2016 Sec 4 Amath

Examguru

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ZHONGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2016

CANDIDATE NAME	

CLASS		REGISTER NUMBER	

ADDITIONAL MATHEMATICS **4047/01**
Paper 1 **2 hours**

Additional Materials: Answer paper
Graph paper (2 sheets)

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

This paper consists of 7 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

Answer **all** the questions.

1. The equation of a curve is given by $f(x) = 2x^3 - 12x - 5$. Find the range of values of x for which $f(x)$ is an increasing function. [3]

2. **(i)** Given that $(3k-5)x^2 + (k-5)x - 2 = 0$ has no real roots, what condition must apply to the constant k? [3]

(ii) From your results in part (i), determine if $y = (3k-5)x^2 + (k-5)x - 2$ has a minimum or maximum point. [2]

3. A sky diver jumps from a certain height above the ground. The downward velocity, $v\ m/s$, of the sky diver at time t seconds is given by $v = 30(1 - e^{-0.2t})$.

(i) Find the initial velocity of the sky diver. [1]

(ii) Find the velocity of the sky diver after 5 seconds. [1]

(iii) Showing your working clearly, explain why the velocity experienced by the sky diver will not exceed 30 m/s. [2]

4. **(i)** Find the values of $\log_4 x$ that will satisfy the equation [3]

$$2(\log_4 x)^2 = \log_4 x + 6.$$

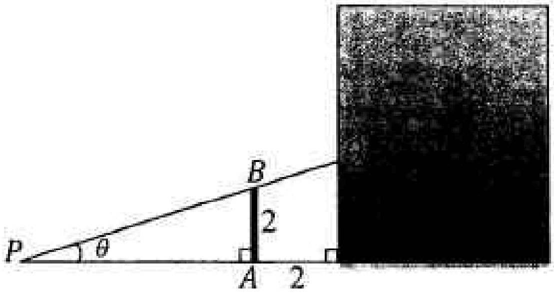
(ii) Sketch the graph of $y = \log_4 x$ and indicate clearly on your graph the location of the values of $\log_4 x$ found in part (i). [2]

Hence, show that the product of the two roots of the equation

$$2(\log_4 x)^2 = \log_4 x + 6$$

is positive. [1]

5. A vertical wall AB is 2 m high and 2 m away from a warehouse. PQ is a ramp resting on the wall AB and just touching the ground at P and the warehouse at Q. The ramp PQ is of length L metres and makes an angle θ with the horizontal.



(i) Show that the length of the ramp, L, is given by

$$L = \frac{2}{sin\theta} + \frac{2}{cos\theta}$$ [1]

(ii) Hence, show that $\frac{dL}{d\theta} = \frac{2sin^3\theta - 2cos^3\theta}{sin^2\theta cos^2\theta}$ [2]

(iii) Given that θ can vary, find the shortest possible length of the ramp. [5]

6 **(i)** Sketch the curve $y^2 = 9x$ for $0 \leq y \leq 12$. [2]

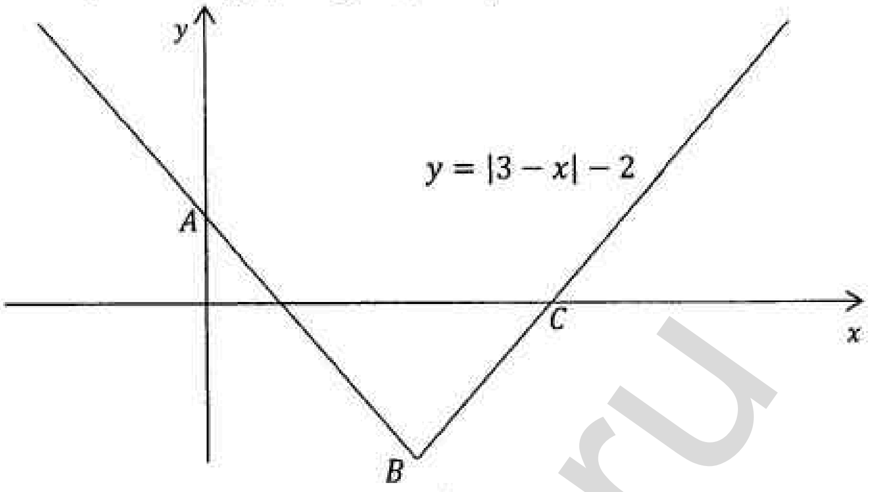
The line $4y - 3x = 9$ intersects the curve $y^2 = 9x$ at two points P and Q.

(ii) Find the coordinates of the midpoint of PQ. [6]

7 **(i)** Given that $\frac{\sin(A-B)}{\sin(A+B)} = \frac{3}{2}$, prove that $tan\, A + 5\, tan\, B = 0$. [3]

(ii) Hence, solve the equation $2\sin(2\theta - 30°) = 3\sin(2\theta + 30°)$ for $0° \leq \theta \leq 360°$. [5]

8 The diagram shows part of the graph of $y = |3 - x| - 2$.



(i) Find the coordinates of A, B and C. [4]

A line QR of gradient 1 cuts the y-axis at $(0, p)$.

(ii) State the number of intersection(s) of the line QR and $y = |3 - x| - 2$ when
(a) $p = 2$ [1]
(b) $p = -6$ [1]

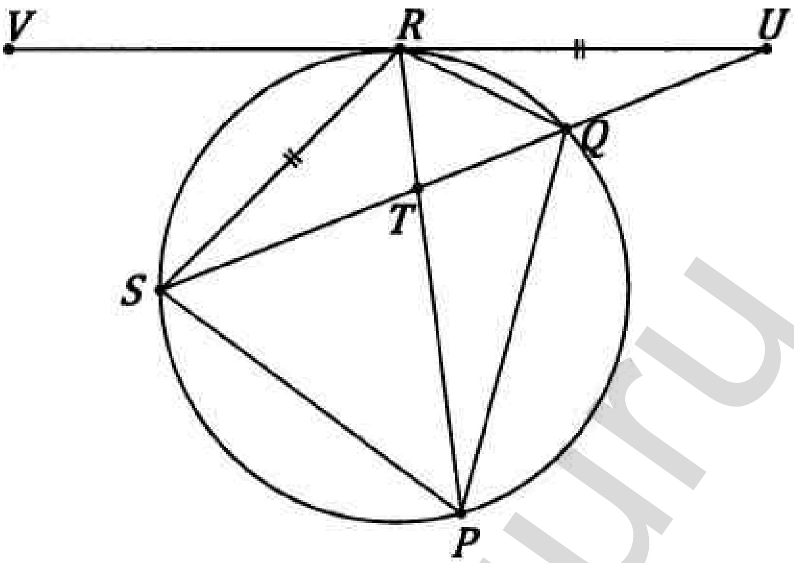
(iii) Determine the set of values of p for which the line QR intersects $y = |3 - x| - 2$ at only one point. [1]

9 A particle travelling in a straight line, passes a fixed point O on the line with a velocity of $9m/s$. The acceleration, $a\ m/s^2$, of the particle $t\ seconds$ after passing through O is given by $a = 8 - 2t$.

(i) Show that the particle comes to instantaneous rest when $t = 9$. [3]

(ii) Find the average speed of the particle for the journey from $t = 0$ to $t = 12$. [5]

10 The diagram shows a circle passing through the points P, Q, R and S. SQU is a straight line that cuts RP at the point T. VRU is a tangent to the circle at R such that $SR = RU$.



Prove that

(i) angle $SPT = 2 \times$ angle QPT, [4]

(ii) triangle QRU is similar to triangle RSU, [2]

(iii) $QR \times SU = (RS)^2$ [2]

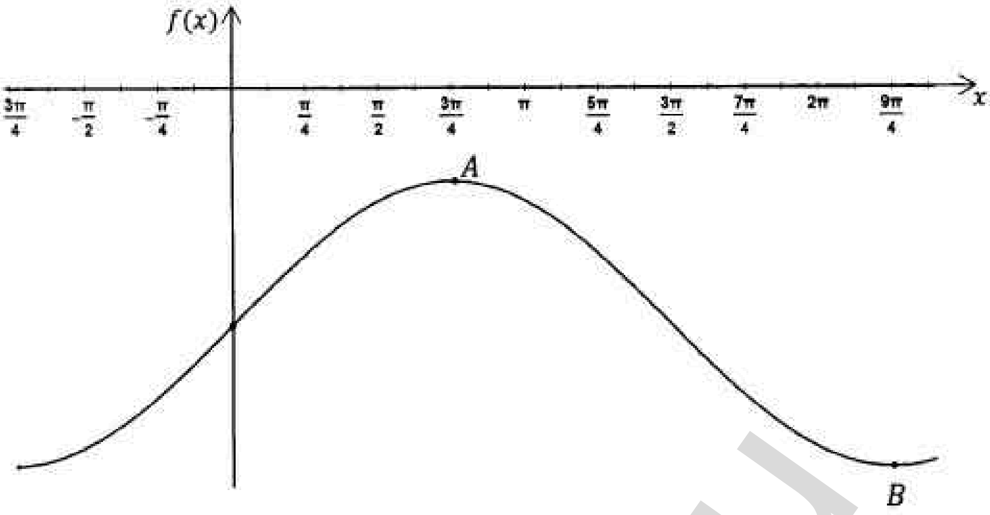
11 A container has a capacity of 960 cm^3 and is initially completely filled with water. The volume, $V\ cm^3$, of water in the container is given by $V = h^2 + 2h$ where $h\ cm$ is the height of the water level in the container. Due to leakage at the bottom of the container, the height of the water level in the container decreases at a rate of $\frac{3t}{2}\ cm/s$.

(i) Find the initial height of the water level in the container. [3]

(ii) Show that the height, h, can be expressed as $-\frac{3t^2}{4} + c$, where c is a constant. [2]

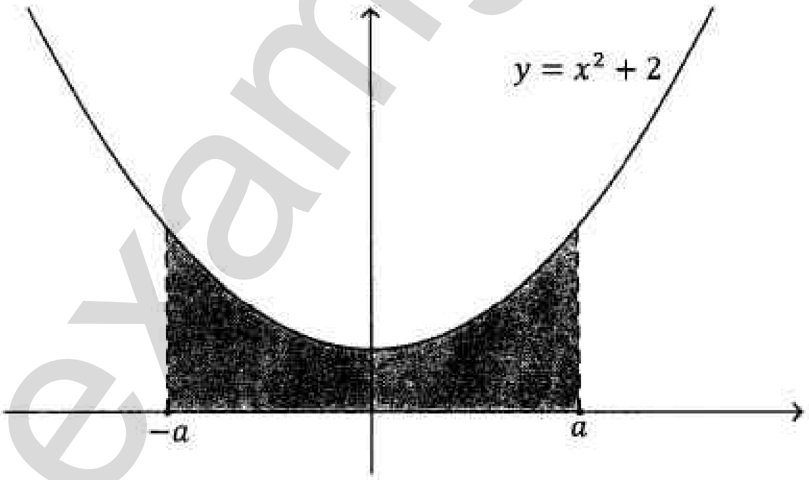
(iii) Find the rate of change of volume when $t = 4$. [3]

12 **(a)** The diagram below shows part of the curve $f(x) = 3\sin(px) - q$.



The coordinates of the turning points are $A(\frac{3\pi}{4}, -2)$ and $B(\frac{9\pi}{4}, -8)$.
Find the values of p and q. [2]

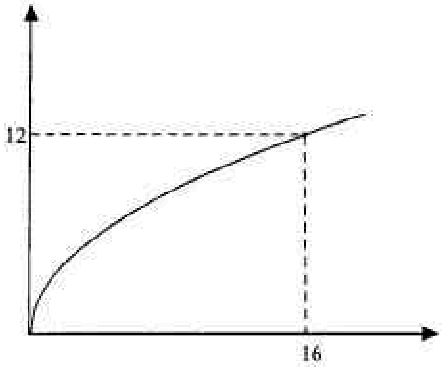
(b) The diagram below shows the graph of $y = x^2+2$. The shaded region from $x = a$ to $x = -a$ has an area of $6a\ units^2$. Find the exact value of a. [5]



END OF PAPER

Answer key:

1. $x < -\sqrt{2}$ or $x > \sqrt{2}$
2. (i) $-15 < k < 1$; (ii) maximum
3. (i) $0\ m/s$ (ii) $19.0\ m/s$
4. (i) $-\frac{3}{2}, 2$
5. (iii) $\frac{\pi}{4}$, 5.66m
6. (i)



(ii) (5,6)

7(ii) 54.6°, 144.6°, 234.6°, 324.6

8(i) (5,0) (ii)(a) 1 (ii)(b) 0 (iii) $p > -5$

9(i) $v = 8t - t^2 + 9$

(ii) $s = 4t^2 - \frac{t^3}{3} + 9t$; $18\ m/s$

11(i) $30cm$ (ii) $h = -\frac{3t^2}{4} + 30$ (iii) $-228\ cm^3/s$

12(a) $p = \frac{2}{3}$; $q = 5$ (b) $a = \sqrt{3}$

2016 ZHSS PRELIM ADD MATHS PAPER 1 MARKING SCHEME

1 $f(x) = 2x^3 - 12x - 5$

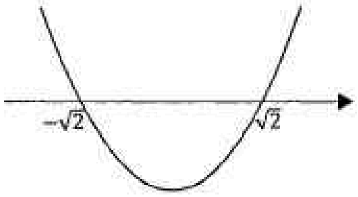
$f'(x) = 6x^2 - 12$

For increasing functions, $f'(x) > 0$

$6x^2 - 12 > 0$

$x^2 - 2 > 0$

$(x + \sqrt{2})(x - \sqrt{2}) > 0$



$\therefore$ the range of values of x is $x < -\sqrt{2}$ or $x > \sqrt{2}$.

2(i) $(3k - 5)x^2 + (k - 5)x - 2 = 0$

No real roots $\Rightarrow$ $discriminant < 0$

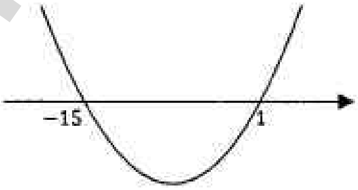
$(k - 5)^2 - 4(3k - 5)(-2) < 0$

$k^2 - 10k + 25 + 24k - 40 < 0$

$k^2 + 14k - 15 < 0$

$(k + 15)(k - 1) < 0$

$-15 < k < 1$



2(ii) coeff of $x^2 = 3k - 5$

From above, $-15 < k < 1$

$-45 < 3k < 3$

$-50 < 3k - 5 < -2$

Since coeff of $x^2 < 0$, the function has a maximum point.

Alternative method:

$y' = 2(3k - 5)x + (k - 5)$

$y'' = 2(3k - 5) = 6k - 10$

From (i), since $-15 < k < 1$, $6k - 10 < 0$

$\Rightarrow$ $y'' < 0$ $\forall x$

$\therefore$ $y = (3k - 5)x^2 + (k - 5)x - 2$ has a max point.

3 $v = 30(1 - e^{-0.2t})$

i) initial velocity, $v = 30(1 - e^{0}) = 0\ m/s$

ii) when $t = 5, v = 30(1 - e^{-1}) = 30\left(1 - \frac{1}{e}\right)$ or $19.0\ m/s$

iii) since $t \geq 0,\ 0 < e^{-0.2t} \leq 1$

$\Rightarrow \max(1 - e^{-0.2t}) < 1$

$\Rightarrow 30(1 - e^{-0.2t}) < 30$

$\therefore$ the velocity will never exceed 30 m/s.

4i) $2(\log_4 x)^2 = (\log_4 x) + 6$

Let $y = \log_4 x$

$2y^2 = y + 6$

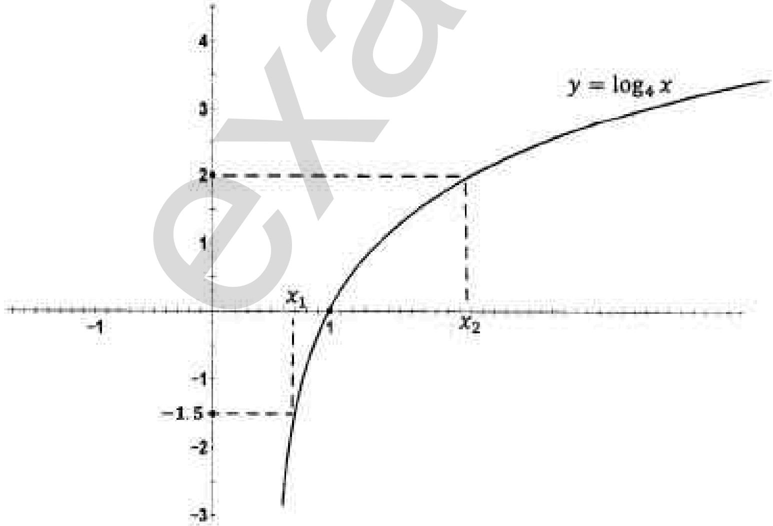
$2y^2 - y - 6 = 0$

$(2y + 3)(y - 2) = 0$

$y = -\frac{3}{2}$ or $y = 2$

$\therefore\ \log_4 x = -\frac{3}{2}$ or $\log_4 x = 2$

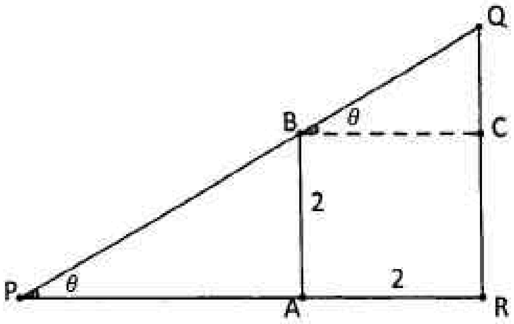
4ii)



From the graph, when $y = -\frac{3}{2}$ and $y = 2$, the x values are both positive.

$\therefore$ the product of the two roots of $2(\log_4 x)^2 = (\log_4 x) + 6$ is positive.

5i) $L = PB + BQ$



$\sin\theta = \frac{2}{PB} \quad \Rightarrow PB = \frac{2}{\sin\theta}$

$\cos\theta = \frac{2}{BQ} \quad \Rightarrow BQ = \frac{2}{\cos\theta}$

$\therefore L = \frac{2}{\sin\theta} + \frac{2}{\cos\theta}$ [AG]

5ii) $\frac{dL}{d\theta} = \frac{-2\cos\theta}{\sin^2\theta} + \frac{2\sin\theta}{\cos^2\theta}$

$= \frac{2\sin^3\theta - 2\cos^3\theta}{\sin^2\theta\cos^2\theta}$ [AG]

5iii) For max/min, $\frac{dL}{d\theta} = 0$

$$2\sin^3\theta - 2\cos^3\theta = 0$$

$$\sin^3\theta = \cos^3\theta$$

$$\tan^3\theta = 1$$

$$\tan\theta = 1$$

$$\theta = \frac{\pi}{4} \qquad 0 < \theta < \frac{\pi}{2}$$

Uisng 1st derivative test,

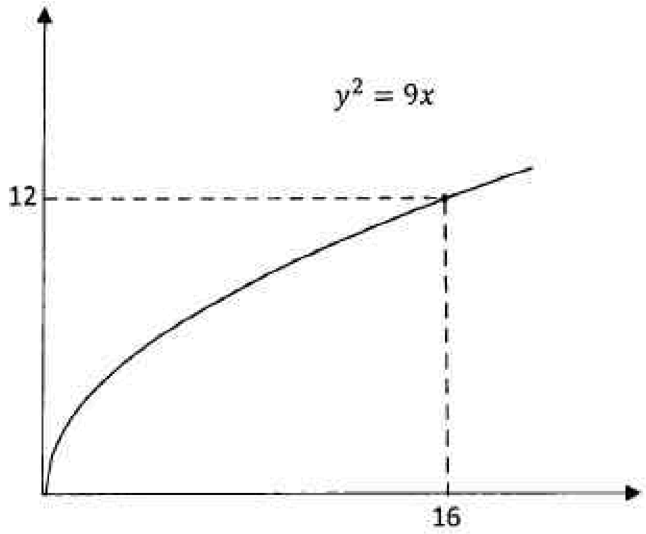
	$\frac{\pi^-}{4}$	$\frac{\pi}{4}$	$\frac{\pi^+}{4}$
$\frac{dL}{d\theta}$	$-$ ↘	0 —	$+$ ↗

$\therefore$ shortest possible length of the ramp

$$= \frac{2}{\sin\frac{\pi}{4}} + \frac{2}{\cos\frac{\pi}{4}}$$

$$= 5.66\, m \qquad [5.6568]$$

6 i)



6ii) $4y - 3x = 9$

Subs $y = \frac{3x+9}{4}$ into $y^2 = 9x$

$\left(\frac{3x+9}{4}\right)^2 = 9x$

$x^2 - 10x + 9 = 0$

$(x-9)(x-1) = 0$

$x = 1 \quad or \quad x = 9$

x-coord of midpoint of PQ $= \frac{1+9}{2} = 5$

y-coord of midpoint of PQ $= \frac{3(5)+9}{4} = 6$

$\therefore$ coords of midpoint of PQ are (5,6)

7i) $\frac{\sin(A-B)}{\sin(A+B)} = \frac{3}{2}$

$\frac{\sin A \cos B - \cos A \sin B}{\sin A \cos B + \cos A \sin B} = \frac{3}{2}$

$2(\sin A \cos B - \cos A \sin B) = 3(\sin A \cos B + \cos A \sin B)$

$\sin A \cos B + 5\cos A \sin B = 0$

Divide throughout by $\cos A \cos B$,

$\therefore \tan A + 5\tan B = 0$ [AG]

7ii) $2\sin(2\theta - 30°) = 3\sin(2\theta + 30°)$ can be written as

$\frac{\sin(2\theta-30°)}{\sin(2\theta+30°)} = \frac{3}{2}$

Compare with (i) and let $A = 2\theta$ and $B = 30°$,

$\therefore \tan 2\theta + 5\tan 30° = 0$ using result from(i)

$\tan 2\theta = -5\left(\frac{1}{\sqrt{3}}\right)$

base angle, $\alpha = \tan^{-1}\left(\frac{5}{\sqrt{3}}\right) = 70.893°$

$2\theta = 109.106°,\ 289.106°,\ 469.106°,\ 649.106°$

$\therefore\ \theta = 54.6°,\ 144.6°,\ 234.6°,\ 324.6°$

8i) $y = |3 - x| - 2$

At A, $x = 0, y = 3 - 2 = 1$

$\therefore A(0,1)$

At B, $\min|3 - x| = 0 \Rightarrow x = 3, y = -2$

$\therefore B(3, -2)$

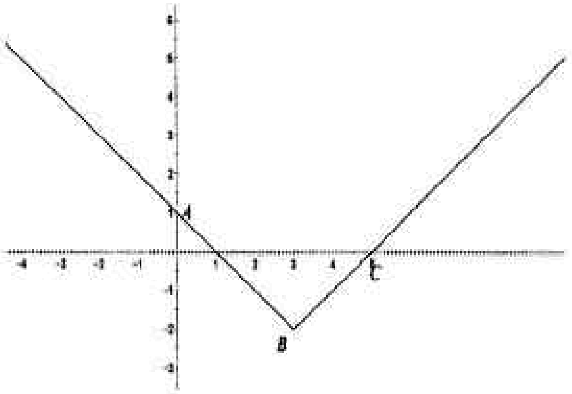
At C, $y = 0$, $|3 - x| - 2 = 0$

$|3 - x| = 2$

$3 - x = 2$ *or* $3 - x = -2$

$x = 1$ *or* $x = 5$

$\therefore C(5,0)$

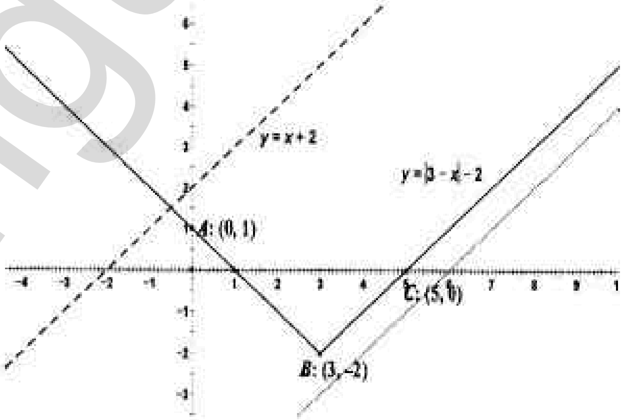


8ii) line QR: $y = x + p$

a) When $p = 2$,
$no.\ of\ intersections = 1$

b) When $p = -6$,
$no.\ of\ intersections = 0$

8iii) set of values of p for which no. of intersections is 1, is $p > -5$



9) $t = 0s,\ v = 9\text{m/s},\ \ a = 8 - 2t$

i) $v = \int a\,dt$

$= \int(8 - 2t)dt$

$= 8t - t^2 + c$

When $t = 0, v = 9$

$8t - t^2 + c = 9$

$c = 9$

$\therefore v = 8t - t^2 + 9$

At instantaneous rest, $v = 0$,

$\therefore 8t - t^2 + 9 = 0$

$t^2 - 8t - 9 = 0$

$(t + 1)(t - 9) = 0$

$t = -1\ (reject)\quad or\quad t = 9s\ \ [AG]$

9ii) $s = \int v\,dt$

$= \int(8t - t^2 + 9)dt$

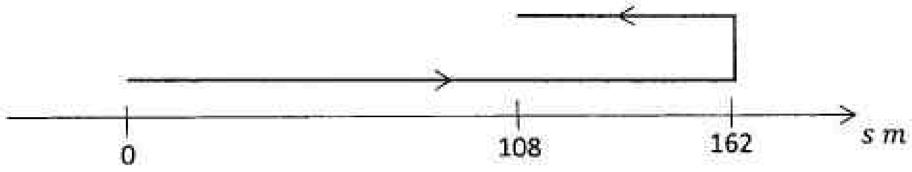
$=4t^2 - \frac{t^3}{3} + 9t + c$

When $t = 0, s = 0\ \ \Rightarrow c = 0$

$\therefore s = 4t^2 - \frac{t^3}{3} + 9t$

At instantaneous rest, $v = 0,\ \ t = 9,\ \ s = 162m$

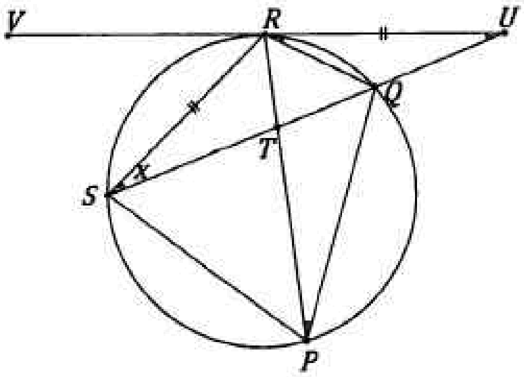
$t = 12,\ \ s = 108m$



Total distance = $162 + (162 - 108) = 216m$

$\therefore$ average speed= $\frac{216m}{12s} = 18\ m/s$

10) Let $\measuredangle RSU = x$



then $\measuredangle RUS = x$ $(base\ \measuredangle s, isos\ \Delta)$

$\measuredangle QPT = \measuredangle RSQ$

$= x$ $(\measuredangle s\ in\ the\ same\ segment)$

$\measuredangle SRV = 2x$ $(ext\ \measuredangle\ of\ \Delta SRU)$

$\measuredangle SPT = \measuredangle SRV$ $(alt\ segment\ thm)$

$= 2x$

$\therefore \measuredangle SPT = 2 \times \measuredangle QPT$ [AG]

10ii) From (i), $\measuredangle QUR = \measuredangle RUS$ $(common\ \measuredangle)$

$\measuredangle QRU = \measuredangle RSU$ $(alt\ segment\ thm)$

$\measuredangle RQU = \measuredangle SRU$ $(\measuredangle\ sum\ of\ \Delta)$

$\therefore \Delta QRU\ is\ similar\ to\ \Delta RSU\ (AAA\ similarity)$

10iii) Using ratio of corresponding sides of similar $\Delta sQRU$ & RSU,

$\frac{QR}{RS} = \frac{RU}{SU}$

$QR \times SU = RU \times RS$

$QR \times SU = (RU)^2$ [AG] $(\because RU = RS\ given)$

11) Given: $Vol = 960cm^3\ at\ t = 0$; $V = h^2 + 2h$; $\frac{dh}{dt} = -\frac{3t}{2}\ cm/s$

11i) $h^2 + 2h = 960$

$$h^2 + 2h - 960 = 0$$
$$(h + 32)(h - 30) = 0$$

$h = 30\ \ or\ \ h = -32\ (rejected)$

$\therefore$ *initial height of water is 30cm.*

11ii) $\frac{dh}{dt} = -\frac{3t}{2}$

$$h = -\frac{3t^2}{4} + c$$

$when\ \ t = 0,\ \ h = 30$

$$\Rightarrow c = 30$$

$$\therefore h = -\frac{3t^2}{4} + 30$$

11iii) $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$

$$= (2h + 2) \times \left(-\frac{3t}{2}\right)$$
$$= \left[2\left(-\frac{3t^2}{4} + 30\right) + 2\right] \times \left(-\frac{3t}{2}\right)$$

$when\ t = 4,\ \ rate\ of\ change\ of\ vol$

$$= \left.\frac{dV}{dt}\right|_{t=4}$$
$$= -228\ cm^3/s$$

12a) $f(x) = 3\sin(px) - q$

$$-q = \frac{-2+(-8)}{2}$$

$$= -5$$

$$\therefore q = 5$$

$$period = \frac{2\pi}{p}$$

From the graph, $period = \left(\frac{9\pi}{4} - \frac{3\pi}{4}\right) \times 2 = 3\pi$

$$\frac{2\pi}{p} = 3\pi$$

$$p = \frac{2}{3}$$

12b) Since graph of $y = x^2 + 2$ is symmetrical about the x-axis,

$$\int_0^a y\,dx = \frac{6a}{2}$$

$$\int_0^a (x^2+2)\,dx = \frac{6a}{2}$$

$$\left[\frac{x^3}{3} + 2x\right]_0^a = 3a$$

$$\frac{a^3}{3} + 2a = 3a$$

$$a^3 + 6a - 9a = 0$$

$$a^3 - 3a = 0$$

$$a(a^2 - 3) = 0$$

$$a = 0(rejected),\ a^2 = 3$$

$$\therefore a = \sqrt{3}\ since\ a > 0$$

ZHONGHUA SECONDARY SCHOOL

2016 Preliminary Examination

CANDIDATE NAME	
CLASS	☐☐☐
INDEX NUMBER	☐☐

ADDITIONAL MATHEMATICS

4047/02

Paper 2

15 Sept 2016

2 hours 30 minutes

Additional Materials: Answer Paper
Graph paper(2 sheets)

READ THESE INSTRUCTONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This question paper consists of **6** printed pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1 **(i)** Sketch the graph of $y = 2x^{\frac{5}{2}}$ for $x > 0$. [1]

(ii) On the same diagram, sketch the graph of $y = 16x^{-\frac{1}{2}}$ for $x > 0$. [1]

(iii) Calculate the x-coordinate of the point of intersection of your graphs. [2]

2 **(a)** A polynomial $f(x)$ has a remainder of -2 when divided by $(2x + 1)$. Showing your method clearly,

(i) find the remainder when $f(x) - 1$ is divided by $(2x + 1)$, [2]

(ii) find in terms of $f(x)$, a polynomial which is completely divisible by $(2x + 1)$. [2]

(b) A polynomial $g(x)$ can be expressed as $g(x) = (x^2 - x - 2)P(x) + ax + b$, where $P(x)$ is a polynomials in x. Given that $g(x)$ leaves a remainder of -7 when divided by $(x - 2)$ and a remainder of -19 when divided by $(x + 1)$

(i) Find the value of a and of b. [5]

(ii) Find the remainder when $g(x)$ is divided by $(x - 2)(x + 1)$. [1]

3 **Do not use a calculator in this question.**

(a) **(i)** Simplify $(2 - \sqrt{5})^2$. [1]

(ii) Given that $x = \frac{1}{2-\sqrt{5}}$, find the exact value of $x^2 + x - 2$ [3]

(b) The volume of a cuboid with a square base is $19 + 11\sqrt{3}$ cm^3. The height of the cuboid is $\sqrt{3} + 1$ cm and the length of each side of the square base is $a + \sqrt{b}$, where a and b are integers. Find the values of a and of b. [6]

4 **(a)** The roots of the quadratic equation $2x^2 + 5x - 1 = 0$ are $\tan A$ and $\tan B$.

(i) Find the value of $\tan(A + B)$. [3]

(ii) Find the value of $sec^2(A + B)$. [2]

(b) **(i)** Show that $\frac{2}{1-\sin 3x} + \frac{2}{1+\sin 3x} = 4\,sec^2 3x$. [2]

(ii) Hence evaluate $\int_0^{\frac{\pi}{12}} \frac{2}{1-\sin 3x} + \frac{2}{1+\sin 3x}\,dx$. [2]

5 A curve has the equation $y = 3x^2e^{-x}$.

(i) Find an expression for $\frac{dy}{dx}$ and obtain the coordinates of the stationary points of the curve. [5]

(ii) Determine the nature of these stationary points. [6]

6 (a) Find in ascending powers of x, the first four terms in the expansion of $(1 + x - x^2)^9$. [4]

(b) (i) Find the term independent of x in the expansion of $\left(2x^2 - \frac{1}{2x}\right)^{12}$. [3]

(ii) Determine the constant term in the expansion of $(3 + 4x^3)\left(2x^2 - \frac{1}{2x}\right)^{12}$. [4]

7 A curve is such that $\frac{d^2y}{dx^2} = \frac{6}{(2x-5)^2}$.

The equation of the tangent to the curve at the point $(3, -1)$ is $y - 2x + 7 = 0$.

(i) Find an expression for $\frac{dy}{dx}$. [4]

(ii) Find the equation of the curve. [5]

8 The table shows experimental values of the variables x and y.

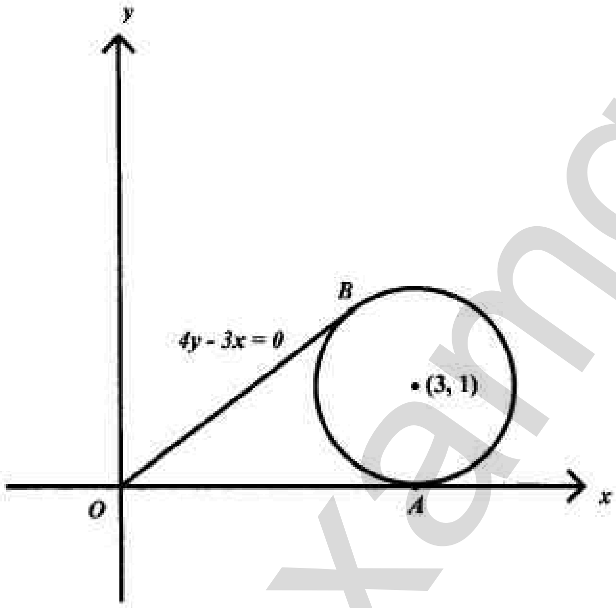
x	1	2	3	4	5
y	0.4	0.6	1.6	3.4	6

It is known that x and y are related by the equation of the form $p(x+y) = pq + qx^2$.

(i) Plot $x + y$ against x^2, draw the straight line graph and use it to estimate the value of p and q. [6]

(ii) Using your values of p and q, find the values of x for which $p(x^2 - 2q) = 2qx^2$. [2]

9 **(a)**



The circle with centre $C(3, 1)$ touches the x-axis at A. The line $4y - 3x = 0$ touches the circle at B.

Find the coordinates of B. [5]

(b) The equation of another circle is $(x-4)^2 + (y+1)^2 = 4$.

The line $y = mx$ is a tangent to the circle. Find the possible exact values of m. [4]

10 **(a)** **(i)** Express $\frac{2x^3+x^2}{x^2+x-2}$ in the form of $ax+b+\frac{cx+d}{x^2+x-2}$. [2]

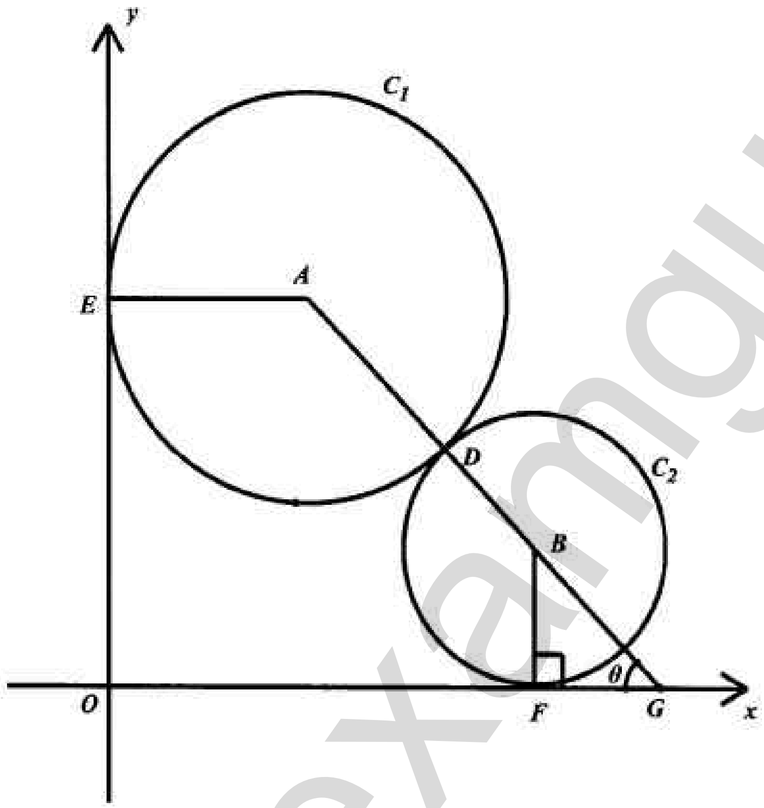
(ii) Using the values of c and d found in (i), express $\frac{cx+d}{x^2+x-2}$ as a sum of two partial fractions. [3]

(b) A curve has the equation $y=\frac{x-1}{\sqrt{4x+1}}$.

(i) Differentiate y with respect to x. [3]

(ii) Using the result in part **b(i)**, determine $\int \frac{2(2x+3)}{(4x+1)^{\frac{3}{2}}}\,dx$. [2]

11.



The diagram shows two circles, C_1 and C_2 with centres A and B respectively. The two circles touch each other at D. C_1 has radius 3 units and touches the y-axis at E. C_2 has radius 2 units and touches the x-axis at F. The lines AB produced meets the x-axis at G and angle $BGO=\theta$ radians.

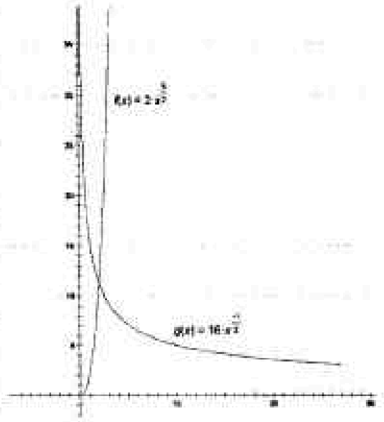
(i) Show with clear explanations, that $OE=5\sin\theta+2$ and $OF=5\cos\theta+3$. [2]

(ii) Show that $EF^2=38+20\sin\theta+30\cos\theta$. [2]

(iii) Express EF^2 in the form $38+R\cos(\theta-\alpha)$, where $R>0$ and α is an acute angle. [3]

(iv) Given that $EF^2=65$, find the value of θ. [2]

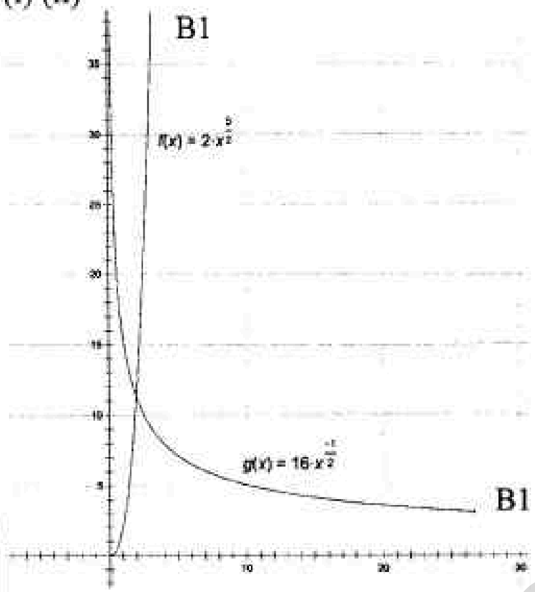
END OF PAPER

	Answer Key		
1	(i) (ii) 		
iii	$x = 2$	8i	$p = 2.5, q = 1$
2i	Remainder $= -3$	ii	$x = \pm\sqrt{10}$ or $x = \pm$ 3.16
ii	A polynomial = $f(x) + 2$, any multiple of $f(x) + 2$	9a	$B(\frac{12}{5}, \frac{9}{5})$
2bi	$a = 4$, $b = -15$	9b	$m = \frac{-2 \pm\sqrt{13}}{6}$
ii	Remainder $= 4x - 15$	10ai	$2x - 1 + \frac{5x-2}{x^2+x-2}$
3ai	$9 - 4\sqrt{5}$	aii	$\frac{5x-2}{x^2+x-2} = \frac{4}{x+2} + \frac{1}{x-1}$
aii	$5 + 3\sqrt{5}$	bi	$\frac{2x+3}{(4x+1)^{\frac{3}{2}}}$
3b	$a = 2$ and $b = 3$	ii)	$\int \frac{2(2x+3)}{(4x+1)^{\frac{3}{2}}}\, dx = \frac{2(x-1)}{\sqrt{4x+1}} + c'$
4ai)	$-\frac{5}{3}$	11iii	$EF^2 = 38 + 10\sqrt{13} cos\,(\theta - 0.58800)$
4aii)	$\frac{34}{9}$	11iv	$\theta = 1.31$
4bii	$\frac{4}{3}$		
5ai	$3xe^{-x}(2 - x)$, (0, 0) and $(2, \frac{12}{e^2})$		
5ii	$(2, \frac{12}{e^2})$ is a maximum point (0, 0) is a minimum point		
6a	$1 + 9x + 27x^2 + 12x^3 + \cdots.)$		
bi)	$\frac{495}{16}$		
bii	$\frac{1265}{16}$		
7i	$\frac{dy}{dx} = -\frac{3}{(2x-5)} + 5$		
ii	$y = -\frac{3\ln(2x-5)}{2} + 5x - 16$		

1 **(i)** Sketch the graph of $y = 2x^{\frac{5}{2}}$ for $x > 0$. [1]

(ii) On the same diagram, sketch the graph of $y = 16x^{-\frac{1}{2}}$ for $x > 0$. [1]

(iii) Calculate the x-coordinate of the point of intersection of your graphs. [2]

1 [2]	(i) (ii)  	
(iii)	$2x^{\frac{5}{2}} = 16x^{-\frac{1}{2}}$	M1 equating with attempt to solve
[2]	$x^3 = 8$	
	$x = 2$	A1

2 **(a)** A polynomial $f(x)$ has a remainder of -2 when divided by $(2x + 1)$. Showing your method clearly,

(i) find the remainder when $f(x) - 1$ is divided by $(2x + 1)$, [2]

(ii) find in terms of $f(x)$, a polynomial which is completely divisible by $(2x + 1)$. [2]

2(a) (i)	Let $f(x) = (2x + 1))Q(x) - 2$	
[2]	$f(x) - 1 = (2x + 1))Q(x) - 2 - 1$	M1
	Remainder $= -3$	B1
(ii)	$f(x) + 2 = (2x + 1))Q(x) - 2 + 2$	M1
[2]	A polynomial = $f(x) + 2$, any multiple of $f(x) + 2$	B1

(b) A polynomial $g(x)$ can be expressed as $g(x) = (x^2 - x - 2)P(x) + ax + b$, where $P(x)$ is a polynomials in x. Given that $g(x)$ leaves a remainder of -7 when divided by $(x - 2)$ and a remainder of -19 when divided by $(x + 1)$

(i) Find the value of a and of b. [5]

(ii) Find the remainder when $g(x)$ is divided by $(x - 2)(x + 1)$. [1]

2(b) (i)	$g(x) = (x^2 - x - 2)P(x) + ax + b,$	
[5]	$= (x-2)(x+1)P(x) + ax + b,$	$(x-2)(x+1)$ seen or
	Substituting $x = -1$ or 2	$(-1)^2 - (-1) - 2$ seen or
	$g(2) = 2a + b = -7$	$2^2 - 2 - 2$ seen B1
	$2a + b = -7$ ……………………(1)	B1
	$g(-1) = -a + b = -19$ ……………(2)	B1
2(b) (i)	(1) − (2), $3a = 12$	
	$a = 4$	A1
	$b = -15$	A1
(b) (ii) [1]	Remainder $= 4x - 15$	A1

3 Do not use a calculator in this question.

(a) (i) Simplify $(2 - \sqrt{5})^2$. [1]

(ii) Given that $x = \frac{1}{2-\sqrt{5}}$, find the exact value of $x^2 + x - 2$ [3]

3(a) (i)	$(2 - \sqrt{5})^2 = 4 - 4\sqrt{5} + 5$	
[1]	$= 9 - 4\sqrt{5}$	A1
(ii)	$x^2 + x - 2 = \dfrac{1}{9 - 4\sqrt{5}} + \dfrac{1}{2 - \sqrt{5}} - 2$	B1
[3]	$= \frac{9+4\sqrt{5}}{81-80} + \frac{2+\sqrt{5}}{-1} - 2$	Rationalising the denominator M1
	$= 5 + 3\sqrt{5}$	A1

(b) The volume of a cuboid with a square base is $19+11\sqrt{3}$ cm^3. The height of the cuboid is $\sqrt{3}+1$ cm and the length of each side of the square base is $a+\sqrt{b}$, where a and b are integers. Find the values of a and of b. [6]

3(b)	Area $=\frac{19+11\sqrt{3}}{\sqrt{3}+1}$	M1
[6]	$=\frac{19+11\sqrt{3}}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$	
	$=\frac{19\sqrt{3}+33-19-11\sqrt{3}}{2}$	
	$(a+\sqrt{b})^2=\frac{14+8\sqrt{3}}{2}$	B1
	$a^2+b+2a\sqrt{b}=7+4\sqrt{3}$	
	$a^2+b=7$---------(1) $2a\sqrt{b}=4\sqrt{3}$ $a\sqrt{b}=2\sqrt{3}$ $a^2b=12$ ---------(2)	Equating rational and irrational parts M1 Do not accept $a\sqrt{b}=2\sqrt{3}$ $a=2, b=3$
	From (1), $a^2=7-b$	
	$(7-b)b=12$	
	$0=b^2-7b+12$	M1 obtain a quadratic equation
	$(b-4)(b-3)=0$	
	$b=3$ or $b=4$	
	when $b=4$, $a^2=7-4=3$ (rejected)	Obtain either both b's or both a's
	when $b=3$, $a^2=7-3=4$	
	$a=2$ or $a=-2$ (rejected)	
	$a=2$ and $b=3$	A1 [given provided M1 has been awarded]

4 **(a)** The roots of the quadratic equation $2x^2+5x-1=0$ are $\tan A$ and $\tan B$.

(i) Find the value of $\tan(A+B)$. [3]

(ii) Find the value of $sec^2(A+B)$. [2]

4(a) (i)	$\tan A+\tan B=-\frac{5}{2}$	Either one B1
	$\tan A\tan B=-\frac{1}{2}$	
	$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$	
	$=\frac{-\frac{5}{2}}{1+\frac{1}{2}}$	B1
	$=-\frac{5}{3}$	A1

4 (a) (ii)	$sec^2(A+\ B) = 1 + \ tan^2(A+B)$	
[2]	$= \ 1 + \frac{25}{9}$	M1
	$= \ \frac{34}{9}$	A1

(b) **(i)** Show that $\frac{2}{1-\sin 3x} + \frac{2}{1+\sin 3x} = 4\,sec^2 3x.$ [2]

(ii) Hence evaluate $\int_0^{\frac{\pi}{12}} \frac{2}{1-\sin 3x} + \frac{2}{1+\sin 3x}\ dx.$ [2]

4(b) (i)	LHS $= \frac{2}{1-\sin 3x} + \frac{2}{1+\sin 3x}$	
[2]	$= \frac{2(1+\sin 3x)+2(1-\sin 3x)}{(1-sin^2 3x)}$	B1
	$= \frac{4}{cos^2 3x}$	B1
	$= 4\,sec^2 3x$ (Shown)	
(ii)	$\int_0^{\frac{\pi}{12}} \frac{2}{1-\sin 3x} + \frac{2}{1+\sin 3x}\ dx$	
[2]	$= \int_0^{\frac{\pi}{12}} 4\,sec^2 3x\ dx$	
	$= \left[\frac{4}{3}\tan 3x\right]_0^{\frac{\pi}{12}}$	B1
	$= \frac{4}{3}$	A1

5 A curve has the equation $y = 3x^2e^{-x}$.

(i) Find an expression for $\frac{dy}{dx}$ and obtain the coordinates of the stationary points of the curve. [5]

(ii) Determine the nature of these stationary points. [6]

5(i) [5]	$\frac{dy}{dx} = 6xe^{-x} + 3x^2(-e^{-x})$	Product rule M1, B1
	$= 3xe^{-x}(2-x)$	
	For stationary points, $\frac{dy}{dx} = 0$	M1
	$3xe^{-x}(2-x) = 0$	
	$e^{-x} \neq 0,\ x = 0$ or $x = 2$	A1[2 values of x]
	(0, 0) and $(2, \frac{12}{e^2})$	Both points A1

5(ii) [6]	$\frac{d^2y}{dx^2} = 6e^{-x} - 6xe^{-x} + 6x(-e^{-x}) + 3x^2(e^{-x})$				Award M1 if there is at most 1 wrong term
	$= 6e^{-x} - 12xe^{-x} + 3x^2(e^{-x})$				A1
	$= 3e^{-x}(2 - 4x + x^2)$				
	when $x = 0$, $\frac{d^2y}{dx^2} = 6 > 0$				B1
	(0, 0) is a minimum point				A1
	when $x = 2$, $\frac{d^2y}{dx^2} = -\frac{6}{e^2} < 0$				B1
	$(2, \frac{12}{e^2})$ is a maximum point				A1
OR	Using $\frac{dy}{dx}$,				
[6]	For (0, 0)				
	x	0^-	0	0^+	
	$\frac{dy}{dx}$	< 0	0	> 0	
	Sketch of tangent	╲	—	╱	B2
	(0, 0) is a minimum point				A1
	For $(2, \frac{12}{e^2})$				
	x	2^-	2	2^+	
	$\frac{dy}{dx}$	> 0	0	< 0	
	Sketch of tangent	╱	—	╲	B2
	$(2, \frac{12}{e^2})$ is a maximum point				A1

6 (a) Find in ascending powers of x, the first four terms in the expansion of $(1+x-x^2)^9$. [4]

6(a)	$(1+x-x^2)^9$	
[4]	$=1+\binom{9}{1}(x-x^2)+\binom{9}{2}(x-x^2)^2+\binom{9}{3}(x-x^2)^3+\ldots$	B1
	$=1+9x-9x^2+36(x^2-2x^3+x^4)+84(x^3+\cdots.)$	
	$=1+9x+27x^2+12x^3+\cdots.)$	A3 deduct 1 mark for every wrong term

(b) (i) Find the term independent of x in the expansion of $\left(2x^2-\frac{1}{2x}\right)^{12}$. [3]

(ii) Determine the constant term in the expansion of $(3+4x^3)\left(2x^2-\frac{1}{2x}\right)^{12}$. [4]

6(b) (i)	$(r+1)^{th}$ term $=\binom{12}{r}(2x^2)^{12-r}\left(-\frac{1}{2x}\right)^r$	M1
[3]	For term independent of x	
	$x^0 = x^{2(12-r)}\times x^{-r}$	
	$0=24-3r$	
	$r=8$	B1
	Term independent of $x=\binom{12}{8}(2x^2)^{12-8}\left(-\frac{1}{2x}\right)^8$	
	$=\binom{12}{8}(2)^4\left(-\frac{1}{2}\right)^8$	
	$=\binom{12}{8}\left(\frac{1}{2}\right)^4$	
	$=\frac{495}{16}$	A1
6(b) (ii)	For x^{-3}, $-3=24-3r$	
[4]	$r=9$	M1
	Term in $x^{-3}=\binom{12}{9}(2x^2)^3\left(-\frac{1}{2x}\right)^9$	
	$=-\binom{12}{9}\left(\frac{1}{2^6}\right)x^{-3}$	
	$=-\frac{220}{64}x^{-3}$	B1
	Constant $=3\times\frac{495}{16}+4\times\left(-\frac{220}{64}\right)$	M1
	$=\frac{1265}{16}$	A1

7 A curve is such that $\frac{d^2y}{dx^2} = \frac{6}{(2x-5)^2}$.

The equation of the tangent to the curve at the point $(3,-1)$ is $y - 2x + 7 = 0$.

(i) Find an expression for $\frac{dy}{dx}$. [4]

(ii) Find the equation of the curve. [5]

7(i)	$\frac{dy}{dx} = \int 6(2x-5)^{-2}\,dx$	M1 attempt to integrate
[4]	$= \frac{6(2x-5)^{-1}}{(-1)(2)} + c$	B1
	$= -\frac{3}{(2x-5)} + c$	
	when $x = 3, \frac{dy}{dx} = 2$	
	$2 = -3 + c$	
	$c = 5$	M1 attempt to find c
	$\frac{dy}{dx} = -\frac{3}{(2x-5)} + 5$	A1
(ii)	$y = \int -\frac{3}{(2x-5)} + 5\ \ dx$	M1 attempt to find y by integrating $\frac{dy}{dx}$.
[5]	$= -\frac{3\ln(2x-5)}{2} + 5x + d$	B1
	substituting $x = 3$ and $y = -1$	
	$-1 = -\frac{3}{2}ln1 + 15 + d$	M1 attempt to find d.
	$d = -16$	B1
	$y = -\frac{3\ln(2x-5)}{2} + 5x - 16$	A1

8 The table shows experimental values of the variables x and y.

x	1	2	3	4	5
y	0.4	0.6	1.6	3.4	6

It is known that x and y are related by the equation of the form $p(x+y)=pq+qx^2$.

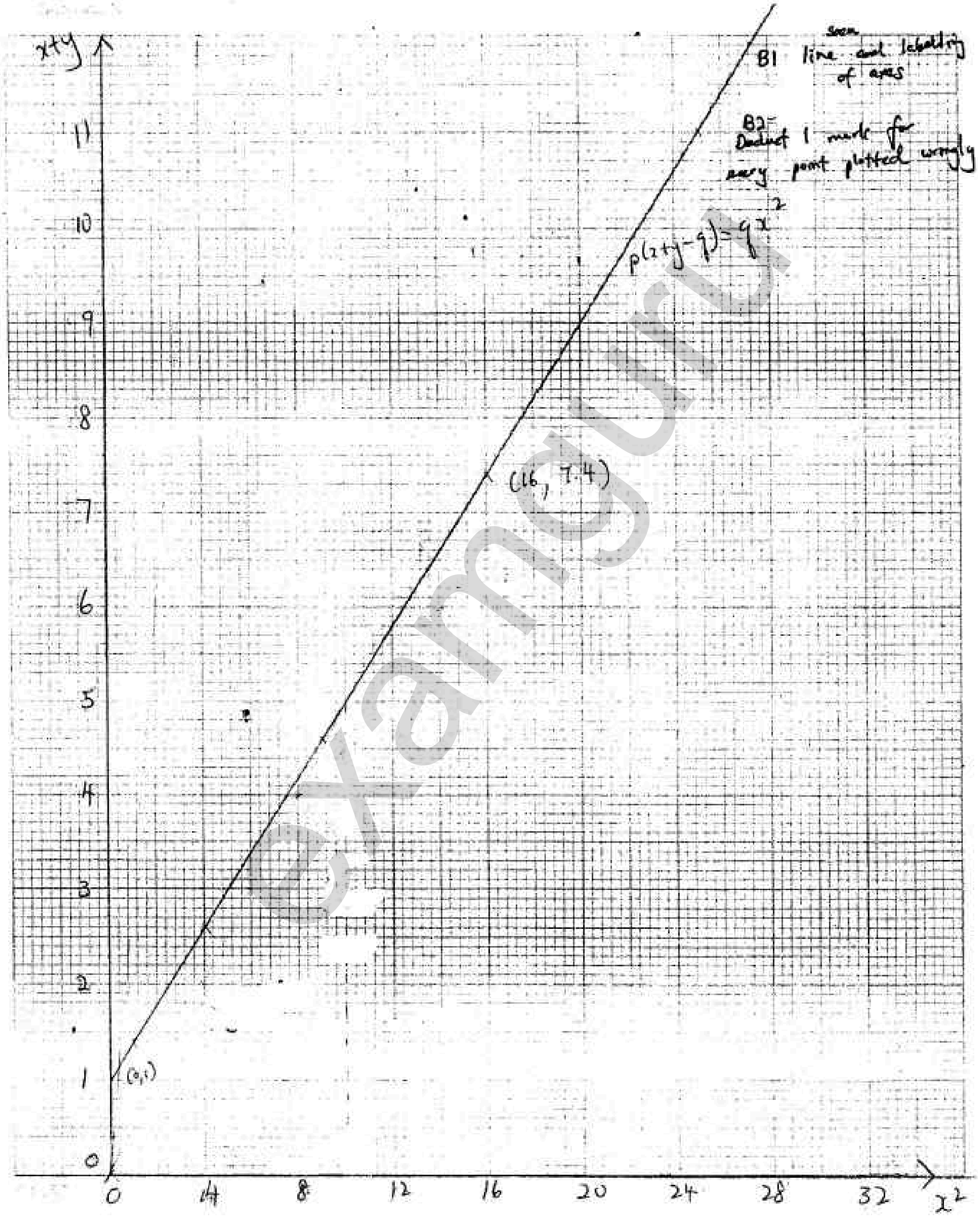
(i) Plot $x+y$ against x^2, draw the straight line graph and use it to estimate the value of p and q. [6]

(ii) Using your values of p and q, find the values of x for which $p(x^2-2q)=2qx^2$. [2]

(i)	x^2	1	4	9	16	25	
[6]	$x+y$	1.4	2.6	4.6	7.4	11	
	$p(x+y)=pq+qx^2$						
	$x+y-q=\frac{q}{p}x^2$						
	$x+y=q+\frac{q}{p}x^2$ ----------(1)						Award B1 either for (1) or (2)
	gradient $=\frac{q}{p}$, $x+y$-intercept $=q$ -------(2)						
	From graph, $x+y$-intercept = 1						
	$q=1$						A1
	gradient $=\frac{7.4-1}{16}=0.4$						
	$\frac{q}{p}=0.4$						
	$\frac{1}{p}=0.4$						
	$p=2.5$						A1
	On graph paper						
	Straight line drawn with correct labelling of axes						B1
	All 5 points correctly plotted						B2 deduct 1 mark for every point plotted wrongly

8(ii)	$\frac{5}{2}(x^2-2)=2x^2$	M1 FT for their answers in (i)
[2]	$\frac{1}{2}x^2=5$	
	$x^2=10$	
	$x=\pm\sqrt{10}$ or $x=\pm 3.16$	A1

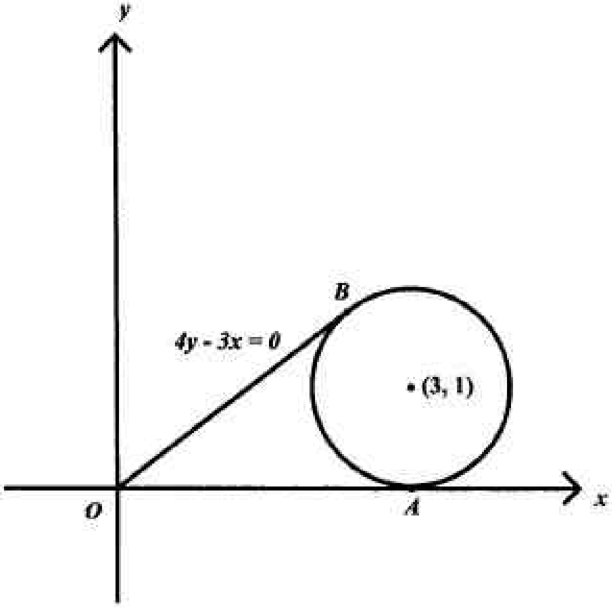
Q 8



B1 line ~~and~~ seen and labelling of axes

B2 – Deduct 1 mark for every point plotted wrongly

9 **(a)**



The circle with centre $C(3, 1)$ touches the x-axis at A. The line $4y - 3x = 0$ touches the circle at B.

Find the coordinates of B. [5]

9(a)	Equation of tangent at B is $y = \frac{3}{4}x$.	
[5]	Gradient of normal at B is $-\frac{4}{3}$	M1
	Equation of normal at B is $y - 1 = -\frac{4}{3}(x - 3)$	
	$y = -\frac{4}{3}x + 5$	B1
	For point of intersection B,	
	$\frac{3}{4}x = -\frac{4}{3}x + 5$	M1
	$\frac{25x}{12} = 5$	
	$x = \frac{12}{5}$	B1 for correct x or y
	$y = \frac{9}{5}$	
	$B(\frac{12}{5}, \frac{9}{5})$	A1

Name: **Register Number:** **Class:**

南僑中學

NAN CHIAU HIGH SCHOOL

PRELIMINARY EXAMINATION (3) 2016
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS **4047/01**
Paper 1 **15 September 2016, Thursday**

Additional Materials : Writing Paper (8 sheets) **2 hours**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on the separate writing papers provided.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

Setter: Mr Tan Beng Guan

This paper consists of 6 printed pages including the coverpage.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\cdots+\binom{n}{r}a^{n-r}b^r+\cdots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}=\frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$
$$\sec^2 A=1+\tan^2 A$$
$$cosec^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$
$$\Delta=\frac{1}{2}bc\sin A$$

Answer ALL Questions

1 Given that $y = \frac{x^4 - 2}{x}$, $x \neq 0$.

(i) Find an expression for $\frac{dy}{dx}$. [2]

(ii) Hence, show that y is an increasing function for all real values of x except zero. [1]

2 (a) Given that $\log_9 m = n$, express each of the following in terms of n.

(i) $\log_9(9m^2)$ [2]

(ii) $\log_3 \frac{1}{m}$ [3]

(b) Solve the equation $2(\ln x)^2 + 3\ln\left(\frac{1}{x}\right) = 5$. [4]

3 On a university campus of 6 000 students, one student returned from vacation with a contagious flu virus. The spread of the virus through the student body is given by

$$f(t) = \frac{6000}{1 + 5999e^{-0.5t}}$$

where f(t) is the total number of students infected after t days. The university will cancel classes when 50% or more of the students are infected. Estimate,

(i) the number of students infected after 5 days, giving your answer to the nearest whole number, [1]

(ii) after how many days will the classes be cancelled. [3]

4 (a) Find the range of values of x for which $(x-2)(x+3) \geq 6$, [3]

(b) Find the range of values of k for which the line $y + kx = 8$ and the curve $x^2 + 4y = 16$ do not intersect. [4]

5 The function f is defined by $f(x) = 4x^2 - 4x - 15$ for $-3 \leq x \leq 4$.

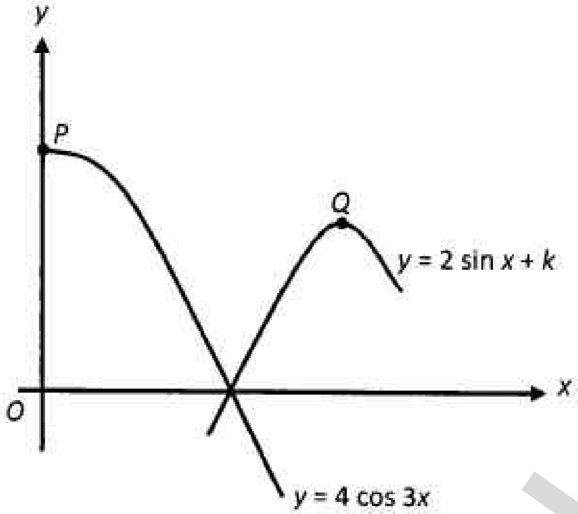
(i) Sketch the graph of $y = |f(x)|$, indicate clearly the x and y intercepts. [4]

(ii) Determine the set of values of $\boldsymbol{m}$ for which there are two or three distinct solutions for the equation $|f(x)| = \boldsymbol{m}$. [2]

6 (a) Prove that $(\sec\theta + \tan\theta)^2 = \frac{1 + \sin\theta}{1 - \sin\theta}$. [4]

(b) Find all the values of t between 0 and 12 for which $\sin(\frac{\pi t}{5}) = \frac{\sqrt{3}}{2}$. [3]

7 The diagram, which is not drawn to scale, shows parts of the graphs of $y = 4\cos 3x$ and $y = 2\sin x + k$.



(i) State the amplitude of $y = 2\sin x + k$ and the period of $y = 4\cos 3x$. [2]

(ii) Points P and Q are the respective maximum points on these graphs. Given that the two graphs intersect at the x-axis, find the value of k and the coordinates of P and of Q. [6]

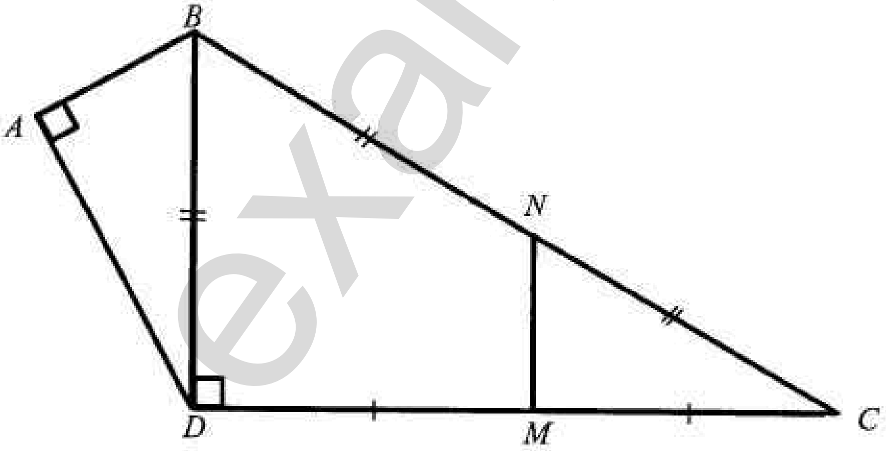
8 A particle P is traveling in a straight line with a velocity v ms^{-1}, given by $v = -2t^2 + 7t + 4$, where t is the number of seconds after passing a fixed point O. Calculate

(i) the value of t at which the particle comes to instantaneous rest, [2]

(ii) the maximum velocity achieved by the particle, [3]

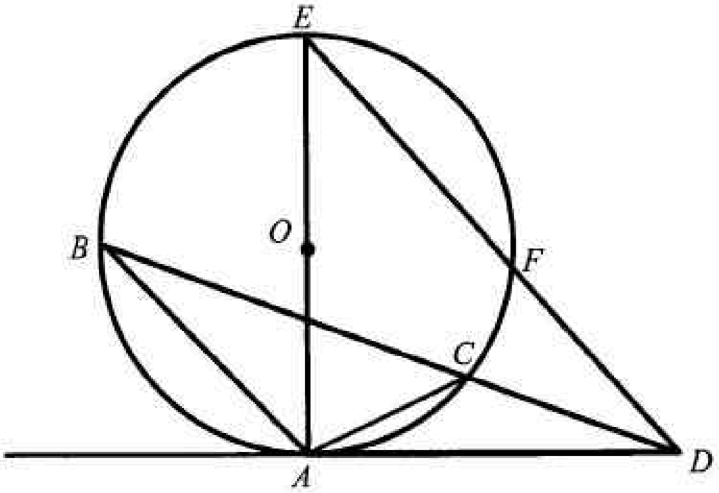
(iii) the total distance travelled by P from $t = 0$ to $t = 5$. [4]

9 (a)



In the diagram, M and N are mid-points of CD and BC respectively. DB bisects $\angle ABC$, $DB = CN$ and $\angle BAD = \angle BDC = 90°$. Prove that ΔABD is congruent to ΔMNC. [4]

(b)



In the diagram, triangle ABC is inscribed in the circle with centre O. The tangent at A meets the line EF and BC produced at D.

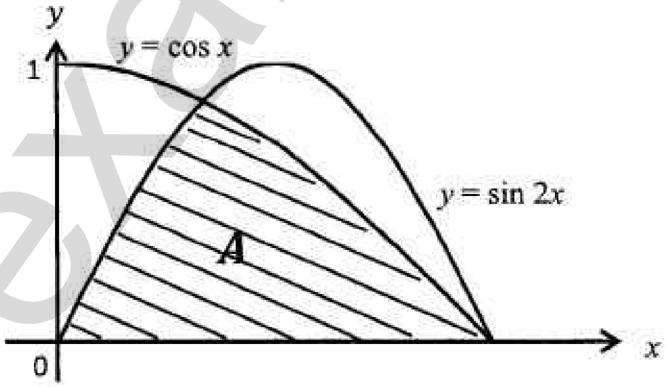
Prove that

(i) ΔADC and ΔBDA are similar. [2]

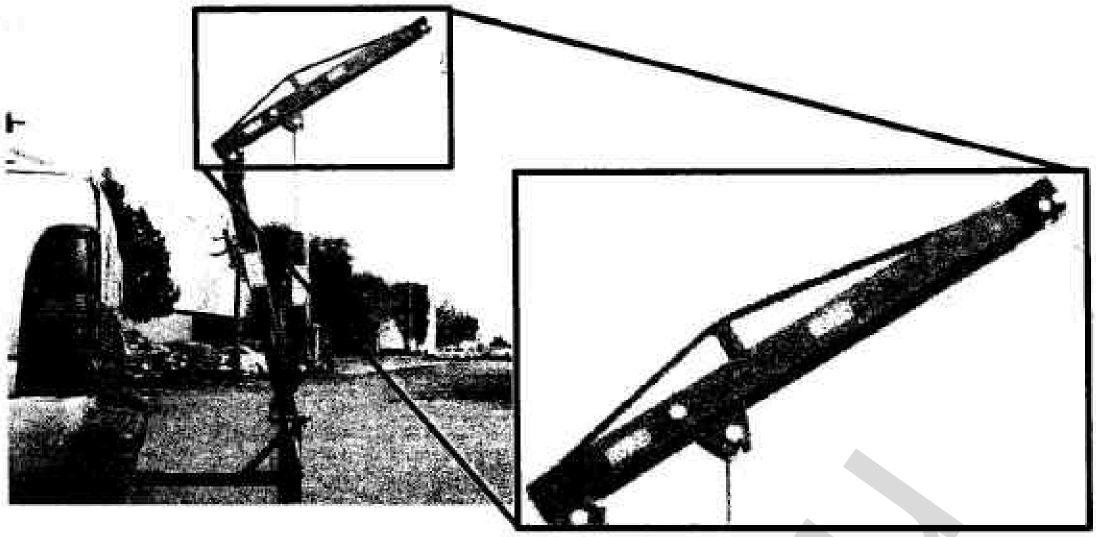
(ii) $BD \times CD = DE^2 - AE^2$ [3]

10 **(a)** It is given that $y=(x-2)\sqrt{2x-1}$. Find the exact value of x when the rate of decrease of y is three times the rate of increase of x. [5]

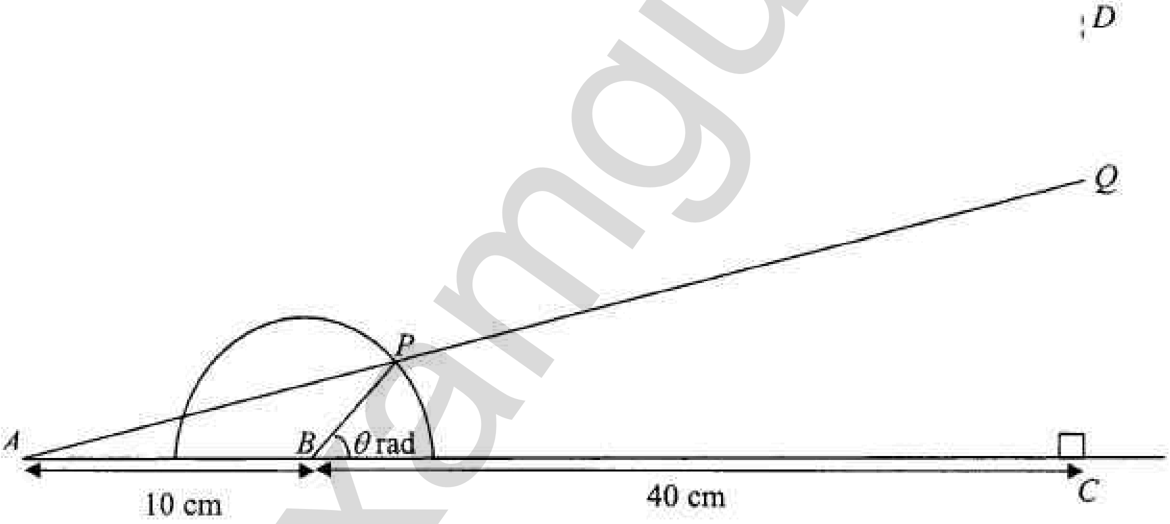
(b) The region A, shown in the diagram is bounded by the curves $y=\sin 2x$, $y=\cos x$ and the x-axis. Find its area. [5]



11 The pictures below show a load lifter and the close-up of its extensible arm.



The movement of the arm can be modelled with the diagram shown below.



(i) In the diagram, APQ is a straight line representing the arm. ABC is a straight line with $AB = 10$ cm and $BC = 40$ cm and CD is perpendicular to ABC. The arm is lifting an object vertically from point C. P is a variable point on the semicircle with centre B, radius 6 cm and $\angle CBP = \theta$. The length of the arm is adjusted so that the point Q lies along the vertical line CD during the lifting of the object.

Show that $CQ = \dfrac{150\sin\theta}{5+3\cos\theta}$. [3]

(ii) Find the value of θ for which CQ is a maximum. [5]

~~~~~ **End of Paper** ~~~~~
~~~~~

Answers

1 **(a)** $\frac{dy}{dx}=3x^2+\frac{2}{x^2}$

(b) Since $3x^2+\frac{2}{x^2}>0$ thus $\frac{dy}{dx}>0$ for all values of x, except $x=0$

$\Rightarrow$ y is an increasing function (shown)

2 **(a)** (i) $1+2n$

(ii) $-2n$

(b) $x=e^{\frac{5}{2}}$ *or* $x=\frac{1}{e}$

$x=12.2$ *or* $x=0.368$ (*to* 3 *s.f.*)

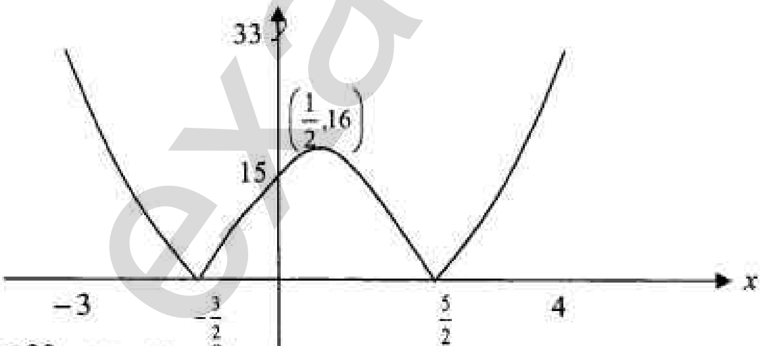
3 **(i)** 12 *student*

(ii) 18 *days*

4 **(a)** $x\le -4$ *or* $x\ge 3$

(b) $-2<k<2$

5 **(i)**



(ii) $16\le m\le 33$ *or* $m=0$

6 **(a)** $LHS=(\sec\theta+\tan\theta)^2$

$$=\sec^2\theta+2\sec\theta\tan\theta+\tan^2\theta$$

$$=\frac{1}{\cos^2\theta}+\frac{2\sin\theta}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}$$

$$=\frac{1+2\sin\theta+\sin^2\theta}{1-\sin^2\theta}$$

$$=\frac{(1+\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$$

$$=\frac{1+\sin\theta}{1-\sin\theta} \text{ (proven)}$$

(b) $t=\frac{5}{3},\frac{10}{3}\ or\ \frac{35}{3}$

7 **(i)** Amplitude = 2 and Period = 120° or $\frac{2\pi}{3}$

(ii) $k=-1 \quad P\,(0,4) \quad Q\,(\frac{\pi}{2},1)$ or $(90°,1)$

8 **(i)** $t=4$

(ii) max $velocity = 10\frac{1}{8}ms^{-1}$

(iii) $34.5\ m$

9 **(a)** Since M and N are mid-points of CD and BC

$MN\ //\ DB$ (Mid-point Theorem)

$\Rightarrow \angle NMC = \angle BDC = 90°$ (Corr. $\angle s\ MN\ //\ DB$)

$\Rightarrow \angle MNC = \angle DBC$ (Corr. $\angle s\ MN\ //\ DB$)

Given DB bisects $\angle ABC$

$\Rightarrow \angle ABD = \angle DBC = \angle MNC$

$DB = CN$ (given)

$\Delta ABD \equiv \Delta MNC$ (AAS) (proven)

(b) (i) $\angle ADC = \angle BDA$ (common angle)
$\angle CAD = \angle ABD$ (alternate segment theorem)
$\therefore \Delta ADC\ and\ \Delta BDA\ are\ similar$ (angle-angle similarity test)

(ii) $\frac{BD}{AD}=\frac{AD}{CD}$ (corr ratios of similar triangles)
$\Rightarrow BD\times CD = AD^2$
Since AD is tangent to circle
$\angle DAE = 90°$ (tangent $\perp$ radius)
$\therefore AD^2 = DE^2 - AE^2$ (pythagoras' theorem)
$\Rightarrow BD\times CD = DE^2 - AE^2$ (proven)

10 **(a)** $x=2-\sqrt{2}$

(b) $\frac{3}{4}\ units^2$

11 **(a)** From the diagram, *PT* is perpendicular to AC

ΔAPT *and* ΔAQC *are similar* (*angle – angle similarity test*)

$$\frac{CQ}{50}=\frac{6\sin\theta}{10+6\cos\theta}$$ (corr ratios of similar triangles)

$$CQ=\frac{150\sin\theta}{5+3\cos\theta}$$ (shown)

(b) $\theta = 2.21\,rad$ $(to\ 3\ s.f.)$

Prelim 3 Add Math P1

Answer Scheme.

1 **(a)** $y = x^3 - 2x^{-1}$ [M1]

$\frac{dy}{dx} = 3x^2 + \frac{2}{x^2}$ [A1]

(b) Since $3x^2 + \frac{2}{x^2} > 0$ thus $\frac{dy}{dx} > 0$ for all values of x, except $x = 0$ [B1]

$\Rightarrow$ y is an increasing function (shown)

2 **(a)** (i) $\log_9(9m^2) = \log_9 9 + 2\log_9 m$ [M1]

$= 1 + 2n$ [A1]

(ii) $\log_3 \frac{1}{m} = \log_3 1 - \log_3 m$ [M1]

$= 0 - \frac{\log_9 m}{\frac{1}{2}}$ [M1]

$= -2n$ [A1]

(b) $2(\ln x)^2 + 3\ln\left(\frac{1}{x}\right) - 5 = 0$

Let $y = \ln x$

$2y^2 - 3y - 5 = 0$ [M1]

$(2y - 5)(y + 1) = 0$ [M1]

$y = \frac{5}{2}$ *or* $y = -1$

$\ln x = \frac{5}{2}$ *or* $\ln x = -1$ [M1]

$x = e^{\frac{5}{2}}$ *or* $x = \frac{1}{e}$ [A1]

Accept $x = 12.2$ *or* $x = 0.368$ (*to* 3 *s.f.*)

3 **(i)** When t = 5

$$f(5)=\frac{6000}{1+5999e^{-0.5(5)}}$$

$$=\frac{6000}{1+5999e^{-0.5(5)}}$$

$$=12.159\approx 12\ \textit{student}$$ [B1]

(ii) For classes to be cancelled, $f(t)\geq 3000$

$$\frac{6000}{1+5999e^{-0.5t}}\geq 3000$$ [M1]

$$2\geq 1+5999e^{-0.5t}$$

$$e^{-0.5t}\leq\frac{1}{5999}$$ [M1]

$$t\geq -2\ln\left(\frac{1}{5999}\right)=17.398$$

$\therefore$ *after* 18 *days* [A1]

4 **(a)** $x^2+x-12\geq 0$ [M1]

$(x+4)(x-3)\geq 0$ [M1]

$x\leq -4\quad or\quad x\geq 3$ [A1]

(b) $y=8-kx$

$x^2+4(8-kx)=16$ [M1]

$x^2-4kx+16=0$

For no intersection, discriminant < 0

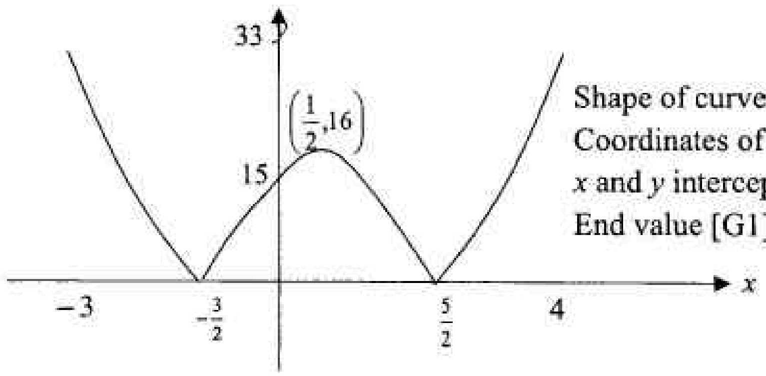
$16k^2-4(1)(16)<0$ [M1]

$k^2-4<0$

$(k-2)(k+2)<0$ [M1]

$-2<k<2$ [A1]

5 **(i)**



Shape of curve [G1]
Coordinates of max pt [G1]
x and y intercepts [G1]
End value [G1]

(ii) $16 \le m \le 33 \quad or \quad m = 0$ [B2]

6 **(a)** $LHS = (\sec\theta + \tan\theta)^2$

$= \sec^2\theta + 2\sec\theta\tan\theta + \tan^2\theta$ [M1]

$= \dfrac{1}{\cos^2\theta} + \dfrac{2\sin\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta}$ [M1]

$= \dfrac{1 + 2\sin\theta + \sin^2\theta}{1 - \sin^2\theta}$

$= \dfrac{(1+\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$ [M1]

$= \dfrac{1+\sin\theta}{1-\sin\theta}$ (proven) [A1]

(b) $\sin\left(\dfrac{\pi t}{5}\right) = \dfrac{\sqrt{3}}{2} \qquad 0 < t < 12 \quad \Rightarrow 0 < \dfrac{\pi t}{5} < \dfrac{12\pi}{5}$

$\alpha = \sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = \dfrac{\pi}{3}$ [M1]

$\dfrac{\pi t}{5} = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ or $\dfrac{7\pi}{3}$ [M1]

$t = \dfrac{5}{3}, \dfrac{10}{3}$ or $\dfrac{35}{3}$ [A1]

7 **(i)** Amplitude = 2 and Period = 120° or $\dfrac{2\pi}{3}$ [B2]

(ii) Coordinates of P (0, 4) [B1]

Since the two curves intersect at the first x-intercept for $y = 4\cos 3x$,

$\Rightarrow x = \dfrac{\pi}{6}$ [M1]

When $x = \dfrac{\pi}{6}, y = 0$ [M1]

$0 = 2\sin\left(\dfrac{\pi}{6}\right) + k$

$\Rightarrow k = -1$ [A1]

For graph of $y = 2\sin x - 1$, first maximum is at $x = \dfrac{\pi}{2}$ [M1]

When $x = \dfrac{\pi}{2}, \quad y = 1$

$\therefore$ coordinates of $Q\ (\frac{\pi}{2}, 1)$ or $(90°, 1)$ [A1]

8 **(i)** For particle at rest, v = 0

$-2t^2 + 7t + 4 = 0$

$(-2t-1)(t-4) = 0 \quad or \quad (2t+1)(t-4) = 0$ [M1]

$t = -\frac{1}{2}\,(rejected) \quad or \quad t = 4$ [A1]

(ii) For maximum velocity, $\frac{dv}{dt} = 0$ [M1]

$-4t + 7 = 0$

$t = \frac{7}{4}s$ [M1]

max $velocity = -2\left(\frac{7}{4}\right)^2 + 7\left(\frac{7}{4}\right) + 4 = \frac{81}{8} = 10\frac{1}{8}\,ms^{-1}$ [A1]

(iii) $s = \int v\,dt = -\frac{2t^3}{3} + \frac{7t^2}{2} + 4t + C$ [M1]

When t=0, s=0 $\Rightarrow C = 0$

When t=4, s = $29\frac{1}{3}$ m [M1]

When t=5, s = 24.17m [M1]

$\therefore total\ distance = 29\frac{1}{3} + \left(29\frac{1}{3} - 24.17\right) = 34.5\ m$ [A1]

9 **(a)** Since M and N are mid-points of CD and BC

$MN \parallel DB$ (Mid-point Theorem) [M1]

$\Rightarrow \angle NMC = \angle BDC = 90°$ (Corr. $\angle s\ MN \parallel DB$)

$\Rightarrow \angle MNC = \angle DBC$ (Corr. $\angle s\ MN \parallel DB$) [M1]

Given DB bisects $\angle ABC$

$\Rightarrow \angle ABD = \angle DBC = \angle MNC$ [M1]

$DB = CN$ (given)

$\Delta ABD \equiv \Delta MNC$ (AAS) (proven) [A1]

(b) (i) $\angle ADC = \angle BDA$ (common angle)
$\angle CAD = \angle ABD$ (alternate segment theorem) [M1]

$\therefore \Delta ADC$ *and* ΔBDA *are similar* (angle-angle similarity test) [A1]

(ii) $\frac{BD}{AD} = \frac{AD}{CD}$ (corr ratios of similar triangles)

$\Rightarrow BD \times CD = AD^2$ [M1]

Since AD is tangent to circle

$\angle DAE = 90°$ (tangent $\perp$ radius)

$\therefore AD^2 = DE^2 - AE^2$ (pythagoras' theorem) [M1]

$\Rightarrow BD \times CD = DE^2 - AE^2$ (proven) [A1]

10 **(a)** $y = (x-2)\sqrt{2x-1}$

$$\frac{dy}{dx} = \sqrt{2x-1} + (x-2)\left(\frac{1}{2\sqrt{2x-1}}\right)(2)$$

$$\frac{dy}{dx} = \frac{2x-1+x-2}{\sqrt{2x-1}} = \frac{3x-3}{\sqrt{2x-1}}$$ [M1]

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dx}$$

$$-3 = \frac{3x-3}{\sqrt{2x-1}}$$ [M1]

$$\sqrt{2x-1} = 1-x$$

$$2x-1 = 1-2x+x^2$$

$$x^2-4x+2=0$$ [M1]

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(2)}}{2}$$ [M1]

$$x = 2 \pm \sqrt{2}$$

Therefore, $x = 2-\sqrt{2}$ since $\frac{dy}{dx} < 0$ [A1]

(b) $\cos x = \sin 2x$ [M1]

$\cos x = 2\sin x \cos x$

$\cos x(2\sin x-1)=0$

$\Rightarrow x=\dfrac{\pi}{6} \quad or \quad \dfrac{\pi}{2}$ [M1]

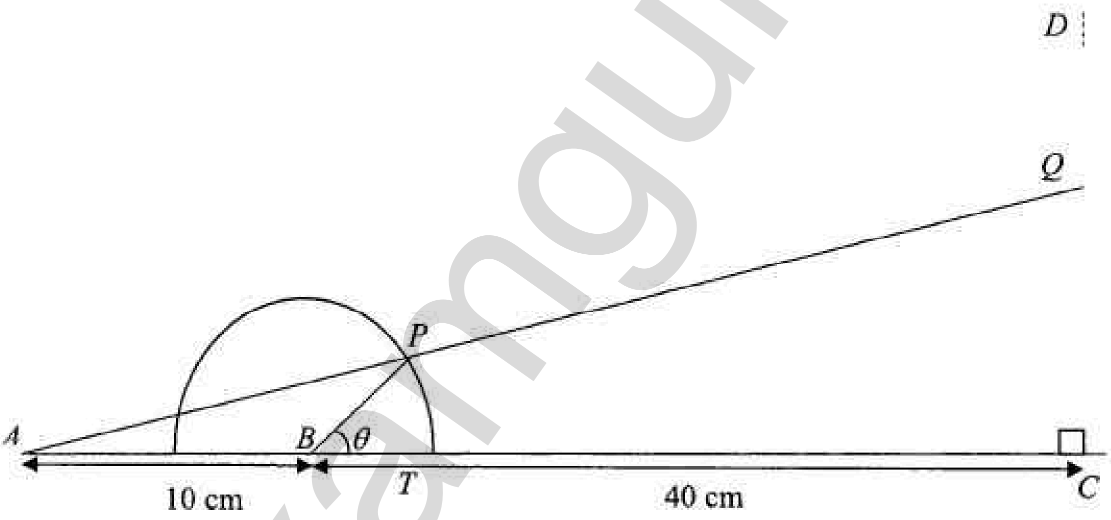
$Area=\int_0^{\frac{\pi}{6}}\sin 2x dx+\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\cos x dx$ [M1]

$=\left[\dfrac{-\cos 2x}{2}\right]_0^{\frac{\pi}{6}}+\left[\sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ [M1]

$=\left[-\dfrac{1}{4}+\dfrac{1}{2}\right]+\left[1-\dfrac{1}{2}\right]$

$=\dfrac{3}{4}\,units^2$ [A1]

11 (a)



From the diagram, PT is perpendicular to AC

ΔAPT and ΔAQC are similar (angle – angle similarity test) [M1]

$\dfrac{CQ}{50}=\dfrac{6\sin\theta}{10+6\cos\theta}$ (corr ratios of similar triangles) [M1]

$CQ=\dfrac{150\sin\theta}{5+3\cos\theta}$ (shown) [A1]

(b) $\dfrac{d}{d\theta}(CQ)=\dfrac{(5+3\cos\theta)(150\cos\theta)-(-3\sin\theta)(150\sin\theta)}{(5+3\cos\theta)^2}$ [M1]

$$= \frac{750\cos\theta + 450}{(5+3\cos\theta)^2}$$ [M1]

For maximum CQ,

$$\frac{d}{d\theta}(CQ) = \frac{750\cos\theta + 450}{(5+3\cos\theta)^2} = 0$$

$750\cos\theta + 450 = 0$

$\cos\theta = -\frac{3}{5}$ [M1]

[A1]

$\theta = 2.21\,rad \quad (to\ 3\ s.f.)$

θ	2.21^-	2.21	2.21^+
$\frac{d}{d\theta}(CQ)$	+	0	-

[A1]

$\therefore when\ \theta = 2.21\,rad, CQ\ is$ max.

Name: **Register Number:** **Class:**

南僑中學

NAN CHIAU HIGH SCHOOL

Preliminary Examination (3) 2016
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS **4047/02**
PAPER 2 **16 September 2016, Friday**

Additional Materials : Writing Papers (8 sheets) **2 hours 30 minutes**
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on the separate writing papers provided.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together. Tie your answer script into 2 separate bundles such as first bundle consists of question1 to 6 and second bundle consists of question 7 to 11.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

Setter: Mdm Chua Seow Ling

This paper consists of **5** printed pages including the coverpage.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$cosec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer ALL Questions

1. The roots of the quadratic equation $3x^2 + \frac{27}{4} = 3x$ are α^2 and β^2.

   (i) Find the value of $\alpha + \beta$ and of $\alpha\beta$ where α and β are both negative. [5]

   (ii) Hence find the quadratic equation whose roots are α^3 and β^3. [4]

2. Given $f(x) = 2 - 24\sin x \cos x$ and $g(x) = 10(1 + \cos^2 x)$.

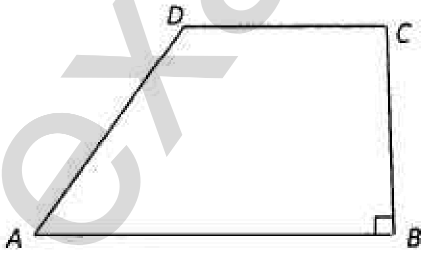
   (i) Express the sum of $f(x)$ and $g(x)$ in the form $R\cos(2x + \alpha) + q$ where R and q are constants and $R > 0$, $0 < \alpha < \frac{\pi}{2}$. [5]

   (ii) Hence find the minimum value of $\frac{2}{f(x) + g(x)}$ and the corresponding values of x for $0 < x < 2\pi$. [3]

3. **(i)** Show $\frac{d}{dx}\ln(\tan^2 3x) = 12\cos ec6x$. [4]

   (ii) Hence integrate $\frac{1}{\sin 6x} + \frac{1}{3e^{2-3x}}$ with respect to x. [4]

4. The diagram shows a right-angled trapezium $ABCD$ such that $2AB = 3CD$ and AB is parallel to DC. Given the height BC of the trapezium is $(3 - \sqrt{3})$cm and area of the trapezium is $(2 + 3\sqrt{3})$cm^2.



   Find length CD in the form $(a + b\sqrt{3})$cm, where a and b are rational numbers. [5]

5. **(i)** The sum of the second and third term of the expansion of $(1 + kx)^n$ is $60x + 1740x^2$. Find the value of k and of n. [5]

   (ii) Hence write down the first 4 terms in the expansion of $(1 + kx)^n$ in ascending powers of x. [2]

   (iii) Hence determine the coefficient of a^3 in the expansion of $(1 + k(a - 2a^2))^n$. [3]

6. An experiment to find the constant acceleration, a m/s^2, of an electric toy car moving in one direction, requires students to measure the speed, v m/s from the speedometer when distance, s m varies. The table below shows the experimental values of v and s, which are connected by the equation $v = \sqrt{e^p + 2as}$, where p is a constant.

s	$4\frac{1}{6}$	$17\frac{1}{2}$	$37\frac{1}{2}$	80
v	3	5	6	10

(i) Plot v^2 against s and draw a straight line graph. Hence determine which value of v, in the table above, is the incorrect recording. Using your graph to estimate the correct v value. [4]

(ii) Use your graph to estimate the value of a and of p. [3]

(iii) Explain what does the value of e^p represents. [1]

(iv) By drawing a suitable straight line on your graph, solve $s = \left(\frac{120 - 2e^p}{4a + 3}\right)$. [2]

Start on a fresh sheet of writing paper and tie answer script from question 7 to 11 together.

7. **(i)** Explain whether the curve $y = 4 - 3e^{2x}$ has any stationary point. [2]

(ii) Sketch the graph $y = 4 - 3e^{2x}$ indicating clearly the asymptote and x and y-intercepts. [3]

(iii) Hence solve $2x = \ln\left(1 - \frac{4}{3}x\right)$ by inserting a straight line on the same graph in part **(ii)**. [3]

8. **(i)** Factorise $8x^3 + 4x^2 - 2x - 1$ completely. [3]

(ii) Hence express $\frac{2x+2}{(8x^3 + 4x^2 - 2x - 1)}$ in partial fractions. [4]

(iii) The polynomial $8x^3 + 4x^2 - 2x - 1$ leaves a remainder of $(px + q)$ when divided by $(x^2 - 1)$. Find the value of p and of q. [4]

9. Given the curve $y=\frac{2}{3}x^{-\frac{1}{2}}$ and $y=\frac{8}{27}x^{\frac{3}{2}}$.

(i) Sketch the two graphs on the same diagram for $x>0$ and label the graphs clearly. [2]

(ii) Calculate the coordinates of the point of intersection of the two graphs drawn in **(i)**. [3]

10. The gradient function of a curve $y=\mathrm{f}(x)$ is given by $m+n(3x-2)^3$. A point *P* lies on the curve and its *x*-coordinate is 2. The equation of the normal to the curve at *P* is given by $37y=9x-129$. The curve has a turning point at *Q* whose *x*-coordinate is $\frac{5}{3}$.

(i) Show that the value of *m* is 3 and *n* is $-\frac{1}{9}$. [3]

(ii) Find the equation of the curve. [4]

(iii) Find the area of triangle *PQR* where *R* is the point the curve intersect the *y*-axis. [4]

11. Given that a circle C_1 passes through the point $A(2,0)$, $B(5,\ 1)$ and $C(6,0)$.

(i) Show that the coordinates of centre *D* of the circle C_1 is $(4,-1)$ and hence find the radius of the circle. [6]

(ii) Find the equation of the circle C_1 in standard form. [1]

(iii) Given 2 tangents are drawn from a point E to touch the circle at point *B* and *C*. Find the coordinates of point *E*. [5]

(iv) Explain why a circle can be drawn to pass through the points *B*, *C*, *D* and *E*. Hence find the coordinate of the centre of this circle. [3]

End of Paper

Answers

1i) $\alpha\beta = \frac{3}{2}$ or $-\frac{3}{2}$ (rej)
$(\alpha+\beta) = 2$ (rej) or -2

1ii) $x^2 - x + \frac{27}{8} = 0$

2i) $f(x) + g(x) = 13\cos(2x + 1.18) + 17$

2ii) min $= \frac{1}{15}$, $x = 2.55,\ \ 5.70$

3ii) $\frac{1}{12}\ln(\tan^2 3x) + \frac{1}{9}e^{3x-2} + c$ OR
$\frac{1}{6}\ln(\tan 3x) + \frac{1}{9}e^{3x-2} + c$

4) $CD = 2 + \frac{22}{15}\sqrt{3}$

5i) $k = 2$
$n = 30$

5ii) $1 + 60x + 1740x^2 + 32480x^3 + \ldots$

5iii) coeff. of $a^3 = 25520$

6i)

S	$4\frac{1}{6}$	$17\frac{1}{2}$	$37\frac{1}{2}$	80
V^2	9	25	36	100

6ii) incorrect $v = 6$ m/s
corrected $v = 7$

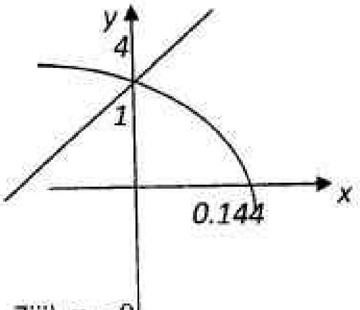
6ii) $p = \ln 4$ or 1.39
$a = 0.605$

6iii) e^p represents the square of initial speed or square of initial velocity

6iv) $s = 20.5$ or $21m$

7i) $\frac{dy}{dx} = -6e^{2x}$
$\frac{dy}{dx} < 0,\ \frac{dy}{dx} \neq 0,$ no stationary point

7ii)



7iii) $x = 0$

8i) $(2x-1)(4x^2+4x+1)=(2x-1)(2x+1)^2$

8ii) $$\frac{2x+2}{(8x^3+4x^2-2x-1)}=\frac{3}{4(2x-1)}-\frac{3}{4(2x+1)}-\frac{1}{2(2x+1)^2}$$

8iii) $q=3$ and $p=6$

9ii) (1.5, 0.544) or $\left(\frac{3}{2}, \frac{2}{9}\sqrt{6}\right)$

10iii) $y=3x-\frac{1}{108}(3x-2)^4-\frac{179}{27}$ 10iii) $\frac{5}{4}$

$R=\sqrt{5}$

$(x-4)^2+(y+1)^2=5$

$E\left(\frac{17}{3}, \frac{2}{3}\right)$

11) Since $\angle DBE = \angle DCE = 90°$

(tangent perpendicular to radius).

$\therefore$ A circle with diameter DE ($\angle$ in semicircle).

$$\text{Centre}\left(\frac{4+\frac{17}{3}}{2}, \frac{-1+\frac{2}{3}}{2}\right)=\left(\frac{29}{6}, -\frac{1}{6}\right)$$

NCHS Prelim Exam (3) 2016 Additional Mathematics Paper 2 – Secondary 4 Express

Qn No	Suggested Solutions	Qn No	Suggested Solutions
1i	$3x^2+\frac{27}{4}=3x$ $3x^2-3x+\frac{27}{4}=0$ $x^2-x+\frac{9}{4}=0$ $\alpha^2+\beta^2=1$ —— M1 $(\alpha+\beta)^2-2\alpha\beta=1$ $(\alpha\beta)^2=\frac{9}{4}$ —— M1 $\alpha\beta=\frac{3}{2}$ or $-\frac{3}{2}$ (rej) —— A1 $(\alpha+\beta)^2-2\left(\frac{3}{2}\right)=1$ —— M1 $(\alpha+\beta)^2=4$ $(\alpha+\beta)=2$ (rej) or -2 —— A1	2i	$f(x)+g(x)=2-24\sin x\cos x+10+10\cos^2 x$ $=12-12\sin 2x+10\left(\frac{\cos 2x+1}{2}\right)$ —— M1 $=12-12\sin 2x+5\cos 2x+5$ $==17+5\cos 2x-12\sin 2x$ —— M1 $R=\sqrt{5^2+12^2}=13$ —— M1 $\tan\alpha=\frac{12}{5},\quad \alpha=1.176$ —— M1 $f(x)+g(x)=13\cos(2x+1.18)+17$ —— A1
1ii	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ $=(-2)^3-3\left(\frac{3}{2}\right)(-2)$ —— M1 $=1$ —— M1 $(\alpha\beta)^3=\left(\frac{3}{2}\right)^3$ —— M1 $=\frac{27}{8}$ $x^2-x+\frac{27}{8}=0$ —— A1	2ii	$\min\left(\frac{2}{f(x)+g(x)}\right)=\frac{2}{13\cos(2x+1.176)+17}$ $=\frac{2}{13+17}=\frac{1}{15}$ —— M1 $\cos(2x+1.176)=1$ —— M1 *basic angle* $=0$ $(2x+1.176)=0$ (rej), 2π, 4π $x=2.55,\ 5.70$ —— A1
3ii	$\int\frac{1}{\sin 6x}+\frac{1}{3e^{2-3x}}dx=\int\frac{1}{\sin 6x}+\frac{1}{3}e^{3x-2}dx$ $=\frac{1}{12}\ln(\tan^2 3x)+\frac{1}{3(3)}e^{3x-2}+c$ —— M2 $=\frac{1}{12}\ln(\tan^2 3x)+\frac{1}{9}e^{3x-2}+c$ OR $=\frac{1}{6}\ln(\tan 3x)+\frac{1}{9}e^{3x-2}+c$ —— A2	3i	$\frac{d}{dx}\ln(\tan^2 3x)=\frac{d}{dx}2\ln(\tan 3x)$ $=\frac{2(3)\sec^2 3x}{\tan 3x}$ —— M1 $=\frac{6\sec^2 3x}{\tan 3x}$ $=\frac{6(\cos 3x)}{\cos^2 3x\sin 3x}$ $=\frac{6}{\cos 3x\ \sin 3x}$ —— M1 $=\frac{12}{\sin 6x}$ —— M1 $=12\cos ec6x$ *(shown)* —— A1

4

Let AB = 3x and CD = 2x

$\frac{1}{2}(3x+2x)(3-\sqrt{3}) = 2+3\sqrt{3}$ — M1

$\frac{5x}{2} = \frac{2+3\sqrt{3}}{3-\sqrt{3}}$

$\frac{5x}{2} = \frac{2+3\sqrt{3}}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}}$ — M1

$\frac{5x}{2} = \frac{6+2\sqrt{3}+9\sqrt{3}+3(3)}{9-3}$

$\frac{5x}{2} = \frac{15+11\sqrt{3}}{6}$ — M1

$x = 1+\frac{11}{15}\sqrt{3}$ — M1

$CD = 2+\frac{22}{15}\sqrt{3}$ — A1

6i

B1

S	$4\frac{1}{6}$	$17\frac{1}{2}$	$37\frac{1}{2}$	80
V^2	9	25	36	100

Straight line graph of correct axes — B1

incorrect v = 6 m/s — M1

$v^2 = 49$

corrected $v = 7$ — A1

6ii

gradient $= \frac{80\text{-}28}{63\text{-}20} = 1.209$ — M1

$v^2 = e^p + 2as$

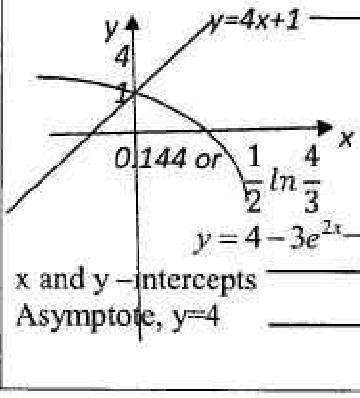
$e^p = 4$

$p = \ln 4$ or 1.39 — B1

$2a = 1.209$

$a = 0.605$ — A1

7ii



y=4x+1 — M1

$y = 4-3e^{2x}$ — B1

x and y – intercepts — B1

Asymptote, y=4 — B1

5i

$(1+kx)^n = \binom{n}{1}(1)(kx)+\binom{n}{2}(1)(kx)^2+..$ — M

$= nkx + \frac{n(n-1)}{2}k^2x^2+...$

$= 60x+1740x^2+...$

$nk = 60$ — M1

$\frac{n(n-1)}{2}k^2 = 1740$ — M1

$n^2k^2 - nk^2 = 3480$

$60^2 - 60k = 3480$

k = 2 — A1

$n = 30$ — A1

5ii

$(1+2x)^{30} = 1+60x+1740x^2+\binom{30}{3}(1)(kx)^3+$

$= 1+60x+1740x^2+32480x^3+...$ — B2

5iii

$(1+k(a-2a^2))^n = 1+60(a-2a^2)$

$+1740(a-2a^2)^2+32480(a-2a^2)^3+...$ — M

$= 1740(2)(a)(-2a^2)+32480a^3$

$= -6960a^3+32480a^3+.$ — M1

$= 25520a^3+...$

coeff. of $a^3 = 25520$ — A1

6iii

e^p represents the square of initial speed or square of initial velocity — B1

6iv

$s = \frac{120\text{-}2e^p}{4a+3}$

$s(4a+3) = 120-2e^p$

$2e^p + 4as = 120-3s$

$e^p + 2as = 60-1.5s$

$v^2 = 60-1.5s$ Draw the line — M1

$s = 20.5$ or $21m$ — A1

7i

$\frac{dy}{dx} = -6e^{2x}$ — M1

$\frac{dy}{dx} < 0,\ \frac{dy}{dx} \neq 0,$ no stationary point — A1

7iii

$2x = \ln\left(1-\frac{4}{3}x\right)$

$e^{2x} = 1-\frac{4}{3}x$

$3e^{2x} = 3-4x$

$4x = 3-3e^{2x}$

$4x+1 = 4-3e^{2x}$

$y = 4x+1$ (Draw this striaght line) — M1

$x = 0$ — A1

8i

$8x^3+4x^2-2x-1$

by trialand error, let x = $\frac{1}{2}$

$8\left(\frac{1}{2}\right)^3+4\left(\frac{1}{2}\right)^2-2\left(\frac{1}{2}\right)-1=0$ — M1

$\therefore (2x-1)$ is a factor

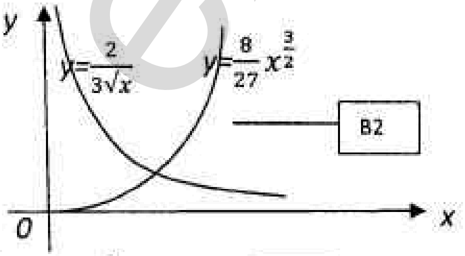
$$\begin{array}{r} 4x^2+4x+1 \\ 2x-1 \;\big)\; 8x^3+4x^2-2x-1 \\ -\left(8x^3-4x^2\right) \\ \hline 8x^2-2x-1 \\ -\left(8x^2-4x\right) \\ \hline 2x-1 \\ -(2x-1) \\ \hline 0 \end{array}$$

M1

A1

$(2x-1)(4x^2+4x+1) = (2x-1)(2x+1)^2$

9i



B2

9ii

$\frac{2}{3}x^{-\frac{1}{2}} = \frac{8}{27}x^{\frac{3}{2}}$ — M1

$x^2 = \frac{9}{4}$ thus $x = \frac{3}{2}$ or $-\frac{3}{2}$ (rej) — M1

(1.5, 0.544) or $\left(\frac{3}{2}, \frac{2}{9}\sqrt{6}\right)$ — A1

8ii

$\frac{2x+2}{(8x^3+4x^2-2x-1)} = \frac{2x+2}{(2x-1)(2x+1)^2}$

Let $\frac{2x+2}{(2x-1)(2x+1)^2} = \frac{A}{2x-1}+\frac{B}{2x+1}+\frac{C}{(2x+1)^2}$ — M1

$2x+2 = A(2x+1)^2+B(2x-1)(2x+1)+C(2x-$

Let $x=\frac{1}{2}$, $2(\frac{1}{2})+2 = A(2(\frac{1}{2})+1)^2$

$A=\frac{3}{4}$ — M1

Let $x=-\frac{1}{2}$, $2\left(-\frac{1}{2}\right)+2 = C\left(2\left(-\frac{1}{2}\right)-1\right)$

$C=-\frac{1}{2}$

Let $x=0$, $2 = A-B-C$

$B=-\frac{3}{4}$ — M1

$\frac{2x+2}{(8x^3+4x^2-2x-1)} = \frac{3}{4(2x-1)}-\frac{3}{4(2x+1)}-\frac{1}{2(2x+1)^2}$ — A1

8iii

Let $x^2-1=0$, $x=1$ or $x=-1$

$8(1)^3+4(1)^2-2(1)-1 = p+q$

$p+q=9$ — M1

$8(-1)^3+4(-1)^2-2(-1)-1 = -p+q$

$q-p=-3$ — M1

$q=3$ and $p=6$ — A2

OR

$$\begin{array}{r} 8x+4 \\ x^2-1 \;\big)\; 8x^3+4x^2-2x-1 \\ -\left(8x^3-8x\right) \\ \hline 4x^2+6x-1 \\ -\left(4x^2-4\right) \\ \hline 6x+3 \end{array}$$

M2

p = 6, q = 3 — A2

10i	$f'(x)=m+n(3x-2)^3$ $m+n\left(3\left(\frac{5}{3}\right)-2\right)^3=0$ $m+n(27)=0$ $m=-27n$ (eqn 1) M1 $y=\frac{9}{37}x-\frac{129}{37}$ gradient of tangent $=-\frac{37}{9}$ $m+n(3(2)-2)^3=-\frac{37}{9}$ $m+64n=-\frac{37}{9}$ (eqn 2) M1 $n=-\frac{1}{9},\ m=3$ (shown) A1
10ii	$f'(x)=3-\frac{1}{9}(3x-2)^3$ $y=3x-\frac{1}{9}\frac{(3x-2)^4}{4(3)}+c$ M1 $y=3x-\frac{1}{108}(3x-2)^4+c$ $37y=9(2)-129,\quad y=-3$ M1 $-3=3(2)-\frac{1}{108}(3(2)-2)^4+c$ $c=-\frac{179}{27}$ M1 $y=3x-\frac{1}{108}(3x-2)^4-\frac{179}{27}$ (eqn of curve) A1
10iii	$x=0,\ y=\frac{-(-2)^4}{108}-\frac{179}{27}=-\frac{61}{9}$ M1 $R\left(0,-\frac{61}{9}\right)$ $y=3\left(\frac{5}{3}\right)-\frac{1}{108}\left(3\left(\frac{5}{3}\right)-2\right)^4-\frac{179}{27}$ $=-\frac{257}{108}$ M1 $Q\left(\frac{5}{3},-\frac{257}{108}\right)$
10iii	$Area=\frac{1}{2}\begin{vmatrix}2 & \frac{5}{3} & 0 & 2\\ -3 & -\frac{257}{108} & -\frac{61}{9} & -3\end{vmatrix}$ M1 $=\frac{1}{2}\left(-\frac{289}{18}-\left(-\frac{167}{9}\right)\right)$ $=\frac{5}{4}$ A1
11i	$x_D=\frac{6+2}{2}=4$ M1 M1 $\sqrt{(y_D-0)^2+(4-2)^2}=\sqrt{(5-4)^2+(1-y_D)^2}$ A1 $y_D=-1$, D(4, -1) (shown) A1 $R=\sqrt{(5-4)^2+(1-(-1))^2}=\sqrt{5}$ A1 M1
11ii	$(x-4)^2+(y+1)^2=5$ B1
11iii	$M_{DB}=\frac{1-(-1)}{5-4}=2,\quad M_{BE}=-\frac{1}{2}$ M1 $y=-\frac{1}{2}x+C$ $1=-\frac{1}{2}(5)+C$ $C=3.5,\quad y=-\frac{1}{2}x+\frac{7}{2}$ (equation BE) M1 $M_{DC}=\frac{-1-0}{4-6}=\frac{1}{2},\quad M_{CE}=-2$ M1 $y=-2x+C$ $0=-2(6)+C$ $C=12,\quad y=-2x+12$ (equation CE) M1 $-\frac{1}{2}x+\frac{7}{2}=-2x+12,\ E\left(\frac{17}{3},\frac{2}{3}\right)$ A1
11iv	Since $\angle DBE=\angle DCE=90°$ M1 (tangent perpendicular to radius). $\therefore$ A circle with diameter DE ($\angle$ in semicircle). A1 Centre $\left(\frac{4+\frac{17}{3}}{2},\frac{-1+\frac{2}{3}}{2}\right)=\left(\frac{29}{6},-\frac{1}{6}\right)$ B1 *** End of Paper ***

O Level Centre/ Index Number /	Class	Name

新加坡海星中学
MARIS STELLA HIGH SCHOOL
PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

ADDITIONAL MATHEMATICS
Paper 2

4047/2
18 August 2016
2 hours 30 minutes

Additional Materials: Answer Paper (7 sheets)
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

For Examiner's Use
/100

This document consists of **6** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = (k-2)e^{3x}$.

(i) For y to be an increasing function of x, what condition must be applied to the constant k? [2]

(ii) Given that $P(0,3)$ is a point on the curve and the gradient of the tangent to the curve at P is 4, find an expression for $\mathrm{f}(x)$. [4]

2 **(i)** Differentiate $\ln(\sin x)$ with respect to x. [2]

(ii) Show that $\frac{\mathrm{d}}{\mathrm{d}x}(x\cot x) = \cot x - x\cos ec^2 x$. [3]

(iii) Using the results from parts **(i)** and **(ii)**, find $\int x\cos ec^2 x \; \mathrm{d}x$. [3]

3 The equation of a curve is $y = 6x^{\frac{2}{3}}$.

(i) Sketch the curve $y = 6x^{\frac{2}{3}}$. [2]

(ii) The point P lies on the curve such that the gradient of the normal to the curve is $-\frac{1}{2}$. The normal at P meets the x-axis at A and the y-axis at B. Find the ratio AP:PB. [6]

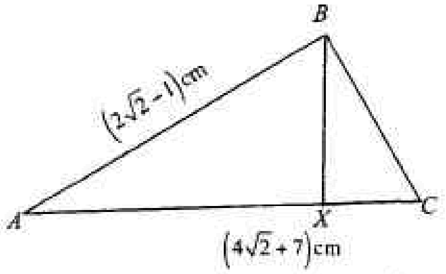
4 **(i)** Given that n is a positive integer, write down, without simplifying, the $(r+1)$th term in the binomial expansion of $\left(\frac{x}{2} - \frac{k}{x^2}\right)^n$. [1]

(ii) The binomial expansion of $\left(\frac{x}{2} - \frac{k}{x^2}\right)^n$ has a constant term. Show that n is a multiple of 3. [1]

(iii) Given that $n = 9$ and that the constant term is $-\frac{2625}{2}$, find the value of k. [3]

(iv) Using the value of k found in part (iii), find the term independent of x in the expansion of $(2 + x^3)\left(\frac{x}{2} - \frac{k}{x^2}\right)^9$. [3]

5



The diagram shows a triangle ABC such that $AB = (2\sqrt{2}-1)$ cm and $AC = (4\sqrt{2}+7)$ cm. The point X lies on AC such that $\angle AXB = \angle ABC$.

(i) Show that $AX \times AC = AB^2$. [2]

(ii) Find an expression for AX in the form $\frac{1}{17}\left(a+b\sqrt{2}\right)$. [4]

(iii) Given that $BC^2 = 72+60\sqrt{2}$, show that $\angle AXB = 90°$. [3]

6 The equation of a curve is $y = \frac{(2x-5)^2}{x-1}$, where $x \neq 1$.

(i) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and obtain the coordinates of the stationary points of the curve. [5]

(ii) Find an expression for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and show that its can be expressed in the form $\frac{k}{(x-1)^3}$. Hence, or otherwise, determine the nature of these stationary points. [4]

7 The highest point on a circle C_1 is $(2,8)$. The line T, $3y = 42-4x$, is a tangent to C_1 at the point $(6,6)$.

(i) Find the coordinates of the centre of C_1. [4]

(ii) Find the equation of C_1. [2]

The circle C_2 is a reflection of C_1 in the line T.

(iii) Find the equation of C_2. [3]

8 **(i)** Show that $3x-1$ is a factor of $3x^3+11x^2+8x-4$ and hence factorise completely the cubic polynomial $3x^3+11x^2+8x-4$. [3]

(ii) Express $\dfrac{5x^2-2x+11}{3x^3+11x^2+8x-4}$ as the sum of 3 partial fractions. [4]

(iii) Hence find $\displaystyle\int \frac{5x^2-2x+11}{3x^3+11x^2+8x-4}\,dx$. [3]

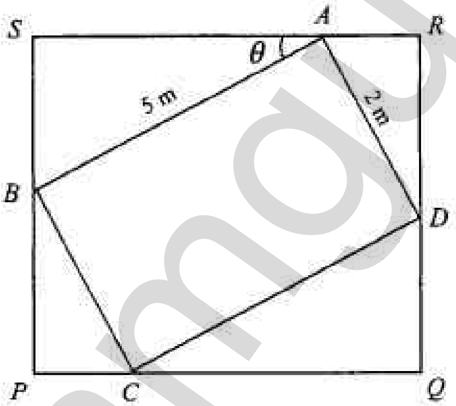
9 The roots of the quadratic equation $4x^2+3x+1=0$ are $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$.

(i) Find the value of $\alpha^2+\beta^2$. [4]

(iii) Show that the value of $\alpha^3+\beta^3$ is 9. [2]

(iii) Find a quadratic equation whose roots are $\alpha^2+\beta$ and $\alpha+\beta^2$. [4]

10



The diagram shows a rug in the shape of a rectangle *ABCD* such that *AB* = 5 m and *AD* = 2 m. The rug is placed inside a rectangular function room *PQRS* such that each of the corners *A*, *B*, *C* and *D* touches the sides of the room *SR*, *SP*, *PQ* and *QR* respectively. The side of the rug *AB* makes an acute angle θ with the side of the room *SR*. The lengths of the room *SR* and *SP* are *L* m and *W* m respectively.

(a) **(i)** Find the values of the integers a and b for which

$$L = a\cos\theta + b\sin\theta.$$ [2]

(ii) Obtain a similar expression for *W*. [1]

(iii) Hence find the perimeter of the room *PQRS* in exact form if *PQRS* is a square. [3]

(b) Using the values of a and b found in (a) part (i),

(i) express *L* in the form $R\cos(\theta-\alpha)$, $R>0$ and $0° < \alpha < 90°$. [2]

(ii) find the value of θ if $L=4$ and the area of the rectangular function room *PQRS*. [4]

11 The amount of expenditure, \$$y$, incurred by a textile company is related to \$$x$, the amount of sales generated. The variables x and y are related by the formula $y=10^k x^a$, where a and k are constants. The following table shows corresponding values of x and y.

x (\$)	6	35	234	1995	6310
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- End of Paper -

O Level Centre/ Index Number /	Class	Name **SOLUTIONS**

新加坡海星中学

MARIS STELLA HIGH SCHOOL
PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

ADDITIONAL MATHEMATICS **4047/2**
Paper 2 **18 August 2016**
2 hours 30 minutes

Additional Materials: Answer Paper (7 sheets)

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

For Examiner's Use
100

This document consists of 6 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = (k-2)e^{3x}$.

(i) For y to be an increasing function of x, what condition must be applied to the constant k? [2]

Solution:

For y is an increasing function of x,

$(k-2)e^{3x} > 0$. [M1]

Since $e^{3x} > 0$, $k-2 > 0$

$\therefore k > 2$. [A1]

(ii) Given that $P(0,3)$ is a point on the curve and the gradient of the tangent to the curve at P is 4, find an expression for $\mathrm{f}(x)$. [4]

Solution:

$\mathrm{f}'(x) = (k-2)e^{3x}$

Subst $x = 0$ and $\mathrm{f}'(x) = 4$,

$4 = k - 2$

$k = 6$ [A1]

$\mathrm{f}(x) = \dfrac{(k-2)e^{3x}}{3} + c$ [M1]

Subst $x = 0$ and $f(x) = 3$,

$3 = \dfrac{4}{3} + c$

$c = 1\frac{2}{3}$ [A1]

$\mathrm{f}(x) = \frac{4}{3}e^{3x} + \frac{5}{3}$ [A1]

2 **(i)** Differentiate $\ln(\sin x)$ with respect to x. [2]

Solution:

$$\frac{d}{dx}(\ln(\sin x)) = \frac{\cos x}{\sin x} \qquad \text{[M1]}$$

$$= \cot x \qquad \text{[A1]}$$

(ii) Show that $\frac{\mathrm{d}}{\mathrm{d}x} x\cot x = \cot x - x\cos ec^2 x$. [3]

Solution:

$$\frac{d}{dx} x\cot x = \frac{d}{dx}\frac{x}{\tan x}$$

$$= \frac{\tan x - x\sec^2 x}{\tan^2 x} \qquad \text{[M1]}$$

$$= \cot x - x\left(\frac{1}{\cos^2 x}\right)\left(\frac{\cos^2 x}{\sin^2 x}\right) \qquad \text{[M1]}$$

$$= \cot x - x\cos ec^2 x \qquad \text{[A1]}$$

(iii) Using the results from parts **(i)** and **(ii)**, find $\int x\cos ec^2 x \ \mathrm{d}x$. [3]

Solution:

$$\int\left(\cot x - x\cos ec^2 x\right) \mathrm{d}x = x\cot x + c \qquad \text{[M1]}$$

$$\int \cot x \ \mathrm{d}x - \int x\cos ec^2 x \ \mathrm{d}x = x\cot x + c$$

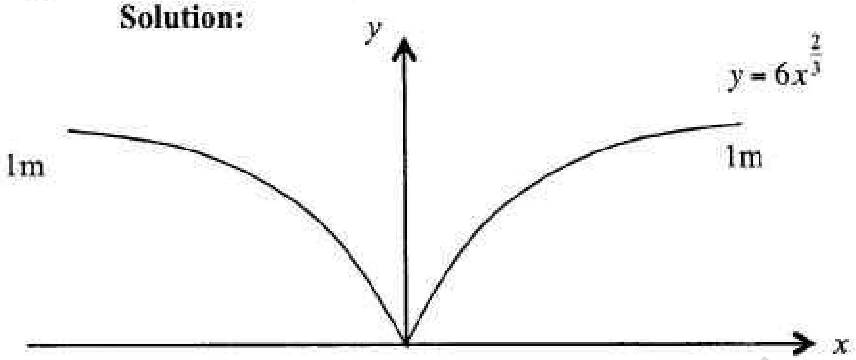
$$\left[\ln(\sin x) + c\right] - \int x\cos ec^2 x \ \mathrm{d}x = x\cot x + c \qquad \text{[M1]}$$

$$\int x\cos ec^2 x \ \mathrm{d}x = \ln(\sin x) - x\cot x + c \qquad \text{[A1]}$$

3 The equation of a curve is $y = 6x^{\frac{2}{3}}$

(i) Sketch the curve $y = 6x^{\frac{2}{3}}$. [2]

Solution:



(ii) The point P lies on the curve such that the gradient of the normal to the curve is $-\frac{1}{2}$. The normal at P meets the x-axis at A and the y-axis at B. Find the ratio AP:PB. [6]

Solution:

$y = 6x^{\frac{2}{3}}$

$\frac{dy}{dx} = 4x^{-\frac{1}{3}}$ [M1]

Gradient of tangent at P $= -1 \div \left(-\frac{1}{2}\right)$

$= 2$

When $\frac{dy}{dx} = 2$, $4x^{-\frac{1}{3}} = 2$ [M1]

$x^{-\frac{1}{3}} = \frac{1}{2}$

$x^{\frac{1}{3}} = 2$

$x = 8$ [A1]

$y = 6(8)^{\frac{2}{3}}$

$= 24$ [A1]

Equation of normal, $y - 24 = -\frac{1}{2}(x - 8)$

$y = -\frac{1}{2}x + 28$ [M1]

$A(56,0)$, $P(8,24)$, $B(0,28)$

$AP : PB = 24 - 0 : 28 - 24$

$= 24 : 4$

$= 6 : 1$ [A1]

4 **(i)** Given that n is a positive integer, write down, without simplifying, the $(r+1)$th term in the binomial expansion of $\left(\frac{x}{2}-\frac{k}{x^2}\right)^n$. [1]

Solution:

$(r+1)\text{th term} = \binom{n}{r}\left(\frac{x}{2}\right)^{n-r}\left(-\frac{k}{x^2}\right)^r$ [B1]

(ii) The binomial expansion of $\left(\frac{x}{2}-\frac{k}{x^2}\right)^n$ has a constant term. Show that n is a multiple of 3. [1]

Solution:

For constant term, $n-r-2r=0$

$n=3r$

Since r is an integer and $n=3r$, n is a multiple of 3. [A1]

(iii) Given that $n=9$ and that the constant term is $-\frac{2625}{2}$, find the value of k. [3]

Solution:

Constant term = $-\frac{2625}{2}$

$\binom{9}{3}\left(\frac{1}{2}\right)^{9-3}(-k)^3 = -\frac{2625}{2}$ [M1]

$84\left(\frac{1}{64}\right)(-k^3) = -\frac{2625}{2}$

$k^3 = 1000$ [M1]

$k = 10$ [A1]

(iv) Using the value of k found in part (iii), find the term independent of x in the expansion of $(2+x^3)\left(\frac{x}{2}-\frac{k}{x^2}\right)^9$. [3]

Solution:

Let $9-3r=-3$

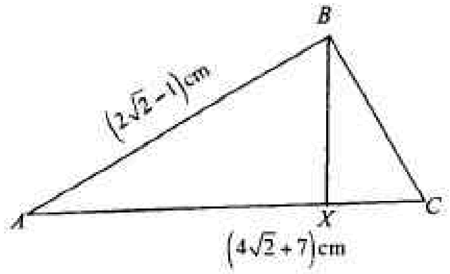
$r=4$

Constant term in the expansion of $(2+x^3)\left(\frac{x}{2}-\frac{10}{x^2}\right)^9$

$= 2\left(-\frac{2625}{2}\right)+x^3\binom{9}{4}\left(\frac{x}{2}\right)^5\left(-\frac{10}{x^2}\right)^4$ [M2]

$= 36750$ [A1]

5



The diagram shows a triangle ABC such that $AB = (2\sqrt{2}-1)$ cm and $AC = (4\sqrt{2}+7)$ cm. The point X lies on AC such that $\angle AXB = \angle ABC$.

(i) Show that $AX \times AC = AB^2$. [2]

Solution:

$\angle AXB = \angle ABC$ (given)

$\angle XAB = \angle BAC$ (common $\angle$)

ΔAXB is similar to ΔABC.

$\frac{AX}{AB} = \frac{AB}{AC}$ [M1]

$\therefore AX \times AC = AB^2$ [A1]

(ii) Find an expression for AX in the form $\frac{1}{17}\left(a+b\sqrt{2}\right)$. [4]

Solution:

$AX \times AC = AB^2$

$AX = \frac{AB^2}{AC}$

$= \frac{\left[2\sqrt{2}-1\right]^2}{7+4\sqrt{2}}$ [M1]

$= \frac{\left(2\sqrt{2}\right)^2 - 4\sqrt{2}+1}{7+4\sqrt{2}}$

$= \frac{9-4\sqrt{2}}{7+4\sqrt{2}} \times \frac{7-4\sqrt{2}}{7-4\sqrt{2}}$ [M1]

$= \frac{63-36\sqrt{2}-28\sqrt{2}+32}{17}$ [M1]

$= \frac{1}{17}\left(95-64\sqrt{2}\right)$ [A1]

(iii) Given that $BC^2 = 72+60\sqrt{2}$, show that $\angle AXB = 90°$. [3]

Solution:

$AB^2 + BC^2 = \left[2\sqrt{2}-1\right]^2 + 72 + 60\sqrt{2}$

$= 8 - 4\sqrt{2} + 1 + 72 + 60\sqrt{2}$

$= 81 + 56\sqrt{2}$ [M1]

$$AC^2 = \left[4\sqrt{2}+7\right]^2$$
$$= 32+56\sqrt{2}+49$$
$$= 81+56\sqrt{2} \qquad \text{[M1]}$$

Since $AC^2 = AB^2 + BC^2$, by Converse of Pythagoras' Theorem,
$\angle ACB = 90°$.
$\therefore \angle AXB = 90°$ (since $\angle AXB = \angle ACB$) [A1]

6 The equation of a curve is $y = \dfrac{(2x-5)^2}{x-1}$.

(i) Find an expression for $\dfrac{dy}{dx}$ and obtain the coordinates of the stationary points of the curve. [5]

Solution:

$$\frac{dy}{dx} = \frac{(x-1)(2)(2x-5)(2)-(2x-5)^2(1)}{(x-1)^2} \quad \text{[M1]}$$
$$= \frac{(2x-5)(4x-4-2x+5)}{(x-1)^2}$$
$$= \frac{(2x-5)(2x+1)}{(x-1)^2} \quad \text{[M1]}$$

When $\dfrac{dy}{dx} = 0$, $(2x-5)(2x+1) = 0$ [M1]

$x = 2.5$ or -0.5 [A1]

When $x = 2.5$, $y = 0$

When $x = -0.5$, $y = -24$

Stationary points are $(2.5, 0)$ and $(-0.5, -24)$ [A1]

(ii) Find an expression for $\dfrac{d^2y}{dx^2}$ and show that its can be expressed in the form $\dfrac{k}{(x-1)^3}$. Hence, or otherwise, determine the nature of these stationary points. [4]

Solution:

$$\frac{d^2y}{dx^2} = \frac{(x-1)^2(8x-8)-(2x-5)(2x+1)(2)(x-1)}{(x-1)^4} \quad \text{[M1]}$$
$$= \frac{(x-1)(8x^2-16x+8-8x^2+16x+10)}{(x-1)^4}$$
$$= \frac{18}{(x-1)^3} \quad \text{[A1]}$$

When $x = -0.5$, $\frac{d^2y}{dx^2} = \frac{18}{(-0.5-1)^3} < 0$

$(-0.5, -24)$ is a maximum point. [A1]

When $x = 2.5$, $\frac{d^2y}{dx^2} = \frac{18}{(2.5-1)^3} > 0$

$(2.5, 0)$ is a minimum point. [A1]

7 The highest point on a circle C_1 is $(2,8)$. The line T, $3y = 42 - 4x$, is a tangent to C_1 at the point $(6,6)$.

(i) Find the coordinates of the centre of C_1. [4]

Solution:

Since the highest point on a circle C_1 is $(2,8)$, the centre is $(2, y)$. [M1]

Gradient of normal at (6, 6) $= 1 \div \left(-\frac{4}{3}\right)$ [M1]

Equation of the normal at $(6,6)$: $(y-6) = \frac{3}{4}(x-6)$

$(y-6) = \frac{3}{4}(x-6)$

$y = \frac{3}{4}x + \frac{3}{2}$ [A1]

When $x = 2$, $y = 3$

The centre of C_1 is (2, 3). [A1]

(ii) Find the equation of C_1. [2]

Solution:

Equation of C_1: $(x-2)^2 + (y-3)^2 = (8-3)^2$[M1]

$(x-2)^2 + (y-3)^2 = 25$ [A1]

The circle C_2 is a reflection of C_1 in the line T.

(iii) Find the equation of C_2. [3]

Solution:

The centre of C_2 is $(2 + 2(6-2), 3 + 2(6-3)) = (10, 9)$. [B2]

Equation of C_2: $(x-10)^2 + (y-9)^2 = 25$ [A1]

8 **(i)** Show that $3x-1$ is a factor of $3x^3+11x^2+8x-4$ and hence factorise completely the cubic polynomial $3x^3+11x^2+8x-4$. [3]

Solution:

Let $\mathrm{f}(x)=3x^3+11x^2+8x-4$

$$\mathrm{f}\left(\frac{1}{3}\right)=3\left(\frac{1}{3}\right)^3+11\left(\frac{1}{3}\right)^2+8\left(\frac{1}{3}\right)-4 \quad \text{[M1]}$$

$$=0$$

Since $\mathrm{f}\left(\frac{1}{3}\right)=0$, $(3x-1)$ is a factor.

$3x^3+11x^2+8x-4=(3x-1)(x^2+bx+4)$

Comparing x term, $12-b=8$

$b=4$

$3x^3+11x^2+8x-4=(3x-1)(x^2+4x+4)$ [M1]

$=(3x-1)(x+2)^2$ [A1]

(ii) Express $\dfrac{5x^2-2x+11}{3x^3+11x^2+8x-4}$ as the sum of 3 partial fractions. [4]

Solution:

$$\frac{5x^2-2x+11}{3x^3+11x^2+8x-4}=\frac{5x^2-2x+11}{(3x-1)(x+2)^2}$$

Let $\dfrac{5x^2-2x+11}{(3x-1)(x+2)^2}=\dfrac{A}{(3x-1)}+\dfrac{B}{(x+2)}+\dfrac{C}{(x+2)^2}$ [M1]

$5x^2-2x+11=A(x+2)^2+B(3x-1)(x+2)+C(3x-1)$

Let $x=-2$, $-7C=35$

$C=-5$ [A1]

Let $x=\frac{1}{3}$, $\frac{49}{9}A=\frac{98}{9}$

$A=2$ [A1]

Let $x=0$, $4A-2B-C=11$

$8-2B-(-5)=11$

$B=1$ [A1]

$$\frac{5x^2-2x+11}{3x^3+11x^2+8x-4}=\frac{2}{(3x-1)}+\frac{1}{(x+2)}-\frac{5}{(x+2)^2}$$

(iii) Hence find $\int \frac{5x^2-2x+11}{3x^3+11x^2+8x-4}\,dx$. [3]

Solution:

$$\int \frac{5x^2-2x+11}{3x^3+11x^2+8x-4}\,dx = \int\left[\frac{2}{(3x-1)}+\frac{1}{(x+2)}-\frac{5}{(x+2)^2}\right]dx$$

$$=\frac{2}{3}\ln(3x-1)+\ln(x+2)-\frac{5}{(-1)}(x+2)^{-1}+c \quad \text{[M2]}$$

$$=\frac{2}{3}\ln(3x-1)+\ln(x+2)+\frac{5}{(x+2)}+c \quad \text{[A1]}$$

9 The roots of the quadratic equation $4x^2+3x+1=0$ are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

(i) Find the value of $\alpha^2+\beta^2$. [4]

Solution:

Sum of roots: $\frac{1}{\alpha}+\frac{1}{\beta}=-\frac{3}{4}$

$$\frac{\alpha+\beta}{\alpha\beta}=-\frac{3}{4}$$

Product of roots: $\frac{1}{\alpha\beta}=\frac{1}{4}$ [M1]

$$\alpha\beta=4$$

$$\alpha+\beta=\frac{\alpha+\beta}{\alpha\beta}\times\alpha\beta$$
$$=-\frac{3}{4}\times 4$$
$$=-3 \quad \text{[M1]}$$
$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$$
$$=(-3)^2-2(4) \quad \text{[M1]}$$
$$=1 \quad [A1]$$

(iii) Show that the value of $\alpha^3+\beta^3$ is 9. [2]

Solution:

$$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$$
$$=(-3)(1-4) \quad \text{[M1]}$$
$$=9 \text{ (shown)} \quad \text{[A1]}$$

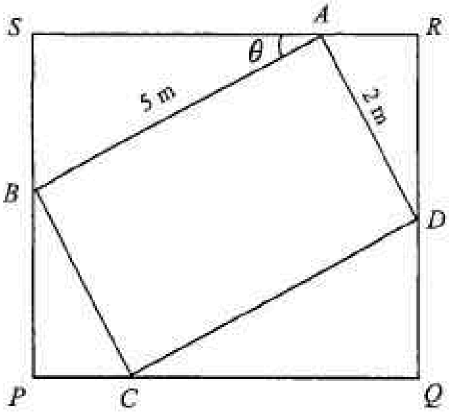
(iii) Find a quadratic equation whose roots are $\alpha^2+\beta$ and $\alpha+\beta^2$. [4]

Solution:

$$\alpha^2+\beta+\alpha+\beta^2=1+(-3)$$
$$=-2 \quad \text{[B1]}$$
$$(\alpha^2+\beta)(\alpha+\beta^2)=\alpha^3+\alpha^2\beta^2+\alpha\beta+\beta^3$$
$$=9+(4)^2+4 \quad \text{[M1]}$$
$$=29 \quad \text{[A1]}$$

The new equation is $x^2+2x+29=0$ [A1]

10



The diagram shows a rug in the shape of a rectangle *ABCD* such that $AB = 5$ m and $AD = 2$ m. The rug is placed inside a rectangular function room *PQRS* such that each of the corners *A*, *B*, *C* and *D* touches the sides of the room *SR*, *SP*, *PQ* and *QR* respectively. The side of the rug *AB* makes an acute angle θ with the side of the room *SR*. The lengths of the room *SR* and *SP* are *L* m and *W* m respectively.

(a) **(i)** Find the values of the integers *a* and *b* for which

$$L = a\cos\theta + b\sin\theta.$$ [2]

Solution:

$L = SA + AR$

$= 5\cos\theta + 2\sin\theta$

$a = 5;\ b = 2$ [B2]

(ii) Obtain a similar expression for *W*. [1]

Solution:

$W = SB + BP$

$= 5\sin\theta + 2\cos\theta$ [B1]

(iii) Hence find the perimeter of the room *PQRS* in exact form if *PQRS* is a square. [3]

Solution:

$W = SB + BP$

$= 5\sin\theta + 2\cos\theta$ [B1]

If *PQRS* is a square, $L = W$

$5\cos\theta + 2\sin\theta = 5\sin\theta + 2\cos\theta$ [M1]

$3\sin\theta = 3\cos\theta$

$\tan\theta = 1$

$\theta = 45°$ [A1]

Perimeter of $PQRS$ $= 4(5\cos 45° + 2\sin 45°)$

$$= 4\left(\frac{5\sqrt{2}}{2} + \frac{2\sqrt{2}}{2}\right)$$

$$= 4\left(\frac{7\sqrt{2}}{2}\right)$$

$$= 14\sqrt{2} \quad \text{m}$$ [A1]

(b) Using the values of a and b found in (a) part (i),

(i) express L in the form $R\cos(\theta - \alpha)$, $R > 0$ and $0° < \alpha < 90°$. [2]

Solution:

$L = 5\cos\theta + 2\sin\theta$

$$= \sqrt{5^2 + 2^2}\cos\left(\theta - \tan^{-1}\frac{2}{5}\right)$$

$$= \sqrt{29}\cos(\theta - 21.801°)$$

$$= \sqrt{29}\cos(\theta - 21.8°) \quad \text{(1 dp)}$$ [B2]

(ii) find the value of θ if $L = 4$ and the area of the rectangular function room $PQRS$. [4]

Solution:

$$L = 4$$

$$\sqrt{29}\cos(\theta - 21.801°) = 4$$

$$\cos(\theta - 21.801°) = \frac{4}{\sqrt{29}}$$ [M1]

$$\theta - 21.801° = 42.031°$$

$$\theta = 63.832°$$

$$= 63.8° \quad \text{(1 dp)}$$ [A1]

Area of room $PQRS = L \times W$

$$= 4 \times (5\sin 63.832° + 2\cos 63.832°)$$ [M1]

$$= 4 \times 5.3695$$

$$= 21.5 \text{ m}^2$$ [A1]

11 The amount of expenditure, \$$y$, incurred by a textile company is related to \$$x$, the amount of sales generated. The variables x and y are related by the formula $y = 10^k x^a$, where incurs a and k are constants. The following table shows corresponding values of x and y.

x (\$)	6	35	234	1995	6310
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(i) Plot $\lg y$ against $\lg x$ for the given data and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of a and of k. [4]

(iii) Estimate the amount of expenditure incurred when the sales generated is \$4000. [2]

(iv) Draw a straight line on the same axes to estimate the amount of sales to be generated in order for the textile company to breakeven. [2]

Name

(i)

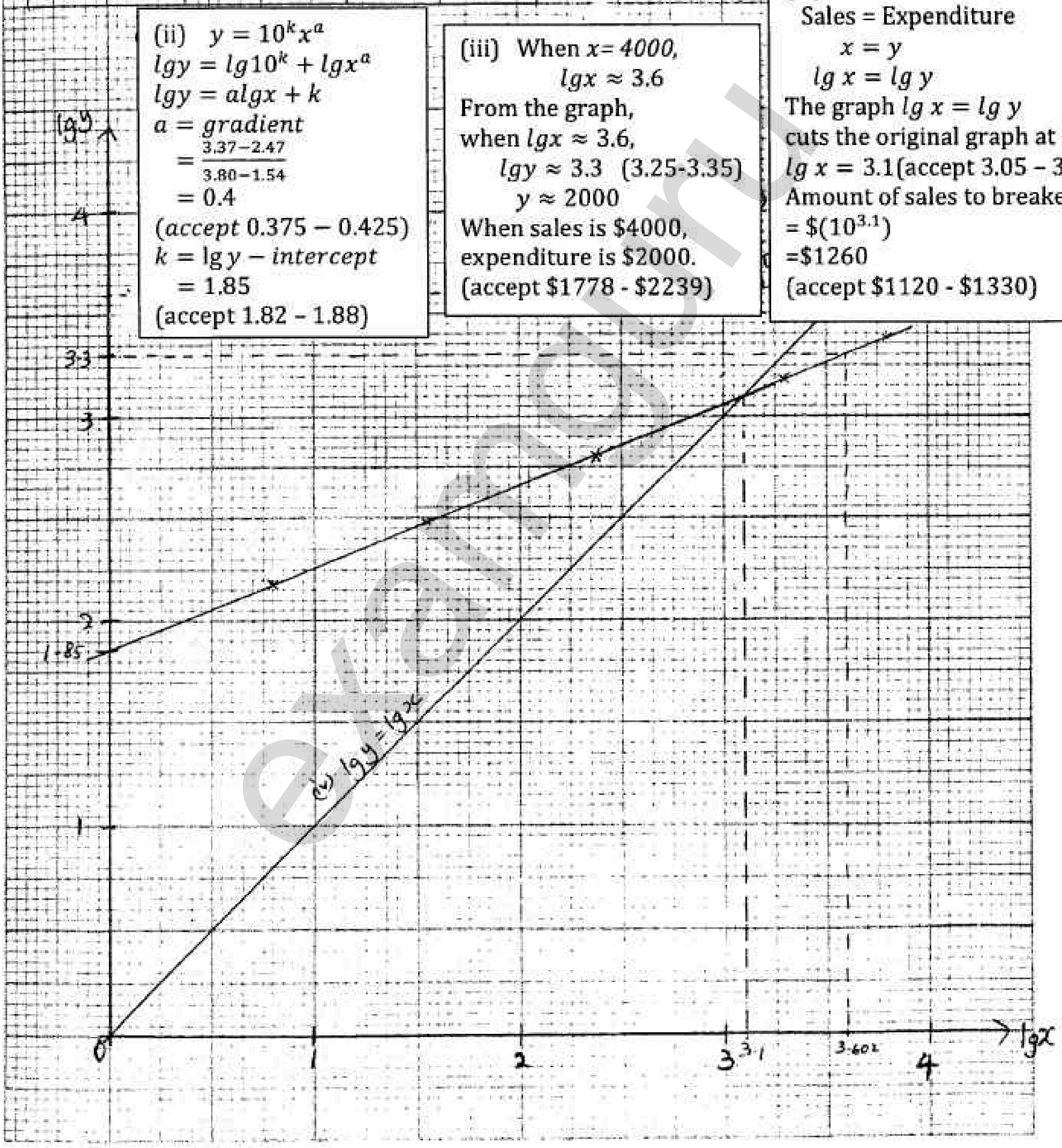
lg x	0.78	1.54	2.37	3.30	3.80
lg y	2.17	2.47	2.80	3.17	3.37

Scale : 4cm rep 1unit on x-axis
4cm rep 1unit on lg y-axis

(ii) $y = 10^k x^a$
$lgy = lg10^k + lgx^a$
$lgy = algx + k$
$a = gradient$
$= \frac{3.37-2.47}{3.80-1.54}$
$= 0.4$
($accept$ 0.375 – 0.425)
k = lg $y - intercept$
= 1.85
(accept 1.82 – 1.88)

(iii) When x= 4000,
$lgx \approx 3.6$
From the graph,
when $lgx \approx 3.6$,
$lgy \approx 3.3$ (3.25-3.35)
$y \approx 2000$
When sales is \$4000,
expenditure is \$2000.
(accept \$1778 - \$2239)

(iv) To breakeven,
Sales = Expenditure
$x = y$
$lg\ x = lg\ y$
The graph $lg\ x = lg\ y$
cuts the original graph at
$lg\ x = 3.1$(accept 3.05 – 3.13)
Amount of sales to breakeven
= \$($10^{3.1}$)
=\$1260
(accept \$1120 - \$1330)



Name:	Class	Class Register Number/ Centre No./Index No.

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2016
SECONDARY 4

ADDITIONAL MATHEMATICS
4047/01

Paper 1
3 August 2016
2 hours

Additional Materials: Answer Paper
Graph Paper (1 Sheet)

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

/ 80

This document consists of 5 printed pages 1 blank page

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

1. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formula for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}ab\sin C$$

1 The area of a triangle is $\left(1+\frac{5\sqrt{5}}{2}\right)$cm^2. If the length of the base of the triangle is $\left(3+2\sqrt{5}\right)$cm, find, without using a calculator, the height of the triangle in the form of $\left(a+b\sqrt{5}\right)$cm, where a and b are integers. [4]

2 Express $\frac{4x^2+6x+5}{2x^2+x-3}$ in partial fractions. [5]

3 The function $\mathrm{f}(x)$ is such that $\mathrm{f}(x)=2x^3+3x^2-x-4$,

(i) find a factor of $\mathrm{f}(x)$. [2]

(ii) Hence, determine the number of solutions in the equation $\mathrm{f}(x)=0$. [4]

4 The roots of the quadratic equation $3x^2-x+5=0$ are α and β.

(i) Evaluate $\alpha^2+\beta^2$. [2]

(ii) Find the quadratic equation whose roots are α^3-1 and β^3-1. [4]

5 The table shows experimental values of 2 variables, R and V, which are connected by an equation of the form $RV^n=k$ where n and k are constants.

R	33	19.95	5.07	2.38
V	2	2.9	8	14

(i) Plot lg R against lg V for the given data and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of k and of n. [3]

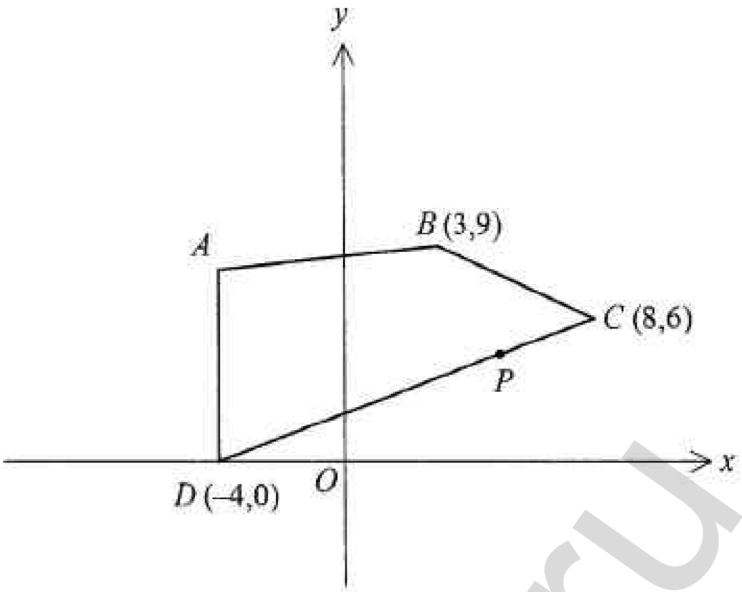
(iii) By drawing a suitable straight line on your graph in **(i)**, find the value of V such that $\frac{R}{V^2}=1$. [3]

6 Given that $y=1-\frac{1}{2}\sin 3x$, $0^\circ \le x \le 240^\circ$.

(i) State the maximum and minimum values of y. [2]

(ii) Sketch the graph of $y=1-\frac{1}{2}\sin 3x$. [3]

7



A quadrilateral *ABCD* passes through vertices *B* (3, 9), *C* (8, 6) and *D* (– 4, 0), line *AD* is parallel to the *y* – axis.

(i) Find the coordinates of *A* given that the length of *AD* is 8 units. [1]

(ii) A point *P* divides the line *DC* in the ratio of 2 : 1. Find the coordinates of *P*. [3]

(iii) Hence, find the area of the quadrilateral *ABPD*. [3]

8 **(a)** Sketch the graph $y^2 = 3x$. [2]

(b) Given that $\mathrm{f}(x) = -2x^3 + 5x^2 + 4x + a$,

(i) find the coordinates of the turning points in terms of *a*. [4]

(ii) Determine the nature of each turning point. [3]

(iii) In the case where $a = 1$, explain why the part of the graph between the turning points lie above the *x* – axis. [1]

9 **(i)** Show that $\sec x + \tan x$ can be expressed as $\dfrac{1+\sin x}{\cos x}$. [1]

(ii) Differentiate $\ln(\sec x + \tan x)$ with respect to *x*. [3]

(iii) Hence, find $\int_{0.25}^{0.5} 2\sec x \, \mathrm{d}x$. [3]

10 The points A and B lie on the circumference of a circle C_1 where A is the point (0, 8) and B is the point (4, 0). The line $y = 2x$ also passes through the centre of the circle C_1.

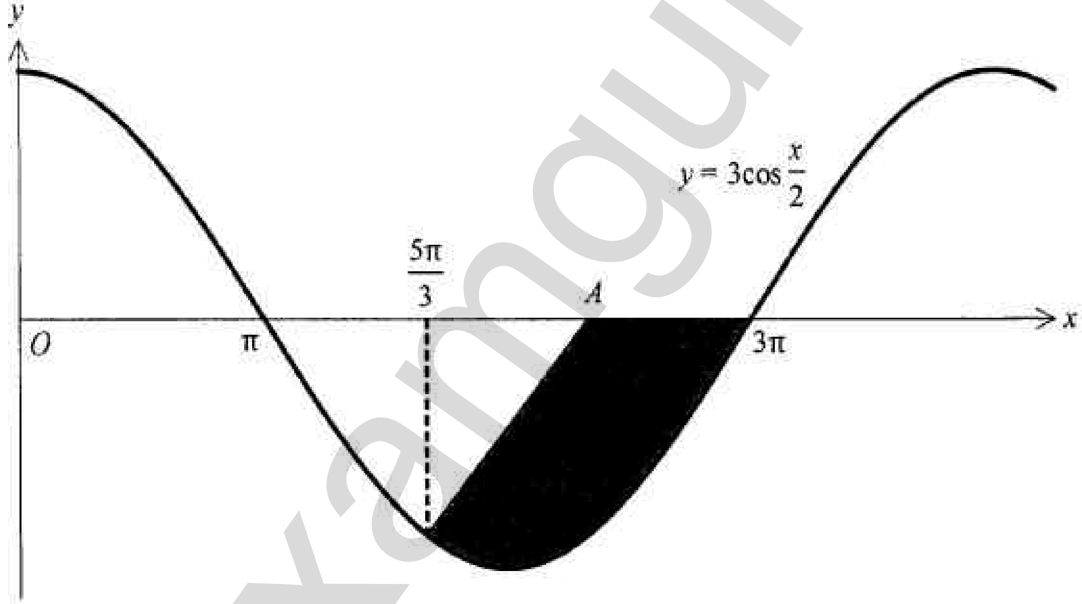
(i) Find the centre and radius of the circle C_1. [4]

(ii) Find the equation of the circle C_1 in the form $x^2 + y^2 + px + qy + r = 0$, where p, q and r are integers. [2]

Another circle C_2 of radius $\sqrt{2}$ units has its centre inside C_1 and it cuts the circle C_1 at the origin and at the point where $x = 2$.

(iii) Find the centre of C_2. [5]

11



The diagram shows part of the curve $y = 3\cos\frac{x}{2}$ that cuts the x – axis at $x = \pi$ and $x = 3\pi$. The normal to the curve at $x = \frac{5\pi}{3}$ cuts the x-axis at A.

(i) Find the coordinates of A, leaving your answer in exact form. [6]

(ii) Hence, find the area of the shaded region. [4]

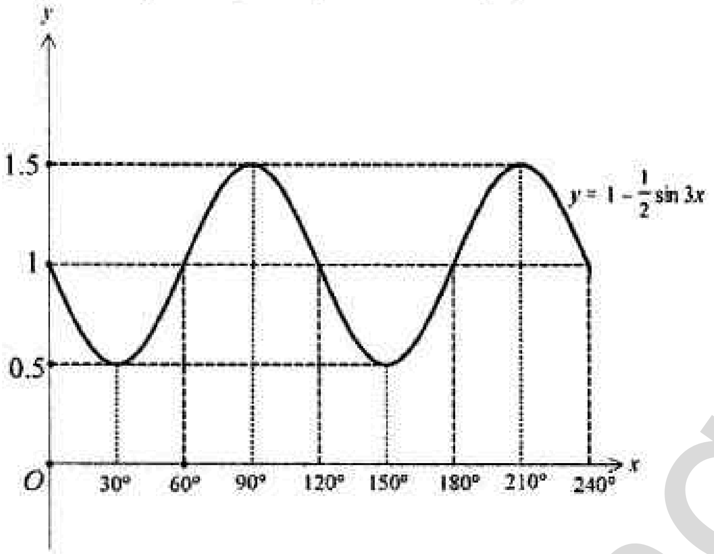
1. $4-\sqrt{5}$

2. $2-\frac{2}{2x+3}+\frac{3}{x-1}$

3. (ii) one solution

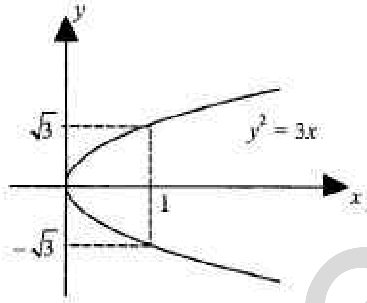
4. (i) $\frac{-29}{9}$ (ii) $27x^2+98x+196=0$

6. (i) Max y = 1.5; Min y = 0.5 (ii)



7. (i) $(-4,8)$ (ii) $P(4,4)$ (iii) 50 units^2

8. (a) (b)(i). $\left(-\frac{1}{3}, a-\frac{19}{27}\right)$ and $(2, 12+a)$ (b)(ii). $\left(-\frac{1}{3}, a-\frac{19}{27}\right)$ min ; $(2,12+a)$ max



9. (ii) $\sec x$ (iii). 0.539

10. (i) Centre (2, 4), Radius $= 2\sqrt{5}$ (ii) $x^2+y^2-4x-8y=0$ (iii) Centre of $C_2(1.22, 0.710)$

11. (i) $A\left(\frac{5\pi}{3}+\frac{9}{8}\sqrt{3}, 0\right)$ (ii) $6\frac{15}{32}$ / 6.47 units^2

	Workings
1	$1+\frac{5\sqrt{5}}{2}=\frac{1}{2}\left(3+2\sqrt{5}\right)\left(a+b\sqrt{5}\right)$ $2+5\sqrt{5}=\left(3+2\sqrt{5}\right)\left(a+b\sqrt{5}\right)$ $a+b\sqrt{5}=\frac{2+5\sqrt{5}}{3+2\sqrt{5}}$ $=\frac{2+5\sqrt{5}}{3+2\sqrt{5}}\times\frac{3-2\sqrt{5}}{3-2\sqrt{5}}$ $=\frac{6-4\sqrt{5}+15\sqrt{5}-50}{9-4(5)}$ $=\frac{-44+11\sqrt{5}}{-11}$ $=4-\sqrt{5}$ The height of the triangle is ($4-\sqrt{5}$) cm
2	Given $\frac{4x^2+6x+5}{2x^2+x-3}$ As this is an improper fraction, **By long division,** $\begin{array}{r} 2 \\ 2x^2+x-3\overline{)4x^2+6x+5} \\ \underline{4x^2+2x-6} \\ 4x+11 \end{array}$ $\frac{4x^2+6x+5}{2x^2+x-3}=2+\frac{4x+11}{(2x+3)(x-1)}$ Let $\frac{4x+11}{(2x+3)(x-1)}=\frac{A}{2x+3}+\frac{B}{x-1}$ $=\frac{A(x-1)+B(2x+3)}{(2x+3)(x-1)}$

$4x+11=A(x-1)+B(2x+3)$

Let $x = 1$,

$15=5B$

$B=3$

Let $x = 0$,

$11=-A+9$

$A=-2$

$$\frac{4x^2+6x=5}{(2x+3)(x-1)}=2-\frac{2}{2x+3}+\frac{3}{x-1}$$

3(i) Given $f(x)=2x^3+3x^2-x-4$

By trial and error,

Consider $(x-1)$

$$f(1)=2(1)^3+3(1)^2-1-4$$
$$=0$$

$\therefore(x-1)$ is a factor.

(ii) $f(x)=2x^3+3x^2-x-4$

By inspection,

$f(x)=(x-1)(2x^2+ax+4)$

By comparing coefficient of

$x^2:3=a-2$

$\therefore a=5$

$f(x)=(x-1)(2x^2+5x+4)$

Applying disciminant for $2x^2+5x+4$,

$$b^2-4ac=5^2-4(2)(4)$$
$$=25-32$$
$$=-7<0$$

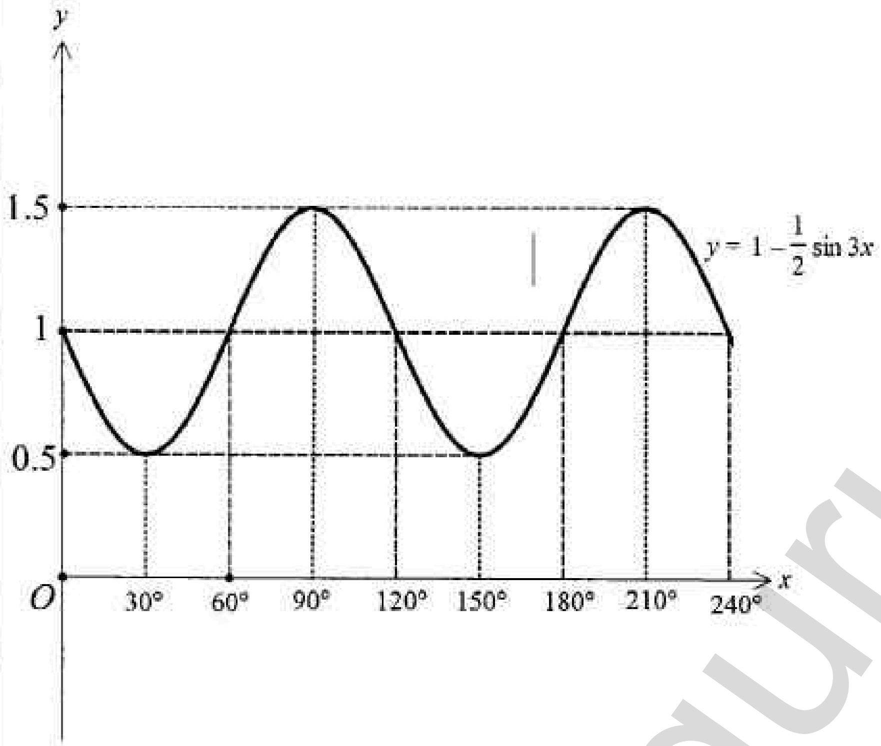
Thus $2x^2+5x+4$ has no real roots.

Therefore, there is only <u>**one solution .**</u>

4(i)	$3x^2 - x + 5 = 0$ $\alpha + \beta = \frac{1}{3}$ $\alpha\beta = \frac{5}{3}$ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $= \left(\frac{1}{3}\right)^2 - 2\left(\frac{5}{3}\right)$ $= \frac{1}{9} - \frac{10}{3}$ $= \frac{-29}{9}$
(ii)	New sum of roots = $\alpha^3 - 1 + \beta^3 - 1$ $= \alpha^3 + \beta^3 - 2$ $= (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) - 2$ $= \left(\frac{1}{3}\right)(\alpha^2 + \beta^2 - \alpha\beta) - 2$ $= \left(\frac{1}{3}\right)\left(\frac{-29}{9} - \frac{5}{3}\right) - 2$ $= \frac{-98}{27}$ New product of roots = $(\alpha^3 - 1)(\beta^3 - 1)$ $= \alpha^3\beta^3 - \beta^3 - \alpha^3 + 1$ $= (\alpha\beta)^3 - (\alpha^3 + \beta^3) + 1$ $= \left(\frac{5}{3}\right)^3 - \left(\frac{-44}{27}\right) + 1$ $= \frac{196}{27}$ Quadratic eqn: $x^2 - \left(\frac{-98}{27}\right)x + \frac{196}{27} = 0$ $27x^2 + 98x + 196 = 0$

6(i) Max y = 1.5; Min y = 0.5

(ii)



7(i) Since line AD is parallel to y – axis,

Coordinates of $A = (-4, 0+8)$

$= (-4, 8)$

7(ii) Since P divides the line DC in ratio 2 : 1,

$P_x = \frac{8+4}{3}\times 2 + (-4); P_y = \frac{6}{3}\times 2 + 0$

$= 4 \qquad ;= 4$

$\therefore P(4,4)$

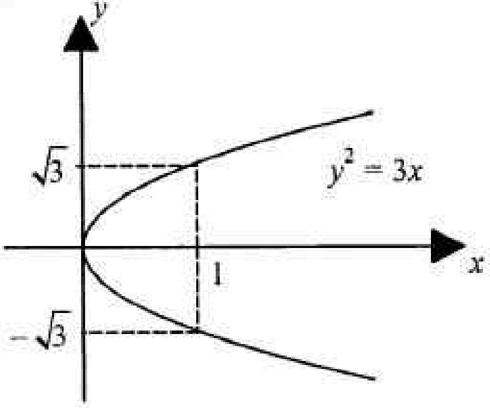
7(iii) Area of quadrilateral $ABPD = \frac{1}{2}\left|\begin{matrix} -4 & 4 & 3 & -4 & -4 \\ 0 & 4 & 9 & 8 & 0 \end{matrix}\right|$

$$= \frac{1}{2}\left[(-16+36+24)-(12-36-32)\right]$$

$$= \frac{1}{2}[44+56]$$

$$= 50unit^2$$

8(a)



8(b)(i)

Given $f(x) = -2x^3 + 5x^2 + 4x + a$

$f'(x) = -6x^2 + 10x + 4$

For stationary point, $f'(x) = 0$

$-6x^2 + 10x + 4 = 0$

$3x^2 - 5x - 2 = 0$

$(3x+1)(x-2) = 0$

$\therefore x = -\frac{1}{3}$ or $x = 2$

$$f(x) = -2\left(-\frac{1}{3}\right)^3 + 5\left(-\frac{1}{3}\right)^2 + 4\left(-\frac{1}{3}\right)^1 + a$$

$$= -2\left(\frac{1}{27}\right) + \frac{5}{9} - \frac{4}{3} + a$$

$$= a - \frac{19}{27}$$

or

$$f(x) = -2(2)^3 + 5(2)^2 + 4(2)^1 + a$$

$$= -16 + 20 + 8 + a$$

$$= a + 12$$

$\left(-\frac{1}{3}, a - \frac{19}{27}\right)$ and $(2, 12+a)$ are turning points

8(b)(ii)

$f''(x) = -12x + 10$

At $x = -\frac{1}{3}, f''(x) = -12\left(-\frac{1}{3}\right) + 10$

$$= 14$$

$$> 0$$

$\therefore \left(-\frac{1}{3}, a - \frac{19}{27}\right)$ is a minimum turning point.

	At $x = 2, \mathrm{f}''(x) = -12(2)+10$ $= -14$ < 0 $\therefore (2,12+a)$ is a maximum turning point.
8(b)(iii)	When $a = 1$, min point $= \left(-\frac{1}{3}, \frac{8}{27}\right)$ is above x - axis max point $= (2,13)$ is above x - axis Since graph has no other turning points, the part of the graph between the 2 turning points lie above x - axis.
9(i)	$\sec x + \tan x = \frac{1}{\cos x} + \frac{\sin x}{\cos x}$ $= \frac{1+\sin x}{\cos x}$
(ii)	$\frac{\mathrm{d}}{\mathrm{d}x}\ln(\sec x + \tan x) = \frac{\mathrm{d}}{\mathrm{d}x}\ln\left(\frac{1+\sin x}{\cos x}\right)$ $= \frac{\mathrm{d}}{\mathrm{d}x}\left[\ln(1+\sin x) - \ln(\cos x)\right]$ $= \frac{\cos x}{1+\sin x} - \frac{-\sin x}{\cos x}$ $= \frac{\cos x(\cos x) + \sin x(1+\sin x)}{(1+\sin x)\cos x}$ $= \frac{\cos^2 x + \sin^2 x + \sin x}{(1+\sin x)\cos x}$ $= \frac{1+\sin x}{(1+\sin x)\cos x}$ $= \frac{1}{\cos x}$ $= \sec x$
(iii)	$\int_{0.25}^{0.5} 2\sec x \ \mathrm{d}x = 2\int_{0.25}^{0.5} \sec x \ \mathrm{d}x$ $= 2\left[\ln\left(\frac{1+\sin x}{\cos x}\right)\right]_{0.25}^{0.5}$ $= 2\left[\ln\left(\frac{1+\sin 0.5}{\cos 0.5}\right) - \ln\left(\frac{1+\sin 0.25}{\cos 0.25}\right)\right]$ $= 0.539184$ $= 0.539$ (3s.f)

10(i)

Midpoint of $AB = \left(\frac{0+4}{2}, \frac{8+0}{2}\right)$

$= (2,4)$

Gradient of $AB = \frac{8-0}{0-4}$

$= -2$

Eqn of perpendicular bisector of AB:

$y - 8 = \frac{1}{2}(x-0)$

$y = \frac{1}{2}x + 3 --- (1)$

$y = 2x --- (2)$

Equating,

$2x = \frac{1}{2}x + 3$

$x = 2$

$y = 4$

$\therefore$ center of $C_1 (2,4)$

Radius $= \sqrt{(2-4)^2 + (4-0)^2}$

$= \sqrt{20}$

$= 2\sqrt{5} units$

10(ii)

Thus eqn of C_1 :

$(x-2)^2 + (y-4)^2 = (2\sqrt{5})^2$

$x^2 - 4x + 4 + y^2 - 8y + 16 = 20$

$x^2 + y^2 - 4x - 8y = 0$

10(iii)

Since $C_1 : x^2 + y^2 - 4x - 8y = 0$

When $x = 2$,

$y^2 - 8y - 4 = 0$

$y = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-4)}}{2(1)}$

$= 4 \pm 2\sqrt{5}$

Use $y = 4 - 2\sqrt{5}$ (C_2 radius is only $\sqrt{2}$*unit* and lies in C_1)

Midpoint $= \left(1, 2 - \sqrt{5}\right)$

$$\text{Gradient} = \frac{4 - 2\sqrt{5} - 0}{2 - 0}$$
$$= 2 - \sqrt{5}$$

Eqn of perpendicular bisector:

$$y - \left(2 - \sqrt{5}\right) = \left(\frac{-1}{2 - \sqrt{5}}\right)(x - 1)$$

$$y = \frac{10 - 4\sqrt{5} - x}{2 - \sqrt{5}} - - - (1)$$

Since equation C_2 is of the form

$(x - a)^2 + (y - b)^2 = 2$ where center is $(a,\ b)$

Using $(0, 0)$,

$a^2 + b^2 = 2 - - - (2)$

By substituting (1) in (2),

$$a^2 + \left(\frac{10 - 4\sqrt{5} - a}{2 - \sqrt{5}}\right)^2 = 2$$

$$a^2 + \frac{a^2 + a(8\sqrt{5} - 20) + 180 - 80\sqrt{5}}{9 - 4\sqrt{5}} = 2$$

$$\left(10 - 4\sqrt{5}\right)a^2 + a(8\sqrt{5} - 20) + 162 - 72\sqrt{5} = 0$$

Solving

$$a = \frac{-(8\sqrt{5} - 20) \pm \sqrt{(8\sqrt{5} - 20)^2 - 4\left(10 - 4\sqrt{5}\right)\left(162 - 72\sqrt{5}\right)}}{2\left(10 - 4\sqrt{5}\right)}$$

$= 1.223$ or 0.7767 (rejected as it outside of C_1)

Hence $b = 0.7101$

Thus center of $C_2(1.22, 0.710)$

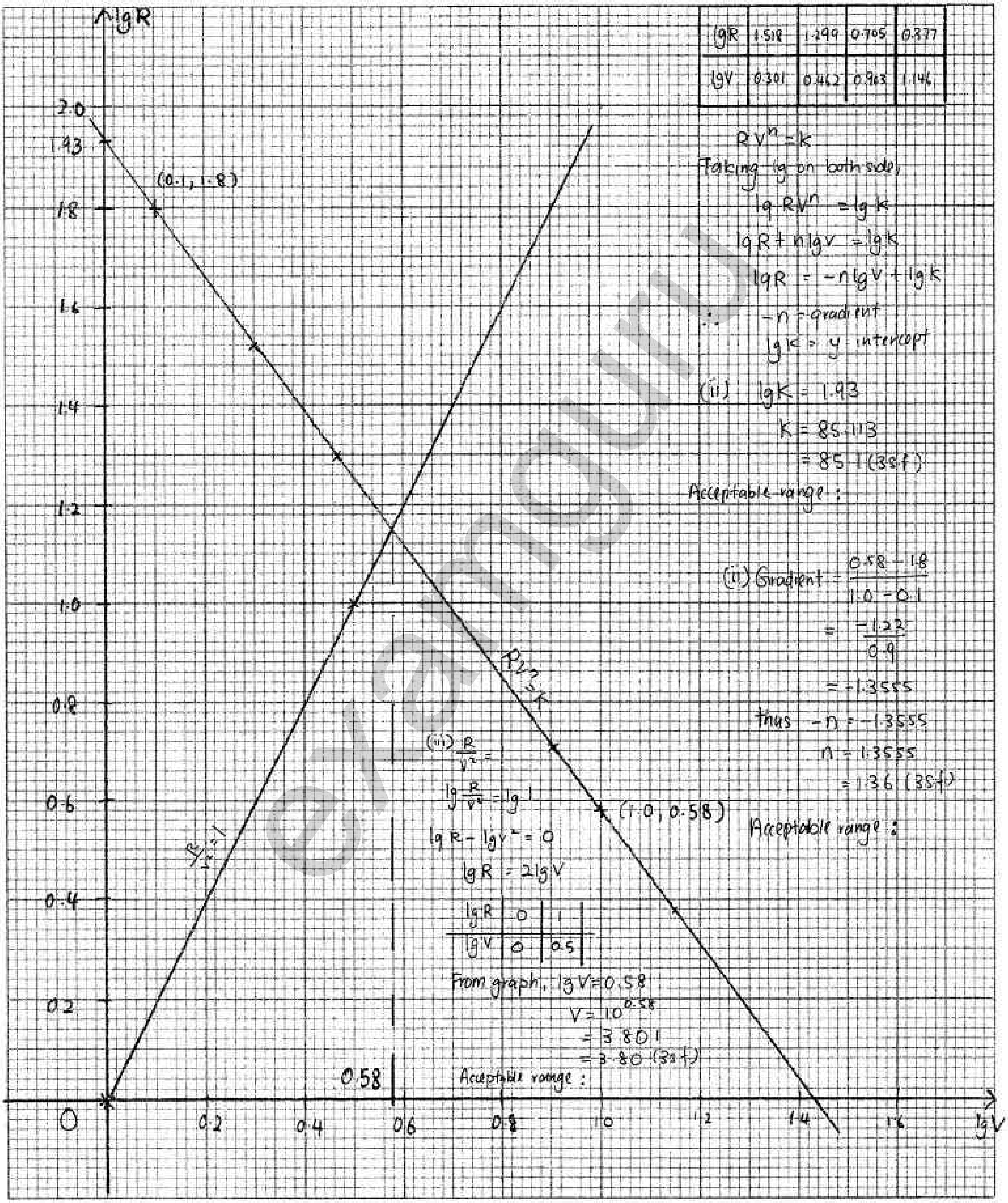
11(i)	Given $y = 3\cos\frac{x}{2}$ $\frac{dy}{dx} = -3\left(\frac{1}{2}\right)\sin\frac{x}{2}$ $= -\frac{3}{2}\sin\frac{x}{2}$ At $x = \frac{5\pi}{3}$, $\frac{dy}{dx} = -\frac{3}{2}\sin\frac{5\pi}{6}$ $= -\frac{3}{4}$ Gradient of normal = $\frac{4}{3}$ At $x = \frac{5\pi}{3}, y = -\frac{3\sqrt{3}}{2}$ Eqn of normal: $y + \frac{3\sqrt{3}}{2} = \frac{4}{3}\left(x - \frac{5\pi}{3}\right)$ $y = \frac{4}{3}x - \frac{20\pi}{9} - \frac{3\sqrt{3}}{2}$ Since the normal cuts x - axis, $y = 0$ $0 = \frac{4}{3}x - \frac{20\pi}{9} - \frac{3\sqrt{3}}{2}$ $x = \frac{5\pi}{3} + \frac{9}{8}\sqrt{3}$ $\therefore A\left(\frac{5\pi}{3} + \frac{9}{8}\sqrt{3}, 0\right)$
11(ii)	Shaded area $= \left\|\int_{\frac{5\pi}{3}}^{3\pi} 3\cos\frac{x}{2}\ dx\right\| - \frac{1}{2} \times \frac{3\sqrt{3}}{2} \times \frac{9\sqrt{3}}{8}$ $= \left\|\left[6\sin\frac{x}{2}\right]_{\frac{5\pi}{3}}^{3\pi}\right\| - \frac{81}{32}$ $= \left\|6\sin\frac{3\pi}{2} - 6\sin\frac{5\pi}{6}\right\| - \frac{81}{32}$ $= \|-6-3\| - \frac{81}{32}$ $= 6\frac{15}{32}\,unit^2 / 6.47unit^2 (3sf)$

Candidate Name ....

Subject .... Paper 01

Centre Number | Index Number

Question No. 5



lgR	1.518	1.299	0.705	0.377
lgV	0.301	0.462	0.963	1.146

$RV^n = k$

Taking lg on both side,

$\lg RV^n = \lg k$

$\lg R + n\lg V = \lg k$

$\lg R = -n\lg V + \lg k$

$\therefore -n = \text{gradient}$

$\lg k = y\ \text{intercept}$

(ii) $\lg K = 1.93$

$K = 85.113$

$= 85.1\ (3sf)$

Acceptable range :

(ii) Gradient $= \frac{0.58 - 1.8}{1.0 - 0.1}$

$= \frac{-1.22}{0.9}$

$= -1.3555$

thus $-n = -1.3555$

$n = 1.3555$

$= 1.36\ (3sf)$

Acceptable range :

(iii) $\frac{R}{V^2} =$

$\lg \frac{R}{V^2} = \lg 1$

$\lg R - \lg V^2 = 0$

$\lg R = 2\lg V$

lgR	0	1
lgV	0	0.5

From graph, $\lg V = 0.58$

$V = 10^{0.58}$

$= 3.801$

$= 3.80\ (3sf)$

Acceptable range :

EX 257 (rev 2012)

Name:	Class	Class Register Number/ Centre No./Index No.

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2016
SECONDARY 4

ADDITIONAL MATHEMATICS

4047/02

Paper 2

5 August 2016

2 hours 30 minutes

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number clearly on all the work you hand in.
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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

$\frac{\quad}{100}$

This document consists of 6 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 **(a)** The equation of a curve is $y = 2x^2 + ax + (6+a)$, where a is a constant. Find the range of values of a for which the curve lies completely above the x-axis. [3]

(b) The equation of a curve is $y = 3x^2 + 4x + 6$.

(i) Find the set of values of x for which the curve is above the line $y = 6$. [3]

(ii) Show that the line $y = -8x - 6$ is a tangent to the curve. [2]

2 **(a)** Given that $\log_a 125 - 3\log_a b + \log_a c = 3$, express a in terms of b and c. [3]

(b) Solve the equation

(i) $\lg 8x - \lg(x^2 - 3) = 2\lg 2$, [3]

(ii) $2\log_5 x = 3 + 7\log_x 5$. [4]

3 The equation of a curve is $y = x^2\sqrt{(5x-1)^3}$, for $x > 0.2$. Given that x is changing at a constant rate of 0.25 units per second, find the rate of change of y when $x = 2$. [4]

4 The graph of $y = |2x^2 - ax - 5|$ passes through the points with coordinates $(-1,\ 0)$ and $(0.75,\ b)$.

(i) Find the value of the constants a and b. [3]

(ii) Sketch the graph of $y = |2x^2 - ax - 5|$. [3]

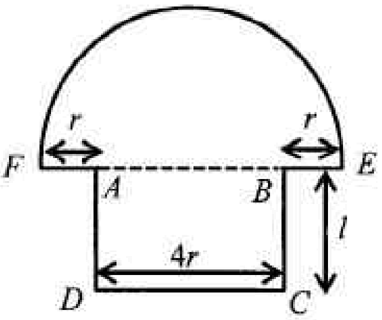
(iii) Determine the set of positive values of m for which the line $y = mx + 2$ intersects the graph of $y = |2x^2 - ax - 5|$ at two points. [2]

5 In the binomial expansion of $\left(2x + \frac{k}{x}\right)^8$, where k is a positive constant, the coefficient of x^2 is 28.

(i) Show that $k = \frac{1}{4}$. [4]

(ii) Hence, determine the term in x in the expansion of $\left(6x - \frac{1}{x}\right)\left(2x + \frac{k}{x}\right)^8$. [4]

6



The diagram shows a design of a bookmark that includes a rectangle *ABCD*, where $BC = l$ cm, $CD = 4r$ cm, a semicircle with radius $3r$ cm, and $AF = BE = r$ cm. The area of the bookmark is 90 cm^2.

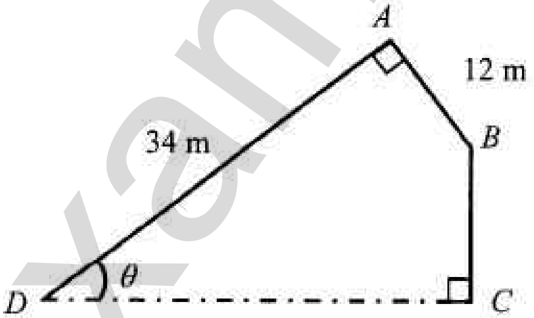
(i) Express l in terms of r. [2]

(ii) Given that the perimeter of the bookmark is P cm, show that

$$P = \left(6 + \frac{3\pi}{4}\right)r + \frac{45}{r}.$$ [2]

(iii) Given that r and l can vary, find the value of r for which P has a stationary value. Explain why this value of r gives the minimum perimeter. [5]

7



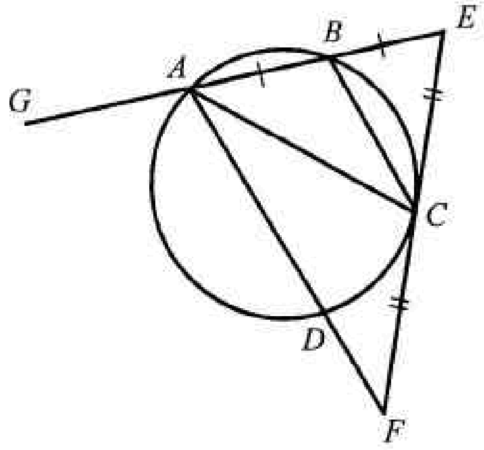
The diagram shows an animal exhibition area that is surrounded by glass panels at *AB*, *BC* and *AD*, where $AB = 12$ m, $AD = 34$ m, angle DAB = angle $BCD = 90°$ and the acute angle $ADC = \theta$ can vary.

(i) Show that L m, the length of the glass panels can be expressed as $L = 46 + 34\sin\theta - 12\cos\theta$. [2]

(ii) Express L in the form $p + R\sin(\theta - \alpha)$, where p and $R > 0$ are constants and α is an acute angle. [4]

(iii) Given that the exact length of the glass panels is 62 m, find the value of θ. [3]

8



The diagram shows points *A*, *B*, *C* and *D* on a circle, line *EF* is tangent to the circle at *C*, lines *ADF* and *EBAG* are straight lines, and points *B* and *C* are the midpoints of *AE* and *EF*.

Prove that

(i) $BC \times EC = AC \times BE$, [3]

(ii) $AF \times EC = AC \times AE$, [2]

(iii) angle GAD = angle ACF. [2]

9 **(a)** **(i)** Show that $\cot 2x = \dfrac{1-\tan^2 x}{2\tan x}$. [2]

(ii) Hence, solve the equation $8\cot 2x \tan x = 1$, for $0° < x < 360°$. [4]

(b) The Ultraviolet Index (UVI) describes the level of solar radiation. The UVI measured from the top of a building is given by $U = 6 - 5\cos qt$, where t is the time in hours from the lowest value of the UVI, $0 \le t \le 10$, and q is a constant. It takes 10 hours for the UVI to reach its lowest value again.

(i) Explain why we are not able to measure a UVI of 12. [1]

(ii) Show that $q = \dfrac{\pi}{5}$. [1]

(iii) The top of the building is equipped with solar panels that supply power to the building when the UVI is at least 3. Find the duration, in hours and minutes, that the building is supplied with power from the solar panels. [4]

10 **(a)** It is given that $y=\dfrac{2x^2}{4x-3}$, where $x>\dfrac{3}{4}$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(ii) Find the range of values of x for which $y=\dfrac{2x^2}{4x-3}$ is a decreasing function. [4]

(b) It is given that $\mathrm{f}(x)$ is such that $\mathrm{f}\,'(x)=\dfrac{1}{2x-5}-\dfrac{4}{(2x-5)^2}$.

Given also that $\mathrm{f}(3)=1.75$, show that $8\mathrm{f}(x)-\left(2x-5\right)^2\mathrm{f}\,''(x)=\ln(2x-5)^4$. [7]

11 A particle moves in a straight line, so that, t seconds after passing a fixed point O, its velocity, v m/s, is given by $v=2e^{0.1t}-10e^{0.1-0.3t}$. The particle comes to an instantaneous rest at the point A.

(i) Show that the particle reaches A when $t=\dfrac{5}{2}\ln 5+\dfrac{1}{4}$. [3]

(ii) Find the acceleration of the particle at A. [3]

(iii) Find the distance OA. [4]

(iv) Explain whether the particle is again at O at some instant during the eleventh second after first passing through O. [2]

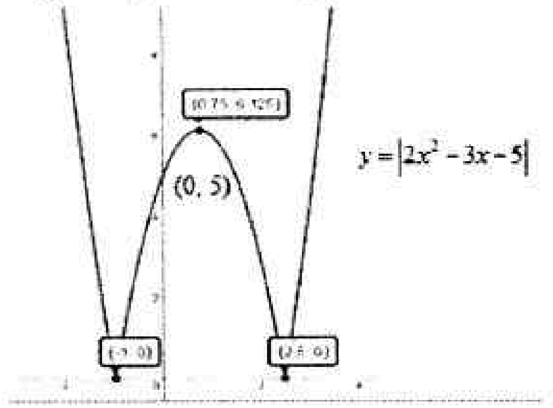
Answer Key

1. (a) $-4 < a < 12$ (b)(i) $x < -1\frac{1}{3}$ or $x > 0$

2. (a) $a = \frac{5\sqrt[3]{c}}{b}$ (b)(i) $x = 3$ (ii) $x = 85.7$ or $x = 0.130$

3. 49.5 units / s

4. (i) $a = 3$, $b = 6.125$ (ii) (iii) $m > 2$



5. (ii) $-1\frac{3}{4}x$

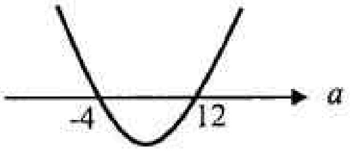
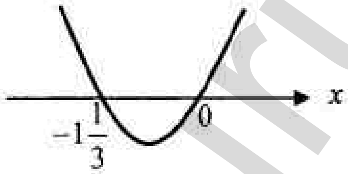
6. (i) $l = \frac{45}{2r} - \frac{9}{8}\pi r$ (iii) $r = 2.32$; min value

7. (ii) $L = 46 + 10\sqrt{13}\sin(\theta - 19.4°)$ (iii) 45.8°

9. (a)(ii) $x = 40.9°, 139.1°, 220.9°, 319.1°$ (b)(iii) 7 hrs and 3 mins

10. (a)(i) $\frac{4x(2x-3)}{(4x-3)^2}$ (ii) $\frac{3}{4} < x < \frac{3}{2}$

11. (ii) 1.23 m/s^2 (iii) 16.0 m (iv) passed through O

		Working
1	(a)	For $y = 2x^2 + ax + (6+a)$ to lie above the x-axis, discriminant $b^2 - 4ac < 0$ $(a)^2 - 4(2)(6+a) < 0$ $a^2 - 8a - 48 < 0$ $(a-12)(a+4) < 0$ $-4 < a < 12$ 
	(b) (i)	$3x^2 + 4x + 6 > 6$ $3x^2 + 4x > 0$ $x(3x+4) > 0$ $x < -1\frac{1}{3}$ or $x > 0$ 
	(ii)	$3x^2 + 4x + 6 = -8x - 6$ $3x^2 + 12x + 12 = 0$ $x^2 + 4x + 4 = 0$ Discriminant $= (4)^2 - 4(1)(4) = 0$ Since discriminant = 0, the line and curve intersects only at one point. Line $y = -8x - 6$ is tangent to the curve. (shown)
2	(a)	$\log_a 125 - 3\log_a b + \log_a c = 3$ $\log_a 125 - \log_a b^3 + \log_a c = 3$ $\log_a \frac{125c}{b^3} = 3$ $a^3 = \frac{125c}{b^3}$ $a = \frac{5\sqrt[3]{c}}{b}$
	(b) (i)	$\lg 8x - \lg(x^2 - 3) = 2\lg 2$ $\lg\left(\frac{8x}{x^2-3}\right) = \lg 2^2$ $\frac{8x}{x^2-3} = 4$

Working

$4x^2 - 8x - 12 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ or -1 (reject $x = -1$ as $\lg 8x$ is undefined)

$x = 3$

(b) (ii)

$2\log_5 x = 3 + 7\log_x 5$

$2\log_5 x = 3 + 7\left(\frac{\log_5 5}{\log_5 x}\right)$

$2(\log_5 x)^2 - 7 - 3\log_5 x = 0$

Let $u = \log_5 x$

$2u^2 - 3u - 7 = 0$

$u = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-7)}}{2(2)}$

$\log_5 x = \frac{3 \pm \sqrt{65}}{4}$

$x = 5^{\frac{1}{4}(3+\sqrt{65})}$ or $x = 5^{\frac{1}{4}(3-\sqrt{65})}$

$x = 85.7$ or $x = 0.130$ (3 sig. fig.)

3

$y = x^2\sqrt{(5x-1)^3}$

$\frac{dy}{dx} = x^2\left(\frac{3}{2}(5x-1)^{\frac{1}{2}}(5)\right) + 2x\sqrt{(5x-1)^3}$

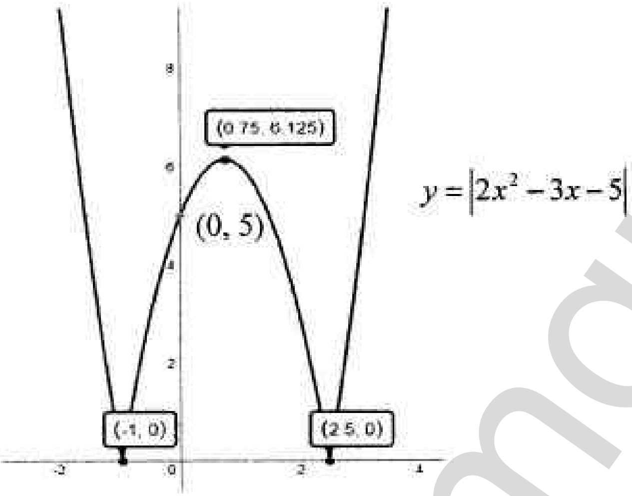
$= (5x-1)^{\frac{1}{2}}\left(\frac{15x^2}{2} + 2x(5x-1)\right)$

$= (5x-1)^{\frac{1}{2}}\left(\frac{35x^2}{2} - 2x\right)$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$= (5(2)-1)^{\frac{1}{2}}\left(\frac{35(2)^2}{2} - 2(2)\right) \times 0.25$

$= 49.5$ units/s

		Working
4	(i)	$y=\|2x^2-ax-5\|$ At $(-1,\ 0)$, $y=\|2(-1)^2-a(-1)-5\|$ $\|a-3\|=0$ $a=3$ At $(0.75,\ b)$, $b=\|2(0.75)^2-3(0.75)-5\|=6.125$
	(ii)	
	(iii)	Line $y=mx+2$ passes through (0, 2) and cuts two points to the right of (0, 2). The line that passes through $(-1,0)$ and (0, 2) has 3 points of intersection. Gradient $=\dfrac{2-0}{0-(-1)}=2$ Lines with $m>2$ intersect the graph at 2 points.
5	(i)	General Term $=\begin{pmatrix}8\\r\end{pmatrix}(2x)^{8-r}\left(\dfrac{k}{x}\right)^r$ $=\begin{pmatrix}8\\r\end{pmatrix}(2)^{8-r}(k)^r x^{8-2r}$ For term in x^2: $8-2r=2$ $r=3$

	Working
	Coefficient $=\binom{8}{3}(2)^{8-3}(k)^3$ $=1792k^3$ $1792k^3=28$ $k^3=\frac{1}{64}$ $k=\frac{1}{4}$
(ii)	$\left(6x-\frac{1}{x}\right)\left(2x+\frac{k}{x}\right)^8$ $=\left(6x-\frac{1}{x}\right)\left(\cdots+28x^2+\cdots+\binom{8}{4}(2x)^4\left(\frac{1}{4x}\right)^4+\cdots\right)$ Term in x $=6\times70(16)\left(\frac{1}{4^4}\right)x-28x$ $=-1\frac{3}{4}x$
6 **(i)**	$\frac{\pi}{2}(3r)^2+4rl=90$ $l=\frac{90-\frac{9\pi r^2}{2}}{4r}$ $l=\frac{45}{2r}-\frac{9}{8}\pi r$
(ii)	$P=4r+2l+2r+\frac{\pi}{2}(6r)$ $=4r+2\left(\frac{45}{2r}-\frac{9}{8}\pi r\right)+2r+3\pi r$ $=6r+\frac{3}{4}\pi r+\frac{45}{r}$ $=\left(6+\frac{3}{4}\pi\right)r+\frac{45}{r}$ (shown)

Working

(iii)

$$P=\left(6+\frac{3}{4}\pi\right)r+\frac{45}{r}$$

$$\frac{\mathrm{d}P}{\mathrm{d}r}=6+\frac{3}{4}\pi-\frac{45}{r^2}$$

For stationary points, $\frac{\mathrm{d}P}{\mathrm{d}r}=0$

$$6+\frac{3}{4}\pi=\frac{45}{r^2}$$

$$r^2=\frac{45\times 4}{24+3\pi}$$

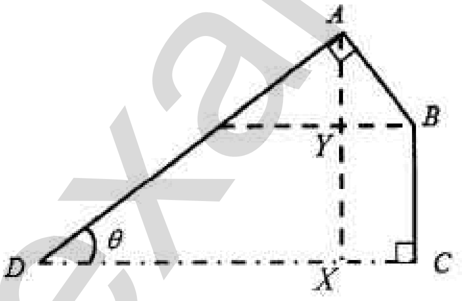
$$r=\sqrt{\frac{45\times 4}{24+3\pi}}\ \text{ since } r>0.$$

$$r=\sqrt{\frac{60}{8+\pi}}\quad \text{or}\quad 2.32\ \text{(3 sig. fig.)}$$

$$\frac{\mathrm{d}^2P}{\mathrm{d}r^2}=\frac{90}{r^3}=\frac{90}{(2.3206)^3}>0$$

Since $\frac{\mathrm{d}^2P}{\mathrm{d}r^2}>0$, this gives a minimum value of P.

7 (i)



$\angle DAX=90°-\theta$

$\angle XAB=\theta$

$AX=34\sin\theta$

$BC=34\sin\theta-12\cos\theta$

$L=AD+AB+BC$

$=46+34\sin\theta-12\cos\theta$

(ii)

$34\sin\theta-12\cos\theta=R\sin(\theta-\alpha)$

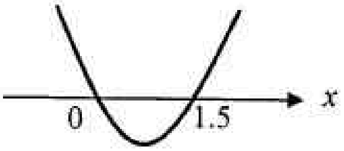
$=R\left(\sin\theta\cos\alpha-\cos\theta\sin\alpha\right)$

Comparing coefficients, $R\sin\alpha=12$ and $R\cos\alpha=34$

$R=\sqrt{12^2+34^2}=\sqrt{1300}=10\sqrt{13}$

		Working
		$\tan\alpha = \dfrac{12}{34} \qquad \alpha = 19.440°$ $L = 46 + 10\sqrt{13}\sin(\theta - 19.4°)$ (to 1 d.p.)
	(iii)	$46 + 10\sqrt{13}\sin(\theta - 19.440°) = 62$ $10\sqrt{13}\sin(\theta - 19.440°) = 16$ $\sin(\theta - 19.440°) = \dfrac{16}{10\sqrt{13}}$ $\theta - 19.440° = 26.344°$ $\theta = 26.344° + 19.440°$ $= 45.8°$
8	**(i)**	$\angle BCE = \angle BAC$ (alternate segment theorem) $\angle BEC = \angle AEC$ (common angle) Triangle BEC is similar to triangle CEA (AA similarity) $\dfrac{BC}{BE} = \dfrac{AC}{CE}$ $BC \times EC = AC \times BE$ (shown)
	(ii)	Since B and C are the midpoints of AE and EF, $BC = \dfrac{1}{2}AF$ $BC \parallel AF$ (midpoint theorem) $\dfrac{1}{2}AF \times EC = AC \times BE$ from (i) $AF \times EC = AC \times 2BE$ $AF \times EC = AC \times AE$ (shown)
	(iii)	$\angle GAD = \angle ABC$ (corr angles, $BC \parallel AF$) $\angle ACF = \angle ABC$ (alternate segment theorem) $\angle ACF = \angle GAD$ (shown)

		Working
9	(a) (i)	LHS: $\cot 2x = \frac{1}{\tan 2x}$ $= \frac{1}{\frac{2\tan x}{1-\tan^2 x}}$ $= \frac{1-\tan^2 x}{2\tan x}$ (RHS) (shown)
	(a) (ii)	From (i), $8\cot 2x \tan x = 4(2\cot 2x \tan x)$ $= 4(1-\tan^2 x)$ $4(1-\tan^2 x) = 1$ $4 - 4\tan^2 x = 1$ $\tan^2 x = \frac{3}{4}$ $\tan x = \pm\frac{\sqrt{3}}{2}$ Basic angle $\alpha = 40.8933°$ $x = 40.8933°, 180° + 40.8933°$ or $x = 180° - 40.8933°, 360° - 40.8933°$ $x = 40.9°, 139.1°, 220.9°, 319.1°$ (1 d.p.)
9	(b) (i)	$U = 6 - 5\cos qt$ Highest value of $-5\cos qt = 5$ when $\cos qt = -1$, highest value is 11, we are not able to measure UVI of 12.
	(b) (ii)	UVI takes 10 hours to reach its lowest again, $10q = 2\pi$ $q = \frac{\pi}{5}$
	(b) (iii)	$3 = 6 - 5\cos\frac{\pi t}{5}$ $5\cos\frac{\pi t}{5} = 3$

		Working
		$\cos\frac{\pi t}{5}=\frac{3}{5}$ Basic angle, $\alpha=0.927295$ $\frac{\pi t}{5}=0.927295 \quad \text{or} \quad 5.35589$ $t=1.47583 \quad \text{or} \quad 8.52416$ Duration of solar power supply $=8.52416-1.47583$ $=7.04833$ hrs = 7 hrs and 3 mins
10	**(a)** **(i)**	$y=\frac{2x^2}{4x-3}$ $\frac{dy}{dx}=\frac{(4x-3)(4x)-2x^2(4)}{(4x-3)^2}$ $=\frac{8x^2-12x}{(4x-3)^2}$ $=\frac{4x(2x-3)}{(4x-3)^2}$
	(a) **(ii)**	For decreasing function, $\frac{dy}{dx}=\frac{8x^2-12x}{(4x-3)^2}<0$ $\frac{4x(2x-3)}{(4x-3)^2}<0$ Denominator $(4x-3)^2>0$ for $x>\frac{3}{4}$, $x(2x-3)<0$ $0<x<\frac{3}{2}$  Since $x>\frac{3}{4}$, y is decreasing function for $\frac{3}{4}<x<\frac{3}{2}$.

Working

10 **(b)**

$$f(x)=\int \frac{1}{2x-5}-\frac{4}{(2x-5)^2}\,dx$$

$$=\frac{1}{2}\ln(2x-5)+\frac{2}{2x-5}+c\text{, where } c \text{ is a constant.}$$

Given $f(3)=1.75$,

$$\frac{1}{2}\ln(2(3)-5)+\frac{2}{2(3)-5}+c=1.75$$

$$c=-0.25$$

$$f''(x)=\frac{d}{dx}\left(\frac{1}{2x-5}-\frac{4}{(2x-5)^2}\right)$$

$$=\frac{-2}{(2x-5)^2}+\frac{16}{(2x-5)^3}$$

$$8f(x)-(2x-5)^2 f''(x)$$

$$=8\left[\frac{1}{2}\ln(2x-5)+\frac{2}{2x-5}-0.25\right]-(2x-5)^2\left(\frac{-2}{(2x-5)^2}+\frac{16}{(2x-5)^3}\right)$$

$$=4\ln(2x-5)$$

$$=\ln(2x-5)^4 \quad \text{(shown)}$$

11 **(i)**

For instantaneous rest, $v=0$

$$2e^{0.1t}-10e^{0.1-0.3t}=0$$

$$2e^{0.1t}=10e^{0.1-0.3t}$$

$$\frac{e^{0.1t}}{e^{-0.3t}}=5e^{0.1}$$

$$e^{0.4t}=5e^{0.1}$$

Taking ln on both sides:

$$0.4t=\ln 5+0.1$$

$$t=\frac{5}{2}\ln 5+\frac{1}{4} \qquad \text{(shown)}$$

(ii)

$$a=\frac{dv}{dt}$$

$$=0.2e^{0.1t}-10(-0.3)e^{0.1-0.3t}$$

$$=0.2e^{0.1t}+3e^{0.1-0.3t}$$

Working

When $t=\frac{5}{2}\ln 5+\frac{1}{4}$,

$$a=0.2e^{0.1\left(\frac{5}{2}\ln 5+\frac{1}{4}\right)}+3e^{0.1-0.3\left(\frac{5}{2}\ln 5+\frac{1}{4}\right)}$$
$$=1.2265$$
$$=1.23\ \text{m/s}^2$$

(iii)

$$s=\int v\ \mathrm{d}t$$
$$=\int 2e^{0.1t}-10e^{0.1-0.3t}\ \mathrm{d}t$$
$$=20e^{0.1t}+\frac{100}{3}e^{0.1-0.3t}+c\quad \text{, where } c \text{ is a constant}$$

Since $s=0$ when $t=0$,

$$s=20+\frac{100}{3}e^{0.1}+c$$
$$c=-\left(20+\frac{100}{3}e^{0.1}\right)$$
$$OA=20e^{0.1\left(\frac{5}{2}\ln 5+\frac{1}{4}\right)}+\frac{100}{3}e^{0.1-0.3\left(\frac{5}{2}\ln 5+\frac{1}{4}\right)}-\left(20+\frac{100}{3}e^{0.1}\right)$$
$$=-15.9535$$
$$=-16.0$$

OA is 16.0 m (3 sig. fig.)

(iv)

When $t=10$,

$$s_{10}=20e^{1}+\frac{100}{3}e^{(0.1-3)}-\left(20+\frac{100}{3}e^{0.1}\right)$$
$$=-0.63928\ \text{m}$$

When $t=11$,

$$s_{11}=20e^{1.1}+\frac{100}{3}e^{(0.1-3.3)}-\left(20+\frac{100}{3}e^{0.1}\right)$$
$$=4.6030\ \text{m}$$

Since the displacement of the particle changes from negative to positive, the particle passed through O during the eleventh second.

O Level Centre / Index Number	Class	Name
/		

新加坡海星中学

MARIS STELLA HIGH SCHOOL
PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

ADDITIONAL MATHEMATICS **4047/1**
Paper 1
19 August 2016
2 hours

Additional Materials: Writing Paper (8 sheets)

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples. paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use
80

This document consists of 6 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Solve the following equations

(a) $5^{2+x} - 3(5^{1-x}) + 10 = 0$, [4]

(b) $\log_9 \sqrt{3-3x} = \frac{1}{2} - \log_{81}(1-2x)$. [4]

2 **(a)** Find the greatest value of the integer k for which $-3x^2 + kx - 5$ is never positive for all values of x. [3]

(b) A curve has an equation $y = \frac{x^2}{2-3x}$, where $x \neq \frac{2}{3}$.

Find the range of values of x for which y is decreasing. [4]

3 **(i)** Prove the identity $1 + \frac{\sin^2 A}{1-\sec^2 A} + \frac{\cos^2 A}{1-\operatorname{cosec}^2 A} = 0$. [3]

(ii) Hence, solve the equation $\frac{\sin^2 A}{1-\sec^2 A} + \frac{\cos^2 A}{1-\operatorname{cosec}^2 A} = \tan(2A + 10°)$

for $-180° < A < 180°$. [4]

4 A curve has the equation $y = 4e^{\tan(\pi - \frac{x}{4})}$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) If x and y vary with time and y increases at the rate of e units per second when $x = \pi$ radian, find the exact value of the rate of decrease of x at this instant. [4]

5 **(a)** Sketch the graph of $f(x) = 2 - |5-3x|$ for $-1 \le x \le 6$.

Indicate clearly the vertex and the intercepts of the axes. [3]

(b) Solve the equation $2 - |5-3x| = x - 1$. [2]

(c) **(i)** State the range of the values of c if there is no solution for the equation $2 - |5-3x| = c$. [1]

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6 The amount of radioactive Sodium-24, M measured in grams, used as a tracer to measure the rate of flow in an artery or vein can be modelled by $M = M_0e^{kt}$, where t is the time in hours, M_0 and k are a constants.

The hospital buys a 40-grams sample of Sodium-24 and will reorder when the sample is reduced to 3 grams.

(i) Given that there are only 20 grams of Sodium-24 left after 14.9 hours. Find the value of M_0 and of k. [3]

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7 **(a)** The function f is defined, for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, by the equation $f(x) = 2\tan 3x$.

(i) State the period of f. [1]

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(b) On the same diagram drawn in part (a), sketch the graph of $g(x) = 1 - 2\sin x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [2]

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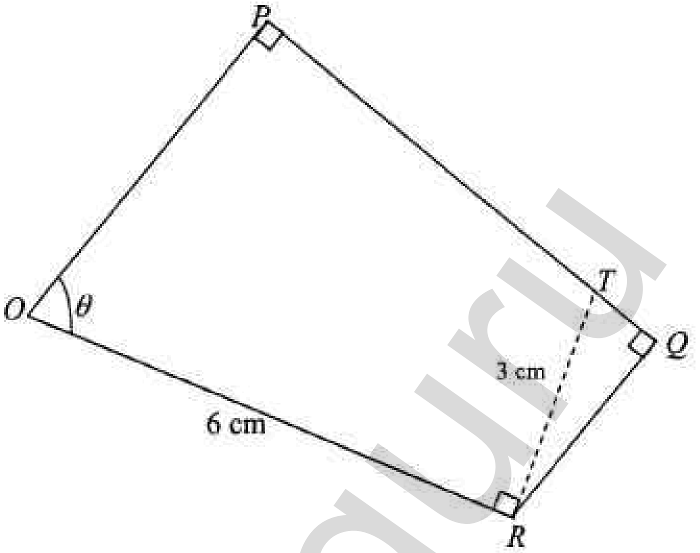
9 The diagram shows a quadrilateral $OPQR$ where $OR = 6$ cm, angle OPQ = angle $PQR = \frac{\pi}{2}$ radian and angle $ROP = \theta$ radian, θ is a variable and an acute angle. T is a point on PQ such that angle $ORT = \frac{\pi}{2}$ radian and $RT = 3$ cm.



(i) Show that the area, A cm^2 of the quadrilateral $OPQR$ is given by

$$A = 9\sin 2\theta + 18\sin^2\theta$$ [3]

(ii) Given that θ can vary, find maximum area of the quadrilateral $OPQR$.

[6]

10 A particle P moves in a straight line so that t seconds after passing through a fixed point O, its velocity, v m/s is given by

$$v_P = 1 - \frac{9}{(3t+1)^2}.$$

(i) Calculate the initial acceleration of the particle P. [2]

(ii) Show that the particle P is at instantaneously rest at $t = \frac{2}{3}$. [2]

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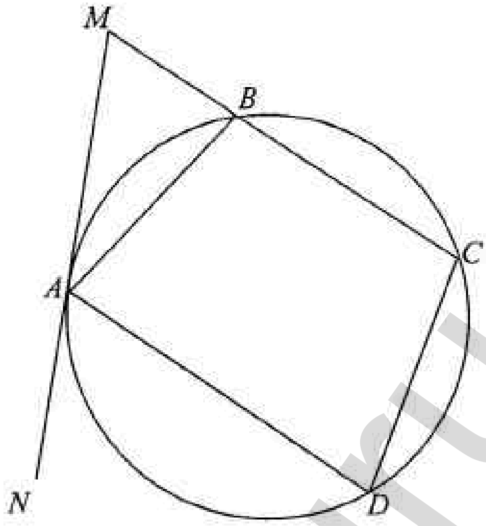
Another particle Q moves in a straight line and its displacement, S meter from O after t seconds is given by $S_Q = t - 1$.

(iv) Find the distance from the fixed point O when P first collides with Q. [2]

11 In the diagram, A, B, C and D are points on the circle. MN is a tangent to the circle at A. MBC is a straight line.



(a) Name a triangle which is similar to triangle CAM. [1]

Hence prove that $\left(\frac{AC}{BA}\right)^2 = \frac{CM}{BM}$. [3]

(b) Given further that AD and BC are parallel, show that

(i) triangle ABM is similar to triangle ADC, [2]

(ii) $AD \times AM = AC \times CD$. [2]

~ End of Paper ~

O Level Centre / Index Number	Class	Solution
/		

新加坡海星中学
MARIS STELLA HIGH SCHOOL
PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

ADDITIONAL MATHEMATICS **4047/1**
Paper 1
19 August 2016
2 hours

Additional Materials: Answer Paper (8 sheets)

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use
80

This document consists of 7 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

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where n is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

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(b) $\log_9 \sqrt{3-3x} = \frac{1}{2} - \log_{81}(1-2x)$ [4]

(a) $5^{2+x} - 3(5^{1-x}) + 10 = 0$

$25(5^x) - \frac{15}{5^x} + 10 = 0$

$5(5^x) - \frac{3}{5^x} + 2 = 0$ [M1]

Let $p = 5^x$

$5p - \frac{3}{p} + 2 = 0$

$5p^2 + 2p - 3 = 0$ [M1]

$(5p-3)(p+1) = 0$

$p = \frac{3}{5}$ or $p = -1$

$5^x = \frac{3}{5}$ or $5^x = -1$ (reject)

$\lg 5^x = \lg(\frac{3}{5})$ [M1] (p if never reject $5^x = -1$)

$x = \frac{\lg(\frac{3}{5})}{\lg 5}$

$x = -0.317$ [A1]

(b) $\log_9 \sqrt{3-3x} = \frac{1}{2} - \log_{81}(1-2x)$

$\log_9 \sqrt{3-3x} = \frac{1}{2} - \log_{81}(1-2x)$

$\log_9 \sqrt{3-3x} = \frac{1}{2} - \frac{\log_9(1-2x)}{\log_9 81}$

$\frac{1}{2}\log_9(3-3x) = \frac{1}{2} - \frac{\log_9(1-2x)}{2}$ [M1 for changing base]

$\log_9(3-3x) + \log_9(1-2x) = 1$

$\log_9(3-3x)(1-2x) = 1$ [M1]

$(3-3x)(1-2x) = 9^1$

$(1-x)(1-2x) = 3$ [M1]

$2x^2 - 3x - 2 = 0$

$(2x+1)(x-2) = 0$

$x = -\frac{1}{2}$ or $x = 2$ (reject) (p if never reject $x = 2$)

$\therefore x = -\frac{1}{2}$ [A1]

2 **(a)** Find the greatest value of the integer k for which $-3x^2 + kx - 5$ is never positive for all values of x. [3]

(b) A curve has an equation $y = \frac{x^2}{2-3x}$, where $x \neq \frac{2}{3}$.

Find the range of values of x for which y is decreasing.

[4]

(a) For all values of x, $-3x^2 + kx - 5$ is never positive,

Discriminant ≤ 0

$k^2 - 4(-3)(-5) \leq 0$ [M1]

$k^2 - 60 \leq 0$

$(k - \sqrt{60})(k + \sqrt{60}) \leq 0$

$-\sqrt{60} \leq k \leq \sqrt{60}$ [A1]

OR $-2\sqrt{15} \leq k \leq 2\sqrt{15}$

OR $-7.7460 \leq k \leq 7.7460$

The greatest integer value of k is 7 [A1]

(b) $y = \dfrac{x^2}{2-3x}, x \neq \dfrac{2}{3}$

$\dfrac{dy}{dx} = \dfrac{2x(2-3x)+3x^2}{(2-3x)^2}$ [M1]

$= \dfrac{4x-3x^2}{(2-3x)^2}$

Since the curve is decreasing, $\dfrac{dy}{dx} < 0$ and $x \neq \dfrac{2}{3}$

$\dfrac{4x-3x^2}{(2-3x)^2} < 0$ [M1]

Since $(2-3x)^2 > 0$, $4x - 3x^2 < 0$

$3x^2 - 4x > 0$ [M1]

$x(3x-4) > 0$

$x < 0$ or $x > \dfrac{4}{3}$ [A1]

3 **(i)** Prove the identity $1 + \dfrac{\sin^2 A}{1-\sec^2 A} + \dfrac{\cos^2 A}{1-\operatorname{cosec}^2 A} = 0.$ [3]

(ii) Hence, solve the equation $\dfrac{\sin^2 A}{1-\sec^2 A} + \dfrac{\cos^2 A}{1-\operatorname{cosec}^2 A} = \tan(2A + 10°)$ for $-180° < A < 180°$. [4]

(i) To prove $1 + \dfrac{\sin^2 A}{1-\sec^2 A} + \dfrac{\cos^2 A}{1-\operatorname{cosec}^2 A} = 0.$

LHS $= 1 + \dfrac{\sin^2 A}{1-\sec^2 A} + \dfrac{\cos^2 A}{1-\operatorname{cosec}^2 A}$

$= 1 + \dfrac{\sin^2 A}{-\tan^2 A} + \dfrac{\cos^2 A}{-\cot^2 A}$ [B1]

$= 1 - \cos^2 A - \sin^2 A$ [B1]

$= 1 - 1$ [B1]

$= 0$

Hence $1 + \dfrac{\sin^2 A}{1-\sec^2 A} + \dfrac{\cos^2 A}{1-\operatorname{cosec}^2 A} = 0.$ (Proved)

(ii) Since $\frac{\sin^2 A}{1-\sec^2 A}+\frac{\cos^2 A}{1-\cos ec^2 A}=\tan(2A+10°)$

$\tan(2A+10°)=-1$ [B1]

Basic angle = 45°

$2A+10°=-45°,-225°,135°,315°$ [M1]

$A=-27.5°,-117.5°,62.5°,152.5°$

[A1 for both] [A1 for both]

4 A curve has the equation $y=4e^{\tan(\pi-\frac{x}{4})}$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) If x and y vary with time and y increases at the rate of e units per second when $x=\pi$ radian. Find the exact value of the rate of decrease of x at this instant. [4]

(i) $\frac{dy}{dx}=4(-\frac{1}{4})\sec^2(\pi-\frac{x}{4})e^{\tan(\pi-\frac{x}{4})}$ [M1]

$\frac{dy}{dx}=-\sec^2(\pi-\frac{x}{4})e^{\tan(\pi-\frac{x}{4})}$ [B1]

(ii) When $x=\pi$,

$\frac{dy}{dx}=-\sec^2(\frac{3\pi}{4})e^{\tan(\frac{3\pi}{4})}$ [M1]

$=-(-\sqrt{2})^2e^{-1}$

$=-\frac{2}{e}$ [A1]

$\frac{dy}{dt}=\frac{dx}{dt}\times\frac{dy}{dx}$

$e=\frac{dx}{dt}\times\left(-\frac{2}{e}\right)$ [M1]

$\frac{dx}{dt}=-\frac{e^2}{2}$

The exact rate of decrease of x is $\frac{e^2}{2}$ $units/s$ [A1]

5 **(a)** Sketch the graph of $f(x) = 2 - |5 - 3x|$ for $-1 \le x \le 6$.

Indicate clearly the vertex and the intercepts of the axes. [3]

(b) Solve the equation $2 - |5 - 3x| = x - 1$ [2]

(c) **(i)** State the range of the values of c if there is no solution for the equation $2 - |5 - 3x| = c$, [1]

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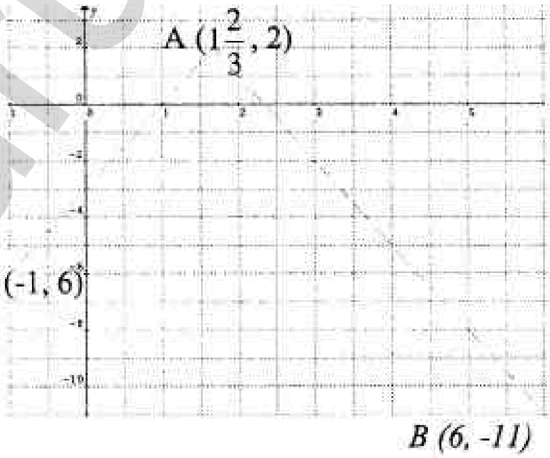
(a) Turning Points = $(1\frac{2}{3}, 2)$ [B1]

Shape - inverted v-shape [B1]

intercepts : $(0, -3)$, $(1, 0)$, $(2\frac{1}{3}, 0)$

terminal points : (-1, -6) , (6, -11) [B1]



(b) $2 - |5 - 3x| = x - 1$

$|5 - 3x| = 3 - x$

$5 - 3x = 3 - x$	or	$5 - 3x = -(3 - x)$	[M1]
$x = 1$		$x = 2$	[A1]

(c) (i) $c > 2$ [B1]

(ii) Gradient of OA = $\frac{6}{5}$

Gradient of AB = -3

The range of values of m : $-3 < m < \frac{6}{5}$ [B1]

6 The amount of radioactive Sodium-24, M measured in grams, used as a tracer to measure the rate of flow in an artery or vein can be modelled by $M = M_0e^{kt}$, where t is the time in hours, M_0 and k are a constants.

The hospital buys a 40-grams sample of Sodium-24 and will reorder when the sample is reduced to 3 grams.

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(ii) Find the amount of Sodium-24 remain after 60 hours. [1]

(iii) Calculate the time taken before the hospital reorders Sodium-24 . [2]

(i) When $t = 0, M = 40$

$M_0 = 40$ [B1]

When $t = 14.9$, $M = 20$

$$20 = 40e^{14.9k}$$

$$e^{14.9k} = \frac{1}{2}$$ [M1]

$$k = \frac{1}{14.9}\ln\frac{1}{2}$$

$$k = -\frac{1}{14.9}\ln 2$$

$$k = -0.046520$$

$k = -0.0465$ $(3s.f.)$ [A1]

(ii) When $t = 60$,

$$M = 40e^{-(\frac{1}{14.9}\ln 2)(60)}$$

$$M = e^{-2.7912}$$

$M = 0.0613\,g$ [A1]

(iii) When $M = 3$,

$$3 = 40e^{-0.04652t}$$

$$\frac{3}{40} = e^{-0.04652t}$$ [M1]

$$\ln\left(\frac{3}{40}\right) = -0.04652t$$

$$t = -\frac{1}{0.04652}\ln\left(\frac{3}{40}\right)$$

t = 55.7 hours [A1]

7 **(a)** The function f is defined, for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, by the equation $f(x) = 2\tan 3x$.

(i) State the period of f. [1]

(ii) Sketch the graph of $y = f(x)$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [2]

(b) On the same diagram drawn in part (a), sketch the graph of $g(x) = 1 - 2\sin x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [2]

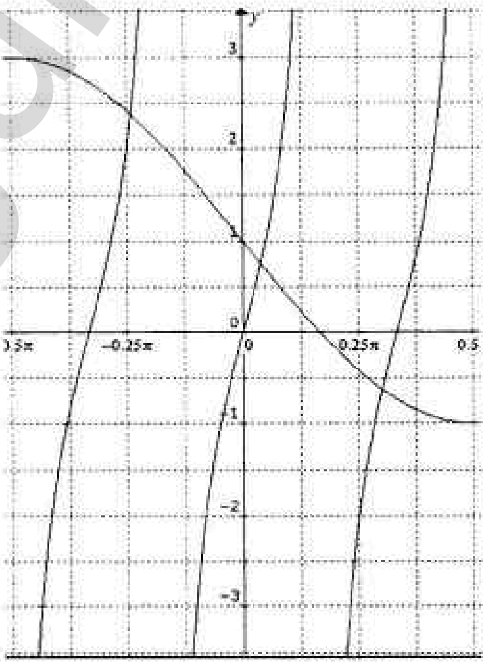
(c) State the number of solutions of the equation $\sin x + \tan 3x = \frac{1}{2}$ in the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [1]

(a) (i) Period = $\frac{\pi}{3}$ [B1]

(ii) Shape [B 1]

4 asymptotes [B 0.5]

x-intercept : $-\frac{\pi}{6}$; 0; $\frac{\pi}{6}$; [B 0.5]



(b) Shape [B1]

turning points $(-\frac{\pi}{2}, 3)$; $(\frac{\pi}{2}, -1)$; [B 0.5]

intercepts : (0, 1), $(\frac{\pi}{6}, 0)$ [B 0.5]

(c)

$$\sin x + \tan 3x = \frac{1}{2}$$
$$2\sin x + 2\tan 3x = 1$$
$$2\tan 3x = 1 - 2\sin x$$

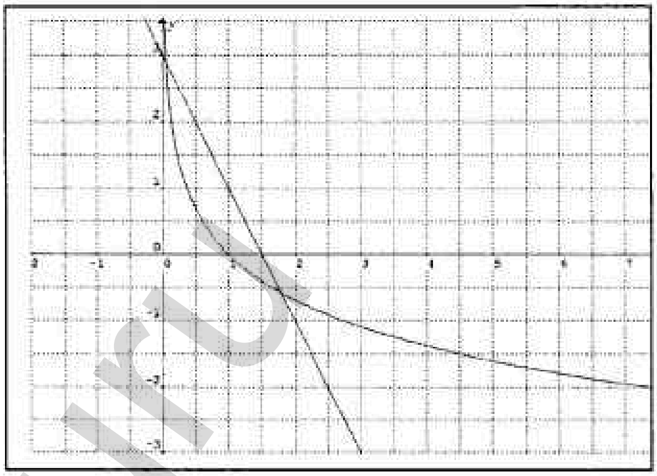
There are 3 solutions for the equation $\sin x + \tan 3x = \frac{1}{2}$ in the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [A1]

8 The function $f(x) = -\ln x$ is defined for $x > k$.

(i) State the value of k. [1]

(ii) Sketch the graph of $f(x) = -\ln x$ for $x > k$. [2]

(iii) Explain how another straight line drawn on your diagram in part **(b)** can lead to the graphical solution of $xe^{2x-3} = 1$. Draw this straight line and state the number of solutions for $xe^{2x-3} = 1$ [3]



(i) $k = 0$ [B1]

(ii) Shape [B1]

Asymptote $x = 0$ [B 0.5]

x-intercept : (1, 0) [B 0.5]

(iii) Since

$$xe^{2x-3} = 1$$
$$\ln(xe^{2x-3}) = 0$$
$$\ln xe^{2x31} = 0$$
$$\ln x + 2x - 3 = 0$$
$$y = 3 - 2x \quad [B1]$$

Hence, by drawing the line $y = 3 - 2x$ on the diagram in part (b), the x-coordinates of the points of intersection would give the solutions for $xe^{2x-3} = 1$. [B1]

From the sketch, we can conclude that there are 2 solutions for $xe^{2x-3} = 1$. [A1]

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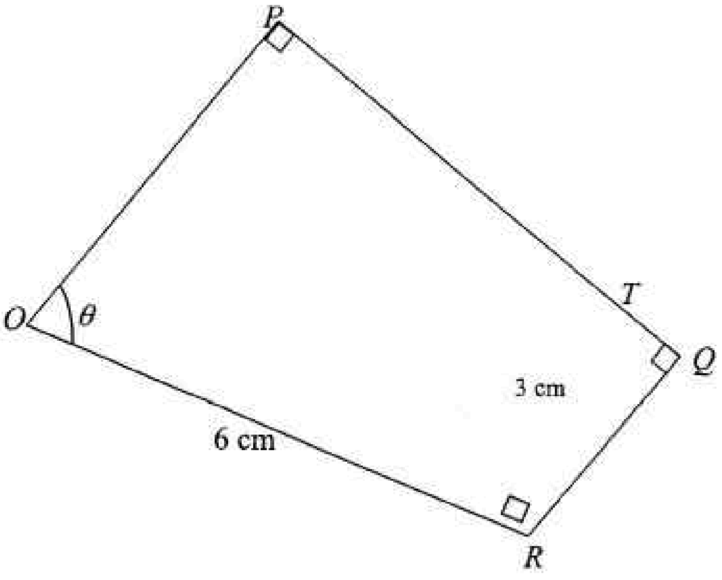


11

(i) Show that the area, A cm^2 of the quadrilateral $OPQR$ is given by

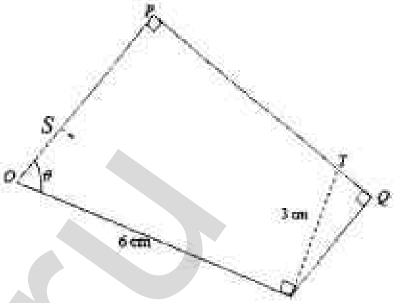
$$A = 9\sin 2\theta + 18\sin^2\theta \qquad [3]$$

(ii) Given that θ can vary, find maximum area of the quadrilateral $OPQR$.

[6]

$P\hat{S}R = \dfrac{\pi}{2}$ rad

$R\hat{T}Q = \theta$ (alt. ∠, $PQ//SR$)



$A = \dfrac{1}{2}(OS)(RS) + (RS)(RQ)$

$A = \dfrac{1}{2}(6\cos\theta)(6\sin\theta) + (6\sin\theta)(3\sin\theta)$ [M1][M1]

$A = 18\sin\theta\cos\theta + 18\sin^2\theta$ [A1]

$A = 9\sin 2\theta + 18\sin^2\theta$ (Shown)

$A = 9\sin 2\theta + 18\sin^2\theta$

$\dfrac{dA}{d\theta} = 18\cos 2\theta + 18(2)\sin\theta\cos\theta)$ [B1] [B1]

$= 18\cos 2\theta + 18\sin 2\theta$

For maximum area, $\dfrac{dA}{d\theta} = 0$.

$\dfrac{dA}{d\theta} = 18\cos 2\theta + 18\sin 2\theta = 0$ [B1]

$\cos 2\theta + \sin 2\theta = 0$

$1 + \tan 2\theta = 0$

$\tan 2\theta = -1$

Basic angle $= \dfrac{\pi}{4}$

$2\theta = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$

(N.A.)

$\theta = \dfrac{3\pi}{8}$ [A1]

$$\frac{d^2A}{d\theta^2} = -36\sin 2\theta + 36\cos 2\theta$$

[B1]

When $\theta = \frac{3\pi}{8}, \frac{d^2A}{d\theta^2} = -36\left(\frac{1}{\sqrt{2}}\right) + 36\left(-\frac{1}{\sqrt{2}}\right)$

$= -36\sqrt{2} < 0$

Therefore, maximum area

$= 9\sin 2\left(\frac{3\pi}{8}\right) + 18\sin^2\left(\frac{3\pi}{8}\right)$

$= \frac{9}{\sqrt{2}} + 18\left(\frac{1}{\sqrt{2}}\right)^2$

$= 9(1 + \frac{\sqrt{2}}{2})$

$= 15.4 \text{ cm}^2$ [A1]

10 A particle P moves in a straight line so that t seconds after passing through a fixed point O, its velocity, v m/s is given by

$$v_P = 1 - \frac{9}{(3t+1)^2}.$$

(i) Calculate the initial acceleration of the particle P. [2]

(ii) Show that the particle P is at instantaneously rest at $t = \frac{2}{3}$. [2]

(iii) Calculate the average speed of the particle P during the first 3 seconds after passing O. [4]

Another particle Q moves in a straight line and its displacement, S m from O after t seconds is given by

$$S_Q = t - 1$$

(iv) Find the distance from the fixed point O when P first collides with Q.

[2]

(i) $v_P = 1 - \frac{9}{(3t+1)^2}$

acceleration, a = $\frac{dv}{dt}$

$$a = \frac{54}{(3t+1)^3} \qquad [M1]$$

Initial acceleration = 54 m/s^2 [A1]

(ii) At instantaneously rest, $v_P = 0$

$1 - \frac{9}{(3t+1)^2} = 0$

$(3t+1)^2 = 9$ [M1]

3t + 1 = ±3

t = $\frac{2}{3}$ or $-\frac{4}{3}$

(reject)

$\therefore$ t = $\frac{2}{3}$ (Shown) [A1]

(iii) $S_P = \int [1 - \frac{9}{(3t+1)^2}]\, dx$

$S_P = t + \frac{3}{3t+1} + c$ [M1]

When $t = 0, S_P = 0,$

$0 = 3 + c$

$c = -3$

$\therefore S_P = t + \frac{3}{3t+1} - 3$ [A1]

When

$$t = 0, S = 0m$$

$$t = \frac{2}{3}, S = -1\frac{1}{3}m$$

$$t = 3, S = \frac{3}{10}m$$

average speed

$= \dfrac{\frac{4}{3} \times 2 + \frac{3}{10}}{3}$ [M1]

$= \dfrac{89}{90}$

$= 0.989$ m/s [A1]

(iv) When P collides with Q, $S_P = S_Q$,

$t + \dfrac{3}{3t+1} - 3 = t - 1$

$\dfrac{3}{3t+1} = 2$

$3t + 1 = \dfrac{3}{2}$

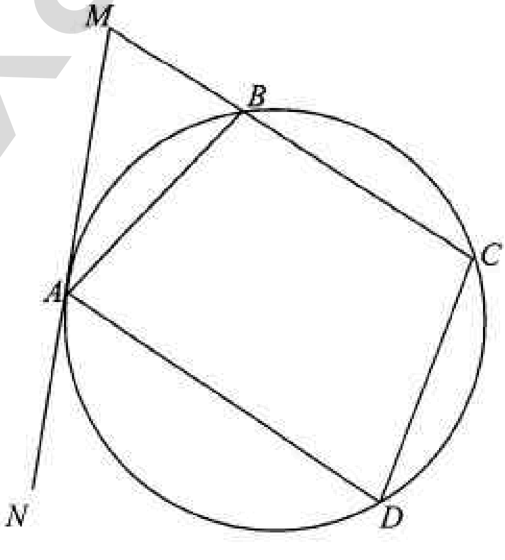
$t = \dfrac{1}{6}$ [M1]

When $t = \dfrac{1}{6}, S_Q = \dfrac{1}{6} - 1$

$S_Q = -\dfrac{5}{6}$ m [A1]

Hence, the particles first collides at $\frac{5}{6}$ m from the fixed point O. [A1]

11 In the diagram,. A, B, C and D are on the circle. MN is a tangent to the circle at A. MBC is a straight line.



15

(a) Name a triangle which is similar to triangle CAM. [1]

Hence prove that $\left(\frac{AC}{BA}\right)^2 = \frac{CM}{BM}$. [3]

(b) Given further that AD and BC are parallel, show that

(i) triangle ABM is similar to triangle ADC. [2]

(ii) $AD \times AM = AC \times CD$. [2]

(a)

$A\hat{M}B = C\hat{M}A$ (common angle)

$M\hat{A}B = M\hat{C}A$ (alternate segment theorem)

triangle CAM is similar to triangle ABM [B1]

$\frac{AC}{BA} = \frac{AM}{BM} = \frac{MC}{MA}$ [B1]

$\left(\frac{AC}{BA}\right)^2 = \left(\frac{AM}{BM}\right)^2$ [B1]

$= \frac{\mathrm{BM} \times \mathrm{MC}}{\mathrm{BM}^2}$ $\left(\mathrm{AM}^2 = MC \times BM\right)$ [B1]

$= \frac{\mathrm{MC}}{\mathrm{BM}}$

$\therefore \left(\frac{AC}{BA}\right)^2 = \frac{CM}{BM}$ (proved) [p if no conclusion]

(b)

$A\hat{B}M = A\hat{D}C$ (angle in opposite segment)

$M\hat{A}B = M\hat{C}A$ (alternate segment theorem)

$= C\hat{A}D$ (alternate angle, AD//BC)

triangle ABM is similar to triangle ADC [B 2,1,0]

$\frac{AD}{AB} = \frac{CD}{MB}$ [B1]

$\frac{AD}{CD} = \frac{AB}{MB}$

$\frac{AD}{CD} = \frac{AC}{AM}$ since $\frac{AB}{MB} = \frac{AC}{AM}$ (from part (a) [B1]

$AD \times AM = AC \times CD$ (Proved) [p if no conclusion]

~ End of Paper

(b) The equation of another circle is $(x-4)^2 + (y+1)^2 = 4$.

The line $y = mx$ is a tangent to the circle. Find the possible exact values of m. [4]

9(b)	For points of intersection,	
[4]	substitute $y = mx$ into $(x-4)^2 + (y+1)^2 = 4$	
	$(x-4)^2 + (mx+1)^2 = 4$	M1
	$x^2 - 8x + 16 + m^2x^2 + 2mx + 1 = 4$	
	$x^2(1+m^2) + x(2m-8) + 13 = 0$	
	For line to be a tangent to the circle, Discriminant = 0	
	$(2m-8)^2 - 4(1+m^2)13 = 0$	M1
	$4m^2 - 32m + 64 - 52 - 52m^2 = 0$	
	$0 = 48m^2 + 32m - 12$	
	$0 = 12m^2 + 8m - 3$	
	$m = \dfrac{-8 \pm \sqrt{64 - 4(12)(-3)}}{2(12)}$	
	$m = \dfrac{-8 \pm 4\sqrt{13}}{24}$	
	$m = \frac{-2 \pm \sqrt{13}}{6}$ also accept $m = -\frac{1}{3} \pm \frac{1}{6}\sqrt{13}$	A1, A1 Deduct 1 mark if answers are not in the lowest terms

10 **(a)** **(i)** Express $\frac{2x^3+x^2}{x^2+x-2}$ in the form of $ax+b+\frac{cx+d}{x^2+x-2}$. [2]

(ii) Using the values of c and d found in (i), express $\frac{cx+d}{x^2+x-2}$ as a sum of two partial fractions. [3]

10(a) (i)	By long division	M1
[2]	$\frac{2x^3+x^2}{x^2+x-2}=2x-1+\frac{5x-2}{x^2+x-2}$	A1
(ii) [3]	$\frac{5x-2}{(x+2)(x-1)}=\frac{A}{x+2}+\frac{B}{x-1}$	
	$5x-2=A(x-1)+B(x+2)$	M1
	Let $x=1$, $3=3B$	
	$B=1$	A1 for either
	Comparing coefficient of x, $A+B=5$	A or B correct
	$A=4$	
	$\frac{5x-2}{x^2+x-2}=\frac{4}{x+2}+\frac{1}{x-1}$	**A1**

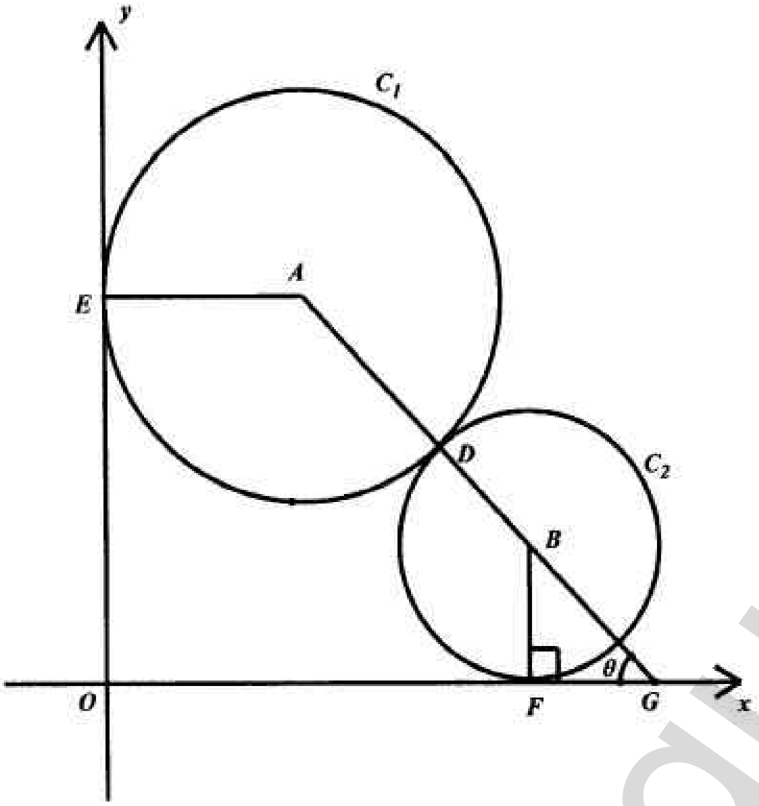
(b) A curve has the equation $y=\frac{x-1}{\sqrt{4x+1}}$.

(i) Differentiate y with respect to x. [3]

(ii) Using the result in part **b(i)**, determine $\int\frac{2(2x+3)}{(4x+1)^{\frac{3}{2}}}\,dx$. [2]

(b) (i) [3]	$\frac{dy}{dx}=\frac{(4x+1)^{\frac{1}{2}}\ (1)-(x-1)\times\frac{1}{2}(4x+1)^{-\frac{1}{2}}\times 4}{(4x+1)}$	M1 quotient rule M1 chain rule
	$=\frac{(4x+1)^{-\frac{1}{2}}[4x+1-2(x-1)]}{(4x+1)}$	
	$=\frac{2x+3}{(4x+1)^{\frac{3}{2}}}$	A1
(ii)	$\int\frac{2x+3}{(4x+1)^{\frac{3}{2}}}\,dx=\frac{x-1}{\sqrt{4x+1}}+c$	M1 Reverse differentiation)
	$\int\frac{2(2x+3)}{(4x+1)^{\frac{3}{2}}}\,dx=\frac{2(x-1)}{\sqrt{4x+1}}+c'$	A1

11.



The diagram shows two circles, C_1 and C_2 with centres *A* and *B* respectively. The two circles touch each other at *D*. C_1 has radius 3 units and touches the y-axis at E. C_2 has radius 2 units and touches the *x*-axis at *F*. The lines *AB* produced meets the *x*-axis at *G* and angle $BGO = \theta$ radians.

(i) Show with clear explanations, that $OE = 5\sin\theta + 2$ and $OF = 5\cos\theta + 3$. [2]

(ii) Show that $EF^2 = 38 + 20\sin\theta + 30\cos\theta$. [2]

(iii) Express EF^2 in the form $38 + R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [3]

(iv) Given that $EF^2 = 65$, find the value of θ. [2]

(i)	$AB = 3 + 2 = 5$cm	
[2]	$OE = AB\sin\theta + BF = 5\sin\theta + 2$	B1
	$OF = AB\cos\theta + AE = 5\cos\theta + 3$	B1

11(ii)	$EF^2 = (5\sin\theta + 2)^2 + (5\cos\theta + 3)^2$	M1
[2]	$= 25sin^2\theta + 20sin\theta + 4 + 25cos^2\theta + 30\cos\theta + 9$	
	$= 25(sin^2\theta + cos^2\theta) + 20sin\theta + 30\cos\theta + 13$	B1
	$= 38 + 20\sin\theta + 30\cos\theta$ (AG)	

11(iii)	$EF^2 = 38 + Rcos\,(\theta - \alpha)$	
[3]	$R = \sqrt{30^2 + 20^2} = 10\sqrt{13}$	B1
	$\alpha = tan^{-1}\left(\frac{20}{30}\right) = 0.58800$	B1
	$EF^2 = 38 + 10\sqrt{13}cos\,(\theta - 0.58800)$	A1

11(iv)	$EF^2 = 65$	
[2]	$65 = 38 + 10\sqrt{13}cos\,(\theta - 0.58800)$	
	$\frac{27}{10\sqrt{13}} = cos\,(\theta - 0.58800)$	M1
	$\theta - 0.58800 = 0.72448$	
	$\theta = 1.31$ (to 3 sig fig)	A1

TANJONG KATONG GIRLS' SCHOOL

PRELIMINARY EXAMINATION 2016
SECONDARY FOUR

4047/01

ADDITIONAL MATHEMATICS
PAPER 1

Thursday **11 August 2016** **2 h**

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper, and use a pencil for drawing graphs and diagrams.
Do not use staples, highlighters or correction fluid.

Answer **all** the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Setter : Ms Yeo
Markers : Mrs Pang / Mrs M Loy / Mdm Tan SE / Ms Yeo

This Question Paper consists of 7 printed pages, including this page.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

*Answer **all** questions*

1 It is given that $\cos A = -\frac{1}{3}$ and $\sin B = \sqrt{\frac{2}{11}}$. A and B are in the same quadrant.
Without using a calculator, find the exact value of $\cot(90° - A - B)$. [5]

2 **(i)** Find the range of values of p for which $(x+1)(x-2) > p(x+2)$ for all real values of x. [4]

(ii) Deduce the number of points at which the line $y = p(x+2)$ intersects the curve $y = (x+1)(x-2)$ for $-1 \leq p < 2$. [1]

3 2000 cm^3 of water is transferred from a rectangular tank to an empty inverted right circular cone in 10 seconds. The ratio of the radius of the cone to the height of the cone is 1 : 3.
Find the rate of change of the horizontal surface area, A cm^2, of the water in the cone, when the height, h cm, of the water in the cone is 12 cm. [6]

4 **(i)** Write down and simplify, the first 3 terms in the expansion of $(2-p)^7$ in ascending powers of p. [2]

(ii) Find the value of n where n is a positive integer, given that the coefficient of x^2 is 96 in the expansion of $(1+x)^n(2-x+x^2)^7$. [4]

5 A curve $y = f(x)$ is such that $f''(x) = 48\sin 4x - 8\cos 2x$. The curve intersects the x-axis at P. The x-coordinate of P is $\frac{\pi}{4}$ and the gradient of the curve at P is 8. Show that $f''(x) + 16f(x) = 24\cos 2x$. [7]

6 The table shows experimental values of two variables x and y.

x	2	4	6	7	8
y	1.33	2.29	3.27	3.77	6.12

It is known that x and y are related by an equation of the form $x^2 + \frac{y}{a} = bxy$, where a and b are constants. An error was made in recording one of the values of y.

(i) Using a scale of 2 cm to represent 1 unit on the horizontal axis and 1 cm to represent 1 unit on the vertical axis, draw a straight line graph for the above given data. The straight line graph is to be drawn with variable x on the horizontal axis. [3]

(ii) Use the graph to estimate

(a) the correct value of y, [2]

(b) the values of a and b. [3]

7 **(i)** Express $\frac{4}{(x-3)x^2}$ in partial fractions. [4]

(ii) Hence evaluate $\int_4^7 \frac{1}{(x-3)x^2}\,dx$. [4]

8 **(i)** Prove that $\frac{1-\sin x}{\cos x}+\frac{\cos x}{1-\sin x}=2\sec x$. [3]

(ii) In the equation

$$\frac{1-\sin x}{\cos x}+\frac{\cos x}{1-\sin x}+\tan^2 x=2,$$

$\cos x=a$ or b where a and b are constants, and $b<0$.

(a) Find the value of a and of b. [2]

(b) Solve the equation $\cos x=b$ for $-\pi\le x\le 2\pi$. [3]

9 The equation of a curve is $y=x\ln(2x-3)$ where $x>\frac{3}{2}$.

(i) Find the equation of the normal to the curve at $x=2$. [4]

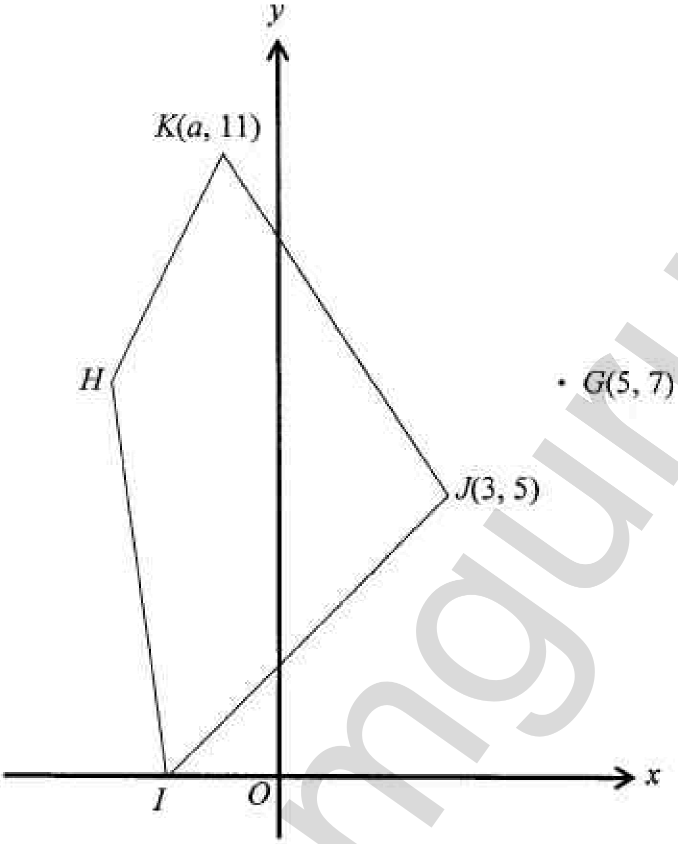
The normal to the curve $y=x\ln(2x-3)$ passes through the vertex of the graph of $y=k-4|2x+1|$ where k is a constant.

(ii) Determine the value of k. [2]

(iii) Sketch the graph of $y=k-4|2x+1|$ for the value of k in part **(ii)**.

Show the vertex and intercepts clearly. [2]

10 Solutions to this question by accurate drawing will not be accepted.



The diagram shows a quadrilateral *HIJK* . *H* is the reflection of point $G(5, 7)$ in the line $x = 1$. Point $K(a, 11)$ is such that the product of the gradients of *HK* and *JK* is -3. The perpendicular bisector of *HJ* intersects the x-axis at *I*.

(i) Deduce the coordinates of *H*. [1]

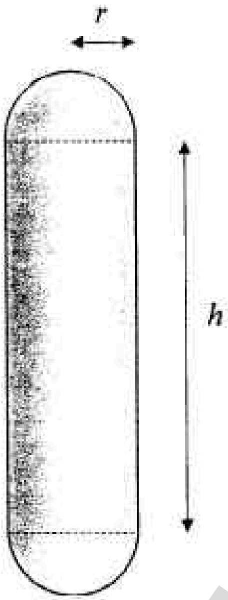
Find

(ii) the value of a given that $a < 0$, [2]

(iii) the equation of the perpendicular bisector of *HJ*, [3]

(iv) the area of quadrilateral *HIJK*. [3]

11



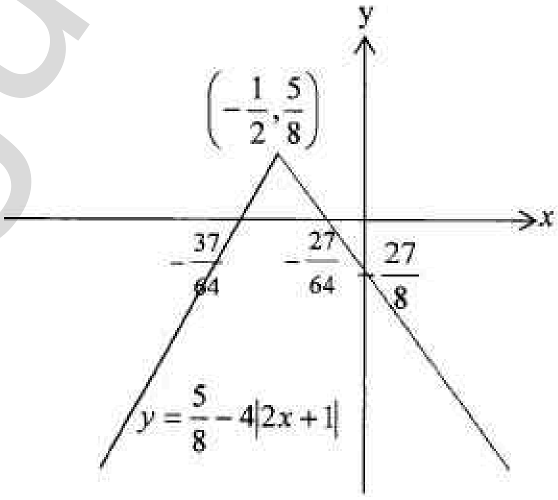
The diagram shows a capsule shaped object with surface area 18π cm^2. It comprised of 2 solid hemispheres of radius r cm joined to the 2 ends of a solid cylinder of radius r cm and height h cm.

(i) Show that the volume, V cm^3, of the object is given by $V = 9\pi r - \frac{2}{3}\pi r^3$. [4]

(ii) Find the stationary value of V, and determine if this stationary value is a maximum or minimum. [6]

THE END

Answer Key to TKGS Prelim 2016 Additional Mathematics Paper 1

1	$7\sqrt{2}$		8(i)	Proof
			(ii)(a)	$a=1$ and $b=-\frac{1}{3}$
2(i)	$-9<p<-1$		(ii)(b)	–1.91, 1.91, 4.37
2 (ii)	1 or 2 points			
			9(i)	$4y=-x+2$
3	$33\frac{1}{3}\text{ cm}^3/\text{s}$		(ii)	$\frac{5}{8}$
			(iii)	
4(i)	$128-448p+672p^2+$			
4(ii)	4			
5	proof			
			10(i)	(–3, 7)
6(ii)(a)	4.24		(ii)	-1
(b)	$a=1,\ \ b=2$		(iii)	$y=3x+6$
			(iv)	34 square units
7(i)	$\frac{4}{(x-3)x^2}=\frac{4}{9(x-3)}-\frac{4}{9x}-\frac{4}{3x^2}$		11(ii)	40.0 cm^3, Stationary value of V is a maximum.

2016 Sec 4 A Math Prelim Exam Paper 1 Marking Scheme

Qn	Solution	Marks	Teaching Points
1	*A* and *B* are in the same quadrant. ∴*A* and *B* are both in 2nd quadrant.		
	$\cos A = -\frac{1}{3}$ $\tan A = -\frac{\sqrt{8}}{1}$ $= -2\sqrt{2}$	B1	Understand how to find ratio of tan *A* from cos *A*.
	$\sin B = \sqrt{\frac{2}{11}}$ $= \frac{\sqrt{2}}{\sqrt{11}}$ $\tan B = -\frac{\sqrt{2}}{3}$	B1	Understand how to find tan *B* from sin *B*.
	$\cot(90° - A - B)$ $= \cot(90° - (A + B))$ $= \tan(A + B)$	B1	Know the relationships $\tan(90° - C) = \frac{1}{\tan C}$ and $\cot(90° - C) = \tan C$
	$= \frac{\tan A + \tan B}{1 - \tan A \tan B}$	M1	Know how to use the addition formula for $\tan(A + B)$
	$= \frac{-2\sqrt{2} - \frac{\sqrt{2}}{3}}{1 - (-2\sqrt{2})\left(-\frac{\sqrt{2}}{3}\right)}$		

	$= \dfrac{\dfrac{-7\sqrt{2}}{3}}{1-\dfrac{4}{3}}$ $= 7\sqrt{2}$	 A1	Able to simplify surds Correct final answer

Qn	Solution	Marks	Teaching Points
2(i)	$(x+1)(x-2) > p(x+2)$ $x^2 - x - 2 > px + 2p$ $x^2 + (-1-p)x - 2 - 2p > 0$		
	$(x+1)(x-2) > p(x+2)$ for all x $\Rightarrow$ discriminant < 0	B1	Know that discriminant < 0 for inequality to be true for all x.
	$\Rightarrow (-1-p)^2 - 4(1)(-2-2p) < 0$ $\Rightarrow 1 + 2p + p^2 + 8 + 8p < 0$	B1	Able to get expression for discriminant
	$\Rightarrow p^2 + 10p + 9 < 0$ $\Rightarrow (p+9)(p+1) < 0$	M1	Able to solve quad inequality
	$\Rightarrow -9 < p < -1$	A1	Correct answer
(ii)	Line $y = p(x+2)$ does not intersect curve $y = (x+1)(x-2)$ when p is in the range $-9 < p < -1$. For $p \geq -1$, line intersects curve at 1 or 2 points.	B1	Able to make a deduction from (i)

Qn	Solution	Marks	Teaching Points
3	V : volume of water in cone A : area of water surface on cone h : height of water in cone r : radius of the water surface t : time		
	$\frac{dV}{dt} = \frac{2000}{10}\text{cm}^3/\text{s}$ $= 200\text{ cm}^3/\text{s}$	B1	Know how to get $\frac{dV}{dt}$

	$\frac{r}{h}=\frac{1}{3}$ $V=\frac{1}{3}\pi r^2 h$ $=\frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$ $=\frac{\pi}{27}h^3$	B1	Know how to express V in terms of h.
	$\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}$ $200=\frac{\pi}{9}h^2\frac{dh}{dt}$ $\frac{dh}{dt}=\frac{1800}{\pi h^2}$	M1	Know how to use Chain Rule to get a relationship between $\frac{dV}{dt}$, $\frac{dV}{dh}$ and $\frac{dh}{dt}$.
	$A=\pi r^2$ $=\pi\left(\frac{1}{3}h\right)^2$ $=\frac{\pi}{9}h^2$	B1	Know how to express A in terms of h.
	$\frac{dA}{dt}=\frac{dA}{dh}\times\frac{dh}{dt}$ $=\frac{2\pi h}{9}\left(\frac{1800}{\pi h^2}\right)$ $=\frac{400}{h}$	M1	Know how to use Chain Rule to get a relationship between $\frac{dA}{dt}$, $\frac{dA}{dh}$ and $\frac{dh}{dt}$.
	When $h=12$, $\frac{dA}{dt}=\frac{400}{12}$ $=33\frac{1}{3}$	A1	Correct final answer.
	Answer : Rate of change of the horizontal surface area of the water $33\frac{1}{3}\text{ cm}^3/\text{s}$.		

Qn	Solution	Marks	Teaching Points
4(i)	$(2-p)^7$		
	$2^7-\binom{7}{1}2^6p+\binom{7}{2}2^5p^2+\ldots$	M1	Know formula for Binomial.expansion
	$=128-448p+672p^2+\ldots$	A1	Able to simplify
(ii)	$(1+x)^n(2-x+x^2)^7$		
	$=\left[1+\binom{n}{1}x+\binom{n}{2}x^2+\ldots\right]\left[2-(x-x^2)\right]^7$	B1	Know $p=x-x^2$
	$=\left[1+nx+\frac{n(n-1))}{2\times1}x^2+\ldots\right]\left[128-448(x-x^2)+672(x-x^2)^2+\ldots\right]$	B1	Able to express $\binom{n}{1}$ and $\binom{n}{2}$ correctly in terms of n.
	$=\left[1+nx+\frac{n(n-1))}{2}x^2+\ldots\right]\left[128-448x+1120x^2+\ldots\right]$		
	Coefficient of $x^2=96$		
	$1(1120)+n(-448)+\frac{n(n-1))}{2}(128)=96$	M1	Able to determine the terms in x^2 in the product of $(1+x)^n$ and $(2-x+x^2)^7$.
	$64n^2-512n+1024=0$		
	$n^2-8n+16=0$		
	$(n-4)(n-4)=0$		
	$n=4$	A1	Final answer

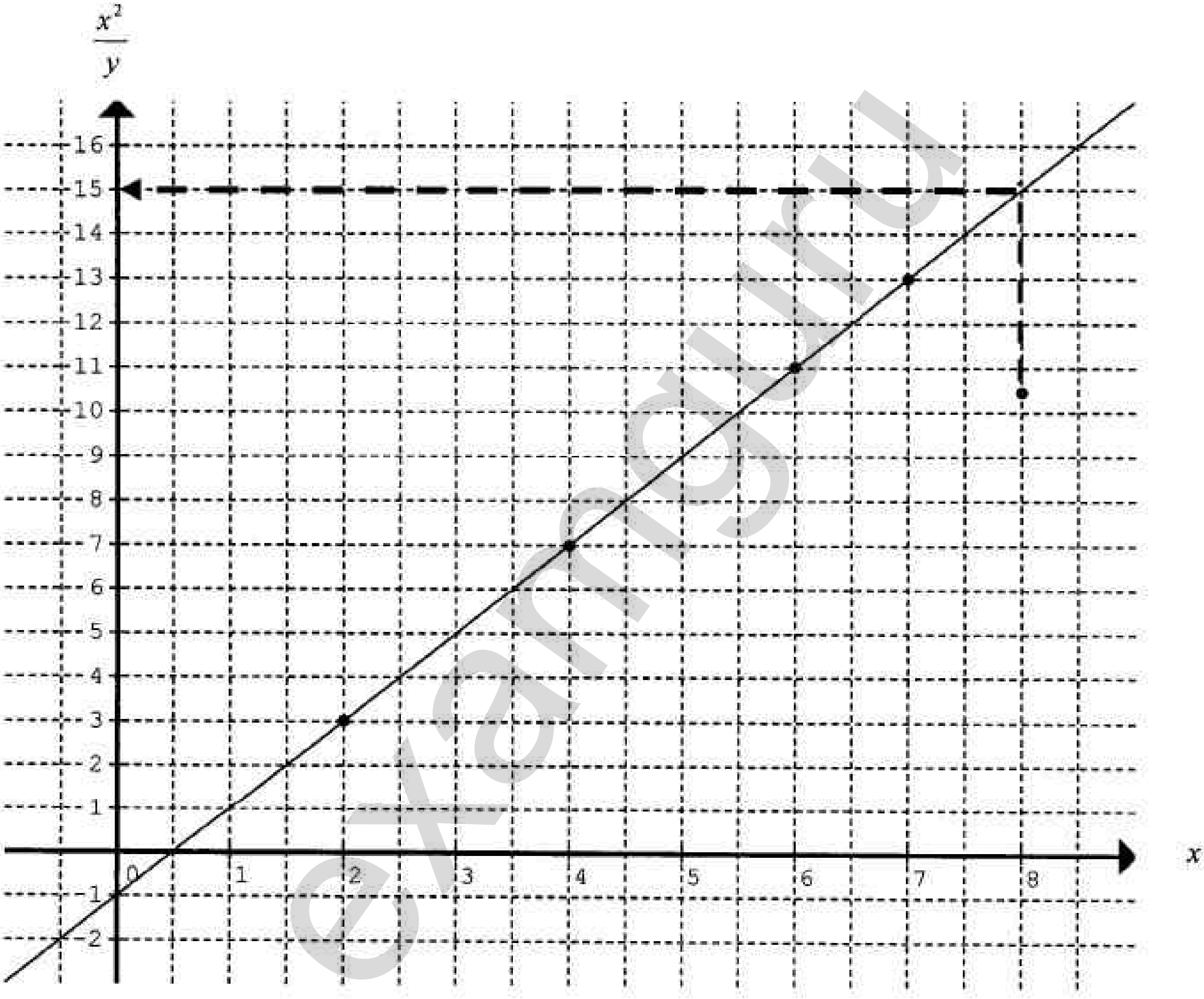
Qn	Solution	Marks	Teaching Points
5	$f''(x) = 48\sin 4x - 8\cos 2x$ $f'(x) = \int (48\sin 4x - 8\cos 2x)\,dx$ $= -12\cos 4x - 4\sin 2x + c_1$	B1	Know how to integrate $f''(x)$ correctly to get $f'(x)$
	$f'\left(\frac{\pi}{4}\right) = 8$ $-12\cos 4\left(\frac{\pi}{4}\right) - 4\sin 2\left(\frac{\pi}{4}\right) + c_1 = 8$ $-12(-1) - 4(1) + c_1 = 8$ $c_1 = 0$	M1	Know how to use the grad at *P* to get $f'(x)$
	$f'(x) = -12\cos 4x - 4\sin 2x$	A1	Correct expression for $f'(x)$
	$f(x) = \int (-12\cos 4x - 4\sin 2x)\,dx$ $= -3\sin 4x + 2\cos 2x + c_2$		Know how to integrate $f'(x)$ correctly to get $f(x)$
	$f\left(\frac{\pi}{4}\right) = 0$ $-3\sin 4\left(\frac{\pi}{4}\right) + 2\cos 2\left(\frac{\pi}{4}\right) + c_2 = 0$ $-3(0) + 2(0) + c_2 = 0$ $c_2 = 0$	M1	Know how to use the *x*-coordinate of *P* to get $f(x)$
	$f(x) = -3\sin 4x + 2\cos 2x$	A1	Correct expression for $f(x)$
	$f''(x) + 16f(x)$ $= (48\sin 4x - 8\cos 2x) + 16(-3\sin 4x + 2\cos 2x)$ $= 24\cos 2x$ (Proved)	M1	sub. expressions for $f(x)$ and $f''(x)$
		A1	Able to get $24\cos 2x$

Qn	Solution	Marks	Teaching Points
6(i)	$x^2+\frac{y}{a}=bxy$ $\frac{x^2}{y}+\frac{1}{a}=bx$ $\frac{x^2}{y}=bx-\frac{1}{a}$	B1	Able to transform given equation into a straight line form with *x* on horizontal axis.
	Graph	B1	Able to plot a straight line passing through all points
		B1	Graph cuts *y*-axis.
(ii)(a)	Correct reading of $\frac{x^2}{y}=15.1$ $\frac{8^2}{y}=15.1$	M1	Know the method to find the correct reading of *y*
	Correct reading of $y=4.24$	A1	Correct final answer
(b)	$-\frac{1}{a}=\frac{x^2}{y}-\text{intercept}$ $=-1$ $a=1$	B1	Understand how to get *a* using the vertical intercept
	$b=\text{Gradient}$ $=\frac{11.01-3.01}{6-2}$	M1	Understand that *b* is the gradient
	$=2$	A1	Correct value of *b*

6(i)

x	2	4	6	7	8
y	1.33	2.29	3.27	3.77	6.12
$\frac{x^2}{y}$	3.01	6.99	11.01	13.00	10.46

Scale :
x-axis : 2 cm to 1 unit
$\frac{x^2}{y}$ axis : 1 cm to 1 unit



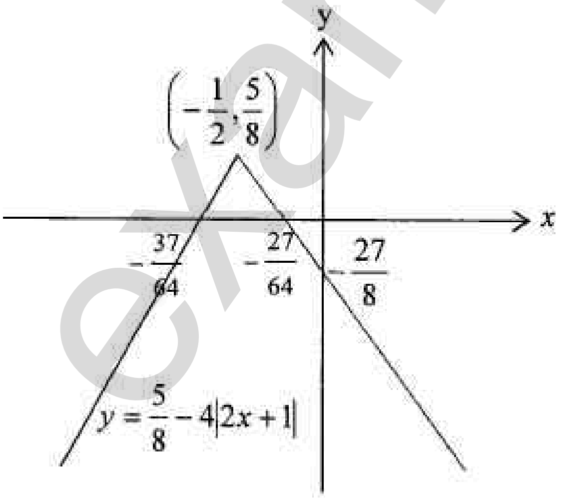
Qn	Solution	Marks	Teaching Points
7(i)	$\frac{4}{(x-3)x^2}=\frac{A}{x-3}+\frac{B}{x}+\frac{C}{x^2}$ $4=Ax^2+Bx(x-3)+C(x-3)$	B1	Know the various partial fraction forms.
	Consider $x = 0$: $4=C(-3)$ $C=-\frac{4}{3}$	B1	Able to use suitable method to find *C*.
	Consider $x=3$: $4=9A$ $A=\frac{4}{9}$	B1	Able to use suitable method to find *A*.
	Compare coefficient of x^2 : $0=A+B$ $B=-A$ $=-\frac{4}{9}$	B1	Able to use suitable method to find *B*.
	Hence $\frac{4}{(x-3)x^2}=\frac{4}{9(x-3)}-\frac{4}{9x}-\frac{4}{3x^2}$		Minus 1 mark if didn't write final line.

(ii)	$\int_4^7 \frac{1}{(x-3)x^2}\mathrm{d}x$ $=\frac{1}{4}\int_4^7 \frac{4}{(x-3)x^2}\mathrm{d}x$		
	$=\frac{1}{4}\int_4^7 \left(\frac{4}{9(x-3)}-\frac{4}{9x}-\frac{4}{3x^2}\right)\mathrm{d}x$	B1	Know the formula $\int \frac{1}{ax+b}dx = \ln x + c$
	$=\frac{1}{4}\left[\frac{4}{9}\ln(x-3)-\frac{4}{9}\ln x-\frac{4}{3}\left(-x^{-1}\right)\right]_4^7$	B1	Know the formula $\int x^n dx = \frac{x^{n+1}}{n+1}+c$
	$=\frac{1}{4}\left(\frac{4}{9}\ln 4-\frac{4}{9}\ln 7+\frac{4}{3}\left(\frac{1}{7}\right)\right)-\frac{1}{4}\left(\frac{4}{9}\ln 1-\frac{4}{9}\ln 4+\frac{4}{3}\left(\frac{1}{4}\right)\right)$	M1	Know how to evaluate a definite integral
	$= 0.0561$	A1	Correct final answer

Qn	Solution	Marks	Teaching Points
8(i)	$\frac{1-\sin x}{\cos x}+\frac{\cos x}{1-\sin x}$ $=\frac{(1-\sin x)^2+\cos^2 x}{\cos x(1-\sin x)}$ $=\frac{1-2\sin x+\sin^2 x+\cos^2 x}{\cos x(1-\sin x)}$		
	$=\frac{1-2\sin x+1}{\cos x(1-\sin x)}$	B1	Knows the identity $\sin^2 x+\cos^2 x=1$
	$=\frac{2(1-\sin x)}{\cos x(1-\sin x)}$	B1	Is aware of 'factorisation' as one technique used in proofs.
	$=\frac{2}{\cos x}$		
	$=2\sec x$	B1	Know the identity $\sec x=\frac{1}{\cos x}$

(ii)(a)	$\frac{1-\sin x}{\cos x}+\frac{\cos x}{1-\sin x}+\tan^2 x=2$		
	$2\sec x+\tan^2 x=2$		
	$2\sec x+\sec^2 x-1=2$	B1	Use identity $1+\tan^2 x=\sec^2 x$
	$\sec^2 x+2\sec x-3=0$		
	$(\sec x-1)(\sec x+3)=0$		
	$\sec x=1$ or $\sec x=-3$		
	$\cos x=1$ or $\cos x=-\frac{1}{3}$		
	Answer : $a=1$ and $b=-\frac{1}{3}$	A1	Correct final answer
(b)	$\cos x=-\frac{1}{3}$ Basic $\angle$ = 1.2310 radians $x=-1.91,\ 1.91,\ 4.37$	B1,B1,B1	Correct angles

Qn	Solution	Marks	Teaching Points
9(i)	$y = x\ln(2x-3)$ $\frac{dy}{dx} = x\left(\frac{2}{2x-3}\right) + \ln(2x-3)$	M1 M1	Use Product Rule to diff $x\ln(2x-3)$ Use Chain Rule to diff $\ln(2x-3)$
	At $x = 2$, $\frac{dy}{dx} = 2\left(\frac{2}{2(2)-3}\right) + \ln(2(2)-3)$ $= 4$ *and* $y = 2\ln(2(2)-3)$ $= 0$	M1	Know how to find gradient, y-coordinate and equation of normal
	Equation of normal : $\frac{y-0}{x-2} = -\frac{1}{4}$ $4y = -x + 2$	A1	Correct answer for equation of normal.

Qn	Solution	Marks	Teaching Points
9(ii)	Equation of normal : $4y=-x+2$ $y=k-4\lvert 2x+1\rvert$ Coordinate of vertex : $\left(-\frac{1}{2},k\right)$ When $x=-\frac{1}{2}$, $4y=-\left(-\frac{1}{2}\right)+2$ $y=\frac{5}{8}$ $k=\frac{5}{8}$	M1 A1	Understand that the x-coordinate of vertex is $-\frac{1}{2}$ and that k is obtained when $\lvert 2x+1\rvert=0$. Correct value of k
(iii)		B1 B1	Shape Critical pts

Qn	Solution	Marks	Teaching Points
10(i)	Coordinates of H are (−3, 7).	B1	Know how to get image of a point given the line of reflection
(ii)	Gradient of HK × Gradient of JK = −3 $\frac{11-7}{a+3}\times\frac{11-5}{a-3}=-3$ $\frac{24}{(a+3)(a-3)}=-3$ $a^2-9=-8$ $a^2=1$ $a=1$ (reject) or -1	M1 A1	Know how to get a relationship between the 2 gradients Correct value of a
(iii)	Midpoint of HJ $=\left(\frac{-3+3}{2},\frac{7+5}{2}\right)$ $=(0, 6)$ Gradient of HJ $=\frac{7-5}{-3-3}$ $=-\frac{1}{3}$ Equation of ⊥ bisector of HJ: $\frac{y-6}{x-0}=3$ $y=3x+6$	B1 M1 A1	Know formula for midpoint Know how to get ⊥ bisector Correct answer

(iv)	$y=3x+6$ When $y=0$, $0=3x+6$ $x=-2$ Coordinates of $I=(-2, 0)$	B1	Know how to find coordinates of *I*
	Area of *HIJK* $=\frac{1}{2}\begin{vmatrix}-2 & 3 & -1 & -3 & -2\\ 0 & 5 & 11 & 7 & 0\end{vmatrix}$ $=\frac{1}{2}\{(-2)5+3(11)+(-1)7+(-3)0-3(0)-(-1)5-(-3)11-(-2)7\}$	M1	Know the formula for area of polygon
	$=34$ square units	A1	Correct final answer

Qn	Solution	Marks	Teaching Points
11(i)	$2\pi rh+2\left(2\pi r^2\right)=18\pi$ $h=\dfrac{18\pi-4\pi r^2}{2\pi r}$	M1	Able to get a relationship between r, h and area.
	$=\dfrac{9}{r}-2r$	A1	Correct expression for h in terms of r.
	$V=\pi r^2h+\left(\dfrac{2}{3}\pi r^3\right)2$ $=\pi r^2h+\dfrac{4}{3}\pi r^3$ $=\pi r^2\left(\dfrac{9}{r}-2r\right)+\dfrac{4}{3}\pi r^3$	M1	Able to get V in terms of r and h.
	$=9\pi r-2\pi r^3+\dfrac{4}{3}\pi r^3$ $=9\pi r-\dfrac{2}{3}\pi r^3$	A1	Correct expression for V in terms of r.
(ii)	$V=9\pi r-\dfrac{2}{3}\pi r^3$ $\dfrac{dV}{dr}=9\pi-2\pi r^2$	B1	Able to differentiate V
	For stationary value of V, $\dfrac{dV}{dr}=0$	M1	Know requirement for stationary pt.
	$9\pi-2\pi r^2=0$ $r=\sqrt{\dfrac{9}{2}}$	A1	Able to get value of r at stationary pt.
	Stationary value of V		

	$= 9\pi\sqrt{\frac{9}{2}} - \frac{2}{3}\pi\left(\sqrt{\frac{9}{2}}\right)^3$ $= 40.0 \text{ cm}^3$ $\frac{d^2V}{dr^2} = -4\pi r$, When $r = \sqrt{\frac{9}{2}}$, $\frac{d^2V}{dr^2} = -4\pi\sqrt{\frac{9}{2}} < 0$ $\therefore$ Stationary value of V is a maximum.	B1 M1 A1	Correct stationary value of V Know the tests to determine nature of stationary pts Able to determine correctly the nature of the stationary value.

TANJONG KATONG GIRLS' SCHOOL

PRELIMINARY EXAMINATION 2016
SECONDARY FOUR

4047/02 **ADDITIONAL MATHEMATICS**

PAPER 2

Friday **5 August 2016** **2 h 30 min**

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper, and use a pencil for for any diagrams or graphs.
Do not use staples, highlighters or correction fluid.

Answer **all** the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

Setter : Mrs M Loy
Markers: Mdm Tan SE, Mrs H Pang, Miss Yeo LS, Mrs M Loy

This Question Paper consists of 7 printed pages, including this page.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

Answer **all** the questions

1. A man buys a new car. The value of the car depreciates with time so that its value, $\$V$, after t months' use is given by $V = 132\,000e^{-pt}$, where p is a constant.
The value of the car is expected to be $122 000 after eight months' use.

(i) Find the value of the car, V when the man bought it. [1]

(ii) Show that $p = 0.01$. [2]

(iii) Using the value of $p = 0.01$, determine the age of the car to the nearest month, when its value reached half of the original value when the man bought it. [2]

2. The function $f(x) = 1 + 2x + Ax^2 - x^3$, where A is a constant, leaves a remainder of $1\frac{3}{8}$ when divided by $(2x-1)$.

(i) Find the value of A. [2]

(ii) Hence solve the equation $f(x) = 0$, giving your answers in the exact form. [4]

3. **(a)** **(i)** Solve $\sqrt{3x+2} - 3x = 0$. [2]

(ii) On the same axes, sketch the graphs of $y = \sqrt{3x+2}$ and $y = 3x$. Indicate clearly all the points of intersections. [2]

(b) The vertical height of a triangle is $\frac{8}{3-\sqrt{5}}$ cm.
Given that the area of the triangle is $\frac{20}{\sqrt{5}-1}$ cm^2, without using a calculator, find the length of the base of the triangle in the form $a + b\sqrt{5}$. [3]

4. The roots of the quadratic equation, $2x^2+4x+5=0$ are $(\alpha+1)$ and $(\beta+1)$.

(i) Show that $\alpha+\beta=-4$ and hence find $\alpha\beta$. [3]

(ii) Find the quadratic equation in x with integer coefficients, whose roots are $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$. [5]

5. **(a)** Given that $\log_2(2x+1)-\log_4(3-x^2)=1$, form a quadratic equation in x and explain with clear working why the roots of the quadratic equation are real and distinct. [5]

(b) Solve $3^{y+2}=2(3^{-y})+17$. [4]

6. The curve $y=\frac{2x^2}{x^2+1}$ has one stationary point $(p\,,q)$.

(i) Find the value of p and of q. [4]

(ii) Determine whether y is increasing or decreasing for

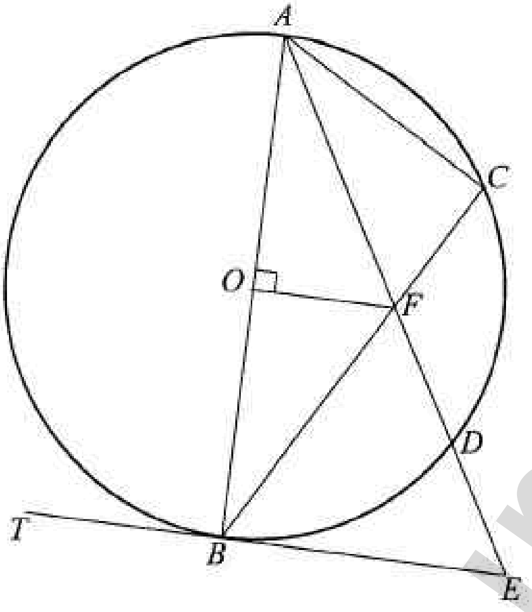
(a) $x>p$, [1]

(b) $x<p$. [1]

Hence state the nature of the stationary point. [1]

(iii) Find $\frac{d^2y}{dx^2}$ at the stationary point and explain how $\frac{d^2y}{dx^2}$ further supports your answer in part **(ii)**. [2]

7.



In the figure, AB is a diameter of the circle with centre O. Chords AD and BC intersect at F. AD produced meets the tangent to the circle, TBE at E. AE is an angle bisector of angle BAC.

(i) Prove that $\angle CBD = \angle DBE$. [3]

Given that $\angle AOF = 90^\circ$, prove that

(ii) triangle AOF is similar to triangle ADB. [2]

(iii) $2(AO)^2 = AF \times (AF + FD)$. [3]

8. A particle moving in a straight line passes through a fixed point O with a speed of 20 m/s. The acceleration, a m/s^2, of the particle, t s after passing through O is given by $a = -100e^{-3t}$. The particle comes to instantaneous rest at point N.

(i) Find the time the particle comes to instantaneous rest at point N. [5]

(ii) Calculate the distance ON. [4]

(iii) Show that the average speed of the particle in the first 2 seconds rounded off to a whole number is 10 m/s. [3]

9. **(i)** Solve the equation $2\sin 2P = 3\cos P$ for $0^\circ \leq P \leq 360^\circ$. [4]

(ii) On the same axes, sketch for $0^\circ \leq x \leq 720^\circ$, the graphs of

$$y = \sin x \quad \text{and} \quad y = \frac{3}{2}\cos\left(\frac{x}{2}\right).$$ [4]

(iii) Using the solutions to part **(i)**, determine the x-coordinates of the points of intersection of the graphs of part **(ii)**. [4]

10. A circle, C_1, has equation $x^2 + y^2 - 14x + 2y = -46$.

(i) Find the coordinates of the centre of the circle and the radius. [3]

The coordinates of the centre of a second circle, C_2, is $(-4, -2)$. The equation of the tangent to the circle, C_2 at a point P is $2y = -2x + 3$.

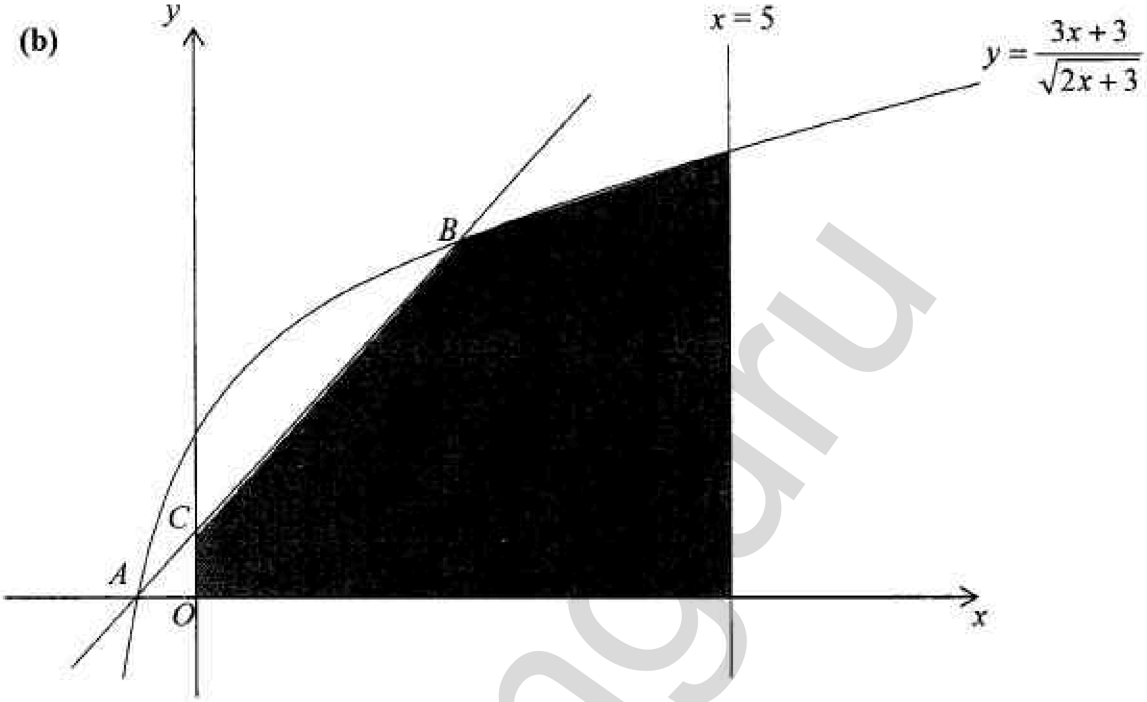
(ii) Find the coordinates of point P. [4]

(iii) Find the exact value of the radius of C_2 and the equation of the circle, C_2. [3]

(iv) Determine whether circles C_1 and C_2 will meet each other, showing your working clearly. [2]

11. **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(2x\sqrt{2x+3}\right)=\frac{6x+6}{\sqrt{2x+3}}$. [3]

(b)



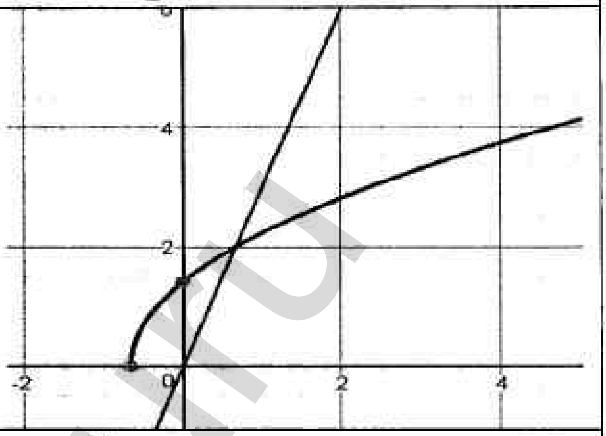
The diagram shows part of the curve $y=\frac{3x+3}{\sqrt{2x+3}}$. The curve intersects the x-axis at point A. The line through A and perpendicular to the line, $y+x=-7$ intersects the curve again at another point, B.

(i) Show that the y-coordinate of point B is 4. [5]

(ii) Given that the line AB intersects the y-axis at C, determine the area of the shaded region bounded by the line CB, the curve, the line $x = 5$, the x-axis and the y-axis. [4]

End of Paper

TKGS S4 PRELIM 2016 Answer Key:

1(i)	$V = 132\ 000$	(ii)	show
(iii)	70 months		
2(i)	$A = -2$	(ii)	$x = 1\ ,\ \frac{-3 \pm \sqrt{5}}{2}$
3(a)i	$x = \frac{2}{3}$	Ii	
(b)	$\frac{5\sqrt{5}}{2} - \frac{5}{2}$	4(i)	$\alpha\beta = \frac{11}{2}$
4(ii)	$1331x^2 - 16x + 8 = 0$	5(a)	Discriminant = 368 Since discriminant > 0, the roots of the quadratic equation are real and distinct.
5(b)	$y = 0.631$	6(i)	$p = 0,\ q = 0$
6(ii)a	$\frac{\mathrm{d}y}{\mathrm{d}x} > 0$, y is increasing	6(ii)b	$\frac{\mathrm{d}y}{\mathrm{d}x} < 0$, y is decreasing
	Since the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ changes from negative to positive value, the stationary point is a minimum point.	6(iii)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 4$, since $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0$, the stationary point is minimum, thus reiterating the result from part **(ii)**.
7	proof	8.(i)	$t = 0.305$ s
8(ii)	Distance = 2.59 m	8(iii)	show
9(i)	48.6°, 90°, 131.4°, 270°	(ii)	
9(iii)	97.2°,180°,262.8°,540°	10(i)	Centre(7, −1), radius = 2 units
10(ii)	$P(-\frac{1}{4}, \frac{7}{4})$	10(iii)	Radius = $\frac{15\sqrt{2}}{4}$ units

			$(x+4)^2+(y+2)^2=(\frac{15\sqrt{2}}{4})^2/\frac{225}{8}$
(iv)	Sum of radii(7.30 units) < distance between the centres (11.0 units) thus the circles will not meet.	11(a)	show
11(b)i	show	11(b)ii	16.5 units^2

4047/02 Prelim 2016 Suggested Solutions

1. A man buys a new car. The value of the car depreciates with time so that its value, $\$V$, after t months' use is given by $V = 132\,000e^{-pt}$, where p is a constant.
The value of the car is expected to be \$122 000 after eight months' use.

(i) Find the value of the car when the man bought it.

$V = 132000e^{-pt}$
When the man bought the car, $t = 0$.
Hence, $V = 132000e^{0}$, $e^{0} = 1$
$\therefore V = 132\,000$.

(ii) Show that $p = 0.01$.

$V = 122000$ when $t = 8$

$$122000 = 132000e^{-8p}$$

$$e^{-8p} = \frac{122000}{132000}$$

$$-8p = \ln\left(\frac{122000}{132000}\right)$$

$$p = -\frac{1}{8}\ln\left(\frac{122000}{132000}\right)$$

$$p = 0.009848$$

$$p = 0.01 \text{ (1 sig fig) (shown)}$$

(iii) Using the value of $p = 0.01$, determine the age of the car to the nearest month, when its value reached half of the original value when the man bought it.

$$132000e^{-0.01t} = \frac{1}{2} \times 132000$$

$$e^{-0.01t} = \frac{1}{2}$$

$$-0.01t = \ln\left(\frac{1}{2}\right)$$

$$t = 69.3147$$

$$t = 70 \text{ months}$$

2. The function $f(x)=1+2x+Ax^2-x^3$, where A is a constant, leaves a remainder of $1\frac{3}{8}$ when divided by $(2x-1)$.

(i) Find the value of A.

$f(x)=1+2x+Ax^2-x^3$

$f(\frac{1}{2})=1\frac{3}{8}$

$1+2(\frac{1}{2})+A(\frac{1}{2})^2-(\frac{1}{2})^3=\frac{11}{8}$

$\frac{1}{4}A=\frac{11}{8}-\frac{15}{8}$

$A=-2$

(ii) Hence, solve the equation $f(x)=0$, giving your answers in the exact form.

$f(x)=1+2x-2x^2-x^3$

$f(1)=1+2-2-1$

$f(1)=0$

$\therefore (x-1)$ is a factor

$f(x)=(x-1)(-x^2+ax-1)$

Compare coefficient of x:

$-1-a=2$

$a=-3$

$f(x)=(x-1)(-x^2-3x-1)$

$f(x)=0$,

$x=1$

$x^2+3x+1=0$

$$x=\frac{-3\pm\sqrt{(-3)^2-4(1)(1)}}{2(1)}$$

$$x=\frac{-3\pm\sqrt{5}}{2}$$

3.(a) **(i)** Solve $\sqrt{3x+2}-3x=0$.

$\sqrt{3x+2}-3x=0$

$\sqrt{3x+2}=3x$

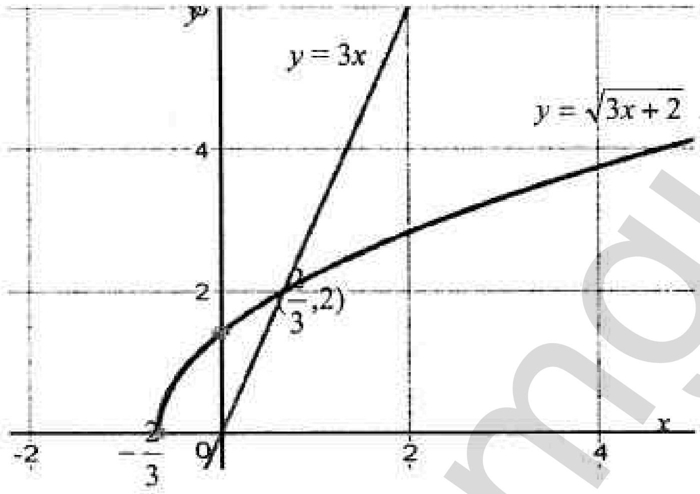
$3x+2=9x^2$

$9x^2-3x-2=0$

$(3x+1)(3x-2)=0$

$x=\frac{2}{3}, \qquad x=-\frac{1}{3}$ (rejected)

(ii) On the same axes, sketch the graphs of $y=\sqrt{3x+2}$ and $y=3x$.
Indicate clearly all the points of intersections.



(b) The vertical height of a triangle is $\frac{8}{3-\sqrt{5}}$ cm. Given that the area of the triangle is $\frac{20}{\sqrt{5}-1}$ cm², without using a calculator, find the length of the base of the triangle in the form $a+b\sqrt{5}$.

$\frac{1}{2}$ base of triangle $\times \frac{8}{3-\sqrt{5}}=\frac{20}{\sqrt{5}-1}$

$$\text{base of triangle}=\frac{20}{\sqrt{5}-1}\times\frac{3-\sqrt{5}}{4}$$

$$=\frac{5(3-\sqrt{5})}{\sqrt{5}-1}\times\frac{\sqrt{5}+1}{\sqrt{5}+1}$$

$$=\frac{5(2\sqrt{5}-2)}{5-1}$$

$$=\frac{5\times2(\sqrt{5}-1)}{4}$$

$$=\frac{5}{2}\sqrt{5}-\frac{5}{2}$$

4. The roots of the quadratic equation, $2x^2 + 4x + 5 = 0$ are $(\alpha + 1)$ and $(\beta + 1)$.

(i) Show that $\alpha + \beta = -4$ and hence find $\alpha\beta$.

Sum of roots :

$(\alpha + 1) + (\beta + 1) = -2$

$\alpha + \beta = -4$ (shown)

Product of roots:

$$(\alpha + 1)(\beta + 1) = \frac{5}{2}$$

$$\alpha\beta + (\alpha + \beta) + 1 = \frac{5}{2}$$

$$\alpha\beta = \frac{5}{2} - 1 - (-4)$$

$$\alpha\beta = \frac{11}{2}$$

(ii) Find the quadratic equation in x with integer coefficients, whose roots are $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$.

Sum of roots of new equation:

$$\begin{aligned}\frac{1}{\alpha^3} + \frac{1}{\beta^3} &= \frac{\alpha^3 + \beta^3}{(\alpha\beta)^3} \\ &= \frac{(\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)}{(\alpha\beta)^3} \\ &= \frac{(\alpha + \beta)[(\alpha + \beta)^2 - 2\alpha\beta - \alpha\beta]}{(\alpha\beta)^3} \\ &= \frac{(-4)[(-4)^2 - 3(\frac{11}{2})]}{(\frac{11}{2})^3} \\ &= \frac{16}{1331}\end{aligned}$$

Product of roots of new equation:

$$\frac{1}{(\alpha\beta)^3}$$

$$= \frac{1}{\left(\frac{11}{2}\right)^3}$$

$$= \frac{8}{1331}$$

Equation is $1331x^2 - 16x + 8 = 0$.

5(a) Given that $\log_2(2x+1)-\log_4(3-x^2)=1$, form a quadratic equation in x and explain why the roots of the quadratic equation are real and distinct.

$$\log_2(2x+1)-\log_4(3-x^2)=1$$

$$\log_2(2x+1)-\frac{\log_2(3-x^2)}{\log_2 2^2}=1$$

$$\log_2(2x+1)-\frac{1}{2}\log_2(3-x^2)=1$$

$$\log_2\frac{(2x+1)}{\sqrt{3-x^2}}=1$$

$$\frac{2x+1}{\sqrt{3-x^2}}=2$$

$$2x+1=2\sqrt{3-x^2}$$

$$(2x+1)^2=4(3-x^2)$$

$$4x^2+4x+1=12-4x^2$$

$$8x^2+4x-11=0$$

$$\text{Discriminant}=4^2-4(8)(-11)$$
$$=368$$

Since the discriminant is greater than 0, the roots of the quadratic equation are real and distinct.

5(b) Solve $3^{y+2}=2(3^{-y})+17$.

$$3^{y+2}=2(3^{-y})+17$$

$$3^{2(y+1)}-17(3^y)=2$$

$$3^2(3^y)^2-17(3^y)=2$$

Let $a=3^y$,

$$9a^2-17a-2=0$$

$$(9a+1)(a-2)=0$$

$a=-\frac{1}{9}$ (rejected), $\quad a=2$

$$3^y=2$$

$$y=\frac{\lg 2}{\lg 3}$$

$$y=0.631$$

6. The curve $y = \frac{2x^2}{x^2+1}$ has one stationary point (p, q).

(i)

Find the value of p and of q .
$y = \frac{2x^2}{x^2+1}$ $\frac{dy}{dx} = \frac{(x^2+1)(4x) - 2x^2(2x)}{(x^2+1)^2}$ $= \frac{4x^3 + 4x - 4x^3}{(x^2+1)^2}$ $= \frac{4x}{(x^2+1)^2}$ For a stationary point, put $\frac{dy}{dx} = 0$. $(x^2+1)^2 > 0, \quad 4x = 0, \therefore x = 0$ $\therefore p = 0$ and $q = 0$

(ii) determine whether y is increasing or decreasing

(a)

for $x > p$,		
For $x > 0$, $(x^2+1)^2 > 0$ and $4x > 0$, $x > 0$ $\therefore \frac{dy}{dx} > 0$, y is increasing		

(b)

for $x < p$.
$x < 0$, $(x^2+1)^2 > 0$ but $4x < 0$, i.e. $x < 0$ $\therefore \frac{dy}{dx} < 0$, y is decreasing

Hence state the nature of the stationary point.
Since the value of $\frac{dy}{dx}$ changes from negative to positive, the stationary point is a minimum point.

(iii) Find $\frac{d^2y}{dx^2}$ at the stationary point and explain how $\frac{d^2y}{dx^2}$ further supports your answer to part **(ii)**.

$$\frac{dy}{dx} = \frac{4x}{(x^2+1)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(x^2+1)^2(4) - 4x(2)(x^2+1)(2x)}{(x^2+1)^4}$$

$$\frac{d^2y}{dx^2} = \frac{4(x^2+1)(x^2+1-4x^4-4x^2)}{(x^2+1)^4}$$

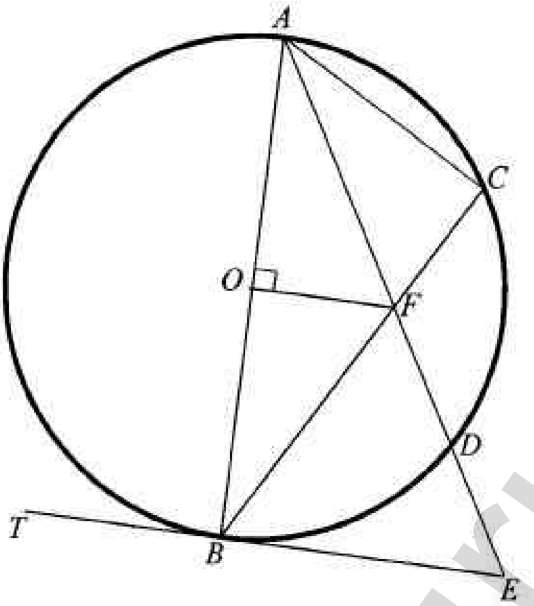
$$= \frac{4(1-3x^2-4x^4)}{(x^2+1)^3}$$

At the stationary point (0 , 0),

$$\frac{d^2y}{dx^2} = 4$$

Since, $\frac{d^2y}{dx^2} > 0$ the stationary point is minimum, thus reiterating the result from part **(ii)**

7.



In the figure, AB is a diameter of the circle with centre O. Chords AD and BC intersect at F. AD produced meets the tangent to the circle, TBE at E. AE is an angle bisector of angle BAC.

(i) Prove that $\angle CBD = \angle DBE$.

$\angle DBE = \angle BAD$ (Alternate segment Thm)

$\angle BAD = \angle CAD$ (given EA is bisector of $\angle BAC$)

$\therefore \angle DBE = \angle CAD$

$\angle CAD = \angle CBD$ (angles in the same segment)

$\angle DBE = \angle CBD$ (proven)

(ii) Given that $\angle AOF = 90°$, prove that

triangle AOF is similar to triangle ADB.

$\angle A$ is a common angle.

$\angle ADB = 90°$ (angle in the semi-circle)

$\angle ADB = \angle AOF$

$\therefore \Delta AOF$ is similar to ΔADB (By AA similarity test)

(iii) $2(AO)^2 = AF \times (AF + FD)$.

Since ΔAOF is similar to ΔADB,

$$\frac{AO}{AD} = \frac{AF}{AB}$$

$$\frac{AO}{AF + FD} = \frac{AF}{AB} \quad (AD = AF + FD)$$

$$\frac{AO}{AF + FD} = \frac{AF}{2AO} \textit{(AO is radius and AB is diameter)}$$

$$2(AO)^2 = AF \times (AF + FD)$$

8. A particle moving in a straight line passes through a fixed point O with a speed of 20 m/s. The acceleration, a m/s^2, of the particle, t s after passing through O is given by $a = -100e^{-3t}$. The particle comes to instantaneous rest at point N.

(i) Find the time the particle comes to instantaneous rest at point N.

$a = -100e^{-3t}$

velocity, $v = \int -100e^{-3t}\,\mathrm{d}t$

$= \frac{100}{3}e^{-3t} + c$, where c is a constant

when $v = 20$ and $t = 0$,

$\frac{100}{3}e^{0} + c = 20$

$\therefore c = -\frac{40}{3}$

$v = \frac{100}{3}e^{-3t} - \frac{40}{3}$

at rest, $v = 0$

$\frac{100}{3}e^{-3t} - \frac{40}{3} = 0$

$e^{-3t} = \frac{40}{3} \times \frac{3}{100}$

$-3t \ln e = \ln\left(\frac{2}{5}\right)$

$t = -\frac{1}{3}\ln\left(\frac{2}{5}\right)$

$t = 0.30543$

The particle comes to rest at $t = 0.305$ s.

(ii) Calculate the distance ON .

$v = \frac{100}{3}e^{-3t} - \frac{40}{3}$

displacement, $s = \int \frac{100}{3}e^{-3t} - \frac{40}{3}\ \mathrm{d}t$

$s = -\frac{100}{9}e^{-3t} - \frac{40}{3}t + c$ where c is a constant

when $s = 0,\ t = 0\ \therefore\ c = \frac{100}{9}$

$\therefore s = -\frac{100}{9}e^{-3t} - \frac{40}{3}t + \frac{100}{9}$

when $t = 0.30543, s = -\frac{100}{9}e^{-3(0.30543)} - \frac{40}{3}(0.30543) + \frac{100}{9}$

$s = 2.5943$

Distance $ON = 2.59$ m

(iii) Show that the average speed of the particle in the first 2 seconds rounded off to whole number is 10 metres per second.

At $t = 2, s = -\frac{100}{9}e^{-3(2)} - \frac{40}{3}(2) + \frac{100}{9}$

$= -15.583$ m

Total distance travelled in the first 2 seconds

= (2)2.5943 + 15.583

=20.7716

Average speed = 20.7716 ÷ 2
= 10.3858
= 10 m/s (whole number) (shown)

9(i) Solve the equation $2\sin 2P = 3\cos P$ for $0^\circ \le P \le 360^\circ$.

$2\sin 2P - 3\cos P = 0$

$2(2\sin P\cos P) - 3\cos P = 0$

$\cos P(4\sin P - 3) = 0$

$\cos P = 0 \qquad ,\sin P = \frac{3}{4}$

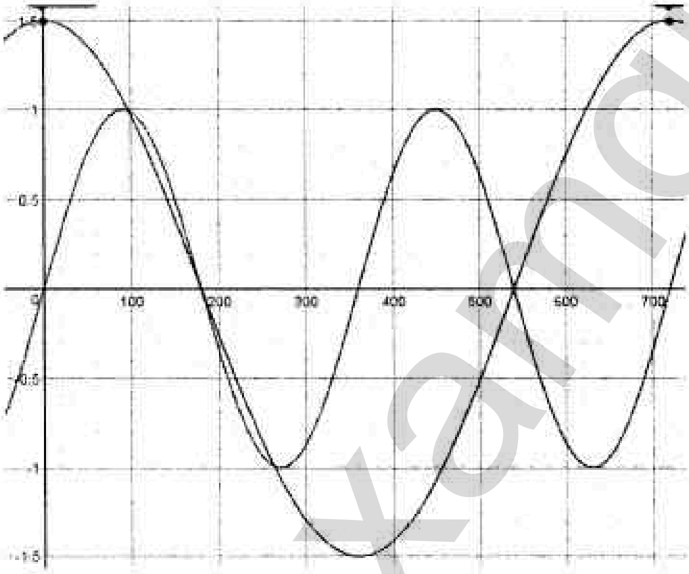
$P = 90^\circ, 270^\circ \qquad \text{basic}\angle = 48.590^\circ$

$P = 48.590^\circ,\ 131.41^\circ$

$\therefore \text{Ans}: P = 48.6^\circ,\ 90^\circ,\ 131.4^\circ, 270^\circ$

(ii) On the same axes, sketch for $0^\circ \le x \le 720^\circ$, the graphs of

$y = \sin x$ and $y = \frac{3}{2}\cos\left(\frac{x}{2}\right)$.



(iii) Using solutions to part **(i)**, determine the x-coordinates of the points of intersections of the graphs of part **(ii)**.

$\sin x = \frac{3}{2}\cos\left(\frac{x}{2}\right)$

$2\sin 2\left(\frac{x}{2}\right) - 3\cos\left(\frac{x}{2}\right) = 0$

Let $P = \left(\frac{x}{2}\right)$, then

$\frac{x}{2} = 48.590^\circ, 90^\circ, 131.41^\circ, 270^\circ$

$x = 97.2^\circ, 180^\circ, 262.8^\circ, 540^\circ$

10. A circle, C_1, has equation $x^2 + y^2 - 14x + 2y = -46$.

(i) Find the coordinates of the centre of the circle and the radius.

Centre $(7, -1)$

Radius $= \sqrt{7^2 + (-1)^2 - 46}$
$= 2$ units

The coordinates of the centre of a second circle, C_2, is $(-4, -2)$. The equation of the tangent to the circle, C_2 at a point P is $2y = -2x + 3$.

(ii) Find the coordinates of point P.

Gradient of tangent to circle at $P = -1$
Equation of the normal at P is

$$\frac{y+2}{x+4} = 1$$

$y + 2 = x + 4$

$y = x + 2$ (1)

$2y = -2x + 3$ (2)

substitute (1) into (2)

$2(x+2) = -2x + 3$

$2x + 4 = -2x + 3$

$x = -\frac{1}{4}$, $y = -\frac{1}{4} + 2$

$y = \frac{7}{4}$

$\therefore P(-\frac{1}{4}, \frac{7}{4})$

(iii) Find the exact value of the radius of C_2 and the equation of the circle, C_2.

Radius of $C_2 = \sqrt{\left(-4 + \frac{1}{4}\right)^2 + \left(-2 - \frac{7}{4}\right)^2}$

$= \frac{15\sqrt{2}}{4}$ units

Equation of C_2 is

$$(x+4)^2 + (y+2)^2 = \frac{225}{8}$$

(iv) Determine whether circles C_1 and C_2 will meet each other, showing your working clearly.

Distance between centres of C_1 and C_2

$$= \sqrt{(7+4)^2 + (-1+2)^2}$$
$$= \sqrt{122}$$
$$=11.0$$

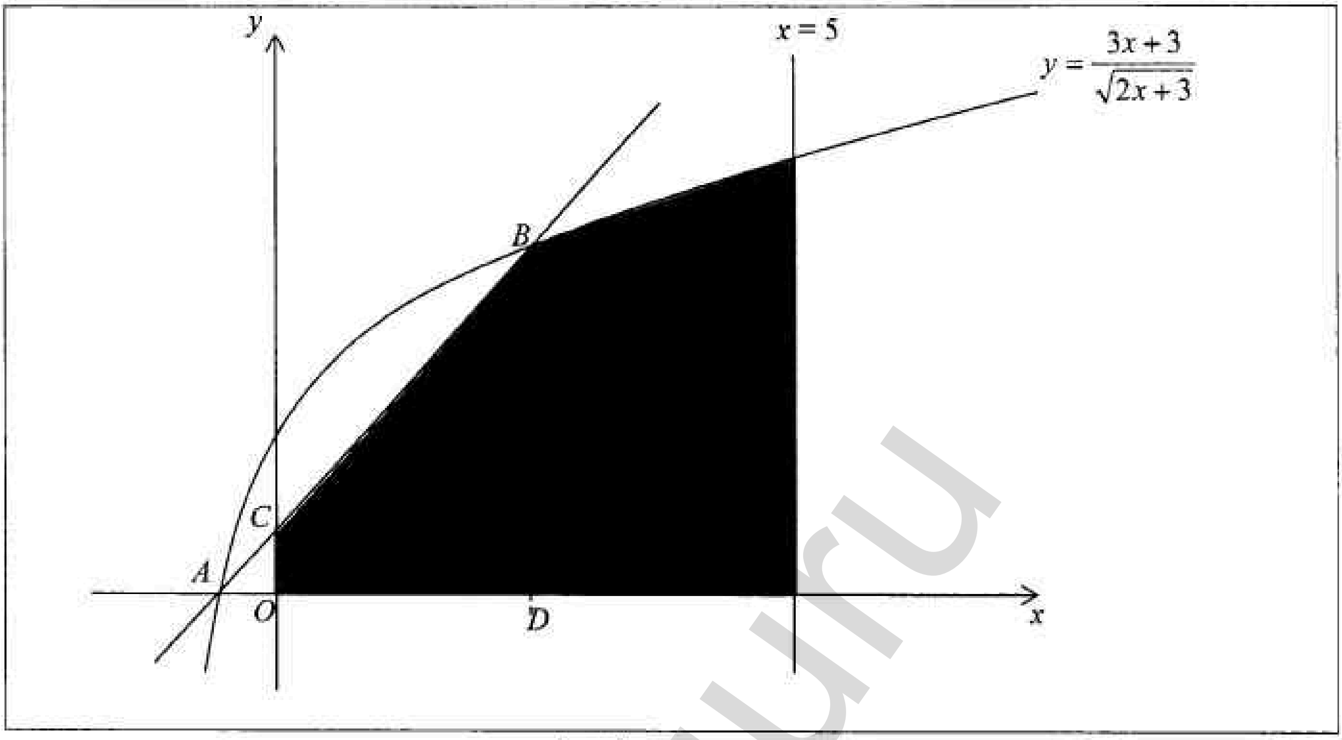
$$\text{Sum of radii} = 2 + \frac{15\sqrt{2}}{4}$$
$$=7.30$$

Since the sum of radii, 7.30 units, is less than the distance between the 2 centres, 11.0 units, the 2 circles C_1 and C_2 will not meet each other.

11(a) Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(2x\sqrt{2x+3}\right) = \frac{6x+6}{\sqrt{2x+3}}$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(2x\sqrt{2x+3}\right)$$
$$= 2x.\frac{1}{2}(2x+3)^{-\frac{1}{2}}(2) + 2\sqrt{2x+3}$$
$$= \frac{2(2x+3)+2x}{\sqrt{2x+3}}$$
$$= \frac{6x+6}{\sqrt{2x+3}} \text{ (shown)}$$

(b)



The diagram shows part of the curve $y = \frac{3x+3}{\sqrt{2x+3}}$. The curve intersects the x-axis at point A. The line through A and perpendicular to the line $y + x = -7$ intersects the curve again at another point, B.

(i)

Show that the y-coordinate of point B is 4.
When $y = 0$, $3x + 3 = 0$. $\therefore$ $A(-1\ ,\ 0)$ Gradient of the line $AB = 1$ Equation of line AB: $\frac{y-0}{x+1} = 1$ $y = x+1 \qquad (1)$ $y = \frac{3x+3}{\sqrt{2x+3}} \qquad (2)$ substitute (1) into (2) $\text{x}+1 = \frac{3(\text{x}+1)}{\sqrt{2\text{x}+3}}$ $(x+1)\left[\frac{\sqrt{2x+3}-3}{\sqrt{2x+3}}\right] = 0$ $x = -1, \quad \sqrt{2x+3} - 3 = 0$ $2x+3 = 9$ $x = 3$ $\therefore y = 3+1$ $y = 4$ Hence the y-coordinate of $B = 4$ (shown)

(ii) Given that the line AB intersects the y-axis at C, determine the area of the shaded region bounded by the line CB, the curve, the line $x = 5$, the x-axis and the y-axis.

For $y = x + 1$

when $x = 0$, $y = 1$

$\therefore C\,(0,1)$

Area of shaded region

= area of trapezium $OCBD$ + area under the curve

$$= \frac{1}{2}(1+4)\times 3 + \int_3^5 \frac{3x+3}{\sqrt{2x+3}}\,\mathrm{d}x$$

$$= \frac{3}{2}(5) + \frac{1}{2}\int_3^5 \frac{6x+6}{\sqrt{2x+3}}\,\mathrm{d}x$$

$$= 7.5 + \frac{1}{2}\Big[2x\sqrt{2x+3}\Big]_3^5$$

$$= 7.5 + \frac{1}{2}\Big[2(5)\sqrt{2(5)+3} - 2(3)\sqrt{2(3)+3}\Big]$$

$= 16.5$ units^2

End of paper

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CONVENT OF THE HOLY INFANT JESUS SECONDARY
Preliminary Examination 1 in preparation for
the General Certificate of Education Ordinary Level 2016

ADDITIONAL MATHEMATICS

4047/01

Paper 1

16 May 2016

2 hours

Additional Materials: Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **6** printed pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ \ldots\ (n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 **(i)** Write down and simplify the first three terms of the expansion, in ascending powers of x, of

(a) $(1+6x)^6$, [1]

(b) $(1-kx)^6$. [1]

(ii) Use the results from part **(i)**, obtain the coefficient of x^2, in terms of k, in the expansion of $\left[1+(6-k)x-6kx^2\right]^6$. [2]

(iii) In the expansion of $\left[1+(6-k)x-6kx^2\right]^6$, where k is an integer, the coefficient of x^2 is 168. Find the value of k. [2]

2 It is given that $\frac{\cos^2\theta}{1+2\sin^2\theta}=\frac{16}{43}$, where $180° < \theta < 270°$. **Without using a calculator**, find the value of

(i) $\sin\theta$, [3]

(ii) $\frac{\cos\theta}{1+2\sin\theta}$. [2]

3 Express $\frac{x^2-3x-6}{(x+1)(x^2-1)}$ as the sum of 3 partial fractions. [5]

4 **(i)** Prove the identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. [3]

(ii) Find all the angles between $0°$ and $360°$ which satisfy the equation $\cos 3\theta + \cos^2\theta = 0$. [5]

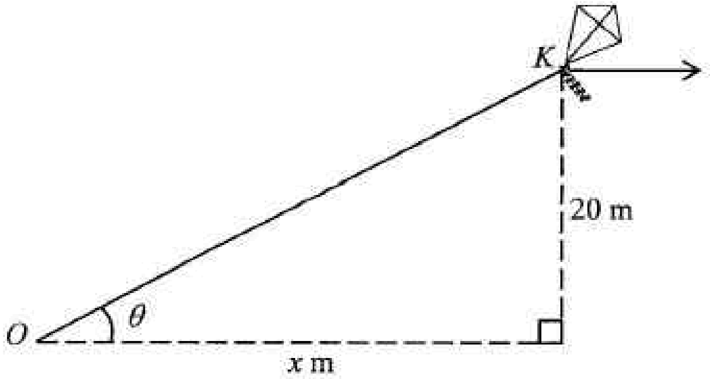
5 It is given that $\frac{d^2y}{dx^2}=2x-1$. Given also that $\frac{dy}{dx}=6$ when $x=2$, find the increase in y as x increases from 2 to 4. [6]

6 The equation of a curve is $y=(1-m)x^2+2(m-1)x+m$, where m is a constant. Find

(i) the range of values of m for which the curve lies completely above the x-axis. [3]

(ii) the values of m for which the line $y=2x-4$ is tangent to the curve. [3]

7



The diagram shows a kite 20 m above the ground. As the string OK is let out, the kite moves horizontally at a constant rate of 0.5 m/s.

(i) Given that θ is the angle of elevation of the string to the horizontal ground, show that the projection of the string on the ground, x m, is given by

$$x = 20\cot\theta .$$ [2]

(ii) Find the rate of change of θ when 50 m of the string has been let out. [4]

(iii) Explain what is meant by your answer in part **(ii)**. [1]

8 In order that each of the equations

(i) $y = a\sqrt{x} + \dfrac{b}{\sqrt{x}}$,

(ii) $y = \dfrac{a}{x+b}$,

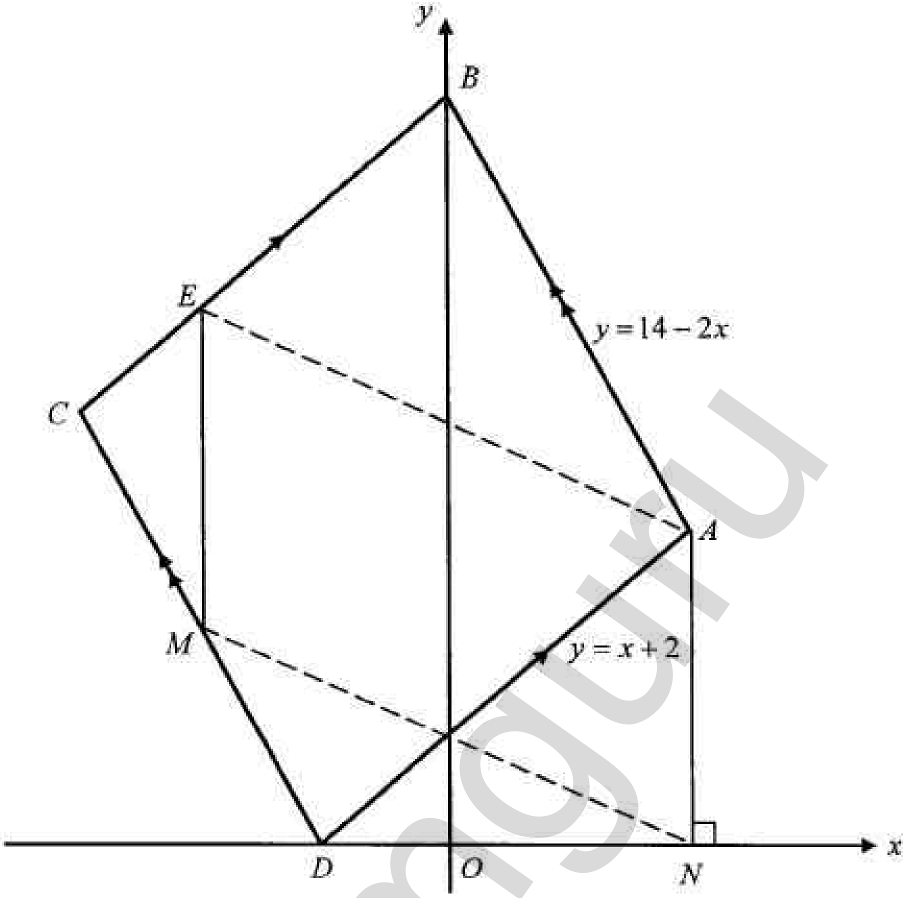
(iii) $y^b = 10^{2x+a}$,

where a and b are unknown constants, may be represented by a straight line, they need to be expressed in the form $Y = mX + c$, where X and Y are each functions of x and/or y, and m and c are constants. Copy the following table and insert in it an expression for Y, X, m and c for each case.

	Y	X	m	c
$y = a\sqrt{x} + \dfrac{b}{\sqrt{x}}$				
$y = \dfrac{a}{x+b}$				
$y^b = 10^{2x+a}$				

[6]

9 **Solutions to this question by accurate drawing will not be accepted.**



The points A, B, C and D $(-2, 0)$ are four points of a parallelogram. The x-coordinate of A is k. Lines are drawn parallel to the y-axis from A to meet the x-axis at N and from E to meet CD at M. $AN = EM$ and $CM = MD$. The y-axis divides the quadrilateral $AEMN$ into two equal halves. The side AB has the equation $y = 14 - 2x$ and the side AD has the equation $y = x + 2$.

(i) Write down the equation of BC. [1]

(ii) Express the coordinates of E and of C in terms of k. [3]

(iii) In the case where $k = 4$, find the area of $AEMN$. [2]

10 The point P lies on the curve $y = \ln(x^2 + 2x)$ where $x > 0$. The normal to the curve at P is parallel to the line $5x + 3 = \pi - 12y$.

(i) Find the coordinates of P. [5]

(ii) Show that the y-coordinate of the point where this normal intersects the y-axis is $\frac{5}{24} + \ln\frac{5}{4}$. [2]

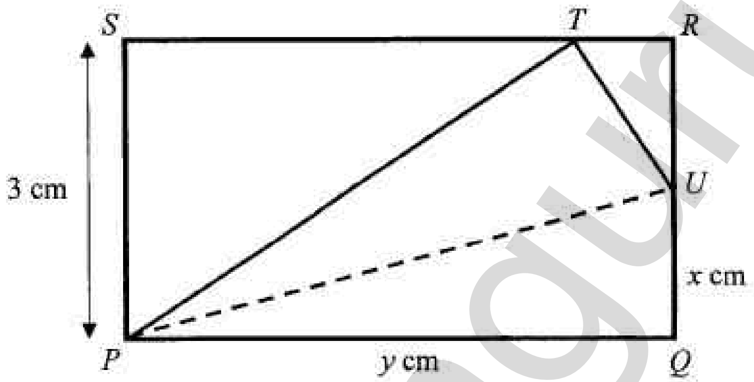
11 A curve has an equation $y=(2x-1)(x-4)$.

(i) Find the minimum value of y and the value of x at which it occurs. [2]

(ii) Sketch the graph of $y=|(2x-1)(x-4)|$. [2]

(iii) A line $y=c$, where c is a constant, intersects the curve at four points.
Using your graph, find the range of values of c. [2]

12 The diagram shows a piece of rectangular paper $PQRS$ such that $PS=3\text{ cm}$, $QU=x\text{ cm}$ and $PQ=y\text{ cm}$. The paper is folded along PU such that Q meets T on SR.



(i) Express TR and PT in terms of x. [4]

(ii) Hence show that the area, A cm^2, of triangle PTU is given by

$$A=\frac{3x^2}{2\sqrt{6x-9}}\ .$$ [1]

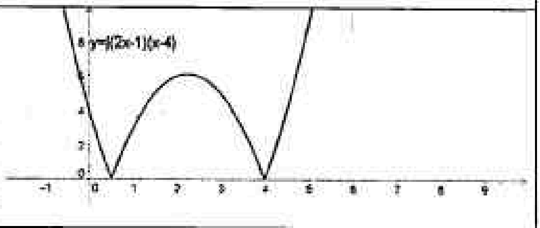
Given that x can vary, find

(iii) the value of x for which A is a minimum, [5]

(iv) the minimum value of A in the form of $a\sqrt{b}$ cm, where a and b are integers. [2]

--- End of Paper 1 ---

Prelim 1 P1 (2016) **Answers**

1ia	$1+36x+540x^2$	8i	$Y=y\sqrt{x},\ X=x,\ m=a,\ c=b$ $Y=\frac{y}{\sqrt{x}},\ X=\frac{1}{x},\ m=b,\ c=a$
1ib	$1-6kx+15k^2x^2$		
1ii	$15k^2-216k+540$		
1iii	$k=\frac{62}{5}$ (reject), $k=2$	8ii	$Y=\frac{1}{y},\ X=x,\ m=\frac{1}{a},\ c=\frac{b}{a}$ $Y=y,\ X=xy,\ m=-\frac{1}{b},\ c=\frac{a}{b}$ $Y=xy,\ X=y,\ m=-b,\ c=a$
2i	$\sin\theta=\frac{3}{5}$ (reject), $\sin\theta=-\frac{3}{5}$		
2ii	4	8iii	$Y=\lg y,\ X=x,\ m=\frac{2}{b},\ c=\frac{a}{b}$ $Y=\ln y,\ X=x,\ m=\frac{2\ln10}{b},\ c=\frac{a\ln10}{b}$
3	$\frac{3}{(x+1)}+\frac{1}{(x+1)^2}-\frac{2}{(x-1)}$	9i	$y=x+14$
4i	$\cos3\theta$ $=\cos(2\theta+\theta)$ $=\cos2\theta\cos\theta-\sin2\theta\sin\theta$ $=(2\cos^2\theta-1)\cos\theta-(2\sin\theta\cos\theta)\sin\theta$ $=2\cos^3\theta-\cos\theta-2\sin^2\theta\cos\theta$ $=2\cos^3\theta-\cos\theta-2(1-\cos^2\theta)\cos\theta$ $=4\cos^3\theta-3\cos\theta$ (Proved)	9ii 9iii	$C(2-2k,\ 16-2k)$ $C(2-2k,\ 4k-8)$ $C(-2-k,\ 12-k)$ $C(-2-k,\ 2k)$ 48 units^2
4ii	$\theta=90^\circ, 270^\circ$, $\theta=41.4^\circ, 31\&6^\circ$, $\theta=180^\circ$	10i	$(\frac{1}{2},\ln\frac{5}{4})$ or $P(\frac{1}{2},0.223)$
5	$y=20\frac{2}{3}$	10ii	$y=\frac{5}{24}+\ln\frac{5}{4}$
6i	$\therefore\frac{1}{2}<m<1$	11i	$x=2\frac{1}{4},\ y=-6\frac{1}{8}$
6ii	$m=0$ or $m=\frac{1}{2}$	11ii	
7i	$\tan\theta=\frac{20}{x}$, $x=\frac{20}{\tan\theta}$, $x=20\cot\theta$	11iii	$0<c<6\frac{1}{8}$

7ii	-0.004	12i	$TR=\sqrt{6x-9}$, $PT=\dfrac{3x}{\sqrt{6x-9}}$
7iii	The negative sign indicates clockwise change in angle size (i.e. reducing angle).	12ii	$\dfrac{1}{2}\cdot x\cdot\dfrac{3x}{\sqrt{6x-9}}=\dfrac{3x^2}{2\sqrt{6x-9}}$ (shown)
12iii	$\dfrac{dy}{dx}=\dfrac{27x^2-54x}{2(6x-9)^{\frac{3}{2}}}$, $x=2$,use table to show min area	12iv	$2\sqrt{3}$ cm^2

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CONVENT OF THE HOLY INFANT JESUS SECONDARY
Preliminary Examination 1 in preparation for
the General Certificate of Education Ordinary Level 2016

ADDITIONAL MATHEMATICS

4047/02

Paper 2

17 May 2016

2 hours 30 Minutes

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
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Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of **5** printed pages and **1** blank page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}ab\sin C$$

1 The number of words per minute, $N(t)$, that Mr Ong can type is given by the function

$$N(t) = 68 - 36\,\mathrm{e}^{-0.6t},$$

where t is the time in months after he begins a computer based typing course.

(i) Find the number of words per minute that Mr Ong can type after 2 months. [2]

(ii) Find the time Mr Ong will take to type at a rate of 58 words per minute. [2]

(iii) Determine whether Mr Ong will be able to type at a rate of 70 words per minute. Explain your answer clearly. [2]

2 When the expression $6x^4 - 5x^3 + 4x^2 + hx + k$ is divided by $3x^2 + 2x - 1$, the remainder is $7 - 7x$. Find the value of h and of k. [6]

3 AC and BD are diagonals of a rhombus $ABCD$. $AC = \left(9 + 2\sqrt{3}\right)$cm and the area of $ABCD$ is $\left(\frac{57}{2} + 14\sqrt{3}\right)$cm^2.

(i) Find the length of the diagonal BD in the form $\left(a + b\sqrt{3}\right)$cm, where a and b are integers. [4]

(ii) Find the value of AB^2, giving your answer in the form $\left(a + b\sqrt{3}\right)$cm^2, where a and b are rational numbers. [3]

4 **(a)** The roots of the equation $2x^2 + 4px + q = 0$ are α and $\alpha + 2$. Express q in terms of p. [4]

(b) The equation $3x^2 - 5x - 7 = 0$ has roots α and β. Form an equation with roots $\alpha + 3$ and $\beta + 3$. [4]

5 **(a)** Given that the equation $2\log_3 x - \frac{3}{\log_3 x} = 5$, find the exact values of x. [4]

(b) Given that $\log_8 x = h$ and $\log_{16} 4x = k$, express h in terms of k. [4]

6 The equation of a curve is $y = 3x + \ln(2x - 5)$.

(i) The line $y = 3x - 2$ intersects the curve at the point *K*. Find the coordinates of *K*, giving your answer correct to 2 decimal places. [3]

(ii) Find the equation of the normal to the curve at the point $x = 3$. [4]

(iii) The normal to the curve at the point $x = 3$ cuts the *x*-axis at the point *H*. Find the coordinates of *H*. [2]

7 Find the coordinates of the stationary point(s) of the curve $y = \frac{x^3 + 16}{x}$. Determine the nature of the turning point(s). Explain clearly why the gradient of the curve is negative when $x < 0$. [7]

8 **(a)** The equation of a circle is $x^2 + 2x + 4y = 20 - y^2$. Given that $A(2, 2)$ is a point on the circle, find the equation of the tangent to the circle at *A*. [5]

(b) $A(0, 2)$, $B(9, 3)$ and $C(1, -7)$ are three points on a circle.

(i) Show that *BC* is the diameter of the circle. [4]

(ii) Find the equation of the circle. [3]

9 **(a)** Solve the equation $\frac{2}{\cos^2 x} = 7\tan x - 3$ for $0 \le x \le 2\pi$. [4]

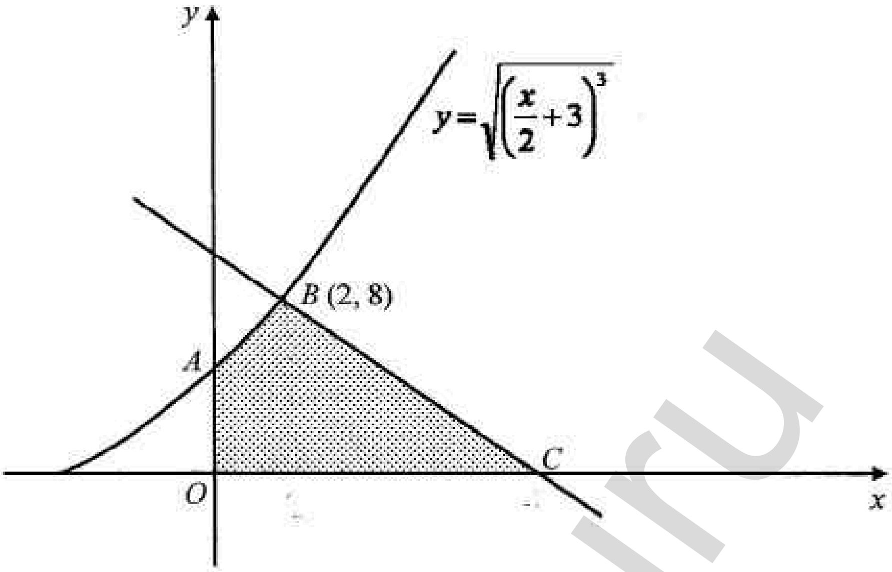
(b) **(i)** Sketch the graphs of $y = 1 - 3\sin x$ and $y = 4\cos 2x - 1$ on the same axes, for $0 \le x \le \pi$. [4]

(ii) Calculate the values of *x* in the given range for which $1 - 3\sin x = 4\cos 2x - 1$. [4]

(iii) Using your graph from part **(b)(i)**, state the range of values of *x* for which $2 - 3\sin x \ge 4\cos 2x$. [1]

10 **(a)** Given that $y = \ln\sqrt{\cos 2x}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and hence find the exact value of $\int_0^{\frac{\pi}{6}} 3\tan 2x\,\mathrm{d}x$. [5]

(b)



The diagram shows part of the curve $y=\sqrt{\left(\frac{x}{2}+3\right)^3}$. The straight line BC is normal to the curve at the point $B(2\ ,\ 8)$. Find

(i) the equation of the line BC, [3]

(ii) the area of the shaded region $OABC$. [5]

11 A particle moves in a straight line and passes through a fixed point O with an initial velocity of 16 cm/s. The acceleration, a cm/s^2, of the particle t seconds after passing O, is given by

$$a = -25e^{-\frac{3t}{2}}.$$

(i) Find an expression, in terms of t, for the velocity of the particle. [3]

(ii) Find the time taken for the particle to come to an instantaneous rest, giving your answer correct to 2 decimal places. [3]

(iii) Calculate the distance moved by the particle in the third second. [5]

--- End of Paper 2 ---

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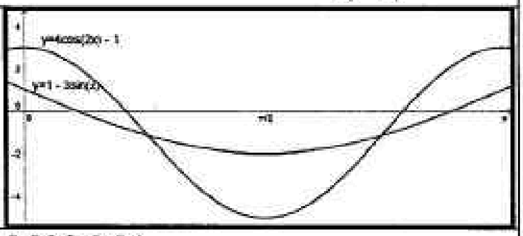
Prelim 1 P2 (2016) **Answers**

1(i)	approx. 57 words/min	9(a)	$x = 1.19$ or 4.33 $\quad x = \frac{\pi}{4}, \frac{5\pi}{4}$
(ii)	2.13 (or 2.14) months	(bi)	
(iii)	As $e^{-0.6t} > 0$ for all values of t, thus $36e^{-0.6t} > 0$, thus N will always be less than 68.	(bii)	0.806, 2.34
2	$h = 4, k = 3$	(biii)	$0.806 \le x \le 2.34$
3(i)	$5 + 2\sqrt{3}$	10(a)	$\frac{dy}{dx} = -\tan 2x$; $-\frac{3}{2}\ln\frac{1}{2}$
(ii)	$32\frac{1}{2} + 14\sqrt{3}$	(bi)	$y = -\frac{2}{3}x + 9\frac{1}{3}$
4(a)	$q = 2p^2 - 2$	(bii)	61.1 sq units
(b)	$x^2 - 7\frac{2}{3}x + \frac{35}{3} = 0$	11(i)	$\therefore v = \frac{50}{3}e^{-\frac{3}{2}t} - \frac{2}{3}$
5(a)	$\frac{1}{\sqrt{3}}$ or 27	(ii)	2.15s
(b)	$h = \frac{4k-2}{3}$	(iii)	0.260 cm
6(i)	$(2.57, 5.70)$		
(ii)	$5y + x = 48$		
(iii)	$(48, 0)$		
7	$(2,12)$ and is a min pt; When $x < 0$, $2x < 0$ and $x^2 > 0$, $2x - \frac{16}{x^2}$ is always –ve;		
8(a)	$y = -\frac{3}{4}x + \frac{7}{2}$		
(bi)	Grad of $AB \times$ Grad of $AC = \frac{1}{9} \times -9 = -1$ $\Rightarrow AB \perp AC$. By the circle property in semicircle is $90°$, $\angle CAB = 90°$ and BC is the diameter.		
(bii)	$(x-5)^2 + (y+2)^2 = 41$		

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HOLY INNOCENTS' HIGH SCHOOL

Name of Student ______

Class ______ Index Number ______

/ 80

PRELIMINARY EXAMINATION 2016
SECONDARY 4 EXPRESS
ADDITIONAL MATHEMATICS PAPER 1

4047/01

Date: 22 Aug 2016
Duration: 2 hours
Time: 0800 – 1000

Additional Materials: **8 sheets of writing paper**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction tape/fluid

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

This document consists of **7** *printed pages (including cover page).*

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer all the questions.

1 A curve has the equation $y = 4x^2 - px + p - 3$, where p is a constant. Find the range of values of p for which the curve lies completely above the x-axis. [4]

2 Solve the equation $\ln(4^x - 4) - x\ln 2 = \ln 3$. [4]

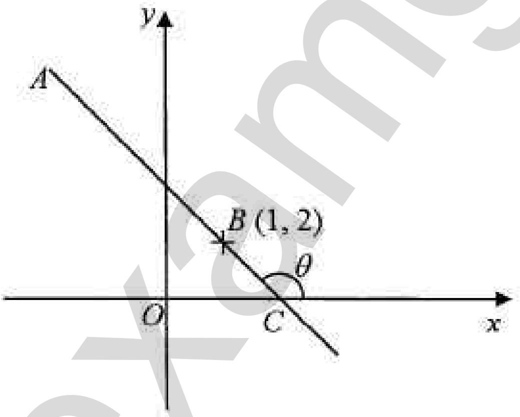
3 A curve has the equation $y = \dfrac{1-x}{3x+4}$ for $x > 0$.

(i) Obtain an expression for $\dfrac{dy}{dx}$. [2]

(ii) Show that y is a decreasing function. [1]

(iii) Given that y decreases at the rate of 0.75 units per second, calculate the rate of change of x at the instant when $x = 3$. [2]

4



The diagram shows a straight line ABC such that $AB : BC = 3 : 1$. The point B is (1, 2) and the point C lies on the x-axis. θ is the angle between the positive x-axis and the line AC. Given that $\tan\theta = -2$, find

(i) the equation of the line AC, [1]

(ii) the coordinates of C and of A. [3]

The point D is such that $ABOD$ is a parallelogram.

(iii) Find the coordinates of D. [2]

5 In an experiment, a scientist started with 5 000 000 cells and observed that 40% of the cells are dying every minute. The number of cells remaining, N, after t minutes, is given by $N = Ae^{kt}$, where A and k are constants.

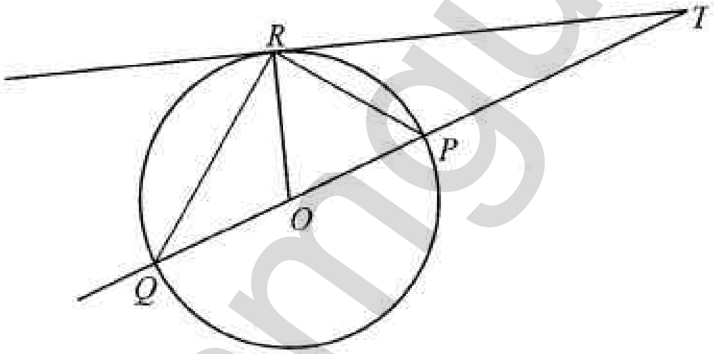
(i) Find the value of A and of k. [4]

(ii) Find the value of t when the number of cells decreases to 2000. [2]

6 **(i)** Sketch the curve $y = \left|x^2 - 4\right|$ for $-2 \leq x \leq 3$. [3]

(ii) Find the x-coordinates of the points of intersection of the curve $y = \left|x^2 - 4\right|$ and the line $y = 6$. [3]

7

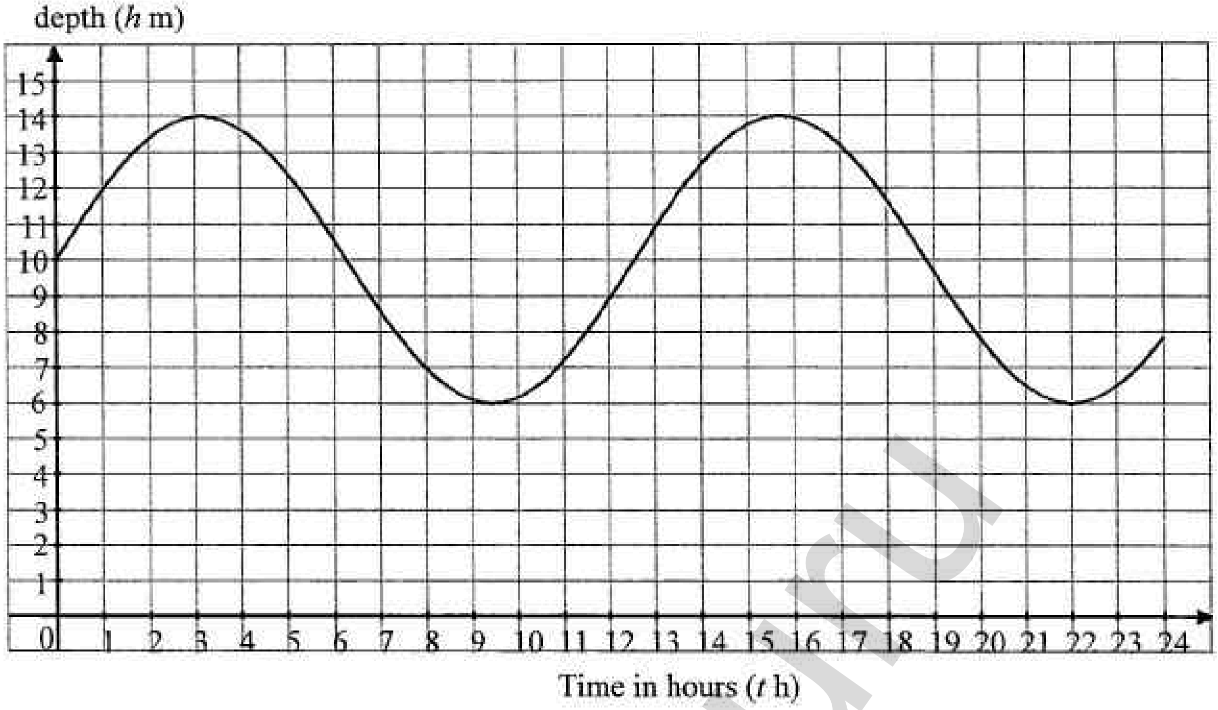


The diagram shows a circle, centre O. The point R lies on the circle and TR is a tangent to the circle. The line TQ passes through O and intersects the circle at P and Q.

(i) Prove that triangles TRP and TQR are similar. [2]

(ii) Prove that $TP \times TQ = OT^2 - OR^2$. [4]

8



The diagram shows the graph of the depth of water, h metres, in a harbour on a particular day, which is modelled by the equation, $h = a\sin\frac{1}{2}t + b$, where a and b are constants and t is the time in hours after midnight.

(i) State the period of h. [1]

(ii) Use the graph to find the value of a and of b. [2]

The harbour gates are closed when the depth of the water is less than seven metres. An alarm rings when the gates are opened or closed.

(iii) Using the values of a and b found in **(ii)**, calculate the values of t when the alarm rings on this particular day. [4]

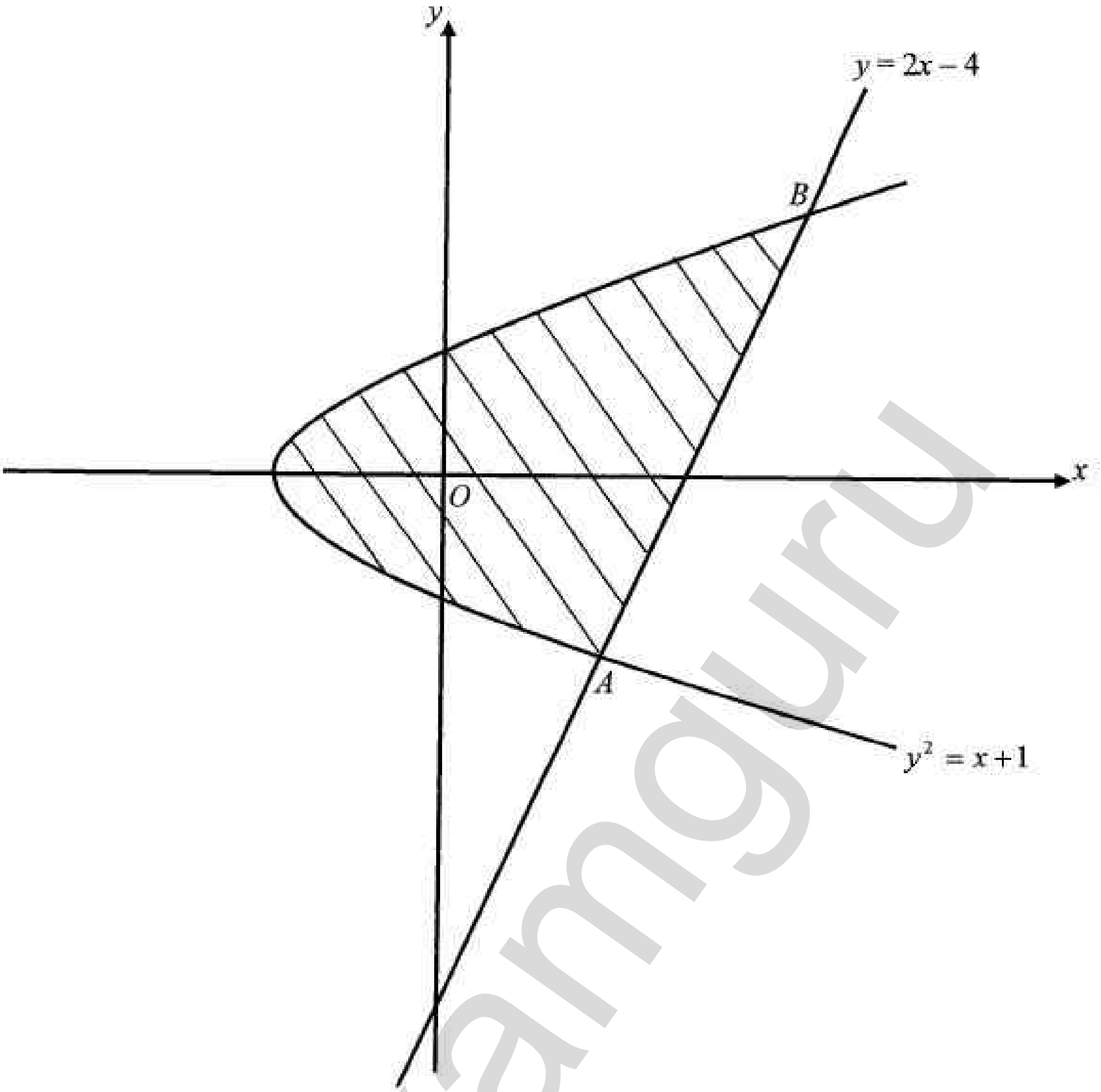
(iv) Hence find the total length of time when the harbour gates are closed. [1]

9 **(i)** Show that $\dfrac{\sin\theta}{1+\dfrac{1}{\sec\theta}} + \cot\theta = \operatorname{cosec}\theta$. [4]

(ii) Hence find, in degrees, the smallest value of θ such that

$$\frac{\sin 2\theta}{1+\dfrac{1}{\sec 2\theta}} + \cot 2\theta = 6\cos 2\theta.$$ [4]

10



The diagram shows part of the curve $y^2 = x + 1$. The line $y = 2x - 4$ intersects the curve at points A and B. Find

(i) the coordinates of A and of B, [4]

(ii) the area of the shaded region. [4]

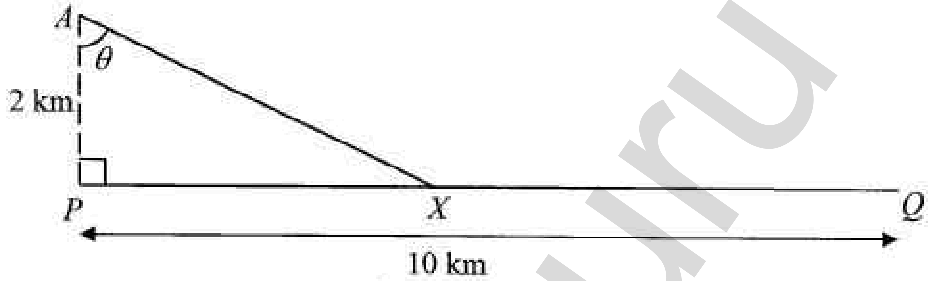
11 A particle moves in a straight line, so that, t seconds after leaving a fixed point O, its velocity, v m s^{-1} is given by $v = 2 + 5t - 3t^2$. The particle comes to instantaneous rest at the point Q. Find

(i) the acceleration of the particle at Q, [4]

(ii) the distance OQ, [3]

(iii) the total distance travelled by the particle in the time interval $t = 0$ to $t = 3$. [2]

12



The diagram shows a straight road PQ, of length 10 km. A man is at point A, where AP is perpendicular to PQ and AP is 2 km. He travels in a straight line to meet the road at point X, where angle $PAX = \theta$ radians. The man travels at 3 km/h along AX and 5 km/h along XQ. He takes T hours to travel from A to Q.

(i) Show that $T = \dfrac{2\sec\theta}{3} + 2 - \dfrac{2\tan\theta}{5}$. [4]

(ii) Given that θ can vary, show that T has a stationary value when $PX = 1.5$ km. [6]

End of paper

Additional Mathematics

Preliminary Examination 2016

Marking Scheme

1 $y = 4x^2 - px + p - 3$

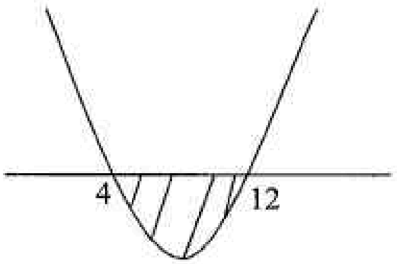
$b^2 - 4ac < 0$

$(-p)^2 - 4(4)(p-3) < 0$ M1

$p^2 - 16p + 48 < 0$ correct quadratic form M1

Finding the solution of quadratic: $p = 4$ or 12 DM1

$(p-12)(p-4) < 0$



$4 < p < 12$ A1

2 $\ln(4^x - 4) - x\ln 2 = \ln 3$

$\ln(4^x - 4) - \ln 2^x = \ln 3$

$\ln\frac{4^x - 4}{2^x} = \ln 3$ applying quotient law M1

$\frac{4^x - 4}{2^x} = 3$

$2^{2x} - 3(2^x) - 4 = 0$ correct quadratic equation M1

Or substituting $y = 2^x$ to get $y^2 - 3y - 4 = 0$

$(y-4)(y+1) = 0$

$y = 4$ or $y = -1$

$2^x = 4$ or $2^x = -1$(rej) M1

$x = 2$ A1

3 (i) $y = \frac{1-x}{3x+4}$

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(-1)(3x+4)-(1-x)(3)}{(3x+4)^2}$ M1

$= \frac{-7}{(3x+4)^2}$ A1

(ii) Since $(3x + 4)^2 > 0$ and $\frac{-7}{(3x+4)^2} < 0$,

y is a decreasing function for all real values of x B1

(iii) $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$-0.75 = \frac{-7}{(3x+4)^2} \times \frac{\mathrm{d}x}{\mathrm{d}t}$ M1

When $x = 3$, $\frac{dx}{dt} = \frac{-3}{4} \times \frac{169}{-7} = 18\frac{3}{28}$ units / sec A1

(or 18.1units / sec)

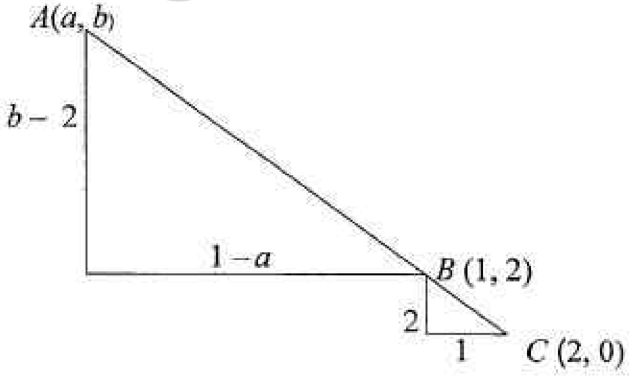
4 (i) $y - 2 = -2(x-1)$

$y = -2x + 4$ B1

(ii) when $y = 0$, $x = 2$

$\therefore$ Coordinates of $C = (2, 0)$ B1

Let the coordinates of A be (a, b).



Apply similar triangle ratios

$\frac{1-a}{1} = \frac{3}{1}$ and $\frac{b-2}{2} = \frac{3}{1}$ M1

$a = -2$ and $b = 8$ A1

$\therefore$ Coordinates of $A = (-2, 8)$

[Or apply distance formula

Subst $x = a$ into $y = -2x + 4$

$y = -2a + 4$

Distance of AB = 3 Distance of BC

$\sqrt{(a-1)^2 + (-2a+4-2)^2} = 3\sqrt{(1-2)^2 + (2-0)^2}$ **M1**

$a^2 - 2a + 1 + 4a^2 - 8a + 4 = 9(5)$
$5a^2 - 10a - 40 = 0$

$5(a-4)(a+2) = 0$

$a = 4$(rej) or $a = -2$

$b = -8$

$\therefore$ Coordinates of $A = (-2, 8)$ A1]

(iii) Let the point D be (h, k)
mid-point of BD = mid-point of AO
$\left(\frac{h+1}{2}, \frac{k+2}{2}\right) = \left(\frac{-2+0}{2}, \frac{8+0}{2}\right)$ M1
$\frac{h+1}{2} = -1, \quad \frac{k+2}{2} = 4$
$h = -3, \quad k = 6$ A1

$D(-3, 6)$

5 (i) $N = Ae^{kt}$

When $t = 0$, $N = 5\ 000\ 000$

$5\ 000\ 000 = Ae^{k(0)}$

$A = 5\ 000\ 000$ B1

When t = 1, $N = \frac{60}{100} \times 5\ 000\ 000$

$= 3\ 000\ 000$

$3\ 000\ 000 = 5\ 000\ 000\,e^{k(1)}$ M1

$e^k = \frac{3}{5}$

$k = \ln\frac{3}{5}$ M1

$= -0.5108 \approx -0.511$ A1

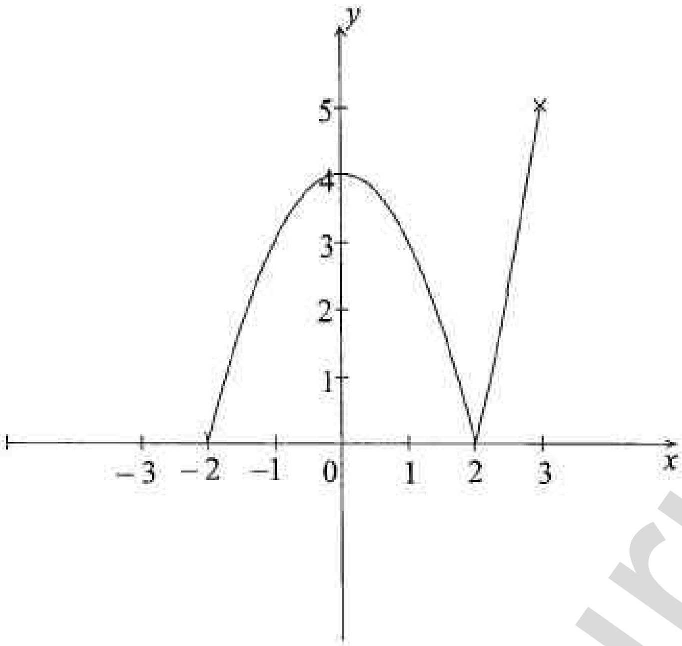
(ii) $2000 = 5000000e^{-0.5108\,t}$

$e^{-0.5108\,t} = \frac{2}{5000}$

$-0.5108t = \ln\frac{2}{5000}$ M1

$t = 15.3$ min A1

6 (i)



Correct shape B1

x - intercepts and turning point shown correctly B1

end point (3, 5) shown clearly B1

(ii) $|x^2 - 4| = 6$

$x^2 - 4 = 6$ or $x^2 - 4 = -6$ M1

$x^2 = 10$ or $x^2 = -2$ (rej)

$x = 3.16$ or -3.16 A2

7 (i) $\angle RTP = \angle QTR$ (common angle)

$\angle TRP = \angle TQR$ ($\angle$s in the alternate segment or tangent chord thm) B1

$\therefore \Delta TRP$ and ΔTQR are similar. (AA similarity) B1

(ii) Since ΔTRP and ΔTQR are similar,

$$\frac{TR}{TQ} = \frac{TP}{TR}$$

$\Rightarrow TR^2 = TP \times TQ$ ------------------------ (1) M1

$\angle ORT = 90°$ (tangent $\perp$ radius) M1

$\Rightarrow \Delta ORT$ is a right angled triangle.

By Pythagoras theorem,

$OT^2 = OR^2 + TR^2$
$TR^2 = OT^2 - OR^2$ ---------------------------------(2) M1

subst (1) into (2)
$OT^2 - OR^2 = TP \times TQ$ (shown) A1

8 (i) period $= \dfrac{2\pi}{\frac{1}{2}} = 4\pi$ B1

(ii) When $t = 0$, $10 = a\sin 0 + b$

$\Rightarrow b = 10$ B1

max value = 14 when $\sin \frac{1}{2}t = 1$

$\Rightarrow a + 10 = 14$
$a = 4$ B1

(iii) $4\sin\frac{1}{2}t + 10 = 7$ M1

$\sin \frac{1}{2}t = -\frac{3}{4}$
$\alpha = 0.8480$ (accept 0.84806)

$\frac{1}{2}t = \pi + 0.8480, 2\pi - 0.8480, \pi + 0.8480 + 2\pi, 2\pi - 0.8480 + 2\pi$ M2

(M1 for each cycle)

$= 3.989, 5.435, 10.27, 11.71$
$t = 7.978, 10.87, 20.54, 23.42$
$\approx$ 7.98 h, 10.9 h, 20.5 h, 23.4 h A1

(iv) Length of time the gates are closed = (10.87 – 7.978) + (23.42 – 20.54)

= 5.772 h ≈ 5.77 h B1

9 (i) $\dfrac{\sin\theta}{1+\dfrac{1}{\sec\theta}}+\cot\theta=\cos ec\theta$

$=\dfrac{\sin\theta}{1+\cos\theta}+\dfrac{\cos\theta}{\sin\theta}$ M1

$=\dfrac{\sin^2\theta+\cos\theta+\cos^2\theta}{\sin\theta(1+\cos\theta)}$ M1

$=\dfrac{1+\cos\theta}{\sin\theta(1+\cos\theta)}$ (Applying the identity $\sin^2\theta+\cos^2\theta=1$) M1

$=\dfrac{1}{\sin\theta}=\cos ec\theta$ A1

(ii) $\dfrac{\sin 2\theta}{1+\dfrac{1}{\sec 2\theta}}+\cot 2\theta=6\cos 2\theta$

$\cos ec\ 2\theta=6\cos 2\theta$ M1

$\dfrac{1}{\sin 2\theta}=6\cos 2\theta$

$6\sin 2\theta\cos 2\theta=1$

$3(2\sin 2\theta\cos 2\theta)=1$

$3\sin 4\theta=1$ (applying double angle formula) M1

$\sin 4\theta=\dfrac{1}{3}$

$\alpha=19.47°$

$4\theta=19.47°$ M1

$\theta=4.87°\approx 4.9°$ A1

10 (i) $y^2 = x + 1$ -------------------------------- (1)

$y = 2x - 4$ -------------------------------- (2)

Subst (2) into (1)

$(2x-4)^2 = x+1$ M1

$4x^2 - 16x + 16 - x - 1 = 0$

$4x^2 - 17x + 15 = 0$ M1

$(4x-5)(x-3) = 0$

$x = 1\frac{1}{4}$ or 3 A1

$y = -1\frac{1}{2}$ or 2

$A\ (1\frac{1}{4}, -1\frac{1}{2})$, $B\ (3, 2)$ A1

(ii) From (2), $x = \frac{y+4}{2}$

$= \frac{y}{2} + 2$

Area $= \int_{-\frac{3}{2}}^{2} \left[\left(\frac{y}{2} + 2 \right) - \left(y^2 - 1 \right) \right] dy$ M2

(**M1** **M1)**

$= \int_{-\frac{3}{2}}^{2} \left[\left(\frac{y}{2} - y^2 + 3 \right) \right] dy$

$= \left[\frac{y^2}{4} - \frac{y^3}{3} + 3y \right]_{-\frac{3}{2}}^{2}$ M1

$= \left(1 - \frac{8}{3} + 6 \right) - \left(\frac{9}{16} + \frac{9}{8} - \frac{9}{2} \right)$

$= 7\frac{7}{48}$ units^2 (Accept 7.15 units^2) A1

Alternative Method

$$[\textbf{Area} = \underbrace{\int_{-1}^{3}(x+1)^{\frac{1}{2}}dx - \frac{1}{2}\times 1\times 2}_{\textbf{M1}} + \underbrace{\left|\int_{-1}^{\frac{5}{4}} -(x+1)^{\frac{1}{2}}\,dx\right| + \frac{1}{2}\times\frac{3}{4}\times\frac{3}{2}}_{\textbf{M1}}$$

$$= \left[\frac{2}{3}(x+1)^{\frac{3}{2}}\right]_{-1}^{3} - 1 + \left[\frac{2}{3}(x+1)^{\frac{3}{2}}\right]_{-1}^{\frac{5}{4}} + \frac{9}{16} \qquad \textbf{M1}$$

$$= \frac{16}{3} - 1 + \frac{9}{4} + \frac{9}{16}$$

$$= 7\frac{7}{48}\text{ units}^2 \qquad \textbf{A1]}$$

Accept other logical methods

11 (i) $v = 2 + 5t - 3t^2$

At instantaneously at rest $\Rightarrow v = 0$

$2 + 5t - 3t^2 = 0$ M1

$3t^2 - 5t - 2 = 0$

$(3t+1)(t-2) = 0$

$t = -\frac{1}{3}$ (rej) or $t = 2$ A1

acceleration $= \frac{dv}{dt}$

$= 5 - 6t$ M1

At $t = 2$, acceleration $= 5 - 6(2) = -7$ m/s^2 A1

(ii) $s = \int\left(2 + 5t - 3t^2\right)dt$

$= 2t + \frac{5t^2}{2} - \frac{3t^3}{3} + c$ M1

when $t = 0$ and $s = 0$, $c = 0$

$s = 2t + \frac{5t^2}{2} - t^3$ M1

At t = 2, $s = \frac{5(2)^2}{2} - (2)^3 + 2(2) = 6$ m A1

[OR $\int_0^2 \left(2+5t-3t^2\right) \mathrm{d}t$

$= \left[2t + \frac{5t^2}{2} - \frac{3t^3}{3}\right]_0^2$ (**M1** for integration, **M1** for the limits)

= 6 m **A1]**

(iii) At t = 3, $s = \frac{5(3)^2}{2} - (3)^3 + 2(3)$

$= 1\frac{1}{2}$ m M1

Total distance travelled = $6 + 6 - 1\frac{1}{2}$

$= 10\frac{1}{2}$ m A1

[OR $\left|\int_2^3 \left(2+5t-3t^2\right) \mathrm{d}t\right|$

$\left|\left[2t + \frac{5t^2}{2} - \frac{3t^3}{3}\right]_2^3\right|$ **M1**

$= 4\frac{1}{2}$ m **M1**

Total distance travelled = $6 + 4\frac{1}{2} = 10\frac{1}{2}$ m **A1]**

12 (i) $\cos\theta = \frac{2}{AX}$

$AX = \frac{2}{\cos\theta}$

$= 2\sec\theta$ km M1

Time taken for $AX = \frac{2\sec\theta}{3}$ h

$\tan\theta = \frac{PX}{2}$

$PX = 2\tan\theta$ km M1

$XQ = 10 - 2\tan\theta$ M1

Time taken for $XQ = \dfrac{10-2\tan\theta}{5}$ h

$$\left.\begin{aligned} T &= \frac{2\sec\theta}{3} + \frac{10-2\tan\theta}{5} \\ &= \frac{2\sec\theta}{3} + 2 - \frac{2\tan\theta}{5} \quad \text{(shown)} \end{aligned}\right\}\text{—}$$ A1

(ii) $T = \dfrac{2\sec\theta}{3} + 2 - \dfrac{2\tan\theta}{5}$

$= \dfrac{2}{3\cos\theta} + 2 - \dfrac{2\tan\theta}{5}$

$\dfrac{dT}{d\theta} = \dfrac{0(\cos\theta)-2(-3\sin\theta)}{9\cos^2\theta} - \dfrac{2}{5}\sec^2\theta$

$= \dfrac{2\sin\theta}{3\cos^2\theta} - \dfrac{2}{5}\sec^2\theta$ M2

M1 **M1**

For stationary value of T, $\dfrac{dT}{d\theta} = 0$

$\dfrac{2\sin\theta}{3\cos^2\theta} - \dfrac{2}{5}\sec^2\theta = 0$ M1

$\dfrac{2\sin\theta}{3\cos^2\theta} - \dfrac{2}{5\cos^2\theta} = 0$

$\dfrac{10\sin\theta - 6}{5\cos^2\theta} = 0$

$\Rightarrow 10\sin\theta - 6 = 0$

$\sin\theta = \dfrac{3}{5}$ M1

$\theta = 0.6435$ M1

$PX = 2\tan 0.6435$

$= 1.5$ m (shown) A1

[OR PX = 1.5

$2\tan\theta = 1.5$ **M1**

$\tan\theta = 0.75$

$\theta = 0.6435$ **M1**

When **$\theta = 0.6435$**, $\dfrac{\mathbf{dT}}{\mathbf{d\theta}} = \dfrac{\mathbf{2\sin 0.6435}}{\mathbf{3\cos^2 0.6435}} - \dfrac{\mathbf{2}}{\mathbf{5\cos^2 0.6435}}$ **M1**

$= 0$ (shown) A1]

聖 嬰 中 學

HOLY INNOCENTS' HIGH SCHOOL

Name of Student			/100
Class		Index Number	

PRELIMINARY EXAMINATION 2016
SECONDARY 4 EXPRESS
ADDITIONAL MATHEMATICS PAPER 2

4047/02

Date: **17 Aug 2016**
Duration: **2 h 30 min**
Time: **1100 – 1330**

Additional Materials: **8 sheets of writing paper**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction tape/fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

This document consists of 7 printed pages (including cover page).

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cos ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

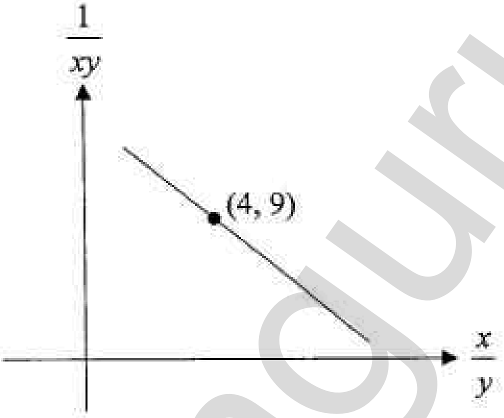
Answer all the questions.

1 Given that, for all values of x, $x^5 - 2x^3 + 2x^2 + 4x - 3 = Ax + B + (x^2 - 1)Q(x)$, where $Q(x)$ is a polynomial,

(i) state the degree of the polynomial, $Q(x)$, [1]

(ii) find the remainder of $x^5 - 2x^3 + 2x^2 + 4x - 3$, when divided by $x^2 - 1$, in terms of x. [5]

2



The diagram shows part of a straight line graph drawn to represent the equation $y = \frac{ax^2 + b}{cx}$, where a, b and c are integers. Given that the line passes through (4, 9) and has gradient $-\frac{1}{4}$, find

(i) the value of $\frac{y}{x}$ where the straight line cuts the horizontal axis, [3]

(ii) the value of a, of b and of c. [3]

3 In the expansion $\left(2x^2 + \frac{3}{x}\right)^n$, in descending powers of x, the ratio of the coefficients of the third and first term is 81 : 1.

(i) Find the value of n. [3]

(ii) Write down the first three terms of the expansion. [2]

(iii) Find the term that is independent of x. [2]

4 **(i)** Express $\dfrac{11-7x}{3x^2+11x-4}$ in partial fractions. [3]

(ii) Hence evaluate $\int_1^2 \dfrac{11-7x}{9x^2+33x-12}\,\mathrm{d}x$. [4]

5 **(i)** Solve $2x^3+x^2-5x+2=0$. [4]

(ii) Hence solve $16\tan^3\theta+4\tan^2\theta-10\tan\theta+2=0$, where $0° \le \theta \le 90°$. [4]

6 A curve is such that $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{e^{5x}+1}{e^{3x}}$ and $(0, \dfrac{1}{2})$ is a point on the curve.

(i) Explain why the curve has no stationary points. [2]

(ii) Find the value of y when $x=2$. [6]

7 The equation of a curve is $y=\dfrac{(x-3)^2}{2x+5}$.

(i) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and obtain the coordinates of the stationary points. [5]

(ii) Find an expression for $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and hence determine the nature of these stationary points. [4]

8

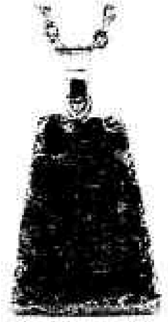


Diagram 1

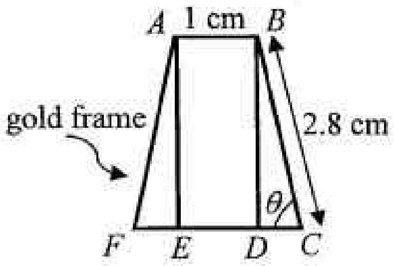


Diagram 2

Diagram 1 shows the front view of a pendant which can be modelled as a regular trapezium. Diagram 2 shows the back view of the modelled pendant with the gold frame that is used to hold the pendant. Trapezium *ABCF*, line *AE* and *BD* form the structure of the gold frame.

$AB = DE = 1$ cm, $AF = BC = 2.8$ cm and $\angle AFE = \angle BCD = \theta$.

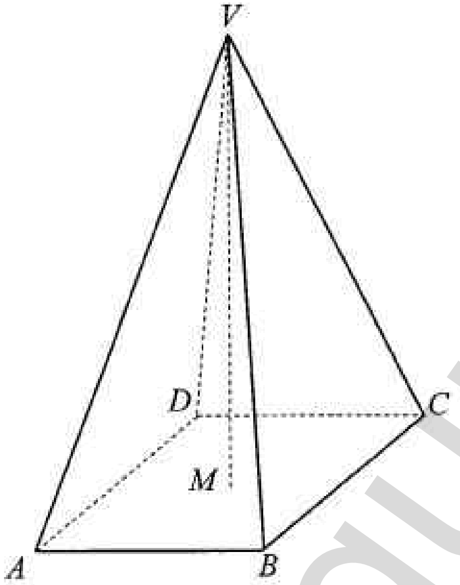
(i) Show that the total length of the structure that form the gold frame, *P*, is $(5.6\sin\theta + 5.6\cos\theta + 7.6)$cm. [2]

(ii) Express *P* in the form $R\sin(\theta+\alpha)+7.6$, where $R > 0$ and α is an acute angle. [4]

(iii) Given that the perimeter of the gold frame is 15 cm, find the values of θ. [3]

9 **Do not use a calculator in this question.**

(i) Express $\frac{7\sqrt{2}}{3\sqrt{2}-2}$ in the form $a+b\sqrt{2}$, where a and b are integers. [2]



The diagram shows a right pyramid with a square base of side $\frac{7\sqrt{2}}{3\sqrt{2}-2}$ cm.

Given that the height, VM, of the pyramid is $\frac{1}{2}BD^2$, find

(ii) an expression for BD^2 in the form $c+d\sqrt{2}$, where c and d are integers, [3]

(iii) the volume of the pyramid in the form $p+q\sqrt{2}$, where p and q are rational numbers. [4]

10 **(a)** A circle, whose equation is $x^2+y^2-10x+8y+5=0$, has centre C.

(i) Find the centre of the circle, C. [1]

(ii) Explain why point P $(4, -11)$ lies outside of the circle. [3]

(iii) A line drawn through P is tangent to the circle at point T.
Find the length of PT. [2]

(b) The equation of a curve is $y=x^2-7x+10$. Point A is a point on the curve and it lies on the y-axis.
Find the equation of the normal at point A. [4]

11 **(a)** Given that $y = \tan x$, show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx}y = 0$. [4]

(b) **(i)** Find $\int_0^\pi 8\cos^2\left(\frac{x}{2}\right)dx$. [3]

(ii) Hence find $\int_0^\pi \left[3 - \sin^2\left(\frac{x}{2}\right)\right]dx$. [3]

12 The roots of the quadratic equation $3x^2 - 7x + 4 = 0$ are $2\alpha + \beta$ and $\alpha + 2\beta$.

(i) Find the value of $\alpha + \beta$. [3]

(ii) Show that the value of $\alpha\beta = \frac{10}{81}$. [3]

(iii) Find a quadratic equation whose roots are $\frac{1}{2}\alpha + \beta$ and $\alpha + \frac{1}{2}\beta$. [5]

~ End of paper ~

Answers

1 (i) 3

(ii) $3x-1$

2 (i) $\frac{y}{x}=\frac{1}{40}$

(ii) $a=1$, $b=4$ and $c=40$

3 (i) $n=-8$ (rejected) or $n=9$

(ii)

$512x^{18}+6912x^{15}+41472x^{12}+\ldots$

(iii) 489888

4 (i)

$$\frac{11-7x}{3x^2+11x-4}=\frac{2}{3x-1}-\frac{3}{x+4}$$

(ii) 0.0213

5 (i) $x=1,\ x=\frac{1}{2}\ ,\ x=-2$

(ii) $\theta\approx 14.0°,\ 26.6°$

6 (i) For all real values of x,

$e^{2x}>0$ and $e^{-3x}>0$,

$\therefore \frac{dy}{dx}>0$, $\frac{dy}{dx}$ can never be zero.

$\therefore$ the curve has no stationary point.

(ii) $y\approx 27.6$

7 (i) $(3,0)$ and $(-8,-11)$

(ii) $(3,0)$ is a min. pt.

$(-8,0)$ is a max. pt.

8 (ii) $P=7.92\sin(\theta+45°)+7.6$

(iii) $\theta\approx 24.1°,\ 65.9°$

9 (i) $3+\sqrt{2}$

(ii) $BD^2=22+12\sqrt{2}$

(iii) $\frac{193}{3}+44\sqrt{2}\quad \text{cm}^3$

10(a) (i) $(5,-4)$

(ii) radius $=6$

Length of $PC=\sqrt{50}$

≈ 7.07

Since **length of *PC* is longer than radius** of circle, thus, the point P is outside of the circle.

(iii) 3.74 units

(b) $y=\frac{1}{7}x+10$

11(a) $y=\tan x$

$$\frac{dy}{dx}=\sec^2 x$$

$$\frac{d^2y}{dx^2}=2\sec x.\sec x.\tan x$$

$$\frac{d^2y}{dx^2}=2\frac{dy}{dx}y$$

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}y=0 \text{ (shown)}$$

11(b) (i) 4π

(ii) $\frac{5\pi}{2}$

12 (i) $\alpha+\beta=\frac{7}{9}$

(iii) $x^2-\frac{7}{6}x+\frac{1}{3}=0$

HIHS 2016 Prelim 4 Express
Additional Mathematics Paper 2 Marking Scheme

Qn	Workings	Marks allocation
1	(i) degree of polynomial, $Q(x) = 3$	B1
	(ii) $x^5 - 2x^3 + 2x^2 + 4x - 3 = Ax + B + (x^2 - 1)Q(x)$ subst. $x = 1$,	M1
	$1 - 2 + 2 + 4 - 3 = A + B + 0$ $A + B = 2$ -------------------- (1)	
	subst. $x = -1$,	M1
	$-1 + 2 + 2 - 4 - 3 = -A + B$ $-A + B = -4$ -------------------- (2) (1) + (2), $2B = -2$ $B = -1$	
	subst. $B = -1$ into (1), $A - 1 = 2$ $A = 3$	A1 each for correct A and B value
	The remainder is $3x - 1$.	A1
	Alternate Method: long division $x^5 - 2x^3 + 2x^2 + 4x - 3 = 3x - 1 + (x^2 - 1)(x^3 - x + 2)$	2 m for remainder 3 m for quotient (1 m for each term)
2	(i) $\frac{1}{xy} = -\frac{1}{4}\left(\frac{x}{y}\right) + C$	M1
	subst. (4, 9), $9 = -\frac{1}{4}(4) + C$ $C = 10$	M1
	Graph cuts at horizontal axis ➔ $\frac{1}{xy} = 0$ $0 = -\frac{1}{4}\left(\frac{x}{y}\right) + 10$ $\frac{y}{x} = \frac{1}{40}$	A1
	(ii) $\frac{1}{xy} = -\frac{1}{4}\left(\frac{x}{y}\right) + 10$ $1 = -\frac{1}{4}(x^2) + 10xy$	

Qn	Workings	Marks allocation
	$10xy=1+\frac{1}{4}x^2$ $40xy=4+x^2$ $y=\frac{4+x^2}{40x}$, thus $a=1$, $b=4$ and $c=40$	B3, 1 m each, working must be seen
3	**(i)** First term = 2^n Coeff. of third term = $\binom{n}{2}(2x^2)^{n-2}\left(\frac{3}{x}\right)^2$ $=\frac{n(n-1)}{2}(2^{n-2}3^2)(x^2)^{n-2}\left(\frac{1}{x}\right)^2$	M1
	Thus, $\frac{\frac{n(n-1)}{2}(2^{n-2}3^2)}{2^n}=81$	M1, o.e., formulating eqn
	$n(n-1)=\frac{81}{2^{-3}3^2}$ $n^2-n-72=0$ $(n+8)(n-9)=0$ $n+8=0$ or $n-9=0$ $n=-8$ (rejected) $n=9$	A1, must reject negative value
	(ii) $\left(2x^2+\frac{3}{x}\right)^9$ $=512x^{18}+\binom{9}{1}(2x^2)^8\left(\frac{3}{x}\right)+\binom{9}{2}(2x^2)^7\left(\frac{3}{x}\right)^2+\ldots$ $=512x^{18}+6912x^{15}+41472x^{12}+\ldots$	B2, minus 1 m for 1 error
	(iii) $T_{r+1}=\binom{9}{r}(2x^2)^{9-r}\left(\frac{3}{x}\right)^r$ $\rightarrow$ $2(9-r)-r=0$ $r=6$	M1, o.e. (e.g. expansion)
	Term independent of $x=\binom{9}{6}(2)^{9-6}(3)^6$ $=489888$	A1
4	**(i)** $\frac{11-7x}{3x^2+11x-4}=\frac{11-7x}{(3x-1)(x+4)}$	

Qn	Workings	Marks allocation
	$\frac{11-7x}{(3x-1)(x+4)}=\frac{A}{3x-1}+\frac{B}{x+4}$	M1
	$11-7x=A(x+4)+B(3x-1)$	
	subst $x=-4$, $11+28=B(-13)$ $B=-3$	A1
	$x=0$, $11=4A+3$ $4A=8$ $A=2$	A1
	Therefore, $\frac{11-7x}{3x^2+11x-4}=\frac{2}{3x-1}-\frac{3}{x+4}$	minus 1m if not written in partial fractions form
	(ii) $\int_1^2 \frac{11-7x}{9x^2+33x-12}\mathrm{d}x$ $=\int_1^2 \frac{11-7x}{3(3x^2+11x-4)}\mathrm{d}x$ $=\int_1^2 \frac{11-7x}{3(3x-1)(x+4)}\mathrm{d}x$	M1, o.e.
	$=\frac{1}{3}\int_1^2 \frac{2}{3x-1}-\frac{3}{x+4}\mathrm{d}x$	M1, integrating ln
	$=\frac{1}{3}\left[\frac{2}{3}\ln(3x-1)-3\ln(x+4)\right]_1^2$	
	$=\frac{1}{3}\left[\frac{2}{3}\ln(5)-3\ln(6)\right]-\frac{1}{3}\left[\frac{2}{3}\ln(2)-3\ln(5)\right]$	[M1, subst]
	$=\frac{1}{3}\left[\frac{2}{3}\ln(\frac{5}{2})+3\ln(\frac{5}{6})\right]$	
	$=\frac{2}{9}\ln(\frac{5}{2})+\ln(\frac{5}{6})$ ≈ 0.0213	A1
5	**(i)** let $f(x) = 2x^3+x^2-5x+2$	
	$f(1)=0$ therefore, $x-1$ is a factor of $f(x)$	M1
	$2x^3+x^2-5x+2=(x-1)(2x^2+ax-2)$ comparing coefficient of x, $-5=-a-2$ $a=3$	
	therefore, $f(x)=(x-1)(2x^2+3x-2)$	M1, o.e.

Qn	Workings	Marks allocation
	$= (x-1)(2x-1)(x+2)$	
	$2x^3 + x^2 - 5x + 2 = 0$ $(x-1)(2x-1)(x+2) = 0$ $x-1=0$ or $2x-1=0$ or $x+2=0$ $x=1$ $\quad x=\frac{1}{2}$ $\quad x=-2$	A2, minus 1m for 1 error
	(ii) $16\tan^3\theta + 4\tan^2\theta - 10\tan\theta + 2 = 0$ $2(2\tan\theta)^3 + (2\tan\theta)^3 - 5(2\tan\theta) + 2 = 0$	M1, or identify $x = 2\tan\theta$
	By comparing, $x = 2\tan\theta$, $(2\tan\theta-1)(4\tan\theta-1)(2\tan\theta+2)=0$	M1 (factorised)
	$2\tan\theta-1=0$ or $4\tan\theta-1=0$ or $2\tan\theta+2=0$ $\tan\theta=\frac{1}{2}$ or $\tan\theta=\frac{1}{4}$ or $\tan\theta=-1$ (rejected) $\theta \approx 26.6°$ $\quad \theta \approx 14.0°$	A2, minus 1 m if $\tan\theta=-1$ not rejected
6	**(i)** $\frac{dy}{dx} = \frac{e^{5x}+1}{e^{3x}}$ $\frac{dy}{dx} = e^{2x} + e^{-3x}$	
	when $\frac{dy}{dx} = 0$, $e^{2x} + e^{-3x} = 0$	M1, o.e.
	$e^{2x} = -e^{-3x}$ $e^{2x} \div e^{-3x} = -1$ $e^{5x} = -1$ x is undefined, thus the curve does not have stationary points.	A1, conclusion
	OR $e^{5x} = -1$ (rejected) Since $e^{5x} > 0$ for all values of x, hence the curve does not have stationary points	
	OR For all real values of x, $e^{2x} > 0$ and $e^{-3x} > 0$,	M1
	$\therefore \frac{dy}{dx} > 0$, $\frac{dy}{dx}$ can never be zero. $\therefore$ the curve has no stationary point.	A1

Qn	Workings	Marks allocation
	(ii) $\frac{dy}{dx}=e^{2x}+e^{-3x}$	
	$y=\frac{e^{2x}}{2}-\frac{e^{-3x}}{3}+c$	M2, integrate exponential
	subst. $(0, \frac{1}{2})$,	
	$\frac{1}{2}=\frac{e^{2(0)}}{2}-\frac{e^{-3(0)}}{3}+c$	M1
	$\frac{1}{2}=\frac{1}{2}-\frac{1}{3}+c$	
	$c=\frac{1}{3}$	M1
	Eqn of curve is $y=\frac{e^{2x}}{2}-\frac{e^{-3x}}{3}+\frac{1}{3}$	
	when $x=2$, $y=\frac{e^{2(2)}}{2}-\frac{e^{-3(2)}}{3}+\frac{1}{3}$	M1, subst into eqn of curve]
	$y=\frac{e^{4}}{2}-\frac{1}{3e^{6}}+\frac{1}{3}$	
	$y\approx 27.6$	A1
7	(i) $y=\frac{(x-3)^2}{2x+5}$	
	$\frac{dy}{dx}=\frac{(2x+5)(2)(x-3)-(x-3)^2(2)}{(2x+5)^2}$	M2
	$=\frac{(2x+5)(2x-6)-(x^2-6x+9)(2)}{(2x+5)^2}$	
	$=\frac{4x^2-12x+10x-30-2x^2+12x-18}{(2x+5)^2}$	
	$=\frac{2x^2+10x-48}{(2x+5)^2}$	
	For stationary points, $\frac{dy}{dx}=0$	M1, o.e.
	$\frac{(2x+5)(2)(x-3)-(x-3)^2(2)}{(2x+5)^2}=0$	
	$(2x+5)(2)(x-3)-(x-3)^2(2)=0$	
	$(x-3)(4x+10-2x+6)=0$	
	$(x-3)(2x+16)=0$	
	$x-3=0$ or $2x+16=0$	A1, for x coordinates
	$x=3$ or $x=-8$	

Qn	Workings	Marks allocation
	subst. $x=3$, into $y=\frac{(x-3)^2}{2x+5}$, $y=0$	
	subst. $x=-8$, into $y=\frac{(x-3)^2}{2x+5}$, $y=\frac{(-8-3)^2}{2(-8)+5}$, $y=-11$ The stationary points are $(3,0)$ and $(-8,-11)$.	A1, for y coordinates [minus 1m if not written in coordinates form]
	(ii) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2x^2+10x-48}{(2x+5)^2}$	
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{(2x+5)^2(4x+10)-(2x^2+10x-48)(2)(2x+5)(2)}{(2x+5)^4}$	M2
	when $x=3$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{2662-0}{14641}=\frac{2}{11}>0$, $(3,0)$ is a min. pt.	A1
	when $x=-8$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{-2662-0}{14641}=-\frac{2}{11}<0$, $(-8,0)$ is a max. pt.	A1
8	**(i)** Perimeter of pendent $=1+1+2\times2.8+2\times2.8\sin\theta+2\times2.8\cos\theta$ $=(5.6\sin\theta+5.6\cos\theta+7.6)\text{cm}$ (Shown)	 M1 A1
	(ii) $R=\sqrt{5.6^2+5.6^2}$ $=\sqrt{62.72}$ ≈7.92	M1 A1
	$\tan\alpha=\frac{5.6}{5.6}$ $\alpha=45°$	M1 A1
	$P=7.92\sin(\theta+45°)+7.6$	(minus 1 m if student did not express in this form)
	(iii) $15=7.92\sin(\theta+45°)+7.6$ $7.4=7.92\sin(\theta+45°)$ $\sin(\theta+45°)=\frac{185}{198}$	 M1
	Basic angle = $69.1223°$ $\theta+45°=69.1223°,180°-69.1223°$	M1 (basic angle)
	$\theta=24.1223°,65.8777°$ $\theta\approx24.1°,\ 65.9°$	 A1

Qn	Workings	Marks allocation
9	**(i)** $\frac{7\sqrt{2}}{3\sqrt{2}-2}\times\frac{3\sqrt{2}+2}{3\sqrt{2}+2}$ $=\frac{7\sqrt{2}\left(3\sqrt{2}+2\right)}{18-4}$ $=\frac{42+14\sqrt{2}}{14}$	M1, rationalise
	$=3+\sqrt{2}$	A1
	(ii) by Pythagoras Theorem, $BD^2=AB^2+AD^2$ Using part (i) answer, $BD^2=\left(3+\sqrt{2}\right)^2+\left(3+\sqrt{2}\right)^2$	M1, formulating
	$BD^2=2\left(3+\sqrt{2}\right)^2$ $BD^2=2\left(9+6\sqrt{2}+2\right)$ $BD^2=22+12\sqrt{2}$	A2, A1 for 22 and A1 for $12\sqrt{2}$
	(iii) Volume of pyramid $=\frac{1}{3}\times\text{base area}\times\text{height}$ $=\frac{1}{3}\times\left(3+\sqrt{2}\right)^2\times\frac{1}{2}\left(22+12\sqrt{2}\right)$	M1, subst. correct values
	$=\frac{1}{3}\times\left(11+6\sqrt{2}\right)\times\left(11+6\sqrt{2}\right)$	M1, correct expansion
	$=\frac{1}{3}\left(121+132\sqrt{2}+72\right)$ $=\frac{1}{3}\left(193+132\sqrt{2}\right)$ $=\frac{193}{3}+44\sqrt{2}\ \text{cm}^3$	A2, A1 for $\frac{193}{3}$, A1 for $44\sqrt{2}$
10	**(a)** **(i)** centre C $=\left(\frac{-10}{-2},\frac{8}{-2}\right)$ $=(5,-4)$	B1
	(ii) radius $=\sqrt{5^2+4^2-5}$ $=6$	M1 (o.e.)
	Length of $PC=\sqrt{(5-4)^2+(-4+11)^2}$ $=\sqrt{50}$ ≈ 7.07	M1

Qn	Workings	Marks allocation
	Since **length of *PC* is longer than radius** of circle, thus, the point *P* is outside of the circle.	A1 (find length PC and conclude)
	(iii) by Pythagoras' Theorem,	
	$PT=\sqrt{50-6^2}$ $=\sqrt{14}$	M1
	≈ 3.74 units	A1
	(b) point A = (0, 10)	
	$\frac{dy}{dx}=2x-7$	M1
	when $x=0$, $\frac{dy}{dx}=-7$	M1
	gradient of normal = $\frac{1}{7}$	M1
	equation of normal is $y=\frac{1}{7}x+10$	A1
11	**(a)** $y=\tan x$	
	$\frac{dy}{dx}=\sec^2 x$	M1
	$\frac{d^2y}{dx^2}=2\sec x.\sec x.\tan x$	M2, 1m for $2\sec x$, 1m for $\sec x.\tan x$ (o.e.)
	$\frac{d^2y}{dx^2}=2\frac{dy}{dx}y$ $\frac{d^2y}{dx^2}-2\frac{dy}{dx}y=0$ (shown)	A1
	<u>Alternate solution</u> $\frac{dy}{dx}=\sec^2 x$ $=\frac{1}{\cos^2 x}$ $\frac{d^2y}{dx^2}=\frac{0-2\cos x(-\sin x)}{\cos^4 x}$ $=2\sec x.\sec x.\tan x$ LHS = $2\sec x.\sec x.\tan x-2\sec^2 x\tan x$ $=0$ = RHS	

Qn	Workings	Marks allocation
	(b) **(i)** $\int_0^\pi 8\cos^2\left(\frac{x}{2}\right)dx = 4\int_0^\pi 2\cos^2\left(\frac{x}{2}\right)dx$	M1, using $\cos x = 2\cos^2\left(\frac{x}{2}\right)-1$]
	$= 4\int_0^\pi (\cos x + 1)dx$	M1, integrate
	$= 4[\sin x + x]_0^\pi$ $= 4[0+\pi-(0\text{-}0)]$ $= 4\pi$	A1
	(ii) $\int_0^\pi \left[3-\sin^2\left(\frac{x}{2}\right)\right]dx = \int_0^\pi \left[2+1-\sin^2\left(\frac{x}{2}\right)\right]dx$	
	$= \int_0^\pi \left[2+\cos^2\left(\frac{x}{2}\right)\right]dx$	M1 (apply identity)
	$= \int_0^\pi 2dx + \int_0^\pi \cos^2\left(\frac{x}{2}\right)dx$ $= [2x]_0^\pi + \frac{4\pi}{8}$	M1
	$= \frac{5\pi}{2}$	A1
12	**(i)** sum of roots, $2\alpha+\beta+\alpha+2\beta = 3\alpha+3\beta$ $= 3(\alpha+\beta)$	M1 $(3(\alpha+\beta))$
	$= -\frac{-7}{3}$ $= \frac{7}{3}$	M1
	product of roots, $(2\alpha+\beta)(\alpha+2\beta) = 2\alpha^2+4\alpha\beta+\alpha\beta+2\beta^2$ $= 2\alpha^2+5\alpha\beta+2\beta^2$ $= \frac{4}{3}$ $\alpha+\beta = \frac{1}{3}\left(\frac{7}{3}\right)$ $= \frac{7}{9}$	A1
	(ii) from product of roots, $2\alpha^2+4\alpha\beta+\alpha\beta+2\beta^2 = \frac{4}{3}$	M1, o.e.
	$2\alpha^2+4\alpha\beta+2\beta^2+\alpha\beta = \frac{4}{3}$ $2(\alpha^2+2\alpha\beta+\beta^2)+\alpha\beta = \frac{4}{3}$	

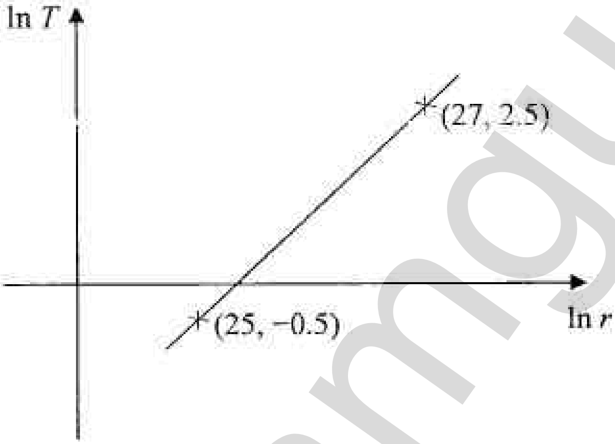
Qn	Workings	Marks allocation
	$2(\alpha+\beta)^2+\alpha\beta=\frac{4}{3}$	M1
	$2\left(\frac{7}{9}\right)^2+\alpha\beta=\frac{4}{3}$	
	$\alpha\beta=\frac{4}{3}-2\left(\frac{7}{9}\right)^2$	
	$\alpha\beta=\frac{10}{81}$ (shown)	A1
	(iii) sum of roots, $\frac{1}{2}\alpha+\beta+\alpha+\frac{1}{2}\beta=\frac{3}{2}(\alpha+\beta)$	
	$=\frac{3}{2}\left(\frac{7}{9}\right)$	
	$=\frac{7}{6}$	M1
	Product of roots,	
	$\left(\frac{1}{2}\alpha+\beta\right)\left(\alpha+\frac{1}{2}\beta\right)=\frac{1}{2}\alpha^2+\frac{1}{4}\alpha\beta+\alpha\beta+\frac{1}{2}\beta^2$	
	$=\frac{1}{2}\alpha^2+\frac{5}{4}\alpha\beta+\frac{1}{2}\beta^2$	
	$=\frac{1}{2}\left(\alpha^2+\beta^2\right)+\frac{5}{4}\alpha\beta$	M1
	$=\frac{1}{2}\left((\alpha+\beta)^2-2\alpha\beta\right)+\frac{5}{4}\alpha\beta$	
	$=\frac{1}{2}\left(\left(\frac{7}{9}\right)^2-2\left(\frac{10}{81}\right)\right)+\frac{5}{4}\left(\frac{10}{81}\right)$	M1
	$=\frac{1}{3}$	M1
	The quadratic equation is $x^2-\frac{7}{6}x+\frac{1}{3}=0$	A1, accept $6x^2-7x+2=0$

1 Express $\dfrac{5x^2+x+6}{(3-2x)(x^2+4)}$ in partial fractions. [5]

2 **(i)** Prove that $\dfrac{1}{\tan\theta+\cot\theta}=\dfrac{\sin 2\theta}{2}$. [4]

(ii) Hence, solve the equation $\dfrac{1}{\tan\dfrac{\theta}{2}+\cot\dfrac{\theta}{2}}=\dfrac{1}{4}$ for $-2\pi\le\theta\le 2\pi$. [3]

3



The period T, in years, of planets' orbit around the Sun is given by $T=kr^n$, where r is the distance, in metres, of the planet from the Sun, and k and n are constants to be determined. The graph of ln T against ln r is given.

(i) Find the value of k and of n. [3]

(ii) Find the period of a planet which is 60×10^9 metres from the Sun. [2]

(iii) On the same axes, a straight line representing the equation ln $T=1$ was drawn. Explain the significance of the intersection of the two lines. [1]

4 **(i)** Expand $\left(x+\frac{1}{x}\right)^4$ in descending powers of x. [2]

(ii) Hence, given that $\left(x+\frac{1}{x}\right)^4-\left(x-\frac{1}{x}\right)^4=ax^2+\frac{b}{x^2}$, find the value of a and of b. [3]

(iii) Given that there is no x term in the expansion of $\left(\frac{4}{3}x+\frac{k}{x}+\frac{x^3}{k}\right)\left(x+\frac{1}{x}\right)^4$, find the value of k. [3]

5 It is given that f (x) is such that f '(x) = 4 cos x + 8 sin $\frac{x}{2}$ + 3.

(i) Find f " (x). [2]

(ii) Given further that f (π)= 0, find f (x). [4]

6 The equation of a curve is $y=ax^2+bx-3$, where a and b are constants and the curve has a minimum turning point.

(i) Explain why the curve cuts the x-axis at two distinct points. [3]

(ii) In the case where a = 1, find the range of values of b for which the curve is above the line $y=x-4$. [4]

(iii) Hence, state the values of b for which the line is a tangent to the curve. [1]

7 A graph has the equation $y=-|3x-9|+6$.

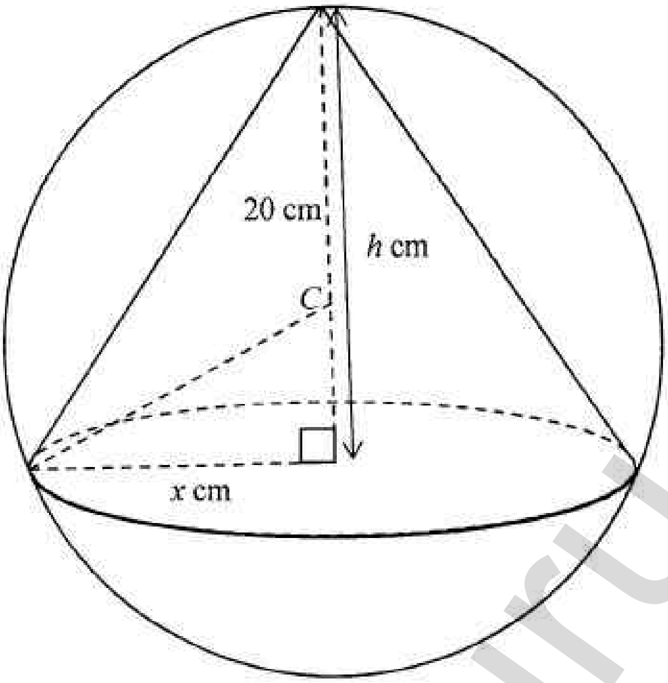
(i) Explain why the highest point on the graph has coordinates (3, 6). [2]

(ii) Find the coordinates at which the graph cuts the x-axis. [2]

(iii) Sketch the graph of $y=-|3x-9|+6$. [2]

(iv) Find the range of values of m such that $-|3x-9|+6=mx$ has 2 solutions. [2]

8



A cone is inscribed in a sphere of radius 20 cm, centre C.
The cone has height, h cm and radius, x cm.

(i) Show that $x = \sqrt{40h - h^2}$. [1]

(ii) Hence, express the volume of the cone in terms of h. [1]

(iii) Given that h can vary, find the value of h for which the volume of the cone is stationary. [3]

(iv) Determine whether this value of h gives the largest cone possible. [1]

9 Given that $\tan 2A = \frac{3}{4}$ and $180° < 2A < 270°$, find, without using a calculator, the exact values of

(i) $\sin 2A$, [2]

(ii) $\sin A$. [3]

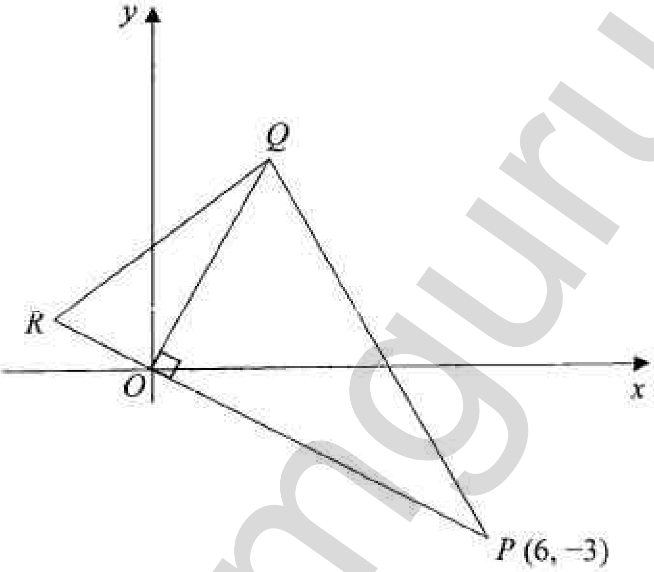
10 The line l, $2x + y = 10$ cuts the curve $xy = 12$ at $T\,(2, 6)$.

(i) Find the equation of the tangent to the curve at T. [2]

(ii) Find the angle, in degrees, between l and the tangent to the curve at T. [2]

(iii) State the gradient of the normal at T. Hence, determine, with reason, whether the normal to the curve will get steeper or gentler as x increases. [2]

11



The diagram show a triangle PQR in which P is the point $(6, -3)$. The line PR passes through the origin O. The line OQ is perpendicular to PR. The area of triangle POQ is 15 units^2.

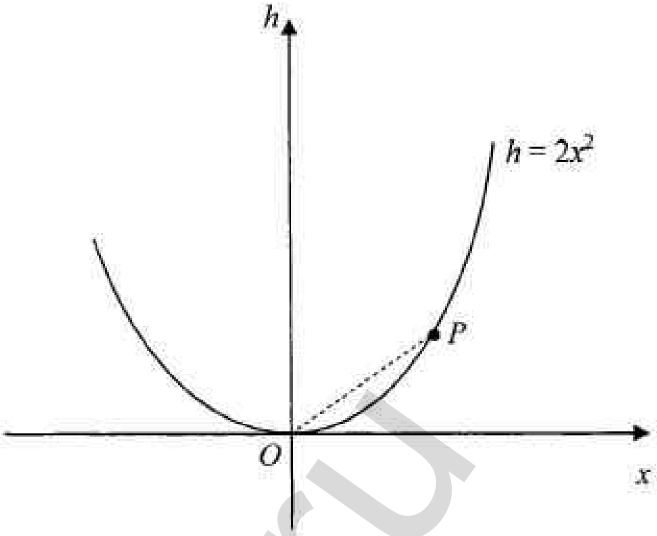
(i) Find the equation of OQ. [2]

(ii) Find the coordinates of Q. [3]

(iii) The length of PO is 3 times the length of OR. Find the coordinates of R. [1]

(iv) The point S is such that any point on the line PR is equidistant from Q and S. Find the coordinates of S. [1]

12



The height above ground level, h m, of a car in a roller coaster is modelled by the equation, $h = 2x^2$, where x is the horizontal distance of the car in metres from a fixed point O.

(i) Given that the horizontal distance of the car is increasing at a constant rate of 2 m/s, find the rate at which the height of the car is increasing when $x = 3$. [3]

(ii) The distance, L, of the car from O is OP, where P is a moving point on the curve. Show that $L = \sqrt{x^2 + 4x^4}$. [1]

(iii) It is possible to take a high definition photograph of the car from the fixed point O if the distance, L is changing at a rate of not more than 20 m/s. Would you be able to take a high definition photograph of the car from the fixed point O when $x = 3$? [4]

End of Paper

Answers:

1. $\frac{5x^2+x+6}{(3-2x)(x^2+4)}=\frac{3}{3-2x}+\frac{-x-2}{x^2+4}$

2ii. $\frac{\pi}{6},\frac{5\pi}{6},\frac{-11\pi}{6},\frac{-7\pi}{6}$

3i. $n=\frac{3}{2}$, $k=e^{-38}$ or 3.14×10^{-17}

3ii. 0.461

4i. $x^4+4x^2+6+4x^{-2}+x^{-4}$

4ii. $a=8, b=8$

4iii. -1

5i. $-4\sin x+4\cos\frac{x}{2}$

5ii. $f(x)=4\sin x-16\cos\frac{x}{2}+3x-3\pi$

6ii. $-1<b<3$

6iii. -1 or 3

7ii. (5.0) and (1,0)

7iv. -3 < m < 2

8ii. $\frac{1}{3}\pi(40h^2-h^3)$

8iii. $h=\frac{80}{3}$

9i. $-\frac{3}{5}$

9ii. $\frac{3}{\sqrt{10}}$

10i. $y=-3x+12$

10ii. 8.14°

11i. $y=2x$, 11ii. (2, 4), 11iii. (−2, 1), 11iv. (−2, −4)

12i. 24 m/s

12iii. $\frac{dL}{dt}=24.0m/s>20$

No

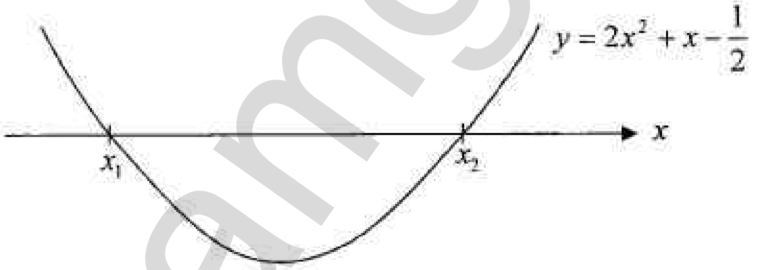
Answer **all** questions.

1 **(i)** Sketch the graph $y = 2x^{\frac{3}{2}}$. [2]

(ii) Find the equation of the graph that has to be drawn in part **(i)** in order to to obtain the graphical solution of $2x^{\frac{11}{6}} = 1$. On the same axes, sketch this graph for $x > 0$. [3]

2 **(a)** The cubic polynomial $\mathrm{f}(x)$ is such that the coefficient of x^3 is 2 and the roots of the equation $\mathrm{f}(x) = 0$ are 2, $-\frac{1}{2}$ and *k*. Given that $\mathrm{f}(x)$ has a remainder of -6 when divided by $x - 1$. Find the value of *k*. [3]

(b) Given that the quadratic curve $y = 2x^2 + x - \frac{1}{2}$ cuts the *x*-axis at x_1 and x_2 as shown in the diagram below. Find the exact value of $\frac{x_1}{x_2}$, leaving your answer in the simplest surd form. [4]



3 The mass, *M* grammes, of a substance, present at the time *t* minutes after first being measured, is given by $M = 10 + 90e^{-0.2t}$. Find

(i) the initial mass of the substance, [1]

(ii) the time taken for the initial mass of the substance to be reduced by 20%, [3]

(iii) the approximate mass of the substance when *t* becomes very large, [1]

(iv) the rate at which the mass is decreasing when $t = 3$ minutes. [3]

Sketch the curve $M = 10 + 90e^{-0.2t}$. [2]

4 **(a)** Solve the following equations.

(i) $3^{\log_4 x} = 729$, [3]

(ii) $\log_2(x-2)+2\log_4(x+1)=\frac{1}{\log_9 3}$. [4]

(b) Given that $x = 3^a$ and $y = 3^b$, express $\log_3\left(\frac{\sqrt{xy^2}}{27}\right)$ in terms of a and of b. [4]

5 **(i)** Solve $-2\sin 2x = 3\cos x$ for $0° \le x \le 360°$. [4]

(ii) On the same diagram, sketch the graphs of $y = -\sin 2x$ and $y = \frac{3}{2}\cos x$ for $0° \le x \le 360°$. [4]

(iii) Hence, explain how parts **(i)** and **(ii)** could be used to deduce the solution(s) of $|-2\sin 2x| = 3\cos x$ for $0° \le x \le 360°$. [2]

6 **(a)** Show that the function $\frac{x^2-4}{x}$ always increases as x increases. [3]

(b) Differentiate $\frac{\sqrt{x}}{1+2x}$ with respect to x. [4]

7 The roots of the quadratic equation $x^2 - 5x + 4 = 0$ are α^2 and β^2, where both α and β are positive.

(i) Show that $\alpha + \beta = 3$. [3]

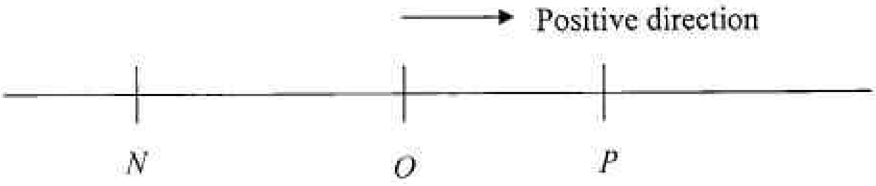
(ii) Find the quadratic equation whose roots are $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$. [4]

8 **(i)** Given that the line $x + y = 2$ is a tangent to a circle with centre $C(0, 6)$

Find the equation of the circle. [6]

(ii) A second circle $x^2 + y^2 = 6y + d$, where d is an integer, is the reflection of the circle in part **(i)** about the line $y = k$. Find the value of k and of d. [5]

9 N, O and P are three fixed points on a straight line as shown in the diagram below. Given that the velocity, v m/s, of a particle travelling on the straight line NP at time t seconds after leaving the fixed point O, is given by $v = t^3 - 10t^2 + 27t - 18$.



(i) Find the initial velocity of the particle at O. Explain the significance of your answer. [2]

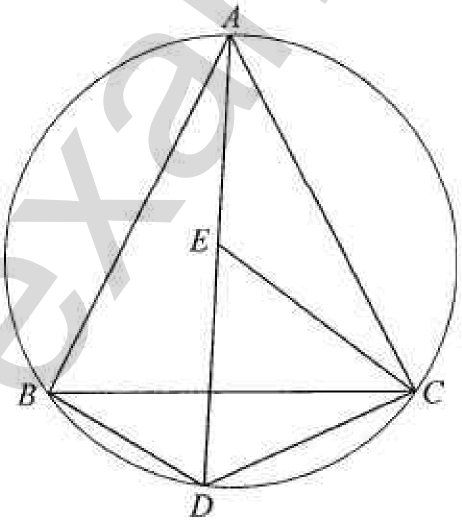
(ii) Find the values of t when the particle comes instantaneously to rest. [4]

(iii) Find the maximum speed attained by the particle for $0 \le t \le 6$. [4]

(iv) Calculate the distance travelled by the particle in the second second. [3]

10 In the diagram, triangle ABC is an equilateral triangle inscribed in a circle.

D is a point on the arc BC, E is a point on AD and $CD = CE$.



Show that

(i) triangle CDE is equilateral, [3]

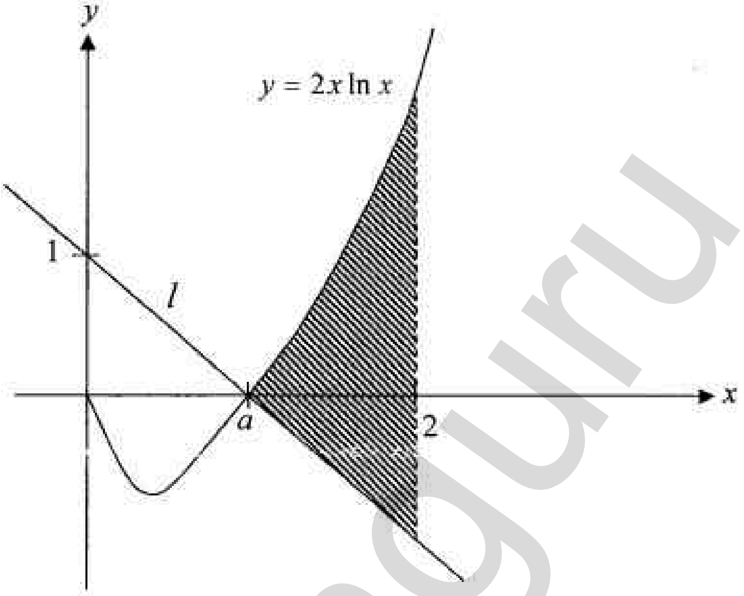
(ii) triangle ACE is congruent to triangle BCD, [3]

(iii) $AD = BD + CD$. [3]

[Turn over

11 **(a)** Differentiate $x^2 \ln x - x$ with respect to x. [3]

(b) The diagram shows the line l and part of the curve $y = 2x \ln x$. Both graphs intersect the x-axis at a. Line l cuts the y-axis at 1.



(i) Find the value of a. [2]

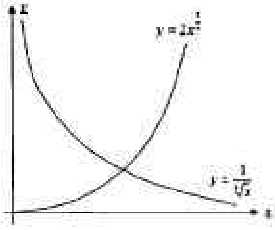
(ii) Find the equation of line l. [1]

(iii) Determine the area of the shaded region bounded by the curve, the line $x = 2$ and the line l. [4]

End of Paper

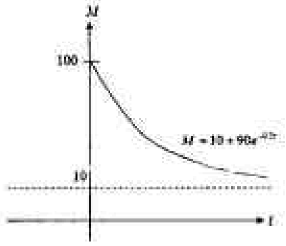
Answers

1(i) 1(ii) $y = \frac{1}{\sqrt[3]{x}}$ (one possible answer)



2(a) $k = -1$ 2(b) $-\frac{3+\sqrt{5}}{2}$

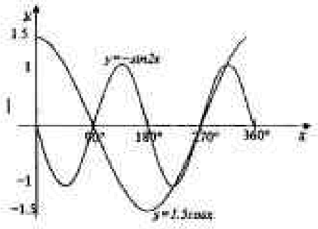
3(i) 100g 3(ii) $t = 1.26$ mins 3(iii) 10 g 3(iv) 9.88 g/min



4(a)(i) 4096 4(a)(ii) 3 4(b) $\frac{1}{2}a + b - 3$

5(i) $x = 90°, 228.6°, 270°, 311.4°$

5(ii)



5(iii) **<u>Reflect the negative parts</u>** of the drawn sine graph in part **(ii)** about the x-axis and relate to **<u>the x-coordinates of the points of intersection</u>** found in part **(i)** give the solution to $|-2\sin 2x| = 3\cos x$.

6(a) Since $\frac{d}{dx}\left(\frac{x^2-4}{x}\right) > 0$, $\therefore$ $\frac{x^2-4}{x}$ increases as x increases. 6(b) $\frac{1-2x}{2\sqrt{x}(1+2x)^2}$

7(ii) $8x^2 - 9x + 1 = 0$ 8(i) $x^2 + (y-6)^2 = 8$ 8(ii) $d = -1$, $k = 4\frac{1}{2}$

9(i) $v = -18$ m/s. The particle is moving in the opposite direction to the positive direction/moving to the left, etc.

(ii) $t = 1$, $t = 3$, $t = 6$ (iii) $= 4.06$ m/s (iv) $= 2.92$ m

11(a) $= x + 2x\ln x - 1$ (b)(i) $a = 1$ (b)(ii) $y = -x + 1$ (b)(iii) $= 1.773$ units^2

1 The area of a triangle is $\left(1+\frac{5\sqrt{5}}{2}\right)$ cm^2. If the length of the base of the triangle is $\left(3+2\sqrt{5}\right)$ cm, find, without using a calculator, the height of the triangle in the form of $\left(a+b\sqrt{5}\right)$ cm, where a and b are integers. [4]

2 Express $\frac{4x^2+6x+5}{2x^2+x-3}$ in partial fractions. [5]

3 The function $f(x)$ is such that $f(x)=2x^3+3x^2-x-4$,

(i) find a factor of $f(x)$. [2]

(ii) Hence, determine the number of solutions in the equation $f(x)=0$. [4]

4 The roots of the quadratic equation $3x^2-x+5=0$ are α and β.

(i) Evaluate $\alpha^2+\beta^2$. [2]

(ii) Find the quadratic equation whose roots are α^3-1 and β^3-1. [4]

5 The table shows experimental values of 2 variables, R and V, which are connected by an equation of the form $RV^n=k$ where n and k are constants.

R	33	19.95	5.07	2.38
V	2	2.9	8	14

(i) Plot lg R against lg V for the given data and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of k and of n. [3]

(iii) By drawing a suitable straight line on your graph in **(i)**, find the value of V such that $\frac{R}{V^2}=1$. [3]

6 Given that $y=1-\frac{1}{2}\sin 3x$, $0^\circ \le x \le 240^\circ$.

(i) State the maximum and minimum values of y. [2]

(ii) Sketch the graph of $y=1-\frac{1}{2}\sin 3x$. [3]

10 The points A and B lie on the circumference of a circle C_1 where A is the point (0, 8) and B is the point (4, 0). The line $y = 2x$ also passes through the centre of the circle C_1.

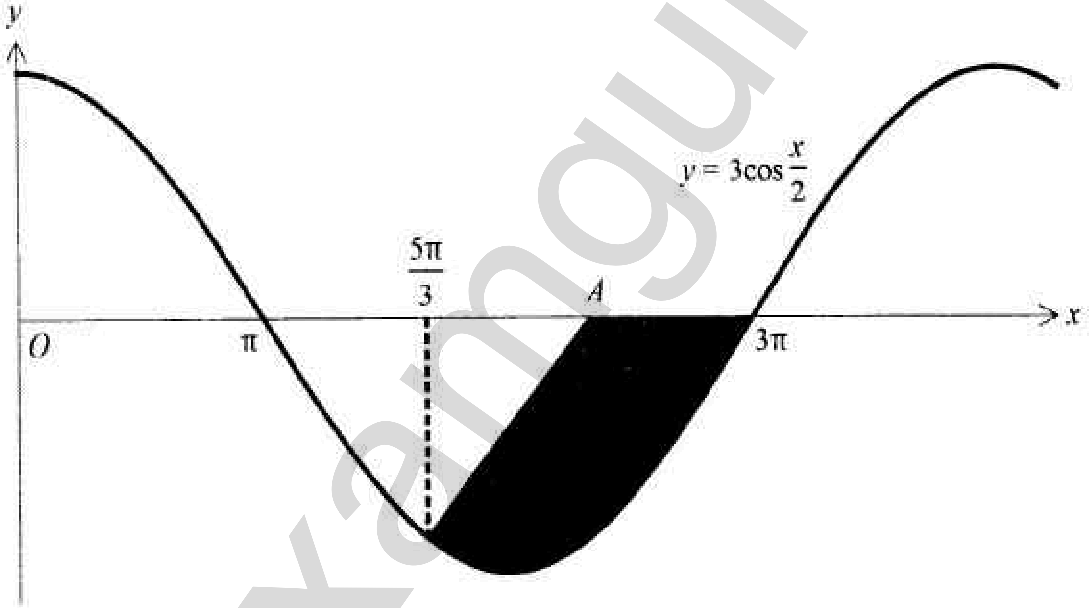
(i) Find the centre and radius of the circle C_1. [4]

(ii) Find the equation of the circle C_1 in the form $x^2 + y^2 + px + qy + r = 0$, where p, q and r are integers. [2]

Another circle C_2 of radius $\sqrt{2}$ units has its centre inside C_1 and it cuts the circle C_1 at the origin and at the point where $x = 2$.

(iii) Find the centre of C_2. [5]

11

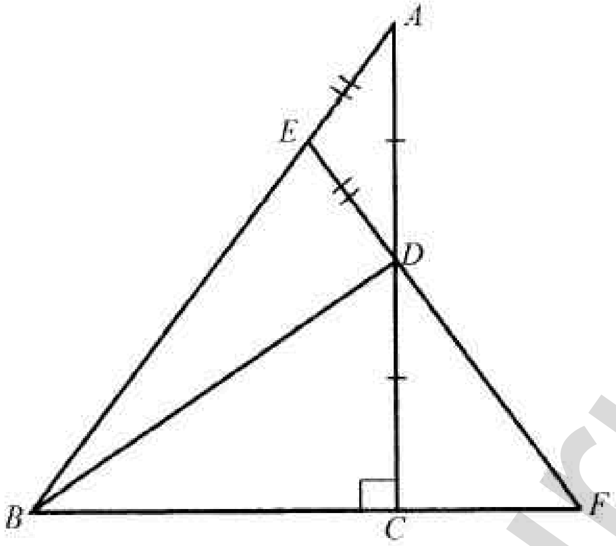


The diagram shows part of the curve $y = 3\cos\frac{x}{2}$ that cuts the x – axis at $x = \pi$ and $x = 3\pi$. The normal to the curve at $x = \frac{5\pi}{3}$ cuts the x-axis at A.

(i) Find the coordinates of A, leaving your answer in exact form. [6]

(ii) Hence, find the area of the shaded region. [4]

11 **(a)** The diagram shows a triangle ABC which has a right angle at C.
The point D is the mid-point of the side AC.
The point E lies on AB such that $AE = DE$.
The line segment ED is produced to meet the line BC produced at F.

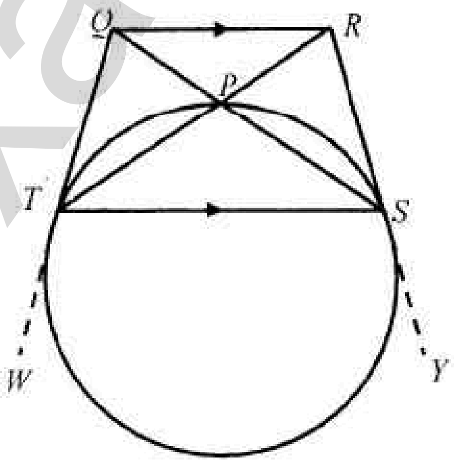


(i) Prove that ΔACB is similar to ΔDCF. [2]

(ii) Explain why ΔEFB is isosceles. [1]

(iii) Show that $EB = 3AE$. [2]

(b) $QRST$ is a trapezium in which QR is parallel to TS and its diagonals meet at P. The circle through T, P and S touches QW, RY at T and S respectively.



Prove that

(i) $\angle RQS = \angle QTR$. [2]

(ii) $QRST$ is a cyclic quadrilateral. [3]

End of Paper

1 **(a)** The equation of a curve is $y = 2x^2 + ax + (6 + a)$, where a is a constant. Find the range of values of a for which the curve lies completely above the x-axis. [3]

(b) The equation of a curve is $y = 3x^2 + 4x + 6$.

(i) Find the set of values of x for which the curve is above the line $y = 6$. [3]

(ii) Show that the line $y = -8x - 6$ is a tangent to the curve. [2]

2 **(a)** Given that $\log_a 125 - 3\log_a b + \log_a c = 3$, express a in terms of b and c. [3]

(b) Solve the equation

(i) $\lg 8x - \lg(x^2 - 3) = 2\lg 2$, [3]

(ii) $2\log_5 x = 3 + 7\log_x 5$. [4]

3 The equation of a curve is $y = x^2\sqrt{(5x-1)^3}$, for $x > 0.2$. Given that x is changing at a constant rate of 0.25 units per second, find the rate of change of y when $x = 2$. [4]

4 The graph of $y = |2x^2 - ax - 5|$ passes through the points with coordinates $(-1,\ 0)$ and $(0.75,\ b)$.

(i) Find the value of the constants a and b. [3]

(ii) Sketch the graph of $y = |2x^2 - ax - 5|$. [3]

(iii) Determine the set of positive values of m for which the line $y = mx + 2$ intersects the graph of $y = |2x^2 - ax - 5|$ at two points. [2]

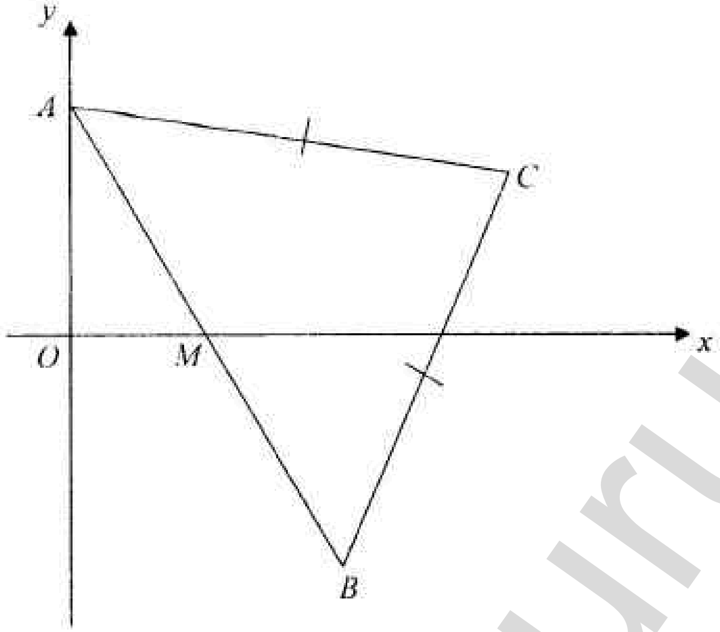
5 In the binomial expansion of $\left(2x + \frac{k}{x}\right)^8$, where k is a positive constant, the coefficient of x^2 is 28.

(i) Show that $k = \frac{1}{4}$. [4]

(ii) Hence, determine the term in x in the expansion of $\left(6x - \frac{1}{x}\right)\left(2x + \frac{k}{x}\right)^8$. [4]

7

12 The diagram, not drawn to scale, shows a triangle ABC, where $AC = BC$ and A lies on the y-axis. M is the mid-point of AB, $OM =$ 2 units and $\tan \angle OMC = -\frac{2}{3}$.



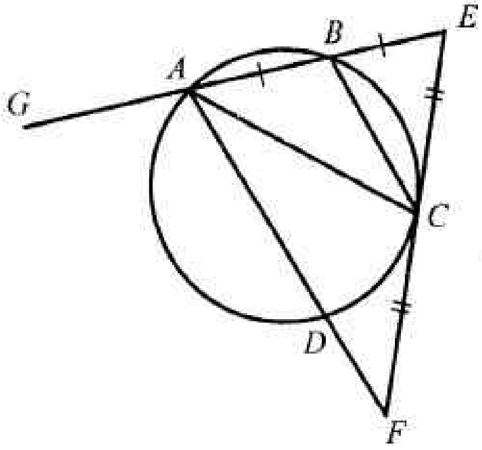
(i) Show that the equation of CM is $3y - 2x + 4 = 0$. [2]

(ii) Find the coordinates of B. [4]

(iii) Given that the area of triangle ABC is $\frac{52}{3}$ square units, find the coordinates of C. [4]

End of Paper

8



The diagram shows points A, B, C and D on a circle, line EF is tangent to the circle at C, lines ADF and $EBAG$ are straight lines, and points B and C are the midpoints of AE and EF.

Prove that

(i) $BC \times EC = AC \times BE$, [3]

(ii) $AF \times EC = AC \times AE$, [2]

(iii) angle GAD = angle ACF. [2]

9 **(a)** **(i)** Show that $\cot 2x = \dfrac{1-\tan^2 x}{2\tan x}$. [2]

(ii) Hence, solve the equation $8\cot 2x \tan x = 1$, for $0° < x < 360°$. [4]

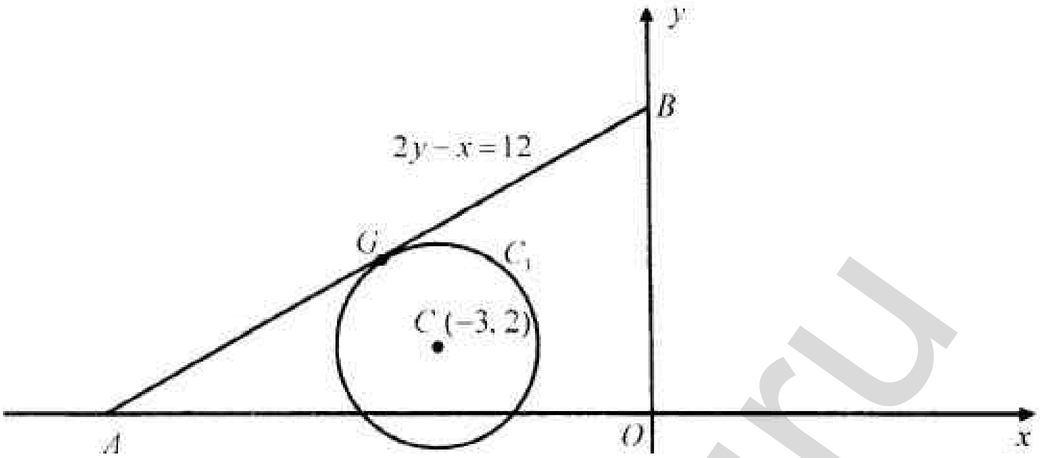
(b) The Ultraviolet Index (UVI) describes the level of solar radiation. The UVI measured from the top of a building is given by $U = 6 - 5\cos qt$, where t is the time in hours from the lowest value of the UVI, $0 \le t \le 10$, and q is a constant. It takes 10 hours for the UVI to reach its lowest value again.

(i) Explain why we are not able to measure a UVI of 12. [1]

(ii) Show that $q = \dfrac{\pi}{5}$. [1]

(iii) The top of the building is equipped with solar panels that supply power to the building when the UVI is at least 3. Find the duration, in hours and minutes, that the building is supplied with power from the solar panels. [4]

5 In the diagram below, a circle C_1, with centre at $C(-3, 2)$, touches the line $2y - x = 12$ at the point G.
The line $2y - x = 12$ intersects the x-axis at A and the y-axis at B.



Find

(i) the coordinates of A and of B. [2]

(ii) the equation of the line CG. [2]

(iii) the equation of the circle C_1. [3]

(iv) the equation of the circle C_2 which is a reflection of the circle C_1 in the line AB. [2]

The acute angle between AG and AC is $\theta°$.

(v) Show that $\theta = \tan^{-1}\frac{1}{4}$. [2]

6 **(i)** Find $\frac{d}{dx}\left[e^{2x}(2-3x) \right]$. [3]

(ii) Hence, find $\int_0^{\ln 2} 5xe^{2x}\, dx$. [5]

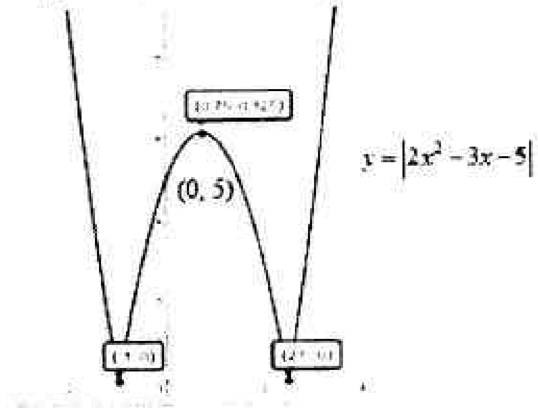
Answer Key

1. (a) $-4 < a < 12$ (b)(i) $x < -1\frac{1}{3}$ or $x > 0$

2. (a) $a = \frac{5\sqrt[3]{c}}{b}$ (b)(i) $x = 3$ (ii) $x = 85.7$ or $x = 0.130$

3. 49.5 units / s

4. (i) $a = 3$, $b = 6.125$ (ii) (iii) $m > 2$



5. (ii) $-1\frac{3}{4}x$

6. (i) $l = \frac{45}{2r} - \frac{9}{8}\pi r$ (iii) $r = 2.32$; min value

7. (ii) $L = 46 + 10\sqrt{13}\sin(\theta - 19.4°)$ (iii) 45.8°

9. (a)(ii) $x = 40.9°, 139.1°, 220.9°, 319.1°$ (b)(iii) 7 hrs and 3 mins

10. (a)(i) $\frac{4x(2x-3)}{(4x-3)^2}$ (ii) $\frac{3}{4} < x < \frac{3}{2}$

11. (ii) 1.23 m/s^2 (iii) 16.0 m (iv) passed through O

8 **(i)** Prove that $\dfrac{1-\tan^2\theta}{1+\tan^2\theta}=\cos 2\theta$. [3]

(ii) Use the result in **(i)** to show that

$$1+x^2=\sqrt{2}x^2-\sqrt{2} \text{ where } x=\tan 67.5^\circ .$$ [2]

(iii) Hence find the values of the constants c and d such that

$$\tan 67.5^\circ = c+d\sqrt{2}\,.$$ [3]

(iv) Hence show that $\tan 7.5^\circ=\dfrac{1+\sqrt{2}-\sqrt{3}}{1+\sqrt{3}+\sqrt{6}}$. [3]

9 The temperature, $x\,^\circ\mathrm{C}$, inside a house t hours after 4 am is given by $x=21-3\cos\left(\dfrac{\pi t}{12}\right)$ for $0\le t\le 24$, and the temperature , $y\,^\circ\mathrm{C}$, outside the house at the same time is given by $y=22-5\cos\left(\dfrac{\pi t}{12}\right)$ for $0\le t\le 24$.

(i) Find the temperature inside the house at 8 am. [2]

The difference between the temperatures inside and outside of the house is given by $D=x-y$.

(ii) Write down and simplify an expression for D in terms of t for $0\le t\le 24$. [1]

(iii) Sketch the graph of D against t for $0\le t\le 24$. [3]

(iv) Determine the time(s) of the day when the temperature inside of the house is equal to the temperature outside the house. Hence find the range of values of t when the temperature inside of the house is less than the temperature outside of the house. [4]

Answer **all** the questions.

1 The equation of the curve is $y = px^q - 8$, where p and q are constants.

Given that the curve passes through the points $(2, -4)$ and $(5, 17)$, find the value of p and of q. [4]

2 The second derivative of y is given by $\frac{d^2y}{dx^2} = 2x + 4$.

Given that $y = 12$ when $x = 3$, and $y = -\frac{1}{3}$ when $x = 2$, find y in terms of x. [4]

3 The equation of a curve is $y = ax^2 - 4x + 2a - 3$, where a is a constant.

Find the range of values of a for which the curve lies completely above the line $y = -1$. [5]

4 The equation of a curve is $y = \frac{3\cos x}{\sin x}$, where $0 < x < \pi$.

(i) Show that the gradient function can be expressed in the form $\frac{k}{\sin^2 x}$, where k is a constant. [2]

(ii) Find the x-coordinates of the points at which the tangents to the curve are perpendicular to the line $2x - 8y = -1$, leaving your answers in exact form. [3]

5 The number of people, N, in a housing estate who contracted influenza during a flu epidemic after t days is modelled by the equation $N = \frac{1000}{1 + 199e^{-0.8t}}$.

(i) Find the initial number of people who contracted influenza during the flu epidemic. [1]

(ii) Given that there are 937 people who contracted influenza after x days, find x correct to the nearest whole number. [3]

(iii) Find the number of people who eventually contracted influenza after a long time. [1]

6 **(i)** Sketch the curve $y = |4x - x^2|$, indicating the coordinates of the maximum point and of the points where the curve meets the x-axis. [3]

(ii) State the value or range of values of m if the equation $|4x - x^2| = m$ has

(a) 2 solutions. [1]

(b) 3 solutions. [1]

(c) 4 solutions. [1]

7 The function P is defined by $P(x) = 2x^3 + (4-2a)x^2 - ax + 6a$, where a is a constant.

$(x+2)(2x^2+bx+3a)$

(i) Show that $x + 2$ is a factor of $P(x)$. [2]

(ii) Find the other quadratic factor of $P(x)$ in terms of a. $0 < a < 6$ [2]

(iii) Find the range of values of a for which the equation $P(x) = 0$ has only 1 real root. [3]

8 The table below shows the experimental values of two variables x and y.
An error was made in recording one of the values of y.

x	2	3	4	5	6
y	5.8	15	30	43.5	74

It is known that x and y are related by an equation $y = ax(x+b)+2$, where a and b are unknown constants.

(i) Express $y = ax(x+b)+2$ in a form suitable for drawing a straight line graph. [1]

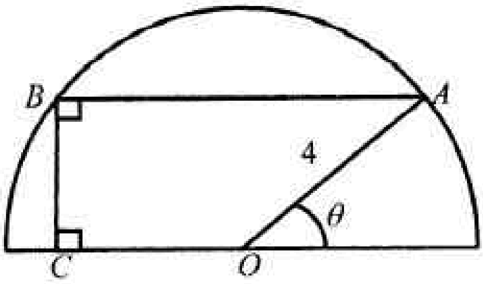
(ii) Draw a straight line graph for the given data. [3]

(iii) Use your graph to estimate

(a) the value of a and of b. [2]

(b) a value of y to replace the incorrect value. [2]

7 The diagram below shows a trapezium $ABCO$ inscribed in a semi-circle with centre O and radius 4 units. OA makes an angle of θ radians with the diameter. AB is parallel to the diameter and BC is perpendicular to both lines AB and OC.



(i) Show that the perimeter, y, of trapezium $ABCO$ is given by

$$y = 4(1 + \sin\theta + 3\cos\theta).$$ [3]

(ii) Find the value of θ for which y has a stationary value and determine whether this value of y is a maximum or a minimum. [4]

(iii) Express the perimeter of the trapezium in the form $y = 4 + R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [2]

(iv) Hence solve the equation $4(1 + \sin\theta + 3\cos\theta) = 12$, for $0 < \theta < \frac{\pi}{2}$. [2]

Answer **all** the questions.

1 It is given that $f(x) = x^3 - 3x^2 + 4x$.

(i) Show that $f(x)$ is an increasing function for all values of x. [3]

(ii) Hence, show that $f(x)$ is positive for all positive values of x. [2]

2 A rectangle has a fixed perimeter of 40 cm. The length of one side, x cm, increases at a constant rate of 0.5 cm/s. Find the rate at which the area is increasing at the instant when $x = 3$. [5]

3 **(a)** Find the term independent of x in the binomial expansion of $\left(x^2 - \frac{1}{2x^3}\right)^{10}$. [3]

(b) Given that the first 4 terms in the binomial expansion of $\left(2x + \frac{1}{4}\right)^9$, in descending powers of x, are $512x^9 + 576x^8 + ax^7 + bx^6 + \dots$, where a and b are constants, find

(i) the value of a and of b, [3]

(ii) the coefficient of x^6 in $\left(2x + \frac{1}{4}\right)^9\left(\frac{4}{x} - 1\right)\left(\frac{4}{x} + 1\right)$. [2]

Begin Question 4 on a fresh piece of paper.

4 **(a)** Given that $\log_3 a = r$, $\log_{27} b = s$ and $\frac{a}{b} = 3^t$, express t in terms of r and s. [3]

(b) Solve $\log_3 x + 3 = 10 \log_x 3$. [5]

10 **(a)** It is given that $y = \dfrac{2x^2}{4x-3}$, where $x > \dfrac{3}{4}$.

(i) Find $\dfrac{dy}{dx}$. [2]

(ii) Find the range of values of x for which $y = \dfrac{2x^2}{4x-3}$ is a decreasing function. [4]

(b) It is given that f(x) is such that $f'(x) = \dfrac{1}{2x-5} - \dfrac{4}{(2x-5)^2}$.

Given also that $f(3) = 1.75$, show that $8f(x) - (2x-5)^2 f''(x) = \ln(2x-5)^4$. [7]

11 A particle moves in a straight line, so that, t seconds after passing a fixed point O, its velocity, v m/s, is given by $v = 2e^{0.2t} - 10e^{0.1-0.2t}$. The particle comes to an instantaneous rest at the point A.

(i) Show that the particle reaches A when $t = \dfrac{5}{2}\ln 5 + \dfrac{1}{4}$. [3]

(ii) Find the acceleration of the particle at A. [3]

(iii) Find the distance OA. [4]

(iv) Explain whether the particle is again at O at some instant during the eleventh second after first passing through O. [2]

9 The roots of the quadratic equation $2x^2 - 4x - 1 = 0$ are α and β.

(i) Find the value of $\alpha^2 + \beta^2$. [2]

(ii) Show that the value of $\alpha^3 + \beta^3$ is 11. [2]

(iii) Find a quadratic equation whose roots are $\left(\alpha^3 + \frac{1}{\beta^3}\right)$ and $\left(\beta^3 + \frac{1}{\alpha^3}\right)$. [4]

10 (i) Express $\frac{14x^2 - 15x + 2}{x(2x-1)^2}$ in partial fractions. [5]

(ii) Hence find $\int \frac{14x^2 - 15x + 2}{x(2x-1)^2}\,dx$. [4]

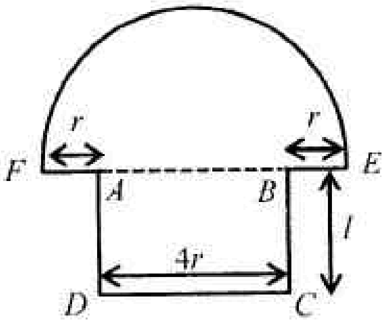
11 A particle P travels in a straight line from a fixed point O with acceleration a m/s^2 given by $a = 8t - k$, where t is the time in seconds after passing O, and k is a constant. When P passes O, its velocity is 5 m/s. At $t = 2$, its velocity is -21 m/s.

(i) Show that the value of k is 21. [2]

(ii) Find the range of values of t during which P is travelling towards O. [3]

(iii) Given that P comes to instantaneous rest at points A and B, find the distance AB. [4]

6



The diagram shows a design of a bookmark that includes a rectangle *ABCD*, where $BC = l$ cm, $CD = 4r$ cm, a semicircle with radius $3r$ cm, and $AF = BE = r$ cm. The area of the bookmark is 90 cm^2.

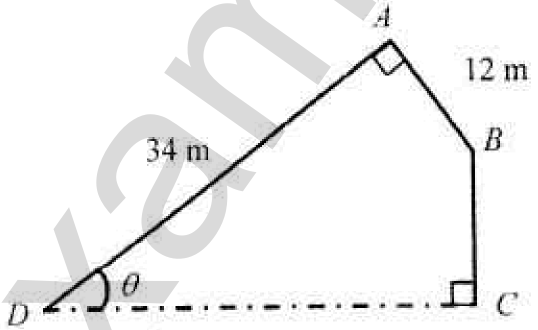
(i) Express l in terms of r. [2]

(ii) Given that the perimeter of the bookmark is P cm, show that

$$P = \left(6 + \frac{3\pi}{4}\right)r + \frac{45}{r}.$$ [2]

(iii) Given that r and l can vary, find the value of r for which P has a stationary value. Explain why this value of r gives the minimum perimeter. [5]

7



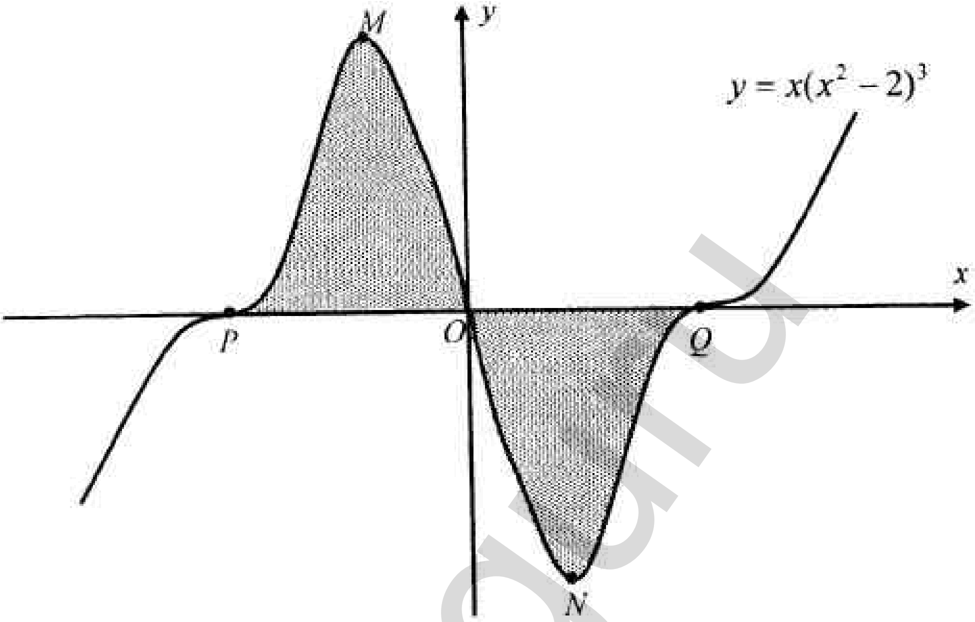
The diagram shows an animal exhibition area that is surrounded by glass panels at *AB*, *BC* and *AD*, where $AB = 12$ m, $AD = 34$ m, angle DAB = angle $BCD = 90^0$ and the acute angle $ADC = \theta$ can vary.

(i) Show that L m, the length of the glass panels can be expressed as $L = 46 + 34\sin\theta - 12\cos\theta$. [2]

(ii) Express L in the form $p + R\sin(\theta - \alpha)$, where p and $R > 0$ are constants and α is an acute angle. [4]

(iii) Given that the exact length of the glass panels is 62 m, find the value of θ. [3]

10 The diagram shows the curve $y = x(x^2 - 2)^3$. P and Q are the points of intersection of the curve with the x-axis. M and N are the maximum and minimum points of the curve respectively.



(i) Find the coordinates of P and of Q. [2]

(ii) Find the x-coordinates of M and of N. [4]

(iii) Show that P and Q are stationary points of inflexion of the curve. [2]

(iv) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left[\left(x^2 - 2\right)^4\right]$. [2]

(v) Hence find the total area of the shaded regions. [3]

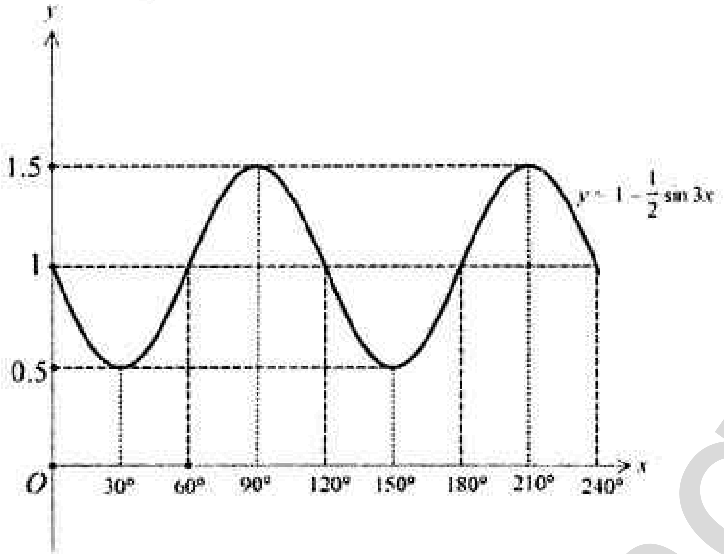
1. $4-\sqrt{5}$

2. $2-\frac{2}{2x+3}+\frac{3}{x-1}$

3. (ii) one solution

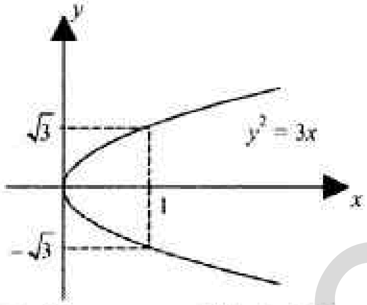
4. (i) $\frac{-29}{9}$ (ii) $27x^2+98x+196=0$

6. (i) Max y = 1.5; Min y = 0.5 (ii)



7. (i) $(-4,8)$ (ii) $P(4,4)$ (iii) 50 units^2

8. (a) (b)(i). $\left(-\frac{1}{3},a-\frac{19}{27}\right)$ and $(2,12+a)$ (b)(ii). $\left(-\frac{1}{3},a-\frac{19}{27}\right)$ min ; $(2,12+a)$ max



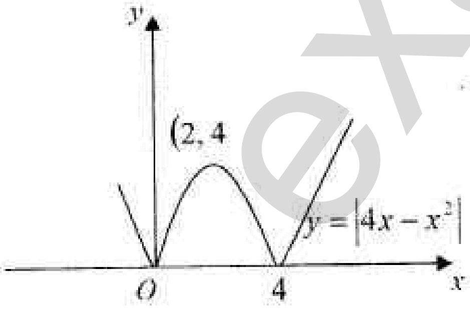
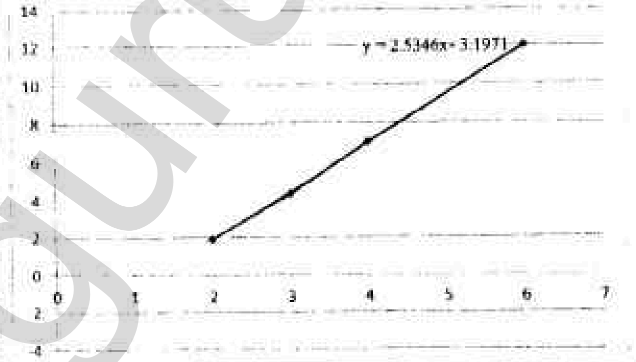
9. (ii) $\sec x$ (iii). 0.539

10. (i) Centre (2, 4), Radius = $2\sqrt{5}$ (ii) $x^2+y^2-4x-8y=0$ (iii) Centre of $C_2(1.22,0.710)$

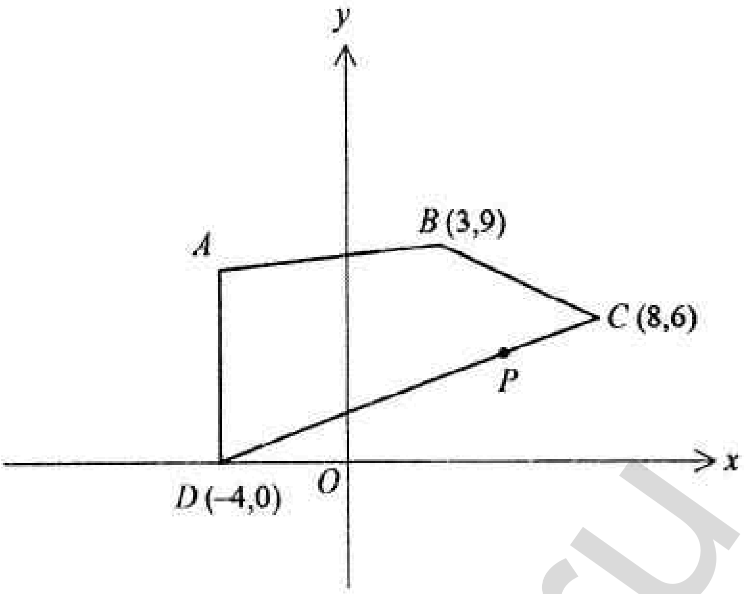
11. (i) $A\left(\frac{5\pi}{3}+\frac{9}{8}\sqrt{3},0\right)$ (ii) $6\frac{15}{32}$ / 6.47 units^2

CEDAR GIRLS' SECONDARY SCHOOL
SECONDARY 4 ADDITIONAL MATHEMATICS
Answer Key for 2016 Preliminary Examination 2

PAPER 4047/1

1	$p = 1$, $q = 2$
2	$y = \frac{x^3}{3} + 2x^2 - 4x - 3$
3	$a > 2$
4	$x = \frac{\pi}{3}$ or $\frac{2\pi}{3}$
5i	5
5ii	$t = 10$
5iii	1000
6i	
6iia	$m = 0$ or $m > 4$
6iib	$m = 4$
6iic	$0 < m < 4$
7ii	$2x^2 - 2ax + 3a$
7iii	$0 < a < 6$
8i	$\frac{y-2}{x} = ax + ab$ where $Y = \frac{y-2}{x}$, $X = x$, $m = a$ and Y-intercept $= ab$ (Accept other correct answers)
8ii	
8iiia	$a = 2.53 \pm 0.2$ $b = -1.26 \pm 0.2$
8iiib	49.5
9i	5
9iii	$8x^2 + 616x - 49 = 0$
10i	$\frac{14x^2 - 15x + 2}{x(2x-1)^2} = \frac{2}{x} + \frac{3}{2x-1} - \frac{4}{(2x-1)^2}$
10ii	$2\ln x + \frac{3}{2}\ln(2x-1) + \frac{2}{2x-1} + C$
11ii	$\frac{1}{4} < t < 5$
11iii	71.4 m
12ii	$B(4, -3)$
12iii	$C\left(6, \frac{8}{3}\right)$

7



A quadrilateral *ABCD* passes through vertices *B* (3, 9), *C* (8, 6) and *D* (– 4, 0), line *AD* is parallel to the *y* – axis.

(i) Find the coordinates of *A* given that the length of *AD* is 8 units. [1]

(ii) A point *P* divides the line *DC* in the ratio of 2 : 1. Find the coordinates of *P*. [3]

(iii) Hence, find the area of the quadrilateral *ABPD*. [3]

8 **(a)** Sketch the graph $y^2 = 3x$. [2]

(b) Given that $\mathrm{f}(x) = -2x^3 + 5x^2 + 4x + a$,

- **(i)** find the coordinates of the turning points in terms of *a*. [4]
- **(ii)** Determine the nature of each turning point. [3]
- **(iii)** In the case where $a = 1$, explain why the part of the graph between the turning points lie above the *x* – axis. [1]

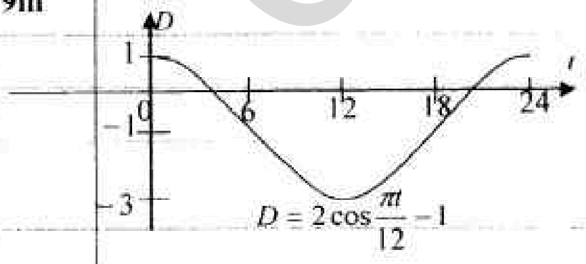
9 **(i)** Show that $\sec x + \tan x$ can be expressed as $\dfrac{1+\sin x}{\cos x}$. [1]

(ii) Differentiate $\ln(\sec x + \tan x)$ with respect to *x*. [3]

(iii) Hence, find $\int_{0.25}^{0.5} 2\sec x \, \mathrm{d}x$. [3]

CEDAR GIRLS' SECONDARY SCHOOL
SECONDARY 4 ADDITIONAL MATHEMATICS
Answer Key for 2016 Preliminary Examination 2

PAPER 4047/2

2	7 cm²/s	**10i**	$P=(-\sqrt{2},0)$ and $Q=(\sqrt{2},0)$
3a	$\frac{105}{8}=13.125$ or $13\frac{1}{8}$	**10ii**	x- coordinate of $N=\sqrt{\frac{2}{7}}$ or 0.535
3bi	$a=288,\ b=84$		x- coordinate of $M=-\sqrt{\frac{2}{7}}$ or -0.535
3bii	9132	**10iv**	$8x\left(x^2-2\right)^3$
4a	$t=r-3s$	**10v**	4 sq. units
4b	$x=\frac{1}{243}$ or $x=9$	**11ai**	(1) $\angle BAC=\angle CDF$
5i	$A=(-12,0),\ B=(0,6)$		(2) $\angle DCF=\angle ACB=90^\circ$ (given)
5ii	$y=-2x-4$		ΔACB is similar to ΔDCF (AA Similarity)
5iii	$(x+3)^2+(y-2)^2=5$	**11aii**	$\angle DFC=\angle ABC$ (Corr angles of similar triangles)
5iv	$(x+5)^2+(y-6)^2=5$		$\therefore\ \Delta EFB$ is isosceles.
6i	$e^{2x}-6xe^{2x}$	**11aiii**	As $AC=2DC$,
6ii	$10\ln 2-\frac{15}{4}$ or 3.18		$\therefore\ AB=2DF$ (ratio of corr sides of similar Δs)
7ii	0.322, y is maximum		$\frac{AE+BE}{DF}=\frac{2}{1}$
7iii	$y=4+\sqrt{160}\cos(\theta-0.322)$ $=4+12.6\cos(\theta-0.322)$		$\frac{AE+BE}{EF-ED}=\frac{2}{1}\Rightarrow AE+BE=2(BE-AE)$
7iv	$\theta=1.21$		$3AE=EB$
8iii	$c=1,\ d=1$	**11bi**	$\angle RQS=\angle QST$ (alt angles, $QR//TS$)
9i	19.5°C		$\angle QST=\angle QTR$ (tan chord theorem)
9ii	$D=2\cos\frac{\pi t}{12}-1$		$\therefore\ \angle RQS=\angle QTR$
9iii		**11bii**	Produce WTQ and YSR to meet at M. $\therefore\ \Delta MTS$ is isos. (tgts from ext pt are equal) $\therefore\ \angle QTS$ and $\angle RST$ are equal. $\therefore\ \angle TQR=180^\circ-\angle QTS$ (corr angles, $QR//TS$)
			Since $\angle TSR+\angle TQR=180^\circ$ $QRST$ is a cyclic quadrilateral. (Angles in opp segments)
9iv	8 am and 12 midnight, $4<t<20$		

1 Express $\dfrac{2x^2+9x+6}{(x+2)(x^2-4)}$ in partial fractions. [4]

2 Given that $(1+ax)^n = 1-24x+252x^2+\ldots$, find the values of a and n. [5]

3 **(a)** Given that $\sin\theta = k$, where θ is an acute angle. Find, in terms of k, the value of $\sin 4\theta$. [3]

(b) Find the exact value of $\tan\left[\cos^{-1}\left(-\dfrac{1}{\sqrt{2}}\right)\right]$ without the use of a calculator. [2]

4 A triangle has vertices $A(-2,2)$, $B(-1,-1)$ and $C(3,-2)$. Given that $ABCD$ is parallelogram, find

(i) the coordinates of the point D, [2]

(ii) the area of the parallelogram $ABCD$. [3]

5 A spherical elastic balloon, with radius r cm, is filled with V cm^3 of helium gas. It was discovered that there is a leakage of helium gas from the balloon at a constant rate of 5 cm^3/s. At the instant when the radius of the spherical elastic balloon is 10 cm, find

(i) the rate at which the radius of the balloon is decreasing, leaving your answer in terms of π, [3]

(ii) the rate of change of the surface area of the spherical elastic balloon. [3]

6 **(a)** Given that $\int_0^4 \mathrm{f}(x)\mathrm{d}x = \int_4^7 \mathrm{f}(x)\mathrm{d}x = \dfrac{2}{5}$, find $\int_7^0 \mathrm{f}(x)\mathrm{d}x$. [2]

(b) **(i)** Show that $\dfrac{d}{dx}\left(\dfrac{x^2}{\sqrt{2x-3}}\right) = \dfrac{3x^2-6x}{(2x-3)^{\frac{3}{2}}}$. [2]

(ii) Hence, or otherwise, find $\int \dfrac{x^2-2x}{(2x-3)^{\frac{3}{2}}}\,\mathrm{d}x$. [2]

[Turn over

7 **(a)** The equation of the curve is $y = (k+4)x^2 + 4x - k$, where k is a constant.

(i) Show that the curve meets the x-axis for all possible values of k. [3]

(ii) Find the value of k for which the x-axis is a tangent to the curve. [1]

(b) Given that $y = px^2 + 4x + q$ is always positive, what conditions must be applied to the constants p and q? [2]

8 **(i)** Show that $\frac{1}{\sec x + 1} + \frac{1}{\sec x - 1} = \frac{2\cos x}{\sin^2 x}$. [3]

(ii) Hence, or otherwise find all the angles which satisfy the equation $\frac{1}{\sec x + 1} + \frac{1}{\sec x - 1} = 8\cos x$, for $0 \le x \le \pi$. [4]

9 A cuboid has a volume of 648 cm^3, a length of 6 cm and a height of x cm.

(i) Find, in terms of x, an expression for the breadth of the cuboid. [1]

(ii) Show that the total external surface area, A cm^2, of the cuboid is given by $A = 12\left(18 + \frac{108}{x} + x\right)$. [2]

(iii) Find the value of x at which A is a minimum. [4]

10 A point H lies on the curve $y = -x^2 + 4x + 7$. The normal to the curve at H is perpendicular to the line $2y - 8x = 4$.

(i) Show that the coordinates of H are $(0, 7)$. [3]

(ii) Find the equation of the normal to the curve at H. [3]

(iii) Find the coordinates of point K, where the tangent to the curve at K is parallel to the normal in part (ii). [3]

11 **(a)** **(i)** Sketch the graph of $y = 0.5x^{-\frac{1}{3}}$, for $x > 0$. [1]

(ii) Determine the equation of the straight line which needs to be drawn on the graph of $y = 0.5x^{-\frac{1}{3}}$ in order to obtain a graphical solution of the equation $1 = 2x^{\frac{4}{3}}$. [1]

(iii) Hence, state the number of solution(s) to the equation $1 = 2x^{\frac{4}{3}}$, for $x > 0$. [1]

(b) **(i)** On the same axes, sketch the graphs of $y = |3x^2 - 6x|$ and $y = 1$. [3]

(ii) State the number of solutions to the equation $|3x^2 - 6x| = 1$. [1]

(iii) Solve the equation $|3x^2 - 6x| = 3x$. [3]

12 **(a)** Variables x and y are related in such a way that when $\frac{y}{x}$ is plotted against x^2, a straight line which passes through the points $(1, 2)$ and $(-4, 17)$ is obtained.

(i) Express y in terms of x. [3]

(ii) Hardev commented that the point $(6, -618)$ can be found on the straight line. Gabriel disagreed. Who do you agree with? Explain your answer. [2]

(b) **Answer the whole of this question part on a sheet of graph paper.**

Two variables x and y are connected by the equation $y = ab^x + 4$. By drawing a suitable straight line graph using the following table of corresponding values of x and y, find the values of a and b. [5]

x	1	2	3	4	5	6
y	16.8	24	37	56	88	138

END OF PAPER

1 A piece of fish fillet is removed from the freezer and left to thaw. After t minutes, its temperature T °C, is given by $T = 33 - 37e^{-0.03t}$. In order to maintain the quality of the fish fillet, Chef Chris needs to marinate the fillet when its temperature reaches 15°C. Find

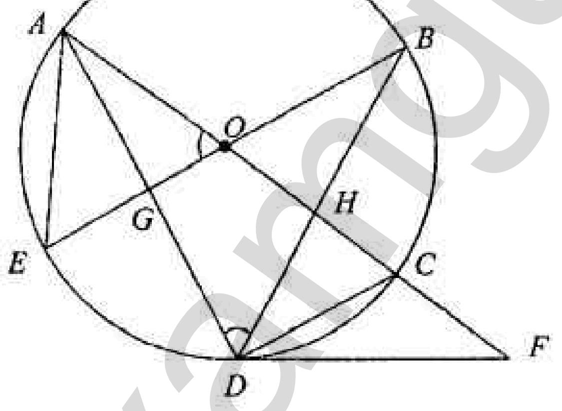
(i) the initial temperature of the fish fillet, [2]

(ii) the waiting time, to the nearest minute, before Chef Chris can start to marinate the fish fillet, [3]

(iii) the value of T as t becomes very large. Explain the significance of this value. [2]

2 The equation $x^2 + 2x - 6 = 0$ has roots α and β and the equation $hx^2 + 2 = kx$ has roots $\dfrac{\alpha}{\beta - 1}$ and $\dfrac{\beta}{\alpha - 1}$. Find the values of h and k. [7]

3



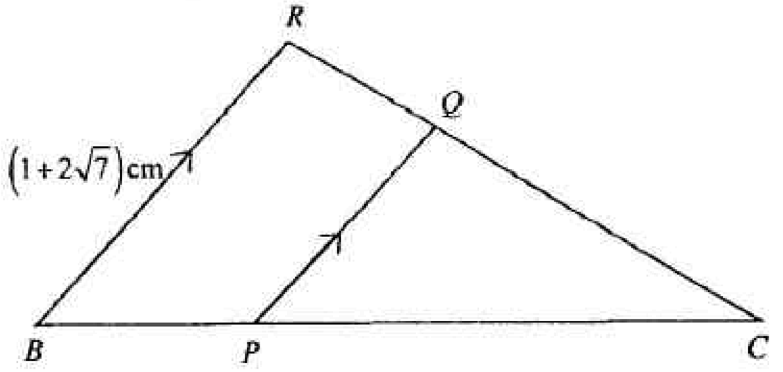
In the diagram, A, B, C, D and E are points on the circle with centre O. The tangent to the circle at D is extended to meet the line AOC at F. BE intersects AD at G and BD intersects AF at H. $\angle ADB = \angle EOA$. Prove that

(i) triangle ADF is similar to triangle DCF, [3]

(ii) $AE \times BH = AG \times BO$. [4]

[TURN OVER

4 **(a)**



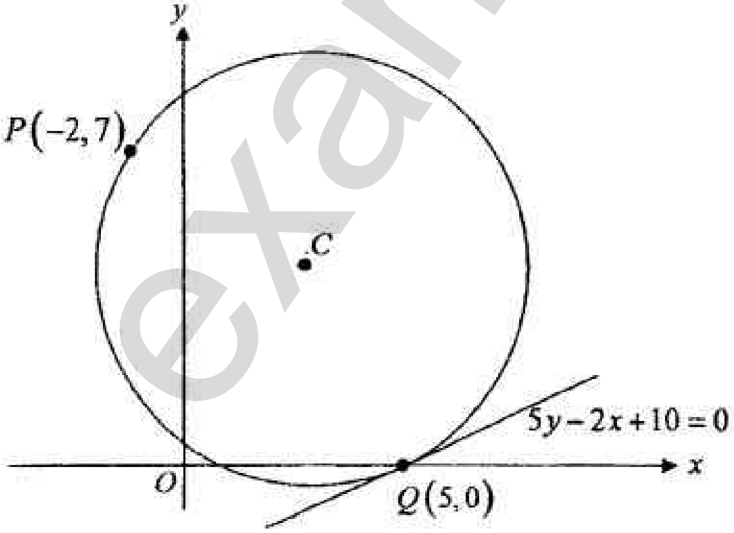
In the diagram, PQ is parallel to BR and BC is divided at P such that $BP:PC=\sqrt{7}:5$. Given that $BR=(1+2\sqrt{7})\text{cm}$, find the length of PQ in the form $(a+b\sqrt{7})\text{cm}$, where a and b are rational numbers. [4]

(b) Solve the equation $4^{n+1}-3(2^{n+3})-64=0$. [3]

5 **(a)** The equation of a curve is $y=4x^3+3px^2+27x-10$. Find the range of p such that y is an increasing function. [4]

(b) The curve $y=(hx^3-1)^2-k$ has a stationary point at $(1,-3)$. Given that h is positive, find the values of h and k. [4]

6



In the diagram, the circle passes through $P(-2,7)$ and touches the line $5y-2x+10=0$ at $Q(5,0)$. The centre of the circle is denoted by C. Find

(i) the equation of the line CQ, [2]

(ii) the coordinates of C, [4]

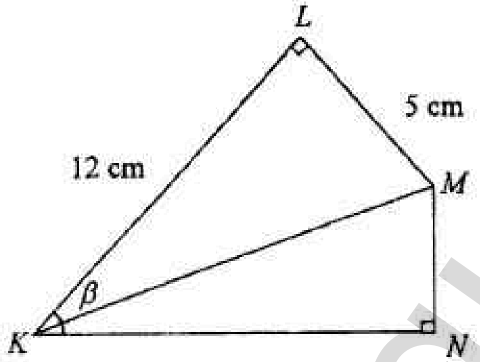
(iii) the equation of the circle. [2]

7 (i) Show that $\sec^2 x - \tan^2 x - 2\cos^2 x = -\cos 2x$. [2]

(ii) Hence, sketch the graph of $y = \sec^2 x - \tan^2 x - 2\cos^2 x + 1$ for $0 \le x \le \pi$. [3]

(iii) On the same axes, sketch a suitable graph to find the number of solutions to the equation $2\left(\sec^2 x - \tan^2 x - 2\cos^2 x\right) - 1 = \frac{x}{\pi}$. [3]

8 The diagram below shows two triangles with right angles at L and N. The length of KL and LM are 12 cm and 5 cm respectively, and $\angle LKN = \beta$, where β is an acute angle.



(i) Express KN in the form $a\cos\beta + b\sin\beta$, where a and b are constants. [2]

(ii) Show that $KN = R\cos(\beta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) Find the value of β for which $KN = 8$ cm. [3]

9 (a) Find the range of values of x for which $3(2x-5)^2 > x(2x-5)$. [3]

(b) The function $g(x) = 3x^3 + x^2 - kx + 4$ has a factor $(x-1)$.

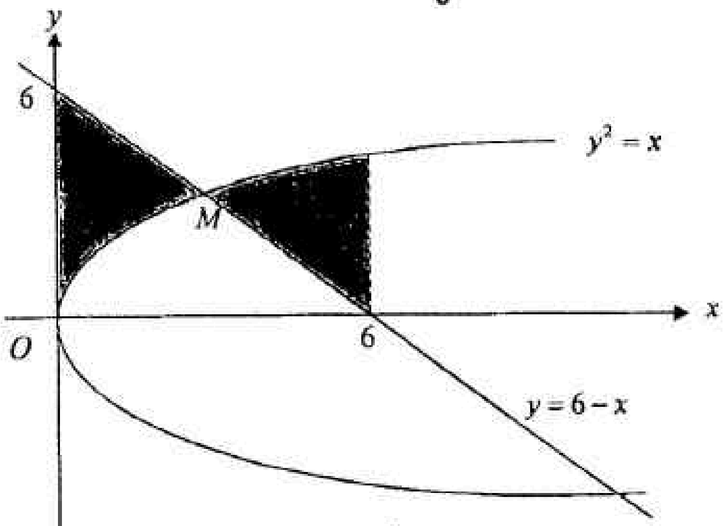
(i) Find the value of k. [1]

(ii) Solve the equation $g(x) = 0$. [3]

(iii) Hence, find the roots of the equation $\frac{y+4}{\sqrt{y}} = 8 - 3y$. [2]

[TURN OVER

10



The diagram shows part of the curve $y^2 = x$ and the line $y = 6 - x$, intersecting at the point M. Find

(i) the coordinates of the point M, [3]

(ii) the total area of the shaded regions. [6]

11 (a) Solve the equation $(\log_{81} x)(\log_3 x) = 4$. [3]

(b) Solve, for x and y, the simultaneous equations

$$e^{x}\left(\frac{1}{e^2}\right)^{1-2y} = e,$$

$$x\ln 32 - y\ln 4 = \ln 16.$$ [4]

(c) Given that $3\lg\sqrt{y} - \lg\frac{y}{100} = 3\lg x$, express y in terms of x. [3]

12 A particle P moves along a horizontal straight line such that at time t seconds after the motion has begun from a fixed point O, its acceleration a m/s² is given by $a = 12t - 18$.

(i) Given that the initial velocity is 12 m/s, find an expression for the displacement of P. [3]

Another particle Q moves along the same line as P at the same instant that P begins to move. The velocity of Q is given by $v = 6t^2 - 16t + 7$.

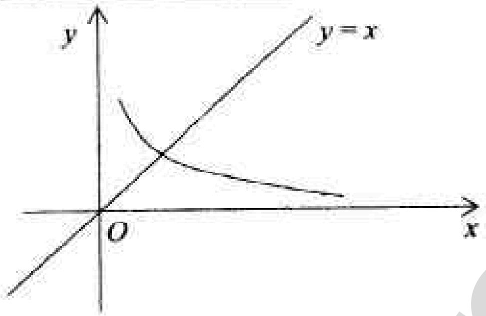
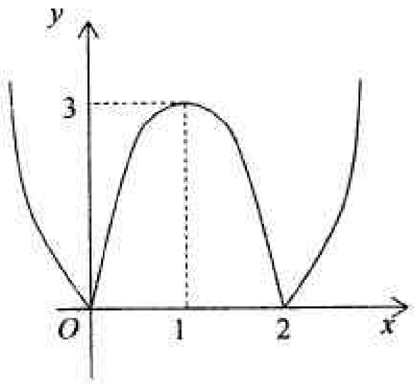
(ii) Given that the initial displacement of Q is -6 m from a fixed point O, find an expression for the displacement of Q. [2]

(iii) Find the total distance travelled by P when it collides with Q. [5]

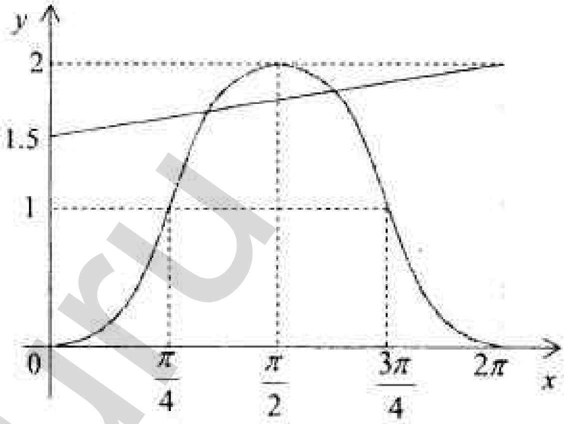
(iv) Determine if P and Q are travelling in the same direction at the instant when P and Q collide. [2]

END OF PAPER (XINMIN)

2016 Xinmin Sec Sch Amath Paper 1 Answer Key:

1	$\frac{2}{x-2}+\frac{1}{(x+2)^2}$	2	$a=-3$, $n=8$
3a	$4k(1-2k^2)(\sqrt{1-2k^2})$	3b	-1
4i	D(2 , 1)	4ii	11 sq units
5i	$-\frac{1}{80\pi}$ cm/s	5ii	-1 cm^2/s
6a	$-\frac{4}{5}$	6bii	$\frac{x^2}{3\sqrt{2x-3}}+c$
7aii	$k=-2$	7b	$pq>4$
8ii	$x=\frac{\pi}{6},\frac{\pi}{2},\frac{5\pi}{6}$		
9i	$b=\frac{108}{x}$ cm	9iii	$x=6\sqrt{3}$
10ii	$y=-\frac{1}{4}x+7$	10iii	$K=(2\frac{1}{8}\ ,\ 10\frac{63}{64})$
11ai		11aii	Line is $y=x$
11aiii	No solution	11bi	
11bii	4	11biii	$x=0$, 1, 3
12ai	$y=-3x^3+5x$	12aii	Yes . When X = 6, Y = −18 which is not equal to −618.
12b	$a=7.96$, $b=1.60$		

2016 Xinmin Sec Sch Amath Paper 2Answer Key:

1i	$-4° C$	1ii	$t = 24$
1iii	Room temperature is 33° C	2	$h = 1$, $k = -6$
4a	$-\frac{5}{2}+\frac{5}{2}\sqrt{7}$	4b	$n = 3$
5a	$-6 < p < 6$	5b	$h = 1$, $k = 3$
6i	$y = -\frac{5}{2}x + \frac{25}{2}$	6ii	$C = (3 , 5)$
6iii	$(x-3)^2 + (y-5)^2 = 29$	7ii	
7iii	2 solutions		
8iii	$\beta = 74.6°$	9a	$x < 2.5$, $x > 3$
9bi	$k = 8$	9bii	$x = 1$, $x = -2$, $x = 2/3$
9biii	$y = 1$, $y = 4/9$		
10i	$M(4 , 2)$	10ii	13.1 units^2
11a	$x = 81$, $x = 1/81$	11b	$x = 1$, $y = 0.5$
11c	$y = \frac{1}{1000}x^6$		
12i	$s = 2t^3 - 9t^2 + 12t$	12ii	$s = 2t^3 - 8t^2 + 7t - 6$
12iii	Total distance = 182 m	12iv	Since the velocities of particles are both positive at $t = 6$, they are travelling in the same direction.

1 Find the set of values of a for which $3ax^2 + 1 > ax$ for all real values of x. [3]

2 The function f is defined by $\mathrm{f}(x) = \tan x \sec x$, where $0° \le x \le 360°$.
Find the values of x for which f is an increasing function. [4]

3 Solve the equation $\log_3(x+4) - \log_3(2x-1) + 2\log_9(x-2) = 1$. [4]

4 The curve $y^2 + 17 = 2x^2$ intersects the straight line $y + 4 = x$ at the points A and B.
Find the equation of the perpendicular bisector of AB. [6]

5 **(i)** Show that $\sin 2x\,(\tan^2 x + 1) = 2\tan x$. [3]

(ii) Hence solve the equation $\sin 4\theta\,(\tan^2 2\theta + 1) = 2\cot\theta$ for $0° < \theta < 360°$. [4]

6 The function f is defined, for $0 \le x \le \pi$, by

$$\mathrm{f}(x) = 3\cos 3x - a,$$

where a is a constant.

Given that the minimum value of $\mathrm{f}(x)$ is -4, find

(i) the value of a, [1]
(ii) the maximum value of $\mathrm{f}(x)$, [1]
(iii) the period and the amplitude of $\mathrm{f}(x)$. [2]

Using the value of a found in part **(i)**,

(iv) find the exact value(s) of x for which $\mathrm{f}(x) = \frac{1}{2}$. [3]

7 **(i)** Sketch the graph of $y = |x^2 - 4x| + 1$. [3]

(ii) It is given that the line $y = mx$, where $m > 0$, does not intersect the graph of $y = |x^2 - 4x| + 1$. Determine the set of possible values of m. [2]

(iii) Find the coordinates of the point(s) of intersection of the line $y = 6$ and the graph of $y = |x^2 - 4x| + 1$. [3]

8 In January 2016, Adam bought an antique vase for \$1500. It was believed that the value of the antique vase will increase continuously with time such that it doubles after every 5 years.

(i) Formulate an expression for \$$V$, the value of the vase after Adam owned it for x years. [2]

(ii) Sketch the graph of V against x. [2]

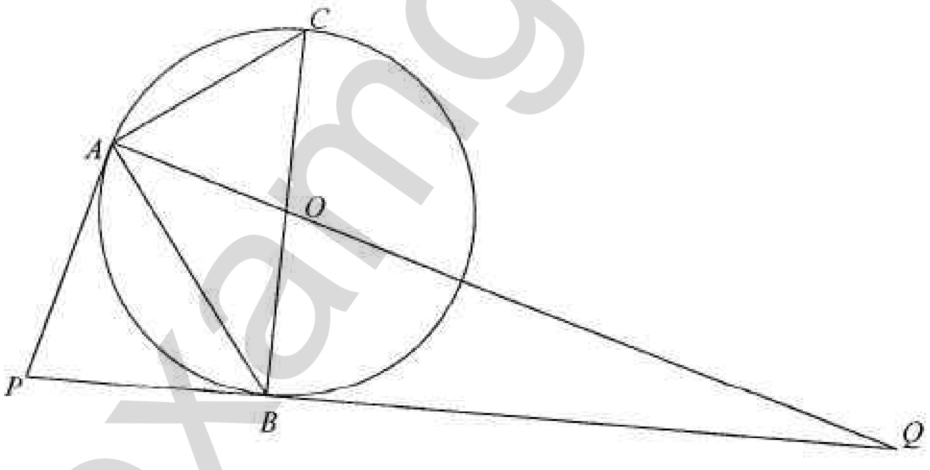
(iii) Using your answer in part **(i)**, find the number of years that Adam has to wait before the value of the vase appreciates to one million dollars. [3]

9 The diagram shows a triangle ABC whose vertices lie on the circumference of a circle with centre O. AP and PB are tangents to the circle at A and B respectively. The tangent to the circle at B meets AO extended at Q.

(i) Show that angle $AOB = 2\times$ angle PAB. [2]

(ii) Hence determine whether it is possible to draw a circle that passes through O, A, P and B? Justify your answer with clear explanations. [3]

(iii) If triangle PAB is equilateral, prove that $OQ = 2OB$. [2]



10 The equation of a curve is $y = -\sqrt{1+3x}$.

(i) A particle P moves along the curve in such a way that the x-coordinate of P decreases at a constant rate of 0.2 units per second. Find the coordinates of P at the instant when the y-coordinate is increasing at a rate of 0.05 units per second. [4]

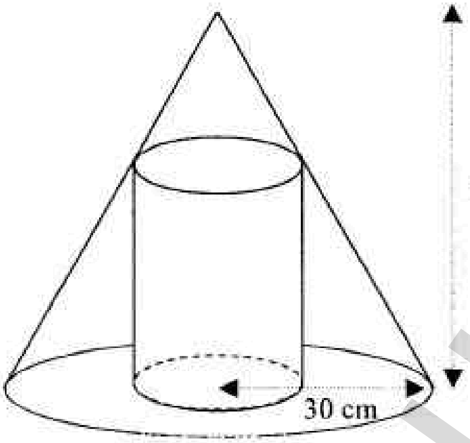
(ii) Find the area enclosed by the curve and the line $y = -3x - 1$. [5]

11 A solid cylinder is cut from a solid cone of height 60 cm and radius 30 cm as shown in the diagram. The cylinder has height h cm, radius r cm and volume V cm^3.

(i) Show that $h = 60 - 2r$. [2]

(ii) Express V in terms of r. [1]

(iii) Determine the value of r for which the volume of the cylinder is maximum. Hence find the maximum volume of the cylinder. [6]



12 A particle travels in a straight line so that, t seconds after passing a fixed point O, its velocity, v m/s, is given by $v = 12t - 2t^2$. The particle comes to an instantaneous rest at A. Find

(i) the acceleration of the particle at A, [3]

(ii) the greatest velocity of the particle, [2]

(iii) the distance travelled by the particle between $t = 0$ and $t = 5$. [4]

1 The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = 3\mathrm{e}^{-x} + \frac{1}{x+1},\ x > 0$.

(i) Explain why the curve $y = \mathrm{f}(x)$has no stationary point. [2]

(ii) Given that the curve passes through the point $(0, 1)$, find an expression for $\mathrm{f}(x)$. [4]

2 (i) Differentiate $\ln(\sin x)$ with respect to x. [2]

(ii) Show that $\frac{\mathrm{d}}{\mathrm{d}x}(x\cot x) = \cot x - x\operatorname{cosec}^2 x$. [2]

(iii) Using the results from parts (i) and (ii), show that

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\cot^2 x\,\mathrm{d}x = \frac{\pi}{4} - \frac{3\pi^2}{32} - \ln\frac{\sqrt{2}}{2}.$$ [4]

3 The equation of a curve is $y = -x^3 - 2x^2 - x - 1$. The point A lies on the curve and has x-coordinate of -2. The normal to the curve at A meets the x-axis at P and the y-axis at Q.

(i) Find the area of triangle POQ, where O is the origin. [6]

The point B also lies on the curve. The tangent to the curve at B is perpendicular to the normal to the curve at A.

(ii) Find the x-coordinate of B. [3]

4 (a) (i) Write down, and simplify, the first four terms in the expansion of $(1-x)^8$ in ascending powers of x. [2]

(ii) Replacing x by $2z - z^2$, determine the coefficient of z^3 in the expansion of $(1 - 2z + z^2)^8$. [3]

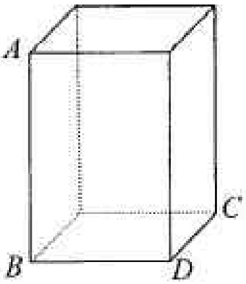
(b) (i) Write down the general term in the binomial expansion of $\left(2x - \frac{1}{3x^3}\right)^6$. [1]

(ii) Determine whether there is a constant term in the expansion. [1]

(iii) Using the general term, or otherwise, determine the coefficient of x^2 in the binomial expansion of $\left(3x^4 + 2 - \frac{3}{x}\right)\left(2x - \frac{1}{3x^3}\right)^6$. [2]

5 **Do not use a calculator in this question.**

The diagram shows a cuboid with a square base. The area of the square base is $\left(7+4\sqrt{3}\right)\text{cm}^2$ and the volume of the cuboid is $\left(26+15\sqrt{3}\right)\text{cm}^3$.



Find

(i) the height of the cuboid in the form $a+b\sqrt{3}$, where a and b are integers, [2]

(ii) an expression for BC^2 in the form $c+d\sqrt{3}$, where c and d are integers, [2]

(iii) the value of m and of n if the length of AC is $\left(\sqrt{m}+\sqrt{n}\right)\text{cm}$, where m and n are integers. [6]

6 The equation of a curve is $y=\dfrac{\sin x}{2-\cos x}$.

(i) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and obtain the coordinates of the stationary point(s) of the curve for $0\le x\le\pi$. [5]

(ii) Find an expression for $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and hence determine the nature of the stationary point(s) for $0\le x\le\pi$. [4]

7 The lines $x=2$ and $y=3$ are tangents to a circle C_1.

Given that the centre of circle C_1 lies on the positive x-axis, find

(i) the equation of C_1. [4]

Circle C_2 is a reflection of circle C_1 along the line $y=x+1$, find

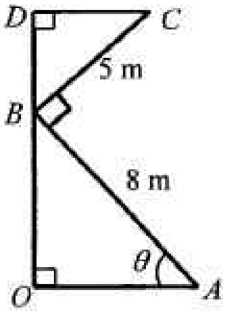
(ii) the equation of C_2. [3]

8 **(a)** **(i)** Find the remainder when $\mathrm{f}(x)=3x^3+x^2+x-4$ is divided by $x+1$. [2]

(ii) Hence find the value of k for which $\mathrm{g}(x)=\mathrm{f}(x)+k$ is divisible by $x+1$ and factorise $g(x)$ completely. [3]

(b) Express $\dfrac{4x+1}{(2x+1)(x-1)^2}$ as the sum of 3 partial fractions. [5]

9 In the diagram, $AB = 8\text{ m}$, $BC = 5\text{ m}$, $\angle AOB = \angle ABC = \angle BDC = 90°$ and $\angle OAB = \theta$ where $0° < \theta < 90°$.



(i) Find OD in terms of θ. [3]

(ii) Express OD in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [4]

(iii) Find the value of θ for which OD has a maximum length. [3]

10 The roots of the quadratic equation $2x^2 - 6x + 1 = 0$ are α and β.

(i) Find the value of $\alpha^2 + \beta^2$. [2]

(ii) Find the value of $\alpha - \beta$ given that $\alpha < \beta$. [2]

(iii) Show that $\alpha^2 - \beta^2 = -3\sqrt{7}$. [1]

(iv) Find a quadratic equation whose roots are $\alpha^2 - \beta$ and $\beta^2 - \alpha$, in the form $ax^2 + bx + c = 0$ where a, b and c are integers. [6]

11 The table below shows experimental values of two variables x and y. It is known that x and y are related by the equation $y = \dfrac{a}{x - b}$ where a and b are constants.

x	-1.0	-0.5	0.5	1.0	1.5
y	0.33	0.40	0.67	1.00	2.00

(i) Express the equation in the form suitable for drawing a straight line graph, with xy as the variable for the horizontal axis.

State clearly the variable(s) used for the vertical axis. [2]

(ii) Using variable xy for the horizontal axis and suitable variable(s) for the vertical axis, draw, on graph paper, a straight line graph and hence estimate the value of a and of b. [6]

(iii) Show that by adding another straight line on your diagram, an estimate of the solutions for the simultaneous equations $y = \dfrac{a}{x - b}$ and $y = \dfrac{2}{x}$ can be obtained.

Write down the solutions for the simultaneous equations. [3]

[End of paper

ANDERSON SECONDARY SCHOOL
2016 Preliminary Examination
Secondary Four Express
ADDITIONAL MATHEMATICS PAPER 1 (4047/01)

Answer Key

1 $0 < a < 12$

2 $0 \leq x < 90°$ or $270° < x \leq 360°$

3 $x = 5$

4 $y = -x - 12$

5 **(ii)** $\theta = 35.3°, 144.7°, 215.3°, 324.7°$

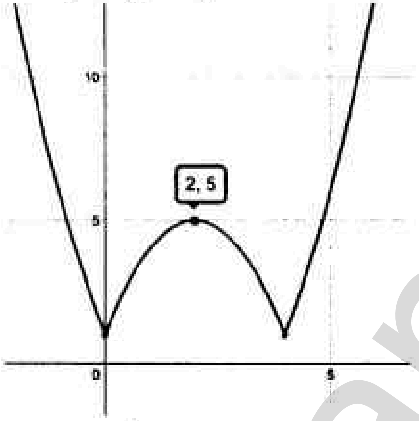
6 **(i)** $a = 1$

(ii) 2

(iii) period = $\dfrac{2\pi}{3}$, amplitude = 3

(iv) $x = \dfrac{\pi}{9}, \dfrac{5\pi}{9}, \dfrac{7\pi}{9}$

7 **(i)**

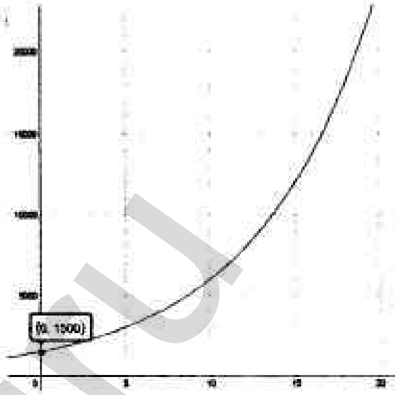


(ii) $0 < m < \dfrac{1}{4}$

(iii) (5, 6) and (–1, 6)

8 **(i)** $V = 1500 \times 2^{\frac{x}{5}}$

(ii)



(iii) 46.9 years

10 **(i)** $x = 11\dfrac{2}{3}$

(ii) $\dfrac{1}{18}$ units^2

11 **(ii)** $V = 60\pi r^2 - 2\pi r^3$

(iii) $r = 20$, 25100 cm^3

ANDERSON SECONDARY SCHOOL
2016 Preliminary Examination
Secondary Four Express
ADDITIONAL MATHEMATICS PAPER 2 (4047/02)

Answer Key

1	**(ii)**	$f(x) = -3e^{-x} + \ln(x+1) + 4$
2	**(i)**	$\cot x$
3	**(i)**	$4\frac{9}{10}$ units^2
	(ii)	$\frac{2}{3}$
4	**(a)(i)**	$1 - 8x + 28x^2 - 56x^3 + \ldots$
	(a)(ii)	-560
	(b)(i)	$\binom{6}{r}(2^{6-r})\left(-\frac{1}{3}\right)^r x^{6-4r}$
	(b)(iii)	-48
5	**(i)**	$2+\sqrt{3}$ cm
	(ii)	$14+8\sqrt{3}$
	(iii)	$m=12$ and $n=9$, or $m=9$ and $n=12$
6	**(i)**	$\frac{dy}{dx} = \frac{2\cos x - 1}{(2-\cos x)^2}$, $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{3}\right)$
	(ii)	$\frac{d^2y}{dx^2} = -\frac{2\sin x(1+\cos x)}{(2-\cos x)^3}$, maximum point
7	**(i)**	$(x-5)^2 + y^2 = 9$
	(ii)	$(x+1)^2 + (y-6)^2 = 9$
8	**(a)(i)**	-7
	(a)(ii)	$k=7$, $g(x) = (x+1)(3x^2 - 2x + 3)$
	(b)	$-\frac{4}{9(2x+1)} + \frac{2}{9(x-1)} + \frac{5}{3(x-1)^2}$
9	**(i)**	$8\sin\theta + 5\cos\theta$
	(ii)	$\sqrt{89}\sin(\theta + 32.0°)$
	(iii)	$58.0°$
10	**(i)**	8
	(ii)	$-\sqrt{7}$
	(iv)	$4x^2 - 20x - 87 = 0$
11	**(i)**	$y = \frac{1}{b}(xy) - \frac{a}{b}$
	(ii)	$b=2$, $a=-1$
	(iii)	$xy=2$, $y=1.5$, $x=1.33$

Name: **Register Number:** **Class:**

南僑中學

NAN CHIAU HIGH SCHOOL

PRELIMINARY EXAMINATION (2) 2016
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS **4047/01**
Paper 1 **10 May 2016, Tuesday**

Additional Materials : Writing Papers (8 sheets) **2 hours**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on the separate writing papers provided.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

Setter: Mdm Chua Seow Ling

This paper consists of 5 printed pages including the coverpage.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$cosec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

Answer ALL Questions

1. Prove the identity $\operatorname{cosec} 2x + \cot 2x = \cot x$. [4]

2. Sketch the two parabola curves $y = -4x^2$ and $x = -4y^2$ on the same diagram. Hence find the equation of the straight line passes through the intersections of the two curves. [5]

3. Solve the following equations

(i) $\log_2 x = \log_{\frac{1}{2}} x + 2$, [4]

(ii) $\log_a 16 = -\dfrac{1}{\log_x a}$ where a is a constant. [3]

4. Given the graph $y = ax^2 + bx + c$ is always greater than the graph $y = 4$, where a, b and c are constant. What conditions must apply to the constants a and c? [3]

5 Sketch the graph $y = 2\tan 3x - 1$ for $0 < x < \dfrac{\pi}{2}$. Hence find the range of values of p such that

$p = 2\tan 3x - 1$ has exactly 2 solutions for $0 < x < \dfrac{\pi}{2}$. [4]

6 **(i)** Sketch the graph of $y = |3 - 2x|$, indicating clearly the x and y-intercepts. [3]

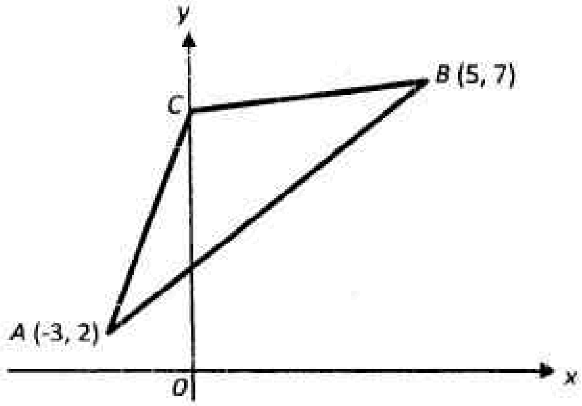
(ii) State the range of values of m for which the line $y = mx + 2$ intersects $y = |3 - 2x|$ at two distinct points. [2]

7 Given $y = \ln(2x+1) + x^2 + x$, state the range of values of x for which y exists. Hence determine whether y is an increasing or decreasing function. Show all your workings clearly. [5]

8 **(i)** Find all the angles between 0 and 2π which satisfy the equation $\sin\left(2x - \dfrac{\pi}{3}\right)\cos x = \cos x$. [5]

(ii) Without using a calculator, find the exact value of $\sin 75° + \cos 15°$. [4]

9 The diagram shows an isosceles triangle ABC which the coordinates of point A and B are $(-3, 2)$ and $(5, 7)$ respectively. C is a point on the y-axis such that $AC = CB$. Find



(i) the coordinates of C, [2]

(ii) the equation of the line which bisects angle ACB. [3]

10. **(i)** Given that $\sin^2 x + 2\cos^2 x - 4$ can be expressed as $a\cos 2x + b$, where a and b are constants. Find the value of a and of b. **[4]**

(ii) Hence for the graph of $y = \sin^2 x + 2\cos^2 x - 4$, state its

- **(a)** amplitude, [1]
- **(b)** period, [1]
- **(c)** greatest value of y, [1]
- **(d)** least value of y. [1]

11 Given that $\sin x = -\frac{2}{\sqrt{5}}$ where $180° < x < 270°$, find

(i) $\cos(-x)$, [2]

(ii) $\sin(x - 45°)$, [3]

(iii) $\sin(2x)$. [2]

12 An experiment was carried out to study the growth of a certain bacteria. It is given the number of bacteria present at t hours after the initial observation, is given by the equation $P = 250 + 420e^{kt}$ where k is a constant.

(i) Find the number of bacteria at the beginning of the experiment. [1]

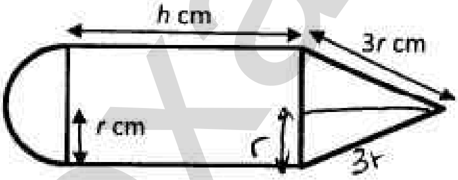
(ii) Find the value of k if the number of bacteria has doubled after 5 h. [3]

(iii) Find the rate of change of the number of bacteria at 10 h. [2]

13 In a Design and Technology competition, students are tasked to design a gigantic pencil. The criteria are shown below:

Surface area of the pencil must be as small as possible
Volume of the pencil as large as possible
Mass of the pencil shou~~ld not~~ exceed 100 g.

Xi Rui shows the cross-section of her design which consists of a hemisphere, a cylinder and a right circular cone, all of their radius are r cm as shown below. She lets the length of the cylinder be h cm and the slant length of the cone be $3r$ cm. She uses wood that has a density of $\frac{3}{3\pi}$ g/cm^3 to make her gigantic pencil.



(i) Show that the greatest volume of the gigantic pencil that Xi Rui can make, is 60π cm^3. [2]

(ii) Using the volume of the pencil in part (i), show $h = \frac{60}{r^2} - \frac{2}{3}r\left(1+\sqrt{2}\right)$. [3]

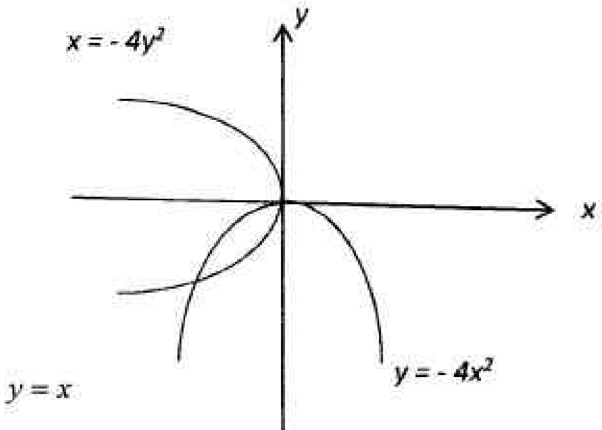
(iii) Show the total surface area, A cm^2, of the pencil is given by $A = \frac{1}{3}\pi r^2\left(11-4\sqrt{2}\right)+\frac{120\pi}{r}$. [3]

(iv) Given r can vary, find the minimum value of A and its corresponding r value that Xi Rui used in her design. [4]

End of Paper

Answers

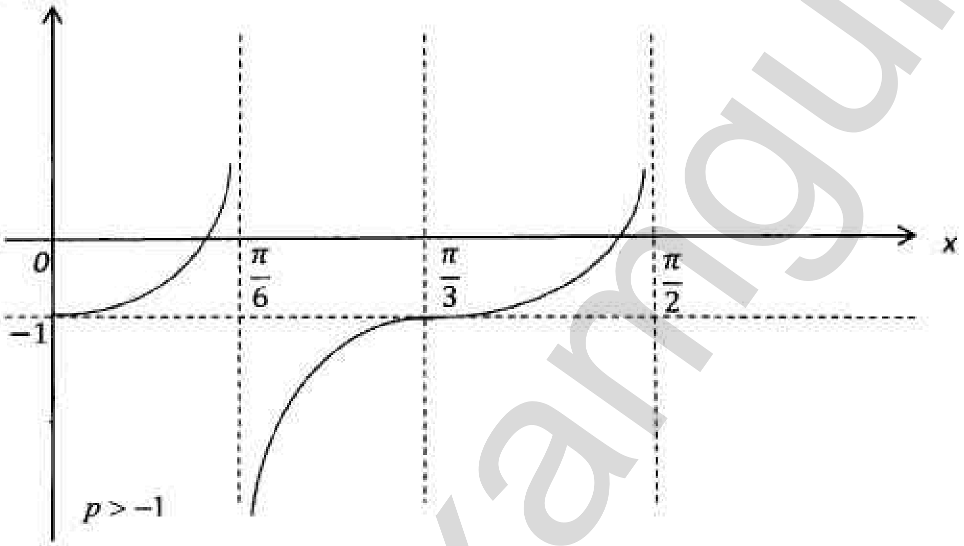
2)



3i) $x = 2$ 3ii) $x = \frac{1}{16}$

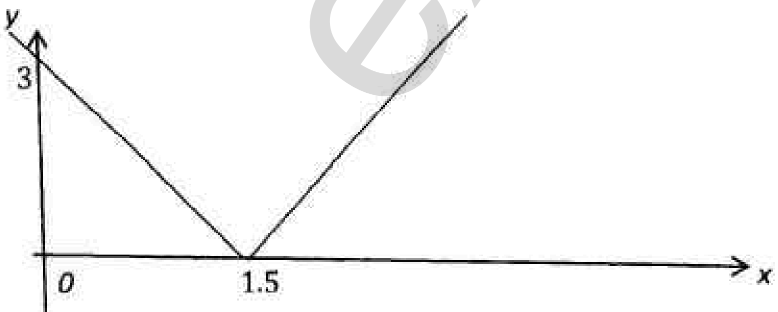
4) $a > 0,\quad c > 4$

5) $y = 2\tan 3x - 1$



$p > -1$

6i) $y = |3 - 2x|$



6ii) $-\frac{4}{3} < m < 2$

7) $x > -\frac{1}{2}$

$$\frac{dy}{dx}=\frac{2}{2x+1}+2x+1$$

Since $2x+1>0,\ \frac{2}{2x+1}>0,\ \frac{dx}{dx}>0$ therefore the y is an increasing function.

8i) $x=\frac{\pi}{2},\frac{3}{2}\pi,\frac{5\pi}{12},\frac{17\pi}{12}$ 8ii) $\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}$

9) $C(0,\ 6.1)$ $y=-\frac{8}{5}x+6.1$

10i) $a=\frac{1}{2}$ $b=-2\frac{1}{2}$ 10ii) $amplitude=\frac{1}{2}$ $period=\pi$ *or* 180° $greatest\ \ y=-2$ $least\ \ y=-3$

11i) $-\frac{\sqrt{5}}{5}$ 11ii) $-\frac{\sqrt{10}}{10}$ 11iii) $\frac{4}{5}$

$P=670$

12) $k=0.191$

$\frac{dP}{dt}=540$ bacteria/h

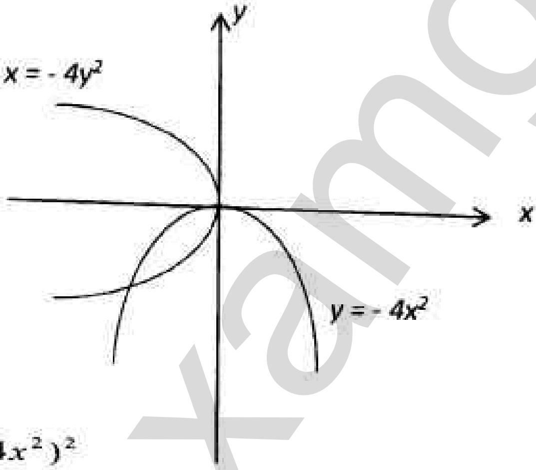
$r=3.23cm$

13iv) $\frac{d^2A}{dx^2}=33.571>0$

therefore $A=175\text{ cm}^2$ is a minimum value.

NCHS Prelim Examination (2) 2016

Additional Mathematics Paper 1 – Secondary 4 Express

Qn No	Suggested Solutions	
1	$\cos ec2x + \cot 2x = \cot x$ $LHS = \cos ec2x + \cot 2x$ $= \frac{1}{\sin 2x} + \frac{\cos 2x}{\sin 2x}$ $= \frac{1 + \cos 2x}{\sin 2x}$ $= \frac{1 + 2\cos^2 x - 1}{2\sin x \cos x}$ $= \frac{2\cos^2 x}{2\sin x \cos x}$ $= \frac{\cos x}{\sin x}$ $= \cot x$ $= RHS$	
2	 $y = -4x^2$ $x = -4y^2$ $x = -4(-4x^2)^2$ $x = -64x^4$ $x + 64x^4 = 0$ $x(1 + 64x^3) = 0$ $64x^3 = -1$ $x^3 = -\frac{1}{64}$ $x = -\frac{1}{4}$ or $x = 0$ $y = -\frac{1}{4}$ or $x = 0$ $\left(-\frac{1}{4}, -\frac{1}{4}\right)$ *and* $(0, 0)$ $y = x$	

3i

$\log_2 x = \log_{\frac{1}{2}} x + 2$

$\dfrac{\log_{\frac{1}{2}} x}{\log_{\frac{1}{2}} 2} = \log_{\frac{1}{2}} x + 2$

$\log_{\frac{1}{2}} x = -\log_{\frac{1}{2}} x - 2$

$2\log_{\frac{1}{2}} x = -2$

$\log_{\frac{1}{2}} x = -1$

$x = 2$

3ii

$\log_a 16 = -\dfrac{1}{\log_x a}$

$\log_a 16 = -\log_a x$

$x^{-1} = 16$

$x = \dfrac{1}{16}$

4

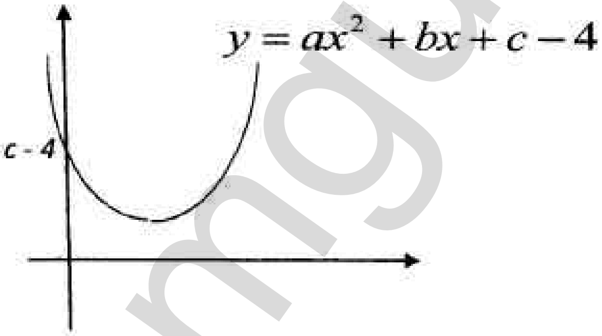
$y = ax^2 + bx + c$

$ax^2 + bx + c > 4$

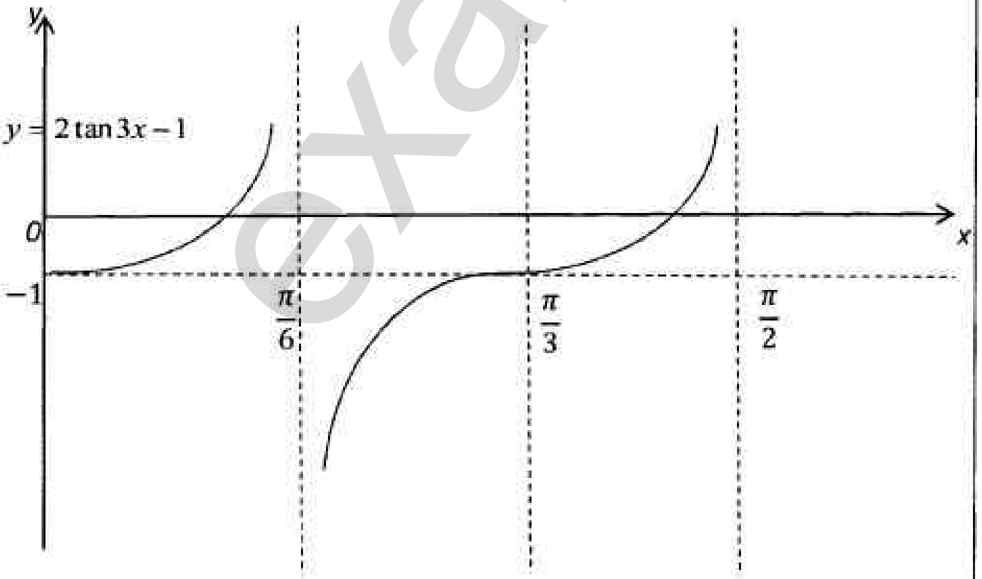
$ax^2 + bx + c - 4 > 0$

$a > 0, \quad c - 4 > 0$

$c > 4$

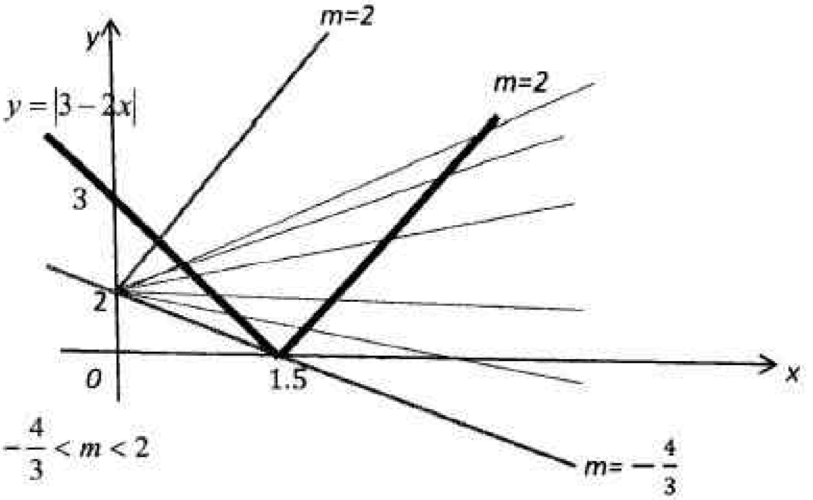


5



$p > -1$

6i



6ii

$-\frac{4}{3}<m<2$

7

$y=\ln(2x+1)+x^2+x$

$2x+1>0$

$x>-\frac{1}{2}$

$\frac{dy}{dx}=\frac{2}{2x+1}+2x+1$

Since $2x+1>0,\ \frac{2}{2x+1}>0,\ \frac{dx}{dx}>0$

therefore the y is an increasing function.

8i

$\sin\left(2x-\frac{\pi}{3}\right)\cos x=\cos x$

$\cos x\left(\sin\left(2x-\frac{\pi}{3}\right)-1\right)=0$

$\sin\left(2x-\frac{\pi}{3}\right)=1 \quad \text{or} \quad \cos x=0$

$basic\ \ angle=\frac{\pi}{2}$

$\left(2x-\frac{\pi}{3}\right)=\frac{\pi}{2},\ 2\frac{1}{2}\pi \quad \text{or} \quad x=\frac{\pi}{2},\ \frac{3}{2}\pi$

$x=\frac{5\pi}{12},\frac{17\pi}{12}$

8ii

$\sin 75°+\cos 15°$

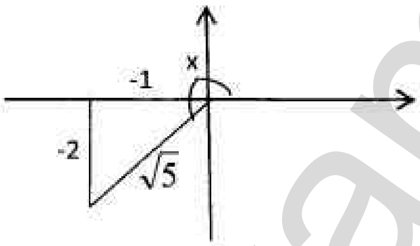
$=\sin(45+30)+\cos(45-30)$

$=\sin 45\cos 30+\sin 30\cos 45+\cos 45\cos 30+\sin 45\sin 30$

$=\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$=\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}$

$=\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}$

9i	$\sqrt{(-3)^2+(2-y)^2}=\sqrt{5^2+(7-y)^2}$ y = 6.1 C$(0,\ 6.1)$	
9ii	$m_1=\frac{7-2}{5+3}$ $=\frac{5}{8}$ $m_2=-\frac{8}{5}$ $y=-\frac{8}{5}x+6.1$	
10i	$\sin^2 x+2\cos^2 x-4=a\cos 2x+b$ $RHS=\sin^2 x+2\cos^2 x-4$ $=\frac{1-\cos 2x}{2}+\cos 2x+1-4$ $=\frac{1}{2}\cos 2x-2\frac{1}{2}$ $a=\frac{1}{2}$ b = $-2\frac{1}{2}$	
10ii	$amplitude=\frac{1}{2}$ $period=\pi$ or 180° $greatest\ \ y=-2$ $least\ \ y=-3$  	
11i	$\cos(-x)=\cos x$ $=-\frac{1}{\sqrt{5}}$ $=-\frac{\sqrt{5}}{5}$	
11ii	$\sin(x-45^\circ)=\sin x\cos 45-\sin 45\cos x$ $=\left(\frac{-2}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right)-\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{\sqrt{5}}\right)$ $=-\frac{\sqrt{2}}{\sqrt{5}}+\frac{\sqrt{2}}{2\sqrt{5}}$ $=-\frac{1}{5}\sqrt{10}+\frac{1}{10}\sqrt{10}$ $=-\frac{\sqrt{10}}{10}$	

11iii	$\sin(2x) = 2\sin x \cos x$ $= 2\left(-\frac{2}{\sqrt{5}}\right)\left(-\frac{1}{\sqrt{5}}\right)$ $= \frac{4}{5}$	
12i	$P = 250 + 420e^{kt}$ $P = 250 + 420$ $= 670$	
12ii	$2(670) = 250 + 420e^{k(5)}$ $k = \frac{1}{5}\ln\frac{109}{42}$ $k = 0.191$	
12iii	$\frac{dP}{dt} = 420ke^{kt}$ $= 420\left(\frac{1}{5}\ln\frac{109}{42}\right)e^{\left(\frac{1}{5}\ln\frac{109}{42}\right)(10)}$ $= 420\left(\frac{1}{5}\ln\frac{109}{42}\right)e^{\left(2\ln\frac{109}{42}\right)}$ $= 420\left(\frac{1}{5}\ln\frac{109}{42}\right)e^{\ln\left(\frac{109}{42}\right)^2}$ $= 420\left(\frac{1}{5}\ln\frac{109}{42}\right)\left(\frac{109}{42}\right)^2$ $= 539.55$ $= 540$ bacteria/h	
13i	$density = \frac{mass}{volume}$ $\frac{5}{3\pi} = \frac{100}{V}$ $V = 60\pi$	

13ii	$V = \frac{2}{3}\pi r^3 + \pi r^2 h + \frac{1}{3}\pi r^2\left(2\sqrt{2}r\right)$ $60\pi = \frac{2}{3}\pi r^3 + \pi r^2 h + \frac{2\sqrt{2}}{3}\pi r^3$ $h = \frac{60 - \frac{2}{3}r^3 - \frac{2\sqrt{2}}{3}r^3}{r^2}$ $h = \frac{60}{r^2} - \frac{2}{3}r - \frac{2\sqrt{2}}{3}r$ $= \frac{60}{r^2} - \frac{2r}{3}\left(1+\sqrt{2}\right)$ *(Shown)*	
13iii	$A = 2\pi r^2 + 2\pi rh + \pi(r)(3r)$ $= 2\pi r^2 + 2\pi r\left(\frac{60}{r^2} - \frac{2}{3}r - \frac{2\sqrt{2}}{3}r\right) + 3\pi r^2$ $= 5\pi r^2 + \frac{120\pi}{r} - \frac{4}{3}\pi r^2 - \frac{4\sqrt{2}}{3}\pi r^2$ $= \frac{1}{3}\pi r^2\left(11-4\sqrt{2}\right) + \frac{120\pi}{r}$ *(shown)*	
13iv	$\frac{dA}{dr} = \frac{2}{3}\pi r\left(11-4\sqrt{2}\right) - \frac{120\pi}{r^2}$ $\frac{dA}{dr} = 0$ $\frac{2}{3}\pi r\left(11-4\sqrt{2}\right) - \frac{120\pi}{r^2} = 0$ $\frac{2}{3}\pi\left(11-4\sqrt{2}\right)r^3 = 120\pi$ $r^3 = \frac{180}{\left(11-4\sqrt{2}\right)}$ $r = 3.2297$ $= 3.23cm$ $\frac{d^2A}{dx^2} = \frac{2}{3}\pi\left(11-4\sqrt{2}\right) + \frac{240\pi}{r^3}$ $= 33.571 > 0$ *therefore* $A = 175\text{ cm}^2$ is a minimum value.	

*** End of Paper ***

Name: **Register Number:** **Class:**

南僑中學

NAN CHIAU HIGH SCHOOL

PRELIMINARY EXAMINATION (2) 2016
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS **4047/02**

Paper 2 **11 May 2016, Wednesday**

$2\frac{1}{2}$ hours

Additional Materials: Writing paper (8 sheets)
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 100.

Setter: Ms Renuka Ramakrishnan

This document consists of **7** printed pages including the coverpage.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$cosec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer all the questions.

1. Find the range of values of m for which the line $x+3y=m$ does not intersect the curve $x(x+y)=-6$. [4]

2. The roots of the quadratic equation $x^2-4x+5=0$ are $\frac{\alpha}{2}$ and $\frac{\beta}{2}$.

(i) Find the value of $\alpha^2+\beta^2$. [3]

(ii) Find a quadratic equation whose roots are α^3 and β^3. [3]

3. The function f is defined by $f(x)=2x^3-4x^2-2x+4$.

(i) Determine, with appropriate workings, whether $(x+2)$ and $(x-2)$ are factors of $f(x)$. [2]

(ii) Hence, by finding the roots of $f(x)=0$, solve the equation $16y^3-16y^2-4y+4=0$. [5]

4. A curve has the equation $y=\ln\left(\cos^2\frac{x}{4}\right)$. Show that the equation of the normal at the point $x=\pi$ is $y=ax+b\pi+c\ln 2$, where a, b and c are constants to be determined. [6]

5. **(a)** **(i)** Find, in ascending powers of x, the expansion of $(2+x)^8$ as far as the term in x^3. [2]

(ii) Hence, determine the coefficient of a^3 in the expansion of $(2+a-5a^2)^8$. [3]

(b) In the expansion of $\left(x^2-\frac{3}{x^4}\right)^{12}$, find

(i) the middle term [2]

(ii) the term independent of x. [3]

6. (a) (i) Differentiate $x^3 \ln 2x$ with respect to x. [1]

(ii) Hence, find $\int x^2 \ln 2x \; dx$. [4]

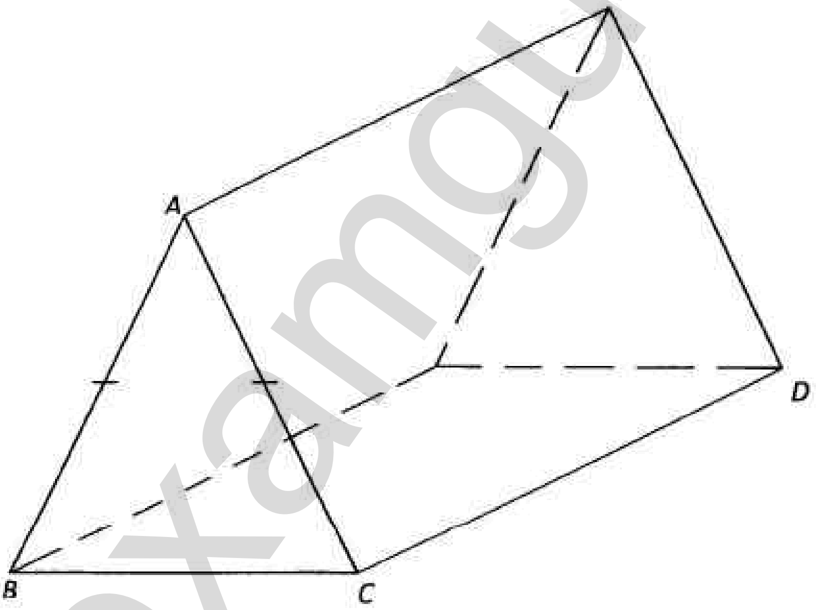
(b) (i) Express $\frac{1}{(x+3)(x+1)^2}$ as partial fractions. [4]

(ii) Hence, show that $\int_0^2 \frac{1}{(x+3)(x+1)^2} dx = \frac{1}{4}\ln\frac{5}{9}+\frac{1}{3}$. [4]

7. **Do not use a calculator in this question.**

(a) Simplify $\frac{4^{3x} \times 8^{x-4}}{2^{7+x}}$. [2]

(b)



The diagram shows a prism where the cross section is an isosceles triangle.

Given that $AB = AC$, the length of BC is $\left(\sqrt{3}-\sqrt{2}\right)$ cm, the length of CD is $\left(3\sqrt{2}+2\sqrt{3}\right)$ cm and the volume of the prism is $100\,\text{cm}^2$, find

(i) the cross-sectional area of the prism, [3]

(ii) the perpendicular height of A from BC. [4]

8. A curve has the equation $y = \sqrt{\dfrac{3-2x}{x^2+2}}$.

(i) Find the range of values of x for which y is defined. [1]

(ii) Calculate the gradient of the curve when $x = 1$. [3]

(iii) Given that x is decreasing at a rate of 0.05 units per second, find the rate of change of y when $x = 1$. [2]

9. A rectangle of area, A m^2, has sides of length x m and $(Mx + N)$ m, where M and N are constants.

Corresponding values of x and A are given in the table below.

x	10	20	30	40	50
A	4600	7400	8700	8000	5500

(i) Using suitable values, draw, on graph paper, a straight line graph. [3]

(ii) Use your graph to estimate the value of M and N. [3]

(iii) On the same diagram, draw the straight line representing the equation $A = x^2$. Explain the significance of the value of x given by the point of intersection of the two lines and state this value of x. [4]

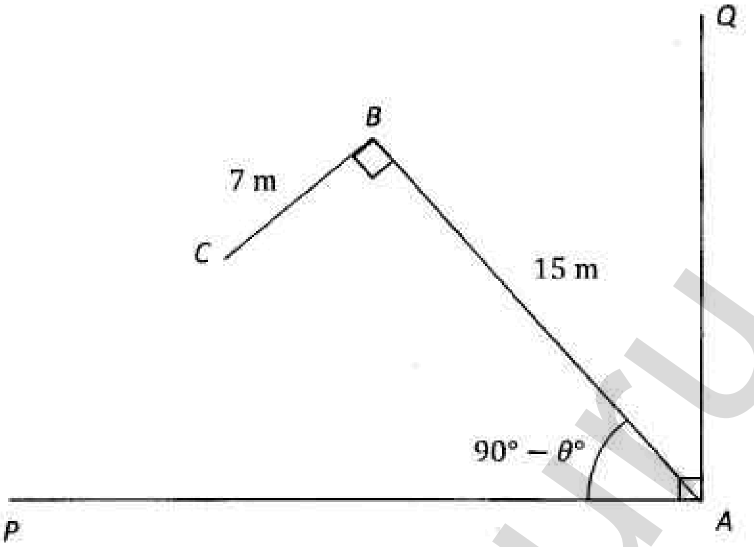
10. The equation of a circle, C_1, is $x^2 + y^2 - 4x - 2y - 20 = 0$.

(i) Find the centre and the radius of the circle. [3]

(ii) Show that the point $P(-2,4)$ is on the circle. [1]

(iii) Find the equation of the smallest circle, C_2, passing through P and having its centre on the line $x + 5y = 2$. [6]

11. In the diagram below, $BC = 7\text{ m}$, $AB = 15\text{ m}$ and angle $PAB = 90° - \theta°$. L is the perpendicular distance from C to AQ.



(i) Show that $L = a\sin\theta + b\cos\theta$, where a, b are constants to be found. [3]

(ii) Express L in the form of $R\sin(\theta + \alpha)$, where $R > 0$ and α is an acute angle. [3]

(iii) Find the maximum value of L and the corresponding value of θ. [2]

(iv) Given that $L = 12$ m, find the value of θ. [3]

12. Figure 1 and Figure 2 shows the graphs of $f'(x)$ and $f''(x)$ respectively.

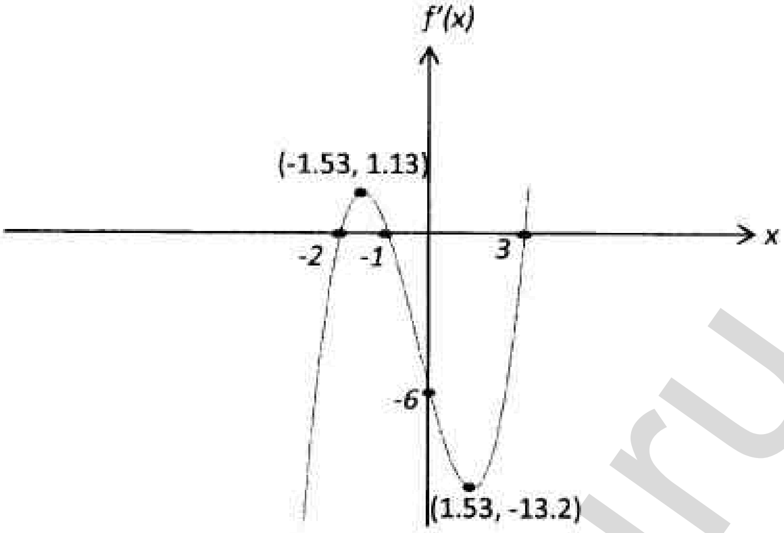


Figure 1

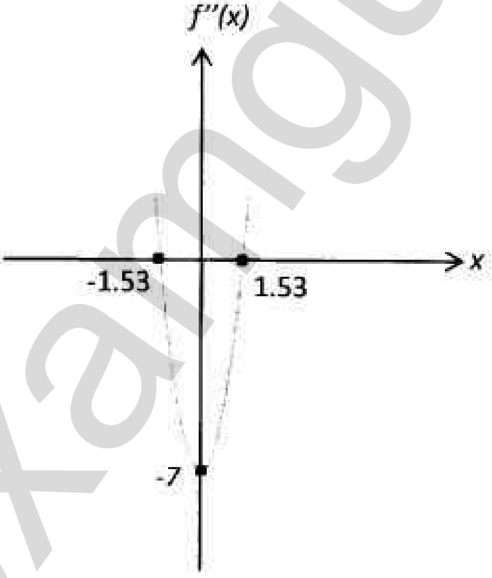


Figure 2

Using the information from figure 1 and/or figure 2,

(i) state the x-coordinates of all the stationary points of the graph $y = f(x)$ and determine the nature of the stationary points. [4]

(ii) find the interval(s) for which $f(x)$ is strictly decreasing. [2]

(iii) find the interval(s) for which $f'(x)$ is strictly increasing. [2]

- End of paper -

Answer Key:

Q1) $-12 < m < 12$

Q2i) 24

Q2ii) $x^2 - 32x + 8000 = 0$

Q3ii) $y = 1, -0.5, 0.5$

Q5ai) $256 + 1024x + 1792x^2 + 1792x^3 + \ldots$

Q5aii) -16128

Q5bi) $673596x^{-12}$

Q5bii) 40095

Q6ai) $3x^2 \ln 2x + x^2$

Q6aii) $\frac{1}{3}x^3 \ln 2x - \frac{1}{9}x^3 + c_2$

Q6bi) $\frac{1}{4(x+3)} - \frac{1}{4(x+1)} + \frac{1}{2(x+1)^2}$

Q7a) 2^{8x-19}

Q7bi) $\frac{150\sqrt{2} - 100\sqrt{3}}{3}$

Q7bii) $\frac{100}{3}\sqrt{6}$

Q8i) $x \leq \frac{3}{2}$

Q8ii) $-\frac{4}{9}\sqrt{3}$

Q8iii) 0.0385 units/sec

Q9ii) M = -8.75, N = 550

Q9iii) The rectangle becomes a square. x = 56.5

Q10i) Centre (2,1) and radius = 5 units

Q10iii) $\left(x + \frac{34}{13}\right)^2 + \left(y - \frac{12}{13}\right)^2 = \frac{128}{13}$

Q11i) $L = 7\cos\theta + 15\sin\theta$

Q11ii) $L = 16.6\cos(\theta + 25.0°)$

Q11iii) Max value = 16.6m
$\theta = 65.0°$

Q11iv) $\theta = 21.4°$

Q12i) At x = -2, minimum point
At x = -1, maximum point
At x = 3, minimum point

Q12ii) $x < -2$
$-1 < x < 3$

Q12iii) $x < -1.53$
$x > 1.53$

Additional Mathematics – Secondary 4 Express

Nan Chiau High School

Prelim Examination (2) 2016

Marking Scheme (Paper 2)

Qn No	Suggested Solutions
1	$x+3y=m$ $y=-\frac{1}{3}x+\frac{1}{3}m$ --(1) $x(x+y)=-6$ --(2) Sub (1) into (2): $x(x-\frac{1}{3}x+\frac{1}{3}m)=-6$ .... M1 $\frac{2}{3}x^2+\frac{1}{3}mx+6=0$ $2x^2+mx+18=0$ .... M1 Since there is no intersection, $discri\min ant<0$ $(m)^2-4(2)(18)<0$ .... M1 $m^2-144<0$ $(m+12)(m-12)<0$ $\therefore -12<m<12$ A1
2i	$\frac{\alpha}{2}+\frac{\beta}{2}=4$ $\alpha+\beta=8$ .... M1 $\left(\frac{\alpha}{2}\right)\left(\frac{\beta}{2}\right)=5$ $\alpha\beta=20$ $\alpha^2+\beta^2$ $=(\alpha+\beta)^2-2\alpha\beta$ $=8^2-2(20)$ .... M1 $=24$ A1

2ii	$\alpha^3+\beta^3$ $=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=8(24-20)$ $=32$ M1 $\alpha^3\beta^3$ $=(\alpha\beta)^3$ $=8000$ M1 New equation: $x^2-32x+8000=0$ A1
3i	$f(x)=2x^3-4x^2-2x+4$ When $x=-2$, $f(x)$ $=2(-2)^3-4(-2)^2-2(-2)+4$ $=-24$ $\therefore (x+2)$ is not a factor of $f(x)$. B1 When $x=2$, $f(x)$ $=2(2)^3-4(2)^2-2(2)+4$ $=0$ $\therefore (x-2)$ is not a factor of $f(x)$. B1
3ii	$f(x)=0$ $(x-2)(2x^2-2)=0$ M1: for long division or synthetic method $(x-2)(x+1)(x-1)=0$ M1 $\therefore x=2,-1,1$ A1 Let $x=2y$, M1 $16y^3-16y^2-4y+4=0$ $2(2y)^3-4(2y)^2-2(2y)+4=0$ $\therefore y=1,-0.5,0.5$ A1

4

$y = \ln(\cos^2\left(\frac{x}{4}\right))$

$\frac{dy}{dx} = \frac{1}{\cos^2\left(\frac{x}{4}\right)} \times 2\cos\left(\frac{x}{4}\right) \times -\frac{1}{4}\sin\left(\frac{x}{4}\right)$ ………… M1

$= -\frac{\sin\left(\frac{x}{4}\right)}{2\cos\left(\frac{x}{4}\right)}$

$= -\frac{1}{2}\tan\left(\frac{x}{4}\right)$ ………… M1

$\frac{dy}{dx}_{x=\pi} = -\frac{1}{2}$

$\therefore$ gradient of normal = 2 ………… M1

When $x = \pi$,

$y = \ln\frac{1}{2} = -\ln 2$ ………… M1

$\therefore$ equation of normal:

$y + \ln 2 = 2(x - \pi)$ ………… M1

$y = 2x - 2\pi - \ln 2$ ………… A1

5ai

$(2+x)^8$

$= 2^8 + \binom{8}{1}2^7 x + \binom{8}{2}2^6 x^2 + \binom{8}{3}2^5 x^3 + \ldots$ ………… M1

$= 256 + 1024x + 1792x^2 + 1792x^3 + \ldots$ ………… A1

aii

Let $x = a - 5a^2$

$(2 + a - 5a^2)^8$

$= 1792(a - 5a^2)^2 + 1792(a - 5a^2)^3 + \ldots.$ ………… M1

$= 1792(-10a^3) + 1792(a^3) + \ldots$ ………… M1

$= -16128a^3 + \ldots$

$\therefore$ coefficient of a^3 = -16128 ………… A1

5bi	T_7 $=\binom{12}{6}(x^2)^6\left(-\frac{3}{x^4}\right)^6$ ………… M1 $=673596x^{-12}$ ………… A1
5bii	T_{r+1} $=\binom{12}{r}(x^2)^{12-r}\left(-\frac{3}{x^4}\right)^r$ $=\binom{12}{r}(-3)^r\left(x^{24-6r}\right)$ ………… M1 For term independent of x: $24-6r=0$ $r=4$ ………… M1 T_5 $=\binom{12}{4}(-3)^4\left(x^0\right)$ $=40095$ ………… A1
6ai	$\frac{d}{dx}x^3\ln 2x = 3x^2\ln 2x + x^2$ ………… B1
aii	$\int 3x^2\ln 2x + x^2dx = x^3\ln 2x + c$ ………… M1 $3\int x^2\ln 2xdx = x^3\ln 2x - \int x^2dx + c$ ………… M1 $3\int x^2\ln 2xdx = x^3\ln 2x - \frac{1}{3}x^3 + c_1$ ………… M1 $\int x^2\ln 2xdx = \frac{1}{3}x^3\ln 2x - \frac{1}{9}x^3 + c_2$ ………… A1

bi

$$\frac{1}{(x+3)(x+1)^2} = \frac{A}{x+3} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$ M1

$$1 = A(x+1)^2 + B(x+1)(x+3) + C(x+3)$$

$$\left.\begin{aligned} A &= \frac{1}{4} \\ B &= -\frac{1}{4} \\ C &= \frac{1}{2} \end{aligned}\right\}$$ M2

$$\therefore \frac{1}{(x+3)(x+1)^2} = \frac{1}{4(x+3)} - \frac{1}{4(x+1)} + \frac{1}{2(x+1)^2}$$ A1

6ii

$$\int_0^2 \frac{1}{(x+3)(x+1)^2}\,dx = \int_0^2 \frac{1}{4(x+3)} - \frac{1}{4(x+1)} + \frac{1}{2(x+1)^2}\,dx$$

$$= \left[\frac{1}{4}\ln(x+3) - \frac{1}{4}\ln(x+1) - \frac{1}{2(x+1)^2}\right]_0^2$$ M2

$$= \frac{1}{4}\ln 5 - \frac{1}{4}\ln 3 - \frac{1}{6} - \frac{1}{4}\ln 3 + \frac{1}{4}\ln 1 + \frac{1}{2}$$ M1

$$= \frac{1}{4}\ln\frac{5}{9} + \frac{1}{3}$$ A1

7a

$$\frac{4^{3x} \times 8^{x-4}}{2^{7+x}}$$

$$= 2^{6x+3x-12-(7+x)}$$ M1

$$= 2^{8x-19}$$ A1

7bi

$$\frac{100}{3\sqrt{2} + 2\sqrt{3}}$$ M1

$$= \frac{100(3\sqrt{2} - 2\sqrt{3})}{(3\sqrt{2})^2 - (2\sqrt{3})^2}$$ M1

$$= \frac{150\sqrt{2} - 100\sqrt{3}}{3}$$ A1

7bii

$$h=\frac{150\sqrt{2}-100\sqrt{3}}{3}\div(\sqrt{3}-\sqrt{2})\times 2$$ M2

$$=\frac{2(150\sqrt{2}-100\sqrt{3})(\sqrt{3}+\sqrt{2})}{3(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$$ M1

$$=\frac{300\sqrt{6}+600-600-200\sqrt{6}}{3}$$

$$=\frac{100}{3}\sqrt{6}$$ A1

8i

$$\left.\begin{array}{l}\because x^2+2>0,\\ 3-2x>0\\ x<\dfrac{3}{2}\end{array}\right]$$ B1

ii

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{3-2x}{x^2+2}\right)^{-\left(\frac{1}{2}\right)}\times\frac{-2(x^2+2)-2x(3-2x)}{(x^2+2)^2}$$ M2

$$\frac{dy}{dx}_{x=1}=-\frac{4}{9}\sqrt{3}$$ A1

iii

$$\frac{dy}{dt}=-\frac{4}{9}\sqrt{3}\times(-0.05)$$ M1

$$=\frac{\sqrt{3}}{45}\, units\,/\,\text{sec}$$ A1

Or = 0.0385 units/sec (3sf)

10i

$$x^2+y^2-4x-2y-20=0$$

$$(x-2)^2+(y-1)^2=25$$ M1

Centre (2,1) and radius = 5 units A2

10ii

When x=-2,

$$\left.\begin{array}{l}(-2)^2+y^2-4(-2)-2y-20=0\\ y^2-2y-8=0\\ (y-4)(y+2)=0\\ y=4,\ y=-2\\ \therefore \text{P lies on the circle}\end{array}\right\}$$ B1

10iii

$$y=-\frac{1}{5}x+\frac{2}{5}$$

Gradient of line $=-\frac{1}{5}$

gradient of perpendicular line = 5 ………… M1

equation of line passing through P with $m=5$,

$y-4=5(x+2)$ ………… M1

$y=5x+14$

$\therefore 5x+14=-\frac{1}{5}x+\frac{2}{5}$ ………… M1

$x=-\frac{34}{13}, y=\frac{12}{13}$ ………… A1

Radius of C₂

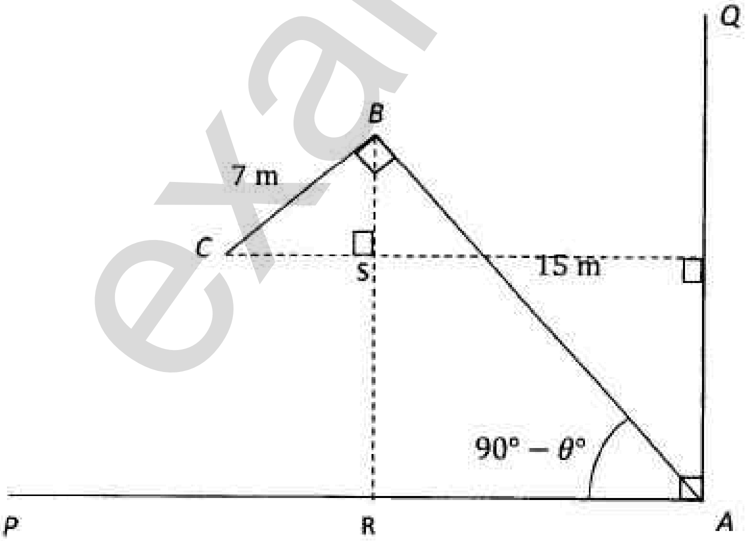
$=\sqrt{\left(-2+\frac{34}{13}\right)^2+\left(4-\frac{12}{13}\right)^2}$ ………… M1

$=\sqrt{\frac{128}{13}}$

$\therefore$ equation of C₂:

$\left(x+\frac{34}{13}\right)^2+\left(y-\frac{12}{13}\right)^2=\frac{128}{13}$ ………… A1

11i



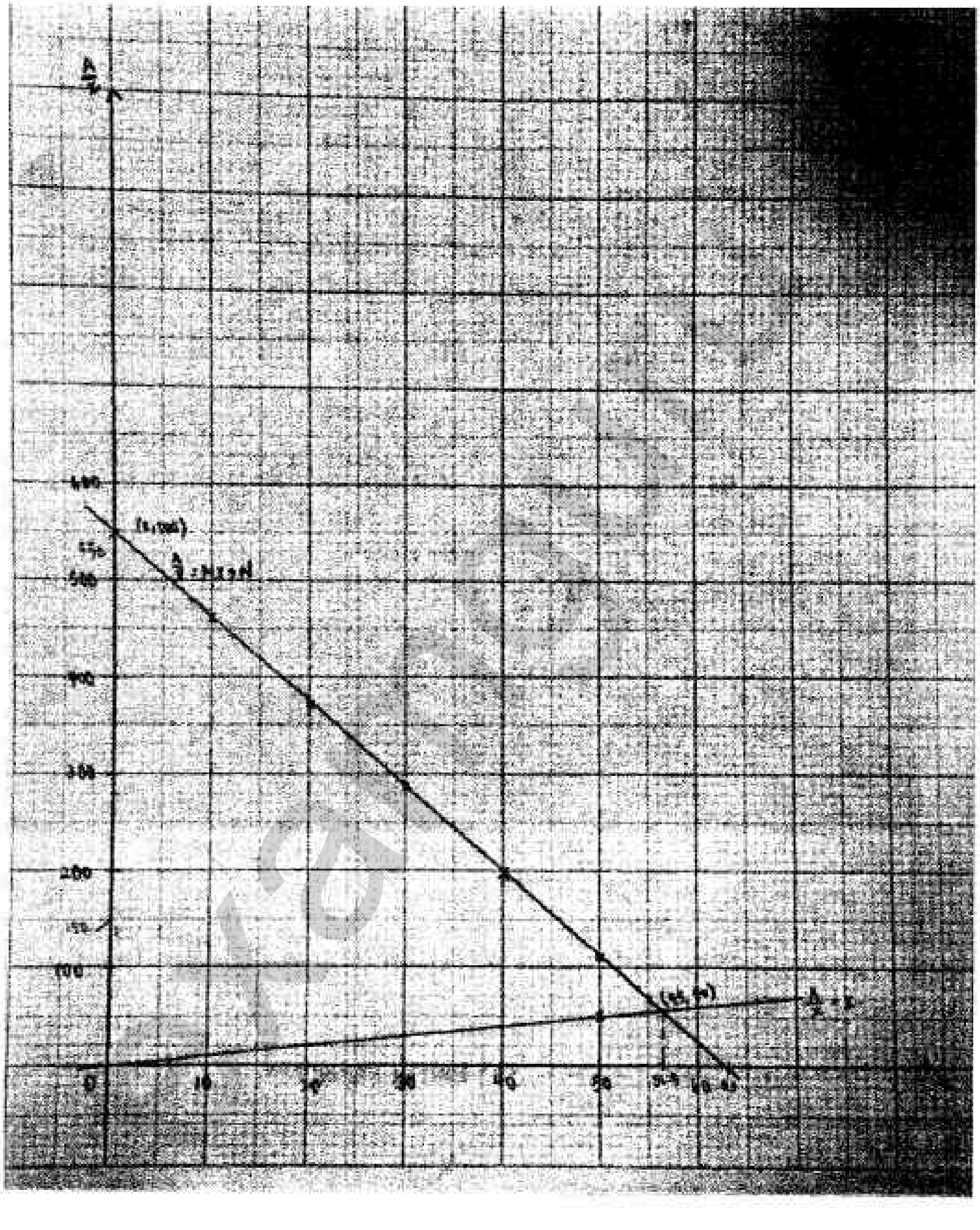
$\cos\theta=\frac{CS}{7}, \sin\theta=\frac{AR}{15}$ ………… M2

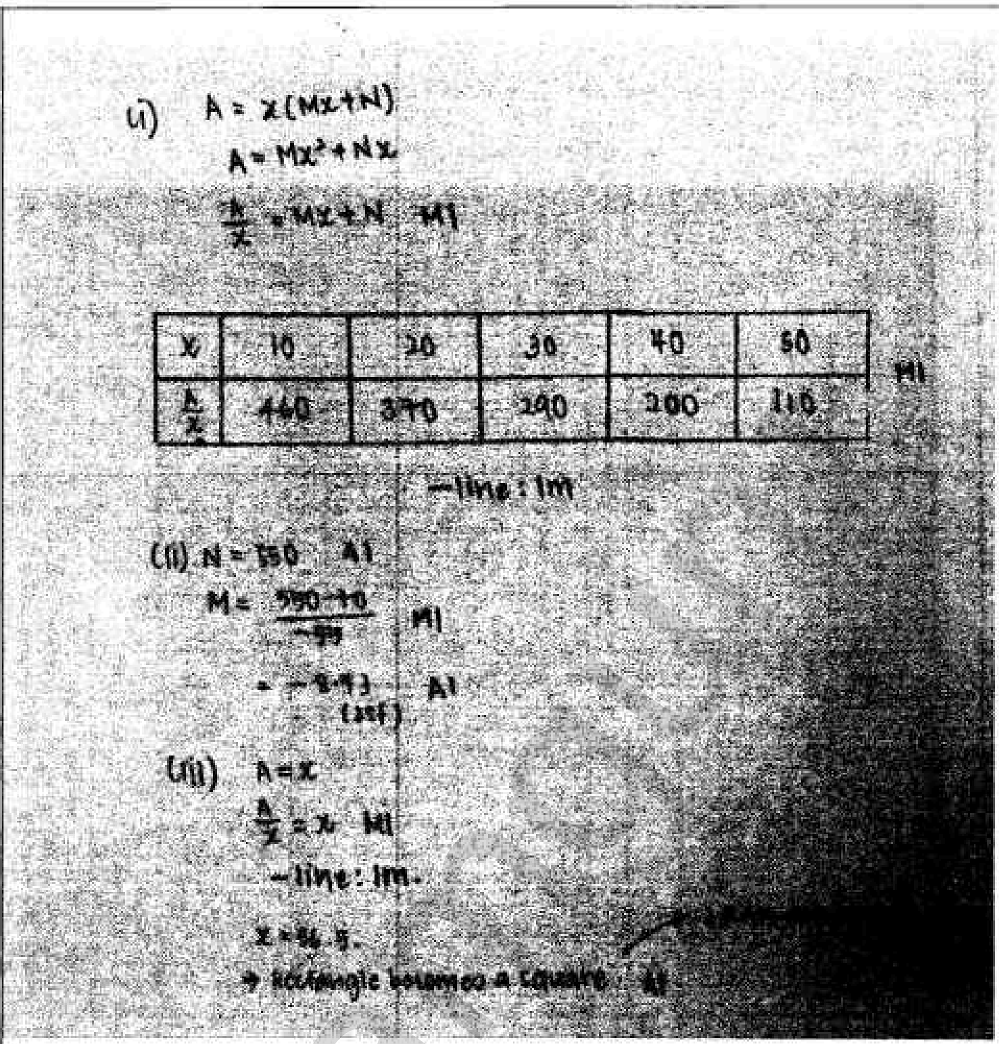
$\therefore L=\text{CS}+\text{AR}$

$L=7\cos\theta+15\sin\theta$ ………… A1

11ii	$L = 7\cos\theta + 15\sin\theta$ $R = \sqrt{274}$ M1 $\tan\alpha = \frac{15}{7}$ $\alpha = 25.017$ M1 $L = 16.6\cos(\theta + 25.0^\circ)$ A1
11iii	Max value = 16.6m B1 $\theta = 65.0^\circ$ B1
11iv	$\sqrt{274}\sin(\theta + 25.017^\circ) = 12$ M1 $\sin(\theta + 25.017^\circ) = 0.72495$ M1 $\alpha = 46.464^\circ$ $\theta + 25.017^\circ = 46.464^\circ$ $\theta = 21.4^\circ$ A1
12i	Stationarypoints: x = -2, x = -1, x = 3 B1 At x = -2, minimum point At x = -1, maximum point At x = 3, minimum point B3
12ii	$x < -2$ $-1 < x < 3$ B2
12iii	$x < -1.53$ $x > 1.53$ B2

9





(i) $A = x(Mx+N)$

$A = Mx^2 + Nx$

$\frac{A}{x} = Mx + N$ M1

x	10	20	30	40	50
$\frac{A}{x}$	460	370	290	200	110

M1

—line : 1m

(ii) $N = 550$ A1

$M = \frac{550-70}{-[illegible]}$ M1

$= -8.93$ (3sf) A1

(iii) $A = x$

$\frac{A}{x} = x$ M1

—line: 1m.

$x = 56.5$.

→ rectangle becomes a square A1

*** End of Paper ***

Name:	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS **4047/01**
Paper 1
17 August 2016
2 hours

Additional Materials: Answer Paper
Mark Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces provided at the top of this page and on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighter, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work and mark sheet securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **5** printed pages and **1** blank page. [Turn Over

1 Sketch the graph of $y = |2x^2 - x - 1|$, indicating the intercepts and the turning point. [3]

2 Find the range of values of c for which the graph $y = x^2 - 3x + cx + 5$ lies entirely above the line $y = x + 1$. [4]

3 Solve the equation $\sin 2x = \sin x$ for $0° \le x \le 360°$. [4]

4 The cubic polynomial $f(x)$ has roots $x = \frac{1}{2}$, -3 and h. Given that the coefficient of x^3 is 6 and $f(x)$ has a remainder of -18 when divided by $x + 1$, find the value of h. Hence, find the remainder when $f(x)$ is divided by $2x - 3$. [4]

5 The sides AB and BC of ΔABC are of length $\left(2+\sqrt{3}\right)$ cm and $\left(4+\frac{2}{\sqrt{3}}\right)$ cm respectively. Given that $\angle ABC = 60°$, find the area of ΔABC in the form $a + b\sqrt{3}$ where a and b are rational numbers. [4]

6 Solve the following simultaneous equations.

$$\frac{1}{x} + \frac{3}{y} = 1$$

$$\left(\frac{1}{e^2}\right)^{1+2x} \times e^y = e$$

[5]

7 Find, without using a calculator, the exact value of $\frac{\tan 49° - \tan 34°}{1 + \tan 49° \tan 34°}$. [5]

8 (a) It is known that x and y are related by the equation $ax^2 + ky^3 - 120 = 0$, where a and k are non-zero constants. Explain how the value of a and k may be obtained from a suitable straight line graph. [3]

(b) A straight line graph is obtained by plotting $\frac{1}{y}$ against x. Given that the graph passes through the point ($\sqrt{3}$, 1) and makes an angle of 60° with the line $y = 1$, express y in terms of x. [4]

9 Given that $\tan^2 A - 2\tan^2 B = 1$,

(i) show that $\cos^2 B = 2\cos^2 A$, [3]

(ii) find the exact value of $\tan B$ given that A and B are acute angles such that $A + B = \frac{\pi}{2}$. [5]

10 (i) Write down and simplify, in descending powers of x, the first three terms in the expansion of $(x^3 + \frac{2}{x^4})^n$, where $n > 0$. [3]

(ii) Hence find the value of n given that the coefficient of the third term is 7 times that of the second term. [2]

(iii) Using the value of n found in (ii), without expanding $(x^3 + \frac{2}{x^4})^n$, show that there is no constant term in the expansion. [3]

11 An object moves in a straight line, so that, t seconds after passing a fixed point O, its velocity, v m/s, is given by $v = 6t^2 - 22t + 9$. Find

(i) an expression for the displacement from O at any time t, [3]

(ii) the acceleration of the object when it comes to momentary rest the second time, [4]

(iii) the total distance travelled in the first two seconds after passing through O. [2]

12 (i) Express $\frac{9x^2 - 15x + 27}{(2x-5)(x^2+9)}$ in partial fractions.
(Hint: use substitution method) [4]

(ii) Differentiate $\ln(x^2 + 9)$ with respect to x. [1]

(iii) Hence find $\int_0^1 \frac{9x^2 - 15x + 27}{(2x-5)(x^2+9)}dx$. Give your answer in the form $a \ln b$ where a and b are rational numbers to be determined. [4]

13

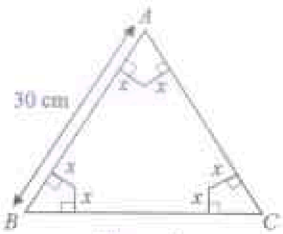


Figure 1

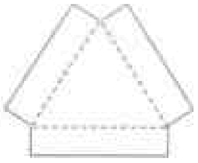


Figure 2

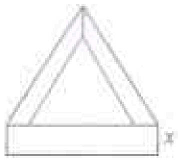


Figure 3

Figure 1 shows a piece of card in the form of an equilateral triangle ABC of side 30 cm. A kite shape is cut from each corner of ΔABC to give the shape as shown in Figure 2. The remaining card shown in Figure 2 is folded along the dotted lines, to form the open triangular box of height x cm, shown in Figure 3.

(i) Show that the volume, V cm^3, of the triangular box is given by

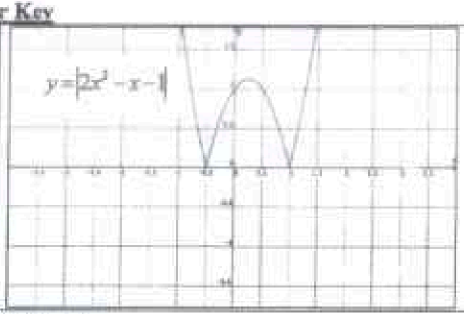
$$V = \frac{\sqrt{3}}{4}x(30 - 2\sqrt{3}x)^2.$$ [4]

(ii) Given that x can vary, find the value of x when V has a stationary value. [4]

(iii) By considering the sign of $\frac{dV}{dx}$, determine whether the volume of the triangular box is a maximum or minimum. [2]

END OF PAPER

Answer Key

1	
2	$0<c<8$
3	0°,60°,180°,300°,360°
4	$94\frac{1}{2}$
5	$4+\frac{5\sqrt{3}}{2}$ cm^2
6	$x=\frac{3}{2}, y=9$ or $x=-\frac{1}{2}, y=1$
7	$2-\sqrt{3}$
8	(a) $y^3=-\frac{a}{k}x^2+\frac{120}{k}$; Plot y^3 against x^2 where gradient $=-\frac{a}{k}$ and y^3-intercept $=\frac{120}{k}$
	(b) $y=\frac{1}{\sqrt{3x-2}}$
9	(i) As shown
	(ii) $\sqrt{\frac{1}{2}}$
10	(i) $x^{3n}+2nx^{3n-7}+2n(n-1)x^{3n-14}+...$
	(ii) 8
	(iii) For constant term, $r=\frac{24}{7}$ is not a positive integer
11	(i) $s=2t^3-11t^2+9t$
	(ii) 16.4 m/s^2
	(iii) 14.0 m
12	(i) $\frac{3}{2x-5}+\frac{3x}{x^2+9}$
	(ii) $\frac{2x}{x^2+9}$
	(iii) $\frac{3}{2}\ln\frac{25}{6}$
13	(i) As shown
	(ii) $\frac{5\sqrt{3}}{3}$
	(iii) Maximum

Name:	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS
Paper 2

4047/02
23 August 2016
2 hours 30 minutes

Additional Materials: Answer Paper
Mark Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces provided at the top of this page and on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighter, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work and mark sheet securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of **8** printed pages.

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1 **(a)** Find the quotient and remainder when $4x^3-12x^2+7$ is divided by $2x^2-3x-2$. [3]

(b) Let $f(x)=4x^3-12x^2+7+(ax+b)$, where a and b are constants. It is given that $f(x)$ is divisible by $2x^2-3x-2$.

(i) State the value of a and of b. [2]

(ii) Deduce the roots of the equation $f(x)=0$. [2]

2 α and β are the roots of the equation $x^2-4x+3=0$, where α and β are positive integers and $\alpha>\beta$.

(i) Express $\alpha-\beta$ in terms of $\alpha+\beta$ and $\alpha\beta$. [2]

(ii) Without finding the value of α and of β, form a quadratic equation whose roots are $\alpha^2\beta$ and $-\alpha\beta^2$. [5]

3 As part of an experiment, a group of students started a rumour in their school and recorded down the number of students who have heard of the rumour after every hour. There are 500 students in the school. After collecting their data, they propose that the spread of the rumour can be modelled by the equation $N=\dfrac{500}{1+99e^{-2t}}$ where N is the number of students who have heard of the rumour and t is the number of hours after the group of students started the rumour.

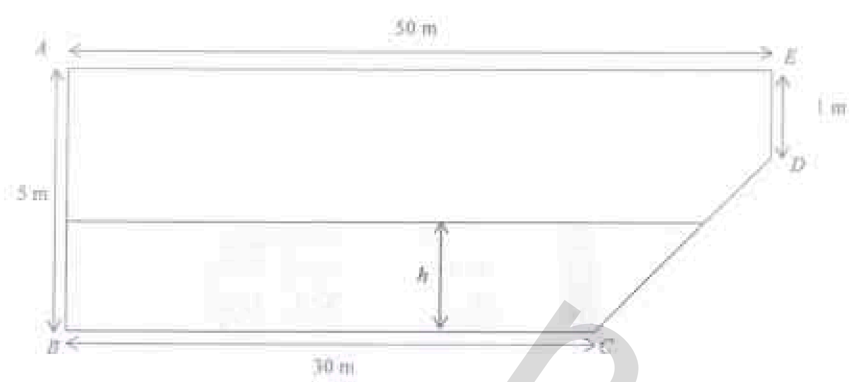
(a) Find the number of students in the group who started the rumour. [1]

(b) How long will it take for the rumour to spread to 300 students? [3]

(c) Find the rate at which the rumour is spreading after 3 hours. [2]

(d) Explain whether the entire school population will hear about the rumour based on the equation modelled by the students. [2]

4



$ABCDE$ is the cross sectional area of a swimming pool with a width of 15 m. AB, BC, DE and AE are 5 m, 30 m, 1 m and 50 m respectively.

(i) Show that the volume of water V, when the swimming pool is filled with water to a depth of h m, is given by $V=\dfrac{900h+75h^2}{2}$. [3]

(ii) Find the rate of change of the depth of water in the swimming pool when $h=3.5$ m, given that the swimming pool is filled with water at a rate of $0.3\ \text{m}^3/\text{min}$. [3]

5 (a) Solve the equation $2(4^x)+3(9^x)=5(6^x)$. [5]

(b) Solve the equation $\log_9(4x^2+3x+5)-\log_3(x+1)=\dfrac{1}{2}$. [5]

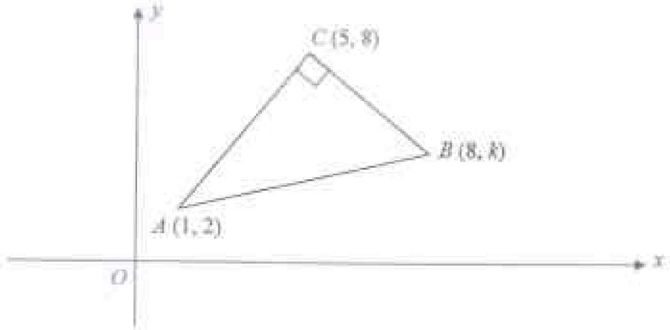
6 Jane researched online for the average monthly temperature at Paradise Island and found that the coldest month on the Island is in January with a temperature of −7°C and the hottest month is in July with a temperature of 45°C. She noticed that the average monthly temperature, T, can be modelled by the equation $T=A\cos Bx+C$, where A, B and C are constants and x is the number of months after January.

(i) Based on the above scenario, show that $T=-26\cos\dfrac{\pi}{6}x+19$. [3]

(ii) Sketch the graph of $T=-26\cos\dfrac{\pi}{6}x+19$ for $0\le x\le 12$. [3]

(iii) Jane would like to visit Paradise Island only when the average monthly temperature is above 25°C. By showing your workings clearly, suggest the months in which Jane should visit the island. [4]

7 The figure shows a right-angled triangle ABC, where the coordinates of A, B and C are $(1, 2)$, $(8, k)$ and $(5, 8)$ respectively and $\angle ACB=90°$.



(i) Find the value of k. [2]

D is the point of intersection of the perpendicular bisector of AC with the y-axis.

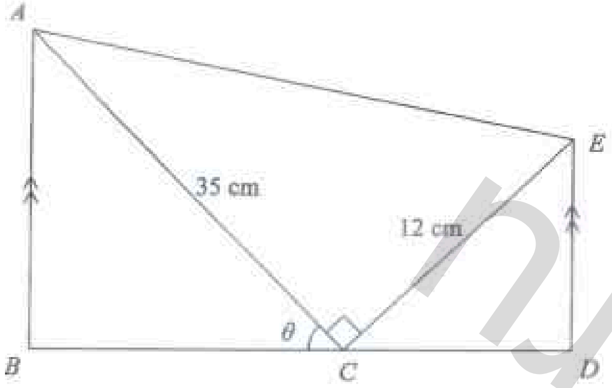
(ii) Find the coordinates of D. [4]

(iii) Determine whether the quadrilateral $ABCD$ is a trapezium. Justify your answer. [2]

$E(-5,-7)$ is a point on CA produced.

(iv) Find the ratio of the area of ΔABC to the area of ΔABE. [2]

8 $ABCDE$ is a trapezium with AB parallel to DE. It is given that $AC = 35$ cm, $CE = 12$ cm, $\angle ACE = 90°$ and $\angle ACB = \theta°$, where θ is an acute angle measured in degrees.



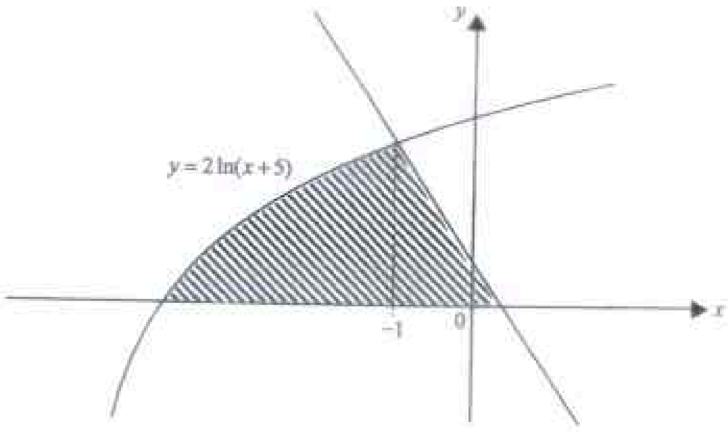
(i) Show that the perimeter, P, of $ABCDE$ is given by $P = 37 + 47\cos\theta + 47\sin\theta$. [3]

(ii) Express P in the form $37 + R\sin(\theta + \alpha)$, where $R > 0$ and α is an acute angle. [3]

(iii) Determine the maximum value of P and the corresponding value of θ. [3]

(iv) Justify with working, if it is possible for the perimeter of $ABCDE$ = 70 cm. [3]

9

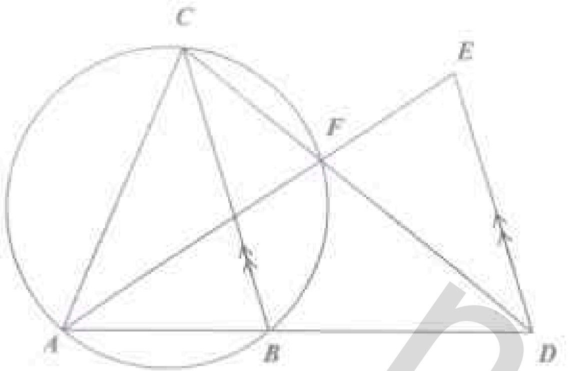


The diagram above shows the curve with equation $y = 2\ln(x+5)$ and the normal to the curve at $x = -1$.

(i) Show that the equation of the normal to the curve $y = 2\ln(x+5)$ at $x = -1$ is $y = -2x + 4\ln 2 - 2$. [3]

(ii) Find the area of the shaded region bounded by the curve, the normal to the curve at $x = -1$ and the x axis, leaving your answer to 2 decimal places. [6]

10



A, B and C are three points on the circumference of a circle. The line AE bisects $\angle BAC$ and intersects the circle ABC at F. D is the point of intersection of AB produced and CF produced. E is a point on AF produced such that DE is parallel to BC.

(i) Prove that DE is a tangent to the circle passing through A, F and D. [3]

(ii) Prove that ΔEDF is similar to ΔEAD. [2]

(iii) Prove that ΔDEF is similar to ΔACF. [2]

(iv) Using your result from (b) and (c), prove that $DE^2 = EF^2 + DF \times CF$. [3]

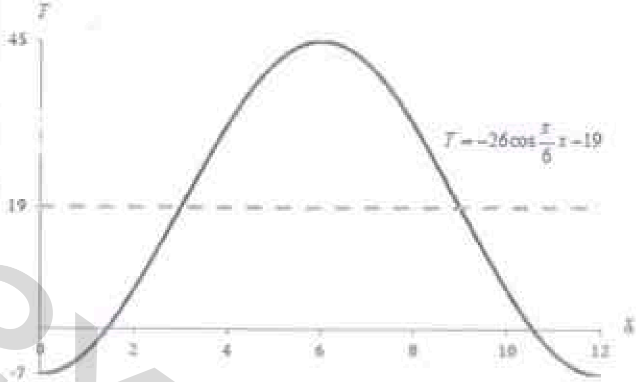
11 (i) Find the equations of the two lines that are tangents to the circle $(x-2)^2 + y^2 = 5$ and pass through the point $(-3,\ 0)$. [6]

(ii) Find the coordinates of the intersection of the circle with the tangent lines. [3]

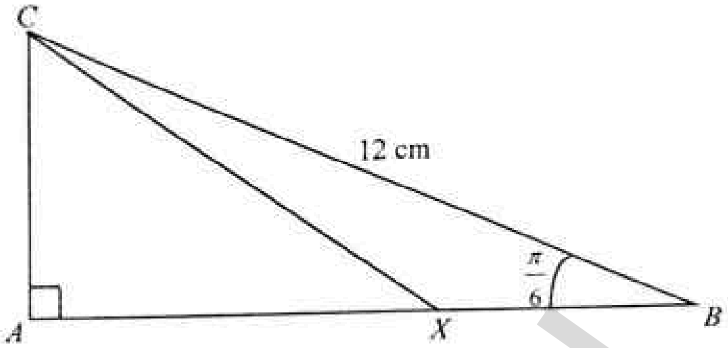
(iii) State the number of intersections between the line $y = \frac{1}{4}(x+3)$ and the circle $(x-2)^2 + y^2 = 5$. Justify your answer without finding the intersections. [2]

END OF PAPER

Answer Key

Question	Answer
1 (a)	Quotient: $2x-3$ Remainder: $-5x+1$
(bi)	$a=5,\ b=-1$
(bii)	$x = 1\frac{1}{2},\ \ 2,\ \ -\frac{1}{2}$
2 (i)	$\therefore \alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$
(ii)	$x^2 - 6x - 27 = 0$
3 (a)	5 students
(b)	2.50 hours
(c)	158
4 (ii)	0.000421m/min
5 (a)	$x=-1$ or 0
(b)	$x=1$ or 2
6 (b)	
(c)	May, June, July, August or September
7 (i)	$k=6$
(ii)	$D(0,7)$
(iii)	$ABCD$ is not a trapezium as it does not have one pair of parallel sides.
(iv)	2 : 3
8 (ii)	$P = 37 + 47\sqrt{2}\sin(\theta + 45°)$
(iii)	Maximum Value = 103 cm; Corresponding value of $\theta = 45°$
(iv)	Not possible
9 (ii)	7.01
11 (i)	$y = \frac{1}{2}x + \frac{3}{2}$ or $y = -\frac{1}{2}x - \frac{3}{2}$
(ii)	(1, 2) or (1, −2)
(iii)	2 intersections

1



In the diagram, the right-angle triangle ABC is such that $BC = 12$ cm, $\angle ABC = \dfrac{\pi}{6}$ and $AX = \dfrac{2}{3}AB$.

Show that $\cos\angle BXC = -\dfrac{2\sqrt{7}}{7}$. [4]

2 Solve the equation $6\cos x = 4\sec x - \tan x$ for $0 < x < 5$. [5]

3 Air leaks from a spherical balloon at a constant rate of 25π cm^3 per second. Given that the initial volume is 5000π cm^3,

(i) calculate the radius of the balloon after 20 seconds, [3]

(ii) find the rate of change of radius at this instant. [2]

4 A curve is such that $\dfrac{d^2y}{dx^2} = 6x - 6$. The gradient of the curve at the point $(2, -1)$ is 4.

(i) Show that y is an increasing function for all real values of x. [4]

(ii) Find the equation of the curve. [2]

[Turn over…

5 Given the cubic expression $f(x) = x^3 + px^2 + qx + 4$ has a factor $(x+2)$ and leaves a remainder of 6 when divided by $(x+1)$,

(i) find the value of p and of q, [4]

(ii) factorize $f(x)$ completely. [2]

6 **(a)** Simplify the expression $\dfrac{3^{n-2} - 3^{n+1}}{3^{n+2} - 3^{n-1}}$. [3]

(b) Solve the equation $\log_2 8x = 4\log_x 2$. [4]

7 Given that the roots of the equation $2x^2 - 2x + 5 = 0$ are α and β.

(i) Show that $\alpha^2 + \beta^2 = -4$. [2]

(ii) Find the value of $\alpha^3 + \beta^3$. [2]

(iii) Find a quadratic equation whose roots are $\dfrac{\alpha}{2\beta^2}$ and $\dfrac{\beta}{2\alpha^2}$. [4]

8 The equation of the curve is given by $y = 3\cos 3x - 2$ for $0 \le x \le \pi$.

(i) Write down the amplitude and period of y. [2]

(ii) Find the coordinates of the maximum and minimum points for $0 < x < \pi$. [2]

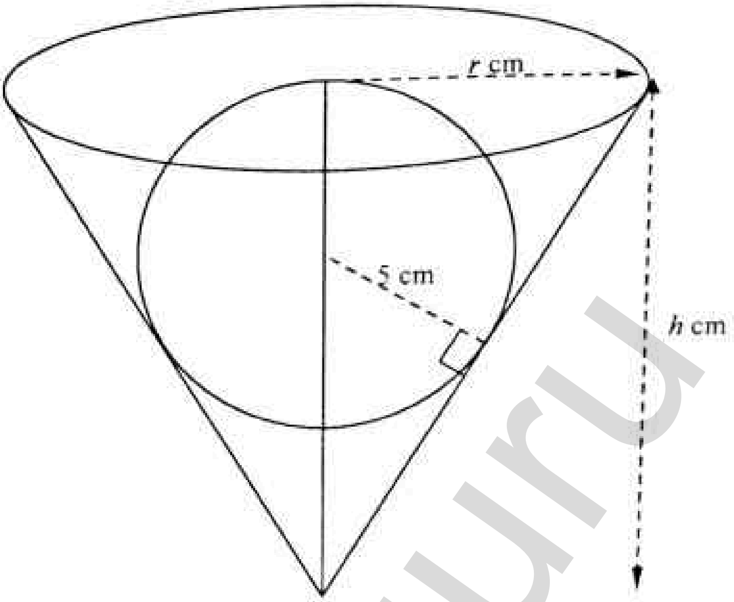
(iii) Calculate the values of x for which the curve cuts the x-axis. [2]

(iv) Sketch the curve $y = 3\cos 3x - 2$ for $0 \le x \le \pi$. [2]

(v) State the range of values of x for which y is decreasing between 0 and π. [2]

[Turn over…

9 A solid spherical ball is dropped into a cone of height h cm and radius r cm.



Given that the radius of the spherical ball is 5 cm,

(i) show that the volume of the cone, V is given by $V = \dfrac{25\pi h^2}{3(h-10)}$. [3]

(ii) Given that h can vary, find the value of h for which V has a stationary value. [3]

(iii) Calculate this stationary value of V and determine if the volume is a maximum or minimum value. [3]

10 **(i)** Express $\dfrac{4x^3+7x^2+4x-2}{(2x-1)(x^2+2)}$ in partial fractions. [5]

(ii) Differentiate $\ln(x^2+2)$ with respect to x. [1]

(iii) Hence evaluate $\displaystyle\int_1^2 \frac{4x^3+7x^2+4x-2}{(2x-1)(x^2+2)}\,dx$. [4]

[Turn over...

11 The table show experimental values of two variables x and y.

x	2	3	4	6	10
y	3.24	5.79	9	17.05	38.43

It is known that x and y are related by the equation $\frac{y-b}{x} = a\sqrt{x} - 1$ for $x > 0$ where a and b are constants.

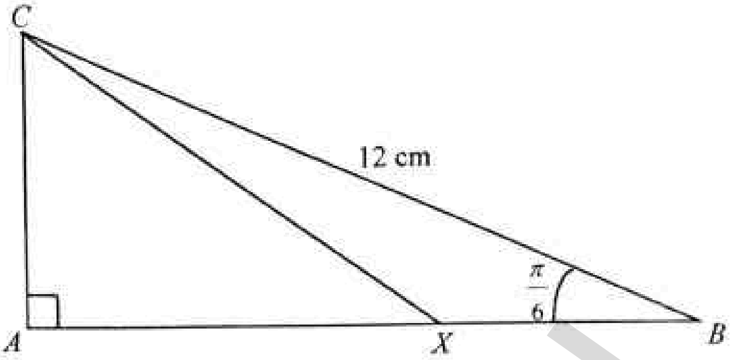
(i) Using a scale of 1 cm to 2 units on the horizontal axis and 2 cm to 5 units on the vertical axis, draw a straight line graph of $x + y$ against $x\sqrt{x}$. [3]

(ii) Use your graph to estimate, to 2 decimal places, the value of a and of b . [4]

(iii) On the same diagram, draw a straight line representing the equation $y + x + 2x\sqrt{x} = 36$.
Hence find the value of x that satisfies the equation $(a+2)x\sqrt{x} = 36 - b$. [3]

~ End of Paper ~

1



In the diagram, the right-angle triangle ABC is such that $BC = 12$ cm, $\angle ABC = \frac{\pi}{6}$ and $AX = \frac{2}{3}AB$.

Show that $\cos\angle BXC = -\frac{2\sqrt{7}}{7}$. [4]

[soln] $\cos\angle BXC = -\cos\angle AXC$

$$\sin\frac{\pi}{6} = \frac{AC}{12} \quad \Rightarrow AC = 6$$
$$AB = \sqrt{144-36} = \sqrt{108} = 6\sqrt{3}$$
$$AX = 4\sqrt{3}$$
$$CX = \sqrt{36+48} = \sqrt{84} = 2\sqrt{21}$$
$$\cos\angle BXC = -\cos\angle AXC = -\frac{4\sqrt{3}}{2\sqrt{21}} = -\frac{2}{\sqrt{7}} = -\frac{2\sqrt{7}}{7}$$

2 Solve the equation $6\cos x = 4\sec x - \tan x$ for $0 < x < 5$. [5]

[soln]
$$6\cos x = \frac{4}{\cos x} - \tan x$$
$$6\cos^2 x = 4 - \sin x$$
$$6\left(1-\sin^2 x\right) = 4 - \sin x$$
$$6\sin^2 x - \sin x - 2 = 0$$
$$(3\sin x - 2)(2\sin x + 1) = 0$$
$$\sin x = \frac{2}{3} \qquad \text{or} \qquad \sin x = -\frac{1}{2}$$

Basic angle = 0.7297 Basic angle = 0.5236

$x = 0.730, 2.41$ $x = 2.62, 5.76$ (NA)

3 Air leaks from a spherical balloon at a constant rate of 25π cm^3 per second. Given that the initial volume is 5000π cm^3,

(i) calculate the radius of the balloon after 20 seconds, [3]

(ii) find the rate of change of radius at this instant. [2]

[soln] $\frac{dV}{dt}-25\pi$

After 20s, volume = $5000\pi - 25\pi \times 20 = 4500\pi$

$$\frac{4}{3}\pi r^3 = 4500\pi$$

$$r^3 = 3375$$

$$r = 15$$

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$-25\pi = 4\pi r^2 \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = -\frac{25}{4 \times 225} = -\frac{25}{900} = -\frac{1}{36} \text{ cm/s}$$

4 A curve is such that $\frac{d^2y}{dx^2} = 6x - 6$. The gradient of the curve at the point $(2, -1)$ is 4.

(i) Show that y is an increasing function for all real values of x. [4]

(ii) Find the equation of the curve. [2]

[soln]

$$\frac{d^2y}{dx^2} = 6x - 6$$

$$\frac{dy}{dx} = 3x^2 - 6x + c$$

At $(2, -1)$, $\frac{dy}{dx} = 4$

$$12 - 12 + c = 4$$

$$c = 4$$

$$\frac{dy}{dx} = 3x^2 - 6x + 4$$

$$\frac{dy}{dx} = 3\left(x^2 - 2x\right) + 4$$

$$\frac{dy}{dx} = 3(x-1)^2 + 1$$

For all values of x, $\frac{dy}{dx} > 0$, y is increasing.

$$y = x^3 - 3x^2 + 4x + d$$

$$8 - 12 + 8 + d = -1$$
$$d = -5$$
$$y = x^3 - 3x^2 + 4x - 5$$

5 Given the cubic expression $f(x) = x^3 + px^2 + qx + 4$ has a factor $(x+2)$ and leaves a remainder of 6 when divided by $(x+1)$,

(i) find the value of p and of q, [4]

(ii) factorize $f(x)$ completely. [2]

[soln]

$$-8 + 4p - 2q + 4 = 0$$
$$2p - q = 2$$

$$-1 + p - q + 4 = 6$$
$$p - q = 3$$
$$p = -1, q = -4$$

$$f(x) = x^3 - x^2 - 4x + 4$$

$$f(x) = (x+2)(x^2 - 3x + 2)$$

$$f(x) = (x+2)(x-2)(x-1)$$

6 **(a)** Simplify the expression $\frac{3^{n-2} - 3^{n+1}}{3^{n+2} - 3^{n-1}}$. [3]

(b) Solve the equation $\log_2 8x = 4\log_x 2$. [4]

[soln]

(a) $$\frac{3^{n-2} - 3^{n+1}}{3^{n+2} - 3^{n-1}} = \frac{3^n\left(\frac{1}{9} - 3\right)}{3^n\left(9 - \frac{1}{3}\right)} = -\frac{1}{3}$$

(b) $$\log_2 8x = 4\log_x 2$$
$$\log_2 8 + \log_2 x = \frac{4\log_2 2}{\log_2 x}$$

$$3+\log_2 x=\frac{4}{\log_2 x}$$

Let $y=\log_2 x$

$$y^2+3y-4=0$$
$$(y+4)(y-1)=0$$

$$\log_2 x=-4 \text{ or } \log_2 x=1$$
$$x=\frac{1}{16} \text{ or } x=2$$

7 Given that the roots of the equation $2x^2-2x+5=0$ are α and β.

(i) Show that $\alpha^2+\beta^2=-4$. [2]

(ii) Find the value of $\alpha^3+\beta^3$. [2]

(iii) Find a quadratic equation whose roots are $\frac{\alpha}{2\beta^2}$ and $\frac{\beta}{2\alpha^2}$. [4]

[soln] $\alpha+\beta=1$ and $\alpha\beta=\frac{5}{2}$

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=1-2\times\frac{5}{2}=-4$$

$$(\alpha+\beta)^3=\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3$$

$$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$$

$$\alpha^3+\beta^3=1-3\times\frac{5}{2}=-\frac{13}{2}$$

$$\frac{\alpha}{2\beta^2}+\frac{\beta}{2\alpha^2}=\frac{\alpha^3+\beta^3}{2(\alpha\beta)^2}=\left(-\frac{13}{2}\right)\div\frac{25}{2}=-\frac{13}{25}$$

$$\frac{\alpha}{2\beta^2}\times\frac{\beta}{2\alpha^2}=\frac{1}{4\alpha\beta}=\frac{1}{10}$$

Quadratic equation is $x^2+\frac{13}{25}x+\frac{1}{10}=0$ or $50x^2+26x+5=0$

8 The equation of the curve is given by $y = 3\cos 3x - 2$ for $0 \le x \le \pi$.

(i) Write down the amplitude and period of y. [2]

(ii) Find the coordinates of the maximum and minimum points for $0 < x < \pi$. [2]

(iii) Calculate the values of x for which the curve cuts the x-axis. [2]

(iv) Sketch the curve $y = 3\cos 3x - 2$ for $0 \le x \le \pi$. [2]

(v) State the range of values of x for which y is decreasing between 0 and π. [2]

[soln] amplitude = 3, period = $\frac{2\pi}{3}$

Minimum point is $\left(\frac{\pi}{3}, -5\right)$ and Maximum point is $\left(\frac{2\pi}{3}, 1\right)$

$\cos 3x = \frac{2}{3}$

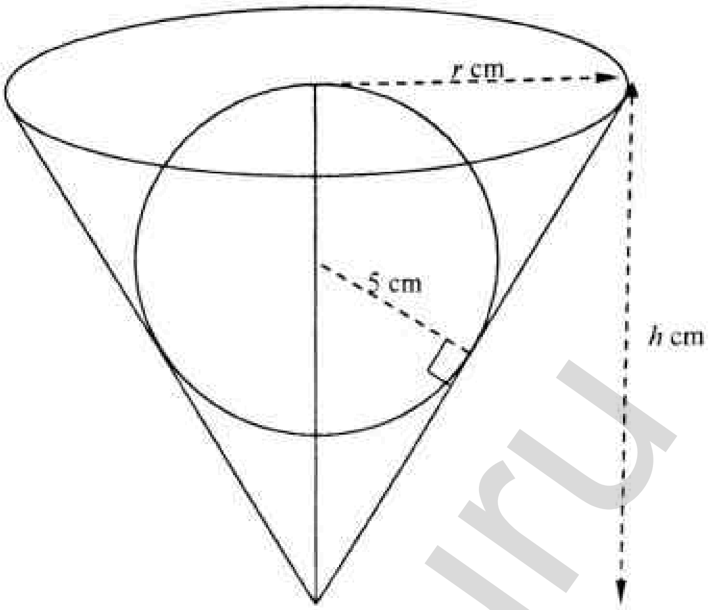
Basic angle = 0.841

$3x = 0.841, 5.4421, 7.124$

$x = 0.280, 1.81, 2.37$

y is decreasing for $0 < x < \frac{\pi}{3}$ and $\frac{2\pi}{3} < x < \pi$

9 A solid spherical ball is dropped into a cone of height h cm and radius r cm.



Given that the radius of the spherical ball is 5 cm,

(i) show that the volume of the cone, V is given by $V=\dfrac{25\pi h^2}{3(h-10)}$. [3]

(ii) Given that h can vary, find the value of h for which V has a stationary value. [3]

(iii) Calculate this stationary value of V and determine if the volume is a maximum or minimum value. [3]

[soln]

$$\frac{r}{\sqrt{h^2+r^2}}=\frac{5}{h-5}$$

$$\frac{r^2}{h^2+r^2}=\frac{25}{h^2-10h+25}$$

$$r^2h^2-10r^2h+25r^2=25h^2+25r^2$$

$$r^2=\frac{25h^2}{h^2-10h}=\frac{25h}{h-10}$$

$$V=\frac{1}{3}\pi h\times\frac{25h}{h-10}=\frac{25\pi h^2}{3(h-10)}$$

$$\frac{dV}{dh}=\frac{25\pi}{3}\left[\frac{(h-10)\times 2h-h^2}{(h-10)^2}\right]$$

$$\frac{dV}{dh}=\frac{25\pi}{3}\left[\frac{h^2-20h}{(h-10)^2}\right]$$

For stationary value,

$$\frac{dV}{dh}=0 \quad \Rightarrow h=20$$

$$V=\frac{25\pi\times400}{3\times10}=\frac{1000\pi}{3}=1047.20 \text{ (minimum volume)}$$

x	<20	20	>20
$\frac{dV}{dh}$	negative	0	positive

10 **(i)** Express $\dfrac{4x^3+7x^2+4x-2}{(2x-1)(x^2+2)}$ in partial fractions. [5]

(ii) Differentiate $\ln(x^2+2)$ with respect to x. [1]

(iii) Hence evaluate $\int_1^2 \dfrac{4x^3+7x^2+4x-2}{(2x-1)(x^2+2)}\,dx$. [4]

[soln]

$$\frac{4x^3+7x^2+4x-2}{(2x-1)(x^2+2)}=2+\frac{9x^2-4x+2}{(2x-1)(x^2+2)}$$

$$\frac{9x^2-4x+2}{(2x-1)(x^2+2)}=\frac{A}{2x-1}+\frac{Bx+C}{x^2+2}$$

$$9x^2-4x+2=A(x^2+2)+(Bx+C)(2x-1)$$

Subst $x=\frac{1}{2}$, $\quad \frac{9}{4}A=\frac{9}{4} \quad A=1$

Coefficient of x^2: $\quad B=4$

Constant term: $\quad C=0$

$$\frac{9x^2-4x+2}{(2x-1)(x^2+2)}=\frac{1}{2x-1}+\frac{4x}{x^2+2}$$

$$\frac{d}{dx}\ln(x^2+2)=\frac{2x}{x^2+2}$$

$$\int_1^2 \frac{4x^3+7x^2+4x-2}{(2x-1)(x^2+2)}dx = \int_1^2 \quad 2+\frac{1}{2x-1}+\frac{4x}{x^2+2}dx$$

$$= \left[2x+\frac{1}{2}\ln(2x-1)+2\ln(x^2+2)\right]_1^2 = \left[4+\frac{1}{2}\ln 3+2\ln 6\right]-\left[2+\frac{1}{2}\ln 1+2\ln 3\right]$$

$$= 2-\frac{3}{2}\ln 3+2\ln 6$$

$$= 3.94$$

11 The table show experimental values of two variables x and y.

x	2	3	4	6	10
y	3.24	5.79	9	17.05	38.43

It is known that x and y are related by the equation $\frac{y-b}{x}=a\sqrt{x}-1$ for $x>0$ where a and b are constants.

(i) Using a scale of 1 cm to 2 units on the horizontal axis and 2 cm to 5 units on the vertical axis, draw a straight line graph of $x+y$ against $x\sqrt{x}$. [3]

(ii) Use your graph to estimate, to 2 decimal places, the value of a and of b . [4]

(iii) On the same diagram, draw a straight line representing the equation $y+x+2x\sqrt{x}=36$.
Hence find the value of x that satisfies the equation $(a+2)x\sqrt{x}=36-b$. [3]

[soln]

$$\frac{y-b}{x}=a\sqrt{x}-1$$
$$y-b=ax\sqrt{x}-x$$
$$x+y=ax\sqrt{x}+b$$

$x\sqrt{x}$	2.83	5.20	8	14.70	31.62
$x+y$	5.24	8.79	13	23.05	48.43

$a=1.5$ and $b=0.994$

$$a\,x\sqrt{x}+2x\sqrt{x}=36-b$$
$$a\,x\sqrt{x}+b=-2x\sqrt{x}+36 \qquad \text{(gradient = −2, intercept = 36)}$$

~ *End of Paper* ~

1. **(a)** **(i)** Sketch the graph of the curve $y^2 = kx$, where k is a positive constant. [1]

(ii) Given that the line $y = 2x + 1$ meets the curve $y^2 = kx$, find the range of values of k. [4]

(b) Determine the conditions for p and q such that the curve $y = px^2 - 2x + 3q$ lies entirely above the x-axis, where p and q are constants. [3]

2. **(i)** Sketch the curve $y = 2\ln(x-3)$ for $x > 3$. [2]

(ii) The tangent to the curve $y = 2\ln(x-3)$ at the point P where $x = 5$ intersects the x-axis at A and the normal to the curve at P intersects the x-axis at B. Calculate the area of ΔAPB. [5]

3. **(a)** Write down and simplify the first three terms in the expansion of $(2-3x)^6$, in ascending powers of x. [2]

(b) Hence

(i) using a suitable value of x, find the estimated value of $(1.997)^6$, correct to 3 decimal places. [2]

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4. A curve has the equation $y = f(x)$, where $f(x) = \dfrac{2+\cos x}{\sin x}$ for $-\pi \le x \le \pi$.

(i) Obtain an expression for $f'(x)$. [2]

(ii) Find the **exact** value of the x-coordinates of the stationary points of the curve, and determine the nature of each stationary point. [6]

5. **(a)** **(i)** Show that $\dfrac{\cot x - \tan x}{\cot x + \tan x} = \cos 2x$. [3]

(ii) Hence solve the equation $\dfrac{\cot x - \tan x}{\cot x + \tan x} = \cos x$ for $0^\circ < x < 360^\circ$. [3]

(b) Without using a calculator, express $\sin 15^\circ$ in the form $\dfrac{1}{k}(\sqrt{a} - \sqrt{b})$, where a, b and k are integers. [3]

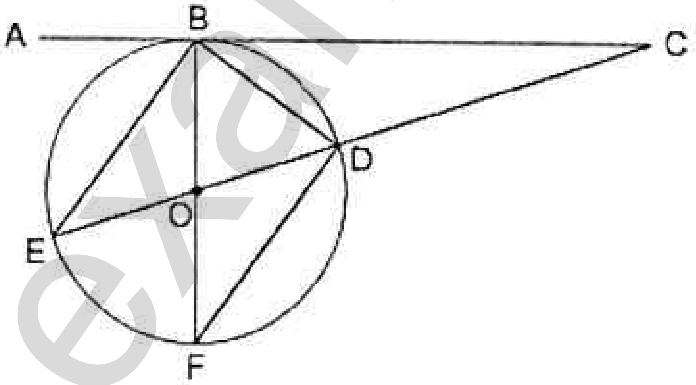
6. **(i)** Sketch the graph of $y = 1 - |x - 3|$. [3]

A line $y = mx + 1$ is drawn on the same axes with the graph $y = 1 - |x - 3|$.

(ii) In the case where $m = 2$, find the coordinates of the point of intersection of the line and the graph of $y = 1 - |x - 3|$. [2]

(iii) Determine the set of values of m for which the line does not intersect the graph of $y = 1 - |x - 3|$. [2]

7.



In the diagram, BF and DE are the diameters of the circle with centre O.

The tangent at B meets ED produced at C. Prove that

(i) $BE = DF$ [3]

(ii) $DF \times BC = BD \times CE$ [3]

(iii) $\angle BCE + 2\angle CBD = 90^\circ$. [2]

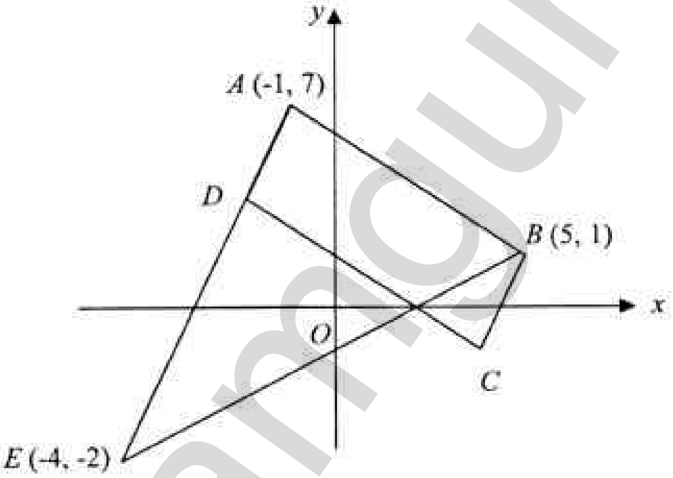
8. The equation of a circle C_1 is $x^2 + y^2 - 4x - 8y + 4 = 0$.

(a) Find the coordinates of the centre and the radius of the circle. [3]

(b) The highest point on the circle is A.
State the coordinates of A. [1]

(c) Another circle, C_2 touches C_1 at the point A. Given that both circles do not overlap and the area of C_2 is four times that of the area of C_1, find the equation of C_2 in the form of $x^2 + y^2 + 2gx + 2fy + c = 0$, stating the value of f, g and c. [4]

9. **Solutions to this question by accurate drawing will not be accepted.**



The diagram, not to scale, shows a parallelogram, $ABCD$. ADE and BE are straight lines. D divides AE such that $AD : DE$ is in the ratio 1 : 2.

A, B and E have coordinates $(-1, 7)$, $(5, 1)$ and $(-4, -2)$ respectively.

(a) **(i)** Find the equation of the perpendicular bisector of AB and show that it passes through E. [3]

(ii) Hence deduce the geometrical property of triangle ABE. [1]

(b) Find the coordinates of D. [2]

(c) Find the area of the parallelogram $ABCD$. [2]

10. A particle starts from rest at 5 m from a fixed point O and moves in a straight line with a velocity, $v = 12t - 3t^2$ m/s where t is the time in seconds after leaving from the initial rest position.

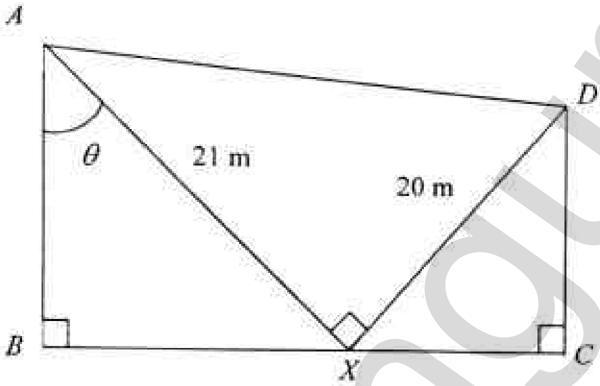
(i) Calculate the acceleration when the particle is instantaneously at rest. [3]

(ii) Calculate the maximum velocity. [2]

(iii) Express the displacement, s, from point O in terms of t. [1]

(iv) Find the average speed of the particle during the first five seconds. [3]

11.



The diagram shows a trapezium field $ABCD$. The point X lies on the side BC such that $AX = 21$ m, $DX = 20$ m, $\angle AXD = \angle ABX = \angle DCX = 90°$ and $\angle BAX = \theta$.

(i) Show that the length of fencing required for the perimeter of the field, L m, can be expressed in the form of $p + q\sin\theta + r\cos\theta$, where p, q and r are constants to be determined. [3]

(ii) Express L in the form $p + R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [2]

(iii) State the maximum value of L and the corresponding value of θ. [2]

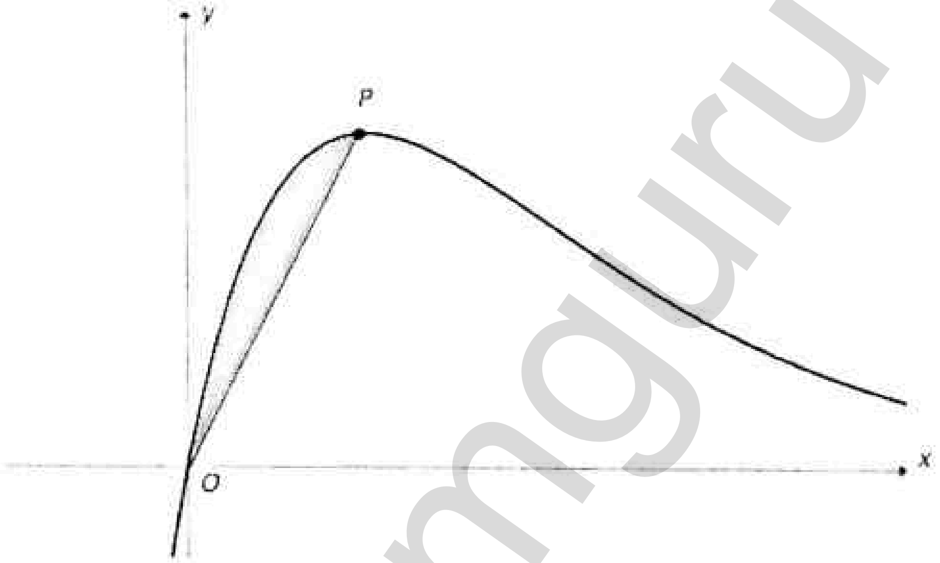
(iv) Given that the fencing used is 80 m, find the value(s) of θ. [3]

12. **(a)** **(i)** Given that $y = xe^{-2x}, x > 0$, show that $\frac{dy}{dx} = (1-2x)e^{-2x}$. [1]

(ii) Hence, find $\int xe^{-2x} dx$. [3]

(b) The diagram, which is not drawn to scale, shows part of the curve $y = xe^{-2x}$

A line drawn from the origin meets the curve at the maximum point P.



(i) Find the coordinates of P. [3]

(ii) Calculate the area of the region bounded by the curve and the line OP. [4]

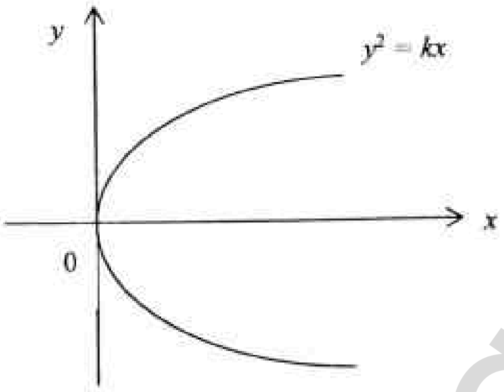
--- END OF PAPER ---

1. **(a)** **(i)** Sketch the graph of the curve $y^2 = kx$, where k is a positive constant. [1]

(ii) Given that the line $y = 2x + 1$ meets the curve $y^2 = kx$, find the range of values of k. [4]

(b) Determine the conditions for p and q such that the curve $y = px^2 - 2x + 3q$ lies entirely above the x-axis, where p and q are constants. [3]

(a)(i)



[D1]

(a)(ii) $y = 2x + 1$ .... (1)

$y^2 = kx$ .... (2)

(1) in (2) : $(2x+1)^2 = kx$

$4x^2 + (4-k)x + 1 = 0$ **[A1]**

For line meets the curve, $D \geq 0$.

$(4-k)^2 - 4(4)(1) \geq 0$ **[M1]**

$16 - 8k + k^2 - 16 \geq 0$

$k(k-8) \geq 0$ **[M1A1]**

$\therefore k \leq 0 (NA) \quad or \quad k \geq 8$

(b) Curve lies entirely above line, $D < 0$ and $p > 0$.

$(-2)^2 - 4p(3q) < 0$

$4 - 12pq < 0$ **[M1]**

$pq > \frac{1}{3}$

$\therefore p > 0 \quad and \quad pq > \frac{1}{3}$ **[A2]**

2. **(i)** Sketch the curve $y = 2\ln(x-3)$ for $x > 3$. [2]

(ii) The tangent to the curve $y = 2\ln(x-3)$ at the point P where $x = 5$ intersects the x-axis at A and the normal to the curve at P intersects the x-axis at B.

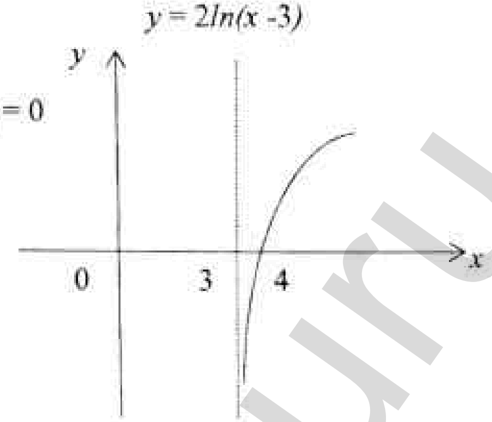
Calculate the area of ΔAPB. [5]

(i) Asymptote : $x = 3$

x-intercept : $2\ln(x-3) = 0$

$x - 3 = 1$

$x = 4$



[D1 : shape]

[D1 : Asymptote, x-int]

(ii) $\frac{dy}{dx} = \frac{2}{x-3}$ **[A1]**

When $x = 5$, gradient of tangent at P = 1

When $x = 5$, $y = 2\ln 2$

$P(1, 2\ln 2)$

Equation of tangent at P : $y - 2\ln 2 = x - 5$

$\therefore y = x - 5 + 2\ln 2$

At x-axis , y = 0 : $x = 5 - 2\ln 2$

$\therefore A(5 - 2\ln 2,\ 0)$ **[A1]**

Gradient of normal at P = -1 **[M1]**

Equation of normal at P : $y - 2\ln 2 = -1(x-5)$

$\therefore y = -x + 5 + 2\ln 2$

At x-axis, $y = 0$: $x = 5 + 2\ln 2$

$\therefore B(5 + 2\ln 2,\ 0)$ **[A1]**

$\therefore Area\ \ of\ \ \Delta APB = \frac{1}{2}(5 + 2\ln 2 - 5 + 2\ln 2)(2\ln 2)$

$= 1.92$ units^2 **[A1]**

3. **(a)** Write down and simplify the first three terms in the expansion of $(2-3x)^6$, in ascending powers of x. [2]

(b) Hence

(i) using a suitable value of x, find the estimated value of $(1.997)^6$, correct to 3 decimal places. [2]

(ii) determine the coefficient of x^2 in the expansion of $(2-3x)^7-(2-3x)^6$. [3]

(a) $(2-3x)^6 = 2^6 + \binom{6}{1}2^5(-3x) + \binom{6}{2}2^4(-3x)^2 + ...$

$= 64-576x+2160x^2 - ....$ (up to 1st 3 terms)**[M1A1]**

(b)(i) Put $2-3x = 1.997$

$x = 0.001$ **[M1]**

$(1.997)^6 = 64-576(0.001)+2160(0.001)^2 + ...$

$= 63.42616 = 63.426$ (correct to 3dp) **[A1]**

(b)(ii) $(2-3x)^7-(2-3x)^6 = (2-3x)^6[2-3x-1]$

$= (1-3x)(2-3x)^6$ **[M1]**

$= (1-3x)(64-576x+2160x^2 - ....)$

Coefficient of x^2 = 1(2160) -3(-576) = 3888 **[M1A1]**

4. A curve has the equation $y = f(x)$, where $f(x) = \dfrac{2+\cos x}{\sin x}$ for $-\pi \le x \le \pi$.

(i) Obtain an expression for $f'(x)$. [2]

(ii) Find the **exact** value of the x-coordinates of the stationary points of the curve, and determine the nature of each stationary point. [6]

(i) $f'(x) = \dfrac{\sin x(-\sin x)-(2+\cos x)(\cos x)}{\sin^2 x}$ **[M1]**

$= \dfrac{-\sin^2 x - 2\cos x - \cos^2 x}{\sin^2 x}$

$= \dfrac{-1-2\cos x}{\sin^2 x}$ **[A1]**

(ii) For stationary points, $f'(x) = 0$.

$$\frac{-1-2\cos x}{\sin^2 x}=0$$

$$-1-2\cos x=0$$ **[M1]**

$$\cos x=-\frac{1}{2}$$

$$x=\frac{2\pi}{3}\ \ or\ \ \pi+\frac{2\pi}{3}-2\pi$$

$$\therefore x=\frac{2\pi}{3}\ \ or\ \ -\frac{2\pi}{3}$$ **[A2]**

x	-2.1	$-\frac{2\pi}{3}$	-2	2	$\frac{2\pi}{3}$	2.1
$f'(x)$	+ve	0	-ve	-ve	0	+ve
Tangent	╱	—	╲	╲	—	╱

[M1]

$\therefore x=-\frac{2\pi}{3}$ is a maximum point and $x=\frac{2\pi}{3}$ is a minimum point. **[A2]**

Alternate Mtd :

$$f''(x)=\frac{\sin^2 x(2\sin x)-(-1-2\cos x)(2\sin x\cos x)}{\sin^4 x}$$

$$=\frac{2(\sin^2 x+\cos x+2\cos^2 x)}{\sin^3 x}$$

$$f''\left(-\frac{2\pi}{3}\right)=-2.31<0\Rightarrow \max\ \ point$$

$$f''\left(\frac{2\pi}{3}\right)=2.31>0\Rightarrow \min\ \ point$$

5. **(a)** **(i)** Show that $\frac{\cot x - \tan x}{\cot x + \tan x} = \cos 2x$. [3]

(ii) Hence solve the equation $\frac{\cot x - \tan x}{\cot x + \tan x} = \cos x$ for $0° < x < 360°$. [3]

(b) Without using a calculator, express $\sin 15°$ in the form $\frac{1}{k}(\sqrt{a} - \sqrt{b})$, where a, b and k are integers. [3]

(a)(i) *LHS* : $\frac{\cot x - \tan x}{\cot x + \tan x} = \frac{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}}$ **[M1]**

$= \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$ **[A1]**

$= \cos 2x = RHS$ **[A1]**

(ii) $\frac{\cot x - \tan x}{\cot x + \tan x} = \cos x$

$\cos 2x = \cos x$

$2\cos^2 x - \cos x - 1 = 0$ **[M1]**

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2}$ *or* $\cos x = 1$

$\therefore x = 120°, 240°$ **[A2]**

(b) $\sin 15° = \sin(45° - 30°)$ Alt Mtd : $\sin 15° = \sin(60° - 45°)$

$= \sin 45° \cos 30° - \sin 30° \cos 45°$ **[M1]**

$= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)$ **[A1]**

$= \frac{\sqrt{6} - \sqrt{2}}{4}$ **[A1]**

6. **(i)** Sketch the graph of $y = 1 - |x-3|$. [3]

A line $y = mx + 1$ is drawn on the same axes with the graph $y = 1 - |x-3|$.

(ii) In the case where $m = 2$, find the coordinates of the point of intersection of the line and the graph of $y = 1 - |x-3|$. [2]

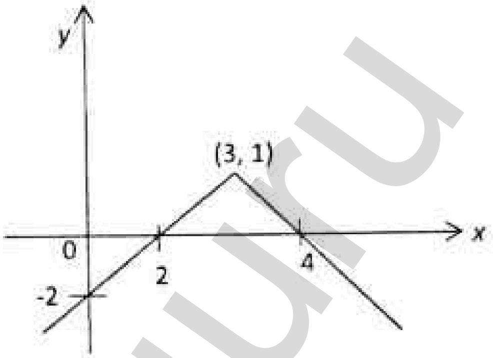
(iii) Determine the set of values of m for which the line does not intersect the graph of $y = 1 - |x-3|$. [2]

(i) y-int : Put $x = 0$: $y = -2$

x-int : $1 - |x-3| = 0$

$x = 4$ or $x = 2$

Max pt = (3, 1)



D1 : Correct shape

D1 : intercepts

D1 : max pt

(ii) $2x + 1 = 1 - |x-3|$

$|x-3| = -2x$

$x - 3 = -2x \quad or \quad x - 3 = 2x$ [M1]

$x = 1 \ (NA) \quad or \quad x = -3$

When $x = -3$, $y = -5$

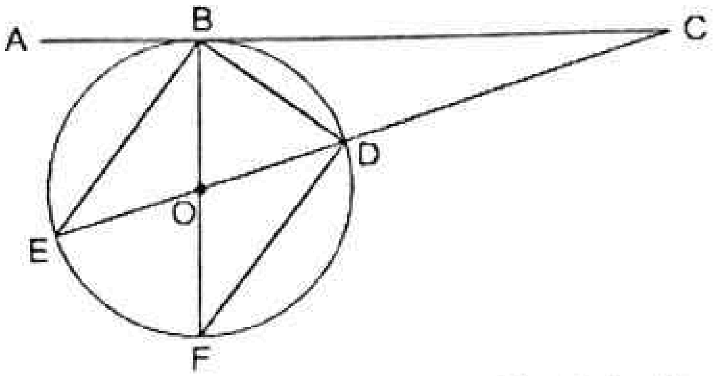
Pt of intersection is (-3, -5) **[A1]**

(iii) For line not to intersect graph of $y = 1 - |x-3|$, line must be parallel to the left arm.

Gradient of left arm $= \dfrac{1-(-2)}{3-0} = 1$

Set of values of m : $0 < m \leq 1$ **[B2]**

7.



In the diagram, BF and DE are the diameters of the circle with centre O.

The tangent at B meets ED produced at C. Prove that

(i) $BE = DF$ [3]

(ii) $DF \times BC = BD \times CE$ [3]

(iii) $\angle BCE + 2\angle CBD = 90^\circ$. [2]

(i) $\angle BED = \angle DFB$ (Angles in the same segment)
$\angle DBE = \angle BDF = 90^\circ$ (right angle in a semi-circle)
$DE = BF$ (diameter) **[M1]**
$\therefore \Delta BDE \equiv \Delta DBF$ (AAS) [M1]

$\therefore BE = DF$ **[A1]**

Alt Mtd : Show $\Delta BOE \equiv \Delta DOF$

(ii) $\angle DBC = \angle BEC$ (Alternate segment theorem)

$\angle DCB = \angle BCE$ (Common angle)

$\therefore \Delta BEC$ is similar to ΔDBC (AA Similarity Test) **[M1A1]**

$$\frac{BE}{DB} = \frac{EC}{BC}$$
$BE \times BC = EC \times DB$ **[M1]**
$\therefore DF \times BC = BD \times CE$

(iii) $\angle BCE + \angle BEC + 90^\circ + \angle CBD = 180^\circ$ **[M1]**

$\angle BCE + 2\angle CBD = 180^\circ - 90^\circ$
$\therefore \angle BCE + 2\angle CBD = 90^\circ$. **[A1]**

8. The equation of a circle C_1 is $x^2 + y^2 - 4x - 8y + 4 = 0$.

(a) Find the coordinates of the centre and the radius of the circle. [3]

(b) The highest point on the circle is A.
State the coordinates of A. [1]

(c) Another circle, C_2 touches C_1 at the point A. Given that both circles do not overlap and the area of C_2 is four times that of the area of C_1, find the equation of C_2 in the form of $x^2 + y^2 + 2gx + 2fy + c = 0$, stating the value of f, g and c. [4]

(a) $C_1 : x^2 + y^2 - 4x - 8y + 4 = 0$.

$$x^2 - 4x + \left(-\frac{4}{2}\right)^2 + y^2 - 8y + \left(-\frac{8}{2}\right)^2 = -4 + \left(-\frac{4}{2}\right)^2 + \left(-\frac{8}{2}\right)^2$$ **[M1]**

$(x-2)^2 + (y-4)^2 = 16$

Centre = (2, 4) and radius = 4 units **[A2]**

(b) x-coordinate of $A = 2$ (radius $\perp$ tangent)

$\therefore A = (2,\ 4+4) = (2,\ 8)$ **[A1]**

(c) Radius of $C_2 = 8$ **[B1]**

Centre of $C_2 = (2, 8+8) = (2, 16)$

Equation of C_2 : $(x-2)^2 + (y-16)^2 = 8^2$ **[M1]**

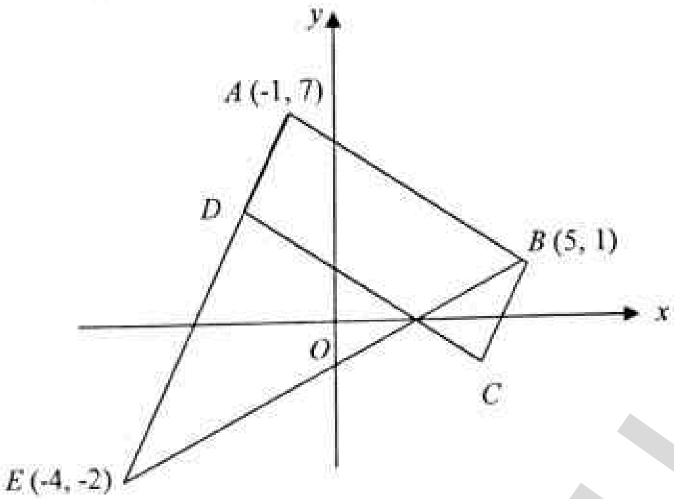
$x^2 - 4x + 4 + y^2 - 32y + 256 = 0$

$x^2 + y^2 - 4x - 32y + 196 = 0$ **[A1]**

$2g = -4$, $2f = -32$ and $c = -196$

$\therefore g = -2,\ f = 16,\ c = 196$ **[A1]**

9. **Solutions to this question by accurate drawing will not be accepted.**



The diagram, not to scale, shows a parallelogram, $ABCD$. ADE and BE are straight lines. D divides AE such that $AD : DE$ is in the ratio 1 : 2.

A, B and E have coordinates $(-1, 7)$, $(5, 1)$ and $(-4, -2)$ respectively.

(a) **(i)** Find the equation of the perpendicular bisector of AB and show that it passes through E. [3]

(ii) Hence deduce the geometrical property of triangle ABE. [1]

(b) Find the coordinates of D. [2]

(c) Find the area of the parallelogram $ABCD$. [2]

(a)(i) Gradient of $AB = \dfrac{7-1}{-1-5} = -1$

Gradient of perpendicular bisector of $AB = 1$

Mid-point of $AB = \left(\dfrac{-1+5}{2},\ \dfrac{7+1}{2}\right) = (2,\ 4)$ **[A1]**

Equation of perpendicular bisector of AB: $y - 4 = x - 2$

$\therefore y = x + 2$ **[A1]**

When $x = -4$, $y = -4 + 2 = -2$.

$\therefore$ perpendicular bisector of AB passes through E. *(Shown)* **[M1]**

(ii) ΔABE is an isosceles triangle. **[A1]**

(b) $\overrightarrow{AD} = \dfrac{1}{3}\overrightarrow{AE} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$

$D = (-1-1\ ,\ 7-3) = (-2\ ,4)$ **[M1A1]**

(c) Area of $\Delta ABD = \dfrac{1}{2}\begin{vmatrix} -1 & -2 & 5 & -1 \\ 7 & 4 & 1 & 7 \end{vmatrix} = 12 \text{ units}^2$ **[M1]**

Area of parallelogram $ABCD = 12 \times 2 = 24 \text{ units}^2$ **[A1]**

10. A particle starts from rest at 5 m from a fixed point O and moves in a straight line with a velocity, $v = 12t - 3t^2$ m/s where t is the time in seconds after leaving from the initial rest position.

(i) Calculate the acceleration when the particle is instantaneously at rest. [3]

(ii) Calculate the maximum velocity. [2]

(iii) Express the displacement, s, from point O in terms of t. [1]

(iv) Find the average speed of the particle during the first five seconds. [3]

(i) $a = \frac{dv}{dt} = 12 - 6t$ **[A1]**

When particle is instantaneously at rest, $v = 0$

$12t - 3t^2 = 0$
$3t(4 - t) = 0$
$t = 0$ (NA) or $t = 4$ **[M1]**

Acceleration = $12 - 6(4) = -12$ m/s^2. **[A1]**

(ii) For max or min velocity, $a = 0$
$12 - 6t = 0$
$t = 2$ **[M1]**

$\frac{d^2v}{dt^2} = -6 < 0 \Rightarrow$ max velocity
Max velocity = 12(2) -3(4) = 12 m/s **[A1]**

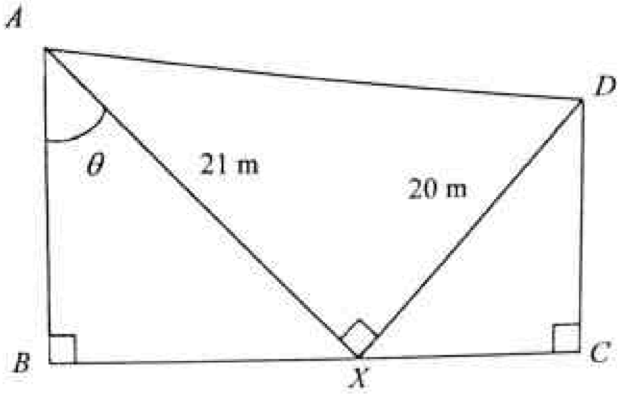
(iii) $S = \int(12t - 3t^2)dt$
$= 6t^2 - t^3 + C$ where C is an arbitrary constant.
Subst $t = 0, s = 5$: $C = 5$.
$\therefore s = 6t^2 - t^3 + 5$ **[A1]**

(iv) When $t = 0$, $s = 5$ m
When $t = 4$, $s = 37$ m
When $t = 5$, $s = 30$ m **[A1]**

Total distance = (37 – 5) + (37 – 30) = 39 m **[M1]**

Average speed = $\frac{39}{5} = 7.8\ m/s$ **[A1]**

11.



The diagram shows a trapezium field $ABCD$. The point X lies on the side BC such that $AX = 21$ m, $DX = 20$ m, $\angle AXD = \angle ABX = \angle DCX = 90°$ and $\angle BAX = \theta$.

(i) Show that the length of fencing required for the perimeter of the field, L m, can be expressed in the form of $p + q\sin\theta + r\cos\theta$, where p, q and r are constants to be determined. [3]

(ii) Express L in the form $p + R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [2]

(iii) State the maximum value of L and the corresponding value of θ. [2]

(iv) Given that the fencing used is 80 m, find the value(s) of θ. [3]

(i) $AD = \sqrt{21^2 + 20^2} = 29m$

$\sin\theta = \frac{BX}{21}$

$BX = 21\sin\theta$

$\cos\theta = \frac{AB}{21}$

$AB = 21\cos\theta$

$\angle DXC = \theta$

$\sin\theta = \frac{DC}{20}$

$DC = 20\sin\theta$ **[M1A1]**

$\cos\theta = \frac{XC}{20}$

$XC = 20\cos\theta$

$L = AB + BC + CD + AD$

$= 21\cos\theta + 21\sin\theta + 20\cos\theta + 20\sin\theta + 29$

$\therefore L = 41\cos\theta + 41\sin\theta + 29$ **[A1]**

(ii) Let $41\cos\theta + 41\sin\theta = R\cos(\theta - \alpha)$

$R = \sqrt{41^2 + 41^2} = \sqrt{3362}$

$\tan\alpha = 1$

$\alpha = 45°$ **[M1A1]**

$\therefore L = 29 + \sqrt{3362}\cos(\theta - 45°)$

(iii) Max value of $L = 29 + \sqrt{3362} = 87.0m$ **[A1]**

$\cos(\theta - 45°) = 1$

$\theta - 45° = 0$ **[A1]**

$\therefore \theta = 45°$

(iv) $29 + \sqrt{3362}\cos(\theta - 45°) = 80$

$\cos(\theta - 45°) = \frac{51}{\sqrt{3362}}$

$\theta - 45° = 28.4°, 331.6°(NA), -28.4°$ **[M1A2]**

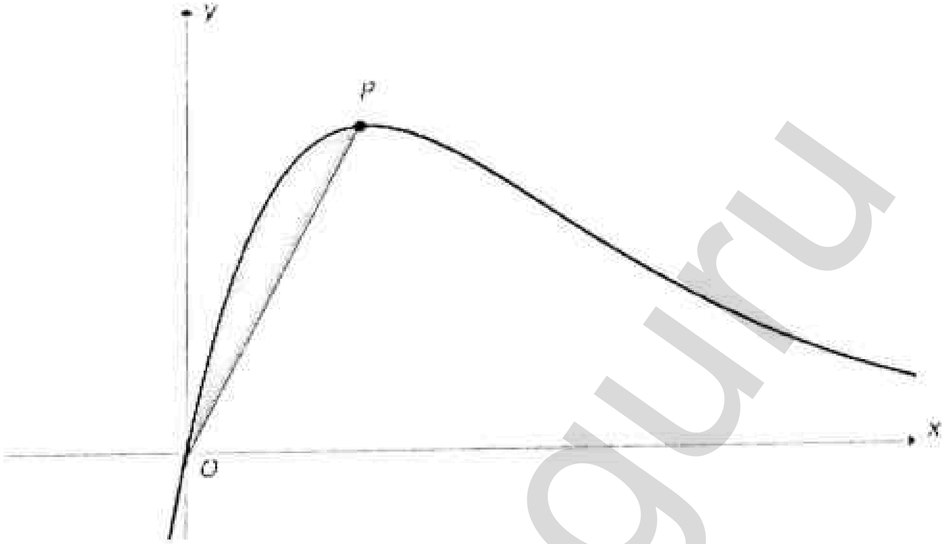
$\therefore \theta = 73.4°, 16.6°$

12. **(a)** **(i)** Given that $y = xe^{-2x}$, $x > 0$, show that $\frac{dy}{dx} = (1-2x)e^{-2x}$. [1]

(ii) Hence, find $\int xe^{-2x}dx$. [3]

(b) The diagram, which is not drawn to scale, shows part of the curve $y = xe^{-2x}$

A line drawn from the origin meets the curve at the maximum point P.



[3]

(ii) Calculate the area of the region bounded by the curve and the line OP. [4]

(a)(i) $y = xe^{-2x}$

$\frac{dy}{dx} = e^{-2x} - 2xe^{-2x}$ **[M1]**

$= (1-2x)e^{-2x}$

(ii) $\int e^{-2x}dx - 2\int xe^{-2x}dx = [xe^{-2x}]$ **[M1]**

$\int xe^{-2x}dx = \frac{1}{2}\int e^{-2x}dx - \frac{1}{2}xe^{-2x}$ **[M1A1]**

$\therefore \int xe^{-2x}dx = -\frac{1}{4}e^{-2x} - \frac{1}{2}xe^{-2x} + C$

(b)(i) For stationary points, $\frac{dy}{dx} = 0$

$(1-2x)e^{-2x} = 0$

$1 - 2x = 0$ **[M1A1]**

$x = \frac{1}{2}$

When $x=\frac{1}{2}, y=\frac{1}{2}e^{-1}=\frac{1}{2e}$

$\therefore P(\frac{1}{2},\frac{1}{2e})$ **[A1]**

(iii) Required area $=\int_0^{\frac{1}{2}} xe^{-2x}\,dx-\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{1}{2e}\right)$ **[M1]**

$=\left[-\frac{1}{4}e^{-2x}-\frac{1}{2}xe^{-2x}\right]_0^{\frac{1}{2}}-\frac{1}{8e}$ **[M1]**

$=\left[-\frac{1}{4}e^{-1}-\frac{1}{4}e\right]-(-\frac{1}{4})-\frac{1}{8e}$ **[M1]**

$=\frac{5}{8}e^{-1}+\frac{1}{4}$ or 0.480 units^2 (3sf) **[A1]**

ST. MARGARET'S SECONDARY SCHOOL

Preliminary Examinations 2016

CANDIDATE NAME

CLASS | | | REGISTER NUMBER | |

ADDITIONAL MATHEMATICS 4047/01

Paper 1 **25 August 2016**

Secondary 4 Express / 5 Normal (Academic) **2 hours**

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST
Write your name and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **7** printed pages

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities $\sin^2 A + \cos^2 A = 1$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 The function f is defined by

$$f(x) = 3 + \frac{1}{2x-1}, \text{ where } x \neq \frac{1}{2}.$$

Show that f is a decreasing function. [3]

2 Find the range of values of p for which $(p+2)x^2 - 12x + 2(p-1)$ is always negative. [4]

3 The line $y = mx + c$ intersects the curve $y^2 = ax$ at $A(4, 4)$ and $B(1, k)$.

B is a point that lies below the x-axis.

(i) Sketch the curve $y^2 = ax$, indicating point A. [1]

(ii) Find the values of a, m, c and k. [4]

4 Sketch the graph of $y = |x-3| + 2$ for $-3 \leq x \leq 6$. [3]

Find the range of values of c for which $|x-3| - c = x - 2$ has

(i) only 1 solution, [1]

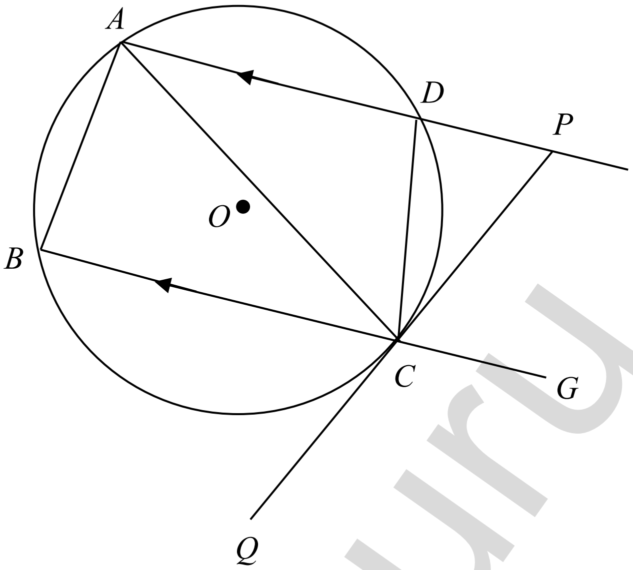
(ii) no solution. [1]

5 Air is pumped into a spherical balloon at a constant rate of 60 cm^3/s.

(i) Find the rate of increase of the radius, at the instant when the radius is 12 cm. [3]

(ii) Hence, find the rate of change of the surface area of the balloon at this instant. [2]

6 In the figure, O is the centre of the circle. PCQ is the tangent to the circle at C and AD is parallel to BC.



(i) Name an angle equal to $\angle BAC$, giving your reason(s) clearly. [1]

(ii) Show that $\angle CPD = \angle BAC$. [2]

(iii) Show that ΔBAC is similar to ΔCPD. [3]

7 Given that $f(x) = \dfrac{\cos^3 x - \sin^3 x}{\cos x - \sin x}$,

(i) express f(x) in the form $a\sin bx + c$, stating the value of each of the integers a, b and c, [4]

(ii) state the greatest and least values of f(x), [2]

(iii) state the period and amplitude of f(x). [2]

8 The decay of a certain radioactive isotope can be modelled by the exponential equation $N = N_o e^{-at}$ after *t* weeks, where *N* represents the amount of radioactive isotope, N_0 and *a* are constants. A sample of this radioactive isotope has a mass of 100.9 g initially.

(i) After 2 weeks, it is found that the amount of this sample left is 84.6 g. Calculate the value of *a*. [3]

(ii) What percentage of this sample has decayed after 5 weeks? [2]

(iii) After 9 weeks, the amount of this sample is found to be only 34.6 g. Suggest a reason why this might be so. [2]

9 **(i)** Show that $\sin^4\theta - \cos^4\theta = 1 - 2\cos^2\theta$. [3]

(ii) Hence solve the equation $\sin^4\theta - \cos^4\theta - 3\cos\theta = 2$ for $0 < \theta < 360°$. [4]

10 A particle moves in a straight line such that, *t* seconds after leaving a fixed point *O*, its velocity, *v* m s^{-1}, is given by $v = 15 - e^{-3t}$.

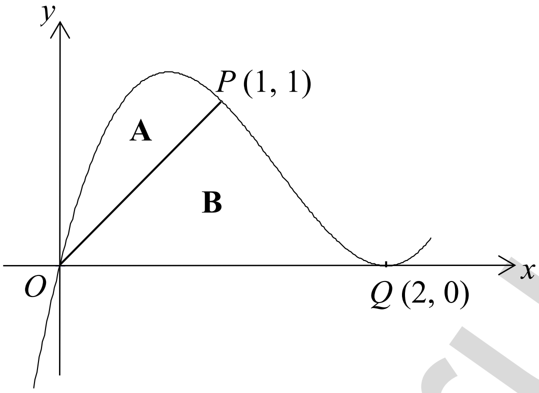
(i) Write down the initial velocity of the particle. [1]

(ii) If *t* becomes very large, what value will *v* approach? Explain your answer clearly and its significance. [2]

(iii) Find the acceleration of the particle when *t* = 3, giving your answer in cm s^{-2} correct to 3 decimal places. [2]

(iv) Find the distance travelled by the particle in the first 4 seconds of its journey, giving your answer correct to 2 decimal places. [2]

11 The diagram above shows part of the curve $y = x(x-2)^2$ which passes through $P\ (1,\ 1)$ and touches the x-axis at $Q\ (2,\ 0)$.



(i) Find the equation of the tangent at *P* and show that line *OP* is the normal to the curve at *P*. [4]

(ii) Show that the area of the region labelled A is $\frac{5}{12}$ unit^2 and determine the ratio of the area of **A** to the area of **B**. [6]

12 In figure 1, *ABCD* is a square plastic plate of side 4 cm and *PQRS* is a square whose centre coincides with that of *ABCD*. The shaded regions are to be cut off and the remaining plastic is folded to form a right pyramid with base *PQRS*, as shown in figure 2.

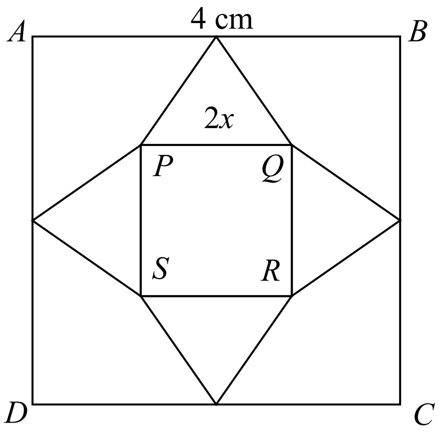


Figure 1

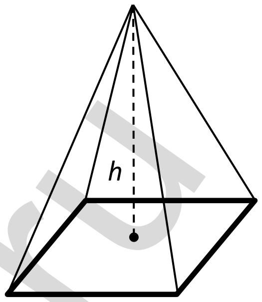


Figure 2

Let $PQ = 2x$ cm and let V be the volume of the pyramid.

(i) Show that the height of the pyramid is $2\sqrt{1-x}$ cm. [2]

(ii) Show that $V = \frac{8}{3}x^2\sqrt{1-x}$ cm^3. [2]

(iii) Find the value of x such that V is maximum. [7]

(iv) Showing your working clearly, explain why the volume of the pyramid will not exceed 0.8 cm^3. [2]

- The End –

Answers

1. $f'(x) = -\frac{2}{(2x-1)^2}$

$(2x-1)^2 > 0$

Therefore, $-\frac{2}{(2x-1)^2} < 0$

Since $f'(x) < 0$, f(*x*) is a decreasing function.

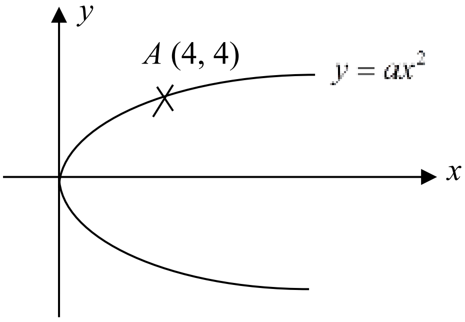
2. $b^2 - 4ac < 0$,

$p < -5$ or $p > 4$

But $p + 2 < 0$,

$\therefore p < -5$

3.



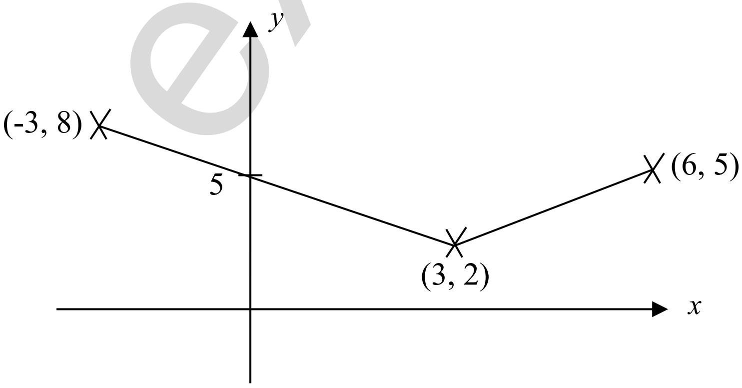
$a = 4$

$k = -2$

$m = 2$

$c = -4$

4



(i) $-1 < c \le 11$

(ii) $c < -1$ or $c > 11$

5(i) $\dfrac{\mathrm{d}r}{\mathrm{d}t} = 0.0332\,\text{cm/s}$

(ii) $\dfrac{\mathrm{d}A}{\mathrm{d}t} = 10.0\,\text{cm}^2\text{/s}$

6(i) $\angle BCQ$.

Alternate Segment Theorem

(ii) from (i),

$\angle BAC = \angle BCQ$.

$\angle BCQ = \angle GCP$ (vert. opp. $\angle$s)

$\therefore \angle CPD = \angle GCP$ (alt. $\angle$s)

$= \angle BAC$

(iii) from (ii),

$\angle BAC = \angle CPD$.

$\angle DCP = \angle DAC$ (alt. seg. thm)

$\angle DAC = \angle BCA$ (alt. $\angle$s)

$\therefore \Delta BAC$ similar to ΔCPD (AA Similarity or 2 pairs of corr. $\angle$s equal)

7. (i) $\mathrm{f}(x) = \dfrac{\cos^3 x - \sin^3 x}{\cos x - \sin x}$

$= \dfrac{1}{2}\sin 2x + 1.$

$\therefore a = \dfrac{1}{2},\ b = 2,\ c = 1$

(ii) greatest = $\dfrac{3}{2}$, least = $\dfrac{1}{2}$

(iii) amplitude = $\dfrac{1}{2}$, period = π or 180°

8(i) $a = 0.0881$

(ii) 35.6%

(iii) Difference = 11.06

Possible reasons:

- Error in data collection
- Due to other external factors that expedited the decay
- Any other logical reasoning with explanation

9(ii) $\theta = 120°,\ 180°,\ 240°$

10(i) initial velocity = 14m/s

(ii) when t is very large, e^{-3t} becomes insignificant,

$\therefore v$ will approach 15 m/s.

Velocity will approach a maximum speed of 15m/s and held constant at 15m/s

(iii) acceleration = 0.037 cm/s^2

(iv) s = 59.67 m

11(i) Equation of tangent at P: $y = -x + 2$

gradient of OP × gradient at P = 1×-1

$= -1$

Since gradient of OP × gradient at P = -1, OP is normal to curve at P.

(ii) 5 : 11

12(ii) $V = \frac{8}{3}x^2\sqrt{1-x}$

(iii) stationary point, $x = \frac{4}{5}$

Use 1st or 2nd derivative test to prove that it is a maximum point.

(iv) When $x = \frac{4}{5}$,

Max $V = 0.763\ \text{cm}^3$, therefore, V will never exceed 0.8cm^3

ST. MARGARET'S SECONDARY SCHOOL

Preliminary Examinations 2016

CANDIDATE NAME

CLASS

REGISTER NUMBER

ADDITIONAL MATHEMATICS **4047/02**

Paper 2 **30 August 2016**

Secondary 4 Express / 5 Normal (Academic) **2 hours 30 minutes**

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST
Write your name and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of **7** printed pages

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$, $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 Differentiate $5xe^{2x}$ with respect to x. Hence evaluate $\int_0^1 3xe^{2x}\,\mathrm{d}x$, giving answer correct to 2 decimal places. [5]

2 **(i)** Given that $y = \dfrac{\sin 2x}{1+\cos 2x}$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ can be written in the form $\dfrac{k}{1+\cos 2x}$ and state the value of k. [4]

(ii) Hence evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \frac{1}{4(1+\cos 2x)}\,\mathrm{d}x$. [3]

3 A curve has the equation $y = px - x\ln x$ for $x > 0$ and p is a constant. Find, in terms of p,

(i) the x-coordinate of the point at which the curve crosses the x-axis, [2]

(ii) the value of x, for which the curve has a turning point, [3]

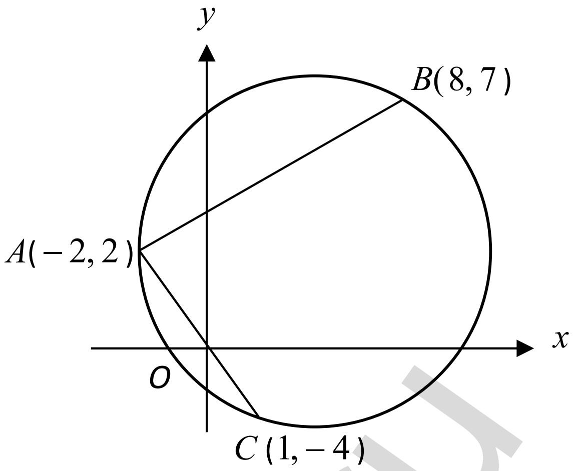
(iii) the coordinates of the turning point and the nature of this point. [3]

4 A curve is such that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{x^2-3}{x^2}$.

(i) Given that the curve passes through the point P (3, 5), find the equation of the curve. [3]

(ii) Find the equation of the tangent at P and determine if this tangent cuts the curve again. [5]

5 In the diagram below, $A(-2, 2)$, $B(8, 7)$ and $C(1, -4)$ are points on a circle.



(i) Find the gradient of AB and of AC. [2]

(ii) Show that BC is a diameter of the circle and hence find the centre of the circle. [4]

(iii) Find the equation of the circle. [2]

6 **(a)** Express $\dfrac{8\sqrt{2}+\sqrt{80}-\sqrt{98}}{\sqrt{18}+2\sqrt{45}-4\sqrt{5}}$ in the form $a+b\sqrt{c}$. [4]

(b) Without using calculators, express the value of $\dfrac{4\cos\left(\dfrac{\pi}{6}\right)}{\sin\left(\dfrac{\pi}{3}\right)-\sin\left(\dfrac{\pi}{6}\right)}$ in the form $a\sqrt{3}+b$, where a and b are integers. [4]

7 **(a)** Express $\frac{2x^2+x+3}{x^3+3x}$ in partial fractions. [4]

(b) A polynomial $\mathrm{P}(x)$ of degree three is exactly divisible by x^2-2.

Given also that $4\mathrm{P}(-1)=\mathrm{P}(2)$, show that x is a factor of $\mathrm{P}(x)$. [4]

8 The roots of the quadratic equation $2x^2-4x+3=0$ are α and β.

(i) Find the value of $\alpha^2+\beta^2$. [2]

(ii) Show that the value of $\alpha^3+\beta^3$ is -1. [2]

(iii) Find a quadratic equation whose roots are $\frac{\alpha}{\beta^2}+1$ and $\frac{\beta}{\alpha^2}+1$. [5]

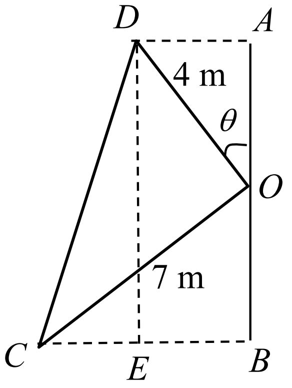
9 **(a)** Find the middle term in the expansion of $\left(x^2-\frac{1}{3x^3}\right)^{10}$. [3]

(b) Write down the first three terms in the expansion, in ascending powers of *x* of $\left(1-\frac{x}{2}\right)^n$, where *a* is a constant and *n* is a positive integer greater than 6. [2]

The first three terms in the expansion, in ascending powers of *x*, of $(2+ax)\left(1-\frac{x}{2}\right)^n$ are $2-6x+7x^2$.

Find the value of *a* and of *n*. [5]

10 In the diagram, $OD = 4$ m, angle DOC = angle DAO = angle $CBO = 90°$, and OC = 7 m. Angle $DOA = \theta$ and varies between 0° and 90° . The point E is on the line CB such that DE is parallel to AB.



(i) Show that $AB = 7\sin\theta + 4\cos\theta$. [2]

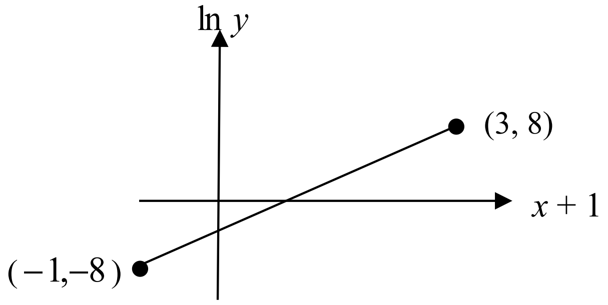
(ii) Express AB in the form $R\sin(\theta+\alpha)$, where R is positive and α is acute. Hence find the value of θ for $AB = 7.5$ m. [4]

(iii) State which line in the diagram has a length of R and which angle in the diagram has a value of α. [2]

(iv) Show that the area of triangle CDE is $\frac{65\sin 2(\theta+\alpha)}{4}$. [3]

(v) Find the maximum value of the area of triangle CDE as θ varies and state the corresponding value of θ. [3]

11 **(a)** The diagram shows a part of a straight line graph obtained by plotting $\ln y$ against $x+1$, together with coordinates of two points on the line. Express y in terms of x. [4]



(b) At time t minutes, the temperature of a liquid, which is left to cool, exceeds room temperature by T°C. The table shows the temperature difference at given times. It is known that one value of T has been recorded incorrectly.

Time, t (min)	5	10	15	20	25
Temperature difference, T°C	14.7	8.1	6.5	2.4	1.3

The variables T and t are related by the equation $T = ke^{at}$, where k and a are constants.

(i) Plot $\ln T$ against t for the given data and draw a straight line graph. [4]

(ii) Use your graph to

(a) identify the abnormal reading and estimate the correct value of T, [2]

(b) estimate the value of k and of a. [3]

(c) explain why the temperature of the liquid will never reach room temperature. [2]

Answer Keys

1 (i) $5(1+2x)e^{2x}$ (ii) 6.26

2 (i) $k=2$ (ii) $\frac{1}{8}$

3 (i) $x=e^{p}$ (ii) $x=e^{p-1}$ (iii) $\left(e^{p-1},e^{p-1}\right)$, max

4 (i) $y=x+\frac{3}{x}+1$ (ii) $y=\frac{2}{3}x+3$, No

5 (i) $\frac{1}{2}$, -2 (ii) $\left(\frac{9}{2},\frac{3}{2}\right)$ (iii) $\left(x-\frac{9}{2}\right)^2+\left(y-\frac{3}{2}\right)^2=\frac{85}{2}$

6 (a) $17-5\sqrt{10}$ (b) $2\sqrt{3}+6$

7 (a) $\frac{2x^2+x+3}{x^3+3x}=\frac{1}{x}+\frac{x+1}{x^2+3}$

8 (i) 1 (iii) $x^2-\frac{14}{9}x+\frac{11}{9}=0$

9 (a) $-\frac{28}{27x^5}$ (b) $1-\frac{n}{2}x+\frac{n(n-1)}{8}x^2+......$, $n=7$, $a=1$

10 (ii) $AB=\sqrt{65}\sin(\theta+29.7°)$ or $AB=8.06\sin(\theta+29.7°)$

(iii) CD has a length of R, $\angle DCO=\alpha$

(v) $16\frac{1}{4}$ m^2, $\theta=15.3°$

11 (a) $y=e^{4x}$

(b) (i) $\ln y=at+\ln k$

(iia) abnormal reading is 6.5, correct reading is 4.5

(iib) $a\approx-0.12$, $k\approx27.1$

(c) $T=0$ at room temperature and $\ln T$ will become undefined. Hence the temperature of the liquid.

TANJONG KATONG GIRLS' SCHOOL

PRELIMINARY EXAMINATION 2016
SECONDARY FOUR

4047/01 **ADDITIONAL MATHEMATICS**
PAPER 1

Thursday **11 August 2016** **2 h**

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper, and use a pencil for drawing graphs and diagrams.
Do not use staples, highlighters or correction fluid.

Answer **all** the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Setter : Ms Yeo
Markers : Mrs Pang / Mrs M Loy / Mdm Tan SE / Ms Yeo

This Question Paper consists of 7 printed pages, including this page.

Answer all questions

1 It is given that $\cos A = -\frac{1}{3}$ and $\sin B = \sqrt{\frac{2}{11}}$. A and B are in the same quadrant.
Without using a calculator, find the exact value of $\cot(90° - A - B)$. [5]

2 (i) Find the range of values of p for which $(x+1)(x-2) > p(x+2)$ for all real values of x. [4]

(ii) Deduce the number of points at which the line $y = p(x+2)$ intersects the curve $y = (x+1)(x-2)$ for $-1 \le p < 2$. [1]

3 2000 cm^3 of water is transferred from a rectangular tank to an empty inverted right circular cone in 10 seconds. The ratio of the radius of the cone to the height of the cone is 1 : 3.
Find the rate of change of the horizontal surface area, A cm^2, of the water in the cone, when the height, h cm, of the water in the cone is 12 cm. [6]

4 (i) Write down and simplify, the first 3 terms in the expansion of $(2-p)^7$ in ascending powers of p. [2]

(ii) Find the value of n where n is a positive integer, given that the coefficient of x^2 is 96 in the expansion of $(1+x)^n(2-x+x^2)^7$. [4]

5 A curve $y = f(x)$ is such that $f''(x) = 48\sin 4x - 8\cos 2x$. The curve intersects the x-axis at P. The x-coordinate of P is $\frac{\pi}{4}$ and the gradient of the curve at P is 8. Show that $f''(x) + 16f(x) = 24\cos 2x$. [7]

6 The table shows experimental values of two variables x and y.

x	2	4	6	7	8
y	1.33	2.29	3.27	3.77	6.12

It is known that x and y are related by an equation of the form $x^2 + \frac{y}{a} = bxy$, where a and b are constants. An error was made in recording one of the values of y.

(i) Using a scale of 2 cm to represent 1 unit on the horizontal axis and 1 cm to represent 1 unit on the vertical axis, draw a straight line graph for the above given data. The straight line graph is to be drawn with variable x on the horizontal axis. [3]

(ii) Use the graph to estimate

(a) the correct value of y, [2]

(b) the values of a and b. [3]

7 (i) Express $\frac{4}{(x-3)x^2}$ in partial fractions. [4]

(ii) Hence evaluate $\int_4^7 \frac{1}{(x-3)x^2}\,dx$. [4]

8 (i) Prove that $\frac{1-\sin x}{\cos x} + \frac{\cos x}{1-\sin x} = 2\sec x$. [3]

(ii) In the equation

$$\frac{1-\sin x}{\cos x} + \frac{\cos x}{1-\sin x} + \tan^2 x = 2,$$

$\cos x = a$ or b where a and b are constants, and $b < 0$.

(a) Find the value of a and of b. [2]

(b) Solve the equation $\cos x = b$ for $-\pi \le x \le 2\pi$. [3]

9 The equation of a curve is $y = x\ln(2x-3)$ where $x > \frac{3}{2}$.

(i) Find the equation of the normal to the curve at $x = 2$. [4]

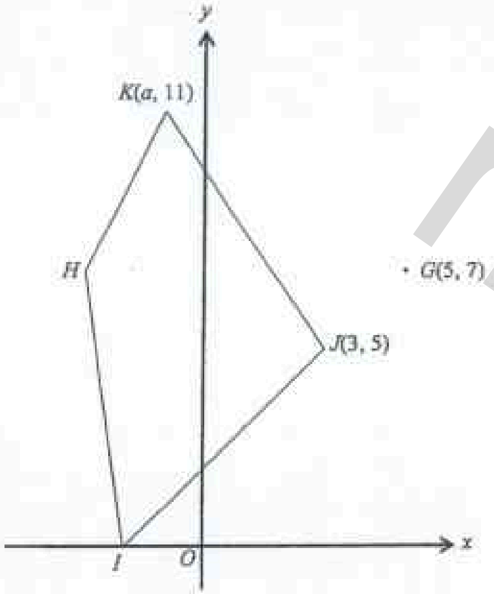
The normal to the curve $y = x\ln(2x-3)$ passes through the vertex of the graph of $y = k - 4|2x+1|$ where k is a constant.

(ii) Determine the value of k. [2]

(iii) Sketch the graph of $y = k - 4|2x+1|$ for the value of k in part (ii).

Show the vertex and intercepts clearly. [2]

10 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a quadrilateral $HIJK$. H is the reflection of point $G(5, 7)$ in the line $x = 1$. Point $K(a, 11)$ is such that the product of the gradients of HK and JK is -3. The perpendicular bisector of HJ intersects the x-axis at I.

(i) Deduce the coordinates of H. [1]

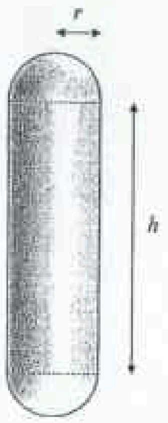
Find

(ii) the value of a given that $a < 0$, [2]

(iii) the equation of the perpendicular bisector of HJ, [3]

(iv) the area of quadrilateral $HIJK$. [3]

11



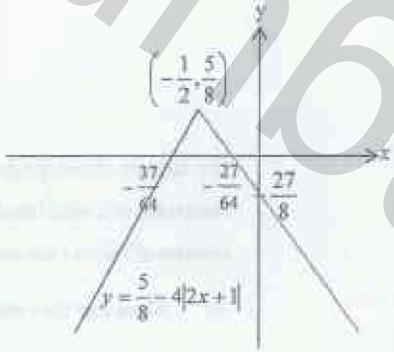
The diagram shows a capsule shaped object with surface area 18π cm^2. It comprised of 2 solid hemispheres of radius r cm joined to the 2 ends of a solid cylinder of radius r cm and height h cm.

(i) Show that the volume, V cm^3, of the object is given by $V = 9\pi r - \frac{2}{3}\pi r^3$. [4]

(ii) Find the stationary value of V, and determine if this stationary value is a maximum or minimum. [6]

THE END

Answer Key to TKGS Prelim 2016 Additional Mathematics Paper 1

1	$7\sqrt{2}$	8(i)	Proof
		(ii)(a)	$a=1$ and $b=-\frac{1}{3}$
2(i)	$-9<p<-1$	(ii)(b)	–1.91, 1.91, 4.37
2 (ii)	1 or 2 points		
		9(i)	$4y=-x+2$
3	$33\frac{1}{3}\text{ cm}^3/s$	(ii)	$\frac{5}{8}$
		(iii)	
4(i)	$128-448p+672p^2+$		
4(ii)	4		
5	proof		
		10(i)	$(-3, 7)$
6(ii)(a)	4.24	(ii)	-1
(b)	$a=1,\ b=2$	(iii)	$y=3x+6$
		(iv)	34 square units
7(i)	$\frac{4}{(x-3)x^2}=\frac{4}{9(x-3)}-\frac{4}{9x}-\frac{4}{3x^2}$	11(ii)	40.0 cm^3. Stationary value of V is a maximum.

TANJONG KATONG GIRLS' SCHOOL

PRELIMINARY EXAMINATION 2016
SECONDARY FOUR

4047/02 **ADDITIONAL MATHEMATICS**
PAPER 2

Friday **5 August 2016** **2 h 30 min**

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper, and use a pencil for for any diagrams or graphs.
Do not use staples, highlighters or correction fluid.

Answer **all** the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

Setter : Mrs M Loy
Markers: Mdm Tan SE, Mrs H Pang, Miss Yeo LS, Mrs M Loy

This Question Paper consists of 7 printed pages, including this page.

Answer **all** the questions

1. A man buys a new car. The value of the car depreciates with time so that its value, $\$V$, after t months' use is given by $V = 132\,000e^{-pt}$, where p is a constant.
The value of the car is expected to be $122 000 after eight months' use.

   (i) Find the value of the car, V when the man bought it. [1]

   (ii) Show that $p = 0.01$. [2]

   (iii) Using the value of $p = 0.01$, determine the age of the car to the nearest month, when its value reached half of the original value when the man bought it. [2]

2. The function $f(x) = 1 + 2x + Ax^2 - x^3$, where A is a constant, leaves a remainder of $1\frac{3}{8}$ when divided by $(2x-1)$.

   (i) Find the value of A. [3]

   (ii) Hence solve the equation $f(x) = 0$, giving your answers in the exact form. [4]

3. (a) (i) Solve $\sqrt{3x+2} - 3x = 0$. [2]

   (ii) On the same axes, sketch the graphs of $y = \sqrt{3x+2}$ and $y = 3x$. Indicate clearly all the points of intersections. [2]

   (b) The vertical height of a triangle is $\frac{8}{3-\sqrt{5}}$ cm.
   Given that the area of the triangle is $\frac{20}{\sqrt{5}-1}$ cm^2, without using a calculator, find the length of the base of the triangle in the form $a + b\sqrt{5}$. [3]

4. The roots of the quadratic equation, $2x^2+4x+5=0$ are $(\alpha+1)$ and $(\beta+1)$.

(i) Show that $\alpha+\beta=-4$ and hence find $\alpha\beta$. [3]

(ii) Find the quadratic equation in x with integer coefficients, whose roots are $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$. [5]

5. (a) Given that $\log_2(2x+1)-\log_4(3-x^2)=1$, form a quadratic equation in x and explain with clear working why the roots of the quadratic equation are real and distinct. [5]

(b) Solve $3^{x+2}=2(3^{-x})+17$. [4]

6. The curve $y=\frac{2x^2}{x^2+1}$ has one stationary point $(p\,,q)$.

(i) Find the value of p and of q. [4]

(ii) Determine whether y is increasing or decreasing for

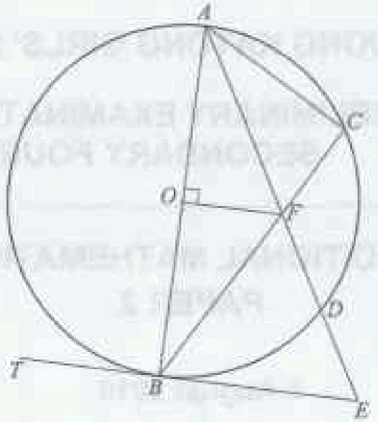
(a) $x>p$, [1]

(b) $x<p$. [1]

Hence state the nature of the stationary point. [1]

(iii) Find $\frac{d^2y}{dx^2}$ at the stationary point and explain how $\frac{d^2y}{dx^2}$ further supports your answer in part (ii). [2]

7.



In the figure, AB is a diameter of the circle with centre O. Chords AD and BC intersect at F. AD produced meets the tangent to the circle, TBE at E. AE is an angle bisector of angle BAC.

(i) Prove that $\angle CBD=\angle DBE$. [3]

Given that $\angle AOF=90°$, prove that

(ii) triangle AOF is similar to triangle ADB, [2]

(iii) $2(AO)^2=AF\times(AF+FD)$. [3]

8. A particle moving in a straight line passes through a fixed point O with a speed of 20 m/s. The acceleration, a m/s^2, of the particle, t s after passing through O is given by $a=-100e^{-3t}$. The particle comes to instantaneous rest at point N.

(i) Find the time the particle comes to instantaneous rest at point N. [5]

(ii) Calculate the distance ON. [4]

(iii) Show that the average speed of the particle in the first 2 seconds rounded off to a whole number is 10 m/s. [3]

9. **(i)** Solve the equation $2\sin 2P = 3\cos P$ for $0^\circ \le P \le 360^\circ$. [4]

(ii) On the same axes, sketch for $0^\circ \le x \le 720^\circ$, the graphs of

$$y = \sin x \quad \text{and} \quad y = \frac{3}{2}\cos\left(\frac{x}{2}\right).$$ [4]

(iii) Using the solutions to part **(i)**, determine the x-coordinates of the points of intersection of the graphs of part **(ii)**. [4]

10. A circle, C_1, has equation $x^2 + y^2 - 14x + 2y = -46$.

(i) Find the coordinates of the centre of the circle and the radius. [3]

The coordinates of the centre of a second circle, C_2, is $(-4, -2)$. The equation of the tangent to the circle, C_2 at a point P is $2y = -2x + 3$.

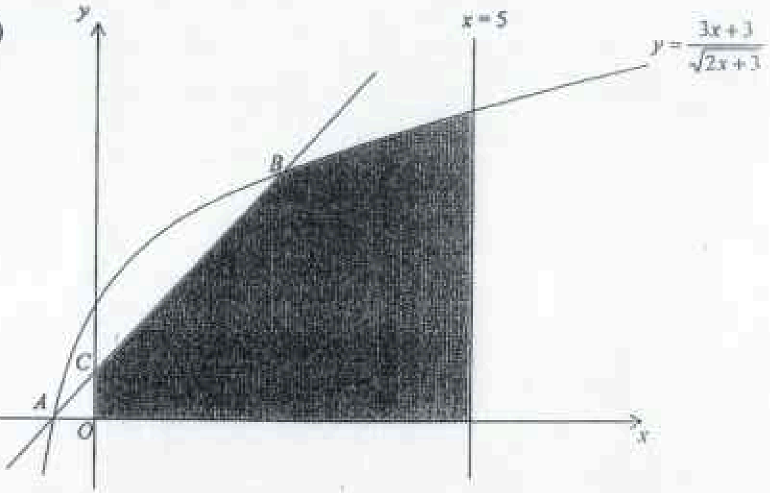
(ii) Find the coordinates of point P. [4]

(iii) Find the exact value of the radius of C_2 and the equation of the circle, C_2. [3]

(iv) Determine whether circles C_1 and C_2 will meet each other, showing your working clearly. [2]

11. **(a)** Show that $\frac{d}{dx}\left(2x\sqrt{2x+3}\right) = \frac{6x+6}{\sqrt{2x+3}}$. [3]

(b)



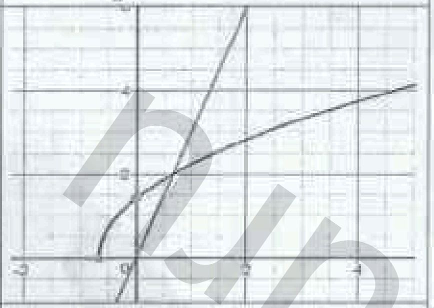
The diagram shows part of the curve $y = \frac{3x+3}{\sqrt{2x+3}}$. The curve intersects the x-axis at point A. The line through A and perpendicular to the line, $y + x = -7$ intersects the curve again at another point, B.

(i) Show that the y-coordinate of point B is 4. [5]

(ii) Given that the line AB intersects the y-axis at C, determine the area of the shaded region bounded by the line CB, the curve, the line $x = 5$, the x-axis and the y-axis. [4]

End of Paper

TKGS S4 PRELIM 2016 Answer Key:

1(i)	$V = 132\ 000$	(ii)	show
(iii)	70 months		
2(i)	$A = -2$	(ii)	$x = 1\ ,\ \frac{-3 \pm \sqrt{5}}{2}$
(a)i	$x = \frac{2}{3}$	Ii	
(b)	$\frac{5\sqrt{5}}{2} - \frac{5}{2}$	4(i)	$\alpha\beta = \frac{11}{2}$
(ii)	$1331x^2 - 16x + 8 = 0$	5(a)	Discriminant = 368 Since discriminant > 0, the roots of the quadratic equation are real and distinct.
(b)	$y = 0.631$	6(i)	$p = 0, q = 0$
(ii)a	$\frac{dy}{dx} > 0$, y is increasing	6(ii)b	$\frac{dy}{dx} < 0$, y is decreasing
	Since the value of $\frac{dy}{dx}$ changes from negative to positive value, the stationary point is a minimum point.	6(iii)	$\frac{d^2y}{dx^2} = 4$, since $\frac{d^2y}{dx^2} > 0$, the stationary point is minimum, thus reiterating the result from part **(ii)**.
7	proof	8.(i)	$t = 0.305$ s
(ii)	Distance = 2.59 m	8(iii)	show
(i)	48.6°, 90°, 131.4°, 270°	(ii)	
(iii)	97.2°,180°,262.8°,540°	10(i)	Centre(7, −1), radius = 2 units
(ii)	$P\,(-\frac{1}{4}, \frac{7}{4})$	10(iii)	Radius = $\frac{15\sqrt{2}}{4}$ units

			$(x+4)^2 + (y+2)^2 = (\frac{15\sqrt{2}}{4})^2 / \frac{225}{8}$
(iv)	Sum of radii(7.30 units) < distance between the centres (11.0 units) thus the circles will not meet.	11(a)	show
11(b)i	show	11(b)ii	16.5 units^2

Name ______________________ Class ______ Register Number ______

4047/01 **16/S4PR2/AM/1**

ADDITIONAL MATHEMATICS **PAPER 1**

Wednesday **3 August 2016** **2 hours**

VICTORIA SCHOOL

PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

Additional Material: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This paper consists of 6 printed pages, including the cover page.

[Turn over

---

1 (i) Simplify $|21-14x|-\left|\frac{2}{3}x-1\right|$. [2]

(ii) Hence, solve $|21-14x|=\left|\frac{2}{3}x-1\right|+40-15x$. [3]

2 The range of solutions for x such that $a+bx-4x^2>0$ is $-\frac{1}{2}<x<3$. Find the value of a and of b, where a and b are real numbers. [4]

3 (a) Solve $4^x-20(4^{-x})=1$. [3]

(b) Given that $\frac{5^{x-\frac{1}{2}}}{625(5^y)}=\frac{25^x}{\sqrt{125^y}}$, find the value of $\frac{x}{y}$. [3]

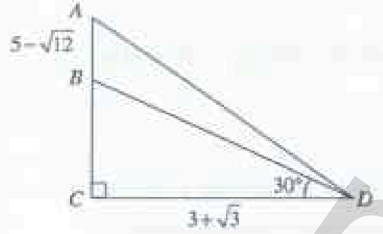
4 In the expansion of $\left(ax+\frac{1}{x}\right)^n$ in descending powers of x, where a and n are positive integers, the fourth term of the expansion is the constant term.

(i) Find the value of n and hence, express the constant term in terms of a. [4]

(ii) Using your value of n in (i), determine if a term in $\frac{1}{x^3}$ in the expansion $(1-x)\left(ax+\frac{1}{x}\right)^n$ exists. [2]

5 (a) Solve $\sqrt{1-2x}-3x=1$. [3]

(b)



In the diagram above, ACD is a triangle such that B lies on AC, $AB=\left(5-\sqrt{12}\right)$cm, $CD=\left(3+\sqrt{3}\right)$cm, $\angle BDC=30°$ and $\angle BCD$ is a right angle. Find AD^2 in the form $p+q\sqrt{3}$, where p and q are constants. [4]

6 (a) Differentiate $\ln\sqrt{\dfrac{1-3x}{e^{-x}}}$. [3]

(b) Given that $\int_1^4 f(x)dx=6$, $\int_3^6 f(x)dx=2$ and $\int_3^4 f(x)dx=-3$, find

(i) $\int_4^3 f(x)dx$, [1]

(ii) $\int_1^3 f(x)dx+\int_6^3 f(x)dx$, [1]

(iii) the value of h, where $\int_1^4 hx^2+2f(x)dx=180$. [2]

7 The equation of a curve is $y=3\left(\dfrac{x}{4}+a\right)^{\frac{5}{6}}$. The normal to the curve at $x=\dfrac{1}{2}$ is parallel to the line $5y+4x=2$.

(i) Show that $a=-\dfrac{7}{64}$. [4]

(ii) Find the equation of the tangent to the curve at $x=\dfrac{1}{2}$. [2]

(iii) Show that the curve is an increasing function for $x>\dfrac{7}{16}$. [2]

---

8 It is given that $\cos 140°=-k$ and $\tan A=-\dfrac{3}{4}$, where k is a positive number and A is a reflex angle. Without finding the value of A or of k,

(i) find the exact value of $\cos 2A$, [2]

(ii) express $\tan 50°$ in terms of k, [3]

(iii) express $\sin(40°+A)$ in terms of k. [3]

9 The points $P(1,\ -2)$ and $Q(1,\ 4)$ lie on the circumference of a circle with centre C. If the circle is reflected in a vertical line, P and Q remain unchanged in the reflection and the x-coordinate of the centre of the reflected circle is 5.

(i) State the equation of the vertical line of reflection. [1]

(ii) Show that the equation of the circle with centre C is $x^2+y^2+6x-2y-15=0$. [3]

(iii) The line $3y+4x=-9$ intersects the circle with centre C at two points, A and B. Find the coordinates of A and of B. [4]

(iv) Determine if AB is a diameter of the circle with centre C. [1]

10 A particle moves in a straight line such that at t seconds after passing point O, its velocity v m/s is given by $v=t-7+\dfrac{12}{t+1}$, where $t>0$.

Find

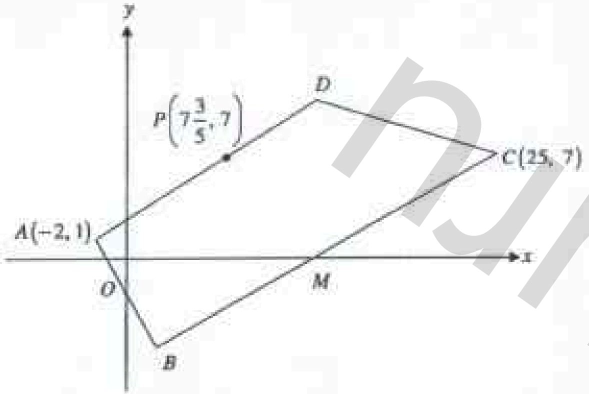
(i) the acceleration of the particle when it is first instantaneously at rest. [3]

(ii) an expression for the displacement of the particle from O. [3]

(iii) the total distance travelled by the particle from $t=0$ to $t=5$. [3]

11 Solutions to this question by accurate drawing will not be accepted.

The diagram shows the quadrilateral $ABCD$ in which point A is $(-2, 1)$ and point C is $(25,\ 7)$. The point $P\left(7\frac{3}{5}, 7\right)$ lies on AD such that $AP : PD = 3 : 2$. The midpoint of BC, point M, lies on the x-axis and directly below point D.



(i) Find the coordinates of points D, M and B. [6]

(ii) Determine if $\angle DAB$ is a right angle. [3]

(iii) Calculate the area of the quadrilateral $ABCD$. [2]

End of Paper

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Answer key

1(i)	$\frac{20}{3}\lvert 3-2x\rvert$ or $\frac{20}{3}\lvert 2x-3\rvert$	**7(ii)**	$y=\frac{5}{4}x-\frac{17}{32}$
1(ii)	$x=2\frac{2}{17}$ or 12 (NA)	**7(iii)**	Show $\frac{dy}{dx}>0$ for $x>\frac{7}{16}$.
2	$a=6,\ b=10$	**8(i)**	$\frac{7}{25}$
3(a)	$x=1.16$	**8(ii)**	$\frac{k}{\sqrt{1-k^2}}$
3(b)	$\frac{x}{y}=3$	**8(iii)**	$\frac{4}{5}\sqrt{1-k^2}-\frac{3}{5}k$
4(i)	Constant term = $20a^3$	**9(i)**	$x=1$
4(ii)	A term exists in $\frac{1}{x^3}$.	**9(iii)**	$(0,-3)$ and $(-6,5)$
5(a)	$x=0$ or $-\frac{8}{9}$ (NA)	**9(iv)**	AB is a diameter of the circle.
5(b)	$AD^2=51-6\sqrt{3}$	**10(i)**	-2 m/s^2
6(a)	$\frac{1}{2}\left(1-\frac{3}{1-3x}\right)$ OR $\frac{2+3x}{2(3x-1)}$	**10(ii)**	$s=\frac{t^2}{2}-7t+12\ln\lvert t+1\rvert$
6(bi)	9	**10(iii)**	4.63 m
6(bii)	7	**11(i)**	$D(14, 11)$, $M(14, 0)$, $B(3, -7)$
6(biii)	$h=8$	**11(ii)**	$\angle DAB$ is a right angle
		11(iii)	210 units^2

	Class	Register Number
Name ______________		

4047/02 **16/S4PR2/AM/2**

ADDITIONAL MATHEMATICS **PAPER 2**

Thursday **4 August 2016** **2 hours 30 minutes**

VICTORIA SCHOOL

PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

Additional Materials:	Answer Paper Graph paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This paper consists of 7 printed pages, including the cover page.

[Turn over

---

1 The variables x and y are connected by the equation $y+a=-x(x+1)$, where a is a constant. Using experimental values of x and y, a graph was drawn in which $y+x^2$ was plotted on the vertical axis against x on the horizontal axis. The straight line obtained passes through the point $(-3,1)$.

Calculate the

(i) value of a, [3]

(ii) coordinates of the point on the line at which $y=x(3-x)$. [2]

2 (a) Find the range of values of k such that the line $y=kx-4$ meets the curve $4x^2-(k-x)=2y+3x$. [3]

(b) The equation $2x^2-7x+6=0$ has roots $2\alpha-1$ and $2\beta-1$ where $\alpha>\beta$.

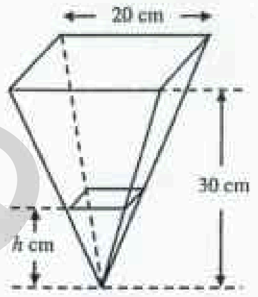
(i) Without solving for α and β, find the value of $\alpha-\beta$. [4]

Hence,

(ii) find the value of $\alpha^3-\beta^3$, [2]

(iii) state the quadratic equation whose roots are α^3 and $-\beta^3$. [1]

3



The diagram shows an inverted square-base pyramidal tank of height 30 cm and base length 20 cm. The tank is filled with water and is held fixed with its square rim horizontal. Water leaks out of the tank at a constant rate of 15 cm^3 s^{-1}. After t seconds, the depth of water is h cm.

(i) Show that the volume of water in the tank, V cm^3, at time t is given by $V=\frac{4}{27}h^3$. [2]

(ii) Find the rate of change of depth when $h=5$. [3]

4 Given that $f(x)=a\left(x^4+1\right)+7x^3-10x^2+bx$ and that $4x^2+7x-2$ is a factor of $f(x)$.

(i) Show that $a=4$ and $b=-14$. [5]

(ii) Find the remainder when $f(x)$ is divided by $x+1$. [2]

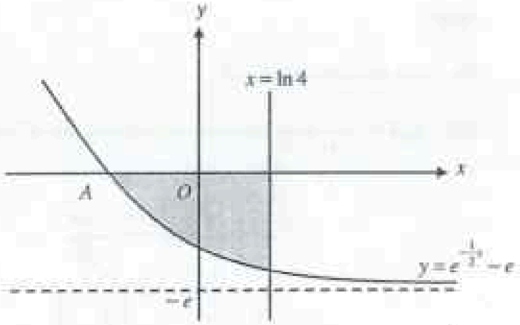
5 (a) The Population White Paper released by the Government of Singapore in 2013, projected that Singapore's population will hit 6.9 million by year 2030. The population of Singapore, P, increased from 5.399 million to 5.535 million from 2013 to 2015. Given that $P=Ae^{kt}$, where A and k are constants and t is the time in years from 2013.

(i) Find the value of A and of k. [3]

If the population continues to increase at the same rate,

(ii) determine if the population trajectory for year 2030 in the Population White Paper is accurate. [2]

(b)



The diagram shows the line $x=\ln 4$ and part of the curve $y=e^{-\frac{1}{2}x}-e$. The curve intersects the x-axis at the point A. Determine the area of the shaded region bounded by the curve, the line $x=\ln 4$ and the x-axis. [4]

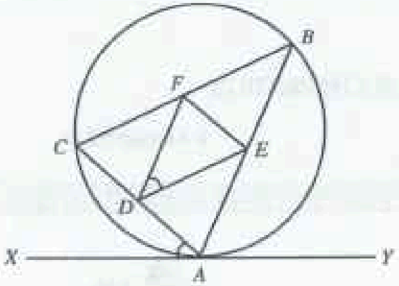
6 (i) Factorise completely the cubic polynomial x^3-x^2+3x-3. [2]

(ii) Express $\dfrac{2x^3-5x^2+10x-3}{x^3-x^2+3x-3}$ in partial fractions. [5]

(iii) Differentiate $\ln\left(x^2+3\right)$ with respect to x. Hence express $\int_2^5 \dfrac{2x^3-5x^2+10x-3}{x^3-x^2+3x-3}\,dx$ in the form $a+b\ln 2$, where a and b are integers. [5]

7 (a) Prove the identity $\dfrac{1}{\operatorname{cosec}\theta+\cot\theta}=\dfrac{1-\cos\theta}{\sin\theta}$. [3]

(b)



The diagram above shows a triangle ABC whose vertices lie on the circumference of a circle. D and E are the mid-points of AC and AB respectively. XY is a tangent to the circle at A. Given that CFB is a straight line and angle FDE = angle DAX.

Prove that

(i) DE is parallel to BC, [1]

(ii) ΔFDE is congruent to ΔEBF, [3]

(iii) $DEBF$ is a parallelogram. [2]

8 (i) On the same axes sketch, for $0\le x\le 2\pi$, the graphs of

$$y_1=2\sin x+1 \quad\text{and}\quad y_2=-\cos x.$$ [4]

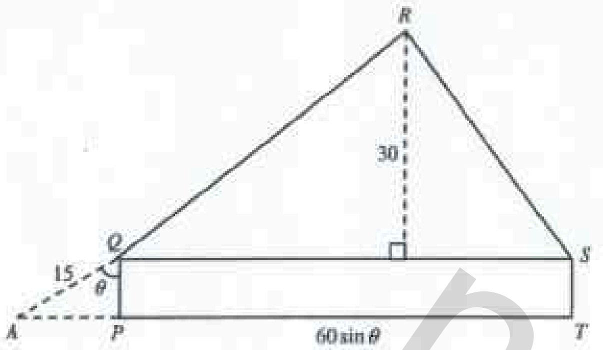
(ii) Given that $f(x)=y_1+y_2$, express $f(x)$ in the form $p\sin(x-q)+r$, where p, q and r are constants to be found. [4]

(iii) State, in exact form, the

(a) greatest and least values of $f(x)$, [2]

(b) amplitude of $f(x)$. [1]

9



The diagram shows the vertical cross-section $PQRST$ of a structure, consisting of a triangle QRS of height 30 m and a rectangle $PQST$. The structure rests with PT on horizontal ground. To hold the structure up, a 15 m rope is secured at Q to a point, A, on the ground. It is given that QA is inclined at an angle, θ radians, to QP and $PT = 60\sin\theta$ m.

(i) Show that the area, A m^2, of the cross-section $PQRST$ is given by

$$A = 900\sin\theta + 450\sin 2\theta.$$

[4]

(ii) Given that θ can vary, find the value of θ for which the maximum amount of paint is required to colour this cross-section. [5]

(iii) Hence, find the maximum value of A. [1]

10 A trapezium of area, A cm^2, has parallel sides of length px^2 cm and q cm and its perpendicular height is x cm. Corresponding values of x and A are shown in the table below.

x	1	2	3	4
A	1.75	5	11.25	22

(i) Using suitable variables, draw, on a graph paper, a straight line graph and hence estimate the value of each of the constants p and q. [6]

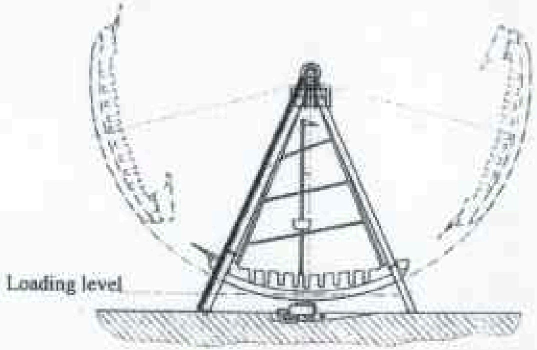
(ii) Using your values of p and q, calculate the value of x for which the trapezium is a rectangle. [2]

(iii) Explain how another straight line drawn on your diagram can lead to an estimate of the value of x for which the trapezium is a rectangle. Draw this line and hence verify your value of x found in part (ii). [3]

11 Gravitational potential energy, measured in kilojoules (kJ), is the energy a body has due to its position. It can be calculated by the following equation:

$$\text{Gravitational potential energy} = \frac{mgh}{1000}$$

where m is the mass of the body in kg, g is the gravitational field strength in N/kg and h is the height of the object in m. The gravitational field strength, g, on Earth is approximately 10 N/kg.



The gravitational potential energy, E, in kJ, of a pirate ship ride can be modelled by the equation, $E = 100(1-\cos kt) + a$, where k and a are constants and t is the time in seconds after starting the ride at loading level.

(i) Given that the mass of the ride is 1000 kg and at an initial loading level of 3 m, show that $a = 30$. [1]

(ii) Explain why this model suggests that the maximum gravitation potential energy possessed by the ride is 230 kJ. [1]

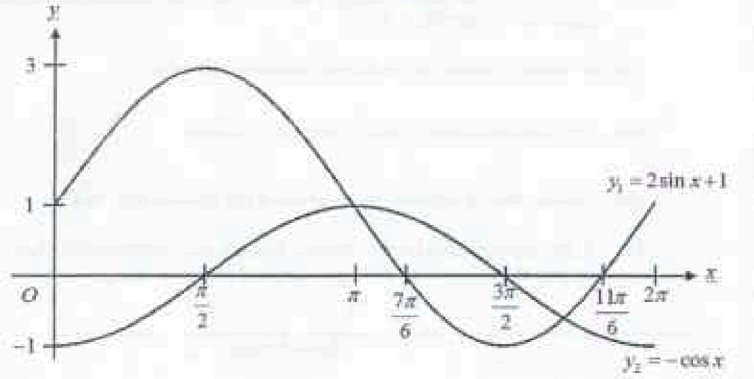
The ride takes 6 seconds to travel from one peak to another.

(iii) Show that the value of k is $\frac{\pi}{3}$ radians per second. [2]

(iv) Calculate the gravitation potential energy of the ride at $t = 8$ s. [2]

(v) If the ride continues for 60 seconds, find the exact duration for which the ride possesses more than 80 kJ of gravitational potential energy. [5]

End of Paper

1	i	$a=2$
	ii	$\left(-\frac{1}{2},-1\frac{1}{2}\right)$
2	a	$k\le -9.32$ or $k\ge 3.32$
	bi	$\frac{1}{4}$
	ii	$1\frac{27}{64}$
	iii	$x^2-\frac{91}{64}x-6\frac{303}{512}=0$
3	i	Show base of similar pyramid $=\frac{2}{3}h$
	ii	-1.35 cm s^{-1}
4	i	Let $f\left(\frac{1}{4}\right)=0$ and $f(-2)=0$
	ii	5
5	ai	$A=5.399\times10^6$ $k=0.0124$
	ii	Show $t=17$; $P=6.67\times10^6$ or $P=6.9\times10^6$, $t=19.72$, Year = 2032 Not accurate
	b	4.77 units^2
6	i	$(x-1)(x^2+3)$
	ii	$2+\frac{1}{x-1}-\frac{4x}{x^2+3}$
	iii	$\frac{2x}{x^2+3}$; $6-2\ln 2$
7	bi	Mid-Point Theorem
	ii	$\Delta FDE\equiv\Delta EBF$ (AAS)
8	i	

8	ii	$2.24\sin(x-0.464)+1$
	iii	a) Greatest value of $f(x)=1+\sqrt{5}$ Least value of $f(x)=1-\sqrt{5}$ b) Amplitude of $f(x)=\sqrt{5}$
9	i	Show $QP=15\cos\theta$
	ii	$\frac{\pi}{3}$
	iii	1170 m^2
10	i	Plot $\frac{2A}{x}=px^2+q$ or $\frac{A}{x}=\frac{1}{2}px^2+\frac{1}{2}q$ $p=0.500$ $q=3$
	ii	$x=2.45$
	iii	Draw $\frac{2A}{x}=x^2$ or $\frac{2A}{x}=6$ or $\frac{A}{x}=0.5x^2$ or $\frac{A}{x}=3$ $x=2.45$
11	i	Sub. $m=1000$, $g=10$, $h=3$
	ii	Sub. $\cos kt=-1$
	iii	Period $=\frac{2\pi}{k}=6$
	iv	180 kJ
	v	40 s

CANDIDATE NAME	CLASS	REGISTER NUMBER

BALESTIER HILL SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2016
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC

ADDITIONAL MATHEMATICS 4047 / 01

19 Aug 2016 **Friday** **2 hours**

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **ALL** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's use:

This paper consists of 6 printed pages, including this cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0,\ a \neq 0,$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}.$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$
$$a^2 = b^2 + c^2 - 2bc \cos A.$$
$$\Delta = \frac{1}{2} ab \sin C.$$

1 The graph $y = x^2 + 3px - 2q$, where p and q are constants, is always positive for all real values of x.

(i) Find an inequality connecting p and q. [2]

(ii) Explain why q cannot be positive. [1]

2 A prism with a trapezium base has a volume of $(14 + 11\sqrt{2})$ cm^3. The trapezium has a height of $(3\sqrt{2} + 2)$ cm and its parallel sides are $\sqrt{2}$ cm and 2 cm respectively. Find the height of the prism, leaving your answer in the form $(\frac{\sqrt{2} + a}{b})$ cm, where a and b are integers. [3]

3 **(i)** Sketch the graph of $y = |x^2 - 9| + 2$. [3]

(ii) Determine the range of values of m for which the line $y = mx$ does not intersect the graph of $y = |x^2 - 9| + 2$. [1]

4 A curve has equation $y = \frac{\sin x}{e^{2x}}$ for $0 \le x \le \frac{\pi}{2}$

(i) Prove that if y is an increasing function, $\tan x < \frac{1}{2}$. [3]

(ii) A point (x, y) moves along the curve $y = \frac{\sin x}{e^{2x}}$ such that the y-coordinate is decreasing at a rate of 0.2 units per second. Find the rate of change of the x-coordinate when $x = 0.5$. [3]

5 Given that $\mathrm{f}(x) = 6x^3 + 3x^2 - x + 2 = 0$,

(i) show that the equation $\mathrm{f}(x) = 0$ has only one real root. Find the value of the real root. [5]

(ii) sketch the curve, showing clearly the x and y intercepts. [2]

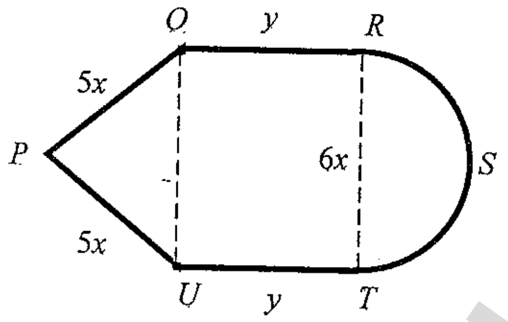
6 A piece of wire, of length 150 cm, is bent into the shape as shown in the diagram. The shape consists of an isosceles triangle PQU where $PQ = PU = 5x$ cm, a rectangle $QRTU$ and a semi-circle RST. Given further that $QR = y$ cm and $RT = 6x$ cm,

(i) show that the enclosed area, A cm^2, is given by

$$A = 450x - 9x^2(2 + \frac{\pi}{2}) .$$ [4]

(ii) Given that x can vary, find the value of x for which the area is stationary. [2]

(iii) Explain why this value of x gives the largest area possible. [1]



7 Given that the first four terms in the expansion of $(1+3x)^2(1+x)^n$ in ascending powers of x is $1 + ax + bx^2 + cx^3 + ...$, where a, b and c are constants, and n is a positive integer.

(i) Express a and b in terms of n. [3]

(ii) If $b = 72$, prove that $n = 7$ and find the value of c. [4]

(iii) Using the value of n found in (ii), find the coefficient of x^2 in the expansion of $(1+3x)^2(1+x)^{n+1}$. [2]

8(i) Show that $\frac{d}{dx}(\ln(\sin^2 x)) = 2\cot x$ [2]

(ii) By expressing $x\cot x$ as $\frac{x}{\tan x}$, differentiate $x\cot x$ with respect to x. [3]

(iii) Using your results from parts (i) and (ii), find $\int x \operatorname{cosec}^2 x \, dx$ and prove that

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} x \operatorname{cosec}^2 x \, dx = \frac{1}{2}\ln 2 - \pi\left(\frac{1}{4} - \frac{\sqrt{3}}{6}\right).$$ [4]

9 A point P is equidistant from A (1, 4) and B (5, 2). Given that P lies on the line $y - x = 1$, find

(i) the co-ordinates of the point P, [3]

(ii) the equation of the perpendicular bisector of AB, [3]

(iii) a point Q such that $APBQ$ is a parallelogram. [2]

(iv) Find the area of triangle AOP, where O is the origin. [2]

10 The table below shows the experimental values of the variables x and y.

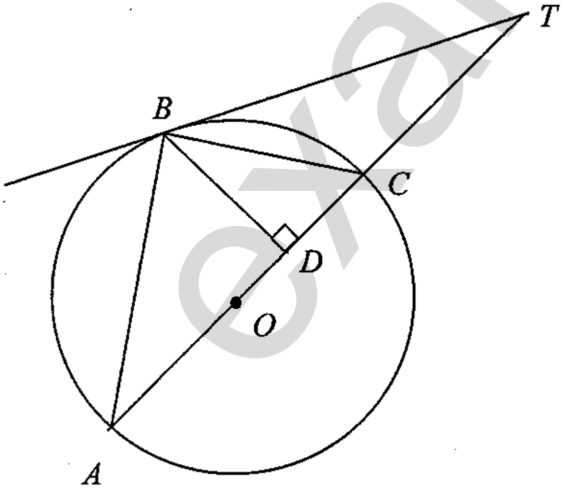
x	1.0	2.0	3.0	4.0	5.0
y	1.10	1.86	2.61	3.44	4.08

It is known that x and y are related by the equation of the form $ay^2 = x(1+bx)$, where a and b are constants. Due to experimental errors, one of the values of y has been recorded incorrectly.

(i) Plot $\left(\frac{y^2}{x}\right)$ against x and use your graph to estimate the value of a and of b. [6]

(ii) State the value of y that has been recorded incorrectly and estimate the correct value. [2]

11 In the diagram, AC is the diameter of the circle with centre O. ACT is a straight line and BT is a tangent to the circle at B. Given that $AB = BT$ and $\angle ADB = 90^o$, prove that



(i) ΔABC is similar to ΔBDC. [2]

(ii) $\angle BTC = \frac{1}{2}(180^o - \angle BCT)$ [3]

(iii) $\angle BAC = 30^o$. [2]

12 The height of water in a harbour changes with tides. The height, h metres, of the water during a particular day can be modelled by the equation, $h = 1.2\cos\left(\frac{\pi x}{6}\right) - 0.4\sin\left(\frac{\pi x}{6}\right) + 1.5$, where x is the number of hours after midnight.

(i) Express h in the form $R\cos\left(\frac{\pi x}{6} + \alpha\right) + 1.5$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3]

(ii) Find the maximum height of the tides. [1]

(iii) At what times are the tides 2.5 m high? Give your answers correct to the nearest minute. [3]

End of Paper 1

CANDIDATE NAME	CLASS	REGISTER NUMBER

BALESTIER HILL SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2016
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC

ADDITIONAL MATHEMATICS 4047 /02

15 Aug 2016 **Monday** **2 hours 30 mins**

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **ALL** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
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Mathematical Formulae

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Quadratic Equation

For the equation $ax^2 + bx + c = 0$, $a \neq 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} ab \sin C.$$

1 **(i)** Sketch the graph of $y = 4\sqrt{x}$. [1]

(ii) On the same axes, sketch the graph of $y = \frac{8}{\sqrt{x^3}}$. [1]

(iii) Calculate the x co-ordinate of the point of intersection of your graphs in exact form. [2]

(iv) Determine, with explanation, whether the tangents to the graphs at the point of intersection are perpendicular. [4]

2 A curve is such that $\frac{d^2y}{dx^2} = 8e^{-2x}$. Given that $\frac{dy}{dx} = 9$ when $x = 0$ and the curve passes through the point (ln 2, 13 ln 2), find the equation of the curve. [4]

3 **(i)** The equation $x^2 + px + q = 0$ has roots α and β. Given that $\alpha^2 + \beta^2 = 85$ and $\alpha - \beta = 1$, find the positive value of p and of q. [4]

(ii) With the values of p and q found in **(i)**, find a quadratic equation with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. [3]

4 The equation of a curve is $\mathrm{f}(x) = x^3 \ln x$.

(i) Show that the curve, $\mathrm{f}(x) = x^3 \ln x$, has only one stationary point. [5]
Find the x-coordinate of the stationary point of the curve in exact form.

(ii) Prove that the value of $\mathrm{f}''(x)$ at the stationary point is $\frac{3}{\sqrt[3]{e}}$. [2]

(iii) What does the result of part (ii) imply about the stationary point? [1]

5 **(i)** Show that $\sin\theta + \sin 3\theta = 4\sin\theta\cos^2\theta$. [3]

(ii) Hence, solve the equation $\sin\theta + \sin 3\theta = \cos\theta$ for $-\pi \le \theta \le \pi$. [5]

6 (i) Given that $\dfrac{4x^3-3x^2+4x+10}{(2x+1)(x^2+2)}=2+\dfrac{A}{2x+1}-\dfrac{Bx+C}{(x^2+2)}$, where A, B and C are constants, find the value of A and of B and show that $C=0$. [6]

(ii) Differentiate $\ln(x^2+2)$ with respect to x. [1]

(iii) Using the results from parts **(i)** and **(ii)**, find $\int \dfrac{4x^3-3x^2+4x+10}{(2x+1)(x^2+2)}\,dx$ [3]

7 (a) Solve $\dfrac{8}{\log_3 x^2}-\dfrac{1}{\log_x 3}=3$. [4]

(b) Miss Gossip started a rumour in a lecture theatre. The spread of the rumour can be modelled by the exponential curve $P=\dfrac{3000}{1+9e^{-kt}}$, where P represents the number of students who heard the rumour at time t, k is a constant and t is time measured in hours.

(i) Two hours after the lecture, 600 students had heard the rumour. Show that $k=\ln\left(\dfrac{3}{2}\right)$ and find the number of students who had heard the rumour after 4 hours. [4]

(ii) If the school has 3000 students, show that it took approximately 5.419 hours for the rumour to spread to half the student population. [3]

8 The function f is defined by $f(x)=a\cos\left(\dfrac{x}{3}\right)+c$ for $0^o \le x \le 540^o$.

Given that the function has a maximum value of 2 and a minimum value of -4,

(i) state values of a and c, [2]

(ii) state the period of f (x), [1]

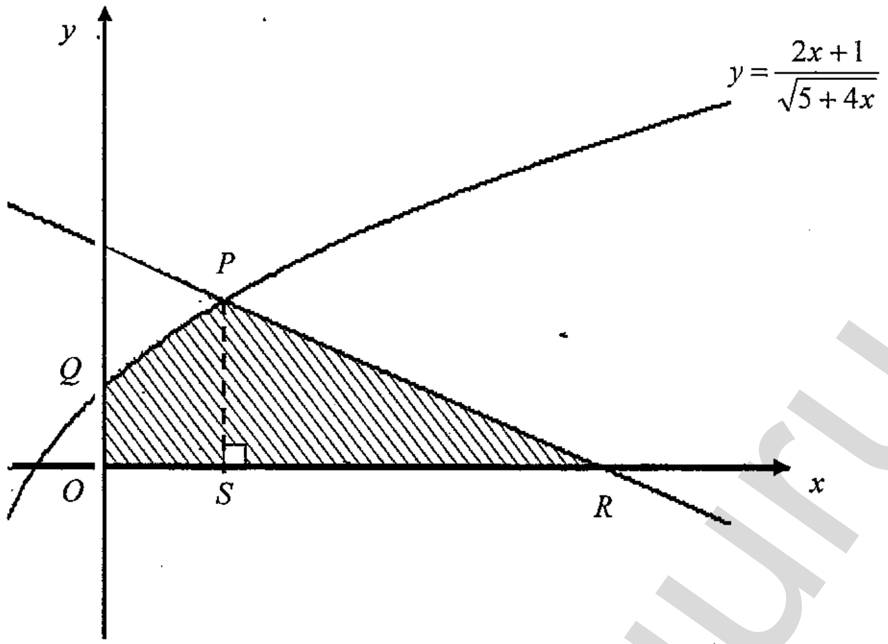
(iii) find the x coordinate(s) of the point(s) where the curve meets the x-axis, [3]

(iv) sketch the graph of $f(x)=a\cos\left(\dfrac{x}{3}\right)+c$ for $0^o \le x \le 540^o$ and [6]

the graph of $g(x)=4-3\sin x$ for $0^o \le x \le 540^o$ on the same axis.

9(a) Show that $\frac{d}{dx}\left[(x-1)\sqrt{5+4x}\right]=\frac{6x+3}{\sqrt{5+4x}}$ [3]

(b)



The diagram shows part of the curve $y=\frac{2x+1}{\sqrt{5+4x}}$. The line PR is a normal to the curve at P. Q is the point where the curve cuts the y-axis and S is a point directly below P.

(i) Given that the x-coordinate of P is 1, find the equation of the line PR. [4]

(ii) Without calculating the area under the curve from $x = 0$ to $x = 1$, explain briefly why $\int_0^1 \frac{2x+1}{\sqrt{5+4x}}\,dx > \frac{1}{2}\left(1+\frac{1}{\sqrt{5}}\right)$. [2]

(iii) Find the area of the shaded region. [3]

10 A particle travels in a straight line such that, t seconds after passing a fixed point O, its acceleration, a m/s^2, is given by $a=200e^{-\frac{t}{2}}$. The particle has an initial velocity of -360 m/s.

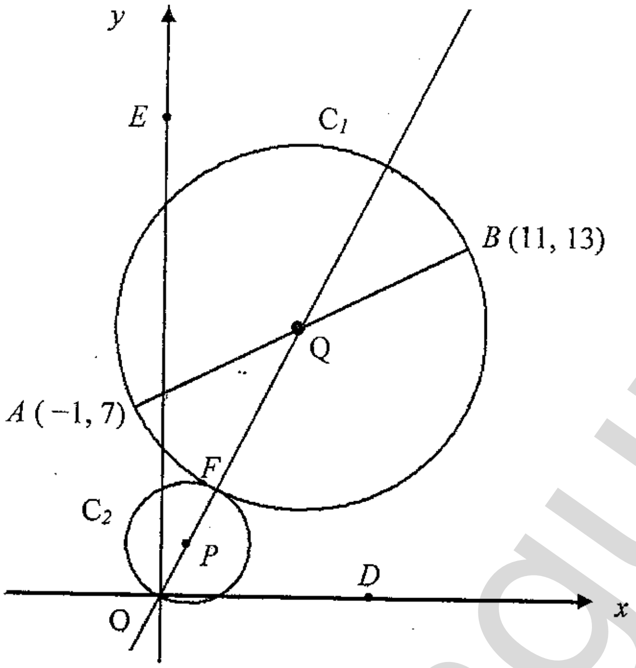
(i) Find an expression for the velocity of the particle. [2]

(ii) Find an expression for the displacement of the particle from O. [2]

(iii) Show that when the particle is instantaneously at rest, $t = \ln 100$. [3]

(iv) Calculate the total distance travelled by the particle for the first 6 seconds. [4]

11 The diagram below shows two circles C_1 and C_2 touching each other at point F. C_1 has centre at Q and C_2 has centre at P. The points A $(-1, 7)$ and B $(11, 13)$ lie on C_1, and AB is the diameter of C_1. The points, O, P and Q lie on a straight line.



(i) Find the equation of C_1. [3]

(ii) Find the equation of the tangent to the 2 circles at F, given that the point F is $(2, 4)$. [3]

(iii) If the co-ordinates of P is $(1, 2)$, determine whether a point $(1, 5)$ lies inside, outside or on circle C_2. [2]

A third circle C_3 is drawn with DE as its diameter, where D and E are points on the x and y axis respectively.

(iv) State whether the origin O lies on C_3. Explain your answer. [1]

End of Paper 2

1 The graph $y = x^2 + 3px - 2q$, where p and q are constants, is always positive for all real values of x.

(i) Find an inequality connecting p and q.

(ii) Explain why q cannot be positive. [3]

(i)
Since $x^2 + 3px - 2q > 0$
For no real roots,
Discriminant, $b^2 - 4ac$
$=(3p)^2 - 4(1)(-2q) < 0$ M1
$9p^2 + 8q < 0$ oe A1

(ii) The graph has y-intercept $-2q$, since graph is always positive, B1
$-2q > 0$
$q < 0$

2 A prism with a trapezium base has a volume of $(14+11\sqrt{2})cm^3$. The trapezium has a height of $(3\sqrt{2}+2)$ cm and its parallel sides are $\sqrt{2}$ cm and 2 cm respectively. Find the height of the prism, leaving your answer in the form $(\frac{\sqrt{2}+a}{b})$ cm, where a and b are integers. [3]

Area of trapezium
$\frac{1}{2}(\sqrt{2}+2)(3\sqrt{2}+2)$
$=\frac{1}{2}(6+2\sqrt{2}+6\sqrt{2}+4)$
$=\frac{1}{2}(10+8\sqrt{2})$
$=5+4\sqrt{2}$ B1
height =
$\frac{14+11\sqrt{2}}{(5+4\sqrt{2})} = \frac{14+11\sqrt{2}}{5+4\sqrt{2}} \times \frac{5-4\sqrt{2}}{5-4\sqrt{2}}$ M1
$=\frac{70-56\sqrt{2}+55\sqrt{2}-88}{(5)^2-(4\sqrt{2})^2}$
$=\frac{-18-\sqrt{2}}{-7}$
$=\frac{\sqrt{2}+18}{7}$ A1

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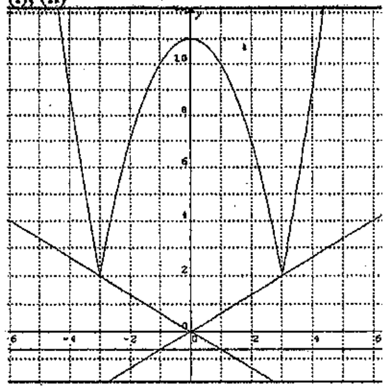
3 (i) Sketch the graph of $y = |x^2 - 9| + 2$. [3]

(ii) Determine the range of values of m for which the line $y = mx$ does not intersect the graph of $y = |x^2 - 9| + 2$. [1]

(i), (ii)



Vertex B1
Shift B1
Shape B1 B3

Critical points $(-3, 2)$ and $(3, 2)$
Gradient of lines through
$(-3, 2)$ and $(3, 2)$
are $-\frac{2}{3}$ & $\frac{2}{3}$ hence

$-\frac{2}{3} < m < \frac{2}{3}$ B1

4 A curve has equation $y = \frac{\sin x}{e^{2x}}$ for $0 \le x \le \frac{\pi}{2}$

(i) Prove that if y is an increasing function, $\tan x < \frac{1}{2}$. [3]

(ii) A point (x, y) moves along the curve $y = \frac{\sin x}{e^{2x}}$ such that the y-coordinate is decreasing at a rate of 0.2 units per second. Find the rate of change of the x-coordinate when x = 0.5. [3]

(i)
$y = \frac{\sin x}{e^{2x}}$
$\frac{dy}{dx} = \frac{e^{2x}\cos x - 2e^{2x}\sin x}{e^{4x}}$ M1
$= \frac{\cos x - 2\sin x}{e^{2x}}$
For increasin g function,
$\frac{dy}{dx} > 0$ M1
$\frac{\cos x - 2\sin x}{e^{2x}} > 0$
$\cos x - 2\sin x > 0$
$\cos x > 2\sin x$
$\tan x < \frac{1}{2}$ B1

(ii)

Given that $\frac{dy}{dt} = -0.2$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$-0.2 = \frac{\cos 0.5 - 2\sin 0.5}{e^1} \times \frac{dx}{dt}$ M1

$-0.2 = -0.029897016 \times \frac{dx}{dt}$ M1

$\frac{dx}{dt} = 6.6896 \approx 6.69$ *units per second*. A1

5 Given that $f(x) = 6x^3 + 3x^2 - x + 2 = 0$,

(i) show that the equation f(x) = 0 has only one real root. Find the value of the real root. [5]

(ii) sketch the curve, showing clearly the x and y intercepts. [2]

(i) Let $f(x) = 6x^3 + 3x^2 - x + 2$

By trial and error,

$f(-1) = 6(-1)^3 + 3(-1)^2 - (-1) + 2$

$= -6 + 3 + 1 + 2$

$= 0$

$(x+1)$ is a factor of $f(x)$. M1

$f(x) = (x+1)(6x^2 + bx + 2)$

By comparing coeffs,

$3 = b + 6$

$b = -3$ M1

$f(x) = (x+1)(6x^2 - 3x + 2)$

$= (x+1)(6x^2 - 2x + 3) = 0$

$x = -1$, *discriminant* $= b^2 - 4ac$ M1

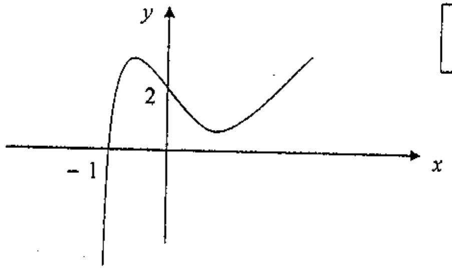
$= (-2)^2 - 4(6)(2)$

$= -39 < 0$

no real roots M1

Hence, $x = -1$ is the only real root. M1

(ii)



B1 Shape of the curve
B1 Intercepts

B2

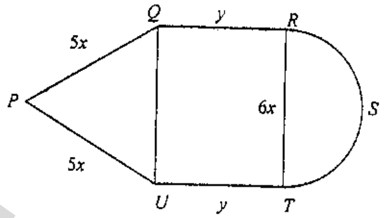
6 A piece of wire, of length 150 cm, is bent into the shape as shown in the diagram. The shape consists of an isosceles triangle PQU where $PQ = PU = 5x$ cm, a rectangle $QRTU$ and a semi-circle RST. Given further that $QR = y$ cm and $RT = 6x$ cm,

(i) show that the enclosed area, A cm^2, is given by [4]

$$A = 450x - 9x^2(2 + \frac{\pi}{2}).$$

(ii) Given that x can vary, find the value of x for which the area is stationary. [2]

(iii) Explain why this value of x gives the largest area possible. [1]



(i) $10x + 2y + 3\pi x = 150$

$2y = 150 - 10x - 3\pi x$ M1

Area of triangle$= \frac{1}{2}(6x)(4x) = 12x^2$ M1

Area of rectangle $= 6xy = 3x(150 - 10x - 3\pi x) = 450x - 30x^2 - 9\pi x^2$

Area of semicircle $= \frac{9\pi x^2}{2}$ M1

Total area $= 12x^2 + 450x - 30x^2 - 9\pi x^2 + \frac{9\pi x^2}{2}$

A$= 450x - 18x^2 - \frac{9\pi x^2}{2}$

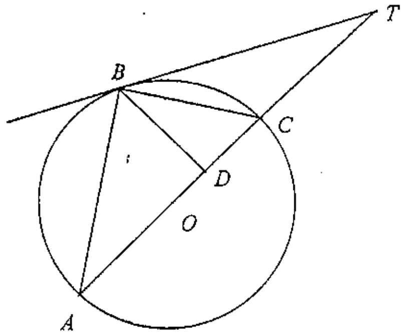
$= 450x - 9x^2\left(2 + \frac{\pi}{2}\right)$ B1

(ii)	$\frac{dA}{dx}=450-18x\left(2+\frac{\pi}{2}\right)=0$ $18x\left(2+\frac{\pi}{2}\right)=450$ $x=\frac{450}{18\left(2+\frac{\pi}{2}\right)}$ $=7.00123...$ $=7.00\ cm$	M1 A1		
(iii)	$\frac{d^2A}{dx^2}=-18\left(2+\frac{\pi}{2}\right)<0$ Hence the stationary value of x gives the maximum area.	B1		
7	Given that the first four terms in the expansion of $(1+3x)^2(1+x)^n$ in ascending powers of x is			
	$1+ax+bx^2+cx^3+...$, where a, b and c are constants, and n is a positive integer.			
(i)	Express a and b in terms of n.			[3]
(ii)	If $b=72$, prove that $n=7$ and find the value of c.			[4]
(iii)	Using the value of n found in (i), find the coefficient of x^2 in the expansion of $(1+3x)^2(1+x)^{n+1}$.			[2]
(iii)	$(1+3x)^2(1+x)^n$ $=(1+6x+9x^2)\left(1+nx+\frac{n(n-1)x^2}{2}+\frac{n(n-1)(n-2)x^3}{6}+...\right)$ $=1+nx+\frac{n(n-1)x^2}{2}+\frac{n(n-1)(n-2)x^3}{6}+...$ $6x+6nx^2 \quad +3n(n-1)x^3+...$ $9x^2 \quad +9nx^3+...$ $a=n+6$ $b=\frac{n(n-1)}{2}+6n+9$	M1 B1 B1		

(ii)	$b=\frac{n(n-1)}{2}+6n+9=72$ $n^2-n+12n+18=144$ $n^2+11n-126=0$ $n=7, n=-18(NA)$ $c=\frac{7(6)(5)}{6}+3(7)(6)+9(7)=224$	M1 M1 A1 B1		
(iii)	$(1+3x)^2(1+x)^{n+1}$ $(1+x)(1+3x)^2(1+x)^n$ $==(1+x)(1+13x+72x^2+...)$ $=...72x^2+13x^2+...$ Coefficient of $x^2=72+13=85$	M1 B1		
8(i)	Show that $\frac{d}{dx}(\ln(\sin^2 x))=2\cot x$			[2]
(ii)	By expressing $x\cot x$ as $\left(\frac{x}{\tan x}\right)$, differentiate $x\cot x$ with respect to x.			[3]
(iii)	Using your results from parts (i) and (ii), find $\int x\operatorname{cosec}^2 x\, dx$ and prove that $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} x\operatorname{cosec}^2 x\, dx=\frac{1}{2}\ln 2-\pi\left(\frac{1}{4}-\frac{\sqrt{3}}{6}\right)$.			[4]
(i)	$\frac{d}{dx}(\ln(\sin^2 x))=\frac{1}{\sin^2 x}(2\sin x\cos x)$ $=\frac{2\cos x}{\sin x}$ $=2\cot x$ (*shown*)	M1 B1		
(ii)	$\frac{d(x\cot x)}{dx}=\frac{d}{dx}\left(\frac{x}{\tan x}\right)$ $=\frac{\tan x(1)-x\sec^2 x}{\tan^2 x}$ $=\frac{\frac{\sin x}{\cos x}-\frac{x}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}}$ $=\frac{\sin x\cos x-x}{\sin^2 x}$ $=\cot x-x\operatorname{cosec}^2 x$	 M1 M1 B1		

(iii)	$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cot x - x\,\mathrm{cosec}^2 x\,dx = [x\cot x]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$	M1	
	$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cot x\,dx - \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} x\,\mathrm{cosec}^2 x\,dx = [x\cot x]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$		
	$\frac{1}{2}[\ln\sin^2 x]_{\frac{\pi}{6}}^{\frac{\pi}{4}} - [x\cot x]_{\frac{\pi}{6}}^{\frac{\pi}{4}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} x\,\mathrm{cosec}^2 x\,dx$	M1	
	$LHS = \frac{1}{2}\left[\ln\left(\frac{1}{\sqrt{2}}\right)^2 - \ln\left(\frac{1}{2}\right)^2\right] - \left[\frac{\pi}{4}(1) - \frac{\pi}{6}(\sqrt{3})\right]$	M1	
	$= \frac{1}{2}[\ln 2] - \frac{\pi}{4} + \frac{\pi\sqrt{3}}{6}$		
	$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} x\,\mathrm{cosec}^2 x\,dx = \frac{1}{2}\ln 2 - \pi\left(\frac{1}{4} - \frac{\sqrt{3}}{6}\right)$ *shown*	B1	
9	A point P is equidistant from A (1, 4) and B (5, 2). Given that P lies on the line $y - x = 1$, find		
	(i) the co-ordinates of the point P,		[3]
	(ii) the equation of the perpendicular bisector of AB,		[3]
	(iii) a point Q such that $APBQ$ is a parallelogram.		[2]
	(iv) Find the area of triangle AOP, where O is the origin.		[2]
(i)	Let P be (x, y)		
	$\sqrt{(x-1)^2 + (y-4)^2} = \sqrt{(x-5)^2 + (y-2)^2}$	M1	
	$(x-1)^2 + (x-3)^2 = (x-5)^2 + (x-1)^2$	M1	
	$x^2 - 6x + 9 = x^2 - 10x + 25$		
	$4x = 16$		
	$x = 4, y = 5$	A1	
(ii)	Gradient of $AB = \frac{4-2}{1-5} = -\frac{1}{2}$		
	Gradient of perpendicular bisector = 2	M1	
	Midpoint of AB , M (3, 3)	M1	
	Equation of the perpendicular bisector AB : $y = 2x + c$ $3 = 6 + c$ $c = -3$		
	$y = 2x - 3$	A1	
(iii)	Let Q be (x, y)		
	$\left(\frac{x+4}{2}, \frac{y+5}{2}\right) = (3,3)$	M1	
	$x = 2, \quad y = 1$ Q (2, 1)	A1	

(iv)	Area $= \frac{1}{2}\left\vert\begin{matrix}0 & 4 & 1 & 0\\ 0 & 5 & 4 & 0\end{matrix}\right\vert$	M1	
	$= 5\frac{1}{2}$ *sq units*	A1	
10	In the diagram, AC is the diameter of the circle with centre O. ACT is a straight line and BT is a tangent to the circle at B. Given that $AB = BT$ and $\angle ADB = 90^\circ$, prove that		



(i)	ΔABC is similar to ΔBDC .		[2]
(ii)	$\angle BTC = \frac{1}{2}(180^o - \angle BCT)$		[3]
(iii)	$\angle BAC = 30^o$.		[2]
(i)	$\angle ABC = \angle BDC$ (*$\angle s$ in a semi circle*)	M1	
	$\angle BCA = \angle DCB$ (*common $\angle s$*)		
	ΔABC is similar ΔBDC	A1	
(ii)	$\angle CBT = \angle BAT$ (*alternate segment theorem*)	M1	
	$\angle BAT = \angle BTA$ (*$AB = BT$, ΔABT is isosc*) $= \angle BTC$		
	Hence $\angle CBT = \angle BTC$		
	ΔBTC is isosceles	M1	
	$\angle BTC = \frac{1}{2}(180^o - BCT)$	B1	

(iii)	Let $\angle BTC = \theta$ Hence $\angle CBT = \theta$ $\angle BCA = 2\theta$ (*ext* $\angle$ *of a triangle*) $\angle BAT = \theta$ (ΔBAT *is isosceles*) *In* ΔABT $3\theta + 90° = 180°$ $3\theta = 90°$ $\theta = 30°$ $\angle BTC = 30°$	M1 B1		

12	The height of water in a harbour changes with tides. The height, h metres, of the water during a particular day can be modelled by the equation, $1.2\cos\left(\frac{\pi x}{6}\right) - 0.4\sin\left(\frac{\pi x}{6}\right) + 1.5$, where x is the number of hours after midnight.		
	(i)	Express h in the form $R\cos\left(\frac{\pi x}{6} + \alpha\right)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.	[3]
	(ii)	Find the maximum height of the tides.	[1]
	(iii)	At what times are the tides 2.5 m high? Give our answers correct to the nearest minute.	[3]

(i)	$h = 1.2\cos\left(\frac{\pi x}{6}\right) - 0.4\sin\left(\frac{\pi x}{6}\right) + 1.5$ $1.2\cos\left(\frac{\pi x}{6}\right) - 0.4\sin\left(\frac{\pi x}{6}\right) = R\cos\left(\frac{\pi x}{6} + \alpha\right)$ $R = \sqrt{1.2^2 + 0.4^2}$ $= \sqrt{1.6}$ $= 1.2649...$ $= 1.26$ $\tan\alpha = \frac{0.4}{1.2}$ $\alpha = 0.321...$ $= 0.322$ $h = \sqrt{1.6}\cos\left(\frac{\pi x}{6} + 0.322\right) + 1.5$ $= 1.26\cos\left(\frac{\pi x}{6} + 0.322\right) + 1.5$	M1 M1 B1		
(ii)	$h = \sqrt{1.6}\cos\left(\frac{\pi x}{6} + 0.322\right) + 1.5$ $h_{\max} = \sqrt{1.6} + 1.5 = 2.76m$	B1		

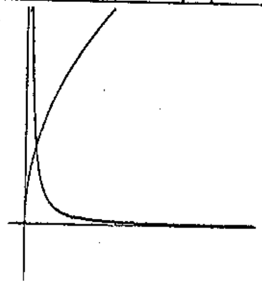
(iii)	$h = \sqrt{1.6}\cos\left(\frac{\pi x}{6} + 0.322\right) + 1.5 = 2.5$ $\cos\left(\frac{\pi x}{6} + 0.322\right) = \frac{1}{\sqrt{1.6}}$ $\cos\left(\frac{\pi x}{6} + 0.322\right) = 0.790569415$ *basic* $\angle, \alpha = 0.659$ $\frac{\pi x}{6} + 0.322 = 0.659, 5.624, 6.942, 11.907$ $x = 00\,39, 10\,08, 12\,39, 22\,08$	M1 M1 A1		

End of Paper 1

Balestier Hill P2.

1	(i)	Sketch the graph of $y = 4\sqrt{x}$.	[1]
	(ii)	On the same axes, sketch the graph of $y = \frac{8}{\sqrt{x^3}}$.	[1]
	(iii)	Calculate the x co-ordinate of the point of intersection of your graphs in exact form.	[2]
	(iv)	Determine, with explanation, whether the tangents to the graphs at the point of intersection are perpendicular.	[4]

(i),(ii)



$4\sqrt{x} = \frac{8}{\sqrt{x^3}}$

$x^{\frac{1}{2}+\frac{3}{2}} = 2$

$x^2 = 2$

$x = \sqrt{2}$

(iii)

$\frac{dy}{dx} = \frac{2}{\sqrt{x}}$

$\frac{dy}{dx} = -\frac{12}{\sqrt{x^5}}$

Product of gradients $= \frac{2}{\sqrt{x}} \times -\frac{12}{\sqrt{x^5}}$

$= -\frac{24}{x^2}$

$= -8.485 \neq -1$

Hence the tangents are not perpendicular at the point of intersection.

2	A curve is such that $\frac{d^2y}{dx^2} = 8e^{-2x}$. Given that $\frac{dy}{dx} = 9$ when $x = 0$ and the curve passes through the point (ln 2, 13 ln 2), find the equation of the curve.	[4]

$\frac{d^2y}{dx^2} = 8e^{-2x}$

$\frac{dy}{dx} = \int 8e^{-2x}\,dx$

$= -4e^{-2x} + c = 9$

$-4 + c = 9$

$c = 13$

$y = \int -4e^{-2x} + 13\,dx$

$y = 2e^{-2x} + 13x + c$

Subst (ln 2,15 ln 2):

$13\ln 2 = 2e^{-2\ln 2} + 13\ln 2 + c$

$0 = 2(\frac{1}{4}) + c$

$c = \frac{1}{2}$

$y = 2e^{-2x} + 13x - \frac{1}{2}$

3	(i)	The equation $x^2 + px + q = 0$ has roots α and β. Given that $\alpha^2 + \beta^2 = 85$ and $\alpha - \beta = 1$, find the positive value of p and of q.	[4]
	(ii)	With the values of p and q found in (i), find a quadratic equation with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.	[3]

(i)

Sum of roots = $\alpha + \beta = -p$

Product of roots = $\alpha\beta = q$

$(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta = 85 + 2q = p^2$ --(1)

$(\alpha-\beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta = 85 - 2q = 1$ ---(2)

(1) + (2)

$170 = p^2 + 1$

$p^2 = 169$

$p = 13$

$q = 42$

(ii)	Sum of new roots = $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{(\alpha\beta)}=-\frac{13}{42}$ Product of new roots = $\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)=\frac{1}{42}$ New equation : $x^2+\frac{13}{42}x+\frac{1}{42}=0$ $42x^2+13x+1=0$	
4	The equation of a curve is $f(x)=x^3\ln x$.	
(i)	Show that the curve, $f(x)=x^3\ln x$, has only one stationary point. Find the x-coordinate of the stationary point of the curve in exact form.	[5]
(ii)	Prove that the value of $f''(x)$ at the stationary point is $\frac{3}{\sqrt[3]{e}}$.	[2]
(iii)	What does the result of part (iii) imply about the stationary point?	[1]
(i)	$f'(x)=x^3\left(\frac{1}{x}\right)+3x^2\ln x$ $=x^2+3x^2\ln x=0$ $x^2(1+3\ln x)=0$ $x=0\quad or\quad 1+3\ln x=0$ $\ln x=-\frac{1}{3}$ $x=e^{-\frac{1}{3}}$ $x=\frac{1}{\sqrt[3]{e}}$ f(x) is not defined for x= 0. Hence f(x) only has one stationary point.	
(ii)	$f''(x)=2x+3x+6x\ln x=5x+6x\ln x$ $\frac{5}{\sqrt[3]{e}}+\frac{6}{\sqrt[3]{e}}\ln e^{-\frac{1}{3}}$ $=\frac{5}{\sqrt[3]{e}}-\frac{1}{3}\left(\frac{6}{\sqrt[3]{e}}\right)$ $=\frac{1}{\sqrt[3]{e}}(5-2)$ $=\frac{3}{\sqrt[3]{e}}$	M1 B1

(iii)	Since $f''(x)=\frac{3}{\sqrt[3]{e}}>0$ The stationary point is a minimum point.		B1
5	(i) Show that $\sin\theta+\sin3\theta=4\sin\theta\cos^2\theta$		[3]
	(ii) Hence, solve the equation $\sin\theta+\sin3\theta=\cos\theta$ for $-\pi\le\theta\le\pi$.		[5]
(i)	$\sin\theta+\sin3\theta$ $=\sin\theta+\sin(\theta+2\theta)$ $=\sin\theta+\sin\theta\cos2\theta+\cos\theta\sin2\theta$ $=\sin\theta+2\sin\theta\cos^2\theta+(2\cos^2\theta-1)\sin\theta$ $=4\sin\theta\cos^2\theta$	M1 M1 B1	
(ii)	$4\sin\theta\cos^2\theta-\cos\theta=0$ $\cos\theta(4\sin\theta\cos\theta-1)=0$ $\cos\theta=0\qquad 4\sin\theta\cos\theta-1=0$ $\theta=\frac{\pi}{2},-\frac{\pi}{2}\qquad 2\sin2\theta=1$ $\sin2\theta=\frac{1}{2}$ $basic\ \angle,\ \alpha=\frac{\pi}{6}$ $2\theta=\frac{\pi}{6},\frac{5\pi}{6},\frac{13\pi}{6},\frac{17\pi}{6}$ $\theta=\frac{\pi}{12},\frac{5\pi}{12},\frac{13\pi}{12},\frac{17\pi}{12}$ $\theta=\frac{\pi}{12},\frac{5\pi}{12},-\frac{11\pi}{12},-\frac{7\pi}{12}$	M1 M1 M1 M1 A1	
6	(i) Given that $\frac{4x^3-3x^2+4x+10}{(2x+1)(x^2+2)}=2+\frac{A}{2x+1}-\frac{Bx+C}{(x^2+2)}$, where A, B and C are constants, find the value of A and of B and show that $C=0$.		[6]
	(ii) Differentiate $\ln(x^2+2)$ with respect to x.		[1]
	(iii) Using the results from parts (i) and (ii), find $\int\frac{4x^3-3x^2+4x+10}{(2x+1)(x^2+2)}dx$		[3]
(i)	$(2x+1)(x^2+2)=2x^3+x^2+4x+2$		

	$2x^3+x^2+4x+2\overline{)4x^3-3x^2+4x+10}$ with quotient 2 $4x^3+2x^2+8x+4$ $-5x^2-4x+6$ $\frac{4x^3-3x^2+4x+10}{(2x+1)(x^2+2)}=2+\frac{6-4x-5x^2}{(2x+1)(x^2+2)}$ $\frac{6-4x-5x^2}{(2x+1)(x^2+2)}=\frac{A}{(2x+1)}-\frac{Bx+C}{(x^2+2)}$ $6-4x-5x^2=A(x^2+2)+(Bx+C)(2x+1)$ $6-4\left(-\frac{1}{2}\right)-5\left(-\frac{1}{2}\right)^2=A\left(2\frac{1}{4}\right)$ $A=3$ *Let* $x=0$ $6=3(2)+C$ $C=0$ *Comparing coeff of x* $B=4$ $\frac{4x^3-3x^2+4x+10}{(2x+1)(x^2+2)}=2+\frac{3}{2x+1}-\frac{4x}{(x^2+2)}$		
(ii)	$\frac{d}{dx}\left[\ln(x^2+2)\right]=\frac{2x}{x^2+2}$		
(iii)	$\int\frac{4x^3-3x^2+4x+10}{(2x+1)(x^2+2)}dx=\int 2+\frac{3}{2x+1}-\frac{4x}{(x^2+2)}dx$ $=2x+\frac{3}{2}\ln(2x+1)-2\ln(x^2+2)+C$		

7	(a)	Solve $\frac{8}{\log_3 x^2}-\frac{1}{\log_x 3}=3.$	[4]
	(b)	Miss Gossip started a rumour in a lecture theatre. The spread of the rumour can be modelled by the exponential curve $P=\frac{3000}{1+9e^{-kt}}$, where P represents the number of students who heard the rumour at time t, k is a constant and t is time measured in hours.	
		(i) Two hours after the lecture, 600 students had heard the rumour. Show that	[4]

		$k=\ln\left(\frac{3}{2}\right)$ and find the number of students who had heard the rumour after 4 hours.	
		(ii) If the school has 3000 students, show that it took approximately 5.419 hours for the rumour to spread to half the student population.	[3]

(i)	$\frac{8}{\log_3 x^2}-\frac{1}{\log_x 3}=3$ $\frac{4}{\log_3 x}-\log_3 x=3$ *let* $u=\log_3 x$ $\frac{4}{u}-u=3$ $4-u^2=3u$ $u^2+3u-4=0$ $(u+4)(u-1)=0$ $u=-4$ *or* $u=1$ $\log_3 x=-4$ *or* $\log_3 x=1$ $x=3^{-4}$ *or* $x=3$ $x=\frac{1}{81}$
b(i)	*At* $t=2$ $600=\frac{3000}{1+9e^{-2k}}$ $1+9e^{-2k}=5$ $e^{-2k}=\frac{4}{9}$ $k=-\frac{1}{2}\ln\left(\frac{4}{9}\right)$ $=\ln\left(\frac{3}{2}\right)$
	After 4 hours, $P=\frac{3000}{1+9e^{-4\ln\left(\frac{3}{2}\right)}}$ $P=\frac{3000}{1+9\left(\frac{16}{81}\right)}=1080$

(ii)

$$P=\frac{3000}{1+9e^{-kt}}$$

$$1500=\frac{3000}{1+9e^{-\ln\left(\frac{3}{2}\right)t}}$$

$$9e^{-\ln\left(\frac{3}{2}\right)t}=1$$

$$e^{-\ln\left(\frac{3}{2}\right)t}=\frac{1}{9}$$

$$-\ln\left(\frac{3}{2}\right)t=\ln\frac{1}{9}$$

$$t=\frac{\ln\left(\frac{1}{9}\right)}{\ln\left(\frac{3}{2}\right)}$$

$$=5.419h$$

8 The function f is defined by $f(x)=a\cos\left(\frac{x}{3}\right)+c$ for $0^\circ \le x \le 540^\circ$.

Given that the function has a maximum value of 2 and a minimum value of -4,

(i) state values of a and c, [2]

(ii) state the period of f (x), [1]

(iii) find the x coordinate(s) of the point(s) where the curve meets the x-axis, [3]

(iv) sketch the graph of $f(x)=a\cos\left(\frac{x}{2}\right)+c$ for $0^\circ \le x \le 540^\circ$ and the graph of $g(x)=4-3\sin x$ for $0^\circ \le x \le 540^\circ$ on the same axis. [6]

(i) $a=3, c=-1$

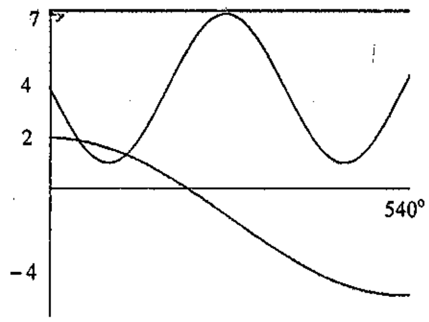
(ii) $$\text{Period}=\frac{360^\circ}{\frac{1}{3}}=1080^\circ$$

(iii) $$3\cos\left(\frac{x}{3}\right)-1=0$$

$$\cos\left(\frac{x}{3}\right)=\frac{1}{3}$$

$$\frac{x}{3}=70.5^\circ, 289.5^\circ$$

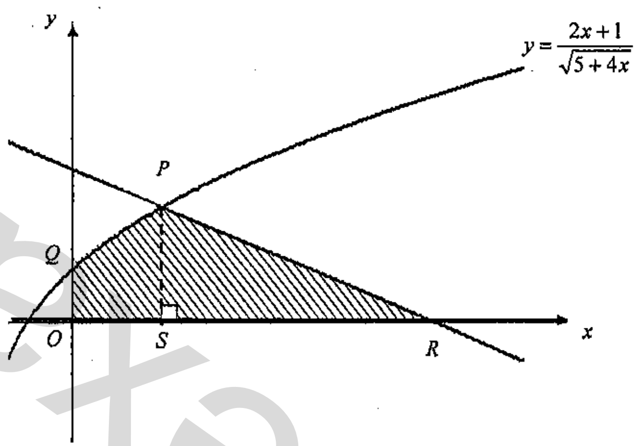
$$x=211.5^\circ, 868.5^\circ\,(rej)$$



B1 – shape, period
B1 – amplitude
B1 – shift

9(a). Show that $\frac{d}{dx}\left[(x-1)\sqrt{5+4x}\right]=\frac{6x+3}{\sqrt{5+4x}}$ [3]

(b)



The diagram shows part of the curve $y=\frac{2x+1}{\sqrt{5+4x}}$. The line PR is a normal to the curve at P. Q is the point where the curve cuts the y-axis and S is a point directly below P.

(i) Given that the x-coordinate of P is 1, find the equation of the line PR. [4]

(ii) Without calculating the area under the curve from $x=0$ to $x=1$, explain briefly why $\int_0^1 \frac{2x+1}{\sqrt{5+4x}}\,dx > \frac{1}{2}\left(1+\frac{1}{\sqrt{5}}\right)$. [2]

(iii) Find the area of the shaded region. [3]

(a)

$$\frac{dy}{dx}=(x-1)\frac{1}{2\sqrt{5+4x}}(4)+\sqrt{5+4x}$$

$$=\frac{1}{\sqrt{5+4x}}(2x-2+5+4x)$$

$$=\frac{6x+3}{\sqrt{5+4x}}$$

(b)(i)

$$y=\frac{2x+1}{\sqrt{5+4x}}$$

$$\frac{dy}{dx}=\frac{8+4x}{(5+4x)\sqrt{5+4x}}$$

$=\frac{12}{27}$ when $x=1$

Equation of PR,

$PR:\quad y=-\frac{27}{12}x+\frac{13}{4}$

$y=-\frac{9}{4}x+\frac{13}{4}$

(ii)

At $x=0$,

$$y=\frac{2(0)+1}{\sqrt{5+4(0)}}=\frac{1}{\sqrt{5}}$$

Area of trapezium $OQPS=\frac{1}{2}\left(1+\frac{1}{\sqrt{5}}\right)\times 1$

Area of shaded region under curve from $x=0$ to $x=1$ is more than area of trapezium.

(iii)

At $y=0$,

$$-\frac{27}{12}x+\frac{13}{4}=0$$

$$x=\frac{13}{9}$$

Area of triangle $=\frac{1}{2}(1)\left(\frac{4}{9}\right)=\frac{2}{9}$

Area of shaded region

$$=\int_0^1\frac{2x+1}{\sqrt{5+4x}}\,dx+\frac{2}{9}$$

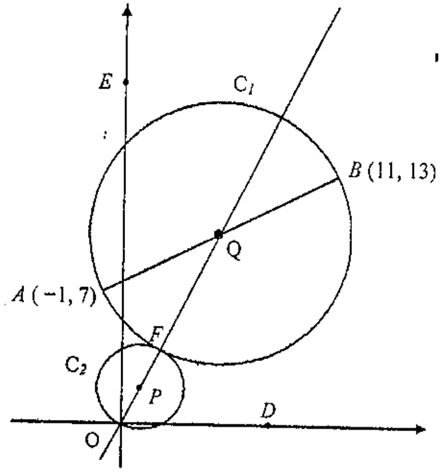
$$=\frac{1}{3}\left[(x-1)\sqrt{5+4x}\right]_0^1+\frac{2}{9}$$

$$=\frac{1}{3}\left[-(-1)\sqrt{5}\right]+\frac{2}{9}$$

$$=\frac{\sqrt{5}}{3}+\frac{2}{9}$$

$=0.968\ (3sf)$

11 (a) The diagram below shows two circles C_1 and C_2 touching each other at point F. C_1 has centre at Q and C_2 has centre at P. The points $A\,(-1, 7)$ and $B\,(11, 13)$ lie on C_1, and AB is the diameter of C_1. The points O,P and Q lie on a straight line.



(i) Find the equation of C_1. [3]

(ii) Find the equation of the tangent to the 2 circles at F, given that F is $(2, 4)$. [3]

(iii) If $P\,(1,2)$, determine whether a point $(1, 5)$ lies inside, outside or on circle C_2. [2]

A third circle C_3 is drawn with DE as its diameter, where D and E are points on the x and y axis respectively.

(iv) State whether the origin O lies on C_3. Explain your answer. [1]

(i)

$$Q=\left(\frac{-1+11}{2},\frac{13+7}{2}\right)$$

$=(5,10)$

Radius $=\sqrt{(5-(-1))^2+(10-7)^2}$

$=\sqrt{36+9}$

$=\sqrt{45}$

$=3\sqrt{5}$

Equation of circle C_1.

$(x-5)^2+(y-10)^2=45$

$x^2+y^2-10x-20y-80=0$

Equation of OQ:

Gradient of $OQ=\frac{10}{5}=2$

Gradient of tangent $=-\frac{1}{2}$

Equation of tangent :

$y=-\frac{1}{2}x+c$

$4=-\frac{1}{2}(2)+c$

$c=5$

$y=-\frac{1}{2}x+5$

(iii) Radius of $C_2=\sqrt{(2-1)^2+(4-2)^2}=\sqrt{5}$

Distance between point (1, 5) and centre =

$\sqrt{(1-1)^2+(5-2)^2}=\sqrt{9}>\sqrt{5}$

Hence point lies outside circle C_2

(iv) Origin lies on C_3 because $\angle DOE=90^\circ$, since DE is the diameter, O must be a point on the circle ($\angle$*in a semicircle*)

10 A particle travels in a straight line such that, t seconds after passing a fixed point O, its acceleration, a m/s^2, is given by $a=200e^{-\frac{t}{2}}$. The particle has an initial velocity of $-360m/s$..

(i) Find an expression for the velocity of the particle. [2]

(ii) Find an expression for the displacement of the particle from O. [2]

(iii) Show that when the particle is instantaneously at rest, $t=\ln 100$. [3]

(iv) Calculate the total distance travelled by the particle for the first 6 seconds. [4]

(i) $v=\int 200e^{-\frac{t}{2}}\,dt$

$v=-400e^{-\frac{t}{2}}+c$

Given $t=0$, $v=-360m/s$

$-360=-400e^0+c$

$-360+400=c$

$c=40$

$v=-400e^{-\frac{t}{2}}+40$

(ii) $s=\int -400e^{-\frac{t}{2}}+40\,dt$

$s=800e^{-\frac{t}{2}}+40t+c$

Given $t=0$, $s=0$

$0=800+c$

$c=-800$

$s=800e^{-\frac{t}{2}}+40t-800$

(iii) $v=-400e^{-\frac{t}{2}}+40=0$

$400e^{-\frac{t}{2}}=40$

$e^{-\frac{t}{2}}=\frac{1}{10}$

$-\frac{t}{2}=\ln\left(\frac{1}{10}\right)$

$t=-2\ln\left(\frac{1}{10}\right)$

$=\ln 100$

(iv) $t=\ln 100$

$s=800\left(\frac{1}{10}\right)+40\ln 100-800$

$=40\ln 100-720$

$=-535.793...$

$t=6$

$s=800e^{-3}+40(6)-800$

$=-520.1703...$

Total distance travelled

=535.793 +(535.793...-520.1703...)

=551.4

=551 m (3sf)

1(a) If n is a positive integer, explain why $8\left(11^{n+1}\right)+7\left(11^{n+2}\right)+11^{n+3}$ is divisible by 103. [2]

(b) Given that $(16t)^{\frac{3}{2}} \times \sqrt{12t} = 2^7 \times 3^x \times t^y$ where t does not have the factor of 2 or 3, find the value of x and of y. [2]

2 The first three terms in the expansion, in ascending powers of x, of $(a-x)(1+2x)^n$ are $3+47x+bx^2$.

(i) By substituting a suitable value of x, find the value of a. [1]

(ii) By considering the coefficient of x, find the value of n. [2]

(iii) Hence, find the value of b. [1]

3(a) It is given that $-3 \le x \le 1$ is the solution of $x^2+px \le q$, find the value of p and of q. [2]

(b) Show that the roots of the equation $x^2+(3k+5)x=3$ are real for all values of k. [2]

4(i) By simplifying $f(x)=5|6x+2|-2|9x+3|$, show that $f(x)=k|3x+1|$, where k is a constant. [2]

(ii) Hence, solve the equation $5|6x+2|=2|9x+3|+6$. [3]

5 A curve has the equation $y=\frac{3x-6}{x+2}$, $x \neq -2$. The curve cuts the x-axis at A. The tangent to the curve at A cuts the y-axis at B.

(a) Find $\frac{dy}{dx}$. [2]

(b) Find the coordinates of A and of B. [4]

6(i) Determine with justification whether $x+2$ is a factor of the polynomial $15x^3+26x^2-11x-6$. [2]

(ii) Find the remainder when $15x^3+26x^2-11x-6$ is divided by $x-3$. [2]

(iii) Find the value of p and of q such that $15x^3+26x^2-11x-6$ is a factor of $15x^4+px^3-37x^2+qx+6$. [2]

7(a) Solve for y in $\log_a 2y^2 + \log_a 8 + \log_a 16y - \log_a 64y = 2\log_a 4$. [3]

(b) If $x = \lg m$ is a solution of the equation $10^{2x+1}+7\left(10^x\right)=26$. Find the value of m. [3]

8(i) On the same axes, sketch and label clearly the graphs of $y=\sqrt[3]{x}$ and $y=\frac{4}{\sqrt{x}}$ for $x>0$. [2]

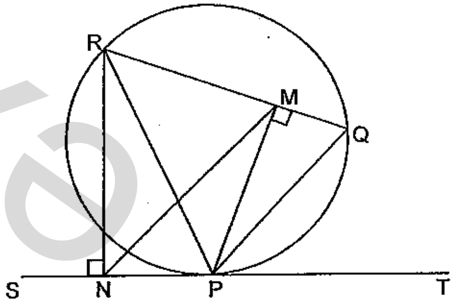
(ii) Solve $\sqrt[3]{x}=\frac{4}{\sqrt{x}}$, leave your answer in exact form. [1]

(iii) Determine with explanation, whether the tangents to the graphs at the point of intersection are perpendicular. [4]

9(a) The gradient of the curve $y=2x^2+mx+n$ at the point (1,5) is 8. Find the value of m and of n. [3]

(b) The variables x and y are related by the equation $y=x^3+\frac{8}{x}$. Given that y is increasing at a rate of 5 units per second when $x=1.6$. Find the corresponding rate of change of x at this instant. Give your answer correct to 2 significant figures. [4]

10



In the diagram above, ST is a tangent to a circle at the point P. The points Q and R lie on the circle. The line PM is perpendicular to the chord QR and the line RN is perpendicular to the tangent ST.

(i) By considering QP as a chord of the circle, find, with explanation, an angle equal to angle QPT. [2]

(ii) Explain why a circle with PR as diameter passes through M and N. [2]

(iii) Prove that the lines MN and QP are parallel. [3]

11(a) Given that $\int e^{4x} \text{f}(x) \, \text{d}x = e^{4x} \sin 3x + c$, where c is an arbitrary constant, find f(x). [3]

(b) (i) By writing $\cos 3x$ as $\cos (2x + x)$, show that $\cos 3x = 4\cos^3 x - 3 \cos x$. [3]

(ii) Hence, find the exact value of $8\cos^3 10° - 6 \cos 10°$. [2]

12(i) Sketch the graph of $y = e^{x+1}$, showing clearly the intercept(s) and asymptote(s), where applicable. [2]

(ii) The equation $\frac{e}{18-9x} = e^{-x}$ can be solved by inserting a straight line to the graph in **(i)**.

(a) Find the equation of the straight line to be added to the graph in **(i)**. [2]

(b) On the graph in **(i)**, sketch the straight line, showing clearly the intercepts. Label your graphs clearly. [2]

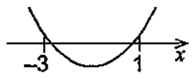
(iii) Hence, determine with justification, the number of solution(s) to the equation $\frac{e}{18-9x} = e^{-x}$. [2]

13(i) Express $\frac{1}{(x+4)(x+1)^2}$ in partial fractions. [4]

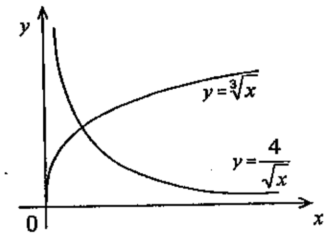
(ii) Hence, find $\int_0^2 \frac{1}{(x+4)(x+1)^2} \, \text{d}x$. [4]

Good Luck! May the Force be with you!

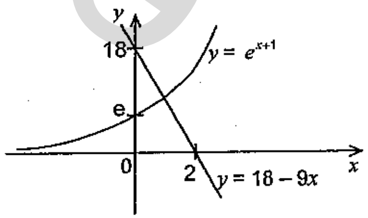
2016 CCHY 4E5N Prelim AMaths P1 Marking Scheme

Qn	Solution	Marks	Remarks
1a	$8\left(11^{n+1}\right)+7\left(11^{n+2}\right)+11^{n+3}$ $=8\left(11^n\times 11\right)+7\left(11^n\times 11^2\right)+\left(11^n\times 11^3\right)$ $=11^n\,(\,88+7\times 11^2+11^3\,)$ $=22\times 103\times 11^n$ when n is a positive integer, 11^n is also an integer, $\therefore$ the expression is divisible by 103 #	M1 R1	must explain 11^n is an integer
1b	$(16t)^{\frac{3}{2}}\times\sqrt{12t}=2^{4\times\frac{3}{2}}\times t^{\frac{3}{2}}\times\left(2^2\times 3\times t\right)^{\frac{1}{2}}$ $=2^7\times 3^{\frac{1}{2}}\times t^2$ Comparing terms, $x=\frac{1}{2}$ and $y=2$ #	M1 A1	
2(i)	$(a-x)(1+2x)^n=3+47x+bx^2+\ldots$ when $x=0$, $a(1)^n=3$ $\therefore\quad a=3$ #	A1	to show working
2(ii)	$(1+2x)^n=1+2nx+\binom{n}{2}(2x)^2+\ldots$ $=1+2nx+2n(n-1)x^2+\ldots$ $(3-x)(\,1+2nx+2n(n-1)x^2+\ldots)$ $=6nx-x+\ldots$ comparing x term, $6n-1=47$ $6n=48$ $\therefore\quad n=8$	M1 A1	
2(iii)	$(3-x)(1+2x)^n$ $=(3-x)(1+16x+112x^2+\ldots)$ $=336x^2-16x^2+\ldots$ $=320x^2+\ldots$ Hence, $b=320$ #	A1	
3(a)	 when $x=-3$ or $x=1$ $x+3=0$ $\quad x-1=0$ $(x+3)(x-1)\le 0$ $x^2-x+3x-3\le 0$ $x^2+2x\le 3$ Comparing terms, $p=2$ and $q=3$ #	B1 A1	for both answers

3(b)	$x^2+(3k+5)x-3=0$ $b^2-4ac=(3k+5)^2-4(1)(-3)$ $=(3k+5)^2+12$ >0 **since $b^2-4ac>0$,** $\Rightarrow$ the quadratic equation has 2 distinct real roots **$\Rightarrow$ roots are real for all values of k** (shown) #	M1 R1	
4(i)	$f(x)=5\lvert 6x+2\rvert-2\lvert 9x+3\rvert$ $=5\times 2\lvert 3x+1\rvert-2\times 3\lvert 3x+1\rvert$ $=4\lvert 3x+1\rvert$ #	M1 A1	
4(ii)	$5\lvert 6x+2\rvert=2\lvert 9x+3\rvert+6$ $5\lvert 6x+2\rvert-2\lvert 9x+3\rvert=6$ $4\lvert 3x+1\rvert=6$ $\lvert 3x+1\rvert=1{\cdot}5$ $\therefore 3x+1=1{\cdot}5$ or $3x+1=-1{\cdot}5$ $3x=0{\cdot}5$ $\quad 3x=-2{\cdot}5$ $x=\frac{1}{6}$ $\quad x=-\frac{5}{6}$ #	M1 M1 A1	For both answers
5a	$y=\frac{3x-6}{x+2}$ $\frac{dy}{dx}=\frac{3(x+2)-(3x-6)(1)}{(x+2)^2}$ $=\frac{12}{(x+2)^2}$	M1 A1	Check for presentation error in brackets
5b	At x-axis, $y=0$ $\therefore\ 3x-6=0$ $3x=6$ $x=2$ when $x=2$, $\frac{dy}{dx}=\frac{12}{(2+2)^2}$ $=\frac{3}{4}$ Equation of tangent : $y=\frac{3}{4}x+c$ At (2,0), $\quad 0=\frac{3}{4}(2)+c$ $c=-\frac{3}{2}$ $\therefore$ A = (2,0) and B = $(0,-1\frac{1}{2})$	B1 M1 M1 A1	Allow follow through error Allow follow through error

6(i)	Let $f(x) = 15x^3 + 26x^2 - 11x - 6$ $f(-2) = 15(-2)^3 + 26(-2)^2 - 11(-2) - 6$ $= 0$ since $f(-2) = 0$, by factor theorem, **$x - 2$ is a factor of $f(x)$**	M1 A1	
6(ii)	Let $f(x) = 15x^3 + 26x^2 - 11x - 6$ $f(3) = 15(3)^3 + 26(3)^2 - 11(3) - 6$ $= 600$ $\therefore$ remainder = 600	M1 A1	
6(iii)	$15x^4 + px^3 - 37x^2 + qx + 6$ $= (x-1)(15x^3 + 26x^2 - 11x - 6)$ $= 26x^3 - 6x - 15x^3 + 11x + ...$ $= 11x^3 + 5x + ...$ Comparing terms, $p = 11$, $q = 5$	M1 A1	
7(a)	$\log_a 2y^2 + \log_a 8 + \log_a 16y - \log_a 64y$ $= 2\log_a 4$ $\log_a 2 + \log_a y^2 + \log_a 2^3 + \log_a 2^4 + \log_a y - \log_a 2^6$ $- \log_a y = 2\log_a 2^2$ $2\log_a 2 + 2\log_a y = 4\log_a 2$ $2\log_a y = 2\log_a 2$ comparing terms, $y = 2$	M2 A1	Apply correctly $\log_a MN$ $= \log_a M + \log_a N$ $\log_a M^r$ $= r \log_a M$
7(b)	$x = \lg m$ $m = 10^x$ $10^{2x+1} + 7(10^x) = 26$ $(10^x)(10^x)(10) + 7(10^x) = 26$ $10m^2 + 7m - 26 = 0$ $(10m - 13)(m + 2) = 0$ $\therefore m = 1{\cdot}3$ or $m = -2$ (N.A.)	B1 B1 A1	A0 if didn't reject the –ve ans
8(i)		[1]	

	correct shape of $y = \sqrt[3]{x}$ correct shape of $y = \frac{4}{\sqrt{x}}$, check asymptote	[1]	
8(ii)	$\sqrt[3]{x} = \frac{4}{\sqrt{x}}$ $x^{\frac{1}{3}+\frac{1}{2}} = 4$ $x^{\frac{5}{6}} = 4$ $\therefore x = 4^{\frac{6}{5}}$	A1	
8(iii)	$y = x^{\frac{1}{3}}$ $\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$ $y = 4x^{-\frac{1}{2}}$ $\frac{dy}{dx} = -2x^{-\frac{3}{2}}$ when $x = 4^{\frac{6}{5}}$, $m_1 = \frac{1}{3}x^{-\frac{2}{3}}$ $= \frac{1}{3}\left(4^{\frac{6}{5}}\right)^{-\frac{2}{3}}$ $= \frac{1}{3}\left(4^{-\frac{4}{5}}\right)$ $m_2 = -2x^{-\frac{3}{2}}$ $= -2\left(4^{\frac{6}{5}}\right)^{-\frac{3}{2}}$ $= -2\left(4^{-\frac{9}{5}}\right)$ $m_1 \times m_2 = -2\left(4^{-\frac{9}{5}}\right) \times \frac{1}{3}\left(4^{-\frac{4}{5}}\right)$ $= -0{\cdot}01813647007$ since $m_1 \times m_2 \neq -1$ $\Rightarrow$the tangents to the graphs at the point of intersection are not perpendicular	B1 M1 R1 A1	accept equivalent form - allow follow thru error - allow working to 4dp

9(a)	$y = 2x^2 + mx + n$ $\frac{dy}{dx} = 4x + m$ At (1,5), $8 = 4(1) + m$ $\therefore\ m = 8 - 4$ $= 4$ # $5 = 2(1)^2 + 4(1) + n$ $= 6 + n$ $\therefore\ n = -1$ #	B1 A1 A1	
9(b)	$y = x^3 + \frac{8}{x}$ $= x^3 + 8x^{-1}$ $\frac{dy}{dx} = 3x^2 - \frac{8}{x^2}$ $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ when $x = 1{\cdot}6$, $\frac{dy}{dx} = 3(1{\cdot}6)^2 - \frac{8}{(1{\cdot}6)^2}$ $= \frac{911}{200}$ $\therefore\ 5 = \frac{911}{200} \times \frac{dx}{dt}$ $\frac{dx}{dt} = 5 \times \frac{200}{911}$ $= 1{\cdot}1$ (2sf) $\therefore\ \frac{dx}{dt} = 1{\cdot}1$ units / s #	B1 M1 M1 A1	Accept equivalent form A0 if omit unit of measurement
10(i)	∠QRP ∠s in alt segment	A1 A1	
10(ii)	Method 1 since PR is the diameter and ∠PMR = 90° and ∠PNR = 90°, **by property of ∠ in a semi-circle**, M and N are points on the circumference of the circle, hence, a circle with PR as diameter passes through M and N	B1 R1	
	Method 2 since PR is the diameter and ∠PMR = 90° and ∠PNR = 90°, **by property of ∠s in opposite segment**, M and N are points on the circumference of the circle, hence, a circle with PR as diameter passes through M and N	B1 R1	
10(iii)	From (ii), ∠MRP = ∠MNP (∠s in the same segment) From (i), ∠QPT = ∠QRP (∠s in alt segment) ∠QRP = ∠MRP (common ∠) ⇒ ∠QPT = ∠MNP **By property of corresponding angles**, MN and PQ are parallel	B1 B1 R1	
11(a)	$\int e^{4x} f(x)\, dx = e^{4x} \sin 3x + c$ $\frac{d}{dx}\left[\int e^{4x} f(x) dx\right] = \frac{d}{dx}\left[e^{4x} \sin 3x + c\right]$ $e^{4x}\ f(x) = e^{4x} 3\cos 3x + 4e^{4x} \sin 3x$ $\div e^{4x}$, $f(x) = 3\cos 3x + 4\sin 3x$	M1 B1 A1	Seen or implied
11b(i)	$\cos 3x = \cos(2x + x)$ $= \cos 2x \cos x - \sin 2x \sin x$ $= (2\cos^2 x - 1)\cos x - 2\sin x \cos x \sin x$ $= 2\cos^3 x - \cos x - 2\sin^2 x\ \cos x$ $= 2\cos^3 x - \cos x - 2(1 - \cos^2 x)\cos x$ $= 4\cos^3 x - 3\cos x$ (shown)	M2 M1	Correct application of cos and sin double angle formula Apply identity
11b(ii)	$4\cos^3 x - 3\cos x = \cos 3x$ x2, $8\cos^3 x - 6\cos x = 2\cos 3x$ $\therefore\ 8\cos^3 10° - 6\cos 10° = 2\cos 3(10°)$ $= 2\left(\frac{\sqrt{3}}{2}\right)$ $= \sqrt{3}$	M1 A1	subst $x = 10°$
12(i)	 correct shape with x-axis as asymptote correct y-intercept	[1] [1]	
12(ii)(b)	correct slope for straight line correct intercepts & label graph	[1] [1]	

12(ii)(a)	$\frac{e}{18-9x}=e^{-x}$ $\frac{e}{e^{-x}}=18-9x$ $e^{1+x}=18-9x$ $y=18-9x$ $\therefore$ equation of straight line is $y=18-9x$ #	M1 A1	
12(iii)	The equation has only **1** solution since there is only **1 intersection** between the curve and the straight line.	A1 R1	
13(i)	Let $\frac{1}{(x+4)(x+1)^2}=\frac{A}{x+4}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$ $\therefore 1=A(x+1)^2+B(x+1)(x+4)+C(x+4)$ when $x=-1$, $1=3C$, $C=\frac{1}{3}$ when $x=-4$, $1=9A$, $A=\frac{1}{9}$ when $x=0$, $1=\frac{1}{9}+4B+4\left(\frac{1}{3}\right)$ $4B=1-\frac{1}{9}-\frac{4}{3}$ $B=-\frac{1}{9}$ $\therefore \frac{1}{(x+4)(x+1)^2}=\frac{1}{9(x+4)}-\frac{1}{9(x+1)}+\frac{1}{3(x+1)^2}$	M1 M1 A2	Accept alternative method [–1] for each error, max 2 errors
13(ii)	$\int_0^2\frac{1}{(x+4)(x+1)^2}dx$ $=\int_0^2\frac{1}{9(x+4)}dx-\int_0^2\frac{1}{9(x+1)}dx+\int_0^2\frac{1}{3(x+1)^2}dx$ $=\frac{1}{9}\left[\ln(x+4)\right]_0^2-\frac{1}{9}\left[\ln(x+1)\right]_0^2+\frac{1}{3}\left[\frac{(x+1)^{-1}}{(-1)(1)}\right]_0^2$ $=\frac{1}{9}\left[\ln6-\ln4\right]-\frac{1}{9}\left[\ln3-\ln1\right]-\frac{1}{3}\left[\frac{1}{x+1}\right]_0^2$ $=\frac{1}{9}\ln6-\frac{1}{9}\ln4-\frac{1}{9}\ln3-\frac{1}{3}\left[\frac{1}{3}-1\right]$	M1 M2	M1 for integration of ln, M1 for integration of polynomials
	$=\frac{1}{9}\ln(2\times3)-\frac{1}{9}\ln2^2-\frac{1}{9}\ln3+\frac{2}{9}$ $=\frac{1}{9}\ln2+\frac{1}{9}\ln3-\frac{2}{9}\ln2-\frac{1}{9}\ln3+\frac{2}{9}$ $=\frac{2}{9}-\frac{1}{9}\ln2$ $=\frac{1}{9}[2-\ln2]$	A1	Accept $\frac{2}{9}-\frac{1}{9}\ln2$ 0·145 (3sf)

1. It is studied that the population, P, of a certain species of butterfly increases exponentially. At the beginning of the experiment, there were 800 butterflies. Given that $P = A(3)^{kt}$, where A and k are constants and t is the time in days after the study is conducted.

    (i) Explain why $A = 800$. [1]

    (ii) Given that the population tripled in 18 days, show that the value of k is $\frac{1}{18}$. [2]

    (iii) Find the number of butterflies after 30 days, giving your answer to the nearest integer. [2]

    (iv) After how many days will the population exceed 100 000? [3]

2. A curve has the equation $y = x\ln x - 3x$, where $x > 0$. The point (p, q) is the stationary point on the curve.

    (i) Find the value of p and of q. [3]

    (ii) Determine whether y is increasing or decreasing

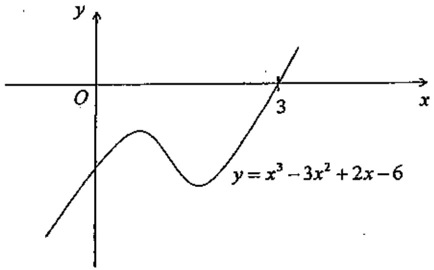
    (a) for values of x less than p, [2]

    (b) for values of x greater than p. [2]

    (iii) What do the results of part **(ii)** imply about the stationary point? [1]

    (iv) Find the value of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at the stationary point. [1]

3. The diagram below shows part of the graph of $y = x^3 - 3x^2 + 2x - 6$.

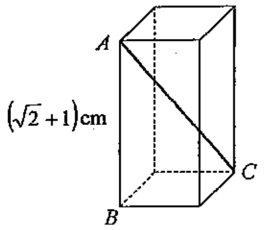


    (i) If $x + k$ is a factor of $x^3 - 3x^2 + 2x - 6$, state the value of k. [1]

    (ii) Hence, factorise $x^3 - 3x^2 + 2x - 6$ completely. [3]

    (iii) Using your answer from **(ii)**, explain why the cubic equation $x^3 - 3x^2 + 2x - 6 = 0$ does not have 3 real roots. [1]

4. The diagram below shows a cuboid with a square base. The height AB of the cuboid is $(\sqrt{2}+1)$ cm and the length of the diagonal AC is $\frac{7\sqrt{2}}{2\sqrt{2}+1}$ cm.



    (i) Express $\frac{7\sqrt{2}}{2\sqrt{2}+1}$ in the form $a + b\sqrt{2}$, where a and b are integers. [2]

    (ii) Find an expression for BC^2 in the form $c + d\sqrt{2}$, where c and d are integers. [4]

    (iii) Express the volume of the cuboid in the form $\frac{5}{2}(\sqrt{2}+k)$ cm^3, where k is an integer. [3]

5. The roots of the quadratic equation $\sqrt{3}x^2 - \sqrt{12}x - 2 = 0$ are α and β.

    (i) Find the values of $\alpha + \beta$ and $\alpha\beta$. [2]

    (ii) Hence, find the quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. [3]

6. The variables x and y are connected by the equation $x + y = e^{a-kx}$, where a and k are constants. The table below shows some values of x and y.

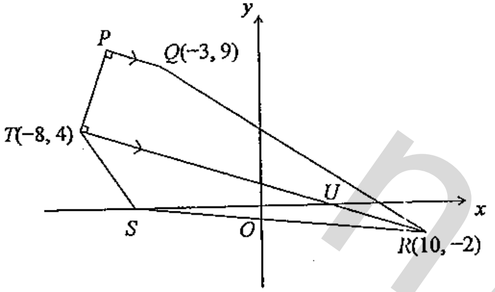
x	1	2	3	4	5	6
y	−0.78	−1.63	−2.39	−3	−3.35	−3.28

    (i) Draw a straight line graph of $\ln(x+y)$ against x, using a scale of 2 cm to represent 1 unit on the x–axis and 1 cm to represent 0.2 units on $\ln(x+y)$ – axis. [2]

    (ii) Use your graph to estimate the value of a and of k. [3]

    (iii) On the same diagram, draw the line representing $y = e^{1-2x} - x$ and hence find the value of x for which $e^{a-kx} = e^{1-2x}$. [3]

7. **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a pentagon $PQRST$ in which PQ is parallel to TR and PT is perpendicular to PQ and TR. The coordinates of Q, R and T are (−3, 9), (10, −2) and (−8, 4) respectively.

Find

(a) the coordinates of U, [2]

(b) the coordinates of P, [4]

(c) the ratio of the area of triangle RSU to the area of triangle STU, [3]

(d) the area of trapezium $PQRT$. [2]

W is a point such that $PQRW$ is a parallelogram.

(e) Find $\dfrac{\text{area of parallelogram } PQRW}{\text{area of trapezium } PQRT}$. [2]

8. The velocity, v m/s of a particle, travelling in a straight line, at time t seconds after leaving a fixed point O is given by $v = 6t^2 + t - 2$. The initial displacement of the particle is 3 m from O.

(i) Find the value of t when the particle comes to an instantaneous rest. [2]

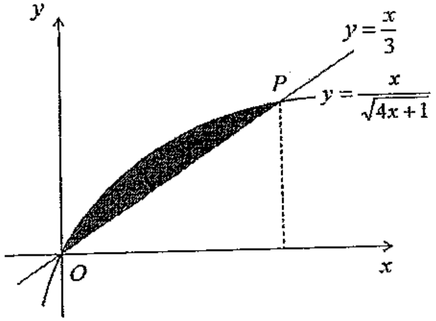
(ii) Find the displacement of the particle when it comes to rest. [2]

(iii) Calculate the average speed of the particle for the first 2 seconds. [3]

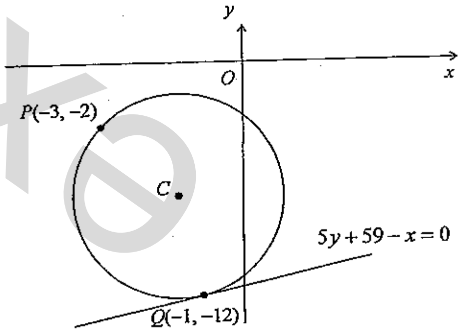
(iv) Will the particle ever achieve constant speed? Explain. [2]

9. (i) Given that $y = (2x-1)\sqrt{4x+1}$, show that $\dfrac{dy}{dx}$ can be written in the form of $\dfrac{kx}{\sqrt{4x+1}}$, where k is a positive constant. [3]

(ii) In the diagram, the curve $y = \dfrac{x}{\sqrt{4x+1}}$ cuts the line $y = \dfrac{x}{3}$ at two points, O and P. Find the area of the shaded region. [4]



10. In the diagram, the circle passes through $P(-3,-2)$ and touches the line $5y + 59 - x = 0$ at $Q(-1,-12)$.



(i) Find the coordinates of C, the centre of the circle. [6]

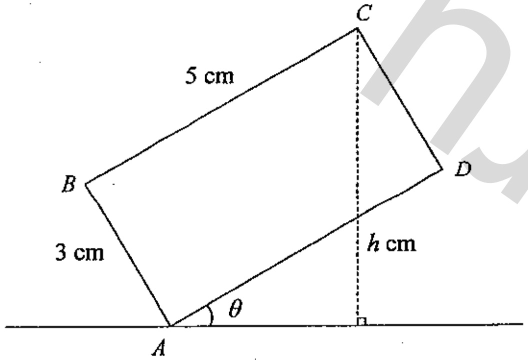
(ii) Hence, or otherwise, find the equation of the circle. [2]

11. (i) Prove the identity $(1-\cos 2x)\cot x = \sin 2x$. [3]

(ii) Sketch the graph of $y = (1-\cos 2x)\cot x$ for $0 \le x \le \frac{3\pi}{2}$. [2]

(iii) Find all the angles between 0 and π which satisfy the equation $(1-\cos 2x)\cot x = -0.2$. [4]

12.



The diagram shows a rectangle $ABCD$ with $AB = 3$ cm and $BC = 5$ cm. The rectangle is hinged to the horizontal ground at A so as to rotate in a vertical plane. The side AD of the rectangle makes an acute angle θ with the horizontal ground.

(i) Show that $h = 3\cos\theta + 5\sin\theta$, where h cm is the height of C above the ground. [2]

(ii) Express h in the form of $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) Find the maximum value of h and the corresponding value of θ. [2]

(iv) Find the value of θ for which C is 4 m above the ground. [3]

1. It is studied that the population, P, of a certain species of butterfly increases exponentially. At the beginning of the experiment, there were 800 butterflies. Given that $P = A(3)^{kt}$, where A and k are constants and t is the time in days after the study is conducted.

(i) Explain why $A = 800$. [1]

(ii) Given that the population tripled in 18 days, show that the value of k is $\frac{1}{18}$. [2]

(iii) Find the number of butterflies after 30 days, giving your answer to the nearest integer. [2]

(iv) After how many days will the population exceed 100 000? [3]

(i) $800 = A(3)^{k(0)}$ $\boxed{\text{B1}}$

$A = 800$

(ii) $P = 800(3)^{kt}$

$2400 = 800(3)^{k(18)}$ $\boxed{\text{M1}}$

$3 = 3^{18k}$

$18k = 1$

$k = \frac{1}{18}$ $\boxed{\text{A1}}$

(iii) When $t = 30$,

$P = 800(3)^{\frac{1}{18}(30)}$ $\boxed{\text{M1}}$

$= 4992$ (nearest integer) $\boxed{\text{A1}}$

(iv) $800\ (3)^{\frac{1}{18}t} > 100000$ $\boxed{\text{M1}}$

$(3)^{\frac{1}{18}t} > 125$

$\frac{1}{18}t > \frac{\lg 125}{\lg 3}$ $\boxed{\text{M1}}$

$t > 79.108$

$t = 80$ $\boxed{\text{A1}}$

---

2. A curve has the equation $y = x\ln x - 3x$, where $x > 0$. The point (p, q) is the stationary point on the curve.

(i) Determine the values of p and q. [3]

(ii) Determine whether y is increasing or decreasing

  (a) for values of x less than p, [2]

  (b) for values of x greater than p. [2]

(iii) What do the results of part **(ii)** imply about the stationary point? [1]

(iv) Find the value of $\frac{d^2y}{dx^2}$ at the stationary point. [1]

(i) $\frac{dy}{dx} = \ln x + \frac{x}{x} - 3$ $\boxed{\text{M1}}$

$= \ln x - 2$

$\ln x - 2 = 0$

$\ln x = 2$

$x = e^2$

$p = e^2$ $\boxed{\text{A1}}$

$y = 2e^2 - 3e^2$

$= -e^2$

$q = -e^2$ $\boxed{\text{A1}}$

(ii) (a) When $x < e^2$,

$\ln x < \ln e^2$

$\ln x < 2$

$\frac{dy}{dx} = \ln x - 2$ $\boxed{\text{M1}}$

< 0

$\therefore y$ is decreasing when $x < e^2$. $\boxed{\text{A1}}$

(b) When $x > e^2$,

$\ln x > \ln e^2$

$\ln x > 2$

$\frac{dy}{dx} = \ln x - 2$ $\boxed{\text{M1}}$

> 0

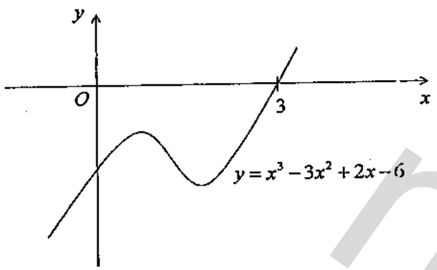
$\therefore y$ is increasing when $x > e^2$. $\boxed{\text{A1}}$

(iii) Stationary point is a minimum point. $\boxed{\text{B1}}$

(iv) $\frac{d^2y}{dx^2} = \frac{1}{x}$

$= \frac{1}{e^2}$ $\boxed{\text{B1}}$

3. The diagram below shows part of the graph of $y = x^3 - 3x^2 + 2x - 6$.



(i) If $x + k$ is a factor of $x^3 - 3x^2 + 2x - 6$, state the value of k. [1]

(ii) Hence, factorise $x^3 - 3x^2 + 2x - 6$ completely. [3]

(iii) Using your answer from (ii), explain why the cubic equation $x^3 - 3x^2 + 2x - 6 = 0$ does not have 3 real roots. [1]

(i) From the graph, $x = 3$, $y = 0$
Hence, $x - 3$ is a factor of $x^3 - 3x^2 + 2x - 6$
$k = -3$ $\boxed{\text{B1 (accept factor theorem)}}$

(ii) Let $x^3 - 3x^2 + 2x - 6 = (x-3)(x^2 + bx + 2)$
By comparing x term, $\boxed{\text{M1 – accept long division}}$
$2x = 2x - 3bx$
$b = 0$
$x^3 - 3x^2 + 2x - 6 = (x-3)(x^2+2)$ $\boxed{\text{A2}}$

(iii) Since $x^2 + 2 > 0$ for all values of x, the cubic equation $x^3 - 3x^2 + 2x - 6 = 0$ has 1 real root and not 3 real roots. $\boxed{\text{R1}}$

OR

$(x-3)(x^2+2) = 0$
$x = 3$ or $x^2 + 2 = 0$
$b^2 - 4ac = 0 - 4(1)(2)$
$= -8 < 0$
$x^2 + 2 = 0$ has no real roots
$\therefore x^3 - 3x^2 + 2x - 6 = 0$ has only 1 real root and not 3 real roots.

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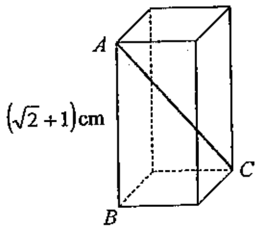
4. The diagram below shows a cuboid with a square base. The height AB of the cuboid is $(\sqrt{2}+1)$ cm and the length of the diagonal AC is $\frac{7\sqrt{2}}{2\sqrt{2}+1}$ cm.



(i) Express $\frac{7\sqrt{2}}{2\sqrt{2}+1}$ in the form $a + b\sqrt{2}$, where a and b are integers. [2]

(ii) Find an expression for BC^2 in the form $c + d\sqrt{2}$, where c and d are integers. [4]

(iii) Express the volume of the cuboid in the form $\frac{5}{2}(\sqrt{2}+k)$ cm³, where k is an integer. [3]

(i)
$$\frac{7\sqrt{2}}{2\sqrt{2}+1} = \frac{7\sqrt{2}}{2\sqrt{2}+1} \times \frac{2\sqrt{2}-1}{2\sqrt{2}-1} \quad \boxed{\text{M1}}$$
$$= \frac{7\sqrt{2}(2\sqrt{2}-1)}{7}$$
$$= 4 - \sqrt{2} \quad \boxed{\text{A1}}$$

(ii)
$$(\sqrt{2}+1)^2 + BC^2 = \left(\frac{7\sqrt{2}}{2\sqrt{2}+1}\right)^2 \quad \boxed{\text{M1}}$$
$$(3+2\sqrt{2}) + BC^2 = (4-\sqrt{2})^2 \quad \boxed{\text{M1}}$$
$$(3+2\sqrt{2}) + BC^2 = 18 - 8\sqrt{2} \quad \boxed{\text{M1}}$$
$$BC^2 = 18 - 8\sqrt{2} - (3+2\sqrt{2})$$
$$BC^2 = 15 - 10\sqrt{2} \quad \boxed{\text{A1}}$$

(iii) Let the length of the base be l cm.
By Pythagoras' Theorem,
$l^2 + l^2 = BC^2$
$BC^2 = 2l^2$ $\boxed{\text{M1}}$
Area of base $= \frac{1}{2}BC^2$
$= \frac{1}{2}(15 - 10\sqrt{2})$

Volume
$= \frac{1}{2}(15 - 10\sqrt{2})(\sqrt{2}+1)$ $\boxed{\text{M1}}$
$= \frac{1}{2}(15\sqrt{2} + 15 - 20 - 10\sqrt{2})$
$= \frac{1}{2}(5\sqrt{2} - 5)$
$= \frac{5}{2}(\sqrt{2}-1)\text{cm}^3$ $\boxed{\text{A1}}$

5. The roots of the quadratic equation $\sqrt{3}x^2 - \sqrt{12}x - 2 = 0$ are α and β.

(i) Find the values of $\alpha + \beta$ and $\alpha\beta$. [2]

(ii) Hence, find the quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. [3]

(i)
$$\alpha + \beta = \frac{\sqrt{12}}{\sqrt{3}} = \sqrt{4} = 2 \quad \boxed{\text{B1}}$$
$$\alpha\beta = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3} \quad \boxed{\text{B1}}$$

(ii)
$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$$
$$= \frac{2}{-\frac{2\sqrt{3}}{3}} \quad \boxed{\text{M1}}$$
$$= -2 \times \frac{3}{2\sqrt{3}}$$
$$= -\frac{3}{\sqrt{3}}$$
$$= -\sqrt{3}$$
$$\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right) = \frac{1}{\alpha\beta}$$
$$= -\frac{3}{2\sqrt{3}} \quad \boxed{\text{M1}}$$
$$= -\frac{3\sqrt{3}}{6}$$
$$= -\frac{\sqrt{3}}{2}$$

Equation: $x^2 + \sqrt{3}x - \frac{\sqrt{3}}{2} = 0$ or $2x^2 + 2\sqrt{3}x - \sqrt{3} = 0$ $\boxed{\text{A1}}$

6. The variables x and y are connected by the equation $x + y = e^{a-kx}$, where a and k are constants. The table below shows some values of x and y.

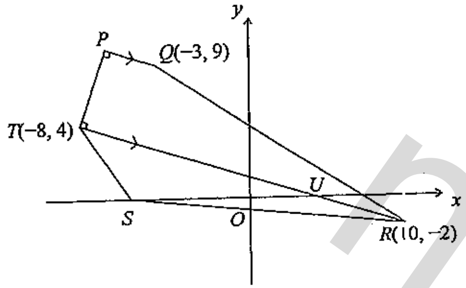
x	1	2	3	4	5	6
y	−0.78	−1.63	−2.39	−3	−3.35	−3.28

(i) Draw a straight line graph of $\ln(x + y)$ against x, using a scale of 2 cm to represent 1 unit on the x–axis and 1 cm to represent 0.2 units on $\ln(x + y)$–axis. [2]

(ii) Use your graph to estimate the value of a and of k. [3]

(iii) On the same diagram, draw the line representing $y = 2 - x$ and hence find the value of x for which $e^{a-kx} = 2$. [3]

7. **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a pentagon *PQRST* in which *PQ* is parallel to *TR* and *PT* is perpendicular to *PQ* and *TR*. The coordinates of *Q*, *R* and *T* are (−3, 9), (10, −2) and (−8, 4) respectively.

Find

(a) the coordinates of *U*, [2]

(b) the coordinates of *P*, [4]

(c) the ratio of the area of triangle *RSU* to the area of triangle *STU*, [3]

(d) the area of trapezium *PQRT*. [2]

W is a point such that *PQRW* is a parallelogram.

(e) Find $\frac{\text{area of parallelogram } PQRW}{\text{area of trapezium } PQRT}$. [2]

(a) Let *U* be (*x*, 0) $\boxed{\text{M1}}$

$$\frac{4+2}{-8-10}=\frac{4-0}{-8-x}$$

$$-\frac{1}{3}=\frac{4}{-8-x}$$

$$8+x=12$$

$$x=4$$

$$\therefore U(4,0) \quad \boxed{\text{A1}}$$

(b)

$$y-9=-\frac{1}{3}(x-(-3))$$

$$y=-\frac{1}{3}x+8\text{-----}(1) \quad \boxed{\text{M1}}$$

Gradient of *PT* = 3

Equation of *PT*:

$$y-4=3(x-(-8))$$

$$y=3x+28\text{----}(2) \quad \boxed{\text{M1}}$$

$$(1)=(2)$$

$$-\frac{1}{3}x+8=3x+28 \quad \boxed{\text{M1}}$$

$$\frac{10}{3}x=-20$$

$$x=-6$$

$$y=10$$

$$\therefore P(-6,10) \quad \boxed{\text{A1}}$$

(c)

$$\left.\begin{aligned}RU&=\sqrt{(10-4)^2+(-2-0)^2}=2\sqrt{10}\\TU&=\sqrt{(-8-4)^2+(4-0)^2}=4\sqrt{10}\end{aligned}\right\}\quad \boxed{\text{M1}}$$

$$\frac{\text{Area of } \Delta RSU}{\text{Area of } \Delta STU}$$

$$=\frac{\frac{1}{2}\times h\times 2\sqrt{10}}{\frac{1}{2}\times h\times 4\sqrt{10}} \quad \boxed{\text{M1}}$$

$$=\frac{1}{2}$$

Ratio of Area of ΔRSU : Area of $\Delta STU = 1:2$ $\boxed{\text{A1}}$

(d) Area of trapezium

$$=\frac{1}{2}\begin{vmatrix}-6 & -8 & 10 & -3 & -6\\ 10 & 4 & -2 & 9 & 10\end{vmatrix} \quad \boxed{\text{M1}}$$

$$=\frac{1}{2}[-24+16+90-30+54-6-40+80]$$

$$=70\text{ units}^2 \quad \boxed{\text{A1}}$$

(e)

$$PQ=\sqrt{(-3+6)^2+(9-10)^2}=\sqrt{10}$$

$$PT=\sqrt{(-8-6)^2+(4-10)^2}=2\sqrt{10}$$

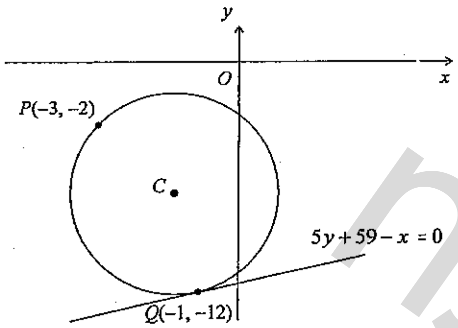
$$\text{Area of } PQRW=\sqrt{10}\times 2\sqrt{10}=20\text{ units}^2 \quad \boxed{\text{M1}}$$

$$\frac{\text{Area of } PQRW}{\text{Area of } PQRT}$$

$$=\frac{20}{70}$$

$$=\frac{2}{7} \quad \boxed{\text{A1}}$$

10. In the diagram, the circle passes through $P(-3,-2)$ and touches the line $5y+59-x=0$ at $Q(-1,-12)$.



(i) Find the coordinates of C, the centre of the circle. [6]

(ii) Hence, or otherwise, find the equation of the circle. [2]

(i) Coordinates of midpoint of PQ

$=\left(\frac{-3+(-1)}{2},\frac{-2+(-12)}{2}\right)$

$=(-2,-7)$

Gradient of $PQ=\frac{-2-(-12)}{-3-(-1)}=-5$ $\boxed{\text{M1}}$

Gradient of perpendicular bisector$=\frac{1}{5}$

Equation of perpendicular bisector:

$y-(-7)=\frac{1}{5}(x-(-2))$

$y=\frac{1}{5}x-6\frac{3}{5}------(1)$ $\boxed{\text{M1}}$

Equation of tangent:

$5y+59-x=0$

$y=\frac{1}{5}x-11\frac{4}{5}$

Gradient of $CQ=-5$ $\boxed{\text{M1}}$

Equation of CQ:

$y-(-12)=-5(x-(-1))$

$y=-5x-17------(2)$ $\boxed{\text{M1}}$

$(1)=(2)$

$\frac{1}{5}x-6\frac{3}{5}=-5x-17$ $\boxed{\text{M1}}$

$x=-2$

$y=-7$

$\therefore C(-2,-7)$ $\boxed{\text{A1}}$

(ii) Radius

$=\sqrt{(-2-(-1))^2+(-7-(-12))^2}$ $\boxed{\text{M1}}$

$=\sqrt{26}$ units

Equation of circle:

$(x+2)^2+(y+7)^2=26$ $\boxed{\text{A1}}$

or

$x^2+y^2+4x+14y+27=0$

11. (i) Prove the identity $(1-\cos 2x)\cot x=\sin 2x$. [3]

(ii) Sketch the graph of $y=(1-\cos 2x)\cot x$ for $0\le x\le\frac{3\pi}{2}$. [2]

(ii) Find all the angles between 0 and π which satisfy the equation $(1-\cos 2x)\cot x=-0.2$. [4]

(i) LHS $=(1-\cos 2x)\cot x$

$=(1-\cos 2x)\left(\frac{\cos x}{\sin x}\right)$ $\boxed{\text{M1}}$

$=\left[1-(1-2\sin^2 x)\right]\left(\frac{\cos x}{\sin x}\right)$ $\boxed{\text{M1}}$

$=2\sin^2 x\left(\frac{\cos x}{\sin x}\right)$

$=2\sin x\cos x$ $\boxed{\text{B1}}$

$=\sin 2x=$ RHS (proven)

(iii) $\sin 2x=-0.2$

$0<2x<2\pi$

Basic $\angle=0.20135792$ $\boxed{\text{M1}}$

$2x=\pi+0.20135792, 2\pi-0.20135792$ $\boxed{\text{M1}}$

$x=1.67,\ 3.04$ (3 sf) $\boxed{\text{A2}}$

8. The velocity, v m/s of a particle, travelling in a straight line, at time t seconds after leaving a fixed point O is given by $v = 6t^2 + t - 2$. The initial displacement of the particle is 3 m from O.

(i) Find the value of t when the particle comes to an instantaneous rest. [2]

(ii) Find the displacement of the particle when it comes to rest. [2]

(iii) Calculate the average speed of the particle for the first 2 seconds. [3]

(iv) Will the particle ever achieve constant speed? Explain. [2]

(i) When $v = 0$,

$6t^2 + t - 2 = 0$ $\boxed{\text{M1}}$

$(3t+2)(2t-1) = 0$

$t = -\frac{2}{3}$ (rej) or $\frac{1}{2}$ $\boxed{\text{A1}}$

(ii) $s = \int 6t^2 + t - 2\,\text{dt}$

$= \frac{6t^3}{3} + \frac{t^2}{2} - 2t + c$

$= 2t^3 + \frac{t^2}{2} - 2t + c$

When $t = 0$,

$2(0)^3 + \frac{(0)^2}{2} - 2(0) + c = 3$

$c = 3$

$\therefore s = 2t^3 + \frac{t^2}{2} - 2t + 3$ $\boxed{\text{M1}}$

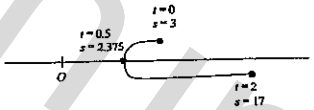
When $t = \frac{1}{2}$,

$\therefore s = 2\left(\frac{1}{2}\right)^3 + \frac{\left(\frac{1}{2}\right)^2}{2} - 2\left(\frac{1}{2}\right) + 3$

$= 2.375$ or $2\frac{3}{8}$

Displacement $= 2.375$ or $2\frac{3}{8}$m $\boxed{\text{A1}}$

(iii)



When $t = 2$,

$s = 2(2)^3 + \frac{(2)^2}{2} - 2(2) + 3$ $\boxed{\text{M1}}$

$= 17m$

Total distance

$= (3 - 2.375) + (17 - 2.375)$ $\boxed{\text{M1}}$

$= 15.25m$

Average speed

$= \frac{15.25}{2}$

$= 7.625$ m/s $\boxed{\text{A1}}$

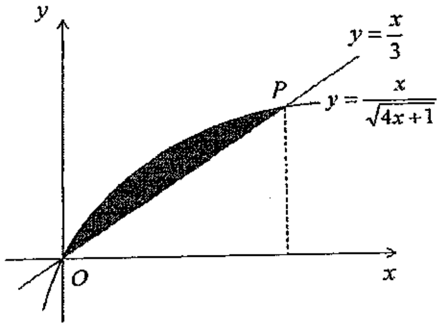
(iv) $a = \frac{dv}{dt}$

$= 12t + 1$ $\boxed{\text{M1}}$

Since $12t + 1 > 0$ for all values of t, particle will accelerate and will not achieve constant speed. $\boxed{\text{R1}}$

9. (i) Given that $y = (2x-1)\sqrt{4x+1}$, show that $\frac{dy}{dx}$ can be written in the form of $\frac{kx}{\sqrt{4x+1}}$, where k is a positive constant. [3]

(ii) In the diagram, the curve $y = \frac{x}{\sqrt{4x+1}}$ cuts the line $y = \frac{x}{3}$ at two points, O and P. Find the area of the shaded region. [4]



(i) $\frac{dy}{dx} = 2(4x+1)^{\frac{1}{2}} + \frac{1}{2}(4x+1)^{-\frac{1}{2}}(4)(2x-1)$ $\boxed{\text{M1}}$

$= 2(4x+1)^{\frac{1}{2}} + 2(4x+1)^{-\frac{1}{2}}(2x-1)$

$= 2(4x+1)^{-\frac{1}{2}}(4x+1+2x-1)$ $\boxed{\text{M1}}$

$= \frac{2(6x)}{\sqrt{4x+1}}$

$= \frac{12x}{\sqrt{4x+1}}$ $\boxed{\text{A1}}$

(ii) $\frac{x}{3} = \frac{x}{\sqrt{4x+1}}$ $\boxed{\text{M1}}$

$x\sqrt{4x+1} - 3x = 0$

$x\left(\sqrt{4x+1} - 3\right) = 0$

$x = 0$ or $\sqrt{4x+1} = 3$

$4x + 1 = 9$

$x = 2$ $\boxed{\text{M1}}$

Area of shaded region

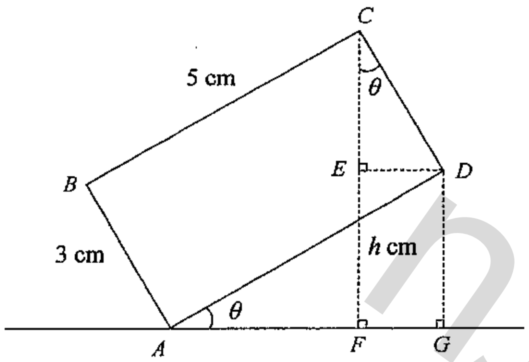
$= \int_0^2 \frac{x}{\sqrt{4x+1}} dx$

$= \frac{1}{12}\int_0^2 \frac{12x}{\sqrt{4x+1}} dx$

$= \frac{1}{12}\left[(2x-1)\sqrt{4x+1}\right]_0^2$ $\boxed{\text{M1}}$

$= \frac{5}{6}$ units^2 $\boxed{\text{A1}}$

12.



The diagram shows a rectangle $ABCD$ with $AB = 3$ cm and $BC = 5$ cm. The rectangle is hinged to the horizontal ground at A so as to rotate in a vertical plane. The side AD of the rectangle makes an acute angle θ with the horizontal ground.

(i) Show that $h = 3\cos\theta + 5\sin\theta$, where h cm is the height of C above the ground. [2]

(ii) Express h in the form of $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) Find the maximum value of h and the corresponding value of θ. [2]

(iv) Find the value of θ for which C is 4 m above the ground. [3]

(i) Using ΔCDE, $CE = 3\cos\theta$ $\boxed{\text{B1}}$

Using ΔADG, $EF = DG = 5\sin\theta$ $\boxed{\text{B1}}$

$\therefore h = CE + EF$

$h = 3\cos\theta + 5\sin\theta$

(ii) $h = R\cos(\theta - \alpha)$, where $R = \sqrt{3^2 + 5^2} = \sqrt{34}$ $\boxed{\text{M1}}$

and $\alpha = \tan^{-1}\left(\frac{5}{3}\right) = 59.036°$ $\boxed{\text{M1}}$

$h = \sqrt{34}\cos(\theta - 59.0°)$ $\boxed{\text{A1 (}R\text{ must be exact and }\alpha\text{ is rounded to 1 dp)}}$

(iii) Max $h = \sqrt{34}$ $\boxed{\text{B1}}$

when $\theta = 59.0°$ (1 dp) $\boxed{\text{B1}}$

(iv) $\sqrt{34}\cos(\theta - 59.036°) = 4$

$\cos(\theta - 59.036°) = \frac{4}{\sqrt{34}}$ $\boxed{\text{M1}}$

Basic angle $= 46.68614334°$ $\boxed{\text{M1}}$

$\theta - 59.036° = -46.68614334°$

$\theta = 12.4°$ (1 dp) $\boxed{\text{A1}}$

COMMONWEALTH SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2016

ADDITIONAL MATHEMATICS
PAPER 2

Name: ______________________________ (   )   Class: ________

SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC
4047/2

Thursday 18 August 2016
08 00 – 10.30
2 h 30 min

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Name of setter: **Ms Lee YJ**

This paper consists of **7** printed pages including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots\ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

1 A curve is such that $\frac{dy}{dx}=x^{\frac{1}{2}}-x^{-\frac{1}{2}}$, $x\neq 0$. The curve passes through the point $\left(4,\frac{2}{3}\right)$.

Find
(a) the equation of the curve, [3]
(b) the coordinates of the stationary point and determine its nature. [4]

2 Given that α and β are the roots of the equation $2x^2-7x+4=0$, form a quadratic equation with integral coefficients, whose roots are $2\alpha^3$ and $2\beta^3$. [5]

3 (a) Find the remainder when $5x^3+6x^2-7x+2$ is divided by $x-3$. [1]
(b) Show that the equation $5x^3+6x^2-7x+2=0$ has only 1 real root. [3]
(c) Find the values of p and of q such that $5x^3+6x^2-7x+2$ is a factor of $10x^4+px^3-20x^2+qx-2$. [3]

4 The function f is defined for all values of x, by

$$f(x)=1+3x^2e^x.$$

Showing your working clearly, determine
(a) the intervals on which f is an increasing function, [3]
(b) the intervals on which f is a decreasing function, [2]
(c) the range of values of $f(x)$. [2]

5 (a) Giving your answer in radians as a multiple of π, state the principal value of
(i) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$, [1]
(ii) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, [1]
(ii) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$. [1]

(b) Solve the equation $\text{cosec}^2\left(2z-\frac{\pi}{3}\right)=4$ for $0<z<\pi$. [4]

(c) On the same axes sketch, for $0°\leq x\leq 240°$, the graphs of

$$y=2\cos 3x+1 \text{ and } y=2-3\sin\frac{3}{2}x.$$ [6]

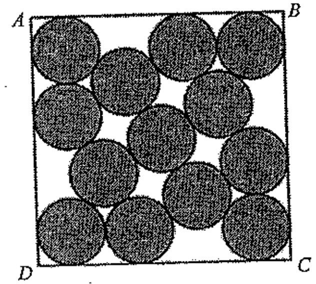
6 (a) Without the use of a calculator, find the value of

$$\frac{(\log_2 11)(\log_3 13)(\log_5 15)}{(\log_5 11)(\log_8 13)(\log_9 15)}.$$ [3]

(b) Solve the equation $2\ln(3-2x)=e$. [3]

(c) By sketching a suitable pair of graphs on the same axes, show that the equation $3\ln x=-\sqrt{x}$ has exactly one real root. [2]

7



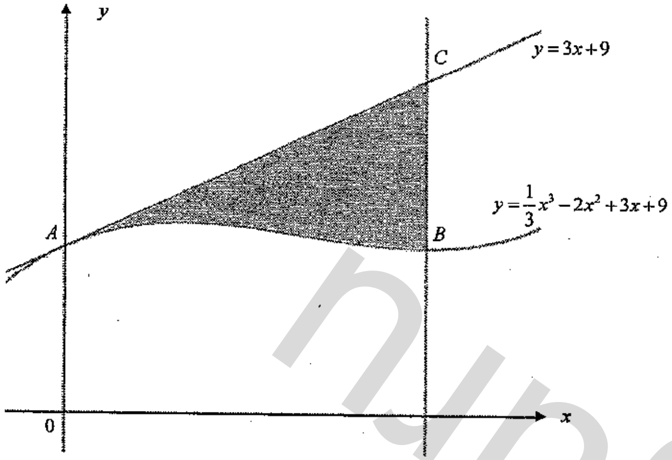
The diagram shows a maximum number of 13 identical circles packed into a square. If the radius of each circle is 1 cm,
(a) find the exact length of AD, [3]
(b) express the area of the square in the form $\left(a+b\sqrt{3}\right)$ cm². [2]

8 A circle, C_1, has equation $x^2+y^2-10x+6y+9=0$.
(a) Find the radius and the coordinates of the centre of C_1. [2]

The circle C_1, crosses the x-axis at the point $P(1,0)$.
(b) Show that the equation of the tangent to the circle at P is $3y-4x=-4$. [2]
(c) State the coordinates of Q, where the circle C_1 crosses the x-axis again. [1]

The normals to the circle C_1, at point P and point Q intersect at the point R.
(d) Calculate the area of the triangle PQR. [3]
(e) Find the equation of another circle C_2 which is a reflection of the circle C_1 in the line $x=1$. [1]

9



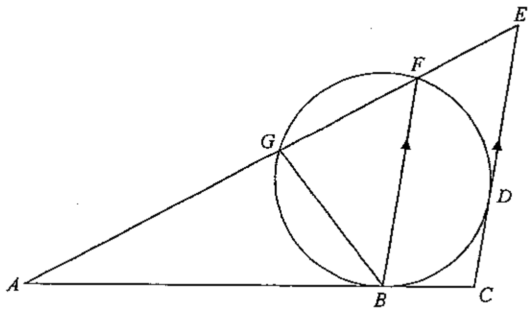
The diagram shows parts of the line $y = 3x + 9$ and the curve $y = \frac{1}{3}x^3 - 2x^2 + 3x + 9$. The line and the curve both pass through the point A on the y-axis. The curve has a minimum at the point B. The line through B, parallel to the y-axis, intersects the line $y = 3x + 9$ at the point C.

(a) Show that the line AC is a tangent to the curve at A. [2]

(b) Find equation of the line BC. [3]

(c) Calculate the area of the shaded region ABC. [5]

10



The diagram shows a triangle BGF inscribed in the circle. The triangle ACE is formed by tangents produced from the circle at points B and D.

Prove that

(a) triangle ABF and triangle ACE are similar, [2]

(b) triangle AGB and triangle ABF are similar, [2]

(c) $AB^2 = AF \times AG$, [2]

(d) $AB \times AE = AC \times AF$. [3]

11 A particle moves in a straight line such that, t s after passing through a fixed point O, its displacement from O is s m.

The velocity v ms^{-1} of the particle is such that $v = 5\cos 4t$.

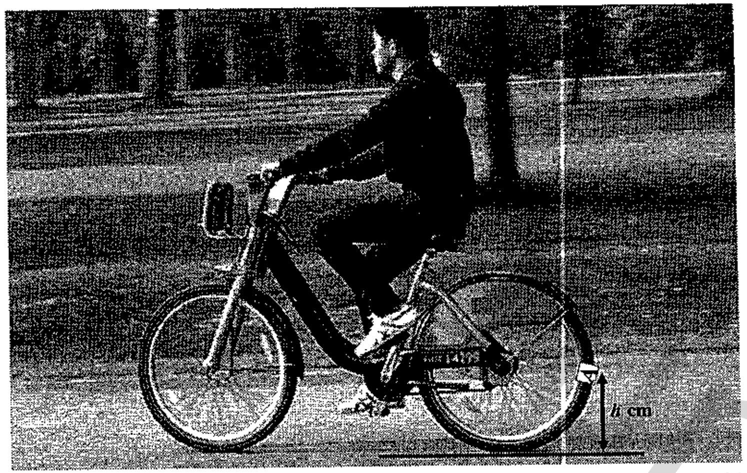
(a) State the initial velocity of the particle. [1]

(b) Determine the value of t when the acceleration of the particle is first equal to 10 ms^{-2}. [3]

(c) Find the displacement of the particle from O when $t = 5$. [3]

(d) Find the total distance travelled by the particle when it comes to instantaneous rest the second time. [5]

12



Source: http://www.telegraph.co.uk/travel/destinations/europe/united-kingdom/england/london/articles/Londons-best-Boris-bike-routes/

In City A, the rear wheel of the city rental bicycle is marked with a white tag with the letter 'A' for easy identification. The height above ground level, h cm, of the white tag on the rear wheel of the bicycle is modelled by the equation $h = 30(1 - \cos pt)$, where p is a constant and t is the time in seconds after a cyclist begins to cycle.

Suppose the cyclist is pedalling at a constant rate of 80 rpm (revolutions per minute) throughout his journey.

(a) Explain why this model suggests that the diameter of the bicycle wheel is 60 cm. [1]

(b) Show that the value of $p = \frac{8\pi}{3}$. [2]

The white tag is completely out of sight at some junctures during the cyclists journey. The white tag first goes out of sight when it is more than 40 cm above ground level and reappears when it is 30 cm above ground level.

(c) Find the length of time for which the white tag will be visible during one revolution. Give your answer in seconds. [5]

END OF PAPER

2016 4E5N Prelim Additional Mathematics Paper 2 Marking Scheme

1(a) $\frac{dy}{dx}=x^{\frac{1}{2}}-x^{-\frac{1}{2}}$

$y=\int\left(x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)dx$

$=\frac{2}{3}x^{\frac{3}{2}}-2x^{\frac{1}{2}}+c$ M1

Sub $\left(4,\frac{2}{3}\right)$

$\frac{2}{3}=\frac{2}{3}(4)^{\frac{3}{2}}-2(4)^{\frac{1}{2}}+c$ M1

$c=-\frac{2}{3}$

Equation of the curve is $y=\frac{2}{3}x^{\frac{3}{2}}-2x^{\frac{1}{2}}-\frac{2}{3}$ A1

1(b) For stationary point, $\frac{dy}{dx}=0$

$x^{\frac{1}{2}}-x^{-\frac{1}{2}}=0$

$\frac{x-1}{\sqrt{x}}=0$ M1

Given $x\neq 0, x=1$

When $x=1$, $y=-2$.

$(1,-2)$ are coordinates of the stationary point. A1

$\frac{d^2y}{dx^2}=\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{-\frac{3}{2}}$ M1

$\left.\frac{d^2y}{dx^2}\right|_{x=1}=\frac{1}{2}+\frac{1}{2}=1>0$

$\therefore$ The stationary point is minimum. A1

---

2 $2x^2-7x+4=0$

$\alpha+\beta=\frac{7}{2};\quad \alpha\beta=2$

$2\alpha^3+2\beta^3=2\left(\alpha^3+\beta^3\right)$

$=2(\alpha+\beta)\left[(\alpha+\beta)^2-3\alpha\beta\right]$

$=2\left(\frac{7}{2}\right)\left[\left(\frac{7}{2}\right)^2-3(2)\right]$ M1

$=\frac{175}{4}$ M1

$\left(2\alpha^3\right)\left(2\beta^3\right)=4(\alpha\beta)^3$

$=4(2)^3$ M1

$=32$

$x^2-\frac{175}{4}x+32=0$ M1

$4x^2-175x+128=0$ A1

3(a) Let $f(x)=5x^3+6x^2-7x+2$

By Remainder Theorem,

$f(3)=5(3)^3+6(3)^2-7(3)+2=170$

$\therefore$ The remainder is 170. B1

3(b) $5x^3+6x^2-7x+2=0$

Let $x=-2$

$f(-2)=5(-2)^3+6(-2)^2-7(-2)+2=0$

By Factor Theorem, since $f(-2)=0$, $(x+2)$ is a factor of $f(x)$.

⇨ $5x^3+6x^2-7x+2=(x+2)(5x^2+bx+1)$

Comparing the coefficients of x,

$-7=1+2b$

$b=-4$

$\therefore (x+2)(5x^2-4x+1)=0$ B1

$x=-2$ or $5x^2-4x+1=0$

For $5x^2-4x+1=0$,

$b^2-4ac=(-4)^2-4(5)(1)$ B1

$=-4$

Since $b^2-4ac=-4<0$, $5x^2-4x+1=0$ has no real roots.

$\therefore 5x^3+6x^2-7x+2=0$ has only one real root. DB1

3(c) $10x^4+px^3-20x^2+qx-2=(5x^3+6x^2-7x+2)(2x-1)$ M1

$=10x^4+7x^3-20x^2+11x-2$

By comparison, $p=7$, $q=11$. A1, A1

4(a) $f(x)=1+3x^2e^x$

$f'(x)=6xe^x+3x^2e^x$

$=3xe^x(2+x)$ M1

For an increasing function,

$3xe^x(2+x)>0$

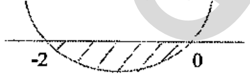
Since $e^x>0$, $3x(2+x)>0$ M1

$\therefore x<-2$ or $x>0$ A1

4(b) For a decreasing function,

$3xe^x(2+x)<0$

Since $e^x>0$, $3x(2+x)<0$ M1



$\therefore -2<x<0$ A1

4(c) Since $e^x>0$ and $x^2\geq 0$,

$3x^2e^x\geq 0$ M1

$1+3x^2e^x\geq 1$

$\therefore f(x)\geq 1$ A1

5(a)(i) principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}$ B1

5(a)(ii) principal value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)=\frac{5\pi}{6}$ B1

5(a)(iii) principal value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)=-\frac{\pi}{6}$ B1

5(b) $\operatorname{cosec}^2\left(2z-\frac{\pi}{3}\right)=4$; $-\frac{\pi}{3}<2z-\frac{\pi}{3}<\frac{5\pi}{3}$

$\sin^2\left(2z-\frac{\pi}{3}\right)=\frac{1}{4}$ M1

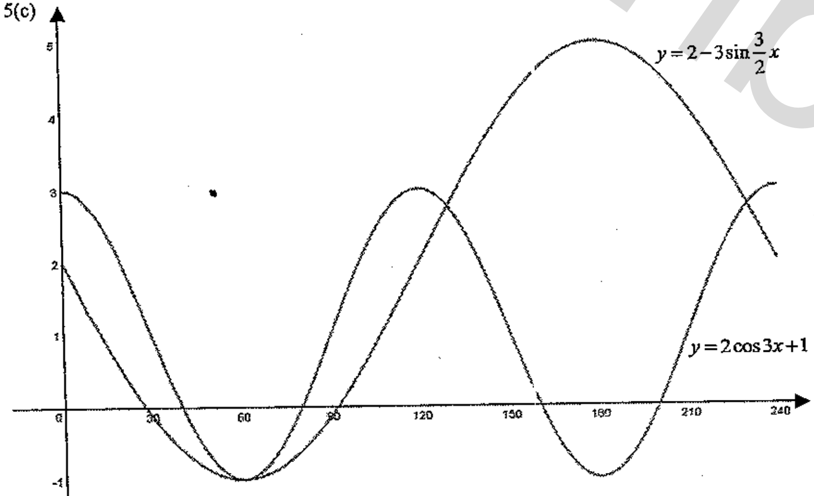
$\sin\left(2z-\frac{\pi}{3}\right)=\pm\frac{1}{2}$ (all quadrants) M1

Basic Angle $=\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$

$2z-\frac{\pi}{3}=-\frac{\pi}{6},\ \frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{7\pi}{6}$

$z=\frac{\pi}{12},\ \frac{\pi}{4},\ \frac{7\pi}{12},\ \frac{3\pi}{4}$ (-1 for each error) A2

5(c)



D3 for amplitude, period and shape of $y=2\cos 3x+1$

D3 for amplitude, period and shape of $y=2-3\sin\frac{3}{2}x$

6(a) $\frac{(\log_2 11)(\log_3 13)(\log_5 15)}{(\log_5 11)(\log_8 13)(\log_9 15)}$

$=\frac{\left(\frac{\lg 11}{\lg 2}\right)\left(\frac{\lg 13}{\lg 3}\right)\left(\frac{\lg 15}{\lg 5}\right)}{\left(\frac{\lg 11}{\lg 5}\right)\left(\frac{\lg 13}{\lg 8}\right)\left(\frac{\lg 15}{\lg 9}\right)}$ M1

$=\frac{\left(\frac{1}{\lg 2}\right)\left(\frac{1}{\lg 3}\right)}{\left(\frac{1}{\lg 8}\right)\left(\frac{1}{\lg 9}\right)}$

$=\frac{(3\lg 2)(2\lg 3)}{(\lg 2)(\lg 3)}$ M1

$=6$ A1

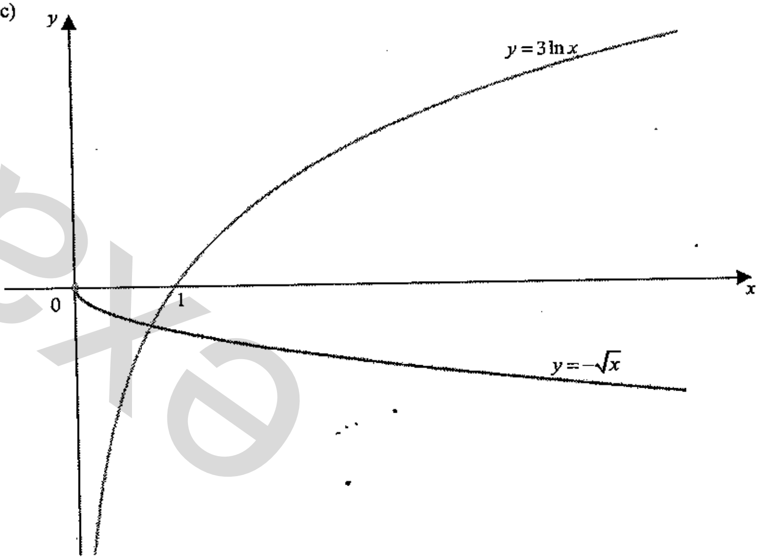
6(b) $2\ln(3-2x)=e$

$\ln(3-2x)=\frac{1}{2}e$ M1

$3-2x=e^{\frac{1}{2}e}$ M1

$x=-0.446$ (3s.f.) A1

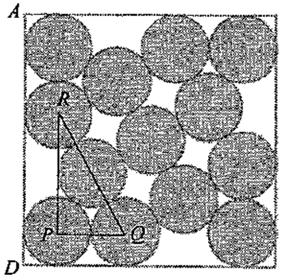
6(c)



D1 for $y=3\ln x$

D1 for $y=-\sqrt{x}$

7(a)



Let the points P, Q and R be the centres of the circles as shown above.

$RQ = 4$ cm

$PQ = 2$cm

By Pythagoras's Theorem,

$PR = \sqrt{4^2 - 2^2}$ M1

$= 2\sqrt{3}$ cm M1

$AD = 4 + PR = \left(4 + 2\sqrt{3}\right)$cm A1

7(b) Area of square $= \left(4 + 2\sqrt{3}\right)^2$

$= \left(16 + 16\sqrt{3} + 12\right)$ cm^2 M1

$= \left(28 + 16\sqrt{3}\right)$cm^2 A1

8(a) $\left(x-5\right)^2 + \left(y+3\right)^2 = 25$

Radius of C$_1$ is 5 units. B1

Centre of C$_1$ is $(5, -3)$. B1

8(b) Let centre of C$_1$ be A.

$m_{AP} = \frac{-3}{5-1} = \frac{-3}{4}$

$m_{\text{tangent at } P} = \frac{4}{3}$ M1

Sub $(1,0)$ into $y = \frac{4}{3}x + c$

$0 = \frac{4}{3} + c$

$c = -\frac{4}{3}$

$y = \frac{4}{3}x - \frac{4}{3}$

$3y - 4x = -4$ (shown) A1

8(c) When $y = 0$,

$x^2 - 10x + 9 = 0$

$(x-1)(x-9) = 0$

$x = 9$ or 1(x-coordinate of P)

$\therefore Q(9,0)$ B1

8(d) The normals to the circle at points P and Q intersect at the centre of the circle.

⇨ R is the centre of the circle. $R(5, -3)$ M1

Area of Triangle $PQR = \frac{1}{2}(9-1)(3)$ M1

$= 12$ units^2 A1

8(e) $(x+3)^2 + (y+3)^2 = 25$ B1

9(a) $\frac{dy}{dx} = x^2 - 4x + 3$ M1

$m_{\text{tangent at } A} = \left.\frac{dy}{dx}\right|_{x=0}$

$= 3$

$= m_{AC}$ (shown) A1

9(b) For stationary point at B, $\frac{dy}{dx} = 0$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x = 1$ or 3 M1

$\frac{d^2y}{dx^2} = 2x - 4$

$\left.\frac{d^2y}{dx^2}\right|_{x=1} = -2 < 0$

The curve has a maximum point at $x = 1$.

$\left.\frac{d^2y}{dx^2}\right|_{x=3} = 2 > 0$ M1

The curve has a minimum point at $x = 3$.

Hence, the equation of line BC is $x = 3$. A1

9(c) When $x = 3$, $y = 18$

⇨ $C(3,18)$ M1

Area of shaded region

$= \frac{1}{2}(3)(9+18) - \int_0^3 \left(\frac{1}{3}x^3 - 2x^2 + 3x + 9\right) dx$ M1

$= \frac{81}{2} - \left[\frac{1}{12}x^4 - \frac{2}{3}x^3 + \frac{3}{2}x^2 + 9x\right]_0^3$ M1

$= \frac{81}{2} - 29\frac{1}{4}$ M1

$= 11\frac{1}{4}$units^2 A1

10(a) $\angle ABF = \angle ACE$ (corr.$\angle$s, $BF//CE$)

$\angle AFB = \angle AEC$ (corr.$\angle$s, $BF//CE$)

$\angle BAF = \angle CAE$ (common $\angle$)

B2 (any 2)

Since all corresponding angles are equal, triangle ABF and triangle ACE are similar.

10(b) $\angle ABG = \angle AFB$ (alt. seg. thm.)

$\angle BAG = \angle FAB$ (common $\angle$) B2

Since all corresponding angles are equal, triangle AGB and triangle ABF are similar.

10(c) Since triangle AGB and triangle ABF are similar,

$\frac{AB}{AF}=\frac{AG}{AB}$ B1

$AB^2=AF\times AG$ (shown) DB1

10(d) Since triangle ABF and triangle ACE are similar,

$\frac{AB}{AC}=\frac{AF}{AE}$ B2

$AB\times AE=AF\times AC$ (shown) DB1

11(a) $v=5\cos 4t$

When $t=0$, $v=5$

Initial velocity of the particle is 5 m/s. B1

11(b) $a=\frac{dv}{dt}=-20\sin 4t$ M1

$-20\sin 4t=10$

$\sin 4t=-\frac{1}{2}$ M1

$4t=\frac{7\pi}{6}$

$t=0.916$ (3 s.f.) A1

11(c) $s=\int 5\cos 4t\ dt$

$=\frac{5}{4}\sin 4t+c$ M1

When $t=0$, $s=0$,

$c=0$

$s=\frac{5}{4}\sin 4t$ M1

When $t=5$,

$s=\frac{5}{4}\sin 20$

$=1.14$ (3s.f.)

Displacement $=1.14$ m A1

11(d) At instantaneous rest, $v=0$

$5\cos 4t=0$

$\cos 4t=0$ M1

$4t=\frac{\pi}{2},\ \frac{3\pi}{2}$

$t=\frac{\pi}{8},\ \frac{3\pi}{8}$ M1

When $t=\frac{\pi}{8}$, $s=\frac{5}{4}\sin\frac{\pi}{2}=1.25$ M1

When $t=\frac{3\pi}{8}$, $s=\frac{5}{4}\sin\frac{3\pi}{2}=-1.25$ M1

Total distance travelled

$=(1.25\times 2)+1.25$

$=3.75$ m A1

12(a) Maximum height occurs when $\cos pt=-1$

Maximum height $=30(1-(-1))=60$ cm B1

12(b) 80 revolutions / minute

= 80 revolutions / 60 seconds

= 1 revolution / 0.75 seconds

Period = 0.75 seconds M1

$p=\frac{2\pi}{0.75}$

$=\frac{8\pi}{3}$ (shown)

OR

Sub $t=0.375$, $h=60$

$60=30(1-\cos(0.375p))$

$\cos(0.375p)=-1$

$0.375p=\pi$

$p=\frac{8\pi}{3}$ (shown) DB1

12(c) When $h=40$,

$40=30\left(1-\cos\left(\frac{8\pi}{3}t\right)\right)$ M1

$\cos\left(\frac{8\pi}{3}t\right)=-\frac{1}{3}$

$\frac{8\pi}{3}t=\pi-1.2310$ M1

$t\approx 0.22807$ A1

When $h=30$,

$t=0.75\times\frac{3}{4}=0.5625$

OR

$30=30\left(1-\cos\left(\frac{8\pi}{3}t\right)\right)$

$\cos\left(\frac{8\pi}{3}t\right)=0$

$\frac{8\pi}{3}t=\frac{\pi}{2},\frac{3\pi}{2}$

$t\approx\frac{3}{16}$ (rej.), $\frac{9}{16}$ M1

Time interval $\approx 0.75-(0.5625-0.22807)=0.416$ seconds (3s.f.) A1

COMMONWEALTH SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2016

ADDITIONAL MATHEMATICS
PAPER 2

Name: ______________________________ (   )   Class: ________

SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC
4047/2

Thursday 18 August 2016
08 00 – 10 30
2 h 30 min

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Name of setter: Ms Lee YJ

This paper consists of 7 printed pages including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots\ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

1 A curve is such that $\frac{dy}{dx} = x^{\frac{1}{2}} - x^{-\frac{1}{2}}$, $x \neq 0$. The curve passes through the point $\left(4, \frac{2}{3}\right)$.

Find

(a) the equation of the curve, [3]

(b) the coordinates of the stationary point and determine its nature. [4]

2 Given that α and β are the roots of the equation $2x^2 - 7x + 4 = 0$, form a quadratic equation with integral coefficients, whose roots are $2\alpha^3$ and $2\beta^3$. [5]

3 **(a)** Find the remainder when $5x^3 + 6x^2 - 7x + 2$ is divided by $x - 3$. [1]

(b) Show that the equation $5x^3 + 6x^2 - 7x + 2 = 0$ has only 1 real root. [3]

(c) Find the values of p and of q such that $5x^3 + 6x^2 - 7x + 2$ is a factor of $10x^4 + px^3 - 20x^2 + qx - 2$. [3]

4 The function f is defined for all values of x, by

$$f(x) = 1 + 3x^2 e^x.$$

Showing your working clearly, determine

(a) the intervals on which f is an increasing function, [3]

(b) the intervals on which f is a decreasing function, [2]

(c) the range of values of $f(x)$. [2]

5 **(a)** Giving your answer in radians as a multiple of π, state the principal value of

(i) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$, [1]

(ii) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, [1]

(ii) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$. [1]

(b) Solve the equation $\operatorname{cosec}^2\left(2z - \frac{\pi}{3}\right) = 4$ for $0 < z < \pi$. [4]

(c) On the same axes sketch, for $0° \leq x \leq 240°$, the graphs of

$$y = 2\cos 3x + 1 \text{ and } y = 2 - 3\sin\frac{3}{2}x.$$ [6]

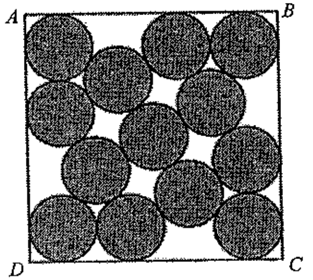
6 **(a)** Without the use of a calculator, find the value of

$$\frac{(\log_2 11)(\log_3 13)(\log_5 15)}{(\log_5 11)(\log_8 13)(\log_9 15)}.$$ [3]

(b) Solve the equation $2\ln(3 - 2x) = e$. [3]

(c) By sketching a suitable pair of graphs on the same axes, show that the equation $3\ln x = -\sqrt{x}$ has exactly one real root. [2]

7



The diagram shows a maximum number of 13 identical circles packed into a square. If the radius of each circle is 1 cm,

(a) find the exact length of AD, [3]

(b) express the area of the square in the form $\left(a + b\sqrt{3}\right)$ cm². [2]

8 A circle, C_1, has equation $x^2 + y^2 - 10x + 6y + 9 = 0$.

(a) Find the radius and the coordinates of the centre of C_1. [2]

The circle C_1, crosses the x-axis at the point $P(1, 0)$.

(b) Show that the equation of the tangent to the circle at P is $3y - 4x = -4$. [2]

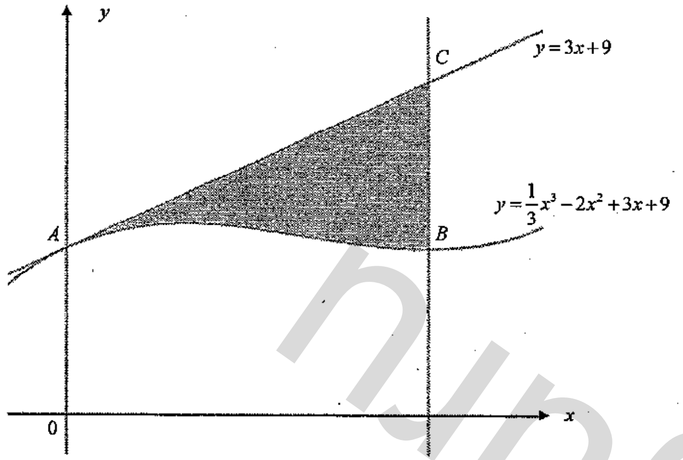
(c) State the coordinates of Q, where the circle C_1 crosses the x-axis again. [1]

The normals to the circle C_1, at point P and point Q intersect at the point R.

(d) Calculate the area of the triangle PQR. [3]

(e) Find the equation of another circle C_2 which is a reflection of the circle C_1 in the line $x = 1$. [1]

9



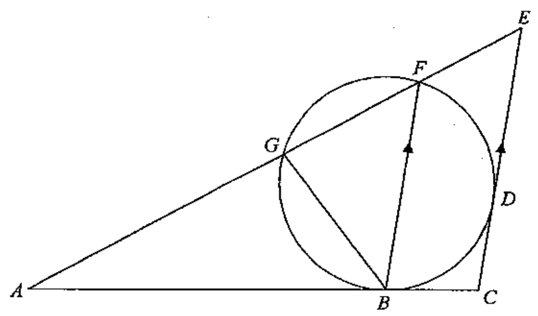
The diagram shows parts of the line $y = 3x + 9$ and the curve $y = \frac{1}{3}x^3 - 2x^2 + 3x + 9$. The line and the curve both pass through the point A on the y-axis. The curve has a minimum at the point B. The line through B, parallel to the y-axis, intersects the line $y = 3x + 9$ at the point C.

(a) Show that the line AC is a tangent to the curve at A. [2]

(b) Find equation of the line BC. [3]

(c) Calculate the area of the shaded region ABC. [5]

10



The diagram shows a triangle BGF inscribed in the circle. The triangle ACE is formed by tangents produced from the circle at points B and D.

Prove that

(a) triangle ABF and triangle ACE are similar, [2]

(b) triangle AGB and triangle ABF are similar, [2]

(c) $AB^2 = AF \times AG$, [2]

(d) $AB \times AE = AC \times AF$. [3]

11 A particle moves in a straight line such that, t s after passing through a fixed point O, its displacement from O is s m.

The velocity v ms^{-1} of the particle is such that $v = 5\cos 4t$.

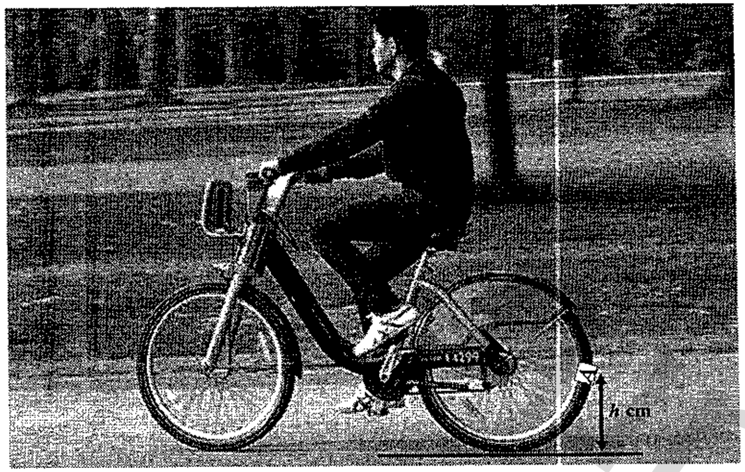
(a) State the initial velocity of the particle. [1]

(b) Determine the value of t when the acceleration of the particle is first equal to 10 ms^{-2}. [3]

(c) Find the displacement of the particle from O when $t = 5$. [3]

(d) Find the total distance travelled by the particle when it comes to instantaneous rest the second time. [5]

12



Source: http://www.telegraph.co.uk/travel/destinations/europe/united-kingdom/england/london/articles/Londons-best-Boris-bike-routes/

In City A, the rear wheel of the city rental bicycle is marked with a white tag with the letter 'A' for easy identification. The height above ground level, h cm, of the white tag on the rear wheel of the bicycle is modelled by the equation $h = 30(1 - \cos pt)$, where p is a constant and t is the time in seconds after a cyclist begins to cycle.

Suppose the cyclist is pedalling at a constant rate of 80 rpm (revolutions per minute) throughout his journey.

(a) Explain why this model suggests that the diameter of the bicycle wheel is 60 cm. [1]

(b) Show that the value of $p = \frac{8\pi}{3}$. [2]

The white tag is completely out of sight at some junctures during the cyclists journey. The white tag first goes out of sight when it is more than 40 cm above ground level and reappears when it is 30 cm above ground level.

(c) Find the length of time for which the white tag will be visible during one revolution. Give your answer in seconds. [5]

END OF PAPER

2016 4E5N Prelim Additional Mathematics Paper 2 Marking Scheme

1(a) $\frac{dy}{dx}=x^{\frac{1}{2}}-x^{-\frac{1}{2}}$

$y=\int\left(x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)dx$

$=\frac{2}{3}x^{\frac{3}{2}}-2x^{\frac{1}{2}}+c$ M1

Sub $\left(4,\frac{2}{3}\right)$

$\frac{2}{3}=\frac{2}{3}(4)^{\frac{3}{2}}-2(4)^{\frac{1}{2}}+c$ M1

$c=-\frac{2}{3}$

Equation of the curve is $y=\frac{2}{3}x^{\frac{3}{2}}-2x^{\frac{1}{2}}-\frac{2}{3}$. A1

1(b) For stationary point, $\frac{dy}{dx}=0$

$x^{\frac{1}{2}}-x^{-\frac{1}{2}}=0$

$\frac{x-1}{\sqrt{x}}=0$ M1

Given $x\neq0, x=1$

When $x=1$, $y=-2$.

(1,−2) are coordinates of the stationary point. A1

$\frac{d^2y}{dx^2}=\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{-\frac{3}{2}}$ M1

$\left.\frac{d^2y}{dx^2}\right|_{x=1}=\frac{1}{2}+\frac{1}{2}=1>0$

∴ The stationary point is minimum. A1

---

2 $2x^2-7x+4=0$

$\alpha+\beta=\frac{7}{2};\qquad \alpha\beta=2$

$2\alpha^3+2\beta^3=2(\alpha^3+\beta^3)$

$=2(\alpha+\beta)\left[(\alpha+\beta)^2-3\alpha\beta\right]$

$=2\left(\frac{7}{2}\right)\left[\left(\frac{7}{2}\right)^2-3(2)\right]$ M1

$=\frac{175}{4}$ M1

$(2\alpha^3)(2\beta^3)=4(\alpha\beta)^3$

$=4(2)^3$ M1

$=32$

$x^2-\frac{175}{4}x+32=0$ M1

$4x^2-175x+128=0$ A1

3(a) Let $f(x)=5x^3+6x^2-7x+2$

By Remainder Theorem,

$f(3)=5(3)^3+6(3)^2-7(3)+2=170$

∴ The remainder is 170. B1

3(b) $5x^3+6x^2-7x+2=0$

Let $x=-2$

$f(-2)=5(-2)^3+6(-2)^2-7(-2)+2=0$

By Factor Theorem, since $f(-2)=0$, $(x+2)$ is a factor of $f(x)$.

⇨ $5x^3+6x^2-7x+2=(x+2)(5x^2+bx+1)$

Comparing the coefficients of x,

$-7=1+2b$

$b=-4$

$\therefore (x+2)(5x^2-4x+1)=0$ B1

$x=-2$ or $5x^2-4x+1=0$

For $5x^2-4x+1=0$,

$b^2-4ac=(-4)^2-4(5)(1)$ B1

$=-4$

Since $b^2-4ac=-4<0$, $5x^2-4x+1=0$ has no real roots.

$\therefore 5x^3+6x^2-7x+2=0$ has only one real root. DB1

3(c) $10x^4+px^3-20x^2+qx-2=(5x^3+6x^2-7x+2)(2x-1)$ M1

$=10x^4+7x^3-20x^2+11x-2$

By comparison, $p=7$, $q=11$. A1, A1

4(a) $f(x)=1+3x^2e^x$

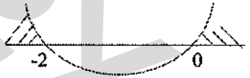
$f'(x)=6xe^x+3x^2e^x$

$=3xe^x(2+x)$ M1

For an increasing function,

$3xe^x(2+x)>0$

Since $e^x>0$, $3x(2+x)>0$ M1

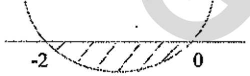


$\therefore x<-2$ or $x>0$ A1

4(b) For a decreasing function,

$3xe^x(2+x)<0$

Since $e^x>0$, $3x(2+x)<0$ M1



$\therefore -2<x<0$ A1

4(c) Since $e^x>0$ and $x^2\geq0$,

$3x^2e^x\geq0$ M1

$1+3x^2e^x\geq1$

$\therefore f(x)\geq1$ A1

5(a)(i) principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}$ B1

5(a)(ii) principal value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)=\frac{5\pi}{6}$ B1

5(a)(iii) principal value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)=-\frac{\pi}{6}$ B1

5(b) $\operatorname{cosec}^2\left(2z-\frac{\pi}{3}\right)=4$; $-\frac{\pi}{3}<2z-\frac{\pi}{3}<\frac{5\pi}{3}$

$\sin^2\left(2z-\frac{\pi}{3}\right)=\frac{1}{4}$ M1

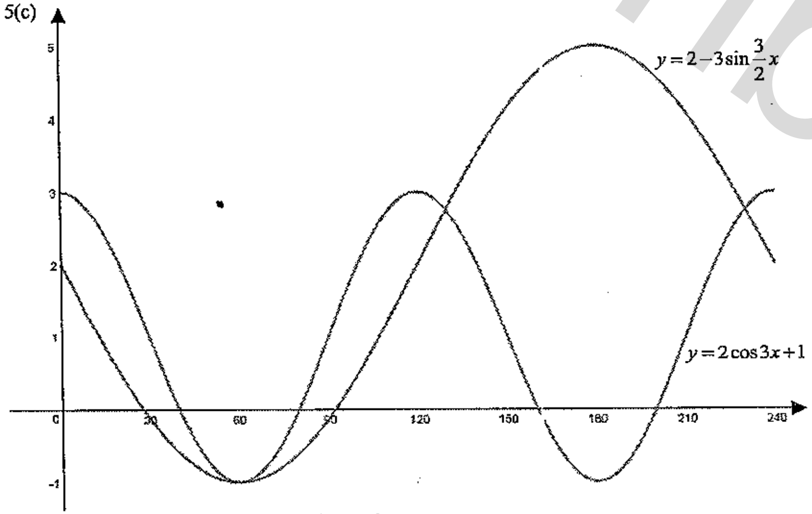
$\sin\left(2z-\frac{\pi}{3}\right)=\pm\frac{1}{2}$ (all quadrants) M1

Basic Angle = $\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$

$2z-\frac{\pi}{3}=-\frac{\pi}{6},\ \frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{7\pi}{6}$

$z=\frac{\pi}{12},\ \frac{\pi}{4},\ \frac{7\pi}{12},\ \frac{3\pi}{4}$ (-1 for each error) A2

5(c)



D3 for amplitude, period and shape of $y=2\cos 3x+1$

D3 for amplitude, period and shape of $y=2-3\sin\frac{3}{2}x$

6(a) $\frac{(\log_2 11)(\log_3 13)(\log_5 15)}{(\log_5 11)(\log_8 13)(\log_9 15)}$

$=\frac{\left(\frac{\lg 11}{\lg 2}\right)\left(\frac{\lg 13}{\lg 3}\right)\left(\frac{\lg 15}{\lg 5}\right)}{\left(\frac{\lg 11}{\lg 5}\right)\left(\frac{\lg 13}{\lg 8}\right)\left(\frac{\lg 15}{\lg 9}\right)}$ M1

$=\frac{\left(\frac{1}{\lg 2}\right)\left(\frac{1}{\lg 3}\right)}{\left(\frac{1}{\lg 8}\right)\left(\frac{1}{\lg 9}\right)}$

$=\frac{(3\lg 2)(2\lg 3)}{(\lg 2)(\lg 3)}$ M1

$=6$ A1

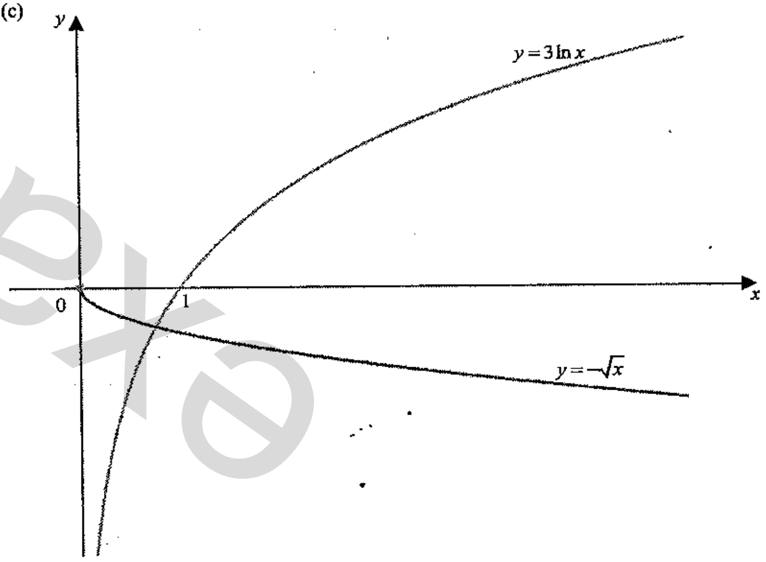
6(b) $2\ln(3-2x)=e$

$\ln(3-2x)=\frac{1}{2}e$ M1

$3-2x=e^{\frac{1}{2}e}$ M1

$x=-0.446$ (3s.f.) A1

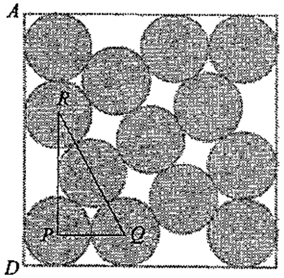
6(c)



D1 for $y=3\ln x$

D1 for $y=-\sqrt{x}$

7(a)



Let the points P, Q and R be the centres of the circles as shown above.
$RQ = 4$ cm
$PQ = 2$cm
By Pythagoras's Theorem,
$PR = \sqrt{4^2 - 2^2}$ M1
$= 2\sqrt{3}$ cm M1
$AD = 4 + PR = \left(4 + 2\sqrt{3}\right)$cm A1

7(b) Area of square $= \left(4 + 2\sqrt{3}\right)^2$
$= \left(16 + 16\sqrt{3} + 12\right)$ cm^2 M1
$= \left(28 + 16\sqrt{3}\right)$cm^2 A1

8(a) $(x-5)^2 + (y+3)^2 = 25$
Radius of C_1 is 5 units. B1
Centre of C_1 is $(5, -3)$. B1

8(b) Let centre of C_1 be A.
$m_{AP} = \frac{-3}{5-1} = \frac{-3}{4}$
$m_{\text{tangent at } P} = \frac{4}{3}$ M1
Sub $(1, 0)$ into $y = \frac{4}{3}x + c$
$0 = \frac{4}{3} + c$
$c = -\frac{4}{3}$
$y = \frac{4}{3}x - \frac{4}{3}$
$3y - 4x = -4$ (shown) A1

8(c) When $y = 0$,
$x^2 - 10x + 9 = 0$
$(x-1)(x-9) = 0$
$x = 9$ or 1 (x-coordinate of P)
$\therefore Q(9, 0)$ B1

8(d) The normals to the circle at points P and Q intersect at the centre of the circle.
⇨ R is the centre of the circle. $R(5, -3)$ M1
Area of Triangle $PQR = \frac{1}{2}(9-1)(3)$ M1
$= 12$ units^2 A1

8(e) $(x+3)^2 + (y+3)^2 = 25$ B1

9(a) $\frac{dy}{dx} = x^2 - 4x + 3$ M1
$m_{\text{tangent at } A} = \left.\frac{dy}{dx}\right|_{x=0}$
$= 3$
$= m_{AC}$ (shown) A1

9(b) For stationary point at B, $\frac{dy}{dx} = 0$
$x^2 - 4x + 3 = 0$
$(x-1)(x-3) = 0$
$x = 1$ or 3 M1
$\frac{d^2y}{dx^2} = 2x - 4$
$\left.\frac{d^2y}{dx^2}\right|_{x=1} = -2 < 0$
The curve has a maximum point at $x = 1$.
$\left.\frac{d^2y}{dx^2}\right|_{x=3} = 2 > 0$ M1
The curve has a minimum point at $x = 3$.
Hence, the equation of line BC is $x = 3$. A1

9(c) When $x = 3, y = 18$
⇨ $C(3, 18)$ M1
Area of shaded region
$= \frac{1}{2}(3)(9+18) - \int_0^3 \left(\frac{1}{3}x^3 - 2x^2 + 3x + 9\right) dx$ M1
$= \frac{81}{2} - \left[\frac{1}{12}x^4 - \frac{2}{3}x^3 + \frac{3}{2}x^2 + 9x\right]_0^3$ M1
$= \frac{81}{2} - 29\frac{1}{4}$ M1
$= 11\frac{1}{4}$ units^2 A1

10(a) $\angle ABF = \angle ACE$ (corr. $\angle$s, $BF // CE$)
$\angle AFB = \angle AEC$ (corr. $\angle$s, $BF // CE$)
$\angle BAF = \angle CAE$ (common $\angle$)
B2 (any 2)
Since all corresponding angles are equal, triangle ABF and triangle ACE are similar.

10(b) $\angle ABG = \angle AFB$ (alt. seg. thm.)
$\angle BAG = \angle FAB$ (common $\angle$)
B2
Since all corresponding angles are equal, triangle AGB and triangle ABF are similar.

10(c) Since triangle AGB and triangle ABF are similar,

$\frac{AB}{AF}=\frac{AG}{AB}$ B1

$AB^2=AF\times AG$ (shown) DB1

10(d) Since triangle ABF and triangle ACE are similar,

$\frac{AB}{AC}=\frac{AF}{AE}$ B2

$AB\times AE=AF\times AC$ (shown) DB1

11(a) $v=5\cos 4t$

When $t=0$, $v=5$

Initial velocity of the particle is 5 m/s. B1

11(b) $a=\frac{dv}{dt}=-20\sin 4t$ M1

$-20\sin 4t=10$

$\sin 4t=-\frac{1}{2}$ M1

$4t=\frac{7\pi}{6}$

$t=0.916$ (3 s.f.) A1

11(c) $s=\int 5\cos 4t\ dt$

$=\frac{5}{4}\sin 4t+c$ M1

When $t=0$, $s=0$,

$c=0$

$s=\frac{5}{4}\sin 4t$ M1

When $t=5$,

$s=\frac{5}{4}\sin 20$

$=1.14$ (3s.f.)

Displacement $=1.14$ m A1

11(d) At instantaneous rest, $v=0$

$5\cos 4t=0$

$\cos 4t=0$ M1

$4t=\frac{\pi}{2},\ \frac{3\pi}{2}$

$t=\frac{\pi}{8},\ \frac{3\pi}{8}$ M1

When $t=\frac{\pi}{8}$, $s=\frac{5}{4}\sin\frac{\pi}{2}=1.25$ M1

When $t=\frac{3\pi}{8}$, $s=\frac{5}{4}\sin\frac{3\pi}{2}=-1.25$ M1

Total distance travelled

$=(1.25\times 2)+1.25$

$=3.75$ m A1

12(a) Maximum height occurs when $\cos pt=-1$

Maximum height $=30(1-(-1))=60$ cm B1

12(b) 80 revolutions / minute

= 80 revolutions / 60 seconds

= 1 revolution / 0.75 seconds

Period = 0.75 seconds M1

$p=\frac{2\pi}{0.75}$

$=\frac{8\pi}{3}$ (shown)

OR

Sub $t=0.375$, $h=60$

$60=30(1-\cos(0.375p))$

$\cos(0.375p)=-1$

$0.375p=\pi$

$p=\frac{8\pi}{3}$ (shown) DB1

12(c) When $h=40$,

$40=30\left(1-\cos\left(\frac{8\pi}{3}t\right)\right)$ M1

$\cos\left(\frac{8\pi}{3}t\right)=-\frac{1}{3}$

$\frac{8\pi}{3}t=\pi-1.2310$ M1

$t\approx 0.22807$ A1

When $h=30$,

$t=0.75\times\frac{3}{4}=0.5625$

OR

$30=30\left(1-\cos\left(\frac{8\pi}{3}t\right)\right)$

$\cos\left(\frac{8\pi}{3}t\right)=0$

$\frac{8\pi}{3}t=\frac{\pi}{2},\frac{3\pi}{2}$

$t\approx\frac{3}{16}$ (rej.) $,\frac{9}{16}$ M1

Time interval $\approx 0.75-(0.5625-0.22807)=0.416$ seconds (3s.f.) A1

COMMONWEALTH SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2016

ADDITIONAL MATHEMATICS
PAPER 1

Name: ______________________ (   )   Class: ________

SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC
4047/1

Wednesday 17 August 2016
08 00 – 10 00
2h

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Name of setter: Mr Eugene Lee

This paper consists of **5** printed pages including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1. The equation of a curve is $y=\frac{\cos^2 x}{2+\sin x}$. Find the equation of the tangent to the curve where the curve meets the y-axis. [4]

2. (i) Express $\frac{2x+8}{(x^2+4)(x-1)}$ in partial fractions. [5]

(ii) Hence, evaluate $\int_2^3 \frac{x+4}{(x^2+4)(x-1)}\,dx$. [5]

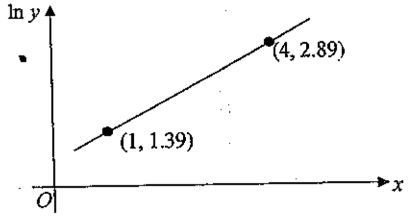
3. (i) Sketch the curve $y=|x^2-2x-3|$ for $-3\le x\le 3$. [3]

(ii) Explain why the equation $|x^2-2x-3|=15$ has no real roots for $-3\le x\le 3$. [1]

(iii) Find the x-coordinates of the points of intersection of the curve $y=|x^2-2x-3|$ and the line $y=1-x$. [3]

4. The first 3 terms in the expansion of $(a+x)^4+(2-bx)^5$ in ascending powers of x are $48+12x+cx^2$, where a, b and c are positive constants. Find the values of a, of b and of c. [5]

5.



The variables x and y satisfy the equation $y=Ae^{k(x-1)}$ where A and k are constants. The graph of $\ln y$ against x is a straight line passing through the points (1,1.39) and (4,2.89) as shown in the diagram. Find the values of A and of k. [4]

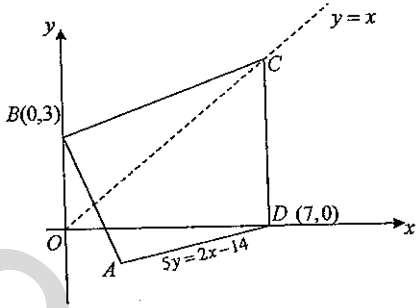
6. A curve has equation $y=x^3+kx^2+kx+8$. Find the set of values of k such that
(i) the curve is a strictly increasing function, [4]
(ii) the curve has exactly 1 stationary point. [2]

7. The point P lies on the curve $y=\ln\left(\frac{x+1}{x-1}\right)$ for $x>1$. The normal to the curve at P is parallel to the line $2y=3x+2$.
(i) Find the coordinates of P. [5]
The tangent at P meets the line $2y=3x+2$ at Q.
(ii) Find the coordinates of Q. [3]

8. (i) Prove that $2\operatorname{cosec}2x\tan x=\sec^2 x$. [3]
Hence
(ii) solve the equation $2\operatorname{cosec}2x\tan x=\sec x+2$ for $0<x<2\pi$, [3]
(iii) find $\int(\operatorname{cosec}2x\tan x+1)\,dx$. [3]

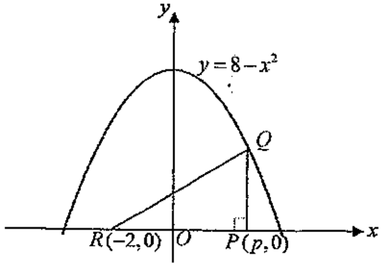
9. The acute angles A and B are such that $\sin A=\frac{1}{5}$ and $\tan B=3$. Without using a calculator, find the exact value of
(a) $\cos A$, [2]
(b) $\sin(A+B)$, [2]
(c) $\cot 2B$. [2]

10. Solutions to this question by accurate drawing will not be accepted.



The diagram shows a quadrilateral $ABCD$ in which the point B is (0, 3) and the point D is (7, 0). The equation of the line AD is $5y=2x-14$ and C lies on the line $y=x$. The line CD is parallel to the y-axis.
Given that A lies on the perpendicular bisector of BD,
(i) find the coordinates of A and of C, [4]
(ii) find the area of the quadrilateral $ABCD$, [2]
(iii) explain clearly whether or not the quadrilateral $ABCD$ is a kite. [3]

11.



The diagram shows the curve $y=8-x^2$ and the points $R(-2,0)$ and $P(p,0)$. The point Q lies on the curve such that PQ is parallel to the y-axis.

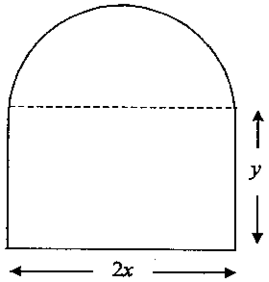
(i) Show that the area, A units^2, of the triangle PQR is given by

$$A=\frac{1}{2}(p+2)(8-p^2).$$ [2]

The point P moves along the x-axis at a constant rate of 0.04 units per second and Q moves along the curve so that PQ remains parallel to the y-axis.

(ii) Find the rate at which A is decreasing when $p=1.5$. [3]

12.



A gardener uses 200 m of fencing to enclose a plot of land in the shape shown above. The shape consists of a semicircle of radius x m and a rectangle with sides $2x$ m and y m.

(i) Show that the area, A m^2, of the plot of land is given by

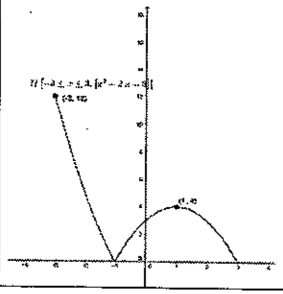
$$A=200x-\left(\frac{\pi+4}{2}\right)x^2.$$ [3]

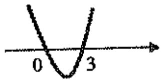
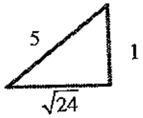
(ii) Given that x can vary, find the value of x for which the area of the plot is the largest possible. [4]

END OF PAPER

2016 4E5N Prelim AM Mark Scheme

1	$\frac{dy}{dx}=\frac{(2+\sin x)(2\cos x(-\sin x))-\cos^3 x}{(2+\sin x)^2}$ when $x=0$, $\frac{dy}{dx}=-\frac{1}{4}$ when $x=0$, $y=\frac{1}{2}$ Equation of tangent: $y-\frac{1}{2}=-\frac{1}{4}x$ $\Rightarrow y=\frac{1}{4}x+\frac{1}{2}$	M1 M1 M1 A1
2i	Let $\frac{2x+8}{(x^2+4)(x-1)}=\frac{Ax+B}{x^2+4}+\frac{C}{x-1}$ $2x+8=(Ax+b)(x-1)+C(x^2+4)$. By substitution or comparison of coefficients: $A=-2$ $B=0$ $C=2$ Hence $\frac{2x+8}{(x^2+4)(x-1)}=\frac{-2x}{x^2+4}+\frac{2}{x-1}$	M1 B1 B1 B1 A1
2ii	$\int_2^3\frac{x+4}{(x^2+4)(x-1)}dx=\int_2^3\left(\frac{-x}{x^2+4}+\frac{1}{x-1}\right)dx$ $=\left[-\frac{1}{2}\ln(x^2+4)+\ln(x-1)\right]_2^3$ $=\left(-\frac{1}{2}\ln 13+\ln 2\right)-\left(-\frac{1}{2}\ln 8\right)$ $=0.450$ (3 s.f.)	M1M1 (each term) M1 M1 A1

3i		D1(x and y intercepts) D1(turning pt) D1(shape)
3ii	The maximum value of y for the given range is 12.	B1
3iii	$\lvert x^2-2x-3\rvert=1-x$ $\Rightarrow x^2-x-4=0$ or $x^2-3x-2=0$ $x=-0.562$ or $x=-1.56$	M1 B1 A1
4	$(a+x)^4+(2-bx)^5=(a^4+4a^3x+6a^2x^2+...)+(32-80bx+80b^2x^2+...)$ Comparing coefficients, $32+a^4=48$ $a=2$ or -2 (NA) Similarly, $b=\frac{1}{4}$ $c=29$	B1 B1 (for each term) B1 B1 B1
5	$y=Ae^{k(x-1)}$ $\ln y=kx+(\ln A-k)$ gradient $=k=\frac{2.89-1.39}{4-1}=0.5$ $2.89=0.5(4)+\ln A-0.5$ $A=4.01$ (3 s.f.)	M1 A1 M1 A1

6i	$\frac{dy}{dx}=3x^2+2kx+k$ If $\frac{dy}{dx}>0$ for all real values of x, $D<0$. $\Rightarrow(2k)^2-4(3)(k)<0$ $4k^2-12k<0$ $k(k-3)<0$ $0<k<3$ 	M1 M1 D1 A1
6ii	Let $\frac{dy}{dx}=3x^2+2kx+k=0$ $k=0$ or $k=3$	M1 A1
7i	$\frac{dy}{dx}=\frac{1}{x+1}-\frac{1}{x-1}$ Let $\frac{dy}{dx}=-\frac{2}{3}$ $\frac{-2}{x^2-1}=-\frac{2}{3}$ $x=2$ or -2 (NA) When $x=2$, $y=\ln 3$ $\therefore P(2,\ln 3)$	M1 M1 M1 M1 A1
7ii	Equation of tangent at P: $y-\ln 3=-\frac{2}{3}(x-2)$ --(1) $2y=3x+2$ --(2) Solving (1) and (2) simultaneously, $x=0.661, y=1.99$ $\therefore Q(0.661,1.99)$	M1 M1 A1
8i	$\text{LHS}=\frac{2\tan x}{\sin 2x}$ $=\frac{\frac{\sin x}{\cos x}}{\sin x\cos x}$ $=\frac{1}{\cos^2 x}$ $=\sec^2 x$ (shown)	M1 M1 A1
8ii	From (i), $\sec^2 x=\sec x+2$ $(\sec x-2)(\sec x+1)=0$ $\cos x=\frac{1}{2}$ or -1 $x=\frac{\pi}{3},\pi,\frac{5\pi}{3}$	M1 M1 A1
8iii	$\int(\operatorname{cosec}2x\tan x+1)\,dx$ $=\int(\frac{1}{2}\sec^2 x+1)\,dx$ from (i) $=\frac{1}{2}\tan x+x+C$	B1 B1B1 for each term
9a	$\cos A=\frac{\sqrt{24}}{5}$ 	D1 B1
9b	$\sin(A+B)=\left(\frac{1}{5}\right)\left(\frac{1}{\sqrt{10}}\right)+\left(\frac{\sqrt{24}}{5}\right)\left(\frac{3}{\sqrt{10}}\right)$ $=\frac{1+3\sqrt{24}}{5\sqrt{10}}$ $=\frac{\sqrt{10}+12\sqrt{15}}{50}$	M1 A1
9c	$\tan 2B=\frac{2(3)}{1-3^2}=-\frac{3}{4}$ $\cot 2B=-\frac{4}{3}$	M1 A1

10i	C(7,7)	B1
	midpoint of $BD = \left(\frac{7}{2}, \frac{3}{2}\right)$. Gradient of BD $= -\frac{3}{7}$ Equation of perpendicular bisector of BD: $y - \frac{3}{2} = \frac{7}{3}(x - \frac{7}{2})$ $3y = 7x - 20$ --(1) $5y = 2x - 14$ --(2) Solving (1) and (2) simultaneously, $x = 2, y = -2$ $\Rightarrow A(2, -2)$.	M1 M1 A1
10ii	Area $= \frac{1}{2}\begin{vmatrix} 2 & 7 & 7 & 0 & 2 \\ -2 & 0 & 7 & 3 & -2 \end{vmatrix}$ $= 39$ units^2	M1 A1
10iii	Gradient of $AC = \frac{7+2}{7-2} = \frac{9}{5}$ $-\frac{3}{7} \times \frac{9}{5} \neq -1$ Since AC and BD are not perpendicular to each other, $ABCD$ is not a kite.	B1 B1 B1
11i	when $x = p$, $y = 8 - p^2$, $\Rightarrow Q(p, 8 - p^2)$ Area of PQR $= \frac{1}{2} \times (8 - p^2) \times (p - (-2))$ $= \frac{1}{2}(p+2)(8 - p^2)$ (Shown)	B1 B1
11ii	$\frac{dA}{dt} = \frac{dA}{dp} \times \frac{dp}{dt}$ $= \frac{1}{2}\left((p+2)(-2p) + 8 - p^2\right) \times 0.04$ $\frac{dA}{dt}\Big\|_{p=1.5} = -0.095$ units^2/s Rate $= 0.095$ units^2/s	M1 M1 A1
12i	Total Perimeter: $200 = 2x + 2y + \frac{1}{2}(2\pi x)$ $y = \frac{200 - (\pi + 2)x}{2}$ $A = \frac{1}{2}\pi x^2 + 2xy$ $= \frac{1}{2}\pi x^2 + 200x - (\pi + 2)x^2$ $= 200x - \left(\frac{\pi + 4}{2}\right)x^2$ (shown)	M1 M1 A1
12ii	$\frac{dA}{dx} = 200 - (\pi + 4)x$ Let $\frac{dA}{dx} = 0$, $x = \frac{200}{\pi + 4}$ or $x = 28.0$ (3 s.f.) $\frac{d^2A}{dx^2} = -\pi - 4 < 0$ By second derivative test, A is maximum when $x = \frac{200}{\pi + 4}$.	M1 M1 M1 A1

Candidate Name: ______________________ Class ______ Index No. ______

FUHUA SECONDARY SCHOOL

Secondary Four Express & Secondary 5 Normal Academic
Preliminary Examination 2016

4E/5N

Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary
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ADDITIONAL MATHEMATICS
PAPER 1

4047/1

Additional Materials:
Writing paper, graph paper & Electronic calculator

DATE **26 August 2016**
TIME **1045 - 1245**
DURATION **2 h**

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.
Write your answers and working on the separate writing paper provided.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

PARENT'S SIGNATURE	FOR EXAMINER'S USE	
		/ 80

This question paper consists of 6 printed pages including this page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 Given that $y = \frac{2x^2}{(x+2)^2}$,

show that for $x > 0$, y is an increasing function. [3]

2 Solve the equation $2\log_4 5x^2 - \log_8 (4-x)^3 = \log_2 (1-x) + 1$. [6]

3 A certain virus increase by 100% at the end of 20 days. It is given that P_o is the number of virus present at a particular time and given that $P = P_o(2^{kd})$, calculate the value of the constant k in the relationship $P = P_o(2^{kd})$, where d is the number of days. [3]

4 (i) Show that $x^2 - x + 1$ is always positive for all real values of x. [2]

(ii) Hence, or otherwise, find the range of values of b if the inequality

$\frac{x^2 + bx - 2}{x^2 - x + 1} < 2$ is satisfied for all real values of x. [3]

5 Sketch the graph of $y = 2x^{-\frac{1}{2}} - 3$ for $x > 0$. From your graph, [3]

(i) find the range of values of y for which $x \geq 2$, [1]

(ii) find the range of values of x for which $y \geq -1$ [1]

6 (i) Express $x^2 - 2x - 6$ in the form $a(x-b)^2 + c$, where a, b and c are constants. [2]

(ii) Hence sketch the graph of $y = |x^2 - 2x - 6|$ for $-4 \leq x \leq 5$ [3]

(iii) By inserting a suitable straight line, find the number of solutions for the equation $|-2x^2 + 4x + 12| = 6 - 4x$ in the given domain [3]

---

7 (a) Prove that $\tan(45° + \theta) + \tan(45° - \theta) = 2\sec 2\theta$. [3]

(b) Hence, solve the equation $\tan(45° + \theta) + \tan(45° - \theta) = 6$ for $0° \leq \theta \leq 360°$. [3]

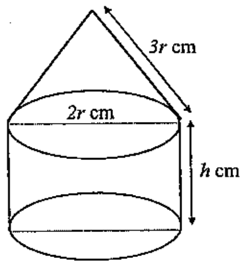
8 A particle Q passes a fixed point B and moves in a straight line such that, t s after leaving B, its velocity, v m/s, is given by $v = 2\cos^2 t - 1$. Find

(a) the acceleration of Q when $t = 2$, [2]

(b) the time when the particle is at instantaneous rest for $0 \leq t \leq 2$, [2]

(c) the total distance travelled by Q in the first 2 second. [3]

9 A solid is made up of a right circular cone and a cylinder with a radius r cm, as shown in the diagram. The slant height of the cone is $3r$ cm and height of cylinder is h cm.



(a) Given that the total surface area of the solid is 500 cm^2, express h in terms of r. [2]

(b) Show that the volume, V cm^3, of the solid is given by $V = 250r + (\frac{2\sqrt{2}}{3} - 2)\pi r^3$. [3]

(c) Given that r and h can vary, find the stationary value of V and determine whether this value of V is maximum or minimum. [5]

10 Liquid is poured into a bucket at a rate of 60 cm^3/s. The volume, V cm^3, of the liquid in the bucket, when the depth is x cm, is given by $V = 0.01x^3 + 2.2x^2 + 200x$.
Find

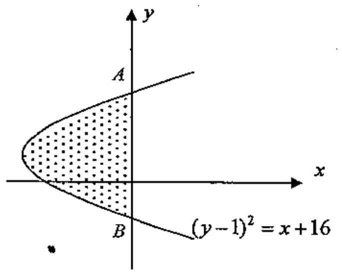
(i) the rate of increase in the depth of the liquid when $x = 10$, and [3]

(ii) the depth of the liquid when the rate of increase in the depth is 0.2 cm/s. [3]

11 The diagram shows the curve $(y-1)^2 = x+16$ which cuts the y-axis at A and B.
Find,

(i) the coordinates of A and of B, [3]

(ii) the area of the region bounded by the curve and the y-axis. [3]



12 The function f(x) is defined by the equation $\text{f}(x) = 3\cos 2x + 1$.

(i) State the period and amplitude of f(x) [2]

(ii) Sketch the graph of f(x) for $0 \le x \le \pi$ [3]

(iii) State the maximum and minimum values of f(x). [2]

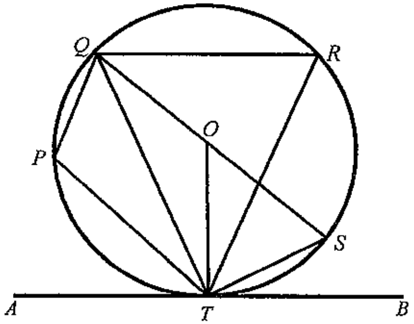
(iv) On the diagram of part (ii), sketch the graph of $y = 4\sin 2x$ for $0 \le x \le \pi$. [2]

(v) Hence, state the number of solutions, for $0 \le x \le \pi$, of the equation
$2\cos 2x + 1 = 4\sin 2x$. [1]

13 In the figure, O is the centre of the circle $PQRST$ with QS as a diameter, ATB is a tangent to the circle at T. Given $QT = RT$,
show that,

(i) $\angle RTO = \angle STB$ [3]

(ii) $\angle QRT = \angle RTB$ [2]



End of Paper

4i $x^2 - x + 1$

$= \left(x - \frac{1}{2}\right)^2 + 1 - \left(-\frac{1}{2}\right)^2$ M1

$= \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$

Since $\left(x - \frac{1}{2}\right)^2 \geq 0$

$\left(x - \frac{1}{2}\right)^2 + \frac{3}{4} > 0$ A1

$\therefore$ x^2 ~~4~~ $- x + 1$ is always positive for all real values of x.

4ii $x^2 + bx - 2 < 2(x^2 - x + 1)$

$x^2 + bx - 2 < 2x^2 - 2x + 2$ M1

$0 < x^2 - 2x - bx + 4$

$x^2 + (-2-b)x + 4 > 0$

$b^2 - 4ac < 0$

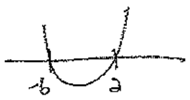
$(-2-b)^2 - 4(1)(4) < 0$ M1

$4 + 4b + b^2 - 16 < 0$

$b^2 + 4b - 12 < 0$

$(b+6)(b-2) < 0$

$-6 < b < 2$ A1



3. $d = 0$, $P = P_0$

$d = \overset{20}{\cancel{8}}$, $P = 2P_0$

$2P_0 = P_0 \left[2^{k(20)}\right]$ M1

$2 = 2^{20k}$

By comparison,

$1 = 20k$ M1

$k = \frac{1}{20}$ A1

2016 Prelim Exams

1 $u = 2x^2$ $v = (x+2)^2$

$\frac{du}{dx} = 4x$ $\frac{dv}{dx} = 2(x+2)(1)$

$$\frac{dy}{dx} = \frac{4x(x+2)^2 - 4x^2(x+2)}{(x+2)^4} \quad \text{M1}$$

$$= \frac{4x\cancel{(x+2)}\left[(x+2) - x\right]}{(x+2)^{\cancel{4}\,3}}$$

$$= \frac{8x}{(x+2)^3} \quad \text{M1}$$

since $8x > 0$; $x > 0$

$(x+2)^3 > 0$, $x > 0$ A1

$\therefore$ y is an increasing function .

2. $2\log_4 5x^2 - \log_8 (4-x)^3 = \log_2 (1-x) + 1$

$$2\left(\frac{\log_2 5x^2}{\log_2 4}\right) - \frac{\log_2 (4-x)^3}{\log_2 8} = \log_2 (1-x) + 1 \quad \text{M1}$$

$$\log_2 5x^2 - \frac{\log_2 (4-x)^3}{3} = \log_2 (1-x) + 1$$

$$\log_2 5x^2 - \log_2 (4-x) = \log_2 (1-x) + 1$$

$$\log_2\left(\frac{5x^2}{4-x}\right) = \log_2 (1-x) + \log_2 2 \quad \text{M1}$$

$$\log_2\left(\frac{5x^2}{4-x}\right) = \log_2 2(1-x)$$

$$\frac{5x^2}{4-x} = 2 - 2x \quad \text{M1}$$

$5x^2 = 8 - 10x + 2x^2$

$3x^2 + 10x - 8 = 0$,

$(3x-2)(x+4) = 0$ M1

$3x - 2 = 0$ or $x + 4 = 0$,

$x = \frac{2}{3}$ A1 or $x = -4$ A1

13i $RT = QT$

$\triangle QRT$ is isos $\triangle$.

$\angle QRT = \angle RQT$

$\angle QTA = \angle QRT = \angle RQT = \angle RTB$ (alt segment $\angle$s) M1

$\angle OTA = \angle OTB = 90°$ (tangent $\perp$ radius)

$\angle RTO = 90° - \angle RTB$

$= 90° - \angle QTA$

$= OTQ$

$= OQT$ M1

$\angle STB = \angle OQT$ (alt segments $\angle$s)

$\therefore \angle RTO = \angle STB$ A1

13ii $\angle RTB = \angle TQR$ (alt segment theorem) M1

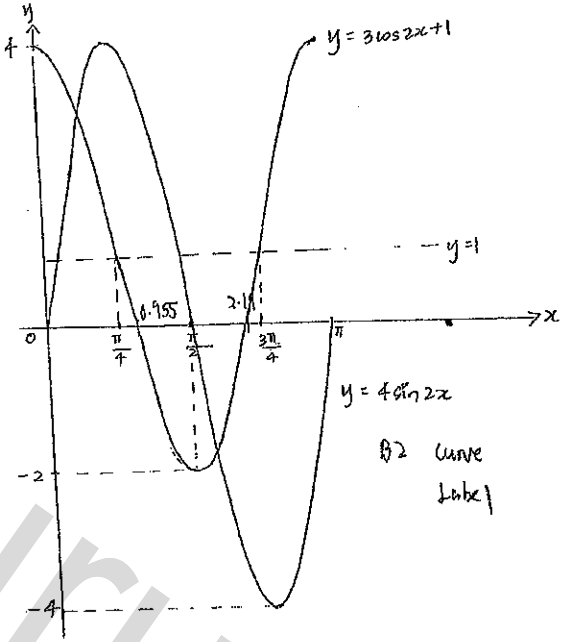
$= \angle TRQ$ (base $\angle$s of an isos $\triangle$) A1

12a period $= \frac{2\pi}{2}$

$= \pi$ B1

amplitude $= 3$ B1

12b



curve B1
label B1
Eqn B1

B2 curve
label

12c max $= 4$
min $= -2$ B2

12d 2 solutions B1

11i $(y-1)^2 = x+16$

$(y-1)^2 = 16$ M1

$y-1 = \sqrt{16}$ or $y-1 = -\sqrt{16}$

$y-1 = 4$ or $y-1 = -4$

$y = 5$ $\quad$ $y = -3$

$\therefore A(0, 5)$ A1 $B(0, -3)$ A1

11ii Area of region $= \int_{-3}^{5} x \, dy$

$-\int_{-3}^{5} x \, dy$

$= \text{Ans}$

$-\int_{-3}^{5} x \, dy$

$= \int_{5}^{-3} x \, dy$

$= \left| \int_{-3}^{5} (y-1)^2 - 16 \, dy \right|$ M1

$= \left| \int_{-3}^{5} y^2 - 2y + 1 - 16 \, dy \right|$

$= \left| \int_{-3}^{5} y^2 - 2y - 15 \, dy \right|$

$= \left| \left(\frac{y^3}{3} - \frac{\cancel{2}y^2}{\cancel{2}} - 15y \right)_{-3}^{5} \right|$ M1

$= \left(\frac{125}{3} - 25 - 75 \right) - (-9 - 9 + 45)$

$= \left| -85\frac{1}{3} \right|$

$= 85\frac{1}{3}$ units^2 A1

10i $\frac{dV}{dt} = 60 \text{ cm}^3/\text{s}$

$V = 0.01x^3 + 2.2x^2 + 200x$

$\frac{dV}{dx} = 0.03x^2 + 4.4x + 200$, M1

$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$

$x = 10$, $\frac{dV}{dx} = 0.03(10)^2 + 4.4(10) + 200$ M1

$= 247$,

$60 = 247 \times \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{60}{247}$ cm/s A1 or 0.243 cm/s (3s.f)

10ii $60 = (0.03x^2 + 4.4x + 200) \times 0.2$ M1

$300 = 0.03x^2 + 4.4x + 200$

$0.03x^2 + 4.4x - 100 = 0$

$(x-20)(x + 166\frac{2}{3}) = 0$ M1

$x - 20 = 0$ $\quad$ $x + 166\frac{2}{3} = 0$

$x = 20$ $\quad$ $x = -166\frac{2}{3}$ (rej) A1

The depth of liquid is 20cm.

$\frac{500-4\pi r^2}{2\pi r}$

9b. $V = \frac{1}{3}\pi r^2 h + \pi r^2 h$

$= \frac{1}{3}\pi r^2 \left(\overset{\sqrt{8}r}{\cancel{\frac{250-2\pi r^2}{\pi r}}}\right) + \pi r^2 \left(\frac{250-2\pi r^2}{\pi r}\right)$ M1

height
$= \sqrt{(3r)^2-(r)^2}$
$= \sqrt{9r^2-r^2}$
$= \sqrt{8r^2}$
$= \sqrt{8}r$ M1

$= \frac{2\sqrt{2}}{3}\pi r^3 + 250r - 2\pi r^3$

$= 250r + \left(\frac{2\sqrt{2}}{3} - 2\right)\pi r^3$ A1 (shown)

9c $\frac{dV}{dr} = 250 + (2\sqrt{2}-6)\pi r^2$, M1

$\frac{dV}{dr} = 0$

$250 + (2\sqrt{2}-6)\pi r^2 = 0$ ~~M1~~ M1

$r^2 = \frac{-250}{(2\sqrt{2}-6)\pi}$

$r^2 = 25.09085387$, M1

$r = 5.00907, -5.00907 \text{ (rej)}$

$r = 5.009$, $V = 250(5.009) + \left(\frac{2\sqrt{2}}{3}-2\right)\pi(5.009)^3$

$= 834.84 \text{ cm}^3$ (5s.f)

$= 835 \text{ cm}^3$ (3s.f) A1

$\frac{d^2V}{dr^2} = (4\sqrt{2}-12)\pi r$, $\therefore$ V is maximum A1

$= -99.8 < 0$

~~9ab height $= \sqrt{(3r)^2-(r)^2}$~~
~~$= \sqrt{9r^2-r^2}$~~
~~$= \sqrt{8r^2}$~~
~~$= \sqrt{8}r$ M1~~

9a
TSA = ~~total~~ curved surface area of cone ✓
+ curved surface area of cylinder ✓
+ area of circle ✓

$= \pi(r)(3r)$, $+ 2\pi rh$, $+ \pi r^2$, ...

$= \cancel{3\pi r} \; 3\pi r^2 + 2\pi rh + \pi r^2$ ...

$= 4\pi r^2 + 2\pi rh$ ....

$4\pi r^2 + 2\pi rh = 500$ M1

$2\pi r^2 + \pi rh = 250$

$\pi rh = 250 - 2\pi r^2$

$h = \frac{250-2\pi r^2}{\pi r}$ A1

8c. $s = \int v\,dt$ $\quad\cos 2A = 2\cos^2 A - 1$

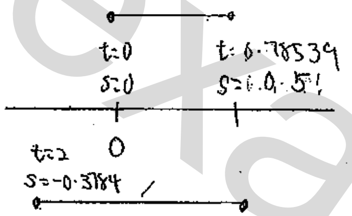
$= \int 2\cos^2 t - 1$

$= \int \cos 2t \cdot dt$ ✓ M1

$= \frac{\sin 2t}{2} + c$ ✓ M1

$t=0,\ s=0,\ c=0$

$s = \frac{\sin 2t}{2}$ ✓



Total distance travelled in 1st 2 second

$= 0.3784 + 0.5 + 0.5$

$= 1.3784$ m (5s.f)

$= 1.38$ m (3s.f) A1

8a. $v = 2\cos^2 t - 1$

$a = \frac{dv}{dt}$

$= 4\cos t\,(-\sin t)$ M1

$= -4\sin t\cos t$ ~~A1~~

$t=2$, $a = -4\sin(2)\cos(2)$ ~~A1~~

$= 1.5136$ m/s² (5s.f)

$= 1.51$ m/s² (3s.f) A1 ✓

8b. $v=0$

$2\cos^2 t - 1 = 0$ M1 or $\cos 2t = 0$ M1

$2\cos^2 t = 1$

$\cos^2 t = \frac{1}{2}$

$\cos t = \sqrt{\frac{1}{2}}$ or $\cos t = -\sqrt{\frac{1}{2}}$

$t = 0.78539$ (5s.f) $\quad t = 3.9269$ (rej), 2.3561 (rej)

$= 0.785$ (3s.f)

$\therefore$ time is 0.785 s when A1

particle at instantaneous

rest

7a. $\tan(45+\theta) + \tan(45-\theta) = 2\sec 2\theta$

LHS : $\frac{1+\tan\theta}{1-\tan\theta} + \frac{1-\tan\theta}{1+\tan\theta}$

$= \frac{(1+\tan\theta)^2 + (1-\tan\theta)^2}{1-\tan^2\theta}$ M1

$= \frac{1+2\tan\theta+\tan^2\theta + 1-2\tan\theta+\tan^2\theta}{1-\tan^2\theta}$

$= \frac{2+2\tan^2\theta}{1-\tan^2\theta}$

$\sec^2\theta = 1+\tan^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$= \frac{2(1+\tan^2\theta)}{1-(\sec^2\theta-1)}$

$= \frac{2\sec^2\theta}{2-\sec^2\theta}$

$= \frac{2}{\cos^2\theta} \div \left(2-\frac{1}{\cos^2\theta}\right)$ M1

$= \frac{2}{\cos^2\theta} \div \frac{2\cos^2\theta-1}{\cos^2\theta}$

$= \frac{2}{\cos^2\theta} \times \frac{\cos^2\theta}{2\cos^2\theta-1}$

$\frac{2}{2\cos^2\theta-1}$

$= \frac{2}{\cos 2\theta}$

$= 2\sec 2\theta$ (shown)

A1

7b. $2\sec 2\theta = 6$

$\frac{2}{\cos 2\theta} = 6$ M1

$0^\circ \leq \theta \leq 360^\circ$

$0^\circ \leq 2\theta \leq 720^\circ$

$\cos 2\theta = \frac{1}{3}$

basic $\measuredangle = 70.5287^\circ$ (4dp) M1

$2\theta = 70.5287^\circ, 289.4713^\circ, 430.5287^\circ, 649.4713^\circ$

$\theta = 35.2643^\circ, 144.7356^\circ, 215.26435^\circ, 324.7356^\circ$

$\theta = 35.3^\circ, 144.7^\circ, 215.3^\circ, 324.7^\circ$ (1dp)

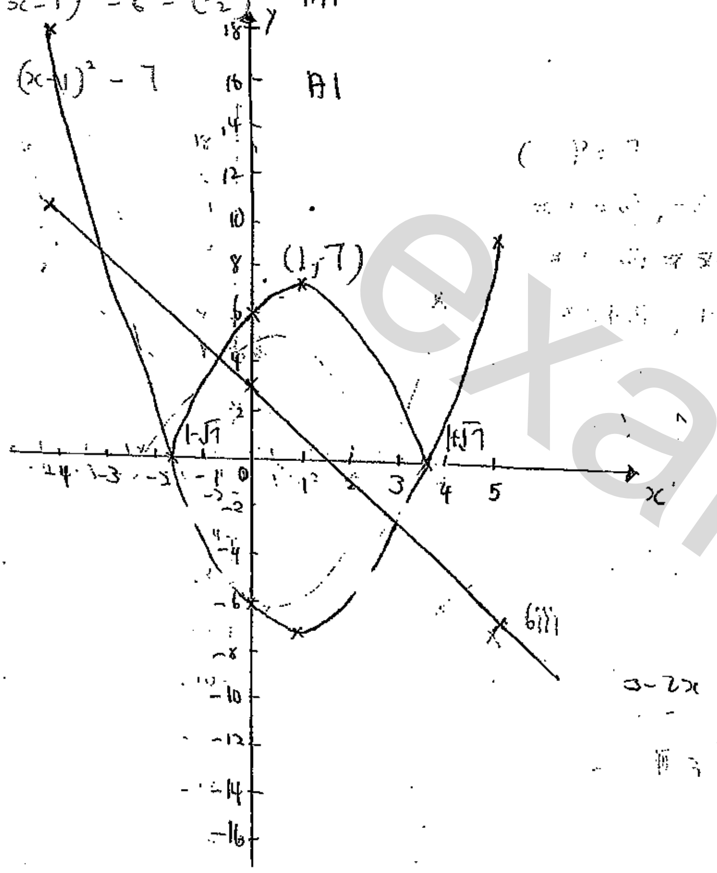
A1

6i $x^2 - 2x - 6$

$= (x-1)^2 - 6 - (\frac{-2}{2})^2$ M1

$= (x-1)^2 - 7$ B1

6ii

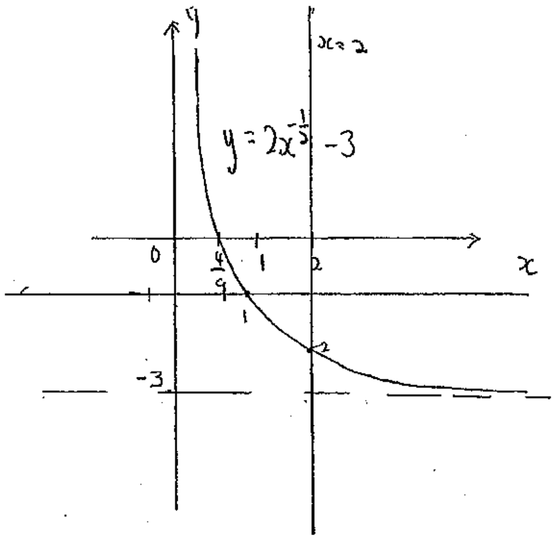


Curve B1
Label B1
Eqn B1

6iii 2 solutions A1 line M1

Eqn B1

5



Curve B1
Label B1
Eqn B1

5i $-3 < y \leq \sqrt{2} - 3$ B1

5ii $0 \leq x \leq 1$ B1

Candidate Name: ____________________

Class	Index No.

FUHUA SECONDARY SCHOOL

Secondary Four Express & Five Normal Academic

Preliminary Examination 2016

4E/5N

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ADDITIONAL MATHEMATICS 4047/2

Additional Materials:
Answer Paper (6), Graph Paper (1)

DATE **24 Aug 2016**
TIME **1045 – 1315**
DURATION **2 h 30 min**

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

PARENT'S SIGNATURE	FOR EXAMINER'S USE		
	Units		/ 100
	Statements/Accuracy		
	Poor Presentation		

This question paper consists of 7 printed pages including this page.

[Turn Over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\dots+\binom{n}{r}a^{n-r}b^r+\dots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}=\frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$cosec^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}bc\sin A$$

1 Given that $\int_m^6 \frac{x-2}{2x^2-x-6}\,dx = \frac{1}{2}\ln\frac{5}{3}$,

(i) state the value(s) of x for the integral to be undefined, [2]

(ii) find the value of m. [4]

2 (i) Show that $\frac{d}{dx}(2x\sin 3x) = 2\sin 3x + 6x\cos 3x$. [2]

(ii) Using the result from part (i), find $\int 2x\cos 3x\ dx$ and hence show that $\int_0^{\frac{\pi}{2}} 2x\cos 3x\,dx = -\frac{\pi}{3} - \frac{2}{9}$. [5]

(iii) Given that $\int_1^5 f(x)\,dx = 7$, evaluate $\int_3^1 \left(\frac{1}{3x^2} - f(x)\right)dx + \int_3^5 f(x)\,dx$. [3]

3 A curve has the equation $y = \frac{1-x}{3x-1}, x \neq \frac{1}{3}$.

(i) Find an expression for $\frac{dy}{dx}$. [2]

(ii) Find the coordinates of the points on the curve where the normal is parallel to the line $2y = 4x + 1$. [5]

The points $P(0.5, 1)$ and $Q(-1, -0.5)$ lie on the curve.

(iii) Find the area of triangle POQ where O is the origin. [2]

[Turn over

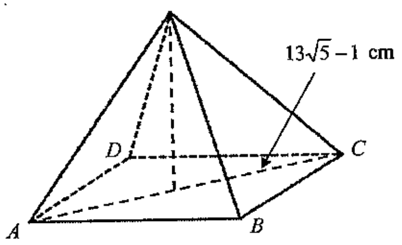
---

4 (a) Write down, and simplify, the first three terms in the expansion of $(3-2x)^5$ in ascending powers of x. [2]

(b) (i) In the binomial expansion of $\left(x + \frac{k}{x^3}\right)^8$, where k is a positive constant, the term independent of x is 112. Show that $k = 2$. [4]

(ii) Hence, find the coefficient of x^4 in the expansion of $\left(1 - \frac{x^4}{4}\right)\left(x + \frac{k}{x^3}\right)^8$. [3]

5 (i) Express $\frac{6\sqrt{5}}{2\sqrt{5}-4}$ in the form $a + b\sqrt{5}$, where a and b are integers. [2]



The diagram shows a pyramid with a square base $ABCD$. The diagonal AC of the square base is $13\sqrt{5} - 1$ cm. Given that the height of the pyramid is $\frac{6\sqrt{5}}{2\sqrt{5}-4}$ cm,

(ii) find an expression for AC^2 in the form $c + d\sqrt{5}$, where c and d are integers, [2]

(iii) express the volume of the pyramid in the form $m + n\sqrt{5}$ cm^3, where m and n are integers. (volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height) [4]

6 The equation of a curve is $y = e^{-2x}\tan x$ where $0 < x < \pi$.

(i) Find the coordinates of the stationary point(s) of the curve, giving your answer in its exact form. [4]

(ii) Determine the nature of the stationary point(s). [2]

7 A circle has the equation $x^2 + y^2 + 6x - 8y + 9 = 0$.

(i) Find the coordinates of the centre of the circle and the radius of the circle. [3]

(ii) Show that the x-axis is a tangent to the circle. [2]

(iii) Show that the point $P(-5, 2)$ lies inside the circle. [2]

(iv) Find the equation of the chord of the circle whose mid-point is P. [2]

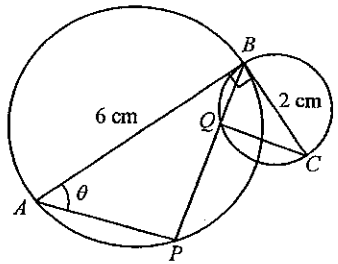
8 **(a)** Find the remainder when $3x^3 - 13x^2 + 3x + 22$ is divided by $x + 1$. [2]

(b) Given that $f(x) = mx^3 - 3x^2 + 5x + 4$ and $g(x) = mx^3 + 4x - 6$ have a common factor $x - a$, where a is an integer, find the value of m. [4]

(c) Express $\dfrac{5x^3 - 9x + 4}{x(x^2 + 3)}$ in partial fractions. [4]

[Turn over

9



The diagram shows two circles ABP and BCQ. AB and BC are the diameters of circles ABP and BCQ respectively. AB = 6 cm, BC = 2 cm and angle ABC = 90°. If P and Q are two variable points on the two circles such that BQP is a straight line and angle $BAP = \theta$,

(i) show that $AP + PQ + QC = 8\sin\theta + 4\cos\theta$, [3]

(ii) express $AP + PQ + QC$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and α is acute, [4]

(iii) find the value(s) of θ for which $AP + PQ + QC$ = 8.8 cm [3]

10 The roots of the quadratic equation $2x^2 - 5x + 1 = 0$ are α and β. Without solving the equation,

(i) find the value of $\alpha^2 + \beta^2$, [3]

(ii) factorise $\alpha^3 + \beta^3$, [1]

(iii) show that the value of $\alpha^3 + \beta^3$ is $\frac{95}{8}$, [2]

(iv) find the quadratic equation whose roots are $\alpha^3 - \alpha$ and $\beta^3 - \beta$. [5]

11 **Answer the whole of this question on a sheet of graph paper.**

(a) The table shows experimental values of two variables, x and y, which are connected by an equation of the form $ay = \frac{1}{x+b}$, where a and b are constants.

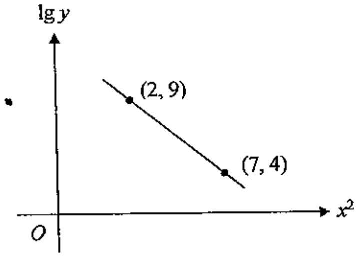
x	1	2	3	4	5	6
y	8	2.67	1.60	1.14	0.89	0.73

(i) Plot $\frac{1}{y}$ against x and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of a and of b. [4]

(iii) Without drawing a second graph, estimate the intercept on the vertical axis of the graph of xy against y. [1]

(b) The diagram below shows part of a straight line graph of lg y against x^2, passing through the points (2, 9) and (7, 4).



(i) Express y in terms of x. [3]

(ii) Find the value of y when $x = \sqrt{13}$ [1]

End of Paper

1. i) $2x^2 - x - 6$

$= (2x+3)(x-2)$

$\begin{array}{r} 2x+3 \\ x-2 \\ \hline 3x-4x \end{array}$

For the integral to be undefined,

$\underline{x = -\frac{3}{2} \text{ or } x = 2}$
B1 B1

ii) $\int_m^6 \frac{x-2}{2x^2-x-6}\,dx$

$= \int_m^6 \frac{x-2}{(2x+3)(x-2)}\,dx$ M1 (either)

$= \int_m^6 \frac{1}{2x+3}\,dx$

$= \left[\frac{1}{2}\ln(2x+3)\right]_m^6$ M1

$\frac{1}{2}\ln[2(6)+3] - \frac{1}{2}\ln(2m+3) = \frac{1}{2}\ln\frac{5}{3}$ M1 (substitution)

$\ln 15 - \ln(2m+3) = \ln\frac{5}{3}$

$\ln\frac{15}{2m+3} = \ln\frac{5}{3}$

Comparing, $2m+3 = 3\times 3$

$2m = 6$

$\underline{m = 3}$ A1

2. i) $\frac{d}{dx}(2x\sin 3x)$

Let $u = 2x$ $v = \sin 3x$

$\frac{du}{dx} = 2$ $\frac{dv}{dx} = 3\cos 3x$

$= 2x(3\cos 3x) + (\sin 3x)(2)$ M1

$= 6x\cos 3x + 2\sin 3x$

$= \underline{2\sin 3x + 6x\cos 3x \text{ (shown)}}$ A1

ii) $\int 2\sin 3x + 6x\cos 3x\,dx = 2x\sin 3x + C$ M1

$\int 2\sin 3x\,dx + \int 6x\cos 3x\,dx = 2x\sin 3x + c$

$\int 6x\cos 3x\,dx = 2x\sin 3x - \int 2\sin 3x\,dx + c$

$\int 2x\cos 3x\,dx = \frac{2}{3}x\sin 3x - \int\frac{2}{3}\sin 3x\,dx + c$

$= \frac{2}{3}x\sin 3x - \left(-\frac{2}{9}\cos 3x\right) + c$ M1

$= \underline{\frac{2}{3}x\sin 3x + \frac{2}{9}\cos 3x + c}$ A1

$\therefore \int_0^{\frac{\pi}{2}} 2x\cos 3x\,dx = \left[\frac{2}{3}x\sin 3x + \frac{2}{9}\cos 3x\right]_0^{\frac{\pi}{2}}$

M1 (substitution)

$= \frac{2}{3}\left(\frac{\pi}{2}\right)\sin\left(3\times\frac{\pi}{2}\right) + \frac{2}{9}\cos 3\left(\frac{\pi}{2}\right) - \frac{2}{3}(0)\sin(3\times 0) - \frac{2}{9}\cos(3\times 0)$

$= \frac{\pi}{3}(-1) + 0 - 0 - \frac{2}{9}$

$= \underline{-\frac{\pi}{3} - \frac{2}{9} \text{ (shown)}}$ A1

iii) $\int_3^1 \frac{1}{3x^2}\,dx - f(x)\,dx + \int_3^5 f(x)\,dx$

$= \int_3^1 \frac{1}{3}x^{-2}\,dx - \int_3^1 f(x)\,dx + \int_3^5 f(x)\,dx$

$= \left[\frac{\frac{1}{3}x^{-2+1}}{-2+1}\right]_3^1 + \left(-\int_3^1 f(x)\,dx\right) + \int_3^5 f(x)\,dx$

$= \left[-\frac{1}{3x}\right]_3^1 + \int_1^3 f(x)\,dx + \int_3^5 f(x)\,dx$
M1 M1

$= \left[-\frac{1}{3(1)} - \left(-\frac{1}{3(3)}\right)\right] + 7$

$= -\frac{1}{3} + \frac{1}{9} + 7$

$= \underline{6\frac{7}{9}}$ A1

3. i) $y = \frac{1-x}{3x-1}$

$\frac{dy}{dx} = \frac{(3x-1)(-1)-(1-x)(3)}{(3x-1)^2}$ M1 (quotient rule)

$= \frac{-3x+1-3+3x}{(3x-1)^2}$

$= \frac{-2}{(3x-1)^2}$ A1

iii) Area of ΔPOQ

$= \frac{1}{2}\begin{vmatrix} 0.5 & -1 & 0 & 0.5 \\ 1 & -0.5 & 0 & 1 \end{vmatrix}$ M1

$= \frac{1}{2}[(0.5)(-0.5)+(-1)(0)+(0)(1)-(0.5)(0)-(0)(-0.5)-(-1)(1)]$

$= \frac{1}{2}[-0.25+1]$

$= 0.375$ units^2 (or $\frac{3}{8}$ units^2) A1

ii) $2y = 4x+1$

$y = 2x+\frac{1}{2}$

Gradient of normal = 2

Gradient of tangent = $-\frac{1}{2}$ M1

$\frac{-2}{(3x-1)^2} = -\frac{1}{2}$ M1 (Equate)

$4 = (3x-1)^2$

$= 9x^2-6x+1$

$9x^2-6x+1-4=0$

$9x^2-6x-3=0$

$3x^2-2x-1=0$

$$\begin{array}{l} 3x+1 \\ x-1 \\ \hline x-3x \end{array}$$

$(3x+1)(x-1)=0$ M1 (Factorise)

$3x=-1$ or $x=1$

$x=-\frac{1}{3}$

When $x=-\frac{1}{3}$,

$y = \frac{1-(-\frac{1}{3})}{3(-\frac{1}{3})-1}$

$= -\frac{2}{3}$

When $x=1$,

$y = \frac{1-1}{3(1)-1}$

$= 0$

A1 A1

$\therefore$ Coordinates of the points are $(-\frac{1}{3}, -\frac{2}{3})$ and $(1,0)$.

4. a) $(3-2x)^5$

$= 3^5 + \binom{5}{1}(3)^{5-1}(-2x)^1 + \binom{5}{2}(3)^{5-2}(-2x)^2 + \ldots$ M1

$= \underline{243 - 810x + 1080x^2 + \ldots}$ A1

b) i) $\left(x + \frac{k}{x^3}\right)^8$

General Term, $T_{r+1} = \binom{8}{r}(x)^{8-r}\left(\frac{k}{x^3}\right)^r$ (or M1)

$= \binom{8}{r} x^{8-r} k^r x^{-3r}$

$= \binom{8}{r} k^r x^{8-4r}$ M1

For term independent of x, $8-4r=0$ M1

$4r=8$

$r=2$

$\therefore$ Term independent of x, $\binom{8}{2}k^2 = 112$ M1

$k^2 = 4$

$\therefore \underline{k = 2 \text{ or } \underset{(\text{rejected})}{-2}}$ A1 (with rejection)

ii) $\left(1-\frac{x^4}{4}\right)\left(x+\frac{2}{x^3}\right)^8$

$= \left(1-\frac{x^4}{4}\right)(x^8 + 16x^4 + 112 + \ldots)$ M1

Coefficient of $x^4 = (1)(16) + \left(-\frac{1}{4}\right)(112)$ M1

$= \underline{-12}$ A1

5. i) $\frac{6\sqrt{5}}{2\sqrt{5}-4}$

$= \frac{6\sqrt{5}}{2\sqrt{5}-4} \times \frac{2\sqrt{5}+4}{2\sqrt{5}+4}$ M1 (rationalise)

$= \frac{12(5)+24\sqrt{5}}{(2\sqrt{5})^2-(4)^2}$

$= \frac{60+24\sqrt{5}}{20-16}$

$= \underline{15+6\sqrt{5}}$ A1

ii) $AC^2 = (13\sqrt{5}-1)^2$

$= (13\sqrt{5})^2 - 2(13\sqrt{5}) + 1$ M1 (expand)

$= 845 - 26\sqrt{5} + 1$

$= \underline{846 - 26\sqrt{5}}$ A1

iii) Let x be the length of the square.

$$\left.\begin{aligned} x^2 + x^2 &= AC^2 \\ 2x^2 &= 846 - 26\sqrt{5} \\ x^2 &= 423 - 13\sqrt{5} \end{aligned}\right\}$$ M2 (finding base area)

Base area $= 423 - 13\sqrt{5}$ cm^2

$\therefore$ Volume of pyramid

$= \frac{1}{3} \times (423 - 13\sqrt{5}) \times (15 + 6\sqrt{5})$ M1 (applying formula)

$= \frac{1}{3}(6345 + 2538\sqrt{5} - 195\sqrt{5} - 78(5))$

$= \frac{1}{3}(5955 + 2343\sqrt{5})$

$= \underline{1985 + 781\sqrt{5} \text{ cm}^3}$ A1

6. i) $y = e^{-2x} \tan x$

$\frac{dy}{dx} = e^{-2x}(\sec^2 x) + \tan x(-2e^{-2x})$ M1

$= e^{-2x}(\sec^2 x - 2\tan x)$

let $u = e^{-2x}$ · $\frac{du}{dx} = -2e^{-2x}$ $v = \tan x$ $\frac{dv}{dx} = \sec^2 x$

When $\frac{dy}{dx} = 0$,

$e^{-2x} = 0$ or $\sec^2 x - 2\tan x = 0$ M1 (with rejection)

(rejected $\because e^{-2x} > 0$)

Method 1: $\frac{1}{\cos^2 x} - \frac{2\sin x}{\cos x} = 0$

$\frac{1 - 2\sin x \cos x}{\cos^2 x} = 0$

$1 - 2\sin x \cos x = 0$

$1 - \sin 2x = 0$

$\sin 2x = 1$

$2x = \frac{\pi}{2}$

$x = \frac{\pi}{4}$ M1

When $x = \frac{\pi}{4}$, $y = e^{-2(\frac{\pi}{4})} \tan\frac{\pi}{4}$

$= e^{-\frac{\pi}{2}}$

$\therefore$ Coordinates of Stationary point is $\underline{(\frac{\pi}{4}, e^{-\frac{\pi}{2}})}$ A1

Method 2: $\sec^2 x - 2\tan x = 0$

$1 + \tan^2 x - 2\tan x = 0$

$\tan^2 x - 2\tan x + 1 = 0$

$(\tan x - 1)^2 = 0$

$\therefore \tan x = 1$

$x = \frac{\pi}{4}$ (M1)

ii)

x	0.7	$\frac{\pi}{4}$	0.8
$\frac{dy}{dx}$	0.00613	0	0.000177
tangent	/	—	/

M1

$\therefore$ $\underline{(\frac{\pi}{4}, e^{-\frac{\pi}{2}})}$ is a point of inflexion. A1

7. i) $x^2 + y^2 + 6x - 8y + 9 = 0$

Method 1:

Comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$,

$2g = 6$, $2f = -8$, $c = 9$

$g = 3$ $f = -4$ M1

$\therefore$ Centre $= (-g, -f)$

$= \underline{(-3, 4)}$ A1

Radius $= \sqrt{f^2 + g^2 - c}$

$= \sqrt{(-4)^2 + (3)^2 - 9}$

$= \underline{4 \text{ units}}$ A1

Method 2: $\underbrace{x^2 + 6x + (\frac{6}{2})^2} - (\frac{6}{2})^2 + \underbrace{y^2 - 8y + (\frac{-8}{2})^2} - (\frac{-8}{2})^2 + 9 = 0$

$(x+3)^2 + (y-4)^2 - 16 = 0$

$(x+3)^2 + (y-4)^2 = 16$ M1

$\therefore$ Centre $= \underline{(-3, 4)}$ A1

Radius $= \sqrt{16}$

$= \underline{4 \text{ units}}$ A1

ii) when $y = 0$,

$x^2 + 0^2 + 6x - 8(0) + 9 = 0$

$x^2 + 6x + 9 = 0$

$(x+3)^2 = 0$

$x + 3 = 0$

$x = -3$ M1

Since $(-3, 0)$ is the only point of intersection,

$\therefore$ the x-axis is a tangent to the circle. (shown) A1

OR Since the $\underline{y\text{-coordinate of the centre is 4}}$ and its $\underline{\text{radius is 4 units}}$, $\therefore$ the circle touches the x-axis and so the x-axis is a tangent to the circle. (shown) (B2)

iii) Distance of P from centre

$= \sqrt{(-5-(-3))^2 + (2-4)^2}$

$= \sqrt{8}$

$= 2\sqrt{2}$ units

$\left.\right\}$ M1

Since $2\sqrt{2} < 4$, $\therefore$ the point P lies inside the circle. (shown). A1

iv) Gradient of line joining P to centre

$= \frac{2-4}{-5-(-3)}$

$= 1$

Gradient of chord

$= \frac{-1}{1}$

$= -1$ M1

$y = -x + c$

Sub P(−5, 2),

$2 = -(-5) + c$

$c = -3$

$\therefore$ Equation of chord is $y = -x - 3$. A1

8. a) let $f(x) = 3x^3 - 13x^2 + 3x + 22$

$\cdot\ f(-1) = 3(-1)^3 - 13(-1)^2 + 3(-1) + 22$ M1

$= \underline{3}$ A1

b) $f(a) = g(a)$

$m(a)^3 - 3(a)^2 + 5(a) + 4 = m(a)^3 + 4(a) - 6$ M1 (Equating)

$-3a^2 + 5a + 4 = 4a - 6$

$-3a^2 + a + 10 = 0$

$3a^2 - a - 10 = 0$

$(3a+5)(a-2) = 0$

$3a = -5$ or $a = 2$ M1 (With rejection)

$a = -\frac{5}{3}$

(rejected $\because$ a is an integer)

$\begin{array}{r} 3a+5 \\ a-2 \\ \hline 5a-6a \end{array}$

When $a = 2$, $x-2$ is a factor.

$f(2) = 0$ (or $g(2) = 0$)

$m(2)^3 - 3(2)^2 + 5(2) + 4 = 0$ M1

$8m + 2 = 0$

$8m = -2$

$m = -\frac{1}{4}$ A1

c)

$$\begin{array}{r|l} & 5 \\ \hline x^3+3x & 5x^3 - 9x + 4 \\ & -(5x^3 + 15x) \\ \hline & -24x + 4 \end{array}$$

$\frac{5x^3 - 9x + 4}{x(x^2+3)} = 5 + \frac{4 - 24x}{x^3 + 3x}$ M1 (expression)

Let $\frac{4-24x}{x(x^2+3)} = \frac{A}{x} + \frac{Bx + C}{x^2+3}$

$4 - 24x = A(x^2+3) + (Bx + C)x$

Sub $x = 0$,

$4 = A(0+3)$

$A = \frac{4}{3}$

Sub $x=1$,

$$4-24(1)=\frac{4}{3}(1+3)+(B+C)$$
$$-20=\frac{16}{3}+B+C$$
$$B+C=-25\tfrac{1}{3} \text{ ——— } (1)$$

Sub $x=2$,

$$4-24(2)=\frac{4}{3}(4+3)+(2B+C)(2)$$
$$-44=9\tfrac{1}{3}+4B+2C$$
$$4B+2C=-53\tfrac{1}{3}$$
$$2B+C=-26\tfrac{2}{3} \text{ ——— } (2)$$

From (1). $C=-B-25\tfrac{1}{3}$ ——— (3)

Sub (3) into (2): $2B-B-25\tfrac{1}{3}=-26\tfrac{2}{3}$

$$B=-\frac{4}{3}$$

Sub $B=-\frac{4}{3}$ into (3)

$$C=-\left(-\frac{4}{3}\right)-25\tfrac{1}{3}$$ M2: A, B and C values

$$=-24$$

$$\therefore \frac{5x^3-9x+4}{x(x^2+3)}=5+\frac{4}{3x}+\frac{-\frac{4}{3}x-24}{x^2+3}$$
$$=5+\frac{4}{3x}+\frac{-4x-72}{3(x^2+3)}$$
$$=\underline{5+\frac{4}{3x}-\frac{4x+72}{3(x^2+3)}} \text{ A1}$$

9. i) $\angle ABP=180^\circ-90^\circ-\theta$

$$=90^\circ-\theta$$
$$\angle QBC=90^\circ-(90^\circ-\theta)$$
$$=\theta$$

$\cos\theta=\frac{AP}{6}$ M1 (Find AP, BP, BQ and QC)

$AP=6\cos\theta$

$\sin\theta=\frac{BP}{6}$

$BP=6\sin\theta$

$\sin\theta=\frac{QC}{2}$ $\cos\theta=\frac{BQ}{2}$

$QC=2\sin\theta$ $BQ=2\cos\theta$

$\therefore$ $AP+PQ+QC$

$$=6\cos\theta+(BP-BQ)+2\sin\theta$$
$$=6\cos\theta+(6\sin\theta-2\cos\theta)+2\sin\theta \text{ M1}$$
$$=\underline{8\sin\theta+4\cos\theta} \text{ A1}$$

ii) $AP+PQ+QC=8\sin\theta+4\cos\theta$

$$=\overset{\text{M1}}{\sqrt{8^2+4^2}}\sin\left(\theta+\overset{\text{M1}}{\tan^{-1}}\left(\frac{4}{8}\right)\right)$$
$$=\sqrt{80}\sin(\theta+26.56505^\circ)$$
$$=\underline{4\sqrt{5}\sin(\theta+26.6^\circ)} \text{ (1dec.pl)}$$

A1 A1

iii) When $AP+PQ+QC=8.8$ cm,

$$4\sqrt{5}\sin(\theta+26.56505^\circ)=8.8$$
$$\sin(\theta+26.56505^\circ)=0.98386 \text{ M1}$$

Basic angle $=\sin^{-1}(0.98386)$

$$=79.69197^\circ$$
$$\theta+26.56505^\circ=79.69197^\circ \text{ or } 100.30802^\circ$$
$$\therefore \theta=53.12692^\circ \text{ or } 73.74297^\circ$$
$$\approx \underline{53.1^\circ \text{ or } 73.7^\circ}$$

A1 A1

10. i) $2x^2-5x+1=0$

$\left.\begin{array}{l}\alpha+\beta=-\frac{-5}{2} \quad , \quad \alpha\beta=\frac{1}{2}\\ \quad =\frac{5}{2}\end{array}\right\}$ B1

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=\left(\frac{5}{2}\right)^2-2\left(\frac{1}{2}\right)$ M1

$=\frac{21}{4}$ A1

ii) $\alpha^3+\beta^3=\underline{(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)}$ or $\underline{(\alpha+\beta)[(\alpha+\beta)^2-3\alpha\beta]}$ B1

ii $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$

$=\left(\frac{5}{2}\right)\left(\frac{21}{4}-\frac{1}{2}\right)$ M1

$=\frac{95}{8}$ (shown) A1

iii) Sum of new roots

$=\alpha^3-\alpha+\beta^3-\beta$

$=\alpha^3+\beta^3-(\alpha+\beta)$

$=\frac{95}{8}-\left(\frac{5}{2}\right)$ M1

$=\frac{75}{8}$ A1

Product of new roots

$=(\alpha^3-\alpha)(\beta^3-\beta)$

$=\alpha^3\beta^3-\alpha^3\beta-\alpha\beta^3+\alpha\beta$

$=(\alpha\beta)^3+\alpha\beta-\alpha\beta(\alpha^2+\beta^2)$

$=\left(\frac{1}{2}\right)^3+\frac{1}{2}-\frac{1}{2}\left(\frac{21}{4}\right)$ M1

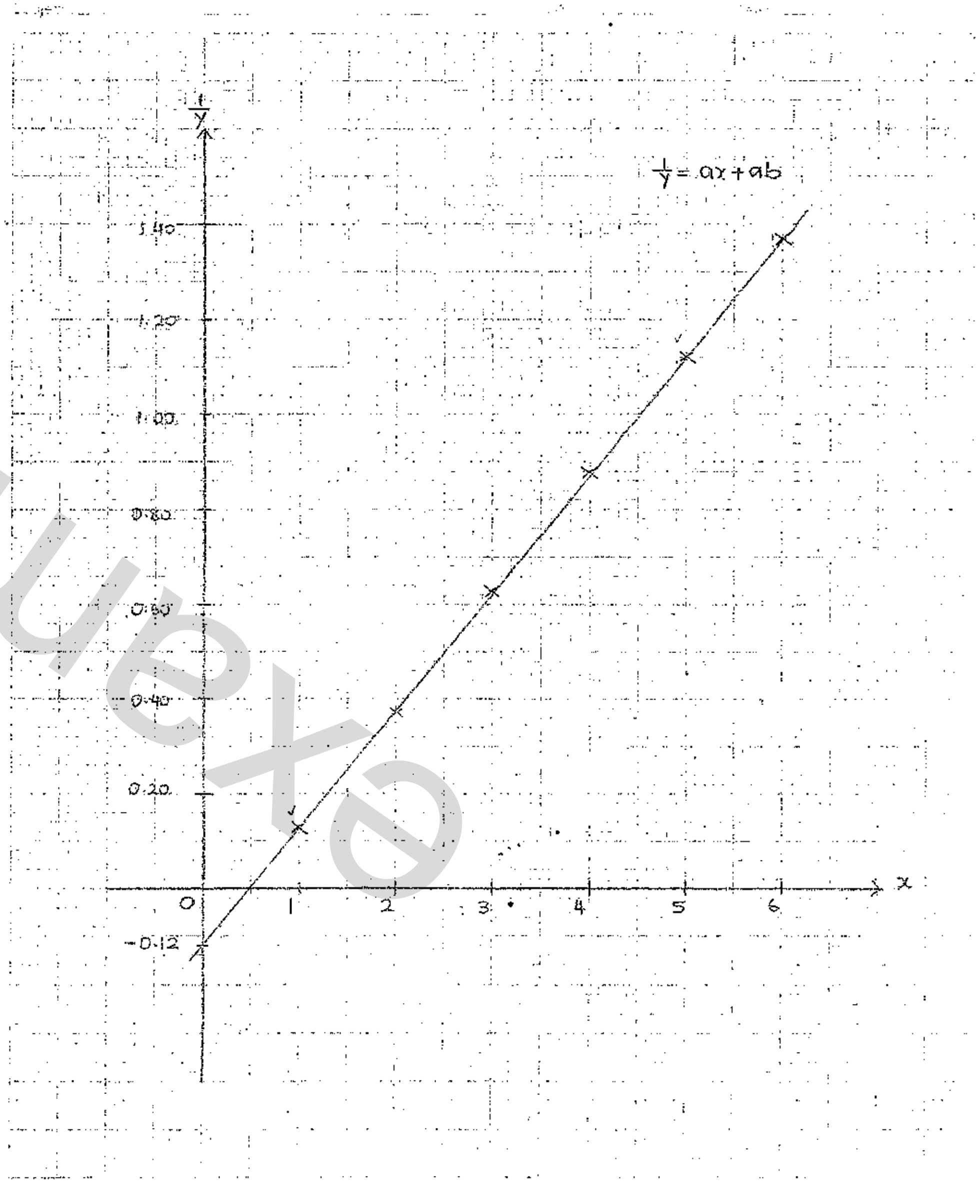
$=-2$ A1

∴ Quadratic equation is $x^2-\frac{75}{8}x-2=0$ A1

or

$\underline{8x^2-75x-16=0}$ A1

11.



11. a) i) $ay = \frac{1}{x+b}$

$axy + aby = 1$

$y(ax + ab) = 1$

B1 (for 11(ii)) $\left\{ \frac{1}{Y} = ax + ab \right.$, where $Y = \frac{1}{y}$, $X = x$, $m = a$ and Y-intercept $= ab$

X	x	1	2	3	4	5	6
Y	$\frac{1}{y}$	0.13	0.37	0.63	0.88	1.12	1.37

} B1

B1 - scale and axes

B1 - points and line

ii) From the graph,

Gradient, $a \approx \frac{1.12 - 0.13}{5 - 1}$ M1

≈ 0.2475 (or $\frac{99}{400}$) A1

Y-intercept, $ab \approx -0.12$

$b \approx \frac{-0.12}{0.2475}$

≈ -0.48484

≈ -0.485 (or $\frac{16}{33}$) B1

$\therefore$ $\underline{a = 0.2475,\ b = -0.485}$

iii) $ay = \frac{1}{x+b}$

$axy + aby = 1$

$xy + by = \frac{1}{a}$

$xy = -by + \frac{1}{a}$

Y-intercept, $\frac{1}{a} \approx \frac{1}{0.2475}$

$\approx 4\ 04040$

$\approx \underline{4.04\ (3\,\text{sig.fig})}$ B1

b) i) $\left.\begin{aligned} \text{Gradient} &= \frac{9-4}{2-7} \\ &= -1 \end{aligned}\right\}$ M1

$Y = -X + c$

$\left.\begin{aligned} \text{Sub } (2,9),\ 9 &= -(2) + c \\ c &= 11 \end{aligned}\right\}$ M1

$\therefore$ $Y = -x + 11$

$\lg y = -x^2 + 11$

$\therefore$ $\underline{y = 10^{-x^2+11}}$ A1

ii) When $x = \sqrt{13}$,

$\left.\begin{aligned} y &= 10^{-(\sqrt{13})^2+11} \\ &= 10^{-2} \\ &= \underline{\frac{1}{100}\ (\text{or } 0.01)} \end{aligned}\right\}$ B1

Geylang Methodist School (Secondary)
Preliminary Examination 2016

ADDITIONAL MATHEMATICS

4047/02

Paper 2

4 Express/ 5 Normal
(Academic)

Additional materials : Writing Paper
Graph Paper

2 hours 30 minutes

Setter : Mr Johney Joseph

05 Aug 2016

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Writing Papers provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of **6** printed pages including the cover page

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

1 The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = 3 \sin x + 5$.

(i) Explain why the curve $y = \mathrm{f}(x)$ has no stationary point. [2]

(ii) Given that the curve passes through the point (0, 5), find an expression for $\mathrm{f}(x)$. [4]

2 **(i)** Differentiate $xe^{\frac{1}{2}x}$ with respect to x. [2]

(ii) Integrate $e^{\frac{1}{2}x}$ with respect to x. [2]

(iii) Using results from part (i) and (ii) show that $\int_0^4 xe^{\frac{1}{2}x}\,dx = 4e^2 + 4$. [4]

3 The equation of a curve is $y = (x + k)^2$.

(i) Show that the equation of the tangent to the curve where $x = 2k$ is $y + 3k^2 = 6kx$. [5]

This tangent meets the x-axis at P and the y-axis at Q.
The mid-point of PQ is M.

(ii) Show that M lies on the curve $y + 24x^2 = 0$. [4]

4 **(a)** (i) Write down, and simplify, the expansion of $(2 - p)^5$. [3]

(ii) Use the result from part **(i)** to find the expansion of $\left(2 - 2x + \frac{x^2}{2}\right)^5$ in ascending powers of x as far as the term in x^2. [3]

(b) (i) Write down the general term in the expansion of $\left(x^2 - \frac{1}{2x^6}\right)^{16}$. [1]

(ii) Hence, or otherwise, evaluate the term independent of x in the expansion of $\left(x^2 - \frac{1}{2x^6}\right)^{16}$. [3]

[Turn over

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5 Given that $k = 3 - 2\sqrt{2}$, express $k - \frac{1}{k^2}$ in the form $a + b\sqrt{2}$, where a and b are integers. [5]

6 **(i)** Prove that $x + 1$ is a factor of $2x^3 - 9x^2 + x + 12$. [2]

(ii) Factorise $2x^3 - 9x^2 + x + 12$ completely and hence solve the equation $2x^3 - 9x^2 + x + 12 = 0$. [4]

(iii) Express $\frac{25}{2x^3 - 9x^2 + x + 12}$ as the sum of three partial fractions. [4]

7 A curve has an equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{(x-3)^2}{x}$ for $x \neq 0$.

(i) Find an expression for $\mathrm{f}'(x)$ and obtain the coordinates of the stationary points on the curve. [4]

(ii) Showing full working, determine the nature of these stationary points. [4]

8 The roots of the quadratic equation $8x^2 - 11x + 67 = 0$ are $\alpha^3 + 1$ and $\beta^3 + 1$.

(i) Find the values of $\alpha^3 + \beta^3$ and $\alpha\beta$. [4]

It is also given that the roots of the quadratic equation $4x^2 - 9x + 16 = 0$ are α^2 and β^2.

(ii) State the value of $\alpha^2 + \beta^2$. [1]

(iii) Use **all** results from **(i)** and **(ii)** to deduce the value of $\alpha + \beta$. [3]

(iv) Form a quadratic equation, with integer coefficients, whose roots are α and β. [2]

(ii)	$f''(x)=\frac{x^2(2x)-(x^2-9)(2x)}{x^4}$ $=\frac{18}{x^3}$ $f''(3)>0$ and $f''(-3)<0$ $\therefore$ (3, 0) Minimum point and (– 3, – 12) Maximum point.	M1 M1 A1A1	[8]	
8(i)	$\alpha^3+1+\beta^3+1=\frac{11}{8}$ $\alpha^3+\beta^3=-\frac{5}{8}$ $(\alpha^3+1)(\beta^3+1)=\frac{67}{8}$ $\alpha^3\beta^3+\alpha^3+\beta^3+1=\frac{67}{8}$ $\alpha^3\beta^3=\frac{67}{8}+\frac{5}{8}-1=8$ $\alpha\beta=2$	M1 A1 M1 A1		
(ii)	$\alpha^2+\beta^2=\frac{9}{4}$ $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$	B1 B1		
(iii)	$-\frac{5}{8}=(\alpha+\beta)(\frac{9}{4}-2)$ $(\alpha+\beta)=-\frac{5}{2}$	M1A1		
(iv)	The quadratic equation is $x^2+\frac{5}{2}x+2=0$ $2x^2+5x+4=0$	M1A1	[10]	
9(i)	$h=5\cos\vartheta+8\sin\vartheta$	B1B1		
(ii)	$R=\sqrt{5^2+8^2}=\sqrt{89}$ $\alpha=\tan^{-1}\left(\frac{8}{5}\right)=1.012197$ h = $\sqrt{89}\cos(\theta-1.01)$	B1 M1A1 A1		
(iii)	Max value of h = 9.43 $\theta=1.01$ $\sqrt{89}\cos(\theta-1.012197)=7.5$ $\theta=0.360$ (accept 0.360 to 0.361)	B1 B1 M1 M1A1	[11]	

10(i)	Mid-point of AB is (2, 3) and Gradient of $AB=-\frac{3}{2}$ Equation of the perpendicular bisector is $y-3=\frac{2}{3}(x-2)$ $3y=2x+5$ Solving $y=x+2$ and $3y=2x+5$, Centre is (– 1, 1)	M1M1 M1A1 M1A1		
(ii)	Radius = $\sqrt{(-1-4)^2+(1-0)^2}=\sqrt{26}$ Equation of the circle is $(x+1)^2+(y-1)^2=26$	M1 M1A1		
(iii)	$a=2$, $b=-2$	B1		
(iv)	Radius of the second circle = $\sqrt{1^2+(-1)^2+23}=5$ $<\sqrt{26}$ $\therefore$ The second circle lies inside the first circle.	M1A1	[12]	

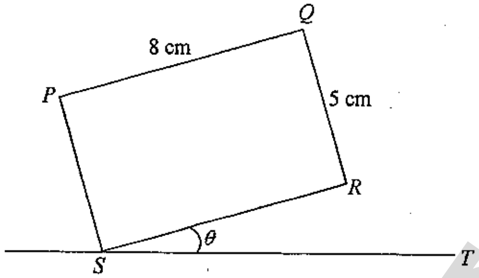
11 (i)	$y=kx^n$ $\lg y=n\lg x+\lg k$ Plot $\lg y$ against $\lg x$ to obtain straight line graph Use graph to find $k\approx 1.43$ and $n\approx 0.563$	M2A1 M1A2		
(ii)	$y=1.43\,x^{0.563}$ $10=1.43\,x^{1.563}$ $x=3.47$	M1A1		
(iii)	xy = 10 $\lg x+\lg y=1$ Plot this straight line using the same axes.	B1 M1A1	[11]	

ADDITIONAL MATHEMATICS Paper 2 (4047/02)

Marking Scheme

Qn	Solution	Marks		Remarks
1 (i)	$3\sin x + 5 = 0$			
	$\sin x = -\frac{5}{3}$ which is not possible as $-1 \le \sin x \le 1$ $f'(x) \ne 0$. ∴ There is no stationary point	M1A1		
(ii)	$y = \int(3\sin x + 5)dx$ $= -3\cos x + 5x + c$	M1A1		
	$x = 0, y = 5 \Rightarrow c = 8$ ∴ $f(x) = -3\cos x + 5x + 8$	M1A1	[6]	
2 (i)	$\frac{d}{dx}\left(xe^{\frac{1}{2}x}\right) = \frac{1}{2}xe^{\frac{1}{2}x} + e^{\frac{1}{2}x}$	M1A1		
(ii)	$\int e^{\frac{1}{2}x}dx = 2e^{\frac{1}{2}x} + c$	B1A1		
(iii)	$\int_0^4 \frac{1}{2}xe^{\frac{1}{2}x}dx + \left[2e^2 - 2\right] = \left[4e^2 - 0\right]$	M1M1		
	$\int_0^4 \frac{1}{2}xe^{\frac{1}{2}x}dx = 2e^2 + 2$	M1		
	$\int_0^4 xe^{\frac{1}{2}x}dx = 4e^2 + 4$	A1	[8]	
3 (i)	$\frac{dy}{dx} = 2(x + k)$	B1		
	Gradient of the tangent $= 2(2k + k) = 6k$	M1		
	When $x = 2k$, $y = (k+2k)^2 = 9k^2$	M1		
	Equation of the tangent is $y - 9k^2 = 6k(x - 2k)$ $y + 3k^2 = 6kx$	M1A1		
(ii)	$P\left(\frac{k}{2}, 0\right)$ and $Q\left(0, -3k^2\right)$	M1		
	Mid-point R is $\left(\frac{k}{4}, -\frac{3k^2}{2}\right)$	M1		
	Substituting in $y + 4x^2 = 0$, $-\frac{3k^2}{2} + 24\left(\frac{k}{4}\right)^2 = 0$ $-\frac{3k^2}{2} + \frac{3k^2}{2} = 0$ $0 = 0$ ∴ M lies on the curve $y + 24x^2 = 0$	M1A1	[9]	
4(a)(i)	$(2 - p)^5 = 32 - 80p + 80p^2 - 40p^3 + 10p^4 - p^5$	M1A2		
(ii)	Let $p = 2x - \frac{x^2}{2}$	B1		
	$\left(2 - 2x + \frac{x^2}{2}\right) = 32 - 80(2x - \frac{x^2}{2}) + 80\,(2x - \frac{x^2}{2})^2 + ..$ $= 32 - 160x + 360x^2 + \ldots\ldots$	M1A1		
(b)(i)	$\binom{16}{r}\left(x^2\right)^{16-r}\left(-\frac{1}{2x^6}\right)^r$	B1		
(ii)	$\binom{16}{r}\left(x^2\right)^{16-r}\left(-\frac{1}{2x^6}\right)^r = \binom{16}{r}\left(-\frac{1}{2}\right)^r x^{32-2r}$	M1		
	$32 - 2r = 0 \Rightarrow r = 4$			
	Term independent of $x = \binom{16}{4}\left(-\frac{1}{2}\right)^4 = \frac{455}{4}$	M1A1	[10]	
5	$k^2 = \left(3 - 2\sqrt{2}\right)^2 = 17 - 12\sqrt{2}$	B1		
	$\frac{1}{k^2} = \frac{1}{17 - 12\sqrt{2}} = \frac{17 + 12\sqrt{2}}{\left(17 - 12\sqrt{2}\right)\left(17 + 12\sqrt{2}\right)} = 17 + 12\sqrt{2}$	M1A1		
	$k - \frac{1}{k^2} = 3 - 2\sqrt{2} - \left(17 + 12\sqrt{2}\right) = -14 - 14\sqrt{2}$	M1A1	[5]	
6(i)	$2(-1)^3 - 9(-1)^2 - (-1) + 12 = 0$ ∴ $x + 1$ is a factor of $2x^3 - 9x^2 + x + 12$.	M1A1		
(ii)	$2x^3 - 9x^2 + x + 12 = (x+1)(2x^2 - 11x + 12)$	B1		
	$= (x+1)(2x-3)(x-4)$	A1		
	$(x+1)(2x-3)(x-4) = 0 \Rightarrow x = -1, \frac{3}{2}$ or 4.	A2		
(iii)	Let $\frac{25}{2x^3 - 9x^2 + x + 12} = \frac{A}{x+1} + \frac{B}{2x-3} + \frac{C}{x-4}$	M1		
	Evaluating A, B and C $A = 1, B = -4, C = 1$	M1A1		
	$\frac{25}{2x^3 - 9x^2 + x + 12} = \frac{1}{x+1} - \frac{4}{2x-3} + \frac{1}{x-4}$	A1	[10]	
7(i)	$f'(x) = \frac{2x(x-3) - (x-3)^2}{x^2}$ $= \frac{x^2 - 9}{x^2}$	M1		
	$\frac{x^2 - 9}{x^2} = 0 \Rightarrow x = \pm 3$	M1		
	The stationary points are (3, 0) and (– 3, – 12)	M1A1		

9



In the figure, *PQRS* is a rectangle of length 8 cm and breadth 5 cm and $\angle RST = \theta$ radians, where θ is acute.

(i) Express h cm, the perpendicular distance from Q to the line ST, in the form $a\cos\theta + b\sin\theta$, where a and b are constants. [2]

(ii) Express h in the form $R\cos(\theta - \alpha)$, where R is a positive constant and α is an acute angle in radians. [4]

(iii) Find the maximum value of h and the corresponding value of θ. [2]

(iv) Find the value of θ for which $h = 7.5$ cm. [3]

10 A circle passes through the points $A(4, 0)$ and $B(0, 6)$. Its centre lies on the line $y = x + 2$.

(i) Find the equation of the perpendicular bisector of AB and hence show that the centre of the circle is $(-1, 1)$. [6]

(ii) Find the equation of the circle. [3]

A second circle with equation $x^2 + y^2 + ax + by - 23 = 0$, has the same centre as the first circle.

(iii) Write down the value of a and of b. [1]

(iv) Show that the second circle lies inside the first circle. [2]

[Turn over

11 The table shows the experimental values of x and y.

x	1.5	2.0	2.5	3	3.5	4.0
y	1.8	2.1	2.4	2.6	2.9	3.1

It is known that x and y are related by the equation $y = kx^n$, where k and n are constants.

(i) Using suitable variables, draw on graph paper, a straight line graph and hence estimate the value of each of the constants k and n. [6]

(ii) Using your values of k and n, calculate the value of x for which $xy = 10$. [2]

(iii) Explain how another straight line drawn on your diagram can lead to an estimate of the value of x for which $xy = 10$. Draw this line. [3]

- End of Paper -

Geylang Methodist School (Secondary)
Preliminary Examination 2016

ADDITIONAL MATHEMATICS **4047/01**

Paper 1

4 Express / 5 Normal (Academic)

Additional materials : Writing Paper **2 hours**

Setter : Mrs Goh Heng Mei **12 August 2016**

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Writing Papers provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of 6 printed pages including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

1 The function f is defined, for all values of x, by

$$f(x) = x(1-x)^2.$$

Find the values of x for which f is an increasing function. [4]

2 (a) Given that $\log_4 p = a$, $\log_{16} q = b$ and $\frac{p}{q} = 2^c$, express c in terms of a and b. [3]

(b) On the same axes, sketch the graphs of $y = \log_4 x$ and $y = \log_{16} x$. [2]

3 The number of bacteria in a culture is given by $N = N_0\, e^{kt}$, where N_0 is the number of bacteria at a particular time and N is the number of bacteria present t hours later. The number of bacteria in the culture triples every 2 hours. Calculate the value of the constant k. [3]

4 (a) Show that the roots of the equation $x^2 + (a-2)x = 2a$ are real for all values of a. [3]

(b) Show that there are no values of b for which the curve $y = (b-3)x^2 - 2bx + (b-2)$ is always positive. [4]

5 The vertices of a parallelogram $ABCD$ are $A(5, 0)$, $B(-3, 4)$, $C(-2, 6)$ and $D(p, q)$ respectively.

(i) Find the mid-point of AC. [1]

(ii) Find the coordinates of D. [2]

Hence show that $ABCD$ is a rectangle. [2]

6 The curve $y = a\sin bx + c$ is defined for $0 \le x \le 2\pi$, where a is a negative integer and b is a positive integer. Given that the amplitude of y is 4 and that the period of y is π,

(i) state the value of a and of b. [2]

Given that the maximum value of y is 6,

(ii) state the value of c, [1]

(iii) Sketch the graph of y, indicating the coordinates of any maximum or minimum points. [3]

7 (a) Show that $|x+5| = x-4$ has no solution. [2]

(b) (i) Sketch the graph of the function $y = |x^2 - 2x - 8|$ for $-6 \le x \le 8$, labelling the turning point and the intercepts of the graph. [4]

(ii) Hence, find the range of values of c if the graph of $y = c$ intersects the graph of $y = |x^2 - 2x - 8|$ at more than 2 points. [2]

8 (i) Find the value of each of the constants a and b for which $\sin 2x(5\tan x + 2\cot x) = a + b\sin^2 x$. [3]

(ii) Hence solve the equation $\sin 4\theta(5\tan 2\theta + 2\cot 2\theta) = 7$, stating the principal values of θ. [3]

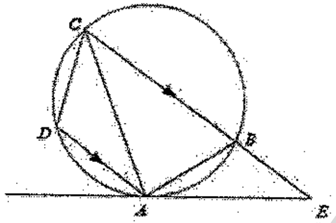
9 A particle starts from rest at a fixed point O and moves in a straight line with its acceleration, a m/s², given by $a = 5 - pt$, where t seconds is the time since leaving O, and p is a real constant.
When $t = 3$, its velocity is 12 m/s.

(i) Find the value of p. [2]

(ii) When does the particle change its direction of motion? [2]

(iii) Show that the particle passes O again when $t = 22.5$. Hence find the total distance travelled by the particle between $t = 0$ and $t = 22.5$. [4]

10



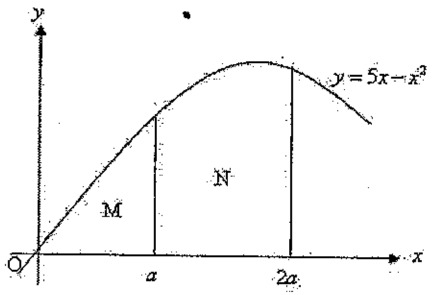
The diagram shows a quadrilateral *ABCD* whose vertices lie on the circumference of the circle. The point *E* lies on *CB* produced such that *AE* is a tangent to the circle.
CE and *AD* are parallel.

(i) Show that angle BAE = angle CAD. [2]

(ii) Show that triangles BAE and DAC are similar. [3]

(iii) Given that $AB = BE$, show that the line AC bisects the angle BCD. [2]

11 The diagram shows part of the curve $y = x(5-x)$.



$y = x(5-x)$

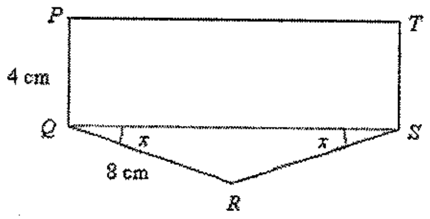
The region M is bounded by the curve $y = x(5-x)$, the x-axis and the line $x = a$.
The region N is bounded by the curve $y = x(5-x)$, the x-axis and the lines $x = a$ and $x = 2a$.
Given that the area of N is twice the area of M, find the value of a. [5]

12 It is given that $y = (x-1)\sqrt{4x+3}$.

(i) Express $\frac{dy}{dx}$ in the form $\frac{px+q}{\sqrt{4x+3}}$ where p and q are integers. [3]

(ii) Given that y is increasing at the rate of 2.5 units per second when $x = 3$, find the rate of change of x at this instant. [2]

13



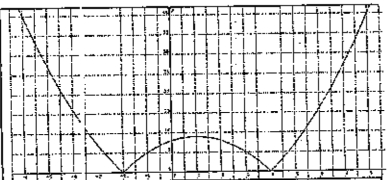
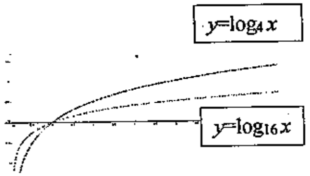
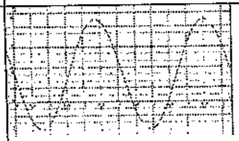
A piece of paper is cut into the shape *PQRST* as shown in the diagram. *PQST* is a rectangle with $PQ = 4$ cm and *QRS* is an isosceles triangle with $QR = 8$ cm.

(i) Given that angle SQR = angle $QSR = x$ radian, show that the area of the paper, A, is given by $A = 64\cos x(1+\sin x)$. [4]

(ii) Find the value of x, in terms of π, for which A has a stationary value. [4]

(iii) Find the exact value of A and determine whether it is a maximum or a minimum. [3]

Answers

1	$x<\frac{1}{3}$ or $x>1$	7(ii)	
2(a)	$c=2a-4b$	7(ii)	$0<c\le 9$
2(b)		8(i) 8(ii)	$a=4,\quad b=6$ Principal values = $-22.5°, 22.5°$
3	0.549	9(i)	$p=\frac{2}{3}$
4(a)	Discriminant = $(a+2)^2\ge 0$	9(ii)	15s
4(b)	$b>3$ and $b<\frac{6}{5}$ There are no real values of b.	9(iii)	375 m
5(i)	$\left(\frac{3}{2},3\right)$	11(i)	$a=\frac{3}{2}$
5(ii)	$(6,2)$ Gradient of AB × gradient of CD = -1 $\Rightarrow\quad AB\perp CD$. $\Rightarrow\quad ABCD$ is a rectangle.	12(i) 12(ii)	$\frac{dy}{dx}=\frac{6x+1}{\sqrt{4x+3}}$ 0.510 units/s
6(i)	$a=-4,\quad b=2$	13(ii)	$x=\frac{\pi}{6}$
6(ii)	$c=2$	13(iii)	A = $48\sqrt{3}$ cm^2 . A is maximum
6(iii)			

Geylang Methodist School (Secondary)
Preliminary Examination 2016

ANSWERS

ADDITIONAL MATHEMATICS **4047/01**

Paper 1

4 Express / 5 Normal (Academic)

Additional materials : Writing Paper

2 hours

Setter : Mrs Goh Heng Mei

12 August 2016

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Writing Papers provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of 6 printed pages including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

1 The function f is defined, for all values of x, by

$$f(x) = x(1-x)^2.$$

Find the values of x for which f is an increasing function. [4]

Solutions

$f(x) = x(1-x)^2$

$= x(1-2x+x^2)$

$= x - 2x^2 + x^3$

$f'(x) = 1 - 4x + 3x^2$

$= (3x-1)(x-1)$

Given that f is an increasing function.

$\Rightarrow \; (3x-1)(x-1) > 0$

$\therefore \; x < \frac{1}{3}$ **or** $x > 1$

2 **(a)** Given that $\log_4 p = a$, $\log_{16} q = b$ and $\frac{p}{q} = 2^c$, [3]

express c in terms of a and b.

(b) On the same axes, sketch the graphs of $y = \log_4 x$ and $y = \log_{16} x$. [2]

Solutions

(a) $\log_4 p = a$, $\log_{16} q = b$

$p = 4^a$, $q = 16^b$

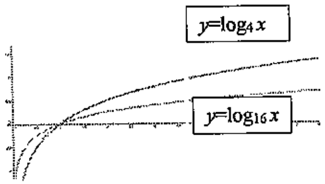
Given $\frac{p}{q} = 2^c$

$\frac{4^a}{16^b} = 2^c$

$\frac{2^{2a}}{2^{4b}} = 2^c$

$2^{2a-4b} = 2^c \quad \therefore \; c = 2a - 4b$

(b)



3

3 The number of bacteria in a culture is given by $N = N_0 e^{kt}$, where N_0 is the number of bacteria at a particular time and N is the number of bacteria present t hours later. The number of bacteria in the culture triples every 2 hours. Calculate the value of the constant k. [3]

Solutions

$N = N_0 e^{kt}$

When t = 0, $N = N_0$

When t = 2, $N = 3N_0$

When t = 2, $N = N_0 e^{k(2)}$

$\therefore \; 3N_0 = N_0 e^{k(2)}$

$3 = e^{2k}$

$2k = \ln 3$

$k = \frac{\ln 3}{2}$

$= 0.5493$

~ 0.549

4 **(a)** Show that the roots of the equation $x^2 + (a-2)x = 2a$ are real for all values of a. [3]

(b) Show that there are no values of b for which the curve $y = (b-3)x^2 - 2bx + (b-2)$ is always positive. [4]

Solutions

(a) $x^2 + (a-2)x - 2a = 0$

Discriminant =

$(a-2)^2 - 4(1)(-2a)$

$= a^2 - 4a + 4 + 8a$

$= a^2 + 4a + 4$

$= (a+2)^2$

Since $(a+2)^2 \geq 0$,

The discriminant ≥ 0

$\therefore$ the roots are real for all real values of a

(b) If $y = (b-3)x^2 - 2bx + (b-2)$ is always positive, then

$b - 3 > 0 \;\Rightarrow\; b > 3$

and discriminant < 0

$(-2b)^2 - 4(b-3)(b-2) < 0$

$4b^2 - 4(b^2 - 5b + 6) < 0$

$4b^2 - 4b^2 + 20b - 24 < 0$

$20b < 24$

$b < \frac{6}{5}$

But from above, b > 3.∴ there are no values of b for which y is always positive.

4

5 The vertices of a parallelogram $ABCD$ are $A(5,0)$, $B(-3,4)$, $C(-2,6)$ and $D(p,q)$ respectively.

(i) Find the mid-point of AC. [1]

(ii) Find the coordinates of D. [2]

Hence show that $ABCD$ is a rectangle. [2]

Solutions

(i) Mid-point of AC =

$$\left(\frac{5-2}{2},\frac{0+6}{2}\right)$$

$$=\left(\frac{3}{2},3\right)$$

(ii)

Mid-point of $BD=\left(\frac{p-3}{2},\frac{q+4}{2}\right)$

Mid-point of BD = mid-point of AC

$$\frac{p-3}{2}=\frac{3}{2} \quad \text{and} \quad \frac{q+4}{2}=3$$

$$p-3=3 \quad , \quad q+4=6$$

$$p=6 \quad , \quad q=2$$

$\therefore$ D is $(6,2)$

Given that $ABCD$ is a parallelogram. Therefore $AB=CD$ and AB // CD.

Gradient of $AB=\frac{4-0}{-3-5}$

$$=-\frac{1}{2}$$

Gradient of $CD=\frac{2-0}{6-5}$

$$=2$$

Gradient of AB × gradient of CD = -1

$\Rightarrow$ $AB \perp CD$.

$\Rightarrow$ $ABCD$ is a rectangle.

6 The curve $y=a\sin bx+c$ is defined for $0\le x\le 2\pi$, where a is a negative integer and b is a positive integer. Given that the amplitude of y is 4 and that the period of y is π,

(i) state the value of a and of b. [2]

Given that the maximum value of y is 6,

(ii) state the value of c, [1]

(iii) Sketch the graph of y, indicating the coordinates of any maximum or minimum points. [3]

Solutions

(i) a is negative and amplitude is 4. Therefore $a=-4$.

Period of y is π . Therefore $b=2$.

(ii)

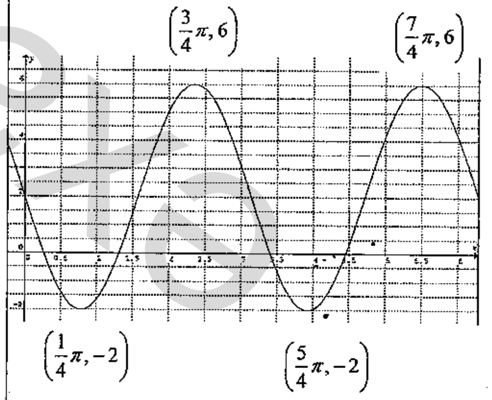
$$y=a\sin bx+c$$

$$y=-4\sin 2x+c$$

When $\sin 2x=-1$,

$$6=-4(-1)+c \quad \Rightarrow c=2$$

(iii)



7 (a) Show that $|x+5|=x-4$ has no solution. [2]

(b) (i) Sketch the graph of the function $y=|x^2-2x-8|$ for $-6\le x\le 8$, labelling the turning point and the intercepts of the graph. [4]

(ii) Hence, find the range of values of c if the graph of $y=c$ intersects the graph of $y=|x^2-2x-8|$ at more than 2 points. [2]

Solutions

(a) $|x+5|=x-4$

$\Rightarrow\ x+5=x-4$ or $x+5=-(x-4)$

NA

$x+5=-x+4$

$2x=-1$

$x=-\dfrac{1}{2}$

But when $x=-\dfrac{1}{2}$, $|x+5|=-\dfrac{1}{2}-4<0$ NA

$\therefore\ |x+5|=x-4$ has no solution.

(b) $y=|x^2-2x-8|$

$=|(x-4)(x+2)|$

When $x=-6$, $y=|(-10)(-4)|$

$=40$

When $x=8$, $y=|(4)(10)|$

$=40$

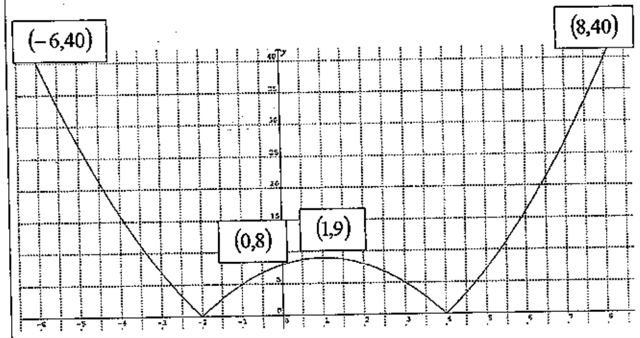
When $x=0$, $y=|-8|$

$=8$

Line of symmetry : $x=\dfrac{-2+4}{2}$

$=1$

When $x=1$, $y=|(-3)(3)|=9$



(ii) $0<c\le 9$

8 (i) Find the value of each of the constants a and b for which $\sin 2x(5\tan x+2\cot x)=a+b\sin^2 x$. [3]

(ii) Hence solve the equation $\sin 4\theta(5\tan 2\theta+2\cot 2\theta)=7$, stating the principal values of θ. [3]

Solutions

(i) $\sin 2x(5\tan x+2\cot x)$

$=$

$2\sin x\cos x\left(\dfrac{5\sin x}{\cos x}+\dfrac{2\cos x}{\sin x}\right)$

$=10\sin^2 x+4\cos^2 x$

$=10\sin^2 x+4(1-\sin^2 x)$

$=4+6\sin^2 x$

$a=4$ and $b=6$

(ii) Let $x=2\theta$

$4+6\sin^2 2\theta=7$

$6\sin^2 2\theta=3$

$\sin^2 2\theta=\dfrac{1}{2}$

$\sin 2\theta=\pm\dfrac{1}{\sqrt{2}}$

Basic angle $=45^0$

Principal value: $-90^0\le\sin^{-1}x\le 90^0$

Principal values of $2\theta=-45^0, 45^0$

Principal values of $\theta=-22.5, 22.5$

9 A particle starts from rest at a fixed point O and moves in a straight line with its acceleration, a m/s^2, given by $a = 5 - pt$, where t seconds is the time since leaving O, and p is a real constant.

When $t = 3$, its velocity is 12 m/s.

(i) Find the value of p. [2]

(ii) When does the particle change its direction of motion? [2]

(iii) Show that the particle passes O again when $t = 22.5$. Hence find the total distance travelled by the particle between $t = 0$ and $t = 22.5$. [4]

Solutions

(i) $a = 5 - pt$

$$v = 5t - \frac{pt^2}{2} + c$$

The particle starts from rest

$\Rightarrow$ when $t = o$, $v = 0$ $\therefore c = 0$

$$v = 5t - \frac{pt^2}{2}$$

When $t = 3$, $v = 12$

$$5(3) - \frac{9p}{2} = 12$$

$$\frac{9p}{2} = 3$$

$$p = \frac{2}{3}$$

(ii)

When particle changes its direction, $v = 0$

$$5t - \frac{2}{3}\left(\frac{t^2}{2}\right) = 0$$

$$t\left(5 - \frac{1}{3}t\right) = 0$$

$T = 0$ (NA) or $5 = \frac{1}{3}t$

t = 15 s

(iii)

$$v = 5t - \frac{t^2}{3}$$

$$s = \frac{5t^2}{2} - \frac{t^3}{9} + c$$

When $t = 0$, $s = 0$ $\therefore c = 0$

$$s = \frac{5t^2}{2} - \frac{t^3}{9}$$

When $t = 22.5$, $s = \frac{5(22.5)^2}{2} - \frac{22.5^3}{9}$

$= 0$

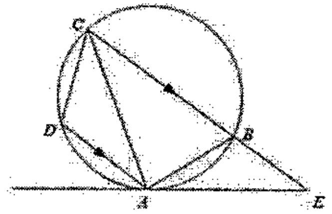
$\Rightarrow$ the particle passes pt O again when t = 22.5 s

When $t = 15$, $s = \frac{5(15)^2}{2} - \frac{15^3}{9} = 187.5$

Dist travelled $= 187.5 \times 2$

$= 375$ m

10



The diagram shows a quadrilateral $ABCD$ whose vertices lie on the circumference of the circle. The point E lies on CB produced such that AE is a tangent to the circle.

CE and AD are parallel.

(i) Show that angle BAE = angle CAD. [2]

(ii) Show that triangles BAE and DAC are similar. [3]

(iii) Given that $AB = BE$, show that the line AC bisects the angle BCD. [2]

Solutions

(i) $\angle BAE = \angle BCA$ (tangent chord thm)

$= \angle CAD$ (alt $\angle s$, $CE // AD$)

(ii) $\angle ABC + \angle ABE = 180^0$ ($\angle s$ in Δ)

$\angle ABC + \angle CDA = 180^0$ ($\angle s$ in opp seg)

$\therefore$ $\angle ABE = \angle CDA$

From (i) $\angle BAE = \angle CAD$

$\therefore \Delta BAE$ is similar to ΔDAC (AA)

(iii)

Given that $AB = BE$.

BAE is an isosceles triangle.

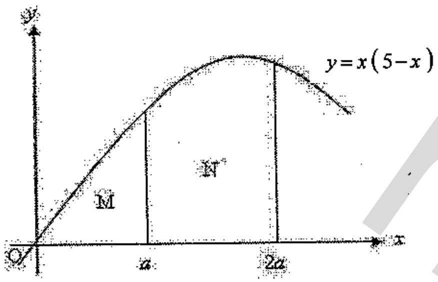
then DAC is also an isosceles Δ.

$\angle DCA = \angle CAD$

$= \angle ACB$

$\therefore$ AC bisects the angle BCD.

11 The diagram shows part of the curve $y=x(5-x)$.



The region M is bounded by the curve $y=x(5-x)$, the x-axis and the line $x=a$.
The region N is bounded by the curve $y=x(5-x)$, the x-axis and the lines $x=a$ and $x=2a$.
Given that the area of N is twice the area of M, find the value of a. [5]

Solutions

$$\text{Area M}=\int_0^a x(5-x)\,dx$$
$$=\int_0^a \left(5x-x^2\right)dx$$
$$=\left[\frac{5x^2}{2}-\frac{x^3}{3}\right]_0^a$$
$$=\frac{5a^2}{2}-\frac{a^3}{3}$$

$$\text{Area N}=\int_0^a x(5-x)\,dx$$
$$=\left[\frac{5x^2}{2}-\frac{x^3}{3}\right]_a^{2a}$$
$$=\frac{5\left(4a^2\right)}{2}-\frac{8a^3}{3}-\left(\frac{5a^2}{2}-\frac{a^3}{3}\right)$$
$$=\frac{15a^2}{2}-\frac{7a^3}{3}$$

Given N = 2 M

$$\frac{15a^2}{2}-\frac{7a^3}{3}=2\left[\frac{5a^2}{2}-\frac{a^3}{3}\right]$$
$$=5a^2-\frac{2a^3}{3}$$
$$\frac{15a^2}{2}-\frac{7a^3}{3}-5a^2+\frac{2a^3}{3}=0$$
$$\frac{5a^2}{2}-\frac{5a^3}{3}=0$$
$$a^2\left(\frac{5}{2}-\frac{5a}{3}\right)=0$$
$$a=0\ (\text{NA}) \quad \text{or} \quad \frac{5a}{3}=\frac{5}{2}$$
$$a=\frac{3}{2}$$

12 It is given that $y=(x-1)\sqrt{4x+3}$.

(i) Express $\frac{dy}{dx}$ in the form $\frac{px+q}{\sqrt{4x+3}}$ where p and q are integers. [3]

(ii) Given that y is increasing at the rate of 2.5 units per second when $x=3$, find the rate of change of x at this instant. [2]

Solutions

(i)
$$y=(x-1)(4x+3)^{\frac{1}{2}}$$
$$\frac{dy}{dx}=(x-1)\frac{1}{2}(4x+3)^{-\frac{1}{2}}(4)+(4x+3)^{\frac{1}{2}}(1)$$
$$=(4x+3)^{-\frac{1}{2}}[2(x-1)+(4x+3)]$$
$$=(4x+3)^{-\frac{1}{2}}(6x+1)$$
$$=\frac{6x+1}{\sqrt{4x+3}}$$

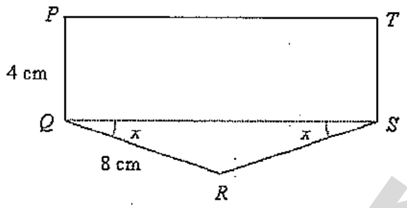
(ii) Given $\frac{dy}{dt}=2.5$

$$\frac{dx}{dt}=\frac{dx}{dy}\cdot\frac{dy}{dt}$$

When $x=3$,

$$\frac{dx}{dt}=\frac{\sqrt{15}}{19}\cdot(2.5)$$
$$=0.5096$$
$$\sim 0.510 \text{ units/s}$$

13



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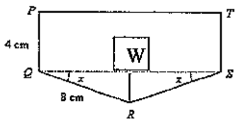
(i) Given that angle SQR = angle $QSR = x$ radian, show that the area of the paper, A, is given by $A = 64\cos x\left(1+\sin x\right)$. [4]

(ii) Find the value of x, in terms of π, for which A has a stationary value. [4]

(iii) Find the exact value of A and determine whether it is a maximum or a minimum. [3]

Solutions

(i)



$RW = 8\sin x$ and $QW = 8\cos x$

Area of rect $PQST = 4(16\cos x)$

Area of $\Delta QRS = \frac{1}{2}(16\cos x)(8\sin x)$

Area of A = $64\cos x + 64\cos x\sin x$

$= 64\cos x(1+\sin x)$

(ii) $A = 64\cos x(1+\sin x)$

$$\frac{dA}{dx} = 64\cos x(\cos x) + 64(1+\sin x)(-\sin x)$$

$$= 64\cos^2 x - 64\sin^2 x - 64\sin x$$

A has stationary value $\Rightarrow \frac{dA}{dx} = 0$

$$64\cos^2 x - 64\sin^2 x - 64\sin x = 0$$

$$64 - 64\sin^2 x - 64\sin^2 x - 64\sin x = 0$$

$$1 - 2\sin^2 x - \sin x = 0$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$\sin x = \frac{1}{2}$ or $\sin x = -1$ (NA)

$$x = \frac{\pi}{6}$$

(iii) $A = 64\cos x(1+\sin x)$

$$A = 64\cos\frac{\pi}{6}\left(1+\sin\frac{\pi}{6}\right)$$

$$= 64\left(\frac{\sqrt{3}}{2}\right)\left(1+\frac{1}{2}\right)$$

$$= 48\sqrt{3}\ \text{cm}^2$$

$$\frac{dA}{dx} = 64 - 128\sin^2 x - 64\sin x$$

$$\frac{d^2A}{dx^2} = -256\sin x\cos x - 64\cos x$$

When $x = \frac{\pi}{6}$, $\frac{d^2A}{dx^2} < 0$

$\therefore$ A is maximum.

Answers

1	3.48 , 5.94	8	Proof
2	3	9(i)	$h = \frac{432}{r^2}$
3	$2 < p < 4$	9(ii)	Proof
4(i)	$\frac{1}{x-3} + \frac{2}{x+1} - \frac{5}{(x+1)^2}$	9(iii)	6 cm
4(ii)	$\ln(x-3) + 2\ln(x+1) + \frac{5}{x+1}$	10(a)	1 , −2
5(i)	$x = -0.8$; $x = 6$ (rej)	10(b)	Proof
5(ii)	graph	11(i)	$\frac{4}{3}$
6(i)	graph	11(ii)	$-\frac{143}{145}$
6(ii)	$x = ^{-\frac{2}{3}(\frac{1}{y}-2)}$	11(iii)	$-\frac{21}{20}$
7(i)	$B = (3 , 0)$	12(i)	Proof
7(ii)	45.5 units^2	12(ii)	0.24 m^2/s
7(iii)	Triangles BCD and BED		
7(iv)	$y = 10x - 76$		

NAVAL BASE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2, 2016

O

Name ____________________________ (   )   **Class** __________

ADDITIONAL MATHEMATICS **4047/01**
Paper 1 **4 Aug 2016**
Additional Materials: Cover Page **2 hours**
Answer Paper
Graph Paper (2 sheets)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number at the top of the page.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

This paper consists of **5** printed pages and **1** blank page. **[Turn over**

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For the equation $ax^2+bx+c=0$, $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$
$$\sec^2 A=1+\tan^2 A$$
$$\operatorname{cosec}^2 A=1+\cot^2 A$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC,

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$
$$a^2=b^2+c^2-2bc\cos A$$
$$\Delta=\frac{1}{2}bc\sin A$$

[Turn over

Answer **all** the questions.

1 Find the value of k for which the coefficient of x^3 in the expansion $(2-x)^3+(4-kx)^5$ is -21. [5]

2 Given that $\cos A=\frac{2}{\sqrt{13}}$ and $\sin B=\sqrt{\frac{3}{4}}$ and that angles A and B are in the same quadrant, find, without using the calculator the value of $\cos(A+B)$. [5]

3 Express $\frac{14+7x-3x^2}{x^2(x+2)}$ as the sum of partial fractions. [5]

4 Two variables, x and y are related by the equation $y=4x+\frac{9}{x-1}, x\neq 1$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) Given that $\frac{dy}{dt}=4$ and $\frac{dx}{dt}=\frac{4}{3}$ find the value of y. [4]

5 The table shows experimental values of the variables x and y.

x	2	3	4	5	6
y	11.9	21.2	32.0	44.1	57.4

It is known that x and y are related by the equation $y-x=kx^n$, where k and n are constants.

(i) Draw the straight line graph and use it to estimate the values of k and n. [4]

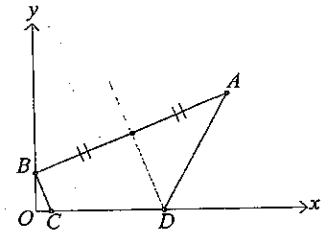
(ii) With the estimated values of k and n, calculate the value of x when $y=x+4.5$ [2]

6 **(i)** Prove that $\frac{\sin x}{\sec x-1}+\frac{\sin x}{\sec x+1}=2\cot x$. [4]

(ii) Find in radians, the acute angle for which $\frac{\sin x}{\sec x-1}+\frac{\sin x}{\sec x+1}=\tan x$. [2]

[Turn over

7 The diagram shows the quadrilateral $ABCD$. The coordinates of A and B are $(3,5)$ and $(0,1)$ respectively.



(i) AB is perpendicular to BC, and C lies on the x-axis. Find the equation of BC and the coordinates of C. [3]

(ii) The point D lies on the x-axis and also on the perpendicular bisector of AB. Find the coordinates of D and the area of the quadrilateral $ABCD$. [4]

8 Given that $y=\cos(\ln(1+x))$, prove that $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=0$. [7]

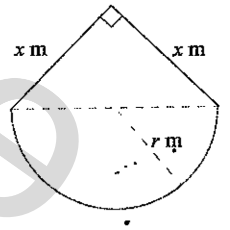
9 The equation of a curve is $y=4x^2+px+p-3$, where p is a constant.

(i) Find the range of values of p for which the curve is always positive. [3]

(ii) **(a)** In the case where $p=12$, show that the x-axis is a tangent to the curve. [2]

(b) Find the coordinates of the point of tangency and state its gradient. [3]

10



A gardener uses 80 m of fencing to enclose a plot of land in the shape shown above. The shape consists of a semicircular arc with radius r m and two sides, each of length x m, of a right-angled triangle.

(i) Show that the area of the plot is $\left(\frac{1}{2}\pi r^2+\frac{1}{8}(80-\pi r)^2\right)$ m^2. [3]

(ii) Given that r can vary, find the value of r for which the area of the plot is stationary. [3]

(iii) Explain why this value of r gives the gardener the minimum area possible. [1]

[Turn over

11 The points $P(1, 2)$ and $Q(7, 14)$ lie on the curve whose equation is $y = x^2 - 6x + 7$.

A is a point on the curve such that the tangent to the curve at A is parallel to PQ.

(i) Find the coordinates of A. [3]

(ii) Find the equation of the normal to the curve at A. [2]

(iii) The normal to the curve at A meets the curve again at B.
Find the coordinates of B. [3]

12 A curve has the equation $y = (x+3)(x-1) - 2$.

(i) Explain why the lowest point on the curve has coordinates $(-1, -6)$. [2]

(ii) Find the coordinates of the points at which the curve intersects the x-axis. [2]

(iii) Sketch the graph of $|(x+3)(x-1) - 2|$. [3]

(iv) Using your graph, state the number of solutions to each of the following equations.

(a) $|(x+3)(x-1) - 2| = 7$ [1]

(b) $|(x+3)(x-1) - 2| = 3$ [1]

(c) $|(x+3)(x-1) - 2| + 2 = 0$ [1]

—— ***End of Paper*** ——

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4E5N AMath Prelim 2 Paper 1, 2016
Answer Scheme

Qn	Answer	Marks
1	term in $x^3 = (-x)^3 + 4^2\binom{5}{3}(-kx)^3$ $= -x^3 - 160k^3x^3$ $-21 = -1 - 160k^3$ $\frac{1}{8} = k^3$ $k = \frac{1}{2}$	M1 M1 M1 M1 A1[5]
2	$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $= \left(\frac{2}{\sqrt{13}}\right)\left(\frac{1}{\sqrt{4}}\right) - \left(\frac{3}{\sqrt{13}}\right)\left(\frac{\sqrt{3}}{\sqrt{4}}\right)$ $= \frac{2-3\sqrt{3}}{\sqrt{52}}$ $= \frac{2\sqrt{13}-3\sqrt{39}}{26}$	B2(value of cos A and sin A) M1 (input in formula) M1 A1[5]
3	$\frac{14+7x-3x^2}{x^2(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$ $14 + 7x - 3x^2 = Ax(x+2) + B(x+2) + Cx^2$ $B = 7$ $A = 0$ $C = -3$ $\therefore \frac{7}{x^2} - \frac{3}{x+2}$	M1 M1 B1 B1 (for A or C) A1[5]
4i	$y = 4x + \frac{9}{x-1}$ $\frac{dy}{dx} = 4 - 9(x-1)^{-2}$ $= 4 - \frac{9}{(x-1)^2}$	M1 or B2 A1[2]
4ii	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ $= 4 \times \frac{3}{4}$ $= 3$ $3 = 4 - \frac{9}{(x-1)^2}$ $x = 4, -2$ $y = 19, -11$	M1 M1 M1 A1[4]

5i	P1: Plot $\ln(y-x)$ against $\ln x$ OR Plot $\lg(y-x)$ against $\lg x$ P1: connect points on a best fit line $\lg k = 0.56 \pm 0.02$ $k = 3.63 \pm 1$ $n = 1\frac{19}{40} \pm 2$	B1 B1[4]
5ii	From graph $\lg x = 0.065$ $x = 1.16$	M1 A1[2]
6i	$LHS = \frac{(\sin x)(\sec x + 1) + (\sin x)(\sec x - 1)}{\sec^2 x - 1}$ $= \frac{2\sin x \sec x}{\tan^2 x}$ $= \frac{2\sin x \frac{1}{\cos x}}{\frac{\sin^2 x}{\cos^2 x}}$ $= 2\cot x (shown)$	M1 M1 M1 A1[4]
6ii	$2\cot x = \tan x$ $2 = \tan^2 x$ $x = 0.955 rad$	M1 A1[2]
7i	gradient BC $= -\frac{3}{4}$ eqn BC : $y = -\frac{3}{4}x + 1$ $C\left(\frac{4}{3}, 0\right)$	M1 A1 B1[3]
7ii	$y = -\frac{3}{4}x + \frac{33}{8}$ $D(5.5, 0)$ $area = \frac{1}{2}\begin{vmatrix} 3 & 0 & \frac{4}{3} & 5.5 & 3 \\ 5 & 1 & 0 & 0 & 5 \end{vmatrix}$ $= 14\frac{7}{12} units^2$	M1 A1 M1 A1[4]

8	$\frac{dy}{dx} = -\sin(\ln(1+x))\left(\frac{1}{1+x}\right)$ $\frac{d^2y}{dx^2} = -\cos(\ln(1+x))\left(\frac{1}{1+x}\right)^2 - \sin(\ln(1+x))(-1)(1+x)^{-2}$ $(1+x)^2\frac{d2y}{dx2} + (1+x)\frac{dy}{dx} + y$ $= -\cos(\ln(1+x)) + \sin(\ln(1+x)) - \sin(\ln(1+x)) + \cos(\ln(1+x))$ $= 0(shown)$	B2 (differentiate cos and ln) B3(differentiate sin, apply pdt and chain rule) M1 A1[7]
9i	$p^2 - 4(4)(p-3) < 0$ $(p-12)(p-4) < 0$ $4 < p < 12$	M1 M1 A1[3]
9iia	$b^2 - 4ac = 12^2 - 4(4)(9)$ $= 0(shown)$	M1 A1[2]
9iib	$0 = 4x^2 + 12x + 9$ $\left(-\frac{3}{2}, 0\right)$ $gradient = 0$	M1 A1 B1[3]
10i	$x = \frac{80 - \pi r}{2}$ $area = \frac{1}{2}\pi r^2 + \frac{1}{2}\left(\frac{80-\pi r}{2}\right)^2$ $= \frac{1}{2}\pi r^2 + \frac{1}{8}(80 - \pi r)^2 m^2 (shown)$	M1 M1 A1[3]
10ii	$0 = \pi r + \frac{1}{4}\pi(80 - \pi r)$ $r = \frac{20\pi}{\pi + \frac{1}{4}\pi^2}$ $= 11.2m$	M1 M1 A1[3]
10iii	$\frac{d^2A}{dr^2}$ $= 5.60899 > 0$ area is minimum	B1[1]
11i	gradient PQ $= \frac{14-2}{7-1}$ $= 2$ $2 = 2x - 6$ $A(4,-1)$	B1 M1 A1[3]
11ii	gradient of normal $= -\frac{1}{2}$ $y = -\frac{1}{2}x + 1$	B1 M1[2]
11iii	$-\frac{1}{2} + 1 = x^2 - 6x + 7$ $x = 4, 1.5$ $y = -1, 0.25$ $B(1.5, 0.25)$	M1 M1 A1[3]
12i	x coordinate of min pt $= \frac{-3+1}{2}$ $= -1$ lowest point (-1,-6)	M1 A1[2]
12ii	$0 = x^2 - x + 3x - 3 - 2$ $x = 1.45, -3.45$	M1 A1[2]
12iii	P1: y intercept P1: x intercepts P1: maximum point	[3]
12iv	**(a)** 2 **(b)** 4 **(c)** 0	B1 B1 B1[3]
	Total	**80**

NAVAL BASE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2, 2016

O

Name ______________________ ( ) **Class** ____________

ADDITIONAL MATHEMATICS **4047/02**
Paper 2 **11 August 2016**
Additional Materials: Cover Page **2 hours 30 minutes**
Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number at the top of the page.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Writing Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

This paper consists of **6** printed pages. **[Turn over**

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$
$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$
$$\sin A + \sin B = 2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$
$$\sin A - \sin B = 2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$
$$\cos A + \cos B = 2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$
$$\cos A - \cos B = -2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

Answer **all** questions.

1 A chef cooks soup until it reaches a temperature of 100°C. The soup is then allowed to cool naturally. The soup cools in such a way that its temperature, T°C, t minutes after it has been removed from the stove, is given by $T = 28 + Ae^{-0.3t}$, where A is a constant.

(i) Explain why $A = 72$. [2]

(ii) The chef would like to serve the soup at 12 noon at a temperature of 40°C. Find the time at which the chef should remove the soup from the stove before he serves it to the customer. [3]

2 Given that $f(x) = 4x^3 + 3x^2 - 16x - 12$.

(i) Find the remainder when $f(x)$ is divided by $x+1$. [2]

(ii) Show that $x+2$ is a factor of $f(x)$. [2]

(iii) Hence, solve the equation $f(x) = 0$. [3]

3 **(i)** The area of a rectangle is $\left(8+2\sqrt{3}\right)$ cm². Given that the width is $\left(4+2\sqrt{3}\right)$ cm.
Find, without using the calculator, the length of the rectangle in the form $\left(a-b\sqrt{3}\right)$ cm. [4]

(ii) The area of a square is $\left(43+30\sqrt{2}\right)$ cm². Given that the length is $\left(3\sqrt{2}+c\right)$ cm.
Find, without using the calculator, the value of c. [3]

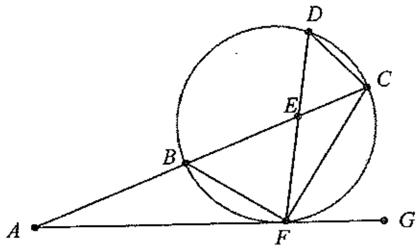
[Turn over

4 The quadratic equation $3x^2 - 6x - 4 = 0$ has roots α and β.

(i) Find the value of $\alpha^2 + \beta^2$. [3]

(ii) Find the quadratic equation whose roots are α^3 and β^3. [5]

5



In the figure, BC is a diameter of the circle. ABC is a straight line and AG is a tangent to the circle at point F. The line DF intersects BC at point E and $3EC = 2EB$.

(i) Prove that triangles ABF and AFC are similar. [3]

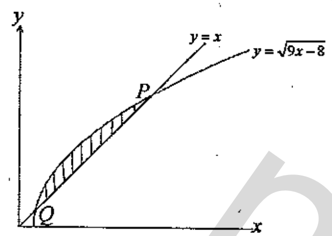
(ii) Show that $AF \times FC = BF \times AC$. [2]

(iii) Given that triangles DEC and BEF are similar, prove that $EF \times ED = \frac{6}{25} BC^2$. [3]

[Turn over

6 **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{4x}{\sqrt{2x-3}}\right)=\frac{4x-12}{\sqrt{(2x-3)^3}}$. [3]

(ii)



The diagram shows part of the curve $y=\sqrt{9x-8}$ intersecting the line $y=x$. The points P and Q are the intersections of the two graphs.

(a) Find the x-coordinates of P and Q. [2]

(b) Find the area of the shaded region. [4]

7 A curve has the equation $y=2x^3-9x^2-8$. The point (p,q) is the stationary point on the curve, where $p>0$.

(i) Determine the value of p and q. [4]

(ii) Determine whether y is increasing or decreasing
  (a) for values of x less than p, [1]
  (b) for values of x greater than p. [1]

(iii) What do the results in part **(ii)** imply about the stationary point? [1]

(iv) What is the value of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at the stationary point? [2]

8 **(i)** Solve $4\log_4 x-9\log_x 4=0$. [4]

(ii) **(a)** Given that $\log_2 x=a$ and $\log_8 y=b$, express x^2y and $\frac{x}{y}$ in terms of a and b. [2]

(b) Given further that $x^2y=32$ and $\frac{x}{y}=0.5$, find the value of a and b. [4]

[Turn over

9 **(i)** Solve the equation $8\cos 2A-\sin A+7=0$ for $0°\le A\le 360°$. [3]

(ii) On the same axes sketch for $0°\le A\le 180°$, the graphs of $y=4\cos 4x+3.5$ and $y=0.5\sin 2x$. [6]

(iii) Show how the solutions of the equation in part **(i)** could be used to find the x-coordinates of the points of intersection of the graphs of part **(ii)**. [2]

10 A particle travelling in a straight line passes through a fixed point O with a velocity of 3.6 m/s. The acceleration, a m/s^2, of the particle, t s, after passing through O, is given by $a=-4e^{-t}$. The particle comes to instantaneous rest at the point P.

(i) Show that the particle reaches P when $t=\ln 10$. [6]

(ii) Calculate the distance OP. [4]

(iii) Show that the particle is again at O at some instant during the tenth second after the first passing through O. [3]

11 A circle has equation $x^2+y^2-4x-2y=20$.

(i) Find the radius and the coordinates of the centre of the circle. [3]

Two diameters lie on the circle, diameter AB and diameter DE. The diameters are perpendicular to each other. Given that diameter AB has equation $4y+3x-10=0$, $A(-2,4)$ and $B(6,-2)$.

(ii) Show that the coordinates of D and E are $(5,5)$ and $(-1,-3)$. [6]

(iii) Determine the type of quadrilateral $ADBE$ and find its area. [4]

End of Paper

4E5N AMath Prelim 2 Paper 2, 2016
Answer Scheme and Markers Report

Qn	Answer	Marks
1(i)	$100 = 28 + Ae^{-0.3(0)}$ $A = 72$	M1 A1[2]
(ii)	$40 = 28 + 72e^{-0.3t}$ $t = 5.9725$ min $12noon - 5.9725$ min $= 11.54am$	M1 A1 A1[3]
2(i)	$f(-1) = 4(-1)^3 + 3(-1)^2 - 16(-1) - 12$ $= 3$	M1 A1[2]
(ii)	$f(-2) = 4(-2)^3 + 3(-2)^2 - 16(-2) - 12$ $= 0$	M1 A1[2]
(iii)	Long division to get $f(x) = (x+2)(4x^2 - 5x - 6)$ $f(x) = (x+2)(x-2)(4x+3)$ $x = -2, 2, -\frac{3}{4}$	M1 M1 A1[3]
3(i)	$length = \frac{8+2\sqrt{3}}{4+2\sqrt{3}} \times \frac{4-2\sqrt{3}}{4-2\sqrt{3}}$ $= \frac{20-8\sqrt{3}}{4}$ $= 5 - 2\sqrt{3}$	M2 A1 A1[4]
(ii)	$43 + 30\sqrt{2} = (3\sqrt{2} + c)^2$ $43 = 18 + c^2$ $c = 5$ or $-5(reject)$ OR $30 = 3c + 3c$ $c = 5$ $c = 5$ OR $43 + 30\sqrt{2} = 9(2) + 6\sqrt{2}c + c^2$ $0 = c^2 + 6\sqrt{2}c - 25 - 30\sqrt{2}$ $c = 5$ or $-13.5(reject)$	M1 M1 A1[3] M1M1 A1[3] M1 M1 A1[3]
4(i)	$\alpha + \beta = 2$ $\alpha\beta = -\frac{4}{3}$ $\alpha^2 + \beta^2 = \frac{20}{3}$	B1 B1 B1[3]

(ii)	sum of roots $= (2)(\frac{20}{3} - (-\frac{4}{3}))$ $= 16$ pdt of roots $= (-\frac{4}{3})^3$ $= -\frac{64}{27}$ $x^2 - 16x - \frac{64}{27} = 0$ OR $27x^2 - 432x - 64 = 0$	M1 M1 M1 M1 A1[5]
5(i)	$\angle CAF = \angle BAF(common)$ $\angle AFB = \angle ACF$(alt segment thm) $\Delta ABF$is similar toΔAFC	M1 M1 A1[3]
(ii)	$\frac{CF}{FB} = \frac{AC}{AF}$ $CF \times AF = AC \times FB(shown)$	M1 A1[2]
(iii)	$BE \times EC = ED \times EF$ $\frac{2}{5}BC \times \frac{3}{5}BC = ED \times EF$ $\frac{6}{25}BC^2 = ED \times EF$	M1 M1 A1[3]
6(i)	$\frac{d}{dx}\left(\frac{4x}{\sqrt{2x-3}}\right) = \frac{(2x-3)^{\frac{1}{2}}(4) - (4x)\left(\frac{1}{2}\right)(2x-3)^{-\frac{1}{2}}(2)}{2x-3}$ $= \frac{(2x-3)^{-\frac{1}{2}}((2x-3)(4) - 4x)}{2x-3}$ $= \frac{4x-12}{\sqrt{(2x-3)^3}}$ OR $\frac{d}{dx}\left(\frac{4x}{\sqrt{2x-3}}\right) = \frac{d}{dx}(4x)(2x-3)^{-\frac{1}{2}}$ $= 4(2x-3)^{-\frac{1}{2}} + (4x)\left(-\frac{1}{2}\right)(2x-3)^{-\frac{3}{2}}(2)$ $= (2x-3)^{-\frac{3}{2}}(4(2x-3) - 4x)$ $= \frac{4x-12}{\sqrt{(2x-3)^3}}$	M1 M1 A1[3] M1

		M1 A1[3
(ii)(a)	$x=\sqrt{9x-8}$ $x=8,1$	M1 A1[2]
(b)	$area=\int_1^8 \sqrt{9x-8}-x\,dx$ $=\left[\frac{2}{27}(9x-8)^{\frac{3}{2}}-\frac{1}{2}x^2\right]_1^8$ $=6\frac{19}{54}units^2$	M2 M1 A1[4]
7(i)	$0=6x^2-18x$ $p=3$ $q=-35$	M2 A1 A1[4]
(ii)(a)	$\frac{dy}{dx}=-12$, decreasing	B1[1]
(ii)(b)	$\frac{dy}{dx}=24$, increasing	B1[1]
(iii)	Minimum point	B1[1]
(iv)	$\frac{d^2y}{dx^2}=12(3)-18$ $=18$	M1 A1[2]
8(i)	$4u-\frac{9}{u}=0$ $u=\pm1.5$ $x=8,\frac{1}{8}$	M1 M1 A2[4]
(ii)(a)	$x^2y=2^{2a+3b}$ $\frac{x}{y}=2^{a-3b}$	B1 B1[2]
(b)	$2a+3b=5$ $a-3b=-1$ $a=\frac{4}{3}$ $b=\frac{7}{9}$	M1 A1 A1 A1[4]
9(i)	$8(1-2\sin^2 A)-\sin A+7=0$ $\sin A=\frac{15}{16},-1$ $A=69.6°,110.4°,270°$	M1 M1 A1[3]
(ii)	P2: axes	

	P2: period P2:shape	
(iii)	$A=2x$ $x=34.8°,55.2°,135°$	M1 A1[2]
10(i)	$3.6=4e^{-t}+c$ $c=-0.4$ $0=4e^{-t}-0.4$ $-\ln 0.1=t$ $\ln 10=t(shown)$	M2 M1 M1 M1 A1[6]
(ii)	$0=-4e^{-t}-0.4t+c$ distance $=-4e^{-\ln 10}-0.4(\ln 10)+4$ $=2.68m$	M2 M1 A1[4]
(iii)	$s=-4e^{-9}-0.4(9)+4$ $=0.3995$ $s=-4e^{-10}-0.4(10)+4$ $=-0.000182$ Since displacement became negative, particle pass O at some instant during the tenth second.	M1 M1 A1[3]
11(i)	$(x-2)^2-4+(y-1)^2-1=20$ $centre(2,1)$ $radius=5$ units	M1 A1 A1[3]
(ii)	gradient $DE=\frac{4}{3}$ $y=\frac{4}{3}x-\frac{5}{3}$ $x^2+\left(\frac{4}{3}x-\frac{5}{3}\right)^2-4x-2\left(\frac{4}{3}x-\frac{5}{3}\right)=20$ $25x^2-100x-125=0$ (5,5) and (−1,−3)	M1 M1 M1 M1 A2[6]
(iii)	length $DE=\sqrt{(5+1)^2+(5+3)^2}$ $=10$ $area=2\times\frac{1}{2}\times10\times5$ $=50units^2$ *square* OR	M1 M1 A1 B1[4] M1 M1 A1

	length $DA = \sqrt{(5+2)^2+(5-4)^2}$ $= \sqrt{50}$ $area = (\sqrt{50})^2$ $= 50 units^2$ *square* OR length $DA = \sqrt{(5+2)^2+(5-4)^2}$ $= \sqrt{50}$ $area = (\sqrt{50})^2$ $= 50 units^2$ *square*	B1[4]
	Total	**100**

1 The area of a triangle is $\left(1+\frac{5\sqrt{5}}{2}\right)$cm². If the length of the base of the triangle is $\left(3+2\sqrt{5}\right)$cm, find, without using a calculator, the height of the triangle in the form of $\left(a+b\sqrt{5}\right)$cm, where a and b are integers. [4]

2 Express $\frac{4x^2+6x+5}{2x^2+x-3}$ in partial fractions. [5]

3 The function $f(x)$ is such that $f(x)=2x^3+3x^2-x-4$,

(i) find a factor of $f(x)$. [2]

(ii) Hence, determine the number of solutions in the equation $f(x)=0$. [4]

4 The roots of the quadratic equation $3x^2-x+5=0$ are α and β.

(i) Evaluate $\alpha^2+\beta^2$. [2]

(ii) Find the quadratic equation whose roots are α^3-1 and β^3-1. [4]

5 The table shows experimental values of 2 variables, R and V, which are connected by an equation of the form $RV^n=k$ where n and k are constants.

R	33	19.95	5.07	2.38
V	2	2.9	8	14

(i) Plot lg R against lg V for the given data and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of k and of n. [3]

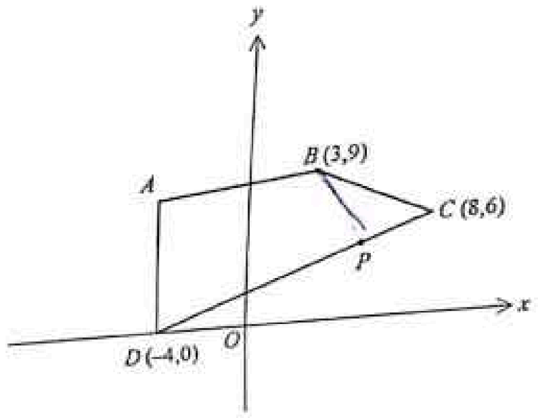
(iii) By drawing a suitable straight line on your graph in (i), find the value of V such that $\frac{R}{V^2}=1$. [3]

6 Given that $y=1-\frac{1}{2}\sin 3x$, $0^\circ \le x \le 240^\circ$.

(i) State the maximum and minimum values of y. [2]

(ii) Sketch the graph of $y=1-\frac{1}{2}\sin 3x$. [3]

7



A quadrilateral $ABCD$ passes through vertices B (3, 9), C (8, 6) and D (– 4, 0), line AD is parallel to the y – axis.

(i) Find the coordinates of A given that the length of AD is 8 units. [1]

(ii) A point P divides the line DC in the ratio of 2 : 1. Find the coordinates of P. [3]

(iii) Hence, find the area of the quadrilateral $ABPD$. [3]

8 (a) Sketch the graph $y^2=3x$. [2]

(b) Given that $f(x)=-2x^3+5x^2+4x+a$,

(i) find the coordinates of the turning points in terms of a. [4]

(ii) Determine the nature of each turning point. [3]

(iii) In the case where $a=1$, explain why the part of the graph between the turning points lie above the x – axis. [1]

9 (i) Show that $\sec x+\tan x$ can be expressed as $\frac{1+\sin x}{\cos x}$. [1]

(ii) Differentiate $\ln(\sec x+\tan x)$ with respect to x. [3]

(iii) Hence, find $\int_{0.25}^{0.5} 2\sec x \, dx$. [3]

10 The points A and B lie on the circumference of a circle C_1 where A is the point (0, 8) and B is the point (4, 0). The line $y = 2x$ also passes through the centre of the circle C_1.

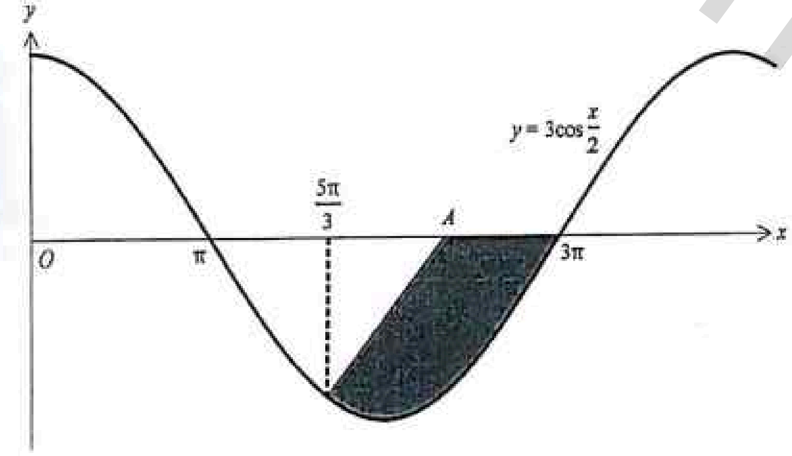
(i) Find the centre and radius of the circle C_1. [4]

(ii) Find the equation of the circle C_1 in the form $x^2 + y^2 + px + qy + r = 0$, where p, q and r are integers. [2]

Another circle C_2 of radius $\sqrt{2}$ units has its centre inside C_1 and it cuts the circle C_1 at the origin and at the point where $x = 2$.

(iii) Find the centre of C_2. [5]

11



The diagram shows part of the curve $y = 3\cos\frac{x}{2}$ that cuts the x-axis at $x = \pi$ and $x = 3\pi$. The normal to the curve at $x = \frac{5\pi}{3}$ cuts the x-axis at A.

(i) Find the coordinates of A, leaving your answer in exact form. [6]

(ii) Hence, find the area of the shaded region. [4]

---

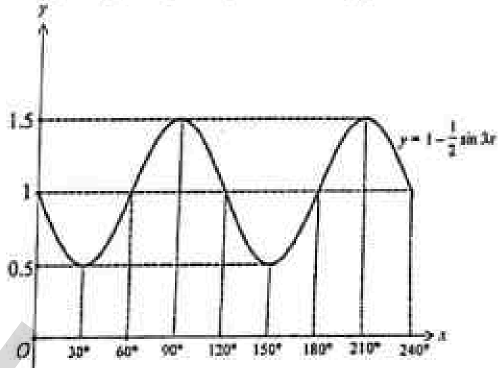
1. $4-\sqrt{5}$

2. $2-\frac{2}{2x+3}+\frac{3}{x-1}$

3. (ii) one solution

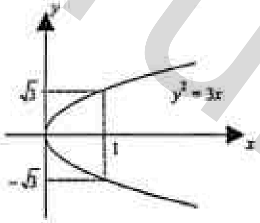
4. (i) $\frac{-29}{9}$ (ii) $27x^2+98x+196=0$

6. (i) Max y = 1.5; Min y = 0.5 (ii)



7. (i) (−4,8) (ii) $P(4,4)$ (iii) 50 units^2

8. (a) (b)(i). $\left(-\frac{1}{3}, a-\frac{19}{27}\right)$ and $(2, 12+a)$ (b)(ii). $\left(-\frac{1}{3}, a-\frac{19}{27}\right)$ min ; $(2, 12+a)$ max



9. (ii) $\sec x$ (iii). 0.539

10. (i) Centre (2, 4), Radius $= 2\sqrt{5}$ (ii) $x^2+y^2-4x-8y=0$ (iii) Centre of $C_2(1.22, 0.710)$

11. (i) $A\left(\frac{5\pi}{3}+\frac{9}{8}\sqrt{3}, 0\right)$ (ii) $6\frac{15}{32}$ / 6.47 units^2

1 (a) The equation of a curve is $y=2x^2+ax+(6+a)$, where a is a constant. Find the range of values of a for which the curve lies completely above the x-axis. [3]

(b) The equation of a curve is $y=3x^2+4x+6$.

(i) Find the set of values of x for which the curve is above the line $y=6$. [3]

(ii) Show that the line $y=-8x-6$ is a tangent to the curve. [2]

2 (a) Given that $\log_a 125-3\log_a b+\log_a c=3$, express a in terms of b and c. [3]

(b) Solve the equation

(i) $\lg 8x-\lg(x^2-3)=2\lg 2$, [3]

(ii) $2\log_3 x=3+7\log_x 5$. [4]

3 The equation of a curve is $y=x^2\sqrt{(5x-1)^3}$, for $x>0.2$. Given that x is changing at a constant rate of 0.25 units per second, find the rate of change of y when $x=2$. [4]

4 The graph of $y=|2x^2-ax-5|$ passes through the points with coordinates $(-1,\ 0)$ and $(0.75,\ b)$.

(i) Find the value of the constants a and b. [3]

(ii) Sketch the graph of $y=|2x^2-ax-5|$. [3]

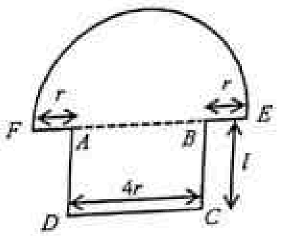
(iii) Determine the set of positive values of m for which the line $y=mx+2$ intersects the graph of $y=|2x^2-ax-5|$ at two points. [2]

5 In the binomial expansion of $\left(2x+\frac{k}{x}\right)^8$, where k is a positive constant, the coefficient of x^2 is 28.

(i) Show that $k=\frac{1}{4}$. [4]

(ii) Hence, determine the term in x in the expansion of $\left(6x-\frac{1}{x}\right)\left(2x+\frac{k}{x}\right)^8$. [4]

6



The diagram shows a design of a bookmark that includes a rectangle $ABCD$, where $BC=l$ cm, $CD=4r$ cm, a semicircle with radius $3r$ cm, and $AF=BE=r$ cm. The area of the bookmark is 90 cm^2.

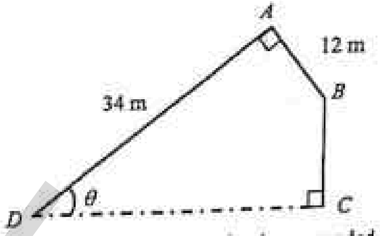
(i) Express l in terms of r. [2]

(ii) Given that the perimeter of the bookmark is P cm, show that

$$P=\left(6+\frac{3\pi}{4}\right)r+\frac{45}{r}.$$ [2]

(iii) Given that r and l can vary, find the value of r for which P has a stationary value. Explain why this value of r gives the minimum perimeter. [5]

7



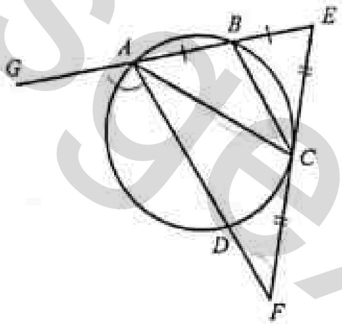
The diagram shows an animal exhibition area that is surrounded by glass panels at AB, BC and AD, where $AB=12$ m, $AD=34$ m, angle DAB = angle $BCD=90°$ and the acute angle $ADC=\theta$ can vary.

(i) Show that L m, the length of the glass panels can be expressed as $L=46+34\sin\theta-12\cos\theta$. [2]

(ii) Express L in the form $p+R\sin(\theta-\alpha)$, where p and $R>0$ are constants and α is an acute angle. [4]

(iii) Given that the exact length of the glass panels is 62 m, find the value of θ. [3]

8



The diagram shows points A, B, C and D on a circle, line EF is tangent to the circle at C, lines ADF and $EBAG$ are straight lines, and points B and C are the midpoints of AE and EF.

Prove that

(i) $BC \times EC = AC \times BE$, [3]

(ii) $AF \times EC = AC \times AE$, [2]

(iii) angle GAD = angle ACF. [2]

9 (a) (i) Show that $\cot 2x = \frac{1-\tan^2 x}{2\tan x}$. [2]

(ii) Hence, solve the equation $8\cot 2x \tan x = 1$, for $0° < x < 360°$. [4]

(b) The Ultraviolet Index (UVI) describes the level of solar radiation. The UVI measured from the top of a building is given by $U = 6 - 5\cos qt$, where t is the time in hours from the lowest value of the UVI, $0 \le t \le 10$, and q is a constant. It takes 10 hours for the UVI to reach its lowest value again.

(i) Explain why we are not able to measure a UVI of 12. [1]

(ii) Show that $q = \frac{\pi}{5}$. [1]

(iii) The top of the building is equipped with solar panels that supply power to the building when the UVI is at least 3. Find the duration, in hours and minutes, that the building is supplied with power from the solar panels. [4]

10 (a) It is given that $y = \frac{2x^2}{4x-3}$, where $x > \frac{3}{4}$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) Find the range of values of x for which $y = \frac{2x^2}{4x-3}$ is a decreasing function. [4]

(b) It is given that f(x) is such that $f'(x) = \frac{1}{2x-5} - \frac{4}{(2x-5)^2}$.

Given also that $f(3) = 1.75$, show that $8f(x) - (2x-5)^2 f''(x) = \ln(2x-5)^4$. [7]

11 A particle moves in a straight line, so that, t seconds after passing a fixed point O, its velocity, v m/s, is given by $v = 2e^{0.2t} - 10e^{0.1-0.2t}$. The particle comes to an instantaneous rest at the point A.

(i) Show that the particle reaches A when $t = \frac{5}{2}\ln 5 + \frac{1}{4}$. [3]

(ii) Find the acceleration of the particle at A. [3]

(iii) Find the distance OA. [4]

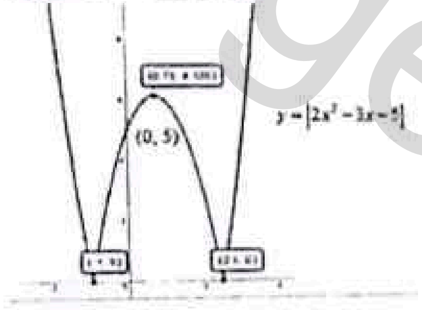
(iv) Explain whether the particle is again at O at some instant during the eleventh second after first passing through O. [2]

1. (a) $-4 < a < 12$ (b)(i) $x < -1\frac{1}{3}$ or $x > 0$

2. (a) $a = \frac{5\sqrt[3]{c}}{b}$ (b)(i) $x = 3$ (ii) $x = 85.7$ or $x = 0.130$

3. 49.5 units / s

4. (i) $a = 3$, $b = 6.125$ (ii) (iii) $m > 2$



5. (ii) $-1\frac{3}{4}x$

6. (i) $l = \frac{45}{2r} - \frac{9}{8}\pi r$ (iii) $r = 2.32$; min value

7. (ii) $L = 46 + 10\sqrt{13}\sin(\theta - 19.4°)$ (iii) 45.8°

9. (a)(ii) $x = 40.9°, 139.1°, 220.9°, 319.1°$ (b)(iii) 7 hrs and 3 mins

10. (a)(i) $\frac{4x(2x-3)}{(4x-3)^2}$ (ii) $\frac{3}{4} < x < \frac{3}{2}$

11. (ii) 1.23 m/s² (iii) 16.0 m (iv) passed through O

1 Prove the identity $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$. [3]

2 The points A, B, and C have coordinates (0, 5), (8, 7) and (4, 1) respectively.

(i) Find the equation of the perpendicular bisector of AB. [3]

(ii) Calculate the area of triangle ABC. [2]

3 The tangent to the curve $y = (x-2)\sqrt{3x+1}$ at $x = 1$, meets the y-axis at A. Find the coordinates of A. [6]

4 (i) Write down the first three terms in the expansion, in descending powers of x, of $\left(2x - \dfrac{1}{x}\right)^7$. [3]

(ii) Find the value of a if the coefficient of x^5 in the expansion of $\left(1 + ax^2\right)\left(2x - \dfrac{1}{x}\right)^7$ is 224. [3]

5 A closed cylindrical can contains 300 cm^3 of liquid when full. The cylinder of radius r cm and height h cm has a total surface area of A cm^2.

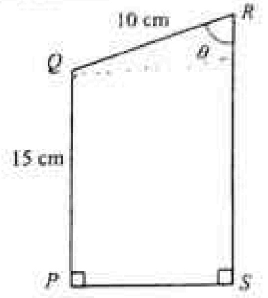
(i) Show that $A = 2\pi r^2 + \dfrac{600}{r}$. [2]

(ii) Given that r can vary, find the stationary value of A and determine if this value of A is a maximum or a minimum. [5]

6 Show that the line $y = 3 - k$ will always intersect the curve $y = x^2 + (1-2k)x$ at two distinct points for all real values of k. [4]

[Turn Over

7 The diagram shows a wooden frame $PQRS$ where QP and RS are perpendicular to PS, PQ = 15 cm and QR = 10 cm. Angle QRS is θ where $0° < \theta < 90°$. The perimeter of the wooden frame is L cm.



(i) Show that $L = 10\cos\theta + 10\sin\theta + 40$. [2]

(ii) Using part (i), express L in the form of $R\cos(\theta - \alpha) + c$ where $R > 0$, α is an acute angle and c is a constant. [2]

(iii) Hence, find the value of θ when L = 53 cm. [2]

8 Given that $\dfrac{3x^3 + 17x^2 + 23x - 12}{x^2 + 6x + 9} = px + q + \dfrac{2x + r}{(x+3)^2}$,

(i) find the value of each of the integers p, q, and r. [4]

(ii) Hence, using partial fractions and the values of p, q, and r found in part (i), find $\displaystyle\int \frac{3x^3 + 17x^2 + 23x - 12}{x^2 + 6x + 9}\,dx$.
[6]

9 A graph has the equation $y = |3x + a| + b$ where a and b are positive constants.

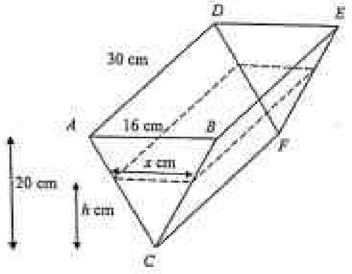
(a) Find, in terms of a and/or b, the coordinates of the minimum point of the graph. [2]

(b) The equation $|3x + a| + b = mx + 1$ has infinite solutions. Write down

(i) the possible values of m, [2]

(ii) the value of $a + b$. [1]

10 An empty trough has the shape of the prism as shown in the diagram. The vertical ends ABC and DEF are identical isosceles triangles of height 20 cm with $AB = 16$ cm and $AC = BC$. The open top $ABED$ is horizontal and rectangular in shape with $AD = 30$ cm. Water is poured into the trough at the rate of 60 cm^{3}s^{-1}. After t seconds, the depth of water is h cm and the breadth of the horizontal surface is x cm.



(i) Show that the volume of water in the trough, V cm^3, at time t is given by $V = 12h^2$. [2]

(ii) Find the rate of change of the depth of water when $h = 4$. [3]

(iii) State, with a reason, whether this rate will increase or decrease as t increases. [2]

11 A prism of volume V cm^3 has height of x cm and a base area of $(mx^2 + n)$. Corresponding values of x and V are shown in the table below.

x	2	4	6	8
V	34	212	678	1576

(i) Using suitable variables, draw, on graph paper, a straight line graph and hence, estimate the value of each of the constants m and n. [6]

(ii) Explain how another straight line drawn on your diagram can lead to an estimate of the height for which the base area is 120 cm^2. Draw this straight line and hence find the height. [3]

12 The height of tides in Singapore for June 2016 is modelled by the equation $h = 1.85 + 1.15\sin\left[\frac{6\pi}{37}(t + 2)\right]$ where t is the time in hours after midnight and h is the height in metres.

(i) Find the height of the tide at midnight. [2]

(ii) What time does the highest tide first occur? [3]

(iii) Show that the time difference between two consecutive lowest tides is $12\frac{1}{3}$ hours. [1]

(iv) The height of the tide was first observed to be 2.9 m at t_1 hours after midnight. It was at least 2.9 m for the next T hours. Find the value of t_1 and hence, calculate the value of T. [6]

End of Paper

Nanyang Girls' High School
Answer Key

2(i)	Equation of perpendicular bisector: $y = -4x + 22$
2(ii)	20 sq units
3	Coordinates of A is $\left(0, -3\frac{1}{4}\right)$
4(i)	$128x^7 - 448x^5 + 672x^3$
4(ii)	$a = 1$
5(ii)	$r = \sqrt[3]{\frac{150}{\pi}} = 3.62783$ Stationary value of A = 248.082 A is a minimum value
7(ii)	$L = 40 + 10\sqrt{2}\cos(\theta - 45°)$
7(iii)	$\theta = 68.2°$
8(i)	$p = 3$ $q = -1$ and $r = -3$
8(ii)	$\frac{3}{2}x^2 - x + 2\ln\lvert x+3\rvert + \frac{9}{x+3} + c$
9(a)	$\left(-\frac{a}{3}, b\right)$
9(b)	(i) 3 or -3 (ii) 1
10(ii)	Rate of change of depth is 0.625 cm/s
10(iii)	As t increases, h increases and therefore $\frac{5}{2h}$ decreases, thus this rate will decrease. Alternatively, as t increases, cross sectional area increases and therefore $\frac{dh}{dt}$ decreases.
11(i)	From the graph, $m = 3$ and $n = 5$
11(ii)	So height is about 6.20 cm
12(i)	Height = 2.83 m
12(ii)	Highest tide first occurs at 1:05 am or 0105 h.
12(iv)	T = 1.908 – 0.2593 = 1.65

Answer all the questions

1 There is a spread of a contagious virus in a high school and the school is closed down. The number of infected students, P, is given by the equation $P = 10 + 200e^{-kt}$, where t is the number of days after the virus is identified and k is a constant.

(i) State the initial number of infected students. [1]

The number of infected students is reduced to half its initial number 5 days after the virus was identified.

(ii) Find the value of k. [3]

The school will only be opened again when the number of infected students is less than 20.

(iii) Determine whether the school will be opened after 20 days. [2]

2 The quadratic equation $x^2 - 4x + 6 = 0$ has roots α and β.

(i) Find the value of $\alpha^2 + \beta^2$. [2]

(ii) Find the quadratic equation whose roots are $\frac{1}{\alpha^2 - 3}$ and $\frac{1}{\beta^2 - 3}$. [4]

(iii) Show that $\alpha^3 = 10\alpha - 24$. [2]

3 (a) (i) Sketch the graph of $y = \frac{1}{2}x^{\frac{1}{2}}$ for $x > 0$. [1]

(ii) On the same diagram, sketch the graph of $y = 8x^{-\frac{3}{2}}$ for $x > 0$. [1]

(iii) Calculate the coordinates of the point of intersection of your graphs. [2]

(b) Given that $a = \log_2 m$ and $b = \log_n 2$, express $\log_2 \frac{4\sqrt{m}}{n}$ in terms of a and b. [4]

4 The diagonal AC of a quadrilateral $ABCD$ is $(4\sqrt{15} - 2\sqrt{6})$ cm.

(i) In the case where the quadrilateral is a rhombus with side $(4\sqrt{5} - 2\sqrt{2})$ cm and AC is the longer diagonal, find, without using a calculator, the exact value of $\sin \angle ABD$. [5]

(ii) In the case where the quadrilateral is a square with area $(a - b\sqrt{10})$ cm², find the value of a and of b. [3]

[Turn Over

5 The function f is defined, for $0 \leq x \leq 720°$, by $f(x) = 4\sin\frac{x}{2} - 2$.

(i) State the amplitude and period of f. [2]

(ii) Find the values of x when $f(x) = 0$. [2]

(iii) Sketch the graph of $y = 4\sin\frac{x}{2} - 2$ for $0 \leq x \leq 720°$, stating clearly the intercepts with the axes. [3]

(iv) State the range of values of k for the equation $\left|4\sin\frac{x}{2} - 2\right| = k$ to have exactly 2 solutions. [2]

6 The function $f(x) = 6x^3 + 11x^2 - 3x - k$, where k is a constant, leaves a remainder of 6 when divided by $x + 1$.

(i) Find the value of k. [2]

(ii) Factorise $f(x)$ completely. [3]

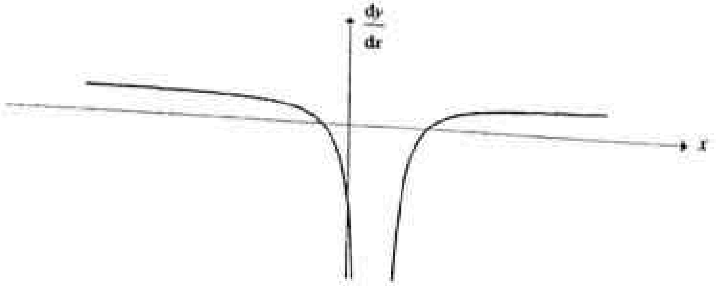
(iii) State the remainder when $f(x) - 8$ is divided by $3x + 1$. [1]

(iv) Using the value of k found in (i), solve the equation $\frac{6}{u^3} + \frac{11}{u^2} = \frac{3}{u} + k$. [2]

7 The equation of a curve is $y = \frac{x^2 - 7x + 10}{x - 1}$.

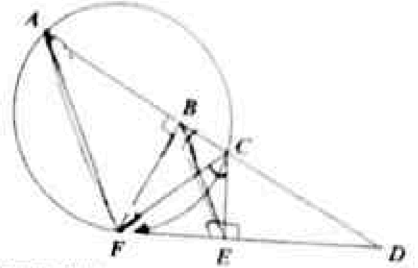
(i) Find the coordinates of the stationary points. [7]

The graph of $\frac{dy}{dx}$ against x is as shown below.



(ii) From the graph, deduce the nature of the stationary points. [2]

8



In the diagram, AD is a straight line intersecting a circle at A and C and DF is a tangent to the circle. $ABCD$ and DEF are straight lines and angle ABF = angle $CED = 90°$.

(i) Explain why $BCEF$ is a cyclic quadrilateral. [2]

(ii) Prove that BE is parallel to AF. [3]

(iii) Show that $DE \times DF = DC \times DB$. [3]

9 The equation of a circle, C_1, with centre P is $x^2 + y^2 - 6x - 4y + 11 = 0$.

(i) Find the coordinates of P and the radius of C_1. [3]

(ii) Find the equation of the tangent to C_1 at the point $Q\ (2, 3)$. [3]

The tangent meets the x-axis at point R.

(iii) State the coordinates of R. [1]

A second circle, C_2, with centre S, passes through P, Q and R.

(iv) State the position of S and hence find the equation of C_2. [4]

(v) Determine, with clear working, whether S lies inside C_1. [2]

10 A particle, moving in a straight line, passes through a point A with a speed of 15 m/s. The acceleration, a m/s^2, of the particle, t s after passing through A, is given by $a = -2e^{-0.1t}$. When $t = 0$, $s = 5$, where s metres is the displacement from a fixed point O. The particle comes to instantaneous rest at the point B.

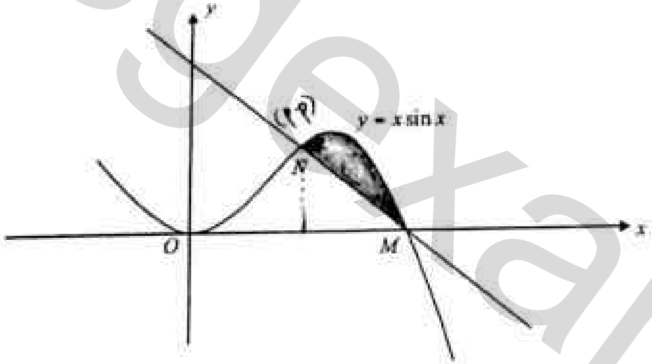
(i) Show that the value of $t = 10\ln 4$ when the particle reaches B. [6]

(ii) Calculate the distance AB. [4]

(iii) Determine if the particle passes through A again at 40 s. [2]

11 **(a)** Differentiate $\sin x - x\cos x$ with respect to x. [2]

(b) The diagram shows part of the curve $y = x\sin x$. M and N are the points of intersection between the curve and a line. M lies on the x-axis and N is (p, p), where p is a constant.



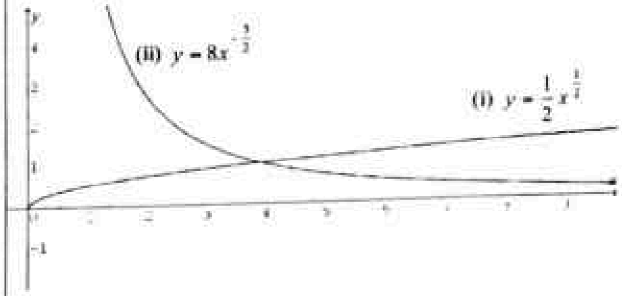
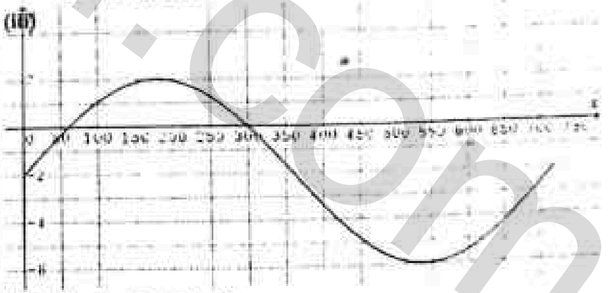
(i) Find the coordinates of M. [2]

(ii) Given the gradient of MN is -1, find the value of p. [2]

(iii) Hence, calculate the area of the shaded region bounded by the curve and the line MN. [5]

End of Paper

ANSWER KEY

Qn	Answers
1	(i) 210 (ii) 0.149 (3 s.f.) (iii) Since the no. of infected students is >20 after 20 days, the school will not be opened yet.
2	(i) $\alpha+\beta=4,\ \alpha\beta=6$ $\alpha^2+\beta^2=4$ (ii) $33x^2+2x+1=0$
3	(a) (i) & (ii)  (iii) (4, 1) (iii) $2+\frac{1}{2}a-\frac{1}{b}$
4	(a) Let M be the point of intersection of the 2 diagonals $\sin\angle ABD = \frac{AM}{AB} = \frac{\sqrt{3}}{2}$ (b) $a = 132,\quad b = 24$
5	(i) Amplitude $= 4$, period $= 720°$ (ii) $x = 60°$ or $300°$ (iii)  (iv) $2 < k < 6$ or $k = 0$

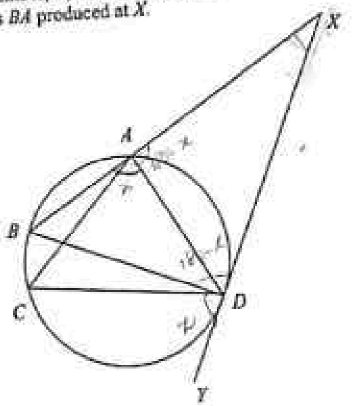
6	(i)	$k = 2$
	(ii)	$f(x) = (x+2)(3x+1)(2x-1)$
	(iii)	remainder = -8
	(iv)	$u = -3, -\frac{1}{2}$ or 2
7	(i)	The stationary points are $(3, -1)$ and $(-1, -9)$
	(ii)	$(3, -1)$ is a min point and $(-1, -9)$ is a max point
9	(i)	centre is $P(3, 2)$ and radius is $\sqrt{2}$ units
	(ii)	eqn of tangent is $y = x + 1$
	(iii)	$R(-1, 0)$
	(iv)	Equation of second circle is $(x-1)^2 + (y-1)^2 = 5$
	(v)	$PS = \sqrt{5}$ units Since $PS >$ radius of C_1, S lies outside of C_1
10	(ii)	$AB = 85.685 - 5 = 80.7$ m (3 s.f.)
	(iii)	particle has passes through A again
11	(a)	$\frac{d}{dx}(\sin x - x\cos x) = x\sin x$
	(b)	(i) M is $(\pi, 0)$ (ii) $p = \frac{\pi}{2}$ (iii) Area $= \pi - 1 - \frac{\pi^2}{8}$ or 0.908 sq units (3 s.f.)

1 (a) Simplify $\frac{49\times 2^{n-2}}{2^{n-3}-2^{n-1}\times 1^{n}}$. [3]

(b) Given that $p=\frac{1}{\sqrt{5}}$ and that $q=\frac{1+p}{1-p}$, express q in the form $\frac{a+\sqrt{5}}{b}$, where a and b are integers. [3]

2 Express $\frac{x^3+3x^2-x-8}{(x+3)(x^2-4)}$ in partial fractions. [6]

3 In the diagram, points A, B, C and D lie on the circumference of the circle such that the tangent at D meets BA produced at X.



Given that $AC = AD$, prove that

(i) angle $CDY = 180° - 2\times$angle ADX, [3]

(ii) $DX^2 = AX\cdot XB$. [3]

4 The function f is defined by $f(x)=3\sin 2x+a$ for $0\le x\le 2\pi$. Given that the maximum value of f is 1,

(a) write down the amplitude, the period of f and the value of a. [3]

(b) Sketch the graph of $y=-f(x)$ for $0\le x\le 2\pi$. [3]

5 (a) Solve $4\operatorname{cosec}^2 x = 7-\cot^2 x+2\cot x$, for $0°\le x\le 360°$. [4]

(b) Find, in radians, the obtuse angle for which $\sin^4 x-\cos^4 x+\cos x=0$. [4]

6 (a) Show that the quadratic equation $2px-x^2-(p^2+1)=0$ is always negative, for all real values of x. [2]

(b) Given that the roots of the equation $2x^2+x-4=0$ are α and β, form the quadratic equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [6]

7 (i) Given that $y=(1+2x)\sqrt{4-3x}$, show that $\frac{dy}{dx}$ can be written in the form $\frac{a+bx}{2\sqrt{4-3x}}$, where a and b are constants. [3]

(ii) Hence, find $\int_{-4}^{0}\frac{17-18x}{2\sqrt{4-3x}}\,dx$. [5]

8 (a) Given that $p=\log_3 x$ and $q=\log_9 y$, find, in terms of p and q,

(i) $\log_3 xy$, [2]

(ii) $\log_y x$. [3]

(b) Solve the equation $\log_3(28-5x)=\log_{\sqrt{3}}(x-2)+1$. [5]

9 (a) The equation of a curve is $y=x\ln(2x+1)$, $x>0$. Show that the curve has no stationary point. [3]

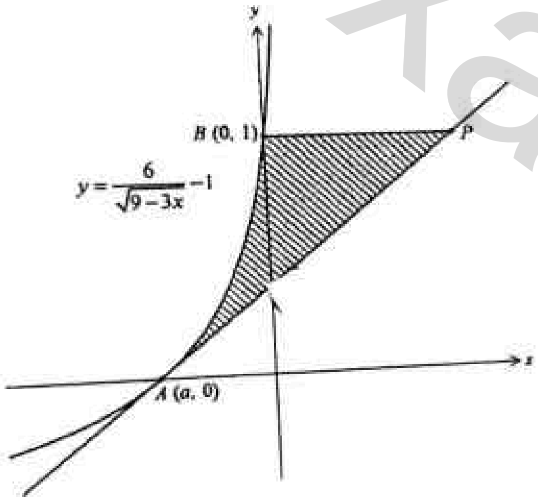
(b) The equation of a curve is $y=3\sin\frac{1}{2}x-4\cos\frac{1}{2}x$, for $0\le x\le 2\pi$. Find the value of x for which the curve has a stationary point and determine the nature of this stationary point. [7]

The diagram shows part of the curve $y = \frac{6}{\sqrt{9-3x}} - 1$.

The curve meets the x-axis at A $(a, 0)$ and the y-axis at B $(0, 1)$. AP is a tangent to the curve at A and PB is parallel to the x-axis.

(i) The normal at A has a gradient of -24. Find the value of a. Hence find the equation of the tangent AP. [3]

(ii) Find the area of the shaded region. [7]



11 A particle moves in a straight line so that, t seconds after passing through a fixed point O, its velocity v ms^{-1}, is given by $v = 2e^{2t} - 15e^{-t}$. Find

(i) the initial velocity of the particle, [1]

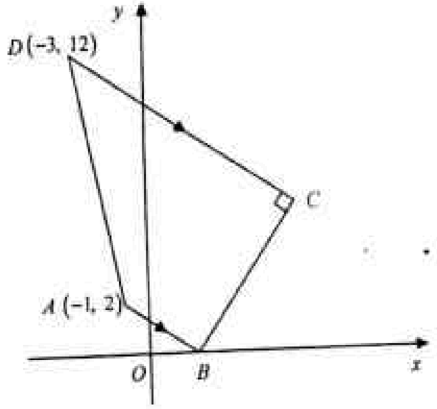
(ii) the value of t when the particle is instantaneously at rest, [3]

(iii) an expression for the displacement in terms of t, [3]

(iv) the distance travelled during the first 2 seconds. [3]

12 **Solutions to this question by accurate drawing is not accepted.**

The diagram shows a trapezium $ABCD$ with AB parallel to DC and BC is perpendicular to CD. The coordinates of A and D are $(-1, 2)$ and $(-3, 12)$ respectively. The point B lies on the x-axis and the equation of CD is $3y + 2x = 30$.



Find

(i) the equation of AB, [2]

(ii) the equation of BC, [3]

(iii) the coordinates of C, [2]

(iv) the area of triangle BCD, [2]

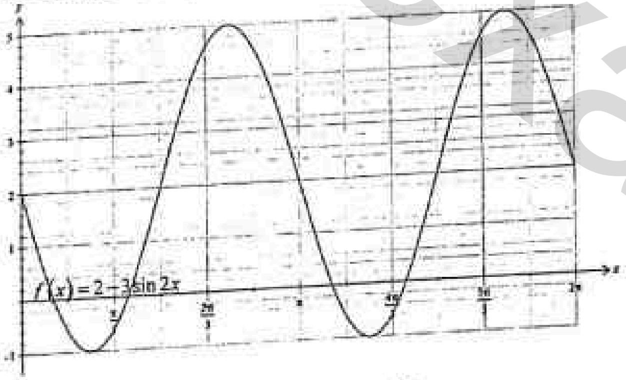
(v) the perpendicular distance from C to the line BD. [3]

1 (a) [illegible] $49 \times 2^{n-1}$

1 (a) −49 (b) $q = \frac{3+\sqrt{5}}{2}$

2 $1 - \frac{1}{(x+3)} + \frac{1}{2(x+2)} + \frac{1}{2(x-2)}$

4 $a = -2$; Period = π; Amplitude = 3 $\therefore f(x) = 3\sin 2x - 2$



5 (a) $x = 45°, 121.0°, 225°$ or $301.0°$ (b) $x = \frac{2\pi}{3}$

6 (b) $8x^2 + 17x + 8 = 0$

7 (i) $\frac{dy}{dx} = \frac{13-18x}{2\sqrt{4-3x}}$ (ii) $32\frac{2}{3}$

8 (a) (i) $p + 2q$ (ii) $\frac{p}{4q}$ (b) $x = 4$

9 (b) the stationary point is a maximum point.

10 (i) Equation of tangent is $y = \frac{1}{24}(x+9)$ or $y = \frac{1}{24}x + \frac{3}{8}$ (ii) 6 units^2

11 (i) -13 ms^{-1} (ii) $t = 0.672$ (iii) $s = e^{2t} + 15e^{-t} - 16$ (iv) 49.6 m

12 (i) $y = -\frac{2}{3}x + \frac{4}{3}$ (ii) $y = \frac{3}{2}x - 3$ (iii) (6,6) (iv) 39 units^2 (vi) 6 units

---

1 The function f is defined by

$$f(x) = \frac{e^{3x}}{7-2x} \text{ where } x \neq \frac{7}{2}.$$

Find the values of x for which f is a decreasing function. [4]

2 Find the range of values of k for which the line $y + kx + 16 = 0$ does not intersect the curve $y = x^2 + 3x$. [4]

3 The equation of a curve is $y = \frac{3x^2}{1+x}$.

(i) Obtain an expression of $\frac{dy}{dx}$ in terms of x. [2]

(ii) A particle moves along the curve. At point T whose x-coordinate is negative, the x-coordinate of the particle is increasing at a rate of 1.5 units/sec and the y-coordinate is increasing at 4 units/sec. Find the coordinates of T. [3]

4 (i) Calculate the term independent of x in the expansion of $\left(x - \frac{1}{25x^3}\right)^{18}$. [2]

(ii) In the binomial expansion of $(1+kx)^n$, where $n \geq 3$ and k is a constant, the coefficient of x^2 and x^3 are equal. Express k in terms of n. [3]

5 Mr. Ng bought a new car. Its expected value \$$V$ would depreciate such that after t months, it is given by $V = 80\,000e^{-kt}$, where k is a constant. The value of the car after ten months is expected to be \$70 000.

(i) Find the initial value of the car. [1]
(ii) Calculate the expected value of the car after twenty months. [3]
(iii) Calculate the age of the car, to the nearest month, when its expected value will be \$30 000. [2]

6 Show that $\frac{\sin x}{1+\sec x} - \frac{\sin x}{1-\sec x}$ can be written in the form $k\cot x$ and find the value of k.

Hence, find the value of x such that $\frac{\sin x}{1+\sec x} - \frac{\sin x}{1-\sec x} = 2$ where $3 < x < 6$. [6]

7 The mean distance R (in millions of kilometres) from the centre of the sun and the time taken T (in years) for a planet to complete one revolution around the sun are recorded in the table below.

Planet	Mercury	Venus	Mars	Jupiter	Saturn
R (in millions of km)	57.9	108.2	227.9	778.3	1427
T (in years)	0.24	0.62	1.88	11.86	29.46

It is given that the planets orbiting around the sun obey Kepler's Law, $T^2 = kR^n$, where k and n are constants.

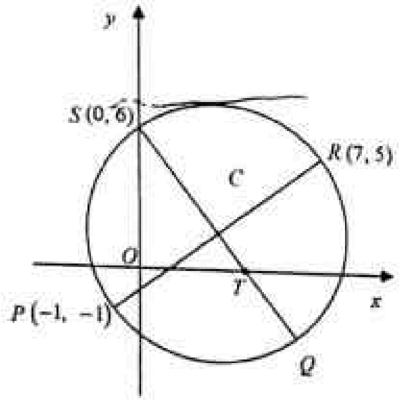
(i) Plot $2\lg T$ against $\lg R$ and draw a straight line graph.

(ii) Use your graph to estimate the value of n correct to 1 decimal place. [2]

(iii) Given that the time taken for Earth to complete one revolution around the sun is exactly 1 year, use your graph to determine the mean distance of Earth from the centre of the sun, in millions of kilometres. [2]

[2]

8 In the diagram, PR and SQ are the diameters of the circle with centre C. The coordinates of P, R and S are $(-1, -1)$, $(7, 5)$ and $(0, 6)$ respectively.



(a) Calculate the coordinates of C. [1]

(b) Show that the lines PR and SQ are perpendicular. [2]

(c) Find the equation of the circle with centre C and passing through P, Q, R and S. [2]

(d) The line $y = k$, where $k > 0$, is a tangent to the circle. State the value of k. [1]

(e) The line SQ cuts the x-axis at T. Find the ratio of $ST : TQ$. [2]

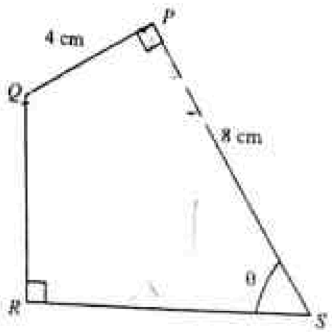
9 It is given that $x^2 + 3x + 2$ is a factor of the polynomial $2x^4 + 3x^3 + px^2 - 12x + q$.

(i) Find the value of p and of q. [3]

(ii) Factorise the polynomial completely. [2]

(iii) Hence, solve the equation $2e^{3y} + 3e^{2y} + pe^{y} + qe^{-y} - 12 = 0$, where y is real. [4]

10



The diagram shows a quadrilateral $PQRS$ in which $\angle QPS$ and $\angle QRS$ are right angles, $\angle PSR = \theta°$, $PQ = 4$ cm and $PS = 8$ cm.

(a) Show that the perimeter, S cm, of the quadrilateral is given by

$$S = 12\sin\theta + 4\cos\theta + 12.$$

[2]

(b) Given further that $0° < \theta < 90°$, express S in the form $A\sin(\theta + \alpha) + k$, where A and k are positive constants and $0° < \alpha < 90°$. Hence find the value of θ for which $S = 19$. [7]

11 A curve has the the equation $y = (3x-2)^2 - 16$.

(i) Explain why the lowest point on the curve has coordinates $\left(\frac{2}{3}, -16\right)$. [1]

(ii) Find the coordinates of the points at which the curve intersects the x-axis. [2]

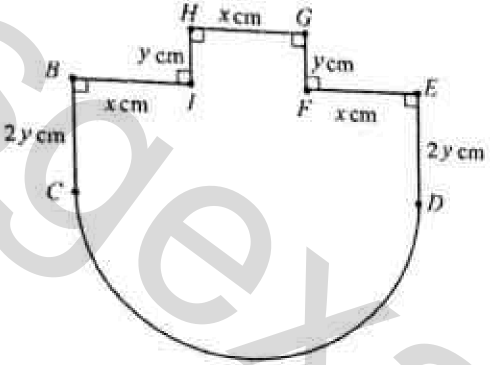
(iii) Sketch the graph of $y = \left|(3x-2)^2 - 16\right|$. [3]

(iv) Use your graph, state the number of solutions to each of the following equations.

(a) $\left|(3x-2)^2 - 16\right| = 8$ [1]

(b) $\left|(3x-2)^2 - 16\right| + 4 = 0$ [1]

12



A piece of wire, length 150 cm, is bent into the shape shown in the diagram, such that $HI = GF = y$ cm, $BI = HG = FE = x$ cm, $BC = ED = 2y$ cm and arc CD is a semi-circle.

(i) Show that the area, enclosed by the wire, A cm^2, is given by

$$A = \frac{1400x - 28x^2 - 5\pi x^2}{8}.$$ [3]

(ii) Given that x and y can vary, find the value of x and of y for which the area A, is stationary. [4]

(iii) Find the stationary value of A, giving your answer to the nearest integer. Determine whether this stationary value is a minimum or maximum. [3]

1 $x > 3\frac{5}{6}$

2 $-11 < k < 5$

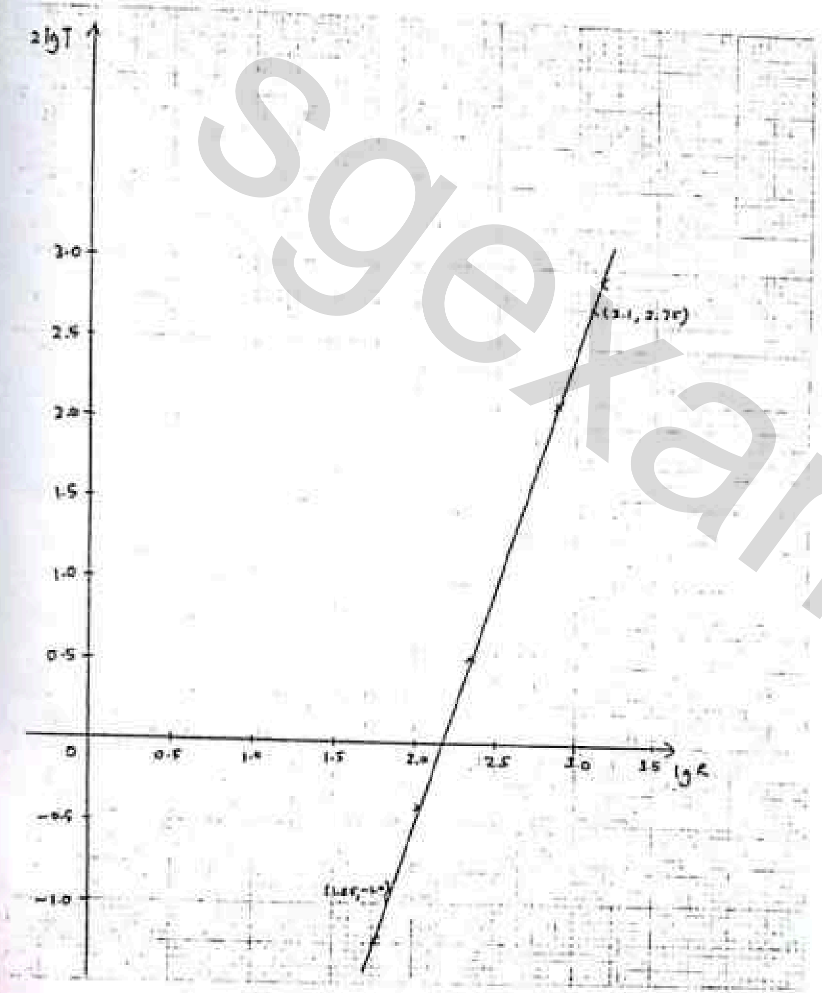
3 (i) $\frac{dy}{dx} = \frac{3x^2 + 6x}{(1+x)^2}$ (ii) $T(-4, -16)$

4 (i) $-\frac{816}{15625}$ (ii) $k = \frac{3}{n-2}$

5 (i) \$80 000 (ii) \$61250 (iii) 73 months old

6 $k = 2$, $x = \frac{5\pi}{4}$

7 (ii) $n = 3.0$ (1 d.p.) ± 0.1 (iii) Distance = 150 million km.



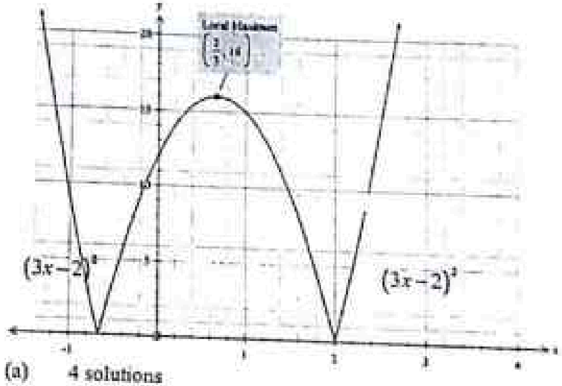
8 (a) $C=(3,2)$ (c) $(x-3)^2+(y-2)^2=25$ (d) $k=7$ (e) The ratio of $ST:TQ$ is 3:1

9 (i) $p=-7$, $q=-4$ (ii) $f(x)=(x+1)(x+2)(x-2)(2x+1)$ (iii) $y=\ln 2$ or 0.693

10 (b) $S=4\sqrt{10}\sin(\theta+18.4°)+12$; $\theta=15.2°$

11 (ii) (2, 0) and $\left(-\frac{2}{3},0\right)$

(iii)



(iv) (a) 4 solutions
(b) 0 solutions / No solutions

12 (ii) $x\approx 16.0$, $y\approx 4.41$ (iii) $A=1401$; maximum

1. (i) **Without using a calculator**, find the exact value of 12^x, given that $3^{\text{[illegible]}} - 4^{\text{[illegible]}} = 0$. [3]

(ii) Solve the equation $2e^{2x} = 13e^x - 15$. [4]

2. (i) Find the range of values of x for which $x(10 - x) > 24$. [2]

(ii) Find the range of values of c for which $x(10 - x) < c^2$. [4]

3. (i) Sketch, on the same diagram, the graphs of $y = x^{-\frac{1}{2}}$ and $y^2 = 4x$ for $x > 0$. [2]

(ii) Find the coordinates of the point of intersection of the graphs. [3]

4. (i) Find the exact value of $\sin 165°$. [3]

(ii) Hence, show that $\cot^2 165°$ can be expressed in the form $a + b\sqrt{3}$ where a and b are integers. [4]

5. Given that the term independent of x in the expansion of $(3 + 5x^2)\left(1 - \frac{1}{2x}\right)^n$ is 38, where n is a constant, find

(a) the value of n, [4]

(b) the coefficient of $\frac{1}{x}$. [3]

6. The population, P, of a certain species of frogs is given by

$$P = Ae^{-kt},$$

where A and k are constants and t is the time in years from 1 January 2000.

Over a period of 10 years from 1 January 2000 to 1 January 2010, P decreased from 90 000 to 40 000.

Find

(i) the value of A and of k, [3]

(ii) the year in which the population will be reduced by 70% as compared to the year 2000. [2]

7. A hemispherical container is shown in the diagram below.

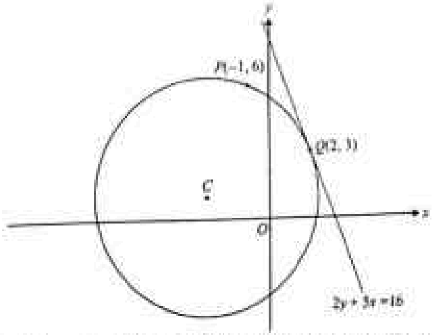
When the depth of the water is h m, the volume, V m^3, is given by

$$V = \frac{1}{12}\pi h(3 - 4h^2), \text{ where } 0 \le h \le 0.5$$

(i) Find the value of h for which $\frac{dV}{dt} = \frac{5\pi}{36}\frac{dh}{dt}$. [3]

(ii) If water is flowing into the bowl at a constant rate of $\frac{\pi}{800}$ m^{3}s^{-1}, find the rate of change of h when $h = 0.25$. [2]

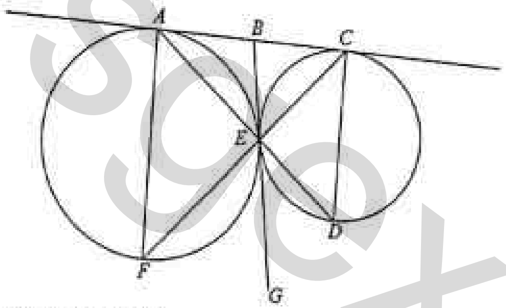
8. A circle, centre C, passes through the point $P(-1, 6)$ and touches the line $2y + 5x = 16$ at the point $Q(2, 3)$.



(i) Find the equation of the perpendicular bisector of PQ. Hence find the equation of the circle. [6]

(ii) Find the coordinates of R such that $CPQR$ is a parallelogram. [2]

9. In the diagram, the two circles touch at E. ABC and BEG are common tangents to the two circles. AE and CE are produced to D and F respectively.

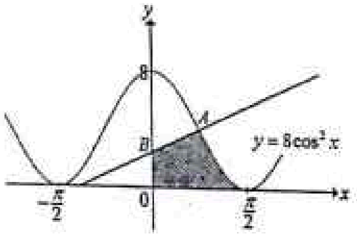


(i) Prove that AF is parallel to CD. [4]

(ii) Prove that AC is a diameter of a circle which passes through A, E and C. [4]

10. (a) Show that $\frac{\mathrm{d}}{\mathrm{d}x}(2x+\sin 2x)=4\cos^2 x$. [2]

(b)



The diagram shows part of the graph of $y=8\cos^2 x$. The normal to the curve at A, where $x=\frac{\pi}{4}$, meets the y-axis at B.

(i) Show that the y-coordinates of B is $\frac{128-\pi}{32}$. [4]

(ii) Determine the area of the shaded region bounded by the curve, the line AB, the x-axis and the y-axis. [5]

11. It is given that $f(x)=x^2-8x+9$ for $2\le x\le 7$.

(i) Find the value of a and of b for which $f(x)=(x-a)^2+b$. [2]

(ii) Find the stationary point of the graph $y=|f(x)|$ and determine its nature. [2]

(iii) Sketch the graph of $y=|f(x)|$. [3]

(iv) Find the range of values of x for which $|f(x)|>6$. [2]

(v) Determine the number of solutions of the equation $|f(x)|=mx+c$ in each of the following cases, when

(a) $m=1$ and $c=2$, [1]

(b) $m=-\frac{1}{2}$ and $c=4$. [1]

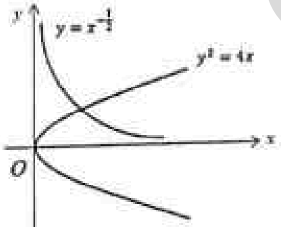
End of Paper 1

Paper 1

1. (i) $12^x = \frac{2}{3}$
(ii) $x = 0.405, 1.61$

1. (i) $4 < x < 6$
(ii) $c < -5$ or $c > 5$

2. (i)



(ii) $\left(\frac{1}{2}, \sqrt{2}\right)$

3. (i) $\frac{\sqrt{6}-\sqrt{2}}{4}$
(ii) $7+4\sqrt{3}$

4. (a) $n = 8$
(b) -47

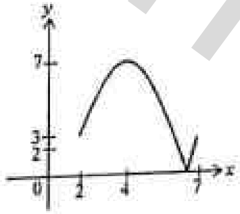
5. (i) $A = 90000$
$k = 0.0811$
(ii) $t = 14.847$
Year = 2014

7. (i) $h = \frac{1}{3}$
(ii) $\frac{1}{150}$ ms^{-1} or 0.00667 ms^{-1}

9. (i) $(x+3)^2+(y-1)^2 = 29$
(ii) $R\ (0, -2)$

10. (a) $4\cos^2 x$
(b)(i) $\frac{128-\pi}{32}$
(ii) 4.24 unit^2

11. (i) $a = 4,\ b = -7$
(ii) (4, 7), Maximum point



(iii)
(iv) $3 < x < 5$
(v)(a) 2
(b) 3

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1. Find a quadratic equation for which the sum of roots is $\frac{1}{2}$ and the sum of the cube of the roots is $\frac{13}{8}$. [5]

2. (a) Variables x and y are connected by the equation $\log_3 y = a\log_3 x + b$, where a and b are constants. Using experimental values of x and y, a graph was drawn in which $\log_3 y$ was plotted on the vertical axis against $\log_3 x$ on the horizontal axis. The straight line which was obtained passed through the points (1, 3) and (−1, 5).

(i) Find the value of a and of b. [3]

(ii) Show that x and y can be expressed in the form $y = kx^n$, where k and n are constants to be found. [3]

(b) Given that $\log_9 x^3 = \log_{\sqrt{3}} u$, express u in terms of x. [3]

3. (i) Show that $\frac{\sin 2x+1-\cos 2x}{\sin 2x-1+\cos 2x} = \frac{1+\tan x}{1-\tan x}$. [3]

(ii) Hence, solve for $-3 < x < 2$, the equation $\frac{\sin 2x+1-\cos 2x}{\sin 2x-1+\cos 2x} = 6\tan x$. [5]

4. (a) Find the value of m, where $m > 0$, for which $2x^2+x+m$ is a factor of $4x^3+5x-3$. [3]

(b) The cubic polynomial $f(x)$ is such that the coefficient of x^3 is 3 and the roots of the equation $f(x) = 0$ are −2, 3 and k. Given that $f(x)$ has a remainder of 42 when divided by $(x+1)$, find
(i) the value of k, [3]
(ii) the remainder when $f(x)$ is divided by x. [2]

5. (i) Express $\frac{-2x-6}{(x+1)(x^2-3)}$ in partial fractions. [4]

(ii) Differentiate $\ln(x^2-3)$. [1]

(iii) Given that $\int_2^3 \frac{-3x-9}{(x+1)(x^2-3)}\,dx = \frac{9}{2}\ln a$, using the results in parts (i) and (ii), find the value of a. [4]

6. A device is used to simulate the breathing patterns of a certain mammal's lungs. The volume, V litres, of air in the lungs of this mammal, t seconds after the beginning of one breath can be modelled by

$$V = 0.45 - 0.4\cos(kt), \quad 0 \le t \le 4.$$

The time for one breath is 4 seconds.

(i) Explain why this model suggests that the maximum capacity of the lungs is 0.85 litres. [1]

(ii) Show that the value of k is $\frac{\pi}{2}$. [2]

(iii) Find the length of time for which the lungs contain at least 0.5 litres of air. [3]

(iv) Sketch the graph of $V = 0.45 - 0.4\cos(kt)$, $0 \le t \le 4$. [2]

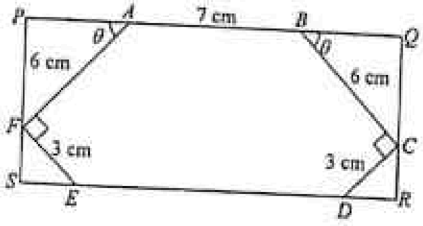
7. A curve has the equation $y = 4x^3e^{2x}$. It has a stationary point at $\left(p, \frac{q}{e^3}\right)$ where $p < 0$.

(i) Find the exact value of p and of q. [5]

(ii) By considering the sign of $\frac{dy}{dx}$, determine the coordinates and the nature of the other stationary point. [2]

(iii) Find the range of values of x for which $y = 4x^3e^{2x}$ is a decreasing function. [2]

8. In the diagram, $PQRS$ is a rectangle. $ABCDEF$ is a hexagon with angle AFE = angle $BCD = 90°$, $AB = 7$ cm, $BC = AF = 6$ cm, $CD = EF = 3$ cm and angle QBC = angle $PAF = \theta$, where $0° \le \theta \le 90°$.

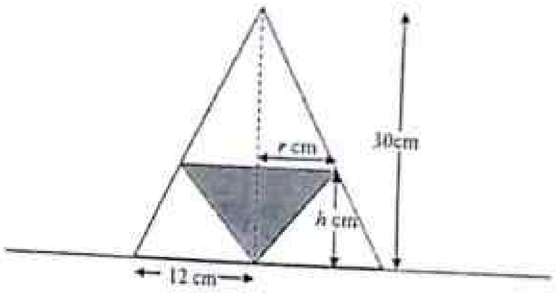


(i) Show that the perimeter, L cm, of $ABCDEF$ is given by $L = 32 + 12\cos\theta - 6\sin\theta$. [3]

(ii) Express L in the form $k + R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(iii) Find the value of θ, if $L = 35$. [3]

9. The diagram shows the cross-section of a hollow cone of height 30 cm and base radius of 12 cm and an inverted cone of radius r cm and height h cm with the inverted cone inside the hollow cone. Both stand on a horizontal surface. The upper circular edge of the inverted cone is in contact with the hollow cone.



(i) Express h in terms of r and hence show that the volume, V cm³ of the inverted cone is given by

$$V = \pi\left(10r^2 - \frac{5r^3}{6}\right)$$ [4]

Given that r can vary,

(ii) find, in terms of π, the volume of the largest inverted cone which can stand inside the hollow cone, and show that, in this case, the inverted cone occupies $\frac{4}{27}$ of the volume of the hollow cone. [7]

10. The population P, in millions, of a country was recorded in various years and the results are shown in the table below.

Year	2000	2005	2010	2015
P	12.88	14.61	17.38	21.88

It is known that P and t are related by an equation of the form $P = 10 + Ab^t$, where t is the time measured in years from January 1995 and A and b are constants.

(i) Using graph paper, draw a straight line graph of $\lg(P - 10)$ against t and use your graph to estimate the value of A and of b. [7]

Use your graph to estimate

(ii) the population, in millions, in the country in January 1995, [2]

(iii) the year in which the population exceeds 35 million. [2]

11. The velocity, $v\,\text{ms}^{-1}$, of a particle, P, moving in a straight line is given by $v = 3t^2 + pt + q$, where t is the time in seconds after the start of motion. At $t = 0$, the displacement of the particle from O is 3 m.

Given also that when $t = 2$, the displacement of the particle from O is 23 m and the acceleration of the particle is $-6\,\text{ms}^{-2}$,

(i) find the value of p and of q. [7]
(ii) explain with clear working whether P will return to its starting point. [5]

End of Paper 2

Solution

1. $2x^2 - x - 2 = 0$

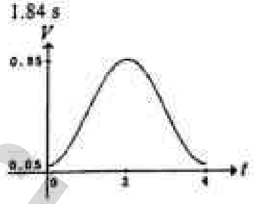
2. (a)(i) $a = -1,\ b = 4$
(a)(ii) $y = 81x^{-4}$
(b) $u = \sqrt{x}$

3. (ii) $x = -2.82, -2.68, 0.322, 0.464$

4. (a) $m = 3$
(b)(i) $k = \frac{5}{2}$
(ii) 45

5. (i) $\frac{2}{x+1} - \frac{2x}{x^2-3}$
(ii) $\frac{2x}{x^2-3}$
(iii) $a = \frac{2}{3}$

6. (iii) 1.84 s
(iv)



7. (i) $p = -\frac{3}{2},\ q = -\frac{27}{2}$
(ii) (0, 0) is a point of inflexion.
(iii) $x < -\frac{3}{2}$

8. (ii) $32 + 6\sqrt{5}\cos(\theta + 26.6°)$
(iii) $\theta = 50.5°$

9. (i) $h = 30 - \frac{5r}{2}$
(ii) $\frac{640\pi}{3}\ \text{cm}^3$

10. (i) $A = 1.82$ (1.55 ~ 2.00)
$b = 1.10$ (1.00 − 1.2)
(ii) $P = 11.8$
(iii) Year 2023

11. (i) $p = -18,\ q = 24$

1 The constant term in the expansion of $(6+x)^4+(x^2+\frac{m}{x})^4$ is 107892.
Find the value of the positive constant m. [4]

2 A is an obtuse angle and B is an acute angle such that $\tan(A-B)=7$ and $\tan B=5$.
Without using a calculator, find the exact value of $\cos A$. [4]

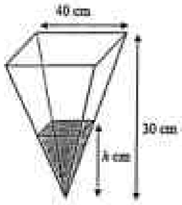
3. The diagram shows a tank in the shape of an inverted right pyramid of height 30 cm and a square base of side 40 cm. Water is poured into the tank at a constant rate of 24 cm^3/s.

After t seconds, the depth of the water is h cm.

(i) Show that, the volume of the water in the tank, V cm^3, after t seconds, is given by $V=\frac{16}{27}h^3$. [3]

(ii) Find the rate of change of the depth of the water when $h=6$. [3]

(iii) State, with a reason, whether $\frac{dh}{dt}$ will increase or decrease as t increases. [1]



4 Express $\frac{1-x^2}{2x^3+x^2}$ in partial fractions. [4]

[Turn over

5 The table shows experimental values of two variables, x and y, which are connected by an equation of the form $\frac{b}{y}+\frac{ab}{x}=2$, where a and b are constants.

x	0.150	0.200	0.250	0.300
y	1.28	0.909	0.512	0.444

An error was made in recording one of the values of y.

(i) Plot $\frac{1}{y}$ against $\frac{1}{x}$ and draw a straight line graph. [2]

(ii) Use your graph to estimate the value of y to replace the incorrect reading. [1]

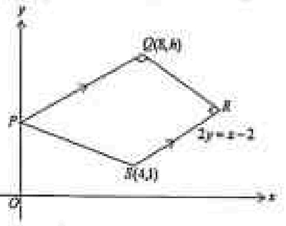
(iii) Use your graph to estimate the value of a and of b. [4]

6 (i) Prove that $\frac{1}{\cos\theta(\cot\theta+\tan\theta)}=\sin\theta$. [3]

(ii) Find, in radians, the exact value of the acute angle θ for which

$$\frac{1}{\cos\theta(\cot\theta+\tan\theta)}=\frac{3}{4}\operatorname{cosec}\theta.$$ [2]

7 **Solution to this question by accurate drawing will not be accepted.**



The diagram shows a trapezium $PQRS$ in which PQ is parallel to SR and angle $QRS=90°$. The point Q is $(8,h)$ and the point S is $(4,1)$.
The equation of SR is $2y=x-2$.

(a) Express, in terms of h,

(i) the equation of PQ, [2]

(ii) the equation of QR, [2]

(iii) the coordinates of P and of R. [3]

(b) In the case where $h=13$, find the area of the trapezium $PQRS$. [2]

8 The function f(x) is such that $f'(x) = 2\sin 2x + 4\cos x$ and $f(\frac{\pi}{6}) = 0$.

Solve the equation $f''(x) - f(x) = \frac{9}{2}$ for $0 < x < 2\pi$. [7]

9 The equation of a curve is $y = 5kx^2 + 21x + 4k - 21$, where k is a constant.

(i) Find the values of k for which the line $y = x - 5$ is a tangent to the curve. [5]

(ii) In the case where $k = 3$, find the set of values of x for which the curve lies above the line $y = -15$. [3]

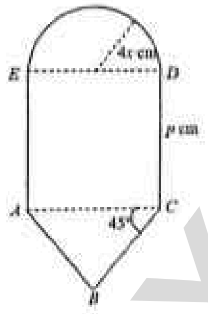
10 A piece of wire of length 250 cm is bent into the shape as shown in the diagram. The shape consists of an isosceles triangle ABC in which angle $ACB = 45°$, a rectangle $ACDE$ of length p cm and a semi-circle of radius $4x$ cm.

(i) Express p in terms of x. [3]

(ii) Show that the area enclosed, A cm², is given by
$A = (16 - 8\pi - 32\sqrt{2})x^2 + 1000x$ [2]

(iii) Given that x can vary, find the value of x for which the area is stationary. [3]

(iv) Explain why this value of x gives the largest area possible. [1]



11 A curve has the equation $y = \frac{2\ln(2x-1)}{x-1}$, where $x > \frac{1}{2}, x \neq 1$. The curve cuts the x-axis at P.

(i) Find the x- coordinate of P. [2]

The equation of the normal to the curve at P cuts the y axis at Q.

(ii) Find the area of the triangle POQ, where O is the origin. [5]

12 A curve has the equation $y = (x-3)^2 - 16$.

(i) Explain why the lowest point of the curve has coordinates $(3, -16)$. [2]

(ii) Find the x-coordinates of the points where the curve intersects the x axis. [2]

(iii) Sketch the graph of $y = |(x-3)^2 - 16|$. [3]

(iv) Using your graph, state the number of solutions to each of the following equations.

(a) $|(x-3)^2 - 16| - 17 = 0$ [1]

(b) $|(x-3)^2 - 16| = -x - 2$ [1]

St Nicholas Girls School Additional Mathematics 2016Preliminary Exam Paper 1

1. $m = 3$

2. $-\frac{17\sqrt{13}}{65}$

3. (ii) $\frac{3}{8} cm/s$ or 0.375 cm/s
   (iii) $\frac{dh}{dt} = \frac{27}{2h^3}$
   As t increases, h increases and $\frac{27}{2h^3}$ decreases. Hence the rate of change of depth of water decreases.

4. $-\frac{2}{x} + \frac{1}{x^2} + \frac{3}{2x+1}$

5. (ii) $y = 0.667$
   (iii) $a = 0.441$, $b = 0.541$

6. (ii) $\theta = \frac{\pi}{3}$

7(a)(i) $y = \frac{1}{2}x + h - 4$
   (ii) $y = -2x + h + 16$
   (iii) $P(0, h-4)$, $R(\frac{2h+34}{5}, \frac{h+12}{5})$

7(b) 80 sq units

8. 0.201, 2.94, $\frac{3\pi}{2}$ or 4.71

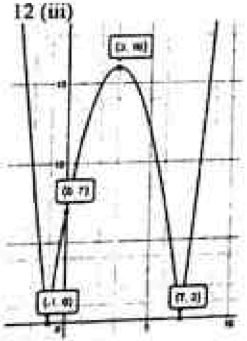
9. (i) $k = -1, 5.$
   (ii) $x < -1$ or $x > -\frac{2}{5}$

10 (i) $p = 125 - 2\pi x - 4\sqrt{2}\,x$
   (iii) $x = 9.19$

11. (i) $x = 1$
   (ii) $\frac{1}{4}$ square units

12. (ii) $x = 7$ or $x = -1$

12 (iii)



12 (iv)(a) 2

12 (iv) (b) 0

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1 A slice of chocolate cake is heated in a convection oven to a temperature of [illegible] °C. It is then left to cool and it is observed that its temperature, T °C, t minutes after removal from the oven, is given by $T = De^{-kt} + 25$, where D and k are constants.

(i) Find the value of D. [2]

(ii) Find the value of k, given that the temperature of the cake is 31 °C after 2 minutes. [2]

(iii) Explain why the temperature of the cake will always be above 25 °C. [1]

2 The function $f(x) = 3x^3 + ax^2 + bx - 16$, where a and b are constants, is exactly divisible by $3x - 4$ and leaves a remainder of -160 when divided by $x + 2$.

(i) Find the value of a and of b. [4]

(ii) Factorise $f(x)$. [2]

(iii) Hence solve the equation $24x^3 + 4ax^2 + 2bx - 16 = 0$. [2]

3 A cuboid has a square base of length $\left(\sqrt{2} + \sqrt{3}\right)$ cm.
The volume of the cuboid is $\left(\sqrt{3} + \sqrt{6}\right)$ cm^3.
Find, **without using a calculator**, the height of the cuboid in the form $\left(a(\sqrt{3} + \sqrt{6}) - b\sqrt{2} - 12\right)$ cm, where a and b are integers. [4]

4 The quadratic equation $x^2 + 4x + 7 = 0$ has roots α and β. Find

(i) the value of $\alpha^3 + \beta^3$, [3]

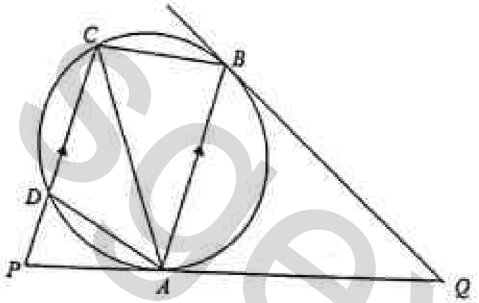
(ii) the quadratic equation whose roots are $2\alpha^3$ and $2\beta^3$. [3]

5 Solve the equation

(i) $\log_3 x^2 - 16\log_x 3 = -4$, [4]

(ii) $e^y - 1 - 6e^{-y} = 0$, [3]

(iii) $\lg z^3 = (\lg z)^3$. [3]



In the diagram, the points A, B, C and D lie on the circle. PQ is a tangent to the circle at point A. PDC is a straight line and is parallel to AB.

(i) Show that triangle ABC is similar to triangle ADP. [3]

(ii) Given further that BQ is a tangent to the circle at B, show that $2 \times \text{angle } CPA = 180° - \text{angle } BQA$ [3]

7 A curve has the equation $y = \frac{(x+2)^2}{2x}$.

(i) Find the coordinates of the stationary points on the curve. [5]

(ii) (a) Find the range of values of x for which y increases as x increases. [2]

(b) What do the results in (a) imply about the stationary points. [2]

(iii) Sketch the curve, indicating clearly the stationary points and asymptotes, if any.

Hence deduce the range of values of k for which the equation $\frac{(x+2)^2}{2x} = k$ has no real roots. [3]

8 Two particles, P and Q, leave a point O at the same time, and travel initially in the same direction along the same straight line.
Particle P starts with a velocity of 6 m/s. Its acceleration a m/s^2, is given by $a = 2 - t$, where t seconds is the time after leaving O.

(i) Find the velocity and distance of the particle P from O in terms of t. [4]

(ii) Find the value of t when P is again at O. [3]

Particle Q moves with a velocity v m/s, where $v = 6t + 2e^{-t} - 1$, and t seconds is the time after leaving O.

(iii) Find the initial acceleration of particle Q. [2]

(iv) Find the distance of the particle Q from O in terms of t. [2]

(v) Show that particle Q overtakes particle P during the third second. [2]

9 (a) (i) Express $12\sin 2\theta - 5\cos 2\theta$ in the form $R\sin(2\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(ii) Solve the equation $12\sin 2\theta - 5\cos 2\theta = 5$ for $-90° \le \theta \le 90°$. [3]

(b) On the same axes, sketch for $-180° \le x \le 180°$ the graphs of

$$y = 12\sin x \quad \text{and} \quad y = 5 + 5\cos x.$$

Hence, state the number of solutions for $12\sin x - 5\cos x = 5$ for $-180° \le x \le 180°$. [4]

10 A circle, C_1, has equation $x^2 + y^2 - 10x + 2y - 10 = 0$.
Point A is the centre of C_1.

(i) Find the radius of C_1 and the coordinates of A. [3]

Point Q lies on C_1. The tangent at Q passes through P (9, 7).

(ii) Find the exact length of PQ. [3]

A second circle, C_2, passes through the points A and P. The centre of C_2 lies on the x-axis.

(iii) Find the equation of the perpendicular bisector of AP. [4]

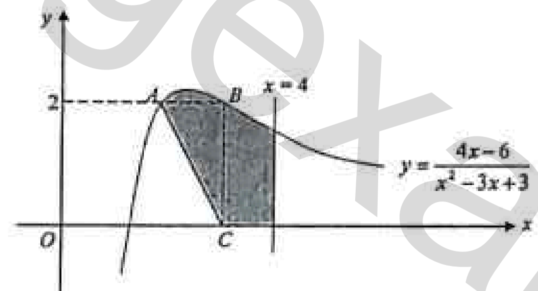
(v) Find the equation of C_2. [3]

11 (a) Differentiate $\ln(x^2-3x+3)$ with respect to x. [1]

(b) Express $\frac{x^2-x}{x^2-3x+3}$ in the form $a+\frac{bx+c}{x^2-3x+3}$, where a, b and c are constants. [2]

(c) Hence, find $\int \frac{x^2-x}{x^2-3x+3}\,dx$. [2]

(d)



The diagram shows part of the curve $y=\frac{4x-6}{x^2-3x+3}$ and the line $x = 4$.

The y-coordinates of points A and B are 2. Point C is vertically below point B. Find

(i) the coordinates of A, B and C. [4]

(ii) the area of the shaded region bounded by the curve, the line $x = 4$, the x-axis and the line AC. [3]

CHIJ St. Nicholas Girls' School Additional Mathematics

2016 Preliminary Examination Paper 2

1. (i) $D=10$

(ii) $k=0.255$

2. (i) $a=-16$, $b=28$

2(ii) $f(x)=(3x-4)(x-2)^2$

(iii) $x=\frac{2}{3}$, $x=1$

3. $(5(\sqrt{3}+\sqrt{6})-6\sqrt{2}-12)$ cm

4. (i) 2

(ii) $x^2-40x+1372=0$

5 (i) $\frac{1}{81}$, 9

(ii) 1.10 or ln3

(iii) 1 , 54.0 , 0.0185

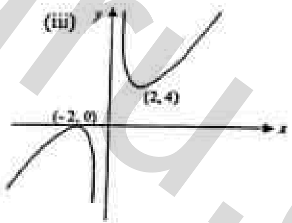
7. (i) (2, 4) and (−2, 0)

(ii)(a) $x<-2$ or $x>2$

(ii) (b) (-2,0) is a maximum point

(2,4)) is a minimum point

(iii)



$\therefore 0<k<4$

8. (i) $v=2t-\frac{t^2}{2}+6$, $s=t^2-\frac{t^3}{6}+6t$

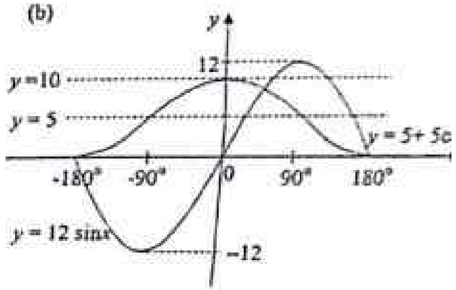
(ii) 9.71 (iii) 4 m/s^2

(iv) $s=3t^2-2e^{-t}-t+2$

9. (a) (i) $13\sin(2\theta-22.6°)$

(a) (ii) −90°, 22.6°, 90°.

(b)



3 solutions

10 (i) radius =6 units , $A(5,-1)$

(ii) $2\sqrt{11}$ units

(iii) $2y=13-x$

(iv) $(x-13)^2+y^2=65$

11 (a) $\frac{2x-3}{x^2-3x+3}$

(b) $1+\frac{2x-3}{x^2-3x+3}$

(c) $x+\ln(x^2-3x+3)+c$

(d)(i) $A(2,2)$, $B(3,2)$, $C(3,0)$

(d)(ii) 2.89 sq units

[Turn over