2015 Sec 4 A-Maths (Total 456 Pages)

1	ACS Independent	SA2
2	Anderson Sec	SA2
3	Anglican High	SA2
4	Catholic High	SA2
5	CHIJ ST Joseph's Convent	SA2
6	Chung Cheng High	SA2
7	Fairfield Methodist	SA2
8	Holy Innocents High	SA2
9	Nanyang Girls High	SA2
10	Paya Lebar Methodist Girl's	SA2
11	Swiss Cottage Sec	SA2
12	Tanjong Katong Sec	SA2
13	Temasek Sec	SA2
14	Victoria	SA2

Name	Class	Index no.	Calculator Model

DEYI SECONDARY SCHOOL

Preliminary Examination 2015
Secondary Four Express / Five Normal Academic

MATHEMATICS **4016/01**

Paper 1

28 Aug 2015
1040 – 1240h
2 hours

Candidates answer on the Question Paper.
No additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen in the spaces provided on the Question Paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid / tape.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.

Calculators should be used where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, the answer should be given to three significant figures.
Answers in degrees should be given to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiner's Use
80

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Calculate $\dfrac{6.7^2 - \sqrt[3]{4.8}}{20.15 - 19.99}$.

Write down the first six digits of your answer.

Answer (*a*) .... [1]

(b) Write down your answer to **part (a)** correct to 2 significant figures.

Answer (*b*) .... [1]

---

2 Write the following in descending order.

$$\frac{54}{67} \qquad \sqrt[3]{0.512} \qquad 0.77^{\frac{5}{6}} \qquad 0.802$$

Answer .... [2]

---

3 Given that $9 \times 27^{-n} = 1$, find the value of n.

Answer $n =$ .... [2]

---

4 The sine of an obtuse angle is 0.6.

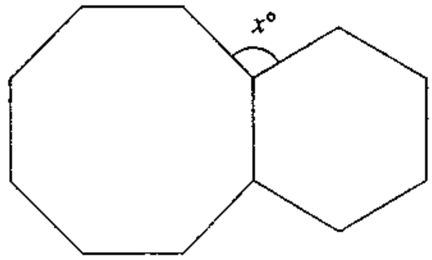
Without using a calculator, find the cosine of this angle.

Answer .... [2]

[Turn over

z

5



The diagram shows a sketch of a regular hexagon and a regular octagon.

Calculate x.

Answer $x =$ .... [2]

6 Brian is leaving Singapore to further his studies in the United Kingdom.
In Singapore, the exchange rate is 1 Singapore Dollar = 0.478 British Pounds.
In the United Kingdom, the exchange rate is 1 British Pound = 2.113 Singapore Dollars.

Brian would like to change 2500 Singapore Dollars into British Pounds.

How many fewer British Pounds will he get by changing his money in the United Kingdom?

Answer .... British Pounds [2]

7 Matthew bought a new car. At the end of each year, the car's value depreciates by 10%.

The value, $\$C$, of the car t years after being bought is given by

$$C = 85000 \times 0.9^t.$$

(a) How much did Matthew pay for his new car?

Answer (*a*) $ .... [1]

(b) Find the percentage decrease in the value of his car at the end of three years.

Answer (*b*) .... % [1]

---

8 Two geometrically similar solids made from the same material have masses 3.60 kg and 12.15 kg respectively.

Calculate the ratio of the area of the smaller solid to the area of the larger solid.

Answer .... : .... [2]

[Turn over

3

9 Dawn invested some money in a savings account that was compounded every six months.

The rate of compound interest was 5% per annum.

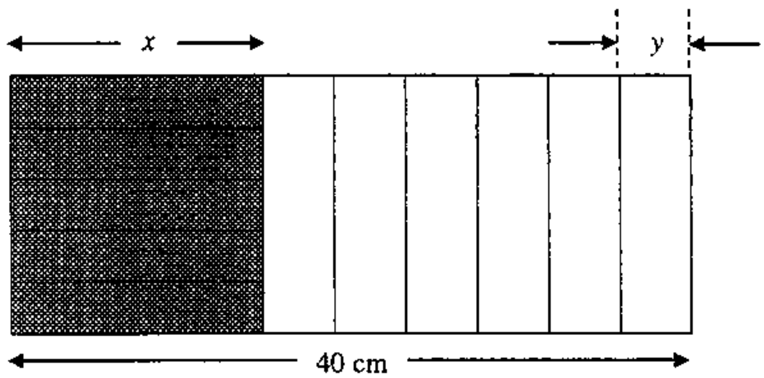
At the end of the 4 years there was $14620.83 in her account.

How much did Dawn invest in the account at first?

Give your answer correct to the nearest dollar.

Answer $ .... [3]

10 A rectangle with length 40 cm is divided into five identical shaded rectangles and another six identical unshaded rectangles.



The shaded area makes up two-thirds of the unshaded area.

Find the lengths labelled x and y.

Answer $x =$ .... cm

$y =$ .... cm [3]

11 Mr Tan decided to buy a laptop under a hire-purchase scheme.
He would have paid \$3906 in total under the scheme, which consists of a deposit of 15% of the selling price of the laptop plus 24 equal monthly payments of \$140.25.

What is the selling price of the laptop?

Answer \$ .... [3]

---

12 Two points P and Q have position vectors **p** and **q** respectively, relative to an origin O.

It is given that $\mathbf{p} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} k \\ 0 \end{pmatrix}$.

Find

(a) $\overrightarrow{PQ}$ in terms of k,

Answer (*a*) $\overrightarrow{PQ}$ = ... [1]

(b) the possible values of k if OP and OQ are two sides of a rhombus.

Answer (*b*) k = ... or .... [2]

4

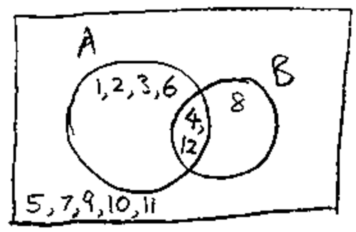
13 ε = { integers x : $1 \le x \le 12$ }

A = { factors of 12 }

B = { multiples of 4 }

(a) Draw a Venn diagram to illustrate this information.

Answer (*a*)



[2]

(b) Describe in words what the set $(A \cup B)'$ represents.

Answer (*b*) .. [1]

---

14 A train 45 m long passes through a tunnel 6 km long.

The average speed of the train is 27 km/h.

(a) Change 27 km/h into m/s.

Answer (*a*) .... m/s [1]

(b) Calculate the time taken for the train to pass completely through the tunnel.

Give your answer in minutes and seconds, to the nearest second.

Answer (*b*) .... minutes ... seconds [3]

15 Simplify

(a) $28x^2y^{-3} \div 16x^3y^{-1}$,

Answer (*a*) .... [2]

(b) $\dfrac{2}{x-3} + \dfrac{3x}{x^2-9}$.

Answer (*b*) .... [2]

16 The numbers 1 to 100 are arranged in a table as shown below.

A *U*-shaped, shaded frame can be placed around various numbers throughout the table.

	2		4	5	6	7	8	9	10
			14	15	16	17	18	19	20
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
91	92	93	94	95	96	97	98	99	100

The ***U*-number** is used to refer to the **shaded frame** that is drawn **around a particular number**.

For example, U_2 refers to the **shaded frame** shown above since it is drawn **around the number 2.**

(a) State the largest possible *U*-number.

Answer (*a*) .... [1]

(b) Write and simplify an expression, in terms of *n*, for the sum of the numbers in U_n.

Answer (*b*) .... [2]

(c) Find the sum of numbers in U_{75}.

Answer (*c*) .... [1]

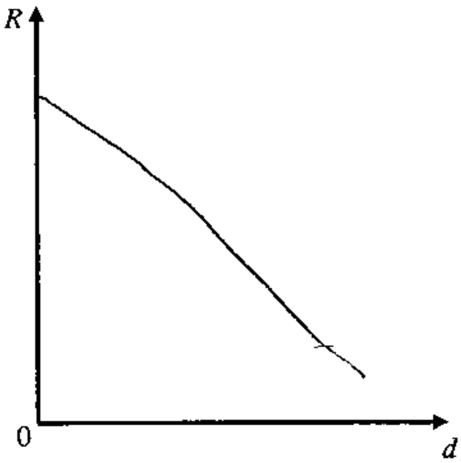
[Turn over

5

17 The resistance of a wire, R ohms, is inversely proportional to the square of its diameter, d μm.

(a) Sketch a resistance-diameter graph for the wire.

Answer (*a*)



[1]

For a fixed length of wire, the resistance is 25.6 ohms when the diameter is 50 μm.

(b) Find the equation for R in terms of d.

Answer (*b*) R = ... [2]

(c) Another wire of the same length has resistance 120 ohms.

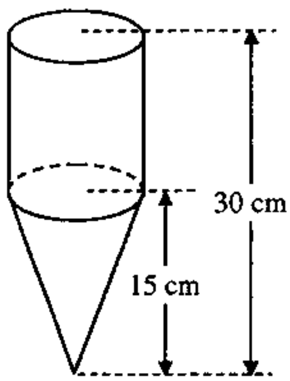
Calculate its diameter.

Answer (*c*) .... μm [1]

18 A composite container made from a cylinder and a cone has a vertical height of 30 cm.

Water is poured into the empty container at a constant rate.

It takes 60 seconds to fill up the entire container.

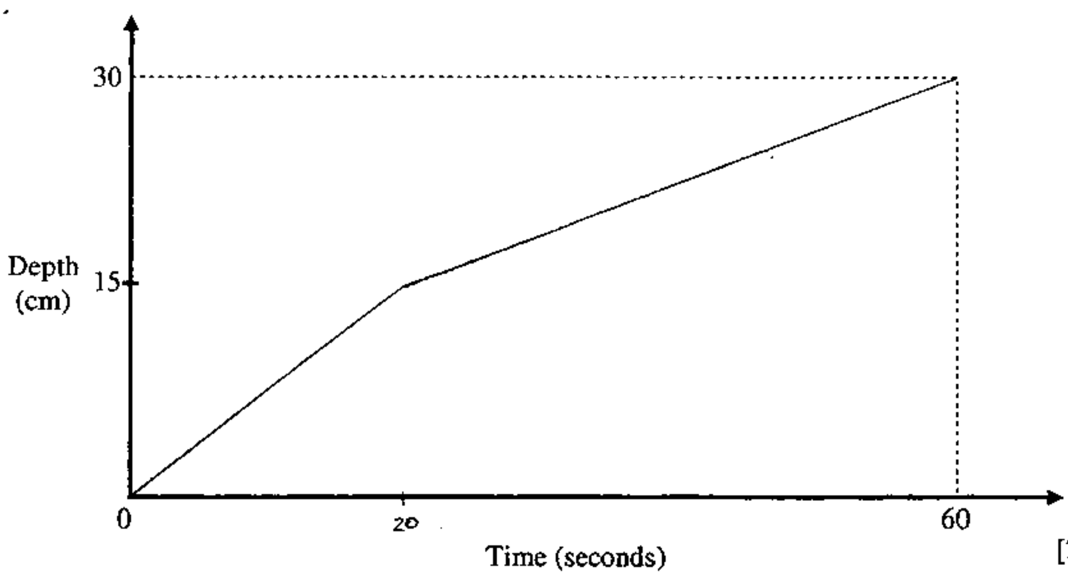


(a) Find the time taken to fill up the cone.

Answer (*a*) .... seconds [2]

(b) Sketch the graph of how the depth of water in the container varies during the 60 seconds.

Answer (*b*)

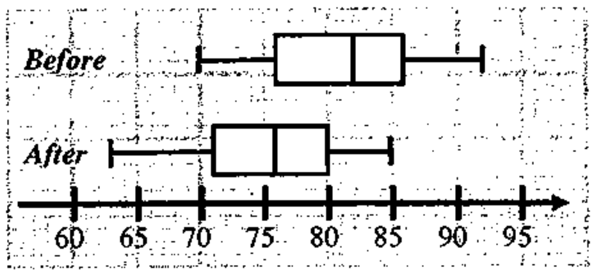


[2]

[Turn over

6

19 The diagram shows the box-and-whisker plots for the distributions of the speeds, in km/h, of 100 vehicles before and after a speed camera was placed on an expressway.



(a) Find the interquartile range of the distribution before the camera was placed.

Answer (*a*) .... km/h [1]

(b) Find the interquartile range of the distribution after the camera was placed.

Answer (*b*) .... km/h [1]

(c) After the camera was placed, 25% of the motorists were issued with traffic summons for exceeding the speed limit of the expressway.
What is the speed limit of the expressway?

Answer (*c*) .... km/h [1]

(d) Has the speed camera been effective in regulating the speed limit of the expressway? Explain your answer by comparing the distributions of the speeds before and after the camera was placed.

Answer (*d*) ...

.... [1]

③

20 Abraham and Lincoln sent out some letters, postcards and greeting cards.

The number of letters, postcards and greeting cards is shown in the table below.

	Letters	Postcards	Greeting cards
Abraham	4	9	2
Lincoln	7	3	3

The postage for each letter, postcard and greeting card is \$0.30, \$0.40 and \$0.50 respectively.

(a) Write out a 2×3 matrix **P** and a column matrix **Q** to represent the above information.

Answer (*a*) **P** =

Q = [2]

(b) Evaluate the matrix $\mathbf{S} = \mathbf{PQ}$.

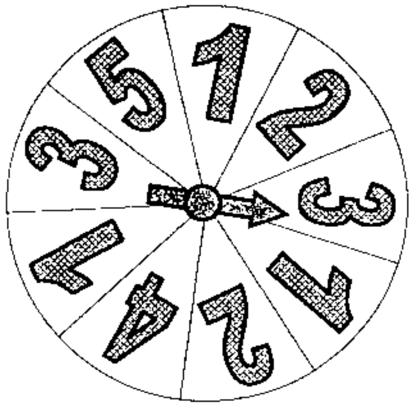
Answer (*b*) **S** = [2]

(c) State what the elements of **S** represent.

[1]

7

21 The diagram shows a spinner with nine numbered sectors of identical sizes.



Each time the pointer is spun, it is equally likely to stop on one of the sectors.

(a) The pointer is spun once.

Find the probability that it stops on an odd number.

Answer (*a*) .... [1]

(b) Aysha spins the pointer twice.

Find the probability that the pointer lands on a prime number **at least once**.

Answer (*b*) .... [2]

(c) Natasha spins the pointer twice.

Her score is found from the difference of the numbers from her two spins.

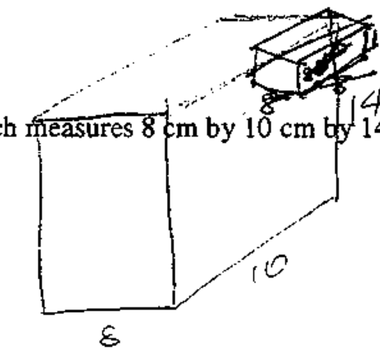
Find the probability that her score is 0.

Answer (*c*) .... [2]

22 Benjamin has 165 identical cubes of sides 2 cm.

(a) He uses some of the cubes to make a cuboid which measures 8 cm by 10 cm by 14 cm.

Calculate the total surface area of the cuboid.



Answer (*a*) .... cm^2 [2]

(b) Benjamin makes the largest cube possible using some of the 165 cubes.

He then makes the largest cube possible from the unused cubes.

How many cubes will he have left over after making the second cube?

Answer (*b*) .... [2]

(c) Benjamin uses all 165 cubes to make a cuboid.

Find the dimensions of the cuboid.

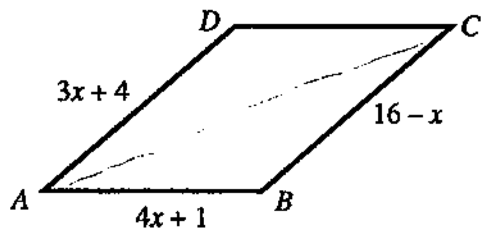
Answer (*c*) .... cm by .... cm by .... . cm [2]

[Turn over

8

23 Expressions for the lengths of three sides of a quadrilateral are shown on the diagram below.

All lengths are in centimetres.



(a) The perimeter of this quadrilateral is given by the expression $(11x+19)$ cm.

Find an expression, in terms of x, for the length of DC.

Give your expression in its simplest form.

Answer (*a*) .... cm [2]

(b) Given that $ABCD$ is a parallelogram and that $AB = AD$, calculate the perimeter of $ABCD$.

Answer (*b*) .... cm [2]

(c) Calculate the area of $ABCD$ if $AC = (10x-6)$ cm.

Answer (*c*) .... cm^2 [5]

END OF PAPER

Name	Class	Index no.	Calculator Model

DEYI SECONDARY SCHOOL

Preliminary Examination 2015
Secondary Four Express / Five Normal Academic

MATHEMATICS **4016/01**

Paper 1

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2 hours

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Answer **all** questions.

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Answers in degrees should be given to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiner's Use
80

This document consists of **16** printed pages including the cover page. **[Turn over**

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Calculate $\dfrac{6.7^2 - \sqrt[3]{4.8}}{20.15 - 19.99}$.

Write down the first six digits of your answer.

Answer (*a*) ....270.019........ [1]

(b) Write down your answer to **part (a)** correct to 2 significant figures.

Answer (*b*) ....270........ [1]

---

2 Write the following in descending order.

$$\frac{54}{67} \qquad \sqrt[3]{0.512} \qquad 0.77^{\frac{5}{6}} \qquad 0.802$$

0.80597 , 0.8 0.804283 0.802

Answer ....$\frac{54}{67}$.... , ...$0.77^{\frac{5}{6}}$.... , ...0.802.... , ...$\sqrt[3]{0.512}$ 0.8.... [2]

---

3 Given that $9 \times 27^{-n} = 1$, find the value of n.

$9 \times 27^{-n} = 1$

$3^2 \times (3^3)^{-n} = 1$

$3^2 \times 3^{-3n} = 3^0$

$2 - 3n = 0$

$2 = 3n$

$n = \frac{2}{3}$

Answer n = ...$\frac{2}{3}$........ [2]

---

4 The sine of an obtuse angle is 0.6.

Without using a calculator, find the cosine of this angle.

$\sin x = 0.6$

~~x = sin~~

S A

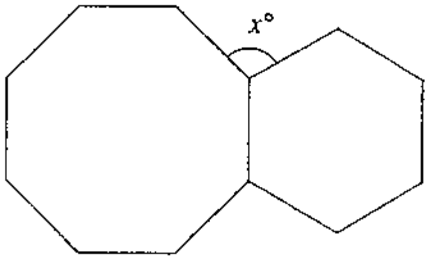
T C

Answer ....$-\frac{4}{5}$........ [2]

10

[Turn over

5



The diagram shows a sketch of a regular hexagon and a regular octagon.

Calculate x.

~~(6−n)~~

$\frac{(6-2)\times180°}{6} = 120°$

~~8−n×~~

$\frac{(8-2)\times180°}{8} = 135°$

$360° - 135° - 120° = 105°$

Answer $x =$ ....105°.............. [2] 2

---

6 Brian is leaving Singapore to further his studies in the United Kingdom.
In Singapore, the exchange rate is 1 Singapore Dollar = 0.478 British Pounds.
In the United Kingdom, the exchange rate is 1 British Pound = 2.113 Singapore Dollars.

Brian would like to change 2500 Singapore Dollars into British Pounds.

How many fewer British Pounds will he get by changing his money in the United Kingdom?

S\$ 2500 ÷ S\$1 × £0.478 = £1195

S\$2500 ÷ \$2.113 × £1 = £1183.151917

£1195 − £1183.151917 = £11.848083

≈ £11.85

Answer ......11.85...... British Pounds [2] 2

(4)

7 Matthew bought a new car. At the end of each year, the car's value depreciates by 10%.
The value, $C, of the car t years after being bought is given by

$$C = 85000 \times 0.9^t.$$

(a) How much did Matthew pay for his new car?

Answer (*a*) $ 85000 ✓ [1]

(b) Find the percentage decrease in the value of his car at the end of three years.

$85000 \times 0.9^t = 61965$

~~81964~~

$85000 - 61965 = 23035$

$\frac{23035}{85000} = 27.1\%$

Answer (*b*) 27.1 ✓ % [1]

8 Two geometrically similar solids made from the same material have masses 3.60 kg and 12.15 kg respectively.
Calculate the ratio of the area of the smaller solid to the area of the larger solid.

$\left(\frac{3.60}{12.15}\right)^2 = \frac{A_1}{A_2}$

$\frac{64}{729} = \frac{A_1}{A_2}$

$A_1 : A_2$

$64 : 729$

$\left(\frac{l_1}{l_2}\right)^3 = \frac{3.6}{12.15}$

$\frac{l_1}{l_2} = \frac{2}{3}$

$l_1 : l_2$

$2 : 3$

$\left(\frac{l_1}{l_2}\right)^2 = \frac{A_1}{A_2}$

$\left(\frac{2}{3}\right)^2 = \frac{4}{9}$

$A_1 : A_2$

$4 : 9$

4 : 9

Answer 64 : 729 ✗ [2]

[Turn over

11

9 Dawn invested some money in a savings account that was compounded every six months.

The rate of compound interest was 5% per annum.

At the end of the 4 years there was \$14620.83 in her account.

How much did Dawn invest in the account at first?

Give your answer correct to the nearest dollar.

$P\left(1+\frac{r}{100}\right)^n = P\left(1+\frac{2.5}{100}\right)^8$

$= \$14620.83$

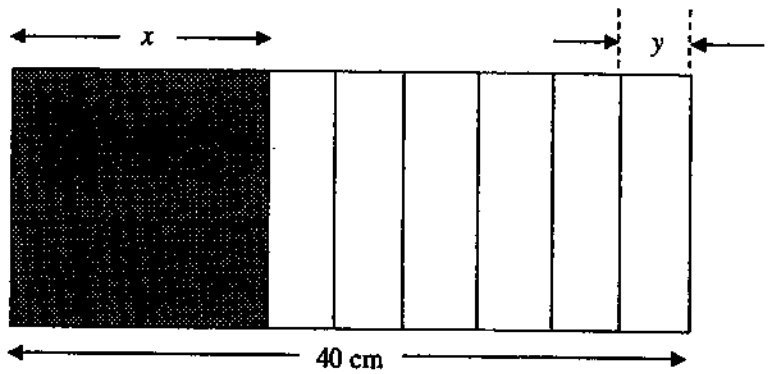
$P = \$11999.99608$

$\approx \$12000$

Answer \$ ......12000...... [3]

---

10 A rectangle with length 40 cm is divided into five identical shaded rectangles and another six identical unshaded rectangles.



The shaded area makes up two-thirds of the unshaded area.

Find the lengths labelled x and y.

$x + 6y = 40$

Let the breadth of each shaded rectangle be z.

$5(xz) + 5(6yz) =$

$5xz \div 2 \times 3 = 5(6yz)$

$7.5xz = 25yz$

$x = 40 - 6y$

$7.5(40-6y)z = 25yz$

$300z - 45yz = 25yz$

$300z - 45yz - 25yz = 0$

$300z - 70yz = 0$

$z(300 - 70y) = 0$

$300 - 70y = 0$

$70y = 300$

$y = 4\frac{2}{7}$

$= 4.285714286$

≈ 4.29

$40 - 6(4.285714286) = 14.28571429$

≈ 14.3

$x = 14.3$

$3x = 2(6y)$

$3x = 12y$

$x = 4y$

$10y = 40$

$y = 4$

$x = 4y$

$= 16$

Answer $x =$ ......14.3 × 16...... cm

$y =$ ......4.29 × 4...... cm [3]

3

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11 Mr Tan decided to buy a laptop under a hire-purchase scheme.
He would have paid \$3906 in total under the scheme, which consists of a deposit of 15% of the selling price of the laptop plus 24 equal monthly payments of \$140.25.

What is the selling price of the laptop?

let selling price be x

$15\% \times x + 24 \times \$140.25 = \$3906$

$15\% \times x = \$3741.75$

$x = \$24945$

Let selling price be x

$15\% \times x + 24 \times \$140.25 = \$3906$

$15\% x = \$540$

$x = \$3600$

Answer \$ ....24945 X 3600.... [3]

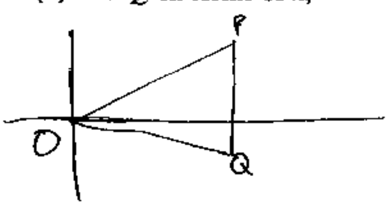
---

12 Two points P and Q have position vectors **p** and **q** respectively, relative to an origin O.

It is given that $\mathbf{p} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} k \\ 0 \end{pmatrix}$.

Find

(a) $\overrightarrow{PQ}$ in terms of k,



$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$

$= -\begin{pmatrix} -3 \\ 4 \end{pmatrix} + \begin{pmatrix} k \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -4 \end{pmatrix} + \begin{pmatrix} k \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 3+k \\ -4 \end{pmatrix}$

Answer (*a*) $\overrightarrow{PQ} =$ .... $\begin{pmatrix} -3k \\ 4 \end{pmatrix} \times \begin{pmatrix} 3+k \\ -4 \end{pmatrix}$ .... [1]

(b) the possible values of k if OP and OQ are two sides of a rhombus.

$|\overrightarrow{OP}| = \sqrt{(-3)^2 + (4)^2}$

$= 5$ or -5

Answer (*b*) $k =$ ....5.... or ....−5.... [2]

[Turn over

12

0

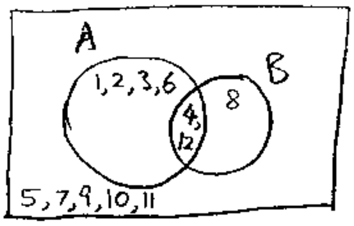
13 $\varepsilon = \{ \text{integers } x : 1 \le x \le 12 \}$ A : { 1, 2, 3, 4, 6, 12 }

$A = \{ \text{factors of 12} \}$ B : { 4, 8, 12 }

$B = \{ \text{multiples of 4} \}$

(a) Draw a Venn diagram to illustrate this information.

Answer (*a*)



[2]

(b) Describe in words what the set $(A \cup B)'$ represents.

Answer (*b*) The set (AUB)' represents all the numbers that are not in Set A and B combined. ✗ It represents the set of numbers that are neither factors of 12 nor multiples of 4. [1]

---

14 A train 45 m long passes through a tunnel 6 km long.

The average speed of the train is 27 km/h.

(a) Change 27 km/h into m/s.

27km ÷ 1h = 27000m ÷ 3600s
= 7.5m/s

Answer (*a*) ....7.5........ m/s [1]

(b) Calculate the time taken for the train to pass completely through the tunnel.

Give your answer in minutes and seconds, to the nearest second.

6000m + 45m = 6045m
6045m ÷ 7.5m/s = 806s
= 13 minutes 26 secs

Answer (*b*) ..13.. minutes ..26.. seconds [3]

---

(6)

15 Simplify

(a) $28x^2y^{-3} \div 16x^3y^{-1}$,

$28x^2y^{-3} \div 16x^3y^{-1} = \dfrac{28x^2y^{-3}}{16x^3y^{-1}}$

$= \dfrac{28x^2y^1}{16x^3y^3}$

$= \dfrac{7}{4xy^2}$

Answer (*a*) $\dfrac{7}{4xy^2}$ [2]

(b) $\dfrac{2}{x-3} + \dfrac{3x}{x^2-9}$.

$\dfrac{2(x+3)}{(x-3)(x+3)} + \dfrac{3x}{x^2-9}$

$= \dfrac{2x+6+3x}{(x-3)(x+3)}$

$= \dfrac{5x+6}{x^2-9}$

Answer (*b*) $\dfrac{5x+6}{x^2-9}$ [2]

---

16 The numbers 1 to 100 are arranged in a table as shown below.

A *U*-shaped, shaded frame can be placed around various numbers throughout the table.

	2		4	5	6	7	8	9	10
			14	15	16	17	18	19	20
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
91	92	93	94	95	96	97	98	99	100

The ***U*-number** is used to refer to the **shaded frame** that is drawn **around a particular number**.

For example, U_2 refers to the **shaded frame** shown above since it is drawn **around the number 2**.

(a) State the largest possible *U*-number.

Answer (*a*) 89 × U_{89} [1]

(b) Write and simplify an expression, in terms of ***n***, for the sum of the numbers in U_n.

$n-1+n+1+n+9+n+10+n+11 = 5n+30$

Answer (*b*) $5n+30$ [2]

(c) Find the sum of numbers in U_{75}.

Answer (*c*) 405 [1]

[Turn over

13

5

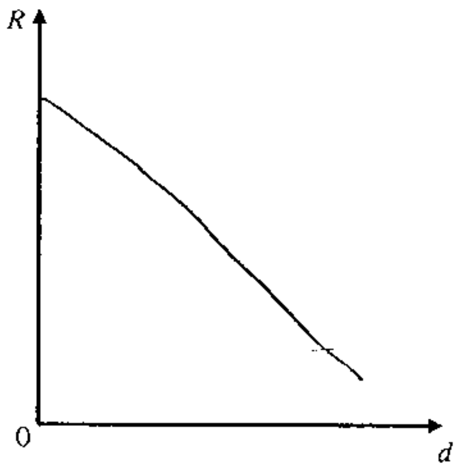
17 The resistance of a wire, R ohms, is inversely proportional to the square of its diameter, d μm.

(a) Sketch a resistance-diameter graph for the wire.

$R = \frac{1}{d^2}$

R	1	$\frac{1}{4}$	$\frac{1}{9}$	$\frac{1}{16}$	$\frac{1}{25}$	$\frac{1}{36}$
d	1	2	3	4	5	6

Answer (*a*)



[1]

For a fixed length of wire, the resistance is 25.6 ohms when the diameter is 50 μm.

(b) Find the equation for R in terms of d.

$R = \frac{k}{d^2}$

$25.6 = \frac{k}{2500}$

$k = 64000$

$R = \frac{64000}{d^2}$

Answer (*b*) $R =$ $\frac{64000}{d^2}$ [2]

(c) Another wire of the same length has resistance 120 ohms.

Calculate its diameter.

$120 = \frac{64000}{d^2}$

$d = 23.09401077$

≈ 23.1

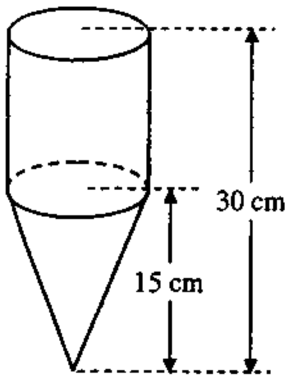
Answer (*c*) 23.1 μm [1]

3

18 A composite container made from a cylinder and a cone has a vertical height of 30 cm.

Water is poured into the empty container at a constant rate.

It takes 60 seconds to fill up the entire container.



$V_{cone} = \frac{1}{3} V_{cylinder}$

Total $V = 4$ units

$V_{cone} = 1$ unit

$\frac{1}{4}$ of time $= 15s$

(a) Find the time taken to fill up the cone.

$\frac{1}{3}\pi r^2 h + \pi r^2 h = \text{Total}$

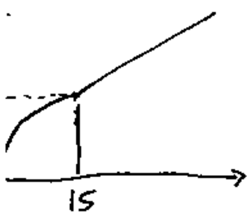
Since the volume of the cone is $\frac{1}{3}$ of the cylinder, the time ~~taken~~ to fill up the cone will be $\frac{1}{3}$ of the total time.

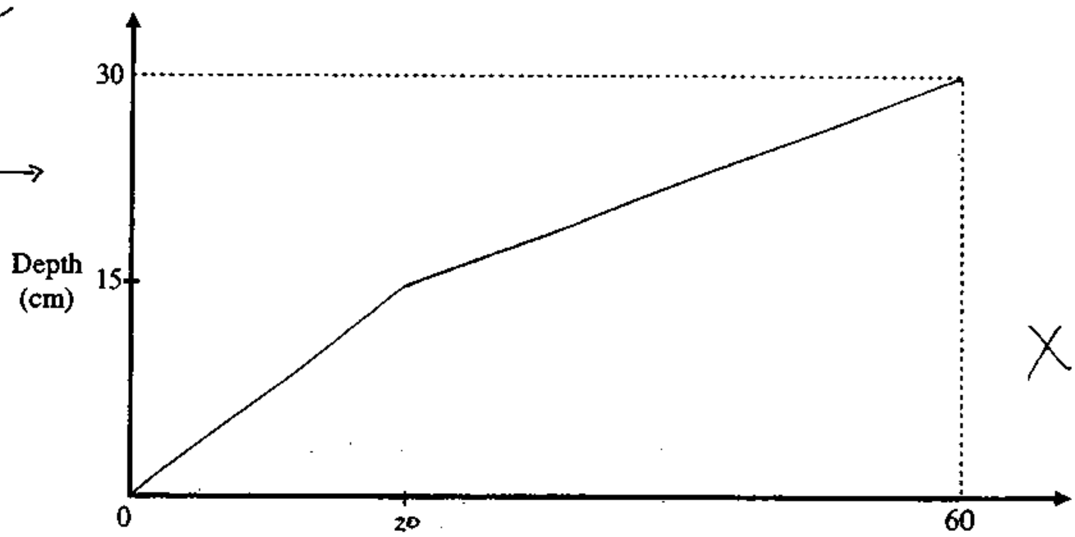
$\frac{1}{3} \times 60s = 20s$

Answer (*a*) .... 20 × 15 .... seconds [2]

(b) Sketch the graph of how the depth of water in the container varies during the 60 seconds.

Answer (*b*)





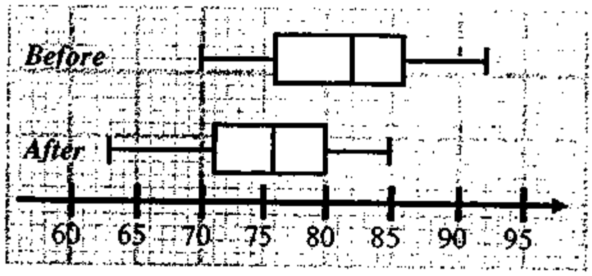
[2]

[Turn over

14

(0)

19 The diagram shows the box-and-whisker plots for the distributions of the speeds, in km/h, of 100 vehicles before and after a speed camera was placed on an expressway.



(a) Find the interquartile range of the distribution before the camera was placed.

Answer (*a*) ....10........ km/h [1]

(b) Find the interquartile range of the distribution after the camera was placed.

Answer (*b*) ....9........ km/h [1]

(c) After the camera was placed, 25% of the motorists were issued with traffic summons for exceeding the speed limit of the expressway.
What is the speed limit of the expressway?

Answer (*c*) ....80........ km/h [1]

(d) Has the speed camera been effective in regulating the speed limit of the expressway? Explain your answer by comparing the distributions of the speeds before and after the camera was placed.

Answer (*d*) Yes, the speed camera has been effective. The upper limit of the motorists' speeds after the camera was placed is 85km/h. Before the camera was placed, the upper limit was 92km/h. × [1]

Yes, it has been effective. The median and the interquartile range are lower after the camera was placed.

3

20 Abraham and Lincoln sent out some letters, postcards and greeting cards.

The number of letters, postcards and greeting cards is shown in the table below.

	Letters	Postcards	Greeting cards
Abraham	4	9	2
Lincoln	7	3	3

The postage for each letter, postcard and greeting card is \$0.30, \$0.40 and \$0.50 respectively.

(a) Write out a 2×3 matrix **P** and a column matrix **Q** to represent the above information.

Answer (*a*) $\mathbf{P} = \begin{pmatrix} 4 & 9 & 2 \\ 7 & 3 & 3 \end{pmatrix}$

$\mathbf{Q} = \cancel{\begin{pmatrix} 5.2 \\ 3.9 \end{pmatrix}} \begin{pmatrix} 0.30 \\ 0.40 \\ 0.50 \end{pmatrix}$ [2]

(b) Evaluate the matrix $\mathbf{S} = \mathbf{PQ}$.

$S = PQ$

$= \begin{pmatrix} 4 & 9 & 2 \\ 7 & 3 & 3 \end{pmatrix} \begin{pmatrix} 0.30 \\ 0.40 \\ 0.50 \end{pmatrix}$

$= \begin{pmatrix} 5.8 \\ 4.8 \end{pmatrix}$

Answer (*b*) $\mathbf{S} = \cancel{\begin{pmatrix} 5.2 \\ 3.9 \end{pmatrix}} \begin{pmatrix} 5.8 \\ 4.8 \end{pmatrix}$ [2]

(c) State what the elements of **S** represent.

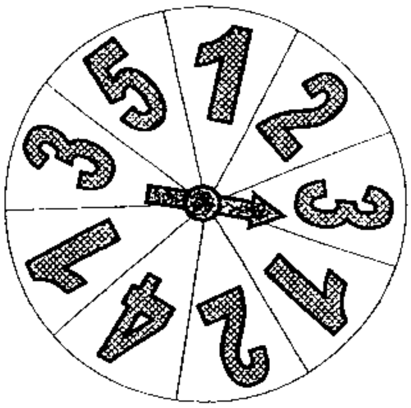
Answer (*c*) The elements of S represents the amount of money Abraham and Lincoln spent on postage respectively. X They represent the total postage [1]
paid by Abraham and Lincoln respectively.

[Turn over

15

4

21 The diagram shows a spinner with nine numbered sectors of identical sizes.



Each time the pointer is spun, it is equally likely to stop on one of the sectors.

(a) The pointer is spun once.

Find the probability that it stops on an odd number.

Answer (*a*) .... $\frac{2}{3}$ .... [1]

(b) Aysha spins the pointer twice.

Find the probability that the pointer lands on a prime number **at least once**.

$1-\left(\frac{1}{9} \times \frac{1}{9}\right) = \frac{80}{81}$

$1-\left(\frac{4}{9} \times \frac{4}{9}\right) = \frac{65}{81}$

Answer (*b*) .... $\frac{80}{81} \times \frac{65}{81}$ .... [2]

(c) Natasha spins the pointer twice.

Her score is found from the difference of the numbers from her two spins.

Find the probability that her score is 0.

$\left(\frac{3}{9} \times \frac{3}{9}\right) + \left(\frac{2}{9} \times \frac{2}{9}\right) + \left(\frac{2}{9} \times \frac{2}{9}\right) + \left(\frac{1}{9} \times \frac{1}{9}\right) + \left(\frac{1}{9} \times \frac{1}{9}\right) = \frac{19}{81}$

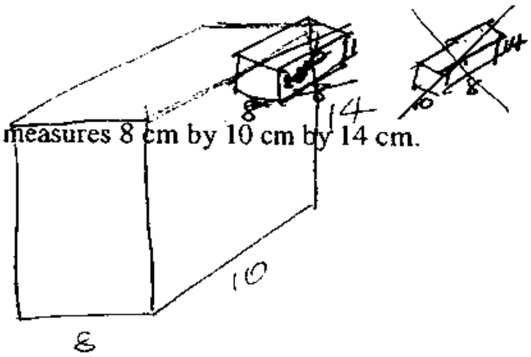
Answer (*c*) .... $\frac{19}{81}$ .... [2]

1

22 Benjamin has 165 identical cubes of sides 2 cm.

(a) He uses some of the cubes to make a cuboid which measures 8 cm by 10 cm by 14 cm.

Calculate the total surface area of the cuboid.



$(8\times14\times4)+(10\times14\times2) = 728$

$10\times14\times2 + 8\times14\times2 + 8\times10\times2$

$= 664\text{cm}^2$

Answer (a) ~~728~~ ✗ 664 cm^2 [2]

(b) Benjamin makes the largest cube possible using some of the 165 cubes.

He then makes the largest cube possible from the unused cubes.

How many cubes will he have left over after making the second cube?

$165\times2 = 330\text{cm}$

$\sqrt{330} = 18.....$

~~$18\div2=9$~~

$330-324=6$

$6\div2=3$

$3\times3=9$ ✗

$\sqrt[3]{165} = 5....$

$165-125=40$

$\sqrt[3]{40} = 3.....$

$40-27=13$

Answer (b) ~~9~~ ✗ 13 [2]

(c) Benjamin uses all 165 cubes to make a cuboid.

Find the dimensions of the cuboid.

3	165
5	55
11	11
	1

Answer (c) ~~2~~ ✗ 6 cm by ~~2~~ ✗ 10 cm by ~~330~~ ✗ 22 cm [2]

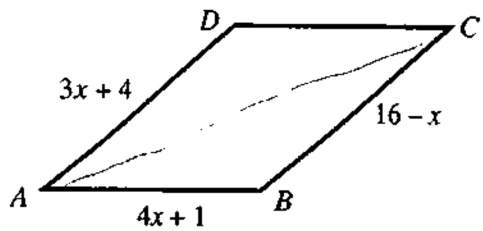
[Turn over

16

0

23 Expressions for the lengths of three sides of a quadrilateral are shown on the diagram below.

All lengths are in centimetres.



(a) The perimeter of this quadrilateral is given by the expression $(11x+19)$ cm.

Find an expression, in terms of x, for the length of DC.

Give your expression in its simplest form.

$11x+19 = 3x+4+16-x+4x+1+DC$

$11x+19 = 6x+21+DC$

$5x-2 = DC$

Answer (*a*) .... $5x-2$ .... cm [2]

(b) Given that $ABCD$ is a parallelogram and that $AB = AD$, calculate the perimeter of $ABCD$.

$AB = AD$

$4x+1 = 3x+4$

$x = 3$

$4(3)+1 = 13$

~~3(3)+4 = 13~~

$13 \times 4 = 52$

Answer (*b*) .... 52 .... cm [2]

(c) Calculate the area of $ABCD$ if $AC = (10x-6)$ cm.

$AC = (10x-6)$ cm

$= [10(3)-6]$ cm

$= 24$ cm

$24 \div 2 = 12$

$(\frac{1}{2} \times 12 \times 24) \times 2 = 288$

$AC = 10(3)-6$

$= 24$

$AB = 4(3)+1$

$= 13$

$24^2 = 13^2 + 13^2 - 2(13)(13)\cos \angle ABC$

$576 = 338 - 338 \cos \angle ABC$

$\angle ABC = 134.7602701$

$\frac{1}{2}(13)(13)\sin 134.7602701 = 60$

$60 \times 2 = 120$

Answer (*c*) .... 288×120 .... cm^2 [5]

END OF PAPER

4

Anglo-Chinese School
(Independent)

THE BEST IS YET TO BE

PRELIMINARY EXAMINATION 2015
YEAR 4 EXPRESS
ADDITIONAL MATHEMATICS **4047/01**
PAPER 1
Thursday **30 July 2015** **2 hours**

Additional Materials:
Answer Paper (8 sheets)

READ THESE INSTRUCTIONS FIRST

Write your candidate number in the spaces provided on the answer paper/answer booklet.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Write your answers on the separate answer paper provided.

If you use more than one sheet of paper, fasten the sheets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

This question paper consists of 6 printed pages

17 **[Turn Over**

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

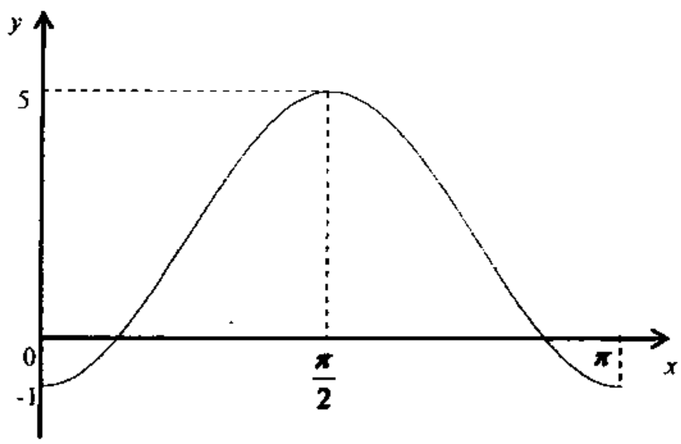
1 The sides AB and BC of a triangle are $(2\sqrt{3} + 2\sqrt{6})\,cm$ and $(8\sqrt{2} - 2)\,cm$ respectively and $\angle ABC$ is $60°$. Show that the area of ΔABC is $(p + q\sqrt{2})\,cm^2$ where p and q are constants to be determined. [3]

2 Find the range of values of k for which $x^2 + 2k(k + x) > 3k + 4$ for all real values of x. [3]

3 Solve the equation $|2x - 3| + 6x = |9 - 6x| + 4$. [4]

4 The polynomial $f(x)$ is divisible by $(2x - 3)$ and leaves a remainder of -2 when divided by $(x - 1)$. Find the remainder when $f(x)$ is divided by $2x^2 - 5x + 3$. [4]

5 The diagram below shows the graph of $y = c + a\cos bx$ where a, b and c are constants.



(i) Use the graph to determine the value of a, of b and of c. [3]

(ii) By using the values of a, b and c found in (i), determine the equation of the straight line that needs to be drawn on the same diagram to solve

$$\sec bx = \frac{a\pi}{x - \pi c}.$$ [2]

6 (i) Given that $\dfrac{x^3-2x^2-x+3}{x^2-2x+1} = x+\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}$, where A and B are constants.

Find the value of A and of B. [4]

(ii) Hence find $\int \dfrac{x^3-2x^2-x-4}{x^2-2x+1}\,dx$. [3]

7 The roots of the equation $2x^2-8x+3=0$ are α and β.

(i) Express $\alpha^2-\alpha\beta+\beta^2$ in terms of $\alpha+\beta$ and $\alpha\beta$. [1]

(ii) Find a quadratic equation with integer coefficients whose roots are α^3 and β^3. [6]

8 (a) Find all the values of x between 0° and 360° for which

$$\frac{1}{\sec^2 x}+3\sin\frac{x}{2}\cos\frac{x}{2}=0.$$ [4]

(b) Find all the exact angles between 0 and π, which satisfy the equation

$$\sin(x-\frac{\pi}{5})-\cos\frac{\pi}{10}=0.$$ [4]

9 A curve is such that $\dfrac{d^2y}{dx^2}=16\cos^2 2x-4\sin 4x-8$ and the gradient of the normal to the curve at $x=\dfrac{\pi}{4}$ is 1.

(i) Find $\dfrac{dy}{dx}$. [3]

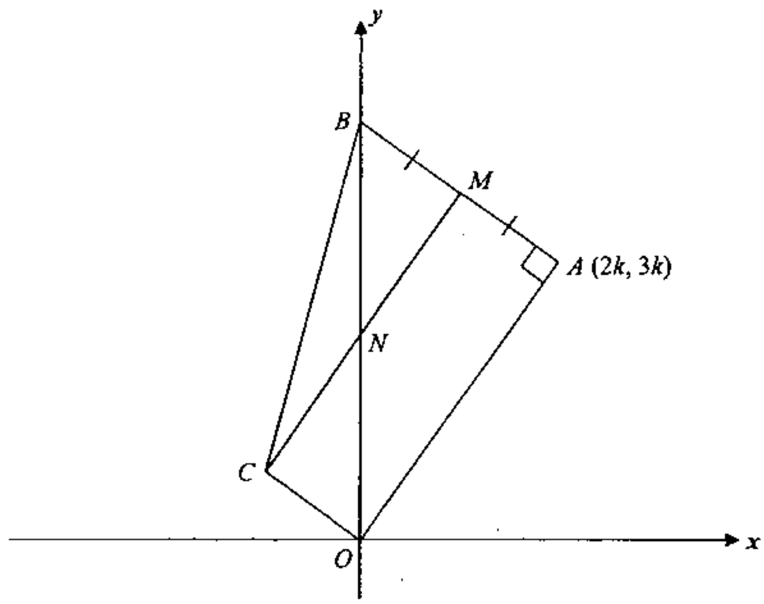
(ii) Hence solve $\dfrac{dy}{dx}=2$ for $0\le x\le 1$. [6]

10 (i) Solve $2+\ln(4-x)=0$. [2]

(ii) Sketch the graph of $y=2+\ln(4-x)$ showing clearly the asymptote and the y-intercept. [3]

(iii) Find the area of the region bounded by the curve $y=2+\ln(4-x)$, the x-axis, the y-axis and the line $x=3$. [5]

11 The diagram shows a quadrilateral $OABC$.



The coordinates of A are $(2k, 3k)$ and the length of OA is $\sqrt{52}$ units.

(i) Calculate the value of k. [2]

AB is perpendicular to OA and B lies on the y-axis.

(ii) Find the coordinates of B. [3]

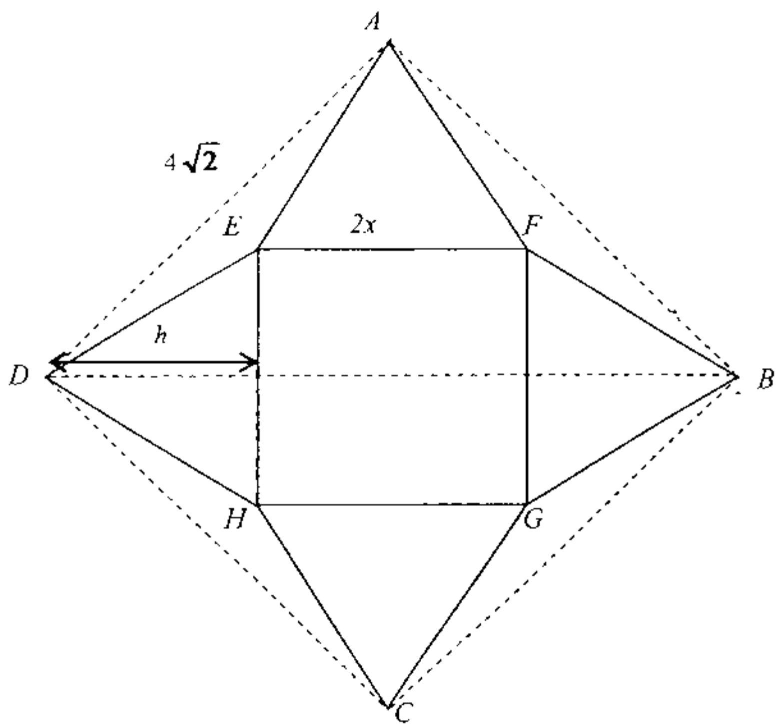
CM, the perpendicular bisector of AB, cuts the y-axis at N and OC is parallel to AB.

Find

(iii) the coordinates of C, [3]

(iv) the ratio of the area of the triangle OCN to the area of the triangle OCB. [2]

12



The diagram above shows a square piece of cardboard paper $ABCD$ of side $4\sqrt{2}$ metres.

Triangles AED, AFB, DHC and BGC are cut off leaving a figure in the shape of a square $EFGH$ of side $2x$ metres with 4 identical isosceles triangles attached to the sides. The height of each triangle is h metres. Mark wants to fold the paper to make a pyramid with $EFGH$ as the base.

(i) Show that $h = 4 - x$. [2]

(ii) Show that the volume of the pyramid, $V\,m^3$, is given by $V = \frac{8}{3}x^2\sqrt{4-2x}$. [4]

(iii) Hence find the maximum volume of the pyramid. [4]

(Proof of maximum is not required)

-- **END OF PAPER 1** --

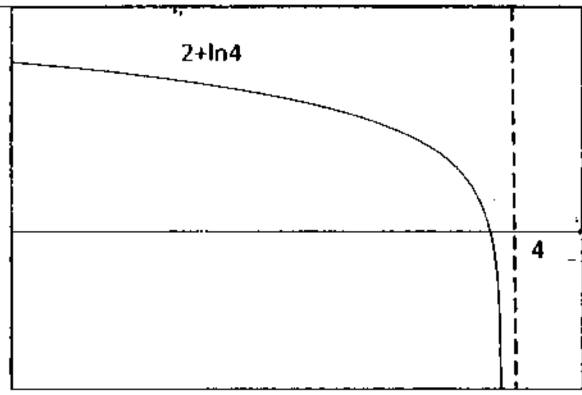
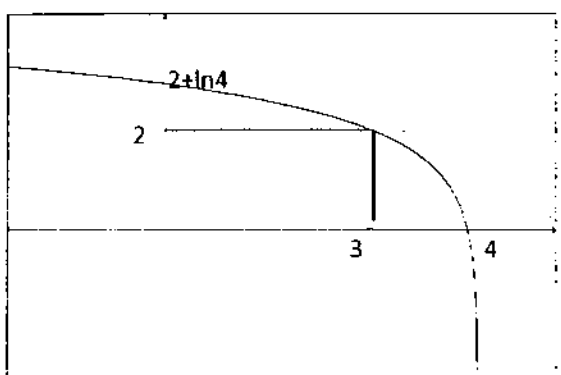
Marking Scheme for Additional Mathematics 2015 Preliminary Examination Paper 1

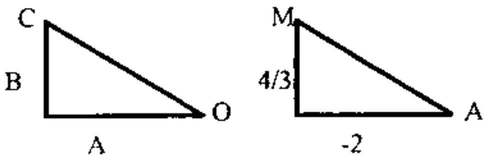
1		$Area = \frac{1}{2}(2\sqrt{3}+2\sqrt{6})(8\sqrt{2}-2)\sin 60$ $= (\sqrt{3}+\sqrt{6})(8\sqrt{2}-2)\frac{\sqrt{3}}{2}$ $= (6\sqrt{6}-2\sqrt{3}+8\sqrt{12})\frac{\sqrt{3}}{2}$ $= 9\sqrt{2}+14\sqrt{3}\times\frac{\sqrt{3}}{2}$ $= 9\sqrt{2}+21$
2		$x^2+2k(k+x)>3k+4$ $D<0$ $x^2+2k^2+2kx-3k-4>0$ $4k^2-4(2k^2-3k-4)<0$ $-4k^2+12k+16<0$ $k^2-3k-4>0$ $k<-1 \ or \ k>4$
3		$\|2x-3\|+6x=\|9-6x\|+4$ $\|2x-3\|-\|9-6x\|=4-6x$ $\|2x-3\|-3\|2x-3\|=4-6x$ $\|2x-3\|=3x-2$ $2x-3=3x-2 \ or \ 2x-3=2-3x$ $x=-1(na) \ or \ x=1$
4		$f(x)=(2x^2-5x+3)Q(x)+ax+b$ $=(2x-3)(x-1)Q(x)+ax+b$ $\frac{3}{2}a+b=0$ $3a+2b=0------(1)$ $a+b=-2------(2)$ $Solve: a=4, \ b=-6$ $The \ remainder \ is \ 4x-6$
5	(i)	$y=c+a\cos bx$ $a=-3$ Period is π. Therefore $b=2$ $c=2$

20

	(ii)	$\sec bx = \frac{a\pi}{x - \pi c}$ $\cos 2x = \frac{x - 2\pi}{-3\pi}$. $-3\pi \cos 2x = x - 2\pi$ $2\pi - 3\pi \cos 2x = x$ $2 - 3\cos 2x = \frac{x}{\pi}$ *Draw* $y = \frac{x}{\pi}$
6	(i)	$\frac{x^3 - 2x^2 - x + 3}{x^2 - 2x + 1} = x + \frac{A}{x-1} + \frac{B}{(x-1)^2}$ *By long div,* $\frac{x^3 - 2x^2 - x + 3}{x^2 - 2x + 1} = x + \frac{3 - 2x}{(x-1)^2}$ $\frac{3-2x}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$ $3 - 2x = A(x-1) + B$ *Sub* $x = 1 : B = 1$ *Compare* $x : A = -2$
	(ii)	$\int \frac{x^3 - 2x^2 - x - 4}{x^2 - 2x + 1} dx = \int (x - \frac{2}{x-1} + \frac{1}{(x-1)^2} - \frac{7}{(x-1)^2}) dx$ $= \frac{x^2}{2} - 2\ln(x-1) + \frac{6}{x-1} + c$
7	(i)	$2x^2 - 8x + 3 = 0$ $\alpha^2 - \alpha\beta + \beta^2 = (\alpha + \beta)((\alpha + \beta)^2 - 3\alpha\beta)$ $\alpha + \beta = 4$ $\alpha\beta = \frac{3}{2}$ $\alpha^3 + \beta^3 = (4)(16 - \frac{9}{2})$ $= 46$
	(ii)	$\alpha^3\beta^3 = \frac{27}{8}$ $x^2 - 46x + \frac{27}{8} = 0$ $8x^2 - 368x + 27 = 0$

8	(a)	$\frac{1}{\sec^2 x} + 3\sin\frac{x}{2}\cos\frac{x}{2} = 0$ $\cos^2 x + \frac{3}{2}\sin x = 0$ $2\sin^2 x - 3\sin x - 2 = 0$ $\sin x = -\frac{1}{2}$ $x = 210°, 330°$
	(b)	$\sin(x - \frac{\pi}{5}) = \cos\frac{\pi}{10}$ $= \sin(\frac{\pi}{2} - \frac{\pi}{10})$ $= \sin\frac{2\pi}{5}$ $x - \frac{\pi}{5} = \frac{2\pi}{5}, \frac{3\pi}{5}$ $x = \frac{3\pi}{5}, \frac{4\pi}{5}$
9	(i)	$\frac{d^2y}{dx^2} = 16\cos^2 2x - 4\sin 4x - 8$ $\frac{dy}{dx} = \int(16\cos^2 2x - 4\sin 4x - 8)\,dx$ $= \int(8\cos 4x - 4\sin 4x)\,dx$ $= 2\sin 4x + \cos 4x + c$ $grad\ of\ \tan gent = -1$ $-1 = 2\sin\pi + \cos\pi + c$ $c = 0$ $\frac{dy}{dx} = 2\sin 4x + \cos 4x$
	(ii)	$2\sin 4x + \cos 4x = R\sin(4x + \alpha)$ $R = \sqrt{5}\quad and\ \ \alpha = 0.4636$ $\sqrt{5}\sin(4x + 0.4636) = 2$ $\sin(4x + 0.4636) = 0.8944$ $4x + 0.4636 = 1.1071,\ 2.0345,$ $x = 0.161,\ 0.393$

10	(i)	$2+\ln(4-x)=0$ $\ln(4-x)=-2$ $4-x=e^{-2}$ $x=3.86$
	(ii)	
	(iii)	$y=2+\ln(4-x)$ $4-x=e^{y-2}$ $x=4-e^{y-2}$ $Area\ =2\times 3+-\int_{2}^{2+\ln 4}(4-e^{y-2})\ dy$ $=6+\left[4y-e^{y-2}\right]$ $=6+\left[(4(2+\ln 4)-e^{\ln 4})-(8-1)\right]$ $=6+[8+4\ln 4-4-7]$ $=8.55\,units^2$ 
11	(i)	$\sqrt{4k^2+9k^2}=\sqrt{52}$ $13k^2=52$ $k=2$

	(ii)	Grad of OA = $\frac{3}{2}$ Grad of AB = $-\frac{2}{3}$ $y = -\frac{2}{3}x + c$ $6 = -\frac{8}{3} + c$ $c = \frac{26}{3}$ $B(0, \frac{26}{3})$
	(iii)	M= $(2, \frac{22}{3})$  a = 2-4 = -2 b = 4/3 C $(-2, \frac{4}{3})$
	(iv)	M is midpt of AB, MN// AO By midpt thm, N is midpt of OB Therefore $\frac{\Delta OCN}{\Delta OCB} = \frac{1}{2}$
12	(i)	$(h+x)^2 + (h+x)^2 = (4\sqrt{2})^2$ $2(h+x)^2 = 32$ $(h+x) = 4$ $h = 4 - x$
	(ii)	Let ht of pyramid be y

	(iii)	$y^2 + x^2 = (4-x)^2$ $y^2 = 16 - 8x$ $y = \sqrt{16-8x}$ $V = \frac{1}{3}4x^2\sqrt{16-8x}$ $= \frac{4}{x}x^2 2\sqrt{4-2x}$ $= \frac{8}{3}x^2\sqrt{4-2x}$ $\frac{dV}{dx} = \frac{8}{3}[x^2(\frac{1}{2})(-2)(4-2x)^{-\frac{1}{2}} + 2x\sqrt{4-2x}]$ $= \frac{8}{3}\frac{x[-x+2(4-2x)]}{\sqrt{4-2x}}$ $-x + 8 - 4x = 0$ $x = 1.6m$ $Max\ V = 6.11$

Anglo-Chinese School (Independent)

PRELIMINARY EXAMINATION 2015
YEAR 4 EXPRESS
ADDITIONAL MATHEMATICS **4047/02**
PAPER 2

Tuesday **4 August 2015** **2 hours 30 minutes**

Additional Materials:
Answer Paper (10 sheets)

READ THESE INSTRUCTIONS FIRST

Write your candidate number in the spaces provided on the answer paper/answer booklet.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.

Write your answers on the separate answer paper provided.

If you use more than one sheet of paper, fasten the sheets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

This question paper consists of 6 printed pages

[Turn Over

23

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

1 Given that $\int_0^5 f(x)\,dx = 12$ and $\int_2^5 f(x)\,dx = 4$, evaluate

$\int_2^1 [2x - f(x)]dx + \int_0^1 f(x)\,dx$. [3]

2 The equations of two curves are $y = \frac{1}{4}x^{\frac{2}{3}}$ for $x > 0$ and $y = 4x^{-\frac{2}{3}}$ for $x > 0$.

(i) Find the coordinates of the point(s) of intersection of the graphs. [2]

(ii) Sketch these graphs on the same axes, indicating the point(s) of intersection clearly. [2]

3 Variables x and y are related by the equation $y = \frac{p - x}{x + q}$, where p and q are constants.

When the graph of $x(1 + y)$ against y is drawn, a straight line is obtained. The line has a gradient of $-1\frac{1}{3}$ and passes through the point (3, 2).

(i) Calculate the value of p and of q. [4]

(ii) Given that this line passes through (6, k), find x in terms of k. [2]

4 The equation of a curve is given by $y = \frac{\ln(x-3)^2}{2x-6}$, $x > 3$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) Find the set of values of x for which y is a decreasing function. [2]

(iii) Evaluate $\int_4^5 \frac{\ln\sqrt{x-3}}{(x-3)^2}\,dx$, leaving your answer in the form $a + b\ln 2$,

where a and b are constants. [3]

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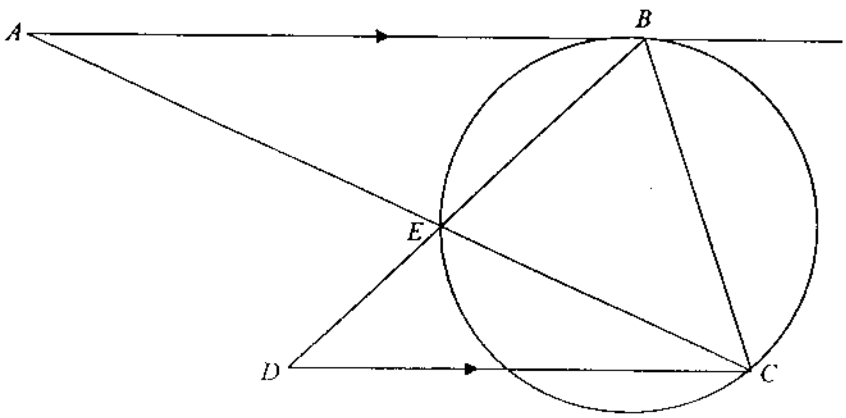
5 In the diagram, AB is a tangent to the circle at the point B, BED is a straight and AB // DC. The points A, E and C lie on a straight line and $AE : EC = 2 : 1$.

(i) Prove that $\angle BCE = \angle BDC$. [2]

(ii) Hence, show that ΔBCE is similar to ΔBDC. [2]

(iii) Prove that $3AE \times CD = AB \times AC$. [3]



6 The equation of a circle, C_1, is $x^2 + y^2 + kx - (k+2)y + 7 = 0$, where k is a constant.

(i) Find the coordinates of the centre in terms of k. [2]

Given that the centre of the circle lies on the line $2x + 5y - 11 = 0$,

(ii) show that $k = 4$. [2]

(iii) Find the equation of the circle, C_2, which is a reflection of C_1 in the line $x = 1$. [3]

(iv) Explain why the two circles do not intersect each other. [1]

7 The height of the tides at a certain place can be modelled by the equation $h = 2(3.25 - \sin kt)$, where k is a constant, and t is the time in hours after midnight. The average time difference between high tides is 14.5 hours.

(i) Explain why this model suggests that the lowest tide for the day is 4.5 m. [1]

(ii) Show that the value of k is $\frac{4\pi}{29}$. [2]

(iii) Find the height of the tide at 2 am. [1]

(iv) Find the time for which the height of the tide first reaches 7.0 m, leaving your answer in 24 hour notation. [4]

8 (a) Given that $\frac{d}{dx}[F(x)] = \frac{9}{2}\sqrt{3x-1} - \frac{3}{\sqrt{3x-1}}$, evaluate $F(3)-F(1)$, giving your answer in the form $k\sqrt{2}$. [4]

(b) The equation of a curve is given by $y = \operatorname{cosec}^2\left(\frac{x}{2} - \frac{\pi}{6}\right)$, where $0 < x < \frac{\pi}{2}$.

Given that x is increasing at 0.3 radian per second, find the rate of change of y with respect to time when $x = \frac{5\pi}{6}$. [4]

9 In the expansion of $\left(x^3 - \frac{k}{x}\right)^9$, where k is a constant, the coefficient of $\frac{1}{x}$ is -4608.

(i) Show that $k = 2$. [3]

(ii) Explain why there is no term independent of x in the expansion of $\left(x^3 - \frac{k}{x}\right)^9$. [1]

(iii) Find the coefficient of x^7 in the expansion of $\left(2x^3 + \frac{1}{x}\right)\left(x^3 - \frac{k}{x}\right)^9$. [4]

10 The gradient of a curve is $\frac{e^{2x}+1}{e^{2x}}$ and $P(0, -1)$ is a point on the curve.

(i) Show that the curve has no stationary point. [2]

(ii) Find the equation of the curve. [2]

The tangent and normal to the curve at P intersect the x-axis at Q and R respectively.

(iii) Find the area of the triangle PQR. [5]

11 (a) Given that $2^{2x-3} = \frac{1}{4^{x-1}}$, evaluate 16^x. [3]

(b) Solve the equation $\log_x 5 - \frac{2}{\log_{\sqrt{x}} 2} = \log_{x^2}\left(\frac{25}{4}\right)$. [6]

12 A particle P travelling in a straight line passes a fixed point O. Its velocity, v ms^{-1}, is given by the equation $v = t^2 - 6t + 8$, where t is the time in seconds after passing O.

(i) Find the times when P is instantaneously at rest. [2]

(ii) Find the total distance travelled by P when its velocity reaches 8 ms^{-1} again. [5]

(iii) Will P return to O in the course of its motion? Explain your answer clearly. [2]

The particle P is at point A when its velocity reaches 8 ms^{-1} again. It continues its motion at this velocity for 1 second and then decelerates uniformly until it comes to a complete rest at point B in another 2 seconds.

(iv) Find the distance AB. [2]

13 (i) Prove that $\tan\frac{\theta}{2} + \cot\frac{\theta}{2} = 2\operatorname{cosec}\theta$. [3]

(ii) Hence, solve $\tan\theta + \cot\theta = (\operatorname{cosec} 4\theta)(\sin 2\theta + \cos 2\theta)$ for $0^o \le \theta \le 180^o$. [5]

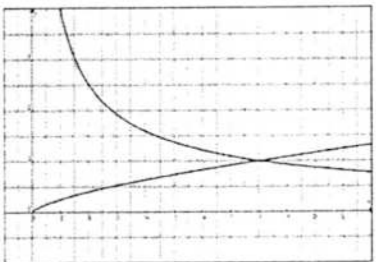
Given that $2\tan A + 2\cot A = 5$ and $0 < A < \frac{\pi}{4}$,

(iii) show that $\cos 2A = \frac{3}{5}$. [2]

(iv) Hence. find the **exact** value of $\cos(2A + \frac{\pi}{6})$. [2]

END OF PAPER 2

Answers

1	5
2(i)	(8, 1)
2(ii)	
3(i)	$p = 6, \quad q = 1\frac{1}{3}$
3(ii)	$x = \frac{k}{7}$
4(i)	$\frac{dy}{dx} = \frac{1 - \ln(x-3)}{(x-3)^2}$
4(ii)	$x > e + 3 = 5.72$
4(iii)	$\frac{1}{2}\left(\frac{1}{2} - \frac{\ln 2}{2}\right) = \frac{1 - \ln 2}{4}$
5	Proof
6(i)	$(-\frac{k}{2}, \frac{k+2}{2})$
6(ii)	Proof
6(iii)	$(x-4)^2 + (y-3)^2 = 6$
6(iv)	$Let\ d = distace\ from\ P_1\ to\ P_2 = 6$ $Let\ R = radius\ of\ C_1 + radius\ of\ C_2 = \sqrt{6} + \sqrt{6} = 4.89$ $Since\ R < d,\ the\ two\ circles\ do\ not\ intersect\ each\ other.$
7(i)	Lowest tide occurs when $\sin kt = 1$, lowest tide = 4.5 m
7(ii)	Proof
7(iii)	$4.98m$
7(iv)	0750
8(a)	$12\sqrt{2}$

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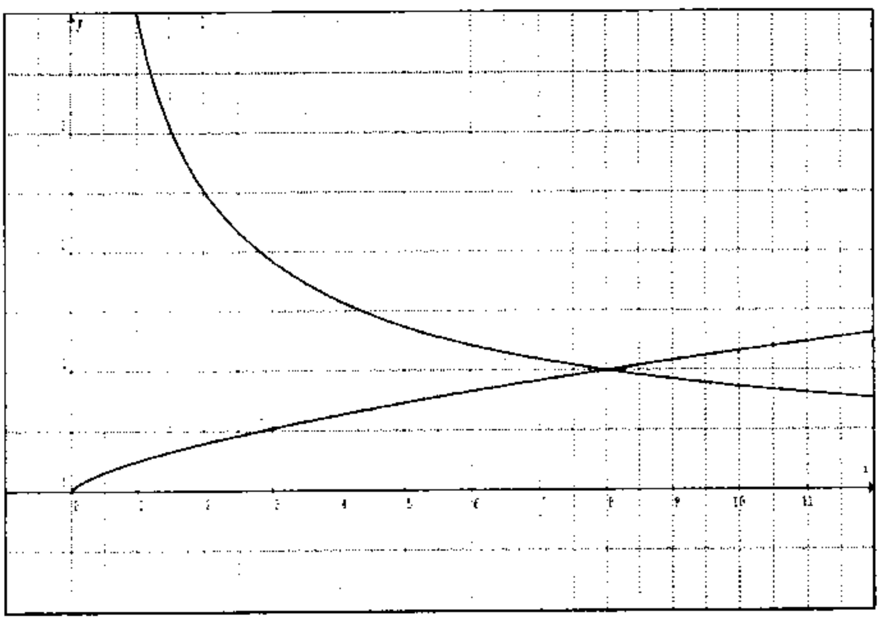
8(b)	$-0.6\ radian/\sec$
9(i)	Proof
9(ii)	$27-4r \neq 0$ *as* r *must be an integer* $\Rightarrow$ *no independent term.*
9(iii)	-3840
10(i)	Proof
10(ii)	$y = x - \frac{1}{2}e^{-2x} - \frac{1}{2}$
10(iii)	$1\frac{1}{4}\ units^2$
11(a)	32
11(b)	$x = 2,\ \frac{1}{2}$
12(i)	$t = 2$ *or* $t = 4$
12(ii)	$14\frac{2}{3}\ m$
12(iii)	Proof
12(iv)	$16m$
13(i)	Proof
13(ii)	$\theta = 35.8°,\ 125.8°$
13(iii)	Proof
13(iv)	$\frac{3\sqrt{3}-4}{10}$

Marking Scheme (Additional Mathematics Paper 2/ Prelim Examination 2015)

Qn No.	Solution
1	$\int_2^1 [2x - f(x)]dx + \int_0^1 f(x)dx$ $= \left[x^2\right]_1^2 + \int_0^2 f(x)\ dx$ $= -3 + \int_0^5 f(x)]dx - \int_2^5 f(x)\ dx$ $= -3 + 12 - 4 = 5$
2(i)	$\frac{1}{4}x^{\frac{2}{3}} = 4x^{-\frac{2}{3}}$ $x^{\frac{4}{3}} = 16$ $x = 8$ $y = \frac{1}{4}(8)^{\frac{2}{3}} = 1$ *Coord* (8, 1)
2(ii)	
3(i)	$y = \frac{p - x}{x + q}$ $x(1 + y) = -qy + p$ – – – – – – – (1) $-q = -1\frac{1}{3} \Rightarrow q = 1\frac{1}{3}$ $2 = -\frac{4}{3}(3) + p$ $\Rightarrow p = 6$

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3(ii)	$x(1+y)=k$ $7x=k$ $x=\dfrac{k}{7}$
4(i)	$y=\dfrac{\ln(x-3)^2}{2x-6}=\dfrac{\ln(x-3)}{x-3}$ $\dfrac{dy}{dx}=\dfrac{(x-3)\left(\dfrac{1}{x-3}\right)-\ln(x-3)}{(x-3)^2}$ $=\dfrac{1-\ln(x-3)}{(x-3)^2}$
4(ii)	$For\ \dfrac{dy}{dx}<0,\quad 1-\ln(x-3)<0$ $\Rightarrow \ln(x-3)>1$ $\Rightarrow x>e+3=5.72$
4(iii)	$\int_4^5 \dfrac{1-\ln(x-3)}{(x-3)^2}dx=\left[\dfrac{\ln(x-3)}{x-3}\right]_4^5$ $\int_4^5 \dfrac{1}{(x-3)^2}dx-\int_4^5\dfrac{\ln(x-3)}{(x-3)^2}dx=\left[\dfrac{\ln(x-3)}{x-3}\right]_4^5$ $\int_4^5 \dfrac{\ln(x-3)}{(x-3)^2}dx=\int_4^5\dfrac{1}{(x-3)^2}dx-\left[\dfrac{\ln(x-3)}{x-3}\right]_4^5$ $=\left[-\dfrac{1}{x-3}\right]_4^5-\left[\dfrac{\ln(x-3)}{x-3}\right]_4^5=\dfrac{1}{2}-\dfrac{\ln 2}{2}$ $\therefore\ \int_4^5\dfrac{\ln\sqrt{(x-3)}}{(x-3)^2}dx=\dfrac{1}{2}\int_4^5\dfrac{\ln(x-3)}{(x-3)^2}=\dfrac{1}{2}\left(\dfrac{1}{2}-\dfrac{\ln 2}{2}\right)=\dfrac{1-\ln 2}{4}$
5(i)	$\angle BDC=\angle ABD$ (*Alternate angles*, $AB\,//\,DC$) $\angle ABD=\angle BCE$ (*Alternate Segment Theorem*) $\therefore\ \angle BCE=\angle BDC$
5(ii)	*In* ΔBCE *and* ΔBDC, $\angle BCE=\angle BDC$ (*From (i)*) $\angle CBE=\angle DBC$ (*Common angles*) $\therefore$ ΔBCE *and* ΔBDC *are similar triangles* (*AAA property*)

5(iii)	ΔAEB *and* ΔCED *are similar triangles* (*AAA property*) $\frac{AE}{CE} = \frac{AB}{CD}$ $\frac{AE}{\frac{1}{3}AC} = \frac{AB}{CD}$ (*Given* AE : EC = 2 : 1) $3AE \times CD = AB \times AC$
6(i)	$a = -\frac{k}{2}$ $b = \frac{k+2}{2}$ $Centre = (-\frac{k}{2}, \frac{k+2}{2})$
6(ii)	$2\left(-\frac{k}{2}\right) + 5\frac{(k+2)}{2} - 11 = 0$ $k = 4$
6(iii)	$C_1 = (-2, 3),\quad r = \sqrt{(-2)^2 + 3^2 - 7} = \sqrt{6}$ *Let* P_2 = *Centre of* C_2 $C_2 = (4, 3)$ *Equation of* C_2: $(x-4)^2 + (y-3)^2 = 6$
6(iv)	*Let* d = *distace from* P_1 *to* $P_2 = 6$ *Let* R = *radius of* C_1 + *radius of* $C_2 = \sqrt{6} + \sqrt{6} = 4.89$ *Since* $R < d$, *the two circles do not intersect each other.*
7(i)	Lowest tide occurs when $\sin kt = 1$, lowest tide = 4.5 m
7(ii)	Period between high tides = 14.5 hours $\frac{2\pi}{k} = 14.5$ $\frac{2\pi}{14.5} = k$ $k = \frac{4\pi}{29}$
7(iii)	$h = 2(3.25 - \sin\frac{8\pi}{29})$ $h = 4.98m$
7(iv)	When h = 7.0 $7.0 = 2(3.25 - \sin\frac{4\pi}{29}t)$ $\sin\frac{4\pi}{29}t = -0.25$ $\frac{4\pi}{29}t = 3.394$ $\Rightarrow\ t = 7.833 = 7h\ 50$ min *The time is* 0750

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8(a)	$F(3)-F(1)=\int_1^3 \left(\frac{9}{2}\sqrt{3x-1}-\frac{3}{\sqrt{3x-1}}\right)dx$ $=\left[\dfrac{\frac{9}{2}(3x-1)^{\frac{3}{2}}}{\frac{9}{2}}-2(3x-1)^{\frac{1}{2}}\right]_1^3$ $=\left[(8)^{\frac{3}{2}}-2(8)^{\frac{1}{2}}\right]-\left[(2)^{\frac{3}{2}}-2(2)^{\frac{1}{2}}\right]$ $=(8\sqrt{8}-2\sqrt{8})-(2\sqrt{2}-2\sqrt{2})=6\sqrt{8}$ $=12\sqrt{2}$
8(b)	$y=\operatorname{cosec}^2\left(\frac{x}{2}-\frac{\pi}{6}\right)=\sin^{-2}\left(\frac{x}{2}-\frac{\pi}{6}\right)$ $\frac{dy}{dx}=-2\left[\sin^{-3}\left(\frac{x}{2}-\frac{\pi}{6}\right)\right]\left[\frac{1}{2}\cos\left(\frac{x}{2}-\frac{\pi}{6}\right)\right]$ $=-\left[\sin^{-3}\left(\frac{x}{2}-\frac{\pi}{6}\right)\right]\left[\cos\left(\frac{x}{2}-\frac{\pi}{6}\right)\right]$ *When* $x=\frac{5\pi}{6}$, $\frac{dy}{dx}=-\left[\sin^{-3}\left(\frac{\pi}{4}\right)\right]\left[\cos\left(\frac{\pi}{4}\right)\right]=-2$ $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}=-2\times 0.3$ $=-0.6$ *radian* / sec
9(i)	$T_{r+1}=\binom{9}{r}(x^3)^{9-r}\left(-\frac{k}{x}\right)^r=(-k)^r\binom{9}{r}(x^{27-4r})$ *For the term in* $\frac{1}{x}$, $27-4r=-1$ $\Rightarrow r=7$ $-\binom{9}{7}(k^7)=-4608$ $\Rightarrow k=2$
9(ii)	$27-4r\neq 0$ *as* r *must be an integer* $\Rightarrow$ *no independent term.*

9(iii)	*For the term in* x^3, $r = 6$ *The term in* $x^3 = (-2)^6\binom{9}{6}(x^3) = 5376x^3$ $(2x^3 + \frac{1}{x})(x^3 - \frac{k}{x})^9 = (2x^3 + \frac{1}{x})(\ldots\ldots - 4608\left(\frac{1}{x}\right) + 5376x^3 + \ldots\ldots)$ $= -9216x^2 + 5376x^2 + \ldots\ldots = -13840x^2 + \ldots\ldots$ $\therefore$ *The coefficient of* x^2 *is* -3840
10(i)	$\frac{dy}{dx} = \frac{e^{2x}+1}{e^{2x}} = 1 + e^{-2x}$; $e^{-2x} > 0$ *for all values of* x $\Rightarrow \frac{dy}{dx} \neq 0.$ $\therefore$ *No stationary pt*
10(ii)	$y = \int 1 + e^{-2x} dx$ $y = x - \frac{e^{-2x}}{2} + c$ *when* $x = 0, y = -1$ $-1 = -\frac{1}{2} + c$ $c = -\frac{1}{2}$ $y = x - \frac{1}{2}e^{-2x} - \frac{1}{2}$
10(iii)	*At* P, $m_{\tan gent} = 2$ *Equation of* tan *gent*: $y = 2x - 1$ $m_{normal} = -\frac{1}{2}$ *Equation of normal*: $y = -\frac{1}{2}x - 1$ $\Rightarrow Q = \left(\frac{1}{2}, 0\right)$ $R = (-2, 0)$ $\therefore$ *Area of* $\Delta PQR = \left(\frac{1}{2}\right)\left(\frac{5}{2}\right)(1) = 1\frac{1}{4}$ *units*^{2}

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11(a)	$2^{2x-3} = \frac{1}{4^{x-1}}$ $\frac{4^x}{8} = \frac{4}{4^x}$ $(4^x)^2 = 32$ $16^x = 32$
11(b)	$\log_x 5 - \frac{2}{\log_{\sqrt{x}} 2} = \log_{x^2}\left(\frac{25}{4}\right)$ $\log_x 5 - \frac{2}{\frac{\log_x 2}{\log_x \sqrt{x}}} = \frac{\log_x\left(\frac{25}{4}\right)}{\log_x x^2}$ $\log_x 5 - \frac{1}{\log_x 2} = \frac{\log_x\left(\frac{25}{4}\right)}{2}$ $2\log_x 5 - \frac{2}{\log_x 2} = \log_x\left(\frac{25}{4}\right)$ $\log_x 25 - \frac{2}{\log_x 2} = \log_x 25 - \log_x 4$ $\log_x 4 = \frac{2}{\log_x 2}$ $2\log_x 2 = \frac{2}{\log_x 2}$ $(\log_x 2)^2 = 1$ $\log_x 2 = \pm 1$ *When* $\log_x 2 = 1$, $x = 2$ *When* $\log_x 2 = -1$, $x = \frac{1}{2}$
12(i)	$t^2 - 6t + 8 = 0$ $t = 2$ *or* $t = 4$

12(ii)	$v = t^2 - 6t + 8$ $s = \frac{t^3}{3} - 3t^2 + 8t + c$ $At\ t = 0, s = 0 \Rightarrow c = 0$ $\therefore\ s = \frac{t^3}{3} - 3t^2 + 8t$ $When\ v = 8,\ t^2 - 6t = 0$ $t = 0\ \ or\ \ t = 6$ $\therefore$ *When the velocity is* $8\ m/s$ *again,* $t = 6$ $S_6 = \frac{216}{3} - 3(36) + 8(6) = 12\ m$ $S_0 = 0$ $S_2 = \frac{8}{3} - 3(4) + 8(2) = 6\frac{2}{3}$ $S_4 = \frac{64}{3} - 3(16) + 8(4) = 5\frac{1}{3}$ *From* $t = 0$ *to* $t = 4$, *distance travelled* $= 6\frac{2}{3} + (6\frac{2}{3} - 5\frac{1}{3}) = 8\ m$ *From* $t = 4$ *to* $t = 6$, *distance travelled* $= 12 - 5\frac{1}{3} = 6\frac{2}{3} m$ *From* $t = 0$ *to* $t = 6$, *distance travelled* $= 14\frac{2}{3}$
12(iii)	*At O,* $s = 0$ $s = \frac{t^3}{3} - 3t^2 + 8t = 0$ $\frac{1}{3}t(t^2 - 9t + 24) = 0$ $\Rightarrow t = 0\ \ \ or\ \ \ t^2 - 9t + 24 = 0$ $t^2 - 3t + 8 = 0 \Rightarrow t = \frac{9 \pm \sqrt{-15}}{2} \Rightarrow$ *No solution* $\Rightarrow$ *The particle is at O when* $t = 0$ *only.Therefore P will not return to O in the course of its motion.*
12(iv)	*From* $t = 6$ *to* $t = 7$, *distance travelled* $= 8\ m$ *From* $t = 7$ *to* $t = 9$, *distance travelled* $= 8\ m$ $\therefore$ *Total distance travelled* $= 8 + 8 = 16 m$

13(i)	$\tan\frac{\theta}{2} + \cot\frac{\theta}{2} = \tan\frac{\theta}{2} + \frac{1}{\tan\frac{\theta}{2}} = \frac{\tan^2\frac{\theta}{2} + 1}{\tan\frac{\theta}{2}}$ $= \frac{\sec^2\frac{\theta}{2}}{\tan\frac{\theta}{2}}$ $= \frac{1}{\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \frac{1}{\frac{1}{2}\sin\theta} = 2\cos ec\theta$
13(ii)	$\tan\theta + \cot\theta = (\cos ec4\theta)(\sin 2\theta + \cos 2\theta)$ $2\cos ec2\theta = (\cos ec4\theta)(\sin 2\theta + \cos 2\theta)$ $= \frac{1}{2\cos 2\theta} + \frac{1}{2\sin 2\theta} = \frac{1}{2\cos 2\theta} + \frac{1}{2}\cos ec2\theta$ $3\cos ec2\theta = \frac{1}{\cos 2\theta}$ $\tan 2\theta = 3$ $\theta = 35.8°,\ \ 125.8°$
13(iii)	$2\tan A + 2\cot A = 5$ $\tan A + \cot A = \frac{5}{2}$ $\cos ec2A = \frac{5}{4}$ $\sin 2A = \frac{4}{5}$ $\cos 2A = \frac{3}{5}$
13(iv)	$\cos(2A + \frac{\pi}{6}) = \cos 2A\cos\frac{\pi}{6} - \sin 2A\sin\frac{\pi}{6}$ $= (\frac{3}{5})(\frac{\sqrt{3}}{2}) - (\frac{4}{5})(\frac{1}{2})$ $= \frac{3\sqrt{3} - 4}{10}$

ANDERSON SECONDARY SCHOOL
Preliminary Examination 2015
Secondary Four Express & Five Normal

CANDIDATE NAME:

CLASS: /   INDEX NUMBER:

ADDITIONAL MATHEMATICS   4047/01

Paper 1   28 August 2015

2 hours

0800 – 1000h

Additional Materials: Writing paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

This document consists of 4 printed pages.

Setter: Miss Leow Hwee Fen & Miss Oh Hui Ying

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1.$$
$$\sec^2 A=1+\tan^2 A.$$
$$\operatorname{cosec}^2 A=1+\cot^2 A.$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}.$$
$$a^2=b^2+c^2-2bc\cos A.$$
$$\Delta=\frac{1}{2}bc\sin A.$$

49

Answer all questions

1 Solve the following equations.

(a) $2(3^x)+3^{-x}=3$ [3]

(b) $2\sqrt{3-2x}=x+1$ [3]

2 The line $2y-x=3$ meets the curve $x^2-xy-y^2=1$ at points A and B. Find the length of AB, giving your answer expressed in the form $a\sqrt{b}$, where a and b are integers. [5]

3 Write down and simplify the first 3 terms, in ascending powers of x, in the expansion of $\left(2-\frac{x}{3}\right)^5$. Given that the first three terms in the expansion of $(1+px+x^2)\left(2-\frac{x}{3}\right)^5$ are $32-qx+2qx^2$, find the value of p. [5]

4 A circle C_1 passes through points $P(0, 2)$, $Q(7, 3)$ and $R(8, -4)$ where $PQRS$ is a square.

(a) Find the coordinates of the centre and the radius of the circle C_1. [2]

(b) Find the equation of another circle C_2, in the form $x^2+y^2+ax+by+c=0$, that is the reflection of the circle C_1, in the line $y=x$. [2]

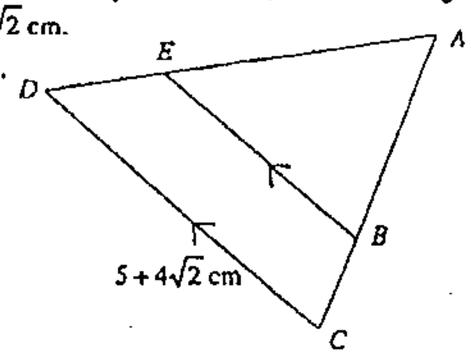
(c) Justify if the point (2, 7) lies inside or outside the circle C_1. [2]

5 Prove the identity $\dfrac{\sec x+2\sin x}{2\cos x-\sec x}=\dfrac{1+\tan x}{1-\tan x}$. [5]

6 The diagram shows a triangle $ABCDE$, such that EB is parallel to DC, the ratio of lengths $AB : BC$ is $4:\sqrt{8}$ and length of DC is $5+4\sqrt{2}$ cm.
By leaving your answer in the form $a+b\sqrt{c}$, calculate

(a) the ratio $\dfrac{BE}{CD}$, [3]

(b) the length of BE. [3]



7 Given that $f(x)=-2+x^2$ and $g(x)=|x+1|-1$.

(a) Find the coordinates of the points of intersection of the graphs $y=f(x)$ and $y=g(x)$. [4]

(b) On the same axes, sketch the graphs of $y=f(x)$ and $y=g(x)$ for $-2\le x\le 2$. [3]

(c) Hence solve the inequality $x^2\le|x+1|+1$. [2]

8 (a) Sketch the graph of $y=2-e^{3x}$ for all real values of x, showing clearly all points of intersection with the axes, if any. [2]

(b) By adding a suitable straight line, explain how the number of solutions to the equation $x=\ln\sqrt[3]{4-x}$ can be obtained. [2]

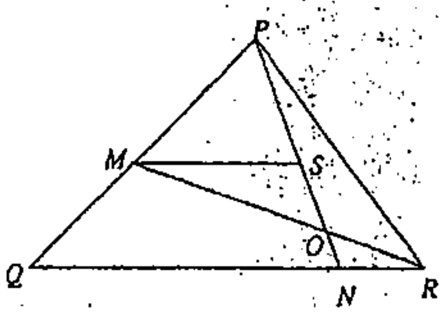
9 A and B lie in the same quadrant such that $\sin A = \frac{3}{5}$ and $\tan B = -\frac{5}{12}$. If the value of A and of B is between 0 and 2π, find, without using the calculator, the values of

(a) $\sin B$, [1]

(b) $\cot(A-B)$, [2]

(c) $\cos\frac{B}{2}$. [3]

10 In ΔPQR, M is the mid-point of PQ. PN and MR intersect at O.



Given that $OR : OM = PS : PN = 1 : 2$, prove that

(a) MS is parallel to QN, [2]

(b) ΔMSO is similar to ΔRNO, [2]

(c) $OP = 5\,NO$. [2]

11 The table shows some experimental values of two variables x and y which are known to be related by the equation $y = ax(x+b)$.

x	1.5	2.5	3.5	4.5	5.5
y	10.1	20.6	34.2	50.7	70.1

Using a suitable scale, plot the graph of $\frac{y}{x}$ against x to represent the above data and use it to estimate

(a) the value of a and of b, [4]

(b) the value of x when $y = 9x$. [1]

12 A function is given by $y = \frac{9x-3b}{4x-1}$ where $x \neq a$ and $x > 0$.

(a) State the value of a. [1]

(b) Determine the range of values of b if y is an increasing function. [3]

(c) Given that $b = 3$ and that x and y vary with time t, find the value(s) of x if $\frac{dy}{dt} = 12\frac{dx}{dt}$. [3]

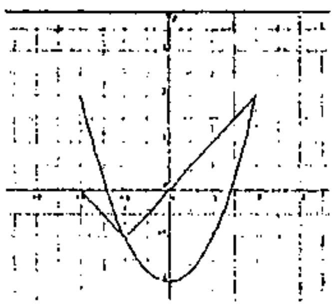
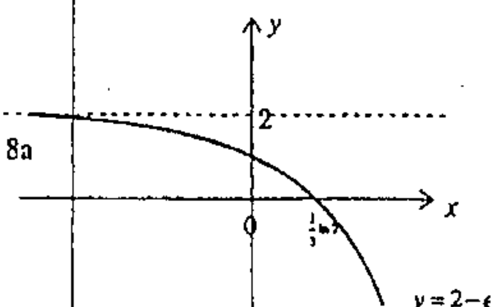
13 An electronic gadget was programmed to travel in a straight line. It started through a fixed point O with a velocity of 3 m/s. Its acceleration, a m/s^2, is given by $a = 2 - 2t$, where t seconds is the time after passing O. Find

(a) its maximum velocity, [3]

(b) its deceleration when it changes its direction of motion, [3]

(c) the total distance travelled during the first four seconds of motion. [4]

Prelim AM Paper 1 Answer Key

1(a)	$x = -0.631$ or $x = 0$	9a	$\frac{5}{13}$
1(b)	$x = 1$	9b	$-3\frac{15}{16}$
2	$7\sqrt{5}$ units	9c	$\frac{\sqrt{26}}{26}$
3	$32-\frac{80}{3}x+\frac{80}{9}x^2+\ldots$; $p=\frac{1}{3}$	10	Proof
4a	$(4,-1)$; 5 units	11a	$a = 1.5$; $b = 3$
4b	$x^2+y^2+2x-8y-8=0$	11b	$x = 3$
4c	(2, 8) lies outside the circle C_1.	12a	$a=\frac{1}{4}$
5	Proof	12b	$b>\frac{3}{4}$
6a	$2-\sqrt{2}$	12c	$x=\frac{5}{8}$
6b	$(2+3\sqrt{2})$ cm	13a	4 m/s
7a	$(-1,-1)$ and $(2, 2)$	13b	4 m/s^2
7b		13c	$11\frac{1}{3}$m
		9a	$\frac{5}{13}$
		9b	$-3\frac{15}{16}$
		9c	$\frac{\sqrt{26}}{26}$
		11a	$a = 1.5$; $b = 3$
7c	$-1 \le x \le 2$	11b	$x = 3$
8a		12a	$a=\frac{1}{4}$
		12b	$b>\frac{3}{4}$
		12c	$x=\frac{5}{8}$
		13a	4 m/s
		13b	4 m/s^2
8b	Add the line $y = x - 2$. The number of intersection points of $y = 2 - e^{3x}$ and $y = x - 2$ gives the number of solutions for $x = \ln\sqrt[3]{4-x}$.	13c	$11\frac{1}{3}$m

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ANDERSON SECONDARY SCHOOL
Preliminary Examination 2015
Secondary Four Express & Five Normal

CANDIDATE NAME:

CLASS: /   INDEX NUMBER:

ADDITIONAL MATHEMATICS 4047/02

Paper 2

27 August 2015

2 hours 30 minutes

0800 – 1030h

Additional Materials: Writing paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

This document consists of 5 printed pages.

Setter: Miss Leow Hwee Fen & Miss Oh Hui Ying

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1.$$
$$\sec^2 A=1+\tan^2 A.$$
$$\operatorname{cosec}^2 A=1+\cot^2 A.$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
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$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}.$$
$$a^2=b^2+c^2-2bc\cos A.$$
$$\Delta=\frac{1}{2}bc\sin A.$$

34

Answer all questions

1 When $(1-2p)^n$ is expanded in ascending powers of p, the sum of the constant term, the coefficients of p and p^2 is 161. If n is a positive integer, find the value of n. [4]

2 (a) Find the smallest positive integer, p such that $4(px-5)=x^2$ has real roots. [4]

(b) Find the range of values of m for which the graph of $y=mx^2-4x+m$ lies entirely below the line $y=3$. [4]

(c) Given that the line $y=4x+k$ is a tangent to the curve $y^2=mx$, where k and m are constants, prove that $\frac{k}{m}=\frac{1}{16}$. [4]

3 Marcus believes that the depth of water, d metres, at the end of a jetty, t hours after low tide, can be modelled by the equation $d=a+b\cos kt$ where a, b and k are constants.

(a) He measures the depth of water at low tide to be 2 metres.
Assuming that low tides occur every 12 hours, show that $k=\frac{\pi}{6}$. [1]

(b) Marcus also measures the depth of water at high tide to be 6 metres.
Calculate the value of a and of b. [2]

(c) Sketch the graph of the equation $d=a+b\cos kt$ for $0<t<2\pi$. [3]

(d) Marcus requires the depth of water at the end of the jetty to be at least 3 metres to sail his boat. Given that the low tide on a particular day was at 0830, find the earliest time after 0830 when Marcus could sail his boat that day. [2]

(e) Marcus measured the depth of water and found that it is 5m. He then claimed that the depth of water at the end of the jetty will reach 5 m again after every 4 hours. Justify if Marcus is right or wrong. [2]

4 (a) (i) Factorise $h(x)=x^3-7x^2+2x+40$ completely. [3]

(ii) Hence, solve the equation $2y^3-7y^2+y+10=0$. [3]

(b) Find the value of n for which the division of $2x^n+3x^2-4x-10$ by $x-2$ gives a remainder of 26. [3]

5 Solve the following equations.

(a) $\log_2\sqrt{5x+1}=\log_4(x-2)+\log_2 4$ [5]

(b) $4\tan^2 x=1-8\sec x$ for $-\pi<x<2\pi$ [5]

6 (a) Consider the equation $kx^2 - k^2x - 3x + 4 = k$.
If the roots of the equation are reciprocal of each other, and β is one of roots,
(i) find the value of k. [2]
(ii) show that $\frac{7}{\beta^2+1} = \frac{2}{\beta}$. [2]

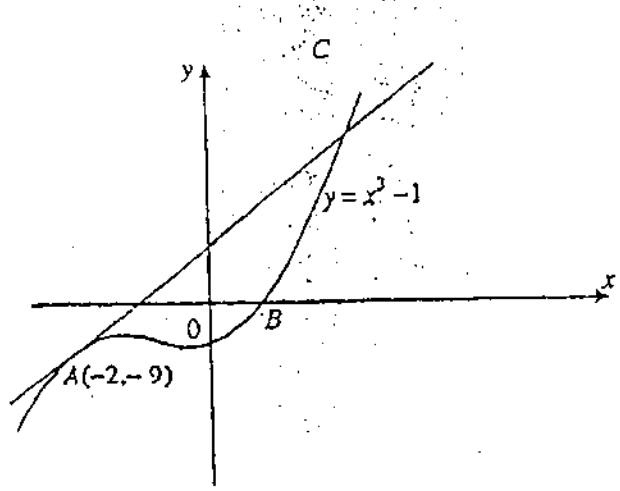
(b) The roots of the quadratic equation $2x^2 - 4x + 5 = 0$ are λ and μ.
Find the quadratic equation whose roots are $\frac{\mu}{\lambda}$ and $\frac{\lambda}{\mu}$. [4]

7 The term containing the highest power of x in the polynomial $f(x)$ is $3x^4$.
$x^2 - 2x + k$ is a quadratic factor of $f(x)$. $x = -1$ and $x = 2$ are roots of the equation $f(x) = 0$. $f(x)$ leaves a remainder of -36 when it is divided by x.
(a) Show that $k = 6$. [2]
(b) Determine the number of real roots of the equation $f(x) = 0$. [2]

8 A curve is defined by $y = (1-2x)^2 e^{2x}$. Find
(a) $\frac{dy}{dx}$, [2]
(b) the equation, in terms of e, of the tangent at the point where $x = 1$, [4]
(c) the x-coordinate(s) of the stationary point(s) on the curve and determine the nature of the point(s). [4]

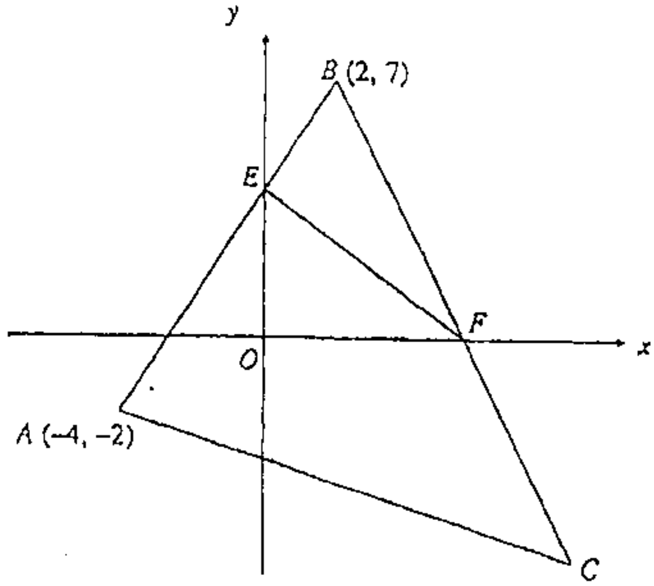
9 (a) Express $\frac{x^2-3x+5}{(x^2+x)(2x-1)}$ in partial fractions. [3]
(b) Hence, evaluate $\int_1^2 \frac{x^2-3x+5}{(x^2+x)(1-2x)}\,dx$. [3]

10 The diagram shows part of the curve $y = x^3 - 1$. The tangent at A $(-2, -9)$ meets the curve again at C. Find the area of the region bounded by the two graphs. [8]



35

11 Solution to this question by accurate drawing will not be accepted.



The diagram shows a triangle ABC where A is $(-4, -2)$, B is $(2, 7)$ and BC is parallel to the line $2y = -4x + 1$. BC cuts the x-axis at F and AB cuts the y-axis at E.

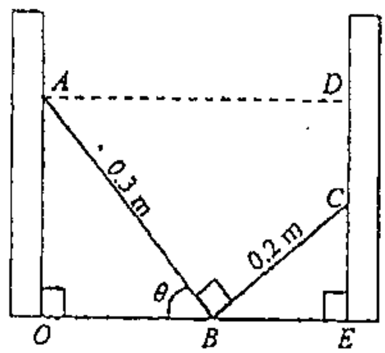
(a) Find the equation of the line BC. [2]

(b) Determine whether if EF is perpendicular to AB. [3]

(c) Given that C is equidistant from A and E, find the coordinates of C. [3]

(d) Find the length of AE, and hence, find the area of ΔAEC. [3]

12



A L-shaped ladder, ABC is wedged in between two pillars AO and DE as shown in the diagram. A and C are the points of contact between the ladder and the pillars while B is the point of contact between the ladder and the ground.

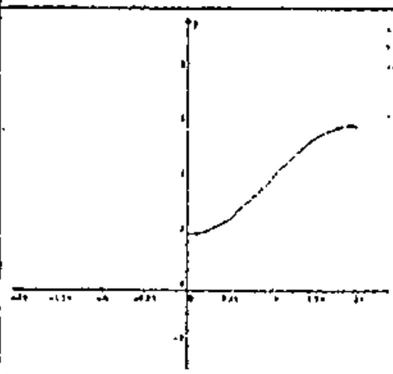
Given that $\angle OBA = \theta$, where $0° < \theta < 90°$, $AB = 0.3$ m, $BC = 0.2$ m and $CD = x$ m,

(a) show that $x = 0.3\sin\theta - 0.2\cos\theta$, [2]

(b) express x in the form $R\sin(\theta - \alpha)$ where $R > 0$ and $0° < \alpha < 90°$, [4]

(c) hence, explain if the length of CD can be 0.45 m. [2]

Prelim AM Paper 2 Answer Key

1	$n=10$	7b	2 real roots
2a	$p=3$	8a	$-2(1-2x)(1+2x)e^{2x}$
2b	$m<-1$	8b	$y=6e^2x-5e^2$
3a	$k=\frac{\pi}{6}$	8c	$x=-\frac{1}{2}$(max) & $x=\frac{1}{2}$(min)
3b	$a=4$	9a	$-\frac{5}{x}+\frac{3}{x+1}+\frac{5}{2x-1}$
3c		9b	-0.497
		10	108 sq units
		11a	$y=-2x+11$
		11b	EF is not perpendicular to AB.
		11c	$\left(\frac{17}{2},-6\right)$
		11d	$45\frac{1}{2}units^2$
		12b	$0.361\sin(\theta-33.7°)$
		12c	Length of CD cannot be 0.45 m.
3d	1030h		
3e	Wrong		
4ai	$(x-4)(x+2)(x-5)$		
4aii	$y=2$ or $y=-1$ or $y=\frac{5}{2}$		
4b	$n=4$		
5a	$x=3$		
5b	$x=-1.98$ or 1.98 or 4.30		
6ai	$k=2$		
6b	$5x^2+2x+5=0$		

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AHS Prelim. Am 2015 Paper 1

1. A curve has the equation $y = \frac{\ln x}{x^2}$.

(i) Find $\frac{dy}{dx}$. [2]

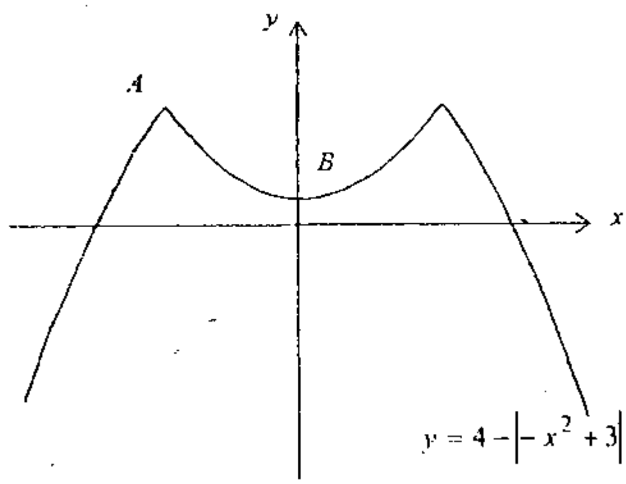
(ii) Hence, find the range of values of x, such that y is increasing. [2]

2. The diagram below shows the graph of $y = 4 - \left|-x^2 + 3\right|$.

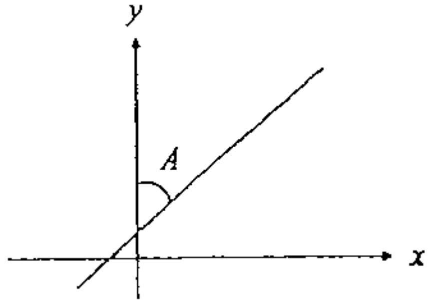
(i) Show that the coordinates of A is $\left(-\sqrt{3}, 4\right)$. [1]

(ii) State the coordinates of B. [2]

(iii) Find the exact value of m, for $m < 0$ for which the equation $mx + 1 = 4 - \left|-x^2 + 3\right|$ has exactly 3 solutions. [2]



3. In the diagram shown, the line forms an angle A with the y-axis. Given that the gradient of the line is 3, without using a calculator, find the exact value of $\cos A$. [3]



37

4. (i) Show that

$$\sqrt{\frac{1-\sin x}{1+\sin x}} = \sec x - \tan x \text{ , when } -90^\circ < x < 90^\circ.$$ [5]

(ii) Hence, explain why x must be acute for the identity to be true. [1]

5. Given that the coefficient of $\frac{1}{x^3}$ is 512 in the expansion $\left(\frac{2}{x}+ax^2\right)^9$, where $a<0$.

(i) Find the value of a [3]

(ii) Hence, using the value of a found in (i), show that the term in $\frac{1}{x^4}$ does not exist in the expansion $\left(\frac{2}{x}+ax^2\right)^9\left(\frac{1}{8x}+\frac{x^2}{12}\right)$. [3]

6. Express $\frac{x^3+5x^2+2x-1}{2x^4+x^2}$ in partial fractions. [6]

7. The equation $6x^2+7x-3=0$ has roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

(i) Find the value of $(\alpha+\beta)$ and of $\alpha\beta$. [3]

(ii) Hence, or otherwise, find the exact value of $\left(\alpha^3+\beta^3\right)$. [3]

8. Solve the equation $\lg\left(4^x-10\right)-x\lg 2=\lg 3$. [5]

9. The voltage V, in volts, of an electrical signal in an electrical system is given by the formula $V = 4\sin \pi t$ where t is in seconds.

(i) Find the exact rate of change of voltage after $\frac{1}{4}$ seconds have elapsed. [2]

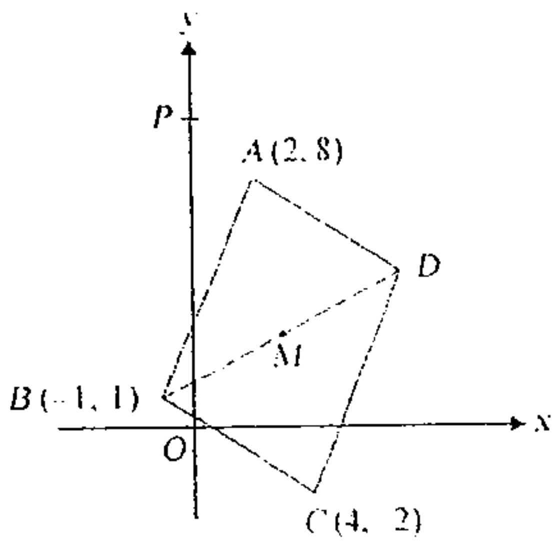
(ii) Find the exact times when the rate of change of voltage is $2\pi\sqrt{3}$ volts per second for $0 < t < 4$. [3]

(iii) Given that current (I in amperes) supplied to the system is governed by the equation $I = \frac{V}{5}$, find the rate of change of current when the rate of change of voltage is 2 volts per second. [2]

10. In the diagram shown below, $ABCD$ is a parallelogram with points $A(2, 8)$, $B(-1, 1)$ and $C(4, -2)$. M is the midpoint of BD and the perpendicular bisector of BD passes through the y-axis at P.

Find

(i) the coordinates of M, [1]
(ii) the coordinates of D, [2]
(iii) the equation of the perpendicular bisector of BD, [2]
(iv) and the area of quadrilateral $ACBP$. [2]



38

11. Given that $y = x^3 + ax^2 + bx + 3$ has a stationary point $(1, 0)$,

(i) find the values of a and of b, [3]

(ii) find the coordinates of the other stationary point, [3]

(iii) and determine the nature of these stationary points. [3]

12. (i) Differentiate the following with respect to x.

(a) $\dfrac{\left(e^{x}\right)^{4} e^{-x}}{e^{x+1}}$ [2]

(b) $\ln\left(\cos^{2} x\right)$ [2]

(ii) Hence, or otherwise, find $\int \dfrac{5}{2x-3} - 4e^{2x+1} - 2\tan x \, dx$ [4]

13. The table shows experimental values of two variables, x and y, which are connected by the equation $y = ae^{bx-1}$.

x	1	2	3	4	5
y	1.89	2.30	2.82	3.44	4.20

(a) Plot $\ln y$ against x and draw a straight line graph. [3]

(b) Use your graph to estimate the value of a and of b. [3]

(c) By drawing a suitable line on your graph, solve the equation $2.46 = ae^{bx-1}$. [2]

END OF PAPER

AHS Prelim AM P1

1. (i) $\frac{1-2\ln x}{x^3}$ (ii) $0 < x < e^{\frac{1}{2}}$

2. (ii) B (0,1) (iii) $-\sqrt{3}$

3. $\frac{3}{\sqrt{10}}$

4. (ii) $\cos x = \sqrt{1-\sin^2 x}$. Since $\cos x$ must be positive, x is in the 1st or 4th quadrant. $\therefore$ x must be acute.

5. (i) $-\frac{1}{3}$ (ii) Term with $\frac{1}{x^4} = \left(-\frac{768}{x^6}\right)\left(\frac{x^2}{12}\right) + \left(\frac{512}{x^3}\right)\left(\frac{1}{8x}\right) = 0$
$\therefore$ it does not exist.

6. $\frac{2}{x} - \frac{1}{x^2} + \frac{7-3x}{2x^2+1}$

7. (i) $\frac{7}{3}$, -2 (ii) $\frac{721}{27}$

8. $x = 2.32$

9. (i) $\frac{dv}{dt} = \frac{4\pi}{\sqrt{2}}$ v/s (ii) $t = \frac{1}{6}, \frac{11}{6}, 2\frac{1}{6}, 3\frac{5}{6}$ sec (iii) $\frac{dI}{dt} = \frac{2}{5}$ Amperes/sec

10. (i) m (3,3) (ii) D (7,5) (iii) $y = -2x + 9$ (iv) $30\frac{1}{2}$ units^2

11. (i) a = 1, b = -5 (ii) $\left(-\frac{5}{3}, \frac{256}{27}\right)$ (iii) max point

12. (i)(a) $2e^{2x-1}$ (b) $-2\tan x$ (ii) $\frac{5}{2}\ln(2x-3) - 2e^{-2x-1} + \ln\cos^2 x + c$

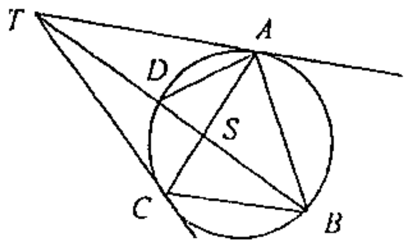
13. (b) a = 4.22, b = 0.2 (c) $x = 2.3$

AHS Prelim. 2015 Am Paper 2

1. The curve $\frac{(x-2)^2}{4}+(y-3)^2=4$ and the line $2y+x=12$ intersect at the points P and Q. Find the exact distance between P and Q. [5]

2. Find the values of a and b for which the function $f(x)=2x^4-7x^3+ax^2+bx-21$ is exactly divisible by x^2-2x-3.
Hence determine, showing all necessary working, the number of real roots of the equation $f(x)=0$. [8]

3.

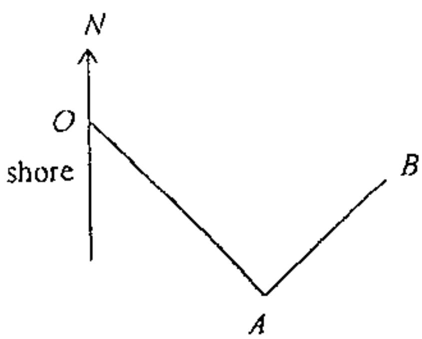


In the diagram, A, B, C and D are points on the circle. $TDSB$ and ASC are straight lines. TA and TC are tangents. Prove that

(a) $\angle ACB=\angle ATD+\angle ABD$, and [3]

(b) $\angle ATC=180°-2\angle ABC$. [3]

4. The diagram shows the route of a fishing boat. The boat leaves the point O from the shore and sails in a straight line for 5 km to a point A, at a bearing of $(090°+\theta)$. At A, the boat makes a right-angled turn and sails for 2 km to the point B to continue fishing. The angle $OAB=90°$ and the shortest distance from B to the shore is L km.



(a) Show that $L=5\cos\theta+2\sin\theta$. [2]

(b) Express L in the form $R\cos(\theta-\alpha)$, where $R>0$ and $0°<\alpha<90°$. [3]

(c) State the maximum value of L and find the corresponding value of θ. [3]

40

5. (a) Express $\tan^2\alpha - \cos^2\beta$ in the form $A\sec^2 B\alpha + C\cos 2\beta + D$ where $A, B, C,$ and D are constants. [3]

(b) Hence, or otherwise, evaluate $\int_1^3 \left(\tan^2 3x - \cos^2\frac{x}{2}\right)dx$. [5]

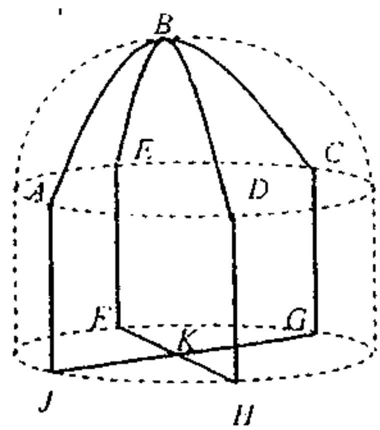
6. The curve $y = P\cos Qx + R$ has a period of $720°$, a maximum value of 8 and a minimum value of -4.

(a) Given that P is a negative constant and Q and R are positive constants, find the value of P, of Q and of R. [4]

(b) Solve the equation $y = 3$ where $0° < x < 360°$. [2]

(c) Sketch the graph of y for $0° \le x \le 360°$. [3]

7. A container in the shape of a cylinder with a hemisphere on top is to be decorated by gold wires.



The wires AC and DE go across the hemisphere and intersect at B, the highest point of the hemisphere. The wires AJ, EF, CG and DH run down the sides of the cylinder. The wires GJ and FH cross at right angles at K where K is the centre of the base. The total length of wire is 30 cm. The height of the cylinder is h cm and the radius of the hemisphere is r cm. The volume of the container is V cm^3.

(a) Express h in terms of r. [2]

(b) Show that $V = \frac{\pi r^2}{6}(45 - 2r - 3\pi r)$. [3]

(c) Find the stationary value of V and determine its nature. [5]

8. (a) Show that the equation $2^{2x} = \frac{1}{2}[3(2^x) + 2]$ is satisfied by only one value of x. [3]

(b) Given that $m = a^s$, $n = a^t$ and $m^t n^s = a^{\frac{2}{u}}$, where $a > 0$ and $a \neq 1$, show that $st = \frac{1}{u}$. [4]

(c) Without using calculators, find the value of k, in the form $\frac{x + y\sqrt{5}}{2}$, such that $k\sqrt{3} - k\sqrt{15} = -2\sqrt{3}$. [4]

(d) Differentiate $e^{-x}\sqrt{1+3x}$ with respect to x. [4]

9. In the Chingay Parade procession held at the heartlands early this year, the Pioneer Generation Float was travelling on a straight road with a velocity, v ms^{-1}, given by the equation $v = 5t - \frac{1}{2}t^2 + 4$, where t is the time after passing a fixed point A.

(a) Show that the maximum velocity is reached 5 s later. [3]

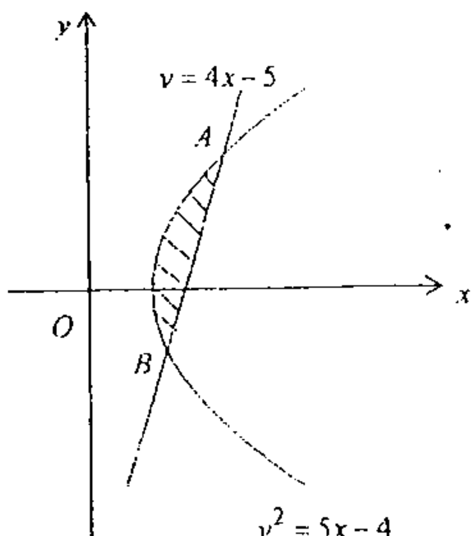
(b) Sketch the velocity-time graph for the first 5 s. [3]

Upon reaching its maximum velocity, the float started to decelerate uniformly at 1.5 ms^{-2}, before coming to a rest at point B to allow residents to take photographs.

(c) Find the time when the float reached B. [2]

(d) Find the total distance travelled from A to B. [3]

10.



The graph above shows part of curve $y^2 = 5x - 4$ and the line $y = 4x - 5$. Find

(a) the coordinates of A and of B, and [3]

(b) the area of the shaded region. [6]

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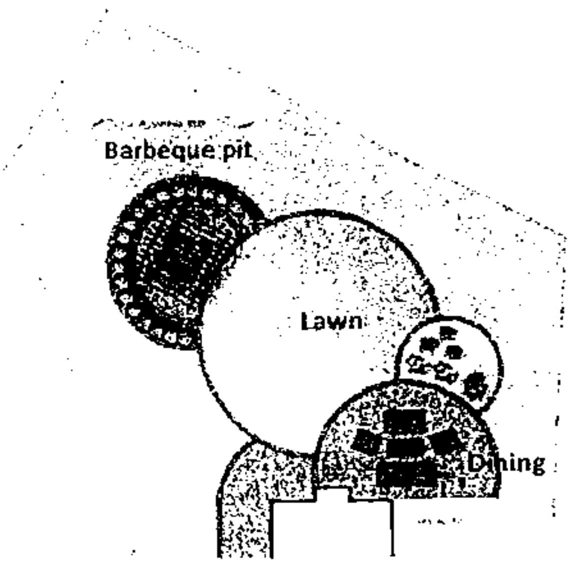
11. A landscaping company has been tasked to design the backyard for a client. The design is made up of overlapping circles as shown below. The circular lawn in the centre will be the focus point of the design and a barbeque pit will be constructed on one side of the lawn.



On the Cartesian plane, the circular lawn can be modelled by the equation of a circle, $x^2 + y^2 + 2x - 6y - 15 = 0$.

(a) Show why this model suggests that the radius of the lawn is 5 m. [2]

(b) A lamp post is positioned at a point $P(-5, 8)$ in the pit area. Determine, with working, if P lies inside or outside the lawn. [3]

(c) Two dustbins, at Q and R, will be placed on the circumference of the lawn such that Q is $(-4, -1)$ and QR is the diameter of the lawn. Find the equation of the tangent to the lawn at R. [6]

END OF PAPER

P2

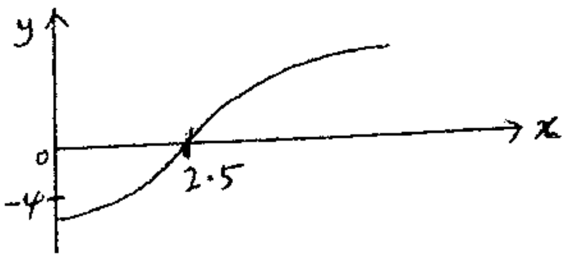
1. P (2 , 5) , Q (6 , 3) , PQ = $2\sqrt{5}$

2. a = 7, b = -5, no real roots

4. (b) $\sqrt{29}\cos(\theta - 21.8^{\circ})$ (c) 21.8°

5. (a) $\sec^2\alpha - \frac{1}{2}\cos 2\beta - \frac{3}{2}$
   (b) -2.75

6. (a) P = -6 , Q = $\frac{1}{2}$, R = 2 (b) $x = 199.2^{\circ}$
   (c)



7. (a) h = $\frac{15-2r-\pi r}{2}$
   (c) v = 54.2 max. value

8. (c) k = $\frac{1+\sqrt{5}}{2}$ (d) $\frac{1-6x}{2e^{x}\sqrt{1+3x}}$

9. (b)

   (c) t = 16
   (d) $152\frac{5}{12}$m

10. (a) A (1.8125 , 2.25) , B (1 , -1)
    (b) 1.14 units²

11. (a) $(x+1)^2 + (y-3)^2 = 5^2$
    (b) Distance = 6.40 m > radius, the lamp post lies outside the lawn
    (c) $y = -\frac{3}{4}x + \frac{17}{2}$

Name:		Index Number:		Class:	

CATHOLIC HIGH SCHOOL
Preliminary Examination 3
Secondary 4

ADDITIONAL MATHEMATICS

4047/1

15 September 2015

2 hour

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
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Answer **All** questions.

Attempt Question 1 to 8 in **Answer Booklet 1A**
Question 9 to 13 in **Answer Booklet 1B**.

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The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

This document consists of **5** printed pages, including this cover page.

43

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \tfrac{1}{2}ab\sin C$$

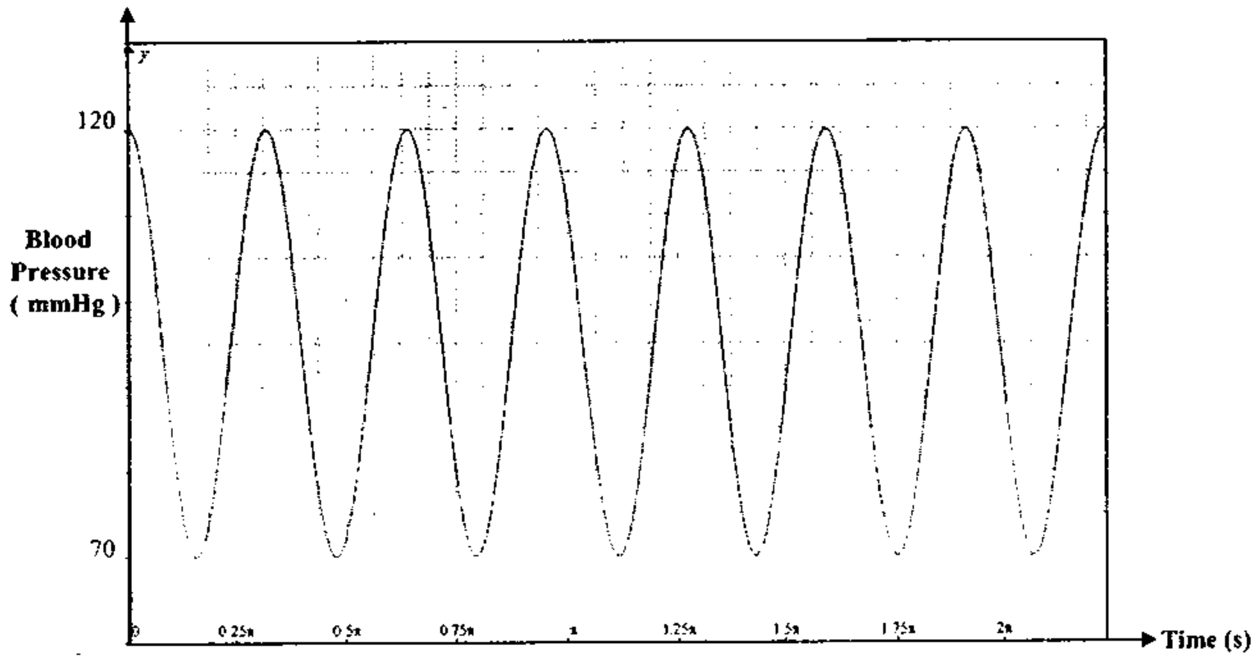
1 Given that $2^{2x+2} \times 5^{x-1} = 8^x \times 5^{2x}$, evaluate 10^x **without using a calculator.** [3]

2 Express $\dfrac{4x+7}{x^2+6x+9}$ in partial fractions. [4]

3 Given that θ is acute and that $\sin\theta = \dfrac{1}{\sqrt{3}}$, express, **without using a calculator,**

$\dfrac{1}{\cos\theta - \sin\theta}$ in the form $\sqrt{a} + \sqrt{b}$ where a and b are integers. [5]

4



The diagram shows a part of the curve of a person's blood pressure, which is modelled using

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Given that the person's heartbeat is 60 beats per minute,

(a) Write down the amplitude of y. [1]

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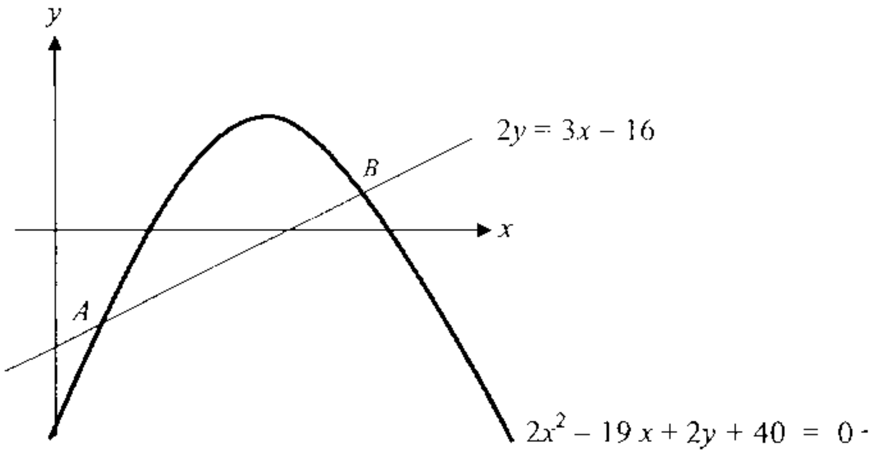
(c) Write down the value of

(i) a,

(ii) b

(iii) c. 44 [3]

5



The straight line $2y = 3x - 16$ intersects the curve $2x^2 - 19x + 2y + 40 = 0$ at the points A and B. Given that A lies below the x-axis and that the point P lies on AB such that $AP : PB = 3 : 1$, find the co-ordinates of P. [6]

6 A curve has the equation $y = \sin x - 3\cos 2x$.

(i) Find the gradient of the curve when $x = \frac{\pi}{6}$. [4]

(ii) Given that x is decreasing at a constant rate of $2\sqrt{3}$ units per second, find the rate of change of y when $x = \frac{\pi}{6}$. [2]

7 **(i)** Given that $y = x^2\sqrt{2x-1}$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x(5x-2)}{\sqrt{2x-1}}$. [2]

(ii) Hence evaluate $\int_1^5 \frac{5x^2 - 2x + 1}{\sqrt{2x-1}}\, \mathrm{d}x$. [4]

8 Find the coordinates of the stationary point on the curve $y = 2x^3 - 6x^2 + 6x - 11$ and determine the nature of the stationary point. [7]

9 **(a)** Show that the roots of the equation $6x^2 + 5(m-1) = 3(x+m)$ are real if $m < 2\frac{11}{16}$. [3]

(b) Find the range of values of k for which $(k+3)x^2 + 4x + k$ is always negative for all real values of x. [4]

10 A particle P moves in a straight line so that t seconds after leaving a fixed point O, its velocity v ms^{-1} is given by $v = (2t-3)^2 - 9$.

(a) Sketch the v-t graph of the particle P for $0 \le t \le 5$. [2]

(b) Hence or otherwise,

(i) find the range of values of t for which the acceleration of P is less than 4 m/s^2. [2]

(ii) find the distance travelled by P in the first 5 seconds. [3]

11 In the expansion of $\left(x^2 - \dfrac{k}{2x}\right)^6$, where k is a positive constant, the term independent of x is 15.

(i) Show that $k = 2$. [4]

(ii) With this value of k, find the coefficient of x^4 in the expansion of $\left(x^2 - \dfrac{k}{2x}\right)^6 (8x+1)$. [3]

12 A circle, C, has equation $x^2 + y^2 - 10x + 6y + 9 = 0$.

(i) Find the coordinates of the centre and radius of C. [3]

(ii) Give a reason why the y-axis is a tangent to C. [1]

The circle C crosses the x-axis at the point $P(1\ .\ 0)$.

(iii) Show that the equation of the tangent to the circle C at P is $3y - 4x = -4$. [3]

(iv) Find the coordinates of the point where the circle C crosses the x-axis again. [1]

13 In a Science experiment, a container of liquid was heated to a temperature of K °C. It was then left to cool in a chiller such that its temperature, T °C, t minutes after removing the heat, is given by $T = Ke^{-qt}$, where q is a constant.

Measured values of t and T are given in the following table.

t (minutes)	2	4	7	10	12
T °C	72.8	60.2	45.2	34.0	28.1

(i) On graph paper, plot $\ln T$ against t and draw a straight line graph. [3]

(ii) Use the graph to estimate the value of K and of q. [4]

(iii) Estimate the temperature of the liquid 5 minutes after it was left to cool. [2]

~ End Of Paper ~ 45

Name:		Index Number:		Class:	

CATHOLIC HIGH SCHOOL
Preliminary Examination 3
Secondary 4

ADDITIONAL MATHEMATICS 4047/1

15 September 2015
2 hour

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1 Given that $2^{2x+2} \times 5^{x-1} = 8^x \times 5^{2x}$, evaluate 10^x **without using a calculator**. [3]

SOLUTION:

1	$2^{2x+2} \times 5^{x-1} = 8^x \times 5^{2x}$
	$4(2^{2x}) \times \dfrac{5^x}{5} = 2^{3x} \times 5^{2x}$
	$\dfrac{(2^{3x})(5^{2x})}{(2^{2x})(5^x)} = \dfrac{4}{5}$
	$(2^x)(5^x) = \dfrac{4}{5}$
	$10^x = \dfrac{4}{5}$

2 Express $\dfrac{4x+7}{x^2+6x+9}$ in partial fractions. [4]

SOLUTION:

2	$\dfrac{4x+7}{x^2+6x+9}$
	$\dfrac{4x+7}{(x+3)^2} = \dfrac{A}{x+3} + \dfrac{B}{(x+3)^2}$
	$4x+7 = A(x+3)+B$
	Let $x=-3$, $B=-5$ Let $x=0$, $7 = 3A-5$ $A=4$
	$\dfrac{4x+7}{x^2+6x+9} = \dfrac{4}{x+3} - \dfrac{5}{(x+3)^2}$

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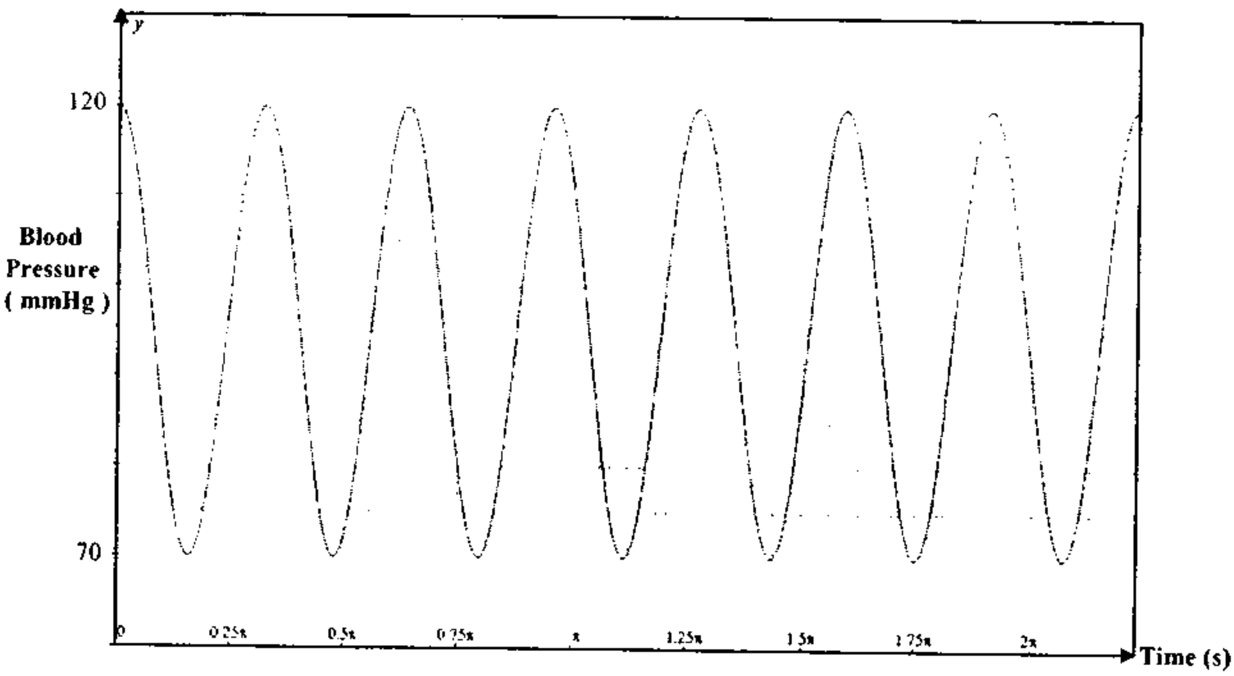
3 Given that θ is acute and that $\sin\theta = \frac{1}{\sqrt{3}}$, express, **without using a calculator,**

$\frac{1}{\cos\theta - \sin\theta}$ in the form $\sqrt{a} + \sqrt{b}$ where a and b are integers. [5]

SOLUTION:

3.	$1^2 + x^2 = \left(\sqrt{3}\right)^2$
	$x = \sqrt{2}$
	$\therefore \cos\theta = \frac{\sqrt{2}}{\sqrt{3}}$
	$\frac{1}{\cos\theta - \sin\theta} = \frac{1}{\frac{\sqrt{2}}{\sqrt{3}} - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{6}+\sqrt{3}}{2-1}$
	$= \sqrt{6} + \sqrt{3}$

4



The diagram shows a part of the curve of a person's blood pressure, which is modelled using

$y = a\cos bt + c$

where t is time in seconds and y is the blood pressure measured in mm (of mercury).

The length of the same person's heartbeat is the time between two consecutive peaks on the curve.

Given that the person's heartbeat is 60 beats per minute,

(a) Write down the amplitude of y. [1]

(b) Explain why the period of the function is 1 second. [1]

(c) Write down the value of

(i) a,

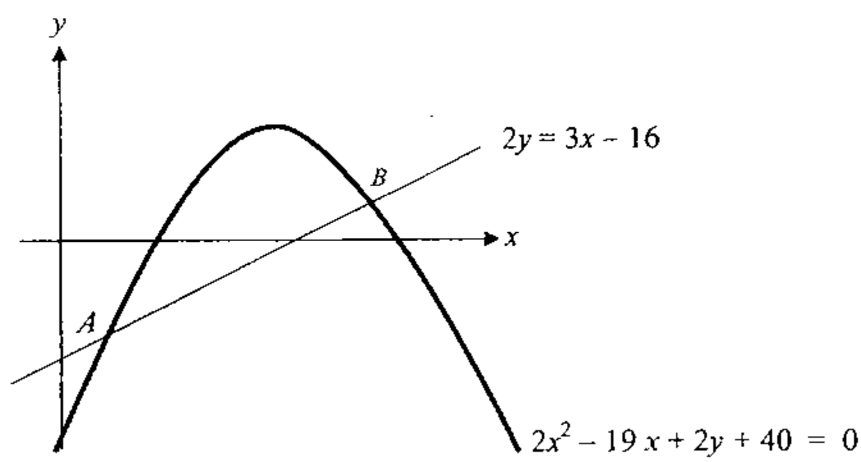
(ii) b

(iii) c. [3]

SOLUTION

4	$y = a\cos bt + c$
(a)	amplitude of y = 25
(b)	60 beats/cycles per 60 seconds. Therefore 1 cycle takes 1 second
(c)	$a = 25$ $b = \frac{2\pi}{1} = 2\pi$ $c = 95$

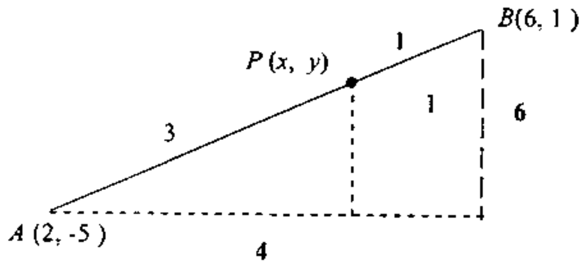
5



The straight line $2y = 3x - 16$ intersects the curve $2x^2 - 19x + 2y + 40 = 0$ at the points A and B. Given that A lies below the x-axis and that the point P lies on AB such that $AP : PB = 3 : 1$, find the co-ordinates of P. [6]

48

SOLUTION

5	$2y = 3x - 16$ into $2x^2 - 19x + 2y + 40 = 0$
	$2x^2 - 19x + 3x - 16 + 40 = 0$ $2x^2 - 16x + 24 = 0$ $(x-2)(x-6) = 0$ $x = 2, y = -5 \quad A(2,-5)$ $x = 6, y = 1 \quad B(6,1)$
	 $\therefore P\left(2 + \frac{3}{4} of\, 4, -5 + \frac{3}{4} of\, 6\right)$ $\therefore P\left(2+3, -5+4\frac{1}{2}\right) \Rightarrow P\left(5, -\frac{1}{2}\right)$

6 A curve has the equation $y = \sin x - 3\cos 2x$.

(i) Find the gradient of the curve when $x = \frac{\pi}{6}$. [4]

(ii) Given that x is decreasing at a constant rate of $2\sqrt{3}$ units per second, find the rate of change of y when $x = \frac{\pi}{6}$. [2]

SOLUTION

6	$y = \sin x - 3\cos 2x$
(i)	$\frac{dy}{dx} = \cos x + 6\sin 2x$ At $x = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{\sqrt{3}}{2} + 6\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{7\sqrt{3}}{2}$ *or* 6.06
(ii)	At $x = \frac{\pi}{6}$, $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $= \frac{7\sqrt{3}}{2} \times \left(-2\sqrt{3}\right)$ $= -21$ units /s **OR** y is decreasing at 21 units /s

7 **(i)** Given that $y = x^2\sqrt{2x-1}$, show that $\frac{dy}{dx} = \frac{x(5x-2)}{\sqrt{2x-1}}$. [2]

(ii) Hence evaluate $\int_1^5 \frac{5x^2-2x+1}{\sqrt{2x-1}}\,dx$. [4]

SOLUTION

7	$y = x^2\sqrt{2x-1}$
(i)	$\frac{dy}{dx} = \left(\sqrt{2x-1}\right)(2x) + x^2\left(\frac{1}{2}(2x-1)^{-\frac{1}{2}}(2)\right)$ $= (2x-1)^{-\frac{1}{2}}(x)\left[2(2x-1)+x\right]$ $= \frac{x(5x-2)}{\sqrt{2x-1}}$ (Shown)
(ii)	$\int \frac{x(5x-2)}{\sqrt{2x-1}}\,dx = x^2\sqrt{2x-1}$ $\int_1^5 \frac{5x^2-2x+1}{\sqrt{2x-1}}\,dx = \left[x^2\sqrt{2x-1}\right]_1^5 + \int_1^5 \frac{1}{\sqrt{2x-1}}\,dx$ $= \left[x^2\sqrt{2x-1}\right]_1^5 + \left[\frac{\left(\sqrt{2x-1}\right)(2)}{2}\right]_1^5$ $= \left[x^2\sqrt{2x-1}\right]_1^5 + \left[\sqrt{2x-1}\right]_1^5$ $= 74 + 2 = 76$

49

8 Find the coordinates of the stationary point on the curve $y = 2x^3 - 6x^2 + 6x - 11$ and determine the nature of the stationary point. [7]

SOLUTION

<table>
<tr><td>8</td><td>$y = 2x^3 - 6x^2 + 6x - 11$</td></tr>
<tr><td></td><td>$\frac{dy}{dx} = 6x^2 - 12x + 6$
At turning point,
$6x^2 - 12x + 6 = 0 \Rightarrow (x-1)^2 = 0$
$x = 1$
<table>
<tr><td>x</td><td>1^-</td><td>1</td><td>1^+</td></tr>
<tr><td>$\frac{dy}{dx}$</td><td>+ve</td><td>0</td><td>+ve</td></tr>
<tr><td>Shape</td><td>/</td><td>--</td><td>/</td></tr>
</table>
At $x = 1$, $y = -9$
$\therefore$ Point of inflexion at $(1, -9)$</td></tr>
</table>

9 **(a)** Show that the roots of the equation $6x^2 + 5(m-1) = 3(x+m)$ are real if $m < 2\frac{11}{16}$. [3]

(b) Find the range of values of k for which $(k+3)x^2 + 4x + k$ is always negative for all real values of x. [4]

SOLUTION

<table>
<tr><td>9</td><td>$6x^2 + 5(m-1) = 3(x+m)$</td></tr>
<tr><td>(a)</td><td>$6x^2 - 3x + 2m - 5 = 0$
$b^2 - 4ac = 9 - 4(6)(2m-5) = 129 - 48m$
For real roots, $129 - 48m > 0$
$48m < 129$
$m < 2\frac{11}{16}$.</td></tr>
<tr><td>(b)</td><td>For function to be negative, $b^2 - 4ac < 0$ and $(k+3) < 0$
$16 - 4k(k+3) < 0$
$-4k^2 - 12k + 16 < 0$
$-4(k+4)(k-1) < 0$
$k < -4$ or $k > 1$ (reject)</td></tr>
</table>

10 A particle P moves in a straight line so that t seconds after leaving a fixed point O, its velocity v ms^{-1} is given by $v=(2t-3)^2-9$.

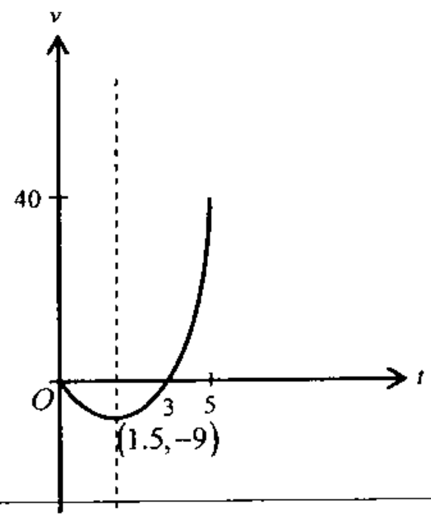
(a) Sketch the v-t graph of the particle P for $0 \le t \le 5$. [2]

(b) Hence or otherwise,

(i) find the range of values of t for which the acceleration of P is less than 4 m/s^2. [2]

(ii) find the distance travelled by P in the first 5 seconds. [3]

SOLUTION

10	$v=(2t-3)^2-9.$
(a)	
b(i)	$\frac{dv}{dt}=4(2t-3)$ $\frac{dv}{dt}<4 \quad \Rightarrow 4(2t-3)<4 \quad \Rightarrow t<2 \qquad \therefore 0 \le t < 2$
(ii)	Distance traveled $=\int_0^3 (2t-3)^2-9\,dt+\int_3^5 (2t-3)^2-9\,dt$ $=\left\lvert\left[\frac{(2t-3)^3}{6}-9t\right]_0^3\right\rvert+\left\lvert\left[\frac{(2t-3)^3}{6}-9t\right]_3^5\right\rvert$ $=18+16\frac{2}{3}+18$ $=52\frac{2}{3}$ m

50

11 In the expansion of $\left(x^2 - \frac{k}{2x}\right)^6$, where k is a positive constant, the term independent of x is 15.

(i) Show that $k = 2$. [4]

(ii) With this value of k, find the coefficient of x^4 in the expansion of

$\left(x^2 - \frac{k}{2x}\right)^6 (8x+1)$. [3]

<u>**SOLUTION**</u>

7	$\left(x^2 - \frac{k}{2x}\right)^6$
(i)	${}^6C_4(x^2)^2\left(-\frac{k}{2x}\right)^4 = 15$ $\frac{k^4}{2^4} \times 15 = 15$ $k^4 = 2^4$ $k = 2$ OR General Term $= {}^6C_r(x^2)^{6-r}\left(-\frac{k}{2x}\right)^r = {}^6C_r\left(-\frac{k}{2}\right)^r (x^2)^{6-r}(x)^{-r}$ Independent of x : $(x^2)^{6-r}(x)^{-r} = x^0$ Therefore $12 - 3r = 0$, $r = 4$ Term = ${}^6C_4\left(-\frac{k}{2}\right)^4 = 15$ $15 \times \left(-\frac{k}{2}\right)^4 = 15$ $k^4 = 2^4$ $k = 2$
(ii)	$(\ldots - 20x^3 + \ldots)(8x+1)$ x^4 term $= -160x^3$ $\therefore$ Coefficient of $x^4 = -160$

12 A circle, C, has equation $x^2+y^2-10x+6y+9=0$.

(i) Find the coordinates of the centre and radius of C. [3]

(ii) Give a reason why the y-axis is a tangent to C. [1]

The circle C crosses the x-axis at the point $P(1\ ,0)$.

(iii) Show that the equation of the tangent to the circle C at P is $3y-4x=-4$. [3]

(iv) Find the coordinates of the point where the circle C crosses the x-axis again. [1]

SOLUTION

12	$x^2+y^2-10x+6y+9=0$
(i)	$x^2-10x+5^2-5^2+y^2+6y+9=0$ $(x-5)^2+(y+3)^2=25$ So, centre is $(5\ ,-3)$ and radius is 5
(ii)	Since radius is 5, leftmost x-coordinate of circle C is $5-5=0$ Hence, the y-axis is a tangent to C.
(iii)	$\text{grad}_{P.centre}=\frac{0+3}{1-5}=-\frac{3}{4}$ Equation of tangent is $y=\frac{4}{3}x+c$ At $P(1\ ,0)$, $0=\frac{4}{3}+c$ $c=-\frac{4}{3}$ $y=\frac{4}{3}x-\frac{4}{3}$ *or* $3y-4x=-4$
(iv)	$x^2-10x+5^2-5^2+y^2+6y+9=0$ sub $y=0$, $x^2-10x+9=0$ $(x-1)(x-9)=0$ Coordinates $=(9,\ \ 0)$

51

13 In a Science experiment, a container of liquid was heated to a temperature of K °C.

It was then left to cool in a chiller such that its temperature, T °C, t minutes after removing the heat, is given by $T = Ke^{-qt}$, where q is a constant.

Measured values of t and T are given in the following table.

t (minutes)	2	4	7	10	12
T °C	72.8	60.2	45.2	34.0	28.1

(i) On graph paper, plot $\ln T$ against t and draw a straight line graph. [3]

(ii) Use the graph to estimate the value of K and of q. [4]

(iii) Estimate the temperature of the liquid 5 minutes after it was left to cool. [2]

SOLUTION

13	$T = Ke^{-qt}$
(i)	Labelling of axes of graph correct plots straight line almost passing all points
(ii)	$T = Ke^{-qt}$ $\ln T = -qt + \ln K$
	$-q = \dfrac{4.2-3.72}{3-8} = -0.096$ $q = 0.096$
	$\ln K = 4.48$ $K = e^{4.48} \approx 88.2$
(iii)	$T = 88.2e^{-0.096(5)}$ ≈ 54.6°C Alternatively from graph, $t = 5, \ln T = 4$ $T = e^4 \approx 54.6$°C

CATHOLIC HIGH SCHOOL, SINGAPORE

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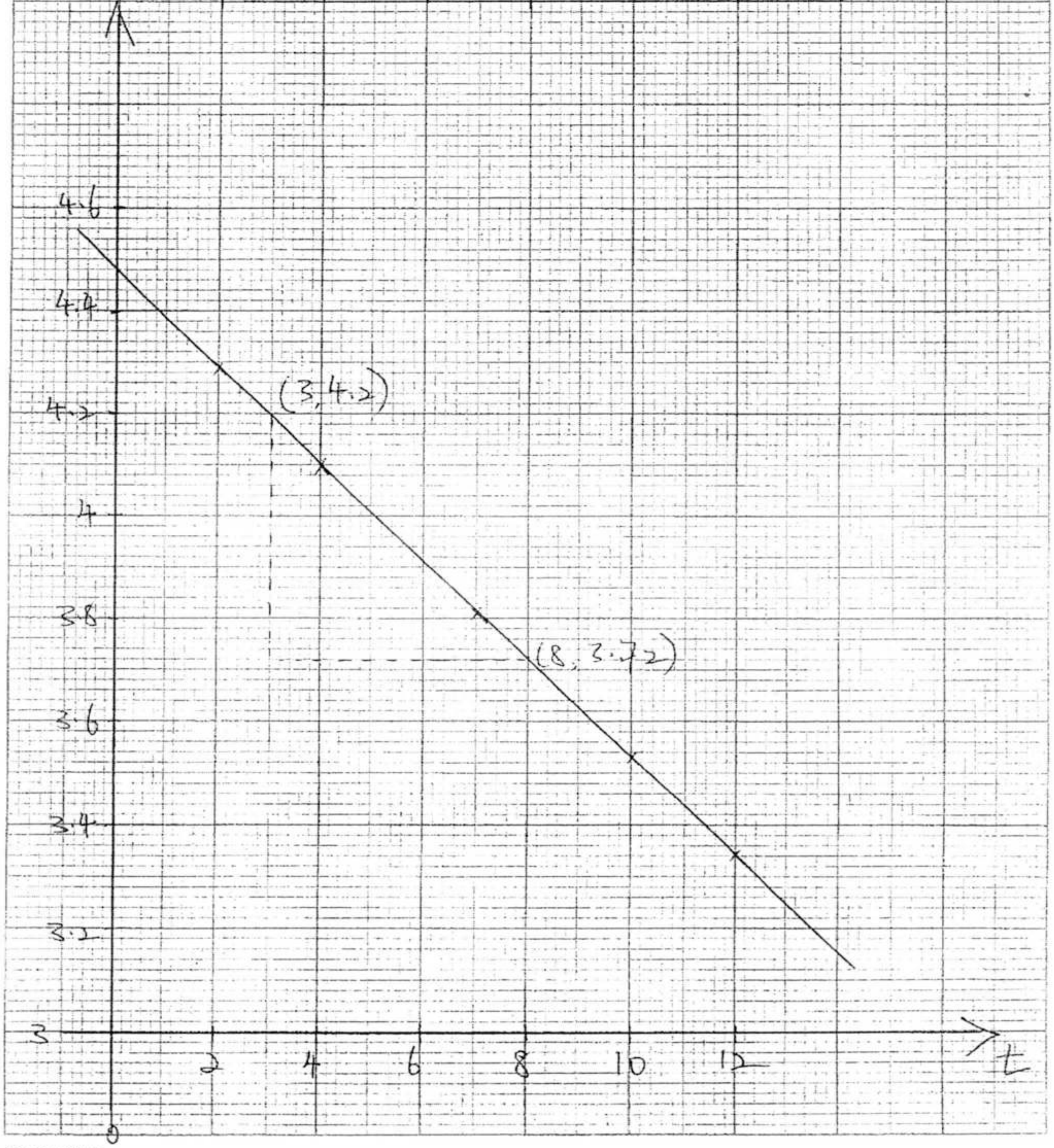
$\ln T$	4.29	4.10	3.81	3.53	3.34
t	2	4	7	10	12



20 cm x 24 cm

52

Name:		Index Number:		Class:	

CATHOLIC HIGH SCHOOL
Preliminary Examination 3
Secondary 4

ADDITIONAL MATHEMATICS **4047/2**

16 September 2015
2 hour 30 min

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **All** questions.

Attempt Question 1 - 4 in **Answer Booklet 2A,**
Question 5 - 8 in **Answer Booklet 2B,**
Question 9 - 12 in **Answer Booklet 2C.**

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **100**.

53

This document consists of 7 printed pages, including this cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

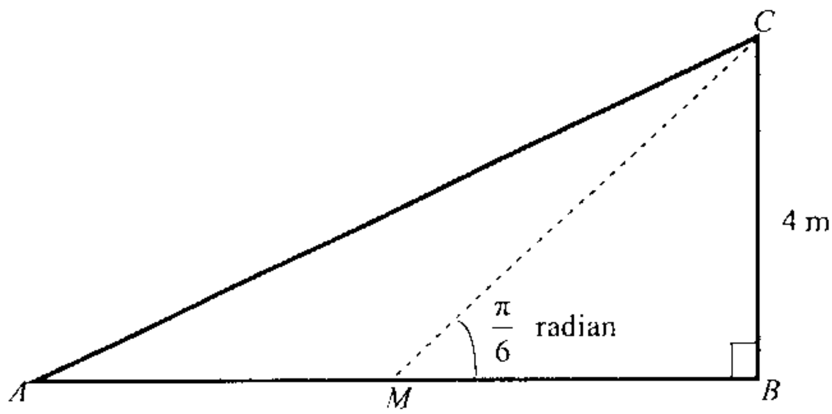
$$\Delta = \tfrac{1}{2} ab \sin C$$

1 The roots of the quadratic equation $4x^2 - 33x + 16 = 0$ are α^2 and β^2. Find the quadratic equation whose roots are α and β, given that $\alpha + \beta > 0$ and $\alpha\beta > 0$. [6]

2 **(a)** Solve the equation $\sin^2 y + 2\cos 2y = 2\cos y$ for $0° \le y \le 360°$. [3]

(b) Prove that $\dfrac{\cos(A+B) + \cos(A-B)}{\sin(A+B) - \sin(A-B)} = \cot B$. [4]

3



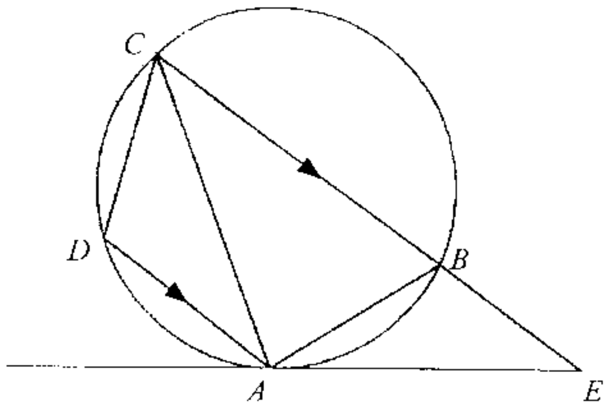
The diagram shows a triangle ABC in which angle CMB is $\dfrac{\pi}{6}$ radians, angle B is a right angle, M is the mid-point of AB and the length of CB is 4 m.

Without using a calculator, find the value of the integer k such that

$\angle ACM = \sin^{-1}\left(\dfrac{\sqrt{k}}{26}\right)$. [6]

54 **[TURN OVER**

4



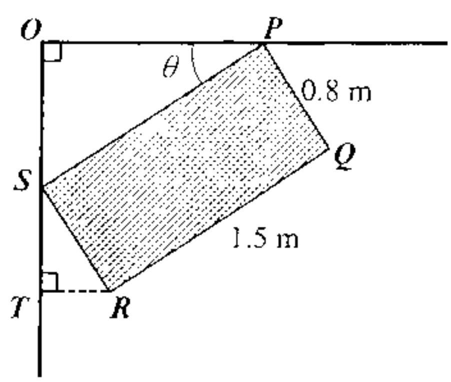
The diagram shows a quadrilateral $ABCD$ whose vertices lie on the circumference of the circle. The point E lies on the extended line CB such that AE is a tangent to the circle. CE and AD are parallel lines.

(i) Explain why angle BAE = angle CAD. [2]

(ii) Show that triangles BAE and DAC are similar. [2]

(iii) Given that $AB = BE$, explain why the line AC bisects the angle BCD. [2]

5



The diagram shows the plan of a rectangular desk, $PQRS$, in a corner of a room. Given that the desk has length 1.5 m and width 0.8 m, and that $\angle POS = \angle STR = 90^\circ$ and $\angle OPS = \theta$.

(i) Show the length of OT, L can be expressed as $L = 1.5\sin\theta + 0.8\cos\theta$. [3]

(ii) Express L in the form $R\sin(\theta + \alpha)$ where $0^\circ < \alpha < 90^\circ$ and $R > 0$. [3]

Hence, find the value of θ for which

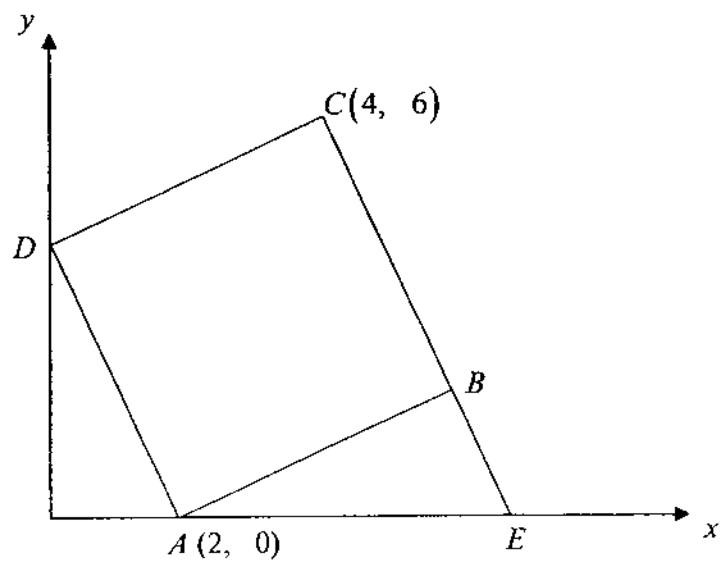
(iii) L has a maximum length, [2]

(iv) $L = 1.2$ m. [2]

6 (a) Simplify $\dfrac{16^{x+1} + 48(4^{2x})}{2^{x+3} \times 8^{x+2}}$. [4]

(b) Solve the equation $5^{x+1} = 8 + 4\left(5^{-x}\right)$. [5]

7



The diagram shows a rhombus $ABCD$ with vertices A and C at the points (2 , 0) and (4 , 6) respectively. D lies on the y-axis and the line CB produced intersects the x-axis at E.

(i) Show that the y-coordinate of D is 4. [3]

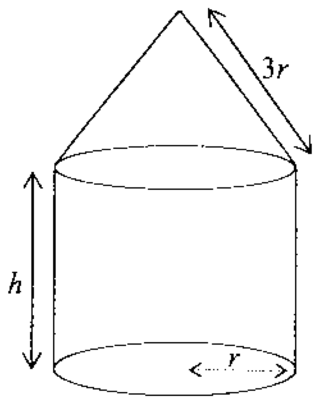
(ii) Explain why the rhombus $ABCD$ is also a square. [2]

(iii) Find the coordinates of E. [2]

(iv) Calculate the area of the quadrilateral $AECD$. [2]

55 [TURN OVER

8



The diagram shows a solid body which consists of a cone fixed to the top of a right circular cylinder of radius r cm and height h cm. The slant edge of the cone is $3r$ cm.

(i) Given that the volume of the cylinder is 108π cm^3, express h in terms of r. [1]

(ii) Show that the total surface area, A cm^2 of the solid is given by $A = 4\pi\left(\frac{54}{r} + r^2\right)$. [3]

(iii) Given that r and h can vary,

(a) find the value of r for which A has a stationary value, [3]

(b) determine whether this stationary value is a maximum or minimum. [2]

9 **(i)** Find the range of values of m for which the curve $y = (x-1)(x-4)$ and the line $y = mx + 3$ do not intersect. [3]

(ii) Sketch the graph of $y = |(x-1)(x-4)|$, showing the coordinates of the turning point and the points where the curve meets the x-axis. [3]

(iii) Find the number of solutions of the equation $|(x-1)(x-4)| = -x + 1$. [2]

10 **(a)** **Without using a calculator**, show that $\frac{\log_2 5 \times \log_5 4}{\log_{25} 5} = 4$. [3]

(b) Given that $y = \ln\sqrt{\frac{2x}{x+4}}$, $x > 0$ and $x < -4$,

(i) find $\frac{dy}{dx}$. [4]

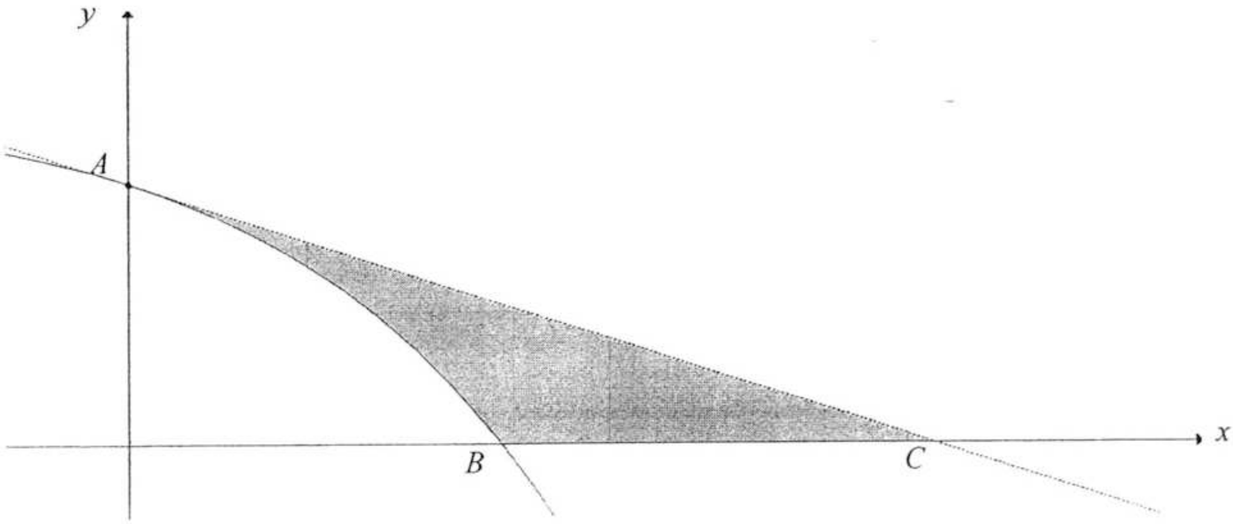
(ii) Hence show that y has no stationary value. [2]

11 The polynomial $P(x) = 2x^3 + ax^2 + bx + 8$, where a and b are constants, leaves a remainder of 10 when divided by $2x - 1$. Given that $x + 2$ is a factor of $P(x)$,

(i) find the value of a and of b. [5]

(ii) Explain why the equation $P(x) = 0$ has only 1 real root. Hence find this root. [4]

12 The diagram shows part of the curve $y = 4 - e^{\frac{1}{2}x}$ which cuts the axes at A and at B.



(i) Find the coordinates of A and of B. [4]

The tangent to the curve at A meets the x-axis at C.

(ii) Find the coordinates of C. [4]

(iii) Find the area of the shaded region. [4]

~ End of Paper ~

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Name:		Index Number:		Class:	

CATHOLIC HIGH SCHOOL
Preliminary Examination 3
Secondary 4

ADDITIONAL MATHEMATICS 4047/2

16 September 2015
2 hour 30 min

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57

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SOLUTION

1	$4x^2 - 33x + 16 = 0$
	$\alpha^2\beta^2 = 4$ $\alpha\beta = 2$ or -2(reject) $\alpha^2 + \beta^2 = \frac{33}{4}$ $(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$ $= \frac{33}{4} + 4 = \frac{49}{4}$ $\alpha + \beta = \frac{7}{2}$ or $-\frac{7}{2}$(reject bec $\alpha+\beta>0$) $\therefore$ Equation: $x^2 - \frac{7}{2}x + 2 = 0 \Rightarrow 2x^2 - 7x + 4 = 0$

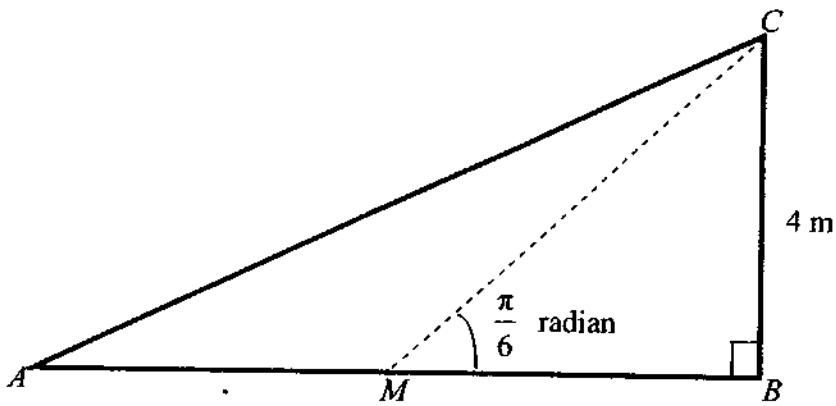
2 **(a)** Solve the equation $\sin^2 y + 2\cos 2y = 2\cos y$ for $0^\circ \le y \le 360^\circ$. [3]

(b) Prove that $\frac{\cos(A+B) + \cos(A-B)}{\sin(A+B) - \sin(A-B)} = \cot B$. [4]

SOLUTION:

2	$\sin^2 y + 2\cos 2y = 2\cos y$
(a)	$1 - \cos^2 y + 4\cos^2 y - 2 - 2\cos y = 0$ $3\cos^2 y - 2\cos y - 1 = 0$ $(3\cos y + 1)(\cos y - 1) = 0$
	$\cos y = -\frac{1}{3}$ or $\cos y = 1$ Basic Angle $= 70.53^\circ$ $y = 0^\circ, 360^\circ$ $y = 109.5^\circ, 250.5^\circ$
(b)	LHS $= \frac{\cos(A+B) + \cos(A-B)}{\sin(A+B) - \sin(A-B)}$ $= \frac{\cos A\cos B + SinA\sin B + \cos A\cos B - \sin A\sin B}{\sin A\cos B + \cos A\sin B - \sin A\cos B + \cos A\sin B}$
	$= \frac{2\cos A\cos B}{2\cos A\sin B}$
	$= \frac{\cos B}{\sin B}$
	$= \cot B$

3



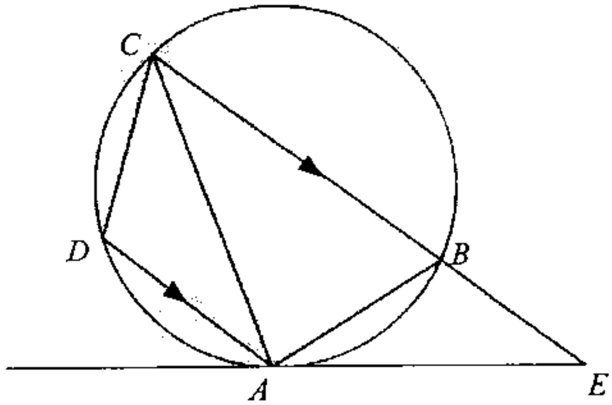
The diagram shows a triangle ABC in which angle CMB is $\frac{\pi}{6}$ radians, angle B is a right angle, M is the mid-point of AB and the length of CB is 4 m.

Without using a calculator, find the value of the integer k such that $\angle ACM = \sin^{-1}\left(\frac{\sqrt{k}}{26}\right)$. [6]

SOLUTION

3	$\tan\frac{\pi}{6} = \frac{4}{MB}$ $AM = MB = \frac{4}{\tan\frac{\pi}{6}} = 4\sqrt{3}$
	$AC = \sqrt{\left(8\sqrt{3}\right)^2 + 4^2} = 4\sqrt{13}$
	$\frac{\sin\angle ACM}{4\sqrt{3}} = \frac{\sin\frac{5\pi}{6}}{4\sqrt{13}}$ $\sin\angle ACM = \frac{\frac{1}{2}}{4\sqrt{13}} \times 4\sqrt{3} = \frac{\sqrt{3}}{2\sqrt{13}}$ $= \frac{\sqrt{3}}{2\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}}$ $\angle ACM = \sin^{-1}\left(\frac{\sqrt{39}}{26}\right)$
	Therefore $k = 39$

4



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(iii) Given that $AB = BE$, explain why the line AC bisects the angle BCD. [2]

SOLUTION:

4	
(i)	$\angle BAE = \angle ACB$ (tangent chord theorem) $= \angle CAD$ (alternate angles)
(ii)	In triangles BAE and DAC, $\angle BAE = \angle CAD$ (part (i)) $\angle CDA = 180° - \angle ABC$ (opposite angles of cyclic quadrilateral) $= \angle ABE$ (angles on straight line) $\angle ACD = \angle AEB$ (angle sum of triangle) Hence, triangles BAE and DAC are similar.
	$AB = BE$, implying that triangles BAE and DAC are similar isosceles triangles. So, $\angle ACD = \angle CAD$ $= \angle BCA$ (alternate angles) Hence, the line AC bisects the angle BCD.

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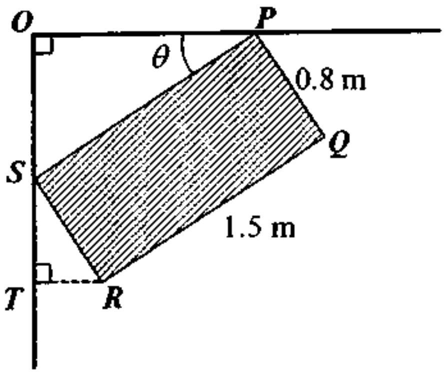
5 (a) Simplify $\dfrac{16^{x+1} + 48(4^{2x})}{2^{x+3} \times 8^{x+2}}$. [4]

(b) Solve the equation $5^{x+1} = 8 + 4\left(5^{-x}\right)$. [5]

SOLUTION:

5 (a)	$\dfrac{16^{x+1} + 48(4^{2x})}{2^{x+3} \times 8^{x+2}}$
	$= \dfrac{2^{4(x+1)} + 48(2^{4x})}{2^{x+3} \times 2^{3(x+2)}}$ $= \dfrac{2^{4x+4} + 48(2^{4x})}{2^{4x+9}}$
	$= \dfrac{2^{4x}(2^4 + 48)}{2^{4x}(2^9)}$
	$= \dfrac{2^6}{2^9} \quad = \dfrac{1}{2^3}$
	$= \dfrac{1}{8}$
(b)	$5^{x+1} = 8 + 4\left(5^{-x}\right)$
	$5\left(5^x\right) = 8 + 4\left(5^x\right)^{-1}$
	Let $u = 5^x$ $5u = 8 + \dfrac{4}{u}$ $5u^2 = 8u + 4$
	$(5u+2)(u-2) = 0$ $u = -\dfrac{2}{5}$ (rejected) or $u = 2$
	$5^x = 2$ $\therefore x = 0.4306 \approx 0.431$ (3 s.f.)

6



The diagram shows the plan of a rectangular desk, *PQRS*, in a corner of a room.

Given that the desk has length 1.5 m and width 0.8 m, and that $\angle POS = \angle STR = 90^\circ$ and $\angle OPS = \theta$.

(i) Show the length of OT, L can be expressed as $L = 1.5\sin\theta + 0.8\cos\theta$. [3]

(ii) Express L in the form $R\sin(\theta+\alpha)$ where $0^\circ < \alpha < 90^\circ$ and $R > 0$. [3]

Hence, find the value of θ for which

(iii) L has a maximum length, [2]

(iv) $L = 1.2$ m. [2]

SOLUTION:

6	
(i)	$\angle TSR = \theta$, $\cos\theta = \dfrac{ST}{0.8} \Rightarrow ST = 0.8\cos\theta$ $\sin\theta = \dfrac{OS}{1.5} \Rightarrow OS = 1.5\sin\theta$ $OT = OS + ST$ $L = 1.5\sin\theta + 0.8\cos\theta$
(ii)	$L = 1.5\sin\theta + 0.8\cos\theta = R\sin(\theta+\alpha)$ where $R = \sqrt{1.5^2 + 0.8^2} = 1.7$ $\tan\alpha = \dfrac{0.8}{1.5}, \Rightarrow \alpha = 28.07^\circ$ $\therefore\ L = 1.7\sin(\theta + 28.07^\circ)$
(iii)	L has maximum length when $\sin(\theta + 28.07) = 1$ $\theta + 28.07^\circ = 90^\circ$ $\theta = 61.9^\circ$ (1 dp)

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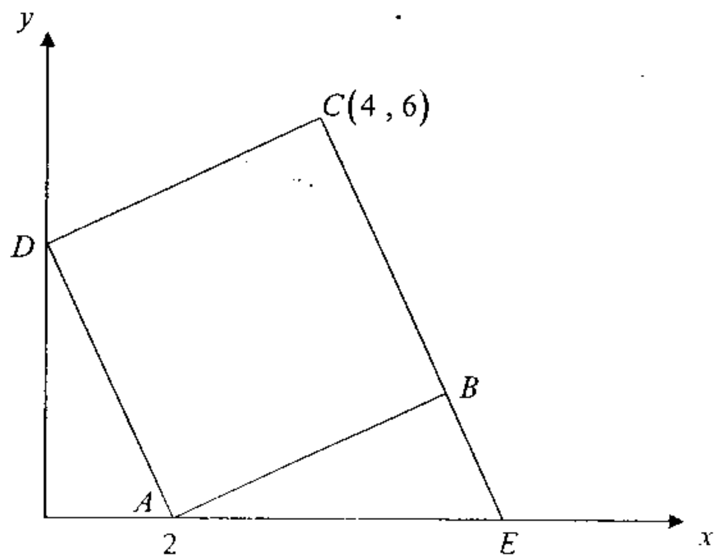
(iv)	$1.7\sin\left(\theta+28.07^\circ\right)=1.2$ $\sin\left(\theta+28.07^\circ\right)=\frac{1.2}{1.7}$ Basic Angle $=44.90^\circ$ $\theta+28.07^\circ=44.9^\circ$ $\theta=16.8^\circ\left(1\text{ dp}\right)$

7



The diagram shows a rhombus $ABCD$ with vertices A and C at the points (2 , 0) and (4 , 6) respectively. D lies on the y-axis and the line BC produced intersects the x-axis at E.

(i) Show that the y-coordinate of D is 4. [3]

(ii) Explain why the rhombus $ABCD$ is also a square. [2]

(iii) Find the coordinates of E. [2]

(iv) Calculate the area of the quadrilateral $AECD$. [2]

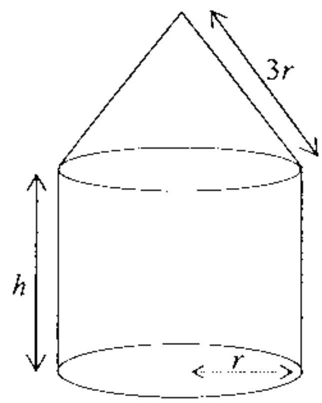
SOLUTION:

7	
(i)	Midpoint of $AC=\left(\frac{2+4}{2},\frac{0+6}{2}\right)$ $=\left(3,\ \ 3\right)$
	$m_{AC}=\frac{6-0}{4-2}=3$

	Equation of perpendicular bisector of AC is $y=-\frac{1}{3}x+c$
	At (3 , 3), $3=-\frac{1}{3}(3)+c$ $c=4$ $\therefore$ y-coordinate of D is 4.
(ii)	$\text{grad}_{AD}=\frac{0-4}{2-0}=-2$ $\text{grad}_{CD}=\frac{6-4}{4-0}=\frac{1}{2}$ $-2\times\frac{1}{2}=-1 \Rightarrow$ AD and CD are perpendicular, hence $ABCD$ is a square.
(iii)	Equation of BC is $y=-2x+c$ At (4 , 6), $6=-2(4)+c$ $c=14$ $\therefore y=-2x+14$ Along x-axis, $y=0$. $0=-2x+14$ $x=7$ $E(7\ ,0)$
(iv)	$\text{Area}=\frac{1}{2}\begin{vmatrix}2&7&4&0&2\\0&0&6&4&0\end{vmatrix}$ $=\frac{1}{2}[58-8]$ $=25$

6

8



The diagram shows a solid body which consists of a cone fixed to the top of a right circular cylinder of radius r cm and height h cm. The slant edge of the cone is $3r$ cm.

(i) Given that the volume of the cylinder is 108π cm^3, express h in terms of r. [1]

(ii) Show that the total surface area, A cm^2 of the solid is given by $A = 4\pi\left(\frac{54}{r} + r^2\right)$. [3]

(iii) Given that r and h can vary,

(a) find the value of r for which A has a stationary value, [3]

(b) determine whether this stationary value is a maximum or minimum. [2]

SOLUTION:

8	
(i)	$\pi r^2 h = 108\pi$ $h = \frac{108}{r^2}$
(ii)	Total surface area = area of cylinder + area of cone $= 2\pi rh + \pi r^2 + \pi rl$ $= 2\pi rh + \pi r^2 + 3\pi r^2$ $= 2\pi r\left(\frac{108}{r^2}\right) + 4\pi r^2$ $= 4\pi\left(\frac{54}{r} + r^2\right)$ (shown)
(iii)	$\frac{dA}{dr} = \frac{-216\pi}{r^2} + 8\pi r$

(a)	$\frac{-216\pi}{r^2}+8\pi r=0$ $\frac{216\pi}{r^2}=8\pi r$ $216=8r^3$ $r=3$
	Sub $r=3$ into $\frac{d^2A}{dr^2}$, $\frac{d^2A}{dr^2}=\frac{432\pi}{r^3}+8\pi$ $=\frac{432\pi}{(3)^3}+8\pi$ $=24\pi$ Since $\frac{d^2A}{dr^2}$ is positive, A is a minimum. (shown)

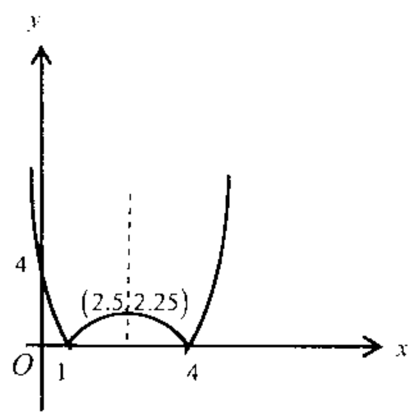
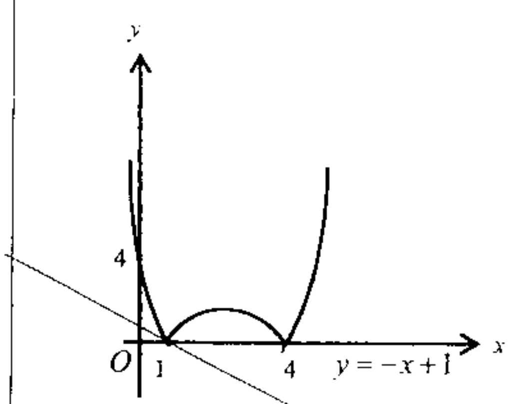
62

9 (i) Find the range of values of m for which the curve $y=(x-1)(x-4)$ and the line $y=mx+3$ do not intersect. [3]

(ii) Sketch the graph of $y=|(x-1)(x-4)|$, showing the coordinates of the turning point and the point where the curve meets the x-axis. [3]

(iii) Find the number of solutions of the equation $|(x-1)(x-4)|=-x+1$. [2]

SOLUTION:

<table>
<tr><td>9</td><td>$y=(x-1)(x-4)$</td></tr>
<tr><td>(i)</td><td>$(x-1)(x-4)=mx+3$
$x^2-5x+4-mx-3=0$
$x^2-(m+5)x+1=0$
$b^2-4ac<0$
$(m+5)^2-4<0$
$(m+7)(m+3)<0$
$\therefore\ \underline{-7<m<-3}$</td></tr>
<tr><td>(ii)</td><td>
</td></tr>
<tr><td>(iii)</td><td>
</td></tr>
<tr><td></td><td>(a) 1 solution</td></tr>
</table>

10 **(a)** **Without using a calculator**, show that $\frac{\log_2 5 \times \log_5 4}{\log_{25} 5} = 4$. [3]

(b) Given that $y = \ln\sqrt{\frac{2x}{x+4}}$, $x > 0$ and $x < -4$,

(i) find $\frac{dy}{dx}$. [4]

(ii) Hence show that y has no stationary value. [2]

SOLUTION:

10	$\frac{\log_2 5 \times \log_5 4}{\log_{25} 5}$
(a)	$\frac{\log_2 5 \times \frac{\log_2 4}{\log_2 5}}{\frac{\log_2 5}{\log_2 25}} = \log_2 4 \div \frac{\log_2 5}{\log_2 25}$
	$2\log_2 2 \div \frac{\log_2 5}{2\log_2 5}$
	$2 \div \frac{1}{2} = 2 \times 2 = 4$
(b) **(i)**	$y = \ln\sqrt{\frac{2x}{x+4}}$ $= \ln\left(\frac{2x}{x+4}\right)^{\frac{1}{2}} = \frac{1}{2}\ln\left(\frac{2x}{x+4}\right)$ $= \frac{1}{2}\left[\ln 2x - \ln(x+4)\right]$
	$\frac{dy}{dx} = \frac{1}{2}\left[\frac{2}{2x} - \frac{1}{x+4}\right]$
	$= \frac{1}{2}\left[\frac{(x+4)-x}{x(x+4)}\right] = \frac{2}{x(x+4)}$
(ii)	$\frac{2}{x(x+4)} \neq 0$ since $\frac{dy}{dx} \neq 0 \Rightarrow$ there is no stationary value

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11 The polynomial $P(x) = 2x^3 + ax^2 + bx + 8$, where a and b are constants, leaves a remainder of 10 when divided by $2x-1$. Given that $x+2$ is a factor of $P(x)$,

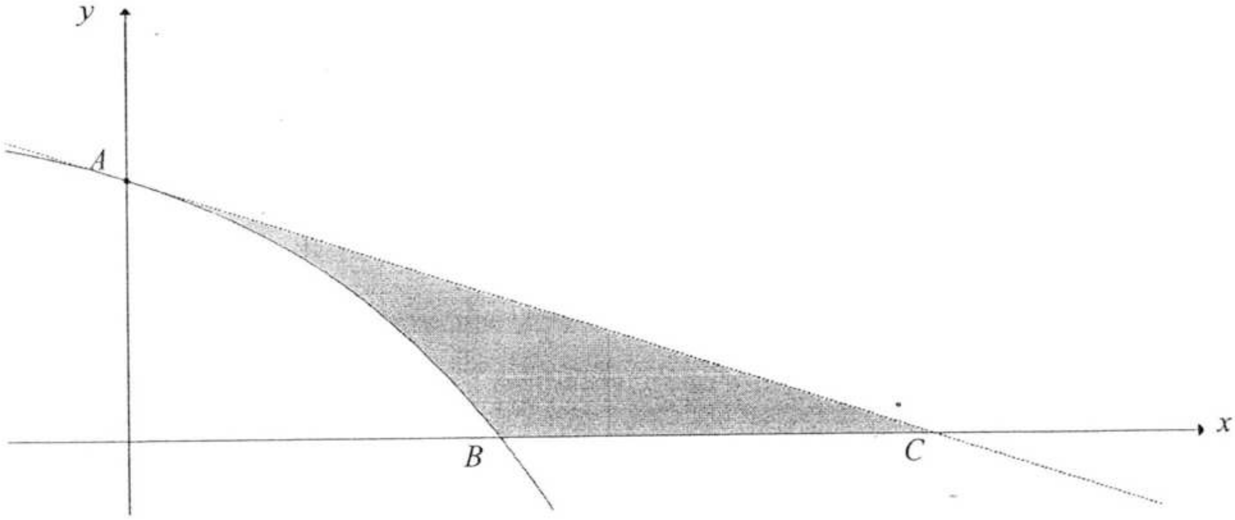
(i) find the value of a and of b. [5]

(ii) Explain why the equation $P(x) = 0$ has only 1 real root. Hence find this root. [4]

SOLUTION:

11	$P(x) = 2x^3 + ax^2 + bx + 8$
(i)	$P\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + b\left(\frac{1}{2}\right) + 8 = 10$ $\frac{1}{4}a + \frac{1}{2}b = \frac{7}{4}$ $a + 2b = 7$
	$P(-2) = 2(-2)^3 + a(-2)^2 + b(-2) + 8 = 0$ $4a - 2b = 8$ $2a - b = 4$
	$2(7-2b) - b = 4 \quad \Rightarrow \quad -5b = -10$
	$b = 2$, $a = 7 - 2(2) = 3$
(ii)	$P(x) = 2x^3 + 3x^2 + 2x + 8$ $= (x+2)(2x^2 + bx + 4)$ term in x^2 : $3x^2 = bx^2 + 4x^2, \quad b = -1$ $P(x) = 2x^3 + 3x^2 + 2x + 8$ $= (x+2)(2x^2 - x + 4)$
	for $2x^2 - x + 4$, $b^2 - 4ac = 1 - 4(2)(4)$ $= -31 < 0$
	Hence, the equation $2x^2 - x + 4 = 0$ has no roots. So $P(x) = 0$ has only 1 real root.
	The root is $x + 2 = 0$ ie $x = -2$

12 The diagram shows part of the curve $y = 4 - e^{\frac{1}{2}x}$ which cuts the axes at A and at B.



(i) Find the coordinates of A and of B. [4]

The tangent to the curve at A meets the x-axis at C.

(ii) Find the coordinates of C. [4]

(iii) Find the area of the shaded region [4]

<u>SOLUTION:</u>

12	$y = 4 - e^{\frac{1}{2}x}$
(i)	When $x = 0$, $y = 4 - e^{\frac{1}{2}(0)} = 3 \Rightarrow A(0, 3)$ When $y = 0$, $0 = 4 - e^{\frac{1}{2}x}$ $e^{\frac{1}{2}x} = 4$ $\frac{1}{2}x = \ln 4$ $x = 2\ln 4$ or $4\ln 2 \Rightarrow B(2\ln 4, 0)$ or $B(4\ln 2, 0)$
(ii)	$\frac{dy}{dx} = -\frac{1}{2}e^{\frac{1}{2}x}$ $= -\frac{1}{2}e^{\frac{1}{2}(0)}$ $= -\frac{1}{2}$

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	Equation of tangent: $y=-\frac{1}{2}x+3$
	When $y=0$, $0=-\frac{1}{2}x+3$ $x=6 \Rightarrow C(6,0)$
(iii)	Shaded area $=\frac{1}{2}\times 6\times 3-\int_0^{4\ln 2} 4-e^{\frac{1}{2}x}\,dx$ $=9-\left[4x-2e^{\frac{1}{2}x}\right]_0^{4\ln 2}$ $=9-\left[4(4\ln 2)-2e^{\frac{1}{2}(4\ln 2)}-(-2)\right]$ $=3.9096$ ≈ 3.91 units^2

~ **End of Paper** ~

Answer all questions.

1 Express $\dfrac{8x^4+1}{x(2x-1)}$ in partial fractions. [5]

2 (i) Prove that $\operatorname{cosec}(60^\circ-\theta)=\dfrac{2}{\cos\theta\,(\sqrt{3}-\tan\theta)}$. [3]

(ii) Hence find, in surd form, the value of $\operatorname{cosec}15^\circ$. [4]

3 (a) Find the term independent of x in the expansion of $(2x^2-\dfrac{1}{\sqrt{x}})^{10}$. [3]

(b) It is given that in the expansion of $(5+px)^n$, the coefficients of x^3 and x^4 are the same. Express n in terms of p. [4]

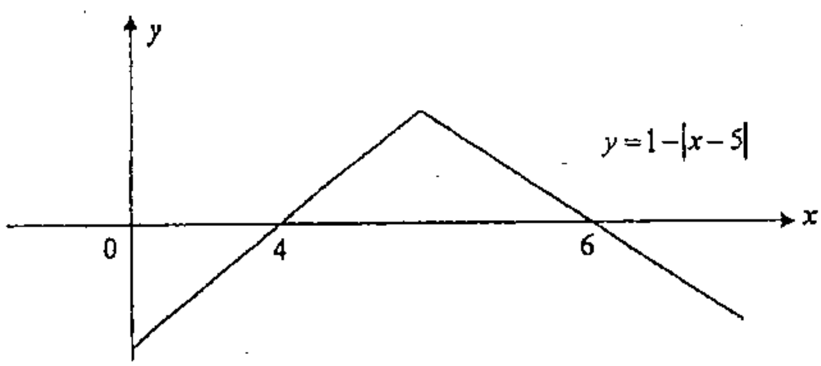
4 (a) The equation of a curve is $y=(a+2)x^2-3x+(a-1)$.
In the case where $a=3$, show that $y=7x-3$ is a tangent to the curve. [3]

P3

(b) Given that $(m-4)x^2<3x-m$, show that m cannot be positive. [4]

5 (a) Sketch $y=|x(1-4x)|$, indicating the coordinates of the maximum point and intercepts.
Hence, state the range of values of k such that $\left|\dfrac{x}{2}-2x^2\right|=k$ gives more than 2 solutions. [4]

(b) The diagram shows the graph of $y=1-|x-5|$ where the x-intercepts are 4 and 6.



If a line $y=mx+c$ is to be added to the diagram above, determine a possible value for m and c if

(i) there is 1 intersection between the 2 graphs, [1]

(ii) there are infinite intersections between the 2 graphs. [2]

65

6 (a) A curve has the equation $y = e\sqrt{x}\ln(3x)$, where $x > 0$. Find an expression for $\frac{dy}{dx}$.

Hence find $\int_1^5 \frac{\ln(3x)+2}{2\sqrt{x}}\,dx$. [4]

(b) It is given that $y = 6e^{\sqrt{x-1}}$. Find, in terms of e, the rate of change of x at the instant when $x = 5$ if y is decreasing at the rate of $\frac{1}{2}e$ units per second at this instant. [4]

7 It is known that x and y are related by the equation $my = n(2^{mx})$, where m and n are constants.

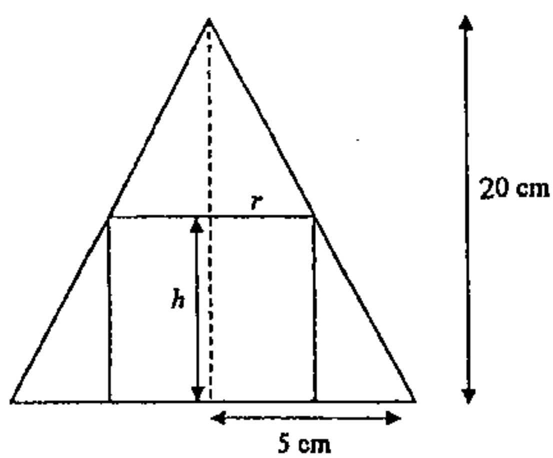
x	1	2	3	4	5
y	0.566	0.80	1.13	1.60	2.26

(i) Plot $\lg y$ against x and draw a straight line graph. [2]

(ii) Use your graph to estimate the values of m and n. [4]

(iii) By drawing a suitable straight line, solve the equation $y = 0.9^x$. [2]

8 A cylinder of radius r cm and height h is inscribed in a cone of base radius 5 cm and height 20 cm. The cross section of the solid is shown in the diagram.



(i) Show that the volume within the cone but not occupied by the cylinder, V, is given by $V = \frac{500}{3}\pi - (5 - \frac{h}{4})^2 \pi h$. [3]

(ii) Find the stationary value of V and determine whether it is maximum or minimum. [6]

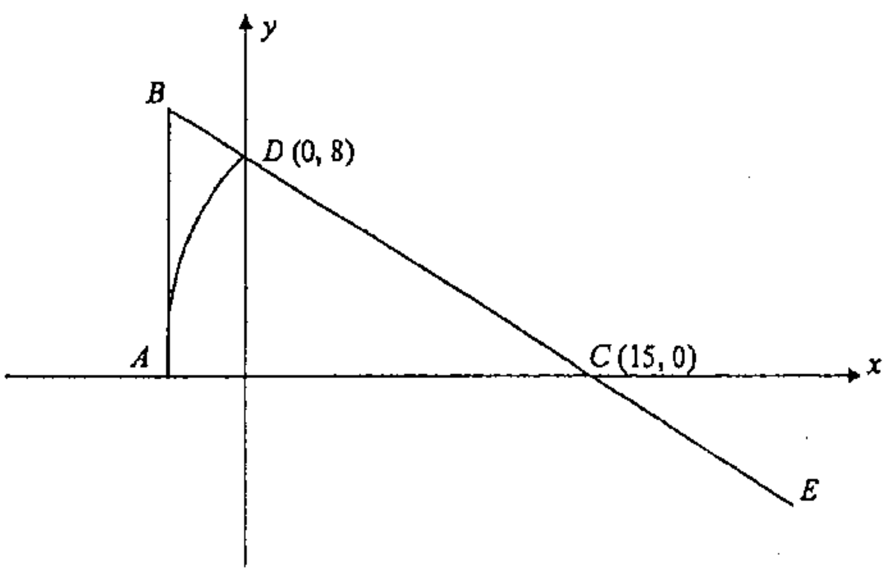
9 (a) The equation of a curve is $y = \frac{5}{kx^2} + 10x^3$. Given that its normal at $x = 2$ will never meet the y-axis, find the value of k. [3]

(b) A curve has the equation $y = 2\cos^2 3x$ for $0 \leq x \leq \pi$. Find

(i) the equation of the normal at $x = \frac{\pi}{12}$, [4]

(ii) the x-values on the curve whose tangents are parallel to the normal at $x = \frac{\pi}{12}$. [4]

10



P4

In the diagram, ADC is a sector of the circle with centre C and $BDCE$ is a straight line. The line AB is parallel to the y-axis and points C and D are (15, 0) and (0, 8) respectively.

(i) Show that coordinates of A is $(-2, 0)$. [2]
(ii) Find the equation of the line that passes through A and perpendicular to the line BC. [2]
(iii) Find the coordinates of E if the ratio of area ABC: area ACE is given to be 2:1. [5]
(iv) Given that $ABFE$ is a kite, find the area of $ABFE$. [2]

~ End of Paper ~

66

Qn	Detailed Working	Mark Distribution
1	$\dfrac{8x^4+1}{x(2x-1)}=4x^2+2x+1+\dfrac{x+1}{x(2x-1)}$	M1 (for correct quotient from long division)
	$\dfrac{x+1}{x(2x-1)}=\dfrac{A}{x}+\dfrac{B}{2x-1}$ $x+1=A(2x-1)+Bx$ Let $x=0$ $A=-1$	M1
	Let $x=0.5$ $0.5B=1.5$ $B=3$	M1, M1 (for A and B)
	$\dfrac{8x^4+1}{x(2x-1)}=4x^2+2x+1-\dfrac{1}{x}+\dfrac{3}{2x-1}$	A1
2(i)	$\operatorname{cosec}(60^\circ-\theta)=\dfrac{2}{\cos\theta\,(\sqrt{3}-\tan\theta)}$ LHS $=\dfrac{1}{\sin(60^\circ-\theta)}$	
	$=\dfrac{1}{\sin 60^\circ\cos\theta-\cos 60^\circ\sin\theta}$	M1 (for applying Addition formula)
	$=\dfrac{1}{(\frac{\sqrt{3}}{2})\cos\theta-(\frac{1}{2})\sin\theta}$ $=\dfrac{2}{\sqrt{3}\cos\theta-\sin\theta}\quad\dfrac{\div\cos\theta}{\div\cos\theta}$	M1
	$=\dfrac{2}{\cos\theta\,(\sqrt{3}-\tan\theta)}=RHS$ (shown)	A1

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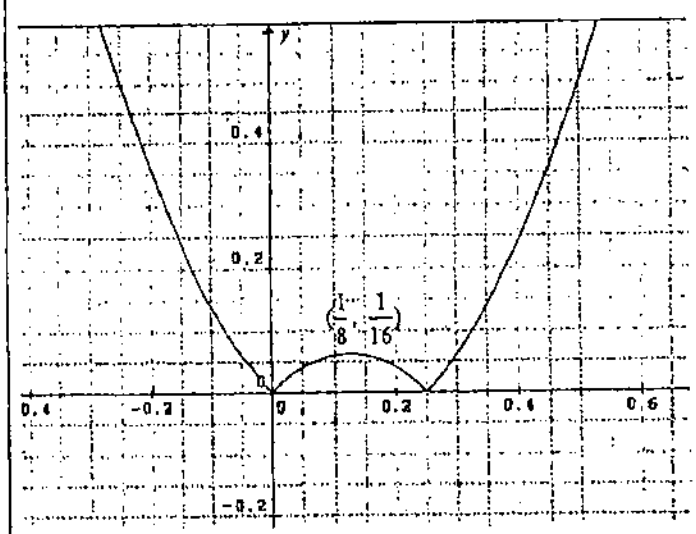
2(ii)	cosec $15°$ $= \text{cosec}\,(60° - 45°)$ $= \dfrac{2}{\cos 45° (\sqrt{3} - \tan 45°)}$ $= \dfrac{2}{\cos 45° (\sqrt{3} - \tan 45°)}$ $= \dfrac{2}{\frac{1}{\sqrt{2}}(\sqrt{3}-1)}$ $= \dfrac{2\sqrt{2}}{(\sqrt{3}-1)} \times \dfrac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$ $= \sqrt{2}(\sqrt{3}+1)$ or $\sqrt{6}+\sqrt{2}$	M1 (for $\theta = -45°$) M1 (for special $\angle$s) M1 (for rationalising) A1
3(a)	$(2x^2 - \frac{1}{\sqrt{x}})^{10}$ T_{r+1} $= 10C_r (2x^2)^{10-r}(-x^{-0.5})^r$ $= 10C_r (2)^{10-r}(-1)^r (x)^{20-2r-0.5r}$ $= 10C_r (2)^{10-r}(-1)^r (x)^{20-2.5r}$ $20 - 2.5r = 0$ $r = 8$ $\therefore T_9 = 10C_8 (2)^2(-1)^8 = 180$	M1 M1 A1
3(b)	$(5+px)^n$ ${}^nC_4 (5)^{n-4} p^4 = {}^nC_3 (5)^{n-3} p^3$ $\dfrac{n(n-1)(n-2)(n-3)}{1\times2\times3\times4}(5)^{n-4}p^4 = \dfrac{n(n-1)(n-2)}{1\times2\times3}(5)^{n-3}p^3$ $\dfrac{n-3}{4}(5)^{n-4}p^4 = (5)^{n-3}p^3$ $\dfrac{n-3}{4}p = 5$ $n = \dfrac{20}{p} + 3$	M1 (for specific term or correct expansion) M1, M1 (for applying ${}^nC_r$ formula to both sides) A1

4(a)	$y=(a+2)x^2-3x+(a-1)$ Let $a=3$, $y=5x^2-3x+2$ $5x^2-3x+2=7x-3$ $5x^2-10x+5=0$ $D=(-10)^2-4(5)(5)=0$ Therefore $y=7x-3$ is a tangent (shown).	M1 M1 B1
4(b)	$(m-4)x^2<3x-m$ $(m-4)x^2-3x+m<0$ $(-3)^2-4(m-4)(m)<0$ $9-4m(m-4)<0$ $4m^2-16m-9>0$ $(2m+1)(2m-9)>0$ $m<-\frac{1}{2}$, $m>4\frac{1}{2}$ Since $m-4<0$, thus $m<-\frac{1}{2}$. $\therefore m$ cannot be positive (shown)	M1 (for D<0) M1 M1 B1
5(a)		shape - B1 turning pt & x-intercepts - B1 68

	$\left\|\frac{x}{2}-2x^2\right\|=k$ $\frac{1}{2}\left\|x-4x^2\right\|=k$ $\left\|x(1-4x)\right\|=2k$ $0<2k\leq\frac{1}{16}$ $0<k\leq\frac{1}{32}$	 **M1** **A1**
5(b)(i)	$m=0,\ c=1$ OR any other relevant answers	**A1**
5(b)(ii)	(5, 1) and (4, 0) $m=\frac{1-0}{5-4}=1$ $y-0=1(x-4)$ $y=x-4$ *OR* (5, 1) and (6, 0) $m=\frac{1-0}{5-6}=-1$ $y-0=-1(x-6)$ $y=-x+6$	 **M1** **A1** *Alternative answer:* **M1** **A1**
6(a)	$y=e\sqrt{x}\ln(3x)$ $\frac{dy}{dx}=\frac{1}{2}ex^{-\frac{1}{2}}\ln(3x)+\frac{1}{x}e\sqrt{x}$ $=\frac{e\ln(3x)}{2\sqrt{x}}+\frac{e}{\sqrt{x}}$ $=\frac{e\ln(3x)+2e}{2\sqrt{x}}$ $=\frac{e[\ln(3x)+2]}{2\sqrt{x}}$	 **M1** **A1**

P6

	$\int_1^5 \frac{e[\ln(3x)+2]}{2\sqrt{x}}\,dx = \quad [e\sqrt{x}\ln(3x)]_1^5$ $\int_1^5 \frac{\ln(3x)+2}{2\sqrt{x}}\,dx$ $=[\sqrt{x}\ln(3x)]_1^5$ $=\sqrt{5}\ln 15-\ln 3$ $=4.96$	M1 A1
6(b)	$y=6e^{\sqrt{x-1}}$ $\frac{dy}{dx}=6e^{\sqrt{x-1}}.\frac{1}{2}(x-1)^{-\frac{1}{2}}=\frac{3e^{\sqrt{x-1}}}{\sqrt{x-1}}$ $-\frac{1}{2}e=\frac{3e^{\sqrt{x-1}}}{\sqrt{x-1}}\times\frac{dx}{dt}$ $-\frac{1}{2}e=\frac{3e^2}{2}\times\frac{dx}{dt}$ $\frac{dx}{dt}=-\frac{1}{3}e$ units/s	 M1 M1 M1 A1
8(i)	$\frac{20}{5}=\frac{20-h}{r}$ $20r=100-5h$ $r=\frac{100-5h}{20}$ $r=5-\frac{1}{4}h$ $V=\frac{1}{3}\pi(5)^2(20)-\pi(5-\frac{1}{4}h)^2h$ $=\frac{500\pi}{3}-(5-\frac{1}{4}h)^2\pi h$ (shown)	M1 M1 A1

8(ii)	$V=\frac{500\pi}{3}-(5-\frac{1}{4}h)^2\pi h$ $=\frac{500\pi}{3}-25\pi h+\frac{5}{2}\pi h^2-\frac{1}{16}\pi h^3$ $\frac{dV}{dh}=-25\pi+5\pi h-\frac{3}{16}\pi h^2$ $\frac{3}{16}h^2-5h+25=0$ $h=20\ (rej),\quad h=6\frac{2}{3}cm$ $\frac{d^2V}{dh^2}=5\pi-\frac{6}{16}\pi h$ $=5\pi-\frac{6}{16}\pi(6\frac{2}{3})>0$ (min) $V=\frac{500\pi}{3}-(5-\frac{1}{4}\times 6\frac{2}{3})^2\pi(6\frac{2}{3})=291cm^2$	 M1 M1 A1 M1 A1 A1
9(a)	$y=\frac{5}{kx^2}+10x^3=\frac{5}{k}x^{-2}+10x^3$ $\frac{dy}{dx}=\frac{-10}{kx^3}+30x^2$ $\frac{-10}{kx^3}+30x^2=0$ $\frac{-10+30kx^5}{kx^3}=0$ $10=30k(2^5)$ $k=\frac{1}{96}$	 M1 M1 A1

P7

9(b)(i)	$y=2\cos^2 3x$ $\frac{dy}{dx}=-12\cos 3x\sin 3x$ $m_t=-12\cos 3(\frac{\pi}{12})\sin 3(\frac{\pi}{12})=-6$ $m_n=\frac{1}{6}$ $x=\frac{\pi}{12}, y=2\cos^2 3(\frac{\pi}{12})=1$ $y-1=\frac{1}{6}(x-\frac{\pi}{12})$ $y=\frac{1}{6}x+\frac{72-\pi}{72}$	M1 M1 M1 A1
9(b)(ii)	$-12\cos 3x\sin 3x=\frac{1}{6}$ $-6(2\sin 3x\cos 3x)=\frac{1}{6}$ $\sin 6x=-\frac{1}{36}$ $\alpha=0.02778$ $6x=3.1694,6.2554,9.4526,12.5386,15.7358,18.8218$ $x=0.528,1.04,1.58,2.09,2.62,3.14$	M1 M1 A1 A1
10(i)	$\sqrt{(15-0)^2+(0-8)^2}=17$ units $A=(15-17,0)=(-2,0)$	M1 A1
10(ii)	$m_{BC}=\frac{8-0}{0-15}=-\frac{8}{15}$ $m_{\perp BC}=\frac{15}{8}$ Equation of line $\perp$ BC: $y-0=\frac{15}{8}(x+2)$ $y=\frac{15}{8}x+\frac{15}{4}$	 M1 A1 70

10(iii)	Equation of line BC: $y-8=-\frac{8}{15}(x+0)$ $y=-\frac{8}{15}x+8$ Let $B(-2,y)$ $y=-\frac{8}{15}(-2)+8=\frac{136}{15}=9\frac{1}{15}$ $B(-2,9\frac{1}{15})$ ABC & ACE share the same base AC. Hence, $\perp$ height of E to x-axis should be $\frac{1}{2}$ of AB. $\perp$ height of E to x-axis $=\frac{1}{2}(9\frac{1}{15})=\frac{68}{15}=4\frac{8}{15}$ Let $E(x,-4\frac{8}{15})$ $-4\frac{8}{15}=-\frac{8}{15}(x)+8$ $x=23\frac{1}{2}$ $E=(23\frac{1}{2},-4\frac{8}{15})$	 M1 M1 M1 M1 A1
10(iv)	Area of $ABFE$ = Area of $2(ABE)$ $=\frac{1}{2}\begin{vmatrix} -2 & -2 & \frac{47}{2} & -2 \\ 0 & \frac{136}{15} & -\frac{68}{15} & 0 \end{vmatrix} \times 2$ $=231.2$ units^2	 M1 A1

1 The curve for which $\frac{dy}{dx}=\frac{k}{(2x+5)^3}-1$, where k is a constant, is such that the tangent to the curve at $(-2, 0)$ is perpendicular to the line $5y=x-1$. Find

(i) the value of k, [2]

(ii) the equation of the curve. [3]

2 The roots of the equation $x^2=mx-2m^2$ are α and β. Find, in terms of m, an equation whose roots are $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$. [6]

3 (a) Without using a calculator, find the value of c given that $34+3\sqrt{128}=\left(\frac{6}{\sqrt{2}}-c\right)^2$. [3]

(b) The volume of a cylinder is $(9+\sqrt{50})\pi$ cm^3. Given that the cylinder has a radius of $(2+\sqrt{2})$ cm, find, without using a calculator, the height of the cylinder in the form $\frac{a+b\sqrt{2}}{c}$. [4]

4 An object is heated in an oven until it reaches the temperature of 90 °C. It is then allowed to cool. Its temperature, T °C, when it has been cooling for time t minutes, is given by the equation $T=k+he^{-\frac{t}{10}}$, where k and h are constants.

Given that the temperature of the object is 40 °C when it has been cooling for exactly $(10\ln 3)$ minutes, show that $k=15$ and $h=75$. [3]

(i) Calculate the value of T when $t=10$. [1]

(ii) Determine the rate at which T is decreasing when $t=25$. [2]

(iii) Find, to the nearest minute, the time taken for the temperature of the object to drop below 35°C. [2]

71

5 (a) Express y in terms of x, $\log_5 x^2y = 3 + \log_5 x - \dfrac{1}{2\log_y 5}$. [4]

(b) Solve the equation $\log_3(x+5) - \log_{\sqrt{3}}(x-1) = \log_3 2$. [4]

6 A cubic polynomial $f(x)$ is such that the coefficient of x^3 is 6. It is given that one of the roots of the equation $f(x) = 0$ is $\dfrac{4}{3}$ and $[2x^2 + (2k+1)x - 3]$ is a quadratic factor of $f(x)$. Given further that $f(x)$ leaves a remainder of 30 when divided by $(x-2)$, find

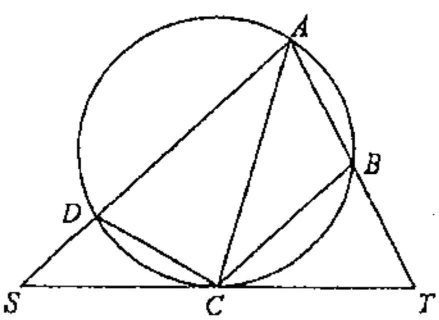
(i) the value of k and hence, factorise $f(x)$ completely, [4]

(ii) by using the result from part (i), the number of real roots of the equation $f(x) = 15(1-2x)$, justifying your answer. [4]

7 The diagram shows a circle $ABCD$ and the tangent ST of the circle at point C. B and C bisect AT and ST respectively. Prove that

(i) ΔABC is similar to ΔSDC, [4]

(ii) $AS = \dfrac{2AC \times DC}{TC}$. [4]



8 (i) A circle passes through the origin O and cuts the x- and y-axes at 3 and 4 respectively. Find the equation of the circle in the general form. [4]

(ii) Given that OP is the diameter of the circle, find the equation of the tangent at P.[3]

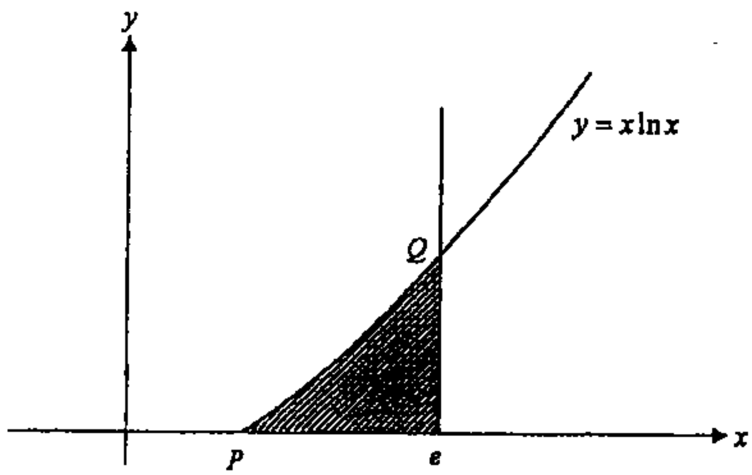
(iii) Another tangent at Q, which is parallel to the y-axis, meets the tangent found in part (ii). Find the points of intersections between the two tangents. [3]

9 (i) Given that $y = x^2 \ln x^3$, show that $\frac{dy}{dx} = 3x(1+2\ln x)$. [3]

(ii) The diagram shows part of the curve $y = x\ln x$ cutting the x-axis at point P. The line $x = e$ intersects the curve at point Q.

(a) Find the x-coordinate of P. [2]

(b) By using the result from part (i), show that the area of the shaded region bounded by the x-axis, the line $x = e$ and the curve is $\frac{1}{4}(e^2+1)$. [4]



10 (i) Express $3\cos 2A + 4\sin 2A$ in the form $R\cos(2A-\alpha)$, where $0° \le \alpha \le 90°$. [2]

(ii) Hence solve $3\cos 2A + 4\sin 2A = 4$ for $0° \le A \le 180°$. [3]

(iii) On the same axes sketch, for $0° \le x \le 60°$, the graphs of [3]

$$y = 2\sin 6x \qquad \text{and} \qquad y = 2 - \frac{3}{2}\cos 6x.$$

(iv) Explain how the solutions of the equation in part (ii) could be used to find the x-coordinates of the points of intersection of the graphs of part (iii). [2]

11 A particle moves in a straight line such that t seconds after passing through a fixed point O, its velocity, v m/s, is given by $v = 24\cos(2t)$. When $t = 0$, its displacement from O is -6 metres. Find

(i) the magnitude of the acceleration when $t = 1$, [2]

(ii) the value of t when the particle first reaches the fixed point O, [4]

(iii) the distance travelled by the particle up to the second instantaneous rest. [4]

12 A curve has the equation $y = (x-2)(x+1)^3$.

(i) Find an expression for $\frac{dy}{dx}$. [2]

(ii) Find the x-coordinates of the stationary points. [2]

(iii) Without determining the nature of the stationary points, show that y decreases as x increases between the stationary points. [3]

(iv) Determine the nature of the stationary points. [4]

P10

Qn	Marking point	Mark Awarded	Remarks
1(a) (5m)	$\left.\frac{dy}{dx}\right\|_{x=-2}=-5$	M1	
	$\frac{k}{(2(-2)+5)^3}-1=-5$		
	$k=-4$	A1	
(b)	$\frac{dy}{dx}=-\frac{4}{(2x+5)^3}-1$		
	$y=\int \frac{-4}{(2x+5)^3}-1\,dx$		
	$=\frac{1}{(2x+5)^2}-x+c$	M1	
	Sub $(-2,0)$, $0=1-(-2)+c$	M1	
	$c=-3$		
	$y=\frac{1}{(2x+5)^2}-x-3$	A1	
2	$x^2-mx+2m^2=0$		
	$\alpha+\beta=m$	B1	Either sum or product
	$\alpha\beta=2m^2$		
	$\alpha^3+\beta^3=(\alpha+\beta)[(\alpha+\beta)^2-3\alpha\beta]$	M1	
	$=m[m^2-3(2m^2)]$		
	$=-5m^3$	A1	
	$\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\alpha^3+\beta^3}{\alpha^3\beta^3}$		
	$=\frac{-5m^3}{(2m^2)^3}$	M1	
	$=-\frac{5}{8m^3}$		
	$\frac{1}{\alpha^3}\times\frac{1}{\beta^3}=\frac{1}{(\alpha\beta)^3}$		
	$=\frac{1}{(2m^2)^3}$	M1	
	$=\frac{1}{8m^6}$		
	$x^2-\left(-\frac{5}{8m^3}\right)x+\frac{1}{8m^6}=0$		
	$8m^6x^2+5m^3x+1=0$	A1	73

Qn	Marking point	Mark Awarded	Remarks
3(i)	$34+3\sqrt{128}=\left(\frac{6}{\sqrt{2}}-c\right)^2$ $34+24\sqrt{2}=(3\sqrt{2}-c)^2$ $34+24\sqrt{2}=18-6c\sqrt{2}+c^2$ $34+24\sqrt{2}=18+c^2-6c\sqrt{2}$ $24=-6c$ $c=-4$	M1 M1 A1	
(ii)	$\pi(2+\sqrt{2})^2h=(9+\sqrt{50})\pi$ $(6+4\sqrt{2})h=9+\sqrt{50}$ $h=\frac{9+5\sqrt{2}}{2(3+2\sqrt{2})}\times\frac{3-2\sqrt{2}}{3-2\sqrt{2}}$ $=\frac{27-3\sqrt{2}-20}{2(9-8)}$ $=\frac{7-3\sqrt{2}}{2}$	M1 M1 M1 A1	
4	$T=k+he^{-\frac{t}{10}}$ $t=0,\quad 90=k+h$ $t=10\ln 3,\quad 40=k+\frac{h}{3}$ $50=\frac{2}{3}h$ $h=75,\quad k=15$	M1 M1 A1	
(i)	$t=10,\quad T=15+75e^{-1}$ $=42.6°$	B1	
(ii)	$T=15+75e^{-\frac{t}{10}}$ $\frac{dT}{dt}=75\left(-\frac{1}{10}\right)e^{-\frac{t}{10}}$ $=-\frac{15}{2}e^{-\frac{t}{10}}$ $\left.\frac{dT}{dt}\right\|_{t=25}=-\frac{15}{2}e^{-\frac{25}{10}}$ $=-0.616$ °C/min	M1 A1	

Qn	Marking point	Mark Awarded	Remarks
(iii)	$15+75e^{-\frac{t}{10}}<35$		Accept working which is in the equation form
	$e^{-\frac{t}{10}}<\frac{4}{15}$		
	$-\frac{t}{10}<\ln\frac{4}{15}$	M1	
	$t>10\ln\frac{4}{15}=13.2$		
	$t=14$	A1	
5(a)	$\log_5 x^2y=3+\log_5 x-\frac{1}{2\log_y 5}$		
	$\log_5 x^2y=3+\log_5 x-\frac{1}{2}\log_5 y$	M1	Change of base
	$2\log_5 x^2y=6+2\log_5 x-\log_5 y$		
	$\log_5(x^2y)^2=6+\log_5\frac{x^2}{y}$		
	$\log_5(x^2y)^2-\log_5\frac{x^2}{y}=6$	M1	Evidence of using Power / Product / Quotient law
	$\log_5\frac{(x^4y^2)y}{x^2}=6$		
	$x^2y^3=5^6$	M1	Index form
	$y=\sqrt[3]{\frac{5^6}{x^2}}$		
	$y=\frac{25}{\sqrt[3]{x^2}}$ o.e.	A1	Accept $x^{\frac{2}{3}}$
(b)	$\log_3(x+5)-\log_{\sqrt{3}}(x-1)=\log_3 2$		
	$\log_3(x+5)-\frac{\log_3(x-1)}{\log_3 3^{\frac{1}{2}}}=\log_3 2$	M1	Change of base
	$\log_3(x+5)-2\log_3(x-1)=\log_3 2$		
	$\log_3\frac{x+5}{(x-1)^2}=\log_3 2$	M1	Quotient rule
	$x+5=2(x^2-2x+1)$	M1	Index form
	$2x^2-5x-3=0$		
	$(2x+1)(x-3)=0$		
	$x=-\frac{1}{2}$ (NA), $x=3$	A1	No A1 if invalid ans is not rejected
			74

Qn	Marking point	Mark Awarded	Remarks
6(i)	$f(x)=(3x-4)[2x^2+(2k+1)x-3]$	M1	
	$f(2)=30$	M1	
	$2[8+(2k+1)2-3]=30$		
	$5+2(2k+1)=15$		
	$2(2k+1)=10$		
	$k=2$	A1	
	$f(x)=(3x-4)(2x^2+5x-3)$		
	$=(3x-4)(2x-1)(x+3)$	A1	No A1 if given as $f(x)=0$
(ii)	$f(x)=15(1-2x)$		
	$(3x-4)(2x-1)(x+3)=-15(2x-1)$		
	$(3x-4)(2x-1)(x+3)+15(2x-1)=0$		
	$(2x-1)[(3x-4)(x+3)+15]=0$	M1	
	$(2x-1)(3x^2+5x+3)=0$	M1	
	$2x-1=0$ $\quad$ $3x^2+5x+3=0$		
	$D=5^2-4(3)(3)<0$	M1	Use of D or quad formula
	No of real roots = 1	A1	
7(i)	$BC//AD$ (mid-point thm)	B1	
	$\angle SCD=\angle CAD$ (alt segment thm)	M1	
	$=\angle ACB$ (alt $\angle$)		
	$\angle ABC=180°-\angle ADC$ ($\angle$ in opp segment)	M1	
	$=\angle SDC$		
	$\therefore$ $\angle ABC$ is similar to $\angle SDC$	A1	
(ii)	$\frac{AC}{SC}=\frac{BC}{CD}$ (part (i))	M1	
	$AC\times CD=\frac{1}{2}AS\times SC$ (mid-pt theorem)	M1	
	$2AC\times DC=AS\times TC$ (C bisects ST)	M1	
	$AS=\frac{2AC\times DC}{TC}$	A1	
8(i)	$A=(3,0)$, $B=(0,4)$	M1	
	$\Rightarrow$ centre lies on $\perp$ bisector of OA & OB		
	$\therefore C=\left(\frac{3}{2},2\right)$		
	Radius $=\sqrt{\left(\frac{3}{2}\right)^2+2^2}$	M1	

P12

Qn	Marking point	Mark Awarded	Remarks
	$=\sqrt{\frac{25}{4}}=\frac{5}{2}$ $\left(x-\frac{3}{2}\right)^2+(y-2)^2=\frac{25}{4}$ $x^2-3x+\frac{9}{4}+y^2-4y+4-\frac{25}{4}=0$ $x^2+y^2-3x-4y=0$	M1 A1	
(ii)	$\left(\frac{x_p+0}{2},\frac{y_p+0}{2}\right)=\left(\frac{3}{2},2\right)$ $(x_p, y_p)=(3,4)$ $m_{OP}=\frac{4}{3}, \qquad m_T=-\frac{3}{4}$ $y-4=-\frac{3}{4}(x-3)$ $4y-16=-3x+9$ $4y=-3x+25$	M1 M1 A1	
(iii)	Since tangent // to y-axis $\Rightarrow x=c$ from centre of circle $x=\frac{3}{2}-\frac{5}{2}=-1, \qquad x=\frac{3}{2}+\frac{5}{2}=4$ $(x,y)=(-1,7), \quad \left(4,\frac{13}{4}\right)$	M1 A1, A1	
9(i)	$y=x^2\ln x^3$ $=3x^2\ln x$ $\frac{dy}{dx}=3\left(x^2\left(\frac{1}{x}\right)+2x\ln x\right)$ $=3x(1+2\ln x)$	M1, M1 A1	
(ii)(a)	$x\ln x=0$ $x=0$ (NA), $\quad \ln x=0$ $x=e^0=1$	M1 A1	
(b)	$\int_1^e 3x(1+2\ln x)\,dx=\left[x^2\ln x^3\right]_1^e$ $\int_1^e 3x+6x\ln x\,dx=e^2\ln e^3$ $\frac{3}{2}\left[x^2\right]_1^e+\int_1^e 6x\ln x\,dx=3e^2$ $\int_1^e 6x\ln x\,dx=3e^2-\frac{3}{2}(e^2-1)$ $\int_1^e 6x\ln x\,dx=\frac{3}{2}(e^2+1)$	M1 M1 M1	For integrating $3x$ For substituting limits

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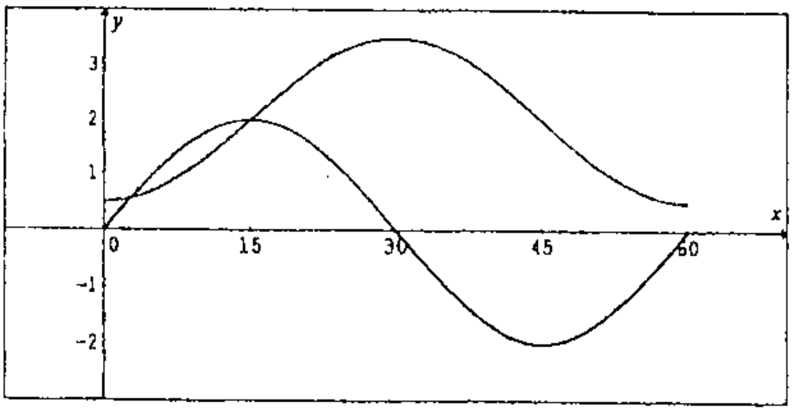
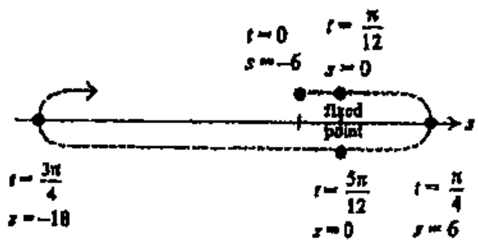
Qn	Marking point	Mark Awarded	Remarks
	$\int_1^e x\ln x\,dx = \frac{1}{4}(e^2+1)$	A1	
10(i)	$R\cos(2A-\alpha)$ $=\sqrt{3^2+4^2}\cos\left(2A-\tan^{-1}\left(\frac{4}{3}\right)\right)$ $=5\cos(2A-53.1°)$	M1 A1	Either R or α No A1 if □ not in 1d.p.
(ii)	$3\cos 2A+4\sin 2A=4$ $5\cos(2A-53.13°)=4$ $\cos(2A-53.13°)=\frac{4}{5}$ $2A-53.13°=-36.87, 36.87°, 323.13°$ $2A=16.26°, 90.0°$ $A\approx 8.13°, 45.0°$	M1 A1, A1	Do not penalise for d.p. if this was done in part (i)
(iii)			B1 for sine graph B1 (amplitude), B1 (shape) for cosine graph
(iv)	$2\sin 6x = 2-\frac{3}{2}\cos 6x$ $4\sin 6x = 4-3\cos 6x$ $3\cos 6x+4\sin 6x=4$ Let $A=3x$ $3\cos 2A+4\sin 2A=4$	M1 A1	
11(i)	$v=24\cos(2t)$ $a=-48\sin(2t)$ $t=1,\quad a=-48\sin(2)$ $=-43.6$ $\lvert a\rvert=43.6$ m/s^2	M1 A1	
(ii)	$s=\int 24\cos(2t)\,dt$ $=12\sin(2t)+c$	M1	

P13

10(iii)	Equation of line BC: $y-8=-\frac{8}{15}(x+0)$ $y=-\frac{8}{15}x+8$ Let $B(-2,y)$ $y=-\frac{8}{15}(-2)+8=\frac{136}{15}=9\frac{1}{15}$ $B(-2,9\frac{1}{15})$ ABC & ACE share the same base AC. Hence, $\perp$ height of E to x-axis should be $\frac{1}{2}$ of AB. $\perp$ height of E to x-axis $=\frac{1}{2}(9\frac{1}{15})=\frac{68}{15}=4\frac{8}{15}$ Let $E(x,-4\frac{8}{15})$ $-4\frac{8}{15}=-\frac{8}{15}(x)+8$ $x=23\frac{1}{2}$ $E=(23\frac{1}{2},-4\frac{8}{15})$	M1 M1 M1 M1 A1
10(iv)	Area of $ABFE$ = Area of $2(ABE)$ $=\frac{1}{2}\begin{vmatrix}-2 & -2 & \frac{47}{2} & -2 \\ 0 & \frac{136}{15} & -\frac{68}{15} & 0\end{vmatrix}\times 2$ $=231.2$ units^2	M1 A1

P8

Qn	Marking point	Mark Awarded	Remarks
	$t=0,\ s=-6 \Rightarrow c=-6$ $s=12\sin(2t)-6$	M1	
	When P first reaches fixed point, $s=0$. $12\sin(2t)-6=0$ $\sin(2t)=0.5$ $\alpha=\frac{\pi}{6}$	M1	
	$t=\frac{\pi}{12},\quad \frac{5\pi}{12}$ (NA)	A1	
(iii)	At inst rest, $v=0$ $24\cos(2t)=0$	M1	
	$2t=\frac{\pi}{2},\quad \frac{3\pi}{2}$ $t=\frac{\pi}{4},\quad \frac{3\pi}{4}$	A1	
	$s=12\sin(2t)-6$ $t=0,\ s=-6$ $t=\frac{\pi}{4},\ s=6$ $t=\frac{3\pi}{4},\ s=-18$ $t=5,\ s=-6$	M1	Either for $t=\frac{\pi}{4}$ or $t=\frac{3\pi}{4}$
	Dist $=6+12+18=36$ m	A1	
			
12(i)	$y=(x-2)(x+1)^3$ $\frac{dy}{dx}=(x-2)[3(x+1)^2]+(x+1)^3$	M1	
	$=(x+1)^2(3x-6+x+1)$ $=(x+1)^2(4x-5)$	A1	
(ii)	$\frac{dy}{dx}=(x+1)^2(4x-5)=0$	M1	
	$x=-1,\quad x=\frac{5}{4}$	A1	

<table>
<tr><th>Qn</th><th>Marking point</th><th>Mark Awarded</th><th>Remarks</th></tr>
<tr><td>(iii)</td><td>For $-1 \le x \le \frac{5}{4}$,
$(x+1)^2 > 0$, $4x-5<0$
$\therefore \quad \frac{dy}{dx} = (x+1)^2(4x-5) < 0$

OR
<table><tr><td>x</td><td>-1^+</td><td>$\frac{5}{4}^-$</td></tr><tr><td>Sign of $\frac{dy}{dx}$</td><td>$-$</td><td>$-$</td></tr></table>Since gradient is negative between 2 stat points, y decreases as x increases between the two stat points.</td><td>M1, M1

A1</td><td></td></tr>
<tr><td>(iv)</td><td><table><tr><td>x</td><td><-1</td><td>-1</td><td>>-1</td></tr><tr><td>$\frac{dy}{dx}$</td><td>-ve</td><td>0</td><td>-ve</td></tr><tr><td></td><td>\</td><td>–</td><td>\</td></tr></table>
$x=-1$ is a point of inflexion

<table><tr><td>x</td><td>$<\frac{5}{4}$</td><td>$\frac{5}{4}$</td><td>$>\frac{5}{4}$</td></tr><tr><td>$\frac{dy}{dx}$</td><td>-ve</td><td>0</td><td>+ve</td></tr><tr><td></td><td>\</td><td>–</td><td>/</td></tr></table>
$x=\frac{5}{4}$ is a min point</td><td>M1

A1

M1

A1</td><td></td></tr>
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中正中学 义顺

CHUNG CHENG HIGH SCHOOL YISHUN

Preliminary Examination (2015)
Secondary 4 Express/ 5 Normal (Academic)

Candidate

Name	Register No	Class

ADDITIONAL MATHEMATICS
4047/01

For examiner's use
/ 80

Date: 26 August 2015
Duration: 2 hours

Additional Materials: Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give your answer in the simplest form. Leave your answer in fraction where applicable. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Setter: Mr Han Ji

This paper consists of 7 printed pages, INCLUDING the cover page. **[Turn over**

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area of } \Delta ABC = \frac{1}{2} ab \sin C$$

1. (i) Sketch the graph of $y = 8x^{-1}$ for $x > 0$. [1]

(ii) On the same diagram, sketch the graph of $y = \frac{1}{4}x^{\frac{3}{2}}$ for $x \geq 0$. [1]

(iii) Calculate the exact coordinates of the point of intersection of the graphs. [2]

(iv) Determine with explanation, whether the normal to the graphs at the point of intersection are perpendicular. [2]

2. The cubic polynomial $f(x)$ is such that the coefficient of x^3 is 1 and the roots of $f(x) = 0$ are -1, m and $2m$, where m is an integer. It is given that $f(x)$ has a remainder of 6 when divided by $x - 1$.

(i) Find an expression for $f(x)$ in descending powers of x. [4]

(ii) Hence or otherwise, solve the equation $y^6 - 5y^4 + 2y^2 + 8 = 0$. [3]

3. A cuboid has a square base of side ($2 + a\sqrt{3}$) cm, where a is an integer. The height of the cuboid is ($1 + \sqrt{3}$) cm and its volume is ($\sqrt{27} - 5$) cm^3.

(i) Find the value of a. [3]

(ii) With the value of a in (i), find the total surface area of the cuboid in the form ($p + q\sqrt{3}$) cm^2, where p and q are integers. [2]

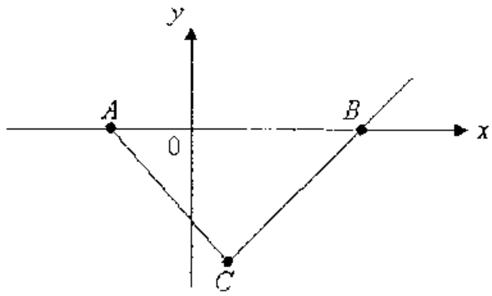
4. The equation of a curve is $y = x^2 + 3x$. A straight line has equation $y = mx - 9$.

(i) Explain why the straight line is a tangent to the curve when $m = 9$. [2]

(ii) Find the other value of m for which the line $y = mx - 9$ is a tangent to the curve. [3]

(iii) State the set of values of m for which the straight line does not intersect the curve. [1]

5. The diagram shows part of the graph of $y = |2x - 1| - 2$.



(i) Find the coordinates of A and of B. [2]

(ii) Explain why the lowest point, C, on the graph has coordinates $(\frac{1}{2}, -2)$. [2]

(iii) In each of the following cases, determine the number of intersections of the line $y = mx + c$ with $y = |2x - 1| - 2$, justifying your answer.

(a) $m = -2$ and $c > -1$ [2]

(b) $m = 1$ and $c < -3$ [2]

6. In a simplified prey-predator model, some wolves were deliberately introduced to an island to curb the population of wild rabbits. The population of rabbits, R, was given by $R = 400 + 6000e^{-0.02t}$, where t is the number of days since the introduction of wolves.

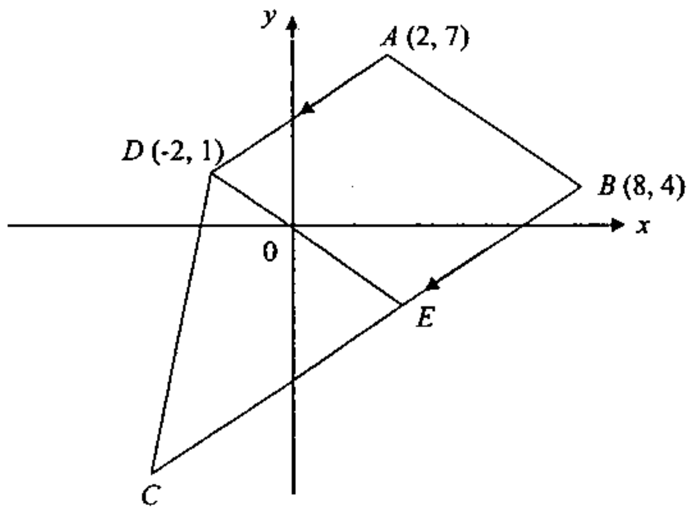
(i) Find the initial population of wild rabbits on the island. [1]

(ii) After how many days would the population of wild rabbits first drop by 40%? [2]

(iii) Explain why the rabbits would never extinct on the island in the long run. [1]

Solutions to this question by accurate drawing will not be accepted.

7.



The diagram shows a trapezium $ABCD$ in which AD is parallel to BC. The points A, B and D are (2, 7), (8, 4) and (-2, 1) respectively. The point E is on BC and DE passes through O.

(i) Show that $ABED$ is a parallelogram. [2]

(ii) Find the coordinates of E. [2]

Given that $CD = CE$,

(iii) find the coordinates of C. [4]

(iv) find the ratio of the area of triangle CDE to the area of $ABED$. [2]

8. (a) (i) Without using a calculator, prove that $\cot(45° - A) = \frac{\cot A + 1}{\cot A - 1}$. [3]

(ii) Hence find the exact value of $\cot 15°$. [2]

(b) Find an expression for $f(x)$ such that

$$f'(x) = 3\sin^2(5x - \frac{\pi}{4}) + \cos^2 x - \tan^2 \frac{1}{2}x.$$ [3]

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9. (i) Express $\dfrac{8x-5}{x^2(1-x)}$ as the sum of 3 partial fractions. [4]

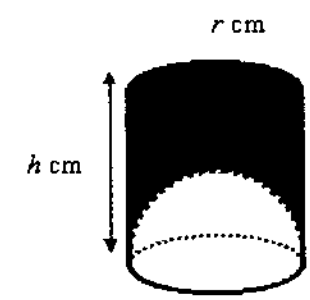
(ii) Hence find $\int \dfrac{8x-5}{x^2(1-x)}\,dx$. [2]

10. The equation of a curve is $y = xe^{-x}$.

(i) Find the set of values of x for which y is an increasing function of x. [2]

(ii) Find the coordinates of the turning point and determine whether the turning point is a maximum or minimum. [2]

11. The diagram shows a solid container consisting of a cylinder with a hemisphere dug out. The radius and height of the cylinder are r cm and h cm respectively.



(i) Express h in terms of r given that the external curved surface area of the cylindrical part of the solid is $1200\pi\ \text{cm}^2$. [2]

(ii) Express the volume, $V\ \text{cm}^3$, of the container in terms of r. [2]

(iii) The solid is heated and it expands at a rate of $0.81\ \text{cm}^3/\text{s}$. Find the rate at which its radius increases when the height is 60 cm. [3]

Answer the whole of this question on a piece of graph paper.

12. The table shows experimental values of the two variables, x and y.

x	0.5	1.5	3	4.5	5.5	6
y	4.43	5.29	7.44	11.4	15.7	18.7

It is known that x and y are related by an equation of the form $y = ab^x + e$, where a and b are constants.

(i) Explain how a straight line graph may be drawn to represent the given data. [2]

(ii) Draw this graph for the given data and use it to estimate the value of a and of b. [4]

(iii) By inserting another suitable line on your graph, solve the equation $ab^x = 5e^{-\frac{x}{2}}$. [3]

END OF PAPER

中正中学 义顺

CHUNG CHENG HIGH SCHOOL YISHUN

Preliminary Examination (2015)

Secondary 4 Express

Candidate			
	Name	Register No	Class

ADDITIONAL MATHEMATICS
Paper 1 (4047)

Date: **27 August 2015**
Duration: **2 hr**

Additional Materials: Answer Paper

For examiner's use
/ 80

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1. Answer **ALL** the questions in this paper.
2. All workings must be clearly shown in the answer space provided.
   Omission of essential working and unit of measurement will result in loss of marks.
3. The use of calculator is expected, where appropriate.
4. Give your answer in the simplest form. Leave your answer in fraction where applicable or correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
5. For π, use your calculator value.
6. The number of marks is given in brackets [ ] at each question or part question.
   The total marks for this paper is **80**.

Setter: Mr Han Ji **[Turn over**

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82

Mathematical Formulae

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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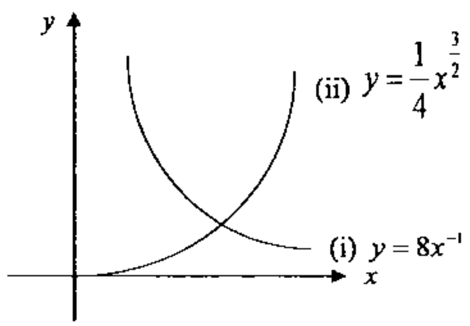
(iii) Calculate the exact coordinates of the point of intersection of the graphs. [2]

(iv) Determine with explanation, whether the normal to the graphs at the point of intersection are perpendicular. [2]

Ans:



1 mark for each graph

(iii) $y = 8x^{-1}$

$y = \frac{1}{4}x^{\frac{3}{2}}$

$8x^{-1} = \frac{1}{4}x^{\frac{3}{2}}$

$x^{\frac{5}{2}} = 32$

$x = 32^{\frac{2}{5}}$ ------ [M1]

$= (2^5)^{\frac{2}{5}} = 2^2 = 4$

$y = 8(4^{-1}) = 2$

The coordinates are (4, 2). ------ [A1]

(iv) $y = 8x^{-1}, \frac{dy}{dx} = -8x^{-2}$

$y = \frac{1}{4}x^{\frac{3}{2}}, \frac{dy}{dx} = \frac{3}{8}x^{\frac{1}{2}}$

When $x = 4$,

$\frac{dy}{dx} = 8x^{-2} = -8(4^{-2}) = -\frac{1}{2}$, gradient of normal = 2

$\frac{dy}{dx} = \frac{3}{8}x^{\frac{1}{2}} = \frac{3\sqrt{4}}{8} = \frac{3}{4}$, gradient of normal = $-\frac{4}{3}$

$2 \times -\frac{4}{3} = -\frac{8}{3} \neq -1$ ------ [M1]

The normals are not perpendicular since the product of their gradients is not -1.

------ [A1]

OR

$\frac{dy}{dx} = 8x^{-2} = -8(4^{-2}) = -\frac{1}{2}$

$\frac{dy}{dx} = \frac{3}{8}x^{\frac{1}{2}} = \frac{3\sqrt{4}}{8} = \frac{3}{4}$

$-\frac{1}{2} \times \frac{3}{4} \neq -1$ ------ [M1]

Since the tangents to the curves at the point of intersection are not perpendicular, the normals at that point are also not perpendicular. ------ [A1]

2. The cubic polynomial f(x) is such that the coefficient of x^3 is 1 and the roots of f(x) = 0 are -1, m and $2m$, where m is an integer. It is given that f(x) has a remainder of 6 when divided by $x-1$.

(i) Find an expression for f(x) in descending powers of x. [4]

(ii) Hence or otherwise, solve the equation $y^6 - 5y^4 + 2y^2 + 8 = 0$. [3]

Ans:

(i) Let f(x) = $k(x+1)(x-m)(x-2m)$.

Since the coefficient of x^3 is 1, k = 1. ------ [M1]

$f(x) = (x+1)(x-m)(x-2m)$

$f(1) = 6$

$(1+1)(1-m)(1-2m) = 6$ ------ [M1]

$(1-m)(1-2m) = 3$

$2m^2 - 3m + 1 = 3$

$2m^2 - 3m - 2 = 0$

$(m-2)(2m+1) = 0$

$m = 2$ or $m = -\frac{1}{2}$ (rej) ------ [M1]

$f(x) = (x+1)(x-2)(x-4)$

$= (x+1)(x^2 - 6x + 8)$

$= x^3 - 5x^2 + 2x + 8$ ------ [A1]

(ii) Let $x = y^2$

$(y^2)^3 - 5(y^2)^2 + 2(y^2) + 8 = 0$ ------ [M1]

$x^3 - 5x^2 + 2x + 8 = 0$

$(x+1)(x-2)(x-4) = 0$

$x = -1$, $x = 2$ or $x = 4$

$y^2 = -1$ (rej), $y^2 = 2$ or $y^2 = 4$ ------ [M1]

$y = \pm\sqrt{2}$, $y = \pm 2$ ------ [A1]

3. A cuboid has a square base of side $(2 + a\sqrt{3})$ cm, where a is an integer. The height of the cuboid is $(1 + \sqrt{3})$ cm and its volume is $(\sqrt{27} - 5)$ cm^3.

(i) Find the value of a. [3]

(ii) With the value of a in (i), find the total surface area of the cuboid in the form $(p + q\sqrt{3})$ cm^2, where p and q are integers. [2]

Ans:

(i) $(2+a\sqrt{3})^2(1+\sqrt{3})=\sqrt{27}-5$

$$(2+a\sqrt{3})^2=\frac{3\sqrt{3}-5}{1+\sqrt{3}}$$

$$=\frac{(3\sqrt{3}-5)(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})} \text{ ------ [M1]}$$

$$=\frac{8\sqrt{3}-14}{-2}$$

$$=7-4\sqrt{3}$$

$4+3a^2+4a\sqrt{3}=7-4\sqrt{3}$ ------ [M1]

$4+3a^2=7, a^2=1, a=\pm 1$

$4a=-4, a=-1$

$\therefore a=-1$ ------ [A1]

(ii) Total surface area $=2(2-\sqrt{3})^2+4(2-\sqrt{3})(1+\sqrt{3})$ ------ [M1]

$=2(7-4\sqrt{3})+4(\sqrt{3}-1)$

$=14-8\sqrt{3}+4\sqrt{3}-4$

$=(10-4\sqrt{3})\text{ cm}^2$ ------ [A1]

4. The equation of a curve is $y=x^2+3x$. A straight line has equation $y=mx-9$.

(i) Explain why the straight line is a tangent to the curve when $m=9$. [2]

(ii) Find the other value of m for which the line $y=mx-9$ is a tangent to the curve. [3]

(iii) State the set of values of m for which the straight line does not intersect the curve. [1]

Ans:

(i) $y = x^2 + 3x$

$y = 9x - 9$

$x^2 + 3x = 9x - 9$

$x^2 - 6x + 9 = 0$

Discriminant = $(-6)^2 - 4(1)(9) = 0$ ------ [M1]

Therefore, the straight line is a tangent to the curve, since they intersect at only one point. ------ [A1]

(ii) $x^2 + 3x = mx - 9$

$x^2 + (3-m)x + 9 = 0$

$(3-m)^2 - 4(1)(9) = 0$ ------ [M1]

$(3-m)^2 = 36$

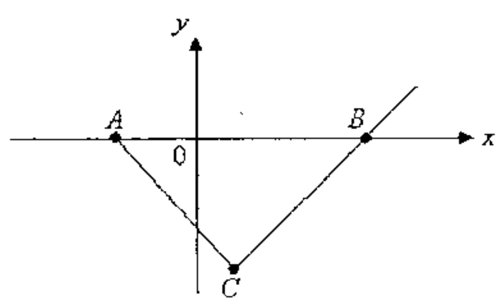
$3 - m = \pm 6$

$m = -3$ or $m = 9$ (rej) ------ [M1]

The other value of m is -3. ------ [A1]

(iii) $-3 < m < 9$ ------ [B1]

5. The diagram shows part of the graph of $y = |2x-1| - 2$.



(i) Find the coordinates of A and of B. [2]

(ii) Explain why the lowest point, C, on the graph has coordinates $(\frac{1}{2}, -2)$. [2]

(iii) In each of the following cases, determine the number of intersections of the line $y = mx + c$ with $y = |2x-1| - 2$, justifying your answer.

(a) $m = -2$ and $c > -1$ [2]

(b) $m = 1$ and $c < -3$ [2]

Ans:

(i) $|2x-1| - 2 = 0$

$|2x-1| = 2$

$2x - 1 = 2$ or $2x - 1 = -2$ ------ [M1]

$2x = 3$ or $2x = -1$

$x = 1\frac{1}{2}$ or $x = -\frac{1}{2}$

A $(-\frac{1}{2}, 0)$, B $(1\frac{1}{2}, 0)$ ------ [A1]

(ii) $|2x-1| \geq 0$, $|2x-1| - 2 \geq -2$

Since C is the lowest point on the graph, $y = -2$. ------ [M1]

$|2x-1| = 0$, $x = \frac{1}{2}$

Therefore the coordinates of C are $(\frac{1}{2}, -2)$. ------ [A1]

OR

At the point where the lines turns, $|2x-1| = 0$, $x = \frac{1}{2}$ ------ [M1]

$y = 0 - 2 = -2$

Therefore the coordinates of C are ($\frac{1}{2}$, -2). ------ [A1]

(iii) (a) For $m=-2$ and $c>-1$, the line $y=mx+c$ is above the left arm and parallel to it. Therefore the line $y=mx+c$ intersects the right arm at one point. ------ [M1]

Number of intersection = 1 ------ [A1]

(b) For $m=1$ and $c<-3$, the line $y=mx+c$ is below C and the gradient is gentler than the right arm. Therefore the line $y=mx+c$ does not intersect the right arm. ------ [M1]

Number of intersection = 0 ------ [A1]

6. In a simplified prey-predator model, some wolves were deliberately introduced to an island to curb the population of wild rabbits. The population of rabbits, R, was given by $R=400+6000e^{-0.02t}$, where t is the number of days since the introduction of wolves.

(i) Find the initial population of wild rabbits on the island. [1]

(ii) After how many days would the population of wild rabbits first drop by 40%? [2]

(iii) Explain why the rabbits would never extinct on the island in the long run. [1]

Ans:

(i) When $t=0$,

$R=400+6000e^{0}=6400$ ------ [A1]

(ii) $6400\times60\%=3840$

$3840=400+6000e^{-0.02t}$

$$3440 = 6000e^{-0.02t}$$

$$e^{-0.02t} = \frac{3440}{6000}$$

$$-0.02t = \ln(\frac{3440}{6000}) \text{ ------ [M1]}$$

$$t = 27.8 \approx 28$$

It takes 28 days for the population of wild rabbits to first drop by 40%. ------ [A1]

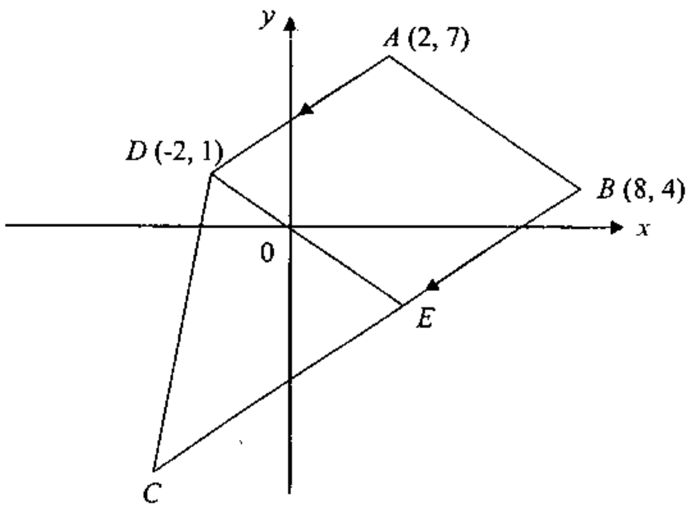
(iii)As $t \to \infty$, $e^{-0.02t} \to 0$

As a result, $R \to 400 + 6000(0) = 400$ ------ [A1]

Therefore, the rabbits would not become extinct in the long run.

Solutions to this question by accurate drawing will not be accepted.

7.



The diagram shows a trapezium $ABCD$ in which AD is parallel to BC. The point A is (2, 7), the point B is (8, 4) and the point D is (-2, 1). The point E is on BC such that DE passes through O.

(i) Show that $ABED$ is a parallelogram. [2]

(ii) Find the coordinates of E. [2]

Given that $CD = CE$,

(iii) find the coordinates of C. [4]

(iv) find the ratio of the area of triangle CDE to the area of $ABED$. [2]

Ans:

(i) $AD // BE$ (given)

Gradient of $AB = \frac{4-7}{8-2} = -\frac{1}{2}$

Gradient of $DE = \frac{0-1}{0-(-2)} = -\frac{1}{2}$

$AB // DE$ ------ [M1]

With two pairs of parallel opposite sides, $ABED$ is a parallelogram. ------ [A1]

(ii) Let the coordinates of E be (x, y)

$(\frac{x+2}{2}, \frac{y+7}{2}) = (\frac{-2+8}{2}, \frac{4+1}{2})$ ------ [M1]

$x+2=6, y+7=5$

$x=4, y=-2$

$E\,(4, -2)$ ------ [A1]

(iii)Since $CD = CE$, C lies on the perpendicular bisector of DE.

Gradient of $DE = -\frac{1}{2}$

Gradient of perpendicular bisector $= \frac{-1}{(-\frac{1}{2})} = 2$ ------ [M1]

Let the equation of the perpendicular bisector be $y = 2x + c$

Midpoint of DE is $(\frac{-2+4}{2}, \frac{1+(-2)}{2}) = (1, -\frac{1}{2})$

$-\frac{1}{2} = 2 \times 1 + c, c = -\frac{5}{2}$

The equation is $y = 2x - \frac{5}{2}$ ------ [M1]

Gradient of $BE = \frac{4-(-2)}{8-4} = \frac{3}{2}$

Let the equation of BE be $y = \frac{3}{2}x + c$

At (4, -2), $-2 = \frac{3}{2}(4) + c, c = -8$

The equation is $y = \frac{3}{2}x - 8$ ------ [M1]

$y = 2x - \frac{5}{2}$

$y = \frac{3}{2}x - 8$

$2x - \frac{5}{2} = \frac{3}{2}x - 8$

$\frac{1}{2}x = -5\frac{1}{2}$

$x = -11$

$y = 2(-11) - \frac{5}{2} = -24\frac{1}{2}$

$C(-11, -24\frac{1}{2})$ ------ [A1]

OR

Gradient of $BE = \frac{4-(-2)}{8-4} = \frac{3}{2}$

Let the equation of BE be $y = \frac{3}{2}x + c$

At (4, -2), $-2 = \frac{3}{2}(4) + c, c = -8$

The equation is $y=\frac{3}{2}x-8$ ------ [M1]

Let the coordinates of C be (x, y).

Since $CD = CE$,

$\sqrt{(x+2)^2+(y-1)^2}=\sqrt{(x-4)^2+(y+2)^2}$ ------ [M1]

$x^2+4x+4+y^2-2y+1=x^2-8x+16+y^2+4y+4$

$12x-6y=15$

$12x-6(\frac{3}{2}x-8)=15$ ------ [M1]

$3x=-33$

$x=-11$

$y=\frac{3}{2}(-11)-8=-24\frac{1}{2}$

$C(-11,-24\frac{1}{2})$ ------ [A1]

(iv) Area of $CDE=\frac{1}{2}\begin{vmatrix}4 & -2 & -11 & 4\\ -2 & 1 & -24\frac{1}{2} & -2\end{vmatrix}$

$=\frac{1}{2}[4+49+22-4-(-11)-(-98)]$

$=90\text{ units}^2$

Area of $ABED=\frac{1}{2}\begin{vmatrix}2 & -2 & 4 & 8 & 2\\ 7 & 1 & -2 & 4 & 7\end{vmatrix}$

$=\frac{1}{2}[2+4+16+56-(-14)-4-(-16)-8]$

$=48\text{ units}^2$ ------ [M1]

Area of CDE : Area of $ABED = 90 : 48 = 15 : 8$ ------ [A1]

8. (a) (i) Without using a calculator, prove that $\cot(45° - A) = \dfrac{\cot A + 1}{\cot A - 1}$. [3]

(ii) Hence find the exact value of $\cot 15°$. [2]

(b) Find an expression for f(x) such that

$$f'(x) = 3\sin^2(5x - \frac{\pi}{4}) + \cos^2 x - \tan^2 \frac{1}{2}x.$$ [3]

Ans:

(a) (i) LHS $= \dfrac{1}{\tan(45° - A)}$

$$= \frac{1 + \tan 45° \tan A}{\tan 45° - \tan A}$$

$$= \frac{1 + \tan A}{1 - \tan A} \text{ ------ [M1]}$$

$$= \frac{1 + \frac{\sin A}{\cos A}}{1 - \frac{\sin A}{\cos A}}$$

$$= \frac{\frac{\cos A + \sin A}{\cos A}}{\frac{\cos A - \sin A}{\cos A}}$$

$$= \frac{\cos A + \sin A}{\cos A - \sin A}$$

$$= \frac{\frac{\cos A + \sin A}{\sin A}}{\frac{\cos A - \sin A}{\sin A}} \text{ ------ [M1]}$$

$$= \frac{\cot A + 1}{\cot A - 1} = \text{RHS (proven)} \text{ ------ [A1]}$$

(ii) $\cot 15° = \cot(45° - 30°)$

$$= \frac{\cot 30° + 1}{\cot 30° - 1}$$

$$= \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$= \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} \text{ ------ [M1]}$$

$$= \frac{4 + 2\sqrt{3}}{2}$$

$$= 2 + \sqrt{3} \text{ ------ [A1]}$$

(b) $f'(x) = 3\sin^2(5x - \frac{\pi}{4}) + \cos^2 x - \tan^2 \frac{1}{2}x$

$$= -\frac{3}{2}[-2\sin^2(5x - \frac{\pi}{4})] + \frac{1}{2}[2\cos^2 x] - (\sec^2 \frac{1}{2}x - 1)$$

$$= -\frac{3}{2}[1 - 2\sin^2(5x - \frac{\pi}{4})] + \frac{3}{2} + \frac{1}{2}[2\cos^2 x - 1] + \frac{1}{2} - \sec^2 \frac{1}{2}x + 1 \text{ ------ [M1]}$$

$$= -\frac{3}{2}\cos(10x - \frac{\pi}{2}) + \frac{1}{2}\cos 2x - \sec^2 \frac{1}{2}x + 3 \text{ ------ [M1]}$$

$$f(x) = \int (3\sin^2(5x - \frac{\pi}{4}) + \cos^2 x - \tan^2 \frac{1}{2}x)dx$$

$$= \int [-\frac{3}{2}\cos(10x - \frac{\pi}{2}) + \frac{1}{2}\cos 2x - \sec^2 \frac{1}{2}x + 3]dx$$

$$= -\frac{3}{20}\cos(10x - \frac{\pi}{2}) + \frac{1}{4}\sin 2x - 2\tan\frac{1}{2}x + 3x + c \text{ ------ [A1]}$$

9. (i) Express $\frac{8x-5}{x^2(1-x)}$ as the sum of 3 partial fractions. [4]

(ii) Hence find $\int \frac{8x-5}{x^2(1-x)}dx$. [2]

Ans:

(i) $\frac{8x-5}{x^2(1-x)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{1-x}$ ------ [M1]

$8x-5 = Ax(1-x) + B(1-x) + Cx^2$

Let $x = 1$, $3 = C$

Let $x = 0$, $-5 = B$ ------ [M1]

Let $x = 2$, $11 = 2A + (-5)(-1) + 3(2^2)$, $A = 3$ ------ [M1]

$\frac{8x-5}{x^2(1-x)} = \frac{3}{x} - \frac{5}{x^2} + \frac{3}{1-x}$ ------ [A1]

(ii) $\int \frac{8x-5}{x^2(1-x)}dx = \int (\frac{3}{x} - \frac{5}{x^2} + \frac{3}{1-x})dx$

$= \int \frac{3}{x}dx - \int \frac{5}{x^2}dx + \int \frac{3}{1-x}dx$

$= 3\ln x - (-5x^{-1}) + [-3\ln(1-x)] + C$ ------ [M1]

$= 3\ln x + \frac{5}{x} - 3\ln(1-x) + C$

$= 3[\ln x - \ln(1-x)] + \frac{5}{x} + C$

$= 3\ln\frac{x}{1-x} + \frac{5}{x} + C$ ------ [A1]

10. The equation of a curve is $y = xe^{-x}$.

(i) Find the set of values of x for which y is an increasing function of x. [2]

(ii) Find the coordinates of the turning point and determine whether the turning point is a maximum or minimum. [2]

Ans:

(i) $\frac{dy}{dx} = e^{-x} + x(-e^{-x})$

$= e^{-x}(1-x)$ ------ [M1]

For increasing function, $\frac{dy}{dx} > 0$

$e^{-x}(1-x) > 0$

Since $e^{-x} > 0, 1-x > 0$

$x < 1$ ------ [A1]

(ii) $\frac{dy}{dx} = 0$

$e^{-x}(1-x) = 0$

$1-x = 0$

$x = 1$

$y = 1(e^{-1}) = \frac{1}{e}$

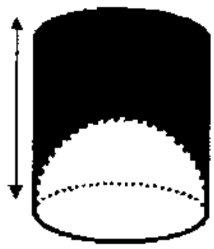
Coordinates of turning point is $(1, \frac{1}{e})$ ------ [A1]

Use 1st derivative test:

x	1-	1	1+
$\frac{dy}{dx}$	+ve	0	-ve
Gradient	/	—	\

The point is a maximum point. ------ [A1]

11. The diagram shows a solid container consisting of a cylinder with a hemisphere dug out. The radius and height of the cylinder are r cm and h cm respectively.



(i) Express h in terms of r given that the external curved surface area of the cylindrical part of the solid is $1200\pi\ \text{cm}^2$. [2]

(ii) Express the volume, $V\ \text{cm}^3$, of the container in terms of r. [2]

(iii) The solid is heated and it expands at a rate of $0.81\ \text{cm}^3/\text{s}$. Find the rate at which its radius increases when the height is 60 cm. [3]

Ans:

(i) $2\pi rh = 1200\pi$ ------ [M1]

$rh = 600$

$h = \dfrac{600}{r}$ ------ [A1]

(ii) $V = \pi r^2 h - \dfrac{2}{3}\pi r^3$ ------ [M1]

$= \pi r^2 \dfrac{600}{r} - \dfrac{2}{3}\pi r^3$

$= 600\pi r - \dfrac{2}{3}\pi r^3$ ------ [A1]

(iii) $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ ------ [M1]

$V = 600\pi r - \frac{2}{3}\pi r^3$

$\frac{dV}{dr} = 600\pi - 2\pi r^2$ ------ [M1]

When $h = 60$, $r = 10$

$\frac{dV}{dr} = 600\pi - 2\pi(10)^2 = 400\pi$

$0.81 = 400\pi \times \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{0.81}{400\pi} = 0.000645$ ------ [A1]

Answer the whole of this question on a piece of graph paper.

12. The table shows experimental values of the two variables, x and y.

x	0.5	1.5	3	4.5	5.5	6
y	4.43	5.29	7.44	11.4	15.7	18.7

It is known that x and y are related by an equation of the form $y = ab^x + e$, where a and b are constants.

(i) Explain how a straight line graph may be drawn to represent the given data. [2]

(ii) Draw this graph for the given data and us it to estimate the values of a and of b. [4]

(iii) By inserting another suitable line on your graph, solve the equation $ab^x = 5e^{-\frac{x}{2}}$. [3]

Ans:

(i) $y = ab^x - e$

$y - e = ab^x$

$\ln(y-e) = \ln(ab^x)$ ------ [M1]

$\ln(y-e) = \ln a + x\ln b$

Let $Y = \ln(y-e)$ and $X = x$

$Y = \ln a + X\ln b$

If $\ln(y-e)$ is plotted against x, a straight line graph can be obtained. ------ [A1]

(ii) From the graph

$\ln a = 0.35$, $a = e^{0.35} = 1.42$ ------ [A1] (Answer range: 0.3352 ± 0.02)

$\ln b = \dfrac{2.35 - 0.75}{5 - 1} = 0.4(\pm 0.02)$, $b = e^{0.4} = 1.49$ ------ [A1]

(iii) $ab^x = 5e^{-\frac{x}{2}}$

$\ln ab^x = \ln 5e^{-\frac{x}{2}}$

$\ln a + x\ln b = \ln 5 - \dfrac{x}{2}$

$\ln(y-e) = \ln 5 - \dfrac{x}{2}$ ------ [M1]

Equation of line to be inserted: $Y = \ln 5 - \dfrac{1}{2}X$

At the point of intersection, $x = 1.40 \pm 0.02$ ----- [A1]

END OF PAPER

中正中学 义顺

CHUNG CHENG HIGH SCHOOL YISHUN

Preliminary Examination (2015)
Secondary 4 Express/ 5 Normal (Academic)

Candidate			
	Name	Register No	Class

For examiner's use
/ 100

ADDITIONAL MATHEMATICS
4047/ 02

Date: 27 August 2015
Duration: 2 hours 30 minutes

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give your answer in the simplest form. Leave your answer in fraction where applicable.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 100.

Setter: Woo Huey Ming

This paper consists of **6** printed pages, INCLUDING the cover page. **[Turn over**

92

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} ab \sin c$$

1. (a) Solve the equation $\log_3 4 - \log_9(x^2+4x+4) = \log_{\frac{1}{3}} x$. [4]

(b) Sketch the graph of $y = e^{-x}$. In order to solve the equation $\ln\left(\frac{1}{\sqrt{x-3}}\right) = \frac{1}{2}x$,

a graph of a suitable straight line is drawn on the same set of axes as the graph of $y = e^{-x}$. Find the equation of the straight line. [3]

2. The roots of the equation $2x^2 - px - q = 0$, where p and q are constants, are α and β.
The roots of the equation $4x^2 + qx - 3x = p - 1$ are $\alpha - 1$ and $\beta - 1$.
(i) Find the value of p and of q. [5]
(ii) Find a quadratic equation whose roots are α^3 and β^3. [3]

3. (i) Given that the coefficient of x^{-2} in the expansion of $\left(\frac{1}{x} + px\right)^8$ is 448, find the value of the positive constant p. [3]
(ii) Using the value of p in part (i), find the term independent of x in the expansion of $\left(\frac{1}{x} + px\right)^8 (5x^2 - 4)$. [4]

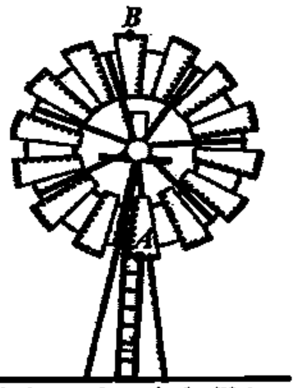
4. A curve has the equation $y = \frac{\ln(2x)^3}{x^2}$ for $x > 0$.

(i) Find an expression for $\frac{dy}{dx}$. [3]

(ii) Hence find $\int \frac{\ln 2x}{x^3}\,dx$. [4]

5. The equation of a circle C_1 is given as $x^2 + y^2 - 16x + 8y + 64 = 0$.
(i) Find the coordinates of the centre and radius of the circle C_1. [3]
(ii) The line $y = k$ is a tangent to the circle at A, where $k \neq 0$. Find the value of k. [2]
(iii) The tangent to the circle at $B(4, -4)$ intersects $y = k$ at point C. Find equation of this tangent. [1]
(iv) Explain why a circle C_2 can be drawn through the points A, B and C with AB being the diameter. [1]
(v) Find the equation of the circle C_2. [3]
(vi) Determine, with working, whether $(3\frac{3}{5}, -6)$ lies within the 2 circles. [2]

6.



The height of a blade on the windmill (measured from the ground) can be modelled by the equation $h = 15 - 7\cos kt$ where k is a constant and t is the time in seconds after the windmill starts moving. The windmill starts rotating from the lowest point, A, when $t = 0$. The windmill rotates at a rate of 12 revolutions per minute.

(i) Explain why this model suggests that the highest point of the windmill, B, is 22 m above the ground level. [1]

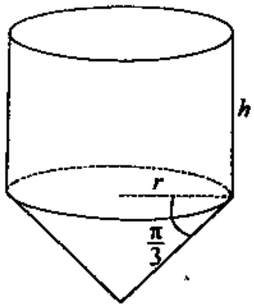
(ii) Find the value of k. [1]

(iii) For how long over the course of one complete revolution will the point A be at least 17 m above ground level? [2]

(iv) Explain how the solution in part (iii) could be used to find the duration of the point A being at least 17 m above ground level over the course of two complete revolutions. [2]

(v) Suggest a possible equation of how the height of a blade varies against time if the windmill starts rotating from the highest point at B. [1]

7.



The diagram shows a solid machine part that is made up of a closed cylinder joined to an inverted right circular cone. The height of the cylinder is h m and the slant height of the cone makes an angle of $\frac{\pi}{3}$ radians to its base radius, r m.

(i) Given that the volume of the machine part is 50π m^3, express h in terms of r. [2]

(ii) Show that the total surface area of the machine part is given by [4]

$$A = \frac{\pi r^2}{3}(9 - 2\sqrt{3}) + \frac{100\pi}{r}.$$

(iii) Given that r can vary, find the value of r for which the total surface area of the machine part is stationary. [3]

(iv) By comparing gradients, explain why this value of r gives the least total surface area possible. [2]

8. The equation of two curves are $y = \cos 2x - 2\sin^2 x$ and $y = \sin 2x$.

(i) Show that the x-coordinate of the points of intersection of the two curves satisfy $2\cos 2x - \sin 2x = 1$. [1]

(ii) On the same axes sketch, for $-\pi < x < \pi$, the graphs of $y = \cos 2x - 2\sin^2 x$ and $y = \sin 2x$. [4]

(iii) Express the equation $2\cos 2x - \sin 2x = 1$ in the form $\cos(2x + \alpha) = k$, where α and k are constants to be found. [4]

(iv) Hence find, in radians, the x-coordinates of the points of intersection for $-\pi < x < \pi$. [3]

9. A particle travels in a straight line from a fixed point O with acceleration a m/s^2, given by $a = 8t - k$ where t is the time in seconds after passing O, and k is a constant. The velocity of the particle is 5 m/s when it passes O, and at $t = 2$, its velocity is -21 m/s.

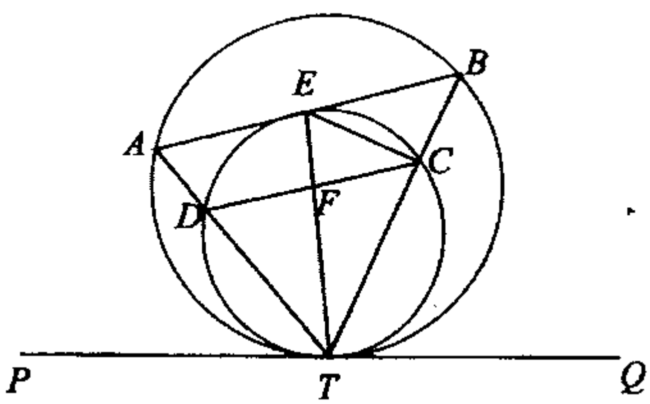
(i) Find the value of k. [3]

(ii) Find the value(s) of t when the particle is instantaneously at rest. [2]

(iii) Calculate the average speed of the particle during the first six seconds. [3]

(iv) Describe completely the motion of the particle in the first six seconds. [2]

10.



In the diagram, two circles touch each other at T and PTQ is their common tangent. AB is a tangent to the smaller circle at E. AT and BT cut the smaller circle at D and C respectively. ET and CD intersect at F. Prove that

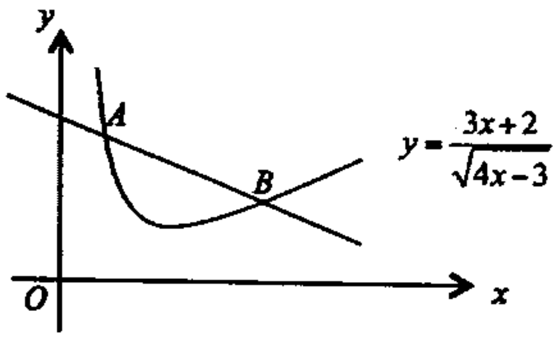
(i) $AB // DC$, [2]

(ii) $\angle ATE = \angle BTE$, [3]

(iii) $ET^2 = CT \times DT + EF \times ET$. [3]

11. **(i)** Differentiate $(x+2)\sqrt{4x-3}$ with respect to x. [2]

(ii)



The diagram shows part of the curve $y=\dfrac{3x+2}{\sqrt{4x-3}}$. A line with gradient $-\dfrac{2}{3}$ intersects the curve at $A(1,5)$ and B.

(a) Verify that the y-coordinate of B is $\dfrac{11}{3}$. [5]

(b) Determine the area of the region bounded by the curve and the line AB. [4]

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95

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$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} ab \sin c$$

1. **(a)** Solve the equation $\log_3 4 - \log_9(x^2+4x+4) = \log_{\frac{1}{3}} x$. [4]

$\log_3 4 - \log_9(x^2+4x+4) = \log_{\frac{1}{3}} x$

$\log_3 4 - \dfrac{2\log_3(x+2)}{\log_3 9} = \dfrac{\log_3 x}{\log_3 \frac{1}{3}}$ M1

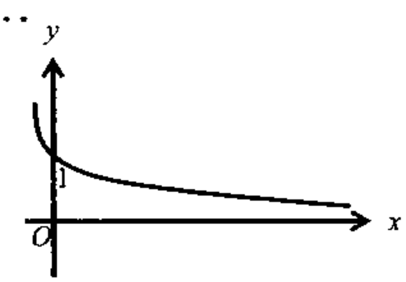
$\log_3 4 - \log_3(x+2) = -\log_3 x$ M1

$\dfrac{4}{x+2} = \dfrac{1}{x}$ M1

$3x = 2$

$x = \dfrac{2}{3}$ A1

(b) Sketch the graph of $y = e^{-x}$. In order to solve the equation $\ln\left(\dfrac{1}{\sqrt{x-3}}\right) = \dfrac{1}{2}x$, a graph of a suitable straight line is drawn on the same set of axes as the graph of $y = e^{-x}$. Find the equation of the straight line. [3]



B1

$\ln\left(\dfrac{1}{\sqrt{x-3}}\right) = \dfrac{1}{2}x$

$\ln 1 - \dfrac{1}{2}\ln(x-3) = \dfrac{1}{2}x$

$\ln(x-3) = -x$ M1

$x - 3 = e^{-x}$

Draw $y = x - 3$. A1

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2. The roots of the equation $2x^2 - px - q = 0$, where p and q are constants, are α and β.
The roots of the equation $4x^2 + qx - 3x = p - 1$ are $\alpha - 1$ and $\beta - 1$.
(i) Find the value of p and of q. [5]

$\alpha + \beta = \dfrac{p}{2} --- (1)$

M1

$\alpha\beta = -\dfrac{q}{2} --- (2)$

$\alpha - 1 + \beta - 1 = \dfrac{3-q}{4}$

$\alpha + \beta = \dfrac{3-q}{4} + 2 --- (3)$

$(\alpha - 1)(\beta - 1) = \dfrac{1-p}{4} --- (4)$ M1

Sub (1) into (4)

$\alpha\beta\text{-}(\alpha + \beta) + 1 = \dfrac{1}{4} - \dfrac{1}{4}(2)(\alpha + \beta)$

$\alpha\beta - \dfrac{1}{2}(\alpha + \beta) + \dfrac{3}{4} = 0 --- (5)$

Sub (2) and (3) into (5)

$-\dfrac{q}{2} - \dfrac{1}{2}\left(\dfrac{3-q}{4} + 2\right) + \dfrac{3}{4} = 0$ M1

$-\dfrac{3q}{8} = \dfrac{5}{8}$

$q = -1\dfrac{2}{3}$ A1

Sub $q = -1\dfrac{2}{3}$ into (3)

$\alpha\text{+}\beta\text{=}\dfrac{3 - \left(-\dfrac{5}{3}\right)}{4} + 2 = 3\dfrac{1}{6}$

$p = 2\left(3\dfrac{1}{6}\right) = 6\dfrac{1}{3}$

$p = 6\dfrac{1}{3}, q = -1\dfrac{2}{3}$ A1

(ii) Find a quadratic equation whose roots are α^3 and β^3. [3]

$(\alpha\beta)^3 = \dfrac{125}{216}$ M1

$\alpha^2 + \beta^2 = \left(3\dfrac{1}{6}\right)^2 - 2\left(\dfrac{5}{6}\right) = \dfrac{301}{36}$

$\alpha^3 + \beta^3 = 3\dfrac{1}{6}\left(\dfrac{301}{36} - \dfrac{5}{6}\right) = \dfrac{5149}{216}$ M1

$x^2 - \dfrac{5149}{216}x + \dfrac{125}{216} = 0$

$216x^2 - 5149x + 125 = 0$ A1

3. **(i)** Given that the coefficient of x^{-2} in the expansion of $\left(\frac{1}{x}+px\right)^8$ is 448, find the value of the positive constant p. [3]

$T_{r+1}=\binom{8}{r}\left(\frac{1}{x}\right)^{8-r}(px)^r$

$=\binom{8}{r}x^{r-8}p^r x^r$

$=\binom{8}{r}p^r x^{2r-8}$ M1

$2r-8=-2$

$r=3$

$T_4=\binom{8}{3}p^3x^{-2}$

$=56p^3x^{-2}$

$56p^3=448$ M1

$p^3=8$

$p=2$ A1

(ii) Using the value of p in part (i), find the term independent of x in the expansion of $\left(\frac{1}{x}+px\right)^8(5x^2-4)$. [4]

$2r-8=0$

$r=4$

$T_5=\binom{8}{4}2^4=1120$ M1

$\left(\frac{1}{x}+2x\right)^8(5x^2-4)$

$=(...+448x^{-2}+1120+...)(5x^2-4)$ M1

Term independent of x

$=448(5)+1120(-4)$ M1

$=-2240$ A1

4. A curve has the equation $y = \frac{\ln(2x)^3}{x^2}$ for $x > 0$.

(i) Find an expression for $\frac{dy}{dx}$. [3]

$\frac{dy}{dx} = \frac{3x - 6x\ln 2x}{x^4}$ M2

$= \frac{3}{x^3} - \frac{6}{x^3}\ln 2x$ A1

(ii) Hence find $\int \frac{\ln 2x}{x^3}\,dx$. [4]

$\int\left(\frac{3}{x^3} - \frac{6\ln 2x}{x^3}\right)dx = \frac{\ln(2x)^3}{x^2} + C_1$ M1

$\int \frac{6\ln 2x}{x^3} = \int \frac{3}{x^3}\,dx - \frac{\ln(2x)^3}{x^2} + C_2$

$\int \frac{\ln 2x}{x^3}\,dx = \frac{1}{6}\int 3x^{-3}\,dx - \frac{\ln 2x}{2x^2} + C$ M1

$= \frac{1}{6}\left(\frac{3x^{-2}}{-2}\right) - \frac{\ln 2x}{2x^2} + C$ M1

$= -\frac{1}{4x^2} - \frac{\ln 2x}{2x^2} + C$ A1

5. The equation of a circle C_1 is given as $x^2+y^2-16x+8y+64=0$.

(i) Find the coordinates of the centre and radius of the circle C_1. [3]

$(x-8)^2-(-8)^2+(y+4)^2-(4)^2=-64$ M1

$(x-8)^2-(y+4)^2=4^2$

Centre of circle= (8, -4) A1

Radius= 4 units A1

(ii) The line $y=k$ is a tangent to the circle at A, where $k\neq 0$. Find the value of k. [2]

The 2 tangents to circle are $y=0$ and $y=-8$. B1

Since $k\neq 0$, $k=-8$. B1

(iii) The tangent to the circle at $B(4,\ -4)$ intersects $y=k$ at point C. Find equation of this tangent. [1]

$x=4$ B1

(iv) Explain why a circle C_2 can be drawn through the points A, B and C with AB being the diameter. [1]

$A(8,-8), B(4,-4), C(4,-8)$

Since $AC\perp BC$, $\angle BCA=90°$

$\therefore$ A circle C_2 can be drawn through the points A, B and C with AB being the diameter.
(Angle in a semicircle) R1

(v) Find the equation of the circle C_2. [3]

Midpoint of AB

$=\left(\frac{8+4}{2},\frac{-8-4}{2}\right)$ M1

$=(6,-6)$

Radius

$=\frac{1}{2}\sqrt{(8-4)^2+(-8-(-4))^2}$ M1

$=2\sqrt{2}$ units A1

$(x-6)^2+(y+6)^2=8$

(vi) Determine, with working, whether $(3\frac{3}{5}, -6)$ lies within the 2 circles. [2]

Length of $(3.6, -6)$ to centre of C_1

$= \sqrt{(8-3.6)^2 + (-4+6)^2}$

$= 4.83$ units

Length of $(3.6, -6)$ to centre of C_2

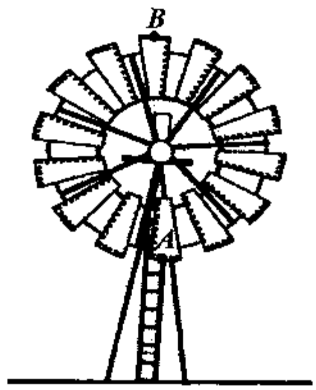
$= \sqrt{(6-3.6)^2 + (-6-(-6))^2}$

$= 2.4$ units

Since length of $(3.6, -6)$ to centre of $C_1 > 4$ units, (3.6, -6) is outside C_1. R1

Since length of $(3.6, -6)$ to centre of $C_2 < 2\sqrt{2}$ units, (3.6, -6) is within C_2. R1

6.



The height of a blade on the windmill (measured from the ground) can be modelled by the equation $h = 15 - 7\cos kt$ where k is a constant and t is the time in seconds after the windmill starts moving. The windmill starts rotating from the lowest point, A, when $t = 0$. The windmill rotates at a rate of 12 revolutions per minute.

(i) Explain why this model suggests that the highest point of the windmill, B, is 22 m above the ground level. [1]

$-1 \le \cos kt \le 1$

$-7 \le -7\cos kt \le 7$

$15 - 7 \le 15 - 7\cos kt \le 15 + 7$

$8 \le 15 - 7\cos kt \le 22$ B1

$\therefore$ The highest point of the windmill is 22 m above the ground level.

(ii) Find the value of k. [1]

Period = 5 seconds

$k = \dfrac{2\pi}{5}$ B1

(iii) For how long over the course of one complete revolution will the point A be at least 17 m above ground level? [2]

$15 - 7\cos \dfrac{2\pi}{5} t = 17$

$\cos \dfrac{2\pi}{5} t = -\dfrac{2}{7}$

Basic angle= 1.281044625

$\dfrac{2\pi}{5} t = 1.860548028, 4.422637279$

$t = 1.480577078, 3.519422922$ M1

Length of time= 3.519422922-1.480577078

=2.04 seconds A1

(iv) Explain how the solution in part **(iii)** could be used to find the duration of the point A being at least 17 m above ground level over the course of **two** complete revolutions. [2]

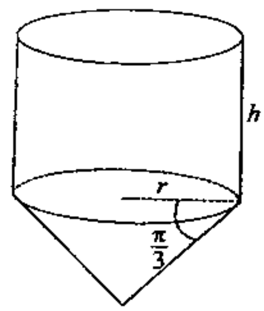
In the first revolution, point A is at least 17 m above the ground level for 2.04 seconds. Since **the second revolution is identical to the first**, the total time for point A to be at least 17 m above the ground = 2(3.519422922 -1.480577078)= 4.08 seconds. R1

B1

(v) Suggest a possible equation of how the height of a blade varies against time if the windmill starts rotating from the highest point at B. [1]

$h = 15 + 7\cos\frac{2\pi}{5}t$ B1

7.



The diagram shows a solid machine part that is made up of a closed cylinder joined to an inverted right circular cone. The height of the cylinder is h m and the slant height of the cone makes an angle of $\frac{\pi}{3}$ radians to its base radius, r m.

(i) Given that the volume of the machine part is 50π m^3, express h in terms of r. [2]

Let the height of the cone be a metre.

$\tan\frac{\pi}{3} = \frac{a}{r}$

$a = \sqrt{3}r$

$50\pi = \pi r^2 h + \frac{1}{3}\pi r^2\left(\sqrt{3}r\right)$ M1

$h = \frac{50}{r^2} - \frac{\sqrt{3}}{3}r$ A1

A1

(ii) Show that the total surface area of the machine part is given by [4]

$$A = \frac{\pi r^2}{3}(9-2\sqrt{3})+\frac{100\pi}{r}.$$

Let the slant height of the cone be p metre.

$\cos\frac{\pi}{3} = \frac{r}{p}$

$p = 2r$

$A = \pi r^2 + 2\pi r h + \pi r(2r)$ M1

$= \pi r^2 + 2\pi r\left(\frac{50}{r^2}-\frac{\sqrt{3}}{3}r\right)+2\pi r^2$ M1

$= 3\pi r^2 + \frac{100\pi}{r} - \frac{2\sqrt{3}}{3}\pi r^2$ M1

$= \frac{\pi r^2}{3}(9-2\sqrt{3})+\frac{100\pi}{r}$ A1

(iii) Given that r can vary, find the value of r for which the total surface area of the machine part is stationary. [3]

$\frac{dA}{dr} = \frac{2\pi r}{3}(9-2\sqrt{3})-\frac{100\pi}{r^2}$ M1

when $\frac{dA}{dr}=0$

$\frac{2r}{3}(9-2\sqrt{3}) = \frac{100}{r^2}$

$r^3 = \frac{300}{2(9-2\sqrt{3})}$ M1

$r = 3.00$ m (3 s.f.) A1

(iv) By comparing gradients, explain why this value of r gives the least total surface area possible. [2]

M1

r	2.99	3.00	3.01
Sketch			

Since $\frac{dA}{dr}$ <u>changes sign from negative to positive</u> as r increases through the stationary point. The total surface area is the least when $r = 3.00$ m. R1

8. The equation of two curves are $y = \cos 2x - 2\sin^2 x$ and $y = \sin 2x$.

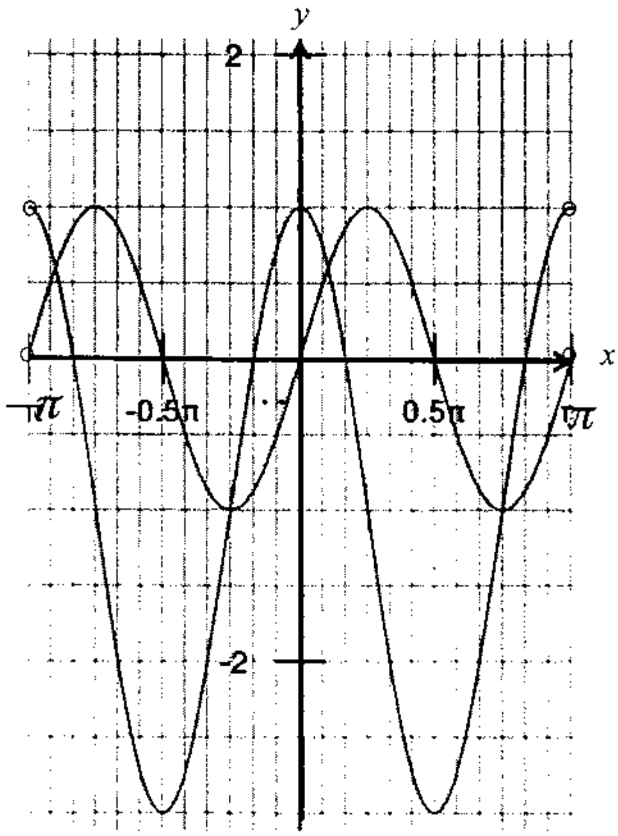
(i) Show that the x-coordinate of the points of intersection of the two curves satisfy $2\cos 2x - \sin 2x = 1$. [1]

$\cos 2x - 2\sin^2 x = \sin 2x$

$\cos 2x + \cos 2x - 1 = \sin 2x$ B1

$2\cos 2x - \sin 2x = 1$

(ii) On the same axes sketch, for $-\pi < x < \pi$, the graphs of $y = \cos 2x - 2\sin^2 x$ and $y = \sin 2x$. [4]



C1: x,y-intercepts
C1: Maximum and Minimum Points

C1: x,y-intercepts
C1: Maximum and Minimum Points

(iii)Express the equation $2\cos 2x - \sin 2x = 1$ in the form $\cos(2x+\alpha) = k$, where α and k are constants to be found. [4]

$2\cos 2x - \sin 2x = 1$

$2\cos 2x - \sin 2x = R\cos(2x+\alpha)$

$= R\cos 2x \cos\alpha - R\sin 2x \sin\alpha$

$R\cos\alpha = 2, R\sin\alpha = 1$ M1

$\tan\alpha = \frac{1}{2} \Rightarrow \alpha = 0.463647609$ M1

$R = \sqrt{5}$ M1

$\sqrt{5}\cos(2x+0.464) = 1$

$\cos(2x+0.464) = \frac{\sqrt{5}}{5}$ A1

(iv)Hence find, in radians, the x-coordinates of the points of intersection for $-\pi < x < \pi$. [3]

$\cos(2x+0.463647609) = \frac{\sqrt{5}}{5}$

Basic angle= 1.107148718

$2x + 0.463647609 = 1.107148718, 5.176036589, -1.107148718, -5.176036589$ M1

$x = 0.322, 2.36, -0.785, -2.82$ A2

Minus 1 for each error

9. A particle travels in a straight line from a fixed point O with acceleration a m/s^2, given by $a = 8t - k$ where t is the time in seconds after passing O, and k is a constant. The velocity of the particle is 5 m/s when it passes O, and at $t = 2$, its velocity is -21 m/s.

(i) Find the value of k. [3]

$a = 8t - k$

$v = \int (8t - k)\,dt$

$= 4t^2 - kt + c$ M1

When $t = 0, v = 5, c = 5$. M1

When $t = 2, v = -21$,

$-21 = 4(2)^2 - 2k + 5$

$k = 21$ A1

(ii) Find the value(s) of t when the particle is instantaneously at rest. [2]

When $v = 0$,

$4t^2 - 21t + 5 = 0$

$(4t-1)(t-5) = 0$ M1

$t = 0.25$ or $t = 5$ A1

(iii)Calculate the average speed of the particle during the first six seconds. [3]

$s = \int (4t^2 - 21t + 5)\,dt$

$= \frac{4}{3}t^3 - \frac{21}{2}t^2 + 5t + C_1$

When $t = 0, s = 0, C_1 = 0$,

$s = \frac{4}{3}t^3 - \frac{21}{2}t^2 + 5t$ M1

When $t = 0, s = 0$ m,

When $t = 0.25, s = 0.164583333$ m,

When $t = 5, s = -70\frac{5}{6}$m,

When $t = 6, s = -60$ m,

Average speed during the first 6 seconds

$= \dfrac{0.614583333 + (0.614583333 + 70\frac{5}{6}) + (70\frac{5}{6} - 60)}{6}$ M1

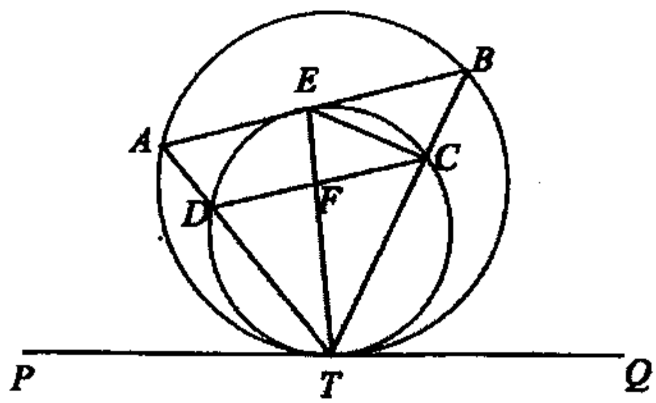
$= 13.8$ m/s A1

(iv)Describe completely the motion of the particle in the first six seconds. [2]

The particle starts at a fixed point O. At $t = 0.25$, the particle stops and reverses its direction of motion. At $t = 5$, the particle stops again and reverses its direction of motion, moving toward O. At $t = 6$, the particle has a displacement of -60 m from O. R2 for 3 points

R1 for 2 points

10.



In the diagram, two circles touch each other at T and PTQ is their common tangent. AB is a tangent to the smaller circle at E. AT and BT cut the smaller circle at D and C respectively. ET and CD intersect at F. Prove that

(i) $AB//DC$, [2]

$\angle BTQ = \angle TAB$ (Alternate Segment Theorem)

$\angle BTQ = \angle TDC$(Alternate Segment Theorem) R1

$\therefore \angle TAB = \angle TDC, AB//DC$ (Corr. $\angle s$) R1

(ii) $\angle ATE = \angle BTE$, [3]

$\angle ATE = \angle ECF$($\angle s$ in same segment) R1

$=\angle BEC$ (alt. $\angle$s, AB//DC) R1

$=\angle BTE$ (alternate segment theorem) R1

(iii) $ET^2 = CT \times DT + EF \times ET$. [3]

$\angle TDF = \angle TEC$ ($\angle$s in same segment)

$\angle DTF = \angle ETC$(part (ii))

ΔDFT and ΔECT are similar. (2 pairs of corr. $\angle$s are equal) R1

$\frac{FT}{CT} = \frac{DT}{ET}$ R1

$ET \times FT = CT \times DT$

$ET(ET - EF) = CT \times DT$ R1

$ET^2 = CT \times DT + EF \times ET$

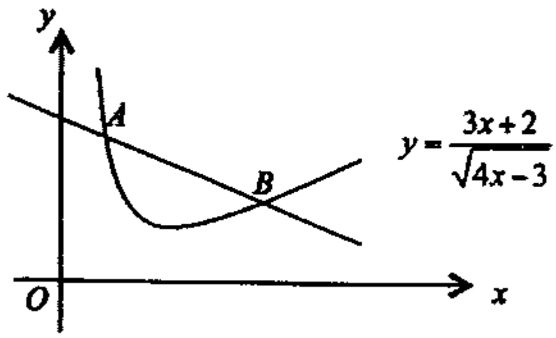
11. **(i)** Differentiate $(x+2)\sqrt{4x-3}$ with respect to x. [2]

$$\frac{d}{dx}(x+2)\sqrt{4x-3} = \frac{2(x+2)}{\sqrt{4x-3}} + \sqrt{4x-3} \qquad \text{M1}$$

$$= \frac{2(x+2)+4x-3}{\sqrt{4x-3}}$$

$$= \frac{6x+1}{\sqrt{4x-3}} \qquad \text{A1}$$

(ii)



The diagram shows part of the curve $y = \frac{3x+2}{\sqrt{4x-3}}$. A line with gradient $-\frac{2}{3}$ intersects the curve at $A(1,5)$ and B.

(a) Verify that the y-coordinate of B is $\frac{11}{3}$. [5]

Equation of AB:

$$y-5 = -\frac{2}{3}(x-1)$$

$$y = -\frac{2}{3}x + \frac{17}{3}$$

$$\frac{3x+2}{\sqrt{4x-3}} = \frac{-2x+17}{3} \qquad \text{M1}$$

$$9x+6 = (-2x+17)\sqrt{4x-3}$$

$$81x^2+108x+36 = 16x^3-12x^2-272x^2+204x+1156x-867$$

$$16x^3-365x^2+1252x-903=0$$

M1

Let $f(x) = 16x^3 - 365x^2 + 1252x - 903$

$f(1) = 0$

$(x-1)$ is a factor of $f(x)$.

$16x^3 - 365x^2 + 1252x - 903 = (x-1)(16x^2 + Bx + 903)$ M1

Comparing coefficient of x^2 :

$-365 = B - 16$

$B = -349$

$(x-1)(16x^2 - 349x + 903) = 0$

$(x-1)(16x - 301)(x-3) = 0$ M1

$x = 1, 3$ or 18.8125(rejected)

when $x = 3, y = \dfrac{3(3)+2}{\sqrt{4(3)-3}} = \dfrac{11}{3}$(shown) A1

(b) Determine the area of the region bounded by the curve and the line AB. [4]

Area of region bounded by curve and line AB

$= \dfrac{1}{2} \times (5 + \dfrac{11}{3}) \times 2 - \int_1^3 \dfrac{3x+2}{\sqrt{4x-3}}\, dx$

$= 8\dfrac{2}{3} - \dfrac{1}{2} \int_1^3 \dfrac{6x+1}{\sqrt{4x-3}} + \dfrac{3}{\sqrt{4x-3}}\, dx$ M1

$= 8\dfrac{2}{3} - \dfrac{1}{2}[(x+2)\sqrt{4x-3}]_1^3 - \dfrac{1}{2}\int_1^3 3(4x-3)^{-\frac{1}{2}}\, dx$

$= 8\dfrac{2}{3} - \dfrac{1}{2}[(x+2)\sqrt{4x-3}]_1^3 - \dfrac{1}{2}\left[\dfrac{3(4x-3)^{\frac{1}{2}}}{4\left(\dfrac{1}{2}\right)}\right]_1^3$ M1

$= 8\dfrac{2}{3} - \dfrac{1}{2}[(x+2)\sqrt{4x-3}]_1^3 - \dfrac{3}{4}\left[\sqrt{4x-3}\right]_1^3$

$= 8\dfrac{2}{3} - \dfrac{1}{2}(5\sqrt{9} - 3\sqrt{1}) - \dfrac{3}{4}(\sqrt{9} - \sqrt{1})$ M1

$= 1\dfrac{1}{6}$ units^2 A1

NAME: ______________________________ (   )   CLASS: _______

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2015
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

ADDITIONAL MATHEMATICS   4047/01

Paper 1

Date: 25 August 2015   **Duration: 2 hours**

Additional Materials: Answer Paper
Graph paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

At the end of the examination, fasten all your work securely together.

For Examiner's Use	
Paper 1	/ 80

Setter: Miss Lee CP

This question paper consists of 6 printed pages including the cover page.

,05

Name: ______________________________ (   )   Class: __________

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Name: ______________________________ (   )   Class: __________

1 Given that $y=\left(1-\tan^2 x\right)\cos^2 x$, show that $\frac{dy}{dx}=-2\sin 2x$. [3]

2 Express $\frac{8-3x}{(1-x)^2(2x+3)}$ as a sum of 3 partial fractions. [5]

3 The pressure, P, and volume, V, of a gas in a container are related by the formula $P=\frac{2500}{\sqrt{V^3}}$. If the pressure increases at a rate of 2.8 units/second, find the rate of change of volume when the pressure of the gas is 50 units. [5]

4 Find the term independent of x in the expansion of $(5-4x)\left(\frac{3x^2}{2}+\frac{2}{3x}\right)^9$. [5]

5 The equation of a curve is $y=\ln(5-2x)$, where $x<\frac{5}{2}$.

(i) Find the coordinates of the point on the curve at which the normal to the curve is parallel to $2y=x+3$. [4]

(ii) Show that as x increases, y is a decreasing function. [2]

6 If $\sin(A+B)=3\sin(A-B)$, show that $\tan A=2\tan B$.
Hence, solve the equation $\sin^2(x+60°)=9\sin^2(x-60°)$ for $0°<x<360°$. [7]

7 Find all angles, leaving your answer in terms of π, between 0 and 6 which satisfy

(i) $4\sin\frac{x}{2}\cos\frac{x}{2}=\sqrt{3}$ [3]

(ii) $\sin^4 x-\cos^4 x-3\cos x=2$. [5]

to2

Name: ______________________________ (   )   Class: __________

8 **(a)** Given that the curve, $y = 4x^2 + px + p - 6$, find the possible range or value(s) of p for which

(i) the curve intersects the line $y = -3$, [3]

(ii) the line $y = -3$ is a tangent to the curve, [1]

(iii) the curve has a positive y-intercept. [1]

(b) Show that $(m + 1)x^2 + (4m + 3)x + 2m - 1 = 0$ has real and distinct roots for all real values of m. [3]

9 **(i)** Sketch the graph of $y = |2x^2 - 3x - 14|$ for $0 \leq x \leq 5$. [3]

(ii) Using your graph, find the range or value(s) of k for each of the number of solutions for the equation $|2x^2 - 3x - 14| = k$.

(a) 3 solutions, [1]

(b) 2 solutions, [2]

(c) 1 solution. [1]

10 **Answer the whole of this question on a graph paper.**

The table below shows experimental values of the variables x and y which are related by an equation of the form $y = a^{b+x}$. One value of y has been recorded incorrectly.

x	0.1	0.2	0.3	0.4	0.5
y	5.9	6.9	7.2	9.4	11.0

(i) Plot $\lg y$ against x and draw a straight line graph. [2]

(ii) Use your graph to estimate the value of a and of b. [4]

(iii) Determine which value of y is inaccurate and estimate the correct value of y. [2]

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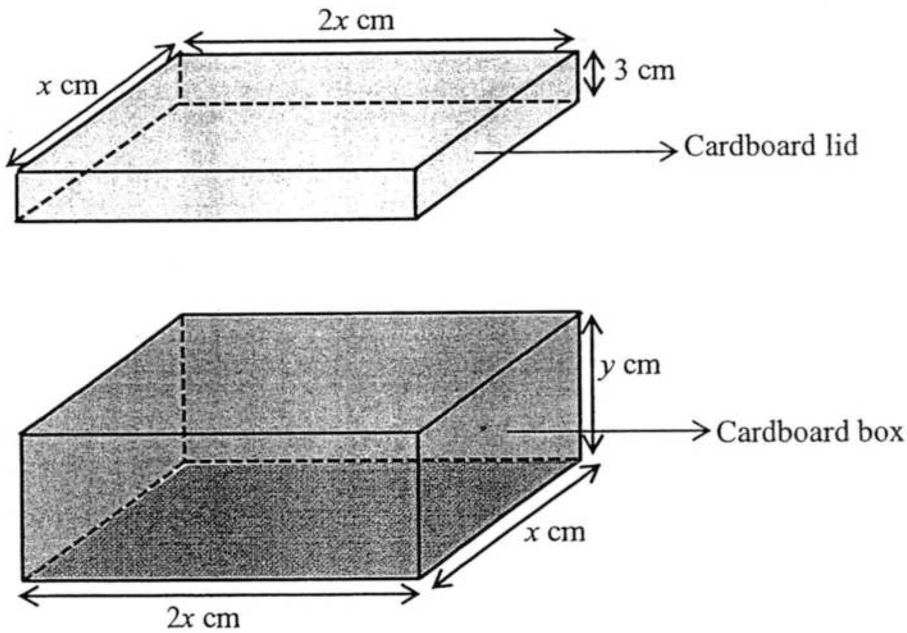

The diagram shows an open cardboard box with a rectangular base and a close fitting cardboard lid which slips over the top of the box.

The dimensions of the lid are $2x$ cm, x cm and 3 cm. The total area of cardboard used in making the box and the lid is 2400 cm^2.

(i) Obtain an expression for y in terms of x, and hence show that the volume, V cm^3 of the box is given by $V = 800x - \frac{4x^3}{3} - 6x^2$. [4]

(ii) Given that x can vary, find the value of x for which volume of the box is stationary. Calculate this stationary value of V. [4]

(iii) Explain why this value of x gives the largest volume of the box. [1]

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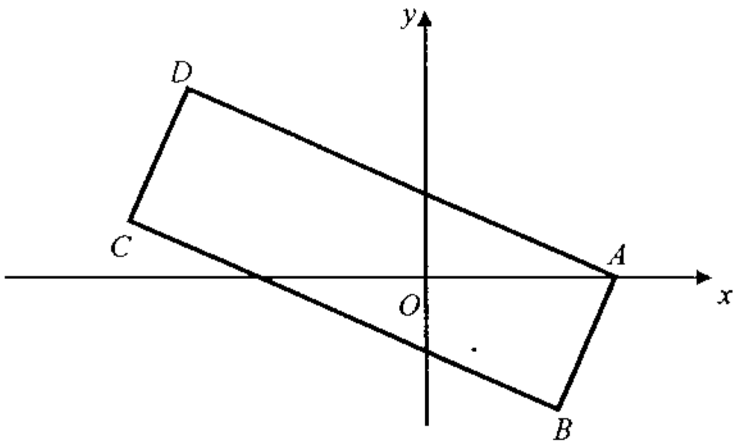
12 Solutions to this question by accurate drawing will not be accepted.



In the rectangle $ABCD$, the coordinates are $A(3, 0)$, $B(-2t-1, t-2)$ and $C(-5, 1)$.

(i) Show that the value of $t=-1$. [3]

Find

(ii) the coordinate of D, [2]

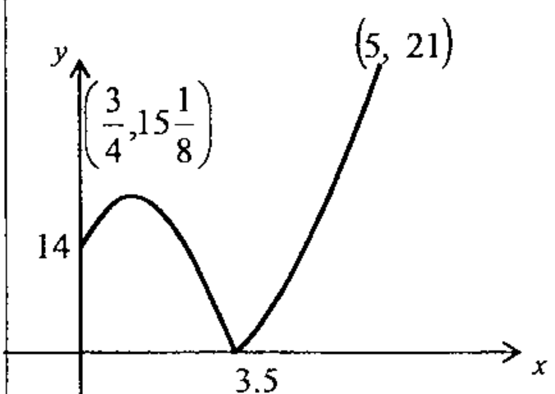
(iii) the equation of perpendicular bisector of AD, [2]

(iv) the area of $ABCD$. [2]

~ **End of Paper** ~

Name: ______________________________ (   )   Class: __________

Sec 4/5 Preliminary Examination 2015
Additional Mathematics Paper 1
Answer Key

2	$\frac{1}{1-x}+\frac{1}{(1-x)^2}+\frac{2}{2x+3}$	3	Rate of change of volume = - 0.507 units/sec
4	$\frac{1190}{9}$	5(i)	(2, 0)
6	73.9°, 253.9°, 40.9°, 220.9°		
7(i)	$\frac{\pi}{3}, \frac{2\pi}{3}$	7(ii)	$x=\frac{2}{3}\pi, \frac{4}{3}\pi$ or $x=\pi$
8(a)(i)	$p\le 0$ or $p\ge 12$	8(a)(ii)	$p = 12$ or $p = 4$
8(a)(iii)	$p > 6$		
9(i)		9(ii)(a) 9(ii)(b) 9(ii)(c)	$14\le k<15\frac{1}{8}$ $0<k<14$ or $k=15\frac{1}{8}$ $k=0$ or $k>15\frac{1}{8}$
10(ii)	When $x = 0.3$, $y = 7.2$ (erroneous) The correct value of $y = 10^{0.905}$ $= 8.04$ (3 s.f.) Accept 7.76 to 8.14	10(iii)	$a = 10^{0.68} = 4.79$ (3 s.f.) Accept 4.68 to 4.82 $b=\frac{0.7}{0.68}=1.03$ Accept 1.02 to 1.06
11(ii)	6460 cm^3 (3 s.f.)	12(ii)	D (- 3, 4)
12(iii)	$y-2=\frac{3}{2}x$ or $y=\frac{3}{2}x+2$	13(iv)	26 $units^2$

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Secondary 4 Express Additional Mathematics
Preliminary Examination 2015
Marking Scheme

No	Working	Description
1	$y=(1-\tan^2 x)\cos^2 x$ $y=\cos^2 x-\tan^2 x\cos^2 x$ $y=\cos^2 x-\sin^2 x$ $y = \cos 2x$ $\frac{dy}{dx}=-2\sin 2x$ = LHS	M1 [expansion] M1 [Substitute identity] AG1
2	$\frac{8-3x}{(1-x)^2(2x+3)}=\frac{A}{1-x}+\frac{B}{(1-x)^2}+\frac{C}{2x+3}$ $8-3x = A(1-x)(2x+3) + B(2x+3) + C(1-x)^2$ When $x = 1$, $8-3 = B(5)$ $B = 1$ When $x = -1.5$, $8-3(-1.5) = C(1-(-1.5))^2$ $12.5 = 6.25C$ $C = 2$ When $x = 0$, $8 = A(1)(3)+B(3)+C(1)$ $8 = 3A + 3 + 2$ $3 = 3A$ $A = 1$ $\frac{8-3x}{(1-x)^2(2x+3)}=\frac{1}{1-x}+\frac{1}{(1-x)^2}+\frac{2}{2x+3}$	B1 [correct partial fraction formula] M1 [Substitution / comparing coefficient method] A1 for value of A / B / C A1 for all values correct B1

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<table>
<tr><th>No</th><th>Working</th><th>Description</th></tr>
<tr><td>3</td><td>

$P = \dfrac{2500}{\sqrt{V^3}}$

$P = 2500V^{-\frac{3}{2}}$

$\dfrac{dP}{dV} = -\dfrac{3}{2} \times 2500 \times V^{-\frac{5}{2}}$

$= -3750V^{-\frac{5}{2}}$ or $-\dfrac{3750}{\sqrt{V^5}}$

When P = 50, $\sqrt{V^3} = \dfrac{2500}{50} = 50$

$V = 50^{\frac{2}{3}}$

$\dfrac{dP}{dt} = \dfrac{dP}{dV} \times \dfrac{dV}{dt}$

$\dfrac{dV}{dt} = \dfrac{dP}{dt} \times \left(\dfrac{dV}{dP}\right)$

$= 2.8 \times -\dfrac{\sqrt{\left(50^{\frac{2}{3}}\right)^5}}{3750} = -0.50669 = -0.507$ units/sec

Rate of change of volume = - 0.507 units/sec

</td><td>

M1 [apply differentiation rule]
A1 [differentiation correctly]

B1 [find corresponding value of V]

M1 [apply chain rule correctly]

A1

</td></tr>
<tr><td>4</td><td>

$(5-4x)\left(\dfrac{3x^2}{2} + \dfrac{2}{3x}\right)^9$

General term for $\left(\dfrac{3x^2}{2} + \dfrac{2}{3x}\right)^9 = \dbinom{9}{r}\left(\dfrac{3x^2}{2}\right)^{9-r}\left(\dfrac{2}{3x}\right)^r$

$= \dbinom{9}{r}\left(\dfrac{3}{2}\right)^{9-r}\left(\dfrac{2}{3}\right)^r x^{18-2r-r}$

$= \dbinom{9}{r}\left(\dfrac{3}{2}\right)^{9-r}\left(\dfrac{3}{2}\right)^{-r} x^{18-3r}$

$= \dbinom{9}{r}\left(\dfrac{3}{2}\right)^{9-2r} x^{18-3r}$

For term independent of x, 18 – 3r = 0 → r = 6

For term with coefficient of x, 18 – 3r = -1 → r = $\dfrac{19}{3}$ → there is no term with $\dfrac{1}{x}$.

$(5-4x)\left(\dfrac{3x^2}{2} + \dfrac{2}{3x}\right)^9 = (5-4x)\left(\ldots + \dbinom{9}{6}\left(\dfrac{3}{2}\right)^{9-12} + \ldots\right)$

Term independent of x = $5 \times \dbinom{9}{6}\left(\dfrac{3}{2}\right)^{-3} = \dfrac{1190}{9}$

</td><td>

M1 [find general term] or [expansion till the 6th and 7th term]

M1 [correct simplification of index for x]

M1 [find the value of r, must equate the index to 0, if r is not a whole number no marks.]

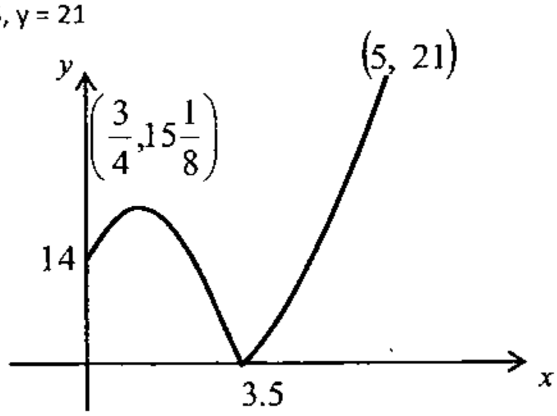
M1 [expansion]

A1

</td></tr>
</table>

No	Working	Description
5(i)	$y = \ln(5-2x)$ $\frac{dy}{dx} = \frac{-2}{5-2x}$ $2y = x + 3$ $y = \frac{x}{2} + \frac{3}{2}$ Gradient of normal $= \frac{1}{2}$ Gradient of tangent = -2 $\frac{-2}{5-2x} = -2$ $5-2x = 1$ $-2x = -4$ $x = 2$ When x = 2, y = ln (5-4) = 0 The coordinates is (2, 0).	B1 [differentiation] B1 [gradient of tangent] M1 [solve for x] A1
5(ii)	$\frac{dy}{dx} = \frac{-2}{5-2x}$ For $x < \frac{5}{2}$, (5 – 2x) > 0 and -2 < 0 Therefore $\frac{-2}{5-2x} < 0$, which implies that dy/dx is <0 Since dy/dx < 0, then y is decreasing.	B1 B1
6	sin $(A + B)$ = 3 sin $(A - B)$, show that tan A = 2 tan B $\sin(A+B) = 3\sin(A-B)$ $\sin A\cos B + \cos A\sin B = 3\sin A\cos B - 3\cos A\sin B$ $4\cos A\sin B = 2\sin A\cos B$ $2\cos A\sin B = \sin A\cos B$ $\frac{2\cos A\sin B}{\cos A\cos B} = \frac{\sin A\cos B}{\cos A\cos B}$ $2\tan B = \tan A$	M1 [simplification] M1 [to get tan function] AG1
	$\sin^2(x+60°) = 9\sin^2(x-60°)$ for $0° < x < 360°$ $\sin(x+60°) = \pm 3\sin(x-60°)$ Case 1: $\sin(x+60°) = 3\sin(x-60°)$ Let A = x and B = 60 Therefore, $\tan x = 2\tan 60°$ $\tan x = 2\sqrt{3}$ Basic angle = 73.898 x =73.898, 180 + 73.898 $= 73.9°,\ 253.9°$	M1 A1
	Case 2: $\sin(x+60°) = -3\sin(x-60°)$ Let A = 60 and B = x Therefore, $2\tan x = \tan 60°$ $\tan x = \frac{\sqrt{3}}{2}$	M1 no

No	Working	Description
	Basic angle =40.893 x =40.893, 180 + 40.893 =40.9°, 220.9°	A1
7(i)	$4\sin\frac{x}{2}\cos\frac{x}{2}=\sqrt{3}$ $2\sin\frac{x}{2}\cos\frac{x}{2}=\frac{\sqrt{3}}{2}$ $\sin x=\frac{\sqrt{3}}{2}$ $\alpha=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{3}$ $x=\frac{\pi}{3},\pi-\frac{\pi}{3}$ $x=\frac{\pi}{3},\frac{2\pi}{3}$	M1 [Apply double angle identity] A1, A1
7(ii)	$\sin^4 x-\cos^4 x-3\cos x=2$ $(\sin^2 x-\cos^2 x)(\sin^2 x+\cos^2 x)-3\cos x=2$ $1-2\cos^2 x-3\cos x=2$ $2\cos^2 x+3\cos x+1=0$ $(2\cos x+1)(\cos x+1)=0$ $\cos x=-0.5$ or $\cos x=-1$ $x=\frac{2}{3}\pi,\ \frac{4}{3}\pi$ or $x=\pi$	B1 [Factorization] M1 [Use identity and simplify to a quadratic function in terms of cos x] M1 [Factorization] A1, A1
8(a)(i)	$y=4x^2+px+p-6$ $y=-3$ $4x^2+px+p-6=-3$ $4x^2+px+p-3=0$ For line intersect the curve, $b^2-4ac\geq 0$ $p^2-4(4)(p-3)\geq 0$ $p^2-16p+48\geq 0$ $(p-12)(p-4)\geq 0$ $p\leq 4$ or $p\geq 12$	M1 [correct discriminant value] M1 [factorization] A1
8(a)(i)	For tangent line, $b^2-4ac=0$ $p^2-4(4)(p-3)=0$ $p^2-16p+48=0$ $(p-12)(p-4)=0$ $p=12$ or $p=4$	B1
8(a)(ii)	$y=4x^2+px+p-6$ For y-intercept is positive, $p-6>0$ $p>6$	B1

No	Working	Description
8(b)	Show that $(m + 1)x^2 + (4m + 3)x + 2m - 1 = 0$ has real and distinct roots for all real values of m.	
	$b^2\text{-}4ac = (4m + 3)^2 - 4(m+1)(2m - 1)$ $= 16m^2 + 24m + 9 - 4(2m^2 + m - 1)$ $= 16m^2 + 24m + 9 - 8m^2 - 4m + 4$ $= 8m^2 + 20m + 13$ $= 8\left[m^2 + \frac{5}{2}m\right] + 13$ $= 8\left(m + \frac{5}{4}\right)^2 + \frac{1}{2}$ Since $\left(m + \frac{5}{4}\right)^2 \geq 0$, therefore, $b^2 - 4ac \geq 0.5$ Therefore, the roots are real and distinct.	M1 [Work out discriminant expression] M1 [Complete the square] A1 [Explanation]
9(i)	$y = \lvert 2x^2 - 3x - 14 \rvert$ for $0 \leq x \leq 5$. $y = 2x^2 - 3x - 14$ y-intercept coordinate is (0, - 14) x-intercept, y = 0, $(2x - 7)(x + 2) = 0$ $x = 3.5$ or $x = -2$ x-coordinate of minimum point $= \frac{3.5 + (-2)}{2} = \frac{3}{4}$ Minimum point is $\left(\frac{3}{4}, -15\frac{1}{8}\right)$ x = 5, y = 21 	Shape of graph [S1] (correct position of maximum point of the graph with one x-intercept P1 [Points of the graph, shows maximum point, and end points] P1 [Show x-intercept and y-intercept]
9(ii)(a)	$14 \leq k < 15\frac{1}{8}$	B1
9(ii)(b)	$0 < k < 14$ or $k = 15\frac{1}{8}$	B1, B1
9(ii)(c)	$k = 0$ or $k > 15\frac{1}{8}$	B1

No	Working	Description
10(i)	<table><tr><td>x</td><td>0.1</td><td>0.2</td><td>0.3</td><td>0.4</td><td>0.5</td><td>0.6</td></tr><tr><td>$\lg y$</td><td>0.771</td><td>0.839</td><td>0.857</td><td>0.973</td><td>1.04</td><td>1.11</td></tr></table>	P1 [Plot points] S1 [Straight line graph
10(ii)	$y = a^{b+x}$ $\lg y = (b+x)\lg a = b\lg a + x\lg a$ $\lg a$ = gradient = $\frac{1.11-0.77}{0.6-0.1} = 0.68$ $a = 10^{0.68} = 4.79$ (3 s.f.) <u>Accept 4.68 to 4.82</u> $b \lg a = 0.7$ which is the $\lg y$-intercept $b = \frac{0.7}{0.68} = 1.03$ <u>Accept 1.02 to 1.06</u>	B1 [convert to straight line graph] M1 A1 B1
10(iii)	When $x = 0.3$, $y = 7.2$ (erroneous) The correct value of $y = 10^{0.905} = 8.04$ (3 s.f.) <u>Accept 7.76 to 8.14</u>	B1 B1
11 (i)	Total surface area of cardboard, A = 2400 $A = 2x^2 + 2(2xy) + 2(xy) + 2x^2 + 2(3x) + 2(6x)$ $A = 6xy + 4x^2 + 18x$ $2400 = 6xy + 4x^2 + 18x$ $y = \frac{2400 - 4x^2 - 18x}{6x}$ $y = \frac{400}{x} - \frac{2x}{3} - 3$ or $y = \frac{1200 - 2x^2 - 9}{3x}$ Volume of box, $V = 2x^2y$ $V = 2x^2\left(\frac{400}{x} - \frac{2x}{3} - 3\right)$ $= 800x - \frac{4x^3}{3} - 6x^2$	B1 [Form correct expression of for surface area] B1 [Form equation for surface area] B1 [make y the subject of formula] AG1
11(ii)	$\frac{dV}{dx} = 800 - 4x^2 - 12x$ At stationary value of V, $\frac{dV}{dx} = 0$ $800 - 4x^2 - 12x = 0$ $4x^2 + 12x - 800 = 0$ $x^2 + 3x - 200 = 0$ $x = \frac{-3 \pm \sqrt{9 - 4(3)(-200)}}{2}$ x = 12.721 or x = - 15.721 x = 12.7 (3 s.f.) Stationary value of V = $800(12.721) - \frac{4}{3}(12.721)^3 - 6(12.721)^2$ = 6461.108846 = 6460 (3 s.f.)	B1 [Differentiate the expression] B1 [equate dy/dx = 0] B1 [solve for x] B1

No	Working	Description
11(iii)	$\frac{d^2V}{dx^2} = -8x - 12$ When x = 12.721, $\frac{d^2V}{dx^2} = -8(12.721) - 12 = -113.768$ Since $\frac{d^2V}{dx^2} < 0$, therefore the volume is maximum.	B1 [explanation with 2^{nd} derivative]
12(i)	gradient of AB × gradient of BC = -1 $\frac{t-2-1}{-2t-1+5} \times \frac{t-2-0}{-2t-1-3} = -1$ $\frac{t-3}{-2t+4} \times \frac{t-2}{-2t-4} = -1$ $\frac{t-3}{-2(t-2)} \times \frac{t-2}{-2(t+2)} = -1$ $t-3 = -4(t+2)$ $t-3 = -4t-8$ $5t = -5$ $t = -1$	M1 M1 [simplification] AG1
12(ii)	Mid-point of AC = mid-point of BD $\left(\frac{3+(-5)}{2}, \frac{0+1}{2}\right) = \left(\frac{1+x}{2}, \frac{-3+y}{.2}\right)$ $x = -3,\ y = 4$ $D(-3,4)$	M1 A1
12(iii)	Gradient of AB = $\frac{-3-0}{1-3} = \frac{3}{2}$ Midpoint of AD = $\left(\frac{3+(-3)}{2}, \frac{0+4}{2}\right) = (0,2)$ Equation of perpendicular bisector of AD: $y - 2 = \frac{3}{2}x$ or y = $\frac{3}{2}x + 2$	M1 [either midpoint or gradient of AB] A1
12(iv)	Area of rectangle ABCD = $\frac{1}{2}\left\lvert \begin{matrix} 3 & 1 & -5 & -3 & 3 \\ 0 & -3 & 1 & 4 & 0 \end{matrix} \right\rvert$ =0.5 |(-9 + 1 – 20) –(15 – 3 + 12)| = 26 units^2 Alternative method: use distance formula Area of rectangle = length x breadth	M1 A1

ıı2

NAME: ________________________ (   )   CLASS: ______

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2015
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

ADDITIONAL MATHEMATICS   4047/02

Paper 2

Date: 26 August 2015   **Duration: 2 hours 30 minutes**

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

At the end of the examination, fasten all your work securely together.

For Examiner's Use	
Paper 2	/ 100

Setter: Mdm Haliza

This question paper consists of 6 printed pages including the cover page.

1/3

Name: ______________________________ (   )   Class: __________

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Name: ______________________________ (   )   Class: __________

1 The roots of the quadratic equation $3x^2 - kx + 4 = 0$, where $k > 0$, are α and β, and that of the equation $12x^2 - x + 12 = 0$ are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. Find the value of k. [7]

2 A weather satellite orbits planet P such that the equation of its path can be represented by the equation

$$x^2 + y^2 - 18x - 14y + 65 = 0$$

where x and y are the longitudinal and the latitudinal distances from the centre of P respectively in kilometres, as shown on an astronomical map.

(i) State the coordinates of the centre and the radius of the orbit. [3]

A second satellite orbits another planet K in the same plane as the first satellite. The diameter of its circular orbit has end points (10, 9) and (22, 3).

(ii) Find the equation of the path of this satellite. [4]

3 Without using a calculator,

(i) find the value of r and of n, given that $\frac{3x^r}{r^2} \times \frac{2(r^{6-r})^2}{27x} = nx^2$, [5]

(ii) simplify $\frac{3+\sqrt{2}}{2\sqrt{2}-1}$ in the form $a + b\sqrt{2}$. [3]

4 (i) Solve the equation $\log_3(x+2) = 3 - \log_3(x-4)$. [4]

(ii) Given that $\log_x y + \log_y x - \frac{5}{\log_x y} = 0$, express y in terms of x. [4]

5 (i) Solve the equation $4\cos 2x + 2\sin x = -2$ for $0 \le x \le 2\pi$. [6]

(ii) On the same axes, sketch the graphs of

$$y = 3\cos 2x \text{ and } y = |\sin x|$$

for the interval $0 \le x \le 2\pi$, labelling each graph clearly.

State the number of solutions in the interval $0 \le x \le 2\pi$ of the equation

$3\cos 2x = |\sin x|$. [4]

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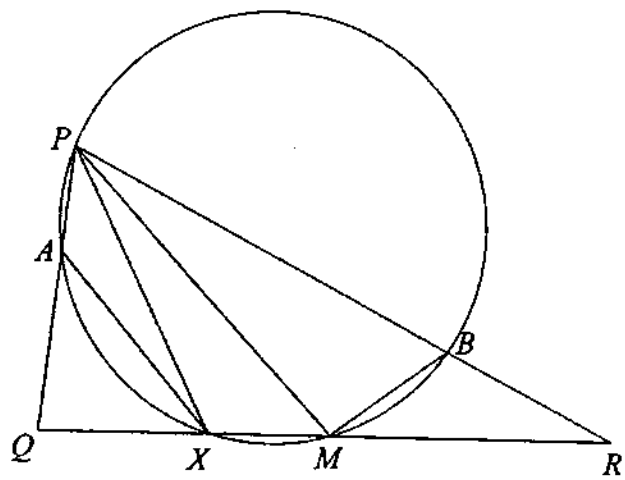
6 In the triangle PQR, M is the mid-point of QR and PX bisects angle QPR.

The circle passing through P, X and M, cuts PQ and PR at A and B respectively.



(i) Explain why $\angle PBM + \angle PXM = 180°$. [1]

(ii) Show that ΔRBM is similar to ΔRXP. [3]

(iii) Given that ΔQXA is also similar to ΔQPM and $\frac{PR}{RX} = \frac{PQ}{QX}$, show that

$RB = QA$. [4]

7 A metal ball is heated to a temperature of $225°C$ before being dropped into a liquid.

As the ball cools, its temperature, $T°C$, t minutes after it enters the liquid is given by

$T = P + 190e^{-kt}$, where P and k are constants.

(i) Explain why $P = 35$. [1]

When $t = 4$, the temperature of the ball reaches $120°C$.

(ii) Find the value of k correct to 3 significant figures. [3]

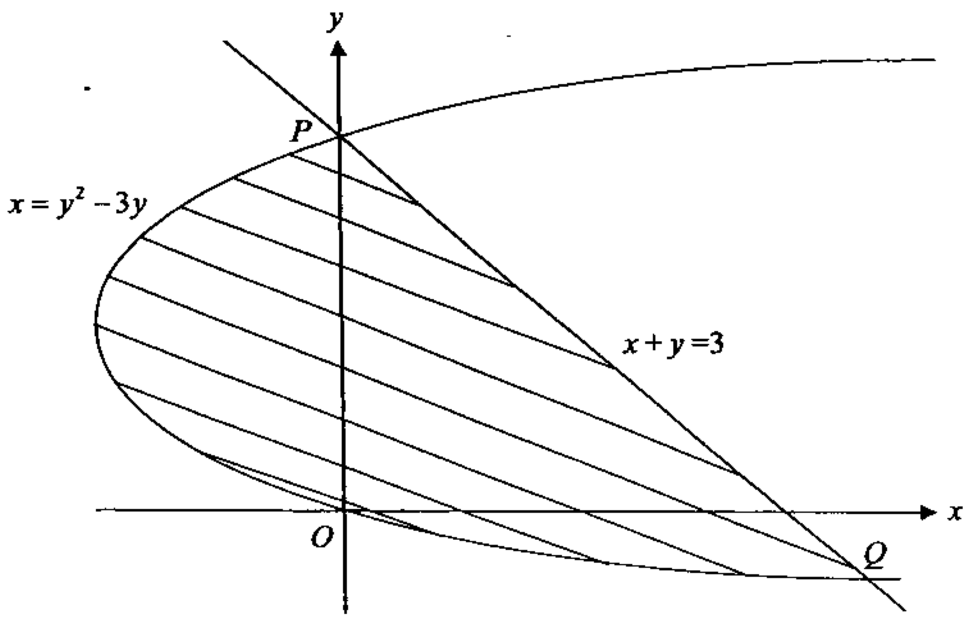
(iii) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 10$. [3]

(iv) From the equation of T given above, explain why the temperature of the ball can never fall below $35°C$. [2]

8 The diagram shows part of the curve $x = y^2 - 3y$ and the line $x + y = 3$. If the line and the curve intersect at P and Q, find

(i) the coordinates of P and Q, [5]

(ii) the area of the shaded region. [5]



9 $f(x) = 6x^3 + ax^2 + bx - 6$ has a factor $x + 2$ but leaves a remainder of -12 when divided by $x - 1$.

(i) Find the value of a and of b. [5]

(ii) Factorise $f(x)$ completely and hence solve the equation

$48x^3 + 4ax^2 = 6 - 2bx$. [6]

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Name: ______________________________ (   )   Class: __________

10 An object at A, with an initial displacement of 3m from a fixed point O, travels in a straight line so that its velocity, v ms^{-1}, is given by $v = t^2 - 5t + 6$ where t is the time in seconds after leaving A.

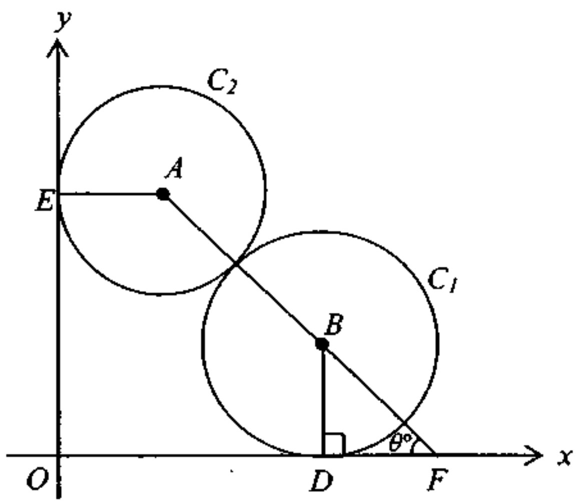
(i) Find the values of t when the object comes to an instantaneous rest. [2]

(ii) Find the acceleration of the object at $t = 5$ s. [2]

(iii) Obtain an expression, in terms of t, for the displacement of the object from O after t seconds. [3]

(iv) Find the average speed of the object in the first 5 seconds. [4]

11 The figure shows two circles C_1 and C_2 which touch each other and lie in the xy-plane as shown below. C_1 has radius 4 units and touches the x-axis at D, C_2 has radius 3 units and touches the y-axis at E. The line AB, joining the centres of C_2 and C_1, meets the x-axis at F and $\angle BFO = \theta°$.



(i) Obtain expressions for OD and OE in terms of θ and show that

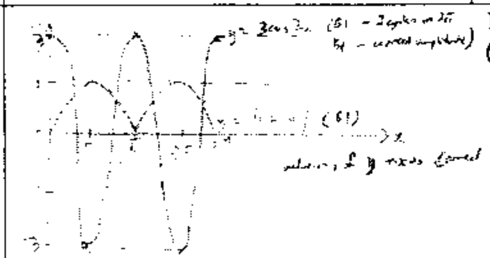
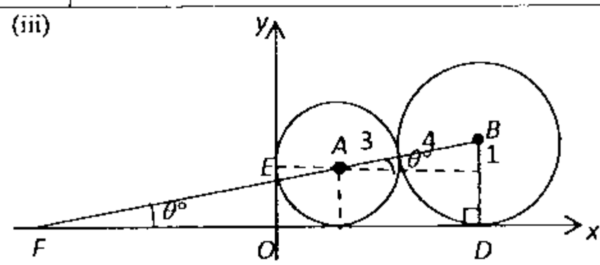
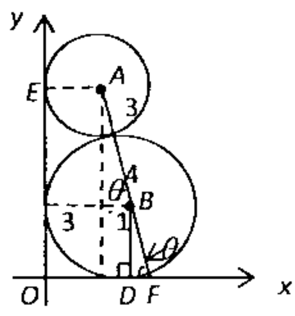
$$ED^2 = 74 + 56\sin\theta + 42\cos\theta. \quad [4]$$

(ii) Express ED^2 in the form $74 + R\cos(\theta - \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [4]

(iii) By considering the extreme positions in which both circles touch the x-axis and both circles touch the y-axis, show that $8.2° \le \theta \le 81.8°$, correct to one decimal place. [3]

~ **End of Paper** ~

FMSS A. Math Preliminary Examination 2015 Paper 2 Answer Key

1.	$k=5$	8(i)	$P(0,3), Q(4,-1)$
2(i)	Centre (9, 7), radius = $\sqrt{65}$ or 8.06km	(ii)	$10\frac{2}{3}$ or 10.7 or $\frac{32}{3}$ sq. units
(ii)	$(x-16)^2+(y-6)^2=45$ or $x^2+y^2-32x-12y+247=0$	9(i)	$a=5, b=-17$
3(i)	$r=3, n=18$	(ii)	$f(x)=(x+2)(3x-1)(2x-3)$ Hence, $(2x+2)(3(2x)-1)(2(2x)-3)=0$ $\therefore x=-1, -\frac{1}{6}, \frac{3}{4}$
(ii)	$1+\sqrt{2}$	10 (i)	$t=2$ or 3
4(i)	$x=-5$ (N.A.) or $x=7$	(ii)	$a=5\ \text{m/s}^2$
(ii)	$y=x^2$ or $y=x^{-2}$	(iii)	$s=\frac{t^3}{3}-\frac{5t^2}{2}+6t+3$
5(i)	$x=\frac{\pi}{2}$ (or 1.57), 3.99, 5.44	(iv)	1.9 m/s
(ii)		11 (i)	$OD=3+7\cos\theta$ $OE=4+7\sin\theta$ $ED^2=OD^2+OE^2$ (By Pythagoras' Theorem)
6(i)	$\angle PBM$ and $\angle PXM$ are <u>angles in opposite segments</u> of the cyclic quadrilateral $PXMB$. Therefore the sum of these two angles is supplementary.	(ii)	$ED^2=74+70\cos(\theta-53.1°)$.
(ii)	$\angle BRM = \angle PRX$ (common angles of ΔRBM and ΔRXP) $\angle XPR = \angle BMR$ (ext. $\angle$ of cyclic quad.) or $\angle RBM = \angle PXM$ (ext. $\angle$ of cyclic quad.) $\therefore \Delta RBM$ is similar to ΔRXP (by AA similarity test)	(iii)	 When both circles touch the x-axis, $\sin\theta=\frac{1}{7} \Rightarrow \theta=8.2°$ (to 1 dec. pl.)  When both circles touch the y-axis, $\cos\theta=\frac{1}{7} \Rightarrow \theta=81.8°$ (to 1 dec. pl.) $\therefore 8.2° \le \theta \le 81.8°$ (Shown)
(iii)	Since ΔRBM is similar to ΔRXP, $\frac{PR}{RX}=\frac{RM}{RB}$. Since ΔQXA is also similar to ΔQPM, $\frac{PQ}{XQ}=\frac{QM}{QA}$ Given that $\frac{PR}{RX}=\frac{PQ}{QX}$, $\therefore \frac{RM}{RB}=\frac{QM}{QA}$. Since $QM=MR$ as M is the mid-point of QR, $\therefore RB=QA$ (Shown)		
7(i)	At $t=0$, $T=225^{0}$C. $\therefore 225=P+190e^{-k(0)}$ $P=225-190=35$		
(ii)	$k=0.201$		
(iii)	5.11^{0}C / min.		
(iv)	As $e^{-0.201t}>0$ for $t\ge 0$, $190e^{-0.201t}>0$ $35+190e^{-0.201t}>35$. $\therefore T>35$. Hence the temperature of the ball can never fall below 35^{0} C.		

u6

FMSS A. Math Preliminary Examination 2015 Paper 2 Marking Scheme

1.	$3x^2-kx+4=0$ have roots α and β. $\alpha+\beta=\frac{k}{3}$ $\alpha\beta=\frac{4}{3}$ $12x^2-x+12=0$ have roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{1}{12}$ $\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{1}{12}$ $\frac{\left(\frac{k}{3}\right)^2-2\left(\frac{4}{3}\right)}{\frac{4}{3}}=\frac{1}{12}$ $k^2-24=1$ $k^2=25$ $k=5$ (since $k>0$)	B1 B1 B1 M1 M1 M1 (simplify eqn) A1
2(i)	$x^2+y^2-18x-14y+65=0$ Centre of orbit (-g, -f) = (9, 7) Radius $=\sqrt{(-9)^2+(-7)^2-65}$ $=\sqrt{65}$ or 8.06 km	B1 M1 A1/ B2
(ii)	Diameter of orbit $=\sqrt{(10-22)^2+(9-3)^2}$ $=\sqrt{180}$ Radius of orbit $=\frac{\sqrt{180}}{2}$ Mid - point of orbit $=\left(\frac{10+22}{2},\frac{9+3}{2}\right)$ $=(16,6)$ $\therefore$ Equation of path of this satellite is $(x-16)^2+(y-6)^2=\left(\frac{\sqrt{180}}{2}\right)^2$ $(x-16)^2+(y-6)^2=45$ or $x^2+y^2-32x-12y+247=0$	M1 A1 B1 B1

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3(i)	$\frac{3x^r}{r^2} \times \frac{2(r^{6-r})^2}{27x} = nx^2$ $\frac{2}{9} r^{12-2r-2} x^{r-1} = nx^2$ $r-1=2$ $r=3$ $\frac{2}{9} r^{10-2r} = n$ Sub. $r=3$, $n = \frac{2}{9}(3)^{10-2(3)}$ $=18$	M1 (Simplify powers of r and x) M1 (Equate powers of x) A1 M1 (Equate scalar) A1
(ii)	$\frac{3+\sqrt{2}}{2\sqrt{2}-1}$ $= \frac{3+\sqrt{2}}{2\sqrt{2}-1} \times \frac{2\sqrt{2}+1}{2\sqrt{2}+1}$ $= \frac{6\sqrt{2}+3+4+\sqrt{2}}{8-1}$ $= \frac{7\sqrt{2}+7}{7}$ $=1+\sqrt{2}$	M1 (rationalise denominator) M1 (simplify) A1
4(i)	$\log_3(x+2) = 3 - \log_3(x-4)$ $\log_3(x+2)(x-4) = 3$ $x^2-2x-8=3^3$ $x^2-2x-35=0$ $(x+5)(x-7)=0$ $x=-5$ (N.A.) or $x=7$	M1 (Apply product law) M1 (Change from log to index form) M1 (factorise) A1

(ii)	$\log_x y + \log_y x - \dfrac{5}{\log_x y} = 0$	
	$\log_x y + \dfrac{\log_x x}{\log_x y} - \dfrac{5}{\log_x y} = 0$	M1 (Change of base)
	$\log_x y + \dfrac{1}{\log_x y} - \dfrac{5}{\log_x y} = 0$	
	$(\log_x y)^2 + 1 - 5 = 0$	M1 (Quadratic form)
	$(\log_x y)^2 = 4$	
	$\log_x y = \pm 2$	
	$y = x^2$ or $y = x^{-2}$	A1, A1 (y in terms of x)
	OR	OR
	$\log_x y + \log_y x - \dfrac{5}{\log_x y} = 0$	
	$\dfrac{\log_y y}{\log_y x} + \log_y x - 5\dfrac{\log_y x}{\log_y y} = 0$	M1 (Change of base)
	$\dfrac{1}{\log_y x} + \log_y x - 5\log_y x = 0$	
	$4(\log_y x)^2 = 1$	M1 (Quadratic form)
	$\log_y x = \pm\dfrac{1}{2}$	
	$x = y^{\frac{1}{2}}$ or $y^{-\frac{1}{2}}$	A1, A1 (y in terms of x)
	$y = x^2$ or $y = x^{-2}$	
5(i)	$4\cos 2x + 2\sin x = -2$ for $0 \le x \le 2\pi$	
	$4(1-2\sin^2 x) + 2\sin x + 2 = 0$	M1 (Apply trigo. identity)
	$4\sin^2 x - \sin x - 3 = 0$	
	$(4\sin x + 3)(\sin x - 1) = 0$	M1 (factorise/general formula)
	$\sin x = -\dfrac{3}{4}$ or $\sin x = 1$	M1 ft (equations)
	Basic $\angle = \sin^{-1}\left(\dfrac{3}{4}\right) = 0.84806$ $\quad x = \dfrac{\pi}{2}$ (or 1.57)	A1
	$x = \pi + 0.84806, 2\pi - 0.84806$	
	$= 3.99, 5.44$	A1, A1

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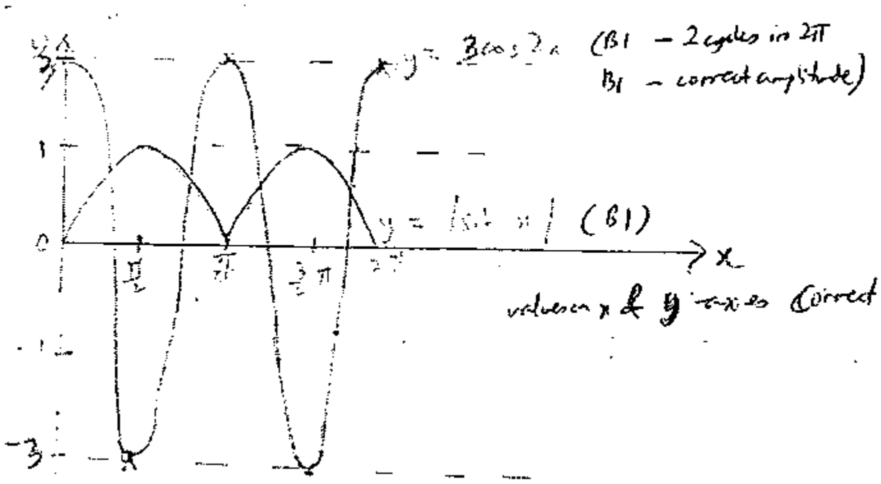
(ii)	 4 solutions B1	
6(i)	$\angle PBM$ and $\angle PXM$ are <u>angles in opposite segments</u> of the cyclic quadrilateral $PXMB$. Therefore the sum of these two angles is supplementary.	B1
(ii)	$\angle BRM = \angle PRX$ (common angles of ΔRBM and ΔRXP) $\angle XPR = \angle BMR$ (ext. $\angle$ of cyclic quad.) or $\angle RBM = \angle PXM$ (ext. $\angle$ of cyclic quad.) $\therefore \Delta RBM$ is similar to ΔRXP (by AA similarity test)	B1 B1 B1
(iii)	Since ΔRBM is similar to ΔRXP, $\frac{PR}{RX} = \frac{RM}{RB}$ Since ΔQXA is also similar to ΔQPM, $\frac{PQ}{XQ} = \frac{QM}{QA}$ Given that $\frac{PR}{RX} = \frac{PQ}{QX}$, $\therefore \frac{RM}{RB} = \frac{QM}{QA}$. Since $QM = MR$ as M is the mid - point of QR, $\therefore RB = QA$ (Shown)	B1 B1 B1 B1
7(i)	At $t = 0$, $T = 225^{0}$ C. $\therefore 225 = P + 190e^{-k(0)}$ $P = 225 - 190$ $= 35$	B1

(ii)	When $t=4$, $T=120$, $120=35+190e^{-k(4)}$ $-4k=\ln\left(\frac{85}{190}\right)$ $k=0.20109$ ≈ 0.201	M1 M1 (change to ln) A1
(iii)	$T=35+190e^{-0.20109t}$ $\frac{dT}{dt}=-0.20109(190e^{-0.20109t})$ $=-38.2077e^{-0.20109t}$ When $t=10$, $\frac{dT}{dt}=-38.2077e^{-0.20109(10)}$ $=-5.1148$ Temperature of the ball is <u>decreasing</u> at a rate of $5.11^{\circ}C$ / min.	M1ft from (ii) value of k M1(ft if value of t and k clearly shown) A1 (positive value)
(iv)	As $e^{-0.201t}>0$ for $t\geq 0$, $190e^{-0.201t}>0$ $35+190e^{-0.201t}>35$. $\therefore T>35$. Hence the temperature of the ball can never fall below $35^{\circ}C$.	B1 B1 (must also include concluding statement)
8(i)	Sub. $x=y^2-3y$ into $x+y=3$, $y^2-3y+y=3$ $y^2-2y-3=0$ $(y-3)(y+1)=0$ $y=3$ or -1 Sub. $y=3$ and -1 into $x=y^2-3y$, $x=3^2-3(3)$ and $x=(-1)^2-3(-1)$ $=0$ $\quad$ $=4$ $\therefore$ P is $(0,3)$ and Q is $(4,-1)$	M1 M1 (factorise/general formula) M1 A1, A1

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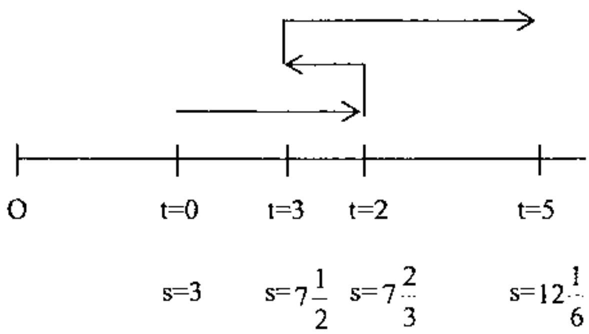
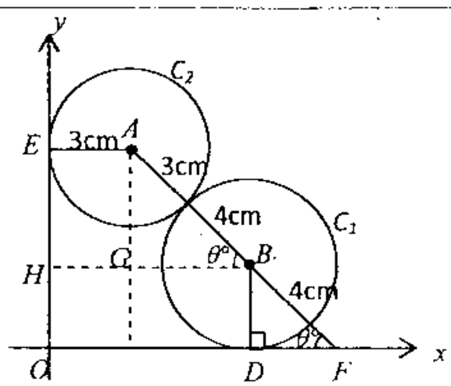
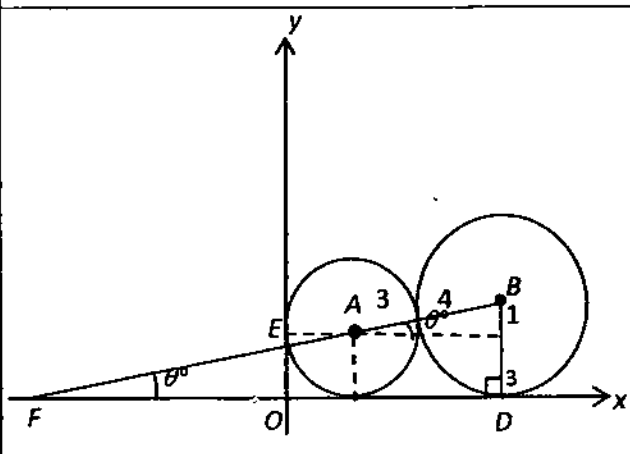
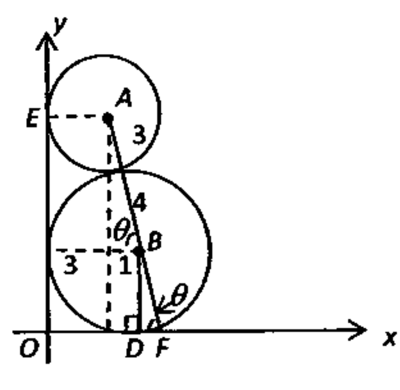
(ii)	Area of shaded region $= \left\| \int_0^3 (y^2 - 3y)\, dy \right\| + \frac{1}{2}(4)(4) - \int_{-1}^0 (y^2 - 3y)\, dy$ $= \left\| \left[\frac{y^3}{3} - \frac{3y^2}{2} \right]_0^3 \right\| + 8 - \left[\frac{y^3}{3} - \frac{3y^2}{2} \right]_{-1}^0$ $= \left\| \frac{3^3}{3} - \frac{3(3)^2}{2} \right\| + 8 - \left[0 - \frac{(-1)^3}{3} + \frac{3(-1)^2}{2} \right]$ $= 4\frac{1}{2} + 8 - 1\frac{5}{6}$ $= 10\frac{2}{3}$ or 10.7 or $\frac{32}{3}$ sq. units or $\frac{1}{2}(4)(4) - \int_{-1}^{3} (y^2 - 3y) dy$ or $\int_{-1}^{3} (3 - y) dy - \int_{-1}^{3} (y^2 - 3y) dy$ or $\left\| \int_0^3 (y^2 - 3y) dy \right\| - \frac{1}{2}(3)(3) + \frac{1}{2}(3+4)(1) - \int_{-1}^0 (y^2 - 3y)\, dy$ or $\left\| \int_0^3 (y^2 - 3y) dy \right\| - \frac{1}{2}(3)(3) + (4)(1) - \frac{1}{2}(1)(1) - \int_{-1}^0 (y^2 - 3y)\, dy$	M1, M1, M1 M1 (integrate) A1
9(i)	$f(x) = 6x^3 + ax^2 + bx - 6 = (x+2)P(x)$ $f(-2) = 6(-2)^3 + a(-2)^2 + b(-2) - 6 = 0$ $-48 + 4a - 2b - 6 = 0$ $2a - b = 27$ --------------(1) $f(x) = 6x^3 + ax^2 + bx - 6 = (x-1)Q(x) - 12$ $f(1) = 6 + a + b - 6 = -12$ $a + b = -12$ ---------------(2) $(1)+(2): 3a = 15$ $a = 5$ Sub. $a = 5$ into (2): $5 + b = -12$ $b = -17$	 M1 (equate to zero) M1 (equate to -12) M1 A1 A1

(ii)	$f(x)=6x^3+5x^2-17x-6=(x+2)(6x^2+kx-3)$ By comparing coefficient of x, $-3+2k=-17$ $k=-7$ $\therefore f(x)=(x+2)(6x^2-7k-3)$ $=(x+2)(3x-1)(2x-3)$ $48x^3+4ax^2+2bx-6=0$ $6(2x)^3+a(2x)^2+b(2x)-6=0$ Hence, $(2x+2)(3(2x)-1)(2(2x)-3)=0$ [or let $u=2x$, $6u^3+au^2+bu-6=0$ Hence, $(u+2)(3u+1)(2u-3)=0$] $u=2x=-2,-\frac{1}{3},\frac{3}{2}$ $\therefore x=-1,-\frac{1}{6},\frac{3}{4}$	M1ft from (i) (compare coeff./ long division/ synthetic division) A1ft A1 M1 A1 A1
10(i)	$v=t^2-5t+6$ When at instantaneous rest, $v=0$. $t^2-5t+6=0$ $(t-3)(t-2)=0$ $t=3$ or 2	M1 (equate to zero) A1
(ii)	$a=\frac{dv}{dt}=2t-5$ When $t=5$, $a=2(5)-5=5$ m/s^2	M1 A1
(iii)	$s=\int(t^2-5t+6)dt$ $=\frac{t^3}{3}-\frac{5t^2}{2}+6t+c$ When $t=0, s=3$, $c=3$. $\therefore s=\frac{t^3}{3}-\frac{5t^2}{2}+6t+3$	M1 M1 (values of t and s indicated) A1

(iv)	When t = 2, $s=\frac{2^3}{3}-\frac{5(2)^2}{2}+6(2)+3=7\frac{2}{3}$ When t = 3, $s=\frac{3^3}{3}-\frac{5(3)^2}{2}+6(3)+3=7\frac{1}{2}$	M1ft from (iii)
	When t = 5, $s=\frac{5^3}{3}-\frac{5(5)^2}{2}+6(5)+3=12\frac{1}{6}$	M1ft from (iii)
		
	Average speed in first 5 s $=\frac{(7\frac{2}{3}-3)+(7\frac{2}{3}-7\frac{1}{2})+(12\frac{1}{6}-7\frac{1}{2})}{5}$	M1ft from (ii)
	$=\frac{9.5}{5}$ $=1.9$ m/s	A1
11(i)		
	$GB=7\cos\theta=JD$ $\therefore OD=OJ+JD=3+7\cos\theta$	B1
	$AG=7\sin\theta=EH$ $\therefore OE=OH+HE=4+7\sin\theta$	B1

	$\therefore$ By Pythagoras' Theorem, $ED^2 = OD^2 + OE^2$ $= (3+7\cos\theta)^2 + (4+7\sin\theta)^2$ $= 9 + 42\cos\theta + 49\cos^2\theta + 16 + 56\sin\theta + 49\sin^2\theta$ $= 25 + 56\sin\theta + 42\cos\theta + 49(\sin^2\theta + \cos^2\theta)$ $= 25 + 56\sin\theta + 42\cos\theta + 49\ (1)$ $= 74 + 56\sin\theta + 42\cos\theta$ (Shown)	M1 AG1
11(ii)	$ED^2 = 74 + 56\sin\theta + 42\cos\theta$ $= 74 + R\cos(\theta - \alpha)$ $= 74 + R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$ By comparing coefficients, $R\sin\alpha = 56$ --------(1) $R\cos\alpha = 42$ -------(2) $(1)\div(2): \tan\alpha = \dfrac{56}{42}$ $\alpha = 53.130°$ $= 53.1°$ (to 1 dec. pl.) $(1)^2 + (2)^2 : R^2 = 56^2 + 42^2$ $R = \sqrt{4900}$ $R = 70$ $\therefore ED^2 = 74 + 70\cos(\theta - 53.1°)$.	M1 B1 B1 B1
(iii)	  $\sin\theta = \dfrac{1}{7}$ (M1) $\qquad$ $\cos\theta = \dfrac{1}{7}$ (M1) $\theta = 8.2°$ (to 1 dec. pl.) $\qquad$ $\theta = 81.8°$ (to 1 dec. pl. $\therefore 8.2° \le \theta \le 81.8°$ (Shown) (AG1)	

聖嬰中學

HOLY INNOCENTS' HIGH SCHOOL

Name of Student			/ 80
Class		Index Number	

PRELIMINARY EXAMINATION 2015
SECONDARY 4 EXPRESS
ADDITIONAL MATHEMATICS

4047/01

Date: 5 August 2015

Duration: 2 hours

Additional Materials: 8 Sheets of Writing Paper
1 Sheet of Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction tape/fluid.

Answer **ALL** questions.

The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The total number of marks for this paper is **80**.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π , use either your calculator value or 3.142.

Set by: Mrs Rajammal Nathan

Vetted by: Ms Tan Bee Choo

Mdm Hayati

This document consists of 6 printed pages (including cover page).

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer all the questions.

1 (i) Find the set of values of k for which the equation $2x^2 + 5x + k = 2kx + 1$ has no real roots. [4]

(ii) Hence state, with a reason, whether the line $y = 4x + 1$ meets the curve $y = 2x^2 + 5x + 2$. [1]

2 (i) Given that sec 200° = – k, where $k > 0$, find an expression, in terms of k, for sin 200°. [2]

(ii) Hence show that $\tan 110° = -\frac{1}{\sqrt{k^2 - 1}}$. [3]

3 The equation of a curve is $y = \ln(5 - 2x)$. Find the coordinates of the point on the curve at which the normal to the curve is parallel to the line $2y = x + 3$. [5]

4 The equation of a curve is $y = \cos^3 x + \sin 3x$. Given that x is changing at a constant rate of 0.56 radians per second, find the rate of change of y when $x = \frac{\pi}{6}$. [5]

5 (i) Express $\frac{2x - 1}{2x^2 - 5x + 3}$ in partial fractions. [3]

(ii) Hence find $\int \frac{2x - 1}{2x^2 - 5x + 3} dx$. [3]

6 A curve is such that $\frac{d^2y}{dx^2} = 16e^{-4x}$. Given that $\frac{dy}{dx} = 3$ when $x = 0$ and that the curve passes through the point $(2, e^{-8})$, find the equation of the curve. [6]

7 (i) Prove that $\dfrac{1-\sin 2\theta}{1+\cos 2\theta}=\dfrac{1}{2}(1-\tan\theta)^2$. [4]

(ii) Hence solve the equation $\dfrac{1-\sin\theta}{1+\cos\theta}=2$, for $0° \le \theta \le 180°$. [3]

8 (i) Write down the first three terms in the expansion, in ascending powers of x, of

(a) $\left(1+\dfrac{3x}{2}\right)^5$, [2]

(b) $(2-x)^5$. [2]

(ii) Hence find the coefficient of x^2 in the expansion $\left(2+2x-\dfrac{3x^2}{2}\right)^5$. [3]

9 (i) Calculate the coordinates of the point of intersection of the graph of $y=3-|2x+1|$ with the coordinate axes. [3]

(ii) Sketch the graph of $y=3-|2x+1|$. [2]

(iii) On the same diagram in part (ii), sketch the graph of $y=x^{\frac{1}{2}}$ for $x \ge 0$. [1]

(iv) State the number of solutions of the equation $3-|2x+1|=x^{\frac{1}{2}}$. [1]

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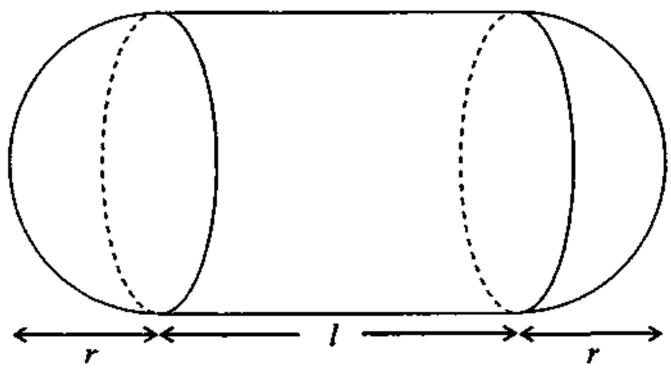

The diagram shows a time capsule consisting of a cylinder of radius r m and length l m, with hemispheres of radius r m attached at each end. The volume of the time capsule is $\frac{\pi}{6}$ m^3.

(i) Show that the surface area of the time capsule, A m^2, is given by

$$A = \frac{4}{3}\pi r^2 + \frac{\pi}{3r}\ .$$ [4]

(ii) Given that r can vary, find the minimum value of A. [4]

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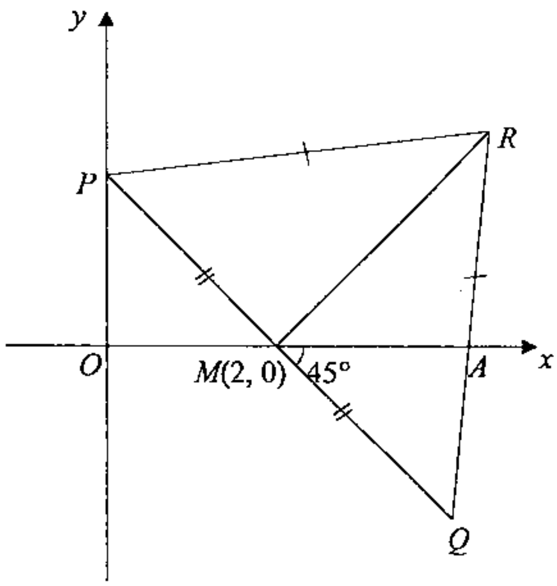
11 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows an isosceles triangle PQR in which $PR = QR$. $M(2, 0)$ is the midpoint of PQ. QR meets the x-axis at A and angle $AMQ = 45°$.

(i) Show that the equation of MR is $y = x - 2$. [2]

(ii) Find the equation of PQ. [2]

(iii) Find the coordinates of Q. [2]

(iv) Given that the area of triangle PQR is 20 units^2, find the coordinates of R. [3]

12 A rectangle of area y m^2 has sides of length x m and $(Ax + B)$ m, where A and B are constants and x and y are variables. Values of x and y are given in the table below.

x	50	100	150	200	250
y	3250	9000	17250	28000	41250

(i) Plot $\dfrac{y}{x}$ against x and draw a straight line graph. [3]

(ii) Use your graph to estimate value of A and of B. [4]

(iii) On the same diagram, draw the straight line representing the equation $y = x^2$ and explain the significance of the value of x given by the point of intersection of the two lines. [3]

End of paper

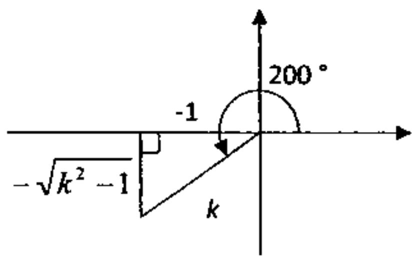
Answers

1 (i) $1\frac{1}{2} < k < 5\frac{1}{2}$

(ii) The equations of the line and the curve are obtained when $k = 2$.
Since $k = 2$ lies in the range, the line does not meet the curve.

2 (i) $\sec 200° = -k$

$$\cos 200° = -\frac{1}{k}$$



$$\sin 200° = -\frac{\sqrt{k^2-1}}{k}$$

[Or $\cos 200° = -\frac{1}{k}$

Applying identity: $\sin^2 200° + \cos^2 200° = 1$

$$\sin 200° = \sqrt{1-\frac{1}{k^2}} \text{ (rej) or } -\sqrt{1-\frac{1}{k^2}}$$

(ii)
$$\tan 110° = \frac{\sin 110°}{\cos 110°}$$
$$= \frac{\sin(200° - 90°)}{\cos(200° - 90°)}$$
$$= \frac{\sin 200° \cos 90° - \cos 200° \sin 90°}{\cos 200° \cos 90° + \sin 200° \sin 90°} \text{ (applying addition formula)}$$
$$= \frac{0-\left(-\frac{1}{k}\right)}{0+\left(-\frac{\sqrt{k^2-1}}{k}\right)(1)} = -\frac{1}{\sqrt{k^2-1}}$$

[OR $\tan 110° = -\tan 70°$

$$= -\frac{1}{\tan 20°}$$

$$= -\frac{1}{\tan 200°}$$

$$= -\frac{1}{\frac{-\sqrt{k^2-1}}{-1}} = -\frac{1}{\sqrt{k^2-1}}$$

3 coordinates of the point = (2, 0)

4 – 0.63 radians/s

5 (i) $\dfrac{2x-1}{2x^2-5x+3} = \dfrac{4}{2x-3} - \dfrac{1}{x-1}$.

(ii) $2\ln(2x-3) - \ln(x-1) + c$

6 $y = e^{-4x} + 7x - 14$

7 (i) $\dfrac{1-\sin 2\theta}{1+\cos 2\theta}$

$= \dfrac{1-2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$ Applying double angle formulas

$= \dfrac{1-2\sin\theta\cos\theta}{2\cos^2\theta}$

$= \dfrac{1}{2\cos^2\theta} - \dfrac{2\sin\theta\cos\theta}{2\cos^2\theta}$

$= \dfrac{1}{2}\sec^2\theta - \tan\theta$

$= \dfrac{1}{2}(1+\tan^2\theta) - \tan\theta$ Applying identity

$= \dfrac{1}{2}(1+\tan^2\theta - 2\tan\theta)$

$= \dfrac{1}{2}(1-\tan\theta)^2$

(ii) $\theta = 143.1°$

8 (i) $1+\frac{15}{2}x+\frac{45}{2}x^2+....$

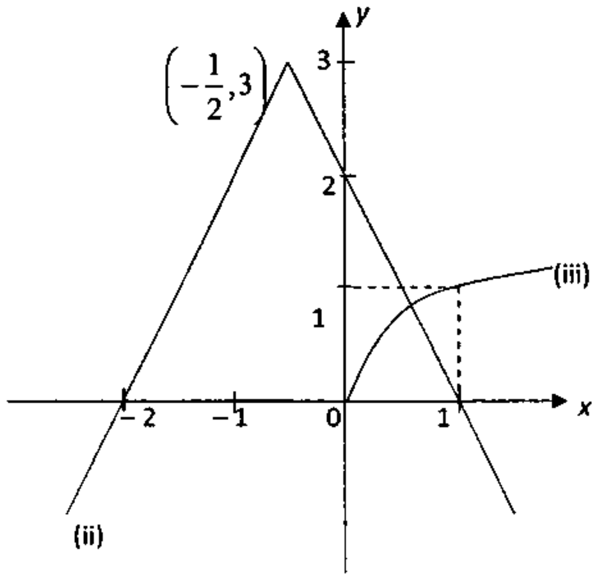
(ii) $32-80x+80x^2+....$

(iii Coefficient of $x^2=200$

9 (i) Intersects the y-axis at (0, 2)
Intersects the x-axis at $(1,0)$ and $(-2,0)$.

(ii) correct shape and passing through the coordinate axes
Coordinates of vertex

(iii) correct shape passing through (1, 1)



(iv) Number of solutions = 1 B1

10 (i) $2\left(\frac{2}{3}\pi r^3\right)+\pi r^2 l=\frac{\pi}{6}$

or $\frac{4}{3}\pi r^3+\pi r^2 l=\frac{\pi}{6}$

$$l=\frac{\frac{\pi}{6}-\frac{4}{3}\pi r^3}{\pi r^2}$$ expressing l in terms of r

$$=\frac{1}{6r^2}-\frac{4}{3}r$$

$A=2(2\pi r^2)+2\pi rl$

$=4\pi r^2+2\pi r\left(\frac{1}{6r^2}-\frac{4}{3}r\right)$ substituting a correct expression for l

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$$= 4\pi r^2 + \frac{\pi}{3r} - \frac{8}{3}\pi r^2$$

$$= \frac{4}{3}\pi r^2 + \frac{\pi}{3r} \qquad \text{(shown)}$$

(ii) 3.14 cm^2

11 (i) $\angle RMA = 90° - 45°$

$= 45°$

Gradient of $MR = \tan 45°$

$= 1$

Equation of MR is $y - 0 = 1(x - 2)$

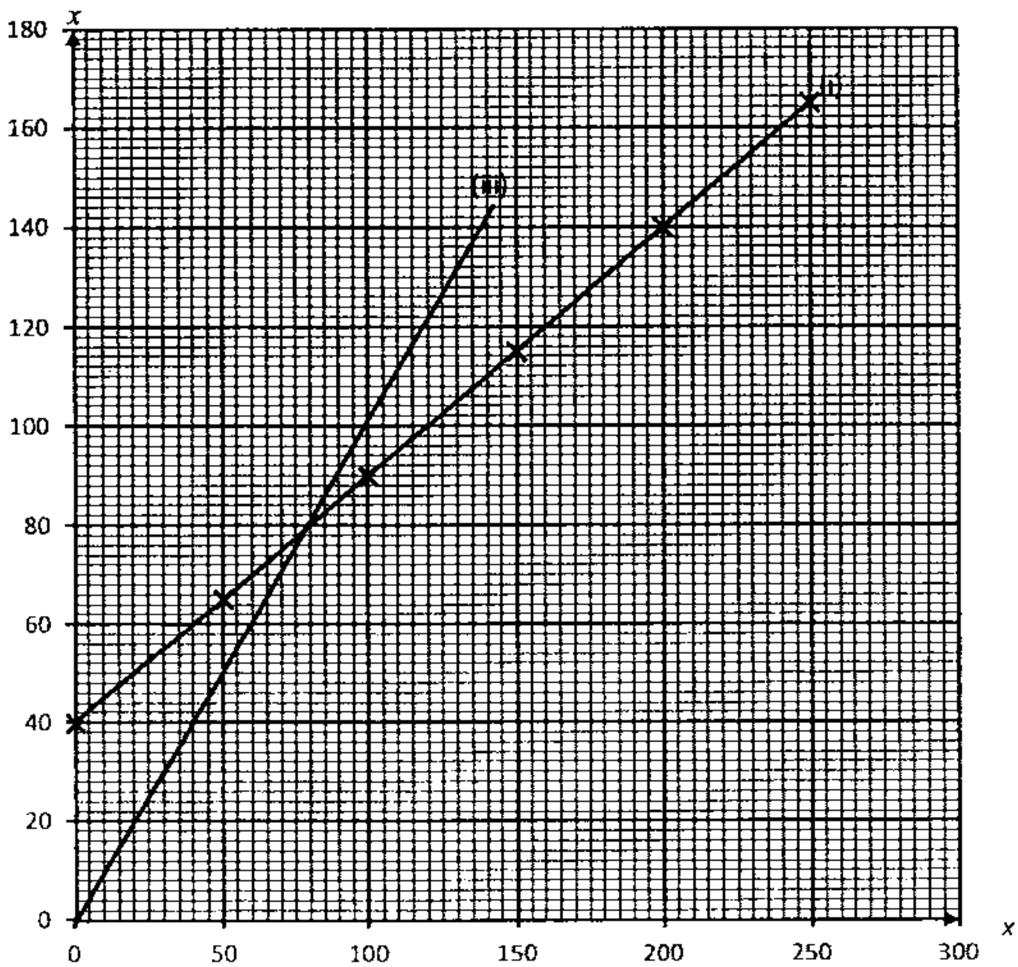
$y = x - 2$

(ii) $y = -x + 2$ (iii) Coordinates of $Q = (4, -2)$ (iv) Coordinates of $R = (7, 5)$

12

x	50	100	150	200	250
y	3250	9000	17250	28000	41250
$\frac{y}{x}$	65	90	115	140	165

(i)



(ii) $A = 0.5 \pm 0.2$ $B = 40 \pm 1$

(iii) Plot $\frac{y}{x}$ against x as a straight line accurately.
The x-value of the point of intersection represents the value where the rectangle becomes a square.

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Marking Scheme
Additional Mathematics
Preliminary Examination 2015
Paper 1

1 (i) $2x^2 + 5x - 2kx + k - 1 = 0$

No real roots $\Rightarrow b^2 - 4ac < 0$ M1

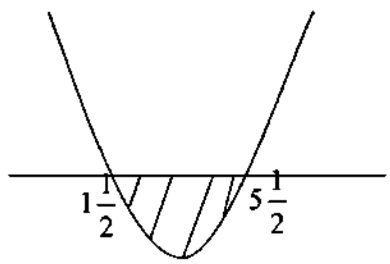
$(5-2k)^2 - 4(2)(k-1) < 0$

$25 - 20k + 4k^2 - 8k + 8 < 0$

$4k^2 - 28k + 33 < 0$ correct quadratic M1

Finding the solution of quadratic: $k = 1\frac{1}{2}$ or $5\frac{1}{2}$ DM1

$(2k-3)(2k-11) < 0$

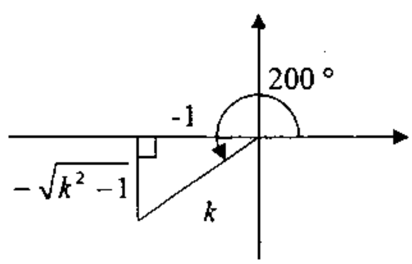


$1\frac{1}{2} < k < 5\frac{1}{2}$ A1

(ii) The equations of the line and the curve are obtained when $k = 2$.
Since $k = 2$ lies in the range, the line does not meet the curve. B1

2 (i) $\sec 200° = -k$

$\cos 200° = -\frac{1}{k}$ M1



$\sin 200° = -\frac{\sqrt{k^2 - 1}}{k}$ A1

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[Or $\cos 200° = -\frac{1}{k}$ **M1**

Applying identity: $\sin^2 200° + \cos^2 200° = 1$

$$\sin 200° = \sqrt{1-\frac{1}{k^2}} \text{ (rej) or } -\sqrt{1-\frac{1}{k^2}} \qquad \textbf{A1}$$

Accept any equivalent form]

(ii) $\tan 110° = \dfrac{\sin 110°}{\cos 110°}$

$= \dfrac{\sin(200°-90°)}{\cos(200°-90°)}$ M1

$= \dfrac{\sin 200° \cos 90° - \cos 200° \sin 90°}{\cos 200° \cos 90° + \sin 200° \sin 90°}$ (applying addition formula) M1

$= \dfrac{0-\left(-\frac{1}{k}\right)}{0+\left(-\frac{\sqrt{k^2-1}}{k}\right)(1)} = -\dfrac{1}{\sqrt{k^2-1}}$ A1

[OR $\tan 110° = -\tan 70°$

$= -\dfrac{1}{\tan 20°}$ M1

$= -\dfrac{1}{\tan 200°}$ M1

$= -\dfrac{1}{\frac{-\sqrt{k^2-1}}{-1}} = -\dfrac{1}{\sqrt{k^2-1}}$ A1]

3 $y = \ln(5-2x)$

$\dfrac{dy}{dx} = \dfrac{-2}{5-2x}$ (**M1** for -2 and **M1** for $\dfrac{1}{5-2x}$) M2

Gradient of line $= \dfrac{1}{2}$

Gradient function of normal $= \dfrac{5-2x}{2}$ M1

$\dfrac{5-2x}{2} = \dfrac{1}{2}$ M1

$x = 2$

$y = 0$

coordinates of the point = (2, 0) A1

4 $y = \cos^3 x + \sin 3x$

$$\frac{dy}{dx} = \underbrace{(3\cos^2 x)}_{\textbf{M1}}\underbrace{(-\sin x)}_{\textbf{M1}} + \underbrace{3\cos 3x}_{\textbf{M1}}$$ M3

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$= \left(-3\cos^2\frac{\pi}{6}\sin\frac{\pi}{6} + 3\cos\frac{3\pi}{6}\right) \times 0.56$$ M1

$= -0.63$ radians/s A1

5 (i) $$\frac{2x-1}{2x^2-5x+3} = \frac{A}{2x-3} + \frac{B}{x-1}$$ M1

$2x - 1 = A(x-1) + B(2x-3)$

Substitute $x = 1$,

$1 = -B$

$B = -1$ A1

Substitute $x = \frac{3}{2}$,

$2 = \frac{1}{2}A$

$A = 4$ A1

$$\therefore \frac{2x-1}{2x^2-5x+3} = \frac{4}{2x-3} - \frac{1}{x-1}.$$

(ii) $$\int \frac{2x-1}{2x^2-5x+3}\,dx = \int\left(\frac{4}{2x-3} - \frac{1}{x-1}\right)dx$$

$$= \frac{4\ln(2x-3)}{2} - \ln(x-1) + c$$

$$= \underbrace{2}_{\textbf{B1}}\ \underbrace{\ln(2x-3)}_{\textbf{B1}} - \underbrace{\ln(x-1)}_{\textbf{B1}} + c$$ B3

−1 for each error

6 $\dfrac{dy}{dx} = \int\left(16e^{-4x}\right)dx$

$= -4e^{-4x} + c$ B1

(for $-4e^{-4x}$)

Substitute $\dfrac{dy}{dx} = 3$ and $x = 0$

$3 = -4e^{-4(0)} + c$ (attempt to find c) M1

$c = 7$

$\dfrac{dy}{dx} = -4e^{-4x} + 7$ A1

$y = \int\left(-4e^{-4x} + 7\right)dx$

$= e^{-4x} + 7x + c_1$ for $e^{-4x} + 7x$ B1

Substitute $x = 2$ and $y = e^{-8}$

$e^{-8} = e^{-4(2)} + 7(2) + c_1$ (attempt to find c_1) M1

$c_1 = -14$

$y = e^{-4x} + 7x - 14$ A1

7 (i) $\dfrac{1 - \sin 2\theta}{1 + \cos 2\theta}$

$= \dfrac{1 - 2\sin\theta\cos\theta}{1 + 2\cos^2\theta - 1}$ M1 M1 Applying double angle formulas M2

$= \dfrac{1 - 2\sin\theta\cos\theta}{2\cos^2\theta}$

$= \dfrac{1}{2\cos^2\theta} - \dfrac{2\sin\theta\cos\theta}{2\cos^2\theta}$

$= \dfrac{1}{2}\sec^2\theta - \tan\theta$

$= \dfrac{1}{2}\left(1 + \tan^2\theta\right) - \tan\theta$ Applying identity M1

$= \dfrac{1}{2}\left(1 + \tan^2\theta - 2\tan\theta\right)$

$= \dfrac{1}{2}(1 - \tan\theta)^2$ A1

(ii) $\dfrac{1-\sin\theta}{1+\cos\theta}=2$

$\dfrac{1}{2}\left(1-\tan\dfrac{\theta}{2}\right)^2=2$ M1

$0^\circ \le \theta \le 180^\circ$

$0^\circ \le \dfrac{\theta}{2} \le 90^\circ$

$1-\tan\dfrac{\theta}{2}=2$ or -2

$\tan\dfrac{\theta}{2}=\ -1$ (reject) or 3

$\dfrac{\theta}{2}=71.56^\circ$ M1

$\theta=143.1^\circ$ A1

8 (i) (a) $\left(1+\dfrac{3x}{2}\right)^5=1+\dbinom{5}{1}\left(\dfrac{3x}{2}\right)+\dbinom{5}{2}\left(\dfrac{3x}{2}\right)^2+\ldots$

$=\underbrace{1+\dfrac{15}{2}x}_{\textbf{B1}}+\underbrace{\dfrac{45}{2}x^2}_{\textbf{B1}}+\ldots.$ B2

(b) $(2-x)^5=(2)^5+\dbinom{5}{1}(2)^4(-x)+\dbinom{5}{2}(2)^3(-x)^2+\ldots$

$=\underbrace{32-80x}_{\textbf{B1}}+\underbrace{80x^2}_{\textbf{B1}}+\ldots.$ B2

(ii) $\left(2+2x-\dfrac{3x^2}{2}\right)^5=\left[\left(1+\dfrac{3x}{2}\right)(2-x)\right]^5$ M1

$=\left(1+\dfrac{3x}{2}\right)^5(2-x)^5$

$=(1+\dfrac{15}{2}x+\dfrac{45}{2}x^2+\ldots.)(32-80x+80x^2+\ldots.)$

$=\ldots.+80x^2-600x^2+720x^2+\ldots.$ M1

$=200x^2+\ldots$

Coefficient of $x^2=200$ A1

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9 (i) Intersects the y-axis at $(0, 2)$ B1

$3-|2x+1|=0$

$|2x+1|=3$

$2x+1=3$ or $2x+1=-3$

$x=1$ or $x=-2$

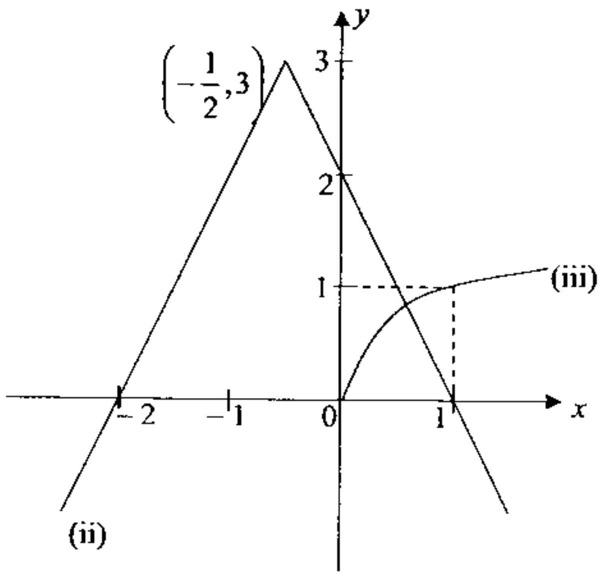
Intersects the x-axis at $(1,0)$ and $(-2,0)$. B2

B1 **B1**

(ii) correct shape and passing through the coordinate axes **B1** B2

Coordinates of vertex **B1**

(iii) correct shape passing through $(1, 1)$ B1



(iv) Number of solutions = 1 B1

10 (i) $2\left(\frac{2}{3}\pi r^3\right)+\pi r^2 l=\frac{\pi}{6}$ M1

or $\frac{4}{3}\pi r^3+\pi r^2 l=\frac{\pi}{6}$

$l=\dfrac{\frac{\pi}{6}-\frac{4}{3}\pi r^3}{\pi r^2}$ expressing l in terms of r M1

$=\frac{1}{6r^2}-\frac{4}{3}r$

$A=2(2\pi r^2)+2\pi rl$

$=4\pi r^2+2\pi r\left(\frac{1}{6r^2}-\frac{4}{3}r\right)$ substituting a correct expression for l M1

$=4\pi r^2+\frac{\pi}{3r}-\frac{8}{3}\pi r^2$

$=\frac{4}{3}\pi r^2+\frac{\pi}{3r}$ (shown) A1

(ii) $\frac{dA}{dr}=\frac{8}{3}\pi r-\frac{\pi}{3r^2}$ M1

For min value, $\frac{dA}{dr}=0$.

$\frac{8}{3}\pi r-\frac{\pi}{3r^2}=0$ M1

$8r^3-1=0$

$r=\frac{1}{2}$ A1

Minimum value of $A=\frac{4}{3}\pi\left(\frac{1}{2}\right)^2+\dfrac{\pi}{3\left(\frac{1}{2}\right)}$ cm^2

$=3.14$ cm^2 A1

[3]

11 (i) $\angle RMA = 90° - 45°$
$= 45°$

Gradient of $MR = \tan 45°$ M1
$= 1$

Equation of MR is $y - 0 = 1(x - 2)$
$y = x - 2$ A1

(ii) Gradient of $PQ = -1$ M1

Equation of PQ is $y - 0 = -1(x - 2)$
$y = -x + 2$ A1

(iii) Coordinates of $P = (0, 2)$ M1

Applying mid point formula:

Coordinates of $Q = (4, -2)$ A1

(iv) Let the coordinates of R be $(k, k - 2)$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 4 & k & 0 & 4 \\ -2 & k-2 & 2 & -2 \end{vmatrix} = 20$$ M1

$$[4(k-2) + 2k + 0] - [8 + 0 - 2k) = 40$$ M1
$$4k = 56$$
$$k = 7$$

Coordinates of $R = (7, 5)$ A1

[OR $PQ = \sqrt{(4-0)^2 + (-2-2)^2}$
$= \sqrt{32}$

$RM = \sqrt{(k-2)^2 + (k-2-0)^2}$
$= \sqrt{2(k-2)^2}$
$= (k-2)\sqrt{2}$

M1

Area $= \frac{1}{2} \times \sqrt{32} \times (k-2)\sqrt{2} = 20$ **M1**
$k = 7$

Coordinates of $R = (7, 5)$ **A1]**

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x	50	100	150	200	250
y	3250	9000	17250	28000	41250
$\frac{y}{x}$	65	90	115	140	165

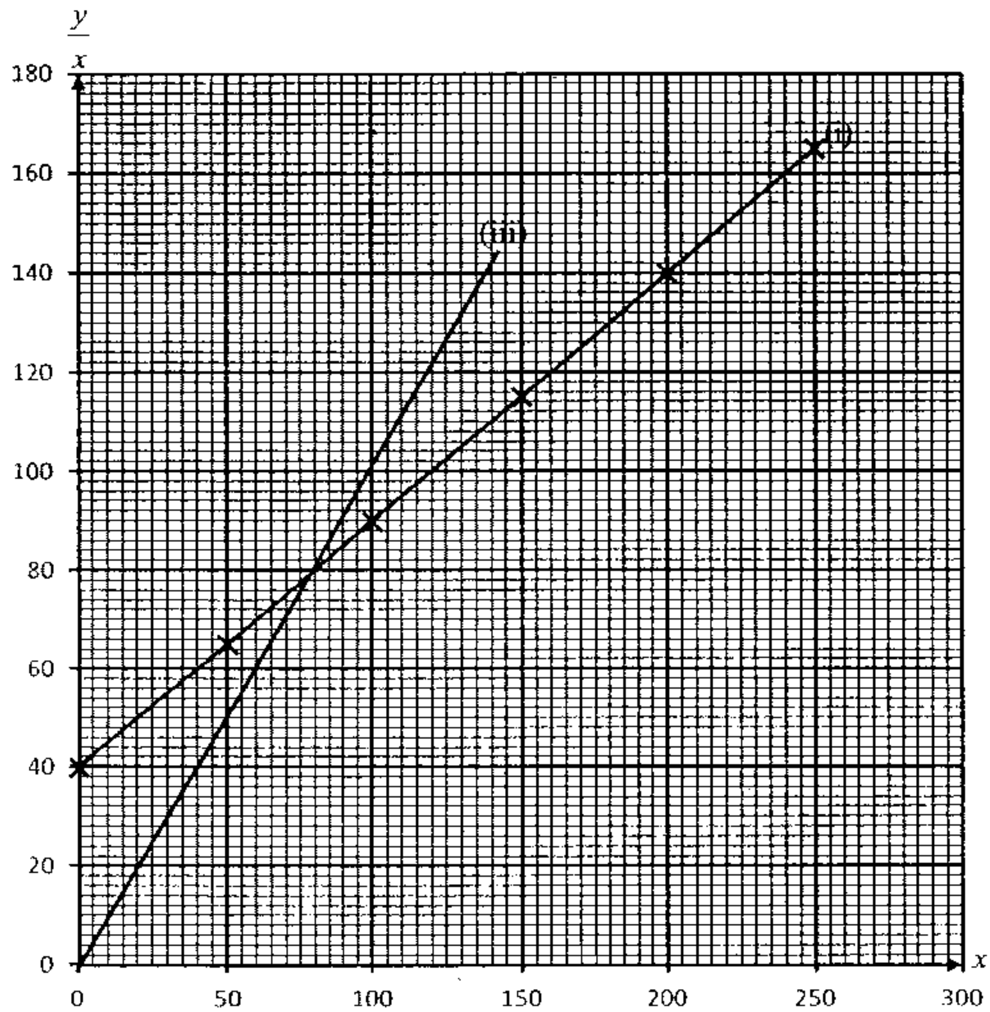
B1

(i) Draw axes and plot all given points. P1

(inaccurate plot: P0)

Drawing a straight line through all plots C1

Deduct 1 mark if a suitable scale is not used.



(ii) $y = x(Ax + B)$

$$\frac{y}{x} = Ax + B$$ M1

$$A = \frac{165 - 65}{250 - 50}$$

M1

$= 0.5 \pm 0.2$ A1

B = 40 ± 1 B1

[5]

(iii) $y = x^2$

$\frac{y}{x} = x$ B1

Plot $\frac{y}{x}$ against x as a straight line accurately. B1

The x-value of the point of intersection represents the value where the rectangle becomes a square. B1

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Answer all questions.

1 A beaker of water is heated until it reaches a temperature of X °C. It is then allowed to cool. It's temperature, θ °C, when it was cooling for time t minutes is given by $\theta = 28 + 60e^{-0.3t}$. Find

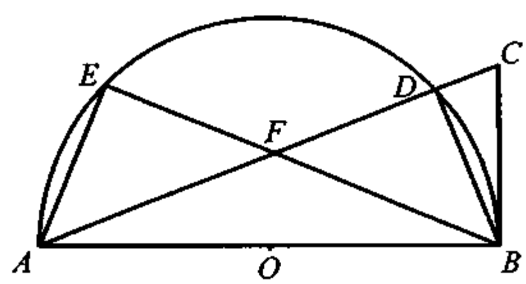
(i) the value of X, [1]

(ii) the value of θ when $t = 6$, [1]

(iii) the value of t when $\theta = \frac{1}{2}X$. [2]

(iv) Explain, with working, if the water will cool to a temperature of 20 °C. [2]

2



The figure shows a semi-circle centre O, with diameter AB. Points D and E lie on the semi-circle. AC is a straight line passing through D. BC is tangent to the semi-circle at B. The tangent to the semi-circle at B meets AD at C. The lines BE and AD intersect at F.

Given that $AE = BD$, prove that

(i) triangle ABD is congruent to triangle BAE, [3]

(ii) triangle ABC is similar to triangle BDC, [2]

(iii) $BC^2 = AC \times DC$. [2]

3 Given that $\log_p(a^3b) = x$, and $\log_p \frac{a}{\sqrt{b}} = y$, express in terms of x and y,

(i) $\log_p a$ and $\log_p b$, [5]

(ii) $\log_b\left(\frac{1}{ab^2}\right)$. [3]

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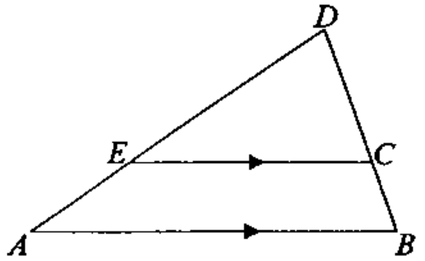
4 In the diagram, AB is parallel to EC.



Given that $AE : ED = 1 : \sqrt{2}$ and $CE = (3+\sqrt{2})$ cm, find in the form $a\sqrt{2}+b$,

(i) $\dfrac{CE}{AB}$, [3]

(ii) $\dfrac{\text{area of } \Delta CDE}{\text{area of } \Delta BDA}$, [2]

(iii) the length of AB. [3]

5 A curve has the equation $y = ax^2 + \dfrac{b}{x^3}$, where a and b are constants.

(i) Given that the curve has a stationary point at $(2, 5)$, find the value of a and of b. [4]

(ii) Determine the nature of the stationary point. [2]

(iii) Explain why the curve is a decreasing function for $x < 0$. [2]

6 The roots of the equation $x^2 - 4x - 8 = 0$ are α^3 and β^3.

(i) Given that $(\alpha+\beta)^3 = -5\alpha - 5\beta$, find the value of $\alpha+\beta$. [5]

(ii) Find the quadratic equation in x whose roots are $\dfrac{\alpha^2}{2}$ and $\dfrac{\beta^2}{2}$. [4]

7 The function $f(x) = 3x^3 + 2x^2 + ax + b$, where a and b are constants, has a factor $(x^2 - 4)$.

(i) Show that $a = -12$ and $b = -8$. [4]

(ii) Factorise $3x^3 + 2x^2 - 12x - 8$ completely. [3]

(iii) Hence solve the equation $-3x^3 + 2x^2 + 12x - 8 = 0$. [2]

8 (i) Find all the angles between $0°$ and $360°$ inclusive which satisfies the equation $2\sin\theta - 3\cos 2\theta - 1 = 0$. [4]

(ii) On the same axes, sketch the graphs of $y = 2\sin\theta - 1$ and $y = 3\cos 2\theta$ for $0° \le \theta \le 360°$. [4]

(iii) Using your answers to part (i) and (ii), state the range of values of θ for which $3\cos 2\theta \ge 2\sin\theta - 1$ for $0° \le \theta \le 360°$. [2]

9 A particle moves in a straight line with a velocity, v m/s, given by $v = -3t^2 + 12t - 13$. The displacement of the particle from a fixed point O after 8 seconds is 400 meters.

Calculate

(i) (a) the displacement of the particle after 5 seconds, [4]

(b) the value of t when the acceleration is zero. [2]

(ii) Explain why the particle will never come to rest.
Hence find the maximum velocity of the particle. [3]

(iii) Will the particle ever return to its starting point? Explain your answer. [2]

10 (i) A circle C_1 has an equation given by $x^2 + y^2 - 2kx + 2y + 1 = 0$, where k is a positive constant.
Given that C_1 has a radius of 2 units, find the value of k. [4]

(ii) The centre of a circle C_2 lies on the line $y = 2x + 2$. Given that C_2 passes through the points $(3, 2)$ and $(0, -1)$, find the equation of C_2. [5]

(iii) Calculate the shortest distance from the centre of C_1 to the circumference of C_2. [3]

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11 (i) Find $\int \frac{1}{\sqrt{2x+14}}\,dx$. [1]

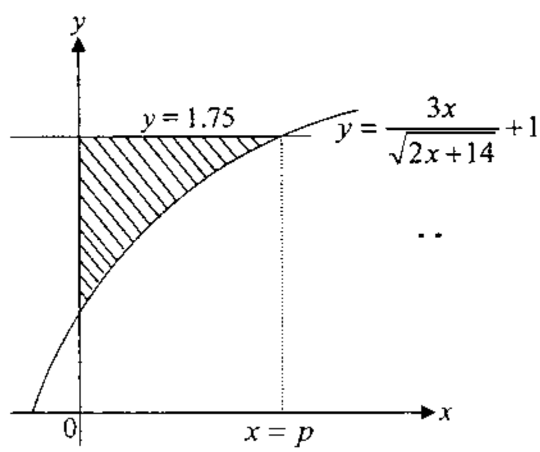
(ii) Show that $\frac{d}{dx}\left[(x-2)\sqrt{2x+14}\right] = \frac{3x+12}{\sqrt{2x+14}}$. [3]

The diagram shows part of the curve $y = \frac{3x}{\sqrt{2x+14}} + 1$ intersecting the line $y = 1.75$ and $x = p$.

(iii) (a) Find the value of p. [2]

(b) Using your results from part (i) and part (ii), find the area bounded by the curve, the line $x = p$, and the coordinate axes. [4]

(c) Find the area of the shaded region. [2]



----------- *End of Paper* -----------

Class	Register Number	Name

南洋女子中学校

NANYANG GIRLS' HIGH SCHOOL

End-of-Year Examination 2015
Secondary 4

INTEGRATED MATHEMATICS 2 **2 hours 30 minutes**

Thursday **8 October 2015** **0800 – 1030**

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Answer **all** the questions.
2. Write your answers and working on the separate writing paper provided.
3. Write your name, register number and class on each separate sheet of paper that you use and fasten the separate sheets together with the string provided. Do not staple your answer sheets together.
4. Omission of essential steps will result in loss of marks.
5. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

1. The number of marks is given in brackets [ ] at the end of each question or part question.
2. The total number of marks for this paper is 100.
3. The use of an electronic calculator is expected, where appropriate.
4. You are reminded of the need for clear presentation in your answers.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin A + \sin B = 2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$

$$\sin A - \sin B = 2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

$$\cos A + \cos B = 2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$

$$\cos A - \cos B = -2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 It is given that $f(x) = 2x^3 + 9x^2 - 2x + 2$.

(i) Find the remainder when $f(x)$ is divided by $x^2 + 4x - 3$. [2]

(ii) Hence solve the equation $f(x) - 5 = 0$, leaving your answers in exact form. [4]

2 Write down and simplify the first four terms in the expansion of $\left(3x - \frac{p}{x^2}\right)^5$, in descending powers of x, where p is a non-zero constant. [3]

Given that the coefficient of $\frac{1}{x}$ in the expansion of $(2x^3 - 1)\left(3x - \frac{p}{x^2}\right)^5$ is $90p^2$, find the value of p. [3]

3 (i) Solve the equation $|10 - 5x| = 10 + 8x - 2x^2$. [4]

(ii) Sketch, on a single diagram, the graphs of $2y = |10 - 5x|$ and $y = 5 + 4x - x^2$, indicating clearly the x- and y-intercepts and the turning points (if any). [4]

(iii) Hence deduce the range of values of x if $10 + 8x - 2x^2 \le |10 - 5x|$. [2]

4 (a) The function f is defined by $f : x \mapsto (x-1)(3x-2), \quad x \in \mathbb{R}$.

(i) Sketch the graph of $y = f(x)$, showing clearly the x- and y-intercepts and the coordinates of the turning point. [4]

(ii) State the range of f. [1]

(iii) The function g is defined by $g : x \mapsto (x-1)(3x-2), \quad x \le k, k \in \mathbb{Z}$. State the maximum value of k such that g^{-1} exists. Hence find an expression for $g^{-1}(x)$. [3]

(b) The graph of $y = h(x)$ undergoes 2 successive transformations

I A translation of $\frac{\pi}{4}$ in the positive x-direction,

II A scaling with a scale factor of 2 along the y-axis.

The resulting graph is $y = 6\cos x$. Find $h(x)$. [2]

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5 **(a)** Without using a calculator, evaluate 6^x, given that

$$3^{2x+3} \times 2^{x+5} = 3^{x+4}.$$ [2]

(b) Solve the simultaneous equations

$$e^2 e^x = e^{4y},$$

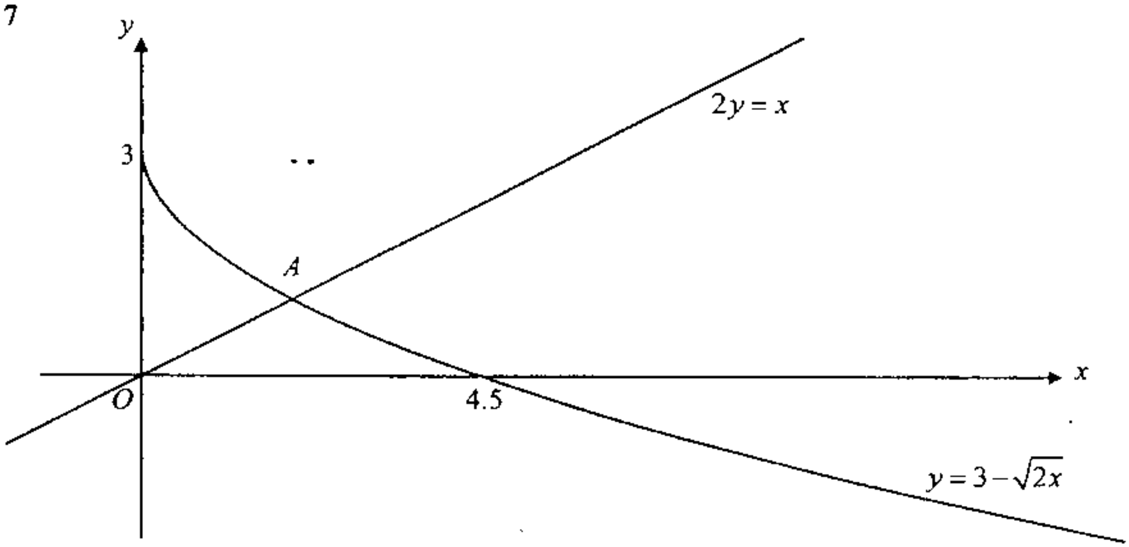
$$\log_4 (x+2) = 1 + \log_2 y.$$ [6]

6 **(i)** Express $\dfrac{4x}{2x+1}$ in the form $a + \dfrac{b}{2x+1}$, where a and b are integers. [2]

(ii) Differentiate $2x\ln(2x+1)$ with respect to x. [2]

(iii) Using the results in part **(i)** and part **(ii)**, determine $\int \ln(2x+1)\, dx$. [4]

7



The diagram shows parts of the curve $y = 3 - \sqrt{2x}$ and the line $2y = x$. The curve and the line intersect at the point A.

(i) Show that the area bounded by the curve $y = 3 - \sqrt{2x}$, the x - axis and the lines $x = 4.5$ and $x = 9$ can be expressed as $\left(a\sqrt{2} + b\right)$ square units, where a and b are constants. [4]

(ii) Find the coordinates of point A. [2]

(iii) Find the area bounded by the straight line $2y = x$, the curve $y = 3 - \sqrt{2x}$ and the y - axis. [3]

8 A circle C_1 has equation given by $(x-1)^2 + y^2 + 6y - 16 = 0$.

(i) Find the radius and the coordinates of the centre of C_1. [3]

(ii) Find the equation of the tangent to the circle at the point $P(4,-7)$. [3]

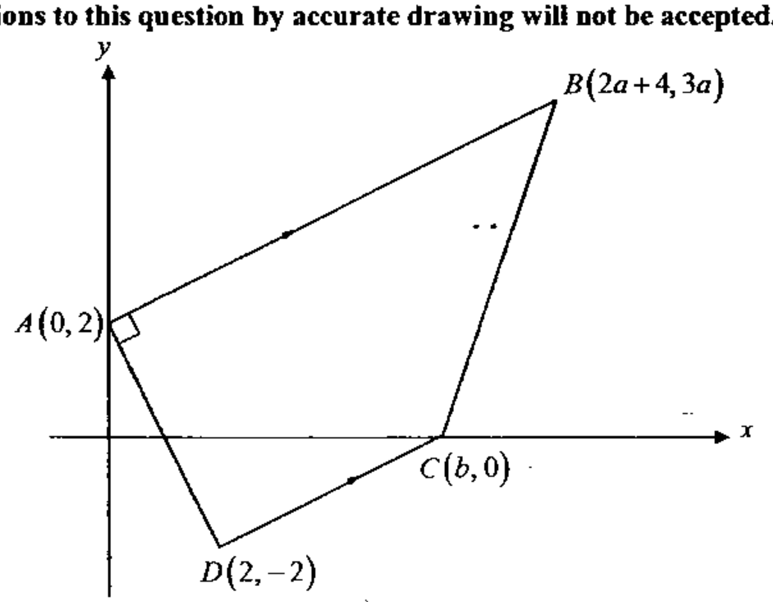
The point Q is such that PQ is the diameter of the circle.

(iii) Find the coordinates of Q. [2]

A new circle C_2 passes through P, Q and R, where R is a point outside the circle C_1 such that angle $PRQ = 45°$.

(iv) Explain briefly if it is possible for the centre of C_2 to lie on the circumference of C_1. [1]

9 **Solutions to this question by accurate drawing will not be accepted.**



The diagram, not drawn to scale, shows a trapezium $ABCD$ in which AB is parallel to DC and angle $BAD = 90°$. The vertices of the trapezium are at the points $A(0,2)$, $B(2a+4, 3a)$, $C(b,0)$ and $D(2,-2)$.

(i) Given that the length of AB is $4\sqrt{5}$ units, find the value of a, where $a > 0$. [3]

(ii) Find the equation of AB. [2]

(iii) Find the value of b. [2]

(iv) Find the perpendicular bisector of AB.
Hence or otherwise, show that C lies on the perpendicular bisector of AB. [3]

(v) Find the area of trapezium $ABCD$. [2]

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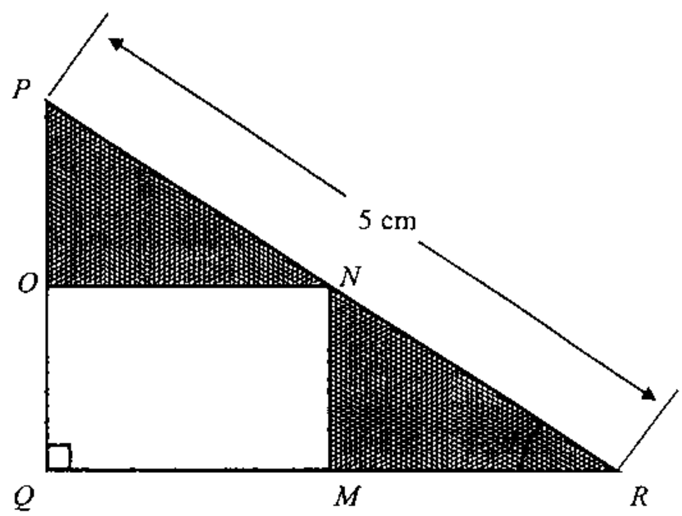
10 **(a)** **(i)** Show that $\dfrac{(\cos\theta+\sin\theta)^2}{\sec^2\theta+2\tan\theta}=\cos^2\theta$. [2]

(ii) Hence find all values of θ, where $0<\theta<2\pi$, which satisfy the equation $\dfrac{\sec^2\theta+2\tan\theta}{(\cos\theta+\sin\theta)^2}=2(2+\tan\theta)$. [4]

(b) The diagram shows a rectangle $MNOQ$ embedded in the triangle PQR. It is given that $PR=5$ cm, $\angle QRP=\theta$ and area of rectangle $MNOQ$ is $\dfrac{25}{4}\cos^2\theta$, where $0°<\theta<90°$.



(i) Show that the shaded area, A, is given by $A=\dfrac{25}{8}(2\sin 2\theta-\cos 2\theta)-\dfrac{25}{8}$.

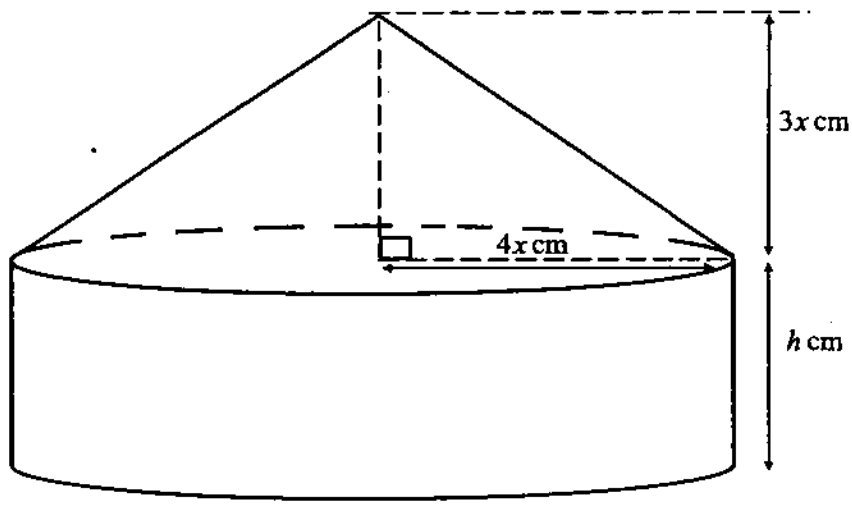
[3]

(ii) Hence, show that A can be expressed in the form $A=R\sin(2\theta-\alpha)-\dfrac{25}{8}$

where $R>0$ and $0°<\alpha<90°$. [2]

(iii) State the exact maximum value of A. [1]

11 The diagram below shows a solid toy which consists of a cone fixed to the end of a right circular cylinder. The cone has a radius of $4x$ cm and a height of $3x$ cm. The cylinder has a radius of $4x$ cm and a height of h cm. It is given that the total volume of the toy is $960\pi \text{ cm}^3$.



(i) Show that $h = \dfrac{60}{x^2} - x$. [2]

(ii) Show that the total surface area, A cm^2, of the toy is given by $A = \dfrac{480\pi}{x} + 28\pi x^2$ cm^2. [3]

(iii) Find, using differentiation, the values of h and x which give the minimum surface area of this toy. You will need to justify that the surface area is a minimum for the values of h and x obtained. [5]

[The area of the curved surface area of a cone of radius r and slant height l is πrl.]

[The volume of a cone $= \dfrac{1}{3} \times$ base area $\times$ height.]

End of Paper

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2015 S4 IM2 Common Paper Solutions

No.	
1(i)	$$\begin{array}{r} 2x+1 \\ x^2+4x-3 \overline{)\,2x^3+9x^2-2x+2} \\ \underline{2x^3+8x^2-6x} \\ x^2+4x+2 \\ \underline{x^2+4x-3} \\ 5 \end{array}$$ Remainder = 5 **Alternatively** Let $f(x)=(x^2+4x-3)(ax+b)+c$ By comparing coefficients, $f(x)=(x^2+4x-3)(2x+1)+5$ Remainder = 5
1(ii)	From $(x^2+4x-3)(2x+1)=0$ $\Rightarrow (x^2+4x-3)=0$ or $(2x+1)=0$ $x=-\frac{1}{2}$ or $x=\frac{-4\pm\sqrt{4^2-4(1)(-3)}}{2(1)}$ $x=\frac{-4\pm\sqrt{28}}{2}$ $x=-2\pm\sqrt{7}$
2	$\left(3x-\frac{p}{x^2}\right)^5$ $=(3x)^5+5(3x)^4\left(-\frac{p}{x^2}\right)+10(3x)^3\left(-\frac{p}{x^2}\right)^2+10(3x)^2\left(-\frac{p}{x^2}\right)^3+\ldots$ $=243x^5+5(81x^4)\left(-\frac{p}{x^2}\right)+10(27x^3)\left(\frac{p^2}{x^4}\right)+10(9x^2)\left(\frac{-p^3}{x^6}\right)+\ldots$ $=243x^5-405px^2+\frac{270p^2}{x}-\frac{90p^3}{x^4}+\ldots$ $(2x^3-1)\left(3x-\frac{p}{x^2}\right)^5=(2x^3-1)\left(243x^5-405px^2+\frac{270p^2}{x}-\frac{90p^3}{x^4}+\ldots\right)$ $=\ldots+(-1)\times\frac{270p^2}{x}+2x^3\times\left(\frac{-90p^3}{x^4}\right)+\ldots$ $=\ldots-\frac{270p^2}{x}-\frac{180p^3}{x}+\ldots$ Thus $-270p^2-180p^3=90p^2$ $\Rightarrow 90p^2(4+2p)=0 \Rightarrow p=0\,(\text{NA})$ or $p=-2$

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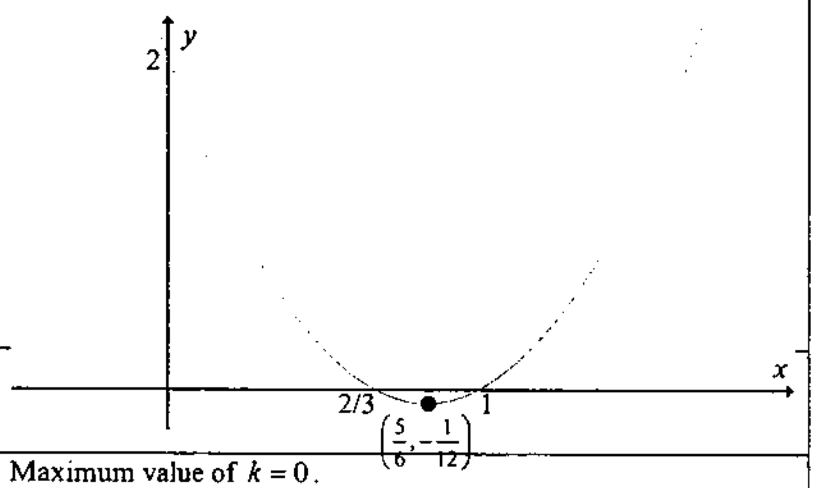
3(i)	From $10+8x-2x^2=\|10-5x\|$ $\Rightarrow 10+8x-2x^2=-(10-5x)$ OR $10+8x-2x^2=10-5x$ $\Rightarrow 2x^2-3x-20=0$ OR $2x^2-13x=0$ $\Rightarrow (2x+5)(x-4)=0$ OR $x(2x-13)=0$ $\Rightarrow x=-2.5, 4$ OR $x=0, 6.5$ (Reject -2.5) (Reject 6.5)
3(ii)	
3 (iii)	The range is $x \le 0$ or $x \ge 4$.
4(a) (i) **4 (a) (ii)**	
4 (a) (iii)	Maximum value of $k=0$. $y=(x-1)(3x-2)=3x^2-5x+2$ $\Rightarrow 3x^2-5x+(2-y)=0$ $x=\dfrac{5\pm\sqrt{25+12(y-2)}}{6}$ Since $x \le 0$, $g^{-1}(x)=\dfrac{5-\sqrt{1+12x}}{6}$
4 (b)	Before II, $y=3\cos x$ Before I, $y=3\cos(x+\frac{\pi}{4})$
5 (a)	$3^{2x}\times 3^3\times 2^x\times 2^5=3^x\times 3^4$ $\dfrac{3^{2x}\times 2^x}{3^x}=\dfrac{3^4}{2^5\times 3^3}$ $6^x=\dfrac{3}{32}$
5(b)	$e^{2+x}=e^{4y}$ $2+x=4y$ $x=4y-2$......(1)

	$\log_4(x+2)=1+\log_2 y$ $\frac{\log_2(x+2)}{\log_2 4}=\log_2 2+\log_2 y$ $\frac{1}{2}\log_2(x+2)=\log_2 2y$ $(x+2)^{\frac{1}{2}}=2y$ $x+2=4y^2$......(2) Substitute (1) into (2), $4y-2+2=4y^2$ $4y(y-1)=0$ Since $y\neq 0$ as it would make $\log_2 y$ undefined. Thus $y=1$ $\therefore x=2$
6 (i)	$\frac{4x}{2x+1}=\frac{2(2x+1)-2}{2x+1}=2-\frac{2}{2x+1}$
6 (ii)	$\frac{d}{dx}\left[2x\ln(2x+1)\right]=2\ln(2x+1)+2x\left(\frac{2}{2x+1}\right)$ $=2\ln(2x+1)+\frac{4x}{2x+1}$
6 (iii)	Integrate both sides in (ii), $\int\frac{d}{dx}\left[2x\ln(2x+1)\right]dx=\int\left[2\ln(2x+1)+\frac{4x}{2x+1}\right]dx$ $2x\ln(2x+1)+c=\int 2\ln(2x+1)\ dx+\int\frac{4x}{2x+1}\ dx$ $=2\int\ln(2x+1)\ dx+\int 2-\frac{2}{2x+1}\ dx$ $2x\ln(2x+1)+c-\int 2-\frac{2}{2x+1}\ dx=\int 2\ln(2x+1)\ dx$ $2x\ln(2x+1)-2x+\ln\lvert 2x+1\rvert+c=\int 2\ln(2x+1)\ dx$ $\therefore\int\ln(2x+1)\ dx=x\ln\lvert 2x+1\rvert-x+\frac{1}{2}\ln\lvert 2x+1\rvert+c$
7 (i)	Area of region $=\int_{4.5}^{9}0-\left(3-\sqrt{2x}\right)dx$

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	$=\left[\frac{1}{3}(2x)^{\frac{3}{2}}-3x\right]_{4.5}^{9}$ $=\left[\left(\frac{1}{3}(18)^{\frac{3}{2}}-27\right)-\left(\frac{1}{3}(9)^{\frac{3}{2}}-\frac{27}{2}\right)\right]$ $=\left[\left(\frac{1}{3}\left(3\sqrt{2}\right)^{3}-27\right)-\left(-4\frac{1}{2}\right)\right]$ $=\left[\frac{1}{3}\left(54\sqrt{2}\right)-22\frac{1}{2}\right]$ $=18\sqrt{2}-22\frac{1}{2}$
7 (ii)	Substitute $2y=x$ into : $\frac{x}{2}=3-\sqrt{2x}$ $\Rightarrow 6-x=2\sqrt{2x}$ $\Rightarrow 36-12x+x^2=8x$ $\Rightarrow x^2-20x+36=0$ $\Rightarrow (x-2)(x-18)=0$ $\Rightarrow x=2,\ 18\ (N.A)$ $\therefore y=1$ $A=(2,1)$
7 (iii)	Area required $=\int_0^2 3-\sqrt{2x}\ \mathrm{d}x-\frac{1}{2}\times 2\times 1$ $=\left[3x-\frac{1}{3}(2x)^{\frac{3}{2}}\right]_0^2-1$ $=\left[\left(6-\frac{1}{3}(4)^{\frac{3}{2}}\right)-0\right]-1$ $=5-\frac{8}{3}=2\frac{1}{3}$ **Alternatively** $=\frac{1}{2}\times 2\times 1+\int_1^3\frac{1}{2}(3-y)^2\ \mathrm{d}y$ $=1+\frac{1}{6}\left[-(3-y)^3\right]_1^3$ $=1-\frac{1}{6}(0-8)=2\frac{1}{3}$
8(i)	$(x-1)^2+y^2+6y-16=0$

	$(x-1)^2+(y+3)^2-9-16=0$ $(x-1)^2+(y+3)^2=25$ Coordinates of the centre of the circle are $(1,-3)$ and the radius is 5 units
8(ii)	Gradient of $CP=\frac{-7+3}{4-1}=-\frac{4}{3}$ Gradient of tangent at $P=\frac{3}{4}$ Equation of tangent at P is: $y-(-7)=\frac{3}{4}(x-4)$ $y=\frac{3}{4}x-10$ **Alternatively,** Sub $x=4, y=-7$ into $y=mx+c$: $-7=3+c \Rightarrow c=-10$ Thus, equation is $y=\frac{3}{4}x-10$
8(iii)	If PQ is the diameter, then C is the midpoint of PQ. Therefore $\frac{4+x}{2}=1$ and $\frac{-7+y}{2}=-3$ $\Rightarrow x=-2$ and $y=1$. Thus coordinates of Q is $(-2,1)$.
8(iv)	Let X be the centre of C_2. If X lies on the circumference of C_1, then $\angle PXQ=90°$. But $\angle PXQ=2\angle PRQ$, thus $\angle PRQ=45°$. Therefore, the centre of C_2 can lie on the circumference of C_1.
9(i)	$\sqrt{(2a+4-0)^2+(3a-2)^2}=4\sqrt{5}$ $4a^2+16a+16+9a^2-12a+4=16(5)$ $13a^2+4a-60=0$ $(13a+30)(a-2)=0$ $a=-\frac{30}{13}$ *or* $a=2$ $(reject\ \because a>0)$
9(ii)	Gradient of $AD==\frac{2-(-2)}{0-2}=-2$. Hence gradient of $AB=\frac{1}{2}$ $\therefore$ Equation of AB is: $y=\frac{1}{2}x+2$
9(iii)	Given $C=(b,0)$ $\frac{0+2}{b-2}=\frac{1}{2}\Rightarrow b=6$
9(iv)	Midpoint of $AB, M\ (4,4)$ Equation of perpendicular bisector of AB is: $y-4=-2(x-4)$ $\Rightarrow y=-2x+12$ Sub $x=6,\ y=-2(6)+12=0$

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	As the point satisfies the equation, point C lies on the perpendicular bisector.
9(v)	Area of trapezium $ABCD=\frac{1}{2}\begin{vmatrix}0 & 2 & 6 & 8 & 0\\ 2 & -2 & 0 & 6 & 2\end{vmatrix}$ $=\frac{1}{2}(36+16-(-12)-4)$ $=\frac{1}{2}(60)$ $=30$ square units
10 (a) (i)	LHS$=\frac{(\cos\theta+\sin\theta)^2}{1+\tan^2\theta+2\tan\theta}$ $=\frac{\left[\cos\theta(1+\tan\theta)\right]^2}{(1+\tan\theta)^2}$ $=\cos^2\theta=$ RHS **Alternatively** LHS$=\frac{\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta}{1+\frac{\sin^2\theta}{\cos^2\theta}+2\frac{\sin\theta}{\cos\theta}}$ $=\frac{\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta}{\frac{\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta}{\cos^2\theta}}$ $=\cos^2\theta=$RHS
10 (a) (ii)	$\frac{1}{\cos^2\theta}=2\tan\theta+4$ $\sec^2\theta=2\tan\theta+4$ $\tan^2\theta-2\tan\theta-3=0$ $(\tan\theta-3)(\tan\theta+1)=0$ $\tan\theta=3$ or $\tan\theta=-1$ $\alpha=1.249$ or $\alpha=\frac{\pi}{4}$ $\theta=1.25,\ 4.39$ or $\theta=\frac{3\pi}{4},\ \frac{7\pi}{4}$ (reject both ans)
10 (b) (i)	Area of shaded region, A $=\frac{1}{2}(5\cos\theta)(5\sin\theta)-\frac{25}{4}\cos^2\theta$ $=\frac{25}{2}\sin\theta\cos\theta-\frac{25}{4}\cos^2\theta$ $=\frac{25}{4}\sin 2\theta-\frac{25}{4}\left(\frac{\cos 2\theta+1}{2}\right)$ $=\frac{50\sin 2\theta-25\cos 2\theta-25}{8}$ $=\frac{25}{8}(2\sin 2\theta-\cos 2\theta)-\frac{25}{8}$.

10 (b) (ii)	Let $2\sin 2\theta - \cos 2\theta = r\sin(2\theta - \alpha)$ $\therefore\ r = \sqrt{2^2 + 1^2} = \sqrt{5}$ $\tan\alpha = \frac{1}{2} \Rightarrow \alpha = 26.5651°$ $A = \frac{25\sqrt{5}}{8}\sin(2\theta - 26.6°) - \frac{25}{8}$ Thus $R = \frac{25\sqrt{5}}{8}$ and $\alpha = 26.6°$
10 (b) (iii)	Maximum $A = \frac{25\sqrt{5} - 25}{8}\ \text{cm}^2$
11 (i)	$\frac{1}{3}\pi(4x)^2(3x) + \pi(4x)^2 h = 960\pi$ $16\pi x^2 h = 960\pi - 16x^3\pi$ $\therefore\quad h = \frac{60}{x^2} - x$ (proven)
11 (ii)	$A = \pi(5x)(4x) + 2\pi(4x)h + \pi(4x)^2$ (slant height of cone = $5x$) $= 20\pi x^2 + \frac{480\pi}{x} - 8\pi x^2 + 16\pi x^2$ $= \frac{480\pi}{x} + 28\pi x^2$ (proven)
11 (iii)	$\frac{\mathrm{d}A}{\mathrm{d}x} = -\frac{480\pi}{x^2} + 56\pi x$ $\frac{-480\pi + 56\pi x^3}{x^2} = 0$ $x^3 = \frac{60}{7}$ $x = \sqrt[3]{\frac{60}{7}} = 2.05\,\text{cm}$ (2.0465) $h = 12.3\,\text{cm}$ (12.28) $\frac{d^2A}{dx^2} = \frac{960\pi}{x^3} + 56\pi$ $\left.\frac{d^2A}{dx^2}\right\|_{x=2.0465} = \frac{960\pi}{(2.0465)^3} + 56\pi > 0$ $\therefore\quad A$ is a minimum.

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Paya Lebar Methodist Girls' School (Secondary)
Preliminary Examination 2015
Secondary 4 Express / 5 Normal Academic

Name: ______________________ (   )   Class: _______

Centre Number	S				

Index Number				

ADDITIONAL MATHEMATICS **4047/01**

Paper 1 29 July 2015

Additional Materials: Answer Paper (8 sheets) 2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

My Target is:

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The acute angle A and obtuse angle B are such that $\tan A = \frac{1}{2}$ and $4\cos(A+B) = 3\sin(A-B)$.
Without using a calculator, find the exact value of $\cos B$. [5]

2 **(i)** Write down the first three terms in the expansion, in ascending powers of x, of $(1+x)^n$, where n is a positive integer greater than 2. [1]

(ii) The coefficient of x^2 in the expansion, in ascending powers of x, of $(1+x)^n(2-3x)^4$, where n is a positive integer greater than 2, is 456. Find the coefficient of x. [4]

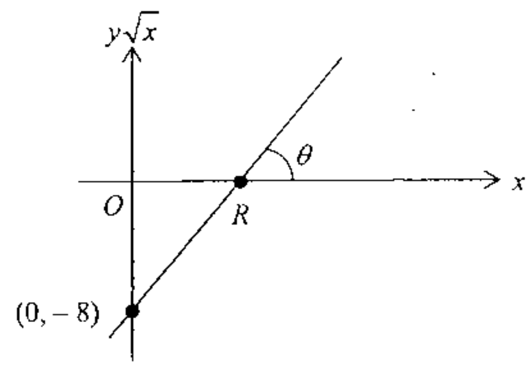
3 The volume of a cube, V cm^3, is increasing at a constant rate of 5 cm^3 per second.
Find the volume of the cube at the instant when the length of the side of the cube, x cm, is increasing at 0.5 cm per second. [5]

4 Given that $\dfrac{2x^3-1}{x^3-x^2} = a + \dfrac{bx^2+c}{x^3-x^2}$, where a, b and c are integers.

(i) Find the value of a, of b and of c. [2]

(ii) Using the values of b and c found in part (i), express $\dfrac{bx^2+c}{x^3-x^2}$ as the sum of 3 partial fractions. [4]

5



The diagram shows part of a straight line graph drawn to represent the equation $y + \dfrac{k}{\sqrt{x}} = 5\sqrt{x}$, where k is a constant. Given that the line passes through the point $(0, -8)$ and makes an angle θ with the x-axis at point R, where $0° < \theta < 90°$, find

(i) the value of k and of θ, [4]

(ii) the coordinates of R. [2]

[Turn over
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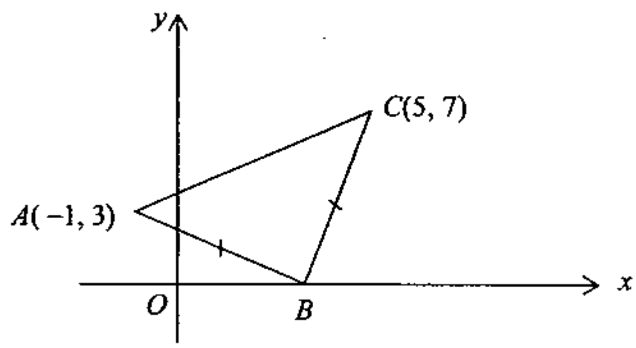
6 (i) Prove that $\cot(\frac{\pi}{4}-\theta)=\frac{\cot\theta+1}{\cot\theta-1}$. [3]

(ii) Hence **without using a calculator**, show that $\cot\frac{\pi}{12}=2+\sqrt{3}$. [3]

7



The diagram shows a triangle ABC in which the coordinates of the points A and C are $(-1, 3)$ and $(5, 7)$ respectively.

Given that $AB = BC$,

(i) find the coordinates of B. [4]

D is a point on the perpendicular bisector of AC.

(ii) Find the equation of BD. [3]

8 (i) Show that $\frac{d}{dx}\left[\ln(\cos 4x)\right]=-4\tan 4x$. [1]

It is given that $\frac{dy}{dx}=\frac{x}{2}-\frac{\tan 4x}{8}$ and $\frac{d^2y}{dx^2}=k\tan^2 4x$.

(ii) Find the value of k. [3]

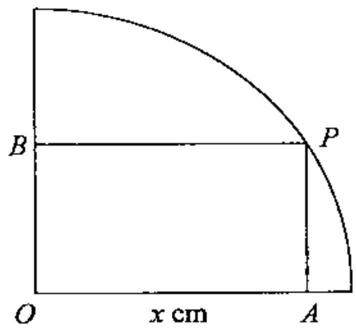
(iii) Using the result in part (i), find y given that $x = 0$ when $y = 1$. [3]

9 The equation of a curve is $y=(k-7)x^2-8x+k$, where k is a constant.

(i) Find the set of values of k for which the curve lies completely above the line $y = 1$. [5]

(ii) In the case where $k = 8$, find the set of values of c for which the line $y = 2x - c$ intersects the curve at two distinct points. [3]

10



The diagram shows a rectangle $OAPB$ inscribed in a quadrant of a circle of radius 5 cm.
The length of OA is x cm.

(i) Show that the area of the rectangle, A cm^2, is given by $A = x\sqrt{25-x^2}$. [2]

(ii) Given that x can vary, find the value of x for which A has a stationary value. [4]

(iii) Determine whether this stationary value of A is a maximum or a minimum. Hence find this value of A. [2]

11 The equation of a curve is $y = -\ln(3 - ax)$, where a is a constant.

(i) Find the value of a if the gradient of the curve at $y = -\ln 5$ is 2. [4]

(ii) Find the value of a if the normal to the curve at $x = 1$ is parallel to the line $2x - y = 5$. [2]

(iii) In the case where $a = 4$, find the coordinates of the point on the curve where the equation of the tangent to the curve is $y = 4x - 2$. [2]

12 **(i)** Find the turning point of the curve $y = x^2 - 4x$. [2]

(ii) Sketch the graph of $y = |x^2 - 4x|$, indicating clearly the coordinates of the turning point and of the points where the graph meets the x-axis. [3]

(iii) Using your graph, find the number of solutions of the equation $|x^2 - 4x| = 2 - mx$ when

(a) $m = -2$, **(b)** $m = \frac{1}{2}$. [4]

End of Paper

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PLMGS (Secondary)
Additional Mathematics Preliminary Examination 2015
Secondary Four Express & Five Normal (Academic)
Additional Mathematics Paper 1 (4047/01) Worked Solutions

1.
$$4\cos(A+B)=3\sin(A-B)$$
$$4(\cos A\cos B-\sin A\sin B=3(\sin A\cos B-\cos A\sin B)$$
$$4(\frac{2}{\sqrt{5}})\cos B-4(\frac{1}{\sqrt{5}})\sin B=3(\frac{1}{\sqrt{5}})\cos B-3(\frac{2}{\sqrt{5}})\sin B$$
$$5\cos B=-2\sin B$$
$$\tan B=-\frac{5}{2}$$
$$\cos B=-\frac{2}{\sqrt{29}}$$
$$=-\frac{2\sqrt{29}}{29}$$

OR

$$4\cos(A+B)=3\sin(A-B)$$
$$4(\cos A\cos B-\sin A\sin B=3(\sin A\cos B-\cos A\sin B)$$
$$\frac{4\cos A\cos B-4\sin A\sin B}{\cos A\cos B}=\frac{3\sin A\cos B-3\cos A\sin B}{\cos A\cos B}$$
$$4-4\tan A\tan B=3\tan A-3\tan B$$
$$4-4(\frac{1}{2})\tan B=3(\frac{1}{2})-3\tan B$$
$$\tan B=-\frac{5}{2}$$
$$\cos B=-\frac{2}{\sqrt{29}}$$
$$=-\frac{2\sqrt{29}}{29}$$

147

2.(i) $(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \ldots$

2.(ii) $(2-3x)^4 = (2)^4 + 4(2)^3(-3x) + 6(2)^2(-3x)^2 + \ldots$

$= 16 - 96x + 216x^2 + \ldots$

$(1+x)^n(2-3x)^4 = [1 + nx + \frac{n(n-1)}{2}x^2 + \ldots][16 - 96x + 216x^2 + \ldots]$

$= -96x + 16nx + (8n^2 - 104n + 216)x^2 + \ldots$

Coefficient of x^2: $8n^2 - 104n + 216 = 456$

$n^2 - 13n - 30 = 0$

$(n-15)(n+2) = 0$

$n = 15$ or $n = -2$ (N.A.)

Coefficient of $x = -96 + 16(15)$

$= 144$

3. $V = x^3$

$\frac{dV}{dx} = 3x^2$

$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$

$5 = 3x^2 \times 0.5$

$x = \sqrt{\frac{10}{3}}$

Volume of cube $= \left(\sqrt{\frac{10}{3}}\right)^3$

$= 6.085806$

$= 6.09\ \text{cm}^3$ (3 s.f.)

4.(i) $\dfrac{2x^3-1}{x^3-x^2}=a+\dfrac{bx^2+c}{x^3-x^2}$

$2x^3-1=ax^3-ax^2+bx^2+c$

$a=2$
$b=a=2$
$c=-1$

4.(ii) $\dfrac{2x^2-1}{x^3-x^2}=\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x-1}$

$2x^2-1=Ax(x-1)+B(x-1)+Cx^2$

Let $x=1$, $C=1$

Let $x=0$, $B=1$

Coefficient of x^2: $A=1$

$\dfrac{2x^2-1}{x^3-x^2}=\dfrac{1}{x}+\dfrac{1}{x^2}+\dfrac{1}{x-1}$

5.(i) $y\sqrt{x}=5x-k$
$k=8$

$\tan\theta=5$
$\theta=78.7°$

5.(ii) when $y\sqrt{x}=0$,
$5x-8=0$
$x=\dfrac{8}{5}$
$R(1\dfrac{3}{5},0)$

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6.(i) $\cot(\frac{\pi}{4}-\theta) = \frac{1}{\tan(\frac{\pi}{4}-\theta)}$

$$= \frac{1 + \tan\frac{\pi}{4}\tan\theta}{\tan\frac{\pi}{4} - \tan\theta}$$

$$= \frac{1 + (1)\tan\theta}{1 - \tan\theta}$$

$$= \frac{\frac{1}{\tan\theta} + 1}{\frac{1}{\tan\theta} - 1}$$

$$= \frac{\cot\theta + 1}{\cot\theta - 1}$$

6.(ii) $\cot\frac{\pi}{12} = \cot(\frac{\pi}{4} - \frac{\pi}{6})$

$$= \frac{\cot\frac{\pi}{6} + 1}{\cot\frac{\pi}{6} - 1}$$

$$= \frac{\frac{1}{\tan\frac{\pi}{6}} + 1}{\frac{1}{\tan\frac{\pi}{6}} - 1}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{3 + 2\sqrt{3} + 1}{3 - 1}$$

$$= 2 + \sqrt{3}$$

7.(i) Let B be $(k, 0)$

$$AB = BC$$

$$\sqrt{(k+1)^2 + (-3)^2} = \sqrt{(k-5)^2 + (-7)^2}$$

$$k^2 + 2k + 10 = k^2 - 10k + 74$$

$$k = \frac{16}{3}$$

$$B(5\frac{1}{3}, 0)$$

7.(ii) Gradient of $AC = \dfrac{7-3}{5+1}$

$$= \frac{2}{3}$$

Gradient of $BD = -\dfrac{3}{2}$

Equation of BD is $\quad y - 0 = -\dfrac{3}{2}(x - \dfrac{16}{3})$

$$y = -\frac{3}{2}x + 8 \qquad \text{OR} \qquad 2y = -3x + 16$$

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8.(i) $\frac{d}{dx}\left[\ln(\cos 4x)\right] = \frac{1}{\cos 4x}(-4\sin 4x)$

$$= -4\tan 4x$$

8.(ii) $\frac{d^2y}{dx^2} = \frac{1}{2} - \frac{1}{8}(4\sec^2 4x)$

$$= \frac{1}{2} - \frac{1}{2}(1 + \tan^2 4x)$$

$$= -\frac{1}{2}\tan^2 4x$$

$$k = -\frac{1}{2}$$

8.(iii) $\int -4\tan 4x \, dx = \ln(\cos 4x) + C$

$$y = \int \left(\frac{x}{2} - \frac{\tan 4x}{8}\right) dx$$

$$= \frac{1}{4}x^2 + \frac{1}{8} \cdot \frac{1}{4}\int -4\tan 4x \, dx$$

$$y = \frac{1}{4}x^2 + \frac{1}{32}\ln(\cos 4x) + C$$

When $x = 0, y = 1$,

$$1 = \frac{1}{32}\ln(\cos 0) + C$$

$$C = 1$$

$$y = \frac{1}{4}x^2 + \frac{1}{32}\ln(\cos 4x) + 1$$

9.(i) $(k-7)x^2 - 8x + k = 1$

$(k-7)x^2 - 8x + (k-1) = 0$

$$b^2 - 4ac < 0$$

$$(-8)^2 - 4(k-7)(k-1) < 0$$

$$64 - 4(k^2 - 8k + 7) < 0$$

$$64 - 4k^2 + 32k - 28 < 0$$

$$4k^2 - 32k - 36 > 0$$

$$k^2 - 8k - 9 > 0$$

$$(k-9)(k+1) > 0$$

$$k < -1 \quad \text{OR} \quad k > 9$$

And $\quad k - 7 > 0$

$$k > 7$$

Since $k > 7$ AND $k > 9$, $\therefore$ $k > 9$.

9.(ii) $y = x^2 - 8x + 8$

$x^2 - 8x + 8 = 2x - c$

$x^2 - 10x + 8 + c = 0$

$$b^2 - 4ac > 0$$

$$(-10)^2 - 4(1)(8 + c) > 0$$

$$c < 17$$

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10.(i) $AP^2 = 5^2 - x^2$

$AP = \sqrt{25 - x^2}$ cm

$A = OA \times AP$

$A = x\sqrt{25 - x^2}$

10.(ii) $\dfrac{dA}{dx} = x \cdot \dfrac{1}{2}(25 - x^2)^{-\frac{1}{2}}(-2x) + (25 - x^2)^{\frac{1}{2}}(1)$

$= (25 - x^2)^{-\frac{1}{2}}[-x^2 + 25 - x^2]$

$= \dfrac{25 - 2x^2}{\sqrt{25 - x^2}}$

For stationary value of A, $\dfrac{dA}{dx} = 0$

$$\frac{25 - 2x^2}{\sqrt{25 - x^2}} = 0$$

$$25 - 2x^2 = 0$$

$$x = \sqrt{\frac{25}{2}}$$

$$= \frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{5\sqrt{2}}{2}$$

10.(iii) For $x < \dfrac{5\sqrt{2}}{2}$, $\dfrac{dA}{dx} > 0$

For $x > \dfrac{5\sqrt{2}}{2}$, $\dfrac{dA}{dx} < 0$

As x increases through $\dfrac{5\sqrt{2}}{2}$, the sign of $\dfrac{dA}{dx}$ changes from positive to negative.

Stationary value of A is a maximum.

Stationary value of $A = \dfrac{5\sqrt{2}}{2}\sqrt{25 - \left(\dfrac{5\sqrt{2}}{2}\right)^2}$

$$= 12\frac{1}{2}$$

11.(i) $y = -\ln(3-ax)$

$$\frac{dy}{dx} = \frac{a}{3-ax}$$

When $y = -\ln 5$, $-\ln(3-ax) = -\ln 5$

$$3 - ax = 5$$

$$x = -\frac{2}{a}$$

When $x = -\frac{2}{a}$, $\frac{dy}{dx} = 2$,

$$\frac{a}{3-a\left(-\frac{2}{a}\right)} = 2$$

$$a = 10$$

11.(ii) $y = 2x - 5$

At $x = 1$, Gradient of normal $= -\frac{3-a(1)}{a}$

$$-\frac{3-a}{a} = 2$$

$$a = -3$$

11.(iii) when $a = 4$, $\frac{dy}{dx} = \frac{4}{3-4x}$

$$\frac{4}{3-4x} = 4$$

$$x = \frac{1}{2}$$

From $y = 4x - 2$, $y = 4\left(\frac{1}{2}\right) - 2$

$$= 0$$

Coordinates of point $= (\frac{1}{2}, 0)$

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12.(i) $y = x(x-4)$

For x-intercepts, $y = 0$,

$x = 0$ or $x = 4$

For minimum y, when $x = \dfrac{4+0}{2}$

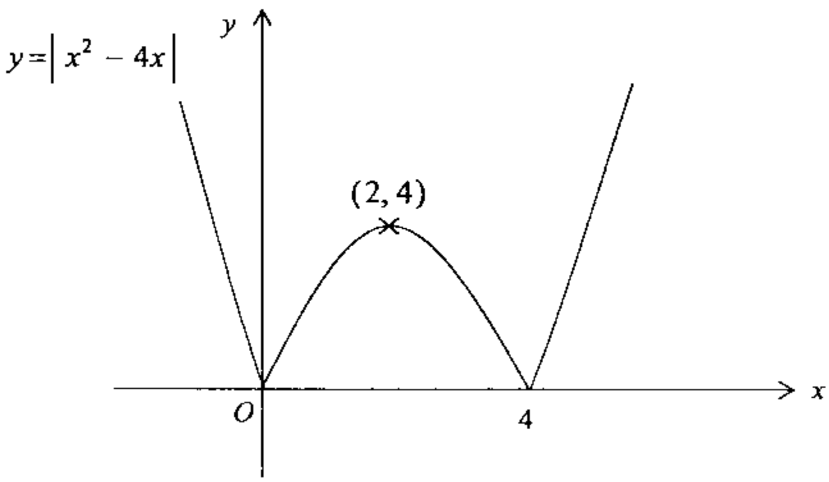
$= 2$

$y = 2^2 - 4(2)$

$= -4$

Turning point $= (2, -4)$

12.(ii)



- Shape
- Turning point
- x-intercepts

12.(iii) (a) $y = 2 + 2x$

2 solutions

12.(iii) (b) $y = 2 - \dfrac{1}{2}x$

3 solutions

Paya Lebar Methodist Girls' School (Secondary)
Preliminary Examination 2015
Secondary 4 Express / 5 Normal Academic

Name: ________________________ (   )   Class: ______

Centre Number	S					Index Number					

ADDITIONAL MATHEMATICS **4047/02**

Paper 2 — 31 July 2015

Additional Materials: Answer Paper (10 sheets) — 2 hours 30 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

HAND IN QUESTIONS 1 TO 9 SEPARATELY FROM QUESTIONS 10 TO 11.

My Target is:

This paper consists of 8 pages.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 In the cubic polynomial f(x), the coefficient of x^3 is a, where $0 < a < 1$.

(i) Given that the repeated root of the equation $f(x) = 0$ is 1, write down an expression for f(x). [2]

(ii) Find the value of a if f(x) has a remainder of 1 when divided by x and f(x) has a remainder of -8 when divided by $x+3$. [4]

2 A triangle ABC in which $AB = AC$ has its base BC of length $\left(4\sqrt{2} - \frac{6}{\sqrt{3}}\right)$ cm.

(i) In the case where the area of the triangle is $\left(3\sqrt{2} - 2\sqrt{3}\right)$ cm^2, find, **without using a calculator**, the length of the height of the triangle in the form $\left(a + b\sqrt{6}\right)$ cm. [4]

(ii) In the case where angle BAC is a right angle, find, **without using a calculator**, the square of the length of AB in the form $\left(c + d\sqrt{6}\right)$ cm^2. [3]

3 (a) Given that $\ln(p^2 q) = a$ and $\ln(pq^2) = b$, express pq in terms of a and b. [2]

(b) Solve the equation $\log_3(2x+1) + \log_{\frac{1}{3}} 3 = \log_9(x-2)^4 - \log_3(x-1)$. [5]

4 A quadratic equation has roots α and β, where $\alpha < \beta$.

(i) Given that $\alpha\beta = -\frac{1}{2}$ and $\alpha^2 + \beta^2 = 5\frac{1}{4}$, **without solving for α and β**, find the value of $\alpha - \beta$. [2]

(ii) Show that $\alpha^3 - \beta^3 = -11\frac{7}{8}$. [2]

(iii) Find the quadratic equation whose roots are $\frac{\alpha^2 - 1}{\beta}$ and $\frac{1 - \beta^2}{\alpha}$. [4]

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[Turn over

5 A circle, centre C, has a diameter AB where A is the point $(-3, 2)$ and B is the point $(5, 8)$.

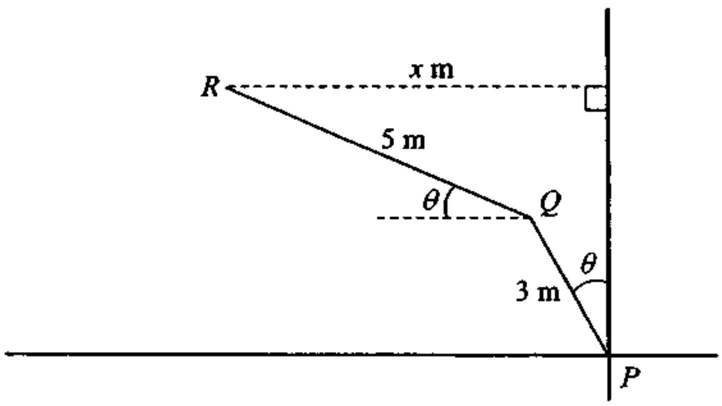
(i) Find the coordinates of C and the radius of the circle. [4]

(ii) Find the equation of the circle. [1]

(iii) Show that the equation of the tangent to the circle at B is $3y + 4x = 44$. [3]

(iv) The highest point on the circle is D. Find the coordinates of the point at which the tangents to the circle at B and D intersect. [2]

6



The diagram shows a rod PQ which is hinged at P, and a rod QR, which is hinged at Q. The rods can only move in the vertical plane as shown. The rod PQ can turn about P and is inclined at an angle θ to the vertical, where $0° \leq \theta \leq 90°$. The rod QR can turn about Q in such a way that its inclination to the horizontal is also θ. The lengths of PQ and QR are 3 m and 5 m respectively.

Given that R is x m from the vertical axis,

(i) find the values of the integers a and b for which $x = a\cos\theta + b\sin\theta$. [2]

Using the values of a and b found in part (i),

(ii) express x in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(iii) Hence state the maximum value of x and find the corresponding value of θ. [3]

(iv) Deduce, with explanation, the value of x when the rod PQ is inclined at $90°$ to the vertical and the rod QR is inclined at $90°$ to the horizontal. [2]

7 A curve is such that $\frac{dy}{dx} = 2(3x-2)(x-3)$.

(i) Given that the curve passes through the point $(0, 9)$, find the equation of the curve. [2]

(ii) The point (p, q) where p and q are integers, is a stationary point on the curve. Find the value of p and of q. [3]

(iii) Determine whether y is increasing or decreasing

(a) for values of x slightly less than p, [1]

(b) for values of x slightly more than p. [1]

(iv) What do the results of part (iii) imply about the stationary point (p, q)? [1]

(v) Find the value of $\frac{d^2y}{dx^2}$ at the stationary point (p, q) and explain how this value supports the conclusion that you have made in part (iv). [2]

8 A particle travelling along a straight line is such that its displacement, s m, from a fixed point O on the line is given by $s = t^3 - 6t^2 + 9t + 18$, where t seconds is the time after motion has begun.

(i) Find the initial displacement of the particle from the fixed point O. [1]

(ii) Find the values of t for which the particle is instantaneously at rest. Show that the particle returns to its starting position at one of the two instances of rest. [4]

(iii) Find the total distance travelled by the particle during the first 4 seconds. [3]

(iv) Find the minimum velocity of the particle. [3]

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9 (i) On the same axes, sketch the graphs of $y = 2\sin t + 2$ and $y = \frac{1}{2}\sin\frac{t}{2} + 2$ for $0 \le t \le 4\pi$. [4]

(ii) It is observed that the height, y m, above sea-level, reached by ocean waves on two particular days during a time interval of 4π minutes can be modelled by trigonometric functions. The function $y = 2\sin t + 2$ models the height of waves on Day 1, and the function $y = \frac{1}{2}\sin\frac{t}{2} + 2$ models the height of waves on Day 2.

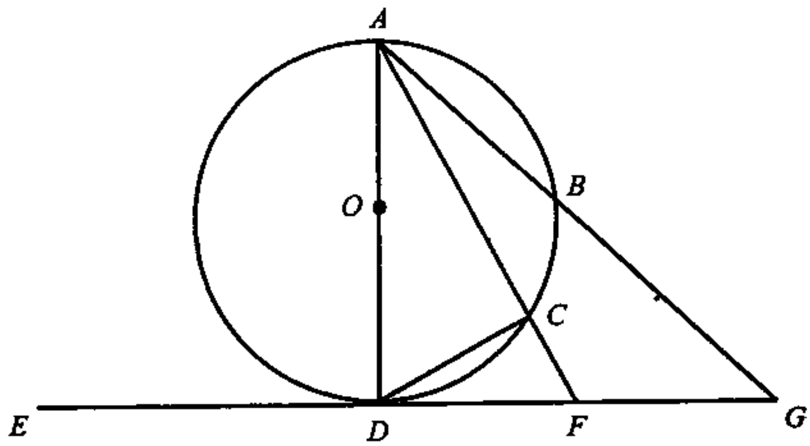
With reference to the graphs that you have sketched in part **(i)**,

(a) state the number of instances when the waves on the two days reached the same height during the time interval $0 < t < 4\pi$. Justify your answer. [2]

(b) Which of the two days would have provided surfers with a more thrilling experience of riding the waves at sea? Explain your answer. [3]

Begin your answer to Question 10 on a fresh sheet of writing paper.

10



In the diagram, A, B, C and D are points on the circumference of the circle with centre O. $EDFG$ is a tangent to the circle at D.

Given that $AB = BG$ and $DF = FG$, prove that

(i) ABD is an isosceles triangle, [3]

(ii) $DB^2 - DF^2 = \frac{1}{4}AD^2$, [2]

(iii) triangle ADF is similar to triangle DCF, [2]

(iv) $GF^2 = AF \times CF$. [2]

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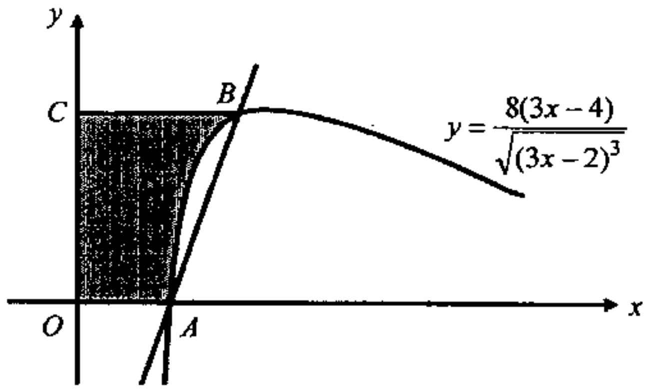
[Turn over

11 **(a)** Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{x}{\sqrt{3x-2}}\right)=\dfrac{3x-4}{2\sqrt{(3x-2)^3}}$. [3]

(b)



The diagram shows part of the curve $y=\dfrac{8(3x-4)}{\sqrt{(3x-2)^3}}$.

The curve intersects the x-axis at the point A. The line through A with gradient 3 intersects the curve again at the point B. The line BC is parallel to the x-axis.

(i) Verify that the y-coordinate of B is 2. [5]

(ii) Determine the area of the shaded region bounded by the curve, the x-axis, the y-axis and the line BC. [4]

End of Paper

2015 Preliminary Examination
Additional Mathematics 4047/2
Worked Solutions

Qn	Working	Marks	Total	Remarks
1	(i) $f(x) = (ax+k)(x-1)^2$ where k is a constant		[2]	Accept $f(x) = (ax-k)(x-1)^2$ where k is a constant
	(ii) By the Remainder Theorem, $f(0) = 1$ $k(-1)^2 = 1$ $k = 1$ By the Remainder Theorem, $f(-3) = -8$ $(-3a+1)(-4)^2 = -8$ $-3a+1 = -\frac{1}{2}$ $3a = \frac{3}{2}$ $a = \frac{1}{2}$		[4] (6 m)	

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Qn	Working	Marks	Total	Remarks
	Alternative Solution:			
1	(i) $f(x) = a(x+k)(x-1)^2$ where k is a constant		[2]	
	(ii) $f(0) = 1$ $a(k)(-1)^2 = 1$ $ak = 1$ ---------- (1)			
	(iii) $f(-3) = -8$ $a(-3+k)(-4)^2 = -8$ $a(-3+k) = -\frac{1}{2}$ ---------- (2) Subst. $k = \frac{1}{a}$ into (2): $a\left(-3+\frac{1}{a}\right) = -\frac{1}{2}$ $-3a+1 = -\frac{1}{2}$ $3a = \frac{3}{2}$ $a = \frac{1}{2}$		[4] (6 m)	

Qn	Working	Marks	Total	Remarks
2	**(i)** Let the height of the triangle be h cm. $\frac{1}{2}\left(4\sqrt{2}-\frac{6}{\sqrt{3}}\right)(h) = 3\sqrt{2}-2\sqrt{3}$ $\left(2\sqrt{2}-\frac{3}{\sqrt{3}}\right)(h) = 3\sqrt{2}-2\sqrt{3}$ $(2\sqrt{2}-\sqrt{3})(h) = 3\sqrt{2}-2\sqrt{3}$ $h = \frac{3\sqrt{2}-2\sqrt{3}}{2\sqrt{2}-\sqrt{3}}$ $= \frac{3\sqrt{2}-2\sqrt{3}}{2\sqrt{2}-\sqrt{3}} \times \frac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}}$ $= \frac{12+3\sqrt{6}-4\sqrt{6}-6}{(2\sqrt{2})^2-(\sqrt{3})^2}$ $= \frac{6-\sqrt{6}}{8-3}$ $= \frac{6}{5}-\frac{1}{5}\sqrt{6}$ $\therefore$ Height of triangle $= \frac{6}{5}-\frac{1}{5}\sqrt{6}$ cm		[4]	
	(ii) By the Pythagoras Theorem, $AB^2+AC^2 = BC^2$ $AB^2+AB^2 = BC^2 \quad (\because AC = AB)$ $2AB^2 = BC^2$ $AB^2 = \frac{1}{2}\left(4\sqrt{2}-\frac{6}{\sqrt{3}}\right)^2$ $= \frac{1}{2}\left[2\left(2\sqrt{2}-\frac{3}{\sqrt{3}}\right)\right]^2$ $= 2(2\sqrt{2}-\sqrt{3})^2$ $= 2(8-4\sqrt{6}+3)$ $= 2(11-4\sqrt{6})$ $= 22-8\sqrt{6}$ cm^2		[3] (7 m)	

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Qn	Working	Marks	Total	Remarks
2	**Alternative Solution**: **(ii)** In rt $-\angle$d ΔABC, $\sin\angle ACB = \dfrac{AB}{BC}$ $\sin 45° = \dfrac{AB}{4\sqrt{2}-\dfrac{6}{\sqrt{3}}}$ $AB = \dfrac{1}{\sqrt{2}} \times (4\sqrt{2}-2\sqrt{3})$ $= 4-\sqrt{6}$ $AB^2 = (4-\sqrt{6})^2$ $= 16-2(4)(\sqrt{6})+6$ $= 22-8\sqrt{6}\ \text{cm}^2$			

Qn	Working	Marks	Total	Remarks
3	(a) $\ln(p^2q) = a$ ---------- (1) $\ln(pq^2) = b$ ---------- (2) (1) + (2): $\ln(p^2q) + \ln(pq^2) = a + b$ $\ln(p^3q^3) = a + b$ $\ln(pq)^3 = a + b$ $3\ln(pq) = a + b$ $\ln(pq) = \dfrac{a+b}{3}$ $pq = e^{\frac{a+b}{3}}$ **Alternative Solution**: $\ln(p^2q) = a \Rightarrow p^2q = e^a$ $\ln(pq^2) = b \Rightarrow pq^2 = e^b$ $(p^2q)(pq^2) = e^a \times e^b$ $(pq)^3 = e^{a+b}$ $pq = e^{\frac{a+b}{3}}$		[2]	

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Qn	Working	Marks	Total	Remarks
3	(b) $\log_3(2x+1)+\log_{\frac{1}{3}} 3 = \log_9(x-2)^4 - \log_3(x-1)$ $\log_3(2x+1)+\dfrac{\log_3 3}{\log_3 \frac{1}{3}} = \dfrac{\log_3(x-2)^4}{\log_3 9} - \log_3(x-1)$ $\log_3(2x+1)-\log_3 3 = \log_3(x-2)^2 - \log_3(x-1)$ $\log_3\left(\dfrac{2x+1}{3}\right) = \log_3\left[\dfrac{(x-2)^2}{x-1}\right]$ $\dfrac{2x+1}{3} = \dfrac{(x-2)^2}{x-1}$ $(2x+1)(x-1) = 3(x-2)^2$ $2x^2-x-1 = 3(x^2-4x+4)$ $x^2-11x+13 = 0$ $x = \dfrac{-(-11)\pm\sqrt{(-11)^2-4(1)(13)}}{2}$ $x = \dfrac{11\pm\sqrt{69}}{2}$ $x \approx 1.35$ or 9.65 (to 3 s.f.) (reject, as $x > 2$) $\therefore x = 9.65$		[5] (7 m)	

Qn	Working	Marks	Total	Remarks
4	(i) $\alpha\beta = -\frac{1}{2}$ $\alpha^2+\beta^2 = \frac{21}{4}$ $(\alpha-\beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2$ $= (\alpha^2+\beta^2) - 2\alpha\beta$ $= \frac{21}{4} - 2\left(-\frac{1}{2}\right)$ $= \frac{25}{4}$ $\therefore \alpha-\beta = -\frac{5}{2} \quad (\because \alpha-\beta<0)$		[2]	
	(ii) $\alpha^3-\beta^3 = (\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)$ $= (\alpha-\beta)[(\alpha^2+\beta^2)+\alpha\beta]$ $= \left(-\frac{5}{2}\right)\left[\frac{21}{4}+\left(-\frac{1}{2}\right)\right]$ $= -11\frac{7}{8}$		[2]	
	(iii) New roots: $\frac{\alpha^2-1}{\beta}, \frac{1-\beta^2}{\alpha}$ $\frac{\alpha^2-1}{\beta}+\frac{1-\beta^2}{\alpha} = \frac{\alpha(\alpha^2-1)+\beta(1-\beta^2)}{\alpha\beta}$ $= \frac{\alpha^3-\alpha+\beta-\beta^3}{\alpha\beta}$ $= \frac{(\alpha^3-\beta^3)-(\alpha-\beta)}{\alpha\beta}$ $= \frac{-11\frac{7}{8}-\left(-\frac{5}{2}\right)}{-\frac{1}{2}}$ $= \frac{75}{4}$			

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Qn	Working	Marks	Total	Remarks
4	(c) $\left(\dfrac{\alpha^2-1}{\beta}\right)\left(\dfrac{1-\beta^2}{\alpha}\right)=\dfrac{\alpha^2-\alpha^2\beta^2-1+\beta^2}{\alpha\beta}$ $=\dfrac{(\alpha^2+\beta^2)-(\alpha\beta)^2-1}{\alpha\beta}$ $=\dfrac{5\frac{1}{4}-\left(-\frac{1}{2}\right)^2-1}{\left(-\frac{1}{2}\right)}$ $=-8$ $\therefore$ The required quadratic equation is $x^2-\dfrac{75}{4}x-8=0$.		[4] (8 m)	

Qn	Working	Marks	Total	Remarks
5	(i) Diameter of circle is AB where $A=(-3,2)$ and $B=(5,8)$ Centre of circle, $C=\left(\frac{-3+5}{2},\frac{2+8}{2}\right)$ $=(1,5)$ Radius of circle $=\frac{1}{2}\sqrt{[5-(-3)]^2+(8-2)^2}$ $=\frac{1}{2}\sqrt{64+36}$ $=\frac{1}{2}(10)$ $=5$ units		[4]	
	Alternatively, Radius of circle $=\sqrt{(5-1)^2+(8-5)^2}$ $=5$ units			
	(ii) Equation of the circle is $(x-1)^2+(y-5)^2=5^2$ i.e. $x^2+y^2-2x-10y+1=0$		[1]	
	(iii) Gradient of $AB=\frac{8-2}{5-(-3)}$ $=\frac{3}{4}$ $\therefore$ Gradient of the tangent at $B=-\frac{4}{3}$ Equation of the tangent to the circle at B is given by $y-8=-\frac{4}{3}(x-5)$ $3(y-8)=-4(x-5)$ $3y-24=-4x+20$ $3y+4x=44$		[3]	

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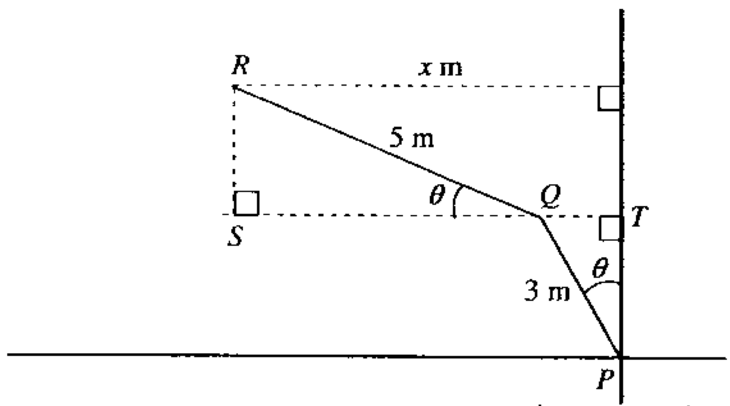
Qn	Working	Marks	Total	Remarks
5	(iv) Highest point on the circle is $D = (1, 10)$ Equation of the tangent at D is $y = 10$ At the point of intersection of the tangents, $3(10) + 4x = 44$ $4x = 14$ $x = 3\frac{1}{2}$ $\therefore$ The tangents intersect at the point $\left(3\frac{1}{2}, 10\right)$		[2] (10 m)	

Qn	Working	Marks	Total	Remarks
6				
	(i) In rt $-\angle$d ΔPQT, $\sin\theta = \frac{QT}{3}$ $QT = 3\sin\theta$ In rt $-\angle$d ΔQRS, $\cos\theta = \frac{QS}{5}$ $QS = 5\cos\theta$ $\therefore\ x = QS + QT$ $= 5\cos\theta + 3\sin\theta$ $= a\cos\theta + b\sin\theta$ $a = 5$ and $b = 3$		[2]	
	(ii) $x = 5\cos\theta + 3\sin\theta$ $= R\cos(\theta - \alpha)$ where $R > 0$ and $0° < \alpha < 90°$ $R = \sqrt{5^2 + 3^2}$ $= \sqrt{34}$ ≈ 5.83 (to 3 s.f.) $\tan\alpha = \frac{3}{5}$ $\alpha = 30.96°$ $\alpha \approx 31.0°$ $\therefore\ x = 5.83\cos(\theta - 31.0°)$		[4]	Accept $x = \sqrt{34}\cos(\theta - 31.0°)$

Qn	Working	Marks	Total	Remarks
6	**(iii)** Maximum value of $x=\sqrt{34}$ ≈ 5.83 Maximum value is attained when $\cos(\theta-30.96°)=1$ where $-30.96° \le \theta-30.96° \le 59.04°$ $\theta-30.96°=0°$ $\theta=30.96°$ $\theta \approx 31.0°$		[3]	
	(iv) When $\theta=90°$, $x=5\cos 90°+3\sin 90°$ $=5(0)+3(1)$ $=3$		[2]	
	Alternatively, When $\theta=90°$, $x=\sqrt{34}\cos(90°-30.96°)$ $=\sqrt{34}\cos 59.04°$ ≈ 3.00		[2] (11 m)	

Qn	Working	Marks	Total	Remarks
7	**(i)** $\frac{dy}{dx} = 2(3x-2)(x-3)$ $= 2(3x^2 - 11x + 6)$ $y = \int 2(3x^2 - 11x + 6)\,dx$ $= 2[x^3 - \frac{11x^2}{2} + 6x] + C$ $= 2x^3 - 11x^2 + 12x + C$ Since the curve passes through the point $(0, 9)$, $9 = 0 + C$ $C = 9$ $\therefore\ y = 2x^3 - 11x^2 + 12x + 9$		[2]	
	(ii) At stationary points, $\frac{dy}{dx} = 0$ $2(3x-2)(x-3) = 0$ $x = \frac{2}{3}$ or $x = 3$ Since p is an integer, $p = 3$ When $x = 3$, $y = 2(3)^3 - 11(3)^2 + 12(3) + 9$ $= 54 - 99 + 36 + 9$ $= 0$ $\therefore\ q = 0$		[3]	

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<table>
<tr><th>Qn</th><th>Working</th><th>Marks</th><th>Total</th><th>Remarks</th></tr>
<tr><td>7</td><td>

(iii) **(a)** For values of x slightly less than p,
let $x = 2.9$

$$\frac{dy}{dx} = 2[3(2.9) - 2](2.9 - 3)$$
$$< 0$$

$\therefore$ y is decreasing for $x = p^-$

</td><td></td><td>[1]</td><td></td></tr>
<tr><td></td><td>

(b) For values of x slightly greater than p,
let $x = 3.1$

$$\frac{dy}{dx} = 2[3(3.1) - 2](3.1 - 3)$$
$$> 0$$

$\therefore$ y is increasing for $x = p^+$

</td><td></td><td>[1]</td><td></td></tr>
<tr><td></td><td>

(iv)

<table>
<tr><td>Value of x</td><td>p^-</td><td>p</td><td>p^+</td></tr>
<tr><td>Sign of $\frac{dy}{dx}$</td><td>$-$</td><td>0</td><td>$+$</td></tr>
<tr><td>Slope</td><td>$\diagdown$</td><td>—</td><td>$\diagup$</td></tr>
</table>

$\therefore$ The stationary point $(3, 0)$ is a minimum point

</td><td></td><td>[1]</td><td></td></tr>
<tr><td></td><td>

(v) $$\frac{d^2y}{dx^2} = 2[3(x-3) + (3x-2)]$$
$$= 2(6x - 11)$$

When $x = 3$, $\frac{d^2y}{dx^2} = 2[6(3) - 11]$
$= 2(7)$
$= 14$

Since $\frac{d^2y}{dx^2} > 0$ at $x = 3$, this supports the conclusion made in part (iv) that $(3, 0)$ is a minimum point

</td><td></td><td>[2]

(10 m)</td><td></td></tr>
</table>

Qn	Working	Marks	Total	Remarks
8	**(i)** Displacement, $s = t^3 - 6t^2 + 9t + 18$ When $t = 0$, $s = 18$ $\therefore$ The initial displacement of the particle from O is 18 m		[1]	
	(ii) Velocity, $v = \frac{ds}{dt}$ $= 3t^2 - 12t + 9$ When the particle is instantaneously at rest, $v = 0$ $3t^2 - 12t + 9 = 0$ $t^2 - 4t + 3 = 0$ $(t-1)(t-3) = 0$ $t = 1$ or $t = 3$ When $t = 3$, $s = (3)^3 - 6(3)^2 + 9(3) + 18$ $= 18$ $\therefore$ The particle will return to its starting position when $t = 3$		[4]	
	(iii) When $t = 0$, $s = 18$ When $t = 1$, $s = 1 - 6 + 9 + 18$ $= 22$ When $t = 3$, $s = 18$ When $t = 4$, $s = (4)^3 - 6(4)^2 + 9(4) + 18$ $= 22$ $\therefore$ The total distance travelled by the particle during the first 4 seconds $= 3 \times 4$ $= 12$ m		[3]	

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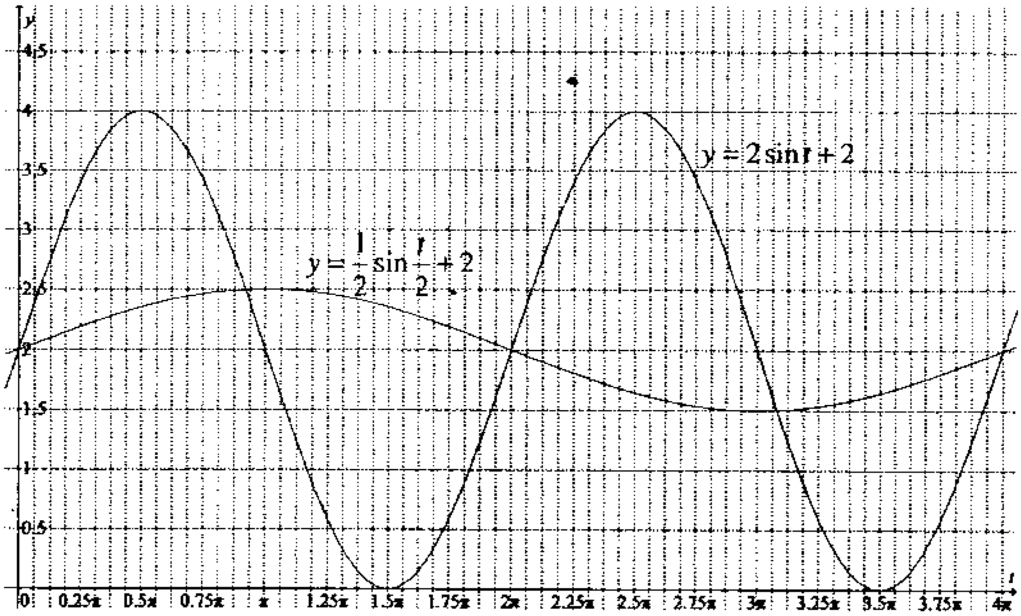
Qn	Working	Marks	Total	Remarks
8	**(iv)** For stationary value of velocity, $\frac{dv}{dt}=0$ $6t-12=0$ $t-2=0$ $t=2$ $\frac{d^2v}{dt^2}=6 \quad (>0)$ $\Rightarrow v$ is minimum when $t=2$ Minimum velocity $=3(2)^2-12(2)+9$ $=-3$ m/s		[3] (11 m)	

Qn	Working	Marks	Total	Remarks
9	(i) 		[4]	
	(ii) (a) In the interval $0 < t < 4\pi$, the two graphs have **3 points of intersection**, showing that there were **3 instances** when the waves on the two days reached the same height.		[2]	
	(b) **Day 1** would have provided surfers with a more thrilling experience of riding the waves at sea. **Period of the waves in Day 1 is half of the period of the waves in Day 2,** showing that for Day 1 there is an additional cycle between the greatest height and the least height of the waves within the same time interval. In addition, **the amplitude of the waves in Day 1 is greater than the amplitude of the waves in Day 2**, showing that the rise and drop in height of the waves in Day 1 is greater.		[3] (9 m)	

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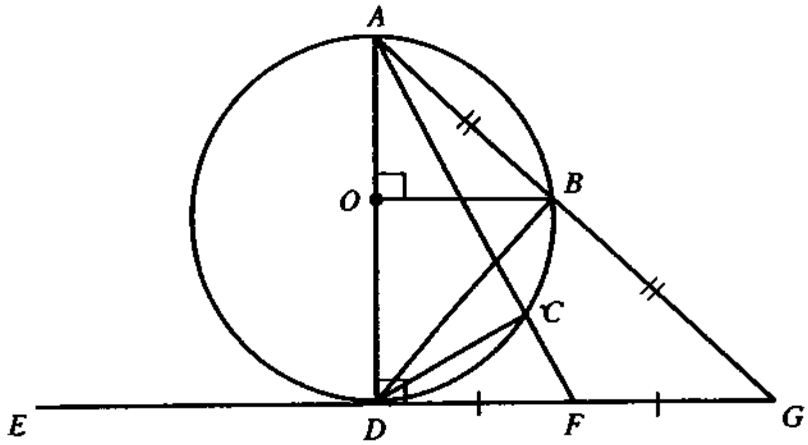
Qn	Working	Marks	Total	Remarks
10				
	(i) $\angle ADG = 90°$ (tan ⊥ rad) $OB // DG$ (Midpoint Theorem) $\angle AOB = \angle ADG = 90°$ (corr. $\angle s$, $OB // DG$) Since OB is the perpendicular bisector of AD, $AB = DB$ $\Rightarrow$ ABD is an isosceles triangle		[3]	
	(ii) In rt $-\angle$d ΔADG, $AG^2 - DG^2 = AD^2$ (Pythagoras Theorem) $(2AB)^2 - (2DF)^2 = AD^2$ (given $AB = BG$ and $DF = FG$) $4(AB^2 - DF^2) = AD^2$ $4(DB^2 - DF^2) = AD^2$ (from (i), $AB = DB$) $\therefore\ DB^2 - DF^2 = \frac{1}{4}AD^2$		[2]	
	(iii) In ΔADF and ΔDCF, $\angle DAF = \angle CDF$ (Tangent-Chord Theorem) $\angle AFD = \angle DFC$ (Common angle) $\therefore\ \Delta ADF$ is similar to ΔDCF. (Two pairs of corresponding angles are equal)		[2]	
	(iv) Since ΔADF and ΔDCF are similar, $\frac{DF}{CF} = \frac{AF}{DF}$ $DF^2 = AF \times CF$ $\therefore\ GF^2 = AF \times CF$ (given, $GF = DF$)		[2]	(9 m)

Qn	Working	Marks	Total	Remarks
11	(a) $\frac{d}{dx}\left(\frac{x}{\sqrt{3x-2}}\right)$ $= \frac{(3x-2)^{\frac{1}{2}}(1) - x \cdot \frac{1}{2}(3x-2)^{-\frac{1}{2}}(3)}{\left(\sqrt{3x-2}\right)^2}$ $= \frac{\frac{1}{2}(3x-2)^{-\frac{1}{2}}\left[\, 2(3x-2) - 3x \,\right]}{3x-2}$ $= \frac{6x-4-3x}{2(3x-2)^{\frac{3}{2}}}$ $= \frac{3x-4}{2\sqrt{(3x-2)^3}}$		[3]	

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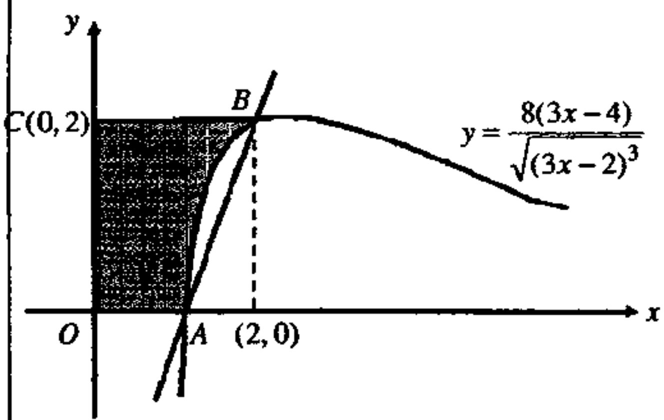
Qn	Working	Marks	Total	Remarks
11	(b) (i) At A, $y=0$, $\frac{8(3x-4)}{\sqrt{(3x-2)^3}}=0$ $3x-4=0$ $x=\frac{4}{3}$ Equation of the line AB is given by $y-0=3\left(x-\frac{4}{3}\right)$ $y=3x-4$ At the points of intersection of the curve and the line AB, $\frac{8(3x-4)}{\sqrt{(3x-2)^3}}=3x-4$ $\frac{8(3x-4)}{\sqrt{(3x-2)^3}}-(3x-4)=0$ $(3x-4)\left[\frac{8}{\sqrt{(3x-2)^3}}-1\right]=0$ $3x-4=0$ or $\frac{8}{\sqrt{(3x-2)^3}}-1=0$ $\frac{8}{\sqrt{(3x-2)^3}}=1$ $\sqrt{(3x-2)^3}=8$ $(3x-2)^3=64$ $3x-2=4$ $x=2$ At B, $x=2$. When $x=2$, $y=3(2)-4$ $y=2$ $\therefore$ The y-coordinate of B is 2.		[5]	

Qn	Working	Marks	Total	Remarks
11				
	(b) (ii) Area of the shaded region $= (2)(2) - \int_{\frac{4}{3}}^{2} \frac{8(3x-4)}{\sqrt{(3x-2)^3}}\,dx$ $= 4 - 16\int_{\frac{4}{3}}^{2} \frac{(3x-4)}{2\sqrt{(3x-2)^3}}\,dx$ $= 4 - 16\left[\frac{x}{\sqrt{3x-2}}\right]_{\frac{4}{3}}^{2}$ $= 4 - 16\left[\frac{2}{\sqrt{3(2)-2}} - \frac{\frac{4}{3}}{\sqrt{3\left(\frac{4}{3}\right)-2}}\right]$ $= 4 - 16\left[1 - \frac{4}{3\sqrt{2}}\right]$ ≈ 3.0849 ≈ 3.08 sq.units (to 3 s.f.)		[4] (12 m)	

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SWISS COTTAGE SECONDARY SCHOOL
SECONDARY FOUR EXPRESS
PRELIMINARY EXAMINATIONS

Name: ______________________________( ) Class: Sec _______

ADDITIONAL MATHEMATICS **4047/01**

Paper 1 **Wednesday 19 August 2015**
2 hour

Additional materials: Answer paper (8 sheets)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This question paper consists of **5** printed pages.

Setter: Ms Zoe Pow
Vetter: Mr Ang Hanping

[Turn over

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*We Nurture Students to **Think, Care** and **Lead** with **P.R.I.D.E.***

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots.. + \binom{n}{r}a^{n-r}b^r + \ldots. + b^n$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{(n-r)!r!} = \dfrac{n(n-1)\ldots\ldots.(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

Answer **all** the questions.

1 The acute angles A and B are such that $\sin(A+B)=0$ and $\sin B=\frac{1}{4}$.

Without using a calculator, find the exact value of $\tan A$. [3]

2 It is given that $y=\frac{2x+16}{x-1}$, where both x and y are positive and vary with time.

Find the value of y when the rate of increase of y is twice the rate of decrease of x. [4]

3 Express $\frac{x+1}{(x+2)(x^2+4)}$ in partial fractions. [4]

4 **(i)** Find the range of values of k for which the line $y=x-k$ intersects the curve $y=kx(x+3)$ at two distinct points. [4]

(ii) Hence or otherwise, find the range of values of k for which $kx(x+3)>x-k$ for all real values of x. [2]

5 Given that the first two non-zero terms of the expansion of $(1-kx)\left(1+\frac{x}{3}\right)^n$ are 1 and $-\frac{7}{3}x^2$, where n is a positive integer, find the value of k and of n. [6]

6 **(i)** Prove that $\frac{\cos^2 x-\sin^2 x}{1+2\sin x\cos x}=\frac{1-\tan x}{1+\tan x}$. [3]

(ii) Find all the angles between $0°$ and $360°$ which satisfy $\frac{\cos^2 x-\sin^2 x}{1+2\sin x\cos x}=\frac{2}{3}\tan x$. [3]

7 It is given that $f(x)$ is such that $f'(x)=\sin 2x+\cos 3x$.

Given also that $f\left(\frac{\pi}{6}\right)=0$, show that $f''(x)+9f(x)=-\frac{3}{4}-\frac{5}{2}\cos 2x$. [6]

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8 Variables x and y are connected by the equation $y = 10^{k-nx}$, where n and k are constants. When a graph of $\lg y$ is plotted against x, a straight line passing through the points $(1, 2)$ and $(4, -7)$ is obtained.

Find

(i) the value of n and of k, [4]

(ii) the coordinates of the point on the line at which $y = 10^x$. [3]

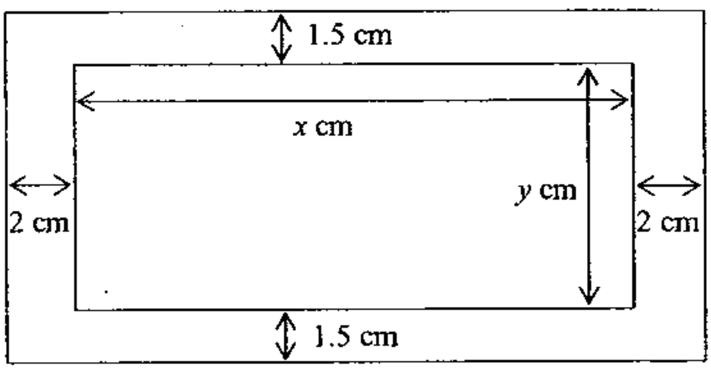
9 The tangent to the curve $y = x\ln 3x$ at point $P(1, \ln 3)$ cuts the x-axis at Q.

(i) Find the angle that PQ makes with the x-axis. [5]

The normal to the curve $y = x\ln 3x$ at R is parallel to the line $y = 5 - 2x$.

(ii) Find the x-coordinate of R. [3]

10 The diagram shows a rectangular poster of area 825 cm^2 with side margins of 2 cm and top and bottom margins of 1.5 cm. The length and breadth of the printing area are x cm and y cm respectively.



(i) Show that the printing area, A cm^2, is given by $A = \dfrac{825x}{x+4} - 3x$. [3]

(ii) Given that x can vary, find the value of x for which the printing area is stationary. [4]

(iii) Explain why this value of x gives the poster the largest printing area possible. [1]

11 **(i)** Sketch the graph of $y = |7x - 2|$, for $-1 \le x \le 2$. [2]

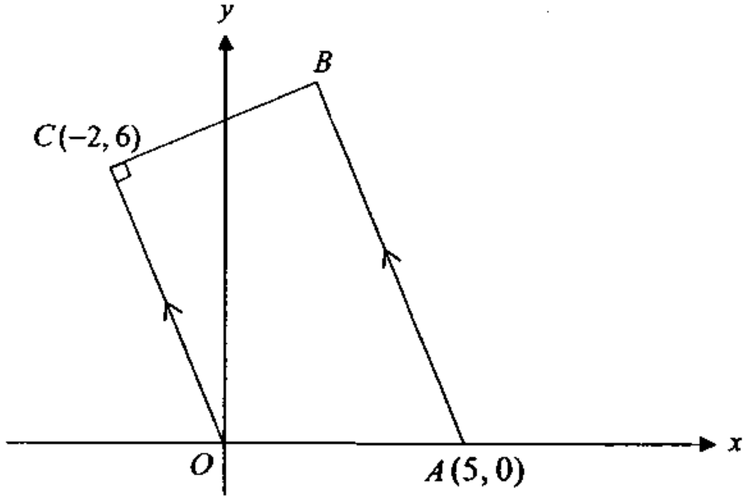
(ii) Hence, find the range of values of x for which $|14x - 4| \ge 3x$. [4]

(iii) Using your graph, determine the number of intersections of the line $y = mx + c$ with $y = |7x - 2|$, justifying your answer in each of the following cases.

(a) $m = -7$ and $c > 2$, [2]

(b) $m = 7$ and $c < -2$. [2]

12 **Solutions to this question by accurate drawing will not be accepted.**



In the trapezium $OABC$, the point A has coordinates $(5, 0)$ and the point C has coordinates $(-2, 6)$. The sides OC and AB are parallel, and BC is perpendicular to OC.

(i) Show that the coordinates of B are $\left(2\frac{1}{2}, 7\frac{1}{2}\right)$. [5]

(ii) OC is produced to D such that $OABD$ is a parallelogram.
Find the coordinates of D. [2]

(iii) Find the equation of the perpendicular bisector of OC. [2]

(iv) E is a point which lies on the perpendicular bisector of OC such that the area of quadrilateral $OAEC$ is 15 units^2. Given that the x-coordinate of E is positive, find the coordinates of E. [3]

END OF PAPER

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Additional Mathematics Paper 1 (80 marks)

Qn	Answer
1	$\tan A = -\dfrac{1}{\sqrt{15}}$
2	$y = 8$
3	$\dfrac{x+6}{8(x^2+4)} - \dfrac{1}{8(x+2)}$
4i	$k < \dfrac{1}{5}$ or $k > 1$
4ii	$\dfrac{1}{5} < k < 1$
5	$\therefore\ n = 6,\ k = 2$
6i	$LHS = \dfrac{\cos^2 x - \sin^2 x}{1 + 2\sin x \cos x}$ $= \dfrac{(\cos x - \sin x)(\cos x + \sin x)}{\sin^2 x + \cos^2 x + 2\sin x \cos x}$ $= \dfrac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x + \sin x)^2}$ $= \dfrac{(\cos x - \sin x)}{(\cos x + \sin x)}$ $= \dfrac{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}$ $= \dfrac{1 - \tan x}{1 + \tan x}$ $= RHS$ (shown)
6ii	$\therefore\ x = 26.6°, 108.4°, 206.6°, 288.4°$
7	$f'(x) = \sin 2x + \cos 3x$ $f(x) = \int (\sin 2x + \cos 3x)\,dx$ $= -\dfrac{1}{2}\cos 2x + \dfrac{1}{3}\sin 3x + c$ $f\left(\dfrac{\pi}{6}\right) = -\dfrac{1}{2}\cos 2\left(\dfrac{\pi}{6}\right) + \dfrac{1}{3}\sin 3\left(\dfrac{\pi}{6}\right) + c$ $-\dfrac{1}{2}\cos\left(\dfrac{\pi}{3}\right) + \dfrac{1}{3}\sin\left(\dfrac{\pi}{2}\right) + c = 0$ $-\dfrac{1}{2}\left(\dfrac{1}{2}\right) + \dfrac{1}{3}(1) + c = 0$ $c = -\dfrac{1}{12}$

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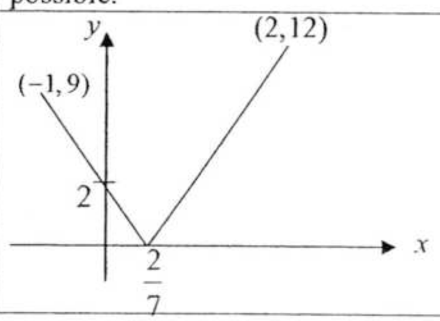
Qn	Answer
	$f(x)=-\frac{1}{2}\cos 2x+\frac{1}{3}\sin 3x-\frac{1}{12}$ $f''(x)=2\cos 2x-3\sin 3x$ $f''(x)+9f(x)=2\cos 2x-3\sin 3x+9\left(-\frac{1}{2}\cos 2x+\frac{1}{3}\sin 3x-\frac{1}{12}\right)$ $=2\cos 2x-3\sin 3x-\frac{9}{2}\cos 2x+3\sin 3x-\frac{3}{4}$ $=-\frac{3}{4}-\frac{5}{2}\cos 2x$ (shown)
8i	$k=5$
8ii	Coordinates are $(1.25,1.25)$
9i	$\theta=64.5°$
9ii	$x=0.202$
10i	$[x+(2\times 2)]\times[y+(1.5\times 2)]=825$ $(x+4)(y+3)=825$ $y+3=\frac{825}{x+4}$ $y=\frac{825}{x+4}-3$ $A=\left(\frac{825}{x+4}-3\right)(x)$ $A=\frac{825x}{x+4}-3x$ (shown)
10ii	$x=29.2$ or -37.2 (N.A.)
10 iii	$\frac{d^2A}{dx^2}=\frac{-2(3300)}{(x+4)^3}$ When $x=29.2$, $\frac{d^2A}{dx^2}=\frac{-6600}{(29.2+4)^3}$ $=-0.181$ Since $\frac{d^2A}{dx^2}<0$, this value of x gives the poster the largest printing area possible.
11i	

Qn	Answer
11 ii	$x \le \frac{4}{17}$ or $x \ge \frac{4}{11}$
11 iiia	1 intersection; Line parallel to L.H. arm cuts R.H. arm at only one point.
11 iiib	0 intersection; Line parallel to R.H. arm; for intersection, $c > -2$.
12i	$m_{AB} = m_{OC}$ $\frac{y-0}{x-5} = \frac{6}{-2}$ $y = -3x + 15$ ----- (1) $m_{BC} = -\frac{1}{m_{OC}}$ $\frac{y-6}{x+2} = \frac{1}{3}$ $y = \frac{1}{3}x + 6\frac{2}{3}$ ----- (2) Sub (1) into (2): $-3x + 15 = \frac{1}{3}x + 6\frac{2}{3}$ $3\frac{1}{3}x = 8\frac{1}{3}$ $x = 2\frac{1}{2}$ Sub $x = 2\frac{1}{2}$ into (1): $y = -3\left(2\frac{1}{2}\right) + 15$ $y = 7\frac{1}{2}$ Coordinates of B are $\left(2\frac{1}{2}, 7\frac{1}{2}\right)$ (shown)
12 ii	$D\left(-2\frac{1}{2}, 7\frac{1}{2}\right)$
12 iii	$y = \frac{1}{3}x + 3\frac{1}{3}$
12 iv	Possible coordinates of E are $(0.8, 3.6)$.

Additional Mathematics Paper 1 (80 marks)

Qn	Solution	Mark Allocation
1	$\sin(A+B)=0$ $\sin A\cos B+\cos A\sin B=0$ $\sin A\left(\frac{\sqrt{15}}{4}\right)+\cos A\left(\frac{1}{4}\right)=0$ $\sqrt{15}\sin A+\cos A=0$ $\sqrt{15}\sin A=-\cos A$ $\frac{\sin A}{\cos A}=-\frac{1}{\sqrt{15}}$ $\tan A=-\frac{1}{\sqrt{15}}$	M1: Addition Formula M1: Find $\cos B$ A1
2	$y=\frac{2x+16}{x-1}$ $\frac{dy}{dx}=\frac{(x-1)(2)-(2x+16)(1)}{(x-1)^2}$ $=\frac{2x-2-2x-16}{(x-1)^2}$ $=-\frac{18}{(x-1)^2}$ $\frac{dy}{dt}=-2\frac{dx}{dt}$ $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $-2\frac{dx}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $\frac{dy}{dx}=-2$ $-\frac{18}{(x-1)^2}=-2$ $(x-1)^2=9$ $x-1=\pm 3$ $x=4$ or -2 (N.A.) When $x=4$, $y=\frac{2(4)+16}{4-1}$ $\therefore\ y=8$	M1 M1 M1 A1

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Qn	Solution	Mark Allocation
3	Let $\dfrac{x+1}{(x+2)(x^2+4)}=\dfrac{A}{(x+2)}+\dfrac{Bx+C}{(x^2+4)}$	
	$x+1=A(x^2+4)+(Bx+C)(x+2)$	M1
	Sub $x=-2$: $-2+1=A\left[(-2)^2+4\right]$ $-1=8A$ $A=-\dfrac{1}{8}$ Comparing coefficients of x^2: $0=A+B$ $B=\dfrac{1}{8}$ Sub $x=0$: $1=4A+2C$ $2C=1-4\left(-\dfrac{1}{8}\right)$ $C=\dfrac{3}{4}$	M2 Any 2 correct 1 mark
	$\dfrac{x+1}{(x+2)(x^2+4)}=-\dfrac{1}{8(x+2)}+\dfrac{\frac{1}{8}x+\frac{3}{4}}{(x^2+4)}$ $=\dfrac{x+6}{8(x^2+4)}-\dfrac{1}{8(x+2)}$	A1
4i	$kx(x+3)=x-k$	
	$kx^2+(3k-1)x+k=0$	M1
	$(3k-1)^2-4(k)(k)>0$	M1: $b^2-4ac>0$
	$9k^2-6k+1-4k^2>0$	
	$5k^2-6k+1>0$	M1: Factorise
	$(5k-1)(k-1)>0$	
	$k<\dfrac{1}{5}$ or $k>1$	A1
4ii	$(3k-1)^2-4(k)(k)<0$	M1: $b^2-4ac<0$
	$(5k-1)(k-1)<0$	
	$\dfrac{1}{5}<k<1$	A1

Qn	Solution	Mark Allocation
5	$\left(1+\frac{x}{3}\right)^n = 1+\binom{n}{1}\left(\frac{x}{3}\right)+\binom{n}{2}\left(\frac{x}{3}\right)^2+\ldots$	
	$= 1+\frac{nx}{3}+\frac{n(n-1)}{2!}\left(\frac{x^2}{9}\right)+\ldots$	
	$= 1+\frac{n}{3}x+\frac{n(n-1)}{18}x^2+\ldots$	M1
	$(1-kx)\left(1+\frac{x}{3}\right)^n = (1-kx)\left(1+\frac{n}{3}x+\frac{n(n-1)}{18}x^2+\ldots\right)$	
	$= 1+\frac{n}{3}x+\frac{n(n-1)}{18}x^2-kx-\frac{nk}{3}x^2+\ldots$	
	$= 1+\left(\frac{n}{3}-k\right)x+\left[\frac{n(n-1)}{18}-\frac{nk}{3}\right]x^2+\ldots$	
	$\frac{n}{3}-k=0$	M1
	$n=3k$ ----- (1)	
	$\frac{n(n-1)}{18}-\frac{nk}{3}=-\frac{7}{3}$ ----- (2)	M1
	Sub (1) into (2):	
	$\frac{3k(3k-1)}{18}-\frac{3k^2}{3}=-\frac{7}{3}$	M1
	$9k^2-3k-18k^2+42=0$	
	$-9k^2-3k+42=0$	
	$3k^2+k-14=0$	
	$(3k+7)(k-2)=0$	M1
	$k=-2\frac{1}{3}$ or $k=2$	
	Sub $k=-2\frac{1}{3}$ into (1): $n=3\left(-2\frac{1}{3}\right)$	
	$n=-7$ (N.A) $\therefore$ Reject $k=-2\frac{1}{3}$	
	Sub $k=2$ into (1): $n=3(2)$	
	$=6$	
	$\therefore n=6,\ k=2$	A1

Qn	Solution	Mark Allocation
6i	$LHS=\dfrac{\cos^2 x-\sin^2 x}{1+2\sin x\cos x}$ $=\dfrac{(\cos x-\sin x)(\cos x+\sin x)}{\sin^2 x+\cos^2 x+2\sin x\cos x}$ $=\dfrac{(\cos x-\sin x)(\cos x+\sin x)}{(\cos x+\sin x)^2}$ $=\dfrac{(\cos x-\sin x)}{(\cos x+\sin x)}$ $=\dfrac{\frac{\cos x}{\cos x}-\frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}}$ $=\dfrac{1-\tan x}{1+\tan x}$ $=RHS$ (shown)	M1: Factorise numerator M1: Factorise denominator A1: Divide by $\cos x$
6ii	$\dfrac{\cos^2 x-\sin^2 x}{1+2\sin x\cos x}=\dfrac{2}{3}\tan x$ $\dfrac{1-\tan x}{1+\tan x}=\dfrac{2\tan x}{3}$ $3-3\tan x=2\tan x+2\tan^2 x$ $2\tan^2 x+5\tan x-3=0$ $(2\tan x-1)(\tan x+3)=0$ $\tan x=\dfrac{1}{2}$ Basic $\angle=26.565$ $x=26.565, 180+26.565$ OR $\tan x=-3$ Basic $\angle=71.565$ $x=180-71.565, 360-71.565$ $\therefore\ x=26.6°, 108.4°, 206.6°, 288.4°$	M1 M1: Find basic angle (both correct) A1
7	$f'(x)=\sin 2x+\cos 3x$ $f(x)=\int(\sin 2x+\cos 3x)\,dx$ $=-\dfrac{1}{2}\cos 2x+\dfrac{1}{3}\sin 3x+c$ $f\left(\dfrac{\pi}{6}\right)=-\dfrac{1}{2}\cos 2\left(\dfrac{\pi}{6}\right)+\dfrac{1}{3}\sin 3\left(\dfrac{\pi}{6}\right)+c$	M1

Qn	Solution	Mark Allocation
	$-\frac{1}{2}\cos\left(\frac{\pi}{3}\right)+\frac{1}{3}\sin\left(\frac{\pi}{2}\right)+c=0$	M1
	$-\frac{1}{2}\left(\frac{1}{2}\right)+\frac{1}{3}(1)+c=0$	
	$c=-\frac{1}{12}$	M1
	$f(x)=-\frac{1}{2}\cos 2x+\frac{1}{3}\sin 3x-\frac{1}{12}$	
	$f''(x)=2\cos 2x-3\sin 3x$	M1
	$f''(x)+9f(x)=2\cos 2x-3\sin 3x+9\left(-\frac{1}{2}\cos 2x+\frac{1}{3}\sin 3x-\frac{1}{12}\right)$	
	$=2\cos 2x-3\sin 3x-\frac{9}{2}\cos 2x+3\sin 3x-\frac{3}{4}$	M1
	$=-\frac{3}{4}-\frac{5}{2}\cos 2x$ (shown)	A1
8i	$y=10^{k-nx}$	
	$\lg y=(k-nx)\lg 10$	
	$\lg y=-nx+k$	
	$m=\frac{2-(-7)}{1-4}$	M1
	$=-3$	
	$-n=-3$	
	$\therefore n=3$	A1
	At $(1,2)$, $2=(-3)(1)+c$	M1
	$c=5$	
	$\therefore k=5$	A1
8ii	$y=10^{5-3x}$ ----- (1)	
	$y=10^{x}$ ----- (2)	
	Sub (1) into (2): $10^{5-3x}=10^{x}$	M1
	$x=5-3x$	
	$4x=5$	
	$x=1\frac{1}{4}$	
	Sub $x=1\frac{1}{4}$ into (2): $y=10^{1\frac{1}{4}}$	
	$y=17.783$	M1
	$\lg y=\lg 17.783$	
	$=1.25$	
	$\therefore$ Coordinates are $(1.25,1.25)$	A1

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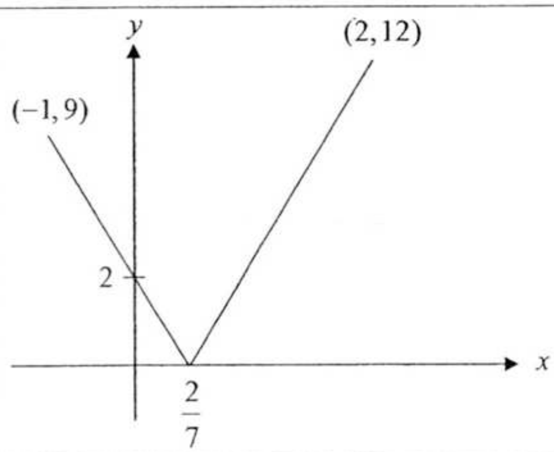
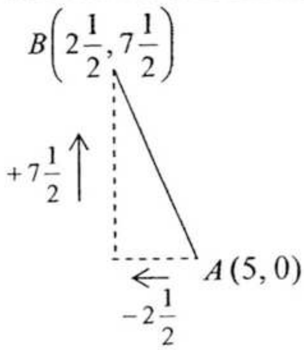
Qn	Solution	Mark Allocation
9i	$y = x\ln 3x$ $\frac{dy}{dx} = x\left(\frac{1}{x}\right) + (\ln 3x)(1)$ $= 1 + \ln 3x$ At $P(1, \ln 3)$, $\frac{dy}{dx} = 1 + \ln 3(1)$ $= 1 + \ln 3$ At $P(1, \ln 3)$, $y - \ln 3 = (1 + \ln 3)(x - 1)$ $y = (1 + \ln 3)x - 1$ When $y = 0$, $(1 + \ln 3)x = 1$ $x = \frac{1}{(1 + \ln 3)}$ $x = 0.477$ Coordinates of Q are $(0.477, 0)$ $\tan\theta = \frac{\ln 3 - 0}{1 - 0.477}$ $= 2.0986$ $\theta = 64.5°$	M1 M1 M1 M1 A1
9ii	Gradient of normal $= -\frac{1}{1 + \ln 3x}$ $-\frac{1}{1 + \ln 3x} = -2$ $1 + \ln 3x = \frac{1}{2}$ $\ln 3x = -\frac{1}{2}$ $3x = e^{-\frac{1}{2}}$ $x = \frac{1}{3}e^{-\frac{1}{2}}$ $= 0.202$	M1 M1 A1
10i	$[x + (2 \times 2)] \times [y + (1.5 \times 2)] = 825$ $(x + 4)(y + 3) = 825$ $y + 3 = \frac{825}{x + 4}$ $y = \frac{825}{x + 4} - 3$	M1 M1

Qn	Solution	Mark Allocation
	$A=\left(\frac{825}{x+4}-3\right)(x)$ $A=\frac{825x}{x+4}-3x$ (shown)	A1
10 ii	$\frac{dA}{dx}=\frac{(x+4)(825)-(825x)(1)}{(x+4)^2}-3$ $=\frac{3300}{(x+4)^2}-3$ $\frac{3300}{(x+4)^2}-3=0$ $(x+4)^2=1100$ $x+4=\pm\sqrt{1100}$ $x=29.2$ or -37.2 (N.A.)	M1 M1 M1 A1
10 iii	$\frac{d^2A}{dx^2}=\frac{-2(3300)}{(x+4)^3}$ When $x=29.2$, $\frac{d^2A}{dx^2}=\frac{-6600}{(29.2+4)^3}$ $=-0.181$ Since $\frac{d^2A}{dx^2}<0$, this value of x gives the poster the largest printing area possible.	B1: Show $\frac{d^2A}{dx^2}<0$
11i		B1: V-shape graph B1: Correct x- and y- intercepts and end-points labelled
11 ii	$\|14x-4\|=3x$ $2\|7x-2\|=3x$ $\|7x-2\|=\frac{3}{2}x$	M1

Qn	Solution	Mark Allocation
	$7x-2=\frac{3}{2}x$ or $7x-2=-\frac{3}{2}x$	M1
	$5\frac{1}{2}x=2$ $\quad$ $8\frac{1}{2}x=2$	
	$x=\frac{4}{11}$ $\quad$ $x=\frac{4}{17}$	M1
	$\therefore\ x\le\frac{4}{17}$ or $x\ge\frac{4}{11}$	A1
11 iiia	1 intersection; Line parallel to L.H. arm cuts R.H. arm at only one point.	B1 B1
11 iiib	0 intersection; Line parallel to R.H. arm; for intersection, $c>-2$.	B1 B1
12i	$m_{AB}=m_{OC}$	
	$\frac{y-0}{x-5}=\frac{6}{-2}$	M1
	$y=-3x+15$ ----- (1)	
	$m_{BC}=-\frac{1}{m_{OC}}$	
	$\frac{y-6}{x+2}=\frac{1}{3}$	M1
	$y=\frac{1}{3}x+6\frac{2}{3}$ ----- (2)	
	Sub (1) into (2): $-3x+15=\frac{1}{3}x+6\frac{2}{3}$	M1
	$3\frac{1}{3}x=8\frac{1}{3}$	
	$x=2\frac{1}{2}$	M1
	Sub $x=2\frac{1}{2}$ into (1): $y=-3\left(2\frac{1}{2}\right)+15$	
	$y=7\frac{1}{2}$	
	Coordinates of B are $\left(2\frac{1}{2},7\frac{1}{2}\right)$	A1

Qn	Solution	Mark Allocation
12 ii	 $D\left(-2\frac{1}{2}, 7\frac{1}{2}\right)$	M1 O.E. A1
12 iii	Midpoint of $OC = \left(\frac{0+(-2)}{2}, \frac{0+6}{2}\right)$ $= (-1, 3)$ Gradient of perpendicular bisector of $OC = -\frac{1}{(-3)}$ $= \frac{1}{3}$ At $(-1, 3)$, $y - 3 = \frac{1}{3}(x+1)$ $y = \frac{1}{3}x + 3\frac{1}{3}$	M1 A1
12 iv	$\frac{1}{2}\begin{vmatrix} 0 & 5 & x & -2 & 0 \\ 0 & 0 & y & 6 & 0 \end{vmatrix} = 15$ $\lvert 5y + 6x - (-2y) \rvert = 30$ $\lvert 7y + 6x \rvert = 30$ $7y + 6x = 30$ ----- (1) or $7y + 6x = -30$ ----- (2) $3y = x + 10$ ----- (3) From (3): $x = 3y - 10$ ----- (4) Sub (4) into (1): $7y + 6(3y-10) = 30$ $7y + 6(3y-10) = -30$ $25y = 90$ $25y = 30$ $y = 3.6$ $y = 1.2$ Sub $y = 3.6$ into (4): Sub $y = 1.2$ into (4): $x = 3(3.6) - 10$ $x = 3(1.2) - 10$ $x = 0.8$ $x = -6.4$ [N.A.] Possible coordinates of E are $(0.8, 3.6)$.	M1 M1 A1 [with rejection]

Additional Mathematics Paper 1 (80 marks)

Qn	Solution	Mark Allocation
1	$\sin(A+B)=0$ $\sin A\cos B+\cos A\sin B=0$ $\sin A\left(\frac{\sqrt{15}}{4}\right)+\cos A\left(\frac{1}{4}\right)=0$ $\sqrt{15}\sin A+\cos A=0$ $\sqrt{15}\sin A=-\cos A$ $\frac{\sin A}{\cos A}=-\frac{1}{\sqrt{15}}$ $\tan A=-\frac{1}{\sqrt{15}}$	M1: Addition Formula M1: Find $\cos B$ A1
2	$y=\frac{2x+16}{x-1}$ $\frac{dy}{dx}=\frac{(x-1)(2)-(2x+16)(1)}{(x-1)^2}$ $=\frac{2x-2-2x-16}{(x-1)^2}$ $=-\frac{18}{(x-1)^2}$ $\frac{dy}{dt}=-2\frac{dx}{dt}$ $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $-2\frac{dx}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $\frac{dy}{dx}=-2$ $-\frac{18}{(x-1)^2}=-2$ $(x-1)^2=9$ $x-1=\pm3$ $x=4$ or -2 (N.A.) When $x=4$, $y=\frac{2(4)+16}{4-1}$ $\therefore\ y=8$	 M1 M1 M1 A1

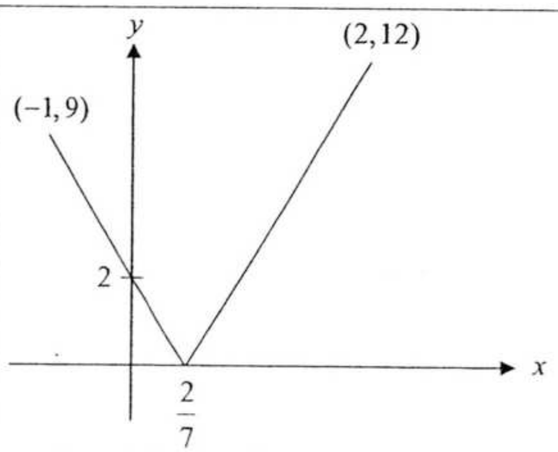
Qn	Solution	Mark Allocation
3	Let $\frac{x+1}{(x+2)(x^2+4)}=\frac{A}{(x+2)}+\frac{Bx+C}{(x^2+4)}$	
	$x+1=A(x^2+4)+(Bx+C)(x+2)$	M1
	Sub $x=-2$: $-2+1=A[(-2)^2+4]$ $-1=8A$ $A=-\frac{1}{8}$ Comparing coefficients of x^2: $0=A+B$ $B=\frac{1}{8}$ Sub $x=0$: $1=4A+2C$ $2C=1-4\left(-\frac{1}{8}\right)$ $C=\frac{3}{4}$	M2. Any 2 correct 1 mark
	$\frac{x+1}{(x+2)(x^2+4)}=-\frac{1}{8(x+2)}+\frac{\frac{1}{8}x+\frac{3}{4}}{(x^2+4)}$ $=\frac{x+6}{8(x^2+4)}-\frac{1}{8(x+2)}$	A1
4i	$kx(x+3)=x-k$ $kx^2+(3k-1)x+k=0$	M1
	$(3k-1)^2-4(k)(k)>0$ $9k^2-6k+1-4k^2>0$	M1: $b^2-4ac>0$
	$5k^2-6k+1>0$ $(5k-1)(k-1)>0$	M1: Factorise
	$k<\frac{1}{5}$ or $k>1$	A1
4ii	$(3k-1)^2-4(k)(k)<0$ $(5k-1)(k-1)<0$	M1: $b^2-4ac<0$
	$\frac{1}{5}<k<1$	A1

Qn	Solution	Mark Allocation
5	$\left(1+\frac{x}{3}\right)^n = 1+\binom{n}{1}\left(\frac{x}{3}\right)+\binom{n}{2}\left(\frac{x}{3}\right)^2+\ldots$	
	$= 1+\frac{nx}{3}+\frac{n(n-1)}{2!}\left(\frac{x^2}{9}\right)+\ldots$	
	$= 1+\frac{n}{3}x+\frac{n(n-1)}{18}x^2+\ldots$	M1
	$(1-kx)\left(1+\frac{x}{3}\right)^n = (1-kx)\left(1+\frac{n}{3}x+\frac{n(n-1)}{18}x^2+\ldots\right)$	
	$= 1+\frac{n}{3}x+\frac{n(n-1)}{18}x^2-kx-\frac{nk}{3}x^2+\ldots$	
	$= 1+\left(\frac{n}{3}-k\right)x+\left[\frac{n(n-1)}{18}-\frac{nk}{3}\right]x^2+\ldots$	
	$\frac{n}{3}-k=0$	M1
	$n=3k$ ----- (1)	
	$\frac{n(n-1)}{18}-\frac{nk}{3}=-\frac{7}{3}$ ----- (2)	M1
	Sub (1) into (2):	
	$\frac{3k(3k-1)}{18}-\frac{3k^2}{3}=-\frac{7}{3}$	M1
	$9k^2-3k-18k^2+42=0$	
	$-9k^2-3k+42=0$	
	$3k^2+k-14=0$	
	$(3k+7)(k-2)=0$	M1
	$k=-2\frac{1}{3}$ or $k=2$	
	Sub $k=-2\frac{1}{3}$ into (1): $n=3\left(-2\frac{1}{3}\right)$	
	$n=-7$ (N.A) $\therefore$ Reject $k=-2\frac{1}{3}$	
	Sub $k=2$ into (1): $n=3(2)$	
	$=6$	
	$\therefore\ n=6,\ k=2$	A1

Qn	Solution	Mark Allocation
6i	$LHS=\dfrac{\cos^2 x-\sin^2 x}{1+2\sin x\cos x}$ $=\dfrac{(\cos x-\sin x)(\cos x+\sin x)}{\sin^2 x+\cos^2 x+2\sin x\cos x}$ $=\dfrac{(\cos x-\sin x)(\cos x+\sin x)}{(\cos x+\sin x)^2}$ $=\dfrac{(\cos x-\sin x)}{(\cos x+\sin x)}$ $=\dfrac{\frac{\cos x}{\cos x}-\frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}}$ $=\dfrac{1-\tan x}{1+\tan x}$ $=RHS$ (shown)	M1: Factorise numerator M1: Factorise denominator A1: Divide by $\cos x$
6ii	$\dfrac{\cos^2 x-\sin^2 x}{1+2\sin x\cos x}=\dfrac{2}{3}\tan x$ $\dfrac{1-\tan x}{1+\tan x}=\dfrac{2\tan x}{3}$ $3-3\tan x=2\tan x+2\tan^2 x$ $2\tan^2 x+5\tan x-3=0$ $(2\tan x-1)(\tan x+3)=0$ $\tan x=\dfrac{1}{2}$ Basic $\angle=26.565$ $x=26.565, 180+26.565$ OR $\tan x=-3$ Basic $\angle=71.565$ $x=180-71.565, 360-71.565$ $\therefore\ x=26.6°, 108.4°, 206.6°, 288.4°$	M1 M1: Find basic angle (both correct) A1
7	$f'(x)=\sin 2x+\cos 3x$ $f(x)=\int(\sin 2x+\cos 3x)\,dx$ $=-\dfrac{1}{2}\cos 2x+\dfrac{1}{3}\sin 3x+c$ $f\left(\dfrac{\pi}{6}\right)=-\dfrac{1}{2}\cos 2\left(\dfrac{\pi}{6}\right)+\dfrac{1}{3}\sin 3\left(\dfrac{\pi}{6}\right)+c$	M1

Qn	Solution	Mark Allocation
	$-\frac{1}{2}\cos\left(\frac{\pi}{3}\right)+\frac{1}{3}\sin\left(\frac{\pi}{2}\right)+c=0$	M1
	$-\frac{1}{2}\left(\frac{1}{2}\right)+\frac{1}{3}(1)+c=0$	
	$c=-\frac{1}{12}$	M1
	$f(x)=-\frac{1}{2}\cos 2x+\frac{1}{3}\sin 3x-\frac{1}{12}$	
	$f''(x)=2\cos 2x-3\sin 3x$	M1
	$f''(x)+9f(x)=2\cos 2x-3\sin 3x+9\left(-\frac{1}{2}\cos 2x+\frac{1}{3}\sin 3x-\frac{1}{12}\right)$	
	$=2\cos 2x-3\sin 3x-\frac{9}{2}\cos 2x+3\sin 3x-\frac{3}{4}$	M1
	$=-\frac{3}{4}-\frac{5}{2}\cos 2x$ (shown)	A1
8i	$y=10^{k-nx}$ $\lg y=(k-nx)\lg 10$ $\lg y=-nx+k$	
	$m=\frac{2-(-7)}{1-4}$ $=-3$	M1
	$-n=-3$ $\therefore\ n=3$	A1
	At $(1,2)$, $2=(-3)(1)+c$	M1
	$c=5$ $\therefore\ k=5$	A1
8ii	$y=10^{5-3x}$ ----- (1) $y=10^{x}$ ----- (2)	
	Sub (1) into (2): $10^{5-3x}=10^{x}$ $x=5-3x$ $4x=5$ $x=1\frac{1}{4}$	M1
	Sub $x=1\frac{1}{4}$ into (2): $y=10^{1\frac{1}{4}}$ $y=17.783$	M1
	$\lg y=\lg 17.783$ $=1.25$ $\therefore$ Coordinates are $(1.25, 1.25)$	A1

Qn	Solution	Mark Allocation
9i	$y = x\ln 3x$ $\frac{dy}{dx} = x\left(\frac{1}{x}\right) + (\ln 3x)(1)$ $= 1 + \ln 3x$ At $P(1, \ln 3)$, $\frac{dy}{dx} = 1 + \ln 3(1)$ $= 1 + \ln 3$ At $P(1, \ln 3)$, $y - \ln 3 = (1 + \ln 3)(x - 1)$ $y = (1 + \ln 3)x - 1$ When $y = 0$, $(1 + \ln 3)x = 1$ $x = \frac{1}{(1 + \ln 3)}$ $x = 0.477$ Coordinates of Q are $(0.477, 0)$ $\tan\theta = \frac{\ln 3 - 0}{1 - 0.477}$ $= 2.0986$ $\theta = 64.5°$	M1 M1 M1 M1 A1
9ii	Gradient of normal $= -\frac{1}{1 + \ln 3x}$ $-\frac{1}{1 + \ln 3x} = -2$ $1 + \ln 3x = \frac{1}{2}$ $\ln 3x = -\frac{1}{2}$ $3x = e^{-\frac{1}{2}}$ $x = \frac{1}{3}e^{-\frac{1}{2}}$ $= 0.202$	M1 M1 A1
10i	$[x + (2\times 2)]\times[y + (1.5\times 2)] = 825$ $(x + 4)(y + 3) = 825$ $y + 3 = \frac{825}{x + 4}$ $y = \frac{825}{x + 4} - 3$	M1 M1

Qn	Solution	Mark Allocation
	$A=\left(\frac{825}{x+4}-3\right)(x)$ $A=\frac{825x}{x+4}-3x$ (shown)	A1
10 ii	$\frac{dA}{dx}=\frac{(x+4)(825)-(825x)(1)}{(x+4)^2}-3$ $=\frac{3300}{(x+4)^2}-3$ $\frac{3300}{(x+4)^2}-3=0$ $(x+4)^2=1100$ $x+4=\pm\sqrt{1100}$ $x=29.2$ or -37.2 (N.A.)	M1 M1 M1 A1
10 iii	$\frac{d^2A}{dx^2}=\frac{-2(3300)}{(x+4)^3}$ When $x=29.2$, $\frac{d^2A}{dx^2}=\frac{-6600}{(29.2+4)^3}$ $=-0.181$ Since $\frac{d^2A}{dx^2}<0$, this value of x gives the poster the largest printing area possible.	B1: Show $\frac{d^2A}{dx^2}<0$
11i		B1: V-shape graph B1: Correct x- and y- intercepts and end-points labelled
11 ii	$\lvert 14x-4\rvert=3x$ $2\lvert 7x-2\rvert=3x$ $\lvert 7x-2\rvert=\frac{3}{2}x$	M1

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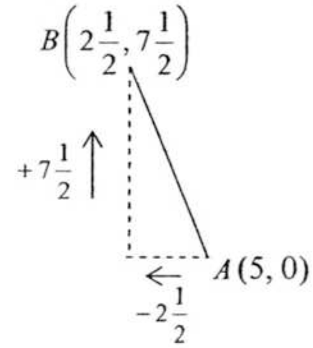
Qn	Solution	Mark Allocation
	$7x-2=\frac{3}{2}x$ or $7x-2=-\frac{3}{2}x$	M1
	$5\frac{1}{2}x=2$ $\quad$ $8\frac{1}{2}x=2$	
	$x=\frac{4}{11}$ $\quad$ $x=\frac{4}{17}$	M1
	$\therefore\ x\le\frac{4}{17}$ or $x\ge\frac{4}{11}$	A1
11 iiia	1 intersection; Line parallel to L.H. arm cuts R.H. arm at only one point.	B1 B1
11 iiib	0 intersection; Line parallel to R.H. arm; for intersection, $c>-2$.	B1 B1
12i	$m_{AB}=m_{OC}$	
	$\frac{y-0}{x-5}=\frac{6}{-2}$	M1
	$y=-3x+15$ ----- (1)	
	$m_{BC}=-\frac{1}{m_{OC}}$	
	$\frac{y-6}{x+2}=\frac{1}{3}$	M1
	$y=\frac{1}{3}x+6\frac{2}{3}$ ----- (2)	
	Sub (1) into (2): $-3x+15=\frac{1}{3}x+6\frac{2}{3}$	M1
	$3\frac{1}{3}x=8\frac{1}{3}$	
	$x=2\frac{1}{2}$	M1
	Sub $x=2\frac{1}{2}$ into (1): $y=-3\left(2\frac{1}{2}\right)+15$	
	$y=7\frac{1}{2}$	
	Coordinates of B are $\left(2\frac{1}{2},7\frac{1}{2}\right)$	A1

Qn	Solution	Mark Allocation
12 ii	 $D\left(-2\frac{1}{2}, 7\frac{1}{2}\right)$	M1 O.E. A1
12 iii	Midpoint of $OC = \left(\frac{0+(-2)}{2}, \frac{0+6}{2}\right)$ $= (-1, 3)$ Gradient of perpendicular bisector of $OC = -\frac{1}{(-3)}$ $= \frac{1}{3}$ At $(-1, 3)$, $y-3 = \frac{1}{3}(x+1)$ $y = \frac{1}{3}x + 3\frac{1}{3}$	M1 A1
12 iv	$\frac{1}{2}\begin{vmatrix} 0 & 5 & x & -2 & 0 \\ 0 & 0 & y & 6 & 0 \end{vmatrix} = 15$ $\lvert 5y+6x-(-2y)\rvert = 30$ $\lvert 7y+6x\rvert = 30$ $7y+6x=30$ ----- (1) or $7y+6x=-30$ ----- (2) $3y = x+10$ ----- (3) From (3): $x = 3y-10$ ----- (4) Sub (4) into (1): $7y+6(3y-10)=30$ $7y+6(3y-10)=-30$ $25y=90$ $25y=30$ $y=3.6$ $y=1.2$ Sub $y=3.6$ into (4): Sub $y=1.2$ into (4): $x=3(3.6)-10$ $x=3(1.2)-10$ $x=0.8$ $x=-6.4$ [N.A.] Possible coordinates of E are $(0.8, 3.6)$.	M1 M1 A1 [with rejection]

SWISS COTTAGE SECONDARY SCHOOL
SECONDARY FOUR EXPRESS
PRELIMINARY EXAMINATIONS

Name: ______________________________( ) Class: Sec _______

ADDITIONAL MATHEMATICS **4047/02**
Paper 2
Friday 21 August 2015
2 hours 30 minutes

Additional materials: Answer paper (8 sheets)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Submit Section A and B separately

This question paper consists of **6** printed pages.

Setter: Mr Ang Hanping
Vetter: Ms Zoe Pow

[Turn over

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*We Nurture Students to **Think, Care** and **Lead** with **P.R.I.D.E.***

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots.. + \binom{n}{r} a^{n-r} b^r + \ldots. + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots\ldots.(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

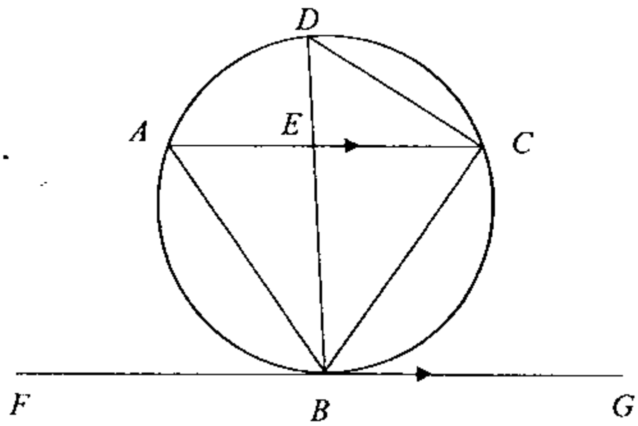
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

Section A (59 marks)

1 The diagram shows triangles ABC and BCD whose vertices lie on the circumference of a circle. The chords BD and AC intersect at E and AC is parallel to FG. FG is a tangent to the circle at B.



Show that

(i) ΔBCD is similar to ΔBEC, [2]

(ii) $BC^2 = BD \times BE$, [1]

(iii) ΔABC is an isosceles triangle. [2]

2 A company buys an engine at a cost of \$120 000. The value of the engine decreases with time so that its value, \$$V$, after t months is given by

$$V = 120\,000e^{-kt},$$

where k is a positive constant. The value of the engine is expected to be \$75 000 after 30 months.

(i) Calculate the value, to the nearest \$100, of the engine after 20 months. [4]

It is only economical to replace the engine after its value reaches $\frac{1}{2}$ of its original value.

(ii) Determine, with working, whether it is economical to replace the engine after 40 months. [2]

3 **(a)** An equilateral triangle has sides $\left(3+\sqrt{7}\right)$ cm in length. Find, without using a calculator, the area of the equilateral triangle. [3]

(b) A cuboid of volume $\left(30+12\sqrt{3}\right)$ cm^3 has a base area of $\left(4-2\sqrt{3}\right)$ cm^2. Find, without using a calculator, the height of the cuboid. [3]

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4 (a) Solve the equation $\log_3(2x-1)-\frac{1}{2}\log_3(x^2+2)=\log_{25}5$. [5]

(b) Evaluate $\log_p 32\times\log_8 p$. [3]

5 It is given that $f(x)=2x^3+ax^2+x+b$.

(i) Find the value of a and of b for which $2x^2+x-1$ is a factor of $f(x)$. [4]

(ii) Solve the equation $f(x)=0$. [2]

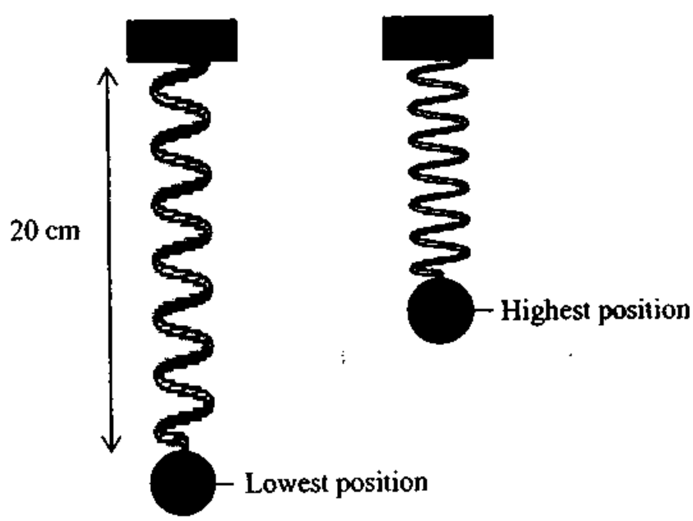
(iii) Hence solve $\frac{1}{4}x^3+\frac{a}{4}x^2+\frac{1}{2}x+b=0$. [2]

6 A curve has the equation $y=(2x+2)\sqrt{2x-1}$.

(i) Show that $\frac{dy}{dx}=\frac{kx}{\sqrt{2x-1}}$, where k is a constant and state the value of k. [4]

(ii) Hence evaluate $\int_5^{13}\frac{3x}{2\sqrt{2x-1}}\,dx$. [4]

7 The diagram below shows an experimental setup where a weighted spring is released from a stretched position and follows a periodic up-down motion. The length of the spring, l cm, during the experiment is modelled by the equation, $l=a\cos kt+16$, where a, k are constants, and t is the time in seconds after releasing the weight from the lowest position.



The length of the spring is 20 cm when the weight is at its lowest position and it takes 2 seconds for the weight to move from the lowest to highest position.

(i) Find the value of a. [1]

(ii) Show that the value of k is $\frac{\pi}{2}$. [2]

(iii) Find the length of the spring when the weight is at its highest position. [1]

(iv) Sketch the graph of $l = a\cos kt + 16$ for $0 \le t \le 4$. [2]

(v) Find the time interval which the length of the spring will be longer than 18 cm for $0 \le t \le 4$. [3]

8 **(a)** The equation of a curve is $y = x^3 + 4x^2 + kx + 3$, where k is a constant. Find the set of values of k for which the curve is always an increasing function. [4]

(b) A curve with equation in the form $y = ax + \dfrac{b}{x^2}$ has a stationary point at $(3, 4)$, where a and b are constants. Find the value of a and of b. [5]

Section B (41 marks)

Begin this section on a fresh sheet of paper.

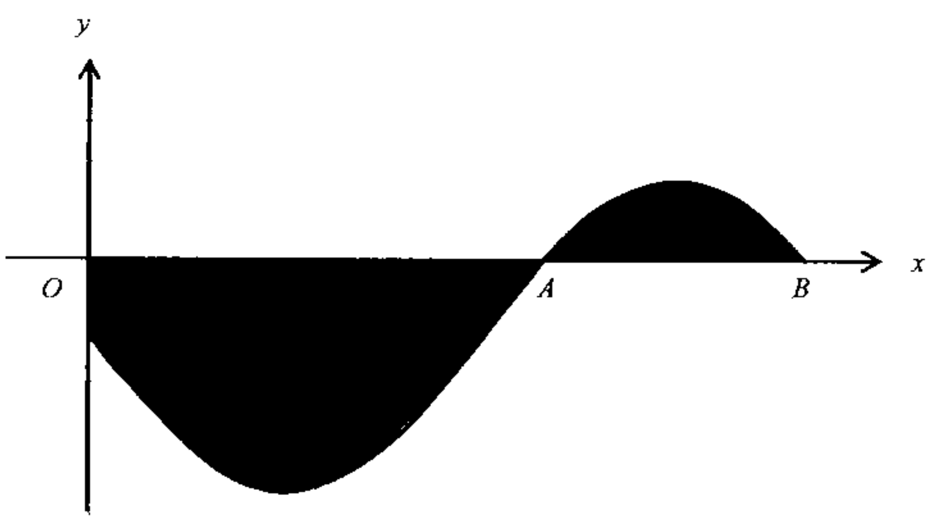
9 The equation $3x^2 + kx + 3 = 0$, where $k > 0$ has roots α and β. A second equation $3x^2 - 2x + 3 = 0$ has roots $\alpha^3\beta$ and $\alpha\beta^3$.

(i) Show that $\alpha^2 + \beta^2 = \dfrac{2}{3}$. [3]

(ii) Find the value of k. [3]

(iii) Form an equation whose roots are α^3 and β^3. [3]

10 The diagram shows part of the curve $y = 2\sin(2x + \pi) - 1$, meeting the x-axis at the points A and B.



(i) Find the x-coordinate of A and of B. [4]

(ii) Find the total area of the shaded regions. 184 [6]

11 A particle, travelling in a straight line, passes a fixed point O on the line with a speed of 2 ms^{-1}. The acceleration, a ms^{-2}, of the particle, t s after passing O, is given by $a = -4e^{-t}$.

(i) Show that the particle comes instantaneously to rest when $t = -\ln\frac{1}{2}$. [4]

(ii) Find the total distance travelled by the particle between $t = 0$ and $t = 2$. [6]

(iii) Find the average velocity of the particle during the first 2 seconds. [1]

12 A circle, C_1, passing through the point A (4, 8) has the same centre as another circle C_2. The equation of C_2 is given by $x^2 + y^2 - 16x - 10y + 5 = 0$.

(i) Find the equation of C_1. [3]

AB is a diameter of C_1.

(ii) Find the coordinates of B. [2]

(iii) Show that the equation of the tangent to C_1 at B is $3y = 4x - 42$. [3]

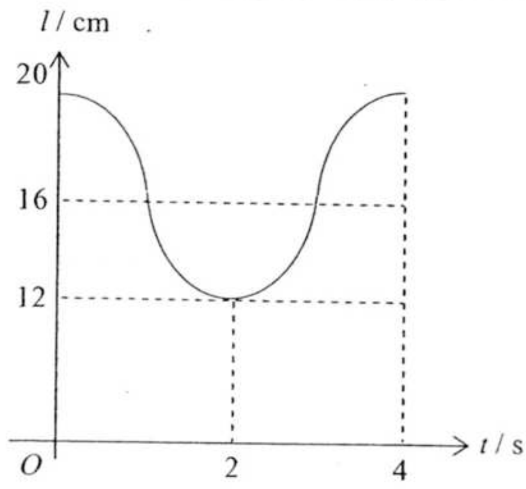
The lowest point on the circle C_1 is D.

(iv) Explain why the x-axis is a tangent to the circle at D. [1]

(v) Find the equation of another tangent to circle C_1 passing through the origin. [2]

END OF PAPER

Additional Mathematics Paper 2 (100 marks)

Qn. #	Solution	Mark Allocation
2i	$V = 87700$	
2ii	$V = 64124.098$ It is not economical to replace the engine after 40 months as the value of the engine has not reached half its original value of \$120000	
3a	Area of equilateral triangle $= \frac{1}{2}\left(8\sqrt{3}+3\sqrt{21}\right)$ cm^2	
3b	Height of cuboid $= 48+27\sqrt{3}$ cm	
4a	$x=5$ or $x=-1$ (rej)	
4b	$\log_p 32 \times \log_8 p = 1\frac{2}{3}$	
5i	$b=-2$ $a=5$	
5ii	$x=\frac{1}{2}$ or $x=-1$ or $x=-2$	
5iii	$x=1$ or $x=-2$ or $x=-4$	
6i	$k=6$	
6ii	$\int_5^{13} \frac{3x}{2\sqrt{2x-1}}\,dx = 26$	
7i	$a=4$	
7ii	$k=\frac{\pi}{2}$	
7iii	Length of string = 12 cm	
7iv		
7v	Time interval $= 0 \le t < \frac{2}{3}$ or $3\frac{1}{3} < t \le 4$	

Qn. #	Solution	Mark Allocation
8a	$k > 5\frac{1}{3}$	
8b	$b = 12$ $a = \frac{8}{9}$	
9i	$\alpha^2 + \beta^2 = \frac{2}{3}$	
9ii	$k = -\frac{4\sqrt{6}}{3}$ (N.A.) or $\frac{4\sqrt{6}}{3}$	
9iii	Equation: $x^2 - \frac{2\sqrt{6}}{9}x + 1 = 0$	
10i	x coordinate of $A = \frac{7\pi}{12}$, x coordinate of $B = \frac{11\pi}{12}$	
10ii	Shaded area $= 4.38$ unit^2	
11i	$t = -\ln\frac{1}{2}$	
11ii	Total distance travelled $= 1.77$ m	
11iii	Average velocity $= -0.271$ m/s	
12i	Equation: $(x-8)^2 + (y-5)^2 = 25$	
12ii	Coordinates of $B = (12, 2)$	
12v	Equation of tangent: $y = \frac{80}{39}x$	

Mid-Year Examination 2015 Solution Level: Sec 4E

Additional Mathematics Paper 2 (100 marks)

Qn. #	Solution	Mark Allocation
1i	$\angle BDC = \angle CBG$ (alternate segment theorem) $\angle BCE = \angle CBG$ (alternate angles, AC//FG) $\angle BDC = \angle BCE$ $\angle CBD = \angle EBC$ (common angle) Since the corresponding angles of the triangles are equal, ΔBCD is similar to ΔBEC	 M1 A1
1ii	Since ΔBCD is similar to ΔBEC $\frac{BC}{BD} = \frac{BE}{BC}$ $BC^2 = BD \times BE$	 B1
1iii	$\angle BDC = \angle BCE$ (from (i)) $\quad \angle CBG = \angle ACB$ (alt $\angle$) $\angle BDC = \angle BAC$ ($\angle$ in same seg) or $\quad \angle CBG = \angle BAC$ (alt seg) $\angle BCE = \angle BAC \quad \angle ACB = \angle BAC$ ΔABC is an isosceles triangle. $\quad \Delta ABC$ is isosceles.	 M1 A1
2i	$75000 = 120\,000e^{-k(30)}$ $\ln\frac{5}{8} = -30k$ $k = -\frac{1}{30}\ln\frac{5}{8}$ After 20 months $V = 120\,000e^{-\left(-\frac{1}{30}\ln\frac{5}{8}\right)(20)}$ $= 87720.53215$ $= 87700$	M1 M1 M1 A1
2ii	$V = 120\,000e^{-\left(-\frac{1}{30}\ln\frac{5}{8}\right)(40)}$ $= 64124.098$ It is not economical to replace the engine after 40 months as the value of the engine has not reached half its original value of \$120000	M1 A1
3a	Area of equilateral triangle $= \frac{1}{2}\left(3+\sqrt{7}\right)^2 \sin 60$ $= \frac{1}{2}\left(9+6\sqrt{7}+7\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{1}{4}\left(16\sqrt{3}+6\sqrt{21}\right)$ $= \frac{1}{2}\left(8\sqrt{3}+3\sqrt{21}\right)$ cm^2	 M1 M1 A1

Qn. #	Solution	Mark Allocation
3b	Height of cuboid $=\dfrac{30+12\sqrt{3}}{4-2\sqrt{3}}$ $=\dfrac{30+12\sqrt{3}}{4-2\sqrt{3}}\times\dfrac{4+2\sqrt{3}}{4+2\sqrt{3}}$ $=\dfrac{120+108\sqrt{3}+72}{16-12}$ $=48+27\sqrt{3}$ cm	M1 M1 A1
4a	$\log_3(2x-1)-\frac{1}{2}\log_3(x^2+2)=\log_{25}5$ $\log_3(2x-1)-\frac{1}{2}\log_3(x^2+2)=\frac{1}{2}$ $2\log_3(2x-1)-\log_3(x^2+2)=1$ $\log_3(2x-1)^2-\log_3(x^2+2)=1$ $\log_3\dfrac{(2x-1)^2}{(x^2+2)}=1$ $\dfrac{(2x-1)^2}{(x^2+2)}=3$ $4x^2-4x+1=3x^2+6$ $x^2-4x-5=0$ $(x+1)(x-5)=0$ $x=5$ or $x=-1$ (rej)	 M1 M1 M1 M1 A1
4b	$\log_p 32\times\log_8 p$ $=\dfrac{\log_2 32}{\log_2 p}\times\dfrac{\log_2 p}{\log_2 8}$ $=\dfrac{\log_2 2^5}{\log_2 2^3}$ $=\dfrac{5}{3}=1\dfrac{2}{3}$	 M1 M1 A1
5i	$2x^2+x-1=(2x-1)(x+1)$ $(2x-1)$ and $(x+1)$ are factors $f\left(\frac{1}{2}\right)=0$ $2\left(\frac{1}{2}\right)^3+a\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)+b=0$	 M1

Qn. #	Solution	Mark Allocation
	$\frac{1}{4}+\frac{1}{4}a+\frac{1}{2}+b=0$	
	$a+4b=-3 \quad ---(1)$	
	$f(-1)=0$	
	$2(-1)^3+a(-1)^2+(-1)+b=0$	M1
	$-2+a-1+b=0$	
	$a+b=3 \quad ---(2)$	
	(1) - (2): $3b=-6$	M1
	$b=-2$	
	$a=5$	A1
5ii	$f(x)=0$	
	$2x^3+5x^2+x-2=0$	
	$(2x^2+x-1)(x+2)=0$	M1
	$(2x-1)(x+1)(x+2)=0$	
	$x=\frac{1}{2}$ or $x=-1$ or $x=-2$	A1
5iii	$\frac{1}{4}x^3+\frac{a}{4}x^2+\frac{1}{2}x+b=0$	
	Let $x=2y$	M1
	$\frac{1}{4}(2y)^3+\frac{a}{4}(2y)^2+\frac{1}{2}(2y)+b=0$	
	$2y^3+ay^2+y+b$	
	From (ii), $y=\frac{1}{2}$ or $y=-1$ or $y=-2$	
	$x=1$ or $x=-2$ or $x=-4$	A1
6i	$\frac{dy}{dx}=(2x+2)\left[\frac{1}{2}(2x-1)^{-\frac{1}{2}}(2)\right]+(2x-1)^{\frac{1}{2}}(2)$	M1
	$=(2x-1)^{-\frac{1}{2}}\{(2x+2)+2(2x-1)\}$	M1
	$=(2x-1)^{-\frac{1}{2}}\{6x\}$	
	$=\frac{6x}{\sqrt{2x-1}}$	A1
	$k=6$	A1

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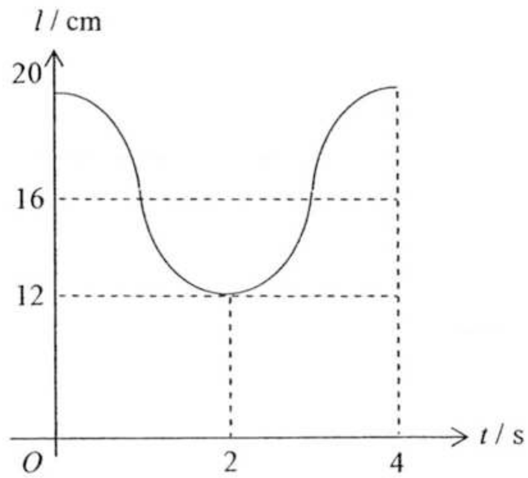
Qn. #	Solution	Mark Allocation
6ii	$\int_5^{13} \frac{6x}{\sqrt{2x-1}}\,dx = \left[(2x+2)\sqrt{2x-1}\right]_5^{13}$ $\frac{1}{4}\int_5^{13} \frac{6x}{\sqrt{2x-1}}\,dx = \frac{1}{4}\left[(2x+2)\sqrt{2x-1}\right]_5^{13}$ $\int_5^{13} \frac{3x}{2\sqrt{2x-1}}\,dx = \frac{1}{4}\left[(28)\sqrt{25}-(12)\sqrt{9}\right]$ $= 26$	M1 M1 M1 A1
7i	$20 = a\cos k(0)+16$ $a = 4$	B1
7ii	$\frac{2\pi}{k} = 4$ $k = \frac{\pi}{2}$	M1 A1
7iii	Length of string = 20 – 8 = 12 cm	B1
7iv		B1 – correct shape B1 – correct values
7v	$4\cos\frac{\pi}{2}t+16=18$ $\cos\frac{\pi}{2}t = \frac{1}{2}$ Basic angle = $\cos^{-1}\frac{1}{2}$ $\frac{\pi}{2}t = \frac{\pi}{3}, 2\pi-\frac{\pi}{3}$ $t = \frac{2}{3}, 3\frac{1}{3}$ Time interval = $0 \le t < \frac{2}{3}$ or $3\frac{1}{3} < t \le 4$	M1 A1 A1

Qn. #	Solution	Mark Allocation
8a	$\frac{dy}{dx} = 3x^2 + 8x + k$	M1
	For curve to be always increasing, $\frac{dy}{dx}$ always > 0	M1
	$b^2 - 4ac < 0$	
	$64 - 4(3)(k) < 0$	M1
	$k > 5\frac{1}{3}$	A1
8b	$\frac{dy}{dx} = a - \frac{2b}{x^3}$	M1
	At stat. pt. $\frac{dy}{dx} = 0$	
	$a - \frac{2b}{x^3} = 0$	
	Sub $x = 3$, $y = 4$	
	$a - \frac{2b}{3^3} = 0$	M1
	$a = \frac{2b}{27}$ --- (1)	
	Sub $x = 3$, $y = 4$ to equation of curve	
	$4 = a(3) + \frac{b}{3^2}$	M1
	$36 = 27a + b$ --- (2)	
	Sub (1) to (2):	
	$36 = 2b + b$	M1
	$b = 12$	
	$a = \frac{24}{27} = \frac{8}{9}$	A1
9i	$\alpha\beta = 1$	
	$\alpha^3\beta + \alpha\beta^3 = \frac{2}{3}$	M1 – either one correct
	$\alpha\beta(\alpha^2 + \beta^2) = \frac{2}{3}$	M1
	$1(\alpha^2 + \beta^2) = \frac{2}{3}$	
	$\alpha^2 + \beta^2 = \frac{2}{3}$	A1

Qn. #	Solution	Mark Allocation
9ii	$(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta$ $=\frac{2}{3}+2(1)$ $=\frac{8}{3}$ $(\alpha+\beta)^2=\frac{8}{3}$ $\alpha+\beta=\frac{2\sqrt{6}}{3}$ or $-\frac{2\sqrt{6}}{3}$ $-\frac{k}{2}=\frac{2\sqrt{6}}{3}$ or $-\frac{2\sqrt{6}}{3}$ $k=-\frac{4\sqrt{6}}{3}$ (N.A.) or $\frac{4\sqrt{6}}{3}$	M1 M1 A1
9iii	$S.O.R.=\alpha^3+\beta^3$ $=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=\left(-\frac{2\sqrt{6}}{3}\right)\left(\frac{2}{3}-1\right)$ $=\frac{2\sqrt{6}}{9}$ $P.O.R.=\alpha^3\beta^3$ $=(\alpha\beta)^3$ $=(1)^3=1$ Equation: $x^2-\frac{2\sqrt{6}}{9}x+1=0$	M1 M1 A1
10i	$2\sin(2x+\pi)-1=0$ $\sin(2x+\pi)=\frac{1}{2}$ $\alpha=\sin^{-1}\frac{1}{2}=\frac{\pi}{6}$ $2x+\pi=\frac{\pi}{6},\quad \pi-\frac{\pi}{6},\quad \frac{\pi}{6}+2\pi,\quad \pi-\frac{\pi}{6}+2\pi$ $x=-\frac{5\pi}{12},\quad -\frac{\pi}{12},\quad \frac{7\pi}{12},\quad \frac{11\pi}{12}$ x coordinate of $A=\frac{7\pi}{12}$, x coordinate of $B=\frac{11\pi}{12}$	M1 M1 M1 A1

Qn. #	Solution	Mark Allocation
10ii	Shaded area $-\int_0^{\frac{7\pi}{12}} 2\sin(2x+\pi)-1\,dx+\int_{\frac{7\pi}{12}}^{\frac{11\pi}{12}} 2\sin(2x+\pi)-1\,dx$ $=-\left[-\cos(2x+\pi)-x\right]_0^{\frac{7\pi}{12}}+\left[-\cos(2x+\pi)-x\right]_{\frac{7\pi}{12}}^{\frac{11\pi}{12}}$ $=-\left[-2.69862-1\right]+\left[-2.01377-(-2.69862)\right]$ $=4.38\text{ unit}^2$	M2 – 1 for limits, 1 for expressions M1 – int. sin -> cos M1 – times 1/2 M1 A1
11i	$v=\int -4e^{-t}\,dt$ $=4e^{-t}+c$ When $t=0$, $v=2$ $2=4e^0+c$ $c=-2$ $v=4e^{-t}-2$ At instantaneous rest, $v=0$ $4e^{-t}-2=0$ $e^{-t}=\frac{1}{2}$ $t=-\ln\frac{1}{2}$	M1 M1 M1 A1
11ii	$s=\int 4e^{-t}-2\,dt$ $s=-4e^{-t}-2t+c$ When $t=0$, $s=0$ $0=-4e^0-2(0)+c$ $c=4$ $s=-4e^{-t}-2t+4$ Distance travelled before instantaneous rest $=-4e^{-\left(-\ln\frac{1}{2}\right)}-2\left(-\ln\frac{1}{2}\right)+4$ $=0.61371$ Distance from instantaneous rest to 2s $=0.61371-\left(-4e^{-(2)}-2(2)+4\right)$ $=0.61371-(-0.54134)$ $=1.15505$ Total distance travelled $=0.61371+1.15505=1.77\text{m}$	M2 M1 M1 M1 A1
11iii	Average velocity $=\frac{-0.54134}{2}=-0.271\text{m/s}$	B1

Qn. #	Solution	Mark Allocation
12i	Centre of C_1 =Centre of C_2 = $(8, 5)$ Radius of $C_1 = \sqrt{(8-4)^2+(5-8)^2} = 5$ Equation: $(x-8)^2+(y-5)^2=25$	M1 M1 A1
12ii	Coordinates of $B = (8+4, 5-3)$ $= (12, 2)$	M1 A1
12iii	Gradient of normal at B $= \frac{5-2}{8-12} = -\frac{3}{4}$ Gradient of tangent $= \frac{4}{3}$ Equation: $y-2=\frac{4}{3}(x-12)$ $y=\frac{4}{3}x-14$ $3y=4x-42$ (shown)	M1 M1 A1
12iv	Centre is at $(8, 5)$ and radius is 5 The lowest point D is at $(8, 0)$ and the circle touches x-axis at D. Thus x-axis is a tangent to the circle at D	B1
12v	$\tan\theta=\frac{5}{8}$ $\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}=\frac{2\left(\frac{5}{8}\right)}{1-\left(\frac{5}{8}\right)^2}$ $=\frac{80}{39}$ Equation of tangent: $y=\frac{80}{39}x$	 M1 A1

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 1 2015
Secondary 4

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS 4047/01

Paper 1

Tues 30 June 2015

2 hours

Additional Materials: Writing Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

190

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Find the range of values of k where $k \neq 0$ if the roots for the equation $kx^2 + (k-2)x + 4k = 0$ are real. [4]

2 A particle moves along the curve $y = 6 + \frac{1}{x^2}$ such that the y-coordinate of the particle is decreasing at a constant rate of 0.04 units per second.

Find the rate of change of the x-coordinate when $x = 2$. [4]

3 Given that $\int_{-1}^{3} [f(x)+1] \mathrm{d}x = 8$, evaluate

(i) $\int_{-1}^{3} f(x) \mathrm{d}x$, [2]

(ii) $\int_{2}^{3} [f(x)+1] \mathrm{d}x - \int_{2}^{-1} [f(x)+1] \mathrm{d}x$. [2]

4 (i) Write down the first three terms in the expansion, in descending powers of x, of $\left(2x - \frac{1}{3x}\right)^7$. [3]

(ii) Hence find the coefficient of x^5 in the expansion of $(x^2+2)\left(2x - \frac{1}{3x}\right)^7$. [2]

5 Given that the roots of $2x^2 + 3x - 6 = 0$ are $2\alpha + \beta$ and $2\beta + \alpha$, find a quadratic equation whose roots are α and β. [6]

[Turn over

6 A bowl of hot soup was left to cool such that t minutes later, its temperature, H°C, is given by $H = 25 + 70e^{-kt}$, where k is a constant.
When $t = 2$, the temperature of the soup is 80°C.

(i) Show that $k = 0.1206$. [2]

(ii) Find the time taken for the soup to reach 40°C. [2]

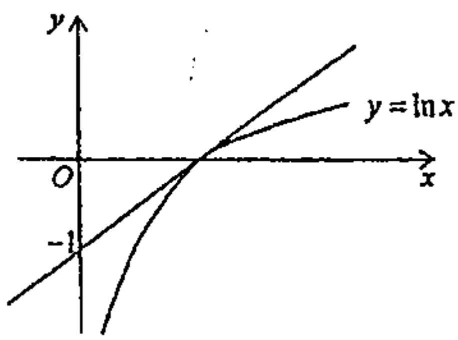
(iii) Explain why the temperature of the soup reaches 25°C after a long time. [1]

(iv) Sketch the graph of H against t. [2]

7 (a) If $a^{3-x}b^{5x} = a^{x+7}b^{3x}$, prove that $(2+x)\lg a = x\lg b$. [3]

(b) Solve $(2\log_3 x + 5)\log_3 x = 3$. [4]

8 The diagram shows part of the curve $y = \ln x$ and a tangent to the curve at $x = k$ which also passes through the point $(0, -1)$.



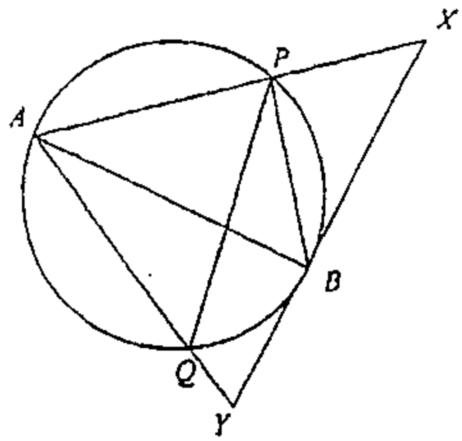
(i) Find the equation of the tangent. [5]

(ii) Write down an inequality for m if the line $y = mx - 1$ where $m > 0$

(a) intersects the curve exactly 2 times, [1]

(b) does not meet the curve. [1]

9



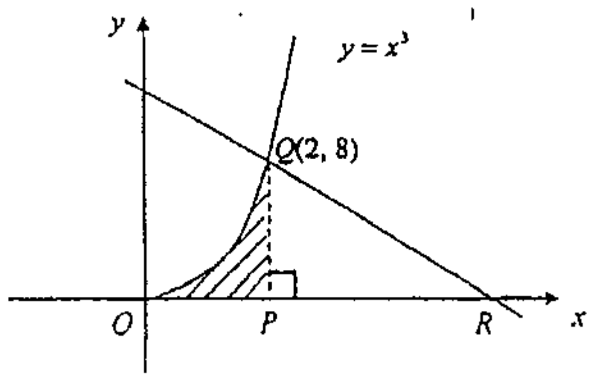
The diagram shows points A, P, B and Q lying on a circle with diameter AB.
The tangent to the circle at B meets AP produced at X and AQ produced at Y.

(i) Prove that triangle APB is similar to triangle ABX.
Hence express AB^2 in terms of AP and AX. [4]

(ii) Express AB^2 in terms of AP and PB. [1]

(iii) Using your answers in (i) and (ii), show that $PB^2 = AP \times PX$. [2]

10



The diagram shows part of the curve $y = x^3$. Points P and R lie on the x-axis.

The line QR intersects the curve at $Q(2, 8)$. QP is perpendicular to the x-axis.

Given that the ratio of the shaded area to the area of triangle PQR is $2 : 5$.

(i) Find the shaded area. [3]

(ii) Find the coordinates of R. [3]

(iii) Determine whether QR is the normal to the curve at Q. [4]

[Turn over

11 The tidal height, y metres, at a jetty on a particular day can be represented by the equation

$$y = 1.6 + 1.4\cos(kt)$$

where t is the time in hours after midnight and k is a constant.

The time between the first high tide and the next high tide is 14 hours.

(i) Show that $k = \frac{\pi}{7}$. [1]

(ii) Find the minimum tidal height for that day and the time it first occured. [3]

(iii) For how long between the first high tide and the next high tide was the tidal height at most 1 m high? [5]

12 Given that $\frac{4x^3+3x^2-8x-1}{x^2+x-2} = ax + b + \frac{x+c}{x^2+x-2}$,

(i) find the value of each of the integers a, b and c. [4]

Hence, using partial fractions and the values of a, b and c obtained in part (i), find

(ii) $\int \frac{4x^3+3x^2-8x-1}{x^2+x-2}\,dx$. [6]

End of paper

Answers

1. $-\frac{2}{3} \le k \le \frac{2}{5}, k \ne 0$

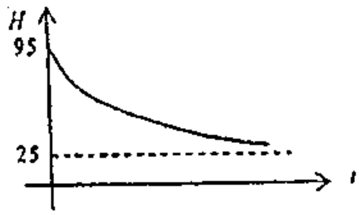
2. 0.16 units/s

3.(i) 4 (ii) 8

4.(i) $128x^7 - \frac{448}{3}x^5 + \frac{224}{3}x^3 + \ldots\ldots$ (ii) -224

5. $x^2 + \frac{1}{2}x - \frac{7}{2} = 0$

6.(ii) 12.8 min (iv)



7.(b) $x = \frac{1}{27}$ or $\sqrt{3}$

8.(i) $y = x - 1$ (ii)(a) $0 < m < 1$ (b) $m > 1$

9.(i) $AB^2 = AP \times AX$

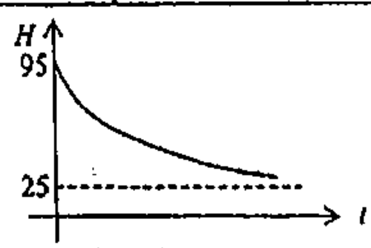
10.(i) 4 units^2 (ii) (4.5, 0) (iii) QR is not the normal to the curve at Q.

11.(ii) 0.2 m at 7 am (iii) 5.03 h

12.(i) $4x - 1 + \frac{x-3}{(x+2)(x-1)}$ (ii) $2x^2 - x + \frac{5}{3}\ln(x+2) - \frac{2}{3}\ln(x-1) + c$

2015 Sec 4 Prelim 1 Add Maths P1

No.	Solution	Remarks	
1	$(k-2)^2-4(k)(4k)\ge 0$ $(k-2+4k)(k-2-4k)\ge 0$ $(5k-2)(-3k-2)\ge 0$ $(5k-2)(3k+2)\le 0$ $-\frac{2}{3}\le k\le\frac{2}{5}, k\ne 0$	M1 Correct sub for discriminant B1 $D \ge 0$ M1 Factorise A1	
		Total	4 m
2	$\frac{dy}{dx}=-\frac{2}{x^3}$ At $x=2$, $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $-0.04=-\frac{2}{2^3}\times\frac{dx}{dt}$ $\frac{dx}{dt}=0.16$ Rate of change of $x = 0.16$ units/s	M1 M1 Eqn with sub B1 $\frac{dy}{dt}=-0.04$ A1	
		Total	4 m
3(i)	$\int_{-1}^{3} f(x)dx=\int_{-1}^{3} f(x)+1dx-\int_{-1}^{3}1dx$ $=8-[x]_{-1}^{3}$ $=4$	M1 For splitting A1	
3(ii)	$\int_{2}^{3} f(x)+1dx+\int_{-1}^{2} f(x)+1dx$ $=\int_{-1}^{3} f(x)+1dx$ $=8$	M1 Change signs & limits A1	
		Total	4 m
4(i)	$\left(2x-\frac{1}{3x}\right)^7=(2x)^7+\binom{7}{1}(2x)^6\left(-\frac{1}{3x}\right)+\binom{7}{2}(2x)^5\left(-\frac{1}{3x}\right)^2$ $=128x^7-\frac{448}{3}x^5+\frac{224}{3}x^3+\ldots\ldots$	M1 B2 Expansion – 1 mark for each error	
4(ii)	$(x^2+2)\left(128x^7-\frac{448}{3}x^5+\frac{224}{3}x^3+\ldots\right)$ $coeff=2\left(-\frac{448}{3}\right)+\frac{224}{3}$ $=-224$	M1 A1	
		Total	5 m

No.	Solution	Remarks	
5	$2\alpha+\beta+2\beta+\alpha=-\frac{3}{2}$ $\alpha+\beta=-\frac{1}{2}$ $(2\alpha+\beta)(2\beta+\alpha)=-3$ $4\alpha\beta+2\beta^2+2\alpha^2+\alpha\beta=-3$ $5\alpha\beta+2[(\alpha+\beta)^2-2\alpha\beta]=-3$ $\alpha\beta=-3-2\left(-\frac{1}{2}\right)^2$ $\alpha\beta=-\frac{7}{2}$ Equation is $x^2+\frac{1}{2}x-\frac{7}{2}=0$	M1 Find sum $=-\frac{b}{a}$ B1 M1 Find prod $=\frac{c}{a}$ M1 Using $(\alpha+\beta)^2$ B1 A1$\sqrt{}$ must $=0$	
		Total	6 m
6(i)	$80=25+70e^{-2k}$ $e^{-2k}=\frac{55}{70}$ $-2k=\ln\frac{55}{70}$ $k=0.1206$	M1 Sub $t=2$, $H=80$ M1 Take ln both sides & result	
6(ii)	$40=25+70e^{-0.1206t}$ $e^{-0.1206t}=\frac{15}{70}$ $-0.1206t=\ln\frac{15}{70}$ $t=12.8\text{min}$	M1 Using ln on both sides A1 or 12 min 46 sec	
6(iii)	As t becomes very large, $70e^{-kt}$ approaches zero, So the temperature reaches 25°C after a long time.	B1	
6(iv)		G1 shape G1 95, 25 seen	
		Total	7 m

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7(a)	$a^{x+7} \div a^{3-x} = b^{5x} \div b^{3x}$ $a^{x+7-3+x} = b^{5x-3x}$ $(4+2x)\lg a = 2x\lg b$ $(2+x)\lg a = x\lg b$	M1 Group/Rearrange M1 Indices or log law B1 Taking lg on both sides and result	
7(b)	Let $a = \log_3 x$ $\quad 2a^2 + 5a - 3 = 0$ $(a+3)(2a-1) = 0$ $a = -3$ *or* $a = \frac{1}{2}$ $\log_3 x = -3$ or $\log_3 x = \frac{1}{2}$ $x = \frac{1}{27}$ or $x = \sqrt{3}$	M1 By sub to get quad eqn B1 A1 A1	
		Total	7 m
8(i)	$\frac{dy}{dx} = \frac{1}{x}$ $x = k, \frac{dy}{dx} = \frac{1}{k}, y = \ln k$ $y - \ln k = \frac{1}{k}(x-k)$ $-1 = \frac{1}{k}(0) - 1 + \ln k$ $\ln k = 0$ $k = 1$ Eqn of tgt: $y = x - 1$	M1 Differentiate M1 Find eqn at $x=k$ M1 Using $(0, -1)$ B1 value of k B1	
8(ii)	(a) $0 < m < 1$ (b) $m > 1$	B1 $\surd$ accept *m <their grad* B1 $\surd$ accept *m >their grad* ** their grad must be > 0*	
		Total	7 r

2015 Sec 4 Prelim 1 Add Maths P1

No.	Solution	Remarks	
9(i)	$\angle PAB = \angle BAX$ (Common angle) $\angle APB = 90°$ ($\angle$ in semi circle) $\angle ABX = 90°$ (tangent perpendicular to radius) So ΔAPB is similar to ΔABX. $\frac{AB}{AP} = \frac{AX}{AB}$ $AB^2 = AP \times AX$	B1 B1 B1 B1	
9(ii)	By Pythagoras Thm, $AB^2 = AP^2 + PB^2$	B1	
9(iii)	$AP^2 + PB^2 = AP \times AX$ $PB^2 = AP \times AX - AP^2$ $PB^2 = AP(AX - AP)$ $\therefore PB^2 = AP \times PX (shown)$	M1 Equating (i) M1 Factorising seen & result	
		Total	7 m
10(i)	Shaded Area $= \int_0^2 x^3 dx$ $= \frac{1}{4}\left[x^4\right]_0^2 = 4 units^2$	B1 M1 correct integration A1	
10(ii)	Area of triangle PQR = 10 units^2 $\frac{1}{2} \times PR \times 8 = 10$ $PR = 2.5$ $R(4.5, 0)$	B1 M1 use area/discriminant mthd A1	
10(iii)	gradient of $QR = -3.2$ $\frac{dy}{dx} = 3x^2$ $x = 2, \frac{dy}{dx} = 12$ Since grad of normal $= -\frac{1}{12} \neq$ gradient of QR, So QR cannot be normal to curve at Q.	M1 B1 M1 grad of normal B1 correct conclusion	
		Total	10 m

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No.	Solution	Remarks	
11(i)	$\frac{2\pi}{k}=14$ $k=\frac{\pi}{7}(shown)$	M1	
11(ii)	$\cos\left(\frac{\pi}{7}t\right)=-1$ Minimal tidal height $=1.6+1.4(-1)$ $=0.2$ m At 7 am	M1 *soi* B1 B1 or 0700	
11(iii)	$1.6+1.4\cos\left(\frac{\pi}{7}t\right)=1$ $\cos\left(\frac{\pi}{7}t\right)=-0.42857$ Basic angle = 1.1279 $\frac{\pi}{7}t=2.0137,\quad 4.2695$ $t=4.4868,\quad 9.5132$ Duration = 5.026 h	M1 Form eqn & = 1 B1 B1 M1 2^{nd} ans – 1^{st} ans A1	
		Total	9 m
12(i)	$4x-1+\frac{x-3}{(x+2)(x-1)}$	M1 use long div A3 value of a,b,c	
12(ii)	$\frac{x-3}{(x+2)(x-1)}=\frac{A}{x+2}+\frac{B}{x-1}$ $x-3=A(x-1)+B(x+2)$ $x=1, B=-\frac{2}{3}$ $x=-2, A=\frac{5}{3}$ $\int 4x-1+\frac{5}{3(x+2)}-\frac{2}{3(x-1)}dx$ $=2x^2-x+\frac{5}{3}\ln(x+2)-\frac{2}{3}\ln(x-1)+c$	M1 correct PF M1 A1 Both answers M1 first 2 terms M1 both ln (...) seen A1 $+c$ seen $\int ...dx$ (–1 mark if dx missing)	
		Total	10 m

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 1 2015
Secondary 4

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS

4047/02

Paper 2

Monday 6 July 2015

2 hours 30 minutes

Additional Materials: Writing Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

196

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

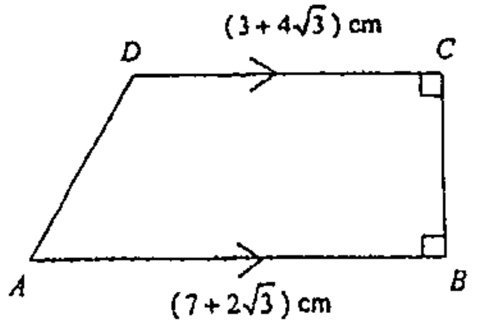
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

45

1



The diagram shows a trapezium $ABCD$ in which $AB = (7+2\sqrt{3})$ cm and $DC = (3+4\sqrt{3})$ cm. AB and DC are perpendicular to CB. Given that the area of the trapezium is $(14+\sqrt{12})$ cm^2, find the exact value of CB in the form of $(a+b\sqrt{3})$ cm. [4]

2 A function has an equation where

$$f(x) = \frac{\ln(4-x)}{x-4}, x < 4.$$

(i) Obtain an expression for $f'(x)$. [3]

(ii) Showing full working, determine whether f is decreasing for $x < 4 - e$, [3]

3 (i) Sketch the graph of $y = |2x-1| - 3$. [2]

(ii) Explain why the minimum value is -3. [1]

(iii) A line $y = kx$, where $k > 0$, is drawn on the same axes with the graph of $y = |2x-1| - 3$. Find the range of values of k for which there is only one point of intersection. [1]

4 (i) Prove the identity $\dfrac{2\cos 2A + \cos A + 2}{2\sin 2A + \sin A} = \cot A$. [4]

(ii) Hence, solve the equation $\dfrac{2\cos 6x + \cos 3x + 2}{2\sin 6x + \sin 3x} = 5$ for $0 \le x \le \pi$. [4]

5 It is given that $\sin A = \frac{1}{\sqrt{5}}$, where A is an acute angle. Without using a calculator,

(i) find $\tan A$. [2]

Given further that $\tan(A+B) = 2$, where B is an acute angle,

(ii) find the exact value of $\tan B$. [4]

6 (i) Sketch the graph of $y = 2x^{\frac{1}{2}}$ for $x > 0$. [1]

(ii) On the same diagram, sketch the graph of $y = \frac{1}{3}x^{\frac{3}{2}}$ for $x > 0$. [1]

(iii) Find the x-coordinate of the point of intersection of your graphs. [2]

7 (i) Differentiate $x\tan^2 x$ with respect to x. [3]

(ii) Show that $\int_0^{\frac{\pi}{4}} \tan^2 x \, dx = 0.2146$. [4]

(iii) Hence, find $\int_0^{\frac{\pi}{4}} x\tan x\sec^2 x \, dx$. [4]

8 Given that $f(x) = 2x^3 + ax^2 + bx - 3$, where a and b are constants, has a factor of $x - 3$ and leaves a remainder of -20 when divided by $x + 1$, find the value of a and of b. [5]

9 The points $A(3, 0)$ and $B(9, 6)$ lie on a circle C_1 such that the x-axis is a tangent to the circle at A.

(i) Find the equation of the perpendicular bisector of AB. [4]

(ii) Hence, or otherwise, find centre of the circle C_1 and the radius. [3]

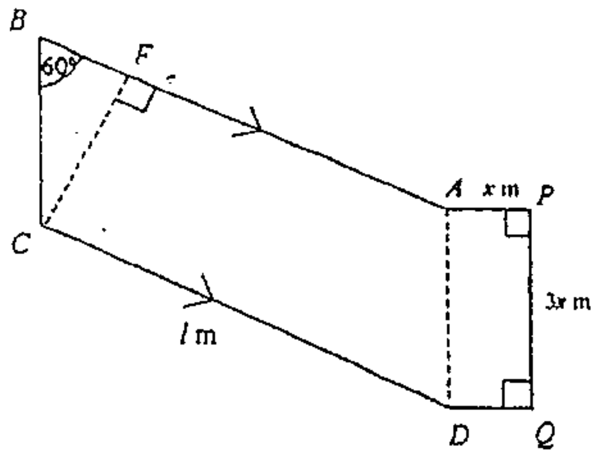
(iii) Show that the equation of the circle is $x^2 + y^2 - 6x - 12y + 9 = 0$. [2]

Another circle C_2 is formed after circle C_1 is being reflected in the line $x = 8$.

(iv) Find the centre of circle C_2. [1]

(v) Explain why the point (12, 9) lies within circle C_2. [2]

10



The diagram consists of a parallelogram $ABCD$ and a rectangle $APQD$.
It is given that $CD = l$ m and that the angle $CBF = 60°$ and angle $CFA = 90°$.
The rectangle has sides $AP = x$ m and $PQ = 3x$ m.
The perimeter of the diagram is 10 m.

(i) Express l in terms of x and show that the area of the diagram is
$3(1 - 2\sqrt{3})x^2 + \dfrac{15\sqrt{3}}{2}x\ \text{m}^2$. [3]

(ii) Given that x can vary, find the value of x for which the area has a stationary value. [3]

(iii) Determine whether this value of area is a maximum or a minimum. [2]

[Turn over

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11 The variables x and y are connected by the equation $y = ax + \frac{b}{x}$, where a and b are constants. Experimental values of x and y were obtained. A graph is drawn in which xy was plotted against x^2. The straight line which was obtained passed through the points (1, 5) and (3, 11).

Find

(i) the value of a and of b, [4]

(ii) the coordinates of the point on the line at which $y = -x + \frac{4}{x}$. [4]

12 A particle travels in a straight line, so that t seconds after passing through a fixed point O, its velocity, v ms^{-1}, is given by $v = \frac{32}{(t+2)^2} - 2$. The particle comes to instantaneous rest at P. Find

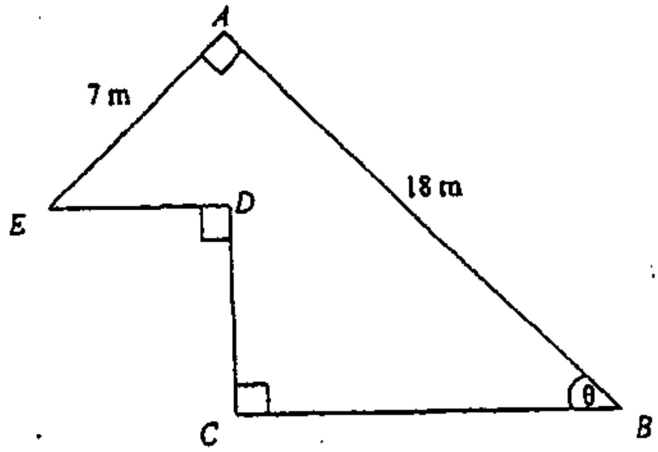
(i) the value of t when the particle is at instantaneous rest, [3]

(ii) distance OP, [4]

(iii) distance travelled for the first 8 seconds, [2]

(iv) the acceleration of the particle at $t = 8$ seconds. [3]

13



A playground is to be built in the shape of the figure shown in which ADC is a straight line and angle EAB = angle DCB = angle $EDC = 90°$. The length of AE is 7 m and AB is 18 m. The angle ABC is θ, where $0° < \theta < 90°$.
The perimeter of the playground is given by L m.

(i) Show that L can be expressed as $p + q\cos\theta + r\sin\theta$, where p, q and r are constants to be found. [3]

(ii) Express L in the form $p + R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [4]

(iii) Given $L = 51$ m, find θ. [2]

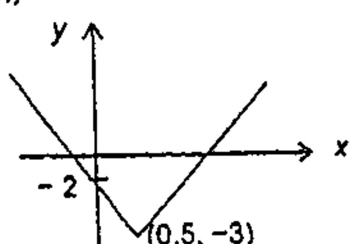
(iv) Find the maximum value of the perimeter and the corresponding value of θ. [3]

End of paper

1) $-26 + 16\sqrt{3}$

2i) $\dfrac{1 - \ln(4 - x)}{(x - 4)^2}$

3i)



3ii) Since

$|2x - 1| \geq 0,$

$|2x - 1| - 3 \geq -3$

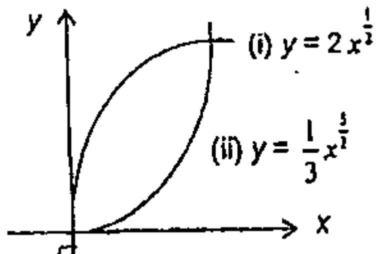
.99

Min value is -3
3iii) $k \geq 2$
4ii) $x = 0.0658, 1.11, 2.16$

5i) $\tan A = \frac{1}{2}$

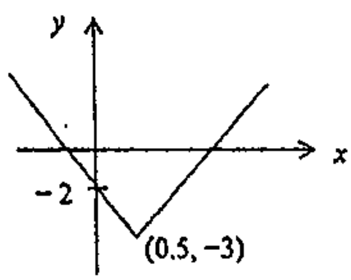
5ii) $\tan B = \frac{3}{4}$

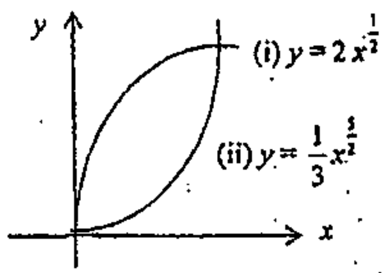
6i)



6iii) $x = \sqrt{6}$
7i) $\tan^2 x + 2x \tan x \sec^2 x$
7ii) 0.2146
7iii) 0.285
8) $a = -8$, $b = 7$
9i) $y = -x + 9$
9ii) Centre is (3,6)
Radius = 6
9v) (13,6)
9iv) $\sqrt{10}$ < radius 6
(12,9) lies within circle
10i) $l = 5 - 4x$
10ii) $x = 0.879$
10iii) $\frac{d^2 A}{dx^2} = 6(1 - 2\sqrt{3}) < 0$
maximum
11i) $a = 3, b = 2$
11ii) $(\frac{1}{2}, \frac{7}{2})$
12i) $t = 2, -6(rej)$
12ii) $OP = 4$ m
12iii) 11.2 m
12iv) $a = -0.064$ m/s^2
13ii) $L = 25 + \sqrt{764}\cos(\theta - 66.25°)$
13iii) 84.1°, 48.4°
13iv) $\theta = 66.3°, 426.3°(rej)$, Max $= 25 + \sqrt{746} = 52.3$ m

[Turn over

No	Solution	Marks	Remarks
1	$14+\sqrt{12}=\frac{1}{2}(7+2\sqrt{3}+3+4\sqrt{3})CB$	B1 Correct eqn	Use of trapezium formula or otherwise
	$CB=\frac{14+2\sqrt{3}}{5+3\sqrt{3}}\times\frac{5-3\sqrt{3}}{5-3\sqrt{3}}$	M1 rationalise	
	$=\frac{70-42\sqrt{3}+10\sqrt{3}-6(3)}{25-9(3)}$	M1 simplify surds	(top and bottom)
	$=-26+16\sqrt{3}$ cm	A1	
			4 marks
2(i)	$f'(x)=\frac{(x-4)\frac{-1}{4-x}-\ln(4-x)(1)}{(x-4)^2}$	M1 use of quotient rule B1 diff. $\ln(4-x)$ correctly	$\frac{-1}{4-x}$ seen
	$=\frac{1-\ln(4-x)}{(x-4)^2}$	A1	
2(ii)	$x<4-e$ $x-4<-e$ $4-x>e$	M1 knowing to show $f'(x)$ +ve and -ve	
	$\ln(4-x)>\ln e$ $\ln(4-x)>1$	M1 manipulate	
	$1-\ln(4-x)<0$ $\therefore f'(x)<0$ f is decreasing	B1 correct conclusion including stating $(\ \ )^2$ is +ve	
			6 marks
3(i)		S1 correct shape/symmetrical B1 vertex and y-int seen	
3(ii)	Since $\|2x-1\|\geq 0$, $\|2x-1\|-3\geq -3$ Min value is -3	B1 with explanation	
3(iii)	Gradient of R.H. arm = 2 $k\geq 2$	A1 2~~0~~	
			4 marks

No	Solution	Marks	Remarks
4(i)	LHS $= \dfrac{2(2\cos^2-1)+\cos A+2}{2(2\sin A\cos A)+\sin A}$ $= \dfrac{4\cos^2 A+\cos A}{\sin A(4\cos A+1)}$ $= \dfrac{\cos A(4\cos A+1)}{\sin A(4\cos A+1)}$ $= \cot A$	B1 double angle for cos 2A B1 double angle for sin 2A M1 factorise both B1 $\dfrac{\cos A}{\sin A} = \cot A$	
4(ii)	$\cot 3x = 5$ $\tan 3x = \dfrac{1}{5}$ $3x = 0.1974, 3.339, 6.481$ $x = 0.0658, 1.11, 2.16$	B1 M1 reciprocal of $\cot 3x$ A2 - 3 correct A1 - 2 correct	Ans must be in rad
			8 marks
5(i)	$\tan A = \dfrac{1}{2}$	M1 use pyt thm to find length A1	Must show working
5(ii)	$\dfrac{\tan A+\tan B}{1-\tan A\tan B} = 2$ $2 = \dfrac{\frac{1}{2}+\tan B}{1-\frac{1}{2}\tan B}$ $2-\tan B = \dfrac{1}{2}+\tan B$ $\tan B = \dfrac{3}{4}$	B1 use of tangent formula SOI M1 subst $\tan A$ M1 simplify A1	
			6 marks
6(i)(ii)		B1 B1	With label Shape must be correct Must touch origin
6(iii)	$\dfrac{1}{3}x^{\frac{3}{2}} = 2x^{\frac{1}{2}}$ $x^2 = 6$ $x = \sqrt{6}$	M1 simplify indices A1 must rej $-\sqrt{6}$	47

No	Solution	Marks	Remarks
			4 marks
7(i)	$\frac{d}{dx}x\tan^2 x = \tan^2 x + x(2\tan x)\sec^2 x$ $= \tan^2 x + 2x\tan x\sec^2 x$	M1 use of pdt rule B1 diff. $\tan^2 x$ correctly B1 presentation	$(2\tan x)\sec^2 x$ seen Eg $\frac{d}{dx}$ = ...
7(ii)	$\int_0^{\frac{\pi}{4}} \tan^2 x dx = \int_0^{\frac{\pi}{4}} \sec^2 x - 1 dx$ $= [\tan x - x]_0^{\frac{\pi}{4}}$ $= 0.2146$ (0.214602)	M1 use of identity B1 integrate $\sec^2 x$ M1 evaluate integral(must show subst) B1 presentation	Eg $\int ...dx$
7(iii)	$\int_0^{\frac{\pi}{4}} \tan^2 x + 2x\tan x\sec^2 x dx = [x\tan^2 x]_0^{\frac{\pi}{4}}$ $2\int_0^{\frac{\pi}{4}} x\tan x\sec^2 x dx = [x\tan^2 x]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \tan^2 x dx$ $= 0.7854 - 0.2146$ $= 0.5708$ $\int_0^{\frac{\pi}{4}} x\tan x\sec^2 x dx = 0.285$	B1 work backwards M1 make subject M1 evaluate integral(don't need show subst) A1 (min 3sf)	ECF
			11 marks
8	$f(3) = 2(3)^3 + 9a + 3b - 3$ $0 = 51 + 9a + 3b$ $-17 = 3a + b$.......(1) $f(-1) = -2 + a - b - 3$ $-20 = -5 + a - b$ $-15 = a - b$.........(2) $a = -8$ $b = 7$	B1 f(3) = 0 with subst B1 f(−1)= −20 with subst M1 solve sim A1, A1	
			5 marks

201

No	Solution	Marks	Remarks
9(i)	Midpoint of AB = (6, 3) Grad AB = 1 Grad of perpendicular bisector = −1 $y-3=-1(x-6)$ $y=-x+9$	B1 M1 find grad. of perpendicular bisector M1 form eqn A1 o.e.	
9(ii)	Let centre be $(3, k)$ $k=-3+9$ Centre is (3,6) Radius = 6	M1 find y-coord of centre A1 A1	Use other mtd such as distance
9(iii)	$(x-3)^2+(y-6)^2=36$ $x^2+y^2-6x-12y+9=0$	B1 seen ECF M1 expand and simplify	
9(iv)	(13,6)	A1	
9(v)	Let point (12,9) be M. Let centre of C_2 be O. $OM=\sqrt{(9-6)^2+(12-13)^2}$ $=\sqrt{10}<$ radius 6 (12,9) lies within circle	M1 find distance B1 comparison made and conclusion seen	
			12 marks
10(i)	$5x+3x+2l=10$ $l=5-4x$ $Area=3x^2+3x\sin 60°(5-4x)$ $=3x^2+15x\frac{\sqrt{3}}{2}-12x^2\frac{\sqrt{3}}{2}$ $=3(1-2\sqrt{3})x^2+15\frac{\sqrt{3}}{2}x$	A1 M1 find area of parallelogram B1 $\sin 60=\frac{\sqrt{3}}{2}$ seen	
10(ii)	$\frac{dA}{dx}=6(1-2\sqrt{3})x+15\frac{\sqrt{3}}{2}$ At stat value, $\frac{dA}{dx}=0$ $x=0.879$	M1 attempt to diff. B1 $\frac{dA}{dx}=0$ seen A1	
10(iii)	$\frac{d^2A}{dx^2}=6(1-2\sqrt{3})<0$ maximum	M1 know 2nd derivative or sign test A1	
			8 marks

No	Solution	Marks	Remarks
11(i)	$xy = ax^2 + b$ $grad = 3$ $\frac{xy-5}{x^2-1} = 3$ $xy = 3x^2 + 2$ $a = 3, b = 2$	B1 manipulate into grad-intercept form M1 using correct subst A1,A1	
11(ii)	$xy = -x^2 + 4 \ldots(1)$ $xy = 3x^2 + 2 \ldots(2)$ $x^2 = \frac{1}{2}$ $xy = \frac{7}{2}$ $(\frac{1}{2}, \frac{7}{2})$	M1 use similar eqn M1 solve simultaneous eqn A1,A1	
			8 marks
12(i)	$\frac{32}{(t+2)^2} - 2 = 0$ $t + 2 = \pm 4$ $t = 2, -6(rej)$	B1 M1 solve eqn A1	
12(ii)	$s = \int \frac{32}{(t+2)^2} - 2dt = -\frac{32}{t+2} - 2t + c$ At $t = 0, s = 0$, $c = 16$ $s = -\frac{32}{t+2} - 2 + 16$ At $t = 2$, $OP = 4$ m	M1 integrate (ok if no + c) B1 M1 find distance (sub t=2) A1	Give marks if use definite integral $\int_0^2$
12(iii)	At $t = 8$, $S = -3.2$m Distance $= 3.2 + 4 + 4 = 11.2$ m	M1 ( ) +2(4) seen A1	
12(iv)	$a = -\frac{64}{(t+2)^3}$ At t = 8, a = -0.064 m/s^2	M1 Knowing to differentiate B1 acc expression seen A1	
			12 marks

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No	Solution	Marks	Remarks
13(i)	$L = 7+18+18\cos\theta+7\sin\theta+18\sin\theta-7\cos\theta$ $= 25+11\cos\theta+25\sin\theta$	B1,B1,B1	
13(ii)	$\tan\alpha = \frac{25}{11}$ $\alpha = 66.25°$ $R = \sqrt{746}$ $L = 25+\sqrt{764}\cos(\theta-66.25°)$	B1 B1 B1 $\sqrt{764}$ or 27.31 seen B1 statement	
13(iii)	When $L = 51$m, $51 = 25+\sqrt{746}\cos(\theta-66.25°)$ $\cos(\theta-66.25°) = 0.95193$ $= 84.1°$ or $48.4°$	M1 solve A1	If use 27.3, will get θ=84.00
13(iv)	$\text{Max} = 25+\sqrt{746} = 52.3$ m At max value, $\cos(\theta-66.25°) = 1$ $\theta-66.25° = 0°, 360°$ $\theta = 66.3°, 426.3°(rej)$	A1 B1 cos(...)=1 seen SOI or ....=0 A1	Penalise if -extra ans -ans in rad.
			12 marks

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TEMASEK SECONDARY SCHOOL
O Level Preliminary Examinations 2015

ADDITIONAL MATHEMATICS 4047/01

Paper 1 2 hours

Question Booklet

Additional Material: Writing paper (8 sheets), Cover page (1 sheet)

READ THESE INSTRUCTIONS FIRST

Do not open the booklet until you are asked to do so.

You are not required to submit this booklet at the end of the paper.

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

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[Turn over

This document consists of 7 printed pages and 1 blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer all the questions.

1 (i) Given that $\sin(A+B)=3\sin(A-B)$, show that $\tan A = 2\tan B$. [2]

(ii) Hence solve the equation $\sin^2(x+30^\circ)=9\sin^2(x-30^\circ)$ for $0^\circ < x < 360^\circ$. [4]

2 (a) The equation $2x^2-2x+1=0$ has roots α and β. Find the quadratic equation whose roots are $\frac{2}{\alpha^3}$ and $\frac{2}{\beta^3}$. [5]

(b) The equation of a curve is $y=(3+m)x^2-(8+4m)x+3+4m$, where m is a constant. For $y=0$, find the value of m for which

(i) one root is the negative of the other, [2]

(ii) one root is the reciprocal of the other. [2]

3 (a) Simplify $\frac{2(4)^{\frac{1}{2}x+2}-2^{x+3}}{6^x \times 3^{1-2x}}$ and express in the form of $k(3)^{nx}$, where k and n are integers. [3]

(b) Find the values of a and b such that $\lg\left(\frac{125}{y}\right)=a\lg(by)-4\lg y$ for all positive values of y. [3]

(c) Solve the equation $2\log_5 e^x+\frac{1}{\log_2 5}=\log_5(2-3e^x)$. [5]

4 Given that x^2+2x-3 is a factor of $f(x)$, where $f(x)=x^4+6x^3+2ax^2+bx-3a$, find

(i) the values of a and b, [4]

(ii) the other quadratic factor of $f(x)$. [3]

Explain why $f(x)=0$ has only two real roots. [1]

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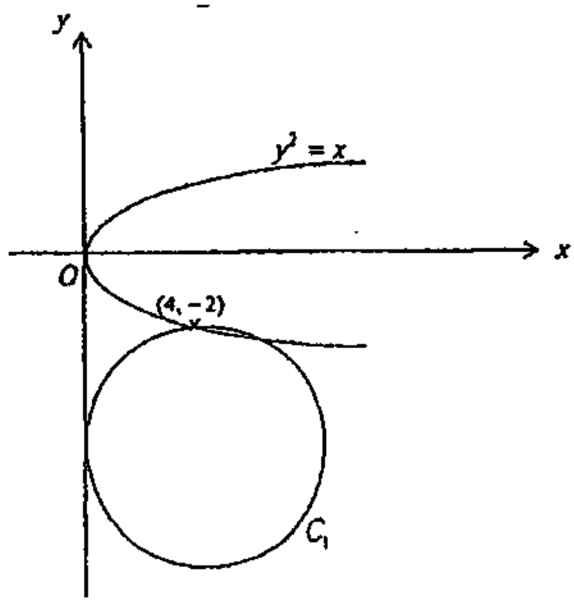
[Turn over

5 The diagram shows a circle C_1 with centre $(4, -6)$.

A curve $y^2 = x$ and the circle C_1 have the y-axis as the common tangent.

Both curves intersect at the point $(4, -2)$.



(i) Write down the radius of circle C_1 and hence the equation of C_1. [2]

(ii) Find the area bounded by the curve $y^2 = x$, the circle C_1 and the y-axis. [3]

(iii) A second circle, C_2 is the reflection of the circle, C_1 in the line $y = 2$.

Write down the equation of the second circle, C_2 in the form

$x^2 + y^2 + 2gx + 2fy + c = 0$. [2]

6 A curve is such that $\frac{dy}{dx} = 4x + \frac{1}{(x+2)^2}$ for $x > 0$ and the curve passes through the point $\left(\frac{1}{2}, \frac{1}{2}\right)$.

(i) Find the equation of the curve. [3]

(ii) Find the equation of the normal to the curve at the point where $x = \frac{1}{2}$. [2]

7 Liquid is poured into a container at a rate of k m^3/s. The volume of liquid in the container is V m^3 where $V = \frac{1}{3}\pi h^2(3k - h)$ and h m is the depth of the liquid in the container. Find, in terms of k, the rate of increase of the liquid level when the depth of the liquid is $\frac{2k}{5}$ m. [4]

8 Given that $\frac{d^2y}{dx^2} = -9y$ and $y = a\cos^3 x + b\cos x$ where a and b are constants, $\cos x \neq 0$, show that $3a + 4b = 0$. [6]

9 The curve $\frac{1}{x} + \frac{2}{y} = \frac{1}{2}$ intersects the line $2x + y + 2 = 0$ at the points A and B. Explain why a line joining points A and B is perpendicular to the line $2y - x - 6 = 0$. [6]

10 On the same axes, sketch the graphs of

$$y = \cos x + 1 \quad \text{and} \quad y = |\tan x|,$$

for $0° \leq x \leq 360°$. [4]

Hence, for $0° \leq x \leq 360°$, state the value or range of values of k for which the equation $|\tan x| = \cos x + k$ has

(i) 2 roots, [1]

(ii) 3 roots, [1]

(iii) 4 roots. [1]

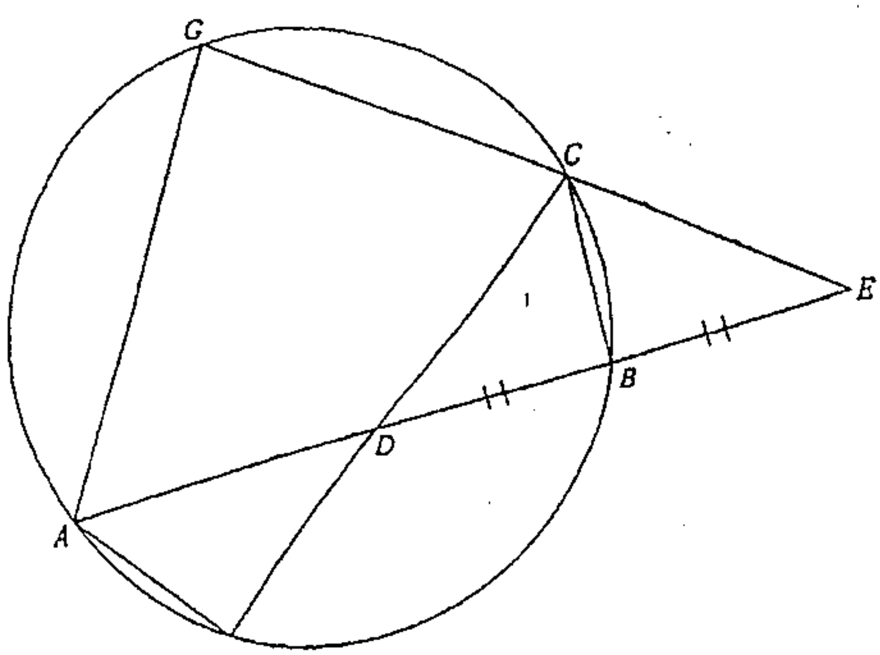
205

[Turn over

11 The diagram shows a triangle AEG which intersects the circle at points A, B, C and G. D is a point on ABE such that $BD = BE$. Show that

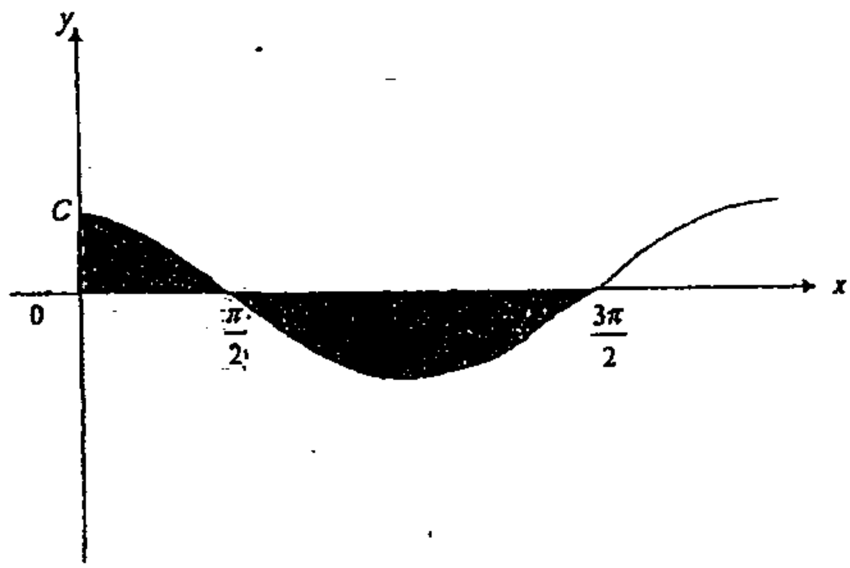
(i) triangle AEG is similar to triangle CEB, [2]

(ii) $AG \times BD = GE \times BC$. [2]



12 (a) Show that $\frac{d}{dx}\left(\sqrt{2+\sin x}\right)=\frac{\cos x}{2\sqrt{2+\sin x}}$. [2]

(b)



The diagram shows part of the curve $y=\frac{\cos x}{2\sqrt{2+\sin x}}$. The curve intersects the x-axis at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ and the y-axis at the point C.

(i) Find the coordinates of point C, in exact form. [1]

(ii) Find the area of the shaded region bounded by the curve, the y-axis and the x-axis. [4]

End of Paper

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Secc 4E/5NA Prelim Exams 2015 AM P1

Marking Scheme

1 (i)

$\sin A\cos B+\cos A\sin B=3(\sin A\cos B-\cos A\sin B)$ M1

$4\cos A\sin B=2\sin A\cos B$

$\frac{\sin A\cos B}{\cos A\sin B}=2$ A1

$\tan A=2\tan B$

(ii)

$(\sin(x+30°))^2=(3\sin(x-30°))^2$ M1

$\sin(x+30°)=\pm3\sin(x-30°)$

$\sin(x+30°)=3\sin(x-30°)$

$\tan x=2\tan 30°$

$x=49.1°$ or $229.1°$ A1

or

$\sin(x+30°)=-3\sin(x-30°)$

$\sin(30°+x)=3\sin(30°-x)$ M1

$\tan 30°=2\tan x$

$\tan x=\frac{\tan 30°}{2}$

$x=16.1°$ or $196.1°$ A1

2 (a)

$2x^2 - 2x + 1 = 0$

$\alpha + \beta = 1$

$\alpha\beta = \frac{1}{2}$ $\boxed{\text{B1}}$

sum of roots $= \frac{2}{\alpha^3} + \frac{2}{\beta^3}$

$= \frac{2(\alpha^3 + \beta^3)}{(\alpha\beta)^3}$

$= \frac{2(\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)}{(\alpha\beta)^3}$ $\boxed{\text{M1}}$

$= \frac{2(\alpha + \beta)((\alpha + \beta)^2 - 3\alpha\beta)}{(\alpha\beta)^3}$

$= \frac{2(1)(1 - \frac{3}{2})}{(\frac{1}{2})^3} = -8$ $\boxed{\text{A1}}$

product of roots $= \left(\frac{2}{\alpha^3}\right)\left(\frac{2}{\beta^3}\right)$

$= \frac{4}{\left(\frac{1}{2}\right)^3} = 32$ $\boxed{\text{A1}}$

equation is $x^2 + 8x + 32 = 0$ $\boxed{\text{A1}}$

(b) (i) $\alpha + (-\alpha) = \frac{8 + 4m}{3 + m} = 0$ $\boxed{\text{M1}}$

$8 + 4m = 0$

$m = -2$ $\boxed{\text{A1}}$

(ii) $(\alpha)\left(\frac{1}{\alpha}\right) = \frac{3 + 4m}{3 + m} = 1$ $\boxed{\text{M1}}$

$3 + 4m = 3 + m$

$m = 0$ $\boxed{\text{A1}}$

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3 (a)

$$\frac{2(4)^{\frac{1}{2}x+2}-2^{x+1}}{6^x\times 3^{1-2x}}=\frac{2(2)^{x+4}-2^{x+1}}{2^x\times 3^x\times\frac{3}{3^{2x}}}$$ M1

$$=\frac{2^x(2^5-2)}{2^x\times\frac{3}{3^x}}$$

$$=10(3^x)$$ A1 for $k=10$, A1 for $n=1$

(b)

$$\lg\frac{125}{y}+\lg y^4=\lg(by)^a$$

$$\lg(125y^3)=\lg(by)^a$$ M1

$$(5y)^3=(by)^a$$

$a=3$ and $b=5$ A2

(c)

$$\log_5 e^{2x}+\frac{1}{\log_2 5}=\log_5(2-3e^x)$$ M1 for changing base

$$\log_5 e^{2x}+\log_5 2=\log_5(2-3e^x)$$

$$\log_5 2e^{2x}=\log_5(2-3e^x)$$ M1 for using correct laws

$$2e^{2x}+3e^x-2=0$$

$$(2e^x-1)(e^x+2)=0$$ M1 for the solving equation

$e^x=\frac{1}{2}$ or $e^x=-2(NA)$ A1 for reject $e^x=-2$

$x=\ln\frac{1}{2}$ or $-\ln 2$ or -0.693 A1

4 $(x^2+2x-3)=(x+3)(x-1)$ $\boxed{B1}$

(i)

f(1) = 0
$a-b=7$..............(1) $\boxed{M1}$
$f(-3)=0$
$5a-b=27$.........(2) $\boxed{M1}$
solve (1) *and* (2)
$a=5$ *and* $b=-2$ $\boxed{A1}$

(ii)

$f(x)=x^4+6x^3+10x^2-2x-15=(x^2+2x-3)Q(x)$
$Q(x)=x^2+4x+5$ $\boxed{A1}$

M1 for long division or any correct method

M1 for correct coef of *x*.

Show that $x^2+4x+5=0$ has no real roots

using $b^2-4ac<0$. $\boxed{A1}$

Therefore f(x) has only 2 real roots

5 (i) radius = 4 units $\boxed{B1}$

Equation of the circle is $(x-4)^2+(y+6)^2=16$. $\boxed{B1}$

(ii)Area $=\int_{-2}^{0} y^2 dy+4^2-\frac{1}{4}\pi(4)^2$ $\boxed{M1}$

$=\left[\frac{y^3}{3}\right]_{-2}^{0}+16-4\pi$ A1 for $\left[\frac{y^3}{3}\right]$

$=2\frac{2}{3}+16-4\pi=6.10$ *sq units* $\boxed{B1}$

(iii) centre is (4, 10) $\boxed{B1}$

Equation is $(x-4)^2+(y-10)^2=16$

$x^2+y^2-8x-20y+100=0$ $\boxed{A1}$

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[Turn over

6 (i) $\frac{dy}{dx} = 4x + (x+2)^{-2}$

$y = 2x^2 - \frac{1}{x+2} + C$ M1

Subst. $\left(\frac{1}{2}, \frac{1}{2}\right)$, $C = \frac{2}{5}$ A1

Equation of the curve is $y = 2x^2 - \frac{1}{x+2} + \frac{2}{5}$. A1

(ii) at $x = \frac{1}{2}$, $\frac{dy}{dx} = \frac{54}{25}$

Gradient of normal $= -\frac{25}{54}$ M1

Equation of normal is $y = -\frac{25}{54}x + \frac{79}{108}$ or $108y = -50x + 79$. A1

7 $V = \frac{1}{3}\pi h^2(3k - h) = \pi k h^2 - \frac{1}{3}\pi h^3$

$\frac{dV}{dh} = 2\pi k h - \pi h^2$ B1

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ M1

$\frac{dh}{dt} = \frac{k}{2\pi k(\frac{2k}{5}) - \pi(\frac{2k}{5})^2}$ M1

$= \frac{25}{16\pi k}$ m/s A1

[Turn over

8

$y = a\cos^3 x + b\cos x$

$\dfrac{dy}{dx} = -3a\cos^2 x\sin x - b\sin x$ M1 for $\dfrac{d}{dx}(a\cos^3 x)$

$\dfrac{d^2 y}{dx^2} = -3a\cos^3 x + 6a\cos x\sin^2 x - b\cos x$ M1 for $\dfrac{d}{dx}(-3a\cos^2 x\sin x)$

$= -3a\cos^3 x + 6a\cos x(1-\cos^2 x) - b\cos x$

$= -3a\cos^3 x + 6a\cos x - 6a\cos^3 x - b\cos x$ M1 for using identity

$= -9a\cos^3 x + (6a - b)\cos x$ A1

$\dfrac{d^2 y}{dx^2} = -9y = -9a\cos^3 x - 9b\cos x$

$(6a - b)\cos x = -9b\cos x$ M1

since $\cos x \neq 0$

$\therefore 6a - b = -9b$

$6a + 8b = 0$ A1

$3a + 4b = 0$

9

$\dfrac{1}{x} + \dfrac{2}{y} = \dfrac{1}{2}$

$2y + 4x = xy$

Subst. $y = -2x - 2$ into above equation M1

$2(-2x-2) + 4x = x(-2x-2)$

$2x^2 + 2x - 4 = 0$

$x^2 + x - 2 = 0$ M1 for forming quad eqn

$(x+2)(x-1) = 0$

$x = -2 \quad or \quad x = 1$ A1 for (-2, 2)

$y = 2 \quad or \quad y = -4$ A1 for (1, -4)

Gradient of the line joining points A and $B = \dfrac{-4-2}{1+2} = -2$ M1

gradient of the line $2y - x - 6 = 0 = \dfrac{1}{2}$

The product of the 2 gradient $= -1$ A1

Or the gradient of one of the line is equal to $-\dfrac{1}{\text{the gradient of the other line}}$

The lines are perpendicular.

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[Turn over

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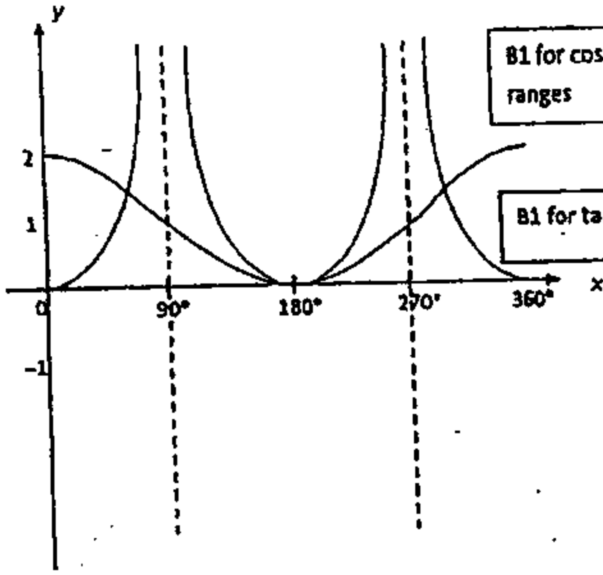

B1 for cosx and B1 for correct ranges

B1 for tanx and B1 for absolute tanx

(i) $-1 \le k < 1$ $\boxed{\text{B1}}$

(ii) $k = 1$ $\boxed{\text{B1}}$

(iii) $k > 1$ $\boxed{\text{B1}}$

11 (i)

$\angle GAE = \angle BCE$ (ext $\angle$, cylic quad) $\boxed{\text{M1 for both conditions}}$

$\angle AEG = \angle CEB$ (common angles)

$\therefore \Delta AEG$ is similar to ΔCEB (AA, similarity) $\boxed{\text{A1 for coding the right test used}}$

(shown)

(ii)

$\frac{AG}{BC} = \frac{GE}{BE}$(from part (i)) $\boxed{\text{M1 for writing the ratios of sides}}$

$\because BD = BE$(given)

$\frac{AG}{BC} = \frac{GE}{BD}$ $\boxed{\text{A1 for using the condition } BD=BE}$

$AG \times BD = GE \times BC$ (shown)

12 (a) $y=(2+\sin x)^{\frac{1}{2}}$

$\frac{dy}{dx}=\frac{1}{2}(2+\sin x)^{-\frac{1}{2}}(\cos x)$ $\boxed{\text{B1B1}}$

$=\frac{\cos x}{2\sqrt{2+\sin x}}$

(b) (i) when $x=0$, $y=\frac{\cos 0}{2\sqrt{2+\sin 0}}=\frac{1}{2\sqrt{2}}$

Point C is $(0,\frac{1}{2\sqrt{2}})$ or $(0,\frac{\sqrt{2}}{4})$ $\boxed{\text{B1}}$

(ii) Area $=\int_0^{\frac{\pi}{2}}\frac{\cos x}{2\sqrt{2+\sin x}}dx+\left|\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\frac{\cos x}{2\sqrt{2+\sin x}}dx\right|$ $\boxed{\text{M1}}$

$=\left[\sqrt{2+\sin x}\right]_0^{\frac{\pi}{2}}+\left|\left[\sqrt{2+\sin x}\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\right|$ $\boxed{\text{A1A1 for integrate correct using (a)}}$

$=\left[\sqrt{3}-\sqrt{2}\right]+\left|\sqrt{2+\sin\frac{3\pi}{2}}-\sqrt{2+\sin\frac{\pi}{2}}\right|$

$=\sqrt{3}-\sqrt{2}+\left|\sqrt{2+(-1)}-\sqrt{3}\right|$

$=1.05$ sq units $\boxed{\text{A1}}$

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TEMASEK SECONDARY SCHOOL
O Level Preliminary Examinations 2015

ADDITIONAL MATHEMATICS 4047/02

Paper 2 2 hour 30 minutes

Question Booklet

Additional Material: Writing paper (8 sheets), Cover page (1 sheet), Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Do not open the booklet until you are told to do so.

You are not required to submit this booklet at the end of the paper.

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

This document consists of 7 printed pages and 1 blank page.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$
$$\sec^2 A=1+\tan^2 A$$
$$\operatorname{cosec}^2 A=1+\cot^2 A$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$
$$a^2=b^2+c^2-2bc\cos A$$
$$\Delta=\frac{1}{2}ab\sin C$$

Answer all the questions.

1 Without using a calculator, find the value of a and of b for which $\dfrac{a\sqrt{14}+b}{335}$ is the solution of the equation $3x\sqrt{2}+x\sqrt{343}=x\sqrt{50}+\sqrt{8}$. [4]

2 (a) Show that the expression $x^2+px-x+p^2+2$, where p is a constant, is always positive for all real values of x.

Hence, find the range of values of x for which $\dfrac{x^2-3x-28}{x^2+px-x+p^2+2}<0$. [7]

(b) Find the range of values of k for which the line $y=2x-k$ cuts the curve $y^2=x+k$ at two different points. [4]

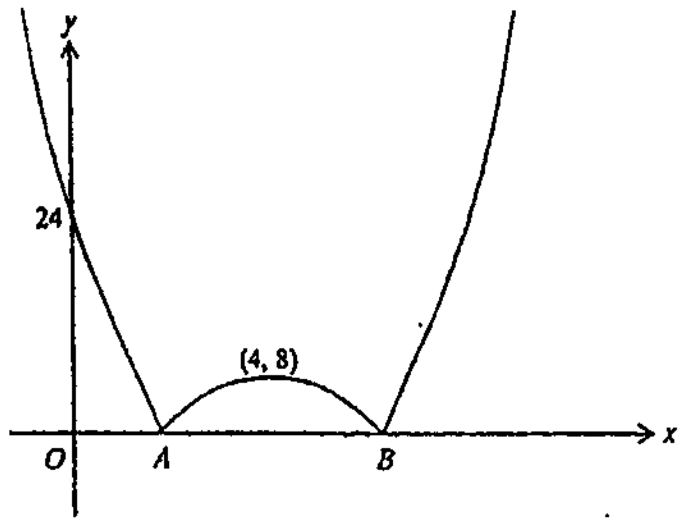
3 (a) Given that the ratio of the coefficients of x^3 and x^9 in the expansion of $\left(x^2-\dfrac{k}{x}\right)^{12}$ is 1 : 4, find the possible values of k. [5]

(b) The first three terms in the expansion of $(2x-3)\left(1+\dfrac{x}{3}\right)^n$, in ascending powers of x, are $p+qx-\dfrac{7}{3}x^2$. Find the values of n, p and q. [5]

4 (a) Express $\dfrac{2x^3-5x^2+11x-3}{(x^2+1)(x-2)}$ in partial fractions. [5]

(b) Prove the identity $\dfrac{\sin x+\cos x}{\sin x-\cos x}-\dfrac{\sin x-\cos x}{\sin x+\cos x}\equiv-2\tan 2x$. [3]

5 The diagram shows part of the curve $y = \left|p(x-r)^2 + q\right|$, where p, q and r are constants and $p > 0$. The curve cuts the y-axis at 24 and (4, 8) is the turning point of the curve.



(i) Find the values of p, q and r. [3]

(ii) Find the coordinates of A and of B. [3]

(iii) Write down, with explanations, the number of solution(s) to the equation $\left|p(x-r)^2 + q\right| = h|x-k|$ for $3 < k < 5$ and

(a) $0 < h < 1$, [2]

(b) $h < 0$. [2]

6 A car P moves in a straight line such that, t seconds after the start of motion, its velocity, v m s^{-1}, is given by $v = t - \dfrac{5}{2t+3}$.

The initial displacement of P is $\left(1 - \dfrac{5}{2}\ln 3\right)$m.

(i) Find the value of t when P is at instantaneous rest. [3]

(ii) Find an expression, in terms of t, for the acceleration of P and determine whether P can attain maximum velocity. [2]

(iii) Find the average speed of P for the first 2 seconds. [4]

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7 **Answer the whole of this question on a piece of graph paper.**

The table shows experimental values of two variables, x and y, which are connected by an equation of the form $y\sqrt{x} = k(\sqrt{x})^3 + nx$, where k and n are constants.

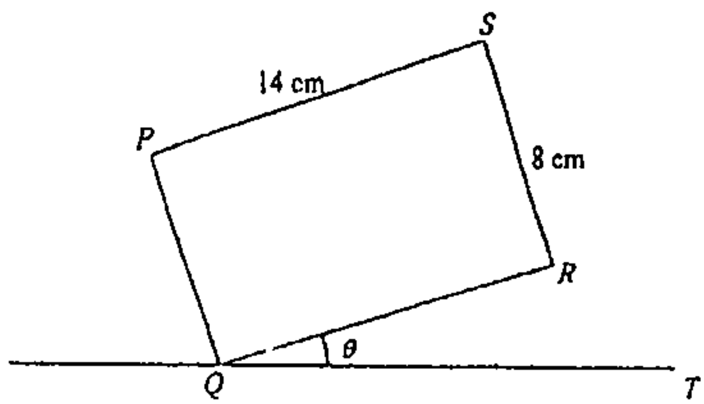
x	1	2	3	4	5
y	3.00	4.53	5.83	7.00	8.09

(i) Using graph paper, plot $\frac{y}{x}$ against $\frac{1}{\sqrt{x}}$ and use your graph to estimate the value of k and of n. [6]

(ii) Use your graph to estimate the value of y when $x = 3.40$. [2]

(iii) By drawing a suitable line on your graph, find the solution to the simultaneous equations $y\sqrt{x} = k(\sqrt{x})^3 + nx$ and $y = \sqrt{x} + x$. [3]

8



The diagram shows a rectangle, $PQRS$, where the QR makes an angle θ with a horizontal line QT.

Given that $PS = 14$ cm, $SR = 8$ cm and $0° < \theta < 90°$, show that the perpendicular distance, H cm, from S to the line QT is given by $H = 8\cos\theta + 14\sin\theta$. [2]

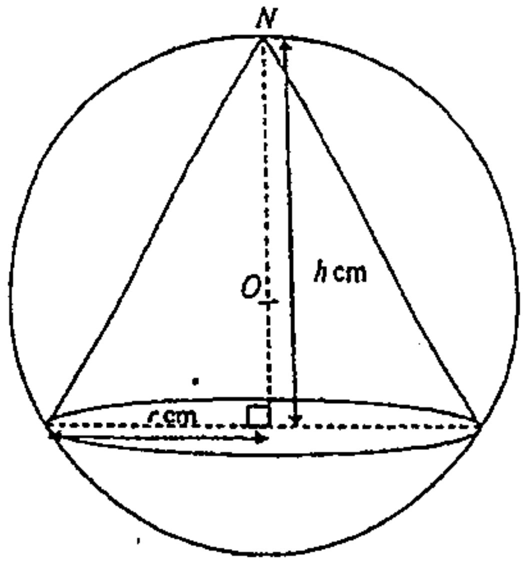
(i) Express H in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(ii) Find the maximum value of H and the corresponding value of θ. [2]

(iii) Find the value of θ when $H = 12$. [2]

[Turn over

9



The diagram shows a right circular cone in a sphere with centre O and radius 40 cm. The vertex of the cone, N, and the circumference of its base lies on the sphere and the centre of the sphere is on the axis of the cone.

(i) Given that the radius, height and volume of the right circular cone are r cm, h cm and V cm^3 respectively, show that $V = \frac{\pi}{3}\left(80h^2 - h^3\right)$. [3]

(ii) Find the stationary value of V and show that this value is a maximum. [5]

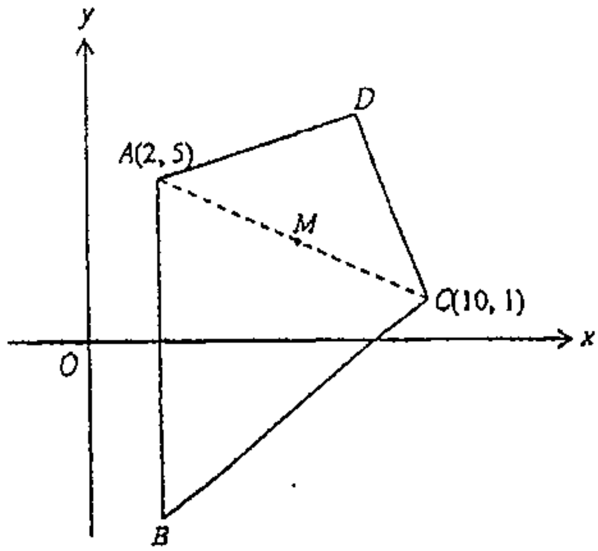
10 (a) Find the range of values of x for which the curve $y = x^3 e^{1-2x}$ is a decreasing function. [4]

(b) Given that $y = \left[\ln(3-4x)\right]^2$, show that $\frac{dy}{dx} = \frac{k\ln(3-4x)}{3-4x}$, where k is a constant to be determined.

Hence, evaluate $\int_{-2}^{-1} \frac{2+3\ln(3-4x)}{3-4x}\,dx$. [7]

Z13

11 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a kite *ABCD*. *M* is the midpoint of *AC* and the coordinates of *A* and *C* are (2, 5) and (10, 1) respectively.

(i) Find the coordinates of *M*. [1]

(ii) Find the equation of *BD*. [2]

(iii) Given that *B* lies on the line $3x+2y+4=0$, find the coordinates of *B*. [2]

(iv) Given that $\frac{BD}{MD}=3$, find the coordinates of *D*. [2]

(v) Find the area of the kite *ABCD*. [2]

End of Paper

[Turn over

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Prelims 2015 Sec 4E/5N –Additional Mathematics Paper 2
Marking Scheme

Qn	Solutions and Marks Allocation
1	$3x\sqrt{2}+x\sqrt{343}=x\sqrt{50}+\sqrt{8}$ $7x\sqrt{7}-2x\sqrt{2}=2\sqrt{2}$ M1 $x=\dfrac{2\sqrt{2}}{7\sqrt{7}-2\sqrt{2}}\times\dfrac{7\sqrt{7}+2\sqrt{2}}{7\sqrt{7}+2\sqrt{2}}$ M1 – *rationalise* $=\dfrac{14\sqrt{14}+8}{335}$ A1 $a=14$ and $b=8$ A1
2(a)	$x^2+px-x+p^2+2$ $=x^2+(p-1)x+p^2+2$ $b^2-4ac=(p-1)^2-4(1)(p^2+2)$ M1 $=-3p^2-2p-7$ $=-3\left(p+\frac{1}{3}\right)^2-6\frac{2}{3}$ A1 Since $\left(p+\frac{1}{3}\right)^2\geq 0$, M1 $b^2-4ac<0$ Since <u>the coefficient of x^2 is positive and the discriminant is less than zero,</u> $x^2+px-x+p^2+2$ is always positive for all real values of x. A1 $x^2-3x-28<0$ M1 $(x-7)(x+4)<0$ M1 $-4<x<7$ A1
2(b)	$y^2=x+k$ ------(1) $y=2x-k$ ------(2) Sub (2) into (1): $(2x-k)^2=x+k$ M1 $4x^2-(4k+1)x+k^2-k=0$ A1 $b^2-4ac>0$ $(4k+1)^2-4(4)(k^2-k)>0$ M1 $24k+1>0$ $k>-\dfrac{1}{24}$ A1

3(a)	$\left(x^2-\frac{k}{x}\right)^{12}$ $T_{r+1}=\binom{12}{r}(-k)^r x^{24-3r}$ A1 for x^{24-3r} Let $24-3r=3$, $r=7$ Coefficient of $x^3=\binom{12}{7}(-k)^7=-792k^7$ A1 Let $24-3r=9$ $r=5$ Coefficient of $x^9=\binom{12}{5}(-k)^5=-792k^5$ A1 $\frac{-792k^7}{-792k^5}=\frac{1}{4}$ M1 $k=\pm\frac{1}{2}$ A1
3(b)	$(2x-3)\left(1+\frac{x}{3}\right)^n$ $=(2x-3)\left(1+\frac{n}{3}x+\frac{n(n-1)}{18}x^2+...\right)$ A1 $=p+qx-\frac{7}{3}x^2+...$ Comparing the constant term: $p=-3$ A1 Comparing the coefficient of x: $q=2-n$ Comparing the coefficient of x^2: $-\frac{7}{3}=\frac{2}{3}n-\frac{n(n-1)}{6}$ M1 $n^2-5n-14=0$ $(n-7)(n+2)=0$ $n=-2$ or $n=7$ A1 (reject) $\therefore q=-5$ A1

Z15

4(a)	$\frac{2x^3-5x^2+11x-3}{(x^2+1)(x-2)}=A+\frac{Bx+C}{x^2+1}+\frac{D}{x-2}$ M1 $2x^3-5x^2+11x-3=A(x^2+1)(x-2)+(Bx+C)(x-2)+D(x^2+1)$ Comparing the coefficient of x^3: $A=2$ Let $x=2$, $D=3$ $\boxed{\text{A3A2A1A0}}$ Let $x=0$, $C=1$ Let $x=1$, $B=-4$ $\frac{2x^3-5x^2+11x-3}{(x^2+1)(x-2)}=2-\frac{4x-1}{x^2+1}+\frac{3}{x-2}$ A1
4(b)	$LHS=\frac{\sin x+\cos x}{\sin x-\cos x}-\frac{\sin x-\cos x}{\sin x+\cos x}$ $=\frac{(\sin x+\cos x)^2-(\sin x-\cos x)^2}{\sin^2 x-\cos^2 x}$ M1 $=\frac{4\sin x\cos x}{\sin^2 x-\cos^2 x}$ $=\frac{2\sin 2x}{-\cos 2x}$ A1 for $2\sin 2x$, A1 for $-\cos 2x$ $=-2\tan 2x$
5(i)	$r=4$ B1 $q=-8$ B1 Let $y=24$ when $x=0$, $24=\|16p-8\|$ $16p-8=24$ or $16p-8=-24$ $p=2$ or $p=-1$ (reject) B1
5(ii)	Let $y=0$, $2(x-4)^2-8=0$ M1 $(x-4)^2=4$ $x-4=\pm 2$ $x=2$ or 6 A1 $A(2,0)$ and $B(6,0)$ A1
5(iii) (a)	The graph of $y=h\|x-k\|$ is V-shaped and the vertex is located between $x=3$ and $x=5$ on the x-axis, thus there will be 4 solutions. B1B1
5(iii) (b)	The graph of $y=h\|x-k\|$ is inverted V-shaped and the vertex is located between $x=3$ and $x=5$ on the x-axis, thus there will be no solution. B1B1

6(i)	Let $v=0$, $t=\frac{5}{2t+3}$ M1 $2t^2+3t-5=0$ M1 $(2t+5)(t-1)=0$ $t=-\frac{5}{2}$ or 1 A1 (reject)
6(ii)	$a=1+\frac{10}{(2t+3)^2}\neq 0$ A1 Since a cannot have a value of zero, P cannot attain maximum velocity. A1
6(iii)	$s=\frac{t^2}{2}-\frac{5}{2}\ln(2t+3)+c$, where c is a constant. M1 Let $s=1-\frac{5}{2}\ln 3$ when $t=0$. $1-\frac{5}{2}\ln 3=-\frac{5}{2}\ln 3+c$ $c=1$ $s=\frac{t^2}{2}-\frac{5}{2}\ln(2t+3)+1$ A1 When $t=1$, $s=\frac{3}{2}-\frac{5}{2}\ln 5=-2.52359$ When $t=2$, $s=3-\frac{5}{2}\ln 7=-1.864775$ When $t=0$, $s=1-\frac{5}{2}\ln 3=-1.74653$ Average speed $=\frac{(2.52359-1.74653)+(2.52359-1.864775)}{2}=0.718$ m/s M1A1

2·6

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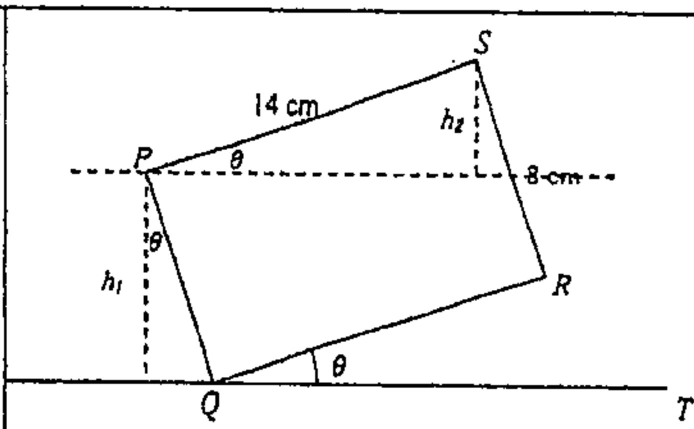
<tr><td>8</td><td>


From the diagram, $h_1 = 8\cos\theta$ and $h_2 = 14\sin\theta$, B1 each (working needed)

$H = 8\cos\theta + 14\sin\theta$
</td></tr>
<tr><td>(i)</td><td>
$R = \sqrt{260} = 2\sqrt{65}$ A1
$\alpha = 60.26°$ A1
$H = 2\sqrt{65}\cos(\theta - 60.26°)$ A1
</td></tr>
<tr><td>(ii)</td><td>
Max value of $H = 2\sqrt{65}$ B1
$\theta = 60.3°$ B1
</td></tr>
<tr><td>(iii)</td><td>
When $H = 12$,
$2\sqrt{65}\cos(\theta - 60.26°) = 12$
$\cos(\theta - 60.26°) = \frac{6}{\sqrt{65}}$ M1
$\theta - 60.26° = -41.91°$
$\theta = 18.4°$ A1
</td></tr>
</table>

9(i)	$(h-40)^2+r^2=40^2$ M1 $r^2=80h-h^2$ A1 $V=\frac{1}{3}\pi r^2 h$ $=\frac{\pi}{3}(80h-h^2)h$ A1 $=\frac{\pi}{3}(80h^2-h^3)$
9(ii)	$\frac{dV}{dh}=\frac{\pi}{3}(160h-3h^2)$ M1 Let $\frac{dV}{dh}=0$, $\frac{\pi}{3}(160h-3h^2)=0$ $h=0$ or $h=53\frac{1}{3}$ A1 (reject) When $h=53\frac{1}{3}$, $V=79431.87\approx 79400$ (3sf) A1 $\frac{d^2V}{dh^2}=\frac{\pi}{3}(160-6h)$ M1 When $h=53\frac{1}{3}$, $\frac{d^2V}{dh^2}=-\frac{160}{3}\pi<0$ The stationary value of V is maximum. A1 [proof is needed]

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10(a)	$y = x^3 e^{1-2x}$ $\frac{dy}{dx} = x^3 e^{1-2x}(-2) + 3x^2 e^{1-2x}$ A2 $= x^2 e^{1-2x}(3-2x)$ Let $\frac{dy}{dx} < 0$, $x^2 e^{1-2x}(3-2x) < 0$ M1 $3 - 2x < 0$ $x > 1\frac{1}{2}$ A1
(b)	$y = [\ln(3-4x)]^2$ $\frac{dy}{dx} = 2\ln(3-4x)\left(\frac{-4}{3-4x}\right)$ M1 $= \frac{-8\ln(3-4x)}{3-4x}$ $\therefore k = -8$ A1 $\int_{-2}^{-1} \frac{2+3\ln(3-4x)}{3-4x}\,dx$ $= \int_{-2}^{-1} \frac{2}{3-4x}\,dx - \frac{3}{8}\int_{-2}^{-1} \frac{-8\ln(3-4x)}{3-4x}\,dx$ M1 $= -\frac{1}{2}[\ln(3-4x)]_{-2}^{-1} - \frac{3}{8}\left[(\ln(3-4x))^2\right]_{-2}^{-1}$ B2 $= -\frac{1}{2}(\ln 7 - \ln 11) - \frac{3}{8}\left[(\ln 7)^2 - (\ln 11)^2\right]$ M1 $= 0.22599 + 0.73625$ $= 0.962 (3sf)$ A1

11(i)	$M(6, 3)$ A1
(ii)	$m_{AC} = -\frac{1}{2}$ $m_{BD} = 2$ M1 Equation of BD: $y-3 = 2(x-6)$ $y = 2x-9$ -------(1) A1
(iii)	$3x + 2y + 4 = 0$ -----(2) Solving (1) and (2): M1 $x = 2$ and $y = -5$ $B(2,-5)$ A1
(iv)	$\frac{D_x-2}{D_x-6} = 3$ $D_x = 8$ $\frac{D_y+5}{D_y-3} = 3$ $D_y = 7$ $D(8,7)$ M1 A1
(v)	Area of kite $ABCD$ $= \frac{1}{2}\begin{vmatrix} 2 & 2 & 10 & 8 & 2 \\ 5 & -5 & 1 & 7 & 5 \end{vmatrix}$ M1 $= \frac{1}{2}(102+18)$ $= 60$ sq.units A1

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Name	Class	Register Number
MARKING SCHEME		

4047/01

15/S4PR2/AM/1

ADDITIONAL MATHEMATICS

PAPER 1

Wednesday

5 August 2015

2 hours

VICTORIA SCHOOL

PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

Additional Material: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

2'9

This paper consists of 5 printed pages, including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} ab \sin C$$

55

1 Find the range of the values of x which satisfy both inequalities
$0 < x^2 - 4x$ and $x^2 - 4x \leq 3x + 10$. [4]

2 Solve

(i) $\dfrac{3^{2-x}}{9^x} = \dfrac{1}{\sqrt{27^x}}$, [2]

(ii) $3e^x - e = 2e^{2-x}$. [3]

3 (i) Find the coefficient of the term in x in the expansion of $\left(x^2 - \dfrac{1}{2x^3}\right)^8$. [3]

(ii) The coefficient of x^2 in the expansion $(5-3x)(1+5x)^n$ is 1785. Find the value of n. [4]

4 The gradient to a curve is given by $\dfrac{dy}{dx} = (kx+3)^2$, where k is a non-zero constant. The equation of the tangent to the curve at the point $(1, 2)$ is $9x - y - 5 = 0$. Find the

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(ii) A cuboid with a square base of length $\sqrt{3} + 1$ cm, has a volume of $\left(5\sqrt{2}\right)^2 - 8\sqrt{3}$ cm^3. Find the height of the cuboid in the form $a + b\sqrt{3}$. [4]

220

7 A curve has the equation $y = xe^{4x}$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) Hence show that $\int_0^{\ln 2} 4xe^{4x}dx = 16\ln 2 - 3\frac{3}{4}$. [4]

(iii) Find the range of values of x for which the function $y = xe^{4x}$ is decreasing. [2]

8 AB is a chord of the circle $x^2 + y^2 - 8x - 2y - 3 = 0$ and $M\left(\frac{4}{5}, 2\frac{2}{5}\right)$ is the midpoint of chord AB. Find the

(i) radius and the coordinates of the centre of the circle, [2]

(ii) equation of chord AB. [3]

If P is a variable point on the circle, find the

(iii) maximum area of triangle ABP. [4]

9 The function f is defined, for $0 \le x \le 2\pi$, by $f(x) = 2\cos ax + b$, where a and b are integers. The minimum value of f is -1 and the period of f is $\frac{4\pi}{3}$.

(i) State the amplitude of f. [1]

(ii) State the values of a and of b. [1]

(iii) Using the values of a and b found in part (ii),

(a) solve $f(x) = 0$ for $0 \le x \le 2\pi$, leaving your answers in terms of π, [4]

(b) sketch the graph of $f(x) = 2\cos ax + b$ for $0 \le x \le 2\pi$. [3]

10 A particle moves in a straight line such that t seconds after leaving a fixed point O, the velocity v m/s, is given by $v = 3t^2 - t - 10$. Find the

(i) initial acceleration of the particle, [2]

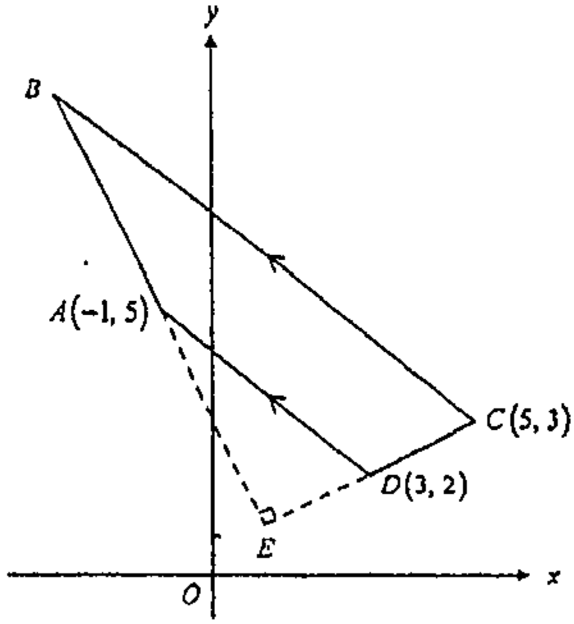
(ii) minimum velocity of the particle, [2]

(iii) total distance travelled by the particle in the first 3 seconds, [4]

(iv) average speed of the particle during the first 3 seconds. [2]

11 **Solutions to this question by accurate drawing will not be accepted.**

The diagram shows the trapezium $ABCD$ in which BC is parallel to AD while BA produced is perpendicular to CD produce at point E. The point A is $(-1, 5)$, C is $(5, 3)$ and D is $(3, 2)$.



(i) Show that the coordinates of B are $(-3, 9)$. [6]

(ii) Find the area of trapezium $ABCD$. [2]

(iii) Given that $\frac{\text{area of } \Delta AED}{\text{area of } \Delta BEC} = \frac{1}{4}$, find the coordinates of E. [3]

End of Paper

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Answer Key

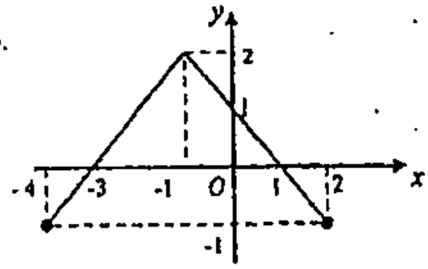
1. $-1.22 \le x < 0$ or $4 < x \le 8.22$

2(i) $1\frac{1}{3}$ 2(ii) $x = 1$

3(i) Coefficient of $x = -7$ 3(ii) $n = 6$

4(i) $k = -6$ 4(ii) $y = \frac{1}{2} - \frac{3(1-2x)^3}{2}$

5.



5(i) $-1 \le p \le 2$

5(ii) There are infinite number of solutions.

6(i) $x = \frac{2\sqrt{7}}{3}$ 6(ii) height $= 62 - 33\sqrt{3}$

7(i) $\frac{dy}{dx} = e^{4x}(4x+1)$ 7(iii) $x < -\frac{1}{4}$

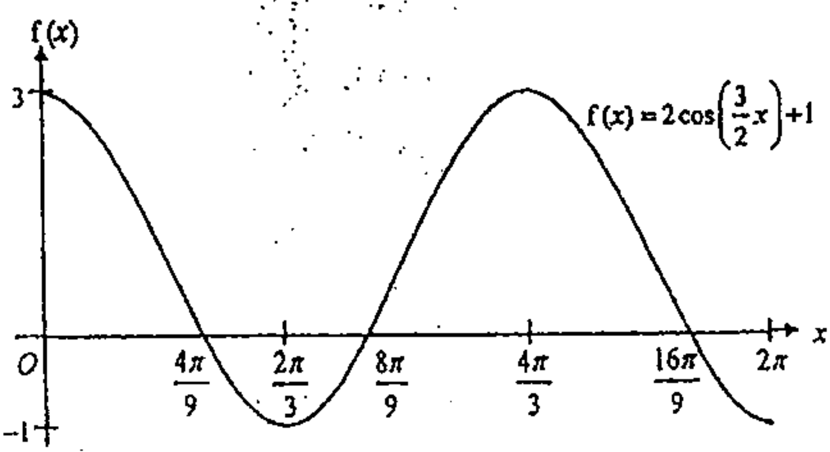
8(i) centre of circle is $(4, 1)$
(ii) radius $= 4.47$ units
(iii) Max area $= 22.2$ units^2

9(i) Amplitude $= 2$, (ii) $a = 1.5$, $b = 1$.
(iii) (a) $x = \frac{4\pi}{9}, \frac{8\pi}{9}, \frac{16\pi}{9}$

10(i) -1 m/s^2
(ii) min velocity $= -10\frac{1}{12}$ m/s
(iii) Total Distance $= 20.5$ m
(iv) Ave Speed $= 6\frac{5}{6}$ or 6.83 m/s

11(ii) Area $= 15$ units^2
(iii) $E(1,1)$

9(iii)(b)



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	Class	Register Number
Name ______________________		

4047/01

15/S4PR2/AM/1

ADDITIONAL MATHEMATICS

PAPER 1

Wednesday　　5 August 2015　　2 hours

VICTORIA SCHOOL

PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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53

1 Find the range of the values of x which satisfy both inequalities $0 < x^2 - 4x$ and $x^2 - 4x \le 3x + 10$. [4]

$0 < x^2 - 4x$ and $x^2 - 4x \le 3x + 10$

$x^2 - 4x > 0$ $\quad$ $x^2 - 7x - 10 \le 0$

$x(x-4) > 0$ $\quad$ for $x^2 - 7x - 10 = 0$

$x < 0$ or $x > 4$ $\quad$ $x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-10)}}{2(1)}$

$= -1.22$ or 8.22

$\therefore\ x^2 - 7x - 10 \le 0$

$-1.22 \le x \le 8.22$

Hence the solution is $-1.22 \le x < 0$ or $4 < x \le 8.22$

2 Solve

(i) $\dfrac{3^{2-x}}{9^x} = \dfrac{1}{\sqrt{27^x}}$, [2]

$\dfrac{3^{2-x}}{9^x} = \dfrac{1}{\sqrt{27^x}}$

$\dfrac{3^{2-x}}{3^{2x}} = \dfrac{1}{3^{\frac{3x}{2}}}$

$2 - x - 2x = -\dfrac{3x}{2}$

$2 = \dfrac{3}{2}x$

$x = \dfrac{4}{3} = 1\dfrac{1}{3}$

(ii) $3e^x - e = 2e^{2-x}$. [3]

$3e^x - e = 2e^{2-x}$

$3e^x - e = \dfrac{2e^2}{e^x}$

$3(e^x)^2 - e \cdot e^x - 2e^2 = 0$

$(e^x - e)(3e^x + 2e) = 0$

$e^x = e$ or $3e^x = -2e$

$x = 1$ $\quad$ (NA)

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3 (i) Find the coefficient of the term in x in the expansion of $\left(x^2 - \frac{1}{2x^3}\right)^8$. [3]

For $\left(x^2 - \frac{1}{2x^3}\right)^8$,

$$T_{r+1} = \binom{8}{r}(x^2)^{8-r}\left(-\frac{1}{2x^3}\right)^r$$

$$= \binom{8}{r}\left(\frac{-1}{2}\right)^r x^{16-2r}x^{-3r}$$

$$= \binom{8}{r}\left(\frac{-1}{2}\right)^r x^{16-5r}$$

For term in x, $16-5r=1$

$5r=15$

$r=3$

Coefficient of $x = \binom{8}{3}\left(\frac{-1}{2}\right)^3$

$= -7$

(ii) The coefficient of x^2 in the expansion $(5-3x)(1+5x)^n$ is 1785. Find the value of n. [4]

$(5-3x)(1+5x)^n$

$= (5-3x)\left(1+\binom{n}{1}(5x)+\binom{n}{2}(5x)^2+\ldots\right)$

$= (5-3x)\left(1+5nx+\frac{n(n-1)}{2}\times 25x^2+\ldots\right)$

coefficient of x^2 in the above expansion $= 1785$

$125\times\frac{n(n-1)}{2}-3(5n)=1785$

$125n(n-1)-30n=3570$

$125n^2-125n-30n-3570=0$

$125n^2-155n-3570=0$

$25n^2-31n-714=0$

$(n-6)(25n+119)=0$

$n-6=0$ or $25n+119=0$

$n=6$ or $n=-\frac{119}{25}$ (N.A)

4 The gradient to a curve is given by $\frac{dy}{dx}=(kx+3)^2$, where k is a non-zero constant. The equation of the tangent to the curve at the point $(1,2)$ is $9x-y-5=0$. Find the

(i) value of k, [2]

$9x-y-5=0$
$y=9x-5$
Gradient of tangent $=9$
At $(1,2)$, $\frac{dy}{dx}=9$
$(k+3)^2=9$
$k+3=3$ or $k+3=-3$
$k=0$ (N.A) or $k=-6$

(ii) equation of the curve. [2]

Equation of curve is,

$$y=\int(-6x+3)^2\,dx$$
$$=\frac{(-6x+3)^3}{3(-6)}+c$$
$$=\frac{(3-6x)^3}{-18}+c$$

At $(1,2)$,

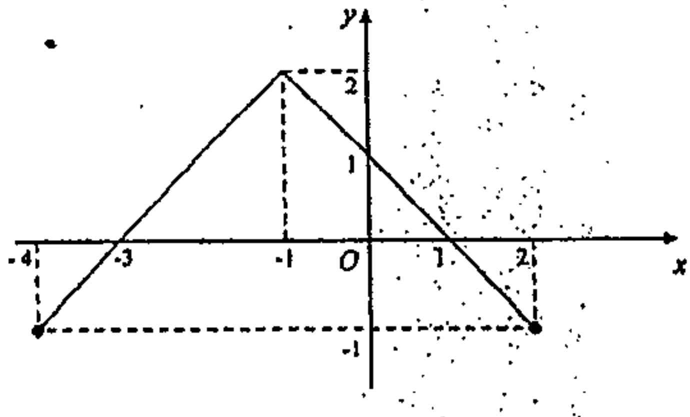
$$2=\frac{(3-6)^3}{-18}+c$$
$$c=\frac{1}{2}$$

$\therefore$ equation of curve is,

$$y=\frac{(3-6x)^3}{-18}+\frac{1}{2}$$
$$y=\frac{1}{2}-\frac{3(1-2x)^3}{2}$$

224

5 Sketch the graph of $y=-|x+1|+2$ for $-4 \le x \le 2$. [3]



(i) State the range of values of p for which the equation $-|x+1|=p-2$ has at least 1 solutions for $-4 \le x \le 2$. [1]

$-1 \le p \le 2$

(ii) Using your graph, state the number of solutions for $-|x+1|+2=x+3$ [1]

There are infinite number of solutions.

56

6 (i) Find the exact value of x in the equation $\sqrt{112}x+5=\sqrt{7}x+19$. [4]

$$\sqrt{112}x+5=\sqrt{7}x+19$$
$$4\sqrt{7}x-\sqrt{7}x=14$$
$$3\sqrt{7}x=14$$
$$x=\frac{14}{3\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}$$
$$=\frac{14\sqrt{7}}{21}$$
$$=\frac{2\sqrt{7}}{3}$$

$$\sqrt{112}x+5=\sqrt{7}x+19$$
$$\sqrt{112}x-\sqrt{7}x=14$$
$$\left(\sqrt{112}-\sqrt{7}\right)x=14$$
$$x=\frac{14}{\sqrt{112}-\sqrt{7}}\times\frac{\sqrt{112}+\sqrt{7}}{\sqrt{112}+\sqrt{7}}$$
$$=\frac{14\cdot4\sqrt{7}+14\sqrt{7}}{112-7}$$
$$=\frac{56\sqrt{7}+14\sqrt{7}}{105}$$
$$=\frac{70\sqrt{7}}{105}$$
$$=\frac{2\sqrt{7}}{3}$$

(ii) A cuboid with a square base of length $\sqrt{3}+1$ cm, has a volume of $\left(5\sqrt{2}\right)^2-8\sqrt{3}$ cm³. Find the height of the cuboid in the form $a+b\sqrt{3}$. [4]

$$\text{Height}=\frac{\left(5\sqrt{2}\right)^2-8\sqrt{3}}{\left(\sqrt{3}+1\right)^2}$$
$$=\frac{25(2)-8\sqrt{3}}{4+2\sqrt{3}}\times\frac{4-2\sqrt{3}}{4-2\sqrt{3}}$$
$$=\frac{200-100\sqrt{3}-32\sqrt{3}+48}{16-12}$$
$$=\frac{248-132\sqrt{3}}{4}$$
$$=62-33\sqrt{3}$$

7 A curve has the equation $y = xe^{4x}$.

(i) Find $\frac{dy}{dx}$. [2]

$y = xe^{4x}$

$\frac{dy}{dx} = x4e^{4x} + e^{4x}(1)$

$= e^{4x}(4x+1)$

(ii) Hence show that $\int_0^{\ln 2} 4xe^{4x} dx = 16\ln 2 - 3\frac{3}{4}$. [4]

$\int_0^{\ln 2} e^{4x}(4x+1)\, dx = \left[xe^{4x}\right]_0^{\ln 2}$

$\int_0^{\ln 2} 4xe^{4x} + e^{4x}\, dx = \ln 2 \times e^{4\ln 2} - 0$

$\int_0^{\ln 2} 4xe^{4x}\, dx + \int_0^{\ln 2} e^{4x}\, dx = \ln 2 \times e^{\ln 16}$

$\int_0^{\ln 2} 4xe^{4x}\, dx = \ln 2 \times 16 - \int_0^{\ln 2} e^{4x}\, dx$

$= 16\ln 2 - \int_0^{\ln 2} e^{4x}\, dx$

$= 16\ln 2 - \left[\frac{e^{4x}}{4}\right]_0^{\ln 2}$

$= 16\ln 2 - \frac{1}{4}\left(e^{4\ln 2} - e^{0}\right)$

$= 16\ln 2 - \frac{1}{4}\left(e^{\ln 16} - 1\right)$

$= 16\ln 2 - \frac{1}{4}(16-1)$

$= 16\ln 2 - 3\frac{3}{4}$

(iii) Find the range of values of x for which the function $y = xe^{4x}$ is decreasing. [2]

For y to be decreasing,

$\frac{dy}{dx} < 0$

$e^{4x}(4x+1) < 0$

Since $e^{4x} > 0$ for all values of x,

then $4x + 1 < 0$

$x < -\frac{1}{4}$

8 AB is a chord of the circle $x^2+y^2-8x-2y-3=0$ and $M\left(\frac{4}{5}, 2\frac{2}{5}\right)$ is the midpoint of chord AB. Find the

(i) radius and the coordinates of the centre of the circle, [2]

$x^2+y^2-8x-2y-3=0$

$x^2+y^2+2(-4)x+2(-1)y+(-3)=0$

$\therefore\ g=-4,\quad f=-1,\quad c=-3$

Hence centre of circle is $(4,1)$

radius of circle is $\sqrt{g^2+f^2-c}$

$=\sqrt{(-4)^2+(-1)^2-(-3)}$

$=\sqrt{20}$

≈ 4.47 units (3 sf)

(ii) equation of chord AB. [3]

Let the centre of circle be C.

$\therefore$ gradient of $CM=\dfrac{2\frac{2}{5}-1}{\frac{4}{5}-4}$

$=\dfrac{-7}{16}$

$\therefore$ gradient of chord $AB=\dfrac{16}{7}$

Hence equation of chord AB is,

$y-2\frac{2}{5}=\frac{16}{7}\left(x-\frac{4}{5}\right)$

$=\frac{16}{7}x-\frac{64}{35}$

$\therefore\ y=\frac{16}{7}x+\frac{4}{7}$

If P is a variable point on the circle, find the

(iii) maximum area of triangle ABP. [4]

Length of $CM=\sqrt{\left(4-\frac{4}{5}\right)^2+\left(1-2\frac{2}{5}\right)^2}$

$=\sqrt{12\frac{1}{5}}$

Length of $BM=\sqrt{BC^2-CM^2}$

$=\sqrt{20-12\frac{1}{5}}$

$=\sqrt{7\frac{4}{5}}$

Length of chord $AB=2\times BM$

$=2\times\sqrt{7\frac{4}{5}}$

Area of ΔABP is maximum when P, C & M are collinear and PM is $\perp AB$.

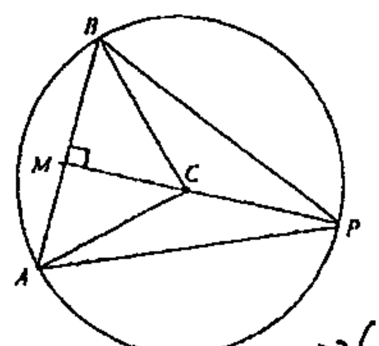
$\therefore$ maximum area of ΔABP

$=\frac{1}{2}\times AB\times PM$

$=\frac{1}{2}\times AB\times(CM+CP)$

$=\frac{1}{2}\times\left(2\times\sqrt{7\frac{4}{5}}\right)\times\left(\sqrt{12\frac{1}{5}}+\sqrt{20}\right)$

≈ 22.2 units^2 (3 sf)



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9 The function f is defined, for $0 \le x \le 2\pi$, by $f(x) = 2\cos ax + b$, where a and b are integers. The minimum value of f is -1 and the period of f is $\frac{4\pi}{3}$.

(i) State the amplitude of f. [1]

Amplitude $= 2$

(ii) State the values of a and of b. [1]

$a = 2\pi \div \frac{4\pi}{3} = 1.5$

$b = 1$

(iii) Using the values of a and b found in part (ii),

(a) solve $f(x) = 0$ for $0 \le x \le 2\pi$, leaving your answers in terms of π, [4]

$$2\cos\left(\frac{3}{2}x\right) + 1 = 0$$

$$\cos\left(\frac{3}{2}x\right) = -\frac{1}{2}$$

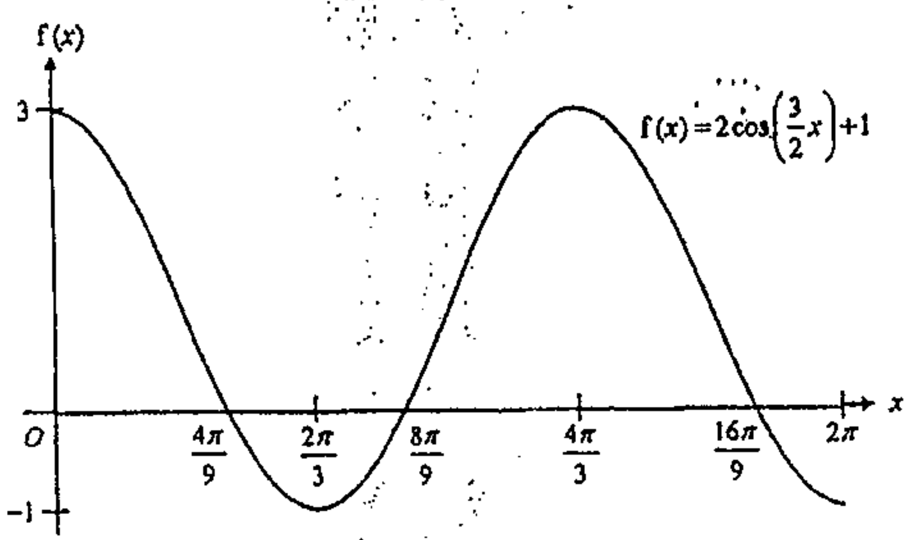
$$\alpha = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\frac{3}{2}x = \pi - \frac{\pi}{3},\ \pi + \frac{\pi}{3},\ 2\pi + \left(\pi - \frac{\pi}{3}\right)$$

$$\frac{3}{2}x = \frac{2\pi}{3},\ \frac{4\pi}{3},\ \frac{8\pi}{3}$$

$$x = \frac{4\pi}{9},\ \frac{8\pi}{9},\ \frac{16\pi}{9}$$

(b) sketch the graph of $f(x) = 2\cos ax + b$ for $0 \le x \le 2\pi$. [3]



57

10 A particle moves in a straight line such that t seconds after leaving a fixed point O, the velocity v m/s, is given by $v = 3t^2 - t - 10$. Find the

(i) initial acceleration of the particle, [2]

$v = 3t^2 - t - 10$

$a = \dfrac{dv}{dt}$

$= 6t - 1$

Initial acceleration $= 6(0) - 1$

$= -1$ m/s^2

(ii) minimum velocity of the particle, [2]

Minimum velocity of the particle occurs when $a = 0$

$6t - 1 = 0$

$t = \dfrac{1}{6}$

$\therefore$ minimum velocity of the particle,

$= 3\left(\dfrac{1}{6}\right)^2 - \left(\dfrac{1}{6}\right) - 10 = -10\dfrac{1}{12}$ m/s

(iii) total distance travelled by the particle in the first 3 seconds, [4]

$s = \int 3t^2 - t - 10 \, dt$

$= t^3 - \dfrac{1}{2}t^2 - 10t + c$

At $t = 0$, $s = 0$, $\therefore c = 0$

Hence, $s = t^3 - \dfrac{1}{2}t^2 - 10t$

When $v = 0$,

$3t^2 - t - 10 = 0$

$(3t + 5)(t - 2) = 0$

$3t + 5 = 0$ or $t - 2 = 0$

$t = \dfrac{-5}{3}$ $(N.A)$ $\quad t = 2$

At $t = 2$,

$s = 2^3 - \dfrac{1}{2}(2)^2 - 10(2) = -14$

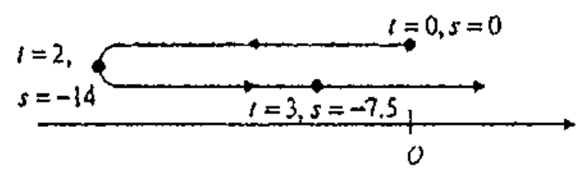
At $t = 3$,

$s = 3^3 - \dfrac{1}{2}(3)^2 - 10(3) = -7\dfrac{1}{2}$

$\therefore$ total distance travelled in the first 3 seconds

$= 14 + (14 - 7.5)$

$= 20.5$ m



(iv) the average speed of the particle during the first 3 seconds. [2]

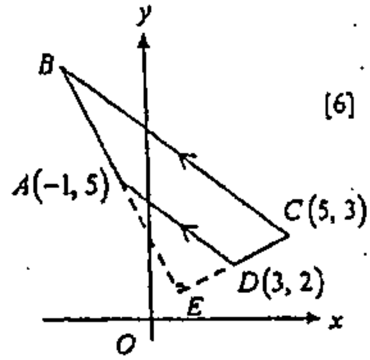
Average speed of the particle during the first 3 seconds

$= \dfrac{\text{total distance}}{\text{total time}} = \dfrac{20.5}{3}$

$= 6\dfrac{5}{6}$ or 6.83 m/s

11 Solutions to this question by accurate drawing will not be accepted.

The diagram shows the trapezium $ABCD$ in which BC is parallel to AD while BA produced is perpendicular to CD produce at point E. The point A is $(-1, 5)$, C is $(5, 3)$ and D is $(3, 2)$.



(i) Show that the coordinates of B are $(-3, 9)$. [6]

Gradient of BC = Gradient of AD

$$= \frac{5-2}{-1-3}$$

$$= -\frac{3}{4}$$

Sub $(5, 3)$ into $y = -\frac{3}{4}x + c$, $\quad 3 = -\frac{3}{4}(5) + c$

$$c = 6\frac{3}{4}$$

Equation of BC is $y = -\frac{3}{4}x + 6\frac{3}{4}$

Gradient of $CD = \frac{3-2}{5-3}$

$$= \frac{1}{2}$$

Gradient of $BA = -2$

Sub $(-1, 5)$ into $y = -2x + d$, $\quad 5 = -2(-1) + d$

$$d = 3$$

Equation of BA is $y = -2x + 3$

$-\frac{3}{4}x + 6\frac{3}{4} = -2x + 3$

$-3x + 27 = -8x + 12$

$5x = -15$

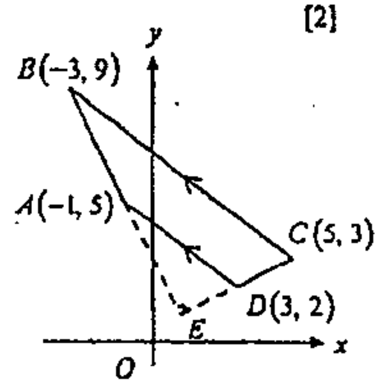
$x = -3$

when $x = -3$, $\quad y = -2(-3) + 3$

$\quad = 9$

$\therefore B(-3, 9)$

11 (ii) Find the area of trapezium $ABCD$. [2]



Area of trapezium $ABCD$

$$=\frac{1}{2}\begin{vmatrix} 5 & -3 & -1 & 3 & 5 \\ 3 & 9 & 5 & 2 & 3 \end{vmatrix}$$

$$=\frac{1}{2}\left[(45-15-2+9)-(-9-9+15+10)\right]$$

$$=\frac{1}{2}(37-7)$$

$$=15 \text{ units}^2$$

(iii) Given that $\frac{\text{area of } \Delta AED}{\text{area of } \Delta BEC}=\frac{1}{4}$, find the coordinates of E. [3]

Since ΔAED and ΔBEC are similar,

$$\frac{ED}{EC}=\frac{EA}{EB}=\sqrt{\frac{1}{4}}=\frac{1}{2}$$

$\therefore D$ and E are the midpoints of EC and EB respectively.

Let $E(m, n)$,

$$\left(\frac{5+m}{2}, \frac{3+n}{2}\right)=(3, 2)$$

$$\frac{5+m}{2}=3 \qquad \frac{3+n}{2}=2$$

$$m=1 \qquad n=1$$

$\therefore E(1, 1) \longleftarrow$ A1

or

$$\left(\frac{-3+m}{2}, \frac{9+n}{2}\right)=(-1, 5)$$

Sub$(3, 2)$ into $y=\frac{1}{2}x+f$,

$$2=\frac{1}{2}(3)+f$$

$$f=\frac{1}{2}$$

Equation of CD is $y=\frac{1}{2}x+\frac{1}{2}$

Equation of BA is $y=-2x+3$

$$\frac{1}{2}x+\frac{1}{2}=-2x+3$$

$$x+1=-4x+6$$

$$5x=5$$

$$x=1$$

when $x=1$, $y=-2(1)+3$
$=1$

$\therefore E(1, 1)$

End of Paper

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Name	Class	Register Number

4047/02 15/S4PR2/AM/2

ADDITIONAL MATHEMATICS PAPER 2

Tuesday 11 August 2015 2 hours 30 minutes

VICTORIA SCHOOL

PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

Additional Materials: Answer Paper
Graph paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This paper consists of 7 printed pages, including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

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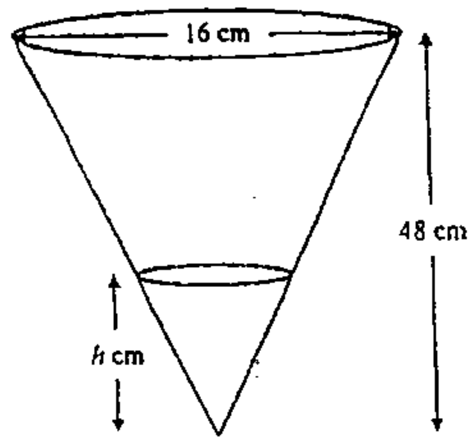
1



Water from a tank in the shape of an inverted cone flows out at the rate of $5\text{ cm}^3/\text{min}$. The height of the cone is 48 cm and the base diameter is 16 cm. After t minutes the water level is h cm.

(i) Show that the volume of water in the tank, $V\text{ cm}^3$, at time t is given by $V=\frac{\pi h^3}{108}$. [2]

(ii) Find the rate of change of the water level when $h=6$. [3]

(iii) State, with a reason, whether this rate will increase or decrease as t increases. [1]

2 The displacement, y mm, of a mass fixed on a vertical spring can be described by the simple harmonic motion equation, $y=A\sin(\omega t)$, where A and ω are constants and t is the time in seconds after the mass is displaced from its equilibrium position, 0 mm.

Given that the maximum displacement of the mass is 20 mm and that the mass first returns to its equilibrium position after 0.25 seconds.

(i) State the positive value of A. [1]

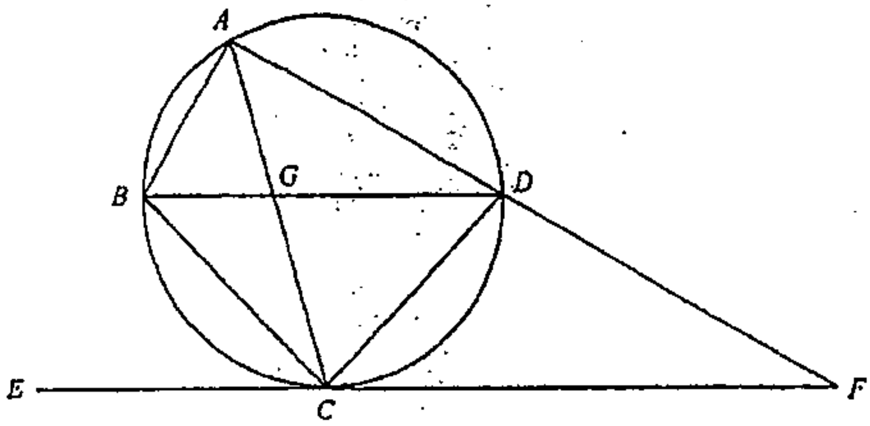
(ii) Show that the value of ω is 4π radians per second. [2]

(iii) Find the exact value of t when the mass first reach a position 10 mm below its equilibrium position. [3]

3 (i) Given that $f(x)=2x^3+ax^2+bx-30$ has a factor $(x+3)$ and leaves a remainder of -28 when divided by $(x-1)$. Find the values of a and of b and solve $f(x)=0$. [6]

(ii) Hence solve $2(y+1)^3+a(y+1)^2+by+b-30=0$. [2]

4



The diagram shows points A, B, C and D lying on a circle. The chords BD and AC intersect at G. EF is a tangent to the circle at C. AD is produced meet the tangent at F and $\angle ABC = \angle BGC$.

Prove that

(i) BD is parallel to EF, [2]

(ii) triangle CFD and triangle AFC are similar, [2]

(iii) $FC^2 - FD^2 = FD \times DA$. [3]

5 (i) Express $\dfrac{3x^2+10x}{(x+2)(x^2-4)}$ in partial fractions. [5]

(ii) Using your answer from (i), find $\int \dfrac{3x^2+10x}{(x+2)(x^2-4)}\,dx$ and hence show that

$$\int_3^4 \frac{3x^2+10x}{(x+2)(x^2-4)}\,dx = \ln\left(\frac{24}{5}\right)+\frac{1}{15}.$$ [4]

6 (a) The quadratic equation $3x^2-2x+4=0$ has roots $3\alpha+\beta$ and $\alpha+3\beta$.

(i) Show that the values of $\alpha+\beta=\frac{1}{6}$ and $\alpha\beta=\frac{5}{16}$. [4]

(ii) If the roots of the equation $gx^2-hx-1=0$ where g and h are constants, are α and β, find the value of g and of h. [2]

(b) Find the range of values of k for which $(k+3)x^2+kx+1$ is always positive for all real values of x. [4]

7 **Answer the whole of this question on a sheet of graph paper.**

The table shows experimental values of two variables, x and y.

x	0.4	0.6	0.8	1.0	1.2
y	2.22	2.13	1.97	1.73	1.37

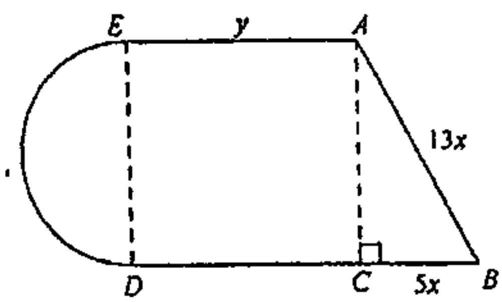
It is known that x and y are related by the equation $y^2 = (ax+1)x - b$, where a and b are constants.

(i) On graph paper, plot $(y^2 - x)$ against x^2, using a scale of 2 cm to represent 0.2 unit on the x^2- axis and 4 cm to represent 1 unit on the $(y^2 - x)$- axis. Draw a straight line graph to represent the equation $y^2 = (ax+1)x - b$. [3]

(ii) Use your graph to estimate the value of a and of b. [4]

(iii) By drawing a suitable straight line on your graph, solve the equation $(a-2) = \dfrac{1+b}{x^2}$. [2]

8 A piece of wire 160 cm long is bent to form the shape shown in the figure. This shape consists of a semi-circular arc whose diameter is given by the length DE, and a right-angled triangle ABC on the opposite ends of a rectangle of length y cm. The length of BC and AB are $5x$ cm and $13x$ cm respectively.



(i) Express y in terms of x. [2]

(ii) Show that the area enclosed, A cm^2, is given by $A = 960x - 6(3\pi + 13)x^2$. [2]

(iii) Determine the value of x for which A has a stationary value. [3]

(iv) Find the stationary value of A and determine if it is a maximum or a minimum value. [3]

9 (a) (i) Prove that $\cos A = \frac{\cos 2A}{\cos A} + \tan A \sin A$. [3]

(ii) Solve, for $0° \le A \le 360°$, $\cos A - \tan A \sin A = -1$. [5]

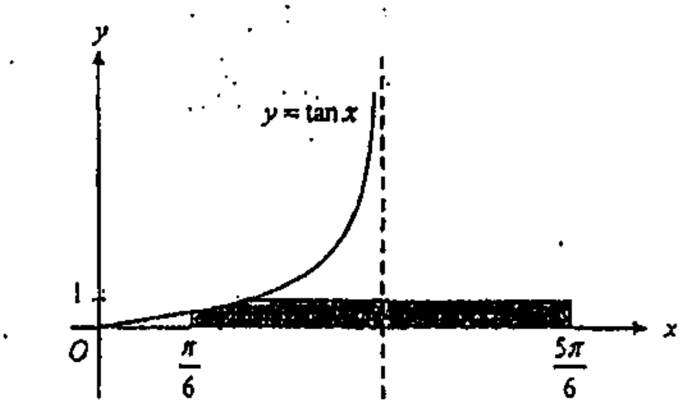
(b) Given $\cos\theta = -\frac{4}{5}$ and θ is in the third quadrant. Without using a calculator, find the value of $\cos\frac{\theta}{2}$. [3]

10 (a) Solve the equation $\log_2 \frac{1}{2} = \log_2 x - \log_4 (9x-2)$. [3]

(b) Given that $\log_2(x+3) - (\log_2 y)(\log_4 2) = 2$, express y in terms of x. [3]

(c) (i) Differentiate $\ln \cos x$. [1]

(ii) State the principal value of $\tan^{-1} 1$, giving your answer as a multiple of π. [1]



The diagram shows part of the graph $y = \tan x$. The shaded region is bounded by the curve, the x-axis, lines $x = \frac{\pi}{6}$, $x = \frac{5\pi}{6}$ and $y = 1$.

(iii) Using your results from (i) and (ii), or otherwise, find the area of the shaded region. [4]

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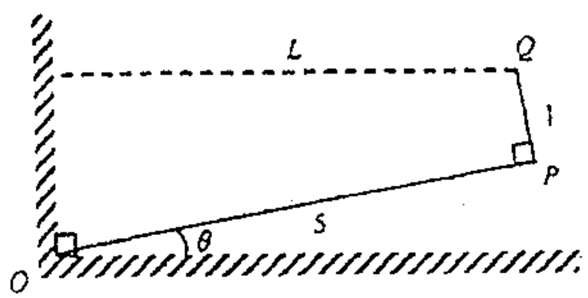
11



A L-shaped structure, OPQ, can be rotated about O. OP and PQ measures 5 m and 1 m respectively. OP makes an acute angle, θ, with the ground. Given that L m is the shortest distance from Q to the wall,

(i) show that $L = 5\cos\theta - \sin\theta$, [2]

(ii) express L in the form $R\cos(\theta+\alpha)$, where $R > 0$ and $0° < \alpha < 90°$, [4]

(iii) state the minimum value of L and find the corresponding value of θ, [3]

(iv) find the value of θ when $L = 3$, [2]

(v) explain why the maximum value of L is not R. [1]

End of Paper

Answer Key

1(ii) $\frac{dh}{dt} = -1.59$ cm / min

(iii) As t increases, h decreases.
Since $\frac{dh}{dt} = \frac{-180}{\pi h^2}$, $\frac{dh}{dt}$ is inversely proportional to h^2,
hence rate of change of water level increases when h decreases.

Or

$\frac{dh}{dt} = \frac{-180}{\pi h^2}$ $\therefore$ $\frac{d^2h}{dt^2} = \frac{360}{\pi h^3}$

Since $h^3 > 0$ for all positive h, then $\frac{d^2h}{dt^2} > 0$.

Hence $\frac{dh}{dt}$ is an increasing function.

2(i) $A = 20$ (iii) $t = \frac{7}{24}$

3(i) $a = 7$, $b = -7$
$x = -3$ or $x = 2$ or $x = -2.5$

(ii) $y = -4$ or $y = 1$ or $y = -3.5$

4 Plane Geometry

5 (i) $\frac{2}{x-2} + \frac{1}{x+2} + \frac{2}{(x+2)^2}$

6a(ii) $g = -3\frac{1}{5}$ $h = -\frac{8}{15}$
6b $-2 < k < 6$

7(ii) $a = -3.00$ $b = -5$
7(iii) Draw $y^2 - x = 2x^2 + 1$
$x = \pm\ 0.894$

8(i) $y = 80 - 3(\pi + 3)x$
8(iii) $x = 3.57$
8(iv) Stationary value of $A = 1710$
A is maximum

9a(ii) $A = 60°, 180°, 300°$
9(b) $\cos\frac{\theta}{2} = -\frac{\sqrt{10}}{10}$

10(a) $x = \frac{1}{4}$ or $x = 2$

(b) $y = \frac{(x+3)^3}{64}$

(c) (i) $-\tan x$

(ii) Principal value of $\tan^{-1} 1 = \frac{\pi}{4}$

(iii) Area = 2.04 units²

11(ii) $L = 5.10\cos(\theta + 11.3°)$

(iii) Min $L = 0$ when $\theta = 78.7°$

(iv) $\theta = 42.7°$

(v) If $L = R$ then $\theta < 0°$.
Since $0° \le \theta < 90°$, $\therefore$ maximum $L \ne R$
[Since $\theta \ge 0°$, maximum L occurs when $\theta = 0°$, maximum $L = 5$.]

	Class	Register Number
Name SOLUTION		

4047/02 **15/S4PR2/AM/2**

ADDITIONAL MATHEMATICS **PAPER 2**

Tuesday 11 August 2015 2 hours 30 minutes

NIL SINE LABORE

VICTORIA SCHOOL

PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

Additional Material: Answer Paper
Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

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[Turn over

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$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

233

1 Water from a tank in the shape of an inverted cone flows out at the rate of $5\text{ cm}^3/\text{min}$. The height of the cone is 48 cm and the base diameter is 16 cm. After t minutes the water level is h cm.

(i) Show that the volume of water in the tank, $V\text{ cm}^3$, at time t is given by $V = \dfrac{\pi h^3}{108}$. [2]

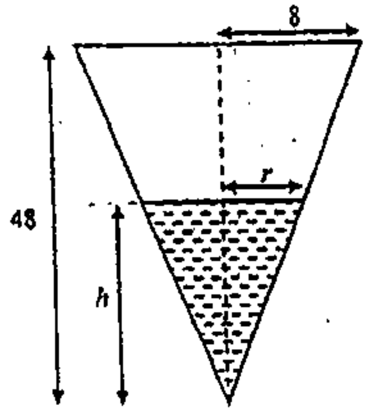
Using similar triangles,

$\dfrac{r}{8} = \dfrac{h}{48} \quad \therefore r = \dfrac{h}{6}$

Volume of water, $V = \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3}\pi\left(\dfrac{h}{6}\right)^2 h$

$\therefore V = \dfrac{\pi h^3}{108}$



(ii) Find the rate of change of the water level when $h = 6$. [3]

$\dfrac{dV}{dh} = \dfrac{\pi h^2}{36}$

$\dfrac{dV}{dt} = \dfrac{dV}{dh} \times \dfrac{dh}{dt}$

$-5 = \dfrac{\pi h^2}{36} \times \dfrac{dh}{dt}$

$\therefore \dfrac{dh}{dt} = \dfrac{-180}{\pi h^2}$

At $h = 6$, $\quad \dfrac{dh}{dt} = \dfrac{-180}{\pi 6^2}$

$= -1.59\text{ cm}^3/\text{min} \quad (3sf)$

(iii) State, with a reason, whether this rate will increase or decrease as t increases. [1]

As t increases, h decreases. Since $\dfrac{dh}{dt} = \dfrac{-180}{\pi h^2}$, $\dfrac{dh}{dt}$ is inversely proportional to h^2, hence rate of change of water level increases when h decreases.

2 The displacement, y mm, of a mass fixed on a vertical spring can be described by the simple harmonic motion equation, $y = A\sin(\omega t)$, where A and ω are constants and t is the time in seconds after the mass is displaced from its equilibrium position, 0 mm.

Given that the maximum displacement of the mass is 20 mm and that the mass first returns to its equilibrium position after 0.25 seconds.

(i) State the positive value of A. [1]

$A = 20$

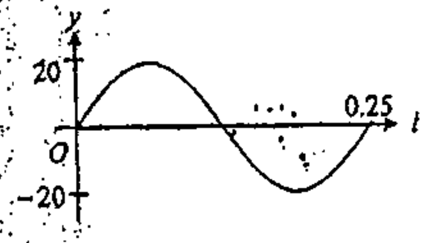
(ii) Show that the value of ω is 4π radians per second. [2]

$0 = 20\sin\omega(0.25)$

$\sin\frac{1}{4}\omega = 0$

$\frac{1}{4}\omega = 0, \pi$

$\omega = 0$ (rej), 4π



(iii) Find the exact value of t when the mass first reach a position 10 mm below its equilibrium position. [3]

$-10 = 20\sin 4\pi t$

$\sin 4\pi t = -\frac{1}{2}$

$\alpha = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$4\pi t = \pi + \frac{\pi}{6}$

$t = \frac{7\pi}{6} \times \frac{1}{4\pi}$

$= \frac{7}{24}$ s

3 (i) Given that $f(x) = 2x^3 + ax^2 + bx - 30$ has a factor $(x+3)$ and leaves a remainder of -28 when divided by $(x-1)$. Find the values of a and of b and solve $f(x) = 0$. [6]

$$f(-3) = 2(-3)^3 + a(-3)^2 + b(-3) - 30 = 0$$
$$-54 + 9a - 3b - 30 = 0$$
$$3a - b = 28 \quad \cdots(1)$$

$$f(1) = 2(1)^3 + a(1) + b - 30 = -28$$
$$a + b = 0 \quad \cdots(2)$$

$$(1)+(2): \quad 4a = 28$$
$$a = 7$$
$$b = -7$$

$$\begin{array}{r} 2x^2 + x - 10 \\ x+3 \overline{)\, 2x^3 + 7x^2 - 7x - 30} \\ \underline{2x^3 + 6x^2 \qquad\qquad} \\ x^2 - 7x \qquad \\ \underline{x^2 + 3x \qquad} \\ -10x - 30 \\ \underline{-10x - 30} \\ 0 \end{array}$$

$$f(x) = 0$$
$$2x^3 + 7x^2 - 7x - 30 = 0$$
$$(x+3)(2x^2 + x - 10) = 0$$
$$(x+3)(x-2)(2x+5) = 0$$
$$x = -3 \quad \text{or} \quad x = 2 \quad \text{or} \quad x = -2.5$$

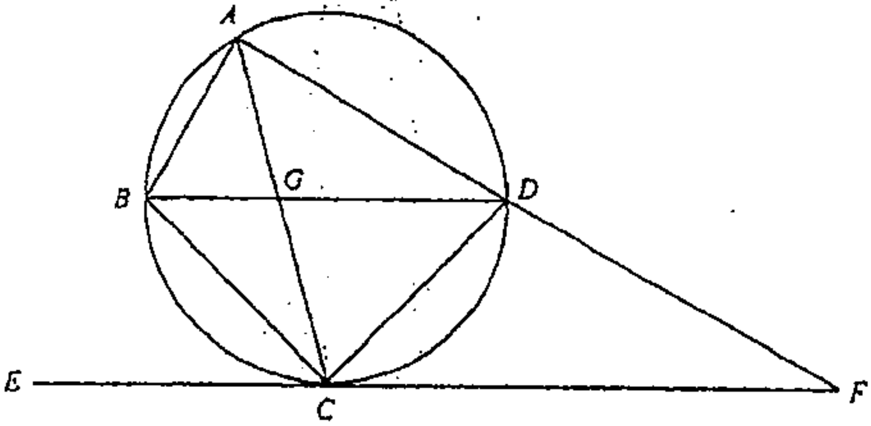
(ii) Hence solve $2(y+1)^3 + a(y+1)^2 + by + b - 30 = 0$. [2]

$$2(y+1)^3 + a(y+1)^2 + by + b - 30 = 0$$
$$2(y+1)^3 + a(y+1)^2 + b(y+1) - 30 = 0$$

Let $x = y + 1$,

$$y+1 = -3 \quad \text{or} \quad y+1 = 2 \quad \text{or} \quad y+1 = -2.5$$
$$y = -4 \qquad y = 1 \qquad y = -3.5$$

4



The diagram shows points A, B, C and D lying on a circle. The chords BD and AC intersect at G. EF is a tangent to the circle at C. AD is produced meet the tangent at F and $\angle ABC = \angle BGC$.

Prove that

(i) BD is parallel to EF, [2]

$\angle ACF = \angle ABC$ ($\angle$s in alternate segment)

$\angle ABC = \angle BGC$ (Given)

$\therefore \angle BGC = \angle ACF$

By the angle property of alternate angles, BD is parallel to EF.

(ii) triangle CFD and triangle AFC are similar, [2]

$\angle CFD = \angle AFC$ (Common $\angle$)

$\angle DCF = \angle CAF$ ($\angle$s in alternate segment)

Hence ΔCFD is similar to ΔAFC.

(iii) $FC^2 - FD^2 = FD \times DA$. [3]

Since ΔCFD is similar to ΔAFC,

$$\left.\begin{aligned} \frac{FD}{FC} &= \frac{CF}{AF} \\ FC^2 &= FD \times AF \end{aligned}\right\}$$

$$\begin{aligned} &= FD \times (FD + DA) \\ &= FD^2 + FD \times DA \end{aligned}$$

$\therefore FC^2 - FD^2 = FD \times DA$ (Proven)

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6 (ii) If the roots of the equation $gx^2 - hx - 1 = 0$ where g and h are constants, are α and β, find the value of g and of h. [2]

$\alpha + \beta = \frac{h}{g}$

$\frac{1}{6} = \frac{h}{g}$

$\therefore h = \frac{g}{6}$ ....(1)

$\alpha\beta = \frac{-1}{g}$

$\frac{5}{16} = \frac{-1}{g}$

$g = \frac{-16}{5}$

$= -3\frac{1}{5}$

Sub $g = -3\frac{1}{5}$ into (1)

$\therefore h = -3\frac{1}{5} \div 6$

$= \frac{-8}{15}$

Alternative solution:

$x^2 - \frac{1}{6}x + \frac{5}{16} = 0$

$-\frac{16}{5}x^2 + \frac{8}{15}x - 1 = 0$

$gx^2 - hx - 1 = 0$

$\therefore g = \frac{-16}{5}$

$= -3\frac{1}{5}$

$\therefore -h = \frac{8}{15}$

$h = -\frac{8}{15}$

(b) Find the range of values of k for which $(k+3)x^2 + kx + 1$ is always positive for all real values of x. [4]

$(k+3)x^2 + kx + 1 > 0$

Since the expression is always positive,

$k + 3 > 0$ and $b^2 - 4ac < 0$

$k > -3$

$k^2 - 4(k+3)(1) < 0$

$k^2 - 4k - 12 < 0$

$(k-6)(k+2) < 0$

$\therefore -2 < k < 6$

Hence $k > -3$ and $-2 < k < 6$

$\therefore$ the solution is $-2 < k < 6$

7 Answer the whole of this question on a sheet of graph paper.

The table shows experimental values of two variables, x and y.

x	0.4	0.6	0.8	1.0	1.2
y	2.22	2.13	1.97	1.73	1.37

It is known that x and y are related by the equation $y^2=(ax+1)x-b$, where a and b are constants.

(i) On graph paper, plot (y^2-x) against x^2, using a scale of 2 cm to represent 0.2 unit on the x^2 axis and 4 cm to represent 1 unit on the (y^2-x) axis. Draw a straight line graph to represent the equation $y^2=(ax+1)x-b$. [3]

x^2	0.160	0.360	0.640	1.00	1.44
y^2-x	4.53	3.94	3.08	1.99	0.677

(ii) Use your graph to estimate the value of a and of b. [4]

$y^2=(ax+1)x-b$
$y^2=ax^2+x-b$
$y^2-x=ax^2-b$

Gradient $=a$

Gradient $=\dfrac{5-3.5}{0-0.5}$

$\therefore a=-3.00$ (3sf)

(y^2-x) – intercept $=-b$
$-b=5$
$\therefore b=-5$

(iii) By drawing a suitable straight line on your graph, solve the equation $(a-2)=\dfrac{1+b}{x^2}$. [2]

$(a-2)=\dfrac{1+b}{x^2}$
$ax^2-2x^2=1+b$
$ax^2-b=2x^2+1$
Draw $y^2-x=2x^2+1$

From graph, $x^2=0.8$
$x=\pm\ 0.894$ (3sf)

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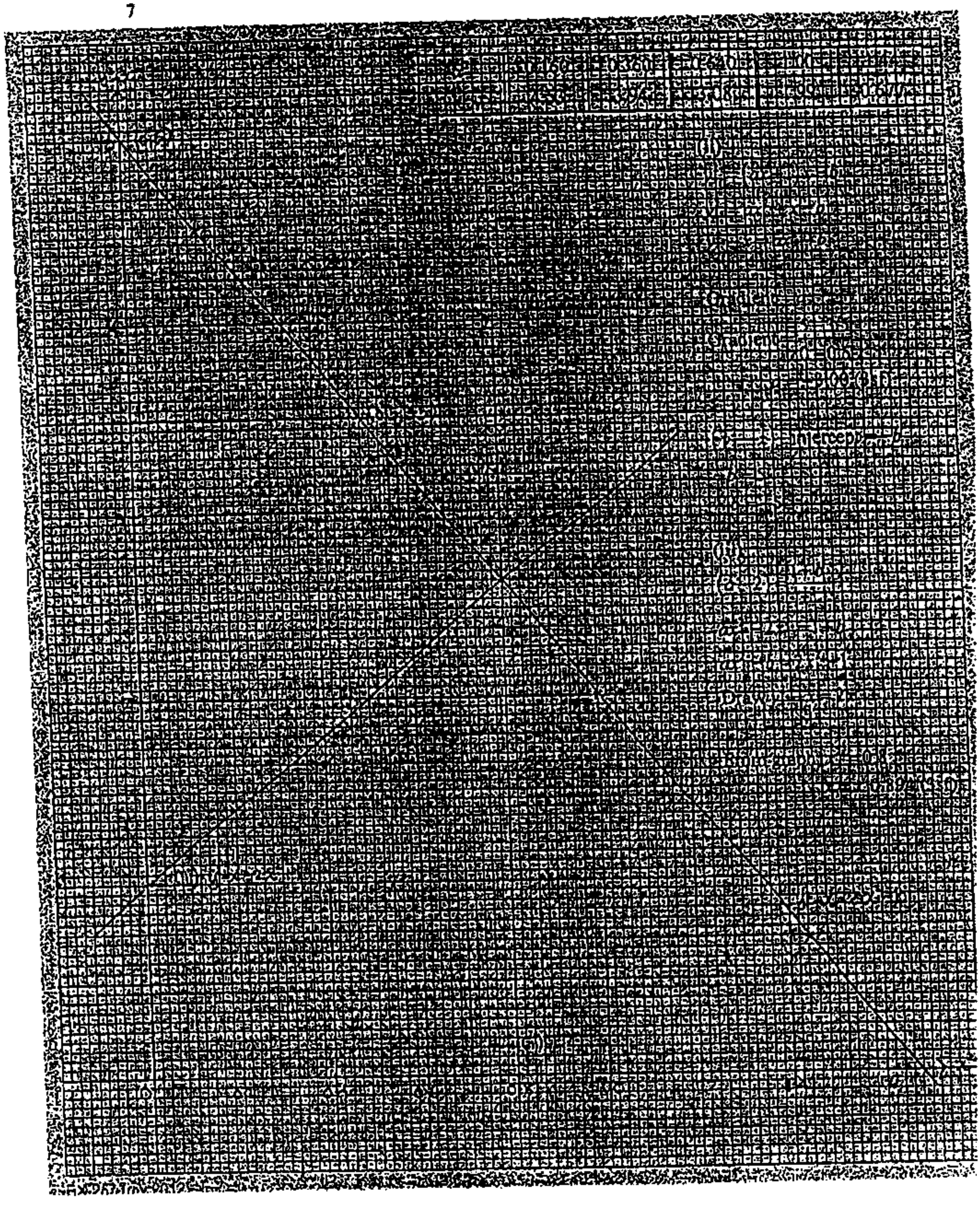
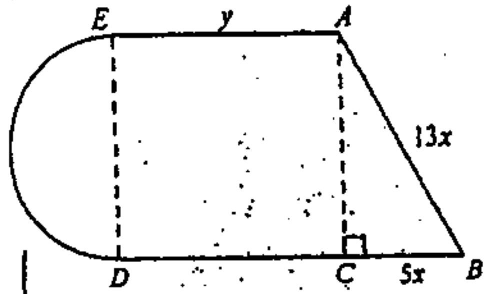
7



8 A piece of wire 160 cm long is bent to form the shape shown in the figure. This shape consists of a semi-circular arc whose diameter is given by the length DE, and a right-angled triangle ABC on the opposite ends of a rectangle of length y cm. The length of BC and AB are $5x$ cm and $13x$ cm respectively.



(i) Express y in terms of x. [2]

$AC = \sqrt{(13x)^2 - (5x)^2} = 12x$

Perimeter of figure = Length of wire

$2y + \frac{\pi 12x}{2} + 13x + 5x = 160$

$2y + 6\pi x + 18x = 160$

$y + 3\pi x + 9x = 80$

$\therefore\ y = 80 - 3(\pi + 3)x$

(ii) Show that the area enclosed, A cm^2, is given by $A = 960x - 6(3\pi + 13)x^2$. [2]

$A = \frac{1}{2}\pi(6x)^2 + y(12x) + \frac{1}{2}(5x)(12x)$

$= 18\pi x^2 + 30x^2 + 12x[80 - 3(\pi + 3)x]$

$= (18\pi + 30)x^2 + 960x - 36(\pi + 3)x^2$

$= [18\pi + 30 - 36(\pi + 3)]x^2 + 960x$

$= (18\pi + 30 - 36\pi - 108)x^2 + 960x$

$= (-18\pi - 78)x^2 + 960x$

$= 960x - 6(3\pi + 13)x^2$

8 (iii) Determine the value of x for which A has a stationary value. [3]

$$A = 960x - 6(3\pi + 13)x^2$$

$$\frac{dA}{dx} = 960 - 12(3\pi + 13)x$$

For stationary value of A,

$$\frac{dA}{dx} = 0$$

$$960 - 12(3\pi + 13)x = 0$$

$$x = \frac{960}{12(3\pi + 13)}$$

$$\approx 3.567$$

$$= 3.57 \text{ (3 sf)}$$

(iv) Find the stationary value of A and determine if it is a maximum or a minimum value. [3]

$$\text{Stationary value of } A = 960(3.567) - 6(3\pi + 13)(3.567)^2$$

$$\approx 1712.39$$

$$= 1710 \text{ (3 sf)}$$

$$\frac{d^2A}{dx^2} = -12(3\pi + 13)$$

since $\frac{d^2A}{dx^2} < 0$, A is a maximum value.

---

9 (a) (i) Prove that $\cos A = \frac{\cos 2A}{\cos A} + \tan A \sin A$. [3]

$$\text{RHS} = \frac{\cos 2A}{\cos A} + \tan A \sin A$$

$$= \frac{2\cos^2 A - 1}{\cos A} + \frac{\sin A}{\cos A} \cdot \sin A$$

$$= \frac{2\cos^2 A - 1 + \sin^2 A}{\cos A}$$

$$= \frac{2\cos^2 A - 1 + 1 - \cos^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\cos A}$$

$$= \cos A$$

$$= \text{LHS}$$

9 (a) (ii) Solve, for $0° \le A \le 360°$, $\cos A - \tan A \sin A = -1$. [5]

$\cos A - \tan A \sin A = -1$

$\frac{\cos 2A}{\cos A} = -1$

$2\cos^2 A - 1 = -\cos A$

$2\cos^2 A + \cos A - 1 = 0$

$(\cos A + 1)(2\cos A - 1) = 0$

$\cos A = -1$ or $\cos A = \frac{1}{2}$

$A = 180°$

$\alpha = 60°$

$A = 60°, 360° - 60°$

$= 60°, 300°$

(b) Given $\cos\theta = -\frac{4}{5}$ and θ is in the third quadrant. Without using a calculator, find the value of $\cos\frac{\theta}{2}$. [3]

$\cos\theta = -\frac{4}{5}$

$180° < \theta < 270°$

$2\cos^2\frac{\theta}{2} - 1 = -\frac{4}{5}$

$90° < \frac{\theta}{2} < 135°$

$\cos^2\frac{\theta}{2} = \frac{1}{10}$

$\cos\frac{\theta}{2} = \pm\frac{1}{\sqrt{10}}$

$= \pm\frac{\sqrt{10}}{10}$

Since $90° < \frac{\theta}{2} < 135°$, $\cos\frac{\theta}{2} = -\frac{\sqrt{10}}{10}$

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10 (a) Solve the equation $\log_2 \frac{1}{2} = \log_2 x - \log_4 (9x-2)$. [3]

$$\log_2 \frac{1}{2} = \log_2 x - \log_4 (9x-2)$$
$$\log_4 (9x-2) = \log_2 x - \log_2 \frac{1}{2}$$
$$\frac{\log_2 (9x-2)}{\log_2 4} = \log_2 \left(x \div \frac{1}{2} \right)$$
$$\frac{\log_2 (9x-2)}{2} = \log_2 2x$$
$$\log_2 (9x-2) = 2\log_2 2x$$
$$\log_2 (9x-2) = \log_2 (2x)^2$$
$$\therefore\ 9x-2 = 4x^2$$
$$4x^2 - 9x + 2 = 0$$
$$(4x-1)(x-2) = 0$$
$$4x-1 = 0 \quad \text{or} \quad x-2 = 0$$
$$x = \frac{1}{4} \quad \text{or} \quad x = 2$$

(b) Given that $\log_2 (x+3) - (\log_2 y)(\log_8 2) = 2$, express y in terms of x. [3]

$$\log_2 (x+3) - (\log_2 y)(\log_8 2) = 2$$
$$\log_2 (x+3) - \log_2 y \times \frac{1}{\log_2 8} = 2$$
$$\log_2 (x+3) - \frac{\log_2 y}{3} = 2$$
$$3\log_2 (x+3) - \log_2 y = 6$$
$$\log_2 (x+3)^3 - \log_2 y = 6$$
$$\log_2 \frac{(x+3)^3}{y} = 6$$
$$\frac{(x+3)^3}{y} = 2^6$$
$$64y = (x+3)^3$$
$$y = \frac{(x+3)^3}{64}$$

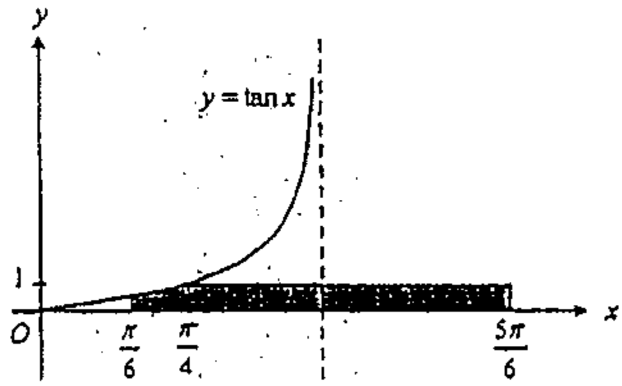
$$\log_2 (x+3) - (\log_2 y)(\log_8 2) = 2$$
$$\log_2 (x+3) - \log_2 y \times \frac{\log_2 2}{\log_2 8} = 2$$
$$\log_2 (x+3) - \frac{\log_2 y}{3} = 2$$
$$\log_2 (x+3) - \log_2 \sqrt[3]{y} = 2$$
$$\log_2 \frac{(x+3)}{\sqrt[3]{y}} = 2$$
$$\frac{(x+3)}{\sqrt[3]{y}} = 2^2$$
$$\frac{(x+3)^3}{y} = 4^3$$
$$y = \frac{(x+3)^3}{64}$$

10 (c) (i) Differentiate $\ln\cos x$. [1]

$$\frac{d}{dx}\ln\cos x = \frac{-\sin x}{\cos x}$$
$$= -\tan x$$

(ii) State the principal value of $\tan^{-1}1$, giving your answer as a multiple of π. [1]

Principal value of $\tan^{-1}1 = \frac{\pi}{4}$

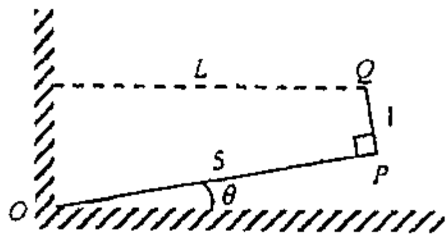


The diagram shows part of the graph $y = \tan x$. The shaded region is bounded by the curve, the x axis, lines $x = \frac{5\pi}{6}$ and $y = 1$.

(iii) Using your results from (i) and (ii), or otherwise, find the area of the shaded region. [4]

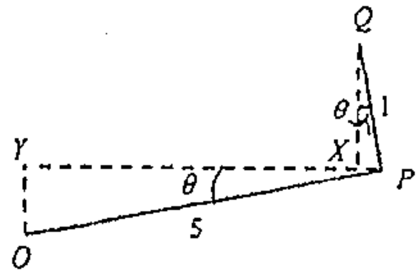
$$\text{Area} = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \tan x\,dx + \left(\frac{5\pi}{6} - \frac{\pi}{4}\right)$$
$$= -\left[\ln\cos x\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}} + \frac{7\pi}{12}$$
$$= -\left[\ln\cos\frac{\pi}{4} - \ln\cos\frac{\pi}{6}\right] + \frac{7\pi}{12}$$
$$= -\ln\frac{1}{\sqrt{2}} + \ln\frac{\sqrt{3}}{2} + \frac{7\pi}{12}$$
$$= \frac{1}{2}\ln\frac{3}{2} + \frac{7\pi}{12}$$
$$= 2.04 \text{ units}^2 \text{ (3sf)}$$

11



A L-shaped structure, OPQ, can be rotated about O. OP and PQ measures 5 m and 1 m respectively. OP makes an acute angle, θ, with the ground. Given that L m is the shortest distance from Q to the wall,

(i) show that $L = 5\cos\theta - \sin\theta$, [2]



$$\cos\theta = \frac{PY}{5}$$
$$PY = 5\cos\theta$$

$$\sin\theta = \frac{PX}{1}$$
$$PX = \sin\theta$$

$$\therefore L = PY - PX$$
$$= 5\cos\theta - \sin\theta$$

(ii) express L in the form $R\cos(\theta+\alpha)$, where $R > 0$ and $0° < \alpha < 90°$, [4]

$$L = 5\cos\theta - \sin\theta$$
$$= R\cos(\theta+\alpha)$$
$$= R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$$
$$R\cos\alpha = 5 \quad \cdots(1) \qquad R\sin\alpha = 1 \quad \cdots(2)$$

$$(1)^2 + (2)^2: \qquad R^2\cos^2\alpha + R^2\sin^2\alpha = 5^2 + 1^2$$
$$R = \sqrt{26}$$
$$= 5.10 \text{ (3sf)}$$

$$\frac{(2)}{(1)}: \qquad \frac{R\sin\alpha}{R\cos\alpha} = \frac{1}{5}$$
$$\tan\alpha = \frac{1}{5}$$
$$\alpha = \tan^{-1}\frac{1}{5}$$
$$= 11.31° \text{ (2dp)}$$

$$\therefore L = 5.10\cos(\theta + 11.3°) \qquad \text{(3sf, 1dp)}$$

11 (iii) state the minimum value of L and find the corresponding value of θ, [3]

Minimum $L=0$, when $\cos(\theta+11.31°)=0$

$\theta+11.31°=90°$

$\theta=78.7°$ (1dp)

(iv) find the value of θ when $L=3$, [2]

$3=\sqrt{26}\cos(\theta+11.31°)$

$\cos(\theta+11.31°)=\frac{3}{\sqrt{26}}$

$\alpha=\cos^{-1}\left(\frac{3}{\sqrt{26}}\right)=53.96°$

$\theta+11.31°=53.96°$

$\theta=42.7°$ (1dp)

(v) explain why the maximum value of L is not R. [1]

If $L=R$ then $\theta<0°$. Since $0°\le\theta<90°$, $\therefore$ maximum $L\neq R$.

[Since $\theta\ge 0°$, maximum L occurs when $\theta=0°$, maximum $L=5$.]

End of Paper

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END