<table>
<thead>
<tr>
<th></th>
<th>School Name</th>
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<tbody>
<tr>
<td>1</td>
<td>Anglo-Chinese School (International)</td>
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<td>2</td>
<td>Anderson Secondary School</td>
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<td>3</td>
<td>Anglican High School</td>
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<td>4</td>
<td>Catholic High School</td>
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<td>5</td>
<td>CHIJ Saint Joseph's Convent</td>
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<td>6</td>
<td>Chung Cheng High School (Main)</td>
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<td>7</td>
<td>Fairfield Methodist Secondary</td>
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<td>8</td>
<td>Holy Innocents' High School</td>
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<td>9</td>
<td>Nanyang Girls' High School</td>
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<td>10</td>
<td>Paya Lebar Methodist Girls' School</td>
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<td>11</td>
<td>Swiss Cottage Secondary School</td>
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<td>12</td>
<td>Tanjong Katong Girls' School</td>
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<td>13</td>
<td>Temasek Secondary School</td>
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<td>14</td>
<td>Victoria School</td>
</tr>
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</table>
DEYI SECONDARY SCHOOL

Preliminary Examination 2015
Secondary Four Express / Five Normal Academic

MATHEMATICS

Paper 1

Candidates answer on the Question Paper.
No additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen in the spaces provided on the Question Paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid / tape.
Answer all questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, the answer
should be given to three significant figures.
Answers in degrees should be given to one decimal place.
For $\pi$, use either your calculator value or 3.142, unless the question requires the answer in terms of $\pi$.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use

80

This document consists of 16 printed pages including the cover page. [Turn over
2

Mathematical Formulae

Compound Interest

Total amount = \( P \left(1 + \frac{r}{100}\right)^n \)

Mensuration

Curved surface area of a cone = \( \pi rl \)

Surface area of a sphere = \( 4\pi r^2 \)

Volume of a cone = \( \frac{1}{3} \pi r^2 h \)

Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

Area of triangle ABC = \( \frac{1}{2} ab \sin C \)

Arc length = \( r\theta \), where \( \theta \) is in radians

Sector area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

Trigonometry

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\( a^2 = b^2 + c^2 - 2bc \cos A \)

Statistics

Mean = \( \frac{\sum fx}{\sum f} \)

Standard deviation = \( \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} \)
Answer all the questions.

1 (a) Calculate \( \frac{6.7^2 - \sqrt{4.8}}{20.15 - 19.99} \)

Write down the first six digits of your answer.

Answer (a) ........................................ [1]

(b) Write down your answer to part (a) correct to 2 significant figures.

Answer (b) ......................... [1]

2 Write the following in descending order.

\( \frac{54}{67} \quad \sqrt[3]{0.512} \quad 0.776 \quad 0.802 \)

Answer ......

[2]

3 Given that \( 9 \times 27^n = 1 \), find the value of \( n \).

Answer \( n = \) ............................................. [2]

4 The sine of an obtuse angle is 0.6.

Without using a calculator, find the cosine of this angle.

Answer ................. ................. [2]
The diagram shows a sketch of a regular hexagon and a regular octagon. Calculate $x$.

Answer $x = \ldots \ldots \ldots \ldots \ldots \ldots \ldots$ [2]

---

6 Brian is leaving Singapore to further his studies in the United Kingdom. In Singapore, the exchange rate is $1$ Singapore Dollar $= 0.478$ British Pounds. In the United Kingdom, the exchange rate is $1$ British Pound $= 2.113$ Singapore Dollars.

Brian would like to change $2500$ Singapore Dollars into British Pounds.

How many fewer British Pounds will he get by changing his money in the United Kingdom?

Answer $\ldots \ldots \ldots \ldots \ldots \ldots \ldots$ British Pounds [2]
7. Matthew bought a new car. At the end of each year, the car's value depreciates by 10%.
   The value, \( C \), of the car \( t \) years after being bought is given by
   \[
   C = 85000 \times 0.9^t.
   \]
   (a) How much did Matthew pay for his new car?

   Answer (a) $ \ldots \ldots \ldots \ldots \ldots \ldots [1]

   (b) Find the percentage decrease in the value of his car at the end of three years.

   Answer (b) \ldots \ldots \ldots \ldots \ldots \ldots \% [1]

8. Two geometrically similar solids made from the same material have masses 3.60 kg and 12.15 kg respectively.
   Calculate the ratio of the area of the smaller solid to the area of the larger solid.

   Answer \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots [2]
9  Dawn invested some money in a savings account that was compounded every six months. The rate of compound interest was 5% per annum. At the end of the 4 years there was $14620.83 in her account.

How much did Dawn invest in the account at first?
Give your answer correct to the nearest dollar.

Answer $\ldots$  \ldots \ldots \ldots \ldots [3]

10  A rectangle with length 40 cm is divided into five identical shaded rectangles and another six identical unshaded rectangles.

The shaded area makes up two-thirds of the unshaded area.

Find the lengths labelled $x$ and $y$.

Answer $x = \ldots \ldots \ldots \ldots \ldots$ cm 
$y = \ldots \ldots \ldots \ldots \ldots$ cm [3]
11  Mr Tan decided to buy a laptop under a hire-purchase scheme.

He would have paid $3906 in total under the scheme, which consists of a deposit of 15% of the
selling price of the laptop plus 24 equal monthly payments of $140.25.

What is the selling price of the laptop?

\[ \text{Answer} \quad \$ \ldots \quad \ldots \ldots \quad [3] \]

12  Two points \( P \) and \( Q \) have position vectors \( \mathbf{p} \) and \( \mathbf{q} \) respectively, relative to an origin \( O \).

It is given that \( \mathbf{p} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \) and \( \mathbf{q} = \begin{pmatrix} k \\ 0 \end{pmatrix} \).

Find

(a) \( \overrightarrow{PQ} \) in terms of \( k \).

\[ \text{Answer} \quad (a) \quad \overrightarrow{PQ} = \ldots \ldots \quad \ldots \quad [1] \]

(b) the possible values of \( k \) if \( OP \) and \( OQ \) are two sides of a rhombus.

\[ \text{Answer} \quad (b) \quad k = \ldots \ldots \quad \text{or} \quad \ldots \ldots \quad [2] \]
13 \( \varepsilon = \{ \text{integers } x : 1 \leq x \leq 12 \} \)

\( A = \{ \text{factors of 12} \} \)

\( B = \{ \text{multiples of 4} \} \)

(a) Draw a Venn diagram to illustrate this information.

\[ \text{Answer (a)} \]

\[ 
\begin{array}{c}
A \\
\{1, 2, 3, 6\} \\
\{5, 7, 9, 10, 11\}
\end{array} \quad B
\]

\( \quad \overline{\bigcup} \quad \)

\[ \] 

[b] [2]

(b) Describe in words what the set \((A \bigcup B)^c\) represents.

\[ \text{Answer (b)} \]

\[ \]

[1]

14 A train 45 m long passes through a tunnel 6 km long.

The average speed of the train is 27 km/h.

(a) Change 27 km/h into m/s.

\[ \text{Answer (a)} \] \hspace{1cm} \ldots \ldots \ldots \ldots \ldots \text{m/s} \] [1]

(b) Calculate the time taken for the train to pass completely through the tunnel.

Give your answer in minutes and seconds, to the nearest second.

\[ \text{Answer (b)} \] \hspace{1cm} \text{minutes} \ldots \text{seconds} \] [3]
15. Simplify
(a) \(28x^2y^{-3} \div 16x^3y^{-1}\).

Answer (a) ............... .... ....... [2]

(b) \(\frac{2}{x-3} + \frac{3x}{x^2-9}\).

Answer (b) ............... .... ....... [2]

16. The numbers 1 to 100 are arranged in a table as shown below.

A U-shaped, shaded frame can be placed around various numbers throughout the table.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
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<td>94</td>
<td>95</td>
<td>96</td>
<td>97</td>
<td>98</td>
<td>99</td>
<td>100</td>
</tr>
</tbody>
</table>

The \(U\)-number is used to refer to the shaded frame that is drawn around a particular number.
For example, \(U_2\) refers to the shaded frame shown above since it is drawn around the number 2.

(a) State the largest possible \(U\)-number.

Answer (a) ......................... [1]

(b) Write and simplify an expression, in terms of \(n\), for the sum of the numbers in \(U_n\).

Answer (b) ............... .... ....... [2]

(c) Find the sum of numbers in \(U_{75}\).

Answer (c) ............... .... ....... [1]

[Turn over]
17. The resistance of a wire, $R$ ohms, is inversely proportional to the square of its diameter, $d$ μm.

(a) Sketch a resistance-diameter graph for the wire.

*Answer (a)*

For a fixed length of wire, the resistance is 25.6 ohms when the diameter is 50 μm.

(b) Find the equation for $R$ in terms of $d$.

*Answer (b) $R = \ldots \ldots$ [2]*

(c) Another wire of the same length has resistance 120 ohms. Calculate its diameter.

*Answer (c) \ldots \ldots \mu m [1]*
A composite container made from a cylinder and a cone has a vertical height of 30 cm. Water is poured into the empty container at a constant rate. It takes 60 seconds to fill up the entire container.

(a) Find the time taken to fill up the cone.

Answer (a) ...... ...... seconds [2]

(b) Sketch the graph of how the depth of water in the container varies during the 60 seconds.

Answer (b)
The diagram shows the box-and-whisker plots for the distributions of the speeds, in km/h, of 100 vehicles before and after a speed camera was placed on an expressway.

(a) Find the interquartile range of the distribution before the camera was placed.

Answer (a) \[\text{km/h}\] [1]

(b) Find the interquartile range of the distribution after the camera was placed.

Answer (b) \[\text{km/h}\] [1]

(c) After the camera was placed, 25% of the motorists were issued with traffic summons for exceeding the speed limit of the expressway.

What is the speed limit of the expressway?

Answer (c) \[\text{km/h}\] [1]

(d) Has the speed camera been effective in regulating the speed limit of the expressway?

Explain your answer by comparing the distributions of the speeds before and after the camera was placed.

Answer (d) ...
Abraham and Lincoln sent out some letters, postcards and greeting cards.

The number of letters, postcards and greeting cards is shown in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Letters</th>
<th>Postcards</th>
<th>Greeting cards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abraham</td>
<td>4</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>Lincoln</td>
<td>7</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

The postage for each letter, postcard and greeting card is $0.30, $0.40 and $0.50 respectively.

(a) Write out a $2 \times 3$ matrix $P$ and a column matrix $Q$ to represent the above information.

Answer: 

(a) $P = \begin{pmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \end{pmatrix}$

$Q = \begin{pmatrix} \_ \end{pmatrix}$

(b) Evaluate the matrix $S = PQ$.

Answer: 

(b) $S = \begin{pmatrix} \_ \end{pmatrix}$

(c) State what the elements of $S$ represent.
21 The diagram shows a spinner with nine numbered sectors of identical sizes.

![Diagram of a spinner with numbers 1 to 9]

Each time the pointer is spun, it is equally likely to stop on one of the sectors.

(a) The pointer is spun once.
    Find the probability that it stops on an odd number.

    \[ \text{Answer (a)} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots [1] \]

(b) Aysha spins the pointer twice.
    Find the probability that the pointer lands on a prime number \textbf{at least once}.

    \[ \text{Answer (b)} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots [2] \]

(c) Natasha spins the pointer twice.
    Her score is found from the difference of the numbers from her two spins.
    Find the probability that her score is 0.

    \[ \text{Answer (c)} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots [2] \]
22 Benjamin has 165 identical cubes of sides 2 cm.

(a) He uses some of the cubes to make a cuboid which measures 8 cm by 10 cm by 14 cm. Calculate the total surface area of the cuboid.

Answer (a) ................. cm$^2$ [2]

(b) Benjamin makes the largest cube possible using some of the 165 cubes. He then makes the largest cube possible from the unused cubes. How many cubes will he have left over after making the second cube?

Answer (b) ................. cm$^2$ [2]

(c) Benjamin uses all 165 cubes to make a cuboid. Find the dimensions of the cuboid.

Answer (c) ................. cm by ................. cm by ................. cm [2]
23 Expressions for the lengths of three sides of a quadrilateral are shown on the diagram below.
All lengths are in centimetres.

\[\begin{array}{c}
A & B & C & D \\
4x + 1 & 16 - x & 3x + 4 & \end{array}\]

(a) The perimeter of this quadrilateral is given by the expression \((11x + 19)\) cm.
Find an expression, in terms of \(x\), for the length of \(DC\).
Give your expression in its simplest form.

Answer (a) .....

(b) Given that \(ABCD\) is a parallelogram and that \(AB = AD\), calculate the perimeter of \(ABCD\).

Answer (b) .....

(c) Calculate the area of \(ABCD\) if \(AC = (10x - 6)\) cm.

Answer (c) .....

END OF PAPER
READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page. Write in dark blue or black pen in the spaces provided on the Question Paper. You may use a pencil for any diagrams or graphs. Do not use staples, paper clips, highlighters, glue or correction fluid / tape.

Answer all questions.

If working is needed for any question, it must be shown with the answer. Omission of essential working will result in loss of marks.

Calculators should be used where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, the answer should be given to three significant figures. Answers in degrees should be given to one decimal place. For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).

The number of marks is given in brackets \((\quad)\) at the end of each question or part question. The total number of marks for this paper is 80.
Mathematical Formulae

Compound Interest

Total amount = \( P \left(1 + \frac{r}{100}\right)^n \)

Mensuration

Curved surface area of a cone = \( \pi rl \)

Surface area of a sphere = \( 4\pi r^2 \)

Volume of a cone = \( \frac{1}{3} \pi r^2 h \)

Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

Area of triangle ABC = \( \frac{1}{2} ab \sin C \)

Arc length = \( r\theta \), where \( \theta \) is in radians

Sector area = \( \frac{1}{2} r^2 \theta \), where \( \theta \) is in radians

Trigonometry

\( \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \)

\( a^2 = b^2 + c^2 - 2bc \cos A \)

Statistics

Mean = \( \frac{\sum fx}{\sum f} \)

Standard deviation = \( \sqrt{\frac{\sum f x^2}{\sum f} - \left(\frac{\sum f x}{\sum f}\right)^2} \)
Answer all the questions.

1 (a) Calculate \( \frac{6.7^2 - \sqrt{4.8}}{20.15 - 19.99} \).

Write down the first six digits of your answer.

Answer (a) \( 270.019 \) \[1\]

(b) Write down your answer to part (a) correct to 2 significant figures.

Answer (b) \( 270 \) \[1\]

2 Write the following in descending order.

\[
\frac{54}{67} \quad \sqrt{0.512} \quad 0.776 \quad 0.802 \\
0.805 \quad 0.2 \quad 0.804 \quad 3.85 \quad 0.802 \\
\sqrt{0.512}
\]

Answer \( \frac{54}{67} \quad 0.776 \quad 0.802 \quad 0.805 \) \[2\]

3 Given that \( 9 \times 27^{-n} = 1 \), find the value of \( n \).

\[
9 \times 27^{-n} = 1 \\
3^2 \times (3^3)^{-n} = 1 \\
3^2 \times 3^{-3n} = 3^0 \\
2 - 3n = 0 \\
2 = 3n \\
n = \frac{2}{3}
\]

Answer \( n = \frac{2}{3} \) \[2\]

4 The sine of an obtuse angle is 0.6.

Without using a calculator, find the cosine of this angle.

\[
\sin \theta = 0.6 \\
\cos \theta = \sqrt{1 - \sin^2 \theta} \\
\cos \theta = \frac{4}{5}
\]

Answer \( \frac{-4}{5} \) \[2\]

[Turn over]
The diagram shows a sketch of a regular hexagon and a regular octagon.

Calculate \( x \).

\[
\frac{(6-2) \times 180}{6} = 120°
\]

\[
\frac{(8-2) \times 180}{8} = 135°
\]

\[
360° - 135° - 120° = 105°
\]

Answer \( x = 105° \). [2]

---

6 Brian is leaving Singapore to further his studies in the United Kingdom.

In Singapore, the exchange rate is 1 Singapore Dollar = 0.478 British Pounds.

In the United Kingdom, the exchange rate is 1 British Pound = 2.113 Singapore Dollars.

Brian would like to change 2500 Singapore Dollars into British Pounds.

How many fewer British Pounds will he get by changing his money in the United Kingdom?

\[
S$ 2500 ÷ S$ 1.80 \times 0.478 = £ 1115
\]

\[
S$ 2500 ÷ \frac{£}{S} 1.155 \times \frac{S}{£} 1 = \frac{£}{S} 1183.15117
\]

\[
\frac{£ 1155}{£ 1183.15117} = \frac{£}{S} 0.84083
\]

Answer \( £ 11.85 \). [2]
7. Matthew bought a new car. At the end of each year, the car’s value depreciates by 10%.
   The value, $C$, of the car $t$ years after being bought is given by
   \[ C = 85000 \times 0.9^t. \]
   (a) How much did Matthew pay for his new car?

   \[ \text{Answer (a)} \quad \$85000 \]

   (b) Find the percentage decrease in the value of his car at the end of three years.
   \[ 85000 \times 0.9^3 = 61965 \]
   \[ 85000 - 61965 = 23035 \]
   \[ \frac{23035}{85000} = 27.1\% \]

   \[ \text{Answer (b)} \quad \ldots \ldots \ldots \ldots \ldots \ldots \% \]

8. Two geometrically similar solids made from the same material have masses 3.60 kg and 12.15 kg respectively.
   Calculate the ratio of the area of the smaller solid to the area of the larger solid.

   \[ \left( \frac{3.60}{12.15} \right)^4 = \frac{A_1}{A_2} \quad \left( \frac{A_1}{A_2} \right)^3 = \frac{3.6}{12.15} \]

   \[ \frac{64}{729} = \frac{A_1}{A_2} \]
   \[ \frac{A_1}{A_2} = \frac{2}{3} \]
   \[ A_1 : A_2 = 2 : 3 \]

   \[ \left( \frac{A_1}{A_2} \right)^2 = \frac{A_1}{A_2} \]
   \[ \left( \frac{2}{3} \right)^2 = \frac{4}{9} \]

   \[ A_1 : A_2 \]

   \[ \frac{4}{9} \]

   \[ \text{Answer} \quad \frac{64}{729} \quad \times \]

   \[ 4 : 9 \]

   \[ [\text{Turn over} \quad 1] \]
Dawn invested some money in a savings account that was compounded every six months.
The rate of compound interest was 5% per annum.
At the end of the 4 years there was $14620.83 in her account.

How much did Dawn invest in the account at first?

\[
P (1 + \frac{r}{100})^n = \frac{S}{P} (1 + \frac{r}{100})^n
\]

\[
P = \frac{11999.97600}{2} = \frac{14620.83}{2}
\]

\[
P = $11,999.97600
\]

Answer: $12,000

A rectangle with length 40 cm is divided into five identical shaded rectangles and another six identical unshaded rectangles.

The shaded area makes up two-thirds of the unshaded area.

Find the lengths labelled \( x \) and \( y \).

\[
x + 6y = 40
\]

Let the length of each shaded rectangle be \( z \).

\[
5(xz) + 5(6yz) = 5x + 30y = 7.5xz = 25yz
\]

\[
x = 40 - 6y
\]

\[
7.5(40 - 6y) = 25yz
300z - 45yz = 25yz
300z - 45yz - 25yz = 0
300z - 70yz = 0
\]

\[
z(300 - 70y) = 0
\]

\[
300 - 70y = 0
y = \frac{300}{70}
\]

Answer: \( x = 14.3 \times 16 \text{ cm} \)

\( y = 4.29 \times 4. \text{ cm} \) [3]
11 Mr Tan decided to buy a laptop under a hire-purchase scheme. He would have paid $3906 in total under the scheme, which consists of a deposit of 15% of the selling price of the laptop plus 24 equal monthly payments of $140.25.

Let selling price be \( x \)

\[ 15\% \times x + 24 \times 140.25 = 3906 \]

\[ 15\% \times x = 540 \]

\[ x = 3600 \]

Answer $3600 \times \frac{264.5}{3000}$ [3]

12 Two points \( P \) and \( Q \) have position vectors \( p \) and \( q \) respectively, relative to an origin \( O \).

It is given that \( p = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \) and \( q = \begin{pmatrix} k \\ 0 \end{pmatrix} \).

Find \( \vec{OQ} \) in terms of \( k \),

(a) \( \vec{PQ} \) in terms of \( k \),

\[ \vec{PQ} = \vec{PO} + \vec{OQ} \]

\[ = -\begin{pmatrix} -3 \\ 4 \end{pmatrix} + \begin{pmatrix} k \\ 0 \end{pmatrix} \]

\[ = \begin{pmatrix} 3 + k \\ 4 \end{pmatrix} \]

Answer (a) \( \vec{PQ} = \begin{pmatrix} -3k/4 \\ 3k/4 \end{pmatrix} \) [1]

(b) the possible values of \( k \) if \( OP \) and \( OQ \) are two sides of a rhombus.

\[ |\vec{OP}| = \sqrt{(-2)^2 + (4)^2} \]

\[ = 5 \text{ or } -5 \]

Answer (b) \( k = \pm 5 \) or \( -5 \) [2]
13 \( \mathcal{E} = \{ \text{integers } x : 1 \leq x \leq 12 \} \)
\( A = \{ \text{factors of 12} \} \)
\( B = \{ \text{multiples of 4} \} \)

(a) Draw a Venn diagram to illustrate this information.

Answer (a)

(b) Describe in words what the set \((A \cup B)\) represents.

Answer (b) The set \((A \cup B)\) represents all the numbers that are not in Set \(A\) and \(B\), combined. \(X\) represents the set of numbers that are neither factors of 12 nor multiples of 4.

14 A train 45 m long passes through a tunnel 6 km long.

The average speed of the train is 27 km/h.

(a) Change 27 km/h into m/s.

\[
27 \text{ km/h} = \frac{27 \times 1000}{3600} \text{ m/s}
\]

\[= 7.5 \text{ m/s} \]

Answer (a) \(7.5\) m/s [1]

(b) Calculate the time taken for the train to pass completely through the tunnel.

Give your answer in minutes and seconds, to the nearest second.

\[
6000\text{m} + 45\text{m} = 6045\text{m}
\]
\[
\frac{6045\text{m}}{7.5\text{m/s}} = 806\text{s}
\]

\[= 13\text{ minutes } 26\text{ secs}\]

Answer (b) \(13\) minutes \(26\) seconds [3]
15. Simplify

(a) \( \frac{28x^2y^{-3} + 16x^3y^{-4}}{28x^2y^{-3} \div 16x^3y^{-4}} = \frac{28x^2y^{-1}}{16x^3y^{-1}} = \frac{7x^2y^{-1}}{4xy} \)

Answer (a) \( \frac{7}{4xy} \) \[2\] 

(b) \( \frac{2}{x-3} + \frac{3x}{x^2-9} = \frac{2(x+3)}{(x-3)(x+3)} + \frac{3x}{x^2-9} \)

\( = \frac{2x+6+3x}{(x-3)(x+3)} \)

\( = \frac{5x+6}{x^2-9} \)

Answer (b) \( \frac{5x+6}{x^2-9} \) \[2\]

16. The numbers 1 to 100 are arranged in a table as shown below.

A U-shaped, shaded frame can be placed around various numbers throughout the table.

<table>
<thead>
<tr>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>91</td>
<td>92</td>
<td>93</td>
<td>94</td>
<td>95</td>
<td>96</td>
<td>97</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
</tr>
</tbody>
</table>

The U-number is used to refer to the shaded frame that is drawn around a particular number.

For example, \( U_2 \) refers to the shaded frame shown above since it is drawn around the number 2.

(a) State the largest possible U-number.

Answer (a) \( \boxed{89} \times U_{89} \) \[1\]

(b) Write and simplify an expression, in terms of \( n \), for the sum of the numbers in \( U_n \).

\( n-1 + n+1 + n+9 + n+10 + n+11 = 5n+30 \)

Answer (b) \( 5n+30 \) \[2\]

(c) Find the sum of numbers in \( U_{75} \).

Answer (c) \( 405 \) \[1\]
The resistance of a wire, \( R \) ohms, is inversely proportional to the square of its diameter, \( d \) \( \mu \text{m} \).

(a) Sketch a resistance-diameter graph for the wire.

\[ R = \frac{1}{d^2} \]

For a fixed length of wire, the resistance is 25.6 ohms when the diameter is 50 \( \mu \text{m} \).

(b) Find the equation for \( R \) in terms of \( d \).

\[ R = \frac{k}{d^2} \]

\[ 25.6 = \frac{k}{25^{10}} \]

\[ k = 64000 \]

\[ R = \frac{64000}{d^2} \]

Answer (b) \( R = \frac{64000}{d^2} \) [2]

(c) Another wire of the same length has resistance 120 ohms. Calculate its diameter.

\[ 120 = \frac{64000}{d^2} \]

\[ d = 12.39401077 \]

\[ \approx 12.3 \]

Answer (c) \( 2.5 \ldots \mu \text{m} \) [1]
18 A composite container made from a cylinder and a cone has a vertical height of 30 cm. Water is poured into the empty container at a constant rate. It takes 60 seconds to fill up the entire container.

\[ V_{\text{cone}} = \frac{1}{3} V_{\text{cylinder}} \]

Total \[ V = 4 \text{ units} \]

\[ V_{\text{cone}} = 1 \text{ unit} \]

\[ \frac{1}{3} \times 60 \text{ s} = 20 \text{ s} \]

(a) Find the time taken to fill up the cone.

\[ \frac{1}{3} \pi r^2 h + \pi r^2 h = \text{Total} \]

Since the volume of the cone is \( \frac{1}{3} \) of the cylinder, the time taken to fill up the cone will be \( \frac{1}{3} \) of the total time.

\[ \frac{1}{3} \times 60 \text{ s} = 20 \text{ s} \]

Answer (a) \( 20 \times 15 \) seconds [2]

(b) Sketch the graph of how the depth of water in the container varies during the 60 seconds.

Answer (b)
The diagram shows the box-and-whisker plots for the distributions of the speeds, in km/h, of 100 vehicles before and after a speed camera was placed on an expressway.

(a) Find the interquartile range of the distribution before the camera was placed.

Answer \( (a) \) \( 10 \) \( \text{km/h} \) [1]

(b) Find the interquartile range of the distribution after the camera was placed.

Answer \( (b) \) \( 9 \) \( \text{km/h} \) [1]

(c) After the camera was placed, 25% of the motorists were issued with traffic summonses for exceeding the speed limit of the expressway.
What is the speed limit of the expressway?

Answer \( (c) \) \( 80 \) \( \text{km/h} \) [1]

(d) Has the speed camera been effective in regulating the speed limit of the expressway?

Explain your answer by comparing the distributions of the speeds before and after the camera was placed.

Answer \( (d) \) Yes, the speed camera has been effective. The upper limit of the motorists' speeds after the camera was placed is 85 km/h. Before the camera was placed, the upper limit was 92 km/h. \( x \) Yes, it has been effective. The median and the interquartile range are lower after the camera was placed.
Abraham and Lincoln sent out some letters, postcards and greeting cards.

The number of letters, postcards and greeting cards is shown in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Letters</th>
<th>Postcards</th>
<th>Greeting cards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abraham</td>
<td>4</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>Lincoln</td>
<td>7</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

The postage for each letter, postcard and greeting card is $0.30, $0.40 and $0.50 respectively.

(a) Write out a $2 \times 3$ matrix $P$ and a column matrix $Q$ to represent the above information.

Answer

(a) $P = \begin{pmatrix} 4 & 9 & 2 \\ 7 & 3 & 3 \end{pmatrix}$

$Q = \begin{pmatrix} 5.2 \\ 3.4 \end{pmatrix}$

(b) Evaluate the matrix $S = PQ$

$S = PQ$

$= \begin{pmatrix} 4 & 9 & 2 \\ 7 & 3 & 3 \end{pmatrix} \begin{pmatrix} 0.30 \\ 0.40 \\ 0.50 \end{pmatrix}$

$= \begin{pmatrix} 5.8 \\ 4.8 \end{pmatrix}$

Answer

(b) $S = \begin{pmatrix} 5.8 \\ 4.8 \end{pmatrix}$

(c) State what the elements of $S$ represent.

Answer

(c) The elements of $S$ represents the amount of money Abraham and Lincoln spent on postage respectively. They represent the total postage paid by Abraham and Lincoln respectively.

[Turn over]
21. The diagram shows a spinner with nine numbered sectors of identical sizes.

Each time the pointer is spun, it is equally likely to stop on one of the sectors.

(a) The pointer is spun once.

Find the probability that it stops on an odd number.

\[
Answer \ (a) \ \frac{2}{3} \quad [1]
\]

(b) Aysha spins the pointer twice.

Find the probability that the pointer lands on a prime number at least once.

\[
1 - \left(\frac{1}{9} \times \frac{1}{9}\right) = \frac{80}{81}
\]

\[
1 - \left(\frac{4}{9} \times \frac{4}{9}\right) = \frac{65}{81}
\]

\[
Answer \ (b) \ \frac{802}{81} \times \frac{65}{81} \quad [2]
\]

(c) Natasha spins the pointer twice.

Her score is found from the difference of the numbers from her two spins.

Find the probability that her score is 0.

\[
\left(\frac{2}{9} \times \frac{1}{9}\right) + \left(\frac{6}{9} \times \frac{2}{9}\right) + \left(\frac{4}{9} \times \frac{4}{9}\right) + \left(\frac{1}{9} \times \frac{2}{9}\right) + \left(\frac{4}{9} \times \frac{1}{9}\right) = \frac{19}{81}
\]

\[
Answer \ (c) \ \frac{19}{81} \quad [2]
\]
22 Benjamin has 165 identical cubes of sides 2 cm.

(a) He uses some of the cubes to make a cuboid which measures 8 cm by 10 cm by 14 cm. Calculate the total surface area of the cuboid.

\[ \frac{8 \times 4 \times 4 + (10 \times 14 \times 2)}{728} \]

\[ 10 \times 14 \times 2 + 8 \times 14 \times 2 + 8 \times 10 \times 2 = 664 \text{ cm}^2 \]

Answer (a) \( 728 \times \text{cm}^2 \) [2]

(b) Benjamin makes the largest cube possible using some of the 165 cubes. He then makes the largest cube possible from the unused cubes.

How many cubes will he have left over after making the second cube?

\[ \sqrt{350} = 18 \ldots \]

\[ 350 - 324 = 6 \]

\[ 6 \div 2 = 3 \]

\[ 3 \times 3 = 9 \]

Answer (b) \( 9 \times 13 \) [2]

(c) Benjamin uses all 165 cubes to make a cuboid.

Find the dimensions of the cuboid.

\[ \begin{array}{c|c}
3 & 165 \\
\hline
5 & 55 \\
11 & 11 \\
\end{array} \]

Answer (c) \( 2 \times \text{cm} \) by \( 2 \times \text{cm} \) by \( 330 \times \text{cm} \) [2]
23. Expressions for the lengths of three sides of a quadrilateral are shown on the diagram below.
All lengths are in centimetres.

(a) The perimeter of this quadrilateral is given by the expression \((11x+19)\) cm.
Find an expression, in terms of \(x\), for the length of \(DC\).
Give your expression in its simplest form.
\[11x+19 = 3x+4 + 16-x + 4x + 1 + DC\]
\[\therefore 5x - 2 = DC\]

Answer (a) \(5x - 2\) cm [2]

(b) Given that \(ABCD\) is a parallelogram and that \(AB = AD\), calculate the perimeter of \(ABCD\).
\[AB = AD\]
\[4x+1 = 3x+4\]
\[x = 3\]
\[4(3)+1 = 13\]
\[3(3)+1 = 10\]
\[13 	imes 4 = 52\]

Answer (b) \(52\) cm [2]

(c) Calculate the area of \(ABCD\) if \(AC = (10x-6)\) cm.
\[AC = 10(3)-6\]
\[= 24\]
\[AB = 4(3)+1\]
\[= 13\]
\[24^2 = 13^2 + b^2 - 2 \times 13 \times b \cos \angle ABC\]
\[576 = 338 - 2 \times 13 \times b \cos \angle ABC\]
\[\angle ABC = 134\.]\]
\[\frac{1}{2} \times (13) \times (13) \sin 134 = 60\]

Answer (c) \(2\times 8 \times 120\) cm² [5]

END OF PAPER
Anglo-Chinese School

(Independent)

PRELIMINARY EXAMINATION 2015
YEAR 4 EXPRESS
ADDITIONAL MATHEMATICS
PAPER 1
30 July 2015

Thursday

2 hours

Additional Materials:
Answer Paper (8 sheets)

READ THESE INSTRUCTIONS FIRST

Write your candidate number in the spaces provided on the answer paper/answer booklet.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all the questions.
Write your answers on the separate answer paper provided.
If you use more than one sheet of paper, fasten the sheets together.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This question paper consists of 6 printed pages
Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion
\[
(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,
\]
where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. TRIGONOMETRY

Identities
\[
\begin{align*}
\sin^2 A + \cos^2 A &= 1 \\
\sec^2 A &= 1 + \tan^2 A \\
\cos ec^2 A &= 1 + \cot^2 A \\
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\tan(A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A \\
\tan 2A &= \frac{2\tan A}{1 - \tan^2 A}
\end{align*}
\]

Formulae for \( \triangle ABC \)
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\Delta = \frac{1}{2} ab \sin C
\]
1. The sides \( AB \) and \( BC \) of a triangle are \((2\sqrt{3} + 2\sqrt{6})\ cm\) and \((8\sqrt{2} - 2)\ cm\) respectively and \(\angle ABC = 60^\circ\). Show that the area of \(\triangle ABC\) is \((p + q\sqrt{2})\ cm^2\) where \(p\) and \(q\) are constants to be determined. [3]

2. Find the range of values of \(k\) for which \(x^2 + 2k(k + x) > 3k + 4\) for all real values of \(x\). [3]

3. Solve the equation \(|2x - 3| + 6x = |9 - 6x| + 4\). [4]

4. The polynomial \(f(x)\) is divisible by \((2x - 3)\) and leaves a remainder of \(-2\) when divided by \((x - 1)\). Find the remainder when \(f(x)\) is divided by \(2x^2 - 5x + 3\). [4]

5. The diagram below shows the graph of \(y = c + a \cos bx\) where \(a, b\) and \(c\) are constants.

   ![Graph of \(y = c + a \cos bx\)]

   (i) Use the graph to determine the value of \(a\), of \(b\) and of \(c\). [3]

   (ii) By using the values of \(a\), \(b\) and \(c\) found in (i), determine the equation of the straight line that needs to be drawn on the same diagram to solve

   \[
   \sec bx = \frac{a \pi}{x - \pi c}
   \]

   [2]
6  (i) Given that \[ \frac{x^3 - 2x^2 - x + 3}{x^2 - 2x + 1} = x + \frac{A}{x-1} + \frac{B}{(x-1)^2} \], where \(A\) and \(B\) are constants. Find the value of \(A\) and of \(B\). [4]

(ii) Hence find \[ \int \frac{x^3 - 2x^2 - x - 4}{x^2 - 2x + 1} \, dx. \] [3]

7  The roots of the equation \(2x^3 - 8x + 3 = 0\) are \(\alpha\) and \(\beta\).

(i) Express \(\alpha^2 - \alpha\beta + \beta^2\) in terms of \(\alpha + \beta\) and \(\alpha\beta\). [1]

(ii) Find a quadratic equation with integer coefficients whose roots are \(\alpha^3\) and \(\beta^2\). [6]

8  (a) Find all the values of \(x\) between \(0^\circ\) and \(360^\circ\) for which
\[ \frac{1}{\sec^2 x} + 3 \sin \frac{x}{2} \cos \frac{x}{2} = 0. \] [4]

(b) Find all the exact angles between \(0\) and \(\pi\), which satisfy the equation
\[ \sin (x - \frac{\pi}{3}) - \cos \frac{\pi}{10} = 0. \] [4]

9  A curve is such that \(\frac{d^2 y}{dx^2} = 16\cos^2 2x - 4\sin 4x - 8\) and the gradient of the normal to the curve at \(x = \frac{\pi}{4}\) is 1.

(i) Find \(\frac{dy}{dx}\). [3]

(ii) Hence solve \(\frac{d^2 y}{dx^2} = 2\) for \(0 \leq x \leq 1\). [6]

10  (i) Solve \(2 + \ln (4 - x) = 0.\) [2]

(ii) Sketch the graph of \(y = 2 + \ln (4 - x)\) showing clearly the asymptote and the \(y\)-intercept. [3]

(iii) Find the area of the region bounded by the curve \(y = 2 + \ln (4 - x)\), the \(x\)-axis, the \(y\)-axis and the line \(x = 3\). [5]
The diagram shows a quadrilateral $OABC$.

The coordinates of $A$ are $(2k, 3k)$ and the length of $OA$ is $\sqrt{52}$ units.

(i) Calculate the value of $k$. [2]

$AB$ is perpendicular to $OA$ and $B$ lies on the $y$-axis.

(ii) Find the coordinates of $B$. [3]

$CM$, the perpendicular bisector of $AB$, cuts the $y$-axis at $N$ and $OC$ is parallel to $AB$.

Find

(iii) the coordinates of $C$. [3]

(iv) the ratio of the area of the triangle $OCN$ to the area of the triangle $OCB$. [2]
The diagram above shows a square piece of cardboard paper $ABCD$ of side $4\sqrt{2}$ metres.

Triangles $AED$, $AFB$, $DHC$ and $BGC$ are cut off leaving a figure in the shape of a square $EFGH$ of side $2x$ metres with 4 identical isosceles triangles attached to the sides. The height of each triangle is $h$ metres. Mark wants to fold the paper to make a pyramid with $EFGH$ as the base.

(i) Show that $h = 4 - x$. [2]

(ii) Show that the volume of the pyramid, $V$ m$^3$, is given by $V = \frac{8}{3} x^2 \sqrt{4 - 2x}$. [4]

(iii) Hence find the maximum volume of the pyramid. [4]

(Proof of maximum is not required.)

END OF PAPER 1
1. \[ \text{Area} = \frac{1}{2} (2\sqrt{3} + 2\sqrt{6})(8\sqrt{2} - 2) \sin 60 \]
\[ = (\sqrt{3} + \sqrt{6})(8\sqrt{2} - 2) \frac{\sqrt{3}}{2} \]
\[ = (6\sqrt{6} - 2\sqrt{3} + 8\sqrt{12}) \frac{\sqrt{3}}{2} \]
\[ = 9\sqrt{2} + 14\sqrt{3} \times \frac{\sqrt{3}}{2} \]
\[ = 9\sqrt{2} + 21 \]

2. \[ x^2 + 2k(k + x) > 3k + 4 \]
   \[ D < 0 \]
   \[ x^2 + 2k^2 + 2kx - 3k - 4 > 0 \]
   \[ 4k^2 - 4(2k^2 - 3k - 4) < 0 \]
   \[ -4k^2 + 12k + 16 < 0 \]
   \[ k^2 - 3k - 4 > 0 \]
   \[ k < -1 \text{ or } k > 4 \]

3. \[ 2x - 3 + 6x = |9 - 6x| + 4 \]
   \[ 2x - 3 - |9 - 6x| = 4 - 6x \]
   \[ 2x - 3 - 3|x - 2| = 4 - 6x \]
   \[ 2x - 3 = 3x - 2 \]
   \[ 2x - 3 = 3x - 2 \text{ or } 2x - 3 = 2 - 3x \]
   \[ x = -1 \text{ (no)} \text{ or } x = 1 \]

4. \[ f(x) = (2x^2 - 5x + 3)Q(x) + ax + b \]
   \[ = (2x - 3)(x - 1)Q(x) + ax + b \]
   \[ \frac{3}{2}a + b = 0 \]
   \[ 3a + 2b = 0 - - - - - - - - (1) \]
   \[ a + b = -2 - - - - - - - (2) \]

Solve: \[ a = 4, \ b = -6 \]
The remainder is \[ 4x - 6 \]

5. (i) \[ y = c + a \cos bx \]
   \[ a = -3 \]
   Period is \[ \pi \]. Therefore \[ b = 2 \]
   \[ c = 2 \]
(ii) \[ \sec bx = \frac{a \pi}{x - \pi c} \\\n\cos 2x = \frac{x - 2\pi}{-3\pi} \\\n-3\pi \cos 2x = x - 2\pi \\\n2\pi - 3\pi \cos 2x = x \\\n2 - 3\cos 2x = \frac{x}{\pi} \\\n\text{Draw } y = \frac{x}{\pi} \]

6 (i) \[ \frac{x^3 - 2x^2 - x + 3}{x^2 - 2x + 1} = x + \frac{A}{x - 1} + \frac{B}{(x - 1)^2} \]
By long div, \[ \frac{x^3 - 2x^2 - x + 3}{x^2 - 2x + 1} = x + \frac{3 - 2x}{(x - 1)^2} \]
\[ \frac{3 - 2x}{(x - 1)^2} = \frac{A}{x - 1} + \frac{B}{(x - 1)^2} \]
\[ 3 - 2x = A(x - 1) + B \]
Sub \( x = 1; B = 1 \)
Compare \( x: A = -2 \)

(ii) \[ \int \frac{x^3 - 2x^2 - x - 4}{x^2 - 2x + 1} \, dx = \int \left( x + \frac{2}{x - 1} + \frac{1}{(x - 1)^2} - \frac{7}{(x - 1)^3} \right) \, dx \]
\[ = \frac{x^2}{2} - 2\ln(x - 1) + \frac{6}{x - 1} + c \]

7 (i) \[ 2x^2 - 8x + 3 = 0 \]
\[ \alpha^2 - \alpha \beta + \beta^2 = (\alpha + \beta)((\alpha + \beta)^2 - 3\alpha \beta) \]
\[ \alpha + \beta = 4 \]
\[ \alpha \beta = \frac{3}{2} \]
\[ \alpha^3 + \beta^3 = (4)(16 - \frac{9}{2}) \]
\[ = 46 \]
\[ \alpha^3 \beta^3 = \frac{27}{8} \]
\[ x^2 - 46x + \frac{27}{8} = 0 \]
\[ 8x^2 - 368x + 27 = 0 \]
(a) \[
\frac{1}{\sec^2 x} + 3\sin \frac{x}{2} \cos \frac{x}{2} = 0
\]
\[
\cos^2 x + \frac{3}{2} \sin x = 0
\]
\[
2\sin^2 x - 3\sin x - 2 = 0
\]
\[
\sin x = -\frac{1}{2}
\]
\[
x = 210^\circ, 330^\circ
\]

(b) \[
\sin(x - \frac{\pi}{5}) = \cos \frac{\pi}{10}
\]
\[
= \sin \left(\frac{\pi}{2} - \frac{\pi}{10}\right)
\]
\[
= \sin \frac{2\pi}{5}
\]
\[
x - \frac{\pi}{5} = \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}
\]
\[
x = \frac{3\pi}{5}, \frac{4\pi}{5}
\]

(i) \[
\frac{d^2 y}{dx^2} = 16\cos^2 2x - 4\sin 4x - 8
\]
\[
\frac{dy}{dx} = \int (16\cos^2 2x - 4\sin 4x - 8) \, dx
\]
\[
= \int (8\cos 4x - 4\sin 4x) \, dx
\]
\[
= 2\sin 4x + \cos 4x + c
\]

**Gradient of tangent = -1**

\[-1 = 2\sin \pi + \cos \pi + c\]
\[c = 0\]
\[
\frac{dy}{dx} = 2\sin 4x + \cos 4x
\]

(ii) \[
2\sin 4x + \cos 4x = R \sin(4x + \alpha)
\]
\[R = \sqrt{5} \quad \text{and} \quad \alpha = 0.4636
\]
\[
\sqrt{5}\sin(4x + 0.4636) = 2
\]
\[
\sin(4x + 0.4636) = 0.8944
\]
\[
4x + 0.4636 = 1.1071, 2.0345,
\]
\[x = 0.161, 0.393
\]
(i) \[2 + \ln(4 - x) = 0\]
\[
\ln(4 - x) = -2
\]
\[4 - x = e^{-2}
\]
\[x = 3.86\]

(ii) \[
\text{Area} = 2 \times 3 + \int_{2}^{3} (4 - e^{-y}) \, dy
\]
\[= 6 + \left[ 4y - e^{-y} \right]_{2}^{3}
\]
\[= 6 + \left[ 4(2 + \ln 4) - e^{-3} - (8 - 1) \right]
\]
\[= 6 + \left[ 8 + 4 \ln 4 - 4 - 7 \right]
\]
\[= 8.55 \text{ units}^2\]

(iii) \[y = 2 + \ln(4 - x)\]
\[4 - x = e^{-y}\]
\[x = 4 - e^{-y}\]

11 (i) \[\sqrt{4k^2 + 9k^2} = \sqrt{52}\]
\[13k^2 = 52\]
\[k = 2\]
(ii) Grad of OA = $\frac{3}{2}$
Grad of AB = $\frac{2}{3}$

\[ y = -\frac{2}{3} x + c \]

\[ 6 = -\frac{8}{3} + c \]

\[ c = \frac{26}{3} \]

\[ B(0, \frac{26}{3}) \]

\[ M = \left(2, \frac{22}{3}\right) \]

(iii) \[ \triangle ABC \]

\[ C (-2, \frac{4}{3}) \]

\[ M \text{ is midpt of } AB, \ MN// AO \]

By midpt thm, \( N \) is midpt of \( OB \)

\[ \frac{\triangle OCN}{\triangle OCB} = \frac{1}{2} \]

(iv) \[ 12 \]

(i) \[ (h + x)^2 + (h + x)^2 = (4\sqrt{2})^2 \]

\[ 2(h + x)^2 = 32 \]

\[ (h + x) = 4 \]

\[ h = 4 - x \]

(ii) Let ht of pyramid be \( y \)
(iii)

\[ y^2 + x^2 = (4 - x)^2 \]
\[ y^2 = 16 - 8x \]
\[ y = \sqrt{16 - 8x} \]

\[ V = \frac{1}{3} 4x^2 \sqrt{16 - 8x} \]
\[ = \frac{4}{3} x^2 \frac{2}{x} \sqrt{4 - 2x} \]
\[ = \frac{8}{3} x^2 \sqrt{4 - 2x} \]

\[ \frac{dV}{dx} = \frac{8}{3} x^2 \left[ \frac{1}{2} (4 - 2x)^{-\frac{1}{2}} + \frac{1}{2} x \sqrt{4 - 2x} \right] \]
\[ = \frac{8}{3} x \left[ -x + 2(4 - 2x) \right] \]
\[ = \frac{8}{3} x \sqrt{4 - 2x} \]
\[-x + 8 - 4x = 0 \]
\[ x = 1.6 \text{ m} \]

\[ \text{Max } V = 6.11 \]
PRELIMINARY EXAMINATION 2015
YEAR 4 EXPRESS
ADDITIONAL MATHEMATICS
PAPER 2
4 August 2015
2 hours 30 minutes

Additional Materials:
Answer Paper (10 sheets)

READ THESE INSTRUCTIONS FIRST

Write your candidate number in the spaces provided on the answer paper/answer booklet.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all the questions.
Write your answers on the separate answer paper provided.
If you use more than one sheet of paper, fasten the sheets together.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This question paper consists of 6 printed pages
Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion
\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]
where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. TRIGONOMETRY

Identities
\[
\sin^2 A + \cos^2 A = 1
\]
\[
\sec^2 A = 1 + \tan^2 A
\]
\[
\csc^2 A = 1 + \cot^2 A
\]
\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
\sin 2A = 2 \sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]
\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \triangle ABC \)
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\Delta = \frac{1}{2} abc \sin C
\]
1. Given that $\int_{0}^{3} f(x)\,dx = 12$ and $\int_{2}^{5} f(x)\,dx = 4$, evaluate

$$\int_{2}^{5} [2x - f(x)]\,dx + \int_{0}^{1} f(x)\,dx.$$  \hspace{1cm} [3]

2. The equations of two curves are $y = \frac{1}{4}x^2$ for $x > 0$ and $y = 4x^{-\frac{1}{3}}$ for $x > 0$.

(i) Find the coordinates of the point(s) of intersection of the graphs.  \hspace{1cm} [2]

(ii) Sketch these graphs on the same axes, indicating the point(s) of intersection clearly.  \hspace{1cm} [2]

3. Variables $x$ and $y$ are related by the equation $y = \frac{p - x}{x + q}$, where $p$ and $q$ are constants.

When the graph of $x(1 + y)$ against $y$ is drawn, a straight line is obtained. The line has a gradient of $-1\frac{1}{3}$ and passes through the point $(3, 2)$.

(i) Calculate the value of $p$ and of $q$.  \hspace{1cm} [4]

(ii) Given that this line passes through $(6, k)$, find $x$ in terms of $k$.  \hspace{1cm} [2]

4. The equation of a curve is given by $y = \frac{\ln(x-3)^2}{2x-6}, x > 3$.

(i) Find $\frac{dy}{dx}$.  \hspace{1cm} [2]

(ii) Find the set of values of $x$ for which $y$ is a decreasing function.  \hspace{1cm} [2]

(iii) Evaluate $\int_{1}^{5} \frac{\ln\sqrt{x-3}}{(x-3)^2} \,dx$, leaving your answer in the form $a + b\ln 2$.  \hspace{1cm} [3]

where $a$ and $b$ are constants.
5 In the diagram, $AB$ is a tangent to the circle at the point $B$, $BED$ is a straight and $AB \parallel DC$. The points $A$, $E$ and $C$ lie on a straight line and $AE : EC = 2 : 1$.

(i) Prove that $\angle BCE = \angle BDC$. [2]

(ii) Hence, show that $\triangle BCE$ is similar to $\triangle BDC$. [2]

(iii) Prove that $3AE \times CD = AB \times AC$. [3]

6 The equation of a circle, $C_1$, is $x^2 + y^2 + kx - (k + 2)y + 7 = 0$, where $k$ is a constant.

(i) Find the coordinates of the centre in terms of $k$. [2]

Given that the centre of the circle lies on the line $2x + 5y - 11 = 0$,

(ii) Show that $k = 4$. [2]

(iii) Find the equation of the circle, $C_2$, which is a reflection of $C_1$ in the line $x = 1$. [3]

(iv) Explain why the two circles do not intersect each other. [1]
7. The height of the tides at a certain place can be modelled by the equation \( h = 2\left(3.25 - \sin kt\right) \), where \( k \) is a constant, and \( t \) is the time in hours after midnight. The average time difference between high tides is 14.5 hours.

(i) Explain why this model suggests that the lowest tide for the day is 4.5 m. [1]

(ii) Show that the value of \( k \) is \( \frac{4\pi}{29} \). [2]

(iii) Find the height of the tide at 2 am. [1]

(iv) Find the time for which the height of the tide first reaches 7.0 m, leaving your answer in 24 hour notation. [4]

8. (a) Given that \( \frac{d}{dx} [F(x)] = \frac{9}{2} \sqrt{3x-1} - \frac{3}{\sqrt{3x-1}} \), evaluate \( F(3) - F(1) \), giving your answer in the form \( k\sqrt{2} \). [4]

(b) The equation of a curve is given by \( y = \csc^2 \left( \frac{x}{2} - \frac{\pi}{6} \right) \), where \( 0 < x < \frac{\pi}{2} \).

Given that \( x \) is increasing at 0.3 radian per second, find the rate of change of \( y \) with respect to time when \( x = \frac{5\pi}{6} \). [4]

9. In the expansion of \( \left(x^2 - \frac{k}{x}\right)^8 \), where \( k \) is a constant, the coefficient of \( \frac{1}{x} \) is -4608.

(i) Show that \( k = 2 \). [3]

(ii) Explain why there is no term independent of \( x \) in the expansion of \( \left(x^2 - \frac{k}{x}\right)^8 \). [1]

(iii) Find the coefficient of \( x^5 \) in the expansion of \( \left(2x^3 + \frac{1}{x}\right) \left(x^4 - \frac{k}{x}\right)^8 \). [4]

10. The gradient of a curve is \( \frac{e^{2x} + 1}{e^{2x}} \) and \( P(0, -1) \) is a point on the curve.

(i) Show that the curve has no stationary point. [2]

(ii) Find the equation of the curve. [2]

The tangent and normal to the curve at \( P \) intersect the x-axis at \( Q \) and \( R \) respectively.

(iii) Find the area of the triangle \( PQR \). [5]
11 (a) Given that \(2^{x-3} = \frac{1}{4^x}\), evaluate \(16^x\). \[3\]

(b) Solve the equation \(\log_a 5 - \frac{2}{\log_a 2} = \log_a \left(\frac{25}{4}\right)\). \[6\]

12 A particle \(P\) travelling in a straight line passes a fixed point \(O\). Its velocity, \(v\) \(\text{ms}^{-1}\), is given by the equation \(v = t^2 - 6t + 8\), where \(t\) is the time in seconds after passing \(O\).

(i) Find the times when \(P\) is instantaneously at rest. \[2\]

(ii) Find the total distance travelled by \(P\) when its velocity reaches \(8\) \(\text{ms}^{-1}\) again. \[5\]

(iii) Will \(P\) return to \(O\) in the course of its motion? Explain your answer clearly. \[2\]

The particle \(P\) is at point \(A\) when its velocity reaches \(8\) \(\text{ms}^{-1}\) again. It continues its motion at this velocity for 1 second and then decelerates uniformly until it comes to a complete rest at point \(B\) in another 2 seconds.

(iv) Find the distance \(AB\). \[2\]

13 (i) Prove that \(\tan \frac{\theta}{2} + \cot \frac{\theta}{2} = 2 \cosec \theta\). \[3\]

(ii) Hence, solve \(\tan \theta + \cot \theta = (\cosec 4\theta)(\sin 2\theta + \cos 2\theta)\) for \(0^\circ \leq \theta \leq 180^\circ\). \[5\]

Given that \(2\tan A + 2\cot A = 5\) and \(0 < A < \frac{\pi}{4}\),

(iii) show that \(\cos 2A = \frac{3}{5}\). \[2\]

(iv) Hence, find the exact value of \(\cos (2A + \frac{\pi}{6})\). \[2\]

END OF PAPER 2
### Answers

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2(i)</td>
<td>8, 1</td>
</tr>
<tr>
<td>2(ii)</td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>3(i)</td>
<td>$p = 6, \quad q = 1 \frac{1}{3}$</td>
</tr>
<tr>
<td>3(ii)</td>
<td>$x = \frac{k}{7}$</td>
</tr>
<tr>
<td>4(i)</td>
<td>$\frac{dy}{dx} = \frac{1 - \ln(x - 3)}{(x - 3)^2}$</td>
</tr>
<tr>
<td>4(ii)</td>
<td>$x &gt; e + 3 = 5.72$</td>
</tr>
<tr>
<td>4(iii)</td>
<td>$\frac{1}{2} \left( \frac{1 + \ln 2}{2} \right) = 1 - \frac{\ln 2}{4}$</td>
</tr>
<tr>
<td>5</td>
<td>Proof</td>
</tr>
<tr>
<td>6(i)</td>
<td>$\left( -\frac{k}{2}, \frac{k + 2}{2} \right)$</td>
</tr>
<tr>
<td>6(ii)</td>
<td>Proof</td>
</tr>
<tr>
<td>6(iii)</td>
<td>$(x - 4)^2 + (y - 3)^2 = 6$</td>
</tr>
</tbody>
</table>
| 6(iv) | Let $d$ = distance from $P_1$ to $P_2 = 6$
Let $R$ = radius of $C_1$ + radius of $C_2 = \sqrt{6} + \sqrt{6} = 4.89$
Since $R < d$, the two circles do not intersect each other. |
<p>| 7(i) | Lowest tide occurs when $\sin kt = 1$, lowest tide $= 4.5$ m |
| 7(ii) | Proof |
| 7(iii) | 4.98 m |
| 7(iv) | 0750 |
| 8(a) | $12\sqrt{2}$ |</p>
<table>
<thead>
<tr>
<th>8(b)</th>
<th>$-0.6 \text{ radian/sec}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9(i)</td>
<td>Proof</td>
</tr>
<tr>
<td>9(ii)</td>
<td>$27 - 4r \neq 0 \text{ as } r \text{ must be an integer } \Rightarrow \text{ no independent term.}$</td>
</tr>
<tr>
<td>9(iii)</td>
<td>$-3840$</td>
</tr>
<tr>
<td>10(i)</td>
<td>Proof</td>
</tr>
<tr>
<td>10(ii)</td>
<td>$y = x - \frac{1}{2} e^{-2x} - \frac{1}{2}$</td>
</tr>
<tr>
<td>10(iii)</td>
<td>$\frac{1}{4} \text{ units}^2$</td>
</tr>
<tr>
<td>11(a)</td>
<td>32</td>
</tr>
<tr>
<td>11(b)</td>
<td>$x = 2, \quad \frac{1}{2}$</td>
</tr>
<tr>
<td>12(i)</td>
<td>$t = 2 \text{ or } t = 4$</td>
</tr>
<tr>
<td>12(ii)</td>
<td>$14\frac{2}{3} m$</td>
</tr>
<tr>
<td>12(iii)</td>
<td>Proof</td>
</tr>
<tr>
<td>12(iv)</td>
<td>$16 m$</td>
</tr>
<tr>
<td>13(i)</td>
<td>Proof</td>
</tr>
<tr>
<td>13(ii)</td>
<td>$\theta = 35.8^\circ, \quad 125.8^\circ$</td>
</tr>
<tr>
<td>13(iii)</td>
<td>Proof</td>
</tr>
<tr>
<td>13(iv)</td>
<td>$\frac{3\sqrt{3} - 4}{10}$</td>
</tr>
</tbody>
</table>
Marking Scheme (Additional Mathematics Paper 2/ Prelim Examination 2015)

| 3(ii) | $x(1 + y) = k$
|       | $7x = k$
|       | $x = \frac{k}{7}$

| 4(i)  | $y = \frac{\ln(x-3)^2}{2x-6} = \frac{\ln(x-3)}{x-3}$
|       | $\frac{dy}{dx} = \frac{(x-3)\left(\frac{1}{x-3}\right) - \ln(x-3)}{(x-3)^2}$
|       | $= \frac{1 - \ln(x-3)}{(x-3)^2}$

| 4(ii)  | For $\frac{dy}{dx} < 0$, $1 - \ln(x-3) < 0$
|       | $\Rightarrow \ln(x-3) > 1$
|       | $\Rightarrow x > e + 3 = 5.72$

| 4(iii) | $\int_{x}^{e} \frac{1 - \ln(x-3)}{(x-3)^2} \, dx = \left[\frac{\ln(x-3)}{x-3}\right]_{4}^{5}$
|       | $\int_{x}^{5} \frac{1}{(x-3)^2} \, dx - \int_{x}^{5} \frac{\ln(x-3)}{(x-3)^2} \, dx = \left[\frac{\ln(x-3)}{x-3}\right]_{4}^{5}$
|       | $\int_{x}^{5} \frac{\ln(x-3)}{x-3} \, dx = \int_{x}^{5} \frac{1}{(x-3)^2} \, dx - \left[\frac{\ln(x-3)}{x-3}\right]_{4}^{5}$
|       | $= \left[-\frac{1}{x-3}\right]_{4}^{5} - \left[\frac{\ln(x-3)}{x-3}\right]_{4}^{5}$
|       | $= \frac{1}{2} - \frac{\ln 2}{2}$

| 5(i)   | $\angle BDC = \angle ABD \quad \text{(Alternate angles, AB // DC)}$
|       | $\angle ABD = \angle BCE \quad \text{(Alternate Segment Theorem)}$
|       | $\therefore \angle BCE = \angle BDC$

| 5(ii)  | In $\triangle BCE$ and $\triangle BDC$,
|       | $\angle BCE = \angle BDC \quad \text{(From (i))}$
|       | $\angle CBE = \angle DBC \quad \text{(Common angles)}$
|       | $\therefore \triangle BCE$ and $\triangle BDC$ are similar triangles \text{ (AAA property)}$
5(iii) \( \triangle AEB \) and \( \triangle CED \) are similar triangles (AAA property)

\[
\begin{align*}
\frac{AE}{CE} &= \frac{AB}{CD} \\
\frac{AE}{3AC} &= \frac{AB}{CD} \quad \text{(Given AE:EC = 2:1)} \\
3AE \times CD &= AB \times AC
\end{align*}
\]

6(i) 
\[
\begin{align*}
a &= -\frac{k}{2} \\
b &= \frac{k+2}{2} \\
Centre &= \left(-\frac{k}{2}, \frac{k+2}{2}\right)
\end{align*}
\]

6(ii) 
\[
2\left(-\frac{k}{2}\right) + 5\left(\frac{k+2}{2}\right) - 11 = 0
\]
\[
k = 4
\]

6(iii) 
\[
C_1 = (-2, 3), \quad r = \sqrt{(-2)^2 + 3^2 - 7} = \sqrt{6}
\]

Let \( P_1 \) = Centre of \( C_2 \)

\[
C_2 = (4, 3)
\]

Equation of \( C_1 \): \((x-4)^2 + (y-3)^2 = 6\)

6(iv) 
Let \( d = \) distance from \( P_1 \) to \( P_2 \) = 6

Let \( R = \) radius of \( C_1 \) + radius of \( C_2 = \sqrt{6} + \sqrt{6} = 4.89 \)

Since \( R < d \), the two circles do not intersect each other.

7(i) Lowest tide occurs when \( \sin kt = 1 \), lowest tide = 4.5 m

7(ii) Period between high tides = 14.5 hours

\[
\frac{2\pi}{k} = 14.5
\]

\[
2\pi = k
\]

\[
k = \frac{4\pi}{29}
\]

7(iii) 
\[
h = 2(3.25 - \sin \frac{8\pi}{29})
\]

\[
h = 4.98 m
\]

7(iv) When \( h = 7.0 \)

\[
7.0 = 2(3.25 - \sin \frac{4\pi t}{29})
\]

\[
\sin \frac{4\pi t}{29} = -0.25
\]

\[
\frac{4\pi}{29} t = 3.394
\]

\[
\Rightarrow t = 7.833 = 7h 50 \text{ min}
\]

The time is 0750
8(a)

\[ F(3) - F(1) = \frac{1}{3} \left( \frac{9}{2} \sqrt{3x-1} - \frac{3}{\sqrt{3x-1}} \right) \]

\[ = \left[ \frac{9}{2} \frac{3}{(3x-1)^2} \right]^{3} - \left[ \frac{9}{2} \frac{1}{-2} \right]^{1} \]

\[ = \left[ \frac{3}{(8)^2 - 2(8)^2} \right] - \left[ \frac{1}{(2)^2 - 2(2)^2} \right] \]

\[ = (8\sqrt{8} - 2\sqrt{8}) - (2\sqrt{2} - 2\sqrt{2}) = 6\sqrt{8} \]

\[ = 12\sqrt{2} \]

8(b)

\[ y = \csc^2 \left( \frac{x}{2} - \frac{\pi}{6} \right) = \sin^{-2} \left( \frac{x}{2} - \frac{\pi}{6} \right) \]

\[ \frac{dy}{dx} = -2 \left[ \sin^{-1} \left( \frac{x}{2} - \frac{\pi}{6} \right) \right] \frac{1}{2} \cos \left( \frac{x}{2} - \frac{\pi}{6} \right) \]

\[ = -\left[ \sin^{-1} \left( \frac{x}{2} - \frac{\pi}{6} \right) \right] \cos \left( \frac{x}{2} - \frac{\pi}{6} \right) \]

When \( x = \frac{5\pi}{6} \),

\[ \frac{dy}{dx} = -\left[ \sin^{-1} \left( \frac{\pi}{4} \right) \right] \cos \left( \frac{\pi}{4} \right) = -2 \]

\[ \frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = -2 \times 0.3 \]

\[ = -0.6 \text{ radian/sec} \]

9(i)

\[ T_{r-1} = \binom{9}{r} (x^3)^{n-r} \left( -\frac{k}{x} \right)^r = (-k)^r \binom{9}{r} (x^{3n-4r}) \]

For the term in \( \frac{1}{x} \), \( 27 - 4r = -1 \)

\[ \Rightarrow r = 7 \]

\[ -\binom{9}{7} (k^7) = -4608 \]

\[ \Rightarrow k = 2 \]

9(ii)

\[ 27 - 4r \neq 0 \text{ as } r \text{ must be an integer } \Rightarrow \text{ no independent term.} \]
9(iii) For the term in $x^3$, $r = 6$

The term in $x^3 = (-2)^6 \binom{9}{6} (x^3) = 5376x^3$

\[(2x^3 + \frac{1}{x})(x^2 - \frac{k}{x})^2 = (2x^3 + \frac{1}{x})(x^2 - \frac{1}{x}) = 4608 + 5376x^3 + \ldots)

\[-9216x^2 + 5376x^2 + \ldots = -13840x^2 + \ldots\]

\[\therefore \text{The coefficient of } x^2 \text{ is } -13840\]

10(i) \[\frac{dy}{dx} = \frac{e^{2x} + 1}{e^{2x}} = 1 + e^{-2x} \Rightarrow e^{-2x} > 0 \text{ for all values of } x\]

\[\Rightarrow \frac{dy}{dx} = 0, \Rightarrow \text{No stationary point}\]

10(ii) \[y = \int 1 + e^{-2x} \, dx\]

\[y = x - \frac{1}{2} e^{-2x} + c\]

when $x = 0$, $y = -1$

\[-1 = -\frac{1}{2} + c, c = \frac{1}{2}, y = x - \frac{1}{2} e^{-2x} - \frac{1}{2}\]

10(iii) At $P$, $m_{\text{tangent}} = 2$

Equation of tangent: $y = 2x - 1$

\[m_{\text{normal}} = -\frac{1}{2}\]

Equation of normal: $y = -\frac{1}{2}x - 1$

\[\Rightarrow Q = \left(\frac{1}{2}, 0\right), R = (-2, 0)\]

\[\therefore \text{Area of } \triangle PQR = \left(\frac{1}{2}\right) \cdot \left(\frac{5}{2}\right) \cdot 1 = \frac{1}{4} \text{ units}^2\]
### Marking Scheme (Additional Mathematics Paper 2/ Prelim Examination 2015)

<table>
<thead>
<tr>
<th>11(a)</th>
<th>(2^{3-3} = \frac{1}{4^{3-1}})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(4' = \frac{4}{8} = \frac{1}{2})</td>
</tr>
<tr>
<td></td>
<td>((4')^2 = 32)</td>
</tr>
<tr>
<td></td>
<td>(16' = 32)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>11(b)</th>
<th>(\log_5 5 - \frac{2}{\log_5 2} = \log_5 \left(\frac{25}{4}\right))</th>
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<tbody>
<tr>
<td></td>
<td>(\log_5 5 - \frac{2}{\log_5 2} = \log_5 \left(\frac{25}{4}\right))</td>
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<tr>
<td></td>
<td>(\log_5 5 - \frac{1}{\log_5 2} = \log_5 \left(\frac{25}{4}\right))</td>
</tr>
<tr>
<td></td>
<td>(\log_5 25 - \frac{2}{\log_5 2} = \log_5 25 - \log_5 4)</td>
</tr>
<tr>
<td></td>
<td>(\log_5 4 = \frac{2}{\log_5 2})</td>
</tr>
<tr>
<td></td>
<td>(2 \log_5 2 = \frac{2}{\log_5 2})</td>
</tr>
<tr>
<td></td>
<td>((\log_5 2)^2 = 1)</td>
</tr>
<tr>
<td></td>
<td>(\log_5 2 = \pm 1)</td>
</tr>
<tr>
<td></td>
<td>(\text{When } \log_5 2 = 1, \quad x = 2)</td>
</tr>
<tr>
<td></td>
<td>(\text{When } \log_5 2 = -1, \quad x = \frac{1}{2})</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>12(i)</th>
<th>(r^2 - 6t + 8 = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(t = 2 \quad \text{or} \quad t = 4)</td>
</tr>
</tbody>
</table>
12(ii) \[ v = t^2 - 6t + 8 \]
\[ s = \frac{t^3}{3} - 3t^2 + 8t + c \]
At \( t = 0 \), \( s = 0 \) \( \Rightarrow \) \( c = 0 \)
\[ s = \frac{t^3}{3} - 3t^2 + 8t \]
When \( v = 8 \), \( t^2 - 6t = 0 \)
\( t = 0 \) or \( t = 6 \)
\( \therefore \) When the velocity is \( 8 \text{ m/s} \) again, \( t = 6 \)
\[ S_1 = \frac{216}{3} - 3(36) + 8(6) = 12 \text{ m} \]
\[ S_2 = \frac{8}{3} - 3(4) + 8(2) = 6 \frac{2}{3} \]
\[ S_4 = \frac{64}{3} - 3(16) + 8(4) = 5 \frac{1}{3} \]
From \( t = 0 \) to \( t = 4 \), distance travelled = \( 6 \frac{2}{3} + (6 \frac{2}{3} - 5 \frac{1}{3}) = 8 \text{ m} \)
From \( t = 4 \) to \( t = 6 \), distance travelled = \( 12 - 5 \frac{1}{3} = 6 \frac{2}{3} \text{ m} \)
From \( t = 0 \) to \( t = 6 \), distance travelled = \( 14 \frac{2}{3} \text{ m} \)

12(iii) At \( O \), \( s = 0 \)
\[ s = \frac{t^3}{3} - 3t^2 + 8t = 0 \]
\[ \frac{1}{3} t(t^2 - 9t + 24) = 0 \]
\( \Rightarrow t = 0 \) or \( t^2 - 9t + 24 = 0 \)
\( t^2 - 3t + 8 = 0 \) \( \Rightarrow \) \( t = \frac{9 \pm \sqrt{15}}{2} \) \( \Rightarrow \) No solution
\( \Rightarrow \) The particle is at \( O \) when \( t = 0 \) only. Therefore \( P \) will not return to \( O \) in the course of its motion.

12(iv) From \( t = 6 \) to \( t = 7 \), distance travelled = \( 8 \text{ m} \)
From \( t = 7 \) to \( t = 9 \), distance travelled = \( 8 \text{ m} \)
\( \therefore \) Total distance travelled = \( 8 + 8 = 16 \text{ m} \)
13(i) \[ \tan \frac{\theta}{2} + \cot \frac{\theta}{2} = \tan \frac{\theta}{2} + \frac{1}{\tan \frac{\theta}{2}} = \frac{\tan \frac{\theta}{2} + 1}{\tan \frac{\theta}{2}} \]

\[ = \sec \frac{\theta}{2} \]

\[ = \frac{1}{\sin \frac{\theta}{2}} \cos \frac{\theta}{2} = \frac{1}{2} \sin \theta = 2 \csc \theta \]

13(ii) \[ \tan \theta + \cot \theta = (\csc \theta)(\sin 2\theta + \cos 2\theta) \]

\[ 2 \csc 2\theta = (\csc \theta)(\sin 2\theta + \cos 2\theta) \]

\[ = \frac{1}{2 \cos 2\theta} + \frac{1}{2 \sin 2\theta} = \frac{1}{2 \cos 2\theta} + \frac{1}{2 \cos 2\theta} \]

\[ 3 \csc 2\theta = \frac{1}{\cos 2\theta} \]

\[ \tan 2\theta = 3 \]

\[ \theta = 35.8^\circ, \quad 125.8^\circ \]

13(iii) \[ 2 \tan A + 2 \cot A = 5 \]

\[ \tan A + \cot A = \frac{5}{2} \]

\[ \csc 2A = \frac{5}{4} \]

\[ \sin 2A = \frac{4}{5} \]

\[ \cos 2A = \frac{3}{5} \]

13(iv) \[ \cos(2A + \frac{\pi}{6}) = \cos 2A \cos \frac{\pi}{6} - \sin 2A \sin \frac{\pi}{6} \]

\[ = \left( \frac{3}{5} \right) \left( \frac{\sqrt{3}}{2} \right) - \left( \frac{4}{5} \right) \left( \frac{1}{2} \right) \]

\[ = \frac{3\sqrt{3} - 4}{10} \]
READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial Theorem

\[(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \).

2. TRIGONOMETRY

Identities

\[
\sin^2 A + \cos^2 A = 1.
\]
\[
\sec^2 A = 1 + \tan^2 A.
\]
\[
\csc^2 A = 1 + \cot^2 A.
\]

\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
\sin 2A = 2 \sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]
\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \triangle ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[\Delta = \frac{1}{2} bc \sin A.\]
Answer all questions

1. Solve the following equations.
   (a) \(2(3^x) + 3^{-x} = 3\) \hspace{1cm} [3]
   (b) \(2\sqrt{3} - 2x = x + 1\) \hspace{1cm} [3]

2. The line \(2y - x = 3\) meets the curve \(x^2 - xy - y^2 = 1\) at points \(A\) and \(B\). Find the length of \(AB\), giving your answer expressed in the form \(a\sqrt{b}\), where \(a\) and \(b\) are integers. \hspace{1cm} [5]

3. Write down and simplify the first 3 terms, in ascending powers of \(x\), in the expansion of \((2-x)^3\). Given that the first three terms in the expansion of \((1 + px + x^2)(2-x)^3\) are \(32 - qx + 2qx^2\), find the value of \(p\). \hspace{1cm} [5]

4. A circle \(C_1\) passes through points \(P(0, 2), Q(7, 3)\) and \(R(8, -4)\) where \(PQRS\) is a square.
   (a) Find the coordinates of the centre and the radius of the circle \(C_1\). \hspace{1cm} [2]
   (b) Find the equation of another circle \(C_2\), in the form \(x^2 + y^2 + ax + by + c = 0\), that is the reflection of the circle \(C_1\), in the line \(y = x\). \hspace{1cm} [2]
   (c) Justify if the point \((2, 7)\) lies inside or outside the circle \(C_1\). \hspace{1cm} [2]

5. Prove the identity \(\frac{\sec x + 2\sin x}{2\cos x - \sec x} = \frac{1 + \tan x}{1 - \tan x}\). \hspace{1cm} [5]

6. The diagram shows a triangle \(ABCDE\), such that \(EB\) is parallel to \(DC\), the ratio of lengths \(AB : BC\) is \(4 : \sqrt{3}\) and length of \(DC\) is \(5 + 4\sqrt{2}\) cm. By leaving your answer in the form \(a + b\sqrt{c}\), calculate
   (a) the ratio \(\frac{BE}{CD}\) \hspace{1cm} [3]
   (b) the length of \(BE\). \hspace{1cm} [3]

7. Given that \(f(x) = -2 + x^3\) and \(g(x) = 1x + 11 - 1\),
   (a) Find the coordinates of the points of intersection of the graphs \(y = f(x)\) and \(y = g(x)\). \hspace{1cm} [4]
   (b) On the same axes, sketch the graphs of \(y = f(x)\) and \(y = g(x)\) for \(-2 \leq x \leq 2\). \hspace{1cm} [3]
   (c) Hence solve the inequality \(x^3 \leq 1x + 11 + 1\). \hspace{1cm} [2]

8. (a) Sketch the graph of \(y = 2 - e^{2x}\) for all real values of \(x\), showing clearly all points of intersection with the axes, if any. \hspace{1cm} [2]
   (b) By adding a suitable straight line, explain how the number of solutions to the equation \(x = \ln\sqrt{4 - x}\) can be obtained. \hspace{1cm} [2]
9. A and B lie in the same quadrant such that \( \sin A = \frac{3}{5} \) and \( \tan B = -\frac{5}{12} \). If the value of A and of B is between 0 and \( 2\pi \), find, without using the calculator, the values of
(a) \( \sin B \),
(b) \( \cos(A - B) \),
(c) \( \cos \frac{B}{2} \).

10. In \( \triangle PQR \), \( M \) is the mid-point of \( PQ \). \( PH \) and \( MR \) intersect at \( O \).

\[ \text{Given that } OR : OM = PS : PN = 1 : 2, \text{ prove that} \]
(a) \( MS \) is parallel to \( QN \),
(b) \( \triangle MSO \) is similar to \( \triangle RNO \),
(c) \( OP = 3 \times NO \).

11. The table shows some experimental values of two variables \( x \) and \( y \) which are known to be related by the equation \( y = ax + b \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>1.5</th>
<th>2.5</th>
<th>3.5</th>
<th>4.5</th>
<th>5.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>10.1</td>
<td>20.6</td>
<td>34.2</td>
<td>50.7</td>
<td>70.1</td>
</tr>
</tbody>
</table>

Using a suitable scale, plot the graph of \( \frac{y}{x} \) against \( x \) to represent the above data and use it to estimate
(a) the value of \( a \) and of \( b \),
(b) the value of \( x \) when \( y = 9x \).

12. A function is given by \( y = \frac{9x - 3b}{4x - 1} \) where \( x \neq a \) and \( x > 0 \).

(a) State the values of \( a \).
(b) Determine the range of values of \( b \) if \( y \) is an increasing function.
(c) Given that \( b = 3 \) and that \( x \) and \( y \) vary with time \( t \), find the value(s) of \( x \)
if \( \frac{dy}{dt} = 12 \times \frac{dx}{dt} \).

13. An electronic gadget was programmed to travel in a straight line. It started through a fixed point \( O \) with a velocity of 3 m/s. Its acceleration, \( a \) m/s\(^2\), is given by \( a = 2 - 2t \), where \( t \) seconds is the time after passing \( O \). Find
(a) its maximum velocity,
(b) its deceleration when it changes its direction of motion,
(c) the total distance travelled during the first four seconds of motion.

ANDSS 4E5N Prelim 2015   Add Math (4047/01)   [End of Paper]
Prelim AM Paper 1 Answer Key

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1(a)</td>
<td>( x = -0.631 \text{ or } x = 0 )</td>
</tr>
<tr>
<td>1(b)</td>
<td>( x = 1 )</td>
</tr>
<tr>
<td>2</td>
<td>7√5 units</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{32 - 80}{3} \frac{x^3 + 80}{9} x + \ldots ); ( p = \frac{1}{3} )</td>
</tr>
<tr>
<td>4a</td>
<td>(4, -1) &amp; 5 units</td>
</tr>
<tr>
<td>4b</td>
<td>( x^2 + y^2 + 2x - 8y - 8 = 0 )</td>
</tr>
<tr>
<td>4c</td>
<td>(2, 8) lies outside the circle ( C_1 ).</td>
</tr>
<tr>
<td>5</td>
<td>Proof</td>
</tr>
<tr>
<td>6a</td>
<td>( 2 - \sqrt{2} )</td>
</tr>
<tr>
<td>6b</td>
<td>((2 + 3\sqrt{2}) \text{ cm} )</td>
</tr>
<tr>
<td>7a</td>
<td>(-1, -1) and (2, 2)</td>
</tr>
<tr>
<td>7b</td>
<td>( y = x - 2 )</td>
</tr>
<tr>
<td>7c</td>
<td>(-1 \leq x \leq 2 )</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>8a</td>
<td>Add the line ( y = x - 2 ). The number of intersection points of ( y = 2 - e^x ) and ( y = x - 2 ) gives the number of solutions for ( x = \ln \sqrt{4-x} ).</td>
</tr>
<tr>
<td>8b</td>
<td></td>
</tr>
</tbody>
</table>
READ THESE INSTRUCTIONS FIRST

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Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[
    x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial Theorem

\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}
\]

2. TRIGONOMETRY

Identities

\[
\begin{align*}
\sin^2 A + \cos^2 A &= 1, \\
\sec^2 A &= 1 + \tan^2 A, \\
\csc^2 A &= 1 + \cot^2 A, \\
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B, \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B,
\end{align*}
\]

\[
\begin{align*}
\tan(A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}, \\
\sin 2A &= 2 \sin A \cos A, \\
\cos 2A &= \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A, \\
\tan 2A &= \frac{2 \tan A}{1 - \tan^2 A}
\end{align*}
\]

Formulae for \( \triangle ABC \)

\[
\begin{align*}
\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C}, \\
a^2 &= b^2 + c^2 - 2bc \cos A, \\
\Delta &= \frac{1}{2}bc \sin A.
\end{align*}
\]
Answer all questions

1. When \((1 - 2p)^2\) is expanded in ascending powers of \(p\), the sum of the constant term, the coefficients of \(p\) and \(p^2\) is 161. If \(n\) is a positive integer, find the value of \(n\). [4]

2. (a) Find the smallest positive integer, \(p\) such that \(4p + 35 = x^2\) has real roots. [4]

   (b) Find the range of values of \(m\) for which the graph of \(y = mx^2 - 4x + m\) lies entirely below the line \(y = 3\). [4]

   (c) Given that the line \(y = 4x + k\) is a tangent to the curve \(y^2 = mx\), where \(k\) and \(m\) are constants, prove that \(k^2 = \frac{1}{16}\). [4]

3. Marcus believes that the depth of water, \(d\) metres, at the end of a jetty, \(t\) hours after low tide, can be modelled by the equation \(d = a + b \cos \frac{\pi}{6}\) where \(a\), \(b\) and \(\theta\) are constants.

   (a) He measures the depth of water at low tide to be 2 metres. Assuming that low tides occur every 12 hours, show that \(k = \frac{\pi}{6}\). [1]

   (b) Marcus also measures the depth of water at high tide to be 6 metres. Calculate the value of \(a\) and of \(\theta\). [2]

   (c) Sketch the graph of the equation \(d = a + b \cos \frac{\pi}{6}\) for \(0 < t < 2\pi\). [3]

   (d) Marcus requires the depth of water at the end of the jetty to be at least 3 metres to sail his boat. Given that the low tide on a particular day was at 0830, find the earliest time after 0830 when Marcus could sail his boat that day. [2]

   (e) Marcus measured the depth of water and found that it is 5m. He then claimed that the depth of water at the end of the jetty will reach 5 m again after every 4 hours. Justify if Marcus is right or wrong. [2]

4. (a) (i) Factorise \(h(x) = x^3 - 7x^2 + 2x + 80\) completely. [3]

   (ii) Hence, solve the equation \(2y^3 - 7y^2 + y + 80 = 0\). [3]

(b) Find the value of \(n\) for which the division of \(2x^n + 3x^2 - 4x - 10\) by \(x - 2\) gives a remainder of 26. [3]

5. Solve the following equations.

(a) \(\log_2 \sqrt{5x + 1} = \log_4(x - 2) + \log_2 4\) [5]

(b) \(4 \tan^2 x - 1 - 8 \sec x\) for \(-\pi < x < 2\pi\) [5]
6 (a) Consider the equation \( kx^2 - k^2x - 3x + 4 = k \).
If the roots of the equation are reciprocal of each other, and \( \beta \) is one of roots,
(i) find the value of \( k \),
(ii) show that \( \frac{7}{\beta^2 + 1} = \frac{2}{\beta} \). \( \text{[2]} \)
(b) The roots of the quadratic equation \( 2x^2 - 4x + 5 = 0 \) are \( \lambda \) and \( \mu \).
Find the quadratic equation whose roots are \( \frac{\lambda}{\mu} \) and \( \frac{\mu}{\lambda} \). \( \text{[4]} \)

7 The term containing the highest power of \( x \) in the polynomial \( f(x) \) is \( 3x^4 \).
\( x^2 - 2x + k \) is a quadratic factor of \( f(x) \). \( x = -1 \) and \( x = 2 \) are roots of the equation \( f(x) = 0 \). \( f(x) \)
leaves a remainder of \( -36 \) when it is divided by \( x \).
(a) Show that \( k = 6 \). \( \text{[2]} \)
(b) Determine the number of real roots of the equation \( f(x) = 0 \). \( \text{[2]} \)

8 A curve is defined by \( y = (1 - 2x)^3 e^{2x} \). Find
(a) \( \frac{dy}{dx} \). \( \text{[2]} \)
(b) the equation, in terms of \( x \), of the tangent at the point where \( x = 1 \). \( \text{[4]} \)
(c) the \( x \)-coordinate(s) of the stationary point(s) on the curve and determine
the nature of the point(s). \( \text{[4]} \)

9 (a) Express \( \frac{x^2 - 3x + 5}{(x^2 + x)(2x - 1)} \) in partial fractions. \( \text{[3]} \)
(b) Hence, evaluate \( \int \frac{x^2 - 3x + 5}{(x^2 + x)(1 - 2x)} \, dx \). \( \text{[3]} \)

10 The diagram shows part of the curve \( y = x^3 - 1 \). The tangent at \( A (-2, -9) \) meets the curve
again at \( C \). Find the area of the region bounded by the two graphs. \( \text{[8]} \)
Solution to this question by accurate drawing will not be accepted.

The diagram shows a triangle $ABC$ where $A$ is $(-4, -2)$, $B$ is $(2, 7)$ and $BC$ is parallel to the line $2y = -4x + 1$. $BC$ cuts the $x$-axis at $F$ and $AB$ cuts the $y$-axis at $E$.

(a) Find the equation of the line $BC$.  
(b) Determine whether if $EF$ is perpendicular to $AB$.  
(c) Given that $C$ is equidistant from $A$ and $E$, find the coordinates of $C$.  
(d) Find the length of $AE$, and hence, find the area of $\triangle ACE$.

A L-shaped ladder, $ABC$ is wedged in between two pillars $AO$ and $DE$ as shown in the diagram. $A$ and $C$ are the points of contact between the ladder and the pillars while $B$ is the point of contact between the ladder and the ground.

Given that $\angle OBA = \theta$, where $0^\circ < \theta < 90^\circ$, $AB = 0.3$ m, $BC = 0.2$ m and $CD = x$ m,

(a) show that $x = 0.3 \sin \theta - 0.2 \cos \theta$,
(b) express $x$ in the form $R \sin(\theta - \alpha)$ where $R > 0$ and $0^\circ < \alpha < 90^\circ$,  
(c) hence, explain if the length of $CD$ can be $0.45$ m.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th>7b</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>2 real roots</td>
<td></td>
</tr>
<tr>
<td>2a</td>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>2b</td>
<td></td>
<td>(-1)</td>
<td></td>
</tr>
<tr>
<td>3a</td>
<td></td>
<td>(\frac{\pi}{6})</td>
<td></td>
</tr>
<tr>
<td>3b</td>
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<td>4</td>
<td></td>
</tr>
<tr>
<td>3c</td>
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<td></td>
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</tr>
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<td>3d</td>
<td></td>
<td>1030h</td>
<td></td>
</tr>
<tr>
<td>4a</td>
<td></td>
<td>Wrong</td>
<td></td>
</tr>
<tr>
<td>4a i</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>4a ii</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>4b</td>
<td></td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5a</td>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>5b</td>
<td></td>
<td>(-1.98) or 1.98 or 4.30</td>
<td></td>
</tr>
<tr>
<td>6a</td>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>6b</td>
<td></td>
<td>(5x^2 + 2x + 5 = 0)</td>
<td></td>
</tr>
</tbody>
</table>

Diagram:

108 sq units

EF is not perpendicular to AB.

\(\left(\frac{17}{2}, -6\right)\)

\(45\frac{1}{2}\) units²

\(0.361\sin(\theta - 33.7°)\)

Length of CD cannot be 0.45 m.
1. A curve has the equation \( y = \frac{\ln x}{x^2} \).
   (i) Find \( \frac{dy}{dx} \). \([2]\)
   (ii) Hence, find the range of values of \( x \), such that \( y \) is increasing. \([2]\)

2. The diagram below shows the graph of \( y = 4 - |x^2 + 3| \).
   (i) Show that the coordinates of \( A \) is \( (-\sqrt{3}, 4) \). \([1]\)
   (ii) State the coordinates of \( B \). \([2]\)
   (iii) Find the exact value of \( m \), for \( m < 0 \) for which the equation
         \( mx + 1 = 4 - |x^2 + 3| \) has exactly 3 solutions. \([2]\)

3. In the diagram shown, the line forms an angle \( A \) with the \( y \)-axis. Given that the gradient of the line is \( 3 \), without using a calculator, find the exact value of \( \cos A \). \([3]\)

4. (i) Show that
\[ \frac{1 - \sin x}{\sqrt{1 + \sin x}} = \sec x - \tan x, \quad \text{when } -90^\circ < x < 90^\circ. \]

(ii) Hence, explain why \( x \) must be acute for the identity to be true.

5. Given that the coefficient of \( \frac{1}{x^3} \) is 312 in the expansion \( \left( \frac{2}{x} + ax^2 \right)^9 \), where \( a < 0 \).

(i) Find the value of \( a \)

(ii) Hence, using the value of \( a \) found in (i), show that the term in \( \frac{1}{x^4} \) does not exist in the expansion \( \left( \frac{2}{x} + ax^2 \right)^9 \left( \frac{1}{8x} + \frac{x^2}{12} \right) \).

6. Express \( \frac{x^3 + 5x^2 + 2x - 1}{2x^4 + x^2 - 2} \) in partial fractions.

7. The equation \( 6x^2 + 7x - 3 = 0 \) has roots \( \frac{1}{\alpha} \) and \( \frac{1}{\beta} \).

(i) Find the value of \( (\alpha + \beta) \) and of \( \alpha \beta \).

(ii) Hence, or otherwise, find the exact value of \( (\alpha^3 + \beta^3) \).

8. Solve the equation \( \log(4^x - 10) - x \log 2 = \log 3 \).
9. The voltage \( V \), in volts, of an electrical signal in an electrical system is given by the formula \( V = 4 \sin \pi t \) where \( t \) is in seconds.

(i) Find the exact rate of change of voltage after \( \frac{1}{4} \) seconds have elapsed. \([2]\)

(ii) Find the exact times when the rate of change of voltage is \( 2\pi\sqrt{3} \) volts per second for \( 0 < t < 4 \). \([3]\)

(iii) Given that current \( I \) in amperes supplied to the system is governed by the equation \( I = \frac{V}{5} \), find the rate of change of current when the rate of change of voltage is 2 volts per second. \([2]\)

10. In the diagram shown below, \( ABCD \) is a parallelogram with points \( A(2, 8), B(-1, 1) \) and \( C(4, -2) \). \( M \) is the midpoint of \( BD \) and the perpendicular bisector of \( BD \) passes through the \( y \)-axis at \( P \).

Find

(i) the coordinates of \( M \), \([1]\)

(ii) the coordinates of \( D \), \([2]\)

(iii) the equation of the perpendicular bisector of \( BD \), \([2]\)

(iv) the area of quadrilateral \( ACBP \). \([2]\)
11. Given that \( y = x^2 + ax^2 + bx + 3 \) has a stationary point \((1, 0)\),
   (i) find the values of \(a\) and of \(b\). \([3]\)
   (ii) find the coordinates of the other stationary point. \([3]\)
   (iii) and determine the nature of these stationary points. \([3]\)

12. (i) Differentiate the following with respect to \(x\):
   
   (a) \( \frac{\left(\frac{e^x}{x}\right)^3}{e^{x+1}} \) \([2]\)
   
   (b) \( \ln(\cos^2 x) \) \([2]\)
   
   (ii) Hence, or otherwise, find \( \int \frac{5}{2x-3} - \frac{4e^{2x-1}}{2 \tan x} \) \([4]\)

13. The table shows experimental values of two variables, \(x\) and \(y\), which are connected by
   the equation \( y = ae^{bx-1} \).

   \[
   \begin{array}{c|c|c|c|c|c}
   x & 1 & 2 & 3 & 4 & 5 \\
   \hline
   y & 1.89 & 2.30 & 2.82 & 3.44 & 4.20 \\
   \end{array}
   \]

   (a) Plot \( \ln y \) against \(x\) and draw a straight line graph. \([3]\)
   
   (b) Use your graph to estimate the value of \(a\) and of \(b\). \([3]\)
   
   (c) By drawing a suitable line on your graph, solve the equation \(2.46 = ae^{bx-1}\). \([2]\)

END OF PAPER
AHS Prelim AM P1

1. (i) \[ \frac{1-2\ln x}{x^3} \] (ii) \[ 0 < x < e^2 \]

2. (ii) B (0,1) (iii) -\sqrt{3}

3. \[ \frac{3}{\sqrt{10}} \]

4. (ii) \[ \cos x - \sqrt{1 - \sin^2 x} \]. Since \( \cos x \) must be positive, \( x \) is in the 1st or 4th quadrant. \( \therefore x \) must be acute.

5. (i) \[ -\frac{1}{3} \] (ii) Term with \[ \frac{1}{x^4} = \left( -\frac{769}{x^6} \right) \left( \frac{x^2}{12} \right) + \left( \frac{512}{x^3} \right) \left( \frac{1}{8x} \right) \] \[ = 0 \]
\( \therefore \) it does not exist.

6. \[ \frac{2}{x} - \frac{1}{x^2} + \frac{7-3x}{2x^2+1} \]

7. (i) \[ \frac{7}{3} \] (ii) \[ \frac{721}{27} \]

8. \[ x = 2.32 \]

9. (i) \[ \frac{\Delta v}{\Delta t} = \frac{4n}{\sqrt{2}} \text{ V/s} \] (ii) \[ t = \frac{1}{6}, \frac{11}{6}, \frac{1}{2}, \frac{5}{6} \text{ sec} \] (iii) \[ \frac{dl}{dt} = \frac{2}{5} \text{ Amperes/sec} \]

10. (i) m (3,3) (ii) D (7,5) (iii) \[ y = -2x + 9 \] (iv) \[ 30 \frac{1}{2} \text{ units}^2 \]

11. (i) \[ a = 1, b = -5 \] (ii) \[ \left( -\frac{5}{3}, \frac{256}{27} \right) \] (iii) max point

12. (i)(a) \[ 2e^{2x-1} \] (b) \[ -2 \tan x \] (ii) \[ \frac{5}{2} \ln(2x - 3) - 2e^{-2x-1} + \ln \cos^2 x + c \]

13. (b) \[ a = 4.22, b = 0.2 \] (c) \[ x = 2.3 \]
1. The curve \[ \frac{(x-2)^2}{4} + (y-3)^2 = 4 \] and the line \( 2y + x = 12 \) intersect at the points \( P \) and \( Q \). Find the exact distance between \( P \) and \( Q \). \[5\]

2. Find the values of \( a \) and \( b \) for which the function \( f(x) = 2x^4 - 7x^3 + ax^2 + bx - 21 \) is exactly divisible by \( x^2 - 2x - 3 \). Hence determine, showing all necessary working, the number of real roots of the equation \( f(x) = 0 \). \[8\]

3. In the diagram, \( A, B, C \) and \( D \) are points on the circle. \( TDSB \) and \( ASC \) are straight lines. \( TA \) and \( TC \) are tangents. Prove that
(a) \[ \angle ACB = \angle ATD + \angle ABD, \] and
(b) \[ \angle ATC = 180^\circ - 2\angle ABC. \]

4. The diagram shows the route of a fishing boat. The boat leaves the point \( O \) from the shore and sails in a straight line for 5 km to a point \( A \), at a bearing of \( (90^\circ + \theta) \). At \( A \), the boat makes a right-angled turn and sails for 2 km to the point \( B \) to continue fishing. The angle \( OAB = 90^\circ \) and the shortest distance from \( B \) to the shore is \( L \) km.

(a) Show that \( L = 5\cos\theta + 2\sin\theta \). \[2\]
(b) Express \( L \) in the form \( R\cos(\theta - \alpha) \), where \( R > 0 \) and \( 0^\circ < \alpha < 90^\circ \). \[3\]
(c) State the maximum value of \( L \) and find the corresponding value of \( \theta \). \[3\]
5. (a) Express \( \tan^2 \alpha - \cos^2 \beta \) in the form \( A \sec^2 B \alpha + C \cos 2 \beta + D \) where \( A, B, C, \) and \( D \) are constants. [3]

(b) Hence, or otherwise, evaluate \( \int \left( \tan^2 3x - \cos^2 \frac{x}{2} \right) \, dx \). [5]

6. The curve \( y = P \cos Qx + R \) has a period of 720\(^\circ\), a maximum value of 8 and a minimum value of \(-4\).
(a) Given that \( P \) is a negative constant and \( Q \) and \( R \) are positive constants, find the value of \( P \), of \( Q \) and of \( R \). [4]
(b) Solve the equation \( y = 3 \) where \( 0^\circ < x < 360^\circ \). [2]
(c) Sketch the graph of \( y \) for \( 0^\circ < x < 360^\circ \). [3]

7. A container in the shape of a cylinder with a hemisphere on top is to be decorated by gold wires.

The wires \( AC \) and \( DF \) go across the hemisphere and intersect at \( B \), the highest point of the hemisphere. The wires \( AJ, EF, CG \) and \( DH \) run down the sides of the cylinder. The wires \( GJ \) and \( FH \) cross at right angles at \( K \) where \( K \) is the centre of the base. The total length of wire is 30 cm. The height of the cylinder is \( h \) cm and the radius of the hemisphere is \( r \) cm. The volume of the container is \( V \) cm\(^3\).
(a) Express \( h \) in terms of \( r \). [2]
(b) Show that \( V = \frac{\pi r^2}{6} (45 - 2r - 3r) \). [3]
(c) Find the stationary value of \( V \) and determine its nature. [5]
8. (a) Show that the equation \(2^{2x} = \frac{1}{2}[3(2^x) + 2]\) is satisfied by only one value of \(x\). 

(b) Given that \(m = a^e\), \(n = a^f\) and \(m^2 n^2 = a^{2u}\), where \(a > 0\) and \(a \neq 1\), show that \(st = \frac{1}{u}\). 

(c) Without using calculators, find the value of \(k\) in the form \(\frac{x + y\sqrt{5}}{2}\), such that \(k\sqrt{3} - k\sqrt{15} = 2\sqrt{5}\). 

(d) Differentiate \(e^{-1}\sqrt{1+3x}\) with respect to \(x\). 

9. In the Chingay Parade procession held at the heartlands early this year, the Pioneer Generation Float was travelling on a straight road with a velocity, \(v\) ms\(^{-1}\), given by the equation \(v = 5t - \frac{1}{2}t^2 + 4\), where \(t\) is the time after passing a fixed point \(A\). 

(a) Show that the maximum velocity is reached 5 s later. 

(b) Sketch the velocity-time graph for the first 5 s. 

Upon reaching its maximum velocity, the float started to decelerate uniformly at 1.5 ms\(^{-2}\), before coming to a rest at point \(B\) to allow residents to take photographs. 

(c) Find the time when the float reached \(B\). 

(d) Find the total distance travelled from \(A\) to \(B\). 

10. 

The graph above shows part of curve \(y^2 = 5x - 4\) and the line \(y = 4x - 5\). Find 

(a) the coordinates of \(A\) and of \(B\), and 

(b) the area of the shaded region.
A landscaping company has been tasked to design the backyard for a client. The design is made up of overlapping circles as shown below. The circular lawn in the centre will be the focus point of the design and a barbeque pit will be constructed on one side of the lawn.

On the Cartesian plane, the circular lawn can be modelled by the equation of a circle, 

\[ x^2 + y^2 + 2x - 6y - 15 = 0. \]

(a) Show why this model suggests that the radius of the lawn is 5 m. [2]

(b) A lamp post is positioned at a point \( P(-5, 8) \) in the pit area. Determine, with working, if \( P \) lies inside or outside the lawn. [3]

(c) Two dustbins, at \( Q \) and \( R \), will be placed on the circumference of the lawn such that \( Q \) is \((-4, -1)\) and \( QR \) is the diameter of the lawn. Find the equation of the tangent to the lawn at \( R \). [6]

END OF PAPER
1. \(P(2, 5), Q(6, 3), PQ = 2\sqrt{5}\)

2. \(a = 7, b = -5,\) no real roots

4. (b) \(\sqrt{29}\cos(\theta - 21.8^\circ)\)  (c) \(21.8^\circ\)

5. (a) \(\sec^2 \alpha - \frac{1}{2}\cos 2\beta - \frac{3}{2}\)
   (b) \(-2.75\)

6. (a) \(P = -6, Q = \frac{1}{2}, R = 2\)  (b) \(x = 199.2^\circ\)
   (c)

7. (a) \(h = \frac{15 - 2r - \pi r}{2}\)
   (c) \(v = 54.2\) max. value

8. (c) \(k = \frac{1 + \sqrt{5}}{2}\)
   (d) \(\frac{1 - 6x}{2e^x\sqrt{1 + 3x}}\)

9. (b)

(c) \(t = 16\)
(d) \(152\frac{5}{12}\) m

10. (a) \(A(1.8125, 2.25), B(1, -1)\)
    (b) 1.14 units

11. (a) \((x + 1)^2 + (y - 3)^2 = 5^2\)
    (b) Distance = 6.40 m > radius, the lamp post lies outside the lawn
    (c) \(y = -\frac{3}{4}x + \frac{17}{2}\)
READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer All questions.

Attempt Question 1 to 8 in Answer Booklet 1A
Question 9 to 13 in Answer Booklet 1B.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the quadratic equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial Expansion

\[(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}
\]

2. TRIGONOMETRY

\[
\sin^2 A + \cos^2 A = 1
\]

\[
\sec^2 A = 1 + \tan^2 A
\]

\[
\cosec^2 A = 1 + \cot^2 A
\]

\[
\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
\sin 2A = 2 \sin A \cos A
\]

\[
\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A
\]

\[
\tan 2A = \frac{2\tan A}{1 - \tan^2 A}
\]

Formulae for \( \triangle ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2} ab \sin C
\]
1. Given that $2^{x^2} \times 5^{x-1} = 8 \times 5^{2x}$, evaluate $10^x$ without using a calculator. [3]

2. Express $\frac{4x+7}{x^2 + 6x + 9}$ in partial fractions. [4]

3. Given that $\theta$ is acute and that $\sin \theta = \frac{1}{\sqrt{3}}$, express, without using a calculator, $\frac{1}{\cos \theta - \sin \theta}$ in the form $\sqrt{a} + \sqrt{b}$ where $a$ and $b$ are integers. [5]

The diagram shows a part of the curve of a person’s blood pressure, which is modelled using $y = a \cos bt + c$

where $t$ is time in seconds and $y$ is the blood pressure measured in mm (of mercury).

The length of the same person's heartbeat is the time between two consecutive peaks on the curve. Given that the person's heartbeat is 60 beats per minute,

(a) Write down the amplitude of $y$. [1]

(b) Explain why the period of the function is 1 second. [1]

(c) Write down the value of
   
   (i) $a$,
   
   (ii) $b$
   
   (iii) $c$. [3]
The straight line $2y = 3x - 16$ intersects the curve $2x^2 - 19x + 2y + 40 = 0$ at the points A and B. Given that A lies below the x-axis and that the point P lies on AB such that $AP : PB = 3 : 1$, find the co-ordinates of P.

6 A curve has the equation $y = \sin x - 3\cos 2x$.

(i) Find the gradient of the curve when $x = \frac{\pi}{6}$. [4]

(ii) Given that $x$ is decreasing at a constant rate of $2\sqrt{3}$ units per second, find the rate of change of $y$ when $x = \frac{\pi}{6}$. [2]

7 (i) Given that $y = x^2 \sqrt{2x-1}$, show that $\frac{dy}{dx} = \frac{x(5x-2)}{\sqrt{2x-1}}$. [2]

(ii) Hence evaluate $\int_1^3 \frac{5x^2 - 2x + 1}{\sqrt{2x-1}} \, dx$. [4]

8 Find the coordinates of the stationary point on the curve $y = 2x^3 - 6x^2 + 6x - 11$ and determine the nature of the stationary point. [7]

9 (a) Show that the roots of the equation $6x^2 + 5(m-1) = 3(x + m)$ are real if $m < 2 \frac{11}{16}$. [3]

(b) Find the range of values of $k$ for which $(k + 3)x^2 + 4x + k$ is always negative for all real values of $x$. [4]
10 A particle $P$ moves in a straight line so that $t$ seconds after leaving a fixed point $O$, its velocity $v$ ms$^{-1}$ is given by 
\[ v = (2t - 3)^3 - 9. \]
(a) Sketch the $v$-$t$ graph of the particle $P$ for $0 \leq t \leq 5$. [2]
(b) Hence or otherwise,
(i) find the range of values of $t$ for which the acceleration of $P$ is less than 4 m/s$^2$. [2]
(ii) find the distance travelled by $P$ in the first 5 seconds. [3]

11 In the expansion of \( \left(x^2 - \frac{k}{2x}\right)^6 \), where $k$ is a positive constant, the term independent of $x$ is 15.
(i) Show that $k = 2$. [4]
(ii) With this value of $k$, find the coefficient of $x^4$ in the expansion of \( \left(x^2 - \frac{k}{2x}\right)^6 (8x+1) \). [3]

12 A circle, $C$, has equation \( x^2 + y^2 - 10x + 6y + 9 = 0 \).
(i) Find the coordinates of the centre and radius of $C$. [3]
(ii) Give a reason why the $y$-axis is a tangent to $C$. [1]

The circle $C$ crosses the $x$-axis at the point $P(1, 0)$.
(iii) Show that the equation of the tangent to the circle $C$ at $P$ is $3y - 4x = -4$. [3]
(iv) Find the coordinates of the point where the circle $C$ crosses the $x$-axis again. [1]

13 In a Science experiment, a container of liquid was heated to a temperature of $K$ °C.
It was then left to cool in a chiller such that its temperature, $T$ °C, $t$ minutes after removing the heat, is given by 
\[ T = Ke^{-q} \], where $q$ is a constant.
Measured values of $t$ and $T$ are given in the following table.

<table>
<thead>
<tr>
<th>$t$ (minutes)</th>
<th>2</th>
<th>4</th>
<th>7</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$ °C</td>
<td>72.8</td>
<td>60.2</td>
<td>45.2</td>
<td>34.0</td>
<td>28.1</td>
</tr>
</tbody>
</table>

(i) On graph paper, plot $\ln T$ against $t$ and draw a straight line graph. [3]
(ii) Use the graph to estimate the value of $K$ and of $q$. [4]
(iii) Estimate the temperature of the liquid 5 minutes after it was left to cool. [2]
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Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer All questions.
Attempt Questions 1 to 7 in Answer Booklet A and Questions 8 to 13 in Answer Booklet B.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
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\[
(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. TRIGONOMETRY

\[
sin^2 A + cos^2 A = 1
\]
\[
sec^2 A = 1 + tan^2 A
\]
\[
cosec^2 A = 1 + cot^2 A
\]
\[
sin (A \pm B) = sin A \cos B \pm \cos A \sin B
\]
\[
\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
\sin 2A = 2 \sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2 \sin^2 A
\]
\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \Delta ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\Delta = \frac{1}{2} ab \sin C
\]
1. Given that \(2^{2n+2} \times 5^{2n+2} = 8^n \times 5^{2n}\), evaluate \(10^n\) without using a calculator. \[\text{[3]}\]

**SOLUTION:**

1. \[
\begin{align*}
2^{2n+2} \times 5^{2n+2} &= 8^n \times 5^{2n} \\
4(2^{2n}) \times \frac{5^{2n}}{5} &= 2^{2n} \times 5^{2n} \\
(2^{2n})(5^{2n}) &= \frac{4}{5} \\
(2^{2n})(5^{2n}) &= \frac{4}{5} \\
10^n &= \frac{4}{5}
\end{align*}
\]

2. Express \(\frac{4x + 7}{x^2 + 6x + 9}\) in partial fractions. \[\text{[4]}\]

**SOLUTION:**

2. \[
\begin{align*}
\frac{4x + 7}{x^2 + 6x + 9} &= \frac{A}{x + 3} + \frac{B}{(x+3)^2} \\
4x + 7 &= A(x + 3) + B \\
\text{Let } x &= -3, \quad B = -5 \\
\text{Let } x &= 0, \quad 7 = 3A - 5 \quad \Rightarrow A = 4
\end{align*}
\]

\[
\begin{align*}
\frac{4x + 7}{x^2 + 6x + 9} &= \frac{4}{x + 3} - \frac{5}{(x+3)^2}
\end{align*}
\]
Given that $\theta$ is acute and that $\sin \theta = \frac{1}{\sqrt{3}}$, express, without using a calculator, 

$$\frac{1}{\cos \theta - \sin \theta}$$

in the form $\sqrt{a} + \sqrt{b}$ where $a$ and $b$ are integers.

**SOLUTION:**

3. 

$$1^2 + x^2 = (\sqrt{3})^2$$

$$x = \sqrt{2}$$

$$\therefore \cos \theta = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\frac{1}{\cos \theta - \sin \theta} = \frac{1}{\frac{\sqrt{2}}{\sqrt{3}} - 1} = \frac{\sqrt{3}}{\sqrt{2} - 1} \frac{\sqrt{2} + 1}{\sqrt{2} + 1} = \frac{\sqrt{6} + \sqrt{3}}{2 - 1}$$

$$= \sqrt{6} + \sqrt{3}$$

4.

The diagram shows a part of the curve of a person’s blood pressure, which is modelled using

$$y = a \cos bt + c$$

where $t$ is time in seconds and $y$ is the blood pressure measured in mm (of mercury).

The length of the same person’s heartbeat is the time between two consecutive peaks on the curve.

Given that the person’s heartbeat is 60 beats per minute,
(a) Write down the amplitude of $y$. \[1\]

(b) Explain why the period of the function is 1 second. \[1\]

(e) Write down the value of

(i) $a$, \[3\]
(ii) $b$ \[3\]
(iii) $c$. \[3\]

**SOLUTION**

<table>
<thead>
<tr>
<th>4</th>
<th>$y = a \cos bt + c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>amplitude of $y = 25$</td>
</tr>
<tr>
<td>(b)</td>
<td>60 beats/cycles per 60 seconds. Therefore 1 cycle takes 1 second</td>
</tr>
</tbody>
</table>
| (c) | $a = 25$  
$b = \frac{2\pi}{1} = 2\pi$  
$c = 98$ |

---

The straight line $2y = 3x - 16$ intersects the curve $2x^2 - 19x + 2y + 40 = 0$ at the points $A$ and $B$. Given that $A$ lies below the $x$-axis and that the point $P$ lies on $AB$ such that $AP : PB = 3 : 1$, find the co-ordinates of $P$. \[6\]
SOLUTION

5 $2y = 3x - 16$ into $2x^2 - 19x + 2y + 40 = 0$

$2x^2 - 19x + 3x - 16 + 40 = 0$
$2x^2 - 16x + 24 = 0$
$(x - 2)(x - 6) = 0$
$x = 2, y = -5 \quad A(2, -5)$
$x = 6, y = 1 \quad B(6, 1)$

6 A curve has the equation $y = \sin x - 3\cos 2x$.

(i) Find the gradient of the curve when $x = \frac{\pi}{6}$.

(ii) Given that $x$ is decreasing at a constant rate of $2\sqrt{3}$ units per second, find the rate of change of $y$ when $x = \frac{\pi}{6}$.

SOLUTION

6 $y = \sin x - 3\cos 2x$

(i) \[ \frac{dy}{dx} = \cos x + 6\sin 2x \]

At $x = \frac{\pi}{6}$:

\[ \frac{dy}{dx} = \frac{\sqrt{3}}{2} + 6 \left( \frac{\sqrt{3}}{2} \right) \]

\[ = \frac{7\sqrt{3}}{2} \quad \text{or} \quad 6.06 \]

(ii) At $x = \frac{\pi}{6}$,

\[ \frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt} \]

\[ = \frac{7\sqrt{3}}{2} \times (-2\sqrt{3}) \]

\[ = -21 \text{ units/s} \]

OR $y$ is decreasing at 21 units/s
(i) Given that $y = x^2 \sqrt{2x-1}$, show that
\[ \frac{dy}{dx} = \frac{x(5x-2)}{\sqrt{2x-1}} \]  \[ \text{[2]} \]

(ii) Hence evaluate \[ \int_{x}^{5} \frac{5x^2 - 2x + 1}{\sqrt{2x-1}} \, dx \]  \[ \text{[4]} \]

**SOLUTION**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>$y = x^2 \sqrt{2x-1}$</td>
</tr>
</tbody>
</table>
| (i) | \[ \frac{dy}{dx} = \left(\sqrt{2x-1}\right)(2x) + x^2 \left( \frac{1}{2} (2x-1)^{-\frac{1}{2}} \right)(2) \]  
|   | \[ = (2x-1)^{\frac{1}{2}} \left( x \right) \left[ 2(2x-1) + x \right] \]  
|   | \[ = x(5x-2) \]  
|   | \[ \frac{1}{\sqrt{2x-1}} \]  (Shown) |
| (ii) | \[ \int \frac{x(5x-2)}{\sqrt{2x-1}} \, dx = x^2 \sqrt{2x-1} \]  
|   | \[ \int_{x}^{5} \frac{5x^2 - 2x + 1}{\sqrt{2x-1}} \, dx = \left[ x^2 \sqrt{2x-1} \right]_{x}^{5} + \int_{x}^{5} \frac{1}{\sqrt{2x-1}} \, dx \]  
|   | \[ = \left[ x^2 \sqrt{2x-1} \right]_{x}^{5} + \left[ \frac{(2x-1)^{-\frac{1}{2}}}{2} \right]_{x}^{5} \]  
|   | \[ = \left[ x^2 \sqrt{2x-1} \right]_{x}^{5} + \left[ \sqrt{2x-1} \right]_{x}^{5} \]  
|   | \[ = 74 + 2 = 76 \]  

49
Find the coordinates of the stationary point on the curve \( y = 2x^3 - 6x^2 + 6x - 11 \) and determine the nature of the stationary point.

**SOLUTION**

<table>
<thead>
<tr>
<th>8</th>
<th>( y = 2x^3 - 6x^2 + 6x - 11 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{dy}{dx} = 6x^2 - 12x + 6 )</td>
<td></td>
</tr>
<tr>
<td>At turning point, ( 6x^2 - 12x + 6 = 0 \Rightarrow (x-1)^2 = 0 )</td>
<td></td>
</tr>
<tr>
<td>( x = 1 )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{dy}{dx} )</td>
<td>+ve</td>
<td>0</td>
</tr>
<tr>
<td>Shape</td>
<td>( / )</td>
<td>( -- )</td>
</tr>
</tbody>
</table>

At \( x = 1 \), \( y = -9 \)

\( \therefore \) Point of inflexion at \((1, -9)\)

9

(a) Show that the roots of the equation \( 6x^2 + 5(m-1) = 3(x + m) \) are real if \( m < \frac{11}{16} \)

(b) Find the range of values of \( k \) for which \( (k+3)x^2 + 4x + k \) is always negative for all real values of \( x \).

**SOLUTION**

<table>
<thead>
<tr>
<th>9</th>
<th>( 6x^2 + 5(m-1) = 3(x + m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>( 6x^2 - 3x + 2m - 5 = 0 )</td>
</tr>
<tr>
<td>( b^2 - 4ac = 9 - 4(6)(2m - 5) = 129 - 48m )</td>
<td></td>
</tr>
<tr>
<td>For real roots, ( 129 - 48m &gt; 0 )</td>
<td></td>
</tr>
<tr>
<td>( 48m &lt; 129 )</td>
<td></td>
</tr>
<tr>
<td>( m &lt; \frac{11}{16} )</td>
<td></td>
</tr>
</tbody>
</table>

(b) For function to be negative, \( b^2 - 4ac < 0 \) and \( (k + 3) < 0 \)

\( 16 - 4k(k + 3) < 0 \)
\( -4k^2 - 12k + 16 < 0 \)
\( -4(k + 4)(k - 1) < 0 \)
\( k < -4 \text{ or } k > 1 \) (reject)
A particle $P$ moves in a straight line so that $t$ seconds after leaving a fixed point $O$, its velocity $v \text{ ms}^{-1}$ is given by $v = (2t - 3)^2 - 9$.

(a) Sketch the $v$-$t$ graph of the particle $P$ for $0 \leq t \leq 5$. 

(b) Hence or otherwise,
   (i) find the range of values of $t$ for which the acceleration of $P$ is less than $4 \text{ m/s}^2$. 
   (ii) find the distance travelled by $P$ in the first 5 seconds.

**SOLUTION**

<table>
<thead>
<tr>
<th>10</th>
<th>$v = (2t - 3)^2 - 9$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>b(i)</td>
<td>$\frac{dv}{dt} = 4(2t - 3)$</td>
</tr>
<tr>
<td></td>
<td>$\frac{dv}{dt} &lt; 4 \Rightarrow 4(2t - 3) &lt; 4 \Rightarrow t &lt; 2 \quad : \quad 0 \leq t &lt; 2$</td>
</tr>
<tr>
<td>(ii)</td>
<td>Distance traveled = $\int_0^1 (2t - 3)^2 - 9 , dt + \int_3^5 (2t - 3)^2 - 9 , dt$</td>
</tr>
<tr>
<td></td>
<td>$= \left[ \frac{(2t - 3)^3}{6} - 9t \right]_0^1 + \left[ \frac{(2t - 3)^3}{6} - 9t \right]_3^5$</td>
</tr>
<tr>
<td></td>
<td>$= 18 + 16 \frac{2}{3} + 18$</td>
</tr>
<tr>
<td></td>
<td>$= 52 \frac{2}{3} \text{ m}$</td>
</tr>
</tbody>
</table>
In the expansion of \( \left( x^2 - \frac{k}{2x} \right)^6 \), where \( k \) is a positive constant, the term independent of \( x \) is 15.

(i) Show that \( k = 2 \).

(ii) With this value of \( k \), find the coefficient of \( x^4 \) in the expansion of \( \left( x^2 - \frac{k}{2x} \right)^6 (8x + 1) \).

**SOLUTION**

\[
\begin{array}{|c|}
\hline
7 & \left( x^2 - \frac{k}{2x} \right)^6 \\
\hline
\text{(i)} & ^6C_4 (x^3)^3 \left( -\frac{k}{2x} \right)^3 = 15 \\
& \frac{k^4}{2^4} \times 15 = 15 \\
& k^4 = 2^4 \\
& k = 2 \\
\text{OR} & \\
\text{General Term} & \begin{align*}
^6C_r (x^3)^{6-r} \left( -\frac{k}{2x} \right)^r &= ^6C_r \left( -\frac{k}{2} \right)^r (x^3)^{6-r} (x)^r \\
\text{Independent of } x & : (x^3)^{6-r} (x)^r = x^0 \\
\text{Therefore} & : 12 - 3r = 0, \quad r = 4
\end{align*}
\end{array}
\]

\[
\begin{aligned}
\text{Term} &= ^6C_4 \left( -\frac{k}{2} \right)^4 = 15 \\
15 \times \left( -\frac{k}{2} \right)^4 &= 15 \\
& \quad k^4 = 2^4 \\
& \quad k = 2
\end{aligned}
\]

\[
\begin{array}{|c|}
\hline
\text{(ii)} & (...) - 20x^3 + ... \right) (8x + 1) \\
& x^4 \text{ term} = -160x^3 \\
& \therefore \text{Coefficient of } x^4 = -160
\end{array}
\]
A circle, $C$, has equation $x^2 + y^2 - 10x + 6y + 9 = 0$.

(i) Find the coordinates of the centre and radius of $C$. [3]

(ii) Give a reason why the $y$-axis is a tangent to $C$. [1]

The circle $C$ crosses the $x$-axis at the point $P(1, 0)$.

(iii) Show that the equation of the tangent to the circle $C$ at $P$ is $3y - 4x = -4$. [3]

(iv) Find the coordinates of the point where the circle $C$ crosses the $x$-axis again. [1]

**SOLUTION**

<table>
<thead>
<tr>
<th>12</th>
<th>$x^2 + y^2 - 10x + 6y + 9 = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>$x^2 - 10x + 5^2 - 5^2 + y^2 + 6y + 9 = 0$</td>
</tr>
<tr>
<td></td>
<td>$(x-5)^2 + (y+3)^2 = 25$</td>
</tr>
<tr>
<td></td>
<td>So, centre is $(5, -3)$ and radius is 5</td>
</tr>
</tbody>
</table>

(ii) Since radius is 5, leftmost $x$-coordinate of circle $C$ is $5 - 5 = 0$
Hence, the $y$-axis is a tangent to $C$.

(iii) \[
\frac{\text{grad}_{r, \text{centre}}}{1-5} = \frac{0+3}{1-5} = -\frac{3}{4}
\]
Equation of tangent is $y = -\frac{4}{3}x + c$
At $P(1, 0)$,
\[
0 = -\frac{4}{3} + c \quad \Rightarrow \quad c = -\frac{4}{3}
\]
\[
y = -\frac{4}{3}x - \frac{4}{3}
\] or $3y - 4x = -4$

(iv) $x^2 - 10x + 5^2 - 5^2 + y^2 + 6y + 9 = 0$
Sub $y = 0$,
\[
x^2 - 10x + 9 = 0
\]
\[
(x-1)(x-9) = 0
\]
Coordinates = $(9, 0)$
In a Science experiment, a container of liquid was heated to a temperature of $K^\circ C$.

It was then left to cool in a chiller such that its temperature, $T^\circ C$, $t$ minutes after removing the heat, is given by $T = Ke^{-qt}$, where $q$ is a constant.

Measured values of $t$ and $T$ are given in the following table.

<table>
<thead>
<tr>
<th>$t$ (minutes)</th>
<th>2</th>
<th>4</th>
<th>7</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T^\circ C$</td>
<td>72.8</td>
<td>60.2</td>
<td>45.2</td>
<td>34.0</td>
<td>28.1</td>
</tr>
</tbody>
</table>

(i) On graph paper, plot $\ln T$ against $t$ and draw a straight line graph.  
(ii) Use the graph to estimate the value of $K$ and of $q$.  
(iii) Estimate the temperature of the liquid 5 minutes after it was left to cool.

**SOLUTION**

<table>
<thead>
<tr>
<th>13</th>
<th>$T = Ke^{-qt}$</th>
</tr>
</thead>
</table>
| (i) | Labelling of axes of graph  
correct plots  
straight line almost passing all points.  

(ii) $T = Ke^{-qt}$  
$\ln T = -qt + \ln K$  

$$-q = \frac{4.2 - 3.72}{3 - 8} = -0.096$$  
$$q = 0.096$$  

$$\ln K = 4.48$$  
$$K = e^{4.48} \approx 88.2$$  

(iii) $T = 88.2e^{-0.096(15)}$  
$$\approx 54.6^\circ C$$  

Alternatively from graph,  
$$t = 5, \ln T = 4$$  
$$T = e^4 \approx 54.6^\circ C$$
CATHOLIC HIGH SCHOOL
Preliminary Examination 3
Secondary 4

ADDITIONAL MATHEMATICS

4047/2
16 September 2015
2 hour 30 min

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer All questions.

Attempt  Question 1 - 4 in Answer Booklet 2A.
          Question 5 - 8 in Answer Booklet 2B.
          Question 9 - 12 in Answer Booklet 2C.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in
the case of angles in degrees, unless a different level of accuracy is specified in the
question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

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The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.
1. ALGEBRA

Quadratic Equation
For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where $n$ is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

2. TRIGONOMETRY

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cosec^2 A = 1 + \cot^2 A$$

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$
The roots of the quadratic equation \( 4x^2 - 33x + 16 = 0 \) are \( \alpha^2 \) and \( \beta^2 \). Find the quadratic equation whose roots are \( \alpha \) and \( \beta \), given that \( \alpha + \beta > 0 \) and \( \alpha\beta > 0 \). [6]

2 (a) Solve the equation \( \sin^2 y + 2\cos 2y = 2\cos y \) for \( 0^\circ \leq y \leq 360^\circ \). [3]

(b) Prove that \( \frac{\cos(A + B) + \cos(A - B)}{\sin(A + B) - \sin(A - B)} = \cot B \). [4]

3

The diagram shows a triangle \( ABC \) in which angle \( CMB \) is \( \frac{\pi}{6} \) radians, angle \( B \) is a right angle, \( M \) is the mid-point of \( AB \) and the length of \( CB \) is 4 m.

Without using a calculator, find the value of the integer \( k \) such that

\[ \angle ACM = \sin^{-1} \left( \frac{\sqrt{k}}{26} \right) \] [6]
The diagram shows a quadrilateral $ABCD$ whose vertices lie on the circumference of the circle. The point $E$ lies on the extended line $CB$ such that $AE$ is a tangent to the circle. $CE$ and $AD$ are parallel lines.

(i) Explain why angle $BAE = \text{angle } CAD$. 

(ii) Show that triangles $BAE$ and $DAC$ are similar. 

(iii) Given that $AB = BE$, explain why the line $AC$ bisects the angle $BCD$. 

The diagram shows the plan of a rectangular desk, $PQRS$, in a corner of a room. Given that the desk has length 1.5 m and width 0.8 m, and that $\angle POS = \angle STR = 90^\circ$ and $\angle OPS = \theta$.

(i) Show the length of $OT$, $L$ can be expressed as $L = 1.5 \sin \theta + 0.8 \cos \theta$. 

(ii) Express $L$ in the form $R \sin (\theta + \alpha)$ where $0^\circ < \alpha < 90^\circ$ and $R > 0$. 

Hence, find the value of $\theta$ for which

(iii) $L$ has a maximum length, 

(iv) $L = 1.2$ m.
6 (a) Simplify \[ \frac{16^{x+1} + 48(4^x)}{2^{x+3} \times 8^{x+2}} \] [4]

(b) Solve the equation \[ 5^{x+1} = 8 + 4(5^{-x}) \] [5]

7

![Diagram of a rhombus with vertices A, B, C, and D.]

The diagram shows a rhombus \(ABCD\) with vertices \(A\) and \(C\) at the points \((2, 0)\) and \((4, 6)\) respectively. \(D\) lies on the \(y\)-axis and the line \(CB\) produced intersects the \(x\)-axis at \(E\).

(i) Show that the \(y\)-coordinate of \(D\) is 4. [3]
(ii) Explain why the rhombus \(ABCD\) is also a square. [2]
(iii) Find the coordinates of \(E\). [2]
(iv) Calculate the area of the quadrilateral \(AECD\). [2]
The diagram shows a solid body which consists of a cone fixed to the top of a right circular cylinder of radius \( r \) cm and height \( h \) cm. The slant edge of the cone is \( 3r \) cm.

(i) \quad \text{Given that the volume of the cylinder is} \ 108\pi \ \text{cm}^3, \ \text{express} \ h \ \text{in terms of} \ r. \quad [1]

(ii) \quad \text{Show that the total surface area,} \ A \ \text{cm}^2 \ \text{of the solid is given by} \ A = 4\pi\left(\frac{54}{r} + r^2\right). \quad [3]

(iii) \quad \text{Given that} \ r \ \text{and} \ h \ \text{can vary,}

(a) \quad \text{find the value of} \ r \ \text{for which} \ A \ \text{has a stationary value}, \quad [3]

(b) \quad \text{determine whether this stationary value is a maximum or minimum}. \quad [2]

9 \quad \text{(i) Find the range of values of} \ m \ \text{for which the curve} \ y = (x-1)(x-4) \ \text{and the line} \ y = mx + 3 \ \text{do not intersect}. \quad [3]

(ii) \quad \text{Sketch the graph of} \ y = (x-1)(x-4), \ \text{showing the coordinates of the turning point and the points where the curve meets the} x\text{-axis}. \quad [3]

(iii) \quad \text{Find the number of solutions of the equation} \ |(x-1)(x-4)| = -x+1. \quad [2]

10 \quad \text{(a) Without using a calculator, show that} \ \frac{\log_5 5 \times \log_5 4}{\log_{25} 5} = 4. \quad [3]

(b) \quad \text{Given that} \ y = \ln \sqrt{\frac{2x}{x+4}}, \ x > 0 \ \text{and} \ x < -4,

(i) \quad \text{find} \ \frac{dy}{dx}. \quad [4]

(ii) \quad \text{Hence show that} \ y \ \text{has no stationary value}. \quad [2]
11 The polynomial $P(x) = 2x^3 + ax^2 + bx + 8$, where $a$ and $b$ are constants, leaves a remainder of 10 when divided by $2x-1$. Given that $x+2$ is a factor of $P(x)$,

(i) find the value of $a$ and of $b$. [5]
(ii) Explain why the equation $P(x) = 0$ has only 1 real root. Hence find this root. [4]

12 The diagram shows part of the curve $y = 4 - e^{2x}$ which cuts the axes at $A$ and at $B$.

(i) Find the coordinates of $A$ and of $B$. [4]

The tangent to the curve at $A$ meets the $x$-axis at $C$.

(ii) Find the coordinates of $C$. [4]

(iii) Find the area of the shaded region. [4]

~ End of Paper ~
CATHOLIC HIGH SCHOOL
Preliminary Examination 3
Secondary 4

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This document consists of 6 printed pages, including this cover page.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation \( ax^2 + bx + c = 0 \),

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Binomial Expansion

\[ (a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!} \)

2. TRIGONOMETRY

Identities

\[ \sin^2 A + \cos^2 A = 1 \]

\[ \sec^2 A = 1 + \tan^2 A \]

\[ \csc^2 A = 1 + \cot^2 A \]

\[ \sin (A \pm B) = \sin A \cos B \pm \cos A \sin B \]

\[ \cos (A \pm B) = \cos A \cos B \mp \sin A \sin B \]

\[ \tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \]

\[ \sin 2A = 2 \sin A \cos A \]

\[ \cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2 \sin^2 A \]

\[ \tan 2A = \frac{2\tan A}{1 - \tan^2 A} \]

Formulae for \( \triangle ABC \)

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

\[ \Delta = \frac{1}{2} ab \sin C \]
1  The roots of the quadratic equation \(4x^2 - 33x + 16 = 0\) are \(\alpha^2\) and \(\beta^2\). Find the quadratic equation whose roots are \(\alpha\) and \(\beta\), given that \(\alpha + \beta > 0\) and \(\alpha\beta > 0\). [6]

**SOLUTION**

<table>
<thead>
<tr>
<th>1</th>
<th>(4x^2 - 33x + 16 = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\alpha^2 \beta^2 = 4)</td>
</tr>
<tr>
<td></td>
<td>(\alpha \beta = 2) or (-2) (reject)</td>
</tr>
<tr>
<td></td>
<td>(\alpha^2 + \beta^2 = \frac{33}{4})</td>
</tr>
<tr>
<td></td>
<td>((\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha \beta)</td>
</tr>
<tr>
<td></td>
<td>(= \frac{33}{4} + 4 = \frac{49}{4})</td>
</tr>
<tr>
<td></td>
<td>(\alpha + \beta = \frac{7}{2}) or (-\frac{7}{2}) (reject bec (\alpha + \beta &gt; 0))</td>
</tr>
<tr>
<td></td>
<td>(\therefore) Equation: (x^2 - \frac{7}{2}x + 2 = 0 \Rightarrow 2x^2 - 7x + 4 = 0)</td>
</tr>
</tbody>
</table>

2  (a)  Solve the equation \(\sin^2 y + 2\cos 2y = 2\cos y\) for \(0^\circ \leq y \leq 360^\circ\). [3]

  (b)  Prove that \(\frac{\cos(A + B) + \cos(A - B)}{\sin(A + B) - \sin(A - B)} = \cot B\). [4]

**SOLUTION:**

<table>
<thead>
<tr>
<th>2</th>
<th>(\sin^2 y + 2\cos 2y = 2\cos y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(1 - \cos^2 y + 4\cos^2 y - 2 - 2\cos y = 0)</td>
</tr>
<tr>
<td></td>
<td>(3\cos^2 y - 2\cos y - 1 = 0)</td>
</tr>
<tr>
<td></td>
<td>((3\cos y + 1)(\cos y - 1) = 0)</td>
</tr>
<tr>
<td></td>
<td>(\cos y = -\frac{1}{3}) or (\cos y = 1)</td>
</tr>
<tr>
<td></td>
<td>Basic Angle = 70.53(^\circ), (y = 0^\circ, 360^\circ)</td>
</tr>
<tr>
<td></td>
<td>(y = 109.5^\circ, 250.5^\circ)</td>
</tr>
<tr>
<td>(b)</td>
<td>LHS = (\frac{\cos(A + B) + \cos(A - B)}{\sin(A + B) - \sin(A - B)})</td>
</tr>
<tr>
<td></td>
<td>= (\frac{\cos A \cos B + \sin A \sin B + \cos A \cos B - \sin A \sin B}{\sin A \cos B + \cos A \sin B - \sin A \cos B + \cos A \sin B})</td>
</tr>
<tr>
<td></td>
<td>= (\frac{2 \cos A \cos B}{2 \cos A \sin B})</td>
</tr>
<tr>
<td></td>
<td>= (\frac{\cos B}{\sin B})</td>
</tr>
<tr>
<td></td>
<td>= (\cot B)</td>
</tr>
</tbody>
</table>
The diagram shows a triangle $ABC$ in which angle $CMB$ is $\frac{\pi}{6}$ radians, angle $B$ is a right angle, $M$ is the mid-point of $AB$ and the length of $CB$ is 4 m.

**Without using a calculator**, find the value of the integer $k$ such that $\angle ACM = \sin^{-1}\left(\frac{\sqrt{k}}{26}\right)$. [6]

**SOLUTION**

<table>
<thead>
<tr>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tan\left(\frac{\pi}{6}\right) = \frac{4}{MB}$</td>
</tr>
<tr>
<td>$AM = MB = \frac{4}{\tan\left(\frac{\pi}{6}\right)} = 4\sqrt{3}$</td>
</tr>
<tr>
<td>$AC = \sqrt{(8\sqrt{3})^2 + 4^2} = 4\sqrt{13}$</td>
</tr>
</tbody>
</table>

\[
\sin \angle ACM = \frac{\sin \frac{5\pi}{6}}{4\sqrt{3}} = \frac{1}{4\sqrt{3}} \times 4\sqrt{3} = \frac{\sqrt{3}}{2\sqrt{13}} = \frac{\sqrt{39}}{26}
\]

$\angle ACM = \sin^{-1}\left(\frac{\sqrt{39}}{26}\right)$

Therefore $k = 39$
The diagram shows a quadrilateral $ABCD$ whose vertices lie on the circumference of the circle. The point $E$ lies on the extended line $CB$ such that $AE$ is a tangent to the circle. $CE$ and $AD$ are parallel lines.

(i) Explain why angle $BAE = \angle CAD$. 
(ii) Show that triangles $BAE$ and $DAC$ are similar. 
(iii) Given that $AB = BE$, explain why the line $AC$ bisects the angle $BCD$.

**SOLUTION:**

<p>| | |</p>
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<tbody>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>
| (i) | $\angle BAE = \angle ACB$ (tangent chord theorem)  
     | $= \angle CAD$ (alternate angles) |
| (ii) | In triangles $BAE$ and $DAC$,  
    | $\angle BAE = \angle CAD$ (part (i))  
    | $\angle CDA = 180^\circ - \angle ABC$ (opposite angles of cyclic quadrilateral)  
    | $= \angle ABE$ (angles on straight line)  
    | $\angle ACD = \angle AEB$ (angle sum of triangle)  
    | Hence, triangles $BAE$ and $DAC$ are similar. |
|   | $AB = BE$, implying that triangles $BAE$ and $DAC$ are similar isosceles triangles.  
|   | $\angle ACD = \angle CAD$  
|   | So,  
|   | $= \angle BCA$ (alternate angles)  
|   | Hence, the line $AC$ bisects the angle $BCD$. |
(a) Simplify \( \frac{16^{x+1} + 48(4^{2x})}{2^{x+3} \times 8^{x+2}} \).

\[
\begin{align*}
&= \frac{2^{4(x+1)} + 48(2^{4x})}{2^{x+3} \times 2^{3(x+2)}} \\
&= \frac{2^{4x+4} + 48(2^{4x})}{2^{4x+9}} \\
&= \frac{2^{4x}(2^4 + 48)}{2^{4x}(2^9)} \\
&= \frac{2^4}{2^9} = \frac{1}{2^5} \\
&= \frac{1}{32} \\
&= \frac{1}{8}
\end{align*}
\]

(b) Solve the equation \( 5^{x+1} = 8 + 4(5^{x}) \).

\[
5(5^x) = 8 + 4(5^x) \Rightarrow 5u = 8 + \frac{4}{u}
\]

Let \( u = 5^x \)

\[
5u = 8 + \frac{4}{u} \Rightarrow 5u^2 = 8u + 4
\]

\[
(5u + 2)(u - 2) = 0
\]

\[
u = -\frac{2}{5} \text{ (rejected)} \quad \text{or} \quad u = 2
\]

\[
5^x = 2
\]

\[
\therefore x = 0.4306 \approx 0.431 \quad (3 \text{ s.f.})
\]
The diagram shows the plan of a rectangular desk, PQRS, in a corner of a room.

Given that the desk has length 1.5 m and width 0.8 m, and that \( \angle POS = \angle STR = 90^\circ \) and \( \angle OPS = \theta \).

(i) Show the length of \( OT \), \( L \) can be expressed as \( L = 1.5 \sin \theta + 0.8 \cos \theta \). [3]

(ii) Express \( L \) in the form \( R \sin (\theta + \alpha) \) where \( 0^\circ < \alpha < 90^\circ \) and \( R > 0 \). [3]

Hence, find the value of \( \theta \) for which

(iii) \( L \) has a maximum length, [2]

(iv) \( L = 1.2 \) m. [2]

**SOLUTION:**

<p>| | | |</p>
<table>
<thead>
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</table>
| (i) \( \angle TSR = \theta \), \( \cos \theta = \frac{ST}{0.8} \) & \( \sin \theta = \frac{OS}{1.5} \) & \( \Rightarrow \) \( ST = 0.8 \cos \theta \)
|   |   | \( \Rightarrow \) \( OS = 1.5 \sin \theta \)
|   |   | \( OT = OS + ST \)
|   |   | \( L = 1.5 \sin \theta + 0.8 \cos \theta \)
| (ii) \( L = 1.5 \sin \theta + 0.8 \cos \theta = R \sin (\theta + \alpha) \) & where \( R = \sqrt{1.5^2 + 0.8^2} = 1.7 \)
|   |   | \( \tan \alpha = \frac{0.8}{1.5} \), \( \Rightarrow \alpha = 28.07^\circ \)
|   |   | \( \therefore L = 1.7 \sin (\theta + 28.07^\circ) \)
| (iii) \( L \) has maximum length when \( \sin (\theta + 28.07) = 1 \)
|   |   | \( \theta + 28.07 = 90^\circ \)
|   |   | \( \theta = 61.9^\circ \) (1 dp)
The diagram shows a rhombus $ABCD$ with vertices $A$ and $C$ at the points $(2, 0)$ and $(4, 6)$ respectively. $D$ lies on the $y$-axis and the line $BC$ produced intersects the $x$-axis at $E$.

(i) Show that the $y$-coordinate of $D$ is 4. [3]

(ii) Explain why the rhombus $ABCD$ is also a square. [2]

(iii) Find the coordinates of $E$. [2]

(iv) Calculate the area of the quadrilateral $AECD$. [2]

**SOLUTION:**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td></td>
</tr>
<tr>
<td><strong>Midpoint of $AC$</strong></td>
<td>$\left(\frac{2 + 4}{2}, \frac{0 + 6}{2}\right)$</td>
</tr>
<tr>
<td></td>
<td>$(3, 3)$</td>
</tr>
<tr>
<td>$m_{AC}$</td>
<td>$\frac{6 - 0}{4 - 2} = 3$</td>
</tr>
</tbody>
</table>
Equation of perpendicular bisector of $AC$ is $y = -\frac{1}{3}x + c$

| At $(3, 3)$, |
| $3 = -\frac{1}{3}(3) + c$ |
| $c = 4$ |

$\therefore$ $y$-coordinate of $D$ is 4.

(ii)

\[
\begin{align*}
\text{grad}_{AD} &= \frac{0 - 4}{2 - 0} = -2 \\
\text{grad}_{CD} &= \frac{6 - 4}{4 - 0} = \frac{1}{2}
\end{align*}
\]

$-2 \times \frac{1}{2} = -1 \implies AD$ and $CD$ are perpendicular, hence $ABCD$ is a square.

(iii)

Equation of $BC$ is $y = -2x + c$

| At $(4, 6)$, |
| $6 = -2(4) + c$ |
| $c = 14$ |

$\therefore y = -2x + 14$

Along $x$-axis, $y = 0$.

$0 = -2x + 14$

$x = 7$

$E (7, 0)$

(iv)

| Area $\quad | \quad 1 \quad 2 \quad 7 \quad 4 \quad 0 \quad 2$ |
| --- | --- | --- | --- | --- | --- |
| $\frac{1}{2} \quad | \quad 0 \quad 0 \quad 6 \quad 4 \quad 0$ |

\[
\begin{align*}
\text{Area} &= \frac{1}{2} \begin{vmatrix} 1 & 2 & 7 & 4 & 0 & 2 \\ 2 & 0 & 0 & 6 & 4 & 0 \end{vmatrix} \\
&= \frac{1}{2} [58 - 8] \\
&= 25
\end{align*}
\]
The diagram shows a solid body which consists of a cone fixed to the top of a right circular cylinder of radius \( r \) cm and height \( h \) cm. The slant edge of the cone is \( 3r \) cm.

(i) Given that the volume of the cylinder is \( 108\pi \text{ cm}^3 \), express \( h \) in terms of \( r \). \[ 1 \]

(ii) Show that the total surface area, \( A \) \( \text{cm}^2 \) of the solid is given by \( A = 4\pi \left( \frac{54}{r} + r^2 \right) \). \[ 3 \]

(iii) Given that \( r \) and \( h \) can vary,

(a) find the value of \( r \) for which \( A \) has a stationary value,

(b) determine whether this stationary value is a maximum or minimum. \[ 2 \]

**SOLUTION:**

| \( h \) | \( \pi r^2 h = 108\pi \) \[ i \]  
|--------|--------------------------|
| \( h \) | \( \frac{108}{r^2} \) \[ ii \]  
| Total surface area = area of cylinder + area of cone | \[ iii \]  
| \( 2\pi rh + \pi r^2 + \pi rl \) | \[ iii \]  
| \( = 2\pi rh + \pi r^2 + 3\pi r^2 \) | \[ iii \]  
| \( = 2\pi r \left( \frac{108}{r^2} \right) + 4\pi r^2 \) | \[ iii \]  
| \( = 4\pi \left( \frac{54}{r} + r^2 \right) \) (shown) | \[ iii \]  
| \( \frac{dA}{dr} = -\frac{216\pi}{r^2} + 8\pi r \) | \[ iii \]  

2015 CHS PRELIM 3 ADDITIONAL MATHEMATICS (4047/2)
(a) \[
\frac{-216\pi}{r^2} + 8\pi r = 0
\]
\[
\frac{216\pi}{r^2} = 8\pi r
\]
\[
216 = 8r^3
\]
\[
r = 3
\]

Sub \( r = 3 \) into \( \frac{d^2 A}{dr^2} \),

\[
\frac{d^2 A}{dr^2} = \frac{432\pi}{r^3} + 8\pi
\]
\[
= \frac{432\pi}{(3)^3} + 8\pi
\]
\[
= 24\pi
\]

Since \( \frac{d^2 A}{dr^2} \) is positive, \( A \) is a minimum. (shown)
9

(i) Find the range of values of \( m \) for which the curve \( y = (x-1)(x-4) \) and the line \( y = mx + 3 \) do not intersect. [3]

(ii) Sketch the graph of \( y = |(x-1)(x-4)| \), showing the coordinates of the turning point and the point where the curve meets the \( x \)-axis. [3]

(iii) Find the number of solutions of the equation \( |(x-1)(x-4)| = -x+1 \). [2]

**SOLUTION:**

\[
9 \quad y = (x-1)(x-4)
\]

\[
(x-1)(x-4) = mx + 3
\]

\[
x^2 - 5x + 4 - mx - 3 = 0
\]

\[
x^2 - (m+5)x + 1 = 0
\]

\[
b^2 - 4ac < 0
\]

\[
(m+5)^2 - 4 < 0
\]

\[
(m+7)(m+3) < 0
\]

\[
\therefore -7 < m < -3
\]

(ii)

(iii)

(a) 1 solution
10  (a) **Without using a calculator**, show that \[ \frac{\log_2 5 \times \log_3 4}{\log_{25} 5} = 4. \]  

(b) Given that \[ y = \ln \sqrt[2]{\frac{2x}{x+4}}, \quad x > 0 \text{ and } x < -4, \]

(i) find \[ \frac{dy}{dx}. \]

(ii) Hence show that \[ y \] has no stationary value.

**SOLUTION:**

<table>
<thead>
<tr>
<th>(a)</th>
<th>[ \frac{\log_2 5 \times \log_3 4}{\log_{25} 5} ]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[ \frac{\log_2 5 \times \log_3 4}{\log_{25} 5} = \log_2 4 \div \log_2 25 ]</td>
</tr>
<tr>
<td></td>
<td>[ \frac{2 \log_2 2 + \log_2 5}{2 \log_2 5} ]</td>
</tr>
<tr>
<td></td>
<td>[ = 2 \times 2 = 4 ]</td>
</tr>
</tbody>
</table>

(b) \[ y = \ln \sqrt[2]{\frac{2x}{x+4}} \]

(i) \[ = \ln \left( \frac{2x}{x+4} \right)^{\frac{1}{2}} = \frac{1}{2} \ln \left( \frac{2x}{x+4} \right) \]

\[ = \frac{1}{2} \left[ \ln 2x - \ln (x+4) \right] \]

\[ \frac{dy}{dx} = \frac{1}{2} \left[ \frac{2}{x} - \frac{1}{x+4} \right] \]

\[ = \frac{1}{2} \left[ \frac{(x+4) - x}{x(x+4)} \right] = \frac{2}{x(x+4)} \]

(ii) \[ \frac{2}{x(x+4)} \neq 0 \]

since \[ \frac{dy}{dx} \neq 0 \Rightarrow \text{there is no stationary value} \]
The polynomial \( P(x) = 2x^3 + ax^2 + bx + 8 \), where \( a \) and \( b \) are constants, leaves a remainder of 10 when divided by \( 2x - 1 \). Given that \( x + 2 \) is a factor of \( P(x) \),

(i) find the value of \( a \) and of \( b \). \[5\]

(ii) Explain why the equation \( P(x) = 0 \) has only 1 real root. Hence find this root. \[4\]

**SOLUTION:**

<table>
<thead>
<tr>
<th></th>
<th>( P(x) = 2x^3 + ax^2 + bx + 8 )</th>
</tr>
</thead>
</table>
| (i) | \[ P \left( \frac{1}{2} \right) = 2 \left( \frac{1}{2} \right)^3 + a \left( \frac{1}{2} \right)^2 + b \left( \frac{1}{2} \right) + 8 = 10 \]  
   | \[ \frac{1}{4}a + \frac{1}{2}b = \frac{7}{4} \]  
   | \[ a + 2b = 7 \]  
   | \[ P(-2) = 2(-2)^3 + a(-2)^2 + b(-2) + 8 = 0 \]  
   | \[ 4a - 2b = 8 \]  
   | \[ 2a - b = 4 \]  
   | \[ 2(7 - 2b) - b = 4 \Rightarrow -5b = -10 \]  
   | \[ b = 2, \]  
   | \[ a = 7 - 2(2) = 3 \]  
| (ii) | \[ P(x) = 2x^3 + 3x^2 + 2x + 8 \]  
   | \[ = (x + 2)(2x^2 + bx + 4) \]  
   | Term in \( x^2 \):  
   | \[ 3x^2 = bx^2 + 4x^2, \ b = -1 \]  
   | \[ P(x) = 2x^3 + 3x^2 + 2x + 8 \]  
   | \[ = (x + 2)(2x^2 - x + 4) \]  
   | for \( 2x^2 - x + 4, \)  
   | \[ b^2 - 4ac = 1 - 4(2)(4) \]  
   | \[ = -31 < 0 \]  
   | Hence, the equation \( 2x^2 - x + 4 = 0 \) has no roots.  
   | So \( P(x) = 0 \) has only 1 real root.  
   | The root is \( x + 2 = 0 \) ie \( x = -2 \) |
12. The diagram shows part of the curve $y = 4 - e^{\frac{1}{2}x}$ which cuts the axes at $A$ and at $B$.

(i) Find the coordinates of $A$ and of $B$. [4]

The tangent to the curve at $A$ meets the $x$-axis at $C$.

(ii) Find the coordinates of $C$. [4]

(iii) Find the area of the shaded region. [4]

SOLUTION:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>12</td>
<td>$y = 4 - e^{\frac{1}{2}x}$</td>
</tr>
<tr>
<td>(i)</td>
<td>When $x = 0$, $y = 4 - e^{\frac{1}{2}(0)} = 3 \Rightarrow A(0, 3)$</td>
</tr>
<tr>
<td></td>
<td>When $y = 0$, $0 = 4 - e^{\frac{1}{2}x}$</td>
</tr>
<tr>
<td></td>
<td>$e^{\frac{1}{2}x} = 4$</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{2}x = \ln 4$</td>
</tr>
<tr>
<td></td>
<td>$x = 2\ln 4$ or $4\ln 2 \Rightarrow B(2\ln 4, 0)$ or $B(4\ln 2, 0)$</td>
</tr>
<tr>
<td>(ii)</td>
<td>$\frac{dy}{dx} = -\frac{1}{2}e^{\frac{1}{2}x}$</td>
</tr>
<tr>
<td></td>
<td>$= -\frac{1}{2}e^{\frac{1}{2}(0)}$</td>
</tr>
<tr>
<td></td>
<td>$= -\frac{1}{2}$</td>
</tr>
</tbody>
</table>
Equation of tangent: \( y = -\frac{1}{2}x + 3 \)

When \( y = 0 \), \( 0 = -\frac{1}{2}x + 3 \)
\[ x = 6 \implies C(6,0) \]

(iii) Shaded area

\[
= \frac{1}{2} \times 6 \times 3 - \int_{\frac{1}{4}}^{4\ln 2} 4 - e^x dx
= 9 - \left[ 4x - e^x \right]_{\frac{1}{4}}^{4\ln 2}
= 9 - \left[ 4(4\ln 2) - 2e^{\frac{1}{4}4\ln 2} - (-2) \right]
= 3.9096
\approx 3.91 \text{ units}^2

~ End of Paper ~
Answer all questions.

1. Express \( \frac{8x^2 + 1}{x(2x - 1)} \) in partial fractions. [5]

2. (i) Prove that \( \csc(60^\circ - \theta) = \frac{2}{\cos \theta (\sqrt{3} - \tan \theta)} \). [3]
(ii) Hence find, in surd form, the value of \( \csc 15^\circ \). [4]

3. (a) Find the term independent of \( x \) in the expansion of \( (2x^2 - \frac{1}{\sqrt{x}})^{10} \). [3]
(b) It is given that in the expansion of \( (5 + px)^n \), the coefficients of \( x^3 \) and \( x^4 \) are the same. Express \( n \) in terms of \( p \). [4]

4. (a) The equation of a curve is \( y = (a + 2)x^3 - 3x + (a - 1) \).
   In the case where \( a = 3 \), show that \( y = 7x - 3 \) is a tangent to the curve. [3]
(b) Given that \( (m - 4)x^2 < 3x - m \), show that \( m \) cannot be positive. [4]

5. (a) Sketch \( y = |x(1 - 4x)| \), indicating the coordinates of the maximum point and intercepts.
   Hence, state the range of values of \( k \) such that \( \left| \frac{x}{2} - 2x \right| = k \) gives more than 2 solutions. [4]
(b) The diagram shows the graph of \( y = 1 - |x - 5| \) where the \( x \)-intercepts are 4 and 6.

If a line \( y = mx + c \) is to be added to the diagram above, determine a possible value for \( m \) and \( c \) if
(i) there is 1 intersection between the 2 graphs. [1]
(ii) there are infinite intersections between the 2 graphs. [2]
6 (a) A curve has the equation \( y = e^{\sqrt{x} \ln(3x)} \), where \( x > 0 \). Find an expression for \( \frac{dy}{dx} \). Hence find \( \int_{1}^{3} \frac{3 \ln(3x) + 2}{2 \sqrt{x}} \, dx \). [4]

(b) It is given that \( y = 6e^{\sqrt{x}-1} \). Find, in terms of \( e \), the rate of change of \( x \) at the instant when \( x = 5 \) if \( y \) is decreasing at the rate of \( \frac{1}{2} \) units per second at this instant. [4]

7 It is known that \( x \) and \( y \) are related by the equation \( m^y = n(2^x) \), where \( m \) and \( n \) are constants.

<table>
<thead>
<tr>
<th></th>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( y )</td>
<td>0.566</td>
<td>0.80</td>
<td>1.13</td>
<td>1.60</td>
<td>2.26</td>
</tr>
</tbody>
</table>

(i) Plot \( \lg y \) against \( x \) and draw a straight line graph. [2]
(ii) Use your graph to estimate the values of \( m \) and \( n \). [4]
(iii) By drawing a suitable straight line, solve the equation \( y = 0.9^x \). [2]

8 A cylinder of radius \( r \) cm and height \( h \) is inscribed in a cone of base radius 5 cm and height 20 cm. The cross section of the solid is shown in the diagram.

(i) Show that the volume within the cone but not occupied by the cylinder, \( V \), is given by \( V = \frac{500}{3} \pi - \left( \frac{h}{4} \right)^3 \pi h \). [3]

(ii) Find the stationary value of \( V \) and determine whether it is maximum or minimum. [6]
9 (a) The equation of a curve is \( y = \frac{5}{kx^3} + 10x^3 \). Given that its normal at \( x = 2 \) will never meet the \( y \)-axis, find the value of \( k \). [3]

(b) A curve has the equation \( y = 2\cos^2 3x \) for \( 0 \leq x \leq \pi \). Find
(i) the equation of the normal at \( x = \frac{\pi}{12} \). [4]
(ii) the \( x \)-values on the curve whose tangents are parallel to the normal at \( x = \frac{\pi}{12} \). [4]

10

In the diagram, \( ADC \) is a sector of the circle with centre \( C \) and \( BDCE \) is a straight line. The line \( AB \) is parallel to the \( y \)-axis and points \( C \) and \( D \) are \((15, 0)\) and \((0, 8)\) respectively.

(i) Show that coordinates of \( A \) is \((-2, 0)\). [2]
(ii) Find the equation of the line that passes through \( A \) and perpendicular to the line \( BC \). [2]
(iii) Find the coordinates of \( E \) if the ratio of area \( ABC \): area \( ACE \) is given to be 2:1. [5]
(iv) Given that \( ABFE \) is a kite, find the area of \( ABFE \). [2]

~ End of Paper ~
<table>
<thead>
<tr>
<th>Qn</th>
<th>Detailed Working</th>
<th>Mark Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{8x^4+1}{x(2x-1)} = 4x^2 + 2x + 1 + \frac{x+1}{x(2x-1)}$</td>
<td>M1 (for correct quotient from long division)</td>
</tr>
<tr>
<td></td>
<td>$\frac{x+1}{x(2x-1)} = \frac{A}{x} + \frac{B}{2x-1}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$x+1 = A(2x-1) + Bx$</td>
<td>M1, M1 (for $A$ and $B$)</td>
</tr>
<tr>
<td></td>
<td>Let $x = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$A = -1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Let $x = 0.5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$0.5B = 1.5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$B = 3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{8x^4+1}{x(2x-1)} = 4x^2 + 2x + 1 - \frac{1}{x} + \frac{3}{2x-1}$</td>
<td>A1</td>
</tr>
<tr>
<td>2(i)</td>
<td>$\csc (60^\circ - \theta) = \frac{2}{\cos \theta (\sqrt{3} - \tan \theta)}$</td>
<td></td>
</tr>
<tr>
<td>LHS</td>
<td>$\frac{1}{\sin (50^\circ - \theta)}$</td>
<td>M1 (for applying Addition formula)</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{\sin 60^\circ \cos \theta - \cos 60^\circ \sin \theta}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{\frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{2 + \cos \theta}{\sqrt{3} \cos \theta - \sin \theta + \cos \theta}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{2}{\cos \theta (\sqrt{3} - \tan \theta)} = \text{RHS (shown)}$</td>
<td>A1</td>
</tr>
</tbody>
</table>
2(ii) \[ \cosec 15^\circ \]
\[ = \cosec (60^\circ - 45^\circ) \]
\[ = \frac{2}{\cos 45^\circ (\sqrt{3} - \tan 45^\circ)} \]
\[ = \frac{2}{\cos 45^\circ (\sqrt{3} - 1)} \]
\[ = \frac{2\sqrt{2}}{(\sqrt{3} - 1)(\sqrt{3} + 1)} \]
\[ = \frac{\sqrt{2}(\sqrt{3} + 1)}{2} \text{ or } \sqrt{6} + \sqrt{2} \]

M1 (for special \( \theta \)s)
M1 (for rationalizing)
A1

3(a) \[(2x^2 - \frac{1}{\sqrt{x}})^9\]
\[T_{r+1} \]
\[= 10C_r (2x^2)^9 (\frac{1}{\sqrt{x}})^r \]
\[= 10C_r (2)^{10r} (-1)^r (x)^{10-1-\frac{r}{2}} \]
\[= 10C_r (2)^{10r} (-1)^r (x)^{10-\frac{3}{4}r} \]
\[20 - 2.5r = 0 \]
\[r = 8 \]
\[\therefore T_9 = 10C_1 (2)^7 (-1)^8 = 180 \]

M1
M1
A1

3(b) \[(5 + px)^n\]
\[^nC_1 (5)^{n-4} p^4 = ^nC_2 (5)^{n-3} p^3 \]
\[\frac{n(n-1)(n-2)(n-3)}{1 \times 2 \times 3 \times 4} (5)^{n-4} p^4 = \frac{n(n-1)(n-2)}{1 \times 2 \times 3} (5)^{n-3} p^3 \]
\[\frac{n-3}{4} p = 5 \]
\[\frac{n-3}{4} = \frac{5}{p} \]
\[n = \frac{20}{p} + 3 \]

M1 (for specific term or correct expansion)
M1, M1 (for applying \( ^nC_r \) formula to both sides)
A1
4(a) \[ y = (a + 2)x^2 - 3x + (a - 1) \]
Let \( a = 3 \),
\[ y = 5x^2 - 3x + 2 \]
\[ 5x^2 - 3x + 2 = 7x - 3 \]
\[ 5x^2 - 10x + 5 = 0 \]

\[ D = (-10)^2 - 4(5)(5) = 0 \]
Therefore \( y = 7x - 3 \) is a tangent (shown).

| M1 | M1 | B1 |

4(b) \[ (m - 4)x^2 < 3x - m \]
\[ (m - 4)x^2 - 3x + m < 0 \]

\[ (-3)^2 - 4(m - 4)(m) < 0 \]
\[ 9 - 4m(m - 4) < 0 \]
\[ 4m^2 - 16m - 9 > 0 \]
\[ (2m + 1)(2m - 9) > 0 \]
\[ m < -\frac{1}{2}, \quad m > 4\frac{1}{2} \]

Since \( m - 4 < 0 \), thus \( m < -\frac{1}{2} \).
\[ \therefore \] \( m \) cannot be positive (shown)

| M1 | M1 | B1 |

5(a) Shape - B1
Turning pt & x-intercepts - B1
\[
\frac{|x - 2x^2|}{2} = k \\
\frac{1}{2}|x - 4x^2| = k \\
|x(1 - 4x)| = 2k
\]

\[
0 < 2k \leq \frac{1}{16} \\
0 < k \leq \frac{1}{32}
\]

**5(b)(i)** \(m = 0, c = 1\)

OR any other relevant answers

**5(b)(ii)** (5, 1) and (4, 0)

\[
m = \frac{1 - 0}{5 - 4} = 1 \\
y = 1(x - 4) \\
y = x - 4
\]

OR

(5, 1) and (6, 0)

\[
m = \frac{1 - 0}{5 - 6} = -1 \\
y = -1(x - 6) \\
y = -x + 6
\]

**6(a)** \(y = e^{\sqrt{x} \ln(3x)}\)

\[
\frac{dy}{dx} = \frac{1}{2} e^{\frac{\ln(3x)}{x}} + e^{\frac{\ln(3x)}{x}} \\
= \frac{e^{\ln(3x)}}{2\sqrt{x}} + \frac{e}{\sqrt{x}} \\
= \frac{e^{\ln(3x) + 2e}}{2\sqrt{x}}
\]

\[
= \frac{e^{\ln(3x) + 2}}{2\sqrt{x}}
\]

M1

A1

Alternative answer:
\[
\int_1^5 \frac{\text{e}^{\ln(3x)+2}}{2\sqrt{x}} \, dx = \left[ \sqrt{x} \ln(3x) \right]_1^5 = \sqrt{5} \ln 15 - \ln 3 = 4.96
\]

### 6(b)

\[
y = 6e^{\sqrt{x}}
\]

\[
\frac{dy}{dx} = 6e^{\sqrt{x}} \cdot \frac{1}{2} (x-1)^{-\frac{1}{2}} = \frac{3e^{\sqrt{x}}}{\sqrt{x-1}}
\]

\[
\frac{1}{2} e^{\sqrt{x}} = \frac{3e^{\sqrt{x}}}{\sqrt{x-1}} \times \frac{dx}{dt}
\]

\[
\frac{1}{2} e^{\sqrt{x}} = \frac{3e^{\sqrt{x}}}{\sqrt{x-1}} \times \frac{dx}{dt}
\]

\[
\frac{dx}{dt} = \frac{1}{3} \text{ units/s}
\]

### 8(i)

\[
\frac{20}{5} = \frac{20 - h}{r}
\]

\[
20r = 100 - 5h
\]

\[
r = \frac{100 - 5h}{20}
\]

\[
r = 5 - \frac{1}{4} h
\]

\[
V = \frac{1}{3} \pi (5)^2 (20) - \pi (5 - \frac{1}{4} h)^2 \frac{h}{3}
\]

\[
= \frac{500\pi}{3} - (5 - \frac{1}{4} h)^2 \pi h \quad \text{(shown)}
\]
8(ii) \[ V = \frac{500\pi}{3} - \left(5 - \frac{1}{4}h\right)^2 \pi h \]
\[ = \frac{500\pi}{3} - 25\pi h + \frac{5}{2} \pi h^2 - \frac{1}{16} \pi h^3 \]
\[
\frac{dV}{dh} = -25\pi + 5\pi h - \frac{3}{16} \pi h^2
\]
\[
\frac{3}{16} h^2 - 5h + 25 = 0
\]
\[
h = 20 \text{ (ref)}, \quad h = \frac{2}{3} \text{ cm}
\]
\[
\frac{d^2V}{dh^2} = 5\pi - \frac{6}{16} \pi h
\]
\[
= 5\pi - \frac{6}{16} \pi \left(h - \frac{2}{3}\right) > 0 \text{ (min)}
\]
\[ V = \frac{500\pi}{3} - (5 - \frac{1}{4} \times \frac{2}{3})^2 \pi \left(\frac{2}{3}\right) = 291 \text{ cm}^2 \]

9(a) \[ y = \frac{5}{kx^2} + 10x^3 = \frac{5}{k}x^{-2} + 10x^3 \]
\[
\frac{dy}{dx} = -\frac{10}{kx^3} + 30x^2
\]
\[
-\frac{10}{kx^3} + 30x^2 = 0
\]
\[
-10 + 30kx^4 = 0
\]
\[
10 = 30k(2^4)
\]
\[ k = \frac{1}{96} \]
9(b)(i) \[ y = 2\cos^2 3x \]
\[
\frac{dy}{dx} = -12\cos 3x \sin 3x 
\]
\[
m_1 = -12 \cos 3\left(\frac{\pi}{12}\right) \sin 3\left(\frac{\pi}{12}\right) = -6 
\]
\[
m_2 = \frac{1}{6} 
\]
\[
x = \frac{\pi}{12}, \quad y = 2\cos^2 3\left(\frac{\pi}{12}\right) = 1 
\]
\[
y - 1 = \frac{1}{6}(x - \frac{\pi}{12}) 
\]
\[
y = \frac{1}{6}x + \frac{72 - \pi}{72} 
\]

9(b)(ii) \[-12\cos 3x \sin 3x = \frac{1}{6} \]
\[
-6(2\sin 3x \cos 3x) = \frac{1}{6} 
\]
\[
\sin 6x = -\frac{1}{36} 
\]
\[
\alpha = 0.02778 
\]
\[
6x = 3.1694, 6.2354, 9.4526, 12.5386, 15.7358, 18.8218 
\]
\[
x = 0.528, 1.04, 1.58, 2.09, 2.62, 3.14 
\]

10(i) \[
\sqrt{(15-0)^2 + (0-8)^2} = 17 \text{ units} 
\]
\[
A = (15-17, 0) = (-2, 0) 
\]

10(ii) \[
m_{BC} = \frac{8-0}{0-15} = -\frac{8}{15} 
\]
\[
m_{\perp BC} = \frac{15}{8} 
\]

Equation of line \( \perp \) \( BC \):
\[
y - 0 = \frac{15}{8}(x + 2) 
\]
\[
y = \frac{15}{8}x + \frac{15}{4} 
\]
10(ii) Equation of line BC:
\[ y - 8 = -\frac{8}{15}(x + 0) \]
\[ y = -\frac{8}{15}x + 8 \]

Let \( B(-2, y) \)
\[ y = -\frac{8}{15}(-2) + 8 = \frac{16}{15} + \frac{120}{15} = \frac{136}{15} \]
\[ B(-2, \frac{136}{15}) \]

\( ABC \) & \( ACE \) share the same base \( AC \).
Hence, \( \perp \) height of \( E \) to \( x \)-axis should be \( \frac{1}{2} \) of \( AB \).
\( \perp \) height of \( E \) to \( x \)-axis \( = \frac{1}{2}(\frac{8}{15}) = \frac{68}{15} = 4 \frac{16}{15} \)

Let \( E(x, -\frac{8}{15}) \)
\[ -\frac{8}{15} = -\frac{8}{15}(x) + 8 \]
\[ x = \frac{23}{2} \]
\[ E = (\frac{23}{2}, -\frac{8}{15}) \]

10(iv) Area of \( ABFE = \) Area of \( 2(ABF) \)
\[
\begin{vmatrix}
-2 & -2 & 47 \\
-2 & 0 & -68 \\
0 & 15 & 15 \\
\end{vmatrix} \\
\times 2
\]
\[ = \frac{1}{2} \times 2 \times 136 = 231.2 \text{ units}^2 \]
1. The curve for which \( \frac{dy}{dx} = \frac{k}{(2x+5)^3} - 1 \), where \( k \) is a constant, is such that the tangent to the curve at \((-2, 0)\) is perpendicular to the line \( 5y = x - 1 \). Find
   (i) the value of \( k \). \[2\]
   (ii) the equation of the curve. \[3\]

2. The roots of the equation \( x^2 = mx - 2m^2 \) are \( \alpha \) and \( \beta \). Find, in terms of \( m \), an equation whose roots are \( \frac{1}{\alpha^3} \) and \( \frac{1}{\beta^3} \). \[6\]

3. (a) Without using a calculator, find the value of \( c \) given that
   \[34 + 3\sqrt{128} = \left( \frac{6}{\sqrt{2}} - c \right)^2. \] \[3\]
   (b) The volume of a cylinder is \( (9 + \sqrt{50})\pi \text{ cm}^3 \). Given that the cylinder has a radius of \( (2 + \sqrt{2}) \text{ cm} \), find, without using a calculator, the height of the cylinder in the form \( \frac{a + b\sqrt{2}}{c} \). \[4\]

4. An object is heated in an oven until it reaches the temperature of 90 °C. It is then allowed to cool. Its temperature, \( T \text{ °C} \), when it has been cooling for time \( t \) minutes, is given by the equation \( T = k + he^{-\frac{T}{10}} \), where \( k \) and \( h \) are constants.

   Given that the temperature of the object is 40 °C when it has been cooling for exactly \( 10 \ln 3 \) minutes, show that \( k = 15 \) and \( h = 75 \). \[3\]
   (i) Calculate the value of \( T \) when \( t = 10 \). \[1\]
   (ii) Determine the rate at which \( T \) is decreasing when \( t = 25 \). \[2\]
   (iii) Find, to the nearest minute, the time taken for the temperature of the object to drop below 35°C. \[2\]
5 (a) Express \( y \) in terms of \( x \), \( \log_{3} x^2 y = 3 + \log_{3} x - \frac{1}{2\log_{3} 5} \) \( \quad [4] \)

(b) Solve the equation \( \log_{5}(x+5) - \log_{5}(x-1) = \log_{5} 2 \). \( \quad [4] \)

6 A cubic polynomial \( f(x) \) is such that the coefficient of \( x^3 \) is 6. It is given that one of the roots of the equation \( f(x) = 0 \) is \( \frac{4}{3} \) and \( [2x^3 + (2k+1)x - 3] \) is a quadratic factor of \( f(x) \). Given further that \( f(x) \) leaves a remainder of 30 when divided by \( (x-2) \), find

(i) the value of \( k \) and hence, factorise \( f(x) \) completely, \( \quad [4] \)

(ii) by using the result from part (i), the number of real roots of the equation \( f(x) = 15(1-2x) \), justifying your answer. \( \quad [4] \)

7 The diagram shows a circle \( ABCD \) and the tangent \( ST \) of the circle at point \( C \). \( B \) and \( C \) bisect \( AT \) and \( ST \) respectively. Prove that

(i) \( \triangle ABC \) is similar to \( \triangle SDC \), \( \quad [4] \)

(ii) \( AS = \frac{2AC \times DC}{TC} \). \( \quad [4] \)

8 (i) A circle passes through the origin \( O \) and cuts the \( x- \) and \( y- \) axes at \( 3 \) and \( 4 \) respectively. Find the equation of the circle in the general form. \( \quad [4] \)

(ii) Given that \( OP \) is the diameter of the circle, find the equation of the tangent at \( P \). \( \quad [3] \)

(iii) Another tangent at \( Q \), which is parallel to the \( y- \) axis, meets the tangent found in part (ii). Find the points of intersections between the two tangents. \( \quad [3] \)
9 (i) Given that \( y = x^2 \ln x^3 \), show that \( \frac{dy}{dx} = 3x(1 + 2 \ln x) \). \[ 3 \]

(ii) The diagram shows part of the curve \( y = x \ln x \) cutting the \( x \)-axis at point \( P \). The line \( x = e \) intersects the curve at point \( Q \).

(a) Find the \( x \)-coordinate of \( P \). \[ 2 \]

(b) By using the result from part (i), show that the area of the shaded region bounded by the \( x \)-axis, the line \( x = e \) and the curve is \( \frac{1}{4}(e^2 + 1) \). \[ 4 \]

10 (i) Express \( 3 \cos 2A + 4 \sin 2A \) in the form \( R \cos(2A - \alpha) \), where \( 0^\circ \leq \alpha \leq 90^\circ \). \[ 2 \]

(ii) Hence solve \( 3 \cos 2A + 4 \sin 2A = 4 \) for \( 0^\circ \leq A \leq 180^\circ \). \[ 3 \]

(iii) On the same axes sketch, for \( 0^\circ \leq x \leq 60^\circ \), the graphs of

\[
y = 2 \sin 6x \quad \text{and} \quad y = 2 - \frac{3}{2} \cos 6x.
\]

(iv) Explain how the solutions of the equation in part (ii) could be used to find the \( x \)-coordinates of the points of intersection of the graphs of part (iii). \[ 2 \]
11 A particle moves in a straight line such that \( t \) seconds after passing through a fixed point \( O \), its velocity, \( v \) m/s, is given by \( v = 24\cos(2t) \). When \( t = 0 \), its displacement from \( O \) is \(-6\) metres. Find

(i) the magnitude of the acceleration when \( t = 1 \), \hspace{1cm} \[2\]
(ii) the value of \( t \) when the particle first reaches the fixed point \( O \), \hspace{1cm} \[4\]
(iii) the distance travelled by the particle up to the second instantaneous rest. \hspace{1cm} \[4\]

12 A curve has the equation \( y = (x - 2)(x + 1)^3 \).

(i) Find an expression for \( \frac{dy}{dx} \). \hspace{1cm} \[2\]
(ii) Find the \( x \)-coordinates of the stationary points. \hspace{1cm} \[2\]
(iii) Without determining the nature of the stationary points, show that \( y \) decreases as \( x \) increases between the stationary points. \hspace{1cm} \[3\]
(iv) Determine the nature of the stationary points. \hspace{1cm} \[4\]
<table>
<thead>
<tr>
<th>Qn</th>
<th>Marking point</th>
<th>Mark Awarded</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(a)</td>
<td>( \frac{dy}{dx} = -5 )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>(5m)</td>
<td>( k \left( \frac{1}{2(-2)+5} \right)^3 - 1 = -5 )</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( k = -4 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>( \frac{dy}{dx} = \frac{4}{(2x+5)^3} - 1 )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( y = \int \frac{-4}{(2x+5)^3} - 1 , dx )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{(2x+5)^3} - x + c )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sub ((-2, 0), 0 = 1 - (-2) + c)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( c = -3 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( y = \frac{1}{(2x+5)^3} - x - 3 )</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>( x^3 - mx + 2m^2 = 0 )</td>
<td>B1</td>
<td>Either sum or product</td>
</tr>
<tr>
<td></td>
<td>( \alpha + \beta = m )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \alpha \beta = 2m^2 )</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \alpha^3 + \beta^3 = (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta] )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = m[(m^3 - 3(2m^2))] )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = -5m^3 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{\alpha^3 + \beta^3}{\alpha^3 \beta^3} )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = -\frac{5}{(2m^2)^3} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = -\frac{5}{8m^2} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{\alpha^3} \times \frac{1}{\beta^3} = \frac{1}{(\alpha\beta)^3} )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = \frac{1}{(2m^2)^3} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = \frac{1}{8m^2} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( x^2 - \left( \frac{5}{8m^3} \right)x + \frac{1}{8m^2} = 0 )</td>
<td>A1</td>
<td>±3</td>
</tr>
<tr>
<td></td>
<td>( 8m^3x^2 + 5m^3x + 1 = 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Qn</td>
<td>Marking point</td>
<td>Mark Awarded</td>
<td>Remarks</td>
</tr>
<tr>
<td>----</td>
<td>---------------</td>
<td>--------------</td>
<td>---------</td>
</tr>
<tr>
<td>3(i)</td>
<td>$34 + 3\sqrt{28} = \left( \frac{6}{\sqrt{2}} - c \right)^2$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$34 + 24\sqrt{2} = (3\sqrt{2} - c)^2$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$34 + 24\sqrt{2} = 18 - 6c\sqrt{2} + c^2$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$24 = -6c$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$c = -4$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>$\pi(2 + \sqrt{2})^2 h = (9 + \sqrt{50})\pi$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(6 + 4\sqrt{2})h = 9 + \sqrt{50}$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$h = \frac{9 + 5\sqrt{2}}{2(3 + 2\sqrt{2})} \times \frac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}}$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \frac{27 - 3\sqrt{2} - 20}{2(9 - 8)}$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \frac{7 - 3\sqrt{2}}{2}$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$T = k + he^{-\frac{t}{10}}$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t = 0, \quad 90 = k + h$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t = 10\ln3, \quad 40 = k + \frac{h}{3}$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$50 = \frac{2}{3}h$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$h = 75, \quad k = 15$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>(i)</td>
<td>$t = 10, \quad T = 15 + 75e^{-\frac{10}{10}}$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 42.6^\circ$</td>
<td>M1</td>
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</tr>
<tr>
<td>(ii)</td>
<td>$T = 15 + 75e^{-\frac{t}{10}}$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{dT}{dt} = 75\left( -\frac{1}{10} \right) e^{-\frac{t}{10}}$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= -\frac{15}{2}e^{-\frac{t}{10}}$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{dT}{dt}_{t=23} = -\frac{15}{2}e^{-\frac{23}{10}}$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= -0.616$ $^\circ$C/min</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>Qn</td>
<td>Marking point</td>
<td>Mark Awarded</td>
<td>Remarks</td>
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</table>
| (iii) | $15 + 75e^{-\frac{t}{15}} < 35$  
$\frac{t}{15} < 4$  
$\frac{t}{10} < \ln 4$  
$t > 10 \ln 4/15 = 13.2$  
$t = 14$ | M1 | Accept working which is in the equation form |
| 5(a) | $\log_x x^2y = 3 + \log_x x - \frac{1}{2\log_x 5}$  
$\log_x x^3y = 3 + \log_x x - \frac{1}{2} \log_x y$  
$2\log_x x^2y = 6 + 2\log_x x - \log_x y$  
$\log_x (x^2y)^2 = 6 + \log_x \frac{x^2}{y}$  
$\log_x (x^2y)^2 - \log_x \frac{x^2}{y} = 6$  
$\log_x \frac{(x^2y)^2}{x^2} = 6$  
$x^2y^2 = 5^6$  
$y = \frac{\sqrt[2]{5^6}}{x^3}$  
$y = \frac{25}{\sqrt[2]{x^3}}$ | A1 | Change of base  
Evidence of using Power/Product/Quotient law  
Index form  
Accept $\frac{25}{\sqrt[2]{x^3}}$ |
| (b) | $\log_x (x + 5) - \log_x (x - 1) = \log_x 2$  
$\log_x (x + 5) - \frac{\log_x (x - 1)}{\log_x 3^2} = \log_x 2$  
$\log_x (x + 5) - 2 \log_x (x - 1) = \log_x 2$  
$\log_x \frac{x + 5}{(x - 1)^2} = \log_x 2$  
$x + 5 = 2(x^2 - 2x + 1)$  
$2x^2 - 5x - 3 = 0$  
$(2x + 1)(x - 3) = 0$  
$x = -\frac{1}{2} (\text{NA}), \quad x = 3$ | A1 | Change of base  
Quotient rule  
Index form  
No A1 if invalid ans is not rejected |
<table>
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<th>Qn</th>
<th>Marking point</th>
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<th>Remarks</th>
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</thead>
</table>
| 6(i) | $f(x) = (3x - 4)[2x^2 + (2k+1)x - 3]$  
$f(2) = 30$  
$2[8 + (2k+1)2 - 3] = 30$  
$5 + 2(2k+1) = 15$  
$2(2k+1) = 10$  
$k = 2$  
$f(x) = (3x - 4)(2x^2 + 5x - 3)$  
$= (3x - 4)(2x - 1)(x + 3)$ | M1  
M1  
A1 | No A1 if given as $f(x) = 0$ |
| 6(ii) | $f(x) = 15(1 - 2x)$  
$(3x - 4)(2x - 1)(x + 3) = -15(2x - 1)$  
$(3x - 4)(2x - 1)(x + 3) + 15(2x - 1) = 0$  
$(2x - 1)(3x^2 + 5x + 3) = 0$  
$2x - 1 = 0$  
$3x^2 + 5x + 3 = 0$  
$D = 5^2 - 4\cdot3(3) < 0$  
No of real roots $= 1$ | M1  
M1  
M1  
A1 | Use of $D$ or quad formula |
| 7(i) | $BC//AD$ (mid-point thm)  
$\angle SCD = \angle CAD$ (alt segment thm)  
$= \angle ACB$ (alt $\angle$)  
$\angle ABC = 180^\circ - \angle ADC$ ( $\angle$ in opp segment)  
$= \angle SDC$  
$\therefore \angle ABC$ is similar to $\angle SDC$ | B1  
M1  
M1 | |
| 7(ii) | $\frac{AC}{SC} = \frac{BC}{CD}$ (part (i))  
$AC \times CD = \frac{1}{2} AS \times SC$ (mid-pt theorem)  
$2AC \times DC = AS \times TC$ (C bisects $ST$)  
$AS = \frac{2AC \times DC}{TC}$ | M1  
M1  
M1  
A1 | |
| 8(i) | $A = (3, 0), \quad B = (0, 4)$  
$\Rightarrow$ centre lies on $\perp$ bisector of $OA \ & OB$  
$\therefore C = \left(\frac{3}{2}, 2\right)$  
$\text{Radius} = \sqrt{\left(\frac{3}{2}\right)^2 + 2^2}$ | M1 |
<table>
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<tr>
<th>Qu.</th>
<th>Marking point</th>
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<th>Remarks</th>
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<tbody>
<tr>
<td></td>
<td>$\sqrt{\frac{25}{4}} = \frac{5}{2}$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(x - \frac{3}{2})^2 + (y - 2)^2 = \frac{25}{4}$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x^2 - 3x + \frac{9}{4} + y^2 - 4y + 4 - \frac{25}{4} = 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x^2 + y^2 - 3x - 4y = 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>$\left(\frac{x_p + 0}{2}, \frac{y_p + 0}{2}\right) = \left(\frac{3}{2}, \frac{1}{2}\right)$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(x_p, y_p) = (3, 4)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$n_{op} = \frac{4}{3}$, $n_p = -\frac{3}{4}$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\mu - 4 = -\frac{3}{4}(x - 3)$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$4y - 16 = -3x + 9$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$4y = -3x + 25$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(iii)</td>
<td>Since tangent $\parallel$ to $y$-axis $\Rightarrow x = c$ from centre of circle</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = \frac{3}{2} - \frac{5}{2} = -1$, $x = \frac{3}{2} + \frac{5}{2} = 4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(x, y) = (-1, \frac{7}{4}), \left(4, \frac{13}{4}\right)$</td>
<td>A1, A1</td>
<td></td>
</tr>
<tr>
<td>9(i)</td>
<td>$y = x^2 \ln x^2$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 3x^2 \ln x$</td>
<td>M1</td>
<td></td>
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<tr>
<td></td>
<td>$\frac{dy}{dx} = 3\left(x^2 \left(\frac{1}{x}\right) + 2x \ln x\right)$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 3x(1 + 2 \ln x)$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>(ii)(a)</td>
<td>$x \ln x = 0$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = 0$ (NA), $\ln x = 0$</td>
<td></td>
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<tr>
<td></td>
<td>$x = e^0 = 1$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>$\int_1^3 3x(1 + 2 \ln x) , dx = \left[x^2 \ln x^2\right]_1^3$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\int_1^3 3x + 6x \ln x , dx = e^2 \ln e^2$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{3}{2}\left[x^2\right]_1^3 + \int_1^3 6x \ln x , dx = 3e^2$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\int_1^3 6x \ln x , dx = 3e^2 - \frac{3}{2}(e^2 - 1)$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\int_1^3 6x \ln x , dx = \frac{3}{2}(e^2 + 1)$</td>
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For integrating $3x$
For substituting limits
<table>
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<th>Remarks</th>
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<tbody>
<tr>
<td></td>
<td>( \int_1^x \ln x , dx = \frac{1}{4} (e^1 + 1) )</td>
<td>A1</td>
<td></td>
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</tbody>
</table>
| 10(i) | \( R \cos(2A - \alpha) \)  
\[ = \sqrt{3^2 + 4^2} \cos \left( 2A - \tan^{-1} \left( \frac{4}{3} \right) \right) \]  
\[ = 5 \cos \left( 2A - 53.1^\circ \right) \] | M1 | Either \( R \) or \( \alpha \)  
A1 | No A1 if \( \alpha \) not in 1 d.p. |
| (ii) | \( 3 \cos 2A + 4 \sin 2A = 4 \)  
\( 5 \cos (2A - 53.13^\circ) = 4 \)  
\( \cos (2A - 53.13^\circ) = \frac{4}{5} \)  
\( 2A - 53.13^\circ = -36.87, 36.87^\circ, 323.13^\circ \)  
\( 2A = 162.65^\circ, 90.0^\circ \)  
\( A \approx 8.13^\circ, 45.0^\circ \) | M1 | Do not penalise for d.p. if this was done in part (i)  
A1, A1 |
| (iii) | B1 for sine graph  
B1 (amplitude), B1 (shape) for cosine graph |
| (iv) | \( 2 \sin 6x = 2 - \frac{3}{2} \cos 6x \)  
\( 4 \sin 6x = 4 - 3 \cos 6x \)  
\( 3 \cos 6x + 4 \sin 6x = 4 \)  
Let \( A = 3x \)  
\( 3 \cos 2A + 4 \sin 2A = 4 \) | M1 | |
| 11(i) | \( v = 24 \cos(2t) \)  
\( a = -48 \sin(2t) \)  
\( t = 1, \quad a = -48 \sin(2) \)  
\[ = -43.6 \]  
\( |a| = 43.6 \text{ m/s}^2 \) | M1 | |
| (ii) | \( s = \int 24 \cos(2t) \, dt \)  
\( = 12 \sin(2t) + c \) | M1 |
10(iii) Equation of line $BC$:

\[ y - 8 = -\frac{8}{15}(x + 0) \]

\[ y = -\frac{8}{15}x + 8 \]

Let $B(-2, y)$

\[ y = -\frac{8}{15}(-2) + 8 = \frac{16}{15} = \frac{9}{15} + \frac{1}{15} \]

$B(-2, 9\frac{1}{15})$ \hspace{1cm} M1

$ABC \& ACE$ share the same base $AC$.

Hence, $\perp$ height of $E$ to $x$-axis should be $\frac{1}{2}$ of $AB$.

$\perp$ height of $E$ to $x$-axis = $\frac{1}{2} (9\frac{1}{15}) = \frac{68}{15} = \frac{4}{15}$ \hspace{1cm} M1

Let $E(x, -\frac{8}{15})$

\[ -\frac{8}{15} = -\frac{8}{15} (x) + 8 \]

\[ x = 23\frac{1}{2} \]

$E = (23\frac{1}{2}, -\frac{8}{15})$ \hspace{1cm} A1

10(iv) Area of $ABFE = 2(\text{Area of } ABE)$

\[
\begin{vmatrix}
1 & -2 & -2 & \frac{47}{2} & -2 \\
-2 & 0 & 136 & \frac{68}{15} & 0 \\
-2 & 0 & 136 & \frac{68}{15} & 0 \\
\end{vmatrix} \times 2
\]

\[ = 231.2 \text{ units}^2 \] \hspace{1cm} A1
<table>
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<th>Question</th>
<th>Marking Points</th>
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<th>Remarks</th>
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</table>
| **t = 0, s = -6 \Rightarrow c = -6**<br>
**s = 12 \sin(2t) - 6**<br>

When P first reaches fixed point, s = 0.<br>
**12 \sin(2t) - 6 = 0**<br>
**\sin(2t) = 0.5** <br>
**a = \frac{\pi}{6}**<br>
**t = \frac{\pi}{12}, \frac{5\pi}{12} (NA)** | M1 | | |

**(iii)**<br>
At inst rest, v = 0<br>
**24 \cos(2t) = 0**<br>
**2t = \frac{\pi}{2}, \frac{3\pi}{2}**<br>
**t = \frac{\pi}{4}, \frac{3\pi}{4}**<br>

**s = 12 \sin(2t) - 6**<br>
**t = 0, s = -6**<br>
**t = \frac{\pi}{4}, s = 6**<br>
**t = \frac{3\pi}{4}, s = -18**<br>
**t = 5, s = -6**<br>

**Dist = 6 + 12 + 18 = 36 m** | M1 | Either for $t = \frac{\pi}{4}$
or $t = \frac{3\pi}{4}$ | |

![Graph showing motion of point P](image)

12(i) $y = (x - 2)(x + 1)^3$

$\frac{dy}{dx} = (x - 2)[3(x + 1)^2] + (x + 1)^3$

$= (x + 1)^2(3x - 6 + x + 1)$

$= (x + 1)^2(4x - 5)$<br>

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</table>
| **(ii)**<br>
**$\frac{dy}{dx} = (x + 1)^2(4x - 5) = 0$** | M1 | | |

**x = -1, x = \frac{5}{4}** | A1 | | |
(iii) For $-1 \leq x \leq \frac{5}{4}$,

$$(x + 1)^2 > 0, \quad 4x - 5 < 0$$

$\therefore \quad \frac{dy}{dx} = (x + 1)^2(4x - 5) < 0$

OR

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-1.1$</th>
<th>$\frac{5}{4}$</th>
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<tbody>
<tr>
<td>Sign of $\frac{dy}{dx}$</td>
<td>$-$</td>
<td>$-$</td>
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Since gradient is negative between 2 stat points, $y$ decreases as $x$ increases between the two stat points.

(iv) $x = -1$ is a point of inflexion

<table>
<thead>
<tr>
<th>$x$</th>
<th>$&lt;-1$</th>
<th>$-1$</th>
<th>$&gt;-1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{dy}{dx}$</td>
<td>-ve</td>
<td>0</td>
<td>-ve</td>
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$x = \frac{5}{4}$ is a min point

<table>
<thead>
<tr>
<th>$x$</th>
<th>$&lt;\frac{5}{4}$</th>
<th>$\frac{5}{4}$</th>
<th>$&gt;\frac{5}{4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{dy}{dx}$</td>
<td>-ve</td>
<td>0</td>
<td>+ve</td>
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M1, M1
Al

M1
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Al
Preliminary Examination (2015)
Secondary 4 Express/5 Normal (Academic)

Candidate

Name

Register No

Class

ADDITIONAL MATHEMATICS
4047/01

Date: 26 August 2015
Duration: 2 hours

Additional Materials: Answer Paper
                           Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give your answer in the simplest form. Leave your answer in fraction where applicable. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Setter: Mr Han Ji

This paper consists of 7 printed pages, INCLUDING the cover page.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial Expansion

\[
(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n
\]

where \( n \) is a positive integer and

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}
\]

2. TRIGONOMETRY

Identities

\[
sin^2 A + cos^2 A = 1
\]
\[
sec^2 A = 1 + tan^2 A
\]
\[
cosec^2 A = 1 + cot^2 A
\]

\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp tan A \tan B}
\]
\[
\sin 2A = 2 \sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]
\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \triangle ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
s^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\text{area of } \triangle ABC = \frac{1}{2} \ ab \ sin \ C
\]
1. (i) Sketch the graph of \( y = 8x^{-1} \) for \( x > 0 \). \([1]\)

(ii) On the same diagram, sketch the graph of \( y = \frac{1}{4}x^2 \) for \( x \geq 0 \). \([1]\)

(iii) Calculate the exact coordinates of the point of intersection of the graphs. \([2]\)

(iv) Determine with explanation, whether the normal to the graphs at the point of intersection are perpendicular. \([2]\)

2. The cubic polynomial \( f(x) \) is such that the coefficient of \( x^3 \) is 1 and the roots of \( f(x) = 0 \) are \( -1, m \) and \( 2m \), where \( m \) is an integer. It is given that \( f(x) \) has a remainder of 6 when divided by \( x - 1 \).

(i) Find an expression for \( f(x) \) in descending powers of \( x \). \([4]\)

(ii) Hence or otherwise, solve the equation \( y^3 - 5y^4 + 2y^2 + 8 = 0 \). \([3]\)

3. A cuboid has a square base of side \((2 + a\sqrt{3})\) cm, where \( a \) is an integer. The height of the cuboid is \((1 + \sqrt{3})\) cm and its volume is \((\sqrt{27} - 5)\) cm\(^3\).

(i) Find the value of \( a \). \([3]\)

(ii) With the value of \( a \) in (i), find the total surface area of the cuboid in the form \((p + q\sqrt{3})\) cm\(^2\), where \( p \) and \( q \) are integers. \([2]\)

4. The equation of a curve is \( y = x^2 + 3x \). A straight line has equation \( y = mx - 9 \).

(i) Explain why the straight line is a tangent to the curve when \( m = 9 \). \([2]\)

(ii) Find the other value of \( m \) for which the line \( y = mx - 9 \) is a tangent to the curve. \([3]\)

(iii) State the set of values of \( m \) for which the straight line does not intersect the curve. \([1]\)
5. The diagram shows part of the graph of \( y = |2x - 1| - 2 \).

![](image)

(i) Find the coordinates of \( A \) and of \( B \). [2]

(ii) Explain why the lowest point, \( C \), on the graph has coordinates \( (\frac{1}{2}, -2) \). [2]

(iii) In each of the following cases, determine the number of intersections of the line \( y = mx + c \) with \( y = |2x - 1| - 2 \), justifying your answer:

(a) \( m = -2 \) and \( c > -1 \) [2]

(b) \( m = 1 \) and \( c < -3 \) [2]

6. In a simplified prey-predator model, some wolves were deliberately introduced to an island to curb the population of wild rabbits. The population of rabbits, \( R \), was given by \( R = 400 + 6000e^{-0.05i} \), where \( i \) is the number of days since the introduction of wolves.

(i) Find the initial population of wild rabbits on the island. [1]

(ii) After how many days would the population of wild rabbits first drop by 40%? [2]

(iii) Explain why the rabbits would never extinct on the island in the long run. [1]
Solutions to this question by accurate drawing will not be accepted.

7.

The diagram shows a trapezium \(ABCD\) in which \(AD\) is parallel to \(BC\). The points \(A, B\) and \(D\) are \((2, 7), (8, 4)\) and \((-2, 1)\) respectively. The point \(E\) is on \(BC\) and \(DE\) passes through \(O\).

(i) Show that \(ABED\) is a parallelogram. \[2\]

(ii) Find the coordinates of \(E\). \[2\]

Given that \(CD = CE\),

(iii) find the coordinates of \(C\). \[4\]

(iv) find the ratio of the area of triangle \(CDE\) to the area of \(ABED\). \[2\]

8. (a) (i) Without using a calculator, prove that \(\cot(45^\circ - \alpha) = \frac{\cot A + 1}{\cot A - 1}\). \[3\]

(ii) Hence find the exact value of \(\cot 15^\circ\). \[2\]

(b) Find an expression for \(f(x)\) such that

\[f'(x) = 3 \sin^2 \left(5x - \frac{\pi}{4}\right) + \cos^2 x - \tan^2 \frac{1}{2} x.\] \[3\]
9. (i) Express $\frac{8x-5}{x^3(1-x)}$ as the sum of 3 partial fractions. [4]

(ii) Hence find $\int \frac{8x-5}{x^2(1-x)} \, dx$. [2]

10. The equation of a curve is $y = xe^{-x}$.

(i) Find the set of values of $x$ for which $y$ is an increasing function of $x$. [2]

(ii) Find the coordinates of the turning point and determine whether the turning point is a maximum or minimum. [2]

11. The diagram shows a solid container consisting of a cylinder with a hemisphere dug out. The radius and height of the cylinder are $r$ cm and $h$ cm respectively.

(i) Express $h$ in terms of $r$ given that the external curved surface area of the cylindrical part of the solid is $1200\pi$ cm$^2$. [2]

(ii) Express the volume, $V$ cm$^3$, of the container in terms of $r$. [2]

(iii) The solid is heated and it expands at a rate of 0.81 cm$^3$/s. Find the rate at which its radius increases when the height is 60 cm. [3]
Answer the whole of this question on a piece of graph paper.

12. The table shows experimental values of the two variables, \( x \) and \( y \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.5</th>
<th>1.5</th>
<th>3</th>
<th>4.5</th>
<th>5.5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>4.43</td>
<td>5.29</td>
<td>7.44</td>
<td>11.4</td>
<td>15.7</td>
<td>18.7</td>
</tr>
</tbody>
</table>

It is known that \( x \) and \( y \) are related by an equation of the form \( y = ab^x + e \), where \( a \) and \( b \) are constants.

(i) Explain how a straight line graph may be drawn to represent the given data. \([2]\)

(ii) Draw this graph for the given data and use it to estimate the value of \( a \) and of \( b \). \([4]\)

(iii) By inserting another suitable line on your graph, solve the equation \( ab^x = 5e^\frac{x}{2} \). \([3]\)

\[ EN D \ OF \ P A P E R \]
Preliminary Examination (2015)
Secondary 4 Express

Candidate

Name  Register No  Class

ADDITIONAL MATHEMATICS
Paper 1 (4047)

Date: 27 August 2015
Duration: 2 hr

Additional Materials: Answer Paper

For examiner’s use

/ 80

READ THESE INSTRUCTIONS FIRST

1. Answer ALL the questions in this paper.
2. All workings must be clearly shown in the answer space provided.
   Omission of essential working and unit of measurement will result in loss of marks.
3. The use of calculator is expected, where appropriate.
4. Give your answer in the simplest form. Leave your answer in fraction where applicable or correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
5. For π, use your calculator value.
6. The number of marks is given in brackets [ ] at each question or part question.
   The total marks for this paper is 80.

Setter: Mr Han Ji

This paper consists of 7 printed pages, INCLUDING the cover page.

CCHY Preliminary Exam (2015)  Additional Mathematics (Sec 4E)  pg 1 of 7
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \(ax^2 + bx + c = 0\),

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Binomial Expansion

\[(a + b)^n = \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n\]

where \(n\) is a positive integer and \(\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}\)

2. TRIGONOMETRY

Identities

\[\sin^2 A + \cos^2 A = 1\]

\[\sec^2 A = 1 + \tan^2 A\]

\[\cosec^2 A = 1 + \cot^2 A\]

\[\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B\]

\[\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B\]

\[\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}\]

\[\sin 2A = 2 \sin A \cos A\]

\[\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A\]

\[\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}\]

Formulæ for \(\triangle ABC\)

\[\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}\]

\[a^2 = b^2 + c^2 - 2bc \cos A\]

area of \(\triangle ABC = \frac{1}{2} ab \sin C\)
1. (i) Sketch the graph of \( y = 8x^{-1} \) for \( x > 0 \). [1]

(ii) On the same diagram, sketch the graph of \( y = \frac{1}{4} x^{\frac{3}{2}} \) for \( x \geq 0 \). [1]

(iii) Calculate the exact coordinates of the point of intersection of the graphs. [2]

(iv) Determine with explanation, whether the normal to the graphs at the point of intersection are perpendicular. [2]

Ans:

\[
\begin{align*}
\text{(i) } y &= 8x^{-1} \\
\text{(ii) } y &= \frac{1}{4} x^{\frac{3}{2}} \\
\end{align*}
\]

\begin{align*}
\text{(i) } y &= 8x^{-1} \\
\frac{dy}{dx} &= -8x^{-2} \\
\text{(ii) } y &= \frac{1}{4} x^{\frac{3}{2}} \\
\frac{dy}{dx} &= \frac{3}{4} x^{\frac{1}{2}} \\
\text{The coordinates are } & (4, 2). [A1]
\end{align*}
\[
 y = \frac{1}{4} \cdot x^3, \quad \frac{dy}{dx} = \frac{3}{8} \cdot x^2
\]

When \( x = 4 \),
\[
 \frac{dy}{dx} = 8x^{-2} = -8(4^{-2}) = -\frac{1}{2}, \quad \text{gradient of normal} = 2
\]
\[
 \frac{dy}{dx} = 8x^{-2} = \frac{3\sqrt{4}}{8} = \frac{3}{4}, \quad \text{gradient of normal} = -\frac{4}{3}
\]
\[
 2 \times -\frac{4}{3} = -\frac{8}{3} \neq -1 \quad \text{----- [M1]}
\]

The normals are not perpendicular since the product of their gradients is not -1. \text{----- [A1]}

OR
\[
 \frac{dy}{dx} = 8x^{-2} = -8(4^{-2}) = -\frac{1}{2}
\]
\[
 \frac{dy}{dx} = \frac{3 \sqrt{4}}{8} = \frac{3}{4}
\]
\[
 -\frac{1}{2} \times -\frac{3}{4} = -1 \quad \text{----- [M1]}
\]

Since the tangents to the curves at the point of intersection are not perpendicular, the normals at that point are also not perpendicular. \text{----- [A1]}

2. The cubic polynomial \( f(x) \) is such that the coefficient of \( x^3 \) is 1 and the roots of \( f(x) = 0 \) are \(-1, m \) and \( 2m \), where \( m \) is an integer. It is given that \( f(x) \) has a remainder of 6 when divided by \( x-1 \).

(i) Find an expression for \( f(x) \) in descending powers of \( x \). \text{[4]}

(ii) Hence or otherwise, solve the equation \( y^4 - 5y^4 + 2y^2 + 8 = 0 \). \text{[3]}

Ans:

(i) Let \( f(x) = k(x+1)(x-m)(x-2m) \).

Since the coefficient of \( x^3 \) is 1, \( k = 1 \). \text{----- [M1]}

CCHY Preliminary Exam (2015) Additional Mathematics (Sec 4E) pg 4 of 7
\[ f(x) = (x + 1)(x - m)(x - 2m) \]
\[ f(1) = 6 \]
\[ (1+1)(1-m)(1-2m) = 6 \quad [M1] \]
\[ (1-m)(1-2m) = 3 \]
\[ 2m^2 - 3m + 1 = 3 \]
\[ 2m^2 - 3m - 2 = 0 \]
\[ (m-2)(2m+1) = 0 \]
\[ m = 2 \text{ or } m = \frac{1}{2} \quad \text{(rej)} \quad [M1] \]
\[ f(x) = (x+1)(x-2)(x-4) \]
\[ = (x+1)(x^2 - 6x + 8) \]
\[ = x^3 - 5x^2 + 2x + 8 \quad [A1] \]

(ii) Let \( x = y^2 \)
\[ (y^2)^3 - 5(y^2)^2 + 2(y^2) + 8 = 0 \quad [M1] \]
\[ x^3 - 5x^2 + 2x + 8 = 0 \]
\[ (x+1)(x-2)(x-4) = 0 \]
\[ x = -1, \ x = 2 \text{ or } x = 4 \]
\[ y^2 = -1 \quad \text{(rej)}, \ y^2 = 2 \text{ or } y^2 = 4 \quad [M1] \]
\[ y = \pm \sqrt{2}, \ y = \pm 2 \quad [A1] \]

3. A cuboid has a square base of side \((2 + a\sqrt{3})\) cm, where \(a\) is an integer. The height of the cuboid is \((1 + \sqrt{3})\) cm and its volume is \((\sqrt{27} - 5)\) cm\(^3\).

(i) Find the value of \(a\). \[ [3] \]

(ii) With the value of \(a\) in (i), find the total surface area of the cuboid in the form \((p + q\sqrt{3})\) cm\(^2\), where \(p\) and \(q\) are integers. \[ [2] \]
Ans:

(i) \((2 + a\sqrt{3})^2(1 + \sqrt{3}) = \sqrt{27} - 5\)

\[ \frac{(2 + a\sqrt{3})^2}{1 + \sqrt{3}} = \frac{3\sqrt{3} - 5}{1 + \sqrt{3}} \]

\[ = \frac{(3\sqrt{3} - 5)(1 - \sqrt{3})}{(1 + \sqrt{3})(1 - \sqrt{3})} \quad [M1] \]

\[ = \frac{8\sqrt{3} - 14}{-2} \]

\[ = 7 - 4\sqrt{3} \]

\[ 4 + 3a^2 + 4a\sqrt{3} = 7 - 4\sqrt{3} \quad [M1] \]

\[ 4 + 3a^2 = 7, \ a^2 = 1, \ a = \pm 1 \]

\[ 4a = -4, \ a = -1 \]

\[ \therefore a = -1 \quad [A1] \]

(ii) Total surface area = \(2(2 - \sqrt{3})^2 + 4(2 - \sqrt{3})(1 + \sqrt{3}) \quad [M1] \]

\[ = 2(7 - 4\sqrt{3}) + 4(\sqrt{3} - 1) \]

\[ = 14 - 8\sqrt{3} + 4\sqrt{3} - 4 \]

\[ = (10 - 4\sqrt{3}) \text{ cm}^2 \quad [A1] \]

4. The equation of a curve is \(y = x^2 + 3x\). A straight line has equation \(y = mx - 9\).

(i) Explain why the straight line is a tangent to the curve when \(m = 9\). [2]

(ii) Find the other value of \(m\) for which the line \(y = mx - 9\) is a tangent to the curve. [3]

(iii) State the set of values of \(m\) for which the straight line does not intersect the curve. [1]

Ans:
(i) \( y = x^2 + 3x \)
\[ y = 9x - 9 \]
\[ x^2 + 3x = 9x - 9 \]
\[ x^2 - 6x + 9 = 0 \]
Discriminant = \((-6)^2 - 4(1)(9) = 0 \) ------ [M1]

Therefore, the straight line is a tangent to the curve, since they intersect at only one point. ------ [A1]

(ii) \( x^2 + 3x = mx - 9 \)
\[ x^2 + (3 - m)x + 9 = 0 \]
\[ (3 - m)^2 - 4(1)(9) = 0 \] ------ [M1]
\[ (3 - m)^2 = 36 \]
\[ 3 - m = \pm 6 \]
\[ m = -3 \] or \( m = 9 \) (rej) ------ [M1]

The other value of \( m \) is -3. ------ [A1]

(iii) \(-3 < m < 9 \) ------ [B1]

5. The diagram shows part of the graph of \( y = |2x - 1| - 2 \).

\[ y \]
\[ A \]
\[ 0 \]
\[ B \]
\[ x \]
\[ C \]

(i) Find the coordinates of \( A \) and of \( B \). [2]

(ii) Explain why the lowest point, \( C \), on the graph has coordinates \( \frac{1}{2}, -2 \). [2]
(iii) In each of the following cases, determine the number of intersections of the line \( y = mx + c \) with \( y = |2x - 1| - 2 \), justifying your answer.

(a) \( m = -2 \) and \( c > -1 \) \hspace{1cm} [2]

(b) \( m = 1 \) and \( c < -3 \) \hspace{1cm} [2]

Ans:

(i) \( |2x - 1| - 2 = 0 \)

\[ |2x - 1| = 2 \]

\( 2x - 1 = 2 \) or \( 2x - 1 = -2 \) \hspace{1cm} [M1]

\( 2x = 3 \) or \( 2x = -1 \)

\( x = \frac{3}{2} \) or \( x = -\frac{1}{2} \)

\[ \bullet \]

A \( (\frac{3}{2}, 0) \), B \( (\frac{1}{2}, 0) \) \hspace{1cm} [A1]

(ii) \( |2x - 1| \geq 0, |2x - 1| - 2 \geq -2 \)

Since C is the lowest point on the graph, \( y = -2 \). \hspace{1cm} [M1]

\[ |2x - 1| = 0, x = \frac{1}{2} \]

Therefore the coordinates of C are \( (\frac{1}{2}, -2) \). \hspace{1cm} [A1]

OR

At the point where the lines turns, \( |2x - 1| = 0, x = \frac{1}{2} \) \hspace{1cm} [M1]

\( y = 0 - 2 = -2 \)
Therefore the coordinates of C are \( \left( \frac{1}{2}, -2 \right) \). ------ [A1]

(iii) (a) For \( m = -2 \) and \( c > -1 \), the line \( y = mx + c \) is above the left arm and parallel to it. Therefore the line \( y = mx + c \) intersects the right arm at one point. ------ [M1]

Number of intersection = 1 ------ [A1]

(b) For \( m = 1 \) and \( c < -3 \), the line \( y = mx + c \) is below C and the gradient is gentler than the right arm. Therefore the line \( y = mx + c \) does not intersect the right arm. ------ [M1]

Number of intersection = 0 ------ [A1]

6. In a simplified prey-predator model, some wolves were deliberately introduced to an island to curb the population of wild rabbits. The population of rabbits, \( R \), was given by \( R = 400 + 6000e^{0.05t} \), where \( t \) is the number of days since the introduction of wolves.

(i) Find the initial population of wild rabbits on the island. [1]

(ii) After how many days would the population of wild rabbits first drop by 40%? [2]

(iii) Explain why the rabbits would never extinct on the island in the long run. [1]

Ans:

(i) When \( t = 0 \),
\[
R = 400 + 6000e^0 = 6400 \quad ------ [A1]
\]

(ii) \( 6400 \times 60\% = 3840 \)
\[
3840 = 400 + 6000e^{-0.05t}
\]

CCHY Preliminary Exam (2015) Additional Mathematics (Sec 4E) pg 9 of 7
\[ 3440 = 6000e^{-0.02t} \]

\[ e^{-0.02t} = \frac{3440}{6000} \]

\[-0.02t = \ln\left(\frac{3440}{6000}\right) \quad \text{[M1]} \]

\[ t = 27.8 \approx 28 \]

It takes 28 days for the population of wild rabbits to first drop by 40%. \quad \text{[A1]}

(iii) As \( t \to \infty \), \( e^{-0.02t} \to 0 \)

As a result, \( R \to 400 + 6000(0) = 400 \) \quad \text{[A1]}

Therefore, the rabbits would not become extinct in the long run.

**Solutions to this question by accurate drawing will not be accepted.**

7.

\[ \begin{array}{c}
\text{The diagram shows a trapezium } ABCD \text{ in which } AD \text{ is parallel to } BC. \text{ The point } A \\
\text{is (2, 7), the point } B \text{ is (8, 4) and the point } D \text{ is (-2, 1). The point } E \text{ is on } BC \text{ such} \\
\text{that } DE \text{ passes through } O. \\
\text{(i) Show that } ABED \text{ is a parallelogram.} \\
\text{(ii) Find the coordinates of } E. \\
\text{Given that } CD = CE, \\
\end{array} \]
(iii) find the coordinates of C. \[4\]

(iv) find the ratio of the area of triangle $CDE$ to the area of $ABED$. \[2\]

Ans:

(i) $AD // BE$ (given)

Gradient of $AB = \frac{4-7}{8-2} = -\frac{1}{2}$

Gradient of $DE = \frac{0-1}{0-(-2)} = -\frac{1}{2}$

$AB // DE$ \[M1\]

With two pairs of parallel opposite sides, $ABED$ is a parallelogram. \[A1\]

(ii) Let the coordinates of $E$ be $(x, y)$

\[
\left(\frac{x+2}{2}, \frac{y+7}{2}\right) = \left(\frac{-2+8}{2}, \frac{4+1}{2}\right) \quad \text{[M1]}
\]

$x + 2 = 6, y + 7 = 5$

$x = 4, y = -2$

$E(4, -2)$ \[A1\]

(iii) Since $CD = CE$, $C$ lies on the perpendicular bisector of $DE$.

Gradient of $DE = -\frac{1}{2}$

Gradient of perpendicular bisector $= \frac{-1}{\left(-\frac{1}{2}\right)} = 2 \quad \text{[M1]}

Let the equation of the perpendicular bisector be $y = 2x + c$

Midpoint of $DE$ is \( \left(\frac{-2+4}{2}, \frac{1+(-2)}{2}\right) = \left(1, \frac{-1}{2}\right) \)

\[-\frac{1}{2} = 2 \times 1 + c, c = \frac{-5}{2}\]
The equation is \( y = 2x - \frac{5}{2} \) \[M1]\)

Gradient of \(BE\) is \( \frac{4 - (-2)}{8 - 4} = \frac{3}{2} \)

Let the equation of \(BE\) be \( y = \frac{3}{2}x + c \)

At \((4, -2)\), \(-2 = \frac{3}{2}(4) + c\), \(c = -8\)

The equation is \( y = \frac{3}{2}x - 8 \) \[M1]\)

\[
y = 2x - \frac{5}{2} \\
y = \frac{3}{2}x - 8 \\
2x - \frac{5}{2} = \frac{3}{2}x - 8 \\
\frac{1}{2}x = -5 + \frac{1}{2} \\
x = -11 \\
y = 2(-11) - \frac{5}{2} = -24 + \frac{1}{2} \\
\]

\(C\) \((-11, -24 + \frac{1}{2})\) \[A1]\)

OR

Gradient of \(BE\) is \( \frac{4 - (-2)}{8 - 4} = \frac{3}{2} \)

Let the equation of \(BE\) be \( y = \frac{3}{2}x + c \)

At \((4, -2)\), \(-2 = \frac{3}{2}(4) + c\), \(c = -8\)
The equation is $y = \frac{3}{2}x - 8$ ------- [M1]

Let the coordinates of $C$ be $(x, y)$.

Since $CD = CE$,

\[ \sqrt{(x+2)^2 + (y-1)^2} = \sqrt{(x-4)^2 + (y+2)^2} \] ------- [M1]

\[ x^2 + 4x + 4 + y^2 - 2y + 1 = x^2 - 8x + 16 + y^2 + 4y + 4 \]

\[ 12x - 6y = 15 \]

\[ 12x - 6\left(\frac{3}{2}x - 8\right) = 15 \] ------- [M1]

\[ 3x = -33 \]

\[ x = -11 \]

\[ y = \frac{3}{2}(-11) - 8 = -24\frac{1}{2} \]

\[ C(-11, -24\frac{1}{2}) \] ------- [A1]

(iv) Area of $CDE = \frac{1}{2} \begin{vmatrix} 4 & -2 & -11 & 4 \\ 2 & 1 & -24 & -2 \end{vmatrix}

= \frac{1}{2} \left[ 4 + 49 + 22 - 4 - (-11) - (-98) \right]

= 90 \text{ units}^2$

Area of $ABED = \frac{1}{2} \begin{vmatrix} 2 & -2 & 4 & 82 \\ 7 & 1 & -24 & 7 \end{vmatrix}

= \frac{1}{2} \left[ 2 + 4 + 16 + 56 - (-14) - 4 - (-16) - 8 \right]

= 48 \text{ units}^2$ ------- [M1]

Area of $CDE : \text{Area of } ABED = 90 : 48 = 15 : 8$ ------- [A1]
8. (a) (i) Without using a calculator, prove that \( \cot(45^\circ - A) = \frac{\cot A + 1}{\cot A - 1} \). 

(ii) Hence find the exact value of \( \cot 15^\circ \). 

(b) Find an expression for \( f(x) \) such that 

\[
f'(x) = 3 \sin^2(5x - \frac{\pi}{4}) + \cos^2 x - \tan \frac{1}{2} x.
\]

Ans:

(a) (i) \( LHS = \frac{1}{\tan(45^\circ - A)} \)

\[
= \frac{1 + \tan 45^\circ \tan A}{\tan 45^\circ - \tan A} \\
= \frac{1 + \tan A}{1 - \tan A} \quad \text{[M1]}
\]

\[
= \frac{\sin A}{\cos A} \quad \text{[M1]}
\]

\[
= \frac{\cos A + \sin A}{\cos A - \sin A}
\]

\[
= \frac{\sin A}{\cos A - \sin A} \quad \text{[M1]}
\]
\[ \frac{\cot A + 1}{\cot A - 1} = \text{RHS (proven)} \quad \text{[A1]} \]

(ii) \[ \cot 15^\circ = \cot (45^\circ - 30^\circ) \]

\[ = \frac{\cot 30^\circ + 1}{\cot 30^\circ - 1} \]

\[ = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \]

\[ = \frac{(\sqrt{3} + 1)(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} \quad \text{[M1]} \]

\[ = \frac{4 + 2\sqrt{3}}{2} \]

\[ = 2 + \sqrt{3} \quad \text{[A1]} \]

(b) \[ f(x) = 3 \sin^2 (5x - \frac{\pi}{4}) + \cos^2 x - \tan^2 \frac{1}{2} x \]

\[ = -\frac{3}{2} \left[-2 \sin^2 (5x - \frac{\pi}{4})\right] + \frac{1}{2} \left[2 \cos^2 x\right] - \left(\sec^2 \frac{1}{2} x - 1\right) \]

\[ = -\frac{3}{2} \left[1 - 2 \sin^2 (5x - \frac{\pi}{4})\right] + \frac{3}{2} + \frac{1}{2} \left[2 \cos^2 x - 1\right] - \frac{1}{2} \left[2 \sec^2 \frac{1}{2} x - 1\right] + \frac{1}{2} - \sec^2 \frac{1}{2} x + 1 \quad \text{[M1]} \]

\[ = -\frac{3}{2} \cos(10x - \frac{\pi}{2}) + \frac{1}{2} \cos 2x - \sec^2 \frac{1}{2} x + 3 \quad \text{[M1]} \]

\[ f(x) = \int (3 \sin^2 (5x - \frac{\pi}{4}) + \cos^2 x - \tan^2 \frac{1}{2} x) dx \]

\[ = \int \left[\frac{3}{2} \cos(10x - \frac{\pi}{2}) + \frac{1}{2} \cos 2x - \sec^2 \frac{1}{2} x + 3\right] dx \]

\[ = -\frac{3}{20} \cos(10x - \frac{\pi}{2}) + \frac{1}{4} \sin 2x - 2 \tan \frac{1}{2} x + 3x + c \quad \text{[A1]} \]
9. (i) Express \( \frac{8x-5}{x^2(1-x)} \) as the sum of 3 partial fractions. [4]

(ii) Hence find \( \int \frac{8x-5}{x^2(1-x)} \, dx \). [2]

Ans:

(i) \( \frac{8x-5}{x^2(1-x)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{1-x} \) \[\text{M1}\]

\[8x-5 = Ax(1-x) + B(1-x) + C x^2\]

Let \( x = 1 \), \( 3 = C \)

Let \( x = 0 \), \( -5 = B \) \[\text{M1}\]

Let \( x = 2 \), \( 11 = 2A + (-5)(-1) + 3(2^2) \), \( A = 3 \) \[\text{M1}\]

\( \frac{8x-5}{x^2(1-x)} = \frac{3}{x} - \frac{5}{x^2} + \frac{3}{1-x} \) \[\text{A1}\]

(ii) \( \int \frac{8x-5}{x^2(1-x)} \, dx = \int \left( \frac{3}{x} - \frac{5}{x^2} + \frac{3}{1-x} \right) \, dx \)

\[= \int \frac{3}{x} \, dx - \int \frac{5}{x^2} \, dx + \int \frac{3}{1-x} \, dx\]

\[= 3 \ln |x| - (5x^{-1}) + [-3 \ln |1-x|] + C \] \[\text{M1}\]

\[= 3 \ln |x| + \frac{5}{x} - 3 \ln |1-x| + C\]

\[= 3 \ln \left| \frac{x}{1-x} \right| + \frac{5}{x} + C \] \[\text{A1}\]

10. The equation of a curve is \( y = xe^{-x} \).

(i) Find the set of values of \( x \) for which \( y \) is an increasing function of \( x \). [2]
(ii) Find the coordinates of the turning point and determine whether the turning point is a maximum or minimum. [2]

Ans:

(i) \( \frac{dy}{dx} = e^{-x} + x(-e^{-x}) \)

\[ = e^{-x}(1-x) \quad \text{[M1]} \]

For increasing function, \( \frac{dy}{dx} > 0 \)

\[ e^{-x}(1-x) > 0 \]

Since \( e^{-x} > 0 \), \( 1-x > 0 \)

\( x < 1 \quad \text{[A1]} \)

(ii) \( \frac{dy}{dx} = 0 \)

\[ e^{-x}(1-x) = 0 \]

\( 1-x = 0 \)

\( x = 1 \)

\( y = 1(e^{-1}) = \frac{1}{e} \)

Coordinates of turning point is \((1, \frac{1}{e})\) \quad \text{[A1]}

Use 1\textsuperscript{st} derivative test:

<table>
<thead>
<tr>
<th>( x )</th>
<th>1-</th>
<th>1</th>
<th>1+</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{dy}{dx} )</td>
<td>+ve</td>
<td>0</td>
<td>-ve</td>
</tr>
<tr>
<td>Gradient</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The point is a maximum point. \quad \text{[A1]}
11. The diagram shows a solid container consisting of a cylinder with a hemisphere dug out. The radius and height of the cylinder are \( r \) cm and \( h \) cm respectively.

![Diagram of a solid container consisting of a cylinder with a hemisphere dug out.](image)

(i) Express \( h \) in terms of \( r \) given that the external curved surface area of the cylindrical part of the solid is 1200\( \pi \) cm\(^2\). [2]

(ii) Express the volume, \( V \) cm\(^3\), of the container in terms of \( r \). [2]

(iii) The solid is heated and it expands at a rate of 0.81 cm\(^3\)/s. Find the rate at which its radius increases when the height is 60 cm. [3]

Ans:

(i) \( 2\pi rh = 1200\pi \) \[M1\]

\[ rh = 600 \]

\[ h = \frac{600}{r} \] \[A1\]

(ii) \( V = \pi r^2 h - \frac{2}{3} \pi r^3 \) \[M1\]

\[ = \pi r^2 \frac{600}{r} - \frac{2}{3} \pi r^3 \]

\[ = 600\pi r - \frac{2}{3} \pi r^3 \] \[A1\]
(iii) \[
\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \quad \text{------ [M1]}
\]

\[
V = 600\pi r - \frac{2}{3} \pi r^3
\]

\[
\frac{dV}{dr} = 600 \pi - 2 \pi r^2 \quad \text{------ [M1]}
\]

When \( h = 60, \ r = 10 \)

\[
\frac{dV}{dr} = 600 \pi - 2 \pi (10)^2 = 400 \pi
\]

\[
0.81 = 400 \pi \times \frac{dr}{dt}
\]

\[
\frac{dr}{dt} = \frac{0.81}{400 \pi} = 0.000645 \quad \text{------ [A1]}
\]

Answer the whole of this question on a piece of graph paper.

12. The table shows experimental values of the two variables, \( x \) and \( y \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.5</th>
<th>1.5</th>
<th>3</th>
<th>4.5</th>
<th>5.5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>4.43</td>
<td>5.29</td>
<td>7.44</td>
<td>11.4</td>
<td>15.7</td>
<td>18.7</td>
</tr>
</tbody>
</table>

It is known that \( x \) and \( y \) are related by an equation of the form \( y = ab^x + e \), where \( a \) and \( b \) are constants.

(i) Explain how a straight line graph may be drawn to represent the given data. \[2\]

(ii) Draw this graph for the given data and use it to estimate the values of \( a \) and of \( b \). \[4\]

(iii) By inserting another suitable line on your graph, solve the equation

\[ ab^x = 5e^x \quad \text{------ [3]} \]
Ans:

(i) \[ y = ab^x - e \]

\[ y - e = ab^x \]

\[ \ln(y - e) = \ln(ab^x) \quad \text{[M1]} \]

\[ \ln(y - e) = \ln a + x \ln b \]

Let \( Y = \ln(y - e) \) and \( X = x \)

\( Y = \ln a + X \ln b \)

If \( \ln(y - e) \) is plotted against \( x \), a straight line graph can be obtained. \( \quad \text{[A1]} \)

(ii) From the graph

\[ \ln a = 0.35, \quad a = e^{0.35} = 1.42 \quad \text{[A1]} \quad (\text{Answer range: } 0.3352 \pm 0.02) \]

\[ \ln b = \frac{2.35 - 0.75}{5 - 1} = 0.4(\pm 0.02), \quad b = e^{0.4} = 1.49 \quad \text{[A1]} \]

(iii) \( ab^x = 5e^{-\frac{x}{2}} \)

\[ \ln(ab^x) = \ln 5e^{-\frac{x}{2}} \]

\[ \ln a + x \ln b = \ln 5 - \frac{x}{2} \]

\[ \ln(y - e) = \ln 5 - \frac{x}{2} \quad \text{[M1]} \]

Equation of line to be inserted: \( Y = \ln 5 - \frac{1}{2}X \)

At the point of intersection, \( x = 1.40 \pm 0.02 \quad \text{[A1]} \)

END OF PAPER
Preliminary Examination (2015)
Secondary 4 Express/ 5 Normal (Academic)

Candidate

Name

Register No

Class

ADDITIONAL MATHEMATICS
4047/ 02

Date: 27 August 2015
Duration: 2 hours 30 minutes

Additional Materials: Answer Paper

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Setter: Woo Huey Ming

This paper consists of 6 printed pages, INCLUDING the cover page. [Turn over
1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion

\[
(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n.
\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \).

2. TRIGONOMETRY

Identities

\[
\sin^2 A + \cos^2 A = 1
\]

\[
\sec^2 A = 1 + \tan^2 A
\]

\[
\csc^2 A = 1 + \cot^2 A
\]

\[
\sin(A + B) = \sin A \cos B + \cos A \sin B
\]

\[
\cos(A + B) = \cos A \cos B - \sin A \sin B
\]

\[
\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}
\]

\[
\sin 2A = 2 \sin A \cos A
\]

\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]

\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \Delta ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2} ab \sin c
\]
1. (a) Solve the equation \( \log_3 4 - \log_3 (x^2 + 4x + 4) = \log_3 x \). \[4\]

(b) Sketch the graph of \( y = e^{-x} \). In order to solve the equation \( \ln \left( \frac{1}{\sqrt{x - 3}} \right) = \frac{1}{2} x \), a graph of a suitable straight line is drawn on the same set of axes as the graph of \( y = e^{-x} \). Find the equation of the straight line. \[3\]

2. The roots of the equation \( 2x^2 - px - q = 0 \), where \( p \) and \( q \) are constants, are \( \alpha \) and \( \beta \).
   The roots of the equation \( 4x^2 + qx - 3x = p - 1 \) are \( \alpha - 1 \) and \( \beta - 1 \).
   
   (i) Find the value of \( p \) and of \( q \). \[5\]
   
   (ii) Find a quadratic equation whose roots are \( \alpha^2 \) and \( \beta^2 \). \[3\]

3. (i) Given that the coefficient of \( x^2 \) in the expansion of \( \left( \frac{1}{x} + px \right)^8 \) is 448, find the value of the positive constant \( p \). \[3\]
   
   (ii) Using the value of \( p \) in part (i), find the term independent of \( x \) in the expansion of \( \left( \frac{1}{x} + px \right)^8 (5x^2 - 4) \). \[4\]

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   (i) Find an expression for \( \frac{dy}{dx} \). \[3\]
   
   (ii) Hence find \( \int \frac{\ln 2x}{x^2} \, dx \). \[4\]

5. The equation of a circle \( C_1 \) is given as \( x^2 + y^2 - 16x + 8y + 64 = 0 \).
   
   (i) Find the coordinates of the centre and radius of the circle \( C_1 \). \[3\]
   
   (ii) The line \( y = k \) is a tangent to the circle at \( A \), where \( k \neq 0 \). Find the value of \( k \). \[2\]
   
   (iii) The tangent to the circle at \( B(4, -4) \) intersects \( y = k \) at point \( C \). Find the equation of this tangent. \[1\]
   
   (iv) Explain why a circle \( C_2 \) can be drawn through the points \( A, B \) and \( C \) with \( AB \) being the diameter. \[1\]
   
   (v) Find the equation of the circle \( C_2 \). \[3\]
   
   (vi) Determine, with working, whether \( (\frac{3}{5}, -6) \) lies within the 2 circles. \[2\]
6.

The height of a blade on the windmill (measured from the ground) can be modelled by the equation \( h = 15 - 7 \cos ki \) where \( k \) is a constant and \( t \) is the time in seconds after the windmill starts moving. The windmill rotates at a rate of 12 revolutions per minute.

(i) Explain why this model suggests that the highest point of the windmill, \( B \), is 22 m above the ground level. [1]

(ii) Find the value of \( k \). [1]

(iii) For how long over the course of one complete revolution will the point \( A \) be at least 17 m above ground level? [2]

(iv) Explain how the solution in part (iii) could be used to find the duration of the point \( A \) being at least 17 m above ground level over the course of two complete revolutions. [2]

(v) Suggest a possible equation of how the height of a blade varies against time if the windmill starts rotating from the highest point at \( B \). [1]

7.

The diagram shows a solid machine part that is made up of a closed cylinder joined to an inverted right circular cone. The height of the cylinder is \( h \) m and the slant height of the cone makes an angle of \( \frac{\pi}{3} \) radians to its base radius, \( r \) m.

(i) Given that the volume of the machine part is \( 50\pi \) m\(^3\), express \( h \) in terms of \( r \). [2]

(ii) Show that the total surface area of the machine part is given by

\[
A = \frac{\pi r^2}{3}(9 - 2\sqrt{3}) + \frac{100\pi}{r}.
\]

(iii) Given that \( r \) can vary, find the value of \( r \) for which the total surface area of the machine part is stationary. [3]

(iv) By comparing gradients, explain why this value of \( r \) gives the least total surface area possible. [2]
8. The equation of two curves are \( y = \cos 2x - 2\sin^2 x \) and \( y = \sin 2x \).
   (i) Show that the \( x \)-coordinates of the points of intersection of the two curves satisfy \( 2\cos 2x - \sin 2x = 1 \). \[1\]
   (ii) On the same axes sketch, for \(-\pi < x < \pi\), the graphs of \( y = \cos 2x - 2\sin^2 x \) and \( y = \sin 2x \). \[4\]
   (iii) Express the equation \( 2\cos 2x - \sin 2x = 1 \) in the form \( \cos(2x + \alpha) = k \), where \( \alpha \) and \( k \) are constants to be found. \[4\]
   (iv) Hence find, in radians, the \( x \)-coordinates of the points of intersection for \(-\pi < x < \pi\). \[3\]

9. A particle travels in a straight line from a fixed point \( O \) with acceleration \( a \) m/s\(^2\), given by \( a = 8t - k \) where \( t \) is the time in seconds after passing \( O \), and \( k \) is a constant. The velocity of the particle is 5 m/s when it passes \( O \), and at \( t = 2 \), its velocity is -21 m/s.
   (i) Find the value of \( k \). \[3\]
   (ii) Find the value(s) of \( t \) when the particle is instantaneously at rest. \[2\]
   (iii) Calculate the average speed of the particle during the first six seconds. \[3\]
   (iv) Describe completely the motion of the particle in the first six seconds. \[2\]

10. \[\text{Diagram of circles touching at } T \text{ and } PTQ \text{ is their common tangent. } AB \text{ is a tangent to the smaller circle at } E. AT \text{ and } BT \text{ cut the smaller circle at } D \text{ and } C \text{ respectively. } ET \text{ and } CD \text{ intersect at } F. \text{ Prove that}

   (i) \( AB \parallel DC \). \[2\]
   (ii) \( \angle ATE = \angle BTE \). \[3\]
   (iii) \( ET^2 = CT \times DT + EF \times ET \). \[3\]
11. (i) Differentiate \((x+2)\sqrt{4x-3}\) with respect to \(x\). 

(ii) 

\[
y = \frac{3x+2}{\sqrt{4x-3}}
\]

The diagram shows part of the curve \(y = \frac{3x+2}{\sqrt{4x-3}}\). A line with gradient \(-\frac{2}{3}\) intersects the curve at \(A(1,5)\) and \(B\).

(a) Verify that the \(y\)-coordinate of \(B\) is \(\frac{11}{3}\). [5]

(b) Determine the area of the region bounded by the curve and the line \(AB\). [4]
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where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{(n-r)! r!} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!} \).

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\]

\[
\csc^2 A = 1 + \cot^2 A
\]

\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
\sin 2A = 2 \sin A \cos A
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\]

\[
\Delta = \frac{1}{2} ab \sin C
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\[
\log_3 4 - \log_3 (x^2 + 4x + 4) = \log_3 x
\]

\[
\log_3 4 - \frac{2 \log_3 (x + 2)}{\log_3 9} = \frac{\log_3 x}{\log_3 3} \quad \text{M1}
\]

\[
\log_3 4 - \log_3 (x + 2) = -\log_3 x \quad \text{M1}
\]

\[
\frac{4}{x + 2} = \frac{1}{x} \quad \text{M1}
\]

\[
3x = 2
\]

\[
x = \frac{2}{3} \quad \text{A1}
\]

(b) Sketch the graph of \( y = e^{-x} \). In order to solve the equation \( \ln \left( \frac{1}{\sqrt{x - 3}} \right) = \frac{1}{2} x \), a graph of a suitable straight line is drawn on the same set of axes as the graph of \( y = e^{-x} \). Find the equation of the straight line.

\[
\ln \left( \frac{1}{\sqrt{x - 3}} \right) = \frac{1}{2} x
\]

\[
\ln 1 - \frac{1}{2} \ln (x - 3) = \frac{1}{2} x \quad \text{M1}
\]

\[
\ln (x - 3) = -x
\]

\[
x - 3 = e^{-x}
\]

\[
\text{Draw } y = x - 3: \quad \text{A1}
\]
2. The roots of the equation $2x^2 - px - q = 0$, where $p$ and $q$ are constants, are $\alpha$ and $\beta$.

The roots of the equation $4x^2 + qx - 3x - p - 1$ are $\alpha - 1$ and $\beta - 1$.

(i) Find the value of $p$ and of $q$.

\[
\begin{align*}
\alpha + \beta &= \frac{p}{2} \quad (1) \\
\alpha\beta &= \frac{-q}{2} \quad (2) \\
\alpha - 1 + \beta - 1 &= \frac{3 - q}{4} \\
\alpha + \beta &= \frac{3 - q}{4} + 2 \quad (3) \\
(\alpha - 1)(\beta - 1) &= \frac{1 - p}{4} \quad (4) \\
\end{align*}
\]

Sub (1) into (4).

\[
\alpha\beta(\alpha + \beta) + 1 = \frac{1}{4} - \frac{1}{4}(\alpha + \beta)
\]

\[
\alpha\beta - \frac{1}{2}(\alpha + \beta) + \frac{3}{4} = 0 \quad (5)
\]

Sub (2) and (3) into (5).

\[
\frac{q}{2} - 1 \left( \frac{3 - q}{4} + 2 \right) + \frac{3}{4} = 0
\]

\[
\frac{3q}{8} = \frac{5}{8}
\]

\[
q = \frac{5}{3}
\]

Sub $q = -1\frac{2}{3}$ into (3).

\[
\alpha + \beta = \frac{3 - \left( -\frac{5}{3} \right)}{4} + 2 = \frac{1}{6}
\]

\[
p = 2 \left( \frac{3}{6} \right) = 6 \frac{1}{3}
\]

\[
p = 6 \frac{1}{3}, q = -1 \frac{2}{3}
\]
(ii) Find a quadratic equation whose roots are $\alpha^3$ and $\beta^3$. 

\[
(\alpha\beta)^3 = \frac{125}{216} \quad \text{M1}
\]

\[
\alpha^2 + \beta^2 = \left(\frac{3}{6}\right)^2 - 2\left(\frac{5}{6}\right) = \frac{301}{36} \quad \text{M1}
\]

\[
\alpha^3 + \beta^3 = 3\left(\frac{301}{36} - \frac{5}{6}\right) = \frac{5149}{216} \quad \text{M1}
\]

\[
x^2 - \frac{5149}{216}x + \frac{125}{216} = 0 \quad \text{A1}
\]

\[
216x^2 - 5149x + 125 = 0
\]
3. (i) Given that the coefficient of $x^{-2}$ in the expansion of $\left(\frac{1}{x} + px\right)^8$ is 448, find the value of the positive constant $p$.

\[
T_{r+1} = \binom{8}{r} \left(\frac{1}{x}\right)^{8-r} (px)^r
\]

\[
= \binom{8}{r} x^{-8+r} p^r x^r
\]

\[
= \binom{8}{r} p^r x^{2r-8}
\]

$2r - 8 = -2$

$r = 3$

$T_3 = \binom{8}{3} p^3 x^{-2}$

$= 56 p^3 x^{-2}$

$56 p^3 - 448$

$p^3 = 8$

$p = 2$

(ii) Using the value of $p$ in part (i), find the term independent of $x$ in the expansion of $\left(\frac{1}{x} + px\right) (5x^2 - 4)$.

\[
2r - 8 = 0
\]

$r = 4$

$T_4 = \binom{8}{4} 2^4 = 1120$

\[
\left(\frac{1}{x} + 2x\right)^8 (5x^2 - 4)
\]

\[
= (... + 448x^{-2} + 1120 + ...)(5x^2 - 4)
\]

Term independent of $x$

$= 448(5) + 1120(-4)$

$= -2240$
4. A curve has the equation \( y = \frac{\ln(2x)^3}{x^2} \) for \( x > 0 \).

(i) Find an expression for \( \frac{dy}{dx} \).

\[
\frac{dy}{dx} = \frac{3x - 6x \ln 2x}{x^4} \quad \text{M2}
\]

\[
= \frac{3}{x^3} - \frac{6}{x^3} \ln 2x \quad \text{A1}
\]

(ii) Hence find \( \int \frac{\ln 2x}{x^3} \, dx \).

\[
\int \left( \frac{3}{x^3} - \frac{6 \ln 2x}{x^3} \right) \, dx = \frac{\ln(2x)^3}{x^2} + C_1 \quad \text{M1}
\]

\[
\int -\frac{6 \ln 2x}{x^2} \, dx = \int \frac{3}{x^2} \, dx - \frac{\ln(2x)^3}{x^2} + C_1
\]

\[
= \frac{1}{6} \int 3x^2 \, dx - \frac{\ln 2x}{2x^2} + C
\]

\[
= \frac{1}{6} \left( \frac{3x^3}{3} \right) - \frac{\ln 2x}{2x^2} + C \quad \text{M1}
\]

\[
= \frac{1}{4x^2} - \frac{\ln 2x}{2x^2} + C \quad \text{A1}
\]
5. The equation of a circle $C_1$ is given as $x^2 + y^2 - 16x + 8y + 64 = 0$.

(i) Find the coordinates of the centre and radius of the circle $C_1$.

\[
(x-8)^2 - (8)^2 + (y+4)^2 - (-4)^2 = -64 \quad \text{M1}
\]

\[
(x-8)^2 - (y+4)^2 = 4^2 \quad \text{A1}
\]

Centre of circle = (8, -4) \quad \text{A1}

Radius = 4 units \quad \text{A1}

(ii) The line $y = k$ is a tangent to the circle at $A$, where $k \neq 0$. Find the value of $k$.

The 2 tangents to circle are $y = 0$ and $y = -8$. \quad \text{B1}

Since $k \neq 0$, $k = -8$. \quad \text{B1}

(iii) The tangent to the circle at $B(4, -4)$ intersects $y = k$ at point $C$. Find the equation of this tangent.

$x = 4$ \quad \text{B1}

(iv) Explain why a circle $C_2$ can be drawn through the points $A$, $B$ and $C$ with $AB$ being the diameter.

$A(8, -8), B(4, -4), C(4, -8)$

Since $AC \perp BC$, $\angle BCA = 90^\circ$

$\therefore$ A circle $C_2$ can be drawn through the points $A$, $B$ and $C$ with $AB$ being the diameter. \quad \text{(Angle in a semicircle)} \quad \text{R1}

(v) Find the equation of the circle $C_2$.

Midpoint of $AB$

\[
= \left( \frac{8 + 4}{2}, \frac{-8 - 4}{2} \right) \quad \text{M1}
\]

\[
= (6, -6) \quad \text{A1}
\]

Radius

\[
= \frac{1}{2} \sqrt{(8-4)^2 + (-8 - (-4))^2} \quad \text{M1}
\]

\[
= 2\sqrt{2} \text{ units} \quad \text{M1}
\]

\[
(x - 6)^2 + (y + 6)^2 = 8 \quad \text{A1}
\]
(vi) Determine, with working, whether $\left(\frac{22}{5}, -6\right)$ lies within the 2 circles. [2]

Length of $\left(3.6, -6\right)$ to centre of $C_1$
$$= \sqrt{(8 - 3.6)^2 + (-4 + 6)^2}$$
$$= 4.83 \text{ units}$$

Length of $\left(3.6, -6\right)$ to centre of $C_2$
$$= \sqrt{(6 - 3.6)^2 + (-6 - (-6))^2}$$
$$= 2.4 \text{ units}$$

Since length of $\left(3.6, -6\right)$ to centre of $C_1 > 4 \text{ units}$, $(3.6, -6)$ is outside $C_1$. R1

Since length of $\left(3.6, -6\right)$ to centre of $C_2 < 2\sqrt{2} \text{ units}$, $(3.6, -6)$ is within $C_2$. R1
The height of a blade on the windmill (measured from the ground) can be modelled by the equation \( h = 15 - 7\cos kt \) where \( k \) is a constant and \( t \) is the time in seconds after the windmill starts moving. The windmill starts rotating from the lowest point, \( A \), when \( t = 0 \). The windmill rotates at a rate of 12 revolutions per minute.

(i) Explain why this model suggests that the highest point of the windmill, \( B \), is 22 m above the ground level.

\[-1 \leq \cos kt \leq 1\]
\[-7 \leq -7\cos kt \leq 7\]
\[15 - 7 \leq 15 - 7\cos kt \leq 15 + 7\]
\[8 \leq 15 - 7\cos kt \leq 22\]

\[\therefore \text{The highest point of the windmill is 22 m above the ground level.}\]

(ii) Find the value of \( k \).

Period = 5 seconds

\[k = \frac{2\pi}{5}\]

(iii) For how long over the course of one complete revolution will the point \( A \) be at least 17 m above ground level?

\[15 - 7\cos \frac{2\pi}{5} t = 17\]
\[\cos \frac{2\pi}{5} t = -\frac{2}{7}\]

Basic angle = 1.281044625

\[\frac{2\pi}{5} t = 1.860548028, 4.422637279\]
\[t = 1.480577078, 3.519422922\]

Length of time = 3.519422922 - 1.480577078
\[= 2.04 \text{ seconds}\]
(iv) Explain how the solution in part (iii) could be used to find the duration of the point A being at least 17 m above ground level over the course of two complete revolutions.

In the first revolution, point A is at least 17 m above the ground level for 2.04 seconds.
Since the second revolution is identical to the first, the total time for point A to be at least 17 m above the ground = 2(3.519422922 - 1.480577078) = 4.08 seconds. [2]

(v) Suggest a possible equation of how the height of a blade varies against time if the windmill starts rotating from the highest point at B.

\[ h = 15 + 7 \cos \frac{2\pi}{5} t \] [1]
The diagram shows a solid machine part that is made up of a closed cylinder joined to an inverted right circular cone. The height of the cylinder is \( h \) m and the slant height of the cone makes an angle of \( \frac{\pi}{3} \) radians to its base radius, \( r \) m.

(i) Given that the volume of the machine part is \( 50\pi \) m\(^3\), express \( h \) in terms of \( r \). \[ \text{[2]} \]

Let the height of the cone be \( a \) metre.

\[
\tan \frac{\pi}{3} = \frac{a}{r} \quad \text{(1)}
\]

\[
a = \sqrt{3}r \
\]

\[ 50\pi = \pi r^2 h + \frac{1}{3} \pi r^2 \left( \sqrt{3}r \right) \quad \text{M1} \]

\[ h = \frac{50}{r^2} \frac{\sqrt{3}}{3} \quad \text{A1} \]
(ii) Show that the total surface area of the machine part is given by
\[
A = \frac{\pi r^2}{3} (9 - 2\sqrt{3}) + \frac{100\pi}{r}.
\]

Let the slant height of the cone be \(p\) metre.
\[
\cos \frac{\pi}{3} = \frac{r}{p}
\]
\[
p = 2r
\]
\[
A = \pi r^2 + 2\pi rh + \pi r(2r) \quad M1
\]
\[
= \pi r^2 + 2\pi r \left( \frac{50 - \sqrt{3}}{r^2} \right) + 2\pi r^2 \quad M1
\]
\[
= 3\pi r^2 + \frac{100\pi}{r} - \frac{2\sqrt{3}}{3}\pi r^2 \quad M1
\]
\[
= \frac{\pi r^2}{3} (9 - 2\sqrt{3}) + \frac{100\pi}{r} \quad A1
\]

(iii) Given that \(r\) can vary, find the value of \(r\) for which the total surface area of the machine part is stationary.
\[
\frac{dA}{dr} = \frac{2\pi r}{3} (9 - 2\sqrt{3}) - \frac{100\pi}{r^2} \quad M1
\]
when \(\frac{dA}{dr} = 0\)
\[
\frac{2r}{3} (9 - 2\sqrt{3}) = \frac{100}{r^2}
\]
\[
r^3 = \frac{300}{2(9 - 2\sqrt{3})} \quad M1
\]
\[
r = 3.00 \text{ m (3 s.f.)} \quad A1
\]

(iv) By comparing gradients, explain why this value of \(r\) gives the least total surface area possible.

<table>
<thead>
<tr>
<th>(r)</th>
<th>2.99</th>
<th>3.00</th>
<th>3.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sketch</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since \(\frac{dA}{dr}\) changes sign from negative to positive as \(r\) increases through the stationary point. The total surface area is the least when \(r\) = 3.00 m. \(\text{R1}\)
8. The equation of two curves are \( y = \cos 2x - 2\sin^2 x \) and \( y = \sin 2x \).

(i) Show that the \( x \)-coordinate of the points of intersection of the two curves satisfy
\[
2\cos 2x - \sin 2x = 1.
\]
\[
\cos 2x - 2\sin^2 x = \sin 2x
\]
\[
\cos 2x + \cos 2x - 1 = \sin 2x
\]
\[
2\cos 2x - \sin 2x = 1
\]

(ii) On the same axes sketch, for \( -\pi < x < \pi \), the graphs of \( y = \cos 2x - 2\sin^2 x \) and \( y = \sin 2x \).
(iii) Express the equation \(2 \cos 2x - \sin 2x = 1\) in the form \(\cos(2x + \alpha) = k\), where \(\alpha\) and \(k\) are constants to be found.
\[
\begin{align*}
2 \cos 2x - \sin 2x &= 1 \\
2 \cos 2x - \sin 2x &= R \cos(2x + \alpha) \\
R \cos \alpha &= 2, R \sin \alpha &= 1 \\
\tan \alpha &= \frac{1}{2} \Rightarrow \alpha &= 0.463647609 \\
R &= \sqrt{5} \\
\sqrt{5} \cos(2x + 0.464) &= 1 \\
\cos(2x + 0.464) &= \frac{\sqrt{5}}{5}
\end{align*}
\]

(iv) Hence find, in radians, the \(x\)-coordinates of the points of intersection for \(-\pi < x < \pi\).
\[
\begin{align*}
\cos(2x + 0.463647609) &= \frac{\sqrt{5}}{5} \\
Basic \ angle &= 1.107148718 \\
2x + 0.463647609 &= 1.107148718, 5.176036589, -1.107148718, -5.176036589 \\
x &= 0.322, 2.36, -0.785, -2.82
\end{align*}
\]
9. A particle travels in a straight line from a fixed point \(O\) with acceleration \(a\) m/s\(^2\),
given by \(a = 8t - k\) where \(t\) is the time in seconds after passing \(O\), and \(k\) is a
canstant. The velocity of the particle is 5 m/s when it passes \(O\), and at \(t = 2\), its
velocity is -21 m/s. 

(i) Find the value of \(k\).

\[
v = \int (8t - k)\, dt
\]

\[
= -4t^2 - kt + c
\]

When \(t = 0, v = 5, c = 5\). M1

When \(t = 2, v = -21,\)

\[-21 = 4(2)^2 - 2k + 5\]

\[k = 21\] A1

(ii) Find the value(s) of \(t\) when the particle is instantaneously at rest. [2]

When \(v = 0,\)

\[4t^2 - 21t + 5 = 0\]

\[(4t - 1)(t - 5) = 0\] M1

\[t = 0.25\] or \(t = 5\). A1

(iii) Calculate the average speed of the particle during the first six seconds. [3]

\[
s = \int (4t^2 - 21t + 5)\, dt
\]

\[
= \frac{4}{3}t^3 - \frac{21}{2}t^2 + 5t + C_i
\]

When \(t = 0, s = 0, C_i = 0\).

\[
s = \frac{4}{3}t^3 - \frac{21}{2}t^2 + 5t\] M1

When \(t = 0, s = 0\) m ,

When \(t = 0.25, s = 0.164583333\) m ,

When \(t = 5, s = -70\) m ,

When \(t = 6, s = -60\) m ,

Average speed during the first 6 seconds

\[
\frac{0.614583333 + (0.614583333 + 70\frac{5}{6}) + (70\frac{5}{6} - 60)}{6}
\]

\[= 13.8\] m/s A1

(iv) Describe completely the motion of the particle in the first six seconds. [2]

The particle starts at a fixed point \(O\). At \(t = 0.25\), the particle stops and reverses its direction of
motion. At \(t = 5\), the particle stops again and reverses its direction of motion, moving toward \(O\).
At \(t = 6\), the particle has a displacement of -60 m from \(O\). 

R2 for 3 points

R1 for 2 points
In the diagram, two circles touch each other at $T$ and $PTQ$ is their common tangent. $AB$ is a tangent to the smaller circle at $E$. $AT$ and $BT$ cut the smaller circle at $D$ and $C$ respectively. $ET$ and $CD$ intersect at $F$. Prove that

(i) $AB \parallel DC$;

\[ \angle BTQ = \angle TAB \text{ (Alternate Segment Theorem)} \quad R1 \]

\[ \angle BTO = \angle TDC \text{ (Alternate Segment Theorem)} \quad R1 \]

\[ \therefore \angle TAB = \angle TDC, AB \parallel DC \text{ (Corr. } \angle \text{s)} \quad R1 \]

(ii) $\angle ATE = \angle BTE$,

\[ \angle ATE = \angle ECF \text{ (} \angle \text{s in same segment)} \quad R1 \]

\[ = \angle BEC \text{ (alt. } \angle \text{s, AB} \parallel DC) \quad R1 \]

\[ = \angle BTE \text{ (alternate segment theorem)} \quad R1 \]

(iii) $ET^2 = CT \times DT + EF \times ET$.

\[ \angle TDF = \angle TEC \text{ (} \angle \text{s in same segment)} \quad R1 \]

\[ \angle DTF = \angle ETC \text{ (part (ii))} \quad R1 \]

$\triangle DFT$ and $\triangle ECT$ are similar. (2 pairs of corr. $\angle \text{s are equal)} \quad R1$

\[ \frac{ET}{CT} = \frac{DT}{ET} \quad R1 \]

\[ ET \times FT = CT \times DT \]

\[ ET(ET - EF) - CT \times DT \quad R1 \]

\[ ET^2 = CT \times DT + EF \times ET \]
11. (i) Differentiate \((x+2)\sqrt{4x-3}\) with respect to \(x\).

\[
\frac{d}{dx}(x+2)\sqrt{4x-3} = \frac{2(x+2)}{\sqrt{4x-3}} + \frac{4x-3}{\sqrt{4x-3}} \quad \text{M1}
\]

\[= \frac{2(x+2)+4x-3}{\sqrt{4x-3}} = \frac{6x+1}{\sqrt{4x-3}} \quad \text{A1}
\]

(ii) 

The diagram shows part of the curve \(y = \frac{3x+2}{\sqrt{4x-3}}\). A line with gradient \(\frac{2}{3}\) intersects the curve at \(A(1,5)\) and \(B\).

(a) Verify that the \(y\)-coordinate of \(B\) is \(\frac{11}{3}\).

Equation of \(AB\):

\[y-5 = \frac{2}{3}(x-1)\]

\[y = \frac{2}{3}x + \frac{17}{3}\]

\[\frac{3x+2}{\sqrt{4x-3}} = \frac{-2x+17}{3} \quad \text{M1}\]

\[9x+6 = (-2x+17)\sqrt{4x-3}\]

\[81x^2+108x+36 = 16x^3-12x^2-272x^2+204x+1156x = 867\]

\[16x^3-365x^2+1252x-903 = 0 \quad \text{M1}\]
Let \( f(x) = 16x^2 - 365x^2 + 1252x - 903 \)

\( f(1) = 0 \)

\((x - 1)\) is a factor of \(f(x)\).

\[ 16x^2 - 365x^2 + 1252x - 903 = (x - 1)(16x^2 + Bx + 903) \]

Comparing coefficient of \(x^2\):

\[-365 = B - 16\]

\[ B = -349 \]

\((x - 1)(16x^2 - 349x + 903) = 0\)

\((x - 1)(16x - 301)(x - 3) = 0\)

\[ x = 1, 3 \text{ or } 18.8125 \text{(rejected)} \]

when \( x = 3, y = \frac{3(3) + 2}{\sqrt[4]{3}} = \frac{11}{3} \) (shown)

**(b) Determine the area of the region bounded by the curve and the line \( AB \).**

Area of region bounded by curve and line \( AB \)

\[
= \frac{1}{2} \times (5 + \frac{11}{3}) \times 2 - \int_{1}^{3} \frac{3x + 2}{\sqrt{4x - 3}} \, dx
\]

\[
= \frac{8}{3} \left[ \frac{1}{2} \int_{1}^{3} \frac{6x + 1}{\sqrt{4x - 3}} + \frac{3}{\sqrt{4x - 3}} \, dx \right]
\]

\[
= \frac{8}{3} \left[ \frac{1}{2} \left[ (x + 2)\sqrt{4x - 3} \right] - \frac{1}{2} \int_{1}^{3} (4x - 3)^{\frac{1}{2}} \, dx \right]
\]

\[
= \frac{8}{3} \left[ \frac{1}{2} \left[ (x + 2)\sqrt{4x - 3} \right] - \frac{3}{4} \left[ \frac{(4x - 3)^{\frac{3}{2}}}{\frac{3}{2}} \right] \right]
\]

\[
= \frac{8}{3} \left[ \frac{1}{2} \left[ (x + 2)\sqrt{4x - 3} \right] - \frac{3}{4} \left[ \sqrt{4x - 3} \right] \right]
\]

\[
= \frac{8}{3} \left[ \frac{1}{2} \left[ (5\sqrt{9} - 3\sqrt{1}) - \frac{3}{4} (\sqrt{9} - \sqrt{1}) \right] \right]
\]

\[
= 1 \frac{1}{6} \text{ units}^2
\]
FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2015
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

ADDITIONAL MATHEMATICS

4047/01

Paper 1

Date: 25 August 2015
Duration: 2 hours

Additional Materials: Answer Paper
Graph paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place
in the case of angles in degrees, unless a different level of accuracy is specified in
the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part
question.
The total number of marks for this paper is 80.

At the end of the examination, fasten all your work securely together.

<table>
<thead>
<tr>
<th>For Examiner’s Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper 1</td>
</tr>
</tbody>
</table>

Setter: Miss Lee CP

This question paper consists of 6 printed pages including the cover page.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion
\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]
where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \)

2. TRIGONOMETRY

Identities
\[
\sin^2 A + \cos^2 A = 1
\]
\[
\sec^2 A = 1 + \tan^2 A
\]
\[
\cosec^2 A = 1 + \cot^2 A
\]
\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
\sin 2A = 2 \sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]
\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \triangle ABC \)
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\Delta = \frac{1}{2} ab \sin C
\]
1. Given that \( y = (1 - \tan^2 x) \cos^2 x \), show that \( \frac{dy}{dx} = -2 \sin 2x \).  

2. Express \( \frac{8 - 3x}{(1-x)^2(2x+3)} \) as a sum of 3 partial fractions.  

3. The pressure, \( P \), and volume, \( V \), of a gas in a container are related by the formula \( P = \frac{2500}{\sqrt{V^3}} \). If the pressure increases at a rate of 2.8 units/second, find the rate of change of volume when the pressure of the gas is 50 units.  

4. Find the term independent of \( x \) in the expansion of \( (5 - 4x) \left( \frac{3x^3}{2} + \frac{2}{3x} \right)^3 \).  

5. The equation of a curve is \( y = \ln(5 - 2x) \), where \( x < \frac{5}{2} \).  
   (i) Find the coordinates of the point on the curve at which the normal to the curve is parallel to \( 2y = x + 3 \).  
   (ii) Show that as \( x \) increases, \( y \) is a decreasing function.  

6. If \( \sin(A + B) = 3 \sin(A - B) \), show that \( \tan A = 2 \tan B \).  
   Hence, solve the equation \( \sin^2(x + 60^\circ) = 9\sin^2(x - 60^\circ) \) for \( 0^\circ < x < 360^\circ \).  

7. Find all angles, leaving your answer in terms of \( \pi \), between 0 and \( \pi \) which satisfy  
   (i) \( 4 \sin \frac{x}{2} \cos \frac{x}{2} = \sqrt{3} \)  
   (ii) \( \sin^4 x - \cos^4 x - 3 \cos x = 2 \).
8 (a) Given that the curve, \( y = 4x^2 + px + p - 6 \), find the possible range or value(s) of \( p \) for which

(i) the curve intersects the line \( y = -3 \), \[3\]  
(ii) the line \( y = -3 \) is a tangent to the curve, \[1\]  
(iii) the curve has a positive \( y \)-intercept. \[1\]  

(b) Show that \( (m + 1)x^2 + (4m + 3)x + 2m - 1 = 0 \) has real and distinct roots for all real values of \( m \). \[3\]  

9 (i) Sketch the graph of \( y = |2x^2 - 3x - 14| \) for \( 0 \leq x \leq 5 \). \[3\]  
(ii) Using your graph, find the range or value(s) of \( k \) for each of the number of solutions for the equation \( |2x^2 - 3x - 14| = k \).

(a) 3 solutions, \[1\]  
(b) 2 solutions, \[2\]  
(c) 1 solution. \[1\]  

10 Answer the whole of this question on a graph paper.  
The table below shows experimental values of the variables \( x \) and \( y \) which are related by an equation of the form \( y = a^x \). One value of \( y \) has been recorded incorrectly.  

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>5.9</td>
<td>6.9</td>
<td>7.2</td>
<td>9.4</td>
<td>11.0</td>
</tr>
</tbody>
</table>

(i) Plot \( \log y \) against \( x \) and draw a straight line graph. \[2\]  
(ii) Use your graph to estimate the value of \( a \) and of \( b \). \[4\]  
(iii) Determine which value of \( y \) is inaccurate and estimate the correct value of \( y \). \[2\]
The diagram shows an open cardboard box with a rectangular base and a close fitting cardboard lid which slips over the top of the box.

The dimensions of the lid are $2x$ cm, $x$ cm and 3 cm. The total area of cardboard used in making the box and the lid is 2400 cm$^2$.

(i) Obtain an expression for $y$ in terms of $x$, and hence show that the volume,

$$V \text{ cm}^3 \text{ of the box is given by } V = 800x - \frac{4x^3}{3} - 6x^2.$$

(ii) Given that $x$ can vary, find the value of $x$ for which volume of the box is stationary. Calculate this stationary value of $V$.

(iii) Explain why this value of $x$ gives the largest volume of the box.
12 Solutions to this question by accurate drawing will not be accepted.

In the rectangle $ABCD$, the coordinates are $A(3, 0), B(-2t - 1, t - 2)$ and $C(-5, 1)$.

(i) Show that the value of $t = -1$. [3]

Find

(ii) the coordinate of $D$, [2]

(iii) the equation of perpendicular bisector of $AD$, [2]

(iv) the area of $ABCD$. [2]

~ End of Paper ~
Sec 4/5 Preliminary Examination 2015
Additional Mathematics Paper 1
Answer Key

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>[ \frac{1}{1-x} + \frac{1}{(1-x)^2} + \frac{2}{2x+3} ]</td>
<td>3</td>
<td>Rate of change of volume = - 0.507 units/sec</td>
</tr>
<tr>
<td>4</td>
<td>[ \frac{1190}{9} ]</td>
<td>5(i)</td>
<td>(2, 0)</td>
</tr>
<tr>
<td>6</td>
<td>73.9(^\circ), 253.9(^\circ), 40.9(^\circ), 220.9(^\circ)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7(i)</td>
<td>[ \frac{\pi}{3}, \frac{2\pi}{3} ]</td>
<td>7(ii)</td>
<td>( x = \frac{2}{3}\pi, \frac{4}{3}\pi ) or ( x = \pi )</td>
</tr>
<tr>
<td>8(a)(i)</td>
<td>( p \leq 0 ) or ( p \geq 12 )</td>
<td>8(a)(ii)</td>
<td>( p = 12 ) or ( p = 4 )</td>
</tr>
<tr>
<td>8(a)(iii)</td>
<td>( p &gt; 6 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9(i)</td>
<td></td>
<td>9(ii)(a)</td>
<td>( 14 \leq k &lt; 15 \frac{1}{8} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9(ii)(b)</td>
<td>( 0 &lt; k &lt; 14 ) or ( k = 15 \frac{1}{8} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9(ii)(c)</td>
<td>( k = 0 ) or ( k &gt; 15 \frac{1}{8} )</td>
</tr>
<tr>
<td>10(ii)</td>
<td>When ( x = 0.3 ), ( y = 7.2 ) (erroneous)</td>
<td>10(iii)</td>
<td>( a = 10^{0.68} = 4.79 ) (3 s.f.)</td>
</tr>
<tr>
<td></td>
<td>The correct value of ( y = 10^{0.503} )</td>
<td></td>
<td>Accept 4.68 to 4.82</td>
</tr>
<tr>
<td></td>
<td>= 8.04 (3 s.f.)</td>
<td></td>
<td>( b = 0.7 )</td>
</tr>
<tr>
<td></td>
<td>Accept 7.76 to 8.14</td>
<td></td>
<td>0.68 ( \approx 1.03 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Accept 1.02 to 1.06</td>
</tr>
<tr>
<td>11(ii)</td>
<td>6460 cm(^2) (3 s.f.)</td>
<td>12(ii)</td>
<td>D (-3, 4)</td>
</tr>
<tr>
<td>12(iii)</td>
<td>( y - 2 = \frac{3}{2}x )</td>
<td>13(iv)</td>
<td>26 units(^2)</td>
</tr>
<tr>
<td></td>
<td>or ( y = \frac{3}{2}x + 2 )</td>
<td></td>
<td></td>
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<tr>
<td>No</td>
<td>Working</td>
<td>Description</td>
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</tr>
</tbody>
</table>
| 1  | \[ y = (1 - \tan^2 x)\cos^2 x \]  
\[ y = \cos^2 x - \tan^2 x \cos^2 x \]  
\[ y = \cos^2 x - \sin^2 x \]  
\[ y = \cos 2x \]  
\[ \frac{dy}{dx} = -2 \sin 2x \]  
\[ = \text{LHS} \] | M1 [expansion]  
M1 [Substitute identity]  
AG1 |
| 2  | \[ \frac{8 - 3x}{(1 - x)^2(2x + 3)} = \frac{A}{1 - x} + \frac{B}{(1 - x)^2} + \frac{C}{2x + 3} \]  
\[ 8 - 3x = A(1 - x)(2x + 3) + B(2x + 3) + C(1 - x)^2 \]  
When \( x = 1 \), \( 8 - 3 = B(5) \)  
\[ B = 1 \]  
When \( x = -1.5 \), \( 8 - 3(-1.5) = C(1 - (-1.5))^2 \)  
\[ 12.5 = 6.25C \]  
\[ C = 2 \]  
When \( x = 0 \), \( 8 = A(1)(3) + B(3) + C(1) \)  
\[ 8 = 3A + 3 + 2 \]  
\[ 3 = 3A \]  
\[ A = 1 \]  
\[ \frac{8 - 3x}{(1 - x)^2(2x + 3)} = \frac{1}{1 - x} + \frac{1}{(1 - x)^2} + \frac{2}{2x + 3} \] | B1 [correct partial fraction formula]  
M1 [Substitution / comparing coefficient method]  
A1 for value of A / B / C  
A1 for all values correct  
B1 |
<table>
<thead>
<tr>
<th>No</th>
<th>Working</th>
<th>Description</th>
</tr>
</thead>
</table>
| 3  | \[ P = \frac{2500}{\sqrt{V^3}} \]  
\[ P = 2500V^{-\frac{3}{2}} \]  
\[ \frac{dP}{dV} = -\frac{3}{2} \times 2500 \times V^{-\frac{5}{2}} \]  
\[ = -3750V^{-\frac{5}{2}} \text{ or } -\frac{3750}{\sqrt{V^5}} \]  
When \( P = 50 \), \( \sqrt{V^3} = \frac{2500}{50} = 50 \)  
\[ V = 50^{\frac{2}{3}} \]  
\[ \frac{dP}{dt} = \frac{dP}{dV} \times \frac{dV}{dt} \]  
\[ \frac{dV}{dt} = \frac{dP}{dt} \times \left( \frac{dV}{dP} \right)^{-\frac{1}{5}} \]  
\[ = 2.8 \times \frac{\frac{1}{50^{\frac{2}{3}}}}{3750} = -0.50669 = -0.507 \text{ units/sec} \]  
Rate of change of volume = -0.507 units/sec | M1 [apply differentiation rule]  
A1 [differentiation correctly]  
B1 [find corresponding value of \( V \)]  
M1 [apply chain rule correctly]  
A1 |
| 4  | \( (5 - 4x) \left( \frac{3x^2}{2} + \frac{2}{3x} \right)^9 \)  
General term for \( \left( \frac{3x^2}{2} + \frac{2}{3x} \right)^9 \)  
\[ = \binom{9}{r} \left( \frac{3x^2}{2} \right)^{(9-r)} \left( \frac{2}{3x} \right)^r \]  
\[ = \frac{9}{r} \left( \frac{3}{2} \right)^{9-r} \left( \frac{2}{3} \right)^r x^{18-3r} \]  
For term independent of \( x \), \( 18 - 3r = 0 \) \( \rightarrow r = 6 \)  
For term with coefficient of \( x \), \( 18 - 3r = -1 \) \( \rightarrow r = \frac{19}{3} \) \( \rightarrow \) there is no term with \( \frac{1}{x} \).  
\( (5 - 4x) \left( \frac{3x^2}{2} + \frac{2}{3x} \right)^9 = (5 - 4x) \binom{9}{r} \left( \frac{3}{6} \right)^{9-r} \left( \frac{2}{2} \right)^r + ... \)  
Term independent of \( x \) = \[ 5 \times \binom{9}{6} \left( \frac{3}{2} \right)^3 = \frac{1190}{9} \] | M1 [find general term]  
M1 [find the value of \( r \), must equate the index to 0, if \( r \) is not a whole number no marks.]  
M1 [correct simplification of index for \( x \)]  
M1 [expansion]  
A1 |
<table>
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<tr>
<th>No</th>
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<th>Description</th>
</tr>
</thead>
</table>
| 5(i) | \( y = \ln (5 - 2x) \)  
\[
\frac{dy}{dx} = \frac{-2}{5 - 2x}
\]  
\(2y = x + 3\)  
\(y = \frac{x}{2} + \frac{3}{2}\)  
Gradient of normal = \(\frac{1}{2}\)  
Gradient of tangent = \(-2\)  
\[
\frac{-2}{5 - 2x} = -2
\]  
\(5 - 2x = 1\)  
\(-2x = -4\)  
\(x = 2\)  
When \(x = 2\), \(y = \ln (5 - 4) = 0\)  
The coordinates is \((2, 0)\). | B1 [differentiation]  
B1 [gradient of tangent]  
M1 [solve for x]  
A1 |
| 5(ii) | \[
\frac{dy}{dx} = \frac{-2}{5 - 2x}
\]  
For \(x < \frac{5}{2}\), \((5 - 2x) > 0\) and \(-2 < 0\)  
Therefore \(\frac{-2}{5 - 2x} < 0\), which implies that \(\frac{dy}{dx}\) is <0.  
Since \(\frac{dy}{dx} < 0\), then \(y\) is decreasing. | B1  
B1 |
| 6 | \[
\sin (A + B) = 3 \sin (A - B), \text{ show that } \tan A = 2 \tan B
\]  
\begin{align*}
\sin A \cos B + \cos A \sin B &= 3 \sin A \cos B - 3 \cos A \sin B \\
4 \cos A \sin B &= 2 \sin A \cos B \\
2 \cos A \sin B &= \sin A \cos B \\
\frac{2 \cos A \sin B}{\cos A \cos B} &= \tan A \\
2 \tan B &= \tan A
\end{align*}
\[
\sin^2(x + 60^\circ) = 9 \sin^2(x - 60^\circ) \text{ for } 0^\circ < x < 360^\circ
\]  
\[
\sin(x + 60^\circ) = \pm 3 \sin(x - 60^\circ)
\]  
Case 1: \(\sin(x + 60^\circ) = 3 \sin(x - 60^\circ)\)  
Let \(A = x\) and \(B = 60\)  
Therefore, \(\tan x = 2 \tan 60^\circ\)  
\[
\tan x = 2 \sqrt{3}
\]  
Basic angle = 73.898  
\(x = 73.898, 180 + 73.898 = 253.9^\circ\)  
AG1 | M1 [simplification]  
M1 [to get tan function]  
AG1 |
| 6 | \[
\sin^2(x + 60^\circ) = -3 \sin(x - 60^\circ)
\]  
Let \(A = 60\) and \(B = x\)  
Therefore, \(2 \tan x = \tan 60^\circ\)  
\[
\tan x = \frac{\sqrt{3}}{2}
\]  | M1  
\(\theta^\circ\)
<table>
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<tr>
<th>No</th>
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</tr>
</thead>
</table>
| 7(i) | Basic angle = 40.893  
\( x = 40.893, 180 + 40.893 \)  
\( = 40.9^\circ, 220.9^\circ \) | A1 |
| 7(ii) |  
\( 4 \sin \frac{x}{2} \cos \frac{x}{2} = \sqrt{3} \)  
\( 2 \sin \frac{x}{2} \cos \frac{x}{2} = \frac{\sqrt{3}}{2} \)  
\( \sin x = \frac{\sqrt{3}}{2} \)  
\( x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3} \)  
\( x = \frac{\pi}{3}, \frac{2\pi}{3} \) | M1 [Apply double angle identity] |
| 7(ii) |  
\( \sin^4 x - \cos^4 x - 3 \cos x = 2 \)  
\( (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) - 3 \cos x = 2 \)  
\( 1 - 2\cos^2 x - 3 \cos x = 2 \)  
\( 2\cos^2 x + 3\cos x + 1 = 0 \)  
\( (2 \cos x + 1)(\cos x + 1) = 0 \)  
\( \cos x = -0.5 \) or \( \cos x = -1 \)  
\( x = -\frac{\pi}{3}, \frac{4\pi}{3} \) or \( x = \pi \) | B1 [Factorization]  
M1 [Use identity and simplify to a quadratic function in terms of \( \cos x \)]  
M1 [Factorization]  
A1, A1 |
| 8(a)(i) |  
\( y = 4x^2 + px + p - 6 \)  
\( y = -3 \)  
\( 4x^2 + px + p - 6 = -3 \)  
\( 4x^2 + px + p - 3 = 0 \)  
\( \text{For line intersect the curve, } b^2 - 4ac \geq 0 \)  
\( p^2 - 4(4)(p - 3) \geq 0 \)  
\( p^2 - 16p + 48 \geq 0 \)  
\( (p - 12)(p - 4) \geq 0 \)  
\( p \leq 4 \text{ or } p \geq 12 \) | M1 [correct discriminant value]  
M1 [factorization]  
A1 |
<table>
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<th>No</th>
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<th>Description</th>
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</thead>
<tbody>
<tr>
<td>8(b)</td>
<td>Show that ((m+1)x^2 + (4m+3)x + 2m - 1 = 0) has real and distinct roots for all real values of (m).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(b^2 - 4ac = (4m + 3)^2 - 4(m+1)(2m - 1))</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= (16m^2 + 24m + 9 - 4(2m^2 + m - 1))</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= (16m^2 + 24m + 9 - 8m^2 - 4m + 4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= (8m^2 + 20m + 13)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(= 8\left(m + \frac{5}{2}\right)^2 + 13)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(= 8\left(m + \frac{5}{4}\right)^2 + \frac{1}{2})</td>
<td></td>
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<tr>
<td></td>
<td>Since (\left(m + \frac{5}{4}\right)^2 \geq 0), therefore, (b^2 - 4ac \geq 0.5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Therefore, the roots are real and distinct.</td>
<td></td>
</tr>
</tbody>
</table>

| 9(i) | \(y = |2x^2 - 3x - 14|\) for \(0 \leq x \leq 5\). |
| | \(y = 2x^2 - 3x - 14\) |
| | \(y\)-intercept coordinate is \((0, -14)\) |
| | \(x\)-intercept, \(y = 0\), \((2x - 7)(x + 2) = 0\) |
| | \(x = 3.5\) or \(x = -2\) |
| | \(x\)-coordinate of minimum point = \(\frac{3.5 + (-2)}{2} = \frac{3}{4}\) |
| | Minimum point is \(\left(\frac{3}{4}, -15\frac{1}{8}\right)\) |
| | \(x = 5, y = 21\) |
| | Shape of graph \([S1]\) \(\text{correct position of maximum point of the graph with one x-intercept}\) |

<p>| 9(ii)(a) | (14 \leq k &lt; 15\frac{1}{8}) |
| 9(ii)(b) | (0 &lt; k &lt; 14) or (k = 15\frac{1}{8}) |
| 9(ii)(c) | (k = 0) or (k &gt; 15\frac{1}{8}) |</p>
<table>
<thead>
<tr>
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<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>10(i)</td>
<td>$y = a^{bix}$</td>
<td>P1 [Plot points]</td>
</tr>
<tr>
<td></td>
<td>$\log y = (b + x) \log a = b \log a + x \log a$</td>
<td>S1 [Straight line graph]</td>
</tr>
<tr>
<td></td>
<td>$\log a = \text{gradient} = \frac{1.11 - 0.77}{0.6 - 0.1} = 0.68$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$a = 10^{0.68} = 4.79$ (3 s.f.)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Accept 4.68 to 4.82</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$b \log a = 0.7$ which is the $\log y$-intercept</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$b = \frac{0.7}{0.68} = 1.03$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Accept 1.02 to 1.06</td>
<td></td>
</tr>
<tr>
<td>10(ii)</td>
<td>$\text{When } x = 0.3, y = 7.2$ (erroneous)</td>
<td>B1 [convert to straight line graph]</td>
</tr>
<tr>
<td></td>
<td>The correct value of $y = 10^{0.905} = 8.04$ (3 s.f.)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Accept 7.76 to 8.14</td>
<td></td>
</tr>
<tr>
<td>10(iii)</td>
<td>$\text{Total surface area of cardboard, } A = 2400$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$A = 2x^2 + 2(2xy) + 2(xy) + 2x^2 + 2(3x) + 2(6x)$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$A = 6xy + 4x^2 + 18x$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$2400 = 6xy + 4x^2 + 18x$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$y = \frac{2400 - 4x^2 - 18x}{6x}$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$y = \frac{400}{x} - \frac{2x}{3} - 3 \text{ or } y = \frac{1200 - 2x^2 - 9}{3x}$</td>
<td>B1 [Form correct expression of for surface area]</td>
</tr>
<tr>
<td></td>
<td>$\text{Volume of box, } V = 2x^2y$</td>
<td>B1 [Form equation for surface area]</td>
</tr>
<tr>
<td></td>
<td>$V = 2x^2 \left( \frac{400}{x} - \frac{2x}{3} - 3 \right)$</td>
<td>B1 [make y the subject of formula]</td>
</tr>
<tr>
<td></td>
<td>$V = 800x - \frac{4x^3}{3} - 6x^2$</td>
<td>AG1</td>
</tr>
<tr>
<td>11(i)</td>
<td>$\frac{dV}{dx} = 800 - 4x^2 - 12x$</td>
<td>B1 [Differentiate the expression]</td>
</tr>
<tr>
<td></td>
<td>At stationary value of $V$, $\frac{dV}{dx} = 0$</td>
<td>B1 [equate dy/dx = 0]</td>
</tr>
<tr>
<td></td>
<td>$800 - 4x^2 - 12x = 0$</td>
<td>B1 [solve for x]</td>
</tr>
<tr>
<td></td>
<td>$4x^2 + 12x - 800 = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x^2 + 3x - 200 = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = \frac{-3 \pm \sqrt{9 - 4(3)(-200)}}{2}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = 12.721$ or $x = -15.721$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = 12.7$ (3 s.f.)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\text{Stationary value of } V = 800(12.721) - \frac{4}{3}(12.721)^3 - 6(12.721)^2$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$= 6461.108846$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 6460$ (3 s.f.)</td>
<td></td>
</tr>
<tr>
<td>No</td>
<td>Working</td>
<td>Description</td>
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</tr>
<tr>
<td>11(iii)</td>
<td>( \frac{d^2V}{dx^2} = -8x - 12 )</td>
<td>B1 [explanation with 2nd derivative]</td>
</tr>
<tr>
<td></td>
<td>When ( x = 12.721 ), ( \frac{d^2V}{dx^2} = -8(12.721) - 12 = -113.768 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Since ( \frac{d^2V}{dx^2} &lt; 0 ), therefore the volume is maximum.</td>
<td></td>
</tr>
<tr>
<td>12(i)</td>
<td>gradient of AB \times gradient of BC = -1 ( \frac{t - 2 - 1}{t - 2 - 1} \times \frac{t - 2 - 0}{t - 2 - 1} = -1 )</td>
<td>M1 [simplification]</td>
</tr>
<tr>
<td></td>
<td>( \frac{t - 3}{t - 2} \times \frac{t - 2}{t - 2} = 1 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \frac{t - 2 - 1}{t - 2} ) ( \frac{t - 2 - 0}{t - 2 - 1} = -1 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \frac{t - 3}{t - 2} \times \frac{t - 2}{t - 2} = 1 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( t - 2 = -4(t + 2) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( t - 3 = -4t - 8 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 5t = -5 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( t = -1 )</td>
<td></td>
</tr>
<tr>
<td>12(ii)</td>
<td>Mid-point of AC = mid-point of BD ( \left( \frac{3 + (-5)}{2}, \frac{0 + 1}{2} \right) = \left( \frac{1 + x - 3 + y}{2}, \frac{2}{2} \right) )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( x = -3 ), ( y = 4 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( D(-3,4) )</td>
<td></td>
</tr>
<tr>
<td>12(iii)</td>
<td>Gradient of AB = ( \frac{-3 - 0}{1 - 3} = \frac{3}{2} )</td>
<td>M1 [either midpoint or gradient of AB]</td>
</tr>
<tr>
<td></td>
<td>Midpoint of AD = ( \left( \frac{3 + (-3)}{2}, \frac{0 + 4}{2} \right) = (0,2) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Equation of perpendicular bisector of AD: ( y - 2 = \frac{3}{2}x )</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>or ( y = \frac{3}{2}x + 2 )</td>
<td></td>
</tr>
<tr>
<td>12(iv)</td>
<td>Area of rectangle ABCD = ( \frac{1}{2} \begin{vmatrix} 3 &amp; 1 &amp; -5 &amp; -3 &amp; 3 \ 2 &amp; 0 &amp; -3 &amp; 1 &amp; 4 &amp; 0 \end{vmatrix} = 0.5</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Alternative method: use distance formula</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Area of rectangle = length \times breadth</td>
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FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2015
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

ADDITIONAL MATHEMATICS

Preliminary Examination 2015
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

ADDITIONAL MATHEMATICS

Paper 2

Date: 26 August 2015

Duration: 2 hours 30 minutes

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place
in the case of angles in degrees, unless a different level of accuracy is specified in
the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part
question.
The total number of marks for this paper is 100.

At the end of the examination, fasten all your work securely together.

For Examiners Use

| Paper 2 | / 100 |

Setter: Mdm Haliza

This question paper consists of 6 printed pages including the cover page.
Mathematical Formulae

1. ALGEBRA

**Quadratic Equation**
For the equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

**Binomial expansion**
\[
(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + ... + \binom{n}{r}a^{n-r}b^r + ... + b^n,
\]
where \( n \) is a positive integer and
\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}
\]

2. TRIGONOMETRY

**Identities**
\[
\sin^2 A + \cos^2 A = 1
\]
\[
\sec^2 A = 1 + \tan^2 A
\]
\[
\cosec^2 A = 1 + \cot^2 A
\]
\[
sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
\sin 2A = 2 \sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]
\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

**Formulae for \( \triangle ABC \)**
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\Delta = \frac{1}{2} ab \sin C
\]
1. The roots of the quadratic equation \(3x^2 - kx + 4 = 0\), where \(k > 0\), are \(\alpha\) and \(\beta\), and that of the equation \(12x^2 - x + 12 = 0\) are \(\frac{\alpha}{\beta}\) and \(\frac{\beta}{\alpha}\). Find the value of \(k\). [7]

2. A weather satellite orbits planet \(P\) such that the equation of its path can be represented by the equation

\[x^2 + y^2 - 18x - 14y + 65 = 0\]

where \(x\) and \(y\) are the longitudinal and the latitudinal distances from the centre of \(P\) respectively in kilometres, as shown on an astronomical map.

(i) State the coordinates of the centre and the radius of the orbit. [3]

A second satellite orbits another planet \(K\) in the same plane as the first satellite. The diameter of its circular orbit has end points \((10, 9)\) and \((22, 3)\).

(ii) Find the equation of the path of this satellite. [4]

3. Without using a calculator,

(i) find the value of \(r\) and of \(n\), given that

\[\frac{3x^r}{r^2} - \frac{2(r^{b+r})^2}{27x} = nx^2,\]  

(ii) simplify \(\frac{3 + \sqrt{2}}{2\sqrt{2} - 1}\) in the form \(a + b\sqrt{2}\). [5]

4. (i) Solve the equation \(\log_3(x + 2) = 3 - \log_3(x - 4)\). [4]

(ii) Given that \(\log_3 y + \log_3 x = \frac{5}{\log_5 y}\), express \(y\) in terms of \(x\). [4]

5. (i) Solve the equation \(4 \cos 2x + 2 \sin x = -2\) for \(0 \leq x \leq 2\pi\). [6]

(ii) On the same axes, sketch the graphs of

\[y = 3 \cos 2x\] 

and \(y = |\sin x|\) for the interval \(0 \leq x \leq 2\pi\), labelling each graph clearly.

State the number of solutions in the interval \(0 \leq x \leq 2\pi\) of the equation \(3 \cos 2x = |\sin x|\). [4]
6 In the triangle $PQR$, $M$ is the mid-point of $QR$ and $PX$ bisects angle $QPR$.
The circle passing through $P$, $X$ and $M$, cuts $PQ$ and $PR$ at $A$ and $B$ respectively.

(i) Explain why $\angle PBM + \angle PXM = 180^\circ$. \hfill [1]
(ii) Show that $\triangle RBM$ is similar to $\triangle RXP$. \hfill [3]
(iii) Given that $\triangle QXA$ is also similar to $\triangle QPM$ and $\frac{PR}{RX} = \frac{PQ}{QX}$, show that $RB = QA$. \hfill [4]

7 A metal ball is heated to a temperature of $225^\circ C$ before being dropped into a liquid.
As the ball cools, its temperature, $T^\circ C$, $t$ minutes after it enters the liquid is given by $T = P + 190e^{-kt}$, where $P$ and $k$ are constants.

(i) Explain why $P = 35$. \hfill [1]
When $t = 4$, the temperature of the ball reaches $120^\circ C$.
(ii) Find the value of $k$ correct to 3 significant figures. \hfill [3]
(iii) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 10$. \hfill [3]
(iv) From the equation of $T$ given above, explain why the temperature of the ball can never fall below $35^\circ C$. \hfill [2]
8 The diagram shows part of the curve \( x = y^2 - 3y \) and the line \( x + y = 3 \). If the line and the curve intersect at \( P \) and \( Q \), find

(i) the coordinates of \( P \) and \( Q \). \[5\]

(ii) the area of the shaded region. \[5\]

9 \( f(x) = 6x^3 + ax^2 + bx - 6 \) has a factor \( x + 2 \) but leaves a remainder of \(-12\) when divided by \( x - 1 \).

(i) Find the value of \( a \) and of \( b \). \[5\]

(ii) Factorise \( f(x) \) completely and hence solve the equation

\[ 48x^3 + 4ax^2 = 6 - 2bx. \] \[6\]
10 An object at A, with an initial displacement of 3m from a fixed point O, travels in a straight line so that its velocity, \( v \) ms\(^{-1} \), is given by \( v = t^2 - 5t + 6 \) where \( t \) is the time in seconds after leaving A.

(i) Find the values of \( t \) when the object comes to an instantaneous rest. \([2]\)

(ii) Find the acceleration of the object at \( t = 5 \) s. \([2]\)

(iii) Obtain an expression, in terms of \( t \), for the displacement of the object from O after \( t \) seconds. \([3]\)

(iv) Find the average speed of the object in the first 5 seconds. \([4]\)

11 The figure shows two circles \( C_1 \) and \( C_2 \) which touch each other and lie in the xy-plane as shown below. \( C_1 \) has radius 4 units and touches the x-axis at \( D \); \( C_2 \) has radius 3 units and touches the y-axis at \( E \). The line \( AB \), joining the centres of \( C_2 \) and \( C_1 \), meets the x-axis at \( F \) and \( \angle BFO = \theta^\circ \).

![Diagram of circles touching each other and touching axes.

(i) Obtain expressions for \( OD \) and \( OE \) in terms of \( \theta \) and show that

\[
ED^2 = 74 + 56 \sin \theta + 42 \cos \theta.
\]

(ii) Express \( ED^2 \) in the form \( 74 + R \cos(\theta - \alpha) \) where \( R > 0 \) and \( 0^\circ < \alpha < 90^\circ \). \([4]\)

(iii) By considering the extreme positions in which both circles touch the x-axis and both circles touch the y-axis, show that \( 8.2^\circ \leq \theta \leq 81.8^\circ \), correct to one decimal place. \([3]\)

~ End of Paper ~
<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1.</td>
<td>( k = 5 )</td>
</tr>
<tr>
<td>2(ii)</td>
<td>Centre ((9, 7)), radius (= \sqrt{65}) or (8.06\text{km})</td>
</tr>
<tr>
<td>(ii)</td>
<td>((x - 16)^2 + (y - 6)^2 = 45) or (x^2 + y^2 - 32x - 12y + 247 = 0)</td>
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<tr>
<td>3(i)</td>
<td>(r = 3, n = 18)</td>
</tr>
<tr>
<td>(ii)</td>
<td>(f(x) = (x + 2)(3x - 1)(2x - 3)) Hence, ((2x + 2)(3(2x) - 1)(2(2x) - 3) = 0) (\therefore x = -1, -\frac{1}{3}, \frac{3}{2})</td>
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<tr>
<td>4(i)</td>
<td>(x = -5) (N.A.) or (x = 7)</td>
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<tr>
<td>(ii)</td>
<td>(a = 5) m/s(^2)</td>
</tr>
<tr>
<td>(iii)</td>
<td>(s = \frac{t^2}{2} - \frac{5t^3}{2} + 6t + 3)</td>
</tr>
<tr>
<td>5(i)</td>
<td>(x = \frac{\pi}{2}) (or (1.57)), 3.99, 5.44</td>
</tr>
<tr>
<td>11(i)</td>
<td>(E_D = 7 + 7\cos{\theta})</td>
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<tr>
<td>(ii)</td>
<td>(O_E = 4 + 7\sin{\theta})</td>
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<tr>
<td>(iii)</td>
<td>(E_D = O_D^2 + O_E^2) (By Pythagoras' Theorem)</td>
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<td>6(ii)</td>
<td>(\angle PBM) and (\angle PXM) are angles in opposite segments of the cyclic quadrilateral (PXMB). Therefore the sum of these two angles is supplementary.</td>
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<td>(ii)</td>
<td>(\angle BRM = \angle PRX) (common angles of (\triangle BRM) and (\triangle RXP)) (\angle XPR = \angle BMR) (ext. (\angle) of cyclic quad.) or (\angle RBM = \angle PXM) (ext. (\angle) of cyclic quad.) (\therefore \triangle BRM) is similar to (\triangle RXP) (by AA similarity test)</td>
</tr>
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<td>(iii)</td>
<td>Since (\triangle BRM) is similar to (\triangle RXP), (\frac{PR}{RX} = \frac{RM}{RB}). Since (\triangle QXA) is also similar to (\triangle QPM), (\frac{PQ}{QX} = \frac{QM}{QA}). Given that (\frac{PR}{RX} = \frac{RM}{RB} = \frac{QM}{QA}). Since (QM = MR) as (M) is the mid-point of (QR), (\therefore RB = QA) (Shown)</td>
</tr>
<tr>
<td>7(i)</td>
<td>At (t = 0), (T = 225^\circ\text{C}). (\therefore 225 = P + 190e^{0.4t}) (P = 225 - 190 = 35)</td>
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<tr>
<td>(ii)</td>
<td>(k = 0.201)</td>
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<tr>
<td>(iii)</td>
<td>(5.11^\circ\text{C} / \text{min.})</td>
</tr>
<tr>
<td>(iv)</td>
<td>As (e^{-2.201} &gt; 0) for (t \geq 0). (190e^{-2.201} &gt; 0) (35 + 190e^{-2.201} &gt; 35). (\therefore T &gt; 35) Hence the temperature of the ball can never fall below (35^\circ\text{C}).</td>
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</tbody>
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1. \(3x^2 - kx + 4 = 0\) have roots \(\alpha\) and \(\beta\).
\[
\frac{\alpha + \beta}{\alpha} = \frac{k}{3}
\]
\[
\frac{\alpha \beta}{\alpha} = \frac{4}{3}
\]
\(12x^2 - x + 12 = 0\) have roots \(\frac{\alpha}{\beta}\) and \(\frac{\beta}{\alpha}\).
\[
\frac{\alpha + \beta}{\beta} = \frac{\alpha^2 + \beta^2}{\alpha \beta} = \frac{1}{12}
\]
\[
\frac{(\alpha + \beta)^2 - 2\alpha \beta}{\alpha \beta} = \frac{1}{12}
\]
\[
\left(\frac{k}{3}\right)^2 - 2\left(\frac{4}{3}\right) = \frac{1}{12}
\]
\[
\frac{4}{3} - \frac{4}{3} = \frac{1}{12}
\]
\(k^2 - 24 = 1\)
\(k^2 = 25\)
\(k = 5\) (since \(k > 0\))

2(i) \(x^2 + y^2 - 18x - 14y + 65 = 0\)
Centre of orbit \((-9, 7)\)

Radius = \(\sqrt{(-9)^2 + (-7)^2 - 65}\)
= \(\sqrt{65}\) or 8.06 km

(ii) Diameter of orbit = \(\sqrt{(10 - 22)^2 + (9 - 3)^2}\)
= \(\sqrt{180}\)

Radius of orbit = \(\frac{\sqrt{180}}{2}\)

Mid-point of orbit = \(\left(\frac{10 + 22}{2}, \frac{9 + 3}{2}\right)\)
= \((16, 6)\)

\therefore\) Equation of path of this satellite is

\((x - 16)^2 + (y - 6)^2 = \left(\frac{\sqrt{180}}{2}\right)^2\)

\((x - 16)^2 + (y - 6)^2 = 45\) or

\(x^2 + y^2 - 32x - 12y + 247 = 0\)
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| **3(i)** | \[
\frac{3x^r}{r^2} \times \frac{2(r^{6-r})^2}{27x} = nx^2
\]
|   | \[
\frac{2}{9} r^{12-2r-2} x^{r-1} = nx^2
\] | M1 (Simplify powers of \( r \) and \( x \))
|   | \( r - 1 = 2 \) | M1 (Equate powers of \( x \))
|   | \( r = 3 \) | A1
|   | \[
\frac{2}{9} r^{18-2r} = n
\] | M1 (Equate scalar)
| Sub. \( r = 3 \), | \[n = \frac{2}{9} (3)^{18-2(3)}\] |   |
|   | \( = 18 \) | A1

| **3(ii)** | \[
\frac{3 + \sqrt{2}}{2\sqrt{2} - 1}
\] | M1 (rationalise denominator)
|   | \[= \frac{3 + \sqrt{2}}{2\sqrt{2} - 1} \times \frac{2\sqrt{2} + 1}{2\sqrt{2} - 1} \] | M1 (simplify)
|   | \[= \frac{6\sqrt{2} + 3 + 4 + \sqrt{2}}{8 - 1} \] |   |
|   | \[= \frac{7\sqrt{2} + 7}{7} \] |   |
|   | \[= 1 + \sqrt{2} \] | A1

| **4(i)** | \[
\log_5(x + 2) = 3 - \log_5(x - 4)
\] | M1 (Apply product law)
| log_5((x + 2)(x - 4)) = 3 | M1 (Change from log to index form)
| \(x^2 - 2x - 8 = 3\) | M1 (factorise)
| \(x^2 - 2x - 35 = 0\) |   |
| \((x + 5)(x - 7) = 0\) | A1
| \(x = -5 \) (N.A.) or \( x = 7 \) |   |
### (ii)

\[
\log_x y + \log_x x - \frac{5}{\log_x y} = 0
\]

\[
\log_x y + \frac{\log_x x}{\log_x y} - \frac{5}{\log_x y} = 0
\]

\[
\log_x y + \frac{1}{\log_x y} - \frac{5}{\log_x y} = 0
\]

\[(\log_x y)^2 + 1 - 5 = 0\]

\[(\log_x y)^2 = 4\]

\[\log_x y = \pm 2\]

\[y = x^2 \text{ or } y = x^{-2}\]

**OR**

\[
\log_x y + \log_x x - \frac{5}{\log_x y} = 0
\]

\[
\frac{\log_x y}{\log_x x} + \log_x x - 5 \frac{\log_x x}{\log_x y} = 0
\]

\[
\frac{1}{\log_x x} + \log_x x - 5 \log_x x = 0
\]

\[4(\log_x x)^2 = 1\]

\[\log_x x = \pm \frac{1}{2}\]

\[x = y^{\frac{1}{2}} \text{ or } y^{\frac{1}{2}}\]

\[y = x^2 \text{ or } y = x^{-2}\]

### 5(i)

\[4 \cos 2x + 2 \sin x = -2 \text{ for } 0 \leq x \leq 2\pi\]

\[4(1 - 2\sin^2 x) + 2 \sin x + 2 = 0\]

\[4\sin^2 x - \sin x - 3 = 0\]

\[(4\sin x + 3)(\sin x - 1) = 0\]

\[
\sin x = \frac{-3}{4} \text{ or } \sin x = 1
\]

Basic \[\sin^{-1}\left(\frac{3}{4}\right) = 0.84806\]

\[x = \frac{\pi}{2} \text{ (or 1.57)}\]

\[x = \pi + 0.84806, 2\pi - 0.84806\]

\[= 3.99, 5.44\]

**M1** (Change of base)

**M1** (Quadratic form)

**A1, A1** (y in terms of x)

**OR**

**M1** (Change of base)

**M1** (Quadratic form)

**A1, A1** (y in terms of x)
ii)

\[ y = \frac{1}{\sin x} \quad (\text{Eqn} \, (b)) \]

\[ y \to \infty \text{ as } x \to \text{Correct} \]

4 solutions B1

6(i)

\( \angle PRM \) and \( \angle PXM \) are angles in opposite segments of the cyclic quadrilateral \( PXMB \).
Therefore the sum of these two angles is supplementary.

B1

(ii)

\[ \angle BRM = \angle PRX \quad (\text{common angles of } \triangle RBM \text{ and } \triangle RXP) \]
\[ \angle XPR = \angle BMR \quad (\text{ext. } \angle \text{ of cyclic quad.)} \]
\[ \text{or } \angle BRM = \angle PXM \quad (\text{ext. } \angle \text{ of cyclic quad.)} \]
\[ \therefore \triangle RBM \text{ is similar to } \triangle RXP \quad (\text{by AA similarity test}) \]

B1 B1 B1

(iii)

Since \( \triangle RBM \) is similar to \( \triangle RXP \),
\[ \frac{PR}{RX} = \frac{RM}{RB} \]

Since \( \triangle QXM \) is also similar to \( \triangle QPM \),
\[ \frac{PQ}{XQ} = \frac{QM}{QA} \]

Given that \[ \frac{PR}{RX} = \frac{PQ}{XQ} \]
\[ \therefore \frac{RM}{RB} = \frac{QM}{QA} \]

Since \( QM = MR \) as \( M \) is the mid-point of \( QR \),
\[ \therefore RB = QA \quad (\text{Shown}) \]

B1 B1 B1 B1

7(i)

At \( t = 0 \), \( T = 225^\circ \text{C} \).
\[ \therefore 225 = P + 190e^{-10(t)} \]
\[ P = 225 - 190 \]
\[ = 35 \]

B1
| (ii) | When \( t = 4 \), \( T = 120 \), 
\[
120 = 35 + 190e^{-k(4)}
\]
\[-4k = \ln\left(\frac{85}{190}\right)\]
\[k = 0.20109\]
\( \approx 0.201 \) | M1
M1 (change to ln)
A1

| (iii) | \( T = 35 + 190e^{-0.20109t} \) 
\[
\frac{dT}{dt} = -0.20109(190e^{-0.20109t})
\]
\[= -38.2077e^{-0.20109t}\]
When \( t = 10 \), 
\[
\frac{dT}{dt} = -38.2077e^{-0.20109(10)}
\]
\[= -5.1148\]
Temperature of the ball is decreasing at a rate of 5.1 \( ^\circ \mathrm{C} \) /min. | M1 from (ii) value of \( k \)
M1 (fi if value of \( t \) and \( k \) clearly shown)
A1 (positive value)

| (iv) | As \( e^{-0.20109t} > 0 \) for \( t \geq 0 \), 
\[
190e^{-3.20109t} > 0
\]
\[
35 + 190e^{-3.20109t} > 35.
\]
\[\therefore T > 35.\]
Hence the temperature of the ball can never fall below 35 \( ^\circ \mathrm{C} \). | B1
---
B1 (must also include concluding statement)

| 8(i) | Sub. \( x = y^2 - 3y \) into \( x + y = 3 \), 
\( y^2 - 3y + y = 3 \)
\( y^2 - 2y - 3 = 0 \)
\((y - 3)(y + 1) = 0\)
\( y = 3 \) or -1
Sub. \( y = 3 \) and -1 into \( x = y^2 - 3y \), 
\( x = 3^2 - 3(3) \) and \( x = (-1)^2 - 3(-1) \)
\( x = 0 \) and \( x = 4 \)
\[\therefore P \text{ is } (0, 3) \text{ and } Q \text{ is } (4, -1)\] | M1
M1 (factorise/general formula)
M1
A1, A1
(ii) Area of shaded region
\[= \int_{0}^{1} (y^2 - 3y) \, dy + \frac{1}{2} (4) - \int_{1}^{3} (y^2 - 3y) \, dy\]
\[= \left[ \frac{y^3}{3} - \frac{3y^2}{2} \right]_{0}^{1} + 8 - \left[ \frac{y^3}{3} - \frac{3y^2}{2} \right]_{1}^{3}\]
\[= \frac{3}{3} - \frac{3(3)^2}{2} + 8 - \left[ \frac{0}{3} - \frac{3(-1)^2}{2} \right]\]
\[= \frac{4}{2} + 8 - \frac{1}{6} = 10 \frac{2}{3} \text{ or } 10.7 \text{ or } \frac{32}{3} \text{ sq. units}\]

or
\[\frac{1}{2} (4) - \int_{1}^{3} (y^2 - 3y) \, dy\]

or
\[\int_{0}^{1} (3 - y) \, dy - \int_{1}^{3} (y^2 - 3y) \, dy\]

or
\[\int_{2}^{3} (y^2 - 3y) \, dy = \frac{1}{2} (3)(3 + 1) - \frac{1}{2} (3)(3) + \frac{1}{2} (3)(1) - \int_{1}^{3} (y^2 - 3y) \, dy\]

or
\[\int_{2}^{3} (y^2 - 3y) \, dy = \frac{1}{2} (3) + (3)(1) - \frac{1}{2} (1)(1) - \int_{1}^{3} (y^2 - 3y) \, dy\]

9(i)
\[f(x) = 6x^3 + ax^2 + bx - 6 = (x + 2)P(x)\]
\[f(-2) = 6(-2)^3 + a(-2)^2 + b(-2) - 6 = 0\]
\[-48 + 4a - 2b - 6 = 0\]
\[2a - b = 27 \quad \text{------------------- (1)}\]

\[f(x) = 6x^3 + ax^2 + bx - 6 = (x - 1)Q(x) - 12\]
\[f(1) = 6 + a + b - 6 = -12\]
\[a + b = -12 \quad \text{------------------- (2)}\]

(1) + (2): 3a = 15
\[a = 5\]

Sub. a = 5 into (2):
\[5 + b = -12\]
\[b = -17\]
(ii) \[ f(x) = 6x^3 + 5x^2 - 17x - 6 = (x + 2)(6x^2 + kx - 3) \]

By comparing coefficients of \( x \),
- \( 3 + 2k = -17 \)
- \( k = -7 \)

\[ \therefore f(x) = (x + 2)(6x^2 - 7k - 3) \]

\[ = (x + 2)(3x - 1)(2x - 3) \]

\[ 48x^3 + 4ax^2 + 2bx - 6 = 0 \]

\[ 6(2x)^3 + a(2x)^2 + b(2x) - 6 = 0 \]

Hence, \((2x + 2)(3(2x) - 1)(2(2x) - 3) = 0 \)

[or let \( u = 2x \),

\[ 6u^3 + au^2 + bu - 6 = 0 \]

Hence, \((u + 2)(3u + 1)(2u - 3) = 0 \)]

\[ u = 2x = -2, -\frac{1}{3}, \frac{3}{2} \]

\[ \therefore x = -1, -\frac{1}{2}, \frac{3}{4} \]

M1 from (i) (compare coeff/ long division/ synthetic division)
A1ft
A1

10(i) \( v = t^2 - 5t + 6 \)

When at instantaneous rest, \( v = 0 \).

\( t^2 - 5t + 6 = 0 \)

\( (t - 3)(t - 2) = 0 \)

\( t = 3 \) or \( 2 \)

M1 (equate to zero)
A1

(ii) \[ a = \frac{dv}{dt} = 2t - 5 \]

When \( t = 5 \),

\[ a = 2(5) - 5 = 5 \text{ m/s}^2 \]

M1
A1

(iii) \[ s = \int (t^2 - 5t + 6)\,dt \]

\[ = \frac{t^3}{3} - \frac{5t^2}{2} + 6t + c \]

When \( t = 0, s = 3 \),

\[ c = 3 \]

\[ \therefore s = \frac{t^3}{3} - \frac{5t^2}{2} + 6t + 3 \]

M1 (values of \( t \) and \( s \) indicated)
A1
(iv) When \( t = 2 \),
\[
s = \frac{2^3}{3} - \frac{5(2)^2}{2} + 6(2) + 3 = 7 \frac{2}{3}
\]
When \( t = 3 \),
\[
s = \frac{3^3}{3} - \frac{5(3)^2}{2} + 6(3) + 3 = 7 \frac{1}{2}
\]
When \( t = 5 \),
\[
s = \frac{5^3}{3} - \frac{5(5)^2}{2} + 6(5) + 3 = 12 \frac{1}{6}
\]

Average speed in first 5 s
\[
= \frac{(7 \frac{2}{3} - 3) + (7 \frac{2}{3} - 7 \frac{1}{2}) + (12 \frac{1}{6} - 7 \frac{1}{2})}{5}
\]
\[
= \frac{9.5}{5}
\]
\[
= 1.9 \text{ m/s}
\]

11(i) \[ GB = 7 \cos \theta = JD \]
\[
\therefore OD = OJ + JD = 3 + 7 \cos \theta
\]
\[
AG = 7 \sin \theta = EH
\]
\[
\therefore OE = OH + HE = 4 + 7 \sin \theta
\]
\[ \therefore \text{By Pythagoras' Theorem,} \\
ED^2 = OD^2 + OE^2 \\
= (3 + 7 \cos \theta)^2 + (4 + 7 \sin \theta)^2 \\
= 9 + 42 \cos \theta + 49 \cos^2 \theta + 16 + 56 \sin \theta + 49 \sin^2 \theta \\
= 25 + 56 \sin \theta + 42 \cos \theta + 49(\sin^2 \theta + \cos^2 \theta) \\
= 25 + 56 \sin \theta + 42 \cos \theta + 49 \quad (1) \\
= 74 + 56 \sin \theta + 42 \cos \theta \quad \text{(Shown)} \quad \text{M1} \]

\[11(ii) \quad \begin{align*}
ED^2 &= 74 + 56 \sin \theta + 42 \cos \theta \\
&= 74 + R \cos(\theta - \alpha) \\
&= 74 + R \cos \theta \cos \alpha + R \sin \theta \sin \alpha \\
\text{By comparing coefficients,} \\
R \sin \alpha &= 56 \quad \ldots \quad (1) \\
R \cos \alpha &= 42 \quad \ldots \quad (2) \\
(1) \div (2) \quad \tan \alpha &= \frac{56}{42} \\
\alpha &= 53.13^\circ \\
&= 53.1^\circ \text{ (to 1 dec. pl.)} \\
(1)^2 + (2)^2 : R^2 &= 56^2 + 42^2 \\
R &= \sqrt{4900} \\
R &= 70 \\
\therefore \quad ED^2 &= 74 + 70 \cos(\theta - 53.1^\circ) \quad \text{M1}\]

\[\text{(iii)} \quad \begin{align*}
\sin \theta &= \frac{1}{7} \quad \text{(M1)} \\
\theta &= 8.2^\circ \text{ (to 1 dec. pl.)} \\
\therefore \quad 8.2^\circ \leq \theta \leq 81.8^\circ \quad \text{(Shown)} \quad \text{(AG1)}
\end{align*} \]

\[\cos \theta = \frac{1}{7} \quad \text{(M1)} \\
\theta &= 81.8^\circ \text{ (to 1 dec. pl.)} \]
PRELIMINARY EXAMINATION 2015
SECONDARY 4 EXPRESS
ADDITIONAL MATHEMATICS

Date: 5 August 2015
Duration: 2 hours

Additional Materials: 8 Sheets of Writing Paper
1 Sheet of Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction tape/liquid.

Answer ALL questions.

The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total number of marks for this paper is 80.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142.

Set by: Mrs Rajammal Nathan
Vetted by: Ms Tan Bee Choo
Mdm Hayati

This document consists of 6 printed pages (including cover page).
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion

\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}
\]

2. TRIGONOMETRY

Identities

\[
\sin^2 A + \cos^2 A = 1
\]

\[
\sec^2 A = 1 + \tan^2 A
\]

\[
\csc^2 A = 1 + \cot^2 A
\]

\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
\sin 2A = 2 \sin A \cos A
\]

\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]

\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \triangle ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2} ab \sin C
\]
Answer all the questions.

1 (i) Find the set of values of \( k \) for which the equation \( 2x^2 + 5x + k = 2kx + 1 \) has no real roots. \([4]\)

(ii) Hence state, with a reason, whether the line \( y = 4x + 1 \) meets the curve \( y = 2x^2 + 5x + 2 \). \([1]\)

2 (i) Given that \( \sec 200^\circ = -k \), where \( k > 0 \), find an expression, in terms of \( k \), for \( \sin 200^\circ \). \([2]\)

(ii) Hence show that \( \tan 110^\circ = \frac{1}{\sqrt{k^2 - 1}} \). \([3]\)

3 The equation of a curve is \( y = \ln(5 - 2x) \). Find the coordinates of the point on the curve at which the normal to the curve is parallel to the line \( 2y = x + 3 \). \([5]\)

4 ** The equation of a curve is \( y = \cos^3 x + \sin 3x \). Given that \( x \) is changing at a constant rate of 0.56 radians per second, find the rate of change of \( y \) when \( x = \frac{\pi}{6} \). \([5]\)

5 (i) Express \( \frac{2x-1}{2x^2-5x+3} \) in partial fractions. \([3]\)

(ii) Hence find \( \int \frac{2x-1}{2x^2-5x+3} \, dx \). \([3]\)

6 A curve is such that \( \frac{d^2 y}{dx^2} = 16e^{-3x} \). Given that \( \frac{dy}{dz} = 3 \) when \( z = 0 \) and that the curve passes through the point \( (2, e^{-3}) \), find the equation of the curve. \([6]\)
7 (i) Prove that \( \frac{1 - \sin 2\theta}{1 + \cos 2\theta} = \frac{1}{2}(1 - \tan \theta)^2 \). \[4\]

(ii) Hence solve the equation \( \frac{1 - \sin \theta}{1 + \cos \theta} = 2 \), for \( 0^\circ \leq \theta \leq 180^\circ \). \[3\]

8 (i) Write down the first three terms in the expansion, in ascending powers of \( x \), of

(a) \( \left(1 + \frac{3x}{2}\right)^5 \). \[2\]

(b) \( (2 - x)^3 \). \[2\]

(ii) Hence find the coefficient of \( x^2 \) in the expansion \( \left(2 + 2x - \frac{3x^2}{2}\right)^3 \). \[3\]

9 (i) Calculate the coordinates of the point of intersection of the graph of \( y = 3 - |2x + 1| \) with the coordinate axes. \[3\]

(ii) Sketch the graph of \( y = 3 - |2x + 1| \). \[2\]

(iii) On the same diagram in part (ii), sketch the graph of \( y = x^2 \) for \( x \geq 0 \). \[1\]

(iv) State the number of solutions of the equation \( 3 - |2x + 1| = x^2 \). \[1\]
The diagram shows a time capsule consisting of a cylinder of radius \( r \) m and length \( l \) m, with hemispheres of radius \( r \) m attached at each end. The volume of the time capsule is \( \frac{\pi}{6} \) m\(^3\).

(i) Show that the surface area of the time capsule, \( A \) m\(^2\), is given by

\[
A = \frac{4}{3} \pi r^2 + \frac{\pi}{3} l.
\]

(ii) Given that \( r \) can vary, find the minimum value of \( A \).
The diagram shows an isosceles triangle $PQR$ in which $PR = QR$. $M(2, 0)$ is the midpoint of $PQ$. $QR$ meets the x-axis at $A$ and angle $AMQ = 45^\circ$.

(i) Show that the equation of $MR$ is $y = x - 2$. [2]

(ii) Find the equation of $PQ$. [2]

(iii) Find the coordinates of $Q$. [2]

(iv) Given that the area of triangle $PQR$ is $20$ units$^2$, find the coordinates of $R$. [3]

12. A rectangle of area $y$ m$^2$ has sides of length $x$ m and $(4x + B)$ m, where $A$ and $B$ are constants and $x$ and $y$ are variables. Values of $x$ and $y$ are given in the table below.

<table>
<thead>
<tr>
<th></th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>3250</td>
<td>9000</td>
<td>17250</td>
<td>28000</td>
<td>41250</td>
</tr>
</tbody>
</table>

(i) Plot $\frac{y}{x}$ against $x$ and draw a straight line graph. [3]

(ii) Use your graph to estimate value of $A$ and of $B$. [4]

(iii) On the same diagram, draw the straight line representing the equation $y = x^2$ and explain the significance of the value of $x$ given by the point of intersection of the two lines. [3]

End of paper
Answers

1 (i) \[ \frac{1}{2} < k < \frac{5}{2} \]

(ii) The equations of the line and the curve are obtained when \( k = 2 \). Since \( k = 2 \) lies in the range, the line does not meet the curve.

2 (i) \[ \sec 200^\circ = -\frac{1}{k} \]

\[ \cos 200^\circ = -\frac{1}{k} \]

\[ \sin 200^\circ = -\frac{\sqrt{k^2 - 1}}{k} \]

Or \[ \cos 200^\circ = -\frac{1}{k} \]

Applying identity: \( \sin^2 200^\circ + \cos^2 200^\circ = 1 \)

\[ \sin 200^\circ = \sqrt{1 - \frac{1}{k^2}} \] (rej) or \[ -\sqrt{1 - \frac{1}{k^2}} \]

(ii) \[ \tan 110^\circ = \frac{\sin 110^\circ}{\cos 110^\circ} = \frac{\sin(200^\circ - 90^\circ)}{\cos(200^\circ - 90^\circ)} \]

\[ = \frac{\sin 200^\circ \cos 90^\circ - \cos 200^\circ \sin 90^\circ}{\cos 200^\circ \cos 90^\circ + \sin 200^\circ \sin 90^\circ} \] (applying addition formula)

\[ = \frac{0 - \left( -\frac{1}{k} \right)}{0 + \left( -\frac{\sqrt{k^2 - 1}}{k} \right)(1)} = -\frac{1}{\sqrt{k^2 - 1}} \]
\[ \text{OR} \quad \tan 110^\circ = -\tan 70^\circ \\
= -\frac{1}{\tan 20^\circ} \\
= -\frac{1}{\tan 200^\circ} \\
= -\frac{1}{-\sqrt{k^2 - 1}} = -\frac{1}{\sqrt{k^2 - 1}} \]

3 coordinates of the point = (2, 0)

4 \quad -0.63 \text{ radians/s}

5 (i) \quad \frac{2x - 1}{2x^2 - 5x + 3} = \frac{4}{2x - 3} \frac{1}{x - 1}.

(ii) \quad 2 \ln(2x - 3) - \ln(x - 1) + c

6 \quad y = e^{-x} + 7x - 14

7 (i) \quad \frac{1 - \sin 2\theta}{1 + \cos 2\theta} = \frac{1 - 2 \sin \theta \cos \theta}{1 + 2 \cos^2 \theta - 1}

\quad = \frac{1 - 2 \sin \theta \cos \theta}{2 \cos^2 \theta}

\quad = \frac{1}{2 \cos^2 \theta} \cdot \frac{2 \sin \theta \cos \theta}{2 \cos^2 \theta}

\quad = \frac{1}{2} \sec^2 \theta - \tan \theta

\quad = \frac{1}{2} \left(1 + \tan^2 \theta\right) - \tan \theta \quad \text{Applying identity}

\quad = \frac{1}{2} \left(1 + \tan^2 \theta - 2 \tan \theta\right)

\quad = \frac{1}{2} \left(1 - \tan \theta\right)^2

(ii) \quad \theta = 143.1^\circ
8. (i) \[ 1 + \frac{15}{2} x + \frac{45}{2} x^2 + \ldots \]
(ii) \[ 32 - 80x + 80x^2 + \ldots \]
(iii) Coefficient of \( x^2 = 200 \)

9. (i) Intersects the \( y \)-axis at \((0, 2)\)
Intersects the \( x \)-axis at \((1, 0)\) and \((-2, 0)\).
(ii) Correct shape and passing through the coordinate axes
Coordinates of vertex
(iii) Correct shape passing through \((1, 1)\)

(iv) Number of solutions = 1

10. (i) \[ \left( \frac{2}{3} \pi r^3 \right) + \pi r^2 l = \frac{\pi}{6} \]
   or \[ \frac{4}{3} \pi r^3 + \pi r^2 l = \frac{\pi}{6} \]
   
   \[ l = \frac{\frac{\pi}{6} - \frac{4}{3} \pi r^3}{\pi r^2} \]

expressing \( l \) in terms of \( r \)

\[ l = \frac{1}{6r^2} - \frac{4}{3} \]

\[ A = 2(2\pi r^2) + 2\pi rl \]

\[ = 4\pi r^2 + 2\pi r \left( \frac{1}{6r^2} - \frac{4}{3} \right) \]

substituting a correct expression for \( l \)
\[ = 4\pi r^2 + \frac{\pi}{3} \cdot \frac{8}{3} \pi r^2 \]
\[ = \frac{4}{3} \pi r^2 + \frac{\pi}{3} \]
(shown)

(ii) \(3.14 \text{ cm}^2\)

11 (i) \(\angle RMA = 90^\circ - 45^\circ\)
\[= 45^\circ\]
Gradient of \(MR = \tan 45^\circ\)
\[= 1\]
Equation of \(MR\) is \(y - 0 = 1(x - 2)\)
\[y = x - 2\]

(ii) \(y = -x + 2\)  
(iii) Coordinates of \(Q = (4, -2)\)  
(iv) Coordinates of \(R = (7, 5)\)
\[\begin{array}{|c|c|c|c|c|c|}
\hline
x & 50 & 100 & 150 & 200 & 250 \\
\hline
y & 3250 & 9000 & 17250 & 28000 & 41250 \\
\hline
\frac{y}{x} & 65 & 90 & 115 & 140 & 165 \\
\hline
\end{array}\]

(i) \[\frac{y}{x}\]

(ii) \[A = 0.5 \pm 0.2 \quad B = 40 \pm 1\]

(iii) Plot \(\frac{y}{x}\) against \(x\) as a straight line accurately.

The \(x\)-value of the point of intersection represents the value where the rectangle becomes a square.
1 (i) \(2x^2 + 5x - 2kx + k - 1 = 0\)

No real roots \(\Rightarrow b^2 - 4ac < 0\)

\((5 - 2k)^2 - 4(2)(k - 1) < 0\)

\[25 - 20k + 4k^2 - 8k + 8 < 0\]

\[4k^2 - 28k + 33 < 0\]  
correct quadratic  

Finding the solution of quadratic: \(k = \frac{1}{2}\) or \(\frac{5}{2}\)

\((2k - 3)(2k - 11) < 0\)

\[1\frac{1}{2} < k < 5\frac{1}{2}\]  

(ii) The equations of the line and the curve are obtained when \(k = 2\).

Since \(k = 2\) lies in the range, the line does not meet the curve.

2 (i) \(\sec 200^\circ = -k\)

\(\cos 200^\circ = -\frac{1}{k}\)

\(\sin 200^\circ = -\frac{\sqrt{k^2 - 1}}{k}\)  

\(\sqrt{k^2 - 1} - 1\)  

\(\frac{\sqrt{k^2 - 1}}{k}\)  

\(\frac{\sqrt{k^2 - 1}}{k}\)  

1
\[ \cos 200^\circ = -\frac{1}{k} \quad \text{M1} \]

Applying identity: \( \sin^2 200^\circ + \cos^2 200^\circ = 1 \)

\[ \sin 200^\circ = \sqrt{1 - \frac{1}{k^2}} \quad \text{(rej)} \quad \text{or} \quad -\sqrt{1 - \frac{1}{k^2}} \quad \text{A1} \]

**Accept any equivalent form**

(ii) \[ \tan 110^\circ = \frac{\sin 110^\circ}{\cos 110^\circ} \]
\[ = \frac{\sin(200^\circ - 90^\circ)}{\cos(200^\circ - 90^\circ)} \]
\[ = \frac{\sin 200^\circ \cos 90^\circ - \cos 200^\circ \sin 90^\circ}{\cos 200^\circ \cos 90^\circ + \sin 200^\circ \sin 90^\circ} \quad \text{(applying addition formula)} \quad \text{M1} \]
\[ = \frac{0 - \left(\frac{1}{k}\right)}{0 + (\frac{-\sqrt{k^2 - 1}}{k})} = -\frac{1}{\sqrt{k^2 - 1}} \quad \text{A1} \]

[OR \[ \tan 110^\circ = -\tan 70^\circ \]
\[ = -\frac{1}{\tan 20^\circ} \quad \text{M1} \]
\[ = -\frac{1}{\tan 200^\circ} \quad \text{M1} \]
\[ = -\frac{1}{-\sqrt{k^2 - 1}} = \frac{1}{\sqrt{k^2 - 1}} \quad \text{A1} \]

3 \[ y = \ln(5 - 2x) \]
\[ \frac{dy}{dx} = \frac{-2}{5 - 2x} \quad \text{(M1 for -2 and M1 for } \frac{1}{5 - 2x} \text{)} \quad \text{M2} \]

Gradient of line \[ = \frac{1}{2} \]

Gradient function of normal \[ = \frac{5 - 2x}{2} \quad \text{M1} \]

\[ \frac{5 - 2x}{2} = \frac{1}{2} \quad \text{M1} \]

\[ x = 2 \]
\[ y = 0 \]

coordinates of the point = (2, 0)

\[ y = \cos^3 x + \sin 3x \]
\[ \frac{dy}{dx} = (3 \cos^2 x)(-\sin x) + 3 \cos 3x \quad \text{M3} \]
\[ \frac{dy}{dr} = \frac{dy}{dx} \times \frac{dx}{dr} \quad \text{M1 M1 M1} \]
\[ = (-3 \cos^2 \frac{\pi}{6} \sin \frac{\pi}{6} + 3 \cos \frac{3\pi}{6}) \times 0.56 \quad \text{M1} \]
\[ = -0.63 \text{ radians/s} \quad \text{A1} \]

5 (i) \[ \frac{2x-1}{2x^2-5x+3} = \frac{A}{2x-3} + \frac{B}{x-1} \quad \text{M1} \]
\[ 2x-1 = A(x-1) + B(2x-3) \]
Substitute \( x = 1 \),
\[ 1 = -B \]
\[ B = -1 \]
Substitute \( x = \frac{3}{2} \),
\[ 2 = \frac{1}{2}A \]
\[ A = 4 \]
\[ \frac{2x-1}{2x^2-5x+3} = \frac{4}{2x-3} - \frac{1}{x-1} \quad \text{A1} \]

(ii) \[ \int \frac{2x-1}{2x^2-5x+3} \, dx = \int \left( \frac{4}{2x-3} - \frac{1}{x-1} \right) \, dx \]
\[ = \frac{4 \ln(2x-3)}{2} - \ln(x-1) + c \]
\[ = 2 \ln(2x-3) - \ln(x-1) + c \quad \text{B3} \]
\[ -1 \text{ for each error} \]

B1 B1 B1
\[
\frac{dy}{dx} = \int (16e^{-4x}) \, dx = -4e^{-4x} + c
\]

(B1) (for \(-4e^{-4x}\))

Substitute \(\frac{dy}{dx} = 3\) and \(x = 0\)

\[
3 = -4e^{-4(0)} + c \quad \text{(attempt to find } c) \quad \text{M1}
\]

\[
e^{-4x} = 7
\]

\[
\frac{dy}{dx} = -4e^{-4x} + 7 \quad \text{A1}
\]

\[
y = \int (-4e^{-4x} + 7) \, dx
\]

\[
e^{-4x} + 7x + c_1 \quad \text{for } e^{-4x} + 7x \quad \text{B1}
\]

Substitute \(x = 2\) and \(y = e^{-4}\)

\[
e^{-8} = e^{-4(i)} + 7(2) + c_1
\]

\[
c_1 = -14 \quad \text{(attempt to find } c_1 \text{)} \quad \text{M1}
\]

\[
y = e^{-4x} + 7x - 14 \quad \text{A1}
\]

7 (i) \[
\frac{1 - \sin 2\theta}{1 + \cos 2\theta} = \frac{1 - 2\sin \theta \cos \theta}{1 + 2\cos^2 \theta - 1} = \frac{1 - 2\sin \theta \cos \theta}{2\cos^2 \theta} \quad \text{M1}
\]

Applying double angle formulas

\[
= \frac{1}{2\cos^2 \theta} - \frac{2\sin \theta \cos \theta}{2\cos^2 \theta} \quad \text{M2}
\]

\[
= \frac{1}{2} \sec^2 \theta - \tan \theta
\]

\[
= \frac{1}{2} \left(1 + \tan^2 \theta\right) - \tan \theta \quad \text{M1}
\]

Applying identity

\[
= \frac{1}{2} \left(1 + \tan^2 \theta - 2\tan \theta\right)
\]

\[
= \frac{1}{2} \left(1 - \tan \theta\right)^2 \quad \text{A1}
\]
(ii) \[
\frac{1 - \sin \theta}{1 + \cos \theta} = 2
\]
\[
\frac{1}{2} \left(1 - \tan \frac{\theta}{2}\right)^2 = 2
\]

\[
0^\circ \leq \theta \leq 180^\circ
\]
\[
0^\circ \leq \frac{\theta}{2} \leq 90^\circ
\]

\[
1 - \tan \frac{\theta}{2} = 2 \text{ or } -2
\]

\[
\tan \frac{\theta}{2} = -1 \text{ (reject) or } 3
\]

\[
\frac{\theta}{2} = 71.56^\circ
\]

\[
\theta = 143.1^\circ
\]

A1

8. (i) (a) \[
\left(1 + \frac{3x}{2}\right)^5 = 1 + \binom{5}{1} \left(\frac{3x}{2}\right) + \binom{5}{2} \left(\frac{3x}{2}\right)^2 + ...
\]

\[
= 1 + \frac{15}{2} x + \frac{45}{2} x^2 + ...
\]

B1

B1

(b) \[(2 - x)^5 = (2)^5 + \binom{5}{1} (2)^4 (-x) + \binom{5}{2} (2)^3 (-x)^2 + ...
\]

\[
= 32 - 80x + 80x^2 + ...
\]

B1

B1

(ii) \[
\left(2 + 2x - \frac{3x^2}{2}\right)^5 = \left[\left(1 + \frac{3x}{2}\right)(2 - x)\right]^5
\]

\[
= \left(1 + \frac{3x}{2}\right)^5 (2 - x)^5
\]

\[
= (1 + \frac{15}{2} x + \frac{45}{2} x^2 + ...) (32 - 80x + 80x^2 + ...)
\]

\[
= ... + 80x^2 - 600x^3 + 720x^2 + ...
\]

M1

Coefficient of \(x^2 = 200\)

A1
9 (i) Intersects the y-axis at (0, 2)

\[3 - |2x + 1| = 0\]
\[|2x + 1| = 3\]
\[2x + 1 = 3 \text{ or } 2x + 1 = -3\]
\[x = 1 \text{ or } x = -2\]
Intersects the x-axis at (1, 0) and (-2, 0).

B1  B1

(ii) correct shape and passing through the coordinate axes Coordinates of vertex.

B1  B2

(iii) correct shape passing through (1, 1)

(iv) Number of solutions = 1

B1
(i) 
\[
2 \left( \frac{2 \pi r^3}{3} \right) + \pi r^2 l = \frac{\pi}{6}
\]

or 
\[
\frac{4}{3} \pi r^3 + \pi r^2 l = \frac{\pi}{6}
\]

expressing \( l \) in terms of \( r \)

\[
l = \frac{\frac{\pi}{6} - \frac{4}{3} \pi r^3}{\pi r^2}
\]

\[
= \frac{1}{6r^3} - \frac{4}{3}
\]

\[
A = 2(2\pi r^2) + 2\pi rl
\]

substituting a correct expression for \( l \)

\[
= 4\pi r^2 + 2\pi r \left( \frac{1}{6r^3} - \frac{4}{3} \right)
\]

\[
= 4\pi r^2 + \frac{\pi}{3r} - \frac{8}{3} \pi r^2
\]

\[
= \frac{4}{3} \pi r^2 + \frac{\pi}{3r}
\]

(shown)

(ii) 
\[
\frac{dA}{dr} = \frac{8}{3} \pi r - \frac{\pi}{3r^2}
\]

For min value, \( \frac{dA}{dr} = 0 \).

\[
\frac{8}{3} \pi r - \frac{\pi}{3r^2} = 0
\]

\[
8r^3 - 1 = 0
\]

\[
r = \frac{1}{2}
\]

Minimum value of \( A = \frac{4}{3} \pi \left( \frac{1}{2} \right)^2 + \frac{\pi}{3} \left( \frac{1}{2} \right) \) cm²

\[
= 3.14 \text{ cm}^2
\]
(i) \[ \angle RMA = 90^\circ - 45^\circ = 45^\circ \]
Gradient of \( MR \) = \( \tan 45^\circ = 1 \)
Equation of \( MR \) is \( y - 0 = 1(x - 2) \)
\[ y = x - 2 \]  
\[ M1 \]
\[ A1 \]

(ii) Gradient of \( PQ \) = \(-1\)
Equation of \( PQ \) is \( y - 0 = -1(x - 2) \)
\[ y = -x + 2 \]  
\[ M1 \]
\[ A1 \]

(iii) Coordinates of \( P = (0, 2) \)
Applying mid point formula:
Coordinates of \( Q = (4, -2) \)  
\[ M1 \]
\[ A1 \]

(iv) Let the coordinates of \( R \) be \((k, k-2)\)

\begin{align*}
\text{Area} & = \frac{1}{2} \left| \begin{array}{ccc} 4 & k & 0 \\ 2 & k-2 & 2 \\ -2 & -2 & -2 \\ \end{array} \right| \ = \ 20 \\
& = \frac{1}{2} \left[ 4(k-2) + 2k + 0 \right] - \left[ 8 + 0 - 2k \right] = 40 \\
& = 4k = 56 \\
& = k = 7 \\
\end{align*}
Coordinates of \( R = (7, 5) \)  
\[ A1 \]

\begin{align*}
\text{OR} \quad PQ & = \sqrt{(4-0)^2 + (-2-2)^2} \\
& = \sqrt{32} \\
RM & = \sqrt{(k-2)^2 + (k-2-0)^2} \\
& = \sqrt{2(k-2)^2} \\
& = (k-2)\sqrt{2} \\
\text{Area} & = \frac{1}{2} \times \sqrt{32} \times (k-2)\sqrt{2} = 20 \\
& = k = 7 \\
\end{align*}
Coordinates of \( R = (7, 5) \)  
\[ A1 \]
(i) Draw axes and plot all given points.

\[
\begin{array}{cccccc}
 x & 50 & 100 & 150 & 200 & 250 \\
 y & 3250 & 9000 & 17250 & 28000 & 41250 \\
 \frac{y}{x} & 65 & 90 & 115 & 140 & 165 \\
\end{array}
\]

(inaccurate plot: P0)

Drawing a straight line through all plots

Deduct 1 mark if a suitable scale is not used.

(ii) \( y = x(Ax + B) \)

\[
\frac{y}{x} = Ax + B
\]

\[
A = \frac{165 - 65}{250 - 50}
\]

\[
= 0.5 \pm 0.2
\]

\[
B = 40 \pm 1
\]
(iii) \( y = x^2 \)

\[
\frac{y}{x} = x
\]

Plot \( \frac{y}{x} \) against \( x \) as a straight line accurately.

The \( x \)-value of the point of intersection represents the value where the rectangle becomes a square.
**Answer all questions.**

1. A beaker of water is heated until it reaches a temperature of $X$ °C. It is then allowed to cool. Its temperature, $\theta$ °C, when it was cooling for time $t$ minutes is given by $\theta = 28 + 60e^{-0.5t}$. Find

(i) the value of $X$, [1]  
(ii) the value of $\theta$ when $t = 6$, [1]  
(iii) the value of $t$ when $\theta = \frac{1}{2}X$. [2]  
(iv) Explain, with working, if the water will cool to a temperature of 20 °C. [2]

2. The figure shows a semi-circle centre $O$, with diameter $AB$. Points $D$ and $E$ lie on the semi-circle. $AC$ is a straight line passing through $D$. $BC$ is tangent to the semi-circle at $B$. The tangent to the semi-circle at $B$ meets $AD$ at $C$. The lines $BE$ and $AD$ intersect at $F$.  

Given that $AE = BD$, prove that

(i) triangle $ABD$ is congruent to triangle $BAE$, [3]  
(ii) triangle $ABC$ is similar to triangle $BDC$, [2]  
(iii) $BC^2 = AC \times DC$. [2]

3. Given that $\log_p(a^rb^s) = x$, and $\log_p \frac{a}{\sqrt{b}} = y$, express in terms of $x$ and $y$,

(i) $\log_p a$ and $\log_p b$, [5]  
(ii) $\log_p \left( \frac{1}{ab^2} \right)$. [3]
4 In the diagram, \( AB \) is parallel to \( EC \).

![](image)

Given that \( AE : ED = 1 : \sqrt{2} \) and \( CE = (3 + \sqrt{2}) \text{ cm} \), find in the form \( a\sqrt{2} + b \),

(i) \( \frac{CE}{AB} \),

(ii) \( \frac{\text{area of } \triangle CDE}{\text{area of } \triangle BDA} \),

(iii) the length of \( AB \).

5 A curve has the equation \( y = ax^2 + \frac{b}{x} \), where \( a \) and \( b \) are constants.

(i) Given that the curve has a stationary point at \( (2, 5) \), find the value of \( a \) and of \( b \).

(ii) Determine the nature of the stationary point.

(iii) Explain why the curve is a decreasing function for \( x < 0 \).

6 The roots of the equation \( x^2 - 4x - 8 = 0 \) are \( \alpha \) and \( \beta \).

(i) Given that \( (\alpha + \beta)^2 = -5\alpha - 5\beta \), find the value of \( \alpha + \beta \).

(ii) Find the quadratic equation in \( x \) whose roots are \( \frac{\alpha^2}{2} \) and \( \frac{\beta^2}{2} \).

7 The function \( f(x) = 3x^3 + 2x^2 + ax + b \), where \( a \) and \( b \) are constants, has a factor \( (x^2 - 4) \).

(i) Show that \( a = -12 \) and \( b = -8 \).

(ii) Factorise \( 3x^3 + 2x^2 - 12x - 8 \) completely.

(iii) Hence solve the equation \( -3x^3 + 2x^2 + 12x - 8 = 0 \).
8 (i) Find all the angles between $0^\circ$ and $360^\circ$ inclusive which satisfies the equation 
$$2 \sin \theta - 3 \cos 2\theta - 1 = 0.$$ 

(ii) On the same axes, sketch the graphs of $y = 2 \sin \theta - 1$ and $y = 3 \cos 2\theta$ for $0^\circ \leq \theta \leq 360^\circ$.

(iii) Using your answers to part (i) and (ii), state the range of values of $\theta$ for which $3 \cos 2\theta \geq 2 \sin \theta - 1$ for $0^\circ \leq \theta \leq 360^\circ$.

9 A particle moves in a straight line with a velocity, $v$ m/s, given by $v = -3t^2 + 12t - 13$. The displacement of the particle from a fixed point $O$ after 8 seconds is 400 meters.

Calculate

(i) (a) the displacement of the particle after 5 seconds,

(b) the value of $t$ when the acceleration is zero.

(ii) Explain why the particle will never come to rest. Hence find the maximum velocity of the particle.

(iii) Will the particle ever return to its starting point? Explain your answer.

10 (i) A circle $C_1$ has an equation given by $x^2 + y^2 - 2kx + 2y + 1 = 0$, where $k$ is a positive constant.

Given that $C_1$ has a radius of 2 units, find the value of $k$.

(ii) The centre of a circle $C_2$ lies on the line $y = 2x + 2$. Given that $C_2$ passes through the points $(3, 2)$ and $(0, -1)$, find the equation of $C_2$.

(iii) Calculate the shortest distance from the centre of $C_1$ to the circumference of $C_2$. 

\[55\]
(i) Find \[ \int \frac{1}{\sqrt{2x+14}} \, dx. \] \[ \text{[1]} \]

(ii) Show that \[ \frac{d}{dx} \left[ (x-2)\sqrt{2x+14} \right] = \frac{3x+12}{\sqrt{2x+14}}. \] \[ \text{[3]} \]

The diagram shows part of the curve \[ y = \frac{3x}{\sqrt{2x+14}} + 1 \] intersecting the line \[ y = 1.75 \] and \[ x = p. \]

(iii) (a) Find the value of \( p. \) \[ \text{[2]} \]

(b) Using your results from part (i) and part (ii), find the area bounded by the curve, the line \( x = p, \) and the coordinate axes. \[ \text{[4]} \]

(c) Find the area of the shaded region. \[ \text{[2]} \]
READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Answer all the questions.
2. Write your answers and working on the separate writing paper provided.
3. Write your name, register number and class on each separate sheet of paper that you use and fasten the separate sheets together with the string provided. Do not staple your answer sheets together.
4. Omission of essential steps will result in loss of marks.
5. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

1. The number of marks is given in brackets [ ] at the end of each question or part question.
2. The total number of marks for this paper is 100.
3. The use of an electronic calculator is expected, where appropriate.
4. You are reminded of the need for clear presentation in your answers.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial expansion

\[
(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots((n-r+1))}{r!}
\]

2. TRIGONOMETRY

\[
\sin^2 A + \cos^2 A = 1
\]

\[
\sec^2 A = 1 + \tan^2 A
\]

\[
cosec^2 A = 1 + \cot^2 A
\]

\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
\sin 2A = 2 \sin A \cos A
\]

\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]

\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

\[
\sin A + \sin B = 2 \sin \frac{1}{2} (A + B) \cos \frac{1}{2} (A - B)
\]

\[
\sin A - \sin B = 2 \cos \frac{1}{2} (A + B) \sin \frac{1}{2} (A - B)
\]

\[
\cos A + \cos B = 2 \cos \frac{1}{2} (A + B) \cos \frac{1}{2} (A - B)
\]

\[
\cos A - \cos B = -2 \sin \frac{1}{2} (A + B) \sin \frac{1}{2} (A - B)
\]

Formulae for \( \triangle ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2} ab \sin C
\]
1. It is given that \( f(x) = 2x^3 + 9x^2 - 2x + 2 \).

(i) Find the remainder when \( f(x) \) is divided by \( x^2 + 4x - 3 \). [2]

(ii) Hence solve the equation \( f(x) - 5 = 0 \), leaving your answers in exact form. [4]

2. Write down and simplify the first four terms in the expansion of \( \left( 3x - \frac{p}{x^2} \right)^5 \), in descending powers of \( x \), where \( p \) is a non-zero constant. [3]

Given that the coefficient of \( \frac{1}{x} \) in the expansion of \( \left( 2x^3 - 1 \right) \left( 3x - \frac{p}{x^2} \right)^5 \) is 90 \( p^2 \), find the value of \( p \). [3]

3. (i) Solve the equation \( |10 - 5x| = 10 + 8x - 2x^2 \). [4]

(ii) Sketch, on a single diagram, the graphs of \( 2y = |10 - 5x| \) and \( y = 5 + 4x - x^2 \), indicating clearly the \( x \)- and \( y \)-intercepts and the turning points (if any). [4]

(iii) Hence deduce the range of values of \( x \) if \( 10 + 8x - 2x^2 \leq |10 - 5x| \). [2]

4. (a) The function \( f \) is defined by \( f : x \mapsto (x - 1)(3x - 2) \), \( x \in \mathbb{R} \).

(i) Sketch the graph of \( y = f(x) \), showing clearly the \( x \)- and \( y \)-intercepts and the coordinates of the turning point. [4]

(ii) State the range of \( f \). [1]

(iii) The function \( g \) is defined by \( g : x \mapsto (x - 1)(3x - 2) \), \( x \leq k, k \in \mathbb{Z} \).

State the maximum value of \( k \) such that \( g^{-1} \) exists. Hence find an expression for \( g^{-1}(x) \). [3]

(b) The graph of \( y = h(x) \) undergoes 2 successive transformations

I A translation of \( \frac{3}{4} \) in the positive \( x \)-direction,

II A scaling with a scale factor of 2 along the \( y \)-axis.

The resulting graph is \( y = 6 \cos x \). Find \( h(x) \). [2]
5 (a) Without using a calculator, evaluate \(6^e\), given that 
\[3^{2x+1} \times 2^{x+2} = 3^{4x+4}.
\]

(b) Solve the simultaneous equations 
\[e^2e^x = e^{4y},
\log_a (x+2) = 1 + \log_2 y.
\] [6]

6 (i) Express \(\frac{4x}{2x+1}\) in the form \(a + \frac{b}{2x+1}\), where \(a\) and \(b\) are integers. [2]

(ii) Differentiate \(2x \ln (2x+1)\) with respect to \(x\). [2]

(iii) Using the results in part (i) and part (ii), determine \(\int \ln (2x+1) \, dx\). [4]

7

The diagram shows parts of the curve \(y = 3 - \sqrt{2x}\) and the line \(2y = x\). The curve and the line intersect at the point \(A\).

(i) Show that the area bounded by the curve \(y = 3 - \sqrt{2x}\), the \(x\)-axis and the lines \(x = 4.5\) and \(x = 9\) can be expressed as \((a\sqrt{2} + b)\) square units, where \(a\) and \(b\) are constants. [4]

(ii) Find the coordinates of point \(A\). [2]

(iii) Find the area bounded by the straight line \(2y = x\), the curve \(y = 3 - \sqrt{2x}\) and the \(y\)-axis. [3]
8 A circle $C_1$ has equation given by $(x-1)^2 + y^2 + 6y - 16 = 0$.

(i) Find the radius and the coordinates of the centre of $C_1$. [3]

(ii) Find the equation of the tangent to the circle at the point $P(4, -7)$. [3]

The point $Q$ is such that $PQ$ is the diameter of the circle.

(iii) Find the coordinates of $Q$. [2]

A new circle $C_2$ passes through $P$, $Q$ and $R$, where $R$ is a point outside the circle $C_1$ such that angle $PRQ = 45^\circ$.

(iv) Explain briefly if it is possible for the centre of $C_2$ to lie on the circumference of $C_1$. [1]

9 Solutions to this question by accurate drawing will not be accepted.

The diagram, not drawn to scale, shows a trapezium $ABCD$ in which $AB$ is parallel to $DC$ and angle $BAD = 90^\circ$. The vertices of the trapezium are at the points $A(0, 2)$, $B(2a + 4, 3a)$, $C(b, 0)$ and $D(2, -2)$.

(i) Given that the length of $AB$ is $4\sqrt{5}$ units, find the value of $a$, where $a > 0$. [3]

(ii) Find the equation of $AB$. [2]

(iii) Find the value of $b$. [2]

(iv) Find the perpendicular bisector of $AB$.

Hence or otherwise, show that $C$ lies on the perpendicular bisector of $AB$. [3]

(v) Find the area of trapezium $ABCD$. [2]
10 (a) (i) Show that \[
\frac{(\cos \theta + \sin \theta)^2}{\sec^2 \theta + 2 \tan \theta} = \cos^2 \theta .
\] [2]

(ii) Hence find all values of \( \theta \), where \( 0 < \theta < 2\pi \), which satisfy the equation \[
\frac{\sec^2 \theta + 2 \tan \theta}{(\cos \theta + \sin \theta)^2} = 2(2 + \tan \theta).\] [4]

(b) The diagram shows a rectangle \( MNOQ \) embedded in the triangle \( PQR \). It is given that \( PR = 5 \text{ cm} \), \( \angle QRP = \theta \) and area of rectangle \( MNOQ \) is \( \frac{25}{4} \cos^2 \theta \), where \( 0^\circ < \theta < 90^\circ \).

(i) Show that the shaded area, \( A \), is given by \[ A = \frac{25}{8} (2 \sin 2\theta - \cos 2\theta) - \frac{25}{8}. \] [3]

(ii) Hence, show that \( A \) can be expressed in the form \[ A = R \sin (2\theta - \alpha) - \frac{25}{8} \] where \( R > 0 \) and \( 0^\circ < \alpha < 90^\circ \). [2]

(iii) State the exact maximum value of \( A \). [1]
The diagram below shows a solid toy which consists of a cone fixed to the end of a right circular cylinder. The cone has a radius of $4x$ cm and a height of $3x$ cm. The cylinder has a radius of $4x$ cm and a height of $h$ cm. It is given that the total volume of the toy is $960\pi$ cm$^3$.

(i) Show that $h = \frac{60}{x} - x$. [2]

(ii) Show that the total surface area, $A$ cm$^2$, of the toy is given by $A = \frac{480\pi}{x} + 28\pi x^2$ cm$^2$. [3]

(iii) Find, using differentiation, the values of $h$ and $x$ which give the minimum surface area of this toy. You will need to justify that the surface area is a minimum for the values of $h$ and $x$ obtained. [5]

[The area of the curved surface area of a cone of radius $r$ and slant height $l$ is $\pi rl$.]
[The volume of a cone $= \frac{1}{3} \times$ base area $\times$ height.]

End of Paper
1(i) 
\[
\frac{2x+1}{x^2+4x-3} \div \frac{2x^3+9x^2-2x+2}{2x^3+8x^2-6x} \div \frac{x^2+4x+2}{x^2+4x-3} \div \frac{5}{5}
\]

Remainder = 5

**Alternatively**
Let \( f(x) = (x^2 + 4x - 3)(ax + b) + c \)

By comparing coefficients, \( f(x) = (x^2 + 4x - 3)(2x + 1) + 5 \)

Remainder = 5

1(ii) 
From \( (x^2 + 4x - 3)(2x + 1) = 0 \)

\[\Rightarrow (x^2 + 4x - 3) = 0 \text{ or } (2x + 1) = 0\]

\[x = -\frac{1}{2} \text{ or } x = -\frac{4 \pm \sqrt{4^2 - 4(1)(-3)}}{2(1)}\]

\[x = -4 \pm \frac{28}{2}\]

\[x = -2 + \sqrt{7} \]

2
\[
\left(3x - \frac{p^2}{x^2}\right)^3
\]

\[= (3x)^3 + 3(3x)^2 \left(-\frac{p}{x^2}\right) + 3(3x) \left(-\frac{p}{x^2}\right)^2 + \frac{p^3}{x^2} + ...
\]

\[= 243x^3 + 91x^4 \left(-\frac{p}{x^2}\right) + 10(27x^4) \left(-\frac{p}{x^2}\right)^2 + \frac{p^3}{x^2} + ...
\]

\[= 243x^3 - 405px^4 + \frac{270p^2}{x} - \frac{90p^3}{x^2} + ...
\]

\[\frac{2x^3 - 1}{3x - \frac{p^2}{x^2}} = \frac{2x^3 - 1}{243x^3 - 405px^4 + \frac{270p^2}{x} - \frac{90p^3}{x^2} + ...}
\]

\[= \frac{1}{x^3} \left(2x^3 - 1\right) \frac{270p^2}{x} - \frac{180p^3}{x^4} + ... \]

Thus \(-270p^2 - 180p^3 = 90p^2\)

\[\Rightarrow 90p^2(4 + 2p) = 0 \Rightarrow p = 0 (\text{NA}) \text{ or } p = -2 \]
3(i) From $10 + 8x - 2x^2 = |10 - 5x|
\Rightarrow 10 + 8x - 2x^2 = -(10 - 5x) \quad \text{OR} \quad 10 + 8x - 2x^2 = 10 - 5x
\Rightarrow 2x^2 - 3x - 20 = 0 \quad \text{OR} \quad 2x^2 - 13x = 0
\Rightarrow (2x + 5)(x - 4) = 0 \quad \text{OR} \quad x(2x - 13) = 0
\Rightarrow x = -2.5, 4 \quad \text{OR} \quad x = 0, 6.5
(Reject -2.5) \quad \text{(Reject 6.5)}

3(ii) The range is $x \leq 0$ or $x \geq 4$.

4(a) 
(i) 
(ii) Maximum value of $k = 0$.

4(a) 
(iii) 
\[ y = (x - 1)(3x - 2) = 3x^2 - 5x + 2 \]
\Rightarrow \[ 3x^2 - 5x + 2 = 0 \]
\Rightarrow \[ x = \frac{5 \pm \sqrt{25 + 12(2 - y)}}{6} \]
Since $x \leq 0$,
\[ g^{-1}(x) = \frac{5 - \sqrt{1 + 12x}}{6} \]

4(b) Before II, \[ y = 3\cos x \]
Before I, \[ y = 3\cos(x + \frac{\pi}{4}) \]

5(a) 
\[ \frac{3^2 \times 3^3 	imes 2^2 \times 2^3}{3^3 	imes 2^2} = \frac{3^4}{3^2} \times 3^3 \]
\[ 6^x = \frac{3}{32} \]

5(b) 
\[ e^{2x} = e^{4y} \]
\[ 2 + x = 4y \]
\[ x = 4y - 2 \ldots \ldots (I) \]
\[
\log_2(x+2) = 1 + \log_2 y \\
\log_2 (x+2) = \log_2 2 + \log_2 y \\
\frac{1}{2} \log_2 (x+2) = \log_2 2y \\
(x+2)^{\frac{1}{2}} = 2y \\
x + 2 = 4y^2 \quad \text{(2)} \\
\text{Substitute (1) into (2),} \\
4y - 2 + 2 = 4y^2 \\
4y(y-1) = 0 \\
\text{Since } y \neq 0 \text{ as it would make } \log_2 y \text{ undefined.} \\
\therefore y = 1 \quad \therefore x = 2
\]

6. (i) \[
\frac{4x}{2x+1} = \frac{2(2x+1)-2}{2x+1} = 2 - \frac{2}{2x+1}
\]

6. (ii) \[
d\left[2x \ln(2x+1)\right] = 2 \ln(2x+1) + 2x \left(\frac{2}{2x+1}\right) \\
= 2 \ln(2x+1) + \frac{4x}{2x+1}
\]

6. (iii) Integrate both sides in (ii),
\[
\int \frac{d}{dx} \left[2x \ln(2x+1)\right] \, dx = \int \left[2 \ln(2x+1) + \frac{4x}{2x+1}\right] \, dx \\
2x \ln(2x+1) + c = \int 2 \ln(2x+1) \, dx + \int \frac{4x}{2x+1} \, dx \\
= 2 \left[ \ln(2x+1) \right] + 2 \int \frac{2}{2x+1} \, dx \\
2x \ln(2x+1) + c = 2 \ln(2x+1) + \int 2 \ln(2x+1) \, dx \\
2x \ln(2x+1) - 2x + \ln |2x+1| + c = \int 2 \ln(2x+1) \, dx \\
\therefore \int \ln(2x+1) \, dx = x \ln |2x+1| - x + \frac{1}{2} \ln |2x+1| + c
\]

7. (i) Area of region = \[
\int_{x_1}^{x_2} \left(3 - \sqrt{2x}\right) \, dx
\]
\[
\begin{align*}
&= \left[ \frac{1}{3} (2x)^\frac{3}{2} - 3x \right]_{45} \\
&= \left( \left( \frac{1}{3} (18)^\frac{3}{2} - 27 \right) - \left( \frac{1}{3} (9)^\frac{3}{2} - \frac{27}{2} \right) \right) \\
&= \left( \left( \frac{1}{3} (3\sqrt{2})^3 - 27 \right) - \left( -4\frac{1}{2} \right) \right) \\
&= \left[ \frac{1}{3} (54\sqrt{2}) - 22\frac{1}{2} \right] \\
&= 18\sqrt{2} - 22\frac{1}{2}
\end{align*}
\]

7 (ii) Substitute $2y = x$ into:

\[
\frac{x}{2} = 3 - \sqrt{2x}
\]

$\Rightarrow 6 - x = 2\sqrt{2x}$

$\Rightarrow 36 - 12x + x^2 = 8x$

$\Rightarrow x^2 - 20x + 36 = 0$

$\Rightarrow (x-2)(x-18) = 0$

$\Rightarrow x = 2, 18$ (N.A)

$\therefore y = 1$

$A = (2, 1)$

7 (iii) Area required = \[\int_0^2 3 - \sqrt{2x} \, dx - \frac{1}{2} \times 2 \times 1\]

\[
= \left[ 3x - \frac{1}{3} (2x)^\frac{3}{2} \right]_0^2 - 1
\]

\[
= \left[ (6 - \frac{1}{3} (4)^\frac{3}{2}) - 0 \right] - 1
\]

\[
= \frac{8}{3} = 2\frac{1}{3}
\]

Alternatively

\[
\begin{align*}
&= \frac{1}{2} \times 2 \times 1 + \int_{\frac{1}{2}}^1 (3 - y)^2 \, dy \\
&= 1 + \frac{1}{6} \left[ - (3 - y)^3 \right]_0^1 \\
&= 1 - \frac{1}{6} (0 - 8) = 2\frac{1}{3}
\end{align*}
\]

8(i) $(x-1)^2 + y^2 + 6y - 16 = 0$
\[(x-1)^2 + (y+3)^2 - 9 - 16 = 0 \]
\[(x-1)^2 + (y+3)^2 = 25 \]
Coordinates of the centre of the circle are \((1, -3)\)
and the radius is 5 units.

**8(ii)**

Gradient of \(CP = \frac{-7 + 3}{4 - 1} = \frac{-4}{3} \)
Gradient of tangent at \(P = \frac{2}{4} \)
Equation of tangent at \(P\) is:
\[y - (-7) = \frac{3}{4}(x-4)\]
\[y = \frac{3}{4}x - 10 \]

**Alternatively,**

Sub \(x = 4, y = -7\) into \(y = mx + c:\)
\[-7 = 3 + c \Rightarrow c = -10 \]
Thus, equation is \(y = \frac{3}{4}x - 10\)

**(8(iii))**

If \(PQ\) is the diameter, then \(C\) is the midpoint of \(PQ\).
Therefore \(\frac{4 + x}{2} = 1\) and \(\frac{-7 + y}{2} = -3\)
\[\Rightarrow x = -2\] and \(y = 1\).
Thus coordinates of \(Q\) is \((-2, 1)\).

**(8(iv))**

Let \(X\) be the centre of \(C_2\). If \(X\) lies on the circumference of \(C_1\),
then \(\angle PXQ = 90^\circ\). But \(\angle PXQ = 2\angle PRQ\), thus \(\angle PRQ = 45^\circ\).
Therefore, the centre of \(C_1\) can lie on the circumference of \(C_4\).

**(9(i))**

\[\sqrt{(2a + 4 - 0)^2 + (3a - 2)^2} = 4\sqrt{5} \]
\[4a^2 + 16a + 16 + 9a^2 - 12a + 4 = 16(5)\]
\[13a^2 + 4a - 60 = 0 \]
\[(13a + 30)(a - 2) = 0 \]
\[a = \frac{-30}{13} \text{ or } a = 2 \]
(reject : \(a > 0\))

**(9(ii))**

Gradient of \(AD = \frac{2 - (-2)}{0 - 2} = -2\). Hence gradient of \(AB = \frac{1}{2}\).
\[
\begin{align*}
\therefore \text{ Equation of } AB & \text{ is: } \ y = \frac{1}{2}x + 2
\end{align*}
\]

**(9(iii))**

Given \(C = (b, 0)\)
\[
\begin{align*}
0 + 2 &= \frac{1}{2} \Rightarrow b = 6
\end{align*}
\]

**(9(iv))**

Midpoint of \(AB, M (4, 4)\)

Equation of perpendicular bisector of \(AB\) is: \(y - 4 = -2(x - 4)\)
\[\Rightarrow y = -2x + 12 \]
Sub \(x = 6, y = -2(6) + 12 = 0\)
As the point satisfies the equation, point $C$ lies on the perpendicular bisector.

<table>
<thead>
<tr>
<th>9(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of trapezium $ABCD = \frac{1}{2} \begin{bmatrix} 0 &amp; 2 &amp; 6 &amp; 8 &amp; 0 \ 2 &amp; -2 &amp; 0 &amp; 6 &amp; 2 \end{bmatrix}</td>
</tr>
<tr>
<td>$= \frac{1}{2} (36 + 16 - (-12) - 4)$</td>
</tr>
<tr>
<td>$= \frac{1}{2} (60)$</td>
</tr>
<tr>
<td>$= 30$ square units</td>
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</tbody>
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<thead>
<tr>
<th>10</th>
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<tbody>
<tr>
<td>(a)</td>
</tr>
<tr>
<td>(i)</td>
</tr>
<tr>
<td>$\text{LHS} = \frac{(\cos \theta + \sin \theta)^2}{1 + \tan^2 \theta + 2 \tan \theta}$</td>
</tr>
<tr>
<td>$= \frac{\cos \theta (1 + \tan \theta)^2}{(1 + \tan \theta)^2}$</td>
</tr>
<tr>
<td>$= \cos^2 \theta = \text{RHS}$</td>
</tr>
</tbody>
</table>

**Alternatively**

| $\text{LHS} = \frac{\cos^2 \theta + \sin^2 \theta + 2 \cos \theta \sin \theta}{1 + \frac{\sin^2 \theta}{\cos^2 \theta} + 2 \frac{\sin \theta}{\cos \theta}}$ |
| $= \frac{\cos^2 \theta + \sin^2 \theta + 2 \cos \theta \sin \theta}{\cos^2 \theta + \sin^2 \theta + 2 \sin \theta \cos \theta}$ |
| $= \cos^2 \theta = \text{RHS}$ |

<table>
<thead>
<tr>
<th>10</th>
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<tbody>
<tr>
<td>(a)</td>
</tr>
<tr>
<td>(ii)</td>
</tr>
<tr>
<td>$\frac{1}{\cos^2 \theta} = 2 \tan \theta + 4$</td>
</tr>
<tr>
<td>$\sec^2 \theta = 2 \tan \theta + 4$</td>
</tr>
<tr>
<td>$\tan^2 \theta - 2 \tan \theta - 3 = 0$</td>
</tr>
<tr>
<td>$(\tan \theta - 3) (\tan \theta + 1) = 0$</td>
</tr>
<tr>
<td>$\alpha = 1.249$ or $\alpha = \frac{\pi}{4}$</td>
</tr>
<tr>
<td>$\theta = 1.25$, $4.39$ or $\theta = \frac{3\pi}{4}$, $\frac{7\pi}{4}$ (reject both ans)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>10</th>
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</thead>
<tbody>
<tr>
<td>(b)</td>
</tr>
<tr>
<td>(i)</td>
</tr>
<tr>
<td>Area of shaded region, $A$</td>
</tr>
<tr>
<td>$= \frac{1}{2} (5 \cos \theta)(5 \sin \theta) - \frac{25}{4} \cos^2 \theta$</td>
</tr>
<tr>
<td>$= \frac{25}{4} \sin \theta \cos \theta - \frac{25}{4} \cos^2 \theta$</td>
</tr>
<tr>
<td>$= \frac{25}{4} \sin 2\theta - \frac{25}{4} \left( \frac{\cos 2\theta + 1}{2} \right)$</td>
</tr>
<tr>
<td>$= \frac{50 \sin 2\theta - 25 \cos 2\theta - 25}{8}$</td>
</tr>
<tr>
<td>$= \frac{25}{8} (2 \sin 2\theta - \cos 2\theta) - \frac{25}{8}$</td>
</tr>
</tbody>
</table>
Let \( 2 \sin 2\theta - \cos 2\theta = r \sin (2\theta - \alpha) \)
\[
\therefore \quad r = \sqrt{2^2 + 1^2} = \sqrt{5}
\]
\[
\tan \alpha = \frac{1}{2} \implies \alpha = 26.5651^\circ
\]
\[
A = \frac{25\sqrt{5}}{8} \sin (2\theta - 26.6^\circ) - \frac{25}{8}
\]
Thus \( R = \frac{25\sqrt{5}}{8} \) and \( \alpha = 26.6^\circ \)

| Maximum \( A = \frac{25\sqrt{5} - 25}{8} \) cm² |

\[
\frac{1}{3} \pi (4x)^2 (3x) + \pi (4x)^3 h = 960\pi
\]
\[
16\pi x^2 h = 960\pi - 16x^3 \pi
\]
\[
\therefore \quad h = \frac{60}{x^3 - x} \quad \text{(proven)}
\]

\[
A = \pi (5x)(4x) + 2\pi (4x)h + \pi (4x)^3 \quad \text{(slant height of cone = 5x)}
\]
\[
= 20\pi x^2 + \frac{480\pi}{x} - 8\pi x^2 + 16\pi x^2
\]
\[
= \frac{480\pi}{x} + 28\pi x^2 \quad \text{(proven)}
\]

\[
\frac{dA}{dx} = -\frac{480\pi}{x^3} + 56\pi x
\]
\[
-480\pi + 56\pi x^3 = 0
\]
\[
x^3 = \frac{60}{7}
\]
\[
x = \sqrt[3]{\frac{60}{7}} = 2.05 \text{ cm (2.0465)}
\]
\[
h = 12.3 \text{ cm (12.28)}
\]
\[
\frac{d^2 A}{dx^2} = \frac{960\pi}{x^3} + 56\pi
\]
\[
\left. \frac{d^2 A}{dx^2} \right|_{x=2.0465} = \frac{960\pi}{(2.0465)^3} + 56\pi > 0
\]
\[
\therefore \quad A \text{ is a minimum.}
\]
READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

My Target is:
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial Expansion

\[
(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \cdots + \binom{n}{r} a^{n-r}b^r + \cdots + b^n,
\]

where \( n \) is a positive integer and

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1) \ldots (n-r+1)}{r!}
\]

2. TRIGONOMETRY

Identities

\[
\sin^2 A + \cos^2 A = 1
\]

\[
\sec^2 A = 1 + \tan^2 A
\]

\[
\csc^2 A = 1 + \cot^2 A
\]

\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
\sin 2A = 2 \sin A \cos A
\]

\[
\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A
\]

\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \Delta ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2} bc \sin A
\]
1. The acute angle \(A\) and obtuse angle \(B\) are such that \(\tan A = \frac{1}{2}\) and \(4 \cos(A + B) = 3 \sin(A - B)\). Without using a calculator, find the exact value of \(\cos B\). [5]

2. (i) Write down the first three terms in the expansion, in ascending powers of \(x\), of \((1 + x)^n\), where \(n\) is a positive integer greater than 2. [1]

(ii) The coefficient of \(x^2\) in the expansion, in ascending powers of \(x\), of \((1 + x)^n(2 - 3x)^4\), where \(n\) is a positive integer greater than 2, is 456. Find the coefficient of \(x\). [4]

3. The volume of a cube, \(V\) cm\(^3\), is increasing at a constant rate of 5 cm\(^3\) per second. Find the volume of the cube at the instant when the length of the side of the cube, \(x\) cm, is increasing at 0.5 cm per second. [5]

4. Given that \(\frac{2x^3 - 1}{x^3 - x^2} = a + \frac{bx^2 + c}{x^3 - x^2}\), where \(a\), \(b\) and \(c\) are integers.

(i) Find the value of \(a\), of \(b\) and of \(c\). [2]

(ii) Using the values of \(b\) and \(c\) found in part (i), express \(\frac{bx^2 + c}{x^3 - x^2}\) as the sum of 3 partial fractions. [4]

5. The diagram shows part of a straight line graph drawn to represent the equation \(y + \frac{k}{\sqrt{x}} = 5\sqrt{x}\), where \(k\) is a constant. Given that the line passes through the point \((0, -8)\) and makes an angle \(\theta\) with the \(x\)-axis at point \(R\), where \(0^\circ < \theta < 90^\circ\), find

(i) the value of \(k\) and of \(\theta\), [4]

(ii) the coordinates of \(R\). [2]
6. (i) Prove that \( \cot \left( \frac{\pi}{4} - \theta \right) = \frac{\cot \theta + 1}{\cot \theta - 1} \)

(ii) Hence without using a calculator, show that \( \cot \frac{\pi}{12} = 2 + \sqrt{3} \). 

7. 

The diagram shows a triangle \( ABC \) in which the coordinates of the points \( A \) and \( C \) are \((-1, 3)\) and \((5, 7)\) respectively.

Given that \( AB = BC \),

(i) find the coordinates of \( B \). 

\( D \) is a point on the perpendicular bisector of \( AC \).

(ii) Find the equation of \( BD \). 

8. (i) Show that \( \frac{d}{dx} \left( \ln(\cos 4x) \right) = -4 \tan 4x \). 

It is given that \( \frac{dy}{dx} = \frac{x - \tan 4x}{2 - \frac{\tan 4x}{8}} \) and \( \frac{d^2y}{dx^2} = k \tan^2 4x \).

(ii) Find the value of \( k \). 

(iii) Using the result in part (i), find \( y \) given that \( x = 0 \) when \( y = 1 \). 

9. The equation of a curve is \( y = (k - 7)x^2 - 8x + k \), where \( k \) is a constant.

(i) Find the set of values of \( k \) for which the curve lies completely above the line \( y = 1 \). 

(ii) In the case where \( k = 8 \), find the set of values of \( c \) for which the line \( y = 2x - c \) intersects the curve at two distinct points.
The diagram shows a rectangle $OAPB$ inscribed in a quadrant of a circle of radius 5 cm.
The length of $OA$ is $x$ cm.

(i) Show that the area of the rectangle, $A$ cm$^2$, is given by $A = x\sqrt{25 - x^2}$. [2]

(ii) Given that $x$ can vary, find the value of $x$ for which $A$ has a stationary value. [4]

(iii) Determine whether this stationary value of $A$ is a maximum or a minimum. Hence find this value of $A$. [2]

11 The equation of a curve is $y = -\ln(3 - ax)$, where $a$ is a constant.

(i) Find the value of $a$ if the gradient of the curve at $y = -\ln 5$ is 2. [4]

(ii) Find the value of $a$ if the normal to the curve at $x = 1$ is parallel to the line $2x - y = 5$. [2]

(iii) In the case where $a = 4$, find the coordinates of the point on the curve where the equation of the tangent to the curve is $y = 4x - 2$. [2]

12 (i) Find the turning point of the curve $y = x^2 - 4x$. [2]

(ii) Sketch the graph of $y = |x^2 - 4x|$, indicating clearly the coordinates of the turning point and of the points where the graph meets the $x$-axis. [3]

(iii) Using your graph, find the number of solutions of the equation $|x^2 - 4x| = 2 - mx$ when

(a) $m = -2$, [4]

(b) $m = \frac{1}{2}$,

End of Paper
1. \[ 4 \cos(A + B) = 3 \sin(A - B) \]
\[ 4 \left( \cos A \cos B - \sin A \sin B \right) = 3 \left( \sin A \cos B - \cos A \sin B \right) \]
\[ 4 \left( \frac{2}{\sqrt{5}} \cos B - \frac{1}{\sqrt{5}} \right) \sin B = 3 \left( \frac{1}{\sqrt{5}} \cos B - \frac{2}{\sqrt{5}} \right) \sin B \]
\[ 5 \cos B = -2 \sin B \]
\[ \tan B = \frac{5}{2} \]
\[ \cos B = -\frac{2}{\sqrt{29}} \]
\[ = -\frac{2\sqrt{29}}{29} \]

OR

\[ 4 \cos(A + B) = 3 \sin(A - B) \]
\[ 4 \left( \cos A \cos B - \sin A \sin B \right) = 3 \left( \sin A \cos B - \cos A \sin B \right) \]
\[ 4 \cos A \cos B - 4 \sin A \sin B = 3 \sin A \cos B - 3 \cos A \sin B \]
\[ \frac{\cos A \cos B}{\cos A \cos B} = \frac{3 \sin A \cos B - 3 \cos A \sin B}{\cos A \cos B} \]
\[ 4 - 4 \tan A \tan B = 3 \tan A - 3 \tan B \]
\[ 4 - 4 \left( \frac{1}{2} \right) \tan B = 3 \left( \frac{1}{2} \right) - 3 \tan B \]
\[ \tan B = \frac{5}{2} \]
\[ \cos B = -\frac{2}{\sqrt{29}} \]
\[ = -\frac{2\sqrt{29}}{29} \]
2. (i) \((1 + x)^n = 1 + nx + \frac{n(n-1)}{2} x^2 + ...\)

2. (ii) \((2 - 3x)^4 = (2)^4 + 4(2)^3(-3x) + 6(2)^2(-3x)^2 + ...\)
   \[= 16 - 96x + 216x^2 + ...\]

\[(1 + x)^n (2 - 3x)^4 = \left[1 + nx + \frac{n(n-1)}{2} x^2 + ...\right]\left[16 - 96x + 216x^2 + ...\right]\]
   \[= -96x + 16nx + (8n^2 - 104n + 216)x^2 + ...\]

Coefficient of \(x^2\):
\[8n^2 - 104n + 216 = 456\]
\[n^2 - 13n - 30 = 0\]
\[(n-15)(n+2) = 0\]
\[n = 15 \quad \text{or} \quad n = -2 \text{ (N.A.)}\]

Coefficient of \(x = -96 + 16(15)\)
\[= 144\]

3. \(V = x^3\)
\[\frac{dV}{dx} = 3x^2\]

\[\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}\]
\[s = 3x^2 \times 0.5\]
\[x = \sqrt{\frac{10}{3}}\]

Volume of cube = \(\left(\frac{10}{\sqrt{3}}\right)^3\)
\[= 6.085806\]
\[= 6.09 \text{ cm}^3 \text{ (3 s.f.)}\]
4. (i) \[
\frac{2x^3 - 1}{x^3 - x^2} = a + \frac{bx^2 + c}{x^3 - x^2}
\]

\[
2x^3 - 1 = ax^3 - ax^2 + bx^2 + c
\]

\[
a = 2
b = a = 2
\]

\[
c = -1
\]

4. (ii) \[
\frac{2x^2 - 1}{x^3 - x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x - 1}
\]

\[
2x^2 - 1 = Ax(x - 1) + B(x - 1) + Cx^2
\]

Let \(x = 1\), \(C = 1\)

Let \(x = 0\), \(B = 1\)

Coefficient of \(x^2\): \(A = 1\)

\[
\frac{2x^2 - 1}{x^3 - x^2} = \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x - 1}
\]

5. (i) \[
y\sqrt{x} = 5x - k
\]

\[k = 8\]

\[
\tan \theta = 5
\]

\[
\theta = 78.7^\circ
\]

5. (ii) when \(y\sqrt{x} = 0\),

\[
5x - 8 = 0
\]

\[
x = \frac{8}{5}
\]

\[
R(\frac{3}{5}, 0)
\]
6.(i) \[ \cot \left( \frac{\pi}{4} - \theta \right) = \frac{1}{\tan \left( \frac{\pi}{4} - \theta \right)} \]
\[ = \frac{1 + \tan \frac{\pi}{4} \tan \theta}{\tan \frac{\pi}{4} - \tan \theta} \]
\[ = \frac{1 + (1) \tan \theta}{1 - \tan \theta} \]
\[ = \frac{\frac{1}{\tan \theta} + 1}{\frac{1}{\tan \theta} - 1} \]
\[ = \frac{\cot \theta + 1}{\cot \theta - 1} \]

6.(ii) \[ \cot \frac{\pi}{12} = \cot \left( \frac{\pi}{4} - \frac{\pi}{6} \right) \]
\[ = \frac{\cot \frac{\pi}{4} + 1}{\cot \frac{\pi}{4} - 1} \]
\[ = \frac{\frac{1}{\tan \frac{\pi}{4}} + 1}{\frac{1}{\tan \frac{\pi}{4}} - 1} \]
\[ = \frac{\tan \frac{\pi}{6} + 1}{\tan \frac{\pi}{6} - 1} \]
\[ = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \]
\[ = \frac{3 + 2 \sqrt{3} + 1}{3 - 1} \]
\[ = 2 + \sqrt{3} \]
7. (i) Let $B$ be $(k, 0)$

\[
AB = BC
\]

\[
\sqrt{(k + 1)^2 + (-3)^2} = \sqrt{(k - 5)^2 + (-7)^2}
\]

\[
k^2 + 2k + 10 = k^2 - 10k + 74
\]

\[
k = \frac{16}{3}
\]

$B(\frac{16}{3}, 0)$

7. (ii) Gradient of $AC = \frac{7 - 3}{5 + 1} = \frac{2}{3}$

Gradient of $BD = -\frac{3}{2}$

Equation of $BD$ is $y - 0 = \frac{3}{2}(x - \frac{16}{3})$

\[
y = \frac{3}{2}x + 8 \quad \text{OR} \quad 2y = -3x + 16
\]
8. (i) \[ \frac{d}{dx} \left[ \ln(\cos 4x) \right] = \frac{1}{\cos 4x} (-4 \sin 4x) \]
\[ = -4 \tan 4x \]

8. (ii) \[ \frac{d^2y}{dx^2} = \frac{1}{2} \left( \frac{1}{8} (4 \sec^2 4x) \right) \]
\[ = \frac{1}{2} \left( \frac{1}{2} (1 + \tan^2 4x) \right) \]
\[ = \frac{1}{2} \tan^2 4x \]
\[ k = \frac{1}{2} \]

8. (iii) \[ \int -4 \tan 4x \, dx = \ln(\cos 4x) + C \]
\[ y = \int \left( \frac{x}{2} - \frac{\tan 4x}{8} \right) \, dx \]
\[ = \frac{1}{4} x^2 + \frac{1}{8} \int -4 \tan 4x \, dx \]
\[ y = \frac{1}{4} x^2 + \frac{1}{32} \ln(\cos 4x) + C \]

When \( x = 0, y = 1, \)
\[ 1 = \frac{1}{32} \ln(\cos 0) + C \]
\[ C = 1 \]
\[ y = \frac{1}{4} x^2 + \frac{1}{32} \ln(\cos 4x) + 1 \]
9.(i) \((k - 7)x^2 - 8x + k = 1\)
\(\Rightarrow (k - 7)x^2 - 8x + (k - 1) = 0\)

\[
b^2 - 4ac < 0
\]
\[
(-8)^2 - 4(k - 7)(k - 1) < 0
\]
\[
64 - 4(k^2 - 8k + 7) < 0
\]
\[
64 - 4k^2 + 32k - 28 < 0
\]
\[
4k^2 - 32k + 36 > 0
\]
\[
k^2 - 8k - 9 > 0
\]
\[
(k - 9)(k + 1) > 0
\]
\[
k < -1 \text{ OR } k > 9
\]

And \(k - 7 > 0\)
\(k > 7\)

Since \(k > 7\) AND \(k > 9\), \(\therefore k > 9\).

9.(ii) \(y = x^2 - 8x + 8\)
\[x^2 - 8x + 8 = 2x - c\]
\[x^2 - 10x + 8 + c = 0\]

\[
b^2 - 4ac > 0
\]
\[
(-10)^2 - 4(1)(8 + c) > 0
\]
\[
c < 17
\]
10.(i) \[ AP^2 = 5^2 - x^2 \]
\[ AP = \sqrt{25 - x^2} \text{ cm} \]
\[ A = OA \times AP \]
\[ A = x\sqrt{25 - x^2} \]

10.(ii) \[ \frac{dA}{dx} = x \cdot \frac{1}{2} \left(25 - x^2\right)^{-\frac{1}{2}} (-2x) + \left(25 - x^2\right)^{\frac{1}{2}} \]  
\[ = (25 - x^2)^{\frac{1}{2}} \left[-x^2 + 25 - x^2\right] \]
\[ = \frac{25 - 2x^2}{\sqrt{25 - x^2}} \]

For stationary value of \( A \), \[ \frac{dA}{dx} = 0 \]
\[ \frac{25 - 2x^2}{\sqrt{25 - x^2}} = 0 \]
\[ 25 - 2x^2 = 0 \]
\[ x = \sqrt{\frac{25}{2}} \]
\[ = \frac{5}{\sqrt{2}} \]
\[ = \frac{5\sqrt{2}}{2} \]

10.(iii) For \( x < \frac{5\sqrt{2}}{2} \), \( \frac{dA}{dx} > 0 \)

For \( x > \frac{5\sqrt{2}}{2} \), \( \frac{dA}{dx} < 0 \)

As \( x \) increases through \( \frac{5\sqrt{2}}{2} \), the sign of \( \frac{dA}{dx} \) changes from positive to negative.

Stationary value of \( A \) is a maximum.

Stationary value of \( A = \frac{5\sqrt{2}}{2} \sqrt{25 - \left(\frac{5\sqrt{2}}{2}\right)^2} \)
\[ = 12 \frac{1}{2} \]
11.(i) \[ y = -\ln(3 - ax) \]
\[ \frac{dy}{dx} = \frac{a}{3 - ax} \]

When \( y = -\ln 5 \), \( -\ln(3 - ax) = -\ln 5 \)
\[ 3 - ax = 5 \]
\[ x = \frac{-2}{a} \]

When \( x = \frac{-2}{a} \), \( \frac{dy}{dx} = 2 \),
\[ \frac{a}{3 - a\left(\frac{-2}{a}\right)} = 2 \]
\[ a = 10 \]

11.(ii) \[ y = 2x - 5 \]
At \( x = 1 \), Gradient of normal = \( -\frac{3 - a(1)}{a} \)
\[ -\frac{3 - a}{a} = 2 \]
\[ a = -3 \]

11.(iii) when \( a = 4 \),
\[ \frac{dy}{dx} = \frac{4}{3 - 4x} \]
\[ \frac{4}{3 - 4x} = 4 \]
\[ x = \frac{1}{2} \]

From \( y = 4x - 2 \), \( y = 4\left(\frac{1}{2}\right) - 2 \)
\[ = 0 \]

Coordinates of point = \( \left(\frac{1}{2}, 0\right) \)
12. (i) \[ y = x(x - 4) \]

For \( x \)-intercepts, \( y = 0 \),

\[ x = 0 \text{ or } x = 4 \]

For minimum \( y \), when \[ x = \frac{4 + 0}{2} \]

\[ = 2 \]

\[ y = 2^2 - 4(2) \]

\[ = -4 \]

Turning point = \((2, -4)\)

12. (ii)

\[ y = \left| x^2 - 4x \right| \]

- Shape
- Turning point
- \( x \)-intercepts

12. (iii) (a) \[ y = 2 + 2x \]

2 solutions

12. (iii) (b) \[ y = 2 - \frac{1}{2}x \]

3 solutions
ADDiTiOnAL MaThEMaTiCS 4047/02

Paper 2 31 July 2015

Additional Materials: Answer Paper (10 sheets) 2 hours 30 minutes

READ THESE INSTRUCTIONS FIRST

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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

HAND IN QUESTIONS 1 TO 9 SEPARATELY FROM QUESTIONS 10 TO 11.

My Target is:

This paper consists of 8 pages.
2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where $n$ is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\csc^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$
1 In the cubic polynomial $f(x)$, the coefficient of $x^3$ is $a$, where $0 < a < 1$.

(i) Given that the repeated root of the equation $f(x) = 0$ is 1, write down an expression for $f(x)$. [2]

(ii) Find the value of $a$ if $f(x)$ has a remainder of 1 when divided by $x$ and $f(x)$ has a remainder of $-8$ when divided by $x + 3$. [4]

2 A triangle $ABC$ in which $AB = AC$ has its base $BC$ of length \(\frac{4\sqrt{2} - \frac{6}{\sqrt{3}}}{\sqrt{3}}\) cm.

(i) In the case where the area of the triangle is $\left(\frac{\sqrt{3} - 2\sqrt{3}}{2}\right)$ cm$^2$, find, without using a calculator, the length of the height of the triangle in the form $(a + b\sqrt{6})$ cm. [4]

(ii) In the case where angle $BAC$ is a right angle, find, without using a calculator, the square of the length of $AB$ in the form $(c + d\sqrt{6})$ cm$^2$. [3]

3 (a) Given that $\ln(p^2 q) = a$ and $\ln(pq^2) = b$, express $pq$ in terms of $a$ and $b$. [2]

(b) Solve the equation $\log_3(2x + 1) + \log_3 3 = \log_9(x - 2)^4 - \log_3(x - 1)$. [5]

4 A quadratic equation has roots $\alpha$ and $\beta$, where $\alpha < \beta$.

(i) Given that $\alpha\beta = -\frac{1}{2}$ and $\alpha^2 + \beta^2 = \frac{9}{4}$, without solving for $\alpha$ and $\beta$, find the value of $\alpha - \beta$. [2]

(ii) Show that $\alpha^3 - \beta^3 = -11\frac{7}{8}$. [2]

(iii) Find the quadratic equation whose roots are $\frac{\alpha^2 - 1}{\beta}$ and $1 - \frac{\beta^2}{\alpha}$. [4]
A circle, centre $C$, has a diameter $AB$ where $A$ is the point $(-3, 2)$ and $B$ is the point $(5, 8)$.

(i) Find the coordinates of $C$ and the radius of the circle. 

(ii) Find the equation of the circle.

(iii) Show that the equation of the tangent to the circle at $B$ is $3y + 4x = 44$.

(iv) The highest point on the circle is $D$. Find the coordinates of the point at which the tangents to the circle at $B$ and $D$ intersect.

The diagram shows a rod $PQ$ which is hinged at $P$, and a rod $QR$, which is hinged at $Q$.

The rods can only move in the vertical plane as shown. The rod $PQ$ can turn about $P$ and is inclined at an angle $\theta$ to the vertical, where $0^\circ \leq \theta \leq 90^\circ$. The rod $QR$ can turn about $Q$ in such a way that its inclination to the horizontal is also $\theta$. The lengths of $PQ$ and $QR$ are $3$ m and $5$ m respectively.

Given that $R$ is $x$ m from the vertical axis,

(i) find the values of the integers $a$ and $b$ for which $x = a\cos\theta + b\sin\theta$.

Using the values of $a$ and $b$ found in part (i),

(ii) express $x$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$.

(iii) Hence state the maximum value of $x$ and find the corresponding value of $\theta$.

(iv) Deduce, with explanation, the value of $x$ when the rod $PQ$ is inclined at $90^\circ$ to the vertical and the rod $QR$ is inclined at $90^\circ$ to the horizontal.
7 A curve is such that \( \frac{dy}{dx} = 2(3x - 2)(x - 3) \).

(i) Given that the curve passes through the point \((0, 9)\), find the equation of the curve. [2]

(ii) The point \((p, q)\) where \(p\) and \(q\) are integers, is a stationary point on the curve. Find the value of \(p\) and of \(q\). [3]

(iii) Determine whether \(y\) is increasing or decreasing

(a) for values of \(x\) slightly less than \(p\), [1]

(b) for values of \(x\) slightly more than \(p\). [1]

(iv) What do the results of part (iii) imply about the stationary point \((p, q)\)? [1]

(v) Find the value of \(\frac{d^2y}{dx^2}\) at the stationary point \((p, q)\) and explain how this value supports the conclusion that you have made in part (iv). [2]

8 A particle travelling along a straight line is such that its displacement, \(s\) m, from a fixed point \(O\) on the line is given by \(s = t^3 - 6t^2 + 9t + 18\), where \(t\) seconds is the time after motion has begun.

(i) Find the initial displacement of the particle from the fixed point \(O\). [1]

(ii) Find the values of \(t\) for which the particle is instantaneously at rest. Show that the particle returns to its starting position at one of the two instances of rest. [4]

(iii) Find the total distance travelled by the particle during the first 4 seconds. [3]

(iv) Find the minimum velocity of the particle. [3]
9  (i) On the **same axes**, sketch the graphs of \( y = 2\sin t + 2 \) and \( y = \frac{1}{2}\sin \frac{t}{2} + 2 \) for \( 0 \leq t \leq 4\pi \). \[4\]

(ii) It is observed that the height, \( y \) m, above sea-level, reached by ocean waves on two particular days during a time interval of \( 4\pi \) minutes can be modelled by trigonometric functions. The function \( y = 2\sin t + 2 \) models the height of waves on Day 1, and the function \( y = \frac{1}{2}\sin \frac{t}{2} + 2 \) models the height of waves on Day 2.

With reference to the graphs that you have sketched in part (i),

(a) state the number of instances when the waves on the two days reached the same height during the time interval \( 0 < t < 4\pi \). Justify your answer. \[2\]

(b) Which of the two days would have provided surfers with a more thrilling experience of riding the waves at sea? Explain your answer. \[3\]
In the diagram, $A$, $B$, $C$ and $D$ are points on the circumference of the circle with centre $O$. $EDFG$ is a tangent to the circle at $D$.

Given that $AB = BG$ and $DF = FG$, prove that

(i) $ABD$ is an isosceles triangle,  

(ii) $DB^2 - DF^2 = \frac{1}{4} AD^2$,  

(iii) triangle $ADF$ is similar to triangle $DCF$,  

(iv) $GF^2 = AF \times CF$. 

[Turn over]
11 (a) Show that \( \frac{d}{dx} \left( \frac{x}{\sqrt{3x-2}} \right) = \frac{3x-4}{2\sqrt{(3x-2)^3}} \). \[3\]

(b) \[Diagram of a curve with points A, B, C labeled.\]

The diagram shows part of the curve \( y = \frac{8(3x-4)}{\sqrt{(3x-2)^3}} \).

The curve intersects the x-axis at the point A. The line through A with gradient 3 intersects the curve again at the point B. The line BC is parallel to the x-axis.

(i) Verify that the y-coordinate of B is 2. \[5\]

(ii) Determine the area of the shaded region bounded by the curve, the x-axis, the y-axis and the line BC. \[4\]

End of Paper
<table>
<thead>
<tr>
<th>Qn</th>
<th>Working</th>
<th>Marks</th>
<th>Total</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(i) $f(x) = (ax + k)(x - 1)^2$ where $k$ is a constant</td>
<td></td>
<td>[2]</td>
<td></td>
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<tr>
<td></td>
<td>(ii) By the Remainder Theorem, $f(0) = 1$</td>
<td></td>
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<tr>
<td></td>
<td>$k(-1)^2 = 1$</td>
<td></td>
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<tr>
<td></td>
<td>$k = 1$</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>By the Remainder Theorem, $f(-3) = -8$</td>
<td></td>
<td>[4]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(-3a + 1)(-4)^2 = -8$</td>
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</tr>
<tr>
<td></td>
<td>$-3a + 1 = -\frac{1}{2}$</td>
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<tr>
<td></td>
<td>$3a = \frac{3}{2}$</td>
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</tr>
<tr>
<td></td>
<td>$a = \frac{1}{2}$</td>
<td></td>
<td>(6 m)</td>
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</table>
**Qn** | **Working** | **Marks** | **Total** | **Remarks**
--- | --- | --- | --- | ---
| 1 | **Alternative Solution:**  
(i) \( f(x) = a(x+k)(x-1)^2 \) where \( k \) is a constant | | [2] | 
(ii) \( f(0) = 1 \)  
\[ a(k)(-1)^2 = 1 \]  
\[ ak = 1 \] \[ \text{(1)} \] | | 
(iii) \( f(-3) = -8 \)  
\[ a(-3+k)(-4)^2 = -8 \]  
\[ a(-3+k) = -\frac{1}{2} \] \[ \text{(2)} \] | | 
Subst. \( k = \frac{1}{a} \) into (2):  
\[ a\left(-3+\frac{1}{a}\right) = -\frac{1}{2} \]  
\[ -3a + 1 = -\frac{1}{2} \]  
\[ 3a = \frac{3}{2} \]  
\[ a = \frac{1}{2} \] | [4] | (6 m)
<table>
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<th>Remarks</th>
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<tbody>
<tr>
<td>2</td>
<td>(i) Let the height of the triangle be $h$ cm.</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>$\frac{1}{2} \left( 4\sqrt{2} - \frac{6}{\sqrt{3}} \right) (h) = 3\sqrt{2} - 2\sqrt{3}$</td>
<td></td>
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<tr>
<td></td>
<td>$\left( 2\sqrt{2} - \frac{3}{\sqrt{3}} \right) (h) = 3\sqrt{2} - 2\sqrt{3}$</td>
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<tr>
<td></td>
<td>$(2\sqrt{2} - \sqrt{3})(h) = 3\sqrt{2} - 2\sqrt{3}$</td>
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<tr>
<td></td>
<td>$h = \frac{3\sqrt{2} - 2\sqrt{3}}{2\sqrt{2} - \sqrt{3}}$</td>
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<tr>
<td></td>
<td>$= \frac{3\sqrt{2} - 2\sqrt{3}}{2\sqrt{2} - \sqrt{3}} \times \frac{2\sqrt{2} + \sqrt{3}}{2\sqrt{2} + \sqrt{3}}$</td>
<td></td>
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<td></td>
<td>$= \frac{12 + 3\sqrt{6} - 4\sqrt{6} - 6}{(2\sqrt{2})^2 - (\sqrt{3})^2}$</td>
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<tr>
<td></td>
<td>$= \frac{6 - \sqrt{6}}{8 - 3}$</td>
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<tr>
<td></td>
<td>$= \frac{6}{5} - \frac{1}{5} \sqrt{6}$ cm</td>
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<td></td>
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<tr>
<td></td>
<td>$\therefore$ Height of triangle $= \frac{6}{5} - \frac{1}{5} \sqrt{6}$ cm</td>
<td></td>
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<tr>
<td></td>
<td>(ii) By the Pythagoras Theorem,</td>
<td></td>
<td></td>
<td>[4]</td>
</tr>
<tr>
<td></td>
<td>$AB^2 + AC^2 = BC^2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$AB^2 + AB^2 = BC^2$ (\because AC = AB)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$2AB^2 = BC^2$</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>$AB^2 = \frac{1}{2} \left( 4\sqrt{2} - \frac{6}{\sqrt{3}} \right)^2$</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>$= \frac{1}{2} \left[ 2 \left( 2\sqrt{2} - \frac{3}{\sqrt{3}} \right)^2 \right]$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 2(2\sqrt{2} - \sqrt{3})^2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 2(8 - 4\sqrt{6} + 3)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 2(11 - 4\sqrt{6})$</td>
<td></td>
<td>[3]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 22 - 8\sqrt{6}$ cm</td>
<td></td>
<td>(7 m)</td>
<td></td>
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</table>
### Qn 2

**Alternative Solution:**

(ii) In rt. \( \triangle ABC \),

\[
\sin \angle ACB = \frac{AB}{BC}
\]

\[
\sin 45^\circ = \frac{AB}{4\sqrt{2} - \frac{6}{\sqrt{3}}}
\]

\[
AB = \frac{1}{\sqrt{2}} \times (4\sqrt{2} - 2\sqrt{3})
\]

\[
= 4 - \sqrt{6}
\]

\[
AB^2 = (4 - \sqrt{6})^2
\]

\[
= 16 - 2(4)(\sqrt{6}) + 6
\]

\[
= 22 - 8\sqrt{6} \text{ cm}^2
\]
<table>
<thead>
<tr>
<th>Question</th>
<th>Working</th>
<th>Marks</th>
<th>Total</th>
<th>Remarks</th>
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</thead>
</table>
| 3 (a)    | \( \ln(p^2q) = a \) \( \text{(1)} \)  \\
|          | \( \ln(pq^2) = b \) \( \text{(2)} \)  \\
|          | \( \ln(p^2q) + \ln(pq^2) = a + b \)  \\
|          | \( \ln(p^3q^3) = a + b \)  \\
|          | \( \ln(pq)^3 = a + b \)  \\
|          | \( 3\ln(pq) = a + b \)  \\
|          | \( \ln(pq) = \frac{a + b}{3} \)  \\
|          | \( pq = e^{\frac{a + b}{3}} \)  \\
|          | [2]     |       |       |         |

**Alternative Solution:**

\( \ln(p^2q) = a \) \( \Rightarrow \) \( p^2q = e^a \)  \\
\( \ln(pq^2) = b \) \( \Rightarrow \) \( pq^2 = e^b \)  \\
\( (p^2q)(pq^2) = e^a \times e^b \)  \\
\( (pq)^3 = e^{a+b} \)  \\
\( pq = e^{\frac{a+b}{3}} \)
Qn | Working | Marks | Total | Remarks
--- | --- | --- | --- | ---
3  | (b) \( \log_3(2x+1) + \log_1 3 = \log_9(x-2)^4 - \log_3(x-1) \)  
   \[\log_3(2x+1) + \frac{\log_3 3}{\log_3 1} = \frac{\log_3(x-2)^4}{\log_2 9} - \log_3(x-1)\]  
   \[\log_3(2x+1) - \log_3 3 = \log_3(x-2)^2 - \log_3(x-1)\]  
   \[\log_3 \left(\frac{2x+1}{3}\right) = \log_3 \left(\frac{(x-2)^2}{x-1}\right)\]  
   \[\frac{2x+1}{3} = \frac{(x-2)^2}{x-1}\]  
   \[(2x+1)(x-1) = 3(x-2)^2\]  
   \[2x^2 - x - 1 = 3(x^2 - 4x + 4)\]  
   \[x^2 - 11x + 13 = 0\]  
   \[x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(1)(13)}}{2}\]  
   \[x = \frac{11 \pm \sqrt{69}}{2}\]  
   \[x = 1.35 \quad \text{or} \quad 9.65 \quad (\text{to 3 s.f.})\]  
   \[\text{rejection, as } x > 2\]  
   \[\therefore x = 9.65\]  
   [5]  
   (7 m)
\[
\frac{4}{7} = \frac{\ell}{S}
\]

\[
\frac{1}{\ell} \left( \frac{\ell}{S} \right) = \frac{8}{L} \frac{11 - \ell}{\ell}
\]

\[
\frac{\ell}{S} \frac{8}{L} \left( \frac{\ell}{S} \right) = \frac{11 - \ell}{\ell}
\]

\[
(g - a) - (\ell g - \ell a) = g \ell a
\]

\[
\frac{\ell}{g - \ell} + \ell - \ell a = \frac{\ell}{g - \ell} + \ell - \ell a
\]

\[
\frac{\ell}{(g - \ell)g + (1 - \ell)a} = \frac{\ell g}{g - 1} + \frac{\ell}{1 - \ell a}
\]

\[
\frac{\ell g}{g - 1} + \frac{\ell}{1 - \ell a}
\]

New Roots:

\[
(\ell) \frac{\ell}{g - 1}
\]

\[
\frac{8}{L} \ell = \frac{11 - \ell}{\ell}
\]

\[
\frac{\ell}{S} \left( \frac{\ell}{S} \right) = \frac{11 - \ell}{\ell}
\]

\[
(g \ell a + (\ell g + \ell a))(g - \ell) = (\ell g + \ell a + \ell a)(g - \ell) = \ell g - \ell a
\]

\[
(0 > \ell - \ell a \therefore) \frac{\ell}{S} = \ell - \ell a \therefore
\]

\[
\frac{4}{\ell} = \frac{\ell}{S}
\]

\[
\left( \frac{\ell}{S} \right) \frac{4}{\ell} = \frac{11 - \ell}{\ell}
\]

\[
g \ell a - (\ell g + \ell a) = \ell g + \ell a - \ell a = (\ell g - \ell a)
\]

\[
\frac{4}{\ell} = \ell g + \ell a
\]

\[
\frac{4}{\ell} = \ell g + \ell a
\]
<table>
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<tr>
<th>Qn</th>
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<th>Total</th>
<th>Remarks</th>
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| 4  | (c) \( \left( \frac{\alpha^2 - 1}{\beta} \right) \left( 1 - \beta^2 \right) = \frac{\alpha^2 - \alpha^2 \beta^2 -1 + \beta^2}{\alpha \beta} \)  
\[ = \frac{(\alpha^2 + \beta^2) - (\alpha \beta)^2 -1}{\alpha \beta} \]
\[ = \frac{5 \cdot \frac{1}{4} - \left( -\frac{1}{2} \right)^2 -1}{\left( -\frac{1}{2} \right)} \]
\[ = -8 \]
\[ \therefore \text{The required quadratic equation is } x^2 - \frac{75}{4}x - 8 = 0. \] | [4] | (8 m) |
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<th>Qn</th>
<th>Working</th>
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<th>Total</th>
<th>Remarks</th>
</tr>
</thead>
</table>
| 5  | (i) Diameter of circle is $AB$ where $A = (-3, 2)$ and $B = (5, 8)$  
Centre of circle, $C = \left( \frac{-3+5}{2}, \frac{2+8}{2} \right)$  
$= (1, 5)$  
Radius of circle $= \frac{1}{2} \sqrt{[5-(-3)]^2 + (8-2)^2}$  
$= \frac{1}{2} \sqrt{64 + 36}$  
$= \frac{1}{2} (10)$  
$= 5 \text{ units}$  
Alternatively,  
Radius of circle $= \sqrt{(5-1)^2 + (8-5)^2}$  
$= 5 \text{ units}$  
(ii) Equation of the circle is $(x-1)^2 + (y-5)^2 = 5^2$  
i.e. $x^2 + y^2 - 2x - 10y + 1 = 0$  
(iii) Gradient of $AB = \frac{8-2}{5-(-3)}$  
$= \frac{3}{4}$  
:. Gradient of the tangent at $B = -\frac{4}{3}$  
Equation of the tangent to the circle at $B$ is given by $y - 8 = -\frac{4}{3}(x - 5)$  
$3(y-8) = -4(x-5)$  
$3y - 24 = -4x + 20$  
$3y + 4x = 44$  | [4] | | | |
<table>
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</tr>
</thead>
</table>
| 5  | (iv) Highest point on the circle is \( D = (1,10) \)  
   
   Equation of the tangent at \( D \) is \( y = 10 \)  
   
   At the point of intersection of the tangents, \( 3(10) + 4x = 44 \)  
   \[ 4x = 14 \]  
   \[ x = 3 \frac{1}{2} \]  
   . The tangents intersect at the point \( \left(3 \frac{1}{2}, 10\right) \) | [2] | (10 m) |
<table>
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<tbody>
<tr>
<td>6</td>
<td><img src="image" alt="Diagram" /></td>
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</tbody>
</table>

(i) In \( \triangle PQT \),
\[
\sin \theta = \frac{QT}{3}
\]
\[
QT = 3 \sin \theta
\]

In \( \triangle QRS \),
\[
\cos \theta = \frac{QS}{5}
\]
\[
QS = 5 \cos \theta
\]

\[
x = QS + QT
\]
\[
x = 5 \cos \theta + 3 \sin \theta
\]
\[
a = 5 \quad \text{and} \quad b = 3
\]

(ii) \[x = 5 \cos \theta + 3 \sin \theta\]
\[= R \cos(\theta - \alpha) \quad \text{where} \quad R > 0 \quad \text{and} \quad 0^\circ < \alpha < 90^\circ\]
\[
R = \sqrt{5^2 + 3^2}
\]
\[
= \sqrt{34}
\]
\[
= 5.83 \quad \text{(to 3 s.f.)}
\]
\[
\tan \alpha = \frac{3}{5}
\]
\[
\alpha = 30.96^\circ
\]
\[
\alpha = 31.0^\circ
\]

\[
x = 5.83 \cos(\theta - 31.0^\circ)
\]

Accept
\[x = \sqrt{34} \cos(\theta - 31.0^\circ)\]

[2]

[4]
<table>
<thead>
<tr>
<th>Qu</th>
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<th>Marks</th>
<th>Total</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>(iii) Maximum value of $x = \sqrt{34}$</td>
<td></td>
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<td></td>
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<tr>
<td></td>
<td>$= 5.83$</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>Maximum value is attained when</td>
<td></td>
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<tr>
<td></td>
<td>$\cos(\theta - 30.96^\circ) = 1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>where $-30.96^\circ \leq \theta - 30.96^\circ \leq 59.04^\circ$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\theta - 30.96^\circ = 0^\circ$</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>$\theta = 30.96^\circ$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\theta \approx 31.0^\circ$</td>
<td>[3]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(iv)</td>
<td>When $\theta = 90^\circ$,</td>
<td></td>
<td>[2]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = 5\cos 90^\circ + 3\sin 90^\circ$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 5(0) + 3(1)$</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>$= 3$</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>Alternatively,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>When $\theta = 90^\circ$,</td>
<td></td>
<td>[2]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = \sqrt{34} \cos(90^\circ - 30.96^\circ)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \sqrt{34} \cos 59.04^\circ$</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>$= 3.00$</td>
<td></td>
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</table>

(11 m)
<table>
<thead>
<tr>
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<th>Total</th>
<th>Remarks</th>
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</thead>
<tbody>
<tr>
<td>7</td>
<td>(i) $\frac{dy}{dx} = 2(3x - 2)(x - 3)$ &lt;br&gt; $= 2(3x^2 - 11x + 6)$ &lt;br&gt; $y = \int 2(3x^2 - 11x + 6) , dx$ &lt;br&gt; $= 2\left[ \frac{x^3}{3} - \frac{11x^2}{2} + 6x \right] + C$ &lt;br&gt; $= 2x^3 - 11x^2 + 12x + C$ &lt;br&gt; Since the curve passes through the point $(0, 9)$, &lt;br&gt; $9 = 0 + C$ &lt;br&gt; $C = 9$ &lt;br&gt; $\therefore y = 2x^3 - 11x^2 + 12x + 9$ [2]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(ii) At stationary points, $\frac{dy}{dx} = 0$ &lt;br&gt; $2(3x - 2)(x - 3) = 0$ &lt;br&gt; $x = \frac{2}{3}$ or $x = 3$ &lt;br&gt; Since $p$ is an integer, $p = 3$ &lt;br&gt; When $x = 3$, $y = 2(3)^3 - 11(3)^2 + 12(3) + 9$ &lt;br&gt; $= 54 - 99 + 36 + 9$ &lt;br&gt; $= 0$ &lt;br&gt; $\therefore q = 0$ [3]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
7 (iii) (a) For values of $x$ slightly less than $p$, let $x = 2.9$

\[
\frac{dy}{dx} = 2[3(2.9) - 2](2.9 - 3)
\]

\[
< 0
\]

\[\therefore \text{ y is decreasing for } x = p^-\]

(b) For values of $x$ slightly greater than $p$, let $x = 3.1$

\[
\frac{dy}{dx} = 2[3(3.1) - 2](3.1 - 3)
\]

\[
> 0
\]

\[\therefore \text{ y is increasing for } x = p^+\]

(iv) | Value of $x$ | $p^-$ | $p$ | $p^+$ |
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Sign of $\frac{dy}{dx}$</td>
<td>$-$</td>
<td>0</td>
<td>$+$</td>
</tr>
<tr>
<td>Slope</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[\therefore \text{ The stationary point (3, 0) is a minimum point}\]

(v) \[
\frac{d^2y}{dx^2} = 2[3(x - 3) + (3x - 2)]
\]

\[
= 2(6x - 11)
\]

When $x = 3$, \[
\frac{d^2y}{dx^2} = 2[6(3) - 11]
\]

\[
= 2(7)
\]

\[
= 14
\]

Since $\frac{d^2y}{dx^2} > 0$ at $x = 3$, this supports the conclusion made in part (iv) that (3, 0) is a minimum point.

[2] (10 m)
<table>
<thead>
<tr>
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<th>Working</th>
<th>Marks</th>
<th>Total</th>
<th>Remarks</th>
</tr>
</thead>
</table>
| 8  | (i) Displacement, \( s = t^3 - 6t^2 + 9t + 18 \)  
   When \( t = 0 \), \( s = 18 \)  
   \( \therefore \) The initial displacement of the particle from \( O \) is 18 m  |  | 1 |  |
|    | (ii) Velocity, \( v = \frac{ds}{dt} \)  
   \[ v = 3t^2 - 12t + 9 \]  
   When the particle is instantaneously at rest, \( v = 0 \)  
   \( 3t^2 - 12t + 9 = 0 \)  
   \( t^2 - 4t + 3 = 0 \)  
   \( (t - 1)(t - 3) = 0 \)  
   \( t = 1 \) or \( t = 3 \)  
   When \( t = 3 \),  
   \[ s = (3)^3 - 6(3)^2 + 9(3) + 18 \]  
   \( = 18 \)  
   \( \therefore \) The particle will return to its starting position when \( t = 3 \)  |  | 4 | |
|    | (iii) When \( t = 0 \), \( s = 18 \)  
   When \( t = 1 \), \( s = 1 - 6 + 9 + 18 \)  
   \( = 22 \)  
   When \( t = 3 \), \( s = 18 \)  
   When \( t = 4 \), \( s = (4)^3 - 6(4)^2 + 9(4) + 18 \)  
   \( = 22 \)  
   \( \therefore \) The total distance travelled by the particle during the first 4 seconds  
   \( = 3 \times 4 \)  
   \( = 12 \text{ m} \)  | [3] | | |
<table>
<thead>
<tr>
<th>Qn</th>
<th>Working</th>
<th>Marks</th>
<th>Total</th>
<th>Remarks</th>
</tr>
</thead>
</table>
| 8  | (iv) For stationary value of velocity,  
    \[
        \frac{dv}{dt} = 0 \\
        6t - 12 = 0 \\
        t - 2 = 0 \\
        t = 2
    \]
    \[
        \frac{d^2v}{dt^2} = 6 (>0)
    \]
    \[\Rightarrow v\text{ is minimum when } t = 2\]
    Minimum velocity = \[3(2)^2 - 12(2) + 9 = -3 \text{ m/s}\] | [3] | (11 m) |
(ii)  (a) In the interval $0 < t < 4\pi$, the two graphs have 3 points of intersection, showing that there were 3 instances when the waves on the two days reached the same height.

(b) Day 1 would have provided surfers with a more thrilling experience of riding the waves at sea.

*Period of the waves in Day 1 is half of the period of the waves in Day 2, showing that for Day 1 there is an additional cycle between the greatest height and the least height of the waves within the same time interval.*

In addition, the amplitude of the waves in Day 1 is greater than the amplitude of the waves in Day 2, showing that the rise and drop in height of the waves in Day 1 is greater.
(i) \( \angle ADG = 90^\circ \) (\( \text{tan} \perp \text{rad} \))
\( OB \parallel DG \) (Midpoint Theorem)
\( \angle AOB = \angle ADG = 90^\circ \) (corr. \( \angle s \), \( OB \parallel DG \))

Since \( OB \) is the perpendicular bisector of \( AD \),
\( AB = DB \)
\( \Rightarrow \) \( ABD \) is an isosceles triangle

(ii) In rt-\( \triangle ADG \),
\( AG^2 - DG^2 = AD^2 \) (Pythagoras Theorem)
\( (2AB)^2 - (2DF)^2 = AD^2 \) (given \( AB = BG \) and \( DF = FG \))
\( 4(AB^2 - DF^2) = AD^2 \)
\( 4(DF^2 - DF^2) = AD^2 \) (from (i), \( AB = DB \))
\( \therefore DB^2 - DF^2 = \frac{1}{4} AD^2 \)

(iii) In \( \triangle ADF \) and \( \triangle DCF \),
\( \angle DAF = \angle CDF \) (Tangent-Chord Theorem)
\( \angle AFD = \angle DFC \) (Common angle)
\( \therefore \) \( \triangle ADF \) is similar to \( \triangle DCF \).
(Two pairs of corresponding angles are equal)

(iv) Since \( \triangle ADF \) and \( \triangle DCF \) are similar,
\( \frac{DF}{AF} = \frac{CF}{DF} \)
\( DF^2 = AF \times CF \)
\( \therefore GF^2 = AF \times CF \) (given, \( GF = DF \))
<table>
<thead>
<tr>
<th>Qn</th>
<th>Working</th>
<th>Marks</th>
<th>Total</th>
<th>Remarks</th>
</tr>
</thead>
</table>
| 11 | (a) \[ \frac{d}{dx} \left( \frac{x}{\sqrt{3x-2}} \right) \]

\[ \begin{align*}
\text{\(1\)} & \quad \frac{1}{(3x-2)^2(1-x)} - \frac{1}{2} \frac{1}{(3x-2)^2} \\
\text{\(2\)} & \quad \frac{1}{2} \frac{1}{(3x-2)^2} [2(3x-2) - 3x] \\
\text{\(3\)} & \quad \frac{6x - 4 - 3x}{3} \\
\text{\(4\)} & \quad \frac{3x - 4}{2(3x-2)^2} \\
\text{\(5\)} & \quad \frac{3x - 4}{2\sqrt{(3x-2)^3}}
\end{align*} \]

\[ [3] \]
<table>
<thead>
<tr>
<th>Qn</th>
<th>Working</th>
<th>Marks</th>
<th>Total</th>
<th>Remarks</th>
</tr>
</thead>
</table>
| 11 | (b) (i) At $A$, $y = 0$,  
\[
\frac{8(3x-4)}{\sqrt{(3x-2)^3}} = 0
\]
\[
3x-4 = 0
\]
\[
x = \frac{4}{3}
\]

Equation of the line $AB$ is given by  
\[
y - 0 = 3 \left( x - \frac{4}{3} \right)
\]
\[
y = 3x - 4
\]

At the points of intersection of the curve and the line $AB$,  
\[
\frac{8(3x-4)}{\sqrt{(3x-2)^3}} = 3x - 4
\]
\[
\frac{8(3x-4)}{\sqrt{(3x-2)^3}} - (3x - 4) = 0
\]
\[
(3x - 4) \left[ \frac{8}{\sqrt{(3x-2)^3}} - 1 \right] = 0
\]
\[
3x - 4 = 0 \quad \text{or} \quad \frac{8}{\sqrt{(3x-2)^3}} - 1 = 0
\]
\[
\frac{8}{\sqrt{(3x-2)^3}} = 1
\]
\[
\sqrt{(3x-2)^3} = 8
\]
\[
(3x-2)^3 = 64
\]
\[
3x-2 = 4
\]
\[
x = 2
\]

At $B$, $x = 2$.

When $x = 2$, $y = 3(2) - 4$
\[
y = 2
\]

$\therefore$ The $y$-coordinate of $B$ is 2.

[5]
<table>
<thead>
<tr>
<th>Qn</th>
<th>Working</th>
<th>Marks</th>
<th>Total</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td><img src="image" alt="Graph" /> [ y = \frac{8(3x - 4)}{\sqrt{(3x - 2)^3}} ] [ C(0, 2) ] [ A(2, 0) ] [ (b) (ii) Area of the shaded region ] [ = (2)(2) - \int_{4}^{2} \frac{8(3x - 4)}{3 \sqrt{(3x - 2)^3}} , dx ] [ = 4 - 16 \int_{4}^{2} \frac{(3x - 4)}{2 \sqrt{(3x - 2)^3}} , dx ] [ = 4 - 16 \left[ \frac{x}{\sqrt{3x - 2}} \right]_{4}^{2} ] [ = 4 - 16 \left[ \frac{2}{\sqrt{3(2) - 2}} - \frac{4}{\sqrt{3 \left( \frac{4}{3} \right) - 2}} \right] ] [ = 4 - 16 \left[ 1 - \frac{4}{3\sqrt{2}} \right] ] [ = 3.0849 ] [ = 3.08 \text{ sq.units (to 3 s.f.)} ]</td>
<td>[4]</td>
<td></td>
<td>(12 m)</td>
</tr>
</tbody>
</table>
READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This question paper consists of 5 printed pages.

Setter: Ms Zoe Pow
Vetter: Mr Ang Hanping

We Nurture Students to Think, Care and Lead with P.R.I.D.E.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial Theorem
\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n
\]

where \( n \) is a positive integer and \[
\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}
\]

2. TRIGONOMETRY

Identities
\[
\sin^2 A + \cos^2 A = 1
\]
\[
\sec^2 A = 1 + \tan^2 A
\]
\[
\csc^2 A = 1 + \cot^2 A
\]
\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
\sin 2A = 2 \sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]
\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \triangle ABC \)
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\Delta = \frac{1}{2} bc \sin A
\]
Answer all the questions.

1. The acute angles $A$ and $B$ are such that $\sin(A + B) = 0$ and $\sin B = \frac{1}{4}$.
   Without using a calculator, find the exact value of $\tan A$. [3]

2. It is given that $y = \frac{2x+16}{x-1}$, where both $x$ and $y$ are positive and vary with time.
   Find the value of $y$ when the rate of increase of $y$ is twice the rate of decrease of $x$. [4]

3. Express $\frac{x+1}{(x+2)(x^2+4)}$ in partial fractions. [4]

4. (i) Find the range of values of $k$ for which the line $y = x-k$ intersects the curve $y = k(x+3)$ at two distinct points. [4]
   (ii) Hence or otherwise, find the range of values of $k$ for which $k(x+3) > x-k$ for all real values of $x$. [2]

5. Given that the first two non-zero terms of the expansion of $(1-kx)^n(1+\frac{x}{3})$ are $1$ and $-\frac{7}{3}x^2$, where $n$ is a positive integer, find the value of $k$ and of $n$. [6]

6. (i) Prove that $\frac{\cos^2 x - \sin^2 x}{1 + 2 \sin x \cos x} = \frac{1 - \tan x}{1 + \tan x}$. [3]
   (ii) Find all the angles between $0^\circ$ and $360^\circ$ which satisfy $\frac{\cos^2 x - \sin^2 x}{1 + 2 \sin x \cos x} = \frac{2}{3} \tan x$. [3]

7. It is given that $f(x)$ is such that $f'(x) = \sin 2x + \cos 3x$.
   Given also that $f\left(\frac{\pi}{6}\right) = 0$, show that $f''(x) + 9f(x) = \frac{3}{4} - \frac{5}{2} \cos 2x$. [6]
8 Variables \( x \) and \( y \) are connected by the equation \( y = 10^{kx} \), where \( n \) and \( k \) are constants. When a graph of \( \ln y \) is plotted against \( x \), a straight line passing through the points \((1, 2)\) and \((4, -7)\) is obtained.

Find

(i) the value of \( n \) and of \( k \), \( [4] \)
(ii) the coordinates of the point on the line at which \( y = 10^k \). \( [3] \)

9 The tangent to the curve \( y = x \ln 3x \) at point \( P(1, \ln 3) \) cuts the \( x \)-axis at \( Q \).

(i) Find the angle that \( PQ \) makes with the \( x \)-axis. \( [5] \)

The normal to the curve \( y = x \ln 3x \) at \( R \) is parallel to the line \( y = 5 - 2x \).

(ii) Find the \( x \)-coordinate of \( R \). \( [3] \)

10 The diagram shows a rectangular poster of area 825 cm\(^2\) with side margins of 2 cm and top and bottom margins of 1.5 cm. The length and breadth of the printing area are \( x \) cm and \( y \) cm respectively.

(i) Show that the printing area, \( A \) cm\(^2\), is given by \( A = \frac{825x}{x + 4} - 3x \). \( [3] \)

(ii) Given that \( x \) can vary, find the value of \( x \) for which the printing area is stationary. \( [4] \)

(iii) Explain why this value of \( x \) gives the poster the largest printing area possible. \( [1] \)

11 (i) Sketch the graph of \( y = |7x - 2| \), for \(-1 \leq x \leq 2\). \( [2] \)

(ii) Hence, find the range of values of \( x \) for which \(|4x - 4| \geq 3x\). \( [4] \)

(iii) Using your graph, determine the number of intersections of the line \( y = mx + c \) with \( y = |7x - 2| \), justifying your answer in each of the following cases.

(a) \( m = -7 \) and \( c > 2 \), \( [2] \)

(b) \( m = 7 \) and \( c < -2 \). \( [2] \)
Solutions to this question by accurate drawing will not be accepted.

In the trapezium $OABC$, the point $A$ has coordinates $(5, 0)$ and the point $C$ has coordinates $(-2, 6)$. The sides $OC$ and $AB$ are parallel, and $BC$ is perpendicular to $OC$.

(i) Show that the coordinates of $B$ are $\left(2 \frac{1}{2}, 7 \frac{1}{2}\right)$. [5]

(ii) $OC$ is produced to $D$ such that $OABD$ is a parallelogram. Find the coordinates of $D$. [2]

(iii) Find the equation of the perpendicular bisector of $OC$. [2]

(iv) $E$ is a point which lies on the perpendicular bisector of $OC$ such that the area of quadrilateral $OAE$ is 15 units$^2$. Given that the $x$-coordinate of $E$ is positive, find the coordinates of $E$. [3]

END OF PAPER
### Additional Mathematics Paper 1 (80 marks)

<table>
<thead>
<tr>
<th>Qn</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \tan A = -\frac{1}{\sqrt{15}} )</td>
</tr>
<tr>
<td>2</td>
<td>( y = 8 )</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{x+6}{8(x^2+4)} - \frac{1}{8(x+2)} )</td>
</tr>
<tr>
<td>4i</td>
<td>( k &lt; -\frac{1}{5} ) or ( k &gt; 1 )</td>
</tr>
<tr>
<td>4ii</td>
<td>( \frac{1}{5} &lt; k &lt; 1 )</td>
</tr>
<tr>
<td>5</td>
<td>( \therefore n = 6, \ k = 2 )</td>
</tr>
</tbody>
</table>
| 6i | \[
LHS = \frac{\cos^2 x - \sin^2 x}{1 + 2\sin x \cos x} = \frac{(\cos x - \sin x)(\cos x + \sin x)}{\sin^2 x + \cos^2 x + 2\sin x \cos x} = \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x + \sin x)^2} = \frac{\cos x - \sin x}{\cos x + \sin x} = \frac{\cos x}{\cos x + \sin x} - \frac{\sin x}{\cos x} = 1 - \tan x \\
= 1 + \tan x = RHS \text{ (shown)}
\] |
| 6ii| \( \therefore x = 26.6^\circ, 108.4^\circ, 206.6^\circ, 288.4^\circ \) |
| 7  | \[
f'(x) = \sin 2x + \cos 3x \\
f(x) = \int (\sin 2x + \cos 3x) \, dx = -\frac{1}{2} \cos 2x + \frac{1}{3} \sin 3x + c \\
f\left(\frac{\pi}{6}\right) = -\frac{1}{2} \cos \left(\frac{\pi}{6}\right) + \frac{1}{3} \sin \left(\frac{\pi}{6}\right) + c \\
= -\frac{1}{2} \cos \left(\frac{\pi}{3}\right) + \frac{1}{3} \sin \left(\frac{\pi}{2}\right) + c = 0 \\
= -\frac{1}{2} \left(\frac{1}{2}\right) + \frac{1}{3} (1) + c = 0 \\
c = -\frac{1}{12}
\] |
### Question Answer

\[ f(x) = -\frac{1}{2} \cos 2x + \frac{1}{3} \sin 3x - \frac{1}{12} \]
\[ f''(x) = 2 \cos 2x - 3 \sin 3x \]
\[ f''(x) + 9f(x) = 2 \cos 2x - 3 \sin 3x + 9 \left( -\frac{1}{2} \cos 2x + \frac{1}{3} \sin 3x - \frac{1}{12} \right) \]
\[ = 2 \cos 2x - 3 \sin 3x - \frac{9}{2} \cos 2x + 3 \sin 3x - \frac{3}{4} \]
\[ = -\frac{3}{4} \cos 2x \text{ (shown)} \]

<table>
<thead>
<tr>
<th>( k )</th>
<th>( k = 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8i</td>
<td>Coordinates are ( (1.25, 1.25) )</td>
</tr>
<tr>
<td>9i</td>
<td>( \theta = 64.5^\circ )</td>
</tr>
<tr>
<td>9ii</td>
<td>( x = 0.202 )</td>
</tr>
</tbody>
</table>

10i

\[ (x + 2 \times 2) \times (y + 1.5 \times 2) = 825 \]
\[ (x + 4)(y + 3) = 825 \]
\[ y + 3 = \frac{825}{x + 4} \]
\[ y = \frac{825}{x + 4} - 3 \]
\[ A = \left( \frac{825}{x + 4} - 3 \right) (x) \]
\[ A = \frac{825x}{x + 4} - 3x \text{ (shown)} \]

| 10ii | \( x = 29.2 \text{ or } -37.2 \text{ (N.A.)} \) |

10 iii

\[ \frac{d^2 A}{dx^2} = -2(3300) \]
\[ \frac{d^2 A}{dx^2} = \frac{(-2)(3300)}{(x + 4)^3} \]

When \( x = 29.2 \),
\[ \frac{d^2 A}{dx^2} = \frac{-6600}{(29.2 + 4)^3} \]
\[ = -0.181 \]

Since \( \frac{d^2 A}{dx^2} < 0 \), this value of \( x \) gives the largest printing area possible.

11i

![Diagram](image-url)

\((1,9)\) to \((2,12)\)
<table>
<thead>
<tr>
<th>Qn</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>11 ii</td>
<td>( x \leq \frac{4}{17} ) or ( x \geq \frac{4}{11} )</td>
</tr>
<tr>
<td>11 iia</td>
<td>1 intersection; Line parallel to L.H. arm cuts R.H. arm at only one point*</td>
</tr>
<tr>
<td>11 iib</td>
<td>0 intersection; Line parallel to R.H. arm; for intersection, ( c &gt; -2 )</td>
</tr>
</tbody>
</table>
| 12i | \( m_{AB} = m_{OC} \)  
\( \frac{y - 6}{x - 5} = \frac{6}{-2} \)  
\( y = -3x + 15 \) ---- (1)  
\( m_{BC} = -\frac{1}{m_{OC}} \)  
\( \frac{y - 6}{x + 2} = \frac{1}{3} \)  
\( y = \frac{x}{3} + 6 \frac{2}{3} \) ---- (2)  
Sub (1) into (2):  
\( -3x + 15 = \frac{1}{3}x + 6 \frac{2}{3} \)  
\( 3x - 8 \frac{1}{3} = 0 \)  
\( x = 2 \frac{1}{2} \)  
Sub \( x = 2 \frac{1}{2} \) into (1):  
\( y = -3 \left(2 \frac{1}{2}\right) + 15 \)  
\( y = 7 \frac{1}{2} \)  
Coordinates of \( B \) are \( \left(2 \frac{1}{2}, 7 \frac{1}{2}\right) \) (shown) |
| 12 ii | \( D \left(-2 \frac{1}{2}, 7 \frac{1}{2}\right) \) |
| 12 iii | \( y = \frac{1}{3}x + 3 \frac{1}{3} \) |
| 12 iv | Possible coordinates of \( E \) are \((0.8, 3.6)\). |
### Additional Mathematics Paper 1 (80 marks)

<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Mark Allocation</th>
</tr>
</thead>
</table>
| 1  | \[
\begin{align*}
\sin(A + B) &= 0 \\
\sin A \cos B + \cos A \sin B &= 0 \\
\sin A \left(\frac{\sqrt{15}}{4}\right) + \cos A \left(\frac{1}{4}\right) &= 0 \\
\sqrt{15} \sin A + \cos A &= 0 \\
\sqrt{15} \sin A &= -\cos A \\
\sin A &= \frac{-1}{\sqrt{15}} \\
\cos A &= \sqrt{15} \\
\tan A &= -\frac{1}{\sqrt{15}}
\end{align*}
\] | M1: Addition Formula  
M1: Find \(\cos B\) |
| 2  | \[
\begin{align*}
y &= \frac{2x + 16}{x - 1} \\
dy &= \frac{(x-1)(2) - (2x+16)(1)}{(x-1)^2} \\
&= \frac{2x - 2 - 2x - 16}{(x-1)^2} \\
&= \frac{-18}{(x-1)^2} \\
dy &= -2 \frac{dx}{dt} \\
dy &= \frac{dy}{dt} \frac{dt}{dx} \\
&= -2 \frac{dx}{dt} \frac{dx}{dt} \\
&= -2 \frac{dx}{dt} \\
&= -2 \\
&= \frac{-18}{(x-1)^2} \\
(x-1)^2 &= 9 \\
x - 1 &= \pm 3 \\
x &= 4 \text{ or } -2 \text{ (N.A.)} \\
\text{When } x = 4, \\
y &= \frac{2(4)+16}{4-1} \\
&= 8
\end{align*}
\] | MI  
MI  
MI  
MI  |
### Question 3

**Solution**

Let \( \frac{x + 1}{(x + 2)(x^2 + 4)} = \frac{A}{x + 2} + \frac{Bx + C}{x^2 + 4} \)

\[ x + 1 = A(x^2 + 4) + (Bx + C)(x + 2) \]

Sub. \( x = -2 \):

\[ -2 + 1 = A((-2)^2 + 4) \]

\[ -1 = 8A \]

\[ A = -\frac{1}{8} \]

Comparing coefficients of \( x^2 \):

\[ 0 = A + B \]

\[ B = \frac{1}{8} \]

Sub. \( x = 0 \):

\[ 1 = 4A + 2C \]

\[ 2C = 1 - 4 \left( -\frac{1}{8} \right) \]

\[ C = \frac{3}{4} \]

\[ \frac{x + 1}{(x + 2)(x^2 + 4)} = \frac{1}{8(x + 2)} + \frac{\frac{1}{8} \cdot \frac{x + 3}{x^2 + 4}}{8(x + 2)} \]

\[ = \frac{x + 6}{8(x^2 + 4)} - \frac{1}{8(x + 2)} \]

### Question 4

**Part i**

\( kx(x + 3) = x - k \)

\( kx^2 + (3k - 1)x + k = 0 \)

\((3k - 1)^2 - 4(k)(k) > 0 \)

\( 9k^2 - 6k + 1 - 4k^2 > 0 \)

\( 5k^2 - 6k + 1 > 0 \)

\( (5k - 1)(k - 1) > 0 \)

\( k < \frac{1}{5} \) or \( k > 1 \)

### Part ii

\((3k - 1)^2 - 4(k)(k) < 0 \)

\((5k - 1)(k - 1) < 0 \)

\( \frac{1}{5} < k < 1 \)
\[
\left(1 + \frac{x}{3}\right)^n = 1 + \binom{n}{1} \left(\frac{x}{3}\right) + \binom{n}{2} \left(\frac{x}{3}\right)^2 + \ldots \\
= 1 + \frac{nx}{3} + \frac{n(n-1)x^2}{2!} \left(\frac{x}{9}\right) + \ldots \\
= 1 + \frac{nx}{3} + \frac{n(n-1)x^2}{18} + \ldots
\]

\[
(1-kx) \left(1 + \frac{x}{3}\right)^n = (1-kx) \left(1 + \frac{n}{3} x + \frac{n(n-1)}{18} x^2 + \ldots\right) \\
= 1 + \frac{n}{3} x + \frac{n(n-1)}{18} x^2 - kx - \frac{nk}{3} x^2 + \ldots \\
= 1 + \left(\frac{n}{3} - k\right) x + \left[\frac{n(n-1)}{18} - \frac{nk}{3}\right] x^2 + \ldots
\]

\[
\frac{n}{3} - k = 0 \\
n = 3k \\
\frac{n(n-1)}{18} - \frac{nk}{3} = \frac{7}{3}
\]

Sub (1) into (2):

\[
\frac{3k(3k-1)}{18} - \frac{3k^2}{3} = \frac{7}{3} \\
9k^2 - 3k - 18k^2 + 42 = 0 \\
-9k^2 - 3k + 42 = 0 \\
3k^2 + k - 14 = 0 \\
(3k+7)(k-2) = 0
\]

\[
k = -2 \frac{1}{3} \quad \text{or} \quad k = 2
\]

Sub \(k = -2 \frac{1}{3}\) into (1): \(n = 3\left(-2 \frac{1}{3}\right)\)

\[
n = -7 \ (\text{N.A.}) \quad \therefore \text{Reject } k = -2 \frac{1}{3}
\]

Sub \(k = 2\) into (1): \(n = 3(2)\)

\[
= 6
\]

\[
\therefore n = 6, \ k = 2
\]
<table>
<thead>
<tr>
<th>Qn</th>
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</thead>
<tbody>
<tr>
<td>6i</td>
<td>[ LHS = \frac{\cos^2 x - \sin^2 x}{1 + 2\sin x \cos x} = \frac{(\cos x - \sin x)(\cos x + \sin x)}{\sin^2 x + \cos^2 x + 2\sin x \cos x} = \frac{(\cos x - \sin x)^2}{(\cos x + \sin x)^2} ]</td>
<td>MI: Factorise numerator MI: Factorise denominator A1: Divide by ( \cos x )</td>
</tr>
<tr>
<td>6ii</td>
<td>[ \frac{\cos^2 x - \sin^2 x}{1 + 2\sin x \cos x} = \frac{2}{3} \tan x ] [1 - \tan x = \frac{2}{3} \tan x ] [1 + \tan x = 3 ] [3 - 3\tan x = 2\tan x + 2\tan^2 x ] [2\tan^2 x + 5\tan x - 3 = 0 ] [(2\tan x - 1)(\tan x + 3) = 0 ] [\tan x = \frac{1}{2} ] Basic ( \leq 26.565 ) ( x = 26.565, 180 + 26.565 ) OR ( \tan x = -3 ) Basic ( \leq 71.565 ) ( x = 180 - 71.565, 360 - 71.565 ) [\therefore x = 26.6^\circ, 108.4^\circ, 206.6^\circ, 288.4^\circ ]</td>
<td>MI A1</td>
</tr>
<tr>
<td>7</td>
<td>( f'(x) = \sin 2x + \cos 3x ) ( f(x) = \int (\sin 2x + \cos 3x) , dx ) [= \frac{1}{2} \cos 2x + \frac{1}{3} \sin 3x + c ] ( f\left(\frac{\pi}{6}\right) = \frac{1}{2} \cos 2\left(\frac{\pi}{6}\right) + \frac{1}{3} \sin 3\left(\frac{\pi}{6}\right) + c )</td>
<td>M1</td>
</tr>
</tbody>
</table>
\[
\begin{align*}
- \frac{1}{2} \cos \left( \frac{\pi}{3} \right) + \frac{1}{3} \sin \left( \frac{\pi}{2} \right) + c &= 0 \\
- \frac{1}{2} \left( \frac{1}{2} \right) + \frac{1}{3} (1) + c &= 0 \\
c &= -\frac{1}{12} \\
f(x) &= -\frac{1}{2} \cos 2x + \frac{1}{3} \sin 3x - \frac{1}{12} \\
f'(x) &= 2 \cos 2x - 3 \sin 3x \\
f''(x) + 9f(x) &= 2 \cos 2x - 3 \sin 3x + 9 \left( -\frac{1}{2} \cos 2x + \frac{1}{3} \sin 3x - \frac{1}{12} \right) \\
&= 2 \cos 2x - 3 \sin 3x - \frac{9}{2} \cos 2x + 3 \sin 3x - \frac{9}{4} \\
&= -\frac{3}{4} \cos 2x 
\end{align*}
\]

<table>
<thead>
<tr>
<th>Qn.</th>
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</thead>
</table>
| 8i  | \( y = 10^{t-nx} \) \\
|     | \( \lg y = (k - nx) \lg 10 \) \\
|     | \( \lg y = -nx + k \) \\
|     | \( m = \frac{2 - (-7)}{1 - 4} \) \\
|     | \( = -3 \) \\
|     | \( n = -3 \) \\
|     | \( \therefore n = 3 \) \\
|     | \( k = 5 \) \\
| 8ii | \( y = 10^{5-3x} \) \text{ (1)} \\
|     | \( y = 10^t \) \text{ (2)} \\
|     | \( \text{Sub (1) into (2)}: \ 10^{5-3x} = 10^t \) \\
|     | \( x = 5 - 3x \) \\
|     | \( 4x = 5 \) \\
|     | \( x = \frac{5}{4} \) \\
|     | \( \text{Sub } x = \frac{5}{4} \text{ into (2)}: \ y = 10^{\frac{5}{4}} \) \\
|     | \( y = 17.783 \) \\
|     | \( \lg y = \lg 17.783 \) \\
|     | \( = 1.25 \) \\
|     | \( \therefore \text{ Coordinates are (0.25, 1.25)} \) | A1
<table>
<thead>
<tr>
<th>Question</th>
<th>Solution</th>
<th>Mark Allocation</th>
</tr>
</thead>
</table>
| 9i       | \[ y = x \ln 3x \]  
           | \[ \frac{dy}{dx} = x \left( \frac{1}{x} \right) + (\ln 3x)(1) \]  
           | \[ = 1 + \ln 3x \]  
           | At \( P(1, \ln 3) \), \[ \frac{dy}{dx} = 1 + \ln 3 \]  
           | \[ = 1 + \ln 3 \]  
           | At \( P(1, \ln 3) \), \[ y - \ln 3 = (1 + \ln 3)(x - 1) \]  
           | \[ y = (1 + \ln 3)x - 1 \]  
           | When \( y = 0 \), \( (1 + \ln 3)x = 1 \)  
           | \[ x = \frac{1}{1 + \ln 3} \]  
           | \[ x = 0.477 \]  
           | Coordinates of \( Q \) are \((0.477, 0)\)  
           | \[ \tan \theta = \frac{\ln 3 - 0}{1 - 0.477} \]  
           | \[ = 2.0986 \]  
           | \[ \theta = 64.5^\circ \]  
           | M1 | M1 | A1 |
| 9ii      | Gradient of normal = \[ \frac{-1}{1 + \ln 3x} \]  
           | \[ \frac{-1}{1 + \ln 3x} = -2 \]  
           | \[ 1 + \ln 3x = \frac{1}{2} \]  
           | \[ \ln 3x = -\frac{1}{2} \]  
           | \[ 3x = e^{-\frac{1}{2}} \]  
           | \[ x = \frac{1}{3} e^{-\frac{1}{2}} \]  
           | \[ x = 0.202 \]  
           | M1 | M1 | A1 |
| 10i      | \([x + (2 \times 2)]x [y + (1.5 \times 2)] = 825\)  
           | \((x + 4)(y + 3) = 825\)  
           | \[ y + 3 = \frac{825}{x + 4} \]  
           | \[ y = \frac{825}{x + 4} - 3 \]  
<pre><code>       | M1 | M1 |
</code></pre>
<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Mark Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>( A = \left( \frac{825}{x+4} - 3 \right) ) ( \left( \frac{825x}{x+4} - 3x \right) ) (shown)</td>
<td>A1</td>
</tr>
<tr>
<td>10</td>
<td>( \frac{dA}{dx} = \frac{825}{(x+4)^2} \left( \frac{825x}{x+4} \right) ) - 3 ( \frac{3300}{(x+4)^2} - 3 ) ( \frac{3300}{(x+4)^2} - 3 = 0 ) ( (x+4)^2 = 1100 ) ( x+4 = \pm \sqrt{1100} ) ( x = 29.2 ) or (-37.2 ) (N.A.)</td>
<td>M1 M1 M1</td>
</tr>
<tr>
<td>10</td>
<td>( \frac{d^2A}{dx^2} = -\frac{2(3300)}{(x+4)^3} )</td>
<td>B1: Show ( \frac{d^2A}{dx^2} &lt; 0 )</td>
</tr>
<tr>
<td></td>
<td>When ( x = 29.2 ), ( \frac{d^2A}{dx^2} = -\frac{-6600}{(29.2+4)^3} ) ( \frac{d^2A}{dx^2} = -0.181 )</td>
<td>B1: Show ( \frac{d^2A}{dx^2} &lt; 0 )</td>
</tr>
<tr>
<td></td>
<td>Since ( \frac{d^2A}{dx^2} &lt; 0 ), this value of ( x ) gives the poster the largest</td>
<td>B1: Show ( \frac{d^2A}{dx^2} &lt; 0 )</td>
</tr>
<tr>
<td></td>
<td>printing area possible.</td>
<td>B1: Show ( \frac{d^2A}{dx^2} &lt; 0 )</td>
</tr>
</tbody>
</table>

11i

\[ y \]

\[ (-1, 9) \]

\[ (2, 12) \]

\[ \frac{2}{7} \]

\[ 2 \]

| 11i | \( |14x - 4| = 3x \) \( |7x - 2| = 3x \) \( |7x - 2| = \frac{3}{2} \) | M1 |
| 11ii | \( 2|7x - 2| = 3x \) | M1 |

B1: Show \( \frac{d^2A}{dx^2} < 0 \)

B1: V-shape graph
B1: Correct x- and y-intercepts and end-points labelled
<table>
<thead>
<tr>
<th>Qn</th>
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</tr>
</thead>
<tbody>
<tr>
<td>7x - 2 = \frac{3}{2}x \text{ or } 7x - 2 = -\frac{3}{2}x</td>
<td></td>
<td>M1</td>
</tr>
<tr>
<td>\frac{1}{2} - x = 2 \text{ or } \frac{1}{2} - x = 2</td>
<td>x = \frac{4}{11} \text{ or } x = \frac{4}{17}</td>
<td>M1</td>
</tr>
<tr>
<td>\therefore x \leq \frac{4}{17} \text{ or } x \geq \frac{4}{11}</td>
<td></td>
<td>A1</td>
</tr>
<tr>
<td>11</td>
<td>1 intersection;</td>
<td>B1</td>
</tr>
<tr>
<td>iia</td>
<td>Line parallel to L.H. arm cuts R.H. arm at only one point.</td>
<td>B1</td>
</tr>
<tr>
<td>11</td>
<td>0 intersection;</td>
<td>B1</td>
</tr>
<tr>
<td>iiib</td>
<td>Line parallel to R.H. arm; for intersection, c &gt; -2.</td>
<td>B1</td>
</tr>
<tr>
<td>12i</td>
<td>\begin{align*} m_{ab} &amp;= m_{oc} \ y - 0 &amp;= 6 \ x - 5 &amp;= -2 \ y &amp;= -3x + 15 \quad \text{(1)} \ m_{bc} &amp;= \frac{1}{m_{oc}} \ y - 6 &amp;= 1 \ x + 2 &amp;= \frac{3}{3} \ y &amp;= \frac{1}{3}x + 6 \quad \text{(2)} \end{align*}</td>
<td>M1</td>
</tr>
<tr>
<td>Sub (1) into (2): \begin{align*} -3x + 15 &amp;= \frac{1}{3}x + 6 \frac{2}{3} \ \frac{3}{3}x &amp;= 8 \frac{1}{3} \ x &amp;= 2 \frac{1}{2} \end{align*}</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>Sub \ x = 2 \frac{1}{2} \text{ into (1)}: \begin{align*} y &amp;= -3 \left(2 \frac{1}{2}\right) + 15 \ y &amp;= 7 \frac{1}{2} \end{align*}</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td>Coordinates of B are \left(2 \frac{1}{2}, 7 \frac{1}{2}\right)</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>Qn</td>
<td>Solution</td>
<td>Mark Allocation</td>
</tr>
<tr>
<td>-----</td>
<td>---------------------------------------------------------------------------------------------------------------</td>
<td>------------------</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ii</td>
<td>$B\left(\frac{1}{2}, \frac{7}{2}\right)$</td>
<td>M1 O.E.</td>
</tr>
<tr>
<td></td>
<td>$D\left(-\frac{1}{2}, \frac{7}{2}\right)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\overrightarrow{AB} = \left(\frac{5}{2}, \frac{1}{2}\right)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\overrightarrow{CD} = \left(-\frac{3}{2}, \frac{1}{2}\right)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Midpoint of $OC = \left(\frac{0 + (-2)}{2}, \frac{0 + 6}{2}\right)$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$= \left(-1, 3\right)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gradient of perpendicular bisector of $OC = -\frac{1}{3}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \frac{1}{3}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>At $(-1, 3)$, $y - 3 = \frac{1}{3}(x + 1)$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$y = \frac{1}{3}x + 3\frac{1}{3}$</td>
<td></td>
</tr>
<tr>
<td>iv</td>
<td>$\begin{vmatrix} 1 &amp; 0 &amp; 5 &amp; x &amp; -2 &amp; 0 \ 2 &amp; 0 &amp; 0 &amp; y &amp; 6 &amp; 0 \end{vmatrix} = 15$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>5y + 6x</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>7y + 6x</td>
</tr>
<tr>
<td></td>
<td>$7y + 6x = 30$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$3y = x + 10$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>From (3): $x = 3y - 10$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sub (4) into (1): $7y + 6(3y - 10) = 30$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$25y = 90$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y = 3.6$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sub $y = 3.6$ into (4): $x = 3(3.6) - 10$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = 0.8$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Possible coordinates of $E$ are $(0.8, 3.6)$.</td>
<td>A1 [with rejection]</td>
</tr>
</tbody>
</table>

76
### Additional Mathematics Paper 1 (80 marks)

<table>
<thead>
<tr>
<th>Qn</th>
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</tr>
</thead>
<tbody>
<tr>
<td>1(\sin(A + B) = 0)</td>
<td>(\sin A \cos B + \cos A \sin B = 0)</td>
</tr>
<tr>
<td></td>
<td>(\sin A \left(\frac{\sqrt{15}}{4}\right) + \cos A \left(\frac{1}{4}\right) = 0)</td>
</tr>
<tr>
<td></td>
<td>(\sqrt{15} \sin A + \cos A = 0)</td>
</tr>
<tr>
<td></td>
<td>(\sqrt{15} \sin A = -\cos A)</td>
</tr>
<tr>
<td></td>
<td>(\sin A = \frac{-1}{\sqrt{15}})</td>
</tr>
<tr>
<td></td>
<td>(\cos A = \frac{\sqrt{15}}{1})</td>
</tr>
<tr>
<td></td>
<td>(\tan A = \frac{-1}{\sqrt{15}})</td>
</tr>
<tr>
<td>2</td>
<td>(y = \frac{2x + 16}{x - 1})</td>
</tr>
<tr>
<td></td>
<td>(\frac{dy}{dx} = \frac{(x - 1)(2) - (2x + 16)(1)}{(x - 1)^2})</td>
</tr>
<tr>
<td></td>
<td>(= \frac{2x - 2 - 2x - 16}{(x - 1)^2})</td>
</tr>
<tr>
<td></td>
<td>(= \frac{18}{(x - 1)^2})</td>
</tr>
<tr>
<td></td>
<td>(\frac{dy}{dt} = -2 \frac{dx}{dt})</td>
</tr>
<tr>
<td></td>
<td>(\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt})</td>
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<tr>
<td></td>
<td>(-2 \frac{dx}{dt} = \frac{dy}{dx} \times \frac{dx}{dt})</td>
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<td></td>
<td>(-2 \frac{dx}{dt} = \frac{dy}{dx} \times \frac{dx}{dt})</td>
</tr>
<tr>
<td></td>
<td>(\frac{dy}{dx} = -2)</td>
</tr>
<tr>
<td></td>
<td>(-\frac{18}{(x - 1)^2} = -2)</td>
</tr>
<tr>
<td></td>
<td>((x - 1)^2 = 9)</td>
</tr>
<tr>
<td></td>
<td>(x - 1 = \pm 3)</td>
</tr>
<tr>
<td></td>
<td>(x = 4 \text{ or } -2 \text{ (N.A.)})</td>
</tr>
<tr>
<td></td>
<td>(\text{When } x = 4,)</td>
</tr>
<tr>
<td></td>
<td>(y = \frac{2(4) + 16}{4 - 1})</td>
</tr>
<tr>
<td></td>
<td>(\therefore y = 8)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mark Allocation</th>
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<tbody>
<tr>
<td>M1: Addition Formula</td>
</tr>
<tr>
<td>M1: Find (\cos B)</td>
</tr>
<tr>
<td>A1</td>
</tr>
<tr>
<td>M1</td>
</tr>
<tr>
<td>M1</td>
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<tr>
<td>M1</td>
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<tr>
<td>A1</td>
</tr>
<tr>
<td>Qn</td>
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<td>----</td>
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</tbody>
</table>
| 3  | Let \( \frac{x+1}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4} \)  
\( x+1 = A(x^2+4) + (Bx+C)(x+2) \)  
Sub \( x=-2 \): \(-2+1 = A\left(\frac{-2}{2} + \frac{4}{4}\right) \)  
\(-1 = 8A \)  
\( A = \frac{1}{8} \)  
Comparing coefficients of \( x^2 \): \( 0 = A + B \)  
\( B = \frac{1}{8} \)  
Sub \( x=0 \): \( 1 = 4A + 2C \)  
\( 2C = 1 - 4\left(-\frac{1}{8}\right) \)  
\( C = \frac{3}{4} \)  
\( \frac{x+1}{(x+2)(x^2+4)} = \frac{1}{8(x+2)} + \frac{x+3}{8(x^2+4)} \)  
\( = \frac{x+6}{8(x^2+4)} - \frac{1}{8(x+2)} \) | M1  
M2.  
Any 2 correct  
1 mark |
| 4i | \( kx(x+3) = x - k \)  
\( kx^2 + (3k-1)x + k = 0 \)  
\( (3k-1)^2 - 4(k)(k) > 0 \)  
\( 9k^2 - 6k + 1 - 4k^2 > 0 \)  
\( 5k^2 - 6k + 1 > 0 \)  
\( (5k-1)(k-1) > 0 \)  
\( k < \frac{1}{5} \text{ or } k > 1 \) | M1  
M1: \( b^2 - 4ac > 0 \)  
M1: Factorise  
A1 |
| 4ii | \( (3k-1)^2 - 4(k)(k) < 0 \)  
\( (5k-1)(k-1) < 0 \)  
\( \frac{1}{5} < k < 1 \) | M1: \( b^2 - 4ac < 0 \)  
A1 |
\[
\left(1 + \frac{x}{3}\right)^n = 1 + \frac{n}{1} \left(\frac{x}{3}\right) + \frac{n(n-1)}{2!} \left(\frac{x^2}{9}\right) + \ldots
\]
\[
= 1 + \frac{nx}{3} + \frac{n(n-1)}{2!} \left(\frac{x^2}{9}\right) + \ldots
\]
\[
= 1 + \frac{n}{3} x + \frac{n(n-1)}{18} x^2 + \ldots
\]

\[
(1 - kx) \left(1 + \frac{x}{3}\right)^n = (1 - kx) \left(1 + \frac{n}{3} x + \frac{n(n-1)}{18} x^2 + \ldots\right)
\]
\[
= 1 + \frac{nx}{3} + \frac{n(n-1)}{18} x^2 - \frac{nk}{3} x^2 + \ldots
\]
\[
= 1 + \left(\frac{n}{3} - k\right) x + \left[\frac{n(n-1)}{18} - \frac{nk}{3}\right] x^2 + \ldots
\]

\[
\frac{n}{3} - k = 0
\]
\[
n = 3k ---- (1)
\]
\[
\frac{n(n-1)}{18} - \frac{nk}{3} = \frac{7}{3} ---- (2)
\]

Sub (1) into (2):
\[
3k(3k-1) - \frac{3k^2}{3} = \frac{7}{3}
\]
\[
9k^2 - 3k - 18k^2 + 42 = 0
\]
\[
-9k^2 - 3k + 42 = 0
\]
\[
3k^2 + k - 14 = 0
\]
\[
(3k + 7)(k - 2) = 0
\]
\[
k = -\frac{7}{3} \text{ or } k = 2
\]

Sub \(k = -\frac{7}{3}\) into (1): \(n = 3 \left(\frac{7}{3}\right)\)
\[
n = 7 \text{ (N.A.) } \therefore \text{ Reject } k = -\frac{7}{3}
\]

Sub \(k = 2\) into (1): \(n = 3(2)\)
\[
n = 6
\]
\[
\therefore n = 6, k = 2.
\]
<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Mark Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>6i</td>
<td>$LHS = \frac{\cos^2 x - \sin^2 x}{1 + 2 \sin x \cos x}$&lt;br&gt;[= \frac{(\cos x - \sin x)(\cos x + \sin x)}{\sin^2 x + \cos^2 x + 2 \sin x \cos x} \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x + \sin x)^2} = \frac{\cos x - \sin x}{\cos x + \sin x} \frac{\cos x - \sin x}{\cos x + \sin x} = \frac{\cos x - \sin x}{\cos x + \sin x} \frac{\cos x - \sin x}{\cos x + \sin x} = \frac{1 - \tan x}{1 + \tan x} = RHS (shown)$</td>
<td>M1: Factorise numerator&lt;br&gt;M1: Factorise denominator&lt;br&gt;A1: Divide by $\cos x$</td>
</tr>
</tbody>
</table>
| 6ii | $\frac{\cos^2 x - \sin^2 x}{1 + 2 \sin x \cos x} = \frac{2 \tan x}{1 - \tan x}$<br>\[1 + \tan x = \frac{3}{2} \tan x = 2 \tan^2 x + 2 \tan x + 3 = 0 \left(\tan x - \frac{1}{2}\right)(\tan x + 3) = 0 \tan x = \frac{1}{2}$<br>Basic $\angle = 26.565$
\[x = 26.565, 180 - 26.565$
OR<br>$\tan x = -3$
Basic $\angle = 71.565$
\[x = 180 - 71.565, 360 - 71.565$
\[\therefore x = 26.6^\circ, 108.4^\circ, 206.6^\circ, 288.4^\circ$ | M1: Factorise numerator<br>M1: Find basic angle (both correct)<br>A1 |
| 7 | $f''(x) = \sin 2x + \cos 3x$
\[f(x) = \int (\sin 2x + \cos 3x) \, dx = -\frac{1}{2} \cos 2x + \frac{1}{3} \sin 3x + c$
\[f\left(\frac{\pi}{6}\right) = -\frac{1}{2} \cos 2\left(\frac{\pi}{6}\right) + \frac{1}{3} \sin 3\left(\frac{\pi}{6}\right) + c$ | M1 |
<table>
<thead>
<tr>
<th>Qn</th>
<th>Solution</th>
<th>Mark Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\frac{1}{2} \cos \left(\frac{\pi}{3}\right) + \frac{1}{3} \sin \left(\frac{\pi}{2}\right) + c = 0)</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>(-\frac{1}{2} \left(\frac{1}{2}\right) + \frac{1}{3} (1) + c = 0)</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>c = \frac{1}{12}</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>(f(x) = \frac{1}{2} \cos 2x + \frac{1}{3} \sin 3x - \frac{1}{12})</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>(f''(x) = 2 \cos 2x - 3 \sin 3x)</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>(f''(x) + 9f(x) = 2 \cos 2x - 3 \sin 3x + 9 \left(\frac{1}{2} \cos 2x + \frac{1}{3} \sin 3x - \frac{1}{12}\right))</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>(= 2 \cos 2x - 3 \sin 3x + \frac{9}{2} \cos 2x + 3 \sin 3x - \frac{3}{4})</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>(= \frac{3}{4} \cos 2x) (shown)</td>
<td>A1</td>
<td></td>
</tr>
</tbody>
</table>

8i

\(y = 10^{k-x}\)

\(\log y = (k - nx) \log 10\)

\(\log y = -nx + k\)

\(m = \frac{2 - (-7)}{1 - 4}\)

\(= \frac{-3}{3}\)

\(-n = -3\)

\(\therefore n = 3\)

\(\therefore k = 5\)

\(\therefore n = 3\)

At \((1, 2)\), \(2 = (-3)(1) + c\)

\(c = 5\)

\(\therefore k = 5\)

8ii

\(y = 10^{5-3x}\) \(---- (1)\)

\(y = 10^x\) \(---- (2)\)

Sub (1) into (2): \(10^{5-3x} = 10^x\)

\(x = 5 - 3x\)

\(4x = 5\)

\(x = \frac{5}{4}\)

Sub \(x = \frac{5}{4}\) into (2): \(y = 10^{\frac{5}{4}}\)

\(y = 17.783\)

\(\log y = \log 17.783\)

\(= 1.25\)

\(\therefore \text{Coordinates are (1.25, 1.25)}\)

\(= 1.25\)
<table>
<thead>
<tr>
<th>Qn</th>
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</tr>
</thead>
</table>
| 9i | \( y = x \ln 3x \)  \\
|    | \( \frac{dy}{dx} = x \left( \frac{1}{x} \right) + (\ln 3x)(1) \)  \\
|    | \( = 1 + \ln 3x \)  \\
|    | At \( P(1, \ln 3) \), \( \frac{dy}{dx} = 1 + \ln 3(1) \)  \\
|    | \( = 1 + \ln 3 \)  \\
|    | At \( P(1, \ln 3) \), \( y - \ln 3 = (1 + \ln 3)(x - 1) \)  \\
|    | \( y = (1 + \ln 3)x - 1 \)  \\
|    | When \( y = 0 \), \( (1 + \ln 3)x = 1 \)  \\
|    | \( x = \frac{1}{(1 + \ln 3)} \)  \\
|    | \( x = 0.477 \)  \\
|    | Coordinates of \( Q \) are \((0.477, 0)\)  \\
|    | \( \tan \theta = \frac{\ln 3 - 0}{1 - 0.477} \)  \\
|    | \( = 2.0986 \)  \\
|    | \( \theta = 64.5^\circ \)  \\
|    | M1  \\
| 9ii| Gradient of normal = \(- \frac{1}{1 + \ln 3x} \)  \\
|    | \( - \frac{1}{1 + \ln 3x} = -2 \)  \\
|    | \( 1 + \ln 3x = \frac{1}{2} \)  \\
|    | \( \ln 3x = \frac{1}{2} \)  \\
|    | \( 3x = e^{\frac{1}{2}} \)  \\
|    | \( x = \frac{1}{3} e^{\frac{1}{2}} \)  \\
|    | \( = 0.202 \)  \\
|    | M1  \\
| 10i| \( (x + (2 \times 2)) \times (y + (1.5 \times 2)) = 825 \)  \\
|    | \( (x + 4)(y + 3) = 825 \)  \\
|    | \( y + 3 = \frac{825}{x + 4} \)  \\
|    | \( y = \frac{825}{x + 4} - 3 \)  \\
|    | M1  \\
|
\[ A = \frac{825}{x+4} - 3 \]

\[ A = \frac{825x - 3x}{x+4} \quad \text{(shown)} \]

\[ \frac{dA}{dx} = \frac{(x+4)(825)-(825x)(1)}{(x+4)^2} - 3 \]

\[ = \frac{3300}{(x+4)^2} - 3 \]

\[ \frac{3300}{(x+4)^2} - 3 = 0 \]

\[ (x+4)^2 = 1100 \]

\[ x+4 = \pm \sqrt{1100} \]

\[ x = 29.2 \quad \text{or} \quad -37.2 \quad \text{(N.A.)} \]

\[ \frac{d^2A}{dx^2} = -2 \left( \frac{3300}{(x+4)^3} \right) \]

When \( x = 29.2 \),

\[ \frac{d^2A}{dx^2} = \frac{-6000}{(29.2 + 4)^3} \]

\[ = -0.181 \]

Since \( \frac{d^2A}{dx^2} < 0 \), this value of \( x \) gives the poster the largest printing area possible.

11i

\[ y = \begin{cases} 
14x - 4 & \text{if } x > 2 \\
2x & \text{if } 2 < x < 7 \\
7x - 2 & \text{if } x < 2 
\end{cases} \]

B1: V-shape graph
B1: Correct \( x \)- and \( y \)-intercepts and end-points labelled
<table>
<thead>
<tr>
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</tr>
</thead>
</table>
| 7x - 2 = \frac{3}{2}x \quad \text{or} \quad 7x - 2 = -\frac{3}{2}x | \begin{align*} \frac{1}{2}x &= 2 \\ \frac{8}{2}x &= 2 \\ x &= \frac{4}{11} \quad \text{or} \quad x = \frac{4}{17} \\
\therefore \quad x \leq \frac{4}{17} \quad \text{or} \quad x \geq \frac{4}{11} | M1 |
| 111 | 1 \text{ intersection;} \\
| 111a | \text{Line parallel to L.H. arm cuts R.H. arm at only one point.} | B1 |
| 111b | 0 \text{ intersection;} \\
| 111b | \text{Line parallel to R.H. arm; for intersection, } c > -2. | B1 |
| 12i | m_{AB} = m_{OC} \quad \frac{y-0}{x-5} = \frac{6}{-2} \quad y = -3x + 15 \quad ----- (1) \quad m_{BC} = -\frac{1}{m_{OC}} \quad \frac{y-6}{x+2} = \frac{1}{3} \quad y = \frac{1}{3}x + 6 \frac{2}{3} \quad ----- (2) \quad \text{Sub (1) into (2): } -3x + 15 = \frac{1}{3}x + 6 \frac{2}{3} \quad 3 \frac{1}{2}x = 8 \frac{1}{3} \quad x = 2 \frac{1}{2} \quad \text{Sub } x = 2 \frac{1}{2} \text{ into (1): } y = -3 \left(2 \frac{1}{2}\right) + 15 \quad y = 7 \frac{1}{2} \quad \text{Coordinates of } B \text{ are } \left(2 \frac{1}{2}, 7 \frac{1}{2}\right) | M1 | A1 |
### Question 12

#### Part ii

- **Solution**:
  
  \[
  B \left( \frac{1}{2}, \frac{1}{2} \right)
  \]

  \[
  A(5, 0)
  \]

  \[
  D \left( \frac{-2}{2}, \frac{1}{2} \right)
  \]

  \[
  \begin{align*}
  \text{Midpoint of } OC &= \left( \frac{0+\left(\frac{-2}{2}\right)}{2}, \frac{0+\left(\frac{6}{2}\right)}{2} \right) \\
  &= \left( -1, 3 \right)
  \\
  \text{Gradient of perpendicular bisector of } OC &= \frac{1}{3}
  \\
  \text{At } (-1, 3), \quad y &= \frac{1}{3} (x + 1)
  \\
  y &= \frac{1}{3} x + \frac{3}{3}
  \\
  
  \end{align*}
  \]

  **Mark Allocation**: M1 O.E.

#### Part iii

- **Solution**:
  
  \[
  \begin{align*}
  \text{Midpoint of } OC &= \left( \frac{0+\left(\frac{-2}{2}\right)}{2}, \frac{0+\left(\frac{6}{2}\right)}{2} \right) \\
  &= \left( -1, 3 \right)
  \\
  \text{Gradient of perpendicular bisector of } OC &= \frac{1}{3}
  \\
  \text{At } (-1, 3), \quad y &= \frac{1}{3} (x + 1)
  \\
  y &= \frac{1}{3} x + \frac{3}{3}
  \\
  
  \end{align*}
  \]

  **Mark Allocation**: M1

#### Part iv

- **Solution**:
  
  \[
  \begin{align*}
  \begin{vmatrix}
  1 & 5 & x & -2 & 0 \\
  2 & 0 & 0 & y & 6 \\
  \end{vmatrix} &= 15 \\
  \end{align*}
  \\
  \begin{align*}
  |5y + 6x - (-2y)| &= 30 \\
  |7y + 6x| &= 30
  \\
  7y + 6x &= 30 \quad ----- (1) \quad \text{or} \quad 7y + 6x = -30 \quad ----- (2)
  \\
  3y = x + 10 \quad ----- (3)
  \\
  \text{From (3): } x = 3y - 10 \quad ----- (4)
  \\
  \text{Sub (4) into (1):} \quad 7y + 6(3y - 10) &= 30 \\
  25y &= 90 \\
  y &= 3.6
  \\
  \text{Sub } y = 3.6 \text{ into (4):} \quad x &= 3(3.6) - 10
  \\
  x &= 0.8
  \\
  \text{Possible coordinates of } E \text{ are } (0.8, 3.6).
  \\
  \end{align*}
  \]

  **Mark Allocation**: M1 [with rejection]
ADDITIONAL MATHEMATICS
Paper 2

Friday 21 August 2015
2 hours 30 minutes

Additional materials: Answer paper (8 sheets)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Submit Section A and B separately

This question paper consists of 6 printed pages.

Setter: Mr Ang Hanping
Verter: Ms Zoe Paw

We Nurture Students to Think, Care and Lead with P.R.I.D.E.
1. ALGEBRA

Quadratic Equation
For the equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.
\]

Binomial Theorem
\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n
\]
where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \).

2. TRIGONOMETRY

Identities
\[
\sin^2 A + \cos^2 A = 1
\]
\[
\sec^2 A = 1 + \tan^2 A
\]
\[
\csc^2 A = 1 + \cot^2 A
\]
\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
\sin 2A = 2 \sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]
\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \triangle ABC \)
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\Delta = \frac{1}{2} bc \sin A
\]
Answer all the questions.

Section A (59 marks)

1. The diagram shows triangles $ABC$ and $BCD$ whose vertices lie on the circumference of a circle. The chords $BD$ and $AC$ intersect at $E$ and $AC$ is parallel to $FG$. $FG$ is a tangent to the circle at $B$.

![Diagram of triangles and circle]

Show that
(i) $\triangle BCD$ is similar to $\triangle BEC$, [2]
(ii) $BC^2 = BD \times BE$, [1]
(iii) $\triangle ABC$ is an isosceles triangle. [2]

2. A company buys an engine at a cost of $120,000. The value of the engine decreases with time so that its value, $V$, after $t$ months is given by

$$V = 120,000e^{-kt},$$

where $k$ is a positive constant. The value of the engine is expected to be $75,000 after 30 months.

(i) Calculate the value, to the nearest $100, of the engine after 20 months. [4]

It is only economical to replace the engine after its value reaches $\frac{1}{2}$ of its original value.

(ii) Determine, with working, whether it is economical to replace the engine after 40 months. [2]

3. (a) An equilateral triangle has sides $(3 + \sqrt{7})$ cm in length. Find, without using a calculator, the area of the equilateral triangle. [3]

(b) A cuboid of volume $(30 + 12\sqrt{3})$ cm$^3$ has a base area of $(4 - 2\sqrt{3})$ cm$^2$. Find, without using a calculator, the height of the cuboid. [3]
4  (a) Solve the equation \( \log_3(2x - 1) - \frac{1}{2} \log_3(x^2 + 2) = \log_5 5 \).  \[5\]

(b) Evaluate \( \log_p 32 \times \log_p p \).  \[3\]

5  It is given that \( f(x) = 2x^3 + ax^2 + x + b \).
(i) Find the value of \( a \) and of \( b \) for which \( 2x^2 + x - 1 \) is a factor of \( f(x) \).  \[4\]
(ii) Solve the equation \( f(x) = 0 \).  \[2\]
(iii) Hence solve \( \frac{1}{4}x^3 + \frac{a}{4}x^2 + \frac{1}{2}x + b = 0 \).  \[2\]

6  A curve has the equation \( y = (2x + 2)\sqrt{2x - 1} \).
(i) Show that \( \frac{dy}{dx} = \frac{kx}{\sqrt{2x - 1}} \), where \( k \) is a constant and state the value of \( k \).  \[4\]
(ii) Hence evaluate \( \int_{\frac{1}{3}}^{\frac{3}{2}} \frac{3x}{2\sqrt{2x - 1}} \, dx \).  \[4\]

7  The diagram below shows an experimental setup where a weighted spring is released from a stretched position and follows a periodic up-down motion. The length of the spring, \( l \) cm, during the experiment is modelled by the equation, \( l = a \cos kt + 16 \), where \( a, k \) are constants, and \( t \) is the time in seconds after releasing the weight from the lowest position.

The length of the spring is 20 cm when the weight is at its lowest position and it takes 2 seconds for the weight to move from the lowest to highest position.

(i) Find the value of \( a \).  \[1\]
(ii) Show that the value of \( k \) is \( \frac{\pi}{2} \).  \[2\]
(iii) Find the length of the spring when the weight is at its highest position. [1]
(iv) Sketch the graph of \( l = a \cos kt + 16 \) for \( 0 \leq t \leq 4 \). [2]
(v) Find the time interval which the length of the spring will be longer than 18 cm for \( 0 \leq t \leq 4 \). [3]

8
(a) The equation of a curve is \( y = x^3 + 4x^2 + kx + 3 \), where \( k \) is a constant. Find the set of values of \( k \) for which the curve is always an increasing function. [4]
(b) A curve with equation in the form \( y = ax + \frac{b}{x^2} \) has a stationary point at \((3, 4)\), where \( a \) and \( b \) are constants. Find the value of \( a \) and of \( b \). [5]

Section B (41 marks)

Begin this section on a fresh sheet of paper.

9
The equation \( 3x^2 + kx + 3 = 0 \), where \( k > 0 \) has roots \( \alpha \) and \( \beta \). A second equation \( 3x^2 - 2x + 3 = 0 \) has roots \( \alpha^3 \beta \) and \( \alpha \beta^3 \).
(i) Show that \( \alpha^2 + \beta^2 = \frac{2}{3} \). [3]
(ii) Find the value of \( k \). [3]
(iii) Form an equation whose roots are \( \alpha^3 \) and \( \beta^3 \). [3]

10
The diagram shows part of the curve \( y = 2 \sin(2x + \pi) - 1 \), meeting the \( x \)-axis at the points \( A \) and \( B \).

(i) Find the \( x \)-coordinate of \( A \) and of \( B \). [4]
(ii) Find the total area of the shaded regions. [6]
11 A particle, travelling in a straight line, passes a fixed point \( O \) on the line with a speed of 2 ms\(^{-1}\). The acceleration, \( a \) ms\(^{-2}\), of the particle, \( t \) s after passing \( O \), is given by \( a = -4e^{-t} \).

(i) Show that the particle comes instantaneously to rest when \( t = -\ln \frac{1}{2} \). \([4]\)

(ii) Find the total distance travelled by the particle between \( t = 0 \) and \( t = 2 \). \([6]\)

(iii) Find the average velocity of the particle during the first 2 seconds. \([1]\)

12 A circle, \( C_t \), passing through the point \( A \) (4, 8) has the same centre as another circle \( C_s \). The equation of \( C_s \) is given by \( x^2 + y^2 - 16x - 10y + 5 = 0 \).

(i) Find the equation of \( C_t \). \([3]\)

\( AB \) is a diameter of \( C_t \).

(ii) Find the coordinates of \( B \). \([2]\)

(iii) Show that the equation of the tangent to \( C_t \) at \( B \) is \( 3y = 4x - 42 \). \([3]\)

The lowest point on the circle \( C_t \) is \( D \).

(iv) Explain why the x-axis is a tangent to the circle at \( D \). \([1]\)

(v) Find the equation of another tangent to circle \( C_t \) passing through the origin. \([2]\)

END OF PAPER
### Additional Mathematics Paper 2 (100 marks)

<table>
<thead>
<tr>
<th>Qn. #</th>
<th>Solution</th>
<th>Mark Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2i</td>
<td>$V = 87700$</td>
<td></td>
</tr>
<tr>
<td>2ii</td>
<td>$V = 64124.098$ It is not economical to replace the engine after 40 months as the value of the engine has not reached half its original value of $120000$</td>
<td></td>
</tr>
<tr>
<td>3a</td>
<td>Area of equilateral triangle $= \frac{1}{2}(8\sqrt{3} + 3\sqrt{21})$ cm²</td>
<td></td>
</tr>
<tr>
<td>3b</td>
<td>Height of cuboid = $48 + 27\sqrt{3}$ cm</td>
<td></td>
</tr>
<tr>
<td>4a</td>
<td>$x = 5$ or $x = -1$ (rej)</td>
<td></td>
</tr>
<tr>
<td>4b</td>
<td>$\log_{10} 32 \times \log_{10} p = 1\frac{2}{3}$</td>
<td></td>
</tr>
<tr>
<td>5i</td>
<td>$b = -2$ $a = 5$</td>
<td></td>
</tr>
<tr>
<td>5ii</td>
<td>$x = \frac{1}{2}$ or $x = -1$ or $x = -2$</td>
<td></td>
</tr>
<tr>
<td>5iii</td>
<td>$x = 1$ or $x = -2$ or $x = -4$</td>
<td></td>
</tr>
<tr>
<td>6i</td>
<td>$k = 6$</td>
<td></td>
</tr>
<tr>
<td>6ii</td>
<td>$\int_{\frac{3\pi}{2}}^{\frac{3\pi}{2}} \frac{3x}{2\sqrt{2x-1}} , dx = 26$</td>
<td></td>
</tr>
<tr>
<td>7i</td>
<td>$a = 4$</td>
<td></td>
</tr>
<tr>
<td>7ii</td>
<td>$k = \frac{\pi}{2}$</td>
<td></td>
</tr>
<tr>
<td>7iii</td>
<td>Length of string = 12 cm</td>
<td></td>
</tr>
<tr>
<td>7iv</td>
<td><img src="image" alt="Graph" /></td>
<td></td>
</tr>
<tr>
<td>7v</td>
<td>Time interval $= 0 \leq t &lt; \frac{2}{3}$ or $3\frac{1}{3} &lt; t \leq 4$</td>
<td></td>
</tr>
<tr>
<td>Qn. #</td>
<td>Solution</td>
<td>Mark Allocation</td>
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<tr>
<td>------</td>
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<td>-----------------</td>
</tr>
<tr>
<td>8a</td>
<td>$k &gt; 5\frac{1}{3}$</td>
<td></td>
</tr>
<tr>
<td>8b</td>
<td>$b = 12$</td>
<td>$a = 8$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9i</td>
<td>$\alpha^2 + \beta^2 = \frac{2}{3}$</td>
<td></td>
</tr>
<tr>
<td>9ii</td>
<td>$k = -\frac{4\sqrt{6}}{3}$ (N.A.) or $\frac{4\sqrt{6}}{3}$</td>
<td></td>
</tr>
<tr>
<td>9iii</td>
<td>Equation: $x^3 - \frac{2\sqrt{6}}{9}x + 1 = 0$</td>
<td></td>
</tr>
<tr>
<td>10i</td>
<td>$x$ coordinate of $A = \frac{7\pi}{12}$, $x$ coordinate of $B = \frac{11\pi}{12}$</td>
<td></td>
</tr>
<tr>
<td>10ii</td>
<td>Shaded area $= 4.38$ unit$^2$</td>
<td></td>
</tr>
<tr>
<td>11i</td>
<td>$t = -\ln \frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td>11ii</td>
<td>Total distance travelled $= 1.77\text{ m}$</td>
<td></td>
</tr>
<tr>
<td>11iii</td>
<td>Average velocity $= -0.271\text{ m/s}$</td>
<td></td>
</tr>
<tr>
<td>12i</td>
<td>Equation: $(x-8)^2 + (y-5)^2 = 25$</td>
<td></td>
</tr>
<tr>
<td>12ii</td>
<td>Coordinates of $B = (12, 2)$</td>
<td></td>
</tr>
<tr>
<td>12v</td>
<td>Equation of tangent: $y = \frac{80}{39}x$</td>
<td></td>
</tr>
</tbody>
</table>
## Additional Mathematics Paper 2 (100 marks)

<table>
<thead>
<tr>
<th>Qn. #</th>
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</tr>
</thead>
</table>
| 1i    | \(\angle BDC = \angle CBG\) (alternate segment theorem)  
      | \(\angle BCE = \angle CBG\) (alternate angles, AC//FG)  
      | \(\angle BDC = \angle BCE\)  
      | \(\angle CBD = \angle EBC\) (common angle)  
      | Since the corresponding angles of the triangles are equal, \(\Delta BCD\) is similar to \(\Delta BEC\) | M1 |
| 1ii   | Since \(\Delta BCD\) is similar to \(\Delta BEC\)  
      | \[
      \frac{BC}{BE} = \frac{BD}{BC}  
      \]
      | \[
      BC^2 = BD \times BE  
      \] | B1 |
| 1iii  | \(\angle BDC = \angle BCE\) (from (i))  
      | \(\angle CBG = \angle ACB\) (alt \(\angle\))  
      | \(\angle BDC = \angle BAC\) (\(\angle\) in same seg) or \(\angle CBG = \angle BAC\) (alt seg)  
      | \(\angle BCE = \angle BAC\)  
      | \(\angle ACB = \angle BAC\)  
      | \(\Delta ABC\) is an isosceles triangle. \(\Delta ABC\) is isosceles. | M1  
      | A1 |
| 2i    | \[
      75000 = 120 000e^{-0.3(t)}  
      \]
      | \[
      \ln \frac{5}{8} = -30k  
      \]
      | \[
      k = -\frac{1}{30} \ln \frac{5}{8}  
      \]
      | After 20 months \(V\)  
      | \[
      V = 120 000e^{-\left(-\frac{1}{30}\right)^{0.3} \times 20}  
      \]
      | \[
      = 87720.53215  
      \]
      | \[
      = 87700  
      \] | M1  
      | M1  
      | A1 |
| 2ii   | \[
      V = 120 000e^{-\left(-\frac{1}{30}\right)^{0.40}}  
      \]
      | \[
      = 64124.098  
      \]
      | It is not economical to replace the engine after 40 months as the value of the engine has not reached half its original value of $120000 | M1  
      | A1 |
| 3a    | Area of equilateral triangle  
      | \[
      = \frac{1}{2} \left(3 + \sqrt{3}\right)^2 \sin 60  
      \]
      | \[
      = \frac{1}{2} \left(9 + 6\sqrt{3} + 7\right) \left(\frac{\sqrt{3}}{2}\right)  
      \]
      | \[
      = \frac{1}{4} \left(16\sqrt{3} + 6\sqrt{21}\right)  
      \]
      | \[
      = \frac{1}{2} \left(8\sqrt{3} + 3\sqrt{21}\right) \text{ cm}^2  
      \] | M1  
      | M1  
      | M1  
<pre><code>  | A1 |
</code></pre>
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</thead>
</table>
| 3b    | Height of cuboid  
\[
\begin{align*}
  &\frac{30 + 12\sqrt{3}}{4 - 2\sqrt{3}} \\
  &= \frac{30 + 12\sqrt{3}}{4 - 2\sqrt{3}} \cdot \frac{4 + 2\sqrt{3}}{4 + 2\sqrt{3}} \\
  &= \frac{120 + 108\sqrt{3} + 72}{16 - 12} \\
  &= 48 + 27\sqrt{3} \\
\end{align*}
\] | M1 |

| 4a    | \[
\begin{align*}
  &\log_3(2x-1) - \frac{1}{2} \log_3(x^2 + 2) = \log_2, 5 \\
  &\log_3(2x-1) = \frac{1}{2} \log_3(x^2 + 2) = \frac{1}{2} \\
  &2\log_3(2x-1) - \log_3(x^2 + 2) = 1 \\
  &\log_3(2x-1)^2 - \log_3(x^2 + 2) = 1 \\
  &\log_3 \left(\frac{(2x-1)^2}{x^2 + 2}\right) = 1 \\
  &\frac{(2x-1)^2}{x^2 + 2} = 3 \\
  &4x^2 - 4x + 1 = 3x^2 + 6 \\
  &x^2 - 4x - 5 = 0 \\
  &(x+1)(x-5) = 0 \\
  &x = 5 \text{ or } x = -1 \text{ (rej)} \\
\end{align*}
\] | M1 |

| 4b    | \[
\begin{align*}
  &\log_3 32 \times \log_8 p \\
  &= \log_3 32 \times \log_2 p \\
  &= \log_3 2^5 \times \log_2 p \\
  &= \log_3 2^5 \\
  &= \log_3 2^5 \\
  &= \frac{5}{3} \\
\end{align*}
\] | M1 |

| 5i    | \[
\begin{align*}
  &2x^2 + x - 1 = (2x-1)(x+1) \\
  &\text{and } (2x - 1) \text{ and } (x + 1) \text{ are factors} \\
  &f\left(\frac{1}{2}\right) = 0 \\
  &2 \left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right) + b = 0 \\
\end{align*}
\] | M1 |
<table>
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<tr>
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</tr>
</thead>
</table>
| i     | \[
\frac{1}{4} + \frac{1}{4}a + \frac{1}{2}b = 0
\]
\[
a + 4b = -3  \tag{1}
\]
\[f(-1) = 0\]  
\[
2(-1)^{\frac{1}{2}} + a(-1)^{\frac{3}{2}} + (-1) + b = 0
\]
\[-2 + a - 1 + b = 0\]
\[a + b = 3  \tag{2}\]
\[(1) - (2): 3b = -6\]
\[b = -2\]
\[a = 5\]  | M1, M1, A1 |
| ii    | \[f(x) = 0\]  
\[
2x^3 + 5x^2 + x - 2 = 0
\]
\[
(2x^3 + x - 1)(x + 2) = 0
\]
\[
(2x - 1)(x + 1)(x + 2) = 0
\]
\[x = \frac{1}{2} \text{ or } x = -1 \text{ or } x = -2\]  | M1, A1 |
| iii   | \[\frac{1}{4}x^3 + \frac{a}{4}x^2 + \frac{1}{2}x + b = 0\]  
Let \(x = 2y\)  
\[
\frac{1}{4}(2y)^3 + \frac{a}{4}(2y)^2 + \frac{1}{2}(2y) + b = 0
\]
\[2y^3 + ay^2 + y + b\]  
From (ii), \(y = \frac{1}{2} \text{ or } y = -1 \text{ or } y = -2\)  
\[x = 1 \text{ or } x = -2 \text{ or } x = -4\]  | M1, A1 |
| iv    | \[
\frac{dy}{dx} = (2x + 2) \left[ \frac{1}{2}(2x - 1)^{\frac{1}{2}} \right] + (2x - 1)^{\frac{1}{2}}
\]
\[= (2x - 1)^{\frac{1}{2}} \left\{ [(2x + 2) + 2(2x - 1)] \right\}
\]
\[= (2x - 1)^{\frac{1}{2}} \left\{ 6x \right\}
\]
\[= \frac{6x}{\sqrt{2x - 1}}\]  
\[k = 6\]  | M1, A1, A1 |
<table>
<thead>
<tr>
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<th>Solution</th>
<th>Mark Allocation</th>
</tr>
</thead>
</table>
| 6ii   | \[
\int_{0}^{3} \frac{6x}{\sqrt{2x-1}} \, dx = \left( \frac{(2x+2)\sqrt{2x-1}}{3} \right)_{0}^{3} \\
\int_{0}^{3} \frac{6x}{\sqrt{2x-1}} \, dx = \frac{1}{4} \left( (2x+2)\sqrt{2x-1} \right)_{0}^{3} \\
\int_{0}^{3} \frac{3x}{2\sqrt{2x-1}} \, dx = \frac{1}{4} \left( 28\sqrt{25} - (12)\sqrt{9} \right) \\
= 26
\] | M1 M1 M1 A1 |
<p>| 7i    | [20 = a \cos k(0) + 16] [a = 4] | B1 |
| 7ii   | [\frac{2\pi}{k} = 4] [k = \frac{\pi}{2}] | M1 A1 |
| 7iii  | Length of string = 20 – 8 = 12 cm | B1 |
| 7iv   | ![Graph of a function showing a wave with peaks at 16 cm and 12 cm, and a trough at 0 cm.] | B1 – correct shape [\text{B1 – correct values}] |
| 7v    | [4 \cos \frac{\pi}{2} t + 16 = 18] [\cos \frac{\pi}{2} t = \frac{1}{2}] [\text{Basic angle} = \cos^{-1} \frac{1}{2}] [\frac{\pi}{2} t = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}] [t = \frac{2}{3}, \frac{1}{3}] [\text{Time interval} = 0 \leq t &lt; \frac{2}{3} \text{ or } \frac{1}{3} &lt; t \leq 4] | M1 A1 A1 |</p>
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</tr>
</thead>
</table>
| 8a    | \[ \frac{dy}{dx} = 3x^2 + 8x + k \]  
For curve to be always increasing, \( \frac{dy}{dx} \) always > 0  
\( b^2 - 4ac < 0 \)  
\( 64 - 4(3)(k) < 0 \)  
\( k > 5\frac{1}{3} \) | M1 |
| 8b    | \[ \frac{dy}{dx} = a - \frac{2b}{x^3} \]  
At stat. pt. \( \frac{dy}{dx} = 0 \)  
\( a - \frac{2b}{x^3} = 0 \)  
Sub \( x = 3, y = 4 \)  
\( a - \frac{2b}{3^3} = 0 \)  
\( a = \frac{2b}{27} \) \( \text{--- (1)} \)  
Sub \( x = 3, y = 4 \) to equation of curve  
\( 4 = a(3) + \frac{b}{3^2} \)  
\( 36 = 27a + b \) \( \text{--- (2)} \)  
Sub (1) to (2):  
\( 36 = 2b + b \)  
\( b = 12 \)  
\( a = \frac{24}{27} = \frac{8}{9} \) | M1 |
| 9i    | \( \alpha \beta = 1 \)  
\( \alpha^3 \beta + \alpha \beta^3 = \frac{2}{3} \)  
\( \alpha \beta (\alpha^2 + \beta^2) = \frac{2}{3} \)  
\( 1(\alpha^2 + \beta^2) = \frac{2}{3} \)  
\( \alpha^2 + \beta^2 = \frac{2}{3} \) | M1 – either one correct |

\( 188 \)
### Solution

#### 9ii
\[(\alpha + \beta)^3 = \alpha^3 + \beta^3 + 2\alpha \beta\]
\[= \frac{2}{3} + 2(1)\]
\[= \frac{8}{3}\]
\[(\alpha + \beta)^3 = \frac{8}{3}\]
\[\alpha + \beta = \frac{2\sqrt{6}}{3} \text{ or } \frac{-2\sqrt{6}}{3}\]
\[-k = \frac{2\sqrt{6}}{3} \text{ or } \frac{-2\sqrt{6}}{3}\]
\[k = -\frac{4\sqrt{6}}{3} \text{ (N.A.) or } \frac{4\sqrt{6}}{3}\]

#### 9iii
S.O.R. = \(\alpha^3 + \beta^3\)
\[= (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)\]
\[= \left(\frac{-2\sqrt{6}}{3}\right)\left(\frac{2}{3} - 1\right)\]
\[= \frac{2\sqrt{6}}{9}\]
P.O.R. = \(\alpha^3 \beta^3\)
\[= (\alpha\beta)^3\]
\[= (1)^3 = 1\]
Equation: \(x^3 - \frac{2\sqrt{6}}{9} \cdot x + 1 = 0\)

#### 10i
\[2\sin(2x + \pi) - 1 = 0\]
\[\sin(2x + \pi) = \frac{1}{2}\]
\[\alpha = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}\]
\[2x + \pi = \frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6} + 2\pi, \frac{\pi}{6} + 2\pi\]
\[x = \frac{5\pi}{12}, \frac{\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}\]
\[x \text{ coordinate of } A = \frac{7\pi}{12}, \text{ x coordinate of } B = \frac{11\pi}{12}\]
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</table>
| 10ii  | Shaded area  
\[
\int_{\pi}^{2\pi} 2\sin(2x + \pi) - 1 \, dx + \int_{\pi}^{2\pi} 2\sin(2x + \pi) - 1 \, dx \\
= -\left[-\cos(2x + \pi) - x\right]_{\pi}^{2\pi} + \left[-\cos(2x + \pi) - x\right]_{\pi}^{2\pi} \\
= \left[-2.69862 - 1\right] + \left[-2.01377 - (-2.69862)\right] \\
= 4.38 \text{ unit}^2 
\] | M2 – 1 for limits, 1 for expressions |
| 11i   | \[v = \int -4e^{-t} \, dt\] 
\[= 4e^{-t} + c\] 
When \(t = 0, v = 2\) 
\[2 = 4e^0 + c\] 
\[c = -2\] 
\[v = 4e^{-t} - 2\] 
At instantaneous rest, \(v = 0\) 
\[4e^{-t} - 2 = 0\] 
\[e^{-t} = \frac{1}{2}\] 
\[t = -\ln\left(\frac{1}{2}\right)\] | M1, M1, M1, A1 |
| 11ii  | \[s = \int 4e^{-t} - 2 \, dt\] 
\[s = -4e^{-t} - 2t + c\] 
When \(t = 0, s = 0\) 
\[0 = -4e^0 - 2(0) + c\] 
\[c = 4\] 
\[s = -4e^{-t} - 2t + 4\] 
Distance travelled before instantaneous rest  
\[= -4e^{-\left(-\frac{1}{2}\right)} - 2\left(-\ln\left(\frac{1}{2}\right)\right) + 4\] 
\[= 0.61371\] 
Distance from instantaneous rest to 2s  
\[= 0.61371 - \left(-4e^{-2(2)} - 2(2) + 4\right)\] 
\[= 0.61371 - (-0.54134)\] 
\[= 1.15505\] 
Total distance travelled  
\[= 0.61371 + 1.15505 = 1.77 \text{ m}\] | M2, M1, A1 |
| 11iii | Average velocity  
\[= \frac{-0.54134}{2} = -0.271 \text{ m/s}\] | B1 |
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<tbody>
<tr>
<td>12i</td>
<td>Centre of $C_1 = Centre of C_2 = (8, 5)$</td>
<td>M1, M1</td>
</tr>
<tr>
<td></td>
<td>Radius of $C_1 = \sqrt{(8-4)^2 + (5-8)^2} = 5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Equation: $(x-8)^2 + (y-5)^2 = 25$</td>
<td>A1</td>
</tr>
<tr>
<td>12ii</td>
<td>Coordinates of $B = (8 + 4, 5 - 3) = (12, 2)$</td>
<td>M1, A1</td>
</tr>
<tr>
<td>12iii</td>
<td>Gradient of normal at $B = \frac{5 - 2}{8 - 12} = \frac{3}{4}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Gradient of tangent $= \frac{4}{3}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Equation: $y - 2 = \frac{4}{3}(x - 12)$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$y = \frac{4}{3}x - 14$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$3y = 4x - 42$ (shown)</td>
<td></td>
</tr>
<tr>
<td>12iv</td>
<td>Centre is at $(8, 5)$ and radius is 5</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>The lowest point $D$ is at $(8, 0)$ and the circle touches $x$-axis at $D$. Thus $x$-axis is a tangent to the circle at $D$</td>
<td></td>
</tr>
<tr>
<td>12v</td>
<td>$\tan \theta = \frac{5}{8}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tan 2\theta = \frac{2\tan \theta}{1 - \tan^2 \theta} = \frac{2\left(\frac{5}{8}\right)}{1 - \left(\frac{5}{8}\right)^2}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{80}{39}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Equation of tangent: $y = \frac{80}{39}x$</td>
<td>A1</td>
</tr>
</tbody>
</table>
READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.
Mathematical Formulae

1. ALGEBRA

For the equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.
\]

Binomial Theorem
\[
(a + b)^n = a^n + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,
\]
where \( n \) is a positive integer and
\[
\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}
\]

2. TRIGONOMETRY

\( \sin^2 A + \cos^2 A = 1 \)
\( \sec^2 A = 1 + \tan^2 A \)
\( \csc^2 A = 1 + \cot^2 A \)
\( \sin (A \pm B) = \sin A \cos B \pm \cos A \sin B \)
\( \cos (A \pm B) = \cos A \cos B \mp \sin A \sin B \)
\( \tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \)
\( \sin 2A = 2 \sin A \cos A \)
\( \cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A \)
\( \tan 2A = \frac{2\tan A}{1 - \tan^2 A} \)

Formulae for \( \triangle ABC \)
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\( a^2 = b^2 + c^2 - 2bc \cos A \)
\( \Delta = \frac{1}{2} bc \sin A \)
1. Find the range of values of $k$ where $k \neq 0$ if the roots for the equation $kx^2 + (k - 2)x + 4k = 0$ are real. [4]

2. A particle moves along the curve $y = 6 + \frac{1}{x^2}$ such that the $y$-coordinate of the particle is decreasing at a constant rate of 0.04 units per second. Find the rate of change of the $x$-coordinate when $x = 2$. [4]

3. Given that $\int_1^2 [f(x) + 1] \, dx = 8$, evaluate
   (i) $\int_1^2 f(x) \, dx$. [2]
   (ii) $\int_1^2 f(x) + 1 \, dx - \int_1^2 [f(x) + 1] \, dx$. [2]

4. (i) Write down the first three terms in the expansion, in descending powers of $x$, of $(2x - \frac{1}{3x})^7$. [3]
   (ii) Hence find the coefficient of $x^3$ in the expansion of $(x^2 + 2)(2x - \frac{1}{3x})^7$. [2]

5. Given that the roots of $2x^2 + 3x - 6 = 0$ are $2\alpha + \beta$ and $2\beta + \alpha$, find a quadratic equation whose roots are $\alpha$ and $\beta$. [6]

[Turn over]
6 A bowl of hot soup was left to cool such that $t$ minutes later, its temperature, $T \, \circ C$, is given by $H = 25 + 70e^{-kt}$, where $k$ is a constant. When $t = 2$, the temperature of the soup is $80 \, \circ C$.

(i) Show that $k = 0.1206$. [2]

(ii) Find the time taken for the soup to reach $40 \, \circ C$. [2]

(iii) Explain why the temperature of the soup reaches $25 \, \circ C$ after a long time. [1]

(iv) Sketch the graph of $H$ against $t$. [2]

7 (a) If $a^{x+y} = a^x b^y$, prove that $(2 + x) \log a = x \log b$. [3]

(b) Solve $(2 \log_b x + 5) \log_b x = 3$. [4]

8 The diagram shows part of the curve $y = \ln x$ and a tangent to the curve at $x = k$ which also passes through the point $(0, -1)$.

(i) Find the equation of the tangent. [5]

(ii) Write down an inequality for $m$ if the line $y = mx - 1$ where $m > 0$

(a) intersects the curve exactly 2 times, [1]

(b) does not meet the curve. [1]
The diagram shows points \( A, P, B \) and \( Q \) lying on a circle with diameter \( AB \). The tangent to the circle at \( B \) meets \( AP \) produced at \( X \) and \( AQ \) produced at \( Y \).

(i) Prove that triangle \( APB \) is similar to triangle \( ABX \). Hence express \( AB^2 \) in terms of \( AP \) and \( AX \). [4]

(ii) Express \( AB^2 \) in terms of \( AP \) and \( PB \). [1]

(iii) Using your answers in (i) and (ii), show that \( PB^2 = AP \times PX \). [2]

The diagram shows part of the curve \( y = x^3 \). Points \( P \) and \( R \) lie on the \( x \)-axis. The line \( QR \) intersects the curve at \( Q(2, 8) \). \( QP \) is perpendicular to the \( x \)-axis. Given that the ratio of the shaded area to the area of triangle \( PQR \) is \( 2 : 5 \).

(i) Find the shaded area. [3]

(ii) Find the coordinates of \( R \). [3]

(iii) Determine whether \( QR \) is the normal to the curve at \( Q \). [4]

[Turn over]
The tidal height, $y$ metres; at a jetty on a particular day can be represented by the equation

$$y = 1.6 + 1.6 \cos(kt)$$

where $t$ is the time in hours after midnight and $k$ is a constant.

The time between the first high tide and the next high tide is 14 hours.

(i) Show that $k = \frac{\pi}{7}$. \hspace{1cm} [1]

(ii) Find the minimum tidal height for that day and the time it first occurred. \hspace{1cm} [3]

(iii) For how long between the first high tide and the next high tide was the tidal height at most 1 m high? \hspace{1cm} [5]

12. Given that \( \frac{4x^3 + 3x^2 - 8x - 1}{x^3 + x - 2} = ax + b + \frac{x + c}{x^3 + x - 2} \):

(i) find the value of each of the integers $a$, $b$ and $c$. \hspace{1cm} [4]

Hence, using partial fractions and the values of $a$, $b$ and $c$ obtained in part (i), find

(ii) \( \int \frac{4x^3 + 3x^2 - 8x - 1}{x^3 + x - 2} \, dx \). \hspace{1cm} [6]

End of paper
Answers

1. \[-\frac{2}{3} \leq k < 0\]

2. 0.16 units/s

3.(i) 4 \hspace{1cm} (ii) 8

4.(i) \[128x^3 - \frac{448}{3}x^2 + \frac{224}{3}x + \ldots .\] \hspace{1cm} (ii) -224

5. \[x^2 + \frac{1}{2}x - \frac{7}{2} = 0\]

6.(ii) 12.8 min \hspace{1cm} (iv)

7.(b) \[x = \frac{1}{27} \text{ or } \sqrt{5}\]

8.(i) \[y = x - 1\] \hspace{1cm} (ii)(a) \(0 < m < 1\) \hspace{1cm} (b) \(m > 1\)

9.(i) \[AB^2 = AP \times AM\]

10.(i) 4 units\(^2\) \hspace{1cm} (ii) (4.5, 0) \hspace{1cm} (iii) \(QK\) is not the normal to the curve at \(Q\).

11.(ii) 0.2 m at 7 am

12.(i) \[4x - 1 + \frac{x - 3}{(x + 2)(x - 1)}\] \hspace{1cm} (ii) \[2x^3 - x + \frac{5}{3}\ln(x + 2) - \frac{2}{3}\ln(x - 1) + c\]
<table>
<thead>
<tr>
<th>No.</th>
<th>Solution</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((k - 2)^2 - 4(k)(4k) \geq 0) &lt;br&gt;((k - 2 + 4k)(k - 2 - 4k) \geq 0) &lt;br&gt;((5k - 2)(-3k - 2) \geq 0) &lt;br&gt;((5k - 2)(3k + 2) \leq 0) &lt;br&gt;(-\frac{2}{3} \leq k \leq \frac{2}{5}, k \neq 0)</td>
<td>M1 Correct sub for discriminant &lt;br&gt;B1 (D \geq 0) &lt;br&gt;M1 Factorise</td>
</tr>
<tr>
<td>2</td>
<td>(\frac{dy}{dx} = -\frac{2}{x^3}) &lt;br&gt;At (x = 2), (\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}) &lt;br&gt;(-0.04 = -\frac{2}{2^3} \times \frac{dx}{dt}) &lt;br&gt;(\frac{dx}{dt} = 0.16)</td>
<td>M1 &lt;br&gt;M1 Eqn with sub &lt;br&gt;B1 (\frac{dy}{dt} = -0.04)</td>
</tr>
<tr>
<td>3(i)</td>
<td>(\int_{-1}^{1} f(x)dx = \int_{-1}^{0} f(x) + 1dx - \int_{0}^{1} f(x)) &lt;br&gt;(= 8 - [x]^1) &lt;br&gt;(= 4)</td>
<td>M1 For splitting</td>
</tr>
<tr>
<td>3(ii)</td>
<td>(\int_{-1}^{1} f(x) + 1dx + \int_{1}^{3} f(x) + 1dx) &lt;br&gt;(= \int_{1}^{3} f(x) + 1dx) &lt;br&gt;(= 8)</td>
<td>M1 Change signs &amp; limits</td>
</tr>
<tr>
<td>4(i)</td>
<td>((2x - \frac{1}{3x})' = (2x) + \left(\frac{7}{1} \times (2x)^2\right) - \left(\frac{1}{3x}\right)^2) &lt;br&gt;(= 128x^2 - \frac{448}{3} x^4 + \frac{224}{3} x^3 + \ldots)</td>
<td>M1 &lt;br&gt;B2 Expansion &lt;br&gt;-1 mark for each error</td>
</tr>
<tr>
<td>4(ii)</td>
<td>((x^2 + 2)(128x^2 - \frac{448}{3} x^4 + \frac{224}{3} x^3 + \ldots)) &lt;br&gt;(\text{coeff} = 2 \left(\frac{448}{3} + \frac{224}{3}\right)) &lt;br&gt;(= -224)</td>
<td>M1 &lt;br&gt;A1</td>
</tr>
</tbody>
</table>
### Question 5

<table>
<thead>
<tr>
<th>No.</th>
<th>Solution</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$2\alpha + \beta + 2\beta + \alpha = \frac{-3}{2}$</td>
<td>M1 Find sum $=- \frac{b}{a}$</td>
</tr>
<tr>
<td></td>
<td>$\alpha + \beta = \frac{-1}{2}$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$(2\alpha + \beta)(2\beta + \alpha) = -3$</td>
<td>M1 Find prod $=- \frac{c}{a}$</td>
</tr>
<tr>
<td></td>
<td>$4\alpha\beta + 2\beta^2 + 2\alpha^2 + \alpha\beta = -3$</td>
<td>M1 Using $(\alpha + \beta)^2$</td>
</tr>
<tr>
<td></td>
<td>$5\alpha\beta + 2[(\alpha + \beta)^2 - 2\alpha\beta] = -3$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$\alpha\beta = 3 - 2 \left( \frac{1}{2} \right)^1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\alpha\beta = \frac{-7}{2}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Equation is $x^2 + \frac{1}{2}x - \frac{7}{2} = 0$</td>
<td>A1, must $= 0$</td>
</tr>
</tbody>
</table>

### Question 6

**6(i)**

|     | $80 = 25 + 70e^{-3t}$ | M1 Sub $t=2$, $H=80$ |
|     | $e^{-3t} = \frac{55}{70}$ | M1 Take ln both sides & result |
|     | $-2k = \ln \frac{55}{70}$ |
|     | $k = 0.1206$ |

**6(ii)**

|     | $40 = 25 + 70e^{-0.1106t}$ | M1 Using ln on both sides |
|     | $e^{-0.1106t} = \frac{15}{70}$ | A1 or 12 min 46 sec |
|     | $-0.1206t = \ln \frac{15}{70}$ |
|     | $t = 12.8$ min |

**6(iii)**

As $t$ becomes very large, $70e^{-t}$ approaches zero, So the temperature reaches $25^\circ C$ after a long time. | B1 |

**6(iv)**

|     | ![Graph](image) | G1 shape |
|     | 95, 25 seen | |

Total 7 m
### 2015 Sec 4 Prelim 1 Add Maths P1

#### 7(a)

<table>
<thead>
<tr>
<th>Expression</th>
<th>Group/Rearrange</th>
<th>Indices or log law</th>
<th>Taking log on both sides and result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a^{1+x} = a^1 \cdot a^x = b^x + b^x)</td>
<td>M1</td>
<td>M1</td>
<td>B1</td>
</tr>
<tr>
<td>(a^{x^{-1}} = b^{x^{-1}})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>((4 + 2x) \log a = 2x \log b)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>((2 + x) \log a = x \log b)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### 7(b)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Group/Rearrange</th>
<th>Indices or log law</th>
<th>Taking log on both sides and result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2a^2 + 5a - 3 = 0)</td>
<td>M1</td>
<td></td>
<td>B1</td>
</tr>
<tr>
<td>((a + 3)(2a - 1) = 0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a = -3) or (a = \frac{1}{2})</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \log_3 x = -3 \quad \text{or} \quad \log_3 x = \frac{1}{2} \]

\[ x = \frac{1}{27} \quad \text{or} \quad x = \sqrt{3} \]

#### 8(i)

<table>
<thead>
<tr>
<th>Expression</th>
<th>Group/Rearrange</th>
<th>Indices or log law</th>
<th>Taking log on both sides and result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{dy}{dx} = \frac{1}{x})</td>
<td>M1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(r = k, \frac{dy}{dx} = \frac{1}{k}, y = \ln k)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(y - \ln k = \frac{1}{k} (x - k))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-1 = \frac{1}{k} (0) - 1 + \ln k)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\ln k = 0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(k = 1)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Eqn of tgf: \(y = x - 1\)

#### 8(ii)

<table>
<thead>
<tr>
<th>Expression</th>
<th>Group/Rearrange</th>
<th>Indices or log law</th>
<th>Taking log on both sides and result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) (0 &lt; m &lt; 1)</td>
<td>B1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b) (m &gt; 1)</td>
<td>B1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ m > \text{their grad} \quad \text{must be} \quad > 0 \]

#### Total

- 7 marks

- 7 marks
<table>
<thead>
<tr>
<th>No.</th>
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</tr>
</thead>
</table>
| 9(i) | $\angle PAB = \angle BAX$ (Common angle)  
$\angle APB = 90^\circ$ ( $\angle$ in semi circle)  
$\angle ABX = 90^\circ$ (tangent perpendicular to radius)  
So $\triangle APB$ is similar to $\triangle ABX$.  
$\frac{AB}{AP} = \frac{AX}{AB}$  
$AB^2 = AP \times AX$ | B1 B1 B1 |
| 9(ii) | By Pythagoras Thm,  
$AB^2 = AP^2 + PB^2$ | B1 |
| 9(iii) | $AP^2 + PB^2 = AP \times AX$  
$PB^2 = AP \times AX - AP^2$  
$PB^2 = AP(AX - AP)$  
$\therefore PB^2 = AP \times PX$ (shown) | M1 Equating (i)  
M1 Factorising seen & result |
| 10(i) | Shaded Area = $\int_0^2 x^2 \, dx$  
= $\frac{1}{4}[x^4]_0^2 = 4 \text{ units}^2$ | B1  
M1 correct integration |
| 10(ii) | Area of triangle PQR = 10 units$^2$  
$\frac{1}{2} \times PR \times 8 = 10$  
$PR = 2.5$  
$R(4.5, 0)$ | B1  
M1 use area/discriminant mthd |
| 10(iii) | gradient of $QR = -3.2$  
$\frac{dy}{dx} = 3x^2$  
$x = 2, \frac{dy}{dx} = 12$  
Since grad of normal = $-\frac{1}{12} \neq$ gradient of $QR$,  
So $QR$ cannot be normal to curve at $Q$. | M1  
M1 grad of normal  
B1 correct conclusion |

Total 7 m  
Total 10 m
<table>
<thead>
<tr>
<th>No.</th>
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</tr>
</thead>
</table>
| 11(i) | \[ \frac{2\pi}{k} = 14 \]  
\[ k = \frac{\pi}{7} (\text{shown}) \] | M1 |
| 11(ii) | \[ \cos \left( \frac{\pi}{7} t \right) = -1 \]  
Minimal tidal height = 1.6 + 1.4(-1)  
At 7 am  
\[ = 0.2 \text{ m} \] | M1 soi |
| 11(iii) | 1.6 + 1.4 \cos \left( \frac{\pi}{7} t \right) = 1 
\[ \cos \left( \frac{\pi}{7} t \right) = -0.42857 \]  
Basic angle = 1.1279  
\[ \frac{\pi}{7} t = 2.0137, 4.2695 \]  
\[ t = 4.4868, 9.5132 \] | M1 Form eqn, \& = 1 |
| | Duration = 5.026 h | B1 B1  
M1 2nd ans - 1st ans  
A1 |

| 12(i) | \[ 4x - 1 + \frac{x - 3}{(x + 2)(x - 1)} \] | M1 use long div  
A3 value of \( a, b, c \) |
| 12(ii) | \[ \frac{x - 3}{(x + 2)(x - 1)} = \frac{A}{x + 2} + \frac{B}{x - 1} \]  
x - 3 = A(x - 1) + B(x + 2)  
x = 1, B = \[ \frac{2}{3} \]  
x = -2, A = \[ \frac{5}{3} \]  
\[ \int 4x - 1 + \frac{5}{3(x + 2)} - \frac{2}{3(x - 1)} \, dx \]  
\[ = 2x^3 - x + \frac{5}{3} \ln(x + 2) - \frac{2}{3} \ln(x - 1) + c \] | M1 correct PF  
M1  
A1 Both answers  
M1 first 2 terms  
M1 both \( \ln (...) \) seen  
A1 + c seen  
\[ \int \ldots dx \] (\(-1\) mark if \( dx \) missing) |

| Total | 9 m | 10 m |
READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
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Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the equation \( ax^2 + bx + c = 0 \),

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Binomial Theorem
\[(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1) \ldots (n-r+1)}{r!} \)

2. TRIGONOMETRY

Identities
\[
\begin{align*}
\sin^2 A + \cos^2 A &= 1 \\
\sec^2 A &= 1 + \tan^2 A \\
\cosec^2 A &= 1 + \cot^2 A \\
\sin (A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos (A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\tan (A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A \\
\tan 2A &= \frac{2 \tan A}{1 - \tan^2 A}
\end{align*}
\]

Formulas for \( \triangle ABC \)
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\Delta = \frac{1}{2} bc \sin A
\]
The diagram shows a trapezium $ABCD$ in which $AB = (7 + 2\sqrt{3})$ cm and $DC = (3 + 4\sqrt{3})$ cm. $AB$ and $DC$ are perpendicular to $CB$. Given that the area of the trapezium is $(14 + \sqrt{12})$ cm$^2$, find the exact value of $CB$ in the form of $(a + b\sqrt{5})$ cm.

2 A function has an equation where

\[ f(x) = \frac{\ln(4 - x)}{x - 4}, \quad x < 4. \]

(i) Obtain an expression for $f'(x)$.

(ii) Showing full working, determine whether $f$ is decreasing for $x < 4 - c$.

3 (i) Sketch the graph of $y = |2x - 1| - 3$.

(ii) Explain why the minimum value is $-3$.

(iii) A line $y = kx$, where $k > 0$, is drawn on the same axes with the graph of $y = |2x - 1| - 3$. Find the range of values of $k$ for which there is only one point of intersection.

4 (i) Prove the identity

\[ \frac{2\cos 2A + \cos A + 2}{2\sin 2A + \sin A} = \cot A. \]

(ii) Hence, solve the equation

\[ \frac{2\cos 6x + \cos 3x + 2}{2\sin 6x + \sin 3x} = 5 \quad \text{for} \quad 0 \leq x \leq \pi. \]
5 It is given that \( \sin A = \frac{1}{\sqrt{5}} \), where \( A \) is an acute angle. Without using a calculator,

(i) find \( \tan A \). \([2]\)

Given further that \( \tan (A+B) = 2 \), where \( B \) is an acute angle,

(ii) find the exact value of \( \tan B \). \([4]\)

6 (i) Sketch the graph of \( y = 2x^3 \) for \( x > 0 \). \([1]\)

(ii) On the same diagram, sketch the graph of \( y = \frac{2}{3}x^{-\frac{1}{2}} \) for \( x > 0 \). \([1]\)

(iii) Find the \( x \)-coordinate of the point of intersection of your graphs. \([2]\)

7 (i) Differentiate \( x \tan^3 x \) with respect to \( x \). \([3]\)

(ii) Show that \( \int_0^2 \tan^3 x \, dx = 0.2146 \). \([4]\)

(iii) Hence, find \( \int_0^2 x \tan x \sec^2 x \, dx \). \([4]\)

8 Given that \( f(x) = 2x^3 + ax^2 + bx - 3 \), where \( a \) and \( b \) are constants, has a factor of \( x - 3 \) and leaves a remainder of \(-20\) when divided by \( x + 1 \),

find the value of \( a \) and of \( b \). \([5]\)
9 The points $A(3, 0)$ and $B(9, 6)$ lie on a circle $C_1$ such that the $x$-axis is a tangent to the circle at $A$.

(i) Find the equation of the perpendicular bisector of $AB$. [4]

(ii) Hence, or otherwise, find centre of the circle $C_1$ and the radius. [3]

(iii) Show that the equation of the circle is $x^2 + y^2 - 6x - 12y + 9 = 0$. [2]

Another circle $C_2$ is formed after circle $C_1$ is being reflected in the line $x = 8$.

(iv) Find the centre of circle $C_2$. [1]

(v) Explain why the point $(12, 9)$ lies within circle $C_2$. [2]

The diagram consists of a parallelogram $ABCD$ and a rectangle $APQD$.

It is given that $CD = l$ m and that the angle $CBF = 60^\circ$ and angle $CPA = 90^\circ$.

The rectangle has sides $AP = x$ m and $PQ = 3x$ m.

The perimeter of the diagram is 10 m.

(i) Express $l$ in terms of $x$ and show that the area of the diagram is

$$3(1 - 2\sqrt{3})x^2 + \frac{15\sqrt{3}}{2} \text{ m}^2.$$ [3]

(ii) Given that $x$ can vary, find the value of $x$ for which the area has a stationary value. [3]

(iii) Determine whether this value of area is a maximum or a minimum. [2]
11 The variables $x$ and $y$ are connected by the equation $y = ax + \frac{b}{x}$, where $a$ and $b$ are constants. Experimental values of $x$ and $y$ were obtained. A graph is drawn in which $xy$ was plotted against $x^2$. The straight line which was obtained passed through the points (1, 5) and (3, 11).

Find

(i) the value of $a$ and of $b$,

(ii) the coordinates of the point on the line at which $y = -x + \frac{4}{x}$.

[4]

12 A particle travels in a straight line, so that $t$ seconds after passing through a fixed point $O$, its velocity, $v$ ms$^{-1}$, is given by $v = \frac{32}{(t + 2)^2} - 2$. The particle comes to instantaneous rest at $P$. Find

(i) the value of $t$ when the particle is at instantaneous rest,

(ii) distance $OP$,

(iii) distance travelled for the first 8 seconds,

(iv) the acceleration of the particle at $t = 8$ seconds.

[3] [4] [2] [3]
A playground is to be built in the shape of the figure shown in which \( ADC \) is a straight line and angle \( EAB = \angle DCB = \angle EDC = 90^\circ \). The length of \( AE \) is 7 m and \( AB \) is 18 m. The angle \( ABC \) is \( \theta \), where \( 0^\circ < \theta < 90^\circ \). The perimeter of the playground is given by \( L \) m.

(i) Show that \( L \) can be expressed as \( p + q \cos \theta + r \sin \theta \), where \( p, q \) and \( r \) are constants to be found. [3]

(ii) Express \( L \) in the form \( p + R \cos(\theta - \alpha) \), where \( R > 0 \) and \( \alpha \) is an acute angle. [4]

(iii) Given \( L = 51 \) m, find \( \theta \). [2]

(iv) Find the maximum value of the perimeter and the corresponding value of \( \theta \). [3]

---

1) \(-25 + 16\sqrt{3}\)
2i) \(\frac{1 - \ln(4 - x)}{(x - 4)^3}\)
3i) 

\[ y \]

\[ \frac{1}{x} \]

\[ -2 \]

\[ (0.5, -3) \]

3ii) Since \( |2x - l| \geq 0\),
\[ |2x - l| - 3 \geq -3 \]
Min value is -3
3iii) \( k \geq 2 \)
4ii) \( x = 0.0658, 1.11, 2.16 \)
5i) \( \tan A = \frac{1}{2} \)
5ii) \( \tan B = \frac{3}{4} \)
6i) \[ y = 2x^\frac{1}{2} \]
\[ y = \frac{1}{3}x^\frac{2}{3} \]
6iii) \( x = \sqrt{6} \)
7i) \( \tan^2 x + 2x \tan x \sec^2 x \)
7ii) 0.2146
7iii) 0.285
8) \( a = -8 \)
\( b = 7 \)
9i) \( y = -x + 9 \)
9ii) Centre is (3,6)
9v) (13,6)
9iv) \( \sqrt{10} < \text{radius 6} \)
(12,9) lies within circle
10i) \( f = 5 - 4x \)
10ii) \( x = 0.879 \)
10iii) \( \frac{d^2A}{dx^2} = 6(1 - 2\sqrt{3}) < 0 \)
maximum
11i) \( a = 3, b = 2 \)
11ii) \( \left( \frac{7}{2}, \frac{7}{2} \right) \)
12i) \( t = 2, -5 \)(ref)
12ii) \( OP = 4 \) m
12iii) 11.2 m
12iv) \( a = -0.064 \) m/s²
13i) \( L = 25 + \sqrt{764\cos(\theta - 66.25°)} \)
13ii) 84.1°, 48.4°
13iv) \( \theta = 66.3°, 426.3° \)(ref), \( \text{Max} = 25 + \sqrt{746} = 52.3 \) m
<table>
<thead>
<tr>
<th>No</th>
<th>Solution</th>
<th>Marks</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 14 + \sqrt{12} = \frac{1}{2}(7 + 2\sqrt{3} + 3 + 4\sqrt{3}) ) ( CB )</td>
<td>B1 Correct eqn</td>
<td>Use of trapezium formula or otherwise (top and bottom)</td>
</tr>
<tr>
<td></td>
<td>( CB = \frac{14 + 2\sqrt{3}}{5 + 3\sqrt{3}} \times \frac{5 - 3\sqrt{3}}{5 - 3\sqrt{3}} )</td>
<td>M1 rationalise</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= ( \frac{70 - 42\sqrt{3} + 10\sqrt{3} - 6(3)}{25 - 9(3)} )</td>
<td>M1 simplify surds</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= (-26 + 16\sqrt{3} \text{ cm} )</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>2(i)</td>
<td>( f'(x) = \frac{(x - 4) - \ln(4 - x)(1)}{(x - 4)^2} )</td>
<td>M1 use of quotient rule</td>
<td>4 marks</td>
</tr>
<tr>
<td></td>
<td>= ( 1 - \frac{\ln(4 - x)}{(x - 4)^2} )</td>
<td>B1 diff. ( \ln(4 - x) ) correctly</td>
<td></td>
</tr>
</tbody>
</table>
| 2(ii) | \( x < 4 - \varepsilon \) | M1 knowing to show \( f'(x) \) +ve and -ve | \(-1 \)
|    | \( x - 4 < -\varepsilon \) | | \( 4 - x \) seen |
|    | \( 4 - x > \varepsilon \) | | |
|    | \( \ln(4 - x) > \ln e \) | | |
|    | \( \ln(4 - x) > 1 \) | | |
|    | \( 1 - \ln(4 - x) < 0 \) | B1 manipulate | |
|    | \( \therefore f'(x) < 0 \) | B1 correct conclusion including stating \((\_\_\_)^2\) is +ve | |
|    | \( f \) is decreasing | | |
| 3(i) | ![Graph](image) | S1 correct shape/symmetrical | 6 marks |
|    | B1 vertex and y-int seen | | |
| 3(ii) | Since | B1 with explanation | |
|    | \( |2x - 1| \geq 0 \) | | |
|    | \( |2x - 1| - 3 \geq -3 \) | | |
|    | Min value is \(-3\) | | |
| 3(iii) | Gradient of R.H. arm = 2 | A1 | 4 marks |
|    | \( k \geq 2 \) | | |
### Marking Scheme 2015 Prelim Add Mathematics P2

<table>
<thead>
<tr>
<th>No</th>
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</tr>
</thead>
</table>
| 4(i) | LHS<br>
\[
\frac{2(2 \cos^2 A - 1) + \cos A + 2}{2(2 \sin A \cos A)^2 + \sin A} = \frac{4 \cos^2 A + \cos A}{\sin A (4 \cos A + 1)} = \frac{\cos A (4 \cos A + 1)}{\sin A (4 \cos A + 1)} = \cot A
\] | B1 double angle for cos 2A<br>B1 double angle for sin 2A<br>M1 factorise both<br>B1 \[
\frac{\cos A}{\sin A} = \cot A
\] |         |
| 4(ii) | \[
\cot 3x = 5<br>\tan 3x = \frac{1}{5}
\]
\[3x = 0.1974, 3.339, 6.481\]
\[x = 0.0658, 1.11, 2.16\] | B1<br>M1 reciprocal of cot3x<br>A2 - 3 correct<br>A1 - 2 correct<br>Ans must be in rad |         |
| 5(i) | \[
tan A = \frac{1}{2}
\] | M1 use pyth thm to find length<br>A1 | Must show working |
| 5(ii) | \[
\frac{\tan A + \tan B}{1 - \tan A \tan B} = 2
\]
\[
\begin{align*}
2 &= \frac{\frac{1}{2} + \tan B}{1 - \frac{1}{2} \tan B} \\
2 - \tan B &= \frac{1}{2} + \tan B \\
\tan B &= \frac{3}{4}
\end{align*}
\] | B1 use of tangent formula SOI<br>M1 subst tan A & B<br>M1 simplify<br>A1 |         |
| 6(i)(ii) | ![Graph of y = 2x^\frac{1}{2} and y = x^\frac{1}{3}](image.png) | B1<br>B1 | With label<br>Shape must be correct Must touch origin |
| 6(iii) | \[
\frac{1}{3} x^{\frac{3}{2}} = 2x^{\frac{1}{2}}
\]
\[
x^2 = 6
\]
\[
x = \sqrt{6}
\] | M1 simplify indices<br>A1 must rej - \sqrt{6} |         |
## Marking Scheme

### 2015 Prelim Add Mathematics P2

<table>
<thead>
<tr>
<th>No</th>
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</tr>
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</table>
| 7(i) | \[
\frac{d}{dx} x \tan^2 x = \tan^2 x + x(2 \tan x) \sec^3 x \\
= \tan^2 x + 2x \tan x \sec^2 x
\] | | M1 use of pdt rule  
B1 diff. \( \tan^2 x \) correctly  
B1 presentation |
| 7(ii) | \[
\int_a^b \tan^2 x \, dx = \int_a^b \sec^2 x \, dx \\
= [\tan x - x]_a^b \\
= 0.2146
\] \( (0.214602) \) | | M1 use of identity  
B1 integrate \( \sec^2 x \)  
M1 evaluate integral (must show subst)  
B1 presentation |
| 7(iii) | \[
\int_a^b \tan^2 x + 2x \tan x \sec^2 x \, dx = [x \tan^2 x]_a^b \\
2 \int_a^b x \tan x \sec^2 x \, dx = [x \tan^2 x]_a^b - \int_a^b \tan^2 x \, dx \\
= 0.7854 - 0.2146 \\
= 0.5708 \\
\int_a^b x \tan x \sec^2 x \, dx = 0.285
\] | | B1 work backwards  
ECF |
| 8 | \[
\begin{align*}
\int (x) &= 2(3)^3 + 9a + 3b - 3 \\
0 &= 51 + 9a + 3b \\
-17 &= 3a + b \\
\end{align*}
\] | | B1 \( \int (3) = 0 \) with subst  
B1 \( \int (-1) = -20 \) with subst |
| \( \int (-1) \) &\( \begin{align*}
-2 &= a - b - 3 \\
-20 &= 5 + a - b \\
-15 &= a - b \\
\end{align*} \) | | M1 solve sim  
A1, A1 |
| | \[ a = -8 \\
b = 7 \] | | |
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<tbody>
<tr>
<td>9(i)</td>
<td>Midpoint of $AB = (6, 3)$</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Grad $AB = 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Grad of perpendicular bisector $= -1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y - 3 = -1(x - 6)$</td>
<td>M1</td>
<td>form eqn</td>
</tr>
<tr>
<td></td>
<td>$y = -x + 9$</td>
<td>M1</td>
<td>find grad. of perpendicular bisector</td>
</tr>
<tr>
<td>9(ii)</td>
<td>Let centre be $(3, k)$</td>
<td>A1</td>
<td>Use other mtd such as distance</td>
</tr>
<tr>
<td></td>
<td>$k = -3 + 9$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Centre is $(3,6)$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Radius $= 6$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9(iii)</td>
<td>$(x - 3)^2 + (y - 6)^2 = 36$</td>
<td>B1</td>
<td>seen ECF</td>
</tr>
<tr>
<td></td>
<td>$x^2 + y^2 - 6x - 12y + 9 = 0$</td>
<td>M1</td>
<td>expand and simplify</td>
</tr>
<tr>
<td>9(iv)</td>
<td>$(13,6)$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>9(v)</td>
<td>Let point $(12,9)$ be $M$. Let centre of $C_2$ be $O$.</td>
<td>M1</td>
<td>find distance</td>
</tr>
<tr>
<td></td>
<td>$OM = \sqrt{(9 - 6)^2 + (12 - 13)^2}$</td>
<td>B1</td>
<td>comparison made and conclusion seen</td>
</tr>
<tr>
<td></td>
<td>$OM = \sqrt{10} &lt; \text{radius 6}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(12,9)$ lies within circle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10(i)</td>
<td>$5x + 3x + 2l = 10$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$l = 5 - 4x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Area $= 3x^2 + 3x \sin 60^\circ (5 - 4x)$</td>
<td>M1</td>
<td>find area of parallelogram</td>
</tr>
<tr>
<td></td>
<td>$= 3x^2 + 15x \cdot \frac{\sqrt{3}}{2} - 12x^2 \cdot \frac{\sqrt{3}}{2}$</td>
<td>B1</td>
<td>$\sin 60 = \frac{\sqrt{3}}{2}$ seen</td>
</tr>
<tr>
<td></td>
<td>$= 3(1 - 2\sqrt{3})x^2 + 15 \frac{\sqrt{3}}{2}x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10(ii)</td>
<td>$\frac{dA}{dx} = 6(1 - 2\sqrt{3})x + 15 \frac{\sqrt{3}}{2}$</td>
<td>M1</td>
<td>attempt to diff.</td>
</tr>
<tr>
<td></td>
<td>At stat value, $\frac{dA}{dx} = 0$</td>
<td>B1</td>
<td>$\frac{dA}{dx} = 0$ seen</td>
</tr>
<tr>
<td></td>
<td>$x = 0.879$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>10(iii)</td>
<td>$\frac{d^2A}{dx^2} = 6(1 - 2\sqrt{3}) &lt; 0$</td>
<td>M1</td>
<td>know $2^{nd}$ derivative or sign test</td>
</tr>
<tr>
<td></td>
<td>maximum</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>8 marks</td>
<td></td>
</tr>
<tr>
<td>No</td>
<td>Solution</td>
<td>Marks</td>
<td>Remarks</td>
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</tbody>
</table>
| 11(i) | \( xy = ax^3 + b \)  
  \( \text{grad} = 3 \)  
  \( xy = \frac{5}{x^3 - 1} \)  
  \( xy = 3x^2 + 2 \)  
  \( a = 3, b = 2 \) | B1 manipulate into 
 grad-intercept form  
 M1 using correct 
 subst  
 A1, A1 | |
| 11(ii) | \( xy = -x^2 + 4 ...(1) \)  
  \( xy = 3x^2 + 2 ...(2) \)  
  \( x^2 = \frac{1}{2} \)  
  \( xy = \frac{7}{2} \)  
  \( \left( \frac{1}{2}, \frac{7}{2} \right) \) | M1 use similar eqn  
 M1 solve 
 simultaneous eqn  
 A1, A1 | |
| 12(i) | \[ \frac{32}{(t+2)^2} - 2 = 0 \]  
  \( t + 2 = \pm 4 \)  
  \( t = 2, -6 \) (ref) | B1  
 M1 solve eqn  
 A1 | 8 marks |
| 12(ii) | \[ s = \int \frac{32}{(t+2)^2} \ dt = -\frac{32}{t+2} - 2r + c \]  
  At \( t = 0, s = 0, c = 16 \)  
  \( s = -\frac{32}{t+2} - 2 + 16 \)  
  At \( t = 2 \),  
  \( OP = 4 \) m | M1 integrate (ok if 
 no + c)  
 B1  
 M1 find distance 
 (sub \( t = 2 \))  
 A1 | Give 
 marks if 
 use 
 definite 
 integral \[ \int_a^b \] |
| 12(iii) | \( t = 8, \)  
  \( S = -3.2 \) m  
  Distance = \( 3.2 + 4 + 4 = 11.2 \) m | M1 ( ) +2(4) seen  
 A1 | |
| 12(iv) | \[ a = \frac{64}{(t+2)^2} \]  
  At \( t = 8, a = -0.064 \) m/s^2 | M1 Knowing to 
 differentiate  
 B1 acc expression 
 seen  
 A1 | 12 marks |
<table>
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</table>
| 13(i) | $L = 7 + 18 + 18 \cos \theta + 7 \sin \theta + 18 \sin \theta - 7 \cos \theta$  
$= 25 + 11 \cos \theta + 25 \sin \theta$ | B1,B1,B1 | |
| 13(ii) | $\tan \alpha = \frac{25}{11}$  
$\alpha = 66.25^\circ$  
$R = \sqrt{746}$  
$L = 25 + \sqrt{746} \cos(\theta - 66.25^\circ)$ | B1 | B1 √764 or 27.31 seen  
B1 statement |
| 13(iii) | When $L = 51m,$  
$51 = 25 + \sqrt{746} \cos(\theta - 66.25^\circ)$  
$\cos(\theta - 66.25^\circ) = 0.95193$  
$= 84.1^\circ$ or $48.4^\circ$ | M1 solve | If use  
27.3, will get  
$\theta = 84.00^\circ$ |
| 13(iv) | Max $= 25 + \sqrt{746} = 52.3 \text{ m}$  
At max value,  
$\cos(\theta - 66.25^\circ) = 1$  
$\theta - 66.25^\circ = 0^\circ, 360^\circ$  
$\theta = 66.3^\circ, 426.3^\circ($ref$)$ | A1 | A1  
Penalise if  
extra ans  
ans in rad.  
SOI or .... = 0 |

|  |  |  | 12 marks |

Page 6
READ THESE INSTRUCTIONS FIRST

Do not open the booklet until you are asked to do so.

You are not required to submit this booklet at the end of the paper.

Write your class, Index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [] at the end of each question or part question.
The total number of marks for this paper is 80.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the equation \( ax^2 + bx + c = 0 \),

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Binomial expansion

\[ (a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n, \]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!} \).

2. TRIGONOMETRY

Identities

\[
\begin{align*}
\sin^2 A + \cos^2 A &= 1 \\
\sec^2 A &= 1 + \tan^2 A \\
\csc^2 A &= 1 + \cot^2 A \\
\sin(A + B) &= \sin A \cos B + \cos A \sin B \\
\cos(A + B) &= \cos A \cos B - \sin A \sin B \\
\tan(A + B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = -1 - 2 \sin^2 A \\
\tan 2A &= \frac{2 \tan A}{1 - \tan^2 A}
\end{align*}
\]

Formule for \( \triangle ABC \)

\[
\begin{align*}
\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\
a^2 &= b^2 + c^2 - 2bc \cos A \\
\Delta &= \frac{1}{2} abc \sin C
\end{align*}
\]
1. (i) Given that \( \sin(A + B) = 3 \sin(A - B) \), show that \( \tan A = 2 \tan B \). [2]
(ii) Hence solve the equation \( \sin^2(x + 30^\circ) = 9 \sin^2(x - 30^\circ) \) for \( 0^\circ < x < 360^\circ \). [4]

2. (a) The equation \( 2x^2 - 2x + 1 = 0 \) has roots \( \alpha \) and \( \beta \). Find the quadratic equation whose roots are \( \frac{2}{\alpha^2} \) and \( \frac{2}{\beta^2} \). [5]
(b) The equation of a curve is \( y = (3 + m)x^3 - (8 + 4m)x + 3 + 4m \), where \( m \) is a constant. For \( y = 0 \), find the value of \( m \) for which
   (i) one root is the negative of the other, [2]
   (ii) one root is the reciprocal of the other. [2]

3. (a) Simplify \( \frac{2(4)\frac{1}{2} - 2\frac{1}{2}}{6^x \times 3^{-2}} \) and express in the form of \( k(3)^n \), where \( k \) and \( n \) are integers. [3]
(b) Find the values of \( a \) and \( b \) such that \( \log\left(\frac{125}{y}\right) = \log(ay) - 4 \log y \) for all positive values of \( y \). [3]
(c) Solve the equation \( 2 \log_6 e^x + \frac{1}{\log_5 3} = \log_6(2 - 3e^x) \). [5]

4. Given that \( x^2 + 2x - 3 \) is a factor of \( f(x) \), where \( f(x) = x^4 + 6x^3 + 2ax^2 + bx - 3a \),
   find
   (i) the values of \( a \) and \( b \), [4]
   (ii) the other quadratic factor of \( f(x) \). [3]

   Explain why \( f(x) = 0 \) has only two real roots. [1]
5 The diagram shows a circle $C_1$ with centre $(4, -6)$.

A curve $y^2 = x$ and the circle $C_1$ have the $y$-axis as the common tangent.

Both curves intersect at the point $(4, -2)$.

(i) Write down the radius of circle $C_1$ and hence the equation of $C_1$. [2]

(ii) Find the area bounded by the curve $y^2 = x$, the circle $C_1$ and the $y$-axis. [3]

(iii) A second circle, $C_2$, is the reflection of the circle, $C_1$ in the line $y = 2$.

Write down the equation of the second circle, $C_2$, in the form $x^2 + y^2 + 2gx + 2fy + c = 0$. [2]

6 A curve is such that \[ \frac{dy}{dx} = 4x + \frac{1}{(x + 2)^2} \] for $x > 0$ and the curve passes through the point \( \left( \frac{1}{2}, \frac{1}{2} \right) \).

(i) Find the equation of the curve. [3]

(ii) Find the equation of the normal to the curve at the point where $x = \frac{1}{2}$. [2]
7 Liquid is poured into a container at a rate of \(k \text{ m}^3/\text{s}\). The volume of liquid in the container is \(V \text{ m}^3\) where \(V = \frac{1}{3} \pi h^3 (3k - h)\) and \(h \text{ m}\) is the depth of the liquid in the container. Find, in terms of \(k\), the rate of increase of the liquid level when the depth of the liquid is \(\frac{2k}{5}\) m. [4]

8 Given that \(\frac{d^2y}{dx^2} = -9y\) and \(y = a \cos^3 x + b \cos x\) where \(a\) and \(b\) are constants, \(\cos x \neq 0\), show that \(3a + 4b = 0\). [6]

9 The curve \(\frac{1}{x} + \frac{2}{y} = \frac{1}{2}\) intersects the line \(2x + y + 2 = 0\) at the points \(A\) and \(B\).

Explain why a line joining points \(A\) and \(B\) is perpendicular to the line \(2y - x - 6 = 0\). [6]

10 On the same axes, sketch the graphs of \(y = \cos x + 1\) and \(y = |\tan x|\), for \(0^\circ \leq x \leq 360^\circ\). [4]

Hence, for \(0^\circ \leq x \leq 360^\circ\), state the value or range of values of \(k\) for which the equation \(|\tan x| = \cos x + k\) has

(i) 2 roots, [1]

(ii) 3 roots, [1]

(iii) 4 roots. [1]
The diagram shows a triangle $AEC$ which intersects the circle at points $A$, $B$, $C$, and $D$ is a point on $ABE$ such that $BD = BE$. Show that

(i) triangle $AEG$ is similar to triangle $CEB$, [2]

(ii) $AC \times BD = GE \times BC$. [2]
12. (a) Show that \[ \frac{d}{dx}(\sqrt{2 + \sin x}) = \frac{\cos x}{2\sqrt{2 + \sin x}}. \] 

(b) \[ y = \frac{\cos x}{2\sqrt{2 + \sin x}}. \] The diagram shows part of the curve \( y = \frac{\cos x}{2\sqrt{2 + \sin x}} \). The curve intersects the 

\( x \)-axis at \( \frac{\pi}{2} \) and \( \frac{3\pi}{2} \) and the \( y \)-axis at the point \( C \).

(i) Find the coordinates of point \( C \), in exact form.

(ii) Find the area of the shaded region bounded by the curve, the \( y \)-axis and the \( x \)-axis.

End of Paper
Marking Scheme

1. (i)  
\[ \sin A \cos B + \cos A \sin B = 3(\sin A \cos B - \cos A \sin B) \]  
\[ 4 \cos A \sin B = 2 \sin A \cos B \]  
\[ \frac{\sin A \cos B}{\cos A \sin B} = 2 \]  
\[ \tan A = 2 \tan B \]  
M1

(ii)  
\[ (\sin(x + 30^\circ))^2 = (3 \sin(x - 30^\circ))^2 \]  
\[ \sin(x + 30^\circ) = \pm 3 \sin(x - 30^\circ) \]  
\[ \sin(x + 30^\circ) = 3 \sin(x - 30^\circ) \]  
\[ \tan x = 2 \tan 30^\circ \]  
\[ x = 49.1^\circ \text{ or } 229.1^\circ \]  
A1

or

\[ \sin(x + 30^\circ) = -3 \sin(x - 30^\circ) \]  
\[ \sin(30^\circ + x) = 3 \sin(30^\circ - x) \]  
\[ \tan 30^\circ = 2 \tan x \]  
\[ \tan x = \frac{\tan 30^\circ}{2} \]  
\[ x = 16.1^\circ \text{ or } 196.1^\circ \]  
A1
2

(a) 

\[2x^2 - 2x + 1 = 0\]
\[\alpha + \beta = 1\]
\[\alpha\beta = \frac{1}{2}\]

sum of roots = \[\frac{2}{\alpha} + \frac{2}{\beta}\]

= \[\frac{2(\alpha^2 + \beta^2)}{(\alpha\beta)^2}\]

= \[\frac{2(\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)}{(\alpha\beta)^2}\]

= \[\frac{2(\alpha + \beta)((\alpha + \beta)^2 - 3\alpha\beta)}{(\alpha\beta)^2}\]

= \[\frac{2(1)(1 - \frac{3}{2})}{\left(\frac{1}{2}\right)^3}\] = -8

product of roots = \[\left(\frac{2}{\alpha^2}\right)\left(\frac{2}{\beta^2}\right)\]

= \[\frac{4}{\left(\frac{1}{2}\right)^4}\] = 32

equation is \[x^2 + 8x + 32 = 0\]

(b) (i) \[\alpha + (-\alpha) = \frac{8 + 4m}{3 + m} = 0\]

\[8 + 4m = 0\]
\[m = -2\]

(ii) \[\alpha\left(\frac{1}{\alpha}\right) = \frac{3 + 4m}{3 + m} = 1\]

\[3 + 4m = 3 + m\]
\[m = 0\]
(a) \[
\frac{2(4)^{k-3} - 2^{k-1}}{6^k \times 3^{k-3}} = \frac{2(2)^{k-3} - 2^{k-1}}{2^k \times 3^k \times \frac{3}{3^k}}
\]
\[A1 \text{ for } k = 10, A1 \text{ for } n = 1\]

(b) \[
\lg \frac{125}{y} + \lg y^4 = \lg (by)^4
\]
\[A1 \]
\[
\lg(125y^4) = \lg (by)^4
\]
\[A2 \; \text{and} \; b = 5\]

(c) \[
\log_5 e^{2x} + \frac{1}{\log_5 5} = \log_5 (2 - 3e^x)
\]
M1 for changing base
\[
\log_5 e^{2x} + \log_5 2 = \log_5 (2 - 3e^x)
\]
M1 for using correct laws
\[
2e^{2x} + 3e^x - 2 = 0
\]
M1 for the solving equation
\[
(2e^x - 1)(e^x + 2) = 0
\]
\[A1 \text{ for reject } e^x = -2\]
\[
e^x = \frac{1}{2} \quad \text{or} \quad e^x = -2 (N/A)
\]
\[A1 \]
\[
x = \ln \frac{1}{2} \quad \text{or} \quad -\ln 2 \quad \text{or} \quad -0.693
\]
4 \quad (x^2 + 2x - 3) = (x+3)(x-1) \quad \boxed{B1}

(i)

\begin{align*}
& f(1) = 0 \quad M1 \\
& a - b = 7 \quad (i) \quad M1 \\
& f(-3) = 0 \\
& 5a + b = 27 \quad (2) \\
& \text{solve (1) and (2)} \\
& a = 5 \quad \text{and} \quad b = -2 \quad A1
\end{align*}

(ii)

\begin{align*}
& f(x) = x^4 + 6x^3 + 10x^2 - 2x - 15 = (x^2 + 2x - 3)Q(x) \quad M1 \\
& Q(x) = x^2 + 4x + 5 \quad A1 \\
& \text{Show that } x^2 + 4x + 5 = 0 \text{ has no real roots using } b^2 - 4ac < 0. \quad A1 \\
& \text{Therefore } f(x) \text{ has only 2 real roots}
\end{align*}

5

(i) radius = 4 units \quad \boxed{B1}

Equation of the circle is \((x-4)^2 + (y+6)^2 = 16\). \quad \boxed{B1}

(ii) \text{Area} = \int_{-2}^{0} y^2 + 4^2 - \frac{1}{4} \pi (4)^2 \quad M1 \\
\begin{align*}
& = \left[\frac{y^3}{3}\right]_{-2}^{0} + 16 - 4\pi \\
& = 2\frac{2}{3} + 16 - 4\pi \\
& = 6.10 \text{ sq units} \quad \boxed{B1}
\end{align*}

(iii) centre is (4, 10) \quad \boxed{B1}

Equation is \((x-4)^2 + (y-10)^2 = 16\)
\begin{align*}
& x^2 + y^2 - 8x - 20y + 100 = 0 \quad A1
\end{align*}
6 (i) \[
\frac{dy}{dx} = 4x + (x+2)^3
\]
\[y = 2x^2 - \frac{1}{x+2} + C \quad \text{M1}\]

Subst. \(\left(\frac{1}{2}, \frac{1}{2}\right), \ C = \frac{2}{5} \quad \text{A1}\)

Equation of the curve is \(y = 2x^2 - \frac{1}{x+2} + \frac{2}{5} \quad \text{A1}\)

(ii) at \(x = \frac{1}{2}, \ \frac{dy}{dx} = \frac{54}{25} \quad \text{M1}\)

Gradient of normal = \(-\frac{25}{54} \quad \text{M1}\)

Equation of normal is \[y = \frac{25}{54}x + \frac{79}{108} \text{ or } 108y = -50x + 79 \quad \text{A1}\)

7 \[V = \frac{1}{3} \pi h^2 (3k - h) = \pi kh^3 - \frac{1}{3} \pi h^3 \]

\[\frac{dV}{dh} = 2\pi kh - \pi h^3 \quad \text{B1}\]

\[\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} \quad \text{M1}\]

\[\frac{dh}{dt} = \frac{k}{2\pi k \left(\frac{2k}{5}\right) - \pi (\frac{2k}{5})^2} \quad \text{M1}\]

\[= \frac{25}{16\pi k} \text{ m/s} \quad \text{A1}\]
\[ y = a \cos^3 x + b \cos x \]

\[ \frac{dy}{dx} = -3a \cos^2 x \sin x - b \sin x \]

\[ \frac{d^2y}{dx^2} = -3a \cos^2 x + 6a \cos x \sin^2 x - b \cos x \]

\[ = -3a \cos^2 x + 6a \cos x - 6a \cos^3 x - b \cos x \]

\[ = -9a \cos^3 x + (6a - b) \cos x \]

\[ \frac{d^3y}{dx^3} = -9y = -9a \cos^2 x - 9b \cos x \]

\[ (6a - b) \cos x = -9b \cos x \]

since \( \cos x \neq 0 \)

\[ 6a - b = -9b \]

\[ 6a + 8b = 0 \]

\[ 2a + 4b = 0 \]

\[ \frac{1}{x} + \frac{2}{y} = \frac{1}{2} \]

\[ 2y + 4x = xy' \]

Subst. \( y = -2x - 2 \) into above equation

\[ 2(-2x - 2) + 4x = x(-2x - 2) \]

\[ 2x^2 + 2x - 4 = 0 \]

\[ x^2 + x - 2 = 0 \]

\[ (x + 2)(x - 1) = 0 \]

\[ x = -2 \text{ or } x = 1 \]

\[ y = 2 \text{ or } y = -4 \]

Gradient of the line joining points A and B

\[ \frac{-4 - 2}{1 + 2} = -2 \]

gradient of the line \( 2y - x - 6 = 0 \)

The product of the 2 gradient = -1

Or the gradient of one of the line is equal to \( \frac{1}{-2} \)

The lines are perpendicular.

\[ \text{Turn over} \]
(i) \(-1 \leq k < 1\)  
(ii) \(k = 1\)  
(iii) \(k > 1\)

11 (i)

\[ \angle CAE = \angle BCA \quad \text{(ext } \angle \text{, cyclic quad)} \]
\[ \angle AEG = \angle CEB \quad \text{(common angles)} \]
\[ \therefore \triangle AEG \text{ is similar to } \triangle CEB \quad \text{(AA, similarity)} \]

(ii)

\[ \frac{AG}{BC} = \frac{GE}{BE} \quad \text{(from part (i))} \]
\[ \therefore BD = BE \quad \text{(given)} \]
\[ \frac{AG}{BC} = \frac{GE}{BD} \]
\[ AG \times BD = GE \times BC \quad \text{(shown)} \]
12. (a) \[ y = (2 + \sin x)^\frac{1}{2} \]

\[
\frac{dy}{dx} = \frac{1}{2} (2 + \sin x)^{-\frac{1}{2}} \cdot \cos x
\]

\[
= \frac{\cos x}{2\sqrt{2} + \sin x}
\]

(b) (i) when \( x = 0 \), \[ y = \frac{\cos 0}{2\sqrt{2} + \sin 0} = \frac{1}{2\sqrt{2}} \]

Point \( C \) is \((0, \frac{1}{2\sqrt{2}})\) or \((0, \frac{\sqrt{2}}{4})\)

(ii) Area = \[
\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{2\sqrt{2} + \sin x} \, dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \frac{\cos x}{2\sqrt{2} + \sin x} \, dx
\]

\[
= \left[ \sqrt{2} + \sin x \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} + \left[ \sqrt{2} + \sin x \right]_{\frac{\pi}{2}}^{\frac{3\pi}{4}}
\]

\[
= \sqrt{3} - \sqrt{2} + \sqrt{2 + \sin \frac{\pi}{2}} - \sqrt{2 + \sin \frac{\pi}{2}}
\]

\[
= \sqrt{3} - \sqrt{2} + \sqrt{2 + (-1) - \sqrt{3}}
\]

\[
= 1.05 \text{ sq units}
\]
READ THESE INSTRUCTIONS FIRST

Do not open the booklet until you are told to do so.

You are not required to submit this booklet at the end of the paper.

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal
place in the case of angles in degrees, unless a different level of accuracy is
specified in the question.
The use of an approved electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part
question.

The total number of marks for this paper is 100.

This document consists of 7 printed pages and 1 blank page.

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[Turn over]
2

Mathematical Formulae

1. ALGEBRA

Quadraatic Equation
For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.
\]

Binomial expansion

\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and \( \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1) \ldots (n-r+1)}{r!} \)

2. TRIGONOMETRY

Identities

\[
\begin{align*}
\sin^2 A + \cos^2 A &= 1 \\
\sec^2 A &= 1 + \tan^2 A \\
\csc^2 A &= 1 + \cot^2 A \\
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
tan(A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A \\
tan 2A &= \frac{2 \tan A}{1 - \tan^2 A}
\end{align*}
\]

Formulae for \( \triangle ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2} abc \sin C
\]
1. Without using a calculator, find the value of $a$ and of $b$ for which \( \frac{a\sqrt[4]{4} + b}{335} \) is the solution of the equation \( 3\sqrt[2]{2} + x\sqrt[3]{343} = x\sqrt[5]{50} + \sqrt[8]{5} \). \[4\]

2. (a) Show that the expression \( x^2 + px - x + p^2 + 2 \), where $p$ is a constant, is always positive for all real values of $x$.

   Hence, find the range of values of $x$ for which \( \frac{x^2 - 3x - 28}{x^2 + px - x + p^2 + 2} < 0 \). \[7\]

   (b) Find the range of values of $k$ for which the line $y = 2x - k$ cuts the curve $y^2 = x + k$ at two different points. \[4\]

3. (a) Given that the ratio of the coefficients of $x^3$ and $x^2$ in the expansion of \( \left( x^2 - \frac{k}{x} \right)^n \) is 1 : 4, find the possible values of $k$. \[5\]

   (b) The first three terms in the expansion of \( (2x - 3) \left( 1 + \frac{x}{3} \right)^6 \), in ascending powers of $x$, are $p + qx - \frac{7}{3} x^2$. Find the values of $n$, $p$ and $q$. \[5\]

4. (a) Express \( \frac{2x^3 - 5x^2 + 11x - 3}{(x^3 + 1)(x - 2)} \) in partial fractions. \[5\]

   (b) Prove the identity \( \frac{\sin x + \cos x}{\sin x - \cos x} = \frac{\sin x - \cos x}{\sin x + \cos x} = -2 \tan 2x \). \[3\]
The diagram shows part of the curve \( y = |p(x-r)^3 + q| \), where \( p, q \) and \( r \) are constants and \( p > 0 \). The curve cuts the \( y \)-axis at 24 and \((4, 8)\) is the turning point of the curve.

(i) Find the values of \( p, q \) and \( r \).
(ii) Find the coordinates of \( A \) and of \( B \).
(iii) Write down, with explanations, the number of solution(s) to the equation \( |p(x-r)^3 + q| = k|x-k| \) for \( 3 < k < 5 \) and

\[
\text{(a) } 0 < h < 1, \\
\text{(b) } h < 0.
\]

A car \( P \) moves in a straight line such that, \( t \) seconds after the start of motion, its velocity, \( v \) m s\(^{-1}\), is given by \( v = t - \frac{5}{2t+3} \).

The initial displacement of \( P \) is \( \left(1 - \frac{5}{2}\right) \) m.

(i) Find the value of \( t \) when \( P \) is at instantaneous rest.
(ii) Find an expression, in terms of \( t \), for the acceleration of \( P \) and determine whether \( P \) can attain maximum velocity.
(iii) Find the average speed of \( P \) for the first 2 seconds.
7 Answer the whole of this question on a piece of graph paper.
The table shows experimental values of two variables, \( x \) and \( y \), which are connected by an equation of the form \( y\sqrt{x} = k\sqrt{x}^b + nx \), where \( k \) and \( n \) are constants.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>3.00</td>
<td>4.53</td>
<td>5.83</td>
<td>7.00</td>
<td>8.09</td>
</tr>
</tbody>
</table>

(i) Using graph paper, plot \( \frac{y}{x} \) against \( \frac{1}{\sqrt{x}} \) and use your graph to estimate the value of \( k \) and of \( n \). [6]

(ii) Use your graph to estimate the value of \( y \) when \( x = 3.40 \). [2]

(iii) By drawing a suitable line on your graph, find the solution to the simultaneous equations \( y\sqrt{x} = k\sqrt{x}^b + nx \) and \( y = \sqrt{x} + x \). [3]

8 The diagram shows a rectangle, \( PQRS \), where the \( QR \) makes an angle \( \theta \) with a horizontal line \( QT \).

Given that \( PS = 14 \text{ cm} \), \( SR = 8 \text{ cm} \) and \( 0^\circ < \theta < 90^\circ \), show that the perpendicular distance, \( H \text{ cm} \), from \( S \) to the line \( QT \) is given by \( H = 8 \cos \theta + 14 \sin \theta \). [2]

(i) Express \( H \) in the form \( R \cos(\theta - \alpha) \), where \( R > 0 \) and \( 0^\circ < \alpha < 90^\circ \). [3]

(ii) Find the maximum value of \( H \) and the corresponding value of \( \theta \). [2]

(iii) Find the value of \( \theta \) when \( H = 12 \). [2]

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[Turn over]
The diagram shows a right circular cone in a sphere with centre O and radius 40 cm.
The vertex of the cone, V, and the circumference of its base lies on the sphere and
the centre of the sphere is on the axis of the cone.

(i) Given that the radius, height and volume of the right circular cone are $r$ cm,
h cm and $V$ cm$^3$ respectively, show that $V = \frac{\pi}{3} (80h^2 - h^3)$. [3]

(ii) Find the stationary value of $V$ and show that this value is a maximum. [5]

10. (a) Find the range of values of $x$ for which the curve $y = x^3 e^{-x}$ is a
decreasing function. [4]

(b) Given that $y = [\ln(3 - 4x)]^2$, show that $\frac{dy}{dx} = \frac{k \ln(3 - 4x)}{3 - 4x}$, where $k$ is a
constant to be determined.

Hence, evaluate $\int_{-3}^{2} \frac{2 + 3\ln(3 - 4x)}{3 - 4x} \, dx$. [7]
11 Solutions to this question by accurate drawing will not be accepted.

The diagram shows a kite $ABCD$. $M$ is the midpoint of $AC$ and the coordinates of $A$ and $C$ are $(2, 5)$ and $(10, 1)$ respectively.

(i) Find the coordinates of $M$. \([1]\)

(ii) Find the equation of $BD$. \([2]\)

(iii) Given that $B$ lies on the line $3x + 2y + 4 = 0$, find the coordinates of $B$. \([2]\)

(iv) Given that $\frac{BD}{MD} = 3$, find the coordinates of $D$. \([2]\)

(v) Find the area of the kite $ABCD$. \([2]\)

End of Paper
<table>
<thead>
<tr>
<th>Qn</th>
<th>Solutions and Marks Allocation</th>
</tr>
</thead>
</table>
| 1  | \[
3x\sqrt{2} + x\sqrt{543} = x\sqrt{50} + \sqrt{8}
7x\sqrt{7} - 2x\sqrt{2} = 2\sqrt{2} \quad M1
\]
|     | \[
x = \frac{2\sqrt{2}}{7\sqrt{7} - 2\sqrt{2}} \cdot \frac{7\sqrt{7} + 2\sqrt{2}}{7\sqrt{7} + 2\sqrt{2}} \quad M1 \text{ - rationalise}
\]
|     | \[
= \frac{14\sqrt{14} + 8}{335} \quad AI
\]
|     | \[
a = 14 \quad \text{and} \quad b = 8 \quad AI
\]
| 2(a) | \[
x^2 + px - x + p^2 + 2
= x^2 + (p - 1)x + p^2 + 2
\]
|     | \[
B^2 - 4ac = (p - 1)^2 - 4(0)(p^2 + 2) \quad M1
\]
|     | \[
= -3p^2 - 2p - 7
\]
|     | \[
= -3\left(p + \frac{1}{3}\right)^2 - \frac{2}{3} \quad AI
\]
|     | Since \[\left(p + \frac{1}{3}\right)^2 \geq 0,\] \[b^2 - 4ac < 0\]
|     | Since the coefficient of \(x^2\) is positive and the discriminant is less than zero, 
\[x^2 + px - x + p^2 + 2\] is always positive for all real values of \(x\). \[AI\]
|     | \[
x^2 - 3x - 28 < 0 \quad M1
\]
|     | \[
(x - 7)(x + 4) < 0 \quad M1
\]
|     | \[-4 < x < 7 \quad AI\]
| 2(b) | \[
j^2 = x + k
y = 2x - k
\]
|     | Sub (2) into (1):
|     | \[
(2x - k)^2 = x + k \quad M1
\]
|     | \[
4x^2 - (4k + 1)x + k^2 - k = 0 \quad AI
\]
|     | \[
b^2 - 4ac > 0
\]
|     | \[
(4k + 1)^2 - 4(4)(k^2 - k) > 0 \quad M1
\]
|     | \[
24k + 1 > 0
\]
|     | \[
k > \frac{-1}{24} \quad AI\]
\[ T_{r+1} = \binom{12}{r} (-k)^r x^{2n-3r} \quad \text{A1 for } x^{2n-3r} \]

Let \( 24 - 3r = 3 \),
\( r = 7 \)

Coefficient of \( x^3 = \binom{12}{7} (-k)^7 = -792k^7 \quad \text{A1} \)

Let \( 24 - 3r = 9 \)
\( r = 5 \)

Coefficient of \( x^9 = \binom{12}{5} (-k)^5 = -792k^5 \quad \text{A1} \)

\[
\frac{-792k^7}{-792k^5} = \frac{1}{4} \quad \text{M1}
\]

\( k = \pm \frac{1}{2} \quad \text{A1} \)

\[ (2x-3 \left(1+\frac{x^2}{3}\right)^3 = (2x-3 \left(1 + \frac{n}{3}x + \frac{n(n-1)}{18}x^2 + \ldots \right) \quad \text{A1} \]

\[ = p + qx - \frac{7}{3} x^3 + \ldots \]

Comparing the constant term:
\( p = -3 \quad \text{A1} \)

Comparing the coefficient of \( x \):
\( q = 2 - n \quad \text{A1} \)

Comparing the coefficient of \( x^2 \):
\[ \frac{7}{3} = \frac{2}{3} - \frac{n(n-1)}{6} \quad \text{M1} \]

\[ n^2 - 5n - 14 = 0 \]
\[ (n - 7)(n + 2) = 0 \]
\[ n = -2 \quad \text{or} \quad n = 7 \quad \text{A1} \]

(reject)

\[ \therefore q = -5 \quad \text{A1} \]
4(a) \[ \frac{2x^3 - 5x^2 + 11x - 3}{(x^2 + 1)(x-2)} = A + \frac{Br+C}{x^2+1} + \frac{D}{x-2} \quad M1 \]

\[ 2x^3 - 5x^2 + 11x - 3 = A(x^2 + 1)(x-2) + (Br+C)(x-2) + D(x^2 + 1) \]

Comparing the coefficient of \( x^3 \): \( A = 2 \)

Let \( x = 2 \), \( D = 3 \) \[ A3A2A1A0 \]

Let \( x = 0 \), \( C = 1 \)

Let \( x = 1 \), \( B = -4 \)

\[ \frac{2x^3 - 5x^2 + 11x - 3}{(x^2 + 1)(x-2)} = 2 - \frac{4x-1}{x^2+1} + \frac{3}{x-2} \quad A1 \]

4(b) \[ LHS = \frac{\sin x + \cos x}{\sin x - \cos x} \]

\[ = \frac{\sin x + \cos x}{\sin x - \cos x} \cdot \frac{\sin x - \cos x}{\sin x - \cos x} \quad M1 \]

\[ = \frac{(\sin x + \cos x)^2 - (\sin x - \cos x)^2}{\sin x - \cos x} \]

\[ = \frac{-1}{\sin x - \cos x} \]

\[ = -2 \tan 2x \]

5(i) \( r = 4 \) \quad B1

\( q = -8 \) \quad B1

Let \( y = 24 \) when \( x = 0 \),

\[ 24 = 16p - 8 \]

\[ 16p = 32 \] or \[ 16p = 8 \]

\[ p = 2 \] or \[ p = -1 \]

(reject)

5(ii) Let \( y = 0 \),

\[ 2(x-4)^2 - 8 = 0 \quad M1 \]

\[ (x-4)^2 = 4 \]

\[ x - 4 = \pm 2 \]

\[ x = 2 \] or \( 6 \) \quad A1

\[ A(2,0) \text{ and } B(6,0) \text{ A1} \]

5(iii)

(a) The graph of \( y = |x - k| \) is V-shaped and the vertex is located between \( x = 3 \) and \( x = 5 \) on the x-axis, thus there will be 4 solutions. \quad B1B1

(b) The graph of \( y = |x - k| \) is inverted V-shaped and the vertex is located between \( x = 3 \) and \( x = 5 \) on the x-axis, thus there will be no solution. \quad B1B1

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[Turn over]
6(i) Let \( v = 0 \),
\[
\begin{align*}
\frac{5}{2t+3} &= 0 & M1 \\
2t^2 + 3t - 5 &= 0 & M1 \\
(2t+5)(t-1) &= 0 \\
t &= \frac{-5}{2} \quad \text{or} \quad 1 & A1 \\
&\quad \text{(reject)}
\end{align*}
\]

6(ii) \[
\begin{align*}
a &= \frac{1 + \frac{10}{(2t+3)^2}}{2} = 0 & A1 \\
\text{Since } a \text{ cannot have a value of zero, } P \text{ cannot attain maximum velocity.} & A1
\end{align*}
\]

6(iii) \[
\begin{align*}
s &= \frac{t^2}{2} - \frac{5}{2} \ln(2t+3) + c, \text{ where } c \text{ is a constant.} & M1 \\
\text{Let } s &= 1 - \frac{5}{2} \ln 3 \text{ when } t = 0. \\
1 - \frac{5}{2} \ln 3 &= -\frac{5}{2} \ln 3 + c \\
c &= 1 \\
s &= \frac{t^2}{2} - \frac{5}{2} \ln(2t+3) + 1 & A1 \\
\text{When } t = 1, s &= \frac{3}{2} - \frac{5}{2} \ln 5 = -2.52359 \\
\text{When } t = 2, s &= 3 - \frac{5}{2} \ln 7 = -1.864775 \\
\text{When } t = 0, s &= 1 - \frac{5}{2} \ln 3 = -1.74653 \\
\text{Average speed} &= \frac{(2.52359 - 1.74653) + (2.52359 - 1.864775)}{2} = 0.718 \text{ m/s} & M1A1
\end{align*}
\]
From the diagram, \( h_1 = 8 \cos \theta \) and \( h_2 = 14 \sin \theta \), B1 each (working needed)

\[ H = 8 \cos \theta + 14 \sin \theta \]

(i) \[ R = \sqrt{260} = 2\sqrt{65} \quad \text{AI} \]
\[ \alpha = 60.26^\circ \quad \text{AI} \]
\[ H = 2\sqrt{65} \cos(\theta - 60.26^\circ) \quad \text{AI} \]

(ii) Max value of \( H = 2\sqrt{65} \quad \text{B1} \)
\[ \theta = 60.3^\circ \quad \text{B1} \]

(iii) When \( H = 12, \)
\[ 2\sqrt{65} \cos(\theta - 60.26^\circ) = 12 \]
\[ \cos(\theta - 60.26^\circ) = \frac{6}{\sqrt{65}} \quad M1 \]
\[ \theta - 60.26^\circ = -41.91^\circ \]
\[ \theta = 18.4^\circ \quad \text{AI} \]
9(i) \[
\begin{align*}
(h-40)^2 + r^2 &= 40^2 \quad M1 \\
r^2 &= 80h - h^3 \quad A1 \\
V &= \frac{1}{3} \pi r^2 h \\
&= \frac{\pi}{3} (80h - h^3) h \quad A1 \\
&= \frac{\pi}{3} (80h^3 - h^3) 
\end{align*}
\]

9(ii) \[
\begin{align*}
\frac{dV}{dh} &= \frac{\pi}{3} (160h - 3h^3) \quad M1 \\
\text{Let } \frac{dV}{dh} &= 0, \\
\frac{\pi}{3} (160h - 3h^3) &= 0 \\
h &= 0 \quad \text{or} \quad h = 53 \frac{1}{3} \quad A1 \\
&\text{(reject)} \\
\text{When } h &= 53 \frac{1}{3}, \\
V &= 79431.87 \approx 79400 \quad (3sf) \quad A1 \\
\frac{d^2V}{dh^2} &= \frac{\pi}{3} (160 - 6h) \quad M1 \\
\text{When } h &= 53 \frac{1}{3}, \\
\frac{d^2V}{dh^2} &= -\frac{160}{3} \pi < 0 \\
\text{The stationary value of } V \text{ is maximum.} \quad A1 \quad \text{[proof is needed]}
\end{align*}
\]
\[
\begin{align*}
10(a) & \quad y = x^2 e^{-2x} \\
& \quad \frac{dy}{dx} = x^2 e^{-2x} (-2) + 3x^3 e^{-2x} M2 \\
& \quad = x^2 e^{-2x} (3 - 2x) \\
& \quad \text{Let } \frac{dy}{dx} < 0, \\
& \quad x^2 e^{-2x} (3 - 2x) < 0 M1 \\
& \quad 3 - 2x < 0 \\
& \quad x > \frac{3}{2} A1
\end{align*}
\]

\[
\begin{align*}
(b) & \quad y = \left[\ln(3 - 4x)\right]^2 \\
& \quad \frac{dy}{dx} = 2 \ln(3 - 4x) \left( \frac{-4}{3 - 4x} \right) M1 \\
& \quad = \frac{-8 \ln(3 - 4x)}{3 - 4x} A1 \\
& \quad \therefore k = -8 A1 \\
& \quad \int_{-3}^{1} \frac{2 + 3 \ln(3 - 4x)}{3 - 4x} \, dx \\
& \quad = \int_{-3}^{1} \frac{2}{3 - 4x} \, dx - \int_{-3}^{1} \frac{8 \ln(3 - 4x)}{3 - 4x} \, dx M1 \\
& \quad = \left[ -\frac{1}{2} \ln(3 - 4x) \right]_{-3}^{1} - \frac{3}{8} \left[ (\ln(3 - 4x))^2 \right]_{-3}^{1} B2 \\
& \quad = \frac{1}{2} (\ln 7 - \ln 11) - \frac{3}{8} \left( (\ln 7)^2 - (\ln 11)^2 \right) M1 \\
& \quad = 0.22599 + 0.73625 \\
& \quad = 0.962 (3sf) A1
\end{align*}
\]
| (ii) | \[ \begin{align*} 
  m_{AC} &= -\frac{1}{2} \\
  m_{BD} &= 2 
\end{align*} \]  
Equation of BD:  
\[ y - 3 = 2(x - 6) \]  
\[ y = 2x - 9 \quad \text{(1)} \]  
\[ (ii) \]  
\[ (iii) \]  
\[ 3x + 2y + 4 = 0 \quad \text{(2)} \]  
Solving (1) and (2):  
\[ x = 2 \text{ and } y = -5 \]  
\[ B(2, -5) \]  
\[ (iii) \]  
| (iv) | \[ \frac{D_x - 2}{D_x - 6} = 3 \]  
\[ D_x = 8 \]  
\[ \frac{D_y + 5}{D_y - 3} = 3 \]  
\[ D_y = 7 \]  
\[ D(8, 7) \]  
\[ (iv) \]  
| (v) | Area of kite ABCD  
\[ \frac{1}{2} \begin{vmatrix} 
 2 & 10 & 8 & 2 \\
 5 & -5 & 1 & 7 
\end{vmatrix} \]  
\[ = \frac{1}{2} \begin{vmatrix} 
 102 + 16 
\end{vmatrix} \]  
\[ = 60 \text{ sq. units} \]  
\[ (v) \]
MARKING SCHEME

Name ____________________________________________

Class ________ Register Number ____________

4047/01

ADDITIONAL MATHEMATICS

PAPER 1

Wednesday 5 August 2015 2 hours

VICTORIA SCHOOL

PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

Additional Material: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical
expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact,
give the answer to three significant figures. Give answers in degrees to one decimal
place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the
answer in terms of \( \pi \).

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part
question.
The total number of marks for this paper is 80.

This paper consists of 5 printed pages, including the cover page.

[Turn over
Mathematical Formulae

1. ALGEBRA

**Quadratic Equation**

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

**Binomial Theorem**

\[
(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}
\]

2. TRIGONOMETRY

**Identities**

\[
\sin^2 A + \cos^2 A = 1
\]

\[
\sec^2 A = 1 + \tan^2 A
\]

\[
\csc^2 A = 1 + \cot^2 A
\]

\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
\sin 2A = 2 \sin A \cos A
\]

\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]

\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

**Formulae for \( \triangle ABC \)**

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
\alpha^2 = b^2 + c^2 - 2bc \cos \alpha
\]

\[
\Delta = \frac{1}{2} abc \sin \alpha
\]
1. Find the range of the values of $x$ which satisfy both inequalities $0 < x^3 - 4x$ and $x^2 - 4x \leq 3x + 10$.

2. Solve
   
   (i) $\frac{3^{x-1}}{y} = \frac{1}{\sqrt{2}^x}$
   
   (ii) $3e^{x} - e = 2e^{2x}$

3. (i) Find the coefficient of the term in $x$ in the expansion of $\left(x^2 - \frac{1}{2x}\right)^4$.
   
   (ii) The coefficient of $x^3$ in the expansion $(5-3x)(1+5x)^n$ is 1785. Find the value of $n$.

4. The gradient to a curve is given by $\frac{dy}{dx} = (kx+3)^2$, where $k$ is a non-zero constant. The equation of the tangent to the curve at the point $(1, 2)$ is $9x - y - 5 = 0$. Find the
   
   (i) value of $k$.
   
   (ii) equation of the curve.

5. Sketch the graph of $y = -|x+1| + 2$ for $-4 \leq x \leq 2$.
   
   (i) State the range of values of $p$ for which the equation $-|x+1| = p - 2$ has at least 1 solution for $-4 \leq x \leq 2$.
   
   (ii) Using your graph, state the number of solutions for $-|x+1| + 2 = x + 3$.

6. (i) Find the exact value of $x$ in the equation $\sqrt{12}x + 5 = \sqrt{x} + 19$.

   (ii) A cuboid with a square base of length $\sqrt{3} + 1$ cm, has a volume of $(5\sqrt{2})^3 - 8\sqrt{3}$ cm$^3$. Find the height of the cuboid in the form $a + b\sqrt{3}$.
7 A curve has the equation \( y = xe^{-x} \).

(i) Find \( \frac{dy}{dx} \). \[2\]

(ii) Hence show that \( \int_{0}^{\ln 2} 4xe^{x} \, dx = 16\ln 2 - \frac{3}{4} \). \[4\]

(iii) Find the range of values of \( x \) for which the function \( y = xe^{-x} \) is decreasing. \[2\]

8 \( AB \) is a chord of the circle \( x^2 + y^2 - 8x - 2y - 3 = 0 \) and \( M \left( \frac{4}{5}, \frac{2}{5} \right) \) is the midpoint of chord \( AB \). Find the

(i) radius and the coordinates of the centre of the circle, \[2\]

(ii) equation of chord \( AB \). \[3\]

If \( P \) is a variable point on the circle, find the

(iii) maximum area of triangle \( AEP \). \[4\]

9 The function \( f \) is defined, for \( 0 \leq x \leq 2\pi \), by \( f(x) = 2\cos ax + b \), where \( a \) and \( b \) are integers. The minimum value of \( f \) is \(-1\) and the period of \( f \) is \( \frac{4\pi}{3} \).

(i) State the amplitude of \( f \). \[1\]

(ii) State the values of \( a \) and of \( b \). \[1\]

(iii) Using the values of \( a \) and \( b \) found in part (ii),

(a) solve \( f(x) = 0 \) for \( 0 \leq x \leq 2\pi \), leaving your answers in terms of \( \pi \). \[4\]

(b) sketch the graph of \( f(x) = 2\cos ax + b \) for \( 0 \leq x \leq 2\pi \). \[3\]

10 A particle moves in a straight line such that \( t \) seconds after leaving a fixed point \( O \), the velocity \( v \) m/s, is given by \( v = 3t^2 - t - 10 \). Find the

(i) initial acceleration of the particle, \[2\]

(ii) minimum velocity of the particle, \[2\]

(iii) total distance travelled by the particle in the first 3 seconds, \[4\]

(iv) average speed of the particle during the first 3 seconds. \[2\]
11 Solutions to this question by accurate drawing will not be accepted.

The diagram shows the trapezium $ABCD$ in which $BC$ is parallel to $AD$ while $BA$ produced is perpendicular to $CD$ produce at point $E$. The point $A$ is $(-1, 5)$, $C$ is $(5, 3)$ and $D$ is $(3, 2)$.

(i) Show that the coordinates of $B$ are $(-3, 9)$. [6]

(ii) Find the area of trapezium $ABCD$. [2]

(iii) Given that $\frac{\text{area of } \triangle MED}{\text{area of } \triangle REC} = \frac{1}{4}$, find the coordinates of $E$. [3]

______________________________

End of Paper
1. $-1.22 \leq x < 0$ or $4 < x \leq 8.22$

2(i) $\frac{1}{3}$ 2(ii) $x = 1$

3(i) Coefficient of $x = -7$ 3(ii) $n = 6$

4(i) $k = -6$ 4(ii) $y = \frac{1}{2} - \frac{3(1-2x)^2}{2}$

5. \[ \begin{array}{c}
\text{Y} \\
\text{X}
\end{array} \]

5(i) $-1 \leq p \leq 2$

5(ii) There are infinite number of solutions.

6(i) $x = \frac{2\sqrt{2}}{3}$ 6(ii) height = $62 - 33\sqrt{3}$

7(i) \[ \frac{dy}{dx} = e^x (4x + 1) \] 7(ii) $x < -\frac{1}{4}$

8(i) Centre of circle is $(4, 1)$

(ii) radius = 4.47 units

(iii) Max area = 22.2 units$^2$

9(i) Amplitude = 2, (ii) $a = 1.5$, $b = 1$

(iii) \[ x = \frac{4\pi}{9}, \frac{8\pi}{9}, \frac{16\pi}{9} \]

9(iii) (b) \[ f(x) = 2\cos\left(\frac{3}{2}x\right) + 1 \]

10(i) $-1$ m/s$^2$

(ii) Min velocity = $-10 \frac{1}{12}$ m/s

(iii) Total Distance = 20.5 m

(iv) Ave Speed = $\frac{5}{6}$ or 6.83 m/s

1(i) Area = 15 units$^2$

(iii) $E(1,1)$
ADDITIONAL MATHEMATICS

PAPER 1

Wednesday 5 August 2015
2 hours

VICTORIA SCHOOL
PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

Additional Material: Answer Paper

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[Turn over
2
Mathematical Formulas

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\[(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}
\]

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\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]

\[
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\]

\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

\[
\sin 2A = 2 \sin A \cos A
\]

\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]

\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulas for \( \Delta ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
\Delta = \frac{1}{2}ab \sin C
\]
1. Find the range of the values of \( x \) which satisfy both inequalities
\[
0 < x^2 - 4x \quad \text{and} \quad x^2 - 4x \leq 3x + 10.
\]
\[
x^2 - 4x > 0 \quad \quad x^2 - 7x - 10 \leq 0
\]
\[
x(x - 4) > 0 \quad \quad \text{for } x^2 - 7x - 10 = 0
\]
\[
x < 0 \text{ or } x > 4
\]
\[
x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-10)}}{2(1)}
\]
\[
= -1.22 \text{ or } 8.22
\]
\[
\therefore x^2 - 7x - 10 \leq 0
\]
\[
-1.22 \leq x \leq 8.22
\]
Hence the solution is \(-1.22 \leq x < 0 \text{ or } 4 < x \leq 8.22\).

2. Solve
(i) \( \frac{3^{x+2}}{9^x} = \frac{1}{\sqrt[3]{27^x}} \) \( \frac{3^{x+2}}{9^x} = \frac{1}{\sqrt[3]{27^x}} \)
\[
\frac{3^{x+2}}{9^x} = \frac{1}{\sqrt[3]{27^x}}
\]
\[
\frac{3^{x+2}}{3^x} = \frac{1}{3^x}
\]
\[
2 - x - 2x = -\frac{3x}{2}
\]
\[
x = \frac{3}{2}
\]
\[
x = \frac{4}{3} \quad \frac{1}{3}
\]
(ii) \( 3e^x - e = 2e^{x^2} \)
\[
3e^x - e = 2e^{x^2}
\]
\[
3e^x - e = \frac{2e^2}{e^x}
\]
\[
3(e^x) - e - e^x - 2e^2 = 0
\]
\[
(e^x - e)(3e^x + 2e) = 0
\]
\[
e^x = e \quad \text{or} \quad 3e^x = -2e
\]
\[
x = 1 \quad \text{(NA)}
\]

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13/54PR2/AM/1
3 (i) Find the coefficient of the term in $x$ in the expansion of $\left(x^2 - \frac{1}{2x^2}\right)^6$. 

For $\left(x^2 - \frac{1}{2x^2}\right)^6$,

$$T_{r+1} = \binom{6}{r} \left(x^2\right)^{6-r} \left(-\frac{1}{2x^2}\right)^r$$

$$= \binom{6}{r} \left(-\frac{1}{2}\right)^r x^{2(6-r)-2r}$$

For term in $x$,

$$16 - 5r = 1$$

$$5r = 15$$

$$r = 3$$

Coefficient of $x = \binom{6}{3} \left(-\frac{1}{2}\right)^3$

$$= -\frac{7}{8}$$

(ii) The coefficient of $x^1$ in the expansion $(5-3x)(1+5x)^n$ is 1785. Find the value of $n$. 

$$(5-3x)(1+5x)^n$$

$$= (5-3x) \left(1 + \binom{n}{1} (5x) + \binom{n}{2} (5x)^2 + ... \right)$$

$$= (5-3x) \left(1 + 5nx + \frac{n(n-1)}{2!} x^2 + ... \right)$$

Coefficient of $x^1$ in the above expansion is 1785

$$125 \times \frac{n(n-1)}{2} - 3(5n) = 1785$$

$$125n(n-1) - 30n = 3570$$

$$125n^2 - 125n - 30n - 3570 = 0$$

$$125n^2 - 155n - 3570 = 0$$

$$25n^2 - 31n - 714 = 0$$

$$(n-6)(25n+119) = 0$$

$$n-6 = 0 \quad \text{or} \quad 25n+119 = 0$$

$$n = 6 \quad \text{or} \quad n = -\frac{119}{25} (\text{N.A.})$$

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15/04/23/AM/1
4 The gradient to a curve is given by \[ \frac{dy}{dx} = (kx + 3)^2 \], where \( k \) is a non-zero constant. The equation of the tangent to the curve at the point \((1, 2)\) is \( 9x - y - 5 = 0 \). Find the

(i) value of \( k \),

\[
9x - y - 5 = 0
\]
\[
y = 9x - 5
\]
Gradients of tangent = 9
At \((1, 2)\), \[ \frac{dy}{dx} = 9 \]
\[
(k + 3)^2 = 9
\]
\[
k + 3 = 3 \quad \text{or} \quad k + 3 = -3
\]
\[
k = 0 \quad \text{(N.A.)} \quad \text{or} \quad k = -6
\]

(ii) equation of the curve.

Equation of curve is,
\[
y = \int (-6x + 3)^2 \, dx
\]
\[
= \frac{(-6x + 3)^3}{3(-6)} + c
\]
\[
= \frac{(3 - 6x)^3}{18} + c
\]
At \((1, 2)\),
\[
2 = \frac{(3 - 6)^3}{18} + c
\]
\[
c = \frac{1}{2}
\]
\[
\therefore \text{equation of curve is},
\]
\[
y = \frac{(3 - 6x)^3}{18} + \frac{1}{2}
\]
\[
y = \frac{1}{2} - \frac{3(1 - 2x)^3}{2}
\]
5. Sketch the graph of \( y = -|x + 1| + 2 \) for \(-4 \leq x \leq 2\). \([3]\)

(i) State the range of values of \( p \) for which the equation \(-|x + 1| = p - 2\) has at least 1 solution for \(-4 \leq x \leq 2\). \([1]\)

\[-1 \leq p \leq 2\]

(ii) Using your graph, state the number of solutions for \(-|x + 1| + 2 = x + 3\). \([1]\)

There are infinite number of solutions.
6. (i) Find the exact value of \( x \) in the equation \( \sqrt{112}x + 5 = \sqrt{7}x + 19 \). \[ \begin{align*}
\sqrt{112}x + 5 &= \sqrt{7}x + 19 \\
4\sqrt{7}x - \sqrt{7}x &= 14 \\
3\sqrt{7}x &= 14 \\
x &= \frac{14}{3\sqrt{7}} \\
x &= \frac{2\sqrt{7}}{3}
\end{align*} \]

(ii) A cuboid with a square base of length \( \sqrt{3} + 1 \) cm, has a volume of \( (\sqrt{3})^3 - 8\sqrt{3} \) cm\(^3\). Find the height of the cuboid in the form \( a + b\sqrt{3} \).

\[
\text{Height} = \frac{(\sqrt{3})^3 - 8\sqrt{3}}{(\sqrt{3} + 1)}
\]
\[
= \frac{25(\sqrt{3})^2 - 8\sqrt{3}}{4 + 2\sqrt{3}} \\
= \frac{25(2) - 8\sqrt{3}}{4 + 2\sqrt{3}}
\]
\[
= \frac{50 - 8\sqrt{3}}{4 + 2\sqrt{3}} \\
= \frac{25 - 4\sqrt{3}}{2}
\]
\[
= \frac{25 - 2\sqrt{3}}{1}
\]
\[
= 25 - 2\sqrt{3}
\]
A curve has the equation \( y = xe^{4x} \).

(i) Find \( \frac{dy}{dx} \).

\[
\begin{align*}
y &= xe^{4x} \\
\frac{dy}{dx} &= xe^{4x} + e^{4x} (i) \\
&= e^{4x} (4x + 1)
\end{align*}
\]

(ii) Hence show that \( \int_{0}^{\ln 2} 4xe^{4x} \, dx = 16\ln 2 - \frac{3}{4} \).

\[
\begin{align*}
\int_{0}^{\ln 2} 4xe^{4x} \, dx &= \left[ xe^{4x} \right]_{0}^{\ln 2} \\
&= 4x e^{4x} + e^{4x} \bigg|_{0}^{\ln 2} - 0 \\
&= 4x e^{4x} + \int_{0}^{\ln 2} e^{4x} \, dx = \ln 2 \times e^{4 \ln 2} - 0 \\
&= \int_{0}^{\ln 2} 4xe^{4x} \, dx + \int_{0}^{\ln 2} e^{4x} \, dx = \ln 2 \times e^{4 \ln 2} \\
&= \int_{0}^{\ln 2} e^{4x} \, dx = \ln 2 \times 16 - \int_{0}^{\ln 2} e^{4x} \, dx \\
&= 16\ln 2 - \int_{0}^{\ln 2} e^{4x} \, dx \\
&= 6\ln 2 - \frac{1}{4} (e^{4 \ln 2} - 1) \\
&= 16\ln 2 - \frac{1}{4} (16 - 1) \\
&= 16\ln 2 - \frac{3}{4}
\end{align*}
\]

(ii) Find the range of values of \( x \) for which the function \( y = xe^{4x} \) is decreasing.

For \( y \) to be decreasing,

\[
\frac{dy}{dx} < 0
\]

\[
e^{4x} (4x + 1) < 0
\]

Since \( e^{4x} > 0 \) for all values of \( x \),

then \( 4x + 1 < 0 \)

\[
x < -\frac{1}{4}
\]

Victoria School 15/4 PR2/ANU1
8. \( AB \) is a chord of the circle \( x^2 + y^2 - 8x - 2y - 3 = 0 \) and \( M\left(\frac{4}{5}, \frac{2}{3}\right) \) is the midpoint of chord \( AB \). Find the

(i) radius and the coordinates of the centre of the circle.

\[
x^2 + y^2 - 8x - 2y - 3 = 0
\]
\[
x^2 + y^2 + 2(-4)x + 2(-1)y + (-3) = 0
\]
\[
\therefore \ g = -4, \ f = -1, \ c = -3
\]

Hence centre of circle is \((4, 1)\)

radius of circle is \(\sqrt{g^2 + f^2 - c} = \sqrt{(-4)^2 + (-1)^2 - (-3)}\)

\[
= \sqrt{16 + 1 + 3} = \sqrt{20}
\]

= 4.47 units (3 sf)

(ii) equation of chord \( AB \).

Let the centre of circle be \( C \).

\[
\therefore \text{gradient of } CM = \frac{\frac{2}{3} - 1}{\frac{4}{5} - 4} = \frac{-7}{16}
\]

\[
\therefore \text{gradient of chord } AB = \frac{16}{7}
\]

Hence equation of chord \( AB \) is,

\[
y - 2 = \frac{16}{7} \left( x - \frac{4}{5} \right)
\]

\[
= \frac{16}{7}x - \frac{64}{35}
\]

\[
\therefore y = \frac{16}{7}x + \frac{4}{7}
\]

If \( P \) is a variable point on the circle, find the

(iii) maximum area of triangle \( ABP \).

Length of \( CM = \sqrt{\left(\frac{4}{5} - \frac{4}{5}\right)^2 + \left(\frac{2}{3} - \frac{9}{5}\right)^2}
\]

\[
= \sqrt{\frac{12}{25} - \frac{12}{25}} = \sqrt{\frac{12}{25}} = \frac{12}{5}
\]

Length of \( BM = \sqrt{BC^2 - CM^2}
\]

\[
= \sqrt{20 - \frac{12}{25}} = \sqrt{\frac{472}{25}} = \frac{4\sqrt{2}}{5}
\]

Length of chord \( AB = 2 \times BM
\]

\[
= 2 \times \sqrt{\frac{4\sqrt{2}}{5}} = \frac{2\sqrt{4\sqrt{2}}}{5} = \frac{2\sqrt{8}}{5}
\]

Area of \( \Delta ABP \) is maximum when \( P, C & M \) are collinear and \( PM \perp AB \).

\[
\therefore \text{maximum area of } \Delta ABP
\]

\[
= \frac{1}{2} \times AB \times PM
\]

\[
= \frac{1}{2} \times AB \times (CM + CP)
\]

\[
= \frac{1}{2} \times \left(2 \times \frac{4\sqrt{2}}{5}\right) \times \left(\sqrt{\frac{12}{5} + \sqrt{20}}\right)
\]

\[
= 2.2 \text{ units}^2 \text{ (3 sf)}
\]

![Diagram of triangle ABC with point P, M, and C labeled.]
The function $f$ is defined, for $0 \leq x \leq 2\pi$, by $f(x) = 2\cos ax + b$, where $a$ and $b$ are integers. The minimum value of $f$ is $-1$ and the period of $f$ is $\frac{4\pi}{3}$.

(i) State the amplitude of $f$.  
Amplitude = 2  

(ii) State the values of $a$ and of $b$.  
$a = 2\pi + \frac{4\pi}{3} = 1.5$  
$b = 1$

(iii) Using the values of $a$ and $b$ found in part (ii),

(a) solve $f(x) = 0$ for $0 \leq x \leq 2\pi$, leaving your answers in terms of $\pi$,  

\[
2\cos\left(\frac{3}{2}x\right) + 1 = 0
\]

\[
\cos\left(\frac{3}{2}x\right) = -\frac{1}{2}
\]

\[
a = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}
\]

\[
\frac{3}{2}x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}, 2\pi + \left(\pi - \frac{\pi}{3}\right)
\]

\[
\frac{3}{2}x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}
\]

\[
x = \frac{4\pi}{9}, \frac{8\pi}{9}, \frac{16\pi}{9}
\]

(b) sketch the graph of $f(x) = 2\cos \frac{3}{2}x + b$ for $0 \leq x \leq 2\pi$.  

![Graph of $f(x) = 2\cos \frac{3}{2}x + 1$]
A particle moves in a straight line such that $t$ seconds after leaving a fixed point $O$, the velocity $v$ m/s, is given by $v = 3t^2 - t - 10$. Find the

(i) initial acceleration of the particle,

\[ \dot{v} = 3t^2 - t - 10 \]
\[ a = \frac{\dot{v}}{\dot{t}} \]
\[ = 6t - 1 \]

Initial acceleration $= 6(0) - 1$
\[ = -1 \text{ m/s}^2 \]

(ii) minimum velocity of the particle,

Minimum velocity of the particle occurs when $a = 0$
\[ 6t - 1 = 0 \]
\[ t = \frac{1}{6} \]

\[ \therefore \text{minimum velocity of the particle,} \]
\[ = 3\left(\frac{1}{6}\right)^2 - \left(\frac{1}{6}\right) - 10 = -10 \frac{1}{12} \text{ m/s} \]

(iii) total distance travelled by the particle in the first 3 seconds,

\[ s = \int (3t^2 - t - 10) \, dt \]
\[ = t^3 - \frac{1}{2} t^2 - 10t + c \]

At $t = 0$, $s = 0$, $\therefore c = 0$

Hence, $s = t^3 - \frac{1}{2} t^2 - 10t$

When $v = 0$,
\[ 3t^2 - t - 10 = 0 \]
\[ (3t + 5)(t - 2) = 0 \]
\[ 3t + 5 = 0 \text{ or } t - 2 = 0 \]
\[ t = -\frac{5}{3} \text{ (N.A.)} \text{ or } t = 2 \]

At $t = 2$,
\[ s = 2^3 - \frac{1}{2} (2)^2 - 10(2) = -14 \]

At $t = 3$,
\[ s = 3^3 - \frac{1}{2} (3)^2 - 10(3) = -\frac{7}{2} \]

\[ \therefore \text{total distance travelled in the first 3 seconds} \]
\[ = 14 + (14 - 7.5) \]
\[ = 20.5 \text{ m} \]

(iv) the average speed of the particle during the first 3 seconds.

Average speed of the particle during the first 3 seconds

\[ = \frac{\text{total distance}}{\text{total time}} \]
\[ = \frac{20.5}{3} \]
\[ = 6.83 \text{ m/s} \]
Solutions to this question by accurate drawing will not be accepted.

The diagram shows the trapezium ABCD in which BC is parallel to AD while BA produced is perpendicular to CD produce at point E. The point A is (-1, 5), C is (5, 3) and D is (3, 2).

(i) Show that the coordinates of B are (-3, 9).

Gradient of BC = Gradient of AD
\[
\frac{5 - 2}{1 - 3} = \frac{3}{4}
\]

Sub (5, 3) into \( y = \frac{3}{4}x + c \),
\[
3 = \frac{3}{4}(5) + c
\]
\[
c = \frac{6}{4}
\]

Equation of BC is \( y = \frac{3}{4}x + \frac{3}{4} \).

Gradient of CD = \( \frac{3 - 2}{5 - 3} = \frac{1}{2} \)

Gradient of BA = -2

Sub (-1, 5) into \( y = -2x + d \),
\[
5 = -2(-1) + d
\]
\[
d = 3
\]

Equation of BA is \( y = -2x + 3 \)

\[
\frac{3}{4}x + \frac{3}{4} = -2x + 3
\]
\[
-3x + 27 = -8x + 12
\]
\[
5x = -15
\]
\[
x = -3
\]

when \( x = -3 \),
\[
y = -2(-3) + 3 = 9
\]

\( \therefore B(-3, 9) \)
11 (ii) Find the area of trapezium $ABCD$.

Area of trapezium $ABCD$

$$\begin{align*}
\frac{1}{2} \begin{vmatrix} 5 & -3 & -1 & 3 & 5 \\ 2 & 3 & 9 & 5 & 2 & 3 \\
\end{vmatrix}
= \frac{1}{2} \left[ (45 - 15 - 2 + 9) - (-9 - 9 + 15 + 10) \right]
= \frac{1}{2} (37 - 7)
= 15 \text{ units}^2
\end{align*}$$

(iii) Given that $\frac{\text{area of } \triangle AED}{\text{area of } \triangle BEC} = \frac{1}{4}$, find the coordinates of $E$.

Since $\triangle AED$ and $\triangle BEC$ are similar,

$$\frac{ED}{EC} = \frac{EA}{EB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{1}{2}$$

$\therefore D$ and $E$ are the midpoints of $EC$ and $EB$ respectively.

Let $E(m, n)$,

$$\left( \frac{5 + m}{2}, \frac{3 + n}{2} \right) = (3, 2)$$

$$\begin{align*}
\frac{5 + m}{2} &= 3 \\
\frac{3 + n}{2} &= 2 \\
m &= 1 \\
n &= 1
\end{align*}$$

$\therefore E(1, 1)$

or

$$\left( \frac{-3 + m}{2}, \frac{9 + n}{2} \right) = (-1, 5)$$

Sub $(3, 2)$ into $y = \frac{1}{2}x + f$,

$$\begin{align*}
2 &= \frac{1}{2}(3) + f \\
f &= \frac{1}{2}
\end{align*}$$

Equation of $CD$ is $y = \frac{1}{2}x + \frac{1}{2}$

Equation of $BA$ is $y = -2x + 3$

$$\begin{align*}
\frac{1}{2}x + \frac{1}{2} &= -2x + 3 \\
x &= \frac{5}{3} \\
x + 1 &= -4x + 6 \\
x &= 1
\end{align*}$$

when $x = 1$, $y = -2(1) + 3$

$\therefore E(1, 1)$

End of Paper
READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact,
give the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This paper consists of 7 printed pages, including the cover page.
2

Mathematical Formulae

I. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial Theorem

\[
(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}
\]

II. TRIGONOMETRY

Identities

\[
\sin^2 A + \cos^2 A = 1
\]
\[
\sec^2 A = 1 + \tan^2 A
\]
\[
\csc^2 A = 1 + \cot^2 A
\]
\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
\sin 2A = 2 \sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]
\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \Delta ABC \)

\[
a = \frac{b}{\sin A} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\Delta = \frac{1}{2} abc \sin C
\]

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Water from a tank in the shape of an inverted cone flows out at the rate of 5 cm³/min. The height of the cone is 48 cm and the base diameter is 16 cm. After t minutes the water level is h cm.

(i) Show that the volume of water in the tank, \( V \) cm³, at time t is given by \( V = \frac{\pi h^3}{108} \) \[2\]

(ii) Find the rate of change of the water level when \( h = 6 \). \[3\]

(iii) State, with a reason, whether this rate will increase or decrease as t increases. \[1\]

2. The displacement, y mm, of a mass fixed on a vertical spring can be described by the simple harmonic motion equation, \( y = Asin(\omega t) \), where A and \( \omega \) are constants and \( t \) is the time in seconds after the mass is displaced from its equilibrium position, 0 mm.

Given that the maximum displacement of the mass is 20 mm and that the mass first returns to its equilibrium position after 0.25 seconds.

(i) State the positive value of \( A \). \[1\]

(ii) Show that the value of \( \omega \) is \( 4\pi \) radians per second. \[2\]

(iii) Find the exact value of \( t \) when the mass first reach a position 10 mm below its equilibrium position. \[3\]

3. Given that \( f(x) = 2x^3 + ax^2 + bx - 30 \) has a factor \( (x + 3) \) and leaves a remainder of -28 when divided by \( (x - 1) \). Find the values of \( a \) and \( b \) and solve \( f(x) = 0 \). \[6\]

(ii) Hence solve \( 2(y+1)^3 + a(y+1)^2 + by + b - 30 = 0 \). \[2\]
The diagram shows points $A$, $B$, $C$ and $D$ lying on a circle. The chords $BD$ and $AC$ intersect at $G$. $EF$ is a tangent to the circle at $C$. $AD$ is produced and meets the tangent at $F$ and $\angle ABC = \angle BGC$.

Prove that

(i) $BD$ is parallel to $EF$, 
(ii) triangle $CFD$ and triangle $AFC$ are similar, 
(iii) $FC^2 - FD^2 = FD \times DA$.

5 (i) Express $\frac{3x^2 + 10x}{(x+2)(x^2-4)}$ in partial fractions.

(ii) Using your answer from (i), find $\int \frac{3x^2 + 10x}{(x+2)(x^2-4)} \, dx$ and hence show that 
$$\int \frac{3x^2 + 10x}{(x+2)(x^2-4)} \, dx = \ln \left( \frac{2x}{5} \right) + \frac{1}{15}.$$ 

6 (a) The quadratic equation $2x^2 - 2x + 4 = 0$ has roots $\alpha + \beta$ and $\alpha + 3\beta$.

(i) Show that the values of $\alpha + \beta = \frac{1}{6}$ and $\alpha \beta = \frac{5}{16}$.

(ii) If the roots of the equation $\alpha x^2 - h\alpha^2 = 0$ where $g$ and $h$ are constants, are $\alpha$ and $\beta$, find the value of $g$ and of $h$.

(b) Find the range of values of $k$ for which $(k + 3)x^2 + kx + 1$ is always positive for all real values of $x$.
5

Answer the whole of this question on a sheet of graph paper.

The table shows experimental values of two variables, x and y.

<table>
<thead>
<tr>
<th>x</th>
<th>0.4</th>
<th>0.6</th>
<th>-0.8</th>
<th>1.0</th>
<th>1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>2.22</td>
<td>2.13</td>
<td>1.97</td>
<td>1.73</td>
<td>1.37</td>
</tr>
</tbody>
</table>

It is known that x and y are related by the equation \( y = (ax + 1)x - b \), where \( a \) and \( b \) are constants.

(i) On graph paper, plot \((y^2 - x)\) against \( x \), using a scale of 2 cm to represent 0.2 unit on the \( x \)-axis and 4 cm to represent 1 unit on the \((y^2 - x)\)-axis. Draw a straight line graph to represent the equation \( y' = (ax + 1)x - b \).

(ii) Use your graph to estimate the value of \( a \) and of \( b \).

(iii) By drawing a suitable straight line on your graph, solve the equation \( (a - 2) = \frac{1 + b}{x^2} \).

---

8

A piece of wire 160 cm long is bent to form the shape shown in the figure. This shape consists of a semi-circular arc whose diameter is given by the length \( DE \), and a right-angled triangle \( ABC \) on the opposite ends of a rectangle of length \( y \) cm. The length of \( BC \) and \( AB \) are 5x cm and 13x cm respectively.

(i) Express \( y \) in terms of \( x \).

(ii) Show that the area enclosed, \( A \) cm\(^2\), is given by \( A = 960x - 6(3x + 13)x^3 \).

(iii) Determine the value of \( x \) for which \( A \) has a stationary value.

(iv) Find the stationary value of \( A \) and determine if it is a maximum or a minimum value.
9 (a) (i) Prove that \( \cos A = \frac{\cos 2A}{\cos A} + \tan A \sin A \). 

(ii) Solve, for \( 0^\circ \leq A \leq 360^\circ \), \( \cos A - \tan A \sin A = -1 \). 

(b) Given \( \cos \theta = -\frac{4}{5} \) and \( \theta \) is in the third quadrant. Without using a calculator, find the value of \( \cos \frac{\theta}{2} \). 

10 (a) Solve the equation \( \log_3 \frac{1}{2} = \log_2 x - \log_4 (9x - 2) \). 

(b) Given that \( \log_3 (x+3) - (\log_2 y) (\log_3 2) = 2 \), express \( y \) in terms of \( x \). 

(c) (i) Differentiate \( \ln \cos x \). 

(ii) State the principal value of \( \tan^{-1} \), giving your answer as a multiple of \( \pi \). 

The diagram shows part of the graph \( y = \tan x \). The shaded region is bounded by the curve, the \( x \)-axis, lines \( x = \frac{\pi}{6} \), \( x = \frac{5\pi}{6} \) and \( y = 1 \). 

(iii) Using your results from (i) and (ii), or otherwise, find the area of the shaded region.
A L-shaped structure, $OPQ$, can be rotated about $O$. $OP$ and $PQ$ measures 5 m and 1 m respectively. $OP$ makes an acute angle, $\theta$, with the ground. Given that $L$ m is the shortest distance from $Q$ to the wall,

(i) show that $L = 5 \cos \theta - \sin \theta$.  

(ii) express $L$ in the form $R \cos (\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. 

(iii) state the minimum value of $L$ and find the corresponding value of $\theta$. 

(iv) find the value of $\theta$ when $L = 3$. 

(v) explain why the maximum value of $L$ is not $R$.  

---

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(ii) \( \frac{dh}{dt} = -1.59 \text{ cm/min} \)

(iii) As \( t \) increases, \( h \) decreases.

Since \( \frac{dh}{dt} = -\frac{180}{\pi h^3} \), \( \frac{dh}{dt} \) is inversely proportional to \( h^3 \), hence rate of change of water level increases when \( h \) decreases.

\[ \frac{dh}{dt} = -\frac{180}{\pi h^3} \]

- Since \( h > 0 \) for all positive \( h \), then \( \frac{d^2h}{dt^2} > 0 \).
- Hence \( \frac{dh}{dt} \) is an increasing function.

2(i) \( A = 20 \) (ii) \( t = \frac{7}{24} \)

3(i) \( a = 7 \), \( b = -7 \)
- \( x = -3 \) or \( x = 2 \) or \( x = -2.5 \)

(ii) \( y = -4 \) or \( y = 1 \) or \( y = -3.5 \)

4 Plane Geometry

5(i) \[ \frac{2}{x-2} + \frac{1}{x+2} = \frac{2}{(x+2)^2} \]

6a(ii) \( g = -3 \frac{1}{5} \) \( h = -\frac{8}{15} \)

6b \(-2 < k < 6\)

7(ii) \( a = -3.00 \) \( b = -5 \)

7(iii) Draw \( y^2 - x = 2x^2 + 1 \)
- \( x = \pm 0.894 \)

8(i) \( y = 80 - 3(\pi + 3)x \)

8(ii) \( x = 3.57 \)

8(iv) Stationary value of \( A = 1710 \)
- \( A \) is maximum

9a(ii) \( \theta = 60^\circ, 180^\circ, 300^\circ \)

9b \( \cos \theta = \frac{\sqrt{10}}{2} \)

10(a) \( x = \frac{1}{4} \) or \( x = 2 \)

(b) \( y = \frac{(x+3)^3}{64} \)

(c)(i) \( -\tan x \)

(ii) Principal value of \( \tan^{-1} 1 = \frac{\pi}{4} \)

(iii) Area = 2.04 units

11(ii) \( L = 5.10 \cos(\theta + 11.3^\circ) \)

(iii) \( \text{Min } L = 0 \) when \( \theta = 78.7^\circ \)

(iv) \( \theta = 42.7^\circ \)

(v) If \( L = R \) then \( \theta < 0^\circ \).
- Since \( 0^\circ \leq \theta < 90^\circ \), maximum \( L = R \).
- Since \( \theta \geq 0^\circ \), maximum \( L \) occurs when \( \theta = 0^\circ \), maximum \( L = 5 \).
READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets \([ \ ] \) at the end of each question or part question.
The total number of marks for this paper is 100.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation \( ax^2 + bx + c = 0 \),

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Binomial Theorem

\[
(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,
\]

where \( n \) is a positive integer and

\[
\binom{n}{r} = \frac{n!}{r! (n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}
\]

2. TRIGONOMETRY

Identities

\[
\sin^2 A + \cos^2 A = 1
\]
\[
\sec^2 A = 1 + \tan^2 A
\]
\[
\csc^2 A = 1 + \cot^2 A
\]
\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]
\[
\sin 2A = 2 \sin A \cos A
\]
\[
\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]
\[
\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}
\]

Formulae for \( \triangle ABC \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\[
a^2 = b^2 + c^2 - 2bc \cos A
\]
\[
\Delta = \frac{1}{2} \cdot ab \cdot \sin C.
\]
1. Water from a tank in the shape of an inverted cone flows out at the rate of 5 cm$^3$/min. The height of the cone is 48 cm and the base diameter is 16 cm. After \( t \) minutes the water level is \( h \) cm.

(i) Show that the volume of water in the tank, \( V \) cm$^3$, at time \( t \) is given by \( V = \frac{\pi h^3}{108} \). \[2\]

Using similar triangles,
\[
\frac{r}{8} = \frac{h}{48} \rightarrow r = \frac{h}{6}
\]
Volume of water,
\[
V = \frac{1}{3} \pi r^2 h
\]
\[
= \frac{1}{3} \pi \left(\frac{h}{6}\right)^2 h
\]
\[= \frac{\pi h^3}{108}\]

(ii) Find the rate of change of the water level when \( h = 6 \).

\[
\frac{dV}{dt} = \frac{\pi h^2}{36}
\]
\[
\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}
\]
\[
-5 = \frac{\pi h^2}{36} \times \frac{dh}{dt}
\]
\[
\frac{dh}{dt} = \frac{-180}{\pi h^2}
\]
At \( h = 6 \),
\[
\frac{dh}{dt} = \frac{-180}{\pi 6^2} = -1.59 \text{ cm}^3/\text{min} \] \( (3sf) \)

(iii) State, with a reason, whether this rate will increase or decrease as \( t \) increases.

As \( t \) increases, \( h \) decreases. Since \( \frac{dh}{dt} = \frac{-180}{\pi h^2} \), \( \frac{dh}{dt} \) is inversely proportional to \( h^2 \), hence rate of change of water level increases when \( h \) decreases.
2. The displacement, \( y \) mm, of a mass fixed on a vertical spring can be described by the simple harmonic motion equation, \( y = A \sin(\omega t) \), where \( A \) and \( \omega \) are constants and \( t \) is the time in seconds after the mass is displaced from its equilibrium position, 0 mm.

Given that the maximum displacement of the mass is 20 mm and that the mass first returns to its equilibrium position after 0.25 seconds.

(i) State the positive value of \( A \). \( [1] \)

\[ A = 20 \]

(ii) Show that the value of \( \omega \) is \( 4\pi \) radians per second. \( [2] \)

\[
\begin{align*}
0 &= 20 \sin \omega(0.25) \\
\sin \frac{1}{4} \omega &= 0 \\
\frac{1}{4} \omega &= 0, \pi \\
\omega &= 0 \text{ (rejected)}, 4\pi
\end{align*}
\]

(iii) Find the exact value of \( t \) when the mass first reaches a position 10 mm below its equilibrium position. \( [3] \)

\[
\begin{align*}
-10 &= 20 \sin 4\pi t \\
\sin 4\pi t &= -\frac{1}{2} \\
4\pi t &= -\frac{\pi}{6} \\
4\pi t &= \pi + \frac{\pi}{6} \\
t &= \frac{7\pi}{6} \times \frac{1}{4\pi} \\
t &= \frac{7}{24} \text{ s}
\end{align*}
\]
3

(i) Given that \( f(x) = 2x^3 + ax^2 + bx - 30 \) has a factor \((x - 3)\) and leaves a remainder of 
\(-28\) when divided by \((x - 1)\). Find the values of \(a\) and \(b\) and solve \( f(x) = 0 \). [6]

\[
\begin{align*}
R(-3) &= 2(-3)^3 + a(-3)^2 + b(-3) - 30 = 0 \\
&= -54 + 9a - 3b - 30 = 0 \\
3a - b &= 28 \quad \cdots (1) \\

R(1) &= 2(1)^3 + a(1)^2 + b - 30 = -28 \\
a + b &= 0 \quad \cdots (2)
\end{align*}
\]

\((1) + (2):\) \(4a = 28\) \(\therefore a = 7\)

\[
\begin{align*}
b &= -7 \\
f(x) &= 0 \\
2x^3 + 7x^2 - 7x - 30 &= 0 \\
(x + 3)(2x^2 + x - 10) &= 0 \\
(x + 3)(x - 2)(2x + 5) &= 0 \\
x &= -3 \quad \text{or} \quad x = 2 \quad \text{or} \quad x = 2.5
\end{align*}
\]

(ii) Hence solve \(2(y + 1)^3 + a(y + 1)^2 + by + b - 30 = 0\). [2]

\[
\begin{align*}
2(y + 1)^3 + a(y + 1)^2 + by + b - 30 &= 0 \\
2(y + 1)^3 + a(y + 1)^2 + b(y + 1) - 30 &= 0
\end{align*}
\]

Let \(x = y + 1\),
\[
\begin{align*}
y + 1 &= -3 \quad \text{or} \quad y + 1 = 2 \quad \text{or} \quad y + 1 = -2.5 \\
y &= -4 \quad \quad y = 1 \quad \quad y = -3.5
\end{align*}
\]
The diagram shows points $A$, $B$, $C$ and $D$ lying on a circle. The chords $BD$ and $AC$ intersect at $G$. $EF$ is a tangent to the circle at $C$. $AD$ is produced to meet the tangent at $F$ and $\angle ABC = \angle BGC$.

Prove that

(i) $BD$ is parallel to $EF$, 

\[ \angle ACF = \angle ABC \quad (\text{\angle s in alternate segment}) \]
\[ \angle ABC = \angle BGC \quad \text{(Given)} \]
\[ \therefore \angle BGC = \angle ACF \]

By the angle property of alternate angles, $BD$ is parallel to $EF$.

(ii) triangle $CFD$ and triangle $AFC$ are similar,

\[ \angle CFD = \angle AFC \quad \text{(Common \angle)} \]
\[ \angle DCF = \angle CAF \quad (\text{\angle s in alternate segment}) \]

Hence $\triangle CFD$ is similar to $\triangle AFC$.

(iii) $FC^2 - FD^2 = FD \times DA$.

Since $\triangle CFD$ is similar to $\triangle AFC$,

\[
\frac{FD}{FC} = \frac{CF}{AF} \\
\therefore \frac{FC}{AF} = \frac{FD}{DF} \times AF \\
\therefore FC^2 = FD \times (FD + DA) \\
\therefore FC^2 = FD^2 + FD \times DA \\
\therefore FC^2 - FD^2 = FD \times DA \quad \text{(Proven)}
\]
(i) If the roots of the equation \( gx^2 - hx - 1 = 0 \) where \( g \) and \( h \) are constants, are \( \alpha \) and \( \beta \), find the value of \( g \) and of \( h \).

\[
\alpha + \beta = \frac{h}{g}
\]
\[
\frac{1}{\alpha} = \frac{g}{h}
\]

\[
.: \alpha = \frac{1}{g}; \beta = \frac{h}{g}
\]

\[
\alpha \beta = \frac{-1}{g}
\]
\[
\frac{5}{16} = \frac{-1}{g}
\]
\[
g = \frac{-16}{5}
\]
\[
= -3 \frac{1}{5}
\]

Sub \( g = -3 \frac{1}{5} \) into (1)

\[
.: \beta = 3 \frac{1}{5}
\]
\[
= -8 \frac{8}{15}
\]

Alternative solution:

\[
x^3 - \frac{1}{6}x + \frac{5}{16} = 0
\]
\[
\frac{16}{5}x^3 + \frac{8}{15}x - 1 = 0
\]
\[
gx^2 - hx - 1 = 0
\]

(b) Find the range of values of \( k \) for which \((k + 3)x^2 + kx + 1\) is always positive for all real values of \( x \).

\((k + 3)x^2 + kx + 1 > 0\)

Since the expression is always positive,

\( k + 3 > 0 \) and \( b^2 - 4ac < 0 \)

\( k > -3 \)

\( k^2 - 4(k + 3) < 0 \)

\( k < 6 \)

\( (k - 6)(k + 2) < 0 \)

\( -2 < k < 6 \)

Hence \( k > -3 \) and \( -2 < k < 6 \)

\( \therefore \) the solution is \( -2 < k < 6 \)
7 c. Answer the whole of this question on a sheet of graph paper.

The table shows experimental values of two variables, $x$ and $y$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
<th>1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>2.22</td>
<td>2.13</td>
<td>1.97</td>
<td>1.73</td>
<td>1.37</td>
</tr>
</tbody>
</table>

It is known that $x$ and $y$ are related by the equation $y^2 = (ax + 1)x - b$, where $a$ and $b$ are constants.

(i) On graph paper, plot $(y^2 - x)$ against $x^2$, using a scale of 2 cm to represent 0.2 unit on the $x^2$ axis and 4 cm to represent 1 unit on the $(y^2 - x)$ axis. Draw a straight line graph to represent the equation $y^2 = (ax + 1)x - b$.

<table>
<thead>
<tr>
<th>$x^2$</th>
<th>0.160</th>
<th>0.360</th>
<th>0.640</th>
<th>1.00</th>
<th>1.44</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y^2 - x$</td>
<td>4.53</td>
<td>3.94</td>
<td>3.08</td>
<td>1.99</td>
<td>0.677</td>
</tr>
</tbody>
</table>

(ii) Use your graph to estimate the value of $a$ and of $b$.

$y^2 = (ax + 1)x - b$
$y^2 = ax^2 + x - b$
$y^2 - x = ax^2 - b$

Gradient $= a$

\[
\frac{5 - 3.5}{0 - 0.5} = a
\]
\[a = -3.00 \text{ (3sf)}.
\]

$(y^2 - x)$ - intercept $=-b$

$-b = 5$
\[\therefore b = -5
\]

(iii) By drawing a suitable straight line on your graph, solve the equation $(a - 2) = \frac{1 + b}{x^2}$.

$(a - 2) = \frac{1 + b}{x^2}$
$ax^2 - 2x^2 = 1 + b$
$ax^2 - b = 2x^2 + 1$

Draw $y^2 - x = 2x^2 + 1$

From graph, $x^2 = 0.8$

$x = \pm 0.894 \text{ (3sf)}$
8 A piece of wire 160 cm long is bent to form the shape shown in the figure. This shape consists of a semi-circular arc whose diameter is given by the length \(DE\), and a right-angled triangle \(ABC\) on the opposite ends of a rectangle of length \(y\) cm. The length of \(BC\) and \(AB\) are 5x cm and 13x cm respectively.

(i) Express \(y\) in terms of \(x\).

\[ AC = \sqrt{(13x)^2 - (5x)^2} = 12x \]

Perimeter of figure = Length of wire
\[ 2y + \frac{\pi 12x}{2} + 13x + 5x = 160 \]
\[ 2y + 6\pi x + 18x = 160 \]
\[ y + 3\pi x + 9x = 80 \]
\[ \therefore y = 80 - 3(\pi + 3)x \]

(ii) Show that the area enclosed, \(A\) cm\(^2\), is given by \(A = 960x - 6(3\pi + 13)x^2\).

\[
A = \frac{1}{2} \pi (5x)^2 + y(12x) + \frac{1}{2} (5x)(12x) \\
= 18\pi x^2 + 30x^2 + 12x[80 - 3(\pi + 3)x] \\
= (18\pi + 30)x^2 + 960x - 36(\pi + 3)x^2 \\
= [18\pi + 30 - 36(\pi + 3)]x^2 + 960x \\
= (18\pi + 30 - 36\pi - 108)x^2 + 960x \\
= (-18\pi - 78)x^2 + 960x \\
= 960x - 6(3\pi + 13)x^2
\]
8  (iii) Determine the value of \( x \) for which \( A \) has a stationary value.

\[ A = 960x - 6(3\pi + 13)x^2 \]
\[ \frac{dA}{dx} = 960 - 12(3\pi + 13)x \]
For stationary value of \( A \),
\[ \frac{dA}{dx} = 0 \]
\[ 960 - 12(3\pi + 13)x = 0 \]
\[ x = \frac{960}{12(3\pi + 13)} \]
\[ \approx 3.567 \]
\[ = 3.57 \text{ (3 sf)} \]

(iv) Find the stationary value of \( A \) and determine if it is a maximum or a minimum value.

Stationary value of \( A = 960(3.567) - 6(3\pi + 13)(3.567)^2 \)
\[ = 1712.39 \]
\[ = 1710 \text{ (3 sf)} \]

\[ \frac{d^2A}{dx^2} = -12(3\pi + 13) \]
since \( \frac{d^2A}{dx^2} < 0 \), \( A \) is a maximum value.

9  (a) (i) Prove that \( \cos A = \frac{\cos 2A}{\cos A} + \tan A \sin A \).

\( \text{RHS} = \frac{\cos 2A}{\cos A} + \tan A \sin A \)
\[ = \frac{2\cos^2 A - 1}{\cos A} + \sin A \cdot \frac{\sin A}{\cos A} \]
\[ = \frac{2\cos^2 A - 1 + \sin^2 A}{\cos A} \]
\[ = \frac{2\cos^2 A - 1 + (1 - \cos^2 A)}{\cos A} \]
\[ = \frac{\cos^2 A}{\cos A} \]
\[ = \cos A \]
\[ = \text{LHS} \]
9. (n) (ii) Solve, for \(0^\circ \leq A \leq 360^\circ\): \( \cos A - \tan A \sin A = -1 \). 

\[
\begin{align*}
\cos A - \tan A \sin A &= -1 \\
\cos 2A &= -1 \\
\cos A &= -1 \\
2 \cos^2 A - 1 &= -\cos A \\
2 \cos^2 A + \cos A - 1 &= 0 \\
(\cos A + 1)(2 \cos A - 1) &= 0 \\
\cos A &= -1 \\
A &= 180^\circ \\
\cos A &= \frac{1}{2} \\
A &= 60^\circ, 360^\circ - 60^\circ \\
&= 60^\circ, 300^\circ \\
\end{align*}
\]

(b) Given \( \cos \theta = -\frac{4}{5} \) and \( \theta \) is in the third quadrant. Without using a calculator, find the value of \( \cos \frac{\theta}{2} \).

\[
\begin{align*}
\cos \theta &= -\frac{4}{5} \\
2 \cos^2 \frac{\theta}{2} - 1 &= -\frac{4}{5} \\
2 \cos^2 \frac{\theta}{2} &= \frac{1}{5} \\
\cos \frac{\theta}{2} &= \frac{1}{\sqrt{10}} \\
&\pm \frac{\sqrt{10}}{10} \\
\text{Since } 90^\circ < \frac{\theta}{2} < 135^\circ, \quad \cos \frac{\theta}{2} &= -\frac{\sqrt{10}}{10} \\
\end{align*}
\]
10 (a) Solve the equation \( \log_2 \frac{1}{2} = \log_2 x - \log_4 (9x - 2) \).

\[
\log_2 \frac{1}{2} = \log_2 x - \log_4 (9x - 2)
\]

\[
\log_4 (9x - 2) = \log_2 x - \log_2 \frac{1}{2}
\]

\[
\log_4 (9x - 2) = \log_2 \left( \frac{x + 1}{2} \right)
\]

\[
\frac{\log_4 (9x - 2)}{2} = \log_2 2x
\]

\[
\log_4 (9x - 2) = 2 \log_2 2x
\]

\[
\log_4 (9x - 2) = \log_4 (2x)^2
\]

\[
\therefore 9x - 2 = 4x^2
\]

\[
4x^2 - 9x + 2 = 0
\]

\[
(4x - 1)(x - 2) = 0
\]

\[
x = \frac{1}{4} \text{ or } x = 2
\]

(b) Given that \( \log_2 (x + 3) - (\log_2 y)(\log_2 2) = 2 \), express \( y \) in terms of \( x \).

\[
\log_2 (x + 3) - (\log_2 y)(\log_2 2) = 2
\]

\[
\log_2 (x + 3) - \log_2 y \times 1 = 2
\]

\[
\log_2 (x + 3) = \frac{\log_2 y}{3}
\]

\[
3 \log_2 (x + 3) = \log_2 y = 6
\]

\[
\log_2 (x + 3)^3 = \log_2 y = 6
\]

\[
\log_2 \frac{(x + 3)^3}{y} = 6
\]

\[
\frac{(x + 3)^3}{y} = 2^6
\]

\[
y = (x + 3)^3
\]

\[
y = \frac{(x + 3)^3}{64}
\]

\[
\frac{(x + 3)}{y} = 2^2
\]

\[
(x + 3)^2 = 4
\]

\[
y = \frac{(x + 3)^2}{64}
\]
10 (c) (i) Differentiate $\ln \cos x$.

$$\frac{d}{dx} \ln \cos x = \frac{-\sin x}{\cos x} = -\tan x$$

(ii) State the principal value of $\tan^{-1} 1$, giving your answer as a multiple of $\pi$.

Principal value of $\tan^{-1} 1 = \frac{\pi}{4}$

The diagram shows part of the graph $y = \tan x$. The shaded region is bounded by the curve, the $x$-axis, lines $x = \frac{5\pi}{6}$ and $y = 1$.

(iii) Using your results from (i) and (ii), or otherwise, find the area of the shaded region.

$$\text{Area} = \int_{\frac{\pi}{4}}^{\frac{5\pi}{6}} \tan x \, dx + \left( \frac{5\pi}{6} - \frac{\pi}{4} \right)$$

$$= -\left[ \ln \cos x \right]^2 + \frac{7\pi}{12}$$

$$= \left[ \ln \cos \frac{\pi}{4} - \ln \cos \frac{\pi}{6} \right] + \frac{7\pi}{12}$$

$$= -\ln \frac{1}{\sqrt{2}} + \ln \frac{\sqrt{3}}{2} + \frac{7\pi}{12}$$

$$= \frac{1}{2} \ln \frac{3}{2} + \frac{7\pi}{12}$$

$$= 2.04 \text{ units}^2 \text{ (3sf)}$$
A L-shaped structure, $OPQ$, can be rotated about $O$. $OP$ and $PQ$ measures $5$ m and $1$ m respectively. $OP$ makes an acute angle, $\theta$, with the ground. Given that $L$ m is the shortest distance from $Q$ to the wall,

(i) show that $L = 5 \cos \theta - \sin \theta$.

\[
\begin{align*}
\cos \theta &= \frac{PY}{S} \\
PY &= 5 \cos \theta \\
\sin \theta &= \frac{PX}{1} \\
PX &= \sin \theta \\
\therefore L &= PY - PX \\
&= 5 \cos \theta - \sin \theta
\end{align*}
\]

(ii) express $L$ in the form $R \cos (\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$.

\[
\begin{align*}
L &= 5 \cos \theta - \sin \theta \\
&= R \cos (\theta + \alpha) \\
&= R \cos \theta \cos \alpha - R \sin \theta \sin \alpha \\
R \cos \alpha &= 5 \quad \ldots (1) \\
R \sin \alpha &= 1 \quad \ldots (2)
\end{align*}
\]

\[
(1)^2 + (2)^2 : \quad R^2 \cos^2 \alpha + R^2 \sin^2 \alpha = 5^2 + 1^2 \\
R &= \sqrt{26} \\
&= 5.10 \, (3sf)
\]

\[
\begin{align*}
\frac{R \sin \alpha}{R \cos \alpha} &= \frac{1}{5} \\
\tan \alpha &= \frac{1}{5} \\
\alpha &= \tan^{-1} \left(\frac{1}{5}\right) \\
&= 11.31^\circ \, (2dp)
\end{align*}
\]

\[
\therefore L = 5.10 \cos (\theta + 11.3^\circ) \quad (3sf, 1dp)
\]

VICTORIA SCHOOL 15/5/PR2/AM/2
11 (iii) state the minimum value of \( L \) and find the corresponding value of \( \theta \).

Minimum \( L = 0 \) when \( \cos(\theta + 11.31^\circ) = 0 \)

\[
\begin{align*}
\theta + 11.31^\circ &= 90^\circ \\
\theta &= 78.7^\circ \text{ (1 dp)}
\end{align*}
\]

(iv) find the value of \( \theta \) when \( L = 3 \),

\[
3 = \sqrt{26} \cos(\theta + 11.31^\circ)
\]
\[
\cos(\theta + 11.31^\circ) = \frac{3}{\sqrt{26}}
\]
\[
\alpha = \cos^{-1}\left(\frac{3}{\sqrt{26}}\right) = 53.96^\circ
\]
\[
\theta + 11.31^\circ = 53.96^\circ
\]
\[
\theta = 42.7^\circ \text{ (1 dp)}
\]

(v) explain why the maximum value of \( L \) is not \( R \).

If \( L = R \) then \( \theta < 0^\circ \). Since \( 0^\circ \leq \theta < 90^\circ \), \( \rightarrow \) maximum \( L \neq R \).

[ Since \( \theta \geq 0^\circ \), maximum \( L \) occurs when \( \theta = 0^\circ \), maximum \( L = 5 \). ]