NAME: _______________________  _____ (          )  CLASS: 4 (          )

11 September 2018 Tuesday  2 hours

Additional Materials: 6 Writing Papers

READ THESE INSTRUCTIONS FIRST
Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the writing paper provided.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer all the questions.
Write your answers on the separate Writing Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case
of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together and attach the question
paper on top of the scripts.
The number of marks is given in brackets [  ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiners’ Use

Table of Penalties

Parent’s Name & Signature:

Date:  Total:  80

This paper consists of 6 printed pages.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion
\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]
where \( n \) is a positive integer and
\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}
\]

2. TRIGONOMETRY

Identities
\[
\sin^2 A + \cos^2 A = 1
\]
\[
\sec^2 A = 1 + \tan^2 A
\]
\[
\cos ec^2 A = 1 + \cot^2 A
\]
\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
\sin 2A = 2\sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A
\]
\[
\tan 2A = \frac{2\tan A}{1 - \tan^2 A}
\]

Formulae for \( \triangle ABC \)
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\text{Area of } \triangle = \frac{1}{2}ab \sin C
\]
Answer ALL questions

1. The product of the two positive numbers, $x$ and $y$, where $x > y$, is 24. The difference between their squares is 14. Form two equations, and hence, find the exact values of the two numbers. [5]

2. Show that \( (2 + \sqrt{7})^2 - \frac{18}{3 - \sqrt{7}} = c + d \sqrt{7} \) where $c$ and $d$ are integers. [4]

3. (a) (i) Sketch the two curves $y = 0.5 \sqrt{x}$ and $y = \frac{8}{x}$ on the same axes for $x > 0$. [3]

   (ii) Find the coordinates of the intersection point. [2]

   (b) Solve the equation $2 = e^{-x} - 3$. [3]

4. (i) Given that the line $y = 2$ intersects the graph of $y = \log_{\frac{1}{5}} x$ at the point $P$, find the coordinates of $P$. [2]

   (ii) Sketch the graph of $y = \log_{\frac{1}{5}} x$. [2]

   (iii) State the range of values of $x$ for which $y < 0$. [1]

5. (i) Sketch the graph of $y^2 = 169x$. [2]

   (ii) Express $4x^2 - 181x = -9$ in the form \( (px + q)^2 = 169x \), where $p$ and $q$ are constants. [2]

   (iii) A suitable straight line can be drawn on the graph in (i) to solve the equation $4x^2 - 181x = -9$.

      (a) State the equation of this straight line. [1]

      (b) On the same axes, sketch the straight line and state the number of solutions of the equation $4x^2 - 181x = -9$. [2]
6. 

The diagram shows a circle passing through points $A$, $B$, $C$ and $D$. The tangents from $E$ meets the circle at $B$ and $D$. Given that $AD = BF$ and triangle $ABD$ is isosceles, where $AB = BD$. Prove that 

(i) $ABFD$ is a parallelogram. [3]

(ii) triangle $BCD$ is similar to triangle $DFE$. [3]

(iii) $BD \times EF = CD \times DE$. [1]

7. (a) Sketch the graph of $y = 2 \cos \left( \frac{x}{2} \right) - 1$, for the interval $0 \leq x \leq 2 \pi$. [2]

(b) In the diagram, triangle $ABC$ is a right-angle triangle, where $\angle ABC = 90^\circ$. $D$ is a point of $AB$ such that $AD$ is 7 cm and $BD$ is 2 cm.

Given that $\cos \angle ADC = -\frac{1}{3}$,

(i) Find the exact length of $BC$. [1]

(ii) Find the value of $\tan \angle ACD$ in the form $a\sqrt{b}$, where $a$ and $b$ are integers. [2]
8. The minute hand of a clock is 50 cm, measured from the centre of the clock, $O$, to the tip of the minute hand, $M$. The displacement, $d$ cm, of $M$ from the vertical line through $O$ is given by $d = a \sin bt$, where $t$ is the time in minutes past the hour.

(i) Find the exact value of $a$ and of $b$. [3]

(ii) Find the duration, in each hour, where $|d| > 25$. [3]

9. (i) Prove that \[
\frac{\cos^4 \theta - \sin^4 \theta}{\sin^2 \theta \cos^2 \theta} = 4 \cot 2\theta \csc^2 2\theta.
\] [3]

(ii) Hence, or otherwise, solve \[
\frac{\cos^4 \theta - \sin^4 \theta}{\sin^2 \theta \cos^2 \theta} = 4 \csc^2 2\theta
\] for $0^\circ \leq \theta \leq 180^\circ$. [3]

10. A curve is such that \[
\frac{d^2 y}{dx^2} = 6x - 2
\] and $P(2, -8)$ is a point on the curve. The gradient of the normal at $P$ is $-\frac{1}{2}$. Find the equation of the curve. [7]

11. Find and simplify $\frac{dy}{dx}$ for the following:

(i) $y = \ln \cos x$

(ii) $y = e^{x^2} \times e^x$ [4]
12. In the diagram, the curve \( x = (y - 1)^2 + 4 \) and the line \( y + x = 7 \) intersect at \( A \) and \( B \).

(i) Find the coordinates of \( A \) and of \( B \). [3]

(ii) Calculate the area of the shaded region. [4]

13. A particle moves in a straight line so that its acceleration, \( a \) ms\(^{-2}\), is given by \( a = 2t - 13 \), where \( t \) is the time in seconds after passing a fixed point \( O \). The particle first comes to instantaneous rest at \( t = 5 \) s. Find,

(i) the velocity when the particle passes through \( O \). [2]

(ii) the total distance travelled by the particle when it next comes to rest. [5]

(iii) the minimum velocity of the particle. [2]

*** End of Paper ***
<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Answer key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( x = 4\sqrt{2} \quad y = 3\sqrt{2} )</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>(a)(i)</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>(ii) ((8, 1))</td>
<td>(iii) (x &gt; 1)</td>
</tr>
<tr>
<td></td>
<td>(b) (x \approx -1.61 ) or 0</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>(i)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(ii) ((2x-3)^2 = 169x)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(iii) (a) (y = 2x - 3) (b) 2 solutions</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>(a)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(b) (i) (4\sqrt{2}) cm (ii) (\frac{14\sqrt{2}}{25})</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>(i) (a = 50) (b = \frac{\pi}{30}) (ii) 40 mins</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>(y = x^3 - 2x^2 - 6x)</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>(i) (-\tan x) (ii) ((2x+1)e^{x^2+x})</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>(i) (A(5, 2), B(8, -1)) (ii) (4 \frac{1}{2}) units²</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>(i) 40 ms⁻¹ (ii) (\frac{2}{3} m) (iii) (-2 \frac{1}{4} ms^{-1})</td>
<td></td>
</tr>
</tbody>
</table>
1. The product of the two positive numbers, \(x\) and \(y\), where \(x > y\), is 24. The difference between their squares is 14. Form two equations, and hence, find the exact values of the two numbers. [5]

<table>
<thead>
<tr>
<th>Solutions</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>(xy = 24)</td>
<td>M1</td>
</tr>
<tr>
<td>(y = \frac{24}{x}) ... (1)</td>
<td></td>
</tr>
<tr>
<td>(x^2 - y^2 = 14) ... (2)</td>
<td>M1</td>
</tr>
<tr>
<td>Sub. (1) into (2):</td>
<td></td>
</tr>
<tr>
<td>(x^2 - \left(\frac{24}{x}\right)^2 = 14)</td>
<td></td>
</tr>
<tr>
<td>(x^2 - \frac{576}{x^2} = 14)</td>
<td></td>
</tr>
<tr>
<td>(x^4 - 14x^2 - 576 = 0)</td>
<td>M1</td>
</tr>
<tr>
<td>((x^2 - 32)(x^2 + 18) = 0)</td>
<td></td>
</tr>
<tr>
<td>(x^2 = 32) or (x^2 = -18) (rejected)</td>
<td></td>
</tr>
<tr>
<td>(x = \sqrt{32}) (-\sqrt{32}) is rejected)</td>
<td>A1</td>
</tr>
<tr>
<td>(=4\sqrt{2})</td>
<td></td>
</tr>
<tr>
<td>(\therefore y = \frac{24}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}})</td>
<td>A1</td>
</tr>
<tr>
<td>(=3\sqrt{2})</td>
<td></td>
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</tbody>
</table>

2. Show that \((2 + \sqrt{7})^2 - \frac{18}{3 - \sqrt{7}} = c + d\sqrt{7}\) where \(c\) and \(d\) are integers. [4]

<p>| | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>((2 + \sqrt{7})^2 - \frac{18}{3 - \sqrt{7}})</td>
<td></td>
</tr>
<tr>
<td>(= 4 + 4\sqrt{7} + 7 - \frac{18(3 + \sqrt{7})}{(3 - \sqrt{7})(3 + \sqrt{7})})</td>
<td>M1, M1</td>
</tr>
<tr>
<td>(= 11 + 4\sqrt{7} - \frac{54 + 18\sqrt{7}}{9 - 7})</td>
<td></td>
</tr>
<tr>
<td>(= 11 + 4\sqrt{7} - \frac{1}{2}(54 + 18\sqrt{7}))</td>
<td>M1</td>
</tr>
<tr>
<td>(= -16 - 5\sqrt{7})</td>
<td>A1</td>
</tr>
</tbody>
</table>
3. (a) (i) Sketch the two curves $y = 0.5\sqrt[3]{x}$ and $y = \frac{8}{x}$ on the same axes for $x > 0$. [3]

(ii) Find the coordinates of the intersection point. [2]

(b) Solve the equation $2 = |e^{-x} - 3|$. [3]
Given that the line $y = 2$ intersects the graph of $y = \log_{\frac{1}{5}} x$ at the point $P$, find the coordinates of $P$. [2]

(ii) Sketch the graph of $y = \log_{\frac{1}{5}} x$. [2]

(iii) State the range of values of $x$ for which $y < 0$. [1]

**[Solution]**

<table>
<thead>
<tr>
<th>(i)</th>
<th>$\log_{\frac{1}{5}} x = 2$</th>
<th>M1 A1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x = 0.2^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 0.04$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Coordinates of $P$ are (0.04, 2)</td>
<td></td>
</tr>
</tbody>
</table>

(ii) [Diagram]

(iii) $y < 0 \Rightarrow \log_{\frac{1}{5}} x < 0$

$\Rightarrow x > 1$ B1
5 (i) Sketch the graph of \( y^2 = 169x \).

(ii) Express \( 4x^2 - 181x = -9 \) in the form \((px + q)^2 = 169x\), where \( p \) and \( q \) are constants.

(iii) A suitable straight line can be drawn on the graph in (i) to solve the equation \( 4x^2 - 181x = -9 \).

(a) State the equation of this straight line.

(b) On the same axes, sketch the straight line and state the number of solutions of the equation \( 4x^2 - 181x = -9 \).

Solution

\[ 5(i) \]

\[ y = 2x - 3 \]

\[ y^2 = 169x \]

(ii) \[
\begin{align*}
4x^2 - 181x &= -9 \\
4x^2 - 181x + 169x &= -9 + 169x \\
4x^2 - 12x + 9 &= 169x \\
(2x - 3)^2 &= 169x 
\end{align*}
\]

\[ \text{M1 A1} \]

(iii) (a) \( y = 2x - 3 \)

\[ \text{B1} \]

2 solutions

\[ \text{A1} \]
The diagram shows a circle passing through points $A$, $B$, $C$ and $D$. The tangents from $E$ meet the circle at $B$ and $D$. Given that $AD = BF$ and triangle $ABD$ is isosceles, where $AB = BD$. Prove that

i) $ABFD$ is a parallelogram. \[3\]

ii) triangle $BCD$ is similar to triangle $DFE$. \[3\]

iii) $BD \times EF = CD \times DE$. \[1\]

Solution

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
| i) | $\angle DBF = \angle BAD$ (alt. seg. thm)  
    = $\angle ADB$ ($\Delta ABD$ is isosceles) |
|   | M1 |
|   | By alternate angles, $AD \parallel BF$ |
|   | M1 |
|   | Since $AD = BF$, $ABFD$ is a parallelogram. |
|   | M1 |
| ii) | $\angle EDF = \angle DBC$ (alt. seg. thm)  
       $\angle DFE = 180^\circ - \angle BFD$ (adj $\angle$ on a str. line)  
       = $180^\circ - \angle BAD$ (opp. $\angle$ in parallelogram)  
       = $180^\circ - (180^\circ - \angle DCB)$ ($\angle$ in opp. seg)  
       = $\angle DCB$ |
|   | M1 |
|   | M1 |
|   | M1 |
|   | M1 |
| iii) | By AA, $\Delta BCD$ is similar to $\Delta DFE$.  
       $\frac{BD}{CD} = \frac{DE}{EF}$  
       $BD \times EF = CD \times DE$ |
|   | M1 |
7. (a) Sketch the graph of $y = 2 \cos \left(\frac{x}{2}\right) - 1$, for the interval $0 \leq x \leq 2\pi$. [2]

(b) In the diagram, triangle $ABC$ is a right-angle triangle, where $\angle ABC = 90^\circ$.

$D$ is a point of $AB$ such that $AD$ is 7 cm and $BD$ is 2 cm.

Given that $\cos \angle ADC = -\frac{1}{3}$,

(i) Find the exact length of $BC$. [1]

(ii) Find the value of $\tan \angle ACD$ in the form $\frac{a}{b}$, where $a$ and $b$ are integers. [2]

Solution

(a) [Graph showing the graph of $y = 2 \cos \left(\frac{x}{2}\right) - 1$ for $0 \leq x \leq 2\pi$.]

G1: Correct shape.  
G1: Label all key points and axes clearly

(bi) $\cos \angle BDC = -\cos \angle ADC$

$$\frac{BD}{CD} = -\left(-\frac{1}{3}\right)$$

$$\frac{2}{CD} = 1$$

$$CD = 6 \text{ cm}$$

$$BC = \sqrt{6^2 - 2^2}$$

$$= 4\sqrt{2} \text{ cm}$$

(bii)
\[
\tan \angle ACD = \tan (\angle ACB - \angle BCD)
\]
\[
= \frac{\tan \angle ACB - \tan \angle BCD}{1 + (\tan \angle ACB)(\tan \angle BCD)}
\]
\[
= \frac{\frac{9}{\sqrt{2}} - \frac{2}{2}}{1 + \left(\frac{9}{\sqrt{2}}\right)\left(\frac{2}{\sqrt{2}}\right)}
\]
\[
= \frac{7}{\frac{4\sqrt{2}}{25}}
\]
\[
= \frac{14\sqrt{2}}{25}
\]

8.

The minute hand of a clock is 50 cm, measured from the centre of the clock, \(O\), to the tip of the minute hand, \(M\). The displacement, \(d\) cm, of \(M\) from the vertical line through \(O\) is given by \(d = a \sin bt\), where \(t\) is the time in minutes past the hour.

i) Find the exact value of \(a\) and of \(b\). [3]

ii) Find the duration, in each hour, where \(|d| > 25\). [3]

Solution

i) \(a = 50\)

Period = 60

\[
\frac{2\pi}{b} = 60
\]

\[
b = \frac{\pi}{30}
\]

B1 -- for \(a\)

M1 -- for period = 60

A1 -- for \(b\)
ii) \( |d| > 25 \)
\[
\begin{align*}
& \text{when } d = 25 \\
& 50 \sin \left( \frac{\pi t}{30} \right) = 25 \\
& 50 \sin \left( \frac{\pi t}{30} \right) = \pm 25 \\
& \sin \left( \frac{\pi t}{30} \right) = \pm \frac{1}{2} \\
& \text{basic angle } = \sin^{-1} \left( \frac{1}{2} \right) = 30^\circ
\end{align*}
\]
\[\begin{align*}
& \pi t = \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \\
& t = 5, 25, 35, 55
\end{align*}\]
For \( |d| > 25 \)
Duration = \((25 - 2) + (55 - 35) = 40 \text{ mins}\)

Alternative Solution:

Observe that at the first instance when \( d = 25 \) at the point \( A \),

\[
\cos \theta = \frac{25}{50} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}.
\]
This happened again at the point \( B \).

Between points \( A \) and \( B \), \( |d| > 25 \).

Time taken from \( A \) to \( B \) = \( \frac{\pi - 2\theta}{\pi} \times 30 = \frac{\pi - \frac{\pi}{3}}{\pi} \times 30 = \frac{2}{3} \times 30 = 20 \text{ minutes} \).

By symmetry, the time for \( |d| > 25 \) in the other half of the clock face would be 20 minutes as well.

Hence total time for \( |d| > 25 \) is \( 20 + 20 = 40 \text{ minutes} \).
9. i) Prove that \( \frac{\cos^4 \theta - \sin^4 \theta}{\sin^2 \theta \cos^2 \theta} = 4 \cot 2 \theta \cos 2 \theta \). [3]

ii) Hence, or otherwise, solve

\[
\frac{\cos^4 \theta - \sin^4 \theta}{\sin^2 \theta \cos^2 \theta} = 4 \csc 2 \theta \text{ for } 0^\circ \leq \theta \leq 180^\circ.
\] [3]

Solution

\[
\text{i) } LHS = \frac{\cos^4 \theta - \sin^4 \theta}{\sin^2 \theta \cos^2 \theta} = \frac{(\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta)}{(\sin \theta \cos \theta)^2} = \frac{\cos 2 \theta}{(\frac{1}{2} \sin 2 \theta)^2} = \frac{\cos 2 \theta}{\frac{1}{4} \sin^2 2 \theta} = 4 \left( \frac{\cos 2 \theta}{\sin 2 \theta} \right) \left( \frac{1}{\sin 2 \theta} \right) = 4 \cot 2 \theta \cos ec 2 \theta \quad (RHS)
\]

\[
\text{ii) } \frac{\cos^4 \theta - \sin^4 \theta}{\sin^2 \theta \cos^2 \theta} = 4 \cot 2 \theta \cos 2 \theta = 4 \cos ec 2 \theta = 4 \cot 2 \theta \cos ec \theta \cos 2 \theta
\]

\[
4 \cos ec 2 \theta (\cot 2 \theta - 1) = 0
\]

\[
\cos ec \theta = 0 \Rightarrow \frac{1}{\sin \theta} = 0 \text{ (no solution)} \quad \text{OR}
\]

\[
\cot 2 \theta = 1
\]

\[
\tan 2 \theta = 1
\]

basic angle = \tan^{-1} 1 = 45^\circ

\[
0^\circ \leq \theta \leq 180^\circ \Rightarrow 0^\circ \leq 2 \theta \leq 360^\circ
\]

\[
2 \theta = 45^\circ \text{ or } 180^\circ + 45^\circ
\]

\[
\theta = 22.5^\circ \text{ or } 112.5^\circ
\] A1

10. A curve is such that \( \frac{d^2 y}{dx^2} = 6x - 2 \) and \( P(2, -8) \) is a point on the curve. The gradient of the normal at \( P \) is \( -\frac{1}{2} \). Find the equation of the curve. [7]

Solution:

Given \( \frac{d^2 y}{dx^2} = 6x - 2 \)
\[
\frac{dy}{dx} = \int (6x - 2)\,dx \\
= 3x^2 - 2x + c
\]

Gradient of normal at \((2, -8)\) = \(-\frac{1}{2}\)

Gradient of tangent at \(P\) = \(-\frac{1}{2}\)

\[
\frac{dy}{dx} = 2
\]

Sub \(x = 2, \quad 3(2)^2 - 2(2) + c = 2\)

\(c = -6\)

\[
\therefore \quad \frac{dy}{dx} = 3x^2 - 2x - 6
\]

\[
y = \int (3x^2 - 2x - 6)\,dx \\
= x^3 - x^2 - 6x + c_1
\]

Sub \((2, -8)\), \(-8 = (2)^3 - (2)^2 - 6(2) + c_1\)

\(c_1 = 0\)

Hence,
the equation of the curve is \(y = x^3 - 2x^2 - 6x\).
11. Find and simplify \( \frac{dy}{dx} \) for the following:

(i) \( y = \ln \cos x \)

(ii) \( y = e^{x^2} \times e^x \)

Solution:

(i) \( y = \ln \cos x \)

\[
\frac{dy}{dx} = \frac{-\sin x}{\cos x} = -\tan x
\]

(ii) \( y = e^{x^2} \times e^x \)

\[
y = e^{x^2+x}
\]

\[
\frac{dy}{dx} = (2x+1)e^{x^2+x}
\]

OR

\[
\frac{dy}{dx} = (2xe^{x^2})(e^x) + (e^{x^2})(e^x)
\]

\[
= (2x+1)e^{x^2+x}
\]

[4]
12. In the diagram, the curve \( x = (y - 1)^2 + 4 \) and the line \( y + x = 7 \) intersect at \( A \) and \( B \).

(i) Find the coordinates of \( A \) and \( B \). \[[3]\]

(ii) Calculate the area of the shaded region. \[[4]\]

Solution:

(i) given \( y + x = 7 \)

\[ y = -x + 7 \quad \text{......} \textcircled{1} \]

sub 1 into \( x = (y - 1)^2 + 4 \)

\[ x = (-x + 7 - 1)^2 + 4 \]

\[ x^2 - 12x + 36 + 4 = 0 \]

\[ x^2 - 13x + 40 = 0 \]

\( (x - 5)(x - 8) = 0 \)

\[ x = 5 \quad \text{or} \quad x = 8 \]

sub \( x \) into 1,

\[ y = 5 + 7 \quad \text{or} \quad y = 8 + 7 \]

\[ y = 2 \quad \text{or} \quad y = 1 \]

\[ A(5, 2) \quad B(8, -1) \]

Area of shaded region

\[ = \frac{1}{2} \int_1^2 ((y - 1)^2 + 4) \, dy \]

\[ = \frac{39}{2} - \left[ \frac{(y-1)^3}{3} + 4y \right]_1^2 \]

\[ = 19\frac{1}{2} - 15 \]

\[ = 4\frac{1}{2} \text{ units}^2 \]
13. A particle moves in a straight line so that its acceleration, $a \text{ ms}^{-2}$, is given by $a = 2t - 13$, where $t$ is the time in seconds after passing a fixed point $O$. The particle first comes to instantaneous rest at $t = 5\text{s}$. Find,

i) the velocity when the particle passes through $O$. [2]

ii) the total distance travelled by the particle when it next comes to rest. [5]

iii) the minimum velocity of the particle. [2]

Solution

i) \[a = 2t - 13\]
\[v = \int 2t - 13 \, dt\]
\[= t^2 - 13t + c\]

When $t = 5$, $v = 0$.
$0 = 5^2 - 13(5) + c$
$c = 40$

Velocity when passes through $O = 40 \text{ ms}^{-1}$

ii) \[t^2 - 13t + 40 = 0\]
\[(t - 5)(t - 8) = 0\]
$t = 5$ or $t = 8$

\[v = t^2 - 13t + 40\]
\[s = \int (t^2 - 13t + 40) \, dt\]
\[= \frac{t^3}{3} - \frac{13t^2}{2} + 40t + c\]

When $t = 0$, $c = 0$,
$s = \frac{t^3}{3} - \frac{13t^2}{2} + 40t$

When $t = 5$,
$s = \frac{5^3}{3} - \frac{13(5)^2}{2} + 40(5) = 79 \frac{1}{6}$

When $t = 8$,
$s = \frac{8^3}{3} - \frac{13(8)^2}{2} + 40(8) = 74 \frac{2}{3}$

![Graph showing distance-time relationship]
Total distance = \( 79 \frac{1}{6} + \left( 79 \frac{1}{6} - 74 \frac{2}{3} \right) \)

\[ = 83 \frac{2}{3} m \]

iii) \[ a = 2t - 13 \]

\[ 2t - 13 = 0 \]

\[ t = \frac{13}{2} \]

\[ v = \left( \frac{13}{2} \right)^2 - 13 \left( \frac{13}{2} \right) + 40 \]

\[ = -2 \frac{1}{4} ms^{-1} \]

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Additional Materials: 8 Writing Papers and 1 Graph Paper

READ THESE INSTRUCTIONS FIRST
Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the writing paper provided.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer all the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together and attach the question paper on top of the scripts.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiners’ Use

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Parent’s Name & Signature:

Date: 100

This paper consists of 6 printed pages.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion
\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]
where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. TRIGONOMETRY

Identities
\[
sin^2 A + \cos^2 A = 1
\]
\[
sec^2 A = 1 + \tan^2 A
\]
\[
\cos ec^2 A = 1 + \cot^2 A
\]
\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
\sin 2A = 2 \sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]
\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \triangle ABC \)
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\text{Area of } \triangle = \frac{1}{2}ab \sin C
\]
Answer all questions.

1 (a) Given that the curve \( y = x^2 + (3k - 1)x + (2k + 10) \) has a minimum value greater than 0, calculate the range of values of \( k \). [4]

(b) Find the range of values of \( x \) for which \((x + 4)(x - 1) - 6 \geq 0\). [2]

(c) The equation \( 2x^2 - x + 18 = 0 \) has roots \( \alpha \) and \( \beta \). Find the quadratic equation whose roots are \( \left( \frac{\alpha}{\beta} \right)^{\frac{1}{2}} \) and \( \left( \frac{\beta}{\alpha} \right)^{\frac{1}{2}} \). [4]

2 (a) Simplify \( \frac{25^p \times 10^{1+p}}{2^{p-1} \times 5^{2+3p}} \). [3]

(b) Given that \( n \) is a positive integer, show that \( 8^n + 8^{n+2} + 8^{n+4} \) is always divisible by 24. [2]

(c) Solve \( 2 - 2^a = 2^{a+3} - 4^{a+1} \). [4]

3 (a) Express \( \frac{2x^3 - 3x - 1}{(x + 3)(x - 1)} \) as partial fractions. [5]

(b) The polynomial \( P(x) = 2x^3 - hx^2 - 48x - 20 \) leaves a remainder of 11 when divided by \( x + 1 \).
   (i) Show that \( h = 15 \). [2]
   (ii) Factorise \( P(x) \) completely. [3]

4 (a) (i) Write down, and simplify, the first 3 terms in the expansion of \( (2 - x)^8 \) in ascending powers of \( x \). [1]
   (ii) Hence, determine the coefficient of \( y^2 \) in the expansion of \( 256(1 - y)^8 \). [3]

(b) (i) Write down the general term in the expansion of \( \left( 3x - \frac{1}{2x^2} \right)^{11} \). [1]
   (ii) Hence, explain why the term in \( x^3 \) does not exist. [2]
5 Solutions to this question by accurate drawing will not be accepted.

The diagram shows a parallelogram with vertices \(A(-1, 4), B(p, 6), C(4, -1)\) and \(D\).

(i) Given that \(AC\) is perpendicular to \(BD\), show that \(p = 6\). [4]

(ii) Find the coordinates of \(D\). [2]

(iii) Find the area of the parallelogram \(ABCD\). [2]

6 A container in the shape of a pyramid has a volume of \(V\) cm\(^3\), given by

\[ V = \frac{1}{3} x(ax^2 + b), \]

where \(x\) is the height of the container in cm, and \((ax^2 + b)\) is the area of the rectangular base, of which \(a\) and \(b\) are unknown constants.

Corresponding values of \(x\) and \(V\) are shown in the table below.

<table>
<thead>
<tr>
<th>(x) (cm)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V) (cm(^3))</td>
<td>150</td>
<td>600</td>
<td>1650</td>
<td>3600</td>
</tr>
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</table>

(i) Using suitable variables, draw on graph paper, a straight line graph. [4]

(ii) Use your graph to estimate the value of \(a\) and of \(b\). [4]

(iii) Explain how another straight line drawn on your graph can lead to an estimate of the value of \(x\) when the base area of the pyramid is three times the square of its height. Draw this line and find an estimate for the value of \(x\). [3]
7 A circle has a diameter $AB$. The point $A$ has coordinates $(1, -6)$ and the equation of the tangent to the circle at $B$ is $3x + 4y = k$.

(i) Show that the equation of the normal to the circle at the point $A$ is $4x - 3y = 22$. [3]

Given also that the line $x = -1$ touches the circle at the point $(-1, -2)$.

(ii) Find the coordinates of the centre and the radius of the circle. [4]

(iii) Find the value of $k$. [3]

8 The diagram shows a lawn made up of two triangles, $ABC$ and $CDE$. Triangle $ABC$ is an isosceles triangle where $AB = AC = 6$ m. $DE = 7$ m, $AE = 3$ m, and $BA$ produced is perpendicular to $DE$. Angle $BAC$ is $\theta$ and the area of the lawn is $S$ m$^2$.

(i) Show that $S = 18 \sin \theta + 31.5 \cos \theta$. [3]

(ii) Hence, express $S$ as a single trigonometric term. [4]

(iii) Given that $\theta$ can vary, find the maximum area of the lawn and the corresponding value of $\theta$. [2]

9 A curve has the equation $y = (1 - x)\sqrt{1 + 2x}$.

(i) Find $\frac{dy}{dx}$ in its simplest form. [3]

Hence,

(ii) determine the interval where $y$ is increasing, [3]

(iii) find the rate of change of $x$ when $x = 4$, given that $y$ is decreasing at a constant rate of 2 units per second, [2]

(iv) evaluate $\int_1^4 \frac{x}{\sqrt{1 + 2x}} dx$. [2]
10 A piece of wire of length 180 cm is bent into the shape $PQRST$ shown in the diagram.

Show that the area, $A$ cm$^2$, enclosed by the wire is given by

$$A = 2160 - 540x^2.$$  

Find the value of $x$ and of $y$ for which $A$ is a maximum. [8]

11 (a) Find the following indefinite integrals.

(i) $$\int \frac{e^{2x}}{2} \, dx$$

(ii) $$\int \left( \frac{4}{x} + \frac{1}{x^2} \right) \, dx$$ [3]

(b) Evaluate $$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{2 \csc^2 x} \, dx$$, leaving your answer in terms of $\pi$. [5]

END OF PAPER.
### ANSWER KEY

| 1 | (a) $-\frac{13}{9} < k < 3$  
(b) $x \leq -5$ or $x \geq 2$  
(c) $x^2 - \frac{1}{6}x + 1 = 0$ or $6x^2 - x + 6 = 0$ |
|---|---|
| 2 | (a) $\frac{4}{5}$  
(b) $24 \times 1387 \times 8^{n-1}$  
Since $n \geq 1$, $8^{n-1} \geq 1$, hence $8^n + 8^{n+2} + 8^{n+4}$ is divisible by 24.  
(c) $a = -2$ or $a = 1$ |
| 3 | (a) $2x - 4 + \frac{23}{2(x+3)} - \frac{1}{2(x-1)}$  
(b)(ii) $(x+2)(2x+1)(x-10)$ |
| 4 | (a)(i) $256 - 1024x + 1792x^2 + \ldots$  
(ii) coefficient of $y^2 = 7168$  
(b)(i) $T_{r+1} = \left(\frac{11}{r}\right)(3x)^{11-r}\left(-\frac{1}{2x^2}\right)^r$  
(ii) $r = \frac{8}{3}$. As $r$ is not a whole number, the term in $x^3$ does not exist. |
| 5 | (ii) $(-3, -3)$  
(iii) $45$ units$^2$ |
| 6 | (ii) centre is $(4, -2)$ radius $= 5$ units  
(iii) $k = 29$ |
| 7 | (ii) $36.3 \sin(\theta + 60.3^\circ)$  
(iii) Max $S \approx 36.3$ m$^2$  
$\theta \approx 29.7^\circ$ |
| 8 | (a) (i) $\int e^{2x} dx = \frac{e^{2x}}{2} + c$  
(ii) $\int \left(\frac{4}{x} + \frac{1}{x^2}\right) dx = 4 \ln x - \frac{1}{x} + c$ |
| 9 | (i) $\frac{dy}{dx} = -\frac{3x}{\sqrt{1+2x}}$  
(ii) $y$ in increasing when $-0.5 < x < 0$.  
(iii) $\frac{dx}{dt} = \frac{1}{2}$ units/sec  
(iv) 3 |
| 10 | $x = 2$ cm and $y = 40$ cm when $A$ is a maximum. |
| 11 | (b) $\frac{\pi}{12} - \frac{\sqrt{3}}{8}$ or $\frac{2\pi - 3\sqrt{3}}{24}$ |
### Solutions

#### 1(a)

\[ x^2 + (3k - 1)x + (2k + 10) > 0 \]

\[ b^2 - 4ac < 0 \]

\[ (3k - 1)^2 - 4(1)(2k + 10) < 0 \]

\[ 9k^2 - 6k + 1 - 8k - 40 < 0 \]

\[ 9k^2 - 14k - 39 < 0 \]

\[ (9k + 13)(k - 3) < 0 \]

\[ \frac{-13}{9} < k < 3 \]

#### (b)

\[(x + 4)(x - 1) - 6 \geq 0\]

\[ x^2 + 3x - 4 - 6 \geq 0 \]

\[ x^2 + 3x - 10 \geq 0 \]

\[ (x + 5)(x - 2) \geq 0 \]

\[ x \leq -5 \text{ or } x \geq 2 \]

#### (c)

\[ 2x^2 - x + 18 = 0 \]

\[ \alpha + \beta = \frac{1}{2} \]

\[ \alpha \beta = 9 \]

\[ \left( \frac{\alpha}{\beta} \right)^{\frac{1}{2}} + \left( \frac{\beta}{\alpha} \right)^{\frac{1}{2}} = \frac{\alpha + \beta}{\alpha^{\frac{1}{2}} \beta^{\frac{1}{2}}} = \frac{(\alpha + \beta)}{(\alpha \beta)^{\frac{1}{2}}} \]

\[ = \frac{1}{2} \]

\[ = \frac{2}{9^{\frac{1}{2}}} \]

\[ = \frac{1}{6} \]

\[ \left( \frac{\alpha}{\beta} \right)^{\frac{1}{2}} \times \left( \frac{\beta}{\alpha} \right)^{\frac{1}{2}} = \left( \frac{\alpha \beta}{\beta \alpha} \right)^{\frac{1}{2}} = 1 \]

Required equation is

\[ x^2 - \frac{1}{6}x + 1 = 0 \text{ or } 6x^2 - x + 6 = 0 \]
2(a) 
\[
\frac{25^p \times 10^{1+p}}{2^{p-1} \times 5^{2+3p}} = \frac{5^{2p} \times (2 \times 5)^{1+p}}{2^{p-1} \times 5^{2+3p}} \\
= \frac{5^{2p} \times 2^{1+p} \times 5^{1+p}}{2^{p-1} \times 5^{2+3p}} \\
= 2^{1+p-(p-1)} \times 5^{2p+1+p-(2+3p)} \\
= 2^2 \times 5^{-1} \\
= \frac{4}{5}
\]

(b) 
\[
8^n + 8^{n+2} + 8^{n+4} = 8^n + 8^n \times 8^2 + 8^n \times 8^4 \\
= 8^n (1 + 64 + 4096) \\
= 8^n (4161) \\
= 8^1 \times 8^{n-1} \times 3 \times 1387 \\
= 24 \times 1387 \times 8^{n-1}
\]
Since \( n \geq 1 \), \( 8^{n-1} \geq 1 \) and \( 24 \times 1387 \times 8^{n-1} \) is divisible by 24. 

(c) 
\[
2 - 2^a = 2^{a+3} - 4^{a+1} \\
2 - 2^a = 2^1 (2^a) - 2^{2(a+1)} \\
2 - 2^a = 8(2^a) - 2^2(2^a) \\
2 - 2^a = 8(2^a) - 4(2^a)^2
\]
Let \( u \) be \( 2^a \). 
\[
2 - u = 8u - 4u^2 \\
4u^2 - 9u + 2 = 0 \\
(4u - 1)(u - 2) = 0 \\
u = \frac{1}{4} \text{ or } u = 2
\]
\[
a = -2 \text{ or } a = 1
\]

3(a) 
\[
\frac{2x^3 - 3x - 1}{(x+3)(x-1)} = \frac{2x^3 - 3x - 1}{x^2 + 2x - 3} \\
= \frac{2x - 4}{x^2 + 2x - 3} - (2x^3 - 4x^2 - 6x) \\
= \frac{-4x^2 + 3x - 1}{x^2 + 2x - 3} - (2x^3 - 8x + 12) \\
= \frac{-4x^2 + 3x - 1}{11x - 13}
\]
\[
\frac{2x^3 - 3x - 1}{(x+3)(x-1)} = 2x - 4 + \frac{11x - 13}{(x+3)(x-1)}
\]
\[
\frac{11x - 13}{(x+3)(x-1)} = \frac{P}{x+3} + \frac{Q}{x-1}
\]

\[
P(x-1) + Q(x+3) = (x+3)(x-1)
\]

\[
\Rightarrow 11x - 13 = P(x-1) + Q(x+3)
\]

\[
(P + Q)x + (-P + 3Q) = 11 \quad \text{...(1)}
\]

\[
-P + 3Q = -13 \quad \text{...(2)}
\]

\[
(1)+(2): \quad 4Q = -2 \quad \Rightarrow Q = -\frac{1}{2}
\]

\[
\therefore P - \frac{1}{2} = 11 \quad \Rightarrow P = \frac{23}{2}
\]

\[
\frac{2x^3 - 3x - 1}{(x+3)(x-1)} = 2x - 4 + \frac{23}{2(x+3)} - \frac{1}{2(x-1)}
\]

(b)

(i)

\[
P(x) = 2x^3 - hx^2 - 48x - 20
\]

\[
P(-1) = 11
\]

\[
2(-1)^3 - h(-1)^2 - 48(-1) - 20 = 11
\]

\[
-2 - h + 48 - 20 = 11
\]

\[
h = 15 \quad \text{(shown)}
\]

(ii)

\[
P(x) = 2x^3 - 15x^2 - 48x - 20
\]

By trial and error, \(x + 2\) is a factor.

\[
2x^3 - 15x^2 - 48x - 20 = (x+2)(ax^2 + bx + c)
\]

\[
= ax^3 + bx^2 + cx + 2ax^2 + 2bx + 2c
\]

\[
= ax^3 + (b+2a)x^2 + (c+2b)x + 2c
\]

By comparing coefficients of

\[
x^3: a = 2
\]

\[
x^2: b + 2(2) = -15
\]

\[
b = -19
\]

constant: \(2c = -20\)

\[
c = -10
\]

\[
\therefore P(x) = (x+2)(2x^2 - 19x - 10)
\]

\[
= (x+2)(2x+1)(x-10)
\]

4(a)

(i)

\[
(2-x)^8 = 2^8 + \binom{8}{1}2^7(-x) + \binom{8}{2}2^6(-x)^2 + \ldots
\]

\[
= 256 - 1024x + 1792x^2 + \ldots
\]

(ii)

\[
256(1-y)^8 = 2^8(1-y)^8
\]

\[
= [2(1-y)]^8
\]

\[
= (2 - 2y)^8
\]

Taking \(x = 2y\),

\[
(2 - 2y)^8 = 256 - 1024(2y) + 1792(2y)^2 + \ldots
\]

Hence, coefficient of \(y^2\) = 1792 \times 2^2 = 7168.
### (b)

#### (i)

\[ T_{r+1} = \left( \frac{11}{r} \right) (3x)^{11-r} \left( -\frac{1}{2x^2} \right)^r \]

#### (ii)

For term in \( x^3 \), \( 11-r - 2r = 3 \)

\[ 3r = 8 \Rightarrow r = \frac{8}{3} \]

As \( r \) is not a whole number, the term in \( x^3 \) does not exist. \hspace{1cm} \text{o.e.}

### 5

#### (i)

\[ \text{Grad } AC = \frac{4-(-1)}{-1-4} = -1 \]

Mid-point of \( AC = \left( \frac{-1+4}{2}, \frac{4-1}{2} \right) = \left( \frac{3}{2}, \frac{3}{2} \right) \]

As \( BD \) and \( AC \) share the same mid-point (property of parallelogram),

\[ \text{Gradient of } BD = \frac{\frac{3}{2}}{p-\frac{3}{2}} = \frac{9}{2p-3} \]

\[ \left( \frac{9}{2p-3} \right)(-1) = -1 \]

\[ 2p - 3 = 9 \Rightarrow 2p = 12 \Rightarrow p = 6 \text{ (shown)} \]

#### (ii)

Let \( D \) be \((a, b)\).

\[ \left( \frac{a+6}{2}, \frac{b+6}{2} \right) = \left( \frac{3}{2}, \frac{3}{2} \right) \]

Comparing coordinates, \( \frac{a+6}{2} = \frac{3}{2} \)

\[ a + 6 = 3 \Rightarrow a = -3 \]

Similarly, \( b = -3 \).

Therefore, coordinates of \( D \) are \((-3, -3)\).

#### (iii)

Area of the parallelogram \( ABCD \)

\[
\begin{vmatrix}
-3 & 4 & 6 & -1 & 3 \\
1 & 2 & -1 & 6 & 4 & -3 \\
\end{vmatrix}
= \frac{1}{2} \{( -3)(-1) + 4(6) + 6(4) + (-1)(-3) \\ -[4(-3) + 6(-1) + (-1)(6) + (-3)(4)] \}
= 45 \text{ units}^2
\]

### 7

#### (i)

The normal to the circle at point \( A \) will pass through the centre of the circle, and point \( B \) also, and is perpendicular to the tangent to the circle at \( B \).

\[ 3x + 4y = k \Rightarrow y = -\frac{3}{4}x + \frac{k}{4} \]

\[ \text{Grad of tangent at } B = -\frac{3}{4} \]

\[ \text{Grad of normal at } A = \frac{4}{3} \]

Equation of normal at \( A \):

\[ y - (-6) = \frac{4}{3}(x - 1) \]

\[ y = \frac{4}{3}x - \frac{22}{3} \]

\[ \Rightarrow 4x - 3y = 22 \text{ (shown)} \]

#### (ii)

Since the line \( x = -1 \) touches the circle at the point \((-1, -2)\), so the equation of the normal at \((-1, -2)\) is \( y = -2 \).

Solving the equations \( 4x - 3y = 22 \) and
\[ y = -2, \]
\[ 4x - 3(-2) = 22 \Rightarrow 4x = 16 \Rightarrow x = 4. \]
Thus the centre is \((4, -2)\).

Radius = √[(4 - 1)^2 + [(-2) - (-6)]^2]
= √9 + 16
= 5 units

(iii) Let the coordinates of \(B\) be \((p, q)\).
\[
\left(\frac{p+1}{2}, \frac{q-6}{2}\right) = (4, -2)
\]
\[ p = 2(4) - 1 = 7 \text{ and } q = 2(-2) + 6 = 2 \]
Therefore, \(B\) is \((7, 2)\).
Sub. \((7, 2)\) into \(3x + 4y = k\),
k = 3(7) + 4(2) = 29

8
(i)
Area \(\Delta ABC = \frac{1}{2}(6)^2 \sin \theta\)
= 18 \sin \theta
Area \(\Delta CDE = \frac{1}{2}(7 \times 9) \sin (90^\circ - \theta)\)
= 31.5 \cos \theta
\(S = 18 \sin \theta + 31.5 \cos \theta\) (shown)

(ii) \(S = 18 \sin \theta + 31.5 \cos \theta\)
\[ R = \sqrt{18^2 + 31.5^2} \]
= 36.28016
\[ \tan \alpha = \frac{31.5}{18} \]
\[ \alpha = \tan^{-1}\left(\frac{31.5}{18}\right) \]
\[ = 60.2551187^\circ \]
\(S = 36.28016 \sin (\theta + 60.2551187^\circ)\)
\[ \approx 36.3 \sin (\theta + 60.3^\circ) \]

(iii) \(S = 36.28016 \sin (\theta + 60.2551187^\circ)\)
Max \(S \approx 36.3\) m²
\[ \sin (\theta + 60.2551187^\circ) = 1 \]
\[ 0^\circ < \theta < 90^\circ \]
\[ 60.2551187^\circ < \theta + 60.2551187^\circ < 150.2551187^\circ \]
\[ \theta + 60.2551187^\circ = 90^\circ \]
\[ \theta = 29.7448813^\circ \]
\[ \approx 29.7^\circ \]
9

(i) \( y = (1-x)\sqrt{1+2x} \)

\[ \frac{dy}{dx} = (1-x)\left(\frac{1}{2}\right)(1+2x)^{-\frac{1}{2}}(1)(2)+(1+2x)^{\frac{1}{2}}(-1) \]

\[ = (1+2x)^{\frac{1}{2}}(1-x-2x) \]

\[ = -\frac{3x}{\sqrt{1+2x}} \]

(ii) For \( \frac{dy}{dx} > 0, \)

\[ -\frac{3x}{\sqrt{1+2x}} > 0 \]

\[ \Rightarrow 1+2x > 0 \quad \text{and} \quad -3x > 0 \]

\[ x > -0.5 \quad x < 0 \]

\[ \therefore y \text{ in increasing when } -0.5 < x < 0. \]

(iii) \( \frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} \)

When \( x = 4, \) \( \frac{dy}{dt} = -2, \)

\[ -2 = -\frac{3(4)}{\sqrt{1+2(4)}} \times \frac{dx}{dt} \]

\[ \frac{dx}{dt} = \frac{1}{2} \text{ units/sec} \]

(iv) \[ \int_{1}^{4} \frac{x}{\sqrt{1+2x}} \, dx \]

\[ = \left[ -\frac{1}{3}(1-x)\sqrt{1+2x} \right]_{1}^{4} \]

\[ = \left( -\frac{1}{3}(1-(4))\sqrt{1+2(4)} \right) \]

\[ - \left( -\frac{1}{3}(1-1)\sqrt{1+2(1)} \right) \]

\[ = 3 \]

10

\[ 13x + 13x + y + 24x + y = 180 \]

\[ 50x + 2y = 180 \]

\[ y = 90 - 25x \]
Let \( h \) cm be the perpendicular distance from \( P \) to \( QT \).

\[
h^2 = (13x)^2 - \left( \frac{24x}{2} \right)^2
\]

\[
= 25x^2
\]

\( h = 5x \)

Area = \( y(24x) + \frac{1}{2} (24x)(5x) \)

\[
A = (90 - 25x)(24x) + 60x^2
\]

\[
= 2160x - 600x^2 + 60x^2
\]

\[
= 2160x - 540x^2 \text{ (shown)}
\]

\[
\frac{dA}{dx} = 2160 - 1080x
\]

When \( \frac{dA}{dx} = 0 \), \( 2160 - 1080x = 0 \)

\[
x = \frac{2160}{1080} = 2
\]

Sub \( x = 2 \), into \( y = 90 - 25x \)

\[
y = 90 - 25(2) = 40
\]

\[
\frac{d^2A}{dx^2} = -1080, \quad \therefore A \text{ is a maximum.}
\]

\[
x = 2 \text{ cm and } y = 40 \text{ cm when } A \text{ is a maximum.}
\]

<table>
<thead>
<tr>
<th>11(a)(i)</th>
<th>[ \int \frac{e^{2x}}{2} ,dx = \frac{e^{2x}}{4} + c ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(ii)</td>
<td>[ \int \left( \frac{4}{x} + \frac{1}{x^2} \right) ,dx = 4 \ln x - \frac{1}{x} + c ]</td>
</tr>
</tbody>
</table>
(b) \[
\int_{\pi/6}^{\pi} \frac{1}{2 \csc^2 x} \, dx
\]
\[
= \int_{\pi/6}^{\pi} \frac{\sin^2 x}{2} \, dx
\]
\[
= \int_{\pi/6}^{\pi} \frac{1}{2} \left( 1 - \cos 2x \right) \, dx
\]
\[
= \int_{\pi/6}^{\pi} \frac{1}{4} \left( 1 - \cos 2x \right) \, dx
\]
\[
= \left[ \frac{1}{4} x - \frac{1}{8} \sin 2x \right]_{\pi/6}^{\pi}
\]
\[
= \left( \frac{1}{4} \left( \frac{\pi}{6} \right) - \frac{1}{8} \sin 2 \left( \frac{\pi}{6} \right) \right) - \left( \frac{1}{4} \left( -\frac{\pi}{6} \right) - \frac{1}{8} \sin 2 \left( -\frac{\pi}{6} \right) \right)
\]
\[
= \frac{\pi}{24} - \frac{\sqrt{3}}{16} + \frac{\pi}{24} - \frac{\sqrt{3}}{16}
\]
\[
= \frac{\pi}{12} - \frac{\sqrt{3}}{8} \quad \text{or} \quad \frac{2\pi - 3\sqrt{3}}{24}
\]
BUKIT PANJANG GOVERNMENT HIGH SCHOOL
Preliminary Examination 2018
SECONDARY 4 EXPRESS/ 5 NORMAL

ADDITIONAL MATHEMATICS 4047/1
Paper 1
Date: 3 August, 2018
Duration: 2 h
Time: 1030 – 1230

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your class, register number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Setter: Mr Choo Kong Lum

This paper consists of 5 printed pages
1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\cosec^2 A = 1 + \cot^2 A$$
$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for $\Delta ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}ab \sin C$$
Answer ALL the questions.

1(a) Given that \((\sqrt{3} + 1)x = \sqrt{3} - 1\), find the value of \(x + \frac{1}{x}\) without using a calculator.

1(b) Given that \(2\sqrt{2} - 3 = \frac{\sqrt{h} - \sqrt{k}}{1 + \sqrt{2}}\), find the values of \(h\) and \(k\).

2(a) Show that for all real values of \(p\) and of \(q\), \(y = -(1 + p^2)x^2 + 2pqx - (2q^2 + 1)\) is always negative for all real values of \(x\).

2(b) Find the range of values of \(m\) for which \(\frac{-4}{m^2 + 3m + 2} < 0\)

3(a) (i) For the function \(y = \sin x\), where \(-1 \leq y \leq 1\), state the principal values of \(x\), in radians.
(ii) For the function \(y = \cos x\), where \(-1 \leq y \leq 1\), state the principal values of \(x\), in radians.
(iii) For the function \(y = \tan x\), state the principal values of \(x\), in radians.

3(b) The diagram shows part of the graph for the function \(y = a \cos bx + c\).

(i) Find the values of \(a\), \(b\) and \(c\).
(ii) Copy the diagram and draw the line \(y = \frac{x}{\pi} - 1\) on the same diagram. Hence state the number of solutions when \(a \cos bx + c = \frac{x}{\pi} - 1\).
4. (i) Sketch the graph of \( y = x^{\frac{2}{3}} \) for \( x \geq 0 \). [1]  
(ii) Find the equation of the line that must be inserted in the graph above in order to solve the equation \( 3x^{\frac{2}{3}} + 9x = 6 \). [2]  

5. Express \( \frac{4x^5 + 2x^4 + 3x^3 - x^2 - x + 1}{x^3 + x} \) in partial fractions. [6]  

6. (i) Sketch the graphs of \( y = |x - 2| + 1 \) and \( y = x^2 + 3 \) on the same diagram. For each graph, indicate the coordinates of the minimum point on the diagram. [4]  
(ii) Find the coordinates of the point of intersection. [4]  

7(a) Given that \( y = \ln \sqrt{\frac{3x+1}{3x-1}} \), find an expression for \( \frac{dy}{dx} \) and simplify your answer as a single fraction. [3]  
7(b) Given that \( y = 2e^{x^2+3} \), find the coordinates of the stationary point, leaving your answer in exact form. Determine the nature of the stationary point. [5]  

8. The diagram shows two lines \( PR \) and \( QS \) which are perpendicular to each other. \( RS = 9 \) cm, \( PQ = 16 \) cm and \( \angle PQT = \angle SRT = \theta \) radians.  

(i) Show that \( QS = 16\cos\theta + 9\sin\theta \). [1]  
(ii) Express \( QS \) in the form of \( R\sin(\theta + \alpha) \). [3]  
(iii) Find the value of \( \theta \) for which \( QS = 12 \) cm. [3]  
(iv) Show that the area of the quadrilateral \( PQRS \) is \( \frac{288 + 337\sin 2\theta}{4} \) cm². [4]
9. (i) Differentiate \((x - 5)\sqrt{2x - 1}\) with respect to \(x\) and simplify your answer as a single fraction. \([2]\)
(ii) Hence evaluate \(\int_{1}^{2} \frac{3x - 9}{\sqrt{2x - 1}} \, dx\), leaving your answer in exact form. \([4]\)

10. (i) Given that \(\frac{dy}{dx} = \frac{5}{1 + \cos 2x}\). Find the equation of the curve if the curve passes through the \(y\)-axis at \(y = 1\). \([4]\)
(ii) Find the equation of the normal to the curve at \(x = \frac{\pi}{4}\). \([3]\)

11. **Solutions to this question by accurate drawing will not be accepted.**
   
   The following diagram shows an isosceles triangle \(PQR\), where \(PR = QR\). It is given that \(M(2, 0)\) is the midpoint of \(PQ\). The line \(QR\) intersects the \(x\)-axis at point \(A\) such that \(\angle AMQ = 45^\circ\).

   (i) Show that the gradient of the line \(MR\) is 1. \([1]\)
   (ii) Find the equation of the line \(PQ\). \([2]\)
   (iii) Find the coordinates of \(Q\). \([2]\)
   (iv) Given that the area of \(\Delta PQR\) is 20 units\(^2\), find the coordinates of \(R\). \([5]\)

**END OF PAPER**
1(a) 4
1(b) $h = 3, \ k = 2$
2(b) $m < -2 \ or \ m > -1$
3(a)(i) $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
3(a)(ii) $0 \leq x \leq \pi$
3(a)(iii) $-\frac{\pi}{2} < x < \frac{\pi}{2}$
3(b)(i) $a = 2, \ b = 2, \ c = 1$
3(b)(ii) 4
4(ii) $y = -3x + 2$
5. $4x^2 + 2x - 1 + \frac{1}{x} - \frac{4x}{x^2+1}$
6(ii) (2, 1) and (0, 3)
6(iii) (0, 3) and (-1, 4)
7(a) $\frac{-3}{(3x+1)(3x-1)} \ or \ \frac{3}{(1+3x)(1-3x)}$
7(b) $(0, 2e^3)$ minimum point
8(ii) $\sqrt{337} \sin(\theta + 1.06)$ or $18.4 \sin(\theta + 1.06)$
8(iii) 1.37 radians
9(i) $\frac{3x-6}{\sqrt{2x-1}}$
9(ii) $7 - 6\sqrt{3}$
10(i) $y = \frac{5}{2}\tan x + 1$
10(ii) $y = -\frac{1}{5}x + \frac{\pi}{20} + \frac{7}{2}$
11(ii) $y = -x + 2$
11(iii) (4, - 2)
11(iv) (7, 5)
BUKIT PANJANG GOVERNMENT HIGH SCHOOL
Preliminary Examinations 2018
SECONDARY FOUR EXPRESS/FIVE NORMAL

ADDITIONAL MATHEMATICS
4047/02
Date: 13 August, 2018
Duration: 2 hours 30 min
Time: 07 45 – 10 15

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [   ] at the end of each question or part question.

The total number of marks for this paper is 100.

This paper consists of 5 printed pages.
1. **ALGEBRA**

**Quadratic Equation**

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

**Binomial Expansion**

\[
(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n
\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. **TRIGONOMETRY**

**Identities**

\[
\sin^2 A + \cos^2 A = 1 \\
\sec^2 A = 1 + \tan^2 A \\
\cosec^2 A = 1 + \cot^2 A
\]

\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
\sin 2A = 2 \sin A \cos A \\
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A \\
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

**Formulae for \( \Delta ABC \)**

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2} ab \sin C
\]
1 Expand \((1 + a \, x)^3(1 - 4x)^3\) in ascending powers of \(x\) up to and including the term containing \(x^2\).

Given that the first two terms in the above expansion are \(p + qx^2\), where \(p\) and \(q\) are constants, find the value of \(p\) and of \(q\).

2 (i) Given that \(u = 4^x\), express \(4^x - 3(4^{1-x}) = 11\) as an equation in \(u\).

(ii) Hence find the value(s) of \(x\) for which \(4^x - 3(4^{1-x}) = 11\).

(iii) Given that \(p > 0\), determine the number of real roots in the equation \(4^x - 3(4^{1-x}) = p\). \textit{Show your working clearly.}

3 (i) Show that \(\frac{1}{\csc x - 1} - \frac{1}{\csc x + 1} = 2 \tan^2 x\).

(ii) Hence solve \(\frac{1}{\csc x - 1} - \frac{1}{\csc x + 1} = 4 + \sec x\) for \(0^\circ < x < 360^\circ\).

4 A curve has the equation \(y = \frac{2x - 7}{x - 1} - 20x\).

(i) Obtain an expression for \(\frac{dy}{dx}\).

(ii) Determine the values of \(x\) for which \(y\) is a decreasing function.

The variables are such that, when \(x = 2\), \(y\) is decreasing at the rate of 1.5 units per second.

(iii) Find the rate of change of \(x\) when \(x = 2\).

It is given further that the variable \(z\) is such that \(z = \frac{2}{y}\).

(iv) Find the rate of change of \(z\) when \(x = 2\).

5 It is given that \(f(x) = (kx + 1)(x^2 - 3x + k)\).

(a) (i) Find the value(s) of \(k\) if \(3 - x\) is a factor of \(f(x)\).

(ii) For the values(s) of \(k\) found in (i), write down an expression for \(f(x)\) with \((3 - x)\) as a factor.

(b) Find the smallest integer value of \(k\) such that there is only one real solution for \((kx + 1)(x^2 - 3x + k) = 0\).
6 The table below shows values of the variables \( x \) and \( y \) which are related by the equation \( ay = x (1 - bx) \) where \( a \) and \( b \) are constants. One of the values of \( y \) is believed to be inaccurate.

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>3.5</th>
<th>4.5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>5.0</td>
<td>9.1</td>
<td>14.0</td>
<td>21.0</td>
<td>26.3</td>
</tr>
</tbody>
</table>

(i) Plot \( \frac{y}{x} \) against \( x \) and draw a straight line graph. [3]

(ii) Determine which value of \( y \) is inaccurate and estimate its correct value. [2]

(iii) Estimate the value of \( a \) and \( b \). [4]

An alternative method for obtaining straight line graph for the equation \( ay = x (1 - bx) \) is to plot \( x \) on the vertical axis and \( \frac{y}{x} \) on the horizontal axis.

(iv) Without drawing a second graph, use your values of \( a \) and \( b \) to estimate the gradient and intercept on the vertical axis of the graph of \( x \) plotted against \( \frac{y}{x} \). [3]

7 The roots of the quadratic equation \( x^2 - 4x + 2 = 0 \) are \( \alpha \) and \( \beta \).

(i) Find the exact value of \( \alpha - \beta \) if \( \alpha < \beta \). [4]

(ii) Form a quadratic equation with roots \( \frac{\alpha - 1}{\beta} \) and \( \frac{\beta - 1}{\alpha} \). [5]

8

The diagram shows the curve \( y = (5 - 2x)^3 \) and the tangent to the curve at the point \( P(1, 27) \).

(i) Find the equation of the tangent to the curve at \( P \). [4]

(ii) Find the area of the shaded region. [5]
9 A particle moves in a straight line so that \( t \) seconds after leaving a fixed point \( O \), its velocity, \( v \) m s\(^{-1}\), is given by \( v = 2 \left( 3 - e^{-t/2} \right) \).

(i) Find the initial velocity of the particle. \([1]\)

(ii) Find the acceleration of the particle when \( v = 5 \). \([3]\)

(iii) Calculate the displacement of the particle from \( O \) when \( t = 10 \). \([3]\)

(iv) Does the particle reverse its direction of motion? Justify your answer with working clearly shown. \([2]\)

10 The diagram shows a point \( C \) on a circle and line \( ACB \) is a tangent to the circle. Points \( F, G \) and \( H \) lie on the circle such that \( FH \) is parallel to \( AB \). The lines \( GC \) and \( FH \) intersect at \( E \).

(i) Prove that triangles \( ECF \) and \( FCG \) are similar.

Hence show that \( EC \times CG = CF^2 \). \([4]\)

(ii) By using similar triangles, show that \( FE \times EH = CF^2 - EC^2 \). \([5]\)

![Diagram](image)

11 The equation of a circle, \( C_1 \), with centre \( A \), is given by \( x^2 + y^2 + 4x + 6y - 12 = 0 \).

(i) Find the coordinates of \( A \) and the radius of \( C_1 \). \([2]\)

Given that the circle passes through a point \( P(-5, -7) \) and a point \( Q \) such that \( PQ \) is the diameter of the circle

(ii) write down the coordinates of \( Q \). \([2]\)

The tangent to the circle at point \( Q \) intersects the x-axis at point \( R \).

A second circle, \( C_2 \), centre \( B \), is drawn passing through \( A, Q \) and \( R \).

(iii) Find the coordinates of \( R \). \([3]\)

(iv) Determine the coordinates of the centre, \( B \) and the radius of \( C_2 \). \([4]\)
1 \( (1 + ax)^4(1 - 4x)^3 = 1 + (4a - 12)x + (48 - 48a + 6a^2)x^2 \)
\( p = 1 \quad a = 3 \quad q = -42 \)

2 (i) \( u - \frac{12}{u} = 11 \)  
(ii) \( x = 1.79, \ 4^x = -1 \) (no real solution)
(iii) \( u - \frac{12}{u} = p \)
\( u^2 - pu - 12 = 0 \)
\( u = \frac{p + \sqrt{p^2 + 48}}{2} \quad \text{or} \quad \frac{p - \sqrt{p^2 + 48}}{2} \)
\( 4^x = \frac{p + \sqrt{p^2 + 48}}{2} \quad \text{or} \quad 4^x = \frac{p - \sqrt{p^2 + 48}}{2} \)
Since \( \frac{p + \sqrt{p^2 + 48}}{2} > 0 \), \( 4^x > 0 \) and there is real solution for \( x \).
Since \( \frac{p - \sqrt{p^2 + 48}}{2} < 0 \), \( 4^x < 0 \) and there is NO real solution for \( x \).
Number of real solutions = 1

3 (i) \( \text{L.H.S} = \frac{1}{\csc x - 1} - \frac{1}{\csc x + 1} \)
\( = \frac{\csc x + 1 - (\csc x - 1)}{\csc^2 x - 1} \)
\( = \frac{2}{\csc^2 x - 1} \)
\( = \frac{2}{\cot^2 x - 1} \)
\( = 2 \tan^2 x \)
(ii) \( x = 30^\circ, 131.8^\circ, 228.2^\circ, 300^\circ \)

4 (i) \( \frac{dy}{dx} = \frac{5}{(x - 1)^2} - 20 \)
(ii) \( x < \frac{1}{2} \quad \text{or} \quad x > \frac{3}{2} \)
(iii) \( \frac{dx}{dt} = 0.1 \text{ units/s} \)
(iv) \( \frac{dz}{dy} = -\frac{2}{y^2} \)
When \( x = 2, y = -43 \)
\( \frac{dz}{dt} = \frac{dz}{dy} \cdot \frac{dy}{dt} = 1.62 \times 10^{-3} \text{ units/s} \)

5 (a) (i) \( k = 0, \ k = -\frac{1}{3} \)
(ii) When \( k = 0 \), \( f(x) = -x(3 - x) \)
\( \text{When} \ k = -\frac{1}{3}, \ f(x) = \frac{1}{3}(3 - x)(x^2 - 3x - \frac{1}{3}) \)
(b) \( x^2 - 3x + k = 0 \)
No real solution when \( b^2 - 4ac < 0 \), \( k > 2\frac{1}{4} \). Smallest integer value of \( k \) is 3.
6  (ii) Inaccurate value of $y = 9.1$

Correct value of $\frac{y}{x} = 2.9$. When $x = 3.5$, correct value of $y = 2.9 \times 3.5 = 10.15$

(iii) Equation is $\frac{y}{x} = \frac{1}{a} - \frac{b}{a}x$

From graph, $\frac{1}{a} = 2$, $a = \frac{1}{2}$

$\frac{-b}{a} = 0.25$, $b = -0.125$

(iv) Equation is $x = \frac{1}{b} - a \left(\frac{y}{x}\right)$

Gradient $= -\frac{a}{b} = 4$

Intercept on vertical axis $= \frac{1}{b} = -8$

7  (i) $\alpha - \beta = -\sqrt{3}$ (given $\alpha < \beta$)

(ii) $\frac{\alpha - 1}{\beta} + \frac{\beta - 1}{\alpha} = 4$, $\left(\frac{\alpha - 1}{\beta}\right) \left(\frac{\beta - 1}{\alpha}\right) = -\frac{1}{2}$

Equation is $x^2 - 4x - \frac{1}{2} = 0$ or $2x^2 - 8x - 1 = 0$

8  (i) $\frac{dy}{dx} = -6(5 - 2x)^2$, equation of tangent is $y = -54x + 81$

(ii) Shaded area $= \int_{0}^{1} (5-2x)^3 dx - \int_{0}^{1} (-54x + 81) dx = 68 - 54 = 14$ units$^2$

9  (i) $v = 4 \text{ m s}^{-1}$

(ii) $a = \frac{1}{2} \text{ m s}^{-2}$

(iii) $s = 6t + 4e^{-t/2} - 4 = 56.0 \text{ m}$ (when $t = 10$ s)

(iv) When $v = 0$, $t = -2.20 \text{ s}$. Since time cannot have a negative value, the particle did not reverse its direction of motion.

10  (i) $\angle ACF = \angle FGC$ (alternate segment theorem/tangent-chord theorem)

$\angle ACF = \angle EFC$ (alternate angles)

$\therefore \angle FGC = \angle EFC$

$\angle EFC = \angle FCG$ (common angle)

$\Delta ECF$ and $\Delta FCG$ are similar triangles (AA similarity test)

$\frac{EC}{CF} = \frac{CF}{CG}$

$(EC)(CG) = (CF)^2$

(ii) $\angle GEF = \angle HEC$ (vertically opposite angles)

$\angle FGE = \angle CHE$ (angles in same segment)

$\Delta FGE$ and $\Delta CHE$ are similar triangles (AA similarity test)

$\frac{FE}{EG} = \frac{EC}{EH}$

$(FE)(EH) = (EG)(EC) = (CG - EC)(EC) = (CG)(EC) - (EC)^2 = CF^2 - EC^2$ $[(EC)(CG) = (CF)^2$ in (i)]

11  (i) Centre, $A = (−2, −3)$, radius = 5 units

(ii) $Q(1, 1)$

(iii) $R\left(\frac{2}{3}, 0\right)$

(iv) $B\left(\frac{1}{6}, -\frac{3}{2}\right)$, radius = 2.64 units
CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2018
SECONDARY 4
ADDITIONAL MATHEMATICS 4047/01
Paper 1
17 September 2018
2 hours

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number clearly on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [_____] at the end of each question or part question.
The total number of marks for this paper is 80.
1. **ALGEBRA**

**Quadratic Equation**

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

**Binomial expansion**

\((a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1) \ldots (n-r+1)}{r!} \).

2. **TRIGONOMETRY**

**Identities**

\[
sin^2 A + cos^2 A = 1
\]

\[
sec^2 A = 1 + tan^2 A
\]

\[
cosec^2 A = 1 + cot^2 A
\]

\[
sin(A \pm B) = sin A \cos B \pm \cos A \sin B
\]

\[
cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
sin 2A = 2 \sin A \cos A
\]

\[
cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]

\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

**Formulae for \( \Delta ABC \)**

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2} ab \sin C
\]
1  A curve is such that $\frac{d^2y}{dx^2} = ax - 2$, where $a$ is a constant. The curve has a minimum gradient at $x = \frac{1}{3}$.

(i)  Show that $a = 6$.  

The tangent to the curve at $(1, 4)$ is $y = 2x + 2$.

(ii)  Find the equation of the curve.

2  The roots of the quadratic equation $3x^2 + 2x + 4 = 0$ are $\alpha$ and $\beta$.

(i)  Show that $\alpha^2 + \beta^2 = -\frac{20}{9}$.  

(ii)  Find a quadratic equation with roots $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$.

3  It is given that $f(x) = (x + h)^2(x - 1) + k$, where $h$ and $k$ are constants and $h < k$. When $f(x)$ is divided by $x + h$, the remainder is 6. It is given that $f(x)$ is exactly divisible by $x + 5$.

(i)  State the value of $k$ and show that $h = 4$.  

(ii)  Find the range of values of the constant $b$ for which the graph of $y = f(x) + bx$ is an increasing function for all values of $x$.

4  Given that $\tan(x + y) = -\frac{120}{119}$ and $\cos x = \frac{5}{13}$, where $x$ and $y$ are acute angles, show that $x = y$ without finding the values of $x$ and $y$.

5  The variables $x$ and $y$ are such that when $\frac{x}{y}$ are plotted against $x$, a straight line $l_1$ of gradient 2 is obtained. It is given that $y = \frac{1}{5}$ when $x = 3$.

(i)  Express $y$ in terms of $x$.

(ii)  When the graph of $x = 2y$ is plotted on the same axes as the line $l_1$, the two lines intersect at one point. Find the coordinates of the point of intersection.
6 The figure shows a semicircle of radius 5 cm and centre, \( O \). A rectangle \( ABCD \) is inscribed in the semicircle such that the four vertices \( A, B, C \) and \( D \) touch the edge of the semicircle. The length of \( AB = x \) cm.

(i) Show that the perimeter, \( P \) cm, of rectangle \( ABCD \) is given by

\[
P = 2x + 4\sqrt{25 - x^2}
\]

(ii) Given that \( x \) can vary, find the value of \( x \) when the perimeter is stationary.

7 In the diagram below, \( PQRS \) is a trapezium where angle \( QRS = \) angle \( TPR = 30^\circ \). \( SQ \) is the height of the trapezium and the length of \( SQ \) is \( \frac{4}{\sqrt{3}+1} \) cm. The length of \( TS \) is \( 2\sqrt{3} \) cm.

By rationalising \( \frac{4}{\sqrt{3}+1} \), find the area of trapezium \( PQRS \) in the form \( (a\sqrt{3} - 12) \) cm\(^2\), where \( a \) is an integer.

8 A particle moving in a straight line passes a fixed point \( A \) with a velocity of \( -8 \) cms\(^{-1}\). The acceleration, \( a \) cms\(^{-2}\) of the particle, \( t \) seconds after passing \( A \) is given by \( a = 10 - kt \), where \( k \) is a constant. The particle first comes to instantaneous rest at \( t = 1 \) and reaches maximum speed at \( T \) seconds (The particle does not come instantaneously to rest at \( 1 < t < T \)).

(i) Find the value of \( k \).

(ii) Find the total distance travelled by the particle when \( t = T \).
9 It is given that \( y = 1 - 3\sin 2x \) for \( -\frac{\pi}{2} \leq x \leq \pi \).

(i) State the period of \( y \). \([1]\)

(ii) Sketch the graph of \( y = 1 - 3\sin 2x \). \([3]\)

(iii) By drawing a straight line on the same diagram as in part (ii), find the number of solutions to the equation \( 3\sin 2x + 1 = \frac{3x}{\pi} \) for \( -\frac{\pi}{2} \leq x \leq \pi \). \([3]\)

10 The diagram shows part of the curve \( y = 4e^{0.5x-1} \). The normal to the curve at point \( A(2, 4) \) cuts the x-axis at point \( B \).

Find

(i) the coordinates of \( B \), \([4]\)

(ii) the area of the shaded region. \([3]\)
In the diagram, $BD$ and $AC$ are chords of the circle. $FD$ is a tangent to the circle at $D$. $AC$ and $FD$ are produced to meet at $G$ such that $CG = CD$. $E$ is a point along $BD$. Triangle $BAE$ is similar to triangle $ADE$.

(i) By showing that triangle $BAD$ and triangle $AED$ are similar, prove that $AB$ is perpendicular to $AD$. [4]

(ii) Show that angle $ADB = 90^\circ - 2 \times (\text{angle } CGD)$. [4]

12 The line $y = \frac{3}{4}x + 19\frac{1}{4}$ is a tangent to the circle, centre $C$. Another line, $l_1$ is tangent to the circle at point $P\left(\frac{2}{5}, \frac{124}{5}\right)$. The two tangents intersect at point $R$, which is directly above the centre of the circle.

(i) Show that the coordinates of $R$ are $\left(5, \frac{151}{2}\right)$. [4]

(ii) Find the equation of the circle. [4]
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(i)</td>
<td>Show question</td>
</tr>
<tr>
<td>2</td>
<td>(ii)</td>
<td>$y = x^3 - x^2 + x + 3$</td>
</tr>
<tr>
<td>2</td>
<td>(i)</td>
<td>Show question</td>
</tr>
<tr>
<td>2</td>
<td>(ii)</td>
<td>$x^2 - \frac{16}{9}x + \frac{4}{3} = 0$ or any other equivalent equation</td>
</tr>
<tr>
<td>3</td>
<td>(i)</td>
<td>$k = 6; h = 4$ (show question)</td>
</tr>
<tr>
<td>3</td>
<td>(ii)</td>
<td>$b &gt; \frac{1}{3}$</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>Show question</td>
</tr>
<tr>
<td>5</td>
<td>(i)</td>
<td>$y = \frac{x}{2x + 9}$</td>
</tr>
<tr>
<td>5</td>
<td>(ii)</td>
<td>$\left(-\frac{3}{2}, 2\right)$</td>
</tr>
<tr>
<td>6</td>
<td>(i)</td>
<td>Show question</td>
</tr>
<tr>
<td>6</td>
<td>(ii)</td>
<td>$x = \sqrt{5}$ or 2.24 (3 s.f.)</td>
</tr>
<tr>
<td>7</td>
<td>(i)</td>
<td>$\left(12\sqrt{3} - 12\right)$ cm$^2$</td>
</tr>
<tr>
<td>8</td>
<td>(i)</td>
<td>$k = 4$</td>
</tr>
<tr>
<td>8</td>
<td>(ii)</td>
<td>$8\frac{1}{6}$ m</td>
</tr>
<tr>
<td>9</td>
<td>(i)</td>
<td>$\pi$</td>
</tr>
<tr>
<td>9</td>
<td>(ii)</td>
<td>$y = \frac{1}{2} - \frac{3x}{\pi}$</td>
</tr>
<tr>
<td>9</td>
<td>(iii)</td>
<td>3 solutions</td>
</tr>
<tr>
<td>10</td>
<td>(i)</td>
<td>$B(10, 0)$</td>
</tr>
<tr>
<td>10</td>
<td>(ii)</td>
<td>$\left(24 - \frac{8}{e}\right)$ units$^2$ or 21.1 units$^2$ (3 s.f)</td>
</tr>
<tr>
<td>11</td>
<td>(i), (ii)</td>
<td>Show question</td>
</tr>
<tr>
<td>12</td>
<td>(ii)</td>
<td>$(x - 5)^2 + (y - 8)^2 = 36$</td>
</tr>
</tbody>
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1. **ALGEBRA**

**Quadratic Equation**

For the equation \( ax^2 + bx + c = 0 \),

\[
 x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

**Binomial expansion**

\[ (a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n, \]

where \( n \) is a positive integer and 

\[
 \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1) \ldots (n-r+1)}{r!}.
\]

2. **TRIGONOMETRY**

**Identities**

\[
 \sin^2 A + \cos^2 A = 1
\]

\[
 \sec^2 A = 1 + \tan^2 A
\]

\[
 \csc^2 A = 1 + \cot^2 A
\]

\[
 \sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
 \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
 \tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
 \sin 2A = 2 \sin A \cos A
\]

\[
 \cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]

\[
 \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

**Formulae for \( \triangle ABC \)**

\[
 \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
 a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
 \Delta = \frac{1}{2} ab \sin C
\]
1. A curve is such that \( \frac{d^2y}{dx^2} = ax - 2 \), where \( a \) is a constant. The curve has a minimum gradient at \( x = \frac{1}{3} \).

(i) Show that \( a = 6 \). \[1\]

The tangent to the curve at \((1, 4)\) is \( y = 2x + 2 \).

(ii) Find the equation of the curve. \[6\]

Marking Scheme

(i) At minimum gradient, \( \frac{d^2y}{dx^2} = 0 \)

\[ a \left( \frac{1}{3} \right) - 2 = 0 \]
\[ \frac{a}{3} = 2 \]
\[ a = 6 \]

(ii) \( \frac{dy}{dx} = \int (6x - 2) \, dx \)

\[ = 3x^2 - 2x + c \quad \text{where } c \text{ is an arbitrary constant} \]

\[ y = 2x + 2 \]

Gradient of tangent = 2

\[ 3(1)^2 - 2(1) + c = 2 \]
\[ c = 1 \]

\[ y = \int (3x^2 - 2x + 1) \, dx \]

\[ = x^3 - x^2 + x + c_1 \quad \text{where } c_1 \text{ is an arbitrary constant} \]

Sub. \((1, 4)\)

\[ 4 = 1^3 - 1^2 + 1 + c_1 \]
\[ c_1 = 3 \]

Equation of curve is \( y = x^3 - x^2 + x + 3 \)
2. The roots of the quadratic equation \(3x^2 + 2x + 4 = 0\) are \(\alpha\) and \(\beta\).

(i) Show that \(\alpha^2 + \beta^2 = -\frac{20}{9}\). [3]

(ii) Find a quadratic equation with roots \(\frac{\alpha^2}{\beta}\) and \(\frac{\beta^2}{\alpha}\). [4]

Marking Scheme

(i) \(\alpha + \beta = -\frac{2}{3}\)
\(\alpha\beta = \frac{4}{3}\)

\(\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta\)
\(= \left(-\frac{2}{3}\right)^2 - 2\left(\frac{4}{3}\right)\)
\(= \frac{4}{9} - \frac{8}{3}\)
\(= -\frac{20}{9}\) (shown)

(ii) Sum of roots \(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\)

\(\alpha^2 \beta^2 = \left(-\frac{2}{3}\right)^2 - 2\left(\frac{4}{3}\right)\)
\(= \frac{4}{9} - \frac{8}{3}\)
\(= -\frac{20}{9}\) (shown)

Product of roots \(\frac{\alpha^2}{\beta} \div \frac{\beta^2}{\alpha}\)

\(= \alpha\beta\)
\(= \frac{4}{3}\)

The quadratic equation is \(x^2 - \frac{16}{9}x + \frac{4}{3} = 0\)

OR \(9x^2 - 16x + 12 = 0\)
3. It is given that \( f(x) = (x + h)^2(x - 1) + k \), where \( h \) and \( k \) are constants and \( h < k \). When \( f(x) \) is divided by \( x + h \), the remainder is 6. It is given that \( f(x) \) is exactly divisible by \( x + 5 \).

(i) State the value of \( k \) and show that \( h = 4 \).

(ii) Find the range of values of the constant \( b \) for which the graph of \( y = f(x) + bx \) is an increasing function for all values of \( x \).

Marking Scheme

(i) \( k = 6 \) \[ B1 \]

\[
\begin{align*}
f(-5) &= 0 \\
(-5 + h)^2(-5 - 1) + 6 &= 0 \\
(h - 5)^2(-6) &= -6 \\
(h - 5)^2 &= 1 \\
h - 5 &= -1 \text{ or } 1 \\
h &= 4 \text{ or } 6 \text{ (rejected as } h < k) \\
\end{align*}
\]

(ii) \( y = (x + 4)^2(x - 1) + 6 + bx \)

\[
\begin{align*}
\frac{dy}{dx} &= 2(x + 4)(x - 1) + (x + 4)^2 + b \\
&= (x + 4)[2(x - 1) + (x + 4)] + b \\
&= (x + 4)(3x + 2) + b
\end{align*}
\]

For increasing function, \( \frac{dy}{dx} > 0 \)

\[
\begin{align*}
(x + 4)(3x + 2) + b &> 0 \\
3x^2 + 14x + 8 + b &> 0 \\
\text{Discriminant } &< 0 \\
(14)^2 - 4(3)(8 + b) &< 0 \\
196 - 96 - 12b &< 0 \\
12b &> 100 \\
b &> 8 \frac{1}{3}
\end{align*}
\]
4. Given that \( \tan(x + y) = -\frac{120}{119} \) and \( \cos x = \frac{5}{13} \), where \( x \) and \( y \) are acute angles, show that \( x = y \) \textbf{without} finding the values of \( x \) and \( y \). \[4\]

**Marking Scheme**

\[
\tan(x + y) = -\frac{120}{119}
\]

\[
\frac{\tan x + \tan y}{1 - \tan x \tan y} = -\frac{120}{119}
\]

\[
\tan x = \frac{12}{5}
\]

\[
\frac{12}{5} + \tan y = -\frac{120}{119}
\]

\[
\frac{12}{5} + \tan y = -\frac{120}{119} + \frac{288}{119} \tan y
\]

\[
\frac{2028}{595} = \frac{169}{119} \tan y
\]

\[
\tan y = \frac{12}{5}
\]

Since \( \tan x = \tan y \) and \( x \) and \( y \) are both acute, \( x = y \).
5. The variables $x$ and $y$ are such that when $\frac{x}{y}$ are plotted against $x$, a straight line $l_1$ of gradient 2 is obtained. It is given that $y = \frac{1}{5}$ when $x = 3$.

(i) Express $y$ in terms of $x$. [3]

(ii) When the graph of $x = 2y$ is plotted on the same axes as the line $l_1$, the two lines intersect at one point. Find the coordinates of the point of intersection. [2]

Marking Scheme

(ii) \[
\frac{x}{y} = 2x + c
\]

\[
\frac{3}{1} = 2(3) + c
\]

\[
c = 9
\]

\[
\frac{x}{y} = 2x + 9
\]

\[
\frac{y}{x} = \frac{1}{2x + 9}
\]

\[
y = \frac{x}{2x + 9}
\]

(iii) \[x = 2y \Rightarrow \frac{x}{y} = 2\]

\[2x + 9 = 2\]

\[x = -3 \frac{1}{2}\]

The point of intersection is \((-3 \frac{1}{2}, 2)\).
6. The figure shows a semicircle of radius 5 cm and centre, $O$. A rectangle $ABCD$ is inscribed in the semicircle such that the four vertices $A$, $B$, $C$ and $D$ touch the edge of the semicircle. The length of $AB = x$ cm.

(i) Show that the perimeter, $P$ cm, of rectangle $ABCD$ is given by

$$P = 2x + 4\sqrt{25 - x^2}$$

(ii) Given that $x$ can vary, find the value of $x$ when the perimeter is stationary.

Marking Scheme

(i) $OB = 5$ cm (radius of circle)  

$OB^2 = OA^2 + AB^2$  

$25 = OA^2 + x^2$  

$OA = \sqrt{25 - x^2}$  

$P = AB + CD + AD + BC$  

$= 2AB + 4OA$  

$= 2x + 4\sqrt{25 - x^2}$ (shown)

(ii) $P = 2x + 4\sqrt{25 - x^2}$  

$$\frac{dP}{dx} = 2 + 4 \left( \frac{1}{2} \right) (25 - x^2)^{-\frac{1}{2}} (-2x)$$  

$$= 2 - \frac{4x}{\sqrt{25 - x^2}}$$

At stationary $P$, $\frac{dP}{dx} = 0$

$$2 - \frac{4x}{\sqrt{25 - x^2}} = 0$$  

$$\frac{4x}{\sqrt{25 - x^2}} = 2$$  

$$16x^2 = 4$$  

$$4x^2 = 25 - x^2$$  

$$5x^2 = 25$$  

$$x^2 = 5$$  

$$x = \sqrt{5} \text{ or } -\sqrt{5} \text{ (rejected)}$$

or 2.24 (3 s.f)
7. In the diagram below, \( PQRST \) is a trapezium where angle \( QRS = \angle TPR = 30^\circ \). \( SQ \) is the height of the trapezium and the length of \( SQ \) is \( \frac{4}{\sqrt{3}+1} \) cm. The length of \( TS \) is \( 2\sqrt{3} \) cm.

By rationalising \( \frac{4}{\sqrt{3}+1} \), find the area of trapezium \( PQRST \) in the form \( (a\sqrt{3} - 12) \) cm\(^2\), where \( a \) is an integer. \[5\]

Marking Scheme

\[
\frac{4}{\sqrt{3}+1} = \frac{4}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}
\]

\[= \frac{4\sqrt{3} - 4}{3-1}\]

\[= 2\sqrt{3} - 2\]

\[\tan 30^\circ = \frac{2\sqrt{3} - 2}{QR}\]

\[\frac{1}{\sqrt{3}} = \frac{2\sqrt{3} - 2}{QR}\]

\[QR = 2(3) - 2\sqrt{3}\]

\[= (6 - 2\sqrt{3}) \text{ cm}\]

Area of trapezium = \[\frac{1}{2} \left[ 2(6 - 2\sqrt{3}) + 2(2\sqrt{3}) \right] \left( 2\sqrt{3} - 2 \right)\]

\[= \frac{1}{2} \left( 12 - 4\sqrt{3} + 4\sqrt{3} \right) \left( 2\sqrt{3} - 2 \right)\]

\[= \frac{1}{2} \left( 12 \right) \left( 2\sqrt{3} - 2 \right)\]

\[= 6 \left( 2\sqrt{3} - 2 \right)\]

\[= (12\sqrt{3} - 12) \text{ cm}^2\]
8. A particle moving in a straight line passes a fixed point $A$ with a velocity of $-8$ cm$^{-1}$. The acceleration, $a$ cm$^{-2}$ of the particle, $t$ seconds after passing $A$ is given by $a = 10 - kt$, where $k$ is a constant. The particle first comes to instantaneous rest at $t = 1$ and reaches maximum speed at $T$ seconds (The particle does not come instantaneous to rest at $1 < t < T$).

(i) Find the value of $k$. [3]

(ii) Find the total distance travelled by the particle when $t = T$. [5]

Marking Scheme

(i) $a = 10 - kt$

$v = \int (10 - kt) \, dt$

$= 10t - \frac{kt^2}{2} + c$ where $c$ is an arbitrary constant

When $t = 0$, $v = -8$

$-8 = c$

$\therefore v = 10t - \frac{kt^2}{2} - 8$

When $t = 1$, $v = 0$

$0 = 10 - \frac{k}{2} - 8$

$k = 4$

(ii) $a = 10 - 4t$

At maximum speed, $a = 0$

$10 - 4t = 0$

$t = 2 \frac{1}{2}$

$s = \int (10t - 2t^2 - 8) \, dt$

$= 5t^2 - \frac{2t^3}{3} - 8t + c_i$ where $c_i$ is an arbitrary constant

When $t = 0$, $s = 0$, $c_i = 0$

$\therefore s = 5t^2 - \frac{2t^3}{3} - 8t$

When $t = 0$, $s = 0$
When $t = 1$, $s = -\frac{11}{3}$

When $t = 2\frac{1}{2}$, $s = \frac{5}{6}$

Total distance travelled = \left(\frac{11}{3}\right) \times 2 + \frac{5}{6}

= 8\frac{1}{6} \text{ m}
9. It is given that \( y = 1 - 3\sin 2x \) for \( \frac{-\pi}{2} \leq x \leq \pi \).

(i) State the period of \( y \). [1]

(ii) Sketch the graph of \( y = 1 - 3\sin 2x \). [3]

(iii) By drawing a straight line on the same diagram as in part (ii), find the number of solutions to the equation \( 3\sin 2x + \frac{1}{2} = \frac{3x}{\pi} \) for \( \frac{-\pi}{2} \leq x \leq \pi \). [3]

Marking Scheme

(i) 180° or \( \pi \)

(ii)

\[
\begin{align*}
3\sin 2x + \frac{1}{2} & = \frac{3x}{\pi} \\
3\sin 2x & = \frac{3x}{\pi} - \frac{1}{2} \\
3\sin 2x - 1 & = \frac{3x}{\pi} - \frac{1}{2} \\
1 - 3\sin 2x & = 2 - \frac{3x}{\pi}
\end{align*}
\]

Draw the line of \( y = 2 - \frac{3x}{\pi} \).

From the graph, there are 3 points of intersections, thus there are 3 solutions.
The diagram shows part of the curve \( y = 4e^{0.5x-1} \). The normal to the curve at point \( A(2, 4) \) cuts the \( x \)-axis at point \( B \).

Find

(i) the coordinates of \( B \), 

(ii) the area of the shaded region.

**Marking Scheme**

(i) \( y = 4e^{0.5x-1} \)

\[
\frac{dy}{dx} = 4(0.5)e^{0.5x-1} = 2e^{0.5x-1}
\]

When \( x = 2 \), \( \frac{dy}{dx} = 2 \)

Gradient of normal = \(- \frac{1}{2} \)

Let \( B(x, 0) \).

\[
\frac{4-0}{2-x} = -\frac{1}{2}
\]

\[
8 = -2 + x
\]

\[
x = 10
\]

\[
\therefore B(10,0)
\]
(ii) Area of shaded region = \[ \int_{0}^{2} 4e^{0.5x-1} \, dx + \frac{1}{2}(10 - 2)(4) \]
\[ = \left[ \frac{4e^{0.5x-1}}{0.5} \right]_0^2 + 16 \]
\[ = 8e^0 - 8e^{-1} + 16 \]
\[ = \left( 24 - \frac{8}{e} \right) \text{ units}^2 \text{ or } 21.1 \text{ units}^2 \text{ (3 s.f)} \]
11.

In the diagram, \( BD \) and \( AC \) are chords of the circle. \( FD \) is a tangent to the circle at \( D \). \( AC \) and \( FD \) are produced to meet at \( G \) such that \( CG = CD \). \( E \) is a point along \( BD \). Triangle \( BAE \) is similar to triangle \( ADE \).

(i) By showing that triangle \( BAD \) and triangle \( AED \) are similar, prove that \( AB \) is perpendicular to \( AD \). [4]

(ii) Show that angle \( ADB = 90^\circ - 2 \times (\text{angle } CGD) \) [4]

Marking Scheme

(i) \( \angle ABE = \angle DAE \) (corresponding angles of similar triangles \( BAE \) and \( ADE \))

\( \angle ADE = \angle BDA \) (common angle)

By AA similarity rule, triangles \( BAD \) and \( AED \) are similar.

\( \angle BEA = \angle AED \) (corresponding angles of similar triangles \( BAE \) and \( ADE \))

\[ = 90^\circ \] (adjacent \( \angle \)'s on straight line)

\( \therefore \angle BAD = \angle AED \) (corresponding angles of similar triangles \( BAD \) and \( AED \))

\[ = 90^\circ \]

\( AB \perp AD \) (shown)
(ii) Let \( \angle CGD = a \).

\[ \angle CDG = \angle CGD \quad \text{(base } \angle \text{s of isosceles } \Delta \text{)} \]

\[ = a \]

\( BD \) is a diameter (right-angle in a semicircle)

\[ \therefore \angle EDG = 90' \quad \text{(tangent } \perp \text{ radius)} \]

\[ \angle DAC = \angle CDG \quad \text{(} \angle \text{s in alternate segment)} \]

\[ = a \]

Consider \( \triangle ADG \),

\[ \angle ADB = 180' - \angle DAC - \angle CGD - \angle EDG \quad \text{(sum of } \angle \text{s in } \Delta \text{)} \]

\[ = 180' - a - a - 90' \]

\[ = 90' - 2a \]

\[ = 90' - 2 \times \angle CGD \quad \text{(shown)} \]
12. The line $y = -\frac{3}{4}x + 19\frac{1}{4}$ is a tangent to the circle, centre $C$. Another line, $l_1$ is tangent to the circle at point $P\left(\frac{2}{5}, \frac{12}{5}\right)$. The two tangents intersect at point $R$, which is directly above the centre of the circle.

(i) Show that the coordinates of $R$ are $\left(5, 15\frac{1}{2}\right)$. [4]

(ii) Find the equation of the circle. [4]
Marking Scheme

1. $y$-coordinate of $S = \frac{12}{5} + \frac{4}{5}$

$\frac{12}{5} + \frac{4}{5} = -\frac{3}{4}x + 19\frac{1}{4}$

$x = 8\frac{3}{5}$

$\therefore S\left(8\frac{3}{5}, 12\frac{4}{5}\right)$

$x_c = \frac{8\frac{3}{5} + \frac{2}{5}}{2}$

$= 5$

$y = -\frac{3}{4}(5) + 19\frac{1}{4}$

$= 15\frac{1}{2}$

$\therefore R\left(5, 15\frac{1}{2}\right)$ (shown)
Alternative Method

Gradient of \( l_1 = \frac{3}{4} \)

Equation of \( l_1 \) is \( y - \frac{4}{5} = \frac{3}{4} \left( x - \frac{2}{5} \right) \) \quad (1)

\[ y = -\frac{3}{4} x + 19 \frac{1}{4} \quad (2) \]

Sub. (2) into (1),

\[ -\frac{3}{4} x + 19 \frac{1}{4} - \frac{12}{5} = \frac{3}{4} \left( x - \frac{2}{5} \right) \]

\[ -\frac{3}{4} x + \frac{129}{20} = \frac{3}{4} x - \frac{21}{20} \]

\[ -\frac{3}{2} x = -\frac{15}{2} \]

\( x = 5 \) sub. into (2)

\[ y = 15 \frac{1}{2} \]

\[ \therefore R \left( 5, 15 \frac{1}{2} \right) \quad (shown) \]
(ii) Gradient of normal at \( S = \frac{4}{3} \)

Equation of normal is \( y - 12 = \frac{4}{5} \left( x - \frac{3}{5} \right) \)

When \( x = 5 \),
\[
y - 12 = \frac{4}{5} \left( 5 - \frac{3}{5} \right)
\]
y = 8
\[
\therefore C(5, 8)
\]

Radius = \( \sqrt{\left( 5 - \frac{3}{5} \right)^2 + \left( 8 - 12 \frac{4}{5} \right)^2} \)

= 6 units

Equation of circle is \( (x - 5)^2 + (y - 8)^2 = 36 \).

Alternative Method

Gradient of normal at \( P = -\frac{4}{3} \)

Gradient of normal at \( P \) is \( y - 12 = \frac{-4}{5} \left( x - \frac{2}{5} \right) \)

Sub. \( x = 5 \),
y = 8

Centre of circle is \( (5, 8) \)

Radius of circle = \( \sqrt{\left( 5 - \frac{2}{5} \right)^2 + \left( 8 - 12 \frac{4}{5} \right)^2} \)

= 6 units

Equation of circle is \( (x - 5)^2 + (y - 8)^2 = 36 \)
PRELIMINARY EXAMINATION 2018
SECONDARY 4
ADDITIONAL MATHEMATICS

4047/02
18 September 2018
2 hours 30 minutes

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number clearly on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of 6 printed pages.
1. ALGEBRA

**Quadratic Equation**

For the equation \( ax^2 + bx + c = 0 \),

\[
    x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

**Binomial expansion**

\[
    (a + b)^n = a^n + \left( \binom{n}{1} \right) a^{n-1} b + \left( \binom{n}{2} \right) a^{n-2} b^2 + \ldots + \left( \binom{n}{n-r} \right) a^{n-r} b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and

\[
    \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1) \ldots (n-r+1)}{r!}.
\]

2. TRIGONOMETRY

**Identities**

\[
    \sin^2 A + \cos^2 A = 1
\]

\[
    \sec^2 A = 1 + \tan^2 A
\]

\[
    \csc^2 A = 1 + \cot^2 A
\]

\[
    \sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
    \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
    \tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
    \sin 2A = 2 \sin A \cos A
\]

\[
    \cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]

\[
    \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

**Formulae for \( \Delta ABC \)**

\[
    \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
    a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
    \Delta = \frac{1}{2} ab \sin C
\]
1 An empty, inverted cone has a height of 600 cm. The radius of the top of the cone is 200 cm.

Water is poured into the cone at a constant rate.

(i) When the depth of the water in the cone is \( h \) cm, find the volume of the water in the cone in terms of \( \pi \) and \( h \).

The water level is rising at a rate of 3 cm per minute when the depth of the water is 120 cm.

(ii) Find the rate at which water is being poured into the cone, leaving your answer in terms of \( \pi \).

2 It is given that \( y = x - \ln(\sec x + \tan x), \ 0 < x < \frac{\pi}{2} \).

(i) Show that \( \frac{dy}{dx} = \sec x \tan x \).

(ii) Hence, express \( \frac{dy}{dx} \) in the form \( a + b \sec x \), where \( a \) and \( b \) are integers.

(iii) Deduce that \( y \) is a decreasing function.

3 (a) Prove that \( \frac{1 + \sin 2x + \cos 2x}{\cos x + \sin x} = 2 \cos x \).

(b) Given that \( \frac{\sec^2 x}{2 \tan^2 x + 1} = \frac{3}{4} \), where \( 180^\circ < x < 270^\circ \), find the exact value of \( \sin x \).

4 (a) Solve, for \( x \) and \( y \), the simultaneous equations

\[
2^x = 8 \left( 2^y \right), \\
\lg (2x + y) = \lg 63 - \lg 3.
\]

(b) Express \( \log_{4\sqrt{6}} y = 3 - \log_2 (y - 6) \) as a cubic equation.

5 (i) Express \( \frac{2x^2 - 7}{(x+1)(x^2 - x - 6)} \) in partial fractions.

(ii) Hence, find \( \int_{-5}^{5} \frac{8x^2 - 28}{(x+1)(x^2 - x - 6)} \, dx \).

2018 Preliminary Exam/CCHMS/Secondary 4/Additional Mathematics/4047/Paper 2
6 (a) (i) Sketch the graph of \( y = [(x-1)(x-5)] \). \[3\]

(ii) Determine the set of values of \( a \) for which the line \( y = a \) intersects the graph of \( y = [(x-1)(x-5)] \) at four points. \[2\]

(b) Find the range of values of \( k \) for which the line \( y = kx - 3 \) does not intersect the curve \( y = 2x^2-6x+5 \). \[4\]

7 (i) Show that \( \frac{d}{dx} \left( \frac{\ln 3x}{2x^2} \right) = \frac{1}{2x^3} - \frac{\ln 3x}{x^3} \). \[3\]

(ii) Hence, integrate \( \frac{\ln 3x}{x^3} \) with respect to \( x \). \[3\]

(iii) Given that the curve \( y = f(x) \) passes through the point \( \left( \frac{1}{3}, \frac{3}{4} \right) \) and is such that \( f'(x) = \frac{\ln 3x}{x^3} \), find \( f(x) \). \[2\]

8 (i) Find the coefficient of \( x^4 \) in the expansion of \( (6-x^2)^5 \left(2x^2+\frac{1}{3}\right) \). \[4\]

(ii) In the expansion of \( (2+x)^n \), the ratio of the coefficients of \( x \) and \( x^2 \) is \( 2:3 \). Find the value of \( n \). \[5\]
9. In the diagram, triangle $ABC$ is a right angle triangle where angle $ACB = \theta$ and $AC = 6$ cm. $R$ is a point on $AB$ and $T$ is the mid-point of $AC$. $RT$ is parallel to $BC$ and $AR$ is a line of symmetry of triangle $AST$.

(a) Show that the perimeter, $P$ cm, of the above diagram is $P = 9 \cos \theta + 3 \sin \theta + 9$. [2]

(b) (i) By expressing $P$ in the form $m + n \cos(\theta - \alpha)$, find the value of $\theta$ for which $P = 15$. [6]

(ii) Hence, state the maximum value of $P$ and find the corresponding value of $\theta$. [3]

10. The diagram shows a quadrilateral $ABCD$ where the coordinates of vertices $A$ and $B$ are $(7,9)$ and $(8,6)$ respectively. Both vertices $C$ and $D$ lie on the line $x = 4$. $AC$ passes through $M$, the midpoint of $BD$.

(i) Given that $AB = AD$, find the coordinates of $C$ and $D$. [7]

(ii) Hence or otherwise, prove that quadrilateral $ABCD$ is a kite. [2]

(iii) Find the area of the kite $ABCD$. [2]
11 (a) The amount of caffeine, $C$ mg, left in the body $t$ hours after drinking a certain cup of coffee is represented by $C = 100e^{-kt}$.

(i) Given that the amount of caffeine left in the body is 20 mg after 2.5 hours, find the value of $k$. \[2\]

(ii) Find the number of hours, correct to 3 significant figures, for half the initial amount of caffeine to be left in the body. \[3\]

(b) The curve $y = ax^4 + bx^3 + 7$, where $a$ and $b$ are constants, has a minimum point at $(1,6)$.

Find

(i) the value of $a$ and of $b$. \[4\]

(ii) the coordinates of the other stationary point on the curve and determine the nature of this stationary point. \[4\]
### Answer Key

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td><strong>1</strong></td>
<td>(i)</td>
<td>[ v = \frac{\pi h^3}{27} ]</td>
</tr>
<tr>
<td></td>
<td>(ii)</td>
<td>4800\pi\text{ cm}^3/\text{min}</td>
</tr>
<tr>
<td><strong>2</strong></td>
<td>(ii)</td>
<td>1 - \sec x</td>
</tr>
<tr>
<td><strong>3</strong></td>
<td>(b)</td>
<td>[ \sin x = -\frac{\sqrt{3}}{3} ]</td>
</tr>
<tr>
<td><strong>4</strong></td>
<td>(a)</td>
<td>[ x = 8, \ y = 5 ]</td>
</tr>
<tr>
<td></td>
<td>(b)</td>
<td>[ y^3 - 6y^2 - 8 = 0 ]</td>
</tr>
<tr>
<td><strong>5</strong></td>
<td>(i)</td>
<td>[ \frac{2x^2 - 7}{(x+1)(x^2-x-6)} = \frac{5}{4(x+1)} + \frac{11}{20(x-3)} + \frac{1}{5(x+2)} ]</td>
</tr>
<tr>
<td></td>
<td>(ii)</td>
<td>2.56</td>
</tr>
<tr>
<td><strong>6</strong></td>
<td>(ai)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(a)</td>
<td>[ 3\sin 3x = -4 ]</td>
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<td></td>
<td>(b)</td>
<td>[ x = 8, \ y = 5 ]</td>
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<tr>
<td><strong>7</strong></td>
<td>(ii)</td>
<td>[ -\frac{1}{4x^2} - \frac{2\ln 3x}{x^2} + c ]</td>
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<tr>
<td></td>
<td>(a)</td>
<td>[ 0 &lt; a &lt; 4 ]</td>
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<tr>
<td></td>
<td>(b)</td>
<td>[ -14 &lt; k &lt; 2 ]</td>
</tr>
<tr>
<td><strong>8</strong></td>
<td>(i)</td>
<td>[ -12440 ]</td>
</tr>
<tr>
<td></td>
<td>(ii)</td>
<td>[ n = 7 ]</td>
</tr>
<tr>
<td><strong>9</strong></td>
<td>(bi)</td>
<td>[ P = 9 + \sqrt{90} \cos(\theta - 18.43495^\circ); \ \theta = 69.2^\circ ]</td>
</tr>
<tr>
<td></td>
<td>(bii)</td>
<td>maximum value of ( P = 9 + \sqrt{90}, ) corresponding value of ( x = 18.4^\circ )</td>
</tr>
<tr>
<td><strong>10</strong></td>
<td>(i)</td>
<td>[ D(4,8), \ C(4,3) ]</td>
</tr>
<tr>
<td></td>
<td>(ii)</td>
<td>Since ( M_{AD} \cdot M_{BD} = -1, ) diagonals ( AC ) and ( BD ) are perpendicular to each other. ( AC ) bisects ( BD. ) [ \therefore \text{quadrilateral } ABCD \text{ is a kite.} ]</td>
</tr>
<tr>
<td></td>
<td>(a)</td>
<td>[ 15 \text{ units}^2 ]</td>
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<tr>
<td><strong>11</strong></td>
<td>(ai)</td>
<td>[ k = 0.644 ]</td>
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<td></td>
<td>(a)</td>
<td>[ 1.08 \text{ hours} ]</td>
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<td></td>
<td>(b)</td>
<td>[ a = 3 \text{ and } b = -4 ]</td>
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<td></td>
<td>(bii)</td>
<td>( (0,7), \text{ point of inflexion} )</td>
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<tr>
<td>Working</td>
<td>Common Issues</td>
<td></td>
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<tr>
<td>---------</td>
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<td></td>
</tr>
<tr>
<td><strong>1 (i)</strong></td>
<td>(ratio of corresponding sides are equal)</td>
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</tr>
</tbody>
</table>
| \[
\frac{h}{600} = \frac{r}{200} \]
| \[
r = \frac{h}{3}
\]
| \[
V = \frac{1}{3} \pi \left( \frac{h}{3} \right)^2 h
\]
| \[
v = \frac{\pi h^3}{27}
\]
| **(ii)** | 
| \[
\frac{dh}{dt} = 3 \text{ cm/s}
\]
| \[
\frac{dV}{dh} = \frac{\pi}{27} (3h^2)
\]
| \[
= \frac{\pi h^2}{9}
\]
| \[
\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}
\]
| \[
= \frac{\pi h^2}{9} \times 3
\]
| \[
= 4800 \pi \text{ cm}^3/\text{min}
\]
<table>
<thead>
<tr>
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</tr>
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<tbody>
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<tr>
<td>( y = x - \ln(\sec x + \tan x) )</td>
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<tr>
<td>( \frac{dy}{dx} = \frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right) )</td>
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<tr>
<td>( = \cos x \left(0 \right) - \left(1 \right)(-\sin x) )</td>
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<tr>
<td>( \cos^2 x )</td>
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<td>( = \frac{\sin x}{\cos^2 x} )</td>
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<td>( = \sec x \tan x )</td>
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<tr>
<td><strong>(ii)</strong></td>
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<tr>
<td>( \frac{dy}{dx} = 1 - \frac{1}{\sec x + \tan x} \left(\sec x \tan x + \sec^2 x \right) )</td>
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<tr>
<td>( = 1 - \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} )</td>
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<td>( = 1 - \frac{\sec x (\tan x + \sec x)}{\sec x + \tan x} )</td>
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<tr>
<td>( = 1 - \frac{1}{\cos x} )</td>
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<tr>
<td>( = \frac{\cos x - 1}{\cos x} )</td>
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<tr>
<td>Numerator: ( 0 &lt; \cos x &lt; 1 )</td>
<td></td>
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<tr>
<td>( \therefore \cos x - 1 ) will always be negative.</td>
<td></td>
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<tr>
<td>Denominator: ( 0 &lt; \cos x &lt; 1 )</td>
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<tr>
<td>( \therefore \cos x ) will always be positive.</td>
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<tr>
<td>( \therefore \frac{dy}{dx} &lt; 0, \ y ) is a decreasing function.</td>
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</table>
### Working

#### (a)

LHS = \[\frac{1 + \sin 2x + \cos 2x}{\cos x + \sin x}\]

= \[\frac{1 + (2\sin x \cos x) + (2\cos^2 x - 1)}{\cos x + \sin x}\]

= \[\frac{2\cos^2 x + 2\sin x \cos x}{\cos x + \sin x}\]

= \[\frac{2\cos x (\cos x + \sin x)}{\cos x + \sin x}\]

= \[2 \cos x\]

= RHS (proven)

#### (b)

\[\frac{\sec^2 x}{2 \tan^2 x + 1} = \frac{3}{4}\]

\[\frac{4}{\cos^2 x} = 6 \tan^2 x + 3\]

\[\frac{4}{\cos^2 x} = 6 \sin^2 x + 3\cos^2 x\]

\[4 = 6 \sin^2 x + 3\cos^2 x\]

\[4 = (3\sin^2 x + 3\cos^2 x) + 3\sin^2 x\]

\[4 = 3 + 3\sin^2 x\]

\[\sin^2 x = \frac{1}{3}\]

\[\sin x = \sqrt{\frac{1}{3}}\]

(reject as \(180^\circ < x < 270^\circ\)) or \(\sin x = -\frac{\sqrt{3}}{3}\)

**Alternative:**

\[1 + \tan^2 x = \frac{3}{4}\]

\[2 \tan^2 x + 1 = 4\]

\[4 + 4 \tan^2 x = 6 \tan^2 x + 3\]

\[\tan^2 x = \frac{1}{2}\]

\[\sin^2 x = \frac{1}{2}\]

\[\cos^2 x = \frac{2}{2}\]

\[\sin^2 x = \frac{1}{2}\]

\[\frac{1 - \sin^2 x}{2}\]

\[2 \sin^2 x = 1 - \sin^2 x\]

\[\sin^2 x = \frac{1}{3}\]

\[\sin x = \sqrt{\frac{1}{3}}\]

(reject as \(180^\circ < x < 270^\circ\)) or \(\sin x = -\frac{\sqrt{3}}{3}\)
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<td></td>
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<tr>
<td>(2^x = 8(2^y)) (\text{----- (1)})</td>
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</tr>
<tr>
<td>(\lg(2x + y) = \lg 63 - \lg 3) (\text{----- (2)})</td>
<td></td>
</tr>
<tr>
<td>From (1), (2^x = 2^3 \times 2^y) (\text{----- (3)})</td>
<td></td>
</tr>
<tr>
<td>(x = 3 + y) (\text{----- (3)})</td>
<td></td>
</tr>
<tr>
<td>From (2), (\lg(2x + y) = \lg \left(\frac{63}{3}\right)) (\text{----- (4)})</td>
<td></td>
</tr>
<tr>
<td>(2x + y = 21) (\text{----- (4)})</td>
<td></td>
</tr>
<tr>
<td>Sub (3) into (4), (2(3 + y) + y = 21)</td>
<td></td>
</tr>
<tr>
<td>(y = 5)</td>
<td></td>
</tr>
<tr>
<td>(x = 8)</td>
<td></td>
</tr>
<tr>
<td>4 (b)</td>
<td></td>
</tr>
<tr>
<td>(\log_{1/2} y = 3 - \log_2 (y - 6))</td>
<td></td>
</tr>
<tr>
<td>(\log_{2^3} y = 3 - \log_2 (y - 6))</td>
<td></td>
</tr>
<tr>
<td>(\frac{\lg y}{1} = 3 - \frac{\lg (y - 6)}{\lg 2})</td>
<td></td>
</tr>
<tr>
<td>(\frac{\lg y}{\frac{1}{2} \lg 2} + \frac{\lg (y - 6)}{\lg 2} = 3)</td>
<td></td>
</tr>
<tr>
<td>(2 \lg y + \lg (y - 6) = 3 \lg 2)</td>
<td></td>
</tr>
<tr>
<td>(\lg y^2 + \lg (y - 6) = \lg 2^3)</td>
<td></td>
</tr>
<tr>
<td>(\lg \left[ y^2 (y - 6) \right] = \lg 8)</td>
<td></td>
</tr>
<tr>
<td>(y^3 - 6y^2 - 8 = 0)</td>
<td></td>
</tr>
<tr>
<td>Working</td>
<td>Common Issues</td>
</tr>
<tr>
<td>---------</td>
<td>---------------</td>
</tr>
<tr>
<td><strong>5 (i)</strong></td>
<td></td>
</tr>
</tbody>
</table>
| \[
\frac{2x^2 - 7}{(x+1)(x^2 - x - 6)} = \frac{2x^2 - 7}{(x+1)(x-3)(x+2)}
\]
| \[
\frac{A}{x+1} + \frac{B}{x-3} + \frac{C}{x+2}
\]
| \[
A(x-3)(x+2) + B(x+1)(x+2) + C(x+1)(x-3) = 2x^2 - 7
\]
| When \(x = 1\),  
\[
A(-4)(1) = 2(-1)^2 - 7
\]
| \(A = \frac{5}{4}\)  
| When \(x = -2\),  
\[
C(-1)(-5) = 2(-2)^2 - 7
\]
| \(C = \frac{1}{5}\)  
| When \(x = 3\),  
\[
B(4)(5) = 2(3)^2 - 7
\]
| \(B = \frac{11}{20}\)  
| \[
\frac{2x^2 - 7}{(x+1)(x^2 - x - 6)} = \frac{5}{4(x+1)} + \frac{11}{20(x-3)} + \frac{1}{5(x+2)}
\]
| **(ii)**  
\[
\int_5^8 \frac{8x^2 - 28}{4(x+1)(x^2 - x - 6)} \, dx
\]
| \[
= 4 \int_5^8 \left[ \frac{5}{4(x+1)} + \frac{11}{20(x-3)} + \frac{1}{5(x+2)} \right] \, dx
\]
| \[
= 4 \left[ \frac{5}{4} \ln(x+1) + \frac{11}{20} \ln(x-3) + \frac{1}{45} \ln(x+2) \right]_5^8
\]
| \[
= \left[ \left( \frac{5 \ln 6 + \frac{11}{5} \ln 2 + \frac{4}{5} \ln 7 \right) - \left( \frac{5 \ln 5 + \frac{11}{5} \ln 1 + \frac{4}{5} \ln 6 \right) \right]
\]
| \(= 2.56 \text{ (3sf)}\)  

<table>
<thead>
<tr>
<th>Working</th>
<th>Common Issues</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6 (a i)</strong></td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Graph" /></td>
<td>$y = [(x-1)(x-5)]$</td>
</tr>
<tr>
<td><strong>(a ii)</strong></td>
<td>$0 &lt; a &lt; 4$</td>
</tr>
</tbody>
</table>
| **(b)** | $2x^2 - 6x + 5 = kx - 3$  
$2x^2 - (6+k)x + 8 = 0$ |
| | $b^2 - 4ac < 0 \therefore$ no intersection |
| | $[-(6+k)]^2 - 4(2)(8) < 0$  
$36 + 12k + k^2 - 64 < 0$  
$k^2 + 12k - 28 < 0$  
$(k+14)(k-2) < 0$ |
<p>| | $-14 &lt; k &lt; 2$ |</p>
<table>
<thead>
<tr>
<th>Working</th>
<th>Common Issues</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7 (i)</strong></td>
<td></td>
</tr>
</tbody>
</table>
\[
\frac{d}{dx} \left( \ln 3x \right) = \frac{1}{2x^3} \quad \frac{d}{dx} \left( \frac{\ln 3x}{x^2} \right) = \frac{1}{2} \left[ \frac{x^2 \left( \frac{3}{3x} \right) - (2x)(\ln 3x)}{x^4} \right] \\
= \frac{1}{2} \left[ \frac{x - 2x(\ln 3x)}{x^4} \right] \\
= \frac{x - 2x(\ln 3x)}{2x^4} \\
= \frac{1}{2x^3} - \frac{\ln 3x}{x^3} \quad \text{(shown)}
\] |
| **(ii)** | 
\[
\int \frac{\ln 3x}{x^3} \, dx = \frac{\ln 3x}{2x^2} + c \\
\int \frac{\ln 3x}{x^3} \, dx = \frac{1}{2} \int x^{-3} \, dx - \frac{\ln 3x}{2x^2} + c \\
= \frac{1}{2} \left( \frac{x^{-2}}{-2} \right) - \frac{\ln 3x}{2x^2} + c \\
= -\frac{1}{4x^2} - \frac{2\ln 3x}{x^2} + c
\] |
| **(iii)** | 
\[
f'(x) = \frac{\ln 3x}{x^3} \\
f(x) = \frac{\ln 3x}{-2x^2} - \frac{1}{4x^2} + c
\] 
Given \( f \left( \frac{1}{3} \right) = \frac{3}{4}, \)
\[
\frac{3}{4} = 0 - \frac{1}{4 \left( \frac{1}{3} \right)^2} + c \\
c = 3
\] 
\[
\therefore f(x) = \frac{\ln 3x}{-2x^2} - \frac{1}{4x^2} + 3
\] |
8

(i) 
\[(6-x^2)^5 = \binom{5}{0}(6)^5(x^2)^0 - \binom{5}{1}(6)^4(x^2)^1 + \binom{5}{2}(6)^3(x^2)^2 + \ldots \]

\[= 7776 - 6480x^2 + 2160x^4 + \ldots \]

Coefficient of \(x^4 = (-6480)(2) + (2160)\left(\frac{1}{3}\right)\]

\[= -12960 + 720 \]

\[= -12240 \]

(ii) 
For \(x\) term, \(r = 1\)

\[T_2 = \binom{n}{1}(2^{n-1})x \]

\[= \frac{2^n (n)}{2} x \]

For \(x^2\) term, \(r = 2\)

\[T_3 = \binom{n}{2}(2^{n-2})x^2 \]

\[= \frac{2^n (n)(n-1)}{8} x^2 \]

Coefficient of \(x = \frac{2^n (n)}{2} \]

Coefficient of \(x^2 = \frac{2^n (n)(n-1)}{8} = \frac{2}{3} \]

\[\frac{2^n (3n)}{2} = \frac{2^n (n)(n-1)}{4} \]

\[2^n (6n) = 2^n (n)(n-1) \]

\[2^n (n)(n-1) - 2^n (6n) = 0 \]

\[2^n (n)[(n-1) - 6] = 0 \]

\[2^n = 0 \text{ (reject as } 2^n > 0) \]

\[n = 0 \text{ (reject as } n \neq 0) \]

\[n = 7 \]
<table>
<thead>
<tr>
<th>Working</th>
<th>Common Issues</th>
</tr>
</thead>
</table>
| **9 (a)**  
$AB = 6 \sin x$
$BC = 6 \cos x$

$RB = \frac{6 \sin x}{2} = 3 \sin x$ (ratio of corresponding sides are equal)

$SR = RT = \frac{6 \cos x}{2} = 3 \cos x$ (ratio of corresponding sides are equal)

$P = 6 + 6 \cos x + 3 \sin x + 3 \cos x + 3$
$= 9 + 9 \cos x + 3 \sin x$ (shown)

**(bi)**  
$P = 9 + 9 \cos \theta + 3 \sin \theta$

$= 9 + n \cos(\theta - \alpha)$

$= 9 + n(\cos \theta \cos \alpha + \sin \theta \sin \alpha)$

Comparing coefficients,

$n \cos \alpha = 9, \quad n \sin \alpha = 3$

$\tan \alpha = \frac{1}{3}$

$\alpha = \tan^{-1} \frac{1}{3}$

$= 18.43495^\circ$

$n^2 = 9^2 + 3^2$

$n = \sqrt{90}$

$\therefore P = 9 + \sqrt{90} \cos(\theta - 18.43495^\circ)$

$9 + \sqrt{90} \cos(\theta - 18.43495^\circ) = 15$

$\cos(\theta - 18.43495^\circ) = \frac{15 - 9}{\sqrt{90}}$

Basic angle $= \cos^{-1} \left( \frac{6}{\sqrt{90}} \right) = 50.7685^\circ$

$\theta - 18.43495^\circ = 50.7685^\circ$ or

$\theta - 18.43495^\circ = 360^\circ - 50.7685^\circ$ (reject)

$\theta = 69.2^\circ$

**(bii)** Maximum value of $P$ is when $\cos(x - 18.43495^\circ) = 1$

$\therefore$ maximum value of $P = 9 + \sqrt{90}$

corresponding value of $x = 18.4^\circ$
<table>
<thead>
<tr>
<th>Working</th>
<th>Common Issues</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>10 (i)</strong></td>
<td></td>
</tr>
<tr>
<td>Length of (AD = \sqrt{(y-9)^2 + (4-7)^2})</td>
<td></td>
</tr>
<tr>
<td>Length of (AB = \sqrt{(6-9)^2 + (8-7)^2})</td>
<td></td>
</tr>
<tr>
<td>((y-9)^2 + 9 = 9 + 1)</td>
<td></td>
</tr>
<tr>
<td>((y-9)^2 = 1)</td>
<td></td>
</tr>
<tr>
<td>(y - 9 = 1) or (y - 9 = -1)</td>
<td></td>
</tr>
<tr>
<td>(y = 10) (reject) or (y = 8)</td>
<td></td>
</tr>
<tr>
<td>(\therefore) coordinates of (D(4,8)).</td>
<td></td>
</tr>
<tr>
<td>Coordinates of (M = \left(\frac{8+4}{2}, \frac{6+8}{2}\right))</td>
<td></td>
</tr>
<tr>
<td>= ((6,7))</td>
<td></td>
</tr>
<tr>
<td>Gradient of (AM = \frac{9-7}{7-6})</td>
<td></td>
</tr>
<tr>
<td>= 2</td>
<td></td>
</tr>
<tr>
<td>Equation of (AC: 9 = 2(7) + c)</td>
<td></td>
</tr>
<tr>
<td>(c = -5)</td>
<td></td>
</tr>
<tr>
<td>(y = 2x - 5)</td>
<td></td>
</tr>
<tr>
<td>When (x = 4, y = 3)</td>
<td></td>
</tr>
<tr>
<td>(\therefore) coordinates of (C(4,3)).</td>
<td></td>
</tr>
</tbody>
</table>

**(ii)**

\(M_{AC} = M_{AM} = 2\)

\(M_{BD} = \frac{6-8}{8-4} = -\frac{1}{2}\)

Since \(M_{AC} \cdot M_{BD} = -1\), diagonals \(AC\) and \(BD\) are perpendicular to each other. \(\therefore\) quadrilateral \(ABCD\) is a kite.

**(iii)**

Area of \(ABCD = \frac{1}{2}\left|\begin{array}{cccc}7 & 4 & 4 & 8 & 7 \\ 9 & 3 & 6 & 9 \end{array}\right|\)

\[= \frac{1}{2}\left[\left(7 \times 8 + 4 \times 3 + 4 \times 6 + 8 \times 9\right) - \left(9 \times 4 + 8 \times 4 + 3 \times 8 + 6 \times 7\right)\right]\]

\[= \frac{1}{2} \times 30\]

\[= 15\text{ units}^2\]
<table>
<thead>
<tr>
<th>Working</th>
<th>Common Issues</th>
</tr>
</thead>
</table>
| **11 (ai)**<br>20 = 100e\(^{-k(2.5)}\)<br>\[ \frac{1}{5} = -2.5k \]<br>\[ k = 0.644 \]<br><br>**(aii)**<br>\[ 100e^{-0.643775t} = \frac{1}{2} \]
\[ e^{\ln \frac{1}{2}} = \frac{1}{2} \]
\[ -0.643775t = \ln \frac{1}{2} \]
\[ t = \frac{1.08}{2} \text{ hours} \] | |
| **(bi)**<br>y = ax\(^4\) + bx\(^3\) + 7<br>\[ \frac{dy}{dx} = 4ax^3 + 3bx^2 \]<br>Sub x = 1 into \( \frac{dy}{dx} \),<br>4a + 3b = 0 ---- (1)<br>Sub (1,6) into curve y,<br>6 = a + b + 7<br>a = -b - 1 ---- (2)<br>Sub (2) into (1),<br>4(-b - 1) + 3b = 0<br>b = -4, \quad a = 3 | |
| **(bii)**<br>\[ \frac{dy}{dx} = 4(3)x^3 + 3(-4)x^2 \]
\[ = 12x^3 - 12x^2 \]
When \( \frac{dy}{dx} = 0 \),
12x\(^3\) - 12x\(^2\) = 0
12x\(^2\)(x - 1) = 0
\[ x = 0 \quad \text{or} \quad x = 1 \]
When x = 0, \( y = 7 \)<br>\[ \therefore \text{ the other stationary point is (0,7).} \] | |
When \( x = 0 \), \( \frac{d^2y}{dx^2} = 0 \) (not conclusive)

<table>
<thead>
<tr>
<th></th>
<th>( x = -0.1 )</th>
<th>( x = 0 )</th>
<th>( x = 0.1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{dy}{dx} )</td>
<td>negative</td>
<td>0</td>
<td>negative</td>
</tr>
</tbody>
</table>

Using the first derivative test, the gradient changes from negative to negative, thus \((0, 7)\) is a point of inflexion.
ADDISONAL MATHEMATICS 4047/01
Paper 1
17 August 2018
2 hours

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.
2

Mathematical Formulae

1. ALGEBRA

**Quadratic Equation**

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

**Binomial expansion**

\[(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,\]

where \( n \) is a positive integer and

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1) \ldots (n-r+1)}{r!}
\]

2. TRIGONOMETRY

**Identities**

\[\sin^2 A + \cos^2 A = 1\]
\[\sec^2 A = 1 + \tan^2 A\]
\[\cosec^2 A = 1 + \cot^2 A\]
\[\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B\]
\[\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B\]

\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[\sin 2A = 2 \sin A \cos A\]
\[\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A\]
\[\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}\]

**Formulae for \( \Delta ABC \)**

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[a^2 = b^2 + c^2 - 2bc \cos A\]
\[\Delta = \frac{1}{2}ab \sin C\]
Answer all the questions.

1 A cone has curved surface area $\pi \left( 17 - \sqrt{3} \right) \text{cm}^2$ and slant height $\left( 7 - 3\sqrt{3} \right) \text{cm}$. **Without using a calculator**, find the diameter of the base of the cone, in cm, in the form of $a + b\sqrt{3}$, where $a$ and $b$ are integers. [4]

2 The roots of the quadratic equation $5x^2 - 3x + 1 = 0$ are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Find a quadratic equation with roots $\alpha^3$ and $\beta^3$. [6]

3 (i) Show that $2x^2 + 1$ is a factor of $2x^3 - 4x^2 + x - 2$. [2]

(ii) Express $\frac{11x - 5x^2 - 11}{2x^3 - 4x^2 + x - 2}$ in partial fractions. [5]

4 (i) Sketch the graph of $y = \frac{4}{\sqrt{x}}$ for $x > 0$. [2]

(ii) Find the coordinates of the point(s) of intersection of $y = \frac{4}{\sqrt{x}}$ and $y^2 = 81x$. [4]

5 The diagram shows a cylinder of height $h$ cm and base radius $r$ cm inscribed in a sphere of radius 35 cm.

(i) Show that the height of the cylinder, $h$ cm, is given by $h = 2\sqrt{1225 - r^2}$. [2]

(ii) Given that $r$ can vary, find the maximum volume of the cylinder. [4]
6 (i) Show that \[
\frac{2 - \sec^2 x}{2 \tan x + \sec^2 x} = \frac{\cos x - \sin x}{\cos x + \sin x}.
\] [3]

(ii) Hence find, for \(0 \leq x \leq 2\pi\), the values of \(x\) for which \[
\frac{6 - 3 \sec^2 x}{2 \tan x + \sec^2 x} = \frac{3}{2}.
\] [3]

7 A curve is such that \[
\frac{d^2 y}{dx^2} = \frac{2}{e^{2x-3}}
\] and the point \(P(1.5, 2)\) lies on the curve.

The gradient of the normal to the curve at \(P\) is 10. Find the equation of the curve. [6]

8 The diagram shows the graph of \(y = x^3 - 4x\) which passes through the origin \(O\) and cuts the \(x\)-axis at the point \(A(16, 0)\). Tangents to the curve at \(O\) and \(A\) meet at the point \(B\).

(i) Show that \(B\) is the point \(\left(\frac{5}{3}, -2\frac{1}{3}\right)\). [3]

(ii) Find the area of the shaded region bounded by the curve and the lines \(OB\) and \(AB\). [4]
9 A tram, moving along a straight road, passes station $O$ with a velocity of 975 m/min. Its acceleration, $a$ m/min$^2$, $t$ mins after passing through station $O$, is given by $a = 2t - 80$. The tram comes to instantaneous rest, first at station $A$ and later at station $B$. Find

(i) the acceleration of the tram at station $A$ and at station $B$, 

(ii) the greatest speed of the tram as it travels from station $A$ to station $B$, 

(iii) the distance between station $A$ to station $B$.

10 (i) By considering the general term in the binomial expansion of $\left(x^4 - \frac{1}{kx^2}\right)^6$, where $k$ is a positive constant, explain why there are only even powers of $x$ in this expansion.

(ii) Given that the term independent of $x$ in this binomial expansion is $\frac{5}{27}$, find the value of $k$.

(iii) Using the value of $k$ found in part (ii), hence obtain the coefficient of $x^{18}$ in $\left(2 - 3x^6\right)\left(x^4 - \frac{1}{kx^2}\right)^6$.

11 $M$ and $N$ are two points on the circumference of a circle, where $M$ is the point (6, 8) and $N$ is the point (10, 16). The centre of the circle lies on the line $y = 2x + 1$.

(i) Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where $a$, $b$ and $c$ are constants.

(ii) Explain whether the point (9, 10) lie inside the circle. Justify your answer with mathematical calculations.
In the diagram, two circles intersect at $B$ and $F$. $BC$ is the diameter of the larger circle and is the tangent to the smaller circle at $B$. Point $A$ lies on the smaller circle such that $AFEC$ is a straight line. Point $D$ lies on the larger circle such that $BHED$ is a straight line. Prove that

(i) $CD$ is parallel to $AH$, \[3\]

(ii) $AB$ is a diameter of the smaller circle, \[2\]

(iii) triangles $ABC$ and $BFC$ are similar, \[2\]

(iv) $AC^2 - AB^2 = CF \times AC$. \[2\]

**End of Paper**
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((10 + 4\sqrt{3})) cm</td>
</tr>
<tr>
<td>2</td>
<td>(x^2 + 18x + 125 = 0)</td>
</tr>
</tbody>
</table>
| 3i | \(2x^3 - 4x^2 + x - 2 = (2x^2 +1)(x - 2)\)  
It is divisible by \(2x^2 + 1\) with no remainder. |
| 3ii | \(-5x^2 + 11x - 11 = -\frac{1}{2x^2 + 1}\)  
\(2x^3 - 4x^2 + x - 2 = \frac{5 - 3x}{2x^2 + 1}\) |
| 4i | \(y = \frac{4}{\sqrt{x}}\) |
| 4ii | \(\left(\frac{4}{9}, 6\right)\) |
| 5i | Using Pythagoras’ Theorem:  
\(\left(\frac{h}{2}\right)^2 + r^2 = 35^2\) |
| 5ii | 104 000 cm\(^3\) (3 s.f.) |
| 6i | \(x = 0.322\) or \(x = 3.46\) (3 s.f.) |
| 7 | \(y = \frac{1}{2}e^{3-2x} + \frac{9}{10}x + \frac{3}{20}\) |
| 8ii | 68.3 units\(^2\) (3 s.f.) |
| 9i | Acceleration at \(A = -50\) m/min\(^2\)  
Acceleration at \(B = 50\) m/min\(^2\) |
| 9ii | Greatest speed = 625 m/min |
| 9iii | 20.8 km (3 s.f.) |
| 10i | General term = \(\binom{6}{r}x^{24-6r}\left(\frac{1}{k}\right)^r\)  
Since \(6r\) is an even number, \(24 - 6r\) will be even. |
## 2018 Preliminary Examination 2
Additional Mathematics 4047 Paper 1
Solutions

<table>
<thead>
<tr>
<th>Qn</th>
<th>Working</th>
</tr>
</thead>
</table>
| 1  | \[\pi rl = \pi \left(17 - \sqrt{3}\right)\]  
    | \[r = \frac{17 - \sqrt{3}}{7 - 3\sqrt{3}}\]  
    | \[r = \frac{17 - \sqrt{3}}{7 - 3\sqrt{3}} \times \frac{7 + 3\sqrt{3}}{7 + 3\sqrt{3}}\]  
    | \[r = \frac{110 + 44\sqrt{3}}{22}\]  
    | \[r = 5 + 2\sqrt{3}\]  
    | Diameter = 10 + 4\sqrt{3} cm |
| 2  | \[\frac{1}{\alpha} + \frac{1}{\beta} = -\frac{3}{5}\]  
    | \[\frac{1}{\alpha\beta} = \frac{1}{5}\]  
    | \[\alpha\beta = 5\]  
    | \[\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}\]  
    | \[\alpha + \beta = 3\]  
    | \[\frac{5}{5} = \frac{3}{5}\]  
    | \[\alpha + \beta = 3\]  
    | \[\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)\]  
    | \[= 3[(\alpha + \beta)^2 - 3\alpha\beta]\]  
    | \[= 3[(3)^2 - 3(5)]\]  
    | \[= -18\]  
    | \[\alpha^3\beta^3 = (\alpha\beta)^3\]  
    | \[= 125\]  
    | Equation: \[x^2 + 18x + 125 = 0\]  

---

Cedar Girls’ Sec School – 2018 Prelim 4047 P1 Solutions Page 1
<table>
<thead>
<tr>
<th>Qn</th>
<th>Working</th>
</tr>
</thead>
</table>
| 3i | \(2x^3 - 4x^2 + x - 2 = (2x^2 + 1)(x - 2)\)  
   It is divisible by \(2x^2 + 1\) with no remainder. |
| 3ii | \[-\frac{5x^2 + 11x - 11}{2x^3 - 4x^2 + x - 2} = \frac{A}{x - 2} + \frac{Bx + C}{2x^2 + 1}\]  
   \[-5x^2 + 11x - 11 = A(2x^2 + 1) + (Bx + C)(x - 2)\]  
   When \(x = 2\),  
   \(A = -1\)  
   Comparing \(x^2\):  
   \(-5 = 2A + B\)  
   \(-5 = -2 + B\)  
   \(B = -3\)  
   Comparing constant:  
   \(-11 = A - 2C\)  
   \(-11 = -1 - 2C\)  
   \(C = 5\)  
   \[-\frac{5x^2 + 11x - 11}{2x^3 - 4x^2 + x - 2} = -\frac{1}{x - 2} + \frac{5 - 3x}{2x^2 + 1}\] |
| 4  | \(y = 4x^{-\frac{1}{2}}\)  
   [Graph showing the function \(y = 4x^{-\frac{1}{2}}\)] |
4ii \[ \left( \frac{4}{\sqrt{x}} \right)^2 = 81x \]
\[ \frac{16}{x} = 81x \]
\[ 81x^2 = 16 \]
\[ x = \pm \frac{4}{9} \]
\[ x = \frac{4}{9} \]
\[ y = 6 \]
Point of intersection = \[ \left( \frac{4}{9}, 6 \right) \]

5i \[ \left( \frac{h}{2} \right)^2 + r^2 = 35^2 \text{ (Pythagoras’ Theorem)} \]
\[ \frac{h^2}{4} = 1225 - r^2 \]
\[ h^2 = 4(1225 - r^2) \]
\[ h = 2\sqrt{1225 - r^2} \]
(shown)

5ii \[ V = \pi r^2 (2\sqrt{1225 - r^2}) \]
\[ V = 2\pi r^2 (1225 - r^2) \frac{1}{2} \]
\[ \frac{dV}{dr} = 2\pi r^2 \left( \frac{1}{2} (-2r)(1225 - r^2)^{-\frac{1}{2}} \right) + (1225 - r^2)^{\frac{1}{2}} (4\pi r) \]
\[ = -2\pi r^3 (1225 - r^2)^{-\frac{1}{2}} + 4\pi r (1225 - r^2)^{\frac{1}{2}} \]
\[ -2\pi r^3 (1225 - r^2)^{-\frac{1}{2}} + 4\pi r (1225 - r^2)^{\frac{1}{2}} = 0 \]
\[ r^3 = 2r(1225 - r^2) \]
\[ 3r^3 = 2450r \]
\[ r = 28.577 \text{ (reject } r = 0 \text{ and } -ve r) \]
Using First Derivative Test,

<table>
<thead>
<tr>
<th>( x )</th>
<th>28.577 (-)</th>
<th>28.577</th>
<th>28.577 (+)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sign of ( \frac{dV}{dr} )</td>
<td>+ve</td>
<td>0</td>
<td>-ve</td>
</tr>
<tr>
<td>slope</td>
<td>/</td>
<td>—</td>
<td>/</td>
</tr>
</tbody>
</table>

\( V \) is maximum at \( r = 28.577 \)
Maximum volume:
\[ V = \pi (28.577)^2 (2\sqrt{1225} - (28.577)^2) \]
= 103 688
= 104 000
= 104 000 cm³ (3 s.f.)

<table>
<thead>
<tr>
<th>Qn</th>
<th>Working</th>
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</thead>
</table>
| 6i | \[ \text{LHS:} \quad \frac{2 - \sec^2 x}{2 \tan x + \sec^2 x} = \frac{2 - (\tan^2 x + 1)}{2 \tan x + (\tan^2 x + 1)} \]  
\[ = \frac{1 - \tan^2 x}{2 \tan x + \tan^2 x + 1} \]  
\[ = \frac{(1 - \tan x)(1 + \tan x)}{(\tan x + 1)^2} \]  
\[ = \frac{1 - \tan x}{1 + \tan x} \]  
\[ = \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}} \]  
\[ = \frac{\cos x - \sin x}{\cos x + \sin x} \]  
\[ = \frac{\cos x - \sin x}{\cos x + \sin x} \times \frac{\cos x}{\cos x} \]  
\[ = \frac{\cos x - \sin x}{\cos x + \sin x} \times \frac{\cos x}{\cos + \sin x} \]  
\[ = \frac{\cos x}{\cos x + \sin x} \times \frac{\sin x}{\cos x} \]  
\[ = \frac{\cos x - \sin x}{\cos x + \sin x} \times \frac{\cos x}{\cos x + \sin x} \]  
\[ \text{(shown)} \]  
\[ 3 \times \frac{\cos x - \sin x}{\cos x + \sin x} = \frac{3}{2} \]  
\[ \frac{\cos x - \sin x}{\cos x + \sin x} = \frac{1}{2} \]  
\[ \frac{2 \cos x - 2 \sin x}{\cos x + \sin x} = \cos x + \sin x \]  
\[ \cos x = 3 \sin x \]  
\[ \tan x = \frac{1}{3} \]  
\[ x = 0.322 \text{ or } x = 3.46 \text{ (3 s.f.)} \] |
Qn 7

\[
\frac{d^2y}{dx^2} = 2e^{3-2x}
\]

\[
\frac{dy}{dx} = 2[-\frac{1}{2}e^{3-2x}] + c
\]

\[
\frac{dy}{dx} = -e^{3-2x} + c
\]

Gradient at tangent at \(P = \frac{1}{10}\)

\[-e^{3-2x} + c = -\frac{1}{10}\]

when \(x = 1.5\)

\(c = \frac{9}{10}\)

\[
\frac{dy}{dx} = -e^{3-2x} + \frac{9}{10}
\]

\[y = \frac{1}{2}e^{3-2x} + \frac{9}{10}x + c\]

\[2 = \frac{1}{2}e^{3-2(1.5)} + \frac{9}{10}(1.5) + c\]

\(c = \frac{3}{20}\)

Eqn: \(y = \frac{1}{2}e^{3-2x} + \frac{9}{10}x + \frac{3}{20}\)
<table>
<thead>
<tr>
<th>Qn</th>
<th>Working</th>
</tr>
</thead>
</table>
| 8i | \( \frac{dy}{dx} = \frac{3}{2} \frac{1}{x^2} - 4 \)  
At \( O, x = 0 \), \( \frac{dy}{dx} = -4 \)  
Equation \( OB: y = -4x \ldots (1) \)  
At \( A, x = 16 \), \( \frac{dy}{dx} = 2 \)  
\( y = 2x + c \)  
\( 0 = 2(16) + c \)  
\( c = -32 \)  
Equation \( AB: y = 2x - 32 \)  
\( 2x - 32 = -4x \)  
\( x = 5 \frac{1}{3} \)  
Sub into (1),  
\( y = -21 + \frac{1}{3} \)  
\( B = \left( 5 \frac{1}{3}, -21 \frac{1}{3} \right) \) (shown)  
Area of curve = \[ \int_{0}^{16} \frac{3}{2} \frac{1}{x^2} - 4x \, dx \]  
= \( \left[ \frac{2}{5} \frac{5}{x^2} - 2x^2 \right]_{0}^{16} \)  
= 102.4 units\(^2\)  
Area of triangle OAB = \( \frac{1}{2} \times 16 \times 21 \frac{1}{3} \)  
= 170 \( \frac{2}{3} \) units\(^2\)  
Area of shaded region = 170 \( \frac{2}{3} \) - 102.4  
= 68.3 units\(^2\) (3 s.f.) |
<table>
<thead>
<tr>
<th>Qn</th>
<th>Working</th>
</tr>
</thead>
</table>
| 9i | $a = 2t - 80$
    | $v = t^2 - 80t + c$
    | $t = 0, v = 975$
    | $975 = (0)^2 - 80(0) + c$
    | $c = 975$
    | $v = t^2 - 80t + 975$
    | When $v = 0$,  
    | $t^2 - 80t + 975 = 0$
    | $(t - 15)(t - 65) = 0$
    | $t = 15, t = 65$
    | Acceleration at $a = 2(15) - 80$
    | $= - 50 \text{ m/min}^2$
    | Acceleration at $a = 2(65) - 80$
    | $= 50 \text{ m/min}^2$
| 9ii | When $a = 0$,  
    | $t = \frac{15 + 65}{2}$
    | $t = 40$
    | $v = (40)^2 - 80(40) + 975$
    | $v = - 625 \text{ m/min}$
    | Greatest speed = 625 m/min
| 9iii | Distance $AB = \int_{15}^{65} (t^2 - 80t + 975) \, dt$
      | $= \left[ \frac{t^3}{3} - 40t^2 + 975t \right]_{15}^{65}$
      | $= 20833 \frac{1}{3} \text{ m}$
      | $= 20800 \text{ m} (3 \text{ s.f.})$
      | $= 20.8 \text{ km}$
### Qn | Working
---|---
**10(i)** | General Term = \[
\binom{6}{r} (x^4)^{6-r} \left(-\frac{1}{k} x^{-2}\right)^r
\]
\[
= \binom{6}{r} x^{24-6r} \left(-\frac{1}{k}\right)^r
\]
Since \(6r\) is an even number, \(24-6r\) will be even.

(ii) | For independent term, \(24-6r = 0 \implies r = 4\)
\[
\binom{6}{4} \left(-\frac{1}{k}\right)^4 = \frac{5}{27}
\]
\[
\frac{15}{k^4} = \frac{5}{27}
\]
\[
k = \pm \sqrt{\frac{27 \times 15}{5}} = 3 \text{ (as } k > 0 \text{)}
\]

(iii) | \((2-3x^6)(\ldots + \text{Term in } x^{18} + \text{Term in } x^{12} + \ldots)\)
For term in \(x^8\), \(24-6r = 18 \implies r = 1\)
Therefore, term in \(x^8 = \binom{6}{1} \left(-\frac{1}{3}\right) x^{18} = -2x^{18}\)
For term in \(x^{12}\), \(24-6r = 12 \implies r = 2\)
Therefore, term in \(x^{12} = \binom{6}{2} \left(-\frac{1}{3}\right)^2 x^{12} = \frac{5}{3}x^{12}\)
Coefficient of \(x^{18} = 2(-2) + (-3) \left(\frac{5}{3}\right) = -9\)
### Working

#### 11i

Let $MN$ be a chord of circle.

Midpoint of $MN = \left( \frac{10+6}{2}, \frac{16+8}{2} \right)$

$= (8, 12)$

Gradient of $MN = \frac{16-8}{10-6}$

$= 2$

Gradient of perpendicular bisector $= -\frac{1}{2}$

Equation of perpendicular bisector of $MN$:

$y - 12 = -\frac{1}{2} (x - 8)$

$y = -\frac{1}{2} x + 16$

$-\frac{1}{2} x + 16 = 2x + 1$

$x = 6$

$y = 13$

Centre of circle $= (6, 13)$

Radius $= 13 - 8$

$= 5$ units

Equation of circle:

$(x - 6)^2 + (y - 13)^2 = 5^2$

$x^2 + y^2 - 12x - 26y + 180 = 0$

#### 11ii

Length of point to centre of circle

$= \sqrt{(9 - 6)^2 + (10 - 13)^2}$

$= \sqrt{18}$

$= 4.24$ units

$< 5$ (radius)

Yes, the point lies inside the circle as its length from the centre of the circle is less than the radius.
<table>
<thead>
<tr>
<th>Qn</th>
<th>Working</th>
</tr>
</thead>
</table>
| 12i | $\angle BDC = 90^\circ$ (in semicircle)  
$\angle BFC = 90^\circ$ (in same segment) or (in semicircle)  
$\angle BFA = 180^\circ - 90^\circ$ (adj $\angle s$ on straight line)  
$= 90^\circ$  
$\angle BHA = \angle BFA = 90^\circ$ (in same segment)  
$\angle AHD = 180^\circ - 90^\circ$ (adj $\angle s$ on straight line)  
$= 90^\circ$  
$\angle AHD = \angle BDC = \angle HDC$ (alternate angles)  
$\therefore CD \parallel AH$ |
| 12ii| $\angle BHA = \angle BFA = 90^\circ$ (in same segment)  
$AB$ is a diameter of the smaller circle (in semicircle). |
| 12iii| Since $AB$ and $BC$ are tangents to the smaller and bigger circle  
respectively, $\angle ABC = 90^\circ$ (tan $\perp$ rad)  
$\angle ABC = \angle BFC$  
$\angle BCA = \angle FCB$ (common $\angle$)  
Triangle $ABC$ is similar to triangle $BFC$ as all corresponding angles are equal. |
| 12iv| $\frac{BC}{FC} = \frac{AC}{CB}$ (ratio of similar triangles)  
$BC^2 = CF \times AC$  
$BC^2 = AC^2 - AB^2$ (Pythagoras' Theorem)  
$\therefore AC^2 - AB^2 = CF \times AC$ (shown) |
This document consists of 8 printed pages and 1 cover page.

[Turn over
Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where $n$ is a positive integer and \(\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1) \ldots (n-r+1)}{r!}\)

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\csc^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin A + \sin B = 2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$$

$$\sin A - \sin B = 2 \cos \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$$

$$\cos A + \cos B = 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$$

$$\cos A - \cos B = -2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$
Answer all the questions.

1 (a) Given that \(3 \log \left(\sqrt[3]{y} \cdot \sqrt{x}\right) = 2 + 2 \log x - \log y\), where \(x\) and \(y\) are positive numbers, express, in its simplest form, \(y\) in terms of \(x\). [3]

(b) Given that \(p = \log_q x\), express, in terms of \(p\),

(i) \(\log_q \left(\frac{1}{q}\right)\), [2]

(ii) \(\log_2 4q\). [2]

2 (i) Show that \(\frac{d}{dx} (\sin x \cos x) = 2 \cos^2 x - 1\). [2]

(ii) Hence, without using a calculator, find the value of each of the constants \(a\) and \(b\) for which

\[
\int_{0}^{\frac{\pi}{2}} \cos^2 x \, dx = a + b\pi.
\] [4]

3 The variables \(x\) and \(y\) are such that when values of \(\frac{1}{y} + \frac{1}{x}\) are plotted against \(\frac{1}{x}\), a straight line with gradient \(m\) is obtained. It is given that \(y = \frac{1}{6}\) when \(x = 1\) and that \(y = \frac{1}{2}\) when \(x = \frac{1}{2}\).

(i) Find the value of \(m\). [4]

(ii) Find the value of \(x\) when \(\frac{3}{y} + \frac{3}{x} = 3\). [2]

(iii) Express \(y\) in terms of \(x\). [2]
4 The equation of a curve is \( y = x^3 + px^2 \), where \( p \) is a positive constant.

(i) Show that the origin is a stationary point on the curve and find the \( x \)-coordinate of the other stationary point in terms of \( p \). \[3\]

(ii) Find the nature of each of the stationary points. \[3\]

Another curve has equation \( y = x^3 + px^2 + px \).

(iii) Find the set of values of \( p \) for which this curve has no stationary points. \[3\]

5 A quadratic function \( f(x) \) is given by \( f(x) = k(x - 2)^2 - (x - 3)(x + 2) \), where \( k \) is a constant and \( k \neq 1 \).

(i) Find the value of \( k \) such that the graph of \( y = f(x) \) touches the \( x \)-axis at one point. \[3\]

(ii) Find the range of values of \( k \) for which the function possesses a maximum point. \[1\]

(iii) Find the range of values of \( k \) for which the value of the function never exceeds 18. \[3\]

6 (a) A substance is decaying in such a way that its mass, \( m \) kg, at a time \( t \) years from now is given by the formula

\[
m = 240e^{-0.04t}.
\]

(i) Find the time taken for the substance to halve its mass. \[2\]

(ii) Find the value of \( t \) for which the mass is decreasing at a rate of 2.1 kg per year. \[3\]

(b) The noise rating, \( N \) and its intensity, \( I \) are connected by the formula

\[
N = 10 \left( \log \frac{I}{k} \right), \text{ where } k \text{ is a constant.}
\]

A hot water pump has a noise rating of 50 decibels.
A dishwasher, however, has a noise rating of 62 decibels.

Find the value of \[
\frac{\text{Intensity of the noise from the dishwasher}}{\text{Intensity of the noise from the hot water pump}}.
\] \[3\]
The diagram shows the curve \( y = \left(6x + 2\right)^{\frac{1}{3}} \) and the point \( A (1, 2) \) which lies on the curve. The tangent to the curve at \( A \) cuts the \( y \)-axis at \( B \) and the normal to the curve at \( A \) cuts the \( x \)-axis at \( C \).

(i) Find the equation of the tangent \( AB \) and the equation of the normal \( AC \). \([4]\)

(ii) Find the length of \( BC \). \([2]\)

(iii) Find the coordinates of the point of intersection, \( E \), of \( OA \) and \( BC \). \([4]\)

It is given that \( y_1 = \tan x \) and \( y_2 = 2 \cos 2x + 1 \).

(i) State the period, in radians, of \( y_1 \) and the amplitude of \( y_2 \). \([2]\)

For the interval \( 0 \leq x \leq 2\pi \),

(ii) sketch, on the same diagram, the graphs of \( y_1 \) and \( y_2 \), \([3]\)

(iii) state the number of roots of the equation \( |\tan x| - 2 \cos 2x = 1 \), \([1]\)

(iv) find the range(s) of values of \( x \) for which \( y_1 \) and \( y_2 \) are both increasing as \( x \) increases. \([2]\)
The diagram shows part of the curve, \( y = \tan x \cos 2x \), and its maximum point \( M \).

(i) Show that \( \frac{dy}{dx} = 4 \cos^2 x - \sec^2 x - 2 \). [5]

(ii) Hence find the \( x \)-coordinate of \( M \). [3]

(b) A particle moves along the line \( y = \ln \sqrt\frac{5x}{x-2} \) in such a way that the \( x \)-coordinate is increasing at a constant rate of 0.4 units per second. Find the rate at which the \( y \)-coordinate of the particle is increasing at the instant when \( x = 2.5 \). [3]
10 (a) The function \( f \) is defined for all real values of \( x \) by \( f(x) = e^{2x} - 3e^{-2x} \).

(i) Show that \( f'(x) > 0 \) for all values of \( x \). \([2]\)

(ii) Show that \( f''(x) = hf(x) \), where \( h \) is an integer. \([2]\)

(iii) Find the value of \( x \) for which \( f''(x) = 0 \) in the form \( x = p \ln q \), where \( p \) and \( q \) are rational numbers. \([2]\)

(b) The function \( g \) is defined for all real values of \( x \) by \( g(x) = e^{2x} + 3e^{-2x} \).

The curve \( y = g(x) \) and the line \( x = \frac{1}{4} \ln 3 \) intersect at point \( Q \).

Show that the \( y \)-coordinate of \( Q \) is \( k\sqrt{3} \), where \( k \) is an integer. \([2]\)

11 Solutions to this question by accurate drawing will not be accepted.

The diagram, which is not drawn to scale, shows a triangle \( ABC \) with vertices \( A(2, 7) \), \( B(1, 0) \) and \( C(6, 5) \) respectively. \( E \) and \( F \) are points on \( BC \) and \( AC \) respectively for which \( AE \) is perpendicular to \( BC \) and \( BF \) bisects \( AC \). \( G \) is the point of intersection of lines \( AE \) and \( BF \).

Find

(i) the coordinates of \( G \), \([4]\)

(ii) the coordinates of the point \( D \) such that \( ABCD \) is a parallelogram, \([2]\)

(iii) the area of \( ABCD \). \([2]\)
The diagram above shows a quadrilateral in which $PX = a \text{ m}$ and $QX = b \text{ m}$. Angle $OQX = \text{Angle } OPX = \theta^\circ$ and $OQ$ is perpendicular to $OP$.

(i) Show that $OP = a \cos \theta + b \sin \theta$. \[3\]

(ii) It is given that the maximum length of $OP$ is $\sqrt{5}$ m and the corresponding value of $\theta$ is $63.43^\circ$. By using $OP = R \cos(\theta - \alpha)$, where $R > 0$ and $\theta$ is acute, find the value of $a$ and of $b$. \[5\]

(iii) Given that $OP = 2.15 \text{ m}$, find the value of $\theta$. \[2\]

End of Paper
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>( y = \frac{10}{\sqrt{x}} )</td>
<td>8(i)</td>
</tr>
<tr>
<td>1bi</td>
<td>(-p)</td>
<td></td>
</tr>
<tr>
<td>bii</td>
<td>( 2 + 3p )</td>
<td>8(ii)</td>
</tr>
<tr>
<td>2ii</td>
<td>( a = \frac{1}{4}, b = \frac{1}{8} )</td>
<td></td>
</tr>
<tr>
<td>3(i)</td>
<td>( m = -3 )</td>
<td></td>
</tr>
<tr>
<td>3(ii)</td>
<td>( x = \frac{1}{3} )</td>
<td></td>
</tr>
<tr>
<td>3(iii)</td>
<td>( y = \frac{x}{10x - 4} )</td>
<td></td>
</tr>
<tr>
<td>4(i)</td>
<td>( x = \frac{-2p}{3} )</td>
<td></td>
</tr>
<tr>
<td>4(ii)</td>
<td>(0, 0) is a minimum point.</td>
<td>8(iii)</td>
</tr>
<tr>
<td></td>
<td>maximum point at ( x = \frac{-2p}{3} )</td>
<td>8(iv)</td>
</tr>
<tr>
<td>4(iii)</td>
<td>( { p : 0 &lt; p &lt; 3 } )</td>
<td>9a(ii)</td>
</tr>
<tr>
<td>5(i)</td>
<td>( k = \frac{25}{16} )</td>
<td>9b</td>
</tr>
<tr>
<td>5(ii)</td>
<td>( k &lt; 1 )</td>
<td>10a(iii)</td>
</tr>
<tr>
<td>5(iii)</td>
<td>( k \leq \frac{47}{56} )</td>
<td>10b</td>
</tr>
<tr>
<td>6ai</td>
<td>17.3 years</td>
<td>11(i)</td>
</tr>
<tr>
<td>6a(ii)</td>
<td>( t = 38.0 )</td>
<td>11(ii)</td>
</tr>
<tr>
<td>6b</td>
<td>15.8</td>
<td>11(iii)</td>
</tr>
<tr>
<td>7(i)</td>
<td>Eqn of ( AB ): ( y = \frac{1}{2}x + \frac{1}{2} )</td>
<td>12(ii)</td>
</tr>
<tr>
<td></td>
<td>Eqn of ( AC ): ( y = -2x + 4 )</td>
<td>12(iii)</td>
</tr>
<tr>
<td>7(ii)</td>
<td>2.5 units</td>
<td></td>
</tr>
<tr>
<td>7(iii)</td>
<td>Coordinates of ( E = \left( \frac{6}{11}, \frac{1}{11} \right) )</td>
<td></td>
</tr>
<tr>
<td>Qn</td>
<td>Working</td>
<td>Marks</td>
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</tr>
<tr>
<td>1a</td>
<td>$3\log \left( x^{\frac{1}{2}}y \right) = 2 + 2\log x - \log y$</td>
<td></td>
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<tr>
<td></td>
<td>$3\log x + \log y = 2 + 2\log x - \log y$</td>
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<tr>
<td></td>
<td>$\log x + 2\log y = 2$</td>
<td></td>
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<tr>
<td></td>
<td>$\log \left( xy^2 \right) = 2$</td>
<td></td>
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<tr>
<td></td>
<td>$xy^2 = 10^2 = 100$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y = \sqrt{\frac{100}{x}} = \frac{10}{\sqrt{x}} = \frac{10\sqrt{x}}{x}$</td>
<td>[3]</td>
</tr>
<tr>
<td>b(i)</td>
<td>$\log_s \frac{1}{q} = \log_s 1 - \log_s q$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 0 - p = -p$</td>
<td></td>
</tr>
<tr>
<td>b(ii)</td>
<td>$\log_2 4q = \log_2 4 + \log_2 q$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 2 + \frac{\log_2 q}{\log_2 2}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 2 + 3p$</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Total</strong></td>
<td><strong>[7]</strong></td>
</tr>
<tr>
<td>2(i)</td>
<td>$\frac{d}{dx} (\sin x \cos x)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \sin x (-\sin x) + \cos x (\cos x)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \cos^2 x - \sin^2 x$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \cos^2 x - (1 - \cos^2 x)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 2\cos^2 x - 1$</td>
<td></td>
</tr>
<tr>
<td>2(ii)</td>
<td>$\int_0^\pi (2\cos^2 x - 1)dx = \left[ \sin x \cos x \right]_0^\pi$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$2\int_0^\frac{\pi}{2} (2\cos^2 x)dx = \frac{1}{2} + \left[ x \right]_0^{\frac{\pi}{2}}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \frac{\pi}{4} + \frac{\pi}{8} \Rightarrow a = \frac{1}{4}, b = \frac{1}{8}$</td>
<td></td>
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<tr>
<td></td>
<td><strong>Total</strong></td>
<td><strong>[6]</strong></td>
</tr>
<tr>
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<td>Marks</td>
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</table>
| 3(i) | The linear equation is \( \frac{1}{y} + \frac{1}{x} = m\left(\frac{1}{x}\right) + c \)  
Subst \( y = \frac{1}{6} \) and \( x = 1 \),  
\( 6 + 1 = m + c \Rightarrow m + c = 7 \)  
Subst \( y = \frac{1}{2} \) and \( x = \frac{1}{2} \),  
\( 2 + 2 = 2m + c \Rightarrow 2m + c = 4 \)  
\( m = -3 \) and \( c = 10 \) | [4] | | |
| (ii) | Since \( \frac{3}{y} + \frac{3}{x} = 3 \Rightarrow \frac{1}{y} + \frac{1}{x} = 1 \),  
\( 1 = \frac{-3}{x} + 10 \Rightarrow x = \frac{1}{3} \) | [2] | | |
| (iii) | \( \frac{1}{y} + \frac{1}{x} = -3\left(\frac{1}{x}\right) + 10 \)  
\( x + y = -3 + 10x \)  
\( xy = \frac{10x}{x} \)  
\( y = \frac{x}{10x - 4} \) | [2] | | |
<p>| | | | | Total [8] |</p>
<table>
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<th>Remarks</th>
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</table>
| 4(i) | \( y = x^3 + px^2 \)  
\[
\frac{dy}{dx} = 3x^2 + 2px = x(3x + 2p) 
\]  
For stationary point, \( \frac{dy}{dx} = 0 \)  
\[
\therefore x = 0 \text{ or } x = -\frac{2p}{3} 
\]  
When \( x = 0 \), \( y = 0 \).  
Therefore, \((0, 0)\) is a stationary point.  
The other \( x \)-coordinate of stationary point is \( x = -\frac{2p}{3} \) | [3] |
| (ii) | \( \frac{d^2y}{dx^2} = 6x + 2p \)  
When \( x = 0 \), \( \frac{d^2y}{dx^2} = 2p > 0 \) as \( p > 0 \)  
Therefore, \((0, 0)\) is a minimum point.  
When \( x = -\frac{2p}{3} \),  
\[
\frac{d^2y}{dx^2} = 6\left(-\frac{2p}{3}\right) + 2p = -2p < 0 \text{ as } p > 0 
\]  
Therefore, there is a maximum point at \( x = -\frac{2p}{3} \) | [3] |
| (iii) | \( y = x^3 + px^2 + px \)  
\[
\frac{dy}{dx} = 3x^2 + 2px + p 
\]  
Since \( \frac{dy}{dx} \neq 0 \), \( b^2 - 4ac < 0 \)  
\[
(2p)^2 - 4(3)(p) < 0 
\]  
\[
4p^2 - 12p < 0 
\]  
\[
4p(p-3) < 0 
\]  
The set is \( \{ p : 0 < p < 3 \} \) | [3] |
<p>| Total | | [9] |</p>
<table>
<thead>
<tr>
<th>Qn</th>
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<th>Marks</th>
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<th>Remarks</th>
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</thead>
</table>
| 5(i) | \( f(x) = k(x-2)^2 - (x-3)(x+2) \)  
= \( k(x^2 - 4x + 4) - (x^2 - x - 6) \)  
= \( kx^2 - 4kx + 4k - x^2 + x + 6 \)  
= \( (k-1)x^2 + (1-4k)x + 4k + 6 \)  
Since it touches the \( x \)-axis at one point, \( b^2 - 4ac = 0 \)  
\( (1-4k)^2 - 4(k-1)(4k+6) = 0 \)  
\( 25-16k = 0 \)  
\( k = \frac{25}{16} \) | [3] | | |
| (ii) | \( k < 1 \) | [1] | | |
| (iii) | \( (k-1)x^2 + (1-4k)x + 4k + 6 \leq 18 \)  
\( (k-1)x^2 + (1-4k)x + 4k - 12 \leq 0 \)  
\( b^2 - 4ac \leq 0 \) and \( k < 1 \)  
\( (1-4k)^2 - 4(k-1)(4k-12) \leq 0 \) and \( k < 1 \)  
\( 56k - 47 \leq 0 \) and \( k < 1 \)  
\( k \leq \frac{47}{56} \) and \( k < 1 \)  
The solution is \( k \leq \frac{47}{56} \) | [3] | | |

Total [7]
<table>
<thead>
<tr>
<th>Qn</th>
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<th>Remarks</th>
</tr>
</thead>
</table>
| 6a(i) | When $t = 0$, $m = 240$  
When $240e^{-0.04t} = 120$  
e^{-0.04t} = 0.5  
$t = \ln 0.5 / -0.04$  
t = 17.3  
No. of years = 17.3 | [2]   |       |        |
| a(ii) | $\frac{dm}{dt} = 240(-0.04)e^{-0.04t} = -9.6e^{-0.04t}$  
$-9.6e^{-0.04t} = -2.1$  
$\ln \left( \frac{2.1}{9.6} \right)$  
t = $\frac{\ln 0.5}{-0.04} = 38.0$ | [3]   |       |        |
| b    | $10\log \left( \frac{I_p}{k} \right) = 50 \Rightarrow \left( \frac{I_p}{k} \right) = 10^5$  
where $I_p$ = intensity of pump  
$\log \left( \frac{I_D}{k} \right) = 6.2 \Rightarrow \left( \frac{I_D}{k} \right) = 10^{6.2}$  
where $I_D$ = intensity of dishwasher  
$\frac{I_D}{I_p} = 10^{6.2 / 3} = 15.8$ | [3]   | Total | [8]    |
<table>
<thead>
<tr>
<th>Qn</th>
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<th>Marks</th>
<th>Total</th>
<th>Remarks</th>
</tr>
</thead>
</table>
| 7(i) | \( y = \left(6x + 2\right)^\frac{1}{3} \)  
\( \frac{dy}{dx} = \frac{1}{3} \left(6x + 2\right)^\frac{-2}{3} \cdot 6 = \frac{2}{\left(6x + 2\right)^\frac{2}{3}} \)  
When \( x = 1 \), \( \frac{dy}{dx} = \frac{2}{\left(6(1) + 2\right)^\frac{2}{3}} = \frac{1}{2} \)  
Eqn of \( AB \): \( y - 2 = \frac{1}{2}(x - 1) \Rightarrow y = \frac{1}{2}x + \frac{3}{2} \)  
Eqn of \( AC \): \( y - 2 = -2(x - 1) \Rightarrow y = -2x + 4 \) | | | | Use of chain rule  
Correct substitution |
| 7(ii) | When \( x = 0 \), \( y = 1.5 \)  
Coordinates of \( B = (0, 1.5) \)  
When \( y = 0 \), \( -2x + 4 = 0 \Rightarrow x = 2 \)  
Coordinates of \( C = (2, 0) \)  
\( BC = \sqrt{1.5^2 + 2^2} = 2.5 \) units | | | |
| 7(iii) | Gradient of \( OA = \frac{2 - 0}{1 - 0} = 2 \)  
Therefore, eqn of \( OA \): \( y = 2x \)  
Gradient of \( BC = \frac{1.5}{-2} = -\frac{3}{4} \)  
Therefore, eqn of \( BC \): \( y = -\frac{3}{4}x + \frac{3}{2} \)  
At \( E \), \( \frac{11x}{4} = \frac{3}{2} \Rightarrow x = \frac{6}{11} \)  
\( y = 2\left(\frac{6}{11}\right) = \frac{12}{11} = 1\frac{1}{11} \)  
Coordinates of \( E = \left(\frac{6}{11}, 1\frac{1}{11}\right) \) | | | |
<p>| | | | | Total 10 |</p>
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<th>Remarks</th>
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</table>
| 8i | Period of $y_1 = \pi$ radians  
Amplitude of $y_2 = 2$ | [2] | | |
<p>| ii |  |  | | |
| iv | $\frac{\pi}{2} &lt; x &lt; \pi$, $\frac{3\pi}{2} &lt; x &lt; 2\pi$ | [2] | | |</p>
<table>
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<th>Remarks</th>
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</thead>
</table>
| 9a(i) | \( y = \tan x \cos 2x \)  
\(\frac{dy}{dx} = \tan x (-2 \sin 2x) + \cos 2x (\sec^2 x)\)  
\(= \frac{\sin x}{\cos x} (-2 \times 2 \sin x \cos x) + \left( 2 \cos^2 x - 1 \right) \left( \frac{1}{\cos^2 x} \right)\)  
\(= -4 \sin^2 x + 2 - \sec^2 x\)  
\(= -4(1 - \cos^2 x) + 2 - \sec^2 x\)  
\(= 4 \cos^2 x - \sec^2 x - 2\) | [5]   |       |         |
| (ii) | When \( \frac{dy}{dx} = 0 \),  
\(4 \cos^2 x - \sec^2 x - 2 = 0\)  
\(4 \cos^4 x - 2 \cos^2 x - 1 = 0\)  
\(\cos^2 x = \frac{2 \pm \sqrt{4 - 4(4)(-1)}}{8}\)  
\(= 0.80902\)  
\(\cos x = 0.89945\)  
\(x = 0.452\) or 25.9°  
The \(x\)-coordinate of \(M\) is 0.452. | [3]   |       |         |
| b   | \( y = \frac{1}{2} \left[ \ln 5x - \ln(x - 2) \right] \)  
\(\frac{dy}{dx} = \frac{1}{2} \left( \frac{5}{5x} \right) - \frac{1}{2} \left( \frac{1}{x - 2} \right)\)  
\(= \frac{1}{2x} - \frac{1}{2(x - 2)}\)  
\(\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}\)  
When \( x = 2.5 \), \( \frac{dy}{dt} = \left( \frac{1}{5} - \frac{1}{2(0.5)} \right) \times 0.4 = -\frac{8}{25} = -0.32\)  
The rate is -0.32 units per second. | [3]   |       |         |

Total [11]
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<tr>
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<th>Remarks</th>
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</thead>
</table>
| 10(a)(i) | \( f(x) = e^{2x} - 3e^{-2x} \)  
\( f'(x) = 2e^{2x} + 6e^{-2x} \)  
Since \( e^{2x} > 0 \) and \( e^{-2x} > 0 \), \( f'(x) > 0 \) | [2] | | |
| (ii) | \( f''(x) = 4e^{2x} - 12e^{-2x} = 4(e^{2x} - 3e^{-2x}) \)  
Therefore \( f''(x) = 4f(x) \) | [2] | | |
| (iii) | \( e^{2x} - 3e^{-2x} = 0 \)  
\( e^{2x} = \frac{3}{e^{2x}} \)  
\( e^{4x} = 3 \)  
\( 4x \ln e = \ln 3 \)  
\( x = \frac{1}{4}\ln 3 \) | [2] | | |
| (b) | \( g(x) = e^{2x} + 3e^{-2x}, \)  
When \( x = \frac{1}{4} \ln 3, \)  
\( g(x) = e^{\frac{2(\frac{1}{4} \ln 3)}{2}} + ke^{\frac{-2(\frac{1}{4} \ln 3)}{2}} = e^{\frac{1}{2} \ln 3} + ke^{\frac{1}{2} \ln 3} \)  
\( = \sqrt{3} + \frac{3}{\sqrt{3}} = 2\sqrt{3} \)  
Therefore the \( y \)-coordinate is \( 2\sqrt{3} \). | [2] | | |
<p>| | | Total | [8] | |</p>
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<tr>
<td>11i</td>
<td><strong>Mid-point of AC, ( F = \left( \frac{2 + 6}{2}, \frac{7 + 5}{2} \right) = (4, 6) )</strong>&lt;br&gt;<strong>Gradient of BF</strong> ( = \frac{6 - 0}{4 - 1} = 2 )&lt;br&gt;<strong>Eqn of BF:</strong> ( y - 0 = 2(x - 1) \Rightarrow y = 2x - 2 )&lt;br&gt;<strong>Gradient of BC</strong> ( = \frac{5 - 0}{6 - 1} = 1 )&lt;br&gt;<strong>Gradient of AE</strong> ( = -1 )&lt;br&gt;<strong>Eqn of AE:</strong> ( y - 7 = -1(x - 2) \Rightarrow y = -x + 9 )&lt;br&gt;(-x + 9 = 2x - 2 )&lt;br&gt;( x = 3\frac{1}{3} )&lt;br&gt;( \therefore y = -3\frac{1}{3} + 9 = 5\frac{1}{3} )&lt;br&gt;( G \left( 3\frac{1}{3}, 5\frac{1}{3} \right) )**</td>
<td></td>
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<tr>
<td>(ii)</td>
<td><strong>Let ((x, y)) be coordinates of D.</strong>&lt;br&gt;( \left( \frac{1 + x}{2}, \frac{0 + y}{2} \right) = (4, 6) )&lt;br&gt;( \Rightarrow x = 7, y = 12 )&lt;br&gt;<strong>Coordinates of D</strong> ( = (7, 12) )</td>
<td></td>
<td></td>
<td>[2]</td>
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<tr>
<td>(iii)</td>
<td><strong>Area of ABCD</strong> ( = \frac{1}{2} \left</td>
<td>\begin{array}{ccc} 1 &amp; 2 &amp; 1 \ 2 &amp; 7 &amp; 0 \ 1 &amp; 6 &amp; 7 \ 2 &amp; 12 &amp; 7 \end{array} \right</td>
<td>= 30 \text{ square units} )**</td>
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<td><strong>Total</strong></td>
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<td>[8]</td>
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<tr>
<td>12</td>
<td><img src="image.png" alt="Diagram" /></td>
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</table>
| (i) | \[
\cos \theta = \frac{SP}{a} \Rightarrow SP = a \cos \theta \\
\sin \theta = \frac{OS}{b} \Rightarrow OS = b \sin \theta \\
OP = SP + OS \\
OP = a \cos \theta + b \sin \theta.
\] | [3] | | |
| (ii) | \[
\sqrt{R} = \sqrt{a^2 + b^2} \Rightarrow a^2 + b^2 = 5
\] | | | |
| | Max. value of \(OP\) occurs at \(\theta = 63.43^\circ\). \\
| | \[\cos(\theta - \alpha) = 1 \Rightarrow \theta - \alpha = 0 \Rightarrow \alpha = \theta = 63.43\] \\
| | \[\tan \alpha = \frac{b}{a} = \frac{b}{a} = \tan 63.43 = 1.9996 \Rightarrow b = 1.9996a\] \\
| | Subst \(b = 1.9996a\) in \(a^2 + b^2 = 5\) \\
| | \[a^2 + (1.9996a)^2 = 5 \Rightarrow a = 1.00\] \\
| | \(\therefore b = 2.00\) | [5] | | |
| (iii) | \[
\cos \theta + 2 \sin \theta = 2.15 \\
\sqrt{5} \cos(\theta - 63.43) = 2.15 \\
(\theta - 63.43) = \cos^{-1} \left( \frac{2.15}{\sqrt{5}} \right) \\
\theta = 79.4^\circ \text{ or } 47.5^\circ
\] | [2] | | |
| | Total | [10] | | |
READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces provided at the top of this page and on all the work you hand in. Write in dark blue or black pen. You may use a soft pencil for any diagrams or graphs. Do not use paper clips, highlighter, glue or correction fluid.

Answer all the questions. Write your answers on the separate Answer Paper provided. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. The use of an electronic calculator is expected, where appropriate. You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work and mark sheet securely together. The number of marks is given in brackets [ ] at the end of each question or part question. The total number of marks for this paper is 80.
2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion

\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]
where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. TRIGONOMETRY

Identities

\[
\sin^2 A + \cos^2 A = 1
\]
\[
\sec^2 A = 1 + \tan^2 A
\]
\[
\csc^2 A = 1 + \cot^2 A
\]
\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
\sin 2A = 2 \sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]
\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \Delta ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\Delta = \frac{1}{2} ab \sin C
\]
1 The straight line \( y - 1 = 2m \) does not intersect the curve \( y = x + \frac{m^2}{x} \).

Find the largest integer value of \( m \). [5]

**Solutions**

\[
\begin{align*}
    y &= 2m + 1 \quad \text{--- (1)} \\
    y &= x + \frac{m^2}{x} \quad \text{--- (2)}
\end{align*}
\]

(1) = (2): \( x + \frac{m^2}{x} = 2m + 1 \) \[M1\]

\[
\begin{align*}
    x^2 - 2mx - x + m^2 &= 0 \\
    x^2 - (2m + 1)x + m^2 &= 0
\end{align*}
\] \[M1\] -- simplification

Line does not intersect curve, \( b^2 - 4ac < 0 \)

\[
\begin{align*}
    \left[-(2m + 1)\right]^2 - 4(1)(m^2) < 0 \\
    (2m + 1 + 2m)(2m + 1 - 2m) < 0 \\
    4m + 1 < 0
\end{align*}
\] \[M1\]

\[
    m < -\frac{1}{4}
\] \[A1\]

The largest integer value of \( m \) is \(-1\). \[A1\]

2 The line \( 2y + x = 5 \) intersects the curve \( y^2 = 6 - xy \) at the points \( P \) and \( Q \).

Determine, with explanation, if the point \((1, 2)\) lies on the line joining the midpoint of \( PQ \) and \((3, 1)\). [5]

**Solutions**

\[
\begin{align*}
    x &= 5 - 2y \quad \text{--- (1)} \\
\end{align*}
\]

Sub (1) into \( y^2 = 6 - xy \)

\[
\begin{align*}
    y^2 &= 6 - (5 - 2y)y \\
    y^2 &= 6 - 5y + 2y^2 \\
    y^2 - 5y + 6 &= 0 \\
    (y - 3)(y - 2) &= 0
\end{align*}
\] \[M1\] – Substitution

Hence \( y = 3 \) or \( y = 2 \) \[A1\]

Correspondingly, \( x = 5 - 2(3) \) or \( x = 5 - 2(2) \)

\[
\begin{align*}
    x &= -1 \quad \text{or} \quad x = 1
\end{align*}
\]

The coordinates of \( P \) and \( Q \) are \((-1, 3)\) and \((1, 2)\).

Midpoint of \( PQ = \left( \frac{-1+1}{2}, \frac{3+2}{2} \right) = (0, 2.5) \) \[A1\]

Equation of line joining midpoint of \( PQ \) and \((3, 1)\) is \[
\frac{y - 1}{2.5 - 1} = \frac{x - 3}{0 - 3}
\] \[M1\]

\[
    y = -\frac{1}{2}x + \frac{5}{2}
\]

When \( x = 1 \), \( y = -\frac{1}{2}(1) + \frac{5}{2} = 2 \)
Therefore, the point \((1, 2)\) lies on the line joining midpoint of \(PQ\) and \((3, 1)\) \([A1]\) – conclusion

**Alternative method**

Let \(R\) be the coordinates of the midpoint of \(PQ\), \(S\) be the point \((3, 1)\) and \(T\) be the point \((1, 2)\). Find gradient of \(RT\) and gradient of \(RS\) and conclude that point \(T\) lies on \(RS\) due to collinearity.

3 (i) Sketch on the same graph \(y = |3 \cos 2x|\) and \(y + \frac{8}{3\pi}x = 2\) for \(0 \leq x \leq \pi\). \([3]\)

(ii) Hence, showing your working clearly, deduce the number of solutions in \(|\cos 2x| - \frac{2}{3} + \frac{8x}{3\pi} = 0\) in the interval \(0 \leq x \leq \pi\). \([2]\)

**Solutions**

(i)

(ii) \(|\cos 2x| - \frac{2}{3} + \frac{8x}{3\pi} = 0\)

\(|3 \cos 2x| = 2 - \frac{8}{\pi}x\)

\(y = 2 - \frac{8}{\pi}x\) \((y\)-intercept = 2; \(x\)-intercept = \(2 + \frac{8}{\pi} = \frac{\pi}{4}\)) \([M1]\) – Si line NOT required

There is one solution. \([A1]\)
4 (i) Find the value of $a$ and of $b$ if the curve $f(x) = ax + \frac{b}{x}$ where $x \neq 0$ has a stationary point at $(-2, -8)$. [4]

(ii) By considering the sign of $f'(x)$, determine the nature of the stationary point. [2]

**Solutions**

(i) $f(x) = ax + \frac{b}{x}$

Sub $x = -2$, $f(x) = -8$

$-8 = -2a - \frac{b}{2}$

$4a + b = 16$ ---- (1) [B1]

$f'(x) = a - \frac{b}{x^2}$. When $x = -2$, $f'(x) = 0$ [M1] – for $f'(x)$

$0 = a - \frac{b}{4} \Rightarrow b = 4a$ ---- (2)

Sub (2) into (1): $4a + 4a = 16$ [M1] – solve simultaneous equations

$a = 2$

Hence $b = 2(4) = 8$ [A1] – both correct

(ii) $f'(x) = 2 - \frac{8}{x^2}$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-2^-$</th>
<th>$-2$</th>
<th>$-2^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sign of $f'(x)$</td>
<td>+</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Sketch of tangent</td>
<td>/</td>
<td>–</td>
<td>\</td>
</tr>
</tbody>
</table>

[M1] – First derivative test

$(-2, -8)$ is a maximum point. [A1] – Awarded only with correct first derivative test

5 It is given that $\int f'(x) dx = \frac{x}{2} - \frac{\sin kx}{8} + c$ where $c$ is a constant of integration, and that $\int_0^\pi f'(x) dx = \frac{\pi}{16} - \frac{1}{8}$.

(i) Show that $k = 4$. [2]

(ii) Hence find $f'(x)$, expressing your answer in $\sin^2 px$, where $p$ is a constant. [2]

(iii) Find the equation of the curve $y = f(x)$ given that the point $\left(\frac{\pi}{4}, 0\right)$ lies on the curve. [2]
Solutions

(i) \[ \int_0^\pi f'(x) \, dx = \frac{\pi}{16} - \frac{1}{8} \]

\[ \frac{\pi}{8} - \frac{\sin k \left( \frac{\pi}{8} \right)}{8} = \frac{\pi}{16} - \frac{1}{8} \] \[ \text{[M1]} \]

\[ \sin \left( \frac{k \pi}{8} \right) = 1 \]

\[ \frac{k \pi}{8} = \frac{\pi}{2} \]

\[ k = 4 \text{ (shown)} \]

(ii) \[ \int f'(x) \, dx = \frac{x}{2} - \frac{\sin 4x}{8} + c \]

\[ f'(x) = \frac{1}{2} - \frac{1}{8} \cos 4x \] \[ \text{[M1] -- Differentiation} \]

\[ = \frac{1}{2} \cos 4x \]

\[ = \frac{1}{2} \left( 1 - 2 \sin^2 2x \right) \]

\[ = \sin^2 2x \] \[ \text{[A1]} -- \text{Upon correct application of double angle formula} \]

(iii) \[ \int f'(x) \, dx = f(x) = \frac{x}{2} - \frac{\sin 4x}{8} + c \]

At \( \left( \frac{\pi}{4}, 0 \right) \), \[ 0 = \frac{\pi}{8} - 0 + c \] \[ \text{[M1]} \]

\[ c = -\frac{\pi}{8} \]

\[ f(x) = \frac{x}{2} - \frac{\sin 4x}{8} - \frac{\pi}{8} \] \[ \text{[A1]} \]

6 (a) The length of each side of a square of area \( (49 + 20\sqrt{6}) \) m\(^2\) can be expressed in the form \( \sqrt{c + \sqrt{d}} \) m where \( c \) and \( d \) are integers and \( c < d \) .

Find the value of \( c \) and of \( d \). \[ [3] \]

(b) A parallelogram with base equals to \( (4 - \sqrt{12}) \) m has an area of \( (22 - \sqrt{48}) \) m\(^2\).

Find, without using a calculator, the height of the parallelogram in the form \( (p + q\sqrt{3}) \) m. \[ [3] \]

Solutions

(a) \( (\sqrt{c + \sqrt{d}})^2 = 49 + 20\sqrt{6} \)

\[ c + d + 2\sqrt{cd} = 49 + 20\sqrt{6} \] \[ \text{[M1] -- correct expansion} \]

\[ c + d = 49 \]
The diagram shows part of the graph \( y = a|x + b| + c \). The graph cuts the \( x \)-axis at \( A(p, 0) \) and at \( B(0.5, 0) \). The graph has a vertex point at \( V(-2, 5) \) and \( y \)-intercept, \( d \).

(i) Explain why \( p = -4.5 \). [1]

(ii) Determine the value of each of \( a \), \( b \) and \( c \). [4]

(iii) State the set of values of \( k \) for which the line \( y = kx + d \) intersects the graph at two distinct points. [2]
Solutions

(i) \( p + 0.5 \) \( \frac{2}{2} = -2 \) \[B1\]

\( p = -4.5 \)

(ii) \( y \)-coordinate of vertex point, \( c = 5 \) \[B1\]

\( b = 2 \) \[B1\]

\( y = a | x + b | + c \)

\( y = a | x + 2 | + 5 \)

At \( B, 0 = a | 0.5 + 2 | + 5 \) \[M1\]

\( a = -2 \) \[A1\]

(iii) Gradient of \( AV = \frac{5 - 0}{-2 + 4.5} = 2 \)

Gradient of \( VB = -2 \) \[B1\] – Any one

Hence \(-2 < k < 2 \) \[B1\]

8 (i) Differentiate \( x^3 \ln x \) with respect to \( x \). \[2\]

(ii) Hence find \( \int \frac{x^2 \ln x}{2} \, dx \). \[4\]

Solutions

(i) \( \frac{d}{dx} \left( x^3 \ln x \right) = x^3 \left( \frac{1}{x} \right) + (\ln x)(3x^2) \) \[M1\] – Product Rule

\( = x^2 + 3x^2 \ln x \) \[A1\]

(ii) \( \frac{d}{dx} \left( x^3 \ln x \right) = x^2 + 3x^2 \ln x \)

\( \frac{x^2 \ln x}{2} = \frac{1}{6} \frac{d}{dx} (x^3 \ln x) - \frac{x^2}{6} \) \[M1\]

\( \int \frac{x^2 \ln x}{2} \, dx = \frac{1}{6} x^3 \ln x - \frac{1}{6} \int x^2 \, dx \) \[M1\]

\( = \frac{1}{6} x^3 \ln x - \frac{1}{18} x^3 + c \) \[A1\]

9 (a) If \( 32^y \times 5^4^y = 2^{4^y+4} \times 5^{3^y-1} \), determine the value of \( 10^y \). \[3\]

(b) (i) Sketch on the same axes, the graphs of \( y = x^{-2} \) and \( y = \sqrt{3}x \). \[2\]

(ii) Find the point of intersection between the graphs. \[3\]
Solutions

\[32^y \times 5^4y = 2^{4y+4} \times 5^{3y-1}\]

\[2^{5y} \times 5^{4y} = 2^{4y} (2^4) \times 5^{3y} \left( \frac{1}{5} \right) \quad [M1] \text{-- splitting}\]

\[\frac{2^{3y}}{2^{4y}} \times 5^{4y} = (2^4) \times \left( \frac{1}{5} \right) \quad [M1] \text{-- using Laws of Indices}\]

\[2^y \times 5^y = \frac{16}{5} \quad [A1]\]

\[
\begin{align*}
\frac{1}{x^2} &= \sqrt{3x} \\
\frac{1}{x^4} &= 3x \\
x &= \sqrt[4]{\frac{1}{3}} = 0.80274 \quad [A1]
\end{align*}
\]

\[y = \frac{1}{0.80274^2} = 1.55\]

The point of intersection is (0.803, 1.55). \( [A1] \text{-- 3 s.f.} \)

10 (i) Express \(\frac{x+1}{x(x+3)^2-(x+3)^2}\) in partial fractions. \( [5] \)

(ii) Hence find the value of \(\int_2^3 \frac{x+1}{x(x+3)^2-(x+3)^2} \, dx\) giving your answer to 2 decimal places. \( [3] \)

Solutions

(i) \[\frac{x+1}{x(x+3)^2-(x+3)^2} = \frac{x+1}{(x-1)(x+3)^2} = \frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{(x+3)^2}\] \[M1\]

\[x + 1 = A(x+3)^2 + B(x-1)(x+3) + C(x-1)\]
Sub $x = -3$: $-2 = C(-4) \Rightarrow C = \frac{1}{2}$ \[A1\]

Sub $x = 1$: $2 = A(16) \Rightarrow A = \frac{1}{8}$ \[A1\]

Sub $x = 0$: $1 = \left(\frac{1}{8}\right)(9) + B(-1)(3) + \left(\frac{1}{2}\right)(-1) \Rightarrow B = -\frac{1}{8}$ \[A1\]

\[
\frac{x+1}{x(x+3)^2 - (x+3)^2} = \frac{x+1}{(x-1)(x+3)^2} = \frac{1}{8(x-1)} - \frac{1}{8(x+3)} + \frac{1}{2(x+3)^2} \] \[A1\]

(ii) \[
\int^3_x \frac{x+1}{x(x+3)^2 - (x+3)^2} \, dx = \int^3_x \frac{1}{8(x-1)} - \frac{1}{8(x+3)} + \frac{1}{2(x+3)^2} \, dx
\]

\[= \left[ \frac{1}{8} \ln(x-1) - \frac{1}{8} \ln(x+3) + \frac{1}{2} \frac{(x+3)^{-1}}{-1} \right]^3_x \]

\[= \left[ \frac{1}{8} \ln \left( \frac{x-1}{x+3} \right) - \frac{1}{2} \frac{1}{(x+3)^2} \right]^3_x \]

\[= \frac{1}{8} \ln \frac{1}{3} - \frac{1}{12} - \left( \frac{1}{8} \ln \frac{1}{5} - \frac{1}{10} \right) \]

\[= 0.08 \text{ (to 2 d.p.)} \] \[A1\]

11 (a) Show that \[\frac{d}{d\theta} (\cot \theta) = -\frac{1}{\sin^2 \theta}.\] \[2\]

(b) In the diagram below, a straight wooden plank $PQ$, of length 12.5 m is supported at an angle $\theta$ to a vertical wall $XY$ by a taut rope fixed to a hook at $H$.

The length of the rope $BH$ from the wall is 2.7 m. The end $P$ of the plank is at a vertical height $h$ m above $H$. 

![Diagram of a wooden plank supported by a rope]
(i) Show that \( h = 12.5 \cos \theta - \frac{2.7 \cos \theta}{\sin \theta} \). \[2\]

(ii) Using part (a), determine the value of \( \sin \theta \) for which \( \frac{dh}{d\theta} = 0 \). \[2\]

(iii) Hence or otherwise, show that as \( \theta \) varies, \( h \) attains a maximum value and find this value. \[3\]

**Solutions**

(a) \[
\frac{d}{d\theta} (\cot \theta) = \frac{d}{d\theta} \left( \frac{\cos \theta}{\sin \theta} \right) = \frac{\sin \theta (- \sin \theta) - \cos \theta (\cos \theta)}{\sin^2 \theta} \]

\[= \frac{- (\sin^2 \theta + \cos^2 \theta)}{\sin^2 \theta} \]

\[= - \frac{1}{\sin^2 \theta} \] (shown) \[M1\] – Quotient Rule

(b)(i) \[
\cos \theta = \frac{XY}{12.5} \Rightarrow XY = 12.5 \cos \theta
\]

\[
\tan \theta = \frac{2.7}{BY} \Rightarrow BY = \frac{2.7}{\tan \theta} = \frac{2.7 \cos \theta}{\sin \theta} \]

\[
h = XY - BY = 12.5 \cos \theta - \frac{2.7 \cos \theta}{\sin \theta} \]

\[A1\] – clear working above

(ii) \[
\frac{dh}{d\theta} = -12.5 \sin \theta - 2.7 \left( \frac{d}{d\theta} \left( \frac{\cos \theta}{\sin \theta} \right) \right) = -12.5 \sin \theta + \frac{2.7}{\sin^2 \theta} \]

\[
\frac{dh}{d\theta} = 0 \Rightarrow -12.5 \sin \theta + \frac{2.7}{\sin^2 \theta} = 0 \]

\[M1\]
\[ \sin \theta = \sqrt{\frac{2.7}{12.5}} = 0.6 \quad [A1] \]

(iii) 
\[ \sin \theta = \frac{3}{5} \] giving rise to \[ \cos \theta = \frac{4}{5} \] \[ [M1] \]

\[ \frac{d^2 h}{d\theta^2} = -12.5 \cos \theta - \frac{5.4 \cos \theta}{\sin^3 \theta} \]
\[ = -12.5 \left( \frac{4}{5} \right) - \frac{5.4 \left( \frac{4}{5} \right)}{\left( \frac{3}{5} \right)^3} = -30 < 0 \quad [M1] \text{ – verify max} \]

Max \( h = 12.5 \left( 0.8 \right) \frac{2.7 \left( 0.8 \right)}{\left( 0.6 \right)} = 6.4 \text{ m} \quad [A1] \]

**Alternative method**

\[ \theta = 36.870^\circ \]
\[ \frac{d^2 h}{d\theta^2} = -12.5 \cos \theta + 2.7(-2)(\sin \theta)^{-3}(\cos \theta) \]
\[ = -12.5 \cos \theta - \frac{5.4 \cos \theta}{\sin^3 \theta} \quad [M1] \text{ – first or second derivative test} \]

When \( \theta = 36.870^\circ \),
\[ \frac{d^2 h}{d\theta^2} = -12.5 \cos 36.870^\circ - \frac{5.4 \cos 36.870^\circ}{\sin^3 36.870^\circ} = -30.0 < 0 \quad [M1] \text{ – verify max} \]

\( h \) is maximum when \( \theta \) is \( 36.870^\circ \).

Maximum \( h = 12.5 \cos 36.870^\circ - \frac{2.7 \cos 36.870^\circ}{\sin 36.870^\circ} \)
\[ = 6.4 \text{ m} \quad [A1] \]
Solutions to this question by accurate drawing will not be accepted.

12 The figure shows a quadrilateral $PTSR$ for which $P$ is $(2, 4)$, $T$ is $(-3, 0)$, $S$ is $(-5, a)$, $R$ is $(-2k, 12-3k)$ and angle $QPT$ is a right angle. $RQP$ is a straight line with point $Q$ lying on the $y$-axis.

(i) Find the value of $k$. [2]

(ii) Given that angle $STU = 45^\circ$, determine the value of $a$. [2]

(iii) A line passing through $Q$ and is perpendicular to $TS$ cuts the $x$-axis at $V$. Find the value of $V R^2$. [5]

**Solutions**

(i) Gradient of $PT = \frac{4}{5}$

Gradient of $PR$, $\frac{12 - 3k - 4}{-2k - 2} = -\frac{5}{4}$ [M1]

$4(8 - 3k) = 5(2k + 2)$

$-22k = -22$

$k = 1$ [A1]

(ii) angle $STU = 45^\circ \Rightarrow$ gradient of $ST = -1$ [M1]

$\frac{a - 0}{-5 + 3} = -1$ [A1]

$a = 2$
(iii) Equation of \( PR \) is \( y - 4 = -\frac{5}{4}(x - 2) \) \[ \text{[M1]} \]

\[
-4(y - 4) = 5(x - 2) \\
4y + 5x = 26
\]

At \( Q, x = 0 \)

\( 4y = 26 \implies y = 6.5 \)

\( Q(0, 6.5) \) \[ \text{[A1]} \]

Equation of line passing through \( Q \) and perpendicular to \( TS \) is

\[
y - 6.5 = \frac{-1}{-1}(x - 0) \\
y = x + 6.5
\]

At \( V, y = 0 \). Hence \( x = -6.5 \)

\( V(-6.5, 0) \) \[ \text{[A1]} \]

\[
VR^2 = (-2 + 6.5)^2 + 9^2 \\
= 101.25
\]

END OF PAPER
READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces provided at the top of this page and on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighter, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work and mark sheet securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.
2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion
\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]
where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. TRIGONOMETRY

Identities
\[
\sin^2 A + \cos^2 A = 1 \\
\sec^2 A = 1 + \tan^2 A \\
\cos ec^2 A = 1 + \cot^2 A \\
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \\
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \\
\sin 2A = 2 \sin A \cos A \\
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A \\
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \triangle ABC \)
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \\
a^2 = b^2 + c^2 - 2bc \cos A \\
\Delta = \frac{1}{2} ab \sin C
\]
1 The straight line \( y - 1 = 2m \) does not intersect the curve \( y = x + \frac{m^2}{x} \).

Find the largest integer value of \( m \). [5]

2 The line \( 2y + x = 5 \) intersects the curve \( y^2 = 6 - xy \) at the points \( P \) and \( Q \).

Determine, with explanation, if the point \((1, 2)\) lies on the line joining the midpoint of \( PQ \) and \((3,1)\). [5]

3 (i) Sketch on the same graph \( y = |3 \cos 2x| \) and \( y + \frac{8}{3\pi} x = 2 \) for \( 0 \leq x \leq \pi \). [3]

(ii) Hence, showing your working clearly, deduce the number of solutions in \( |\cos 2x| - \frac{2}{3} + \frac{8x}{3\pi} = 0 \) in the interval \( 0 \leq x \leq \pi \). [2]

4 (i) Find the value of \( a \) and of \( b \) if the curve \( f(x) = ax + \frac{b}{x} \) where \( x \neq 0 \) has a stationary point at \((-2, -8)\). [4]

(ii) By considering the sign of \( f'(x) \), determine the nature of the stationary point. [2]

5 It is given that \( \int f'(x) \, dx = \frac{x}{2} - \frac{\sin kx}{8} + c \) where \( c \) is a constant of integration, and that \( \int_0^\pi f'(x) \, dx = \frac{\pi}{16} - \frac{1}{8} \).

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6 (a) The length of each side of a square of area \((49 + 20\sqrt{6})\) m\(^2\) can be expressed in the form \(\sqrt{c + \sqrt{d}}\) m where \(c\) and \(d\) are integers and \(c < d\).
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Find, without using a calculator, the height of the parallelogram in the form \((p + q\sqrt{3})\) m. \[3\]

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(i) Explain why \(p = -4.5\). \[1\]

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(iii) State the set of values of \(k\) for which the line \(y = kx + d\) intersects the graph at two distinct points. \[2\]

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(i) Show that \( h = 12.5 \cos \theta - \frac{2.7 \cos \theta}{\sin \theta} \). [2]

(ii) Using part (a), determine the value of \( \sin \theta \) for which \( \frac{dh}{d\theta} = 0 \). [2]

(iii) Hence or otherwise, show that as \( \theta \) varies, \( h \) attains a maximum value and find this value. [3]
Solutions to this question by accurate drawing will not be accepted.

12 The figure shows a quadrilateral \( PTSR \) for which \( P \) is (2, 4), \( T \) is (−3, 0), \( S \) is (−5, \( a \)), \( R \) is (−2\( k \), 12−3\( k \)) and angle \( QPT \) is a right angle. \( RQP \) is a straight line with point \( Q \) lying on the \( y \)-axis.

(i) Find the value of \( k \). [2]

(ii) Given that angle \( STU = 45^\circ \), determine the value of \( a \). [2]

(iii) A line passing through \( Q \) and is perpendicular to \( TS \) cuts the \( x \)-axis at \( V \). Find the value of \( VR^2 \). [5]

END OF PAPER
2018 CGS A Math Prelim Paper 1 Answer Key

<table>
<thead>
<tr>
<th>Qn</th>
<th>Ans Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( m = -1 )</td>
</tr>
<tr>
<td>2</td>
<td>Yes</td>
</tr>
</tbody>
</table>

3(i) \[
y = |3 \cos 2x|
\]

3(ii) 1 solution

4(i) \( a = 2 ; \ b = 8 \)

4(ii) Maximum point

5(ii) \( \sin^2 2x \)

5(iii) \[
f(x) = \frac{x}{2} - \frac{\sin 4x}{8} - \frac{\pi}{8}
\]

6(a) \( c = 24 ; \ d = 25 \)

6(b) \( h = (16 + 7\sqrt{3}) \text{ m} \)

7(ii) \( a = -2 ; \ b = 2 ; \ c = 5 \)

7(iii) \(-2 < k < 2 \)

8(i) \( x^2 + 3x^2 \ln x \)

8(ii) \[
\frac{1}{6} x^3 \ln x - \frac{1}{18} x^3 + c
\]

9(a) 3.2
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>9(b)(i)</td>
<td>$(0.803, 1.55)$</td>
</tr>
<tr>
<td>10(i)</td>
<td>$\frac{1}{8(x-1)} - \frac{1}{8(x+3)} + \frac{1}{2(x+3)^2}$</td>
</tr>
<tr>
<td>10(ii)</td>
<td>0.08</td>
</tr>
<tr>
<td>11(b)(ii)</td>
<td>$\sin \theta = \frac{3}{5}$</td>
</tr>
<tr>
<td>11(b)(iii)</td>
<td>$h = 6.4 \text{ m}$</td>
</tr>
<tr>
<td>12(i)</td>
<td>$k = 1$</td>
</tr>
<tr>
<td>12(ii)</td>
<td>$a = 2$</td>
</tr>
<tr>
<td>12(iii)</td>
<td>101.25</td>
</tr>
</tbody>
</table>
READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces provided at the top of this page and on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighter, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work and mark sheet securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.
1. ALGEBRA

**Quadratic Equation**
For the equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

**Binomial expansion**
\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]
where \( n \) is a positive integer and 
\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}
\]

2. TRIGONOMETRY

**Identities**
\[
\begin{align*}
\sin^2 A + \cos^2 A &= 1 \\
\sec^2 A &= 1 + \tan^2 A \\
\csc^2 A &= 1 + \cot^2 A \\
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\tan(A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A \\
\tan 2A &= \frac{2 \tan A}{1 - \tan^2 A}
\end{align*}
\]

**Formulae for \(\triangle ABC\)**
\[
\begin{align*}
\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\
\sin A &= \sqrt{b^2 + c^2 - 2bc \cos A} \\
\Delta &= \frac{1}{2} ab \sin C
\end{align*}
\]
1. (i) Write down and simplify the first four terms in the expansion \((2x - \frac{p}{x^2})^5\) in descending powers of \(x\), where \(p\) is a non-zero constant. [3]

(ii) Given that the coefficient of \(x^{-1}\) in the expansion \((4x^3 - 1)(2x - \frac{p}{x^2})^5\) is \(-160p^2\), find the value of \(p\). [4]

2. Variables \(x\) and \(y\) are related by the equation \(y = ax^b + 3\) where \(a\) and \(b\) are constants. When \(\log(y - 3)\) is plotted against \(\log x\), a straight line is obtained. The straight line passes through \((-2.5, 8)\) and \((3.5, -4)\). Find

(i) the value of \(a\) and of \(b\), [5]

(ii) the coordinates of the point on the line when \(x = 10^6\). [3]

3. (a) Given that \(x = \log_3 a\) and \(y = \log_3 b\), express \(\log_3 \frac{\sqrt{b^5}}{27a^4}\) in terms of \(x\) and \(y\). [3]

(b) Solve the equation \(\log_2 (5x + 3)^2 - \log_{5x+3} 2 = 1\). [5]

4. (i) The roots of the equation \(2x^2 + px - 8 = 0\), where \(p\) is a constant, are \(\alpha\) and \(\beta\).

The roots of the equation \(4x^2 - 24x + q = 0\), where \(q\) is a constant, are \(\alpha + 2\beta\) and \(2\alpha + \beta\). Find the values of \(p\) and \(q\). [6]

(ii) Hence form the quadratic equation whose roots are \(\alpha^3\) and \(\beta^3\). [3]

5. The equation of a circle \(C\) is \(x^2 + y^2 - 12x - 8y - 13 = 0\).

(i) Find the centre and radius of \(C\). [3]

(ii) Find the equation of the line which passes through the centre of \(C\) and is perpendicular to the line \(4x + 7y = 117\). [3]

(iii) Show that the line \(4x + 7y = 117\) is a tangent to \(C\) and state the coordinates of the point where the line touches \(C\). [5]
6  (a) A car travelling on a straight road passes through a traffic light \(X\) with speed of \(90 \text{ m/s}\). The acceleration, \(a \text{ m/s}^2\) of the car, \(t\) seconds after passing \(X\), is given by \(a = 20 - 8t\). Determine with working whether the car is travelling towards or away from \(X\) when it is travelling at maximum speed. \([4]\)

(b) A particle moving in a straight line such that its displacement, \(s \text{ m}\), from the fixed point \(O\) is given by \(s = 7 \sin t - 2 \cos 2t\), where \(t\) is the time in seconds, after passing through a point \(A\).

(i) Find the value of \(t\) when the particle first comes to instantaneous rest. \([5]\)

(ii) Find the total distance travelled by the particle during the first 4 seconds of its motion. \([3]\)

7  (i) Show that \(\frac{d}{dx} \left( \frac{x + 2}{\sqrt{x - 1}} \right) = \frac{x - 4}{2\sqrt{(x - 1)^3}}\). \([3]\)

The diagram shows the line \(x = 15\) and part of the curve \(y = \frac{12(x - 4)}{\sqrt{(x - 1)^3}}\). The curve intersect the \(x\)-axis at the point \(A\). The line through \(A\) with gradient \(\frac{4}{9}\) intersects the curve again at the point \(B\).

(ii) Verify that the \(y\)-coordinate of \(B\) is \(2\frac{2}{3}\). \([4]\)

(iii) Determine the area of the region bounded by the curve, the \(x\)-axis, the line \(x = 15\) and the line \(AB\). \([4]\)
8 A curve has equation given by  \( y = \frac{e^{4x-3}}{8e^{2x}} \).

(i) Show that \( \frac{dy}{dx} = \frac{e^{2x-3}}{4} \). \[2\]

(ii) Given that \( x \) is decreasing at a rate of \( 4e^2 \) units per second, find the exact rate of change of \( y \) when \( x = 1 \). \[3\]

(iii) The curve passes through the \( y \)-axis at \( P \). Find the equations of the tangent and normal to the curve at point \( P \). \[4\]

(iv) The tangent and normal to the curve at point \( P \) meets the \( x \)-axis at \( Q \) and \( R \) respectively. Show that the area of the triangle \( PQR \) is \( \frac{1+16e^6}{512e^9} \) units\(^2\). \[3\]

9 (a) Prove that \( \csc^4 x - \cot^4 x = 2 \csc^2 x - 1 \). \[3\]

(b) Solve the equation \( 6 \tan 2x + 1 = \cot 2x \) , for the interval \( 0 \leq x \leq 180^\circ \). \[5\]

(c) \[
\begin{align*}
\text{An object is connected to the wall with a spring that has a original horizontal length of 20 cm. The object is pulled back 8 cm past the original length and released. The object completes 4 cycles per second.} \\
(\text{i}) \text{ Given that the function } x = 8\cos(a \pi t) + b, \text{ where } x \text{ is the horizontal distance, in centimetres, of the object from the wall and } t \text{ is the time in seconds after releasing the object, find the values of } a \text{ and } b. \quad [2] \\
(\text{ii}) \text{ Find the duration of time for each cycle such that the object is more than 27 cm from the wall.} \quad [3]
\end{align*}
\]
Given that $AD$ and $BC$ are straight lines, $AC$ bisects angle $DAY$ and $AB$ bisects angle $DAX$, show that

(i) $AC^2 = EC \times BC$, \hspace{1cm} [3]

(ii) $BC$ is a diameter of the circle, \hspace{1cm} [3]

(iii) $AD$ and $BC$ are perpendicular to each other. \hspace{1cm} [3]

END OF PAPER
### Answer Key for Paper 2

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1(i)</strong></td>
<td>32x^4 - 80px^3 + \frac{80p^2}{x} - \frac{40p^3}{x^4} + ...</td>
</tr>
<tr>
<td>(ii)</td>
<td>p = 0.5</td>
</tr>
<tr>
<td><strong>2(ii)</strong></td>
<td>a = 1000, b = -2</td>
</tr>
<tr>
<td>(ii)</td>
<td>(6, -9)</td>
</tr>
<tr>
<td><strong>3(a)</strong></td>
<td>\frac{5}{2}y - 4x - 3</td>
</tr>
<tr>
<td>(b)</td>
<td>x = -0.459 or -0.2</td>
</tr>
<tr>
<td><strong>4(i)</strong></td>
<td>p = -4, q = 16</td>
</tr>
<tr>
<td>(ii)</td>
<td>x^2 - 32x - 64 = 0</td>
</tr>
<tr>
<td><strong>5(i)</strong></td>
<td>Centre = (6, 4), Radius = \sqrt{65} units</td>
</tr>
<tr>
<td>(ii)</td>
<td>4y = 7x - 26</td>
</tr>
<tr>
<td>(iii)</td>
<td>(10, 11)</td>
</tr>
<tr>
<td><strong>6(a)</strong></td>
<td>Travelling away from X</td>
</tr>
<tr>
<td>(b)(i)</td>
<td>\frac{\pi}{2} s</td>
</tr>
<tr>
<td>(b)(ii)</td>
<td>25.0 m</td>
</tr>
<tr>
<td><strong>7(iii)</strong></td>
<td>21.0 units^2</td>
</tr>
<tr>
<td><strong>8 (ii)</strong></td>
<td>-e units/s</td>
</tr>
<tr>
<td>(iii)</td>
<td>y = \frac{x}{4e^3} + \frac{1}{8e^3}, \ y = -4e^3x + \frac{1}{8e^3}</td>
</tr>
<tr>
<td><strong>9(b)</strong></td>
<td>x = 9.2^\circ, 76.7^\circ, 99.2^\circ, 166.7^\circ</td>
</tr>
<tr>
<td>(c)(i)</td>
<td>a = 8, b = 20</td>
</tr>
<tr>
<td>(c)(ii)</td>
<td>0.0402 s</td>
</tr>
</tbody>
</table>
CRESCENT GIRLS’ SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS
4047/02
17 August 2018
2 hours 30 minutes

Additional Materials: Answer Paper
Mark Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces provided at the top of this page and on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
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Answer all the questions.
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The total number of marks for this paper is 100.
1. **ALGEBRA**

*Quadratic Equation*

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

*Binomial expansion*

\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. **TRIGONOMETRY**

*Identities*

\[
sin^2 A + cos^2 A = 1
\]
\[
sec^2 A = 1 + tan^2 A
\]
\[
sec^2 A = 1 + cot^2 A
\]
\[
sin(A \pm B) = sin A \cos B \pm cos A \sin B
\]
\[
cos(A \pm B) = cos A \cos B \mp sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
sin 2A = 2 \sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]
\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

*Formulae for \( \triangle ABC \)*

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\Delta = \frac{1}{2}ab \sin C
\]
1 (i) Write down and simplify the first four terms in the expansion \( \left( 2x - \frac{p}{x^2} \right)^5 \) in descending powers of \( x \), where \( p \) is a non-zero constant. [3]

(ii) Given that the coefficient of \( x^{-1} \) in the expansion \( (4x^3 - 1) \left( 2x - \frac{p}{x^2} \right)^5 \) is \(-160p^2\), find the value of \( p \). [4]

Solution:
(i) \( \left( 2x - \frac{p}{x^2} \right)^5 = (2x)^5 + 5(2x)^4\left( -\frac{p}{x^2} \right) + 10(2x)^3\left( -\frac{p}{x^2} \right)^2 + 10(2x)^2\left( -\frac{p}{x^2} \right)^3 + ... \) [M1]
\[ = 32x^5 - 80px^3 + \frac{80p^2}{x} - \frac{40p^3}{x^2} + ... \] [A2]

(ii) \( (4x^3 - 1) \left( 2x - \frac{p}{x^2} \right)^5 = \left( 4x^3 - 1 \right) \left( 32x^5 - 80px^3 + \frac{80p^2}{x} - \frac{40p^3}{x^2} + ... \right) \) [M1]
Coefficient of \( x^{-1} = 4\left( -40p^3 \right) + \left( -1 \right)\left( 80p^2 \right) \) [M1]
\[ = -160p^3 - 80p^2 \]
\[ -160p^3 - 80p^2 = -160p^2 \] [M1]
\[ 80p^2 \left( 2p - 1 \right) = 0 \]
\[ p = 0 \text{ (NA)} \text{ or } p = 0.5 \] [A1]
Variables \( x \) and \( y \) are related by the equation \( y = ax^b + 3 \) where \( a \) and \( b \) are constants. When \( \log(y - 3) \) is plotted against \( \log x \), a straight line is obtained. The straight line passes through \((-2.5, 8)\) and \((3.5, -4)\). Find

(i) the value of \( a \) and of \( b \), \[5\]

(ii) the coordinates of the point on the line when \( x = 10^6 \). \[3\]

**Solution:**

(i) \( y = ax^b + 3 \)

\( y - 3 = ax^b \)

\( \log(y - 3) = \log a + b \log x \) \[M1\]

Gradient \( = \frac{8 - (-4)}{-2.5 - 3.5} \)

\( = -2 \) \[M1\]

\( b = -2 \) \[A1\]

Sub \( \log x = -2.5 \), \( \log(y - 3) = 8 \) and \( b = -2 \),

\( 8 = -2(-2.5) + \log a \) \[M1\]

\( \log a = 3 \)

\( a = 10^3 = 1000 \) \[A1\]

(ii) \( \log(y - 3) = -2 \log x + 3 \)

\( x = 10^6 \)

\( \log x = 6 \) \[M1\]

\( \log(y - 3) = -2(6) + 3 = -9 \) \[M1\]

Coordinates \( = (6, -9) \) \[A1\]
3 (a) Given that \( x = \log_3 a \) and \( y = \log_3 b \), express \( \log_3 \frac{\sqrt{b^5}}{27a^4} \) in terms of \( x \) and \( y \). [3]

(b) Solve the equation \( \log_3 (5x + 3)^2 - \log_{5x+3} 2 = 1 \). [5]

Solution

(a) \[
\log_3 \frac{\sqrt{b^5}}{27a^4} = \log_3 \sqrt{b^5} - \log_3 27 - \log_3 a^4 \\
= \frac{5}{2} \log_3 b - 3 - 4 \log_3 a \\
= \frac{5}{2} y - 4x - 3
\] [M1]

(b) \[
\log_3 (5x + 3)^2 - \log_{5x+3} 2 = 1 \\
2 \log_3 (5x + 3) - \frac{\log_3 2}{\log_3 (5x + 3)} = 1 \\
2 \left[ \log_3 (5x + 3) \right]^2 - 1 = \log_3 (5x + 3) \\
2 \left[ \log_3 (5x + 3) \right]^2 - \log_3 (5x + 3) - 1 = 0
\] [M1]

Let \( y = \log_3 (5x + 3) \).

\[
2y^2 - y - 1 = 0 \\
(2y + 1)(y - 1) = 0
\] [M1]

\[
y = -0.5 \quad \text{or} \quad y = 1
\]

\[
\log_3 (5x + 3) = -0.5 \quad \log_3 (5x + 3) = 1
\] [M1]

\[
5x + 3 = 2^{-0.5} \quad 5x + 3 = 2
\]

\[
x = -0.459 \quad x = -0.2
\] [A1]
4 (i) The roots of the equation $2x^2 + px - 8 = 0$, where $p$ is a constant, are $\alpha$ and $\beta$. The roots of the equation $4x^2 - 24x + q = 0$, where $q$ is a constant, are $\alpha + 2\beta$ and $2\alpha + \beta$. Find the values of $p$ and $q$. \[6\]

(ii) Hence form the quadratic equation whose roots are $\alpha^3$ and $\beta^3$. \[3\]

**Solution:**

(i) $2x^2 + px - 8 = 0$

$$\alpha + \beta = -\frac{p}{2}, \quad \alpha\beta = -4$$

$4x^2 - 24x + q = 0$

$$\alpha + 2\beta + 2\alpha + \beta = 6$$

$$3(\alpha + \beta) = 6$$

Sub $\alpha + \beta = -\frac{p}{2}$,

$$-\frac{p}{2} = 2 \quad \Rightarrow \quad p = -4$$

$$(\alpha + 2\beta)(2\alpha + \beta) = \frac{q}{4}$$

$$2(\alpha^2 + \beta^2) + 5\alpha\beta = \frac{q}{4}$$

$$2[2(\alpha + \beta)^2 - 2\alpha\beta] + 5\alpha\beta = \frac{q}{4}$$

$$2(\alpha + \beta)^2 + \alpha\beta = \frac{q}{4}$$

Sub $\alpha + \beta = 2, \quad \alpha\beta = -4$,

$$2\left(2^2 - 4\right) = \frac{q}{4}$$

$$q = 16$$

(ii) $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$

$$= (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$$

$$= 2[2^2 - 3(-4)]$$

$$= 32$$

$$(\alpha\beta)^3 = (-4)^3 = -64$$

$$\therefore \quad x^2 - 32x - 64 = 0$$
The equation of a circle \( C \) is \( x^2 + y^2 - 12x - 8y - 13 = 0 \).

(i) Find the centre and radius of \( C \). [3]

(ii) Find the equation of the line which passes through the centre of \( C \) and is perpendicular to the line \( 4x + 7y = 117 \). [3]

(iii) Show that the line \( 4x + 7y = 117 \) is a tangent to \( C \) and state the coordinates of the point where the line touches \( C \). [5]

Solution:
(i) \( x^2 + y^2 - 12x - 8y - 13 = 0 \)
\((x - 6)^2 + (y - 4)^2 - 36 - 16 - 13 = 0 \) [M1]
\((x - 6)^2 + (y - 4)^2 = 65 \)
Centre = (6, 4) [A1]
Radius = \( \sqrt{65} \) units [A1]

(ii) For \( 4x + 7y = 117 \),
Gradient of the line = \(-\frac{4}{7}\) [M1]
Gradient of the line passing through \( C = \frac{7}{4} \) [M1]
Equation of the line:
\( y - 4 = \frac{7}{4}(x - 6) \) [M1]
\( 4y - 16 = 7x - 42 \)
\( 4y = 7x - 26 \) [A1]

(iii) \( 4x + 7y = 117 \) ------ (1)
\( 4y = 7x - 26 \) \( \Rightarrow y = \frac{7}{4}x - \frac{26}{4} \) ------ (2)
Sub (2) into (1):
\( 4x + 7\left(\frac{7}{4}x - \frac{26}{4}\right) = 117 \) [M1]
\( 16.25x = 162.5 \)
\( x = 10 \) [M1]
\( y = 11 \)
Distance between (10, 11) and centre of circle = \( \sqrt{(10 - 6)^2 + (11 - 4)^2} \) [M1]
\( = \sqrt{65} \) units [A1]
Since distance from the point and the centre of circle equals to the radius, the line is a tangent to the circle.
Coordinates of the point = (10, 11) [A1]
Alternative Solution:

4x + 7y = 117 \implies x = \frac{117 - 7y}{4} \quad \text{(1)}

x^2 + y^2 - 12x - 8y - 13 = 0 \quad \text{(2)}

Sub (1) into (2):

\[ \left( \frac{117 - 7y}{4} \right)^2 + y^2 - 12\left( \frac{117 - 7y}{4} \right) - 8y - 13 = 0 \]

\[ \frac{13689 - 1638y + 49y^2}{16} + y^2 - 351 + 21y - 8y - 13 = 0 \]

\[ 13689 - 1638y + 49y^2 + 16y^2 - 5616 + 336y - 128y - 208 = 0 \]

\[ 65y^2 - 1430y + 7865 = 0 \]

\[ y^2 - 22y + 121 = 0 \]

\[ b^2 - 4ac = (-22)^2 - 4(1)(121) = 0 \]

Since \( b^2 - 4ac = 0 \), the line is a tangent to \( C \).

\[ y^2 - 22y + 121 = 0 \]

\[ (y - 11)^2 = 0 \]

\[ y = 11 \]

\[ x = 10 \]

Coordinate of the point = \( (10, 11) \)
6  (a)  A car travelling on a straight road passes through a traffic light \( X \) with speed of 90 m/s. The acceleration, \( a \) m/s\(^2\) of the car, \( t \) seconds after passing \( X \), is given by \( a = 20 - 8t \). Determine with working whether the car is travelling towards or away from \( X \) when it is travelling at maximum speed. \[4\]

(b)  A particle moving in a straight line such that its displacement, \( s \) m, from the fixed point \( O \) is given by \( s = 7\sin t - 2\cos 2t \), where \( t \) is the time in seconds, after passing through at a point \( A \).

(i)  Find the value of \( t \) when the particle first comes to instantaneous rest. \[5\]

(ii)  Find the total distance travelled by the particle during the first 4 seconds of its motion. \[3\]

Solution:

(a)  \( a = 20 - 8t \)

\[ v = \int 20 - 8t \, dt = 20t - 4t^2 + c, \text{ where } c \text{ is a constant} \]

[M1]

When \( t = 0 \), \( v = 90 \), \( c = 90 \).

\[ v = 20t - 4t^2 + 90 \]

When the car is travelling at max speed, \( a = 0 \).

\[ a = 20 - 8t \quad \Rightarrow \quad t = 2.5 \]

[M1]

\[ v = 20(2.5) - 4(2.5)^2 + 90 = 115 \]

\[ s = \int 20t - 4t^2 + 90 \, dt = 10t^2 - \frac{4}{3}t^3 + 90t + d, \text{ where } d \text{ is a constant} \]

[M1]

When \( t = 0 \), \( s = 0 \), \( d = 0 \).

\[ s = 10t^2 - \frac{4}{3}t^3 + 90t \]

When \( t = 2.5 \), \( s = 10(2.5)^2 - \frac{4}{3}(2.5)^3 + 90(2.5) = 266\frac{2}{3} \]

Since \( s > 0 \) and \( v > 0 \), the car is travelling away from \( X \) at maximum speed. \[A1\]

Alternative Solution:

When the car is at instantaneous rest, \( v = 0 \).

\[ 20t - 4t^2 + 90 = 0 \]

[M1]

\[ t = \frac{-20 \pm \sqrt{(-20)^2 - 4(-4)(90)}}{2(-4)} = -2.8619 \text{ or } 7.8619 \]

Since there is no change of direction from \( t = 0 \) to \( t = 7.86 \) s , the car is travelling away from \( X \) at maximum speed. \[B1\]
(b)(i) \[ s = 7 \sin t - 2 \cos 2t \]
\[ v = 7 \cos t + 4 \sin 2t \]
When the particle is at instantaneous rest, \( v = 0 \).
\[ 7 \cos t + 4 \sin 2t = 0 \]
\[ 7 \cos t + 8 \sin t \cos t = 0 \]
\[ \cos t (7 + 8 \sin t) = 0 \]
\[ \cos t = 0 \quad \text{or} \quad \sin t = -\frac{7}{8} \]
\[ t = \frac{\pi}{2}, \quad \frac{3\pi}{2} \]
\[ t = 4.2069, \quad 5.2177 \]
Time when particle first comes to instantaneous rest \( = \frac{\pi}{2} \) s

(b)(ii) When \( t = 0 \), \( s = -2 \).
When \( t = \frac{\pi}{2} \), \( s = 9 \).
When \( t = 4 \), \( s = -5.0066 \).
Total distance travelled \( = 2 + 2(9) + 5.0066 \)
\[ = 25.0 \text{ m} \]
7 (i) Show that \[ \frac{d}{dx} \left( \frac{x+2}{\sqrt{x-1}} \right) = \frac{x-4}{2\sqrt{(x-1)^3}}. \] [3]

The diagram shows the line \( x = 15 \) and part of the curve \( y = \frac{12(x-4)}{\sqrt{(x-1)^3}} \). The curve intersect the \( x \)-axis at the point \( A \). The line through \( A \) with gradient \( \frac{4}{9} \) intersects the curve again at the point \( B \).

(ii) Verify that the \( y \)-coordinate of \( B \) is \( 2 \frac{2}{3} \). [4]

(iii) Determine the area of the region bounded by the curve, the \( x \)-axis, the line \( x = 15 \) and the line \( AB \). [4]

Solution:
(i) \[ \frac{d}{dx} \left( \frac{x+2}{\sqrt{x-1}} \right) = \frac{\sqrt{x-1} -(x+2)\left[ \frac{1}{2} (x-1)^{\frac{1}{2}} \right]}{x-1} \]

\[ = \frac{\left[ \frac{1}{2} (x-1)^{-\frac{1}{2}} \right] [2x - 2 - x - 2]}{x-1} \]

\[ = \frac{x-4}{2\sqrt{(x-1)^3}} \] [M1] [A1]
(ii) \( A = (4, 0) \)

Equation of \( AB : y = \frac{4}{9}(x - 4) \) \[1\] \[M1\]

\[ y = \frac{12(x - 4)}{\sqrt{(x - 1)^3}} \] \[2\] \[M2\]

(1) = (2):

\[ \frac{4}{9}(x - 4) = \frac{12(x - 4)}{\sqrt{(x - 1)^3}} \] \[M1\]

\((x - 4)(x - 1)^{\frac{3}{2}} = 27(x - 4)\)

\[ (x - 4) [(x - 1)^{\frac{3}{2}} - 27] = 0 \] \[M1\]

\( x = 4 \) or \( (x - 1)^{\frac{3}{2}} = 27 \)

\( x = 10 \)

Sub \( x = 10 \) in (1):

\[ y = \frac{4}{9}(10 - 4) = 2 \frac{2}{3} \] \[A1]\n
\( y \) – coordinate of \( B = 2 \frac{2}{3} \) (shown)

(iii)

\[ \text{Area} = \frac{1}{2} \left( 2 \frac{2}{3} \right)(10 - 4) + \int_{10}^{15} \frac{12(x - 4)}{\sqrt{(x - 1)^3}} \, dx \] \[M1\]

\[ = 8 + 24 \int_{10}^{15} \frac{x - 4}{2\sqrt{(x - 1)^3}} \, dx \] \[M1\]

\[ = 8 + 24 \left[ \frac{x + 2}{\sqrt{x - 1}} \right]_{10}^{15} \] \[M1\]

\[ = 8 + 24 \left( \frac{17}{\sqrt{14}} - \frac{12}{\sqrt{9}} \right) \]

\[ = 21.0 \text{ units}^2 \] \[A1\]
A curve has equation given by \( y = \frac{e^{4x-3}}{8e^{3x}} \).

(i) Show that \( \frac{dy}{dx} = \frac{e^{2x-3}}{4} \). [2]

(ii) Given that \( x \) is decreasing at a rate of \( 4e^2 \) units per second, find the exact rate of change of \( y \) when \( x = 1 \). [3]

(iii) The curve passes through the \( y \)-axis at \( P \). Find the equations of the tangent and normal to the curve at point \( P \). [4]

(iv) The tangent and normal to the curve at point \( P \) meets the \( x \)-axis at \( Q \) and \( R \) respectively. Show that the area of the triangle \( PQR \) is \( \frac{1+16e^6}{512e^3} \) units\(^2\). [3]

**Solution:**

(i) \( y = \frac{e^{2x-3}}{8} \) [M1]

\[ \frac{dy}{dx} = \frac{e^{2x-3}}{4} \] [A1]

(ii) \[ \frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} \]

\[ = \frac{e^{2x-3}}{4} \times (-4e^2) \] [M1]

\[ = -e^{2x-4} \] [M1]

When \( x = 1 \), \( \frac{dy}{dt} = -e \) units/s [A1]

(iii) When \( x = 0 \), \( y = \frac{1}{8e^3} \)

Gradient of tangent at \( P = \frac{1}{4e^3} \) [M1]

Equation of tangent at \( P \):

\[ y - \frac{1}{8e^3} = \frac{1}{4e^3}(x) \quad \Rightarrow \quad y = \frac{x}{4e^3} + \frac{1}{8e^3} \] [M1]

Gradient of normal at \( P = -4e^3 \) [M1]

Equation of normal at \( P \):

\[ y - \frac{1}{8e^3} = -4e^3(x) \quad \Rightarrow \quad y = -4e^3x + \frac{1}{8e^3} \] [A1]
(iv) Equation of tangent at \( P \): 
\[ y = \frac{x}{4e^3} + \frac{1}{8e^3} \]

When \( y = 0 \), \( x = -\frac{1}{2} \). 
\[ \therefore Q = \left( -\frac{1}{2}, 0 \right) \]

Equation of normal at \( P \): 
\[ y = -4e^3x + \frac{1}{8e^3} \]

When \( y = 0 \), \( x = \frac{1}{32e^6} \). 
\[ \therefore R = \left( \frac{1}{32e^6}, 0 \right) \]  

Area of triangle \( PQR \) = 
\[ \frac{1}{2} \left( \frac{1}{8e^3} \right) \left( \frac{1}{32e^6} - \left( -\frac{1}{2} \right) \right) \]

\[ = \frac{1}{16e^3} \left( \frac{1 + 16e^6}{32e^6} \right) \]  

\[ = \frac{1 + 16e^6}{512e^6} \text{ units}^2 \]
9 (a) Prove that \( \csc^4 x - \cot^4 x = 2 \csc^2 x - 1 \). [3]

(b) Solve the equation \( 6 \tan 2x + 1 = \cot 2x \), for the interval \( 0 \leq x \leq 180^\circ \). [5]

(c) An object is connected to the wall with a spring that has a original horizontal length of 20 cm. The object is pulled back 8 cm past the original length and released. The object completes 4 cycles per second.

(i) Given that the function \( x = 8 \cos (a \pi t) + b \), where \( x \) is the horizontal distance, in centimetres, of the object from the wall and \( t \) is the time in seconds after releasing the object, find the values of \( a \) and \( b \). [2]

(ii) Find the duration of time for each cycle such that the object is more than 27 cm from the wall. [3]

Solution:

(a) \[
\text{LHS} = (\csc^2 x - \cot^2 x)(\csc^2 x + \cot^2 x) \\
= \csc^2 x + \cot^2 x \\
= \csc^2 x + \csc^2 x - 1 \\
= 2 \csc^2 x - 1 \\
= \text{RHS}
\]

(b) \[
6 \tan 2x + 1 = \cot 2x \\
6 \tan^2 2x + \tan 2x - 1 = 0 \\
(3 \tan 2x - 1)(2 \tan 2x + 1) = 0 \\
0 \leq x \leq 360^\circ \Rightarrow 0 \leq 2x \leq 720^\circ \\
\tan 2x = \frac{1}{3} \quad \text{or} \quad \tan 2x = -\frac{1}{2} \\
\alpha = 18.435^\circ \quad \alpha = 26.565^\circ \\
x = 9.2^\circ, 99.2^\circ \ (1 \text{ dp}) \quad 2x = 153.43^\circ, 333.43^\circ \\
x = 76.7^\circ, 166.7^\circ \ (1 \text{ dp})
\]

(c)(i) \( b = 20 \)

\[
\text{Period} = \frac{2\pi}{a\pi} \\
\frac{1}{4} = \frac{2\pi}{a\pi} \Rightarrow a = 8
\]
(c)(ii) \(27 = 8 \cos(8\pi t) + 20\)

\[
\cos(8\pi t) = \frac{7}{8}
\]

\(\alpha = 0.50536\)  

\(8\pi t = 0.50536\)  

\(t = 0.020107\)

Duration of time \(= 0.020107 \times 2\)

\(= 0.0402\) s
Given that \(AD\) and \(BC\) are straight lines, \(AC\) bisect angle \(DAY\) and \(AB\) bisects angle \(DAX\), show that

(i) \(AC^2 = EC \times BC\), \[3\]

(ii) \(BC\) is a diameter of the circle, \[3\]

(iii) \(AD\) and \(BC\) are perpendicular to each other. \[3\]

Solution:

(i) \(\angle BCA = \angle ACE\) (Common angle)

\(\angle ABC = \angle CAY\) (Angles in the alternate segments)

\(\angle CAY = \angle EAC\) (\(AC\) bisects \(\angle DAY\))

\(\therefore \triangle BAC\) and \(\triangle EAC\) are similar.

\[
\frac{AC}{EC} = \frac{BC}{AC} \quad \text{(corresponding sides of similar triangles)}
\]

\[AC^2 = EC \times BC \quad \text{(shown)}\]

(ii) \(\angle CAY = \angle EAC\) (\(AC\) bisects \(\angle DAY\))

\(\angle BAX = \angle EAB\) (\(AB\) bisects \(\angle BAX\))

\(\angle BAX + \angle EAB + \angle EAC + \angle CAY = 180^\circ\) (angles on a straight line)

\[2\angle EAB + 2\angle EAC = 180^\circ\]

\[\angle EAB + \angle EAC = \angle BAC = 90^\circ\]

Since \(\angle BAC = 90^\circ\), \(BC\) is a diameter of the circle. \[B1\]

(iii) \(\angle ABE = \angle CAY\) (Angles in the alternate segments)

\(\angle CAY = \angle EAC\) (\(AC\) bisects \(\angle BAY\))

\(\therefore \angle ABE = \angle EAC\)

\(\angle EAB + \angle EAC = \angle EAB + \angle ABE = 90^\circ\) (from (ii))

\(\angle AEB = 90^\circ\) (sum of \(\angle s\) in a triangle)

\(\therefore AD\) and \(BC\) are perpendicular. \[B1\]

END OF PAPER
CHIJ KATONG CONVENT
PRELIMINARY EXAMINATION 2018
SECONDARY 4 EXPRESS /
5 NORMAL (ACADEMIC)

ADDITIONAL MATHEMATICS                                      4047/01
PAPER 1

Classes: 403, 405, 406, 502

READ THESE INSTRUCTIONS FIRST

Write your name, class and registration number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of
angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
Omission of essential working will result in loss of marks.

There are two sections in this paper.
At the end of the examination, fasten sections A and B separately.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Binomial expansion

\[(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. TRIGONOMETRY

Identities

\[ \sin^2 A + \cos^2 A = 1 \]
\[ \sec^2 A = 1 + \tan^2 A \]
\[ \cosec^2 A = 1 + \cot^2 A \]
\[ \sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \]
\[ \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \]
\[ \tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \]
\[ \sin 2A = 2 \sin A \cos A \]
\[ \cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A \]
\[ \tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \]

Formulae for \( \triangle ABC \)

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]
\[ a^2 = b^2 + c^2 - 2bc \cos A \]
\[ \Delta = \frac{1}{2} bc \sin A \]
Answer all questions.

Section A

1. A metal cube with sides 2x mm is heated. The sides are expanding at a rate of 0.05 mm/s. Calculate the rate of change of the total surface area of the cube when x = 0.57 mm. [3]

2. Without using a calculator, find the integer value of a and of b for which the solution of the equation \(2x\sqrt{5} = x\sqrt{2} + \sqrt{18}\) is \(\frac{\sqrt{a+b}}{3}\). [4]

3. The equation of a curve is \(y = \frac{3x^2}{\sqrt{4x-h}}\). Given that the x-coordinate of the stationary point is 1, find the value of h. [4]

4. The roots of the quadratic equation \(8x^2 - 49x + c = 0\) are \(\frac{2\alpha}{\beta}\) and \(\frac{2\beta}{\alpha}\).
   
   (i) Show that \(c = 32\). [1]

   (ii) Given that \(\alpha\beta = 4\), find two distinct quadratic equations whose roots are \(\alpha\) and \(\beta\). [4]

5. Given that \(y = \frac{2 - 3\sec^2 2x}{\tan^2 2x + 1}\),

   (i) express \(y\) in the form \(\cos 4x + k\), [2]

   (ii) sketch the graph of \(|y|\) for \(-\frac{\pi}{2} \leq x \leq \pi\) and state the value of \(n\) when \(|y| = n\) has four solutions. [3]

6. The polynomial \(f(x) = px^3 + 3x^2 + qx - 6\) is divisible by \(x^2 + x - 6\).

   (i) Find the value of \(p\) and of \(q\). [4]

   (ii) Find the remainder in terms of \(x\) when \(f(x)\) is divided by \(x^2 - 1\). [2]
7 Given the equation \( \frac{2}{\sin^2 \theta} = 5 - \cot \theta \) where \( 0^\circ < \theta < 360^\circ \), find

(i) the values of \( \theta \), [4]

(ii) the exact values of \( \cos \theta \). [2]

8 (i) Express \( \frac{2x - 1}{x^2(x + 1)} \) in partial fractions. [4]

(ii) Hence, determine \( \int \frac{2x - 1}{x^2(x + 1)} \, dx \). [2]

Section B

Begin this section on a new sheet of writing paper.

9 Given the curve \( y = (m + 1)x^2 - 8x + 3m \) has a minimum value, find the range of values of \( m \)

(i) for which the line \( y = m - 4mx \) meets the curve, [5]

(ii) for which the \( y \)-intercept of the curve is greater than \( -\frac{5}{2} \). [2]

10 (i) Solve the equation

\[
3 \log_{27} \left[ \log_{1000} (x^2 + 9) - \log_{1000} x \right] = -1.
\] [3]

(ii) (a) On the same axes, sketch the graphs of \( y = \log_{\frac{1}{\sqrt{3}}} x - 1 \) and \( y = \log_{2} x + 1 \). [2]

(b) Explain why the two graphs are symmetrical about the \( x \)-axis. [2]
A piece of wire of length 80 cm is bent into the shape of a trapezium $ABCD$.
$AB = CD = x$ cm and angle $BAD = angle ADC = 120^\circ$.

(i) Show that the area of the trapezium $ABCD$ is given by $\frac{\sqrt{3}}{2}x(40 - x)$ cm$^2$.

(ii) Given that $x$ can vary, find the value of $x$ for which the area has a stationary value.

(iii) Determine whether this stationary value is a maximum or a minimum.

12 A particle moves in a straight line so that its velocity, $v$ m/s, is given by $v = 2 - \frac{18}{(t + 2)^2}$ where $t$ is the time in seconds, after leaving a fixed point $O$.

Its displacement from $O$ is 9 m when it is at instantaneous rest.

Find

(i) the value of $t$ when it is at instantaneous rest,

(ii) the distance travelled during the first 4 seconds.

At $t = 7$, the particle starts with a new velocity, $V$ m/s, given by $V = -h(t^2 - 7t) + k$.

(iii) Find the value of $k$.

(iv) Given that the deceleration is 0.9 m/s$^2$ when $t = 8$, find the value of $h$. 
In the diagram, $PQ$ is parallel to $SR$ and the coordinates of $P$, $Q$ and $S$ are $(3, 2)$, $(9, 4)$ and $(5, 7)$ respectively.

The gradient of the line $OR$ is 1.

Find

(i) the coordinates of $R$, [4]

(ii) the area of the quadrilateral $PQRS$, [2]

(iii) the coordinates of the point $H$ on the line $y = 1$ which is equidistant from $P$ and $Q$. [4]

*End of Paper*
A metal cube with sides $2x$ mm is heated. The sides are expanding at a rate of 0.05 mm/s.

Calculate the rate of change of the total surface area of the cube when $x = 0.57$ mm. \[3\]

**Solution**

Let $l = 2x$

Area $A = 6l^2$

\[
\frac{dA}{dt} = \frac{dA}{dl} \times \frac{dl}{dt}
\]

\[
= 12l \times 0.05
\]

\[
= 12(2(0.57)) \times 0.05
\]

\[
= 0.684
\]

Answer: 0.684 mm$^2$/s.

**OR**

Most students applied this method but used 0.05 wrongly for $\frac{dx}{dt}$.

Some students used wrong formula for SA.

Area $A = 6(2x)^2 = 24x^2$

\[
\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}
\]

\[
= 48x \times 0.025
\]

\[
= 48(0.57) \times 0.025
\]

\[
= 0.684
\]

Answer: 0.684 mm$^2$/s.
2 Without using a calculator, find the integer value of $a$ and of $b$ for which the solution of the equation $2x\sqrt{5} = x\sqrt{2} + \sqrt{18}$ is $\frac{\sqrt{a+b}}{3}$. [4]

**Solution**

$$x(2\sqrt{5} - \sqrt{2}) = \sqrt{18}$$

$$x = \frac{\sqrt{18}}{2\sqrt{5} - \sqrt{2}} \times \frac{2\sqrt{5} + \sqrt{2}}{2\sqrt{5} + \sqrt{2}}$$

conjugate surds

$$= \frac{2\sqrt{90} + 6}{2\sqrt{5} + \sqrt{2}}$$

$$= \frac{6\sqrt{10} + 6}{18}$$

$$= \frac{\sqrt{10} + 1}{3}$$

$$a = 10, \ b = 1$$

**OR**

A handful used this method but did not reject one answer/ did not know why one of the answers is not acceptable.

$$\left(2x\sqrt{5}\right)^2 = \left(x\sqrt{2} + \sqrt{18}\right)^2$$

$$20x^2 = 2x^2 + 2\sqrt{36x} + 18$$

$$18x^2 - 12x - 18 = 0$$

$$3x^2 - 2x - 3 = 0$$

$$x = \frac{2 + \sqrt{4 - 4(2)(-3)}}{2(3)}$$

$$= \frac{1 + \sqrt{10}}{3} \text{ or } \frac{1 - \sqrt{10}}{3} \text{ (reject)}$$

$$a = 10, \ b = 1$$

3 The equation of a curve is $y = \frac{3x^2}{\sqrt{4x-h}}$. Given that the $x$-coordinate of the stationary point is 1, find the value of $h$. [4]

**Solution**

$$\frac{dy}{dx} = \frac{\sqrt{4x-h}(6x) - 3x^2\left(\frac{1}{2}\right)(4x-h)^{\frac{1}{2}}(4)\frac{1}{2}}{4x-h}$$

quotient OR product rule

$$= \frac{\left(4x-h\right)^{\frac{1}{2}}\left[(6x)(4x-h) - 6x^2\right]}{4x-h}$$

$$= \frac{18x^2 - 6hx}{4x-h^{\frac{1}{2}}}$$

At stationary point, $\frac{dy}{dx} = 0$.

When $x = 1$, $\frac{18(1)^2 - 6h(1)}{(4(1)-h)^{\frac{1}{2}}} = 0$

$$h = 3$$
The roots of the quadratic equation \( 8x^2 - 49x + c = 0 \) are \( \frac{2\alpha}{\beta} \) and \( \frac{2\beta}{\alpha} \).

(i) Show that \( c = 32 \). [1]

(ii) Given that \( \alpha\beta = 4 \), find two distinct quadratic equations whose roots are \( \alpha \) and \( \beta \). [4]

**Solution**

(i)

\[
\left( \frac{2\alpha}{\beta} \right) \left( \frac{2\beta}{\alpha} \right) = \frac{c}{8}
\]

\[
4 = \frac{c}{8}
\]

\[
c = 32
\]

(ii)

\[
\frac{2\alpha}{\beta} + \frac{2\beta}{\alpha} = \frac{49}{8}
\]

Simplifying:

\[
\frac{2\alpha^2 + 2\beta^2}{\alpha\beta} = \frac{49}{8}
\]

\[
\frac{2\alpha^2 + 2\beta^2}{4} = \frac{49}{8}
\]

\[
\alpha^2 + \beta^2 = \frac{49}{4}
\]

Apply perfect square:

\[
(\alpha + \beta)^2 - 2\alpha\beta = \frac{49}{4}
\]

\[
(\alpha + \beta)^2 - 8 = \frac{49}{4}
\]

\[
(\alpha + \beta)^2 = \frac{81}{4}
\]

\[
\alpha + \beta = \pm \frac{9}{2}
\]

Eqns are \( 2x^2 - 9x + 8 = 0 \), \( 2x + 9x + 8 = 0 \).

Both eqns, accept fractional coefficients.

---

**Given that** \( y = \frac{2 - 3\sec^2 2x}{\tan^2 2x + 1} \),
(i) express \( y \) in the form \( \cos 4x + k \), \[2\]

(ii) sketch the graph of \(|y|\) for \(-\frac{\pi}{2} \leq x \leq \pi\) and state the value of \( n \) when \(|y| = n\) has four solutions. \[3\]

Solution

\[(i)\] \[
\frac{2 - 3 \sec^2 2x}{\tan^2 2x + 1} = \frac{2 - 3 \sec^2 2x}{\sec^2 2x} = 2 \cos^2 2x - 3 = 2 \cos^2 2x - 1 - 2 = \cos 4x - 2
\]

(iii) graph

\[n = 1\]

(iii) graph

\[n = 1\]

6 The polynomial \( f(x) = px^3 + 3x^2 + qx - 6 \) is divisible by \( x^2 + x - 6 \).

(i) Find the value of \( p \) and of \( q \). \[4\]
(ii) Find the remainder in terms of \(x\) when \(f(x)\) is divided by \(x^2 - 1\).

Solution

(i) \(x^2 + x - 6 = (x - 2)(x + 3)\)

By the factor thm, \(f(2) = 0\)

\[
p(2)^2 + 3(2)^2 + q(2) - 6 = 0 \quad \text{factor thm}
\]

\[
8p + 2q + 6 = 0
\]

\[
4p + q = -3 \quad \ldots \ldots \text{(1)}
\]

\(f(-3) = 0\)

\[
p(-3)^2 + 3(-3)^2 + q(-3) - 6 = 0 \quad \text{factor thm}
\]

\[
-27p - 3q + 21 = 0
\]

\[
9p + q = 7 \quad \ldots \ldots \text{(2)}
\]

Solve (1) and (2); \(p = 2, q = -11\)

(ii) Using \(x^2 = 1\),

\[
f(x) = 2x^3 + 3x^2 - 11x - 6
\]

\[
= 2x^2(x) + 3x^2 - 11x - 6
\]

\[
= 2x + 3 - 11x - 6
\]

\[
= -9x - 3
\]

Given the equation \(\frac{2}{\sin^2 \theta} = 5 - \cot \theta\) where \(0^0 < \theta < 360^0\), find

(i) the values of \(\theta\).

(ii) the exact values of \(\cos \theta\).

Solution

(i) \(2 \cos ec^2 \theta = 5 - \cot \theta\)

\[
2(1 + \cot^2 \theta) - 5 + \cot \theta = 0 \quad \text{identity}
\]

\[
2 \cot^2 \theta + \cot \theta - 3 = 0 \quad \text{factorisation}
\]

\[
(2 \cot \theta + 3)(\cot \theta - 1) = 0
\]

\[\cot \theta = -\frac{3}{2} \quad \text{or} \quad \cot \theta = 1\]

\[\tan \theta = -\frac{2}{3} \quad \text{or} \quad \tan \theta = 1\]

Basic angle = \(33.69^0, 45^0\)

\[
\theta = 146.3^0, 326.3^0, 45^0, 225^0
\]

(ii) \(\tan \theta = -\frac{2}{3}\) \(\text{ (quadrants 2, 4) or} \tan \theta = 1\) \(\text{ (quadrants 1, 3)}\)
\[ \cos \theta = \pm \frac{3}{\sqrt{13}}, \quad \cos \theta = \pm \frac{1}{\sqrt{2}} \]

8. (i) Express \( \frac{2x - 1}{x^2(x + 1)} \) in partial fractions. [4]

(ii) Hence, determine \( \int \frac{2x - 1}{x^2(x + 1)} \, dx \). [2]

Solution

(i) \( \frac{2x - 1}{x^2(x + 1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x + 1} \)

\[ 2x - 1 = Ax(x + 1) + B(x + 1) + Cx^2 \]

Let \( x = -1 \), \( -3 = C(-1)^2 \implies C = -3 \)

Let \( x = 0 \), \( B = -1 \)

Let \( x = 1 \), \( 1 = 2A - (2) - 3(1)^2 \implies A = 3 \)

Hence, \( \frac{2x - 1}{x^2(x + 1)} = \frac{3}{x} - \frac{1}{x^2} - \frac{3}{x + 1} \)

(ii) \( \int \frac{2x - 1}{x^2(x + 1)} \, dx = \int \left( \frac{3}{x} - \frac{1}{x^2} - \frac{3}{x + 1} \right) \, dx \)

9. Given the curve \( y = (m + 1)x^2 - 8x + 3m \) has a minimum value, find the range of values of \( m \)

   (i) for which the line \( y = m - 4mx \) meets the curve, [5]

   (ii) for which \( y \) - intercept of the curve is greater than \( -\frac{5}{2} \). [2]

Solution

(i) \( (m + 1)x^2 - 8x + 3m = m - 4mx \)

\( (m + 1)x^2 + 4mx - 8x + 2m = 0 \)

Quadratic Eqn

\[ b^2 - 4ac \geq 0 \]

\( (4m - 8)^2 - 4(m + 1)(2m) \geq 0 \) 

Expansion, Simplify

\( 2(m^2 - 4m + 4) - m^2 - 8 \geq 0 \)

\( m^2 - 9m + 8 \geq 0 \)

Factorisation

\( m \leq 1 \) or \( m \geq 8 \)

Since it is a minimum graph, \( m + 1 > 0 \), ie \( m > -1 \)

So \( -1 < m \leq 1 \) or \( m \geq 8 \)
(ii) At $y$-intercept, $x = 0$,
\[
(m + 1)x^2 - 8x + 3m > \frac{-5}{2}
\]
\[
m > \frac{-5}{6}
\]

10  (i)  Solve the equation $3\log_{27} \left[ \log_{1000} (x^2 + 9) - \log_{1000} x \right] = -1$.  
Solution

\[
\log_{1000} \frac{x^2 + 9}{x} = 27^{\frac{1}{3}}
\]
\[
\log_{1000} \frac{x^2 + 9}{x} = \frac{1}{3}
\]
\[
x^{\frac{1}{3}} = \frac{x^2 + 9}{x}
\]
\[
x = 1000^{\frac{1}{3}}
\]
\[
x = x^2 + 9 = 10x
\]
\[
x^2 - 10x + 9 = 0
\]
\[
(x - 1)(x - 9) = 0
\]
\[
x = 1 \text{ or } 9
\]

(ii)  (a)  On the same axes, sketch the graphs of $y = \log_{\frac{1}{2}} x - 1$ and $y = \log_{\frac{7}{2}} x + 1$.  
(b)  Explain why the two graphs are symmetrical about the $x$-axis.  
Solution
(ii) \(-(\log_\frac{1}{x} x - 1) = -\frac{\log_2 x}{\log_2 2^{-1}} + 1\) [M1]

\[
= -\frac{\log_2 x}{\log_2 2^{-1}} + 1
= \log_2 x + 1
\]

The functions are negative of each other. [A1]

11

A piece of wire of length 80 cm is bent into the shape of a trapezium \(ABCD\).
\(AB = CD = x\) cm and \(\angle BAD = \angle ADC = 120^\circ\).

(i) Show that the area of the trapezium \(ABCD\) is given by \(\frac{\sqrt{3}}{2} x (40 - x)\) cm\(^2\). [4]

(ii) Given that \(x\) can vary, find the value of \(x\) for which the area has a stationary value. [2]

(iii) Determine whether this stationary value is a maximum or a minimum. [2]

Solution
\[\angle ABC = 180 - 120(\text{int. } \angle x, AD // BC)\]
\[= 60^\circ\]

(i) \(\cos \angle ABC = \frac{BQ}{x}\)

\(BQ = \frac{x}{2}\)
Perimeter = \(BC + 2x + AD\)

\(80 = \frac{x}{2} + AD + \frac{x}{2} + x + AD + x\)
\(AD = \frac{80 - 3x}{2}\)

\(\sin 60^\circ = \frac{AQ}{x} \Rightarrow AQ = \frac{\sqrt{3}}{2} x\)

Area = \(\frac{1}{2} (AD + BC) \left(\frac{\sqrt{3}}{2} x\right)\)

OR (Most used this method)

\[\angle ABC = 180 - \angle BAD\] (int. angles, \(AD // BC\))
\[= 60^\circ\]

\(AQ = \text{height of the trapezium}\)
\(\sin 60^\circ = \frac{AQ}{x}\)

\(AQ = \frac{\sqrt{3}}{2} x\)

Area = \(\frac{1}{2} \left(\frac{\sqrt{3}}{2} x\right) (AD + BC)\)
\[= \frac{1}{2} \left(\frac{\sqrt{3}}{2} x\right) (80 - 2x)\]
\[= \frac{\sqrt{3}}{2} x (40 - x)\] (shown)
\[
= \frac{1}{4}(80 - 3x + x)\sqrt{3}x
\]
\[
= \frac{\sqrt{3}}{4}x(80 - 2x)
\]
\[
= \frac{\sqrt{3}}{2}x(40 - x) \quad \text{(Shown)}
\]

\[\text{(ii)} \quad \frac{dA}{dx} = 0 \quad \text{when the area has a stationary value}\]
\[20\sqrt{3} - \frac{\sqrt{3}}{2}(2x) = 0 \quad \text{differentiation}\]
\[x = 20\]

\[\text{(iii)} \quad \frac{d^2A}{dx^2} = -\sqrt{3} < 0 \quad \text{second derivative or using first derivative}\]
\[\text{Area is a maximum}\]

12

A particle moves in a straight line so that its velocity, \(v\) m/s, is given by \(v = 2 - \frac{18}{(t + 2)^2}\) where \(t\) is the time in seconds, after leaving a fixed point \(O\).

Its displacement from \(O\) is 9 m when it is at instantaneous rest.

\(\text{(i)}\) the value of \(t\) when it is at instantaneous rest, \(\text{[2]}\)

\(\text{(ii)}\) the distance travelled during the first 4 seconds. \(\text{[4]}\)

At \(t = 7\), the particle starts with a new velocity, \(V\) ms\(^{-1}\), given by \(V = -h(t^2 - 7t) + k\).

\(\text{(iii)}\) Find the value of \(k\). \(\text{[1]}\)

\(\text{(iv)}\) Given that the deceleration is 1.9 m/s\(^2\) when \(t = 8\), find the value of \(h\). \(\text{[2]}\)

**Solution**

(i) At turning pt, \(v = 0\)
\[
2 - \frac{18}{(t + 2)^2} = 0
\]
\[t = 1 \quad \text{or} \quad -5 \text{ (NA)}\]

(ii)
\[s = \int \frac{dv}{dt} \ dt = 2t + \frac{18}{t + 2} + c\]

When \(t = 1\), \(s = 9\)
\[2(1) + \frac{18}{1 + 2} + c = 9\]
\[c = 1, \text{ so} \quad s = 2t + \frac{18}{t + 2} + 1\]
When \( t = 0 \), \( s = 10 \text{ m} \)
When \( t = 1 \), \( s = 9 \text{ m} \)
When \( t = 4 \), \( s = 12 \text{ m} \)

Total distance travelled = \( 10 - 9 + 12 - 9 = 4 \text{ m} \)

(iii) When \( t = 7 \), \( v = 2 - \frac{18}{(7+2)^2} = \frac{16}{9} \)
\[ V = -h(t-7) + k = \frac{16}{9}, \text{ hence } k = \frac{16}{9} \]

(iv) \( V = -h(t^2 - 7t) + k = -ht^2 + 7ht + k \)
\[ a = \frac{dV}{dt} = -2ht + 7h \]
\[ -2h(8) + 7h = -0.9 \]
\[ -16h + 7h = -0.9 \]
\[ -9h = -0.9 \]
\[ h = 0.1 \]

13 Solutions to this question by accurate drawing will not be accepted.

In the diagram, \( PQ \) is parallel to \( SR \) and the coordinates of \( P, Q \) and \( S \) are \( (3, 2), (9, 4) \) and \( (5, 7) \) respectively. The gradient of the line \( OR \) is 1.

Find
(i) the coordinates of \( R \), \([4]\)
(ii) the area of the quadrilateral \( PQRS \), \([2]\)
(iii) the coordinates of the point \( H \) on the line \( y = 1 \) which is equidistant from \( P \) and \( Q \). \([4]\)

Solution
(i) \( m_{PQ} = \frac{1}{3} \)
Since \( PQ \parallel SR \), \( m_{SR} = \frac{1}{3} \)
Eqn of SR, \( (y - 7) = \frac{1}{3}(x - 5) \)  \( \Rightarrow \)  \( y = \frac{x}{3} + \frac{16}{3} \)

Sub. \( R(a, a) \) into \( y = \frac{x}{3} + \frac{16}{3} \),  \( a = 8 \) \( \Rightarrow \) use eqn of OR as \( y = x \)

\( \therefore R = (8, 8) \)

(ii) \( \text{Area of } PQRS = \frac{1}{2} \begin{vmatrix} 3 & 9 & 8 & 5 & 3 \\ 2 & 4 & 8 & 7 & 2 \end{vmatrix} \quad [M1] \)

\[ = \frac{1}{2} (39) = 19.5 \text{ units}^2 \quad [A1] \]

(iii) Since the point \( H \) lies on the line \( y = 1 \) and is equidistant from \( P \) and \( Q \), \( H \) must lie on the \( \perp \) bisector of \( PQ \).

Mid-point of \( PQ = (6, 3) \)

gradient of \( \perp \) bisector = \(-3\).

Equation, \( (y - 3) = -3(x - 6) \)

\[ y = -3x + 21 \]

Since \( y = 1 \),

\[ 1 = -3x + 21, \quad x = 6 - \frac{2}{3} \]

\( \therefore H(6 - \frac{2}{3}, 1) \)

OR

\[ PH = QH \]

\[ \sqrt{(2-1)^2 + (3-x)^2} = \sqrt{(4-1)^2 + (9-x)^2} \quad \text{using length} \]

\[ 1 + 9 - 6x + x^2 = 9 + 81 - 18x + x^2 \quad \text{expansion} \]

\[ 12x = 80 \]

\[ x = \frac{20}{3} \]

\[ H = \left( \frac{20}{3}, 1 \right) \]
CHIJ KATONG CONVENT
PRELIMINARY EXAMINATION 2018
SECONDARY 4 EXPRESS/
5 NORMAL (ACADEMIC)

ADDITIONAL MATHEMATICS
PAPER 2

4047/02
Duration: 2 hours 30 minutes

Classes: 403, 405, 406, 502

READ THESE INSTRUCTIONS FIRST

Write your name, class and registration number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
Omission of essential working will result in loss of marks.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion

\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]
where \( n \) is a positive integer and
\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}
\]

2. TRIGONOMETRY

Identities

\[
\sin^2 A + \cos^2 A = 1
\]
\[
\sec^2 A = 1 + \tan^2 A
\]
\[
\cosec^2 A = 1 + \cot^2 A
\]
\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
\sin 2A = 2 \sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]
\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \Delta ABC \)

\[
a \cdot \sin A = b \cdot \sin B = c \cdot \sin C
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\Delta = \frac{1}{2} bc \sin A
\]
1 A rectangular garden, with length \( x \) m and breadth \( y \) m, has an area of 270 m\(^2\). It has a path of width 2.5 m all round it. Given that the outer perimeter of the path is 87 m, find the length and breadth of the garden. [5]

2 (a) Solve \( 2(9^{x-1}) - 5(3^x) = 27 \). [4]

(b) Given that \( f(x) = \ln(5x - 2)^3 \),
(i) State the range of \( x \) for \( f(x) \) to be defined. [1]
(ii) Show that \( 5f''(x) + (5x - 2)f''(x) = 0 \). [4]

3 (a) (i) Write down the first four terms in the expansion of \( (1 + x)^{50} \) and \( (1 - x)^{50} \).

Hence, write down the first two terms for \( (1 + x)^{50} - (1 - x)^{50} \). [3]

(ii) Without the use of calculator, deduce if \( 1.01^{50} \) or \( 1.99^{50} + 0.99^{50} \) is larger. [3]

(b) The term independent of \( x \) in \( x^{11}(2x + \frac{k}{x^2})^7 \) is 896.

Find the two possible values of \( k \). [4]

4 (i) Prove that \( \tan A + \cot A = \frac{2}{\sin 2A} \). [4]

(ii) Hence, or otherwise, solve \( \tan A + \cot A = 2.5 \) for \( 0^\circ < A < 270^\circ \). [4]
5 In the diagram, not drawn to scale, \( P(a, b) \) is a point on the graph \( y^2 = 16x \), and \( Q \) is a point on the line \( x = -4 \). \( PQ \) is the perpendicular distance from \( P \) to this line. \( F(4, 0) \) is a point on the \( x \)-axis.

\[ y^2 = 16x \]

\[ x = -4 \]

\[ P(a, b) \]

\[ Q \]

\[ F(4, 0) \]

(i) Find the length \( PF \) in terms of \( a \). [3]

(ii) Given that the tangent to the curve at \( P \) cuts the \( y \)-axis at \( G \), find the coordinates of \( G \) in terms of \( a \). [4]

(iii) Show that \( G \) is the the mid-point of \( QF \). [2]

(iv) Find the equation of the normal at \( P \) in terms of \( a \). [2]

6 (a) Evaluate \( \int_{0}^{\frac{\pi}{6}} \sin \left( 2x + \frac{\pi}{6} \right) \, dx \), leaving your answer in surd form. [3]

(b) (i) Find \( \frac{d}{dx} \left[ e^{2x} \left( \cos 3x + \frac{3}{2} \sin 3x \right) \right] \). [4]

(ii) Hence find \( \int e^{2x} \cos 3x \, dx \). [2]
7. The diagram shows a point $P$ on a circle and $PQ$ is a tangent to the circle. Points $A$, $B$ and $C$ lie on the circle such that $PA$ bisects angle $QPB$ and $QAC$ is a straight line. The lines $QC$ and $PB$ intersect at $D$.

(i) Prove that $AP = AB$. [4]

(ii) Prove that $CD$ bisects angle $PCB$. [4]

(iii) Prove that triangles $CDP$ and $CBA$ are similar. [2]

8. The table below shows experimental values of two variables $x$ and $y$ obtained from an experiment.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>$y$</td>
<td>5.1</td>
<td>17.5</td>
<td>37.5</td>
<td>60.5</td>
<td>98</td>
<td>137</td>
</tr>
</tbody>
</table>

It is also given that $x$ and $y$ are related by the equation $y = ax + bx^2$, where $a$ and $b$ are constants.

(i) Plot $\frac{y}{x}$ against $x$ and draw a straight line graph. Use 2 cm to represent 1 unit on the horizontal axis and 4 cm to represent 10 units on the vertical axis. [4]

(ii) Use the graph to estimate the value of $a$ and of $b$. [2]

(iii) By drawing a suitable straight line, estimate the value of $x$ for which $(b+5)x = 38 - a$. [4]
9 The figure below shows two circles, $C_1$ and $C_2$, touching each other in the first quadrant of the Cartesian plane. $C_1$ has radius 5 and touches the $y$-axis at $D$. $C_2$ has radius 4 and touches the $x$-axis at $E$. The line $AB$ joining the centre of $C_1$ and $C_2$, meets the $x$-axis at $F$. Angle $BFO$ is $\theta$.

(i) Find expressions for $OD$ and $OE$ in terms of $\theta$ and show that

$$DE^2 = 122 + 90\cos \theta + 72\sin \theta.$$  

(ii) Hence express $DE^2$ in the form $122 + R\cos(\theta - \alpha)$, where $R > 0$ and $\alpha$ is acute.

(iii) Calculate the greatest possible length of $DE$ and state the corresponding value of $\theta$. 


10 The population of a town is estimated to increase by \( k \) \% per year. The population at the end of 2017 was 20000. The population, \( y \), after \( x \) years can be modelled by

\[ y = A(1.11)^x. \]

(i) Deduce the value of \( A \) and of \( k \) with the information provided. [2]

(ii) Sketch the graph of \( y \). [1]

(iii) Find the value of \( x \) when \( y = 9600 \).

Explain the meaning of this value of \( x \). [3]

(iv) Calculate the population of the town at the end of 2027. [2]

11 Given that \( y = 2x^3 + 3x^2 + 11x + 5 \),

(i) show that

(a) \( y \) is an increasing function for all values of \( x \), [2]

(b) \( y \) has only one real root at \( x = -\frac{1}{2} \). [3]

(ii) sketch the graph of \( y \), [2]

(iii) hence, calculate the area bounded by \( y = 2x^3 + 3x^2 + 11x + 5 \), the \( x \)-axis and the lines \( x = -1 \) and \( x = 1 \). [4]

End of paper
1. \( xy = 270 \)
\[
\frac{y}{x} = \frac{270}{x} \quad \text{.......... (1)}
\]
\[2(x + 5 + y + 5) = 87\]
\[x + y = \frac{67}{2} \quad \text{.......... (2)}
\]
Substituting (1) into (2),
\[
x + \frac{270}{x} = \frac{67}{2}
\]
\[2x^2 - 67x + 540 = 0\]
\[(2x - 27)(x - 20) = 0\]
\[2x - 27 = 0 \quad \text{or} \quad x - 20 = 0\]
\[x = 13.5 \quad \text{or} \quad x = 20\]

When \( x = 13.5 \), \( y = 20 \)
When \( x = 20 \), \( y = 13.5 \)

Since \( x \) is the length, then \( x = 20 \text{ m} \) and \( y = 13.5 \text{ m} \).

2a. \[2 \left( 3^{2x} \cdot \frac{1}{9} \right) - 5(3^x) = 27\]
Let \( 3^x \),
\[
\frac{2}{9} y^2 - 5y - 27 = 0
\]
\[2y^2 - 45y - 243 = 0\]
\[(2y + 9)(y - 27) = 0\]
\[y = -\frac{9}{2} \quad \text{or} \quad y = 27\]
\[3^x = -\frac{9}{2} \quad \text{(rejected)} \quad \text{or} \quad 3^x = 3^3\]
\[\therefore \ x = 3\]

2bi.
\[5x - 2 > 0\]
\[x > \frac{2}{5}\]

\[f'(x) = \frac{3(5x - 2)^2 \cdot 5}{(5x - 2)^3}\]
\[= \frac{15}{5x - 2}\]
OR
\[ f'(x) = \frac{3 \cdot 5}{(5x - 2)} \]
\[ = \frac{15}{5x - 2} \]

\[ f''(x) = -\frac{15}{(5x - 2)^2} \cdot 5 \]
\[ = -\frac{75}{(5x - 2)^2} \]

\[ \therefore 5f'(x) + (5x - 2)f''(x) = \frac{75}{5x - 2} - \frac{75}{5x - 2} = 0 \]
(shown)

3ai
\[ (1 + x)^{50} = 1^{50} + 50x + \frac{50 \cdot 50}{2} x^2 + \frac{50 \cdot 50 \cdot 49}{3!} x^3 + \ldots + x^{50} \]
\[ = 1 + 50x + 1225x^2 + 19600x^3 + \ldots + x^{50} \]
\[ (1 - x)^{50} = 1 - 50x + 1225x^2 - 19600x^3 + \ldots - x^{50} \]
\[ (1 + x)^{50} - (1 - x)^{50} = 100x + 39200x^3 \]

ii Let \( x = 0.01 \),
\[ 1.01^{50} - 0.99^{50} = 100(0.01) + 39200(0.01)^3 \]
\[ = 1 + 0.0392 \]
\[ 1.01^{50} = 1 + 0.0392 + 0.99^{50} \]
\[ > 1 + 0.99^{50} \]

Hence, \( 1.01^{50} \) is larger.

3b
\[ T_{r+1} = ^7C_r (2x)^{7-r} \left( \frac{k}{x^r} \right)^r \]
\[ = ^7C_r 2^{7-r} k^r x^{7-3r} \]
For \( 7 - 3r = -11 \)
\[ r = 6 \]
\[ x^{11}T_{r+1} = \binom{7}{r} (2x)^{7-r} \left( \frac{k}{x^2} \right)^r x^{11} = \binom{7}{r} 2^{7-r} k^r x^{18-3r} \]

For \( 18 - 3r = 0 \)
\[ r = 6 \]

Term independent of \( x = 896 \)
\[ x^{11} \left( 2x + \frac{k}{x^2} \right)^7 = 896 \]
\[ \binom{7}{6} 2^7 k^6 = 896 \]
\[ k^6 = 64 \]
\[ k = \pm 2 \]

Alternative method:
\[ x^{11} \left( 2x + \frac{k}{x^2} \right)^7 \]
\[ = x^{11} \left( 2^7 x^7 + 7 \left( 2x \right)^6 \left( \frac{k}{x^2} \right) + \binom{7}{2} \left( 2x \right)^5 \left( \frac{k}{x^2} \right)^2 \right) \]
\[ + \binom{7}{4} \left( 2x \right)^4 \left( \frac{k}{x^2} \right)^3 + \binom{7}{5} \left( 2x \right)^3 \left( \frac{k}{x^2} \right)^4 \]
\[ + \binom{7}{6} \left( 2x \right)^2 \left( \frac{k}{x^2} \right)^5 \]
\[ + \left( \frac{k}{x^2} \right)^7 \]
\[ = x^{11} \left( 2^7 x^7 + 7 \left( 2^6 \right) k x^4 + \binom{7}{2} 2^5 k^2 x^5 + \binom{7}{4} 2^4 k^3 x^2 \right) \]
\[ = x^{11} \left( \binom{7}{6} 2k^6 x^{-11} \right) \]
\[ 896 = x^{11} \left( \binom{7}{6} 2k^6 x^{-11} \right) \]
\[ 896 = 14k^6 \]
\[ k^6 = 64 \]
\[ k = \pm 2 \]
4i  \[ LHS = \tan A + \cot A \]
\[ = \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \]
\[ = \frac{\sin^2 A + \cos^2 A}{\sin A \cos A} \]
\[ = \frac{1}{\frac{1}{2} (2 \sin A \cos A)} \]
\[ = \frac{2}{\sin 2A} \]
\[ = RHS \quad \text{(shown)} \]

OR

\[ LHS = \tan A + \cot A \]
\[ = \tan A + \frac{1}{\tan A} \]
\[ = \frac{\tan^2 A + 1}{\tan A} \]
\[ = \frac{\sec^2 A}{\tan A} \]
\[ = \frac{1}{\cos^2 A} \cdot \frac{\cos}{\sin A} \]
\[ = \frac{1}{\sin A \cos A} \]
\[ = \frac{2}{2 \sin A \cos A} \]
\[ = \frac{2}{\sin 2A} \]
\[ = RHS \quad \text{(shown)} \]

4ii  \[ \frac{2}{\sin 2A} = \frac{5}{2} \]
\[ \sin 2A = \frac{4}{5} \]
\[ \alpha = 53.13^\circ \]
\[ 2A = 53.13^\circ, 126.87^\circ, 413.13^\circ, 486.67^\circ \]
\[ A = 26.6^\circ, 63.4^\circ, 206.6^\circ, 243.4^\circ \]
5i \[ y^2 = 16x \]
At P, \( b^2 = 16a \)

PF = \( \sqrt{(a - 4)^2 + b^2} \)
= \( \sqrt{a^2 - 8a + 16 + 16a} \)
= \( \sqrt{(a + 4)^2} \)
= \( a + 4 \)

ii \[ y^2 = 16x \]
\[ y = 4\sqrt{x} \]

\[ \frac{dy}{dx} = \frac{2}{\sqrt{x}} \]
At P,
\[ \frac{dy}{dx} = \frac{2}{\sqrt{a}} \]

Equation of tangent at P,
\[ y - b = \frac{2}{\sqrt{a}}(x - a) \]
\[ y = \frac{2x}{\sqrt{a}} - 2\sqrt{a} + 4\sqrt{a} \]
\[ = \frac{2x}{\sqrt{a}} + 2\sqrt{a} \]

When \( x = 0, \ y = 2\sqrt{a} \)
\[ \therefore G (0, 2\sqrt{a}) \]

iii Mid-point of QF
\[ = \left( -\frac{4 + 4}{2}, \frac{b + 0}{2} \right) \]
\[ = \left( 0, \frac{4\sqrt{a}}{2} \right) \]
\[ = \left( 0, 2\sqrt{a} \right) \]

Hence, G lies in the centre of QF.
OR find lengths of QG and GP.
Gradient of normal at $P = -\frac{\sqrt{a}}{2}$

Equation of normal at $P$:

$$y - b = -\frac{\sqrt{a}}{2}(x - a)$$

$$y = -\frac{\sqrt{a}}{2}x + \frac{a\sqrt{a}}{2} + 4a$$

\[6a\]

\[
\int_0^\pi \sin\left(2x + \frac{\pi}{6}\right)dx
\]

\[= \left[ \cos\left(2x + \frac{\pi}{6}\right) \right]_0^\pi
\]

\[= -\frac{\cos \frac{\pi}{2}}{2} - \left( \frac{\cos \frac{\pi}{6}}{2} \right)
\]

\[= 0 + \frac{\sqrt{3}}{4}
\]

\[= \frac{\sqrt{3}}{4}
\]

\[6bi\]

\[
\frac{d}{dx}\left[ e^{2x} \left( \cos 3x + \frac{3}{2} \sin 3x \right) \right]
\]

\[= 2e^{2x} \left( \cos 3x + \frac{3}{2} \sin 3x \right) + e^{2x} \left( -3 \sin 3x + \frac{9}{2} \cos 3x \right)
\]

\[= e^{2x} \left( 2 \cos 3x + 3 \sin 3x - 3 \sin 3x + \frac{9}{2} \cos 3x \right)
\]

\[= \frac{13}{2} e^{2x} \cos 3x
\]

\[6bii\]

\[
\int e^{2x} \cos 3x \, dx = \frac{2}{13} \int \frac{13}{2} e^{2x} \cos 3x \, dx
\]

\[= \frac{2}{13} e^{2x} \left( \cos 3x + \frac{3}{2} \sin 3x \right) + C
\]

\[7i\]

$\angle ABP = \angle APQ$ (alt. segment theorem)

Since PA bisects $\angle QPB$,

$\angle APQ = \angle APB$

$\therefore \angle ABP = \angle APB$ (base $\angle$s of isosceles triangle APB)

Hence,

$AP = AB.$
7ii \[ \angle ACB = \angle APB \quad (\text{s in the same segment}) \]
\[ \angle ACP = \angle ABP \quad (\text{s in the same segment}) \]
\[ = \angle APB \quad \text{(shown)} \]
\[ \angle ACB = \angle ACP \]

Hence, CD bisects \( \angle PCB \).

7iii \[ \angle ACB = \angle ACP \quad \text{(from ii)} \]
\[ \angle CPD = \angle CAB \quad (\text{s in the same segment}) \]

Hence, \( \triangle CDX \) and \( \triangle CBA \) are similar.

8i \[ \frac{y}{x} = bx + a \]

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y/x</td>
<td>5.1</td>
<td>8.75</td>
<td>12.5</td>
<td>15.13</td>
<td>19.6</td>
<td>22.83</td>
</tr>
</tbody>
</table>
ii

\( a = \frac{y}{x} \) - intercept

\[ a = \frac{1}{1.5} \]

\( b = \text{gradient} \)

\[ b = \frac{13.5}{3.8} = 3.55 \]

iii

\((b + 5) x = 38 - a\)

\[ bx + 5x = 38 - a \]

\[ bx + a = 38 - 5x \]

Draw \( \frac{y}{x} = 38 - 5x \),

at point of intersection, \( x = 4.25 \)

9i

\[ OE = 5 + 9 \cos \theta \]

\[ OD = 4 + 9 \sin \theta \]

\[ DE^2 = OE^2 + OD^2 \]

\[ = (5 + 9 \cos \theta)^2 + (4 + 9 \sin \theta)^2 \]

\[ = 25 + 90 \cos \theta + 81 \cos^2 \theta + 16 + 72 \sin \theta + 81 \sin^2 \theta \]

\[ = 41 + 81 + 90 \cos \theta + 72 \sin \theta \]

\[ = 122 + 90 \cos \theta + 72 \sin \theta \]

ii

Let \( 90 \cos \theta + 72 \sin \theta = R \cos (\theta - \alpha) \).

\[ R = \sqrt{90^2 + 72^2} \]

\[ = \sqrt{13284} \]

\[ = 115 \quad (3 \text{ s.f.}) \]

\[ \theta = \tan^{-1} \frac{72}{90} \]

\[ = 38.65^\circ \]

\[ DE^2 = 122 + 115 \cos (\theta - 38.7^\circ) \]

OR

\[ 122 + \sqrt{13284} \cos (\theta - 38.7^\circ) \]
iii DE is greatest when $\cos(\theta - 38.7^\circ) = 1$

$$DE = \sqrt{122 + 115}$$
$$= 15.4 \text{ units (3 s.f.)}$$

Corresponding $\theta$ is $38.7^\circ$.

10i $A = 20000, k = 11$

ii When $y = 9600$,

$$9600 = 20000(1.11)^x$$
$$x = \log \frac{9600}{20000} + \log 1.11$$
$$= -7.03 \text{ (3 s.f.)}$$

The population of the town was 9600 approximately 7 years ago.

iv When $x = 10$,

$$y = 20000(1.11)^{10}$$
$$= 56788$$

The population of the town would be 56788 (or 56800) at the end of 2027.

11i $y = 2x^3 + 3x^2 + 11x + 5$

$$\frac{dy}{dx} = 6x^2 + 6x + 11$$
$$= 6\left(x + \frac{1}{2}\right)^2 + \frac{19}{2}$$

$$\frac{dy}{dx} > 0 \text{ as } \left(x + \frac{1}{2}\right)^2 \geq 0 \text{ for all values of } x,$$ hence $y$ is an increasing function for all values of $x$. 
ii) Using long division,

\[
y = (2x + 1)(x^2 + x + 5)
\]

But for \(x^2 + x + 5\), discriminant = -19 < 0, hence \(x^2 + x + 5\) has no real roots. Therefore, \(y\) has only one real root at

\[
x = \frac{-1}{2}.
\]

iii) Area required

\[
= \int_{-1}^{1} y \, dx
= \left| \int_{-0.5}^{1} 2x^3 + 3x^2 + 11x + 5 \, dx \right|
+ \int_{-0.5}^{1} 2x^3 + 3x^2 + 11x + 5 \, dx
= \left[ \frac{x^4}{2} + x^3 + \frac{11}{2}x + 5x \right]_{-0.5}^{1} + \left[ \frac{x^4}{2} + x^3 + \frac{11}{2}x + 5x \right]_{-0.5}^{1}
= \left[ \frac{39}{32} + 12 \left( -\frac{39}{32} \right) \right]
= 14.7
or
= 14.4 \text{ sq. units (3 s.f.)}
READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Name of setter: Mrs Margaret Loh

This paper consists of 7 printed pages including the cover page.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$, 

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$ 

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where $n$ is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cosec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\Delta ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$
1. The slope at any point \((x, y)\) of a curve is given by \(\frac{dy}{dx} = \frac{k}{(2x + 3)^2} - 1\) where \(k\) is a constant. If the tangent to the curve at \((-1, 0)\) is perpendicular to the line \(3y = x + 1\), find
   (i) the value of \(k\),
   (ii) the equation of the curve.

2. (i) On the same axes, sketch the curves \(y = -8x^{-\frac{1}{2}}\) and \(y^2 = \frac{1}{4}x\).
   (ii) Find the equation of the line passing through the origin and the point of intersection of the two curves.

3. The equation \(y = \frac{x + c}{x + d}\), where \(c\) and \(d\) are constants, can be represented by a straight line when \(xy - \alpha\) is plotted against \(y\). The line passes through the points \((0, 4)\) and \((0.2, 0)\).
   (i) Find the value of \(c\) and of \(d\),
   (ii) If \((2.5, a)\) is a point on the straight line, find the value of \(a\).

4. The roots of a quadratic equation are \(\alpha\) and \(\beta\), where \(\alpha + \beta = 0\), \(\alpha \beta = \frac{27}{64}\), \(\alpha + \beta > 0\).
   (i) Find this quadratic equation with integral coefficient.
   The roots of another quadratic equation \(x^2 + px + q = 0\) are \(\alpha - \beta\) and \(\beta - \alpha\).
   (ii) Find the value of \(p\) and of \(q\).

5. (i) Prove the identity \(\sin^2 2x(\cot^2 x - \tan^2 x) = 4\cos 2x\).
   (ii) Hence find, for \(0 \leq x \leq 2\pi\), the values of \(x\) for which \(\sin^2 2x = \frac{e}{\cot^2 x - \tan^2 x}\).
(a) The diagram shows a cylinder of height \( h \) cm and base radius \( r \) cm inscribed in a cone of height 28 cm and base radius 10 cm. Show that

(i) the height, \( h \) cm, of the cylinder is given by

\[
h = 28 - \frac{14}{5} r. \tag{1}\]

(ii) the volume, \( V \) cm\(^3\), of the cylinder is given by

\[
V = 14\pi r^2 \left(2 - \frac{r}{5}\right). \tag{1}\]

(b) (i) Given that \( r \) can vary, find the maximum volume of the cylinder. \hfill [5]

(ii) Show that, in this case, the cylinder occupies \( \frac{4}{9} \) of the volume of the cone. \hfill [2]
7. (a) A circle with centre \( P \) lies in the first quadrant of the Cartesian plane. It is tangential to the \( x \)-axis and the \( y \)-axis, and passes through the points \( A(4, 18) \) and \( B(18, 16) \).

Find

(i) the equation of the perpendicular bisector of the line segment \( AB \),

(ii) the coordinates of the centre \( P \),

(iii) the equation of the circle,

The tangent at \( A \) touches the \( x \)-axis at \( R \). The line joining \( A \) and \( P \) is produced to touch the \( x \)-axis at \( S \).

(b) Find the area of triangle \( ARS \).

8. Use the result \((\sqrt{x} + \sqrt{y})^2 = x + y + 2\sqrt{xy}\), or otherwise, find the square root of \(12 + \sqrt{140}\) in the form \(\sqrt{a} + \sqrt{b}\), where \(a\) and \(b\) are constants to be determined.

9. Given that \( P(x) = 2x^4 - 5x^3 + 5x^2 - x - 10 \),

(i) find the quotient when \( P(x) \) is divided by \((2x-1)(x^2+3)\),

(ii) hence express \( \frac{P(x)}{(2x-1)(x^2+3)} \) in partial fractions.

10. The velocity, \( v \) ms\(^{-1}\), of a particle travelling in a straight line at time \( t \) seconds after leaving a fixed point \( O \), is given by

\[
v = 2t^2 + (1 - 3k)t + 8k - 1,
\]

where \( k \) is a constant. The velocity is a minimum at \( t = 5 \).

(i) Show that \( k = 7 \).

(ii) Show that the particle will never return to \( O \) with time.

(iii) Find the duration when its velocity is less than 13 ms\(^{-1}\).

(iv) Find the distance travelled by the particle during the third second.
The diagram shows part of curve $y = \frac{3}{1-2x}$ intersecting with a straight line $y = 2x + 3$ at the point $A$. Find

(i) the coordinates of $A$. \hfill [2]

(ii) the area of the shaded region bounded by the line and the curve. \hfill [4]
In the diagram, two circles touch each other at $A$. $TA$ is tangent to both circles at $A$ and $FE$ is a tangent to the smaller circle at $C$. Chords $AE$ and $AF$ intersect the smaller circle at $B$ and $D$ respectively. Prove that

(i) line $BD$ is parallel to line $FE$,  \[2\]
(ii) $\angle FAC = \angle CAE$,  \[3\]
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(i) $-2$  (ii) $y = \frac{1}{2x+3} - x - 2$</td>
<td>10. (iii) $4s$  (iv) $17\frac{2}{3}$ m or 17.7 m</td>
</tr>
<tr>
<td>2.</td>
<td>(ii) $y = -\frac{1}{x}$</td>
<td>11. (i) $A = (-1,1)$  (ii) $0.352$ units$^2$</td>
</tr>
</tbody>
</table>
| 3. | (i) $c = 4$ ; $d = 20$  (ii) $-46$ | 12. (i) **To prove:** $BD \parallel FE$  
   Proof: Let $\angle TAF$ be $\theta$. |
| 4. | (i) $64x^2 - 72x + 27 = 0$  
   (ii) $p = 0$;  $q = \frac{27}{64}$ |   |
| 5. | (ii) $0.412$, $2.73$, $3.55$, $5.87$ | Using property of corresponding angles, $BD \parallel EF$ (shown) |
| 6. | b(i) $\frac{11200\pi}{27}$ cm$^2$ or $1300$ cm$^2$ | (ii) **To prove:** $\angle FAC = \angle CAE$  
   Proof: Let $\angle BCE = \alpha$ |
| 7. | a(i) $y = 7x - 60$  (ii) $(10, 10)$ |   |
| 8. |   | $\sqrt{7} + \sqrt{5}$ |
| 9. | (i) $x - 2$  (ii) $x - 2 = \frac{3}{2x-1} + \frac{7}{x^2+3}$ |   |

END
<table>
<thead>
<tr>
<th>Qn No</th>
<th>Solutions</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3(y = x + 1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(y = \frac{1}{3}x + \frac{1}{3})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>∴ grad of tangent = −3</td>
<td>M1</td>
</tr>
<tr>
<td>(i)</td>
<td>(-3 = \frac{k}{(2x + 3)^2} - 1)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(k = -2)</td>
<td>A1</td>
</tr>
<tr>
<td>(ii)</td>
<td>(\frac{dy}{dx} = \frac{-2}{(2x + 1)^2} - 1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(y = \int[-2(2x + 1)^{-2} - 1]dx)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(= -\frac{2(2x + 1)^{-1}}{-1} - x + c)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(= \frac{1}{2x + 3} - x + c)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>When (y = 0, x = -1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 = (\frac{1}{-2 + 3}) + 1 + c</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c = -2)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>∴ (y = \frac{1}{(2x + 3)} - x - 2)</td>
<td>A1</td>
</tr>
</tbody>
</table>

2(i)

Graphs are [B1] & [B1]

(ii) \((-8x^{-\frac{1}{2}})^2 = \frac{1}{4}x\) | M1 |
| \(64x^{-1} = \frac{1}{4}x\) |       |
| \(256 = x^2\) |       |
| \(x = 16\) or \(-16\) (NA) | M1   |
| When \(x = 16, y = \frac{-8}{\sqrt{16}} = -2\) |       |
\[
\text{Grad of line } = \frac{-2}{16} = -\frac{1}{8}
\]
\[
\therefore \text{ Eqn of line is } y = -\frac{1}{8}x \quad \text{A1}
\]

3(i) \[
y(x + d) = x + c
\]
\[
xy - x = -yd + c \quad \text{M1}
\]
\[
\therefore \ c = 4 \quad \text{B1}
\]
\[
\text{Grad } = -\frac{4}{0.2} = -20 \quad \text{M1}
\]
\[
\therefore \ -d = -20 \quad d = 20 \quad \text{A1}
\]

(ii) \[
\therefore \ xy - x = -20y + 4
\]
\[
a = -20(2.5) + 4 = -46 \quad \text{B1}
\]

4 \[
\alpha^2 + \beta^2 = 0
\]
\[
(\alpha + \beta)((\alpha + \beta)^2 - 3\alpha\beta) = 0
\]
\[
(\alpha + \beta)((\alpha + \beta)^2 - 3\left(\frac{27}{64}\right)) = 0 \quad \text{M1}
\]
\[
\text{Since } \alpha \neq -\beta, \quad (\alpha + \beta) = \frac{81}{64}
\]
\[
\alpha + \beta = \frac{9}{8} \quad \text{or} \quad -\frac{9}{8} \quad \text{(NA)} \quad \text{A1}
\]

(i) \[
\text{Quad eqn is } x^2 - \frac{9}{8}x + \frac{27}{64} = 0
\]
\[
64x^2 - 72x + 27 = 0 \quad \text{B1}
\]

(ii) \[
\text{Sum of roots } = \alpha - \beta + \beta - \alpha = 0
\]
\[
\text{Prod of roots } = (\alpha - \beta)(\beta - \alpha)
\]
\[
= \alpha\beta - \alpha^2 - \beta^2 + \alpha\beta
\]
\[
= 2\alpha\beta - (\alpha^2 + \beta^2)
\]
\[
= 2\alpha\beta - [(\alpha + \beta)^2 - 2\alpha\beta]
\]
\[
= 4\alpha\beta - (\alpha + \beta)^2
\]
\[
= 4\left(\frac{27}{64}\right) - \left(\frac{9}{8}\right)^2
\]
\[
= \frac{108}{64} - \frac{81}{64} = \frac{27}{64}
\]
\[
\therefore \ p = 0 \quad \text{&} \quad q = \frac{27}{64} \quad \text{B1, B1}
\]
5(i) To prove: \( \sin^2 2x (\cot^2 x - \tan^2 x) = 4 \cos 2x \)

Proof: LHS = \( \sin^2 2x (\cot^2 x - \tan^2 x) \)

\[
= \sin^2 2x \left( \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x} \right)
\]

\[
= \sin^2 2x \left( \frac{\cos^4 x - \sin^4 x}{\sin^2 x \cos^2 x} \right)
\]

\[
= 4 \sin^2 x \cos^2 x \left( \frac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{\sin^2 x \cos^2 x} \right)
\]

\[
= 4(\cos^2 x - \sin^2 x)
\]

\[
= 4 \cos 2x
\]

= RHS (proved)

(ii) \( \sin^2 2x (\cot^2 x - \tan^2 x) = e \)

\[4 \cos 2x = e\]

\[
\cos 2x = \frac{e}{4}
\]

\[2x \approx 0.8236, 5.4596, 7.1068, 11.743\]

\[x \approx 0.412, 2.73, 3.55, 5.87\]

A1, A1

6a(i) Using Similar triangles,

\[
\frac{28 - h}{28} = \frac{r}{10}
\]

\[
28 - h = \frac{28r}{10}
\]

\[
h = 28 - \frac{14}{5} r \quad \text{(shown)}
\]

(ii) Vol of cylinder = \( \pi r^2 h \)

\[
V = \pi r^2 \left( 28 - \frac{14}{5} r \right)
\]

\[
V = 14 \pi r^2 \left( 2 - \frac{1}{5} r \right) \quad \text{(shown)}
\]

b(i) \[
\frac{dV}{dr} = 56 \pi r - \frac{14}{5} \pi (3r^2)
\]

\[
= 14 \pi r (4 - \frac{3}{5} r)
\]

At stat pt, \( \frac{dV}{dr} = 0 \)

\[
14 \pi r (4 - \frac{3}{5} r) = 0
\]

M1
\[ r = 0 \text{ (NA), } 4 - \frac{3}{5}r = 0 \quad \Rightarrow \quad r = 6\frac{2}{3} \]

\[ \frac{d^2V}{dr^2} = 56\pi - \frac{84}{5}\pi r \]

\[ = 56\pi - \frac{84}{5}\pi \left( 6\frac{2}{3} \right) \]

\[ = -175.93(2dp) < 0 \]

Since \( \frac{d^2V}{dr^2} < 0 \), \( r = 6\frac{2}{3} \) will make \( V \) a maximum.

Max volume = \[ 14\pi \left( \frac{20}{3} \right) \left( \frac{20}{3} \right) \left( 2 - \frac{1}{5} \left[ \frac{20}{3} \right] \right) \]

\[ = \frac{11200}{27}\pi \text{ cm}^3 \quad \text{or} \quad 1300 \text{ cm}^3 \quad (3sf) \]

(ii) To show: Vol of cylinder = \( \frac{4}{9} \) (Vol of cone)

Proof: Vol of cone = \( \frac{1}{3}\pi(10)^2(28) = \frac{2800}{3}\pi \text{ cm}^3 \)

\[ \frac{\text{Vol of cylinder}}{\text{Vol of cone}} = \frac{11200\pi}{27} \times \frac{3}{2800\pi} = \frac{4}{9} \]

\[ \therefore \text{ Vol of cylinder} = \frac{4}{9} \text{ (Vol of cone) (shown)} \]

7a(i)

Mid-pt of \( AB \) = \( \left( \frac{4+18}{2}, \frac{18+16}{2} \right) \) = (11, 17)

Grad of \( AB \) = \( \frac{18-16}{4-18} = -\frac{1}{7} \)

Grad of perpendicular bisector = 7

Eqn of perpendicular bisector is \( y - 17 = 7(x - 11) \)

\[ y = 7x - 60 \]

(ii) Let the centre \( P \) be \( (m, m) \).

\[ m = 7m - 60 \]

\[ m = 10 \]

\[ \therefore P = (10, 10) \]

(iii) Eqn of circle is \( (x-10)^2 + (y-10)^2 = 100 \)

Or \( x^2 + y^2 - 20x - 20y + 100 = 0 \)

(b) Grad of \( AP \) = \( \frac{18-10}{4-10} \)

\[ = -\frac{4}{3} \]
\( \text{Grad of tangent at } A = \frac{3}{4} \)

Eqn of tangent at \( A \) is \( y - 18 = \frac{3}{4}(x - 4) \)

\[ y = \frac{3}{4}x + 15 \]

\( R = ( -20, 0 ) \)

Eqn of \( AP \) is \( y - 10 = -\frac{4}{3}(x - 10) \)

\[ y = -\frac{4}{3}x + 23\frac{1}{3} \]

\( S = \left( 17\frac{1}{2}, 0 \right) \)

Area of \( \triangle ARS = \frac{1}{2} \left( 20 + 17\frac{1}{2} \right)(18) \)

\[ = 337.5 \text{ units}^2 \]

\[
8 \quad x + y = 12 \quad \text{----------(1)}
\]

\[ 4xy = 140 \quad \text{----------(2)} \]

From eqn (1): \( y = 12 - x \) sub\( \text{sti into eqn (2)} \)

\[ 4x(12 - x) = 140 \]

\[ x^2 - 12x + 35 = 0 \]

\[ (x - 7)(x - 5) = 0 \]

\( \therefore \ x = 7 \quad \text{or} \quad x = 5 \)

When \( x = 7, \quad y = 5 \)

When \( x = 5, \quad y = 7 \)

\( \therefore \sqrt{12 + \sqrt{140}} = \left( \sqrt{7} + \sqrt{5} \right) \)

9(i) \( (2x - 1)(x^2 + 3) = 2x^3 - x^2 + 6x - 3 \)

\[
\begin{align*}
2x^3 - x^2 + 6x - 3 &= 2x^3 - 5x^3 + 5x^2 - x - 10 \\
&\quad - (2x^3 - x^2 + 6x^2 - 3x) \\
&\quad - 4x^3 - x^2 + 2x - 10 \\
&\quad - ( -4x^3 + 2x^2 - 12x + 6) \\
&\quad - 3x^2 + 14x - 16
\end{align*}
\]

\( \therefore \text{Quotient} = x - 2 \)

(ii) \[ \frac{P(x)}{(2x - 1)(x^2 + 3)} = x - 2 + \frac{-3x^3 + 14x - 16}{(2x - 1)(x^2 + 3)} \]
\[ \frac{(-3x^3 + 14x - 16)}{(2x - 1)(x^2 + 3)} = \frac{A}{2x - 1} + \frac{(Bx + C)}{(x^2 + 3)} \]
where \(A\), \(B\) and \(C\) are constants

\[-3x^3 + 14x - 16 = A(x^2 + 3) + (Bx + C)(2x - 1)\]  

When \(x = \frac{1}{2}\), 
\[-3 \left(\frac{1}{4}\right) + 14 \left(\frac{1}{2}\right) - 16 = A \left(\frac{3}{4}\right)\]
\[A = -3\]  

When \(x = 0\), 
\[-16 = 3A - C\]
\[-16 = -9 - C\]
\[C = 7\]

Comparing coeff of \(x^1\): 
\[-3 = A + 2B\]
\[-3 = -3 + 2B\]
\[B = 0\]

\[P(x) = x - 2 - \frac{3}{2x - 1} + \frac{7}{x^2 + 3}\]

\[\frac{dv}{dt} = 4t + (1 - 3k)\]

When vel is minimum, \(\frac{dv}{dt} = 0\)
\[4(5) + (1 - 3k) = 0\]
\[3k = 21\]
\[k = 7\] (shown)

(ii) When \(k = 7\), \(v = 2t^2 - 20t + 55\)

\[\text{Discriminant} = (-20)^2 - 4(2)(55)\]
\[= 400 - 440\]
\[= -40\]
\[< 0\]

\(\Rightarrow\) there is no real values of \(t\) such that \(v = 0\), also \(v > 0\) hence particle will never return to \(O\) with time.

(iii) \(2t^2 - 20t + 55 < 13\)
\[2t^2 - 20t + 42 < 0\]
\[t^2 - 10t + 21 < 0\]
\[(t - 7)(t - 3) < 0\]

\[\therefore \ 3 < t < 7\]

Duration = \(7 - 3 = 4\ s\)
(iv) \[ s = \frac{1}{2} \int (2t^3 - 20t + 55)\,dt \]

\[ = \left[ \frac{2t^4}{3} - 10t^2 + 55t \right] \]

\[ = [18 - 90 + 165] - \left[ \frac{16}{3} - 40 + 110 \right] \]

\[ = 17\frac{2}{3} \text{ m or } 17.7 \text{ m}(3sf) \quad \text{A1} \]

11(i) \[ \frac{3}{1 - 2x} = 2x + 3 \quad \text{M1} \]

\[ 3 = (2x + 3)(1 - 2x) \]

\[ 3 = 2x - 4x^2 + 3 - 6x \]

\[ 4x^2 + 4x = 0 \]

\[ 4x(x + 1) = 0 \]

\[ x = 0 \text{ or } x = -1 \]

For pt A: When \( x = -1, \) \( y = -2 + 3 = 1 \)

\[ \therefore \quad A = (-1,1) \quad \text{A1} \]

(ii) Area of shaded region = \[ \frac{1}{2} (1 + 3) - \int_{1 - 2x}^{0} \frac{3}{1 - 2x} \, dx \quad \text{M1, M1} \]

\[ = 2 - \left[ \frac{3\ln(1 - 2x)}{-2} \right]_1^0 \quad \text{M1} \]

\[ = 2 - \left[ 0 + \frac{3}{2}\ln 3 \right] \]

\[ = 2 - 1.6479 \approx 0.352 \text{ units}^2 \quad \text{A1} \]

12(i) To prove: \( BD \parallel FE \)

Proof: Let \( \angle TAF \) be \( \theta. \)

\[ \angle ABD = \angle TAF = \theta \quad \text{(alt seg thm)} \quad \text{M1} \]

\[ \angle AEF = \angle TAF = \theta \quad \text{(alt seg thm)} \]

\[ \therefore \quad \angle ABD = \angle AEF = \theta \]

Using property of corresponding angles, \( BD \parallel EF \) (shown) \quad \text{A1} \]

(ii) To prove: \( \angle FAC = \angle CAE \)

Proof: Let \( \angle BCE = \alpha \)

\[ \angle CBD = \angle BCE = \alpha \quad \text{(alt \ angle, } BD//EF) \quad \text{B1} \]

\[ \angle FAC = \angle CBD = \alpha \quad \text{(\angle s in same segment)} \quad \text{B1} \]

Also, \( \angle CAE = \angle BCE = \alpha \quad \text{(alt seg thm)} \quad \text{B1} \]

\[ \therefore \quad \angle FAC = \angle CAE = \alpha \quad \text{(shown)} \]

END
1 (i) A particle moves along the curve \( y = \ln \left( x^2 + 1 \right) \) in such a way that the \( y \)-coordinate of the particle is decreasing at a constant rate of 0.2 units per second. Find the rate at which the \( x \)-coordinate of the particle is changing at the instant when \( x = -0.5 \). [3]

(ii) Find the \( x \)-coordinates of the point on the curve where the gradient is stationary. [3]

2 (i) Solve the equation \( \log_3 (2x+1) - \log_3 (2x-3) = 1 + \log_3 \frac{2}{5} \). [4]

(ii) Solve the equation \( \ln y + 1 = 2 \log_3 e \), giving your answer(s) in terms of \( e \). [5]

3 Given that \( y = e^x \sin x \),

(i) show that \( \frac{dy}{dx} - \frac{d^2 y}{dx^2} = 2y \). [4]

(ii) Hence, or otherwise, find the value of \( \int_0^\frac{\pi}{2} e^x \sin x \, dx \). [4]

4 Given that the first three terms, in ascending powers of \( y \), of the expansion of \( (a + y)^n \), where \( a \) and \( n \) are positive real constants, are \( 64 + 192y + 240y^2 \).

(i) By considering the ratio of the coefficients of the first two terms, show that \( a = \frac{1}{3} n \). [3]

(ii) Find the value of \( a \) and of \( n \). [4]

5 (a) Using the substitution \( u = 2^x \), solve the equation \( 4^{x+1} = 2^x + 3 \). [4]

(b) The quantity, \( N \), of a radioactive substance, at time \( t \) years, is given by \( N = N_0 e^{-kt} \), where \( N_0 \) and \( k \) are positive constants.

(i) Sketch the graph of \( N \) against \( t \), labelling any axes intercepts. [2]

(ii) State the significance of \( N_0 \). [1]

(iii) The quantity halves every 5 years. Calculate the value of \( k \). [3]
6 Solutions to this question by accurate drawing will not be accepted.

The coordinates of the points $P$ and $Q$ are $(-5, 2)$ and $(7, 6)$ respectively. Find

(i) the equation of the line parallel to $PQ$ and passing through the point $(-2, 3)$, [3]

(ii) the equation of the perpendicular bisector of $PQ$. [3]

A point $R$ is such that the shortest distance of $R$ from the line passing through $P$ and $Q$ is $\sqrt{10}$ units.

(iii) Find the area of triangle $PQR$. [3]

7 The diagram shows a sketch of the curve $y = f(x)$. The $x$-coordinates of the maximum and minimum points are $-\alpha$ and $\alpha$, where $k > 0$.

![Diagram of a curve with maximum and minimum points]

It is given that $f'(x) = ax^2 + bx + c$, where $a$, $b$ and $c$ are real constants. For each of the following, state, with reasons, whether they are positive, negative or zero.

(i) $b^2 - 4ac$, [2]

(ii) $\frac{b}{a}$, [2]

(iii) $\frac{c}{a}$, [2]
The diagram shows the cross-section of a house with a rooftop $BAC$. The length of $AB$ and $AC$ are 10 m and 24 m respectively. The angle between $AB$ and the horizontal through $A$ is $\theta$ degrees and $\angle BAC = 90^\circ$.

The base of the house is of length $L$ m.

(i) Show that $L = 10 \cos \theta + 24 \sin \theta$. [2]

(ii) Express $L$ in the form $R \sin (\theta + \alpha)$, where $R > 0$ and $\alpha$ is an acute angle. [4]

(iii) Find the longest possible base of the house and the corresponding value of $\theta$. [3]

9 (a) The equation of a curve is $y = \frac{2x}{1+x}$.

(i) Find the equation of the tangent to the curve at point $P(1,1)$. [4]

(ii) The tangent cuts the axes at $Q$ and $R$ respectively. Find the area of triangle $OPQ$. [2]

(b) A curve has equation $y = f(x)$, where $f(x) = \frac{1}{3}x^3 - 2x^2 + 13x + 5$.

Determine, with explanation, whether $f$ is an increasing or decreasing function. [4]
10 (a) (i) Solve the equation $|x^2 - 3x + 2| + x = 1$. 
(ii) What can be deduced about the number of points of intersection of the graphs of $y = |x^2 - 3x + 2|$ and $y = -x + 1$? 
(iii) Hence, on a single diagram, sketch the graphs of $y = |x^2 - 3x + 2|$ and $y = -x + 1$, indicating the coordinates of any axial intercepts and turning point.

(b) The diagram shows part of the graph of $y = |k - x|$, where $k$ is a constant.

A line $y = mx + c$ is drawn to determine the number of solutions to the equation $|k - x| = mx + c$.
(i) If $m = 1$, state the range of values of $c$, in terms of $k$, such that the equation has one solution.
(ii) If $c = 0$, state the range of values of $m$ such that the equation has no solutions.

11 (a) State the principal range of $\sin^{-1} x$, leaving your answers in terms of $\pi$.
(b) (i) Prove that $\frac{1 + \tan x}{1 - \tan x} = \sec 2x + \tan 2x$.
(ii) Hence find the reflex angle $x$ such that $3\sec 2x + 3\tan 2x = 1$.
(c) A buoy floats and its height above the seabed, $h$ m, is given by $h = a \cos bt + c$, where $t$ is time measured in hours from 0000 hours and $a$, $b$ and $c$ are constants. The least height of the buoy above seabed is 180 metres and is recorded at 0000 hours. The greatest height of the buoy above seabed is 196 metres and is first recorded at 0600 hours.
(i) Find the values of $a$, $b$ and $c$.
(ii) Using values found in (i), sketch the graph of $h = a \cos bt + c$ for $0 \leq t \leq 24$.
(iii) The buoy floats above the top of a huge rock first at 0500 hours. State the number of hours in each day that the buoy is above the rock.

END OF PAPER
Question 1

(i) 0.25 units/s
(ii) \( x = \pm 1 \)

Question 2

(i) \( x = \frac{23}{2} \)
(ii) \( y = e^{-x} \) or \( y = e \)

Question 3

(ii) 1.02 (3s.f.)

Question 4

(ii) \( n = 6, a = 2 \)

Question 5

(a) \( x = 0 \)

(a)(i)

(ii) It represents the initial amount of radioactive substance.
(iii) 0.139

Question 6

(i) \( y = \frac{1}{3}x + 3 \)
(ii) \( y = -3x + 7 \)
(iii) 20 units²

Question 7

(i) \( b^2 - 4ac > 0 \)
(ii) \( \frac{b}{a} > 0 \)
(iii) \( \frac{c}{a} < 0 \)

Question 8

(ii) \( L = 26 \sin (\theta + 22.6^\circ) \)
(iii) Longest possible base is 26 m.
\[ \theta = 67.4^\circ \text{(1 d.p.)} \]

**Question 9**

(a)(i) \[ y = \frac{1}{2}x + \frac{1}{2} \]

(ii) \[ \frac{1}{4} \text{ units}^2 \]

**Question 10**

(a)(i) \[ x = 1 \]

(ii) The line \( y = -x + 1 \) is tangential to \( y = |x^2 - 3x + 2| \).

(b)(i) \( c > -k \)

(ii) \( -1 \leq m < 0 \)

**Question 11**

(a) \[ -\frac{\pi}{2} \leq \sin^{-1}x \leq \frac{\pi}{2} \]

(b)(ii) \[ x = 333.3^\circ \text{(1 d.p.)} \]

(c)(i) \[ a = -8, \ b = \frac{\pi}{6}, \ c = 188 \]

(iii) 4 hours
1 (i) A particle moves along the curve \( y = \ln(x^2 + 1) \) in such a way that the \( y \)-coordinate of the particle is decreasing at a constant rate of 0.2 units per second. Find the rate at which the \( x \)-coordinate of the particle is changing at the instant when \( x = -0.5 \). [3]

(ii) Find the \( x \)-coordinates of the point on the curve where the gradient is stationary. [3]

| (i) | \( \frac{dy}{dx} = \frac{2x}{x^2 + 1} \) | B1 \\
| d\( \frac{dy}{dx} \) = \( \frac{dy}{dt} \times \frac{dx}{dt} \) | M1 \\
| \(-0.2 = \frac{2(-0.5)}{(-0.5)^2 + 1} \times \frac{dx}{dt} \) | A1 \\
| \( \Rightarrow \frac{dx}{dt} = 0.25 \) units/s |

| (ii) | \( \frac{d^2 y}{dx^2} = \frac{(x^2 + 1)(2) - 2x(2x)}{(x^2 + 1)^2} \) | \( \sqrt{M1} \) \\
| = \frac{2 - 2x^2}{(x^2 + 1)^2} | M1 \\
| \( \frac{d^2 y}{dx^2} = 0 \) | A1 \\
| \( 2 - 2x^2 = 0 \) | \\
| \( x = \pm 1 \) |
2  (i)  Solve the equation $\log_3 (2x + 1) - \log_3 (2x - 3) = 1 + \log_3 \frac{2}{5}$.

(ii) Solve the equation $\ln y + 1 = 2 \log_y e$, giving your answer(s) in terms of $e$.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>(i)</td>
<td>$\log_3 (2x + 1) - \log_3 (2x - 3) = 1 + \log_3 \frac{2}{5}$</td>
</tr>
<tr>
<td></td>
<td>$\log_3 \frac{2x + 1}{2x - 3} = \log_3 \left( \frac{3 \times \frac{2}{5}}{5} \right)$</td>
</tr>
<tr>
<td></td>
<td>$\frac{2x + 1}{2x - 3} = 6$</td>
</tr>
<tr>
<td></td>
<td>$\frac{2x + 1}{2x - 3} = 5$</td>
</tr>
<tr>
<td></td>
<td>$10x + 5 = 12x - 18$</td>
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<tr>
<td></td>
<td>$2x = 23$</td>
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<tr>
<td></td>
<td>$x = \frac{23}{2}$</td>
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<tr>
<td></td>
<td>B1, B1</td>
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<tr>
<td></td>
<td>M1 – remove log</td>
</tr>
<tr>
<td>(ii)</td>
<td>$\ln y + 1 = 2 \log_y e$</td>
</tr>
<tr>
<td></td>
<td>$\ln y + 1 = \frac{2}{\ln y}$</td>
</tr>
<tr>
<td></td>
<td>$(\ln y)^2 + \ln y - 2 = 0$</td>
</tr>
<tr>
<td></td>
<td>$(\ln y + 2)(\ln y - 1) = 0$</td>
</tr>
<tr>
<td></td>
<td>$\ln y = -2$ or $1$</td>
</tr>
<tr>
<td></td>
<td>$y = e^{-2}$ or $y = e$</td>
</tr>
<tr>
<td></td>
<td>B1 – change base</td>
</tr>
<tr>
<td></td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>M1 – attempt to solve</td>
</tr>
<tr>
<td></td>
<td>A2</td>
</tr>
</tbody>
</table>
3 Given that \( y = e^x \sin x \),

(i) show that \( \frac{2dy}{dx} - \frac{d^2y}{dx^2} = 2y \). [4]

(ii) Hence, or otherwise, find the value of \( \int_0^\frac{\pi}{3} e^x \sin x \, dx \). [4]

\[
\begin{array}{|c|c|}
\hline
(i) & (ii) \\
\hline
y = e^x \sin x & \frac{-d^2y}{dx^2} + 2 \frac{dy}{dx} = 2y \\
\frac{dy}{dx} = e^x \sin x + e^x \cos x & \therefore \frac{dy}{dx} + 2y = 2 \int e^x \sin x \, dx \\
\frac{d^2y}{dx^2} = e^x \sin x + e^x \cos x - e^x \sin x + e^x \cos x & \Rightarrow -e^x \sin x - e^x \cos x + 2e^x \sin x = 2 \int e^x \sin x \, dx \\
& \therefore \int e^x \sin x \, dx = \frac{1}{2} (e^x \sin x - e^x \cos x) + c \\
& \int_0^\frac{\pi}{3} e^x \sin x \, dx = \left[ \frac{1}{2} (e^x \sin x - e^x \cos x) \right]_0^{\frac{\pi}{3}} \\
& = 1.02 (3.s.f.) \\
\hline
M1 – product rule & M1 – integration \\
B1 & B1 – making integral the subject \\
M1 – product rule & M1 – substitution of limits \\
a.g. & A1 \\
\end{array}
\]
Given that the first three terms, in ascending powers of \( y \), of the expansion of \((a + y)^n\), where \( a \) and \( n \) are positive real constants, are \( 64 + 192y + 240y^2 \).

(i) By considering the ratio of the coefficients of the first two terms, show that \( a = \frac{1}{3}n \). [3]

(ii) Find the value of \( a \) and of \( n \). [4]

<table>
<thead>
<tr>
<th>(i)</th>
<th>((a + y)^n = a^n + na^{n-1}y + \frac{n(n-1)}{2}a^{n-2}y^2 + \ldots)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>By comparing coefficients,</td>
</tr>
<tr>
<td></td>
<td>(a^n = 64)</td>
</tr>
<tr>
<td></td>
<td>(na^{n-1} = 192)</td>
</tr>
<tr>
<td></td>
<td>(\frac{n(n-1)}{2}a^{n-2} = 240)</td>
</tr>
</tbody>
</table>
|     | \(\frac{1}{3}\cdot \frac{a}{n} = 64\) 
|     | \(\frac{a}{n} = 192\)                                            |
|     | \(\Rightarrow a = \frac{1}{3}n\)                                 |

| (ii) | \(2a = 192\) 
|      | \(n - 1 = 240\) 
|      | \(\Rightarrow a = \frac{2}{5}(n-1)\)  
|      | \(\frac{1}{3}n = \frac{2}{5}(n-1)\) 
|      | \(5n = 6n - 6\) 
|      | \(n = 6\) 
|      | \(\Rightarrow a = 2\)  

\(B1\) – award for first two terms

\(M1\), \(A1\)

\(\sqrt{M1}\)

\(\sqrt{M1} – \) simultaneous eqn

\(A1\)

\(A1\)
5 (a) Using the substitution \( u = 2^x \), solve the equation \( 4^{x+1} = 2^x + 3 \). [4]

(b) The quantity, \( N \), of a radioactive substance, at time \( t \) years, is given by \( N = N_0 e^{-kt} \), where \( N_0 \) and \( k \) are positive constants.

(i) Sketch the graph of \( N \) against \( t \), labelling any axes intercepts. [2]

(ii) State the significance of \( N_0 \). [1]

(iii) The quantity halves every 5 years. Calculate the value of \( k \). [3]

<table>
<thead>
<tr>
<th>(a)</th>
<th>( 4u^2 = u + 3 )</th>
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<tbody>
<tr>
<td></td>
<td>( 4u^2 - u - 3 = 0 )</td>
</tr>
<tr>
<td></td>
<td>((4u + 3)(u - 1) = 0 )</td>
</tr>
<tr>
<td></td>
<td>( u = 1 ) or ( u = -\frac{3}{4} )</td>
</tr>
<tr>
<td></td>
<td>( x = 0 ) or ( 2^x = -\frac{3}{4} ) (no solutions)</td>
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<table>
<thead>
<tr>
<th>(a)(i)</th>
<th>B1 – shape</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>B1 – ( t &gt; 0 ) and label ( N_0 )</td>
</tr>
</tbody>
</table>

| (ii) | It represents the initial amount of radioactive substance. B1 |

| (iii) | \( \frac{1}{2} N_0 = N_0 e^{-k(5)} \) |
|       | \( \frac{1}{2} = e^{-5k} \) |
|       | \(-5k = \ln \frac{1}{2} = -\ln 2 \) |
|       | \( t = \frac{\ln 2}{5} \approx 0.139 \) |

\( t \leq \frac{\ln 2}{5} \approx 0.139 \)
6 Solutions to this question by accurate drawing will not be accepted.

The coordinates of the points \( P \) and \( Q \) are \((-5,2)\) and \((7,6)\) respectively. Find

(i) the equation of the line parallel to \( PQ \) and passing through the point \((-2,3)\). \[3\]

(ii) the equation of the perpendicular bisector of \( PQ \). \[3\]

A point \( R \) is such that the shortest distance of \( R \) from the line passing through \( P \) and \( Q \) is \( \sqrt{10} \) units.

(iii) Find the area of triangle \( OQR \). \[3\]

\[
\begin{array}{|c|c|}
\hline
(i) & (ii) \\
\hline
\text{Midpoint of } PQ & \text{Gradient of perpendicular bisector} \\
\hline
\frac{-5+7}{2}, \frac{2+6}{2} = (1,4) & -3 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
(iii) & \\
\hline
\sqrt{(7-(-5))^2 + (6-2)^2} = 4\sqrt{10} \text{ units} & \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
\text{Area} & \\
\hline
\frac{1}{2}(4\sqrt{10})\sqrt{10} = 20 \text{ units}^2 & \\
\hline
\end{array}
\]
7 The diagram shows a sketch of the curve \( y = f(x) \). The \( x \)-coordinates of the minimum and maximum points are \( \alpha \) and \( -\alpha \), where \( \alpha > 0 \).

It is given that \( f'(x) = ax^2 + bx + c \), where \( a \), \( b \) and \( c \) are real constants. For each of the following, state, with reasons, whether they are positive, negative or zero.

(i) \( b^2 - 4ac \), \[2\]
(ii) \( \frac{b}{a} \), \[2\]
(iii) \( \frac{c}{a} \). \[2\]

| (i) | Since there are two stationary points, \( f'(x) = 0 \) has two real roots, therefore \( b^2 - 4ac > 0 \). | M1 | A1 |
| (ii) | Since \( |\alpha| > |\beta| \) and \( \alpha < 0, \alpha + \beta < 0 \), \[ \frac{b}{a} = -(\alpha + \beta) > 0 \] | M1 | A1 |
| (iii) | Since \( \alpha < 0 \) and \( \beta > 0 \), \( \alpha \beta < 0 \), \[ \frac{c}{a} = \alpha \beta < 0 \] | M1 | A1 |
The diagram shows the cross-section of a house with a rooftop $BAC$. The length of $AB$ and $AC$ are 10 m and 24 m respectively. The angle between $AB$ and the horizontal through $A$ is $\theta$ degrees and $\angle BAC = 90^\circ$.

The base of the house is of length $L$ m.

(i) Show that $L = 10 \cos \theta + 24 \sin \theta$. \[2\]

(ii) Express $L$ in the form $R \sin (\theta + \alpha)$, where $R > 0$ and $\alpha$ is an acute angle. \[4\]

(iii) Find the longest possible base of the house and the corresponding value of $\theta$. \[3\]
9 (a) The equation of a curve is \( y = \frac{2x}{1+x} \).

(i) Find the equation of the tangent to the curve at point \( P(1,1) \). \( \text{[4]} \)

(ii) The tangent cuts the axes at \( Q \) and \( R \) respectively. Find the area of triangle \( PQR \). \( \text{[2]} \)

(b) A curve has equation \( y = f(x) \), where \( f(x) = \frac{1}{3}x^3 - 2x^2 + 13x + 5 \).

Determine, with explanation, whether \( f \) is an increasing or decreasing function. \( \text{[4]} \)

\[
\begin{align*}
| (a) & | & \frac{dy}{dx} &= \frac{(1+x)(2)-(2x)(1)}{(1+x)^2} \\
& | & = \frac{2}{(1+x)^2} \\
& | & \frac{dy}{dx} \bigg|_{x=-1} &= \frac{1}{2} \\
\text{Equation of Tangent: } y &= \frac{1}{2}(x-1) \Rightarrow y = \frac{1}{2}x + \frac{1}{2} \\
& & \text{M1} & \text{M1 - substitution of point A1} \\
| (ii) & | & Q(-1,0) \text{ and } R\left(0,\frac{1}{2}\right) \\
& | & \text{Area of Triangle} = \frac{1}{2} \left(1 \cdot \frac{1}{2}\right) = \frac{1}{4} \text{ units}^2 \\
& & \text{B1} & \text{B1} \\
| (b) & | & f'(x) = x^2 - 4x + 13 \\
& & = (x - 2)^2 - 2^2 + 13 \\
& & = (x - 2)^2 + 9 \\
& & (x - 2)^2 \geq 0 \Rightarrow (x - 2)^2 + 9 > 0 \\
& & \therefore f'(x) > 0, f \text{ is an increasing function.} \\
& | & \text{B1} & \text{B1 - complete the square} \\
& | & \text{M1} & \text{M1} \\
& | & \text{A1} & \text{A1} \\
\end{align*}
\]
10 (a) (i) Solve the equation \( |x^2 - 3x + 2| + x = 1 \).

(ii) What can be deduced about the number of points of intersection of the graphs of \( y = |x^2 - 3x + 2| \) and \( y = -x + 1 \)?

(iii) Hence, on a single diagram, sketch the graphs of \( y = |x^2 - 3x + 2| \) and \( y = -x + 1 \), indicating any axial intercepts.

(b) The diagram shows part of the graph of \( y = |k - x| \), where \( k \) is a constant.

A line \( y = mx + c \) is drawn to determine the number of solutions to the equation \( |k - x| = mx + c \).

(i) If \( m = 1 \), state the range of values of \( c \), in terms of \( k \), such that the equation has one solution.

(ii) If \( c = 0 \), state the range of values of \( m \) such that the equation has no solutions.

(a) (i) \[
\begin{align*}
    x^2 - 3x + 2 &= -x + 1 \\
    x^2 - 2x + 1 &= 0 \\
    (x - 1)^2 &= 0 \\
    x &= 1
\end{align*}
\] or

\[
\begin{align*}
    x^2 - 3x + 2 &= -(-x + 1) \\
    x^2 - 4x + 3 &= 0 \\
    (x - 3)(x - 1) &= 0 \\
    x &= 1 \text{ or } x = 3 \text{ (rejected)}
\end{align*}
\] M1

A1, A1

(ii) The line \( y = -x + 1 \) is tangential to \( y = |x^2 - 3x + 2| \). B1
(b)(i) $c > -k$

(ii) $-1 \leq m < 0$

**B1** - modulus graph
**B1** – axial intercepts & turning point
**B1** – line with intercepts
**B1** – line tangent to modulus

A1, A1
11 (a) State the principal range of $\sin^{-1} x$, leaving your answers in terms of $\pi$. [1]

(b) (i) Prove that $\frac{1 + \tan x}{1 - \tan x} = \sec 2x + \tan 2x$. [5]

(ii) Hence find the reflex angle $x$ such that $\sec 2x + \tan 2x = \frac{1}{3}$. [3]

(c) A buoy floats and its height above the seabed, $h$ m, is given by $h = a \cos bt + c$, where $t$ is time measured in hours from 0000 hours and $a$, $b$ and $c$ are constants. The least height of the buoy above seabed is 180 metres and is recorded at 0000 hours. The greatest height of the buoy above seabed is 196 metres and is first recorded at 0600 hours.

(i) Find the values of $a$, $b$ and $c$. [3]

(ii) Using values found in (i), sketch the graph of $h = a \cos bt + c$ for $0 \leq t \leq 24$. [2]

(ii) The buoy floats above the top of a huge rock first at 0500 hours. State the number of hours in each day that the buoy is above the rock. [1]

<table>
<thead>
<tr>
<th>(a)</th>
<th>$-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$</th>
<th>B1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td>$1 + \tan x = \frac{1 + \sin x}{\cos x}$</td>
<td>M1</td>
</tr>
<tr>
<td>(i)</td>
<td>$1 - \tan x = \frac{1 - \sin x}{\cos x}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{\cos x + \sin x}{\cos x - \sin x}$</td>
<td>M1 – double angle</td>
</tr>
<tr>
<td></td>
<td>$= \frac{(\cos x + \sin x)^2}{\cos^2 x - \sin^2 x}$</td>
<td>M1 – double angle</td>
</tr>
<tr>
<td></td>
<td>$= \frac{1 + 2\sin x \cos x}{\cos 2x}$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{1 + \sin 2x}{\cos 2x}$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$= \sec 2x + \tan 2x$</td>
<td>A1</td>
</tr>
</tbody>
</table>

(ii) $1 + \tan x = \frac{1}{3}$

$3 + 3\tan x = 1 - \tan x$

$4 \tan x = -2$

$\tan x = -\frac{1}{2}$

$\alpha = 26.565^\circ$ (3 d.p.)

$x = 333.3^\circ$ (1 d.p.)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>1+ tan x</td>
<td>1/3</td>
<td>M1</td>
</tr>
<tr>
<td>3 + 3 tan x</td>
<td>1 - tan x</td>
<td>B1</td>
</tr>
<tr>
<td>4 tan x</td>
<td>-1/2</td>
<td>A1</td>
</tr>
<tr>
<td>\alpha</td>
<td>26.565° (3 d.p.)</td>
<td></td>
</tr>
<tr>
<td>x</td>
<td>333.3° (1 d.p.)</td>
<td></td>
</tr>
</tbody>
</table>
(c) 
(i) \[
\frac{196 - 180}{2} = 8 \Rightarrow a = -8 \\
c = \frac{196 + 180}{2} = 188 \\
b = \frac{2\pi}{12} = \frac{\pi}{6}
\]

(ii) \[
B_1 - \text{shape} \\
B_1 - \text{points}
\]

(iii) 4 hours

<table>
<thead>
<tr>
<th>(c)</th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
</tr>
</thead>
</table>
|     | \[
\frac{196 - 180}{2} = 8 \Rightarrow a = -8 \\
c = \frac{196 + 180}{2} = 188 \\
b = \frac{2\pi}{12} = \frac{\pi}{6}
\] | \[
B_1 - \text{shape} \\
B_1 - \text{points}
\] | 4 hours | B1 |
INSTRUCTIONS TO CANDIDATES

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion
$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2)(n-3)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities
$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\csc^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}ab \sin C$$
1 The function $f$ is defined, for all values of $x$, by

$$f(x) = x^2 e^{2x}.$$  

Find the values of $x$ for which $f$ is a decreasing function. [4]

2 A man buys an antique porcelain at the beginning of 2015. After $t$ years, its value, $V$, is given by $V = 15000 + 3000e^{0.2t}$.

(i) Find the value of the porcelain when the man first bought it. [1]

(ii) Find the year in which the value of the porcelain first reached $50000$. [3]

3 Given the identity $\cos 3x = 4\cos^3 x - 3\cos x$, find the value of $\int_0^\frac{\pi}{6} \cos^3 x \, dx$. [3]

4 (i) Sketch the graph of $y = 4x^\frac{1}{3}$ for $x \geq 0$. [2]

The line $y = x$ intersects the curve $y = 4x^\frac{1}{3}$ at the points $A$ and $B$.

(ii) Show that the perpendicular bisector of $AB$ passes through the point $(5, 3)$. [4]

5 Solve the following equations:

(i) $\log_8 y + \log_2 y = 4$  

(ii) $10^{2x+1} = 7(10^x) + 26$  

[4]

6 (i) Show that $(\cosec x - 1)(\cosec x + 1)(\sec x - 1)(\sec x + 1) = 1$.  

(ii) Hence solve $(\cosec x - 1)(\cosec x + 1)(\sec x - 1)(\sec x + 1) = 2\tan^2 2x - 5\sec 2x$ for $0 \leq x \leq 360^\circ$. [4]
The function $f(x) = \sin^2 x + 2 \cos^2 x$ is defined for $0 \leq x \leq \pi$.

(i) Express $f(x)$ in the form $a + b \cos 2x$, stating the values of $a$ and $b$. [2]

(ii) State the period and amplitude of $f(x)$. [2]

(iii) Sketch the graph of $y = f(x)$ and hence state the number of solutions of the equation $\frac{1}{2} \frac{x}{2} + \cos 2x = 0$. [4]

A particle moves in a straight line passes through a fixed point $X$ with velocity 5 m/s. Its acceleration is given by $a = 4 - 2t$, where $t$ is the time in seconds after passing $X$. Calculate

(i) the value of $t$ when the particle is instantaneously at rest, [4]

(ii) the total distance travelled by the particle in the first 6 seconds. [4]

The diagram shows part of the graph of $y = 1 - 2x - 6$. Find the coordinates of $A$ and $B$. [3]

A line of gradient $m$ passes through the point (4, 1).

(ii) In the case where $m = 2$, find the coordinates of the points of intersection of the line and the graph of $y = 1 - 2x - 6$. [4]

(iii) Determine the sets of values of $m$ for which the line intersects the graph of $y = 1 - 2x - 6$ in two points. [1]
10 An equilateral triangle $ABC$ is inscribed in a circle. $PT$ is a tangent to the circle at $B$. It is given that $AS = QC$. $PQA$ is a straight line and $BS$ meets $AQ$ at $R$.

(i) Show that $AC$ is parallel to $PB$. [2]
(ii) Prove that $ABS$ is congruent to $CAQ$. [2]
(iii) Prove that $PBQ = BRQ$. [3]

11 In the diagram, $PQRST$ is a piece of cardboard. $PQST$ is a rectangle with $PQ = 2$ cm and $QRS$ is an isosceles triangle with $QR = RS = 4$ cm. $RSQ = RQS = \theta$ radians.

(i) Show that the area, $A$ cm$^2$, of the cardboard is given by $A = 8\sin 2\theta + 16\cos \theta$. [3]
(ii) Given that $\theta$ can vary, find the stationary value of $A$ and determine whether it is a maximum or a minimum. [6]
12 (a) The diagram shows part of a curve \( y^2 = 4x \). The point \( P \) is on the \( x \)-axis and the point \( Q \) is on the curve. \( PQ \) is parallel to the \( y \)-axis and \( k \) is units in length. Given \( R \) is \((2, 0)\), express the area, \( A \), of the \( \text{D}\text{PQR} \) in terms of \( k \) and hence show that \( \frac{dA}{dk} = \frac{3k^2}{8} - \frac{8}{8} \).

The point \( P \) moves along the \( x \)-axis and the point \( Q \) moves along the curve in such a way that \( PQ \) remains parallel to the \( y \)-axis. \( k \) increases at the rate of 0.2 units per second.

Find the rate of increase of \( A \) when \( k = 6 \) units. [5]

(b) The diagram shows part of the curve \( y = 2x^2 + 3 \).

The tangent to the curve at the point \( A( -2, 11) \) intersects the \( y \)-axis at \( B \). Find the area of the shaded region \( ABC \). [6]

~ End of Paper ~
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1. **ALGEBRA**

   **Quadratic Equation**
   For the quadratic equation \( ax^2 + bx + c = 0 \),
   \[
   x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
   \]

   **Binomial Expansion**
   \[
   (a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,
   \]
   where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!} \).

2. **TRIGONOMETRY**

   **Identities**
   \[
   \sin^2 A + \cos^2 A = 1
   \]
   \[
   \sec^2 A = 1 + \tan^2 A
   \]
   \[
   \csc^2 A = 1 + \cot^2 A
   \]
   \[
   \sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
   \]
   \[
   \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
   \]
   \[
   \tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
   \]
   \[
   \sin 2A = 2 \sin A \cos A
   \]
   \[
   \cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
   \]
   \[
   \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
   \]

   **Formulae for \( \Delta ABC \)**
   \[
   \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
   \]
   \[
   a^2 = b^2 + c^2 - 2bc \cos A
   \]
   \[
   \Delta = \frac{1}{2} ab \sin C
   \]
1. The function \( f \) is defined, for all values of \( x \), by

\[ f(x) = x^2 e^{2x}. \]

Find the values of \( x \) for which \( f \) is a decreasing function. [4]

\[ f(x) = x^2 e^{2x} \]

\[ f'(x) = e^{2x} (2x) + x^2 (2e^{2x}) \]

\[ f'(x) = 2xe^{2x} (1 + x) \]

For increasing function,

\[ f'(x) < 0 \]

\[ 2xe^{2x} (1 + x) < 0 \]

Since \( e^{2x} > 0 \)

\[ x(1 + x) < 0 \]

Ans : \( -1 < x < 0 \)

2. A man buys an antique porcelain at the beginning of 2015. After \( t \) years, its value, \( V \), is given by \( V = 15000 + 3000e^{0.2t} \).

(i) Find the value of the porcelain when the man first bought it. [1]

(ii) Find the year in which the value of the porcelain first reached $50,000. [3]

(i) at \( t = 0 \),

\[ V = 15000 + 3000e^{0} = 18000 \]

(ii) \[ 50000 = 15000 + 3000e^{0.2t} \]

\[ 35000 = 3000e^{0.2t} \]

\[ \frac{35}{3} = e^{0.2t} \]

\[ 0.2t = \ln \left( \frac{35}{3} \right) \]

\[ t = 12.283... \]

Ans : 2027
Given the identity \( \cos 3x = 4\cos^3 x - 3\cos x \), find the value of \( \frac{1}{6} \int_0^\pi \cos^3 x \, dx \). \[3\]

\[
\frac{1}{6} \int_0^\pi \cos^3 x \, dx = \frac{1}{4} \left[ \frac{\sin 3x}{3} + 3\sin x \right]_0^\pi = \frac{1}{4} \left[ \left( \frac{1}{3} + 3 \right) \left( \frac{1}{3} + \frac{3}{2} \right) \right] = \frac{5}{24}
\]
4 (i) Sketch the graph of \( y = 4x^{\frac{1}{3}} \) for \( x \geq 0 \). [2]

The line \( y = x \) intersects the curve \( y = 4x^{\frac{1}{3}} \) at the points \( A \) and \( B \).

(ii) Show that the perpendicular bisector of \( AB \) passes through the point \( (5, 3) \). [4]

\[
x = 4x^{\frac{1}{3}}
\]

\[
x \cdot 4x^{\frac{1}{3}} = 0
\]

\[
x^{\frac{1}{3}} \left( x^{\frac{2}{3}} - 4 \right) = 0
\]

\[
x^{\frac{1}{3}} = 0 \quad \text{or} \quad x^{\frac{2}{3}} = 4
\]

\[
x = 0 \quad \text{or} \quad x = 4^{\frac{3}{2}}
\]

\[
x = 0 \quad \text{or} \quad x = 8 \quad (x \geq 0)
\]

\( A(0,0), \quad B(8,8) \)

mid-point of \( AB = (4, 4) \)

gradient \( AB = 1 \)

eqn of perpendicular bisector ,

\[
y = 4 = 1(x - 4)
\]

\[
y = x + 8
\]

when \( x = 5, \ y = 3 \).

Therefore the perpendicular bisector passes through \( (5, 3) \).
5 Solve the following equations:

(i) \[ \log_8 y + \log_2 y = 4 \] [2]

(ii) \[ 10^{2x+1} = 7(10^x) + 26 \] [4]

(i) \[ \log_8 y + \log_2 y = 4 \]

\[ \frac{\log_2 y}{\log_2 8} + \log_2 y = 4 \]

\[ \frac{\log_2 y}{3} + \log_2 y = 4 \]

\[ \frac{4}{3} \log_2 y = 4 \]

\[ \log_2 y = 3 \]

\[ y = 8 \]

(ii) \[ 10^{2x+1} = 7(10^x) + 26 \]

\[ 10^{2x}(10^1) = 7(10^x) + 26 \]

let \( p = 10^x \),

\[ 10p^2 - 7p - 26 = 0 \]

\[ (10p+13)(p-2) = 0 \]

\[ p = \frac{13}{10} \quad \text{or} \quad p = 2 \]

\[ 10^x = \frac{13}{10} \quad \text{or} \quad 10^x = 2 \]

(NA) \quad \text{or} \quad x = \log_2 2 = 0.301
6  (i)  Show that \((\csc x - 1)(\csc x + 1)(\sec x - 1)(\sec x + 1) = 1\). \[2\]

(ii) Hence solve \((\csc x - 1)(\csc x + 1)(\sec x - 1)(\sec x + 1) = 2\tan^2 2x \quad 5\sec 2x\) for \(0 \leq x \leq 360^\circ\). \[4\]

(i) LHS,

\[(\csc x - 1)(\csc x + 1)(\sec x - 1)(\sec x + 1) = (\csc^2 x - 1)(\sec^2 x - 1) = (\cot^2 x)(\tan^2 x) = 1\]

(ii) \((\csc x - 1)(\csc x + 1)(\sec x - 1)(\sec x + 1) = 2\tan^2 2x \quad 5\sec 2x \quad 1 = 2\tan^2 2x \quad 5\sec 2x \quad 2(\sec^2 2x - 1) \quad 5\sec 2x \quad 1 = 0 \quad 2\sec^2 2x \quad 5\sec 2x \quad 3 = 0 \quad (\sec 2x - 3)(2\sec 2x + 1) = 0 \quad \sec 2x = 3 \quad \text{or} \quad 2x = \frac{1}{2} \quad \cos 2x = \frac{1}{3} \quad \text{or} \quad \cos 2x = 2 \quad \text{basic angle,} \quad = 70.529\ldots \quad \text{or} \quad \text{NA} \quad 2x = 360^\circ, 360^\circ, 720^\circ \quad 2x = \frac{1}{3}, 144.7^\circ, 215.3^\circ, 324.7^\circ \quad x = 35.3^\circ, 144.7^\circ, 215.3^\circ, 324.7^\circ
7 The function \( f(x) = \sin^2 x + 2 \cos^2 x \) is defined for \( 0 \leq x \leq 2\pi \).

(i) Express \( f(x) \) in the form \( a + b\cos 2x \), stating the values of \( a \) and \( b \). [2]

(ii) State the period and amplitude of \( f(x) \). [2]

(iii) Sketch the graph of \( y = f(x) \) and hence state the number of solutions of the equation \( \frac{1}{2} \frac{x}{2} + \cos 2x = 0 \). [4]

\( f(x) = \sin^2 x + 2 \cos^2 x \)

\( f(x) = \sin^2 x + \cos^2 x + 2 \cos^2 x \)

\( f(x) = 3 - 2\cos^2 x \)

\( f(x) = 1 - 2\cos^2 x \)

\( f(x) = 1 - 2\cos 2x \)

(ii) Amplitude = 2

Period = \( \frac{2\pi}{2} = \pi \)

(iii) \( \frac{1}{2} \frac{x}{2} + \cos 2x = 0 \)

\( 1 - x = 2\cos 2x \)

\( 2 - x = 1 \ 2\cos 2x \)

No. of solutions = 4
A particle moves in a straight line passes through a fixed point $X$ with velocity 5 m/s. Its acceleration is given by $a = 4 - 2t$, where $t$ is the time in seconds after passing $X$. Calculate

(i) the value of $t$ when the particle is instantaneously at rest, [4]

(ii) the total distance travelled by the particle in the first 6 seconds. [4]

(i) \[ a = 4 - 2t \]
\[ v = (4 - 2t) \, dt \]
\[ v = 4t \, t^2 + c \]

at $t = 0$, $v = 5$,
\[ 5 = c \]

\[ v = 4t \, t^2 + 5 \]

at $v = 0$,
\[ 0 = 4t \, t^2 + 5 \]
\[ t^2 - 4t - 5 = 0 \]
\[ (t - 5)(t + 1) = 0 \]
\[ t = 5 \text{ or } t = -1 \]
(NA)

(ii) \[ s = (4t \, t^2 + 5) \, dt \]
\[ s = 2t^2 \, \frac{t^3}{3} + 5t + c_1 \]

at $t = 0$, $s = 0$,
\[ c_1 = 0 \]

\[ s = 2t^2 \, \frac{t^3}{3} + 5t \]

at $t = 0$, $s = 0$

at $t = 5$, $s = \frac{100}{3}$

at $t = 6$, $s = 30$

Total Distance = $\left(2 \times \frac{100}{3}\right)$ = 30 = $36\frac{2}{3}$
9 (i) The diagram shows part of the graph of \( y = 1 - 2x - 6 \). Find the coordinates of \( A \) and \( B \). [3]

\[
2x - 6 = 0 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \qa
10 An equilateral triangle $ABC$ is inscribed in a circle. $PT$ is a tangent to the circle at $B$. It is given that $AS = QC$. $PQA$ is a straight line and $BS$ meets $AQ$ at $R$.

(i) Show that $AC$ is parallel to $PB$. [2]

(ii) Prove that $ABS$ is congruent to $CAQ$. [2]

(iii) Prove that $PBQ = BRQ$. [3]

(i) $ACB = BAC = 60^\circ$ (equilateral triangle)
$PBC = BAC$ (Alternate Segment Theorem)
Since $PBC = ACB$,
$AC$ is parallel to $PB$ (alternate angle)

(ii) $AS = CQ$ (given)
$BAS = ACQ = 60^\circ$ (equilateral triangle)
$AB = AC$ (sides of a equilateral triangle)
$ABS \equiv CAQ$ (SAS)

(iii) let $RBQ = x$,
$RBA = 60^\circ + x$ (equilateral triangle)
$ASB = 180^\circ - (60^\circ + x)$

$RBA = RAS = 60^\circ + x$ (ABS $\equiv$ CAQ)

$ARS = 180^\circ - (60^\circ + x) = 60^\circ$ (angle sum of triangle)
$BRQ = 60^\circ$ (vertically opposite angle)
s, $PBQ = BRQ$
In the diagram, $PQRST$ is a piece of cardboard. $PQST$ is a rectangle with $PQ = 2$ cm and $QRS$ is an isosceles triangle with $QR = RS = 4$ cm. $RSQ = RQS = \theta$ radians.

(i) Show that the area, $A \text{ cm}^2$, of the cardboard is given by $A = 8 \sin^2 \theta + 16 \cos \theta$. [3]

(ii) Given that $\theta$ can vary, find the stationary value of $A$ and determine whether it is a maximum or a minimum. [6]

(i) \[ QS = 2(4 \cos \theta) = 8 \cos \theta \]
\[ RX = 4 \sin \theta \]

Area, $A = \frac{1}{2} (4 \sin \theta)(8 \cos \theta) + 2(8 \cos \theta) = 16 \sin \theta \cos \theta + 16 \cos \theta = 8 \sin 2 \theta + 16 \cos \theta$

(ii) \[ \frac{dA}{d\theta} = (8 \cos^2 \theta)(2) + 16(\sin \theta) \]
\[ \frac{dA}{d\theta} = 16(\cos^2 \theta \sin \theta) \]

For $\frac{dA}{d\theta} = 0,
\[ \cos 2 \theta \sin \theta = 0 \]
\[ 1 \ 2 \sin^2 \theta \sin \theta = 0 \]
\[ 2 \sin^2 \theta + \sin \theta = 0 \]
\[ (2 \sin \theta + 1)(\sin \theta + 1) = 0 \]
\[ \sin \theta = 0.5 \quad \text{or} \quad \sin \theta = 1 \]
\[ \theta = \frac{\pi}{6} \quad \text{or} \quad \text{NA} \]

\[ A = 12 \sqrt{3} = 20.8 \]
The diagram shows part of a curve \( y^2 = 4x \). The point \( P \) is on the \( x \)-axis and the point \( Q \) is on the curve. \( PQ \) is parallel to the \( y \)-axis and \( k \) is units in length. Given \( R \) is \((2, 0)\), express the area, \( A \), of the \( PQR \) in terms of \( k \) and hence show that \( \frac{dA}{dk} = \frac{3k^2}{8} - \frac{8}{8} \).

The point \( P \) moves along the \( x \)-axis and the point \( Q \) moves along the curve in such a way that \( PQ \) remains parallel to the \( y \)-axis. \( k \) increases at the rate of 0.2 units per second.

Find the rate of increase of \( A \) when \( k = 6 \) units. 

\[
\begin{align*}
\frac{dA}{dt} & = \frac{dA}{dk} \cdot \frac{dk}{dt} \\
\text{at } p = 6, \quad & \frac{dA}{dt} = \left( \frac{3(6)^2}{8} \right) \times 0.2 = 2.5
\end{align*}
\]
The diagram shows part of the curve \( y = 2x^2 + 3 \).

The tangent to the curve at the point \( A(-2, 11) \) intersects the \( y \)-axis at \( B \). Find the area of the shaded region \( ABC \).   

\[
\frac{dy}{dx} = 4x
\]

at \( A \), \( m = 8 \)

let \( B(0, y) \)   

\[
m_{AB} = \frac{11 - 0}{y - 0} = \frac{11}{y}
\]

\[
y = 5
\]

\( B(0, -5) \)

\[
\text{eqn } AB
\]

\[
y = 8x + 5
\]

\[
\text{Area} = \int_{-2}^{0} \left[ (2x^2 + 3)(8x + 5) \right] \, dx
\]

\[
= \int_{-2}^{0} [2x^3 + 8x + 8] \, dx
\]

\[
= \left[ \frac{2x^3}{3} + 4x^2 + 8x \right]_{-2}^{0}
\]

\[
= 0 \left[ \frac{16}{3} + 16 \right] = \frac{16}{3}
\]

~ End of Paper ~
**Mathematical Formulae**

1. **ALGEBRA**

   **Quadratic Equation**

   For the quadratic equation \( ax^2 + bx + c = 0 \),
   
   \[
   x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
   \]

   **Binomial Expansion**

   \[
   (a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
   \]

   where \( n \) is a positive integer and 

   \[
   \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}.
   \]

2. **TRIGONOMETRY**

   **Identities**

   \[
   \sin^2 A + \cos^2 A = 1
   \]

   \[
   \sec^2 A = 1 + \tan^2 A
   \]

   \[
   \csc^2 A = 1 + \cot^2 A
   \]

   \[
   \sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
   \]

   \[
   \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
   \]

   \[
   \tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
   \]

   \[
   \sin 2A = 2 \sin A \cos A
   \]

   \[
   \cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
   \]

   \[
   \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
   \]

   **Formulae for \( \triangle ABC \)**

   \[
   \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
   \]

   \[
   a^2 = b^2 + c^2 - 2bc \cos A
   \]

   \[
   \Delta = \frac{1}{2}ab \sin C
   \]
1. The equation \( 2x^2 + px + 3 = 0 \), where \( p > 0 \), has roots \( \alpha \) and \( \beta \).

(i) Given that \( \alpha^2 + \beta^2 = 1 \), show that \( p = 4 \). \[3\]

(ii) Find the value of \( \alpha^3 + \beta^3 \). \[2\]

(iii) Find a quadratic equation with roots \( \frac{2\alpha}{\beta^2} \) and \( \frac{2\beta}{\alpha^2} \). \[3\]

2. (a) Find the term independent of \( x \) in the expansion of \( 2x \left( 2x - \frac{1}{x^2} \right)^8 \). \[4\]

(b) The first 3 terms in the binomial expansion \( (1 + kx)^n \) are \( 1 + 5x + \frac{45}{4}x^2 + \ldots \). Find the value of \( n \) and of \( k \). \[4\]

3. The diagram shows an isosceles triangle \( ABC \), where \( AB = AC \). The point \( M \) is the mid-point of \( BC \). Given that \( AM = (3 + 2\sqrt{5}) \text{cm} \) and \( BC = (4 + 6\sqrt{5}) \text{cm} \).

Without the use of a calculator, find

(i) the area of triangle \( ABC \), \[2\]

(ii) \( AB^2 \), \[3\]

(iii) \( \sin \angle BAC \), giving your answer in the form \( \frac{p + q\sqrt{5}}{r} \) where \( p \), \( q \) and \( r \) are positive integers. \[3\]
4. (i) Given that \(\frac{6x^3 - 15x^2 + 6x - 5}{2x^2 - x} = ax + b + \frac{c}{2x^2 - x}\), where \(a\), \(b\) and \(c\) are integers, express \(\frac{6x^3 - 15x^2 + 6x - 5}{2x^2 - x}\) in partial fractions. \([5]\)

(ii) Hence find \(\int \frac{6x^3 - 15x^2 + 6x - 5}{2x^2 - x} \, dx\). \([3]\)

5. The term containing the highest power of \(x\) and the term independent of \(x\) in the polynomial \(f(x)\) are \(2x^4\) and \(-3\) respectively. It is given that \((2x^2 + x - 1)\) is a quadratic factor of \(f(x)\) and the remainder when \(f(x)\) is divided by \((x - 1)\) is \(4\).

(i) Find an expression for \(f(x)\) in descending powers of \(x\), \([5]\)

(ii) Explain why the equation \(f(x) = 0\) has only 2 real roots and state the values. \([4]\)

6. \(PQRS\) is a rectangle. A line through \(Q\), intersects \(PS\) at \(N\) and \(RS\) produced at \(T\), where \(QN = 7\) cm, \(NT = 17\) cm, \(\angle NTS = \theta\), and \(\theta\) varies.

(i) Show that the perimeter of \(PQRS\), \(P\) cm, is given by \(P = 14\cos \theta + 48\sin \theta\). \([2]\)

(ii) Express \(P\) in the form of \(R \cos(\theta - \alpha)\) and state the value of \(R\) and \(\alpha\) in degree. \([3]\)

(iii) Without evaluating \(\theta\), justify with reasons if \(P\) can have a value of \(48\) cm. \([1]\)

(iv) Find the value of \(P\) for which \(QR = 12\) cm. \([2]\)
7. Variables $x$ and $y$ are related by the equation \( \frac{x + sy}{t} = xy \), where $s$ and $t$ are constants.

The table below shows the measured values of $x$ and $y$ during an experiment.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1.00</th>
<th>1.50</th>
<th>2.00</th>
<th>2.50</th>
<th>3.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0.48</td>
<td>0.65</td>
<td>0.85</td>
<td>1.00</td>
<td>1.13</td>
</tr>
</tbody>
</table>

(i) On graph paper, draw a straight line graph of $\frac{x}{y}$ against $x$, using a scale of 4 cm to represent 1 unit on the $x$–axis. The vertical $\frac{x}{y}$–axis should start at 1.5 and have a scale of 1 cm to 0.1 units. \[3\]

(ii) Determine which value of $y$ is inaccurate and estimate its correct value. \[1\]

(iii) Use your graph to estimate the value of $s$ and of $t$. \[2\]

(iv) By adding a suitable straight line on the same axes, find the value of $x$ and $y$ which satisfy the following pair of simultaneous equations.

\[
\frac{x + sy}{t} = xy \\
5y - 2x = 2xy.
\] \[3\]

8. The equation of a circle $C_1$, is $x^2 + y^2 - 2x - y - 10 = 0$.

(i) Find the centre and the radius of the circle. \[3\]

(ii) The equation of a tangent to the circle $C_1$ at the point $A$ is $y + 2x = k$, where $k > 0$.

Find the value of the constant $k$. \[4\]

A second circle $C_2$ has its centre at point $A$ and its lowest point $B$ lies on the $x$-axis.

(iii) Find the equation of the circle $C_2$. \[2\]

9. (a) The curve $y = \frac{2x - 5}{1 - 2x}$ passes through the point $A$ where $x = 1$.

(i) Find the equation of the normal to the curve at the point $A$. \[4\]

(ii) Find the acute angle the tangent makes with the positive $x$-axis. \[2\]
9. (b) The curve \( y = f(x) \) is such that \( f''(x) = 3(e^x - e^{-3x}) \) and the point \( P(0, 2) \) lies on the curve. Given that the gradient of the curve at \( P \) is 5, find the equation of the curve. \([6]\)

10. The diagram (not drawn to scale) shows a trapezium \( OPQR \) in which \( PQ \) is parallel to \( OR \) and \( \angle ORQ = 90^\circ \). The coordinates of \( P \) and \( R \) are \((-4, 3)\) and \((4, 2)\) respectively and \( O \) is the origin.

\[
\begin{array}{c}
\text{\( P(\text{-}4, 3) \)} \\
\text{\( Q \)} \\
\text{\( T \)} \\
\text{\( R(4, 2) \)} \\
\text{\( \text{\textbullet} \)} \\
\text{\( O \)} \\
\end{array}
\]

(i) Find the coordinates of \( Q \). \([3]\)

(ii) \( PQ \) meets the \( y \)-axis at \( T \). Show that triangle \( ORT \) is isosceles. \([2]\)

(iii) Find the area of the trapezium \( OPQR \). \([2]\)

(iv) \( S \) is a point such that \( ORPS \) forms a parallelogram, find the coordinates of \( S \). \([2]\)

11. (a) Given that \( y = x^2 \sqrt{2x+1} \), show that \( \frac{dy}{dx} = \frac{x(5x+2)}{\sqrt{2x+1}} \). \([3]\)

(b) Hence

(i) find the coordinates of the stationary points on the curve \( y = x^2 \sqrt{2x+1} \) and determine the nature of these stationary points. \([5]\)

(ii) evaluate \( \int_1^5 \frac{5x^2 + 2x - 3}{\sqrt{2x+1}} \, dx \). \([4]\)

~~ End of Paper ~~
READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in. Write in dark blue or black pen on both sides of the paper. You may use an HB pencil for any diagrams or graphs. Do not use staples, paper clips, glue or correction fluid.

Answer all the questions. Write your answers on the separate Answer Paper provided. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. The use of an approved scientific calculator is expected, where appropriate. You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together. The number of marks is given in brackets [ ] at the end of each question or part question. The total number of marks for this paper is 100.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the quadratic equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial Expansion
\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]
where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!} \).

2. TRIGONOMETRY

Identities
\[
\sin^2 A + \cos^2 A = 1
\]
\[
\sec^2 A = 1 + \tan^2 A
\]
\[
\csc^2 A = 1 + \cot^2 A
\]
\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
\sin 2A = 2\sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A
\]
\[
\tan 2A = \frac{2\tan A}{1 - \tan^2 A}
\]
\[
\sin A + \sin B = 2\sin \frac{1}{2}(A + B)\cos \frac{1}{2}(A - B)
\]
\[
\sin A - \sin B = 2\cos \frac{1}{2}(A + B)\sin \frac{1}{2}(A - B)
\]
\[
\cos A + \cos B = 2\cos \frac{1}{2}(A + B)\cos \frac{1}{2}(A - B)
\]
\[
\cos A - \cos B = -2\sin \frac{1}{2}(A + B)\sin \frac{1}{2}(A - B)
\]

Formulae for \( \Delta ABC \)
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\Delta = \frac{1}{2}ab \sin C
\]
1. The equation $2x^2 + px + 3 = 0$, where $p > 0$, has roots $\alpha$ and $\beta$.

(i) Given that $\alpha^2 + \beta^2 = 1$, show that $p = 4$. [3]

(ii) Find the value of $\alpha^3 + \beta^3$. [2]

(iii) Find a quadratic equation with roots $\frac{2\alpha}{\beta^2}$ and $\frac{2\beta}{\alpha^2}$. [3]

(i) $\alpha + \beta = -\frac{p}{2}$ and $\alpha\beta = \frac{3}{2}$

$\alpha^2 + \beta^2 = 1$

$(\alpha + \beta)^2 - 2\alpha\beta = 1$

$\frac{p^2}{4} - 3 = 1$

$p^2 = 16$

$p = 4$ or $p = -4$

Since $p > 0$, $p = 4$ (Shown)

(ii) $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= -2(1 - \frac{3}{2})$

$= 1$

(iii) $\frac{2\alpha}{\beta^2} \times \frac{2\beta}{\alpha^2} = \frac{2(\alpha^3 + \beta^3)}{\alpha^2 \beta^2}$

$= \frac{8}{9}$

$\frac{2\alpha}{\beta^2} \times \frac{2\beta}{\alpha^2} = \frac{4}{\alpha\beta}$

$= \frac{8}{3}$

Required quadratic equation: $x^2 - \frac{8}{9} x + \frac{8}{3} = 0$ or $9x^2 - 8x + 24 = 0$
2. (a) Find the term independent of \( x \) in the expansion of \( 2x \left( 2x - \frac{1}{x^2} \right)^8 \). [4]

(b) The first 3 terms in the binomial expansion \( (1 + kx)^n \) are \( 1 + 5x + \frac{45}{4}x^2 + ... \)

Find the value of \( n \) and of \( k \). [4]

(a) For \( \left( 2x - \frac{1}{x^2} \right)^8 \), \( T_{r+1} = \binom{8}{r} \left( 2x \right)^{8-r} \left( -\frac{1}{x^2} \right)^r \)

For \( x^{-1} \), \( 8 - r - 2r = -1 \)

\( r = 3 \)

Coefficient of \( x^{-1} = \binom{8}{3} \left( 2 \right)^5 \left( -1 \right)^3 = -1792 \)

Term independent of \( x \) in \( 2x \left( 2x - \frac{1}{x^2} \right)^8 = -3584 \).

(b) \( (1 + kx)^n = 1 + \binom{n}{1} kx + \binom{n}{2} k^2 x^2 + ... \)

\( = 1 + nkx + \frac{n(n-1)k^2}{2} x^2 + ... \)

Comparing coefficients: \( nk = 5 \) ............ (1)

\( \frac{n(n-1)k^2}{2} = \frac{45}{4} \)

\( 2n^2k^2 - 2nk^2 = 45 \) ............ (2)

Subst (1) in (2): \( 50 - 10k = 45 \)

\( \therefore k = \frac{1}{2} \) and \( n = 10 \)
3. The diagram shows an isosceles triangle $ABC$, where $AB = AC$. The point $M$ is the midpoint of $BC$. Given that $AM = (3 + 2\sqrt{5}) \text{cm}$ and $BC = (4 + 6\sqrt{5}) \text{cm}$.

Without the use of a calculator, find
(i) the area of triangle $ABC$, [2]
(ii) $AB^2$, [3]
(iii) $\sin \angle BAC$, giving your answer in the form $\frac{p + q\sqrt{5}}{r}$ where $p$, $q$ and $r$ are positive integers. [3]

(i) Area of triangle $ABC = \frac{1}{2} (4 + 6\sqrt{5})(3 + 2\sqrt{5})$

\[= (2 + 3\sqrt{5})(3 + 2\sqrt{5})\]

\[= (36 + 13\sqrt{5}) \text{ cm}^2\]

(ii) $AB^2 = (3 + 2\sqrt{5})^2 + (2 + 3\sqrt{5})^2$

\[= 9 + 12\sqrt{5} + 20 + 4 + 12\sqrt{5} + 45\]

\[= (78 + 24\sqrt{5}) \text{ cm}^2\]

(iii) $\frac{1}{2} (78 + 24\sqrt{5}) \sin \angle BAC = 36 + 13\sqrt{5}$

\[
\sin \angle BAC = \frac{36 + 13\sqrt{5}}{39 + 12\sqrt{5}}
\]

\[= \frac{36 + 13\sqrt{5}}{39 + 12\sqrt{5}} \times \frac{39 - 12\sqrt{5}}{39 - 12\sqrt{5}}\]

\[= \frac{1404 - 432\sqrt{5} + 507\sqrt{5} - 780}{801}\]

\[= \frac{624 + 75\sqrt{5}}{801}\]

\[= \frac{208 + 25\sqrt{5}}{267}\]
4. (i) Given that \( \frac{6x^3-15x^2+6x-5}{2x^2-x} = ax + b + \frac{c}{2x^2-x} \), where \( a, b \) and \( c \) are integers, express \( \frac{6x^3-15x^2+6x-5}{2x^2-x} \) in partial fractions. \([5]\]

(ii) Hence find \( \int \frac{6x^3-15x^2+6x-5}{2x^2-x} \, dx \). \([3]\]

(i) Using long division, \( \frac{6x^3-15x^2+6x-5}{2x^2-x} = 3x - 6 - \frac{5}{2x^2-x} \)

Let \( \frac{-5}{x(2x-1)} = \frac{A}{x} + \frac{B}{2x-1} \)

\(-5 = A(2x-1) + Bx \)

Put \( x = 0 \) : \( A = 5 \)

Put \( x = \frac{1}{2} \) : \( B = -10 \)

\[ -5 = A(2x-1) + Bx \]

\[ \frac{6x^3-15x^2+6x-5}{2x^2-x} = 3x - 6 - \frac{5}{2x^2-x} \]

(ii) \[ \int \frac{6x^3-15x^2+6x-5}{2x^2-x} \, dx = \int \left(3x - 6 + \frac{5}{x} - \frac{10}{2x-1}\right) \, dx \]

\[ = \frac{3x^2}{2} - 6x + 5 \ln x - 5 \ln(2x-1) + C \]
5. The term containing the highest power of \( x \) and the term independent of \( x \) in the polynomial \( f(x) \) are \( 2x^4 \) and \(-3\) respectively. It is given that \((2x^2 + x - 1)\) is a quadratic factor of \(f(x)\) and the remainder when \( f(x) \) is divided by \( (x - 1) \) is 4.

(i) Find an expression for \( f(x) \) in descending powers of \( x \), [5]

(ii) Explain why the equation \( f(x) = 0 \) has only 2 real roots and state the values. [4]

(i) \[ f(x) = (2x^2 + x - 1)(x^2 + bx + 3) \]

\[ f(1) = 4 \]

\[ 2(4 + b) = 4 \]

\[ b = -2 \]

\[ f(x) = (2x^2 + x - 1)(x^2 - 2x + 3) \]

\[ = 2x^4 - 4x^3 + 6x^2 + x^3 - 2x^2 + 3x - x^2 + 2x - 3 \]

\[ = 2x^4 - 3x^3 + 3x^2 + 5x - 3 \]

(ii) \[ f(x) = (2x^2 + x - 1)(x^2 - 2x + 3) \]

\[ = (2x - 1)(x + 1)(x^2 - 2x + 3) \]

\[ (2x - 1)(x + 1)(x^2 - 2x + 3) = 0 \]

\[ x = \frac{1}{2} \text{ or } x = -1 \]

\[ x^2 - 2x + 3 = 0 \]

\[ D = (-2)^2 - 4(1)(3) = -8 < 0 \]

\[ \therefore f(x) = 0 \text{ has only 2 real roots (Shown)} \]
6. \( PQRS \) is a rectangle. A line through \( Q \), intersects \( PS \) at \( N \) and \( RS \) produced at \( T \), where \( QN=7 \text{cm}, NT=17 \text{cm}, \angle NTS=\theta \), and \( \theta \) varies.

\( T \)

\( N \)

\( S \)

\( R \)

\( Q \)

\( 7 \text{cm} \)

\( 17 \text{cm} \)

(i) Show that the perimeter of \( PQRS \), \( P \) cm, is given by \( P=14\cos\theta+48\sin\theta \).

(ii) Express \( P \) in the form of \( R\cos(\theta-\alpha) \) and state the value of \( R \) and \( \alpha \) in degrees.

(iii) Without evaluating \( \theta \), justify with reasons if \( P \) can have a value of 48 cm.

(iv) Find the value of \( P \) for which \( QR=12 \text{cm} \).

\( P = 2(7\cos\theta) + 2(48\sin\theta) \)

\[ P = 14\cos\theta + 48\sin\theta \]

\[ 14\cos\theta + 48\sin\theta = R\cos(\theta-\alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha \]

\( R\cos\alpha = 14 \) and \( R\sin\alpha = 48 \)

\[ R = \sqrt{14^2 + 48^2} = \sqrt{2500} = 50 \]

\[ \tan\alpha = \frac{48}{14} \]

\[ \alpha = 73.74^\circ \]

\( 48^\circ \)

\( 14\cos\theta + 48\sin\theta = 50\cos(\theta-73.74^\circ) \)

(ii) Since maximum value of \( P = 50 \), \( P \) can have a value of 48 cm.

\[ \text{Or} \quad \cos(\theta-73.74^\circ) = \frac{48}{50} < 1 \quad \text{P can have a value of 48 cm.} \]

When \( QR=12 \text{cm} \), \( \sin\theta = \frac{12}{24} = \frac{1}{2} \)

\[ \theta = 30^\circ, 150^\circ (\text{NA} \quad \therefore \theta < 90^\circ) \]

\[ \therefore P = 50\cos(30^\circ - 73.74^\circ) \]

\[ = 36.1 \text{ cm (3sf)} \]
7. Variables $x$ and $y$ are related by the equation $\frac{x + sy}{t} = xy$, where $s$ and $t$ are constants. The table below shows the measured values of $x$ and $y$ during an experiment.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1.00</th>
<th>1.50</th>
<th>2.00</th>
<th>2.50</th>
<th>3.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0.48</td>
<td>0.65</td>
<td>0.85</td>
<td>1.00</td>
<td>1.13</td>
</tr>
</tbody>
</table>

(i) On graph paper, draw a straight line graph of $\frac{x}{y}$ against $x$, using a scale of 4 cm to represent 1 unit on the $x$ – axis. The vertical $\frac{x}{y}$ – axis should start at 1.5 and have a scale of 1 cm to 0.1 units. [3]

(ii) Determine which value of $y$ is inaccurate and estimate its correct value. [1]

(iii) Use your graph to estimate the value of $s$ and of $t$. [2]

(iv) By adding a suitable straight line on the same axes, find the value of $x$ and $y$ which satisfy the following pair of simultaneous equations.

$$\frac{x + sy}{t} = xy$$
$$5y - 2x = 2xy.$$ [3]

(i) $x + sy = xyt$

$$\frac{x}{y} = tx - s$$

Gradient = $t$ and $\frac{x}{y} - \text{intercept} = -s$

(ii) Incorrect value of $y = 0.65$.

From graph, correct value of $\frac{x}{y} = 2.2$

Estimated correct value of $y = 0.68$

(iii) From the graph,

$s = -1.75$ (-1.82 ~ -1.72)
$t = 0.3$ (0.28 ~ 0.32)

(iv) Draw the line : $\frac{x}{y} = -x + \frac{5}{2}$

From graph, $x = 0.575$ (0.55 ~ 0.60)

and $\frac{x}{y} = 1.93 (1.92 ~ 1.95) \Rightarrow y = 0.30$
8. The equation of a circle \( C_1 \), is \( x^2 + y^2 - 2x - y - 10 = 0 \).

(i) Find the centre and the radius of the circle. [3]

(ii) The equation of a tangent to the circle \( C_1 \) at the point \( A \) is \( y + 2x = k \), where \( k > 0 \).

Find the value of the constant \( k \). [4]

A second circle \( C_2 \) has its centre at point \( A \) and its lowest point \( B \) lies on the \( x \)-axis.

Find the equation of the circle \( C_2 \). [2]

(i) \[ x^2 + y^2 - 2x - y - 10 = 0 \]

\[ (x-1)^2 - 1 + \left( y - \frac{1}{2} \right)^2 - \frac{1}{4} - 10 = 0 \]

\[ (x-1)^2 + \left( y - \frac{1}{2} \right)^2 = 11 + \frac{1}{4} \]

\[ \therefore \text{centre of circle} = \left( 1, \frac{1}{2} \right) \text{ and radius} = \frac{\sqrt{45}}{2} = \frac{3\sqrt{5}}{2} \text{ units} \]

(ii) \[ x^2 + (k - 2x)^2 = 2x - (k - 2x) - 10 = 0 \]

\[ 5x^2 - 4kx + k^2 - 2x - k + 2x - 10 = 0 \]

\[ 5x^2 - 4kx + k^2 - k - 10 = 0 \]

Since line is a tangent to the circle, Discriminant = 0

\[ (-4k)^2 - 4(5)(k^2 - k - 10) = 0 \]

\[ -4k^2 + 20k + 200 = 0 \]

\[ k^2 - 5k - 5 = 0 \]

\[ k = 10 \quad \text{or} \quad k = -5 \ (NA \therefore k > 0) \]

(iii) When \( k = 10 \), \[ 5x^2 - 40x + 80 = 0 \]

\[ x^2 - 8x + 16 = 0 \]

\[ \therefore x = 4 \quad \text{and} \quad y = 2 \]

\( A(4, \ 2) \)

Since lowest point lies on \( x \)-axis, radius of circle \( C_2 = 2 \) units

Equation of circle \( C_2 \) : \( (x - 4)^2 + (y - 2)^2 = 4 \).
9. (a) The curve \( y = \frac{2x - 5}{1 - 2x} \) passes through the point \( A \) where \( x = 1 \).

(i) Find the equation of the normal to the curve at the point \( A \).

(ii) Find the acute angle the tangent makes with the positive \( x \)-axis.

(a)(i) \( y = \frac{2x - 5}{1 - 2x} \)

\[
\frac{dy}{dx} = \frac{(1-2x)(2) - (2x-5)(-2)}{(1-2x)^2}
\]

\[
= \frac{2 - 4x + 4x - 10}{(1-2x)^2}
\]

\[
= \frac{-8}{(1-2x)^2}
\]

\( m_{\text{tangent}} = -8 \)

\( m_{\text{normal}} = \frac{1}{8} \)

\( y = 3 \)

\( y - 3 = \frac{1}{8} (x - 1) \)

\( y = \frac{1}{8} x + \frac{23}{8} \) or \( 8y = x + 23 \)

(ii) \( \tan \theta = 8 \)

\( \theta = 82.9^\circ \) or \( 1.45 \text{ rad} \)
9. (b) The curve \( y = f(x) \) is such that \( f''(x) = 3(e^x - e^{-3x}) \) and the point \( P(0, 2) \) lies on the curve. Given that the gradient of the curve at \( P \) is 5, find the equation of the curve.

\[
f'(x) = 3e^x + e^{-3x} + C, \text{ where } C \text{ is an arbitrary constant.}
\]

\[
f'(0) = 5
\]

\[
3e^0 + e^0 + C = 5 \quad \Rightarrow C = 1
\]

\[
\therefore f'(x) = 3e^x + e^{-3x} + 1
\]

\[
f(x) = \int (3e^x + e^{-3x} + 1) \, dx
\]

\[
= 3e^x - \frac{e^{-3x}}{3} + x + D, \text{ where } D \text{ is an arbitrary constant.}
\]

\[
f(0) = 2
\]

\[
3 - \frac{1}{3} + 0 + D = 2 \quad \Rightarrow D = -\frac{2}{3}
\]

Equation of curve : \( y = 3e^x - \frac{1}{3e^{3x}} + x - \frac{2}{3} \).
10. The diagram (not drawn to scale) shows a trapezium $OPQR$ in which $PQ$ is parallel to $OR$ and $\angle ORQ = 90^\circ$. The coordinates of $P$ and $R$ are $(-4, 3)$ and $(4, 2)$ respectively and $O$ is the origin.

(i) Find the coordinates of $Q$. [3]

(ii) $PQ$ meets the $y$-axis at $T$. Show that triangle $ORT$ is isosceles. [2]

(iii) Find the area of the trapezium $OPQR$. [2]

(iv) $S$ is a point such that $ORPS$ forms a parallelogram, find the coordinates of $S$. [2]

(i) Gradient of $PQ$ = gradient of $OR$ = 0.5

Eqn of $PQ$: $y - 3 = \frac{1}{2}(x + 4)$

$y = \frac{1}{2}x + 5 \quad (1)$

Gradient of $QR = -2$

Eqn of $QR$: $y - 2 = -2(x - 4)$

$y = -2x + 10 \quad (2)$

$(1)=(2)$

$-2x + 10 = \frac{1}{2}x + 5$

$\frac{5}{2}x = 5$

$x = 2$

$y = -2(2) + 10 = 6$

$\therefore Q(2, 6)$
(ii) In eqn (1), let \( x = 0, \ y = 5, \therefore \ OT = 5 \text{units} \)

\[
RT = \sqrt{(4-0)^2 + (2-5)^2} \\
RT = \sqrt{25} = 5
\]

Since \( OT = RT = 5 \text{ units} \)

\[\therefore \Delta ORT \text{ is isosceles.}\]

Area of trapezium \( OPQR \)

\[
\frac{1}{2} \begin{vmatrix} 0 & -4 & 2 & 4 & 0 \\ 0 & 3 & 6 & 2 & 0 \end{vmatrix} \\
= \frac{1}{2} \begin{vmatrix} -24 + 4 - 24 - 6 \end{vmatrix} \\
= \frac{1}{2} \begin{vmatrix} -50 \end{vmatrix} \\
= 25 \text{units}^2
\]

(iii) Let \( S (a, b) \)

Midpoint of \( RS = \) Midpoint of \( OP \)

\[
\left( \frac{a+4}{2}, \frac{b+2}{2} \right) = \left( \frac{-4}{2}, \frac{3}{2} \right) \\
\frac{a+4}{2} = -4 \quad \& \quad \frac{b+2}{2} = 3 \\
a + 4 = -8 \quad \& \quad b + 2 = 3 \\
a = -8 \quad \& \quad b = 1
\]

Hence coordinates of \( S (-8,1) \)
11. (a) Given that \( y = x^2\sqrt{2x+1} \), show that \( \frac{dy}{dx} = \frac{x(5x+2)}{\sqrt{2x+1}} \). [3]

(a) \( y = x^2\sqrt{2x+1} \)

\[
\frac{dy}{dx} = x^2[\frac{1}{2}(2x+1)^{-\frac{1}{2}}(2)] + 2x(2x+1)^{\frac{1}{2}}
\]

\[
= x(2x+1)^{\frac{1}{2}}(x+4x+2)
\]

\[
= x(5x+2)(2x+1)^{\frac{1}{2}}
\]

\[
\therefore \frac{dy}{dx} = \frac{x(5x+2)}{\sqrt{2x+1}} \text{ (shown)}
\]

(b) Hence

(i) find the coordinates of the stationary points on the curve \( y = x^2\sqrt{2x+1} \) and determine the nature of these stationary points. [5]

(ii) evaluate \( \int_0^1 \frac{5x^2 + 2x - 3}{\sqrt{2x+1}} \, dx \). [4]

(b)(i) For stationary points, \( \frac{dy}{dx} = \frac{x(5x+2)}{\sqrt{2x+1}} = 0 \)

\[ x = 0 \text{ or } x = -\frac{2}{5} \]

Stationary points are \((0, 0)\) and \((-\frac{2}{5}, 0.0716)\)

Using 1\textsuperscript{st} derivative test:

<table>
<thead>
<tr>
<th>x</th>
<th>-0.5</th>
<th>-0.4</th>
<th>-0.3</th>
<th>-0.1</th>
<th>0</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>dy/dx</td>
<td>&gt;0</td>
<td>0</td>
<td>&lt;0</td>
<td>&lt;0</td>
<td>0</td>
<td>&gt;0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sketch of tangent</th>
</tr>
</thead>
<tbody>
<tr>
<td>( / )</td>
</tr>
<tr>
<td>( / )</td>
</tr>
<tr>
<td>( / )</td>
</tr>
<tr>
<td>( / )</td>
</tr>
</tbody>
</table>

\((-\frac{2}{5}, 0.0716)\) is a maximum point and \((0, 0)\) is a minimum point.

(ii) \[
\int_0^5 \frac{5x^2 + 2x - 3}{\sqrt{2x+1}} \, dx = \int_0^5 \frac{x(5x+2)}{\sqrt{2x+1}} \, dx - \int_0^5 \frac{3}{\sqrt{2x+1}} \, dx
\]

\[
= \left[ x^2\sqrt{2x+1} \right]^5_0 - 3 \left[ \sqrt{2x+1} \right]^5_0
\]

\[
= 76.4
\]
<table>
<thead>
<tr>
<th>Qn (ii)</th>
<th>Answer Key</th>
<th>Qn (iii)</th>
<th>Answer Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^2 + b^2 = 1$</td>
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<td></td>
</tr>
<tr>
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<td></td>
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<td></td>
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<td>$k = \frac{1}{2}$ and $n = 10$</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$(36 + 13\sqrt{5}) \text{ cm}^2$</td>
<td></td>
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<td></td>
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<td>$(78 + 24\sqrt{5}) \text{ cm}^2$</td>
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<td></td>
</tr>
<tr>
<td>$208 + 25\sqrt{5} \quad \frac{257}{2}$</td>
<td></td>
<td>$\tan \theta = 8$ $\theta = 82.9^\circ \text{ or } 1.45 \text{ rad}$</td>
<td></td>
</tr>
<tr>
<td>$3x - 6 + \frac{5}{x} - \frac{10}{2x - 1}$</td>
<td></td>
<td>$Q(2, 6)$</td>
<td></td>
</tr>
<tr>
<td>$\frac{3x^2 - 6x + 5 \ln x - 5 \ln(2x - 1) + C}{2}$</td>
<td></td>
<td>Since $OT = RT = 5 \text{ units}$ $\Rightarrow \Delta ORT \text{ is isosceles}$</td>
<td></td>
</tr>
<tr>
<td>$f(x) = 2x^4 - 3x^3 + 3x^2 + 5x - 3$</td>
<td></td>
<td>$\text{Area of trapezium } OPQR = 25 \text{ units}^2$</td>
<td></td>
</tr>
<tr>
<td>$x^2 - 2x + 3 = 0$</td>
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<td>$14 \cos \theta + 48 \sin \theta = 50 \cos(\theta - 73.74^\circ)$</td>
<td></td>
<td>$S(-8, 1)$</td>
<td></td>
</tr>
<tr>
<td>$\text{Since maximum value of } P = 50, P \text{ can have a value of } 48 \text{ cm.}$</td>
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<td></td>
</tr>
<tr>
<td>$\text{Or } \cos(\theta - 73.74^\circ) = \frac{48}{50} &lt; 1, P \text{ can have a value of } 48 \text{ cm.}$</td>
<td></td>
<td>$\left( -\frac{2}{5}, 0.0716 \right) \text{ is a maximum point and } (0, 0) \text{ is a minimum point.}$</td>
<td></td>
</tr>
<tr>
<td>$36.1 \text{ cm} \text{ (3sf)}$</td>
<td></td>
<td>$76.4$</td>
<td></td>
</tr>
<tr>
<td>$7 \text{ (ii)}$</td>
<td>Incorrect value of $y = 0.65$. Estimated correct value of $y = 0.68$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(i) \( x + 2y = 3y + 6 \)
\[
\frac{x}{2} + \frac{2y}{6} = 3
\]

Gradient = \( \frac{2}{2} \) = 1

(ii) \( \text{Mean value of } y = 0.65 \)

Correct value of \( y = 0.68 \)

(iii) \( \text{From the graph, } S = 1.7 \times 10^{-2} \)

Gradient = \( \frac{2.5}{2.5 - 0.5} \) = 0.5

(iv) \( \frac{5x - 2x + 2x}{2} = x \)

\( \frac{5x}{2} = 2x \)

\( \frac{5x}{2} - 2x + 5 \)

\( \frac{x}{y} = -x + \frac{5}{2} \)

\( \frac{x}{y} = 1.93 \)

\( \frac{5x}{2} = 0.30 \times \)

\( x = 0.675 \)
READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on the separate writing papers provided.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

Setter: Ms Renuka Ramakrishnan

This paper consists of 6 printed pages including the coverpage.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion

\[(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,
\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. TRIGONOMETRY

Identities

\[
\begin{align*}
\sin^2 A + \cos^2 A &= 1 \\
\sec^2 A &= 1 + \tan^2 A \\
cosec^2 A &= 1 + \cot^2 A
\end{align*}
\]

\[
\begin{align*}
\sin(A + B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A + B) &= \cos A \cos B \mp \sin A \sin B \\
\tan(A + B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\end{align*}
\]

\[
\begin{align*}
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A \\
\tan 2A &= \frac{2\tan A}{1 - \tan^2 A}
\end{align*}
\]

Formulae for \( \triangle ABC \)

\[
\begin{align*}
\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\
a^2 &= b^2 + c^2 - 2bc \cos A \\
\Delta &= \frac{1}{2}bc \sin A
\end{align*}
\]
1. (i) On the same axes, sketch the curves \( y = \frac{2}{x^2} \) and \( y^2 = 128x \). [2]

(ii) Find the coordinates of the point of intersection of the two curves. [2]

2. (i) Factorise completely the cubic polynomial \( 2x^3 - 11x^2 + 12x + 9 \). [3]

(ii) Hence, express \( \frac{6x^3 - 33x^2 + 35x + 51}{2x^3 - 11x^2 + 12x + 9} \) in partial fractions. [5]

3. A quadratic curve passes through \((0, -1)\) and \((2, 7)\). The gradient of the curve at \(x = -2\) is \(-8\). Find the equation of the curve. [5]

4. (i) Show that \( \cos 3\theta - \cos \theta = -2 \sin 2\theta \sin \theta \). [3]

(ii) Hence find the values of \(\theta\) between \(0^\circ\) and \(360^\circ\) for which \( \cos 3\theta - \cos \theta = \sin 2\theta \). [3]

5. The volume of a right square pyramid of length \((3 + \sqrt{2})\) cm is \(\frac{1}{3}(29 - 2\sqrt{2})\) cm\(^3\). Without using a calculator, find the height of the pyramid in the form \((a + b\sqrt{2})\) cm, where \(a\) and \(b\) are integers. [5]

6. The roots of the quadratic equation \(6x^2 - 5x + 2 = 0\) are \(\frac{2}{\alpha}\) and \(\frac{2}{\beta}\).

(i) Find the value of \(\frac{\alpha}{\alpha + 2} + \frac{\beta}{\beta + 2}\). [5]

(ii) Find a quadratic equation whose roots are \(\frac{\alpha}{\alpha + 2}\) and \(\frac{\beta}{\beta + 2}\). [2]
A particle moves in a straight line such that, \( t \) seconds after leaving a fixed point \( O \), its velocity, \( v \) ms\(^{-1}\), is given by \( v = t^2 - 5t + 4 \).

(i) Find the acceleration of the particle when it first comes to an instantaneous rest.  
(ii) Find the average speed of the particle for the first 5 seconds.

The following table shows the experimental values of two variables, \( x \) and \( y \), which are related by the equation \( y = ab^{x+1} \), where \( a \) and \( b \) are constants.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>10.12</td>
<td>10.23</td>
<td>10.35</td>
<td>10.47</td>
</tr>
</tbody>
</table>

(i) On graph paper, plot \( \lg y \) against \( x \) and draw a straight line graph. The vertical \( \lg y \) – axis should start from 0.995 and have a scale of 4 cm to 0.005.  
(ii) Use your graph to estimate the value of \( a \) and of \( b \).  
(iii) Explain how the value of \( a \) and of \( b \) will change if a graph of \( \ln y \) against \( x \) was plotted instead.

In the diagram, \( A, B \) and \( C \) are three points on the circle such that \( AB \) is the diameter of the circle and \( W \) is the midpoint of \( AC \). \( AB \) and \( CK \) are parallel to each other and \( KL \) is a tangent to the circle at \( A \).

(i) Prove that \( OW \) is parallel to \( BC \).  
(ii) Prove that \( \angle AWO = \angle AKC \).
Diagram I shows a right angled $\Delta ABC$, with hypotenuse $AB$ of length 4 m. This triangle is revolved around $BC$ to generate a right circular cone as shown in Diagram II.

Diagram I

Diagram II

(i) Find the exact height that gives the maximum volume of the cone. [6]

(ii) Show that this maximum volume is obtained when $BC: CA = 1: \sqrt{2}$. [2]

The equation of a curve is $y = \frac{4-5x+x^2}{5-x}, x \neq 5$.

(i) Find the set of values of $x$ for which $y$ is an increasing function of $x$. [3]

(ii) The diagram below shows part of the curve $y = \frac{4-5x+x^2}{5-x}, x \neq 5$.

By expressing $\frac{4-5x+x^2}{5-x}$ in the form $ax + \frac{b}{5-x}$, where $a$ and $b$ are constants, find the total area of the shaded regions. [5]
A circle $C_1$, with centre $C$, passes through four points $A, B, F$ and $G$. The coordinates of $A$ and $B$ are $(0, 4)$ and $(8, 0)$ respectively. The equation of the normal to the circle at $F$ is $y = -\frac{4}{3}x + 4$.

(i) Show that the coordinates of $C$ is $(3, 0)$.

(ii) Hence find the equation of the circle.

Another circle $C_2$ passes through the points $C, F$ and $G$.

(iii) Given that $GF$ is the diameter of the circle, calculate the radius of $C_2$. 

- End of Paper -

Answer Key

1) $\left(\frac{1}{2}, 8\right)$
2) $(x - 3)^2(2x + 1)$
2ii) $3 + \frac{2}{2x+1} - \frac{1}{x-3} + \frac{3}{(x-3)^2}$
3) $y = 2x^2 - 1$
4) $\theta = 90^\circ, 180^\circ, 210^\circ, 270^\circ, 330^\circ$
5) $(7 - 4\sqrt{2})\text{cm}$
6) $\frac{17}{13}$
6ii) $x^2 - \frac{17}{13}x + \frac{6}{13} = 0$
7) $-\frac{3m}{s^2}$
7ii) $1.63\text{ m/s}$
8) $b = 1.01, a = 9.89$
8iii) remain unchanged
10) $\frac{4\sqrt{3}}{3}\text{ cm}$
11) $3 < x < 7, x \neq 5$
11ii) $2.35\text{ units}^2$
12) $(x - 3)^2 + y^2 = 25$
12ii) $3.54\text{ units}$
**G1:** graph of $y = \frac{2}{x^2}$

**G1:** graph of $y^2 = 128x$

1i

Let $f(x) = 2x^3 - 11x^2 + 12x + 9$

$f(3) = 0$

$\therefore (x - 3)$ is a factor of $f(x)$.

$f(x) = (x - 3)(2x^2 - 5x - 3)$

$= (x - 3)^2(2x + 1)$

2i

$\frac{-x + 24}{(x-3)^2(2x+1)} = \frac{A}{2x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

$-x + 24 = A(x-3)^2 + B(x-3)(2x+1) + C(2x + 1)$

$\Rightarrow$ Using substitution/comparing coefficient

$A = 2, B = -1, C = 3$

$\therefore \frac{6x^3 - 33x^2 + 35x + 51}{2x^3 - 11x^2 + 12x + 9} = 3 + \frac{1}{x-3} + \frac{3}{(x-3)^2}$
### Question 3

\( y = ax^2 + bx + c \)

When \( x = 0 \), \( y = -1 \) → \( c = -1 \)

\[
\frac{dy}{dx} = 2ax + b
\]

When \( x = -2 \), \( \frac{dy}{dx} = -8 \)

\[-4a + b = -8
\]

\[b = 4a - b \quad --- (1)\]

Sub \( y = 7 \) and \( x = 2 \) into \( y = ax^2 + bx - 1 \)

\[7 = 4a + 2b - 1
\]

\[b = 4 - 2a \quad --- (2)\]

From (1) and (2),

\[4a - 8 = 4 - 2a\]

\[a = 2\]

\[\rightarrow b = 0\]

\[\therefore \text{equation of curve: } y = 2x^2 - 1\]

### Question 4i

\[
\cos 3\theta - \cos \theta = \cos(2\theta + \theta) - \cos \theta
\]

\[
= \cos 2\theta \cos \theta - \sin 2\theta \sin \theta - \cos \theta
\]

\[
= \cos \theta (\cos 2\theta - 1) - \sin 2\theta \sin \theta
\]

\[
= -2 \sin^2 \theta \cos 2\theta - \sin 2\theta \sin \theta
\]

\[
= -2 \sin 2\theta \sin \theta
\]

\[
= (\sin 2\theta \cos \theta + \cos 2\theta \sin \theta)
\]

\[
= \sin 2\theta (1 + 2 \sin \theta)
\]

\[
\sin 2\theta(1 + 2 \sin \theta) = 0
\]

\[
\sin 2\theta = 0 \quad \text{or} \quad \sin \theta = -0.5
\]

\[2\theta = 180, 360, 540 \quad \text{or} \quad \theta = 210, 330\]

\[\theta = 90, 180, 270\]

\[\therefore \theta = 90^\circ, 180^\circ, 210^\circ, 270^\circ, 330^\circ\]

### Question 4ii

\[
\cos 3\theta - \cos \theta = \sin 2\theta
\]

\[
\sin 2\theta + 2 \sin 2\theta \sin \theta = 0
\]

\[
\sin 2\theta(1 + 2 \sin \theta) = 0
\]

\[
\sin 2\theta = 0 \quad \text{or} \quad \sin \theta = -0.5
\]

\[2\theta = 180, 360, 540 \quad \text{or} \quad \theta = 210, 330\]

\[\theta = 90, 180, 270\]

\[\therefore \theta = 90^\circ, 180^\circ, 210^\circ, 270^\circ, 330^\circ\]

### Question 5

\[
\frac{1}{3}(3 + \sqrt{2})^2 h = \frac{1}{3}(29 - 2\sqrt{2})
\]

\[
h = \frac{29 - 2\sqrt{2}}{11 + 6\sqrt{2}}
\]

\[
= \frac{(29 - 2\sqrt{2})(11 - 6\sqrt{2})}{49}
\]

\[
= \frac{319 - 174\sqrt{2} - 22\sqrt{2} + 24}{49}
\]

\[
= \frac{343 - 196\sqrt{2}}{49}
\]

\[= (7 - 4\sqrt{2})\text{cm}\]
\[
\begin{align*}
6i \quad \frac{2}{\alpha} + \frac{2}{\beta} &= \frac{5}{6} \\
\frac{2(\alpha + \beta)}{\alpha \beta} &= \frac{5}{6} \quad \text{--- (1)} \\
\left(\frac{2}{\alpha}\right) \left(\frac{2}{\beta}\right) &= \frac{1}{3} \\
\alpha \beta &= 12 \quad \text{--- (2)} \\
\substitute \text{Sub (2) into (1)} \\
\alpha + \beta &= 5 \\
\frac{\alpha + \beta}{\alpha + 2 + \beta + 2} &= \frac{\alpha(\beta + 2) + \beta(\alpha + 2)}{(\alpha + 2)(\beta + 2)} \\
&= \frac{\alpha \beta + 2(\alpha + \beta) + 4}{17} \\
&= \frac{13}{13} \\
\therefore \text{Equation : } x^2 - \frac{17}{13} x + \frac{6}{13} &= 0 \\
or \quad 13x^2 - 17x + 6 &= 0 \\
\end{align*}
\]

M1: applying concept of sum and product of roots
A1: \(\alpha \beta\)

A1: \(\alpha + \beta\)

\[
\begin{align*}
6ii \quad \left(\frac{\alpha}{\alpha + 2}\right) \left(\frac{\beta}{\beta + 2}\right) &= \frac{\alpha \beta}{(\alpha + 2)(\beta + 2)} \\
&= \frac{6}{13} \\
\therefore \text{Equation : } x^2 - \frac{17}{13} x + \frac{6}{13} &= 0 \\
or \quad 13x^2 - 17x + 6 &= 0 \\
\end{align*}
\]

M1

A1

\[
\begin{align*}
7i \quad \text{When } v = 0, \\
t^2 - 5t + 4 &= 0 \\
(t - 4)(t - 1) &= 0 \\
t &= 1 \quad \text{or} \quad t &= 4 \\
\text{Therefore, } a &= \frac{dv}{dt} \\
&= 2t - 5 \\
\text{When } t = 1, a &= -3 \text{ m/s}^2 \\
\end{align*}
\]

M1

A1: Differentiate correctly
A1

\[
\begin{align*}
7ii \quad s &= \int \left( t^2 - 5t + 4 \right) \text{ d}t \\
&= \frac{1}{3} t^3 - \frac{5}{2} t^2 + 4t + c \\
\text{When } t = 0, s = 0 \Rightarrow c = 0 \\
\therefore \quad s &= \frac{1}{3} t^3 - \frac{5}{2} t^2 + 4t \\
\text{When } t = 1, s &= \frac{11}{6} \text{ m} \\
t &= 4, s &= -\frac{8}{3} \text{ m} \\
t &= 5, s &= -\frac{5}{6} \text{ m} \\
\end{align*}
\]

A1: integrate correctly
(look out for +c, unless definite integral)
\[
\therefore \text{Average Speed} = \frac{\frac{11}{6} + \left(\frac{11}{6} + \frac{8}{3}\right) + \left(\frac{8}{3} - \frac{5}{6}\right)}{5}
\]
\[
= 1\frac{19}{30} \text{ m/s or } 1.63 \text{ m/s}
\]

M1: Calculating distance in 5\textsuperscript{th} sec
M1: formula for average speed
A1

8

\[y = ab^{x+1}\]
\[\log y = (x + 1) \log b + \log a\]
\[\log y = x \log b + (\log b + \log a)\]

\[
\begin{array}{c|c|c|c|c}
 x & 1 & 2 & 3 & 4 \\
\hline
 \log y & 1.005 & 1.010 & 1.015 & 1.020 \\
\end{array}
\]

M1: table of values

8ii

\[
\log b = \frac{1.0175 - 1}{3.5} = 0.005
\]
\[b = 1.01 \text{ (3sf)}\]
\[
\log b + \log a = 1
\]
\[\log a = 1 - 0.005\]
\[a = 9.89 \text{ (3sf)}\]

M1
A1

8iii

The values of \(a\) and \(b\) will remain unchanged.
B1

9i

O is the midpoint of AB and W is the midpoint of AC,
By Midpoint Theorem, BC parallel to OW.
M1
A1

9ii

\[\text{Angle } AOW = \text{Angle } ABC \text{ (corr angles, OW//BC)}\]
\[\text{Angle } ABC = \text{Angle } CAK \text{(alt segment theorem)}\]
\[\Rightarrow \text{Angle } AOW = \text{Angle } CAK\]
\[\text{Angle } BAC = \text{Angle } ACK \text{ (alt angles, AB//CK)}\]
\[\therefore \text{Angle } AWO\]
\[= 180° - \text{Angle } BAC - \text{Angle } AOW \text{ (Angle sum of } \Delta)\]
\[= 180° - \text{Angle } ACK - \text{Angle } CAK\]
\[= \text{Angle } AKC \text{ (shown)}\]
M1
M1
A1
### 10i

Let $AC = r$ and $BC = h$

$r^2 = 16 - h^2$

$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi (16 - h^2)h$$
$$= \frac{16}{3}\pi h - \frac{1}{3}\pi h^3$$

$$\frac{dV}{dh} = \frac{16}{3}\pi - \pi h^2$$

When $\frac{dV}{dh} = 0$,

$$\frac{16}{3}\pi = \pi h^2$$

$$h = \frac{4}{\sqrt{3}} \left(\text{rej } h = -\frac{4}{\sqrt{3}} \text{ since } h > 0\right)$$

$$\frac{d^2V}{dh^2} = -2\pi h$$

$$= -\frac{8}{\sqrt{3}}\pi \ (< 0)$$

$$\therefore h = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3} \text{ cm}$$

M1: Finding $r/s$ between $h$ and $r$

M1: finding $V$ in terms of one variable

M1: differentiation

M1: Stationary point

M1: Prove Max

A1

### 10ii

$$r^2 = 16 - \left(\frac{4}{\sqrt{3}}\right)^2$$

$$r = \frac{4\sqrt{2}}{\sqrt{3}}$$

$$h = \frac{4}{\sqrt{3}}$$

$$\frac{h}{r} = \frac{4\sqrt{2}}{\sqrt{3}}$$

$$\frac{h}{r} = \frac{1}{\sqrt{2}}$$

$$\therefore BC:CA = 1:\sqrt{2}$$

M1

A1

### 11i

$$\frac{dy}{dx} = \frac{(2x - 5)(5 - x) - (4 - 5x + x^2)(-1)}{(5 - x)^2}$$

$$= \frac{10x - 2x^2 + 5x - 25 + 4 - 5x + x^2}{(5 - x)^2}$$

$$= \frac{-x^2 + 10x - 21}{(5 - x)^2}$$

Since $(5 - x)^2 > 0$, for $y$ to be an increasing function,

$-x^2 + 10x - 21 > 0$

$x^2 - 10x + 21 < 0$

$(x - 3)(x - 7) < 0$

$3 < x < 7, x \neq 5$

M1: Applying quotient rule

A1
### 11ii

When \( y = 0 \):

\[
4 - 5x + x^2 = 0
\]

\((x - 4)(x - 1) = 0\)

\(x = 4 \text{ or } x = 1\)

\[
\frac{x^2 - 5x + 4}{5 - x} = \frac{-x(5 - x) + 4}{5 - x} = -x + \frac{4}{5 - x}
\]

Area of shaded region:

\[
\int_{0}^{1} -x + \frac{4}{5 - x} \, dx + \left| \int_{1}^{4} -x + \frac{4}{5 - x} \, dx \right|
\]

\[
= \left[ -0.5x^2 - 4 \ln(5 - x) \right]_{0}^{1} + \left| \left[ -0.5x^2 - 4 \ln(5 - x) \right]_{1}^{4} \right|
\]

\[
= 0.39257 + | -1.95482 |
\]

\[
= 2.35 \text{ units}^2 \quad (3sf)
\]

### 12i

**Gradient of AB** = \(-\frac{1}{2}\)

Midpoint of AB = (4, 2)

**Eqn of perpendicular bisector of AB:**

\[y - 2 = 2(x - 4)\]

\[y = 2x - 6\]

Sub \( y = 2x - 6 \) into \( y = -\frac{4}{3} + 4 \),

\[2x - 6 = -\frac{4}{3} + 4\]

\[x = 3\]

\[\rightarrow y = 0\]

\(C (3, 0)\)

### 12ii

**Radius** = \(\sqrt{(3 - 0)^2 + (0 - 4)^2}\)

= 5 units

Equation of circle: \((x - 3)^2 + y^2 = 25\)

Or \(x^2 + y^2 - 6x - 16 = 0\)

### 12iii

*Angle GCF = 90° (Angle in Semicircle)*

\(GF^2 = 5^2 + 5^2\)

\(GF = \sqrt{50}\)

**Radius of \(C_2\)** = \(\frac{1}{2} \sqrt{50}\)

\[= \frac{5}{2} \sqrt{2} \text{ units}\]

or

\[= 3.54 \text{ units} \quad (3sf)\]
READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on the separate writing papers provided.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142, unless the question requires the answer in terms of $\pi$.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [   ] at the end of each question or part question.
The total of the marks for this paper is 100.

Setter: Mdm Chua Seow Ling
**Mathematical Formulae**

1. **ALGEBRA**

   **Quadratic Equation**

   For the equation \( ax^2 + bx + c = 0 \),

   \[
   x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
   \]

   **Binomial expansion**

   \[(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,\]

   where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \).

2. **TRIGONOMETRY**

   **Identities**

   \[
   \begin{align*}
   \sin^2 A + \cos^2 A &= 1 \\
   \sec^2 A &= 1 + \tan^2 A \\
   \csc^2 A &= 1 + \cot^2 A \\
   \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
   \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
   \tan(A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
   \end{align*}
   \]

   \[
   \begin{align*}
   \sin 2A &= 2 \sin A \cos A \\
   \cos 2A &= \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A \\
   \tan 2A &= \frac{2 \tan A}{1 - \tan^2 A}
   \end{align*}
   \]

   **Formulae for \( \triangle ABC \)**

   \[
   \begin{align*}
   \frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\
   a^2 &= b^2 + c^2 - 2bc \cos A \\
   \Delta &= \frac{1}{2} bc \sin A
   \end{align*}
   \]
Answer ALL Questions.

1. (i) Given \( \frac{3 \log 3x - 2 \log x}{4} = \log 3 \), find the value of \( x \). [3]

   (ii) Given \( \log_{(x-2)} y = 2 \) and \( \log_y (x + k) = \frac{1}{2} \), find the value of \( k \) if \( k \) is an integer. [3]

2. (i) Show that \( \frac{d}{dx} \left[ \ln \left( \frac{\sin x}{1 - \cos x} \right) \right] = -\frac{1}{\sin x} \). [4]

   (ii) Hence evaluate \( \int \sin^2 x + \frac{2}{\sin x} \, dx \). [4]

3. It is given that \( y_1 = -2\cos x + 1 \) and \( y_2 = \sin \frac{1}{2} x \).

   For the interval \( 0 < x < 2\pi \),

   (i) state the amplitude and period of \( y_1 \) and of \( y_2 \), [2]

   (ii) sketch, on the same diagram, the graphs of of \( y_1 \) and \( y_2 \), [4]

   (iii) find the \( x \)-coordinate of the points of intersection of the two graphs drawn in (ii), [3]

   (iv) hence, find the range of values of \( x \) for which \( y_1 \leq y_2 \). [2]

4. Some liquid is poured onto a flat surface and formed a circular patch. This circular patch is left to dry and its surface area decreases at a constant rate of 4 cm\(^2\)/s. The patch remains circular during the drying process. Find the rate of change of the circumference of the circular patch at the instant when the area of the patch is 400 cm\(^2\). [4]

5. (i) In the expansion of \( \left( 2 + \frac{4}{x^4} \right) \left( kx^3 - \frac{2}{x} \right)^{13} \) where \( k \) is a constant and \( k \neq 0 \), find the value of \( k \) if there is no coefficient of \( \frac{1}{x} \). [5]

   (ii) Given the coefficients of \( \frac{1}{x} \) and \( \frac{1}{x^2} \) in the expansion of \( \left( 1 - \frac{c}{x} \right)^n \) are \(-80\) and \(3000\) respectively. Find the value of \( c \) and of \( n \) where \( n \) is a positive integer greater than 2 and \( c \) is a constant. [5]
6. Curve A is such that \( \frac{dy}{dx} = 27(2x - 1)^2 \) and curve B is such that \( \frac{dy}{dx} = -27(2x - 1)^3 \), and the y-coordinates of the stationary points for both curves are \(-4\).

(i) Find the coordinates of the stationary points for curve A and B. [2]

(ii) Determine the nature of the stationary points for curve A and B. [4]

(iii) Find the equations of curve A and B. [4]

7. The diagram shows a triangle ABC. The mid-points of the sides of the triangle are \( M(1, 5) \), \( N(8, 1) \) and \( P \left( -\frac{1}{2}, 2 \right) \).

(i) State and explain which line is parallel to \( AB \). [1]

(ii) Find the equation of the line \( AB \). [3]

(iii) Find the equation of the line \( AC \). [3]

(iv) Show the coordinates of \( A \) is \( \left( -\frac{1}{2}, 6 \right) \). [3]

(v) Find the area of the quadrilateral \( AMNP \). [2]
8. Diagram I and II show two types of isosceles triangular cards, $\triangle OAB$ with $\angle AOB = \theta$, $OA = OB = 3\text{ cm}$ and $\triangle OMN$ with $OM = ON = 2\text{ cm}$. These two types of cards are connected as shown in diagram III where $\angle AON = 120^\circ$.

$\quad$

Three sets of cards from diagram III are connected as shown in diagram IV.

(i) Show that the area of all the connected cards in diagram IV, $A$ cm$^2$ is given by $A = \frac{33}{2} \sin \theta + 3\sqrt{3} \cos \theta$. \[3\]

(ii) Express $A$ in the form $A = R \cos(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. \[3\]

(iii) Find the value of $\theta$ for which $A = 15$, where $0^\circ < \theta < 90^\circ$. \[3\]

(iv) Find the maximum value of $A$ and the corresponding value of $\theta$. \[2\]
In an experiment to study the growth of a certain type of bacteria, the bacteria are injected into a mouse and the mouse’s blood samples are collected at various time interval for testing. The blood test result shows that the population, \( P \), of the bacteria is related to the time, \( t \) hours, after the injection, by the equation \( P = 550 + 200e^{kt} \), where \( k \) is a constant. It takes **one day** for the population of bacteria to double.

(i) Find the population of the bacteria at the start of the experiment. \([1]\)

(ii) Find the value of \( k \). \([2]\)

(iii) Find the percentage increase of the population of the bacteria when \( t = 30 \). \([4]\)

(iv) The line \( P = mt + c \) is a tangent to the curve \( P = 550 + 200e^{kt} \) at the point where \( t = 30 \). Find the constant value of \( m \) and of \( c \). \([3]\)

(v) At \( t = 50 \), an antibiotics dosage is injected into the same mouse to stop the growth of bacteria. The dosage is able to kill the bacteria at a constant rate of 25 bacteria per hour. How much time needed for the dosage fully take its effectiveness? Hence sketch the graph of \( P \) against \( t \) for the whole experiment. \([4]\)

A curve has the equation of \( y = p(x - 2)^2 - (x - 3)(x + 2) \) where \( p \) is a constant and \( p \neq 1 \).

(i) Find the range of values of \( p \) for which curve has a minimum point. \([2]\)

Given that the curve touches the \( x \)-axis at point \( A \).

(ii) Show that \( p = \frac{25}{16} \). \([3]\)

(iii) Find the coordinates of point \( A \). \([4]\)

(iv) Given that the line \( y = mx + 2 \) intersects the curve \( y = p(x - 2)^2 - (x - 3)(x + 2) \) at two distinct points where one of the points is at point \( A \). Another line of the equation \( y = mx + c \), is a tangent to the same curve at point \( B \). Find the value of \( c \) where \( m \) and \( c \) are constants. \([5]\)
Answers
1i) \( x = 3, \)
1ii) \( k = -2 \)
2ii) \( \frac{1}{2}x - \frac{1}{4} \sin 2x - 2 \ln \left( \frac{\sin x}{1 - \cos x} \right) + c \)
3ii)

Amplitude of \( y_1 = 2, \) Period of \( y_1 = 2\pi \)
Amplitude of \( y_2 = 1, \) Period of \( y_2 = 4\pi \)

3iii) \( x = 1.39, 4.89 \)
3iv) \( 0 < x \leq 1.39 \text{ or } 4.89 \leq x < 2\pi \)

4) \(-k = \frac{2}{5}\)
5i) \( k = \frac{2}{5} \)
5ii) \( c = 5, n = 16 \)
6i) \( \left( \frac{1}{2}, -4 \right) \)
6ii) \( \left( \frac{1}{2}, -4 \right) \) is point of inflexion – Curve \( A, \left( \frac{1}{2}, -4 \right) \) is a max stationary point – curve \( B \)
6iii) \( y_A = \frac{9}{2}(2x - 1)^3 - 4, \quad y_B = -\frac{27}{8}(2x - 1)^4 - 4 \)

7i) Line PN is parallel to AB (Mid-Point Theorem)
7ii) \( y = -\frac{2}{17}x + 5 \frac{2}{17} \)
7iii) \( y = -\frac{4}{7}x + \frac{12}{7} \)
7v) \( A = 27 \)
8ii) \( A = \frac{3\sqrt{133}}{2} \cos(\theta - 72.5^\circ) = 17.3 \cos(\theta - 72.5^\circ) \)
8iii) \( 42.6^\circ \)
8iv) Max \( A = \frac{3\sqrt{133}}{2} = 17.3, \quad \theta = 72.5^\circ \)
9i) 750

9ii) \( k = \frac{1}{24} \ln \frac{19}{4} = 0.0649 \)

9iii) 160 \%

9iv) \( m = 91.1, \ c = -779 \)

9v)

![Graph showing a point (50, 5690) on a t-axis ranging from 50 to 278, with a line segment connecting the point to the t-axis at 50 and 278.]  

10i) \( p > 1 \)

10iii) \( A \left( \frac{14}{3}, 0 \right) \)

10iv) \( c = \frac{94}{49} \)
### 2018 NCHS A-Math Prelim 2/ Paper 2 solution

#### (1i)
\[
\frac{3 \log 3x - 2 \log x}{4} = \log 3
\]
\[
3 \log 3x - 2 \log x = 4 \log 3
\]
\[
\frac{(3x)^3}{x^2} = 3^4
\]
\[
27x = 81
\]
x = 3

#### (1ii)
\[
\log (x - 2) y = 2
\]
\[
y = (x - 2)^2
\]
\[
\frac{1}{2} y^2 = x - 2
\]
\[
\log y(x + k) = \frac{1}{2}
\]
x + k = \frac{1}{2} y^2
k = -2

#### (2i)
\[
\int \frac{2}{\sin x} - \cos x - \cos^2 x + 2 \sin x \, dx = \int 1 - \cos^2 x + 2 \sin x \, dx
\]
\[
\frac{1}{2} x - \frac{1}{4} \sin 2x - 2 \ln \frac{\sin x}{1 - \cos x} + C
\]

#### (2ii)
Amplitude of \( y_1 \) = 2, Period of \( y_1 \) = 2\pi
Amplitude of \( y_2 \) = 1, Period of \( y_2 \) = 4\pi

#### (3i)
Amplitude of \( y_1 \) = 2
Period of \( y_1 \) = 2\pi
Amplitude of \( y_2 \) = 1
Period of \( y_2 \) = 4\pi

#### (3ii)
\[-2 \cos x + 1 = \sin \frac{1}{2} x
\]
\[-2 \left( 1 - 2 \sin^2 \frac{x}{2} \right) + 1 = \sin \frac{1}{2} x
\]
\[4 \sin^2 \frac{x}{2} - \sin \frac{1}{2} x - 1 = 0
\]
\[\sin \frac{1}{2} x = 0.6403882
\]
\[\alpha = 0.69500
\]
\[
\frac{1}{2} x = 0.69500, \quad \pi - 0.69500
\]
= 0.695 or 2.4466
\[x = 1.39, 4.89
\]
\[0 < x \leq 1.39 \quad \text{or} \quad 4.89 \leq x < 2\pi
\]

#### (4)
\[
A = \pi r^2
\]
\[
\frac{dA}{dr} = 2\pi r
\]
\[
C = 2\pi r
\]
\[
\frac{dC}{dr} = 2\pi
\]
\[
\pi r^2 = 400
\]
\[
r = \frac{20}{\sqrt{\pi}}
\]
\[
\frac{dA}{dr} \frac{dr}{dt} = \frac{dA}{dt}
\]
\[
2\pi r \frac{dr}{dt} = -4
\]
\[
\frac{dr}{dt} = -4
\]
\[
\frac{dC}{dt} = \frac{dC}{dr} \times \frac{dr}{dt}
\]
\[
2\pi \times \frac{dr}{2\pi r}
\]
\[
= -4
\]
\[
= \frac{20}{5} \text{ cm/s}
\]
\[= -0.354 \text{ cm/s}
\]
\[ \begin{align*}
(5i) & \quad \left(2 + \frac{4}{\sqrt{x}}\right) \left(kx^3 - \frac{2}{x}\right)^{13} \\
& = \left(2 + \frac{4}{\sqrt{x}}\right) \left(\frac{13}{r}\right) \left(kx^3\right)^{13-r} \left(-\frac{2}{x}\right)^r + \ldots \\
& = \frac{13}{9} \left(kx^3\right)^{13} \left(-\frac{2}{x}\right)^9 + \frac{13}{10} \left(kx^3\right)^{12} \left(-\frac{2}{x}\right)^{10} + \ldots \\
& = 715k^4x^{12} \left(\frac{512}{x^7} + 286k^3x^3 \left(\frac{1024}{x^{10}}\right) + \ldots \\
& = -366080k^4x^3 + \frac{x^{292864}}{k^3} + \ldots \\
& = \left(2 + \frac{4}{\sqrt{x}}\right)(-366080k^4x^3 + \frac{x^{292864}}{k^3} + \ldots) \\
& \quad \frac{585728k^3}{x} - 1464320k^4 + \ldots \\
& \quad k = \frac{2}{5} \\
\end{align*} \]

\[ \begin{align*}
(5ii) & \quad \left(1 - \frac{c}{x}\right)^n = \left(\frac{n}{x}\right) - \left(\frac{n}{x}\right)^2 + \ldots \\
& \quad = -\frac{nc}{x} + \frac{n(n-1)c^2}{2}x^2 + \ldots \\
& \quad - nc = -80 \\
& \quad nc = 80 \\
& \quad \frac{n(n-1)c^2}{2} = 3000 \\
& \quad n^2c^2 - nc^2 = 6000 \\
& \quad 80^2 - 80c = 6000 \\
& \quad c = 5, n = 16 \\
\end{align*} \]

\[ \begin{align*}
(6i) & \quad \text{Curve A --- } \frac{dy}{dx} = 27(2x - 1)^2 \\
& \quad 27(2x - 1)^2 = 0 \\
& \quad x = \frac{1}{2} \\
& \quad (\frac{1}{2}, -4) \text{ is a point of inflexion} \\
\end{align*} \]

\[ \begin{align*}
(6ii) & \quad \text{Curve A --- } x = 0.4, x = 0.5, x = 0.6 \\
& \quad \frac{dy}{dx} > 0 \\
& \quad \frac{dy}{dx} = 0 \\
& \quad \frac{dy}{dx} > 0 \\
\end{align*} \]

\[ \begin{align*}
(6iii) & \quad \text{y_A} = \frac{27(2x-1)^3}{3(2)} + c \\
& \quad c = -4 \\
& \quad \text{y_A} = \frac{9}{2}(2x - 1)^3 - 4 \\
\end{align*} \]

\[ \begin{align*}
(7i) & \quad \text{Line PN is parallel to AB (Mid-Point Theorem)} \\
\end{align*} \]

\[ \begin{align*}
(7ii) & \quad m_{PN} = \frac{2 - 1}{\frac{1}{2} - \frac{5}{2}} = -\frac{2}{17}, \quad m_{AB} = -\frac{2}{17} \\
& \quad y = -\frac{2}{17}x + c_1 \\
& \quad 5 = -\frac{2}{17}(1) + c_1 \quad \Rightarrow \quad c_1 = \frac{5}{2} \\
& \quad y = -\frac{2}{17}x + \frac{5}{2} \\
\end{align*} \]

\[ \begin{align*}
(7iv) & \quad -\frac{2}{17}x + \frac{5}{2} = -\frac{4}{7}x + \frac{12}{7} \\
& \quad x = -\frac{7}{1} \\
& \quad y = -\frac{4}{7}\left(-\frac{15}{2}\right) + \frac{12}{7} \\
& \quad \text{y} = 6 \\
& \quad (-7\frac{1}{2}, 6) \text{ Shown} \\
\end{align*} \]
\( A = 3\left(\frac{1}{2}(3)(3)\sin\theta + \frac{1}{2}(2)(2)\sin(120 - \theta)\right) \)
\(= \frac{9}{2}\sin\theta + 2\left(\frac{\sqrt{3}}{2}\cos\theta - \sin\theta\cos120\right) \)
\(= \frac{9}{2}\sin\theta + 3\sqrt{3}\cos\theta \quad \text{(shown)} \)

\(R = \sqrt{\left(\frac{9}{2}\sin\theta + 3\sqrt{3}\cos\theta\right)^2 + (3\sqrt{3})^2} \)
\(= \sqrt{\frac{1197}{4}} = \frac{3\sqrt{133}}{2} \)
\(\tan\alpha = \frac{\frac{9}{2}}{3\sqrt{3}} \)
\(\alpha = 72.5198^\circ \)
\(A = \frac{3\sqrt{133}}{2} \cos(\theta - 72.5^\circ) \)

\(P = 550 + 200e^{kt} \)
\(P = 550 + 200 \)
\(= 750 \)

\(2(750) = 550 + 200e^{k(24)} \)
\(e^{24k} = \frac{19}{4} \)
\(24k = \ln\frac{19}{4} \)
\(k = \frac{1}{24}\ln\frac{19}{4} = 0.0649 \)

\(P = 550 + 200e^{\left(\frac{1}{24}\ln\frac{19}{4}\right)(30)} \)
\(= 1952.4811 \)
\(\frac{1952.4811 - 750}{750} \times 100 = 160\% \)

\(y = p(x - 2)^2 - (x - 3)(x + 2) \)
\(\frac{dy}{dx} = 2p(x - 2) - [(x - 3) + (x + 2)] \)
\(= 2p(x - 2) - 2x + 1 \)
\(\frac{d^2y}{dx^2} = 2p - 2 > 0 \)
\(p > 1 \quad \text{(happy face since it is a min quadratic curve)} \)

\(y = px^2 - x^2 - 4px + x + 4p + 6 \)
\(y = px^2 - x^2 - 4px + x + 4p + 6 \)
\(y = \frac{25}{16}x^2 - x^2 - 4\left(\frac{15}{8}\right)x + x + 4\left(\frac{5}{16}\right) + 6 \)
\(y = \frac{9}{16}x^2 - \frac{3}{4}x + \frac{49}{4} \)
\(y = \frac{9}{16}x^2 - \frac{3}{4}x + \frac{49}{4} \)
\(= 0 \)
\(9x^2 - 84x + 196 = 0 \)
\(3(x - 14)^2 = 0 \)
\(x = \frac{14}{3} \)
\(A \left(\frac{14}{3}, 0\right) \)
\[ y = mx + 2 \]
\[ 0 = m \left( \frac{14}{3} \right) + 2 \]
\[ m = -\frac{3}{7} \]
\[ y = -\frac{3}{7}x + c \]
\[ \frac{dy}{dx} = -\frac{3}{7} \]
\[ 2p(x - 2) - 2x + 1 = -\frac{3}{7} \]
\[ 2\left( \frac{25}{16} \right)(x - 2) - 2x + 1 = -\frac{3}{7} \]
\[ x = \frac{30}{7} \]
\[ y = \frac{9}{16} x^2 - \frac{21}{4} x + \frac{49}{4} \]
\[ y = \frac{9}{16} \left( \frac{30}{7} \right)^2 - \frac{21}{4} \left( \frac{30}{7} \right) + \frac{49}{4} \]
\[ y = \frac{4}{49} \]
\[ y = -\frac{3}{7}x + c \]
\[ \frac{4}{49} = -\frac{3}{7} \left( \frac{30}{7} \right) + c \]
\[ c = \frac{94}{49} \]
READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved electronic scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the equation $ax^2 + bx + c = 0$, 
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion
$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$
where $n$ is a positive integer and
$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities
$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\cosec^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for \(\Delta ABC\)
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}bc \sin A$$
1. A rectangle has a length of \((6\sqrt{3} + 3)\) cm and an area of \(66\) cm\(^2\). Find the perimeter of the rectangle in the form \((a + b\sqrt{3})\) cm, where \(a\) and \(b\) are integers. [3]

2. On the same axes sketch the curves \(y^2 = 225x\) and \(y = 15x^3\). [3]

3. (i) Find the exact value of \(15^x\), given that \(25x^2 = 36 \times 9^{1-x}\). [3]
   (ii) Hence, find the value of \(x\), giving your answer to 2 decimal places. [2]

4. (a) Given that \(\log_3 y - \log_3 x = 1 + \log_3 (x + y)\), express \(y\) in terms of \(x\). [3]
   (b) Solve the equation \(\log_3 (8 - x) + \log_3 x = 2 \log_3 15\). [4]

5. The equation of a curve is \(y = \frac{x - 4}{\sqrt{2x + 5}}\).
   (i) Show that \(\frac{dy}{dx}\) can be expressed in the form \(\frac{ax + b}{(2x + 5)^{3/2}}\) where \(a\) and \(b\) are constants. [3]
   (ii) Given that \(y\) is increasing at a rate of 0.4 units per second, find the rate of change of \(x\) when \(x = 2\). [2]

6. The roots of the quadratic equation \(4x^2 + x - m = 0\), where \(m\) is a constant, are \(\alpha\) and \(\beta\).
   The roots of the quadratic equation \(8x^2 + nx + 1 = 0\), where \(n\) is a constant, are \(\frac{1}{\alpha^3}\) and \(\frac{1}{\beta^3}\).
   (i) Show that \(m = -8\) and hence find the value of \(n\). [5]
   (ii) Find a quadratic equation whose roots are \(\alpha + 2\) and \(\beta + 2\). [4]
7. (i) Show that \( \frac{2 - 2 \sec^2 x}{(1 + \cos x)(1 - \cos x)} = -2 \sec^2 x \). \[3\]

(ii) Hence find, for \(-\pi \leq x \leq \pi\), the values of \(x\) in radians for which
\( \frac{2 - 2 \sec^2 x}{(1 + \cos x)(1 - \cos x)} = 4 \tan x \). \[4\]

8. The temperature, \(T \, ^\circ C\), of a container of liquid decreases with time, \(t\) minutes. Measured values of \(T\) and \(t\) are given in the table below.

<table>
<thead>
<tr>
<th>(t) (min)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T) ((^\circ C))</td>
<td>58.5</td>
<td>41.6</td>
<td>34.7</td>
<td>31.9</td>
</tr>
</tbody>
</table>

It is known that \(T\) and \(t\) are related by the equation \(T = 30 + pe^{-qt}\), where \(p\) and \(q\) are constants.

(i) On a graph paper, plot \(\ln(T - 30)\) against \(t\) for the given data and draw a straight line graph. \[3\]

(ii) Use your graph to estimate the value of \(p\) and of \(q\). \[4\]

(iii) Explain why the temperature of the liquid can never drop to 30\(^\circ\)C. \[1\]

9. Given that \(y = 2xe^{1-x}\), find

(i) \(\frac{dy}{dx}\), \[2\]

(ii) \(p\) for which \(\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + pe^{1-x} = 0\), \[4\]

(iii) the range of values of \(x\) for which \(y\) is an increasing function. \[3\]

10. An open rectangular cake tin is made of thin sheets of steel which costs $2 per 1000 cm\(^2\). The tin has a square base of length \(x\) cm, a height of \(h\) cm and a volume of 4000 cm\(^3\).

(i) Show that the cost of steel, \(C\), in dollars, for making the cake tin is given by
\[C = \frac{x^2}{500} + \frac{32}{x}.\] \[2\]

Given that \(x\) can vary,

(ii) find the value of \(x\) for which \(C\) has a stationary value, \[3\]

(iii) explain why this value of \(x\) gives the minimum value of \(C\). \[3\]
11. The diagram shows a kite $EFGH$ with $EF = EH$ and $GF = GH$. The point $G$ lies on the $y$-axis and the coordinates of $F$ and $H$ are $(2, 1)$ and $(6, 9)$ respectively.

![Diagram of a kite EFGH]

The equation of $EF$ is $y = \frac{x}{8} + \frac{3}{4}$.

Find

(i) the equation of $EG$, [4]
(ii) the coordinates of $E$ and $G$, [3]
(iii) the area of the kite $EFGH$. [2]

12. The diagram shows a circle passing through points $D, E, C$ and $F$, where $FC = FD$. The point $D$ lies on $AP$ such that $AD = DP$. $DC$ and $EF$ cut $PB$ at $T$ such that $PT = TB$.

![Diagram of a circle and triangles]

(i) Show that $AB$ is a tangent to the circle at point $F$. [3]

(ii) By showing that triangle $DFT$ and triangle $EFD$ are similar, show that $DF^2 - FT^2 = FT \times ET$. [4]

End of Paper 1
1. Breadth = \frac{66}{6\sqrt{3} + 3} \quad \text{or} \quad \frac{22}{2\sqrt{3} + 1}
\quad = \frac{66}{6\sqrt{3} + 3} \times \frac{6\sqrt{3} - 3}{6\sqrt{3} - 3} \quad \frac{22}{2\sqrt{3} + 1} \times \frac{2\sqrt{3} - 1}{2\sqrt{3} - 1}
\quad = \frac{66(6\sqrt{3} - 3)}{99} \quad \frac{22(2\sqrt{3} - 1)}{11}
\quad = 4\sqrt{3} - 2 \text{ cm}

Perimeter = 2(6\sqrt{3} + 3 + 4\sqrt{3} - 2)
\quad = 20\sqrt{3} + 2 \text{ cm}

2. \[ y = 15x^3 \]

3. (i) \[ 25^{x+2} = 36 \times 9^{1-x} \]
\[ (5^{2x})(5^4) = \frac{2^2 \times 9^2}{3^{2x}} \]
\[ (3^{2x})(3^{2x}) = \frac{2^2 \times 9^2}{25^2} \]
\[ (15^x)^2 = \frac{2^2 \times 9^2}{25^2} \]
\[ 15^x > 0, \quad 15^x = \frac{18}{25} \]

(ii) \[ 15^x = \frac{18}{25} \]
\[ x \log_{15} 15 = \log_{15} \left( \frac{18}{25} \right) \]
\[ x = \frac{\log_{15} \left( \frac{18}{25} \right)}{\log_{15} 15} \]
\[ = -0.12 \]

4. (a) \[ \log_3 y - \log_3 x = 1 + \log_3 (x + y) \]
\[\log_3 \frac{y}{x} = \log_3 3 + \log_3(x + y)\]
\[\frac{y}{x} = 3(x + y)\]
\[y = 3x^2 + 3xy\]
\[y - 3xy = 3x^2\]
\[y(1 - 3x) = 3x^2\]
\[y = \frac{3x^2}{1 - 3x}\]

\[\log_3 (8 - x) + \log_3 x = 2 \log_9 15\]
\[\log_3 [x(8 - x)] = \frac{2 \log_3 15}{\log_9 9}\]
\[\log_3 [x(8 - x)] = \frac{2 \log_3 15}{2 \log_3 3}\]
\[8x - x^2 = 15\]
\[x^2 - 8x + 15 = 0\]
\[(x - 3)(x - 5) = 0\]
\[x = 3, \ 5\]

5. (i) \[\frac{dy}{dx} = \frac{(2x + 5)^{\frac{1}{2}}(1) - \frac{1}{2}(x - 4)(2x + 5)^{-\frac{1}{2}}(2)}{2x + 5}\]
\[= \frac{(2x + 5)^{\frac{1}{2}}(2x + 5 - x + 4)}{2x + 5}\]
\[= \frac{x + 9}{(2x + 5)^{\frac{3}{2}}}\]

(ii) When \(x = 2\), \[\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}\]
\[0.4 = \frac{2 + 9}{(4 + 5)^{-\frac{3}{2}}} \times \frac{dx}{dt}\]
\[\frac{dx}{dt} = \frac{0.4 \times 27}{11}\]
\[= \frac{54}{55}\] or 0.982 unit per second
6. (i) \( \alpha + \beta = -\frac{1}{4} \)
\( \alpha \beta = -\frac{m}{4} \)
\( \frac{1}{(\alpha \beta)^3} = \frac{1}{8} \)
\( \alpha \beta = 2 \)
\( \therefore -\frac{m}{4} = 2 \)
\( m = -8 \)

\( \frac{1}{\alpha^3} + \frac{1}{\beta^3} = -\frac{n}{8} \)
\( \alpha^3 + \beta^3 = -\frac{n}{8} \)
\( \frac{\alpha^3 + \beta^3}{\alpha \beta^3} = \frac{n}{8} \)
\( \alpha^3 + \beta^3 = -n \)
\( -n = (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha \beta] \)
\( n = -\left(-\frac{1}{4}\right)[\left(-\frac{1}{4}\right)^2 - 6] \)
\( n = -\frac{95}{64} \)

(ii) Sum of roots \( = \alpha + \beta + 4 \)
\( = \frac{15}{4} \)

Product of roots \( = (\alpha + 2)(\beta + 2) \)
\( = \alpha \beta + 2(\alpha + \beta) + 4 \)
\( = 2 + 2\left(-\frac{1}{4}\right) + 4 \)
\( = \frac{11}{2} \)

New equation: \( x^2 - \frac{15}{4} x + \frac{11}{2} = 0 \) or \( 4x^2 - 15x + 22 = 0 \)
7. (i) \[
\frac{2 - 2\sec^2 x}{(1 + \cos x)(1 - \cos x)} = \frac{-2(\sec^2 x - 1)}{\sin^2 x} = \frac{-2\tan^2 x}{\sin^2 x} = \frac{-2}{\cos^2 x} = -2\sec^2 x
\]
(ii) \[
\frac{2 - 2\sec^2 x}{(1 + \cos x)(1 - \cos x)} = 4\tan x
\]
\[-2\sec^2 x = 4\tan x \]
\[-\frac{1}{\cos^2 x} = \frac{2\sin x}{\cos x} \]
\[-1 = 2\sin x\cos x \]
\[\sin 2x = -1 \]
\[2x = -\frac{\pi}{2}, \quad \frac{3\pi}{2} \]
\[x = -\frac{\pi}{4}, \quad \frac{3\pi}{4} \]

8. (i) \[
\begin{array}{|c|c|c|c|c|}
\hline
\text{t (min)} & 10 & 20 & 30 & 40 \\
\hline
\ln(T - 30) & 3.35 & 2.45 & 1.55 & 0.64 \\
\hline
\end{array}
\]
(ii) \[
T = 30 + pe^{-qt} \\
\ln(T - 30) = \ln p - qt \\
\ln p = 4.25 \\
p = e^{4.25} = 70.1 \\
-q = \text{gradient} \\
= \frac{0.65 - 4.25}{40} \\
= -0.09
\]
(ii) Since \(e^{-qt} > 0\), \(T = 30 + 70e^{-0.09t} > 30\) 
Hence, \(T > 30\) for all values of \(t\).
9. (i) \[ \frac{dy}{dx} = 2e^{1-x} - 2xe^{1-x} \]

(ii) \[ \frac{d^2 y}{dx^2} = -2e^{1-x} - 2e^{1-x} + 2xe^{1-x} \]

\[ = -4e^{1-x} + 2xe^{1-x} \]

\[ \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + pe^{1-x} = 0 \]

\[ -pe^{1-x} = \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} \]

\[ = -4e^{1-x} + 2xe^{1-x} + 2(2e^{1-x} - 2xe^{1-x}) \]

\[ = -4e^{1-x} + 2xe^{1-x} + 4e^{1-x} - 4xe^{1-x} \]

\[ = -2xe^{1-x} \]

\[ p = 2x \]

(iii) When \( \frac{dy}{dx} > 0 \), \( 2e^{1-x} - 2xe^{1-x} > 0 \)

\( 2e^{1-x}(1-x) > 0 \)

Since \( e^{1-x} > 0 \) for all \( x \), \( 1-x > 0 \)

\( x < 1 \)

10. (i) \[ x^2h = 4000 \Rightarrow h = \frac{4000}{x^2} \]

\[ C = \frac{2}{1000} \times (x^2 + 4hx) \]

\[ = \frac{2}{1000} \left( x^2 + 4x \times \frac{4000}{x^2} \right) \]

\[ = \frac{x^2 + 32}{500} \cdot \frac{1}{x} \]

(ii) \[ \frac{dC}{dx} = \frac{x}{250} - \frac{32}{x^2} \]

When \( \frac{dC}{dx} = 0 \), \( \frac{x}{250} - \frac{32}{x^2} = 0 \)

\( x^3 = 8000 \)

\( x = 20 \)

(iii) \[ \frac{d^2 C}{dx^2} = \frac{1}{250} + \frac{64}{x^3} \]

When \( x = 20 \), \( \frac{d^2 C}{dx^2} = \frac{3}{250} > 0 \)

Since, \( \frac{d^2 C}{dx^2} > 0 \) when \( x = 20 \), \( C \) has a minimum value.
11. (i) Gradient of $FH = \frac{9-1}{6-2} = 2$
Gradient of $EG = -\frac{1}{2}$

(ii) Midpoint of $FH = \left(\frac{2+6}{2}, \frac{1+9}{2}\right) = (4, 5)$
Equation of $EG$: $y - 5 = -\frac{1}{2}(x - 4)$
$y = -\frac{x}{2} + 7$

(iii) $\frac{-x}{2} + 7 = \frac{x}{4} + \frac{3}{8}$
$5x = 25 \Rightarrow x = 10$
$y = 2$
Coordinate of $E = (10, 2)$
$y = -\frac{x}{2} + 7$
When $x = 0$, $y = 7$
Coordinate of $G = (0, 7)$

(iv) Area of $EFGH$
$= \frac{1}{2} \begin{vmatrix} 0 & 2 & 10 & 6 & 0 \\ 2 & 7 & 1 & 9 & 7 \end{vmatrix}$
$= \frac{1}{2} \left[ (4 + 90 + 42) - (14 + 10 + 12) \right]$
$= 50 \text{ unit}^2$

**Alternative Method**
Area of $EFGH = \frac{1}{2} \times HF \times GE$
$= \frac{1}{2} \times \sqrt{4^2 + 8^2} \times \sqrt{10^2 + 5^2}$
$= 50 \text{ units}^2$
12. (i) $DT$ is parallel to $AB$. (Midpoint Theorem)  
$\angle AFD = \angle TDF$ (alt angles)  
$= \angle FED$  
Since $\angle AFD$ and $\angle FED$ satisfies the alternate segment theorem, $AB$ is a tangent at $F$.

(ii) $\angle DFE$ is common.  
$\angle TDF = \angle DCF$ (base angles of an isos triangle)  
$\angle DCF = \angle DEF$ (angles in the same segment)  
$\therefore DFT$ and $EFD$ are similar triangles (AA)

\[
\frac{DF}{EF} = \frac{FT}{FD}  
DF^2 = FT \times EF  
= FT \times (ET + TF)  
= FT^2 + FT \times ET  
DF^2 - FT^2 = FT \times ET
\]
READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved electronic scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the equation $ax^2 + bx + c = 0$,
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Binomial expansion
\[(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,\]
where $n$ is a positive integer and
\[ \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \]

2. TRIGONOMETRY

Identities
\[-\sin^2 A + \cos^2 A = 1 \]
\[\sec^2 A = 1 + \tan^2 A \]
\[\csc^2 A = 1 + \cot^2 A \]
\[\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \]
\[\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \]
\[\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \]
\[\sin 2A = 2\sin A \cos A \]
\[\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A \]
\[\tan 2A = \frac{2\tan A}{1 - \tan^2 A} \]

Formulae for $\triangle ABC$
\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]
\[ a^2 = b^2 + c^2 - 2bc \cos A \]
\[ \Delta = \frac{1}{2}bc \sin A \]
1. (a) In the expansion of $(3x - 1)(1-kx)^7$ where $k$ is a non-zero constant, there is no term in $x^2$. Find the value of $k$. [4]

(b) In the binomial expansion of $\left(\frac{2}{x^3} - x^2\right)^{12}$, in ascending powers of $x$, find the term in which the power of $x$ first becomes positive. [4]

2. (a) Explain why the curve $y = px^2 + 2x - p$ will always cut the line $y = -1$ at two distinct points for all real values of $p$. [4]

(b) Find the values of $a$ such that the curve $y = ax^2 + x + a$ lies below the $x$-axis. [4]

3. (a) The diagram shows two right-angled triangles with the same height $x$ cm. One triangle has a base of 4 cm and the other triangle has a base of 6 cm. Angles $A$ and $B$ are such that $A + B = 135^\circ$.

Find the value of $x$. [4]

(b) The current $y$ (in amperes), in an alternating current (A.C.) circuit, is given by $y = 170\sin(kt)$, where $t$ is the time in seconds.

The period of this function is \(\frac{1}{60}\) second.

(i) Find the amplitude of $y$. [1]

(ii) Find the exact value of $k$ in radians per second. [1]

(iii) For how long in a period is $y > 85$? [3]
4. The function $g(x) = 2x^4 + x^3 + 4x^2 + hx - k$ has a quadratic factor $2x^2 + 3x + 1$.
   (i) Find the value of $h$ and of $k$. [5]
   (ii) Determine, showing all necessary working, the number of real roots of the equation $g(x) = 0$. [4]

5. The function $f$ is defined by $f(x) = 4 + 2x - 3x^2$.
   (i) Find the value of $a$, of $b$ and of $c$ for which $f(x) = a + b(x + c)^2$. [4]
   (ii) State the maximum value of $f(x)$ and the corresponding value of $x$. [2]
   (iii) Sketch the curve of $y = f(x)$ for $-1 \leq x \leq 2$, indicating on your graph the coordinates of the maximum point. [3]
   (iv) State the value(s) of $k$ for which $|f(x)| = k$ has
      (a) 1 solution, [1]
      (b) 3 solutions. [1]

6. (i) Find $\frac{d}{dx} \left[ (\ln x)^2 \right]$. [2]
   (ii) Using the result from part (i), find $\int \frac{3x^3 - 5\ln x}{x} \, dx$ and hence show that
        $$\int_1^e \frac{3x^3 - 5\ln x}{x} \, dx = e^3 - \frac{7}{2}.$$ [4]

7. (i) Show that $\frac{d}{dx} (\sec x) = \sec x \tan x$. [2]
   (ii) Given that $-\frac{\pi}{2} < x < \frac{\pi}{2}$, find the value of $n$ for which $y = e^{\tan x}$ is a solution of the equation
        $$\frac{d^2 y}{dx^2} = (1 + \tan x)^n \frac{dy}{dx}.$$ [7]

8. A circle passes through the points $A(2, 6)$ and $B(5, 5)$, with its centre lying on the line $3y = -x + 5$.
   (i) Find the perpendicular bisector of $AB$. [3]
   (ii) Find the equation of the circle. [4]

   $CD$ is a diameter of the circle and the point $P$ has coordinates $(-2, -1)$.
   (iii) Determine whether the point $P$ lies inside the circle. [2]
   (iv) Is angle $CPD$ a right angle? Explain. [1]
9.  (i) Given that \( \frac{x^2 - 4x + 1}{x^3 - 6x + 9} = A + \frac{B}{x - 3} + \frac{C}{(x - 3)^2} \), where \( A, B \) and \( C \) are constants, find the value of \( A \), of \( B \) and of \( C \). [4]

(ii) Hence, find the coordinates of the turning point on the curve, \( y = \frac{x^2 - 4x + 1}{x^2 - 6x + 9} \) and determine the nature of this turning point. [6]

10. A particle starts from rest at \( O \) and moves in a straight line with an acceleration of \( a \) ms\(^{-2} \), where \( a = 2t - 1 \) and \( t \) is the time in seconds since leaving \( O \).

(i) Find the value of \( t \) for which the particle is instantaneously at rest. [4]

(ii) Show that the particle returns to \( O \) after \( 1\frac{1}{2} \) seconds. [4]

(iii) Find the distance travelled in the first 4 seconds. [2]

11. The diagram below shows part of a curve \( y = f(x) \). The curve is such that \( f'(x) = x^2 - x^{-\frac{1}{2}} \) and it passes through the point \( Q(4, 0) \). The tangent at \( Q \) meets the \( y \)-axis at the point \( P \).

(i) Find \( f(x) \). [3]

(ii) Show that the \( y \)-coordinate of \( P \) is \(-6\). [3]

(iii) Find the area of the shaded region. [4]
Solution

1. (a) \[(3x - 1)(1 - kx)^7 = (3x - 1)(1 - 7kx + 21k^2x^2 + ...)
-21k - 21k^2 = 0
-21k(1 + k) = 0
k \neq 0, k = -1\]

(b) \[T_{r+1} = \binom{12}{r} \left(\frac{2}{x^3}\right)^{12-r} (-x^2)^r\]
\[= \binom{12}{r} (2^{12-r})(-1)^r x^{5r-36}\]
\[5r - 36 > 0\]
\[r > 7.2\]
\[r = 8\]
\[T_8 = \binom{12}{8} (2^4)(-1)^8 x^{40-36}\]
\[= 7920x^4\]

2. (a) \[px^2 + 2x - p = -1\]
\[px^2 + 2x + 1 - p = 0\]
\[D = 4 - 4(p)(1 - p)\]
\[= 4p^2 - 4p + 4\]
\[= 4(p^2 - p + 1)\]
\[or\]
\[4p^2 - 4p + 1 + 3\]
\[= 4 \left( p - \frac{1}{2} \right)^2 + \frac{3}{4}\]
\[= 4 \left( p - \frac{1}{2} \right)^2 + 3 > 0\]
\[(2p - 1)^2 + 3 > 0\]
Since \( \left( p - \frac{1}{2} \right)^2 \geq 0 \) or \((2p - 1)^2 \geq 0\),
the discriminant > 0, the curve will always cut the line at two distinct points for all real values of \(p\).

(b) \[D = 1 - 4a^2 < 0\]
\[D = 1 - 4a^2 < 0\]
\[(1 + 2a)(1 - 2a) < 0\]
\[4a^2 - 1 > 0\]
\[(2a - 1)(2a + 1) > 0\]
\[a < -\frac{1}{2} or a > \frac{1}{2}\]
\[\therefore a < -\frac{1}{2}\]
3. (a) \( \tan A = \frac{x}{4}, \tan B = \frac{x}{6} \)
\( \tan(A + B) = \tan 135^\circ \)
\( \tan A + \tan B = 1 - \tan A \tan B \)
\( \frac{x}{4} + \frac{x}{6} = -1 + \left( \frac{x}{4} \right) \left( \frac{x}{6} \right) \)
\( 6x + 4x = -24 + x^2 \)
\( x^2 - 10x - 24 = 0 \)
\( (x - 12)(x + 2) = 0 \)
\( x = 12, -2 \) (NA)

(b) \( y = 170 \sin(kt) \)
(i) Amplitude = 170 or 170 A
(ii) \( k = \frac{2\pi}{60} \)
\( = \frac{120\pi}{60} \)
(iii) When \( y = 85 \), \( 170 \sin(120\pi t) = 85 \)
\( \sin(120\pi t) = \frac{85}{170} = \frac{1}{2} \)
\( 120\pi t = \frac{\pi}{6}, \frac{5\pi}{6} \)
\( t = \frac{1}{720}, \frac{5}{720} \)
Duration \( = \frac{5}{720} - \frac{1}{720} \)
\( = \frac{1}{180} \) seconds
4. (i) \[ g(x) = 2x^4 + x^3 + 4x^2 + hx - k \]
\[ 2x^2 + 3x + 1 = (2x + 1)(x + 1) \]
\[ g\left(-\frac{1}{2}\right) = 0 \]
\[ 2\left(-\frac{1}{2}\right)^4 + \left(-\frac{1}{2}\right)^3 + 4\left(-\frac{1}{2}\right)^2 + h\left(-\frac{1}{2}\right) - k = 0 \]
\[ 1 - \frac{h}{2} - k = 0 \]
\[ k + \frac{h}{2} = 1 \quad \ldots \ldots (1) \]
\[ g(-1) = 0 \]
\[ 2 - 1 + 4 - h - k = 0 \]
\[ h + k = 5 \quad \ldots \ldots (2) \]
\[ \frac{h}{2} = 4 \]
\[ h = 8 \]
\[ k = -3 \]

**Alternative method**

\[ 2x^4 + x^3 + 4x^2 + hx - k = (2x^2 + 3x + 1)(x^2 + bx - k) \]

Comparing coefficient of \( x^3 \), \[ 1 = 2b + 3 \]
\[ b = -1 \]

Comparing coefficient of \( x^2 \), \[ 4 = -2k + 3b + 1 \]
\[ k = -3 \]

Comparing coefficient of \( x \), \[ h = b - 3k \]
\[ h = 8 \]

(ii) Let \( g(x) = (2x^2 + 3x + 1)(x^2 + bx + 3) \)

Comparing coefficient of \( x \), \[ 8 = 9 + b \]
\[ b = -1 \]

\[ g(x) = (2x^2 + 3x + 1)(x^2 - x + 3) \]

\[ g(x) = 0 \]
\[ (2x + 1)(x + 1)(x^2 - x + 3) = 0 \]
\[ x = -\frac{1}{2}, \quad x = -1 \quad \text{or} \quad x^2 - x + 3 = 0 \]
\[ b^2 - 4ac = 1 - 12 < 0 \]
\[ = -11 < 0 \]

No real roots.

Hence, \( g(x) = 0 \) has only 2 real roots
5. (i) \[ f(x) = 4 + 2x - 3x^2 \]
\[ = -3 \left( x^2 - \frac{2x}{3} \right) + 4 \]
\[ = -3 \left( x - \frac{1}{3} \right)^2 - \frac{1}{9} \right] + 4 \]
\[ = -3 \left( x - \frac{1}{3} \right)^2 + \frac{13}{3} \]
\[ a = \frac{13}{3}, \ b = -3, \ c = -\frac{1}{3} \]

(ii) Max value = \( \frac{13}{3} \) or \( 4\frac{1}{3} \)

at \( x = \frac{1}{3} \)

(iii) \[ \left( \frac{1}{3}, \frac{13}{3} \right) \]

(iv)(a) \( k = \frac{13}{3} \)

(b) \( 1 < k \leq 4 \)
### 6. (i)
\[ \frac{d}{dx} (\ln x)^2 = 2 \ln x \left( \frac{1}{x} \right) \]
\[ = \frac{2 \ln x}{x} \]

(ii)
\[ \int 3x^3 - 5 \ln x \frac{dx}{x} = \int 3x^2 \, dx - \int \frac{5 \ln x}{x} \, dx \]
\[ = x^3 - \frac{5}{2} (\ln x)^2 + C \]
\[ \int_1^e 3x^3 - 5 \ln x \frac{dx}{x} = \left[ x^3 - \frac{5}{2} (\ln x)^2 \right]_1^e \]
\[ = e^3 - \frac{5}{2} (\ln e)^2 - 1 \]
\[ = e^3 - \frac{7}{2} \]

### 7. (i)
\[ \frac{d}{dx} (\sec x) = \frac{d}{dx} [(\cos x)^{-1}] \]
\[ = (-1)(\cos x)^{-2} (-\sin x) \]
\[ = \frac{1}{\cos x} \frac{\sin x}{\cos x} \]
\[ = \sec x \tan x \]

(ii)
\[ \frac{dy}{dx} = \frac{d}{dx} (e^{\tan x}) \]
\[ = \sec^2 x e^{\tan x} \]
\[ \frac{d^2 y}{dx^2} = \frac{d}{dx} (\sec^2 x e^{\tan x}) \]
\[ = e^{\tan x} (2 \sec x)(\sec x \tan x) + \sec^2 x (\sec^2 x e^{\tan x}) \]
\[ = \sec^2 x e^{\tan x} (2 \tan x + \sec^2 x) \]
\[ = (1 + \tan x)^2 \frac{dy}{dx} \]
\[ \therefore n = 2 \]
8. (i) Midpoint of $AB = \left( \frac{2+5}{2}, \frac{6+5}{2} \right) = \left( \frac{7}{2}, \frac{11}{2} \right)$

Gradient of $AB = \frac{5-6}{5-2} = -\frac{1}{3}$

Gradient of perpendicular bisector = 3

Equation of perpendicular bisector,

$$y - \frac{11}{2} = 3 \left( x - \frac{7}{2} \right)$$

$$y = 3x - 5$$

(ii) From (i) \(y = 3x - 5\) \(\ldots (1)\)

The centre also lies on \(3y = -x + 5\) \(\ldots (2)\)

Substitute (1) into (2),

\(3(3x - 5) = -x + 5\)

\(x = 2\)

\(y = 1\)

Centre of circle, \((2, 1)\)

Radius of circle \(= \sqrt{(2 - 5)^2 + (1 - 5)^2}\)

\(= \sqrt{25}\)

\(= 5\) units

Equation of circle,

\((x - 2)^2 + (y - 1)^2 = 25\)

Or \(x^2 + y^2 - 4x - 2y - 20 = 0\)

(iii) Distance between the Centre and \(P\)

\(= \sqrt{(2+2)^2 + (1+1)^2}\)

\(= 2\sqrt{5}\) units < 5 units

\(\therefore P\) lies inside the circle.

If angle $CPD = 90^\circ$, $P$ should lie on the circle.

(Right angle in a semicircle)

Hence, angle $CPD$ cannot be $90^\circ$
Using Long Division, \[ \frac{x^2-4x+1}{x^2-6x+9} = 1 + \frac{2x-8}{(x-3)^2} \]

Let \[ \frac{2x-8}{(x-3)^2} = \frac{B}{x-3} + \frac{C}{(x-3)^2} \cdot \]

\[ 2x-8 = B(x-3) + C \]

Comparing coefficient of \( x \), \( B = 2 \)

Let \( x = 3 \), \( 6 - 8 = C \)

\[ C = -2 \]

\[ A = 1 \]

\[ \frac{dy}{dx} = \frac{d}{dx}\left[1+2(x-3)^{-1} - 2(x-3)^{-2}\right] \]

\[ = -2(x-3)^{-2} - 2(-2)(x-3)^{-3} \]

\[ = -2(x-3)^{-3}(x-3-2) \]

\[ = \frac{2(x-5)}{(x-3)^3} \text{ or } 10 - 2x \]

\[ \frac{(x-3)^3}{(x-3)^3} \text{ or } -\frac{2}{(x-3)^2} + \frac{4}{(x-3)^3} \]

When \( \frac{dy}{dx} = 0 \), \[ -\frac{2(x-5)}{(x-3)^3} = 0 \]

\[ x = 5 \]

When \( x = 5 \), \( y = \frac{3}{2} \)

Turning point, \( \left( 5, \frac{3}{2} \right) \).

\[ \text{Alternative Method} \]

When \( \frac{dy}{dx} = 0 \), \[ -\frac{2}{(x-3)^2} + \frac{4}{(x-3)^3} = 0 \]

\[ 4 = \frac{2}{(x-3)^2} \]

\[ 2(x-3)^2 = (x-3)^3 \]

Since \( x \neq 3 \), \[ 2 = x - 3 \]

\[ x = 5 \]

When \( x = 5 \), \( y = \frac{3}{2} \)

Turning point, \( \left( 5, \frac{3}{2} \right) \).
\[
\frac{d^2y}{dx^2} = \frac{-2(x-3)^3 + 2(3)(x-5)(x-3)^2}{(x-3)^6} \\
= \frac{-2(x-3) + 6(x-5)}{(x-3)^4} \\
= \frac{4x-24}{(x-3)^3} \quad \text{or} \quad \frac{4}{(x-3)^3} - \frac{12}{(x-3)^3}
\]

When \( x = 5, \ \frac{d^2y}{dx^2} = -\frac{1}{4} < 0 \)

\( \left(5, \frac{3}{2}\right) \) is maximum point.

**Alternative method**

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<tbody>
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<td>( \frac{dy}{dx} )</td>
<td>+</td>
<td>0</td>
<td>–</td>
</tr>
<tr>
<td>Slope</td>
<td>/</td>
<td>–</td>
<td>|</td>
</tr>
</tbody>
</table>

\( \left(5, \frac{3}{2}\right) \) is maximum point.
10. (i) \( v = \int (2t - 1)\,dt \)
\[ = t^2 - t + C \]
When \( t = 0, \ v = 0, \ C = 0 \)
\[ \therefore v = t^2 - t \]
When \( v = 0, \quad t^2 - t = 0 \)
\[ t(t^2 - 1) = 0 \]
\[ t = 0 \text{ (NA)}, \ 1 \]

(ii) \( s = \int (t^2 - t)\,dt \)
\[ = \frac{t^3}{3} - \frac{t^2}{2} + D \]
When \( t = 0, \ s = 0, \ D = 0 \)
\[ \therefore s = \frac{t^3}{3} - \frac{t^2}{2} \]
When \( s = 0, \quad \frac{t^3}{3} - \frac{t^2}{2} = 0 \)
\[ 2t^3 - 3t^2 = 0 \]
\[ t^2 (2t - 3) = 0 \]
\[ t = 0, \frac{3}{2} \]
Hence, the particle returns to \( O \) after \( 1\frac{1}{2} \) seconds.

**Alternative method**

When \( t = \frac{3}{2} \), \( s = \frac{1.5^3}{3} - \frac{1.5^2}{2} = 0 \)
Hence, the particle returns to \( O \) after \( 1\frac{1}{2} \) seconds.

(iii) When \( t = 1, \quad s = \frac{1^3}{3} - \frac{1^2}{2} = -\frac{1}{6} \)
When \( t = 4, \quad s = \frac{4^3}{3} - \frac{4^2}{2} = 13\frac{1}{3} \)
Distance travelled
\[ = 13\frac{1}{3} + 2\left(\frac{1}{6}\right) \]
\[ = 13\frac{2}{3} \text{ m} \]

(10 marks)
### Question 11.

| (i) | $f'(x) = x^2 - x^{-\frac{1}{2}}$

$f(x) = \int (x^2 - x^{-\frac{1}{2}}) \, dx$

$= \frac{2}{3} x^3 - 2x^{\frac{1}{2}} + C$

At $(4, 0)$,

$$\frac{2}{3} (4)^2 - 2(4)^{\frac{1}{2}} + C = 0$$

$C = -\frac{4}{3}$

$f(x) = \frac{2}{3} x^3 - 2x^{\frac{1}{2}} - \frac{4}{3}$ | **M1**

| **A1**

| (ii) | At $Q$, $f'(x) = \frac{dy}{dx} = 4^2 - 4^{-\frac{1}{2}} = \frac{3}{2}$

Equation of $PQ$, $y = \frac{3}{2} (x - 4)$

$y = \frac{3}{2} x - 6$

∴ at $P$, $y = -6$ | **B1**

| **M1**

| (iii) | Area of shaded region

$$= \frac{1}{2} \times 4 \times 6 + \int_0^4 \left( \frac{2}{3} x^3 - 2x^{\frac{1}{2}} - \frac{4}{3} \right) \, dx$$

or

$$= \frac{1}{2} \times 4 \times 6 - \left[ \int_0^4 \left( \frac{2}{3} x^3 - 2x^{\frac{1}{2}} - \frac{4}{3} \right) \, dx \right]$$

$$= 12 + \left[ \frac{2}{3} x^2 \left| \begin{array}{c} 4 \\ 0 \end{array} \right. - 2x \left| \begin{array}{c} 4 \\ 2 \end{array} \\ \frac{4}{3} x \left| \begin{array}{c} 4 \\ 0 \end{array} \right. - 4 \right]$$

$$= 12 + \left[ \frac{4}{15} x^2 \left| \begin{array}{c} 4 \\ 0 \end{array} \right. - \frac{4}{3} x^2 \left| \begin{array}{c} 4 \\ 3 \end{array} \right. - \frac{4}{3} \right]$$

$$= 12 + \left[ \frac{4}{15} (4)^{\frac{5}{2}} - \frac{4}{3} (4)^{\frac{3}{2}} - \frac{4}{3} (4) \right]$$

$$= 12 - \frac{112}{15}$$

$$= \frac{68}{15} \text{ unit}^2 \text{ or } \frac{8}{15} \text{ unit}^2 \text{ or } 4.53 \text{ unit}^2$$ | **M1**

| **A1**

(10 marks) |
SECONDARY 4
2018 Preliminary Examinations

ADDITIONAL MATHEMATICS
Paper 1

10 September 2018 (Monday)                        2 hours

CANDIDATE NAME

CLASS INDEX NUMBER

READ THESE INSTRUCTIONS FIRST
Do not turn over the page until you are told to do so.
Write your name, class and index number in the spaces provided above.

Write in dark blue or black pen on both sides of the paper. You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

INFORMATION FOR CANDIDATES
Answer all the questions.
Write your answers on the separate writing paper provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your answer scripts securely together.

The number of marks is given in brackets [   ] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiner’s Use
Q1 3
Q2 6
Q3 5
Q4 5
Q5 5
Q6 6
Q7 6
Q8 8
Q9 9
Q10 9
Q11 9
Q12 9
Total /80

This document consists of 7 printed pages including the cover page.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial Expansion
\[
(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n
\]
where \( n \) is a positive integer and 
\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1) \ldots (n-r+1)}{r!}
\]

2. TRIGONOMETRY

Identities
\[
\sin^2 A + \cos^2 A = 1
\]
\[
\sec^2 A = 1 + \tan^2 A
\]
\[
\cosec^2 A = 1 + \cot^2 A
\]
\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]
\[
\sin 2A = 2 \sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]
\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \Delta ABC \)
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\Delta = \frac{1}{2} ab \sin C
\]
1. Given that \( a = \sqrt{2} - \sqrt{3} \), find the value of \( a^2 \), leaving your answer in exact form. Hence, or otherwise, and without the use of a calculator, find the exact value of \( 2a^4 - 16a^2 + 5 \). [3]

2. A curve, for which \( \frac{dy}{dx} = kx^2 - 8 \), has a gradient of \(-4\) at \( x = 2 \).
   (i) State the value of \( k \). [1]
   
   With this value of \( k \), find
   (ii) the equation of the normal at point \( P(3, -2) \), [2]
   (iii) the equation of the curve \( y \). [3]

3. (i) Sketch the graph of \( y^2 = 9x \). [2]
   
   (ii) You were going through your old notes and happen to come across the following graph sketched on a piece of paper. It brought back some memories of your time in SST because you had to draw that graph in a Mathematics quiz. However, the equation of the function is missing from the graph. You decided to complete the equation before putting the graph back into the pile.

Given that the \( y \)-intercept of the graph is \( \frac{5}{3} \) and that the equation is of the curve is of the form \( y = \frac{k}{(x-h)} + c \), where \( h, k, c \) are constants that need to be determined, find the value of \( h \), of \( k \) and of \( c \). [3]
4. Express \( \frac{2x^2 + x + 1}{(x+1)(x-2)} \) in partial fractions. [5]

5. **Answer the whole of this question on a piece of graph paper.**

Variables \( x \) and \( y \) are known to be related by an equation of the form \( y = a\sqrt{x} + \frac{b}{\sqrt{x}} \), where \( a \) and \( b \) are constants. The table shows experimental values of the two variables.

<table>
<thead>
<tr>
<th></th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>2.4</td>
<td>3.9</td>
<td>5.1</td>
<td>6.4</td>
<td>7.4</td>
<td>8.3</td>
</tr>
</tbody>
</table>

(i) Plot \( y\sqrt{x} \) against \( x \) and draw a straight-line graph. [3]

(ii) Use the graph to estimate the values of \( a \) and of \( b \). [2]

6. Given that the roots of the quadratic equation \( 2x^2 + x + 6 = 0 \) are \( \alpha \) and \( \beta \).

(i) Find the quadratic equation whose roots are \( \left( \alpha + \frac{1}{2\beta} \right) \) and \( \left( \beta + \frac{1}{2\alpha} \right) \). [4]

(ii) Explain why the value for \( \alpha - \beta \) is undefined. [2]

7. (i) Prove the following trigonometric identity:

\[
\left( \frac{1-\cos\theta}{1+\cos\theta} \right) = (\csc\theta - \cot\theta)^2.
\]

(ii) Hence, for \( -\pi \leq \theta \leq \pi \), solve the equation

\[
(\csc\theta - \cot\theta)^2 = 5.
\]

[Turn over]
8. \(AB\) is a diameter of the circle with centre \(O\). \(C\) is a point on \(OG\) produced and \(CB\) intersects the circle at \(D\). \(OG\) is perpendicular to \(AB\) and \(OG\) intersects the chord \(AD\) at \(E\),

(i) Prove that \(AE \times ED = OE \times EC\). \([4]\)

(ii) Explain why \(C\) is at an equal distance from \(A\) and \(B\). \([2]\)

(iii) Explain why a circle with \(BC\) as a diameter passes through \(O\). \([2]\)

9. The straight line \(3x - y + 5 = 0\) and the curve \(x^2 + y^2 - 2x - 6y + 5 = 0\) intersect at two points, \(A\) and \(B\).

(i) Find the coordinates of \(A\) and of \(B\). \([3]\)

(ii) Find the equation of the perpendicular bisector of \(AB\). \([3]\)

(iii) Find the coordinates of the centre of the circle \(x^2 + y^2 - 2x - 6y + 5 = 0\) and determine whether the point \((1, 1)\) lies within, outside or on the circumference of the circle. \([3]\)
10. A piece of wire of length 680m is bent to form an enclosure consisting of a trapezium $ABCD$ and a quadrant $ADE$ with $AB = y$ m, $DE = x$ m and $B\hat{C}D = 45^\circ$.

(i) Show that the area $A$ m$^2$ of the enclosure is given by

\[ A = 340x - \frac{\sqrt{2} + 1}{2} x^2. \]  

(ii) Find the value of $x$, correct to 2 decimal places, for which there is a stationary value for $A$ and determine whether it is a maximum or a minimum.

11. A particle starts from a point $O$ and moves in a straight line so that its velocity, $v$ m/s, is given by $v = (3t + 5)(t - 5)$ where $t$ is the time in seconds after leaving $O$.

Find,

(i) the time(s) when the particle is at rest,  

(ii) the time when the particle passes through $O$ again,  

(iii) the distance travelled during the third second,  

(iv) the time interval during which the velocity is decreasing.
12.

In the diagram above the line $PQ$ is normal to the curve $y = \frac{1}{(2x-1)^3}$ at the point $P \left(\frac{3}{2}, \frac{1}{8}\right)$.

(i) Find the length of $OQ$. \[4\]

(ii) Find the area bounded by the line $PQ$, the curve $y = \frac{1}{(2x-1)^3}$ and the line $x = 3$. \[5\]

END OF PAPER
a = \sqrt{2} - \sqrt{3}

a^2 = (\sqrt{2} - \sqrt{3})^2

= 2 - 2\sqrt{2}\sqrt{3} + 3

= 5 - 2\sqrt{6}

dot 2a^4 - 16a^2 + 5 = 2(a^2)^2 - 16(a^2) + 5

= 2[5 - 2\sqrt{6}]^2 - 16(5 - 2\sqrt{6}) + 5

= 2[25 - 2(5)(2\sqrt{6}) + 24] - 80 + 32\sqrt{6} + 5

= 2[49 - 2\sqrt{6}] - 75 + 32\sqrt{6}

= 23 - 8\sqrt{6} \#
(ii) \( \frac{dy}{dx} = x^2 - 8 \)

@ \( x = 3 \), \( \frac{dy}{dx} = 3^2 - 8 = 1 \)

\[ y = 1 \]

:. The equation of the normal

\[ y - (-2) = \frac{1}{1} (x - 3) \]

\[ y = -x + 1 \]

(iii) \( y = \int x^2 - 8 \, dx \)

\[ = \frac{1}{3} x^3 - 8x + C \]

@ \( P(3, -2) \)

\[ -2 = \frac{1}{3}(3)^3 - 8(3) + C \]

\[ C = 13 \]

\[ y = \frac{1}{3} x^3 - 8x + 13 \]
(ii) Asymptote @ \( x = 3/2 \)
\[ y = 1 \]
\[ \therefore h = 3/2 \]
\[ c = 1 \]
\[ \therefore y = \frac{k}{x - 3/2} + 1 \]
At \((0, \frac{5}{3})\):
\[ \frac{5}{3} = \frac{k}{0 - 3/2} + 1 \]
\[ k = -\frac{1}{6} \]
\[ \therefore y = \frac{-1}{x - 3/2} + 1 \]
\[
\frac{2x^2 + x + 1}{(x+1)(x-2)} = \frac{2x^2 + x + 1}{x^2 - x - 2}
\]

\[
= \frac{2(x^2 - x - 2) + 3x + 5}{(x+1)(x-2)}
\]

\[
= 2 + \frac{3x + 5}{(x+1)(x-2)}
\]

\[
= 2 + \frac{B(x-2) + C(x+1)}{(x+1)(x-2)}
\]

\[
= 2 + \frac{(B+C)x + (-2B+C)}{(x+1)(x-2)}
\]

\[
\therefore \quad B + C = 3 \quad \text{(1)}
\]

\[
-2B + C = 5 \quad \text{(2)}
\]

\[
\begin{align*}
\text{(1) - (2)} & \\
B + C - (-2B + C) &= 3 - 5 \\
3B &= -2 \\
B &= \frac{-2}{3}
\end{align*}
\]

\[
\therefore \quad \frac{-2}{3} + C = 3
\]

\[
C = \frac{11}{3}
\]
\( y = ax + \frac{b}{5}x \) 

\[ y_{5x} = ax + b \]

\[ Y = ax + b \]

<table>
<thead>
<tr>
<th>( x = x )</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y = y_{5x} )</td>
<td>2.4</td>
<td>4.777</td>
<td>7.212</td>
<td>10.119</td>
<td>12.817</td>
<td>15.528</td>
</tr>
</tbody>
</table>

(i) Draw the graph:

\( b = -3 \) \( y \) 

\( a = \frac{8.04}{1.5} \)

\( = 5.36 \) \( \# \)
Scale:
X: 4 cm = 1 unit
Y: 2 cm = 1 unit
\[ 2x^2 + x + 6 = 0 \]

\[ : \alpha + \beta = -\frac{1}{2} \]
\[ \alpha \beta = \frac{6}{2} = 3 \]

\[ : (\alpha + \frac{1}{2\beta}) + (\beta + \frac{1}{2\alpha}) = (\alpha + \beta) + \frac{1}{2\alpha} + \frac{1}{2\beta} \]
\[ = (\alpha + \beta) + \frac{\beta + \alpha}{2\alpha \beta} \]
\[ = (\alpha + \beta) + \frac{\beta + \alpha}{2(3)} \]
\[ = -\frac{1}{12} \]

\[ \therefore (\alpha + \frac{1}{\alpha}) (\beta + \frac{1}{\beta}) = \alpha \beta + \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\alpha \beta} \]
\[ = \alpha \beta + \frac{1}{\alpha \beta} + 1 \]
\[ = 2 + \frac{1}{4(3)} + 1 \]
\[ = \frac{49}{12} \]

\[ \therefore \text{the equation} \]
\[ \lambda^2 - \left( -\frac{1}{12} \right) \lambda + \frac{49}{12} = 0 \]
\[ \lambda^2 + \frac{\lambda}{12} + \frac{49}{12} = 0 \]
\[ \alpha \approx 12 \lambda^2 + 7\lambda + 49 = 0 \]
(ii) \[(d - B)^2 = (x + B)^2 - 4\alpha \beta\]
\[= \left( -\frac{1}{2} \right)^2 - 4(3)\]
\[= -\frac{47}{4}\]
\[< 0\]

\[\therefore (d - B)^2 > 0 \text{ for all values of } \alpha \neq B,\]

\[\therefore d - B \text{ is undefined.}\]

\[\text{Q.E.D.} \]

\[2x^2 + 2x + 6 = 0\]

\[\text{Roots: } x = -1 \pm \sqrt{-47}\]

\[= \frac{-1 + \sqrt{-47}}{4} \quad \text{or} \quad \frac{-1 - \sqrt{-47}}{4}\]

\[\therefore d - B = \frac{-1 + \sqrt{-47}}{4} - \frac{-1 - \sqrt{-47}}{4}\]

\[= \frac{2\sqrt{-47}}{4}\]

\[\therefore \text{undefined because no real value exists for } \sqrt{-47}.\]
3) \[ \text{LHS:} \]
\[ (\cos(\theta) - \sin(\theta))^2 \]
\[ = \left( \frac{1}{\sin(\theta)} - \frac{\cos(\theta)}{\sin(\theta)} \right)^2 \]
\[ = \frac{(1 - \cos(\theta))^2}{\sin^2(\theta)} \]
\[ = \frac{(1 - \cos(\theta))}{1 - \cos^2(\theta)} \]
\[ = \frac{(1 - \cos(\theta))}{(1 + \cos(\theta))(1 - \cos(\theta))} \]
\[ = \frac{1 - \cos(\theta)}{1 + \cos(\theta)} \]
\[ = \text{LHS (Prove)} \]

(ii) \[ (\sin(\theta) - \cos(\theta))^2 = 5 \]
\[ \frac{1 - \sin(\theta)}{1 + \sin(\theta)} = 5 \]
\[ 5 + 5\sin(\theta) = 1 - \sin(\theta) \]
\[ 6\sin(\theta) = -4 \]
\[ \sin(\theta) = -\frac{2}{3} \]

Hence \[ x > 0.84107 \text{ rad \ (1.500)} \]

\[ \theta = (-\pi + 0.84107) \ & \ (\pi - 0.84107) \]
\[ = -2.30 \text{ rad} \ & \ 2.30 \text{ rad \ (t. 3sf)} \]
(i) $\angle ADB = 90^\circ$ (in a semicircle)

$\triangle AFO$ is similar $\triangle CED$

$\therefore \angle AFO = \angle CEO$ (Vertically opposite angles)

$\angle EAO = 90^\circ - \angle ABO$ (Sum of angles)

$= 90^\circ - \angle CED$

$= \angle ECD$

$\therefore \frac{AE}{CE} = \frac{BO}{EO} = \frac{AO}{CD}$

$\Rightarrow AE \times ED = OE \times EC$ (Proportions)

(ii) $OG$ is perpendicular to $AB$ (Given)

& $OG$ passes through the centre, i.e. its equidistant from $A, B$

$\therefore$ all points along $OG$ will be equidistant from $A, B$

& $C$ lies on $OG$ produced

$\Rightarrow C$ will be equidistant from $A$ & $B$ (shown)
\( \text{iii) } \therefore \angle COB = 90^\circ \text{ (given) } \\
\therefore \text{ angles is semicircle } = 90^\circ \\
\therefore \text{ there is a circle, with CB as its diameter that passes through point O. } \)
\[ 3x - y + 5 = 0 \\
\quad y = 2x + 5 \quad (1) \]

\[ x^2 + y^2 - 2x - 6y + 5 = 0 \quad (2) \]

Subtract (1) from (2):

\[ x^2 + (2x + 5)^2 - 2x - 6(2x + 5) + 5 = 0 \]

\[ x^2 + 4x^2 + 5x + 25 - 2x - 12x - 30 + 5 = 0 \]

\[ 10x^2 + 10x = 0 \]

\[ 10x(x + 1) = 0 \]

\[ x = 0 \quad \text{or} \quad x = -1 \]

\[ @ x = 0, \quad y = 3(0) + 5 \]

\[ = 5 \]

\[ \therefore A(0, 5) \quad \text{and} \quad B(-1, 2) \]

(ii) gradient \((AB) = \frac{2 - 5}{-1 - 0} = 3\)

midpoint \((AB) = \left( \frac{0 + (-1)}{2}, \frac{5 + 2}{2} \right) = \left( \frac{-1}{2}, \frac{7}{2} \right)\)

\[ \therefore \text{equation of the perpendicular bisector of } AB \]

\[ y - \frac{7}{2} = \frac{1}{3} (x - (-\frac{1}{2})) \]

\[ y = \frac{1}{3} x + \frac{10}{3} \]
(i) \( x^2 + y^2 - 2x - 6y + 5 = 0 \)

\( x^2 - 2x + y^2 - 6y + 5 = 0 \)

\( x^2 - 2x + 1^2 + (y^2 - 2(3y) + 3^2) + 5 = 0 \)

\( (x - 1)^2 + (y - 3)^2 = 5 \)

At \((1,1)\)

\[ (1 - 1)^2 + (1 - 3)^2 = 4 < 5 \]

\[ \therefore \text{point } (1,1) \text{ lies inside the circumference of the circle}. \]
\[ \overline{AB} = \frac{1}{4} [2\pi (x)] \]

\[ = \frac{\pi}{2} x \]

\[ \overline{BC} = x^2 + y^2 \text{ (Pythagorean Theorem)} \]

\[ = \sqrt{2x^2} \quad (BC > 0) \]

\[ = x\sqrt{2} \]

\[ \therefore x + y + x + x\sqrt{2} + y + \frac{\pi}{2} x = 680 \]

\[ 2y + (2 + \frac{\pi}{2} + \sqrt{2}) x = 680 \]

\[ y = \frac{680 - (2 + \frac{\pi}{2} + \sqrt{2}) x}{2}. \quad (1) \]

\[ \Delta Amn = \frac{1}{4} [\pi (x)^2] + \frac{1}{2} (y + y + x)(x) \]

\[ = \frac{\pi}{4} x^2 + \frac{x}{2} (2y + x) \]

\[ = \frac{\pi}{4} x^2 + \frac{x}{2} \left[ 2 \left( \frac{680 - (2 + \frac{\pi}{2} + \sqrt{2}) x}{2} \right) + x \right] \]

\[ = \frac{\pi}{4} x^2 + \frac{x}{2} \left[ 680 - (2 + \frac{\pi}{2} + \sqrt{2}) x + x^2 \right] \]

\[ = \frac{\pi}{4} x^2 + 340 x - \frac{x^2}{2} \left( 2 + \frac{\pi}{2} + \sqrt{2} \right) + \frac{x^2}{2} \]

\[ = 340 x + \left[ \frac{\pi}{4} - \frac{1}{2} \left( 2 + \frac{\pi}{2} + \sqrt{2} \right) + \frac{1}{2} \right] x^2 \]

\[ > 340 x + \left[ -\frac{1}{2} - \frac{\sqrt{2}}{2} \right] x^2 \]

\[ > 340 x - \left( \frac{1 + \sqrt{2}}{2} \right) x^2 \quad (\text{shown}) \]
(ii) 
\[ \text{Area} = 340x - \left( \frac{1 + \sqrt{2}}{2} \right)x^2 \]

\[ \frac{d\text{Area}}{dx} = 340 - 2 \left( \frac{1 + \sqrt{2}}{2} \right)x \]

\[ = 340 - (1 + \sqrt{2})x \]

\[ \frac{d^2\text{Area}}{dx^2} = -(1 + \sqrt{2}) < 0 \]

\[ \therefore \text{Area will be maximized at stationary point.} \]

\[ \therefore \text{Maximum value of } x \text{ at } \frac{d\text{Area}}{dx} = 0 \]

\[ 340 - (1 + \sqrt{2})x = 0 \]

\[ x = \frac{340}{1 + \sqrt{2}} \]

\[ = 140.83 \text{ m (to 2 DP)} \]
(i) \( V = (3t+5)(t-5) = 3t^2 - 10t - 25 \)

@ \( V = 0 \)

\((3t+5)(t-5) = 0\)

\( t = \frac{-5}{3} \) & \( t = 5 \) sec

\((\text{NG}: t > 0)\)

(ii) \( S = \int 3t^2 - 10t - 25 \, dt \)

\[ = t^3 - 5t^2 - 25t + C \]

@ \( t = 0, \, S = 0 \), \( C = 0 \)

\[ \therefore S = t^3 - 5t^2 - 25t = t(t^2 - 5t - 25) \]

@ \( S = 0 \)

\[ t = 0 \, \text{ or } \, t^2 - 5t - 25 = 0 \]

\[ t = \frac{5 \pm \sqrt{125}}{2} \]

\[ = \frac{5 \pm 5\sqrt{5}}{2} \]

\[ = 8.09 \, \text{sec} \, \text{ or } \, -3.09 \, (t < 25) \]

\((\text{NG})\)

\(- \text{ particle will pass through the origin again } @ \, t = 8.09 \, \text{sec} \)
@ t = 2, \quad s = 2(2^2 - 5(2) - 25) = -62 \, m

@ t = 3, \quad s = 3(3^2 - 5(3) - 25) = -93 \, m

\therefore \text{ distance travelled } = 31 \, m

\text{(ii)} \quad \text{Distance} = \int_{2}^{3} (2t^2 - 10t - 25) \, dt

= \left[ \frac{2t^3}{3} - 5t^2 - 25t \right]_{2}^{3}

= \left[ -62 \right] - \left[ -93 \right]

= 31 \, m

\text{(iii)} \quad \text{Velocity decreasing } \Rightarrow a < 0

\therefore a = \text{acceleration}

= \frac{dv}{dt}

= 6t - 10

\therefore 6t - 10 < 0

\therefore t < \frac{5}{3}

\therefore 0 < t < \frac{5}{3} \, \text{h}
(i) \( y = (2x-1)^{-3} \)

\[
\frac{dy}{dx} = -3(2x-1)^{-4} \quad [2]
\]

\[
= \frac{-6}{(2x-1)^4}
\]

\( x = \frac{3}{2}, \)

gradient \( = \frac{dy}{dx} = \frac{-6}{[2(\frac{3}{2})-1]^4} \)

\[
= \frac{-3}{8}
\]

\[ \therefore \text{the equation of the normal at } P \]

\[
y - \frac{1}{8} = \frac{6}{5}(x - \frac{3}{2})
\]

\[
y = \frac{6}{5}x - \frac{3y}{8}
\]

(ii) \( @ y = 0 \)

\[
x = \frac{93}{64}
\]

\[ \therefore Q \left( \frac{93}{64}, 0 \right) \]

\[ \therefore OQ = \frac{93}{64} \text{ units} \]
\[ A(x) = \frac{1}{2} \left( \frac{3}{2} - \frac{97}{8} \right) \left( \frac{1}{8} \right) + \int_{3/\nu}^{3} \left( 2 \chi - 1 \right)^{-3} d\chi \]

\[ \Rightarrow \frac{3}{1024} + \left[ \left( \frac{(2\chi - 1)^{-2}}{(-2)(2)} \right) \right]_{3/\nu}^{3} \]

\[ = \frac{3}{1024} + \left[ \frac{-1}{4 \left( 2\chi - 1 \right)^{2}} \right]_{3/\nu}^{3} \]

\[ \Rightarrow \frac{3}{1024} - \left( \frac{-1}{4 \left( 2(3) - 1 \right)^{2}} - \frac{-1}{4 \left( 2(\frac{3}{2}) - 1 \right)^{2}} \right) \]

\[ = \frac{3}{1024} + \left[ \frac{-1}{100} + \frac{1}{10} \right] \]

\[ = \frac{14.19}{256 \, \nu} \]

\[ = 0.0554 \, \text{unit}^{-1} \, ( \text{to 3SF} \, \text{(show)} ) \]

\[ \frac{1}{n} \]
SECONDARY 4
2018 Preliminary Examinations

ADDITIONAL MATHEMATICS
Paper 2

12 September 2018 (Wednesday) 2 hours 30 minutes

CANDIDATE
NAME

CLASS
INDEX
NUMBER

READ THESE INSTRUCTIONS FIRST
Do not turn over the page until you are told to do so.
Write your name, class and index number in the spaces above.
Write in dark blue or black pen in the writing papers provided.

You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

INFORMATION FOR CANDIDATES
Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your answer scripts securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

For Examiner's Use

<table>
<thead>
<tr>
<th>Q1</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q2</td>
<td>6</td>
</tr>
<tr>
<td>Q3</td>
<td>6</td>
</tr>
<tr>
<td>Q4</td>
<td>6</td>
</tr>
<tr>
<td>Q5</td>
<td>7</td>
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<tr>
<td>Q6</td>
<td>8</td>
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<td>10</td>
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<td>Q10</td>
<td>10</td>
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<tr>
<td>Q11</td>
<td>13</td>
</tr>
<tr>
<td>Q12</td>
<td>13</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
</tr>
</tbody>
</table>

This document consists of 8 printed pages including the cover page.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial Expansion
\[
(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n
\]
where \( n \) is a positive integer and
\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}
\]

2. TRIGONOMETRY

Identities
\[
\sin^2 A + \cos^2 A = 1
\]
\[
\sec^2 A = 1 + \tan^2 A
\]
\[
\csc^2 A = 1 + \cot^2 A
\]
\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
\sin 2A = 2 \sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]
\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \triangle ABC \)
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\Delta = \frac{1}{2} ab \sin C
\]
1 Find the value of the constant $k$ for which $y = x^2e^{1-2x}$ is a solution of the equation
\[
\frac{d^2y}{dx^2} - \frac{2y}{x^2} = k \left( \frac{dy}{dx} + y \right).
\]

2 (i) Find $\frac{d}{dx} \left( \frac{\ln x}{x} \right)$.

(ii) Hence find $\int \frac{\ln x}{x^2} \, dx$.

The curve $y = f(x)$ is such that $f(x) = \frac{\ln x}{x}$, for $x > 0$.

(iii) Explain why the curve $y = f(x)$ has only one stationary point.

3 The expression $2x^3 + ax^2 + bx - 35$, where $a$ and $b$ are constants, has a factor of $2x - 7$ and leaves a remainder of $-36$ when divided by $x + 1$.

(i) Find the value of $a$ and of $b$.

(ii) Using the values of $a$ and $b$ found in part (i), explain why the equation $2x^3 + ax^2 + bx - 35 = 0$ has only one real root.

4 As part of his job in a restaurant, John learned to cook a hot pot of soup late at night so that there would be enough for sale the next day. While refrigeration was essential to preserve the soup overnight, the soup was too hot to be put directly in the refrigerator when it was ready at $100 \degree C$. The soup subsequently cooled in such a way that its temperature, $x \degree C$ after $t$ minutes, was given by the expression $x = 20 + Ae^{-kt}$, where $A$ and $k$ are constants.

(i) Explain why $A = 80$.

(ii) When $t = 15$, the temperature of the soup is $58 \degree C$. Find the value of $k$.

(iii) Deduce the temperature of the soup if it is left unattended for a long period of time, giving a reason for your answer.

(iv) For the soup to be refrigerated, its temperature should be less than $35 \degree C$. What is the shortest possible time, correct to the nearest minute, that John has to wait before he could refrigerate the soup?
5 \hspace{1cm} (a) The function \( f \) is defined, for all values of \( x \), by

\[
f(x) = x^2(3 - 4x).
\]

Find the range of values of \( x \) for which \( f \) is an increasing function. \hspace{1cm} [3]

(b) A particle moves along the curve \( y = \frac{16}{(3 - 4x)^2} \) in such a way that the \( y \)-coordinate of the particle is increasing at a constant rate of 0.03 units per second. Find the exact \( y \)-coordinate of the particle at the instant that the \( x \)-coordinate of the particle is decreasing at 0.12 units per second. \hspace{1cm} [4]

6 \hspace{1cm} (a) (i) Sketch the graph of \( y = 10^x \). \hspace{1cm} [1]

(ii) Given that \( \frac{4^x}{2^{5x}} = \frac{3}{5^x} \), find the value of \( 10^x \). \hspace{1cm} [2]

(b) Solve the equation \( \log_2 \sqrt{5x + 1} + 2 \log_5 3 = \log_4 (2x - 3) + \log_3 27 \). \hspace{1cm} [5]

7 \hspace{1cm} The population of a herd of deer can be modelled by the function

\[
D = 400 + 40 \sin \left( \frac{\pi}{6} t \right),
\]

where \( D \) is the deer population in week \( t \) of the year for \( 0 \leq t \leq 24 \).

Using the model,

(i) state the amplitude of the function, \hspace{1cm} [1]

(ii) state the period of the function, \hspace{1cm} [1]

(iii) find the maximum and minimum values of \( D \), \hspace{1cm} [1]

(iv) sketch the function \( D = 400 + 40 \sin \left( \frac{\pi}{6} t \right) \) for \( 0 \leq t \leq 24 \). \hspace{1cm} [2]

(v) estimate the number of weeks for \( 0 \leq t \leq 24 \) that the population is greater than 420. \hspace{1cm} [3]
The diagram shows a quadrilateral $ABCD$ in which $A$ is $(6, 0)$, $C$ is $(3k, 5k + 10)$ and $D$ is $(-2, 4)$. The equation of line $AB$ is $y = 2x - 12$ and angle $ADC = 90^\circ$.

(i) Find the value of $k$. [3]

Given that the perpendicular bisector of $CD$ passes through $B$, find

(ii) the coordinates of $B$, [4]

(iii) the area of the quadrilateral $ABCD$. [2]
(a) The first three terms in the binomial expansion of \((1 + px)^n\) are \(1 - 48x + 960x^2\). Find the value of \(p\) and of \(n\). [4]

(b) In the expansion of \(\left(2x^2 + \frac{a}{x}\right)^8\), where \(a\) is a non-zero real number, the ratio of the coefficient of the 3\(^{rd}\) term to that of the 5\(^{th}\) term is 5 : 2.

(i) Find the possible values of \(a\). [4]

(ii) Explain whether the term independent of \(x\) exists for the expansion of \(\left(2x^2 + \frac{a}{x}\right)^8\). [2]

The diagram shows part of the curve \(y = \mid ax^2 + bx + c \mid\) where \(a < 0\).
The curve touches the \(x\)-axis at \(A(p, 0)\) and at \(B(1, 0)\).
The curve touches the \(y\)-axis at \(C(0, q)\) and has a maximum point at \(T(-1, 8)\).

(i) Explain why \(p = -3\). [1]

(ii) Determine the value of \(a, b, c\) and \(q\). [4]

(iii) State the range of values of \(r\) for which the line \(y = r\) intersects the curve \(y = \mid ax^2 + bx + c \mid\) at four distinct points. [1]

(iv) In the case where \(r = 2\), find the exact \(x\)-coordinates of all points of intersection of the line \(y = r\) and the curve \(y = \mid ax^2 + bx + c \mid\). [4]
The diagram shows a circular garden. A farmer decides to fence part of the garden. He puts fences around the perimeter $ABCD$ such that $BC = 8 \text{ m}$, $CD = 5 \text{ m}$, angle $DAB = 90^\circ$ and angle $ABC = \theta$ where $0^\circ < \theta < 90^\circ$.

(i) Given that $CM$ is perpendicular to $AB$, express $CM$ and $AB$ in terms of $\theta$. [4]

(ii) Show that $L \text{ m}$, the length of fencing needed for perimeter $ABCD$, is given by $L = 13 + 3\cos \theta + 13\sin \theta$. [2]

(iii) Express $L$ in the form $13 + R\cos(\theta - \alpha)$ where $R > 0$ and $\alpha$ is an acute angle. [4]

(iv) Given that the farmer uses exactly 26.2 m of fencing, find the possible values of $\theta$. [3]
12 (a) It is given that \( \int f(x)dx = k \cos 2x - \sin 3x + c \), where \( c \) is a constant of integration, and that \( \int_{0}^{\frac{\pi}{6}} f(x)dx = \frac{1}{3} \).

(i) Show that \( k = -2 \frac{2}{3} \). [1]

(ii) Find \( f(x) \). [2]

(b) A curve has the equation \( y = g(x) \), where \( g(x) = 2\sin^2 x - \sin 2x \) for \( 0 \leq x \leq \pi \).

(i) Find the \( x \)-coordinates of the stationary points of the curve. [3]

(ii) Use the second derivative test to determine the nature of each of these points. [3]

(iii) Given that \( \int g(x)dx = ax + b\sin x \cos x + \cos^2 x + k \), where \( k \) is a constant of integration, find the value of \( a \) and of \( b \). [4]

END OF PAPER
SECONDARY 4
2018 Preliminary Examinations

ADDITIONAL MATHEMATICS
Paper 2 4047/2

12 September 2018 (Wednesday) 2 hours 30 minutes

CANDIDATE NAME

CLASS INDEX NUMBER

READ THESE INSTRUCTIONS FIRST
Do not turn over the page until you are told to do so.
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INFORMATION FOR CANDIDATES
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The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

For Examiner’s Use

| Q1 | 4 |
| Q2 | 6 |
| Q3 | 6 |
| Q4 | 6 |
| Q5 | 7 |
| Q6 | 8 |
| Q7 | 8 |
| Q8 | 9 |
| Q9 | 10 |
| Q10 | 10 |
| Q11 | 13 |
| Q12 | 13 |
| Total | /100 |

This document consists of 8 printed pages including the cover page.
Mathematical Formulae

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\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

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\[(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n\]
where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. TRIGONOMETRY

Identities
\[ \sin^2 A + \cos^2 A = 1 \]
\[ \sec^2 A = 1 + \tan^2 A \]
\[ \csc^2 A = 1 + \cot^2 A \]
\[ \sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \]
\[ \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \]
\[ \tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \]
\[ \sin 2A = 2 \sin A \cos A \]
\[ \cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A \]
\[ \tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \]

Formulæ for \( \Delta \ ABC \)
\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]
\[ a^2 = b^2 + c^2 - 2bc \cos A \]
\[ \Delta = \frac{1}{2} ab \sin C \]
Find the value of the constant \( k \) for which \( y = x^2 e^{1-2x} \) is a solution of the equation

\[
\frac{d^2 y}{dx^2} - \frac{2y}{x^2} = k \left( \frac{dy}{dx} + y \right).
\]

### Solution

\( y = x^2 e^{1-2x} \)

\[
\frac{dy}{dx} = x^2 (-2e^{1-2x}) + e^{1-2x} (2x)
\]

\[
= -2x^2 e^{1-2x} + 2xe^{1-2x}
\]

\[
= -2y + 2xe^{1-2x} = -2y + \frac{2y}{x}
\]

\[
\frac{d^2 y}{dx^2} = -2 \frac{dy}{dx} + 2x(-2e^{1-2x}) + 2e^{1-2x}
\]

\[
= -2 \frac{dy}{dx} - 4xe^{1-2x} + 2e^{1-2x}
\]

\[
= -2 \frac{dy}{dx} - 4y + 2y
\]

\[
\frac{d^2 y}{dx^2} - \frac{2y}{x^2} = -2 \frac{dy}{dx} - 4y
\]

\[
= -2 \frac{dy}{dx} - 2(\frac{dy}{dx} + 2y)
\]

\[
= -4 \frac{dy}{dx} - 4y
\]

\[
k = -4
\]
2 (i) Find \( \frac{d}{dx} \left( \frac{\ln x}{x} \right) \). 

**Solution**

\[
\frac{d}{dx} \left( \frac{\ln x}{x} \right) = \frac{x \left( \frac{1}{x} \right) - \ln x}{x^2} = \frac{1 - \ln x}{x^2}
\]

(ii) Hence find \( \int \frac{\ln x}{x^2} \, dx \).

**Solution**

From (i),

\[
\int \frac{1 - \ln x}{x^2} \, dx = \frac{\ln x}{x} + C
\]

\[
\int \frac{1}{x^2} \, dx - \int \frac{\ln x}{x^2} \, dx = \frac{\ln x}{x} + C
\]

\[
- \frac{1}{x} \int \frac{\ln x}{x^2} \, dx = \frac{\ln x}{x} + C
\]

\[
\int \frac{\ln x}{x^2} \, dx = \frac{-1}{x} \frac{\ln x}{x} + D
\]

The curve \( y = f(x) \) is such that \( f(x) = \frac{\ln x}{x} \), for \( x > 0 \).

(iii) **Explain** why the curve \( y = f(x) \) has only one stationary point.

**Solution**

\( f(x) = \frac{\ln x}{x} \)

\( f'(x) = \frac{1 - \ln x}{x^2} \)

For stationary point to exist, \( f'(x) = 0 \)

\( 1 - \ln x = 0 \)

\( \ln x = 1 \)

\( x = e \)

For \( x > 0 \), \( y = f(x) \) has only 1 stationary point at \( x = e \).
The expression \(2x^3 + ax^2 + bx - 35\), where \(a\) and \(b\) are constants, has a factor of \(2x - 7\) and leaves a remainder of -36 when divided by \(x + 1\).

(i) Find the value of \(a\) and of \(b\).

(ii) Using the values of \(a\) and \(b\) found in part (i), explain why the equation \(2x^3 + ax^2 + bx - 35 = 0\) has only one real root.

**Solution**

(i) 
\[
f(x) = 2x^3 + ax^2 + bx - 35
\]
\[
f\left(\frac{7}{2}\right) = 0
\]
\[
2\left(\frac{7}{2}\right)^3 + a\left(\frac{7}{2}\right)^2 + b\left(\frac{7}{2}\right) - 35 = 0
\]
\[
\frac{343}{4} + \frac{49a}{4} + \frac{7b}{2} - 35 = 0
\]
\[
\frac{49a}{4} + \frac{7b}{2} = \frac{-203}{4}
\]
\[
49a + 14b = -203 \quad \text{(1)}
\]
\[
f(-1) = -36
\]
\[
2(-1)^3 + a(-1)^2 + b(-1) - 35 = -36
\]
\[
-2 + a - b - 35 = -36
\]
\[
a - b = 1 \quad \text{(2)}
\]
\[
49(1+b) + 14b = -203
\]
\[
49 + 63b = -203
\]
\[
63b = -252
\]
\[
b = -4
\]
\[
a = b + 1 = -4 + 1 = -3
\]

(ii) \(2x^3 - 3x^2 - 4x - 35 = 0\)
\[
2x^3 + ax^2 + bx - 35 = (2x - 7)(x^2 + 2x + 5)
\]
For \(x^2 + 2x + 5\), since \((2)^2 - 4(1)(20) < 0\) and the coefficient of \(x^2\) is always positive, \(x^2 + 2x + 5\) is always positive.

As part of his job in a restaurant, John learned to cook a hot pot of soup late at night so that there would be enough for sale the next day. While refrigeration was essential to preserve the soup overnight, the soup was too hot to be put directly in the refrigerator when it was ready at 100°C. The soup subsequently cools in such
a way that its temperature, \(x \, ^\circ\text{C}\) after \(t\) minutes, is given by the expression 
\[x = 20 + Ae^{-kt},\] where \(A\) and \(k\) are constants.

(i) **Explain** why \(A = 80\).  

**Solution**

Since the soup is ready at 100\(^{\circ}\)C initially,
At \(t = 0\), \(x = 20 + Ae^0 = 100\)
\[A = 80\]

(ii) When \(t = 15\), the temperature of the soup is 58\(^{\circ}\)C.  
Find the value of \(k\).

**Solution**

\[
58 = 20 + 80e^{-k(15)} \\
38 = 80e^{-15k} \\
e^{-15k} = \frac{38}{80} \\
-15k = \ln\frac{38}{80} \\
k = 0.0496
\]

(iii) Deduce the temperature of the soup if it is left unattended for a long period of time, giving a reason for your answer.

**Solution**

For \(x = 20 + 80e^{-kt}\), as \(t \to \infty\), \(e^{-kt} \to 0\)  
Temperature of the soup approaches 20\(^{\circ}\)C  
if it is left unattended for a long period of time.
4 (iv) For the soup to be refrigerated, its temperature should be less than 35 °C.
What is the shortest possible time, correct to the nearest minute that John has to wait before he can refrigerate the soup? [2]

Solution

\[ 20 + 80e^{-\frac{\ln 38}{15}t} = 35 \]
\[ 80e^{-\frac{\ln 38}{80}15} = 15 \]
\[ e^{\frac{\ln 38}{15}} = 15 \]
\[ \ln \frac{38}{80} t = \ln \frac{15}{80} \]
\[ t = 33.7 \]

Shortest possible time = 34 minutes

5 (a) The function \( f \) is defined, for all values of \( x \), by

\[ f(x) = x^2(3 - 4x). \]

Find the values of \( x \) for which \( f \) is an increasing function. [3]

Solution

\[ f(x) = 3x^2 - 4x^3 \]
\[ f'(x) = 6x - 12x^2 \]

For \( f \) to be an increasing function,
\[ f'(x) > 0 \]
\[ 6x - 12x^2 > 0 \]
\[ 6x(1 - 2x) > 0 \]
\[ 0 < x < \frac{1}{2} \]
5 (b) A particle moves along the curve \( y = \frac{16}{(3 - 4x)^2} \) in such a way that the y-coordinate of the particle is increasing at a constant rate of 0.03 units per second. Find the exact y-coordinate of the particle at the instant that the x-coordinate of the particle is decreasing at 0.12 units per second.

**Solution**

\[
y = \frac{16}{(3 - 4x)^2} = 16(4 - 4x)^{-2}
\]

\[
\frac{dy}{dx} = -32(3 - 4x)^{-3}(-4) = \frac{128}{(3 - 4x)^3}
\]

\[
\frac{dy}{dt} = 0.03
\]

\[
\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}
\]

\[
0.03 = \frac{128}{(3 - 4x)^3} (-0.12)
\]

\[
(3 - 4x)^3 = -512
\]

\[
3 - 4x = -8
\]

\[
-4x = -11
\]

\[
x = \frac{11}{4}
\]

\[
y = \frac{16}{(3 - 4(\frac{11}{4}))^2} = \frac{1}{4}
\]
6  (a) (i) Sketch the graph of \( y = 10^x \). [1]

(ii) Given that \( \frac{4^x}{2^{x+2}} = \frac{3}{5^x} \), find the value of \( 10^x \). [2]

**Solution**

(i) ![Graph of \( y = 10^x \)]

(ii) \[
\frac{4^x}{2^{x+2}} = \frac{3}{5^x} \\
\frac{2^x}{2^{x+2}} = \frac{3}{5^x} \\
2^{x-(x+2)}5^x = 3 \\
2^{x}5^x = 3 \\
\frac{2^x}{4} (5^x) = 3 \\
10^x = 12
\]

(b) Solve the equation \( \log_2 \sqrt{5x+1} + 2 \log_9 3 = \log_4 (2x - 3) + \log_3 27 \). [5]

**Solution**

\[
\log_2 \sqrt{5x+1} + 2 \log_9 3 = \log_4 (2x - 3) + \log_3 27 \\
\log_2 \sqrt{5x+1} = \log_4 (2x - 3) + 3 - 1 \\
\log_2 \sqrt{5x+1} = \frac{\log_2 (2x - 3)}{\log_2 2^2} + \log_2 2^2 \\
\log_2 \sqrt{5x+1} = \log_2 (2x - 3)^{\frac{1}{2}} + \log_2 4 \\
\sqrt{5x+1} = 4\sqrt{2x - 3} \\
5x + 1 = 16(2x - 3) \\
5x + 1 = 32x - 48 \\
27x = 49 \\
x = \frac{49}{27}
\]

or \( x = 1.81 \) (3sf)

[Turn over]
The population of a herd of deer can be modelled by the function
\[ D = 400 + 40\sin\left(\frac{\pi}{6} t\right), \]
where \( D \) is the deer population in week \( t \) of the year for \( 0 \leq t \leq 24 \).

Using the model,

(i) state the amplitude of the function, [1]
(ii) state the period of the function, [1]
(iii) find the maximum and minimum values of \( D \), [2]
(iv) sketch the function \( D = 400 + 40\sin\left(\frac{\pi}{6} t\right) \) for \( 0 \leq t \leq 24 \). [2]
(v) estimate the number of weeks for \( 0 \leq t \leq 24 \) that the population is greater than 420. [3]

<table>
<thead>
<tr>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) ( D = 400 + 40\sin\left(\frac{\pi}{6} t\right) ) for ( 0 \leq t \leq 24 )</td>
</tr>
<tr>
<td>Amplitude = 40</td>
</tr>
<tr>
<td>(ii) Period = ( \frac{2\pi}{\frac{\pi}{6}} = 12 )</td>
</tr>
<tr>
<td>(iii) Maximum ( D = 400 + 40 = 440 )</td>
</tr>
<tr>
<td>Minimum ( D = 400 - 40 = 360 )</td>
</tr>
<tr>
<td>(iv)</td>
</tr>
</tbody>
</table>

\[ 400 + 40\sin\left(\frac{\pi}{6} t\right) = 420 \]
\[40 \sin \left( \frac{\pi}{6} t \right) = 20\]
\[\sin \left( \frac{\pi}{6} t \right) = 0.5\]

Basic angle = \( \frac{\pi}{6} \)

\[
\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}
\]

\[t = 1, 5, 13, 17\]

No of weeks = (5-1) + (17-13) = 8
The diagram shows a quadrilateral $ABCD$ in which $A$ is $(6, 0)$, $C$ is $(3k, 5k + 10)$ and $D$ is $(-2, 4)$. The equation of line $AB$ is $y = 2x - 12$ and angle $ADC = 90^\circ$.

(i) Find the value of $k$.

<table>
<thead>
<tr>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gradient of line $AD = \frac{4 - 0}{-2 - 6} = \frac{-1}{2}$</td>
</tr>
<tr>
<td>Gradient of line $CD = 2$</td>
</tr>
<tr>
<td>$\frac{5k + 10 - 4}{3k + 2} = 2$</td>
</tr>
<tr>
<td>$5k + 6 = 6k + 4$</td>
</tr>
<tr>
<td>$k = 2$</td>
</tr>
</tbody>
</table>

Given that the perpendicular bisector of $CD$ passes through $B$, find
(ii) the coordinates of $B$.

**Solution**

Midpoint of line $CD = \left( \frac{6-2}{2}, \frac{24}{2} \right) = (2, 12)$

Gradient of perpendicular bisector of $CD = -\frac{1}{2}$

Equation of perpendicular bisector of $CD$:

$$y - 12 = -\frac{1}{2}(x - 2)$$

$$y = -\frac{1}{2}x + 13$$

To find intersection point between equation of line $AB$ with perpendicular bisector of $CD$: solve simultaneously

$$y = -\frac{1}{2}x + 13$$

$$y = 2x - 12$$

$$-\frac{1}{2}x + 13 = 2x - 12$$

$$2.5x = 25$$

$$x = 10,$$

$$y = 8$$

$B = (10, 8)$

(iii) the area of the quadrilateral $ABCD$.

**Solution**

Area of $ABCD$

$$\frac{1}{2} \left| 6 \quad 10 \quad 6 - 2 \quad 6 \right|$$

$$= \frac{1}{2} \left| 6 \quad 10 \quad 6 \quad 0 \right|$$

$$= \frac{1}{2} \left| 6 \quad 10 \quad 6 \quad 0 \right|$$

$$= \frac{1}{2} \left| 48 + 200 + 24 - (48 - 40 + 24) \right|$$

$$= 120 \text{ units}^2$$
(a) The first three terms in the binomial expansion of \((1 + px)^n\) are \(1 - 48x + 960x^2\).

Find the value of \(p\) and of \(n\).

**Solution**

\[
(1 + px)^n = \binom{n}{0} (px)^0 + \binom{n}{1} (px)^1 + \binom{n}{2} (px)^2 + \cdots
= 1 + npx + \frac{n(n-1)}{2} p^2 x^2 + \cdots
\]

Comparing coefficients of

\(x\) \quad \Rightarrow \quad np = -48

\(x^2\) \quad \Rightarrow \quad \frac{n(n-1)}{2} p^2 = 960

Solving by substitution: \(p = \frac{-48}{n}\)

\[
\frac{n(n-1)}{2} \left(\frac{-48}{n}\right)^2 = 960
\]

\[
n-1 = 5
\]

\[
n = 6
\]

\[
p = \frac{-48}{6} = -8
\]

(b) In the expansion of \(\left(2x^2 + \frac{a}{x}\right)^8\), where \(a\) is a non-zero real number, the ratio of the coefficient of the 3rd term to that of the 5th term is 5 : 2.

(i) Find the possible values of \(a\).

**Solution**

General Term, \(T_{r+1} = \binom{8}{r} (2x^2)^{8-r} \left(\frac{a}{x}\right)^r\)

\[
T_3 = \binom{8}{2} (2x^2)^6 \left(\frac{a}{x}\right)^2 = \binom{8}{2} (2)^6 (a)^2 (x)^2
\]

\[
T_5 = \binom{8}{4} (2x^2)^4 \left(\frac{a}{x}\right)^4 = \binom{8}{4} (2)^4 (a)^4 (x)^4
\]

\[
28(64)a^2 = 5
\]

\[
70(16)a^4 = \frac{5}{2}
\]

\[
3584a^2 = 5600a^4
\]

\[
5600a^4 - 3584a^2 = 0
\]

\[
a^2(5600a^2 - 3584) = 0
\]

\[
a = 0 \text{ (Rejected)} \quad \text{or} \quad a^2 = \frac{3584}{5600} \Rightarrow a = \pm \frac{4}{5}
\]
(ii) Explain whether the term independent of $x$ exists for the expansion of \( \left( 2x^2 + \frac{a}{x} \right)^8 \). [2]

**Solution**

For term independent of $x$, power of $x = 0$

Considering the terms in $x$ of the general term,
\[
(x^2)^{8-r} (x)^{-r} = x^{16-3r}
\]

Supposing $16 - 3r = 0$, $r = \frac{16}{3}$ (not a positive integer/whole number)

Term independent of $x$ does not exist.

10

The diagram shows part of the curve $y = |ax^2 + bx + c|$ where $a < 0$.

The curve touches the $x$-axis at $A (p, 0)$ and at $B (1, 0)$.

The curve touches the $y$-axis at $C (0, q)$ and has a maximum point at $T (-1, 8)$.

(i) Explain why $p = -3$. [1]

**Solution**

The curve is symmetrical about the line $x = -1$.

$x$-coord of $A = p = -1 - 2 = -3$
(ii) Determine the value of each of \( a, b, c \) and \( q \).

Solution

\[ y = |m(x + 3)(x - 1)| \]

At \( x = -1 \), \( y = 8 \)

\[ 8 = |m(2)(-2)| \]

\[ m = 2 \text{ or } -2 \]

For \( y = |ax^2 + bx + c| \) where \( a < 0 \), \( a = -2 \)

\[ y = | -2x^2 + bx + c | \]

\[ -2x^2 + bx + c \]

\[ = -2(x - 1)(x + 3) \]

\[ = -2x^2 + 2x - 6 \]

\[ b = -4, \ c = 6 \]

At \( x = 0 \), \( y = 6 \). Therefore \( q = 6 \).

(iii) State the set of values of \( r \) for which the line \( y = r \) intersects the curve \( y = |ax^2 + bx + c| \) at four distinct points.

Solution

\[ 0 < r < 8 \]

(iv) In the case where \( r = 2 \), find the exact \( x \)-coordinates of all points of intersection of the line \( y = r \) and the curve \( y = |ax^2 + bx + c| \).

Solution

Line: \( y = 2 \)

Curve: \( y = | -2x^2 - 4x + 6 | \)

\[ -2x^2 - 4x + 6 = 2 \quad \text{or} \quad 2x^2 + 4x - 8 = 0 \]

\[ x^2 + 2x - 2 = 0 \quad \text{or} \quad x^2 + 2x - 4 = 0 \]

\[ x = \frac{-2 \pm \sqrt{4(1)(-2)}}{2(1)} \quad \text{or} \quad x = \frac{-2 \pm \sqrt{4(1)(-4)}}{2(1)} \]

\[ x = \frac{-2 \pm \sqrt{12}}{2} \quad \text{or} \quad x = \frac{-2 \pm \sqrt{20}}{2} \]

\[ x = -1 \pm \sqrt{3} \quad x = -1 \pm \sqrt{5} \]
The diagram shows a circular garden. A farmer decides to fence part of the garden. He puts fences around the perimeter $ABCD$ such that $BC = 8$ m, $CD = 5$ m, angle $DAB = 90^0$ and angle $ABC = \theta$ where $0^0 < \theta < 90^0$.

(i) Given that $CM$ is perpendicular to $AB$, express $CM$ and $AB$ in terms of $\theta$.  

**Solution**

<table>
<thead>
<tr>
<th>Trigonometric Ratio</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin \theta$</td>
<td>$\frac{CM}{8}$</td>
</tr>
<tr>
<td>$CM$</td>
<td>$8 \sin \theta$</td>
</tr>
<tr>
<td>$\cos \theta$</td>
<td>$\frac{BM}{8}$</td>
</tr>
<tr>
<td>$BM$</td>
<td>$8 \cos \theta$</td>
</tr>
<tr>
<td>$\sin \theta$</td>
<td>$\frac{DP}{5}$</td>
</tr>
<tr>
<td>$DP$</td>
<td>$5 \sin \theta$</td>
</tr>
<tr>
<td>$AB$</td>
<td>$5 \sin \theta + 8 \cos \theta$</td>
</tr>
</tbody>
</table>
(ii) Show that $L$ m, the length of fencing needed for perimeter $ABCD$, is given by

$$L = 13 + 3\cos\theta + 13\sin\theta.$$  [2]

**Solution**

<table>
<thead>
<tr>
<th></th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\cos\theta = \frac{CP}{5}$</td>
<td></td>
</tr>
<tr>
<td>$CP = 5\cos\theta$</td>
<td></td>
</tr>
<tr>
<td>$MP = 8\sin\theta - 5\cos\theta = AD$</td>
<td></td>
</tr>
</tbody>
</table>

Perimeter $ABCD$

$$= 5\sin\theta + 8\cos\theta + 8 + 5 + 8\sin\theta - 5\cos\theta$$

$$= 13 + 3\cos\theta + 13\sin\theta$$

(iii) Express $L$ in the form $13 + R\cos(\theta - \alpha)$ where $R > 0$ and $\alpha$ is an acute angle.  [4]

**Solution**

$$L = 13 + \sqrt{3^2 + 13^2}\cos(\theta - \alpha)$$

$$= 13 + \sqrt{178}\cos(\theta - 77.0^\circ)$$

$tana = \frac{13}{3}$

$\alpha = 77.0^\circ$

(iv) Given that the farmer uses exactly 26.2 m of fencing, find the possible values of $\theta$.  [3]

**Solution**

$$13 + \sqrt{178}\cos(\theta - 77.0^\circ) = 26.2$$

$$\sqrt{178}\cos(\theta - 77.0^\circ) = 13.2$$

$$\cos(\theta - 77.0^\circ) = \frac{13.2}{\sqrt{178}}$$

Basic Angle = $8.4^\circ$

$\theta - 77.0^\circ = 8.4^\circ, -8.4^\circ$

$\theta = 85.4^\circ, 68.6^\circ$
12  (a) It is given that \( \int_0^\pi f(x) \, dx = k \cos 2x - \sin 3x + c \), where \( c \) is a constant of integration, and that \( \int_0^\pi f(x) \, dx = \frac{1}{3} \).

(i) Show that \( k = -2\frac{2}{3} \).

Solution

\[
[k\cos 2x - \sin 3x]_0^\pi = \frac{1}{3}
\]
\[k\cos \frac{\pi}{3} - \sin \frac{\pi}{2} - (k\cos 0) = \frac{1}{3}
\]
\[
k\left(-1 - k\right) = \frac{1}{3}
\]
\[
-k = \frac{4}{3}
\]
\[
k = -\frac{8}{3} = -2\frac{2}{3}
\]

(ii) Find \( f(x) \).

Solution

\[
f(x) = \frac{d}{dx} \left(-2\frac{2}{3} \cos 2x - \sin 3x\right)
\]
\[
= -2\frac{2}{3}(-2 \sin 2x) - 3 \cos 3x
\]
\[
= \frac{16}{3} \sin 2x - 3 \cos 3x
\]
(b) A curve has the equation $y = g(x)$, where $g(x) = 2\sin^2 x - \sin 2x$ for $0 \leq x \leq \pi$.

(i) Find the $x$-coordinates of the stationary points of the curve.

**Solutions**

\[
y = 2\sin^2 x - \sin 2x
\]

\[
\frac{dy}{dx} = 4\sin x \cos x - 2 \cos 2x = 0
\]

\[
2\sin 2x - 2 \cos 2x = 0
\]

\[
\sin 2x = \cos 2x
\]

\[
\tan 2x = 1
\]

Basic Angle $= \frac{\pi}{4}$

\[
2x = \frac{\pi}{4}, \frac{5\pi}{4}
\]

\[
x = \frac{\pi}{8}, \frac{5\pi}{8}
\]

(ii) Use the second derivative test to determine the nature of each of these points.

**Solution**

\[
\frac{d^2y}{dx^2} = 4 \cos 2x - 2(-2 \sin 2x)
\]

\[
= 4 \cos 2x + 4 \sin 2x
\]

At $x = \frac{\pi}{8}$,

\[
\frac{d^2y}{dx^2} = 4 \cos \frac{\pi}{4} + 4 \sin \frac{\pi}{4} > 0
\]

Minimum point at $x = \frac{\pi}{8}$.

At $x = \frac{5\pi}{8}$,

\[
\frac{d^2y}{dx^2} = 4 \cos \frac{10\pi}{8} + 4 \sin \frac{10\pi}{8} < 0
\]

Maximum point at $x = \frac{5\pi}{8}$. 
(iii) Given that \( \int g(x) \, dx = ax + b \sin x \cos x + \cos^2 x + k \), where \( k \) is a constant of integration, find the value of \( a \) and of \( b \).

**Solutions**

\[
\int 2\sin^2 x - \sin 2x \, dx \\
= \int 1 - \cos 2x - \sin 2x \, dx \\
= x - \frac{\sin 2x}{2} + \frac{\cos 2x}{2} + C \\
= x - \sin x \cos x + \frac{\cos^2 x}{2} + C \\
a = 1, \ b = -1
\]

END OF PAPER
READ THESE INSTRUCTIONS FIRST

Write your name, class, and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue, or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, staple all your work together with this cover sheet.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

FOR EXAMINER’S USE

<table>
<thead>
<tr>
<th>Q1</th>
<th>Q6</th>
<th>Q11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q2</td>
<td>Q7</td>
<td></td>
</tr>
<tr>
<td>Q3</td>
<td>Q8</td>
<td></td>
</tr>
<tr>
<td>Q4</td>
<td>Q9</td>
<td></td>
</tr>
<tr>
<td>Q5</td>
<td>Q10</td>
<td></td>
</tr>
</tbody>
</table>

80

This document consists of 5 printed pages.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where $n$ is a positive integer and

$$\binom{n}{r} = \frac{n!}{(n-r)! r!} = \frac{n(n-1) \ldots (n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\csc^2 A = 1 + \cot^2 A.$$
$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Delta = \frac{1}{2} bc \sin A$$
1 Express \( \frac{3x^3 + 2x^2 + 4x - 1}{x^3 + x^2} \) in partial fractions. \[4\]

2 A cylinder has a radius of \( (1 + 2\sqrt{2}) \) cm and its volume is \( \pi (84 + 21\sqrt{2}) \) cm\(^3\). Find, without using a calculator, the exact length of the height of the cylinder in the form \( (a + b\sqrt{2}) \) cm, where \( a \) and \( b \) are integers. \[5\]

3 (i) Sketch the graph of \( y = 4 - 3\sin 2x \) for \( 0 \leq x \leq \pi \). \[3\]
(ii) State the range of values of \( k \) for which \( 4 - 3\sin 2x = k \) has two roots for \( 0 \leq x \leq \pi \). \[2\]

4 Solutions to this question by accurate drawing will not be accepted.

\( PQRS \) is a parallelogram in which the coordinates of the points \( P \) and \( R \) are \((-5, 8)\) and \((6, -2)\) respectively. Given that \( PQ \) is perpendicular to the line \( y = -\frac{1}{2}x + 3 \) and \( QR \) is parallel to the \( x \) axis, find

(i) the coordinates of \( Q \) and of \( S \), \[5\]
(ii) the area of \( PQRS \). \[2\]

5 (i) Differentiate \( \frac{\ln x}{x} \) with respect to \( x \). \[3\]
(ii) Hence find \( \int \frac{\ln x^2}{x^2} \, dx \). \[4\]
6 (i) Show that \(\frac{2}{\tan \theta + \cot \theta} = \sin 2\theta\). [3]

(ii) Hence find the value of \(p\), giving your answer in terms of \(\pi\), for which
\[
\int_0^p \frac{4}{\tan 2x + \cot 2x} \, dx = \frac{1}{4}, \text{ where } 0 < p < \frac{\pi}{4}.
\] [4]

7

In the diagram \(XBY\) is a structure consisting of a beam \(XB\) of length 35 cm attached at \(B\) to another beam \(BY\) of length 80 cm so that angle \(XBY = 90^\circ\). Small rings at \(X\) and \(Y\) enable \(X\) to move along the vertical wire \(AP\) and \(Y\) to move along the vertical wire \(CQ\). There is another ring at \(B\) that allows \(B\) to move along the horizontal line \(AC\). Angle \(ABX = \theta\) and \(\theta\) can vary.

(i) Show that \(AC = (35\cos \theta + 80\sin \theta)\) cm. [2]

(ii) Express \(AC\) in the form of \(R\sin(\theta + \alpha)\), where \(R > 0\) and \(0^\circ < \alpha < 90^\circ\). [4]

(iii) Tom claims that the length of \(AC\) is 89 cm. Without measuring, Mary said that this was not possible. Explain how Mary came to this conclusion. [1]

8 (a) Find the range of values of \(p\) for which \(px^2 + 4x + p > 3\) for all real values of \(x\). [5]

(b) Find the range of values of \(k\) for which the line \(5y = k - x\) does not intersect the curve \(5x^2 + 5xy + 4 = 0\). [5]
9 The diagram shows part of the graph of \( y = 4 - |x + 1| \).

(i) Find the coordinates of the points \( A, B, C \) and \( D \). \[5\]

(ii) Find the number of solutions of the equation \( 4 - |x + 1| = mx + 3 \) when

(a) \( m = 2 \)

(b) \( m = -1 \) \[2\]

(iii) State the range of values of \( m \) for which the equation \( 4 - |x + 1| = mx + 3 \) has two solutions. \[1\]

10 The diagram shows a cone of radius \( r \) cm and height \( h \) cm. It is given that the volume of the cone is \( 10\pi \) cm\(^3\).

(i) Show that the curved surface area, \( A \) cm\(^2\), of the cone, is \( A = \frac{\pi \sqrt{r^6 + 900}}{r} \). \[3\]

(ii) Given that \( r \) can vary, find the value of \( r \) for which \( A \) has a stationary value. \[4\]

(iii) Determine whether this value of \( A \) is a maximum or a minimum. \[2\]

11 The equation of a curve is \( y = x(2 - x)^3 \).

(i) Find the range of values of \( x \) for which \( y \) is an increasing function. \[5\]

(ii) Find the coordinates of the stationary points of the curve. \[3\]

(iii) Hence, sketch the graph of \( y = x(2 - x)^3 \). \[3\]
St Nicholas Girls School Additional Mathematics Preliminary Examination Paper I 2018

Answers

Paper 1

1. \[ 3 + \frac{5}{x} - \frac{1}{x^2} - \frac{6}{x+1} \]

2. \((12 - 3\sqrt{2})\) cm

3 (i)

(ii) \(1 < k < 4\) or \(4 < k < 7\)

4 (i) \(Q(-10, -2), S(11, 8)\) (ii) 160 units²

5 (i) \(\frac{1-\ln x}{x^3}\) (ii) \(2\left(-\frac{1}{x} - \frac{\ln x}{x}\right) + c\)

6 (ii) \(\frac{\pi}{12}\)

7 (ii) \(5\sqrt{305}\sin(\theta + 23.6°)\) cm or \(87.3\sin(\theta + 23.6°)\) cm

(iii) The maximum value of \(AC=87.3\text{cm} < 89\text{ cm}\)

8 (a) \(p > 4\) (b) \(-8 < k < 8\)

9 (i) \(A(-5,0), B(-1,4), C(3,0), D(0,3)\) (ii) (a) 1 (b) infinite (iii) 2

10 (ii) 2.77 (iii) minimum

11 (i) \(x < \frac{1}{2}\) (ii) \((2,0)\left(\frac{1}{2}, \frac{27}{16}\right)\) (iii)
PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

ADDITIONAL MATHEMATICS

Paper 1

Marking Scheme

Thursday 16 August 2018

2 hours

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

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At the end of the examination, staple all your work together with this cover sheet.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

FOR EXAMINER’S USE

Q1  Q6  Q11

Q2  Q7

Q3  Q8

Q4  Q9

Q5  Q10

This document consists of 5 printed pages.

[Turn over
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial Theorem

\[
(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{(n-r)! r!} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!} \)

2. TRIGONOMETRY

Identities

\[
\sin^2 A + \cos^2 A = 1.
\]

\[
\sec^2 A = 1 + \tan^2 A.
\]

\[
\cosec^2 A = 1 + \cot^2 A.
\]

\[
\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
\sin 2A = 2 \sin A \cos A
\]

\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]

\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \DeltaABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2} bc \sin A
\]
1. Express \( \frac{3x^3 + 2x^2 + 4x - 1}{x^3 + x^2} \) in partial fractions. [4]

\[
x^3 + x^2 \quad \overline{\frac{3}{3x^3 + 2x^2 + 4x - 1}} \quad \frac{3x^3 + 3x^2}{-x^2 + 4x - 1}
\]

\[
\frac{3x^3 + 2x^2 + 4x - 1}{x^2 + x^3} = -3 + \frac{-x^2 + 4x - 1}{x^2(x + 1)}
\]

\[
\frac{-x^2 + 4x - 1}{x^2(x + 1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{c}{x + 1}
\]

Let \( x = -1 \) \( \Rightarrow -1 - 4 - 1 = c \)
\( c = -6 \)

Let \( x = 0 \) \( \Rightarrow B = -1 \)

\[
-3 + \frac{-x^2 + 4x - 1}{x^2(x + 1)} = \frac{Ax + B}{x^2} + \frac{c}{x + 1}
\]

\[
\frac{3x^3 + 2x^2 + 4x - 1}{x^2 + x^3} = 3 + \frac{5}{x} - \frac{1}{x^3} - \frac{6}{x + 1}
\]

[4]

**Max 3m**

**3m**

**2m**

If

- \( \frac{3x^3 + 2x^2 + 4x - 1}{x^2 + x^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{c}{x + 1} \)
- \( \frac{3x^3 + 2x^2 + 4x - 1}{x^2 + x^3} = 3 + \frac{Ax + B}{x^2} + \frac{c}{x + 1} \)
- \( \frac{3x^3 + 2x^2 + 4x - 1}{x^2 + x^3} = \frac{Ax + B}{x^2} + \frac{c}{x + 1} \)
2. A cylinder has a radius of \((1 + 2\sqrt{2})\) cm and its volume is \(\pi(84 + 21\sqrt{2})\) cm\(^3\). Find, without using a calculator, the exact length of the height of the cylinder in the form \((a + b\sqrt{2})\) cm, where \(a\) and \(b\) are integers. [5]

\[
\pi(84 + 21\sqrt{2}) = \pi(1 + 2\sqrt{2})^2 \times h
\]

\[
h = \frac{84 + 21\sqrt{2}}{(1 + 2\sqrt{2})^2}
\]

\[
h = \frac{84 + 21\sqrt{2}}{1 + 4\sqrt{2} + 8}
\]

\[
h = \frac{(84 + 21\sqrt{2})(4\sqrt{2} - 9)}{(4\sqrt{2} + 9)(4\sqrt{2} - 9)}
\]

\[
h = \frac{756 - 336\sqrt{2} + 189\sqrt{2} - 168}{81 - 32}
\]

\[
h = \frac{588 - 147\sqrt{2}}{49}
\]

\[
h = (12 - 3\sqrt{2}) \text{ cm}
\]

[5]

3. (i) Sketch the graph of \(y = 4 - 3\sin 2x\) for \(0 \leq x \leq \pi\). [3]

(ii) State the range of values of \(k\) for which \(4 - 3\sin 2x = k\) has two roots for \(0 \leq x \leq \pi\). [2]

3 (a)

B1 any one pt
B1 2nd pt
The 2 pts must be different nature
B1 perfect sine shape
-ve sine shape
1 cycle
Amplitude
shift +4 up
ignoring no labelling of axes

[3]

3 (b) \(1 < k < 4\) or \(4 < k < 7\) [2]

Alternative \(1 < k < 7, k \neq 4\) [5]
4 Solutions to this question by accurate drawing will not be accepted.

PQRS is a parallelogram in which the coordinates of the points P and R are (−5, 8) and (6, −2) respectively. Given that PQ is perpendicular to the line $y = -\frac{1}{2}x + 3$ and QR is parallel to the x axis, find

(i) the coordinates of Q and of S, [5]

(ii) the area of PQRS. [2]

1(i) Since QR parallel to the x axis, $y_Q = -2$.

Since PQ is perpendicular to the line $y = -\frac{1}{2}x + 3$,

gradient of $PQ = 2$

\[
\frac{(-2) - (8)}{x_Q - (-5)} = 2
\]

$-10 = 2x_Q + 10$

$x_Q = -10$

$Q(-10, -2)$

Midpoint of PR = Midpoint of QS or by inspection

\[
\left(\frac{-5 + 6}{2}, \frac{8 + (-2)}{2}\right) = \left(\frac{-10 + x_s}{2}, \frac{-2 + y_s}{2}\right)
\]

1 = $-10 + x_s$

$x_s = 11$

6 = $-2 + y_s$

$y_s = 8$

$S(11, 8)$ [5] B1

(ii) Area of PQRS

\[
= \frac{1}{2} \begin{vmatrix} -5 & -10 & 6 & 11 & -5 \\ 8 & -2 & -2 & 8 & 8 \end{vmatrix}
\]

\[
= \frac{1}{2} [(10 + 20 + 48 + 88) - (-80 - 12 - 22 - 40)]

= \frac{1}{2} |320|

= 160 \text{ units}^2 [7]

B1

√M1

A1 no unit overall -1m
5 (i) Differentiate \( \frac{\ln x}{x} \) with respect to \( x \). [3]

\[
\frac{d}{dx}\left( \frac{\ln x}{x} \right) = \frac{x \left( \frac{1}{x} \right) - \ln x}{x^2}
\]

\[
= \frac{1 - \ln x}{x^2}
\]

(ii) Hence find \( \int \frac{\ln x^2}{x^2} \, dx \). [4]

\[
\int 1 - \ln \frac{x}{x^2} \, dx = \ln \frac{x}{x}
\]

\[
\int \frac{1}{x^2} \, dx - \int \frac{\ln x}{x^2} \, dx = \frac{\ln x}{x}
\]

\[
\int x^{-2} \, dx - \frac{\ln x}{x} = \int \frac{\ln x}{x^2} \, dx
\]

\[
x^{-1} - \frac{\ln x}{x} = \int \frac{\ln x}{x^2} \, dx
\]

\[
\int \frac{\ln x}{x^2} \, dx = -\frac{1}{x} - \frac{\ln x}{x}
\]

\[
\int \frac{\ln x^2}{x^2} \, dx = 2 \int \frac{\ln x}{x^2} \, dx
\]

\[
= 2 \left( -\frac{1}{x} - \frac{\ln x}{x} \right) + c
\]

<table>
<thead>
<tr>
<th>Marking Scheme</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>Either ( \frac{du}{dx} ) or ( \frac{dv}{dx} ) with the use of quotient rule/product rule</td>
</tr>
<tr>
<td>+B1</td>
<td>perfect</td>
</tr>
<tr>
<td>A1</td>
<td>Integration is the reverse process of differentiation</td>
</tr>
<tr>
<td>M1</td>
<td>Making ( \int \frac{\ln x}{x^2} , dx ) the subject or split the expression</td>
</tr>
<tr>
<td>B1</td>
<td>Integration of ( x^{-2} )</td>
</tr>
<tr>
<td>A1</td>
<td>With ( c )</td>
</tr>
</tbody>
</table>
6 (i) Show that \( \frac{2}{\tan \theta + \cot \theta} = \sin 2\theta \). \[3\]

(ii) Hence find the value of \( p \), giving your answer in terms of \( \pi \), for which
\[
\int_0^p \frac{4}{\tan 2x + \cot 2x} \, dx = \frac{1}{4}, \text{ where } 0 < p < \frac{\pi}{4}.
\] \[4\]

\[(i)\] \[
\frac{2}{\tan \theta + \cot \theta} = 2 + \frac{\sin \theta + \cos \theta}{\cos \theta + \sin \theta} = 2 + \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} = 2 + \frac{1}{\cos \theta \sin \theta} = 2 \sin \theta \cos \theta = \sin 2\theta
\] \[3\]

B1 change to sin and cos
M1 combine terms
M1 for identity …to the end. (must show “1”)

\[(ii)\]  
\[
\int_0^p \frac{4}{\tan 2x + \cot 2x} \, dx = 2 \int_0^p \sin 4x \, dx = 2 \left[ -\frac{1}{4} \cos 4x \right]_0^p = \left( -\frac{1}{2} \cos 4p \right) - \left( -\frac{1}{2} \cos 0 \right) = -\frac{1}{2} \cos 4p + \frac{1}{2}
\]
\[
\int_0^p \frac{4}{\tan 2x + \cot 2x} \, dx = \frac{1}{4}
\]
\[
-\frac{1}{2} \cos 4p + \frac{1}{2} = \frac{1}{4}
\]
\[
-\frac{1}{2} \cos 4p = -\frac{1}{4}
\]
\[
\cos 4p = \frac{1}{2}
\]
\[
4p = \frac{\pi}{3}
\]
\[
p = \frac{\pi}{12}
\]

A1

[4] [7]
In the diagram $XY$ is a structure consisting of a beam $XB$ of length 35 cm attached at $B$ to another beam $BY$ of length 80 cm so that angle $XY = 90^\circ$. Small rings at $X$ and $Y$ enable $X$ to move along the vertical wire $AP$ and $Y$ to move along the vertical wire $CQ$. There is another ring at $B$ that allows $B$ to move along the horizontal line $AC$. Angle $ABX = \theta$ and $\theta$ can vary.

(i) Show that $AC = (35 \cos \theta + 80 \sin \theta)$ cm.

(ii) Express $AC$ in the form of $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$.

(iii) Tom claims that the length of $AC$ is 89 cm. Without measuring, Mary said that this was not possible. Explain how Mary came to this conclusion.
7 (i) \[ AB = 35 \cos \theta \]
\[ \angle YBC = 90^\circ - \theta \]
\[ \angle BYC = \theta \]
\[ BC = 80 \sin \theta \]
\[ AC = (35 \cos \theta + 80 \sin \theta) \text{ cm} \]

7 (ii) \[ R \sin(\theta + \alpha) = R \sin \theta \cos \alpha + R \cos \theta \sin \alpha \]
\[ AC = 35 \cos \theta + 80 \sin \theta \]
\[ R \sin \alpha = 35 \]
\[ R \cos \alpha = 80 \]
\[ R^2 \cos^2 \alpha + R^2 \sin^2 \alpha = 80^2 + 35^2 \]
\[ R = \sqrt{80^2 + 35^2} \]
\[ R^2 = 7625 \]
\[ R = 87.3 \text{ or } 5\sqrt{305} \]
\[ R \sin \alpha = \frac{35}{80} \]
\[ R \cos \alpha = \frac{80}{80} \]
\[ \tan \alpha = \frac{35}{80} \]
\[ \alpha = 23.6^\circ \]
\[ AC = 35 \cos \theta + 80 \sin \theta \]
\[ 35 \cos \theta + 80 \sin \theta = 5\sqrt{305} \sin(\theta + 23.6^\circ) \text{ cm} \]
\[ \text{or } 87.3 \sin(\theta + 23.6^\circ) \text{ cm} \]

7 (iii) The maximum value of \(AC\) is 87.3 cm

Therefore it is not possible for the length to be more than that.

Alternative
\[ 5\sqrt{305} \sin(\theta + 23.6^\circ) = 89 \]
\[ \sin(\theta + 23.6^\circ) = \frac{89}{5\sqrt{305}} \]

No Solution
Therefore it is not possible for the length to be more than that.
8 (a) Find the range of values of $p$ for which $px^2 + 4x + p > 3$ for all real values of $x$. \[5\]

(b) Find the range of values of $k$ for which the line $5y = k - x$ does not intersect the curve $5x^2 + 5xy + 4 = 0$. \[5\]

(a) \[
px^2 + 4x + p > 3 \text{ for all real values of } x \\
px^2 + 4x + p - 3 > 0 \text{ for all real values of } x,
\]

D<0 \[
4^2 - 4(p)(p - 3) < 0
\]

\[
16 - 4p^2 + 12p < 0 \\
4p^2 - 12p - 16 > 0 \\
p^2 - 3p - 4 > 0 \\
(p - 4)(p + 1) > 0 \\
p < -1, \ p > 4 \text{ NA}
\]

As $p > 0$

M1 D<0 with substitution

M1 For $b^2 - 4ac$

M1 For factorisation

DA1+DA1 Upon correct factorisation

Ignore “and” and no $p > 0$

(b) \[
5y = k - x \\
5x^2 + 5xy + 4 = 0 \\
5x^2 + 5x\left(\frac{k - x}{5}\right) + 4 = 0 \\
5x^2 + kx - x^2 + 4 = 0 \\
4x^2 + kx + 4 = 0 \\
k^2 - 4(4)(4) < 0
\]

\[
5(k - 5y)^2 + 5(k - 5y)y + 4 = 0 \\
5k^2 - 50ky + 125y^2 + 5ky - 25y^2 + 4 = 0 \\
100y^2 - 45ky + 5k^2 + 4 = 0 \\
(-45k)^2 - 400(5k^2 + 4) < 0 \\
2025k^2 - 2000k^2 - 1600 < 0 \\
k^2 - 64 < 0 \\
(k - 8)(k + 8) < 0
\]

\[
-8 < k < 8
\]

M1 For substitution

D<0 with substitution

For $b^2 - 4ac$

factorisation

M1 \[+M1\sqrt{\text{factorisation}}\]

[5] [10] DA1 Upon correct factorisation
The diagram shows part of the graph of \( y = 4 - |x + 1| \).

(i) Find the coordinates of the points \( A, B, C \) and \( D \). \[5\]

(ii) Find the number of solutions of the equation \( 4 - |x + 1| = mx + 3 \) when

(a) \( m = 2 \) \[2\]

(b) \( m = -1 \) \[2\]

(iii) State the range of values of \( m \) for which the equation \( 4 - |x + 1| = mx + 3 \) has two solutions. \[1\]
10 The diagram shows a cone of radius $r$ cm and height $h$ cm. It is given that the volume of the cone is $10\pi$ cm$^3$.

(i) Show that the curved surface area, $A$ cm$^2$, of the cone, is 

$$A = \frac{\pi \sqrt{r^6 + 900}}{r}$$

(ii) Given that $r$ can vary, find the value of $r$ for which $A$ has a stationary value. 

(iii) Determine whether this value of $A$ is a maximum or a minimum.
10(i) \[ \text{Volume} = \frac{1}{3} \pi r^2 h = 10\pi \]
\[
h = \frac{30}{r^2} \]
\[
l^2 = r^2 + h^2 \]
\[
= r^2 + \left( \frac{30}{r^2} \right)^2 \]
\[
l = \sqrt{r^2 + \frac{900}{r^4}} \]
\[
A = \pi rl = \pi r \sqrt{r^2 + \frac{900}{r^4}} \]
\[
A = \pi r \sqrt{\frac{(r^6 + 900)}{r^4}} \]
\[
A = \frac{\pi r \sqrt{(r^6 + 900)}}{r^2} \]
\[
A = \frac{\pi \sqrt{(r^6 + 900)}}{r} \]

(ii) \[ u = \pi \sqrt{r^6 + 900}, \quad v = r \]
\[
\frac{du}{dr} = \frac{1}{2} \times \pi \times (r^6 + 900)^{\frac{1}{2}} \times 6r^5 \]
\[
\frac{dv}{dr} = 1 \]
\[
\frac{dA}{dr} = 3\pi r^6 (r^6 + 900)^{\frac{1}{2}} \]

When \( \frac{dA}{dr} = 0 \)
\[
\frac{\pi (r^6 + 900)^{\frac{1}{2}} [3r^6 - r^6 - 900]}{r^2} = 0 \]
\[
\pi [3r^6 - r^6 - 900] \]
\[
r^2 (r^6 + 900)^{\frac{1}{2}} \]
\[
2r^6 - 900 = 0 \]
\[
r^6 = 450 \]
\[
r = 2.77 \]
(iii) 

<table>
<thead>
<tr>
<th>$r$</th>
<th>$r &lt; 2.768$</th>
<th>$r = 2.768$</th>
<th>$r &gt; 2.768$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{dA}{dr}$</td>
<td>-</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>Sketch</td>
<td>_</td>
<td>–</td>
<td>/</td>
</tr>
</tbody>
</table>

$A$ is a minimum when $r = 2.77$

M1

DA1

For subst with $+ r$

Upon correct $\frac{dA}{dr}$

[2]

[9]
11. The equation of a curve is \( y = x(2 - x)^3 \).

(i) Find the range of values of \( x \) for which \( y \) is an increasing function. \([5]\)

(ii) Find the coordinates of the stationary points of the curve. \([3]\)

(iii) Hence, sketch the graph of \( y = x(2 - x)^3 \). \([3]\)
READ THESE INSTRUCTIONS FIRST

Write your name, class, and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue, or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
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The total number of marks for this paper is 100.

FOR EXAMINER’S USE

<table>
<thead>
<tr>
<th>Q1</th>
<th>Q5</th>
<th>Q9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q2</td>
<td>Q6</td>
<td>Q10</td>
</tr>
<tr>
<td>Q3</td>
<td>Q7</td>
<td>Q11</td>
</tr>
<tr>
<td>Q4</td>
<td>Q8</td>
<td>Q12</td>
</tr>
</tbody>
</table>

This document consists of 5 printed pages.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial Theorem
\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]
where \( n \) is a positive integer and
\[
\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}
\]

2. TRIGONOMETRY

Identities
\[
\sin^2 A + \cos^2 A = 1.
\]
\[
\sec^2 A = 1 + \tan^2 A.
\]
\[
\cosec^2 A = 1 + \cot^2 A.
\]
\[
\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
\sin 2A = 2\sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A
\]
\[
\tan 2A = \frac{2\tan A}{1 - \tan^2 A}
\]

Formulae for \( \Delta ABC \)
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\Delta = \frac{1}{2}bc \sin A
\]
1 (i) On the same axes sketch the curves \( y^2 = 64x \) and \( y = -x^2 \). \([2]\)

(ii) Find the equation of the line passing through the points of intersection of the two curves. \([4]\)

2 The roots of the equation \( x^2 + 2x + p = 0 \), where \( p \) is a constant, are \( \alpha \) and \( \beta \).
The roots of the equation \( x^2 + qx + 27 = 0 \), where \( q \) is a constant, are \( \alpha^3 \) and \( \beta^3 \).
Find the value of \( p \) and of \( q \). \([6]\)

3 (a) Given that \( 3^{2x-2} \times 5^{-2x} = 27^x / 5^{x+1} \), evaluate the exact value of \( 15^x \). \([3]\)

(b) Given that \( \log_{27} y = 64 \log_{27} x \), express \( y \) in terms of \( x \). \([4]\)

4 (i) Write down, and simplify, the first three terms in the expansion of \( \left(1 - \frac{x^2}{2}\right)^n \), in ascending powers of \( x \), where \( n \) is a positive integer greater than 2. \([2]\)

(ii) The first three terms in the expansion, in ascending powers of \( x \), of \( (2 + 3x^3)(1 - \frac{x^2}{2})^n \) are \( 2 - px^2 + 2x^4 \), where \( p \) is an integer. Find the value of \( n \) and of \( p \). \([5]\)

5 In the figure, \( XYZ \) is a straight line that is tangent to the circle at \( X \).
\( XQ \) bisects \( \angle RXZ \) and cuts the circle at \( S \). \( RS \) produced meets \( XZ \) at \( Y \) and \( ZR = XR \).
Prove that

(a) \( SR = SX \), \([3]\)

(b) a circle can be drawn passing through \( Z \), \( Y \), \( S \) and \( Q \). \([4]\)
6 The expression \(3x^3 + ax^2 + bx + 4\), where \(a\) and \(b\) are constants, has a factor of \(x - 2\) and leaves a remainder of \(-9\) when divided by \(x + 1\).

(i) Find the value of \(a\) and of \(b\). [4]

(ii) Using the values of \(a\) and \(b\) found in part (i), solve the equation \(3x^3 + ax^2 + bx + 4 = 0\), expressing non-integer roots in the form \(\frac{c \pm \sqrt{d}}{3}\), where \(c\) and \(d\) are integers. [4]

7 (a) Prove that \(\sec \theta + 1 = \frac{\tan \theta \sin \theta}{1 - \cos \theta}\). [4]

(b) Hence or otherwise, solve \(\frac{\tan \theta \sin \theta}{1 - \cos \theta} = \frac{3}{4} \sec^2 \theta\) for \(0 \leq \theta \leq 2\pi\). [4]

8 The temperature, \(A \, ^\circ C\), of an object decreases with time, \(t\) hours. It is known that \(A\) and \(t\) can be modelled by the equation \(A = A_0 e^{-kt}\), where \(A_0\) and \(k\) are constants.

Measured values of \(A\) and \(t\) are given in the table below.

<table>
<thead>
<tr>
<th>(t) (hours)</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) ((^\circ)C)</td>
<td>49.1</td>
<td>40.2</td>
<td>32.9</td>
<td>26.9</td>
</tr>
</tbody>
</table>

(i) Plot \(\ln A\) against \(t\) for the given data and draw a straight line graph. [2]

(ii) Use your graph to estimate the value of \(A_0\) and of \(k\). [4]

(iii) Assuming that the model is still appropriate, estimate the number of hours for the temperature of the object to be halved. [2]

9 The curve \(y = f(x)\) passes through the point \((0, 3)\) and is such that \(f'(x) = \left(e^x + \frac{1}{e^x}\right)^2\).

(i) Find the equation of the curve. [4]

(ii) Find the value of \(x\) for which \(f''(x) = 3\). [4]
10 A circle has the equation \( x^2 + y^2 + 4x + 6y - 12 = 0 \).

(i) Find the coordinates of the centre of the circle and the radius of the circle. [3]

The highest point of the circle is A.

(ii) State the equation of the tangent to the circle at A. [1]

(iii) Determine whether the point \((0, -7)\) lies within the circle. [2]

The equation of a chord of the circle is \( y = 7x - 14 \).

(iv) Find the length of the chord. [5]

11

The diagram shows part of the curve of \( y = x^2 - 7x + 12 \) passing through the point B and meeting the x-axis at the point A.

(i) Find the gradient of the curve at A. [4]

The normal to the curve at A intersects the curve at B.

(ii) Find the coordinates of B. [4]

The line BC is perpendicular to the x-axis.

(iii) Find the area of the shaded region. [4]

12 A particle P moves in a straight line, so that, \( t \) seconds after passing through a fixed point O, its velocity, \( v \) m s\(^{-1}\), is given by \( v = \cos t - \sin 2t \), where \( 0 \leq t \leq \frac{\pi}{2} \). Find

(i) in terms of \( \pi \), the values of \( t \), when P is at instantaneous rest, [5]

(ii) the distance travelled by P from \( t = 0 \) to \( t = \frac{\pi}{2} \), [6]

(iii) an expression for the acceleration of P in terms of \( t \). [1]
St Nicholas Girls School Additional Mathematics Preliminary Examination Paper II 2018

Answers

1. (i) \( y = -4x \)

2. \( p = 3, \; q = -10 \)

3. (a) \( \frac{5}{9} \) \hspace{1cm} (b) \( y = x^9, \; y = x^{-8} \)

4. (i) \( 1 - n \left( \frac{x^2}{2} \right) + \frac{n(n-1)}{8} x^4 + \cdots \)

5. (i) \( a = -8, \; b = 2 \)

6. (i) \( n = 8, \; p = 5 \)

7. (b) \( \frac{\pi}{3}, \; \frac{5\pi}{3} \)

8. (ii) \( A_v = 59.7, \; k = 0.1 \)

9. (i) \( y = \frac{1}{2} e^{2x} + 2x - \frac{1}{2} e^{-2x} + 3 \)

10. (i) Centre = \((-2, \; -3)\), Radius = 5 units \hspace{1cm} (ii) \( y = 2 \)

11. (ii) \( B(5, 2) \)

12. (i) \( \frac{\pi}{2}, \; \frac{\pi}{6} \)

CHIJ SNGS Preliminary Examinations 2018 - Additional Mathematics 4047/02
PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

ADDITIONAL MATHEMATICS 4047/02

Paper 2
Marking Scheme

Friday 17 August 2018
2 hours 30 minutes

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class, and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue, or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, staple all your work together with this cover sheet.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

FOR EXAMINER’S USE

<table>
<thead>
<tr>
<th>Q1</th>
<th>Q5</th>
<th>Q9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q2</td>
<td>Q6</td>
<td>Q10</td>
</tr>
<tr>
<td>Q3</td>
<td>Q7</td>
<td>Q11</td>
</tr>
<tr>
<td>Q4</td>
<td>Q8</td>
<td>Q12</td>
</tr>
</tbody>
</table>

This document consists of 5 printed pages.
Formulae

1. ALGEBRA

Quadratic Equation
For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where $n$ is a positive integer and

$$\binom{n}{r} = \frac{n!}{(n-r)! r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$  
$$\sec^2 A = 1 + \tan^2 A.$$  
$$\csc^2 A = 1 + \cot^2 A.$$  
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$  
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$  
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$  
$$\sin 2A = 2 \sin A \cos A$$  
$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$  
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\Delta ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$
1 (i) On the same axes sketch the curves $y^2 = 64x$ and $y = -x^2$. 

(ii) Find the equation of the line passing through the points of intersection of the two curves. 

\[ y^2 = 64 \]

\[ y = -x^2 \]

Sub (2) into (1),

\[ x^4 = 64 \]

\[ x^4 - 64x = 0 \]

\[ x(x^3 - 64) = 0 \]

\[ x = 0 \text{ or } x^3 - 64 = 0 \]

\[ x = 4 \]

\[ y = -16 \]

\[ m = \frac{-16 - 0}{4 - 0} = -4 \]

\[ y = -4x \]
The roots of the equation \( x^2 + 2x + p = 0 \), where \( p \) is a constant, are \( \alpha \) and \( \beta \).

The roots of the equation \( x^2 + qx + 27 = 0 \), where \( q \) is a constant, are \( \alpha^3 \) and \( \beta^3 \).

Find the value of \( p \) and of \( q \). [6]

<table>
<thead>
<tr>
<th>2</th>
<th>( x^2 + 2x + p = 0 )</th>
<th>( x^2 + qx + 27 = 0 )</th>
<th>( \alpha + \beta = -2 )</th>
<th>( \alpha^3 + \beta^3 = -q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha \beta = p )</td>
<td>( \alpha^3 \beta^3 = 27 )</td>
<td>( \alpha \beta = 3 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p = 3 )</td>
<td></td>
<td></td>
<td>B1 For both sum of roots or first pair of sum &amp; product of roots</td>
<td></td>
</tr>
<tr>
<td>( (\alpha + \beta)(\alpha^2 - \alpha \beta + \beta^2) = -q ) or ( (\alpha + \beta)^3 - 3\alpha^2 \beta + 3\beta^2 \alpha = -q )</td>
<td></td>
<td></td>
<td>B1 For both product of roots or 2nd pair of product and sum of roots</td>
<td></td>
</tr>
<tr>
<td>( (\alpha + \beta)((\alpha + \beta)^2 - 2\alpha \beta - \alpha \beta) = -q ) or ( (\alpha + \beta)^3 - 3\alpha \beta(\alpha + \beta) = -q )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( (-2)[4 - 9] = -q ) or ( (-2)^3 - 3p(-2) = -q )</td>
<td></td>
<td></td>
<td>M1 or A1</td>
<td></td>
</tr>
<tr>
<td>( q = -10 )</td>
<td></td>
<td></td>
<td>[6]</td>
<td></td>
</tr>
</tbody>
</table>
3 (a) Given that $3^{2x-2} \times 5^{-2x} = 27^x + 5^{x+1}$, evaluate the exact value of $15^x$. [3]

(b) Given that $\log_x y = 64 \log_x x$, express $y$ in terms of $x$. [4]

(a) $3^{2x-2} \times 5^{-2x} = 27^x + 5^{x+1}$
Method (i)

\[
\begin{align*}
3^{2x-2} \times 5^{-2x} &= 3^{3x} \times 5^{-1-x} \\
3^{2x-2} &= \frac{5^{x-1}}{5^{2x}} \\
3^{2x-2-3x} &= 5^{-1-3x+2x} \\
3^{-x-2} &= 5^{-x+1} \\
3^{-x} \times 3^{-2} &= 5^x \times 5^{-1} \\
3^x \times 5^x &= 5^{-1} + 3^{-2} \\
15^x &= \frac{5^9}{9}
\end{align*}
\]

Method (ii)

\[
\begin{align*}
3^{2x} \times 3^{-2} \times 5^{-2x} &= 3^{3x} \times 5^{-x} \times 5^{-1} \\
3^x \times 5^{x} &= 5^{-1} + 3^{-2} \\
15^x &= \frac{5^9}{9}
\end{align*}
\]

(b) $\log_x y = 64 \log_x x$

\[
\begin{align*}
\log_x y &= \frac{64 \log_x x}{\log_x y} \\
(\log_x y)^2 &= 64 \\
\log_x y &= \pm 8 \\
y &= x^8, \quad y = x^{-8}
\end{align*}
\]

change of base

$\log_x y = \pm 8$

$y = x^8, \quad y = x^{-8}$

[7]
4  (i) Write down, and simplify, the first three terms in the expansion of \((1 - \frac{x^2}{2})^n\), in ascending powers of \(x\), where \(n\) is a positive integer greater than 2.  

(ii) The first three terms in the expansion, in ascending powers of \(x\), of \((2 + 3x^2)(1 - \frac{x^2}{2})^n\) are \(2 - px^2 + 2x^4\), where \(p\) is an integer. Find the value of \(n\) and of \(p\).  

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| (i) | \[
\left(1 - \frac{x^2}{2}\right)^n = 1 - n\left(\frac{x^2}{2}\right) + \frac{n(n-1)}{8}x^4 + \cdots \cdots \cdots
\]
|   | M1 | B1 |
| Or any two terms 1m, perfect 2m |
|   |   |   |
| (ii) | \[
(2 + 3x^2)(1 - \frac{x^2}{2})^n = (2 + 3x^2)(1 - \frac{nx^2}{2} + \frac{n(n-1)}{8}x^4 + \cdots \).
\]
|   | M1 | A1 |
|   | \[
= 2 - nx^2 + \frac{n(n-1)}{4}x^4 + 3x^2 - \frac{3n}{2}x^4 + \cdots \cdots \cdots
\]
|   | \[
= 2 - (n - 3)x^2 + \left(\frac{n^2 - 7n}{4}\right)x^4 + \cdots \cdots
\]
|   | M1 | M1 | DA1 |
|   | \[
= 2, \quad \frac{n^2 - 7n}{4} = 2
\]
|   | \[
\frac{n^2 - 7n}{4} - 8 = 0
\]
|   | \[
(n - 8)(n + 1) = 0
\]
|   | \[
n = 8, n = -1(NA)
\]
|   | \[
-n + 3 = -p
\]
|   | \[
-8 + 3 = -p
\]
|   | M1 | M1 | A1 |
|   | \[
p = 5
\]

\[\text{Turn over}\]
In the figure, $XYZ$ is a straight line that is tangent to the circle at $X$.  
$XQ$ bisects $\angle RXZ$ and cuts the circle at $S$.  $RS$ produced meets $XZ$ at $Y$ and $ZR = XR$.

Prove that

(a) $SR = SX$,  

(b) a circle can be drawn passing through $Z, Y, S$ and $Q$.  

(a) $\angle ZXQ = \angle SRX$ (Alternate Segment Theorem)  
$\angle ZXQ = \angle QXR$ (XQ is the angle bisector of $\angle RXZ$)  
$\angle QXR = \angle SRX$  
By base angles of isosceles triangles, $SR = SX$  

(b) Let $\angle QXR$ be $x$  
$\angle RSX = 180^\circ - 2x$ (Isosceles Triangle)  
$\angle YSQ = 180^\circ - 2x$ (Vertically Opposite Angles)  
$\angle RZX = \angle ZXR = 2x$ (Base angles of Isosceles Triangle)  
$\angle RZX + \angle YSQ = 180^\circ - 2x + 2x = 180^\circ$  
Since opposite angles are supplementary in cyclic quadrilaterals, a circle that passes through $Z, Y, S$ and $Q$ can be drawn

Alternative  
Similar but use of tangent secant theorem.
6. The expression \(3x^3 + ax^2 + bx + 4\), where \(a\) and \(b\) are constants, has a factor of \(x - 2\) and leaves a remainder of \(-9\) when divided by \(x + 1\).

(i) Find the value of \(a\) and of \(b\). \([4]\)

(ii) Using the values of \(a\) and \(b\) found in part (i), solve the equation \(3x^3 + ax^2 + bx + 4 = 0\), expressing non-integer roots in the form \(\frac{c \pm \sqrt{d}}{3}\), where \(c\) and \(d\) are integers. \([4]\)

\[
\begin{array}{|l|l|}
\hline
\text{(i)} & f(x) = 3x^3 + ax^2 + bx + 4 \\
& \text{x-2 is a factor } f(2) = 0 \\
& 3(8) + 4a + 2b + 4 = 0 \\
& 4a + 2b + 28 = 0 \\
& 2a + b + 14 = 0 \quad \text{--------(1)} \\
& f(-1) = -9 \\
& -3 + a - b + 4 = -9 \\
& a - b = -10 \quad \text{--------(2)} \\
& (1)+(2) \quad 3a = -24 \\
& a = -8 \\
& \text{Sub into (2)} \quad -8 - b = -10 \\
& b = 2 \\
\hline
\end{array}
\]

\[
\begin{array}{|l|l|}
\hline
\text{(ii)} & 3x^3 - 8x^2 + 2x + 4 = 0 \\
& (x - 2)(3x^2 - 2x - 2) = 0 \\
& x - 2 = 0 \quad 3x^2 - 2x - 2 = 0 \\
& x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 3 \times (-2)}}{2 \times 3} \\
& x = \frac{2 \pm \sqrt{28}}{6} \\
& x = \frac{2(1 \pm \sqrt{7})}{6} \\
& x = \frac{1 \pm \sqrt{7}}{3} \\
\hline
\end{array}
\]
7  (a) Prove that  \[ \sec \theta + 1 = \frac{\tan \theta \sin \theta}{1 - \cos \theta}. \]  \[ \text{[4]} \]

(b) Hence or otherwise, solve  \[ \frac{\tan \theta \sin \theta}{1 - \cos \theta} = \frac{3}{4} \sec^2 \theta \] for  \( 0 \leq \theta \leq 2\pi \).  \[ \text{[4]} \]

\begin{align*}
\text{(a)} & \quad \text{RHS} = \frac{\tan \theta \sin \theta}{1 - \cos \theta} \\
& \quad = \frac{\sin \theta}{\cos \theta} \frac{1}{1 - \cos \theta} \\
& \quad = \frac{1 - \cos^2 \theta}{(1 - \cos \theta) \cos \theta} \\
& \quad = \frac{(1 - \cos \theta)(1 + \cos \theta)}{(1 - \cos \theta)(1 + \cos \theta)} \\
& \quad = 1 + \cos \theta \\
& \quad = \frac{1}{\cos \theta} + 1 \\
& \quad = \sec \theta + 1 \\
& \quad \text{(B1) change tan} \\
& \quad \text{(B1) change sin}^2 \text{ to cos}^2 \\
& \quad \text{(B1) identity for a}^2 - b^2 \\
& \quad \text{(B1) split and bring to answer} \\
\end{align*}

\begin{align*}
\text{(b)} & \quad \frac{\tan \theta \sin \theta}{1 - \cos \theta} = \frac{3}{4} \sec^2 \theta \\
& \quad 1 + \sec \theta = \frac{3}{4} \sec^2 \theta \\
& \quad 3 \sec^2 \theta - 4 \sec \theta - 4 = 0 \\
& \quad (\sec \theta - 2)(3 \sec \theta + 2) = 0 \\
& \quad \sec \theta = 2 \quad \text{or} \quad \sec \theta = -\frac{2}{3} \\
& \quad \cos \theta = \frac{1}{2} \quad \text{or} \\
& \quad \theta = \frac{\pi}{3}, \frac{5\pi}{3} \quad \cos \theta = -\frac{3}{2} \text{ (No Solution)} \\
& \quad \text{(B1) substitution} \\
& \quad \text{(M1) factorization} \\
& \quad \text{(DA1+)} \text{ for change to cos & no soln} \\
& \quad \text{Upon correct factorisation} \\
\end{align*}
The temperature, \( A \degree C \), of an object decreases with time, \( t \) hours. It is known that \( A \) and \( t \) can be modelled by the equation \( A = A_0 e^{-kt} \), where \( A_0 \) and \( k \) are constants.

Measured values of \( A \) and \( t \) are given in the table below.

<table>
<thead>
<tr>
<th>( t ) (hours)</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A ) (( ^\circ)C)</td>
<td>49.1</td>
<td>40.2</td>
<td>32.9</td>
<td>26.9</td>
</tr>
</tbody>
</table>

(i) Plot \( \ln A \) against \( t \) for the given data and draw a straight line graph. 

(ii) Use your graph to estimate the value of \( A_0 \) and of \( k \).

(iii) Assuming that the model is still appropriate, estimate the number of hours for the temperature of the object to be halved.
\[ \ln A \]
9 The curve \( y = f(x) \) passes through the point \((0,3)\) and is such that \( f'(x) = \left( e^x + \frac{1}{e^x} \right)^2 \).

(i) Find the equation of the curve. [4]  

(ii) Find the value of \( x \) for which \( f''(x) = 3 \). [4]  

(i) \[
y = \int \left( e^x + \frac{1}{e^x} \right)^2 \, dx \\
= \int e^{2x} + 2 + e^{-2x} \, dx \\
= \frac{e^{2x}}{2} + 2x + \frac{e^{-2x}}{-2} + c \\
\text{at } (0,3), \quad 3 = \frac{1}{2} e^0 + 2(0) - \frac{1}{2} e^0 + c \\
c = 3 \\
y = \frac{1}{2} e^{2x} + 2x - \frac{1}{2} e^{-2x} + 3
\]

(ii) \[
f'(x) = e^{2x} + 2 + e^{-2x} \\
f''(x) = 2e^{2x} - 2e^{-2x} \\
\text{when } f''(x) = 3, \quad 2e^{2x} - 2e^{-2x} = 3 \\
\text{Let } e^{2x} = a, \quad 2a - \frac{2}{a} = 3 \\
\quad 2a^2 - 2 = 3a \\
\quad 2a^2 - 3a - 2 = 0 \\
\quad (2a + 1)(a - 2) = 0 \\
\quad a = -\frac{1}{2}, \quad a = 2 \\
\quad e^{2x} = -\frac{1}{2}, \quad e^{2x} = 2 \\
\text{no solution} \quad 2x = \ln 2 \\
x = \frac{1}{2} \ln 2 = \ln \sqrt{2} = 0.347
10 A circle has the equation \( x^2 + y^2 + 4x + 6y - 12 = 0 \).

(i) Find the coordinates of the centre of the circle and the radius of the circle. [3]

The highest point of the circle is \( A \).

(ii) State the equation of the tangent to the circle at \( A \). [1]

(iii) Determine whether the point \( (0, -7) \) lies within the circle. [2]

The equation of a chord of the circle is \( y = 7x - 14 \).

(iv) Find the length of the chord. [5]
### (i)

\[ x^2 + y^2 + 4x + 6y - 12 = 0 \]
\[ x^2 + y^2 + 2gx + 2fy + c = 0 \]
\[ 2g = 4 \quad 2f = 6 \]
\[ g = 2 \quad f = 3 \]

Centre = \((-2, -3)\)

Radius = \[\sqrt{g^2 + f^2 - c}\]
\[= \sqrt{(-2)^2 + (-3)^2 - (-12)}\]
\[= \sqrt{4 + 9 + 12}\]
\[= \sqrt{25}\]
\[= 5 \text{ units}\]

### (ii)

\[ y = 2 \quad (y = \text{their y coord of centre} + \text{radius}) \]

### (iii)

The distance of the point from the centre of the circle
\[= \sqrt{(0 - (-2))^2 + (-7 - (-3))^2}\]
\[= \sqrt{20} < \sqrt{25}\]

Since it is lesser than the radius of the circle, it lies within the circle.

### (iv)

\[ y = 7x - 14 \quad \text{(1)} \]
\[ x^2 + y^2 + 4x + 6y - 12 = 0 \quad \text{(2)} \]

Sub (1) into (2),
\[ x^2 + (7x - 14)^2 + 4x + 6(7x - 14) - 12 = 0 \]
\[ x^2 + 49x^2 - 196x + 196 + 4x + 42x - 84 - 12 = 0 \]
\[ 50x^2 - 150x + 100 = 0 \]
\[ x^2 - 3x + 2 = 0 \]
\[ (x - 1)(x - 2) = 0 \]
\[ x = 1 \quad \text{or} \quad x = 2 \]

Sub into (1),
\[ y = -7 \quad \text{or} \quad y = 0 \]

The length of the chord = \[\sqrt{(1 - 2)^2 + (-7 - 0)^2}\]
\[= \sqrt{50}\]
\[= 5\sqrt{2} \text{ units}\]
11

The diagram shows part of the curve of \( y = x^2 - 7x + 12 \) passing through the point \( B \) and meeting the \( x \)-axis at the point \( A \).

(i) Find the gradient of the curve at \( A \). \([4]\)

The normal to the curve at \( A \) intersects the curve at \( B \).

(ii) Find the coordinates of \( B \). \([4]\)

The line \( BC \) is perpendicular to the \( x \)-axis.

(iii) Find the area of the shaded region. \([4]\)
### (i)

\[ y = x^2 - 7x + 12 \]
\[ = (x - 3)(x - 4) \]

\[ \frac{dy}{dx} = 2x - 7 \]

*when* \( x = 3 \), \( \frac{dy}{dx} = 2(3) - 7 = -1 \)

- **M1**
- **B1**

### (ii)

\[ \perp m = 1 \]

sub \( m = 1 \) and \((3,0)\) into \( y = mx + c \)

\[ 0 = 1(3) + c \]
\[ c = -3 \]

equation of normal: \( y = x - 3 \)

\[ x^2 - 7x + 12 = x - 3 \quad \text{or} \quad (x - 3)(x - 4) = x - 3 \]
\[ x^2 - 8x + 15 = 0 \quad \text{or} \quad (x - 3)(x - 5) = 0 \]
\[ x = 3 \quad x = 5 \]

\( y = 2 \)

\( B(5,2) \)

- **M1**
- **A1**

### (iii)

\[ \text{Area} = \int_3^4 x^2 - 7x + 12 \, dx + \int_4^5 x^2 - 7x + 12 \, dx \]

\[ = \left[ \frac{x^3}{3} - \frac{7x^2}{2} + 12x \right]_3^4 + \left[ \frac{x^3}{3} - \frac{7x^2}{2} + 12x \right]_4^5 \]

\[ = \left( \frac{64}{3} - \frac{7(16)}{2} + 12(4) \right) - \left( \frac{27}{3} - \frac{7(9)}{2} + 12(3) \right) \]

\[ + \left( \frac{125}{3} - \frac{7(25)}{2} + 12(5) \right) - \left( \frac{64}{3} - \frac{7(16)}{2} + 12(4) \right) \]

\[ = \left( 13\frac{1}{3} - 13\frac{1}{2} \right) + 14\frac{1}{6} - 13\frac{1}{3} \]

\[ = \frac{1}{6} + \frac{5}{6} \]

\[ = 1 \text{sq unit} \]

- **M1**
- **B1**
- **M1**
- **A1**

[Turn over]
A particle $P$ moves in a straight line, so that, $t$ seconds after passing through a fixed point $O$, its velocity, $v$ m s$^{-1}$, is given by $v = \cos t - \sin 2t$, where $0 \leq t \leq \frac{\pi}{2}$. Find

(i) in terms of $\pi$, the values of $t$, when $P$ is at instantaneous rest, [5]

(ii) the distance travelled by $P$ from $t = 0$ to $t = \frac{\pi}{2}$, [6]

(iii) an expression for the acceleration of $P$ in terms of $t$. [1]
### (i) \( v = \cos t - \sin 2t \)

When \( v = 0 \), \( \cos t - \sin 2t = 0 \)

\[
\begin{align*}
\cos t - 2 \sin t \cos t &= 0 \\
\cos t (1 - 2 \sin t) &= 0 \\
\cos t &= 0 \quad \sin t = \frac{1}{2} \\
t &= \frac{\pi}{2} \quad t = \frac{\pi}{6}
\end{align*}
\]

\[\text{B1} \quad \text{B1} \quad \text{M1} \quad \text{A1+A1} \quad \text{For v=0 for double angle factorisation}\]

### (ii) \( s = \int \cos t - \sin 2t \, dt \)

\[
= \sin t + \frac{1}{2} \cos 2t + c
\]

When \( t = 0, s = 0 \) \( 0 = \sin 0 + \frac{1}{2} \cos 0 + c \) \( c = -\frac{1}{2} \)

\[s = \sin t + \frac{1}{2} \cos 2t - \frac{1}{2}\]

When \( t = \frac{\pi}{6}, s = \sin \frac{\pi}{6} + \frac{1}{2} \cos \frac{\pi}{3} - \frac{1}{2} \)

\[
= \frac{1}{2} + \frac{1}{2} \left( \frac{1}{2} \right) - \frac{1}{2} \\
= \frac{1}{4}
\]

When \( t = \frac{\pi}{2}, s = \sin \frac{\pi}{2} + \frac{1}{2} \cos \frac{\pi}{2} - \frac{1}{2} \)

\[
= 1 + \frac{1}{2} (-1) - \frac{1}{2} \\
= 0
\]

Distance travelled \( = 2 \left( \frac{1}{4} \right) \)

\[= \frac{1}{2} \text{m}\]

\[\text{B1} \quad \text{B1+B1} \quad \text{M1} \quad \text{M1} \quad \text{Sub either} \quad t = \frac{\pi}{6} \text{ or } t = \frac{\pi}{2} \]

### (iii) \( a = \frac{dv}{dt} = (-\sin t - 2 \cos 2t) \frac{m}{s^2} \)

\[\text{[1]} \quad \text{[12]}\]
1 Express \( \frac{8x^2 - 2x + 19}{(1-x)(4 + x^2)} \) in partial fractions. \[5\]

2 (i) On the same axes sketch the curves \( y = -\sqrt{x} \) and \( y = -\sqrt{32x^3} \). \[2\]

(ii) Find the \( x \)-coordinates of the points of intersection of the two curves. \[2\]

3 (a) Given that \( \theta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) \), express \( \theta \) in terms of \( \pi \).

Hence, find the exact value of \( \sin 2\theta + \tan \theta \). \[4\]

(b)

The figure shows part of the graph of \( y = a \tan (bx) \) and a point \( P\left(\frac{3\pi}{2}, -2\right) \) marked. Find the value of each of the constants \( a \) and \( b \). \[2\]

4 The equation of a curve is \( y = e^x + 2e^{-x} \).

(i) Find the coordinates of the stationary point of the curve, leaving your answer in exact form. \[4\]

(ii) Determine the nature of this point. \[2\]
5 (i) Sketch the graph of \( y = \left| 4 - \frac{x}{2} \right| - 1 \), indicating clearly the vertex and the intercepts on the coordinate axes. [3]

(ii) State the range of \( y \). [1]

(iii) Find the values of \( x \) for \( \left| 4 - \frac{x}{2} \right| - 1 = 6 \). [2]

(iv) The graph \( y = \left| 4 - \frac{x}{2} \right| - 1 \) is reflected in the \( y \)-axis. Write down the equation of the new graph. [1]

6 (a) Find the maximum and minimum values of \((1 - \cos A)^2 - 5\) and the corresponding value(s) of \( A \) where each occurs for \( 0^\circ \leq A \leq 360^\circ \). [4]

(b) \( A, B \) and \( C \) are angles of a triangle such that \( \cos A = -\frac{1}{\sqrt{5}} \) and \( \sin B = \frac{5}{13} \).

(i) State the range of values for \( A \). [1]

(ii) Find the exact value of \( \cos (A + B) \). Hence find the exact value of \( \cos C \). [4]

7 (a) (i) Show that \( \frac{d}{dx} \left( \frac{\ln x}{4x} \right) = \frac{1 - \ln x}{4x^2} \). [3]

(ii) Integrate \( \frac{\ln x}{x^2} \) with respect to \( x \). [4]

(b) Given that \( \int_1^5 f(x) \, dx = 8 \), find \( \int_1^2 f(x) \, dx - \int_5^2 [f(x) + 3x] \, dx \). [3]
8 (a) A curve $C$ is such that $\frac{dy}{dx} = 8 \cos 2x$ and $P \left( \frac{\pi}{4}, 2\sqrt{3} - 3 \right)$ is a point on $C$.

(i) The normal to the curve at $P$ crosses the $x$-axis at $Q$.

Find the coordinates of $Q$. [3]

(ii) Find the equation of $C$. [3]

(b) Given that $y = \sin 4x$, show that $\frac{d^2y}{dx^2} \times \frac{dy}{dx} = -32 \sin 8x$. [4]

9 (a) Find the range of values of $k$ for which $2x(2x + k) + 6 = 0$ has no real roots. [4]

(b) If $p$ and $q$ are roots of the equation $x^2 + 2x - 1 = 0$ and $p > q$, express $\frac{g}{p^2}$ in the form $a + b \sqrt{2}$, where $a$ and $b$ are integers. [5]

10 A hexagon $ABCDEF$ has a fixed perimeter of 210 cm. $BCD$ and $AFE$ are 2 equilateral triangles and $ABDE$ is a rectangle. The length of $BC$ is represented as $x$ cm.

(i) Express $AB$ in terms of $x$. [1]

(ii) Show that the area of the hexagon, $H$ is given by

$$H = \left( \frac{\sqrt{3}}{2} - 2 \right) x^2 + 105x.$$ [2]

(iii) Find the value of $x$ for which $H$ is a maximum. [4]
The diagram shows triangle $PQR$ in which the point $P$ is $(8, -8)$ and angle $PQR$ is $90^\circ$. The gradient of $PR$ is $\frac{-13}{8}$ and the equation of $QR$ produced is $y = 2x + 1$.

The line $PR$ makes an angle $\theta$ with $QR$ produced.

(i) Find the coordinates of $Q$. [4]

(ii) Find the value of $\theta$. [3]
### Answers

<p>| | |</p>
<table>
<thead>
<tr>
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</thead>
</table>
| **1** | \[
\frac{5 - 3x + 1}{1 - x - 4 + x^2}
\] |
| **2(i)** | ![Graph of y = -\sqrt{32x^3}
and y = -\sqrt{x}]

<p>| <strong>2(ii)</strong> | (x = 0) or (\frac{1}{2}) |
| <strong>3(a)</strong> | (\theta = -\frac{\pi}{3}) (2\sin \theta \cos \theta + \tan \theta = -\frac{\sqrt{3}}{2}) |
| <strong>3(b)</strong> | (a = 2; \ b = \frac{1}{2}) |
| <strong>4(i)</strong> | ((\ln \sqrt{2}, 2\sqrt{2})) (ii) Minimum point |
| <strong>5(i)</strong> | ![Graph of (y = \ln x) and (y = \frac{1}{x^2} + \frac{1}{2})] |
| <strong>5(ii)</strong> | (y \geq -1) (iii) (x = -6) or (22) |
| <strong>5(iv)</strong> | (y = \left| 4 + \frac{x}{2}\right| - 1) |
| <strong>6(a)</strong> | Max value = (-1) when (A = 180^\circ) Min value = (-5) when (A = 0^\circ, 360^\circ) |
| <strong>6(b)(i)</strong> | (90^\circ &lt; A &lt; 180^\circ) or (\frac{\pi}{2} &lt; A &lt; \pi) |
| <strong>6(b)(ii)</strong> | (\cos(A + B) = -\frac{22}{13\sqrt{5}}) (\cos C = \frac{22}{13\sqrt{5}}) |
| <strong>7(a)(ii)</strong> | (\int \frac{\ln x}{x^2} , dx = -\frac{\ln x}{x} + c) (b) (39\frac{1}{2}) |
| <strong>8(a)(i)</strong> | (Q(12 - 8\sqrt{3} + \frac{\pi}{3}, 0)) or ((-0.809, 0)) |
| <strong>8(ii)</strong> | (y = 4\sin 2x - 3) |
| <strong>9(a)</strong> | (-\sqrt{24} &lt; k &lt; \sqrt{24}) |
| <strong>9(b)</strong> | (p = -1 + \sqrt{2}), (q = -1 - \sqrt{2}) (\frac{q}{p^2} = -7 - 5\sqrt{2}) |
| <strong>10(i)</strong> | (AB = 105 - 2x) |
| <strong>10(iii)</strong> | (x = 46.3) Maximum (H) |
| <strong>11(i)</strong> | (Q(-2, -3)) (ii) (\theta = 121.8^\circ) |</p>
<table>
<thead>
<tr>
<th>Qn</th>
<th>Working</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$8x^2 - 2x + 19 = \frac{A}{1-x} + \frac{Bx + C}{4 + x^2}$</td>
<td>B1 correct PF</td>
</tr>
<tr>
<td></td>
<td>$8x^2 - 2x + 19 = A(4 + x^2) + (Bx + C)(1 - x)$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Sub $x = 1$, $8 - 2 + 19 = 5A$ $A = 5$</td>
<td>A2 For all 3 correct</td>
</tr>
<tr>
<td></td>
<td>Sub $x = 0$, $19 = 4(5) + C$ $C = -1$</td>
<td>A1 For 2 correct</td>
</tr>
<tr>
<td></td>
<td>Compare coeff of $x^2$, $8 = A - B$ $B = -3$</td>
<td>√ A1 Only if B1 awarded</td>
</tr>
<tr>
<td></td>
<td>$8x^2 - 2x + 19 = \frac{5}{1-x} - \frac{3x + 1}{4 + x^2}$</td>
<td></td>
</tr>
<tr>
<td>2(i)</td>
<td>$y = -\sqrt{32}x^3$</td>
<td>G1</td>
</tr>
<tr>
<td></td>
<td>$y = -\sqrt{x}$</td>
<td>G1</td>
</tr>
<tr>
<td>2(ii)</td>
<td>$x^{\frac{3}{2}} = \sqrt{32}x^3$ $x = 32x^6$ $x(1 - 32x^6) = 0$ $x = 0$ or $\frac{1}{2}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$x = 0$ or $\frac{1}{2}$</td>
<td>A1</td>
</tr>
<tr>
<td>3(a)</td>
<td>$\theta = -\frac{\pi}{3}$ $2\sin \theta \cos \theta + \tan \theta = 2\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) + \left(-\sqrt{3}\right)$ $= -\frac{3}{2}\sqrt{3}$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$\theta = -\frac{\pi}{3}$</td>
<td>B1 value of $\cos \theta$</td>
</tr>
<tr>
<td></td>
<td>$2\sin \theta \cos \theta + \tan \theta = 2\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) + \left(-\sqrt{3}\right)$</td>
<td>B1 value of $\tan \theta$</td>
</tr>
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<td></td>
<td>$\frac{3}{2}\sqrt{3}$</td>
<td>B1</td>
</tr>
<tr>
<td>3(b)</td>
<td>$a = 2$ Period $= 2\pi = \frac{\pi}{b}$ $b = \frac{1}{2}$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$a = 2$</td>
<td>B1</td>
</tr>
<tr>
<td>4(i)</td>
<td>$\frac{dy}{dx} = e^x - 2e^{-x} = 0$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\frac{d^2y}{dx^2} = e^x = 2$</td>
<td>B1 differentiating $\frac{dy}{dx}$</td>
</tr>
<tr>
<td></td>
<td>$x = \ln \sqrt{2}$</td>
<td>A1 value of $x$</td>
</tr>
<tr>
<td></td>
<td>$y = e^{\ln \sqrt{2}} + 2e^{-\ln \sqrt{2}}$</td>
<td>B1 o.e.</td>
</tr>
<tr>
<td></td>
<td>$= \sqrt{2} + \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 2\sqrt{2}$ Point is $\left(\ln \sqrt{2}, 2\sqrt{2}\right)$</td>
<td></td>
</tr>
<tr>
<td>4(ii)</td>
<td>$\frac{d^2y}{dx^2} = e^x + 2e^{-x}$</td>
<td>M1 Knowing test</td>
</tr>
<tr>
<td></td>
<td>$x = \ln \sqrt{2}, \frac{d^2y}{dx^2} = 2 + \frac{2}{\sqrt{2}} &gt; 0$</td>
<td>Correct conclusion based on test</td>
</tr>
<tr>
<td></td>
<td>Minimum point</td>
<td>$\sqrt{A1}$</td>
</tr>
</tbody>
</table>
### 2018 Add Math Prelim Paper 1 Mark Scheme

<table>
<thead>
<tr>
<th>Qn</th>
<th>Working</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>5(i)</td>
<td><img src="image" alt="Graph" /> G1 vertex G1 x ints G1 y int</td>
<td></td>
</tr>
<tr>
<td>5(ii)</td>
<td>( y \geq -1 )</td>
<td>B1</td>
</tr>
</tbody>
</table>
| 5(iii) | \[
\begin{align*}
4 - \frac{x}{2} - 1 &= 6 \\
4 - \frac{x}{2} &= 7 \\
x &= -6 \text{ or } 22
\end{align*}
\]  | M1 or by counting A1 |
| 5(iv) | \( y = \left| 4 + \frac{x}{2} \right| - 1 \) | B1 |

**Total 7 marks**

| 6(a) | \((1 - \cos A)^2 - 5\)  
Max value = \((1-(-1))^2 - 5 = -1\)  
When \( \cos A = -1, A = 180^\circ \) | B1 |
|      | Min value = \((1-1)^2 - 5 = -5\)  
When \( \cos A = 1, A = 0^\circ, 360^\circ \) | B1 |
|      | \((1 - \cos A)^2 - 5\) | B1 |

**Total 9 marks**

| 6(b)(i) | \(90^\circ < A < 180^\circ \) or \( \frac{\pi}{2} < A < \pi \) | B1 |
| 6(b)(ii) | \[
\begin{align*}
\cos (A + B) &= \cos A \cos B - \sin A \sin B \\
&= -\frac{1}{\sqrt{2}} \left( \frac{12}{13} \right) - \frac{2}{\sqrt{5}} \left( \frac{5}{13} \right) \\
&= -\frac{13\sqrt{5}}{22} \\
\cos C &= \cos (180^\circ - (A + B)) \\
&= -\cos (A + B) \\
&= \frac{22}{13\sqrt{5}}
\end{align*}
\]  | B1 value of \( \cos B \)  
B1 value of \( \sin A \)  
\( \sqrt{B1} e \) |

**Total 9 marks**
<table>
<thead>
<tr>
<th>Qn</th>
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<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>7(a)(i)</td>
<td>( \frac{d}{dx} \left( \frac{\ln x}{4x} \right) = \frac{4x \left( \frac{1}{x} \right) - 4 \ln x}{(4x)^2} = \frac{4 - 4\ln x}{16x^2} = \frac{1 - \ln x}{4x^2} ) (shown)</td>
<td>M1 quotient rule M1 diff ( \ln x ) B1 working seen</td>
</tr>
<tr>
<td>7(a)(ii)</td>
<td>( \int \frac{1 - \ln x}{4x^2} , dx = \frac{\ln x}{4x} + c_1 )</td>
<td>B1 use integ( n ) as reverse of diff Ignore if +c is missing</td>
</tr>
<tr>
<td>7(b)</td>
<td>( \int_1^2 f(x) , dx + \int_2^5 [f(x) + 3x] , dx )</td>
<td>M1 switch limits and –ve becomes +ve B1 correct integral</td>
</tr>
<tr>
<td>8(a)(i)</td>
<td>When ( x = \frac{\pi}{3}, \frac{dy}{dx} = 8\cos^2 \frac{2\pi}{3} = -4 )</td>
<td>B1 M1</td>
</tr>
<tr>
<td>8(ii)</td>
<td>( y = 4\sin 2x + c ) ( \sub \left( \frac{\pi}{3}, 2\sqrt{3} - 3 \right) ) ( 2\sqrt{3} - 3 = 4\sin \frac{2\pi}{3} + c ) ( 2\sqrt{3} - 3 = 4 \left( \frac{\sqrt{3}}{2} \right) + c )</td>
<td>B1 ignore if +c missing M1 A1</td>
</tr>
<tr>
<td>8(iii)</td>
<td>( \frac{dy}{dx} = 4 \cos 4x ) ( \frac{d^2y}{dx^2} = -16 \sin 4x )</td>
<td>B1 ( \frac{d}{dx} \sin x = \cos x ) B1 ( \frac{d}{dx} \cos x = -\cos x )</td>
</tr>
<tr>
<td></td>
<td>( \frac{d^2y}{dx^2} \times \frac{dy}{dx} = (-16 \sin 4x) (4 \cos 4x) = -32(2 \sin 4x \cos 4x) = -32 \sin 8x )</td>
<td>B1 use of chain rule B1 2sin4xcos4x seen</td>
</tr>
<tr>
<td>Total</td>
<td>10 marks</td>
<td></td>
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</tbody>
</table>


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<thead>
<tr>
<th>Qn</th>
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<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>9(a)</td>
<td>[2x(2x + k) + 6 = 0] [4x^2 + 2kx + 6 = 0]&lt;br&gt;Discriminant &lt; 0&lt;br&gt;[(2k)^2 - 4(4)(6) &lt; 0]&lt;br&gt;[k^2 - 24 &lt; 0] [(k - \sqrt{24})(k + \sqrt{24}) &lt; 0]&lt;br&gt;[-\sqrt{24} &lt; k &lt; \sqrt{24}]</td>
<td>B1 For D &lt; 0&lt;br&gt;M1 correct sub&lt;br&gt;M1 Solve ineq&lt;br&gt;A1 (M0 if (k &lt; \pm \sqrt{24}))</td>
</tr>
<tr>
<td>9(b)</td>
<td>[x^2 + 2x - 1 = 0] [x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2}]&lt;br&gt;[p = -1 + \sqrt{2}, \ q = -1 - \sqrt{2}]&lt;br&gt;[q^2 = \frac{-1 - \sqrt{2}}{(1 + \sqrt{2})^2}]&lt;br&gt;[= \frac{-1 - \sqrt{2}}{3 - 2\sqrt{2}} \times \frac{3 + 2\sqrt{2}}{3 + 2\sqrt{2}}]&lt;br&gt;[= \frac{-3 - 2(2) - 3\sqrt{2} - 2\sqrt{2}}{9 - 4(2)}]&lt;br&gt;[= -7 - 5\sqrt{2}]</td>
<td>M1&lt;br&gt;A1 p &gt; q&lt;br&gt;M1 rationalise&lt;br&gt;M1 simplify&lt;br&gt;A1</td>
</tr>
<tr>
<td>10(i)</td>
<td>[4x + 2(AB) = 210] [AB = 105 - 2x]</td>
<td>B1&lt;br&gt;B1 Area of Δ&lt;br&gt;B1 sub &amp; working</td>
</tr>
<tr>
<td>10(ii)</td>
<td>[H = 2 \left(\frac{1}{2}\right) x^2 \sin 60 + (105 - 2x)x]&lt;br&gt;[= \frac{\sqrt{3}}{2} x^2 + 105x - 2x^2]&lt;br&gt;[= \left(\frac{\sqrt{3}}{2} - 2\right) x^2 + 105x] (shown)</td>
<td>B1&lt;br&gt;B1 test &amp; concl</td>
</tr>
<tr>
<td>10(iii)</td>
<td>[\frac{dH}{dx} = 2 \left(\frac{\sqrt{3}}{2} - 2\right) x + 105]&lt;br&gt;[\frac{dx}{dv} = 0]&lt;br&gt;[x = 46.3]&lt;br&gt;[\frac{d^2H}{dx^2} = \sqrt{3} - 4 &lt; 0] Maximum H</td>
<td>B1&lt;br&gt;M1&lt;br&gt;A1&lt;br&gt;B1 test &amp; concl</td>
</tr>
<tr>
<td>11(i)</td>
<td>Eqn of PQ: [y - (-8) = -\frac{1}{2}(x - 8)]&lt;br&gt;[y = -\frac{1}{2}x - 4] [--------(1)]&lt;br&gt;[QR:] [y = 2x + 1] [--------(2)]&lt;br&gt;Solving simultaneously&lt;br&gt;Q (-2, -3)</td>
<td>B1 correct m_PQ&lt;br&gt;B1 form eqn&lt;br&gt;M1&lt;br&gt;A1</td>
</tr>
<tr>
<td>11(ii)</td>
<td>[\tan \alpha = 2]&lt;br&gt;[\alpha = 63.43^\circ]&lt;br&gt;[\tan \beta = \frac{13}{8}]&lt;br&gt;[\beta = 58.39^\circ]&lt;br&gt;[\theta = 63.43^\circ + 58.39^\circ] (ext (\angle) of Δ)&lt;br&gt;[= 121.8^\circ]</td>
<td>M1 use grads to&lt;br&gt;Find angles&lt;br&gt;M1 manipulate (\angle)s&lt;br&gt;A1</td>
</tr>
</tbody>
</table>

Total 9 marks

Total 7 marks

Total 7 marks
READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.
2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the equation $ax^2 + bx + c = 0$, 
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$ 

Binomial Theorem
$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$
where $n$ is a positive integer and $$\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}.$$

2. TRIGONOMETRY

Identities
$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\cosec^2 A = 1 + \cot^2 A$$
$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\Delta ABC$
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$
Answer all questions.

1. The amount of energy, $E$ erg, generated in an earthquake is given by the equation $E = 10^a + bM$, where $a$ and $b$ are constants and $M$ is the magnitude of the earthquake.

The table below shows some corresponding values of $M$ and $E$.

<table>
<thead>
<tr>
<th>$M$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ (erg)</td>
<td>$2.0 \times 10^{13}$</td>
<td>$6.3 \times 10^{14}$</td>
<td>$2.0 \times 10^{16}$</td>
<td>$6.3 \times 10^{17}$</td>
<td>$2.0 \times 10^{19}$</td>
</tr>
</tbody>
</table>

(i) Plot $\log E$ against $M$. [2]

(ii) Using your graph, find an estimate for the value of $a$ and of $b$. [3]

(iii) Using your answers from (ii), find the amount of energy generated, in erg, by an earthquake of magnitude 7. [2]

2. (i) Write down the expansion of $(3 - x)^3$ in ascending powers of $x$. [1]

(ii) Expand $(3 + 2x)^8$, in ascending powers of $x$, up to the term in $x^3$. [3]

(iii) Write down the expansion of $(3 - x)^3 (3 + 2x)^8$ in ascending powers of $x$, up to $x^2$. [2]

(iv) By letting $x = 0.01$ and your expansion in (iii), find the value of $2.99^3 \times 3.02^8$, giving your answer correct to 3 significant figures. Show your workings clearly. [2]

(v) Explain clearly why the expansion in (iii) is not suitable for finding the value of $2^3 \times 5^8$. [2]

3. (i) By writing $3 \theta$ as $(2 \theta + \theta)$, show that $\sin(3 \theta) = 3 \sin \theta - 4 \sin^3 \theta$. [3]

(ii) Solve $\sin(3 \theta) = 3 \sin \theta \cos \theta$ for $0^\circ < \theta < 360^\circ$. [5]

4. The equation $x^2 + bx + c = 0$ has roots $\alpha$ and $\beta$, where $b > 0$.

(i) Write down, in terms of $b$ and/or $c$, the value of $\alpha + \beta$ and of $\alpha \beta$. [1]

(ii) Find a quadratic equation with roots $\alpha^2$ and $\beta^2$, in terms of $a$ and $b$. [3]

(iii) Find the relation between $b$ and $c$ for which the equation found in (ii) has two distinct roots. [2]

(iv) Give an example of values of $b$ and $c$ which satisfy the relation found in (iii). [1]
In the diagram, $A$, $B$, $C$ and $D$ are points on the circle centre $O$. $AP$ and $BP$ are tangents to the circle at $A$ and $B$ respectively. $DQ$ and $CQ$ are tangents to the circle at $D$ and $C$ respectively. $POQ$ is a straight line.

(i) Prove that angle $COD = 2 \times$ angle $CDQ$. [3]

(ii) Make a similar deduction about angle $AOB$. [1]

(iii) Prove that $2 \times$ angle $OAD = \text{angle } CDQ + \text{angle } BAP$. [4]

(i) Differentiate $y = 2e^{3x}(1 - 2x)$ with respect to $x$. [3]

(ii) Find the range of values of $x$ for which $y$ is decreasing. [1]

(iii) Given that $x$ is decreasing at a rate of 5 units per second, find the rate of change of $y$ at the instant when $x = -1.5$. [3]

(i) By using an appropriate substitution, express $2^{3a+1} - 2^{2a+2} + 2^a$ as a cubic function. [3]

(ii) Solve the equation $2^{3a+1} - 2^{2a+2} + 2^a = 0$. [5]

(iii) Find the range of values of $k$ for which $2^{3a+1} - 2^{2a+2} + k(2^a) = 0$ has at least one real solution. [3]
8 The diagram shows the graphs of \( y = f(x) \) and \( y = f'(x) \).

The function \( f(x) = ax^3 + bx^2 + 24x + 16 \) has stationary points at \( x = p \) and \( x = 4 \).

(i) Find an expression for \( f'(x) \), in terms of \( a \) and \( b \). [1]

(ii) Find the value of \( a \) and of \( b \). [3]

(iii) Find the value of \( p \).
State the range of values of \( k \), where \( k > 0 \) and \( y = f(x) - k \) has only one real root. [3]

(iv) Find the minimum value of the gradient of \( f(x) \). [2]

9 The diagram shows the graph of
\[
y = -\frac{1}{2} (x - 2)^4 + 16.
\]

\( AB \) and \( AC \) are tangents to the curve at \( B \) and \( C \) respectively.

\( B \) lies on the \( y \)-axis and \( AB = AC \).

(i) Find the gradient function of the curve. [1]

(ii) Find the equation of the tangent at \( B \).
Hence, state the coordinates of \( A \). [3]

(iii) Find the area of the shaded region. [6]
10 A particle, \( P \), travels along a straight line so that, \( t \) seconds after passing a fixed point \( O \), its velocity, \( v \) m/s is given by
\[
v = (12e^{kt} + 18),
\]
where \( k \) is a constant.

(i) Find the initial velocity of the particle. [1]

Two seconds later, its velocity is 40 m/s.

(ii) Show that \( k = 0.3031 \), correct to 4 significant figures. [3]

(iii) Sketch the graph of \( v = 12e^{kt} + 18 \), for \( 0 \leq t \leq 4 \). [3]

(iv) Explain why the distance travelled by \( P \) during the 4 seconds does not exceed 180 metres. [2]

(v) Find the maximum acceleration of \( P \) during the interval \( 0 \leq t \leq 4 \). [2]

11 A circle, \( C_1 \), with centre \( A \), has equation \( x^2 + y^2 - 8x - 4y - 5 = 0 \).

(i) Find the coordinates of \( A \) and the radius of \( C_1 \). [3]

(ii) Show that \((1, 6)\) lies on the circle. [1]

(iii) Find the equation of the tangent to the circle at \((1, 6)\). [3]

The equation of the tangent to the circle at \((1, 6)\) cuts the \( x \)-axis at \( B \).

(iv) Find the coordinates of \( B \). [2]

Another circle, \( C_2 \), has centre at \( B \) and radius \( r \).

(v) Find the exact value of \( r \) given that circle \( C_2 \) touches circle \( C_1 \). [3]
Answers:

1. (i) $a = 11.7$ to $11.9$, $b = 1.49$ to $1.51$  
   (ii) $E = 2.0 \times 10^{22}$ Erg

2. (i) $27 - 9x + 3x^2 - x^3$  
   (ii) $6561 + 34992x + 81648x^2 + 108864x^3 + \ldots$  
   (iii) $177147 + 885735x + 1909251x^2 + \ldots$  
   (iv) $186000$
   (v) For $2^3 \times 5^8$, need to use $x = 1$
       Since $1$ is large in comparison to $0.01$, the value is inaccurate because a
       significantly large value is removed after the 3rd term.

3. (ii) $104.5^\circ, 255.5^\circ, 180^\circ$

4. (i) $\alpha + \beta = -b$, $\alpha \beta = c$  
   (iii) $b^2 - 4c > 0$  
   (ii) $x^2 - (b^2 - 2c)x + c^2 = 0$  
   (iv) $b = 5$, $c = 2$
   (o.e.)

6. (i) $\frac{dy}{dx} = 2e^{3x}(1 - 6x)$  
   (ii) $x > \frac{1}{6}$  
   (iii) $-1.11$ units/sec

7. (i) $2x^3 - 4x^2 + x$  
   (o.e.)  
   (ii) $a = 0.7771$ or $-1.77$  
   (iii) $k \leq 2$

8. (i) $f'(x) = 3ax^2 + 2bx + 24$  
   (ii) $a = 2$, $b = -15$
   (iii) $p = 1$, $k > 27$
   (iv) $-13.5$

9. (i) $\frac{dy}{dx} = -2(x - 2)^3$  
   (ii) Eq $AB$: $y = 16x + 8$, $A$ is $(2, 40)$
   (iii) $38.4$ units$^2$

10. (i) $30$ m/s
   (iv) area of trapezium $< 0.5(30 + 60) \times 4 = 180$
   (iii) $v (m/s)$
   (v) max $a = 12.23$ m/s$^2$

11. (i) $A$ is $(4, 2)$, Radius = $5$ units
   (iii) $4v - 3x = 21$ (o.e.)
   (iv) $(-7, 0)$
   (v) $r = 5\sqrt{5} - 5$
<table>
<thead>
<tr>
<th>Qn</th>
<th>Key Steps</th>
<th>Marks / Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(i)</td>
<td></td>
<td>B1 TOV</td>
</tr>
<tr>
<td>ln $E$</td>
<td>13.3</td>
<td>14.8</td>
</tr>
<tr>
<td>$\ln E$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M$</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$\ln E$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Line passes through pts</td>
<td>B1</td>
<td></td>
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<tr>
<td>(ii)</td>
<td>$\lg E = a + bM$</td>
<td>B1</td>
</tr>
<tr>
<td>$a = \text{vertical intercept} = 11.8$</td>
<td></td>
<td>M1</td>
</tr>
<tr>
<td>$b = \text{gradient (their rise/run)} = 1.5$</td>
<td></td>
<td>A1</td>
</tr>
<tr>
<td>(iii)</td>
<td>$\lg E = 11.8 + 1.5(7) = 22.3$</td>
<td>M1</td>
</tr>
<tr>
<td>$E = 2.0 \times 10^{22}$ Erg</td>
<td>A1</td>
<td>1.34$\times10^{22}$ to $2.95 \times 10^{22}$</td>
</tr>
<tr>
<td>2(i)</td>
<td>$(3 - x)^3 = 27 - 27x + 9x^2 - x^3$</td>
<td>B1</td>
</tr>
<tr>
<td>(ii)</td>
<td>$(3 + 2x)^8$</td>
<td>B3</td>
</tr>
<tr>
<td>$= 3^8 + \binom{8}{1}(3)^7(2x) + \binom{8}{2}(3)^6(2x)^2 + \binom{8}{3}(3)^5(2x)^3$</td>
<td></td>
<td>-1m if 1st term missing</td>
</tr>
<tr>
<td>$= 6561 + 34992x + 81648x^2 + 108864x^3 + \ldots$</td>
<td></td>
<td>B0 is all not evaluated</td>
</tr>
<tr>
<td>(iii)</td>
<td>$(3 - x)^3 (3 + 2x)^8$</td>
<td>M1</td>
</tr>
<tr>
<td>= their (i) $\times$ their (ii)</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>= $177147 + 767637x + 2854035x^2 + \ldots$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(iv)</td>
<td>$2.99^3 \times 3.02^8$</td>
<td>B1</td>
</tr>
<tr>
<td>$= 177147 + 767637(0.01) + 2854035(0.01)^2$</td>
<td></td>
<td>B1</td>
</tr>
<tr>
<td>$= 185108.7735$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$= 185000$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(v)</td>
<td>For $2^3 \times 5^8$, need to use $x = 1$</td>
<td>B1</td>
</tr>
<tr>
<td>Since 1 is large in comparison to 0.01, the value is inaccurate because a significantly large value is removed after the 3rd term</td>
<td></td>
<td>B1</td>
</tr>
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<td>----</td>
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</tbody>
</table>
| 3(i) | \[
\sin (\theta + 2\theta) = \sin \theta \cos 2\theta + \cos \theta \sin 2\theta \\
= \sin \theta (1 - 2 \sin^2 \theta) + \cos \theta (2 \sin \theta \cos \theta) \\
= \sin \theta (1 - 2 \sin^2 \theta) + 2 \sin \theta \cos^2 \theta \\
= \sin \theta (1 - 2 \sin^2 \theta) + 2 \sin \theta (1 - \sin^2 \theta) \\
= 3 \sin \theta - 4 \sin^3 \theta 
\] | B1 Use compound angle  
B1 Any double angle seen  
B1 Use identity  
AG |
| (ii) | \[
\sin (3\theta) = 3 \sin \theta \cos \theta \\
3 \sin \theta - 4 \sin^3 \theta = 3 \sin \theta \cos \theta \\
\sin \theta (3 - 4 \sin^2 \theta - 3 \cos \theta) = 0 \\
\sin \theta = 0 \quad \therefore \theta = 180^\circ \\
\text{or} \\
3 - 4 \sin^2 \theta - 3 \cos \theta = 0 \\
3 - 4(1 - \cos^2 \theta) - 3 \cos \theta = 0 \\
4 \cos^2 \theta - 3 \cos \theta - 1 = 0 \\
(4 \cos \theta + 1)(\cos \theta - 1) = 0 \\
\cos \theta = -\frac{1}{4} \text{ or } \cos \theta = 1 \text{ (NA)} \\
\text{Hence, } \theta = 104.5^\circ, 255.5^\circ 
\] | B2 \(-1m\) for extra answer  
8 |
| 4(i) | \[
\alpha + \beta = -b \\
\alpha \beta = c 
\] | B1 Both correct |
| (ii) | \[
\begin{align*}
\alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2\alpha\beta \\
&= b^2 - 2c \\
\alpha^2 \beta^2 &= c^2 \\
\text{Eqn: } x^2 - (b^2 - 2c)x + c^2 &= 0 \\
\end{align*}
\] | B1 Correct sum  
B1 Correct product  
B1 Equation seen |
| (iii) | For 2 distinct roots,  
\((b^2 - 2c)^2 - 4c^2 > 0\) \(\checkmark\)  
\(b^2(b^2 - 4c) > 0\) \(\checkmark\)  
Since \(b^2 > 0\), hence \(b^2 - 4c > 0\) \(\checkmark\)  
| B1 Correct D  
ok if \([-\left(b^2 - 2c\right)]^2\) or \(\left(b^2 - 2c\right)^2\)  
B1 o.e. |
| (iv) | \[
b = 5, c = 2 
\] | B1 o.e.  
7 |
<table>
<thead>
<tr>
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<th>Marks / Remarks</th>
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</thead>
</table>
| 5(i) | Let $\angle CDQ = a$
   $\angle ODQ = 90^\circ$ (tan $\perp$ rad)
   $\therefore \angle ODC = 90^\circ - a$
   $\therefore \angle COD = 180^\circ - 2(90^\circ - a)$ ($\angle$ sum, $\triangle COD$) | B1 with reason |

| (ii) | $\angle AOB = 2 \times \angle BAP$ | B1 |

| (iii) | From (i) and (ii), $2(\angle CDQ + \angle BAP) = \angle COD + \angle AOB$
   $\angle CDQ + \angle BAP = \frac{1}{2}(\angle COD + \angle AOB)$
   $= \angle AOP + \angle DOQ$ ($\perp$ prop of chord)
   $= 180^\circ - \angle AOD$
   $= 2\angle OAD$ | B1 attempt to use (i) and (ii) |
|       | | B1B1 1m for reason |

| 6(i) | $y = 2e^{3x} (1 - 2x)$
   $\frac{dy}{dx} = 2e^{3x}(-2) + 6e^{3x}(1 - 2x)$
   $= 2e^{3x}(1 - 6x)$ | B1 Product Rule |

| (ii) | For decreasing function, $\frac{dy}{dx} < 0$
   $\therefore 1 - 6x < 0$
   $x > \frac{1}{6}$ | B1 |

| (iii) | Given that $\frac{dy}{dx} = -5$ units/s
   $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$
   $= 2e^{3x}(1 - 6x)(-5)$
   $= 2e^{3(-1.5)}(1 + 6 \times 1.5)(-5)$
   $= -1.11$ units/sec | B1 with negative seen |

|       | | B1 with subs seen |

Total: 8 and 7
<table>
<thead>
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</table>
| 7(i) | \(2^{3a+1} - 2^{2a+2} + 2^a\) Let \(2^a = x\)  
\(= 2 \times 2^{3a} - 4 \times 2^{2a} + 2^a\)  
\(= 2x^3 - 4x^2 + x\) | B1 Use of: \(2^p + q = 2^p \times 2^q\)  
B1 Use of: \((2^p)^q = 2^{pq}\)  
B1 |
| (ii) | \(2x^3 - 4x^2 + x = 0\)  
\(x(2x^2 - 4x + 1) = 0\)  
\(x = 0, \quad \therefore 2^a = 0\) (rej)  
or \(2x^2 - 4x + 1 = 0\)  
\(x = \frac{-4 \pm \sqrt{16 - 4 \times 2 \times 1}}{4}\)  
\(= 1.707\) or 0.2929  
\(2^a = 1.707\) or 0.2929  
\(a = \frac{\lg 1.707}{\lg 2}\) or \(\frac{\lg 0.2929}{\lg 2}\)  
\(= 0.7771\) or \(-1.77\) | M1 Solving quad with working seen  
A1 Both \(x\) |
| (iii) | \(2^{3a+1} - 2^{2a+2} + (k)2^a = 0\) has at least one root  
\(\therefore 2x^2 - 4x + k = 0\) has at least one root  
\(\therefore 16 - 4 \times 2 \times k \geq 0\)  
\(k \leq 2\) | M1 Using quad part of eqn  
B1 Correct D with subs  
A1 Both \(a\) |
| 8(i) | \(f(x) = ax^3 + bx^2 + 24x + 16\)  
\(f'(x) = 3ax^2 + 2bx + 24\) | B1 |
| (ii) | Sub (4, 0) into \(f'(x) = 0\)  
\(3a(16) + 2b(4) + 24 = 0\)  
\(\therefore 48a + 8b + 24 = 0\) ..................(1)  
Sub (4, 0) into \(f(x)\)  
\(a(64) + 16b + 24(4) + 16 = 0\)  
\(\therefore 64a + 16b + 96 + 16 = 0\) ..................(2)  
\(a = 2, b = -15\) | B1 Sub into their \(f'(x)\) and \(f(x)\)  
M1 Solve simul eqn  
A1 Both |
| (iii) | \(f'(x) = 6x^2 - 30x + 24\)  
\(= 6(x^2 - 5x + 4)\)  
\(= 6(x - 1)(x - 4)\)  
\(\therefore p = 1\)  
At \(x = 1, \ f(x) = 2(1) - 15(1) + 24(1) + 16 = 27\)  
Hence, \(k > 27\) | B1 Using their \(p\)  
M1 Use \(x = 2.5\)  
A1 |
<table>
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<tr>
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<tbody>
<tr>
<td>9(i)</td>
<td>( y = -\frac{1}{2} (x - 2)^4 + 16 ), ( \therefore \frac{dy}{dx} = -2(x - 2)^3 )</td>
<td>B1 o.e.</td>
</tr>
<tr>
<td></td>
<td>Grad of ( AB = -2(-8) = 16 )</td>
<td>B1 Grad ( AB ) seen</td>
</tr>
<tr>
<td></td>
<td>At ( B ), ( x = 0 ), ( \therefore y = 8 )</td>
<td>B1 Eqn ( AB ) seen</td>
</tr>
<tr>
<td></td>
<td>Eqn ( AB: y = 16x + 8 )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( \therefore A ) is (2, 40)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Area ( OBACD = (8 + 40) \times 2 )</td>
<td>M1 Using composite figures</td>
</tr>
<tr>
<td></td>
<td>= 96 units(^2)</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Area bounded by curve and axes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \frac{\pi}{4} \left( -\frac{1}{2} (x - 2)^4 + 16 \right) dx )</td>
<td>B1 Knowing to use integral for area</td>
</tr>
<tr>
<td></td>
<td>( \left( -\frac{1}{10} (x - 2)^5 + 16x \right) ) (^4 )</td>
<td>B1 Correct integration</td>
</tr>
<tr>
<td></td>
<td>( = (-\frac{1}{10} \times 32 + 64) - (\frac{1}{10} \times 32) )</td>
<td>B1 Subs seen</td>
</tr>
<tr>
<td></td>
<td>= 57.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \therefore ) shaded area – 96 – 57.6 = 38.4 units(^2)</td>
<td>B1</td>
</tr>
<tr>
<td>10(i)</td>
<td>( v_0 = 12e^{k(0)} + 18 = 30 ) m/s</td>
<td>B1 Sub need not be seen</td>
</tr>
<tr>
<td></td>
<td>( v_2 = 40 ) ( \therefore 40 = 12e^{k(2)} + 18 )</td>
<td>B1 Sub into eqn</td>
</tr>
<tr>
<td></td>
<td>( e^{2k} = \frac{11}{6} )</td>
<td>B1 Using logarithm</td>
</tr>
<tr>
<td></td>
<td>( 2k = \ln \left( \frac{11}{6} \right) )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( k = 0.3031 )</td>
<td></td>
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<td></td>
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<td></td>
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<tr>
<td></td>
<td>(iii)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( v ) (m/s) ( (4, 58.3) )</td>
<td>B1 Shape</td>
</tr>
<tr>
<td></td>
<td>( t ) (s)</td>
<td>B1 Label ( y )-intercept</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>B1 Label (4, 58.3)</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Area under curve &lt; Area of trapezium</td>
<td>B1 Find relevant distance travelled using any suitable method</td>
</tr>
<tr>
<td></td>
<td>Area of trapezium = ( 0.5(30 + 60) \times 4 = 180 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \therefore ) distance travelled &lt; 180 m</td>
<td>B1 Making conclusion</td>
</tr>
<tr>
<td></td>
<td>(iv)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Max accn occurs at ( t = 4 ) where the gradient is most steep</td>
<td>M1 Knowing to differentiate</td>
</tr>
<tr>
<td></td>
<td>Max accn = ( 0.3031 \times 12 e^{0.3031(4)} )</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>= 12.23 m/s(^2)</td>
<td></td>
</tr>
<tr>
<td>Qn</td>
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</tbody>
</table>
| 11(i) | $x^2 + y^2 - 8x - 4y - 5 = 0$  
A is (4, 2)  
Radius $= \sqrt{4^2 + 2^2 + 5} = 5$ (units) | B1  
M1A1 |
| 11(ii) | $1^2 + 6^2 - 8(1) - 4(6) - 5 = 0$  
Hence, (1, 6) lies on the circle. | B1  
Subs seen and statement |
| 11(iii) | Gradient of line joining (4, 2) and (1, 6)  
$\frac{4}{3}$  
Eqn of tangent at (1, 6) is  
$y - 6 = \frac{4}{3} (x - 1)$  
$4y - 3x = 21$ | B1  
$\perp$ grad seen  
B1  
Find eqn  
B1  
o.e. |
| 11(iv) | At B, $y = 0$  
$\therefore x = -7$  
$\therefore B$ is ($-7$, 0) | M1  
Finding x  
A1  
Ordered pair seen |
| 11(v) | Distance between centres  
$= \sqrt{11^2 + 2^2}$  
$= \sqrt{125}$  
$\therefore$ radius of $C_2 = \sqrt{125} - 5$  
$= 5 \sqrt{5} - 5$ | M1  
Find dist between centres  
M1  
Using sum radii = distance  
A1 |
1. ALGEBRA

Quadratic Equation
For the quadratic equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.
\]

Binomial Expansion
\[
(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n.
\]
where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\cdots(n-r+1)}{r!} \).

2. TRIGONOMETRY

Identities
\[
sin^2 A + cos^2 A = 1
\]
\[
sec^2 A = 1 + tan^2 A
\]
\[
\text{cosec}^2 A = 1 + \cot^2 A
\]
\[
sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
\sin 2A = 2\sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A
\]
\[
\tan 2A = \frac{2\tan A}{1 - \tan^2 A}
\]

Formulae for \( \Delta ABC \)
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\text{Area of } \Delta = \frac{1}{2} bc \sin A
\]
1 Find the range of values of \( a \) for which \( x^2 + ax + 2(a - 1) \) is always greater than 1. [4]

2 Find the distance between the points of intersection of the line \( 2x + 3y = 8 \) and the curve \( y = 2x^2 \), leaving your answer in 2 significant figures. [5]

3 Express \( \frac{x^2 - 2x - 6}{x(x^2 - x - 6)} \) as a sum of 3 partial fractions. [5]

4 Triangle \( ABC \) is an right angled isosceles triangle with angle \( ABC \) as the right angle. The height from point \( B \) to the base \( AC \) is \( \frac{3\sqrt{2}}{\sqrt{3} + \sqrt{6}} \). **Without using a calculator**, express the area of the triangle \( ABC \) in the form \( a + b\sqrt{2} \), where \( a \) and \( b \) are integers.

5 (i) Given \( \sin(A + B) + \sin(A - B) = k \sin A \cos B \), find \( k \). [2]

(ii) Hence, find the exact value of \( \int_{0}^{\pi/2} \sin 2x \cos x \, dx \). [4]

6 (a) State the values between which the principal value of \( \tan^{-1} x \) must lie. [1]

(b) The function \( f \) is defined by \( f(x) = 3\cos ax + 1 \), where \( a \) is a positive integer and \( -\pi \leq x \leq \pi \).

(i) State the amplitude and the minimum value of \( f \). [2]

(ii) Given that \( f(x) = 1 \) when \( x = \frac{\pi}{4} \), find the smallest possible value of \( a \) [1]

(iii) Using the value of \( a \) found in part (ii), state the period of \( f \) and sketch the graph of \( y = f(x) \). [4]
The function \( f \) is defined by \( f(x) = 6x^3 - kx^2 + 3x + 10 \), where \( k \) is a constant.

(i) Given that \( 2x + 1 \) is a factor of \( f(x) \), find the value of \( k \). \([2]\)

(ii) Using the value of \( k \) found in part (i), solve the equation \( f(x) = 0 \). \([4]\)

Solve the equation

(i) \( 3 \log_3 x - \log_3 3 = 2 \), \([5]\)

(ii) \( 2 \log_2(1 - 2x) - \log_2(6 - 5x) = 0 \). \([4]\)

The equation of a curve is \( y = \frac{2x^2}{x-1}, \quad x > 1 \).

(i) Find the coordinates of the stationary point of the curve. \([4]\)

(ii) Use the second derivative test to determine the nature of the point. \([3]\)

The diagram shows part of the graph \( y = c - |ax + b| \) where \( a > 0 \). The graph has a maximum point \((12, 2)\) and passes through the point \((18, -1)\).

(i) Determine the value of each of \( a, b \) and \( c \). \([4]\)

(ii) State the set of value(s) of \( m \) for which the line \( y = mx + 4 \) cuts the graph \( y = c - |ax + b| \) at exactly one point. \([3]\)
The diagram shows a triangle $ABC$ in where points $B$ and $C$ are on the $y$-axis. The line $AC$ cuts the $x$-axis at point $D$ and the coordinates of point $C$ and $D$ are $(0, -10)$ and $(5, 0)$ respectively. $AD = \frac{2}{7} AC$ and points $A$, $B$ and $D$ are vertices of a rhombus $ABDE$.

(i) Show that the coordinates of $A$ is $(7, 4)$. [1]

(ii) Find the coordinates of $B$ and $E$. [5]

(iii) Calculate the area of the quadrilateral $ABOD$. [2]

The diagram above shows part of the curve $y = x^2 + 1$. $P$ is the point on the curve where $x = p$, $p > 0$. The tangent at $P$ cuts the $x$-axis at point $Q$ and the foot of the perpendicular from $P$ to $x$-axis is $R$.

(i) Show that the area $A$ of the triangle $PQR$ is given by $A = \frac{p^3}{4} + \frac{p}{2} + \frac{1}{4p}$. [5]

(ii) Obtain an expression for $\frac{dA}{dp}$. [1]

(iii) Find the least area of the triangle $PQR$, leaving your answer in 2 decimal places. [4]

**End of Paper**
### Answer Scheme

<table>
<thead>
<tr>
<th>Qn</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2 &lt; a &lt; 6$</td>
</tr>
<tr>
<td>2</td>
<td>2.8 units</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{x^2 - 2x - 6}{x(x^2 - x - 6)} = \frac{1}{x} - \frac{1}{5(x - 3)} + \frac{1}{5(x + 2)}$</td>
</tr>
<tr>
<td>4</td>
<td>$18 - 12\sqrt{2}$</td>
</tr>
<tr>
<td>5(i)</td>
<td>$k = 2$</td>
</tr>
<tr>
<td>5(ii)</td>
<td>$\frac{4 - \sqrt{2}}{6}$</td>
</tr>
<tr>
<td>6(a)</td>
<td>$-\frac{\pi}{2} &lt; \tan^{-1} x &lt; \frac{\pi}{2}$ or $-90^0 &lt; \tan^{-1} x &lt; 90^0$</td>
</tr>
</tbody>
</table>
| 6(b) | (i) Amplitude = 3, minimum value = $-2$  
(ii) $a = 2$ |
|     | (iii) Period = $\pi$ |
| 7(i) | $k = 31$  
(ii) | $x = 5$ or $\frac{2}{3}$ or $-\frac{1}{2}$ |
| 8(i) | $x = 0.693$ or 3 |
|     | (ii) $x = -\frac{5}{4}$ |
| 9(i) | $(2, 8)$ |
|     | (ii) $\frac{d^2y}{dx^2} = \frac{4}{(x-1)^3}$ Min point |
| 10(i) | $a = \frac{1}{2}$, $b = -6$, $c = 2$ |
|     | (ii) $m = -\frac{1}{6}$ or $m > \frac{1}{2}$ or $m \leq \frac{1}{2}$ |
| 11(i) | $(7, 4)$ |
|     | (ii) $B(0, 5)$, $E(12, -1)$ |
|     | (iii) 27.5 units$^2$ |
| 12(ii) | $\frac{dA}{dp} = \frac{3}{4}p^2 + \frac{1}{2} - \frac{1}{4p^2}$ |

12(ii) 0.77 units$^2$
YISHUN TOWN SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2018
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC
ADDITIONAL MATHEMATICS PAPER 2 (4047/02)

DATE : 16 AUGUST 2018
DURATION: 2 h 30 min
DAY : THURSDAY
MARKS: 100

ADDITIONAL MATERIALS
Writing Paper x 8
Mathematics Cover Sheet x 1

READ THESE INSTRUCTIONS FIRST

Do not turn over the cover page until you are told to do so.
Write your name, class and class index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/ tape.
Write your answers on the writing papers provided.

Answer all the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [   ] at the end of each question or part question.

This question paper consists of 6 printed pages and 1 blank page.

Yishun Town Secondary School 4E/5N/AMaths(4047/02)/2018/Prelim

[Turn over
1. **ALGEBRA**

**Quadratic Equation**

For the quadratic equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.
\]

**Binomial Expansion**

\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and

\[
\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}.
\]

2. **TRIGONOMETRY**

**Identities**

\[\sin^2 A + \cos^2 A = 1\]
\[\sec^2 A = 1 + \tan^2 A\]
\[\cos^2 A = 1 + \cot^2 A\]
\[\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B\]
\[\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B\]
\[\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}\]
\[\sin 2A = 2 \sin A \cos A\]
\[\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A\]
\[\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}\]

**Formulae for \( \Delta ABC \)**

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[a^2 = b^2 + c^2 - 2bc \cos A\]
\[\text{Area of } \Delta = \frac{1}{2} bc \sin A\]
1. (a) Given that the roots of the equation \( x^2 - 6x + k = 0 \) differ by 2, find the value of \( k \). \([3]\]

(b) If \( \alpha \) and \( \beta \) are the roots of the equation \( x^2 + bx + 1 = 0 \), where \( b \) is a non-zero constant, show that the equation with roots \( \frac{\alpha}{\beta} \) and \( \frac{\beta}{\alpha} \) is \( x^2 - (b^2 - 2)x + 1 = 0 \). \([4]\)

2. If the first three terms in the expansion of \( \left(1 - \frac{x}{2}\right)^n \) is \( 1 - 6x + ax^2 \), find the value of \( n \) and of \( a \). \([4]\)

3. (a) Solve the equation \( \sqrt{4 + \frac{3}{x}} = \frac{1}{\sqrt{x}} + 2 \). \([5]\)

(b) Given that \( \frac{4}{n} (3x)^{n-2} \left( \frac{2}{9x^2} \right)^{n-2} = \frac{m}{x^7} \), where \( x \neq 0 \), find the values of the constants \( m \) and \( n \). \([4]\)

4. A precious stone was purchased by a jeweler in the beginning of January 2003. The expected value, \( V \), of the stone may be modelled by the equation \( V = 6000(4^t) - 1000(16^t) \), where \( t \) is the number of years since the time of purchase. Find

(i) the expected value of the stone when \( t = \frac{3}{4} \). \([1]\)

(ii) the value(s) of \( t \) for which the expected value of the stone is \$8000. \([3]\)

(iii) the range of values of \( t \) for which the expected value of the stone exceeds \$8000. \([1]\)

5. The equation of a circle, \( C \), is \( x^2 + y^2 - 4ux + 2uy + 5(u^2 - 20) = 0 \) where \( u \) is a positive constant.

(a) Given that \( u = 6 \), find the coordinates of the centre and the radius of the circle \( C \). \([3]\)

(b) Determine the value of \( u \) for which

(i) the circle, \( C \), passes through the point \((-4, 4)\), \([2]\)

(ii) the line \( x = 2 \) is a tangent to the circle, \( C \). \([4]\)
6. The variables \( x \) and \( y \) are related by the equation \( mx + ny - 3xy = 0 \), where \( m \) and \( n \) are non-zero constants. When \( \frac{1}{y} \) is plotted against \( \frac{1}{x} \), a straight line is obtained. Given that the line passes through the points \((1,0)\) and \((-5,9)\), find the values of \( m \) and of \( n \).

[6]

7. (i) Prove that \( \sin^4 \theta - \cos^4 \theta = 1 - 2 \cos^2 \theta \).

(ii) Hence solve \( \sin^4 \theta - \cos^4 \theta = 2 \cos \theta \) for \( 0 < \theta < 2\pi \).

[3] [4]

8. In the diagram, \( OS = 3 \text{ m}, \ OR = 7 \text{ m} \) and angle \( SOR = \) angle \( SPO = \) angle \( RQO = 90^\circ \).

It is given that angle \( SOP \) is a variable angle \( \theta \) where \( 0^\circ < \theta < 90^\circ \). The point \( T \) is on the line \( RQ \) such that \( ST \) is parallel to \( PQ \).

(i) Show that \( PQ = 7 \sin \theta + 3 \cos \theta \).

(ii) Show that the area of triangle \( RST \) is \( \frac{21}{2} \cos 2\theta + 10 \sin 2\theta \).

(iii) Express the area of the triangle \( RST \) as \( k \cos (2\theta - \alpha) \), where \( k > 0 \) and \( 0^\circ < \alpha < 90^\circ \).

(iv) Hence find the maximum area of triangle \( RST \) and the corresponding value of \( \theta \).

[1] [3] [4] [3]
9. The diagonals of a cyclic quadrilateral $PQRS$ intersect at a point $U$. The circle’s tangent at $R$ meets the line $PS$ produced at $T$.

If $QR = RS$, prove the following.

(i) $QS$ is parallel to $RT$. [3]

(ii) Triangles $PUS$ and $QUR$ are similar. [3]

(iii) $PU^2 - QU^2 = (PU \times PR) - (QU \times QS)$. [3]

10. It is given that $y = xe^{-x} - 2e^{-2x}$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) If $x$ and $y$ can vary with time and $x$ increases at the rate of 1.5 units per second at the instant when $x = \ln 2$, find the exact value of the rate of increase of $y$ at this instant. [3]

11. A curve has the equation $y = \frac{\ln x}{x^2} - 2$.

(i) Show that $\frac{dy}{dx} = \frac{1 - 2\ln x}{x^3}$. [2]

(ii) (a) The $x$-coordinate of a point $P$ on the curve is 1. Find the equation of the tangent to the curve at $P$. [2]

(b) The tangent to the curve at the point $P$ intersects the $x$-axis at $Q$ and the $y$-axis at $R$. Calculate the shortest distance from the origin $O$ to the line $QR$. [4]

(iii) Given that another curve $y = f(x)$ passes through the point $(1, -0.25)$ and is such that $f'(x) = \frac{\ln x}{x^3}$, find the function $f(x)$. [3]
12. The diagram shows the graphs of $y = e^x - 1$ and $y = e - x$. $P$ is the point of intersection of the two graphs.

![Graph of $y = e^x - 1$ and $y = e - x$]

(i) Show that $\alpha = 1$ is a root to the equation $e \left(1 - e^{\alpha - 1}\right) - \alpha + 1 = 0$. [1]

(ii) Hence, find the coordinates of $P$. [2]

(iii) Find the area of region $A$, which is enclosed by the two graphs and the $y$-axis. [4]

(iv) Find the exact value of the area of region $A$, given that the area of region $B$ is enclosed by the two graphs and the $x$-axis. [2]

13. A particle moves pass a point $A$ in a straight line with a displacement of $-4$ m from a fixed point $O$. Its acceleration, $a$ m/s$^2$, is given by $a = \frac{t}{2}$, where $t$ seconds is the time elapsed after passing through point $A$.

Given that the initial velocity is $-1$ m/s, find,

(i) the velocity when $t = 2$, [3]

(ii) the distance travelled by the particle in the first 5 seconds. [5]

**END OF PAPER**
1(a) \( k = 8 \)

1(b) Show \( x^2 - (b^2 - 2) x + 1 = 0 \)

2 \( n = 12, a = \frac{33}{2} \)

3(a) \( x = \frac{1}{4} \)

3(b) \( n = 4, m = \frac{4}{9} \)

4(i) $8970$

4(ii) \( t = \frac{1}{2} \cdot 1 \)

4(iii) \( \frac{1}{2} < t < 1 \)

5(i) Centre is \((12, -6)\)
Radius = 10 units

5(ii)(a) \( u = 2 \)

5(ii)(b) \( u = 6 \)

6 \( m = 2, n = 3 \)

7(i) Show \( \sin^4 \theta - \cos^4 \theta = 1 - 2 \cos^2 \theta \)

7(ii) \( \theta = \frac{2\pi}{3}, \frac{4\pi}{3} \)

8(i) Show \( PQ = 7 \sin \theta + 3 \cos \theta \)

8(ii) Show Area = \( \frac{21}{2} \cos 2\theta + 10 \sin 2\theta \)

8(iii) Area = \( \frac{29}{2} \cos (2\theta - 43.6^\circ) \)

8(iv) Max area of triangle \( RST \) = \( \frac{29}{2} \) m², \( \theta = 21.8^\circ \)

9(i) Show \( QS \) is parallel to \( RT \)

9(ii) Show Triangles \( PUS \) and \( QUR \) are similar

9(iii) Show \( PU^2 - QU^2 = (PU \times PR) - (QU \times QS) \)

10(i) \( \frac{dy}{dx} = (1-x)e^{-x} + 4e^{-2x} \)

10(ii) \( \frac{dy}{dt} \bigg|_{x=2} = \frac{9}{4} - \frac{3}{4} \ln 2 \)

11(i) \( \frac{dy}{dx} = \frac{1 - 2 \ln x}{x^3} \)

11(ii)(a) \( y = x - 3 \)

11(ii)(b) \( h = \frac{3\sqrt{2}}{2} \) units

11(iii) \( f(x) = -\frac{1 + 2 \ln x}{4x^2} \)

12(ii) \( P(1, e-1) \)

12(iii) Area of Region \( A = \frac{3}{2} \) units²

12(iv) Area of Region \( A = \frac{3}{e^2 - 3} \)

Area of Region \( B = \frac{3}{e^2 - 3} \)

13(i) Velocity = 0 m/s

13(ii) Total distance travelled = \( 8 \frac{1}{12} \) m
READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the presentation, fasten all your work securely together.
The number of marks is given in brackets [   ] at the end of each question or part question.

The total number of marks for this paper is 80.
Mathematical Formulae

1. **ALGEBRA**

   **Quadratic Equation**
   For the equation \( ax^2 + bx + c = 0 \),
   \[
   x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
   \]

   **Binomial Theorem**
   \[
   (a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,
   \]
   where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. **TRIGONOMETRY**

   **Identities**
   \[
   \sin^2 A + \cos^2 A = 1.
   \]
   \[
   \sec^2 A = 1 + \tan^2 A.
   \]
   \[
   \csc^2 A = 1 + \cot^2 A.
   \]
   \[
   \sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
   \]
   \[
   \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
   \]
   \[
   \tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
   \]
   \[
   \sin 2A = 2 \sin A \cos A
   \]
   \[
   \cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
   \]
   \[
   \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
   \]

   **Formulae for \( \Delta ABC \)**
   \[
   \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
   \]
   \[
   a^2 = b^2 + c^2 - 2bc \cos A
   \]
   \[
   \Delta = \frac{1}{2} ab \sin C
   \]
Answer all the questions

1. \(AB\) is parallel to \(EC\) and \(AB = (1 + 3\sqrt{5})\) cm. \(E\) is a point on \(AD\) such that \(AE : ED = \sqrt{5} : 3\). Find
   (i) \(\frac{EC}{AB}\) in the form of \(a + b\sqrt{5}\), where \(a\) and \(b\) are rational numbers. \[3\]
   (ii) the length of \(EC\) in the form of \(c + d\sqrt{5}\), where \(c\) and \(d\) are integers. \[3\]

2. The equation of a curve is \(y = (k + 2)x^3 - 10x + 2k + 1\), where \(k\) is a constant.
   (i) In the case where \(k = 1\), sketch the graph of \(y = (k + 2)x^3 - 10x + 2k + 1\), showing the \(x\)- and \(y\)-intercepts and its turning point clearly. \[3\]
   (ii) Find the range of values of \(k\) for which the curve meets the line \(y = 2x + 3\). \[5\]

3. (a) Express \(\frac{3x^3 - 5}{x^2 - 1}\) in partial fractions. \[5\]
   (b) Solve the equation \(|21 - 18x| - |7 - 6x| = 4x - 1\). \[4\]

4. The equation of a curve is \(y = 2x(x - 1)^3\).
   (i) Find the coordinates of the stationary points of the curve. \[5\]
   (ii) Determine the nature of each of these points using the first derivative test. \[3\]

5. (i) On the same diagrams, sketch the graphs \(y = \frac{4}{x^2}\), \(x > 0\) and \(y = 3x^{\frac{1}{3}}\), \(x \geq 0\). \[2\]
   (ii) Find the value of the constant \(k\) for which the \(x\)-coordinate of the point of intersection of your graphs is the solution to the equation \(x^3 = k\). \[2\]
6. (i) Prove that \( \frac{1}{3 \tan^2 \theta + 3} = \frac{\cos^2 \theta}{3} \). \[2\]

(ii) Show that \( \int_0^\frac{\pi}{3} \frac{\sec^2 \theta \cos 2\theta}{3 \tan^2 \theta + 3} \, d\theta = \frac{\sqrt{3}}{12} \). \[4\]

7. **Solutions to this question by accurate drawing will not be accepted.**

The diagram above shows a quadrilateral \(ABCD\). Point \(B\) is \((2, 8)\) and point \(C\) is \((8, 6)\). The point \(D\) lies on the perpendicular bisector of \(BC\) and the point \(A\) lies on the \(y\)-axis. The equation of \(CD\) is \(3y = 4x - 14\) and angle \(ABC = 90^\circ\). Find

(i) the equation of \(AB\), \[2\]

(ii) the coordinates of \(A\), \[1\]

(iii) the equation of the perpendicular bisector of \(BC\), \[3\]

(iv) the coordinates of \(D\), \[3\]

8. (i) Show that \( \frac{d}{dx} (x^2 \ln x - 3x) = x + 2x \ln x - 3 \). \[2\]

(ii) Evaluate \( \int_1^4 x \ln x \, dx \). \[4\]

9. A curve is such that the gradient function is \( 1 + \frac{1}{2x^2} \). The equation of the tangent at point \(P\) on the curve is \(y = 3x + 1\). Given that the \(x\)-coordinate of \(P\) is positive, find the equation of the curve. \[7\]
10. A right circular cone, \( ABC \), is inscribed in a sphere of radius 10 cm and centre \( O \).
The perpendicular distance from \( O \) to the base of the cone is \( x \) cm.

\[
\text{Volume of cone} = \frac{1}{3} \pi r^2 h
\]

(i) Show that volume, \( V \), of the cone is \( V = \frac{1}{3} \pi (100 - x^2)(10 + x) \). [2]

(ii) If \( x \) can vary, find the value of \( x \) for which \( V \) has a stationary value. [3]

(iii) Find this stationary volume. [1]

(iv) Determine whether the volume is a maximum or minimum. [2]

11. (a) Find, in radians, the two principal values of \( y \) for which \( 2 \tan^2 y + \tan y - 6 = 0 \). [4]

(b) The height, \( h \) m, above the ground of a carriage on a carnival ferris wheel is modelled by the equation \( h = 7 - 5 \cos(8t) \), where \( t \) in the time in minutes after the wheel starts moving.

(i) State the initial height of the carriage above ground. [1]

(ii) Find the greatest height reached by the carriage. [1]

(iii) Calculate the duration of time when the carriage is 9 m above the ground. [3]
1i $\triangle ABD$ is similar to $\triangle ECD$.

\[
\frac{CE}{BA} = \frac{DE}{DA}
\]

\[
CE = \frac{3}{3 + \sqrt{5}} \quad \text{[M1] ratio seen}
\]

\[
= \frac{3}{3 + \sqrt{5}} \times \frac{3 - \sqrt{5}}{3 - \sqrt{5}} \quad \text{[B1] correct conjugate surd}
\]

\[
= \frac{9 - 3\sqrt{5}}{3^2 - 5}
\]

\[
= \frac{9 - 3\sqrt{5}}{4} \quad \text{[A1]}
\]

ii $EC = \frac{9 - 3\sqrt{5}}{4} \times (1 + 3\sqrt{5})$  

\[
= \frac{1}{4} \left[9(1 + 3\sqrt{5}) - 3\sqrt{5}(1 + 3\sqrt{5})\right] 
\]

\[
= \frac{1}{4} (-36 + 24\sqrt{5}) 
\]

\[
= -9 + 6\sqrt{5} \quad \text{[A1]}
\]

2i When $k = 1$,

\[
y = 3x^2 - 10x + 3
\]

\[
= 3 \left( x^2 - \frac{10}{3} \right) + 3
\]

\[
= 3 \left( x - \frac{10}{6} \right)^2 - \left( \frac{10}{6} \right)^2 + 3
\]

\[
= 3 \left( x - \frac{5}{3} \right)^2 - \frac{25}{3} + 3
\]

\[
= 3 \left( x - \frac{5}{3} \right)^2 - \frac{16}{3}
\]

Turning point $\left( \frac{5}{3}, -\frac{16}{3} \right)$

When $y = 0, x = 3$ or $\frac{1}{3}$
### ii

\[(k + 2)x^2 - 10x + 2k + 1 = 2x + 3\]  
[M1] substitution

\[(k + 2)x^2 - 12x + 2k - 2 = 0\]

\[b^2 - 4ac \geq 0\]  
[B1]

\[-12)^2 - 4(k + 2)(2k - 2) \geq 0\]

\[144 - 8(k^2 + k - 2) \geq 0\]

\[-8k^2 - 8k + 160 \geq 0\]

\[k^2 + k - 20 \leq 0\]

\[(k + 5)(k - 4) \leq 0\]  
[M1] factorisation

\[-5 \leq k \leq 4\] and \(k \neq -2\)

[A1] [A1]

### 3i

By long division [M1]

\[
\frac{3x^3 - 5}{x^2 - 1} = 3x + \frac{3x - 5}{x^2 - 1}
\]

\[
\frac{3x - 5}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}
\]

\[3x - 5 = A(x-1) + B(x+1)\]  
[M1] any acceptable method to find A and B

\[x = 1:\quad 3(1) - 5 = 2B\]

\[B = -1\]

\[x = -1:\quad -3 - 5 = -2A\]

\[A = 4\]  
[A1] correct A and B

\[
\therefore \frac{3x^3 - 5}{x^2 - 1} = 3x + \frac{4}{x+1} - \frac{1}{x-1}
\]

[A1]

### ii

\[
\left| 21 - 18x \right| - \left| 7 - 6x \right| = 4x - 1
\]

[B1] factorise

\[
\frac{3(7 - 6x) - 7 - 6x}{4x - 1}
\]

2

\[
\frac{7 - 6x}{4x - 1}
\]

\[
7 - 6x = \frac{4x - 1}{2} \text{ or } 7 - 6x = \frac{-4x + 1}{2}
\]

[B1] either one seen

\[x = \frac{15}{16} \text{ or } x = \frac{13}{8}\]

[A1] [A1]
4i

\[ y = 2x(x-1)^3 \]

\[ \frac{dy}{dx} = 2x \left[ 3(x-1)^2 \right] + 2(x-1)^3 \quad [M1] \text{product rule} \]

\[ = 6x(x-1)^2 + 2(x-1)^3 \quad [A1] \]

\[ = 2(x-1)^2 (3x + x - 1) \]

\[ = 2(x-1)^2 (4x - 1) \]

For \( \frac{dy}{dx} = 0 \)

\[ 2(x-1)^2 (4x - 1) = 0 \quad [M1] \]

\[ x = 1 \quad \text{or} \quad x = \frac{1}{4} \]

\[ y = 0 \quad \text{or} \quad y = -\frac{27}{128} \]

\[ (1,0) \quad \text{and} \quad \left( \frac{1}{4}, -\frac{27}{128} \right) \quad [A1] \]

ii

By first derivative test, [M1]

\( (1,0) \) is a point of inflexion and \( \left( \frac{1}{4}, -\frac{27}{128} \right) \) is a min. point \([A1], [A1]\)

5i

\[ y = 3x^2 \quad [B1] \]

\[ y = \frac{4}{x^2} \quad [B1] \]
| ii | $3x^2 = \frac{4}{x}$ \hspace{1cm} [M1] substitution 
$\frac{1}{x^2}, x^2 = \frac{4}{3}$ 
$x^5 = \frac{4}{3}$ 
$x^5 = (\frac{4}{3})^2$ \hspace{1cm} [M1] squaring 
$= \frac{16}{9}$ 
$\therefore k = \frac{16}{9}$ \hspace{1cm} [A1] |
|---|---|
| 6i | LHS $= \frac{1}{3\tan^2 \theta + 3}$ \hspace{1cm} [B1] apply correct identity 
$= \frac{1}{3(\sec^2 \theta - 1) + 3}$ 
$= \frac{1}{3\sec^2 \theta}$ \hspace{1cm} [B1] able to simplify 
$= \frac{\cos^2 \theta}{3}$ 
$= \text{RHS}$ |
| ii | $\int_0^\pi \frac{\sec^2 \theta \cos 2\theta}{3\tan^2 \theta + 3} \, d\theta = \int_0^\pi \frac{\cos^2 \theta}{3} \left( \frac{1}{\cos^2 \theta} \right) \cos 2\theta \, d\theta$ \hspace{1cm} [M1] substitution of $\frac{1}{3\tan^2 \theta + 3}$ 
$= \frac{1}{3} \int_0^\pi \cos 2\theta \, d\theta$ \hspace{1cm} [B1] $\sec^2 \theta = \frac{1}{\cos^2 \theta}$ 
$= \frac{1}{3} \left[ \sin 2\theta \right]_0^\pi$ \hspace{1cm} [B1] correct integration of $\cos 2\theta$
$= \frac{1}{6} \left( \sin \frac{2\pi}{3} - \sin 0 \right)$ 
$= \frac{1}{6} \left( \sin \frac{\pi}{3} - 0 \right)$ 
$= \frac{1}{6} \left( \frac{\sqrt{3}}{2} \right)$ \hspace{1cm} [B1] $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ 
$= \frac{\sqrt{3}}{12}$ (shown) |
7i \[ \text{Grad. } BC = \frac{8 - 6}{2 - 8} = -\frac{1}{3} \]
Grad. \( AB = 3 \) \[ \text{Eqn } AB \text{ is } \frac{y - 8}{x - 2} = 3 \]
\[ \therefore y = 3x + 2 \] [B1]

ii When \( x = 0 \), \( y = 2 \)
\( A (0, 2) \) \[ \text{[B1]} \]

iii \[ \text{Grad. of perpendicular bisector } = 3 \]
Midpt. \( BC = \left( \frac{2 + 8}{2}, \frac{8 + 6}{2} \right) \)
\[ = (5, 7) \] [M1]
Eqn is \[ \frac{y - 7}{x - 5} = 3 \]
\[ y = 3x - 8 \] [A1]

iv \[ 3y = 4x - 14 \]
\[ 3(3x - 8) = 4x - 14 \] [M1]
\[ 9x - 24 = 4x - 14 \]
\[ 5x = 10 \]
\[ x = 2 \] [A1]
\[ y = 3(2) - 8 \]
\[ = -2 \]
\[ D (2, -2) \] [A1]

8i \[ \frac{d}{dx} (x^2 \ln x - 3x) = x^2 \left( \frac{1}{x} \right) + 2x \ln x - 3 \] [B1]
\[ \frac{1}{x} \] seen
\[ = x + 2x \ln x - 3 \] [B1]

ii \[ \int_{1}^{4} x + 2 \ln x - 3 \, dx = \left[ x^2 \ln x - 3x \right]_{1}^{4} \] [M1]
\[ \int_{1}^{4} x - 3 \, dx + \int_{1}^{4} 2 \ln x \, dx = 4^2 \ln 4 - 3(4) - (0 - 3) \]
\[ \left[ \frac{x^2}{2} - 3x \right]_{1}^{4} + 2 \int_{1}^{4} \ln x \, dx = 16 \ln 4 - 12 + 3 \] [A1]
\[ \left[ \frac{x^2}{2} - 3x \right] \] seen
\[ 2 \int_{1}^{4} \ln x \, dx = 16 \ln 4 - 9 - \left[ \frac{4^2}{2} - 3(4) - \frac{1}{2} + 3 \right] \] [A1]
\[ \text{simplification} \]
\[ = 16 \ln 4 - \frac{15}{2} \text{ or } -14.7 \text{ (3s.f.)} \] [A1]
\[ \frac{dy}{dx} = 1 + \frac{1}{2x^2} \]
\[ = 1 + \frac{1}{2x^2} \]
\[ y = \int \left(1 + \frac{1}{2x^2}\right) \, dx \quad \text{[M1]} \]
\[ = x + \frac{1}{2} \left( \frac{x^{-1}}{-1} \right) + c \]
\[ = x - \frac{1}{2x} + c \quad \text{[A1]} \]

Since \( \frac{dy}{dx} = 3 \)
\[ 1 + \frac{1}{2x^2} = 3 \quad \text{[M1]} \]
\[ \frac{1}{2x^2} = 2 \]
\[ x^2 = \frac{1}{4} \]
\[ x = \pm \frac{1}{2} \quad \text{(reject } - \frac{1}{2}\text{)} \quad \text{[A1]} \]

When \( x = \frac{1}{2}, \)
\[ y = 3 \left( \frac{1}{2} \right) + 1 \]
\[ = \frac{5}{2} \quad \text{[A1]} \]

At \( \left( \frac{1}{2}, \frac{5}{2} \right) \), \( \frac{5}{2} = \frac{1}{2} \left( \frac{1}{2} \right) - \frac{1}{2(0.5)} + c \quad \text{[M1] attempt to find } c \)
\[ c = 3 \]
\[ y = x - \frac{1}{2x} + 3 \quad \text{[A1]} \]

10i Radius of cone = \( \sqrt{10^2 - x^2} \)
\[ = \sqrt{100 - x^2} \quad \text{[B1]} \]

Volume of cone
\[ = \frac{1}{3} \pi r^2 h \]
\[ = \frac{1}{3} \pi \left( \sqrt{100 - x^2} \right)^2 (x + 10) \]
\[ = \frac{1}{3} \pi (100 - x^2)(x + 10) \quad \text{[B1] application of formula and substitution} \]
ii \[
\frac{dV}{dx} = \frac{1}{3} \pi \left[-2x(x+10) + 100 - x^2 \right]
\]
[M1] product rule

\[
= \frac{1}{3} \pi \left[-20x - 2x^2 + 100 - x^2 \right]
\]
\[
= \frac{1}{3} \pi \left(-3x^2 - 20x + 100 \right)
\]

For stationary \(V\), \(\frac{dV}{dx} = 0\)  
[M1]

\[
\frac{1}{3} \pi \left(-3x^2 - 20x + 100 \right) = 0
\]

\[
x = -10 \text{ (rejected), } x = \frac{10}{3}
\]
[A1]

iii \(V = \frac{1}{3} \pi \left(100 - \frac{100}{9} \right) \left(\frac{10}{3} + 10 \right)
\)

\[
= 1241.123
\]

\[
= 1240 \text{ cm}^3 \text{ (3s.f.)} \quad \text{[B1]}
\]

iv \[
\frac{d^2V}{dx^2} = \frac{1}{3} \pi (-6x - 20)
\]
[M1]

Since \(\frac{d^2V}{dx^2} < 0\), \(V\) is a maximum.  
[A1]

11a \(2\tan^2 y + \tan y - 6 = 0\)  
[M1] factorisation

\((2\tan y - 3)(\tan y + 2) = 0\)

\(\tan y = \frac{3}{2} \quad \text{or} \quad \tan y = -2\)  
[A1] either one

\(y = \tan^{-1} \left(\frac{3}{2} \right) \quad y = \tan^{-1} (-2)\)

\(= 0.9827 \quad = -1.1071\)

\(= 0.983 \text{ (3s.f.)} \quad \approx -1.11 \text{ (3s.f.)}\)
[A1]  
[A1]

bi Initial height = 2 m  
[B1]

ii Greatest height = 7 - 5(-1)  
= 12 m  
[A1]
<table>
<thead>
<tr>
<th>iii</th>
<th>(7 - 5 \cos 8t = 9)</th>
<th>[M1]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\cos 8t = -\frac{2}{5})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\alpha = 1.1592)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(8t = 1.9823, 4.300)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(t = 0.2477, 0.5375)</td>
<td>[A1]</td>
</tr>
<tr>
<td></td>
<td>Duration = 0.5375 – 0.2477</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 0.2898</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\approx 0.290 \text{ minutes (3s.f.)})</td>
<td>[A1]</td>
</tr>
</tbody>
</table>
READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the presentation, fasten all your work securely together.
The number of marks is given in brackets [   ] at the end of each question or part question.
The total number of marks for this paper is 100.
Mathematical Formulae

1. **ALGEBRA**

*Quadratic Equation*

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

*Binomial Theorem*

\[
(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. **TRIGONOMETRY**

*Identities*

\[
\sin^2 A + \cos^2 A = 1.
\]

\[
\sec^2 A = 1 + \tan^2 A.
\]

\[
\cosec^2 A = 1 + \cot^2 A.
\]

\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
\sin 2A = 2 \sin A \cos A
\]

\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]

\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

*Formulae for \( \Delta ABC \)*

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2} ab \sin C
\]
1. (i) Given that \( u = 4^x \), express \( 4^x = 9 - 5 \times 4^{1-x} \) as a quadratic equation in \( u \). [2]

(ii) Hence find the values of \( x \) for which \( 4^x = 9 - 5 \times 4^{1-x} \), giving your answer, where appropriate, to 1 decimal place. [4]

(iii) Determine the values of \( k \) for which \( 4^x = k - 5 \times 4^{1-x} \) has no solution. [3]

2. (i) By using long division, divide \( 2x^4 + 5x^3 - 8x^2 - 8x + 3 \) by \( x^2 + 3x - 1 \). [2]

(ii) Factorise \( 2x^4 + 5x^3 - 8x^2 - 8x + 3 \) completely. [2]

(iii) Hence find the exact solutions to the equation

\[
32p^4 + 40p^3 - 32p^2 - 16p + 3 = 0.
\] [4]

3. The roots of the quadratic equation \( 8x^2 - 4x + 1 = 0 \) are \( \frac{1}{\alpha^2} \beta \) and \( \frac{1}{\alpha} \beta^2 \). Find a quadratic equation with roots \( \alpha^3 \) and \( \beta^3 \). [7]

4. (i) Write down the general term in the binomial expansion of

\[
\left( \frac{2x^2 - \frac{p}{x}}{x} \right)^{10},
\]

where \( p \) is a constant. [1]

(ii) Given that the coefficient of \( x^8 \) in the expansion of \( \left( \frac{2x^2 - \frac{p}{x}}{x} \right)^{10} \) is

negative \( \frac{10}{3} \) times the coefficient of \( x^5 \). Show that the value of \( p \) is \( \frac{1}{2} \). [5]

(iii) Showing all your working, use the value of \( p \) in part (ii), to find the constant term in the expansion of \( (2x-1) \left( \frac{2x^2 - \frac{p}{x}}{x} \right)^{10} \). [5]

5. (a) (i) Show that \( \sin 3x = \sin x(4 \cos^2 x - 1) \) [3]

(ii) Solve the equation \( 3 \sin 3x = 16 \cos x \sin x \) for \( 0 \leq x \leq 2\pi \) [5]

(b) Differentiate \( \cos 2x (\tan^2 x - 1) \) with respect to \( x \). No simplification is required. [3]
The equation of a curve is \( y = x^3 - 4x^2 + px + q \) where \( p \) and \( q \) are constants. The equation of the tangent to the curve at the point \( A(-1, 5) \) is \( 15x - y + 20 = 0 \).

(i) Find the values of \( p \) and of \( q \). [4]
(ii) Determine the values of \( x \) for which \( y \) is an increasing function. [3]
(iii) Find the range of values of \( x \) for which the gradient is decreasing. [2]
(iv) A point \( P \) moves along the curve in such a way that the \( x \)-coordinate of \( P \) increases at a constant rate of 0.02 units per second. Find the possible \( x \)-coordinates of \( P \) at the instant that the \( y \)-coordinate of \( P \) is increasing at 1.9 units per second. [4]

7.

The diagram shows two intersecting circles, \( C_1 \) and \( C_2 \). \( C_1 \) passes through the vertices of the triangle \( AB \). The tangents to \( C_1 \) at \( A \) and \( B \) intersect at the point \( Q \) on \( C_2 \). A line is drawn from \( Q \) to intersect the line \( AD \) at \( E \) on \( C_2 \).

Prove that
(i) \( QE \) bisects angle \( AEB \), [4]
(ii) \( EB = ED \), [2]
(iii) \( BD \) is parallel to \( QE \). [2]
8. The number, \(N\), of E. Coli bacteria increases with time, \(t\) minutes. Measured values of \(N\) and \(t\) are given in the following table.

<table>
<thead>
<tr>
<th>(t)</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N)</td>
<td>3215</td>
<td>3446</td>
<td>3693</td>
<td>3959</td>
<td>4243</td>
</tr>
</tbody>
</table>

It is known that \(N\) and \(t\) are related by the equation \(N = N_0 (2)^{kt}\), where \(N_0\) and \(k\) are constants.

(i) Plot \(\lg N\) against \(t\) and draw a straight line graph. The vertical \(\lg N\) axis should start at 3.40 and have a scale of 2 cm to 0.02.

(ii) Use your graph to estimate the values of \(N_0\) and \(k\).

(iii) Estimate the time taken for the number of bacteria to increase by 25%.

9. A man was driving along a straight road, towards a traffic light junction. When he saw that the traffic light had turned amber, he applied the brakes to his car and it came to a stop just before the traffic light junction. The velocity, \(v\) m/s, of the car after he applied the brakes is given by \(v = 40e^{-\frac{t}{3}} - 15\), where \(t\), the time after he applied the brakes, is measured in seconds.

(i) Calculate the initial acceleration of the car.

(ii) Calculate the time taken to stop the car.

(iii) Obtain an expression, in terms of \(t\), for the displacement of the car, \(t\) seconds after the brakes has been applied.

(iv) Calculate the braking distance.

10. The points \(P(4, 6)\), \(Q(-3, 5)\) and \(R(4, -2)\) lie on a circle.

(i) Find the equation of the perpendicular bisector of \(PQ\).

(ii) Show that the centre of the circle is \((1, 2)\) and find the radius of the circle.

(iii) State the equation of the circle.

(iv) Find the equation of the tangent to the circle at \(R\).
11. The diagram shows part of the curve \( y = x \left( \frac{1}{16} x^2 - 1 \right) \). The curve cuts the x-axis at \( P(4, 0) \). The tangent to the curve at \( P \) meets the vertical line \( x = 6 \) at \( T(6, 4) \).

Showing all your workings, find the total area of the shaded regions. [6]
1  (i)  \( u^2 - 9u + 20 = 0 \)

(ii)  \( x = 1 \)

\( x = 1.2 \)  

(iii)  \(-\sqrt{80} < k < \sqrt{80}\)  

2  (i)  \( 2x^2 - x - 3 \)

(ii)  \( (x^2 + 3x - 1)(2x - 3)(x + 1) \)

(iii)  

\[
p = \frac{-3 \pm \sqrt{13}}{4} \text{ M1}
\]

\[
p = \frac{3}{4} \text{ or } p = -\frac{1}{2}
\]

3  \( x^2 + 4x + 8 = 0 \)

4  (i)  \[
\left(\frac{10}{r}\right)(2x^2)^{10-r}\left(-\frac{P}{x}\right)^{r}
\]

(ii)  

\[
\left(\frac{10}{4}\right)^{2^6} \times \frac{3}{10} = p
\]

\[
p = \frac{1}{2} \text{ AG}
\]

(iii)  -15

5a  (ii)  \( x = 0, \pi, 2\pi \) or \( x = 1.74 \) or \( 4.54 \)

5b  \( 2\cos 2x \tan x \sec^2 x - 2\sin 2x \left( \tan^2 x - 1 \right) \)

6  (i)  \( p = 4 \) \( q = 14 \)

(ii)  \( x < \frac{2}{3} \) or \( x > 2 \)

(iii)  \( x < \frac{4}{3} \)

(iv)  \( x = -\frac{13}{3} \) or \( x = 7 \)
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
</table>
| 8 | (ii) | \( N_o = 2992 \) accept also 2990  
\( k = 0.05 \) |
| (iii) | time taken= 6.4 mins |
| 9 | (i) | \(-\frac{40}{3}\) m/s² |
| (ii) | 2.94s |
| (iii) | \( s = -120e^{-\frac{t}{3}} - 15t + 120 \) |
| (iv) | 30.9m |
| 10 | (i) | \( y = -7x + 9 \) |
| (ii) | \( r = 5 \) units |
| (iii) | \( (x-1)^2 + (y-2)^2 = 25 \) |
| (iv) | \( y = \frac{3}{4}x - 5 \) |
| 11 | \( \frac{25}{4} \) units² |
READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the presentation, fasten all your work securely together.
The number of marks is given in brackets [   ] at the end of each question or part question.
The total number of marks for this paper is 100.
Mathematical Formulae

1. **ALGEBRA**

**Quadratic Equation**

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Binomial Theorem**

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where $n$ is a positive integer and

$$\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

2. **TRIGONOMETRY**

**Identities**

\[
\begin{align*}
\sin^2 A + \cos^2 A &= 1. \\
\sec^2 A &= 1 + \tan^2 A. \\
\csc^2 A &= 1 + \cot^2 A. \\
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\tan(A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A \\
\tan 2A &= \frac{2 \tan A}{1 - \tan^2 A}
\end{align*}
\]

**Formulae for \(\Delta ABC\)**

\[
\begin{align*}
\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\
a^2 &= b^2 + c^2 - 2bc \cos A \\
\Delta &= \frac{1}{2} ab \sin C
\end{align*}
\]
Answer all the questions

<table>
<thead>
<tr>
<th></th>
<th>Solutions</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>(i) $u = 9 - 5 \times \frac{4}{u}$</td>
<td>M1</td>
</tr>
<tr>
<td>[2]</td>
<td>$u^2 - 9u + 20 = 0$</td>
<td>A1</td>
</tr>
<tr>
<td>(ii)</td>
<td>(ii) $(u - 4)(u - 5) = 0$</td>
<td>M1</td>
</tr>
<tr>
<td>[4]</td>
<td>$u = 4$ or $u = 5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$4^x = 4$ or $4^x = 5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = 1$ A1 or $x \log 4 = \log 5$</td>
<td>M1 taking $\log$</td>
</tr>
<tr>
<td></td>
<td>$x = \frac{\log 5}{\log 4} = 1.16$</td>
<td>A1</td>
</tr>
<tr>
<td>(iii)</td>
<td>(iii) $u = k - 5 \times \frac{4}{u}$</td>
<td></td>
</tr>
<tr>
<td>[3]</td>
<td>$u^2 - ku + 20 = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>For no real roots, $(-k)^2 - 4(1)(20) &lt; 0$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$(k - \sqrt{80})(k + \sqrt{80}) &lt; 0$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$-\sqrt{80} &lt; k &lt; \sqrt{80}$</td>
<td>A1</td>
</tr>
</tbody>
</table>
2. (i) By using long division, divide $2x^4 + 5x^3 - 8x^2 - 8x + 3$ by $x^2 + 3x - 1$. [2]

<table>
<thead>
<tr>
<th>2</th>
<th>(i)</th>
<th>$2x^2 - x - 3$</th>
<th>M1 A1</th>
</tr>
</thead>
<tbody>
<tr>
<td>[2]</td>
<td>$x^2 + 3x - 1$</td>
<td>$2x^4 + 5x^3 - 8x^2 - 8x + 3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$- (2x^4 + 6x^3 - 2x^2)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$- x^3 - 6x^2 - 8x$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-(- x^3 - 3x^2 + x)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-3x^2 - 9x + 3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-(-3x^2 - 9x + 3)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

(ii) Factorise $2x^4 + 5x^3 - 8x^2 - 8x + 3$ completely. [2]

<table>
<thead>
<tr>
<th>2</th>
<th>(ii)</th>
<th>$2x^4 + 5x^3 - 8x^2 - 8x + 3 = (x^2 + 3x - 1)(2x^2 - x - 3)$</th>
<th>B1</th>
</tr>
</thead>
<tbody>
<tr>
<td>[2]</td>
<td></td>
<td>$= (x^2 + 3x - 1)(2x - 3)(x+1)$</td>
<td>A1</td>
</tr>
</tbody>
</table>

(iii) Hence find the exact solutions to the equation $32p^4 + 40p^3 - 32p^2 - 16p + 3 = 0$. [4]

<table>
<thead>
<tr>
<th>2</th>
<th>(iii)</th>
<th>Let $x = 2p$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>[4]</td>
<td></td>
<td>$2(2p)^4 + 5(2p)^3 - 8(2p)^2 - 8(2p) + 3 = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$((2p)^2 + 3(2p) - 1)(2(2p) - 3)(2p + 1) = 0$</td>
<td>either B1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(4p^2 + 6p - 1)(4p - 3)(2p + 1) = 0$</td>
<td>or</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(4p^2 + 6p - 1) = 0$ or $(4p - 3) = 0$ or $(2p + 1) = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$p = \frac{-6 \pm \sqrt{36 - 4(4)(-1)}}{2(4)} = \frac{-3 \pm \sqrt{13}}{4}$ A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$p = \frac{3}{4}$ or $p = -\frac{1}{2}$ [A1 for both ans]</td>
<td></td>
</tr>
</tbody>
</table>
3. The roots of the quadratic equation $8x^2 - 4x + 1 = 0$ are $\frac{1}{\alpha^2 \beta}$ and $\frac{1}{\alpha \beta^2}$. Find a quadratic equation with roots $\alpha^3$ and $\beta^3$.

3. [7] \[
\frac{1}{\alpha^2 \beta} + \frac{1}{\alpha \beta^2} = \frac{1}{2} \quad \text{(1)}
\]
\[
\frac{1}{\alpha^3 \beta^3} = \frac{1}{8} \quad \text{(2)}
\]

From (2), $\alpha \beta = \sqrt[3]{8} = 2$ B1

From (1), $\frac{\beta + \alpha}{\alpha^2 \beta^2} = \frac{1}{2}$

$\alpha + \beta = \frac{1}{2} \times 4$

$= 2$ B1

$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha \beta + \beta^2)$ B1

$= (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha \beta]$ B1

$= 2[2^2 - 3 \times 2]$ B1

$= -4$ B1

$\alpha^3 \beta^3 = 8$

Equation is $x^2 + 4x + 8 = 0$ A1

4. (i) Write down the general term in the binomial expansion of $\left(2x^2 - \frac{p}{x}\right)^{10}$.

4 [ 1 ] (i) General term $= \binom{10}{r} \left(2x^2\right)^{10-r} \left(-\frac{p}{x}\right)^r$ A1
(ii) Given that the coefficient of \( x^8 \) in the expansion of \( \left( 2x^2 - \frac{P}{x} \right)^{10} \) is negative \( \frac{10}{3} \) times the coefficient of \( x^5 \). Show that the value of \( p \) is \( \frac{1}{2} \). [5]

4 (ii) For \( x^3 \), \( x^{20-2r} = x^3 \),

\[
20 - 3r = 8
\]

\[ r = 4 \quad \text{seen or any method (M1)} \]

For \( x^5 \), \( x^{20-2r} = x^5 \),

\[
20 - 3r = 5
\]

\[ r = 5 \quad \text{A1 for any correct value of } r \]

\[
\binom{10}{4}(2)^{10-4}\left(-\frac{1}{2}\right)^4 = -\frac{10}{3}\binom{10}{5}(2)^{10-5}\left(-\frac{1}{2}\right)^5
\]

\[
\frac{10}{4}\frac{2^5}{10^5} \times \frac{3}{10} = p \quad \text{M1}
\]

\[ p = \frac{1}{2} \quad \text{AG} \]
### Question 4 (iii) [5]

Showing all your working, use the value of $p$ found in part (i), find the constant term in the expansion of \((2x - 1) \left(2x^2 - \frac{p}{x}\right)^{10}\).

**Solution:**

\[
\left(2x - \frac{1}{2x}\right)^{10}
\]

For \(x^0\), \(20 - 3r = 0\)

\[
r = \frac{20}{3} \text{ (not an integer)}
\]

No constant term in \(\left(2x^2 - \frac{1}{2x}\right)^{10}\)

**For \(x^{-1}\), \(20 - 3r = -1\)**

\[
r = 7
\]

\[
(2x+1)\left(\begin{array}{c} 10 \\ 7 \end{array}\right)*(2x^2)^3\left(-\frac{1}{2x}\right)^7 + \ldots
\]

**Constant term** = \[2x\left(\begin{array}{c} 10 \\ 7 \end{array}\right)\left(2x^2\right)^3\left(-\frac{1}{2x}\right)^7\]

= \(-15\) A1

### Question 5 (a) (i) [3]

Show that \(\sin 3x = \sin x(4 \cos^2 x - 1)\)

**Solution:**

\[
LHS = \sin(x + 2x) \quad \text{Addition formula} \quad M1
\]

\[
= \sin x \cos 2x + \cos x \sin 2x
\]

using \(\cos 2x = 2\cos^2 x - 1\)

or \(2x = 2 \sin x \cos x\) B1

\[
= \sin x(2 \cos^2 x - 1) + \cos x \times 2 \sin x \cos x
\]

Factorisation B1

\[
= \sin x \left(2 \cos^2 x - 1 + 2 \cos^2 x\right)
\]

\[
= \sin x \left(4 \cos^2 x - 1\right)
\]
Solve the equation \( 3 \sin 3x = 16 \cos x \sin x \) for \( 0 \leq x \leq 2\pi \) [5]

5(a) (ii) \[ 3 \sin 3x = 16 \cos x \sin x \]

\[ 3 \sin x \left( 4 \cos^2 x - 1 \right) = 16 \cos x \sin x \]

\[ \sin x \left( 12 \cos^2 x - 16 \cos x - 3 \right) = 0 \quad \text{factorisation with \( \sin x \) seen} \quad \text{M1} \]

\[ \sin x \left( 6 \cos x + 1 \right)(2 \cos x - 3) = 0 \quad \text{correct factorisation of quad} \quad \text{exp B1} \]

\[ \sin x = 0 \quad \text{or} \quad \cos x = -\frac{1}{6} \quad \text{or} \quad \cos x = \frac{3}{2} \quad \text{(rejected)} \quad \text{A1} \]

\[ x = 0, \pi, 2\pi \quad \text{or} \quad x = \pi - 1.40335, \pi + 1.40335 \]

\[ = 1.74 \quad \text{or} \quad 4.54 \quad \text{A1} \quad \text{A1} \]

5(b) Differentiate \( \cos 2x (\tan^2 x - 1) \) with respect to \( x \). No simplification is required [3]

\[ \frac{d}{dx} \left[ \cos 2x (\tan^2 x - 1) \right] \]

\[ = \cos 2x \left( 2 \tan x \sec^2 x \right) + \left( \tan^2 x - 1 \right)(-2 \sin 2x) \quad \text{M1 product rule} \]

\[ = 2 \cos 2x \tan x \sec^2 x - 2 \sin 2x \left( \tan^2 x - 1 \right) \quad \text{B1} \]
The equation of a curve is \( y = x^3 - 4x^2 + px + q \) where \( p \) and \( q \) are constants. The equation of the tangent to the curve at the point \( A(-1, 5) \) is \( 15x - y + 20 = 0 \).

(i) Find the values of \( p \) and of \( q \). [4]

\[
\frac{dy}{dx} = 3x^2 - 8x + p \quad \text{B1}
\]

At \( A(-1, 5) \), equation of the tangent is \( y = 15x + 20 \)

gradient = 15

\( 3(-1)^2 - 8(-1) + p = 15 \) \quad \text{M1}

\( 11 + p = 15 \)

\( p = 4 \quad \text{A1} \)

substitute \( p = 4, x = -1, y = 5 \) into equation of curve

\( 5 = -1 - 4 + 4 + q \)

\( q = 14 \quad \text{A1} \)

(ii) Determine the values of \( x \) for which \( y \) is an increasing function. [3]

For \( y \) to be an increasing function,

\[
\frac{dy}{dx} > 0
\]

\( 3x^2 - 8x + 4 > 0 \quad \text{B1 (with value of } p \text{ substituted)} \)

\( (3x - 2)(x - 2) > 0 \quad \text{M1} \)

\[
\begin{align*}
& \frac{2}{3} < x < 2 \quad \text{A1} \\
& x > 2
\end{align*}
\]

(iii) Find range of values of \( x \) for which the gradient is decreasing. [2]

For decreasing gradient,

\[
\frac{d^2y}{dx^2} < 0 \quad \text{either}
\]

\[
6x - 8 < 0 \quad \text{or} \quad \text{M1}
\]

\( x < \frac{4}{3} \quad \text{A1} \)
A point \( P \) moves along the curve in such a way that the \( x \)-coordinate of \( P \) increases at a constant rate of 0.02 units per second. Find the possible \( x \)-coordinates of \( P \) at the instant that the \( y \)-coordinate of \( P \) is increasing at 1.9 units per second.

\[
\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}
\]

\[
1.9 = \frac{dy}{dx} \times (0.02) \quad \text{M1}
\]

\[
\frac{dy}{dx} = \frac{1.9}{0.02} = 95
\]

\[
3x^2 - 8x + 4 = 95 \quad \text{M1 (quadratic equation in } x)\\
3x^2 - 8x - 91 = 0
\]

\[
(3x + 13)(x - 7) = 0
\]

\[
x = -\frac{13}{3} \text{ or } x = 7 \quad \text{A2}
\]
The diagram shows two intersecting circles, $C_1$ and $C_2$. $C_1$ passes through the vertices of the triangle $ABD$. The tangents to $C_1$ at $A$ and $B$ intersect at the point $Q$ on $C_2$. A line is drawn from $Q$ to intersect the line $AD$ at $E$ on $C_2$.

Prove that

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>(i)</td>
<td>$QE$ bisects angle $AEB$</td>
</tr>
<tr>
<td>(i)</td>
<td>$EB = ED.$</td>
</tr>
<tr>
<td>(ii)</td>
<td>$BD$ is parallel to $QE.$</td>
</tr>
</tbody>
</table>

7.(i)[4] Let $\angle QEA = x^\circ$

$\angle QBA = \angle QEA$ (angles in same segment in $C_2$) B1

$= x^\circ$

$QB = QA$ (tangents to $C_1$ from external point $Q$) B1

$\angle QAB = \angle QBA$ (base angles of isosceles triangle) B1

$= x^\circ$

$\angle QEB = \angle QAB$ (angles in the same segment in $C_2$)

$= x^\circ$

$\therefore \angle QEB = \angle QEA$

Hence $QE$ bisects angle $AEB$. [B1]
7(ii) \( \angle QBA = x^\circ \) (from (i))

\[
\angle ADB = \angle QBA \text{ (angles in alternate segment in } C_1) \text{ either}
\hspace{1cm} = x^\circ
\]

\( \angle AEB = 2x^\circ \) (from (i))

\[
\angle DBE = \angle AEB - \angle ADB \text{ (exterior angle of triangle } BDE) \text{ or B1}
= 2x^\circ - x^\circ
= x^\circ
\]

\( \therefore \angle ADB = \angle EDB = \angle DBE = x^\circ \) (base angles of isosceles triangle BDE) B1

Hence \( EB = ED \)

(iii) [2] From (i) \( \angle EBD = \angle QEB = x^\circ \) B1

\( \therefore \angle EBD \) and \( \angle QEB \) are alternate angles of parallel lines. (alternate angles are equal) B1

BD is parallel to QE
The number, $N$, of E. Coli bacteria increases with time, $t$ minutes. Measured values of $N$ and $t$ are given in the following table.

<table>
<thead>
<tr>
<th>$t$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>3215</td>
<td>3446</td>
<td>3693</td>
<td>3959</td>
<td>4243</td>
</tr>
</tbody>
</table>

It is known that $N$ and $t$ are related by the equation $N = N_o (2)^{kt}$, where $N_o$ and $k$ are constants.

(i) Plot $\lg N$ against $t$ and draw a straight line graph. The vertical $\lg N$ axis should start at 3.40 and have a scale of 2 cm to 0.02.

(ii) Use your graph to estimate the values of $N_o$ and $k$.

(iii) Estimate the time taken for the number of bacteria to increase by 25%.

8. (i) [3] On graph paper

\[ N = N_o (2)^{kt} \]

\[ \lg N = \lg N_o + kt \lg 2 \]

\[ \lg N \text{-intercept} = 3.476 \quad \text{M1} \]

\[ \lg N_o = 3.476 \]

\[ N_o = 2992 \text{ accept also 2990} \quad \text{A1} \]

\[ \text{gradient} = \frac{3552 - 3476}{5 - 0} = 0.0152 \quad \text{M1(with points used to find gradient labelled on graph)} \]

\[ k \lg 2 = 0.0152 \]

\[ k = \frac{0.0152}{\lg 2} \]

\[ = 0.05 \quad \text{A1} \]

(iii) [2] when $N = 125\%$ of 2992

\[ = 3740 \text{ (to 4 sf)} \]

\[ \lg N = \lg 3740 \]

\[ = 3.573 \text{(M1)} \]

From graph, time taken = 6.4 mins \quad \text{A1}
A man was driving along a straight road, towards a traffic light junction. When he saw that the traffic light had turned amber, he applied the brakes to his car and it came to a stop just before the traffic light junction. The velocity, \( v \) m/s, of the car after he applied the brakes is given by \( v = 40e^{-\frac{1}{3}t} - 15 \), where \( t \) is the time after he applied the brakes, is measured in seconds.

(i) Calculate the initial acceleration of the car.

(ii) Calculate the time taken to stop the car.

(iii) Obtain an expression, in term of \( t \), for the displacement of the car, \( t \) seconds after the brakes has been applied.

(iv) Calculate the braking distance.

\[
\begin{align*}
(i) & \quad v = 40e^{-\frac{1}{3}t} - 15 \\
& \quad a = \frac{dv}{dt} = -\frac{40}{3}e^{-\frac{1}{3}t} \quad \text{B1}
\end{align*}
\]

Initial acceleration = \(-\frac{40}{3}\) m/s\(^2\) \quad \text{A1}

(ii) when \( v = 0 \)

\[
\begin{align*}
40e^{-\frac{1}{3}t} - 15 &= 0 \\
\quad e^{-\frac{1}{3}t} &= \frac{3}{8} \\
\quad -\frac{t}{3} &= \ln\frac{3}{8} \quad \text{(M1 taking logarithm)} \\
\quad t &= -3\ln\frac{3}{8} \\
&= 2.94 \text{s} \quad \text{(A1)}
\end{align*}
\]

(iii) \( s = \int \left(40e^{-\frac{1}{3}t} - 15\right) dt \quad \text{M1} \)

\[
\begin{align*}
&= -120e^{-\frac{1}{3}t} - 15t + c \quad \text{B1} \\
\quad \text{when } t = 0, s = 0, \text{ where } s \text{ is the displacement from the point where the brakes was applied.} \\
\quad c &= 120 \\
\quad s &= -120e^{-\frac{1}{3}t} - 15t + 120 \quad \text{A1}
\end{align*}
\]

(iv) Substitute \( t = -3\ln\frac{3}{8} \), Braking distance = \(-120 \left(\frac{3}{8}\right) - 15 \left(-3\ln\frac{3}{8}\right) + 120 \)

\[
\begin{align*}
&= 30.9 \text{m (to 3 sf)} \quad \text{A1}
\end{align*}
\]
10. The points \(P(4, 6), Q(-3, 5)\) and \(R(4, -2)\) lie on a circle.

(i) Find the equation of the perpendicular bisector of \(PQ\). [3]

(ii) Show that the centre of the circle is \((1, 2)\) and find the radius of the circle. [3]

(iii) State the equation of the circle. [1]

(iv) Find the equation of the tangent to the circle at \(R\). [3]

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<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>10.</td>
<td></td>
</tr>
<tr>
<td>(i)</td>
<td>midpoint of (PQ = \left(\frac{1}{2}, \frac{11}{2}\right)) B1</td>
</tr>
<tr>
<td></td>
<td>gradient of (PQ = \frac{1}{7}) B1</td>
</tr>
<tr>
<td></td>
<td>gradient of perpendicular bisector of (PQ = -7) B1</td>
</tr>
<tr>
<td></td>
<td>Equation of perpendicular bisector of (PQ) is</td>
</tr>
<tr>
<td></td>
<td>(y - \frac{11}{2} = -7\left(x - \frac{1}{2}\right))</td>
</tr>
<tr>
<td></td>
<td>(y = -7x + 9) A1</td>
</tr>
<tr>
<td>(ii)</td>
<td>Equation of perpendicular bisector of (PR) is (y = 2) B1</td>
</tr>
<tr>
<td></td>
<td>Alternatively use : Equation of perpendicular bisector of (QR) is (y = x + 1)</td>
</tr>
<tr>
<td></td>
<td>Since perpendicular bisector of chords passes through centre of circle,</td>
</tr>
<tr>
<td></td>
<td>for centre of circle, substitute (y = 2) into (y = -7x + 9)</td>
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<td></td>
<td>(2 = -7x + 9) M1 solving simultaneous equations</td>
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<td>(7x = 7)</td>
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<td></td>
<td>(x = 1)</td>
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<td></td>
<td>centre = ((1, 2)) AG</td>
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<td>Alternative method : centre = ((a, -7a + 9)) B1</td>
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<td>(RC = PC) M1 forming an equation in (a)</td>
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<td></td>
<td>(r = ) distance between centre and (P)</td>
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<td></td>
<td>(= \sqrt{(4 - 1)^2 + (6 - 2)^2})</td>
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<td>(= 5) units A1</td>
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<td>(iii)</td>
<td>Equation of circle is ((x - 1)^2 + (y - 2)^2 = 25) A1</td>
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<tr>
<td>(iv)</td>
<td>gradient of normal at (R = \frac{2 - (-2)}{1 - 4} = -\frac{4}{3}) M1</td>
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<td>gradient of tangent at (R = \frac{3}{4}) M1</td>
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<td></td>
<td>Equation of tangent at (R) is (y + 2 = \frac{3}{4}(x - 4))</td>
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<td>(y = \frac{3}{4}x - 5) A1</td>
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</table>
The diagram shows part of the curve $y = x\left(\frac{1}{16}x^2 - 1\right)$. The curve cuts the $x$-axis at $P(4, 0)$. The tangent to the curve at $P$ meets the vertical line $x = 6$ at $T(6, 4)$.

Showing all your workings, find the total area of the shaded regions. [6]

Area of total shaded regions $= \left[ -\int_0^4 \left(\frac{x^3}{16} - x\right) \, dx + \int_4^6 \left(\frac{x^3}{16} - x\right) \, dx - \frac{1}{2} \times 2 \times 4 \right]$

$= \left[ \left. -\frac{1}{16} \times \frac{x^4}{4} + \frac{x^2}{2} \right|_0^4 + \left. \frac{1}{16} \times \frac{x^4}{4} - \frac{x^2}{2} \right|_4^6 - 4 \right]$

$= -\frac{1}{64} \times 4^4 + \frac{1}{2} \times 4^2 + \left(\frac{6^4}{64} - \frac{6^2}{2}\right) - \left(\frac{4^4}{64} - \frac{4^2}{2}\right) - 4$

M1 correct substitution of upper and lower limits

$= \frac{25}{4}$ units$^2$ A1

End of paper