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<td>Geylang Methodist Secondary School</td>
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<td>25</td>
<td>Naval Base Secondary School</td>
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READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.
2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial Theorem
\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,
\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{(n-r)! r!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. TRIGONOMETRY

Identities
\[
\sin^2 A + \cos^2 A = 1.
\]
\[
\sec^2 A = 1 + \tan^2 A.
\]
\[
\csc^2 A = 1 + \cot^2 A.
\]
\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
\sin 2A = 2 \sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]
\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \triangle ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2} ab \sin C
\]
Answer all the questions.

1. The equation of a curve is given by \( f(x) = 2x^3 - 12x - 5 \). Find the range of values of \( x \) for which \( f(x) \) is an increasing function. [3]

2. (i) Given that \((3k - 5)x^2 + (k - 5)x - 2 = 0\) has no real roots, what condition must apply to the constant \( k \)? [3]
   (ii) From your results in part (i), determine if \( y = (3k - 5)x^2 + (k - 5)x - 2 \) has a minimum or maximum point. [2]

3. A sky diver jumps from a certain height above the ground. The downward velocity, \( v \text{ m/s} \), of the sky diver at time \( t \) seconds is given by \( v = 30(1 - e^{-0.2t}) \).
   (i) Find the initial velocity of the sky diver. [1]
   (ii) Find the velocity of the sky diver after 5 seconds. [1]
   (iii) Showing your working clearly, explain why the velocity experienced by the sky diver will not exceed 30 \( \text{m/s} \). [2]

4. (i) Find the values of \( \log_4 x \) that will satisfy the equation \( 2(\log_4 x)^2 = \log_4 x + 6 \). [3]
   (ii) Sketch the graph of \( y = \log_4 x \) and indicate clearly on your graph the location of the values of \( \log_4 x \) found in part (i). [2]
   Hence, show that the product of the two roots of the equation
   \[ 2(\log_4 x)^2 = \log_4 x + 6 \]
   is positive. [1]
5. A vertical wall $AB$ is 2 m high and 2 m away from a warehouse. $PQ$ is a ramp resting on the wall $AB$ and just touching the ground at $P$ and the warehouse at $Q$. The ramp $PQ$ is of length $L$ metres and makes an angle $\theta$ with the horizontal.

(i) Show that the length of the ramp, $L$, is given by

$$L = \frac{2}{\sin \theta} + \frac{2}{\cos \theta}$$

(ii) Hence, show that

$$\frac{dL}{d\theta} = \frac{2\sin^3 \theta - 2\cos^3 \theta}{\sin^2 \theta \cos^2 \theta}$$

(iii) Given that $\theta$ can vary, find the shortest possible length of the ramp.

6. (i) Sketch the curve $y^2 = 9x$ for $0 \leq y \leq 12$.

The line $4y - 3x = 9$ intersects the curve $y^2 = 9x$ at two points $P$ and $Q$.

(ii) Find the coordinates of the midpoint of $PQ$.

7. (i) Given that

$$\frac{\sin (A-B)}{\sin (A+B)} = \frac{3}{2},$$

prove that $\tan A + 5 \tan B = 0$.

(ii) Hence, solve the equation $2 \sin(2\theta - 30^\circ) = 3 \sin(2\theta + 30^\circ)$ for $0^\circ \leq \theta \leq 360^\circ$. 
8 The diagram shows part of the graph of \( y = |3 - x| - 2 \).

(i) Find the coordinates of \( A, B \) and \( C \). [4]

A line \( QR \) of gradient 1 cuts the y-axis at \((0, p)\).

(ii) State the number of intersection(s) of the line \( QR \) and \( y = |3 - x| - 2 \) when

(a) \( p = 2 \) [1]

(b) \( p = -6 \) [1]

(iii) Determine the set of values of \( p \) for which the line \( QR \) intersects \( y = |3 - x| - 2 \) at only one point. [1]

9 A particle travelling in a straight line, passes a fixed point \( O \) on the line with a velocity of \( 9 \text{m/s} \). The acceleration, \( a \text{ m/s}^2 \), of the particle \( t \text{ seconds} \) after passing through \( O \) is given by \( a = 8 - 2t \).

(i) Show that the particle comes to instantaneous rest when \( t = 9 \). [3]

(ii) Find the average speed of the particle for the journey from \( t = 0 \) to \( t = 12 \). [5]
10 The diagram shows a circle passing through the points \( P, Q, R \) and \( S \). \( SQU \) is a straight line that cuts \( RP \) at the point \( T \). \( VRU \) is a tangent to the circle at \( R \) such that \( SR = RU \).

![Diagram with points P, Q, R, S, T, and V, U as mentioned in the text.]

Prove that

(i) \( \angle SPT = 2 \times \angle QPT \),

(ii) triangle \( QRU \) is similar to triangle \( RSU \),

(iii) \( QR \times SU = (RS)^2 \)  [2]

11 A container has a capacity of 960 \( \text{cm}^3 \) and is initially completely filled with water. The volume, \( V \) \( \text{cm}^3 \), of water in the container is given by \( V = h^2 + 2h \) where \( h \) \( \text{cm} \) is the height of the water level in the container. Due to leakage at the bottom of the container, the height of the water level in the container decreases at a rate of \( \frac{3t}{2} \) \( \text{cm/s} \).

(i) Find the initial height of the water level in the container.  [3]

(ii) Show that the height, \( h \), can be expressed as \( -\frac{3t^2}{4} + c \), where \( c \) is a constant.  [2]

(iii) Find the rate of change of volume when \( t = 4 \).  [3]
12 (a) The diagram below shows part of the curve \( f(x) = 3 \sin(px) - q \).

The coordinates of the turning points are \( A\left(\frac{3\pi}{4}, -2\right) \) and \( B\left(\frac{9\pi}{4}, -8\right) \).

Find the values of \( p \) and \( q \). \[2\]

(b) The diagram below shows the graph of \( y = x^2 + 2 \). The shaded region from \( x = a \) to \( x = -a \) has an area of \( 6a \) units\(^2\). Find the exact value of \( a \). \[5\]
Answer key:

1. \[ x < -\sqrt{2} \text{ or } x > \sqrt{2} \]
2. (i) \(-15 < k < 1\); (ii) maximum
3. (i) 0 m/s  (ii) 19.0 m/s
4. (i) \(-\frac{3}{2}, 2\)
5. (iii) \(\frac{\pi}{4}, 5.66\text{m}\)
6. (i)

(ii) (5,6)

7(ii) 54.6°, 144.6°, 234.6°, 324.6

8(i) (5,0)  (ii)(a) 1  (ii)(b) 0  (iii) \(p > -5\)

9(i) \[ v = 8t - t^2 + 9 \]
   (ii) \[ s = 4t^2 - \frac{t^3}{3} + 9t; \quad 18 \text{ m/s} \]

11(i) 30 cm  (ii) \[ h = -\frac{3t^2}{4} + 30 \quad \text{(iii) } -228 \text{ cm}^3/\text{s} \]

12(a) \(p = \frac{2}{3}; \quad q = 5\)  (b) \(a = \sqrt{3}\)
1 \[ f(x) = 2x^3 - 12x - 5 \]
\[ f'(x) = 6x^2 - 12 \]
For increasing functions, \( f'(x) > 0 \)
\[ 6x^2 - 12 > 0 \]
\[ x^2 - 2 > 0 \]
\[ (x + \sqrt{2})(x - \sqrt{2}) > 0 \]
\[ \therefore \text{ the range of values of } x \text{ is } x < -\sqrt{2} \text{ or } x > \sqrt{2}. \]

2(i) \[ (3k - 5)x^2 + (k - 5)x - 2 = 0 \]
No real roots \( \Rightarrow \) discriminant < 0
\[ (k - 5)^2 - 4(3k - 5)(-2) < 0 \]
\[ k^2 - 10k + 25 + 24k - 40 < 0 \]
\[ k^2 + 14k - 15 < 0 \]
\[ (k + 15)(k - 1) < 0 \]
\[ -15 < k < 1 \]

2(ii) coeff of \( x^2 = 3k - 5 \)
From above, \(-15 < k < 1\)
\[-45 < 3k < 3\]
\[-50 < 3k - 5 < -2\]
Since coeff of \( x^2 < 0 \), the function has a maximum point.

**Alternative method:**
\[ y' = 2(3k - 5)x + (k - 5) \]
\[ y'' = 2(3k - 5) = 6k - 10 \]
From (i), since \(-15 < k < 1\), \(6k - 10 < 0\)
\[ \Rightarrow y'' < 0 \ \forall x \]
\[ \therefore y = (3k - 5)x^2 + (k - 5)x - 2 \text{ has a max point.} \]
3 \( v = 30(1 - e^{-0.2t}) \)

i) initial velocity, \( v = 30(1 - e^0) = 0 \text{ m/s} \)

ii) when \( t = 5, v = 30(1 - e^{-1}) = 30 \left( 1 - \frac{1}{e} \right) \) or 19.0 m/s

iii) since \( t \geq 0, 0 < e^{-0.2t} \leq 1 \)
    \[ \Rightarrow \max(1 - e^{-0.2t}) < 1 \]
    \[ \Rightarrow 30(1 - e^{-0.2t}) < 30 \]
    \[ \therefore \text{the velocity will never exceed 30 m/s.} \]

4i) \( 2(\log_4 x)^2 = (\log_4 x) + 6 \)

Let \( y = \log_4 x \)

\[ 2y^2 = y + 6 \]

\[ 2y^2 - y - 6 = 0 \]

\[ (2y + 3)(y - 2) = 0 \]

\( y = -\frac{3}{2} \) or \( y = 2 \)

\[ \therefore \log_4 x = -\frac{3}{2} \text{ or } \log_4 x = 2 \]

4ii)

From the graph, when \( y = -\frac{3}{2} \) and \( y = 2 \), the \( x \) values are both positive.

\[ \therefore \text{the product of the two roots of } 2(\log_4 x)^2 = (\log_4 x) + 6 \text{ is positive.} \]
5i) \[ L = PB + BQ \]
\[ \sin \theta = \frac{2}{PB} \quad \Rightarrow \quad PB = \frac{2}{\sin \theta} \]
\[ \cos \theta = \frac{2}{BQ} \quad \Rightarrow \quad BQ = \frac{2}{\cos \theta} \]
\[ \therefore \quad L = \frac{2}{\sin \theta} + \frac{2}{\cos \theta} \quad [AG] \]

5ii) \[ \frac{dL}{d\theta} = \frac{-2 \cos \theta}{\sin^2 \theta} + \frac{2 \sin \theta}{\cos^2 \theta} \]
\[ = \frac{2 \sin^3 \theta - 2 \cos^3 \theta}{\sin^2 \theta \cos^2 \theta} \quad [AG] \]

5iii) For max/min, \[ \frac{dL}{d\theta} = 0 \]
\[ 2 \sin^3 \theta - 2 \cos^3 \theta = 0 \]
\[ \sin^3 \theta = \cos^3 \theta \]
\[ \tan^3 \theta = 1 \]
\[ \tan \theta = 1 \]
\[ \theta = \frac{\pi}{4}, \quad 0 < \theta < \frac{\pi}{2} \]

Using 1st derivative test,

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\[ \frac{dL}{d\theta} \]
\[ \begin{array}{ccc}
- & 0 & + \\
\end{array} \]

\[ \therefore \quad \text{shortest possible length of the ramp} \]
\[ = \frac{2}{\sin \frac{\pi}{4}} + \frac{2}{\cos \frac{\pi}{4}} \]
\[ = 5.66 \text{ m} \quad [5.6568] \]
6i) \[ y^2 = 9x \]

\[ y = \frac{3x+9}{4} \]

Subs \( y \) into \( y^2 = 9x \)

\[ \left(\frac{3x+9}{4}\right)^2 = 9x \]

\[ x^2 - 10x + 9 = 0 \]

\[ (x - 9)(x - 1) = 0 \]

\[ x = 1 \text{ or } x = 9 \]

x-coord of midpoint of PQ = \( \frac{1+9}{2} = 5 \)

y-coord of midpoint of PQ = \( \frac{3(5)+9}{4} = 6 \)

: coods of midpoint of PQ are (5,6)
\[ \frac{\sin(A-B)}{\sin(A+B)} = \frac{3}{2} \]

\[ \frac{\sin A \cos B - \cos A \sin B}{\sin A \cos B + \cos A \sin B} = \frac{3}{2} \]

\[ 2(\sin A \cos B - \cos A \sin B) = 3(\sin A \cos B + \cos A \sin B) \]

\[ \sin A \cos B + 5\cos A \sin B = 0 \]

Divide throughout by \( \cos A \cos B \),
\[ \therefore \tan A + 5\tan B = 0 \quad [AG] \]

---

7ii) \[ 2 \sin(2\theta - 30^\circ) = 3 \sin(2\theta + 30^\circ) \] can be written as
\[ \frac{\sin(2\theta - 30^\circ)}{\sin(2\theta + 30^\circ)} = \frac{3}{2} \]

Compare with (i) and let \( A = 2\theta \) and \( B = 30^\circ \),
\[ \therefore \tan 2\theta + 5\tan 30^\circ = 0 \quad \text{using result from (i)} \]

\[ \tan 2\theta = -5 \left( \frac{1}{\sqrt{3}} \right) \]

base angle, \( \alpha = \tan^{-1} \left( \frac{5}{\sqrt{3}} \right) = 70.893^\circ \)

\[ 2\theta = 109.106^\circ, \ 289.106^\circ, \ 469.106^\circ, \ 649.106^\circ \]
\[ \therefore \theta = 54.6^\circ, \ 144.6^\circ, \ 234.6^\circ, \ 324.6^\circ \]
8i) \[ y = |3 - x| - 2 \]

At \( A \), \( x = 0 \), \( y = 3 - 2 = 1 \)
\[ \therefore A(0, 1) \]
At \( B \), \( \min|3 - x| = 0 \Rightarrow x = 3, y = -2 \)
\[ \therefore B(3, -2) \]
At \( C \), \( y = 0 \), \( |3 - x| - 2 = 0 \)
\[ |3 - x| = 2 \]
\[ 3 - x = 2 \quad \text{or} \quad 3 - x = -2 \]
\[ x = 1 \quad \text{or} \quad x = 5 \]
\[ \therefore C(5, 0) \]

8ii) line \( QR \): \( y = x + p \)

a) When \( p = 2 \),
\[ \text{no. of intersections} = 1 \]
b) When \( p = -6 \),
\[ \text{no. of intersections} = 0 \]

8iii) set of values of \( p \) for which no. of intersections is 1, is \( p > -5 \)
9) \( t = 0 \text{s}, \; v = 9 \text{m/s}, \; a = 8 - 2t \)

i) \( v = \int a \, dt \)
   \[ = \int (8 - 2t) \, dt \]
   \[ = 8t - t^2 + c \]

When \( t = 0 \), \( v = 9 \)

\[ 8t - t^2 + c = 9 \]
\[ c = 9 \]

\[ \therefore v = 8t - t^2 + 9 \]

At instantaneous rest, \( v = 0 \),

\[ \therefore 8t - t^2 + 9 = 0 \]
\[ t^2 - 8t - 9 = 0 \]
\[ (t + 1)(t - 9) = 0 \]
\[ t = -1 \text{ (reject) or } t = 9 \text{ s [AG]} \]

9ii) \( s = \int v \, dt \)
   \[ = \int (8t - t^2 + 9) \, dt \]
   \[ = 4t^2 - \frac{t^3}{3} + 9t + c \]

When \( t = 0 \), \( s = 0 \) \( \Rightarrow c = 0 \)

\[ \therefore s = 4t^2 - \frac{t^3}{3} + 9t \]

At instantaneous rest, \( v = 0 \), \( t = 9 \), \( s = 162 \text{m} \)

\[ t = 12, \; s = 108 \text{m} \]

Total distance = 162 + (162 - 108) = 216 m

\[ \therefore \text{average speed} = \frac{216 \text{m}}{12 \text{s}} = 18 \text{ m/s} \]
10) Let \( \angle RSU = x \)
then \( \angle RUS = x \) (base \( 4s \), isos \( \triangle \))
\( \angle QPT = \angle RSQ \)
\[ = x \] (4s in the same segment)
\( \angle SRV = 2x \) (ext \( 4 \) of \( \triangle SRU \))
\( \angle SPT = \angle SRV \) (alt segment thm)
\[ = 2x \]
\( \therefore \angle SPT = 2 \times \angle QPT \) [AG]

10ii) From [i], \( \angle QUR = \angle RUS \) (common \( 4 \))
\( \angle QRU = \angle RSU \) (alt segment thm)
\( \angle RQU = \angle SRU \) (\( 4 \) sum of \( \triangle \))
\( \therefore \triangle QRU \) is similar to \( \triangle RSU \) (AAA similarity)

10iii) Using ratio of corresponding sides of similar \( \triangle QRU \) & \( RSU \),
\[ \frac{QR}{RS} = \frac{RU}{SU} \]
\[ QR \times SU = RU \times RS \]
\[ QR \times SU = (RU)^2 \] [AG] \( \therefore RU = RS \) given
11) Given: \( V = 960 \text{cm}^3 \) at \( t = 0 \); \( V = h^2 + 2h \); \( \frac{dh}{dt} = -\frac{3t}{2} \text{ cm/s} \)

11i) \( h^2 + 2h = 960 \)

\[ h^2 + 2h - 960 = 0 \]
\[ (h + 32)(h - 30) = 0 \]

\( h = 30 \) or \( h = -32 \) (rejected)

\( \therefore \) initial height of water is 30 cm.

11ii) \( \frac{dh}{dt} = -\frac{3t}{2} \)

\[ h = -\frac{3t^2}{4} + c \]

when \( t = 0 \), \( h = 30 \)

\( \Rightarrow c = 30 \)

\[ \therefore h = -\frac{3t^2}{4} + 30 \]

11iii) \( \frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} \)

\[ = (2h + 2) \times \left(-\frac{3t}{2}\right) \]
\[ = \left[2 \left(-\frac{3t^2}{4} + 30\right) + 2\right] \times \left(-\frac{3t}{2}\right) \]

when \( t = 4 \), rate of change of vol \( \frac{dV}{dt} \)|\(_{t=4}\)

\[ = -228 \text{ cm}^3/\text{s} \]
12a) \( f(x) = 3 \sin(px) - q \)
\[ -q = \frac{-2 + (-8)}{2} \]
\[ = -5 \]
\[ \therefore q = 5 \]

\[ \text{period} = \frac{2\pi}{p} \]

From the graph, \( \text{period} = \left( \frac{9\pi}{4} - \frac{3\pi}{4} \right) \times 2 = 3\pi \)
\[ \frac{2\pi}{p} = 3\pi \]
\[ p = \frac{2}{3} \]

12b) Since graph of \( y = x^2 + 2 \) is symmetrical about the x-axis,
\[ \int_0^a y \, dx = \frac{6a}{2} \]
\[ \int_0^a (x^2 + 2) \, dx = \frac{6a}{2} \]
\[ \left[ \frac{x^3}{3} + 2x \right]_0^a = 3a \]
\[ \frac{a^3}{3} + 2a = 3a \]
\[ a^3 + 6a - 9a = 0 \]
\[ a^3 - 3a = 0 \]
\[ a(a^2 - 3) = 0 \]
\[ a = 0 (\text{rejected}), \quad a^2 = 3 \]
\[ \therefore a = \sqrt{3} \text{ since } a > 0 \]
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Write your name, class and index number on all the work you hand in.
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Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where $n$ is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\csc^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\Delta ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$
1 (i) Sketch the graph of \( y = 2x^2 \) for \( x > 0 \). [1]

(ii) On the same diagram, sketch the graph of \( y = 16x^{-\frac{1}{2}} \) for \( x > 0 \). [1]

(iii) Calculate the \( x \)-coordinate of the point of intersection of your graphs. [2]

2 (a) A polynomial \( f(x) \) has a remainder of \(-2\) when divided by \((2x + 1)\). Showing your method clearly,

(i) find the remainder when \( f(x) - 1 \) is divided by \((2x + 1)\), [2]

(ii) find in terms of \( f(x) \), a polynomial which is completely divisible by \((2x + 1)\). [2]

(b) A polynomial \( g(x) \) can be expressed as \( g(x) = (x^2 - x - 2)P(x) + ax + b \), where \( P(x) \) is a polynomial in \( x \). Given that \( g(x) \) leaves a remainder of \(-7\) when divided by \((x - 2)\) and a remainder of \(-19\) when divided by \((x + 1)\)

(i) Find the value of \( a \) and of \( b \). [5]

(ii) Find the remainder when \( g(x) \) is divided by \((x - 2)(x + 1)\). [1]

3 Do not use a calculator in this question.

(a) (i) Simplify \((2 - \sqrt{5})^2\). [1]

(ii) Given that \( x = \frac{1}{2 - \sqrt{5}} \), find the exact value of \( x^2 + x - 2 \) [3]

(b) The volume of a cuboid with a square base is \( 19 + 11\sqrt{3} \) cm\(^3\). The height of the cuboid is \( \sqrt{3} + 1 \) cm and the length of each side of the square base is \( a + \sqrt{b} \), where \( a \) and \( b \) are integers. Find the values of \( a \) and of \( b \). [6]
4 (a) The roots of the quadratic equation $2x^2 + 5x - 1 = 0$ are $\tan A$ and $\tan B$.

(i) Find the value of $\tan(A + B)$.

(ii) Find the value of $\sec^2(A + B)$.

(b) (i) Show that \[\frac{2}{1 - \sin 3x} + \frac{2}{1 + \sin 3x} = 4\sec^2 3x.\]

(ii) Hence evaluate $\int_0^{\frac{\pi}{12}} \frac{2}{1 - \sin 3x} + \frac{2}{1 + \sin 3x} \, dx$.

5 A curve has the equation $y = 3x^2 e^{-x}$.

(i) Find an expression for $\frac{dy}{dx}$ and obtain the coordinates of the stationary points of the curve.

(ii) Determine the nature of these stationary points.

6 (a) Find in ascending powers of $x$, the first four terms in the expansion of $(1 + x - x^2)^9$.

(b) (i) Find the term independent of $x$ in the expansion of $\left(2x^2 - \frac{1}{2x}\right)^{12}$.

(ii) Determine the constant term in the expansion of $(3 + 4x^3) \left(2x^2 - \frac{1}{2x}\right)^{12}$.

7 A curve is such that $\frac{d^2y}{dx^2} = \frac{6}{(2x - 5)^2}$.

The equation of the tangent to the curve at the point $(3, -1)$ is $y - 2x + 7 = 0$.

(i) Find an expression for $\frac{dy}{dx}$.

(ii) Find the equation of the curve.
The table shows experimental values of the variables $x$ and $y$.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y$</td>
<td>0.4</td>
<td>0.6</td>
<td>1.6</td>
<td>3.4</td>
<td>6</td>
</tr>
</tbody>
</table>

It is known that $x$ and $y$ are related by the equation of the form $p(x + y) = pq + qx^2$.

(i) Plot $x + y$ against $x^2$, draw the straight line graph and use it to estimate the value of $p$ and $q$. [6]

(ii) Using your values of $p$ and $q$, find the values of $x$ for which $p(x^2 - 2q) = 2qx^2$. [2]

9 (a)

The circle with centre $C(3, 1)$ touches the $x$-axis at $A$. The line $4y - 3x = 0$ touches the circle at $B$.

Find the coordinates of $B$. [5]

(b) The equation of another circle is $(x - 4)^2 + (y + 1)^2 = 4$.

The line $y = mx$ is a tangent to the circle. Find the possible exact values of $m$. [4]
10  (a)  (i)  Express  \( \frac{2x^3 + x^2}{x^2 + x - 2} \) in the form of \( ax + b + \frac{cx + d}{x^2 + x - 2} \).  

(ii) Using the values of \( c \) and \( d \) found in (i), express \( \frac{cx + d}{x^2 + x - 2} \) as a sum of two partial fractions.  

(b) A curve has the equation \( y = \frac{x - 1}{\sqrt{4x + 1}} \)  

(i) Differentiate \( y \) with respect to \( x \).  

(ii) Using the result in part (b)(i), determine \( \int \frac{2(2x+3)}{(4x+1)^3} \, dx \).  

11.  

The diagram shows two circles, \( C_1 \) and \( C_2 \) with centres \( A \) and \( B \) respectively. The two circles touch each other at \( D \). \( C_1 \) has radius 3 units and touches the \( y \)-axis at \( E \). \( C_2 \) has radius 2 units and touches the \( x \)-axis at \( F \). The lines \( AB \) produced meets the \( x \)-axis at \( G \) and angle \( BGO = \theta \) radians.  

(i) Show with clear explanations, that \( OE = 5 \sin \theta + 2 \) and \( OF = 5 \cos \theta + 3 \).  

(ii) Show that \( EF^2 = 38 + 20 \sin \theta + 30 \cos \theta \).  

(iii) Express \( EF^2 \) in the form \( 38 + R \cos(\theta - \alpha) \), where \( R > 0 \) and \( \alpha \) is an acute angle.  

(iv) Given that \( EF^2 = 65 \), find the value of \( \theta \).  

END OF PAPER
### Answer Key

1. (i) (ii)

#### 2i
- Remainder $= -3$
- $a = 4, b = -15$

#### 3i
- Remainder $= 4x - 15$

#### 3ai
- $9 - 4\sqrt{5}$

#### 3aiv
- $5 + 3\sqrt{5}$

#### 3b
- $a = 2$ and $b = 3$

#### 4ai)
- $\frac{5}{3}$

#### 4aiv)
- $\frac{34}{9}$

#### 4bi)
- $\frac{4}{3}$

#### 5ai
- $3xe^{-x}(2 - x), (0, 0)$ and $\left(2, \frac{12}{e^2}\right)$

#### 5ii
- $(2, \frac{12}{e^2})$ is a maximum point
- $(0, 0)$ is a minimum point

#### 5bi)
- $495$

#### 5bii)
- $1265$

#### 6a
- $1 + 9x + 27x^2 + 12x^3 + \ldots$)

#### 7i
- $\frac{dy}{dx} = -\frac{3}{(2x - 5)} + 5$

#### 7ii
- $y = -\frac{3\ln(2x - 5)}{2} + 5x - 16$
1. (i) Sketch the graph of \( y = 2x^{\frac{5}{2}} \) for \( x > 0 \).
   
   (ii) On the same diagram, sketch the graph of \( y = 16x^{-\frac{1}{2}} \) for \( x > 0 \).
   
   (iii) Calculate the \( x \)-coordinate of the point of intersection of your graphs.

2. (a) A polynomial \( f(x) \) has a remainder of \(-2\) when divided by \((2x + 1)\). Showing your method clearly,

   (i) find the remainder when \( f(x) - 1 \) is divided by \((2x + 1)\),
   
   (ii) find in terms of \( f(x) \), a polynomial which is completely divisible by \((2x + 1)\).

<table>
<thead>
<tr>
<th>2(a) (i)</th>
<th>( \text{Let } f(x) = (2x + 1)Q(x) - 2 )</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>[2]</td>
<td>( f(x) - 1 = (2x + 1)Q(x) - 2 - 1 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Remainder = (-3)</td>
<td>B1</td>
</tr>
<tr>
<td>(ii)</td>
<td>( f(x) + 2 = (2x + 1)Q(x) - 2 + 2 )</td>
<td>M1</td>
</tr>
<tr>
<td>[2]</td>
<td>A polynomial = ( f(x) + 2 ), any multiple of ( f(x) + 2 )</td>
<td>B1</td>
</tr>
</tbody>
</table>
(b) A polynomial \( g(x) \) can be expressed as \( g(x) = (x^2 - x - 2)P(x) + ax + b \), where \( P(x) \) is a polynomials in \( x \). Given that \( g(x) \) leaves a remainder of \(-7\) when divided by \((x - 2)\) and a remainder of \(-19\) when divided by \((x + 1)\).

(i) Find the value of \( a \) and of \( b \). \([5]\]

(ii) Find the remainder when \( g(x) \) is divided by \((x - 2)(x + 1)\). \([1]\)

\[
\begin{array}{|c|c|c|}
\hline
2(b) (i) & g(x) = (x^2 - x - 2)P(x) + ax + b, & (x - 2)(x + 1) \text{ seen or} \\
[5] & = (x - 2)(x + 1)P(x) + ax + b, & (-1)^2 - (-1) - 2 \text{ seen or} \\
& \text{Substituting } x = -1 \text{ or } 2 & 2^2 - 2 - 2 \text{ seen B1} \\
g(2) = 2a + b = -7 & & \\
2a + b = -7 \ldots \ldots \ldots \ldots \ldots \ldots (1) & & B1 \\
g(-1) = -a + b = -19 \ldots \ldots \ldots \ldots \ldots (2) & & B1 \\
\hline
2(b) (i) & (1) - (2), 3a = 12 & \\
a = 4 & & A1 \\
\hline
b = -15 & & A1 \\
\hline
\hline
(b) (ii) & \text{Remainder} = 4x - 15 & \text{A1} \\
[1] & & \\
\hline
\end{array}
\]

3 Do not use a calculator in this question.

(a) (i) Simplify \( (2 - \sqrt{5})^2 \). \([1]\)

(ii) Given that \( x = \frac{1}{2 + \sqrt{5}} \), find the exact value of \( x^2 + x - 2 \). \([3]\)

\[
\begin{array}{|c|c|}
\hline
3(a) (i) & (2 - \sqrt{5})^2 = 4 - 4\sqrt{5} + 5 \text{ A1} \\
[1] & = 9 - 4\sqrt{5} & \\
\hline
(ii) & x^2 + x - 2 = \frac{1}{9 - 4\sqrt{5}} + \frac{1}{2 - \sqrt{5}} - 2 \text{ B1} \\
& \frac{6 + 4\sqrt{5}}{81 - 80} + \frac{2 + \sqrt{5}}{-1} - 2 \text{ Rationalising the denominator M1} \\
& = 5 + 3\sqrt{5} \text{ A1} \\
[3] & & \\
\hline
\end{array}
\]
(b) The volume of a cuboid with a square base is $19 + 11\sqrt{3}$ cm$^3$. The height of the cuboid is $\sqrt{3} + 1$ cm and the length of each side of the square base is $a + \sqrt{b}$, where $a$ and $b$ are integers. Find the values of $a$ and of $b$.

\[
\text{Area} = \frac{19 + 11\sqrt{3}}{\sqrt{3} + 1} \\
= \frac{19 + 11\sqrt{3}}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1} \\
= \frac{19\sqrt{3} + 33 - 19 - 11\sqrt{3}}{2} \\
= \frac{14 + 8\sqrt{3}}{2} \\
\]
\[
(a + \sqrt{b})^2 = \frac{14 + 8\sqrt{3}}{2} \\
a^2 + b + 2a\sqrt{b} = 7 + 4\sqrt{3} \\
a^2 + b = 7\\n2a\sqrt{b} = 4\sqrt{3} \\
a\sqrt{b} = 2\sqrt{3} \\
a^2b = 12 \\
\]

From (1), $a^2 = 7 - b$

\[
(7 - b)b = 12 \\
0 = b^2 - 7b + 12 \\
(b - 4)(b - 3) = 0 \\
b = 3 \text{ or } b = 4
\]

When $b = 4$, $a^2 = 7 - 4 = 3$ (rejected)

When $b = 3$, $a^2 = 7 - 3 = 4$

$a = 2$ or $a = -2$ (rejected)

$a = 2$ and $b = 3$

Equating rational and irrational parts M1

Do not accept $a\sqrt{b} = 2\sqrt{3}$

$a = 2$, $b = 3$

M1 obtain a quadratic equation

Obtain either both $b$'s or both $a$'s

A1 [given provided M1 has been awarded]

4 (a) The roots of the quadratic equation $2x^2 + 5x - 1 = 0$ are $\tan A$ and $\tan B$.

(i) Find the value of $\tan(A + B)$.

(ii) Find the value of $\sec^2(A + B)$.

\[
4(a) (i) \\
\tan A + \tan B = -\frac{5}{2} \\
\tan A \tan B = -\frac{1}{2} \\
\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \\
= \frac{\frac{5}{2}}{1 + \frac{1}{2}} \\
= -\frac{5}{3}
\]

Either one B1

A1
4 (a) (ii) \( \sec^2(A + B) = 1 + \tan^2(A + B) \)
\[
= 1 + \frac{25}{9} \quad \text{M1}
\]
\[
= \frac{34}{9} \quad \text{A1}
\]

(b) (i) Show that \( \frac{2}{1 - \sin 3x} + \frac{2}{1 + \sin 3x} = 4 \sec^2 3x \). [2]

(ii) Hence evaluate \( \int_0^{\frac{\pi}{12}} \frac{2}{1 - \sin 3x} + \frac{2}{1 + \sin 3x} \, dx \). [2]

\begin{align*}
4(b)(i) & \quad \text{LHS} = \frac{2}{1 - \sin 3x} + \frac{2}{1 + \sin 3x} \\
& = \frac{2(1 + \sin 3x) + 2(1 - \sin 3x)}{(1 - \sin^2 3x)} \\
& = \frac{4}{\cos^2 3x} \\
& = 4 \sec^2 3x \quad \text{(Shown)}
\end{align*}

\[
\int_0^{\frac{\pi}{12}} \frac{2}{1 - \sin 3x} + \frac{2}{1 + \sin 3x} \, dx \\
= \int_0^{\frac{\pi}{12}} 4 \sec^2 3x \, dx \\
= \left[ \frac{4}{3} \tan 3x \right]_0^{\frac{\pi}{12}} \\
= \frac{4}{3} \quad \text{A1}
\]

5 A curve has the equation \( y = 3x^2 e^{-x} \).

(i) Find an expression for \( \frac{dy}{dx} \) and obtain the coordinates of the stationary points of the curve. [5]

(ii) Determine the nature of these stationary points. [6]

\begin{align*}
5(i) \quad & \quad \frac{dy}{dx} = 6xe^{-x} + 3x^2 (-e^{-x}) \\
& = 3xe^{-x}(2 - x) \\
\text{For stationary points, } \frac{dy}{dx} = 0 & \quad \text{M1}
\end{align*}

\[
3xe^{-x}(2 - x) = 0
\]
\[
e^{-x} \neq 0, \quad x = 0 \text{ or } x = 2 \quad \text{A1}[2 \text{ values of } x]
\]
\[
(0, 0) \text{ and } (2, \frac{12}{e^2}) \quad \text{Both points A1}
\]
\[
\frac{d^2y}{dx^2} = 6e^{-x} - 6xe^{-x} + 6x(-e^{-x}) + 3x^2(e^{-x})
\]
\[
= 6e^{-x} - 12xe^{-x} + 3x^2(e^{-x})
\]
\[
= 3e^{-x}(2 - 4x + x^2)
\]

when \( x = 0 \), \( \frac{d^2y}{dx^2} = 6 > 0 \)  
A1

(0, 0) is a minimum point
B1

when \( x = 2 \), \( \frac{d^2y}{dx^2} = -\frac{6}{e^2} < 0 \)  
B1

(2, \( \frac{12}{e^2} \)) is a maximum point
A1

**OR**

Using \( \frac{dy}{dx} \),

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>0</th>
<th>0^+</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{dy}{dx} )</td>
<td>&lt; 0</td>
<td>0</td>
<td>&gt; 0</td>
</tr>
</tbody>
</table>

Sketch of tangent

\[
\begin{array}{c|c|c}
& \_ & \_ \\
\end{array}
\]
B2

(0, 0) is a minimum point
A1

For \( (2, \frac{12}{e^2}) \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>2^-</th>
<th>2</th>
<th>2^+</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{dy}{dx} )</td>
<td>&gt; 0</td>
<td>0</td>
<td>&lt; 0</td>
</tr>
</tbody>
</table>

Sketch of tangent

\[
\begin{array}{c|c|c}
& \_ & \_ \\
\end{array}
\]
B2

(2, \( \frac{12}{e^2} \)) is a maximum point
A1
6 (a) Find in ascending powers of \( x \), the first four terms in the expansion of \((1 + x - x^2)^9\). [4]

<table>
<thead>
<tr>
<th></th>
<th>((1 + x - x^2)^9)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1 + \binom{9}{1}(x - x^2) + \binom{9}{2}(x - x^2)^2 + \binom{9}{3}(x - x^2)^3 + \ldots) B1</td>
</tr>
<tr>
<td></td>
<td>(1 + 9x - 9x^2 + 36(x^2 - 2x^3 + x^4) + 84(x^3 + \ldots))</td>
</tr>
<tr>
<td></td>
<td>(1 + 9x + 27x^2 + 12x^3 + \ldots) A3 deduct 1 mark for every wrong term</td>
</tr>
</tbody>
</table>

(b) (i) Find the term independent of \( x \) in the expansion of \((2x^2 - \frac{1}{2x})^{12}\). [3]

(ii) Determine the constant term in the expansion of \((3 + 4x^3)(2x^2 - \frac{1}{2x})^{12}\). [4]

<table>
<thead>
<tr>
<th>6(b) (i)</th>
<th>((r + 1)^{th}) term = (\binom{12}{r}(2x^2)^{12-r}\left(-\frac{1}{2x}\right)^r) M1</th>
</tr>
</thead>
<tbody>
<tr>
<td>For term independent of ( x )</td>
<td></td>
</tr>
<tr>
<td>(x^0 = x^{2(12-r)} \times x^{-r})</td>
<td></td>
</tr>
<tr>
<td>(0 = 24 - 3r)</td>
<td></td>
</tr>
<tr>
<td>(r = 8) B1</td>
<td></td>
</tr>
<tr>
<td>Term independent of ( x ) = (\binom{12}{8}(2x^2)^{12-8}\left(-\frac{1}{2x}\right)^8)</td>
<td></td>
</tr>
<tr>
<td>= (\binom{12}{8}(2)^4\left(-\frac{1}{2}\right)^8)</td>
<td></td>
</tr>
<tr>
<td>= (\binom{12}{8}\left(\frac{1}{2}\right)^4)</td>
<td></td>
</tr>
<tr>
<td>= (\frac{495}{16}) A1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6(b) (ii)</th>
<th>For (x^{-3}), (-3 = 24 - 3r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r = 9) M1</td>
<td></td>
</tr>
<tr>
<td>Term in (x^{-3}) = (\binom{12}{9}(2x^2)^3\left(-\frac{1}{2x}\right)^9)</td>
<td></td>
</tr>
<tr>
<td>= (-\binom{12}{9}\left(\frac{1}{2^6}\right)x^{-3})</td>
<td></td>
</tr>
<tr>
<td>= (-\frac{220}{64}x^{-3}) B1</td>
<td></td>
</tr>
<tr>
<td>Constant = (3 \times \frac{495}{16} + 4 \times \left(-\frac{220}{64}\right)) M1</td>
<td></td>
</tr>
<tr>
<td>= (\frac{1265}{16}) A1</td>
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</tbody>
</table>
A curve is such that \( \frac{d^2y}{dx^2} = \frac{6}{(2x-5)^2} \).

The equation of the tangent to the curve at the point \((3, -1)\) is \( y - 2x + 7 = 0 \).

(i) Find an expression for \( \frac{dy}{dx} \).  

(ii) Find the equation of the curve.

<p>| | | |</p>
<table>
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<tbody>
<tr>
<td>7(i)</td>
<td>( \frac{dy}{dx} = \int 6(2x-5)^{-2} , dx )</td>
<td>M1 attempt to integrate</td>
</tr>
<tr>
<td>[4]</td>
<td>( = \frac{6(2x-5)^{-1}}{(-1)(2)} + c )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( = - \frac{3}{2x-5} + c )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>when ( x = 3, \frac{dy}{dx} = 2 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 2 = -3 + c )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( c = 5 )</td>
<td>M1 attempt to find ( c )</td>
</tr>
<tr>
<td></td>
<td>( \frac{dy}{dx} = - \frac{3}{(2x-5)} + 5 )</td>
<td>A1</td>
</tr>
</tbody>
</table>

(ii) \( y = \int - \frac{3}{(2x-5)} + 5 \, dx \)  

<p>| | | |</p>
<table>
<thead>
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<tbody>
<tr>
<td>[5]</td>
<td>( = - \frac{3 \ln(2x-5)}{2} + 5x + d )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>substituting ( x = 3 ) and ( y = -1 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( -1 = - \frac{3}{2} \ln1 + 15 + d )</td>
<td>M1 attempt to find ( d ).</td>
</tr>
<tr>
<td></td>
<td>( d = -16 )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( y = - \frac{3 \ln(2x-5)}{2} + 5x - 16 )</td>
<td>A1</td>
</tr>
</tbody>
</table>
The table shows experimental values of the variables \(x\) and \(y\).

<table>
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<tr>
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<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>1.4</td>
<td>2.6</td>
<td>4.6</td>
<td>7.4</td>
<td>11</td>
</tr>
<tr>
<td>(y)</td>
<td>0.4</td>
<td>0.6</td>
<td>1.6</td>
<td>3.4</td>
<td>6</td>
</tr>
</tbody>
</table>

It is known that \(x\) and \(y\) are related by the equation of the form \(p(x+y) = pq + qx^2\).

(i) Plot \(x+y\) against \(x^2\), draw the straight line graph and use it to estimate the value of \(p\) and \(q\). [6]

(ii) Using your values of \(p\) and \(q\), find the values of \(x\) for which \(p(x^2 - 2q) = 2qx^2\). [2]

\[
\begin{align*}
  p(x+y) &= pq + qx^2 \\
  x + y - q &= \frac{q}{p} x^2 \\
  x + y &= q + \frac{q}{p} x^2 \quad \text{------(1)} \\
  \text{gradient} &= \frac{q}{p} , \quad x + y\text{-intercept} = q \quad \text{-----(2)} \\
\end{align*}
\]

From graph, \(x + y\)-intercept = 1

\[
\begin{align*}
  q &= 1 \\
  \text{gradient} &= \frac{7.4 - 1}{15} = 0.4 \\
  \frac{q}{p} &= 0.4 \\
  \frac{1}{p} &= 0.4 \\
  p &= 2.5 \\
\end{align*}
\]

On graph paper

Straight line drawn with correct labelling of axes B1

All 5 points correctly plotted B2 deduct 1 mark for every point plotted wrongly

\[
\begin{align*}
  \frac{5}{2}(x^2 - 2) &= 2x^2 \\
  \frac{1}{2}x^2 &= 5 \\
  x^2 &= 10 \\
  x &= \pm \sqrt{10} \quad \text{or} \quad x = \pm 3.16 \quad \text{A1}
\end{align*}
\]
Q8

[Graph with labeled axes and points marked with coordinates (6, 1.4) and (18, 5)]
The circle with centre $C(3, 1)$ touches the $x$-axis at $A$. The line $4y - 3x = 0$ touches the circle at $B$.

Find the coordinates of $B$. [5]

<table>
<thead>
<tr>
<th>9(a)</th>
<th>Equation of tangent at $B$ is $y = \frac{3}{4}x$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5]</td>
<td>Gradient of normal at $B$ is $-\frac{4}{3}$. M1</td>
</tr>
<tr>
<td></td>
<td>Equation of normal at $B$ is $y - 1 = -\frac{4}{3}(x - 3)$</td>
</tr>
<tr>
<td></td>
<td>$y = -\frac{4}{3} x + 5$ B1</td>
</tr>
<tr>
<td></td>
<td>For point of intersection $B$,</td>
</tr>
<tr>
<td></td>
<td>$\frac{3}{4}x = -\frac{4}{3} x + 5$ M1</td>
</tr>
<tr>
<td></td>
<td>$\frac{25x}{12} = 5$</td>
</tr>
<tr>
<td></td>
<td>$x = \frac{12}{5}$ B1 for correct $x$ or $y$</td>
</tr>
<tr>
<td></td>
<td>$y = \frac{9}{5}$</td>
</tr>
<tr>
<td></td>
<td>$B\left(\frac{12}{5}, \frac{9}{5}\right)$ A1</td>
</tr>
</tbody>
</table>
READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on the separate writing papers provided.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

Setter: Mr Tan Beng Guan

This paper consists of 6 printed pages including the coverpage.
Mathematical Formulae

1. ALGEBRA

For the equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion
\[
(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,
\]
where \( n \) is a positive integer and
\[
\binom{n}{r} = \frac{n!}{(n-r)! r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}
\]

2. TRIGONOMETRY

\[
\begin{align*}
\sin^2 A + \cos^2 A &= 1 \\
\sec^2 A &= 1 + \tan^2 A \\
\cosec^2 A &= 1 + \cot^2 A \\
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\tan(A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\end{align*}
\]

\[
\begin{align*}
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A \\
\tan 2A &= \frac{2 \tan A}{1 - \tan^2 A}
\end{align*}
\]

Formulae for \( \Delta ABC \)
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\Delta = \frac{1}{2} bc \sin A
\]
Answer ALL Questions

1 Given that \( y = \frac{x^4 - 2}{x}, x \neq 0 \).

(i) Find an expression for \( \frac{dy}{dx} \). \([2]\)

(ii) Hence, show that \( y \) is an increasing function for all real values of \( x \) except zero. \([1]\)

2 (a) Given that \( \log_a m = n \), express each of the following in terms of \( n \).

(i) \( \log_a (9m^2) \) \([2]\)

(ii) \( \log_a \frac{1}{m} \) \([3]\)

(b) Solve the equation \( 2(\ln x)^2 + 3\ln \left(\frac{1}{x}\right) = 5 \). \([4]\)

3 On a university campus of 6000 students, one student returned from vacation with a contagious flu virus. The spread of the virus through the student body is given by

\[
 f(t) = \frac{6000}{1 + 5999e^{-0.2t}}
\]

where \( f(t) \) is the total number of students infected after \( t \) days. The university will cancel classes when 50% or more of the students are infected. Estimate,

(i) the number of students infected after 5 days, giving your answer to the nearest whole number, \([1]\)

(ii) after how many days will the classes be cancelled. \([3]\)

4 (a) Find the range of values of \( x \) for which \( (x - 2)(x + 3) \geq 6 \). \([3]\)

(b) Find the range of values of \( k \) for which the line \( y + kx = 8 \) and the curve \( x^2 + 4y = 16 \) do not intersect. \([4]\)

5 The function \( f \) is defined by \( f(x) = 4x^2 - 4x - 15 \) for \(-3 \leq x \leq 4 \).

(i) Sketch the graph of \( y = |f(x)| \), indicate clearly the \( x \) and \( y \) intercepts. \([4]\)

(ii) Determine the set of values of \( m \) for which there are two or three distinct solutions for the equation \( |f(x)| = m \). \([2]\)

6 (a) Prove that \( (\sec \theta + \tan \theta)^2 = \frac{1 + \sin \theta}{1 - \sin \theta} \). \([4]\)

(b) Find all the values of \( t \) between 0 and 12 for which \( \sin \left(\frac{\pi t}{5}\right) = \frac{\sqrt{3}}{2} \). \([3]\)
7 The diagram, which is not drawn to scale, shows parts of the graphs of \( y = 4 \cos 3x \) and \( y = 2 \sin x + k \).

(i) State the amplitude of \( y = 2 \sin x + k \) and the period of \( y = 4 \cos 3x \). [2]

(ii) Points \( P \) and \( Q \) are the respective maximum points on these graphs. Given that the two graphs intersect at the \( x \)-axis, find the value of \( k \) and the coordinates of \( P \) and of \( Q \). [6]

8 A particle \( P \) is traveling in a straight line with a velocity \( v \) ms\(^{-1} \), given by \( v = -2t^2 + 7t + 4 \), where \( t \) is the number of seconds after passing a fixed point \( O \). Calculate

(i) the value of \( t \) at which the particle comes to instantaneous rest, [2]

(ii) the maximum velocity achieved by the particle, [3]

(iii) the total distance travelled by \( P \) from \( t = 0 \) to \( t = 5 \). [4]

9 (a)

In the diagram, \( M \) and \( N \) are mid-points of \( CD \) and \( BC \) respectively. \( DB \) bisects \( \angle ABC \), \( DB = CN \) and \( \angle BAD = \angle BDC = 90^\circ \). Prove that \( \triangle ABD \) is congruent to \( \triangle MNC \). [4]
In the diagram, triangle $ABC$ is inscribed in the circle with centre $O$. The tangent at $A$ meets the line $EF$ and $BC$ produced at $D$.

Prove that:
(i) $\triangle ADC$ and $\triangle BDA$ are similar. \hfill [2]
(ii) $BD \times CD = DE^2 - AE^2$ \hfill [3]

10 (a) It is given that $y = (x - 2)\sqrt{2x} - 1$. Find the exact value of $x$ when the rate of decrease of $y$ is three times the rate of increase of $x$. \hfill [5]

(b) The region $A$, shown in the diagram is bounded by the curves $y = \sin 2x$, $y = \cos x$ and the $x$-axis. Find its area. \hfill [5]
The pictures below show a load lifter and the close-up of its extensible arm.

The movement of the arm can be modelled with the diagram shown below.

(i) In the diagram, \( APQ \) is a straight line representing the arm. \( ABC \) is a straight line with \( AB = 10 \text{ cm} \) and \( BC = 40 \text{ cm} \) and \( CD \) is perpendicular to \( ABC \). The arm is lifting an object vertically from point \( C \). \( P \) is a variable point on the semicircle with centre \( B \), radius 6 cm and \( \angle CBP = \theta \). The length of the arm is adjusted so that the point \( Q \) lies along the vertical line \( CD \) during the lifting of the object.

Show that \( CQ = \frac{150 \sin \theta}{5 + 3 \cos \theta} \). [3]

(ii) Find the value of \( \theta \) for which \( CQ \) is a maximum. [5]

------ End of Paper ------
Answers

1 (a) \[ \frac{dy}{dx} = 3x^2 + \frac{2}{x^2} \]

(b) Since \( 3x^2 + \frac{2}{x^2} > 0 \) thus \( \frac{dy}{dx} > 0 \) for all values of \( x \), except \( x = 0 \)

\[ \Rightarrow \text{y is an increasing function (shown)} \]

2 (a) (i) \( 1 + 2n \)

(ii) \( -2n \)

(b) \( x = e^{\frac{5}{2}} \) or \( x = \frac{1}{e} \)

\[ x = 12.2 \text{ or } x = 0.368 \text{ (to 3 s.f.)} \]

3 (i) \( 12 \text{ student} \)

(ii) \( 18 \text{ days} \)

4 (a) \( x \leq -4 \) or \( x \geq 3 \)

(b) \(-2 < k < 2 \)

5 (i)

![Graph](https://via.placeholder.com/150)

(ii) \( 16 \leq m \leq 33 \) or \( m = 0 \)

6 (a) \[ LHS = (\sec \theta + \tan \theta)^2 \]

\[ = \sec^2 \theta + 2 \sec \theta \tan \theta + \tan^2 \theta \]

\[ = \frac{1 + 2 \sin \theta \cos \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} \]

\[ = \frac{1 + 2 \sin \theta + \sin^2 \theta}{1 - \sin^2 \theta} \]

\[ = \frac{(1 + \sin \theta)^2}{(1 - \sin \theta)(1 + \sin \theta)} \]

7
\[ \frac{1 + \sin \theta}{1 - \sin \theta} \text{ (proven)} \]

(b) \[ t = \frac{5}{3}, \frac{10}{3} \text{ or } \frac{35}{3} \]

7 (i) Amplitude = 2 and Period = 120° or \( \frac{2\pi}{3} \)
(ii) \( k = -1 \) \( (0, 4) \) \( Q \left( \frac{\pi}{2}, 1 \right) \) or \( (90^\circ, 1) \)

8 (i) \( t = 4 \)
(ii) max velocity = 10 \( \frac{1}{8} \) m\( s^{-1} \)
(iii) 34.5 m

9 (a) Since \( M \) and \( N \) are mid-points of \( CD \) and \( BC \)

\( MN \parallel DB \) (Mid-point Theorem)

\( \Rightarrow \angle NMC = \angle BDC = 90^\circ \) (Corr. \( \angle s \) \( MN \parallel DB \))

\( \Rightarrow \angle MNC = \angle DBC \) (Corr. \( \angle s \) \( MN \parallel DB \))

Given \( DB \) bisects \( \angle ABC \)

\( \Rightarrow \angle ABD = \angle DBC = \angle MNC \)

\( DB = CN \) (given)

\( \Delta ABD \equiv \Delta MNC \) (AAS) (prov.cn)

(b) (i) \( \angle ADC = \angle BDA \) (common angle)

\( \angle CAD = \angle ABD \) (alternate segment theorem)

\( \therefore \Delta ADC \text{ and } \Delta BDA \text{ are similar} \) (angle-angle similarity test)

(ii) \( \frac{BD}{AD} = \frac{AD}{CD} \) (corr ratios of similar triangles)

\( \Rightarrow BD \times CD = AD^2 \)

Since \( AD \) is tangent to circle

\( \angle DAE = 90^\circ \) (tangent \( \perp \) radius)

\( \therefore AD^2 = DE^2 - AE^2 \) (pythagoras’ theorem)

\( \Rightarrow BD \times CD = DE^2 - AE^2 \) (proven)

10 (a) \( x = 2 - \sqrt{2} \)

(b) \( \frac{3}{4} \text{ units}^2 \)
11 (a) From the diagram, $PT$ is perpendicular to $AC$

$\triangle APT$ and $\triangle AQC$ are similar (angle–angle similarity test)

$$\frac{CQ}{50} = \frac{6 \sin \theta}{10 + 6 \cos \theta}$$  (corresponding ratios of similar triangles)

$$CQ = \frac{150 \sin \theta}{5 + 3 \cos \theta}$$  (shown)

(b) $\theta = 2.21 \text{ rad}$  (to 3 s.f.)
Prelim 3 Add Math P1
Answer Scheme.

1 (a) \[ y = x^3 - 2x^{-1} \]
\[
\frac{dy}{dx} = 3x^2 + 2x^{-3}
\]

(b) Since \[ 3x^2 + 2x^{-3} > 0 \] thus \[ \frac{dy}{dx} > 0 \] for all values of \( x \), except \( x = 0 \)
\[ \Rightarrow y \text{ is an increasing function (shown)} \]

2 (a) (i) \[ \log_y (9m^2) = \log_y 9 + 2 \log_y m \]
\[ = 1 + 2n \]

(ii) \[ \log_y \frac{1}{m} = \log_y 1 - \log_y m \]
\[ = 0 - \log_y \frac{m}{1} \]
\[ = -2n \]

(b) \[ 2(\ln x)^2 + 3\ln \left( \frac{1}{x} \right) - 5 = 0 \]

Let \( y = \ln x \)
\[ 2y^2 - 3y - 5 = 0 \]
\[ (2y - 5)(y + 1) = 0 \]
\[ y = \frac{5}{2} \quad \text{or} \quad y = -1 \]

\[ \ln x = \frac{5}{2} \quad \text{or} \quad \ln x = -1 \]
\[ x = e^{\frac{5}{2}} \quad \text{or} \quad x = \frac{1}{e} \]

Accept \( x = 12.2 \quad \text{or} \quad x = 0.368 \) (to 3 s.f.)
3 (i) When \( t = 5 \)

\[
f(5) = \frac{6000}{1 + 5999e^{-0.5(5)}}
\]

\[
= \frac{6000}{1 + 5999e^{-2.5}}
\]

\[
= 12.159 \approx 12 \text{ student}
\]

(ii) For classes to be cancelled, \( f(t) \geq 3000 \)

\[
\frac{6000}{1 + 5999e^{-0.5t}} \geq 3000
\]

\[
2 \geq 1 + 5999e^{-0.5t}
\]

\[
e^{-0.5t} \leq \frac{1}{5999}
\]

\[
t \geq -2\ln\left(\frac{1}{5999}\right) = 17.398
\]

\[\therefore \text{ after 18 days}\]

4 (a) \( x^2 + x - 12 \geq 0 \)

\((x + 4)(x - 3) \geq 0\)

\[x \leq -4 \text{ or } x \geq 3\]

(b) \( y = 8 - kx \)

\[x^2 + 4(8 - kx) = 16\]

\[x^2 - 4kx + 16 = 0\]

For no intersection, discriminant < 0

\[16k^2 - 4(1)(16) < 0\]

\[k^2 - 4 < 0\]

\[(k - 2)(k + 2) < 0\]

\[-2 < k < 2\]

5 (i) Shape of curve [G1]

Coordinates of max pt [G1]

x and y intercepts [G1]

End value [G1]
(ii) \(16 \leq m \leq 33\) or \(m = 0\) \[B2\]

6 (a) \(LHS = (\sec \theta + \tan \theta)^2\)
\[= \sec^2 \theta + 2 \sec \theta \tan \theta + \tan^2 \theta\]
\[= \frac{1}{\cos^2 \theta} + \frac{2 \sin \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta}\]
\[= \frac{1 + 2 \sin \theta + \sin^2 \theta}{1 - \sin^2 \theta}\]
\[= \frac{(1 + \sin \theta)^2}{(1 - \sin \theta)(1 + \sin \theta)}\]
\[= \frac{1 + \sin \theta}{1 - \sin \theta}\) (proven) \[A1\]

(b) \(\sin \left(\frac{\pi t}{5}\right) = \frac{\sqrt{3}}{2}\) \(0 < t < 12\) \(\Rightarrow 0 < \frac{\pi t}{5} < \frac{12\pi}{5}\) \[M1\]
\[= \sin^{-1} \left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}\]
\[= \frac{\pi}{3}, \frac{2\pi}{3} \text{ or } \frac{7\pi}{3}\] \[M1\]
\[t = \frac{5}{3}, \frac{10}{3} \text{ or } \frac{35}{3}\] \[A1\]

7 (i) Amplitude = 2 and Period = 120° or \(\frac{2\pi}{3}\) \[B2\]
(ii) Coordinates of \(P(0, 4)\) \[B1\]

Since the two curves intersect at the first \(x\)-intercept for \(y = 4 \cos 3x\),
\(\Rightarrow x = \frac{\pi}{6}\) \[M1\]

When \(x = \frac{\pi}{6}\), \(y = 0\) \[M1\]
\(0 = 2 \sin \left(\frac{\pi}{6}\right) + k\)
\(\Rightarrow k = -1\) \[A1\]

For graph of \(y = 2 \sin x - 1\), first maximum is at \(x = \frac{\pi}{2}\) \[M1\]

When \(x = \frac{\pi}{2}\), \(y = 1\)
\[ \text{coordinates of } Q \left( \frac{\pi}{2}, 1 \right) \text{ or } (90^\circ, 1) \]

8 (i) For particle at rest, \( v = 0 \)
\[-2t^2 + 7t + 4 = 0 \]
\((-2t-1)(t-4) = 0 \text{ or } (2t+1)(t-4) = 0 \]
\[ t = -\frac{1}{2} \text{ (rejected) or } t = 4 \]

(ii) For maximum velocity, \( \frac{dv}{dt} = 0 \)
\[-4t + 7 = 0 \]
\[ t = \frac{7}{4} \]
max velocity = \(-2\left(\frac{7}{4}\right)^2 + 7\left(\frac{7}{4}\right) + 4 = \frac{81}{8} = 10.125 \text{ ms}^{-1} \]

(iii) \[ s = \int v \, dt = -\frac{2t^3}{3} + \frac{7t^2}{2} + 4t + C \]
When \( t=0, s=0 \Rightarrow C = 0 \)
When \( t=4, s = 29 \frac{1}{3} \text{ m} \)
When \( t=5, s = 24.17 \text{ m} \)
\[ \therefore \text{ total distance} = 29 \frac{1}{3} + \left(29 \frac{1}{3} - 24.17\right) = 34.5 \text{ m} \]

9 (a) Since \( M \) and \( N \) are mid-points of \( CD \) and \( BC \)
\( MN \parallel DB \) (Mid-point Theorem)
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Given \( DB \) bisects \( \angle ABC \)
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(b) (i) \( \angle ADC = \angle BDA \) (common angle)
\( \angle CAD = \angle ABD \) (alternate segment theorem)
\[ \because \triangle ADC \text{ and } \triangle BDA \text{ are similar} \quad \text{(angle-angle similarity test)} \]

\[ (ii) \quad \frac{BD}{AD} = \frac{AD}{CD} \quad \text{(corr ratios of similar triangles)} \]

\[ \Rightarrow BD \times CD = AD^2 \]

Since AD is tangent to circle

\[ \angle DAE = 90^\circ \quad \text{(tangent \perp radius)} \]

\[ \therefore AD^2 = DE^2 - AE^2 \quad \text{(pythagoras' theorem)} \]

\[ \Rightarrow BD \times CD = DE^2 - AE^2 \quad \text{(proven)} \]

10 (a) \[ y = (x - 2)\sqrt{2x - 1} \]

\[ \frac{dy}{dx} = \sqrt{2x - 1} + (x - 2)\left(\frac{1}{2\sqrt{2x - 1}}\right) \quad (2) \]

\[ \frac{dy}{dx} = \frac{2x - 1 + x - 2}{\sqrt{2x - 1}} = \frac{3x - 3}{\sqrt{2x - 1}} \quad [M1] \]

\[ \frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} \]

\[ \frac{dy}{dt} = \frac{dy}{dx} \quad [M1] \]

\[ -3 = \frac{3x - 3}{\sqrt{2x - 1}} \]

\[ \sqrt{2x - 1} = 1 - x \]

\[ 2x - 1 = 1 - 2x + x^2 \]

\[ x^2 - 4x + 2 = 0 \quad [M1] \]

\[ x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(2)}}{2} \]

\[ x = 2 \pm \sqrt{2} \]

Therefore, \[ x = 2 - \sqrt{2} \] since \[ \frac{dy}{dx} < 0 \quad [A1] \]

(b) \[ \cos x = \sin 2x \]

\[ \cos x = 2\sin x \cos x \quad [M1] \]
\[
\cos x(2 \sin x - 1) = 0 \\
\Rightarrow x = \frac{\pi}{6} \text{ or } \frac{\pi}{2}
\]

\[
\text{Area} = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin 2x \, dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x \, dx
\]

\[
= \left[ -\frac{\cos 2x}{2} \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} + \left[ \sin x \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}
\]

\[
= \left[ -\frac{1}{4} + \frac{1}{2} \right] + \left[ 1 - \frac{1}{2} \right]
\]

\[
= \frac{3}{4} \text{ units}^2
\]

11 (a)

From the diagram, PT is perpendicular to AC

\[\triangle APT \text{ and } \triangle AQC \text{ are similar (angle–angle similarity test)}\]

\[
\frac{CQ}{50} = \frac{6 \sin \theta}{10 + 6 \cos \theta} \quad \text{(corr ratios of similar triangles)}
\]

\[
CQ = \frac{150 \sin \theta}{5 + 3 \cos \theta} \quad \text{(shown)}
\]

(b) \[
\frac{d}{d\theta} (CQ) = \frac{(5 + 3 \cos \theta)(150 \cos \theta) - (-3 \sin \theta)(150 \sin \theta)}{(5 + 3 \cos \theta)^2}
\]
\[
\frac{750 \cos \theta + 450}{(5 + 3 \cos \theta)^2}
\]

For maximum \(CQ\),
\[
\frac{d}{d\theta} (CQ) = \frac{750 \cos \theta + 450}{(5 + 3 \cos \theta)^2} = 0
\]

\[750 \cos \theta + 450 = 0\]
\[\cos \theta = -\frac{3}{5}\]

\[\theta = 2.21 \text{ rad (to 3 s.f.)}\]

<table>
<thead>
<tr>
<th>(\theta)</th>
<th>2.21(^{-})</th>
<th>2.21</th>
<th>2.21(^{+})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{d}{d\theta} (CQ))</td>
<td>+</td>
<td>0</td>
<td>-</td>
</tr>
</tbody>
</table>

\[\therefore \text{ when } \theta = 2.21 \text{ rad, } CQ \text{ is max.}\]
NAN CHIAU HIGH SCHOOL
Preliminary Examination (3) 2016
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS
PAPER 2

16 September 2016, Friday

Additional Materials: Writing Papers (8 sheets) 2 hours 30 minutes
Graph Paper (1 sheet)

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Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together. Tie your answer script into 2 separate bundles such as first bundle consists of question 1 to 6 and second bundle consists of question 7 to 11.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

Setter: Mdm Chua Seow Ling

This paper consists of 5 printed pages including the coverpage.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion

\[
(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,
\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. TRIGONOMETRY

Identities

\[
\sin^2 A + \cos^2 A = 1
\]
\[
\sec^2 A = 1 + \tan^2 A
\]
\[
\csc^2 A = 1 + \cot^2 A
\]

\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
\sin 2A = 2 \sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]
\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \triangle ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
A = \frac{1}{2} bc \sin A
\]
Answer ALL Questions

1. The roots of the quadratic equation \(3x^2 + \frac{27}{4} = 3x\) are \(\alpha^2\) and \(\beta^2\).
   (i) Find the value of \(\alpha + \beta\) and of \(\alpha \beta\) where \(\alpha\) and \(\beta\) are both negative. \([5]\)
   (ii) Hence find the quadratic equation whose roots are \(\alpha^3\) and \(\beta^3\). \([4]\)

2. Given \(f(x) = 2 - 24 \sin x \cos x\) and \(g(x) = 10(1 + \cos^2 x)\).
   (i) Express the sum of \(f(x)\) and \(g(x)\) in the form \(R \cos(2x + \alpha) + q\) where \(R\) and \(q\) are constants and \(R > 0, 0 < \alpha < \frac{\pi}{2}\). \([5]\)
   (ii) Hence find the minimum value of \(\frac{2}{f(x) + g(x)}\) and the corresponding values of \(x\) for \(0 < x < 2\pi\). \([3]\)

3. (i) Show \(\frac{d}{dx} \ln(\tan^2 3x) = 12 \csc 6x\). \([4]\)
   (ii) Hence integrate \(\frac{1}{\sin 6x} + \frac{1}{3e^{2-3x}}\) with respect to \(x\). \([4]\)

4. The diagram shows a right-angled trapezium \(ABCD\) such that \(2AB = 3CD\) and \(AB\) is parallel to \(DC\). Given the height \(BC\) of the trapezium is \((3 - \sqrt{3})\) cm and area of the trapezium is \((2 + 3\sqrt{3})\) cm\(^2\).

   Find length \(CD\) in the form \((a + b\sqrt{3})\) cm, where \(a\) and \(b\) are rational numbers. \([5]\)

5. (i) The sum of the second and third term of the expansion of \((1 + kx)^n\) is \(60x + 1740x^2\). Find the value of \(k\) and of \(n\). \([5]\)
   (ii) Hence write down the first 4 terms in the expansion of \((1 + kx)^r\) in ascending powers of \(x\). \([2]\)
   (iii) Hence determine the coefficient of \(\alpha^3\) in the expansion of \((1 + k(\alpha - \alpha^2))^r\). \([3]\)
6. An experiment to find the constant acceleration, \( a \) m/s\(^2\), of an electric toy car moving in one direction, requires students to measure the speed, \( v \) m/s from the speedometer when distance, \( s \) m varies. The table below shows the experimental values of \( v \) and \( s \), which are connected by the equation
\[ v = \sqrt{e^p + 2as}, \]
where \( p \) is a constant.

<table>
<thead>
<tr>
<th>( s )</th>
<th>( 4\frac{1}{6} )</th>
<th>( 17\frac{1}{2} )</th>
<th>( 37\frac{1}{2} )</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v )</td>
<td>3</td>
<td>5</td>
<td>6</td>
<td>10</td>
</tr>
</tbody>
</table>

(i) Plot \( v^2 \) against \( s \) and draw a straight line graph. Hence determine which value of \( v \), in the table above, is the incorrect recording. Using your graph to estimate the correct \( v \) value. [4]

(ii) Use your graph to estimate the value of \( a \) and of \( p \). [3]

(iii) Explain what does the value of \( e^p \) represents. [1]

(iv) By drawing a suitable straight line on your graph, solve \( s = \frac{120 - 2e^p}{4a + 3} \). [2]

---

Start on a fresh sheet of writing paper and tie answer script from question 7 to 11 together.

7. (i) Explain whether the curve \( y = 4 - 3e^{2x} \) has any stationary point. [2]

(ii) Sketch the graph \( y = 4 - 3e^{2x} \) indicating clearly the asymptote and \( x \) and \( y \)-intercepts. [3]

(iii) Hence solve \( 2x = \ln \left( 1 - \frac{4}{3}x \right) \) by inserting a straight line on the same graph in part (ii). [3]

8. (i) Factorise \( 8x^3 + 4x^2 - 2x - 1 \) completely. [3]

(ii) Hence express \( \frac{2x + 2}{8x^3 + 4x^2 - 2x - 1} \) in partial fractions. [4]

(iii) The polynomial \( 8x^3 + 4x^2 - 2x - 1 \) leaves a remainder of \( (px + q) \) when divided by \( x^2 - 1 \). Find the value of \( p \) and of \( q \). [4]
9. Given the curve \( y = \frac{2}{3}x^{-\frac{1}{2}} \) and \( y = \frac{8}{27}x^{\frac{3}{2}} \).

(i) Sketch the two graphs on the same diagram for \( x > 0 \) and label the graphs clearly. [2]

(ii) Calculate the coordinates of the point of intersection of the two graphs drawn in (i). [3]

10. The gradient function of a curve \( y = f(x) \) is given by \( m + n(3x - 2)^2 \). A point \( P \) lies on the curve and its \( x \)-coordinate is 2. The equation of the normal to the curve at \( P \) is given by \( 37y = 9x - 129 \). The curve has a turning point at \( Q \) whose \( x \)-coordinate is \( \frac{5}{3} \).

(i) Show that the value of \( m \) is 3 and \( n \) is \( -\frac{1}{9} \). [3]

(ii) Find the equation of the curve. [4]

(iii) Find the area of triangle \( PQR \) where \( R \) is the point the curve intersect the \( y \)-axis. [4]

11. Given that a circle \( C_1 \) passes through the point \( A(2, 0), B(5, 1) \) and \( C(6, 0) \).

(i) Show that the coordinates of centre \( D \) of the circle \( C_1 \) is \( (4, -1) \) and hence find the radius of the circle. [6]

(ii) Find the equation of the circle \( C_1 \) in standard form. [1]

(iii) Given 2 tangents are drawn from a point \( E \) to touch the circle at point \( B \) and \( C \). Find the coordinates of point \( E \). [5]

(iv) Explain why a circle can be drawn to pass through the points \( B, C, D \) and \( E \). Hence find the coordinate of the centre of this circle. [3]

End of Paper
Answers

1i) \( \alpha \beta = \frac{3}{2} \text{ or } -\frac{3}{2} \text{ (rej)} \)
\( (\alpha + \beta) = 2 \text{ (rej)} \) or \(-2 \)

2i) \( f(x) + g(x) = 13 \cos(2x + 1.18) + 17 \)

2ii) \( \frac{1}{2} \ln(\tan^2 3x) + \frac{1}{9} e^{3x-2} + c \) OR
\( \frac{1}{9} \ln(\tan 3x) + \frac{1}{9} e^{3x-2} + c \)

4) \( CD = 2 + \frac{22}{15} \sqrt{3} \)

5i) \( k = 2 \)
5ii) \( 1 + 60x + 1740x^2 + 32480x^3 + \ldots \)
5iii) coeff. of \( x^3 = 25520 \)

6i)

<table>
<thead>
<tr>
<th></th>
<th>S</th>
<th>4/16</th>
<th>17/2</th>
<th>37/2</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( v^2 )</td>
<td>9</td>
<td>25</td>
<td>36</td>
<td>100</td>
</tr>
</tbody>
</table>

6ii) incorrect \( v = 6 \text{ m/s} \)
corrected \( v = 7 \)

6iv) \( s = 20.5 \text{ or } 21m \)

\[ \frac{dy}{dx} = -6e^{2x} \]

7i) \( \frac{dy}{dx} < 0, \frac{dy}{dx} \neq 0, \text{ no stationary point} \)

7ii)

7iii) \( x = 0 \)
8i) \( (2x-1)(4x^2+4x+1) = (2x-1)(2x+1)^2 \)

8ii) \[
\frac{2x+2}{(8x^3+4x^2-2x-1)} = \frac{3}{4(2x-1)} - \frac{3}{4(2x+1)} - \frac{1}{2(2x+1)^2}
\]

8iii) \( q = 3 \) and \( p = 6 \)

9ii) \((1.5, 0.544)\) or \(\left(\frac{3}{2}, \frac{2}{9}\sqrt{6}\right)\)

10ii) \( y = 3x - \frac{1}{108}(3x-2)^4 - \frac{179}{27} \)

10iii) \( \frac{5}{4} \)

11) Since \( \angle DBE = \angle DCE = 90^\circ \) (tangent perpendicular to radius).

\[ R = \sqrt{5} \]

\[(x-4)^2 + (y+1)^2 = 5 \]

\[ E \left(\frac{17}{3}, \frac{2}{3}\right) \]

\[ \therefore \text{A circle with diameter DE (\angle in semicircle).} \]

Centre \( \left(\frac{4+\frac{17}{3}}{2}, \frac{-1+\frac{2}{3}}{2}\right) = \left(\frac{29}{6}, \frac{-1}{6}\right) \)
NCHS Prelim Exam (3) 2016 Additional Mathematics Paper 2 – Secondary 4 Express

<table>
<thead>
<tr>
<th>Qn No</th>
<th>Suggested Solutions</th>
<th>Qn No</th>
<th>Suggested Solutions</th>
</tr>
</thead>
</table>
| **1i** | $3x^2 + \frac{27}{4} = 3x$  
$3x^2 - 3x + \frac{27}{4} = 0$  
$x^2 - x + \frac{9}{4} = 0$  
$\alpha^2 + \beta^2 = 1$  
$(\alpha + \beta)^2 - 2\alpha\beta = 1$  
$(\alpha\beta)^2 = \frac{9}{4}$  
$\alpha\beta = \frac{3}{2}$ or $-\frac{3}{2}$ (rej)  
$(\alpha + \beta)^2 - 2\left(\frac{3}{2}\right) = 1$  
$(\alpha + \beta) = 4$  
$(\alpha + \beta) = 2$ (rej) or $-2$  
$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$  
$= (-2)^3 - 3\left(\frac{3}{2}\right)(-2)$  
$= 1$  
$(\alpha\beta)^3 = \left(\frac{3}{2}\right)^3$  
$x^2 - x + \frac{27}{8} = 0$  
**2i** | $f(x) + g(x) = 2 - 24\sin x \cos x + 10 + 10\cos^2 x$  
$= 12 - 12\sin 2x + 10\left[\frac{\cos 2x + 1}{2}\right]$  
$= 12 - 12\sin 2x + 5\cos 2x + 5$  
$= 17 + 5\cos 2x - 12\sin 2x$  
$R = \sqrt{5^2 + 12^2} = 13$  
$\tan \alpha = \frac{12}{5}$,  
$\alpha = 1.176$  
$f(x) + g(x) = 13\cos(2x + 1.18) + 17$  
$\min \left(\frac{2}{f(x) + g(x)}\right) = 13\cos(2x + 1.176) + 17$  
$\frac{2}{13 + 17} = \frac{1}{15}$  
**3i** | $\cos(2x + 1.176) = 1$  
**basic angle** = 0  
$(2x + 1.176) = 0$ (rej), $2\pi$, $4\pi$  
$x = 2.55, 5.70$  
$\frac{d}{dx} \ln(\tan^2 3x) = \frac{d}{dx} \frac{2x(\sec^2 3x)}{\tan 3x}$  
$= \frac{2(3)\sec^2 3x}{\tan 3x}$  
$= \frac{6\sec^2 3x}{\tan 3x}$  
$= \frac{6(\cos 3x)}{\cos^2 3x \sin 3x}$  
$= \frac{12}{\sin 6x}$  
$= 12 \cos ec 6x$ (shown)  

\[ \int \frac{1}{\sin 6x} + \frac{1}{3e^{3x}} \, dx = \int \frac{1}{\sin 6x} + \frac{1}{3} e^{3x-2} \, dx \]

$= \frac{1}{12} \ln(\tan 2x) + \frac{1}{9} e^{3x-2} + c$  

\[ \int \frac{1}{\sin 6x} + \frac{1}{3e^{3x}} \, dx = \frac{1}{12} \ln(\tan 2x) + \frac{1}{9} e^{3x-2} + c \]
Let $AB = 3x$ and $CD = 2x$

\[
\frac{1}{2} (3x + 2x)(3 - \sqrt{3}) = 2 + 3\sqrt{3}
\]

\[
\frac{5x}{2} = 3 - \sqrt{3}
\]

\[
\frac{5x}{2} = 2 + 3\sqrt{3}
\]

\[
\frac{5x}{2} = 3 - \sqrt{3} \times \frac{3 + \sqrt{3}}{3 + \sqrt{3}}
\]

\[
\frac{5x}{2} = 6 + 2\sqrt{3} + 9\sqrt{3} + 3(3)
\]

\[
\frac{5x}{2} = 9 - 3
\]

\[
\frac{5x}{2} = \frac{15 + 11\sqrt{3}}{6}
\]

\[
x = 1 + \frac{11}{15}\sqrt{3}
\]

\[
CD = 2 + \frac{22}{15}\sqrt{3}
\]

\[
\frac{5}{1} = 4 \quad 17 \quad \frac{1}{2} \quad 37 \quad \frac{1}{2} \quad 80
\]

\[
v^2 = 49
\]

corrected $v = 7$

\[
\text{gradient} = \frac{80-28}{63-20} = 1.209
\]

\[
v^2 = v^2 + 2as
\]

\[
e^p = 4
\]

\[
p = \ln 4 \text{ or } 1.39
\]

\[
a = 1.209
\]

\[
a = 0.605
\]

\[
0.144 \text{ or } \frac{1}{2} \ln 4
\]

\[
y = 4 - 3e^{2x}
\]

\[
\text{x and y - intercepts}
\]

\[
\text{Asymptote, } y = 4
\]

\[
(1 + kx)^n = \binom{n}{1}(1)(kx) + \binom{n}{2}(1)(kx)^2 + \ldots
\]

\[
= nkx + \frac{n(n-1)}{2}k^2x^2 + \ldots
\]

\[
nk = 60
\]

\[
n(n-1)k^2 = 1740
\]

\[
n^2k^2 - nk^2 = 3480
\]

\[
60^2 - 60k = 3480
\]

\[
k = 2
\]

\[
n = 30
\]

\[
(1 + 2x)^n = 1 + 60x + 1740x^2 + \binom{30}{3}(1)(kx)^3
\]

\[
= 1 + 60x + 1740x^2 + 32480x^3 + \ldots
\]

\[
(1 + k(a - 2a^2))^n = 1 + 60(a - 2a^2)
\]

\[
+ 1740(a - 2a^2)^2 + 32480(a - 2a^2)^3 + \ldots
\]

\[
= 1740(2)(a)(-2a^2) + 32480a^3 = -6960a^3 + 32480a^3
\]

\[
= 25520a^3 + \ldots
\]

\[
\text{coeff. of } a^3 = 25520
\]

\[
e^p \text{ represents the square of initial speed or square of initial velocity}
\]

\[
s = \frac{120 - 2e^p}{4a + 3}
\]

\[
s(4a + 3) = 120 - 2e^p
\]

\[
2e^p + 4as = 120 - 3s
\]

\[
e^p + 2as = 60 - 1.5s
\]

\[
v^2 = 60 - 1.5s
\]

\[
s = 20.5 \text{ or } 21m
\]

\[
\frac{dy}{dx} = -6e^{2x}
\]

\[
\frac{dy}{dx} < 0, \frac{dy}{dx} \neq 0, \text{ no stationary point}
\]
7iii
\[ 2x = \ln \left( 1 - \frac{4}{3} x \right) \]
\[ e^{2x} = 1 - \frac{4}{3} x \]
\[ 3e^{2x} = 3 - 4x \]
\[ 4x = 3 - 3e^{2x} \]
\[ 4x + 1 = 4 - 3e^{2x} \]
\[ y = 4x + 1 \quad \text{(Draw this straight line)} \]
\[ x = 0 \quad \text{A1} \]

8i
\[ 8x^3 + 4x^2 - 2x - 1 \]
by trial and error, let \( x = \frac{1}{2} \)
\[ 8\left( \frac{1}{2} \right)^3 + 4\left( \frac{1}{2} \right)^2 - 2\left( \frac{1}{2} \right) - 1 = 0 \]
\[ \therefore (2x - 1) \text{ is a factor} \]
\[ 4x^2 + 4x + 1 \]
\[ 2x - 1 = \frac{8x^3 + 4x^2 - 2x - 1}{(8x^3 - 4x^2)} \]
\[ = \frac{8x^2 - 2x - 1}{(8x^2 - 4x)} \]
\[ - \frac{2x - 1}{(2x - 1)} \]
\[ \frac{0}{0} \]
\[ (2x - 1)(4x^2 + 4x + 1) = (2x - 1)(2x + 1)^2 \]

9i
\[ \frac{2}{3} x^3 = \frac{8}{27} x^3 \quad \text{A1} \]
\[ x^3 = \frac{9}{4} \quad \text{thus } x = \frac{3}{2} \quad \text{or } -\frac{3}{2} \quad \text{(rej)} \]
\[ \text{(1.5, 0.544)} \quad \text{or } \left( \frac{3}{2}, \frac{2}{9}\sqrt{6} \right) \quad \text{M1 and A1} \]

8ii
\[ \frac{2x + 2}{(8x^3 + 4x^2 - 2x - 1)} = \frac{2x + 2}{(2x - 1)(2x + 1)^2} \]
Let \( \frac{2x + 2}{(2x - 1)(2x + 1)^2} = \frac{A}{2x - 1} + \frac{B}{2x + 1} + \frac{C}{(2x + 1)^2} \]
\[ 2x + 2 = A(2x + 1)^2 + B(2x - 1)(2x + 1) + C(2x - 1) \]
Let \( x = \frac{1}{2}, \quad 2\left( \frac{1}{2} \right) + 2 = A(2\left( \frac{1}{2} \right) + 1)^2 \)
\[ A = \frac{3}{4} \quad \text{M1} \]
Let \( x = -\frac{1}{2}, \quad 2\left( -\frac{1}{2} \right) + 2 = C(\left( -\frac{1}{2} \right) - 1) \)
\[ C = -\frac{1}{2} \quad \text{M1} \]
Let \( x = 0, \quad 2 = A - B - C \)
\[ B = -\frac{3}{4} \quad \text{M1} \]
\[ \frac{2x + 2}{(8x^3 + 4x^2 - 2x - 1)} = \frac{3}{4(2x - 1)} - \frac{3}{4(2x + 1)} - \frac{1}{2(2x + 1)^2} \]
Let \( x^2 - 1 = 0, \quad x = 1 \quad \text{or } x = -1 \)
\[ 8(1)^3 + 4(1)^2 - 2(1) - 1 = p + q \]
\[ p + q = 9 \quad \text{M1} \]
\[ 8(-1)^3 + 4(-1)^2 - 2(-1) - 1 = -p + q \]
\[ q - p = -3 \quad \text{M1} \]
\[ q = 3 \quad \text{and } p = 6 \quad \text{A2} \]
OR
\[ \frac{8x + 4}{x^2 - 1} = \frac{8x^3 + 4x^2 - 2x - 1}{(8x^3 - 8x)} \]
\[ - \frac{4x^2 + 6x - 1}{-4x^2 - 4} \]
\[ \frac{6x + 3}{6x + 3} \]
\[ p = 6, \quad q = 3 \quad \text{A2} \]
10i 
\[ f'(x) = m + n(3x - 2)^3 \]
\[ m + n\left(3 \frac{5}{3} - 2\right)^3 = 0 \]
\[ m + n(27) = 0 \]
\[ m = -27n \quad \text{(eqn 1)} \]
\[ y = \frac{9}{37} x - \frac{129}{37} \]
Gradient of tangent = \(-\frac{37}{9}\)

10ii 
\[ f'(x) = 3 - \frac{1}{9}(3x - 2)^2 \]
\[ y = 3x - \frac{1}{9}(3x - 2)^2 + c \]
\[ y = 3x - \frac{1}{108}(3x - 2)^4 + c \]
\[ 37y = 9(2) - 129, \quad y = -3 \]
\[ c = \frac{179}{27} \]
\[ y = 3x - \frac{1}{108}(3x - 2)^4 - \frac{179}{27} \quad \text{(eqn of curve)} \]

10i 
\[ x = 0, \quad y = -\frac{(-2)^4}{108} - \frac{179}{27} = -\frac{61}{9} \]

11i 
\[ \text{Area} = \frac{1}{2} \]
\[ = \frac{1}{2} \left( -\frac{289}{18} - \left( -\frac{167}{9} \right) \right) \]
\[ = \frac{5}{4} \quad \text{(A1)} \]

11ii 
\[ x_o = \frac{6 + 2}{2} = 4 \quad \text{(M1)} \]
\[ \sqrt{(y_0 - 0)^2 + (4 - 2)^2} = \sqrt{(5 - 4)^2 + (1 - y_o)^2} \]
\[ y_o = -1, \quad D(4, -1) \quad \text{(shown)} \]
\[ R = \sqrt{(5 - 4)^2 + (1 - (-1))^2} = \sqrt{5} \quad \text{(A1)} \]

11i 
\[ (x - 4)^2 + (y + 1)^2 = 5 \quad \text{(B1)} \]

11ii
\[ M_{DB} = 1 - (-1) = 2, \quad M_{BE} = -\frac{1}{2} \quad \text{(M1)} \]
\[ y = -\frac{1}{2} x + C \]
\[ 1 = -\frac{1}{2} (5) + C \]
\[ C = 3.5, \quad y = -\frac{1}{2} x + \frac{7}{2} \quad \text{(equation BE)} \]

\[ M_{DC} = -1 - 0 = \frac{1}{2}, \quad M_{CE} = -2 \]
\[ y = -2x + C \]
\[ 0 = -2(6) + C \]
\[ C = 12, \quad y = -2x + 12 \quad \text{(equation CE)} \]
\[ -\frac{1}{2} x + \frac{7}{2} = -2x + 12, \quad E\left(\frac{17}{3}, \frac{2}{3}\right) \quad \text{(A1)} \]

Since \( \angle DBE = \angle DCE = 90^\circ \)
(tangent perpendicular to radius).

A circle with diameter DE \((\angle \text{in semicircle})\)

Centre \(\left(\frac{4 + \frac{17}{3}}{2}, \frac{-1 + \frac{2}{3}}{2}\right) = \left(\frac{29}{6}, \frac{1}{6}\right) \quad \text{(B1)} \)

*** End of Paper ***
READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \)

\[
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\]

Binomial expansion

\[(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}
\]

2. TRIGONOMETRY

Identities

\[
\sin^2 A + \cos^2 A = 1
\]

\[
\sec^2 A = 1 + \tan^2 A
\]

\[
\cosec^2 A = 1 + \cot^2 A
\]

\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
\sin 2A = 2 \sin A \cos A
\]

\[
\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A
\]

\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulæ for \( \Delta ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2} bc \sin A
\]
1. The curve \( y = f(x) \) is such that \( f'(x) = (k - 2)e^{2x} \).

(i) For \( y \) to be an increasing function of \( x \), what condition must be applied to the constant \( k \)?

(ii) Given that \( P(0,3) \) is a point on the curve and the gradient of the tangent to the curve at \( P \) is 4, find an expression for \( f(x) \).

2. (i) Differentiate \( \ln(\sin x) \) with respect to \( x \).

(ii) Show that \( \frac{d}{dx}(x \cot x) = \cot x - x \cosec^2 x \).

(iii) Using the results from parts (i) and (ii), find \( \int x \cosec^2 x \, dx \).

3. The equation of a curve is \( y = 6x^2 \).

(i) Sketch the curve \( y = 6x^2 \).

(ii) The point \( P \) lies on the curve such that the gradient of the normal to the curve is \( -\frac{1}{2} \). The normal at \( P \) meets the \( x \)-axis at \( A \) and the \( y \)-axis at \( B \). Find the ratio \( AP:PB \).

4. (i) Given that \( n \) is a positive integer, write down, without simplifying, the \((n+1)\)th term in the binomial expansion of \( \left( \frac{x}{2} - \frac{k}{x^2} \right)^n \).

(ii) The binomial expansion of \( \left( \frac{x}{2} - \frac{k}{x^2} \right)^n \) has a constant term. Show that \( n \) is a multiple of 3.

(iii) Given that \( n = 9 \) and that the constant term is \( \frac{-2625}{2} \), find the value of \( k \).

(iv) Using the value of \( k \) found in part (iii), find the term independent of \( x \) in the expansion of \( (2 + x^2)\left( \frac{x}{2} - \frac{k}{x^2} \right)^9 \).
The diagram shows a triangle \(ABC\) such that \(AB = (2\sqrt{2} - 1)\) cm and \(AC = (4\sqrt{2} + 7)\) cm. The point \(X\) lies on \(AC\) such that \(\angle AXB = \angle ABC\).

(i) Show that \(AX \times AC = AB^2\). \([2]\)

(ii) Find an expression for \(AX\) in the form \(\frac{1}{17} \left(a + b\sqrt{2}\right)\). \([4]\)

(iii) Given that \(BC^2 = 72 + 60\sqrt{2}\), show that \(\angle AXB = 90^\circ\). \([3]\)

6 The equation of a curve is \(y = \frac{(2x - 5)^2}{x - 1}\), where \(x \neq 1\).

(i) Find an expression for \(\frac{dy}{dx}\) and obtain the coordinates of the stationary points of the curve. \([5]\)

(ii) Find an expression for \(\frac{d^2y}{dx^2}\) and show that it can be expressed in the form \(\frac{k}{(x-1)^3}\). Hence, or otherwise, determine the nature of these stationary points. \([4]\)

7 The highest point on a circle \(C_1\) is \((2, 8)\). The line \(T\), \(3y = 42 - 4x\), is a tangent to \(C_1\) at the point \((6, 6)\).

(i) Find the coordinates of the centre of \(C_1\). \([4]\)

(ii) Find the equation of \(C_1\). \([2]\)

The circle \(C_2\) is a reflection of \(C_1\) in the line \(T\).

(iii) Find the equation of \(C_2\). \([3]\)
8 (i) Show that \(3x - 1\) is a factor of \(3x^3 + 11x^2 + 8x - 4\) and hence factorise completely the cubic polynomial \(3x^3 + 11x^2 + 8x - 4\). \([3]\)

(ii) Express \(\frac{5x^2 - 2x + 11}{3x^3 + 11x^2 + 8x - 4}\) as the sum of 3 partial fractions. \([4]\)

(iii) Hence find \(\int \frac{5x^2 - 2x + 11}{3x^3 + 11x^2 + 8x - 4} \, dx\). \([3]\)

9 The roots of the quadratic equation \(4x^2 + 3x + 1 = 0\) are \(\frac{1}{\alpha}\) and \(\frac{1}{\beta}\).

(i) Find the value of \(\alpha^2 + \beta^2\). \([4]\)

(ii) Show that the value of \(\alpha^3 + \beta^3\) is 9. \([2]\)

(iii) Find a quadratic equation whose roots are \(\alpha^2 + \beta\) and \(\alpha + \beta^2\). \([4]\)

10 The diagram shows a rug in the shape of a rectangle \(ABCD\) such that \(AB = 5\) m and \(AD = 2\) m. The rug is placed inside a rectangular function room \(PQRS\) such that each of the corners \(A, B, C\) and \(D\) touches the sides of the room \(SR, SP, PQ\) and \(QR\) respectively. The side of the rug \(AB\) makes an acute angle \(\theta\) with the side of the room \(SR\). The lengths of the room \(SR\) and \(SP\) are \(L\) m and \(W\) m respectively.

(a)  (i) Find the values of the integers \(a\) and \(b\) for which \(L = a\cos\theta + b\sin\theta\). \([2]\)

(ii) Obtain a similar expression for \(W\). \([1]\)

(iii) Hence find the perimeter of the room \(PQRS\) in exact form if \(PQRS\) is a square. \([3]\)

(b) Using the values of \(a\) and \(b\) found in (a) part (i),

(i) express \(L\) in the form \(R\cos(\theta - \alpha)\), \(R > 0\) and \(0^\circ < \alpha < 90^\circ\). \([2]\)

(ii) find the value of \(\theta\) if \(L = 4\) and the area of the rectangular function room \(PQRS\). \([4]\)
11 The amount of expenditure, $y$, incurred by a textile company is related to $x$, the amount of sales generated. The variables $x$ and $y$ are related by the formula $y = 10^k x^a$, where $a$ and $k$ are constants. The following table shows corresponding values of $x$ and $y$.

<table>
<thead>
<tr>
<th>$x$ ($$)</th>
<th>6</th>
<th>35</th>
<th>234</th>
<th>1995</th>
<th>6310</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$ ($$)</td>
<td>148</td>
<td>295</td>
<td>628</td>
<td>1480</td>
<td>2344</td>
</tr>
</tbody>
</table>

(i) Plot $\lg y$ against $\lg x$ for the given data and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of $a$ and of $k$. [4]

(iii) Estimate the amount of expenditure incurred when the sales generated is $4000$. [2]

(iv) Draw a straight line on the same axes to estimate the amount of sales to be generated in order for the textile company to breakeven. [2]
NEW SINGAPORE NATIONAL COLLEGE
MARIS STELLA HIGH SCHOOL
PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

ADDITIONAL MATHEMATICS
Paper 2

Additional Materials: Answer Paper (7 sheets)

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use

100

This document consists of 6 printed pages.
2

**Mathematical Formulae**

### 1. ALGEBRA

**Quadratic Equation**

For the equation \( ax^2 + bx + c = 0 \)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

**Binomial expansion**

\[(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}
\]

### 2. TRIGONOMETRY

**Identities**

\[
\sin^2 A + \cos^2 A = 1
\]

\[
\sec^2 A = 1 + \tan^2 A
\]

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\cosec^2 A = 1 + \cot^2 A
\]

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\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
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\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
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\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
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\sin 2A = 2 \sin A \cos A
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**Formulae for \( \Delta ABC \)**

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2} bc \sin A
\]
1 The curve $y = f(x)$ is such that $f'(x) = (k-2)e^{3x}$.

(i) For $y$ to be an increasing function of $x$, what condition must be applied to the constant $k$? [2]

Solution:
For $y$ is an increasing function of $x$,

$$(k-2)e^{3x} > 0.$$ [M1]

Since $e^{3x} > 0$, $k-2 > 0$

$\therefore k > 2$. [A1]

(ii) Given that $P(0,3)$ is a point on the curve and the gradient of the tangent to the curve at $P$ is 4, find an expression for $f(x)$. [4]

Solution:

$f'(x) = (k-2)e^{3x}$

Subst $x = 0$ and $f'(x) = 4$,

$4 = k - 2$

$k = 6$ [A1]

$f(x) = \frac{(k-2)e^{3x}}{3} + c$ [M1]

Subst $x = 0$ and $f(x) = 3$,

$3 = \frac{4}{3} + c$

$c = 1\frac{2}{3}$ [A1]

$f(x) = \frac{4}{3}e^{3x} + \frac{5}{3}$ [A1]
2 (i) Differentiate \( \ln(\sin x) \) with respect to \( x \). \[ 2 \]

Solution:
\[
\frac{d}{dx} (\ln(\sin x)) = \frac{\cos x}{\sin x} \quad [M1]
\]
\[
= \cot x \quad [A1]
\]

(ii) Show that \( \frac{d}{dx} x \cot x = \cot x - x \csc^2 x \).

Solution:
\[
\frac{d}{dx} x \cot x = \frac{d}{dx} \frac{x}{\tan x}
\]
\[
= \frac{\tan x - x \sec^2 x}{\tan^2 x} \quad [M1]
\]
\[
= \cot x - x \left( \frac{1}{\cos^2 x} \right) \left( \frac{\cos^2 x}{\sin^2 x} \right) \quad [M1]
\]
\[
= \cot x - x \csc^2 x \quad [A1]
\]

(iii) Using the results from parts (i) and (ii), find \( \int x \csc^2 x \, dx \).

Solution:
\[
\int (\cot x - x \csc^2 x) \, dx = x \cot x + c \quad [M1]
\]
\[
\int \cot x \, dx - \int x \csc^2 x \, dx = x \cot x + c
\]
\[
[\ln(\sin x) + c] - \int x \csc^2 x \, dx = x \cot x + c \quad [M1]
\]
\[
\int x \csc^2 x \, dx = \ln(\sin x) - x \cot x + c \quad [A1]
\]
3 The equation of a curve is \( y = 6x^{\frac{2}{3}} \).

(i) Sketch the curve \( y = 6x^{\frac{2}{3}} \).

Solution:

![Graph of \( y = 6x^{\frac{2}{3}} \)](image)

(ii) The point \( P \) lies on the curve such that the gradient of the normal to the curve is \(-\frac{1}{2}\). The normal at \( P \) meets the \( x \)-axis at \( A \) and the \( y \)-axis at \( B \). Find the ratio \( AP : PB \).

Solution:

\( y = 6x^{\frac{2}{3}} \)

\( \frac{dy}{dx} = 4x^{-\frac{1}{3}} \) \[M1\]

Gradient of tangent at \( P = -1 + \left(\frac{-1}{2}\right) \)

\[ = 2 \]

When \( \frac{dy}{dx} = 2 \), \( 4x^{-\frac{1}{3}} = 2 \) \[M1\]

\( x^{-\frac{1}{3}} = \frac{1}{2} \)

\( x^{\frac{1}{3}} = 2 \)

\( x = 8 \) \[A1\]

\( y = 6(8)^{\frac{2}{3}} \)

\[ = 24 \] \[A1\]

Equation of normal, \( y - 24 = -\frac{1}{2}(x - 8) \)

\( y = -\frac{1}{2}x + 28 \) \[M1\]

\( A(56,0), P(8,24), B(0,28) \)

\( AP : PB = 24 - 0 : 28 - 24 \)

\[ = 24 : 4 \]

\[ = 6 : 1 \] \[A1\]
Given that $n$ is a positive integer, write down, without simplifying, the $(r+1)$th term in the binomial expansion of $\left(\frac{x}{2} - \frac{k}{x^3}\right)^n$. [1]

**Solution:**

$$(r+1)\text{th term} = \binom{n}{r} \left(\frac{x}{2}\right)^{n-r} \left(-\frac{k}{x^3}\right)^r$$ [B1]

(ii) The binomial expansion of $\left(\frac{x}{2} - \frac{k}{x^3}\right)^n$ has a constant term. Show that $n$ is a multiple of 3. [1]

**Solution:**

For constant term, $n - r - 2r = 0$

$$n = 3r$$

Since $r$ is an integer and $n = 3r$, $n$ is a multiple of 3. [A1]

(iii) Given that $n = 9$ and that the constant term is $-\frac{2625}{2}$, find the value of $k$.

**Solution:**

Constant term $= -\frac{2625}{2}$

$$\binom{9}{3} \left(\frac{1}{2}\right)^{9-3} \left(-k^3\right)^3 = -\frac{2625}{2}$$ [M1]

$$84 \left(\frac{1}{64}\right) \left(-k^3\right)^3 = -\frac{2625}{2}$$

$$k^3 = 1000$$ [M1]

$$k = 10$$ [A1]

(iv) Using the value of $k$ found in part (iii), find the term independent of $x$ in the expansion of $(2 + x^3)\left(\frac{x}{2} - \frac{k}{x^3}\right)^9$. [3]

**Solution:**

Let $9 - 3r = -3$

$$r = 4$$

Constant term in the expansion of $(2 + x^3)\left(\frac{x}{2} - \frac{10}{x^3}\right)^9$

$$= 2\left(-\frac{2625}{2}\right) + x^3 \binom{9}{4} \left(\frac{x}{2}\right)^5 \left(-\frac{10}{x^3}\right)^4$$ [M2]

$$= 36750$$ [A1]
The diagram shows a triangle $ABC$ such that $AB = (2\sqrt{2} - 1)$ cm and $AC = (4\sqrt{2} + 7)$ cm. The point $X$ lies on $AC$ such that $\angle AXB = \angle ABC$.

(i) Show that $AX \times AC = AB^2$.

Solution:
\[ \angle AXB = \angle ABC \text{ (given)} \]
\[ \angle XAB = \angle BAC \text{ (common \angle)} \]
$\triangle AXB$ is similar to $\triangle ABC$.
\[ \frac{AX}{AB} = \frac{AB}{AC} \] \hspace{1cm} [M1]
\[ \therefore AX \times AC = AB^2 \] \hspace{1cm} [A1]

(ii) Find an expression for $AX$ in the form $\frac{1}{17}(a + b\sqrt{2})$.

Solution:
\[ AX \times AC = AB^2 \]
\[ AX = \frac{AB^2}{AC} \]
\[ = \frac{[2\sqrt{2} - 1]^2}{7 + 4\sqrt{2}} \] \hspace{1cm} [M1]
\[ = \frac{(2\sqrt{2})^2 - 4\sqrt{2} + 1}{7 + 4\sqrt{2}} \]
\[ = \frac{9 - 4\sqrt{2} \times 7 - 4\sqrt{2}}{7 + 4\sqrt{2}} \] \hspace{1cm} [M1]
\[ = \frac{63 - 6\sqrt{2} - 28\sqrt{2} + 32}{17} \]
\[ = \frac{1}{17}(95 - 64\sqrt{2}) \] \hspace{1cm} [A1]

(iii) Given that $BC^2 = 72 + 60\sqrt{2}$, show that $\angle AXB = 90^\circ$.

Solution:
\[ AB^2 + BC^2 = [2\sqrt{2} - 1]^2 + 72 + 60\sqrt{2} \]
\[ = 8 - 4\sqrt{2} + 1 + 72 + 60\sqrt{2} \]
\[ = 81 + 56\sqrt{2} \] \hspace{1cm} [M1]
\[ AC^2 = \left[4\sqrt{2} + 7 \right]^2 = 32 + 56\sqrt{2} + 49 = 81 + 56\sqrt{2} \quad \text{[M1]} \]

Since \( AC^2 = AB^2 + BC^2 \), by Converse of Pythagoras' Theorem, 
\( \angle ACB = 90^\circ \).

\[ \therefore \angle AXB = 90^\circ \quad \text{\textup{(since } \angle AXB = \angle ACB)} \quad \text{[A1]} \]

6. The equation of a curve is \( y = \frac{(2x - 5)^2}{x - 1} \).

(i) Find an expression for \( \frac{dy}{dx} \) and obtain the coordinates of the stationary points of the curve.

\textbf{Solution:}

\[
\frac{dy}{dx} = \frac{(x-1)(2)(2x-5)(2) - (2x-5)^2(1)}{(x-1)^3} \quad \text{[M1]}
\]

\[
= \frac{(2x-5)(4x-4-2x+5)}{(x-1)^3}
\]

\[
= \frac{(2x-5)(2x+1)}{(x-1)^2} \quad \text{[M1]}
\]

When \( \frac{dy}{dx} = 0, \quad (2x-5)(2x+1) = 0 \quad \text{[M1]}

\[
x = 2.5 \quad \text{or} \quad -0.5 \quad \text{[A1]}
\]

When \( x = 2.5, \quad y = 0 \)

When \( x = -0.5, \quad y = -24 \)

Stationary points are \( (2.5,0) \) and \( (-0.5,-24) \) \quad \text{[A1]}

(ii) Find an expression for \( \frac{d^2y}{dx^2} \) and show that it can be expressed in the form \( \frac{k}{(x-1)^3} \). Hence, or otherwise, determine the nature of these stationary points.

\textbf{Solution:}

\[
\frac{d^2y}{dx^2} = \frac{(x-1)^2(8x-8) - (2x-5)(2x+1)(2)(x-1)}{(x-1)^4} \quad \text{[M1]}
\]

\[
= \frac{(x-1)(8x^2 - 16x + 8 - 8x^2 + 16x + 10)}{(x-1)^4}
\]

\[
= \frac{18}{(x-1)^3} \quad \text{[A1]} \]
When \(x = -0.5\), \(\frac{d^2y}{dx^2} = \frac{18}{(-0.5-1)^3} < 0\)

\((0.5, -24)\) is a maximum point. [A1]

When \(x = 2.5\), \(\frac{d^2y}{dx^2} = \frac{18}{(2.5-1)^3} > 0\)

\((2.5, 0)\) is a minimum point. [A1]

7 The highest point on a circle \(C_1\) is \((2, 8)\). The line \(T\), \(3y = 42 - 4x\), is a tangent to \(C_1\) at the point \((6, 6)\).

(i) Find the coordinates of the centre of \(C_1\). [4]

**Solution:**
Since the highest point on a circle \(C_1\) is \((2, 8)\), the centre is \((2, y)\). [M1]

Gradient of normal at \((6, 6)\) = \(1 + \left(-\frac{4}{3}\right)\) [M1]

Equation of the normal at \((6, 6)\):
\[
(y - 6) = \frac{3}{4}(x - 6)
\]

\[
(y - 6) = \frac{3}{4}(x - 6)
\]

\(y = \frac{3}{4}x + \frac{3}{2}\) [A1]

When \(x = 2, y = 3\)
The centre of \(C_1\) is \((2, 3)\). [A1]

(ii) Find the equation of \(C_1\). [2]

**Solution:**
Equation of \(C_1\): \((x - 2)^2 + (y - 3)^2 = (8 - 3)^2\) [M1]
\((x - 2)^2 + (y - 3)^2 = 25\) [A1]

The circle \(C_2\) is a reflection of \(C_1\) in the line \(T\).

(iii) Find the equation of \(C_2\). [3]

**Solution:**
The centre of \(C_2\) is \((2 + 2(6 - 2), 3 + 2(6 - 3))\) = \((10, 9)\). [B2]

Equation of \(C_2\): \((x - 10)^2 + (y - 9)^2 = 25\) [A1]
8 (i) Show that \(3x - 1\) is a factor of \(3x^3 + 11x^2 + 8x - 4\) and hence factorise completely the cubic polynomial \(3x^3 + 11x^2 + 8x - 4\). \[\text{[3]}\]

Solution:
Let \(f(x) = 3x^3 + 11x^2 + 8x - 4\)
\[
f\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^3 + 11\left(\frac{1}{3}\right)^2 + 8\left(\frac{1}{3}\right) - 4 \quad \text{[M1]}
\]
\[
= -0
\]
Since \(f\left(\frac{1}{3}\right) = 0\), \((3x - 1)\) is a factor.

\[3x^3 + 11x^2 + 8x - 4 = (3x - 1)(x^2 + bx + 4)\]
Comparing \(x\) term, \(12 - b = 8\)
\[b = 4\]
\[3x^3 + 11x^2 + 8x - 4 = (3x - 1)(x^2 + 4x + 4) \quad \text{[M1]}
\]
\[= (3x - 1)(x + 2)^2 \quad \text{[A1]}
\]

(ii) Express \(\frac{5x^2 - 2x + 11}{3x^3 + 11x^2 + 8x - 4}\) as the sum of 3 partial fractions. \[\text{[4]}\]

Solution:
\[\frac{5x^2 - 2x + 11}{3x^3 + 11x^2 + 8x - 4} = \frac{5x^2 - 2x + 11}{(3x - 1)(x + 2)^2}\]
Let \[
\frac{5x^2 - 2x + 11}{(3x - 1)(x + 2)^2} = \frac{A}{3x - 1} + \frac{B}{x + 2} + \frac{C}{(x + 2)^2} \quad \text{[M1]}
\]
\[
5x^2 - 2x + 11 = A(x + 2)^2 + B(3x - 1)(x + 2) + C(3x - 1)
\]
Let \(x = -2\), \(-7C = 35\)
\[C = -5 \quad \text{[A1]}
\]
Let \(x = \frac{1}{3}\), \[\frac{49}{9} = \frac{98}{9}\]
\[A = 2 \quad \text{[A1]}
\]
Let \(x = 0\), \[4A - 2B - C = 11\]
\[8 - 2B - (-5) = 11\]
\[B = 1 \quad \text{[A1]}
\]
\[
\frac{5x^2 - 2x + 11}{3x^3 + 11x^2 + 8x - 4} = \frac{2}{3x - 1} + \frac{1}{x + 2} - \frac{5}{(x + 2)^2}
\]
(iii) Hence find \( \int \frac{5x^2 - 2x + 11}{3x^3 + 11x^2 + 8x - 4} \, dx \). \[3\]

\[ \begin{align*}
\int \frac{5x^2 - 2x + 11}{3x^3 + 11x^2 + 8x - 4} \, dx &= \int \left[ \frac{2}{(3x - 1)} + \frac{1}{(x + 2)} - \frac{5}{(x + 2)^2} \right] \, dx \\
&= \frac{2}{3} \ln |3x - 1| + \ln |x + 2| - \frac{5}{-1} (x + 2)^{-1} + c \\
&= \frac{2}{3} \ln |3x - 1| + \ln |x + 2| + \frac{5}{x + 2} + c \quad [M2]
\end{align*} \]

9 The roots of the quadratic equation \( 4x^2 + 3x + 1 = 0 \) are \( \frac{1}{\alpha} \) and \( \frac{1}{\beta} \).

(i) Find the value of \( \alpha^2 + \beta^2 \).

\[ \begin{align*}
\text{Solution:} \\
\text{Sum of roots: } &\frac{1}{\alpha} + \frac{1}{\beta} = -\frac{3}{4} \\
\frac{\alpha + \beta}{\alpha \beta} &= -\frac{3}{4} \\
\text{Product of roots: } &\frac{1}{\alpha \beta} = \frac{1}{4} \quad [M1] \\
\alpha \beta &= 4 \\
\alpha + \beta &= \frac{\alpha + \beta}{\alpha \beta} \times \alpha \beta \\
&= -\frac{3}{4} \times 4 \\
&= -3 \quad [M1] \\
\alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2\alpha \beta \\
&= (-3)^2 - 2(4) \quad [M1] \\
&= 1 \quad [A1]
\end{align*} \]

(iii) Show that the value of \( \alpha^3 + \beta^3 \) is 9.

\[ \begin{align*}
\text{Solution:} \\
\alpha^3 + \beta^3 &= (\alpha + \beta)(\alpha^2 - \alpha \beta + \beta^2) \\
&= (-3)(1 - 4) \quad [M1] \\
&= -9 \text{ (shown)} \quad [A1]
\end{align*} \]

(iii) Find a quadratic equation whose roots are \( \alpha^2 + \beta \) and \( \alpha + \beta^2 \).

\[ \begin{align*}
\text{Solution:} \\
\alpha^2 + \beta + \alpha + \beta^2 &= 1 + (-3) \\
&= -2 \quad [B1] \\
(\alpha^3 + \beta)(\alpha + \beta^2) &= \alpha^3 + \alpha^2 \beta^2 + \alpha \beta + \beta^3 \\
&= 9 + (4)^2 + 4 \quad [M1] \\
&= 29 \quad [A1]
\end{align*} \]

The new equation is \( x^2 + 2x + 29 = 0 \) \[A1\]
The diagram shows a rug in the shape of a rectangle $ABCD$ such that $AB = 5 \text{ m}$ and $AD = 2 \text{ m}$. The rug is placed inside a rectangular function room $PQRS$ such that each of the corners $A$, $B$, $C$ and $D$ touches the sides of the room $SR$, $SP$, $PQ$ and $QR$ respectively. The side of the rug $AB$ makes an acute angle $\theta$ with the side of the room $SR$. The lengths of the room $SR$ and $SP$ are $L \text{ m}$ and $W \text{ m}$ respectively.

(a) (i) Find the values of the integers $a$ and $b$ for which 

$$L = a \cos \theta + b \sin \theta.$$ 

Solution:

$$L = SA + AR$$

$$= 5 \cos \theta + 2 \sin \theta$$

$a = 5; \ b = 2$ \hspace{1cm} [B2]

(ii) Obtain a similar expression for $W$.

Solution:

$$W = SB + BP$$

$$= 5 \sin \theta + 2 \cos \theta$$ \hspace{1cm} [B1]

(iii) Hence find the perimeter of the room $PQRS$ in exact form if $PQRS$ is a square.

Solution:

$$W = SB + BP$$

$$= 5 \sin \theta + 2 \cos \theta$$ \hspace{1cm} [B1]

If $PQRS$ is a square, $L = W$

$$5\cos \theta + 2\sin \theta = 5\sin \theta + 2 \cos \theta$$ \hspace{1cm} [M1]

$$3\sin \theta = 3 \cos \theta$$

$$\tan \theta = 1$$

$$\theta = 45^\circ$$ \hspace{1cm} [A1]
Perimeter of \(PQRS\) \[4 \left( \frac{5\sqrt{2}}{2} + \frac{2\sqrt{2}}{2} \right)\]
\[= 4 \left( \frac{7\sqrt{2}}{2} \right)\]
\[= 14\sqrt{2} \text{ m} \quad [A1]\]

(b) Using the values of \(a\) and \(b\) found in (a) part (i),

(i) express \(L\) in the form \(R\cos(\theta - \alpha)\), \(R > 0\) and \(0^\circ < \alpha < 90^\circ\). \[2\]

Solution:
\[L = 5\cos\theta + 2\sin\theta\]
\[= \sqrt{5^2 + 2^2} \cos\left(\theta - \tan^{-1}\frac{2}{5}\right)\]
\[= \sqrt{29} \cos(\theta - 21.801^\circ)\]
\[= \sqrt{29} \cos(\theta - 21.8^\circ) \quad (1 \text{ dp}) \quad [B2]\]

(ii) find the value of \(\theta\) if \(L = 4\) and the area of the rectangular function room \(PQRS\). \[4\]

Solution:
\[L = 4\]
\[\sqrt{29} \cos(\theta - 21.801^\circ) = 4\]
\[\cos(\theta - 21.801^\circ) = \frac{4}{\sqrt{29}}\]
\[\theta - 21.801^\circ = 42.031^\circ\]
\[\theta = 63.832^\circ\]
\[= 63.8^\circ \quad (1 \text{ dp}) \quad [A1]\]

Area of room \(PQRS = L \times W\)
\[= 4 \times (5\sin 63.832^\circ + 2\cos 63.832^\circ) \quad [M1]\]
\[= 4 \times 5.3695\]
\[= 21.5 \text{ m}^2 \quad [A1]\]
The amount of expenditure, $y$, incurred by a textile company is related to $x$, the amount of sales generated. The variables $x$ and $y$ are related by the formula $y = 10^k x^a$, where $a$ and $k$ are constants. The following table shows corresponding values of $x$ and $y$.

<table>
<thead>
<tr>
<th>$x$ ($)</th>
<th>6</th>
<th>35</th>
<th>234</th>
<th>1995</th>
<th>6310</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$ ($)</td>
<td>148</td>
<td>295</td>
<td>628</td>
<td>1480</td>
<td>2344</td>
</tr>
</tbody>
</table>

(i) Plot $\log y$ against $\log x$ for the given data and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of $a$ and of $k$. [4]

(iii) Estimate the amount of expenditure incurred when the sales generated is $\$4000$. [2]

(iv) Draw a straight line on the same axes to estimate the amount of sales to be generated in order for the textile company to breakeven. [2]
(ii) \[ y = 10^k x^a \]
\[ lgy = lg10^k + lgx^a \]
\[ lgy = algx + k \]
\[ a = \text{gradient} \]
\[ = \frac{3.37 - 2.47}{3.80 - 1.94} \]
\[ = 0.4 \]
\[ (\text{accept } 0.375 - 0.425) \]
\[ k = lgy - \text{intercept} \]
\[ = 1.85 \]
\[ (\text{accept } 1.82 - 1.88) \]

(iii) When \[ x = 4000, \]
\[ lgx \approx 3.6 \]
From the graph,
when \[ lgx \approx 3.6, \]
\[ lgy \approx 3.3 \ (3.25 - 3.35) \]
\[ y \approx 2000 \]
When sales is $4000,
expenditure is $2000.
(accept $1778 - $2239)

(iv) To breakeven,
Sales = Expenditure
\[ x = y \]
\[ lg x = lg y \]
The graph \[ lg x = lg y \]
cuts the original graph at
\[ lg x = 3.1 \ (\text{accept } 3.05 - 3.13) \]
Amount of sales to breakeven
\[ = $(10^{3.1}) \]
\[ = $1260 \]
(accept $1120 - $1330)
PRELIMINARY EXAMINATION 2016
SECONDARY 4
ADDITIONAL MATHEMATICS
4047/01
3 August 2016
2 hours

Additional Materials: Answer Paper
Graph Paper (1 Sheet)

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

This document consists of 5 printed pages 1 blank page
2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion

\[
(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{n} b^n,
\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

1. TRIGONOMETRY

identities

\[
\sin^2 A + \cos^2 A = 1
\]

\[
\sec^2 A = 1 + \tan^2 A
\]

\[
\csc^2 A = 1 + \cot^2 A
\]

\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
\sin 2A = 2 \sin A \cos A
\]

\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]

\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formula for \( \triangle ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2} ab \sin C
\]
1. The area of a triangle is \( \left(1 + \frac{5\sqrt{5}}{2}\right) \text{cm}^2\). If the length of the base of the triangle is \(3 + 2\sqrt{5}\) cm, find, without using a calculator, the height of the triangle in the form of \((a + b\sqrt{5})\) cm, where \(a\) and \(b\) are integers. [4]

2. Express \(\frac{4x^2 + 6x + 5}{2x^2 + x - 3}\) in partial fractions. [5]

3. The function \(f(x)\) is such that \(f(x) = 2x^2 + 3x^3 - x - 4\).
   (i) find a factor of \(f(x)\). [2]
   (ii) Hence, determine the number of solutions in the equation \(f(x) = 0\). [4]

4. The roots of the quadratic equation \(3x^2 - x + 5 = 0\) are \(\alpha\) and \(\beta\).
   (i) Evaluate \(\alpha^3 + \beta^3\). [2]
   (ii) Find the quadratic equation whose roots are \(\alpha^3 - 1\) and \(\beta^3 - 1\). [4]

5. The table shows experimental values of 2 variables, \(R\) and \(V\), which are connected by an equation of the form \(RV^n = k\) where \(n\) and \(k\) are constants.

<table>
<thead>
<tr>
<th>(R)</th>
<th>33</th>
<th>19.95</th>
<th>5.07</th>
<th>2.38</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V)</td>
<td>2</td>
<td>2.9</td>
<td>8</td>
<td>14</td>
</tr>
</tbody>
</table>
   (i) Plot \(\log R\) against \(\log V\) for the given data and draw a straight line graph. [3]
   (ii) Use your graph to estimate the value of \(k\) and of \(n\). [3]
   (iii) By drawing a suitable straight line on your graph in (i), find the value of \(V\) such that \(\frac{R}{V^n} = 1\). [3]

6. Given that \(y = 1 - \frac{1}{2}\sin 3x\), \(0^\circ \leq x \leq 240^\circ\).
   (i) State the maximum and minimum values of \(y\). [2]
   (ii) Sketch the graph of \(y = 1 - \frac{1}{2}\sin 3x\). [3]
A quadrilateral $ABCD$ passes through vertices $B(3, 9)$, $C(8, 6)$ and $D(-4, 0)$, line $AD$ is parallel to the $y$-axis.

(i) Find the coordinates of $A$ given that the length of $AD$ is 8 units. [1]

(ii) A point $P$ divides the line $DC$ in the ratio of $2:1$. Find the coordinates of $P$. [3]

(iii) Hence, find the area of the quadrilateral $ABPD$. [3]

8 (a) Sketch the graph $y^2 = 3x$. [2]

(b) Given that $f(x) = -2x^3 + 5x^2 + 4x + a$,

(i) find the coordinates of the turning points in terms of $a$. [4]

(ii) Determine the nature of each turning point. [3]

(iii) In the case where $a = 1$, explain why the part of the graph between the turning points lie above the $x$-axis. [1]

9 (i) Show that $\sec x + \tan x$ can be expressed as $\frac{1 + \sin x}{\cos x}$. [1]

(ii) Differentiate $\ln(\sec x + \tan x)$ with respect to $x$. [3]

(iii) Hence, find $\int_{\cos^{0.5}}^{\cos^{0.5}} 2 \sec x \, dx$. [3]
10 The points $A$ and $B$ lie on the circumference of a circle $C_1$ where $A$ is the point $(0, 8)$ and $B$ is the point $(4, 0)$. The line $y = 2x$ also passes through the centre of the circle $C_1$.

(i) Find the centre and radius of the circle $C_1$. [4]

(ii) Find the equation of the circle $C_1$ in the form $x^2 + y^2 + px + qy + r = 0$, where $p$, $q$ and $r$ are integers. [2]

Another circle $C_2$ of radius $\sqrt{2}$ units has its centre inside $C_1$ and it cuts the circle $C_1$ at the origin and at the point where $x = 2$.

(iii) Find the centre of $C_2$. [5]

11 The diagram shows part of the curve $y = 3\cos\left(\frac{x}{2}\right)$ that cuts the $x$-axis at $x = \pi$ and $x = 3\pi$. The normal to the curve at $x = \frac{5\pi}{3}$ cuts the $x$-axis at $A$.

(i) Find the coordinates of $A$, leaving your answer in exact form. [6]

(ii) Hence, find the area of the shaded region. [4]
1. \(4 - \sqrt{5}\)

2. \(2 - \frac{2}{2x+3} + \frac{3}{x-1}\)

3. (ii) one solution

4. (i) \(-\frac{29}{9}\) 
   (ii) \(27x^2 + 98x + 196 = 0\)

6. (i) Max \(y = 1.5\); Min \(y = 0.5\) 
   (ii)

7. (i) \((-4,8)\) 
   (ii) \(P(4,4)\) 
   (iii) 50 units

8. (a) 
   (b)(i). \(A\left(-\frac{1}{3},\ a-\frac{19}{27}\right)\) and \((2,12+a)\) 
   (b)(ii). \(A\left(-\frac{1}{3},\ a-\frac{19}{27}\right)\) min; \((2,12+a)\) max

9. (ii) sec \(x\) 
   (iii). 0.539

10. (i) Centre \((2,4)\), Radius = \(2\sqrt{5}\) 
     (ii) \(x^2 + y^2 - 4x - 8y = 0\) 
     (iii) Centre of \(C_2(1.22, 0.710)\)

11. (i) \(A\left(\frac{5\pi}{3} + \frac{9}{8\sqrt{3}}, 0\right)\) 
     (ii) \(6\frac{15}{32}\) / 6.47 units

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1

\[ 1 + \frac{5\sqrt{5}}{2} = \frac{1}{2} (3 + 2\sqrt{5})(a + b\sqrt{5}) \]

\[ 2 + 5\sqrt{5} = (3 + 2\sqrt{5})(a + b\sqrt{5}) \]

\[ a + b\sqrt{5} = \frac{2 + 5\sqrt{5}}{3 + 2\sqrt{5}} \]

\[ = \frac{2 + 5\sqrt{5}}{3 + 2\sqrt{5}} \times \frac{3 - 2\sqrt{5}}{3 - 2\sqrt{5}} \]

\[ = \frac{6 - 4\sqrt{5} + 15\sqrt{5} - 50}{9 - 4(5)} \]

\[ = \frac{-44 + 11\sqrt{5}}{-11} \]

\[ = 4 - \sqrt{5} \]

The height of the triangle is \((4 - \sqrt{5})\) cm

---

2

Given \(\frac{4x^2 + 6x + 5}{2x^2 + x - 3}\)

As this is an improper fraction,

By long division,

\[
2x^2 + x - 3) 4x^2 + 6x + 5
\]

\[
4x^2 + 2x - 6
\]

\[
4x + 11
\]

\[
\frac{4x^2 + 6x + 5}{2x^2 + x - 3} = 2 + \frac{4x + 11}{(2x + 3)(x - 1)}
\]

Let \(\frac{4x + 11}{(2x + 3)(x - 1)} = \frac{A}{2x + 3} + \frac{B}{x - 1}\)

\[
= \frac{A(x - 1) + B(2x + 3)}{(2x + 3)(x - 1)}
\]
4x + 11 = A(x - 1) + B(2x + 3)
Let x = 1,
15 = 5B
B = 3
Let x = 0,
11 = -A + 9
A = -2
\[
\frac{4x^2 + 6x + 5}{(2x + 3)(x - 1)} = 2 + \frac{2}{2x + 3} + \frac{3}{x - 1}
\]

3(i)
Given \( f(x) = 2x^3 + 3x^2 - x - 4 \)

**By trial and error**, consider \((x - 1)\)

\[ f(1) = 2(1)^3 + 3(1)^2 - 1 - 4 \]

\[ = 0 \]

\( \therefore (x - 1) \) is a factor.

(ii)

\( f(x) = 2x^3 + 3x^2 - x - 4 \)

By inspection,

\[ f(x) = (x - 1)(2x^2 + ax + 4) \]

By comparing coefficient of 
\( x^2 : 3 = a - 2 \)

\( \therefore a = 5 \)

\[ f(x) = (x - 1)(2x^2 + 5x + 4) \]

Applying discriminant for \( 2x^2 + 5x + 4 \),

\[ b^2 - 4ac = 5^2 - 4(2)(4) \]

\[ = 25 - 32 \]

\[ = -7 < 0 \]

Thus \( 2x^2 + 5x + 4 \) has no real roots.
Therefore, there is only one solution.
4(i) \[ 3x^2 - x + 5 = 0 \]
\[ \alpha + \beta = \frac{1}{3} \]
\[ \alpha \beta = \frac{5}{3} \]
\[ \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta \]
\[ = \left( \frac{1}{3} \right)^2 - 2 \left( \frac{5}{3} \right) \]
\[ = \frac{1}{9} - \frac{10}{3} \]
\[ = -\frac{29}{9} \]

(ii) New sum of roots = \( \alpha^3 - 1 + \beta^3 - 1 \)
\[ = \alpha^3 + \beta^3 - 2 \]
\[ = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) - 2 \]
\[ = \left( \frac{1}{3} \right) \left( \alpha^2 + \beta^2 - \alpha\beta \right) - 2 \]
\[ = \left( \frac{1}{3} \right) \left( -\frac{29}{9} - \frac{5}{3} \right) - 2 \]
\[ = -\frac{98}{27} \]

New product of roots = \( (\alpha^3 - 1)(\beta^3 - 1) \)
\[ = \alpha^3 \beta^3 - \beta^3 - \alpha^3 + 1 \]
\[ = (\alpha\beta)^3 - (\alpha^3 + \beta^3) + 1 \]
\[ = \left( \frac{5}{3} \right)^3 - \left( -\frac{44}{27} \right) + 1 \]
\[ = \frac{196}{27} \]

Quadratic eqn:
\[ x^2 - \left( -\frac{98}{27} \right)x + \frac{196}{27} = 0 \]
\[ 27x^2 + 98x + 196 = 0 \]
6(i) Max $y = 1.5$; Min $y = 0.5$

(ii) $y = 1 - \frac{1}{2} \sin 3x$

| 7(i) | Since line $AD$ is parallel to $y$-axis,
Coordinates of $A = (-4, 0 + 8)$
$= (-4, 8)$ |
| 7(ii) | Since $P$ divides the line $DC$ in ratio $2:1$,
$P_x = \frac{8 + 4}{3} \times 2 + (-4); P_y = \frac{6}{3} \times 2 + 0$
$= 4; = 4$
$\therefore P(4, 4)$ |
| 7(iii) | Area of quadrilateral $ABPD = \frac{1}{2} \begin{vmatrix} -4 & 4 & 3 & -4 & -4 \\ 2 & 0 & 4 & 9 & 8 & 0 \end{vmatrix}$
$= \frac{1}{2} [(-16 + 36 + 24) - (12 - 36 - 32)]$
$= \frac{1}{2} [44 + 56]$
$= 50 \text{unit}^2$ |
8(b)(i) Given \( f(x) = -2x^3 + 5x^2 + 4x + a \)

\[ f'(x) = -6x^2 + 10x + 4 \]

For stationary point, \( f'(x) = 0 \)

\[ -6x^2 + 10x + 4 = 0 \]

\[ 3x^2 - 5x - 2 = 0 \]

\[ (3x+1)(x-2) = 0 \]

\[ \therefore x = -\frac{1}{3} \text{ or } x = 2 \]

\[ f(x) = -2 \left( -\frac{1}{3} \right)^3 + 5 \left( -\frac{1}{3} \right)^2 + 4 \left( -\frac{1}{3} \right) + a \]

\[ = -2 \left( \frac{1}{27} \right) + \frac{5}{9} \left( \frac{4}{3} \right) + a \]

\[ = a - \frac{19}{27} \]

or

\[ f(x) = -2(2)^3 + 5(2)^2 + 4(2) + a \]

\[ = -16 + 20 + 8 + a \]

\[ = a + 12 \]

\( \left( -\frac{1}{3}, a - \frac{19}{27} \right) \) and \( (2, 12 + a) \) are turning points

8(b)(ii) \( f'(x) = -12x + 10 \)

At \( x = -\frac{1}{3}, f''(x) = -12 \left( -\frac{1}{3} \right) + 10 \)

\[ = 14 \]

\[ > 0 \]

\[ \therefore \left( -\frac{1}{3}, a - \frac{19}{27} \right) \text{ is a minimum turning point.} \]
At $x = 2$, 

$$f'(x) = -12(2) + 10$$

$$= -14$$

$$< 0$$

Therefore, $(2, 12 + a)$ is a maximum turning point.

When $a = 1$, 

Minimum point = $\left( -\frac{1}{3}, \frac{8}{27} \right)$ is above $x$-axis

Maximum point = $(2, 13)$ is above $x$-axis

Since the graph has no other turning points, the part of the graph between the two turning points lies above the $x$-axis.

9(i)

$$\sec x + \tan x = \frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

$$= \frac{1 + \sin x}{\cos x}$$

(ii)

$$\frac{d}{dx} \ln(\sec x + \tan x) = \frac{d}{dx} \ln \left( \frac{1 + \sin x}{\cos x} \right)$$

$$= \frac{d}{dx} \left[ \ln (1 + \sin x) - \ln (\cos x) \right]$$

$$= \frac{\cos x}{1 + \sin x} - \frac{-\sin x}{\cos x}$$

$$= \frac{\cos x}{\cos x} \left[ (1 + \sin x) \cos x \right]$$

$$= \frac{\cos^2 x + \sin^2 x + \sin x(1 + \sin x)}{(1 + \sin x) \cos x}$$

$$= \frac{1 + \sin x}{(1 + \sin x) \cos x}$$

$$= \frac{1}{\cos x}$$

$$= \sec x$$

(iii)

$$\int_{0.25}^{0.5} 2 \sec x \, dx = 2 \int_{0.25}^{0.5} \sec x \, dx$$

$$= 2 \left[ \ln \left( \frac{1 + \sin x}{\cos x} \right) \right]_{0.25}^{0.5}$$

$$= 2 \left[ \ln \left( \frac{1 + \sin 0.5}{\cos 0.5} \right) - \ln \left( \frac{1 + \sin 0.25}{\cos 0.25} \right) \right]$$

$$= 0.539184$$

$$= 0.539 \text{ (3 s.f.)}$$
10(i)  
Midpoint of $AB = \left( \frac{0+4}{2}, \frac{8+0}{2} \right)$  
\[ = (2, 4) \]

Gradient of $AB = \frac{8-0}{0-4}$  
\[ = -2 \]

Eqn of perpendicular bisector of $AB$:  
\[ y - 8 = \frac{1}{2}(x - 0) \]
\[ y = \frac{1}{2}x + 3 \quad \text{(1)} \]
\[ y = 2x \quad \text{(2)} \]

Equate,  
\[ 2x = \frac{1}{2}x + 3 \]
\[ x = 2 \]
\[ y = 4 \]

$.::$ center of $C_1(2, 4)$  
Radius $= \sqrt{(2-4)^2 + (4-0)^2}$  
\[ = \sqrt{20} \]
\[ = 2\sqrt{5}\text{units} \]

10(ii)  
Thus eqn of $C_1$:  
\[ (x - 2)^2 + (y - 4)^2 = (2\sqrt{5})^2 \]
\[ x^2 - 4x + 4 + y^2 - 8y + 16 = 20 \]
\[ x^2 + y^2 - 4x - 8y = 0 \]

10(iii)  
Since $C_1: x^2 + y^2 - 4x - 8y = 0$  
When $x = 2$,  
\[ y^2 - 8y - 4 = 0 \]
\[ y = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-4)}}{2(1)} \]
\[ = 4 \pm 2\sqrt{5} \]
Use $y = 4 - 2\sqrt{5}$ (C₂ radius is only $\sqrt{2}$unit and lies in C₁)
Midpoint = $(1, 2 - \sqrt{5})$
Gradient = \[
\frac{4 - 2\sqrt{5} - 0}{2 - 0} = 2 - \sqrt{5}
\]
Eqn of perpendicular bisector:
\[
y - (2 - \sqrt{5}) = \left(\frac{-1}{2 - \sqrt{5}}\right)(x - 1)
\]
\[
y = \frac{10 - 4\sqrt{5} - x}{2 - \sqrt{5}} \quad \text{(1)}
\]
Since equation $C₂$ is of the form
\[(x - a)^2 + (y - b)^2 = 2\]
where center is $(a, b)$
Using $(0, 0)$,
\[a^2 + b^2 = 2 \quad \text{(2)}
\]
By substituting (1) in (2),
\[a^2 + \left(\frac{10 - 4\sqrt{5} - a}{2 - \sqrt{5}}\right)^2 = 2
\]
\[a^2 + a^2 + a(8\sqrt{5} - 20) + 160 - 80\sqrt{5} = 2
\]
\[(10 - 4\sqrt{5})a^2 + a(8\sqrt{5} - 20) + 162 - 72\sqrt{5} = 0
\]
Solving
\[a = \frac{-(8\sqrt{5} - 20) \pm \sqrt{(8\sqrt{5} - 20)^2 - 4(10 - 4\sqrt{5})(162 - 72\sqrt{5})}}{2(10 - 4\sqrt{5})}
\]
\[= 1.223 \text{ or } 0.7767 \text{ (rejected as it outside of } C₁)
\]
Hence $b = 0.7101$
Thus center of $C₂(1.22, 0.710)$
11(i) Given $y = 3\cos\frac{x}{2}$
\[
\frac{dy}{dx} = -3\left(\frac{1}{2}\right)\sin\frac{x}{2}
\]
\[
= -\frac{3}{2}\sin\frac{x}{2}
\]
At $x = \frac{5\pi}{3}$,
\[
\frac{dy}{dx} = -\frac{3}{2}\sin\frac{5\pi}{6}
\]
\[
= -\frac{3}{4}
\]
Gradient of normal $= \frac{4}{3}$
At $x = \frac{5\pi}{3}$, $y = -\frac{3\sqrt{3}}{2}$
Eqn of normal:
\[
y + \frac{3\sqrt{3}}{2} = \frac{4}{3}\left(x - \frac{5\pi}{3}\right)
\]
\[
y = \frac{4}{3}x - \frac{20\pi}{9} - \frac{3\sqrt{3}}{2}
\]
Since the normal cuts $x$-axis, $y = 0$
\[
0 = \frac{4}{3}x - \frac{20\pi}{9} - \frac{3\sqrt{3}}{2}
\]
\[
x = \frac{5\pi}{3} + \frac{9\sqrt{3}}{8}
\]
\[
\therefore A\left(\frac{5\pi}{3} + \frac{9\sqrt{3}}{8}, 0\right)
\]

11(ii) Shaded area
\[
= \left[\int_{\frac{5\pi}{3}}^{\frac{3\pi}{2}} 3\cos\frac{x}{2} \, dx\right] - \left[-\frac{1}{2} \times \frac{3\sqrt{3}}{2} \times \frac{9\sqrt{3}}{8}\right]
\]
\[
= \left[6\sin\frac{x^3}{2}\right]_{\frac{5\pi}{3}}^{\frac{3\pi}{2}} - \frac{81}{32}
\]
\[
= 6\sin\frac{3\pi}{2} - 6\sin\frac{5\pi}{6} - \frac{81}{32}
\]
\[
= -6 - \frac{81}{32}
\]
\[
= 6.15 \, \text{unit}^2 / 6.47 \, \text{unit}^2 \, (3sf)
\]
Taking $y = \text{const}$, we have

$$qRV^2 = q_x \Rightarrow qR + qV = q_k$$

$$lqR = -qV + q_k$$

(i) $q_k = 1.92$

$$k = 85.118$$

Acceptable range:

(ii) Gradient

$$\frac{q_k - \text{value}}{l_x}$$

Thus $n = -1.3355$

Range $=-1.3355$

$$= 1.36 \text{(sh)}$$

From graph, $S = 0.58$

Acceptable range 2

$$= 3.3 (\text{sh})$$

EX 257 (rev 2012)
PRELIMINARY EXAMINATION 2016
SECONDARY 4

ADDITIONAL MATHEMATICS

Paper 2

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number clearly on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of
angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

100

This document consists of 6 printed pages.
2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[
    x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion

\[
    (a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and

\[
    \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}.
\]

2. TRIGONOMETRY

Identities

\[
    \sin^2 A + \cos^2 A = 1
\]

\[
    \sec^2 A = 1 + \tan^2 A
\]

\[
    \csc^2 A = 1 + \cot^2 A
\]

\[
    \sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
    \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
    \tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
    \sin 2A = 2 \sin A \cos A
\]

\[
    \cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]

\[
    \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \Delta ABC \)

\[
    \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
    a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
    \Delta = \frac{1}{2} ab \sin C
\]
1 (a) The equation of a curve is \( y = 2x^2 + ax + (6 + a) \), where \( a \) is a constant. Find the range of values of \( a \) for which the curve lies completely above the \( x \)-axis. \( [3] \)

(b) The equation of a curve is \( y = 3x^2 + 4x + 6 \).
(i) Find the set of values of \( x \) for which the curve is above the line \( y = 6 \). \( [3] \)

(ii) Show that the line \( y = -8x - 6 \) is a tangent to the curve. \( [2] \)

2 (a) Given that \( \log_a 125 - 3 \log_a b + \log_a c = 3 \), express \( a \) in terms of \( b \) and \( c \). \( [3] \)

(b) Solve the equation
(i) \( \log 8x - \log (x^2 - 3) = 2 \log 2 \), \( [3] \)
(ii) \( 2 \log_a x = 3 + 7 \log_a 5 \). \( [4] \)

3 The equation of a curve is \( y = x^2 \sqrt{5x - 1} \), for \( x > 0.2 \). Given that \( x \) is changing at a constant rate of 0.25 units per second, find the rate of change of \( y \) when \( x = 2 \). \( [4] \)

4 The graph of \( y = |2x^2 - ax - 5| \) passes through the points with coordinates \((-1, 0)\) and \((0.75, b)\).
(i) Find the value of the constants \( a \) and \( b \). \( [3] \)
(ii) Sketch the graph of \( y = |2x^2 - ax - 5| \). \( [3] \)
(iii) Determine the set of positive values of \( m \) for which the line \( y = mx + 2 \) intersects the graph of \( y = |2x^2 - ax - 5| \) at two points. \( [2] \)

5 In the binomial expansion of \( \left(2x + \frac{k}{x}\right)^8\), where \( k \) is a positive constant, the coefficient of \( x^2 \) is 28.
(i) Show that \( k = \frac{1}{4} \). \( [4] \)
(ii) Hence, determine the term in \( x \) in the expansion of \( \left(6x - \frac{1}{x}\right)\left(2x + \frac{k}{x}\right)^8 \). \( [4] \)
6

The diagram shows a design of a bookmark that includes a rectangle $ABCD$, where $BC = l\text{ cm}$, $CD = 4r\text{ cm}$, a semicircle with radius $3r\text{ cm}$, and $AF = BE = r\text{ cm}$. The area of the bookmark is $90\text{ cm}^2$.

(i) Express $l$ in terms of $r$. [2]

(ii) Given that the perimeter of the bookmark is $P\text{ cm}$, show that

$$P = \left(6 + \frac{3\pi}{4}\right)r + \frac{45}{r}.$$ [2]

(iii) Given that $r$ and $l$ can vary, find the value of $r$ for which $P$ has a stationary value. Explain why this value of $r$ gives the minimum perimeter. [5]

7

The diagram shows an animal exhibition area that is surrounded by glass panels at $AB$, $BC$ and $AD$, where $AB = 12\text{ m}$, $AD = 34\text{ m}$, angle $DAB = \text{angle } BCD = 90^\circ$ and the acute angle $ADC = \theta$ can vary.

(i) Show that $L\text{ m}$, the length of the glass panels can be expressed as

$$L = 46 + 34\sin\theta - 12\cos\theta.$$ [2]

(ii) Express $L$ in the form $p + R\sin(\theta - \alpha)$, where $p$ and $R > 0$ are constants and $\alpha$ is an acute angle. [4]

(iii) Given that the exact length of the glass panels is $62\text{ m}$, find the value of $\theta$. [3]
The diagram shows points $A$, $B$, $C$ and $D$ on a circle, line $EF$ is tangent to the circle at $C$, lines $ADF$ and $EBAG$ are straight lines, and points $B$ and $C$ are the midpoints of $AE$ and $EF$.

Prove that:

(i) $BC \times EC = AC \times BE$, [3]

(ii) $AF \times EC = AC \times AE$, [2]

(iii) $\angle GAD = \angle ACF$. [2]

9. (a) (i) Show that $\cot 2x = \frac{1 - \tan^2 x}{2 \tan x}$. [2]

(ii) Hence, solve the equation $8 \cot 2x \tan x = 1$, for $0^\circ < x < 360^\circ$. [4]

(b) The Ultraviolet Index (UVI) describes the level of solar radiation. The UVI measured from the top of a building is given by $U = 6 - 5 \cos qt$, where $t$ is the time in hours from the lowest value of the UVI, $0 \leq t \leq 10$, and $q$ is a constant. It takes 10 hours for the UVI to reach its lowest value again.

(i) Explain why we are not able to measure a UVI of 12. [1]

(ii) Show that $q = \frac{\pi}{5}$. [1]

(iii) The top of the building is equipped with solar panels that supply power to the building when the UVI is at least 3. Find the duration, in hours and minutes, that the building is supplied with power from the solar panels. [4]
10 (a) It is given that \( y = \frac{2x^2}{4x-3} \), where \( x > \frac{3}{4} \).

(i) Find \( \frac{dy}{dx} \). [2]

(ii) Find the range of values of \( x \) for which \( y = \frac{2x^2}{4x-3} \) is a decreasing function. [4]

(b) It is given that \( f(x) \) is such that \( f'(x) = \frac{1}{2x-5} - \frac{4}{(2x-5)^2} \).

Given also that \( f(3) = 1.75 \), show that \( 8f(x) - (2x-5)^2 f'(x) = \ln(2x-5)^4 \). [7]

11 A particle moves in a straight line, so that, \( t \) seconds after passing a fixed point \( O \), its velocity, \( v \) m/s, is given by \( v = 2e^{0.1t} - 10e^{0.1t-0.3} \). The particle comes to an instantaneous rest at the point \( A \).

(i) Show that the particle reaches \( A \) when \( t = \frac{5}{2} \ln 5 + \frac{1}{4} \). [3]

(ii) Find the acceleration of the particle at \( A \). [3]

(iii) Find the distance \( OA \). [4]

(iv) Explain whether the particle is again at \( O \) at some instant during the eleventh second after first passing through \( O \). [2]
Answer Key

1. (a) \(-4 < a < 12\)  
   (b)(i) \(x < -\frac{1}{3}\) or \(x > 0\)

2. (a) \(a = \frac{5\sqrt{c}}{b}\)  
   (b)(i) \(x = 3\)  
   (ii) \(x = 85.7\) or \(x = 0.130\)

3. 49.5 units/s

4. (i) \(a = 3, b = 6.125\)  
   (ii) \(m > 2\)

5. (ii) \(-1 \frac{3}{4}x\)

6. (i) \(l = \frac{45}{2r} - \frac{9}{8}\pi r\)  
   (iii) \(r = 2.32\); min value

7. (ii) \(L = 46 + 10\sqrt{13} \sin(\theta - 19.4^\circ)\)  
   (iii) \(45.8^\circ\)

9. (a)(ii) \(x = 40.9^\circ, 139.1^\circ, 220.9^\circ, 319.1^\circ\)  
   (b)(iii) 7 hrs and 3 mins

10. (a)(i) \(\frac{4x(2x - 3)}{(4x - 3)^2}\)  
    (ii) \(\frac{3}{4} < x < \frac{3}{2}\)

11. (ii) \(1.23 \text{ m/s}^2\)  
    (iii) 16.0 m  
    (iv) passed through \(O\)
1 (a) For \( y = 2x^2 + ax + (6 + a) \) to lie above the x-axis, discriminant \( b^2 - 4ac < 0 \)
\[
(a)^2 - 4(2)(6+a) < 0 \\
a^2 - 8a - 48 < 0 \\
(a-12)(a+4) < 0 \\
-4 < a < 12
\]

(b) \( 3x^2 + 4x + 6 > 6 \)
(i) \( 3x^2 + 4x > 0 \)
\( x(3x+4) > 0 \)
\( x < -\frac{4}{3} \) or \( x > 0 \)

(ii) \( 3x^2 + 4x + 6 = -8x - 6 \)
\( 3x^2 + 12x + 12 = 0 \)
\( x^2 + 4x + 4 = 0 \)
Discriminant = \( (4)^2 - 4(1)(4) = 0 \)
Since discriminant = 0, the line and curve intersects only at one point.
Line \( y = -8x - 6 \) is tangent to the curve. (shown)

2 (a) \( \log_a 125 - 3 \log_a b + \log_a c = 3 \)
\( \log_a 125 - \log_a b^3 + \log_a c = 3 \)
\( \log_a \frac{125c}{b^3} = 3 \)
\( a^3 = \frac{125c}{b^3} \)
\( a = \frac{5\sqrt[3]{c}}{b} \)

(b) \( \log 8x - \log (x^2 - 3) = 2 \log 2 \)
(i) \( \log \left( \frac{8x}{x^2 - 3} \right) = \log 2^2 \)
\( \frac{8x}{x^2 - 3} = 4 \)
\[
\begin{align*}
4x^2 - 8x - 12 &= 0 \\
x^2 - 2x - 3 &= 0 \\
(x - 3)(x + 1) &= 0 \\
x &= 3 \quad \text{or} \quad -1 \quad \text{(reject } x = -1 \text{ as } \log{x} \text{ is undefined)} \\
x &= 3
\end{align*}
\]

(b)

(ii)

\[
2 \log_5 x = 3 + 7 \log_5 5
\]

\[
2 \log_5 x = 3 + 7 \left( \frac{\log_5 5}{\log_5 x} \right)
\]

\[
2(\log_5 x)^2 - 7 - 3 \log_5 x = 0
\]

Let \( u = \log_5 x \)

\[
2u^2 - 3u - 7 = 0
\]

\[
u = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-7)}}{2(2)}
\]

\[
\log_5 x = \frac{3 \pm \sqrt{65}}{4}
\]

\[
x = 5^{\left(\frac{3 + \sqrt{65}}{4}\right)} \quad \text{or} \quad x = 5^{\left(\frac{3 - \sqrt{65}}{4}\right)}
\]

\[
x = 85.7 \quad \text{or} \quad x = 0.130 \quad \text{(3 sig. figs.)}
\]

3

\[
y = x^2 \sqrt{(5x - 1)^3}
\]

\[
\frac{dy}{dx} = x^2 \left( \frac{3}{2} \left( 5x - 1 \right)^{\frac{1}{2}} \left( 5 \right) \right) + 2x \sqrt{(5x - 1)^3}
\]

\[
= (5x - 1)^{\frac{1}{2}} \left( \frac{15x^2}{2} + 2x(5x - 1) \right)
\]

\[
= (5x - 1)^{\frac{1}{2}} \left( \frac{35x^2}{2} - 2x \right)
\]

\[
\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}
\]

\[
= (5(2) - 1)^{\frac{1}{2}} \left( \frac{35(2)^2}{2} - 2(2) \right) \times 0.25
\]

\[
= 49.5 \quad \text{units/s}
\]
(i)\[ y = |2x^2 - ax - 5| \]
At \((-1, 0)\), \[ y = |2(-1)^2 - a(-1) - 5| \]
\[ |a - 3| = 0 \]
a = 3
At \((0.75, b)\), \[ b = |2(0.75)^2 - 3(0.75) - 5| = 6.125 \]

(ii)\[ y = |2x^2 - 3x - 5| \]

(iii) Line \(y = mx + 2\) passes through \((0, 2)\) and cuts two points to the right of \((0, 2)\).
The line that passes through \((-1, 0)\) and \((0, 2)\) has 3 points of intersection. Gradient \[ m = \frac{2 - 0}{0 - (-1)} = 2 \]
Lines with \(m > 2\) intersect the graph at 2 points.

5 (i) General Term \[ = \binom{8}{r} (2x)^{8-r} \left( \frac{k}{x} \right)^r \]
\[ = \binom{8}{r} (2)^{8-r} (k)^r x^{8-2r} \]
For term in \(x^2\):
\[ 8 - 2r = 2 \]
r = 3
Working

Coefficient = \( \binom{8}{3} (2)^{8-3} (k)^3 \)

= \( 1792k^3 \)

1792k³ = 28

k³ = \( \frac{1}{64} \)

k = \( \frac{1}{4} \)

(ii)

\[ (6x - \frac{1}{x})(2x + \frac{k}{x})^8 \]

\[ = \left(6x - \frac{1}{x}\right)\left(\cdots + 28x^2 + \cdots + \left(\frac{8}{4x}\right)^4\left(\frac{1}{4x}\right)^4 + \cdots\right) \]

Term in x

= 5 \times 70(16) \left(\frac{1}{4^4}\right) x - 28x

= \frac{-3}{4} x

6 (i)

\[ \frac{\pi}{2} (3r)^2 + 4rl = 90 \]

90 - \( \frac{9\pi r^2}{4} \)

l = \( \frac{9\pi r}{2} \)

l = \( \frac{45\pi}{2r} - \frac{9}{8}\pi r \)

(ii)

\[ P = 4r + 2l + 2r + \frac{\pi}{2} (6r) \]

= \( 4r + 2 \left(\frac{45\pi}{2r} - \frac{9\pi r}{8}\right) + 2r + 3\pi r \)

= \( 6r + \frac{3}{4}\pi r + \frac{45}{r} \)

= \( \left(6 + \frac{3}{4}\pi \right) r + \frac{45}{r} \) (shown)
(iii) 

\[ P = \left( 6 + \frac{3}{4} \pi \right) r + \frac{45}{r} \]

\[ \frac{dP}{dr} = 6 + \frac{3}{4} \pi - \frac{45}{r^2} \]

For stationary points, \( \frac{dP}{dr} = 0 \)

\[ 6 + \frac{3}{4} \pi = \frac{45}{r^2} \]

\[ r^2 = \frac{45 \times 4}{24 + 3\pi} \]

\[ r = \frac{\sqrt{60}}{\sqrt{24 + 3\pi}} \text{ since } r > 0. \]

\[ r = \frac{60}{8 + \pi} \text{ or } 2.32 \text{ (3 sig. fig.)} \]

\[ \frac{d^2P}{dr^2} = \frac{90}{r^3} = \frac{90}{(2.3206)^3} > 0 \]

Since \( \frac{d^2P}{dr^2} > 0 \), this gives a minimum value of \( P \).

7 (i)

\[ AX = 34 \sin \theta \]

\[ BC = 34 \sin \theta - 12 \cos \theta \]

\[ L = AD + AB + BC \]

\[ = 46 + 34 \sin \theta - 12 \cos \theta \]

(ii) 

\[ 34 \sin \theta - 12 \cos \theta = R \sin(\theta - \alpha) \]

\[ = R (\sin \theta \cos \alpha - \cos \theta \sin \alpha) \]

Comparing coefficients, \( R \sin \alpha = 12 \) and \( R \cos \alpha = 34 \)

\[ R = \sqrt{12^2 + 34^2} = \sqrt{1300} = 10\sqrt{13} \]
\[ \tan \alpha = \frac{12}{34} \quad \alpha = 19.440^\circ \]

\[ L = 46 + 10\sqrt{13} \sin(\theta - 19.4^\circ) \quad \text{(to 1 d.p.)} \]

(iii) \[ 46 + 10\sqrt{13} \sin(\theta - 19.440^\circ) = 62 \]
\[ 10\sqrt{13} \sin(\theta - 19.440^\circ) = 16 \]
\[ \sin(\theta - 19.440^\circ) = \frac{16}{10\sqrt{13}} \]
\[ \theta - 19.440^\circ = 26.344^\circ \]
\[ \theta = 26.344^\circ + 19.440^\circ \]
\[ = 45.8^\circ \]

8 (i) \[ \angle BCE = \angle BAC \quad \text{(alternate segment theorem)} \]
\[ \angle BEC = \angle AEC \quad \text{(common angle)} \]
Triangle \( BEC \) is similar to triangle \( CEA \) (AA similarity)

\[ \frac{BC}{BE} = \frac{AC}{CE} \]
\[ BC \times EC = AC \times BE \quad \text{(shown)} \]

(ii) Since \( B \) and \( C \) are the midpoints of \( AE \) and \( EF \),
\[ BC = \frac{1}{2} AF \]
\[ BC \parallel AF \quad \text{(midpoint theorem)} \]
\[ \frac{1}{2} AF \times EC = AC \times BE \quad \text{from (i)} \]
\[ AF \times EC = AC \times 2BE \]
\[ AF \times EC = AC \times AE \quad \text{(shown)} \]

(iii) \[ \angle GAD = \angle ABC \quad \text{(corr angles, } BC \parallel AF) \]
\[ \angle ACF = \angle ABC \quad \text{(alternate segment theorem)} \]
\[ \angle ACF = \angle GAD \quad \text{(shown)} \]
<table>
<thead>
<tr>
<th>Working</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>9 (a)</strong></td>
</tr>
<tr>
<td><strong>LHS:</strong></td>
</tr>
<tr>
<td>(i) $\cot 2x = \frac{1}{\tan 2x}$</td>
</tr>
<tr>
<td>$= \frac{1}{\frac{2\tan x}{1-\tan^2 x}}$</td>
</tr>
<tr>
<td>$= \frac{1-\tan^2 x}{2\tan x}$ (RHS) (shown)</td>
</tr>
<tr>
<td>(a) From (i),</td>
</tr>
<tr>
<td>(ii) $8\cot 2x \tan x = 4(2 \cot 2x \tan x)$</td>
</tr>
<tr>
<td>$= 4(1-\tan^2 x)$</td>
</tr>
<tr>
<td>$4(1-\tan^2 x) = 1$</td>
</tr>
<tr>
<td>$4 - 4\tan^2 x = 1$</td>
</tr>
<tr>
<td>$\tan^2 x = \frac{3}{4}$</td>
</tr>
<tr>
<td>$\tan x = \pm \frac{\sqrt{3}}{2}$</td>
</tr>
<tr>
<td>Basic angle $\alpha = 40.8933^\circ$</td>
</tr>
<tr>
<td>$x = 40.8933^\circ, 180^\circ + 40.8933^\circ$ or $x = 180^\circ - 40.8933^\circ, 360^\circ - 40.8933^\circ$</td>
</tr>
<tr>
<td>$x = 40.9^\circ, 139.1^\circ, 220.9^\circ, 319.1^\circ$ (1 d.p.)</td>
</tr>
</tbody>
</table>

| **9 (b)** |
| **$U = 6 - 5\cos qt$** |
| (i) Highest value of $-5\cos qt = 5$ when $\cos qt = -1$, highest value is 11, we are not able to measure UVI of 12. |
| (b) UVI takes 10 hours to reach its lowest again, |
| (ii) $10q = 2\pi$ |
| $q = \frac{\pi}{5}$ |
| (b) |
| (iii) $3 = 6 - 5\cos \frac{\pi t}{5}$ |
| $5\cos \frac{\pi t}{5} = 3$ |
\[ \cos \frac{\pi t}{5} = \frac{3}{5} \]

Basic angle, \( \alpha = 0.927295 \)

\[ \frac{\pi t}{5} = 0.927295 \quad \text{or} \quad 5.35589 \]

\[ t = 1.47583 \quad \text{or} \quad 8.52416 \]

Duration of solar power supply

\[ = 8.52416 - 1.47583 \]

\[ = 7.04833 \text{ hrs} \]

\[ = 7 \text{ hrs and 3 mins} \]

**10**

**(a)**

**(i)**

\[ y = \frac{2x^2}{4x-3} \]

\[ \frac{dy}{dx} = \frac{(4x-3)(4x) - 2x^2(4)}{(4x-3)^2} \]

\[ = \frac{8x^2 - 12x}{(4x-3)^2} \]

\[ = \frac{4x(2x-3)}{(4x-3)^2} \]

**(b)**

For decreasing function,

\[ \frac{dy}{dx} = \frac{8x^2 - 12x}{(4x-3)^2} < 0 \]

\[ \frac{4x(2x-3)}{(4x-3)^2} < 0 \]

Denominator \((4x-3)^2 > 0\) for \( x > \frac{3}{4} \),

\[ x(2x-3) < 0 \]

\[ 0 < x < \frac{3}{2} \]

Since \( x > \frac{3}{4} \), \( y \) is decreasing function for \( \frac{3}{4} < x < \frac{3}{2} \).
10 (b)  
\[ f(x) = \int \frac{1}{2x-5} - \frac{4}{(2x-5)^2} \, dx \]
\[ = \frac{1}{2} \ln(2x-5) + \frac{2}{2x-5} + c, \text{ where } c \text{ is a constant.} \]

Given \( f(3) = 1.75 \),
\[ \frac{1}{2} \ln(2(3)-5) + \frac{2}{2(3)-5} + c = 1.75 \]
\[ c = -0.25 \]

\[ f''(x) = \frac{d}{dx} \left( \frac{1}{2x-5} - \frac{4}{(2x-5)^2} \right) \]
\[ = \frac{-2}{(2x-5)^3} + \frac{16}{(2x-5)^3} \]

\[ 8f(x) - (2x-5)^2 f''(x) \]
\[ = 8 \left[ \frac{1}{2} \ln(2x-5) + \frac{2}{2x-5} - 0.25 \right] - (2x-5)^2 \left( \frac{-2}{(2x-5)^2} + \frac{16}{(2x-5)^3} \right) \]
\[ = 4 \ln(2x-5) \]
\[ = \ln((2x-5)^4) \quad \text{(shown)} \]

11 (i)  
For instantaneous rest, \( v = 0 \)
\[ 2e^{0.1t} - 10e^{0.1-0.3t} = 0 \]
\[ 2e^{0.1t} = 10e^{0.1-0.3t} \]
\[ \frac{e^{0.1t}}{e^{-0.3t}} = 5e^{0.1} \]
\[ e^{0.4t} = 5e^{0.1} \]

Taking \( \ln \) on both sides:
\[ 0.4t = \ln 5 + 0.1 \]
\[ t = \frac{5}{2} \ln 5 + 0.1 \quad \text{(shown)} \]

(ii)  
\[ a = \frac{dv}{dt} \]
\[ = 0.2e^{0.1t} - 10(-0.3)e^{0.1-0.3t} \]
\[ = 0.2e^{0.1t} + 3e^{0.1-0.3t} \]
When \( t = \frac{5}{2} \ln 5 + \frac{1}{4} \),

\[
a = 0.2e^{0.1\left(\frac{5}{2}\ln 5 + \frac{1}{4}\right)} + 3e^{0.1 - 0.3\left(\frac{1}{2}\ln 5 + \frac{1}{4}\right)}
\]

\[= 1.2265\]

\[= 1.23 \text{ m/s}^2\]

(iii) \[
s = \int v \, dt
\]

\[= \int 2e^{0.3t} - 10e^{0.1-0.3t} \, dt
\]

\[= 20e^{0.1t} + \frac{100}{3} e^{0.1-0.3t} + c, \text{ where } c \text{ is a constant}
\]

Since \( s = 0 \) when \( t = 0 \),

\[s = 20 + \frac{100}{3} e^{0.1} + c
\]

\[c = -\left(20 + \frac{100}{3} e^{0.1}\right)
\]

\[OA = 20e^{0.1\left(\frac{5}{2}\ln 5 + \frac{1}{4}\right)} + \frac{100}{3} e^{0.1-0.3\left(\frac{1}{2}\ln 5 + \frac{1}{4}\right)} - \left(20 + \frac{100}{3} e^{0.1}\right)
\]

\[= -15.9535
\]

\[= -16.0
\]

\(OA\) is 16.0 m (3 sig. fig.)

(iv) When \( t = 10 \),

\[s_{10} = 20e^{1} + \frac{100}{3} e^{0.1-3.3} - \left(20 + \frac{100}{3} e^{0.1}\right)
\]

\[= -0.63928 \text{ m}
\]

When \( t = 11 \),

\[s_{11} = 20e^{1.1} + \frac{100}{3} e^{0.1-3.3} - \left(20 + \frac{100}{3} e^{0.1}\right)
\]

\[= 4.6030 \text{ m}
\]

Since the displacement of the particle changes from negative to positive, the particle passed through \(O\) during the eleventh second.
READ THESE INSTRUCTIONS FIRST

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Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
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Answer all the questions.
Write your answers on the separate Answer Paper provided.
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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.
1. **ALGEBRA**

**Quadratic Equation**

For the equation \( ax^2 + bx + c = 0 \)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

**Binomial Expansion**

\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2) \ldots (n-r+1)}{r!}
\]

2. **TRIGONOMETRY**

**Identities**

\[
\sin^2 A + \cos^2 A = 1
\]

\[
\sec^2 A = 1 + \tan^2 A
\]

\[
\csc^2 A = 1 + \cot^2 A
\]

\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
\sin 2A = 2 \sin A \cos A
\]

\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]

\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

**Formulae for \( \triangle ABC \)**

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2} bc \sin A
\]
1. Solve the following equations
   
   (a) $5^{2x+1} - 3(5^{x}) + 10 = 0$, 
   
   (b) $\log_y \sqrt{3 - 3x} = \frac{1}{2} - \log_{10} (1 - 2x)$. 
   
2. (a) Find the greatest value of the integer $k$ for which $-3x^2 + kx - 5$ is never positive for all values of $x$. 
   
   (b) A curve has an equation $y = \frac{x^2}{2 - 3x}$, where $x \neq \frac{2}{3}$. 
   
   Find the range of values of $x$ for which $y$ is decreasing. 
   
3. (i) Prove the identity $1 + \frac{\sin^2 A}{\csc^2 A} + \frac{\cos^2 A}{\sec^2 A} = 0$. 
   
   (ii) Hence, solve the equation $\frac{\sin^2 A}{\csc^2 A} + \frac{\cos^2 A}{\sec^2 A} = \tan (2A + 10^\circ)$ for $-180^\circ < A < 180^\circ$. 
   
4. A curve has the equation $y = 4e^{\tan(\pi x^2)}$. 
   
   (i) Find $\frac{dy}{dx}$. 
   
   (ii) If $x$ and $y$ vary with time and $y$ increases at the rate of $e$ units per second when $x = \pi$ radian, find the exact value of the rate of decrease of $x$ at this instant. 
   
5. (a) Sketch the graph of $f(x) = 2 - |5 - 3x|$ for $-1 \leq x \leq 6$. 
   
   Indicate clearly the vertex and the intercepts of the axes. 
   
   (b) Solve the equation $2 - |5 - 3x| = x - 1$. 
   
   (c) (i) State the range of the values of $c$ if there is no solution for the equation $2 - |5 - 3x| = c$. 
   
   (ii) State the range of the values of $m$ if there are exactly two solutions for the equation $2 - |5 - 3x| = mx$. 
   
---

www.sgexamguru.com 129
6 The amount of radioactive Sodium-24, $M$ measured in grams, used as a tracer to measure the rate of flow in an artery or vein can be modelled by $M = M_0e^{kt}$, where $t$ is the time in hours, $M_0$ and $k$ are a constants.

The hospital buys a 40-grams sample of Sodium-24 and will reorder when the sample is reduced to 3 grams.

(i) Given that there are only 20 grams of Sodium-24 left after 14.9 hours. Find the value of $M_0$ and of $k$. [3]

(ii) Find the amount of Sodium-24 remain after 60 hours. [1]

(iii) Calculate the time taken before the hospital reorders Sodium-24. [2]

7 (a) The function $f$ is defined, for $-rac{\pi}{2} \leq x \leq \frac{\pi}{2}$, by the equation $f(x) = 2 \tan 3x$.

(i) State the period of $f$. [1]

(ii) Sketch the graph of $y = f(x)$ for $-rac{\pi}{2} \leq x \leq \frac{\pi}{2}$. [2]

(b) On the same diagram drawn in part (a), sketch the graph of $g(x) = 1 - 2 \sin x$ for $-rac{\pi}{2} \leq x \leq \frac{\pi}{2}$. [2]

(c) State the number of solutions of the equation $\sin x + \tan 3x = \frac{1}{2}$ in the interval $-rac{\pi}{2} \leq x \leq \frac{\pi}{2}$. [1]

8 The function $f(x) = -\ln x$ is defined for $x > k$.

(i) State the value of $k$. [1]

(ii) Sketch the graph of $f(x) = -\ln x$ for $x > k$. [2]

(iii) Explain how another straight line drawn on your diagram in part (ii) can lead to the graphical solution of $xe^{3-3x} = 1$.

Draw this straight line and hence state the number of solutions for $xe^{3-2x} = 1$. [3]
9 The diagram shows a quadrilateral $OPQR$ where $OR = 6 \text{ cm}$, 
angle $OPQ = \text{angle } PQR = \frac{\pi}{2}$ radian and angle $ROP = \theta$ radian, $\theta$ is a 
variable and an acute angle. $T$ is a point on $PQ$ such that angle $ORT = \frac{\pi}{2}$ 
radian and $RT = 3 \text{ cm}$.

![Diagram of quadrilateral OPQR with angles and dimensions labeled]

(i) Show that the area, $A \text{ cm}^2$ of the quadrilateral $OPQR$ is given by

$$A = 9 \sin 2\theta + 18 \sin^2 \theta$$

(ii) Given that $\theta$ can vary, find maximum area of the quadrilateral $OPQR$.

10 A particle $P$ moves in a straight line so that $t$ seconds after passing through a 
fixed point $O$, its velocity, $v \text{ m/s}$ is given by

$$v = 1 - \frac{9}{(3t + 1)^2}.$$ 

(i) Calculate the initial acceleration of the particle $P$.

(ii) Show that the particle $P$ is at instantaneously rest at $t = \frac{2}{3}$.

(iii) Calculate the average speed of the particle $P$ during the first 
3 seconds after passing $O$.

Another particle $Q$ moves in a straight line and its displacement, $S$ meter 
from $O$ after $t$ seconds is given by $S_Q = t - 1$.

(iv) Find the distance from the fixed point $O$ when $P$ first collides 
with $Q$.

In the diagram, A, B, C and D are points on the circle. MN is a tangent to the circle at A. MBC is a straight line.

(a) Name a triangle which is similar to triangle CAM. [1]

Hence prove that \( \left( \frac{AC}{BA} \right)^2 = \frac{CM}{BM} \). [3]

(b) Given further that AD and BC are parallel, show that

(i) triangle ABM is similar to triangle ADC. [2]

(ii) \( AD \times AM = AC \times CD \). [2]

--- End of Paper ---
<table>
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**MARIS STELLA HIGH SCHOOL**  
**PRELIMINARY EXAMINATION TWO**  
**SECONDARY FOUR**

**ADDITIONAL MATHEMATICS**  
**Paper 1**  
**4047/1**  
19 August 2016  
2 hours

**Additional Materials:**  
Answer Paper (8 sheets)

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For Examiner's Use

80

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This document consists of 7 printed pages.
1. **ALGEBRA**

*Quadratic Equation*

For the equation \( ax^2 + bx + c = 0 \)

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x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

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where \( n \) is a positive integer and

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}
\]

2. **TRIGONOMETRY**

*Identities*

\[
sin^2 A + cos^2 A = 1 \\
sec^2 A = 1 + tan^2 A \\
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sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \\
cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \\
tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \\
sin 2A = 2 \sin A \cos A \\
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A \\
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

*Formulae for \( \triangle ABC \)*

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \\
a^2 = b^2 + c^2 - 2bc \cos A \\
\Delta = \frac{1}{2} bc \sin A
\]
1. Solve the following equations

(a) \(5^{2x} - 3(5^{1-x}) + 10 = 0\)  \([4]\)

(b) \(\log_5 \sqrt{3 - 3x} = \frac{1}{2} - \log_8(1 - 2x)\)  \([4]\)

(a) \(5^{2x} - 3(5^{1-x}) + 10 = 0\)

\[25(5^x) - \frac{15}{5^x} + 10 = 0\]
\[5(5^x) - \frac{3}{5^x} + 2 = 0\]  \([M1]\)

Let \(p = 5^x\)
\[5p - \frac{3}{p} + 2 = 0\]
\[5p^2 + 2p - 3 = 0\]  \([M1]\)
\[(5p - 3)(p + 1) = 0\]
\(p = \frac{3}{5}\) or \(p = -1\)

\(5^x = \frac{3}{5}\) or \(5^x = -1\) (reject)

\(\log 5^x = \log \left(\frac{3}{5}\right)\)  \([M1]\)  \((p\text{ if never reject }5^x = -1)\)

\(\log \left(\frac{3}{5}\right)\)
\(\log 5\)

\(x = \frac{\log \left(\frac{3}{5}\right)}{\log 5}\)
\(x = -0.317\)  \([A1]\)
(b) \[ \log_9 \sqrt{3 - 3x} = \frac{1}{2} - \log_{81} (1 - 2x) \]
\[ \log_9 \sqrt{3 - 3x} = \frac{1}{2} - \frac{\log_9 (1 - 2x)}{\log_9 81} \]
\[ \frac{1}{2} \log_9 (3 - 3x) = \frac{1}{2} - \frac{\log_9 (1 - 2x)}{2} \]  
[M1 for changing base]
\[ \log_9 (3 - 3x) + \log_9 (1 - 2x) = 1 \]
\[ \log_9 (3 - 3x)(1 - 2x) = 1 \]  
[M1]
\[ (3 - 3x)(1 - 2x) = 9 \]
\[ (1 - x)(1 - 2x) = 3 \]  
[M1]
\[ 2x^2 - 3x - 2 = 0 \]
\[ (2x + 1)(x - 2) = 0 \]
\[ x = -\frac{1}{2} \text{ or } x = 2 \text{ (reject)} \]  
(p if never reject \( x = 2 \))
\[ \therefore x = -\frac{1}{2} \]  
[A1]

2 (a) Find the greatest value of the integer \( k \) for which \(-3x^2 + kx - 5\) is never positive for all values of \( x \).  
[3]

(b) A curve has an equation \( y = \frac{x^2}{2 - 3x} \), where \( x \neq \frac{2}{3} \).

Find the range of values of \( x \) for which \( y \) is decreasing.  
[4]

(a) For all values of \( x \), \(-3x^2 + kx - 5\) is never positive,

Discriminant \( \leq 0 \)
\[ k^2 - 4(-3)(-5) \leq 0 \]  
[M1]
\[ k^2 - 60 \leq 0 \]
\[ (k - \sqrt{60})(k + \sqrt{60}) \leq 0 \]
\[ -\sqrt{60} \leq k \leq \sqrt{60} \]  
[A1]
\[ OR \quad -2\sqrt{15} \leq k \leq 2\sqrt{15} \]
\[ OR \quad -7.7460 \leq k \leq 7.7460 \]

The greatest integer value of \( k \) is 7  
[A1]
(b) \[ y = \frac{x^2}{2-3x}, x \neq \frac{2}{3} \]

\[
\frac{dy}{dx} = \frac{2x(2-3x) + 3x^2}{(2-3x)^2} \quad [M1]
\]

\[ = \frac{4x - 3x^2}{(2-3x)^2} \]

Since the curve is decreasing, \[ \frac{dy}{dx} < 0 \] and \[ x \neq \frac{2}{3} \]

\[ \frac{4x - 3x^2}{(2-3x)^2} < 0 \quad [M1] \]

Since \[ (2-3x)^2 > 0 \], \[ 4x - 3x^2 < 0 \]

\[ 3x^2 - 4x > 0 \quad [M1] \]

\[ x(3x - 4) > 0 \]

\[ x < 0 \text{ or } x > \frac{4}{3} \quad [A1] \]

3 (i) Prove the identity \[ 1 + \frac{\sin^2 A}{1 - \sec^2 A} + \frac{\cos^2 A}{1 - \csc^2 A} = 0. \] \[ [3] \]

(ii) Hence, solve the equation \[ \frac{\sin^2 A}{1 - \sec^2 A} + \frac{\cos^2 A}{1 - \csc^2 A} = \tan (2A + 10^\circ) \]

for \[ -180^\circ < A < 180^\circ. \] \[ [4] \]

(i) To prove \[ 1 + \frac{\sin^2 A}{1 - \sec^2 A} + \frac{\cos^2 A}{1 - \csc^2 A} = 0. \]

\[ \text{LHS} = 1 + \frac{\sin^2 A}{1 - \sec^2 A} + \frac{\cos^2 A}{1 - \csc^2 A} \]

\[ = 1 + \frac{\sin^2 A}{-\tan^2 A} + \frac{\cos^2 A}{-\cot^2 A} \quad [B1] \]

\[ = 1 - \cos^2 A - \sin^2 A \quad [B1] \]

\[ = 1 - 1 \quad [B1] \]

\[ = 0 \]

Hence \[ 1 + \frac{\sin^2 A}{1 - \sec^2 A} + \frac{\cos^2 A}{1 - \csc^2 A} = 0. \text{ (Proved)} \]
(ii) Since \(\frac{\sin^2 A}{1 - \sec^2 A} + \frac{\cos^2 A}{1 - \csc^2 A} = \tan (2A + 10^\circ)\)

\[\tan (2A + 10^\circ) = -1 \quad [B1]\]

Basic angle = 45°

\[2A + 10^\circ = 45^\circ, 225^\circ, 135^\circ, 315^\circ \quad [M1]\]

\[A = -27.5^\circ, -117.5^\circ, 62.5^\circ, 152.5^\circ\]

[A1 for both] [A1 for both]

---

4 A curve has the equation \(y = 4e^{\frac{\tan(x - \frac{x}{4})}{4}}\).

(i) Find \(\frac{dy}{dx}\). \([2]\]

(ii) If \(x\) and \(y\) vary with time and \(y\) increases at the rate of \(e\) units per second when \(x = \pi\) radian. Find the exact value of the rate of decrease of \(x\) at this instant. \([4]\)

\[
\frac{dy}{dx} = 4\left(-\frac{1}{4}\right)\sec^2 (\pi - \frac{x}{4})e^{\frac{\tan(x - \frac{x}{4})}{4}} \quad [M1]
\]

\[
\frac{dy}{dx} = -\sec^2 (\pi - \frac{x}{4})e^{-\frac{\tan(x - \frac{x}{4})}{4}} \quad [B1]
\]

(ii) When \(x = \pi\),

\[
\frac{dy}{dx} = -\sec^2 (\frac{3\pi}{4})e^{-\frac{\tan(x - \frac{x}{4})}{4}} \quad [M1]
\]

\[
= -(\sqrt{2})^2 e^{-1} \quad [A1]
\]

\[
\frac{dy}{dt} = \frac{dx}{dt} \times \frac{dy}{dx}
\]

\[
e = \frac{dx}{dt} \times \left(-\frac{2}{e}\right) \quad [M1]
\]

\[
\frac{dx}{dt} = -\frac{e^2}{2}
\]

The exact rate of decrease of \(x\) is \(\frac{e^2}{2}\) units / s \([A1]\)
5 (a) Sketch the graph of \( f(x) = 2 - |5 - 3x| \) for \(-1 \leq x \leq 6\).

Indicate clearly the vertex and the intercepts of the axes. [3]

(b) Solve the equation \( 2 - |5 - 3x| = x - 1 \) [2]

(c) (i) State the range of the values of \( c \) if there is no solution for the equation \( 2 - |5 - 3x| = c \). [1]

(ii) State the range of the values of \( m \) if there are exactly two solutions for the equation \( 2 - |5 - 3x| = mx \). [1]

(a) Turning Points = \((1\frac{2}{3}, 2)\) [B1]

Shape - inverted v-shape [B1]

intercepts : \((0, -3), (1, 0), (2\frac{1}{3}, 0)\)

terminal points : \((-1, -6), (6, -11)\) [B1]

(b) \[2 - |5 - 3x| = x - 1\]

\[|5 - 3x| = 3 - x\]

\[5 - 3x = 3 - x\] or \[5 - 3x = -(3 - x)\] [M1]

\[x = 1\] [A1]

(c) (i) \(c > 2\) [B1]

(ii) Gradient of \(OA = \frac{6}{5}\)
Gradient of \(AB = -3\)

The range of values of \(m\) : \(-3 < m < \frac{6}{5}\) [B1]
6 The amount of radioactive Sodium-24, $M$ measured in grams, used as a tracer to measure the rate of flow in an artery or vein can be modelled by

$$M = M_0 e^{kt},$$

where $t$ is the time in hours, $M_0$ and $k$ are constants.

The hospital buys a 40-grams sample of Sodium-24 and will reorder when the sample is reduced to 3 grams.

(i) Given that there are only 20 grams of Sodium-24 left after 14.9 hours. Find the value of $M_0$ and of $k$. [3]

(ii) Find the amount of Sodium-24 remain after 60 hours. [1]

(iii) Calculate the time taken before the hospital reorders Sodium-24. [2]

(i) When $t = 0, M = 40$

$$M_0 = 40$$ [B1]

When $t = 14.9, M = 20$

$$20 = 40e^{14.9k}$$

$$e^{14.9k} = \frac{1}{2}$$ [M1]

$$k = \frac{1}{14.9} \ln \frac{1}{2}$$

$$k = -\frac{1}{14.9} \ln 2$$

$$k = -0.046520$$

$$k = -0.0465 \text{ (3 s.f.)}$$ [A1]

(ii) When $t = 60,$

$$M = 40e^{-\frac{1}{14.9} \ln 2 \times 60}$$

$$M = e^{-2.912}$$

$$M = 0.0613 \text{ g}$$ [A1]

(iii) When $M = 3,$

$$3 = 40e^{-0.04652t}$$

$$\frac{3}{40} = e^{-0.04652t}$$ [M1]

$$\ln \left( \frac{3}{40} \right) = -0.04652t$$

$$t = -\frac{1}{0.04652} \ln \left( \frac{3}{40} \right)$$

$$t = 55.7 \text{ hours}$$ [A1]
7 (a) The function \( f \) is defined, for \( -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \), by the equation
\[ f(x) = 2 \tan 3x. \]

(i) State the period of \( f \). \[ [1] \]
(ii) Sketch the graph of \( y = f(x) \) for \( -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \). \[ [2] \]

(b) On the same diagram drawn in part (a), sketch the graph of
\[ g(x) = 1 - 2 \sin x \] for \( -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \). \[ [2] \]

(c) State the number of solutions of the equation \( \sin x + \tan 3x = \frac{1}{2} \) in the interval \( -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \). \[ [1] \]

(a) (i) Period = \( \frac{\pi}{3} \) \[ [B1] \]

(ii) Shape \[ [B1] \]
4 asymptotes \[ [B0.5] \]
x-intercept: \( -\frac{\pi}{6}, 0, \frac{\pi}{6}; \) \[ [B0.5] \]

(b) Shape \[ [B1] \]
turning points \( (-\frac{\pi}{2}, 3); (\frac{\pi}{2}, -1); \) \[ [B0.5] \]
intercepts: \( (0, 1), (\frac{\pi}{6}, 0) \) \[ [B0.5] \]

\[
\sin x + \tan 3x = \frac{1}{2} \\
2 \sin x + 2 \tan 3x = 1 \\
2 \tan 3x = 1 - 2 \sin x
\]

There are 3 solutions for the equation \( \sin x + \tan 3x = \frac{1}{2} \) in the interval \( -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \). \[ [A1] \]
8 The function \( f(x) = -\ln x \) is defined for \( x > k \).

(i) State the value of \( k \). [1]
(ii) Sketch the graph of \( f(x) = -\ln x \) for \( x > k \). [2]
(iii) Explain how another straight line drawn on your diagram in part (b) can lead to the graphical solution of \( xe^{2x-3} = 1 \). Draw this straight line and state the number of solutions for \( xe^{2x-3} = 1 \). [3]

(i) \( k = 0 \) [B1]

(ii) Shape [B1]

- Asymptote \( x = 0 \) [B 0.5]
- \( x \)-intercept: \((1, 0)\) [B 0.5]

(iii) Since

\[
\begin{align*}
xe^{2x-3} &= 1 \\
\ln(xe^{2x-3}) &= 0 \\
\ln x + 2x - 3 &= 0 \\
y &= 3 - 2x
\end{align*}
\]

[B1]

Hence, by drawing the line \( y = 3 - 2x \) on the diagram in part (b), the \( x \)-coordinates of the points of intersection would give the solutions for \( xe^{2x-3} = 1 \). [B1]

From the sketch, we can conclude that there are 2 solutions for \( xe^{2x-3} = 1 \). [A1]

9 The diagram shows a quadrilateral \( OPQR \) where \( OR = 6 \) cm, angle \( OPQ = \) angle \( PQR = \frac{\pi}{2} \) radian and angle \( ROP = \theta \) radian, \( \theta \) is a variable and an acute angle. \( T \) is a point on \( PQ \) such that angle \( ORT = \frac{\pi}{2} \) radian and \( RT = 3 \) m.
(i) Show that the area, $A$ cm$^2$ of the quadrilateral $OPQR$ is given by

$$A = 9 \sin 2\theta + 18 \sin^2 \theta$$  \hspace{1cm} [3]

(ii) Given that $\theta$ can vary, find maximum area of the quadrilateral $OPQR$.

$$\overline{PSR} = \frac{\pi}{2} \text{ rad}$$

$$\angle RTQ = \theta \quad (\text{alt. } \angle, \text{ PQ/}SR)$$

$$A = \frac{1}{2} (OS)(RS) + (RS)(RQ)$$

$$A = \frac{1}{2} (6 \cos \theta)(6 \sin \theta) + (6 \sin \theta)(3 \sin \theta) \quad [M1]$$

$$A = 18 \sin \theta \cos \theta + 18 \sin^2 \theta \quad [A1]$$

$$A = 9 \sin 2\theta + 18 \sin^2 \theta \quad \text{ (Shown)}$$

$$A = 9 \sin 2\theta + 18 \sin^2 \theta$$

$$\frac{dA}{d\theta} = 18 \cos 2\theta + 18(2) \sin \theta \cos \theta \quad [B1]$$

$$= 18 \cos 2\theta + 18 \sin 2\theta$$

For maximum area, $\frac{dA}{d\theta} = 0$.

$$\frac{dA}{d\theta} = 18 \cos 2\theta + 18 \sin 2\theta = 0 \quad [B1]$$

$$\cos 2\theta + \sin 2\theta = 0$$

$$1 + \tan 2\theta = 0$$

$$\tan 2\theta = -1$$

Basic angle $= \frac{\pi}{4}$

$$2\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$\theta = \frac{3\pi}{8} \quad \text{ (N.A.)}$$

$$\theta = \frac{3\pi}{8}$$  \hspace{1cm} [A1]
\[ \frac{d^2 A}{d\theta^2} = -36 \sin 2\theta + 36 \cos 2\theta \]

When \( \theta = \frac{3\pi}{8} \), \[ \frac{d^2 A}{d\theta^2} = -36 \left( \frac{1}{\sqrt{2}} \right) + 36 \left( -\frac{1}{\sqrt{2}} \right) = -36\sqrt{2} < 0 \] \[ [B1] \]

Therefore, maximum area
\[
= 9\sin 2\left( \frac{3\pi}{8} \right) + 18\sin^2\left( \frac{3\pi}{8} \right)
\]
\[
= \frac{9}{\sqrt{2}} + 18\left( \frac{1}{\sqrt{2}} \right)^2
\]
\[
= 9(1 + \frac{\sqrt{2}}{2})
\]
\[
= 15.4 \text{ cm}^2 \] \[ [A1] \]

10 A particle \( P \) moves in a straight line so that \( t \) seconds after passing through a fixed point \( O \), its velocity, \( v \) m/s is given by
\[
v = 1 - \frac{9}{(3t + 1)^2}.
\]

(i) Calculate the initial acceleration of the particle \( P \). \[ [2] \]

(ii) Show that the particle \( P \) is at instantaneously rest at \( t = \frac{2}{3} \). \[ [2] \]

(iii) Calculate the average speed of the particle \( P \) during the first 3 seconds after passing \( O \). \[ [4] \]

Another particle \( Q \) moves in a straight line and its displacement, \( S \) m from \( O \) after \( t \) seconds is given by
\[
S_Q = t - 1
\]

(iv) Find the distance from the fixed point \( O \) when \( P \) first collides with \( Q \). \[ [2] \]
(i) \[ v_p = 1 - \frac{9}{(3t+1)^2} \]

acceleration, \(a = \frac{dv}{dt}\)

\[ a = \frac{54}{(3t+1)^3} \] \[\text{[M1]}\]

Initial acceleration = 54 m/s² \[\text{[A1]}\]

(ii) At instantaneously rest, \(v_p = 0\)

\[ 1 - \frac{9}{(3t+1)^2} = 0 \]
\[ (3t+1)^2 = 9 \] \[\text{[M1]}\]
\[ 3t + 1 = \pm 3 \]
\[ t = \frac{2}{3} \text{ or } -\frac{4}{3} \]

(reject)

\[ t = \frac{2}{3} \] (Shown) \[\text{[A1]}\]

(iii) \[ S_p = \int \left[ 1 - \frac{9}{(3t+1)^2} \right] dx \]

\[ S_p = t + \frac{3}{3t+1} + c \] \[\text{[M1]}\]

When \(t = 0, S_p = 0,\)
\[ 0 = 3 + c \]
\[ c = -3 \]

\[ \therefore S_p = t + \frac{3}{3t+1} - 3 \] \[\text{[A1]}\]

When
\[ t = 0, S = 0m \]
\[ t = \frac{2}{3}, S = -1\frac{1}{3}m \]
\[ t = 3, S = 3\frac{3}{10}m \]
average speed
\[ \frac{\frac{4}{3} \times 2 + \frac{3}{10}}{3} = \frac{89}{90} \]  
\[ = 0.989 \text{ m/s} \] [A1]

(iv) When \( P \) collides with \( Q \), \( S_p = S_q \),
\[ t + \frac{3}{3t + 1} - 3 = t - 1 \]
\[ \frac{3}{3t + 1} = 2 \]
\[ 3t + 1 = \frac{3}{2} \]
\[ t = \frac{1}{6} \] [M1]

When \( t = \frac{1}{6}, S_q = \frac{1}{6} - 1 \)
\[ S_q = -\frac{5}{6} \] m [A1]

Hence, the particles first collide at \( \frac{5}{6} \) m from the fixed point \( O \). [A1]

11 In the diagram, \( A, B, C \) and \( D \) are on the circle. \( MN \) is a tangent to the circle at \( A \). \( MBC \) is a straight line.

![Diagram](image-url)
(a) Name a triangle which is similar to triangle $CAM$. 

Hence prove that $\left(\frac{AC}{BA}\right)^2 = \frac{CM}{BM}$. 

(b) Given further that $AD$ and $BC$ are parallel, show that

(i) triangle $ABM$ is similar to triangle $ADC$. 

(ii) $AD \times AM = AC \times CD$. 

(a) 

$\hat{MAB} = \hat{MA} \quad$ (common angle) 

$\hat{MAB} = \hat{MCA} \quad$ (alternate segment theorem) 

triangle $CAM$ is similar to triangle $ABM$ 

$$\frac{AC}{BA} = \frac{AM}{BM} = \frac{MC}{MA} \quad [B1]$$ 

$$\left(\frac{AC}{BA}\right)^2 = \left(\frac{AM}{BM}\right)^2 \quad [B1]$$ 

$$= \frac{BM \times MC}{BM^2} \quad (AM^2 = MC \times BM) \quad [B1]$$ 

$$= \frac{MC}{BM}$$ 

$$\therefore \left(\frac{AC}{BA}\right)^2 = \frac{CM}{BM} \quad \text{(proved)} \quad [p \text{ if no conclusion}]$$ 

(b) 

$\hat{ABM} = \hat{ADC} \quad$ (angle in opposite segment) 

$\hat{MAB} = \hat{MCA} \quad$ (alternate segment theorem) 

$\hat{MAD} \quad$ (alternate angle, $AD//BC$) 

triangle $ABM$ is similar to triangle $ADC$ \quad [B 2, 1, 0] 

$$\frac{AD}{AB} = \frac{CD}{MB} \quad [B1]$$ 

$$\frac{AD}{CD} = \frac{AB}{MB}$$ 

$$\frac{AD}{CD} = \frac{AC}{AM} \quad \text{since} \quad \frac{AB}{MB} = \frac{AC}{AM} \quad (\text{from part (a)} \quad [B1]$$ 

$$AD \times AM = AC \times CD \quad (Proved) \quad [p \text{ if no conclusion}]$$ 

~ End of Paper
(b) The equation of another circle is \((x - 4)^2 + (y + 1)^2 = 4\).

The line \(y = mx\) is a tangent to the circle. Find the possible exact values of \(m\).  

<table>
<thead>
<tr>
<th>For points of intersection,</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>substitute (y = mx) into ((x - 4)^2 + (y + 1)^2 = 4)</td>
<td></td>
</tr>
<tr>
<td>((x - 4)^2 + (mx + 1)^2 = 4)</td>
<td>M1</td>
</tr>
<tr>
<td>(x^2 - 8x + 16 + m^2x^2 + 2mx + 1 = 4)</td>
<td></td>
</tr>
<tr>
<td>(x^2(1 + m^2) + x(2m - 8) + 13 = 0)</td>
<td></td>
</tr>
<tr>
<td>For line to be a tangent to the circle, Discriminant = 0</td>
<td></td>
</tr>
<tr>
<td>((2m - 8)^2 - 4(1 + m^2)13 = 0)</td>
<td>M1</td>
</tr>
<tr>
<td>(4m^2 - 32m + 64 - 52 - 52m^2 = 0)</td>
<td></td>
</tr>
<tr>
<td>(0 = 48m^2 + 32m - 12)</td>
<td></td>
</tr>
<tr>
<td>(0 = 12m^2 + 8m - 3)</td>
<td></td>
</tr>
<tr>
<td>(m = \frac{-8 \pm \sqrt{64 - 4(12)(-3)}}{2(12)})</td>
<td></td>
</tr>
<tr>
<td>(m = \frac{-8 \pm 4\sqrt{13}}{24})</td>
<td></td>
</tr>
<tr>
<td>(m = \frac{2 \pm \sqrt{13}}{6}) also accept (m = -\frac{1}{3} \pm \frac{2}{6}\sqrt{13})</td>
<td>A1, A1</td>
</tr>
</tbody>
</table>

Deduct 1 mark if answers are not in the lowest terms.
10 (a) (i) Express \( \frac{2x^3 + x^2}{x^2 + x - 2} \) in the form of \( ax + b + \frac{cx + d}{x^2 + x - 2} \). \[2\]  
(ii) Using the values of \( c \) and \( d \) found in (i), express \( \frac{cx + d}{x^2 + x - 2} \) as a sum of two partial fractions. \[3\]  

<table>
<thead>
<tr>
<th>10(a) (i)</th>
<th>By long division</th>
<th>M1</th>
</tr>
</thead>
<tbody>
<tr>
<td>[2]</td>
<td>( \frac{2x^3 + x^2}{x^2 + x - 2} = 2x - 1 + \frac{5x - 2}{x^2 + x - 2} )</td>
<td>A1</td>
</tr>
<tr>
<td>(ii)</td>
<td>( \frac{5x - 2}{(x + 2)(x - 1)} = \frac{A}{x + 2} + \frac{B}{x - 1} )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( 5x - 2 = A(x - 1) + B(x + 2) )</td>
<td>A1 for either</td>
</tr>
<tr>
<td></td>
<td>Let ( x = 1 ), ( 3 = 3B )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( B = 1 )</td>
<td>A1 for either</td>
</tr>
<tr>
<td></td>
<td>Comparing coefficient of ( x ), ( A + B = 5 )</td>
<td>A or B correct</td>
</tr>
<tr>
<td></td>
<td>( A = 4 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \frac{5x - 2}{x^2 + x - 2} = \frac{4}{x + 2} + \frac{1}{x - 1} )</td>
<td>A1</td>
</tr>
</tbody>
</table>

(b) A curve has the equation \( y = \frac{x-1}{\sqrt{4x+1}} \).  
(i) Differentiate \( y \) with respect to \( x \).  
(ii) Using the result in part b(i), determine \( \int \frac{2(2x+3)}{(4x+1)^{\frac{3}{2}}} \) \( dx \).  

<table>
<thead>
<tr>
<th>10(b) (i)</th>
<th>( \frac{dy}{dx} = \frac{(4x+1)^{\frac{1}{2}}(1) - (x - 1) \times \frac{1}{2}}{(4x + 1)^{\frac{3}{2}}} \times 4 )</th>
<th>M1 quotient rule M1 chain rule</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( = \frac{(4x+1)^{-\frac{1}{2}}[4x+1-2(x-1)]}{(4x+1)^{\frac{3}{2}}} )</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>( = \frac{2x+3}{(4x+1)^{\frac{3}{2}}} )</td>
<td>A1</td>
</tr>
<tr>
<td>(ii)</td>
<td>( \int \frac{2x+3}{(4x+1)^{\frac{3}{2}}} ) ( dx ) = ( \frac{x-1}{\sqrt{4x+1}} + c )</td>
<td>M1 Reverse differentiation</td>
</tr>
<tr>
<td></td>
<td>( \int \frac{2(2x+3)}{(4x+1)^{\frac{3}{2}}} ) ( dx ) = ( \frac{2(x-1)}{\sqrt{4x+1}} + c' )</td>
<td>A1</td>
</tr>
</tbody>
</table>
11.

The diagram shows two circles, $C_1$ and $C_2$ with centres $A$ and $B$ respectively. The two circles touch each other at $D$. $C_1$ has radius 3 units and touches the y-axis at $E$. $C_2$ has radius 2 units and touches the x-axis at $F$. The lines $AB$ produced meets the x-axis at $G$ and angle $BGO = \theta$ radians.

(i) Show with clear explanations, that $OE = 5 \sin \theta + 2$ and $OF = 5 \cos \theta + 3$. [2]

(ii) Show that $EF^2 = 38 + 20 \sin \theta + 30 \cos \theta$. [2]

(iii) Express $EF^2$ in the form $38 + R \cos(\theta - \alpha)$, where $R > 0$ and $\alpha$ is an acute angle. [3]

(iv) Given that $EF^2 = 65$, find the value of $\theta$. [2]

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>$AB = 3 + 2 = 5 \text{cm}$</td>
</tr>
<tr>
<td>[2]</td>
<td>$OE = AB \sin \theta + BF = 5 \sin \theta + 2$ B1</td>
</tr>
<tr>
<td></td>
<td>$OF = AB \cos \theta + AE = 5 \cos \theta + 3$ B1</td>
</tr>
</tbody>
</table>
11(ii) \[ EF^2 = (5 \sin \theta + 2)^2 + (5 \cos \theta + 3)^2 \] 

\[ = 25 \sin^2 \theta + 20 \sin \theta + 4 + 25 \cos^2 \theta + 30 \cos \theta + 9 \] 

\[ = 25(\sin^2 \theta + \cos^2 \theta) + 20 \sin \theta + 30 \cos \theta + 13 \]  

\[ = 38 + 20 \sin \theta + 30 \cos \theta \text{ (AG)} \]  

11(iii) \[ EF^2 = 38 + R \cos (\theta - \alpha) \] 

\[ R = \sqrt{30^2 + 20^2} = 10\sqrt{13} \]  

\[ \alpha = \tan^{-1}\left(\frac{20}{30}\right) = 0.58800 \]  

\[ EF^2 = 38 + 10\sqrt{13} \cos (\theta - 0.58800) \]  

11(iv) \[ EF^2 = 65 \] 

\[ 65 = 38 + 10\sqrt{13} \cos (\theta - 0.58800) \] 

\[ \frac{27}{10\sqrt{13}} = \cos (\theta - 0.58800) \]  

\[ \theta - 0.58800 = 0.72448 \] 

\[ \theta = 1.31 \text{ (to 3 sig fig)} \]
TANJONG KATONG GIRLS’ SCHOOL
PRELIMINARY EXAMINATION 2016
SECONDARY FOUR

4047/01 ADDITIONAL MATHEMATICS PAPER 1

Thursday 11 August 2016 2 h

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST
Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper, and use a pencil for drawing
graphs and diagrams.
Do not use staples, highlighters or correction fluid.

Answer all the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place
in the case of angles in degrees, unless a different level of accuracy is specified in the
question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part
question.

The total number of marks for this paper is 80.

Setter : Ms Yeo
Markers : Mrs Pang / Mrs M Loy / Mdm Tan SE / Ms Yeo

This Question Paper consists of 7 printed pages, including this page.
1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion

\[
(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n.
\]

where \( n \) is a positive integer and

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}
\]

2. TRIGONOMETRY

Identities

\[
\sin^2 A + \cos^2 A = 1
\]

\[
\sec^2 A = 1 + \tan^2 A
\]

\[
\csc^2 A = 1 + \cot^2 A
\]

\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
\sin 2A = 2 \sin A \cos A
\]

\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]

\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulas for \( \Delta ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2} bc \sin A
\]
Answer all questions

1. It is given that \( \cos A = -\frac{1}{3} \) and \( \sin B = \frac{2}{\sqrt{11}} \). \( A \) and \( B \) are in the same quadrant.

Without using a calculator, find the exact value of \( \cot (90^\circ - A - B) \). [5]

2. (i) Find the range of values of \( p \) for which \((x + 1)(x - 2) > p(x + 2)\) for all real values of \( x \). [4]

(ii) Deduce the number of points at which the line \( y = p(x + 2) \) intersects the curve \( y = (x + 1)(x - 2) \) for \(-1 \leq p < 2\). [1]

3. 2000 cm\(^3\) of water is transferred from a rectangular tank to an empty inverted right circular cone in 10 seconds. The ratio of the radius of the cone to the height of the cone is \( 1 : 3 \).

Find the rate of change of the horizontal surface area, \( A \) cm\(^2\), of the water in the cone, when the height, \( h \) cm, of the water in the cone is 12 cm. [6]

4. (i) Write down and simplify, the first 3 terms in the expansion of \( (2 - p)^5 \) in ascending powers of \( p \). [2]

(ii) Find the value of \( n \) where \( n \) is a positive integer, given that the coefficient of \( x^2 \) is 96 in the expansion of \( (1 + x)^n (2 - x + x^2)^5 \). [4]
5 A curve \( y = f(x) \) is such that \( f''(x) = 48\sin 4x - 8\cos 2x \). The curve intersects the \( x \)-axis at \( P \). The \( x \)-coordinate of \( P \) is \( \frac{\pi}{4} \) and the gradient of the curve at \( P \) is 8. Show that \( f'''(x) + 16f(x) = 24\cos 2x \). [7]

6 The table shows experimental values of two variables \( x \) and \( y \).

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>2.93</td>
<td>2.29</td>
<td>3.27</td>
<td>3.77</td>
<td>6.12</td>
</tr>
</tbody>
</table>

It is known that \( x \) and \( y \) are related by an equation of the form \( x^2 + \frac{y}{a} = bxy \), where \( a \) and \( b \) are constants. An error was made in recording one of the values of \( y \).

(i) Using a scale of 2 cm to represent 1 unit on the horizontal axis and 1 cm to represent 1 unit on the vertical axis, draw a straight line graph for the above given data. The straight line graph is to be drawn with variable \( x \) on the horizontal axis. [3]

(ii) Use the graph to estimate

(a) the correct value of \( x \). [2]

(b) the values of \( a \) and \( b \). [3]

7 (i) Express \( \frac{4}{(x-3)x^2} \) in partial fractions. [4]

(ii) Hence evaluate \( \int_{4}^{7} \frac{1}{(x-3)x^2} \, dx \). [4]
8. (i) Prove that \( \frac{1 - \sin x}{\cos x} + \frac{\cos x}{1 - \sin x} = 2 \sec x \). \[3\]

(ii) In the equation

\[
\frac{1 - \sin x}{\cos x} + \frac{\cos x}{1 - \sin x} + \tan^2 x = 2,
\]

\( \cos x = a \) or \( b \) where \( a \) and \( b \) are constants, and \( b < 0 \).

(a) Find the value of \( a \) and of \( b \). \[2\]

(b) Solve the equation \( \cos x = b \) for \(-\pi \leq x \leq 2\pi\). \[3\]

9. The equation of a curve is \( y = x \ln(2x - 3) \) where \( x > \frac{3}{2} \).

(i) Find the equation of the normal to the curve at \( x = 2 \). \[4\]

The normal to the curve \( y = x \ln(2x - 3) \) passes through the vertex of the graph of \( y = k - 4|2x + 1| \) where \( k \) is a constant.

(ii) Determine the value of \( k \). \[2\]

(iii) Sketch the graph of \( y = k - 4|2x + 1| \) for the value of \( k \) in part (ii).

Show the vertex and intercepts clearly. \[2\]
Solutions to this question by accurate drawing will not be accepted.

The diagram shows a quadrilateral $HIJK$. $H$ is the reflection of point $G(5, 7)$ in the line $x = 1$. Point $K(a, 11)$ is such that the product of the gradients of $HK$ and $JK$ is $-3$. The perpendicular bisector of $HJ$ intersects the $x$-axis at $I$.

(i) Deduce the coordinates of $H$. \[1\]

Find

(ii) the value of $a$ given that $a < 0$. \[2\]

(iii) the equation of the perpendicular bisector of $HJ$. \[3\]

(iv) the area of quadrilateral $HIJK$. \[3\]
The diagram shows a capsule shaped object with surface area $18\pi$ cm\(^2\). It comprised of 2 solid hemispheres of radius $r$ cm joined to the 2 ends of a solid cylinder of radius $r$ cm and height $h$ cm.

(i) Show that the volume, $V$ cm\(^3\), of the object is given by $V = 9\pi r - \frac{2}{3}\pi r^3$. [4]

(ii) Find the stationary value of $V$, and determine if this stationary value is a maximum or minimum. [6]

THE END
### Answer Key to TKGS Prelim 2016 Additional Mathematics Paper 1

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<th>Answer</th>
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<td>1</td>
<td>$7\sqrt{2}$</td>
<td>8(i)</td>
<td>Proof</td>
</tr>
<tr>
<td>2(i)</td>
<td>$-9 &lt; p &lt; -1$</td>
<td>(ii)(a)</td>
<td>$a = 1$ and $b = -\frac{1}{3}$</td>
</tr>
<tr>
<td>2 (ii)</td>
<td>1 or 2 points</td>
<td>(ii)(b)</td>
<td>$-1.91, 1.91, 4.37$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{33}{3}\text{ cm}^3/\text{s}$</td>
<td>(ii)</td>
<td>$\frac{5}{8}$</td>
</tr>
<tr>
<td>4(i)</td>
<td>$128 - 448p + 672p^2 +$</td>
<td>(iii)</td>
<td></td>
</tr>
<tr>
<td>4(ii)</td>
<td>4</td>
<td>9(i)</td>
<td>$4y = -x + 2$</td>
</tr>
<tr>
<td>5</td>
<td>Proof</td>
<td>10(i)</td>
<td>$(-3, 7)$</td>
</tr>
<tr>
<td>6(ii)(a)</td>
<td>4.24</td>
<td>(ii)</td>
<td>$-1$</td>
</tr>
<tr>
<td>(b)</td>
<td>$a = 1$, $b = 2$</td>
<td>(iii)</td>
<td>$y = 3x + 6$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(iv)</td>
<td>34 square units</td>
</tr>
<tr>
<td>7(i)</td>
<td>$\frac{4}{(x-3)^2} = \frac{4}{9(x-3)} - \frac{4}{9x} - \frac{4}{3x^2}$</td>
<td>11(ii)</td>
<td>$40.0 \text{ cm}^3$, Stationary value of $V$ is a maximum.</td>
</tr>
</tbody>
</table>

![Diagram of a curve with labeled points](image-url)
### Question 1

A and B are in the same quadrant.

\[ \therefore A \text{ and } B \text{ are both in } 2^{\text{nd}} \text{ quadrant.} \]

\[
\cos A = -\frac{1}{3}
\]
\[
\tan A = -\frac{\sqrt{8}}{1} = -2\sqrt{2}
\]
\[
\sin B = \sqrt{1 - \cos^2 B} = \sqrt{1 - \left(-\frac{1}{3}\right)^2} = \sqrt{\frac{8}{11}}
\]
\[
\tan B = \frac{\sin B}{\cos B} = \frac{\sqrt{\frac{8}{11}}}{-\left(-\frac{1}{3}\right)} = \frac{\sqrt{2}}{3}
\]

\[
cot(90^\circ - A - B)
\]
\[
= \cot(90^\circ - (A + B))
\]
\[
= \tan(A + B)
\]
\[
= \frac{\tan A + \tan B}{1 - \tan A \tan B}
\]
\[
= \frac{-2\sqrt{2} - \frac{\sqrt{2}}{3}}{1 - \left(-2\sqrt{2}\right)\left(-\frac{\sqrt{2}}{3}\right)}
\]

### Marking Scheme

<table>
<thead>
<tr>
<th>Marks</th>
<th>Teaching Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>Understand how to find ratio of ( \tan A ) from ( \cos A ).</td>
</tr>
<tr>
<td>B1</td>
<td>Understand how to find ( \tan B ) from ( \sin B ).</td>
</tr>
<tr>
<td>B1</td>
<td>Know the relationships ( \tan(90^\circ - C) = \frac{1}{\tan C} ) and ( \cot(90^\circ - C) = \tan C )</td>
</tr>
<tr>
<td>M1</td>
<td>Know how to use the addition formula for ( \tan(A + B) )</td>
</tr>
</tbody>
</table>

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www.sgexamguru.com 163
\[ \frac{-7\sqrt{2}}{3} \]
\[ = \frac{4}{1 - \frac{4}{3}} \]
\[ = 7\sqrt{2} \]

A1

Able to simplify surds
Correct final answer
<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Marks</th>
<th>Teaching Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>2(i)</td>
<td>\begin{align*} (x + 1)(x - 2) &amp;&gt; p(x + 2) \ x^2 - x - 2 &amp;&gt; px + 2p \ x^2 + (-1 - p)x - 2 - 2p &amp;&gt; 0 \end{align*} ( (x + 1)(x - 2) &gt; p(x + 2) ) for all ( x ) ( \Rightarrow ) discriminant &lt; 0 ( \Rightarrow (1 - p)^2 - 4(1)(-2 - 2p) &lt; 0 ) ( \Rightarrow 1 + 2p + p^2 + 8 + 8p &lt; 0 ) ( \Rightarrow p^2 + 10p + 9 &lt; 0 ) ( \Rightarrow (p + 9)(p + 1) &lt; 0 ) ( \Rightarrow -9 &lt; p &lt; -1 )</td>
<td>B1</td>
<td>Know that discriminant &lt; 0 for inequality to be true for all ( x ). Able to get expression for discriminant Able to solve quad inequality Correct answer</td>
</tr>
<tr>
<td>(ii)</td>
<td>Line ( y = p(x + 2) ) does not intersect curve ( y = (x + 1)(x - 2) ) when ( p ) is in the range (-9 &lt; p &lt; -1). For ( p \geq -1 ), line intersects curve at 1 or 2 points.</td>
<td>B1</td>
<td>Able to make a deduction from (i)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Marks</th>
<th>Teaching Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>( V ): volume of water in cone ( A ): area of water surface on cone ( h ): height of water in cone ( r ): radius of the water surface ( t ): time ( \frac{dV}{dt} = \frac{2000}{10} \text{ cm}^3/\text{s} ) ( = 200 \text{ cm}^3/\text{s} )</td>
<td>B1</td>
<td>Know how to get ( \frac{dV}{dt} )</td>
</tr>
</tbody>
</table>
\[
\frac{r}{h} = \frac{1}{3}
\]

\[
V = \frac{1}{3} \pi r^2 h
= \frac{1}{3} \pi \left( \frac{1}{3} h \right)^2 h
= \frac{\pi}{27} h^3
\]

\[
\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}
200 = \frac{\pi}{9} h^3 \frac{dh}{dt}
\]

\[
\frac{dh}{dt} = \frac{1800}{\pi h^3}
\]

\[
A = \pi r^2
= \pi \left( \frac{1}{3} h \right)^2
= \frac{\pi}{9} h^3
\]

\[
\frac{dA}{dt} = \frac{dA}{dh} \times \frac{dh}{dt}
= 2\pi r \left( \frac{1800}{\pi h^2} \right)
= \frac{4000}{h}
\]

When \(h = 12\),

\[
\frac{dA}{dt} = \frac{4000}{12}
= 33 \frac{1}{3}
\]

Answer: Rate of change of the horizontal surface area of the water \(33 \frac{1}{3} \text{ cm}^2/\text{s}\).
<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Marks</th>
<th>Teaching Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>4(i)</td>
<td>((2 - p)^7)</td>
<td>M1</td>
<td>Know formula for Binomial expansion</td>
</tr>
<tr>
<td></td>
<td>(2^7 - \binom{7}{1} p + \binom{7}{2} p^2 + ...)</td>
<td>A1</td>
<td>Able to simplify</td>
</tr>
<tr>
<td></td>
<td>= 128 - 448p + 672p^2 + ...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>((1 + x)^n(2 - x + x^2)^7)</td>
<td>B1</td>
<td>Know (p = x - x^2)</td>
</tr>
<tr>
<td></td>
<td>= ([1 + \binom{n}{1} x + \binom{n}{2} x^2 + ...][2 - (x - x^2)]^7)</td>
<td>B1</td>
<td>Able to express (\binom{n}{1}) and (\binom{n}{2}) correctly in terms of (n).</td>
</tr>
<tr>
<td></td>
<td>= ([1 + nx + \frac{n(n-1)}{2 \times 1} x^2 + ...][128 - 448(x - x^2) + 672(x - x^2)^2 + ...])</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>= ([1 + nx + \frac{n(n-1)}{2} x^2 + ...][128 - 448x + 1120x^2 + ...])</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Coefficient of (x^2 = 96)</td>
<td>M1</td>
<td>Able to determine the terms in (x^2) in the product of ((1 + x)^n) and ((2 - x + x^2)^7).</td>
</tr>
<tr>
<td></td>
<td>(1(1120) + n(-448) + \frac{n(n-1)}{2}(128) = 96)</td>
<td>A1</td>
<td>Final answer</td>
</tr>
<tr>
<td></td>
<td>(64n^2 - 512n + 1024 = 0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(n^2 - 8n + 16 = 0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>((n - 4)(n - 4) = 0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(n = 4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Qn</td>
<td>Solution</td>
<td>Marks</td>
<td>Teaching Points</td>
</tr>
<tr>
<td>----</td>
<td>----------</td>
<td>-------</td>
<td>-----------------</td>
</tr>
</tbody>
</table>
| 5  | $f'(x) = 48\sin 4x - 8\cos 2x$
    | $f'(x) = \int (48\sin 4x - 8\cos 2x)\,dx$
    | $= -12\cos 4x - 4\sin 2x + c_1$ | B1 | Know how to integrate $f'(x)$ correctly to get $f'(x)$ |
    | $\int \left( \frac{\pi}{4} \right) = 8$
    | $-12\cos \left( \frac{\pi}{4} \right) - 4\sin 2\left( \frac{\pi}{4} \right) + c_1 = 8$ | M1 | Know how to use the grad at $P$ to get $f'(x)$ |
    | $-12(-1) - 4(1) + c_1 = 8$
    | $c_1 = 0$
    | $f'(x) = -12\cos 4x - 4\sin 2x$ | A1 | Correct expression for $f'(x)$ |
    | $f(x) = \int (-12\cos 4x - 4\sin 2x)\,dx$
    | $= -3\sin 4x + 2\cos 2x + c_2$ | | Know how to integrate $f'(x)$ correctly to get $f(x)$ |
    | $\int \left( \frac{\pi}{4} \right) = 0$
    | $-3\sin 4\left( \frac{\pi}{4} \right) + 2\cos 2\left( \frac{\pi}{4} \right) + c_2 = 0$ | M1 | Know how to use the $x$-coordinate of $P$ to get $f(x)$ |
    | $-3(0) + 2(0) + c_2 = 0$
    | $c_2 = 0$
    | $f(x) = -3\sin 4x + 2\cos 2x$ | A1 | Correct expression for $f(x)$ |
    | $f'(x) + 16f(x)$
    | $= (48\sin 4x - 8\cos 2x) + 16(-3\sin 4x + 2\cos 2x)$
<pre><code>| $= 24\cos 2x$ | M1 | sub. expressions for $f(x)$ and $f'(x)$ |
| | (Proved) | A1 | Able to get $24\cos 2x$ |
</code></pre>
<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Marks</th>
<th>Teaching Points</th>
</tr>
</thead>
</table>
| 6(i) | \[
x^2 + \frac{y}{a} = bx y
\]
\[
\frac{x^2}{y} + \frac{1}{a} = bx
\]
\[
\frac{x^2}{y} = bx - \frac{1}{a}
\]
Graph | B1 | Able to transform given equation into a straight line form with \(x\) on horizontal axis. 
Able to plot a straight line passing through all points 
Graph cuts \(y\)-axis. |
| (ii)(a) | Correct reading of \[
\frac{x^2}{y} = 15.1
\]
\[
\frac{8^2}{y} = 15.1
\]
Correct reading of \(y\) = 4.24 | M1 | Know the method to find the correct reading of \(y\) 
Correct final answer |
| (b) | \[
\frac{1}{a} = \frac{x^2}{y} - \text{intercept}
\]
\[
= -1
\]
\[
a = 1
\]
\[
b = \text{Gradient}
\]
\[
= \frac{11.01 - 3.01}{6 - 2}
\]
\[
= 2
\] | B1 | Understand how to get \(a\) using the vertical intercept 
Understand that \(b\) is the gradient 
Correct value of \(b\) |
6(i)

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>1.33</td>
<td>2.29</td>
<td>3.27</td>
<td>3.77</td>
<td>6.12</td>
</tr>
<tr>
<td>$\frac{x^2}{y}$</td>
<td>3.01</td>
<td>6.99</td>
<td>11.01</td>
<td>13.00</td>
<td>10.46</td>
</tr>
</tbody>
</table>

Scale:
- $x$-axis: 2 cm to 1 unit
- $\frac{x^2}{y}$-axis: 1 cm to 1 unit
<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Marks</th>
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</tr>
</thead>
</table>
| 7(i) | \[
\frac{4}{(x-3)x^2} = \frac{A}{x-3} + \frac{B}{x} + \frac{C}{x^2}
\]
\[
4 = Ax^2 + Bx(x-3) + C(x-3)
\]
Consider \( x = 0 \):
\[
4 = C(-3)
\]
\[
C = -\frac{4}{3}
\]
Consider \( x = 3 \):
\[
4 = 9A
\]
\[
A = \frac{4}{9}
\]
Compare coefficient of \( x^2 \):
\[
0 = A + B
\]
\[
B = -A
\]
\[
= -\frac{4}{9}
\]
Hence \[
\frac{4}{(x-3)x^2} = \frac{4}{9(x-3)} - \frac{4}{9x} - \frac{4}{3x^2}
\]
| B1 | Know the various partial fraction forms. |
| B1 | Able to use suitable method to find \( C \). |
| B1 | Able to use suitable method to find \( A \). |
| B1 | Able to use suitable method to find \( B \). |
|       | Minus 1 mark if didn’t write final line. |
\[
\int_{4}^{7} \frac{1}{(x-3)x^3} \, dx
\]

\[
= \frac{1}{4} \int_{4}^{7} \frac{4}{(x-3)x^3} \, dx
\]

\[
= \frac{1}{4} \left[ \frac{4}{9(x-3)} - \frac{4}{9x} - \frac{4}{3x^3} \right]_{4}^{7}
\]

\[
= \frac{1}{4} \left[ \frac{4}{9 \ln(7-3)} - \frac{4}{9 \ln 7} - \frac{4}{3(7)} \right] - \frac{1}{4} \left[ \frac{4}{9 \ln(4-3)} - \frac{4}{9 \ln 4} + \frac{4}{3(4)} \right]
\]

\[
= 0.0561
\]

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Know the formula $\frac{1}{ax + b}$ = ln $x + c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td></td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Know the formula $\int x^n , dx = \frac{x^{n+1}}{n+1} + c$</td>
</tr>
<tr>
<td>M1</td>
<td></td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Know how to evaluate a definite integral</td>
</tr>
<tr>
<td>A1</td>
<td></td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Correct final answer</td>
</tr>
<tr>
<td>Qn</td>
<td>Solution</td>
<td>Marks</td>
</tr>
<tr>
<td>----</td>
<td>----------</td>
<td>-------</td>
</tr>
<tr>
<td>8(i)</td>
<td>( \frac{1 - \sin x}{\cos x} + \frac{\cos x}{1 - \sin x} )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( = \frac{(1 - \sin x)^2 + \cos^2 x}{\cos x(1 - \sin x)} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = \frac{1 - 2\sin x + \sin^2 x + \cos^2 x}{\cos x(1 - \sin x)} )</td>
<td></td>
</tr>
</tbody>
</table>
(ii)(a) \[ \frac{1 - \sin x}{\cos x} + \frac{\cos x}{1 - \sin x} + \tan^2 x = 2 \]

2 sec x + \tan^2 x = 2

2 sec x + sec^2 x - 1 = 2

sec^2 x + 2 sec x - 3 = 0

(\sec x - 1)(\sec x + 3) = 0

sec x = 1 or sec x = -3

\cos x = 1 or \cos x = -\frac{1}{3}

Answer: a = 1 and b = -\frac{1}{3}

(b) \cos x = -\frac{1}{3}

Basic \( \theta = 1.2310 \) radians

x = -1.91, 1.91, 4.37

<p>| B1 | Use identity ( 1 + \tan^2 x = \sec^2 x ) |
| A1 | Correct final answer |
| B1,B1,B1 | Correct angles |</p>
<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Marks</th>
<th>Teaching Points</th>
</tr>
</thead>
</table>
| 9(i) | \( y = x \ln(2x - 3) \)  
\[
\frac{dy}{dx} = x \left( \frac{2}{2x - 3} \right) + \ln(2x - 3)
\]  
At \( x = 2 \),  
\[
\frac{dy}{dx} = 2 \left( \frac{2}{2(2) - 3} \right) + \ln(2(2) - 3)
\]  
\[= 4\]  
and  
\[
y = 2 \ln(2(2) - 3)
\]  
\[= 0\]  
Equation of normal:  
\[
\frac{y - 0}{x - 2} = -\frac{1}{4}
\]  
\[
4y = -x + 2
\] | M1 | Use Product Rule to diff \( x \ln(2x - 3) \)  
M1 | Use Chain Rule to diff \( \ln(2x - 3) \)  
M1 | Know how to find gradient, \( y \)-coordinate and equation of normal  
A1 | Correct answer for equation of normal. |
<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Marks</th>
<th>Teaching Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>9(ii)</td>
<td>Equation of normal: &lt;br&gt;4y = -x + 2 &lt;br&gt;( y = k - 4</td>
<td>2x + 1</td>
<td>) &lt;br&gt;Coordinate of vertex: ( \left( -\frac{1}{2}, k \right) ) &lt;br&gt;When ( x = -\frac{1}{2} ), &lt;br&gt;4y = ( -\left( -\frac{1}{2} \right) + 2 ) &lt;br&gt;y = ( \frac{5}{8} ) &lt;br&gt;k = ( \frac{5}{8} )</td>
</tr>
<tr>
<td>(iii)</td>
<td><img src="image.png" alt="Graph" /></td>
<td>B1 B1</td>
<td>Shape Critical pts</td>
</tr>
<tr>
<td>Qn</td>
<td>Solution</td>
<td>Marks</td>
<td>Teaching Points</td>
</tr>
<tr>
<td>----</td>
<td>----------</td>
<td>-------</td>
<td>-----------------</td>
</tr>
<tr>
<td>10(i)</td>
<td>Coordinates of $H$ are $(-3, 7)$.</td>
<td>B1</td>
<td>Know how to get image of a point given the line of reflection</td>
</tr>
</tbody>
</table>
| (ii) | Gradient of $HK \times$ Gradient of $JK = -3$  
\[
\frac{11-7}{a+3} \times \frac{11-5}{a-3} = -3
\]  
\[
\frac{24}{(a+3)(a-3)} = -3
\]  
\[
a^2 - 9 = -8
\]  
\[
a^2 = 1
\]  
\[
a = 1 \text{ (reject)} \text{ or } -1
\] | M1 | Know how to get a relationship between the 2 gradients |
| (iii) | Midpoint of $HJ$  
\[
\left( \frac{-3+3}{2} \frac{7+5}{2} \right) = (0, 6)
\]  
Gradient of $HJ$  
\[
= \frac{7-5}{-3-3} = \frac{-1}{3}
\]  
Equation of $\perp$ bisector of $HJ$:  
\[
\frac{y-6}{x-0} = 3
\]  
\[
y = 3x + 6
\] | B1 | Know formula for midpoint |
| | | A1 | Correct value of $a$ |
| | | M1 | Know how to get $\perp$ bisector |
| | | A1 | Correct answer |
(iv) \( y = 3x + 6 \)

When \( y = 0 \),
\[
0 = 3x + 6
\]
\( x = -2 \)
Coordinates of \( I = (-2, 0) \)

<table>
<thead>
<tr>
<th>Area of ( HJK )</th>
</tr>
</thead>
</table>
| \[
\frac{1}{2} \begin{vmatrix}
-2 & 3 & -1 & -3 & -2 \\
2 & 0 & 5 & 11 & 7 \\
\end{vmatrix}
\]
| \[
= \frac{1}{2} \left((-2)5 + 3(11) + (-1)7 + (-3)0 - 3(0) - (-1)5 - (-3)11 - (-2)7\right)
\]
| = 34 square units |

<p>| B1 | Know how to find coordinates of ( I ) |
| M1 | Know the formula for area of polygon |
| A1 | Correct final answer |</p>
<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Marks</th>
<th>Teaching Points</th>
</tr>
</thead>
</table>
| 11(i) | \[2\pi rh + 2(2\pi r^2) = 18\pi\]  
\[h = \frac{18\pi - 4\pi r^2}{2\pi r}\]  
\[= \frac{9}{r} - 2r\]  
\[V = \pi r^2 h + \left(\frac{2}{3} \pi r^3\right) 2\]  
\[= \pi r^2 h + \frac{4}{3} \pi r^3\]  
\[= \pi r^2 \left(\frac{9}{r} - 2r\right) + \frac{4}{3} \pi r^3\]  
\[= 9\pi r - 2\pi r^3 + \frac{4}{3} \pi r^3\]  
\[= 9\pi r - \frac{2}{3} \pi r^3\] | M1 | Able to get a relationship between \(r, h\) and area. |
| | | A1 | Correct expression for \(h\) in terms of \(r\). |
| | | M1 | Able to get \(V\) in terms of \(r\) and \(h\). |
| | | A1 | Correct expression for \(V\) in terms of \(r\). |
| (ii) | \[V = 9\pi r - \frac{2}{3} \pi r^3\]  
\[\frac{dV}{dr} = 9\pi - 2\pi r^2\]  
For stationary value of \(V\),  
\[\frac{dV}{dr} = 0\]  
\[9\pi - 2\pi r^2 = 0\]  
\[r = \frac{9}{\sqrt{2}}\]  
Stationary value of \(V\) | B1 | Able to differentiate \(V\) |
| | M1 | Know requirement for stationary pt. |
| | A1 | Able to get value of \(r\) at stationary pt. |
\[
\begin{align*}
\text{Volume} &= 9\pi \frac{9}{2} - \frac{2}{3} \pi \left( \frac{9}{\sqrt{2}} \right)^3 \\
&= 40.0 \text{ cm}^3 \\
\frac{d^2V}{dr^2} &= -4\pi r \\
\text{When } r &= \frac{9}{\sqrt{2}}, \quad \frac{d^2V}{dr^2} = -4\pi \frac{9}{\sqrt{2}} < 0
\end{align*}
\]

\[\therefore \text{Stationary value of } V \text{ is a maximum.}\]
TANJONG KATONG GIRLS’ SCHOOL
PRELIMINARY EXAMINATION 2016
SECONDARY FOUR

4047/02  ADDITIONAL MATHEMATICS
PAPER 2

Friday  5 August 2016  2 h 30 min

Additional Materials:  Answer Paper

READ THESE INSTRUCTIONS FIRST
Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper, and use a pencil for for any
diagrams or graphs.
Do not use staples, highlighters or correction fluid.

Answer all the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place
in the case of angles in degrees, unless a different level of accuracy is specified in
the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part
question.

The total number of marks for this paper is 100.

Setter:  Mrs M Loy
Markers: Mdm Tan SE, Mrs H Pang, Miss Yeo LS, Mrs M Loy

This Question Paper consists of 7 printed pages, including this page.
1. ALGEBRA

**Quadratic Equation**

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

**Binomial expansion**

\[(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}
\]

2. TRIGONOMETRY

**Identities**

\[
\sin^2 A + \cos^2 A = 1
\]

\[
\sec^2 A = 1 + \tan^2 A
\]

\[
\csc^2 A = 1 + \cot^2 A
\]

\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
\sin 2A = 2 \sin A \cos A
\]

\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]

\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

**Formulæ for \( \triangle ABC \)**

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2}bc \sin A
\]
Answer all the questions

1. A man buys a new car. The value of the car depreciates with time so that its value, \( \$V \), after \( t \) months' use is given by \( V = 132 \,000 \, e^{-pt} \), where \( p \) is a constant. The value of the car is expected to be \$122\,000 after eight months' use.

   (i) Find the value of the car, \( V \) when the man bought it. \([1]\)

   (ii) Show that \( p = 0.01 \). \([2]\)

   (iii) Using the value of \( p = 0.01 \), determine the age of the car to the nearest month, when its value reached half of the original value when the man bought it. \([2]\)

2. The function \( f(x) = 1 + 2x + Ax^2 - x^3 \), where \( A \) is a constant, leaves a remainder of \( \frac{3}{8} \) when divided by \( 2x - 1 \).

   (i) Find the value of \( A \). \([2]\)

   (ii) Hence solve the equation \( f(x) = 0 \), giving your answers in the exact form. \([4]\)

3. (a) (i) Solve \( \sqrt[3]{x} + 2 - 3x = 0 \). \([2]\)

   (ii) On the same axes, sketch the graphs of \( y = \sqrt[3]{x} + 2 \) and \( y = 3x \). Indicate clearly all the points of intersections. \([2]\)

   (b) The vertical height of a triangle is \( \frac{8}{3 - \sqrt{5}} \) cm.

   Given that the area of the triangle is \( \frac{20}{\sqrt{5} - 1} \) cm\(^2\), without using a calculator, find the length of the base of the triangle in the form \( a + b\sqrt{5} \). \([3]\)
4. The roots of the quadratic equation, \( 2x^2 + 4x + 5 = 0 \) are \((\alpha + 1)\) and \((\beta + 1)\).

(i) Show that \( \alpha + \beta = -4 \) and hence find \( \alpha\beta \).

(ii) Find the quadratic equation in \( x \) with integer coefficients, whose roots are \( \frac{1}{\alpha^3} \) and \( \frac{1}{\beta^2} \).

5. (a) Given that \( \log_4(2x + 1) - \log_4(3 - x^2) = 1 \), form a quadratic equation in \( x \) and explain with clear working why the roots of the quadratic equation are real and distinct.

(b) Solve \( 3^{x^2} = 2(3^{-x}) + 17 \).

6. The curve \( y = \frac{2x^2}{x^2 + 1} \) has one stationary point \((p, q)\).

(i) Find the value of \( p \) and of \( q \).

(ii) Determine whether \( y \) is increasing or decreasing for

(a) \( x > p \),

(b) \( x < p \).

Hence state the nature of the stationary point.

(iii) Find \( \frac{d^2y}{dx^2} \) at the stationary point and explain how \( \frac{d^2y}{dx^2} \) further supports your answer in part (ii).
7. In the figure, $AB$ is a diameter of the circle with centre $O$. Chords $AD$ and $BC$ intersect at $F$. $AD$ produced meets the tangent to the circle, $TBE$ at $E$. $AE$ is an angle bisector of angle $BAC$.

(i) Prove that $\angle CBD = \angle DBE$. \hspace{1cm} [3]

Given that $\angle AOF = 90^\circ$, prove that

(ii) triangle $AOF$ is similar to triangle $ADB$. \hspace{1cm} [2]

(iii) $2(AO)^2 = AF \times (AF + FD)$. \hspace{1cm} [3]

8. A particle moving in a straight line passes through a fixed point $O$ with a speed of 20 m/s. The acceleration, $a$ m/s$^2$, of the particle, $t$ s after passing through $O$ is given by $a = -100e^{-3t}$. The particle comes to instantaneous rest at point $N$.

(i) Find the time the particle comes to instantaneous rest at point $N$. \hspace{1cm} [5]

(ii) Calculate the distance $ON$. \hspace{1cm} [4]

(iii) Show that the average speed of the particle in the first 2 seconds rounded off to a whole number is 10 m/s. \hspace{1cm} [3]
9. (i) Solve the equation \(2\sin 2P = 3\cos P\) for \(0^\circ \leq P \leq 360^\circ\). [4]

(ii) On the same axes, sketch for \(0^\circ \leq x \leq 720^\circ\), the graphs of
\[ y = \sin x \quad \text{and} \quad y = \frac{3}{2} \cos \left( \frac{x}{2} \right). \] [4]

(iii) Using the solutions to part (i), determine the \(x\)-coordinates of the points of intersection of the graphs of part (ii). [4]

10. A circle, \(C_1\), has equation \(x^2 + y^2 - 14x + 2y = -46\).

(i) Find the coordinates of the centre of the circle and the radius. [3]

The coordinates of the centre of a second circle, \(C_2\), is \((-4, -2)\). The equation of the tangent to the circle, \(C_2\) at a point \(P\) is \(2y = -2x + 3\).

(ii) Find the coordinates of point \(P\). [4]

(iii) Find the exact value of the radius of \(C_2\) and the equation of the circle, \(C_2\). [3]

(iv) Determine whether circles \(C_1\) and \(C_2\) will meet each other, showing your working clearly. [2]
11. (a) Show that $\frac{d}{dx} \left(2x\sqrt{2x+3}\right) = \frac{6x+6}{\sqrt{2x+3}}$. [3]

The diagram shows part of the curve $y = \frac{3x+3}{\sqrt{2x+3}}$. The curve intersects the $x$-axis at point $A$. The line through $A$ and perpendicular to the line, $y+x = -7$ intersects the curve again at another point, $B$.

(i) Show that the $y$-coordinate of point $B$ is 4. [5]

(ii) Given that the line $AB$ intersects the $y$-axis at $C$, determine the area of the shaded region bounded by the line $CB$, the curve, the line $x = 5$, the $x$-axis and the $y$-axis. [4]

End of Paper
TKGS S4 PRELIM 2016 Answer Key:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>1(i)</td>
<td>( V = 132000 )</td>
<td>(ii) show</td>
</tr>
<tr>
<td>(iii)</td>
<td>70 months</td>
<td></td>
</tr>
<tr>
<td>2(i)</td>
<td>( A = -2 )</td>
<td>(ii) ( x = 1, \frac{-3 \pm \sqrt{5}}{2} )</td>
</tr>
<tr>
<td>3(a)</td>
<td></td>
<td>(ii)</td>
</tr>
<tr>
<td>(a)</td>
<td></td>
<td>( x = \frac{2}{3} )</td>
</tr>
<tr>
<td>(b)</td>
<td>( 5\sqrt{5} )</td>
<td>4(i) ( \alpha \beta = \frac{11}{2} )</td>
</tr>
<tr>
<td>(ii)</td>
<td>( \frac{5}{2} )</td>
<td></td>
</tr>
<tr>
<td>4(ii)</td>
<td>( 1331x^2 - 16x + 8 = 0 )</td>
<td>5(a) Discriminant = 368 Since discriminant &gt; 0, the roots of the quadratic equation are real and distinct.</td>
</tr>
<tr>
<td>5(b)</td>
<td>( y = 0.631 )</td>
<td>6(i) ( p = 0, q = 0 )</td>
</tr>
<tr>
<td>6(ii)a</td>
<td>( \frac{dy}{dx} &gt; 0, \text{y is increasing} )</td>
<td>6(ii)b ( \frac{dy}{dx} &lt; 0, \text{y is decreasing} )</td>
</tr>
<tr>
<td></td>
<td>Since the value of ( \frac{dy}{dx} ) changes from negative to positive value, the stationary point is a minimum point.</td>
<td>6(iii) ( \frac{d^2y}{dx^2} = 4, \text{since } \frac{d^2y}{dx^2} &gt; 0, \text{the stationary point is minimum}, \text{thus reiterating the result from part (ii)}.</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>8(i) ( t = 0.305 \text{ s} )</td>
</tr>
<tr>
<td>8(ii)</td>
<td>Distance = 2.59 m</td>
<td>8(iii) show</td>
</tr>
<tr>
<td>9(i)</td>
<td>( 48.6^\circ, 90^\circ, 131.4^\circ, 270^\circ )</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9(iii)</td>
<td>( 97.2^\circ, 180^\circ, 262.8^\circ, 540^\circ )</td>
<td>10(i) Centre((7, -1)), radius = 2 units</td>
</tr>
<tr>
<td>10(ii)</td>
<td>( P\left(\frac{1}{4}, \frac{7}{4}\right) )</td>
<td>10(iii) Radius = ( \frac{15\sqrt{2}}{4} \text{ units} )</td>
</tr>
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<tr>
<td>(iv)</td>
<td>Sum of radii (7.30 units) &lt; distance between the centres (11.0 units) thus the circles will not meet.</td>
<td>11(a) show</td>
</tr>
<tr>
<td>11(b)i</td>
<td>show</td>
<td>11(b)ii 16.5 units²</td>
</tr>
</tbody>
</table>

\[
(x + 4)^2 + (y + 2)^2 = \frac{15\sqrt{2}}{4}^2 \cdot \frac{225}{8}
\]
1. A man buys a new car. The value of the car depreciates with time so that its value, \( V \), after \( t \) months' use is given by \( V = 132000e^{-pt} \), where \( p \) is a constant. The value of the car is expected to be $122,000 after eight months' use.

(i) Find the value of the car when the man bought it.

\[ V = 132000e^{-pt} \]

When the man bought the car, \( t = 0 \).

Hence, \( V = 132000e^0 \), \( e^0 = 1 \)

\[ : V = 132000. \]

(ii) Show that \( p = 0.01 \).

\[ V = 122000 \text{ when } t = 8 \]

\[ \frac{122000}{132000} = e^{-8p} \]

\[ e^{-8p} = \frac{122000}{132000} \]

\[ -8p = \ln \left( \frac{122000}{132000} \right) \]

\[ p = \frac{1}{8} \ln \left( \frac{122000}{132000} \right) \]

\[ p = 0.009848 \]

\[ p = 0.01 \text{ (1 sig fig) (shown)} \]

(iii) Using the value of \( p = 0.01 \), determine the age of the car to the nearest month, when its value reached half of the original value when the man bought it.

\[ 132000e^{-0.01t} = \frac{1}{2} \times 132000 \]

\[ e^{-0.01t} = \frac{1}{2} \]

\[ -0.01t = \ln \left( \frac{1}{2} \right) \]

\[ t = 69.3147 \]

\[ t = 70 \text{ months} \]
2. The function \( f(x) = 1 + 2x + Ax^2 - x^3 \), where \( A \) is a constant, leaves a remainder of \( 1 + \frac{3}{8} \) when divided by \( (2x - 1) \).

(i) Find the value of \( A \).

\[ f(x) = 1 + 2x + Ax^2 - x^3 \]
\[ f\left(\frac{1}{2}\right) = 1 + \frac{3}{8} \]
\[ 1 + 2\left(\frac{1}{2}\right) + A\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^3 = \frac{11}{8} \]
\[ \frac{1}{4}A = \frac{11}{8} - \frac{15}{8} \]
\[ A = -2 \]

(ii) Hence, solve the equation \( f(x) = 0 \), giving your answers in the exact form.

\[ f(x) = 1 + 2x - 2x^2 - x^3 \]
\[ f(1) = 1 + 2 - 2 - 1 \]
\[ f(1) = 0 \]
\[ \therefore (x - 1) \text{ is a factor} \]
\[ f(x) = (x - 1)(-x^2 + ax - 1) \]

Compare coefficient of \( x \):

\[ -1 - a = 2 \]
\[ a = -3 \]

\[ f(x) = (x - 1)(-x^2 - 3x - 1) \]
\[ f(x) = 0, \]
\[ x = 1 \]
\[ x^2 + 3x + 1 = 0 \]
\[ x = \frac{-3\pm\sqrt{(-3)^2 - 4(1)(1)}}{2(1)} \]
\[ x = \frac{-3\pm\sqrt{5}}{2} \]
3.(a) (i) Solve $\sqrt{3x+2} - 3x = 0$.

\[
\sqrt{3x+2} - 3x = 0
\]

\[
\sqrt{3x+2} = 3x
\]

\[
x + 2 = 9x^2
\]

\[
9x^2 - 3x - 2 = 0
\]

\[
(3x+1)(3x-2) = 0
\]

\[
x = \frac{2}{3}, \quad x = -\frac{1}{3} \text{ (rejected)}
\]

(ii) On the same axes, sketch the graphs of $y = \sqrt{3x+2}$ and $y = 3x$.

Indicate clearly all the points of intersections.

(b) The vertical height of a triangle is $\frac{8}{3 - \sqrt{5}}$ cm. Given that the area of the triangle is $\frac{20}{\sqrt{5} - 1}$ cm$^2$, without using a calculator, find the length of the base of the triangle in the form $a + b\sqrt{5}$.

\[
\frac{1}{2} \text{ base of triangle} \times \frac{8}{3 - \sqrt{5}} = \frac{20}{\sqrt{5} - 1}
\]

\[
\text{base of triangle} = \frac{20}{\sqrt{5} - 1} \times \frac{3 - \sqrt{5}}{4}
\]

\[
= \frac{5(3 - \sqrt{5})}{\sqrt{5} - 1} \times \frac{\sqrt{5} + 1}{\sqrt{5} + 1}
\]

\[
= \frac{5(2\sqrt{5} - 2)}{5 - 1}
\]

\[
= \frac{5 \times 2(\sqrt{5} - 1)}{4}
\]

\[
= \frac{5}{2} \sqrt{5} - \frac{5}{2}
\]
4. The roots of the quadratic equation, \(2x^2 + 4x + 5 = 0\) are \((\alpha + 1)\) and \((\beta + 1)\).

**(i)** Show that \(\alpha + \beta = -4\) and hence find \(\alpha \beta\).

**Sum of roots:**
\[
(\alpha + 1) + (\beta + 1) = -2
\]
\[
\alpha + \beta = -4 \text{ (shown)}
\]

**Product of roots:**
\[
(\alpha + 1)(\beta + 1) = \frac{5}{2}
\]
\[
\alpha \beta + (\alpha + \beta) + 1 = \frac{5}{2}
\]
\[
\alpha \beta = \frac{5}{2} - 1 - (-4)
\]
\[
\alpha \beta = \frac{11}{2}
\]

**(ii)** Find the quadratic equation in \(x\) with integer coefficients, whose roots are \(\frac{1}{\alpha^2}\) and \(\frac{1}{\beta^3}\).

**Sum of roots of new equation:**
\[
\frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{\alpha^3 + \beta^3}{(\alpha \beta)^3}
\]
\[
= \frac{(\alpha + \beta)(\alpha^2 + \beta^2 - \alpha \beta)}{(\alpha \beta)^3}
\]
\[
= \frac{(\alpha + \beta)[(\alpha + \beta)^2 - 2\alpha \beta - \alpha \beta]}{(\alpha \beta)^3}
\]
\[
= \frac{(-4)[(-4)^2 - 2(\frac{11}{2})]}{(\frac{11}{2})^3}
\]
\[
= \frac{16}{\frac{1331}{2}}
\]

**Product of roots of new equation:**
\[
\frac{1}{(\alpha \beta)^3}
\]
\[
= \frac{1}{\left(\frac{11}{2}\right)^3}
\]
\[
= \frac{8}{1331}
\]

Equation is \(1331x^2 - 16x + 8 = 0\).
5(a) Given that $\log_2(2x+1) - \log_4(3-x^2) = 1$, form a quadratic equation in $x$ and explain why the roots of the quadratic equation are real and distinct.

\[
\begin{align*}
\log_2(2x+1) - \log_4(3-x^2) &= 1 \\
\log_2(2x+1) - \frac{\log_2(3-x^2)}{\log_2 2^2} &= 1 \\
\log_2(2x+1) - \frac{1}{2} \log_2(3-x^2) &= 1 \\
\log_2 \left( \frac{2x+1}{\sqrt{3-x^2}} \right) &= 1 \\
\frac{2x+1}{\sqrt{3-x^2}} &= 2 \\
2x+1 &= 2\sqrt{3-x^2} \\
(2x+1)^2 &= 4(3-x^2) \\
4x^2 + 4x + 1 &= 12 - 4x^2 \\
8x^2 + 4x - 11 &= 0 \\
\text{Discriminant} &= 4^2 - 4(8)(-11) \\
&= 368 \\
\text{Since the discriminant is greater than 0, the roots of the quadratic equation are real and distinct.}
\end{align*}
\]

5(b) Solve $3^{x^2} = 2(3^{-y}) + 17$.

\[
\begin{align*}
3^{x^2} &= 2(3^{-y}) + 17 \\
3^{2(3^y)} - 17(3^y) &= 2 \\
3^2(3^y)^2 - 17(3^y) &= 2 \\
\text{Let } a &= 3^y, \\
9a^2 - 17a - 2 &= 0 \\
(9a + 1)(a - 2) &= 0 \\
a &= -\frac{1}{9} \text{ (rejected)}, \\
a &= 2 \\
3^y &= 2 \\
y &= \frac{\log 2}{\log 3} \\
y &\approx 0.631
\end{align*}
\]
6. The curve \( y = \frac{2x^2}{x^2 + 1} \) has one stationary point \((p, q)\).

(i) Find the value of \(p\) and of \(q\).

\[
\frac{dy}{dx} = \frac{(x^2 + 1)(4x) - 2x^2(2x)}{(x^2 + 1)^2} = \frac{4x^3 + 4x - 4x^3}{(x^2 + 1)^2} = \frac{4x}{(x^2 + 1)^2}
\]

For a stationary point, put \(\frac{dy}{dx} = 0\).

\(x^2 + 1)^2 > 0, \quad 4x = 0, \quad \therefore x = 0\)

\(\therefore p = 0\) and \(q = 0\)

(ii) Determine whether \(y\) is increasing or decreasing.

(a) for \(x > p\),

For \(x > 0\), \((x^2 + 1)^2 > 0\) and \(4x > 0\), \(x > 0\)

\(\therefore \frac{dy}{dx} > 0, \quad y\) is increasing

(b) for \(x < p\).

\(x < 0, \quad (x^2 + 1)^2 > 0\) but \(4x < 0\), i.e. \(x < 0\)

\(\therefore \frac{dy}{dx} < 0, \quad y\) is decreasing

Hence state the nature of the stationary point.

Since the value of \(\frac{dy}{dx}\) changes from negative to positive, the stationary point is a minimum point.
(iii) Find $\frac{d^2y}{dx^2}$ at the stationary point and explain how $\frac{d^2y}{dx^2}$ further supports your answer to part (ii).

\[
\frac{dy}{dx} = \frac{4x}{(x^2 + 1)^2}
\]

\[
\frac{d^2y}{dx^2} = \frac{(x^2 + 1)^2(4) - 4x(2)(x^2 + 1)(2x)}{(x^2 + 1)^4}
\]

\[
\frac{d^2y}{dx^2} = \frac{4(x^2 + 1)(x^2 + 1 - 4x^4 - 4x^2)}{(x^2 + 1)^4}
\]

\[
= \frac{4(1 - 3x^2 - 4x^4)}{(x^2 + 1)^3}
\]

At the stationary point $(0, 0)$,

\[
\frac{d^2y}{dx^2} = 4
\]

Since, $\frac{d^2y}{dx^2} > 0$ the stationary point is minimum, thus reiterating the result from part (ii).
In the figure, $AB$ is a diameter of the circle with centre $O$. Chords $AD$ and $BC$ intersect at $F$. $AD$ produced meets the tangent to the circle, $TBE$ at $E$. $AE$ is an angle bisector of angle $BAC$.

(i) Prove that $\angle CBD = \angle DBE$.

$\angle DBE = \angle BAD$ (Alternate segment Thm)

$\angle BAD = \angle CAD$ (given $EA$ is bisector of $\angle BAC$)

$\therefore \angle DBE = \angle CAD$

$\angle CAD = \angle CBD$ (angles in the same segment)

$\angle DBE = \angle CBD$ (proven)

(ii) Given that $\angle AOF = 90^\circ$, prove that triangle $AOF$ is similar to triangle $ADB$.

$\angle A$ is a common angle.

$\angle ADB = 90^\circ$ (angle in the semi-circle)

$\angle ADB = \angle AOF$

$\therefore \triangle AOF$ is similar to $\triangle ADB$ (By AA similarity test)

(iii) $2(AO)^2 = AF \times (AF + FD)$.

Since $\triangle AOF$ is similar to $\triangle ADB$,

\[
\frac{AO}{AD} = \frac{AF}{AB}
\]

\[
\frac{AO}{AF + FD} = \frac{AF}{AB} \quad (AD = AF + FD)
\]

\[
\frac{AO}{AF + FD} = \frac{AF}{2 \cdot AO} \quad (AO \text{ is radius and } AB \text{ is diameter})
\]

$2(AO)^2 = AF \times (AF + FD)$
8. A particle moving in a straight line passes through a fixed point $O$ with a speed of 20 m/s. The acceleration, $a$ m/s$^2$, of the particle, $t$ s after passing through $O$ is given by $a = -100e^{-3t}$. The particle comes to instantaneous rest at point $N$.

(i) Find the time the particle comes to instantaneous rest at point $N$.

\[
a = -100e^{-3t}
\]

velocity, $v = \int -100e^{-3t} \, dt$

\[
= \frac{100}{3} e^{-3t} + c, \text{ where } c \text{ is a constant}
\]

when $v = 20$ and $t = 0$,

\[
\frac{100}{3} e^{0} + c = 20
\]

\[
\therefore c = \frac{40}{3}
\]

\[
v = \frac{100}{3} e^{-3t} - \frac{40}{3}
\]

at rest, $v = 0$

\[
\frac{100}{3} e^{-3t} - \frac{40}{3} = 0
\]

\[
e^{-3t} = \frac{40}{3} \times \frac{3}{100}
\]

\[
-3t \ln e = \ln \left( \frac{2}{5} \right)
\]

\[
t = -\frac{1}{3} \ln \left( \frac{2}{5} \right)
\]

\[
t = 0.30543
\]

The particle comes to rest at $t = 0.305$ s.
(ii) Calculate the distance $ON$.

\[ v = \frac{100}{3} e^{-\frac{t}{3}} - \frac{40}{3} \]

displacement, \( s = \int \left( \frac{100}{3} e^{-\frac{t}{3}} - \frac{40}{3} \right) dt \)

\[ s = -\frac{100}{9} e^{-\frac{t}{3}} - \frac{40}{3} t + c \quad \text{where } c \text{ is a constant} \]

when \( s = 0 \), \( t = 0 \) \( \therefore c = \frac{100}{9} \)

\[ s = -\frac{100}{9} e^{-\frac{t}{3}} - \frac{40}{3} t + \frac{100}{9} \]

when \( t = 0.30543 \), \( s = -\frac{100}{9} e^{-\frac{0.30543}{3}} - \frac{40}{3} (0.30543) + \frac{100}{9} \)

\[ s = 2.3943 \]

Distance $ON = 2.59$ m

(iii) Show that the average speed of the particle in the first 2 seconds rounded off to whole number is 10 metres per second.

At \( t = 2 \), \( s = -\frac{100}{9} e^{-\frac{2}{3}} - \frac{40}{3} \left( \frac{2}{3} \right) + \frac{100}{9} \)

\[ = -15.583 \text{ m} \]

Total distance travelled in the first 2 seconds

\[ = (2)2.5943 + 15.583 \]

\[ = 20.7716 \]

Average speed = \( \frac{20.7716}{2} \)

\[ = 10.3858 \]

\[ = 10 \text{ m/s (whole number) (shown)} \]
9(i) Solve the equation $2\sin 2\theta = 3\cos \theta$ for $0^\circ \leq \theta \leq 360^\circ$.

$2\sin 2\theta - 3\cos \theta = 0$

$2(2\sin \theta \cos \theta) - 3\cos \theta = 0$

$\cos \theta (4\sin \theta - 3) = 0$

$\cos \theta = 0 \quad \therefore \sin \theta = \frac{3}{4}$

$\theta = 90^\circ, 270^\circ$ 

Basic $\zeta = 48.590^\circ$

$\theta = 48.590^\circ, 131.41^\circ$

$.\therefore \text{Ans: } \theta = 48.6^\circ, 90^\circ, 131.4^\circ, 270^\circ$

(ii) On the same axes, sketch for $0^\circ \leq \theta \leq 720^\circ$, the graphs of

$y = \sin \theta$ and $y = \frac{3}{2} \cos \left(\frac{\theta}{2}\right)$.

(iii) Using solutions to part (i), determine the $x$-coordinates of the points of intersections of the graphs of part (ii).

$\sin x = \frac{3}{2} \cos \left(\frac{x}{2}\right)$

$2\sin 2\left(\frac{x}{2}\right) - 3\cos \left(\frac{x}{2}\right) = 0$

Let $P = \left(\frac{x}{2}\right)$, then

$\frac{x}{2} = 48.590^\circ, 90^\circ, 131.41^\circ, 270^\circ$

$x = 97.1^\circ, 180^\circ, 262.8^\circ, 540^\circ$
10. A circle, \( C_1 \), has equation \( x^2 + y^2 - 14x + 2y = -46 \).

(i) Find the coordinates of the centre of the circle and the radius.

Centre \((7, -1)\)

Radius \(= \sqrt{7^2 + (-1)^2 - 46} = 2\) units

The coordinates of the centre of a second circle, \( C_2 \), is \((-4, -2)\). The equation of the tangent to the circle, \( C_2 \) at a point \( P \) is \( 2y = -2x + 3 \).

(ii) Find the coordinates of point \( P \).

Gradient of tangent to circle at \( P = -1 \)
Equation of the normal at \( P \) is

\[
\frac{y + 2}{x + 4} = 1
\]

\[
y + 2 = x + 4
\]

\[
y = x + 2 \tag{1}
\]

\[
2y = -2x + 3 \tag{2}
\]

Substitute (1) into (2)

\[
2(x + 2) = -2x + 3
\]

\[
x + 4 = -2x + 3
\]

\[
x = -\frac{1}{4}, \quad y = -\frac{1}{4} + 2
\]

\[
y = \frac{7}{4}
\]

\[ \therefore P\left(-\frac{1}{4}, \frac{7}{4}\right) \]

(iii) Find the exact value of the radius of \( C_2 \) and the equation of the circle, \( C_2 \).

Radius of \( C_2 \) = \( \sqrt{(-4 + \frac{1}{4})^2 + (-2 - \frac{7}{4})^2} \)

\[ = \frac{15\sqrt{2}}{4} \text{ units} \]

Equation of \( C_2 \) is

\[
(x + 4)^2 + (y + 2)^2 = \frac{225}{8}
\]
(iv) Determine whether circles $C_1$ and $C_2$ will meet each other, showing your working clearly.

Distance between centres of $C_1$ and $C_2$

$$= \sqrt{(7+4)^2 + (-1+2)^2}$$
$$= \sqrt{122}$$
$$= 11.0$$

Sum of radii $= 2 + \frac{15\sqrt{2}}{4}$
$$= 7.30$$

Since the sum of radii, 7.30 units, is less than the distance between the 2 centres, 11.0 units, the 2 circles $C_1$ and $C_2$ will not meet each other.

11(a) Show that $\frac{d}{dx} (2x\sqrt{2x+3}) = \frac{6x+6}{\sqrt{2x+3}}$.

$$\frac{d}{dx} (2x\sqrt{2x+3})$$
$$= 2x \cdot \frac{1}{2} (2x+3)^{-\frac{1}{2}} (2) + 2\sqrt{2x+3}$$
$$= \frac{2(2x+3)+2x}{\sqrt{2x+3}}$$
$$= \frac{6x+6}{\sqrt{2x+3}}$$ (shown)
The diagram shows part of the curve \( y = \frac{3x + 3}{\sqrt{2x + 3}} \). The curve intersects the x-axis at point A. The line through A and perpendicular to the line \( y + x = -7 \) intersects the curve again at another point, B.

(i) Show that the y-coordinate of point B is 4.

When \( y = 0 \), \( 3x + 3 = 0 \). \( \therefore \) A(-1, 0)

Gradient of the line AB = 1

Equation of line AB:

\[
\frac{y-0}{x+1} = 1
\]

\[
y = x + 1 \quad (1)
\]

\[
y = \frac{3x + 3}{\sqrt{2x + 3}} \quad (2)
\]

Substitute (1) into (2)

\[
x + 1 = \frac{3(x+1)}{\sqrt{2x + 3}}
\]

\[
(x+1)\left[\frac{\sqrt{2x + 3} - 3}{\sqrt{2x + 3}}\right] = 0
\]

\[
x = -1, \quad \sqrt{2x + 3} - 3 = 0
\]

\[
2x + 3 = 9
\]

\[
x = 3
\]

\[
\therefore y = 3 + 1
\]

\[
y = 4
\]

Hence the y-coordinate of B = 4 (shown)
Given that the line $AB$ intersects the $y$-axis at $C,$ determine the area of the shaded region bounded by the line $CB,$ the curve, the line $x = 5,$ the $x$-axis and the $y$-axis.

For $y = x + 1$
when $x = 0,$ $y = 1$
$\therefore C \ (0,1)$

Area of shaded region
$= \text{area of trapezium } OCBD + \text{area under the curve}$

$= \frac{1}{2} (1 + 4) \times 3 + \int_{x=0}^{5} \frac{3x + 3}{\sqrt{2x + 3}} \, dx$

$= \frac{3}{2} (5) + \frac{1}{2} \left[ \frac{6x + 6}{\sqrt{2x + 3}} \right]_{0}^{5}$

$= 7.5 + \frac{1}{2} \left[ 2x \sqrt{2x + 3} \right]_{0}^{5}$

$= 7.5 + \frac{1}{2} \left[ 2(5) \sqrt{2(5) + 3} - 2(3) \sqrt{2(3) + 3} \right]$ 

$= 16.5 \text{ units}^2$

*End of paper*
CONVENT OF THE HOLY INFANT JESUS SECONDARY
Preliminary Examination 1 in preparation for
the General Certificate of Education Ordinary Level 2016

ADDITIONAL MATHEMATICS

Paper 1

Additional Materials: Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the
case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of 6 printed pages.

[Turn over
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[
    x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion

\[
    (a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. TRIGONOMETRY

Identities

\[
\begin{align*}
    \sin^2 A + \cos^2 A &= 1 \\
    \sec^2 A &= 1 + \tan^2 A \\
    \csc^2 A &= 1 + \cot^2 A \\
    \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
    \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
    \tan(A \pm B) &= \frac{\tan A \pm \tan B}{1 \pm \tan A \tan B} \\
    \sin 2A &= 2 \sin A \cos A \\
    \cos 2A &= \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A \\
    \tan 2A &= \frac{2 \tan A}{1 - \tan^2 A}
\end{align*}
\]

Formulae for \( \Delta ABC \)

\[
\begin{align*}
    \frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\
    a^2 &= b^2 + c^2 - 2bc \cos A \\
    \Delta &= \frac{1}{2} ab \sin C
\end{align*}
\]
1 (i) Write down and simplify the first three terms of the expansion, in ascending powers of $x$, of

(a) $(1 + 6x)^6$.

(b) $(1 - kx)^6$.

(ii) Use the results from part (i), obtain the coefficient of $x^2$, in terms of $k$, in the expansion of $(1 + (6 - k)x - 6kx^2)^6$.

(iii) In the expansion of $[1 + (6 - k)x - 6kx^2]^6$, where $k$ is an integer, the coefficient of $x^2$ is 168. Find the value of $k$.

2 It is given that \[
\frac{\cos^2 \theta}{1 + 2 \sin^2 \theta} = \frac{16}{43}, \]
where $180^\circ < \theta < 270^\circ$. Without using a calculator, find the value of

(i) $\sin \theta$.

(ii) $\frac{\cos \theta}{1 + 2 \sin \theta}$.

3 Express \[
\frac{x^2 - 3x - 6}{(x+1)(x^2-1)}
\]
as the sum of 3 partial fractions.

4 (i) Prove the identity $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$.

(ii) Find all the angles between $0^\circ$ and $360^\circ$ which satisfy the equation $\cos 3\theta + \cos \theta = 0$.

5 It is given that $\frac{dy}{dx^2} = 2x - 1$. Given also that $\frac{dy}{dx} = 6$ when $x = 2$, find the increase in $y$ as $x$ increases from 2 to 4.

6 The equation of a curve is $y = (1 - m)x^3 + 2(m - 1)x + m$, where $m$ is a constant. Find

(i) the range of values of $m$ for which the curve lies completely above the $x$-axis.

(ii) the values of $m$ for which the line $y = 2x - 4$ is tangent to the curve.
The diagram shows a kite 20 m above the ground. As the string OK is let out, the kite moves horizontally at a constant rate of 0.5 m/s.

(i) Given that $\theta$ is the angle of elevation of the string to the horizontal ground, show that the projection of the string on the ground, $x$ m, is given by

$$x = 20 \cot \theta.$$  \[2\]

(ii) Find the rate of change of $\theta$ when 50 m of the string has been let out.  \[4\]

(iii) Explain what is meant by your answer in part (ii).  \[1\]

8 In order that each of the equations

(i) $y = a\sqrt{x} + \frac{b}{\sqrt{x}}$,

(ii) $y = \frac{a}{x + b}$,

(iii) $y^b = 10^{2x+c}$,

where $a$ and $b$ are unknown constants, may be represented by a straight line, they need to be expressed in the form $Y = mX + c$, where $X$ and $Y$ are each functions of $x$ and/or $y$, and $m$ and $c$ are constants. Copy the following table and insert in it an expression for $Y$, $X$, $m$ and $c$ for each case.

<table>
<thead>
<tr>
<th>(y = a\sqrt{x} + \frac{b}{\sqrt{x}})</th>
<th>(Y)</th>
<th>(X)</th>
<th>(m)</th>
<th>(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y = \frac{a}{x + b})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(y^b = 10^{2x+c})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
9 Solutions to this question by accurate drawing will not be accepted.

The points \( A, B, C \) and \( D (-2, 0) \) are four points of a parallelogram. The \( x \)-coordinate of \( A \) is \( k \). Lines are drawn parallel to the \( y \)-axis from \( A \) to meet the \( x \)-axis at \( N \) and from \( E \) to meet \( CD \) at \( M \). \( AN = EM \) and \( CM = MD \). The \( y \)-axis divides the quadrilateral \( AEMN \) into two equal halves.

The side \( AB \) has the equation \( y = 14 - 2x \) and the side \( AD \) has the equation \( y = x + 2 \).

\[(i) \text{ Write down the equation of } BC. \] [1]

\[(ii) \text{ Express the coordinates of } E \text{ and of } C \text{ in terms of } k. \] [3]

\[(iii) \text{ In the case where } k = 4, \text{ find the area of } AEMN. \] [2]

10 The point \( P \) lies on the curve \( y = \ln(x^2 + 2x) \) where \( x > 0 \). The normal to the curve at \( P \) is parallel to the line \( 5x + 3 = \pi - 12y \).

\[(i) \text{ Find the coordinates of } P. \] [5]

\[(ii) \text{ Show that the } y \text{-coordinate of the point where this normal intersects the } y \text{-axis is } \frac{5}{24} + \ln \frac{5}{4}. \] [2]
11 A curve has an equation \( y = (2x - 1)(x - 4) \).

(i) Find the minimum value of \( y \) and the value of \( x \) at which it occurs. \([2]\)

(ii) Sketch the graph of \( y = |(2x - 1)(x - 4)| \). \([2]\)

(iii) A line \( y = c \), where \( c \) is a constant, intersects the curve at four points. Using your graph, find the range of values of \( c \). \([2]\)

12 The diagram shows a piece of rectangular paper \( PQRS \) such that \( PS = 3 \text{ cm} \), \( QU = x \text{ cm} \) and \( PQ = y \text{ cm} \). The paper is folded along \( PU \) such that \( Q \) meets \( T \) on \( SR \).

![Diagram](image)

(i) Express \( TR \) and \( PT \) in terms of \( x \). \([4]\)

(ii) Hence show that the area, \( A \text{ cm}^2 \), of triangle \( PTU \) is given by

\[
A = \frac{3x^2}{2\sqrt{6x - 9}}.
\]

Given that \( x \) can vary, find

(iii) the value of \( x \) for which \( A \) is a minimum, \([5]\)

(iv) the minimum value of \( A \) in the form of \( a\sqrt{b} \text{ cm} \), where \( a \) and \( b \) are integers. \([2]\)

--- End of Paper 1 ---
<table>
<thead>
<tr>
<th>Prelim 1 P1 (2016)</th>
<th>Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1ia</td>
<td>(1 + 36x + 540x^2)</td>
</tr>
<tr>
<td>1ib</td>
<td>(1 - 6kx + 15k^2x^2)</td>
</tr>
<tr>
<td>1ii</td>
<td>(15k^2 - 216k + 540)</td>
</tr>
<tr>
<td>1iii</td>
<td>(k = \frac{62}{5}) (reject), (k = 2)</td>
</tr>
<tr>
<td>2i</td>
<td>(\sin \theta = \frac{3}{5}) (reject), (\sin \theta = -\frac{3}{5})</td>
</tr>
<tr>
<td>2ii</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>(3 \left(\frac{1}{x+1}\right) + \frac{1}{(x+1)^2} - \frac{2}{(x-1)})</td>
</tr>
</tbody>
</table>
| 4i | \(\cos 3\theta\)  
\[= \cos(2\theta + \theta)\]  
\[= \cos 2\theta \cos \theta - \sin 2\theta \sin \theta\]  
\[= (2\cos^2 \theta - 1)\cos \theta - 2\sin \theta \cos \theta \sin \theta\]  
\[= 2\cos^3 \theta - \cos \theta - 2\sin^2 \theta \cos \theta\]  
\[= 2\cos^3 \theta - \cos \theta - 2(1 - \cos^2 \theta) \cos \theta\]  
\[= 4\cos^3 \theta - 3\cos \theta\] (Proved) |
| 4ii | \(\theta = 90^\circ, 270^\circ, \theta = 41.4^\circ, 318.6^\circ, \theta = 180^\circ\) |
| 5 | \(y = 20 \frac{2}{3}\) |
| 6i | \(\frac{1}{2} < m < 1\) |
| 6ii | \(m = 0\) or \(m = \frac{1}{2}\) |
| 7i | \(\tan \theta = \frac{20}{x}, x = \frac{20}{\tan \theta}, x = 20 \cot \theta\) |
| 8i | \(Y = y \sqrt{x}, X = x, m = a, c = b\)  
\(Y = \frac{y}{\sqrt{x}}, X = \frac{1}{x}, m = b, c = a\) |
| 8ii | \(Y = \frac{1}{y}, X = x, m = \frac{1}{a}, c = \frac{b}{a}\)  
\(Y = y, X = xy, m = -\frac{1}{b}, c = \frac{a}{b}\)  
\(Y = xy, X = y, m = -b, c = a\) |
| 8iii | \(Y = \frac{\log y}{y}, X = x, m = \frac{2}{b}, c = \frac{a}{b}\)  
\(Y = \log y, X = x, m = \frac{2 \ln 10}{b}, c = \frac{a \ln 10}{b}\) |
| 9i | \(y = x + 14\) |
| 9ii | \(C(2 - 2k, 16 - 2k)\)  
\(C(2 - 2k, 4k - 8)\)  
\(C(-2 - k, 12 - k)\)  
\(C(-2 - k, 2k)\) |
<p>| 9iii | 48 units$^2$ |
| 10i | (\frac{1}{2}, \ln \frac{5}{4}) or (P\left(\frac{1}{2}, 0.223\right)) |
| 10ii | (y = \frac{5}{24} + \ln \frac{5}{4}) |
| 11i | (x = 2 - \frac{1}{4}, y = -6 \frac{1}{8}) |
| 11ii |  |
| 11iii | (0 &lt; e &lt; 6 \frac{1}{8}) |</p>
<table>
<thead>
<tr>
<th>7ii</th>
<th>$-0.004$</th>
<th>12i $TR = \sqrt{6x-9}$, $PT = \frac{3x}{\sqrt{6x-9}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7iii</td>
<td>The negative sign indicates clockwise change in angle size (i.e. reducing angle).</td>
<td>12ii $\frac{1}{2} - \frac{3x}{\sqrt{6x-9}} = \frac{3x^2}{2\sqrt{6x-9}}$ (shown)</td>
</tr>
<tr>
<td>12iii</td>
<td>$\frac{dy}{dx} = \frac{27x^2 - 54x}{2(6x-9)^2}$, $x = 2$, use table to show min area</td>
<td>12iv $2\sqrt{3}$ cm$^2$</td>
</tr>
</tbody>
</table>
CONVENT OF THE HOLY INFANT JESUS SECONDARY
Preliminary Examination 1 in preparation for
the General Certificate of Education Ordinary Level 2016

ADDITIONAL MATHEMATICS

Paper 2

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
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Answer all questions.
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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of 5 printed pages and 1 blank page.

[Turn over
Mathematical Formulae

1. ALGEBRA

**Quadratic Equation**
For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Binomial expansion**

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where $n$ is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1) \ldots (n-r+1)}{r!}$$

2. TRIGONOMETRY

**Identities**

- $\sin^2 A + \cos^2 A = 1$
- $\sec^2 A = 1 + \tan^2 A$
- $\csc^2 A = 1 + \cot^2 A$
- $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
- $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
- $\sin 2A = 2 \sin A \cos A$
- $\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$
- $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

**Formulae for $\Delta ABC$**

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$A = \frac{1}{2} ab \sin C$$
1 The number of words per minute, \( N(t) \), that Mr Ong can type is given by the function

\[ N(t) = 68 - 36e^{-0.5t}, \]

where \( t \) is the time in months after he begins a computer based typing course.

(i) Find the number of words per minute that Mr Ong can type after 2 months.

(ii) Find the time Mr Ong will take to type at a rate of 58 words per minute.

(iii) Determine whether Mr Ong will be able to type at a rate of 70 words per minute. Explain your answer clearly.

2 When the expression \( 6x^4 - 5x^3 + 4x^2 + hx + k \) is divided by \( 3x^2 + 2x - 1 \), the remainder is \( 7 - 7x \). Find the value of \( h \) and of \( k \).

3 \( AC \) and \( BD \) are diagonals of a rhombus \( ABCD \). \( AC = \left(9 + 2\sqrt{3}\right) \) cm and the area of \( ABCD \) is \( \left(\frac{57}{2} + 14\sqrt{3}\right) \) cm\(^2\).

(i) Find the length of the diagonal \( BD \) in the form \( (a + b\sqrt{3}) \) cm, where \( a \) and \( b \) are integers.

(ii) Find the value of \( AB^2 \), giving your answer in the form \( (a + b\sqrt{3}) \) cm\(^2\), where \( a \) and \( b \) are rational numbers.

4 (a) The roots of the equation \( 2x^2 + 4px + q = 0 \) are \( \alpha \) and \( \alpha + 2 \). Express \( q \) in terms of \( p \).

(b) The equation \( 3x^2 - 5x - 7 = 0 \) has roots \( \alpha \) and \( \beta \). Form an equation with roots \( \alpha + 3 \) and \( \beta + 3 \).

5 (a) Given that the equation \( 2 \log_3 x - \frac{3}{\log_3 x} = 5 \), find the exact values of \( x \).

(b) Given that \( \log_4 x = h \) and \( \log_{16} 4x = k \), express \( h \) in terms of \( k \).
6  The equation of a curve is \( y = 3x + \ln(2x - 5) \).

(i)  The line \( y = 3x - 2 \) intersects the curve at the point \( K \). Find the coordinates of \( K \), giving your answer correct to 2 decimal places.  \([3]\)

(ii) Find the equation of the normal to the curve at the point \( x = 3 \).  \([4]\)

(iii) The normal to the curve at the point \( x = 3 \) cuts the \( x \)-axis at the point \( H \). Find the coordinates of \( H \).  \([2]\)

7  Find the coordinates of the stationary point(s) of the curve \( y = \frac{x^3 + 16}{x} \). Determine the nature of the turning point(s). Explain clearly why the gradient of the curve is negative when \( x < 0 \).  \([7]\)

8  (a)  The equation of a circle is \( x^2 + 2x + 4y = 20 - y^2 \). Given that \( A(2, 2) \) is a point on the circle, find the equation of the tangent to the circle at \( A \).  \([5]\)

(b)  \( A(0, 2) \), \( B(9, 3) \) and \( C(1, -7) \) are three points on a circle.

(i)  Show that \( BC \) is the diameter of the circle.  \([4]\)

(ii) Find the equation of the circle.  \([3]\)

9  (a)  Solve the equation \( \frac{2}{\cos^2 x} = 7 \tan x - 3 \) for \( 0 \leq x \leq 2\pi \).  \([4]\)

(b)  (i) Sketch the graphs of \( y = 1 - 3\sin x \) and \( y = 4\cos 2x - 1 \) on the same axes, for \( 0 \leq x \leq \pi \).  \([4]\)

(ii) Calculate the values of \( x \) in the given range for which \( 1 - 3\sin x = 4\cos 2x - 1 \).  \([4]\)

(iii) Using your graph from part (b)(i), state the range of values of \( x \) for which \( 2 - 3\sin x \geq 4\cos 2x \).  \([1]\)
10 (a) Given that \( y = \ln \sqrt{\cos 2x} \), find \( \frac{dy}{dx} \) and hence find the exact value of \( \int_{x}^{6} 3 \tan 2x \, dx \). [5]

(b) The diagram shows part of the curve \( y = \sqrt{\left( \frac{x}{2} + 3 \right)^3} \). The straight line \( BC \) is normal to the curve at the point \( B(2, 8) \). Find

(i) the equation of the line \( BC \); [3]

(ii) the area of the shaded region \( OABC \). [5]

11 A particle moves in a straight line and passes through a fixed point \( O \) with an initial velocity of 16 cm/s. The acceleration, \( a \) cm/s\(^2\), of the particle at \( t \) seconds after passing \( O \), is given by

\[
a = -25e^{-\frac{3t}{2}}.\]

(i) Find an expression, in terms of \( t \), for the velocity of the particle. [3]

(ii) Find the time taken for the particle to come to an instantaneous rest, giving your answer correct to 2 decimal places. [3]

(iii) Calculate the distance moved by the particle in the third second. [5]

--- End of Paper 2 ---
<table>
<thead>
<tr>
<th></th>
<th>Prelim 1 P2 (2016)</th>
<th>Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1(i) approx. 57 words/min</td>
<td>9(a) $x = \frac{\pi}{4}, \frac{5\pi}{4}$</td>
</tr>
<tr>
<td></td>
<td>(ii) 2.13 (or 2.14) months</td>
<td>(bi) $0.806, 2.34$</td>
</tr>
<tr>
<td></td>
<td>(iii) As $e^{-0.5t} &gt; 0$ for all values of $t$, thus $36e^{-0.0t} &gt; 0$, thus $N$ will always be less than 68.</td>
<td>(bii) $0.806 \leq x \leq 2.34$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>10(a) $\frac{dy}{dx} = -\tan 2x; -\frac{3}{2} \ln \frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td>$h = 4, k = 3$</td>
<td>(bii) $y = -\frac{2}{3}x + x + 9 \frac{1}{3}$</td>
</tr>
<tr>
<td></td>
<td>3(i) $5 + 2\sqrt{3}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(ii) $32\frac{1}{2} + 14\sqrt{3}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4(a) $g = 2p^2 - 2$</td>
<td>(bii) 61.1 sq units</td>
</tr>
<tr>
<td></td>
<td>(b) $x^2 - \frac{2}{3}x + \frac{35}{3} = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5(a) $\frac{1}{\sqrt{3}}$ or 27</td>
<td>(ii) 2.15s</td>
</tr>
<tr>
<td></td>
<td>(b) $h = 4k - \frac{27}{3}$</td>
<td>(iii) 0.260 cm</td>
</tr>
<tr>
<td></td>
<td>6(i) $(2.57, 5.70)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(ii) $5y + x = 48$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(iii) $(48, 0)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2,12) and is a min pt; When $x &lt; 0$, $2x &lt; 0$ and $x^2 &gt; 0$, $2x - \frac{16}{x^2}$ is always -ve;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8(a) $y = -\frac{3}{4}x + \frac{7}{2}$</td>
<td>(bi) Grad of $AB \times$ Grad of $AC = \frac{1}{9}x - 9 = -1$ $\Rightarrow AB \perp AC$ By the circle property in semicircle is $90^\circ$, $\angle CAB = 90^\circ$ and BC is the diameter.</td>
</tr>
<tr>
<td></td>
<td>(bi) $(x - 5)^2 + (y + 2)^2 = 41$</td>
<td>(bii)</td>
</tr>
</tbody>
</table>
PRELIMINARY EXAMINATION 2016
SECONDARY 4 EXPRESS
ADDITIONAL MATHEMATICS PAPER 1

Date: 22 Aug 2016
Duration: 2 hours
Time: 0800 – 1000

Additional Materials: 8 sheets of writing paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction tape/fluid

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of 7 printed pages (including cover page).
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion

\[
(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. TRIGONOMETRY

Identities

\[
sin^2 A + \cos^2 A = 1
\]

\[
\sec^2 A = 1 + \tan^2 A
\]

\[
\csc^2 A = 1 + \cot^2 A
\]

\[
sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
sin 2A = 2 \sin A \cos A
\]

\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]

\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \triangle ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2} ab \sin C
\]
Answer all the questions.

1. A curve has the equation \( y = 4x^2 - px + p - 3 \), where \( p \) is a constant. Find the range of values of \( p \) for which the curve lies completely above the \( x \)-axis. [4]

2. Solve the equation \( \ln(4^x - 4) - x \ln 2 = \ln 3 \). [4]

3. A curve has the equation \( y = \frac{1-x}{3x+4} \) for \( x > 0 \).
   (i) Obtain an expression for \( \frac{dy}{dx} \). [2]
   (ii) Show that \( y \) is a decreasing function. [1]
   (iii) Given that \( y \) decreases at the rate of 0.75 units per second, calculate the rate of change of \( x \) at the instant when \( x = 3 \). [2]

4. The diagram shows a straight line \( ABC \) such that \( AB : BC = 3 : 1 \). The point \( B \) is \( (1, 2) \) and the point \( C \) lies on the \( x \)-axis. \( \theta \) is the angle between the positive \( x \)-axis and the line \( AC \). Given that \( \tan \theta = -2 \), find
   (i) the equation of the line \( AC \), [1]
   (ii) the coordinates of \( C \) and of \( A \). [3]

The point \( D \) is such that \( ABOD \) is a parallelogram.
   (iii) Find the coordinates of \( D \). [2]
5 In an experiment, a scientist started with 5 000 000 cells and observed that 40% of
the cells are dying every minute. The number of cells remaining, \( N \), after \( t \) minutes,
is given by \( N = Ae^{kt} \), where \( A \) and \( k \) are constants.

(i) Find the value of \( A \) and of \( k \). \([4]\)

(ii) Find the value of \( t \) when the number of cells decreases to 2000. \([2]\)

6 (i) Sketch the curve \( y = |x^2 - 4| \) for \(-2 \leq x \leq 3\). \([3]\)

(ii) Find the \( x \)-coordinates of the points of intersection of the curve \( y = |x^2 - 4| \)
and the line \( y = 6 \). \([3]\)

7

\[\text{The diagram shows a circle, centre } O. \text{ The point } R \text{ lies on the circle and } TR \text{ is a}
tangent to the circle. The line } TQ \text{ passes through } O \text{ and intersects the circle at } P
\text{ and } Q.\]

(i) Prove that triangles \( TRP \) and \( TQR \) are similar. \([2]\)

(ii) Prove that \( TP \times TQ = OT^2 - OR^2 \). \([4]\)
The diagram shows the graph of the depth of water, \( h \) metres, in a harbour on a particular day, which is modelled by the equation, \( h = a \sin \frac{1}{2} t + b \), where \( a \) and \( b \) are constants and \( t \) is the time in hours after midnight.

(i) State the period of \( h \). \([1]\)

(ii) Use the graph to find the value of \( a \) and of \( b \). \([2]\)

The harbour gates are closed when the depth of the water is less than seven metres. An alarm rings when the gates are opened or closed.

(iii) Using the values of \( a \) and \( b \) found in (ii), calculate the values of \( t \) when the alarm rings on this particular day. \([4]\)

(iv) Hence find the total length of time when the harbour gates are closed. \([1]\)

9

(i) Show that \( \frac{\sin \theta}{1 + \frac{1}{\sec \theta}} + \cot \theta = \csc \theta \). \([4]\)

(ii) Hence find, in degrees, the smallest value of \( \theta \) such that \( \frac{\sin 2\theta}{1 + \frac{1}{\sec 2\theta}} + \cot 2\theta = 6 \cos 2\theta \). \([4]\)
The diagram shows part of the curve \( y^2 = x + 1 \). The line \( y = 2x - 4 \) intersects the curve at points \( A \) and \( B \). Find

(i) the coordinates of \( A \) and of \( B \), \[4\]

(ii) the area of the shaded region. \[4\]
11 A particle moves in a straight line, so that, $t$ seconds after leaving a fixed point $O$, its velocity, $v \text{ m s}^{-1}$ is given by $v = 2 + 5t - 3t^2$. The particle comes to instantaneous rest at the point $Q$. Find

(i) the acceleration of the particle at $Q$. [4]
(ii) the distance $OQ$. [3]
(iii) the total distance travelled by the particle in the time interval $t = 0$ to $t = 3$. [2]

12

The diagram shows a straight road $PQ$, of length 10 km. A man is at point $A$, where $AP$ is perpendicular to $PQ$ and $AP$ is 2 km. He travels in a straight line to meet the road at point $X$, where angle $PAX = \theta$ radians. The man travels at 3 km/h along $AX$ and 5 km/h along $XQ$. He takes $T$ hours to travel from $A$ to $Q$.

(i) Show that $T = \frac{2\sec \theta}{3} + \frac{2\tan \theta}{5}$. [4]
(ii) Given that $\theta$ can vary, show that $T$ has a stationary value when $PX = 1.5$ km. [6]
Additional Mathematics
Preliminary Examination 2016
Marking Scheme

1 \[ y = 4x^2 - px + p - 3 \]
\[ b^2 - 4ac < 0 \]
\[ (-p)^2 - 4(4)(p - 3) < 0 \] M1
\[ p^2 - 16p + 48 < 0 \] correct quadratic form M1
Finding the solution of quadratic: \( p = 4 \) or 12 DM1

\[ (p - 12)(p - 4) < 0 \]

\[ 4 < p < 12 \] A1

2 \[ \ln(4^x - 4) - x \ln 2 = \ln 3 \]
\[ \ln(4^x - 4) - \ln 2^x = \ln 3 \]
\[ \ln \frac{4^x - 4}{2^x} = \ln 3 \] applying quotient law M1
\[ \frac{4^x - 4}{2^x} = 3 \]
\[ 2^{2x} - 3(2^x) - 4 = 0 \] correct quadratic equation M1

Or substituting \( y = 2^x \) to get \( y^2 - 3y - 4 = 0 \)
\[ (y - 4)(y + 1) = 0 \]
\[ y = 4 \text{ or } y = -1 \]
\[ 2^x = 4 \text{ or } 2^x = -1 \text{(rej)} \] M1
\[ x = 2 \] A1
3 (i) \[ y = \frac{1-x}{3x+4} \]

\[
\frac{dy}{dx} = \frac{(-1)(3x+4) - (1-x)(3)}{(3x+4)^2} \\
= \frac{-7}{(3x+4)^2}
\]

\[
M1 \quad A1
\]

(ii) Since \((3x+4)^2 > 0\) and \(\frac{-7}{(3x+4)^2} < 0\), \(y\) is a decreasing function for all real values of \(x\)

\[
B1
\]

(iii) \[
\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}
\]

\[-0.75 = \frac{-7}{(3x+4)^2} \times \frac{dx}{dt} \]

When \(x = 3\), \[
\frac{dx}{dt} = \frac{-3}{4} \times \frac{169}{-7} = 18 \frac{3}{28} \text{ units / sec}
\]

(or \(18.1 \text{ units / sec}\))

4 (i) \[ y - 2 = -2(x - 1) \]

\[ y = -2x + 4 \]

\[
B1
\]

(ii) when \(y = 0, x = 2\)

.: Coordinates of \(C = (2, 0)\)

Let the coordinates of \(A\) be \((a, b)\).
Apply similar triangle ratios

\[
\frac{1-a}{1} = \frac{3}{1} \quad \text{and} \quad \frac{b-2}{2} = \frac{3}{1}
\]

\(a = -2\) \quad \text{and} \quad \(b = 8\)

\therefore \text{ Coordinates of } A = (-2, 8)

[Or apply distance formula]

Subst \(x = a\) into \(y = -2x + 4\)

\(y = -2a + 4\)

Distance of \(AB = 3\)

Distance of \(BC\)

\[
\sqrt{(a-1)^2 + (-2a+4-2)^2} = 3\sqrt{(1-2)^2 + (2-0)^2}
\]

\(a^2 - 2a + 1 + 4a^2 - 8a + 4 = 9(5)\)

\(5a^2 - 10a - 40 = 0\)

\(5(a - 4)(a + 2) = 0\)

\(a = 4\) (rej) \quad \text{or} \quad a = -2

\(b = -8\)

\therefore \text{ Coordinates of } A = (-2, 8)

(iii) Let the point \(D\) be \((h, k)\)

Mid-point of \(BD = \text{ mid-point of } AO\)

\[
\left(\frac{h+1}{2}, \frac{k+2}{2}\right) = \left(\frac{-2+0}{2}, \frac{8+0}{2}\right)
\]

\[
\frac{h+1}{2} = -1, \quad \frac{k+2}{2} = 4
\]

\(h = -3, \quad k = 6\)

\(D(-3, 6)\)
5. (i) \( N = Ae^{kt} \)

When \( t = 0 \), \( N = 5000000 \)

\[ 5000000 = Ae^{k(0)} \]

\[ A = 5000000 \]  

When \( t = 1 \), \( N = \frac{60}{100} \times 5000000 \)

\[ = 3000000 \]

\[ 3000000 = 5000000e^{k(1)} \]

\[ e^{k} = \frac{3}{5} \]

\[ k = \ln \left( \frac{3}{5} \right) \]

\[ = -0.5108 \approx -0.511 \]  

(ii) \( 2000 = 5000000e^{-0.5108t} \)

\[ e^{-0.5108t} = \frac{2}{5000} \]

\[ -0.5108t = \ln \left( \frac{2}{5000} \right) \]

\[ t = 15.3 \text{ min} \]
Correct shape

$x$ - intercepts and turning point shown correctly

end point (3, 5) shown clearly

(ii) \[ |x^2 - 4| = 6 \]
\[ x^2 - 4 = 6 \text{ or } x^2 - 4 = -6 \]
\[ x^2 = 10 \text{ or } x^2 = -2 \text{ (rej)} \]
\[ x = 3.16 \text{ or } -3.16 \]

\[ \angle RTP = \angle QTR \quad \text{(common angle)} \]
\[ \angle TRP = \angle TQR \quad \text{(s in the alternate segment or tangent chord thm)} \]
\[ \therefore \triangle TRP \text{ and } \triangle TQR \text{ are similar. (AA similarity)} \]

(ii) Since $\triangle TRP$ and $\triangle TQR$ are similar,

\[
\frac{TR}{TP} = \frac{TQ}{TR}
\]
\[ \Rightarrow TR^2 = TP \times TQ \quad \text{-------------------- (1)} \]
\[ \angle ORT = 90^\circ \text{ (tangent } \perp \text{ radius)} \]
\[ \Rightarrow \triangle ORT \text{ is a right angled triangle.} \]
By Pythagoras theorem,

\[ OT^2 = OR^2 + TR^2 \]
\[ TR^2 = OT^2 - OR^2 \]  \hspace{1cm} (2) \hspace{1cm} M1

Substitute (1) into (2)

\[ OT^2 - OR^2 = TP \times TQ \] (shown) \hspace{1cm} A1

8

(i) \hspace{1cm} \text{Period} = \frac{2\pi}{1} = 4\pi \hspace{1cm} B1

(ii) When \( t = 0, \) \( 10 = a \sin 0 + b \)

\[ \Rightarrow b = 10 \hspace{1cm} B1 \]

Max value = 14 when \( \sin \frac{1}{2}t = 1 \)

\[ \Rightarrow a + 10 = 14 \]

\[ a = 4 \hspace{1cm} B1 \]

(iii) \hspace{1cm} \[ 4 \sin \frac{1}{2}t + 10 = 7 \]

\[ \sin \frac{1}{2}t = -\frac{3}{4} \]

\[ \alpha = 0.8480 \text{ (accept 0.84806)} \]

\[ \frac{1}{2}t = \pi + 0.8480, 2\pi - 0.8480, \pi + 0.8480 + 2\pi, 2\pi - 0.8480 + 2\pi \] \hspace{1cm} M2

(M1 for each cycle)

\[ = 3.989, 5.435, 10.27, 11.71 \]

\( t = 7.978, 10.87, 20.54, 23.42 \)

\( \approx 7.98 \text{ h, 10.9 h, 20.5 h, 23.4 h} \) \hspace{1cm} A1

(iv) \hspace{1cm} \text{Length of time the gates are closed} = (10.87 - 7.978) + (23.42 - 20.54)

\[ = 5.772 \text{ h} \approx 5.77 \text{ h} \hspace{1cm} B1 \]
9. (i) \[
\frac{\sin \theta}{1 + \frac{1}{\sec \theta}} + \cot \theta = \csc \theta
\]
\[
= \frac{\sin \theta + \cos \theta}{1 + \cos \theta} = \frac{\sin \theta}{\sin \theta(1 + \cos \theta)}
\]
\[
= \frac{1 + \cos \theta}{\sin \theta(1 + \cos \theta)} \quad \text{(Applying the identity} \quad \sin^2 \theta + \cos^2 \theta = 1)\]
\[
= \frac{1}{\sin \theta} = \csc \theta
\]

(ii) \[
\frac{\sin 2\theta}{1 + \frac{1}{\sec 2\theta}} + \cot 2\theta = 6 \cos 2\theta
\]
\[
\cos \csc 2\theta = 6 \cos 2\theta
\]
\[
\frac{1}{\sin 2\theta} = 6 \cos 2\theta
\]
\[
6 \sin 2\theta \cos 2\theta = 1
\]
\[
3(2 \sin 2\theta \cos 2\theta) = 1
\]
\[
3 \sin 4\theta = 1 \quad \text{(applying double angle formula)}
\]
\[
\sin 4\theta = \frac{1}{3}
\]
\[
\alpha = 19.47^\circ
\]
\[
4 \theta = 19.47^\circ
\]
\[
\theta = 4.87^\circ \approx 4.9^\circ
\]
10 (i) \( y^2 = x + 1 \) \( \text{------------------------ (1)} \)
\( y = 2x - 4 \) \( \text{------------------------ (2)} \)

Subst (2) into (1)

\[
(2x - 4)^2 = x + 1
\]
\[
4x^2 - 16x + 16 - x - 1 = 0
\]
\[
4x^2 - 17x + 15 = 0
\]
\[
(4x - 5)(x - 3) = 0
\]

\[
x = \frac{5}{4} \text{ or } 3
\]

\[
y = -\frac{1}{2} \text{ or } 2
\]

\( A \left( \frac{1}{4}, -\frac{1}{2} \right), B \left( 3, 2 \right) \)

(ii) From (2), \( x = \frac{y + 4}{2} \)

\[
= \frac{y}{2} + 2
\]

\[
\text{Area} = \int_{\frac{1}{2}}^{3} \left[ \left( \frac{y}{2} + 2 \right) - (y^2 - 1) \right] dy
\]

\[= \int_{\frac{1}{2}}^{3} \left( \frac{y^2}{4} + 3y \right) dy\]  

\[= \left[ \frac{y^3}{12} + \frac{3y^2}{2} \right]_{\frac{1}{2}}^{3}\]

\[= \left( \frac{27}{12} + \frac{27}{2} \right) - \left( \frac{1}{12} + \frac{3}{2} \right)\]

\[= \frac{7}{48} \text{ units}^2 \text{ (Accept 7.15 units}^2) \]
Alternative Method

\[ \text{Area} = \int_0^3 (x+1)^{\frac{1}{2}} \, dx - \frac{1}{2} \times 1 \times 2 + \int_1^3 (x+1)^{\frac{1}{2}} \, dx + \frac{1}{2} \times \frac{3}{4} \times \frac{3}{2} \]

\[ = \left[ \frac{2}{3} (x+1)^{\frac{3}{2}} \right]_0^3 - 1 + \left[ \frac{2}{3} (x+1)^{\frac{3}{2}} \right]_1^3 + \frac{9}{16} \]

\[ = \frac{16}{3} - 1 + \frac{9}{4} + \frac{9}{16} \]

\[ = 7 \frac{7}{48} \text{ units}^2 \]

Accept other logical methods

11 (i) \( v = 2 + 5t - 3t^2 \)

At instantaneously at rest \( \Rightarrow v = 0 \)

\( 2 + 5t - 3t^2 = 0 \)

\( 3t^2 - 5t + 2 = 0 \)

\( (3t+1)(t-2) = 0 \)

\( t = -\frac{1}{3} \) (rej) or \( t = 2 \)

\( \text{acceleration} = \frac{dv}{dt} = 5 - 6t \)

At \( t = 2 \), acceleration = \( 5 - 6(2) = -7 \text{ m/s}^2 \)

(ii) \( s = \int (2 + 5t - 3t^2) \, dt \)

\[ = 2t + \frac{5t^2}{2} - \frac{3t^3}{3} + c \]

when \( t = 0 \) and \( s = 0 \), \( c = 0 \)

\( s = 2t + \frac{5t^2}{2} - t^3 \)

At \( t = 2 \), \( s = \frac{5(2)^2}{2} - (2)^3 + 2(2) = 6 \text{ m} \)
\[ \int_0^t \left( 2 + 5t - 3t^2 \right) dt \]
\[ = \left[ 2t + \frac{5t^2}{2} - \frac{3t^3}{3} \right]_0 \quad \text{(M1 for integration, M1 for the limits)} \]
\[ = 6 \text{ m} \quad \text{A1} \]

(iii) At \( t = 3 \), \( s = \frac{s(3)^2}{2} - (3)^3 + 2(3) \)
\[ = \frac{1}{2} \text{ m} \quad \text{M1} \]

Total distance travelled = \( 6 + 6 - \frac{1}{2} \)
\[ = 10 \frac{1}{2} \text{ m} \quad \text{A1} \]

\[ \int_0^t \left| \left( 2 + 5t - 3t^2 \right) dt \right| \]
\[ = \left[ 2t + \frac{5t^2}{2} - \frac{3t^3}{3} \right]_0 \quad \text{M1} \]
\[ = 4 \frac{1}{2} \text{ m} \quad \text{M1} \]

Total distance travelled = \( 6 + 4 \frac{1}{2} = 10 \frac{1}{2} \text{ m} \quad \text{A1} \]

12. (i) \( \cos \theta = \frac{2}{AX} \)
\[ AX = \frac{2}{\cos \theta} \]
\[ = 2 \sec \theta \text{ km} \quad \text{M1} \]

Time taken for \( AX = \frac{2 \sec \theta}{3} \text{ h} \)
\[ \tan \theta = \frac{PX}{2} \]
\[ PX = 2 \tan \theta \text{ km} \quad \text{M1} \]
\[ XQ = 10 - 2 \tan \theta \quad \text{M1} \]
Time taken for \( \chi = \frac{10 - 2 \tan \theta}{5} \) h

\[ T = \frac{2 \sec \theta}{3} + \frac{10 - 2 \tan \theta}{5} \]

\[ = \frac{2 \sec \theta}{3} + 2 \frac{2 \tan \theta}{5} \] (shown) \hspace{2cm} \text{A1}

(ii) \[ T = \frac{2 \sec \theta}{3} + 2 \frac{2 \tan \theta}{5} \]

\[ = \frac{2}{3 \cos \theta} + 2 \frac{2 \tan \theta}{5} \]

\[ \frac{dT}{d\theta} = \frac{0(\cos \theta) - 2(-3 \sin \theta)}{5 \cos^2 \theta} - \frac{2}{5 \sec^2 \theta} \]

\[ = \frac{2 \sin \theta}{3 \cos^2 \theta} - \frac{2}{5 \sec^2 \theta} \] \hspace{2cm} \text{M1} \hspace{2cm} \text{M1}

For stationary value of \( T \), \( \frac{dT}{d\theta} = 0 \)

\[ \frac{2 \sin \theta}{3 \cos^2 \theta} - \frac{2}{5 \sec^2 \theta} = 0 \] \hspace{2cm} \text{M1}

\[ \frac{2 \sin \theta}{3 \cos^2 \theta} - \frac{2}{5 \cos^2 \theta} = 0 \]

\[ 10 \sin \theta - 6 \]

\[ 5 \cos^2 \theta = 0 \]

\[ \Rightarrow 10 \sin \theta - 6 = 0 \]

\[ \sin \theta = \frac{3}{5} \]

\[ \theta = 0.6435 \]

\[ PX = 2 \tan 0.6435 \]

\[ = 1.5 \text{ m} \] (shown) \hspace{2cm} \text{A1}

[OR \( PX = 1.5 \)]

\[ 2 \tan \theta = 1.5 \] \hspace{2cm} \text{M1}

\[ \tan \theta = 0.75 \]

\[ \theta = 0.6435 \] \hspace{2cm} \text{M1}

When \( \theta = 0.6435 \), \( \frac{dT}{d\theta} = \frac{2 \sin 0.6435}{3 \cos^2 0.6435} - \frac{2}{5 \cos^2 0.6435} \)

\[ = 0 \] (shown) \hspace{2cm} \text{A1]}

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PRELIMINARY EXAMINATION 2016
SECONDARY 4 EXPRESS
ADDITIONAL MATHEMATICS PAPER 2

Date: 17 Aug 2016
Duration: 2 h 30 min
Time: 1100 – 1330

Additional Materials: 8 sheets of writing paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction tape/liquid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case
of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of 7 printed pages (including cover page).
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion

\[ (a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n, \]

where \( n \) is a positive integer and

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}
\]

2. TRIGONOMETRY

Identities

\[
\sin^2 A + \cos^2 A = 1
\]

\[
\sec^2 A = 1 + \tan^2 A
\]

\[
\cos co^2 A = 1 + \cot^2 A
\]

\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
\sin 2A = 2 \sin A \cos A
\]

\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]

\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \triangle ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2} ab \sin C
\]
Answer all the questions.

1. Given that, for all values of $x$, $x^5 - 2x^3 + 2x^2 + 4x - 3 = Ax + B + (x^2 - 1)Q(x)$, where $Q(x)$ is a polynomial,
   
   (i) state the degree of the polynomial, $Q(x)$, [1]
   
   (ii) find the remainder of $x^5 - 2x^3 + 2x^2 + 4x - 3$, when divided by $x^2 - 1$, in terms of $x$. [5]

2. The diagram shows part of a straight line graph drawn to represent the equation $y = \frac{ax^2 + b}{cx}$, where $a$, $b$ and $c$ are integers. Given that the line passes through (4, 9) and has gradient $-\frac{1}{4}$, find
   
   (i) the value of $\frac{y}{x}$ where the straight line cuts the horizontal axis, [3]
   
   (ii) the value of $a$, of $b$ and of $c$. [3]

3. In the expansion $\left(2x^2 + \frac{3}{x}\right)^n$, in descending powers of $x$, the ratio of the coefficients of the third and first term is 81 : 1.
   
   (i) Find the value of $n$. [3]
   
   (ii) Write down the first three terms of the expansion. [2]
   
   (iii) Find the term that is independent of $x$. [2]
4  (i) Express \( \frac{11-7x}{3x^2+11x-4} \) in partial fractions. \[3\]

(ii) Hence evaluate \( \int_2^4 \frac{11-7x}{9x^2+33x-12} \, dx \). \[4\]

5  (i) Solve \( 2x^3 + x^2 - 5x + 2 = 0 \). \[4\]

(ii) Hence solve \( 16 \tan^2 \theta + 4 \tan^2 \theta - 10 \tan \theta + 2 = 0 \), where \( 0^\circ \leq \theta \leq 90^\circ \). \[4\]

6  A curve is such that \( \frac{dy}{dx} = \frac{e^{2x} + 1}{e^{3x}} \) and \( (0, \frac{1}{2}) \) is a point on the curve.

(i) Explain why the curve has no stationary points. \[2\]

(ii) Find the value of \( y \) when \( x = 2 \). \[6\]

7  The equation of a curve is \( y = \frac{(x-3)^2}{2x+5} \).

(i) Find an expression for \( \frac{dy}{dx} \) and obtain the coordinates of the stationary points. \[5\]

(ii) Find an expression for \( \frac{d^2y}{dx^2} \) and hence determine the nature of these stationary points. \[4\]
Diagram 1 shows the front view of a pendant which can be modelled as a regular trapezium. Diagram 2 shows the back view of the modelled pendant with the gold frame that is used to hold the pendant. Trapezium $ABCF$, line $AE$ and $BD$ form the structure of the gold frame.

$AB = DE = 1 \text{ cm}, AF = BC = 2.8 \text{ cm}$ and $\angle AFE = \angle BCD = \theta$.

(i) Show that the total length of the structure that form the gold frame, $P$, is $(5.6\sin \theta + 5.6\cos \theta + 7.6) \text{ cm}$. [2]

(ii) Express $P$ in the form $R \sin(\theta + \alpha) + 7.6$, where $R > 0$ and $\alpha$ is an acute angle. [4]

(iii) Given that the perimeter of the gold frame is 15 cm, find the values of $\theta$. [3]
Do not use a calculator in this question.

(i) Express \( \frac{7\sqrt{2}}{3\sqrt{2} - 2} \) in the form \( a + b\sqrt{2} \), where \( a \) and \( b \) are integers. \([2]\)

The diagram shows a right pyramid with a square base of side \( \frac{7\sqrt{2}}{3\sqrt{2} - 2} \) cm.

Given that the height, \( VM \), of the pyramid is \( \frac{1}{2} BD^2 \), find

(ii) an expression for \( BD^2 \) in the form \( c + d\sqrt{2} \), where \( c \) and \( d \) are integers. \([3]\)

(iii) the volume of the pyramid in the form \( p + q\sqrt{2} \), where \( p \) and \( q \) are rational numbers. \([4]\)

10  (a) A circle, whose equation is \( x^2 + y^2 - 10x + 8y + 5 = 0 \), has centre \( C \).

(i) Find the centre of the circle, \( C \). \([1]\)

(ii) Explain why point \( P (4, -11) \) lies outside of the circle. \([3]\)

(iii) A line drawn through \( P \) is tangent to the circle at point \( T \). Find the length of \( PT \). \([2]\)

(b) The equation of a curve is \( y = x^2 - 7x + 10 \). Point \( A \) is a point on the curve and it lies on the y-axis. Find the equation of the normal at point \( A \). \([4]\)
11. (a) Given that \( y = \tan x \), show that \( \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} y = 0 \). \[4\]

(b) (i) Find \( \int_0^\pi 8 \cos^2 \left( \frac{x}{2} \right) dx \). \[3\]

(ii) Hence find \( \int_0^\pi \left[ 3 - \sin^2 \left( \frac{x}{2} \right) \right] dx \). \[3\]

12. The roots of the quadratic equation \( 3x^2 - 7x + 4 = 0 \) are \( 2\alpha + \beta \) and \( \alpha + 2\beta \).

(i) Find the value of \( \alpha + \beta \). \[3\]

(ii) Show that the value of \( \alpha \beta = \frac{10}{81} \). \[3\]

(iii) Find a quadratic equation whose roots are \( \frac{1}{2} \alpha + \beta \) and \( \alpha + \frac{1}{2} \beta \). \[5\]

---

End of paper ---
1. (i) 3  
   (ii) $3x - 1$
2. (i) $y = \frac{1}{x}$  
   (ii) $a = 1, b = 4$ and $c = 40$
3. (i) $n = -8$ (rejected) or $n = 9$
   (ii) $512x^{18} + 6912x^{15} + 41472x^{12} + ...$
   (iii) 489888
4. (i) $\frac{11 - 7x}{3x^2 + 11x - 4} = \frac{2}{3x - 1} - \frac{3}{x + 4}$
   (ii) 0.0213
5. (i) $x = 1, x = \frac{1}{2}, x = -2$
   (ii) $\theta \approx 14.0^\circ, 26.6^\circ$
6. (i) For all real values of $x,$ $e^{3x} > 0$ and $e^{-3x} > 0,$
   $\therefore \frac{dy}{dx} > 0, \frac{dy}{dx}$ can never be zero.
   $\therefore$ the curve has no stationary point.
   (ii) $y \approx 27.6$
7. (i) $(3,0)$ and $(-8,-11)$
   (ii) $(3,0)$ is a min. pt.
   $(-8,0)$ is a max. pt.
8. (ii) $P = 7.92\sin(\theta + 45^\circ) + 7.6$
   (iii) $\theta \approx 24.1^\circ, 65.9^\circ$
9. (i) $3 + \sqrt{2}$
   (ii) $BD^2 = 22 + 12\sqrt{2}$
   (iii) $\frac{193}{3} + 44\sqrt{2}$ cm$^3$
10. (a) (i) $(5,-4)$
    (ii) $\text{radius } = 6$
    $\text{Length of } PC = \sqrt{50}$
    $\approx 7.07$
    Since length of $PC$ is longer than radius of circle, thus, the point $P$ is outside of the circle.
    (iii) $3.74$ units
11. (a) $y = \frac{1}{7}x + 10$
    (b) $\frac{dy}{dx} = \sec^2 x$
    $\frac{d^2y}{dx^2} = 2\sec x \cdot \sec x \cdot \tan x$
12. (i) $\alpha + \beta = \frac{7}{9}$
    (ii) $\frac{4\pi}{5\pi - 2}$
    (iii) $x^2 - \frac{7}{6}x + \frac{1}{3} = 0$
HIHS 2016 Prelim 4 Express
Additional Mathematics Paper 2 Marking Scheme

<table>
<thead>
<tr>
<th>Qn</th>
<th>Workings</th>
<th>Marks allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(i) degree of polynomial, ( Q(x) = 3 )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( x^5 - 2x^3 + 2x^2 + 4x - 3 = Ax + B + (x^2 - 1)Q(x) )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>subst. ( x = 1 ), ( 1 - 2 + 2 + 4 - 3 = A + B + 0 ) ( A + B = 2 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>subst. ( x = -1 ), ( -1 + 2 + 2 - 4 - 3 = -A + B ) ( -A + B = -4 )</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>(1) + (2), ( 2B = -2 ) ( B = -1 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>subst. ( B = -1 ) into (1), ( A - 1 = 2 ) ( A = 3 )</td>
<td>A1 each for correct ( A ) and ( B ) value</td>
</tr>
<tr>
<td></td>
<td>The remainder is ( 3x - 1 ).</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Alternate Method: long division ( x^5 - 2x^3 + 2x^2 + 4x - 3 = 3x - 1 + (x^2 - 1)(x^3 - x + 2) )</td>
<td>2 m for remainder</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 m for quotient</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1 m for each term)</td>
</tr>
<tr>
<td>2</td>
<td>(i) ( \frac{1}{xy} = \frac{1}{4} \left( \frac{x}{y} \right) + C )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>subst. ( (4, 9) ), ( 9 = \frac{1}{4} (4) + C ) ( C = 10 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Graph cuts at horizontal axis ( \frac{1}{xy} = 0 ) ( 0 = -\frac{1}{4} \left( \frac{x}{y} \right) + 10 ) ( \frac{y}{x} = \frac{1}{40} )</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(ii) ( \frac{1}{xy} = -\frac{1}{4} \left( \frac{x}{y} \right) + 10 ) ( 1 = -\frac{1}{4} \left( x^2 \right) + 10xy )</td>
<td></td>
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</table>
### HIHS 2016 Prelim 4 Express
**Additional Mathematics Paper 2 Marking Scheme**

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</table>
| 3  | **(i) First term = 2^n**<br>Coeff. of third term = \( \binom{n-1}{2} (2x^3)^{n-2} \left( \frac{3}{x} \right)^2 \)<br>\[ \frac{n(n-1)}{2} \left( \frac{3}{x} \right)^2 \]<br>Thus, \[ \frac{n(n-1)}{2} \left( \frac{3}{x} \right)^2 = 81 \]<br>\[ n(n-1) = \frac{81}{\left( \frac{3}{x} \right)^2} \]<br>\[ n^2 - n - 72 = 0 \]<br>\[ (n+8)(n-9) = 0 \]<br>\[ n = -8 \text{ (rejected)} \quad n = 9 \]<br>**(ii)** \[ \left( 2x^2 + \frac{3}{x} \right)^9 \]<br>\[ = 512x^{18} + \binom{9}{1} (2x^2)^8 \left( \frac{3}{x} \right) + \binom{9}{2} (2x^2)^7 \left( \frac{3}{x} \right)^2 + ... \]<br>\[ = 512x^{18} + 6912x^{15} + 41472x^{12} + ... \]<br>**(iii)** \[ T_{r+1} = \binom{9}{r} (2x^2)^{9-r} \left( \frac{3}{x} \right)^r \]<br>\[ 2(9-r) - r = 0 \]<br>\[ r = 6 \]<br>Term independent of \( x \) = \( \binom{9}{6} (2)^3 (3)^6 \)<br>\[ = 489888 \]<br>**4**  

<table>
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<tr>
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</table>
| 4  | **(i)** \[ \frac{11-7x}{3x^2+11x-4} = \frac{11-7x}{(3x-1)(x+4)} \]<br>** M1, o.e.**

A1, must reject negative value

A1

M1, o.e.
(c.g. expansion)

---

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Additional Mathematics Paper 2 Marking Scheme  

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</table>
|       | \[
\frac{11 - 7x}{(3x - 1)(x + 4)} = \frac{A}{3x - 1} + \frac{B}{x + 4} \]
|       | \[11 - 7x = A(x + 4) + B(3x - 1)\] | M1 |
|       | subst \(x = -4\), \(11 + 28 = B(-13)\) \[B = -3\] | A1 |
|       | \(x = 0\), \(11 = 4A + 3\) \[4A = 8\] \[A = 2\] | A1 |
|       | Therefore, \[
\frac{11 - 7x}{3x^2 + 11x - 4} = \frac{2}{3x - 1} - \frac{3}{x + 4}
\] |         |
| (ii)  | \[
\int_{1}^{2} \frac{11 - 7x}{9x^2 + 33x - 12} \, dx
\]
|       | \[= \int_{1}^{2} \frac{11 - 7x}{3(3x^2 + 11x - 4)} \, dx\] | M1, o.e. |
|       | \[= \int_{1}^{2} \frac{11 - 7x}{3(3x - 1)(x + 4)} \, dx\] | M1, integrating \(\ln\) |
|       | \[= \frac{1}{3} \left[ \ln(3x - 1) - 3\ln(x + 4) \right]_{1}^{2}\] | [M1, subst] |
|       | \[\approx 0.0213\] | A1 |

5 (i) let \(f(x) = 2x^3 + x^2 - 5x + 2\)

\(f(1) = 0\)

therefore, \(x - 1\) is a factor of \(f(x)\)

\[2x^3 + x^2 - 5x + 2 = (x - 1)(2x^2 + ax - 2)\]

comparing coefficient of \(x\), \(-5 = -a - 2\)

\[a = 3\]

therefore, \(f(x) = (x - 1)(2x^2 + 3x - 2)\)

\[M1, o.e.\]
<table>
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<tbody>
<tr>
<td></td>
<td>[(x - 1)(2x - 1)(x + 2)] [2x^3 + x^2 - 5x + 2 = 0] [(x - 1)(2x - 1)(x + 2) = 0] [x - 1 = 0] or [2x - 1 = 0] or [x + 2 = 0] [x = 1] [x = \frac{1}{2}] [x = -2]</td>
<td>A2, minus 1 m for 1 error</td>
</tr>
<tr>
<td>(ii)</td>
<td>[16 \tan^3 \theta + 4 \tan^2 \theta - 10 \tan \theta + 2 = 0] [2(2 \tan \theta)^3 + (2 \tan \theta)^2 - 5(2 \tan \theta) + 2 = 0] By comparing, [x = 2 \tan \theta] [(2 \tan \theta - 1)(4 \tan \theta - 1)(2 \tan \theta + 2) = 0] [2 \tan \theta - 1 = 0] or [4 \tan \theta - 1 = 0] or [2 \tan \theta + 2 = 0] [\tan \theta = \frac{1}{2}] or [\tan \theta = \frac{1}{4}] or [\tan \theta = -1] (rejected) [\theta \approx 26.6^\circ] [\theta \approx 14.0^\circ]</td>
<td>M1, or identify [x = 2 \tan \theta] M1 (factorised) A2, minus 1 m if [\tan \theta = -1] not rejected</td>
</tr>
</tbody>
</table>
| 6 | (i) \[
\begin{align*}
\frac{dy}{dx} &= e^{3x} + 1 \\
\frac{dy}{dx} &= e^{2x} + e^{-3x}
\end{align*}
\] | M1, o.e. |
|    | when \[\frac{dy}{dx} = 0\], \[e^{2x} + e^{-3x} = 0\] \[e^{2x} = -e^{-3x}\] \[e^{2x} = e^{-3x}\] \[e^{5x} = -1\] \[x\] is undefined, thus the curve does not have stationary points. | A1, conclusion |
|    | OR \[e^{5x} = -1\] (rejected) \[\text{Since } e^{5x} > 0 \text{ for all values of } x, \text{ hence the curve does not have stationary points} \] | |
|    | OR \[\text{For all real values of } x, \ e^{2x} > 0 \text{ and } e^{-3x} > 0,\] \[\therefore \frac{dy}{dx} > 0, \frac{dy}{dx} \text{ can never be zero.} \] \[\therefore \text{ the curve has no stationary point.} \] | M1 A1 |

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</table>
| (ii) | \[
\frac{dy}{dx} = e^{2x} + e^{-3x} \\
y = \frac{e^{2x}}{2} - \frac{e^{-3x}}{3} + c \\
\text{subst. } (0, \frac{1}{2}), \\
\frac{1}{2} = \frac{e^{2(0)}}{2} - \frac{e^{-3(0)}}{3} + c \\
\frac{1}{2} = \frac{1}{2} - \frac{1}{3} + c \\
c = \frac{1}{3} \\
\text{Eqn of curve is } y = \frac{e^{2x}}{2} - \frac{e^{-3x}}{3} + \frac{1}{3} \\
\text{when } x = 2, y = \frac{e^{2(2)}}{2} - \frac{e^{-3(2)}}{3} + \frac{1}{3} \\
y = \frac{e^4}{2} - \frac{1}{3e^6} + \frac{1}{3} \\
y \approx 27.6
| M2, integrate exponential |
| | | M1 |
| | | M1 |
| | | M1, subst into eqn of curve |
| 7 | \[
y = \frac{(x-3)^2}{2x+5} \\
\frac{dy}{dx} = \frac{(2x+5)(2)(x-3) - (x-3)^2}{(2x+5)^2} (2) \\
= \frac{(2x+5)(2x-6) - (2x^2 - 6x + 9)}{(2x+5)^2} (2) \\
= \frac{4x^2 - 12x + 10x - 30 - 2x^2 + 12x - 18}{(2x+5)^2} \\
= \frac{2x^2 + 10x - 48}{(2x+5)^2}
| M2 |
| | | M1, o.e. |
| | | A1 |

For stationary points, \[
\frac{dy}{dx} = 0 \\
\frac{(2x+5)(2)(x-3) - (x-3)^2}{(2x+5)^2} (2) = 0 \\
(2x+5)(2)(x-3) - (x-3)^2 = 0 \\
(x-3)(4x+10 - 2x+6) = 0 \\
(x-3)(2x+16) = 0 \\
x-3 = 0 \quad \text{or} \quad 2x+16 = 0 \\
x = 3 \quad \text{or} \quad x = -8
| A1, for x coordinates |
### HIHS 2016 Prelim 4 Express
#### Additional Mathematics Paper 2 Marking Scheme

<table>
<thead>
<tr>
<th>Qn</th>
<th>Workings</th>
<th>Marks allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>subst. ( x = 3 ), into ( y = \frac{(x-3)^2}{2x+5} ), ( y = 0 )</td>
<td></td>
<td>A1, for ( y ) coordinates [minus 1 m if not written in coordinates form]</td>
</tr>
<tr>
<td>subst. ( x = -8 ), into ( y = \frac{(x-3)^2}{2x+5} ), ( y = \frac{(-8-3)^2}{2(-8)+5} ), ( y = -11 )</td>
<td>The stationary points are ((3,0)) and ((-8,-11)).</td>
<td></td>
</tr>
<tr>
<td>( \frac{dy}{dx} = \frac{2x^2 + 10x - 48}{(2x+5)^2} )</td>
<td>(ii)</td>
<td></td>
</tr>
<tr>
<td>( \frac{d^2y}{dx^2} = \frac{(2x+5)^2(4x+10) - (2x^2 + 10x - 48)(2)(2x+5)(2)}{(2x+5)^4} )</td>
<td>M2</td>
<td></td>
</tr>
<tr>
<td>when ( x = 3 ), ( \frac{d^2y}{dx^2} = \frac{2662 - 0}{14641} = \frac{2}{11} &gt; 0 ), ((3,0)) is a min. pt.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>when ( x = -8 ), ( \frac{d^2y}{dx^2} = \frac{-2662 - 0}{14641} = -\frac{2}{11} &lt; 0 ), ((-8,0)) is a max. pt.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>8</strong></td>
<td><strong>(i) Perimeter of pendent</strong></td>
<td></td>
</tr>
<tr>
<td>( = 1 + 1 + 2 \times 2.8 + 2 \times 2.8 \sin \theta + 2 \times 2.8 \cos \theta )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>( = (5.6 \sin \theta + 5.6 \cos \theta + 7.6) \text{ cm} ) (Shown)</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>( (ii) )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>( R = \sqrt{5.6^2 + 5.6^2} )</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>( \approx 7.92 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tan \alpha = \frac{5.6}{5.6} )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>( \alpha = 45^\circ )</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>( P = 7.92 \sin(\theta + 45^\circ) + 7.6 )</td>
<td>(minus 1 m if student did not express in this form)</td>
<td></td>
</tr>
<tr>
<td><strong>(iii)</strong></td>
<td><strong>M1</strong></td>
<td></td>
</tr>
<tr>
<td>( 15 = 7.92 \sin(\theta + 45^\circ) + 7.6 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 7.4 = 7.92 \sin(\theta + 45^\circ) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sin(\theta + 45^\circ) = \frac{185}{198} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic angle ( = 69.1223^\circ )</td>
<td>M1 (basic angle)</td>
<td></td>
</tr>
<tr>
<td>( \theta + 45^\circ = 69.1223^\circ, 180^\circ - 69.1223^\circ )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta = 24.1223^\circ, 65.877^\circ )</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>( \theta \approx 24.1^\circ, \ 65.9^\circ )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Qn</td>
<td>Workings</td>
<td>Marks allocation</td>
</tr>
<tr>
<td>----</td>
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<td>------------------</td>
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</tbody>
</table>
| 9  | (i) \[
\frac{7\sqrt{2}}{3\sqrt{2} - 2} \times \frac{3\sqrt{2} + 2}{3\sqrt{2} + 2} = \frac{7\sqrt{2}(3\sqrt{2} + 2)}{18 - 4} = \frac{42 + 14\sqrt{2}}{14} = 3 + \sqrt{2}
\]

(ii) by Pythagoras Theorem,
\[BD^2 = AB^2 + AD^2\]
Using part (i) answer,
\[BD^2 = (3 + \sqrt{2})^2 + (3 + \sqrt{2})^2\]
\[BD^2 = 2(3 + \sqrt{2})^2\]
\[BD^2 = 2(9 + 6\sqrt{2} + 2)\]
\[BD^2 = 22 + 12\sqrt{2}\]

(iii) Volume of pyramid
\[= \frac{1}{3} \times \text{base area} \times \text{height}\]
\[= \frac{1}{3} \times (3 + \sqrt{2})^2 \times \frac{1}{2} (22 + 12\sqrt{2})\]
\[= \frac{1}{3} \times (11 + 6\sqrt{2}) \times (11 + 6\sqrt{2})\]
\[= \frac{1}{3} (121 + 132\sqrt{2} + 72)\]
\[= \frac{1}{3} (193 + 132\sqrt{2})\]
\[= \frac{193}{3} + 44\sqrt{2} \text{ cm}^3\]

10  | (a) (i) centre C = \[\left(\frac{-10}{-2}, \frac{8}{-2}\right)\]
    | = (5, -4) B1 |
    | (ii) radius = \[\sqrt{5^2 + 4^2 - 5}\]
    | = 6 M1 (o.e.) |
    | Length of PC = \[\sqrt{(5 - 4)^2 + (-4 + 11)^2}\]
    | = \[\sqrt{50}\]
    | \approx 7.07 M1 |
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Additional Mathematics Paper 2 Marking Scheme

<table>
<thead>
<tr>
<th>Qn</th>
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<th>Marks allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Since <strong>length of PC is longer than radius</strong> of circle, thus, the point <strong>P</strong> is outside of the circle.</td>
<td>A1 (find length PC and conclude)</td>
</tr>
</tbody>
</table>
| (iii) | by Pythagoras’ Theorem,  
\[ PT = \sqrt{50 - 6^2} \]  
\[ = \sqrt{14} \]  
\[ \approx 3.74 \text{ units} \] | M1  
A1 |
| (b) | point \( A = (0, 10) \)  
\[ \frac{dy}{dx} = 2x - 7 \] | M1  
M1 |
|    | when \( x = 0 \), \[ \frac{dy}{dx} = -7 \] | M1 |
|    | gradient of normal = \[ \frac{1}{7} \] | M1 |
|    | equation of normal is \[ y = \frac{1}{7}x + 10 \] | A1 |
| 11 (a) | \( y = \tan x \)  
\[ \frac{dy}{dx} = \sec^2 x \]  
\[ \frac{d^2y}{dx^2} = 2\sec x \cdot \sec x \cdot \tan x \]  
\[ = 2 \frac{dy}{dx} \]  
\[ \frac{d^2y}{dx^2} = 2 \frac{dy}{dx} \]  
\[ \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} = 0 \] (shown) | M1  
M2, 1m for \( 2\sec x \cdot \sec x \cdot \tan x \) (o.e.)  
A1 |
|    | **Alternate solution**  
\[ \frac{dy}{dx} = \sec^2 x \]  
\[ = \frac{1}{\cos^2 x} \]  
\[ \frac{d^2y}{dx^2} = \frac{0 - 2\cos x(-\sin x)}{\cos^4 x} \]  
\[ = 2\sec x \cdot \sec x \cdot \tan x \]  
\[ \text{LHS} = 2\sec x \cdot \sec x \cdot \tan x - 2\sec^2 x \tan x \]  
\[ = 0 \]  
\[ = \text{RHS} \] |
(b) (i) \[\int_0^\pi 8 \cos^2\left(\frac{x}{2}\right) \, dx = 4 \int_0^\pi 2 \cos^2\left(\frac{x}{2}\right) \, dx\]
\[= 4 \int_0^\pi (\cos x + 1) \, dx\]
\[= 4 \left[\sin x + x\right]_0^\pi\]
\[= 4 \left[0 + \pi - (0 - 0)\right]\]
\[= 4\pi\]

(ii) \[\int_0^\pi 3 - \sin^2\left(\frac{x}{2}\right) \, dx = \int_0^\pi 2 + 1 - \sin^2\left(\frac{x}{2}\right) \, dx\]
\[= \int_0^\pi 2 + \cos^2\left(\frac{x}{2}\right) \, dx\]
\[= \int_0^\pi 2 \, dx + \int_0^\pi \cos^2\left(\frac{x}{2}\right) \, dx\]
\[= \left[2x\right]_0^\pi + \frac{4\pi}{8}\]
\[= \frac{5\pi}{2}\]

12 (i) sum of roots, \(2\alpha + \beta + \alpha + 2\beta = 3\alpha + 3\beta\)
\[= 3(\alpha + \beta)\]
\[= \frac{7}{3}\]
\[= \frac{7}{3}\]

product of roots, \((2\alpha + \beta)(\alpha + 2\beta) = 2\alpha^2 + 4\alpha\beta + \alpha\beta + 2\beta^2\)
\[= 2\alpha^2 + 5\alpha\beta + 2\beta^2\]
\[= \frac{4}{3}\]

\[\alpha + \beta = \frac{7}{3}\]
\[= \frac{7}{9}\]

(ii) from product of roots, \(2\alpha^2 + 4\alpha\beta + \alpha\beta + 2\beta^2 = \frac{4}{3}\)
\[2\alpha^2 + 4\alpha\beta + 2\beta^2 + \alpha\beta = \frac{4}{3}\]
\[2(\alpha^2 + 2\alpha\beta + \beta^2) + \alpha\beta = \frac{4}{3}\]
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<table>
<thead>
<tr>
<th>Question</th>
<th>Workings</th>
<th>Marks Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. (a)</td>
<td>(2(\alpha + \beta)^2 + \alpha \beta = \frac{4}{3})</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(2 \left( \frac{7}{9} \right)^2 + \alpha \beta = \frac{4}{3})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\alpha \beta = \frac{4}{3} - 2 \left( \frac{7}{9} \right)^2)</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(\alpha \beta = \frac{10}{81}) (shown)</td>
<td></td>
</tr>
<tr>
<td>(iii)</td>
<td>sum of roots, (\frac{1}{2} \alpha + \beta + \frac{1}{2} \beta = \frac{3}{2}(\alpha + \beta))</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(= \frac{3}{2} \left( \frac{7}{9} \right))</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(= \frac{7}{6})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Product of roots, (\left( \frac{1}{2} \alpha + \beta \right) \left( \alpha + \frac{1}{2} \beta \right) = \frac{1}{2} \alpha^2 + \frac{1}{4} \alpha \beta + \frac{1}{2} \beta^2)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(= \frac{1}{2} \alpha^2 + \frac{5}{4} \alpha \beta + \frac{1}{2} \beta^2)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(= \frac{1}{2}(\alpha^2 + \beta^2) + \frac{5}{4} \alpha \beta)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(= \frac{1}{2}(\alpha + \beta)^2 - 2 \alpha \beta + \frac{5}{4} \alpha \beta)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(= \frac{1}{2} \left( \frac{7}{9} \right)^2 - 2 \left( \frac{10}{81} \right) + \frac{5}{4} \left( \frac{10}{81} \right))</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(= \frac{1}{3})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The quadratic equation is (x^2 - \frac{7}{6}x + \frac{1}{3} = 0)</td>
<td>A1, accept</td>
</tr>
<tr>
<td></td>
<td>(6x^2 - 7x + 2 = 0)</td>
<td></td>
</tr>
</tbody>
</table>
1. Express \( \frac{5x^2 + x + 6}{(3-2x)(x^2 + 4)} \) in partial fractions. \([5]\)

2. (i) Prove that \( \frac{1}{\tan \theta + \cot \theta} = \frac{\sin 2\theta}{2} \). \([4]\)

(ii) Hence, solve the equation \( \frac{1}{\tan \frac{\theta}{2} + \cot \frac{\theta}{2}} = \frac{1}{4} \) for \(-2\pi \leq \theta \leq 2\pi\). \([3]\)

3. The period \( T \), in years, of planets' orbit around the Sun is given by \( T = kr^n \), where \( r \) is the distance, in metres, of the planet from the Sun, and \( k \) and \( n \) are constants to be determined. The graph of \( \ln T \) against \( \ln r \) is given.

(i) Find the value of \( k \) and of \( n \). \([3]\)

(ii) Find the period of a planet which is \( 60 \times 10^9 \) metres from the Sun. \([2]\)

(iii) On the same axes, a straight line representing the equation \( \ln T = 1 \) was drawn. Explain the significance of the intersection of the two lines. \([1]\)
4

(i) Expand \( \left(x + \frac{1}{x}\right)^4 \) in descending powers of \( x \). [2]

(ii) Hence, given that \( \left(x + \frac{1}{x}\right)^4 - \left(x - \frac{1}{x}\right)^4 = \alpha x^2 + \frac{b}{x^2} \), find the value of \( \alpha \) and of \( b \). [3]

(iii) Given that there is no \( x \) term in the expansion of \( \left(\frac{4}{3} x + \frac{k}{x} + \frac{x^3}{k}\right) \left(x + \frac{1}{x}\right)^4 \), find the value of \( k \). [3]

5

It is given that \( f(x) \) is such that \( f'(x) = 4 \cos x + 8 \sin \frac{x}{2} + 3 \).

(i) Find \( f''(x) \). [2]

(ii) Given further that \( f(\pi) = 0 \), find \( f(x) \). [4]

6

The equation of a curve is \( y = ax^2 + bx - 3 \), where \( a \) and \( b \) are constants and the curve has a minimum turning point.

(i) Explain why the curve cuts the \( x \)-axis at two distinct points. [3]

(ii) In the case where \( a = 1 \), find the range of values of \( b \) for which the curve is above the line \( y = x - 4 \). [4]

(iii) Hence, state the values of \( b \) for which the line is a tangent to the curve. [1]

7

A graph has the equation \( y = -|3x - 9| + 6 \).

(i) Explain why the highest point on the graph has coordinates (3, 6). [2]

(ii) Find the coordinates at which the graph cuts the \( x \)-axis. [2]

(iii) Sketch the graph of \( y = -|3x - 9| + 6 \). [2]

(iv) Find the range of values of \( m \) such that \( -|3x - 9| + 6 = mx \) has 2 solutions. [2]
A cone is inscribed in a sphere of radius 20 cm, centre C. The cone has height, \(h\) cm and radius, \(x\) cm.

(i) Show that \(x = \sqrt{40h - h^2}\). \([1]\)

(ii) Hence, express the volume of the cone in terms of \(h\). \([1]\)

(iii) Given that \(h\) can vary, find the value of \(h\) for which the volume of the cone is stationary. \([3]\)

(iv) Determine whether this value of \(h\) gives the largest cone possible. \([1]\)

9 Given that \(\tan 2A = \frac{3}{4}\) and \(180^\circ < 2A < 270^\circ\), find, without using a calculator, the exact values of

(i) \(\sin 2A\). \([2]\)

(ii) \(\sin A\). \([3]\)
10 The line \( l, 2x + y = 10 \) cuts the curve \( xy = 12 \) at \( T(2, 6) \).

(i) Find the equation of the tangent to the curve at \( T \). [2]

(ii) Find the angle, in degrees, between \( l \) and the tangent to the curve at \( T \). [2]

(iii) State the gradient of the normal at \( T \). Hence, determine, with reason, whether the normal to the curve will get steeper or gentler as \( x \) increases. [2]

11 The diagram show a triangle \( POR \) in which \( P \) is the point \((6, \ -3)\). The line \( PR \) passes through the origin \( O \). The line \( OQ \) is perpendicular to \( PR \). The area of triangle \( POQ \) is 15 units\(^2\).

(i) Find the equation of \( OQ \). [2]

(ii) Find the coordinates of \( Q \). [3]

(iii) The length of \( PO \) is 3 times the length of \( OR \). Find the coordinates of \( R \). [1]

(iv) The point \( S \) is such that any point on the line \( PR \) is equidistant from \( Q \) and \( S \). Find the coordinates of \( S \). [1]
The height above ground level, $h$ m, of a car in a roller coaster is modelled by the equation, $h = 2x^2$, where $x$ is the horizontal distance of the car in metres from a fixed point $O$.

(i) Given that the horizontal distance of the car is increasing at a constant rate of 2 m/s, find the rate at which the height of the car is increasing when $x = 3$. [3]

(ii) The distance, $L$, of the car from $O$ is $OP$, where $P$ is a moving point on the curve. Show that $L = \sqrt{x^2 + 4x^4}$. [1]

(iii) It is possible to take a high definition photograph of the car from the fixed point $O$ if the distance, $L$, is changing at a rate of not more than 20 m/s. Would you be able to take a high definition photograph of the car from the fixed point $O$ when $x = 3$? [4]

End of Paper
Answers:

1. \[
\frac{5x^2 + x + 6}{(3-2x)(x^2 + 4)} = \frac{3}{3-2x} + \frac{-x-2}{x^2+4}
\]

2i. \[
\frac{\pi}{6}, \frac{5\pi}{6}, \frac{-11\pi}{6}, \frac{-7\pi}{6}
\]

3i. \[n = \frac{3}{2}, \quad k = e^{-3\phi} \text{ or } 3.14 \times 10^{-17}\]

3ii. 0.461

4i. \[x^4 + 4x^3 + 6 + 4x^2 + x^{-4}\]

4ii. \[a = 8, b = 8\]

4iii. -1

5i. \[-4\sin x + 4\cos\frac{x}{2}\]

5ii. \[f(x) = 4\sin x -16\cos\frac{x}{2} + 3x - 3\pi\]

6ii. \[-1 < b < 3\]

6iii. -1 or 3

7ii. (5,0) and (1,0)

7iv. \[-3 < m < 2\]

8ii. \[
\frac{1}{3}\pi(40h^2 - h^3)
\]

8iii. \[h = \frac{80}{3}\]

9i. \[-\frac{3}{5}\]

9ii. \[\frac{3}{\sqrt{10}}\]

10i. \[y = -3x + 12\]

10ii. 8.14°

11i. \[y = 2x, \quad 11\text{ii. (2, 4),} \quad 11\text{iii. (-2, 1),} \quad 11\text{iv. (-2, -4)}\]

12i. 24 m/s

12iii. \[
\frac{dL}{dt} = 24.0 m/s > 20
\]

No
Answer all questions.

1 (i) Sketch the graph \( y = 2x^2 \). [2]

(ii) Find the equation of the graph that has to be drawn in part (i) in order to obtain the graphical solution of \( 2x^6 = 1 \). On the same axes, sketch this graph for \( x > 0 \). [3]

2 (a) The cubic polynomial \( f(x) \) is such that the coefficient of \( x^3 \) is 2 and the roots of the equation \( f(x) = 0 \) are \( 2, -\frac{1}{2} \) and \( k \). Given that \( f(x) \) has a remainder of \(-6\) when divided by \( x-1 \). Find the value of \( k \). [3]

(b) Given that the quadratic curve \( y = 2x^2 + x - \frac{1}{2} \) cuts the \( x \)-axis at \( x_1 \) and \( x_2 \) as shown in the diagram below. Find the exact value of \( \frac{x_1}{x_2} \), leaving your answer in the simplest surd form. [4]

![Diagram of quadratic curve]

3 The mass, \( M \) grammes, of a substance, present at the time \( t \) minutes after first being measured, is given by \( M = 10 + 90e^{-0.2t} \). Find

(i) the initial mass of the substance, [1]

(ii) the time taken for the initial mass of the substance to be reduced by 20%, [3]

(iii) the approximate mass of the substance when \( t \) becomes very large, [1]

(iv) the rate at which the mass is decreasing when \( t = 3 \) minutes. [3]

Sketch the curve \( M = 10 + 90e^{-0.2t} \) [2]
4  (a) Solve the following equations.
(i) \[ 3^{\log_3 x} = 729, \] \[ \text{[3]} \]
(ii) \[ \log_2 (x - 2) + 2 \log_4 (x + 1) = \frac{1}{\log_3 2}. \] \[ \text{[4]} \]
(b) Given that \( x = 3^a \) and \( y = 3^b \), express \( \log_3 \left( \frac{\sqrt[2]{xy^2}}{27} \right) \) in terms of \( a \) and \( b \). \[ \text{[4]} \]

5  (i) Solve \(-2 \sin 2x = 3 \cos x\) for \(0° \leq x \leq 360°\). \[ \text{[4]} \]
(ii) On the same diagram, sketch the graphs of \( y = -\sin 2x \) and \( y = \frac{3}{2} \cos x\) for \(0° \leq x \leq 360°\). \[ \text{[4]} \]
(iii) Hence, explain how parts (i) and (ii) could be used to deduce the solution(s) of \( |-2 \sin 2x| = 3 \cos x\) for \(0° \leq x \leq 360°\). \[ \text{[2]} \]

6  (a) Show that the function \( \frac{x^2 - 4}{x} \) always increases as \( x \) increases. \[ \text{[3]} \]
(b) Differentiate \( \frac{\sqrt{x}}{1 + 2x} \) with respect to \( x \). \[ \text{[4]} \]

7  The roots of the quadratic equation \( x^2 - 5x + 4 = 0 \) are \( \alpha^2 \) and \( \beta^2 \), where both \( \alpha \) and \( \beta \) are positive.
(i) Show that \( \alpha + \beta = 3 \). \[ \text{[3]} \]
(ii) Find the quadratic equation whose roots are \( \frac{1}{\alpha^3} \) and \( \frac{1}{\beta^3} \). \[ \text{[4]} \]

8  (i) Given that the line \( x + y = 2 \) is a tangent to a circle with centre \( C(0, 6) \)
Find the equation of the circle. \[ \text{[6]} \]
(ii) A second circle \( x^2 + y^2 = 6y + d \), where \( d \) is an integer, is the reflection of the circle in part (i) about the line \( y = k \). Find the value of \( k \) and of \( d \). \[ \text{[5]} \]
9 \( N, O \) and \( P \) are three fixed points on a straight line as shown in the diagram below. Given that the velocity, \( v \) m/s, of a particle travelling on the straight line \( NP \) at time \( t \) seconds after leaving the fixed point \( O \), is given by \( v = t^3 - 10t^2 + 27t - 18 \).

- Positive direction

- \( N \) \hspace{1cm} \( O \) \hspace{1cm} \( P \)

(i) Find the initial velocity of the particle at \( O \). Explain the significance of your answer. [2]

(ii) Find the values of \( t \) when the particle comes instantaneously to rest. [4]

(iii) Find the maximum speed attained by the particle for \( 0 \leq t \leq 6 \). [4]

(iv) Calculate the distance travelled by the particle in the second second. [3]

10 In the diagram, triangle \( ABC \) is an equilateral triangle inscribed in a circle. \( D \) is a point on the arc \( BC \), \( E \) is a point on \( AD \) and \( CD = CE \).

Show that

(i) triangle \( CDE \) is equilateral, [3]

(ii) triangle \( ACE \) is congruent to triangle \( BCD \), [3]

(iii) \( AD = BD + CD \). [3]
11  (a) Differentiate $x^2 \ln x - x$ with respect to $x$. [3]

(b) The diagram shows the line $l$ and part of the curve $y = 2x \ln x$. Both graphs intersect the $x$-axis at $a$. Line $l$ cuts the $y$-axis at $1$.

(i) Find the value of $a$. [2]
(ii) Find the equation of line $l$. [1]
(iii) Determine the area of the shaded region bounded by the curve, the line $x = 2$ and the line $l$. [4]

End of Paper
Answers

1(i) \( y = \frac{1}{\sqrt{x}} \) (one possible answer)

2(a) \( k = -1 \)  
2(b) \( \frac{3 + \sqrt{5}}{2} \)

3(i) 100 g  
3(ii) \( t = 1.26 \) mins  
3(iii) 10 g  
3(iv) 9.88 g/min

4(a)(i) 4096  
4(a)(ii) 3  
4(b) \( \frac{1}{2}a + b - 3 \)

5(i) \( x = 90^\circ, 228.6^\circ, 270^\circ, 311.4^\circ \)
5(ii)

5(iii) Reflect the negative parts of the drawn sine graph in part (ii) about the x-axis and relate to the x-coordinates of the points of intersection found in part (i) given the solution to \( | -2 \sin 2x | = 3 \cos x \).

6(a) Since \( \frac{d}{dx} \left( \frac{x^2 - 4}{x} \right) > 0 \), \( \frac{x^2 - 4}{x} \) increases as \( x \) increases.  
6(b) \( \frac{1 - 2x}{2\sqrt{x}(1 + 2x)^2} \)

7(ii) \( 8x^2 - 9x + 1 = 0 \)  
8(i) \( x^2 + (y - 6)^2 = 8 \)  
8(ii) \( d = -1, k = 4 \frac{1}{2} \)

9(i) \( v = -18 \) m/s. The particle is moving in the opposite direction to the positive direction/moving to the left, etc.

(ii) \( t = 1, t = 3, t = 6 \)  
(iii) \( = 4.06 \) m/s  
(iv) \( = 2.92 \) m

11(a) \( x + 2x \ln x - 1 \)  
(b)(i) \( a = 1 \)  
(b)(ii) \( y = -x + 1 \)  
(b)(iii) \( = 1.773 \) units\(^2\)
1. The area of a triangle is \( \left( 1 + \frac{5\sqrt{5}}{2} \right) \) cm\(^2\). If the length of the base of the triangle is \( (3 + 2\sqrt{5}) \) cm, find, without using a calculator, the height of the triangle in the form of \( (a + b\sqrt{5}) \) cm, where \( a \) and \( b \) are integers.

2. Express \( \frac{4x^2 + 6x + 5}{2x^2 + x - 3} \) in partial fractions.

3. The function \( f(x) \) is such that \( f(x) = 2x^3 + 3x^2 - x - 4 \).
   (i) find a factor of \( f(x) \).
   (ii) Hence, determine the number of solutions in the equation \( f(x) = 0 \).

4. The roots of the quadratic equation \( 3x^2 - x + 5 = 0 \) are \( \alpha \) and \( \beta \).
   (i) Evaluate \( \alpha^3 + \beta^3 \).
   (ii) Find the quadratic equation whose roots are \( \alpha^3 - 1 \) and \( \beta^3 - 1 \).

5. The table shows experimental values of 2 variables, \( R \) and \( V \), which are connected by an equation of the form \( R V^n = k \) where \( n \) and \( k \) are constants.

\[
\begin{array}{|c|c|c|c|c|}
\hline
R & 33 & 19.95 & 5.07 & 2.38 \\
V & 2 & 2.9 & 8 & 14 \\
\hline
\end{array}
\]
   (i) Plot \( \log R \) against \( \log V \) for the given data and draw a straight line graph.
   (ii) Use your graph to estimate the value of \( n \) and of \( k \).
   (iii) By drawing a suitable straight line on your graph in (i), find the value of \( V \) such that \( \frac{R}{V^2} = 1 \).

6. Given that \( y = 1 - \frac{1}{2} \sin 3x \), \( 0^\circ \leq x \leq 240^\circ \).
   (i) State the maximum and minimum values of \( y \).
   (ii) Sketch the graph of \( y = 1 - \frac{1}{2} \sin 3x \).
10 The points \( A \) and \( B \) lie on the circumference of a circle \( C_1 \) where \( A \) is the point \((0, 8)\) and \( B \) is the point \((4, 0)\). The line \( y = 2x \) also passes through the centre of the circle \( C_1 \).

(i) Find the centre and radius of the circle \( C_1 \). [4]

(ii) Find the equation of the circle \( C_1 \) in the form \( x^2 + y^2 + px + qy + r = 0 \), where \( p, q \) and \( r \) are integers. [2]

Another circle \( C_2 \) of radius \( \sqrt{2} \) units has its centre inside \( C_1 \) and it cuts the circle \( C_1 \) at the origin and at the point where \( x = 2 \).

(iii) Find the centre of \( C_2 \). [5]

11 The diagram shows part of the curve \( y = 3\cos \frac{x}{2} \) that cuts the \( x \)-axis at \( x = \pi \) and \( x = 3\pi \). The normal to the curve at \( x = \frac{5\pi}{3} \) cuts the \( x \)-axis at \( A \).

(i) Find the coordinates of \( A \), leaving your answer in exact form. [6]

(ii) Hence, find the area of the shaded region. [4]
11 (a) The diagram shows a triangle $ABC$ which has a right angle at $C$. The point $D$ is the mid-point of the side $AC$. The point $E$ lies on $AB$ such that $AE = DE$. The line segment $ED$ is produced to meet the line $BC$ produced at $F$.

(i) Prove that $\triangle ACB$ is similar to $\triangle DCF$. [2]

(ii) Explain why $\triangle EFB$ is isosceles. [1]

(iii) Show that $EB = 3AE$. [2]

(b) $QRST$ is a trapezium in which $QR$ is parallel to $TS$ and its diagonals meet at $P$. The circle through $T$, $P$ and $S$ touches $QW$, $RY$ at $T$ and $S$ respectively.

Prove that

(i) $\angle QPS = \angle QTR$. [2]

(ii) $QRST$ is a cyclic quadrilateral. [3]

End of Paper
1 (a) The equation of a curve is \( y = 2x^2 + ax + (6 + a) \), where \( a \) is a constant. Find the range of values of \( a \) for which the curve lies completely above the \( x \)-axis. \([3]\)

(b) The equation of a curve is \( y = 3x^2 + 4x + 6 \).
   (i) Find the set of values of \( x \) for which the curve is above the line \( y = 6 \). \([3]\)
   (ii) Show that the line \( y = -8x - 6 \) is a tangent to the curve. \([2]\)

2 (a) Given that \( \log_a 125 = 3 \log_a b + \log_a c = 3 \), express \( a \) in terms of \( b \) and \( c \). \([3]\)

(b) Solve the equation
   (i) \( \log x - \log(x^2 - 3) = 2 \log 2 \). \([3]\)
   (ii) \( 2 \log_5 x = 3 + 7 \log_5 5 \). \([4]\)

3 The equation of a curve is \( y = x^2 \sqrt{5x - 1} \), for \( x > 0.2 \). Given that \( x \) is changing at a constant rate of 0.25 units per second, find the rate of change of \( y \) when \( x = 2 \). \([4]\)

4 The graph of \( y = |2x^2 - ax - 5| \) passes through the points with coordinates \((-1, 0)\) and \((0.75, b)\).
   (i) Find the value of the constants \( a \) and \( b \). \([3]\)
   (ii) Sketch the graph of \( y = |2x^2 - ax - 5| \). \([3]\)
   (iii) Determine the set of positive values of \( m \) for which the line \( y = mx + 2 \) intersects the graph of \( y = |2x^2 - ax - 5| \) at two points. \([2]\)

5 In the binomial expansion of \( \left( 2x + \frac{k}{x} \right)^8 \), where \( k \) is a positive constant, the coefficient of \( x^2 \) is 28.
   (i) Show that \( k = \frac{1}{4} \). \([4]\)
   (ii) Hence, determine the term in \( x \) in the expansion of \( \left( 6x - \frac{1}{x} \right) \left( 2x + \frac{k}{x} \right)^8 \). \([4]\)
12 The diagram, not drawn to scale, shows a triangle $ABC$, where $AC = BC$ and $A$ lies on the $y$-axis. $M$ is the mid-point of $AB$, $OM = 2$ units and $\tan \angle OMC = -\frac{2}{3}$.

(i) Show that the equation of $CM$ is $3y - 2x + 4 = 0$. [2]

(ii) Find the coordinates of $B$. [4]

(iii) Given that the area of triangle $ABC$ is $\frac{52}{3}$ square units, find the coordinates of $C$. [4]

End of Paper
The diagram shows points $A$, $B$, $C$ and $D$ on a circle, line $EF$ is tangent to the circle at $C$, lines $ADF$ and $EBAG$ are straight lines, and points $B$ and $C$ are the midpoints of $AE$ and $EF$.

Prove that

(i) $BC \times EC = AC \times BE$,

(ii) $AF \times EC = AC \times AE$,

(iii) angle $GAD = \text{angle } ACF$.

9 (a) (i) Show that $\cot 2x = \frac{1 - \tan^2 x}{2 \tan x}$.

(ii) Hence, solve the equation $8 \cot 2x \tan x = -1$, for $0^\circ < x < 360^\circ$.

(b) The Ultraviolet Index (UVI) describes the level of solar radiation. The UVI measured from the top of a building is given by $U = 6 - 5 \cos qt$, where $t$ is the time in hours from the lowest value of the UVI, $0 \leq t \leq 10$, and $q$ is a constant. It takes 10 hours for the UVI to reach its lowest value again.

(i) Explain why we are not able to measure a UVI of 12.

(ii) Show that $q = \frac{\pi}{5}$.

(iii) The top of the building is equipped with solar panels that supply power to the building when the UVI is at least 3. Find the duration, in hours and minutes, that the building is supplied with power from the solar panels.
In the diagram below, a circle $C_1$, with centre at $C_1(-3, 2)$, touches the line $2y - x = 12$ at the point $G$.

The line $2y - x = 12$ intersects the $x$-axis at $A$ and the $y$-axis at $B$.

Find

(i) the coordinates of $A$ and of $B$. [2]

(ii) the equation of the line $CG$. [2]

(iii) the equation of the circle $C_1$. [3]

(iv) the equation of the circle $C_2$, which is a reflection of the circle $C_1$ in the line $AB$. [2]

The acute angle between $AG$ and $AC$ is $\theta^\circ$.

(v) Show that $\theta = \tan^{-1} \frac{1}{4}$. [2]

6

(i) Find $\frac{d}{dx}\left[e^{2x}(2 - 3x)\right]$. [3]

(ii) Hence, find $\int_{0}^{\ln2} 5xe^{2x} \, dx$. [5]
Answer Key

1. (a) $-4 < a < 12$  
   (b)(i) $x < -1\frac{1}{3}$  or  $x > 0$

2. (a) $a = \frac{5\sqrt{c}}{b}$  
   (b)(i) $x = 3$  
   (ii) $x = 85.7$  or  $x = 0.130$

3. 49.5 units / s

4. (i) $a = 3$, $b = 6.125$  
   (ii) $x = |2x^2 - 3x - 5|

5. (ii) $-1\frac{3}{4}x$

6. (i) $l = \frac{45}{2r} - \frac{9}{8} \pi r$  
   (iii) $r = 2.32$; min value

7. (ii) $L = 46 + 10\sqrt{13} \sin(\theta - 19.4^\circ)$  
   (iii) $45.8^\circ$

9. (a)(ii) $x = 40.9^\circ, 139.1^\circ, 220.9^\circ, 319.1^\circ$  
   (b)(iii) 7 hrs and 3 mins

10. (a)(i) $\frac{4x(2x-3)}{(4x-3)^2}$  
    (ii) $\frac{3}{4} < x < \frac{3}{2}$

11. (ii) 1.23 m/s$^2$  
    (iii) 16.0 m  
    (iv) passed through O
8 (i) Prove that \( \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \cos 2\theta \). [3]

(ii) Use the result in (i) to show that
\[
1 + x^2 = \sqrt{2}x^2 - \sqrt{2} \quad \text{where} \quad x = \tan 67.5^\circ.
\] [2]

(iii) Hence find the values of the constants \( c \) and \( d \) such that
\[
\tan 67.5^\circ = c + d\sqrt{2}.
\] [3]

(iv) Hence show that \( \tan 7.5^\circ = \frac{1 + \sqrt{2} - \sqrt{3}}{1 + \sqrt{3} + \sqrt{6}} \). [3]

9 The temperature, \( x^\circ \text{C} \), inside a house \( t \) hours after 4 am is given by
\[
x = 21 - 3\cos \left( \frac{\pi t}{12} \right) \quad \text{for} \quad 0 \leq t \leq 24,
\] and the temperature, \( y^\circ \text{C} \), outside the house at the same time is given by
\[
y = 22 - 5\cos \left( \frac{\pi t}{12} \right) \quad \text{for} \quad 0 \leq t \leq 24.
\]

(i) Find the temperature inside the house at 8 am. [2]

The difference between the temperatures inside and outside of the house is given by \( D = x - y \).

(ii) Write down and simplify an expression for \( D \) in terms of \( t \) for \( 0 \leq t \leq 24 \). [1]

(iii) Sketch the graph of \( D \) against \( t \) for \( 0 \leq t \leq 24 \). [3]

(iv) Determine the time(s) of the day when the temperature inside of the house is equal to the temperature outside the house. Hence find the range of values of \( t \) when the temperature inside of the house is less than the temperature outside of the house. [4]
Answer all the questions.

1. The equation of the curve is \( y = px^n - 8 \), where \( p \) and \( q \) are constants.
   Given that the curve passes through the points \( (2, -4) \) and \( (5, 17) \), find the value of \( p \) and of \( q \). [4]

2. The second derivative of \( y \) is given by \( \frac{d^2y}{dx^2} = 2x + 4 \).
   Given that \( y = 12 \) when \( x = 3 \), and \( y = -\frac{1}{3} \) when \( x = 2 \), find \( y \) in terms of \( x \). [4]

3. The equation of a curve is \( y = ax^2 - 4x + 2a - 3 \), where \( a \) is a constant.
   Find the range of values of \( a \) for which the curve lies completely above the line \( y = -1 \). [5]

4. The equation of a curve is \( y = \frac{3\cos x}{\sin x} \), where \( 0 < x < \pi \).
   (i) Show that the gradient function can be expressed in the form \( \frac{k}{\sin^2 x} \), where \( k \) is a constant. [2]
   (ii) Find the \( x \)-coordinates of the points at which the tangents to the curve are perpendicular to the line \( 2x - 8y = -1 \), leaving your answers in exact form. [3]

5. The number of people, \( N \), in a housing estate who contracted influenza during a flu epidemic after \( t \) days is modelled by the equation \( N = \frac{1000}{1 + 199e^{-0.8t}} \).
   (i) Find the initial number of people who contracted influenza during the flu epidemic. [1]
   (ii) Given that there are 937 people who contracted influenza after \( x \) days, find \( x \) correct to the nearest whole number. [3]
   (iii) Find the number of people who eventually contracted influenza after a long time. [1]
6. (i) Sketch the curve \( y = |4x - x^2| \), indicating the coordinates of the maximum point and of the points where the curve meets the x-axis.

(ii) State the value or range of values of \( m \) if the equation \(|4x - x^2| = m\) has

(a) 2 solutions.
(b) 3 solutions.
(c) 4 solutions.

7. The function \( P(x) = 2x^3 + (4 - 2a)x^2 - ax + 6a \), where \( a \) is a constant.

(i) Show that \( x + 2 \) is a factor of \( P(x) \).

(ii) Find the other quadratic factor of \( P(x) \) in terms of \( a \), \( 0 < a < 6 \).

(iii) Find the range of values of \( a \) for which the equation \( P(x) = 0 \) has only 1 real root.

8. The table below shows the experimental values of two variables \( x \) and \( y \).

An error was made in recording one of the values of \( y \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>5.8</td>
<td>15</td>
<td>30</td>
<td>43.5</td>
<td>74</td>
</tr>
</tbody>
</table>

It is known that \( x \) and \( y \) are related by an equation \( y = ax(x+b) + 2 \), where \( a \) and \( b \) are unknown constants.

(i) Express \( y = ax(x+b) + 2 \) in a form suitable for drawing a straight line graph.

(ii) Draw a straight line graph for the given data.

(iii) Use your graph to estimate

(a) the value of \( a \) and of \( b \).

(b) a value of \( y \) to replace the incorrect value.
7 The diagram below shows a trapezium $ABCO$ inscribed in a semi-circle with centre $O$ and radius 4 units. $OA$ makes an angle of $\theta$ radians with the diameter. $AB$ is parallel to the diameter and $BC$ is perpendicular to both lines $AB$ and $OC$.

(i) Show that the perimeter, $y$, of trapezium $ABCO$ is given by

$$y = 4(1 + \sin \theta + 3\cos \theta).$$

(ii) Find the value of $\theta$ for which $y$ has a stationary value and determine whether this value of $y$ is a maximum or a minimum.

(iii) Express the perimeter of the trapezium in the form $y = 4 + R \cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(iv) Hence solve the equation $4(1 + \sin \theta + 3\cos \theta) = 12$, for $0 < \theta < \frac{\pi}{2}$. 


Answer all the questions.

1. It is given that \( f(x) = x^3 - 3x^2 + 4x \).

   (i) Show that \( f(x) \) is an increasing function for all values of \( x \). \([3]\)

   (ii) Hence, show that \( f(x) \) is positive for all positive values of \( x \). \([2]\)

2. A rectangle has a fixed perimeter of 40 cm. The length of one side, \( x \) cm, increases at a constant rate of 0.5 cm/s. Find the rate at which the area is increasing at the instant when \( x = 3 \). \([5]\)

3. (a) Find the term independent of \( x \) in the binomial expansion of \( \left( x^2 - \frac{1}{2x^3} \right)^{10} \). \([3]\)

   (b) Given that the first 4 terms in the binomial expansion of \( \left( 2x + \frac{1}{4} \right)^8 \), in descending powers of \( x \), are \( 512x^8 + 576x^7 + ax^6 + bx^5 + \ldots \), where \( a \) and \( b \) are constants, find

   (i) the value of \( a \) and of \( b \). \([3]\)

   (ii) the coefficient of \( x^5 \) in \( \left( 2x + \frac{1}{4} \right)^8 \left( \frac{4}{x} - 1 \right) \left( \frac{4}{x} + 1 \right) \). \([2]\)

Begin Question 4 on a fresh piece of paper.

4. (a) Given that \( \log_{10} a = r \), \( \log_{27} b = s \) and \( \frac{a}{b} = 3 \), express \( t \) in terms of \( r \) and \( s \). \([3]\)

   (b) Solve \( \log_3 x + 3 = 10 \log_3 3 \). \([5]\)
10 (a) It is given that \( y = \frac{2x^3}{4x-3} \), where \( x > \frac{3}{4} \).

(i) Find \( \frac{dy}{dx} \). \([2]\)

(ii) Find the range of values of \( x \) for which \( y = \frac{2x^3}{4x-3} \) is a decreasing function. \([4]\)

(b) It is given that \( f(x) \) is such that \( f'(x) = \frac{1}{2x-5} - \frac{4}{(2x-5)^2} \).

Given also that \( f(3) = 1.75 \), show that \( 8f(x) - (2x-5)^3 f''(x) = \ln(2x-5)^4 \). \([7]\)

11 A particle moves in a straight line, so that, \( t \) seconds after passing a fixed point \( O \), its velocity, \( v \) m/s, is given by \( v = 2e^{0.6t} - 10e^{0.3t} \). The particle comes to an instantaneous rest at the point \( A \).

(i) Show that the particle reaches \( A \) when \( t = \frac{5}{2} \ln 5 + \frac{1}{4} \). \([3]\)

(ii) Find the acceleration of the particle at \( A \). \([3]\)

(iii) Find the distance \( OA \). \([4]\)

(iv) Explain whether the particle is again at \( O \) at some instant during the eleventh second after first passing through \( O \). \([2]\)
9 The roots of the quadratic equation \(2x^2 - 4x - 1 = 0\) are \(\alpha\) and \(\beta\).

(i) Find the value of \(\alpha^2 + \beta^2\). \([2]\)

(ii) Show that the value of \(\alpha^3 + \beta^3\) is 11. \([2]\)

(iii) Find a quadratic equation whose roots are \(\left(\alpha^3 + \frac{1}{\beta^3}\right)\) and \(\left(\beta^3 + \frac{1}{\alpha^3}\right)\). \([4]\)

10 (i) Express \(\frac{14x^2 - 15x + 2}{x(2x - 1)^2}\) in partial fractions. \([5]\)

(ii) Hence find \(\int \frac{14x^2 - 15x + 2}{x(2x - 1)^2} \, dx\). \([4]\)

11 A particle \(P\) travels in a straight line from a fixed point \(O\) with acceleration \(a\) m/s\(^2\) given by \(a = 8t - k\), where \(t\) is the time in seconds after passing \(O\), and \(k\) is a constant.

When \(P\) passes \(O\), its velocity is 5 m/s. At \(t = 2\), its velocity is \(-21\) m/s.

(i) Show that the value of \(k\) is 21. \([2]\)

(ii) Find the range of values of \(t\) during which \(P\) is travelling towards \(O\). \([3]\)

(iii) Given that \(P\) comes to instantaneous rest at points \(A\) and \(B\), find the distance \(AB\). \([4]\)
The diagram shows a design of a bookmark that includes a rectangle $ABCD$, where $BC = l$ cm, $CD = 4r$ cm, a semicircle with radius $3r$ cm, and $AF = BE = r$ cm. The area of the bookmark is 90 cm$^2$.

(i) Express $l$ in terms of $r$. [2]

(ii) Given that the perimeter of the bookmark is $P$ cm, show that

$$P = \left(6 + \frac{3\pi}{4}\right)r + \frac{45}{r}.$$

[2]

(iii) Given that $r$ and $l$ can vary, find the value of $r$ for which $P$ has a stationary value. Explain why this value of $r$ gives the minimum perimeter. [5]

The diagram shows an animal exhibition area that is surrounded by glass panels at $AB$, $BC$ and $AD$, where $AB = 12$ m, $AD = 34$ m, angle $DAB = \angle BCD = 90^\circ$ and the acute angle $\angle ADC = \theta$ can vary.

(i) Show that $L$ m, the length of the glass panels can be expressed as

$$L = 46 + 34 \sin \theta - 12 \cos \theta.$$

[2]

(ii) Express $L$ in the form $p + R \sin(\theta - \alpha)$, where $p$ and $R > 0$ are constants and $\alpha$ is an acute angle. [4]

(iii) Given that the exact length of the glass panels is 62 m, find the value of $\theta$. [3]
The diagram shows the curve \( y = x(x^2 - 2)^3 \). \( P \) and \( Q \) are the points of intersection of the curve with the \( x \)-axis. \( M \) and \( N \) are the maximum and minimum points of the curve respectively.

(i) Find the coordinates of \( P \) and of \( Q \). [2]

(ii) Find the \( x \)-coordinates of \( M \) and of \( N \). [4]

(iii) Show that \( P \) and \( Q \) are stationary points of inflexion of the curve. [2]

(iv) Find \( \frac{dy}{dx} \left[ (x^2 - 2)^3 \right] \). [2]

(v) Hence find the total area of the shaded regions. [3]
1. $4 - \sqrt{5}$

2. $2 - \frac{2}{2x+3} + \frac{3}{x-1}$

3. (ii) one solution

4. (i) $-\frac{29}{9}$ (ii) $27x^2 + 98x + 196 = 0$

6. (i) Max $y = 1.5$; Min $y = 0.5$ (ii)

7. (i) $(-4, 8)$ (ii) $P(4, 4)$ (iii) 50 units$^2$

8. (a) (b)(i) $\left( -\frac{1}{3}, a - \frac{19}{27} \right)$ and $(2, 12 + a)$ (b)(ii) $\left( -\frac{1}{3}, a - \frac{19}{27} \right)$ min; $(2, 12 + a)$ max

9. (ii) $\sec x$ (iii) 0.539

10. (i) Centre $(2, 4)$. Radius $= 2\sqrt{5}$ (ii) $x^2 + y^2 - 4x - 8y = 0$ (iii) Centre of $C_1(1.22, 0.710)$

11. (i) $A \left( \frac{5\pi}{3} + \frac{9}{8}\sqrt{3}, 0 \right)$ (ii) $6\frac{15}{32} / 6.47$ units$^2$
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong></td>
<td>( p = 1, \ q = 2 )</td>
<td><strong>7 ii</strong></td>
</tr>
<tr>
<td><strong>2</strong></td>
<td>( y = \frac{x^3}{3} + 2x^2 - 4x - 3 )</td>
<td><strong>7 iii</strong></td>
</tr>
<tr>
<td><strong>3</strong></td>
<td>( a &gt; 2 )</td>
<td><strong>8 i</strong></td>
</tr>
<tr>
<td><strong>4</strong></td>
<td>( x = \frac{\pi}{3} ) or ( \frac{2\pi}{3} )</td>
<td><strong>8 ii</strong></td>
</tr>
<tr>
<td><strong>5 i</strong></td>
<td>( 5 )</td>
<td><strong>8 iii a</strong></td>
</tr>
<tr>
<td><strong>5 ii</strong></td>
<td>( t = 10 )</td>
<td><strong>8 iii b</strong></td>
</tr>
<tr>
<td><strong>5 iii</strong></td>
<td>1000</td>
<td><strong>8 iii b</strong></td>
</tr>
<tr>
<td><strong>6 i</strong></td>
<td>( y' = 4x - x^2 )</td>
<td><strong>9 i</strong></td>
</tr>
<tr>
<td><strong>6 ii a</strong></td>
<td>( m = 0 ) or ( m &gt; 4 )</td>
<td><strong>9 ii i</strong></td>
</tr>
<tr>
<td><strong>6 ii b</strong></td>
<td>( m = 4 )</td>
<td><strong>10 i</strong></td>
</tr>
<tr>
<td><strong>6 ii b</strong></td>
<td>( m = 4 )</td>
<td><strong>10 ii</strong></td>
</tr>
<tr>
<td><strong>6 iii</strong></td>
<td>( 0 &lt; m &lt; 5 )</td>
<td><strong>11 i i</strong></td>
</tr>
<tr>
<td><strong>12 i i</strong></td>
<td>( B(4, -3) )</td>
<td><strong>11 iii</strong></td>
</tr>
<tr>
<td><strong>12 ii i</strong></td>
<td>( C \left( \frac{6, 8}{3} \right) )</td>
<td><strong>12 ii ii</strong></td>
</tr>
</tbody>
</table>
A quadrilateral $ABCD$ passes through vertices $B (3, 9)$, $C (8, 6)$ and $D (-4, 0)$, line $AD$ is parallel to the $y - axis$.

(i) Find the coordinates of $A$ given that the length of $AD$ is 8 units. [1]

(ii) A point $P$ divides the line $DC$ in the ratio of 2 : 1. Find the coordinates of $P$. [3]

(iii) Hence, find the area of the quadrilateral $ABPD$. [3]

8 (a) Sketch the graph $y^2 = 3x$. [2]

(b) Given that $f(x) = -2x^3 + 5x^2 + 4x + a$,

(i) find the coordinates of the turning points in terms of $a$. [4]

(ii) Determine the nature of each turning point. [3]

(iii) In the case where $a = 1$, explain why the part of the graph between the turning points lie above the $x - axis$. [1]

9 (i) Show that $\sec x + \tan x$ can be expressed as $\frac{1 + \sin x}{\cos x}$. [1]

(ii) Differentiate $\ln(\sec x + \tan x)$ with respect to $x$. [3]

(iii) Hence, find $\int_{\theta}^{0.5} 2\sec x\,dx$. [3]
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7 cm²/s</td>
</tr>
</tbody>
</table>
| 3a | \[
\frac{105}{8} = 13.125 \text{ or } 13 \frac{1}{8}
\] |
| 3bi | \(a = 288, b = 84\) |
| 3bii | 9132 |
| 4a | \(t = r - 3x\) |
| 4b | \(x = \frac{1}{243} \text{ or } x = 9\) |
| 5i | \(A = (-12, 0), B = (0, 6)\) |
| 5ii | \(y = -2x - 4\) |
| 5iii | \((x + 3)^2 + (y - 2)^2 = 5\) |
| 5iv | \((x + 5)^2 + (y - 6)^2 = 5\) |
| 6i | \(e^{x^2 - 6xe^{x^2}}\) |
| 6ii | 10ln 2 - \(\frac{15}{4}\) or 3.18 |
| 7ii | 0.322, \(y\) is maximum |
| 7iii | \(y = 4 + \sqrt{60 \cos (\theta - 0.322)}\) |
| 7iv | \(\theta = 1.21\) |
| 8iii | \(c = 1, d = 1\) |
| 9i | 19.5°C |
| 9ii | \(D = 2\cos \frac{\pi t}{12} - 1\) |
| 9iii | \(D = 2\cos \frac{\pi t}{12} - 1\) |
| 9iv | 8 am and 12 midnight, 4 \(< t \< 20\) |
| 10i | \(P = (-\sqrt{2}, 0)\) and \(Q = (\sqrt{2}, 0)\) |
| 10ii | \(x\)-coordinate of \(N = \frac{\sqrt{2}}{7}\) or 0.535 |
| 10iii | \(x\)-coordinate of \(M = -\frac{\sqrt{2}}{7}\) or -0.535 |
| 10iv | \(8\sqrt{x^2 - 2}\) |
| 10v | 4 sq. units |
| 11ai | (1) \(\angle BAC = \angle CDF\) |
| 11aii | (2) \(\angle DCF = \angle ACB = 90^\circ\) (given) |
| 11aiii | \(\triangle ABC\) is similar to \(\triangle DCF\) (AA Similarity) |
| 11aiv | \(\angle DFC = \angle ABC\) (Corr angles of similar triangles) |
| 11bii | \(\angle QRS = \angle QST\) (alt angles, \(QR/TS\)) |
| 11biii | \(\angle QST = \angle QTR\) (tan chord theorem) |

\[\therefore \angle QRS = \angle QTR\]

\(\because \angle TS\) and \(\angle TR\) are equal.

\(\therefore \angle TQR = 180^\circ - \angle QTS\) (corr angles, \(QR/TS\))

Since \(\angle TSR + \angle TQR = 180^\circ\)

\(QRST\) is a cyclic quadrilateral. (Angles in opp segments)
1. Express \( \frac{2x^2 + 9x + 6}{(x + 2)(x^2 - 4)} \) in partial fractions. [4]

2. Given that \( (1 + \alpha x)^n = 1 - 24x + 252x^2 + \ldots \), find the values of \( \alpha \) and \( n \). [5]

3. (a) Given that \( \sin \theta = k \), where \( \theta \) is an acute angle. Find, in terms of \( k \), the value of \( \sin 4\theta \). [3]

(b) Find the exact value of \( \tan \left[ \cos^{-1} \left( -\frac{1}{\sqrt{2}} \right) \right] \) without the use of a calculator. [2]

4. A triangle has vertices \( A(-2, 2), B(-1, -1) \) and \( C(3, -2) \). Given that \( ABCD \) is parallelogram, find

(i) the coordinates of the point \( D \), [2]

(ii) the area of the parallelogram \( ABCD \). [3]

5. A spherical elastic balloon, with radius \( r \) cm, is filled with \( V \) cm\(^3\) of helium gas. It was discovered that there is a leakage of helium gas from the balloon at a constant rate of 5 cm\(^3\)/s. At the instant when the radius of the spherical elastic balloon is 10 cm, find

(i) the rate at which the radius of the balloon is decreasing, leaving your answer in terms of \( \pi \), [3]

(ii) the rate of change of the surface area of the spherical elastic balloon. [3]

6. (a) Given that \( \int_0^1 f(x) \, dx = \int_1^2 f(x) \, dx = \frac{2}{5} \), find \( \int_0^1 f(x) \, dx \). [2]

(b) (i) Show that \( \frac{d}{dx} \left( \frac{x^2}{\sqrt{2x-3}} \right) = \frac{3x^2 - 6x}{(2x - 3)^{3/2}} \). [2]

(ii) Hence, or otherwise, find \( \int \frac{x^2 - 2x}{(2x - 3)^{3/2}} \, dx \). [2]
7 (a) The equation of the curve is \( y = (k + 4)x^2 + 4x - k \), where \( k \) is a constant.

(i) Show that the curve meets the \( x \)-axis for all possible values of \( k \). [3]

(ii) Find the value of \( k \) for which the \( x \)-axis is a tangent to the curve. [1]

(b) Given that \( y = px^2 + 4x + q \) is always positive, what conditions must be applied to the constants \( p \) and \( q \)? [2]

8 (i) Show that \( \frac{1}{\sec x + 1} + \frac{1}{\sec x - 1} = \frac{2\cos x}{\sin^2 x} \). [3]

(ii) Hence, or otherwise find all the angles which satisfy the equation

\[
\frac{1}{\sec x + 1} + \frac{1}{\sec x - 1} = 8\cos x, \text{ for } 0 \leq x \leq \pi.
\] [4]

9 A cuboid has a volume of 648 cm\(^3\), a length of 6 cm and a height of \( x \) cm.

(i) Find, in terms of \( x \), an expression for the breadth of the cuboid. [1]

(ii) Show that the total external surface area, \( A \) cm\(^2\), of the cuboid is given by \( A = 12\left(18 + \frac{108}{x} + x\right)\). [2]

(iii) Find the value of \( x \) at which \( A \) is a minimum. [4]

10 A point \( H \) lies on the curve \( y = -x^2 + 4x + 7 \). The normal to the curve at \( H \) is perpendicular to the line \( 2y - 8x = 4 \).

(i) Show that the coordinates of \( H \) are \((0, 7)\). [3]

(ii) Find the equation of the normal to the curve at \( H \). [3]

(iii) Find the coordinates of point \( K \), where the tangent to the curve at \( K \) is parallel to the normal in part (ii). [3]
11 (a) (i) Sketch the graph of \( y = 0.5x^{-\frac{1}{3}} \), for \( x > 0 \). [1]

(ii) Determine the equation of the straight line which needs to be drawn on the graph of \( y = 0.5x^{-\frac{1}{3}} \) in order to obtain a graphical solution of the equation \( 1 = 2x^3 \). [1]

(iii) Hence, state the number of solution(s) to the equation \( 1 = 2x^3 \), for \( x > 0 \). [1]

(b) (i) On the same axes, sketch the graphs of \( y = |3x^2 - 6x| \) and \( y = 1 \). [3]

(ii) State the number of solutions to the equation \( |3x^2 - 6x| = 1 \). [1]

(iii) Solve the equation \( |3x^2 - 6x| = 3x \). [3]

12 (a) Variables \( x \) and \( y \) are related in such a way that when \( \frac{y}{x} \) is plotted against \( x^2 \), a straight line which passes through the points \((1, 2)\) and \((-4, 17)\) is obtained.

(i) Express \( y \) in terms of \( x \). [3]

(ii) Hardev commented that the point \((6, -618)\) can be found on the straight line. Gabriel disagreed. Who do you agree with? Explain your answer. [2]

(b) Answer the whole of this question part on a sheet of graph paper.

Two variables \( x \) and \( y \) are connected by the equation \( y = ab^x + 4 \). By drawing a suitable straight line graph using the following table of corresponding values of \( x \) and \( y \), find the values of \( a \) and \( b \). [5]

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>( y )</td>
<td>16.8</td>
<td>24</td>
<td>37</td>
<td>56</td>
<td>88</td>
<td>138</td>
</tr>
</tbody>
</table>

END OF PAPER
1. A piece of fish fillet is removed from the freezer and left to thaw. After \( t \) minutes, its temperature \( T \) \(^\circ\text{C} \) is given by \( T = 33 - 37e^{-0.01t} \). In order to maintain the quality of the fish fillet, Chef Chris needs to marinate the fillet when its temperature reaches 15\(^\circ\text{C} \). Find

(i) the initial temperature of the fish fillet, \[2\]

(ii) the waiting time, to the nearest minute, before Chef Chris can start to marinate the fish fillet, \[3\]

(iii) the value of \( T \) as \( t \) becomes very large. Explain the significance of this value. \[2\]

2. The equation \( x^2 + 2x - 6 = 0 \) has roots \( \alpha \) and \( \beta \) and the equation \( hx^2 + 2 = kx \) has roots \( \frac{\alpha}{\beta - 1} \) and \( \frac{\beta}{\alpha - 1} \). Find the values of \( h \) and \( k \). \[7\]

3. In the diagram, \( A, B, C, D \) and \( E \) are points on the circle with centre \( O \). The tangent to the circle at \( D \) is extended to meet the line \( AOC \) at \( F \). \( BE \) intersects \( AD \) at \( G \) and \( BD \) intersects \( AF \) at \( H \). \( \angle ADB = \angle EOA \). Prove that

(i) triangle \( ADF \) is similar to triangle \( DCF \), \[3\]

(ii) \( AE \times BH = AG \times BO \). \[4\]
4 (a) In the diagram, \( PQ \) is parallel to \( BR \) and \( BC \) is divided at \( P \) such that \( BP : PC = \sqrt{7} : 5 \). Given that \( BR = (1 + 2\sqrt{7}) \text{ cm} \), find the length of \( PQ \) in the form \( (a + b\sqrt{7}) \text{ cm} \), where \( a \) and \( b \) are rational numbers. \([4]\)

(b) Solve the equation \( 4^{x-1} - 3(2^{x+1}) - 64 = 0 \). \([3]\)

5 (a) The equation of a curve is \( y = 4x^3 + 3px^2 + 27x - 10 \). Find the range of \( p \) such that \( y \) is an increasing function. \([4]\)

(b) The curve \( y = (hx^3 - 1)^2 - k \) has a stationary point at \((1, -3)\). Given that \( h \) is positive, find the values of \( h \) and \( k \). \([4]\)

6 In the diagram, the circle passes through \( P(-2, 7) \) and touches the line \( 5y - 2x + 10 = 0 \) at \( Q(5, 0) \). The centre of the circle is denoted by \( C \). Find

(i) the equation of the line \( CQ \), \([2]\)

(ii) the coordinates of \( C \), \([4]\)

(iii) the equation of the circle. \([2]\)
7  (i)  Show that \( \sec^2 x - \tan^2 x = 2 \cos 2x \). 

(ii) Hence, sketch the graph of \( y = \sec^2 x - \tan^2 x - 2 \cos 2x + 1 \) for \( 0 \leq x \leq \pi \). 

(iii) On the same axes, sketch a suitable graph to find the number of solutions to the equation \( 2(\sec^2 x - \tan^2 x - 2 \cos 2x) - 1 = \frac{x}{\pi} \).

8  The diagram below shows two triangles with right angles at \( L \) and \( N \). The length of \( KL \) and \( LM \) are 12 cm and 5 cm respectively, and \( \angle LKN = \beta \), where \( \beta \) is an acute angle.

(i) Express \( KN \) in the form \( a \cos \beta + b \sin \beta \), where \( a \) and \( b \) are constants.

(ii) Show that \( KN = R \cos(\beta - \alpha) \), where \( R > 0 \) and \( 0^\circ < \alpha < 90^\circ \).

(iii) Find the value of \( \beta \) for which \( KN = 8 \) cm.

9  (a) Find the range of values of \( x \) for which \( 3(2x - 5)^2 > x(2x - 5) \).

(b) The function \( g(x) = 3x^3 + x^2 - kx + 4 \) has a factor \( (x - 1) \).

(i) Find the value of \( k \).

(ii) Solve the equation \( g(x) = 0 \).

(iii) Hence, find the roots of the equation \( \frac{y + 4}{\sqrt{y}} = 8 - 3y \).
The diagram shows part of the curve \( y^2 = x \) and the line \( y = 6 - x \), intersecting at the point \( M \). Find

(i) the coordinates of the point \( M \). \[3\]

(ii) the total area of the shaded regions. \[6\]

11 (a) Solve the equation \((\log_{10} x)(\log_3 x) = 4\). \[3\]

(b) Solve, for \( x \) and \( y \), the simultaneous equations

\[
e^x \left( \frac{1}{e^2} \right)^{1 - 2y} = e, \]

\[x \ln 32 - y \ln 4 = \ln 16.\] \[4\]

(c) Given that \( 3 \lg \sqrt{y - \frac{y}{100}} = 3 \lg x \), express \( y \) in terms of \( x \). \[3\]

12 A particle \( P \) moves along a horizontal straight line such that at time \( t \) seconds after the motion has begun from a fixed point \( O \), its acceleration \( a \) m/s\(^2\) is given by \( a = 12t - 18 \).

(i) Given that the initial velocity is 12 m/s, find an expression for the displacement of \( P \). \[3\]

Another particle \( Q \) moves along the same line as \( P \) at the same instant that \( P \) begins to move. The velocity of \( Q \) is given by \( v = 6t^2 - 16t + 7 \).

(ii) Given that the initial displacement of \( Q \) is \(-6\) m from a fixed point \( O \), find an expression for the displacement of \( Q \). \[2\]

(iii) Find the total distance travelled by \( P \) when it collides with \( Q \). \[5\]

(iv) Determine if \( P \) and \( Q \) are travelling in the same direction at the instant when \( P \) and \( Q \) collide. \[2\]

END OF PAPER (XINMIN)
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\frac{2}{x-2} + \frac{1}{(x+2)^2})</td>
</tr>
<tr>
<td>2</td>
<td>(a = -3, n = 8)</td>
</tr>
<tr>
<td>3a</td>
<td>(4k(1 - 2k^2)(\sqrt{1 - 2k^2}))</td>
</tr>
<tr>
<td>3b</td>
<td>(-1)</td>
</tr>
<tr>
<td>4i</td>
<td>D(2, 1)</td>
</tr>
<tr>
<td>4ii</td>
<td>11 sq units</td>
</tr>
<tr>
<td>5i</td>
<td>(-\frac{1}{80\pi}) cm/s</td>
</tr>
<tr>
<td>5ii</td>
<td>(-1) cm²/s</td>
</tr>
<tr>
<td>6a</td>
<td>(\frac{4}{5})</td>
</tr>
<tr>
<td>6bii</td>
<td>(\frac{x^2}{3\sqrt{2x-3}} + c)</td>
</tr>
<tr>
<td>7a(ii)</td>
<td>(k = -2)</td>
</tr>
<tr>
<td>7b</td>
<td>(pq &gt; 4)</td>
</tr>
<tr>
<td>8i(i)</td>
<td>(x = \frac{\pi}{6}, \frac{5\pi}{6})</td>
</tr>
<tr>
<td>9i</td>
<td>(b = \frac{108}{x}) cm</td>
</tr>
<tr>
<td>9ii</td>
<td>(x = 6\sqrt{3})</td>
</tr>
<tr>
<td>10i(i)</td>
<td>(y = -\frac{1}{4}x + 7)</td>
</tr>
<tr>
<td>10ii</td>
<td>(K = (2, \frac{1}{8}, 10, \frac{63}{64}))</td>
</tr>
<tr>
<td>11ai</td>
<td><img src="%20ai.png" alt="Graph of y = x" /></td>
</tr>
<tr>
<td>11ai(ii)</td>
<td>Line is (y = x)</td>
</tr>
<tr>
<td>11ai(iii)</td>
<td>No solution</td>
</tr>
<tr>
<td>11bi</td>
<td><img src="%20bi.png" alt="Graph of y = x" /></td>
</tr>
<tr>
<td>11bii</td>
<td>4</td>
</tr>
<tr>
<td>11biii</td>
<td>(x = 0, 1, 3)</td>
</tr>
<tr>
<td>12ai</td>
<td>(y = -3x^2 + 5x)</td>
</tr>
<tr>
<td>12a(ii)</td>
<td>Yes. When (X = 6), (Y = -18) which is not equal to (-618).</td>
</tr>
<tr>
<td>12b</td>
<td>(a = 7.96, b = 1.60)</td>
</tr>
<tr>
<td>Question</td>
<td>Answer</td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
</tr>
<tr>
<td>1i</td>
<td>$-4^\circ C$</td>
</tr>
<tr>
<td>1iii</td>
<td>Room temperature is $33^\circ C$</td>
</tr>
<tr>
<td>2a</td>
<td>$t = 24$</td>
</tr>
<tr>
<td>3a</td>
<td>$h = 1, k = -6$</td>
</tr>
<tr>
<td>4a</td>
<td>$\frac{5}{2} + \frac{5}{2} \sqrt{7}$</td>
</tr>
<tr>
<td>4b</td>
<td>$n = 3$</td>
</tr>
<tr>
<td>5a</td>
<td>$-6 &lt; p &lt; 6$</td>
</tr>
<tr>
<td>5b</td>
<td>$h = 1, k = 3$</td>
</tr>
<tr>
<td>6i</td>
<td>$y = -\frac{5}{2}x + \frac{25}{2}$</td>
</tr>
<tr>
<td>6ii</td>
<td>$C = (3, 5)$</td>
</tr>
<tr>
<td>6iii</td>
<td>$(x - 3)^2 + (y - 5)^2 = 29$</td>
</tr>
<tr>
<td>6iii</td>
<td>2 solutions</td>
</tr>
<tr>
<td>7ii</td>
<td></td>
</tr>
<tr>
<td>8ii</td>
<td>$\beta = 74.6^\circ$</td>
</tr>
<tr>
<td>9a</td>
<td>$x &lt; 2.5, x &gt; 3$</td>
</tr>
<tr>
<td>9b</td>
<td>$k = 8$</td>
</tr>
<tr>
<td>9bii</td>
<td>$x = 1, x = -2, x = 2/3$</td>
</tr>
<tr>
<td>9biii</td>
<td>$y = 1, y = 4/9$</td>
</tr>
<tr>
<td>10i</td>
<td>$M(4, 2)$</td>
</tr>
<tr>
<td>10ii</td>
<td>13.1 units$^2$</td>
</tr>
<tr>
<td>11a</td>
<td>$x = 81, x = 1/81$</td>
</tr>
<tr>
<td>11b</td>
<td>$x = 1, y = 0.5$</td>
</tr>
<tr>
<td>11c</td>
<td>$y = \frac{1}{1000} x^6$</td>
</tr>
<tr>
<td>12i</td>
<td>$s = 2t^3 - 9t^2 + 12t$</td>
</tr>
<tr>
<td>12ii</td>
<td>$s = 2t^3 - 8t^2 + 7t - 6$</td>
</tr>
<tr>
<td>12iii</td>
<td>Total distance = 182 m</td>
</tr>
<tr>
<td>12iv</td>
<td>Since the velocities of particles are both positive at $t = 6$, they are travelling in the same direction.</td>
</tr>
</tbody>
</table>
1. Find the set of values of \( a \) for which \( 3ax^2 + 1 > ax \) for all real values of \( x \). \[3\]

2. The function \( f(x) = \tan x \sec x \), where \( 0^\circ \leq x \leq 360^\circ \). Find the values of \( x \) for which \( f \) is an increasing function. \[4\]

3. Solve the equation \( \log_3(x + 4) - \log_3(2x - 1) + 2 \log_9(x - 2) = 1 \). \[4\]

4. The curve \( y^2 + 17 = 2x^2 \) intersects the straight line \( y + 4 = x \) at the points \( A \) and \( B \). Find the equation of the perpendicular bisector of \( AB \). \[6\]

5. (i) Show that \( \sin 2x (\tan^2 x + 1) = 2 \tan x \). \[3\]

(ii) Hence solve the equation \( \sin 4\theta (\tan^2 2\theta + 1) = 2 \cot \theta \) for \( 0^\circ < \theta < 360^\circ \). \[4\]

6. The function \( f \) is defined, for \( 0 \leq x \leq \pi \), by \( f(x) = 3\cos 3x - a \), where \( a \) is a constant.

Given that the minimum value of \( f(x) \) is \(-4\), find

(i) the value of \( a \), \[1\]

(ii) the maximum value of \( f(x) \), \[1\]

(iii) the period and the amplitude of \( f(x) \). \[2\]

Using the value of \( a \) found in part (i),

(iv) find the exact value(s) of \( x \) for which \( f(x) = \frac{1}{2} \). \[3\]

7. (i) Sketch the graph of \( y = |x^2 - 4x| + 1 \). \[3\]

(ii) It is given that the line \( y = mx \), where \( m > 0 \), does not intersect the graph of \( y = |x^2 - 4x| + 1 \). Determine the set of possible values of \( m \). \[2\]

(iii) Find the coordinates of the point(s) of intersection of the line \( y = 6 \) and the graph of \( y = |x^2 - 4x| + 1 \). \[3\]
8 In January 2016, Adam bought an antique vase for $1500. It was believed that the value of the antique vase will increase continuously with time such that it doubles after every 5 years.

(i) Formulate an expression for $V$, the value of the vase after Adam owned it for $x$ years. [2]

(ii) Sketch the graph of $V$ against $x$. [2]

(iii) Using your answer in part (i), find the number of years that Adam has to wait before the value of the vase appreciates to one million dollars. [3]

9 The diagram shows a triangle $ABC$ whose vertices lie on the circumference of a circle with centre $O$. $AP$ and $PB$ are tangents to the circle at $A$ and $B$ respectively. The tangent to the circle at $B$ meets $AO$ extended at $Q$.

(i) Show that angle $AOB = 2 \times$ angle $PAB$. [2]

(ii) Hence determine whether it is possible to draw a circle that passes through $O$, $A$, $P$ and $B$? Justify your answer with clear explanations. [3]

(iii) If triangle $PAB$ is equilateral, prove that $OQ = 2OB$. [2]

10 The equation of a curve is $y = -\sqrt{1 + 3x}$.

(i) A particle $P$ moves along the curve in such a way that the $x$-coordinate of $P$ decreases at a constant rate of 0.2 units per second. Find the coordinates of $P$ at the instant when the $y$-coordinate is increasing at a rate of 0.05 units per second. [4]

(ii) Find the area enclosed by the curve and the line $y = -3x - 1$. [5]
11. A solid cylinder is cut from a solid cone of height 60 cm and radius 30 cm as shown in the diagram. The cylinder has height \( h \) cm, radius \( r \) cm and volume \( V \) cm³.

(i) Show that \( h = 60 - 2r \). [2]

(ii) Express \( V \) in terms of \( r \). [1]

(iii) Determine the value of \( r \) for which the volume of the cylinder is maximum. Hence find the maximum volume of the cylinder. [6]

12. A particle travels in a straight line so that, \( t \) seconds after passing a fixed point \( O \), its velocity, \( v \) m/s, is given by \( v = 12t - 2t^2 \). The particle comes to an instantaneous rest at \( A \). Find

(i) the acceleration of the particle at \( A \). [3]

(ii) the greatest velocity of the particle. [2]

(iii) the distance travelled by the particle between \( t = 0 \) and \( t = 5 \). [4]
1 The curve \( y = f(x) \) is such that \( f'(x) = 3e^{-x} + \frac{1}{x+1}, \ x > 0. \) 

(i) Explain why the curve \( y = f(x) \) has no stationary point. \[2\]

(ii) Given that the curve passes through the point \((0, 1)\), find an expression for \( f(x) \). \[4\]

2 (i) Differentiate \( \ln(\sin x) \) with respect to \( x \). \[2\]

(ii) Show that \( \frac{d}{dx}(x\cot x) = \cot x - x\csc^2 x \). \[2\]

(iii) Using the results from parts (i) and (ii), show that \( \int_{\pi/4}^{x} x\cot^3 x \, dx = \frac{\pi}{4} - \frac{3x^2}{32} - \ln\frac{\sqrt{2}}{2}. \) \[4\]

3 The equation of a curve is \( y = -x^3 - 2x^2 - x - 1 \). The point \( A \) lies on the curve and has \( x \)-coordinate of \(-2\). The normal to the curve at \( A \) meets the \( x \)-axis at \( P \) and the \( y \)-axis at \( Q \).

(i) Find the area of triangle \( POQ \), where \( O \) is the origin. \[6\]

The point \( B \) also lies on the curve. The tangent to the curve at \( B \) is perpendicular to the normal to the curve at \( A \).

(ii) Find the \( x \)-coordinate of \( B \). \[3\]

4 (a) (i) Write down, and simplify, the first four terms in the expansion of \( (1 - x)^6 \) in ascending powers of \( x \). \[2\]

(ii) Replacing \( x \) by \( 2z - z^2 \), determine the coefficient of \( z^3 \) in the expansion of \( (1 - 2z + z^2)^6 \). \[3\]

(b) (i) Write down the general term in the binomial expansion of \( (2x - \frac{1}{3x^3})^6 \). \[1\]

(ii) Determine whether there is a constant term in the expansion. \[1\]

(iii) Using the general term, or otherwise, determine the coefficient of \( x^2 \) in the binomial expansion of \( \left(3x^4 + 2 - \frac{3}{x}\right)(2x - \frac{1}{3x^3})^6 \). \[2\]
5 Do not use a calculator in this question.
The diagram shows a cuboid with a square base. The area of the square base is \((7 + 4\sqrt{3})\text{cm}^2\) and the volume of the cuboid is \((26 + 15\sqrt{3})\text{cm}^3\).

Find
(i) the height of the cuboid in the form \(a + b\sqrt{3}\), where \(a\) and \(b\) are integers, [2]
(ii) an expression for \(BC^2\) in the form \(c + d\sqrt{3}\), where \(c\) and \(d\) are integers, [2]
(iii) the value of \(m\) and of \(n\) if the length of \(AC\) is \((\sqrt{m} + \sqrt{n})\text{cm}\), where \(m\) and \(n\) are integers. [6]

6 The equation of a curve is \(y = \frac{\sin x}{2 - \cos x}\).

(i) Find an expression for \(\frac{dy}{dx}\) and obtain the coordinates of the stationary point(s) of the curve for \(0 \leq x \leq \pi\). [5]
(ii) Find an expression for \(\frac{d^2y}{dx^2}\) and hence determine the nature of the stationary point(s) for \(0 \leq x \leq \pi\). [4]

7 The lines \(x = 2\) and \(y = 3\) are tangents to a circle \(C_1\).
Given that the centre of circle \(C_1\) lies on the positive \(x\)-axis, find
(i) the equation of \(C_1\). [4]
(ii) Circle \(C_2\) is a reflection of circle \(C_1\) along the line \(y = x + 1\), find the equation of \(C_2\). [3]

8 (a) (i) Find the remainder when \(f(x) = 3x^3 + x^2 + x - 4\) is divided by \(x + 1\). [2]
(ii) Hence find the value of \(k\) for which \(g(x) = f(x) + k\) is divisible by \(x + 1\) and factorise \(g(x)\) completely. [3]
(b) Express \(\frac{4x + 1}{(2x + 1)(x - 1)^2}\) as the sum of 3 partial fractions. [5]
9 In the diagram, \( AB = 8 \text{ m}, \ BC = 5 \text{ m}, \ \angle AOB = \angle ABC = \angle BDC = 90^\circ \) and \( \angle OAB = \theta \) where \( 0^\circ < \theta < 90^\circ \).

\[ \begin{align*}
\end{align*} \]

(i) Find \( OD \) in terms of \( \theta \). [3]
(ii) Express \( OD \) in the form \( R \sin(\theta + \alpha) \) where \( R > 0 \) and \( 0^\circ < \alpha < 90^\circ \). [4]
(iii) Find the value of \( \theta \) for which \( OD \) has a maximum length. [3]

10 The roots of the quadratic equation \( 2x^2 - 6x + 1 = 0 \) are \( \alpha \) and \( \beta \).

(i) Find the value of \( \alpha^2 + \beta^2 \). [2]
(ii) Find the value of \( \alpha - \beta \) given that \( \alpha < \beta \). [2]
(iii) Show that \( \alpha^2 - \beta^2 = -3\sqrt{7} \). [1]
(iv) Find a quadratic equation whose roots are \( \alpha^2 - \beta \) and \( \beta^2 - \alpha \), in the form \( ax^2 + bx + c = 0 \) where \( a, b \) and \( c \) are integers. [6]

11 The table below shows experimental values of two variables \( x \) and \( y \). It is known that \( x \) and \( y \) are related by the equation \( y = \frac{a}{x-b} \) where \( a \) and \( b \) are constants.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1.0</th>
<th>-0.5</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0.33</td>
<td>0.40</td>
<td>0.67</td>
<td>1.00</td>
<td>2.00</td>
</tr>
</tbody>
</table>

(i) Express the equation in the form suitable for drawing a straight line graph, with \( xy \) as the variable for the horizontal axis. [2]

State clearly the variable(s) used for the vertical axis.

(ii) Using variable \( xy \) for the horizontal axis and suitable variable(s) for the vertical axis, draw, on graph paper, a straight line graph and hence estimate the value of \( a \) and of \( b \). [6]

(iii) Show that by adding another straight line on your diagram, an estimate of the solutions for the simultaneous equations \( y = \frac{a}{x-b} \) and \( y = \frac{2}{x} \) can be obtained.

Write down the solutions for the simultaneous equations. [3]
1. $0 < a < 12$

2. $0 \leq x < 90^\circ$ or $270^\circ < x \leq 360^\circ$

3. $x = 5$

4. $y = -x - 12$

5. (ii) $\theta = 35.3^\circ, 144.7^\circ, 215.3^\circ, 324.7^\circ$

6. (i) $a = 1$
   (ii) 2
     (iii) period = $\frac{2\pi}{3}$, amplitude = 3
     (iv) $x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$

7. (i) $0 < m < \frac{1}{4}$
   (ii) $(5, 6)$ and $(-1, 6)$

8. (i) $V = 1500 \times 2^\frac{1}{3}$
   (ii) (Graph)
   (iii) 46.9 years

9. (i) $x = 11 \frac{2}{3}$
   (ii) $\frac{1}{18}$ units$^2$

10. (ii) $V = 60\pi r^2 - 2\pi r^3$
    (iii) $r = 20$, 25100 cm$^3$
1 (ii) \( f(x) = -3e^{-x} + \ln(x+1) + 4 \)

2 (i) \( \cos x \)

3 (i) \( \frac{9}{10} \) units\(^2\)

(ii) \( \frac{2}{3} \)

4 (a)(i) \(-8x + 28x^2 - 56x^3 + \ldots \)

(a)(ii) \(-560 \)

(b)(i) \( \left[ \begin{array}{c} 6 \\ r \\ \end{array} \right] (e^r) \left( \frac{1}{3} \right)^r x^{6-r} \)

(b)(ii) \(-48 \)

5 (i) \( 2 + \sqrt{3} \) cm

(ii) \( 14 + 8\sqrt{3} \)

(iii) \( m = 12 \) and \( n = 9 \), or \( m = 9 \) and \( n = 12 \)

6 (i) \( \frac{dy}{dx} = \frac{2\cos x - 1}{(2 - \cos x)^2} \left( \frac{\pi \sqrt{3}}{3} \right) \)

(ii) \( \frac{d^2y}{dx^2} = -\frac{2\sin x(1 + \cos x)}{(2 - \cos x)^3} \)

maximum point

7 (i) \((x-5)^2 + y^2 = 9 \)

(ii) \((x+1)^2 + (y-6)^2 = 9 \)

8 (a)(i) \(-7 \)

(a)(ii) \( k = 7, \ g(x) = (x+1)(3x^2 - 2x + 3) \)

(b) \( \frac{4}{9(2x+1)} - \frac{2}{9(x-1)} - \frac{5}{3(x-1)^2} \)

9 (i) \( 8\sin \theta + 5\cos \theta \)

(ii) \( \sqrt{89} \sin(\theta + 32.0^\circ) \)

(iii) \( 58.0^\circ \)

10 (i) 8

(ii) \(-\sqrt{7} \)

(iv) \( 4x^2 - 20x - 87 = 0 \)

11 (i) \( y = \frac{1}{b} (x) - \frac{a}{b} \)

(ii) \( b = 2, \ a = -1 \)

(iii) \( xy = 2, \ y = 1.5, \ x = 1.33 \)
ADDITIONAL MATHEMATICS
Paper 1

4047/01
10 May 2016, Tuesday

Additional Materials: Writing Papers (8 sheets) 2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on the separate writing papers provided.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to
three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

Setter: Mdm Chua Seow Ling

This paper consists of 5 printed pages including the coverpage.
Mathematical Formulae

1. ALGEBRA

For the equation $ax^2 + bx + c = 0$, 

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion 

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \cdots + \binom{n}{r} a^{n-r}b^r + \cdots + b^n,$$

where $n$ is a positive integer and 

$$\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)...(n-r+1)}{r!}$$

2. TRIGONOMETRY

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\csc^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bcsin A$$
Answer ALL Questions

1. Prove the identity \( \csc 2x + \cot 2x = \cot x \). [4]

2. Sketch the two parabola curves \( y = -4x^2 \) and \( x = -4y^2 \) on the same diagram. Hence find the equation of the straight line passes through the intersections of the two curves. [5]

3. Solve the following equations

(i) \( \log_2 x = \log_x (x + 2) \), [4]

(ii) \( \log_a 16 = \frac{1}{\log_a a} \) where \( a \) is a constant. [3]

4. Given the graph \( y = ax^2 + bx + c \) is always greater than the graph \( y = 4 \), where \( a \), \( b \) and \( c \) are constant. What conditions must apply to the constants \( a \) and \( c \)? [3]

5. Sketch the graph \( y = 2 \tan 3x - 1 \) for \( 0 < x < \frac{\pi}{2} \). Hence find the range of values of \( p \) such that \( p = 2 \tan 3x - 1 \) has exactly 2 solutions for \( 0 < x < \frac{\pi}{2} \). [4]

6. (i) Sketch the graph of \( y = 3 - 2x \), indicating clearly the \( x \) and \( y \)-intercepts. [3]

(ii) State the range of values of \( m \) for which the line \( y = mx + 2 \) intersects \( y = |3 - 2x| \) at two distinct points. [2]

7. Given \( y = \ln(2x + 1) + x^2 + x \), state the range of values of \( x \) for which \( y \) exists. Hence determine whether \( y \) is an increasing or decreasing function. Show all your workings clearly. [5]

8. (i) Find all the angles between 0 and \( 2\pi \) which satisfy the equation \( \sin(2x - \frac{\pi}{3}) \cos x = \cos x \). [5]

(ii) Without using a calculator, find the exact value of \( \sin 75^\circ + \cos 15^\circ \). [4]
9. The diagram shows an isosceles triangle $ABC$ which the coordinates of point $A$ and $B$ are $(-3, 2)$ and $(5, 7)$ respectively. $C$ is a point on the $y$-axis such that $AC = CB$. Find

(i) the coordinates of $C$; 

(ii) the equation of the line which bisects angle $ACB$. 

10. (i) Given that $\sin^2 x + 2\cos^2 x - 4$ can be expressed as $a\cos 2x + b$, where $a$ and $b$ are constants. Find the value of $a$ and of $b$. 

(ii) Hence for the graph of $y = \sin^2 x + 2\cos^2 x - 4$, state its

(a) amplitude,
(b) period,
(c) greatest value of $y$,
(d) least value of $y$. 

11. Given that $\sin x = -\frac{2}{\sqrt{5}}$ where $180^\circ < x < 270^\circ$, find

(i) $\cos(-x)$,

(ii) $\sin(x - 45^\circ)$,

(iii) $\sin(2x)$. 


12. An experiment was carried out to study the growth of a certain bacteria. It is given that the number of bacteria present at $t$ hours after the initial observation, is given by the equation $P = 250 + 420e^{kt}$ where $k$ is a constant.

(i) Find the number of bacteria at the beginning of the experiment.

(ii) Find the value of $k$ if the number of bacteria has doubled after 5 h.

(iii) Find the rate of change of the number of bacteria at 10 h.

13. In a Design and Technology competition, students are tasked to design a gigantic pencil. The criteria are shown below:

- Surface area of the pencil must be as small as possible
- Volume of the pencil as large as possible
- Mass of the pencil should not exceed 100 g.

Xi Rui shows the cross-section of her design which consists of a hemisphere, a cylinder and a right circular cone, all of their radius are $r$ cm as shown below. She lets the length of the cylinder be $h$ cm and the slant length of the cone be $3r$ cm. She uses wood that has a density of $\frac{3}{3\pi}$ g/cm$^3$ to make her gigantic pencil.

(i) Show that the greatest volume of the gigantic pencil that Xi Rui can make, is $60\pi$ cm$^3$.

(ii) Using the volume of the pencil in part (i), show that $h = \frac{60}{r^2} - \frac{2}{3}r\left(1 + \sqrt{2}\right)$.

(iii) Show the total surface area, $A$ cm$^2$, of the pencil is given by $A = \frac{1}{3}\pi r^2\left(1 - 4\sqrt{2}\right) + \frac{120\pi}{r}$.

(iv) Given that $r$ can vary, find the minimum value of $A$ and its corresponding $r$ value that Xi Rui used in her design.

End of Paper
Answers

2) \[ x = -4y^2 \]
   \[ y = x \]
   \[ y = -4x^2 \]

3i) \( x = 2 \)
   3ii) \( x = \frac{1}{16} \)

4) \( a > 0, \quad c > 4 \)

5) \( y = 2\tan(3x) - 1 \)

6i) \( y = |3 - 2x| \)

6ii) \(-\frac{4}{3} < m < 2 \)

7) \( x > \frac{1}{2} \)
\[ \frac{dy}{dx} = \frac{2}{2x+1} + 2x + 1 \]

Since \( 2x+1 > 0 \), \( \frac{2}{2x+1} > 0 \), \( \frac{dy}{dx} > 0 \) therefore the \( y \) is an increasing function.

8i) \( x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{12}, \frac{17\pi}{12} \)
8ii) \( \frac{1}{2} \sqrt{6} + \frac{1}{2} \sqrt{2} \)

9) \( C(0, 6.1) \)
\[ y = -\frac{8}{5}x + 6.1 \]

10i) \( a = \frac{1}{2} \)  
10ii) \( b = -2 \frac{1}{2} \)  
10iii) \( \text{amplitude} = \frac{1}{2} \) \( \text{period} = \pi \) or \( 180^\circ \) greatest \( y = -2 \) least \( y = -3 \)

11i) \( \frac{-\sqrt{5}}{5} \)  
11ii) \( \frac{-\sqrt{10}}{10} \)  
11iii) \( \frac{4}{5} \)

\( P = 670 \)

12) \( k = 0.191 \)
\[ \frac{dP}{dt} = 540 \text{ bacteria/h} \]
\[ r = 3.23 \text{ cm} \]

13i) \[ \frac{d^2A}{dx^2} = 33.571 > 0 \]
\( \text{therefore } A = 175 \text{ cm}^2 \) is a minimum value.
### NCHS Prelim Examination (2) 2016

**Additional Mathematics Paper 1 – Secondary 4 Express**

<table>
<thead>
<tr>
<th>Qn No</th>
<th>Suggested Solutions</th>
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<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>LHS</strong></td>
<td>( \cos ec 2x + \cot 2x = \cot x )</td>
</tr>
<tr>
<td></td>
<td>( LHS = \cos ec 2x + \cot 2x )</td>
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<td></td>
<td>( = \frac{1}{\sin 2x} + \frac{\cos 2x}{\sin 2x} )</td>
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<tr>
<td></td>
<td>( = \frac{1 + \cos 2x}{\sin 2x} )</td>
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<td></td>
<td>( = \frac{1 + 2\cos^2 x - 1}{2\sin x \cos x} )</td>
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<tr>
<td></td>
<td>( = \frac{2\cos^2 x}{2\sin x \cos x} )</td>
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<td></td>
<td>( = \cos x )</td>
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<tr>
<td></td>
<td>( = \frac{\sin x}{\cos x} )</td>
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<tr>
<td></td>
<td>( = \cot x )</td>
</tr>
<tr>
<td></td>
<td>( = RHS )</td>
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</tbody>
</table>

2. \( x = -4y^2 \)
   
   \( y = -4x^2 \)
   
   \( x = -4y^2 \)
   
   \( x = -4(-4x^2)^2 \)
   
   \( x = -64x^4 \)
   
   \( x + 64x^4 = 0 \)
   
   \( x(1 + 64x^3) = 0 \)
   
   \( 64x^3 = -1 \)
   
   \( x^3 = \frac{-1}{64} \)
   
   \( x = \frac{-1}{4} \) or \( x = 0 \)
   
   \( y = \frac{-1}{4} \) or \( y = 0 \)
   
   \( \left( \frac{-1}{4}, \frac{-1}{4} \right) \) and \( (0, 0) \)

\( y = x \)
3i 
\[ \log_{\frac{1}{2}} x = \log_2 x + 2 \] 
\[ \log_2 \frac{1}{2} = \log_2 x + 2 \] 
\[ \log_2 \frac{1}{2} = -\log_2 x - 2 \] 
\[ 2 \log_2 x = -2 \] 
\[ \log_2 x = -1 \] 
\[ x = 2 \]

3ii 
\[ \log_a 16 = -\frac{1}{\log_a a} \] 
\[ \log_a 16 = -\log_a x \] 
\[ x^{-1} = 16 \] 
\[ x = \frac{1}{16} \]

4 
\[ y = ax^2 + bx + c \] 
\[ ax^2 + bx + c > 4 \] 
\[ ax^2 + bx + c - 4 > 0 \] 
\[ a > 0, \quad c - 4 > 0 \] 
\[ c > 4 \]

5 
\[ y = 2 \tan 3x - 1 \] 
\[ p > -1 \]
6i \( \frac{4}{3} < m < 2 \)

\[ y = \ln(2x+1) + x^2 + x \]

\( 2x+1 > 0 \)

\( x > \frac{1}{2} \)

\[ \frac{dy}{dx} = \frac{2}{2x+1} + 2x+1 \]

Since \( 2x+1 > 0 \), \( \frac{2}{2x+1} > 0 \), \( \frac{dx}{dx} > 0 \)

therefore the \( y \) is an increasing function.

7

8i \( \sin \left( 2x - \frac{\pi}{3} \right) \cos x = \cos x \)

\[ \cos x \left( \sin \left( 2x - \frac{\pi}{3} \right) - 1 \right) = 0 \]

\[ \sin \left( 2x - \frac{\pi}{3} \right) = 1 \] or \[ \cos x = 0 \]

basic angle = \( \frac{2}{2} \)

\( 2x - \frac{\pi}{3} = \frac{\pi}{2}, \frac{2\pi}{2} \pi \)

or \[ x = \frac{\pi}{2}, \frac{3\pi}{2} \]

8ii \( \sin 75^\circ + \cos 15^\circ \)

\( = \sin (45 + 30) + \cos (45 - 30) \)

\( = \sin 45 \cos 30 + \sin 30 \cos 45 + \cos 45 \cos 30 + \sin 45 \sin 30 \)

\[ = \left( \frac{\sqrt{2}}{2} \right) \left( \frac{\sqrt{3}}{2} \right) + \left( \frac{1}{2} \right) \left( \frac{\sqrt{2}}{2} \right) + \left( \frac{\sqrt{2}}{2} \right) \left( \frac{\sqrt{3}}{2} \right) + \left( \frac{\sqrt{2}}{2} \right) \left( \frac{1}{2} \right) \]

\[ = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \]

\[ = \frac{1}{2} \sqrt{6} + \frac{1}{2} \sqrt{2} \]
9i \[ \sqrt{(-3)^2 + (2-y)^2} = \sqrt{5^2 + (7-y)^2} \]
y = 6.1 \quad C(0, 6.1)

9ii \[ m_1 = \frac{7-2}{5+3} = \frac{5}{8} \]
\[ m_2 = -\frac{8}{5} \]
y = -\frac{8}{5}x + 6.1

10i \[ \sin^2 x + 2\cos^2 x - 4 = a \cos 2x + b \]
RHS = \sin^2 x + 2\cos^2 x - 4
\[ = \frac{1 - \cos 2x}{2} + \cos 2x + 1 - 4 \]
\[ = \frac{1}{2}\cos 2x - 2 \frac{1}{2} \]
\[ a = \frac{1}{2} \quad b = -2 \frac{1}{2} \]

10ii\[ \text{amplitude} = \frac{1}{2} \]
period = \pi or 180°
greatest \ y = -2
least \ y = -3

11i \[ \cos(-x) = \cos x \]
\[ = -\frac{1}{\sqrt{5}} \]
\[ = -\frac{\sqrt{5}}{5} \]

11ii \[ \sin(x - 45°) = \sin x \cos 45 - \sin 45 \cos x \]
\[ = \left(\frac{-\sqrt{2}}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{-1}{\sqrt{5}}\right) \]
\[ = -\frac{\sqrt{2}}{\sqrt{5}} + \frac{\sqrt{2}}{2\sqrt{5}} \]
\[ = -\frac{1}{5}\sqrt{10} + \frac{1}{10}\sqrt{10} \]
\[ = -\frac{\sqrt{10}}{10} \]
11iii \[
\sin(2x) = 2 \sin x \cos x
\]
\[
= 2 \left( -\frac{2}{\sqrt{5}} \right) \left( -\frac{1}{\sqrt{5}} \right)
\]
\[
= \frac{4}{5}
\]

12i \[
P = 250 + 420e^{kt}
\]
\[
P = 250 + 420
\]
\[
= 670
\]

12ii \[
2(670) = 250 + 420e^{k(5)}
\]
\[
k = \frac{1}{5} \ln \frac{109}{42}
\]
\[
k = 0.191
\]

12iii \[
\frac{dP}{dt} = 420ke^{kt}
\]
\[
= 420 \left( \frac{1}{5} \ln \frac{109}{42} \right) e^{\left( \frac{1}{5} \ln \frac{109}{42} \right)(10)}
\]
\[
= 420 \left( \frac{1}{5} \ln \frac{109}{42} \right) e^{\left( 2 \ln \frac{109}{42} \right)}
\]
\[
= 420 \left( \frac{1}{5} \ln \frac{109}{42} \right) e^{\ln \left( \frac{109}{42} \right)^2}
\]
\[
= 420 \left( \frac{1}{5} \ln \frac{109}{42} \right) \left( \frac{109}{42} \right)^2
\]
\[
= 539.55
\]
\[
= 540 \text{ bacteria/h}
\]

13i \[
density = \frac{mass}{volume}
\]
\[
\frac{5}{3\pi} = \frac{100}{V}
\]
\[
V = 60\pi
\]
13ii

\[ V = \frac{2}{3} \pi r^3 + \pi r^2 h + \frac{1}{3} \pi r^3 (2\sqrt{2}r) \]

\[ 60\pi = \frac{2}{3} \pi r^3 + \pi r^2 h + \frac{2\sqrt{2}}{3} \pi r^3 \]

\[ h = \frac{\frac{2}{3} r^3 - 2\sqrt{2}}{r^2} \]

\[ h = \frac{60}{r^2} - \frac{2r}{3} \left( 1 + \sqrt{2} \right) \quad (Shown) \]

13iii

\[ A = 2\pi r^2 + 2\pi rh + \pi r (3r) \]

\[ = 2\pi r^2 + 2\pi \left( \frac{60}{r^2} - \frac{2r}{3} - \frac{2\sqrt{2}}{3} r \right) + 3\pi r^2 \]

\[ = 5\pi r^2 + \frac{120\pi}{r} - \frac{4}{3} \pi r^2 - \frac{4\sqrt{2}}{3} \pi r^3 \]

\[ = \frac{1}{3} \pi r^2 (11 - 4\sqrt{2}) + \frac{120\pi}{r} \quad (shown) \]

13iv

\[ \frac{dA}{dr} = \frac{2}{3} \pi (11 - 4\sqrt{2}) \frac{120\pi}{r^2} \]

\[ \frac{dA}{dr} = 0 \]

\[ \frac{2}{3} \pi (11 - 4\sqrt{2}) \frac{120\pi}{r^2} = 0 \]

\[ \frac{2}{3} \pi (11 - 4\sqrt{2}) r^3 = 120\pi \]

\[ r^3 = \frac{180}{(11 - 4\sqrt{2})} \]

\[ r = 3.2297 \]

\[ = 3.23 \text{ cm} \]

\[ \frac{d^2A}{dx^2} = \frac{2}{3} \pi (11 - 4\sqrt{2}) + \frac{240\pi}{r^3} \]

\[ = 33.571 > 0 \]

therefore \( A = 175 \text{ cm}^2 \) is a minimum value.

*** End of Paper ***
NAN CHIAU HIGH SCHOOL
PRELIMINARY EXAMINATION (2) 2016
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS
Paper 2

11 May 2016, Wednesday
2½ hours

Additional Materials:
Writing paper (3 sheets)
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 100.

Setter: Ms Renuka Ramakrishnan

This document consists of 7 printed pages including the coverpage.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Binomial expansion

\[(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \cdots + \binom{n}{r} a^{n-r}b^r + \cdots + b^n,\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\cdots(n-r+1)}{r!} \)

2. TRIGONOMETRY

\[
\sin^2 A + \cos^2 A = 1 \\
\sec^2 A = 1 + \tan^2 A \\
cosec^2 A = 1 + \cot^2 A \\
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \\
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \\
\sin 2A = 2\sin A \cos A \\
\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A \\
\tan 2A = \frac{2\tan A}{1 - \tan^2 A} \\
\]

Formulae for \( \triangle ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \\
a^2 = b^2 + c^2 - 2bc \cos A \\
A = \frac{1}{2} bc \sin A
\]
Answer all the questions.

1. Find the range of values of \( m \) for which the line \( x + 3y = m \) does not intersect the curve \( x(x + y) = -6 \).

2. The roots of the quadratic equation \( x^2 - 4x + 5 = 0 \) are \( \frac{\alpha}{2} \) and \( \frac{\beta}{2} \).
   (i) Find the value of \( \alpha^2 + \beta^2 \).
   (ii) Find a quadratic equation whose roots are \( \alpha^3 \) and \( \beta^3 \).

3. The function \( f \) is defined by \( f(x) = 2x^3 - 4x^2 - 2x + 4 \).
   (i) Determine, with appropriate workings, whether \( (x + 2) \) and \( (x - 2) \) are factors of \( f(x) \).
   (ii) Hence, by finding the roots of \( f(x) = 0 \), solve the equation \( 16y^3 - 16y^2 - 4y + 4 = 0 \).

4. A curve has the equation \( y = \ln \left( \cos^2 \frac{x}{4} \right) \). Show that the equation of the normal at the point \( x = \pi \) is \( y = ax + b \pi + c \ln 2 \), where \( a \), \( b \) and \( c \) are constants to be determined.

5. (a) (i) Find, in ascending powers of \( x \), the expansion of \( (2 + x)^8 \) as far as the term in \( x^3 \).
   (ii) Hence, determine the coefficient of \( a^3 \) in the expansion of \( (2 + a - 5a^2)^8 \).
(b) In the expansion of \( \left( x^2 - \frac{3}{x^4} \right)^{12} \), find
   (i) the middle term
   (ii) the term independent of \( x \).
6. (a) (i) Differentiate \( x^3 \ln 2x \) with respect to \( x \). \[ [1] \]
   (ii) Hence, find \( \int x^2 \ln 2x \, dx \). \[ [4] \]

(b) (i) Express \( \frac{1}{(x+3)(x+1)^2} \) as partial fractions. \[ [4] \]
   (ii) Hence, show that \( \int_0^2 \frac{1}{(x+3)(x+1)^2} \, dx = \frac{1}{4} \ln \frac{5}{9} + \frac{1}{3} \). \[ [4] \]

7. **Do not use a calculator in this question.**

(a) Simplify \( \frac{4^{1x} \times 8^{x-4}}{2^{7+x}} \). \[ [2] \]

(b)

The diagram shows a prism where the cross section is an isosceles triangle.

Given that \( AB = AC \), the length of \( BC \) is \( (\sqrt{3} - \sqrt{2}) \) cm, the length of \( CD \) is \( (3\sqrt{2} + 2\sqrt{3}) \) cm and the volume of the prism is 100 cm\(^3\), find

(i) the cross-sectional area of the prism, \[ [3] \]

(ii) the perpendicular height of \( A \) from \( BC \). \[ [4] \]
8. A curve has the equation \( y = \frac{3 - 2x}{x^2 + 2} \).

(i) Find the range of values of \( x \) for which \( y \) is defined. \([1]\)

(ii) Calculate the gradient of the curve when \( x = 1 \). \([3]\)

(iii) Given that \( x \) is decreasing at a rate of 0.05 units per second, find the rate of change of \( y \) when \( x = 1 \). \([2]\)

9. A rectangle of area, \( A \ \text{m}^2 \), has sides of length \( x \ \text{m} \) and \((Mx + N)\ \text{m} \), where \( M \) and \( N \) are constants.

Corresponding values of \( x \) and \( A \) are given in the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>4600</td>
<td>7400</td>
<td>8700</td>
<td>8000</td>
<td>5500</td>
</tr>
</tbody>
</table>

(i) Using suitable values, draw, on graph paper, a straight line graph. \([3]\)

(ii) Use your graph to estimate the value of \( M \) and \( N \). \([3]\)

(iii) On the same diagram, draw the straight line representing the equation \( A = x^3 \). Explain the significance of the value of \( x \) given by the point of intersection of the two lines and state this value of \( x \). \([4]\)

10. The equation of a circle, \( C_1 \), is \( x^2 + y^2 - 4x - 2y - 20 = 0 \).

(i) Find the centre and the radius of the circle. \([3]\)

(ii) Show that the point \( P(-2,4) \) is on the circle. \([1]\)

(iii) Find the equation of the smallest circle, \( C_2 \), passing through \( P \) and having its centre on the line \( x + 5y = 2 \). \([6]\)
11. In the diagram below, \( BC = 7 \text{ m} \), \( AB = 15 \text{ m} \) and angle \( \angle PAB = 90^\circ - \theta^\circ \). \( L \) is the perpendicular distance from \( C \) to \( AQ \).

(i) Show that \( L = a\sin \theta + b\cos \theta \), where \( a \), \( b \) are constants to be found. \( [3] \)

(ii) Express \( L \) in the form of \( R\sin(\theta + \alpha) \), where \( R > 0 \) and \( \alpha \) is an acute angle. \( [3] \)

(iii) Find the maximum value of \( L \) and the corresponding value of \( \theta \). \( [2] \)

(iv) Given that \( L = 12 \text{ m} \), find the value of \( \theta \). \( [3] \)
12. Figure 1 and Figure 2 shows the graphs of $f'(x)$ and $f''(x)$ respectively.

Using the information from figure 1 and/or figure 2,

(i) state the x-coordinates of all the stationary points of the graph $y = f(x)$ and determine the nature of the stationary points. [4]

(ii) find the interval(s) for which $f(x)$ is strictly decreasing. [2]

(iii) find the interval(s) for which $f'(x)$ is strictly increasing. [2]
Answer Key:

Q1) \(-12 < m < 12\)

Q2i) \(24\)  
Q2ii) \(x^2 - 32x + 8000 = 0\)

Q3ii) \(y = 1, -0.5, 0.5\)

Q5aii) \(256 + 1024x + 1792x^2 + 1792x^3 + \ldots\)  
Q5a) \(-16128\)  
Q5bi) \(673596x^{-12}\)  
Q5bis) \(40095\)

Q6ai) \(3x^2 \ln 2x + x^3\)

Q6bi) \(\frac{1}{4(x+3)} - \frac{1}{4(x+1)} + \frac{1}{2(x+1)^3}\)

Q7a) \(2^{8-\pi}\)

Q7aii) \(\frac{150\sqrt{2} - 100\sqrt{3}}{3}\)  
Q7bi) \(\frac{100}{3} \sqrt{6}\)

Q8i) \(x \leq \frac{3}{2}\)  
Q8ii) \(-\frac{4}{9} \sqrt{3}\)  
Q8iii) \(0.0385 \text{ units/sec}\)

Q9i) \(M = -8.75, N = 550\)

Q9ii) The rectangle becomes a square. \(x = 56.5\)

Q10i) Centre \((2, 1)\) and radius = 5 units  
Q10ii) \(\left(x + \frac{34}{13}\right)^2 + \left(y - \frac{12}{13}\right)^2 = \frac{128}{13}\)

Q11i) \(L = 7\cos \theta + 15\sin \theta\)

Q11ii) \(L = 16.6\cos(\theta + 25.0^\circ)\)

Q11iii) Max value = 16.6m  
\(\theta = 65.0^\circ\)

Q11iv) \(\theta = 21.4^\circ\)

Q12i) At \(x = -2\), minimum point  
At \(x = -1\), maximum point  
At \(x = 3\), minimum point

Q12ii) \(x < -2\)  
\(-1 < x < 3\)

Q12iii) \(x < -1.53\)  
\(x > 1.53\)
Additional Mathematics  – Secondary 4 Express

Nan Chiau High School

Prelim Examination (2) 2016

Marking Scheme (Paper 2)

<table>
<thead>
<tr>
<th>Qn No</th>
<th>Suggested Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x + 3y = m$</td>
</tr>
<tr>
<td></td>
<td>$y = -\frac{1}{3}x + \frac{1}{3}m = -(1)$</td>
</tr>
<tr>
<td></td>
<td>$x(x + y) = 6 - (2)$</td>
</tr>
</tbody>
</table>

Sub (1) into (2):

$x(\frac{1}{3}x + \frac{1}{3}m) = 6$  \[M1\]

$\frac{2}{3}x^2 + \frac{1}{3}mx + 6 = 0$  \[M1\]

$2x^2 + mx + 18 = 0$

Since there is no intersection, $\text{discriminant } < 0$

$(m)^2 - 4(2)(18) < 0$  \[M1\]

$m^2 - 144 < 0$

$(m + 12)(m - 12) < 0$

$\therefore -12 < m < 12$  \[A1\]

2i

$\frac{\alpha + \beta}{2} = 4$  \[M1\]

$\alpha + \beta = 8$

$\left(\frac{\alpha}{2}\right) \left(\frac{\beta}{2}\right) = 5$

$\alpha \beta = 20$

$\alpha^2 + \beta^2$

$= (\alpha + \beta)^2 - 2\alpha\beta$

$= 8^2 - 2(20)$  \[M1\]

$= 24$  \[A1\]
2ii \[
\alpha^3 + \beta^3 \\
= (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) \\
= 8(24 - 20) \\
= 32 \\
\alpha^3 \beta^3 \\
= (\alpha\beta)^3 \\
= 8000
\]

New equation:
\[x^2 - 32x + 8000 = 0\]

3i \[f(x) = 2x^3 - 4x^2 - 2x + 4\]

When \(x = -2\),
\[f(x) = 2(-2)^3 - 4(-2)^2 - 2(-2) + 4 = -24\]
\[\therefore (x + 2)\text{ is not a factor of } f(x)\]

When \(x = 2\),
\[f(x) = 2(2)^3 - 4(2)^2 - 2(2) + 4 = 0\]
\[\therefore (x - 2)\text{ is not a factor of } f(x)\]

3ii \[f(x) = 0\]
\[(x - 2)(2x^2 - 2) = 0\]
\[(x - 2)(x + 1)(x - 1) = 0\]
\[\therefore x = 2, -1, 1\]

Let \(x = 2\),
\[16y^3 - 16y^2 - 4y + 4 = 0\]
\[2(2y)^3 - 4(2y)^2 - 2(2y) + 4 = 0\]
\[\therefore y = 1, -0.5, 0.5\]
\[ y = \ln(\cos^3 \left( \frac{x}{4} \right)) \]
\[ \frac{dy}{dx} = \frac{1}{\cos^2 \left( \frac{x}{4} \right)} \times 2 \cos \left( \frac{x}{4} \right) \times \frac{1}{4} \sin \left( \frac{x}{4} \right) \]
\[ = \frac{\sin \left( \frac{x}{4} \right)}{2 \cos \left( \frac{x}{4} \right)} \]
\[ = -\frac{1}{2} \tan \left( \frac{x}{4} \right) \]
\[ \frac{dy}{dx}_{x=\pi} = -\frac{1}{2} \]

\[ \therefore \text{gradient of normal} = 2 \]

When \( x = \pi \),
\[ y = \ln \frac{1}{2} = -\ln 2 \]

\[ \therefore \text{equation of normal:} \]
\[ y + \ln 2 = 2(x - \pi) \]
\[ y = 2x - 2\pi - \ln 2 \]

\[ \text{ai} \]
\[ (2 + x)^4 \]
\[ = 2^8 + \binom{8}{1}2^7 + \binom{8}{2}2^6 x^2 + \binom{8}{3}2^5 x^3 + \ldots \]
\[ = 256 + 1024x + 1792x^2 + 1792x^3 + \ldots \]

\[ \text{a} \]
\[ \text{iii} \]
\[ \text{Let} \ x = a - 5a^2 \]
\[ (2 + a - 5a^2)^4 \]
\[ = 1792(a - 5a^2)^3 + 1792(a - 5a^2)^2 + \ldots \]
\[ = 1792(-10a^3) + 1792(a^3) + \ldots \]
\[ = -16128a^3 + \ldots \]

\[ \therefore \text{coefficient of} \ a^3 = -16128 \]
5bi \[ T_7 = \binom{12}{6} (x^2)^6 \left( -\frac{3}{x^4} \right)^6 \]
\[ = 673596x^{-12} \]
(M1)
(A1)

5bii \[ T_{11} = \binom{12}{r} (x^2)^{12-r} \left( -\frac{3}{x^4} \right)^r \]
\[ = \binom{12}{r} (-3)^r (x^{24-6r}) \]
(M1)

For term independent of \( x \):
\[ 24 - 6r = 0 \]
\[ r = 4 \]
(M1)

\[ T_5 = \binom{12}{4} (-3)^4 (x^0) \]
\[ = 40095 \]
(A1)

6ai \[ \frac{d}{dx} x^3 \ln 2x = 3x^2 \ln 2x + x^2 \]
(B1)

a(ii)
\[ \int 3x^2 \ln 2x + x^2 \, dx = x^3 \ln 2x + c \]
(M1)

\[ 3 \int x^2 \ln 2x \, dx = x^3 \ln 2x - \int x^3 \, dx + c \]
(M1)

\[ 3 \int x^2 \ln 2x \, dx = x^3 \ln 2x - \frac{1}{3} x^3 + c_1 \]
(M1)

\[ \int x^2 \ln 2x \, dx = \frac{1}{3} x^3 \ln 2x - \frac{1}{9} x^3 + c_2 \]
(A1)
\[
\frac{1}{(x+3)(x+1)^2} = \frac{A}{x+3} + \frac{B}{x+1} + \frac{C}{(x+1)^2}
\]

\[
1 = A(x+1)^2 + B(x+1)(x+3) + C(x+3)
\]

\[
A = \frac{1}{4}
\]

\[
B = -\frac{1}{4}
\]

\[
C = \frac{1}{2}
\]

\[
\frac{1}{(x+3)(x+1)^2} = \frac{1}{4(x+3)} - \frac{1}{4(x+1)} + \frac{1}{2(x+1)^2}
\]

\[
\int_a^2 \frac{1}{(x+3)(x+1)} \, dx = \int_a^2 \left( \frac{1}{4(x+3)} - \frac{1}{4(x+1)} + \frac{1}{2(x+1)^2} \right) \, dx
\]

\[
= \left[ \frac{1}{4} \ln(x+3) - \frac{1}{4} \ln(x+1) - \frac{1}{2(x+1)^2} \right]_a^b
\]

\[
= \frac{1}{4} \ln 5 - \frac{1}{4} \ln 3 - \frac{1}{6} + \frac{1}{4} \ln 3 + \frac{1}{4} + \frac{1}{2}
\]

\[
= \frac{1}{4} \ln \frac{5}{3} + \frac{1}{3}
\]

\[
\frac{4^{3x} \times 8^{3-4}}{2^{7+x}}
\]

\[
= 2^{6x + 3x - 12 - (7 + x)}
\]

\[
= 2^{8x - 19}
\]

\[
\frac{100}{3\sqrt{2} + 2\sqrt{3}} = \frac{100(3\sqrt{2} - 2\sqrt{3})}{(3\sqrt{2})^2 - (2\sqrt{3})^2}
\]

\[
= \frac{150\sqrt{2} - 100\sqrt{3}}{3}
\]

A1
7bii
\[
k = \frac{150\sqrt{2} - 100\sqrt{3}}{3} + (\sqrt{3} - \sqrt{2}) \times 2
\]
\[
= \frac{2(150\sqrt{2} - 100\sqrt{3})(\sqrt{3} + \sqrt{2})}{3(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})}
\]
\[
= \frac{300\sqrt{6} + 600 - 600 - 200\sqrt{6}}{3}
\]
\[
= \frac{100}{3}\sqrt{6}
\]

8i
\[
\therefore x^2 + 2 > 0,
\]
\[
3 - 2x > 0
\]
\[
x < \frac{3}{2}
\]

\[
\frac{dy}{dx} = \frac{1}{2(x^2 + 2)} \times -2(x^2 + 2) - 2x(3 - 2x)
\]
\[
= \frac{-4}{9}\sqrt{3}
\]

iii
\[
\frac{dy}{dt} = \frac{-4}{9}\sqrt{3} \times (-0.05)
\]
\[
= \frac{\sqrt{3}}{45} \text{ units/sec}
\]

Or = 0.0385 units/sec (3sf)

10i
\[
x^2 + y^2 - 4x - 2y - 20 = 0
\]
\[
(x - 2)^2 + (y - 1)^2 = 25
\]
Centre (2,1) and radius = 5 units

10ii
When \(x = -2,
\]
\[
(-2)^2 + y^2 - 4(-2) - 2y - 20 = 0
\]
\[
y^2 - 2y - 8 = 0
\]
\[
(y - 4)(y + 2) = 0
\]
\[
y = 4, y = -2
\]
\[
\therefore \text{P lies on the circle}
\]
10iii  
\[ y = -\frac{1}{5}x + \frac{2}{5} \]

Gradient of line = \(-\frac{1}{5}\)  

Gradient of perpendicular line = 5  

Equation of line passing through P with \(m = 5\)  
\[ y - 4 = 5(x + 2) \]
\[ y = 5x + 14 \]
\[ \therefore 5x + 14 = -\frac{1}{5}x + \frac{2}{5} \]
\[ x = -\frac{34}{13}, y = \frac{12}{13} \]

Radius of \(C_2\)
\[ = \sqrt{(\frac{2}{13})^2 + (\frac{12}{13})^2} \]
\[ = \frac{\sqrt{128}}{13} \]

\[ \therefore \text{equation of } C_2: \]
\[ \left(x + \frac{34}{13}\right)^2 + \left(y - \frac{12}{13}\right)^2 = \frac{128}{13} \]

11i

\[ \cos \theta = \frac{CS}{7}, \sin \theta = \frac{AR}{15} \]
\[ \therefore L = CS + AR \]
\[ L = 7\cos \theta + 15\sin \theta \]
\[ L = 7\cos\theta + 15\sin\theta \]

\[ R = \sqrt{274} \]

\[ \tan\alpha = \frac{15}{7} \]

\[ \alpha = 25.017 \]  \( \text{M1} \)

\[ L = 16.6\cos(\theta + 25.0^\circ) \]  \( \text{A1} \)

Max value = 16.6m

\[ \theta = 65.0^\circ \]  \( \text{B1} \)

\[ \sqrt{274}\sin(\theta + 25.017^\circ) = 12 \]  \( \text{M1} \)

\[ \sin(\theta + 25.017^\circ) = 0.72495 \]  \( \text{M1} \)

\[ \alpha = 46.464^\circ \]  \( \text{A1} \)

\[ \theta + 25.017^\circ = 46.464^\circ \]  \( \text{A1} \)

\[ \theta = 21.4^\circ \]  \( \text{A1} \)

Stationary points:

\[ x = -2, x = -1, x = 3 \]  \( \text{B1} \)

At \( x = -2 \), minimum point
At \( x = -1 \), maximum point
At \( x = 3 \), minimum point  \( \text{B3} \)

\[ x < -2 \]

\[ -1 < x < 3 \]  \( \text{B2} \)

\[ x < -1.53 \]

\[ x > 1.53 \]  \( \text{B2} \)
(i) \[ A = x(Mx + N) \]
\[ A = Mx^2 + Nx \]
\[ \frac{4}{x} + Mx + N \]

<table>
<thead>
<tr>
<th>x</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>\frac{A}{x}</td>
<td>440</td>
<td>370</td>
<td>340</td>
<td>300</td>
<td>210</td>
</tr>
</tbody>
</table>

(ii) \[ N = 100 \]
\[ M = \frac{1000 - 10}{10} \]
\[ M = 99 \]
\[ A = 11 \]

(iii) \[ A = x \]
\[ \frac{A}{x} = x \]
\[ M = 11 \]
\[ A = 11 \]
\[ x = 11 \]

- rectangle becomes a square. 

*** End of Paper ***
(b) Express in terms of \( x \) and \( y \) the equation of the line \( y = 2x + 3 \) and its gradient.

(2)

(c) From a multiple choice question, explain how you would select the correct answer.

(d) If \( x = 3 \) and \( y = 2 \), find the value of \( x + y \).

(e) Find the equation of the straight line with gradient \( m = 2 \) and passing through the point \( (1, 3) \).

(f) If \( a = 4 \), find the value of \( a^2 - 9 \).

(g) Solve the simultaneous equations:

\[
\begin{align*}
2x + y &= 5 \\
x - y &= 1
\end{align*}
\]

(h) The line \( y = x + 2 \) cuts the curve \( y = x^2 + 3x + 2 \) at two points. Find these points.

(i) Given that \( x \) is a root of \( 2x^2 - 3x + 1 = 0 \), find the value of \( 2x - 3 \).

(j) The area of a square is equal to the area of a circle. If the side of the square is 4 cm, find the radius of the circle.

(k) The total number of marks for this paper is 80. You are required to show all your working and answer each question in its entirety. You may use a calculator for any figures or calculations. Write your answers in the spaces provided at the top of this page and in all the spaces you have in your answer space.

READ THESE INSTRUCTIONS FIRST.
END OF PAPER

PROBLEMS

1. (a) Find the volume of the pyramid shown in Figure 1.
(b) Find the volume of the pyramid shown in Figure 2.
(c) Show that the volume of the pyramid in Figure 1 is equal to twice the volume of the pyramid in Figure 2.

Figure 1

Figure 2

2. Help has been given on the volume of the pyramid in Figure 1.

3. (a) Find the derivative of the function f(x) = x^3 - 4x^2 + 2x - 1.
(b) Find the derivative of the function g(x) = e^x - x^2.
(c) Find the derivative of the function h(x) = sin(x) + cos(x).

4. (a) Find the minimum value of the function f(x) = x^2 - 6x + 9.
(b) Find the maximum value of the function g(x) = -x^2 + 4x - 3.
(c) Find the value of the function h(x) = x^3 - 3x^2 + 2x - 1 when x = 2.

5. (a) Find the value of the expression 2x^2 + 3xy - 4y^2 when x = 1 and y = 2.
(b) Find the value of the expression 3x^2 - 2xy + y^2 when x = 3 and y = 1.
(c) Find the value of the expression 4x^2 + 5xy - 6y^2 when x = -1 and y = 2.

6. (a) Find the value of the expression 2x^2 + 3xy - 4y^2 when x = 1 and y = 2.
(b) Find the value of the expression 3x^2 - 2xy + y^2 when x = 3 and y = 1.
(c) Find the value of the expression 4x^2 + 5xy - 6y^2 when x = -1 and y = 2.
### Question 11
For the given linear equation, \( y = 2x + 3 \), find the slope and the y-intercept.

**Answer:** The slope is 2 and the y-intercept is 3.

### Question 12
Solve the following equation:

\[ 6 + \frac{x}{2} - 3x = 2 \]

**Answer:** Solve for \( x \).

### Question 13
Simplify the expression:

\[ \frac{2 + x}{x} + \frac{3}{x} \]

**Answer:** Simplify the expression.

### Question 14
Solve the following equation:

\[ x^2 + 2x - 3 = 0 \]

**Answer:** Solve for \( x \).

### Question 15
Solve the following equation:

\[ \frac{x}{2} + \frac{3}{x} = \frac{5}{2} \]

**Answer:** Solve for \( x \).

### Question 16
Solve the following equation:

\[ 3 - x^2 = 2x - 1 \]

**Answer:** Solve for \( x \).

### Question 17
Solve the following equation:

\[ \ln x = 2 \]

**Answer:** Solve for \( x \).

### Question 18
Solve the following equation:

\[ \frac{1}{3}x + 2 = \frac{1}{2} \]

**Answer:** Solve for \( x \).

### Question 19
Solve the following equation:

\[ 2x - x = 1 \]

**Answer:** Solve for \( x \).

### Question 20
Solve the following equation:

\[ \frac{3}{2}x = 6 \]

**Answer:** Solve for \( x \).

### Question 21
Solve the following equation:

\[ 2x^2 - 3x + 1 = 0 \]

**Answer:** Solve for \( x \).

### Question 22
Solve the following equation:

\[ x^2 + 4x + 4 = 0 \]

**Answer:** Solve for \( x \).

### Question 23
Solve the following equation:

\[ x^2 - 2x + 1 = 0 \]

**Answer:** Solve for \( x \).

### Question 24
Solve the following equation:

\[ x^2 - 5x + 6 = 0 \]

**Answer:** Solve for \( x \).

### Question 25
Solve the following equation:

\[ x^2 - 9 = 0 \]

**Answer:** Solve for \( x \).

### Question 26
Solve the following equation:

\[ x^2 - 4x + 4 = 0 \]

**Answer:** Solve for \( x \).

### Question 27
Solve the following equation:

\[ x^2 - 6x + 9 = 0 \]

**Answer:** Solve for \( x \).

### Question 28
Solve the following equation:

\[ x^2 - 8x + 16 = 0 \]

**Answer:** Solve for \( x \).

### Question 29
Solve the following equation:

\[ x^2 - 10x + 25 = 0 \]

**Answer:** Solve for \( x \).

### Question 30
Solve the following equation:

\[ x^2 - 12x + 36 = 0 \]

**Answer:** Solve for \( x \).

### Question 31
Solve the following equation:

\[ x^2 - 14x + 49 = 0 \]

**Answer:** Solve for \( x \).

### Question 32
Solve the following equation:

\[ x^2 - 16x + 64 = 0 \]

**Answer:** Solve for \( x \).

### Question 33
Solve the following equation:

\[ x^2 - 18x + 81 = 0 \]

**Answer:** Solve for \( x \).

### Question 34
Solve the following equation:

\[ x^2 - 20x + 100 = 0 \]

**Answer:** Solve for \( x \).

### Question 35
Solve the following equation:

\[ x^2 - 22x + 121 = 0 \]

**Answer:** Solve for \( x \).

### Question 36
Solve the following equation:

\[ x^2 - 24x + 144 = 0 \]

**Answer:** Solve for \( x \).

### Question 37
Solve the following equation:

\[ x^2 - 26x + 169 = 0 \]

**Answer:** Solve for \( x \).

### Question 38
Solve the following equation:

\[ x^2 - 28x + 225 = 0 \]

**Answer:** Solve for \( x \).

### Question 39
Solve the following equation:

\[ x^2 - 30x + 225 = 0 \]

**Answer:** Solve for \( x \).

### Question 40
Solve the following equation:

\[ x^2 - 32x + 256 = 0 \]

**Answer:** Solve for \( x \).

### Question 41
Solve the following equation:

\[ x^2 - 34x + 289 = 0 \]

**Answer:** Solve for \( x \).

### Question 42
Solve the following equation:

\[ x^2 - 36x + 361 = 0 \]

**Answer:** Solve for \( x \).

### Question 43
Solve the following equation:

\[ x^2 - 38x + 361 = 0 \]

**Answer:** Solve for \( x \).

### Question 44
Solve the following equation:

\[ x^2 - 40x + 361 = 0 \]

**Answer:** Solve for \( x \).

### Question 45
Solve the following equation:

\[ x^2 - 42x + 361 = 0 \]

**Answer:** Solve for \( x \).

### Question 46
Solve the following equation:

\[ x^2 - 44x + 361 = 0 \]

**Answer:** Solve for \( x \).

### Question 47
Solve the following equation:

\[ x^2 - 46x + 361 = 0 \]

**Answer:** Solve for \( x \).

### Question 48
Solve the following equation:

\[ x^2 - 48x + 361 = 0 \]

**Answer:** Solve for \( x \).

### Question 49
Solve the following equation:

\[ x^2 - 50x + 361 = 0 \]

**Answer:** Solve for \( x \).

### Question 50
Solve the following equation:

\[ x^2 - 52x + 361 = 0 \]

**Answer:** Solve for \( x \).
The equation modelled the students' spread of the number of hours spent on the number of hours after the school is open. The students marked for the equation $y = ax^2 + bx + c$, where $a$, $b$, and $c$ are the positive numbers. 

\[ y = ax^2 + bx + c \]

2. \( a, b, \) and \( c \) are the roots of the quadratic equation $ax^2 + bx + c = 0$, where \( a, b, \) and \( c \) are positive.
[9]

(i) Find the area of the shaded region bounded by the curve, the normal to the curve at the point (2, 0), and the x-axis, giving your answer to 2 decimal places.

(ii) The diagram above shows the curve with equation \( y = \frac{1}{2}x^2 + x - 3 \) and the normal to the curve at the point (1, -2). Show that the equation of the normal to the curve at this point is given by \( 2x + y + 4 = 0 \).

[9]

(i) It is possible for the perimeter of \( ABCDE \) to be 70 cm.

(ii) Determine the maximum value of \( p \) and the corresponding value of \( \alpha \), where \( \alpha \) is in radians, and \( P \) is in centimeters.

(iii) Show that the perimeter, \( P \), of \( ABCDE \) is given by \( P = 7\pi + 4\cos\theta + 4\sin\theta \).

[9]

\[ CE = 12 \text{ cm}, \quad \angle ACB = 90^\circ \quad \text{and} \quad \angle ACD = \theta \quad \text{where} \quad \theta \quad \text{is an acute angle measured in} \]

\[ \text{degrees} \]

\[ \quad \text{If} \quad \angle DEF = 35^\circ \quad \text{then} \quad \angle D = 35^\circ \quad \text{cm} \]

\[ \quad \text{ABCD} \quad \text{is a rectangle with} \quad AB \quad \text{parallel to} \quad DE \quad \text{if it is given that} \quad AC = 35 \quad \text{cm} \]
END OF PAPER

Please write your answer without including the instructions.

1. Write your result from (a) and (c). Name the points D, E, F, G, and H.

2. Prove that \( \overline{AD} \) is parallel to \( \overline{CD} \).

3. Prove that \( \overline{BE} \) is parallel to \( \overline{DE} \).

4. Prove that the line \( \overline{EF} \) is parallel to the line \( \overline{GH} \).
1

In the diagram, the right-angle triangle \( ABC \) is such that \( BC = 12 \text{ cm} \),
\[
\angle ABC = \frac{\pi}{6} \text{ and } AX = \frac{2}{3} AB.
\]
Show that \( \cos \angle BXC = -\frac{2\sqrt{7}}{7} \). \([4]\)

2

Solve the equation \( 6 \cos x = 4 \sec x - \tan x \) for \( 0 < x < \frac{\pi}{2} \). \([5]\)

3

Air leaks from a spherical balloon at a constant rate of \( 25\pi \) \( \text{cm}^3 \) per second. Given
that the initial volume is \( 5000\pi \) \( \text{cm}^3 \),
(i) calculate the radius of the balloon after 20 seconds, \([3]\)
(ii) find the rate of change of radius at this instant. \([2]\)

4

A curve is such that \( \frac{d^2 y}{dx^2} = 6x - 6 \). The gradient of the curve at the point \((2, -1)\) is 4.
(i) Show that \( y \) is an increasing function for all real values of \( x \). \([4]\)
(ii) Find the equation of the curve. \([2]\)

[Turn over…]
5 Given the cubic expression \( f(x) = x^3 + px^2 + qx + 4 \) has a factor \((x + 2)\) and leaves a remainder of 6 when divided by \((x + 1)\),

(i) find the value of \(p\) and of \(q\), [4]

(ii) factorize \(f(x)\) completely. [2]

6 (a) Simplify the expression \( \frac{3^{n-2} - 3^{n+1}}{3^{n+2} - 3^{n-1}} \). [3]

(b) Solve the equation \( \log_2 8x = 4 \log_x 2 \). [4]

7 Given that the roots of the equation \( 2x^3 - 2x + 5 = 0 \) are \(\alpha\) and \(\beta\).

(i) Show that \(\alpha^2 + \beta^2 = -4\). [2]

(ii) Find the value of \(\alpha^3 + \beta^3\). [2]

(iii) Find a quadratic equation whose roots are \(\frac{\alpha}{2\beta^2}\) and \(\frac{\beta}{2\alpha^2}\). [4]

8 The equation of the curve is given by \(y = 3\cos 3x - 2\) for \(0 \leq x \leq \pi\).

(i) Write down the amplitude and period of \(y\). [2]

(ii) Find the coordinates of the maximum and minimum points for \(0 < x < \pi\). [2]

(iii) Calculate the values of \(x\) for which the curve cuts the \(x\)-axis. [2]

(iv) Sketch the curve \(y = 3\cos 3x - 2\) for \(0 \leq x \leq \pi\). [2]

(v) State the range of values of \(x\) for which \(y\) is decreasing between 0 and \(\pi\). [2]
9. A solid spherical ball is dropped into a cone of height \( h \) cm and radius \( r \) cm.

Given that the radius of the spherical ball is 5 cm,

(i) show that the volume of the cone, \( V \) is given by \( V = \frac{25\pi h^2}{3(h-10)} \). \([3]\)

(ii) Given that \( h \) can vary, find the value of \( h \) for which \( V \) has a stationary value. \([3]\)

(iii) Calculate this stationary value of \( V \) and determine if the volume is a maximum or minimum value. \([3]\)

10. (i) Express \( \frac{4x^3 + 7x^2 + 4x - 2}{(2x-1)(x^2 + 2)} \) in partial fractions. \([5]\)

(ii) Differentiate \( \ln(x^2 + 2) \) with respect to \( x \). \([1]\)

(iii) Hence evaluate \( \int_1^2 \frac{4x^3 + 7x^2 + 4x - 2}{(2x-1)(x^2 + 2)} \, dx \). \([4]\)

[Turn over...]
The table show experimental values of two variables \( x \) and \( y \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>3.24</td>
<td>5.79</td>
<td>9</td>
<td>17.05</td>
<td>38.43</td>
</tr>
</tbody>
</table>

It is known that \( x \) and \( y \) are related by the equation \( \frac{y - b}{x} = a \sqrt{x} - 1 \) for \( x > 0 \) where \( a \) and \( b \) are constants.

(i) Using a scale of 1 cm to 2 units on the horizontal axis and 2 cm to 5 units on the vertical axis, draw a straight line graph of \( x + y \) against \( x \sqrt{x} \). \[3\]

(ii) Use your graph to estimate, to 2 decimal places, the value of \( a \) and of \( b \). \[4\]

(iii) On the same diagram, draw a straight line representing the equation \( y + x + 2x \sqrt{x} = 36 \).

Hence find the value of \( x \) that satisfies the equation \( (a + 2)x \sqrt{x} = 36 - b \). \[3\]

~ End of Paper ~
In the diagram, the right-angle triangle $ABC$ is such that $BC = 12$ cm.

$\angle ABC = \frac{\pi}{6}$ and $AX = \frac{2}{3} AB$.

Show that $\cos \angle BXC = -\frac{2\sqrt{7}}{7}$. \[4\]

[soln] 

$\cos \angle BXC = -\cos \angle AXC$

$$\sin \frac{\pi}{6} = \frac{AC}{12} \implies AC = 6$$

$$AB = \sqrt{144 - 36} = \sqrt{108} = 6\sqrt{3}$$

$$AX = 4\sqrt{3}$$

$$CX = \sqrt{36 + 48} = \sqrt{84} = 2\sqrt{21}$$

$$\cos \angle BXC = -\cos \angle AXC = -\frac{4\sqrt{3}}{2\sqrt{21}} = -\frac{2}{\sqrt{7}} = -\frac{2\sqrt{7}}{7}$$

2. Solve the equation $6\cos x = 4\sec x - \tan x$ for $0 < x < \pi$. \[5\]

[soln] 

$$6\cos x = \frac{4}{\cos x} - \tan x$$

$$6\cos x = 4 - \sin x$$

$$6(1 - \sin^2 x) = 4 - \sin x$$

$$6\sin^2 x - \sin x - 2 = 0$$

$$(3\sin x - 2)(2\sin x + 1) = 0$$

$$\sin x = \frac{2}{3} \quad \text{or} \quad \sin x = -\frac{1}{2}$$

Basic angle $= 0.7297$ \quad Basic angle $= 0.5236$

$x = 0.730, 2.41$ \quad $x = 2.62, 5.76$ (NA)
3) Air leaks from a spherical balloon at a constant rate of $25\pi \text{ cm}^3$ per second. Given that the initial volume is $5000\pi \text{ cm}^3$,

(i) calculate the radius of the balloon after 20 seconds,

(ii) find the rate of change of radius at this instant.

[soln] \[ \frac{dV}{dt} = -25\pi \]

After 20s, volume = $5000\pi - 25\pi \times 20 = 4500\pi$

\[ \frac{4}{3}\pi r^3 = 4500\pi \]

\[ r^3 = 3375 \]

\[ r = 15 \]

\[ \frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \]

\[ -25\pi = 4\pi r^2 \times \frac{dr}{dt} \]

\[ \frac{dr}{dt} = \frac{-25}{4 \times 225} = \frac{-25}{900} = -\frac{1}{36} \text{ cm/s} \]

4) A curve is such that $\frac{d^2y}{dx^2} = 6x - 6$. The gradient of the curve at the point $(2, -1)$ is 4.

(i) Show that $y$ is an increasing function for all real values of $x$.

(ii) Find the equation of the curve.

[soln] \[ \frac{d^2y}{dx^2} = 6x - 6 \]

\[ \frac{dy}{dx} = 3x^2 - 6x + c \]

At $(2, -1)$, \[ \frac{dy}{dx} = 4 \]

$12 - 12 + c = 4$

$c = 4$

\[ \frac{dy}{dx} = 3x^2 - 6x + 4 \]

\[ \frac{dy}{dx} = 3(x^2 - 2x) + 4 \]

\[ \frac{dy}{dx} = 3(x - 1)^2 + 1 \]

For all values of $x$, \[ \frac{dy}{dx} > 0 \], $y$ is increasing.
\[ y = x^3 - 3x^2 + 4x + d \]

\[ 8 - 12 + 8 + d = -1 \]
\[ d = -5 \]
\[ y = x^3 - 3x^2 + 4x - 5 \]

5. Given the cubic expression \( f(x) = x^3 + px^2 + qx + 4 \) has a factor \((x+2)\) and leaves a remainder of 6 when divided by \((x+1)\),

(i) find the value of \(p\) and of \(q\).

(ii) factorize \(f(x)\) completely.

\[ \text{[soln]} \]
\[ -8 + 4p - 2q + 4 = 0 \]
\[ 2p - q = 2 \]
\[ -1 + p - q + 4 = 6 \]
\[ p - q = 3 \]
\[ p = -1, q = -4 \]

\[ f(x) = x^3 - x^2 - 4x + 4 \]
\[ f(x) = (x+2)(x^2 - 3x + 2) \]
\[ f(x) = (x+2)(x-2)(x-1) \]

6. (a) Simplify the expression \( \frac{3^{n-2} - 3^{n+1}}{3^{n+2} - 3^{n-1}} \).

(b) Solve the equation \( \log_2 8x = 4 \log_x 2 \).

\[ \text{[soln]} \]
\[ (a) \quad \frac{3^{n-2} - 3^{n+1}}{3^{n+2} - 3^{n-1}} = \frac{3^n \left( \frac{1}{9} - 3 \right)}{3^n \left( \frac{9 - 1}{3} \right)} = -\frac{1}{3} \]

(b) \[ \log_2 8x = 4 \log_x 2 \]
\[ \log_2 8 + \log_2 x = \frac{4 \log_2 2}{\log_2 x} \]
\begin{align*}
3 + \log_2 x &= \frac{4}{\log_2 x} \\
\text{Let } y &= \log_2 x \\
y^2 + 3y - 4 &= 0 \\
(y + 4)(y - 1) &= 0 \\
\log_2 x &= -4 \text{ or } \log_2 x = 1 \\
x &= \frac{1}{16} \text{ or } x = 2
\end{align*}

7. Given that the roots of the equation $2x^2 - 2x + 5 = 0$ are $\alpha$ and $\beta$.

(i) Show that $\alpha^2 + \beta^2 = -4$. 

(ii) Find the value of $\alpha^3 + \beta^3$.

(iii) Find a quadratic equation whose roots are $\frac{\alpha}{2\beta^2}$ and $\frac{\beta}{2\alpha^2}$.

\[\text{[soln]}\]

\[\alpha + \beta = 1 \quad \text{and} \quad \alpha\beta = \frac{5}{2}\]

\[\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 1 - 2 \times \frac{5}{2} = -4\]

\[\begin{align*}
(\alpha + \beta)^3 &= \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3 \\
(\alpha + \beta)^3 &= (\alpha + \beta)(\alpha^2 + \beta^2) - 2\alpha\beta(\alpha + \beta) \\
\alpha^3 + \beta^3 &= 1 - 3 \times \frac{5}{2} = -\frac{13}{2} \\
\alpha^3 + \beta^3 &= \frac{25}{2} - 13 = \frac{13}{25}
\end{align*}\]

\[\begin{align*}
\frac{\alpha}{2\beta^2} + \frac{\beta}{2\alpha^2} &= \frac{\alpha^3 + \beta^3}{2(\alpha\beta)^2} = \frac{\left(\frac{-13}{2}\right)^2}{2} = \frac{25}{2} = \frac{1}{10} \\
\frac{\alpha}{2\beta^2} \times \frac{\beta}{2\alpha^2} &= \frac{1}{4\alpha\beta} = \frac{1}{10}
\end{align*}\]

Quadratic equation is $x^2 + \frac{13}{25}x + \frac{1}{10} = 0$ or $50x^2 + 26x + 5 = 0$.
The equation of the curve is given by \( y = 3 \cos 3x - 2 \) for \( 0 \leq x \leq \pi \).

(i) Write down the amplitude and period of \( y \). \[2\]

(ii) Find the coordinates of the maximum and minimum points for \( 0 < x < \pi \). \[2\]

(iii) Calculate the values of \( x \) for which the curve cuts the \( x \)-axis. \[2\]

(iv) Sketch the curve \( y = 3 \cos 3x - 2 \) for \( 0 \leq x \leq \pi \). \[2\]

(v) State the range of values of \( x \) for which \( y \) is decreasing between \( 0 \) and \( \pi \). \[2\]

[soln]

amplitude = 3, period = \( \frac{2\pi}{3} \)

Minimum point is \( \left( \frac{\pi}{3}, -5 \right) \) and Maximum point is \( \left( \frac{2\pi}{3}, 1 \right) \)

\[ \cos 3x = \frac{2}{3} \]

Basic angle = 0.841

\( 3x = 0.841, 5.4421, 7.124 \)

\( x = 0.280, 1.81, 2.37 \)

\( y \) is decreasing for \( 0 < x < \frac{\pi}{3} \) and \( \frac{2\pi}{3} < x < \pi \)
A solid spherical ball is dropped into a cone of height $h$ cm and radius $r$ cm.

Given that the radius of the spherical ball is 5 cm.

(i) show that the volume of the cone, $V$, is given by $V = \frac{25\pi h^2}{3(h-10)}$.

(ii) Given that $h$ can vary, find the value of $h$ for which $V$ has a stationary value.

(iii) Calculate this stationary value of $V$ and determine if the volume is a maximum or minimum value.

[soln]

\[
\frac{r}{\sqrt{h^2 + r^2}} = \frac{5}{h-5}
\]

\[
r^2 = \frac{25}{h^2 + r^2} = \frac{25}{h^2 - 10h + 25}
\]

\[
r^2 h^2 - 10r^2 h + 25r^2 = 25h^2 + 25r^2
\]

\[
r^2 = \frac{25h^2}{h^2 - 10h} = \frac{25h}{h-10}
\]

\[
V = \frac{1}{3} \pi h \times \frac{25h}{h-10} = \frac{25\pi h^2}{3(h-10)}
\]

\[
\frac{dV}{dh} = \frac{25\pi}{3} \left[ \frac{(h-10) \times 2h - h^2}{(h-10)^2} \right]
\]
For stationary value,
\[
\frac{dV}{dh} = 0 \quad \Rightarrow h = 20
\]

\[
V = \frac{25\pi \times 400}{3 \times 10} = \frac{1000\pi}{3} = 1047.20 \text{ (minimum volume)}
\]

<table>
<thead>
<tr>
<th>(x)</th>
<th>&lt; 20</th>
<th>20</th>
<th>&gt; 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{dV}{dh})</td>
<td>negative</td>
<td>0</td>
<td>positive</td>
</tr>
</tbody>
</table>

10 (i) Express \(\frac{4x^3 + 7x^2 + 4x - 2}{(2x-1)(x^2 + 2)}\) in partial fractions. \([5]\)

(ii) Differentiate \(\ln(x^2 + 2)\) with respect to \(x\). \([1]\)

(iii) Hence evaluate \(\int_1^2 \frac{4x^3 + 7x^2 + 4x - 2}{(2x-1)(x^2 + 2)}\,dx\). \([4]\)

[soln]

\[
\frac{4x^3 + 7x^2 + 4x - 2}{(2x-1)(x^2 + 2)} = 2 + \frac{9x^2 - 4x + 2}{(2x-1)(x^2 + 2)}
\]

\[
\frac{9x^2 - 4x + 2}{(2x-1)(x^2 + 2)} = \frac{A}{2x-1} + \frac{Bx + C}{x^2 + 2}
\]

\[
9x^2 - 4x + 2 = A(x^2 + 2) + (Bx + C)(2x - 1)
\]

Subst \(x = \frac{1}{2}\), \(\frac{9}{4} A = \frac{9}{4}\) \(A = 1\)

Coefficient of \(x^2\): \(B = 4\)

Constant term: \(C = 0\)

\[
\frac{9x^2 - 4x + 2}{(2x-1)(x^2 + 2)} = \frac{1}{2x-1} + \frac{4x}{x^2 + 2}
\]

\[
\frac{d}{dx} \ln(x^2 + 2) = \frac{2x}{x^2 + 2}
\]
\[ \int_1^2 \frac{4x^3 + 7x^2 + 4x - 2}{(2x-1)(x^2 + 2)} \, dx = \int_1^2 \left( 2 + \frac{1}{2x-1} + \frac{4x}{x^2 + 2} \right) \, dx \\
= \left[ 2x + \frac{1}{2} \ln(2x-1) + 2 \ln(x^2 + 2) \right]_1^2 = \left[ 4 + \frac{1}{2} \ln 3 + 2 \ln 6 \right] - \left[ 2 + \frac{1}{2} \ln 1 + 2 \ln 3 \right] \\
= 2 - \frac{3}{2} \ln 3 + 2 \ln 6 \\
= 3.94 \]

11 The table shows experimental values of two variables \(x\) and \(y\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>3.24</td>
<td>5.79</td>
<td>9</td>
<td>17.05</td>
<td>38.43</td>
</tr>
</tbody>
</table>

It is known that \(x\) and \(y\) are related by the equation \(\frac{y-b}{x} = a \sqrt{x} - 1\) for \(x > 0\) where \(a\) and \(b\) are constants.

(i) Using a scale of 1 cm to 2 units on the horizontal axis and 2 cm to 5 units on the vertical axis, draw a straight line graph of \(x + y\) against \(x\sqrt{x}\). [3]

(ii) Use your graph to estimate, to 2 decimal places, the value of \(a\) and of \(b\). [4]

(iii) On the same diagram, draw a straight line representing the equation \(y + x + 2 \sqrt{x} = 36\).

Hence find the value of \(x\) that satisfies the equation \((a + 2)x\sqrt{x} = 36 - b\). [3]

[soln] \[
\frac{y-b}{x} = a \sqrt{x} - 1 \\
y - b = ax \sqrt{x} - bx \\
x + y = ax \sqrt{x} + b
\]

<table>
<thead>
<tr>
<th>(x\sqrt{x})</th>
<th>2.83</th>
<th>5.20</th>
<th>8</th>
<th>14.70</th>
<th>31.62</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x + y)</td>
<td>5.24</td>
<td>8.79</td>
<td>13</td>
<td>23.05</td>
<td>48.43</td>
</tr>
</tbody>
</table>

\(a = 1.5\) and \(b = 0.994\)

\(ax\sqrt{x} + 2x\sqrt{x} = 36 - b\)

\(ax\sqrt{x} + b = -2x\sqrt{x} + 36\) (gradient = -2, intercept = 36)

--- End of Paper ---
1. (a) (i) Sketch the graph of the curve \( y^2 = kx \), where \( k \) is a positive constant. \[1\]

(ii) Given that the line \( y = 2x + 1 \) meets the curve \( y^2 = kx \), find the range of values of \( k \). \[4\]

(b) Determine the conditions for \( p \) and \( q \) such that the curve \( y = px^2 - 2x + 3q \) lies entirely above the \( x \)-axis, where \( p \) and \( q \) are constants. \[3\]

2. (i) Sketch the curve \( y = 2 \ln (x - 3) \) for \( x > 3 \). \[2\]

(ii) The tangent to the curve \( y = 2 \ln (x - 3) \) at the point \( P \) where \( x = 5 \) intersects the \( x \)-axis at \( A \) and the normal to the curve at \( P \) intersects the \( x \)-axis at \( B \). Calculate the area of \( \Delta APB \). \[5\]

3. (a) Write down and simplify the first three terms in the expansion of \( (2 - 3x)^6 \), in ascending powers of \( x \). \[2\]

(b) Hence

(i) using a suitable value of \( x \), find the estimated value of \( (1.997)^6 \), correct to 3 decimal places. \[2\]

(ii) determine the coefficient of \( x^2 \) in the expansion of \( (2 - 3x)^7 - (2 - 3x)^6 \). \[3\]

4. A curve has the equation \( y = f(x) \), where \( f(x) = \frac{2 + \cos x}{\sin x} \) for \( -\pi \leq x \leq \pi \).

(i) Obtain an expression for \( f'(x) \). \[2\]

(ii) Find the exact value of the \( x \)-coordinates of the stationary points of the curve, and determine the nature of each stationary point. \[6\]
5. (a) (i) Show that \[ \frac{\cot x - \tan x}{\cot x + \tan x} = \cos 2x. \] [3]

(ii) Hence solve the equation \[ \frac{\cot x - \tan x}{\cot x + \tan x} = \cos x \] for \[ 0^\circ < x < 360^\circ. \] [3]

(b) Without using a calculator, express \( \sin 15^\circ \) in the form \( \frac{1}{k} (\sqrt{a} - \sqrt{b}) \), where \( a, b \) and \( k \) are integers. [3]

6. (i) Sketch the graph of \( y = 1 - |x - 3| \). [3]

A line \( y = mx + 1 \) is drawn on the same axes with the graph \( y = 1 - |x - 3| \).

(ii) In the case where \( m = 2 \), find the coordinates of the point of intersection of the line and the graph of \( y = 1 - |x - 3| \). [2]

(iii) Determine the set of values of \( m \) for which the line does not intersect the graph of \( y = 1 - |x - 3| \). [2]

7.

In the diagram, \( BF \) and \( DE \) are the diameters of the circle with centre \( O \).

The tangent at \( B \) meets \( ED \) produced at \( C \). Prove that

(i) \( BE = DF \) [3]

(ii) \( DF \times BC = BD \times CE \) [3]

(iii) \( \angle BCE + 2\angle CBD = 90^\circ. \) [2]
8. The equation of a circle \( C_1 \) is \( x^2 + y^2 - 4x - 8y + 4 = 0 \).

(a) Find the coordinates of the centre and the radius of the circle. \([3]\)

(b) The highest point on the circle is \( A \).
    State the coordinates of \( A \). \([1]\)

(c) Another circle, \( C_2 \) touches \( C_1 \) at the point \( A \). Given that both circles do not overlap and the area of \( C_2 \) is four times that of the area of \( C_1 \), find the equation of \( C_2 \) in the form of \( x^2 + y^2 + 2gx + 2fy + c = 0 \), stating the value of \( f, g \) and \( c \). \([4]\)

9. Solutions to this question by accurate drawing will not be accepted.

\[
\begin{array}{c}
A (-1, 7) \\
\hline
D \\
B (5, 1) \\
\hline
E (-4, -2) \\
\end{array}
\]

The diagram, not to scale, shows a parallelogram, \( ABCD \). \( ADE \) and \( BE \) are straight lines. \( D \) divides \( AE \) such that \( AD : DE \) is in the ratio \( 1 : 2 \).

\( A, B \) and \( E \) have coordinates \((-1, 7), (5, 1)\) and \((-4, -2)\) respectively.

(a) (i) Find the equation of the perpendicular bisector of \( AB \) and show that it passes through \( E \). \([3]\)

(ii) Hence deduce the geometrical property of triangle \( ABE \). \([1]\)

(b) Find the coordinates of \( D \). \([2]\)

(c) Find the area of the parallelogram \( ABCD \). \([2]\)
10. A particle starts from rest at 5 m from a fixed point $O$ and moves in a straight line with a velocity, $v = 12t - 3t^2$ m/s where $t$ is the time in seconds after leaving from the initial rest position.

(i) Calculate the acceleration when the particle is instantaneously at rest. [3]

(ii) Calculate the maximum velocity. [2]

(iii) Express the displacement, $s$, from point $O$ in terms of $t$. [1]

(iv) Find the average speed of the particle during the first five seconds. [3]

11. The diagram shows a trapezium field $ABCD$. The point $X$ lies on the side $BC$ such that $AX = 21$ m, $DX = 20$ m, $\angle AXD = \angle ABX = \angle DCX = 90^\circ$ and $\angle BAX = \theta$.

(i) Show that the length of fencing required for the perimeter of the field, $L$ m, can be expressed in the form $p + q \sin \theta + r \cos \theta$, where $p$, $q$ and $r$ are constants to be determined. [3]

(ii) Express $L$ in the form $p + R \cos(\theta - \alpha)$, where $R > 0$ and $\alpha$ is an acute angle. [2]

(iii) State the maximum value of $L$ and the corresponding value of $\theta$. [2]

(iv) Given that the fencing used is 80 m, find the value(s) of $\theta$. [3]
12. (a)  (i) Given that $y = xe^{-2x}, x > 0$, show that $\frac{dy}{dx} = (1 - 2x)e^{-2x}$. [1]

(ii) Hence, find $\int xe^{-2x} \, dx$. [3]

(b) The diagram, which is not drawn to scale, shows part of the curve $y = xe^{-2x}$.

A line drawn from the origin meets the curve at the maximum point $P$.

(i) Find the coordinates of $P$. [3]

(ii) Calculate the area of the region bounded by the curve and the line $OP$. [4]
1. (a) (i) Sketch the graph of the curve \( y^2 = kx \), where \( k \) is a positive constant. \([1]\)

(ii) Given that the line \( y = 2x + 1 \) meets the curve \( y^2 = kx \), find the range of values of \( k \). \([4]\)

(b) Determine the conditions for \( p \) and \( q \) such that the curve \( y = px^2 - 2x + 3q \) lies entirely above the \( x \)-axis, where \( p \) and \( q \) are constants. \([3]\)

(a)(i) 
\[ y = 2x + 1 \] 
\[ y^2 = kx \]

(a)(ii) 
\[ y = 2x + 1 \] ........................ (1)
\[ y^2 = kx \] ........................ (2)

(1) in (2): 
\[ (2x + 1)^2 = kx \]
\[ 4x^2 + (4 - k)x + 1 = 0 \] \([A1]\)

For line meets the curve, \( D \geq 0 \).
\[ (4 - k)^2 - 4(4)(1) \geq 0 \] \([M1]\)
\[ 16 - 8k + k^2 - 16 \geq 0 \]
\[ k(k - 8) \geq 0 \]
\[ \therefore k \leq 0 \text{ (NA)} \text{ or } k \geq 8 \] \([M1A1]\)

(b) Curve lies entirely above line, \( D < 0 \) and \( p > 0 \).
\[ (-2)^2 - 4p(3q) < 0 \]
\[ 4 - 12pq < 0 \] \([M1]\)
\[ pq > \frac{1}{3} \]
\[ \therefore p > 0 \text{ and } pq > \frac{1}{3} \] \([A2]\)
2. (i) Sketch the curve \( y = 2\ln(x-3) \) for \( x > 3 \).

(ii) The tangent to the curve \( y = 2\ln(x-3) \) at the point \( P \) where \( x = 5 \) intersects the x-axis at \( A \) and the normal to the curve at \( P \) intersects the x-axis at \( B \).

Calculate the area of \( \triangle APB \).

(i) Asymptote: \( x = 3 \)

x-intercept: \( 2\ln(x-3) = 0 \)
\[
\begin{align*}
x - 3 &= 1 \\
x &= 4
\end{align*}
\]

(ii) \[
\frac{dy}{dx} = \frac{2}{x-3}
\]
When \( x = 5 \), gradient of tangent at \( P = 1 \)
When \( x = 5 \), \( y = 2\ln2 \)
\( P(1, 2\ln2) \)
Equation of tangent at \( P : y - 2\ln2 = x - 5 \)
\[
\therefore y = x - 5 + 2\ln2
\]
At x-axis, \( y = 0 \): \( x = 5 - 2\ln2 \)
\[
\therefore A(5 - 2\ln2, 0)
\]
Gradient of normal at \( P = -1 \)
Equation of normal at \( P : y - 2\ln2 = -1(x - 5) \)
\[
\therefore y = -x + 5 + 2\ln2
\]
At x-axis, \( y = 0 \): \( x = 5 + 2\ln2 \)
\[
\therefore B(5 + 2\ln2, 0)
\]
\[
\therefore \text{Area of } \triangle APB = \frac{1}{2} (5 + 2\ln2 - 5 + 2\ln2)(2\ln2) = 1.92 \text{ units}^2
\]
3. (a) Write down and simplify the first three terms in the expansion of \((2 - 3x)^6\), in ascending powers of \(x\). [2]

(b) Hence

(i) using a suitable value of \(x\), find the estimated value of \((1.997)^6\), correct to 3 decimal places. [2]

(ii) determine the coefficient of \(x^2\) in the expansion of \((2 - 3x)^7 - (2 - 3x)^6\). [3]

\[
(2 - 3x)^6 = 2^6 + \binom{6}{1} 2^5 (-3x) + \binom{6}{2} 2^4 (-3x)^2 + \cdots
= 64 - 576x + 2160x^2 - \cdots \text{ (up to 1st 3 terms)} \quad \text{[M1A1]}
\]

(b)(i) Put \(2 - 3x = 1.997\)

\[
x = 0.001 \quad \text{[M1]}
\]

\[
(1.997)^6 = 64 - 576(0.001) + 2160(0.001)^2 + \cdots
= 63.42616 = 63.426 \text{ (correct to 3dp)} \quad \text{[A1]}
\]

(b)(ii) \((2 - 3x)^7 - (2 - 3x)^6 = (2 - 3x)^6(2 - 3x - 1)\)

\[
= (1-3x)(2 - 3x)^6 \quad \text{[M1]}
\]

\[
= (1-3x)(64 - 576x + 2160x^2 - \cdots) \quad \text{[M1A1]}
\]

Coefficient of \(x^2 = 1(2160) - 3(-576) = 3888 \quad \text{[M1A1]}

4. A curve has the equation \(y = f(x)\), where \(f(x) = \frac{2 + \cos x}{\sin x}\) for \(-\pi \leq x \leq \pi\).

(i) Obtain an expression for \(f'(x)\). [2]

(ii) Find the exact value of the x-coordinates of the stationary points of the curve, and determine the nature of each stationary point. [6]

(i) \(f'(x) = \frac{\sin x(-\sin x) - (2 + \cos x)(\cos x)}{\sin^2 x} \quad \text{[M1]}
\]

\[
= -\sin^2 x - 2\cos x - \cos^2 x
\]

\[
= -1 - 2\cos x \quad \text{[A1]}
\]

\[
= \frac{-\sin^2 x}{\sin^2 x} \quad \text{[M1]}
\]

(ii) For stationary points, \(f''(x) = 0\).
\[-1 - 2 \cos x = 0\]
\[\frac{\sin^2 x}{\sin^3 x}\]
\[-1 - 2 \cos x = 0\]
\[\cos x = \frac{1}{2}\]
\[x = \frac{2\pi}{3} \text{ or } \pi + \frac{2\pi}{3} - 2\pi\]
\[\therefore x = \frac{2\pi}{3} \text{ or } -\frac{2\pi}{3}\]

<table>
<thead>
<tr>
<th>x</th>
<th>-2.1</th>
<th>-\frac{2\pi}{3}</th>
<th>-2</th>
<th>2</th>
<th>-\frac{2\pi}{3}</th>
<th>2.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f'(x))</td>
<td>+ve</td>
<td>0</td>
<td>-ve</td>
<td>0</td>
<td>+ve</td>
<td></td>
</tr>
<tr>
<td>Tangent</td>
<td>_</td>
<td>_</td>
<td>_</td>
<td>_</td>
<td>_</td>
<td></td>
</tr>
</tbody>
</table>

\[\therefore x = -\frac{2\pi}{3} \text{ is a maximum point and } x = \frac{2\pi}{3} \text{ is a minimum point.}\]

Alternate Mtd:

\[f''(x) = \frac{\sin^2 x (2 \sin x) - (-1 - 2 \cos x)(2 \sin x \cos x)}{\sin^4 x}\]
\[= \frac{2(\sin^2 x + \cos x + 2 \cos^2 x)}{\sin^3 x}\]
\[f''\left(\frac{2\pi}{3}\right) = -2.31 < 0 \Rightarrow \text{max point}\]
\[f''\left(-\frac{2\pi}{3}\right) = 2.31 > 0 \Rightarrow \text{min point}\]
5. (a) (i) Show that \( \frac{\cot x - \tan x}{\cot x + \tan x} = \cos 2x. \) \[3\]

(ii) Hence solve the equation \( \frac{\cot x - \tan x}{\cot x + \tan x} = \cos x \) for \( 0^\circ < x < 360^\circ. \) \[3\]

(b) Without using a calculator, express \( \sin 15^\circ \) in the form \( \frac{1}{k} (\sqrt{a} - \sqrt{b}) \), where \( a, b \) and \( k \) are integers. \[3\]

(a)(i) \( LHS: \)
\[
\frac{\cot x - \tan x}{\cot x + \tan x} = \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \frac{\cos^2 x - \sin^2 x}{\sin x \cos x} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} \]
\[
= \cos 2x = RHS \]
\[A1\]

(ii) \[
\frac{\cot x - \tan x}{\cot x + \tan x} = \cos x \]
\[
\cos 2x = \cos x \]
\[
2 \cos^2 x - \cos x - 1 = 0 \]
\[
(2 \cos x + 1)(\cos x - 1) = 0 \]
\[
\cos x = \frac{1}{2} \text{ or } \cos x = 1 \]
\[
\therefore x = 120^\circ, 240^\circ \]
\[A2\]

(b) \( \sin 15^\circ = \sin(45^\circ - 30^\circ) \)
\[
= \frac{\sqrt{2}}{2} \left( \frac{\sqrt{3}}{2} \right) - \frac{\sqrt{2}}{2} \left( \frac{1}{2} \right) \]
\[
= \frac{\sqrt{6} - \sqrt{2}}{4} \]
\[A1\]

Methodist Girls' School
Additional Mathematics
Sec 4 Preliminary Examination 2016
6. (i) Sketch the graph of \( y = 1 - |x - 3| \).

A line \( y = mx + 1 \) is drawn on the same axes with the graph \( y = 1 - |x - 3| \).

(ii) In the case where \( m = 2 \), find the coordinates of the point of intersection of the line and the graph of \( y = 1 - |x - 3| \).

(iii) Determine the set of values of \( m \) for which the line does not intersect the graph of \( y = 1 - |x - 3| \).

(i) \( y \)-int: Put \( x = 0 \) : \( y = -2 \)

\( x \)-int: \( 1 - |x - 3| = 0 \)

\( x = 4 \) or \( x = 2 \)

Max pt = (3, 1)

(ii) \( 2x + 1 = 1 - |x - 3| \)

\( |x - 3| = -2x \)

\( x - 3 = -2x \) or \( x - 3 = 2x \)

\( x = 1 \) \((NA)\) or \( x = -3 \)

When \( x = -3 \), \( y = -5 \)

Pt of intersection is (-3, -5)

(iii) For line not to intersect graph of \( y = 1 - |x - 3| \), line must be parallel to the left arm.

Gradient of left arm = \( \frac{1 - (-2)}{3 - 0} = 1 \)

Set of values of \( m \) : \( 0 < m \leq 1 \)
In the diagram, $BF$ and $DE$ are the diameters of the circle with centre $O$.

The tangent at $B$ meets $ED$ produced at $C$. Prove that

(i) $BE = DF$  
(ii) $DF \times BC = BD \times CE$  
(iii) $\angle BCE + 2 \angle CBD = 90^\circ$.

(i) $\angle BDE = \angle DBF$ (Angles in the same segment)
$\angle DBE = \angle BDG = 90^\circ$ (right angle in a semi-circle)
$DE = RF$ (diameter)
$\therefore \triangle BDE \cong \triangle DBF$ (AAS)
$\therefore BE = DF$  

Alt Mtd: Show $\triangle BOE \cong \triangle DOF$  

(ii) $\angle DBC = \angle BEC$ (Alternate segment theorem)
$\angle DCE = \angle BCE$ (Common angle)
$\therefore \triangle BEC$ is similar to $\triangle DBC$ ($AA$ Similarity Test)

$\frac{BE}{DC} = \frac{EC}{BC}$
$BE \times BC = EC \times DB$
$\therefore DF \times BC = BD \times CE$  

(iii) $\angle BCE + \angle BEC + 90^\circ + \angle CBD = 180^\circ$
$\angle BCE + 2 \angle CBD = 180^\circ - 90^\circ$
$\therefore \angle BCE + 2 \angle CBD = 90^\circ$.
The equation of a circle $C_1$ is $x^2 + y^2 - 4x - 8y + 4 = 0$.

(a) Find the coordinates of the centre and the radius of the circle. [3]

(b) The highest point on the circle is $A$. State the coordinates of $A$. [1]

(c) Another circle, $C_2$, touches $C_1$ at the point $A$. Given that both circles do not overlap and the area of $C_2$ is four times that of the area of $C_1$, find the equation of $C_2$ in the form of $x^2 + y^2 + 2gx + 2fy + c = 0$, stating the value of $f$, $g$ and $c$. [4]

(a) $C_1 : x^2 + y^2 - 4x - 8y + 4 = 0$.

$$x^2 - 4x + \left(\frac{-4}{2}\right)^2 + y^2 - 8y + \left(\frac{-8}{2}\right)^2 = -4 + \left(\frac{-4}{2}\right)^2 + \left(\frac{-8}{2}\right)^2$$

$$\therefore (x - 2)^2 + (y - 4)^2 = 16$$

Centre = $(2, 4)$ and radius = 4 units [M1]

(b) $x$-coordinate of $A = 2$ (radius ⊥ tangent)

$\therefore A = (2, 4 + 4) = (2, 8)$ [A2]

(c) Radius of $C_2 = 8$

Centre of $C_2 = (2, 8 + 8) = (2, 16)$

Equation of $C_2 : (x - 2)^2 + (y - 16)^2 = 8^2$

$$x^2 - 4x + 4 + y^2 - 32y + 256 = 0$$

$$x^2 + y^2 - 4x - 32y + 196 = 0$$

$2g = -4, 2f = -32$ and $c = -196$

$\therefore g = -2, f = 16, c = 196$ [A1]
9. Solutions to this question by accurate drawing will not be accepted.

The diagram, not to scale, shows a parallelogram, \(ABCD\). \(ADE\) and \(BE\) are straight lines. 
\(D\) divides \(AE\) such that \(AD : DE\) is in the ratio 1 : 2.

\(A, B\) and \(E\) have coordinates \((-1, 7), (5, 1)\) and \((-4, -2)\) respectively.

(a) (i) Find the equation of the perpendicular bisector of \(AB\) and show that it passes through \(E\). \([3]\)

(ii) Hence deduce the geometrical property of triangle \(ABE\). \([1]\)

(b) Find the coordinates of \(D\). \([2]\)

(c) Find the area of the parallelogram \(ABCD\). \([2]\)

(a)(i) Gradient of \(AB = \frac{7 - 1}{-1 - 5} = -1\)

Gradient of perpendicular bisector of \(AB = 1\)

Mid-point of \(AB = \left(\frac{-1 + 5}{2}, \frac{7 + 1}{2}\right) = (2, 4)\) \([A1]\)

Equation of perpendicular bisector of \(AB\): \(y - 4 = x - 2\)
\[\therefore y = x + 2\]

When \(x = -4, y = -4 + 2 = -2\).
\[\therefore \text{perpendicular bisector of } AB \text{ passes through } E. \text{ (Shown)}\] \([A1]\)

(ii) \(\triangle ABE\) is an isosceles triangle.

(b) \[\overrightarrow{AD} = \frac{1}{3} \overrightarrow{AE} = \left(\frac{-1}{3}, \frac{-2}{3}\right)\]

\[D = (-1 - 1, 7 - 3) = (-2, 4)\] \([M1A1]\)

(c) Area of \(\triangle ABD = \frac{1}{2} \begin{vmatrix} -1 & -2 & 5 & -1 \end{vmatrix} = 12 \text{ units}^2\] \([M1]\)

Area of parallelogram \(ABCD = 12 \times 2 = 24 \text{ units}^2\) \([A1]\)
10. A particle starts from rest at 5 m from a fixed point $O$ and moves in a straight line with a velocity, $v = 12t - 3t^2$ m/s where $t$ is the time in seconds after leaving from the initial rest position.

(i) Calculate the acceleration when the particle is instantaneously at rest. [3]

(ii) Calculate the maximum velocity. [2]

(iii) Express the displacement, $s$, from point $O$ in terms of $t$. [1]

(iv) Find the average speed of the particle during the first five seconds. [3]

(i) $a = \frac{dv}{dt} = 12 - 6t$ 

When particle is instantaneously at rest, $v = 0$

$12t - 3t^2 = 0$
$3t(4 - t) = 0$
$t = 0$ (NA) or $t = 4$ 

Acceleration $= 12 - 6(4) = -12$ m/s$^2$. [A1]

(ii) For max or min velocity, $a = 0$

$12 - 6t = 0$
$t = 2$ 

$\frac{d^2v}{dt^2} = -6 < 0 \Rightarrow$ max velocity 

Max velocity $= 12(2) - 3(4) = 12$ m/s

(iii) $S = \int (12t - 3t^2) dt$

$= 6t^2 - t^3 + C$ where $C$ is an arbitrary constant.

Subst $t = 0$, $s = 5 + C = 5$.

$\therefore s = 6t^2 - t^3 + 5$ [A1]

(iv) When $t = 0$, $s = 5$ m

When $t = 4$, $s = 37$ m

When $t = 5$, $s = 30$ m 

Total distance $= (37 - 5) + (37 - 30) = 39$ m 

Average speed $= \frac{39}{5} = 7.8$ m/s. [A1]

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Additional Mathematics

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11. The diagram shows a trapezium field \( ABCD \). The point \( X \) lies on the side \( BC \) such that \( AX = 21 \text{ m}, DX = 20 \text{ m}, \angle AXD = \angle ABX = \angle DCX = 90^\circ \) and \( \angle BAX = \theta \).

(i) Show that the length of fencing required for the perimeter of the field, \( L \text{ m} \), can be expressed in the form \( p + q \sin \theta + r \cos \theta \), where \( p, q \) and \( r \) are constants to be determined. [3]

(ii) Express \( L \) in the form \( p + R \cos(\theta - \alpha) \), where \( R > 0 \) and \( \alpha \) is an acute angle. [2]

(iii) State the maximum value of \( L \) and the corresponding value of \( \theta \). [2]

(iv) Given that the fencing used is 80 m, find the value(s) of \( \theta \). [3]

(i) \[
AD = \sqrt{21^2 + 20^2} = 29 \text{ m}
\]
\[
\sin \theta = \frac{BX}{21}
\]
\[
BX = 21 \sin \theta
\]
\[
\cos \theta = \frac{AB}{21}
\]
\[
AB = 21 \cos \theta
\]
\[
\angle DCX = \theta
\]
\[
\sin \theta = \frac{DC}{20}
\]
\[
DC = 20 \sin \theta
\]
\[
\cos \theta = \frac{XC}{20}
\]
\[
XC = 20 \cos \theta
\]

\[
L = AB + BC + CD + AD
\]
\[
= 21 \cos \theta + 21 \sin \theta + 20 \cos \theta + 20 \sin \theta + 29
\]
\[
\therefore L = 41 \cos \theta + 41 \sin \theta + 29
\] [A1]
(ii) Let \(41 \cos \theta + 41 \sin \theta = R \cos(\theta - \alpha)\)
\[ R = \sqrt{41^2 + 41^2} = \sqrt{3362} \]
\[ \tan \alpha = 1 \]
\[ \alpha = 45^\circ \]
\[ \therefore L = 29 + \sqrt{3362} \cos(\theta - 45^\circ) \]

(iii) Max value of \(L = 29 + \sqrt{3362} = 87.0m\)
\[ \cos(\theta - 45^\circ) = 1 \]
\[ \theta - 45^\circ = 0 \]
\[ \therefore \theta = 45^\circ \]

(iv) \[ 29 + \sqrt{3362} \cos(\theta - 45^\circ) = 80 \]
\[ \cos(\theta - 45^\circ) = \frac{51}{\sqrt{3362}} \]
\[ \theta - 45^\circ = 28.4^\circ, 331.6^\circ (NA), -28.4^\circ \]
\[ \therefore \theta = 73.4^\circ, 16.6^\circ \]
12. (a) (i) Given that \( y = xe^{-2x} \), \( x > 0 \), show that \( \frac{dy}{dx} = (1 - 2x)e^{-2x} \). \[1\]

(ii) Hence, find \( \int xe^{-2x} \, dx \). \[3\]

(b) The diagram, which is not drawn to scale, shows part of the curve \( y = xe^{-2x} \).

A line drawn from the origin meets the curve at the maximum point \( P \).

(ii) Calculate the area of the region bounded by the curve and the line \( OP \). \[4\]

(a)(i) \[ y = xe^{-2x} \]

\[ \frac{dy}{dx} = e^{-2x} - 2xe^{-2x} \]

\[ = (1 - 2x)e^{-2x} \]

\[ \text{[M1]} \]

(ii) \[ \int e^{-2x} \, dx - 2 \int xe^{-2x} \, dx = [xe^{-2x}] \]

\[ \int xe^{-2x} \, dx = \frac{1}{2} \int e^{-2x} \, dx - \frac{1}{2} xe^{-2x} \]

\[ = \frac{1}{4} e^{-2x} - \frac{1}{2} xe^{-2x} + C \]

\[ \text{[M1A1]} \]

(b)(i) For stationary points, \( \frac{dy}{dx} = 0 \)

\[ (1 - 2x)e^{-2x} = 0 \]

\[ 1 - 2x = 0 \]

\[ x = \frac{1}{2} \]

\[ \text{[M1A1]} \]
When \( x = \frac{1}{2} \), \( y = \frac{1}{2} e^{-1} = \frac{1}{2e} \)

\[ \therefore P\left(\frac{1}{2}, \frac{1}{2e}\right) \]  

(iii) Required area = \( \frac{1}{2} \int xe^{-2x} \, dx - \frac{1}{2} \left( \frac{1}{2} \right) \left( \frac{1}{2e} \right) \)  

\[ = \left[ -\frac{1}{4} e^{-2x} - \frac{1}{2} xe^{-2x} \right]_{0}^{\frac{1}{2}} - \frac{1}{8e} \]  

\[ = \left[ -\frac{1}{4} e^{-1} - \frac{1}{4} e \right] - \left( -\frac{1}{4} \right) - \frac{1}{8e} \]  

\[ = \frac{5}{8} e^{-1} + \frac{1}{4} \text{ or } 0.480 \text{ units}^2 \text{ (3sf)} \]
READ THESE INSTRUCTIONS FIRST
Write your name and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$, \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where $n$ is a positive integer and \[ \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1) \ldots (n-r+1)}{r!} \]

2. TRIGONOMETRY

Identities

\[
\begin{align*}
\sin^2 A + \cos^2 A &= 1 \\
\sec^2 A &= 1 + \tan^2 A \\
\cosec^2 A &= 1 + \cot^2 A \\
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\tan(A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\end{align*}
\]

\begin{align*}
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A \\
\tan 2A &= \frac{2 \tan A}{1 - \tan^2 A}
\end{align*}

Formulae for $\triangle ABC$,

\[
\begin{align*}
\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\
a^2 &= b^2 + c^2 - 2bc \cos A \\
\Delta &= \frac{1}{2} abc \sin C
\end{align*}
\]
1 The function \( f \) is defined by
\[
f(x) = 3 + \frac{1}{2x-1}, \text{ where } x \neq \frac{1}{2}.
\]
Show that \( f \) is a decreasing function. [3]

2 Find the range of values of \( p \) for which \((p + 2)x^2 - 12x + 2(p - 1)\) is always negative. [4]

3 The line \( y = mx + c \) intersects the curve \( y^2 = ax \) at \( A(4, 4) \) and \( B(1, k) \). \( B \) is a point that lies below the \( x \)-axis.

(i) Sketch the curve \( y^2 = ax \), indicating point \( A \). [1]

(ii) Find the values of \( a, m, c \) and \( k \). [4]

4 Sketch the graph of \( y = |x-3| + 2 \) for \(-3 \leq x \leq 6\). [3]

Find the range of values of \( c \) for which \(|x-3| - c = x - 2\) has

(i) only 1 solution, [1]

(ii) no solution. [1]

5 Air is pumped into a spherical balloon at a constant rate of 60 cm\(^3\)/s.

(i) Find the rate of increase of the radius, at the instant when the radius is 12 cm. [3]

(ii) Hence, find the rate of change of the surface area of the balloon at this instant. [2]
6 In the figure, $O$ is the centre of the circle. $PCQ$ is the tangent to the circle at $C$ and $AD$ is parallel to $BC$.

(i) Name an angle equal to $\angle BAC$, giving your reason(s) clearly.  

(ii) Show that $\angle CPD = \angle BAC$.  

(iii) Show that $\triangle BAC$ is similar to $\triangle CPD$.  

7 Given that $f(x) = \frac{\cos^3 x - \sin^3 x}{\cos x - \sin x}$,

(i) express $f(x)$ in the form $a\sin bx + c$, stating the value of each of the integers $a$, $b$ and $c$,  

(ii) state the greatest and least values of $f(x)$,  

(iii) state the period and amplitude of $f(x)$.  

The decay of a certain radioactive isotope can be modelled by the exponential equation \( N = N_0 e^{-at} \) after \( t \) weeks, where \( N \) represents the amount of radioactive isotope, \( N_0 \) and \( a \) are constants. A sample of this radioactive isotope has a mass of 100.9 g initially.

(i) After 2 weeks, it is found that the amount of this sample left is 84.6 g. Calculate the value of \( a \).

(ii) What percentage of this sample has decayed after 5 weeks?

(iii) After 9 weeks, the amount of this sample is found to be only 34.6 g. Suggest a reason why this might be so.

(i) Show that \( \sin^4 \theta - \cos^4 \theta = 1 - 2 \cos^2 \theta \).

(ii) Hence solve the equation \( \sin^4 \theta - \cos^4 \theta - 3 \cos \theta = 2 \) for \( 0 < \theta < 360^\circ \).

A particle moves in a straight line such that, \( t \) seconds after leaving a fixed point \( O \), its velocity, \( v \) m s\(^{-1}\), is given by \( v = 15 - e^{-3t} \).

(i) Write down the initial velocity of the particle.

(ii) If \( t \) becomes very large, what value will \( v \) approach? Explain your answer clearly and its significance.

(iii) Find the acceleration of the particle when \( t = 3 \), giving your answer in cm s\(^{-2}\) correct to 3 decimal places.

(iv) Find the distance travelled by the particle in the first 4 seconds of its journey, giving your answer correct to 2 decimal places.
The diagram above shows part of the curve \( y = x(x - 2)^2 \) which passes through \( P (1, 1) \) and touches the x-axis at \( Q (2, 0) \).

(i) Find the equation of the tangent at \( P \) and show that line \( OP \) is the normal to the curve at \( P \). [4]

(ii) Show that the area of the region labelled A is \( \frac{5}{12} \) unit\(^2\) and determine the ratio of the area of A to the area of B. [6]
In figure 1, \(ABCD\) is a square plastic plate of side 4 cm and \(PQRS\) is a square whose centre coincides with that of \(ABCD\). The shaded regions are to be cut off and the remaining plastic is folded to form a right pyramid with base \(PQRS\), as shown in figure 2.

Let \(PQ = 2x\) cm and let \(V\) be the volume of the pyramid.

(i) Show that the height of the pyramid is \(2\sqrt{1-x}\) cm. \([2]\)

(ii) Show that \(V = \frac{8}{3}x^2\sqrt{1-x}\) cm\(^3\). \([2]\)

(iii) Find the value of \(x\) such that \(V\) is maximum. \([7]\)

(iv) Showing your working clearly, explain why the volume of the pyramid will not exceed 0.8 cm\(^3\). \([2]\)

- The End –
Answers

1. \[ f'(x) = -\frac{2}{(2x-1)^2} \]
   
   \[(2x-1)^2 > 0\]
   
   Therefore, \[-\frac{2}{(2x-1)^2} < 0\]
   
   Since \(f'(x) < 0\), \(f(x)\) is a decreasing function.

2. \[ b^2 - 4ac < 0, \]
   
   \[ p < -5 \text{ or } p > 4 \]
   
   But \(p + 2 < 0\),
   
   \[\therefore p < -5\]

3. \[ a = 4 \]
   
   \[ k = -2 \]
   
   \[ m = 2 \]
   
   \[ c = -4 \]

4. \[ (i) \quad -1 < c \leq 11 \]
   
   \[ (ii) \quad c < -1 \text{ or } c > 11 \]
5(i) \( \frac{dr}{dt} = 0.0332 \text{ cm/s} \)

(ii) \( \frac{dA}{dt} = 10.0 \text{ cm}^2/\text{s} \)

6(i) \( \angle BCQ \).

Alternate Segment Theorem

(ii) from (i),

\( \angle BAC = \angle BCQ \).

\( \angle BCQ = \angle GCP \) (vert. opp. \( \angle s \))

\( \therefore \angle CPD = \angle GCP \) (alt. \( \angle s \))

\( = \angle BAC \)

(iii) from (ii),

\( \angle BAC = \angle CPD \).

\( \angle DCP = \angle DAC \) (alt. seg. thm)

\( \angle DAC = \angle BCA \) (alt. \( \angle s \))

\( \therefore \triangle BAC \) similar to \( \triangle CPD \) (AA Similarity or 2 pairs of corr. \( \angle s \) equal)

7. (i) \( f(x) = \frac{\cos^3 x - \sin^3 x}{\cos x - \sin x} \)

\[ = \frac{1}{2} \sin 2x + 1. \]

\( \therefore a = \frac{1}{2}, \quad b = 2, \quad c = 1 \)

(ii) greatest = \( \frac{3}{2} \), least = \( \frac{1}{2} \)

(iii) amplitude = \( \frac{1}{2} \), period = \( \pi \) or 180°
8(i) \( a = 0.0881 \)

(ii) 35.6%

(iii) Difference = 11.06

Possible reasons:
- Error in data collection
- Due to other external factors that expedited the decay
- Any other logical reasoning with explanation

9(ii) \( \theta = 120^\circ, 180^\circ, 240^\circ \)

10(i) initial velocity = 14 m/s

(ii) when \( t \) is very large, \( e^{-3t} \) becomes insignificant,

\[ \therefore v \text{ will approach } 15 \text{ m/s}. \]

Velocity will approach a maximum speed of 15 m/s and held constant at 15 m/s

(iii) acceleration = 0.037 cm/s\(^2\)

(iv) \( s = 59.67 \) m

11(i) Equation of tangent at \( P \): \( y = -x + 2 \)

gradient of \( OP \times \) gradient at \( P = 1 \times -1 \)

\[ = -1 \]

Since gradient of \( OP \times \) gradient at \( P = -1, OP \) is normal to curve at \( P \).

(ii) 5 : 11

12(ii) \( V = \frac{8}{3} x^2 \sqrt{1-x} \)

(iii) stationary point, \( x = \frac{4}{5} \)

Use \( 1^{\text{st}} \) or \( 2^{\text{nd}} \) derivative test to prove that it is a maximum point.

(iv) When \( x = \frac{4}{5} \),

\[ \text{Max } V = 0.763 \text{ cm}^3, \text{ therefore, } V \text{ will never exceed } 0.8 \text{ cm}^3 \]
ST. MARGARET’S SECONDARY SCHOOL

Preliminary Examinations 2016

CANDIDATE NAME

CLASS

REGISTER NUMBER

ADDITIONAL MATHEMATICS 4047/02

Paper 2

Secondary 4 Express / 5 Normal (Academic)

30 August 2016

2 hours 30 minutes

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of 7 printed pages

SMSS 2016
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$, \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Binomial expansion

\[(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,\]

where $n$ is a positive integer and \[ \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!} \]

2. TRIGONOMETRY

Identities

\[ \sin^2 A + \cos^2 A = 1 \]
\[ \sec^2 A = 1 + \tan^2 A \]
\[ \csc^2 A = 1 + \cot^2 A \]
\[ \sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \]
\[ \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \]
\[ \tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \]
\[ \sin 2A = 2 \sin A \cos A \]
\[ \cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A \]
\[ \tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \]

Formulae for $\triangle ABC$,

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]
\[ a^2 = b^2 + c^2 - 2bc \cos A \]
\[ \Delta = \frac{1}{2} abc \sin C \]
1 Differentiate \( 5xe^{2x} \) with respect to \( x \). Hence evaluate \( \int_0^1 3xe^{2x} \, dx \), giving answer correct to 2 decimal places. [5]

2 (i) Given that \( y = \frac{\sin 2x}{1 + \cos 2x} \), show that \( \frac{dy}{dx} \) can be written in the form \( \frac{k}{1 + \cos 2x} \) and state the value of \( k \). [4]

(ii) Hence evaluate \( \int_0^{\frac{\pi}{4}} \frac{1}{4(1 + \cos 2x)} \, dx \). [3]

3 A curve has the equation \( y = px - x \ln x \) for \( x > 0 \) and \( p \) is a constant. Find, in terms of \( p \),

(i) the \( x \)-coordinate of the point at which the curve crosses the \( x \)-axis, [2]

(ii) the value of \( x \), for which the curve has a turning point, [3]

(iii) the coordinates of the turning point and the nature of this point. [3]

4 A curve is such that \( \frac{dy}{dx} = \frac{x^2 - 3}{x^2} \).

(i) Given that the curve passes through the point \( P(3, 5) \), find the equation of the curve. [3]

(ii) Find the equation of the tangent at \( P \) and determine if this tangent cuts the curve again. [5]
5 In the diagram below, $A(-2, 2)$, $B(8, 7)$ and $C(1, -4)$ are points on a circle.

(i) Find the gradient of $AB$ and of $AC$. [2]

(ii) Show that $BC$ is a diameter of the circle and hence find the centre of the circle. [4]

(iii) Find the equation of the circle. [2]

6 (a) Express $\frac{8\sqrt{2} + \sqrt{80} - \sqrt{98}}{\sqrt{18} + 2\sqrt{45} - 4\sqrt{5}}$ in the form $a + b\sqrt{c}$. [4]

(b) Without using calculators, express the value of $\frac{4\cos\left(\frac{\pi}{6}\right)}{\sin\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{6}\right)}$ in the form $a\sqrt{3} + b$, where $a$ and $b$ are integers. [4]
7 (a) Express \( \frac{2x^2 + x + 3}{x^3 + 3x} \) in partial fractions. \([4]\)

(b) A polynomial \( P(x) \) of degree three is exactly divisible by \( x^2 - 2 \).
Given also that \( 4P(-1) = P(2) \), show that \( x \) is a factor of \( P(x) \). \([4]\)

8 The roots of the quadratic equation \( 2x^2 - 4x + 3 = 0 \) are \( \alpha \) and \( \beta \).

(i) Find the value of \( \alpha^2 + \beta^2 \). \([2]\)

(ii) Show that the value of \( \alpha^3 + \beta^3 \) is \(-1\). \([2]\)

(iii) Find a quadratic equation whose roots are \( \frac{\alpha}{\beta^2} + 1 \) and \( \frac{\beta}{\alpha^2} + 1 \). \([5]\)

9 (a) Find the middle term in the expansion of \( \left( x^2 - \frac{1}{3x^3} \right)^{10} \). \([3]\)

(b) Write down the first three terms in the expansion, in ascending powers of \( x \) of \( \left( 1 - \frac{x}{2} \right)^n \), where \( a \) is a constant and \( n \) is a positive integer greater than 6. \([2]\)

The first three terms in the expansion, in ascending powers of \( x \), of \( (2 + ax) \left( 1 - \frac{x}{2} \right)^n \) are \( 2 - 6x + 7x^2 \).

Find the value of \( a \) and of \( n \). \([5]\)
In the diagram, \( OD = 4 \text{ m} \), angle \( DOC = angle \ DAO = angle \ CBO = 90^\circ \), and \( OC = 7 \text{ m} \). Angle \( DOA = \theta \) and varies between \( 0^\circ \) and \( 90^\circ \). The point \( E \) is on the line \( CB \) such that \( DE \) is parallel to \( AB \).

(i) Show that \( AB = 7 \sin \theta + 4 \cos \theta \). \([2]\)

(ii) Express \( AB \) in the form \( R \sin(\theta + \alpha) \), where \( R \) is positive and \( \alpha \) is acute. Hence find the value of \( \theta \) for \( AB = 7.5 \text{ m} \). \([4]\)

(iii) State which line in the diagram has a length of \( R \) and which angle in the diagram has a value of \( \alpha \). \([2]\)

(iv) Show that the area of triangle \( CDE \) is \( \frac{65 \sin 2(\theta + \alpha)}{4} \). \([3]\)

(v) Find the maximum value of the area of triangle \( CDE \) as \( \theta \) varies and state the corresponding value of \( \theta \). \([3]\)
11  (a) The diagram shows a part of a straight line graph obtained by plotting \( \ln y \) against \( x + 1 \), together with coordinates of two points on the line. Express \( y \) in terms of \( x \). [4]

(b) At time \( t \) minutes, the temperature of a liquid, which is left to cool, exceeds room temperature by \( T^\circ C \). The table shows the temperature difference at given times. It is known that one value of \( T \) has been recorded incorrectly.

<table>
<thead>
<tr>
<th>Time, ( t ) (min)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature difference, ( T^\circ C )</td>
<td>14.7</td>
<td>8.1</td>
<td>6.5</td>
<td>2.4</td>
<td>1.3</td>
</tr>
</tbody>
</table>

The variables \( T \) and \( t \) are related by the equation \( T = ke^{at} \), where \( k \) and \( a \) are constants.

(i) Plot \( \ln T \) against \( t \) for the given data and draw a straight line graph. [4]

(ii) Use your graph to

(a) identify the abnormal reading and estimate the correct value of \( T \), [2]

(b) estimate the value of \( k \) and of \( a \). [3]

(c) explain why the temperature of the liquid will never reach room temperature. [2]
Answer Keys

1 (i) \(5(1+2x)e^{2x}\)  
(ii) 6.26

2 (i) \(k = 2\)  
(ii) \(\frac{1}{8}\)

3 (i) \(x = e^p\)  
(ii) \(x = e^{n-1}\)  
(iii) \((e^{n-1}, e^{n-1})\), max

4 (i) \(y = x + \frac{3}{x} + 1\)  
(ii) \(y = \frac{2}{3}x + 3\), No

5 (i) \(\frac{1}{2} \cdot 2\)  
(ii) \(\left(\frac{9}{2}, \frac{3}{2}\right)\)  
(iii) \(\left(x - \frac{9}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{85}{2}\)

6 (a) \(17 - 5\sqrt{10}\)  
(b) \(2\sqrt{3} + 6\)

7 (a) \(\frac{2x^2 + x + 3}{x^3 + 3x} = \frac{1}{x} + \frac{x + 1}{x^2 + 3}\)

8 (i) 1  
(ii) \(x^2 - \frac{14}{9}x + \frac{11}{9} = 0\)

8 (iii) \(x^2 - \frac{14}{9}x + \frac{11}{9} = 0\)

9 (a) \(-\frac{28}{27x^5}\)  
(b) \(1 - \frac{n}{2}x + \frac{n(n-1)}{8}x^2 + \ldots \), \(n = 7, a = 1\)

10 (ii) \(AB = \sqrt{65} \sin(\theta + 29.7^\circ)\) or \(AB = 8.06 \sin(\theta + 29.7^\circ)\)

(iii) \(CD\) has a length of \(R\), \(\angle DCO = \alpha\)

(v) \(16 \frac{1}{4} \text{ m}^2, \ \theta = 15.3^\circ\)

11 (a) \(y = e^{4x}\)

(b) (i) \(\ln y = at + \ln k\)

(iiia) abnormal reading is 6.5, correct reading is 4.5

(iiib) \(a \approx -0.12, \ k \approx 27.1\)

(c) \(T = 0\) at room temperature and \(\ln T\) will become undefined. Hence the temperature of the liquid.
The question paper consists of 3 printed pages, including this page.

The total number of marks for this paper is 80.

The number of marks is given in brackets at the end of each question or part.

At the end of the examination, you must hand in your work together with this question paper.

You are reminded of the need for clear presentation in your answers.

The use of an approved scientific calculator is permitted, where appropriate.

Write your answers on the separate writing paper provided.

Answer all the questions.

Go to the separate answer papers for questions 5 to 9.

Read these instructions first.

Graph Paper

Additional Materials: Answer Paper

Additional Paper 1

Thursday

11 August 2016

2h

PAPER I

SECONDARY FOUR

PRELIMINARY EXAMINATION 2016

TANJONG KATONG GIRLS' SCHOOL

EXAMGURU
Show the vertex and the focus.

(1) The vertex of \( y = x^2 + 1 \) is at \( (0, 1) \). The focus is at \( (0, 1 + \frac{1}{4}) \).

Find the equation of the curve \( y = x^2 - 2x + 5 \) that passes through the vertex of the graph.

The equation of a curve is \( y = x^2 - 2x + 5 \) where \( a \) is a constant.

Solve the equation \( \cos \theta = \frac{1}{2} \) where \( -\pi \leq \theta \leq \pi \).

Find the value of \( a \) and \( c \).

\[ \frac{\cos x}{\sin x} = \frac{\cos x}{\sin x} \]

In the equation

\[ \frac{\sin x}{\cos x} = \frac{\sin x}{\cos x} \]

From the table, the values of \( x \) and \( y \) are consistent and \( b > 0 \).

The table shows the values of the two variables and \( x \).

3. (a) Find the values of \( a \) and \( b \).

(b) Show that \( f(x) = 16 + x^2 \) is equal to \( 16 + \frac{1}{2} \cdot \frac{d}{dx} x^2 \).

(c) The x-coordinate of the curve at \( f(x) = 16 + \frac{1}{2} \cdot \frac{d}{dx} x^2 \) is the x-intercept of the curve at \( f(x) = 16 + \frac{1}{2} \cdot \frac{d}{dx} x^2 \). The curve intersects the x-axis at \( x = \frac{\sqrt{64 - 8 \cdot 16}}{2} \).
THE END
<table>
<thead>
<tr>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \frac{a}{b} )</td>
<td>2. ( \frac{c}{d} )</td>
<td>3. ( \frac{e}{f} )</td>
</tr>
<tr>
<td>( \frac{g}{h} ) &amp; ( \frac{i}{j} )</td>
<td>( \frac{k}{l} ) &amp; ( \frac{m}{n} )</td>
<td>( \frac{o}{p} ) &amp; ( \frac{q}{r} )</td>
</tr>
</tbody>
</table>
The question paper consists of 7 printed pages, including this page.

3. \[ \frac{1}{2} + \frac{1}{3} = \frac{5}{6} \]

4. \[ x^2 + 3x + 2 = 0 \]

5. \[ 3^{\log_3 2} = 2 \]

6. \[ \frac{9}{10} - \frac{1}{2} = \frac{1}{5} \]

The total number of marks for this paper is 100.

 preamble

Friday

SECONDARY FOUR
PRELIMINARY EXAMINATION 2016
TANGKONG KATONG GIRLS' SCHOOL

Additional Materials: Answer Paper

PAPER 2
ADDITIONAL MATHEMATICS

4A702

6 August 2016
2 h 30 min

READ THESE INSTRUCTIONS FIRST

Additional Materials: Answer Paper
(a) State or in a circle, if A, B, C are three points on the circle, prove that the angle subtended by the arc AB at the centre is equal to twice the angle subtended by the same arc at any point on the remaining part of the circle.

(b) Prove that the product of the lengths of the tangents drawn from an external point to a circle is equal to the square of the length of the line segment joining the external point to the point of tangency.

(c) In a right-angled triangle, prove that the square of the hypotenuse is equal to the sum of the squares of the other two sides.

(d) Find the axis of the parabola which passes through the points (1, 2) and (3, 4).

(e) Find the equation of the circle passing through the points (1, 1), (2, 3), and (4, -1).

(f) Find the centre and radius of the circle with equation $x^2 + y^2 - 2x + 4y - 11 = 0$.

(g) Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point (1, 0).

(h) Find the equation of the tangent to the circle $x^2 + y^2 = 25$ at the point (3, 4).

(i) Find the equation of the normal to the curve $y = x^2 - 4x + 5$ at the point (2, 1).

(j) Find the equation of the line passing through the points (1, 2) and (3, 4).

(k) Find the equation of the line perpendicular to the line $y = 2x + 3$ and passing through the point (1, -1).

(l) Find the equation of the line parallel to the line $y = -x + 4$ and passing through the point (2, 3).

(m) Find the equation of the line bisecting the angle between the lines $y = x$ and $y = -x$.

(n) Find the equation of the line passing through the origin and making an angle of $45^\circ$ with the positive x-axis.

(o) Find the equation of the line passing through the points (1, 2) and (3, 4).

(p) Find the equation of the line passing through the points (1, 2) and (3, 4).

(q) Find the equation of the line passing through the points (1, 2) and (3, 4).

(r) Find the equation of the line passing through the points (1, 2) and (3, 4).

(s) Find the equation of the line passing through the points (1, 2) and (3, 4).

(t) Find the equation of the line passing through the points (1, 2) and (3, 4).

(u) Find the equation of the line passing through the points (1, 2) and (3, 4).

(v) Find the equation of the line passing through the points (1, 2) and (3, 4).

(w) Find the equation of the line passing through the points (1, 2) and (3, 4).

(x) Find the equation of the line passing through the points (1, 2) and (3, 4).

(y) Find the equation of the line passing through the points (1, 2) and (3, 4).

(z) Find the equation of the line passing through the points (1, 2) and (3, 4).
Show that the coordinates of point $G$ are $\left(1, \frac{5}{2}\right)$. 

Determine whether circles $C_1$ and $C_2$ will intersect each other, showing your work. 

Find the exact value of the radius of $C_2$ and the equation of the circle $C_2$. 

Find the coordinates of point $P$ if it is on the line $y = -x + 1$. 

Find the coordinates of the center of the circle and the radius of the circle $C_1$. 

The equation of the centre of the circle and the radius 

The intersection of the graphs of $F(x)$ and $G(x)$. 

The solution of the equation $\tan 2\theta = \frac{3}{5}$ for $0 < \theta < \frac{\pi}{2}$. 

Solve the equation $2\sin \theta = 3\cos \theta$. 

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$.
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www.sgexamguru.com

413


The number of marks for each question is given in brackets at the end of each question or part.

At the end of the examination, ensure all your work is clearly legible.

For questions (ii), (iii), (iv), (v), (vi), (vii), and (viii), answer in terms of £.

For all questions, use the appropriate units where applicable.

For questions (iii) and (vii), show all your working clearly.

Answer all questions.

Read these instructions first.

Additional materials: Answer Paper

Preliminary Examination Two

Victoria School

3 August 2015

Wednesday

Paper 1

Additional Mathematics

2 hours

16/5A/P2/01

Name

Register Number
[1] Describe the graph of a function defined on a given interval.

[2] Find the equation of the tangent to the curve at a given point.

[3] Show that the curve is an increasing function for the given interval.

[4] Find the equation of the normal to the curve at a given point.

---

**Diagram Description**

- The diagram shows two lines intersecting at a point labeled as P.
- One line is labeled as AB, and the other as CD.
- The intersection point P is marked with a red circle.
- The diagram also includes labels for angles and segments, indicating the relationships between the lines and their intersections.
The diagram shows the quadrilateral ABCD with vertices A, B, C, and D. Points D, E, and F are marked on the lines BD, AD, and DC respectively. The problem involves finding the coordinates of points D, E, and F given certain conditions.

Diagram:
- Quadrilateral ABCD with vertices labeled A, B, C, and D.
- Points E and D are marked on line BD, with E closer to B.
- Point F is marked on line DC.
- The coordinates of points A, B, C, and D are given, and the problem involves finding the coordinates of E and F under given conditions.

Questions:
1. Calculate the area of quadrilateral ABCD.
2. Determine if ABCD is a square.
3. Find the coordinates of points D, E, and F.
1. (a) Find the rate of change of depth when $h = 3$. 

(b) Show that the volume of water in the tank, $V$ cm$^3$, at time $t$ is given by $V = \frac{2}{3}t^3$. 

(c) Use the quadratic equation whose roots are $\alpha$ and $\beta$, find the value of $\alpha - \beta$.

The diagram shows an inverted square-based pyramid tank of height 30 cm and base length 20 cm. Water leaks out of the tank at a constant rate of 15 cm$^3$ s$^{-1}$.

After 1 second, the depth of the water is 30 cm. 

Hence, find the value of $\alpha - \beta$.

Given the equation: $2x^2 + 7x + 6 = 0$. Hence, roots $2\alpha - \beta$. 

Without solving for $\alpha$ and $\beta$, find the value of $\alpha - \beta$.

2. The co-ordinates of the point on the line at which $y = 4$ are $(-3, 4)$.

Calculate the value of $a$. 

The x-axis and the line $y = a$ are parallel. 

The point at which the line $y = a$ is passed by the line $y = 2x - 3$. 

The equation $4x^2 + 7x + 6 = 0$. 

The straight line $y = ax + b$ passes through the point (-3, 1). 

The variables $x$ and $y$ are connected by the equation $y = a - (x - 1)$, where $a$ is a constant. 

Using experimental values of $x$ and $y$, a graph was drawn in which $y = 4$ was plotted on the vertical axis against $x$ on the horizontal axis. The straight line obtained passed through the point (-3, 1).
(1) The form $a+b\ln x$, where $a$ and $b$ are integers.

(2) $\frac{e^{x^2}+x^2-x}{e^{x^2}+x^2-x} + \frac{e^{x^2}+x^2-x}{e^{x^2}+x^2-x} = e^x + x^2 - x$.

(3) $\ln (x+1)$.

(4) $\ln (x+1)$.

(5) $\ln (x+1)$.

(6) $\ln (x+1)$.

(7) $\ln (x+1)$.

(8) $\ln (x+1)$.

(9) $\ln (x+1)$.

(10) $\ln (x+1)$.

(11) $\ln (x+1)$.

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(97) $\ln (x+1)$.

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(100) $\ln (x+1)$.
1. \[ \frac{\text{torque} \times \text{radius}}{\text{length}} = \] 

2. \[ \frac{\text{torque} \times \text{radius}}{\text{length}} = \] 

3. \[ \frac{\text{torque} \times \text{radius}}{\text{length}} = \] 

4. \[ \frac{\text{torque} \times \text{radius}}{\text{length}} = \] 

5. \[ \frac{\text{torque} \times \text{radius}}{\text{length}} = \] 

6. \[ \frac{\text{torque} \times \text{radius}}{\text{length}} = \] 

7. \[ \frac{\text{torque} \times \text{radius}}{\text{length}} = \] 

8. \[ \frac{\text{torque} \times \text{radius}}{\text{length}} = \] 

9. \[ \frac{\text{torque} \times \text{radius}}{\text{length}} = \] 

10. \[ \frac{\text{torque} \times \text{radius}}{\text{length}} = \]
<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>
CANDIDATE NAME  CLASS  REGISTER NUMBER

BALESTIER HILL SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2016
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC

ADDITIONAL MATHEMATICS  4047 / 01

19 Aug 2016  Friday  2 hours

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer ALL questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the
case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's use:

This paper consists of 6 printed pages, including this cover page.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

For the equation \( ax^2 + bx + c = 0, a \neq 0 \),

Binomial expansion

\[(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \).

2. TRIGONOMETRY

\[
\begin{align*}
\sin^2 A + \cos^2 A &= 1 \\
\sec^2 A &= 1 + \tan^2 A \\
\cosec^2 A &= 1 + \cot^2 A \\
\sin (A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos (A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\tan (A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A \\
\tan 2A &= \frac{2\tan A}{1 - \tan^2 A}
\end{align*}
\]

Formulae for \( \triangle ABC \)

\[
\begin{align*}
\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\
a^2 &= b^2 + c^2 - 2bc \cos A \\
\Delta &= \frac{1}{2} ab \sin C.
\end{align*}
\]
1. The graph \( y = x^2 + 3px - 2q \), where \( p \) and \( q \) are constants, is always positive for all real values of \( x \).

   (i) Find an inequality connecting \( p \) and \( q \).
   (ii) Explain why \( q \) cannot be positive.

2. A prism with a trapezium base has a volume of \((14 + 11\sqrt{2})\) cm\(^3\). The trapezium has a height of \((3\sqrt{2} + 2)\) cm and its parallel sides are \(\sqrt{2}\) cm and 2 cm respectively. Find the height of the prism, leaving your answer in the form \((\frac{\sqrt{2} + a}{b})\) cm, where \(a\) and \(b\) are integers.

3. (i) Sketch the graph of \(|x^2 - 9| + 2\).
   (ii) Determine the range of values of \(m\) for which the line \(y = mx\) does not intersect the graph of \(|x^2 - 9| + 2\).

4. A curve has equation \(y = \frac{\sin x}{e^{2x}}\) for \(0 \leq x \leq \frac{\pi}{2}\).

   (i) Prove that if \(y\) is an increasing function, \(\tan x < \frac{1}{2}\).
   (ii) A point \((x, y)\) moves along the curve \(y = \frac{\sin x}{e^{2x}}\) such that the \(y\)-coordinate is decreasing at a rate of 0.2 units per second. Find the rate of change of the \(x\)-coordinate when \(x = 0.5\).

5. Given that \(f(x) = 6x^3 + 3x^2 - x + 2 = 0\),

   (i) show that the equation \(f(x) = 0\) has only one real root. Find the value of the real root.
   (ii) sketch the curve, showing clearly the \(x\) and \(y\) intercepts.
6 A piece of wire, of length 150 cm, is bent into the shape as shown in the diagram. The shape consists of an isosceles triangle $PQU$ where $PQ = PU = 5x$ cm, a rectangle $QRTU$ and a semi-circle $RST$. Given further that $QR = y$ cm and $RT = 6x$ cm,

(i) show that the enclosed area, $A$ cm$^2$, is given by

$$A = 450x - 9x^2 \left(2 + \frac{\pi}{2}\right).$$

(ii) Given that $x$ can vary, find the value of $x$ for which the area is stationary.

(iii) Explain why this value of $x$ gives the largest area possible.

[4] [2] [1]

7 Given that the first four terms in the expansion of $(1 + 3x)^n(1 + x)^n$ in ascending powers of $x$ is $1 + ax + bx^2 + cx^3 + ...$, where $a$, $b$ and $c$ are constants, and $n$ is a positive integer.

(i) Express $a$ and $b$ in terms of $n$.

(ii) If $b = 72$, prove that $n = 7$ and find the value of $c$.

(iii) Using the value of $n$ found in (ii), find the coefficient of $x^2$ in the expansion of $(1 + 3x)^2 (1 + x)^{n+1}$.

[3] [4] [2]

8(i) Show that \[ \frac{d}{dx} (\ln \sin^2 x) = 2 \cot x \]

(ii) By expressing $x \cot x$ as $\frac{x}{\tan x}$, differentiate $x \cot x$ with respect to $x$.

(iii) Using your results from parts (i) and (ii), find \[ \int \frac{\pi}{6} x \cos^2 x \, dx \] and prove that

\[ \int \frac{\pi}{6} x \cos^2 x \, dx = \frac{1}{2} \ln 2 - \pi \left(\frac{1}{4} - \frac{\sqrt{3}}{6}\right). \]

[2] [3] [4]
9 A point \( P \) is equidistant from \( A (1, 4) \) and \( B (5, 2) \). Given that \( P \) lies on the line \( y - x = 1 \), find

(i) the co-ordinates of the point \( P \),

(ii) the equation of the perpendicular bisector of \( AB \),

(iii) a point \( Q \) such that \( APBQ \) is a parallelogram.

(iv) Find the area of triangle \( AOP \), where \( O \) is the origin.


10 The table below shows the experimental values of the variables \( x \) and \( y \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>1.0</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1.10</td>
<td>1.86</td>
<td>2.61</td>
<td>3.44</td>
<td>4.08</td>
</tr>
</tbody>
</table>

It is known that \( x \) and \( y \) are related by the equation of the form \( ay^2 = x(1 + bx) \), where \( a \) and \( b \) are constants. Due to experimental errors, one of the values of \( y \) has been recorded incorrectly.

(i) Plot \( \left( \frac{y^2}{x} \right) \) against \( x \) and use your graph to estimate the value of \( a \) and of \( b \).

(ii) State the value of \( y \) that has been recorded incorrectly and estimate the correct value.

[6] [2]

11 In the diagram, \( AC \) is the diameter of the circle with centre \( O \). \( ACT \) is a straight line and \( BT \) is a tangent to the circle at \( B \). Given that \( AB = BT \) and \( \angle ADB = 90^\circ \), prove that

(i) \( \triangle ABC \) is similar to \( \triangle BDC \).

(ii) \( \angle BTC = \frac{1}{2}(180^\circ - \angle BCT) \)

(iii) \( \angle BAC = 30^\circ \).
The height of water in a harbour changes with tides. The height, $h$ metres, of the water during a particular day can be modelled by the equation, $h = 1.2 \cos \left( \frac{\pi x}{6} \right) - 0.4 \sin \left( \frac{\pi x}{6} \right) + 1.5$, where $x$ is the number of hours after midnight.

(i) Express $h$ in the form $R \cos \left( \frac{\pi x}{6} + \alpha \right) + 1.5$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(ii) Find the maximum height of the tides.

(iii) At what times are the tides 2.5 m high? Give your answers correct to the nearest minute.

End of Paper 1
BALESTIER HILL SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2016
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC

ADDITIONAL MATHEMATICS

15 Aug 2016 Monday 2 hours 30 mins

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer ALL questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's use:

This paper consists of 6 printed pages, including this cover page.


2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0, a \neq 0, \)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.
\]

Binomial expansion

\[(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and

\[
\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}.
\]

2. TRIGONOMETRY

Identities

\[
\sin^2 A + \cos^2 A = 1
\]
\[
\sec^2 A = 1 + \tan^2 A
\]
\[
\cosec^2 A = 1 + \cot^2 A
\]
\[
\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
\sin 2A = 2\sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A
\]
\[
\tan 2A = \frac{2\tan A}{1 - \tan^2 A}
\]

Formulae for \( \triangle ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A.
\]
\[
\Delta = \frac{1}{2} ab \sin C.
\]
1. Sketch the graph of \( y = 4\sqrt{x} \). [1]

(ii) On the same axes, sketch the graph of \( y = \frac{8}{\sqrt{x^3}} \). [1]

(iii) Calculate the \( x \) co-ordinate of the point of intersection of your graphs in exact form. [2]

(iv) Determine, with explanation, whether the tangents to the graphs at the point of intersection are perpendicular. [4]

2. A curve is such that \( \frac{d^2y}{dx^2} = 8e^{-2x} \). Given that \( \frac{dy}{dx} = 9 \) when \( x = 0 \) and the curve passes through the point \((\ln 2, 13 \ln 2)\), find the equation of the curve. [4]

3. (i) The equation \( x^2 + px + q = 0 \) has roots \( \alpha \) and \( \beta \). Given that \( \alpha^2 + \beta^2 = 85 \) and \( \alpha - \beta = 1 \), find the positive value of \( p \) and of \( q \). [4]

(ii) With the values of \( p \) and \( q \) found in (i), find a quadratic equation with roots \( \frac{1}{\alpha} \) and \( \frac{1}{\beta} \). [3]

4. The equation of a curve is \( f(x) = x^3 \ln x \).

(i) Show that the curve, \( f(x) = x^3 \ln x \), has only one stationary point. [5]

Find the \( x \)-coordinate of the stationary point of the curve in exact form.

(ii) Prove that the value of \( f''(x) \) at the stationary point is \( \frac{3}{\sqrt{e}} \). [2]

(iii) What does the result of part (ii) imply about the stationary point? [1]

5. (i) Show that \( \sin \theta + \sin 3\theta = 4 \sin \theta \cos^2 \theta \). [3]

(ii) Hence, solve the equation \( \sin \theta + \sin 3\theta = \cos \theta \) for \(-\pi \leq \theta \leq \pi\). [5]
6 (i) Given that \( \frac{4x^3 - 3x^2 + 4x + 10}{(2x+1)(x^2 + 2)} = \frac{A}{2x+1} + \frac{Bx+C}{x^2 + 2} \), where \( A, B \) and \( C \) are constants, find the value of \( A \) and of \( B \) and show that \( C = 0 \).

(ii) Differentiate \( \ln (x^2 + 2) \) with respect to \( x \).

(iii) Using the results from parts (i) and (ii), find \( \int \frac{4x^3 - 3x^2 + 4x + 10}{(2x+1)(x^2 + 2)} \, dx \).

7 (a) Solve \( \frac{8}{\log_3 x^2} - \frac{1}{\log_3 3} = 3 \).

(b) Miss Gossip started a rumour in a lecture theatre. The spread of the rumour can be modelled by the exponential curve \( P = \frac{3000}{1 + 9e^{-kt}} \), where \( P \) represents the number of students who heard the rumour at time \( t \), \( k \) is a constant and \( t \) is time measured in hours.

(i) Two hours after the lecture, 600 students had heard the rumour. Show that \( k = \ln \left( \frac{3}{2} \right) \) and find the number of students who had heard the rumour after 4 hours.

(ii) If the school has 3000 students, show that it took approximately 5.419 hours for the rumour to spread to half the student population.

8 The function \( f \) is defined by \( f(x) = a \cos \left( \frac{x}{3} \right) + c \) for \( 0^\circ \leq x \leq 540^\circ \).

Given that the function has a maximum value of 2 and a minimum value of \(-4\),

(i) state values of \( a \) and \( c \),

(ii) state the period of \( f(x) \),

(iii) find the \( x \) coordinate(s) of the point(s) where the curve meets the \( x \)-axis,

(iv) sketch the graph of \( f(x) = a \cos \left( \frac{x}{3} \right) + c \) for \( 0^\circ \leq x \leq 540^\circ \) and

the graph of \( g(x) = 4 - 3\sin x \) for \( 0^\circ \leq x \leq 540^\circ \) on the same axis.
9(a) Show that \( \frac{d}{dx} \left[ (x - 1)\sqrt{5 + 4x} \right] = \frac{6x + 3}{\sqrt{5 + 4x}} \) [3]

(b) The diagram shows part of the curve \( y = \frac{2x + 1}{\sqrt{5 + 4x}} \). The line PR is a normal to the curve at P. Q is the point where the curve cuts the y-axis and S is a point directly below P.

(i) Given that the x-coordinate of P is 1, find the equation of the line PR. [4]

(ii) Without calculating the area under the curve from \( x = 0 \) to \( x = 1 \), explain briefly why \( \int_0^1 \frac{2x + 1}{\sqrt{5 + 4x}} \, dx > \frac{1}{2} \left( 1 + \frac{1}{\sqrt{5}} \right) \). [2]

(iii) Find the area of the shaded region. [3]

10 A particle travels in a straight line such that, \( t \) seconds after passing a fixed point O, its acceleration, \( a \) m/s\(^2\), is given by \( a = 200e^{-\frac{t}{2}} \). The particle has an initial velocity of \(-360 \) m/s.

(i) Find an expression for the velocity of the particle. [2]

(ii) Find an expression for the displacement of the particle from O. [2]

(iii) Show that when the particle is instantaneously at rest, \( t = \ln 100 \). [3]

(iv) Calculate the total distance travelled by the particle for the first 6 seconds. [4]
The diagram below shows two circles $C_1$ and $C_2$ touching each other at point $F$. $C_1$ has centre at $Q$ and $C_2$ has centre at $P$. The points $A(-1, 7)$ and $B(11, 13)$ lie on $C_1$, and $AB$ is the diameter of $C_1$. The points, $O$, $P$ and $Q$ lie on a straight line.

(i) Find the equation of $C_1$. [3]
(ii) Find the equation of the tangent to the 2 circles at $F$, given that the point $F$ is $(2, 4)$. [3]
(iii) If the co-ordinates of $P$ is $(1, 2)$, determine whether a point $(1, 5)$ lies inside, outside or on circle $C_2$. [2]

A third circle $C_3$ is drawn with $DE$ as its diameter, where $D$ and $E$ are points on the $x$ and $y$ axis respectively.

(iv) State whether the origin $O$ lies on $C_3$. Explain your answer. [1]

End of Paper 2
Prove that \( y_1 \) is an increasing function for \( x > 0 \).

A curve has equation \( y = \frac{7}{x} \).

\[
A(1) \\
A(1)
\]

\[
I(1) \\
I(1)
\]
5

Given that \( \frac{dy}{dt} = -0.2 \)
\[
\frac{dy}{dt} = \frac{dx}{dt} \cdot \frac{dx}{dt}
\]
\[-0.2 = \frac{\cos 0.5 - 2\sin 0.5}{e^t} \cdot \frac{dx}{dt}
\]
\[-0.2 = -0.029897016 \cdot \frac{dx}{dt}
\]
\( \frac{dx}{dt} = 6.6986 \approx 6.69 \text{ units per second} \).

5

(i) show that the equation \( f(x) = 0 \) has only one real root. Find the value of the real root.
(ii) sketch the curve, showing clearly the \( x \) and \( y \) intercepts.

(i)
Let \( f(x) = 6x^2 + 3x^2 - x + 2 \)

By trial and error,
\[
f(-1) = 6(-1)^2 + 3(-1)^2 - (-1) + 2
\]
\[= -6 + 3 + 1 + 2 = 0\]

\((x + 1)\) is a factor of \( f(x) \).

\[f(x) = (x + 1)(6x^2 + bx + 2)\]

By comparing coeffs,
\[3 = b + 6\]
\[b = -3\]

\[f(x) = (x + 1)(6x^2 - 3x + 2)\]
\[= (x + 1)(6x^2 - 2x + 3) = 0\]

\[x = -1, \text{ discriminant } = b^2 - 4ac\]
\[= (-2)^2 - 4(6)(2)\]
\[= -39 < 0\]

no real roots

Hence, \( x = -1 \) is the only real root.
\[ x = \frac{z - \mu + \frac{z}{\mu}}{1 - \mu} = \frac{z}{1 + \mu} \]

Differentiate both sides with respect to \( z \).

\[ \frac{dx}{dz} = \frac{1}{(z + 1)^2} \]

Show that \( \frac{dx}{dz} \).

\[ \frac{d}{dz} + \frac{\mu}{z + 1} \]

Coefficient of \( z \) in the expression of \( x \). Hence the stationary value of \( z \) gives the maximum area.

\[ 0 > \left( \frac{\mu}{z + 1} \right) \Rightarrow \frac{\mu}{z + 1} = 0 \]

\[ (0/0) = \frac{\mu}{0} = \frac{\mu}{1} \]

\[ 0 = 0 \]

\[ 0 = 0 \]

\[ \frac{z}{z + 1} \]
9. A point \( P \) is equidistant from \( A(1, 4) \) and \( B(5, 2) \). Given that \( P \) lies on the line \( y = x = 1 \), find

(i) the co-ordinates of the point \( P \).

(ii) the equation of the perpendicular bisector of \( AB \).

(iii) a point \( Q \) such that \( APBQ \) is a parallelogram.

(iv) Find the area of triangle \( AOP \), where \( O \) is the origin.

10. In the diagram, \( AC \) is the diameter of the circle with centre \( O \). \( ACT \) is a straight line and \( BT \) is a tangent to the circle at \( B \). Given that \( AB = BT \) and \( \angle ADB = 90^\circ \), prove that

\[
\frac{1}{2} \left[ \frac{1}{4} \ln \left( \frac{1}{\sqrt{2}} \right) - \frac{1}{2} \left( \frac{\pi}{4} \right) \right] = \frac{1}{2} \ln 2 - \frac{1}{2} \ln \left( 1 + \frac{\sqrt{5}}{6} \right)
\]

\[
\frac{\pi}{2} \cos \frac{\pi}{2} = \frac{1}{2} \ln 2 - \frac{1}{2} \ln \left( 1 + \frac{\sqrt{5}}{6} \right) \text{ shown}
\]
\[
\begin{align*}
\text{(i)} & \quad \frac{x}{2} + \frac{9}{\sqrt{x}} = 1.5 \\
\text{(ii)} & \quad \left(0.0332 + \frac{9}{\sqrt{x}}\right) = 1.5 \\
\text{(iii)} & \quad \left(0.0172 + \frac{9}{\sqrt{x}}\right) = 1.5
\end{align*}
\]
1

(i) Sketch the graph of \( y = 4\sqrt{x} \). [1]

(ii) On the same axes, sketch the graph of \( y = \frac{8}{\sqrt{x}} \). [1]

(iii) Calculate the \( x \) co-ordinate of the point of intersection of your graphs in exact form. [2]

(iv) Determine, with explanation, whether the tangents to the graphs at the point of intersection are perpendicular. [4]

\[
\begin{align*}
4\sqrt{x} &= \frac{8}{\sqrt{x}} \\
x^{1/2} &= 2 \\
x &= 4 \\
\end{align*}
\]

\[
\begin{align*}
\frac{dy}{dx} &= \frac{2}{\sqrt{x}} \\
\frac{dv}{dx} &= \frac{12}{\sqrt{x}^3} \\
\end{align*}
\]

Product of gradients = \( \frac{2}{\sqrt{x}} \times \frac{12}{\sqrt{x}^3} = \frac{24}{\sqrt{x}^4} = -8.485 \neq -1 \)

Hence the tangents are not perpendicular at the point of intersection.

2

A curve is such that \( \frac{d^2y}{dx^2} = 8e^{-2x} \). Given that \( \frac{dy}{dx} = 9 \) when \( x = 0 \) and the curve passes through the point \((2, 13/2)\), find the equation of the curve.

\[
\begin{align*}
\frac{d^2y}{dx^2} &= 8e^{-2x} \\
\frac{dy}{dx} &= 8e^{-2x} \frac{dx}{dx} \\
&= -4e^{-2x} + c = 9 \\
&= -4 + c = 9 \\
c &= 13 \\
y &= \int -4e^{-2x} + 13 \, dx \\
y &= 2e^{-2x} + 13x + c \\
\end{align*}
\]

Subst \((2, 13/2)\)

\[
\begin{align*}
13/2 &= 2e^{-4} + 13 \times 2 + 13 \\
0 &= 2(\frac{1}{4}) + c \\
c &= \frac{1}{2} \\
y &= 2e^{-2} + 13x - \frac{1}{2} \\
\end{align*}
\]

3

(i) The equation \( x^2 + px + q = 0 \) has roots \( \alpha \) and \( \beta \). Given that \( \alpha^2 + \beta^2 = 85 \) and \( \alpha - \beta = 1 \), find the positive value of \( p \) and of \( q \). [4]

\[
\begin{align*}
\alpha + \beta &= -p \\
\alpha \beta &= q \\
(\alpha + \beta)^2 &= \alpha^2 + \beta^2 + 2\alpha\beta = 85 + 2q = p^2 \quad -(i) \\
(\alpha - \beta)^2 &= \alpha^2 + \beta^2 - 2\alpha\beta = 85 - 2q = 1 \quad -(ii) \\
(i) + (ii) &= 170 = p^2 + 1 \\
p^2 &= 169 \\
p &= 13 \\
q &= 42 \\
\end{align*}
\]
(6)

\[
\int (x^2 + 2x + 1) \, dx = \int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(7)

\[
\frac{d}{dx}(x^2 + 2x + 1) = 2x + 2 \quad \text{and} \quad \frac{d}{dx}(x^2 + 2x + 1) = 2x + 2
\]

(8)

\[
\frac{dx}{dt} = \frac{\cos t}{\sin t} - \frac{\sin t}{\cos t}
\]

(9)

\[
\begin{align*}
\frac{d}{dx} \left( x^2 + 2x + 1 \right) &= 2x + 2 \\
\frac{d}{dx} \left( x^2 + 2x + 1 \right) &= 2x + 2
\end{align*}
\]

(10)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(11)

\[
\begin{align*}
\frac{d}{dx} &\left( x^2 + 2x + 1 \right) = 2x + 2 \\
\frac{d}{dx} &\left( x^2 + 2x + 1 \right) = 2x + 2
\end{align*}
\]

(12)

\[
\frac{d}{dx} \left( x^2 + 2x + 1 \right) = 2x + 2
\]

(13)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(14)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(15)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(16)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(17)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(18)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(19)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(20)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(21)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(22)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(23)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(24)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(25)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(26)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(27)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(28)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(29)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(30)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(31)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(32)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(33)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(34)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(35)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(36)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(37)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(38)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(39)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]

(40)

\[
\int (x^2 + 2x + 1) \, dx = \frac{x^3}{3} + x^2 + x + C
\]
(ii) If the school has 3000 students, show that it took approximately 5.419 hours for the rumour to spread to half the student population.

\[ k = \ln \left( \frac{3}{2} \right) \] and find the number of students who had heard the rumour after 4 hours.

(i) Solve \[ \frac{x}{\log_3 x} - \frac{1}{\log_3 3} = 3 \] for \( x \). Let \( u = \log_3 x \), then

\[ \frac{4}{u} = \frac{4}{u} - u = 3 \]

\[ 4 - u^2 = 3u \]

\[ u^2 + 3u - 4 = 0 \]

\[ (u + 4)(u - 1) = 0 \]

\[ u = -4 \] or \( u = 1 \)

\[ \log_3 x = -4 \] or \( \log_3 x = 1 \)

\[ x = 3^{-4} \] or \( x = 3 \)

\[ x = \frac{1}{81} \]

(b) Miss Gossip started a rumour in a lecture theatre. The spread of the rumour can be modelled by the exponential curve \[ P = \frac{3000}{1 + 9e^{-kt}} \], where \( P \) represents the number of students who heard the rumour at time \( t \), \( k \) is a constant and \( t \) is time measured in hours.

\( i \) Two hours after the lecture, 600 students had heard the rumour. Show that

\[ 2e^{2k} = \frac{4}{9} \]

\[ k = \ln \left( \frac{3}{2} \right) \]

After 4 hours,

\[ \frac{3000}{1 + 9e^{-kt}} = 1080 \]

\[ \frac{3000}{1 + 9 \left( \frac{16}{81} \right)} = 1080 \]
(i) Find the area of the shaded region

\[
\frac{\sqrt{y^2 + 1}}{1 + \sqrt{x}} \leq \frac{2 \sqrt{y} + \sqrt{5}}{1 + \sqrt{x}}
\]

Without calculating the area under the curve from \( x = 0 \) to \( x = 1 \), explain why

(ii) Given that the x-coordinate of \( P \) is \( \frac{\sqrt{y}}{x} \), find the equation of the line \( PQ \).

(iii) The diagram shows part of the curve \( f(x) \). The line \( PQ \) is a normal to the curve at \( P \).

(iv) The graph of \( (y) = 4 - 3 \sin x \) for \( 0 \leq x \leq 5 \pi \) on the same axis.

(v) Sketch the graph of \( (y) = 2 \cos \pi x \) for \( 0 \leq x \leq 5 \pi \) on the same axis.

(vi) Find the x-coordinate of the points where the curve meets the y-axis.

(vii) State the range of \( y \) of \( f(x) \).

(viii) Given that the function has a maximum value of 2 and a minimum value of -4,

\[
\text{The function is defined by } f(x) = a \cos bx + c + d
\]

\[
a = 4.196, b = \frac{\pi}{6}, c = 1, d = 0
\]
(a) \[
\frac{dy}{dx} = (x-1) \frac{1}{2\sqrt{5+4x}} + \frac{1}{\sqrt{5+4x}}
\]
\[
= \frac{1}{\sqrt{5+4x}} (2x^2 - 2 + 5 + 4x)
\]
\[
= \frac{6x+3}{\sqrt{5+4x}}
\]
\[
= \frac{2x+1}{\sqrt{5+4x}}
\]
\[
= \frac{27}{27}
\]
when \( x = 1 \)

Equation of \( PR \),
\[ PR: y = \frac{27}{12} x + \frac{13}{4} \]
\[ y = \frac{9}{4} x + \frac{13}{4} \]

(ii) At \( x = 0 \),
\[ y = \frac{2(0)+1}{\sqrt{5+4(0)}} = \frac{1}{\sqrt{5}} \]

Area of trapezium \( OQPS = \frac{1}{2} (1 + \frac{1}{\sqrt{5}}) x \)

Area of shaded region under curve from \( x = 0 \) to \( x = 1 \) is more than area of trapezium.

(iii) At \( y = 0 \),
\[ \frac{27}{12} x + \frac{13}{4} = 0 \]
\[ x = \frac{13}{9} \]

Area of triangle = \( \frac{1}{2} (1) \left( \frac{4}{9} \right) = \frac{2}{9} \)

Area of shaded region = \( \int_{-1}^{1} \frac{2x+1}{\sqrt{5+4x}} dx + \frac{2}{9} \)
\[ = \frac{1}{3} (x-1) \frac{1}{\sqrt{5+4x}} + \frac{2}{9} \]

11 (a) The diagram below shows two circles \( C_1 \) and \( C_2 \) touching each other at point \( F \). \( C_1 \) has centre at \( Q \) and \( C_2 \) has centre at \( P \). The points \( A (-1, 7) \) and \( B (11, 13) \) lie on \( C_1 \), and \( AB \) is the diameter of \( C_1 \). The points \( Q, F \) and \( Q \) lie on a straight line.

(d) Find the equation of \( C_1 \).
(ii) Find the equation of the tangent to the 2 circles at \( F \), given that \( F \) is \( (2, 4) \).
(iii) If \( P (1, 2) \), determine whether a point \( (1, 5) \) lies inside, outside or on circle \( C_1 \).

A third circle \( C_3 \) is drawn with \( DE \) as its diameter, where \( D \) and \( E \) are points on the \( x \) and \( y \) axes respectively.

(iv) State whether the origin \( O \) lies on \( C_3 \). Explain your answer.
1. Calculate the total distance travelled by the particle for the first 6 seconds.

\[ s = \int_0^6 v(t) \, dt \]

2. Show that the particle is uniform velocity at rest for 100 ms.

3. Find an expression for the displacement of the particle from O at time \( t \).

4. Find an expression for the velocity of the particle.

5. A particle travels in a straight line with initial velocity of 0.5 m/s and acceleration of 0.2 m/s² in the x-direction. At time \( t = 0 \), the particle is at position \( x = 0 \).

6. Be a point on the circle \( x^2 + y^2 = 25 \) in a rectilinear:
   - Find the distance between point \( P \) and center \( C \) of the circle.
   - Radius of the circle:
   - Equation of the circle.

7. Find the center of the circular track at point \( P \):
   - Equation of the circular track.
   - Equation of the chord at point \( P \).
[1] Prove that the lines MN and OP are parallel.

[2] Explain why a circle with a diameter passes through M and N.

[3] By considering OP as a chord of the circle, find the equation of its perpendicular at V.

[4] The diagram shows ST is a tangent to a circle at the point P. The points Q and R are on the circumference of ST.

[5] Find the equation of the line QR.

[6] Find the equation of the line ST.

[7] Find the equation of the line OP.

[8] Find the equation of the line MN.

[9] Given that the equation of the line OP is 2x + 3y = 6, find the value of p.

[10] Solve for x when x + y = 2.


[12] Solve for x when 3x + y = 4.


[16] Solve for x when 7x + y = 8.

[17] Solve for x when 8x + y = 9.

[18] Solve for x when 9x + y = 10.

[19] Solve for x when 10x + y = 11.

[20] Given that the line 2x + 3y = 6 is not a factor of 2x + y = 3, determine if the line 2x + y = 3 is a factor of the polynomial x^2 + xy + y^2 = 0.

[21] Determine whether a circle with center at (2, 3) and radius 4 is a factor of the polynomial x^2 + xy + y^2 = 0.

[22] Find the coordinates of A and B.

[23] Find the coordinates of C and D.

[24] The equation of the line ST is x + y = 2. A curve has the equation x^2 + xy + y^2 = 3. Show that the line ST intersects the curve x^2 + xy + y^2 = 3 at two distinct points.

[25] Find the points of intersection of the lines 2x + 3y = 6 and 4x + 5y = 7.

[26] Find the points of intersection of the lines 2x + 3y = 6 and 5x + 7y = 9.

[27] Find the points of intersection of the lines 2x + 3y = 6 and 7x + 9y = 11.

[28] Find the points of intersection of the lines 2x + 3y = 6 and 9x + 11y = 13.

[29] Find the points of intersection of the lines 2x + 3y = 6 and 11x + 13y = 15.

[30] Find the points of intersection of the lines 2x + 3y = 6 and 13x + 15y = 17.
Given that \( \int_{a}^{b} f(x) \, dx = \alpha \) and \( b - a = \omega \), where \( \alpha \) is an arbitrary constant, find \( \int_{a}^{b} (a + x) \, dx \).
### Question 1

#### (a) Algebra

For the function: 
\[ f(x) = x^3 - 3x^2 + 2x - 1 \]

1. Find the derivative, \( f'(x) \).
2. Determine the critical points of the function.
3. Classify the critical points as local maxima, local minima, or neither.

#### (b) Calculus

For the function: 
\[ g(x) = x^4 - 2x^3 + x^2 - x + 1 \]

1. Find the derivative, \( g'(x) \).
2. Determine the intervals where the function is increasing or decreasing.
3. Find the points of inflection and classify them as points of concavity change.

### Question 2

#### (a) Geometry

Given a circle with radius \( r \) and center \( (h, k) \), find the equation of the circle.

#### (b) Trigonometry

For the function: 
\[ y = \sin(x) \cos(x) \]

1. Find the derivative, \( y' \).
2. Determine the intervals where the function is increasing or decreasing.
3. Find the points of inflection and classify them as points of concavity change.
### Question 1

**Diagram:**
- A circle with a diameter labeled as $D$.
- The center of the circle is marked as $O$.
- A line segment $AB$ is drawn from one point on the circumference to another, with a point $P$ on the circumference.

**Equation:**
- $\sin \theta = \frac{AB}{AO}$

**Solution:**
- Since $AB$ is a chord of the circle, $\angle AOB = 90^\circ$.
- Using the Pythagorean theorem, $AO^2 = AB^2 + BO^2$.

**Result:**
- $AO = \sqrt{AB^2 + BO^2}$.

---

**Question 2

**Diagram:**
- A triangle with sides $a$, $b$, and $c$.
- A line segment $MN$ is drawn parallel to $BC$.

**Equation:**
- $\frac{MN}{BC} = \frac{MD}{BD}$

**Solution:**
- By the Basic Proportionality Theorem (Thales' theorem), $\frac{MN}{BC} = \frac{MD}{BD}$.

**Result:**
- The line segment $MN$ divides the triangle into two similar triangles.

---

**Question 3

**Diagram:**
- A right-angled triangle with sides $a$, $b$, and $c$ (hypotenuse).

**Equation:**
- $c^2 = a^2 + b^2$

**Solution:**
- Using the Pythagorean theorem, $c$ is the hypotenuse of the right-angled triangle.

**Result:**
- The triangle is a right-angled triangle with sides $a$, $b$, and $c$.

---

**Question 4

**Diagram:**
- A parallelogram with sides $AB$, $BC$, $CD$, and $DA$.
- A diagonal $AC$ is drawn.

**Equation:**
- $AB = CD$, $BC = DA$

**Solution:**
- Opposite sides of a parallelogram are equal.

**Result:**
- The parallelogram has sides $AB$, $BC$, $CD$, and $DA$.

---

**Question 5

**Diagram:**
- A circle with a radius $r$.
- A line segment $AB$ is drawn from one point on the circumference to another, with a center $O$.

**Equation:**
- $\angle AOB = 90^\circ$

**Solution:**
- Since $AB$ is a chord of the circle, $\angle AOB = 90^\circ$.

**Result:**
- The triangle formed by the radius and the chord is a right-angled triangle.
<table>
<thead>
<tr>
<th>Table</th>
<th>Equation</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>( g = \frac{1}{2} (v_0^2 + 2a \cdot \Delta t) )</td>
<td>( g ) is the acceleration due to gravity, ( v_0 ) is the initial velocity, and ( \Delta t ) is the change in time.</td>
</tr>
<tr>
<td>A2</td>
<td>( \Delta x = v_0 \Delta t + \frac{1}{2} a \Delta t^2 )</td>
<td>( \Delta x ) is the change in position, ( v_0 ) is the initial velocity, ( a ) is the acceleration, and ( \Delta t ) is the change in time.</td>
</tr>
<tr>
<td>A3</td>
<td>( \Sigma F = ma )</td>
<td>The principle of conservation of momentum, ( \Sigma F ) is the sum of all forces, ( m ) is the mass, and ( a ) is the acceleration.</td>
</tr>
<tr>
<td>A4</td>
<td>( E = mc^2 )</td>
<td>The equivalence of mass and energy, ( E ) is the energy, ( m ) is the mass, and ( c ) is the speed of light.</td>
</tr>
<tr>
<td>A5</td>
<td>( I = \int \vec{P} \cdot d\vec{L} )</td>
<td>The principle of conservation of angular momentum, ( I ) is the angular momentum.</td>
</tr>
<tr>
<td>A6</td>
<td>( E = \frac{1}{2} mv^2 )</td>
<td>The kinetic energy, ( E ) is the energy, ( m ) is the mass, and ( v ) is the velocity.</td>
</tr>
<tr>
<td>A7</td>
<td>( \theta = \arcsin \left( \frac{\text{opposite}}{\text{hypotenuse}} \right) )</td>
<td>The relationship between the angle ( \theta ), the opposite side, and the hypotenuse.</td>
</tr>
<tr>
<td>A8</td>
<td>( \text{area} = \frac{1}{2} \text{base} \times \text{height} )</td>
<td>The formula for the area of a triangle.</td>
</tr>
<tr>
<td>A9</td>
<td>( \text{circumference} = 2\pi r )</td>
<td>The formula for the circumference of a circle.</td>
</tr>
<tr>
<td>A10</td>
<td>( \text{volume} = \frac{4}{3} \pi r^3 )</td>
<td>The formula for the volume of a sphere.</td>
</tr>
</tbody>
</table>
1. Find the value of x for which \( y = x^2 + 2x \) is a perfect square.

2. On the same diagram, draw the graph of \( y = ax^2 + bx + c \) and state the nature of its roots.

3. The graph below shows some points of \( y = ax^2 + bx + c \), where \( a \neq 0 \).

4. The diagram below shows part of the graph of \( y = ax^2 + bx + c \), where \( a > 0 \).

5. The diagram below shows part of the graph of \( y = ax^2 + bx + c \), where \( a < 0 \).

6. The diagram below shows some points of \( y = ax^2 + bx + c \), where \( a \neq 0 \).
10. The equation of the circle passing through \(-3,2\) and having the line \(x+y=0\) as a diameter is

\[ (x+3)^2 + (y-2)^2 = r^2 \]

11. Find the equation of the line passing through \((-1,2)\) and tangent to the line \(x+y=2\).

\[ y = mx + c \]

where \(m\) is the slope of the line and \(c\) is the y-intercept.

12. Find the equation of the line passing through \((1,2)\) and parallel to the line \(y = 2x + 3\).

\[ y = 2x + b \]

where \(b\) is the y-intercept.

13. Find the point of intersection of the lines \(y = mx + c\) and \(y = nx + d\).

\[ mx + c = nx + d \]

14. Find the point of intersection of the lines \(y = mx + c\) and \(y = nx + d\) if \(m = n\).

\[ 2mx + 2c = 2nx + 2d \]

15. Find the point of intersection of the lines \(y = mx + c\) and \(y = nx + d\) if \(m = -n\).

\[ -mx + c = nx + d \]

16. Find the point of intersection of the lines \(y = mx + c\) and \(y = nx + d\) if \(m = n\) and \(c = d\).

\[ mx + c = nx + c \]

17. Find the point of intersection of the lines \(y = mx + c\) and \(y = nx + d\) if \(m = -n\) and \(c = d\).

\[ -mx + c = nx + d \]

18. Find the point of intersection of the lines \(y = mx + c\) and \(y = nx + d\) if \(m = n\) and \(c = -d\).

\[ mx + c = nx - d \]

19. Find the point of intersection of the lines \(y = mx + c\) and \(y = nx + d\) if \(m = -n\) and \(c = -d\).

\[ -mx + c = nx - d \]
[1] Find the value of \( \theta \) for which \( C \) is above the ground.

[2] Find the maximum value of \( r \) and the corresponding angle of \( \theta \).

[3] Express \( \tan \) in the form of \( \frac{a}{b} \) where \( a > 0 \) and \( 0 < \theta < 90^\circ \).

[4] Draw where \( h = a + \frac{z}{x} \) when \( h \) can be the height of \( C \) above the

The diagram shows a rectangular ABCD with \( AD = x \) cm and \( BC = z \) cm. The area of the rectangle is given by \( A = x \cdot z \). The length of the diagonal AC is \( \sqrt{x^2 + z^2} \). The point P is on the line BC, and the length of BP is \( y \) cm. The angle \( \theta \) is the angle between the line AC and the line BP.

The equation of the line AC is given by \( y = \frac{z}{x}x + b \). The equation of the line BP is given by \( y = \frac{y}{z}x + c \). The point of intersection of the two lines is the point P.

[5] Solve the equation \( 2x + 3y = 5 \) for \( y \).

[6] Find the solution of \( 2x^2 + 3x + 1 = 0 \) using the quadratic formula.

[7] Prove the identity \( 1 - \cos 2x = 2 \sin^2 x \).

[8] Sketch the graph of \( f(x) = \sin x \) for \( 0 \leq x \leq 2\pi \).

[9] Sketch the graph of \( g(x) = \cos x \) for \( 0 \leq x \leq 2\pi \).

[10] Sketch the graph of \( h(x) = \tan x \) for \( 0 < x < \pi \).

[11] Sketch the graph of \( j(x) = \sec x \) for \( 0 < x < \frac{\pi}{2} \).
Let the length of the edge be \( l \). Then the height of the pyramid is \( \sqrt{l^2 - \frac{1}{4}} \).

The base of the pyramid is a square with side length \( l \).

The volume of the pyramid is \( \frac{1}{3} \times \text{base area} \times \text{height} \).

The volume of the pyramid is \( \frac{1}{3} \times l^2 \times \sqrt{l^2 - \frac{1}{4}} \).

Express the volume of the pyramid in the form \( \frac{1}{3} (x + y) \), where \( x \) and \( y \) are integers.

[Diagram of a pyramid]

\[ \begin{align*}
\text{Volume} &= \frac{1}{3} \times l^2 \times \sqrt{l^2 - \frac{1}{4}} \\
&= \frac{1}{3} \left( l^2 \sqrt{l^2 - \frac{1}{4}} \right)
\end{align*} \]

By the Pythagorean Theorem, \( l^2 = x^2 + y^2 \).

(iii) The volume of the pyramid is \( \frac{1}{3} (x + y) \), where \( x \) and \( y \) are integers.

[Diagram of a right triangle]
[A1] Given: \( f(x) = x^2 + 2x + 3 \)

(a) Find the derivative of \( f(x) \). 

(b) Determine the equation of the tangent line to the graph of \( f(x) \) at the point where \( x = -1 \). 

(c) Draw a sketch of the graph of \( f(x) \) and the tangent line found in (b) on the same set of axes.

---

[Table]

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-3)</th>
<th>(-2)</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
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</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>(-1)</td>
<td>(1)</td>
<td>(3)</td>
<td>(4)</td>
<td>(6)</td>
</tr>
</tbody>
</table>

---

**Solution:**

(a) The derivative of \( f(x) = x^2 + 2x + 3 \) is \( f'(x) = 2x + 2 \).

(b) The equation of the tangent line at \( x = -1 \) is found using the formula \( y - y_1 = m(x - x_1) \), where \( m = f'(-1) = 2(-1) + 2 = 0 \) and \( (x_1, y_1) = (-1, 3) \).

Thus, the equation of the tangent line is \( y = 3 \).

(c) Sketch of the graph and tangent line.

---

**Note:** The table shows some values of \( x \) and \( f(x) \), which are connected by the equation \( y = x^2 + 2x + 3 \).
The given problem requires understanding of geometry and algebra. The problem involves finding the area of a region defined by certain lines and curves. The solution involves calculating the area under the curves and between the lines. The final answer is presented in a clear and concise manner, ensuring that all steps are logically connected.
(2) Find the equation of the circle with center $(2, -1)$ and radius 3.

\[ (x-2)^2 + (y+1)^2 = 9 \]

(3) The circle with center $(1, -2)$ and radius 5.

\[ (x-1)^2 + (y+2)^2 = 25 \]

(4) Find the equation of the circle with center $(3, -4)$ and radius 2.

\[ (x-3)^2 + (y+4)^2 = 4 \]

(5) Find the equation of the circle with center $(-2, 1)$ and radius 3.

\[ (x+2)^2 + (y-1)^2 = 9 \]

(6) Find the equation of the circle with center $(0, 0)$ and radius 4.

\[ x^2 + y^2 = 16 \]
8. The velocity of a particle moving in a straight line at time $t$ sec is given by $v = 6 - 2t^2$. Find the acceleration of the particle at 3 sec. (2)

7. Find the derivative of the function with respect to $x$.

6. Find the area of the shaded region.

5. Find the area of the shaded region.

4. Find the area of the shaded region.

3. Find the area of the shaded region.

2. Find the area of the shaded region.

1. Find the area of the shaded region.
Find the value of \( \theta \) for which \( C \) is in line with the ground.

Find the maximum value of \( h \) and the corresponding value of \( \theta \) in the form of \( (a - \theta) \), where \( a > \theta \) and \( a < 90^\circ \).

Examine the figure above to confirm that \( z \) cm is the height of \( C \) above the ground.

The diagram shows a rectangular ABCD with \( AD = z \) cm and \( BC = 5 \) cm. The diagram also shows a segment HPD with \( HP = 3 \) cm.
2. TRIGONOMETRY

\[
\frac{\sin x}{\cos x} = \tan x
\]

Formula for tan x

1. ALGEBRA

Homework assignment

Name of Student: [Student Name]

This page consists of 2 printed pages including the cover pages.

TOTAL 100 MARKS

The total number of marks for the paper is 100.

Formula for trigonometric ratios:

\[
\sin x = \frac{\text{opposite}}{\text{hypotenuse}}
\]

Problem:

Given \( \sin x = \frac{3}{5} \), find the value of \( \cos x \).

Solution:

Using the Pythagorean identity:

\[
\sin^2 x + \cos^2 x = 1
\]

\[
\left(\frac{3}{5}\right)^2 + \cos^2 x = 1
\]

\[
\frac{9}{25} + \cos^2 x = 1
\]

\[
\cos^2 x = 1 - \frac{9}{25}
\]

\[
\cos^2 x = \frac{16}{25}
\]

\[
\cos x = \pm \frac{4}{5}
\]

Since sine is positive in the first quadrant, \( \cos x = \frac{4}{5} \).

Do not use calculator for algebra questions. Show all working.

PLEASE-answer-first-questions-1-2.

Name of Student: [Student Name]

Class: [Class]

REMEMBER TO READ THESE INSTRUCTIONS FIRST

Secondary Paper Expression

Commonwealth Secondary School

Paper 2

ADDITIONAL MATHEMATICS

Preliminary Examination 2016
By comparison, we have 

\[ t = \frac{1}{2} \left( \frac{1}{x} + 1 \right) \]

Then, we get 

\[ t = \frac{1}{x} \]

The coordinates of the secondary point 

\[ (x', y') = (x, y) \]

The equation of the curve is 

\[ \frac{x}{t} + \frac{y}{2} = \frac{t}{x} \]
Since the corresponding angles are equal, triangles $ABC$ and $DEF$ are similar.

$p = \frac{2x}{x - 3}$

$x + x = 5$

$2x + 2x + 3x = 18$

$6x = 18$

$x = 3$

$x = 3$

$z_1 = 3$

$z_2 = 3$

$x = 3$

$y = 3$

$x = 3$

$z_1 = 3$

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This paper consists of 3 printed pages including the cover page.

Name of other...

The total number of marks for this paper is 70.

The number of questions is 7 given in Question 1 to 7. All questions carry equal marks.

At the end of each question, leave the space for answer.

You are allowed to use one set of additional materials. Write your answers in your own words.

Give your answers in the space provided. Write your answers in the space provided.

Answer all questions.

Do not use a sharp pencil, destroy the question papers.

Your answers must be clear and neat.

Write your answers as quickly as possible on both sides of the paper.

Turn over

1. ALGEBRA

A) Algebraic Formulae

b) Trigonometry

\[
\begin{align*}
\frac{\sin \theta}{1} &= \sin \theta \\
\cos \theta &= \frac{\cos \theta}{1} \\
\tan \theta &= \frac{\tan \theta}{1}
\end{align*}
\]

B) Geometric Progression

\[
\frac{a}{1} - \frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \ldots = \frac{a}{r^2} \cdot \frac{1}{1 - \frac{1}{r}} \quad \text{Sum to } \infty
\]

C) Arithmetic Progression

\[
\frac{a}{1} + \frac{a}{2} + \frac{a}{3} + \ldots = \frac{a}{x} \quad \text{Sum to } \infty
\]

D) Logarithms

\[
\log_a b = \frac{\log c}{\log d}
\]

E) Calculus

\[
\frac{d}{dx} \sin x = \cos x
\]

F) Differential Equations

\[
\frac{dy}{dx} + 2y = 0
\]
The diagram above shows a maximum number of 13 hexagonal circles packed into a square. If the radius of each circle is 1 cm, find the equation of the circle C which is a reflection of the circle C_1 in the line y = 1.
The given figure is a triangle ABC with a parallelogram GDEF inscribed within it. The problem involves finding the area of the shaded region.

**Problem Statement:**

Find the area of the shaded region by the parallelogram GDEF inscribed in the triangle ABC.

**Solution:**

1. **Step 1:** Calculate the area of the triangle ABC using the formula for the area of a triangle, which is 
   \[ \frac{1}{2} \times \text{base} \times \text{height} \].

2. **Step 2:** Calculate the area of the parallelogram GDEF using the formula for the area of a parallelogram, which is 
   \[ \text{base} \times \text{height} \].

3. **Step 3:** Subtract the area of the parallelogram GDEF from the area of the triangle ABC to get the area of the shaded region.

**Diagram:**

The diagram shows a triangle ABC with a parallelogram GDEF inscribed within it. The shaded region is the area to be calculated.
END OF PAPER

[5] Show that the value of $d$ is 60.

[6] Explain why the shaded region is the region of the enclosed area where is 60 cm.

Explain the equation is satisfied in a constant term of 60 (accelerations are constant).

[7] Consider the case of the enclosed area which is 60 cm.

For example, the equation is modified by the acceleration $a = -g(t - c)^2$, where $g$ is 60.

V. The float is illustrated that the enclosed shaded area, given by the solution $60 cm$. In the float is with the

In the City A, the area shaded the enclosed region is modified which is 60 cm. The float is with the other

L.S. You need to illustrate that the enclosed shaded area is modified.

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examguru
The areas of the circles are equal. \( A_F = A_G \). Let \( F \) be the point of contact.

\[
S_1 = S_2 = \frac{1}{4} \pi r^2
\]

\[
x^2 + y^2 = r^2
\]

The condition for the circles to touch each other is:

\[
x^2 + (y - a)^2 = r^2
\]

Solving these equations simultaneously gives us the coordinates of the points of contact.

\[
A = \left( r \cos \theta, r \sin \theta \right)
\]

\[
B = \left( r \cos (\theta + \frac{\pi}{2}), r \sin (\theta + \frac{\pi}{2}) \right)
\]

The distance between the centers of the circles is:

\[
d = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}
\]

The area of the annulus between the two circles is:

\[
A_{\text{annulus}} = \pi r^2 - \pi r^2 = \pi (r^2 - r^2)
\]

The area of the shaded region is:

\[
A_{\text{shaded}} = A_F + A_G
\]
\[
\begin{align*}
\text{If } \sin(\theta) = \frac{1}{2}, & \quad \text{then } \theta = \frac{\pi}{6} \\
\text{or } \theta = \frac{5\pi}{6}. \\
\text{If } 0 < \theta < \frac{\pi}{2}, & \quad \text{then } \theta = \frac{\pi}{6}. \\
\text{If } \frac{\pi}{2} < \theta < \pi, & \quad \text{then } \theta = \frac{5\pi}{6}.
\end{align*}
\]
2. TRIGONOMETRY

\[ \frac{\sin^{-1} \theta}{\cos^{-1} \theta} = \frac{\sin \theta}{\cos \theta} = \tan \theta \]

where \( n \) is a positive integer and

\[ \begin{align*}
\sin^{-1} \theta &= \frac{1}{\cos^{-1} \theta} \\
\cos^{-1} \theta &= \frac{1}{\sin^{-1} \theta}
\end{align*} \]

\[ \tan^{-1} \theta = \frac{1}{\sin^{-1} \theta} \]

Identity

2. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
1. The equation of the parabola is given by

\[ y = x^2 - 4x + 4 \]

2. The equation of the tangent to the parabola at the point (2, 0) is given by

\[ y - 0 = m(x - 2) \]

3. The equation of the normal to the parabola at the point (2, 0) is given by

\[ y = 2 - 2x \]

4. The equation of the curve is given by

\[ y = x^2 + 2x + 1 \]

5. The equation of the tangent to the curve at the point (1, 4) is given by

\[ y - 4 = 2(x - 1) \]

6. The equation of the normal to the curve at the point (1, 4) is given by

\[ y - 4 = -\frac{1}{2}(x - 1) \]

7. The equation of the curve is given by

\[ y = x^3 - 3x^2 + 3x - 1 \]

8. The equation of the tangent to the curve at the point (1, 0) is given by

\[ y - 0 = 2(x - 1) \]

9. The equation of the normal to the curve at the point (1, 0) is given by

\[ y - 0 = -\frac{1}{2}(x - 1) \]
11. The diagram shows the curve \( y = 8 - x^2 \) and the points \( R(-2, 0) \) and \( P(p, 0) \). The point \( Q \) lies on the curve such that \( PQ \) is parallel to the \( y \)-axis.

(i) Show that the area, \( A \) units\(^2\), of the triangle \( PQR \) is given by

\[
A = \frac{1}{2} (p + 2)(8 - p^2).
\]

(ii) The point \( P \) moves along the \( x \)-axis at a constant rate of 0.04 units per second and \( Q \) moves along the curve so that \( PQ \) remains parallel to the \( y \)-axis.

Find the rate at which \( A \) is decreasing when \( p = 1.5 \). [3]

12. A gardener uses 200 m of fencing to enclose a plot of land in the shape shown above. The shape consists of a semicircle of radius \( x \) m and a rectangle with sides \( 2x \) m and \( y \) m.

(i) Show that the area, \( A \) m\(^2\), of the plot of land is given by

\[
A = 200x - \left( \frac{x + 4}{2} \right) x^2.
\]

(ii) Given that \( x \) can vary, find the value of \( x \) for which the area of the plot is the largest possible. [4]

END OF PAPER
2016 4ESN Prelim AM Mark Scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[ \frac{dy}{dx} = \frac{(2 \cos x)(2 \cos x - \sin x) - \cos^2 x}{(2 \sin x)^2} ] When ( x = 0 ), ( \frac{dy}{dx} = \frac{1}{4} ) When ( x = \frac{\pi}{2} ), ( y = \frac{1}{2} ) Equation of tangent: [ y - \frac{1}{2} = \frac{1}{4} x ] [ \Rightarrow y = \frac{1}{4} x + \frac{1}{2} ]</td>
</tr>
<tr>
<td>2i</td>
<td>Let [ \frac{2x+8}{(x^2+4)(x-1)} = \frac{Ax+B}{x^2+4} + \frac{C}{x-1} ] [ 2x+8 = (Ax+B)(x-1)+C(x^2+4) ] By substitution or comparison of coefficients: ( A = -2 ) ( B = 0 ) ( C = 2 ) Hence [ \frac{2x+8}{(x^2+4)(x-1)} = \frac{-2x+2}{x^2+4} - \frac{2}{x-1} ]</td>
</tr>
<tr>
<td>3ii</td>
<td>The maximum value of ( y ) for the given range is 12. [ x = -0.562 \text{ or } x = 1.56 ]</td>
</tr>
<tr>
<td>3iii</td>
<td>[</td>
</tr>
<tr>
<td>4</td>
<td>[ (a+x)^3 + (2-bx)^3 - (a^3 + 4a^2x + 6a^2x^2 + ...) + (32 - 80bx + 80b^2x^2 + ...) ] Comparing coefficients, ( 32 + a^3 = 48 ) ( a = 2 ) or ( b = -2 ) (NA) Similarly, ( b = \frac{1}{4} ) ( c = 29 )</td>
</tr>
<tr>
<td>5</td>
<td>( y = Ae^{x-a} ) [ \ln y = \ln A + (x-a) ] Gradient: ( A = 2.89 - 1.39 = 0.5 ) ( x = 4.01 ) (3 s.f.) [ A = 4.01 ] (3 s.f.)</td>
</tr>
</tbody>
</table>
### 10k C(7,7)

- Midpoint of BD = \( \left( \frac{7}{2}, \frac{3}{2} \right) \).
- Gradient of BD = \( \frac{3}{7} \).
- Equation of perpendicular bisector of BD:
  \[
  y - \frac{3}{2} = \frac{3}{7} (x - \frac{7}{2})
  \]
  \[
  3y = 7x - 20 \quad \text{-- (1)}
  \]
  \[
  5y = 2x - 14 \quad \text{-- (2)}
  \]

Solving (1) and (2) simultaneously:
\[
  x = 2, \, y = -2
  \]
\[
  \Rightarrow A(2, -2).
  \]

### 10ii

- Area = \( \frac{1}{2} \begin{vmatrix} 1 & 2 & 7 & 7 & 0 & 2 \\ 2 & -2 & 0 & 7 & 3 & -2 \end{vmatrix} \)
  \[
  \approx 39 \text{ units}^2
  \]

### 10iii

- Gradient of AC = \( \frac{7 + 2}{7 - 2} = \frac{9}{5} \)
- \( \frac{3}{5} \times \frac{9}{5} \neq -1 \)
- Since AC and BD are not perpendicular to each other, ABCD is not a kite.

### 11i

- When \( x = p \), \( y = 8 - p^2 \), \( \Rightarrow Q(p, 8 - p^2) \)
- Area of \( PQR \)
  \[
  = \frac{1}{2} \times (8 - p^2) \times (p - 0)
  \]
  \[
  = \frac{1}{2} (p + 2)(8 - p^2) \text{ (Shown)}
  \]

### 11ii

\[
\begin{align*}
11ii & \quad \frac{dA}{dt} = \frac{dA}{dp} \cdot \frac{dp}{dt} \\
& \quad = \frac{1}{2} \left( \frac{1}{2} \left( p + 2 \right)(-2p) + 8 - p^2 \right) \times 0.04 \\
& \quad \frac{dA}{dt} \bigg|_{t=1} = -0.095 \text{ units}^2/\text{s} \\
& \quad \text{Rate} = 0.095 \text{ units}^3/\text{s}
\end{align*}
\]

### 11iii

- Total Perimeter:
  \[
  200 = 2x + 2y + \frac{1}{2} (2xx)
  \]
  \[
  y = 200 - (x + 2)x
  \]
  \[
  A = \frac{1}{2} \left( x^2 + 2xy \right)
  \]
  \[
  = \frac{1}{2} \pi x^2 + 200x - (x + 2)x^2
  \]
  \[
  = 200x - \left( \frac{\pi + 4}{2} \right)x^2 \text{ (shown)}
  \]

### 12ii

- \( \frac{dA}{dx} = 200 - (x + 4)x \)

Let \( \frac{dA}{dx} = 0 \),
\[
\frac{200}{\pi + 4} \text{ or } x = 28.0 \text{ (s.f.)}
\]

\[
\frac{d^2A}{dx^2} = -x - 4 < 0
\]

By second derivative test, \( A \) is maximum when \( x = \frac{200}{\pi + 4} \).
2. TRIGONOMETRY

\[ \frac{x}{(1 + x)(1 - u)} = \frac{1}{u} \]

where \( u \) is a positive integer and 

\[ a + b + c = \frac{1}{u} \] 

\[ a + b + c = \rho = (q + v) \]

\[ \frac{z}{w} = \frac{x}{y + z} \]

For the equation \( ax + bx + c = 0 \),

Quadratic Equation

1. ALGEBRA

1. Algebraic Formulae

2. Quadratic Formula

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
1. Find the volume of the cylinder.

2. Given that the volume of the solid is 700 cm³, express the volume by

3. Given the height of the can is 3 cm and the radius of the cylinder is

4. Show that \( x > 3 \) is always positive for real values of \( x \).

5. Find the range of values of \( x \) for which \( \log_{\frac{1}{x}} x > 0 \).

6. Show that the function \( f(x) = \frac{1}{x} - x \) is satisfied for all real values of \( x \).

7. Find the number of values of \( x \) for which \( \log_{\frac{1}{x}} x \) is defined.

8. Find the value of the constant \( c \) in the relationship

9. Solve the equation for all \( x \) such that \( 0 < x < 2 \).

10. Find the point of tangency of the circle with a radius of 3 cm at

11. The radius of the circle is 2 cm and the height of the cylinder is

12. Find the area of the triangle formed by the points \( A \), \( B \), and \( C \).

13. Find the area of the triangle formed by the points \( D \), \( E \), and \( F \).

14. Find the area of the triangle formed by the points \( G \), \( H \), and \( I \).

15. Find the area of the triangle formed by the points \( J \), \( K \), and \( L \).

16. Find the area of the triangle formed by the points \( M \), \( N \), and \( O \).

17. Find the area of the triangle formed by the points \( P \), \( Q \), and \( R \).

18. Find the area of the triangle formed by the points \( S \), \( T \), and \( U \).

19. Find the area of the triangle formed by the points \( V \), \( W \), and \( X \).

20. Find the area of the triangle formed by the points \( Y \), \( Z \), and \( a \).
12. The function \( f(x) \) is defined by the equation \( f(x) = 2 \cos 2x + 1 \).

\[ f(x) = 2 \cos 2x + 1 \]

\[ f' = 2 \cdot (-2 \sin 2x) = -4 \sin 2x \]

\[ f'' = -4 \cdot 2 \cos 2x = -8 \cos 2x \]

Show each of the following.

(i) The roots of the equation \( f(x) = 0 \) are \( \pm \frac{\pi}{4} \).

(ii) The intervals of increase of \( f(x) \) are \( (0, \pi) \) and \( (\pi, 2\pi) \).

(iii) The intervals of decrease of \( f(x) \) are \( (-\pi, 0) \) and \( (-2\pi, -\pi) \).

(iv) The maximum value of \( f(x) \) occurs at \( x = \frac{\pi}{2} \).

(v) The minimum value of \( f(x) \) occurs at \( x = \frac{3\pi}{2} \).

13. In the figure, \( O \) is the centre of the circle, \( PQR \) is a diameter, \( AB \) is a tangent at \( A \), and \( PQ = 6 \).

\[ \angle QOR = 90^\circ \]

\[ \angle APQ = 15^\circ \]

Find the distance of the point where the line of the depth of the circle is the depth of the circle.

\[ \text{Distance} = \sqrt{6^2 + 2^2} = \sqrt{40} \]

\[ \text{Volume} = \pi \cdot 6^2 \cdot \sqrt{40} \]

\[ V = 36 \pi \sqrt{40} \]

\[ \text{Volume} = 36 \pi \cdot 6.32455 \approx 716.8 \]

\[ \text{Volume} = 716.8 \text{ cm}^3 \]
\[ x^2 - x + 1 = (x - \frac{1}{2})^2 + 1 - \left(\frac{1}{4}\right)^2 \]

Since \((x - \frac{1}{2})^2 \geq 0\),

\[(x - \frac{1}{2})^2 + \frac{3}{4} > 0 \quad \text{(A1)}
\]

\[ x^2 - x + 1 \] is always positive for all real values of \(x\).

4(i)
\[ x^2 + bx - 2 < 2 (x^2 - x + 1) \]
\[ x^2 + bx - 2 < 2x^2 - 2x + 2 \quad \text{(A1)} \]
\[ 0 < x^2 - 2x - bx + 4 \]
\[ x^2 + (-2-b)x + 4 > 0 \]
\[ b^2 - 4ac < 0 \]
\[ (-2-b)^2 - 4(1)(4) < 0 \quad \text{(A1)} \]
\[ 4 + 4b + b^2 - 16 < 0 \]
\[ b^2 + 4b - 12 < 0 \]
\[ (b + 6)(b - 2) < 0 \]
\[ -6 < b < 2 \quad \text{(A1)} \]
\[ RL = R1 \]
\[ \angle QRT = \angle LRT = \angle LQT = \angle LTB \quad (\text{Alt. segment}) \]
\[ \angle QTA = \angle LTB = 90^\circ \quad (\text{tangent} \perp \text{radius}) \]
\[ LR1 = 90^\circ - \angle LTB = 90^\circ - \angle LTA = OTQ = \angle QtQ \quad m1 \]
\[ LSTB = LQT \quad (\text{Alt. segment}) \]
\[ : LR1 = LSTB \quad m1 \]
\[ LR1 = LQTR \quad (\text{Alt. segment}) \]
\[ = \angle LRQ \quad (\text{base of a } \triangle \text{ and opposite}) \quad m1 \]

12a. \[ \text{Period } = \frac{2\pi}{\frac{2}{4}} = \pi \]
\[ \text{Amplitude } = 3 \]

12b. \[ y = 3 \cos(2x + 1) \]

Curve B1
Label B1
Exp B1

12c. \[ \text{Max } = 4 \quad \text{B2} \]
\[ \text{Min } = -2 \]

12d. \[ 2 \text{ solutions} \quad \text{B1} \]
\[ \frac{dy}{dx} = \frac{4x^3 + 2x^2 + 20x}{0.01x^2 + 4x + 20} \]

\[ y = \frac{1}{0.01x^2 + 4x + 20} \]

\[ \frac{dy}{dx} = \frac{d}{dx} \left( \frac{1}{0.01x^2 + 4x + 20} \right) = \frac{-0.02x - 4}{(0.01x^2 + 4x + 20)^2} \]

\[ x = 1 \]

\[ y = \frac{1}{0.01(1)^2 + 4(1) + 20} = \frac{1}{7} \]

\[ \frac{dy}{dx} = \frac{-0.02(1) - 4}{(0.01(1)^2 + 4(1) + 20)^2} = \frac{-4.02}{7^2} \]

The depth of fluid at point \( x = 20 \) cm is...

\[ x = 0 \]

\[ y = \frac{0}{16.3} \]

\[ x = 20 \]

\[ y = \frac{40}{16.3} \]

\[ \text{Depth} = 20 \text{ cm} \]
\[
V = \frac{1}{3} \pi r^2 h + \pi r^2 \frac{250 - 8r^2}{18r}
\]
\[
= \frac{1}{3} \pi r^2 \left( \frac{250 - 8r^2}{18r} \right) + \pi r^2 \left( \frac{250 - 8r^2}{18r} \right)
\]
\[
\text{Height}
\]
\[
= \frac{250 - 8r^2}{18r} \pi r^3 + \pi 8r^2 - 2\pi r^3
\]
\[
= \frac{250}{18} r + \left( \frac{250}{18} - 2 \right) \pi r^2
\]
\[
= \frac{250}{18} \pi r^2
\]
\[
\frac{dv}{dr} = 0
\]
\[
250 + (250 - 6) \pi r^2 = 0
\]
\[
r^2 = \frac{-250}{(250 - 6) \pi}
\]
\[
r = \frac{25.0695387}{\pi}
\]
\[
r = 0.0695927
\]
\[
r = 5.009
\]
\[
V = 250 \left( 5.009 \right) + \left( \frac{250}{5.009} - 2 \right) \pi (5.009)^3
\]
\[
= 834.89 \text{ cm}^3 (5.009)
\]
\[
= 835 \text{ cm}^3 (5.009)
\]
\[
\frac{dv}{dr} = (455 - 12) \pi r
\]
\[
= 44 \pi r < 0
\]
}\[
\text{QA}
\]
\[
\text{TSF} = \text{curved surface area of cone} + \text{curved surface area of cylinder} + \text{area of circle}
\]
\[
= \pi r (3r) + 2\pi rh + \pi r^2
\]
\[
= 3\pi r^2 + 2\pi rh + \pi r^2
\]
\[
= 4\pi r^2 + 2\pi rh
\]
\[
4\pi r^2 + 2\pi rh = 500
\]
\[
2\pi r^2 + \pi rh = 250
\]
\[
h = \frac{250 - 2\pi r^2}{\pi r}
\]
\[ s = \int v \, dt \]
\[ = \int (2u^2 \cdot t - 1) \, dt \]
\[ = \int (2u^2 \cdot t) \, dt - \int 1 \, dt \]
\[ = \frac{u^2 t^2}{2} - t \]

At \( t = 0 \), \( d = 0 \) \( c = 0 \)

\[ s = \frac{u^2 t^2}{2} \]
\[ \text{Total distance travelled in } 1 \text{ ft } 2 \text{ second} \]
\[ = 0.3784 \text{ ft} \]
\[ = 0.5 \text{ ft} \]
\[ = 1.3784 \text{ ft} \]
\[ = 1.5 \text{ ft} \]

At \( t = 2 \), \( a = -4 \sin(2) \cos(2) \)
\[ = -1.5136 \text{ m/s}^2 \]

At \( t = 0 \), \( v = 2u^2 t - 1 \)
\[ = 4 \cos t (-3 \sin t) \]
\[ = -4 \sin t \cos t \]

At \( t = 2 \), \( a = -4 \sin(2) \cos(2) \)
\[ = -1.5136 \text{ m/s}^2 \]

At \( t = 0 \), \( v = 2u^2 t - 1 \)
\[ = 2u^2 t = 1 \]
\[ a = \frac{1}{t} \]

At \( t = \sqrt{2} \), \( \cos t = -\frac{1}{\sqrt{2}} \)

At \( t = 3.9269 \), \( 2.3561 \)

Time is 0.715 s when \( a = 0 \)

Particle at instantaneous rest.
\[ 2 \sec 2\theta = 6 \]
\[ \frac{2}{\cos 2\theta} = 6 \]
\[ \cos 2\theta = \frac{1}{3} \]
\[ \sin \theta = 70.5287^\circ \quad \text{(dp)} \]
\[ \tan \theta = 70.5287^\circ \]
\[ \theta = 70.5287^\circ, 297.4713^\circ, 430.5287^\circ, 569.4713^\circ \]
\[ \theta = 35.2643^\circ, 144.7356^\circ, 215.2643^\circ, 324.7356^\circ \]
\[ \theta = 10\sqrt{3}^\circ, 194.7^\circ, 215.3^\circ, 324.7^\circ \quad \text{(dp)} \]

\[ \tan (45^\circ + \theta) + \tan (45^\circ - \theta) = 2 \sec 2\theta \]

\[ \frac{1 + \tan \theta}{1 - \tan \theta} + \frac{1 - \tan \theta}{1 + \tan \theta} \]
\[ = \left( \frac{1 + \tan \theta}{1 - \tan \theta} \right)^2 + \left( \frac{1 - \tan \theta}{1 + \tan \theta} \right)^2 \quad \text{(dp)} \]
\[ = \frac{1 + 2\tan \theta + \tan^2 \theta + 1 - 2\tan \theta + \tan^2 \theta}{1 - \tan^2 \theta} \]
\[ = \frac{2 + 2\tan^2 \theta}{1 - \tan^2 \theta} \]

\[ \sec \theta = |1 + \tan \theta| \]
\[ \tan \theta = \sec^2 \theta - 1 \]

\[ \frac{2 \sec^2 \theta}{2 - \sec^2 \theta} \]

\[ \frac{2}{\cos^2 \theta} \quad \text{(dp)} \]

\[ = \frac{2}{\cos^2 \theta} \quad \text{and} \quad 2 \sec 2\theta \]
\[ x^2 - 2x - 6 \]
\[ = (x-1)^2 - 4 - (\frac{5}{2})^2 \]
\[ = (x-1)^2 - 7 \]

**Curve B1**

**Label B1**

**Eqn B1**

\[ y = 2x^\frac{1}{2} - 3 \]

\[ -3 < y \leq \sqrt{5} - 3 \]

\[ 0 \leq x \leq 1 \]
1. Find the area of triangle $POD$ where $O$ is the origin.

2. Find the coordinates of the points on the curve where the normal is parallel to $x$.

3. Express the expression $\frac{2+y}{x}$ in the form $a + \sqrt{x}$, where $a$ and $b$ are integers.

4. Hence, find the coordinates of $x$ in the expression of $\left(\frac{2+y}{x}\right)^{\frac{1}{2}}$.

5. Show that $x = 2$.

6. Show that $x = 2y$.

7. Find the value of $x$ for which the normal to the curve is parallel to the line $y = mx$.

8. Show that $x = 2e^{2x}$.
[f] Find the equation of the circle whose centre is (a, b) and passes through the point (c, d).

\[ (x-a)^2 + (y-b)^2 = r^2 \]

[2] Find the equation of the circle whose centre is (c, d) and passes through the point (a, b).

\[ (x-c)^2 + (y-d)^2 = r^2 \]

[3] Find the equation of the circle whose centre is (a, b) and passes through the point (c, d).

\[ (x-a)^2 + (y-b)^2 = r^2 \]

[4] Find the equation of the circle whose centre is (c, d) and passes through the point (a, b).

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[5] Find the equation of the circle whose centre is (a, b) and passes through the point (c, d).

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\[ (x-a)^2 + (y-b)^2 = r^2 \]

[8] Find the equation of the circle whose centre is (c, d) and passes through the point (a, b).

\[ (x-c)^2 + (y-d)^2 = r^2 \]

[9] Find the equation of the circle whose centre is (a, b) and passes through the point (c, d).

\[ (x-a)^2 + (y-b)^2 = r^2 \]

[10] Find the equation of the circle whose centre is (c, d) and passes through the point (a, b).

\[ (x-c)^2 + (y-d)^2 = r^2 \]
End of Paper

[1] Find the value of \( x \) when \( y = 4 \).

[2] Express \( 9 \) in terms of \( x \).

The diagram below shows part of a straight line graph of \( y = \frac{1}{x} \). "Passes through the points \((1, 2)\) and \((2, 1)\)."


[4] "Write down the gradient of the vertical axis of a graph."

[5] Plan - vertical x and draw a diagonal line through graph.

\[ \begin{array}{c|c|c|c|c|c|c}
 x & 0 & 0.89 & 1.14 & 1.49 & 2.86 & \frac{9}{x} \\
- & 0 & 4 & 3 & 2 & 1 & \frac{9}{x} \\
\end{array} \]

Connected by an equation of the form \( y = \frac{9}{x} \). Where a and b are constants.

The table shows exponential values of two variables \( x \) and \( y \) which are

Answer the whole of this question on a sheet of graph paper.
\[
\begin{align*}
A &= \frac{L}{\frac{\pi^2}{8}} \\
L &= \frac{1}{\Psi + \frac{\pi^2}{8}} \\
L &= \frac{1}{\left[ \frac{\pi^2}{8} - \frac{11\pi^4}{16} \right]} \\
\exp \left( \frac{x+y}{2} \right) + \exp \left( \frac{x+y}{2} \right) &= \frac{1}{4} \\
\exp \left( \frac{x+y}{2} \right) + \left( \exp \left( \frac{x+y}{2} \right) \right)_x - \exp \left( \frac{x+y}{2} \right)_x &= \frac{1}{4} \\
\exp \left( \frac{x+y}{2} \right) + \left( \exp \left( \frac{x+y}{2} \right) \right)_y - \exp \left( \frac{x+y}{2} \right)_y &= \frac{1}{4} \\
\exp \left( \frac{x+y}{2} \right) + \left( \exp \left( \frac{x+y}{2} \right) \right)_x - \exp \left( \frac{x+y}{2} \right)_x &= \frac{1}{4} \\
\exp \left( \frac{x+y}{2} \right) + \left( \exp \left( \frac{x+y}{2} \right) \right)_y - \exp \left( \frac{x+y}{2} \right)_y &= \frac{1}{4} \\
\exp \left( \frac{x+y}{2} \right) + \left( \exp \left( \frac{x+y}{2} \right) \right)_x - \exp \left( \frac{x+y}{2} \right)_x &= \frac{1}{4} \\
\exp \left( \frac{x+y}{2} \right) + \left( \exp \left( \frac{x+y}{2} \right) \right)_y - \exp \left( \frac{x+y}{2} \right)_y &= \frac{1}{4} \tag{1}
\end{align*}
\]
The coordinates of the points are \( (\frac{1}{3}, 2) \) and \( (\frac{2}{3}, 1) \).

1. Determine \( M \):
   
   \[
   M \left( \frac{1}{3}, 2 \right), \quad M \left( \frac{2}{3}, 1 \right)
   \]

2. Find the equation of the line passing through \( M \):
   
   \[
   \frac{x - \frac{1}{3}}{\frac{2}{3} - \frac{1}{3}} = \frac{y - 2}{1 - 2}
   \]

3. Solve for \( x \) and \( y \):
   
   \[
   x = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}, \quad y = \frac{1}{3}
   \]

4. Find the gradient of the line:
   
   \[
   \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 2}{\frac{2}{3} - \frac{1}{3}} = -1
   \]

5. The equation of the line can be written as:
   
   \[
   y - \frac{1}{3} = -1 \left( x - \frac{1}{3} \right)
   \]

6. Simplify to find the final equation of the line:
   
   \[
   y = -x + \frac{1}{3}
   \]
4. (a) \( \frac{b}{x} - 3 \cdot \frac{a}{x^2} = \frac{b - 3a}{x^2} \)...

(b) The term independent of \( x \) in \( x^2 - 4x + 5 \) is 5.

For the term independent of \( x \), \( x - 4x = 0 \)

\( k = -4 \) (constant term)

5. Let \( z \) be the length of the opposite

\[ z^2 = x^2 + 2 \text{cm}^2 \]

\[ z = \sqrt{x^2 + 2} \text{ cm} \]
\[
\begin{align*}
\text{Given:} & \quad a = 2b + 2c + 2d + 2e \\
\text{Required:} & \quad x, y, z, t
\end{align*}
\]
\[
\frac{x}{c + x} + \frac{y}{c + y} = \frac{x}{c + x} + \frac{y}{c + y} = \frac{c + x + x}{c + x} + \frac{c + y + y}{c + y} = \frac{c + 2x}{c + x} + \frac{c + 2y}{c + y} = \frac{c + 2x + c + 2y}{c + x + c + y} = \frac{2(c + x + y)}{c + x + y} = 2
\]

Equation of circle is \( x^2 + y^2 = r^2 \).

Given: \( c = 3 \), \( e = 2 \).

Since \( 2e = 4 \), the point of inverse is outside the circle. (Proved).

Hence, \( \frac{a}{b} = \frac{e}{f} \) from centre. (Given)
\[ y = -x + \frac{4}{3} \]

The quadratic equation is:

\[ x^2 - 2x + \frac{4}{3} = 0 \]

Using the quadratic formula, we find the roots:

\[ x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot \frac{4}{3}}}{2 \cdot 1} = \frac{2 \pm \sqrt{4 - \frac{16}{3}}}{2} = \frac{2 \pm \sqrt{\frac{12 - 16}{3}}}{2} = \frac{2 \pm \sqrt{-\frac{4}{3}}}{2} = \frac{2 \pm i\sqrt{\frac{4}{3}}}{2} = 1 \pm i\frac{\sqrt{3}}{3} \]

Therefore, the roots are:

\[ x_1 = 1 + i\frac{\sqrt{3}}{3}, \quad x_2 = 1 - i\frac{\sqrt{3}}{3} \]

The product of the roots is:

\[ x_1 \cdot x_2 = (1 + i\frac{\sqrt{3}}{3})(1 - i\frac{\sqrt{3}}{3}) = 1^2 - (i\frac{\sqrt{3}}{3})^2 = 1 + \frac{1}{3} = \frac{4}{3} \]

This matches the constant term in the quadratic equation.

The sum of the roots is:

\[ x_1 + x_2 = (1 + i\frac{\sqrt{3}}{3}) + (1 - i\frac{\sqrt{3}}{3}) = 2 \]

This matches the coefficient of the linear term in the quadratic equation.
1. (a)

\[ a = \frac{\text{rise}}{\text{run}} = \frac{1}{2} \]

(b) \[ \text{Gradient} = \frac{3-4}{1-0} = -1 \]

\[ y = 0 \]

1. \[ \frac{\partial}{\partial x} \left( \frac{4}{3}x^3 + 5 \right) = \frac{4}{3} \cdot 3x^2 = 4x^2 \]

\[ y' = 4x^2 \]

\[ \text{Gradient}, \quad a = \frac{1.12 - 0.15}{1} = 1 \]

From the graph,

\[ y = 0.5x^3 + 5 \left( 1 \right) \]

1. \[ a = \frac{1}{2}, \quad b = -\frac{1}{2} \]

\[ y = 0.5x^3 + 5(x - 1) \]

\[ (1, 4) \]

\[ B \]

\[ \text{Gradient}, \quad a = \frac{1.12 - 0.15}{1} = 1 \]

\[ \text{Gradient}, \quad a = \frac{1}{2}, \quad b = -\frac{1}{2} \]

\[ y = 0.5x^3 + 5(x - 1) \]

\[ B \]

\[ y = 0.5x^3 + 5(x - 1) \]
The document consists of printed pages including the cover page.
Given the quadratic equation with integer coefficients, where $a$ and $c$ are integers, find the values of $a$ and $c$.

The roots of the quadratic equation \( ax^2 + 2x + c = 0 \) are \( x = \frac{-1 \pm \sqrt{1 - 4ac}}{2a} \). Therefore, the expression of the form \( \frac{x}{a} \) where \( a \neq 0 \) can be written as \( \frac{x}{a} = \frac{-1 \pm \sqrt{1 - 4ac}}{2a} \).

1. Find the roots of the quadratic equation.
2. Write the values of $a$ and $c$.
3. Find the values of $a$ and $c$.
4. Write the expression of the form \( \frac{x}{a} \) where \( a \neq 0 \).
5. Find the expression \( \frac{x}{a} \) where \( a \neq 0 \).
6. Write the expression \( \frac{x}{a} \) where \( a \neq 0 \).
7. Find the expression \( \frac{x}{a} \) where \( a \neq 0 \).
8. Write the expression \( \frac{x}{a} \) where \( a \neq 0 \).
(ii) \[ f''(x) = \frac{x^4 - 3x^2 - 9x^2}{x^2} \]
\[ = \frac{18}{x^2} \]
\[ f''(3) > 0 \text{ and } f''(-3) < 0 \]
\( : (3, 0) \text{ Minimum point and } (-3, -12) \text{ Maximum point.} \)

8(i) \[ \alpha^2 + 1 + \beta^2 + 1 = \frac{11}{8} \]
\[ \alpha^2 + \beta^2 = \frac{5}{8} \]
\[ (\alpha^2 + 1)(\beta^2 + 1) = \frac{67}{8} \]
\[ \alpha^2 \beta^2 + \alpha^2 + \beta^2 + 1 = \frac{67}{8} \]
\[ \alpha^2 \beta^2 = \frac{67}{8} - \frac{5}{8} = 8 \]
\[ \alpha \beta = -2 \]
\[ \alpha^2 + \beta^2 = \frac{9}{4} \]
\[ \alpha^2 + \beta^2 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha \beta) \]
\[ \frac{5}{8} = (\alpha + \beta)(\frac{9}{4} - 2) \]
\[ (\alpha + \beta) = \frac{5}{2} \]
(iv) The quadratic equation is

9(i) \[ h = 5 \cos \theta + 8 \sin \theta \]

(ii) \[ \sqrt{5^2 + 8^2} = \sqrt{89} \]
\[ \alpha = \tan^{-1} \left( \frac{8}{5} \right) = 1.012197 \]
\[ h = \sqrt{89} \cos(\theta - 1.01) \]

(iii) Max value of \( h = 9.43 \)
\[ \theta = 1.01 \]
\[ \sqrt{89} \cos(\theta - 1.012197) = 7.5 \]
\[ \theta = 0.360 \text{ (accept 0.360 to 0.361)} \]

10(i) Mid-point of \( AB \) is \( (2, 3) \) and Gradient of \( AB = -\frac{3}{2} \)
Equation of the perpendicular bisector is
\[ y - 3 = \frac{2}{3}(x - 2) \]
\[ 3y = 2x + 5 \]
Solving \( y = x + 2 \) and \( 3y = 2x + 5 \), Centre is \( (-1, 1) \)

(ii) Radius = \( \sqrt{(-1 - 4)^2 + (1 - 0)^2} = \sqrt{26} \)
Equation of the circle is
\[ (x + 1)^2 + (y - 1)^2 = 26 \]

(iii) \( a = -2, b = -2 \)

(iv) Radius of the second circle = \( \sqrt{1^2 + (-1)^2 + 23} = 5 < \sqrt{26} \)
The second circle lies inside the first circle.

11(i) \[ y = ax^2 \]
\[ \log y = \log a + \log x^2 \]
Plot \( \log y \) against \( \log x \) to obtain straight line graph
Use graph to find \( a = 1.43 \) and \( n = 0.563 \)

(ii) \[ y = 1.43 x^{2.563} \]
\[ 10 = 1.43 x^{2.563} \]
\[ x = 3.47 \]

(iii) \[ xy = 10 \]
\[ \log x + \log y = 1 \]
Plot this straight line using the same axes.
### ADDITIONAL MATHEMATICS Paper 2 (4047/02)

#### Marking Scheme

<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Marks</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(i)</td>
<td>$3\sin x + 5 = 0$&lt;br&gt;$\sin x = -\frac{5}{3}$ which is not possible as $-1 \leq \sin x \leq 1$&lt;br&gt;$f''(x) \neq 0$. There is no stationary point</td>
<td>M1A1</td>
<td></td>
</tr>
<tr>
<td>1(ii)</td>
<td>$y = \int (2 \sin x + 5) , dx$&lt;br&gt;$= -2 \cos x + 5x + c$&lt;br&gt;$x = 0, y = 5 \Rightarrow c = 8$&lt;br&gt;$\therefore f(x) = -2 \cos x + 5x + 8$</td>
<td>M1A1 [6]</td>
<td></td>
</tr>
<tr>
<td>2(i)</td>
<td>$\frac{d}{dx} \left( xe^{\frac{1}{2}} \right) = \frac{1}{2} xe^{\frac{1}{2}} + e^{\frac{1}{2}}$</td>
<td>M1A1</td>
<td></td>
</tr>
<tr>
<td>2(ii)</td>
<td>$\int e^{\frac{1}{2}} , dx = 2e^{\frac{1}{2}} + c$</td>
<td>B1A1</td>
<td></td>
</tr>
<tr>
<td>2(iii)</td>
<td>$\frac{1}{2} xe^{\frac{1}{2}} + [2e^{\frac{1}{2}} - 2] = [4e^{\frac{1}{2}} - 0]$</td>
<td>M1M1</td>
<td></td>
</tr>
<tr>
<td>3(i)</td>
<td>$\frac{dy}{dx} = 2(x + k)$&lt;br&gt;Gradient of the tangent = $2(2k + k) = 6k$&lt;br&gt;When $x = 2k, y = (k+2k)^2 = 9k^2$&lt;br&gt;Equation of the tangent is $y - 9k^2 = 6k(x - 2k)$&lt;br&gt;$6k^3 = 6kx$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>3(ii)</td>
<td>$P \left( \frac{k}{2}, 0 \right)$ and $Q \left( 0, -3k^2 \right)$&lt;br&gt;Mid-point $R$ is $\left( \frac{k}{4}, \frac{-3k^2}{2} \right)$&lt;br&gt;Substituting in $y + 4x^2 = 0$,&lt;br&gt;$-\frac{3k^2}{2} + 2x \left( \frac{k}{4} \right) = 0$&lt;br&gt;$-\frac{3k^2}{2} + \frac{3k^2}{2} = 0$&lt;br&gt;$0 = 0$&lt;br&gt;. $M$ lies on the curve $y + 4x^2 = 0$</td>
<td>M1A1 [9]</td>
<td></td>
</tr>
<tr>
<td>4(a)(i)</td>
<td>$(2 - p)^2 - 32 - 80p + 80p^2 - 40p^3 + 10p^4 - p^5$</td>
<td>M1A2</td>
<td></td>
</tr>
<tr>
<td>4(a)(ii)</td>
<td>Let $p = -x - \frac{x^3}{2}$&lt;br&gt;$\left( 2 - 2x + \frac{x^3}{2} \right) = 32 - 80(2x - \frac{x^3}{2}) + 80(x - \frac{x^3}{2})^2 + ...$&lt;br&gt;$= 32 - 160x + 360x^2 + ...$</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td>4(b)(i)</td>
<td>$\left( \frac{16}{r} \right) [x - \left( \frac{1}{r} \right)^{\frac{1}{3}}]^{\frac{1}{2}} \cdot \left( \frac{-1}{2x^2} \right)$</td>
<td>M1A1</td>
<td></td>
</tr>
<tr>
<td>4(b)(ii)</td>
<td>$\left( \frac{16}{r} \right) [x - \left( \frac{1}{r} \right)^{\frac{1}{3}}]^{\frac{1}{2}} = 16 \left( \frac{1}{r} \right)^{\frac{1}{3}} x^{12-2r}$&lt;br&gt;$32 - 2x = 0 \Rightarrow r = 4$&lt;br&gt;Term independent of $x = \left( \frac{16}{4} \right)^{\frac{1}{3}} = \frac{455}{4}$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$k^3 = (9 - 2\sqrt{2})^2 = 17 - 12\sqrt{2}$&lt;br&gt;$\frac{1}{k^3} = \frac{1}{17 - 12\sqrt{2}} = \frac{17 + 12\sqrt{2}}{(17 - 12\sqrt{2})(17 + 12\sqrt{2})} = 17 + 12\sqrt{2}$&lt;br&gt;$\frac{1}{k} = 3 - 2\sqrt{2} - (7 + 12\sqrt{2}) = -14 - 14\sqrt{2}$</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td>6(i)</td>
<td>$2(-1)^2 - 9(-1)^2 + (-1) + 12 = 0$&lt;br&gt;$x + 1$ is a factor of $2x^3 - 9x^2 + x + 12$ .</td>
<td>M1A1</td>
<td></td>
</tr>
<tr>
<td>6(ii)</td>
<td>$2x^3 - 9x^2 + x + 12 = (x+1)(2x^2 - 13x + 12)$&lt;br&gt;$= (x+1)(2x-3)(x-4)$&lt;br&gt;$(x+1)(2x-3)(x-4) = 0 \Rightarrow x = -1, \frac{3}{2}$ or 4.</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td>6(iii)</td>
<td>$\frac{25}{2x^2 - 9x^2 + x + 12} = \frac{A}{x+1} + \frac{B}{2x-3} + \frac{C}{x-4}$&lt;br&gt;Evaluating $A, B$ and $C$. $A = -1, B = -4, C = 1$</td>
<td>M1A1</td>
<td></td>
</tr>
<tr>
<td>7(i)</td>
<td>$f''(x) = 2(2x-3) - (x-3)^2$&lt;br&gt;$= \frac{x^2 - 9}{x^2}$&lt;br&gt;$\frac{x^2 - 9}{x^2} = 0 \Rightarrow x = \pm 3$&lt;br&gt;The stationary points are $(3, 0)$ and $(-3, -12)$</td>
<td>M1</td>
<td></td>
</tr>
</tbody>
</table>
(i) Find the equation of the circle.

(ii) Find the equation of the line through (1, 2).

(iii) Find the equation of the perpendicular bisector of AB and hence show that $x + y = 1$. 

(iv) Find the point $(a, 0)$ and $(b, 0)$ whose distance is the same.

(v) Find the value of $\theta$ for which $f(\theta)$ can be.

(vi) Find the maximum value of $f$ and the corresponding value of $f$.
2. \( \frac{a}{b^2} = \frac{c}{d^2} \)

For the equation \( ax + bx + c = 0 \),

Quadratic Equation

Mathematical Formulae

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Additional Mathematics

Preliminary Examination 2016
Ge Yang Methodist School (Secondary)

Exam Guru

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Read These Instructions First

12 August 2016
Z hours
4 Express / 5 Normal (academic)
4047/01

Additional Mathematics

Section A: This question paper will contain 30 questions, each carrying 1 mark. The total marks for this paper are 30.

1. Integer

2. Algebra

Mathematical Formulae
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Given that the area of the region bounded by the curve \( y = x^2 \) and the x-axis and the line \( x = 5 \) is equal to \( 5 \pi \), find the value of \( a \) and \( b \).

The region is bounded by the curve \( y = x^2 \) and the x-axis and the line \( x = 5 \).

The x-intercepts of the curve \( y = x^2 \) are \( x = 0 \) and \( x = 5 \).

\( \int_{0}^{5} \sqrt{5-x} \, dx = 5 \pi \)

The diagram shows part of the curve \( y = x^2 \) and the x-axis.

Given that \( AD = BE \), show that the line AC bisects the angle BCD.

Show that triangles \( ABE \) and \( BCD \) are similar.

Show that \( \angle BAC = \angle DBC \).

The diagram shows a quadrilateral \( ABCD \) whose vertices lie on the circumference of the circle. The point \( E \) lies on \( CD \) produced such that \( AE \) is a tangent to the circle.

The diagram shows a quadrilateral \( ABCD \) whose vertices lie on the circumference of the circle.
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
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</thead>
<tbody>
<tr>
<td>1(a)</td>
<td>$V = \frac{49J}{5cm}$, $V$ is maximum</td>
</tr>
<tr>
<td>1(b)</td>
<td>$c = 2$</td>
</tr>
<tr>
<td>1(c)</td>
<td>$a = -4$, $q = 2$</td>
</tr>
<tr>
<td>2</td>
<td>$ABCD$ is a rectangle</td>
</tr>
<tr>
<td>3</td>
<td>Gradient of $AB$ is $1$, gradient of $CD$ is $-1$</td>
</tr>
<tr>
<td>4</td>
<td>There are no real values of $x$</td>
</tr>
<tr>
<td>5</td>
<td>$g = 980$</td>
</tr>
<tr>
<td>6</td>
<td>$x = d = 2$</td>
</tr>
<tr>
<td>7</td>
<td>$x = 2a - 4b$</td>
</tr>
</tbody>
</table>

Graphs and diagrams are shown for questions 2 and 3.
2. **Algebra**

\[ \frac{1}{a + b} - \frac{1}{a - b} = \frac{1}{a^2 - b^2} \]

The document consists of printed pages including the cover page.

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**Additional Mathematics**

**Preliminary Examination 2016**

Georgetown Methodist School (Secondary)

**Answers**

**Paper 1**

4. Express \( \frac{1}{a^2 + b^2} \) in the form \( \frac{a}{a^2 - c} + \frac{b}{b^2 - c} \).
Solution

Since 0 < q < 1, the derivative is always positive.

The graph is always positive.

Show that there are no values of q for which the area

of the region in the x-y plane is equal to

\( \int_{0}^{q} (x^2 - x^3) \, dx = \frac{1}{4} \)

Calculation of the roots of the equation x^2 - x^3 = \frac{1}{4}

The function is negative for all values of x by

\( x(1-x) = \frac{1}{4} \)
The curve \( y = ax^2 + bx + c \) is defined for \( 0 \leq x \leq 2 \). Verify in a negative investigation.
The diagram shows a quadrilateral $ABCD$ where vertices lie on the circle. The opposite angles of the cyclic quadrilateral $ABCD$ are equal.

1. **Calculation**
   - $A = (x, y)$
   - $D = (x', y')$

2. **Equation**
   - $y - y' = k(x - x')$
   - $y - y' = m(x - x')$

3. **Solution**
   - $x = \frac{m}{m-k}y + \frac{k}{m-k}x' - \frac{km}{m-k}$
   - $y = \frac{k}{k-m}x + \frac{m}{k-m}y' - \frac{mk}{k-m}$

4. **Diagram**
   - Diagram showing the quadrilateral $ABCD$ with vertices $A$, $B$, $C$, and $D$.
Given that the area of the circle is equal to the area of the rectangle, the equation is $\pi r^2 = 2r$. Simplifying, we get $\pi r = 2$. Then, the length of the rectangle is $2\pi r = 4\pi$, and its breadth is $2r = 2$. The area of the rectangle is $2 \times 2 = 4$. Therefore, the area of the circle is $4\pi$. The diagram shows the given geometric figures and their relationships.
<p>| | | | | |</p>
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</table>

### Answers

1. 3.68, 5.94

2. 3.46, 8.92

3. 9.02

4. 0.49

5. 0.38

6. 0.43

7. 0.44

8. 0.41

9. 0.40

10. 0.39

11. 0.38

12. 0.37

13. 0.36

14. 0.35

15. 0.34

16. 0.33

17. 0.32

18. 0.31

19. 0.30

20. 0.29

21. 0.28

22. 0.27

23. 0.26

24. 0.25

25. 0.24

26. 0.23

27. 0.22

28. 0.21

29. 0.20

30. 0.19

31. 0.18

32. 0.17

33. 0.16

34. 0.15

35. 0.14

36. 0.13

37. 0.12

38. 0.11

39. 0.10

40. 0.09

41. 0.08

42. 0.07

43. 0.06

44. 0.05

45. 0.04

46. 0.03

47. 0.02

48. 0.01

49. 0.00

50. 0.00
1. ALGEBRA

a) Quadratic equation

\[ \frac{\sqrt{a}}{\sqrt{b}} = x \]

For the equation \( ax^2 + bx + c = 0 \),

\[ a = 0 \]

b) Formulas

\[ \frac{\sqrt{a}}{\sqrt{b}} = x \]

2. TRIGONOMETRY

\[ \tan \theta = \frac{\sin \theta}{\cos \theta} \]

\[ \sin \theta + \cos \theta = \sqrt{2} \cdot \sin \left( \frac{\pi}{4} + \theta \right) \]

\[ \cos \theta - \sin \theta = \sqrt{2} \cdot \cos \left( \frac{\pi}{4} - \theta \right) \]
The diagram shows the quadrilateral $ABCD$. The coordinates of $A$ are $(x,y)$ and of $D$ are $(y,x)$. The diagonal $AC$ is expressed by the equation $y = x + 2$. Find the coordinates of $C$.
(a) $0 = 2 + (1-x)(x+y)
(b) \varepsilon = (1-x)(x+y)
(c) \lambda = (1-x)(x+y)

Sketch the graph. Write the number of solutions to each of the equations.

(i) Sketch the graph of $(x+y)$ at which the graph intersects the x-axis.
(ii) Find the coordinates of the points at which the curve touches the x-axis.
(iii) Find the lowest point on the curve that contains $(x^0, y^0)$.
(iv) Find the equation of the line $y = x - 2$.
(v) Find the coordinates of the point where the line with the equation $y = 1$ intersects the curve at $x = 1$.

The points $P(3, 1)$ and $Q(1, 1)$. Find the curve whose equation is $y = x - 2$, and $L = 1$. The parameter $t$ is in the interval $[0, 1]$. The point $P(1, 1)$ and $Q(1, 1)$.
[3] The area of a square is \( (x+2)^2 \) cm². Given that the length is \( x+2 \) cm,
find without using the calculator, the value of \( x \).

\[ x = 2 \]

[4] Find, without using the calculator, the length of \( CD \).

\[ CD = 2 \times AC \]

[5] Prove that triangles \( AFB \) and \( AEC \) are similar.

\[ \triangle AFB \sim \triangle AEC \]

[6] Construct \( DC = 2AB \).

In the figure, \( E \) is a point on the circle \( ABC \) and \( D \) is a point on the circle \( ADE \).

\[ \angle AED = \angle BCD \]

[7] Find the length of \( AB \), where \( x \) is the length of \( AC \).

\[ AB = x + 1 \]

[8] Find the remainder when \( f(x) \) is divided by \( x + 1 \).

\[ f(1) = \]

[9] Plot on the coordinate plane the graph of \( f(x) \) and \( g(x) \).

\[ f(x), \ g(x) \]

[10] Solve the equation \( f(x) = 0 \).

\[ x = \]

[11] Show that \( x + 2 \) is a factor of \( f(x) \).

\[ f(x) = \]

[12] The graph of \( f(x) \) is similar to the graph of \( g(x) \), where \( a \) is a constant.

\[ f(x) \sim g(ax) \]

[13] Explain why \( a = 7 \).

\[ a = 7 \]

[14] Explain why \( a = 7 \).

\[ a = 7 \]

[15] Explain why \( a = 7 \).

\[ a = 7 \]

Answer all questions.
Given further that \( x' = 12 \) and \( y' = \frac{1}{2} \), find the value of \( a \) and \( b \).

(iii) Solve for \( y' \) in the equation \( 2y - 3x = 2 \).

(iv) What do the results in part (ii) imply about the stationary point?

(v) Determine the value of \( y' \) at the stationary point in part (i).

(vi) Determine the value of \( x' \) and \( y' \) for values of \( a \) and \( b \) given above.

(vii) Find the area of the shaded region.

The diagram shows part of the curve \( y = x^2 - 8 \). The points \( P \) and \( Q \) are the intersections of the two graphs.

(i) Find the coordinates of \( P \) and \( Q \).

(ii) Find the area of the shaded region.

The diagram shows part of the curve \( y = \sqrt{x} - 8 \). The points \( P \) and \( Q \) are the intersections of the two graphs.

(i) Find the coordinates of \( P \) and \( Q \).

(ii) Find the area of the shaded region.

A particle is travelling in a straight line passes through a fixed point \( O \) with a velocity of \( 3 \) m/s. The acceleration \( a \) in the direction of the particle is given by \( a = -4t \). The particle's distance from its initial position at time \( t \) is given by \( t^2 - 2t \). Find the coordinates of the points of intersection of the graphs of part (ii).
\[\begin{align*}
\frac{\varepsilon}{02} &= \varepsilon + \varepsilon_0 \\
\varepsilon &= \varepsilon_0 \\
\varepsilon &= \varepsilon_0
\end{align*}\]
<table>
<thead>
<tr>
<th>Term</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{student} )</td>
<td>( {05} )</td>
</tr>
<tr>
<td>( \text{teacher} )</td>
<td>( {05} )</td>
</tr>
<tr>
<td>( \text{length} )</td>
<td>( 2(4) )</td>
</tr>
<tr>
<td>( \text{length} )</td>
<td>( {4(3) + {3 \times 3} } )</td>
</tr>
<tr>
<td>( \text{OR} )</td>
<td>( \text{same} )</td>
</tr>
<tr>
<td>( \text{same} )</td>
<td>( {05} )</td>
</tr>
<tr>
<td>( \text{same} )</td>
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<tr>
<td>( \text{length} )</td>
<td>( 2{4} )</td>
</tr>
<tr>
<td>( \text{length} )</td>
<td>( {4(3) + {3 \times 3} } )</td>
</tr>
</tbody>
</table>
A. The area of a triangle is \( \frac{1 + \frac{3\sqrt{3}}{2}}{2} \) cm\(^2\). If the length of the base of the triangle is \(3 + 2\sqrt{3}\) cm, find, without using a calculator, the height of the triangle in the form of \((n + \frac{b\sqrt{3}}{2})\) cm, where \(n\) and \(b\) are integers. [4]

2. Express \(\frac{4x^2 + 6x + 5}{2x^2 + x - 3}\) in partial fractions. [5]

3. The function \(f(x)\) is such that \(f(x) = 2x^3 + 3x^2 - x - 4\).
   
   (i) Find a factor of \(f(x)\). [2]
   
   (ii) Hence, determine the number of solutions in the equation \(f(x) = 0\). [4]

4. The roots of the quadratic equation \(3x^2 - x + 1 = 0\) are \(\alpha\) and \(\beta\).
   
   (i) Evaluate \(\alpha^2 + \beta^2\). [2]
   
   (ii) Find the quadratic equation whose roots are \(\alpha^2 - 1\) and \(\beta^2 - 1\). [4]

5. The table shows experimental values of 2 variables, \(R\) and \(V\), which are connected by an equation of the form \(RV^2 = k\) where \(n\) and \(k\) are constants.

<table>
<thead>
<tr>
<th>(R)</th>
<th>33</th>
<th>19.95</th>
<th>5.07</th>
<th>2.38</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V)</td>
<td>2</td>
<td>2.9</td>
<td>8</td>
<td>14</td>
</tr>
</tbody>
</table>

   
   (i) Plot \(R\) against \(V\) for the given data and draw a straight line graph. [3]
   
   (ii) Use your graph to estimate the value of \(k\) and of \(n\). [3]
   
   (iii) By drawing a suitable straight line on your graph in (i), find the value of \(V\) such that \(\frac{R}{V^2} = 1\). [3]

6. Given that \(y = 1 - \frac{1}{2}\sin 3x\), \(0^\circ \leq x \leq 240^\circ\).
   
   (i) State the maximum and minimum values of \(y\). [3]
   
   (ii) Sketch the graph of \(y = 1 - \frac{1}{2}\sin 3x\). [3]

7. A quadrilateral \(ABCD\) passes through vertices \(B(3, 9), C(8, 6)\) and \(D(-4, 0)\), line \(AD\) is parallel to the \(y\)-axis.
   
   (i) Find the coordinates of \(A\) given that the length of \(AD\) is 8 units. [1]
   
   (ii) A point \(P\) divides the line \(DC\) in the ratio 2 : 1. Find the coordinates of \(P\). [3]
   
   (iii) Hence, find the area of the quadrilateral \(ABCD\). [3]

8. (a) Sketch the graph \(y^2 = 3x\). [2]

   (b) Given that \(f(x) = -2x^2 + 5x^2 + 4x + a\),

   (i) Find the coordinates of the turning points in terms of \(a\). [4]
   
   (ii) Determine the nature of each turning point. [3]
   
   (iii) In the case where \(a = 1\), explain why the part of the graph between the turning points lie above the \(x\)-axis. [1]

9. (i) Show that \(sec x + tan x\) can be expressed as \(1 + \frac{1}{\cos x}\) [1]

   (ii) Differentiate \(ln(sec x + tan x)\) with respect to \(x\). [3]

   (iii) Hence, find \(\int_{\frac{\pi}{2}}^{\pi} sec x\ dx\). [3]
10. The points $A$ and $B$ lie on the circumference of a circle $C_1$ where $A$ is the point $(0, 8)$ and $B$ is the point $(4, 0)$. The line $y = 2x$ also passes through the centre of the circle $C_1$.

(i) Find the centre and radius of the circle $C_1$. [4]

(ii) Find the equation of the circle $C_1$ in the form $x^2 + y^2 + px + qy + r = 0$, where $p$, $q$, and $r$ are integers. [2]

Another circle $C_2$ of radius $\sqrt{2}$ units has its centre inside $C_1$ and it cuts the circle $C_1$ at the origin and at the point where $x = 2$.

(iii) Find the centre of $C_2$. [5]

The diagram shows part of the curve $y = 3\cos\frac{x}{2}$ that cuts the $x$-axis at $x = \pi$ and $x = 3\pi$. The normal to the curve at $x = \frac{5\pi}{3}$ cuts the $x$-axis at $A$.

(i) Find the coordinates of $A$, leaving your answer in exact form. [6]

(ii) Hence, find the area of the shaded region. [4]

\[ 4 - \frac{27}{5} \]
\[ 2 \times 3 + \frac{3}{x-1} \]
\[ 27x^2 + 98x + 196 = 0 \]
\[ (i) \frac{-29}{9} \]
\[ (ii) \]
\[ 6. (i) \text{Max } y = 1.5; \text{Min } y = 0.5 \]

\[ 7. (i) (-4,8) \]
\[ (ii) P(4,4) \]
\[ (iii) 50 \text{ units}^2 \]

\[ 8. (a) \]
\[ (b)(i) \left(-\frac{1}{2}, -\frac{19}{27}\right) \text{ and } (2, 12 + c) \]
\[ (b)(ii) \left(-\frac{1}{3}, -\frac{19}{27}\right) \text{ min; } (2, 12 + c) \text{ max} \]

\[ 9. (ii) \sec x \]
\[ (iii) 0.539 \]

10. (i) Centre $(2, 4)$, Radius $= 2\sqrt{5}$ (ii) $x^2 + y^2 - 4x - 8y = 0$ (iii) Centre of $C_2(1.22, 0.710)$

11. (i) $A\left(\frac{5\pi}{3}, \frac{9}{8}\sqrt{5}, 0\right)$ (ii) $6\frac{15}{32}$ / $6.47$ units$^2$. 
(a) The equation of a curve is \( y = 2x^2 + ax + (6+a) \), where \( a \) is a constant. Find the range of values of \( a \) for which the curve lies completely above the \( x \)-axis.

(b) The equation of a curve is \( y = 3x^2 + 4x + 6 \).
   (i) Find the set of values of \( x \) for which the curve is above the line \( y = 6 \).
   (ii) Show that the line \( y = -8x - 6 \) is a tangent to the curve.

(a) Given that \( \log_2 125 - 3 \log_2 b + \log_2 c = 3 \), express \( c \) in terms of \( b \) and \( c \).

(b) Solve the equation
   (i) \( 1 \log_b 8x - \log_b (x^2 - 2) = 2 \log_b 2 \),
   (ii) \( 2 \log_b x = 3 + 7 \log_b 2 \).

The equation of a curve is \( y = x^2 \sqrt{(5x - 1)} \), for \( x > 0.2 \). Given that \( x \) is changing at a constant rate of 0.25 units per second, find the rate of change of \( y \) when \( x = 2 \).

4 The graph of \( y = |2x^2 - ax - 5| \) passes through the points with coordinates \((-1, 0)\) and \((0.75, b)\).
   (i) Find the value of the constants \( a \) and \( b \).
   (ii) Sketch the graph of \( y = |2x^2 - ax - 5| \).
   (iii) Determine the set of positive values of \( m \) for which the line \( y = mx + 2 \) intersects the graph of \( y = |2x^2 - ax - 5| \) at two points.

5 In the binomial expansion of \( (2x + \frac{k}{x})^4 \), where \( k \) is a positive constant, the coefficient of \( x^3 \) is 28.
   (i) Show that \( k = \frac{1}{4} \).
   (ii) Hence, determine the term in \( x \) in the expansion of \( \left( 6x - \frac{1}{x} \right)^{2x + \frac{k}{x}} \).
The diagram shows points A, B, C and D on a circle, line EF is tangent to the circle at C, lines ADF and EBAG are straight lines, and points B and C are the midpoints of AE and EF.

Prove that:
(i) \[ BC \times EC = AC \times BE, \] \[ \text{[3]} \]
(ii) \[ AF \times EC = AC \times AE, \] \[ \text{[2]} \]
(iii) \[ \angle GAD = \angle ACF. \] \[ \text{[2]} \]

9 (a) Show that \[ \cot 2x = \frac{1 - \tan^2 x}{2 \tan x}. \] \[ \text{[2]} \]
Hence, solve the equation \[ 3 \cot 2x \tan x = 1, \text{ for } 0^\circ < x < 360^\circ. \] \[ \text{[4]} \]

(b) The Ultraviolet Index (UVI) describes the level of solar radiation. The UVI measured from the top of a building is given by \[ U = 6 - 5 \cos qt, \] where \( t \) is the time in hours from the lowest value of the UVI, \( 0 \leq t \leq 10, \) and \( q \) is a constant. It takes 10 hours for the UVI to reach its lowest value again.

(i) Explain why we are not able to measure a UVI of 12. \[ \text{[1]} \]
(ii) Show that \[ q = \frac{\pi}{5}. \] \[ \text{[1]} \]
(iii) The top of the building is equipped with solar panels that supply power to the building when the UVI is at least 3. Find the duration, in hours and minutes, that the building is supplied with power from the solar panels. \[ \text{[4]} \]

10 (a) It is given that \[ y = \frac{2x^2}{4x-3}, \text{ where } x > \frac{3}{4}. \]

(i) Find \[ \frac{dy}{dx}. \] \[ \text{[2]} \]
(ii) Find the range of values of \( x \) for which \( y = \frac{2x^2}{4x-3} \) is a decreasing function. \[ \text{[4]} \]

(b) It is given that \( f(x) \) is such that \[ f'(x) = \frac{1}{2x-5} - \frac{4}{(2x-5)^2}. \]

Given also that \( f(3) = 1.75, \) show that \( 3f(x) - (2x-5) f'(x) = \ln(2x-5). \] \[ \text{[7]} \]

11 A particle moves in a straight line, so that, \( t \) seconds after passing a fixed point \( O, \) its velocity, \( v \text{ m/s}, \) is given by \[ v = 2e^{0.5t} - 10e^{-t}. \] The particle comes to an instantaneous rest at the point \( A. \)

(i) Show that the particle reaches \( A \) when \( t = \frac{5}{2} \ln 5 + \frac{1}{4}. \) \[ \text{[3]} \]
(ii) Find the acceleration of the particle at \( A. \) \[ \text{[3]} \]
(iii) Find the distance \( OA. \) \[ \text{[4]} \]
(iv) Explain whether the particle is again at \( O \) at some instant during the eleventh second after first passing through \( O. \) \[ \text{[2]} \]
1. (a) $-4 < a < 12$
   (b) $x = -\frac{1}{3}$
   (c) $x = 3$
   (d) $a = 3$
   (e) $b = 0.125$
   (f) $m = 2$
   (g) $x = 2$

2. (a) $a = \sqrt{2}x$
   (b) $y = x$
   (c) $y = 3$

3. $4.95$ units
   (b) $3.6$

4. $x = 3$

5. (a) $x = \frac{1}{2}$
   (b) $y = 2$
   (c) $r = 2$
   (d) $L = 4 + 10 \sqrt{2}$
   (e) $x = 4\sqrt{2} - 3$

6. (a) $u = \frac{1}{2}$
   (b) $v = 2$
   (c) $w = 5$
   (d) $r = \frac{4L}{5}

7. (a) $x = 0.9^{139.1} - 220.8^{319.1}$
   (b) $r = 2$
   (c) $x = \frac{3}{2}$

8. (a) $x = 0.9^{139.1} - 220.8^{319.1}$
   (b) $x = 2$
   (c) $x = \frac{3}{2}$

9. (a) $x = 4\sqrt{2} - 3$
   (b) $x = 2$
   (c) $x = \frac{3}{2}$

10. (a) $u = \frac{1}{2}$
    (b) $v = 2$
    (c) $w = 5$

11. (a) $L = 4 + 10 \sqrt{2}$
    (b) $r = \frac{4L}{5}$
    (c) $x = \frac{3}{2}$

12. (a) $u = \frac{1}{2}$
    (b) $v = 2$
    (c) $w = 5$

13. (a) $x = 4\sqrt{2} - 3$
    (b) $x = 2$
    (c) $x = \frac{3}{2}$

14. (a) $u = \frac{1}{2}$
    (b) $v = 2$
    (c) $w = 5$

15. (a) $x = 4\sqrt{2} - 3$
    (b) $x = 2$
    (c) $x = \frac{3}{2}$

16. (a) $u = \frac{1}{2}$
    (b) $v = 2$
    (c) $w = 5$

17. (a) $x = 4\sqrt{2} - 3$
    (b) $x = 2$
    (c) $x = \frac{3}{2}$
3

1. Prove the identity \( \cos \theta - \cot \theta = \frac{1 - \cos \theta}{1 + \cos \theta} \).

2. The points A, B, and C have coordinates (0, 5), (8, 7) and (4, 1) respectively.
   (i) Find the equation of the perpendicular bisector of AB.
   (ii) Calculate the area of triangle ABC.

   The tangent to the curve \( y = (x - 2)^3 - 3x^2 + x + 1 \) at \( x = 1 \), meets the y-axis at A.
   Find the coordinates of A.

3. Write down the first three terms in the expansion, in descending powers of x, of \( \left( \frac{2x - 1}{x} \right)^7 \).

4. Find the value of \( x \) if the coefficient of \( x^3 \) in the expansion of \( (1 + \alpha x^2)^5 \) is 224.

5. A closed cylindrical container contains 300 cm\(^3\) of liquid when full. The cylinder of radius \( r \) cm and height \( h \) cm has a total surface area of \( A \) cm\(^2\).
   (i) Show that \( A = 2\pi r^2 + \frac{300}{r} \).
   (ii) Given that \( r \) can vary, find the stationary value of \( A \) and determine if this value of \( A \) is a maximum or a minimum.

6. Show that the line \( y = 3 - k \) will always intersect the curve \( y = x^2 + (1 - 2k)x \) at two distinct points for all real values of \( k \).

7. The diagram shows a wooden frame PQRS where QR and PS are perpendicular to PS, PQ = 15 cm and QR = 10 cm. Angle QRS is \( \theta \) where \( \theta = 0 \) to \( 90^\circ \). The perimeter of the wooden frame is 10 cm.

   (i) Show that \( L = 10 \cos \theta + 10 \sin \theta + 40 \).
   (ii) Using part (i), express \( L \) in the form \( A \cos(\theta - \alpha) + c \) where \( A > 0 \), \( \alpha \) is an acute angle and \( c \) is a constant.
   (iii) Hence, find the value of \( \theta \) when \( L = 53 \) cm.

8. Given that \( \frac{3x^3 + 17x^2 + 22x + 12}{x^2 + 6x + 9} = \frac{3x + r}{x} + q + \frac{2x + r}{(x + 3)^2} \).
   (i) find the value of each of the integers \( p, q, \) and \( r \).
   (ii) Hence, using partial fractions and the values of \( p, q, \) and \( r \) found in part (i), find \( \int \frac{3x^3 + 17x^2 + 22x + 12}{x^2 + 6x + 9} \) dx.

9. A graph has the equation \( y = |3x + d| + b \) where \( a \) and \( b \) are positive constants.
   (a) Find, in terms of \( a \) and/or \( b \), the coordinates of the minimum point of the graph.
   (b) The equation \( |3x + a| + b = 3x + 1 \) has infinite solutions. Write down
      (i) the possible values of \( m \),
      (ii) the value of \( a + b \).
11. The output design is to be used for June 2016, which is available from the supplier. The input design is to be used for July 2016, which is available from the supplier.

12. A triangle with sides of length 3 cm, 4 cm, and 5 cm is drawn on graph paper. The area of the triangle is measured to be 6 square cm. The actual area of the triangle is 6.25 square cm. The scale used on the graph paper is 1 cm = 2 cm in real life. What is the error in the measurement of the area of the triangle on the graph paper?
[Image of a page with mathematical problems and diagrams]
5. The function $f(x)$ is defined, for $0 \leq x \leq 720^\circ$, by $f(x) = 4 \sin \frac{x}{2} - 2$.

(i) State the amplitude and period of $f$.

(ii) Find the values of $x$ when $f(x) = 0$.

(iii) Sketch the graph of $y = 4 \sin \frac{x}{2} - 2$ for $0 \leq x \leq 720^\circ$, stating clearly the intercepts with the axes.

(iv) State the range of values of $k$ for the equation $4 \sin \frac{x}{2} - 2 = k$ to have exactly 2 solutions.

6. The function $f(x) = 6x^3 + 11x^2 - 3x - k$, where $k$ is a constant, leaves a remainder of 6 when divided by $x + 1$.

(i) Find the value of $k$.

(ii) Factorise $f(x)$ completely.

(iii) State the remainder when $f(x) - 8$ is divided by $3x + 1$.

(iv) Using the value of $k$ found in (i), solve the equation $\frac{6}{u^2} + \frac{11}{u} = \frac{3}{u} + k$.

9. The equation of a circle, $C_1$, with centre $P$ is $x^2 + y^2 - 6x - 4y + 11 = 0$.

(i) Find the coordinates of $P$ and the radius of $C_1$.

(ii) Find the equation of the tangent to $C_1$ at the point $Q(2, 3)$.

(iii) The tangent meets the $x$-axis at point $R$.

(iv) State the coordinates of $R$.

A second circle, $C_2$, with centre $S$, passes through $P$, $Q$, and $R$.

(v) State the position of $S$ and hence find the equation of $C_2$.

(vi) Determine, with clear working, whether $S$ lies inside $C_1$.

10. A particle, moving in a straight line, passes through a point $A$ with a speed of 15 m/s. The acceleration, $a$ m/s$^2$, of the particle, is after passing through $A$, is given by $a = -2e^{2t}$ m/s$^2$. When $t = 0$, $s = S_0$, where $s$ metres is the displacement from a fixed point $O$. The particle comes to instantaneous rest at the point $B$.

(i) Show that the value of $t = 10 \ln 4$ when the particle reaches $B$.

(ii) Calculate the distance $AB$.

(iii) Determine if the particle passes through $A$ again at $40$ s.
(a) Differentiate $\sin x - x \cos x$ with respect to $x$.

(b) The diagram shows part of the curve $y = x \sin x$. $M$ and $N$ are the points of intersection between the curve and a line. $M$ lies on the $x$-axis and $N$ is $(p, p)$, where $p$ is a constant.

(i) Find the coordinates of $M$.
(ii) Given the gradient of $MN$ is $-1$, find the value of $p$.
(iii) Hence, calculate the area of the shaded region bounded by the curve and the line $MN$.

End of Paper
6
(i) \( k = 2 \)
(ii) \( f(x) = (x + 2)(3x + 1)(2x - 1) \)
(iii) remainder = -8
(iv) \( u = \frac{-3}{2} \) or 2

7
(i) The stationary points are \((3, -1)\) and \((-1, -9)\)
(ii) \((3, -1)\) is a min point and \((-1, -9)\) is a max point

9
(i) centre is \(P(3, 2)\) and radius is \(\sqrt{2}\) units
(ii) eqn of tangent is \(y = x + 1\)
(iii) \(R(-1, 0)\)
(iv) Equation of second circle is \((x - 1)^2 + (y - 1)^2 = 5\)
(v) \(PS = \sqrt{5}\) units
Since \(PS >\) radius of \(C_1\), \(S\) lies outside of \(C_1\)

10
(i) \(AB = 85.685 - 5 = 80.7\) m (3 s.f.)
(ii) particle has passes through \(A\) again

11
(a) \(\frac{d}{dx} (\sin x - x \cos x) = x \sin x\)
(b) (i) \(M\) is \((\pi, 0)\)
(ii) \(p = \frac{\pi}{2}\)
(iii) Area \(-x - \frac{x^2}{8}\) or 0.508 sq units (3 s.f.)
1. (a) Simplify \( \frac{49 + 2^2}{2^2 - 2} \). [3]

(b) Given that \( p = \frac{1}{\sqrt{5}} \) and that \( q = \frac{1 + p}{1 - p} \), express \( p \) in the form \( a + \sqrt{b} \), where \( a \) and \( b \) are integers. [3]

2. Express \( \frac{x^2 + 3x^2 - x - 8}{(x+3)(x^2 - 4)} \) in partial fractions. [6]

3. In the diagram, points \( A, B, C \) and \( D \) lie on the circumference of the circle such that the tangent at \( D \) meets \( BA \) produced at \( X \).

4. The function \( f \) is defined by \( f(x) = 3\sin 2x + a \) for \( 0 \leq x \leq 2\pi \). Given that the maximum value of \( f \) is 1,
   (a) write down the amplitude, the period of \( f \) and the value of \( a \). [3]
   (b) Sketch the graph of \( y = -f(x) \) for \( 0 \leq x \leq 2\pi \). [3]

5. (a) Solve \( 4\cos^2 x = 7 - \cos^2 x + 2\cos x \), for \( 0^\circ \leq x \leq 360^\circ \). [4]
   (b) Find, in radians, the obtuse angle for which \( \sin x - \cos x + \cos x = 0 \). [4]

6. (a) Show that the quadratic equation \( 2px^2 - x^2 - (p^2 + 1) = 0 \) is always negative, for all real values of \( x \).
   (b) Given that the roots of the equation \( 2x^2 + x - 4 = 0 \) are \( \alpha \) and \( \beta \), form the quadratic equation whose roots are \( \frac{\alpha}{\beta} \) and \( \frac{\beta}{\alpha} \). [6]

7. (i) Given that \( y = (1 + 2x)^2 \sqrt{4 - 3x} \), show that \( \frac{dy}{dx} \) can be written in the form \( \frac{n + bx}{2\sqrt{4 - 3x}} \), where \( a \) and \( b \) are constants. [3]
   (ii) Hence, find \( \int \frac{17 - 18x}{2\sqrt{4 - 2x}} \, dx \). [5]

8. (a) Given that \( p = \log_2 x \) and \( q = \log_4 y \), find, in terms of \( p \) and \( q \),
   (i) \( \log_2 y \), [2]
   (ii) \( \log_4 x \). [3]
   (b) Solve the equation \( \log_2 (28 - 5x) = \log_3 (x - 2) + 1 \). [3]

9. (a) The equation of a curve is \( y = x \ln(2x + 1), \ x > 0 \). Show that the curve has no stationary point. [3]
   (b) The equation of a curve is \( y = \frac{3}{2} \sin \frac{1}{2} x - 4 \cos \frac{1}{2} x, \ 0 \leq x \leq 2\pi \). Find the value of \( x \) for which the curve has a stationary point and determine the nature of this stationary point. [7]
The diagram shows part of the curve \( y = \frac{5}{\sqrt{9-3x}} - 1 \).

The curve meets the x-axis at \( A (a, 0) \) and the y-axis at \( B (0, 1) \). \( AP \) is a tangent to the curve at \( A \) and \( PB \) is parallel to the x-axis.

(i) The normal at \( A \) has a gradient of \(-24\). Find the value of \( a \). Hence find the equation of the tangent \( AP \). \[3\]

(ii) Find the area of the shaded region. \[7\]

12 Solutions to this question by accurate drawing is not accepted.

The diagram shows a trapezium \( ABCD \) with \( AB \) parallel to \( BC \) and \( BC \) perpendicular to \( CD \). The coordinates of \( A \) and \( D \) are \((-1, 2)\) and \((-3, 12)\) respectively. The point \( B \) lies on the x-axis and the equation of \( CD \) is \( 3y + 2x = 30 \).

Find

(i) the equation of \( AB \),
(ii) the equation of \( BC \),
(iii) the coordinates of \( C \),
(iv) the area of triangle \( BCD \),
(v) the perpendicular distance from \( C \) to the line \( BD \). \[3\]

11 A particle moves in a straight line so that, \( t \) seconds after passing through a fixed point

\( O \), its velocity \( v \) m/s\(^2\), is given by \( v = 2e^{2t} - 15e^{-t} \). Find

(i) the initial velocity of the particle,
(ii) the value of \( t \) when the particle is instantaneously at rest,
(iii) an expression for the displacement in terms of \( t \),
(iv) the distance travelled during the first 2 seconds. \[1\] \[3\] \[3\] \[3\]
1. (a) $-9$  
(b) $q = \frac{3 + \sqrt{5}}{2}$

2. 
$$\frac{1}{2(x+3)} + \frac{1}{2(x+2)} + \frac{1}{2(x-2)}$$

3. $x = -2$; Period = $\pi$; Amplitude = 3; $f(x) = 3 \sin 2x - 2$

4. 
$$f(x) = 2 - \sin 2x$$

5. (a) $x = 45^\circ, 121.0^\circ, 225^\circ$ or $301.0^\circ$  
(b) $x = \frac{2\pi}{3}$

6. $8x^2 + 17x + 8 = 0$

7. (i) $\frac{dy}{dx} = \frac{13 - 18x}{2x^2 - 3x}$  
(ii) $32 \frac{2}{3}$

8. (a) $p + 2q$  
(b) $\frac{p}{4q}$

9. (b) the stationary point is a maximum point

10. (i) Equation of tangent is $y = \frac{1}{24}(x + 3)$ or $y = \frac{1}{24}x - \frac{3}{8}$  
(ii) $6$ units

11. (i) $-13$ min$^{-1}$  
(ii) $t = 0.672$  
(iii) $x = 2 + 15e^{-t} - 16$  
(iv) 49.6 m

12. (i) $y = -\frac{2}{3}x + \frac{4}{3}$  
(ii) $y = \frac{3}{2}x - 3$  
(iii) $6, 0, 4$  
(iv) $39$ units$^2$  
(vi) $6$ units

---

The function $f$ is defined by
$$f(x) = \frac{e^{3x}}{7 - 2x}$$
where $x \neq \frac{7}{2}$.

Find the values of $x$ for which $f$ is a decreasing function.

2. Find the range of values of $k$ for which the line $y + 4x + 16 = 0$ does not intersect the curve $y = x^2 + 3x$.

3. The equation of a curve is $y = \frac{3x^2}{1 + x}$.

(i) Obtain an expression of $\frac{dy}{dx}$ in terms of $x$.

(ii) A particle moves along the curve. At point $T$ whose $x$-coordinate is negative, the $x$-coordinate of the particle is increasing at a rate of $1.5$ units/sec and the $y$-coordinate is increasing at $4$ units/sec. Find the coordinates of $T$.

4. (i) Calculate the term independent of $x$ in the expansion of $\left(x - \frac{1}{25x^2}\right)^8$.

(ii) In the binomial expansion of $(1 + kx)^n$, where $n \geq 3$ and $k$ is a constant, the coefficient of $x^3$ and $x^4$ are equal. Express $k$ in terms of $n$.

5. Mr. Ng bought a new car. Its expected value $V$ would depreciate such that after $t$ months, it is given by $V = 80000e^{-k}$, where $k$ is a constant. The value of the car after ten months is expected to be $570,000$.

(i) Find the initial value of the car.

(ii) Calculate the expected value of the car after twenty months.

(iii) Calculate the age of the car, to the nearest month, when its expected value will be $300,000$.

6. Show that $\frac{\sin x}{1 + \sec x} - \frac{\sin x}{1 - \sec x}$ can be written in the form $k \cot x$ and find the value of $k$.

Hence, find the value of $x$ such that $\frac{\sin x}{1 + \sec x} - \frac{\sin x}{1 - \sec x} = 2$ where $1 < x < 5$. [6]
The mean distance $R$ (in millions of kilometres) from the centre of the sun and the time taken $T$ (in years) for a planet to complete one revolution around the sun are recorded in the table below.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Mercury</th>
<th>Venus</th>
<th>Mars</th>
<th>Jupiter</th>
<th>Saturn</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$ (in millions of km)</td>
<td>57.9</td>
<td>108.2</td>
<td>227.9</td>
<td>778.3</td>
<td>1427</td>
</tr>
<tr>
<td>$T$ (in years)</td>
<td>0.24</td>
<td>0.62</td>
<td>1.88</td>
<td>11.86</td>
<td>29.46</td>
</tr>
</tbody>
</table>

It is given that the planets orbiting the sun obey Kepler's Law, $T^2 = kR^n$, where $k$ and $n$ are constants.

(i) Plot $2\log T$ against $\log R$ and draw a straight line graph.
(ii) Use your graph to estimate the value of $n$ to 1 decimal place.
(iii) Given that the time taken for Earth to complete one revolution around the sun is exactly 1 year, use your graph to determine the mean distance of Earth from the centre of the sun, in millions of kilometres.

In the diagram, $PR$ and $SQ$ are the diameters of the circle with centre $C$. The coordinates of $P$, $R$ and $S$ are $(-1, -1)$, $(7, 5)$ and $(0, 6)$ respectively.

The diagram shows a quadrilateral $PQRS$ in which $\angle QPS$ and $\angle QRS$ are right angles, $\angle PSR = \theta$, $PQ = 4$ cm and $PS = 8$ cm.

(a) Show that the perimeter, $S$, cm, of the quadrilateral is given by $S = 12\sin \theta + 4\cos \theta + 12$.
(b) Given further that $0^\circ < \theta < 90^\circ$, express $S$ in the form $A\sin(0 + \alpha) + k$, where $A$ and $k$ are positive constants and $0^\circ < \alpha < 90^\circ$. Hence find the value of $\theta$ for which $S = 19$.

A curve has the equation $y = (3x - 2)^2 - 16$.

(i) Explain why the lowest point on the curve has coordinates $\left(\frac{2}{3}, -16\right)$.
(ii) Find the coordinates of the point at which the curve intersects the x-axis.
(iii) Sketch the graph of $y = (3x - 2)^2 - 16$.
(iv) Use your graph, state the number of solutions to each of the following equations.
(a) $(3x - 2)^2 - 16 = 8$
(b) $(3x - 2)^2 - 16 + 4 = 0$
A piece of wire, length 150 cm, is bent into the shape shown in the diagram, such that $HI = GF = y$ cm, $BJ = HG = FE = x$ cm, $BC = ED = 2y$ cm and arc $CD$ is a semi-circle.

(i) Show that the area, enclosed by the wire, $A$ cm$^2$, is given by

$$A = \frac{1400x - 28x^2 - 5\pi x^2}{8}.$$  \[3\]

(ii) Given that $x$ and $y$ can vary, find the value of $x$ and of $y$ for which the area $A$, is stationary.

(iii) Find the stationary value of $A$, giving your answer to the nearest integer. Determine whether this stationary value is a minimum or maximum. \[3\]
8. (a) \( c = (3, 2) \)  
(b) \( (x - 2)^2 + (y - 1)^2 = 25 \)  
(c) \( a = 5 \)  
(d) \( k = 7 \)  
(e) \( \pi = 3.14 \)  
(f) \( \text{The graph is a circle.} \)

9. (a) \( p = \frac{1}{2} \)  
(b) \( y = 4 \)  
(c) \( f(x) = \frac{1}{x^2 - 2} \)  
(d) \( (x^2 - 2)(x - 1) \)  
(e) \( y = 2x^2 - 0.63 \)

10. (a) \( \frac{1}{2} \)  
(b) \( \cdot \)  
(c) \( (x, 0) \)  
(d) \( x = \frac{4}{3} \)  
(e) \( (6, 0) \)  
(f) \( y = 0 \)  
(g) \( 3, 2, 1 \)  
(h) \( 4, 2, 1 \)  
(i) \( y = 1, x = 2 \)

11. (a) \( (1, 2) \)  
(b) \( \frac{1}{2} \)  
(c) \( 2, 0 \)  
(d) \( 3, 0 \)  
(e) \( 4 \)  
(f) \( 0 \)  
(g) \( 4 \)  
(h) \( 0 \)  
(i) \( 4 \)  
(j) \( 0 \)  
(k) \( 4 \)  
(l) \( 0 \)  
(m) \( 4 \)  
(n) \( 0 \)  
(o) \( 4 \)  
(p) \( 0 \)  
(q) \( 4 \)  
(r) \( 0 \)  
(s) \( 4 \)  
(t) \( 0 \)  
(u) \( 4 \)  
(v) \( 0 \)  
(w) \( 4 \)  
(x) \( 0 \)  
(y) \( 4 \)  
(z) \( 0 \)  
{[diagram]}

12. (a) \( x = 16.0, y = 43.0 \)  
(b) \( A = 100 \)  
(c) \( \text{maximum} \)
1. Without using a calculator, find the exact value of \(12^\circ\), given that \(\tan 12^\circ = \tan x\). 

2. Solve the equation \(2x^2 = 1 + 3x - 5x^2\).

3. Find the range of values of \(x\) for which \(x(0 - x) = 24\).

4. Find the range of values of \(x\) for which \(x(0 - x) = 16\).

5. Sketch, on the same diagram, the graphs of \(y = x^2\) and \(y = 4x\) for \(x > 0\).

6. Find the coordinates of the point of intersection of the graphs.

7. Find the exact value of \(\sin 165^\circ\).

8. Hence, show that \(\sin 165^\circ\) can be expressed in the form \(a \cos b\), where \(a\) and \(b\) are integers.

9. Given that the term independent of \(x\) in the expansion of \((1 + 3x)^{11/2} = a + bx\) is 30, where \(a\) is a constant, find:
   (a) the value of \(a\).
   (b) the coefficient of \(\frac{1}{x}\).

10. The population, \(P_t\), of a certain species of frog is given by
    \[P_t = Ae^{-kt}\]
    where \(A\) and \(k\) are constants and \(t\) is the time in years from 1 January 2000.

    Over a period of 18 years from 1 January 2001 to 1 January 2019, \(P_t\) decreased from 90 000 to 45 000.

    Find:
    (i) the value of \(A\) and \(k\).
    (ii) the year in which the population will be reduced by 70% as compared to the year 2000.
11. It is given that \( f(x) = x^2 - 8x + 9 \) for \( 2 \leq x \leq 7 \):

(i) Find the value of \( a \) and \( b \) for which \( f(x) = (x - a)^2 + b \).

(ii) Find the stationary point of the graph \( y = f'(x) \) and determine its nature.

(iii) Sketch the graph of \( y = |f'(x)| \).

(iv) Find the range of values of \( x \) for which \( |f'(x)| > 6 \).

(v) Determine the number of solutions of the equation \( f(x) = mx + c \) in each of the following cases, when

(a) \( m = 1 \) and \( c = -2 \),

(b) \( m = \frac{1}{2} \) and \( c = 4 \).

End of Paper 1
1. Find a quadratic equation for which the sum of roots is \( \frac{1}{2} \) and the sum of the cubes of the roots is \( \frac{13}{8} \). [5]

2. (a) Variables \( x \) and \( y \) are connected by the equation \( y = a \log x + b \), where \( a \) and \( b \) are constants. Using experimental values of \( x \) and \( y \), a graph was drawn in which \( \log y \) was plotted on the vertical axis against \( \log x \) on the horizontal axis. The straight line which was obtained passed through the points \((1, 3)\) and \((-1, 5)\).

(i) Find the value of \( a \) and of \( b \). [3]

(ii) Show that \( x \) and \( y \) can be expressed in the form \( y = kx^n \), where \( k \) and \( n \) are constants to be found. [3]

(b) Given that \( \log x^2 = \log y \), express \( y \) in terms of \( x \). [2]

3. (i) Show that

\[
\frac{\sin 2x + 1 - \cos 2x}{\sin 2x - 1 + \cos 2x} = \frac{1 + \tan x}{1 - \tan x}
\]

(ii) Hence, solve for \(-3 < x < 2\), the equation

\[
\frac{\sin 2x + 1 - \cos 2x}{\sin 2x - 1 + \cos 2x} = 6 \tan x
\]

[5]

4. (a) Find the value of \( m \), where \( m > 0 \), for which \( 2x^3 + x + m \) is a factor of \( 4x^3 - 2x^2 - 3x - 3 \). [3]

(b) The cubic polynomial \( f(x) \) is such that the coefficient of \( x^3 \) is 3 and the roots of the equation \( f(x) = 0 \) are \(-2, 3 \) and \( k \). Given that \( f(x) \) has a remainder of 42 when divided by \( (x + 1) \), find

(i) the value of \( k \), [3]

(ii) the remainder when \( f(x) \) is divided by \( x \). [2]

5. (i) Express \( \frac{-2x - 6}{(x + 1)(x^2 - 3)} \) in partial fractions. [4]

(ii) Differentiate \( \ln(x^2 - 3) \). [1]

(iii) Given that \( \int_{-3}^{3} \frac{-3x - 9}{x(x^2 - 3)} \, dx = \frac{9}{2} \ln a \), using the results in parts (ii) and (iii), find the value of \( a \). [4]
6. A device is used to simulate the breathing patterns of a certain mammal's lungs. The volume, \( V \) litres, of air in the lungs of this mammal, \( t \) seconds after the beginning of one breath can be modelled by
\[
V = 0.45 - 0.4\cos(tk), \quad 0 \leq t \leq 4.
\]
(i) Explain why this model suggests that the maximum capacity of the lungs is 0.85 litres.
(ii) Show that the value of \( k \) is \( \frac{\pi}{2} \).
(iii) Find the length of time for which the lungs contain at least 0.5 litres of air.
(iv) Sketch the graph of \( V = 0.45 - 0.4\cos(tk), \quad 0 \leq t \leq 4 \).

7. A curve has the equation \( y = 4e^{x^2} \). It has a stationary point at \( \left( p, \frac{y}{e^2} \right) \) where \( p < 0 \).
(i) Find the exact value of \( p \) and \( q \).
(ii) By considering the sign of \( \frac{dy}{dx} \), determine the coordinates and the nature of the other stationary point.
(iii) Find the range of values of \( x \) for which \( y = 4e^{x^2} \) is a decreasing function.

8. In the diagram, \( PQRS \) is a rectangle. \( ABCDEF \) is a hexagon with angle \( AFE = \angle BCD = 90^\circ \). \( AB = 7 \text{ cm} \), \( BC = AF = 6 \text{ cm} \), \( CD = EF = 3 \text{ cm} \) and angle \( BHC = \angle PAF = \theta \), where \( 0^\circ \leq \theta \leq 90^\circ \).
(i) Show that the perimeter, \( L \text{ cm} \), of \( ABCDEF \) is given by \( L = 32 + 12\cos\theta - 6\sin\theta \).
(ii) Express \( L \) in the form \( kR \cos(\theta + a) \), where \( R > 0 \) and \( 0^\circ < a < 90^\circ \).
(iii) Find the value of \( \theta \), if \( L = 35 \).

9. The diagram shows the cross-section of a hollow cone of height 12 cm and base radius of 15 cm and an inverted cone of radius \( r \) cm and height \( h \) cm. Both stand on a horizontal surface with the inverted cone inside the hollow cone. The upper circular edge of the inverted cone is in contact with the hollow cone.
(i) Express \( h \) in terms of \( r \) and hence show that the volume, \( V \text{ cm}^3 \), of the inverted cone is given by
\[
V = 6\left(10 - \frac{5r^3}{3}\right).
\]
Given that \( r \) can vary,
(ii) find, in terms of \( r \), the volume of the largest inverted cone which can stand inside the hollow cone, and show that, in this case, the inverted cone occupies \( \frac{4}{27} \) of the volume of the hollow cone.

10. The population \( P \) in millions, of a country was recorded in various years and the results are shown in the table below.

<table>
<thead>
<tr>
<th>Year</th>
<th>2000</th>
<th>2005</th>
<th>2010</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>12.88</td>
<td>14.61</td>
<td>17.38</td>
<td>21.88</td>
</tr>
</tbody>
</table>

It is known that \( P \) and \( t \) are related by an equation of the form \( P = 10 - At^2 \), where \( r \) is the time measured in years from January 1995 and \( A \) and \( b \) are constants.
(i) Using graph paper, draw a straight line graph of \( \log(P - 10) \) against \( t \) and use your graph to estimate the value of \( A \) and of \( b \).
(ii) the population, in millions, in the country in January 1995,
(iii) the year in which the population exceeds 35 million.
11. The velocity, \( \text{ms}^{-1} \), of a particle, \( P \), moving in a straight line is given by \( v = 3t^2 + pt + q \), where \( t \) is the time in seconds after the start of motion. At \( t = 0 \), the displacement of the particle from \( O \) is 3 m.

Given also that when \( t = 2 \), the displacement of the particle from \( O \) is 23 m and the acceleration of the particle is \(-6 \text{ m} \text{s}^{-2}\).

(i) find the value of \( p \) and of \( q \),
(ii) explain with clear working whether \( P \) will return to its starting point.

[7]

\[ 2x^2 - x - 2 = 0 \]

\[ \begin{align*}
  2. & \quad a = -1, \quad b = 4 \\
  3. & \quad \gamma = 81x^3 \\
  4. & \quad u = \sqrt{x} \\
  5. & \quad \begin{aligned}
  & \frac{2}{x+1} \\
  & \frac{2x}{x^2 - 3} \\
  & \frac{x}{x^2 - 3} \\
  & \frac{2}{3} \\
  & \frac{2}{3} \\
  & \frac{2}{3} \\
  & \frac{2}{3} \\
  & \frac{2}{3} \\
  & \frac{2}{3} \\
  & \frac{2}{3} \\
  & 1.84 \text{ s} \\
  & 2.98 \text{ m} \text{s}^{-1} \\
  & \begin{cases}
  0, 1 & \text{is a point of inflexion.} \\
  x < \frac{3}{2} & \text{is a point of inflexion.} \\
  x > \frac{3}{2} & \text{is a point of inflexion.} \\
  \theta = 50.5^\circ \\
  \theta = 26.6^\circ \\
  h = 30 - \frac{5p}{2} \\
  h = 30 - \frac{5p}{2} \\
  \end{cases}
\end{aligned} \]
The constant term in the expansion of \( (6 + x)^4 \) is \( 6x^4 \). Find the value of the positive constant \( m \). [4]

\( A \) is an acute angle and \( \theta \) is an acute angle such that \( \tan(\theta - A) = 7 \) and \( \tan A = 3 \). Without using a calculator, find the exact value of \( \tan \theta \). [4]

3. The diagram shows a tank in the shape of an inverted right pyramid of height 30 cm and a square base of side 40 cm. Water is poured into the tank at a constant rate of 2 cubic cm/s.

(a) After 1 second, the depth of the water is 1 cm.

(b) Show that, the volume of the water in the tank, \( V \), after \( t \) seconds, is given by \( \frac{125}{3} t^3 \). [3]

(c) Find the rate of change of the depth of the water when \( t = 6 \). [3]

(d) State, with a reason, whether \( \frac{dV}{dt} \) will increase or decrease as \( t \) increases. [3]

\[ \text{Diagram:} \]

\[ \text{40 cm} \]

\[ \text{30 cm} \]

\[ \text{h cm} \]

\[ \text{Express } \frac{1 - x^2}{2x^3 + 3x^2} \text{ in partial fractions.} \] [4]

The table shows experimental values of two variables, \( x \) and \( y \), which are connected by an equation of the form \( y = ax + 2 \) where \( a \) is a constant.

\[ \begin{array}{cccc}
    x & 2 & 1.5 & 1.25 \\
    y & 3.00 & 3.20 & 3.40 \\
\end{array} \]

An error was made in rounding one of the values only.

(i) Plot \( y \) against \( x \) and draw a straight line graph.

(ii) Use your graph to estimate the value of \( y \) when \( x = 1 \).

(iii) Use your graph to estimate the value of \( x \) when \( y = 3 \).

(iv) Prove that \( \tan \theta = \frac{\sin \theta}{\cos \theta} \).

(v) Find, in radians, the exact value of the acute angle \( \theta \) for which \( \frac{1}{\tan \theta} - \tan \theta = 3 \).

Solutions to this question by accurate drawing will not be accepted.

\[ \text{Diagram:} \]

\[ (0,4) \]

\[ (4,0) \]

The diagram shows a triangle \( PQR \) in which \( PQ \) is parallel to \( XY \) and \( QR \parallel XY \). The point \( Q \) is \((3,6)\) and the point \( R \) is \((4,1)\).

The equation of \( XR \) is \( y - x - 2 \).

(a) Express, in terms of \( h \), the equation of \( PQ \). [3]

(b) the equation of \( QR \). [3]

(c) the coordinates of \( P \) and of \( R \). [3]

(d) In the case where \( h = 1 \), find the area of the triangle \( PQR \). [3]
The function $f(x)$ is such that $f'(x) = 2\cos 2x + 4\cos 3x$ and $f\left(\frac{\pi}{6}\right) = 0$.

Solve the equation $f'(x) - f(x) = \frac{9}{2}$ for $0 < x < 2\pi$.

The equation of a curve is $y = 5x^3 + 21x + 44 - 21$, where $a$ is a constant.

(i) Find the values of $a$ for which the line $y = x - 1$ is a tangent to the curve.

(ii) In the case where $a = 3$, find the set of values of $x$ for which the curve lies above the line $y = 15$.

A piece of wire of length 220 cm is bent into the shape as shown in the diagram. The shape consists of an isosceles triangle $ABC$ in which angle $ACB = 45^\circ$, a rectangle $ACDE$ of length $p$ cm and a semi-circle of radius $x$ cm.

(i) Express $p$ in terms of $x$.

(ii) Show that the area enclosed, $A$ cm$^2$, is given by

$A = (16 - 8x - 4x^2) + 1000\pi$.

(iii) Given that $x$ can vary, find the value of $x$, for which the area is stationary.

(iv) Explain why this value of $x$ gives the greatest area possible.

A curve has the equation $y = \frac{2x}{x - 3}$, where $x > \frac{1}{2}$ and $x \neq 1$. The curve cuts the x-axis at $P$.

(i) Find the x-coordinates of $P$.

(ii) The equation of the normal to the curve at $P$ cuts the y-axis at $Q$.

(iii) Find the area of the triangle $POQ$, where $O$ is the origin.

A curve has the equation $y = (x - 1)^3 - 16$.

(i) Find the coordinates of the turning point of the curve.

(ii) Find the x-coordinates of the points where the curve intersects the x-axis.

(iii) Sketch the graph of $y = (x - 1)^3 - 16$.

(iv) State the number of solutions to each of the following equations.

(a) $(x - 3)^2 - 16 = 0$

(b) $(x - 3)^2 - 16 = x - 1$
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1. \( m = 3 \)

2. \(-\frac{17\sqrt{3}}{65}\)

3. (i) \( h = \frac{2}{5} \) cm/s or 0.375 cm/s
   (ii) \( P(0, h - 4), R\left(\frac{2h + 34}{5}, h + 12\right)\)
   (iii) \( \frac{dh}{dt} = \frac{27}{x^4} \)
   As \( t \) increases, \( h \) increases and \( \frac{dh}{dt} \) decreases. Hence the rate of change of depth of water decreases.

4. \( \frac{-2 + 1}{x} + \frac{3}{2x + 1} \)

5. (i) \( y = 0.667 \)
   (ii) \( x < -\frac{3}{2} \) or \( x > \frac{5}{2} \)
   (iii) \( a = 0.441, b = 0.541 \)

6. (i) \( \theta = \frac{\pi}{3} \)

7. (a) (i) \( y = \frac{1}{2}x + h - 4 \)
   (ii) \( y = -2x + h + 16 \)
   (iii) \( P(0, h - 4), R\left(\frac{2h + 34}{5}, h + 12\right)\)

8. \( 0.201, 2.94, \frac{m}{3} \) or 4.71

9. (i) \( k = -1, 5 \)
   (ii) \( x < -1 \) or \( x > 2 \)

10. (i) \( P = 125 - 2nx - 4\sqrt{2}x \)
    (ii) \( x = 9.19 \)

11. (i) \( f(x) = 1 \)
    (ii) \( \frac{1}{4} \) square units

12. (i) \( x = 7 \) or \( x = -1 \)
    (ii) (iv) (a) 2
    (v) (b) 0

A slice of chocolate cake is heated in a convection oven to a temperature of \( t \) °C. It is then left to cool and it is observed that its temperature, \( T^\circ C \), \( t \) minutes after removal from the oven, is given by \( T = De^{-\frac{t}{5}} + 25 \), where \( D \) and \( e \) are constants.

(i) Find the value of \( D \).
(ii) Find the value of \( e \), given that the temperature of the cake is 31 °C after 2 minutes.
(iii) Explain why the temperature of the cake will always be above 25 °C.

The function \( f(x) = 3x^2 + ax + bx - 15 \), where \( a \) and \( b \) are constants, is exactly divisible by \( 3x - 4 \) and leaves a remainder of -160 when divided by \( x + 2 \).

(i) Find the value of \( \alpha \) and \( \beta \).
(ii) Factorise \( f(x) \).
(iii) Hence solve the equation \( 24x^3 + 4ax^2 + 20x - 16 = 0 \).

A cuboid has a square base of length \( \sqrt{5} + \sqrt{3} \) cm.
The volume of the cuboid is \( (\sqrt{5} + \sqrt{3})^3 \) cm³. Find, without using a calculator, the height of the cuboid in the form \( a(\sqrt{5} + \sqrt{3}) + b\sqrt{2} - 12 \) cm, where \( a \) and \( b \) are integers.

The quadratic equation \( x^2 + 4x + 7 = 0 \) has roots \( \alpha \) and \( \beta \). Find
(i) the value of \( \alpha^2 + \beta^2 \),
(ii) the quadratic equation whose roots are \( 2\alpha \) and \( 2\beta \).

Solve the equation
(i) \( \log_{3} x^2 - 16 \log_{3} x = 4 \)
(ii) \( e^x - 1 - 6e^{-x} = 0 \)
(iii) \( 9x^2 - (\log x)^2 \).
5

Two particles, \( P \) and \( Q \), leave a point \( O \) at the same time, and travel initially in the same direction along the same straight line. Particle \( P \) starts with a velocity of 6 m/s. Its acceleration \( a \text{ m/s}^2 \) is given by \( a = 2 - t \), where \( t \) seconds is the time after leaving \( O \).

(i) Find the velocity and distance of the particle \( P \) from \( O \) in terms of \( t \). [4]

(ii) Find the value of \( t \) when \( P \) is again at \( O \). [3]

Particle \( Q \) moves with a velocity \( v \) m/s, where \( v = 6t + 2e^{-t} - 1 \). and \( t \) seconds is the time after leaving \( O \).

(iii) Find the initial acceleration of particle \( Q \). [2]

(iv) Find the distance of the particle \( Q \) from \( O \) in terms of \( t \). [2]

(v) Show that particle \( Q \) overtakes particle \( P \) during the third second. [2]

7

A curve has the equation \( y = \frac{(x + 2)^2}{2x} \).

(i) Find the coordinates of the stationary points on the curve. [5]

(ii) Find the range of values of \( x \) for which \( y \) increases as \( x \) increases. [2]

(a) Find the results in (a) imply about the stationary points. [2]

(iii) Sketch the curve, indicating clearly the stationary points and asymptotes, if any.

Hence deduce the range of values of \( k \) for which the equation \( \frac{(x + 2)^2}{2x} = k \) has no real roots. [3]

8

A circle, \( C_1 \), has equation \( x^2 + y^2 - 10x + 2y - 10 = 0 \). Point \( A \) is the centre of \( C_1 \).

(i) Find the radius of \( C_1 \) and the coordinates of \( A \). [3]

Point \( Q \) lies on \( C_1 \). The tangent at \( Q \) passes through \( P (9, 7) \).

(ii) Find the exact length of \( PQ \). [3]

A second circle, \( C_2 \), passes through the points \( A \) and \( P \). The centre of \( C_2 \) lies on the x-axis.

(iii) Find the equation of the perpendicular bisector of \( AP \). [4]

(iv) Find the equation of \( C_2 \). [3]
11 (a) Differentiate \( \ln(x^2 - 3x + 3) \) with respect to \( x \).

(b) Express \( \frac{x^2 - x}{x^2 - 3x + 3} \) in the form \( \frac{a}{x^2 - 3x + 3} + c \), where \( a \), \( b \) and \( c \) are constants.

(c) Hence, find \( \int \frac{x^2 - x}{x^2 - 3x + 3} \, dx \).

(d) The diagram shows part of the curve \( y = \frac{4x - 6}{x^2 - 3x + 3} \) and the line \( x = 4 \).

The \( y \)-coordinates of points \( A \) and \( B \) are 2. Point \( C \) is vertically below point \( B \). Find

(i) the coordinates of \( A \), \( B \) and \( C \).

(ii) the area of the shaded region bounded by the curve, the line \( x = 4 \), the \( x \)-axis and the line \( AC \).

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1. (i) \( D = 10 \)

(ii) \( k = 0.255 \)

2. (i) \( a = -16 \), \( b = 28 \)

(ii) \( f(x) = (3x - 4)(x - 2) \)

(iii) \( x = \frac{5}{3} \), \( x = 1 \)

3. \( (5\sqrt{3} + \sqrt{6}) - 6\sqrt{2} - 12 \) cm

4. (i) 2

(ii) \( x^2 - 40x + 1372 = 0 \)

5. (i) \( \frac{1}{B!} = 9 \)

(ii) \( 1.10 \) or 1.1

(iii) \( 1.54 \) or 0.0155

7. (i) \( (2, 4) \) and \( (-2, 0) \)

(ii) \( x < -2 \) or \( x > 2 \)

(iii) \( (4, 4) \) is a maximum point

(iv) \( (2, 4) \) is a minimum point

8. (i) \( v = 21 - \frac{t^2}{2} + 6 \)

(ii) \( 9.71 \)

(iii) \( 4 \) m/s

(iv) \( s = 3t^2 - 2e^{-t} - t + 2 \)

9. (a) \( \sin(2\theta - 22.6^\circ) \)

(b) \( -90^\circ, 22.6^\circ, 90^\circ \)

10. (i) \( 10 \)

(ii) \( 5 \times 5 \) units

(iii) \( 2y = 13 - x \)

(iv) \( (x - 13)^2 + y^2 = 65 \)

11. (a) \( \frac{2x - 3}{x^2 - 3x + 3} \)

(b) \( \frac{2x - 3}{x^2 - 3x + 3} \)

(c) \( x + \ln(x^2 - 3x + 3) + c \)

(d) \( A(2, 2), B(3, 2), C(3, 9) \)

(e) \( 2.39 \) sq units