axis at -6 and 2 and the y-axis at A. B is the minimum point of the curve.



(a) Express the equation of the curve in the form of $y = a(x + h)^2 - 5$, where a and h are constants

(b) A straight line cuts the curve at $x = -6$ and point A. Find the equation of the straight line.

Answer (a) Equation of curve: $y =$ ..... [3]

(b) Equation of straight line: $y =$ ..... [2]

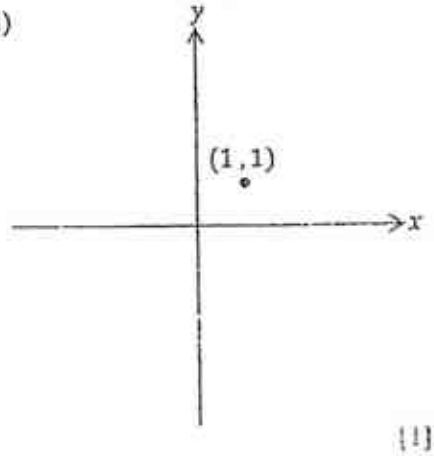
13 In the axes provided below, the point (1, 1) has been marked. Indicating clearly any intercepts, sketch the graph of

(a) $yx = 2x - 1$

(b) $2y + x = 4$

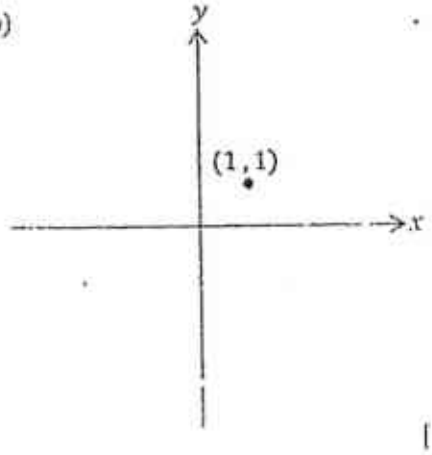
Answer

(a)



[1]

(b)



[1]

14 The equation of a curve can be expressed as $y = ax^n$. State the possible value of a and of n in each case.

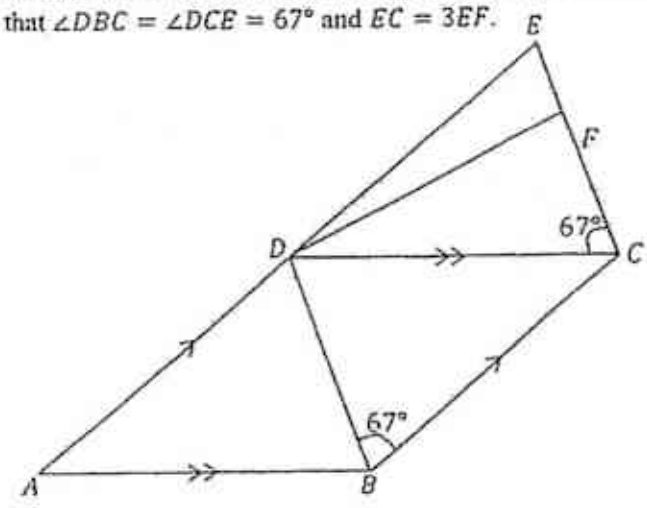
(a) y is the volume of a hemisphere with radius x.

(b) y is the acceleration of a car that has travelled a distance of 100 km in x hours.

Answer (a) $a =$ ..... $n =$ ..... [1]

(b) $a =$ ..... $n =$ ..... [1]

15 In the diagram below, $ABCD$ is a rhombus. AD is produced to point E and EFC is a straight line. It is given that $\angle DBC = \angle DCE = 67°$ and $EC = 3EF$.



(a) Name one triangle which is congruent to triangle EDC. Explain your reasons clearly.

(b) Find the numerical value of

(i) $\frac{\text{Area of } \Delta EDF}{\text{Area of } \Delta EDC}$,

(ii) $\frac{\text{Area of } \Delta DFC}{\text{Area of quadrilateral } ABCD}$.

Answer

(a) .....

.....

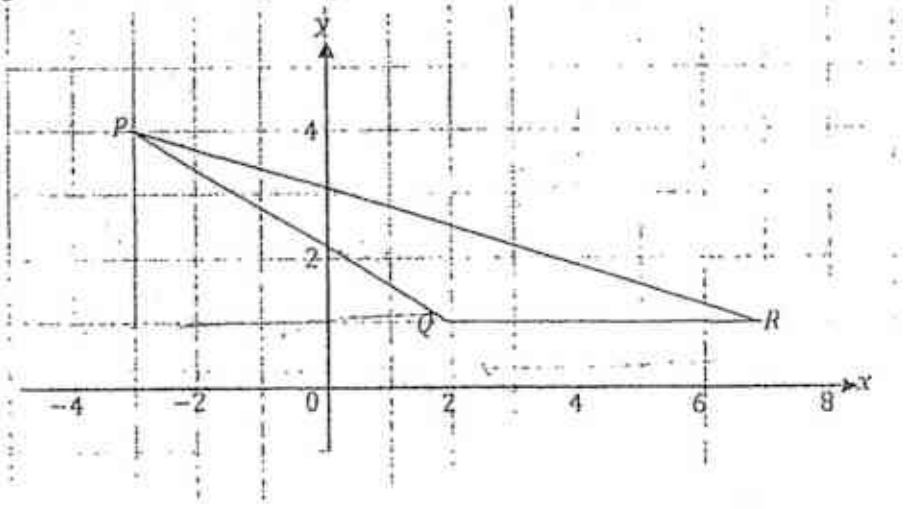
.....

..... [3]

(b) (i) ..... [1]

(ii) ..... [2]

16 The figure below shows triangle PQR with vertices $P(-3, 4), Q(2, 1)$ and $R(7, 1)$.



(a) Find the exact value of

(i) $\tan \angle PQR$,

(ii) $\sin \angle PRQ$.

(b) Hence, find the exact value of the perpendicular distance from Q to PR.

(c) $PQRS$ is a trapezium with an area that is 3 times the area of triangle PQR and PS is parallel to QR. Find the coordinates of point S.

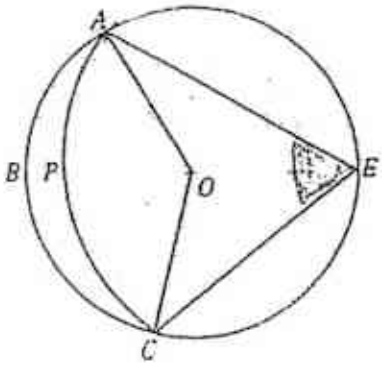
Answer (a) (i) ..... [1]

(ii) ..... [1]

(a) Perpendicular distance = ... units [2]

(b) Coordinates of S = ... [1]

17 In the figure below, $ABCE$ lies on the circumference of a circle with centre O and radius 10 cm. APC is the minor arc of a circle with centre E and radius 15 cm.



(a) Show that $\angle AEC = 1.445$ radians, correct to 3 decimal places. [2]

(b) Find $\angle AOC$ in radians.

(c) Find the area of the shaded region $AOCP$.

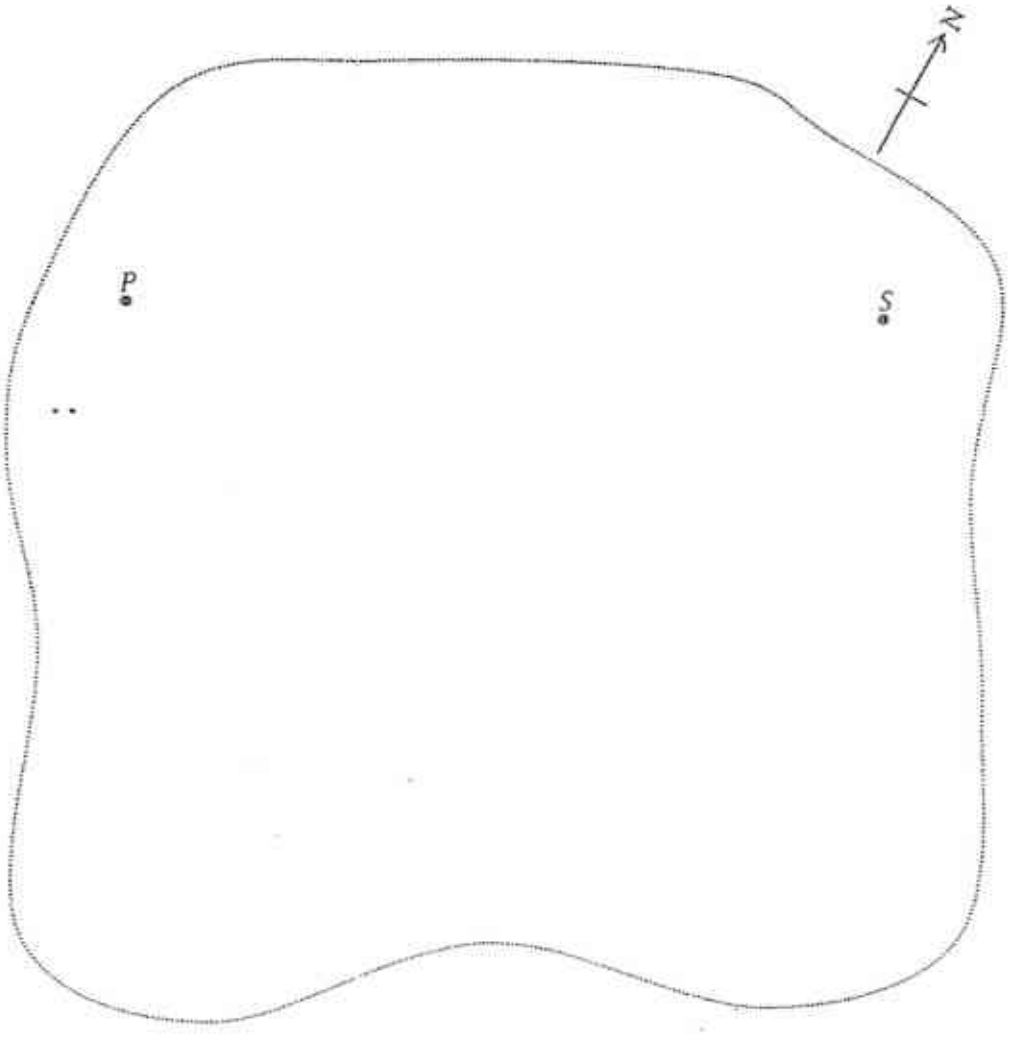
Answer (b) $\angle AOC =$ ..... [1]

(c) Area of shaded region $AOCP =$ ..... cm^2 [3]

18 The diagram below represent the map of Town Crescent drawn to the scale of 1 cm to 2 km. P and S are the positions of a primary and a secondary school respectively and the North direction is indicated in the diagram.

(a) The bearing of a community centre from the primary school is 085° and the distance between the community centre and the primary school is 17 km. By making appropriate measurements, find and label the location of the community centre. [2]

(b) A police station is located south of the community centre and is equidistant from the two schools. Find the distance in km, from the police station to the primary school.

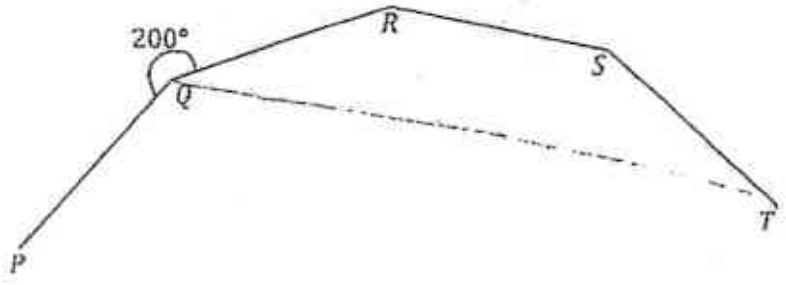


Answer (b) Distance = ... km [3]

19 PQ, QR, RS and ST are adjacent sides of a regular polygon.
Given that reflex angle $PQR = 200°$, find

(a) the number of sides of the polygon,

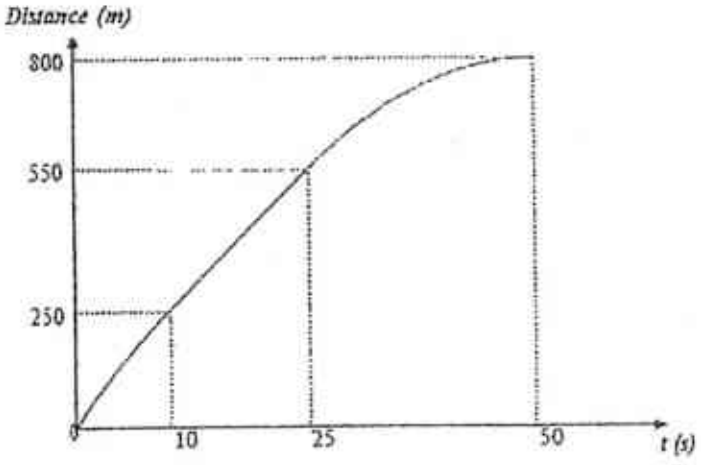
(b) angle PQT.



Answer (a) Number of sides = ... [2]

(b) $\angle PQT$ = ... [2]

20 The diagram shows the distance-time graph for the first 50 seconds of a car's journey. The car moves at a constant speed from $t = 10$s to $t = 25$ s and comes to rest after 50 seconds



(a) Find the speed of the car when $t = 15$.

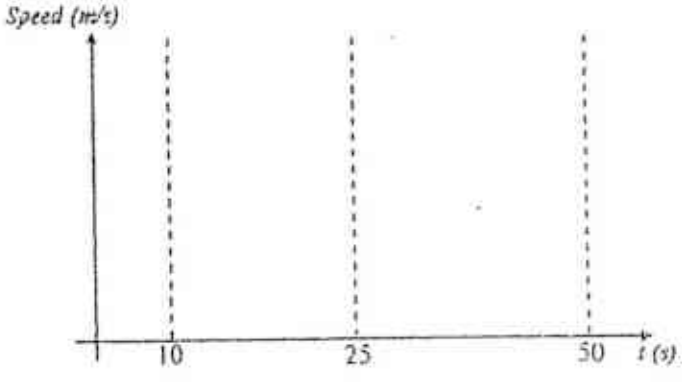
(b) Given that the car moves with a constant deceleration for the last 25 seconds, find its deceleration for the last 25 seconds.

(c) The car starts at 30m/s and decelerates uniformly for the first 10 seconds. On the axes below, sketch the speed-time graph for the car's journey.

Answer (a) ..... m/s [1]

(b) ..... m/s^2 [1]

(c)



[1]

21 Figure 1 below shows a hollow square base pyramid with base length $2l$ and height $2h$ cut into 2 parts. The lower part of the pyramid is attached to a cuboid with a square base of $2l$ and height h, as shown in Figure 2.

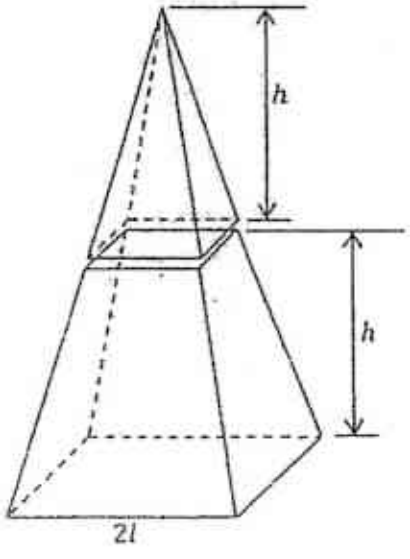


Figure 1

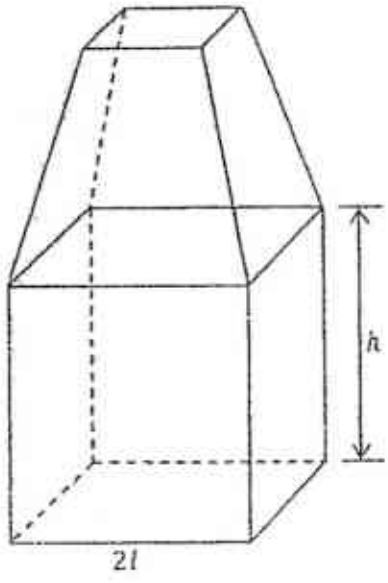


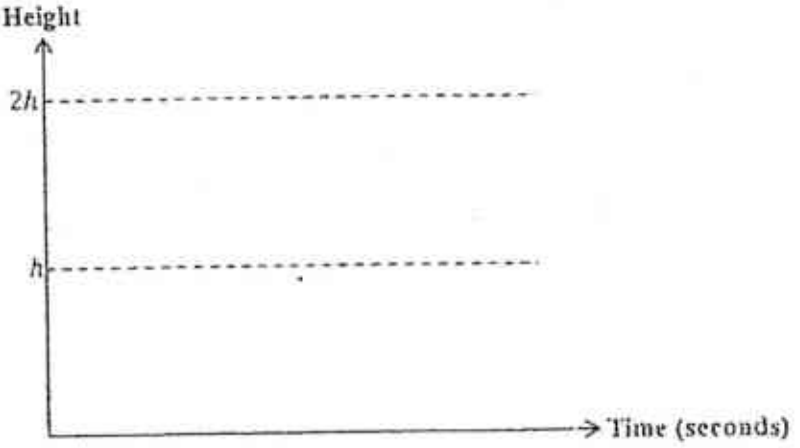
Figure 2

(a) If it takes 1 minute to fill up the cuboid, find the time taken, in seconds, to fill up the whole container in Figure 2.

(b) On the answer grid given on the next page, sketch the graph of the depth of water against time.

Answer (a) ..... seconds [2]

(b)



[1]

22 The table below shows the age distribution of customers in an electronic store.

Age (years)	$0 < x \leq 10$	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$
Frequency	2	7	37	25	9

(a) Find the
(i) mean,
(ii) standard deviation

(b) Find the modal group of the above distribution.

Answer (a)(i) ..... [1]

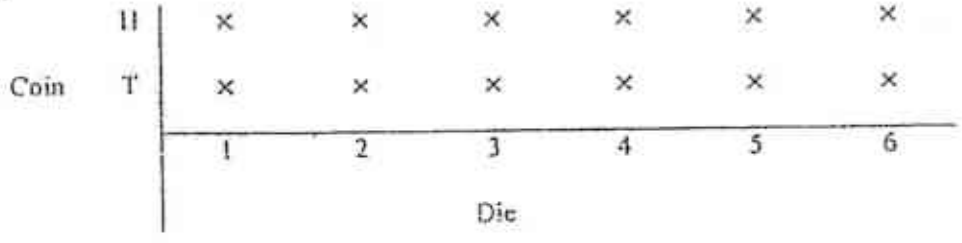
(ii) ..... [2]

(b) ..... [1]

END OF PAPER

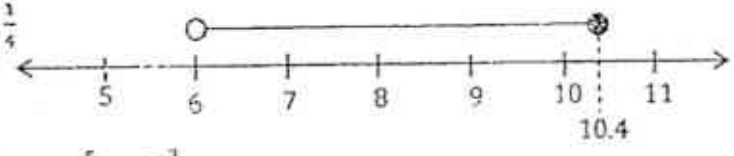
ANSWER KEY

1. (a) 100 (b) 180
2. $\frac{48}{x-1}$
3. 14
4. (a) 1 : 150 000 (b) 6.75
5. 16
6. (a) 2660 : 167 (b) 2.994×10^{-26}
7. (a) $(3x+y)(2a+b)(2a-b)$ (b) $a=-3\frac{1}{3}$ or $-3\frac{1}{2}$ or $3\frac{1}{2}$
8. $y=7$ and $x=-25$
9.



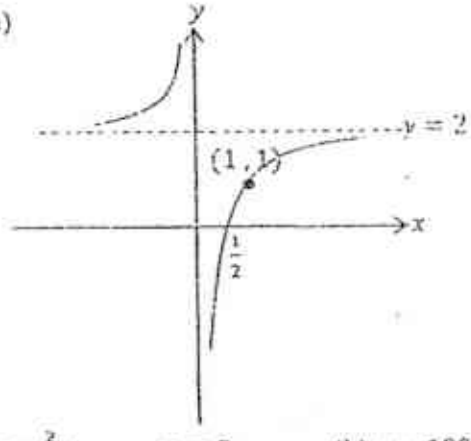
(a) $\frac{1}{3}$ (b) (i) $\frac{1}{6}$ (ii) $\frac{1}{4}$

10. (a) $6 < x \leq 10.4$ (b)

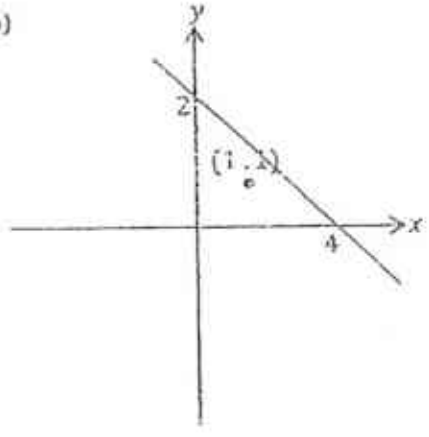


(c) $x=7$

11. $y=4p$
12. (a) $y=\frac{5}{16}(x+2)^2-5$ (b) $y=-\frac{5}{8}x-3\frac{1}{4}$
13. (a)



(b)



14. (a) $a=\frac{2}{3}\pi$ $n=3$ (b) $a=100$ $n=-2$
15. (a) $\angle CDB=\angle CBD=\angle ADB=67°$ (diagonal of rhombus bisects interior angle)
$\angle DCB=180°-67°-67°=46°$ (sum of angles in triangle BCD)
$\angle EDC=180°-67°-67°=46°$ (sum of angles on straight line ADE)
DC is a common length
$\therefore$ Triangle EDC is congruent to triangle DCB. (ASA)
(b) (i) $\frac{1}{3}$ (ii) $\frac{1}{3}$

16. (a)(i) $-\frac{3}{5}$ (ii) $\frac{3}{\sqrt{109}}$ (b) $\frac{15}{\sqrt{109}}$ (c) $(7, 4)$
17. (b) 2.89 radians (c) 63.4 cm^2
18. (b) 9.8 km (± 0.2) km
19. (a) 18 (b) 140°
20. (a) 20 m/s (b) 0.8 m/s^2
21. (a) 95 seconds
22. (a) (i) 29 (ii) 8.89 (c) $20 < x \leq 30$

NAME: _____ ( ) CLASS: _____

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2015
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

MATHEMATICS 4016/01

Paper 1

Date: 20 August 2015 **Duration: 2 hours**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use	
Paper 1	/ 80
Paper 2	/ 100
Total	%

Setters: Mr Alester Tan

75

This question paper consists of 19 printed pages including the cover page.

Name: _____ ( ) Class: _____

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of a triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Name: _____ ( ) Class: _____

Answer **all** the questions.

1 Calculate $\sqrt{\dfrac{28\cdot047}{3\cdot743\times10^{-4}}}$.

(a) Give your answer as displayed in the calculator.

(b) Write down your answer in part (a) correct to 2 significant figures.

Answer (a) ..... [1]

(b) ..... [1]

---

2 Alan bought a smart watch for \$240. When he sold it, he made a profit of 115% of his cost.

Find the selling price.

Answer \$..... [2]

---

3 **(a)** Given that $a^5 \times a^2 \div \sqrt[3]{a} = a^k$, find the value of k.

(b) Simplify $\left(\dfrac{2b}{3}\right)^{-3}$.

Answer (a) k = ... [1]

(b) ..... [1]

Name: _____ ( ) Class: _____

4 Trevor bought p apples at q cents per half a dozen. He sold them for r cents each.
Write down, in its simplest form, an expression in terms of p, q and r, for the profit, in dollars, that he made.

Answer $..... [2]

---

5 The scale of the floor plan of a house can be expressed as $\frac{1}{2}$ cm represents 2 m.

(a) Express this scale in the form 1 : n.

(b) Calculate, in cm^2, the area of a room on the map which has an actual area of 32 m^2.

Answer (a) 1 : ..... [1]

(b) ..... cm^2 [2]

6 The number of books that students borrowed is shown in the table below.

No. of books	0	1	2	3	4	5	6
No. of students	5	6	x	11	5	2	1

Find

(a) the largest possible value of x if the median is 3,

(b) the value of x if the mean number of books borrowed is $2\frac{1}{3}$.

Answer (a) ..... [1]

(b) ..... [2]

---

7 Express as a single fraction in its simplest form.

$$\frac{16x^2-1}{(4x-1)^2}-\frac{2x}{4x-1}$$

Answer ..... [3]

177

Name: _____ ( ) Class: _____

8 The thickness of a sheet of paper is 0.05 millimetre.

(a) Express 0.05 millimetre in metre in standard form.

(b) Find the total number of sheets of paper from a pile of paper, which has a height of 6×10^{-2} m.

Answer (a) ..... [1]

(b) ..... sheets [2]

---

9 Given a sequence

$$1, \frac{2}{3}, \frac{4}{7}, \frac{8}{15}, \frac{16}{31}, \ldots$$

(a) Write down the next term in the sequence.

(b) One of the terms in the sequence is $\frac{x}{y}$. Find in **terms of** x and y, the term which comes immediately after $\frac{x}{y}$.

(c) Write down an expression in terms of n, for the nth term of the sequence.

Answer (a) ..... [1]

(b) ..... [1]

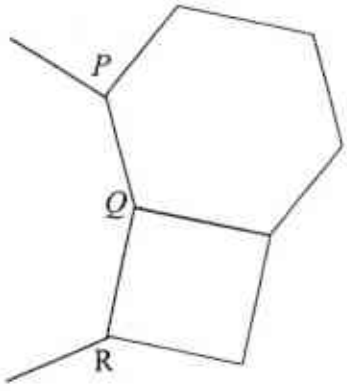
(c) ..... [1]

Name: _____ ( ) Class: _____

10 The figure is made up of a square and a regular hexagon. PQ and QR are two sides of another regular polygon. Find

(a) the reflex $\angle PQR$,

(b) the number of sides of this polygon.



Answer (a) .....° [1]

(b) ..... [2]

---

11 **(i)** Express 14520 as a product of prime factors.

(ii) Given that $14520k$ is a perfect square, write down the smallest positive integer k.

(iii) Find the largest integer which is a factor of 14520 and 4400.

Answer (i) ..... [1]

(ii) ..... [1]

(iii) ..... [1]

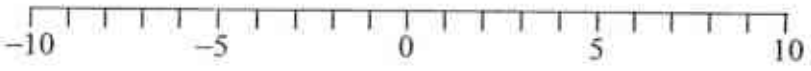
178

Name: _____ ( ) Class: _____

12 **(a)** Solve the inequality $-2-3x \le 8+2x \le x+12$.

Show your solution on the number line in the answer space.

Answer (a)



[2]

(b) Write down all the prime numbers which satisfy $-2-3x \le 8+2x \le x+12$.

Answer (b) ..... [1]

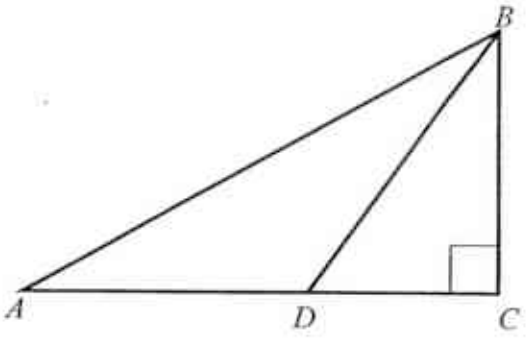
---

13 ABC is a right-angled triangle where $AB = 17$ cm, $BC = 8$ cm and $\angle ACB = 90°$.

Given that D is a point on AC such that $\frac{CD}{AD} = \frac{2}{3}$, find, giving your answer as a fraction in its simplest form, the value of

(a) tan $\angle CBD$,

(b) cos $\angle ADB$.



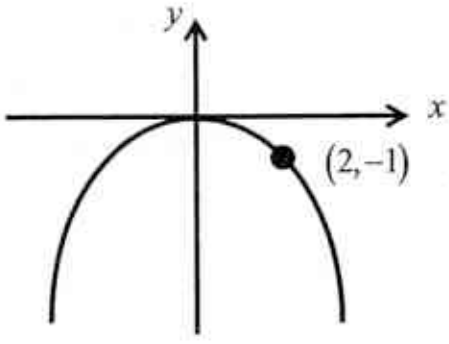
Answer (a) tan $\angle CBD$ =..... [2]

(b) cos $\angle ADB$ =..... [2]

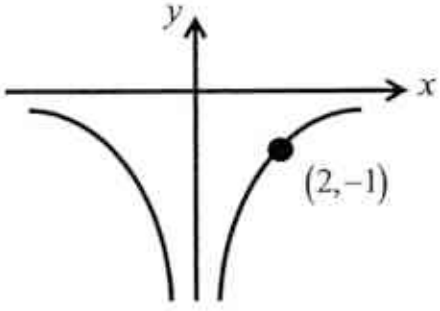
Name: _____ ( ) Class: _____

14 The equation of the following curves is of the form $y = ax^n$, where n is an integer. Each of the curves passes through the point $(2,-1)$. State a possible value for a and n for each curve.

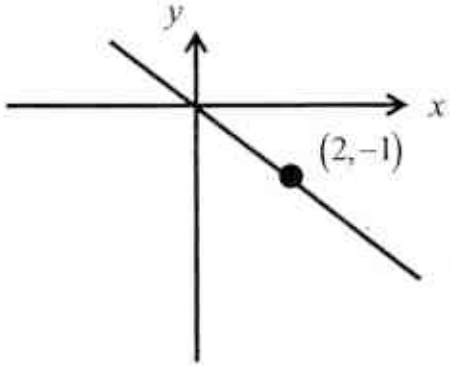
(a)



(b)



(c)



Answer (a) $a =$ ....., $n =$ ..... [1]

(b) $a =$ ....., $n =$ ..... [1]

(c) $a =$ ....., $n =$ ..... [1]

179

Name: _____ ( ) Class: _____

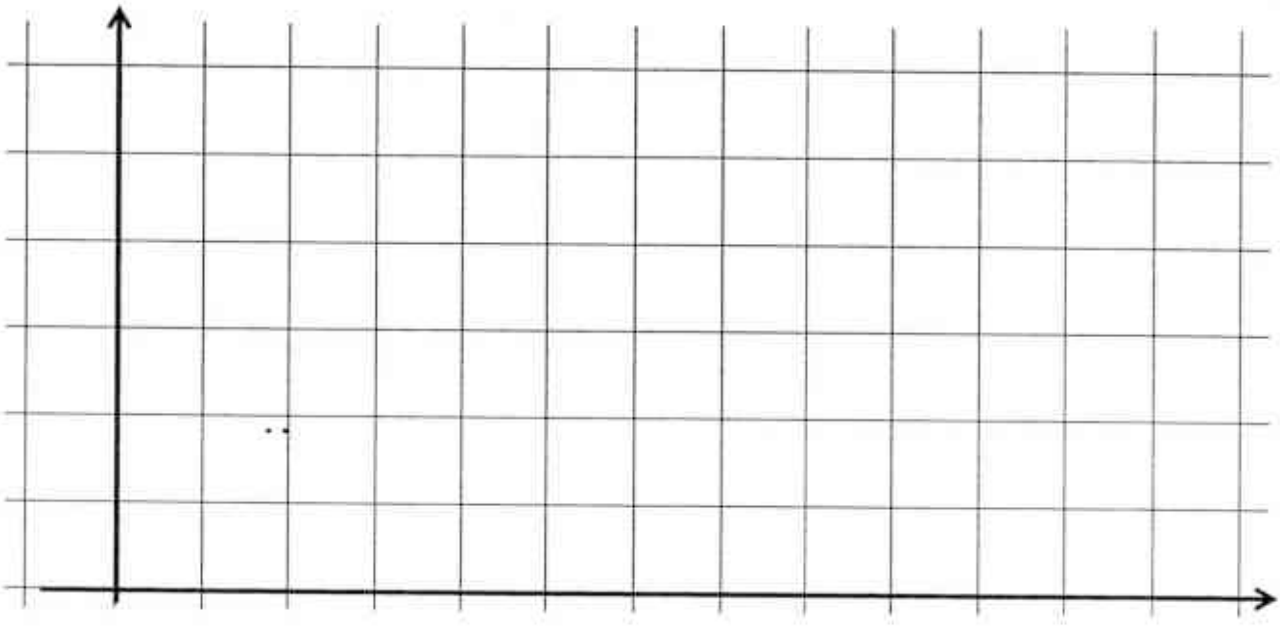
15 The table below shows the number of goals scored by a soccer team in 16 matches.

No. of goals	0	1	2	3	4	5
No. of matches	3	5	1	4	2	1

(a) Complete, with labels, the bar chart below using the data from the table above.

(b) Find the median of the distribution.

Answer (a) [2]



Answer (b) .....goal(s) [1]

Name: _____ ( ) Class: _____

16 **(a)** Express $x^2-6x-16$ in the form $(x-a)^2+b$.

(b) Hence solve the equation $\frac{x^2-16}{2}=3x$.

Answer (a) ..... [1]

(b) x = ... or x = ... [2]

---

17 **(a)** 6 men can paint a building in 15 days.

Find the number of days taken for 5 men to paint the same building.

(b) y is directly proportional to x^2. It is known that $y = 14$ for a particular value of x.

Find the value of y when this value of x is halved.

Answer (a) ..... days [1]

(b) y = ... [2]

180

Name: _____ ( ) Class: _____

18 Solve the following simultaneous equations

$$3a+2b=3$$
$$4a-3b=-13$$

Answer a = ...

b = ... [3]

Name: _____ ( ) Class: _____

19 Two coins are placed in a bag. One of the coins is fair while the other is biased such that when tossed, the probability of obtaining a tail is $\frac{2}{3}$. A coin is chosen at random from the bag and is tossed twice.

Calculate the probability that

(i) two tails are obtained,

(ii) a head is obtained at second toss.

Answer (i) ..... [2]

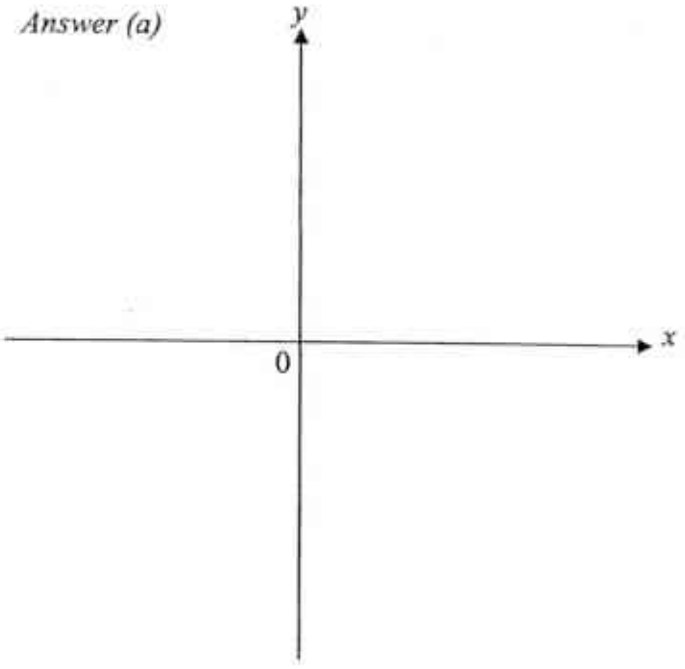
(ii) .....[2]

181

Name: _____ ( ) Class: _____

20 **(a)** Sketch the graph of $y = (x-2)(x+3)$.

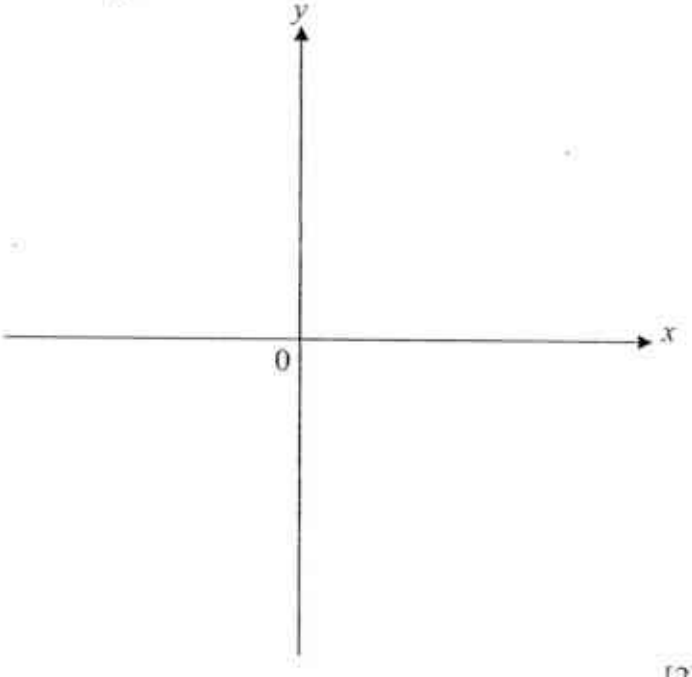
Answer (a)



[2]

(b) Sketch the graph of $y = 5-(x+1)^2$.

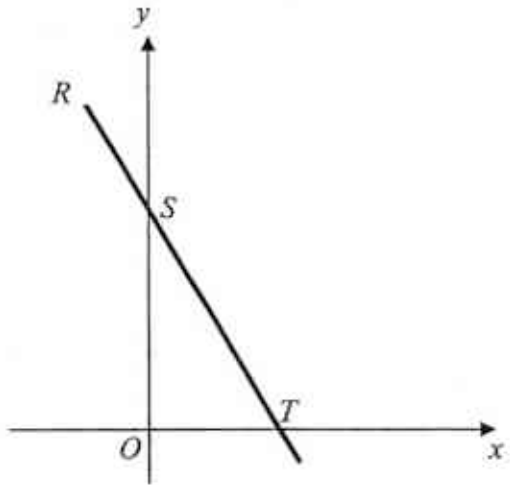
Answer *(b)*



[2]

Name: _____ ( ) Class: _____

21 In the diagram, which is not drawn to scale, the line RST has the equation $y+2x=5$.



(a) Write down the coordinates of the points S and T.

(b) The y-coordinate of the point R is 9. Find

(i) the x-coordinate of R,

(ii) the area of triangle RSO.

Answer (a) S (,) [1]

T (,) [1]

(b)(i) x = ... [1]

(ii) ..... units^2 [1]

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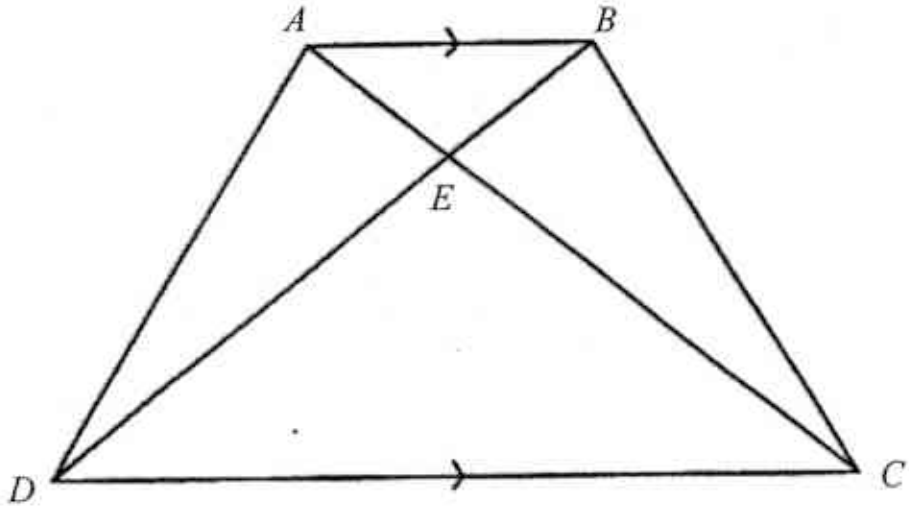
Name: _____ ( ) Class: _____

22



The diagram shows a trapezium $ABCD$ where $AB = 3$ cm and $DC = 9$ cm.
The diagonals AC and BD meet at E.

(a) Show that triangle AEB is similar to triangle CED.

.....

.....

.....

.....[2]

(b) Given that the area of triangle AEB is 7 cm^2, find the area of triangle CED.

Answer (b) .....cm^2 [1]

(c) Find $\dfrac{\text{area of triangle } DEC}{\text{area of trapezium } ABCD}$.

Answer (c) ..... [2]

Name: _____ ( ) Class: _____

23 Three shops A, B and C are located such that B is 500 m due South of A and C is 1000 m from B on a bearing of 220°.

A map is being drawn to a scale of 1 cm to 100 m. The positions of A, B and C are shown below.

Another shop D is 1200 m from C on a bearing of 330°.

(a) Complete the map to show the position of D. [2]

(b) A new shop is to be built [3]

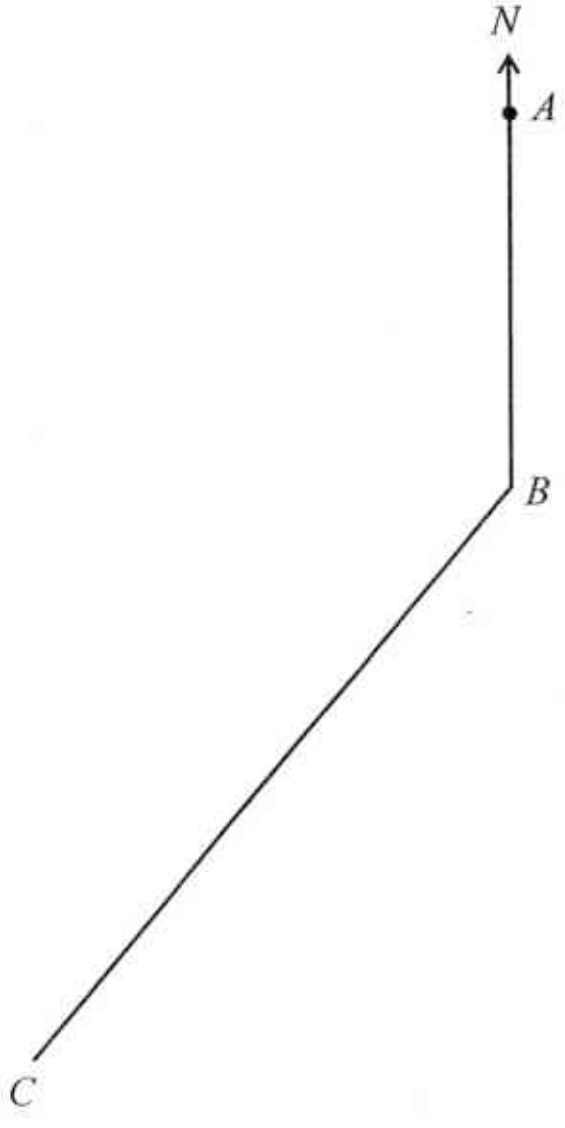
I equidistant from B and C,

II equidistant from BC and DC.

Using the ruler and compasses only, find and label the position of the new shop X.

(c) Find the bearing of shop X from D.

Answer (a) and *(b)*



Answer (c) ..... ° [1]

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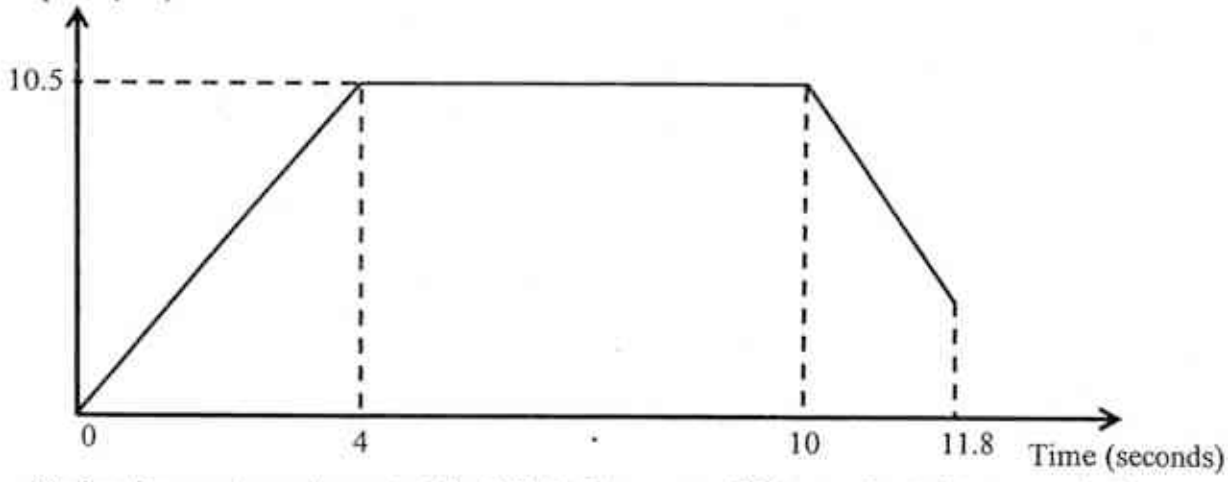
Name: _____ ( ) Class: _____

24 Speed (m/s)



During the recent sports meet at the S.E.A. Games, an athlete accelerated to a speed of 10.5 m/s in 4 seconds. She ran at the same speed for the next 6 seconds and slowed down over the last 1.8 seconds as shown in the speed-time graph. She crossed the finish line after 11.8 seconds.

(a) Find the acceleration of the athlete in the first 3 seconds.

(b) If the total distance she ran was 100 m, calculate her speed when she crossed the finishing line.

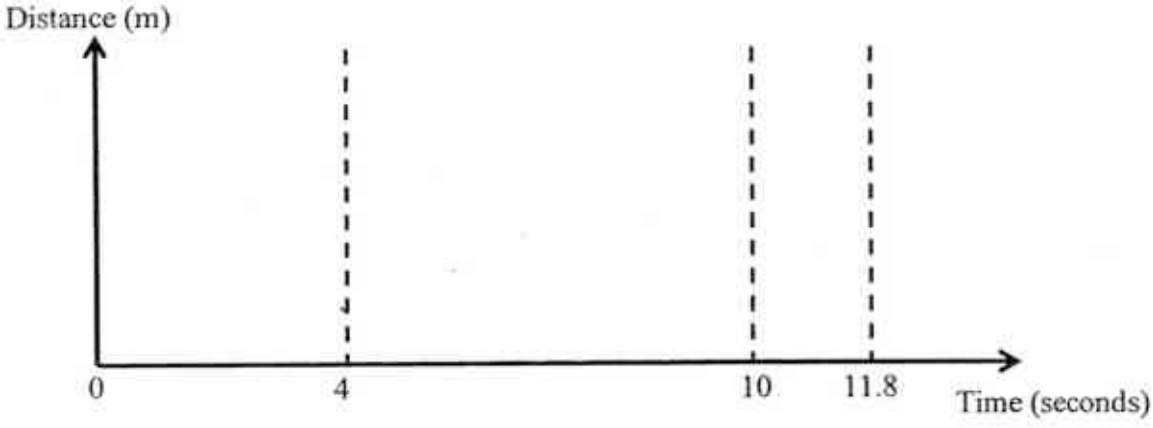
Answer (a) ..... m/s^2 [1]

(b) ..... m/s [2]

Name: _____ ( ) Class: _____

24 **(c)** Use the grid below to sketch the distance-time graph for the first 11.8 seconds.

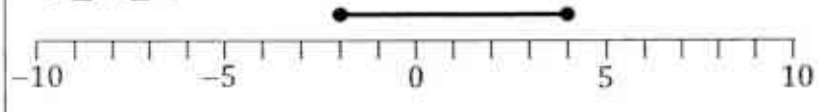
Answer (c) [3]

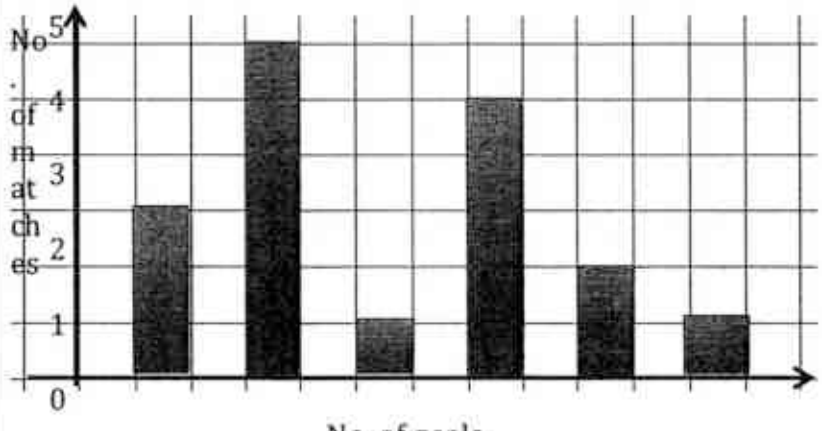
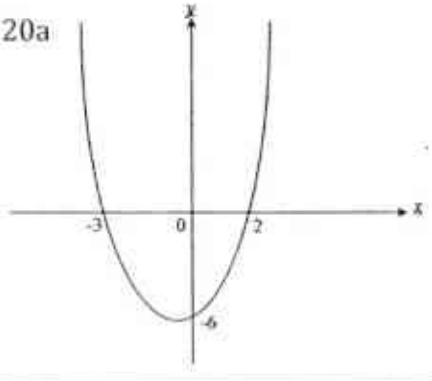
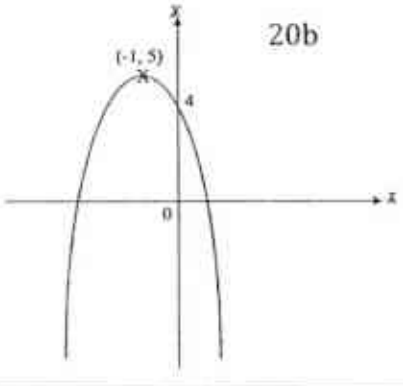


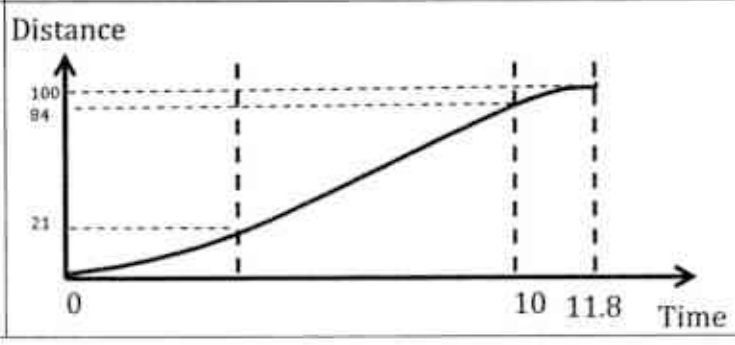
End of paper

184

Answer Key for FMS(S) Sec 4/5 Prelim 2015 Maths P1

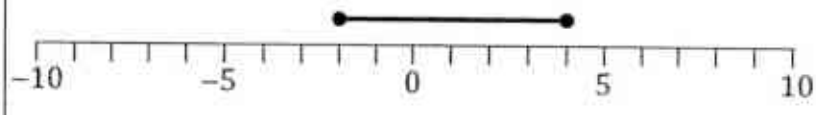
1a	273.7368679
1b	270
2	\$516
3a	$k = 6\frac{2}{3}$
3b	$\frac{27}{8b}$
4	$\$\frac{1}{100}(pr - \frac{pq}{6})$
5a	1 : 400
5b	2 cm^2
6a	7
6b	$x = 15$
7	$\frac{2x+1}{4x-1}$
8a	5×10^{-5}
8b	1200
9a	$\frac{32}{63}$
9b	$\frac{2x}{2x+y}$ or $\frac{2x}{2y+1}$
9c	$\frac{2^{n-1}}{2^n - 1}$
10a	210°
10b	n = 12
11i	$2^3 \times 3 \times 5 \times 11^2$
11ii	30
11iii	440
12a	$-2 \leq x \leq 4$ 
12b	2, 3
13a	$\frac{3}{4}$
13b	$-\frac{3}{5}$

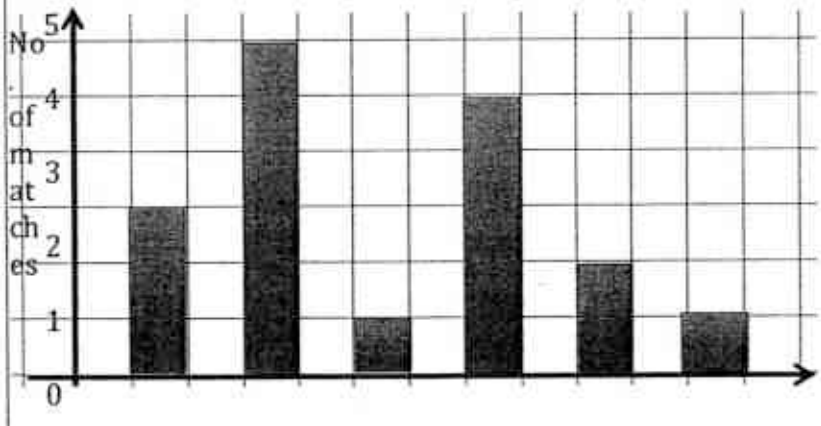
14a	$a=-\frac{1}{4}, n=2$
14b	$a=-4, n=-2$
14c	$a=-\frac{1}{2}, n=1$
15a	 No. of goals
15b	1.5
16a	$(x-3)^2-25$
	$x=8 \text{ or } -2$
17a	18
17b	3.5
18	$a=-1$ $b=3$
19i	$\frac{25}{36}$
19ii	$\frac{5}{6}$
20	20a  20b 
21a	$S(0,5)$ $T(2.5,0)$

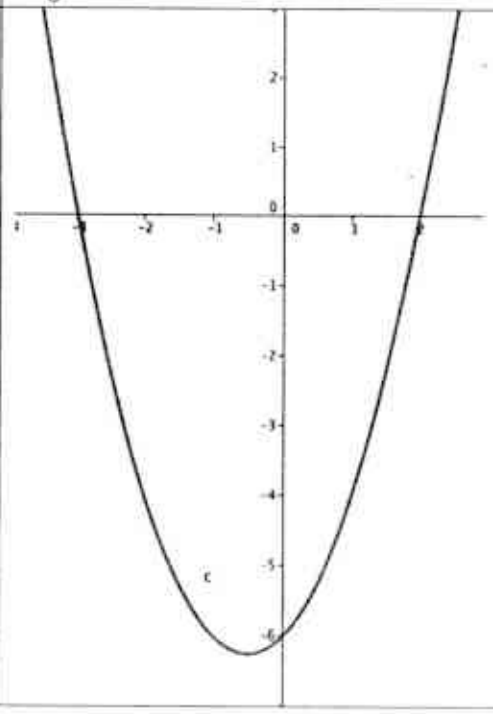
21bi	$x = -2$
21bii	5 units^2
22a	$\angle AEB = \angle CEB$ (*vert opp* $\angle s$) $\angle ABE = \angle CDE$ (*alt* $\angle s$) $\angle BAE = \angle DCE$ (*alt* $\angle s$) ΔAEB *is similar to* ΔCED (*AAA*)
22b	63
22c	$\frac{9}{16}$
23c	123.5±1
24a	2.625
24b	7.28 (3*s.f.*)
24c	

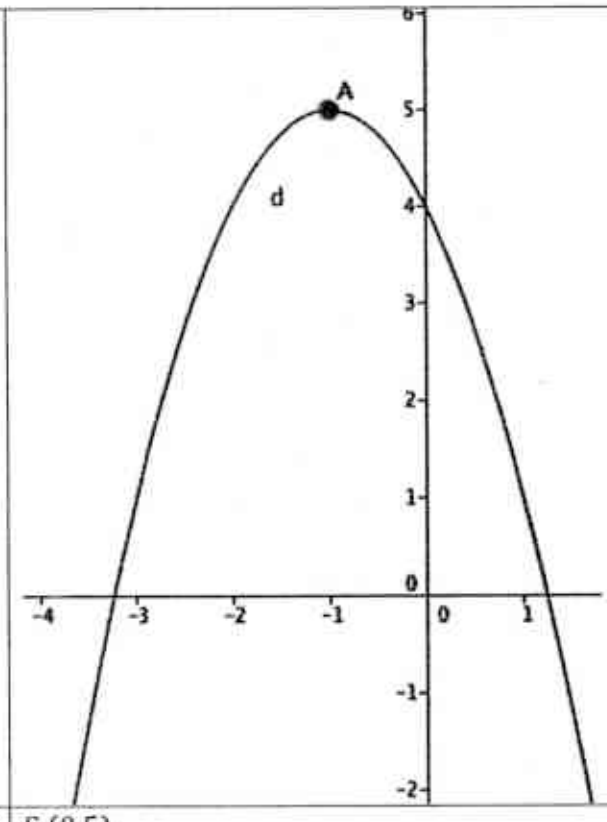
Marking scheme for FMS(S) Sec 4/5 Prelim 2015 Maths P1

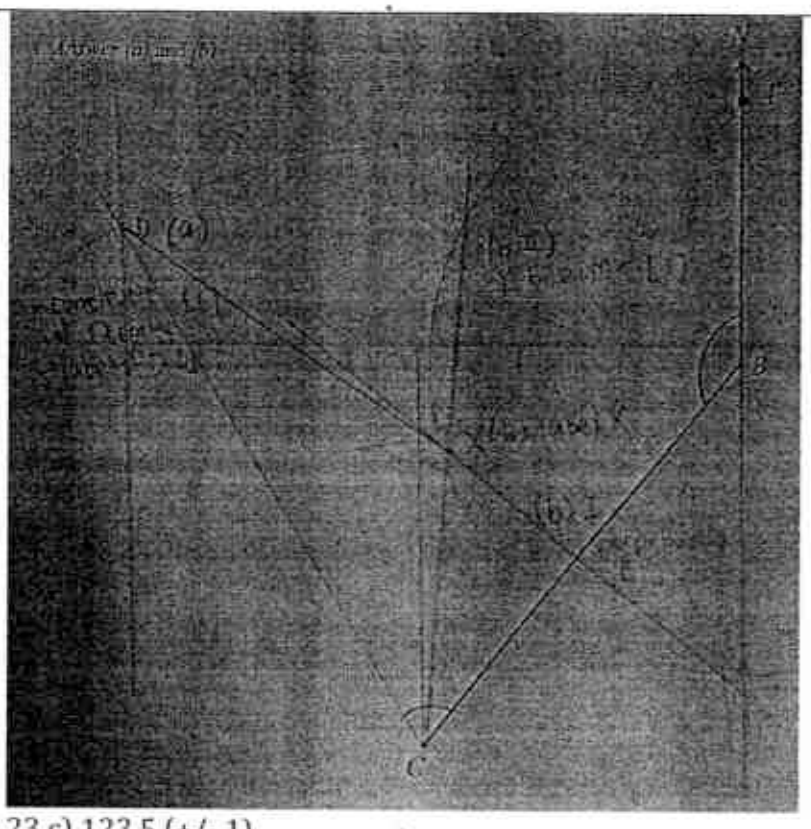
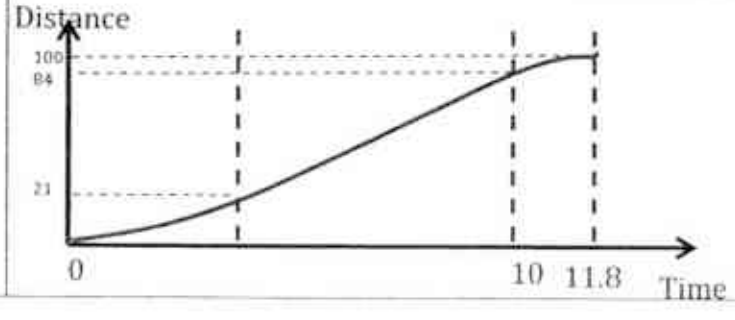
1a	273.7368679	B1
1b	270	B1
2	240 x 2.15 = $516	M1 A1
3a	$k = 6\frac{2}{3}$	B1
3b	$\frac{27}{8b}$	B1
4	Cost of apples = $\frac{pq}{6}$ cents Selling price = pr cents Profit in dollars = $\$\frac{1}{100}(pr - \frac{pq}{6})$	B1 B1
5a	1 : 400	B1
5b	$1\ cm^2 : 16\ m^2$ Area = 32 / 16 $= 2\ cm^2$	M1 – finding area ratio A1 or B2
6a	7	B1
6b	Mean = $\frac{2x+75}{x+30} = \frac{7}{3}$ $x = 15$	B1 – form an equation B1
7	$\frac{(4x-1)(4x+1)}{(4x-1)^2} - \frac{2x}{4x-1}$ $=\frac{4x+1-2x}{4x-1}$ $=\frac{2x+1}{4x-1}$	M1 – factorise $16x^2-1$ M1 – Common denominator A1
8a	5×10^{-5}	B1
8b	$\frac{6 \times 10^{-2}}{5 \times 10^{-5}}$ =1200	M1 A1
9a	$\frac{32}{63}$	B1
9b	$\frac{2x}{2x+y}$ or $\frac{2x}{2y+1}$	B1
9c	$\frac{2^{n-1}}{2^n - 1}$	B1
10a	210^o	B1
10b	150n = (n-2) 180 n = 12	M1 – form the equation A1
11i	$2^3 \times 3 \times 5 \times 11^2$	B1
11ii	$2 \times 3 \times 5 = 30$	B1
11iii	$2^3 \times 5 \times 11 = 440$	B1

12a	$-2-3x \le 8+2x \quad 8+2x \le x+12$ $-10 \le 5x \quad x \le 4$ $-2 \le x \le 4$ 	B1 – correct inequality B1 for correct number line drawn
12b	2, 3	B1
13a	$AC=\sqrt{17^2-8^2}=15$ $DC=\frac{15\times 2}{5}=6$ $\tan\angle CBD=\frac{3}{4}$	 M1 A1
13b	$BD=\sqrt{8^2-6^2}=10$ $\cos\angle ABD=-\cos\angle BDC$ $=-\frac{3}{5}$	M1 A1
14a	$a=-\frac{1}{4}, n=2$ ..	B1 (n can be any even positive number; a is the corresponding value to n)
14b	$a=-4, n=-2$	B1 (n can be any even negative number; a is the corresponding value to n)
14c	$a=-\frac{1}{2}, n=1$	B1

15a		[B2] - 1 per mistake from bar chart - 1 for no label
15b	1.5	B1
16a	$(x-3)^2-25$	B1
	$\frac{x^2-16}{2}=3x$ $x^2-16=6x$ $x-6x-16=0$ $(x-3)^2-25=0$ $(x-3)^2=25$ $x-3=\pm\sqrt{25}$ $x=3+\sqrt{25}$ *or* $3-\sqrt{25}$ $x=8$ *or* -2	M1 A1
17a	D = 18	B1
17b	$k=\frac{14}{x^2}$ $y=\frac{14}{x^2}(\frac{1}{2}x)^2$ =3.5	M1 A1

18	$3a+2b=3 \quad -(1)$ $4a-3b=-13 \quad -(2)$ $(1)\times 3,$ $9a+6b=9 \quad -(3)$ $(2)\times 2,$ $8a-6b=-26 \quad -(4)$ $(3)+(4),$ $17a=-17$ $a=-1$ *Sub* $a=-1$ *into* o (1) $3a+2b=3$ $3(-1)+2b=3$ $b=3$	M1- Using elimination or substitution method A1 A1
19i	$P(2\ tails)=\frac{1}{2}\times\frac{1}{2}+\frac{2}{3}\times\frac{2}{3}$ $=\frac{25}{36}$	M1 A1 or B2
19ii	$P(H\ at\ 2nd\ toss)=\left[\left(\frac{1}{2}\times\frac{1}{2}\right)\times 2\right]+\left(\frac{1}{3}\times\frac{1}{3}\right)+\left(\frac{2}{3}\times\frac{1}{3}\right)$ $=\frac{5}{6}$	M1 A1
20a		B1 – "U" curve B1- mark and label correct x-intercepts (x=2 and x=-3) and y-intercept (y=-6)

20b		B1 – "∩" curve B1- mark & label correct maximum point (-1,5) and y-intercept (y=4)
21a	S (0,5) T (2.5, 0)	B1 B1
21bi	X = -2	B1
21bii	Area = 0.5 x 5 x 2 =5 units²	B1
22a	$\angle AEB = \angle CEB$ (*vert opp* $\angle s$) $\angle ABE = \angle CDE$ (*alt* $\angle s$) $\angle BAE = \angle DCE$ (*alt* $\angle s$) ΔAEB *is similar to* ΔCED (*AAA*)	} } B1 – Show } 2 angles with valid reasons B1- Final statement with valid test of similarity
22b	$\frac{A_1}{A_2} = \left(\frac{1}{3}\right)^2$ $\frac{1}{9} = \frac{7}{A_2}$ $A_2 = 63$	B1

22c	$\frac{Area\ of\ triangle\ DEC}{Area\ of\ trapezium\ ABCD} = \frac{\text{Area of DEC}}{\text{Area of DBC}} \times \frac{\text{Area of DBC}}{\text{Area of Trapezium}}$ $= \frac{3}{4} \times \frac{\frac{1}{2} \times DC \times h}{\frac{1}{2} \times (DC + AB) \times h}$ $= \frac{3}{4} \times \frac{3}{4}$ $= \frac{9}{16}$	M1 A1
23	 23 c) 123.5 (+/- 1)	
24a	2.625	B1
24b	$\frac{1}{2}(4 \times 10.5) + (6 \times 10.5) + Area\ of\ trapezium = 100$ $Area\ of\ trapezium = 100 - 84 = 16$ $Speed = \left[16 \div (\frac{1}{2} \times 1.8)\right] - 10.5 = 7.28\ (3s.f.)$	M1 A1
24c	 	Acurate sketch of graph with labelling of distance for time: 0-4 [B1], 4-10 [B1], 10-11.8 [B1] -1 if no label of distance/the graph is not smooth

NAME: _____ ( ) CLASS: _____

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2015
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

MATHEMATICS 4016/02

Paper 2

Date: 24 August 2015 **Duration: 2 hours 30 minutes**

Additional Materials: Answer Paper
Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

At the end of the examination, fasten all your work securely together.

For Examiner's Use	
Total	/ 100

Setters: Mdm Haliza and Mr Joel Li

This question paper consists of 13 printed pages including the cover page.

[9]

Name: _____ ( ) Class: _____

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of a triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Name: _____ ( ) Class: _____

Answer **all** the questions.

1 A bag contains 10 identical balls of which 4 are red and 6 are blue. Peter and Glen take turns to draw a ball from the bag **without** replacement. The first student to take a blue ball wins the game and the game ends.

(a) Given that Peter takes the first draw, Glen takes the second draw, Peter then takes the third draw and so on, state, as a fraction, the probability that

(i) Peter wins at the first draw, [1]

(ii) Glen wins at the second draw, [1]

(iii) no fourth draw is required. [2]

(b) If the game ends at the nth draw, state the largest value of n. [1]

---

2 **(a)** Simplify $\dfrac{2ab-a^2}{a^2-4b^2}$. [2]

(b) Given that $b=\dfrac{2a-3}{4+a}$, express a in terms of b. [2]

(c) It is given that

$$\xi=\{x: x \text{ is an integer, } 1<x<16\},$$

$$D=\{x: x \text{ is a perfect square}\},$$

$$E=\{x: x \text{ is a multiple of } 3\},$$

$$F=\{x: x \text{ is a prime number}\}.$$

(i) List the elements of D. [1]

(ii) Find $n(D \cap E \cap F)$. [1]

(iii) Draw a Venn diagram to represent the given sets. [1]

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Name: _____ ( ) Class: _____

3 During weekdays, a restaurant serves set lunches and set dinners. 2 variations of lunches and dinners are served. The number of meals sold from Monday to Wednesday, for each type of meal is as shown in the table.

	Set Lunch *A*	**Set Lunch *B***	**Set Dinner *C***	**Set Dinner *D***
Monday	56	54	60	68
Tuesday	63	69	67	74
Wednesday	70	66	78	68

The prices of the set lunches and set dinners are as follows:

Set Lunch *A*	$ 12.50
Set Lunch *B*	$ 13.20
Set Dinner *C*	$ 19.90
Set Dinner *D*	$ 22.50

(a) Write down a 3×4 matrix, **M**, that represents the number of set meals sold from Monday to Wednesday. [1]

(b) Write down a 4×1 matrix, **N**, to represent the prices of the set meals. [1]

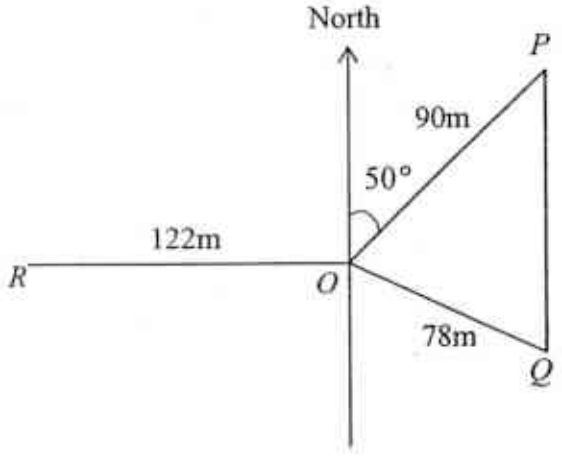
(c) Using matrix multiplication, evaluate matrix **MN**. [2]

(d) Explain what the matrix **MN** represents. [1]

(e) Using matrix multiplication, calculate the total earnings for the set lunches sold for the 3 days. [1]

Name: _____ ( ) Class: _____

4



(a) A surveyor is carrying out a survey on horizontal ground. From a point O, she observes a point P which is 90 m from O on a bearing of $050°$ and a point Q which is 78 m from O and due south of the point P.

Calculate

(i) the bearing of O from P, [1]

(ii) the angle OQP. [2]

(b) The point R is 122 m due west of O.

The surveyor walks directly from R to P. Calculate the distance she walked. [3]

(c) The surveyor then walks from P towards Q until she reaches a point T, where RT is a minimum. Calculate RT. [2]

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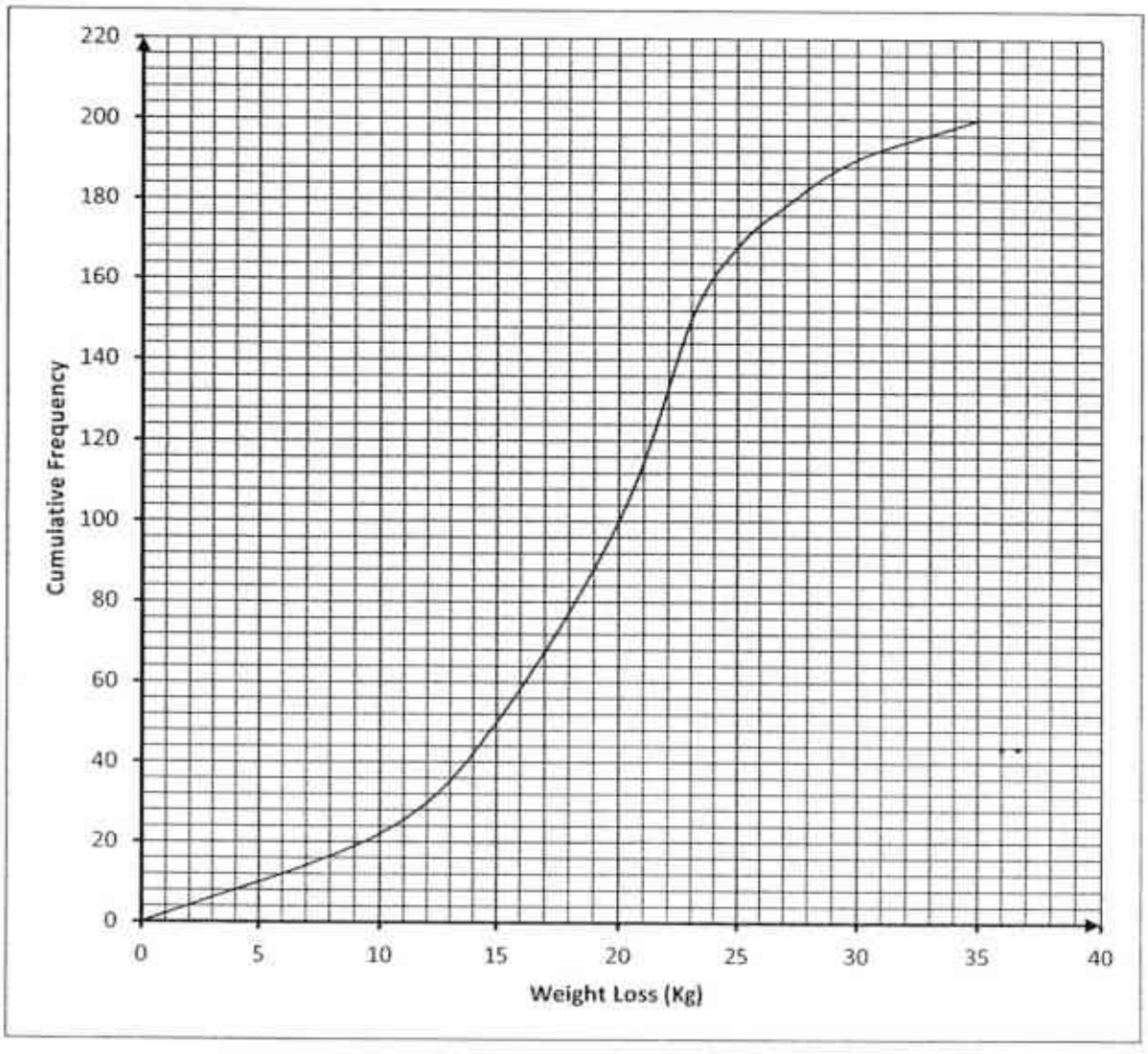
Name: _____ ( ) Class: _____

5 The cumulative frequency curve below shows the amount of weight loss, measured to the nearest kg, of 200 men in Glow Fitness Centre.



The corresponding cumulative frequency table for this distribution is as shown below.

Weight loss (x kg)	≤ 10	≤ 15	≤ 20	≤ 25	≤ 30	≤ 35
Cumulative Frequency	22	a	100	168	190	200

(a) Determine the value of a. [1]

(b) Use your graph to find

(i) the median weight loss, [1]

(ii) the interquartile range, [2]

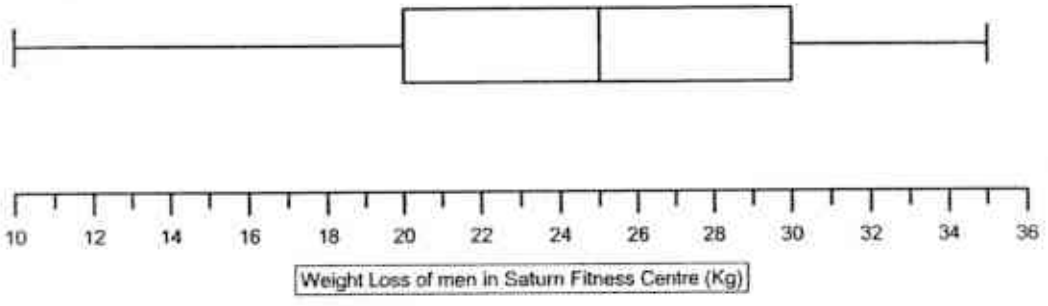
(iii) the 44^{th} percentile weight loss. [1]

Name: _____ ( ) Class: _____

5 (c) A second fitness centre, Saturn Fitness Centre also measured the amount of weight loss by 200 of their men. The box-and-whisker diagram below illustrates their weight loss achieved.

(i) Find the interquartile range. [1]

(ii) A physical instructor claimed that the weight loss program at Glow Fitness Centre is more effective than Saturn Fitness Centre. Explain if his statement is true. [2]



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6 (a) It is given that $\mathbf{u} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$, find

(i) $|\mathbf{u}|$, [1]

(ii) $2\mathbf{u} - \mathbf{v}$. [1]

(b) $ABCD$ is a quadrilateral in which $\overrightarrow{AB} = 3\mathbf{p}$, $\overrightarrow{DC} = 4.5\mathbf{p}$ and $\overrightarrow{DA} = 2\mathbf{t}$.

(i) Sketch the quadrilateral $ABCD$. [1]

(ii) Write down two important facts about the pair of sides AB and DC. [2]

(iii) Express $\overrightarrow{BC}$, as simply as possible, in terms of $\mathbf{p}$ and/or $\mathbf{t}$. [1]

(iv) The sides DA and CB, when produced, meet at X.

Express $\overrightarrow{XA}$, as simply as possible, in terms of $\mathbf{p}$ and/or $\mathbf{t}$. [1]

(v) Write down the value of

(a) $\dfrac{\text{Area of } \Delta ABD}{\text{Area of } \Delta BDC}$, [1]

(b) $\dfrac{\text{Area of } \Delta ABX}{\text{Area of } ABCD}$. [2]

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7 **Answer the whole of this question on a graph paper.**

The variables x and y are connected by the equation $y = \frac{1}{2}x\left(x^2 - 4\right)$.

The table below shows some values of x and the corresponding values of y correct to 1 decimal place.

x	-0.5	0	0.5	1	1.5	2	2.5
y	0.9	0	-0.9	-1.5	-1.3	0	2.8

(a) Taking 4 cm to represent 1 unit on each axis, draw a horizontal x-axis for $-1 \le x \le 3$ and a vertical y-axis for $-2 \le y \le 3$. Draw the graph of $y = \frac{1}{2}x\left(x^2 - 4\right)$ for $-0.5 \le x \le 2.5$. [3]

(b) Use your graph to solve the following equation in the range $-0.5 \le x \le 2.5$.

$$x\left(x^2 - 4\right) = 2 \ldots$$ [1]

(c) By drawing a suitable tangent, find the gradient of the curve at (0.5, -0.9). [2]

(d) **(i)** On the same axes, draw the graph of $y = \frac{x}{2} + 1$. [1]

(ii) Write down the x coordinates of the points at which the two graphs intersect. [1]

(iii) Find the equation, in the form $\frac{1}{2}x^3 + bx^2 + cx + d = 0$, which is satisfied by the values of x found in part **(d)(ii)**. [1]

Name: _____ ( ) Class: _____

8 (a) Mr Ang went to Australia for a short holiday in 2013.

(i) He exchanged some Singapore dollars (S$) for 3000 Australian dollars (A$) from a money changer at an exchange rate of S$1.00 = A$0.92. The money changer also charges a commission of 0.6% on the amount exchanged.

Calculate the amount of Singapore dollars he has to pay. [2]

(ii) He booked a hotel at a cost of A$232 ++ per night. This means that he would have to pay a basic charge of A$232 per night plus a service charge of 10%. On top of these, he would have to pay an additional Goods and Service Tax (GST) of 10% on the sum of the basic charge and service charge.

If he stayed a total of 6 nights at the hotel, calculate the amount of A$ he has to pay for the hotel stay. [2]

(b) (i) Peter deposits $7680 in a fund that pays compound interest of 3.35% per annum, compounded half-yearly. Calculate the total amount of money that Peter will have in the fund at the end of five years. [2]

(ii) At the end of five years, Peter withdrew $6500 from the fund and invested this amount in a bank which offered simple interest rate of 4.8% per annum. Find the minimum number of full years he had to leave the money in the bank in order for it to be more than $12000. [3]

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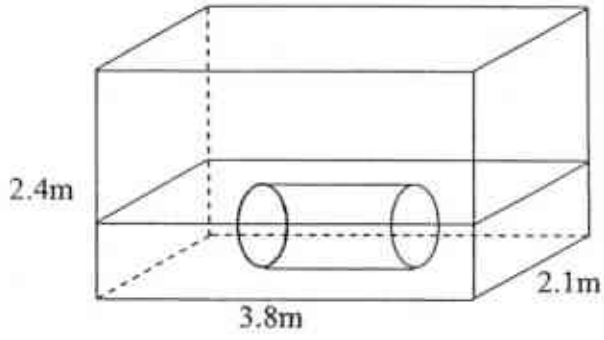
Name: _____ ( ) Class: _____

9



An open rectangular tank of depth 2.4 m has a horizontal base of length 3.8 m and breadth 2.1 m.

A solid metal cylinder of volume 0.865 m^3 rests with its curved surface on the base of the tank as shown in the figure above.

6 400 litres of water is poured into the tank at a rate of 900 cm^3/s.

(a) Calculate how many hours and minutes it takes for all the water to be poured in, giving your answer to the nearest minute. [2]

(b) Given that the water just covers the cylinder as shown in the figure, calculate, in metres,

(i) the depth of the water, [3]

(ii) the radius of the cylinder, [1]

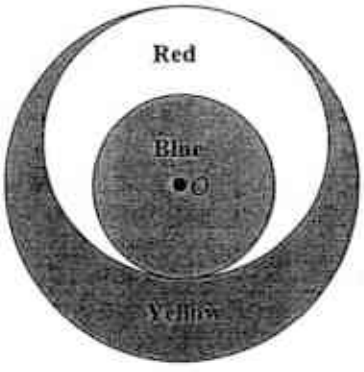
(iii) the length of the cylinder. [2]

(c) The cylinder is now removed from the tank.

Calculate by how much the water level falls. [2]

Name: _____ ( ) Class: _____

10 The diagram below shows the design of a company logo. It consists of three circles.



The largest and the smallest circles have the same centre O.

The largest circle has radius $2x$ cm and the smallest circle has radius $2y$ cm.

The third circle touches both the other two circles.

The three regions formed are coloured red, blue and yellow.

(a) Explain fully why the radius of the third circle is $(x+y)$ centimetres. [2]

(b) Write down and simplify, in terms of π, x and y, expressions for the area of the region that is coloured

(i) red, [1]

(ii) yellow. [1]

(c) The area of the yellow region is twice the area of the red region.

Use this information to write down an equation involving x and y, and show that it simplifies to $x^2 - 6xy + 5y^2 = 0$. [3]

(d) Solve the equation $x^2 - 6xy + 5y^2 = 0$, expressing x in terms of y. [3]

(e) Calculate the fraction of the logo that is coloured yellow. [2]

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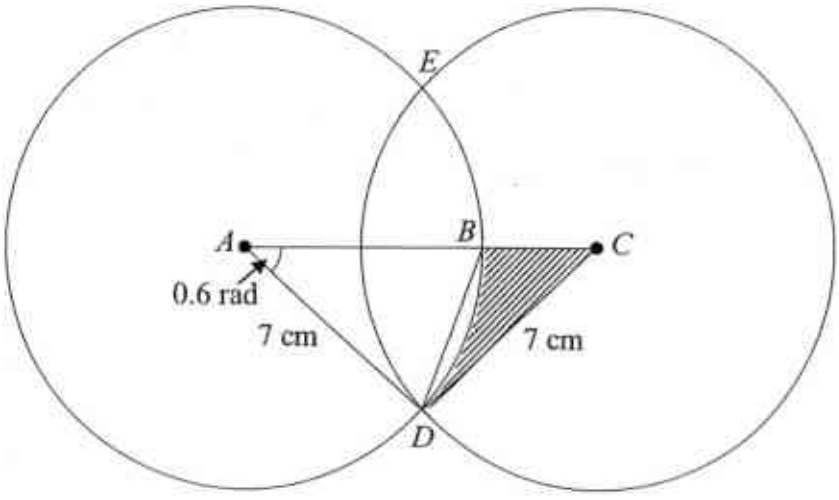
11 **(a)** The diagram shows two circles, centres A and C, with equal radii of 7 cm. The circles meet at D and E. AC meets the circle with centre A at B. Angle $DAB = 0.6$ radians.



Find

(i) the area of sector ABD, [2]

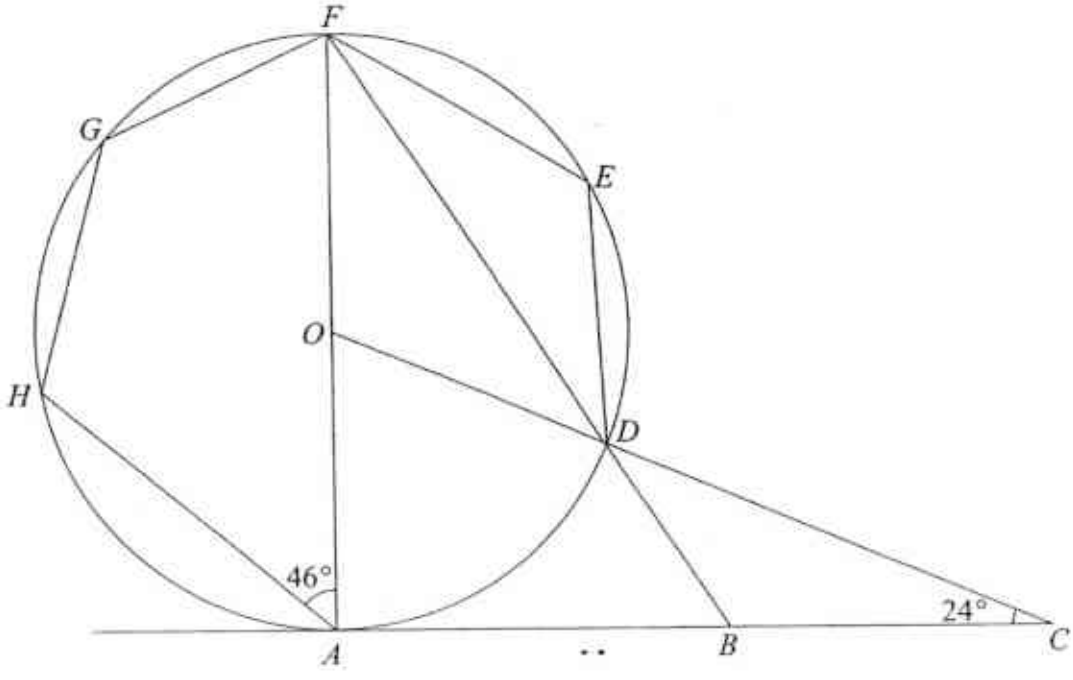
(ii) the area of shaded region. [3]

(iii) the length of arc BD, [2]

(iv) the perimeter of the shaded region, [3]

Name: _____ ( ) Class: _____

11 (b) The diagram shows a circle with centre O. AC is a tangent to the circle. AOF, FDB and ODC are straight lines. Given that $\angle ACO = 24°$ and $\angle HAF = 46°$.



(i) Write down why $\angle FAB = 90°$. [1]

(ii) Calculate

(a) $\angle HEF$, [1]

(b) $\angle AFB$, [1]

(c) $\angle HGF$, [1]

(d) $\angle FED$. [2]

– End of Paper –

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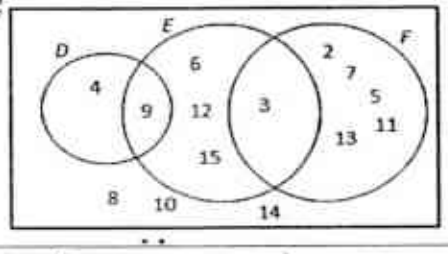
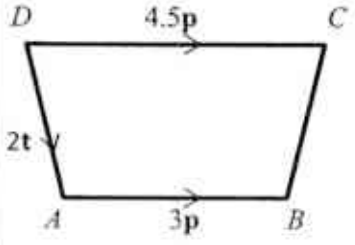
SEC 4 MATHEMATICS PRELIMINARY EXAM PAPER 2 2015 ANSWER KEY

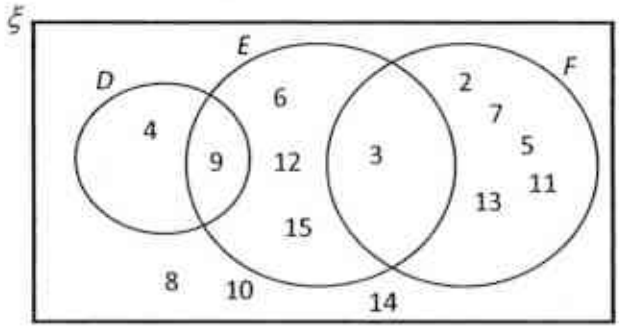
1	(a)(i)	$\frac{3}{5}$
	(ii)	$\frac{4}{15}$
	(iii)	$\frac{29}{30}$
	(iv)	n = 5
2	(a)	$\frac{-a}{a+2b}$
	(b)	$a = \frac{3+4b}{2-b}$
	(c)(i)	D = {4, 9}
	(c)(ii)	$n(D \cap E \cap F) = 0$
	(c)(iii)	
3	(i)	$M = \begin{pmatrix} 56 & 54 & 60 & 68 \\ 63 & 69 & 67 & 74 \\ 70 & 66 & 78 & 68 \end{pmatrix}$
	(ii)	$N = \begin{pmatrix} 12.50 \\ 13.20 \\ 19.90 \\ 22.50 \end{pmatrix}$
	(iii)	$\begin{pmatrix} 4136.80 \\ 4696.60 \\ 4828.40 \end{pmatrix}$
	(iv)	Matrix MN represents the total amount of money (revenue) collected for all set meals for each day respectively.
	(v)	(13661.80)
4	(a)(i)	230°
	(ii)	62.1°
	(b)	200 m
	(c)	191 m
5	(a)	a = 50
	(b)(i)	20kg
	(b)(ii)	Interquartile Range = 23 – 15 = 8 kg

	(b)(iii)	44th Percentile = 19 kg
	(c)(i)	Interquartile Range = 30 – 20 = 10 kg
	(c)(ii)	The median weight loss of Saturn Fitness Centre is higher than Glow Fitness Centre (25kg > 20 kg) The Interquartile Range of Glow Fitness Centre is slightly lower than Saturn Fitness Centre (8kg < 10 kg) Since the median weight loss of Saturn Fitness Centre is higher and the interquartile range difference between 2 centres is comparable. It is untrue that Glow Fitness Centre is more effective at weight loss.
6(a)	(i)	7.62 units
	(a)(ii)	$\begin{pmatrix} 19 \\ -12 \end{pmatrix}$
	(b)(i)	
	(b)(ii)	AB is parallel to DC. DC is 1.5 times of AB or $\lvert\overrightarrow{DC}\rvert = \frac{3}{2}\lvert\overrightarrow{AB}\rvert$.
	(b)(iii)	$1.5\mathbf{p} - 2\mathbf{t}$
	(b)(iv)	$-4\mathbf{t}$
	(b)(v)(a)	$\frac{2}{3}$
	(b)(v)(b)	$\frac{4}{5}$
7	(a)	Draw axes and plot all given points Draw a smooth curve
	(b)	Draw y = 1, x = -0.55 or 2.25 (±0.1)
	(c)	Gradient = $\frac{-1.95}{1.25} = -1.56$ (Accept – 1.425 to – 1.825)
	(d)(i)	Plot and draw $y = \frac{x}{2} + 1$
	(d)(ii)	x = 2.45 or x = -0.425 (±0.1)
	(d)(iii)	$\frac{1}{2}x^3 - \frac{5}{2}x - 1 = 0$
8	(a)(i)	S$3280.43 (2dp)
	(a)(ii)	A$1684.32
	(b)(i)	$9067.82
	(b)(ii)	T = 17.628 years Minimum number of full years = 18 years

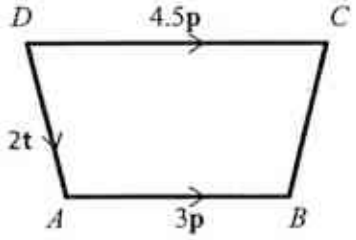
9	(a)(i)	1 h 59 min
	(b)(i)	0.910 m
	(b)(ii)	0.455 m
	(b)(iii)	1.33 m
	(c)	0.108 m
10	(a)	Diameter of 3rd circle is $2x + 2y$ cm. Therefore its radius is $\frac{2x+2y}{2} = (x+y)$ cm (Shown)
	(b)(i)	$\pi(x^2 + 2xy - 3y^2)\,\text{cm}^2$
	(b)(ii)	$\pi(3x^2 - 2xy - y^2)\,\text{cm}^2$
	(c)	$\pi(3x^2 - 2xy - y^2) = 2\pi(x^2 + 2xy - 3y^2)$
	(d)	$x = 5y$ or x = y
	(e)	$\frac{16}{25}$
11	(a)(i)	14.7 cm^2
	(a)(ii)	8.13 cm^2
	(a)(iii)	4.2 cm
	(a)(iv)	15.8 cm
	(b)(i)	AC is a tangent to circle Therefore $\angle FAB = 90°$
	(ii)(a)	$\angle HEF = 46°$ (Angles in same segment)
	(ii)(b)	$\angle AFB = 66° \div 2$ $= 33°$ (Angle at centre = 2 x angle at circumference)
	(ii)(c)	$\angle HCF = 180° - 46°$ $= 134°$ (opp angles of cyclic quad)
	(ii)(d)	$\angle FED = 246° \div 2$ (Angle at centre = 2 x angle at circumference) $= 123°$

SEC 4 MATHEMATICS PELIMINARY EXAM MARKING SCHEME 2015 Maths P2

1.	(a)(i)	$\frac{3}{5}$	B1
	(ii)	P(Student B wins at 2nd draw) $= \frac{4}{10} \times \frac{6}{9}$ $= \frac{4}{15}$	 B1
	(iii)	P(No fourth draw is required) = P(student A wins 1st draw) + P(student B wins 2nd draw) + P(student A wins 3rd draw) $= \frac{3}{5} + \frac{4}{15} + \left(\frac{4}{10} \times \frac{3}{9} \times \frac{6}{8}\right)$ $= \frac{29}{30}$	 M1 A1
	(iv)	n = 5	B1
2	(a)	$\frac{2ab-a^2}{a^2-4b^2} = \frac{a(2b-a)}{(a+2b)(a-2b)}$ $= \frac{-a(a-2b)}{(a+2b)(a-2b)}$ $= \frac{-a}{a+2b}$	M1 – factorize both numerator and denominator A1
	(b)	$b = \frac{2a-3}{4+a}$ $4b+ab = 2a-3$ $ab-2a = -3-4b$ $2a-ab = 3+4b$ $a(2-b) = 3+4b$ $a = \frac{3+4b}{2-b}$	 M1 – factorize A1
	(c)(i)	D = {4, 9}	B1
	(ii)	$n(D \cap E \cap F) = 0$	B1

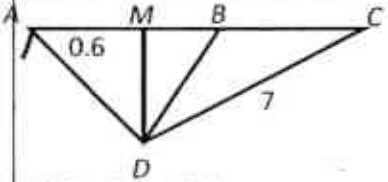
	(iii)		B1 (Must label clearly D,E, F) Zero marks if all 3 sets intersect each other
3	(i)	$M = \begin{pmatrix} 56 & 54 & 60 & 68 \\ 63 & 69 & 67 & 74 \\ 70 & 66 & 78 & 68 \end{pmatrix}$	B1
	(ii)	$N = \begin{pmatrix} 12.50 \\ 13.20 \\ 19.90 \\ 22.50 \end{pmatrix}$	
	(iii)	$MN = \begin{pmatrix} 56 & 54 & 60 & 68 \\ 63 & 69 & 67 & 74 \\ 70 & 66 & 78 & 68 \end{pmatrix}\begin{pmatrix} 12.50 \\ 13.20 \\ 19.90 \\ 22.50 \end{pmatrix}$ $= \begin{pmatrix} 700+712.8+1194+1530 \\ 787.5+910.8+1333.3+1665 \\ 875+871.2+1552.2+1530 \end{pmatrix}$ $= \begin{pmatrix} 4136.80 \\ 4696.60 \\ 4828.40 \end{pmatrix}$	M1 A1
	(iv)	Matrix MN represents the total amount of money (revenue) collected for all set meals for each day respectively.	B1
	(v)	$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 4136.80 \\ 4696.60 \\ 4828.40 \end{pmatrix} = \begin{pmatrix} 13661.80 \end{pmatrix}$	B1

4	(a)(i)	Bearing of O from $P = 230°$	B1
	(ii)	$\sin \angle OQP = \frac{\sin 50°}{78} \times 90$ $\angle OQP = 62.116$ $= 62.1°$ (to 1 d.p.)	M1 A1
	(b)	$RP^2 = 90^2 + 122^2 - 2(90)(122)\cos 140°$ $RP = \sqrt{39806.33597}$ $= 199.52$ $= 200$ m	M1 M1 A1
	(c)	$OT = 90 \sin 50°$ $= 68.944$ $\therefore RT = 122 + 68.944$ $= 190.944$ $= 191$ m	M1 A1
5	(a)	a = 50	B1
	(b)(i)	20kg	B1
	(b)(ii)	Lower Quartile = 15 kg Upper Quartile = 23 kg Interquartile Range = 23 – 15 = 8 kg	M1 A1
	(b)(iii)	44^{th} Percentile = 19 kg	B1
	(c)(i)	Interquartile Range = 30 – 20 = 10 kg	B1
	(c)(ii)	The median weight loss of Saturn Fitness Centre is higher than Glow Fitness Centre (25kg > 20 kg) The Interquartile Range of Glow Fitness Centre is slightly lower than Saturn Fitness Centre (8kg < 10 kg) Since the median weight loss of Saturn Fitness Centre is higher and the interquartile range difference between 2 centres is comparable. It is untrue that Glow Fitness Centre is more effective at weight loss.	M1 for both median and IQR A1 for conclusion
6(a)	(i)	$\lvert\mathbf{u}\rvert = \sqrt{7^2 + (-3)^2} = 7.62$ units	B1

6	(a)(ii)	$2\mathbf{u}-\mathbf{v}=2\begin{pmatrix}7\\-3\end{pmatrix}-\begin{pmatrix}-5\\6\end{pmatrix}$ $=\begin{pmatrix}19\\-12\end{pmatrix}$	B1
	(b)(i)		B1 (2 // sides, correct labelling of vertices)
	(b)(ii)	AB is parallel to DC. DC is 1.5 times of AB. $\lvert\overrightarrow{DC}\rvert=\frac{3}{2}\lvert\overrightarrow{AB}\rvert$.	Any 2 facts. [B1,1]
	(b)(iii)	$\overrightarrow{BC}=-3\mathbf{p}-2\mathbf{t}+4.5\mathbf{p}$ $=1.5\mathbf{p}-2\mathbf{t}$	B1
	(b)(iv)	$\overrightarrow{XA}=-4\mathbf{t}$	B1
	(b)(v)(a)	$\frac{\text{Area of }\Delta ABD}{\text{Area of }\Delta BDC}=\frac{2}{3}$	B1
	(b)(v)(b)	$\frac{\text{Area of }\Delta ABX}{\text{Area of }\Delta DCX}=\left(\frac{2}{3}\right)^2=\frac{4}{9}$ $\therefore\frac{\text{Area of }\Delta ABX}{\text{Area of }ABCD}=\frac{4}{5}$	M1 A1
7	(a)	Draw axes and plot all given points Draw a smooth curve	P2 S1
	(b)	Draw y = 1, x = -0.55 or 2.25 (± 0.1)	B1
	(c)	Gradient = $\frac{-1.95}{1.25}=-1.56$ (Accept – 1.425 to – 1.825)	B1 – Draw tangent B1 – Gradient
	(d)(i)	Plot and draw $y=\frac{x}{2}+1$	B1
	(d)(ii)	x = 2.45 or x = -0.425 (± 0.1)	B1

	(d)(iii)	$\frac{1}{2}x(x^2-4)=\frac{1}{2}x+1$ $\frac{1}{2}x^3-2x-\frac{1}{2}x-1=0$ $\frac{1}{2}x^3-\frac{5}{2}x-1=0$	B1
8	(a)(i)	A\$3000 = $\frac{3000}{0.92}$ = S\$ 3260.8696 Commission = $\frac{0.6}{100}\times 3260.8696$ = \$19.5652 Amount paid = \$3260.8696 + \$19.5652 = S\$3280.43 (2dp)	M1 A1
	(a)(ii)	Cost of Service Charge = $\frac{10}{100}\times A\$232$ = A\$23.20 Cost of room and Service Charge = A\$232 + A\$23.20 = A\$255.20 Cost of GST = 0.1 x A\$255.20 = A\$25.52 Total cost for 1 night = A\$255.20 + A\$25.32 = A\$280.72 Total cost for 6 nights = A\$280.72 x 6 = A\$1684.32	M1 A1
	(b)(i)	Total amount = $7680\left(1+\frac{1.675}{100}\right)^{5\times 2}$ = \$9067.8229 = \$9067.82	M1 A1
	(b)(ii)	Amount of interest to earn for it to be \$12 000 = 12 000 – 6500 = \$5500 $5500=\frac{(6500)(4.8)(T)}{100}$ T = 17.628 years Minimum number of full years = 18 years	M1 M1 A1
9	(a)(i)	$\frac{6400000}{900}$ = 7111.11s = 1h 59 min	M1 A1

9	(b)(i)	Depth of water $= \dfrac{6.4+0.865}{3.8\times 2.1}$ $= 0.91040$ $= 0.910\,\text{m}$	M1 (Total vol.) + M1 (Base area) + units must be the same A1
	(b)(ii)	Radius of cylinder $= \dfrac{0.910040}{2}$ $= 0.4552$ $= 0.455\,\text{m}$	B1 ft from (b)(i)
	(b)(iii)	Length of cylinder $= \dfrac{0.865}{\pi(0.4552)^2}$ $= 1.3288$ $= 1.33\,\text{m}$	M1 A1
	(c)	Ht. of water level after cylinder is removed $= \dfrac{6.4}{3.8\times 2.1}$ $= 0.80201$ .. Water level falls by $0.91040 - 0.80201 = 0.10839$ $= 0.108\,\text{m}$ OR $= \dfrac{0.865}{(3.8\times 2.1)}$ $= 0.108\,\text{m}$	M1 A1 any unit
10	(a)	Diameter of 3rd circle is $2x + 2y$ cm. Therefore its radius is $\dfrac{2x+2y}{2} = (x+y)$ cm (Shown)	M1 A1
	(b)(i)	Area of red region $= \pi(x+y)^2 - \pi(2y)^2$ $= \pi(x^2+2xy-3y^2)\,\text{cm}^2$	B1
	(b)(ii)	Area of yellow region $= \pi(2x)^2 - \pi(x+y)^2$ $= \pi(3x^2-2xy-y^2)\,\text{cm}^2$	B1

	(c)	$\pi(3x^2-2xy-y^2)=2\pi(x^2+2xy-3y^2)$ $3x^2-2xy-y^2-2x^2-4xy+6y^2=0$ $x^2-6xy+5y^2=0$ (Shown)	M1 M1 AG1
	(d)	$x^2-6xy+5y^2=0$ $(x-5y)(x-y)=0$ $x=5y$ or $x=y$	 M1 A1, A1
	(e)	Yellow $=\frac{\pi[3(5y)^2-2(5y)y-y^2]}{\pi[2(5y)]^2}$ $=\frac{75-10-1}{100}$ $=\frac{64}{100}$ $=\frac{16}{25}$	M1 A1
11	(a)(i)	Area of sector ABD $=\frac{1}{2}(7)^2(0.6)$ $=14.7\ \text{cm}^2$	M1 A1
	(a)(ii)	Area of shaded region = Area of Triangle ADC – Area of Sector Area of Triangle ADC $=\frac{1}{2}(7)(2AM)\sin 0.6$ $=22.83476\ \text{cm}^2$ Shaded Area $=22.83476-14.7$ $=8.13476$ $=8.13\ \text{cm}^2$	 M1 M1 A1
	(a)(iii)	Arc Length BD $=r\theta$ $=(7)(0.6)$ $=4.2$ cm	 M1 A1
	(a)(iv)	 AM $=7\cos 0.6$ $=5.7773$ cm BM $=7-5.7773$ $=1.2227$ cm BC $=7-2$BM $=4.5546$	 M1 M1

		Perimeter of shaded region $= 7 + 4.2 + 4.5546$ $= 15.7546$ $= 15.8$ cm	 A1
11	(b)(i)	AC is a tangent to circle Therefore $\angle FAB = 90°$	B1
	(ii)(a)	$\angle HEF = 46°$ (Angles in same segment)	B1
	(ii)(b)	$\angle AOC = 90° - 24°$ $= 66°$ (Sum of Angles in Triangle) $\angle AFB = 66° \div 2$ $= 33°$ (Angle at centre = 2 x angle at circumference)	 B1
	(ii)(c)	$\angle HCF = 180° - 46°$ $= 134°$ (opp angles of cyclic quad)	 B1
	(ii)(d)	$\angle FOD = 180° - 66°$ (Angles on straight line) $= 114°$ Reflex $\angle FOD = 246°$ (Angles at a pt) $\angle FED = 246° \div 2$ (Angle at centre = 2 x angle at circumference) $= 123°$	 M1 A1

Answer all the questions

For Examiner's use

1. By rounding each number to 1 significant figure, estimate, to the nearest hundred, the value of $\frac{17543 \times 9.32}{714.2 + 103}$.

Answer _____ [2]

For Examiner's use

2. (a) Write down all the irrational numbers from the set given below.

$$-3,\ 1.2,\ \sqrt[3]{3},\ \frac{3\pi}{2},\ 4.\dot{6}\dot{2},\ -\sqrt{16}.$$

(b) Consider the sequence $1,\ 1,\ \frac{3}{4},\ \frac{1}{2},\ \frac{5}{16},\ \ldots$.

(i) Write down the next term.

(ii) Express in terms of n, the n^{th} term of the sequence.

Answer (a) _____ [1]

(b) (i) _____ [1]

(ii) _____ [1]

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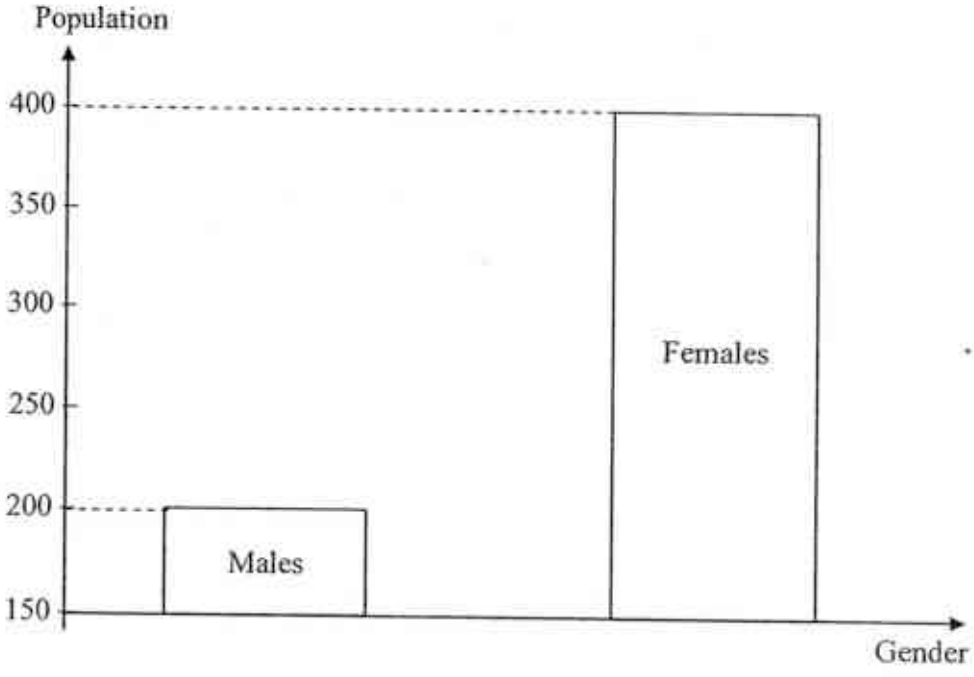
For Examiner's use

3. The population of a small town is surveyed.
The results are shown in the bar chart below.



Explain why the bar chart is considered to be misleading.

Answer _____

_____

_____ [2]

For Examiner's use

4. The sine ratio of an angle is 0.6431.
Find the possible values of the angle.

Answer _____ [2]

For Examiner's use

5. John wrote in his Math journal that it is not possible to form a regular polygon with an interior angle of 130°.
Explain why and show all essential working clearly.

Answer _____

_____

_____

_____

_____

_____ [2]

6. The cash price of a car is $62 400.
The hire purchase price is a deposit of 20% of the cash price and equal monthly instalments for 6 years.

(a) Agnes bought the car at the hire purchase price.
If she paid a total of $75 840, calculate her monthly instalment.

(b) If Agnes had bought the car at the cash price, she would have received a 5% discount.
Express the amount saved from the hired purchase price as a percentage of the hire purchase price.
Give your answer correct to the nearest percent.

Answer (a) $ _____ [1]

(b) _____ % [2]

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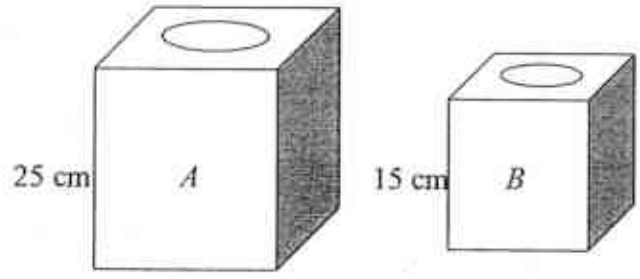
For Examiner's use

7. The heights of two geometrically similar containers are 25 cm and 15 cm respectively.



If the container A is filled with rice, and Benny wishes to transfer all of the rice to container B, how many of container B are required, assuming each container is filled with the rice?

Answer _____ [3]

8. Charles just returned from his trip to London.
He planned to travel to Canada in two weeks' time.
He had £2000 left and wanted to change part of it to C$2000.
The balance was changed to Singapore dollars (S$).
He went to a money changer and saw the following currency table.

Singapore dollar (S$) to 1 unit of foreign currency	**Currency**	**Buying**	**Selling**
	Sterling Pound (£)	2.1059	2.1184
	Canadian Dollar (C$)	1.0726	1.0875

Calculate the amount of Singapore dollars he changed.

Answer S$ _____ [3]

For Examiner's use

9. A grocer bought p dozens of apples at q dollars.
He found that r apples were rotten.

(a) Express, in terms of p and r, the number of good apples he had.

(b) If he sold the good apples at 3 for t dollars, write an algebraic expression for his profit if he sold all his apples.

Answer (a) _____ [1]

(b) $ _____ [2]

10. Darby invested $5 400 in a bank that pays m% interest per annum which is compounded half yearly.

If she received $5 847.89 after $2\frac{1}{2}$ years, calculate the value of m.

Answer $m =$ _____ [3]

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For Examiner's use

11. Two variables G and H are related such that G is inversely proportional to the square of H.
If H is increased by 400%, find the percentage change in G.

Answer _____ % [3]

12. It is given that
$\xi = \{x : x \text{ is an integer such that } 1 \le x \le k\}$,
$M = \{x : x \text{ is a prime number}\}$, and
$N = \{x : \text{the unit digit of } x \text{ is } 7\}$.

Given that $n(M \cap N) = 4$,

(a) state the maximum value of k,

(b) find maximum $n(M \cup N)$,

(c) find the element x such that $x \in N$ and $x \notin M$.

Answer (a) _____ [1]

(b) _____ [1]

(c) _____ [1]

For Examiner's use

13. Shinkansen trains, Japan's high speed bullet trains, can reach a speed of up to 320 km/h.

(a) Express 320 km/h in m/s.

(b) The distance between Tokyo and Kyoto is 513.6 km.
Calculate the time taken, in hours and minutes, to travel from Tokyo to Kyoto by the Shinkansen train.

Answer (a) _____ m/s [1]

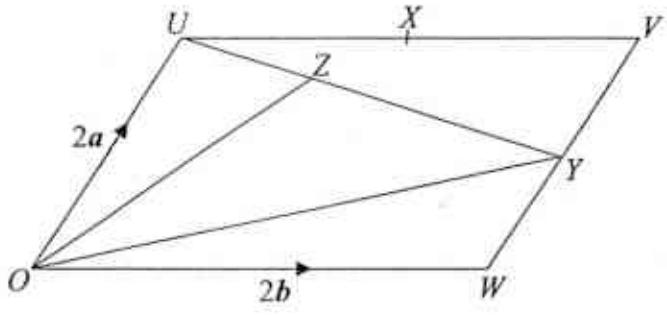
(b) _____ seconds [2]

14. Express $\frac{3}{d+3e} - \frac{d-15e}{d^2-9e^2}$ as a single fraction in its simplest form.

Answer _____ [3]

For Examiner's use

15. In the diagram, $OUVW$ is a parallelogram.
X and Y are midpoints on UV and VW respectively.
Z is a point on UY such that $5UZ = 2UY$.
$\overrightarrow{OU} = 2\boldsymbol{a}$ and $\overrightarrow{OW} = 2\boldsymbol{b}$.



(a) Express, in terms of $\boldsymbol{a}$ and/or $\boldsymbol{b}$,

(i) $\overrightarrow{OX}$,

(ii) $\overrightarrow{OZ}$.

(b) Prove that OZ, when produced, will pass through X.

Answer (a) (i) _____ [1]

(ii) _____ [2]

(b) _____

_____

_____ [1]

For Examiner's use

16. (a) Simplify $\left(\frac{z^8}{196}\right)^{-\frac{1}{2}}$, leaving your answer in positive index.

(b) If $2^x 3^y = 3^9 + 3^8$, find the value of x and of y.

Answer (a) _____ [1]

(b) $x =$ _____, $y =$ _____ [3]

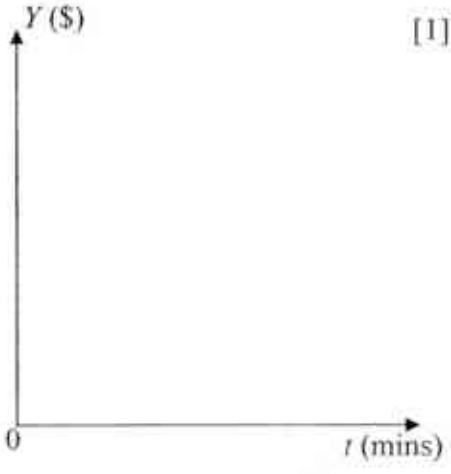
17. The available balance, \$$Y$, of a pre-paid mobile card changes with the duration of local calls made in t minutes.
The initial value of the card was \$38.
After a total of 1.25 hours of local calls, the balance in the card is \$25.25.

(a) Sketch the graph of Y against t.

(b) Calculate the gradient of the graph and explain its significance.

(c) Determine the available balance if local calls of another 100 minutes are made.

Answer (a)



[1]

(b) The gradient is _____ and it represents _____

_____. [2]

(c) \$ _____ [1]

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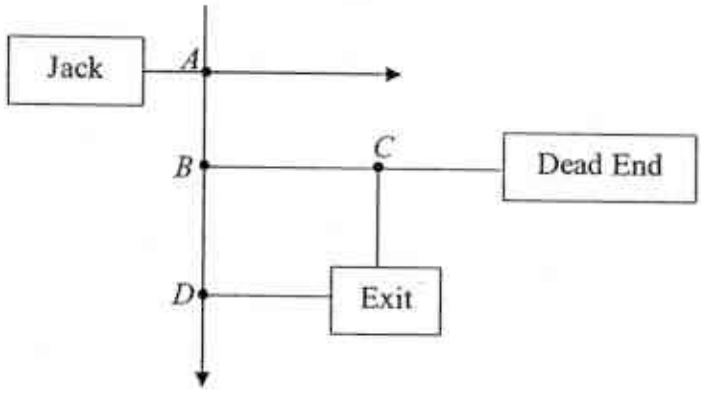
For Examiner's use

18. The diagram shows four junctions A, B, C and D in the game '*Maze Runner*' played at a carnival.



Jack decided to participate in the game.
He had to find the exit in the shortest time to win a laptop.
Once Jack runs pass a junction, he is not able to make a turnaround.

The probability that Jack goes straight at every junction is $\frac{2}{5}$.

Calculate the probability that Jack

(a) hits the dead end,

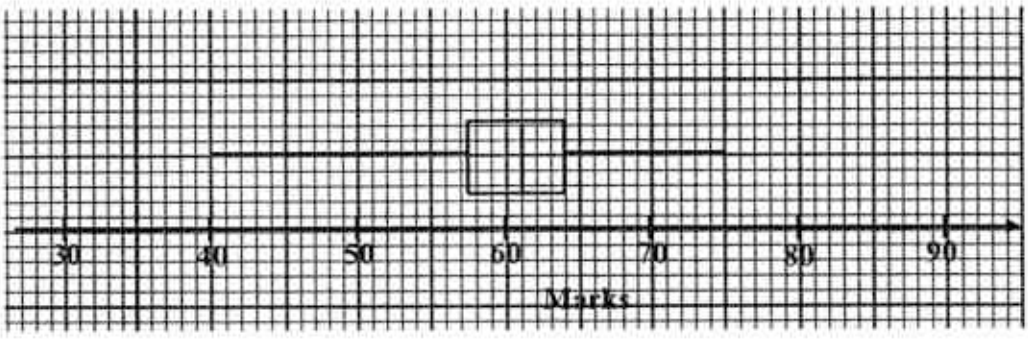
(b) wins the laptop.

Answer (a) _____ [1]

(b) _____ [3]

For Examiner's use

19. A group of students from Class A took a Mathematics test and their results are shown in the box-and-whisker plot.



(a) Find the

(i) median score,

(ii) inter-quartile range of Class A.

(b) Another group of students from Class B took the same test and their results are summarised in the table below.

Median	65 marks
Minimum score	55 marks
Maximum score	85 marks
Lower quartile	60 marks
Upper quartile	75 marks

State two differences in the box-and-whisker plot for Class B, as compared to that of Class A.

Answer (a) (i) _____ [1]

(ii) _____ [1]

(b) 1st difference : _____

_____

2nd difference : _____

_____ [2]

For Examiner's use

20. When a group of children shared a box of 200 beads equally, there were 4 beads left.
When the remaining beads were added to another box of 80 beads and shared equally, there was no remainder.

Calculate the number of beads left if 250 beads are shared equally among the children.

Answer _____ [5]

21. A cylindrical tank is filled with water to three-quarter of its height.
Water is then pumped into the tank at a rate of 20 millimetres per second.
However, there is water leaking through a hole at the base at a rate of 10 millilitres per second.
After 1.5 minutes, there are 5.7 litres of water in the tank.

Calculate how much more time needed to fill the tank completely.

Answer _____ seconds [5]

For Examiner's use

22. Lisa and Nancy wanted to surprise their parents by buying them a present for their 15th wedding anniversary.
Every day, they saved the balance of their pocket money in their piggy banks.
After 3 months, they opened their piggy banks to count how much they have saved.
The number of coins they have saved is given in the table below.

	50 cents	20 cents	10 cents
Lisa	30	35	40
Nancy	24	40	50

(a) Write a matrix S of order 2×3 to represent the above data.

(b) (i) Write a matrix T to represent the denomination of the coins such that ST exists.
Hence evaluate ST.

(ii) Explain the significance of ST.

(c) If the girls have planned a budget of \$50 for the gift, using matrix multiplication, determine if they can achieve this target.
Explain your reasons.

Answer (a) $S =$ _____ [1]

(b) (i) $T =$ _____ [1]

$ST =$ _____ [1]

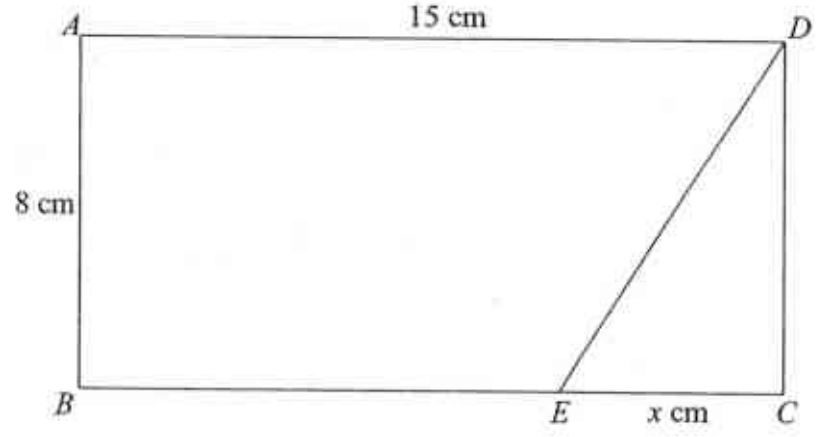
(b) (ii) _____

_____ [1]

(c) _____

_____ [2]

23. The diagram below shows $ABCD$ as the cross section of a solid cuboid. $AB = 8$ cm, $AD = 15$ cm and $EC = x$ cm.



(a) Explain why $\angle ADE = \angle CED$.

(b) Write down an expression, in terms of x, for the area of quadrilateral $ABED$.

(c) Given that the area of triangle DEC is $\frac{1}{6}$ of the area of quadrilateral $ABED$, express EC as a percentage of BC.

Answer (a) _____ [1]

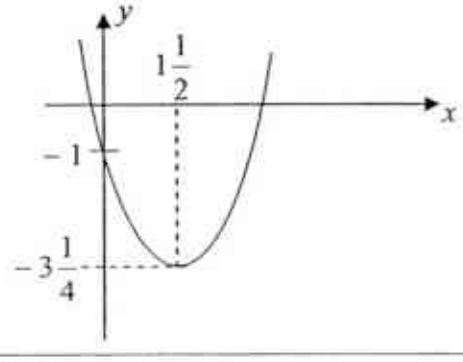
(b) _____ cm^2 [2]

(c) _____ % [3]

~ End of Paper 1 ~

Answer

Qn	Asnwer
1	-0.010
2	$=(2a+5b+2c)(2a+5b-2c)$
3a	2.63×10^{8} m
3b	1 hr 10 min 8 sec
4	$-1\frac{3}{4}$
5ai	3
5aii	2
5b	$H \subset I$
6a	$\therefore 3\frac{1}{4} \le k < 22$
6b	16
7ai	4, 10, 20, 34
7aii	$2n^2+2=2(n^2+1)$, which contains a factor of 2. Or $2n^2$ is an even number and when added to 2 (an even number), $2n^2+2$ is even.
7b	$2n^2-n-2$
8	$\frac{-5}{x-2}$ or $\frac{5}{2-x}$
9a	$\frac{8}{17}$
9b	$-\frac{8}{17}$
10a	18
10b	$\frac{1}{2}$
11a	$180=2^2\times3^2\times5$
11b	20 cm
11c	45
12 a	$0 \le x \le 8$ or $0 \le x < 9$
12b	11
12c	16

Qn	Answer
13a	15 km
13b	10 cm^2
13c	1 : 25 000
14	See attached
14cii	11.4 m $\le y <$ 14.6 m
15ai	e.g. $y=-\frac{1}{2}x-1$ negative gradient with numerical value less than 1, y-intercept of -1
15aii	$y=a^{-x}+2$
15bi	$\left(x-1\frac{1}{2}\right)^2-3\frac{1}{4}$
15bii	
16ai	210°
16aii	30°
16b	$\angle BAR = (180° - 150°) \div 2 = 15°$ (base $\angle$ of isos. Δ) $\angle PAR = 45° + 120° + 15° = 180°$ $\therefore PAR$ is a straight line Alternatively $\angle BAR = (180° - 150°) \div 2 = 15°$ (base $\angle$ of isos. Δ) $\angle QAR = 150° - 15° = 135°$ $\angle PAQ + \angle QAR = 45° + 135° = 180°$
16c	4

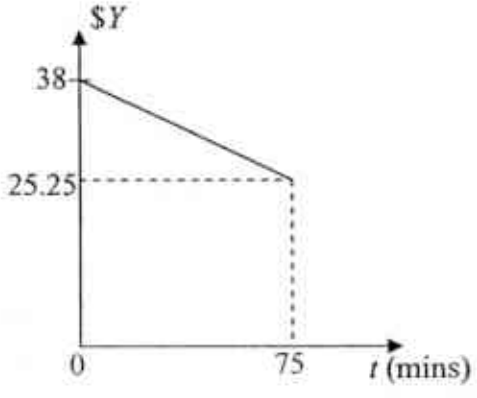
PRELIMINARY EXAM 2015

SECONDARY 4 EXPRESS 5 NORMAL (ACADEMIC)

Mathematics Paper 1

Qn	Solution and Answer	Marks allocation
1	$\approx \frac{20000 \times 9}{700+100} = \ldots$ ≈ 200 (to nearest hundred)	**M1:** rounding off each number to 1 s.f. **A1**
2(a)	$\sqrt[3]{3}$, $\frac{3\pi}{2}$	**B1**: c.a.o
2(bi)	$\frac{6}{2^5} = \frac{3}{16}$	**B1**
(ii)	$\frac{n}{2^{n-1}}$	**B1**
3	From the bar chart, the number of females in the small town seems to be 5 times that of the male, when it is actually just twice instead.	**B1** **B1**
4	Let the angle be θ $\sin\theta = 0.6431 \Rightarrow \theta = \sin^{-1} 0.6431$ $\therefore \theta = 40.0°$ or $140.0°$	**M1** **A1:** both answers
5	If each interior angle = 130°, each exterior angle = 50° $\Rightarrow$ number of sides $= \frac{360°}{50°} = 7.2$, which is not possible. Hence it is not possible to have a regular polygon with an interior angle of 130°.	**B1:** each exterior angle **B1:** number of sides not integer value
6(a)	$\frac{75840 - 0.2 \times 62400}{6 \times 12} = \frac{63360}{72} = \$880$	**B1**
6(b)	Amount saved = $\$75840 - 0.95 \times 62400 = \$16560$ $\therefore$% saved $= \frac{16560}{75840} \times 100\% = 21.83\ldots \approx 22\%$	**M1** **A1**: final answer
7	$\frac{V_A}{V_B} = \left(\frac{25}{15}\right)^3 = \frac{125}{27}$ $\therefore$No. of filled smaller containers $= \frac{125}{27} = 4.629\ldots \approx 5$	**M1**: ratio of the 2 volumes **M1, A1**
8	£2000 = 2000×2.1059 = S$4211.80 C$2000 = 2000×1.0875 = S$2175 $\therefore$Amount of S$ kept = 4211.80 − 2175 = S$2036.80	**M1** **M1** **A1**
9(a)	$12p - r$	**B1**
9(b)	Total selling price $= \frac{12p-r}{3} \times \$t = \$\left(\frac{12pt-rt}{3}\right)$ $\therefore$Profit $= \$\left(\frac{12pt-rt}{3} - q\right)$	**M1** **B1**

Qn	Solution and Answer	Marks allocation
10	$5847.89 = 5400\left(1+\frac{m/2}{100}\right)^5$ $1+\frac{m}{200} = \sqrt[5]{\frac{5847.89}{5400}}$ $\therefore m = 3.21$	**M1:** using compound interest formula **M1:** simplification **A1**
11	$G = \frac{k}{H^2}$ When H is increased by 400%, $H_{new} = 5H_{original}$ $\Rightarrow G_{new} = \frac{k}{(5H_{original})^2} = \frac{k}{25H_{original}^2}$ $G_{new} = \frac{1}{25}G_{original} \quad \Rightarrow \frac{G_{new}}{G_{original}} = \frac{1}{25} = \frac{4}{100}$ % change in $G = \frac{4-100}{100} \times 100\% = -96\%$	**M1:** finding new G **M1:** ratio of the 2 G values **A1**
12(a)	$M = \{x : x \text{ is a prime number}\}$ and $N = \{x : \text{the unit digit is 7}\}$ Since $n(M \cap N) = 4 \Rightarrow M \cap N = \{7,17,37,47\}$ $\Rightarrow M = \{2,3,5,7,11,...,47,53\}$ and $N = \{7,17,27,37,47\}$ $\therefore$ maximum of $k =$ 56	**B1**
12(b)	Maximum $n(M \cup N) = 17$ ..	**B1**
12(c)	$x = 27$	**B1**
13(a)	$320\text{km/h} = \frac{320000m}{3600s} = 88\frac{8}{9}\text{m/s}$	**B1**
13(b)	$\frac{513.6}{320} = 1.605\text{ h} = 1\text{ h } 36.3\text{ mins}$	**M1, A1**
14	$\frac{3}{d+3e} - \frac{d-15e}{d^2-9e^2} = \frac{3(d-3e)-d+15e}{(d+3e)(d-3e)}$ $= \frac{2d+6e}{(d+3e)(d-3e)}$ $= \frac{2}{d-3e}$	**M1:** LCM as common denominator **M1:** as 1 fraction **A1:** simplest form
15(ai)	$\overrightarrow{OX} = \overrightarrow{OU} + \overrightarrow{UX} = \overrightarrow{OU} + \frac{1}{2}\overrightarrow{UV} = 2\boldsymbol{a} + \boldsymbol{b}$	**B1**
15(aii)	$\overrightarrow{UY} = \overrightarrow{UV} + \overrightarrow{VY} = \overrightarrow{OU} + \frac{1}{2}\overrightarrow{VW} = 2\boldsymbol{b} - \boldsymbol{a}$ $\overrightarrow{UZ} = \frac{2}{5}\overrightarrow{UY} = \frac{2}{5}(2\boldsymbol{b} - \boldsymbol{a})$ $\overrightarrow{OZ} = \overrightarrow{OU} + \overrightarrow{UZ} = 2\boldsymbol{a} + \frac{2}{5}(2\boldsymbol{b} - \boldsymbol{a}) = \ldots = \frac{8}{5}\boldsymbol{a} + \frac{4}{5}\boldsymbol{b}$	**M1:** $\overrightarrow{UZ}$ **A1**

Qn	Solution and Answer	Marks allocation
15(aiii)	$\overrightarrow{OZ} = \frac{8}{5}\boldsymbol{a} + \frac{4}{5}\boldsymbol{b} = \frac{4}{5}(2\boldsymbol{a} + \boldsymbol{b}) = \frac{4}{5}\overrightarrow{OX}$ Since $\overrightarrow{OZ}$ and $\overrightarrow{OX}$ are in the same direction, O, Z and X are collinear points. $\therefore$ OZ when produced will pass through X. **Alternative approach**: Find $\overrightarrow{ZX} = \frac{2}{5}\boldsymbol{a} + \frac{1}{5}\boldsymbol{b} = \frac{1}{5}(2\boldsymbol{a} + \boldsymbol{b})$	**B1**
16(a)	$\left(\frac{z^8}{196}\right)^{-\frac{1}{2}} = \left(\frac{196}{z^8}\right)^{\frac{1}{2}} = \frac{14}{z^4}$	**B1**
16(b)	$2^x 3^y = 3^9 + 3^8 = 3^8(3+1) = 3^8 \times 2^2$ $\therefore x = 2,\ y = 8$	**M1** **A1 each**
17(a)		**B1:** linear graph with given information shown clearly
17(b)	Gradient $= -\frac{12.75}{75} = -\frac{17}{100}$ It represents the cost deducted per minute for local calls.	**B1** **B1**
17(c)	Available balance in card $= 38 - \frac{17}{100} \times (175) = \$8.25$	**B1**
18(a)	P (hits the dead end) = P (turns at A, turn at B, straight at C) $= \frac{3}{5} \times \frac{3}{5} \times \frac{2}{5} = \frac{18}{125}$	**M1, B1**
18(b)	P (wins the laptop) = P (hits exit) = P (turns at A, straight at B and turns at D, or turns at A, B and C) $= \frac{3}{5} \times \frac{2}{5} \times \frac{3}{5} + \left(\frac{3}{5}\right)^3 = \frac{9}{25}$	**M2, A1**
19(ai)	Median = 61 marks	**B1**
19(aii)	IQR = 64 – 57.5 =6.5 marks	**B1**
19(b)	For Class B: IQR = 75 – 60 =15 marks 1st difference: the length of the box is longer as the IQR is wider. 2nd difference: the left whisker is shorter as the difference between the lower quartile and minimum value is smaller.	**B1** **B1**

Qn	Solution and Answer	Marks allocation
20	$196 = 2^2 \times 7^2$ and $84 = 2^2 \times 3 \times 7$ No. of children in the group = HCF of 196 and 84 = $2^2 \times 7 = 28$ No. of beads each child has if 250 beads are shared equally $= \frac{250}{28} = 8.92... \approx 8$ $\therefore$ No. of beads left = $250 - 28 \times 8 = 26$	**M2:** prime factorisation **M1:** no. of children **M1:** no. of beads each child has **A1**
21	Pumped in 20 ml/sec and leaked 10 ml/sec $\Rightarrow$ 10ml of water added in 1 sec $\Rightarrow$Amount of water added in 1.5 mins = $1.5 \times 60 \times 10 = 900$ ml Initial volume of water in tank = $5700 - 900 = 4800$ ml $\Rightarrow$ Capacity of tank = $\frac{4}{3} \times 4800 = 6400$ ml $\therefore$ Time needed to fill tank = $\frac{6400 - 5700}{10} = 70$ sec	**M1** **M1** **M1** **M1** **A1**
22(a)	$\boldsymbol{S} = \begin{pmatrix} 30 & 35 & 40 \\ 24 & 40 & 50 \end{pmatrix}$	**B1**
22(bi)	$\boldsymbol{T} = \begin{pmatrix} 50 \\ 20 \\ 10 \end{pmatrix}$ $\boldsymbol{ST} = \begin{pmatrix} 30 & 35 & 40 \\ 24 & 40 & 50 \end{pmatrix}\begin{pmatrix} 50 \\ 20 \\ 10 \end{pmatrix} = ... = \begin{pmatrix} 2600 \\ 2500 \end{pmatrix}$	**B1:** matrix $\boldsymbol{T}$ **B1:** $\boldsymbol{ST}$
22(bii)	The total amount of money, in cents, saved by each girl in 3 months	**B1**
22(c)	Let $\boldsymbol{M} = \begin{pmatrix} 1 & 1 \end{pmatrix}$ Total amount saved by both girls = $\boldsymbol{MST} = \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 2600 \\ 2500 \end{pmatrix}$ $= \begin{pmatrix} 5100 \end{pmatrix}$ The girls can achieve this target as they saved a total of \$51.	**M1** **B1**
23(a)	$\angle ADE = \angle CED$ as they are alternate angles since AD and EC are parallel to each other.	**B1**
23(b)	Area of quadrilateral $ABED = \frac{1}{2} \times [15 + (15 - x)] \times 8$ $= 4(30 - x) = 120 - 4x$	**M1** **A1**
23(c)	Since area of triangle DEC is $\frac{1}{6}$ the area of $ABED$, $\Rightarrow \frac{1}{2} \times 8 \times x = \frac{1}{6}(120 - 4x) \quad \Rightarrow 24x = 120 - 4x$ $\Rightarrow 28x = 120 \quad \Rightarrow x = 4\frac{2}{7}$ $\therefore$ EC as a percentage of BC $\frac{4\frac{2}{7}}{15} \times 100\% = 28\frac{4}{7}\%$	**M1:** form eqn in x **M1:** value of x **A1**

聖 嬰 中 學

HOLY INNOCENTS' HIGH SCHOOL

Name of Student

Class

Index Number

100

PRELIMINARY EXAMINATION 2015
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)
MATHEMATICS PAPER 2

4016 / 02

Date: **31 July 2015**

Duration: **2 h 30 min**

Additional Materials: 1 string
1 sheet of graph paper
6 sheets of writing paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction tape/fluid.

Answer **ALL** questions.

The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown in the space below the question.

Omission of essential working will result in loss of marks.

The total of the marks for this paper is **100**.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

Set by: Ms Lua Bee Hian
Vetted by: Mdm Hayati & Ms Goh Lay Ching

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Geometry and Measurement

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{ , where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1. **(a)** Solve the inequality $1 > \frac{7x+2}{3}$. [2]

(b) **(i)** Factorise completely $x^2 + 4xy + 4y^2 - 9$. [2]

(ii) Simplify $\frac{2x^2 + 2x - 4}{x+2}$. [2]

(c) It is given that $v^2 = u^2 + 2as$.

(i) Find v^2 when $u = 0$, $a = 9.8$ and $s = 3$. [1]

(ii) Express u in terms of a, s and v. [2]

---

2. Food waste is one of the top waste types in Singapore.

The table shows an overview of the waste figures for food waste in megatonnes in 2014.

Waste Type	Waste Disposed	Waste Recycled	Waste Generated
Food waste	0.6872	0.1014	0.7886

(a) Convert the total amount of food waste generated into tonnes.
Give your answer in standard form. [2]

(b) Calculate the percentage of the food waste that was recycled. [2]

(c) The population of Singapore in 2014 was 5.47 million.

Calculate the average amount of food waste generated by per person, in tonnes.
Give your answer correct to 3 significant figures. [2]

(d) The amount of food waste generated in Singapore has increased by about 48 per cent over the past 10 years.

Calculate the amount of food waste generated, in tonnes, in 2005.
Give your answer correct to the nearest kilotonnes. [2]

(e) Food waste accounts for about 10.5 per cent of the total waste generated in Singapore. In 2014, 2.7 per cent of the total waste generated was used as fuel in biomass plants.

Calculate the amount of waste, in tonnes, that was used as fuel in biomass plants.
Give your answer correct to 3 significant figures. [2]

3. Answer the whole of this question on a sheet of graph paper.

A pebble was thrown from the top of a vertical building.

Its position during the flight is represented by the equation $y = 30 + 4x - x^2$, where y metres is the height of the pebble above the ground and x metres is its horizontal distance from the foot of the building.

Some corresponding values of x and y are given in the table below.

x	0	1	2	3	4	5	6	7
y	30	33	34	33	30	25	k	9

(a) Find the value of k. [1]

(b) Using a scale of 2 cm to represent 1 metres, draw a horizontal x-axis for $0 \le x \le 7$.
Using a scale of 2 cm to represent 5 metres, draw a vertical y-axis for $0 \le y \le 40$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

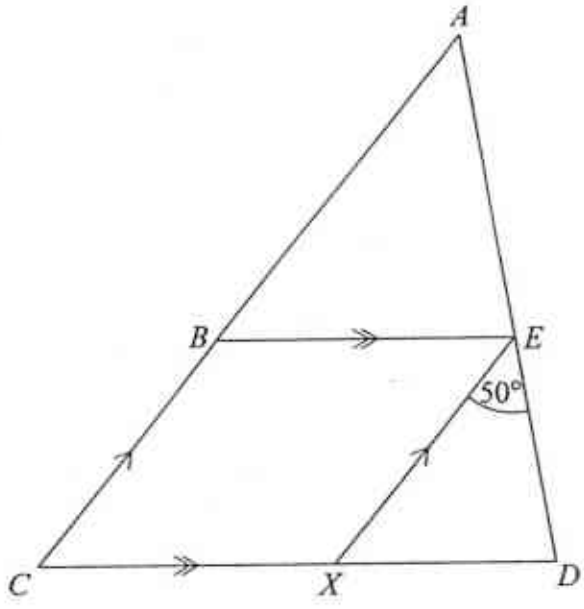
(c) Use your graph to find the greatest height reached by the pebble. [1]

(d) Find the horizontal distances of the pebble when $y = 33.5$ m. [2]

(e) **(i)** By drawing a tangent, find the gradient of the graph at the point (5, 25). [2]

(ii) Use your answer to (e)(i) to explain what was happening to the stone at $x = 5$. [1]

4.



The diagram shows triangle ACD.
$BCXE$ is a parallelogram such that XE bisects angle BED and $\angle XED = 50°$.

(a) Find angle EDX. [1]

(b) Show that triangle ABE is similar to triangle EXD. [2]

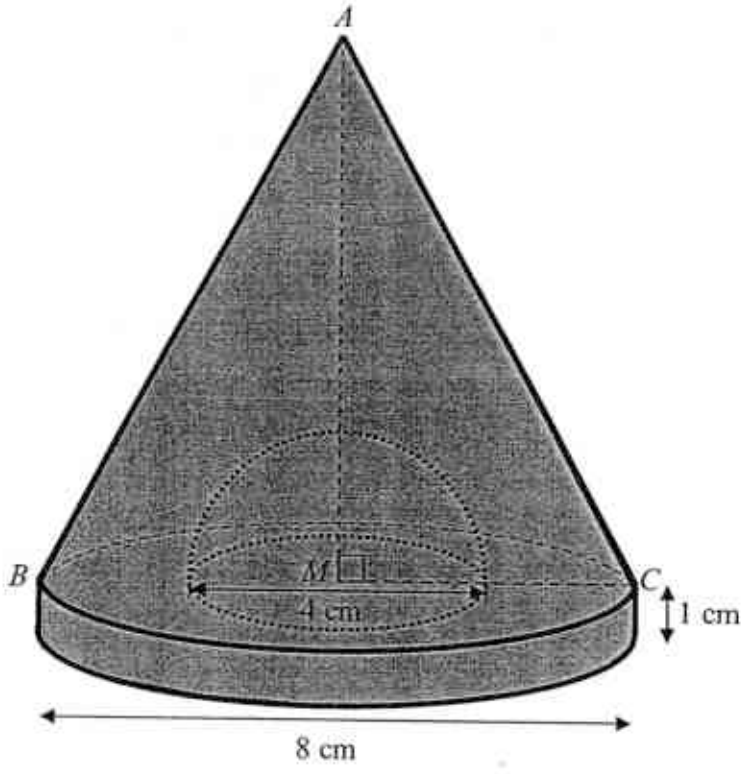
(c) State another triangle that is similar to ABE and EXD. [1]

(d) The ratio $AE : ED = 4 : 3$.

(i) Find the ratio $CD : XD$. [1]

(ii) Given that the area of triangle $ABE = 36\ \text{cm}^2$, find the area of triangle BXE. [3]

5.



The figure shows a trophy that is made up of a circular base and conical top.
The circular base of the trophy has a diameter of 8 cm and thickness of 1 cm.
A hemisphere of diameter 4 cm is removed from the centre of the trophy as shown in the diagram.
The ratio $AM : MC = 3 : 2$.
[Take $\pi = 3.142$]

(a) Show that the angle $ACM = 56.3°$, correct to 3 significant figures. [2]

(b) Find the height of the trophy. [2]

(c) The exterior surface area of the trophy is coated with rose gold.
Find the total surface area that is coated with rose gold. [4]

(d) Find the volume of the trophy. [3]

6. **(a)** C is the point $(-3, 5)$ and D is the point $(4, 3)$.

(i) Write down the column vector $\overrightarrow{CD}$. [1]

(ii) Find $\left|\overrightarrow{CD}\right|$. [2]

(iii) Given that $4\overrightarrow{DE} = 3\overrightarrow{DC}$, find the coordinates of point E. [2]

(b) The position vector of a point Q is $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\overrightarrow{QR} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$.

(i) Find the equation of the line QR. [2]

(ii) The equation of the line ST is $4y - 3x = 18$.

Find the coordinates of the point of intersection of QR and ST. [3]

---

7. A printer prints pages on either single or double sides.

(a) In one minute, it prints x pages on single side.

Write down an expression, in terms of x, for the number of seconds it takes to print one page on single side. [1]

(b) In one minute, it prints 3 more copies on single side than it does on double sides. Write down an expression, in terms of x, for the number of seconds it takes to print one page on double sides. [1]

(c) It takes 4 seconds shorter to print single side than it takes to print double sides.

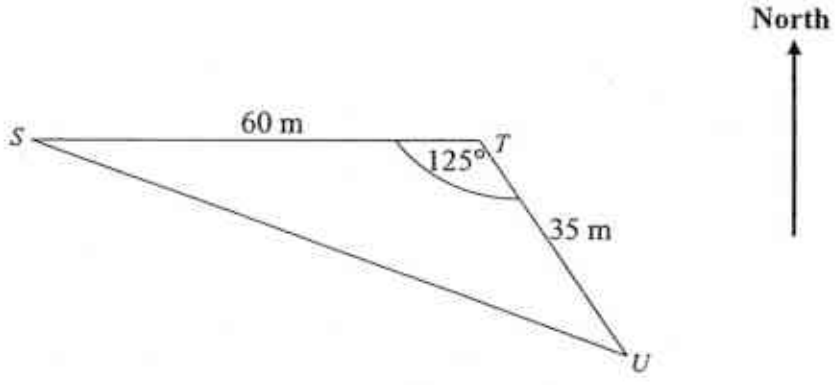
(i) Write down an equation in x to represent this information, and show that it reduces to $x^2 - 3x - 45 = 0$. [3]

(ii) Solve the equation $x^2 - 3x - 45 = 0$, giving your solutions correct to 1 decimal places. [3]

(iii) Find the time taken to print 20 pages on double sides. [2]

216

8.



The positions of 3 islands on a map are represented by S, T and U as shown in the diagram.
Angle $STU = 125°$ and island T is due east of island S.
$ST = 60$ m and $TU = 35$ m.

(a) Show that $SU = 85.1$ m. [3]

(b) Calculate the bearing of T from U. [2]

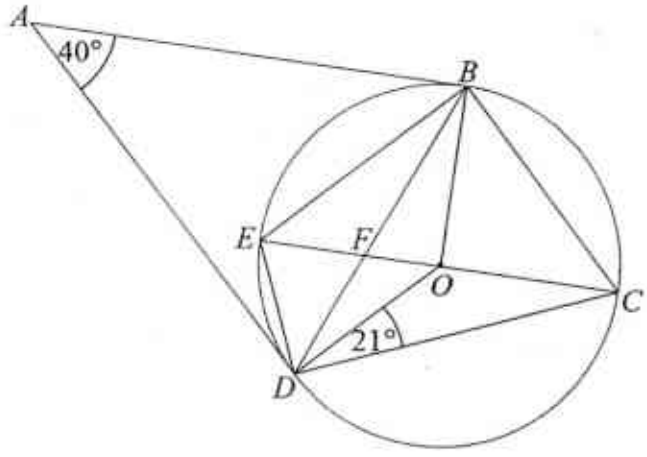
(c) The area of the triangle STU is 860.11 m^2.
Find the shortest distance to sail from island T to SU. [2]

(d) An eagle is hovering directly above T.
The angle of depression of point U viewed from the eagle is $65°$.

(i) Find the distance between the eagle and point T. [2]

(ii) Calculate the greatest possible angle of elevation of the eagle when viewed from a ship that sails along SU. [2]

9.



(a) The diagram shows a circle $BCDE$, centre O.
F is a point of intersection of BD and diameter EC.
AB and AD are tangents to the circle such that angle $BAD = 40°$.
Angle $ODC = 21°$.

Find, giving reasons for each answer,

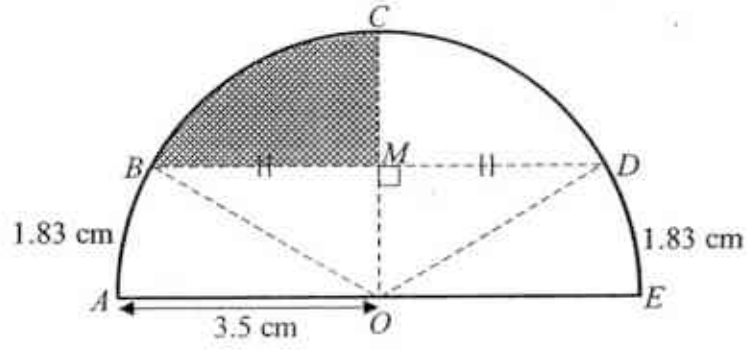
(i) obtuse angle BOD, [1]

(ii) angle BED, [2]

(iii) angle ODF, [1]

(iv) angle ABE. [1]

(b)



$OABCDE$ is a semi circle, centre O, of radius 3.5 cm.
M is the midpoint of BD.
Length of arc AB = Length of arc DE = 1.83 cm.

(i) Show that angle $BOD = 2.096$ radians. [2]

(ii) Find the area of the shaded region. [3]

2.7

10. **(a)** 40 students from Class 4 Excel were surveyed on the number of hours they spent on revision the previous week.
The results of this survey are summarised in the table below.

No. of hours spent on revision (x)	Number of people (f)	fx
0	1	0
1	3	3
2	2	4
3	a	18
4	2	8
5	b	c
6	10	60
7	9	63
8	3	24
Total	40	d

(i) Find

(a) the values of a, b, c and d, [2]

(b) the mean, [1]

(c) the standard deviation. [2]

(b) In a group, there are 15 students with dimples and 9 students without dimples. Two students are selected at random.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

(ii) Find the probability that

(a) both students selected are with dimples, [1]

(b) one student with dimples and the other without dimples are selected. [1]

(iii) A third student is selected.
Find the probability that at least one of the students selected is with dimples. [2]

~ End of paper ~

Answers:

Q1. (a) $x < \frac{1}{7}$

(b) (i) $(x+2y+3)(x+2y-3)$

(ii) $2x-2$

(c) (i) $v^2 = 58.8$

(ii) $u = \pm\sqrt{v^2 - 2as}$

Q2. (a) 7.886×10^5 tonnes

(b) 12.9%

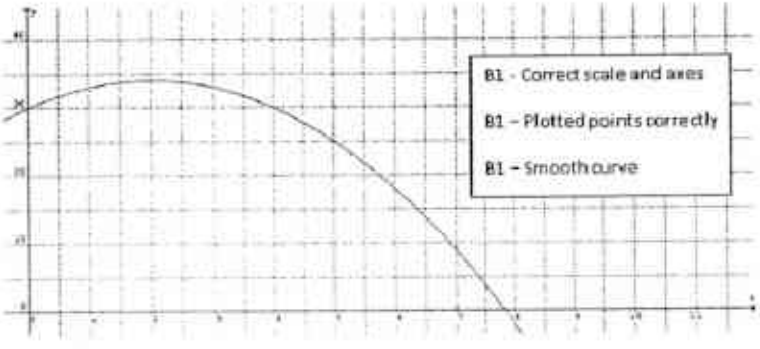
(c) 0.144 tonnes

(d) 533 kilotonnes

(e) 203000 tonnes

Q3. (a) $k = 18$

(b)



(c) 34 m

(d) 1.3 m and 2.7 m

(e) (i) gradient = -6 ± 0.5

(ii) The height of the pebble above the ground decreases at a rate of 6 m for every 1 m it travels horizontally / falls.

Q4. (a) $\angle EDX = 80°$

(b) $\angle AEB = \angle EDX = 80°$ (corresponding angles)
$\angle ABE = \angle EXD = 50°$ (alternate angles)
$\angle BAE = \angle XED = 50°$ (sum of angles in triangle). $\therefore \Delta ABE$ is similar to ΔEXD.

(c) ΔACD

(d) (i) 7 : 3

(ii) Area of $\Delta BXE = 27\ cm^2$

Q5. (a) $\tan \angle ACM = \frac{3}{2}$

$\angle ACM = \tan^{-1}\left(\frac{3}{2}\right)$
$= 56.309932$
$\approx 56.3°$ (shown)

(b) Height of trophy = 7 cm

(c) $166\ cm^2$

(d) $134\ cm^3$

Q6. (a) (i) $\overrightarrow{CD} = \begin{pmatrix} 7 \\ -2 \end{pmatrix}$

(ii) 7.28

(iii) $(-1.25, 4.5)$

(b) (i) $3y = 5 - 2x$

(ii) $(-2, 3)$

Q7. (a) $\frac{60}{x}$

(b) $\frac{60}{x-3}$

(c) (i) $\frac{60}{x-3} - \frac{60}{x} = 4$

$60x - 60(x-3) = 4x(x-3)$
$60x - 60x + 180 = 4x^2 - 12x$
$4x^2 - 12x - 180 = 0$
$x^2 - 3x - 45 = 0$ (shown)

(ii) $x \approx 8.4$ or $x \approx -5.4$

(iii) 223 seconds

Q8. (a)

$SU^2 = 60^2 + 35^2 - 2(60)(35)\cos 125°$
$SU^2 = 6^2 + 3.5^2 - 2(6)(3.5)\cos 125°$
$SU^2 = 7234.0210$
$SU = \sqrt{7234.0210}$
≈ 85.1 km

(b) Bearing of T from $U = 325°$

(c) 20.2 m

(d) (i) 75.1 m

(ii) 74.9°

Q9. (a) (i) 140°

(ii) angle $BED = 110°$

(iii) angle $ODF = 20°$

(iv) angle $ABE = 49°$

(b) (i) $\theta = \frac{s}{r}$

$= 1.83 \div 3.5$

$= \frac{183}{350}$

Angle $BOD = \pi - 2\left(\frac{183}{350}\right)$

≈ 2.096

(shown)

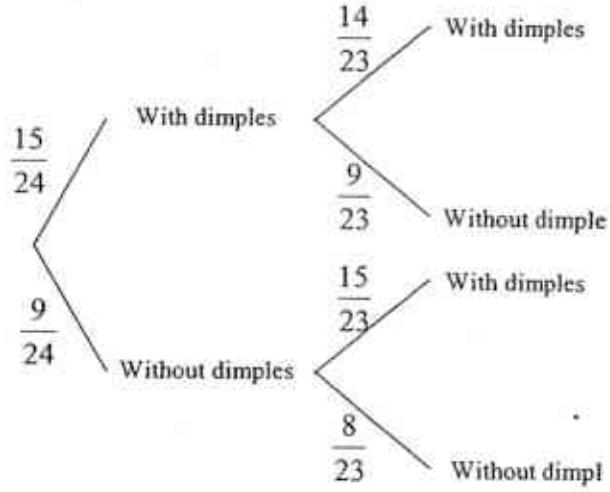
(ii) 3.77 cm^2

10. (a) (i) (a) $a = 6$, $b = 4$, $c = 20$ and $d = 200$

(b) 5

(c) 2.18

(b) (i)



(b) (ii) (a) $\frac{35}{92}$

(b) $\frac{45}{92}$

(iii) $\frac{485}{506}$

Q1. (a) $1 > \frac{7x+2}{3}$

$3 > 7x+2$

$1 > 7x$ [M1]

$7x < 1$

$x < \frac{1}{7}$ [A1]

(b) (i) $x^2+4xy+4y^2-9$

$=(x+2y)^2-9$ [M1]

$=(x+2y+3)(x+2y-3)$ [A1]

(ii) $\frac{2x^2+2x-4}{x+2}$

$=\frac{2(x^2+x-2)}{x+2}$ [M1, factorising numerator correctly]

$=\frac{2(x+2)(x-1)}{x+2}$

$=2x-2$ [A1, accept $2(x-1)$]

(c) (i) $v^2=0^2+2(9.8)(3)$

$v^2=58.8$ [B1]

(ii) $v^2=u^2+2as$

$u^2=v^2-2as$ [M1]

$u=\pm\sqrt{v^2-2as}$ [A1]

Q2. (a) 788600 [M1, convert to tonnes]

$=7.886\times10^5$ tonnes [A1or s.o.i. B2]

(b) Percentage of food waste that was recycled

$=0.1014\div0.7886\times100$ [M1 o.e.]

$=12.85822$

$\approx 12.9\%$ [A1]

(c) Average amount of food waste generated by per person

$=0.7886\div5.47$ [M1 o.e.]

≈ 0.144 tonnes [A1]

(d) Amount of food waste generated in 2005

$=0.7886\div1.48\times10^6$ [M1 o.e.]

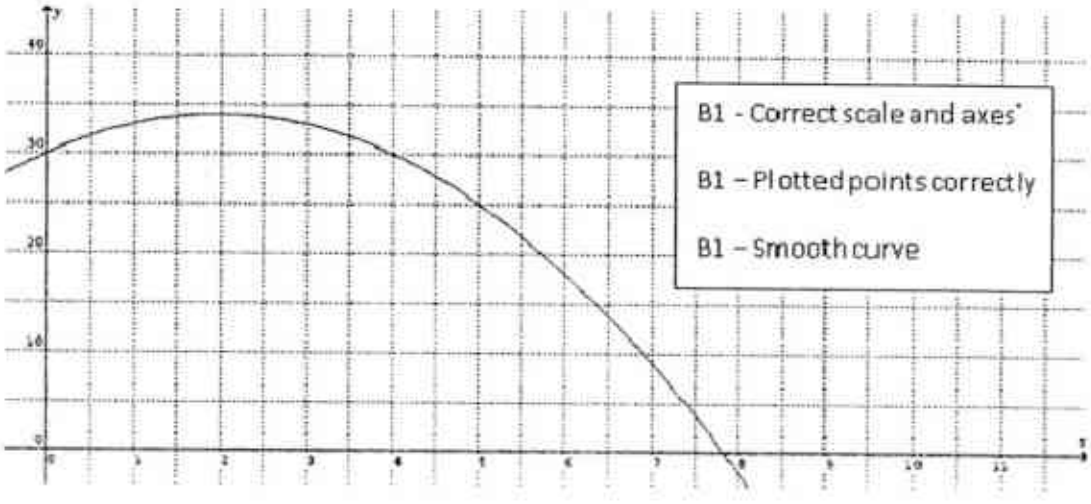
≈ 532838

≈ 533 kilotonnes [A1]

Q2. (e) Amount of waste used as fuel in biomass plants
$= 0.7886 \div 10.5 \times 2.7 \times 10^6$ [M1]
≈ 202782
≈ 203000 tonnes [A1]

Q3. (a) $k = 18$ [B1]

(b)



(c) 34 m [B1]

(d) 1.3 m and 2.7 m [B1 each]

(e) (i) B1 – Draw tangent
B1 – gradient $= -6 \pm 0.5$

(ii) The height of the pebble above the ground decreases at a rate of 6 m for every 1 m it travels horizontally / falls. [B1]

Q4. (a) $\angle EDX = 180° - 50° - 50°$
$= 80°$ [B1]

(b) $\angle AEB = \angle EDX = 80°$ (corresponding angles)
$\angle ABE = \angle EXD = 50°$ (alternate angles)
$\angle BAE = \angle XED = 50°$ (sum of angles in triangle) [M1]

$\therefore \Delta ABE$ is similar to ΔEXD. [A1]

(c) ΔACD [B1]

(d) (i) 7 : 3 [B1]

Q4. (d) (ii) Area of $\Delta EXD = \frac{36 \times 3^2}{4^2}$ [M1]

$= 20.25 \text{ cm}^2$

Area of $\Delta ACD = \frac{36 \times 7^2}{4^2}$ [M1]

$= 110.25 \text{ cm}^2$

Area of $\Delta BXE = \frac{110.25 - 36 - 20.25}{2}$

$= 27 \text{ cm}^2$ [A1]

Q5. (a) $\tan \angle ACM = \frac{3}{2}$ [M1]

$\angle ACM = \tan^{-1}\left(\frac{3}{2}\right)$

$= 56.309932$

$\approx 56.3°$ (shown) [A1]

(b) $\frac{3}{2} = \frac{AM}{4}$

$AM = \frac{3}{2} \times 4$

$= 6 \text{ cm}$ [M1]

Height of trophy = 6 + 1

$= 7 \text{ cm}$ [A1]

(c) $\cos 56.3° = \frac{4}{AC}$

$AC = \frac{4}{\cos 56.3°}$ [M1]

$= 7.209228$

total surface area coated with rose gold

$= 3.142 \times 4 \times \frac{4}{\cos 56.3°} + 2 \times 3.142 \times 4 \times 1 + 3.142 \times 4^2$ [M2, correct substitution]

$= 166.013$

$\approx 166 \text{ cm}^2$ [A1]

(d) Volume of trophy

= Vol. of cone + Vol. of cylindrical base – Vol. of hemisphere

$= \frac{1}{3} \times 3.142 \times 4^2 \times 6 + 3.142 \times 4^2 \times 1 - \frac{2}{3} \times 3.142 \times 2^3$ [M2, minus 1m for one error]

$= 134.05866$

$\approx 134 \text{ cm}^3$ [A1]

Q6. (a) (i) $\overrightarrow{CD} = \begin{pmatrix} 4+3 \\ 3-5 \end{pmatrix}$

$= \begin{pmatrix} 7 \\ -2 \end{pmatrix}$ [B1]

(ii) $\left|\overrightarrow{CD}\right| = \sqrt{7^2 + (-2)^2}$ [M1, allow e.c.f.]

$= \sqrt{53}$

≈ 7.28 [A1]

(iii) $4\overrightarrow{DE} = 3\overrightarrow{DC}$

$\overrightarrow{DE} = \frac{3}{4}\begin{pmatrix} -7 \\ 2 \end{pmatrix}$ [M1, recognised $\overrightarrow{DC} = -\overrightarrow{CD}$]

$\overrightarrow{OE} = \frac{3}{4}\begin{pmatrix} -7 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -1.25 \\ 4.5 \end{pmatrix}$

Coordinates of point E is $(-1.25, 4.5)$ [A1]

(b) (i) $\frac{-4}{6} = \frac{y-3}{x+2}$ [M1]

$-4x - 8 = 6y - 18$

$6y = 10 - 4x$

$3y = 5 - 2x$ [A1 o.e.]

(ii) $4y - 3x = 18$ ------ (1)

$3y = 5 - 2x$ ------ (2)

From (2), $y = \frac{5-2x}{3}$ ------ (3)

Subst. (3) into (1),

$4\left(\frac{5-2x}{3}\right) - 3x = 18$ [M1]

$20 - 8x - 9x = 54$

$17x = -34$

$x = -2$ [A1]

$y = \frac{5+4}{3} = 3$ [A1]

Coordinates of point of intersection is (− 2 , 3).

Q7. (a) $\frac{60}{x}$ [B1]

(b) $\frac{60}{x-3}$ [B1]

(c) (i) $\frac{60}{x-3}-\frac{60}{x}=4$ [M1, formulate equation]

$60x-60(x-3)=4x(x-3)$ [M1, remove denominator]
$60x-60x+180=4x^2-12x$
$4x^2-12x-180=0$
$x^2-3x-45=0$ (shown) [A1]

(ii) $x=\frac{3\pm\sqrt{(-3)^2-4(1)(-45)}}{2(1)}$

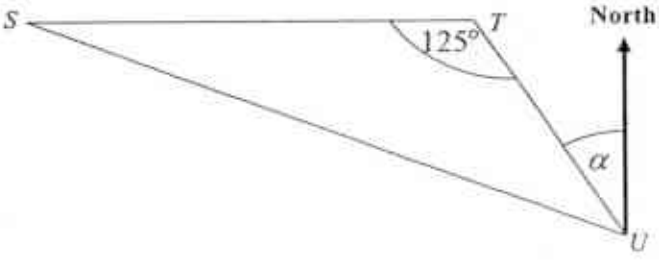
$x=\frac{3\pm\sqrt{189}}{2}$ [M1, for $\sqrt{189}$]

$x=8.37386$ or $x=-5.37386$
$x\approx 8.4$ $x\approx -5.4$ [A2, 1m each]

(iii) time taken to print 20 pages in double side
$=\frac{60}{8.37386-3}\times 20$ [M1]
≈ 223 seconds [A1]

Q8. (a) $SU^2=60^2+35^2-2(60)(35)\cos 125°$ [M1, subst. in correctly]
$SU^2=6^2+3.5^2-2(6)(3.5)\cos 125°$
$SU^2=7234.0210$
$SU=\sqrt{7234.0210}$ [M1]
≈ 85.1 km [A1]

(b) $\alpha=35°$ [M1]
Bearing of T from $U=325°$ [A1]



Q8. (c) Shortest distance
$=\frac{860.11}{0.5\times\sqrt{7234.0210}}$ [M1]

$\approx\frac{860.11}{0.5\times\sqrt{7234.0210}}$
$=20.2249$
≈ 20.2 m [A1]

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Q8. (d) (i) $\tan 65° = \frac{Dis\tan ce}{35}$

$Dis\tan ce = 35\tan 65°$ [M1]

≈ 75.1 m [A1]

(ii) greatest possible angle of elevation

$= \tan^{-1}\left(\frac{35\tan 65°}{20.2249}\right)$ [M1]

$\approx 74.9°$ [A1]

Q9. (a) (i) obtuse angle BOD = $180° - 40°$ (tangent to ext. pt.)

$= 140°$ [B1]

(ii) reflex angle BOD = $360° - 140°$

angle $BED = \frac{360° - 140°}{2}$ (angle at centre = two angles at circumference) [M1]

$= 110°$ [A1]

(iii) angle $ODF = \frac{180° - 220°}{2}$

$= 20°$ [B1]

(iv) angle $ABE = 90° - 20° - 21°$

$= 49°$ [B1]

(b) (i) $\theta = \frac{s}{r}$

$= 1.83 \div 3.5$

$= \frac{183}{350}$ [M1]

Angle $BOD = \pi - 2\left(\frac{183}{350}\right)$

≈ 2.096 [A1]

(ii) area of triangle $OMB = \frac{1}{2} \times 3.5 \times 3.5 \times \sin 2.096 \times \frac{1}{2}$

$= 2.6497\,\text{cm}^2$ [M1, use any method to find the area]

Area of shaded region = Area of sector OBC – area of triangle OMB

$= \frac{1}{2} \times 3.5^2 \times 2.096 \times \frac{1}{2} - 2.6497$ [M1, correct subst.]

$= 3.7693$

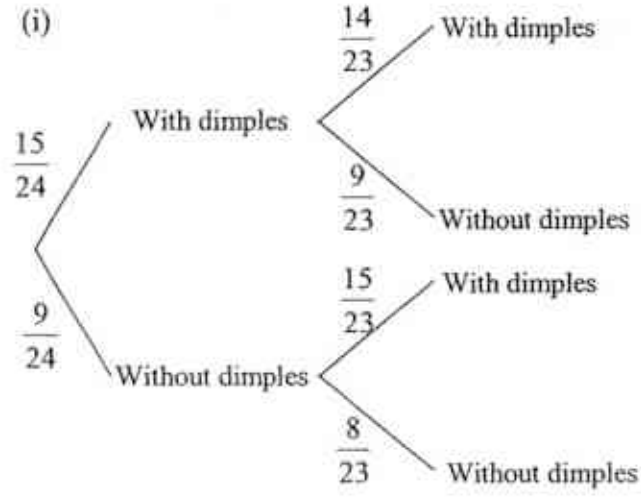
$\approx 3.77\,\text{cm}^2$ [A1]

Q10. (a) (i) (a) $a = 6$, $b = 4$, $c = 20$ and $d = 200$ [B1 for every two correct]

(b) Mean = 5 [B1]

(c) Standard deviation = 2.17944
$= \approx 2.18$ [B2]

(b) (i)



[B2, every mistake minus 1m]

(b) (ii) (a) P(both students with dimples)

$$= \frac{15}{24} \times \frac{14}{23}$$

$$= \frac{35}{92}$$ [B1]

(b) P(one student with dimples and the other without dimples)

$$= \frac{15}{24} \times \frac{9}{23} + \frac{9}{24} \times \frac{15}{23}$$

$$= \frac{45}{92}$$ [B1]

(iii) P(at least one student with dimples)
= 1 – P(all three students without dimples)

$$= 1 - \frac{9}{24} \times \frac{8}{23} \times \frac{7}{22}$$ [M1 o.e.]

$$= \frac{485}{506}$$ [A1]

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Class	Register Number	Name

南洋女子中学校

NANYANG GIRLS' HIGH SCHOOL

End-of-Year Examination 2015
Secondary Four

INTEGRATED MATHEMATICS 1 — **2 hours**

Monday — **12 October 2015** — **0845 – 1045**

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Write your name, register number and class in the spaces at the top of this page.
2. Answer questions 1 - 11 before attempting question 12 (Bonus Question).
3. Write your answers and working on the separate writing paper provided.
4. Omission of essential working will result in loss of marks.
5. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

1. The number of marks is given in brackets [ ] at the end of each question or part question.
2. The total number of marks for this paper is 80.
3. You are reminded of the need for clear presentation in your answers.

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Mathematical Formulae

1. MENSURATION

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

2. TRIGONOMETRY

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

1 $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}$.

(i) Find the matrix $\mathbf{C}$ such that $2\mathbf{A} + \mathbf{C} = \mathbf{B}^2$. [3]

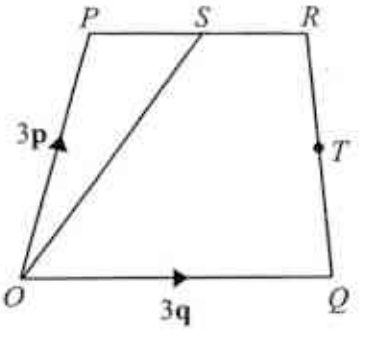
(ii) Given that $\mathbf{ABD} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, find the matrix $\mathbf{D}$. [3]

2 In the diagram, $\overrightarrow{OP} = 3\mathbf{p}$ and $\overrightarrow{OQ} = 3\mathbf{q}$.

PR is parallel to OQ and $3PR = 2OQ$.

S is the midpoint of PR and T is the midpoint of QR.

U is a point on OQ such that $OU = 2UQ$.



(i) Find $\overrightarrow{OS}$ in terms of $\mathbf{p}$ and $\mathbf{q}$. [2]

(ii) Use vectors to determine if OS and UT are parallel to each other. [3]

3 **(a)** Simplify the expression

$$27a^3(b-c)^2 \div 18a^{-2}(b^2-c^2),$$

giving your answer in positive indices only. [2]

(b) Write the following expression as a single fraction in its simplest form:

$$\frac{4x}{(2x-1)^2} - \frac{3}{2x-1} + \frac{1}{4x-2}.$$ [4]

(c) Factorise $3p(2q-r) - (r-2q)^2$. [2]

4 On separate axes, sketch the graphs of the following functions, indicating clearly the intercepts and asymptotes where applicable.

(i) $y = (x+3)(3-2x)$, [2]

(ii) $y = \frac{2}{x-3}$. [2]

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5 A cookie factory produces cookies in three flavours and delivers them to two outlets. The number of cookies supplied in a single delivery is given by the matrix **P**.

$$\mathbf{P} = \begin{matrix} \text{Outlet 1} & \text{Outlet 2} \\ \begin{pmatrix} 80 & 60 \\ 30 & 40 \\ 50 & 20 \end{pmatrix} & \begin{matrix} \text{Chocolate Deluxe} \\ \text{Peanut Crunch} \\ \text{Zesty Orange} \end{matrix} \end{matrix}$$

(i) The cost price of a Chocolate Deluxe cookie is \$1.30.
The cost price of a Peanut Crunch cookie is \$0.80.
The cost price of a Zesty Orange cookie is \$1.10.
Represent these prices in a 1×3 row matrix **C**. [1]

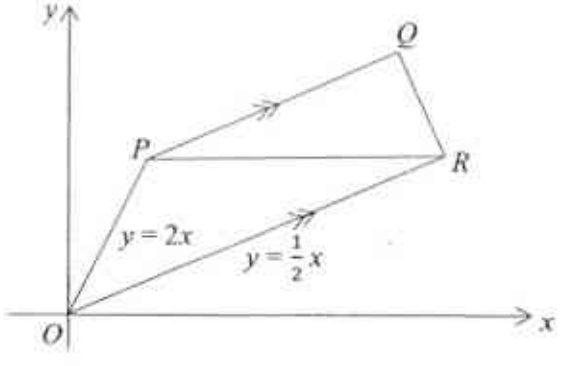
(ii) Evaluate the matrix **Q**, where $\mathbf{Q} = \mathbf{CP}$. [1]

(iii) State what the elements of **Q** represent. [1]

In a particular month, there were 18 deliveries to Outlet 1 and 13 deliveries to Outlet 2.

(iv) The elements of the matrix **N**, where $\mathbf{N} = \mathbf{PR}$, represent the total number of cookies of each flavour delivered to the two outlets in that particular month.
Write down the matrix **R**. [1]

6



The diagram shows a trapezium $OPQR$ in which PQ is parallel to OR and O is the origin. QR is perpendicular to PQ and the diagonal PR is parallel to the x-axis. The side OP has equation $y = 2x$ and the side OR has equation $y = \frac{1}{2}x$. The y-coordinate of P is k.

(i) Express the coordinates of P and R in terms of k. [2]

(ii) In the case where $k = 6$, find

(a) the coordinates of Q, [4]

(b) the coordinates of the point S which lies on PR produced such that $PR : PS = 2 : 5$. [2]

7 The table shows the number of books donated by each of 30 students in a class in a book donation drive.

1	2	1	4	3
1	2	0	3	2
0	1	6	3	2
0	3	2	1	1
2	4	1	0	3
3	0	2	1	2

(a) A student is chosen at random.

Find the probability that a student donated 2 books. [1]

(b) Two students are chosen at random.

Find the probability that

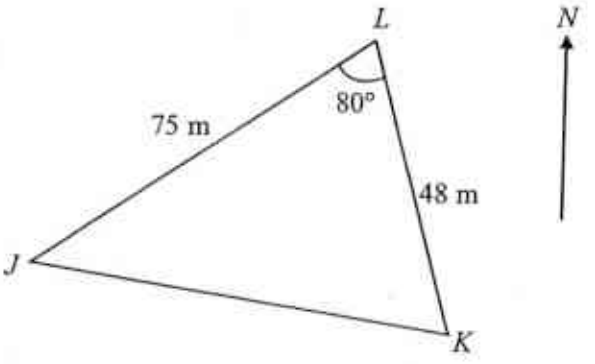
(i) one student donated two books and the other donated 4 books, [2]

(ii) both the students donated at least one book, [2]

(c) A book is chosen at random.

Find the probability that it was donated by a student who donated 3 books. [2]

8



J, K and L are three points on level ground. $JL = 75$ m, $KL = 48$ m and angle $JLK = 80°$. The bearing of K from J is 110°.

Calculate

(i) the distance JK, [2]

(ii) the area of the field JKL, [2]

(iii) the bearing of J from L. [3]

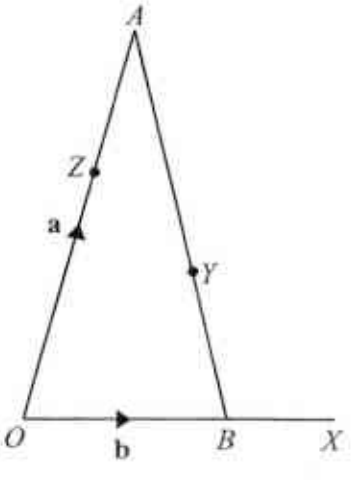
A vertical tree with height 23 m, has it base at L. A man walks from J to K. Find the greatest angle of elevation of the top of the tree when viewed from any point during his walk. [3]

9 **(a)** The points A, B, C and D are the vertices of a parallelogram $ABCD$. Given that $\overrightarrow{AB} = 6\mathbf{i} + 8\mathbf{j}$ and that $\overrightarrow{AD} = 11\mathbf{i} - 4\mathbf{j}$, find a unit vector in the direction of $\overrightarrow{BD}$. [3]

(b) In the diagram, $\overrightarrow{OA}$ and $\overrightarrow{OB}$ represent vectors **a** and **b** respectively.

X, Y and Z are points such that $\overrightarrow{OX} = \frac{3}{2}\overrightarrow{OB}$,

$\overrightarrow{AY} = \frac{3}{5}\overrightarrow{AB}$ and $\overrightarrow{OZ} = \lambda\overrightarrow{OA}$.



(i) Express $\overrightarrow{OX}$ and $\overrightarrow{OY}$ in terms of **a** and/or **b**. [2]

(ii) Express $\overrightarrow{XZ}$ in terms of λ, **a** and **b**.

Given that X, Y and Z are collinear,

evaluate the value of λ and

the ratio $XY:XZ$. [5]

(iii) Find $\frac{\text{area of triangle } OYZ}{\text{area of triangle } AYZ}$. [1]

10

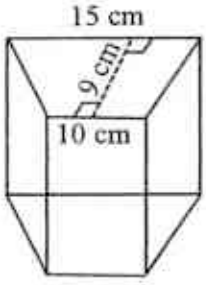


Diagram 1

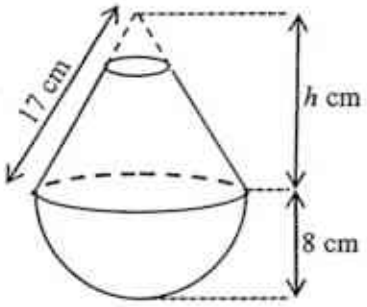


Diagram 2

In a restaurant, the fruit punch is mixed in a container with no lid as shown in Diagram 1. The container is a prism whose cross-section is a trapezium. The lengths of the parallel sides of the trapezium are 10 cm and 15 cm and the distance between the parallel sides is 9 cm. It is given that the capacity of the container is 1.8 litres.

(a) Show that the height of the container is 16 cm. [1]

(b) All the fruit punch in one full container is transferred to a jar as shown in Diagram 2. The jar is made by joining a hemisphere of radius 8 cm to a part of a right circular cone of height, h cm.

(i) Given that the slant height of the cone is 17 cm, show that $h = 15$ cm. [1]

(ii) Find the depth of the fruit punch in this jar after all the fruit punch has been transferred from the container in Diagram 1. [6]

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11 ***Answer the whole of this question on a sheet of graph paper.***

The variables x and y are connected by the equation $y = \frac{1}{10}x^2(5-x)$.

Some of the corresponding values of x and y are given in the table below.

x	−2	−1	0	1	2	3	4	5
y	a	0.6	0	0.4	1.2	1.8	1.6	0

(a) Find the value of a. [1]

(b) Taking 2 cm to represent 1 unit on each axis, draw the graph of $y = \frac{1}{10}x^2(5-x)$ for $-2 \le x \le 5$. [3]

(c) Use your graph to find

(i) the range of values of x for which $x^2(5-x) > 10$, [2]

(ii) the values of k, where k is a constant, for which the equation $\frac{1}{10}x^2(5-x) = k$ has exactly 2 solutions. [1]

(d) By drawing a tangent, find the gradient of the curve at the point where $x = 4$. [2]

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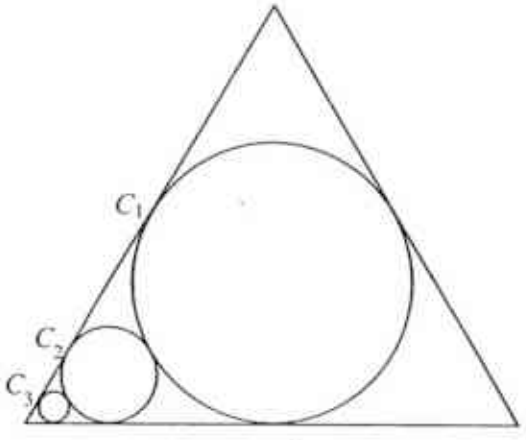
Bonus Question

12



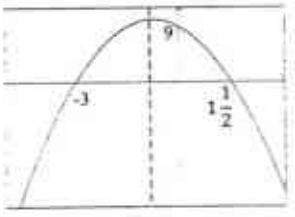
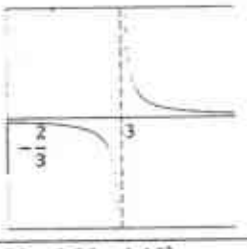
A circle C_1 is inscribed in an equilateral triangle as shown in the diagram. A second circle C_2 is tangent to the circle C_1 and to the two sides of the triangle. A third circle C_3 is tangent to the circle C_2 and to the two sides of the triangle.

Find the ratio of the radius of circle C_3 to the radius of circle C_1. [2]

END OF PAPER

Sec 4 EOY IM1 2015

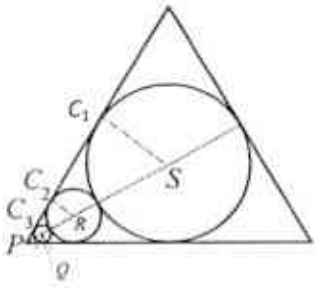
1(i)	$\mathbf{C} = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} - 2\begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$ $= \begin{pmatrix} -1 & -2 \\ 2 & 3 \end{pmatrix} - \begin{pmatrix} 4 & -2 \\ 0 & 2 \end{pmatrix}$ $= \begin{pmatrix} -5 & 0 \\ 2 & 1 \end{pmatrix}$
1(ii)	$\mathbf{AB} = \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}$ $= \begin{pmatrix} 1 & 4 \\ -1 & -2 \end{pmatrix}$ $\det \mathbf{AB} = 2$ $\mathbf{D} = (\mathbf{AB})^{-1}$ $= \frac{1}{2}\begin{pmatrix} -2 & -4 \\ 1 & 1 \end{pmatrix}$ or $\begin{pmatrix} -1 & -2 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$
2(i)	$\overrightarrow{PS} = \frac{1}{2}\overrightarrow{PS} = \frac{1}{2} \times \frac{2}{3}(3\mathbf{q}) = \mathbf{q}$ $\overrightarrow{OS} = 3\mathbf{p} + \mathbf{q}$
2(ii)	$\overrightarrow{UT} = \overrightarrow{UQ} + \frac{1}{2}\overrightarrow{QR}$ $= \mathbf{q} + \frac{1}{2}(3\mathbf{p} - \mathbf{q})$ $= \frac{3}{2}\mathbf{p} + \frac{1}{2}\mathbf{q}$ $= \frac{1}{2}(3\mathbf{p} + \mathbf{q})$ $\overrightarrow{UT} = \frac{1}{2}\overrightarrow{OS}$ OS and UT are parallel to each other.
3(a)	$\frac{27a^3(b-c)^2}{18a^{-2}(b+c)(b-c)}$ $= \frac{3a^5(b-c)}{2(b+c)}$
3(b)	$= \frac{8x - 3(2)(2x-1) + 2x - 1}{2(2x-1)^2}$ $= \frac{8x - 12x + 6 + 2x - 1}{2(2x-1)^2}$ $= \frac{-2x+5}{2(2x-1)^2}$ or $\frac{5-2x}{2(2x-1)^2}$
3(c)	$3p(2q-r) - (2q-r)^2$ $= (2q-r)[3p - (2q-r)]$ $= (2q-r)(3p - 2q + r)$ or $= (r-2q)[-3p - (r-2q)]$ $= (r-2q)(-3p - r + 2q)$

4(i)	
4(ii)	
5(i)	(1.30 0.80 1.10)
5(ii)	$(1.30 \quad 0.80 \quad 1.10)\begin{pmatrix} 80 & 60 \\ 30 & 40 \\ 50 & 20 \end{pmatrix}$ $= (183 \quad 132)$
5(iii)	The **total cost** price of the cookies for **each** of the outlet
5(iv)	$\mathbf{R} = \begin{pmatrix} 18 \\ 13 \end{pmatrix}$
6(i)	Subs $y = k$ $P = \left(\frac{k}{2}, k\right)$ $R = (2k, k)$
(ii)(a)	$6 = \frac{1}{2}(3) + c$ $c = 4\frac{1}{2}$ $PQ: y = \frac{1}{2}x + 4\frac{1}{2}$ $6 = -2(12) + c$ $c = 30$ $QR: y = -2x + 30$ $PQ: y = \frac{1}{2}x + 4\frac{1}{2}$ Solve simultaneous equations. $x = 10\frac{1}{5}$ $y = 9\frac{3}{5}$ Q is $\left(10\frac{1}{5}, 9\frac{3}{5}\right)$

6(ii)(b)	$P(3,6)\quad R(12,6)$ Let S be $(x,6)$. $\frac{x-12}{12-3}=\frac{3}{2}$ $x=25\frac{1}{2}$ $S\left(25\frac{1}{2},6\right)$
7(a)	$\frac{8}{30}=\frac{4}{15}$
(b)(i)	$\frac{8}{30}\times\frac{2}{29}+\frac{2}{30}\times\frac{8}{29}$ $=\frac{16}{435}$
(b)(ii)	$\frac{25}{30}\times\frac{24}{29}$ $=\frac{20}{29}$
(c)	$\frac{18}{56}$ $=\frac{9}{28}$

8(i)	$JK=\sqrt{75^2+48^2-2(75)(48)\cos 80°}$ $=81.72=81.7$ m(3 sf)
(ii)	$\frac{1}{2}(75)(48)\sin 80°$ = 1773 sq m or 1770 sq m (3sf)
(iii)	$\frac{\sin\angle KJL}{48}=\frac{\sin 80°}{81.72}$ $\sin\angle KJL=\frac{48\sin 80°}{81.72}=0.5784$ $\angle KJL=35.34°$ $180°+110°-35.34°=254.7°$ Bearing $=254.7°$
	Let the shortest distance be d m. $\frac{d}{75}=\sin 35.34°$ $d=75\sin 35.34°=43.38$ $\tan\theta=\frac{23}{43.38}$ $\theta=27.9°$
9(a)	$\overrightarrow{BD}=-(6\mathbf{i}+8\mathbf{j})+11\mathbf{i}-4\mathbf{j}$ $=5\mathbf{i}-12\mathbf{j}$ $\lvert\overrightarrow{BD}\rvert=\sqrt{5^2+12^2}=13$ unit vector is $\frac{1}{13}(5\mathbf{i}-12\mathbf{j})$

9(b)(i)	$\overrightarrow{OX}=\frac{3}{2}\mathbf{b}$ $\overrightarrow{OY}=\frac{2}{5}\mathbf{a}+\frac{3}{5}\mathbf{b}$
(b)(ii)	$\overrightarrow{XZ}=-\frac{3}{2}\mathbf{b}+\lambda\mathbf{a}$ $\overrightarrow{XY}=-\frac{3}{2}\mathbf{b}+\frac{2}{5}\mathbf{a}+\frac{3}{5}\mathbf{b}$ $=-\frac{9}{10}\mathbf{b}+\frac{2}{5}\mathbf{a}$ Let $\overrightarrow{XY}=h\overrightarrow{XZ}$ $-\frac{9}{10}\mathbf{b}+\frac{2}{5}\mathbf{a}=h(-\frac{3}{2}\mathbf{b}+\lambda\mathbf{a})$ $\frac{3h}{2}=\frac{9}{10}$ $h=\frac{3}{5}$ $\lambda h=\frac{2}{5}$ $\lambda=\frac{2}{5}\times\frac{5}{3}$ $=\frac{2}{3}$ $XY:XZ=3:5$ Or alternatively, $\overrightarrow{YZ}=\frac{3}{5}(\mathbf{a}-\mathbf{b})+(1-\lambda)(-\mathbf{a})$ $=\left(\lambda-\frac{2}{5}\right)\mathbf{a}-\frac{3}{5}\mathbf{b}$ Let $\overrightarrow{XZ}=k\overrightarrow{YZ}$ $-\frac{3}{2}\mathbf{b}+\lambda\mathbf{a}=k\left[\left(\lambda-\frac{2}{5}\right)\mathbf{a}-\frac{3}{5}\mathbf{b}\right]$ $\frac{3}{5}k=\frac{3}{2}$ $k=\frac{5}{2}$ $\lambda=\frac{5}{2}\left(\lambda-\frac{2}{5}\right)$ $2\lambda=5\lambda-2$ $\lambda=\frac{2}{3}$ $XY:XZ=3:5$
(b)(iii)	$\frac{OZ}{OA}=\frac{2}{3}$ $\frac{\text{area of triangle } OYZ}{\text{area of triangle } AYZ}=\frac{OZ}{AZ}=2$
10(a)	$\left(\frac{10+15}{2}\right)(9H)=1800$ $H=16$
(c)(i)	$h^2+8^2=17^2$ Or $h=\sqrt{17^2-8^2}$ $h=15$ cm

(c)(ii)	Volume of hemisphere $= \frac{2}{3}\pi(8^3)$ $= \frac{1024\pi}{3}$ or 1072.33 cm^3 Volume of cone $= \frac{1}{3}\pi(8^2)(15)$ $= 320\pi$ or 1005.31 cm^3 Total volume $= \frac{1984\pi}{3}$ or 2077.64 cm^3 $\frac{1}{3}\pi r^2 h = 2077.64 - 1800$ $\frac{1}{3}\pi\left(\frac{8h}{15}\right)^2 h = 277.64$ $\frac{64\pi h^3}{675} = 277.64$ $h^3 = 932.086$ $h = 9.768$ depth of water = $8 + (15 - 9.768)$ = 13.2 cm (3 sf)
Bonus Qn 12	Let P be one vertex of the triangle and Q, R and S be the centres of the circles C_3, C_2 and C_1 respectively. Let the radii of the circles C_1, C_2 and C_3 be a, b and c respectively.  $\frac{a}{PS} = \cos 60° = \frac{1}{2} \Rightarrow PS = 2a$ $PR = 2b$ $PQ = 2c$ Length of PS: $2c + c + 2b + a = 2a$ $\Rightarrow 3c + 2b = a$ ____(1) Length of PS: $2b + b + a = 2a$ $\Rightarrow 3b = a$ $\Rightarrow b = \frac{a}{3}$ ____(2) Subs (2) in (1) $3c + 2\left(\frac{a}{3}\right) = a$ $9c + 2a = 3a$ $\frac{c}{a} = \frac{1}{9}$

Name	Class	Register Number

4016/01 **15/S4PR2/EM/1**

MATHEMATICS PAPER 1

Friday 31 July 2015 2 hours

VICTORIA SCHOOL

PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer all the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 A school with an area of 0.4 km^2 is represented by an area of 2.5 cm^2 on a map. Find the scale of the map, in the form of $1:n$.

Answer 1 : .....[2]

---

2 Three light bulbs flicker at intervals of 4 seconds, 12 seconds and 15 seconds respectively. If all three bulbs flicker together at 0915, at what time will they next flicker together again?

Answer .....[2]

---

3 21 tins of biscuits can last 3 days for a camp of 70 students. What is the number of additional tins of biscuits required if 10 extra students join in the last 2 days of the camp?

Answer .....additional tins [2]

---

4 Write the following numbers in order of size, starting with the smallest.

$$\frac{1}{3}, \quad \sqrt{0.01}, \quad 0.33, \quad 0.5^{\frac{1}{3}}$$

Answer ....., ..., ..., ... [2]

5 A bookshop sells two types of drawing papers, each with thickness 1.6×10^{-3} cm and 2 mm. If Victor bought 2 sheets of drawing papers of the thicker type and 25 sheets of drawing paper of the other type, what is the total thickness of the papers, in cm, when bound together? Express your answer in standard form.

Answer .....cm [2]

---

6 It is given that y varies inversely as the square root of x. It is known that $x = 32$ for a particular value of y.

(a) Describe the change in y when x increases by 300%.

Answer (a).....[2]

(b) Find the value of x when the value of y is doubled.

Answer (b) $x =$ .....[2]

7 (a) Find the integer values of x which satisfy the inequalities $\frac{3}{2}x+4<2x-9\leq\frac{3x+64}{3}$.

Answer (a) $x=$ .....[2]

(b) If p and q are two of the values of x found in (a), find the least value of $\frac{1}{p^2-q^2}$.

Answer (b) .....[1]

8 Simplify $\frac{\left(3\sqrt{x}\right)^3\div\left(xy^3\right)^{-\frac{4}{3}}}{\left(2x^3y^2\right)^2}$, leaving your answer in positive indices.

Answer .....[3]

9

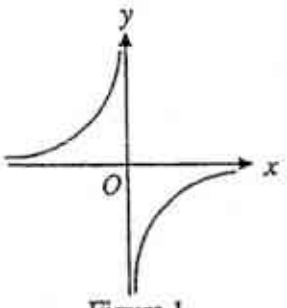


Figure 1

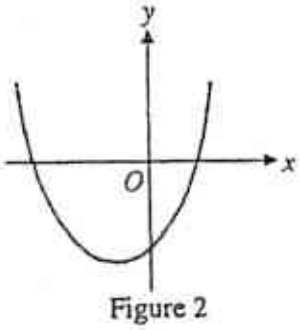


Figure 2

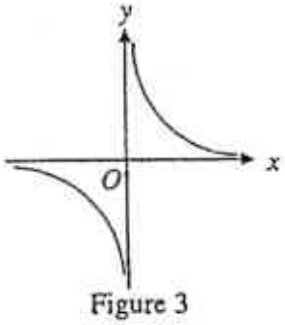


Figure 3

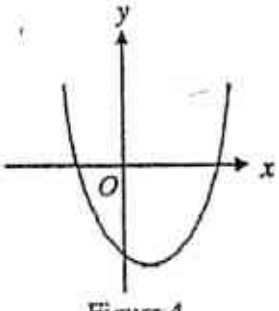


Figure 4

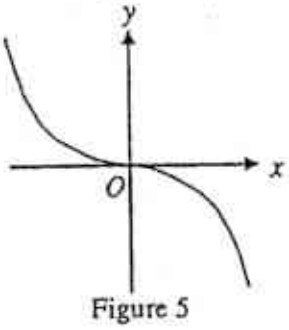


Figure 5

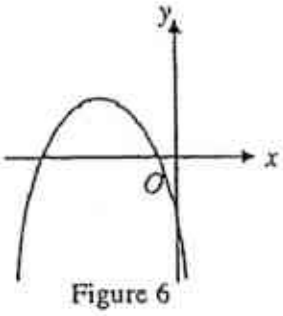


Figure 6

Which of the graphs shown above could be the graph of

(a) $y = 2x^2 + 4x - 3$,

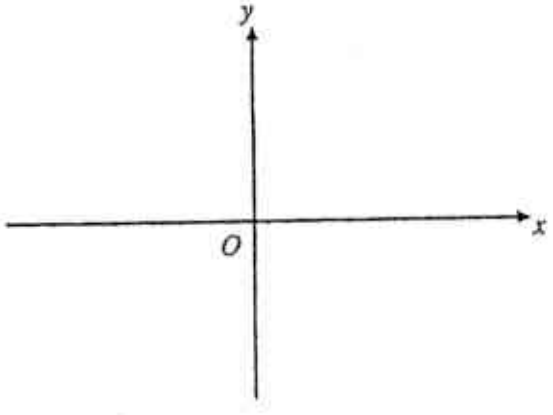
Answer (a) Figure ..... [1]

(b) $y = x^{-1}$?

Answer (b) Figure ..... [1]

(c) Sketch the graph of $y = 3^{-x}$.

Answer (c)



[1]

10 Solve the simultaneous equations

$$x+2y=4,$$
$$-0.1x+0.9y=2.35.$$

Answer $x =$ ....., $y =$ ..... [3]

---

11 Dylan would like to accumulate an interest of \$800 in his account after $3\frac{1}{2}$ years. Given that a bank pays 2.8% per annum interest compounded half yearly, how much does Dylan need to deposit in his account? Give your answer to the nearest dollar.

Answer \$ .....[3]

---

12 In 15 years' time from 2015, Valerie's age will be the square of her age 15 years ago from 2015. Find Valerie's age in 2015.

Answer ..... years old [3]

13 (a) Given that A is the point $(1,3)$ and B is the point $(-2,9)$, find the equation of the line PQ that is parallel to line AB and passes through the point $(3,2)$.

Answer (a).....[2]

(b) Determine the type of solution (I, II, III) when the line PQ intersects the line $x+\frac{1}{2}y=4$.

Type I 1 unique solution
Type II No solution
Type III Infinite number of solutions

Answer (b) Type.....[1]

---

14 Shops A and B are having sales on a particular jacket. Shop A offers a 35% discount on the original price, p, while Shop B offers a 20% discount on the same original price, with a further reduction of x % on the discounted price for members.

(i) Given that Ali holds the membership of Shop B, the total amount of discount offered to him from Shop B can be written as $\frac{p}{125}(a+x)$. Find the value of a.

Answer (i) $a =$.....[1]

(ii) Given that Ali is offered a higher discount from Shop B, form an inequality in x and show that it reduces to $x > k$, where k is a constant to be determined.

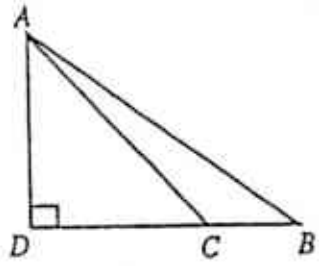
Answer (ii).....

.....

.....

.....[2]

15 In triangle ADB, $\angle ADB = 90°$ and $DC = 2CB$. Given that $\cos^2 \angle ACB = \frac{16}{25}$, express as a fraction in its lowest term.



(i) $\tan \angle ACB$.

Answer (i) $\tan \angle ACB =$ .....[2]

(ii) $\tan \angle ABD$.

Answer (ii) $\tan \angle ABD =$ .....[2]

---

16 The queuing time, in minutes, of 20 customers queuing at the cashier during peak hours is shown in the table below.

Time (minutes)	1–4	5–8	9–12	13–16
Number of customers	9	7	3	1

Calculate the

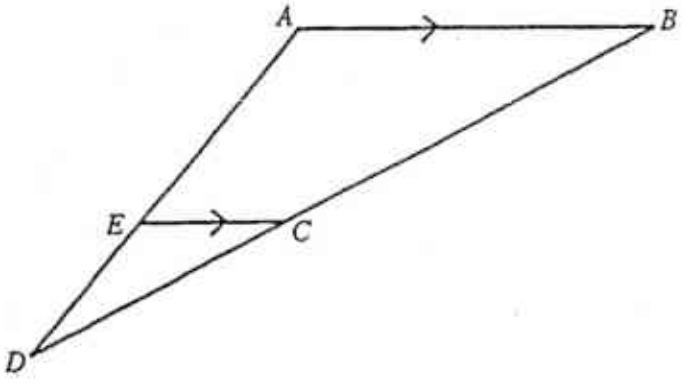
(i) mean queuing time,

Answer (i) .....min [1]

(ii) standard deviation.

Answer (ii) .....min [2]

17 In the diagram, lines AB and EC are parallel, and $DE : EA = 2 : 3$.



(a) Prove that triangles ABD and ECD are similar.

Answer (a).....

.....

.....

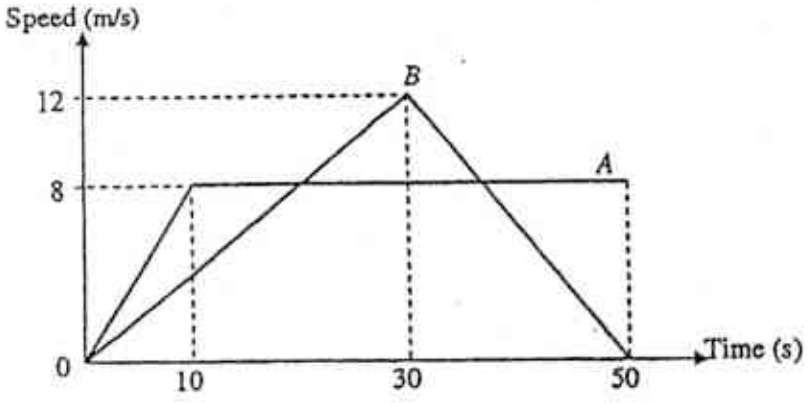
.....

..... [2]

(b) Find $\dfrac{\text{area of } \Delta ECD}{\text{area of } ABCE}$.

Answer (b) .....[2]

18 The diagram shows the speed-time graph for the motion of two objects A and B travelling in opposite directions. Both objects are 1 km apart at $t = 0$ s.



Calculate the

(i) deceleration of object B in the last 5 seconds of the motion,

Answer (i) .....m/s^2 [1]

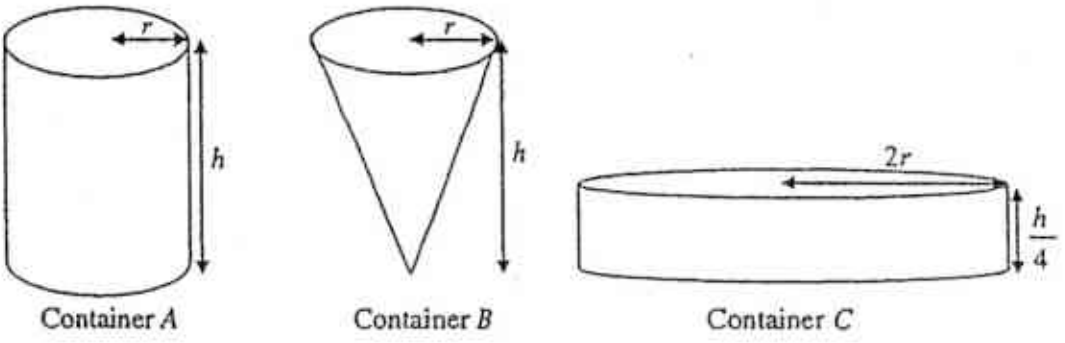
(ii) time t, where $t > 30$, at which both objects have the same speed,

Answer (ii) .....s [2]

(iii) distance between both objects at 28 s.

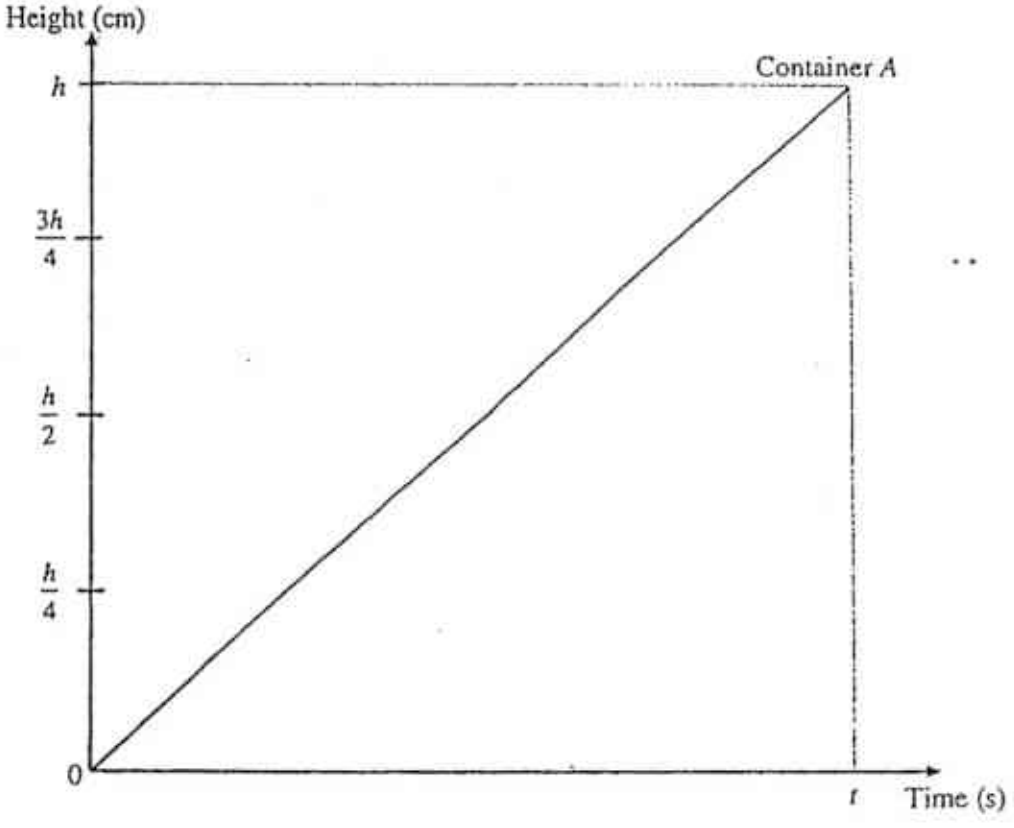
Answer (iii) .....m [2]

19 The diagram shows three open containers with varying heights and radii, in cm. Water is poured into the containers from the top at a constant rate until the containers are filled completely.



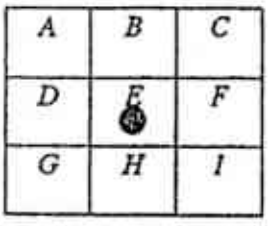
Container A Container B Container C

The graph below shows the height of the water against time as container A is filled. On the same diagram below, draw the corresponding graphs for containers B and C, labeling each graph clearly.



[2]

20 A coin is placed on grid square E as shown in the diagram below. A fair die is thrown, and the number on the surface is noted after each throw.
If the number is 1 or 2, the coin is moved one square to the right.
If the number is 3, the coin is moved one square to the left.
If the number is 4, the coin is moved one square up.
If the number is 5 or 6, the coin is moved one square down.



The die is thrown two times. Find the probability that the coin is moved to grid square

(a) A,

Answer (a) .....[1]

(b) E,

Answer (b) .....[2]

(c) F.

Answer (c) .....[1]

(d) Justify your answer in (c).

Answer (d).....

.....[1]

21 $OPQR$ is a rhombus. Points R and P have position vectors $\begin{pmatrix}4\\2\end{pmatrix}$ and $\begin{pmatrix}p+1\\p-1\end{pmatrix}$, $p<0$.

(i) Calculate the value of p.

Answer (i) $p =$ .....[2]

(ii) Find the coordinates of Q.

Answer (ii) $Q($.....,.....$)$ [2]

(iii) Given that D is the point on PQ produced such that $PD = 3QD$, find $\overrightarrow{RD}$.

Answer (iii) $\overrightarrow{RD} =$ .....[2]

22 The masses, in grams, of 20 packs of sweets are shown in the ordered stem-and-leaf diagram.

1	3 8 9
2	1 x 2 3 3 y
3	1 2 2 4 5 5 6 7 8
4	3 9

key 1 | 3 means 13

(a) Find the median mass.

Answer (a) .....g [1]

(b) Find the value of $x + y$ if the mean mass is 29.55 grams.

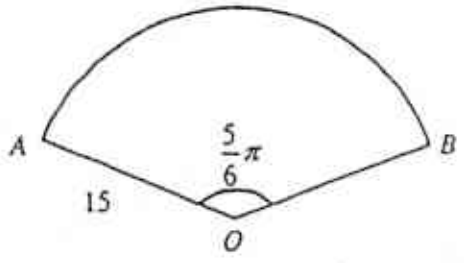
Answer (b) .....g [2]

(c) Find the value of x and of y if the lower quartile is 21.5 grams.

Answer (c) x=....., y=..... [2]

236

23 The diagram shows the sector AOB of a circle, that represents a piece of paper. The radius of the sector is 15 cm and the angle AOB is $\frac{5}{6}\pi$ radians.



(a) Calculate the length of arc AB.

Answer (a) ..... cm [2]

(b) The piece of paper is used to make a hollow cone by joining the edges OA and OB. Calculate the volume of the cone.

Answer (b) ..... cm^3 [3]

24 Three toddlers, Ahmad, Betty and Charlie, were playing on a big triangular play-mat ABC, where $AB = 4$ m, $BC = 6$ m and $AC = 5$ m. After some time, Ahmad and Charlie rested at Points A and C respectively, while Betty sat at a position equidistant from Ahmad and Charlie, and equidistant from AB and AC.

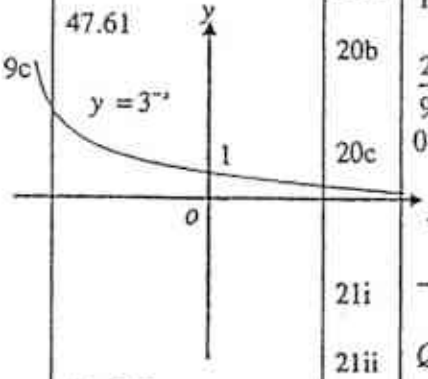
(a) Using a scale of 1: 50, construct the triangular play-mat ABC. [1]

(b) By constructing suitable lines, locate and label the position of Betty. Label this position Q. [2]

Answer (a) and (b)

End of Paper

2015 Victoria School Prelim 2 Mathematics Paper 1 Answer Key

1	40 000
2	0916
3	2
4	$\sqrt{0.01}, 0.5^{\frac{8}{5}}, 0.33, \frac{1}{3}$
5	4.4×10^{-1}
6a	The new value of y is halved.
6b	8
7a	27, 28, 29, 30
7b	$-\frac{1}{55}$
8	$\frac{27}{4x^{\frac{19}{6}}}$
9a	2
9b	3
9c	47.61 
10	$y=2.5$ $x=-1$
11	7827
12	21
13a	$y=-2x+8$
13b	III
14i	25
14ii	$k=18.75$
15i	$-\frac{3}{4}$
15ii	$\frac{1}{2}$
16i	5.7
16ii	3.49
17a	$\angle DAB = \angle DEC$ (corresponding angles, $AB//EC$) $\angle ADB = \angle EDC$ (common angle) $\therefore$ Triangles ABD and ECD are similar.
17b	$\frac{4}{21}$
18i	$\frac{3}{5}$
18ii	$36\frac{2}{3}$
18iii	659.2 or 1340.8
20a	$\frac{1}{18}$
20b	$\frac{2}{9}$
20c	0
21i	-3
21ii	$Q(2,-2)$
21iii	$\begin{pmatrix}0\\-3\end{pmatrix}$
22a	31.5
22b	10
22c	$x=1$ $y=9$
23a	39.3
23b	558

MARKING SCHEME

Class	Register Number

Name _____

4016/01 **15/S4PR2/EM/1**

MATHEMATICS **PAPER 1**

Friday 31 July 2015 2 hours

VICTORIA SCHOOL

PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer all the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

238

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 A school with an area of 0.4 km^2 is represented by an area of 2.5 cm^2 on a map. Find the scale of the map, in the form of $1:n$.

Area scale = 2.5 cm^2 : 0.4 km^2
= 1 cm^2 : 0.16 km^2

Linear Scale = 1 cm : 0.4 km
= 1 : 40 000

Answer 1 : 40 000 [2]

---

2 Three light bulbs flicker at intervals of 4 seconds, 12 seconds and 15 seconds respectively. If all three bulbs flicker together at 0915, at what time will they next flicker together again?

LCM of 4s, 12s and 15s = $2^2 \times 3 \times 5$
= 60s
= 1 min
0915 + 0001 = 0916

Answer 0916 [2]

---

3 21 tins of biscuits can last 3 days for a camp of 70 students. What is the number of additional tins of biscuits required if 10 extra students join in the last 2 days of the camp?

No. of tins required for 70 students in 2 days of camp $= \frac{2}{3} \times 21$
$= 14$

No. of additional tins required for 10 students in 2 days of camp $= \frac{10}{70} \times 14$
$= 2$

Answer 2 additional tins [2]

---

4 Write the following numbers in order of size, starting with the smallest.

$\frac{1}{3}$, $\sqrt{0.01}$, 0.33, $0.5^{\frac{1}{5}}$

$\sqrt{0.01}, 0.5^{\frac{1}{5}}, 0.33, \frac{1}{3}$

Answer $\sqrt{0.01}$, $0.5^{\frac{1}{5}}$, 0.33, $\frac{1}{3}$ [2]

5 A bookshop sells two types of drawing papers, each with thickness 1.6×10^{-3} cm and 2 mm. If Victor bought 2 sheets of drawing papers of the thicker type and 25 sheets of drawing paper of the other type, what is the total thickness of the papers, in cm, when bound together? Express your answer in standard form.

$$\begin{aligned}\text{Total thickness} &= 2\left(2\times10^{-1}\right)+25\left(1.6\times10^{-3}\right)\\ &= 0.44 \text{ cm}\\ &= 4.4\times10^{-1} \text{ cm}\end{aligned}$$

Answer 4.4×10^{-1} cm[2]

---

6 It is given that y varies inversely as the square root of x. It is known that $x=32$ for a particular value of y.

(a) Describe the change in y when x increases by 300%.

$$\begin{aligned}y &\propto \frac{1}{\sqrt{x}}\\ y &= \frac{k}{\sqrt{x}}, \text{ where } k \text{ is a constant.}\\ x_{new} &= x+3x\\ &= 4x\\ y_{new} &= \frac{k}{\sqrt{4x}}\\ &= \frac{k}{2\sqrt{x}}\\ &= \frac{1}{2}y\end{aligned}$$

Answer (a) The value of new y is halved [2]

(b) Find the value of x when the value of y is doubled.

$$\begin{aligned}y &= \frac{k}{\sqrt{x}}, \text{ where } k \text{ is a constant.}\\ y_1\sqrt{x_1} &= y_2\sqrt{x_2}\\ y_1\sqrt{32} &= 2y_1\times\sqrt{x_2}\\ \sqrt{x_2} &= \frac{\sqrt{32}}{2}\\ x_2 &= 8\end{aligned}$$

Answer (b) $x=$ 8 [2]

7 (a) Find the integer values of x which satisfy the inequalities $\frac{3}{2}x+4<2x-9\leq\frac{3x+64}{3}$.

$\frac{3}{2}x+4<2x-9$ and $2x-9\leq x+21\frac{1}{3}$

$13<\frac{x}{2}$ and $x\leq 30\frac{1}{3}$

$26<x$

$\therefore x=27,28,29,30$

Answer (a) $x=$ 27, 28, 29, 30 [2]

(b) If p and q are two of the values of x found in (a), find the least value of $\frac{1}{p^2-q^2}$.

$$\frac{1}{p^2-q^2}=\frac{1}{(p-q)(p+q)}$$
$$=\frac{1}{(27-28)(27+28)}$$
$$=-\frac{1}{55}$$

Answer (b) $-\frac{1}{55}$ [1]

8 Simplify $\frac{\left(3\sqrt{x}\right)^3+\left(xy^3\right)^{-\frac{4}{3}}}{\left(2x^3y^2\right)^2}$, leaving your answer in positive indices.

$$\frac{\left(3\sqrt{x}\right)^3+\left(xy^3\right)^{-\frac{4}{3}}}{\left(2x^3y^2\right)^2}=\frac{\left(3^3x^{\frac{3}{2}}\right)}{x^{-\frac{4}{3}}y^{-4}}\times\frac{1}{2^2x^6y^4}$$
$$=3^3x^{\frac{3}{2}}x^{\frac{4}{3}}\times\frac{1}{2^2x^6}$$
$$=\frac{3^3x^{-\frac{19}{6}}}{2^2}$$
$$=\frac{27}{4x^{\frac{19}{6}}}$$

Answer $\frac{27}{4x^{\frac{19}{6}}}$ [3]

9

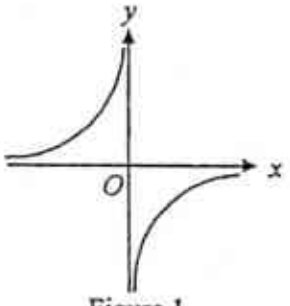


Figure 1

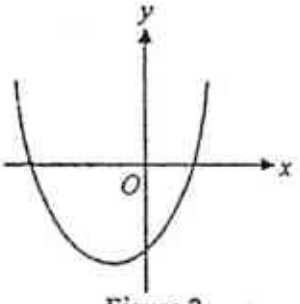


Figure 2

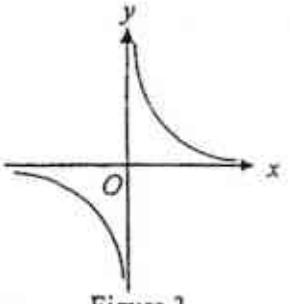


Figure 3

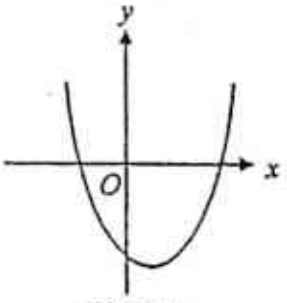


Figure 4

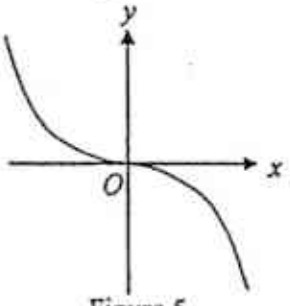


Figure 5

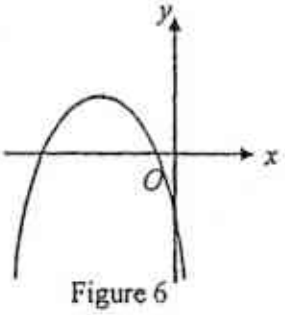


Figure 6

Which of the graphs shown above could be the graph of

(a) $y = 2x^2+4x-3$,

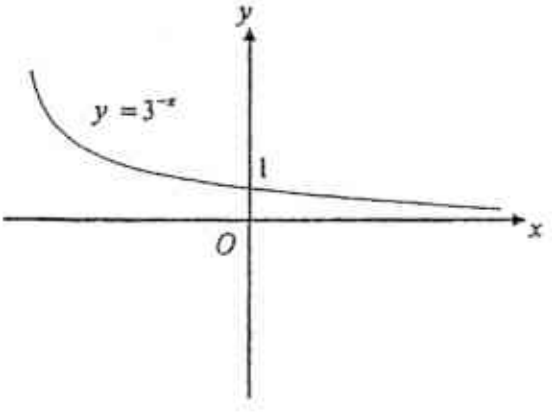
Answer (a) Figure ....2....[1]

(b) $y = x^{-1}$?

Answer (b) Figure ....3....[1]

(c) Sketch the graph of $y = 3^{-x}$.

Answer (c)



[1]

10 Solve the simultaneous equations

$$x+2y=4,$$
$$-0.1x+0.9y=2.35.$$

$x+2y=4$ -----(1)
$-x+9y=23.5$ ----(2)
(1)+(2): $11y=27.5$
$y=2.5$
$x=-1$

Answer $x=$ ..−1..., $y=$ ..2.5... [3]

---

11 Dylan would like to accumulate an interest of \$800 in his account after $3\frac{1}{2}$ years. Given that a bank pays 2.8% per annum interest compounded half yearly, how much does Dylan need to deposit in his account? Give your answer to the nearest dollar.

Let x be the original amount.
$$x+800=\left(1+\frac{2.8+2}{100}\right)^{7}\times x$$
$0.10221x=800$
$x=\$7826.76\approx\$7827$ (nearest dollar)

Answer \$ 7827 [3]

---

12 In 15 years' time from 2015, Valerie's age will be the square of her age 15 years ago from 2015. Find Valerie's age in 2015.

Let x be Valerie's current age.
$x+15=(x-15)^2$
$x+15=x^2-30x+15^2$
$x^2-31x+210=0$
$$x=\frac{-(-31)\pm\sqrt{(-31)^2-4(1)(210)}}{2(1)}$$
$=21$ or 10 (rej. $\because 10-15<0$)
Valerie's current age is 21.

Answer ..21... years old [3]

13 (a) Given that A is the point $(1,3)$ and B is the point $(-2,9)$, find the equation of the line PQ that is parallel to line AB and passes through the point $(3,2)$.

$$\text{gradient of AB} = \frac{9-3}{-2-1}$$
$$= -2$$
$$y-2 = -2(x-3)$$
$$y = -2x+8$$

Answer (a) $y = -2x+8$ [2]

(b) Determine the type of solution (**I, II, III**) when the line PQ intersects the line $x+\frac{1}{2}y=4$.

Type I 1 unique solution
Type II No solution
Type III Infinite number of solutions

Answer (b) Type III [1]

---

14 Shops A and B are having sales on a particular jacket. Shop A offers a 35% discount on the original price, p, while Shop B offers a 20% discount on the same original price, with a further reduction of x % on the discounted price for members.

(i) Given that Ali holds the membership of Shop B, the total amount of discount offered to him from Shop B can be written as $\frac{p}{125}(a+x)$. Find the value of a.

$$\text{Total discount from Shop B} = 0.2p + \frac{x}{100}(0.8p)$$
$$= p\left(0.2 + \frac{0.8}{100}x\right)$$
$$= \frac{p}{125}(25+x)$$

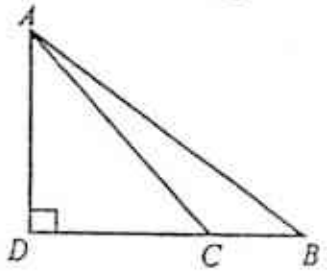
Answer (i) $a =$ 25 [1]

(ii) Given that Ali is offered a higher discount from Shop B, form an inequality in x and show that it reduces to $x > k$, where k is a constant to be determined.

$$0.35p < \frac{p}{125}(25+x)$$
$$0.35(125) < 25+x$$
$$x > 18.75$$
$$k = 18.75$$

[2]

15 In triangle ADB, $\angle ADB = 90°$ and $DC = 2CB$. Given that $\cos^2 \angle ACB = \frac{16}{25}$, express as a fraction in its lowest term,



(i) $\tan \angle ACB$,

$\cos \angle ACB = \pm\frac{4}{5}$

Since $\angle ACB$ is obtuse, $\cos \angle ACB = -\frac{4}{5}$.

$AD = \sqrt{5^2 - 4^2} = 3$

$\tan \angle ACB = -\frac{3}{4}$

Answer (i) $\tan \angle ACB =$ $-\frac{3}{4}$ [2]

(ii) $\tan \angle ABD$.

$\tan \angle ACD = \frac{3}{4}$

$\tan \angle ABD = \frac{3}{4+2}$

$= \frac{1}{2}$

Answer (ii) $\tan \angle ABD =$ $\frac{1}{2}$ [2]

---

16 The queuing time, in minutes, of 20 customers queuing at the cashier during peak hours is shown in the table below.

Time (minutes)	1–4	5–8	9–12	13–16
Number of customers	9	7	3	1

Calculate the

(i) mean queuing time,

Mean $= \frac{114}{20}$

$= 5.7$ min

Answer (i) 5.7 min [1]

(ii) standard deviation.

Standard deviation $= \sqrt{\frac{893}{20} - (5.7)^2}$

$= 3.49$ min (to 3sf)

Answer (ii) 3.49 min [2]

28

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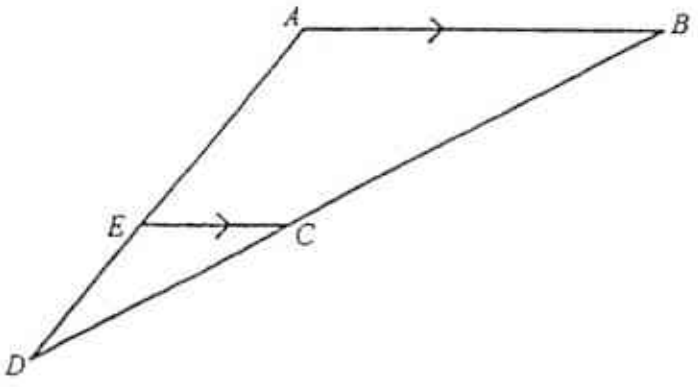
17 In the diagram, lines AB and EC are parallel, and $DE : EA = 2 : 3$.



(a) Prove that triangles ABD and ECD are similar.

Answer (a)

$\angle DAB = \angle DEC$ (corresponding angles, $AB // EC$)

$\angle ADB = \angle EDC$ (common angle)

$\therefore$ Triangles ABD and ECD are similar.

[2]

(b) Find $\dfrac{\text{area of } \Delta ECD}{\text{area of } ABCE}$.

$$\frac{\text{area of } \Delta ECD}{\text{area of } \Delta ABD} = \left(\frac{2}{5}\right)^2$$

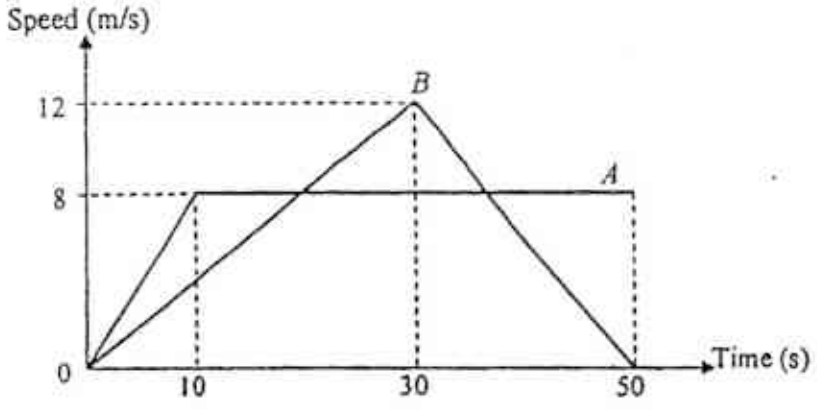
$$= \frac{4}{25}$$

$$\frac{\text{area of } \Delta ECD}{\text{area of } ABCE} = \frac{4}{25-4}$$

$$= \frac{4}{21}$$

Answer (b) $\dfrac{4}{21}$ [2]

18 The diagram shows the speed-time graph for the motion of two objects A and B travelling in opposite directions. Both objects are 1 km apart at $t = 0$ s.



Calculate the

(i) deceleration of object B in the last 5 seconds of the motion,

$$\text{Acceleration} = \frac{0-12}{50-30}$$
$$= -\frac{3}{5}\ \text{m/s}^2$$
$$\text{Deceleration} = \frac{3}{5}\ \text{m/s}^2$$

Answer (i) $\frac{3}{5}$ or 0.6 m/s² [1]

(ii) time t, where $t > 30$, at which both objects have the same speed,

$$\frac{8-12}{t-30} = -\frac{3}{5}$$
$$t = 36\frac{2}{3}\ s$$

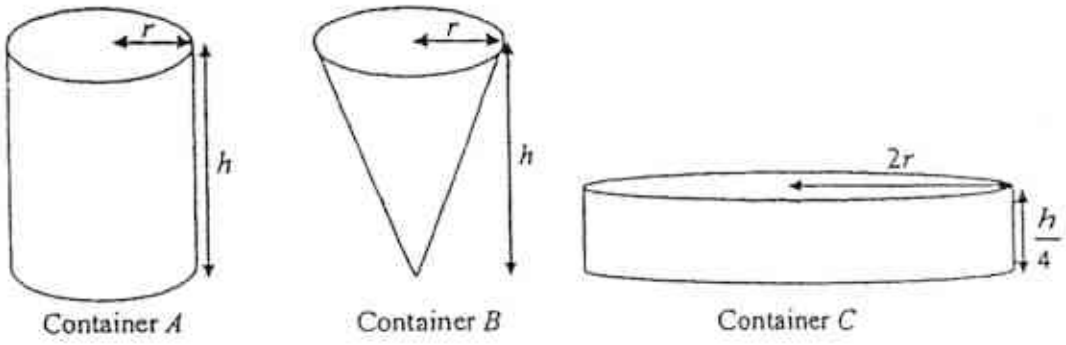
Answer (ii) $36\frac{2}{3}$ s [2]

(iii) distance between both objects at 28 s.

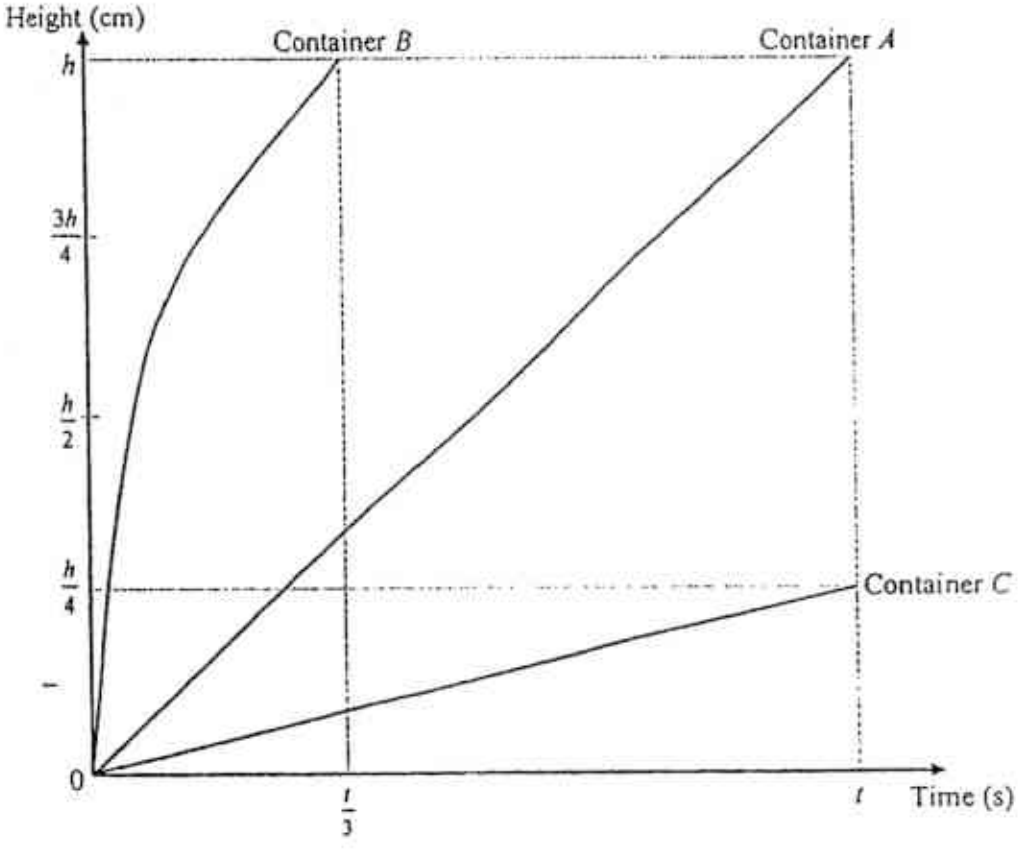
$$\text{Distance covered by } A = \frac{1}{2}(10)(8) + (18 \times 8) = 184\text{m}$$
$$\text{Distance covered by } B = \frac{1}{2}(28)\left(\frac{12}{30} \times 28\right) = 156.8\text{m}$$
$$\text{Distance apart} = 1000 - 184 - 156.8 = 659.2\text{m}$$
Or
$$\text{Distance apart} = 1000 + 184 + 156.8 = 1340.8\text{m}$$

Answer (iii) 659.2 or 1340.8 m [2]

19 The diagram shows three open containers with varying heights and radii, in cm. Water is poured into the containers from the top at a constant rate until the containers are filled completely.

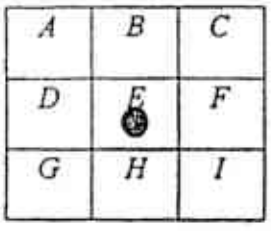


The graph below shows the height of the water against time as container A is filled. On the same diagram below, draw the corresponding graphs for containers B and C, labeling each graph clearly.



[2]

20 A coin is placed on grid square E as shown in the diagram below. A fair die is thrown, and the number on the surface is noted after each throw.
If the number is 1 or 2, the coin is moved one square to the right.
If the number is 3, the coin is moved one square to the left.
If the number is 4, the coin is moved one square up.
If the number is 5 or 6, the coin is moved one square down.



The die is thrown two times. Find the probability that the coin is moved to grid square

(a) A,

(a) Required prob. = P(up, left) + P(left, up)

$$= \frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}$$

$$= \frac{1}{18}$$

Answer (a) ..... $\frac{1}{18}$ ..... [1]

(b) E,

(b) Required prob. = P(left, right) + P(right, left) + P(up, down) + P(down, up)

$$= 4\left(\frac{1}{6}\times\frac{2}{6}\right)$$

$$= \frac{2}{9}$$

Answer (b) ..... $\frac{2}{9}$ ..... [2]

(c) F.

(c) Required prob. = 0

Answer (c) ..... 0 ..... [1]

(d) Justify your answer in (c).

Answer (d).....Odd number of throws is required to move coin to square F. [1]

21 $OPQR$ is a rhombus. Points R and P have position vectors $\begin{pmatrix}4\\2\end{pmatrix}$ and $\begin{pmatrix}p+1\\p-1\end{pmatrix}$, $p<0$.

(i) Calculate the value of p.

$\sqrt{(p+1)^2+(p-1)^2}=\sqrt{4^2+2^2}$

$p^2+2p+1+p^2-2p+1=20$

$p^2=9$

$p=\pm 3$

Since $p<0$, $p=-3$

Answer (i) $p=$ ..-3.. [2]

(ii) Find the coordinates of Q.

$\overrightarrow{OQ}=\overrightarrow{OP}+\overrightarrow{PQ}$

$=\begin{pmatrix}-3+1\\-3-1\end{pmatrix}+\begin{pmatrix}4\\2\end{pmatrix}$

$=\begin{pmatrix}2\\-2\end{pmatrix}$

$Q(2,-2)$

Answer (ii) ..Q(2, −2).. [2]

(iii) Given that D is the point on PQ produced such that $PD=3QD$, find $\overrightarrow{RD}$.

$\overrightarrow{RD}=\overrightarrow{OD}-\overrightarrow{OR}$

$=\overrightarrow{OP}+\overrightarrow{PD}-\overrightarrow{OR}$

$=\begin{pmatrix}-2\\-4\end{pmatrix}+\frac{3}{2}\overrightarrow{PQ}-\begin{pmatrix}4\\2\end{pmatrix}$

$=\begin{pmatrix}-2\\-4\end{pmatrix}+\frac{3}{2}\begin{pmatrix}4\\2\end{pmatrix}-\begin{pmatrix}4\\2\end{pmatrix}$

$=\begin{pmatrix}0\\-3\end{pmatrix}$

Answer (iii) $\overrightarrow{RD}=$ ..$\begin{pmatrix}0\\-3\end{pmatrix}$.. [2]

22 The masses, in grams, of 20 packs of sweets are shown in the ordered stem-and-leaf diagram.

1	3 8 9
2	1 x 2 3 3 y
3	1 2 2 4 5 5 6 7 8
4	3 9

key 1 | 3 means 13

(a) Find the median mass.

median mass $= \frac{31+32}{2} = 31.5$ grams

Answer (*a*) ..... 31.5 ..... g [1]

(b) Find the value of $x+y$ if the mean mass is 29.55 grams.

$$29.55 = \frac{13+18+19+21+20+x+22+23+23+20+y+31+32+32+34+35+35+36+37+38+43+49}{20}$$

$x+y=10$

Answer (*b*) ..... 10 ..... g [2]

(c) Find the value of x and of y if the lower quartile is 21.5 grams.

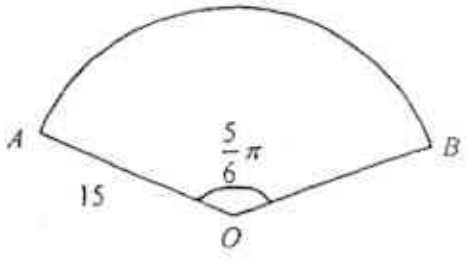
$$21.5 = \frac{20+x+22}{2}$$

$x=1$

$y=9$

Answer (*c*) $x=$ ..... 1 ....., $y=$ ..... 9 ..... [2]

23 The diagram shows the sector AOB of a circle, that represents a piece of paper. The radius of the sector is 15 cm and the angle AOB is $\frac{5}{6}\pi$ radians.



(a) Calculate the length of arc AB.

$$S = r\theta$$
$$= 15\left(\frac{5}{6}\pi\right)$$
$$= 39.3 \text{ cm (to 3sf)}$$

Answer (a) ....39.3..... cm [2]

(b) The piece of paper is used to make a hollow cone by joining the edges OA and OB. Calculate the volume of the cone.

Let R be the radius of cone

$$15\left(\frac{5}{6}\pi\right) = 2\pi R$$
$$R = 6.25$$
$$\text{Volume of cone} = \frac{1}{3}\pi 6.25^2\sqrt{15^2 - 6.25^2}$$
$$= 558 \text{ cm}^3 \text{ (to 3sf)}$$

Answer (b) .....558..... cm^3 [3]

24 Three toddlers, Ahmad, Betty and Charlie, were playing on a big triangular play-mat ABC, where $AB = 4$ m, $BC = 6$ m and $AC = 5$ m. After a while, Ahmad and Charlie rested at Points A and C respectively, while Betty sat at a position equidistant from Ahmad and Charlie, and equidistant from AB and AC.

(a) Using a scale of 1: 50, construct the triangular play-mat ABC. [1]

(b) By constructing suitable lines, locate and label the position of Betty. Label this position Q. [2]

Answer (a) and (b)

End of Paper

48

246

17

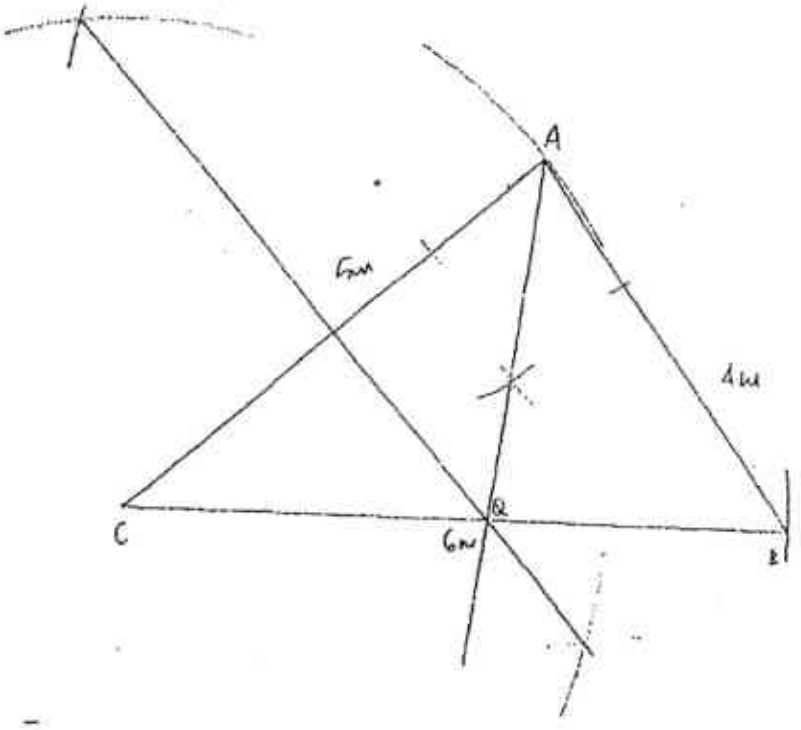
24 Three toddlers, Ahmad, Betty and Charlie, were playing on a big triangular play-mat ABC, where $AB = 4$ m, $BC = 6$ m and $AC = 5$ m. After some time, Ahmad and Charlie rested at Points A and C respectively, while Betty sat at a position equidistant from Ahmad and Charlie, and equidistant from AB and AC.

(a) Using a scale of 1: 50, construct the triangular play-mat ABC. [1]

(b) By constructing suitable lines, locate and label the position of Betty. Label this position Q. [2]

1cm : 50cm 4m : 8cm
1m : 0.5m 6m : 12cm
2m : 1m 5m : 10cm

Answer (a) and (b)



End of Paper

VICTORIA SCHOOL 15/S4PR2/EM/1

Name _____ Class _____ Register Number _____

4016/02

15/S4PR2/EM/2

MATHEMATICS

PAPER 2

Monday 3 August 2015 2 hours 30 minutes

VICTORIA SCHOOL

PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

247

This paper consists of 12 printed pages, including the cover page.

[Turn over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

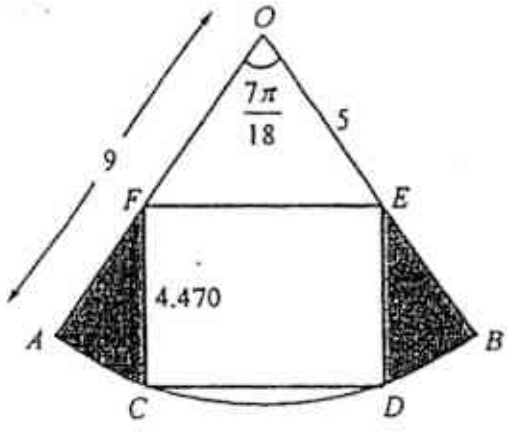
Answer all the questions.

1 (a) Solve the equation $x(3x-8)=-3$. [2]

(b) Given that $\frac{v}{3}=\sqrt{\frac{2(2s-v)}{s+3}}$, express s in terms of v. [3]

(c) Express $\frac{1}{4x^2-11x-3}-\frac{3}{4x+1}-\frac{2}{3-x}$ as a single fraction in its simplest form. [4]

2



In the diagram, OAB is a sector of a circle, centre O and radius 9 cm. Rectangle $CDEF$ is inscribed in the sector. Given that angle $AOB=\frac{7\pi}{18}$ radians, $OE=OF=5$ cm and $CF=DE=4.470$ cm,

(a) find the length of CD, [2]

(b) show that angle COD is approximately 0.6486 radians, [2]

(c) find the arc length DB, [2]

(d) find the area of the shaded region. [3]

3 Given that $\varepsilon = \{x : x \text{ is an integer}, 20 \le x \le 99\}$

$A = \{x : x = 2m + 1, \text{ where } m \text{ is a positive integer}\}$

$B = \{x : x \text{ is divisible by } 7\}$

$C = \{x : x \text{ is a multiple of } 6\}$

$D = \{x : x \text{ has a remainder of } 2 \text{ when divided by } 4\}$

(a) State the value of $n(\varepsilon)$ and $n(A \cap D)$. [2]

(b) List the elements in B. [1]

(c) Describe, as simply as possible, the meaning of $B \cap C \neq 0$ in words. [1]

---

4 At a fund raising event, a class ran a fast food store and had 4 serving counters A, B, C and D. The store sold cheeseburgers, French fries and drinks. Each cheeseburger was sold at \$2.00, a packet of French fries for \$1.50 and a drink for \$1.20. The table below shows the number of items sold by each counter during the event.

Items \ Counter	A	B	C	D
Cheeseburger	61	55	58	84
French fries	74	81	84	77
Drinks	59	52	64	58

(a) Write down a 1×3 matrix P to represent the selling price of each cheeseburger, French fries and drink sold. [1]

(b) Write down a matrix Q such that PQ will give the total sales at each serving counter. [1]

(c) Evaluate PQ. [1]

Matrix C is given by $C = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$.

(d) Evaluate PQC. [1]

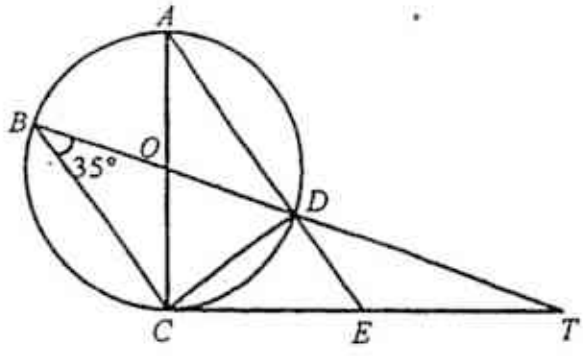
(e) State what the element of PQC represent. [1]

(f) If the total profit made by the class is 45% of the total sales at the event, calculate the total profit made. [2]

5 (a) In a n-sided regular polygon, its exterior angle is $\frac{1}{11}$ of its interior angle.

Find n. [2]

(b)



The diagram shows a circle $ABCD$, centre O.
CET is a tangent to the circle at C.
AC and BD passes through O.
AE and BT intersect at D.
Angle $CBD = 35°$.

(i) Find, giving reasons for each answer,

(a) angle COD, [1]

(b) angle ODA, [1]

(c) angle BTC. [1]

(ii) Given that the chord $CD = 3.3$ cm.

(a) Show that the radius of the circle is 2.877 cm, correct to 4 significant figures. [2]

(b) Calculate the length of the chord AD. [2]

6 Consider the following pattern.

$R_1: \quad 1 \times \frac{1}{2} = 1 - \frac{1}{2} = \frac{1}{2}$

$R_2: \quad 2 \times \frac{2}{3} = 2 - \frac{2}{3} = \frac{4}{3}$

$R_3: \quad 3 \times \frac{3}{4} = 3 - \frac{3}{4} = \frac{9}{4}$

$R_4: \quad 4 \times \frac{4}{5} = 4 - \frac{4}{5} = \frac{16}{5}$

.

.

.

$R_9: \quad 9 \times \frac{9}{y} = 9 - \frac{9}{y} = \frac{z}{y}$

(a) Write down the value of y and of z. [1]

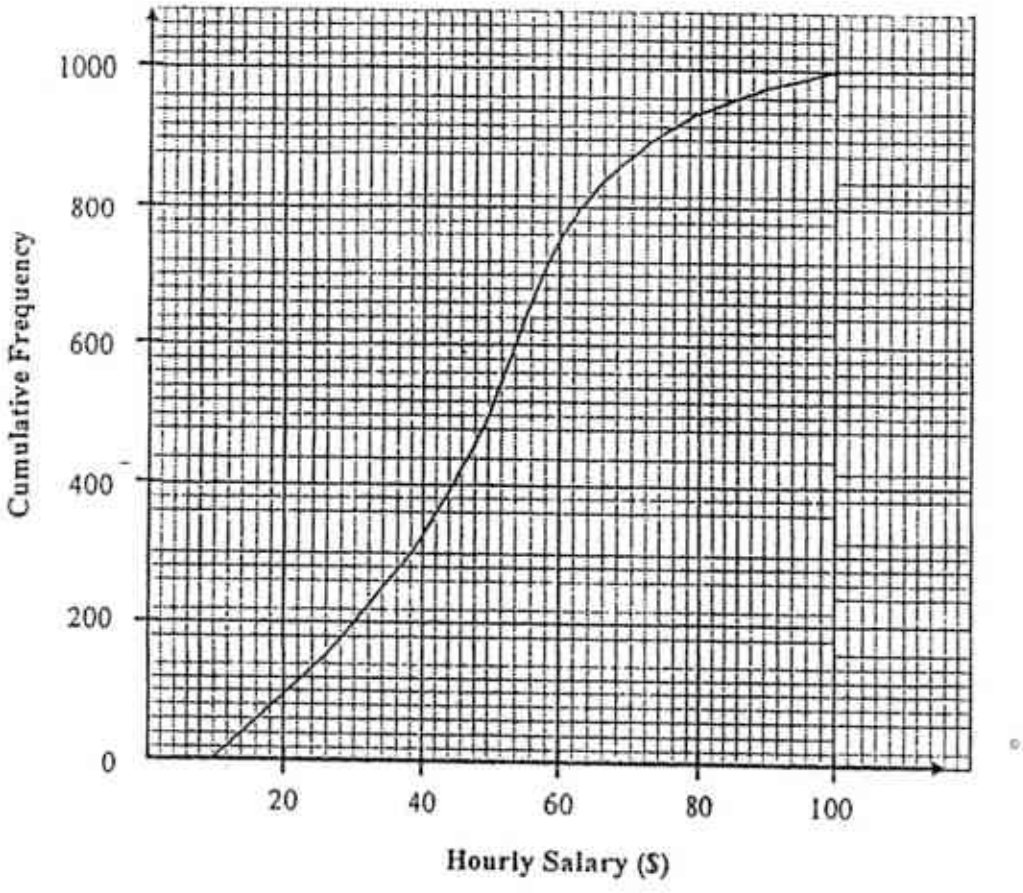
(b) If $R_a = \frac{3136}{b}$, find the value of a and of b. [2]

(c) Write down the n^{th} line in this pattern. [1]

(d) Given that the result of R_n is $\frac{c}{d}$, express $\frac{c}{d}$ in the form of $e + \frac{f}{n+1}$, where e is in terms of n and f is a constant. [1]

(e) From the above pattern, express q in terms of p if $p \times q = p - q$. [1]

7 The cumulative frequency graph below shows the distribution of hourly salary of 1000 workers in Company *V*.



(a) Use the graph to estimate the

(i) median hourly salary, [1]

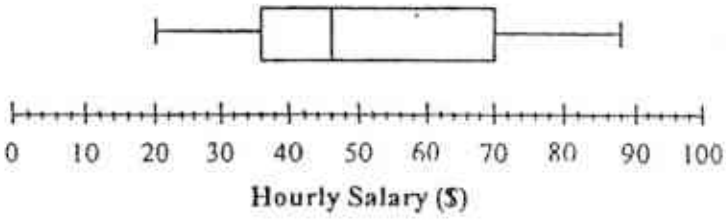
(ii) interquartile range of the distribution, [1]

(iii) 60^{th} percentile, [1]

(iv) value of g if 14% of the workers earn more than $\$g$ hourly. [1]

(b) Two workers were selected at random from this distribution. Find the probability that one worker selected earns not more than \$30 and the other earns more than \$80 hourly. Express your answer as a fraction in its lowest terms. [2]

(c) The hourly salary of 1000 workers in Company S is illustrated in a box-and-whisker diagram as shown below.



(i) Find the median hourly salary and interquartile range of the distribution in Company S. [2]

(ii) Compare and comment on the hourly salary of the workers from Company V and Company S in two different ways. [2]

8

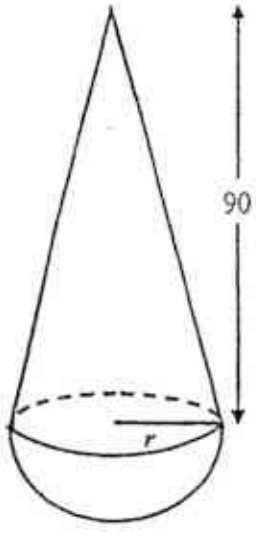


Figure *A*

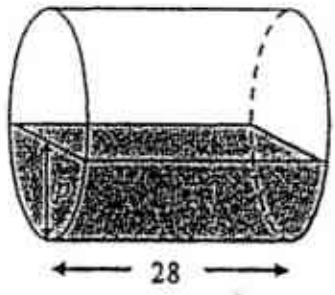


Figure *B*

Figure *A* shows a hollow container which is made by joining a hemispherical bowl of radius r cm to a right circular cone of radius r cm and height 90 cm. The capacity of the right circular cone is 4 times that of the hemispherical bowl.

(a) Calculate

(i) the value of r, [2]

(ii) the total exterior surface area of the container. [3]

(b) The container is completely filled with water. Calculate the volume of the water in the container. [2]

Figure *B* shows a hollow cylindrical container which is resting on a horizontal surface. The cylinder has radius p cm and length 28 cm.

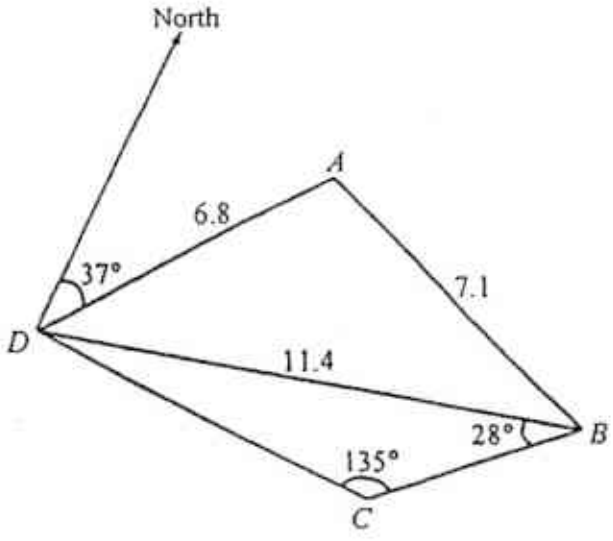
(c) (i) If all the water from the container in **Figure *A*** is poured into the container in **Figure *B***, it will fill up to exactly half of the capacity of **Figure *B***. Calculate the value of p. [2]

(ii) Find the total surface area of the inside of the container in **Figure *B*** which is in contact with the water. [2]

18

251

9



The diagram shows a quadrilateral garden $ABCD$ on horizontal ground.
$AD = 6.8$ m, $AB = 7.1$ m, $BD = 11.4$ m, angle $DBC = 28°$ and angle $DCB = 135°$.
A is on a bearing of 037° from D.

(a) Calculate

(i) angle DAB, [2]

(ii) the bearing of B from A, [2]

(iii) DC, [2]

(iv) the area of the garden. [3]

(b) A vertical flagpole of height 18 m stands at A. Find the greatest possible angle of elevation of the top of the flagpole viewed from a point along BD. [3]

10 (a) Amos changed 4319.70 Singapore Dollars (S$) into Japanese Yen (¥) at the bank for his trip to Tokyo. Upon returning from the trip, he has a remainder of ¥18760. The bank's exchange rates are as follows:

Singapore Dollar (S$) to 100 Japanese Yen (¥)	Selling	Buying
	1.120	1.100

(i) Calculate the amount of Japanese Yen Amos spent on the trip. [2]

(ii) Find the percentage of his money that was spent on the trip. [2]

Amos decided to convert all his remaining Japanese Yen from the trip to Euro (€) at a money changer.

(iii) Given that the exchange rate is ¥ 100 to € 0.7334, calculate the amount of euros he received from the transaction, correct to the nearest euro. [2]

(b) The cash price of a home entertainment system is $5799.
Zayn paid for it by hire purchase, with a deposit of 15% followed by 24 equal monthly instalments, with a simple interest charged at 3.5% per annum.

Calculate the

(i) simple interest charged, [2]

(ii) total amount Zayn will have to pay for the home entertainment system, [2]

(iii) amount of each instalment. [2]

11 Answer the whole of this question on a sheet of graph paper.

The table below gives some values of x and the corresponding values of y for $y = \frac{1}{4}x^2(x-3)$.

x	-2	-1	0	1	2	3	4	5
y	-5	-1	0	p	-1	0	4	q

(a) Calculate the value of p and of q. [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-2 \le x \le 5$.
Using a scale of 1 cm to represent 1 unit, draw a vertical y-axis for $-6 \le y \le 14$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the three solutions of $\frac{1}{4}x^2(x-3) = -\frac{4}{5}$. [2]

(d) Determine graphically the coordinate of the point S, for $-2 \le x \le 0$, such that the gradient of the curve at S is 1. [1]

(e) (i) On the same axes, draw the line $y = 12 - \frac{8}{5}x$ for $0 \le x \le 5$. [1]

(ii) Write down the x-coordinate of the point where this line intersects the curve. [1]

(iii) This value of x is a solution of the equation $x^3 - 3x^2 + Ax + B = 0$.
Find the value of A and of B. [2]

End of Paper

Answer Key:

1 (a) $x = 2.22$ or $x = 0.451$ (3 s.f.)

(b) $s = \dfrac{3v}{6 - v}$

(c) $\dfrac{5x + 12}{(4x + 1)(x - 3)}$

2 (a) $CD = 5.74$ cm (3 s.f.)

(c) Arc length $DB = 2.58$ cm (3 s.f.)

(d) 10.5 cm^2 (3 s.f.)

3 (a) $n(\varepsilon) = 80$ and $n(A \cap D) = 0$

(b) $B = \{21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98\}$

(c) There are elements which are multiples of 42.

4 (a) $\mathbf{P} = (2 \quad 1.50 \quad 1.20)$ or $\mathbf{P} = (2.00 \quad 1.50 \quad 1.20)$

(b) $\mathbf{Q} = \begin{pmatrix} 61 & 55 & 58 & 84 \\ 74 & 81 & 84 & 77 \\ 59 & 52 & 64 & 58 \end{pmatrix}$

(c) $\mathbf{PQ} = (303.80 \quad 293.90 \quad 318.80 \quad 353.10)$

(d) $\mathbf{PQC} = (1269.60)$

(e) The element of PQC represents the total sales from selling the cheeseburgers, French fries and drinks at all the 4 counters at the event.

(f) $571.32

5 (a) $n = 24$

(b) (i) (a) angle $COD = 70°$

(b) angle $ODA = 35°$

(c) angle $BTC = 20°$

(ii) (b) $AD = 4.71$ cm (3 s.f.)

88

253

6 (a) $y = 10$ and $z = 81$

(b) $a = 56$ and $b = 57$

(c) R_n : $n \times \frac{n}{n+1} = n - \frac{n}{n+1} = \frac{n^2}{n+1}$

(d) $\frac{c}{d} = n - 1 + \frac{1}{n+1}$

(e) $q = \frac{p}{p+1}$

7 (a) (i) Median hourly salary = \$50

(ii) Interquartile range = \$26

(iii) 60^{th} percentile = \$54

(iv) $g = 68$

(b) Probability $= \frac{8}{333}$

(c) (i) Median hourly salary = \$46

Interquartile range $= 70 - 36$
$= \$34$

(ii) Workers in Company *S* earned less per hour than workers in Company *V* since the median hourly salary in Company *S* is less than that of Company *V*.

The spread of hourly salary in Company *S* is wider than that of Company *V* since the interquartile range is higher for Company *S* than that for Company *V*.

8 (a) (i) $r = 11\frac{1}{4}$ or $r = 11.25$

(ii) Total exterior surface area of container = 4000 cm^2 (3 s.f.)

(b) Total volume of water = 14900 cm^2 (3 s.f.)

(c) (i) $p = 18.4$ (3 s.f.)

(ii) Total surface area in contact with water = 2680 cm^2 (3 s.f.)

9 (a) (i) $\angle DAB = 110.2°$ (1 d.p.)

(ii) Bearing of B from A is 106.8°.

(iii) $DC = 7.57$ m (3 s.f.)

(iv) Area of garden $= 35.3\ m^2$ (3 s.f.)

(b) Greatest possible angle of elevation $= 77.5°$ (1 d.p.)

10 (a) (i) Amount of Yen he spent on the trip = 366 927.50

(ii) Percentage = 95.1% (3 s.f.)

(iii) Amount of Euros received = 138 (nearest Euro)

(b) (i) Simple interest = $ 345.04 (2 d.p.)

(ii) Total amount to be paid = $ 6144.04 (2 d.p.)

(iii) Amount of each instalment = $ 219.76 (2 d.p.)

11 (a) $p = -0.5$ and $q = 12.5$

(c) $x = -0.9$ or $x = 1.4$ or $x = 2.5$

(d) $S(-0.5, -0.2)$

(e) (ii) $x = 4.2$

(iii) $A = 6\frac{2}{5}$ and $B = -48$

Name	Marking Scheme	Class	Register Number

4016/02 **15/S4PR2/EM/2**

MATHEMATICS

PAPER 2

Monday 3 August 2015 2 hours 30 minutes

NIL SINE LABORE

VICTORIA SCHOOL

PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

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255

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$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer all the questions.

1 (a) Solve the equation $x(3x-8)=-3$. [2]

$x(3x-8)=-3$

$3x^2-8x+3=0$

$x=\dfrac{-(-8)\pm\sqrt{(-8)^2-4(3)(3)}}{2(3)}$

$=\dfrac{8\pm\sqrt{64-36}}{6}$

$=\dfrac{8\pm\sqrt{28}}{6}$

$x=\dfrac{8+\sqrt{28}}{6}$ or $x=\dfrac{8-\sqrt{28}}{6}$

$x=2.22$ (3 s.f.) $x=0.451$ (3 s.f.)

(b) Given that $\dfrac{v}{3}=\sqrt{\dfrac{2(2s-v)}{s+3}}$, express s in terms of v. [3]

$\dfrac{v}{3}=\sqrt{\dfrac{2(2s-v)}{s+3}}$

$\dfrac{v^2}{9}=\dfrac{2(2s-v)}{s+3}$

$v^2(s+3)=18(2s-v)$

$sv^2+3v^2=36s-18v$

$sv^2-36s=-3v^2-18v$

$s(v^2-36)=-3v(v+6)$

$s=\dfrac{-3v(v+6)}{v^2-36}$

$s=\dfrac{-3v(v+6)}{(v+6)(v-6)}$

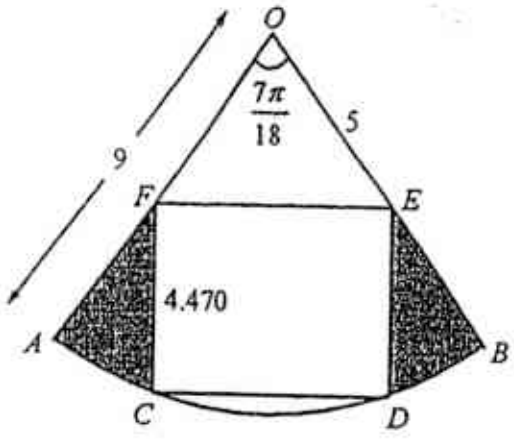
$s=\dfrac{-3v}{v-6}$

$s=\dfrac{3v}{6-v}$

(c) Express $\frac{1}{4x^2-11x-3}-\frac{3}{4x+1}-\frac{2}{3-x}$ as a single fraction in its simplest form. [4]

$$\frac{1}{4x^2-11x-3}-\frac{3}{4x+1}-\frac{2}{3-x}$$
$$=\frac{1}{(4x+1)(x-3)}-\frac{3}{4x+1}-\frac{2}{3-x}$$
$$=\frac{1}{(4x+1)(x-3)}-\frac{3}{4x+1}+\frac{2}{x-3}$$
$$=\frac{1-3(x-3)+2(4x+1)}{(4x+1)(x-3)}$$
$$=\frac{1-3x+9+8x+2}{(4x+1)(x-3)}$$
$$=\frac{5x+12}{(4x+1)(x-3)}$$

2



In the diagram, OAB is a sector of a circle, centre O and radius 9 cm. Rectangle $CDEF$ is inscribed in the sector. Given that angle $AOB = \frac{7\pi}{18}$ radians, $OE = OF = 5$ cm and $CF = DE = 4.470$ cm,

(a) find the length of CD, [2]

$FE = \sqrt{5^2 + 5^2 - 2(5)(5)\cos\frac{7\pi}{18}}$

≈ 5.736 cm

$CD = FE$

$CD = 5.74$ cm (3 sig. fig.)

(b) show that angle COD is approximately 0.6486 radians, [2]

$5.736^2 = 9^2 + 9^2 - 2(9)(9)\cos COD$

$\cos COD = \frac{9^2 + 9^2 - 5.736^2}{2(9)(9)}$

$COD \approx 0.6486$ (shown)

(c) find the arc length DB, [2]

$BOD = \frac{1}{2}\left(\frac{7\pi}{18} - 0.6486\right)$

≈ 0.28657

Arc length $DB = (9)(0.2866)$

$= 2.58$ cm (3 sig. fig.)

(d) find the area of the shaded region. [3]

Area of shaded region

$= (\text{area of sector } AOB) - (\text{area of sector } COD) - 2(\text{area of } \Delta\ FOC)$

$= \left(\frac{1}{2} \times 9^2 \times \frac{7\pi}{18}\right) - \left(\frac{1}{2} \times 9^2 \times 0.6486\right) - 2\left(\frac{1}{2} \times 9 \times 5 \times \sin 0.2866\right)$

$= 10.5 \text{ cm}^2$ (3 sig.fig.)

3 Given that $\varepsilon = \{x : x \text{ is an integer}, 20 \leq x \leq 99\}$

$A = \{x : x = 2m + 1, \text{ where } m \text{ is a positive integer}\}$

$B = \{x : x \text{ is divisible by } 7\}$

$C = \{x : x \text{ is a multiple of } 6\}$

$D = \{x : x \text{ has a remainder of 2 when divided by } 4\}$

(a) State the value of $n(\varepsilon)$ and $n(A \cap D)$. [2]

$n(\varepsilon) = 99 - 20 + 1 = 80$ and $n(A \cap D) = 0$

(b) List the elements in B. [1]

$B = \{21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98\}$

(c) Describe, as simply as possible, the meaning of $B \cap C \neq 0$ in words. [1]

There are elements which are multiples of 42.

4 At a fund raising event, a class ran a fast food store and had 4 serving counters *A*, *B*, *C* and *D*. The store sold cheeseburgers, French fries and drinks. Each cheeseburger was sold at \$2.00, a packet of French fries for \$1.50 and a drink for \$1.20. The table below shows the number of items sold by each counter during the event.

Items \ Counter	*A*	*B*	*C*	*D*
Cheeseburger	61	55	58	84
French fries	74	81	84	77
Drinks	59	52	64	58

(a) Write down a 1×3 matrix P to represent the selling price of each cheeseburger, French fries and drink sold. [1]

$\mathbf{P} = (2 \quad 1.50 \quad 1.20)$ or $\mathbf{P} = (2.00 \quad 1.50 \quad 1.20)$

(b) Write down a matrix Q such that PQ will give the total sales at each serving counter. [1]

$$\mathbf{Q} = \begin{pmatrix} 61 & 55 & 58 & 84 \\ 74 & 81 & 84 & 77 \\ 59 & 52 & 64 & 58 \end{pmatrix}$$

(c) Evaluate PQ. [1]

$$\mathbf{PQ} = (2 \quad 1.50 \quad 1.20)\begin{pmatrix} 61 & 55 & 58 & 84 \\ 74 & 81 & 84 & 77 \\ 59 & 52 & 64 & 58 \end{pmatrix}$$

$$= (303.80 \quad 293.90 \quad 318.80 \quad 353.10)$$

Matrix C is given by $\mathbf{C} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$.

(d) Evaluate PQC.

[1]

$$PQC = \begin{pmatrix} 2 & 1.50 & 1.20 \end{pmatrix} \begin{pmatrix} 61 & 55 & 58 & 84 \\ 74 & 81 & 84 & 77 \\ 59 & 52 & 64 & 58 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 303.80 & 293.90 & 318.80 & 353.10 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1269.60 \end{pmatrix}$$

(e) State what the element of PQC represent.

[1]

The element of PQC represents the total sales from selling the cheeseburgers, French fries and drinks at all the 4 counters at the event.

(f) If the total profit made by the class is 45% of the total sales at the event, calculate the total profit made.

[2]

$$\text{Total profit made} = \frac{45}{100} \times 1269.60$$
$$= \$571.32$$

5 (a) In a n-sided regular polygon, its exterior angle is $\frac{1}{11}$ of its interior angle.

Find n. [2]

Let the exterior angle and interior angle be x and $11x$ respectively.

$x + 11x = 180$ (adj. $\angle$s on a str. line)

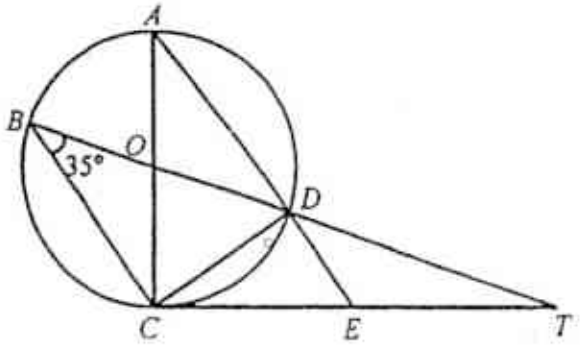
$12x = 180$

$x = 15$

$n = \frac{360}{15}$

$n = 24$

(b)



The diagram shows a circle $ABCD$, centre O.
CET is a tangent to the circle at C.
AC and BD passes through O.
AE and BT intersect at D.
Angle $CBD = 35°$.

(i) Find, giving reasons for each answer,

(a) angle COD, [1]

angle $COD = 2 \times 35°$ ($\angle$ at centre $= 2\angle$ at circumference)

$= 70°$

(b) angle ODA, [1]

angle $OAD = 35°$ ($\angle$s in the same segment)

angle ODA = angle OAD (base $\angle$s of isos. Δ)

$= 35°$

(c) angle BTC. [1]

angle $OCT = 90°$ (tan $\perp$ rad)

angle $BTC = 180° - 90° - 70°$ ($\angle$ sum of Δ)
$= 20°$

(ii) Given that the chord $CD = 3.3$ cm.

(a) Show that the radius of the circle is 2.877 cm, correct to 4 significant figures. [2]

angle $ADC = 90°$ (rt. $\angle$ in a semicircle)

$$\sin 35° = \frac{3.3}{AC}$$
$$AC = \frac{3.3}{\sin 35°}$$
$$AC \approx 5.75337 \text{ cm}$$

$$\text{Radius} = \frac{5.75337}{2}$$
Radius $= 2.877$ cm (4 sig. fig.)

Alternative working:

Let the radius be r cm.

$$3.3^2 = r^2 + r^2 - 2(r)(r)\cos 70°$$
$$2r^2 - 2r^2 \cos 70° = 3.3^2$$
$$2r^2(1 - \cos 70°) = 3.3^2$$
$$r^2 = \frac{3.3^2}{2(1 - \cos 70°)}$$
$$r \approx 2.877 \text{ (4 sig. fig.)}$$

(b) Calculate the length of the chord AD. [2]

$$AD = \sqrt{5.753^2 - 3.3^2}$$
$= 4.71$ cm (3 sig. fig.)

6 Consider the following pattern.

$$R_1: \quad 1\times\frac{1}{2}=1-\frac{1}{2}=\frac{1}{2}$$

$$R_2: \quad 2\times\frac{2}{3}=2-\frac{2}{3}=\frac{4}{3}$$

$$R_3: \quad 3\times\frac{3}{4}=3-\frac{3}{4}=\frac{9}{4}$$

$$R_4: \quad 4\times\frac{4}{5}=4-\frac{4}{5}=\frac{16}{5}$$

$$R_9: \quad 9\times\frac{9}{y}=9-\frac{9}{y}=\frac{z}{y}$$

(a) Write down the value of y and of z. [1]

$y=10$ and $z=81$

(b) If $R_a=\frac{3136}{b}$, find the value of a and of b. [2]

$a=\sqrt{3136}$ and $b=56+1$

$a=56$ $\quad$ $b=57$

(c) Write down the n^{th} line in this pattern. [1]

Row n: $\quad n\times\frac{n}{n+1}=n-\frac{n}{n+1}=\frac{n^2}{n+1}$

(d) Given that the result of R_n is $\frac{c}{d}$, express $\frac{c}{d}$ in the form of $e+\frac{f}{n+1}$, where e is in terms of n and f is a constant. [1]

$$\frac{c}{d}=\frac{n^2}{n+1}$$
$$=\frac{n^2-1+1}{n+1}$$
$$=\frac{n^2-1}{n+1}+\frac{1}{n+1}$$
$$=\frac{(n+1)(n-1)}{n+1}+\frac{1}{n+1}$$
$$=n-1+\frac{1}{n+1}$$

(e) From the above pattern, express q in terms of p if $p\times q=p-q$. [1]

$$q=\frac{p}{p+1}$$

20

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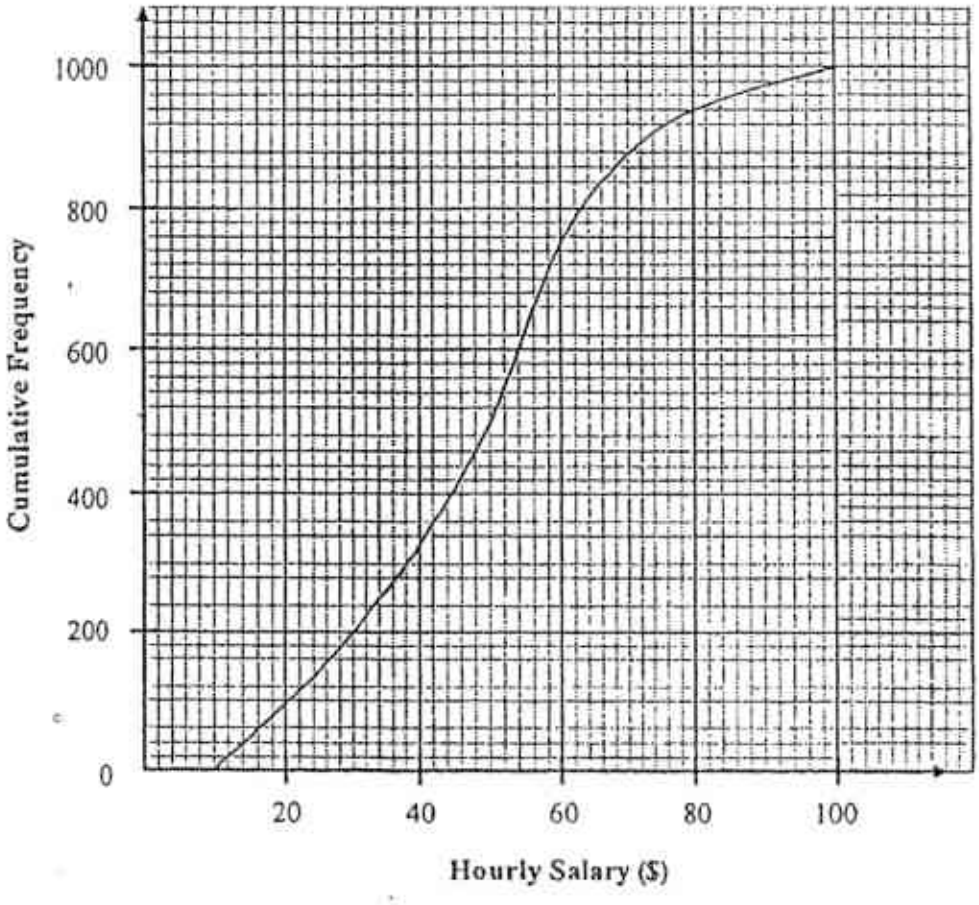
7 The cumulative frequency graph below shows the distribution of hourly salary of 1000 workers in Company *V*.



(a) Use the graph to estimate the

(i) median hourly salary, [1]

Median hourly salary = \$50

(ii) interquartile range of the distribution, [1]

Interquartile range = 60 – 34
= \$26

(iii) 60^{th} percentile, [1]

60% of workers = $\frac{60}{100} \times 1000$
= 600

60^{th} percentile = \$54

(iv) value of g if 14% of the workers earn more than $\$g$ hourly. [1]

No. of workers who earn more than $\$g = \frac{14}{100} \times 1000$

$= 140$

$g = 68$

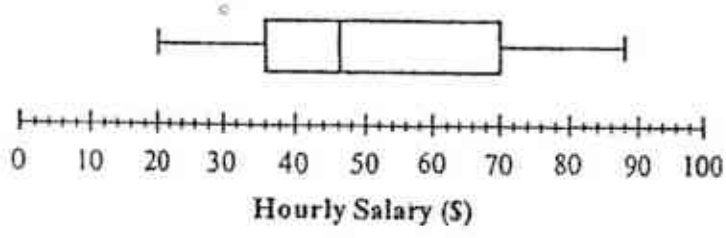
(b) Two workers were selected at random from this distribution. Find the probability that one worker selected earns not more than \$30 and the other earns more than \$80 hourly. Express your answer as a fraction in its lowest terms. [2]

$$\text{Probability} = \frac{200}{1000} \times \frac{1000-940}{999} \times 2$$

$$= \frac{200}{1000} \times \frac{60}{999} \times 2$$

$$= \frac{8}{333}$$

(c) The hourly salary of 1000 workers in Company S is illustrated in a box-and-whisker diagram as shown below.



(i) Find the median hourly salary and interquartile range of the distribution in Company S. [2]

Median hourly salary = \$46

Interquartile range = $70 - 36$

= \$34

(ii) Compare and comment on the hourly salary of the workers from Company V and Company S in two different ways. [2]

Workers in Company S earned lesser per hour than workers in Company V since the median hourly salary in Company S is lesser than that of Company V.

The spread of hourly salary in Company S is wider than that of Company V since the interquartile range is higher for Company S than that for Company V.

8

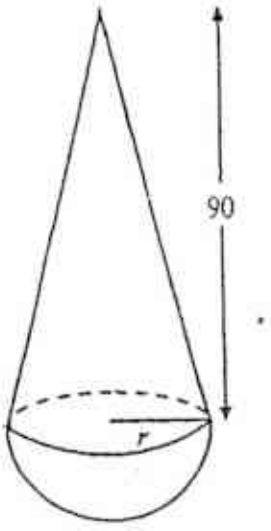


Figure *A*

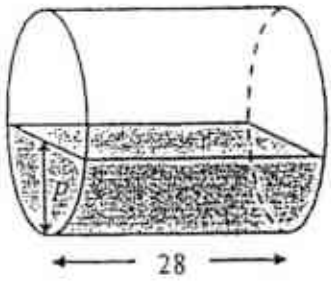


Figure *B*

Figure *A* shows a hollow container which is made by joining a hemispherical bowl of radius r cm to a right circular cone of radius r cm and height 90 cm. The capacity of the right circular cone is 4 times that of the hemispherical bowl.

(a) Calculate

(i) the value of r, [2]

volume of cone = 4 × volume of hemisphere

$$\frac{1}{3}\pi r^2(90) = 4 \times \frac{1}{2} \times \frac{4}{3}\pi r^3$$

$$30 = \frac{8}{3}r$$

$$r = 30 \times \frac{3}{8}$$

$$r = 11\frac{1}{4} \text{ or } r = 11.25$$

(ii) the total exterior surface area of the container. [3]

Total exterior surface area of container

$$= \left[\pi(11.25)\left(\sqrt{90^2 + 11.25^2}\right)\right] + \left[\frac{1}{2} \times 4\pi\left(11.25^2\right)\right]$$

$$\approx 4000.83237$$

$$= 4000 \text{ cm}^2 \quad (3 \text{ sig.fig.})$$

(b) The container is completely filled with water. Calculate the volume of the water in the container. [2]

Total volume of water

$= 5 \times \frac{1}{2} \times \frac{4}{3} \pi (11.25)^3$

≈ 14910.29326

$= 14900 \text{ cm}^3$ (3 sig.fig.)

Alternative working:

Total volume of water

$= \frac{1}{3} \pi (11.25)^2 (90) + \frac{1}{2} \times \frac{4}{3} \pi (11.25)^3$

≈ 14910.29326

$= 14900 \text{ cm}^3$ (3 sig.fig.)

Figure *B* shows a hollow cylindrical container which is resting on a horizontal surface. The cylinder has radius p cm and length 28 cm.

(c) (i) If all the water from the container in Figure *A* is poured into the container in Figure *B*, it will fill up to exactly half of the capacity of Figure *B*. Calculate the value of p. [2]

volume of water in cylinder = 14910

$\frac{1}{2} \pi p^2 (28) = 14910$

$p^2 = \frac{14910}{14\pi}$

$p \approx 18.41195$

$p = 18.4$ (3 sig.fig.)

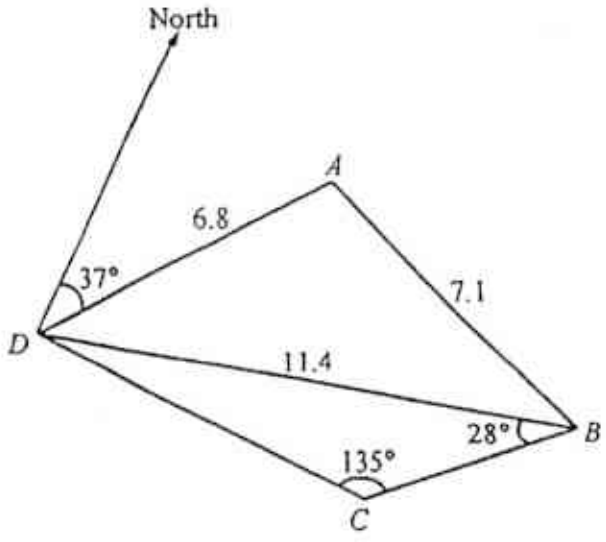
(ii) Find the total surface area of the inside of the container in Figure *B* which is in contact with the water. [2]

Total surface area in contact with water

$= \pi (18.41)^2 + \frac{1}{2} (2 \times \pi \times 18.41)(28)$

$= 2680 \text{ cm}^2$ (3 sig.fig.)

9



The diagram shows a quadrilateral garden $ABCD$ on horizontal ground.
$AD = 6.8$ m, $AB = 7.1$ m, $BD = 11.4$ m, angle $DBC = 28°$ and angle $DCB = 135°$.
A is on a bearing of 037° from D.

(a) Calculate

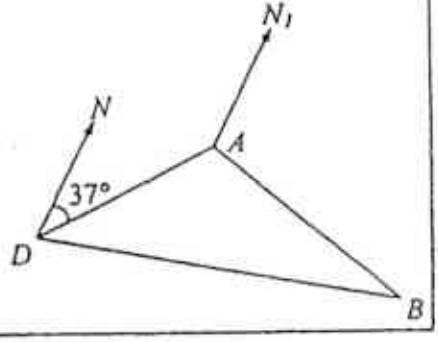
(i) angle DAB, [2]

$$11.4^2 = 6.8^2 + 7.1^2 - 2(6.8)(7.1)\cos DAB$$
$$\cos DAB = \frac{6.8^2 + 7.1^2 - 11.4^2}{2(6.8)(7.1)}$$
$$\angle DAB \approx 110.18°$$
$$= 110.2° \text{ (1 dec. pl.)}$$

(ii) the bearing of B from A, [2]

$\angle N_1AD = 180° - 37°$ (int. $\angle$s, $DN // AN_1$)
$= 143°$

$\angle N_1AB = 360° - 143° - 110.18°$ ($\angle$s at a pt.)
$= 106.82°$
$= 106.8°$ (1 d.p.)



Bearing of B from A is 106.8°.

(iii) DC. [2]

$$\frac{DC}{\sin 28°} = \frac{11.4}{\sin 135°}$$
$$DC \approx 7.5688$$
$$DC = 7.57 \text{ m} \quad (3 \text{ s.f.})$$

(iv) the area of the garden. [3]

$$\angle BDC = 180° - 135° - 28° \quad (\angle \text{ sum of } \Delta)$$
$$= 17°$$

Area of garden

$$= \left[\frac{1}{2}(6.8)(7.1)\sin 110.18°\right] + \left[\frac{1}{2}(11.4)(7.569)\sin 17°\right]$$
$$\approx 35.272$$
$$= 35.3 \text{ m}^2 \quad (3 \text{ s.f.})$$

(b) A vertical flagpole of height 18 m stands at A. Find the greatest possible angle of elevation of the top of the flagpole viewed from a point along BD. [3]

Let the shortest distance from A to BD be h m.

$$\frac{1}{2} \times 11.4 \times h = \frac{1}{2} \times 6.8 \times 7.1 \times \sin 110.18°$$
$$h \approx 3.975$$

Let the greatest possible angle of elevation be θ.

$$\tan \theta = \frac{18}{3.975}$$
$$\theta = 77.5° \quad (1 \text{ d.p.})$$

10 (a) Amos changed 4319.70 Singapore Dollars (S$) into Japanese Yen (¥) at the bank for his trip to Tokyo. Upon returning from the trip, he has a remainder of ¥18760. The bank's exchange rates are as follows:

Singapore Dollar (S$) to 100 Japanese Yen (¥)	Selling	Buying
	1.120	1.100

(i) Calculate the amount of Japanese Yen Amos spent on the trip. [2]

$$\text{Amount of Yen he received} = \frac{100}{1.12} \times 4\,319.70$$
$$= 385\,687.50$$

$$\text{Amount of Yen he spent on the trip} = 385\,687.50 - 18\,760$$
$$= 366\,927.50$$

(ii) Find the percentage of his money that was spent on the trip. [2]

$$\text{Percentage} = \frac{366\,927.50}{385\,687.50} \times 100\%$$
$$= 95.1\% \text{ (3 s.f.)}$$

Amos decided to convert all his remaining Japanese Yen from the trip to Euro (€) at a money changer.

(iii) Given that the exchange rate is ¥ 100 to € 0.7334, calculate the amount of euros he received from the transaction, correct to the nearest euro. [2]

$$\text{Amount of Euros received} = \frac{0.7334}{100} \times 18760$$
$$\approx 137.59$$
$$= 138 \text{ (nearest Euro)}$$

(b) The cash price of a home entertainment system is $5799.
Zayn paid for it by hire purchase, with a deposit of 15% followed by 24 equal monthly instalments, with a simple interest charged at 3.5% per annum.

Calculate the

(i) simple interest charged, [2]

$$\text{Simple interest} = \left(\frac{85}{100} \times 5799\right) \times \frac{3.5}{100} \times 2$$
$$\approx 345.0405$$
$$= \$\,345.04 \text{ (2 d.p.)}$$

(ii) total amount Zayn will have to pay for the home entertainment system, [2]

$$\text{Total amount to be paid} = 5799 + 345.0405$$
$$= 6144.0405$$
$$= \$\,6144.04 \text{ (2 d.p.)}$$

(iii) amount of each instalment. [2]

$$\text{Amount of each instalment} = \frac{\left(\frac{85}{100} \times 5799\right) + 345.0405}{24}$$
$$= \frac{4929.15 + 345.0405}{24}$$
$$= \frac{5274.1905}{24}$$
$$= \$\,219.76 \text{ (2 d.p.)}$$

Alternative working:

$$\text{Amount of each instalment} = \frac{6144.0405 - \frac{15}{100} \times 5799}{24}$$
$$= \frac{6144.0405 - 869.85}{24}$$
$$= \frac{5274.1905}{24}$$
$$= \$\,219.76 \text{ (2 d.p.)}$$

11 **Answer the whole of this question on a sheet of graph paper.**

The table below gives some values of x and the corresponding values of y for $y = \frac{1}{4}x^2(x-3)$.

x	−2	−1	0	1	2	3	4	5
y	−5	−1	0	p	−1	0	4	q

(a) Calculate the value of p and of q. [1]

$p = -0.5$ and $q = 12.5$

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-2 \le x \le 5$.
Using a scale of 1 cm to represent 1 unit, draw a vertical y-axis for $-6 \le y \le 14$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

Correct points are plotted

Smooth curve drawn

Labelling of x-axis, y-axis and equation of graph

(c) Use your graph to find the three solutions of $\frac{1}{4}x^2(x-3) = -\frac{4}{5}$. [2]

$x = -0.9$ or $x = 1.4$ or $x = 2.5$

(d) Determine graphically the coordinate of the point S, for $-2 \le x \le 0$, such that the gradient of the curve at S is 1. [1]

$x = -0.5$

(e) (i) On the same axes, draw the line $y = 12 - \frac{8}{5}x$ for $0 \le x \le 5$. [1]

B1 for the line $y = 12 - \frac{8}{5}x$ drawn.

(ii) Write down the x-coordinate of the point where this line intersects the curve. [1]

$x = 4.2$

(iii) This value of x is a solution of the equation $x^3 - 3x^2 + Ax + B = 0$.
Find the value of A and of B. [2]

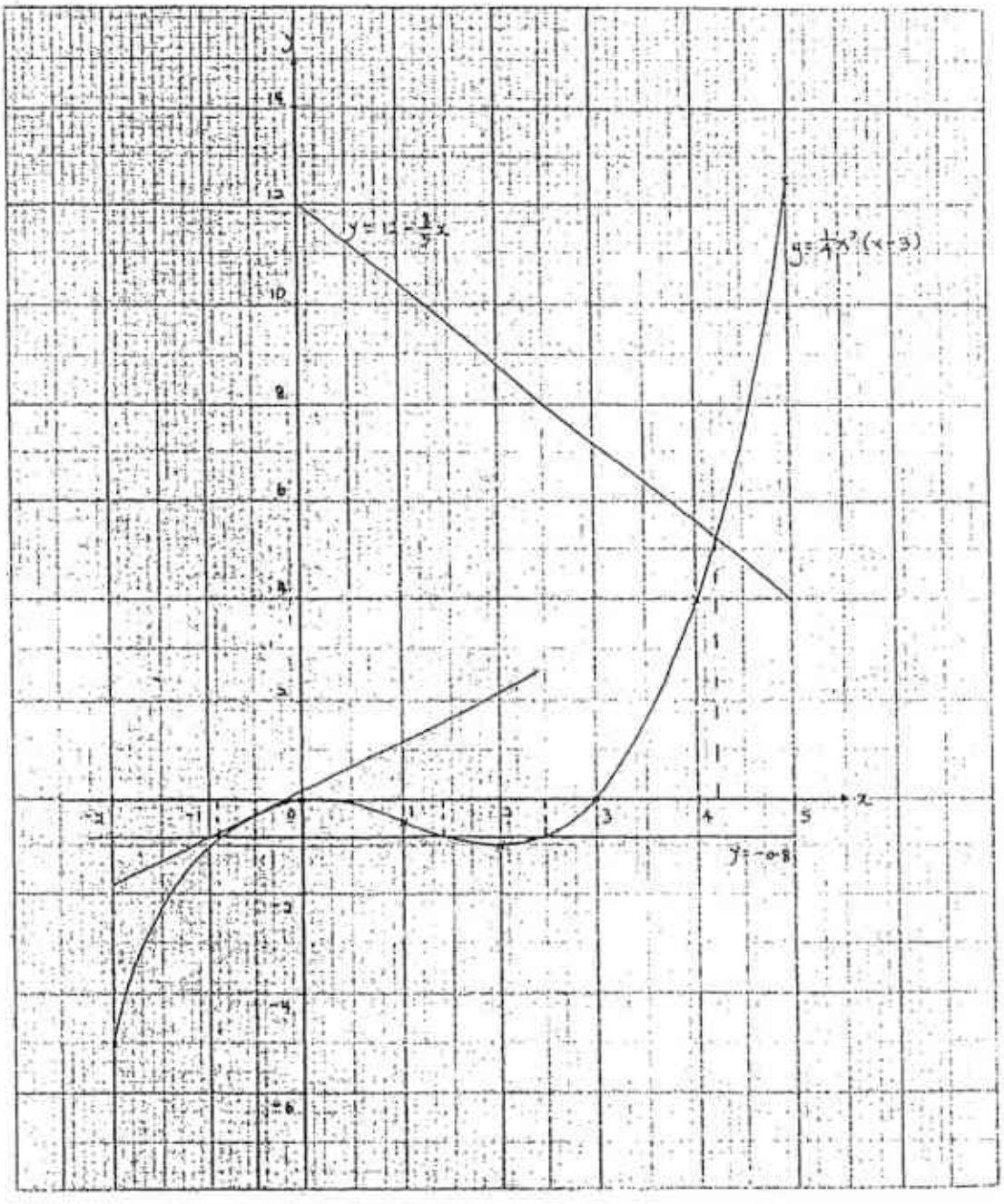
$\frac{1}{4}x^3(x-3) = 12 - \frac{8}{5}x$

$x^3 - 3x^2 = 48 - \frac{32}{5}x$

$x^3 - 3x^2 + \frac{32}{5}x - 48 = 0$

$A = \frac{32}{5}$ and $B = -48$

$A = 6\frac{2}{5}$



End of Paper

SWISS COTTAGE SECONDARY SCHOOL
SECONDARY FOUR EXPRESS
PRELIMINARY EXAMINATIONS

Name: _____( ) Class: Sec _____

MATHEMATICS

Paper 1

4016/01
Friday 14 August 2015
2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and is the answer is not exact, give your answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use
80

This question paper consists of **18** printed pages.

Setter: Ms Yeo Koon Koon
Vetter : Mr Ang Hanping

[Turn over

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Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Measurement

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistic

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Calculate $\dfrac{3+\sqrt[3]{27-8\times\sqrt{16}\times(-3)}}{4^{\frac{5}{2}}}$,

showing the first five digits on your calculator display.

Answer ..... [1]

(b) Give your answer to part **(a)** correct to 3 significant figures.

Answer ..... [1]

---

2 These are the first five terms in a sequence.

1 3 7 15 31

(a) Write down the sixth and seventh term in the sequence.

Answer ..... , ... [1]

(b) Write down an expression, in terms of n, for the nth term in the sequence.

Answer ..... [1]

---

3 **(a)** Given that $7^{2n} \div 49^{-2} = 7^{5}$, find the value of n.

Answer $n =$ ..... [2]

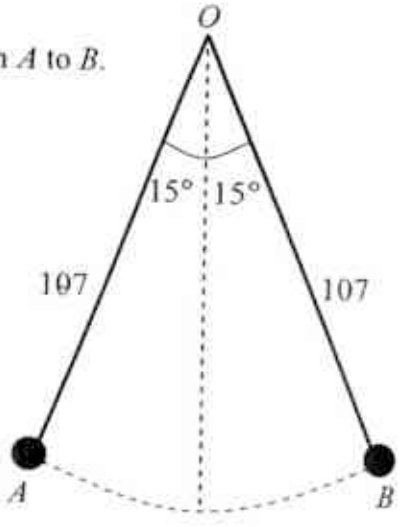
(b) Given that $\frac{1}{500}\pi = k \times 10^{-3}$, find the value of k, rounding your answer to 1 significant figure.

Answer $k =$ ..... [1]

---

4 A pendulum of length 107 cm is suspended from O.
Its end swings 15° on either side of the vertical from A to B.

Calculate the length of the arc AB.



Answer ..... cm [2]

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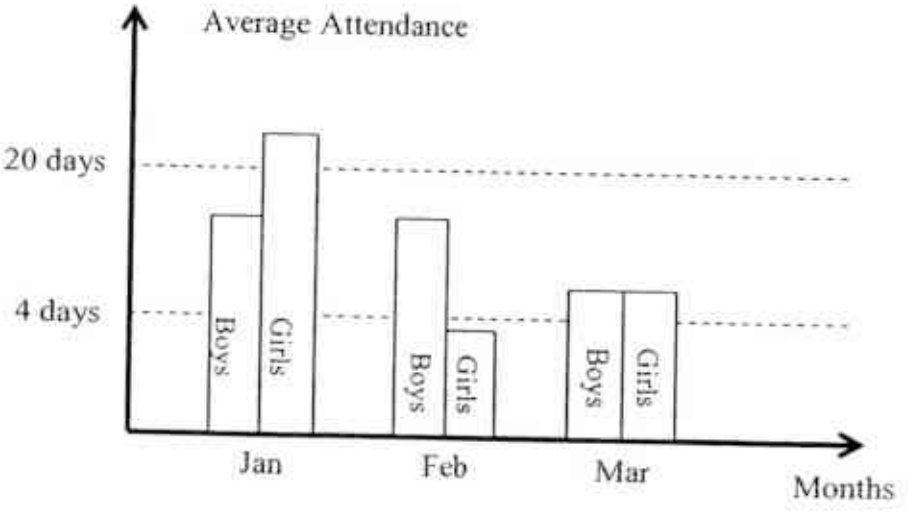
5 The graph shows the average attendance of boys and girls respectively for a school over three months.



Explain one way in which the graph is misleading.

Answer .....

.....

.....

..... [1]

---

6 Simplify $(x-4)^2 - 16(x-4) + 64$.

Answer ..... [2]

---

7 V is inversely proportional to the cube of T. Given that V is increased by 237.5%, calculate the percentage change in T.

Answer ..... % [2]

---

8 $\varepsilon = \{ \text{integers} \quad x : 0 < x \leq 16 \}$
$A = \{ \text{multiples of } 3 \}$
$B = \{ \text{perfect squares} \}$
$C = \{ \text{integers} \quad x : x > \sqrt{90} \}$

List the elements in

(a) A',

Answer ..... [1]

(b) $A \cap B$,

Answer ..... [1]

(c) $(A \cup B \cup C)'$.

Answer ..... [1]

9 A polygon has n sides. When the number of sides is doubled, the interior angle is increased by $30°$.

Find the value of n.

Answer n = ... [3]

10 The volume of a cone of radius r cm and height h cm is 400 cm^3.

(a) Calculate the volume of a similar cone whose height is thrice that of the given cone.

Answer ..... cm^3 [1]

(b) Find the volume of a cone whose radius is $\frac{1}{2}r$ cm and whose height is $\frac{3}{5}h$ cm.

Answer ..... cm^3 [2]

---

11 **(a)** Factorise completely $9x^2 + 12x + 4 - 25y^2$.

Answer ..... [2]

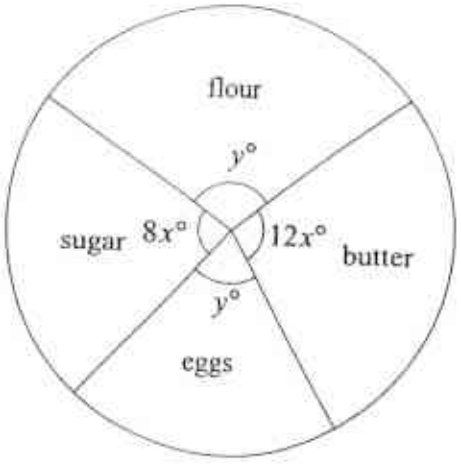
(b) Simplify $\frac{x^{2n} - y^{2n}}{x^{n+2} - x^2 y^n}$.

Answer ..... [2]

---

12 The mass of ingredients used for making a cake are represented on the pie chart. Given that the ratio of the mass of butter to eggs is 5 : 2, find the value of x.

If 360 grams of butter is used, calculate the mass of eggs used.

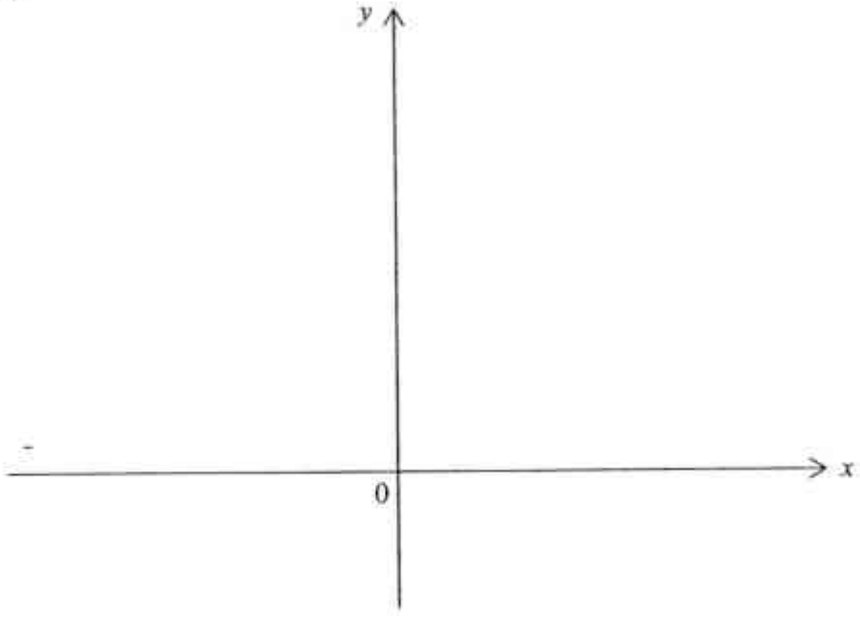


Answer $x =$ ..... [2]

..... g [2]

13 **(a)** Sketch the graph of $y = -(x+1)(x-3)$.

Answer



[2]

(b) Write down the equation of the line of symmetry of the graph of $y = -(x+1)(x-3)$.

Answer ..... [1]

---

14 **(a)** Solve $\frac{3}{5}x(x-4)(3x+5) = 0$.

Answer x = ... [2]

(b) Solve the equation $2x^2 - 6x - 9 = 0$, giving your answers correct to two decimal places.

Answer x = ... , ... [2]

---

15 (a) Alan bought an Ipad mini for \$329. He sold it online and made a profit of 135% on the price he paid for it.

Find the price at which he sold the Ipad Mini.

Answer \$ ..... [2]

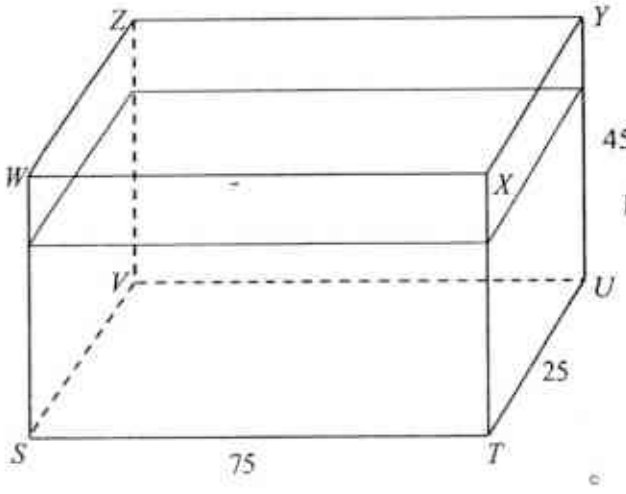
(b) The proposed area of a pond of 0.00078 km^2 is represented by 2 cm^2 on the floor plan of a garden.

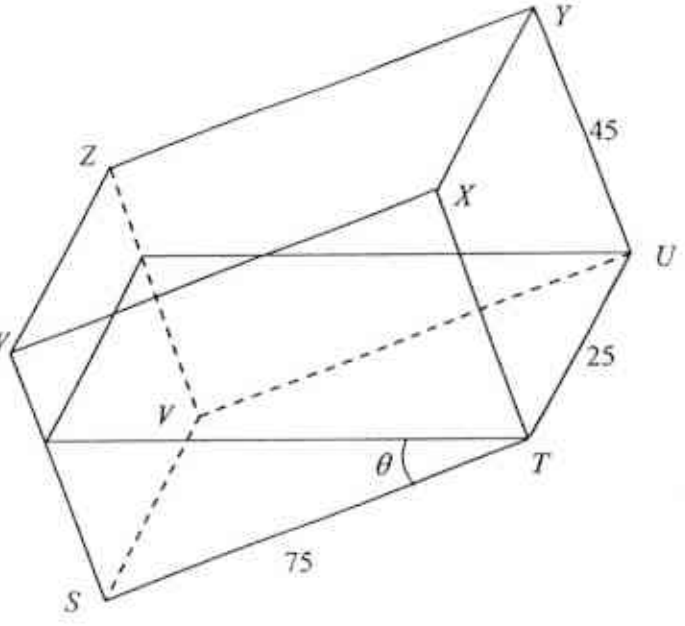
Given that the floor plan is drawn to a scale of $1 : n$, find the value of n.

Answer $n =$ .....[2]

16 Figure *A* shows an open rectangular container *STUVWXYZ* which is $\frac{4}{5}$ filled with water. The internal dimensions of the container are 75 cm long, 25 cm wide and 45 cm high. The base, *STUV*, of the container is resting on a flat surface.
In figure *B*, the container is tilted about the edge *SV* and 40% of the amount of water has been poured out.

Calculate the angle θ.

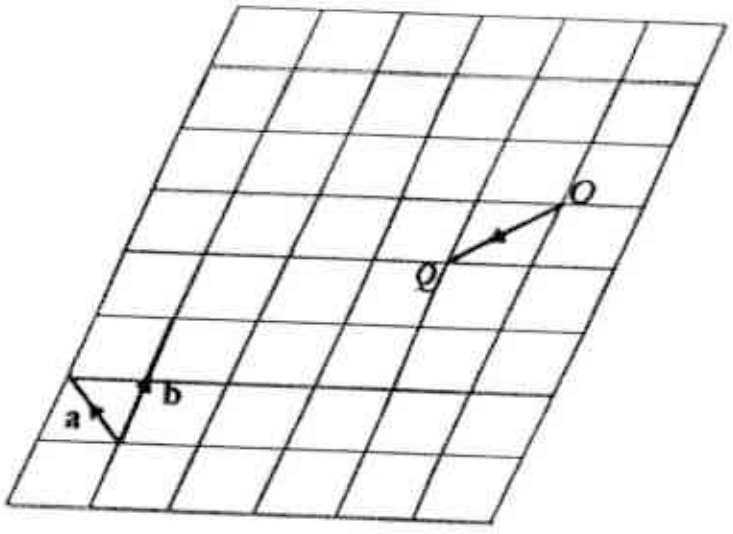




Answer $\theta =$ .....[4]

17 The grid in the answer space below shows the vectors **a** and **b**.
O and Q are two points on the given grid.

Answer



(i) Write down an expression for $\overrightarrow{OQ}$ in terms of **a** and **b.**

Answer $\overrightarrow{OQ}$ = ... [1]

(ii) Mark and label clearly on the grid the point P such that $\overrightarrow{OP} = 3\mathbf{a} - 2\mathbf{b}$.

Answer On the grid [1]

(iii) A point R is such that $\overrightarrow{OR} = h\ \overrightarrow{OQ}$ and $\overrightarrow{PR} = k\ \mathbf{b}$, where h and k are scalars.

(a) Mark and label clearly on the grid the point R.

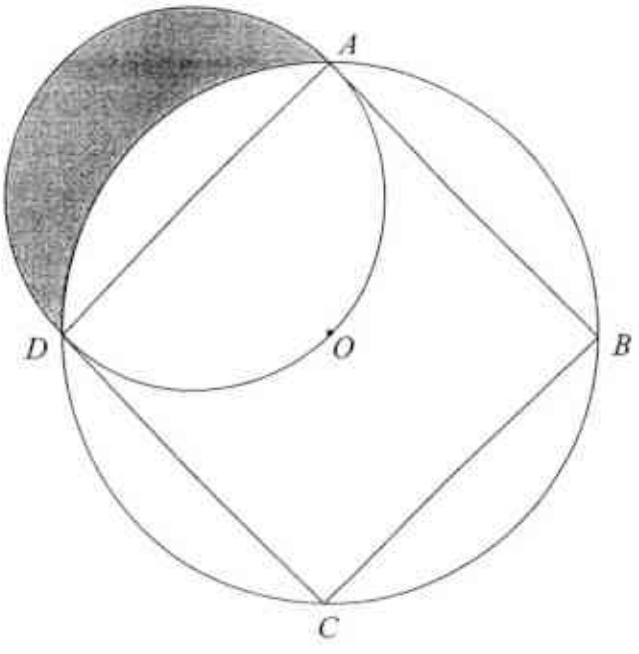
Answer On the grid [1]

(b) Find the value of h and of k.

Answer h = ... , k = ... [1]

18 In the diagram, $ABCD$ is a square whose diagonals are each 2 cm long. Given that O is the centre of the bigger circle and BD and AD are the diameters of the bigger and smaller circle respectively.

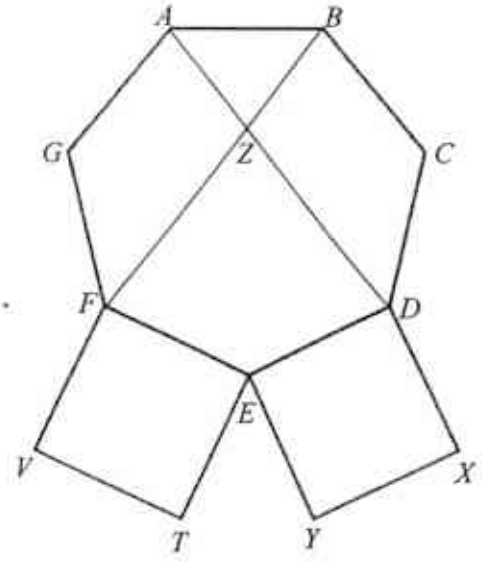
Find the area of the shaded region.



Answer ..... cm^2 [4]

19 **(a)** Given that figure *ABCDEFG* is a regular polygon.

Name the figure *ABCDEFG*.



Answer ..... [1]

(b) It is given that figure *FETV* and figure *EDXY* are squares, find

(i) angle *YET*,

Answer $\angle YET =$ ..... [2]

(ii) angle *AZF*.

Answer $\angle AZF =$ ..... [2]

20 **(a)** Solve the equation $\frac{3x}{4}+\frac{2x-9}{7}=5$.

Answer x = ... [2]

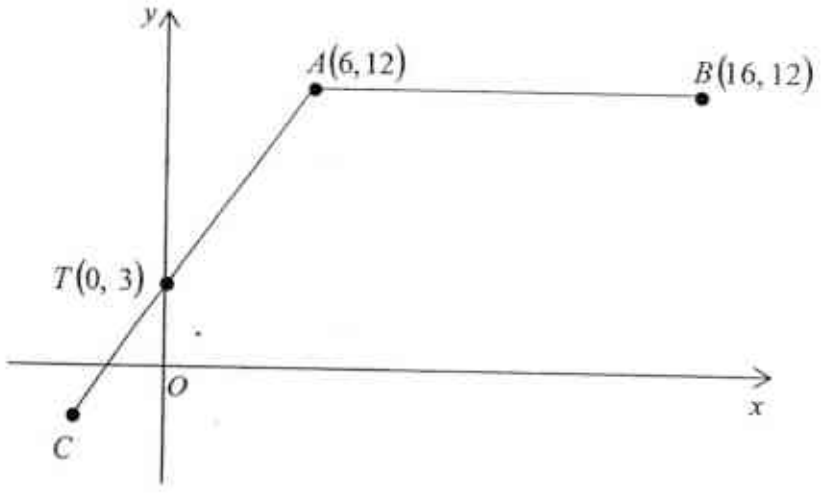
(b)(i) Express x^2+6x-7 in the form of $(x+p)^2+q$.

Answer ..... [2]

(ii) Hence solve $x^2+6x-7=0$.

Answer x = ... or ..... [1]

21 The diagram shows a sketch of two straight lines ATC and AB.
It is given that $AC = 3TC$ and $A(6, 12)$, $B(16, 12)$ and $T(0, 3)$.



(a) Find the coordinates of C.

Answer C (..... , ...) [1]

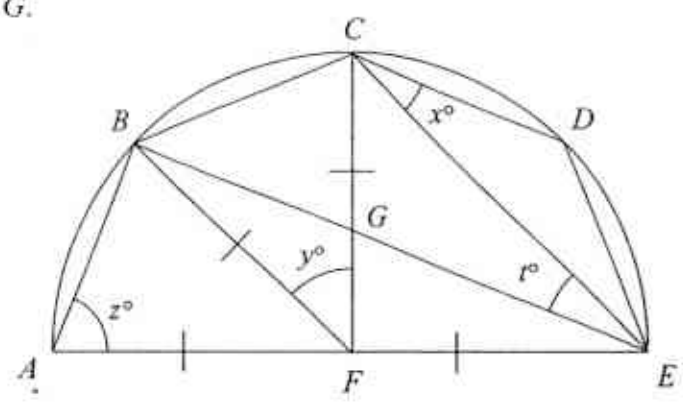
(b) Find the equation of the line ATC.

Answer ..... [2]

(c) Calculate the area of triangle ATB.

Answer ..... units^2 [2]

22 In the diagram, AB, BC, CD and DE are four adjacent sides of a regular octagon.
$FA = FB = FC = FE = 5$ cm.
CF meets BE at G.



(i) Calculate

(a) x,

Answer $x =$ ..... [1]

(b) y,

Answer $y =$ ..... [1]

(c) z,

Answer $z =$ ..... [1]

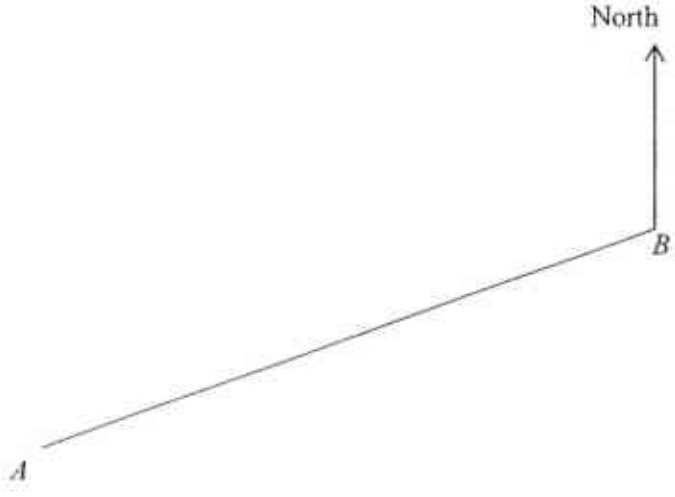
(d) t.

Answer $t =$ ..... [1]

(ii) If semi-circle $ABCDE$ is a dartboard, and a dart is thrown towards the board at random, calculate the probability that the dart lands on the octagonal region. (Assume that the dart will always land on the dartboard.)

Answer ..... [2]

23 A map is drawn to a scale of 1 cm to 3 km.
The diagram below shows the positions of two villages A and B on the map.



(a) A third village, C lies north of the line AB.
It is 21 km from A and 18 km from B.

Using ruler and compasses only, construct triangle ABC. [2]

(b) Construct the perpendicular bisector of AB. [1]

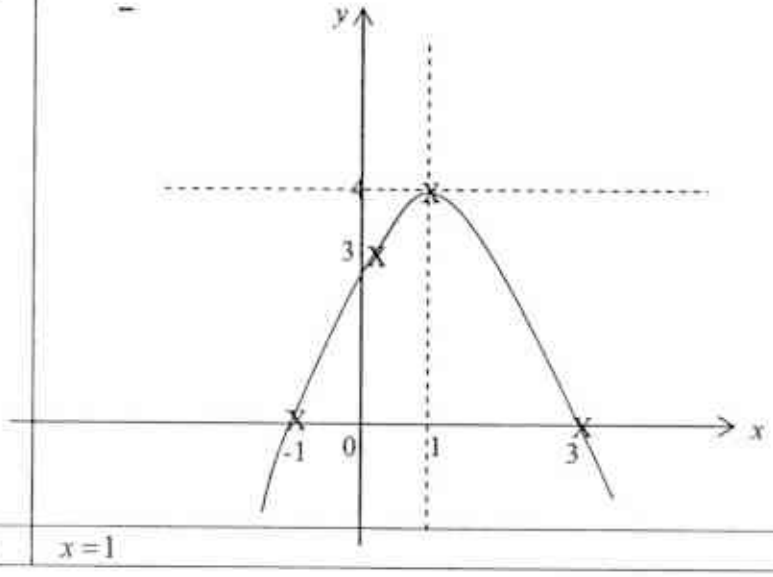
(c) Draw a circle of radius 9 km taking village C as the centre of the circle. [1]

(d) A petrol station is to be built so that it is equidistant from A and B and 9 km from village C.

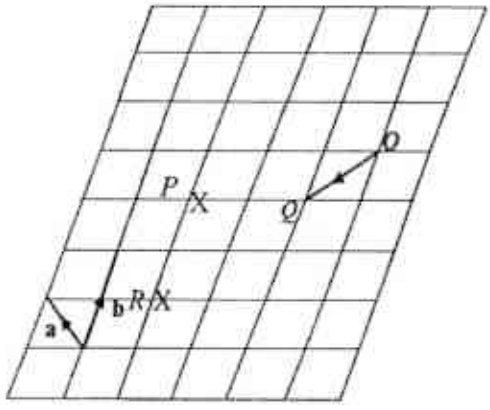
Mark, with letters F and G, the two possible positions of the petrol station. [1]

End of Paper

Answers to 2015 S4E EM PRELIMINARY EXAMINATION Paper 1

1a	0.2491
1b	0.249
2a	63, 127
2b	nth term : $2^n - 1$
3a	$n = -\frac{5}{34}$
3b	$k = 6$ (1 sf)
4	56.0 cm
5	The vertical interval range is not proportionate thus giving people wrong impression. or The vertical axis should be measured in terms of percentage as it gives a clearer and consistent projection of the average attendance.
6	$x^2 - 24x + 144$
7	-70.4%
8a	$A' = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16\}$
8b	$A \cap B = \{9\}$
8c	$(A \cup B \cup C)' = \{2, 5, 7, 8\}$
9	$n = 6$
10a	10800 cm^3
10b	60 cm^3
11a	$(3x + 2 + 5y)(3x + 2 - 5y)$
11b	$\frac{(x^n + y^n)}{x^2}$
12	$y + 10x = 180$ ------ (1) $y = \frac{24x}{5}$ ----- (2) $x = 12\frac{6}{37}$ $y = 58\frac{14}{37}$ 144 grams
13a	
13b	$x = 1$

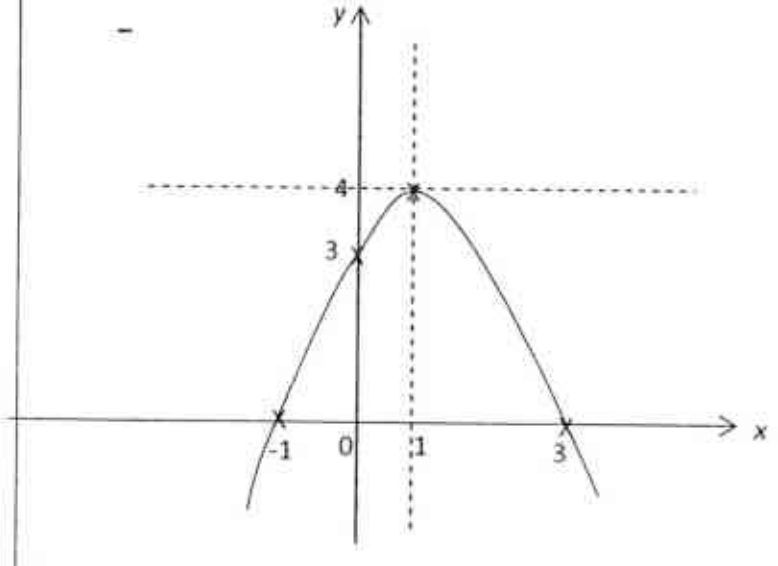
276

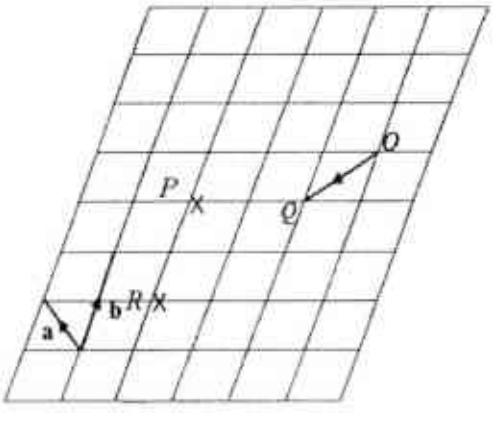
14a	$x = 0, \quad 4, \quad -\frac{5}{3}$
14b	$x = 4.10 \quad \text{or} \quad -1.10$
15a	\$773.15
15b	1 cm : 1970 cm
16	$\theta = 29.9°$
17i	$\mathbf{a} - \mathbf{b}$
17iiib	$h = 3, \quad k = -1$
	
18	0.5 cm^2
19a	Heptagon
19bi	51.4°
19bii	102.9°
20a	$x = 6\frac{2}{29} \quad \text{or} \quad 6.07$
20bi	$(x+3)^2 - 16$
20bii	$x = -7 \quad \text{or} \quad 1$
21a	$(-3, \quad -1.5)$
21b	Equation of ATC : $y = \frac{3}{2}x + 3 \quad \text{or} \quad 2y = 3x + 6$
21c	Area of triangle $ATB = 45$ units^2
22ia	22.5°
22ib	45°
22ic	67.5°
22id	22.5°
22ii	0.900

Answers to 2015 S4E EM PRELIMINARY EXAMINATION Paper 1

1a	$\frac{3+\sqrt[3]{27-8\times\sqrt{16}\times(-3)}}{4^{\frac{5}{2}}}$ =0.249162182 =0.2491 (5 digits)	B1
1b	0.249 (3 sf)	B1
2a	63, 127	B1
2b	nth term : 2^n-1	B1
3a	$7^{\frac{1}{2n}}\div 49^{-2}=7^{\frac{3}{5}}$ $\frac{1}{2n}+4=\frac{3}{5}$ $n=-\frac{5}{34}$	M1 A1
3b	$\frac{1}{500}\pi=k\times10^{-3}$ $k\times10^{-3}=6.283185307\times10^{-3}$ $k=6$ (1 sf)	B1
4	Arc length AB $=\frac{30}{360}\times2\pi(107)$ = 56.02507 cm = 56.0 cm	M1 A1
5	The vertical interval range is not proportionate thus giving people wrong impression. or The vertical axis should be measured in terms of percentage as it gives a clearer and consistent projection of the average attendance.	B1
6	$(x-4)^2-16(x-4)+64$ $=(x-12)^2$ or $x^2-8x+16-16x+64+64$ $=x^2-24x+144$	M1 A1

7	$V=\frac{k}{t^3} \Rightarrow k=Vt^3$ $3.375\,V=\frac{k}{T^3}$ $T^3=\frac{Vt^3}{3.375\,V}$ $T^3=\frac{8}{27}t^3$ Percentage change in $T=\left(1-\frac{8}{27}\right)\times 100=-70.4\%$	M1 A1
8a	$A' = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16\}$	B1
8b	$A\cap B = \{9\}$	B1
8c	$(A\cup B\cup C)' = \{2, 5, 7, 8\}$	B1
9	$\frac{(2n-2)\times 180}{2n}-\frac{(n-2)\times 180}{n}=30$ $\frac{360n-360-360n+720}{2n}=30$ $60n=360$ $n=6$	M1 M1 A1
10a	Volume of new cone $=\left(\frac{3}{1}\right)^3\times 400$ $=10800\ \text{cm}^3$	B1
10b	Volume of the new cone $=\frac{\frac{1}{3}\times\pi\times\left(\frac{1}{2}r\right)^2\times\frac{3}{5}h}{\frac{1}{3}\times\pi\times(r)^2\times h}\times 400$ $=\frac{3}{20}\times 400$ $=60\ \text{cm}^3$	M1 A1
11a	$9x^2+12x+4-25y^2$ $=(3x+2)^2-25y^2$ $=(3x+2)^2-(5y)^2$ $=(3x+2+5y)(3x+2-5y)$	M1 A1

11b	$\dfrac{x^{2n}-y^{2n}}{x^{n+2}-x^2y^n}$ $=\dfrac{(x^n)^2-(y^n)^2}{x^2(x^n-y^n)}$ $=\dfrac{(x^n+y^n)(x^n-y^n)}{x^2(x^n-y^n)}$ $=\dfrac{(x^n+y^n)}{x^2}$	M1 A1
12	$2y+20x=360$ $y+10x=180$ ------ (1) $\dfrac{12x}{y}=\dfrac{5}{2}$ $y=\dfrac{24x}{5}$ ----- (2) Sub (2) into (1), $\dfrac{24x}{5}+10x=180$ $x=12\dfrac{6}{37}$ $y=58\dfrac{14}{37}$ $12x=145\dfrac{35}{37}$: 360 grams $58\dfrac{14}{37}$: 144 grams	M1 M1 M1 A1
13a		M1 – x & y-intercept A1 – shape & critical pts

13b	$x = 1$	A1
14a	$x = 0, \quad 4, \quad -\frac{5}{3}$	M1 – Any 2 answers A1 – All answers
14b	$x = \frac{6 \pm \sqrt{108}}{4}$ $x = 4.10 \quad \text{or} \quad -1.10$	M1 A1
15a	Price that he sold $= \frac{235}{100} \times 329$ $= \$773.15$	M1 – ratio A1
15b	$2\ \text{cm}^2 : 0.00078\ \text{km}^2$ $1\ \text{cm} : 0.019748417\ \text{km}$ $1\ \text{cm} : 1970\ \text{cm}$	 M1 A1
16	Amount of water in container $= 0.6 \times 0.8 \times (75 \times 25 \times 45)$ $= 40\ 500\ \text{cm}^3$ Volume of water in container $= 40\ 500\ \text{cm}^3$ $\frac{1}{2} \times 75 \times 25 \times h = 40500$ $h = 43.2\ \text{cm}$ $\tan\theta = \frac{43.2}{75}$ $\theta = 29.9°$	M1 M1 M1 A1
17i	**a** – **b**	B1
17iiib	$h = 3, \quad k = -1$	B1
17ii 17iiia	 	

	B1, B1	

18	By pythagoras' thm $AD = \sqrt{2} \Rightarrow$ Radius of small circle $= \frac{\sqrt{2}}{2}$ Area of semi-circle $= \frac{1}{2}\pi\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$ / 0.785398163 cm^2 Area of segment $AD = \frac{1}{2}(1)^2[\frac{\pi}{2} - \sin\frac{\pi}{2}]$ = 0.285398163 cm^2 Area of shaded region $= \frac{\pi}{4} - 0.285398163 =$ 0.5 cm^2	M1 M1 M1 A1
19a	Heptagon	B1
19bi	One interior angle $= \frac{(7-2)\times 180}{7} = 128.5714286°$ Angle YET $= 360° - 128.57° - 90° - 90°$ $= 51.4°$	M1 A1
19bii	Angle ABF $= \frac{360° - 128.57° - 128.57°}{2}$ $= 51.43°$ Angle AZF $= 51.43° \times 2$ $= 102.9°$	M1 A1
20a	$\frac{3x}{4} + \frac{2x-9}{7} = 5$ $\frac{21x + 8x - 36}{28} = 5$ $29x = 176$ $x = 6\frac{2}{29}$ or 6.07	M1 A1
20bi	$x^2 + 6x - 7$ $= x^2 + 6x - 7$ $= (x+3)^2 - (3)^2 - 7$ $= (x+3)^2 - 9 - 7$ $= (x+3)^2 - 16$	M1 A1

20bii	$(x+3)^2-16=0$ $(x+3)^2=16$ $x=-7$ or 1	 A1

21a	Coordinates C $(0-3,\ 3-4.5)$ $(-3,\ -1.5)$	B1
21b	Gradient $=\frac{3}{2}$ Equation of ATC: $y=\frac{3}{2}x+3$ or $2y=3x+6$	M1 A1
21c	Area of triangle $ATB=\frac{1}{2}\times 10\times 9$ $=45$ units^2	M1 A1
22ia	One interior angle $=135°$ Angle $x=\frac{180-135}{2}$ $=22.5°$	B1
22ib	$y=\frac{360}{8}$ $=45°$	B1
22ic	$z=\frac{180-45}{2}$ $=67.5°$	B1
22id	$t=\frac{1}{2}y=22.5°$	B1
22ii	Area of dartboard $=4\times\frac{1}{2}\times 5\times 5\times\sin 45°=35.35533906$ cm^2 Area of semi-circle $=\frac{1}{2}\pi(5)^2=\frac{25}{2}\pi$ cm^2 P(dart land on $ABCDE$) $=\frac{35.35533906}{\frac{25}{2}\pi}=0.900$	M1 A1

SWISS COTTAGE SECONDARY SCHOOL
SECONDARY FOUR EXPRESS
PRELIMINARY EXAMINATIONS

Name: _____( ) Class: Sec _____

MATHEMATICS

Paper 2

4016/02
Monday 17 August 2015
2 hours 30 minutes

Additional materials: Answer paper (8 sheets)
Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.

SUBMIT SECTION A AND B SEPERATELY

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This question paper consists of **10** printed pages.

Setter: Mr Ang Hanping
Vetter: Ms Zoe Pow

[Turn over

280

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

Section A (61 marks)

1 **(a)** Factorise completely $a^3b + 2ab^3 - 2a^2 - 4b^2$. [2]

(b) Express as a single fraction in its simplest form $\frac{2}{3x^2 + 11x - 20} - \frac{3}{3x - 4}$. [3]

(c) Given that $2xy + 3x = \sqrt{3yz - x^2}$, express x in terms of y and z. [3]

(d) **(i)** Simplify $(2x + y)(2x - y) - 4x^2$. [1]

(ii) **Hence** evaluate $23 \times 17 - 400$. [1]

---

2 **(a)** **(i)** Express 3780 as the product of its prime factors. [1]

(ii) Find the smallest integer p such that $3780p$ is a perfect cube. [1]

(iii) Find the smallest integer q such that $3780q$ is a multiple of 350. [1]

(iv) The lowest common multiple of 3780 and integer k is 7560.
The highest common factor of 3780 and integer k is 60.
Find the smallest possible value of integer k. [1]

(b) The first four terms in a sequence of numbers are

3, 6, 11, 18, ...

(i) Write down an expression, in terms of n, for the nth term of the sequence. [1]

(ii) Find and simplify an expression for the difference between two consecutive terms in the sequence. [2]

(iii) Hence explain why the difference between any two consecutive terms in the sequence is always an odd number. [1]

---

3 The fares for taxis from 3 companies are shown in the table below.

Company	Epic	Toyo	Crystal
Flag-Down (inclusive of first km or less)	\$3.60	\$3.80	\$5.00
Every 1 km thereafter or less up to 10 km	\$0.40	\$0.40	\$0.60
Every 1 km thereafter or less after 10 km	\$0.80	\$0.80	\$1.20

The information can be represented by matrix $\mathbf{A} = \begin{pmatrix} 3.6 & 3.8 & 5.0 \\ 0.4 & 0.4 & 0.6 \\ 0.8 & 0.8 & 1.2 \end{pmatrix}$.

Bobby travels a distance of 13 km by taxi. The breakdown of the distance he travelled can be represented by matrix $\mathbf{B} = \begin{pmatrix} 1 & 9 & 3 \end{pmatrix}$.

(a) Evaluate matrix **C** such that $\mathbf{C} = \mathbf{BA}$. [1]

(b) Explain what the elements in **C** represent. [1]

Clara travels a distance of 20 km by taxi every day. During peak hour, the peak hour surcharge for Epic, Toyo and Crystal are 30%, 20% and 10% of the total fare respectively.

(c) **(i)** Write down a matrix **D** similar to **B** to represent the breakdown of the distance she travelled. [1]

(ii) Evaluate **DA**. [1]

(d) **(i)** Write down a matrix **E** such that the product **DAE** will give the amount after surcharge that Clara will have to pay for a taxi from each of the companies. [1]

(ii) Evaluate **DAE**. [1]

(e) Explain which company would be the most economical choice during peak hour and non-peak hour respectively. [1]

4 **(a)** Solve the simultaneous equations

$$3x + 5y = 5,$$
$$\frac{1}{2}x = \frac{1}{3}y + 2.$$ [3]

(b) Andy, Bernard and Charlie were running on a 400 m circular track. Andy started running at point O in an anti-clockwise direction at a speed of x m/s. At the same time, Bernard and Charlie also started running at point O, but moved in a clockwise direction at speeds of $(x+3)$ m/s and $(x-4)$ m/s respectively.

(i) Show that the time passed before Andy and Bernard meet each other on the track is $\frac{400}{2x+3}$ seconds. [1]

(ii) Find, in terms of x, the time passed before Andy and Charlie meet each other on the track. [1]

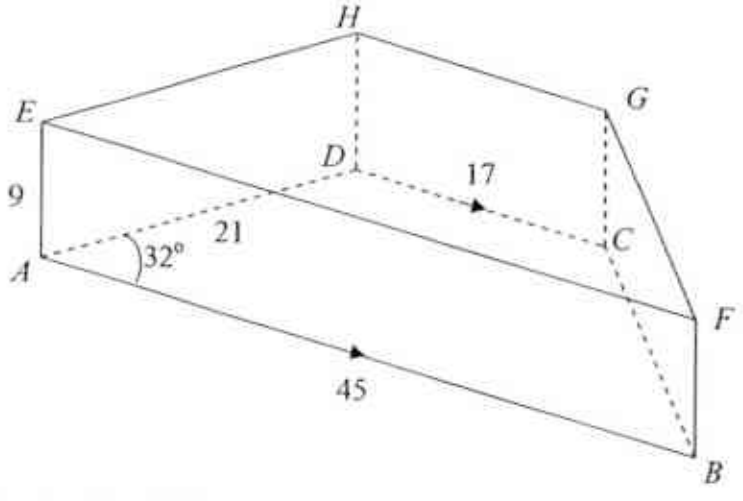
(iii) Given that Andy meets Charlie $23\frac{1}{3}$ seconds after passing Bernard, form an equation in terms of x and show that it simplifies to $2x^2 - x - 66 = 0$. [3]

(iv) Solve the equation $2x^2 - x - 66 = 0$. [3]

(v) Find the time taken for Andy to run one round around the track. [1]

---

5 In the diagram, $ABCDEFGH$ shows a lecture theatre in the shape of a trapezoidal prism. The floor of the theatre, $ABCD$, is on flat ground and $ABFE$, $BCGF$, $DCGH$ and $ADHE$ are vertical walls. AB = 45 m, CD = 17 m, AD = 21 m, AE = 9 m, $\angle BAD = 32°$ and AB is parallel to DC.



(a) Find the length of BD. [3]

(b) Find the angle of depression of D from F. [2]

(c) Show that $\angle ADB = 125.74°$, correct to two decimal places. [2]

(d) Calculate the area of ΔBCD. [2]

(e) J is a point on AB such that DJ and AB are perpendicular. Find $\angle DFJ$. [3]

---

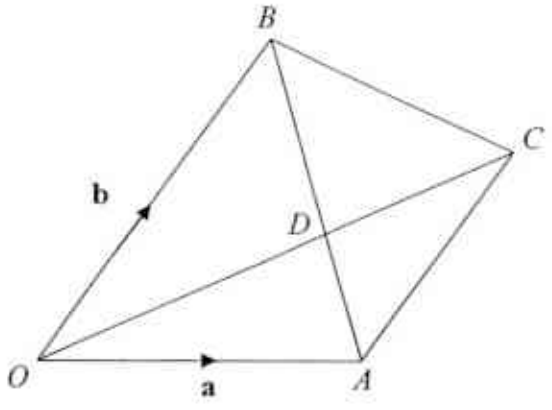
6 **(a)** A is the point $(4, 6)$, B is the point $(6, k)$ and C is the point $(-3, 5)$.

(i) Find $\left|\overrightarrow{AC}\right|$. [2]

(ii) Express $\overrightarrow{AC}$ as a column vector. [1]

(iii) Find the value of k if OA and OB are parallel. [2]

(b) In the diagram, OC and AB intersect at D, $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{DC} = \frac{2}{5}\overrightarrow{OC}$ and $3AD = 2DB$.



(i) Express, as simply as possible, in terms of **a** and **b**,

(a) $\overrightarrow{AB}$, [1]

(b) $\overrightarrow{OD}$, [1]

(c) $\overrightarrow{AC}$. [2]

(ii) Write down two facts about OB and AC. [1]

(iii) Find $\dfrac{\text{area of } \Delta OAD}{\text{area of } \Delta CBD}$. [2]

Section B (39 marks)

Begin this section on a fresh sheet of paper.

7 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation

$$y = 5x - 3 + \frac{1}{2x}.$$

The table below shows some values of x and the corresponding values of y correct to 2 decimal places.

x	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7
y	7.25	2.50	0.50	0.17	0.25	a	0.83	1.21

(a) Calculate the value of a. [1]

(b) Using a scale of 2 cm to represent 0.1 unit, draw a horizontal x-axis for $0 < x \leq 0.7$. Using a scale of 2 cm to represent 1 unit, draw a vertical y-axis for $0 \leq y \leq 8$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

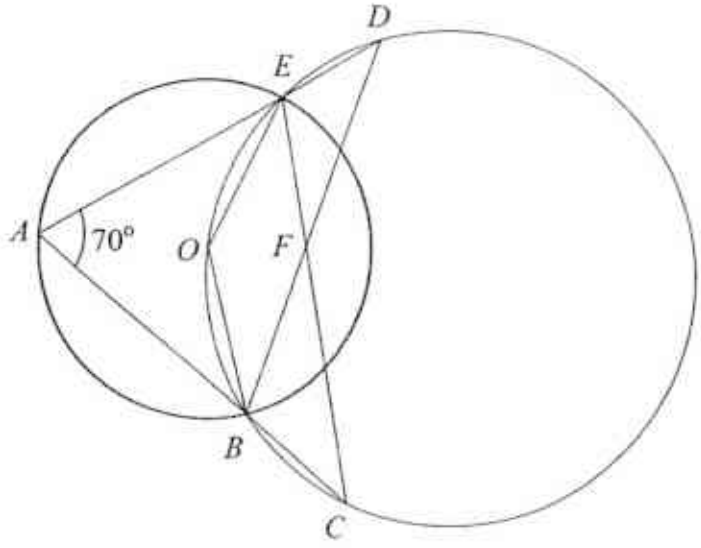
(c) By drawing a tangent, find the gradient of the curve at $(0.2, 0.5)$. [2]

(d) By drawing a suitable line, find the solutions of $10x^2 - 8x + 1 = 0$ in the range $0 < x \leq 0.7$. [2]

(e) The line $y = kx$ touches the curve $y = 5x - 3 + \frac{1}{2x}$ at exactly one point in the range $0 < x \leq 0.7$.

By drawing a suitable straight line on the same axes, use your graph to find the value of k. [2]

8 The diagram shows two circles with the bigger circle passing through the centre, O, of the smaller circle. The circles intersect at B and E. A lies on the smaller circle while C and D lie on the bigger circle. AED and ABC are straight lines. CE and BD intersect at F and $\angle BAE = 70°$.



(a) Find, stating the reasons clearly,

(i) $\angle BOE$, [1]

(ii) $\angle BDE$, [1]

(iii) $\angle ABD$, [1]

(iv) $\angle CED$, [2]

(v) $\angle DFC$. [1]

(b) A point G lies on the same side of BE as A. Given that $\angle BGE = 75°$, explain whether point G lies inside, outside or on the circumference of the smaller circle. [1]

9 **(a)** The table below shows the marks obtained in a Mathematics test by 200 students.

Mark (x)	$0 < x \leq 20$	$20 < x \leq 40$	$40 < x \leq 60$	$60 < x \leq 80$	$80 < x \leq 100$
Frequency	4	43	55	64	34

(i) State the median class. [1]

(ii) Calculate

(a) the mean mark, [1]

(b) the standard deviation. [2]

(b) Asha and Brennan are sitting for a test. The probability that Asha passes the test is 0.9 and the probability that Brennan passes the test is 0.8.

(i) Find the probability that both Asha and Brennan pass the test. [1]

A similar test is set every week and Asha and Brennan can take the test as many times as they like. They decide to stop taking the test only when both of them succeed in passing the test in the same week.

(ii) Find the probability that they

(a) do not succeed in the first week but succeed in the second, [1]

(b) succeed in either the first or second week, [1]

(c) do not succeed in the first three weeks, [1]

(d) succeed in one of the first three weeks. [1]

(iii) Find the probability, in terms of n, that they

(a) do not succeed in the first n weeks, [1]

(b) succeed in one of the first n weeks. [1]

10 The corn in Diagram I is modelled as a frustum with a hemisphere on top as shown in Diagram II. The diameter of the top of the frustum is 4 cm and the diameter of the bottom of the frustum is 6 cm. The height of the frustum is 20 cm.

Diagram I

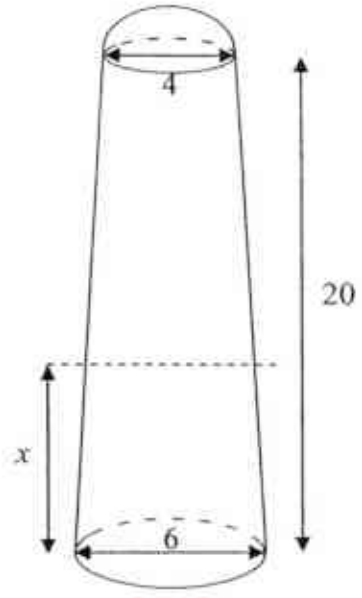


Diagram II

(a) Calculate the volume of the corn. [4]

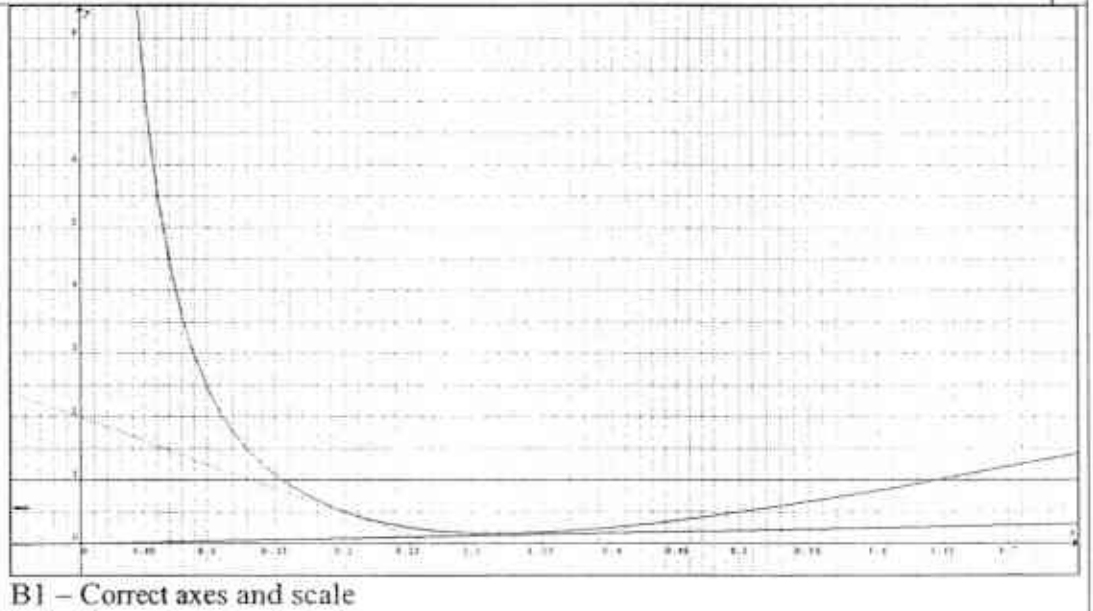
(b) The curved surface area of the corn is covered by grains but the circular base is not. Find the total curved surface area of the corn. [3]

(c) The number of grains on the corn is directly proportional to the curved surface area of the corn. The corn is to be cut horizontally into two lengths such that the number of grains on both lengths is the same. Calculate the height from the base, x cm, at which the corn should be cut. [4]

End of paper

Mathematics Paper 2 (100 marks)

Qn. #	Solution	
1a	$(ab-2)(a^2+2b^2)$	
1b	$\dfrac{-3x-13}{(3x-4)(x+5)}$	
1c	$x=\pm\sqrt{\dfrac{3yz}{4y^2+12y+10}}$	
1di	$-y^2$	
1dii	-9	
2ai	$3780=2^2\times3^3\times5\times7$	
2aii	$p=2450$	
2aiii	$q=5$	
2aiv	$k=120$	
2bi	n^2+2	
2bii	$2n+1$	
2biii	Since n is an integer, 2n is always even and 2n+1 will always be an odd number	
3a	$\mathbf{C}=\begin{pmatrix}9.6 & 9.8 & 14\end{pmatrix}$	
3b	Elements in C represent the taxi fare Bobby would have paid if he took a taxi from Epic, Toyo or Crystal respectively.	
3ci	$\mathbf{D}=\begin{pmatrix}1 & 9 & 10\end{pmatrix}$	
3cii	$\mathbf{DA}=\begin{pmatrix}15.2 & 15.4 & 22.4\end{pmatrix}$	
3di	$\mathbf{E}=\begin{pmatrix}1.3 & 0 & 0\\ 0 & 1.2 & 0\\ 0 & 0 & 1.1\end{pmatrix}$	
3dii	$\mathbf{DAE}=\begin{pmatrix}19.76 & 18.48 & 24.64\end{pmatrix}$	
3e	Epic will be the most economical during non-peak hour and Toyo most economical during peak hours as their prices are cheapest during the respective hours	
4a	$y=-1,\ x=3\dfrac{1}{3}$	
4bi	$t=\dfrac{400}{2x+3}$	
4bii	$t=\dfrac{400}{2x-4}$	
4biv	$x=-5.5$ or $x=6$	
4bv	Time taken = 66.7 s	
5a	$BD=29.4$ m	
5b	Angle of depression = 17.0°	
5c	$\angle ADB=125.74$ (2 d.p.)	
5d	Area $BCD=94.6$ m^2	

Qn. #	Solution	
5e	$\angle DFJ = 21.2°$	
6ai	$\left\|\overrightarrow{AC}\right\| = 7.07$ units	
6aii	$\overrightarrow{AC} = \begin{pmatrix} -7 \\ -1 \end{pmatrix}$	
6aiii	$k = 9$	
6bia	$\overrightarrow{AB} = \mathrm{b} - \mathrm{a}$	
6bib	$\overrightarrow{OD} = \frac{1}{5}(3\mathrm{a} + 2\mathrm{b})$	
6bic	$\overrightarrow{AC} = \frac{2}{3}\mathrm{b}$	
6bii	$OB // AC$ $OB = \frac{3}{2}AC$	
6biii	$\frac{\text{area of } \Delta OAD}{\text{area of } \Delta CBD} = 1$	
7a	$a = 0.5$	
7b	 B1 – Correct axes and scale B1 – Correct points plotted B1 – Smooth curve	
7c	Gradient = –7.5 ±1.5	
7d	$x = 0.155 \pm 0.01$ or $x = 0.64 \pm 0.01$	
7e	$k = 0.409 \pm 0.15$	
8ai	$\angle BOE = 140°$	
8aii	$\angle BDE = 40°$	
8aiii	$\angle ABD = 70°$	
8aiv	$\angle CED = 110°$	

Qn. #	Solution	
8av	$\angle DFC = 150°$	
8b	Any point X on the same side of BE as A, lying on the circumference of the circle will form an angle $\angle BXE = 70°$ (angles in same segment). Since $\angle BGE = 75°$ which is larger than $70°$, G lies in the circle.	
9ai	$40 < x \leq 60$	
9aiia	Mean mark = 58.1 marks	
9aiib	Standard deviation = 21.3 marks	
9bi	0.72	
9biia	0.2016	
9biib	0.9216	
9biic	0.021952	
9biid	0.978048	
9biiia	0.28"	
9biiib	$1 - 0.28^n$	
10a	Total volume = 415 cm^3	
10b	Total curved surface area= 340 cm^2	
10c	$x = 9.80$	

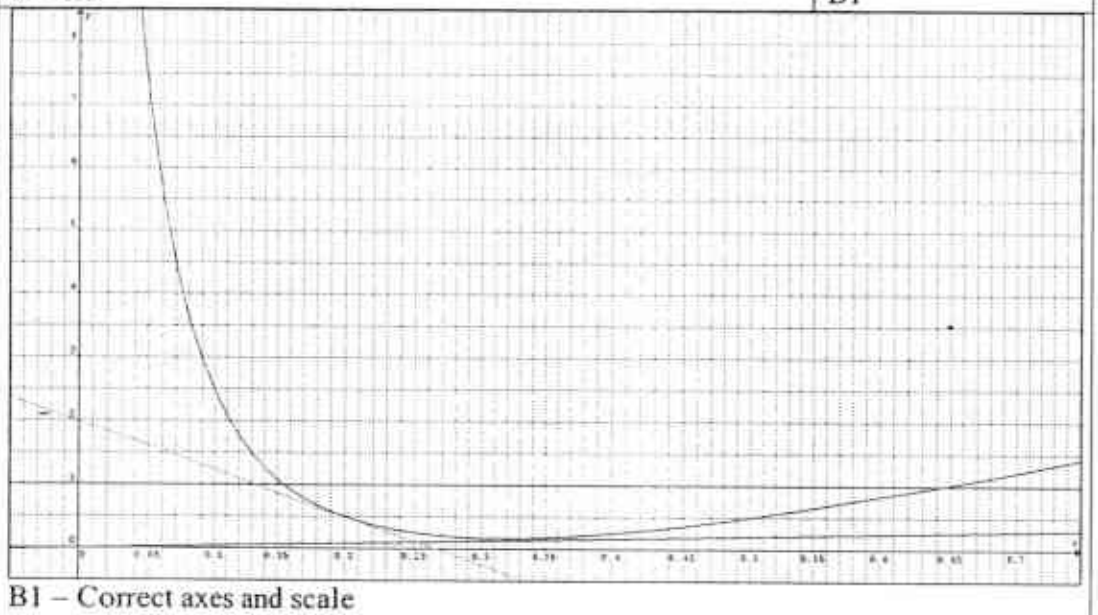
Mathematics Paper 2 (100 marks)

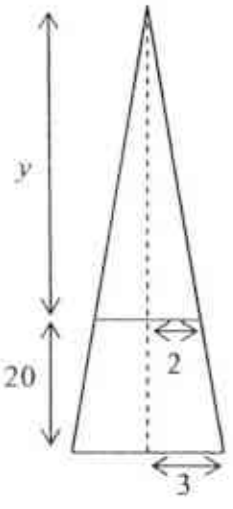
Qn. #	Solution	Mark Allocation
1a	$a^3b+2ab^3-2a^2-4b^2$ $=ab(a^2+2b^2)-2(a^2+2b^2)$ $=(ab-2)(a^2+2b^2)$	 M1 A1
1b	$\frac{2}{3x^2+11x-20}-\frac{3}{3x-4}$ $=\frac{2}{(3x-4)(x+5)}-\frac{3}{3x-4}$ $=\frac{2-3(x+5)}{(3x-4)(x+5)}$ $=\frac{-3x-13}{(3x-4)(x+5)}$	 M1 M1 A1
1c	$2xy+3x=\sqrt{3yz-x^2}$ $4x^2y^2+12x^2y+9x^2=3yz-x^2$ $4x^2y^2+12x^2y+10x^2=3yz$ $x^2(4y^2+12y+10)=3yz$ $x^2=\frac{3yz}{4y^2+12y+10}$ $x=\pm\sqrt{\frac{3yz}{4y^2+12y+10}}$	 M1 M1 A1
1di	$(2x+y)(2x-y)-4x^2$ $=4x^2-y^2-4x^2$ $=-y^2$	 B1
1dii	$23\times17-400$ $=[2(10)+3][2(10)-3]-4(10)^2$ $=-(3)^2$ $=-9$	 A1
2ai	$3780=2^2\times3^3\times5\times7$	B1
2aii	$p=2\times5^2\times7^2=2450$	B1
2aiii	$q=5$	B1
2aiv	$k=2^3\times3\times5=120$	B1
2bi	n^2+2	B1
2bii	$(n+1)^2+2-(n^2+2)$ $=2n+1$	M1 A1
2biii	Since n is an integer, 2n is always even and 2n+1 will always	

Qn. #	Solution	Mark Allocation
	be an odd number	B1
3a	$\mathbf{C} = \begin{pmatrix}1 & 9 & 3\end{pmatrix}\begin{pmatrix}3.6 & 3.8 & 5.0\\ 0.4 & 0.4 & 0.6\\ 0.8 & 0.8 & 1.2\end{pmatrix} = \begin{pmatrix}9.6 & 9.8 & 14\end{pmatrix}$	B1
3b	Elements in C represent the taxi fare Bobby would have paid if he took a taxi from Epic, Toyo or Crystal respectively.	B1
3ci	$\mathbf{D} = \begin{pmatrix}1 & 9 & 10\end{pmatrix}$	B1
3cii	$\mathbf{DA} = \begin{pmatrix}1 & 9 & 10\end{pmatrix}\begin{pmatrix}3.6 & 3.8 & 5.0\\ 0.4 & 0.4 & 0.6\\ 0.8 & 0.8 & 1.2\end{pmatrix} = \begin{pmatrix}15.2 & 15.4 & 22.4\end{pmatrix}$	B1
3di	$\mathbf{E} = \begin{pmatrix}1.3 & 0 & 0\\ 0 & 1.2 & 0\\ 0 & 0 & 1.1\end{pmatrix}$	B1
3dii	$\mathbf{DAE} = \begin{pmatrix}15.2 & 15.4 & 22.4\end{pmatrix}\begin{pmatrix}1.3 & 0 & 0\\ 0 & 1.2 & 0\\ 0 & 0 & 1.1\end{pmatrix}$ $= \begin{pmatrix}19.76 & 18.48 & 24.64\end{pmatrix}$	
3e	Epic will be the most economical during non-peak hour and Toyo most economical during peak hours as their prices are cheapest during the respective hours	B1
4a	$3x + 5y = 5$ --- (1) $\frac{1}{2}x = \frac{1}{3}y + 2$ --- (2) (2) x 6: $3x = 2y + 12$ --- (3) Sub (3) to (1): $2y + 12 + 5y = 5$ $7y = -7$ $y = -1$ $3x = 2(-1) + 12$ $x = 3\frac{1}{3}$	M1 A1 A1
4bi	$xt + (x+3)t = 400$ $t = \frac{400}{2x+3}$	A1
4bii	$xt + (x-4)t = 400$ $t = \frac{400}{2x-4}$	

Qn. #	Solution	Mark Allocation
4biii	$\frac{400}{2x-4}-\frac{400}{2x+3}=23\frac{1}{3}$	M1
	$1200+1600=\frac{70}{3}\left(4x^2-2x-12\right)$	M1
	$280x^2-140x-9240=0$	
	$2x^2-x-66=0$	A1
4biv	$2x^2-x-66=0$	
	$(2x+11)(x-6)=0$	M1
	$x=-5.5$ or $x=6$	A2
4bv	Time taken $=\frac{400}{6}=66.7$ s	B1
5a	$BD^2=45^2+21^2-2(45)(21)\cos 32$	M2
	$BD=29.38008$	
	$BD=29.4$ m	A1
5b	tan (angle of depression) $=\frac{9}{29.38008}$	M1
	Angle of depression $=17.03140=17.0^\circ$	A1
5c	$\frac{\sin\angle ADB}{45}=\frac{\sin 32}{29.38008}$	
	$\sin\angle ADB=\frac{45\sin 32}{29.38008}$	M1
	$\angle ADB=54.2575$ (N.A.) or $180-54.2575$	
	$\angle ADB=125.74$ (2 d.p.)	A1
5d	Area BCD	
	$=\frac{1}{2}\times 17\times 29.38008\times\sin(180-32-125.74)$	M1
	$=94.6052$	
	$=94.6$ m^2	A1
5e	$DJ=21\sin 32=11.12830$	
	$JF=\sqrt{(45-21\cos 32)^2+9^2}$	M1
	$=28.64175$	
	$\tan\angle DFJ=\frac{11.12839}{28.64175}$	M1
	$\angle DFJ=21.2^\circ$	A1
6ai	$\left\lvert\overrightarrow{AC}\right\rvert=\sqrt{(4+3)^2+(6-5)^2}$	M1
	$=7.07$ units	A1
6aii	$\overrightarrow{AC}=\begin{pmatrix}-7\\-1\end{pmatrix}$	B1

Qn. #	Solution	Mark Allocation
6aiii	$\begin{pmatrix}4\\6\end{pmatrix} = m\begin{pmatrix}6\\k\end{pmatrix}$ $4 = 6m$ $m = \frac{2}{3}$ $6 = \frac{2}{3}k$ $k = 9$	M1 A1
6bia	$\overrightarrow{AB} = \mathrm{b} - \mathrm{a}$	B1
6bib	$\overrightarrow{OD} = \mathrm{a} + \frac{2}{5}(\mathrm{b} - \mathrm{a})$ $= \frac{1}{5}(3\mathrm{a} + 2\mathrm{b})$	 B1
6bic	$\overrightarrow{AC} = -\mathrm{a} + \frac{5}{3} \times \frac{1}{5}(3\mathrm{a} + 2\mathrm{b})$ $= \frac{2}{3}\mathrm{b}$	M1 A1
6bii	$OB \parallel AC$ $OB = \frac{3}{2}AC$	 B1
6biii	$\Delta OAD : \Delta ADC : \Delta CBD$ $3 : 2$ $: 2 : 3$ $3 : 2 : 3$ $\frac{\text{area of } \Delta OAD}{\text{area of } \Delta CBD} = 1$	M1 A1

Qn. #	Solution	Mark Allocation
7a	$a = 0.5$	B1
7b	 B1 – Correct axes and scale B1 – Correct points plotted B1 – Smooth curve	
7c	Draw tangent Gradient $= -7.5 \pm 1.5$	M1 A1
7d	$10x^2 - 8x + 1 = 0$ $5x - 4 + \frac{1}{2x} = 0$ $5x - 3 + \frac{1}{2x} = 1$ Draw $y = 1$ $x = 0.155 \pm 0.01$ or $x = 0.64 \pm 0.01$	 M1 A1
7e	At $x = 0.33$, tangent passes through origin Gradient $= 0.409 \pm 0.2$ $k = 0.409 \pm 0.15$	 M1 – Draw tangent A1
8ai	$\angle BOE = 2 \times 70 = 140°$ (angle at centre is twice angle at circumference)	B1
8aii	$\angle BDE = 180 - 140 = 40°$ (opp. angles of cyclic quad)	B1
8aiii	$\angle ABD = 180 - 70 - 40 = 70°$ (angle sum of triangle)	B1
8aiv	$\angle CBD = 180 - 70 = 110°$ (angles on a straight line) $\angle CED = 110°$ (angles in same segment)	M1 A1
8av	$\angle DFC = 40 + 110 = 150°$ (ext. angle of a triangle)	B1
8b	Any point X on the same side of BE as A, lying on the circumference of the circle will form an angle $\angle BXE = 70°$ (angles in same segment). Since $\angle BGE = 75°$ which is larger than $70°$, G lies in the circle.	B1

Qn. #	Solution	Mark Allocation
9ai	$40 < x \leq 60$	B1
9aiia	Mean mark $= \dfrac{11620}{200} = 58.1$ marks	B1
9aiib	Standard deviation $= \sqrt{\dfrac{765600}{200} - \left(\dfrac{11620}{200}\right)^2}$ $= 21.3$ marks	M1 A1
9bi	$0.9 \times 0.8 = 0.72$	B1
9biia	$0.28 \times 0.72 = 0.2016$	B1
9biib	$0.72 + 0.28 \times 0.72 = 0.9216$	B1
9biic	$0.28^3 = 0.021952$	B1
9biid	$1 - 0.021952 = 0.978048$	B1
9biiia	0.28^n	B1
9biiib	$1 - 0.28^n$	B1
10a	Let the height of imaginary cone above frustum be y $\dfrac{y}{2} = \dfrac{y+20}{3}$ $y = 40$ Volume of frustum $= \dfrac{1}{3} \times \pi \times (3)^2 (60) - \dfrac{1}{3} \times \pi \times (2)^2 (40)$ $= 397.93507$ Volume of hemisphere $= \dfrac{2}{3} \times \pi \times (2)^3$ $= 16.75516$ Total volume = 397.93507 + 16.75516 $= 414.69023$ $= 415 \text{ cm}^3$  	M1 M1 M1 A1
10b	Curved surface area of frustum $\pi \times (3)(\sqrt{3^2 + 60^2}) - \pi \times (2)(\sqrt{2^2 + 40^2}))$ $= 314.55172$ Total curved surface area $= 2\pi(2)^2 + 314.55172$ $= 339.68446$ $= 340 \text{ cm}^2$	M1 – Pythagoras M1 – large curved surface – small curved surface A1
10c	Curved area of bottom part $= \pi \times (3)(\sqrt{3^2 + 60^2}) - \left(\dfrac{60-x}{60}\right)^2 \left[\pi \times (3)(\sqrt{3^2 + 60^2})\right]$ $= 18.87310x - 0.15728x^2$ Curved area of bottom part = $339.68446 \div 2$	M1 – Area of top or bottom in terms of a variable M1 – Equate to half

Qn. #	Solution	Mark Allocation
	$18.87310x - 0.15728x^2 = 169.84223$ $0.15728x^2 - 18.87310x + 169.84223 = 0$ Using quadratic formula, $x = 110.19739$ *or* 9.79943 cm $x = 9.80$	of earlier area found M1 – quadratic formula A1
10c	Curved area of bottom part $= \pi \times (3)(\sqrt{3^2 + 60^2}) - \left(\frac{60 - x}{60}\right)^2 \left[\pi \times (3)(\sqrt{3^2 + 60^2})\right]$ $= 18.87310x - 0.15728x^2$ Curved area of top part $= 2\pi \times (2)^2 + (314.55172 - (18.87310x - 0.15728x^2))$ $= 0.15728x^2 - 18.87310x + 339.68446$ Curved area of top = Curved area of bottom $0.15728x^2 - 18.87310x + 339.68446 = 18.87310x - 0.15728x^2$ $0.31456x^2 - 37.7462x + 339.68446 = 0$ Using quadratic formula, $x = 110.19739$ *or* 9.79943 cm $x = 9.80$	M1 M1 M1 A1

Answer all questions

1 (a) Calculate $\frac{\pi - 56\%}{2 - 2.31^2}$, giving your answer correct to 4 significant figures.

(b) Write your answer to part (a) correct to 3 significant figures.

Answer (a) _____ [1]

(b) _____ [1]

2 The first five terms of a sequence are 19, 15, 11, 7, 3.

(a) Write down the 8th term of this sequence.

(b) Write down an expression, in terms of n, for the nth term in the sequence.

Answer (a) _____ [1]

(b) _____ [1]

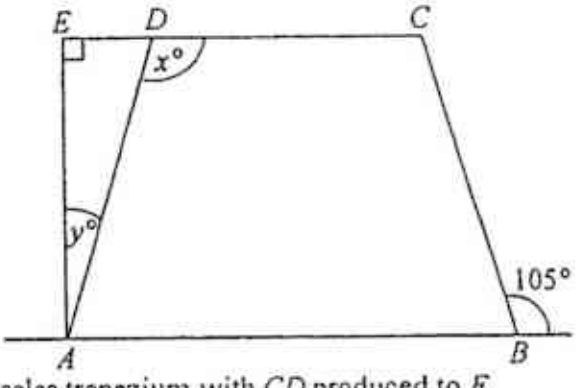
3 (a) Given that $9^3 \div 9^{-2n} = 9^7$, find the value of n.

(b) Calculate $3.4 \times 10^7 - 5 \times 10^6$, giving your answer in standard form.

Answer (a) $n =$ _____ [1]

(b) _____ [1]

4



ABCD is an isosceles trapezium with *CD* produced to *E*.
Find

(a) x,

(b) y.

Answer (a) $x =$ _____ [1]

(b) $y =$ _____ [1]

5 Simplify $(3x-5)^2-(-3x-5)^2$.

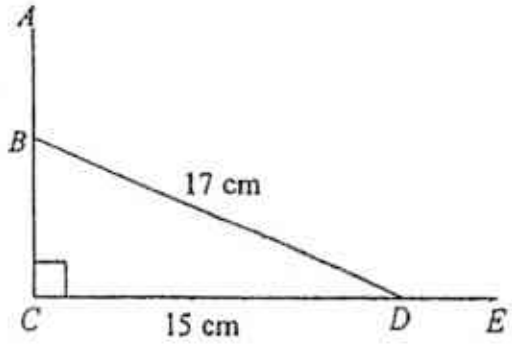
Answer _____ [2]

6 BCD is a right-angled triangle. $CD = 15$ cm and $BD = 17$ cm. ABC and CDE are straight lines. Express, as a fraction, the value of

(a) sin $\angle CBD$,

(b) cos $\angle BDE$,

(c) tan $(90° - \angle CBD)$.



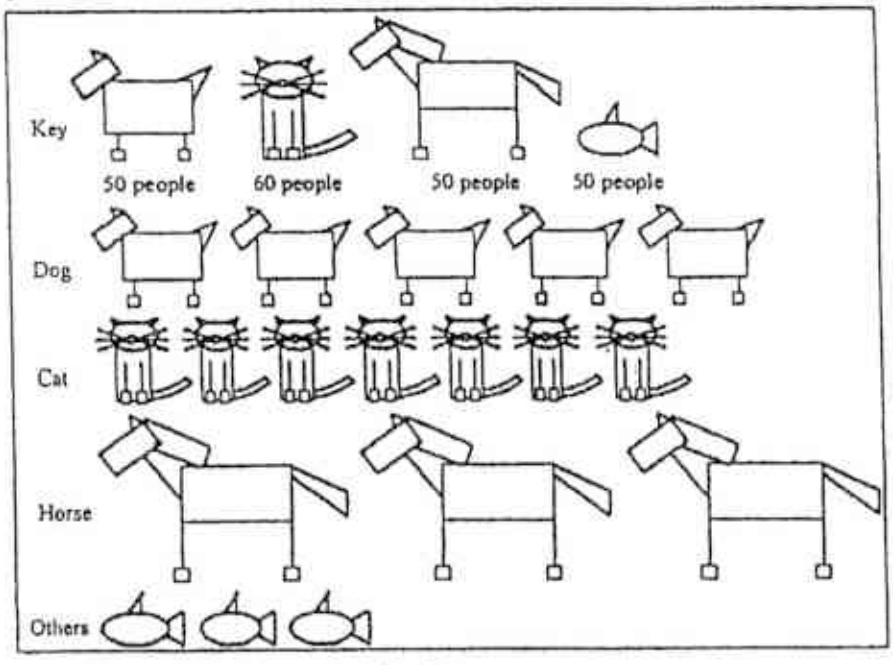
Answer (a) _____ [1]

(b) _____ [1]

(c) _____ [1]

For Examiner's use

7 The pictogram shows the number of people who own different types of pets in a town.



List two things misleading with this pictogram. [2]

Answer 1. _____

2. _____

8 A bag contains 1 red, 3 blue and 6 white balls. Two balls are taken from the bag, at random, without replacement.
Find the probability that

(a) the first ball is red,

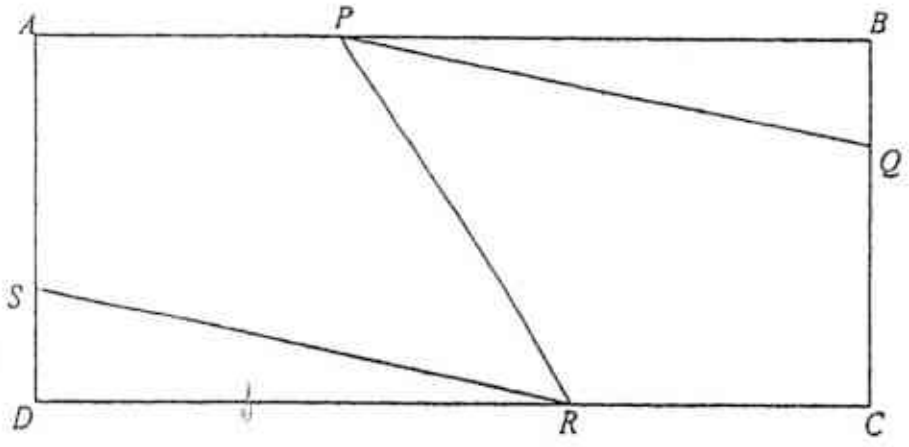
(b) none of the balls are blue.

Answer (a) _____ [1]

(b) _____ [2]

For Examiner's use

9



$ABCD$ is a rectangle.
Points P, Q, R and S lie on AB, BC, CD and DA respectively such that triangle PBQ is congruent to triangle RDS.

(a) Show that $\angle RPQ = \angle PRS$. [3]

Answer _____

_____

_____

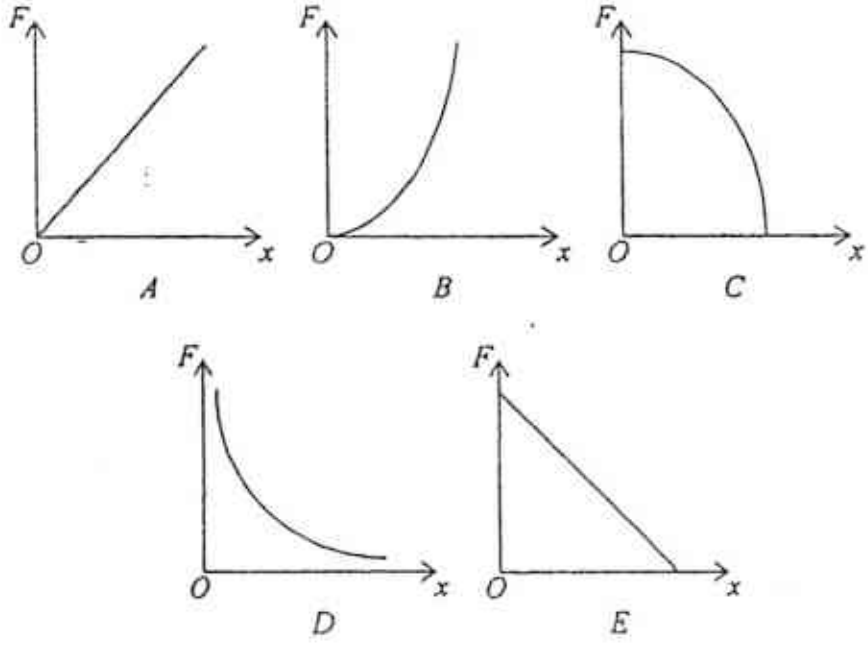
_____

(b) Given further triangle ASP is congruent to triangle CQR.
State the name of the quadrilateral $PQRS$.

Answer (b) _____ [1]

10 The force (F units) between two particles is inversely proportional to the square of the distance (x units) between them.

(a) Select one of the graphs below that could represent the relation between the force and the distance.



Answer (a) _____ [1]

(b) When the distance between the two particles is x, the force is F.
Find the percentage change in the value of the force when the distance is reduced to 50%.

Answer (b) _____ % [2]

For Examiner's use

Exc

11

A company sells cereals in boxes which measure 18 cm by 12 cm by 30 cm.

(a) Calculate the volume of the box.

(b) They make a special edition box which is geometrically similar to the original box. The volume of the special edition box is 1920 cm^3.
Find the dimensions of this box.

Answer (a) _____ cm^3 [1]

(b) _____ cm by _____ cm by _____ cm [3]

12 (a) Factorise $(8x+13)^2 - 12(8x+13) + 36$ completely.

(b) Hence find the value of
$(48+13)^2 - 12(48+13) + 36$.

Answer (a) _____ [2]

(b) _____ [2]

294

For Examiner's use

13 20 students from Class 3A and 20 students from 3B took a Mathematics Test. The marks are shown in the stem-and-leaf diagram.

Class 3A						Class 3B					
		9	7	6	0	5	7				
9	8	6	5	4	1	1	8	8	9		
5	5	4	2	2	2	5	6	7	7	7	7
			8	3	3	2	5	8	9		
	7	4	4	4	4	5	5	5			
				0	5	0					4

Key (Class 3A)
0 | 5 means 50

Key (Class 3B)
5 | 0 means 50

(a) (i) Find the median marks for Class 3A.

(ii) Find the range for the top 25% students marks in Class 3A.

Answer *(a) (i)* _____ marks [1]

(ii) _____ marks [1]

(b) Here are two statements comparing the performance for the two classes. For each one, write whether you agree or disagree. Give a reason for each answer, stating clearly which statistic you use to make your decision.

(i) On the average, Class 3A performed better than Class 3B. [1]

Answer _____ because _____

_____

_____

_____

(ii) The top 25% of Class 3A results are closer together. [1]

Answer _____ because _____

_____

_____

_____

For Examiner's use

14 (a) The value of Ms Tan's car is $120 000. By the end of each year, the value of the car decreases by 10 % of its value at the start of the year.
Find the value of the car at the end of 2 years.

Answer (a) $ _____ [2]

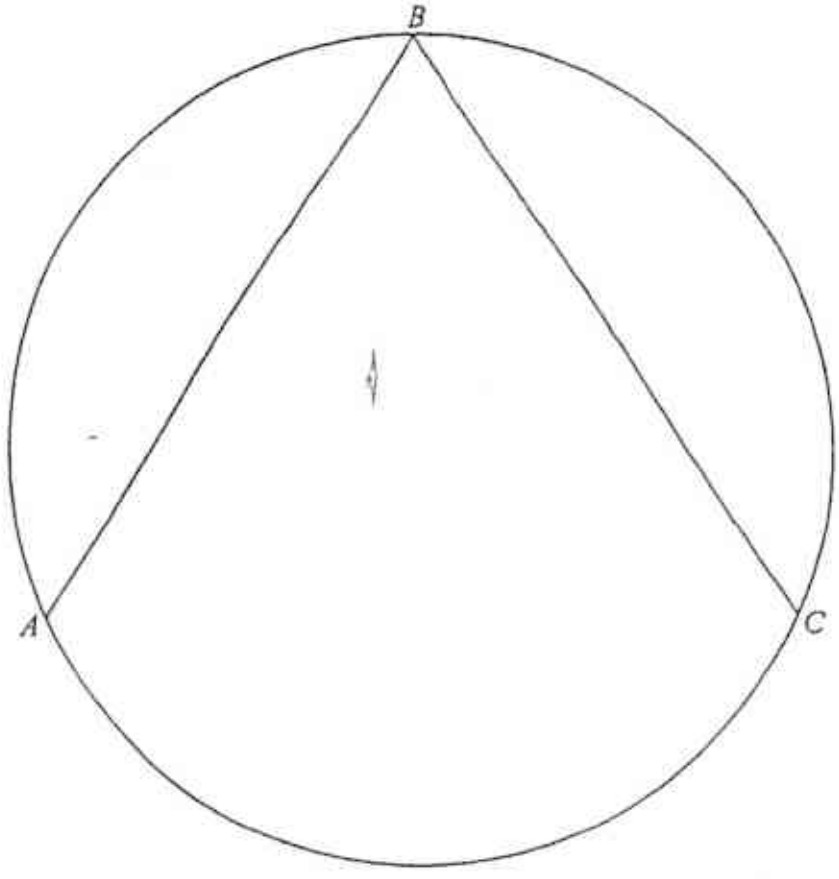
(b) A car is priced at $200 000. It can be bought on hire purchase with a down payment of $110 000, interest rate of 2% per annum over 7 years and equal monthly repayment. Find the amount of each monthly installment.

Answer (b) $ _____ [2]

>95

For Examiner's use

15 The points A, B, and C lie on the circle and $AB = BC$.



(a) Construct

(i) the bisector of angle ABC, [1]

(ii) the perpendicular bisector of AB. [1]

(b) (i) Mark clearly a point, P, on the circle along minor arc AC such that ΔBPC is a right-angled triangle. [1]

(ii) State the name of the line BP.

Answer (ii) _____ [1]

(iii) Hence explain why ΔBPC is a right–angle triangle. [1]

Answer (iii) _____

_____

_____

16 (a) A race is x km long. A runner ran $12\frac{1}{2}$ km/h for 50% of the race, 10 km/h for $\frac{1}{5}$ of it and 5 km/h for the remaining distance.

(i) Find, in terms of x, the total time taken for the race.

(ii) Calculate the average speed, in kilometres/hour of the runner for the whole journey.

Answer (i) _____ h [2]

(ii) _____ km/h [2]

(b) The Dead Sea is 380 m below the sea level.
A helicopter hovers above the Dead Sea at 70 m above the sea level before lowering 150 m.
How far is the helicopter above the Dead Sea?

Answer (b) _____ m [2]

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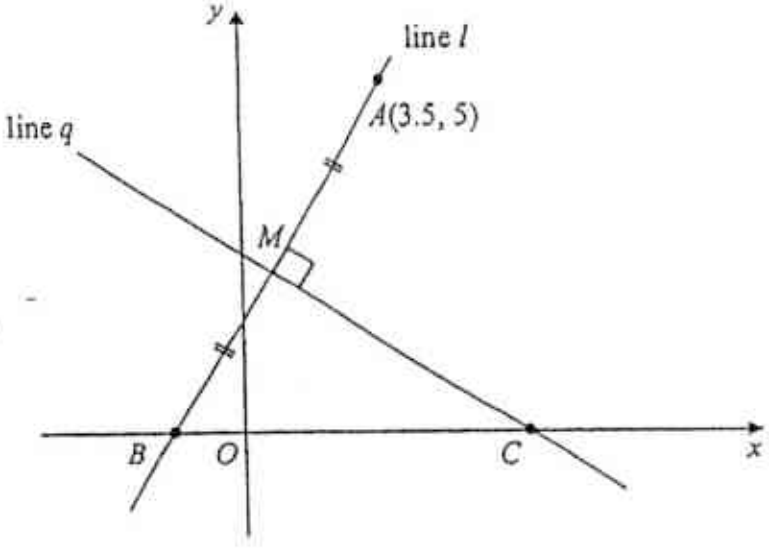
For Examiner's use

17 The diagram shows line l with the equation $2y = 3 + 2x$ passing through point $A(3.5, 5)$ and the x-axis at B.
Another line q is perpendicular to line l. It passes through M, the mid-point of AB and cuts the x-axis at C.



Find

(a) the coordinates of B,

(b) the mid-point of AB,

(c) the equation of the line q,

(d) the ratio of area of ΔBMC to area of ΔABC.

Answer (a) _____ [1]

(b) _____ [1]

(c) _____ [2]

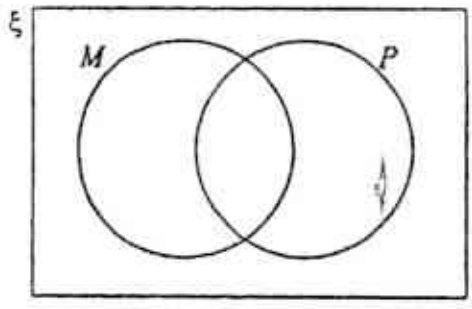
(d) _____ : _____ [2]

For Examiner's use

18 ξ = {Secondary 4 students who study Physics, Mathematics or both}
P = {Students who study Physics}
M = {Students who study Mathematics}

(a) On the Venn Diagram below, shade the region representing $M \cap P'$.

Answer [1]



(b) Describe the elements of $M \cap P'$. [1]

Answer _____

_____

_____

(c) There are 138 students who study Physics out of 220 students in Secondary 4 and $n(P) = n(M)$.

(i) Find the number of students who study both Physics and Mathematics.

(ii) If one student is chosen at random from the Secondary 4 students, find the probability that the student does not study Mathematics.

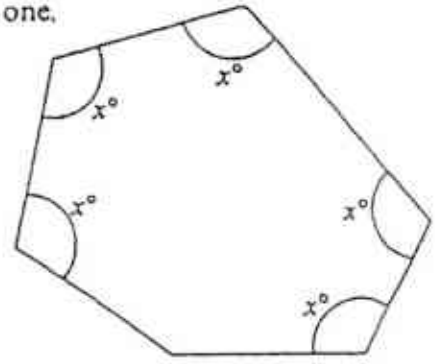
Answer (i) _____ [2]

(ii) _____ [1]

297

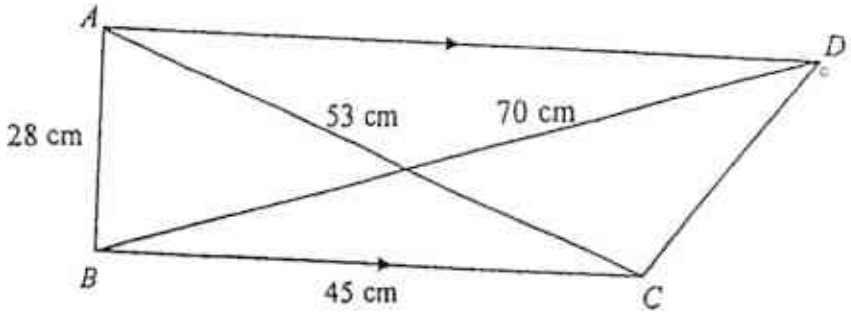
For Examiner's use

19 In a hexagon, all the interior angles are $x°$ except one.
One of the exterior angles is 36°.
Find the value of x.



Answer _____ [3]

20 The diagram shows a trapezium $ABCD$ with AD parallel to BC.



Given $AB = 28$ cm, $BC = 45$ cm, $AC = 53$ cm and $BD = 70$ cm,
Find

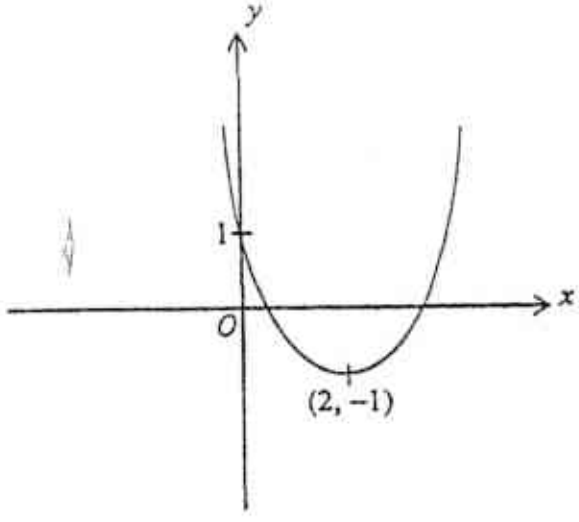
(a) angle ABC,

(b) the shortest distance of C to BD.

Answer (a) _____ ° [1]

(b) _____ cm [3]

21 (a) The diagram shows a quadratic curve which can be expressed in the form $y = a(x + b)^2 + c$.
Find the values of a, b and c.

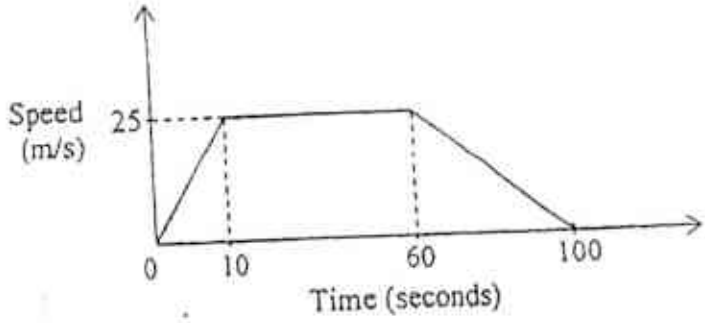


Answer (a) $a =$ ____ $b =$ ____ $c =$ ____ [2]

(b) Hence find the x-intercepts, giving your answers correct to two decimal places.

Answer (b) $x =$ _____ or _____ [3]

22 The diagram shows the speed-time graph for a cyclist's journey that started from Point A to Point B.



(a) Calculate the acceleration during the first 10 seconds.

(b) Calculate the total distance travelled for the journey.

Answer (a) _____ m/s^2 [1]

(b) _____ m [2]

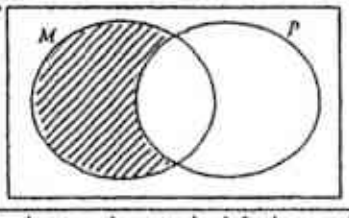
(c) A rider on an electric scooter started at the same time as the cyclist. It travelled at a constant speed of 20 m/s and was overtaken by the cyclist some time later. Find the time taken for the cyclist to overtake the rider.

Answer (c) _____ seconds [2]

END OF PAPER

Answer

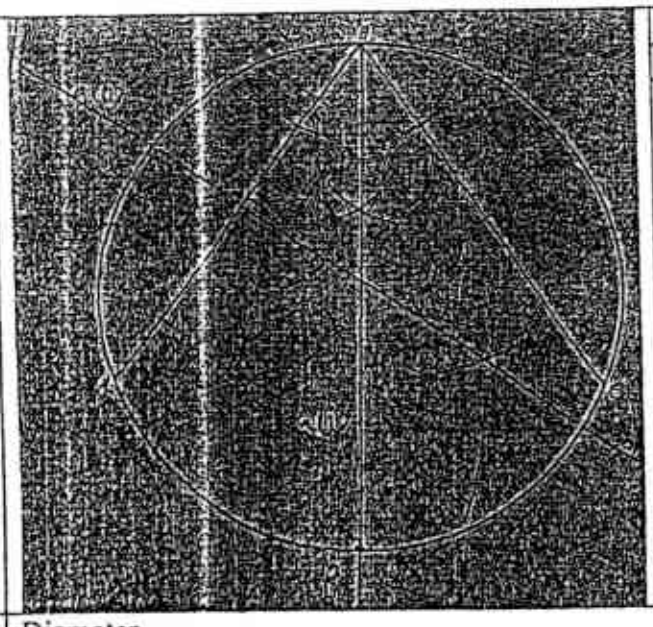
Qn	Solution
1(a)	–0.7738
(b)	–0.774
2(a)	–9
(b)	$23 - 4n$
3(a)	2
(b)	2.9×10^7
4(a)	105
(b)	15
5	$= -60x$
6(a)	$\frac{15}{17}$
(b)	$-\frac{15}{17}$
(c)	$\frac{8}{15}$
8(a)	$\frac{1}{10}$
(b)	$\frac{7}{15}$
9(b)	Parallelogram
10(a)	D
(b)	% changes = 300
T1(a)	6480
(b)	12 by 8 by 20 cm
12(a)	$(8x+7)^2$
(b)	3025
13(ai)	23
(aii)	50 – 44 = 6
14(a)	97 200
(b)	Monthly instalment = 1220
15(bii)	Diameter

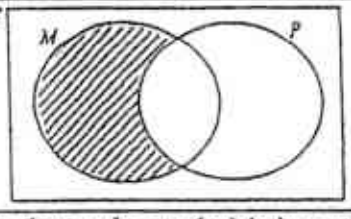
Qn	Solution
16(ai)	$\frac{3x}{25}$
(aii)	$8\frac{1}{3}$
(b)	300
17(a)	$\left(-1\frac{1}{2},0\right)$
(b)	$\left(1,2\frac{1}{2}\right)$
(c)	$y=-x+3\frac{1}{2}$
(d)	1 : 2
18(a)	
(b)	Sec 4 students who study Mathematics only
(ci)	$x=56$
(cii)	$\frac{41}{110}$
19	115.2
20(a)	$\angle ABC=90°$
(b)	18
21(a)	$a=\frac{1}{2}$ $b=-2$ $c=-1$
(b)	$x=0.59, 3.41$
22(a)	2.5
(b)	Dist = 1875
(c)	$x=25$

2015 Prelim 1 Maths Paper 1 Marking Scheme

Qn	Solution	Marks
1(a)	-0.7738	A1
(b)	-0.774	A1
2(a)	-9	A1
(b)	$23-4n$	A1
3(a)	2	A1
(b)	2.9×10^{7}	A1
4(a)	105	A1
(b)	15	A1
5	$(3x-5)^2-(-3x-5)^2$ $=(3x-5+3x+5)(3x-5-3x-5)$ $=-60x$	M1 $(a+b)(a-b)$ or correct expansion A1
6(a)	$\frac{15}{17}$	A1
(b)	$-\frac{15}{17}$	A1
(c)	$\frac{8}{15}$	A1
7	Inconsistent pictorial representation. The picture of the cat should represent 50 people instead of 60. Size of each type of pictorial representation should be consistent. For example, the horses are bigger than the rest of the pictures	B1 o.e scale B1 o.e size
8(a)	$\frac{1}{10}$	A1
(b)	$\frac{7}{10}\times\frac{6}{9}$ $=\frac{7}{15}$	M1 $\frac{6}{9}$ or o.e seen A1
9(a)	$\angle BPR=\angle DRP$ (alt $\angle$s) $\angle DRS=\angle BPQ$ ($\Delta PBQ\equiv\Delta RDS$) $\angle BPR-\angle BPQ=\angle DRP-\angle DRS$ $\angle RPQ=\angle PRS$	B1 B1 B1 Answer Given
(b)	Parallelogram	B1

10(a)	D	A1
(b)	$F = \frac{k}{x^2}$ $k = Fx^2$ $F_1 = \frac{Fx^2}{\frac{1}{4}x^2}$ $= 4F$ % changes = 300	M1 $\frac{1}{4}$ seen A1
11(a)	6480	A1
(b)	$\left(\frac{1920}{6480}\right)^{\frac{1}{3}} = \frac{2}{3}$ $\frac{2}{3} \times 18 = 12$ $\frac{2}{3} \times 12 = 8$ $\frac{2}{3} \times 30 = 20$	M1 $\left(\frac{1920}{6480}\right)^{\frac{1}{3}}$ SOI. Using similarity ratio of volume A2 if All 3 answers correct, A1 if 2 answers correct, A1 correct but wrong order.
12(a)	$(8x+13)^2 - 12(8x+13) + 36$ $= (8x+13)^2 - 2(8x+13)(6) + (6)^2$ $= (8x+13-6)^2$ $= (8x+7)^2$	M1 $a^2 - 2ab + b^2$ seen or implied or expansion A1
(b)	$x = 6$ $(8(6) + 7)^2$ $= 3025$	M1 sub $x = 6$ into answer of 12(a) A1
13(ai)	23	A1
(aii)	$50 - 44 = 6$	A1
(bi)	Disagree because it has a lower median than Class 3B	B1
(bii)	Agree because spread of the top 25% of Class 3A is smaller than Class 3B	B1 spread or range
14(a)	$120\,000 \times 0.9 \times 0.9$ $= 97\,200$	M1 $\times 0.9 \times 0.9$ seen A1
(b)	Monthly instalment $= \frac{90000 \times 2\% \times 7 + 90000}{7(12)}$ $= 1220$	B1 Interest and principal seen A1 at least 3 s.f.

15(ai)		C1
(aii)		C1
(bi)		P1
(bii)	Diameter	B1
(biii)	Since BP is a diameter, ΔBPC is a right–angle triangle given the property of angle in semi-circle.	B1 Not Pythagoras Thm
16(ai)	$\frac{\frac{1}{2}x}{12.5} + \frac{\frac{1}{5}x}{10} + \frac{\frac{3}{10}x}{5}$ $= \frac{2x}{50} + \frac{x}{50} + \frac{3x}{50}$ $= \frac{3x}{25}$	M1 eqn, o.e use T=D/S A1
(aii)	$\frac{x}{\frac{3x}{25}}$ $= 8\frac{1}{3}$	M1 $\frac{x}{\text{their answer}}$ seen A1 reject 8.33
(b)	$380 + 70 - 150$ $= 300$	M1 + 70 – 150 seen or use of diagram A1
17(a)	$\left(-1\frac{1}{2}, 0\right)$	A1
(b)	$\left(1, 2\frac{1}{2}\right)$	A1
(c)	$y - 2\frac{1}{2} = -(x-1)$ $y = -x + 3\frac{1}{2}$	M1 $y - y_1 = m(x - x_1)$ seen A1
(d)	Ratio $= \frac{BM}{AM}$ $= \frac{1}{2}$ 1 : 2	M1 ratio of bases seen. A1 3 01

18(a)		A1
(b)	Sec 4 students who study Mathematics only	A1 o.e students ...but not physics
(ci)	$138-x+x+138-x=220$ $x=56$	M1 A1
(cii)	$\frac{138-56}{220}$ $=\frac{41}{110}$	A1 No decimals
19	Int $\angle = 180-36$ $=144$ $x=\frac{4(180)-144}{5}$ $=115.2$	M1 correct deduction M1 Using int or ext $\angle$ formula A1 Ignore ° symbol
20(a)	$28^2+45^2=2808$ $=53^2$ $\angle ABC=90°$	A1
(b)	$\frac{1}{2}\times 45\times 28$ $=630$ Shortest Dist $=\frac{630}{\frac{1}{2}\times 70}$ $=18$	M1 Area of triangle Using $\Delta ABC=\Delta PBC$ M1 $\frac{1}{2}\times 70\times$ shortest dist seen (Form Eqn) A1
21(a)	$a=\frac{1}{2}$ $b=-2$ $c=-1$	A2 A2 all answers correct, A1 2 answers correct
(b)	$0=\frac{1}{2}(x-2)^2-1$ $(x-2)^2=2$ $x-2=\pm\sqrt{2}$ $x=0.59, 3.41$	M1 Eqn = 0 A1 A1
22(a)	2.5	A1
(b)	Dist $=\frac{1}{2}\times 25\times(50+100)$ $=1875$	M1 area of trapezium or o.e A1

(c)	$125 + 25(x - 10) = 20x$ $x = 25$	M1 A1	Dist of cyc = Dist of rider

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 1 2015
Secondary 4

CANDIDATE NAME

CLASS | INDEX NUMBER

MATHEMATICS

4016/02

Paper 2

Thursday 2 July 2015

2 hours 30 minutes

Additional Materials: Writing Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

22

303

Mathematical Formulae

Compound Interest

$$\text{Total Amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r\ell$

Curved surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

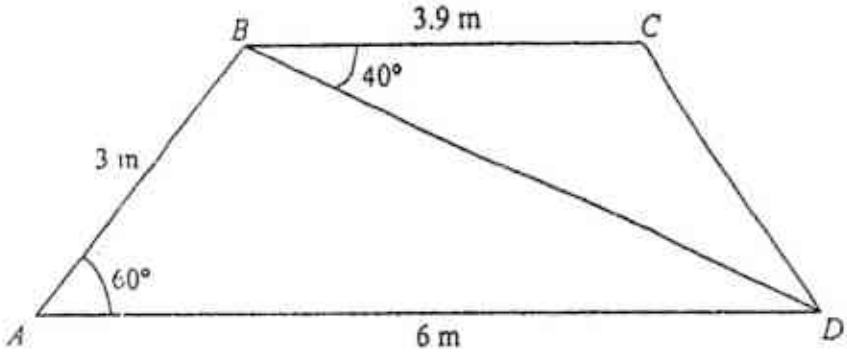
$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 (a) Bob has $x and Kim has half as much as Bob. Bob receives $10 each month and Kim receives 4 times as much as Bob. In two months' time, Kim will have 3 times as much as Bob. How much money does Bob have now? [3]

(b) Given that $\frac{x+3y}{5x-4y}=\frac{1}{3}$, find the ratio of $x : y$. [3]

(c) Solve the equation $2x^2=-3(2x-11)$, giving your answers to two decimal places. [3]

2



The diagram shows the points A, B, C and D on level ground. It is given that $AB = 3$ m, $AD = 6$ m, $BC = 3.9$ m, $\angle BAD = 60°$ and $\angle CBD = 40°$.

(a) Calculate

(i) the distance from B to D, [2]

(ii) the shortest distance from C to BD, [2]

(iii) the area of $ABCD$. [3]

(b) A vertical pole stands at point A and the angles of elevation of the top of the pole from both points B and D are measured. Given that the smaller of these two angles is 18°, find the height of the pole. [2]

304

3 (a) A shop sells two brands of mattresses, Hoiland and Sleeping King. Each brand of mattress comes in three different sizes, King, Queen and Single. The sales of the two brands of mattresses over a period of two months are given in the table below.

	November			December		
	King	Queen	Single	King	Queen	Single
Hoiland	10	15	25	12	20	40
Sleeping King	8	5	12	10	10	25

Given that the above sales table can be represented by matrix $P = \begin{pmatrix} 10 & 15 & 25 \\ 8 & 5 & 12 \end{pmatrix}$

and matrix $Q = \begin{pmatrix} 12 & 20 & 40 \\ 10 & 10 & 25 \end{pmatrix}$.

(i) Calculate $P + Q$. [1]

(ii) Describe what is represented by the elements of $P + Q$. [1]

(iii) Find matrix R such that $S = RP$ and S represents the total number of King, Queen and Single sizes mattresses sold by both Hoiland and Sleeping King in November. [1]

(iv) Evaluate matrix S. [1]

(v) The profit made for each mattress regardless of the brands are \$350, \$500 and \$250 for King, Queen and Single sizes respectively. Using matrix multiplication involving two matrices only, find the profit made by the shop in November. [2]

(b) On a map depicting part of a country, the scale is 1 : 250 000.

(i) Calculate the distance between the Town A to Town B, given that the distance on the map is 1.5 cm. Leave your answer in kilometres. [2]

(ii) Ali walks from Town B to Town A at the speed of 1 m/s. Calculate the time Ali takes for the walk, giving your answer to the nearest minute. [2]

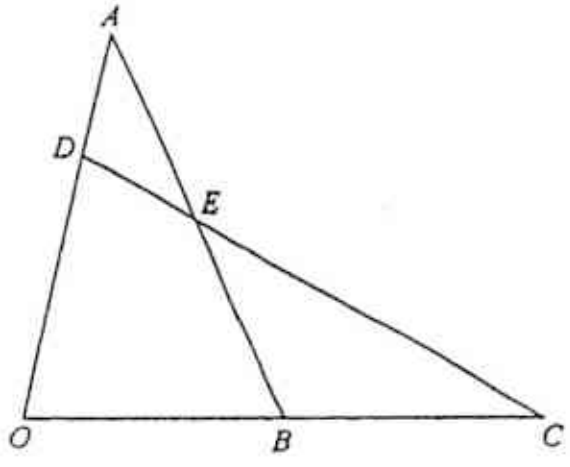
(iii) Kim walks from Town B to Town A at the speed of 4 km/h. She started her journey at the same time as Ali.
Who will reach Town A first? Justify your answer. [2]

4 (a) Solve the inequality $\frac{2x-1}{5} \geq \frac{x-3}{9}$. [2]

(b) Simplify $\frac{20x-15y}{32x^2-18y^2}$. [3]

(c) Express $\frac{5x+6}{4x^2-9} + \frac{2}{3-2x}$ as a single fraction in its simplest form. [3]

5



(a) In the diagram, B is the midpoint of OC and $OA = 3AD$. It is also given that $\overrightarrow{OA} = 3\mathbf{a} + \mathbf{b}$ and $\overrightarrow{OB} = 6\mathbf{a} - \mathbf{b}$. E is the point of intersection of AB and CD.

(i) Express as simply as possible, in terms of a and/or b,

(a) $\overrightarrow{AB}$, [1]

(b) $\overrightarrow{DC}$. [2]

(ii) Find the ratio of $\frac{\text{area of triangle } ADE}{\text{area of triangle } DEO}$. [1]

(b) The vector $\overrightarrow{OP}$ has a magnitude of 55 units and has the same direction as $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$. Express $\overrightarrow{OP}$ as a column vector. [2]

3 05

6 The variables x and y are connected by the equation

$$y = 2x - x^2 + 6.$$

Some corresponding values of x and y are given in the following table.

x	-2	-1	-0.5	0.5	2	3	4
y	-2	3	4.75	p	6	3	-2

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $-2 \le x \le 4$.
Using a scale of 2 cm to 1 unit, draw a vertical y-axis for $-2 \le y \le 8$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

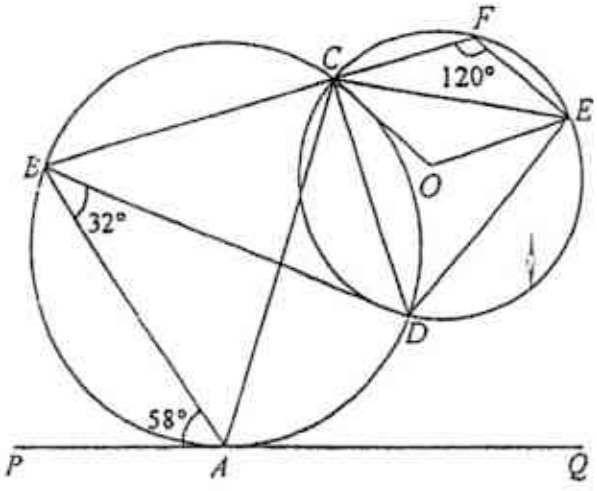
(c) Find the equation of the line of symmetry. [1]

(d) Use your graph to solve $4x + 7 = 2x^2$. [3]

(e) By drawing a tangent, find the gradient of the curve at the point where $x = 2$. [2]

(f) A straight line has the equation $y = ax + b$. Given that $ax + b = 2x - x^2 + 6$ has solutions at $x = 2$ and $x = -1$, find the value of a and of b. [2]

7 In the diagram, PAQ is a tangent to the circle $ABCD$ at A. O is the centre of the circle $CDEF$ and BCF is a straight line. It is given that $\angle PAB = 58°$, $\angle ABD = 32°$ and $\angle CFE = 120°$.



(a) Find

(i) angle ACD, [1]

(ii) angle ACB. [1]

(b) Explain why BD is a diameter of circle $ABCD$. [2]

(c) Given that $FC = FE$, show that triangle CDE is equilateral. [3]

(d) What is the special name given to quadrilateral $COEF$? Explain. [3]

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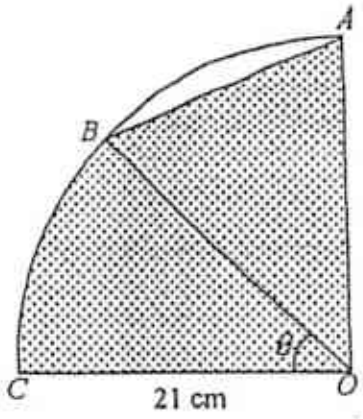
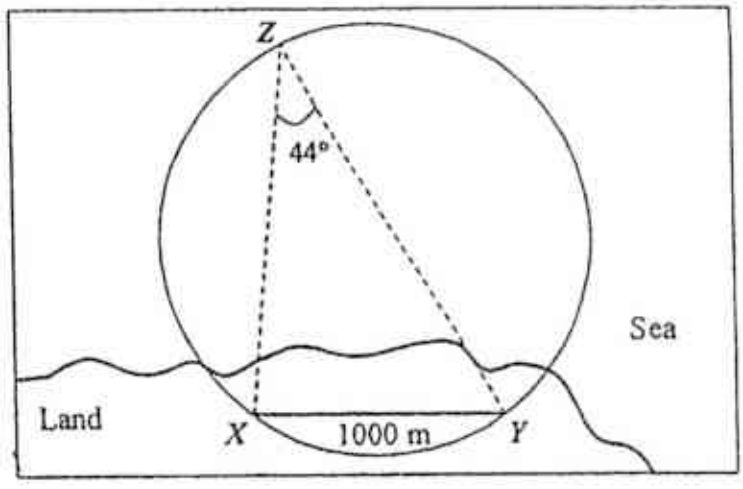


(a) $OABC$ is a quadrant of a circle with centre O and radius 21 cm. Given that $\angle BOC = \theta$ rad and Arc AB : Arc BC = 4 : 3, calculate

(i) the value of θ, [3]

(ii) the area of the shaded region. [3]

(b) A ship at sea is represented by point Z and it is travelling in a circular arc as shown in the diagram such that $\angle XZY = 44°$. The two landmarks X and Y are such that Y is 1000 m due east of X.

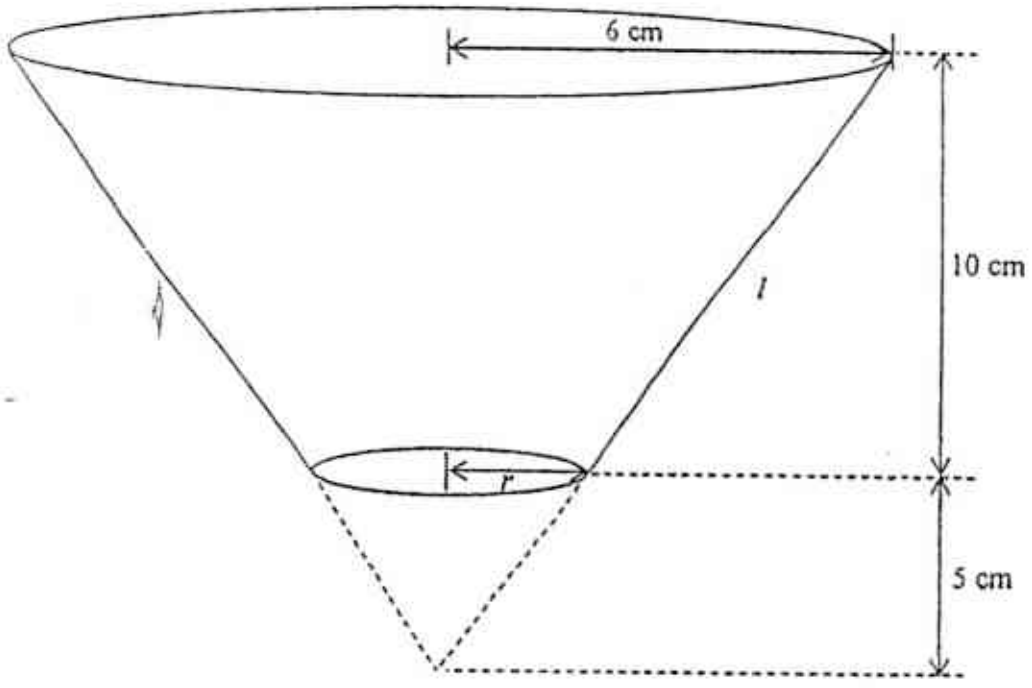


(i) Find the bearing of X from Z, when Z is due north of Y. [1]

(ii) Find the bearing of Z from Y, when Z is equidistant from X and Y. [2]

(iii) Calculate the distance from Z to X when the ship is 1000 m from Y. [3]

9 The bulb end of a torchlight consists of an open frustum as shown in the diagram. The frustum is formed by cutting off the bottom 5 cm from an inverted cone of radius 6 cm and height 15 cm.



(a) Find the base radius r of the frustum. [2]

(b) Find l, the slant height of the frustum. [2]

(c) Calculate the volume of the frustum. [2]

(d) The inner surface of the frustum, which will eventually contain the bulb, has to be lined with a reflective material. Find the amount of the material required. [2]

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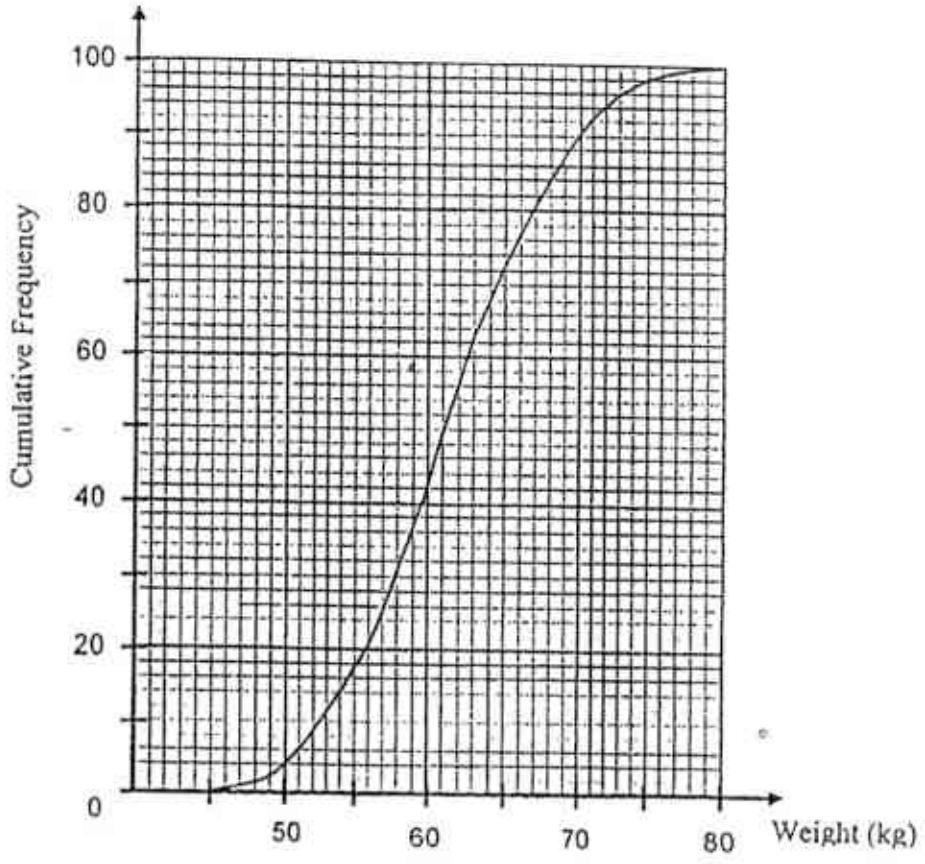
10 The cumulative frequency curve below illustrates the weights of 100 students in Shine Secondary School.



(a) Use the graph to find

(i) the median weight of the students, [1]

(ii) the interquartile range. [2]

(b) The grouped frequency table of the weights of the same 100 students in Shine Secondary School is shown below. Find p. [1]

Weight (x kg)	$45 < x \leq 50$	$50 < x \leq 55$	$55 < x \leq 60$	$60 < x \leq 65$	$65 < x \leq 70$	$70 < x \leq 75$	$75 < x \leq 80$
No. of students	4	14	24	30	p	8	2

Using the table, calculate an estimate of

(i) the mean weight of students, [2]

(ii) the standard deviation. [2]

The students are classified into three categories:
Students who weigh 50 kg or less are in the ***underweight*** category
Students who are heavier than 70 kg are in the ***overweight*** category
The rest of the students are in the ***normal*** weight category

(c) A student is selected at random from the 100 students.

(i) Find the probability that the student is underweight. [1]

(ii) If the probability that the student weighs more than y kg is $\frac{1}{10}$, find y. [1]

Two students are selected at random from the group of 100 students.

(iii) Find the probability that at least one of the students is of normal weight. [2]

(d) The weights of 100 students in Oxford Secondary School have the same median but a smaller standard deviation.
Describe how the cumulative frequency curve will differ from the given curve. [2]

----End of Paper ----

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Answers:

1(a) \$8 (b) $x : y = 13 : 2$ (c) $x = 2.83$ or $x = -5.83$

2(a)(i) 5.20 m (ii) 2.51 m (iii) 14.3 m^2 (b) $h = 1.95$ m

3(a)(i) $\begin{pmatrix} 22 & 35 & 65 \\ 18 & 15 & 37 \end{pmatrix}$ (iii) $R = \begin{pmatrix} 1 & 1 \end{pmatrix}$ (iv) $\begin{pmatrix} 18 & 20 & 37 \end{pmatrix}$ (v) \$25 550

(iii) It represents the <u>total</u> number of the King, Queen and Single sizes mattresses <u>sold over the two months</u> for <u>both brands</u> respectively.

(b)(i) 3.75 km (ii) $T = 63$ mins (iii) $1\frac{1}{9}$ m/s . Kim will reach Town A first as her speed is faster.

4(a) $x \geq -\frac{6}{13}$ (b) $\frac{5}{2(4x+3y)}$ (c) $\frac{x}{(2x+3)(2x-3)}$

5(a)(i)(a) 3a – 2b (b) $10a - \frac{8}{3}b$ (ii) $\frac{\text{area of triangle } ADE}{\text{area of triangle } DEO} = \frac{1}{2}$ (b) $\overrightarrow{OP} = \begin{pmatrix} 33 \\ -44 \end{pmatrix}$

6(a) $p = 6.75$ (c) $x = 1$ (d) $y = 2.5$ (to be plotted), $x = -1.1$ or $x = -3.1$ (e) $= -2.4$ (f) $a = 1, b = 4$

7(a)(i) $\angle ACD = 32°$ (ii) $\angle ACB = 58°$ (b) $\angle BCD = 32° + 58° = 90°$

8(a)(i) $\theta = 0.673$ (ii) 32 cm^2 (b)(i) 224° (ii) 338° (iii) $XZ = 1440$ m

9(a) $r = 2$ cm (b) 10.8 (c) 545 cm^3 (d) 271 cm^2

10(a)(i) 61 (ii) 8.5 (b) 18 (i) 61.3 (ii) 6.68 (c)(i) $\frac{1}{25}$

(ii) 70 (iii) $\frac{4859}{4950}$ (d) The graph will be <u>steeper</u> but they will <u>intersect at the median mark</u>.

No	Solution	Marks	Remarks
1(a)	Bob = $\$x$ Kim = $\$\frac{1}{2}x$ In 2 months, Bob = $\$(x+20)$ Kim = $\$\left(\frac{1}{2}x+80\right)$ $\frac{1}{2}x+80=3(x+20)$ $x+160=6x+120$ $x=8$ Bob = \$8	B1 soi (Kim) M1 equation A1	3 m
(b)	$\frac{x+3y}{5x-4y}=\frac{1}{3}$ $3(x+3y)=5x-4y$ $13y=2x$ $\frac{x}{y}=\frac{13}{2}$ $\therefore x:y=13:2$	M1 cross-multiply M1 grouping x & y A1 required form	3 m
(c)	$2x^2=-3(2x-11)$ $2x^2+6x-33=0$ $x=\frac{-6\pm\sqrt{(6)^2-4(2)(-33)}}{2(2)}$ $x=2.83$ (2 dp) or $x=-5.83$ (2 dp)	B1 equation (=0) M1 formula + sub A1 must indicate x somewhere	3 m
			9 marks
2 (a)(i)	$BD=\sqrt{3^2+6^2-2(3)(6)\cos 60°}$ $=5.19615$ m $=5.20$ m (3sf)	M1 cosine rule A1	2 m
(ii)	Let the shortest distance be CX. $\sin 40°=\frac{CX}{3.9}$ $CX=2.50687$ m $CX=2.51$ m (3 sf)	M1 trigo ratios A1 30	2 m

No	Solution	Marks	Remarks
(iii)	$A_{\Delta ABD} = \frac{1}{2}(3)(6)\sin 60°$ $= 7.7942 \text{ m}^2$ $A_{\Delta BCD} = \frac{1}{2}(3.9)(5.19615)\sin 40°$ $= 6.51304 \text{ m}^2$ $A_{ABCD} = 7.7942 + 6.51304$ $= 14.307 \text{ m}^2$ $= 14.3 \text{ m}^2$ (3 sf)	M1 area $A_{\Delta ABD}$ M1 area $A_{\Delta BCD}$ (5.20 not acceptable although ans is correct) A1	3 m
(b)	Let the height of the pole be h. $\tan 18° = \frac{h}{6}$ $h = 1.9495$ m $h = 1.95$ m (3 sf)	M1 identifying AD A1	2 m
			9 marks
3(a)(i)	$P+Q = \begin{pmatrix} 10 & 15 & 25 \\ 8 & 5 & 12 \end{pmatrix} + \begin{pmatrix} 12 & 20 & 40 \\ 10 & 10 & 25 \end{pmatrix}$ $= \begin{pmatrix} 22 & 35 & 65 \\ 18 & 15 & 37 \end{pmatrix}$	A1 correct order	1 m
(ii)	It represents the <u>total</u> number of the King, Queen and Single sizes mattresses <u>sold over the two months</u> for <u>both brands</u> respectively.	B1 oe (use diff)	1 m
(iii)	$R = \begin{pmatrix} 1 & 1 \end{pmatrix}$	B1 correct order	1 m
(iv)	$S = \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 10 & 15 & 25 \\ 8 & 5 & 12 \end{pmatrix}$ $= \begin{pmatrix} 18 & 20 & 37 \end{pmatrix}$	A1	1 m
(v)	$\begin{pmatrix} 18 & 20 & 37 \end{pmatrix}\begin{pmatrix} 350 \\ 500 \\ 250 \end{pmatrix}$ $= (25550)$ $\therefore$ Profit = \$25 550	M1 2 correct matrices No marks if matrix has unit B1 statement seen	2 m
(b)(i)	Distance $= 1.5 \times 2.5$ km $= 3.75$ km	M1 x scale A1	2 m
(ii)	$T = \frac{3750}{1}$ $T = 3750$ secs $T = 63$ mins (nearest mins)	B1 convert (b)(i) to metres A1	2 m
(iii)	$\frac{4 \times 1000}{3600} = 1.11$ m/s or $1\frac{1}{9}$ m/s $\therefore$ Kim will reach Town A first as her speed is faster.	M1 conversion for both num and den B1	2 m
			12 marks

No	Solution	Marks	Remarks
4(a)	$\frac{2x-1}{5} \geq \frac{x-3}{9}$ $9(2x-1) \geq 5(x-3)$ $13x \geq -6$ $x \geq -\frac{6}{13}$	M1 cross-multiply A1	2 m
(b)	$\frac{20x-15y}{32x^2-18y^2}$ $= \frac{5(4x-3y)}{2(16x^2-9y^2)}$ $= \frac{5(4x-3y)}{2(4x-3y)(4x+3y)}$ $= \frac{5}{2(4x+3y)}$	M1 taking out both common factors M1 diff of 2 sq A1	3 m
(c)	$\frac{5x+6}{4x^2-9} + \frac{2}{3-2x}$ $= \frac{5x+6}{(2x+3)(2x-3)} - \frac{2}{2x-3}$ $= \frac{5x+6-2(2x+3)}{(2x+3)(2x-3)}$ $= \frac{x}{(2x+3)(2x-3)}$	M1 one factor common (den $2x-3$) M1 combining term A1 no marks if den expanded	3 m
			8 marks
5(a)(i) (a)	$\overrightarrow{AB} = (6a-b)-(3a+b)$ $= 3a-2b$	No vector symbol [-1m] B1	1 m
(b)	$\overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD}$ $= 2(6a-b) - \frac{2}{3}(3a+b)$ $= 10a - \frac{8}{3}b$	M1 $\overrightarrow{OC}$ soi A1	2 m
(ii)	$\frac{\text{area of triangle } ADE}{\text{area of triangle } DEO} = \frac{1}{2}$	A1	1 m
(b)	Let $OP = \begin{pmatrix} 3a \\ -4a \end{pmatrix}$. $\sqrt{(3a)^2+(-4a)^2} = 55$ $a = 11$ (same direction) $\therefore \overrightarrow{OP} = \begin{pmatrix} 33 \\ -44 \end{pmatrix}$	M1 using magnitude A1	2m
			6 marks

No	Solution	Marks	Remarks
5(a)	$p = 6.75$	A1	1 m
(b)	Refer to graph.	P2 pts. correct G1 smoothness (diff scale -1m)	3 m
(c)	$x = 1$	A1	1 m
(d)	$4x + 7 = 2x^2$ $4x - 2x^2 + 7 = 0$ $2x - x^2 + \frac{7}{2} = 0$ $2x - x^2 + 6 = -\frac{7}{2} + 6$ $\therefore y = 2.5$ (to be plotted) $x = -1.1$ or $x = -3.1$	M1 attempt to make y the subject L1 line to be Plotted A1	3 m
(e)	Gradient $= \frac{8.5 - 1.3}{1 - 4}$ $= -2.4$	M1 tangent correct A1	2 m
(f)	$a = \frac{6 - 3}{2 - (-1)}$ $= 1$ $b = 4$	A1 A1	2 m
			12 marks
7(a)(i)	$\angle ACD = 32°$	A1 $\angle$ in same seg.	1 m
(ii)	$\angle ACB = 58°$	A1 $\angle$ in alt. seg.	1m
(b)	$\angle BCD = 32° + 58°$ $= 90°$ Since it obeys $\angle$s in semicircle property, $\Rightarrow BD$ is a diameter. (shown)	B1 proving 90° B1	2 m
(c)	$\angle FCE = \angle FEC = \frac{180° - 120°}{2}$ (base $\angle$s of isos. Δ) $= 30°$ $\angle ECD = 180° - 90° - 30°$ (adj. $\angle$s) $= 60°$ $\angle FED = 90°$ ($\angle$s in opp. seg.) $\angle CED = 90° - 30°$ $= 60°$ $\therefore \angle CDE = 60°$ $\Rightarrow \Delta CDE$ is an equilateral Δ. (shown)	M1 attempt to show $\angle ECD = 60°$ M1 $\angle$s in opp. seg. M1 attempt to show $\angle CED = 60°$	3 m
(d)	$\angle COE = 120°$ ($\angle$ at centre = 2 $\angle$s at circumference) $\Rightarrow \angle OCE = \angle OEC = 30°$ CE is common $\therefore \Delta OCE \equiv \Delta FCE$ (AAS) $\angle OCF + \angle CFE = 180°$ (int. $\angle$s) $\Rightarrow COEF$ is a rhombus.	M1 proving congruency M1 int. $\angle$s /alt $\angle$s B1	3 m
			10 marks

No	Solution	Marks	Remarks
8(a)(i)	$\frac{Arc_{AB}}{Arc_{BC}} = \frac{4}{3}$ $\frac{r\left(\frac{\pi}{2}-\theta\right)}{r\theta} = \frac{4}{3}$ $3\left(\frac{\pi}{2}-\theta\right) = 4\theta$ $7\theta = \frac{3\pi}{2}$ $\theta = \frac{3}{14}\pi$ $\theta = 0.673198$ $\theta = 0.673$ (3 sf)	M1 $\frac{\pi}{2}$ seen M1 $s = r\theta$ soi A1 oe	3 m
(ii)	Area of sector$_{BOC} = \frac{1}{2}(21)^2\left(\frac{3}{14}\pi\right)$ $= 47\frac{1}{4}\pi$ cm^2 Area of $\Delta_{OAB} = \frac{1}{2}(21)^2 \sin\left(\frac{\pi}{2}-\frac{3}{14}\pi\right)$ $= \frac{1}{2}(21)^2 \sin\frac{2}{7}\pi$ $= 172.3938$ cm^2 $\therefore$ Shaded Area $= 172.3938 + 47\frac{1}{4}\pi$ $= 320.834$ cm^2 $= 321$ cm^2 (3 sf)	M1 Area of sector M1 Area of Δ A1	3 m
(b)(i)	Bearing $= 180° + 44°$ $= 224°$	A1	1 m
(ii)	$\angle ZYX = \frac{180° - 44°}{2}$ (base $\angle$s of isos. Δ) $= 68°$ $\therefore$ Bearing $= 270° + 68°$ $= 338°$	M1 Isos. Δ A1	2 m
(iii)	$\angle ZXY = 44°$ (base $\angle$s of isos. Δ) $\angle ZYX = 180° - 2(44°)$ ($\angle$s sum in Δ) $= 92°$ $\frac{XZ}{\sin 92°} = \frac{1000}{\sin 44°}$ $XZ = 1438.68$ m $XZ = 1440$ m (3 sf)	B1 Isos. Triangle / finding $\angle ZYX$ M1 sine rule A1	3 m
			12 marks

No	Solution	Marks	Remarks
9(a)	By similar Δs, $\frac{r}{6}=\frac{5}{15}$ $r=2$ cm	M1 similar Δ A1	2 m
(b)	$l=\sqrt{15^2+6^2}-\sqrt{5^2+2^2}$ $=10.7703$ $=10.8$ (3 sf)	M1 pythagoras' thm A1	2 m
(c)	Volume of frustum $=\frac{1}{3}\pi(6)^2(15)-\frac{1}{3}\pi(2)^2(5)$ $=544.5427$ cm^3 $=545$ cm^3 (3 sf)	M1 vol of cone A1	2 m
(d)	Amount of material required $=\pi(6)\left(\sqrt{15^2+6^2}\right)-\pi(2)\left(\sqrt{5^2+2^2}\right)$ $=270.688$ cm^2 $=271$ cm^2 (3 sf)	M1 surface area pf cone A1	2 m
			8 marks
10(a)(i)	Median weight = 61	A1	1 m
(ii)	Interquartile range $=65.5-57$ $=8.5$	M1 UQ - LQ A1	2 m
(b)	$p=90-72$ $=18$	B1	1 m
(i)	Mean $=\frac{47.5\times4+\cdots+77.5\times2}{100}$ $=61.3$ (3 sf)	M1 Formulae A1	2 m
(ii)	Standard deviation $=\sqrt{\frac{4(47.5-61.3)^2+\cdots+2(77.5-61.3)^2}{100}}$ $=6.68$ (3 sf)	M1 Formulae A1	2 m
(c)(i)	P (student is underweight) $=\frac{4}{100}$ $=\frac{1}{25}=0.04$	A1 oe	1 m

No	Solution	Marks	Remarks
(ii)	P (student weighs more than y kg) $= \frac{1}{10} = \frac{10}{100}$ From graph, $y = 70$	A1	1 m
(iii)	P (out of 2 students, at least one of the student is of normal weight) $= \frac{86}{100} \times \frac{14}{99} \times 2 + \frac{86}{100} \times \frac{85}{99}$ $= \frac{4859}{4950}$	M1 A1	2 m
(d)	The graph will be steeper but they will intersect at the median mark.	B2 or graphs drawn [B1] Label [B1]	2m
			14 marks

3,2
END