SA1 Bowen Secondary School 2
SA1 Bukit Merah Secondary School 8
SA1 Chij Katong School 14
SA1 Maris Stella High School 30
SA1 Tanjong Katong Girl's School 41
SA2 Cedar Girls' Secondary School 59
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Calculator model	

Class	Full Name	Index Number

bowen SECONDARY

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MID YEAR EXAMINATION
2016

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4047

ADDITIONAL MATHEMATICS **2 hours**

Secondary 3 Exp
12th May 2016

Additional Materials : Writing Papers

INSTRUCTIONS TO CANDIDATES:

1. Write your name, index number, class in the spaces provided at the top of this page and on all the work you hand in.
2. Answer **ALL** the questions on the writing papers provided.
3. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
4. The use of a scientific calculator is expected, where appropriate.
5. You are reminded of the need for clear presentation in your answers.
6. At the end of the test, fasten all your work securely together.
7. The number of marks is given in brackets [ ] at the end of each question or part question.
8. The total number of marks for this paper is 80.

DO NOT OPEN THIS PAPER UNTIL YOU ARE TOLD TO DO SO.

Setter: Mrs Li Seow Koon

For Examiner's use
80

This document consists of **5** printed pages (including the cover page).

[Turn over

Mathematical Formula :

Quadratic Equation   For the equation $ax^2 + bx + c = 0$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1 The curve $\dfrac{5}{x} - \dfrac{2}{y} = 2$ and the line $x + 2y = 1$ intersect at points A and B.
Calculate the coordinates of A and of B. [4]

2 It is given that $4x^3 - 9x = Ax(x^2 - 1) + B(x + 1) + C - 3$ for all real values of x.
Find the values of A, B and C. [3]

3 The polynomial $2x^4 - 18x^2 + 5x - 17$ can be expressed as $(x^2 - 3x - 2)\,Q(x) + R(x)$.
Determine $Q(x)$ and $R(x)$. [4]

4 The function f is defined by $f(x) = 2x^3 + ax^2 + bx - 12$.
Given $f(x)$ has a factor of $(x - 3)$ and leaves a remainder of -14 when divided by $(2x + 1)$,

(i) find the value of a and of b [4]

(ii) find the remainder when $f(x)$ is divided by x. [1]

5 (a) (i) Factorise $x^3 - 3x^2 + x + 2$. [2]

(ii) Hence, solve the equation $x^3 - 3x^2 + x = -2$. Leave your answers in surd form where necessary. [3]

(b) Factorize completely $16x^3 - 250y^6$ [3]

6 Given that the roots of the equation $2x^2 - 3 = 6x$ are α and β ,

(i) find the values of

(a) $\alpha^2 + \beta^2$ [2]

(b) $(\alpha + 1)(\beta + 1)$ [2]

(ii) form an equation whose roots are $\dfrac{\alpha}{\beta + 1}$ and $\dfrac{\beta}{\alpha + 1}$. [3]

7 (a) Given that the equation $kx^2 - kx - 1 = 3x - 5$ has equal real roots, find the values of k . [3]

(b) Find the range of values of m for which $2mx^2 + 1 = 4x - m$ has no real roots. [3]

(c) Find the range of values of x for which $(2x-1)^2 \le 25$. [3]

(d) Show that the equation $x^2 + px = 4 - 2p$ has real roots for all real values of p. [3]

8 Solve the following equations.

(a) $4\sqrt{2x-3}=\sqrt{24}$ [3]

(b) $x-\sqrt{2x+2}=3$ [3]

(c) $27\left(\sqrt{3}\right)^{x}=\dfrac{3^{x}}{\sqrt[3]{9}}$ [3]

9 (a) Given that $2^{2x-4}\times 40=5^{2-x}$, evaluate 20^{x}. [3]

(b) By using the substitution $u=5^{x}$, solve $5^{x-1}+5^{2-x}=6$. [4]

10 (a) The mass, m grams, of a radioactive substance, present at time t days after first being observed, is given by the formula $m=38(2.5)^{-0.04t}$.

Find

(i) the initial mass of the radioactive substance. [1]

(ii) the mass of the radioactive substance after 4 weeks. [2]

(iii) the mass of the radioactive substance that has decayed after 10 days. [2]

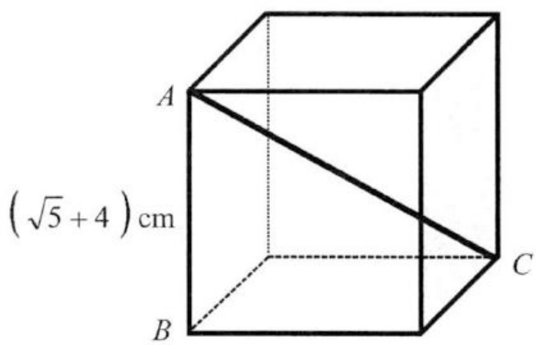
(b) Sketch the graph of $y=4\,(6.3)^{x}$, indicating the y – intercept on the graph. [2]

11 (a) Express $\dfrac{5-7x}{x^2-4}$ in partial fractions. [3]

(b) Given $\dfrac{2x^3-4x^2+19x+4}{x\left(x^2+4\right)} = A+\dfrac{B}{x}+\dfrac{Cx+D}{x^2+4}$, find the values of A, B, C and D. [4]

12 (i) Express $\dfrac{41\sqrt{5}}{3\sqrt{5}+2}$ in the form $a+b\sqrt{5}$, where a and b are integers. [2]

(ii) The diagram below shows a cuboid with a square base. The height AB of the cuboid is $\left(\sqrt{5}+4\right)$ cm. Given that the length of the diagonal AC is $\dfrac{41\sqrt{5}}{3\sqrt{5}+2}$ cm,



(a) find an expression for BC^2 in the form $c+d\sqrt{5}$, where c and d are integers. [3]

(b) hence, find the area of the square base in the exact form. [2]

~ End of Paper ~

Answer key for Bowen Sec Sch Sec 3 Exp A Maths MYE 2016

1	$(5, -2)$ and $\left(\frac{1}{2}, \frac{1}{4}\right)$		8a	$\frac{9}{4}$
2	$A = 4$, $B = -5$, $C = 8$		8b	$x = 7$, $x = -1$ (rej)
3	$Q(x) = 2x^2 + 6x + 4$, $R(x) = 29x - 9$		8c	$\frac{22}{3}$
4i, ii	$a = -5$, $b = 1$, remainder $= -12$		9a	10
5ai	$(x-2)(x^2 - x - 1)$		9b	1 , 2
5aii	$x = 2$, $x = \frac{1 \pm \sqrt{5}}{2}$		10ai	38
5b	$2\left(2x - 5y^2\right)\left(4x^2 + 10xy^2 + 25y^4\right)$,		10aii	13.6
6ia	12		10aiii	11.7
6ib	$\frac{5}{2}$		11a	$\frac{-19}{4(x+2)} - \frac{9}{4(x-2)}$
6ii	$5x^2 - 30x - 3 = 0$		11b	$A = 2, B = 1, C = -5, D = 11$
7a	$k = 1$, 9		12i	$15 - 2\sqrt{5}$
7b	m < -2 or m > 1		12iia	$\left(224 - 68\sqrt{5}\right) cm^2$
7c	$-2 \leq x \leq 3$		12iib	$\left(112 - 34\sqrt{5}\right) cm^2$
7d	To prove $b^2 - 4ac \geq 0$			

Class	Register No	Name

Bukit Merah Secondary School
Mid-Year Examination 2016
Secondary 3 Express

E

ADDITIONAL MATHEMATICS

4047

10 May 2016

Additional Materials: Writing Paper (8 sheets)
Cover Page

2 hours

READ THESE INSTRUCTIONS FIRST

Write your class, register number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of the marks for this paper is **80**.

This document consists of 5 printed pages.

[Turn Over

Mathematical Formulae

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \text{ ,}$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

---

1. $P(x)$ is a cubic polynomial. The graph of $y = P(x)$ intersects the x-axis at -4, -1 and $\frac{1}{2}$ and passes through the point (0, 16). Find an expression for $P(x)$. [3]

2. Solve the simultaneous equations

$$0.5^x(4^{3y}) = 16$$

$$\log_4 2x + \log_4(x+3y) = 1$$ [5]

3. The polynomial $f(x)$ is such that $f(x) = 3x^3 + ax^2 + bx + c$, where a, b and c are constants, is divisible by $x-1$ but leaves a remainder of 3 when divided by $x+2$.

(i) Show that $a - b = 10$. [3]

Given also that the remainder is 15 when $f(x)$ is divided by $x+1$.

(ii) Find the value of c. [2]

4. Express the equation $2(9^x) - 75(3^{x-1}) + 63 = 0$ as a quadratic equation in 3^x. Hence solve the equation for x. [5]

5. Express $\dfrac{7-3x}{(1-x)(1+x-2x^2)}$ as a sum of 3 partial fractions. [7]

6. (a) The length of the two shorter sides of a right-angled triangle are $(2\sqrt{2}+\sqrt{3})$ cm and $(4\sqrt{3}-\sqrt{8})$ cm respectively. Find, without using a calculator, the exact value of the square of the longest side of the triangle in the form $(c + d\sqrt{6})$cm², where c and d are integers. [3]

(b) The solution of the equation $x\sqrt{2} = \sqrt{135} - x\sqrt{5}$ is $a\sqrt{3} - \sqrt{b}$.
Without using a calculator, find the values of the integers a and b. [4]

[Turn Over

7. (a) Given that $\log_x 8 = \frac{3}{4}$, evaluate $\log_4\left(\frac{1}{x}\right)$. [3]

(b) Given that $\log_x y + \log_y x - \frac{10}{\log_x y} = 0$, express y in terms of x. [4]

8. (a) One root of the equation $2x^2 + px + 3q = 0$ is three times the other root. Express p in terms of q. [4]

(b) Given that α and β are the roots of the equation $3x^2 - 2x + 3 = 0$, form an equation whose roots are $4\alpha\beta$ and $\alpha^3 + \beta^3$. [5]

9. Solve the equations

(a) $\sqrt{3\sqrt{2x-3}} = 2$ [3]

(b) $\sqrt{5^x} \div \left(\frac{1}{5}\right)^{2x+1} = 25^{x+3}$ [3]

(c) $\log_3(10 - 9x) - 4\log_9 x = 2$ [4]

10. Solve the equation $8x^3 - 6x^2 - 5x + 3 = 0$. **Hence** solve [5]

(i) $3x^3 - 5x^2 = 6x - 8$ [3]

(ii) $8(2^{3y}) - 3(2^{2y+1}) - 5(2^y + 1) + 8 = 0$ [3]

11. (a) Given that the coefficient of x in the binomial expansion of $\left(x+\frac{k}{x^3}\right)^9$ is 9,

find the negative value of the constant k. [4]

(b) Write down, and simplify, the first three terms in the expansion of $\left(2-\frac{x}{5}\right)^8$

in ascending powers of x.

Given that the first three non-zero terms in the expansion of $(3+ax)\left(2-\frac{x}{5}\right)^8$ are

$b-512x+cx^2$. Find the values of a, b and c. [7]

* * * End-of-Paper * * *

BUKIT MERAH SECONDARY SCHOOL
2016 MID-YEAR EXAMINATION

ADDITIONAL MATHEMATICS (4047) – Answer Keys

1.	$P(x) = -4(x+4)(x+1)(2x-1)$
2.	$x = \frac{2}{3}$, $y = \frac{7}{9}$
3.	(ii) $c = 8$
4.	$2(3^x)^2 - 25(3^x) + 63 = 0$, $x = 1.14$ or 2
5.	$\frac{7-3x}{(1-x)^2(1+2x)} = \frac{17}{9(1-x)} + \frac{4}{3(1-x)^2} + \frac{34}{9(1+2x)}$
6.	(a) $(67 - 12\sqrt{6})\, cm^2$ (b) $a = 5$ and $b = 30$
7.	(a) -2 (b) $y = x^3$ or $y = \frac{1}{x^3}$
8.	(a) $p = \pm 4\sqrt{2q}$ (b) Quad equation is $x^2 - \frac{62}{27}x - \frac{184}{27} = 0$ or $27x^2 - 62x - 184 = 0$
9.	(a) $2\frac{7}{18}$ (b) $x = 10$ (c) $x = \frac{2}{3}$
10.	$x = 1$, $x = -\frac{3}{4}$ or $x = \frac{1}{2}$ (i) $x = 1$, $-\frac{4}{3}$ or 2 (ii) $y = 0$, or $y = -1$
11.	(a) $k = -\frac{1}{2}$ (b) $256 - \frac{1024}{5}x + \frac{1792}{25}x^2 + \ldots$, $a = \frac{2}{5}$, $b = 768$ and $c = 133\frac{3}{25}$

1 The line $3x+2y=12$ meets the curve $x^2-y+2y^2=12$ at the points A and B.
Calculate the length of AB. [7]

2 If the difference between the roots of the equation $x^2+px+q=0$ is 3, show that
$p^2=4q+9$. [4]

3 The roots of the equation $100x^2-29x+1=0$ are α^2 and β^2. Find the quadratic equation whose roots are α and β, such that α and β are positive. [5]

4 The line $y=mx+1$, where m is a constant, intersect the curve $y=x^2-3x+2$ at two distinct points. Find the range of values of m. [5]

5 A piece of wire of length of 24 cm is bent into a rectangle. Let x cm be the length of one side of the rectangle and A cm^2 be the area of the rectangle.

(i) Express A in terms of x. [1]

(ii) Find the range of values of x such that the area of the rectangle is greater than 27 cm^2. [5]

(iii) Hence, find the maximum of the rectangle. [1]

6 Jane threw a ball such that the height, s metres, of the ball at time t seconds is given by the equation $s=-5.1t^2+vt+2.5$, where v m/s is the speed at which she threw the ball. Use the discriminate to determine whether the ball could reach a height of 15 m if it is thrown at speed of 20 m/s. [4]

7 Solve the following equations.

(a) $\sqrt{5x+2}-\sqrt{3x-8}=0$ [2]

(b) $3\sqrt{x-1}=2\sqrt{x+4}$ [2]

(c) $\sqrt{7-6x}+x=-3x$ [4]

8 It is given that x and y are rational numbers. Find the values of x and y in

$(6-3\sqrt{5})(x+y\sqrt{5})=81-30\sqrt{5}$. [7]

9 An open cuboid bin has a square base of side $(\sqrt{7}-\sqrt{5})$ m. The capacity of the bin is $(90\sqrt{5}-76\sqrt{7})$ m^3. Find the exact value of

(a) the base area of the bin, [2]
(b) the height of the bin, [3]
(c) the total surface area of the bin. [3]

10 **Solve the following equations.**

(a) $4^{x+1}+8=33(2^x)$ [5]

(b) $7^{2x+3}\div 7^{x^2}=1$ [4]

11 **(a)** In a lucky draw, $(x-3)$ winners shared a sum of \$$(3x^3-5x^2+6x-54)$ equally. Find the share of each winner. [3]

(b) In a given cubic polynomial $f(x)$, the coefficient of x^3 is 1 and the roots of $f(x)=0$ are -2, 2 and k. When $f(x)$ is divided by $x+1$, the remainder is -6.

(i) Find the value of k. [4]
(ii) Find the remainder when $f(x)$ is divided by x^2-3. [4]

12 Express $\dfrac{5x^2-4x+2}{(3x-4)(x^2+1)}$ in partial fraction. [8]

13 **(a)** Factorise each of the following

(i) $1000a^3-b^3$ [2]
(ii) $3x^4+81x$ [2]

(b) **(i)** Sketch the graph of $y=4e^x$. [2]

(ii) Add the line $y=4+x$ to your graph. [1]

(iii) Hence state the number of solutions of the equation

$4e^x=5+x$. [1]

14 **(a)** **(i)** Expand $\left(1+\frac{x}{4}\right)^9$ up to the first 3 terms. [1]

(ii) Hence, given that $\left(8-2x-3x^2\right)\left(1+\frac{x}{4}\right)^9 = 8+hx+kx^2+\ldots\ldots,$

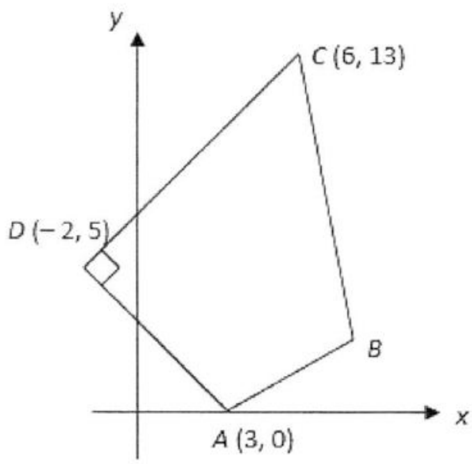
find the values of h and k. [4]

(b) Evaluate the coefficient of x^7 in the binomial expansion of $\left(x^2-\frac{1}{2x}\right)^{14}$. [3]

15 **Solutions to this equation by accurate drawing will not be accepted.**

The diagram shows a quadrilateral $ABCD$ in which $A(3,0)$, $C(6,13)$ and $D(-2,5)$.

The equation of AB is $5y=3x-9$ and $\angle ADC = 90^0$.



Find

(i) the equation of AD, [2]

(ii) the perpendicular bisector of CD. [3]

The perpendicular bisector of CD passes through B.

(iii) Find the coordinates of B. [2]

(iv) Find the area of the quadrilateral $ABCD$. [2]

End of Paper

MYE 3E AM 2016 Solution

1 The line $3x+2y=12$ meets the curve $x^2-y+2y^2=12$ at the points A and B. Calculate the length of AB.

Solution

$$x=\frac{12-2y}{3}$$

$$\left(\frac{12-2y}{3}\right)^2-y+2y^2=12$$

Some students expressed

$(12-2x)^2=144-4y^2$ **X**

$$\left(\frac{144-48y+4y^2}{9}\right)-y+2y^2=12$$

$$22y^2-57y+36=0$$

$$(11y-12)(2y-3)=0$$

$$y=\frac{12}{11},\ y=\frac{3}{2}$$

$$x=3\frac{3}{11},\ \ x=3$$

$$AB=\sqrt{\left(3\frac{3}{11}-3\right)^2+\left(\frac{12}{11}-\frac{3}{2}\right)^2}=0.492\text{ units}$$

Some students wrote cm instead units. Some left units out.

A lot of students used $(x_1+x_2)^2$ instead $(x_1-x_2)^2$ Or some did not square the brackets. **X** **Some did not know the formula.**

2 The roots of the equation $100x^2 - 29x + 1 = 0$ are α^2 and β^2. Find the quadratic equation whose roots are α and β, such that α and β are positive.

$$\alpha^2 + \beta^2 = \frac{29}{100}$$

$$(\alpha\beta)^2 = \frac{1}{100}$$

$$\alpha\beta = \frac{1}{10}$$

A lot students did not reject $\alpha\beta = -\frac{1}{10}$

$$(\alpha+\beta)^2 - 2\alpha\beta = \frac{29}{100}$$

$$(\alpha+\beta)^2 - 2(\frac{1}{10}) = \frac{29}{100}$$

$$(\alpha+\beta) = \frac{7}{10}$$

A lot students did not reject $\alpha + \beta = -\frac{7}{10}$

$$x^2 - \left(\frac{7}{10}\right)x + \frac{1}{10} = 0$$

$$10x^2 - 7x + 1 = 0$$

Some students did not write (= 0) for the required equation.

3 The line $y = mx + 1$, where m is a constant, intersect the curve $y = x^2 - 3x + 2$ at <u>two distinct points</u>. Find the range of values of m.

$$x^2 - 3x + 2 - mx - 1 = 0$$

b = - 3 - m, some student identified b = 3 + m **X**

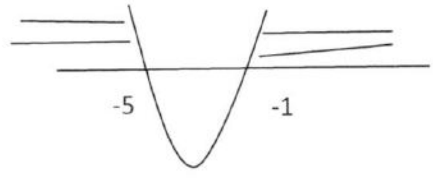
$$(-3-m)^2 - 4(1)(1) > 0$$

$$9 + 6m + m^2 - 4 > 0$$

$$m^2 + 6m + 5 > 0$$

$$(m+5)(m+1) > 0$$

$$m < -5, \quad m > -1$$



4 **A piece of wire of length of 24 cm is bent into a rectangle. Let x cm be the length of one side of the rectangle and A cm^2 be the area of the rectangle.**

(i) Express A in terms of x.

(ii) Find the range of values of x such that the area of the rectangle is greater than 27 cm^2 .

(iii) Hence, find the maximum area of the rectangle.

This question is not well-done.

(i) $A = x(12-x)$

Some students left this question totally blank. Very few students drew a rectangle to analyse the question

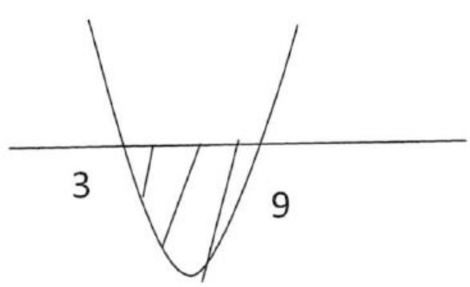
$$\frac{24-2x}{2} = 12 - x$$

A simple diagram will help a lot !!!!!!!!!!!!

(ii) $x(12-x) > 27$

$x^2 - 12x + 27 < 0$

$(x-9)(x-3) < 0$



$$3 < x < 9$$

Only a few students were able to use the mid-value to locate the Maximum Area.

(iii) Maximum A occurs at $x = 6$

Maximum A = (12 - 6)(6) = 36 cm^2

5 Jane threw a ball such that the height, s metres, of the ball at time t seconds is given by the equation $s = -5.1t^2 + vt + 2.5$, where v m/s is the speed at which she threw the ball. Use the discriminant to determine whether the ball could reach a height of 15 m if it is thrown at speed of 20 m/s.

This question is badly done despite the hint to use the discriminate to show the conclusion.

$15 = -5.1t^2 + 20t + 2.5$ **Many did not equate S = 15.**

$20^2 - 4(-5.1)(-12.5) = 145 > 0$

Hence the ball could reach a height of 15 m at 20 m/s.

6 Solve the following equations.

(a) $\sqrt{5x+2} - \sqrt{3x+8} = 0$

(b) $\sqrt{7-6x} + x = -3x$

Solution

(a) $\left(\sqrt{5x+2}\right)^2 = \left(\sqrt{3x+8}\right)^2$

$5x + 2 = 3x + 8$

$x = 3$

This was well-done.

(b)

$$7-6x=16x^2$$

$$16x^2+6x-7=0$$

$$(2x-1)(8x+7)=0$$

$$x=\frac{1}{2}, \quad x=-\frac{7}{8}$$

(rejected)

Many students did check the feasibility of x = ½ thus many did not reject x = ½ .

7 It is given that *x* and *y* are rational numbers. Find the values of x and y in

$$\left(6-3\sqrt{5}\right)\left(x+y\sqrt{5}\right)=81-30\sqrt{5}.$$

Solution

$$6x+6y\sqrt{5}-3x\sqrt{5}-3y(5)=81-30\sqrt{5}$$

$$6x-15y+\sqrt{5}\left(6y-3x\right)=81-30\sqrt{5}$$

$$6x-15y=81$$

$$6y-3x=-30$$

Solving

2(2y +10) - 5y = 27

$$y=-7$$

$$x=-4$$

This was well-done.

8 **An** open **cuboid bin has a square base of side $\left(\sqrt{7}-\sqrt{5}\right)$ m. The capacity of the bin is $\left(90\sqrt{5}-76\sqrt{7}\right)$ m^3. Find the exact value of**

(a) the base area of the bin,
(b) the height of the bin,
(c) the total surface area of the bin.

Solution

(a) Base area $=\left(\sqrt{7}-\sqrt{5}\right)^2=12-2\sqrt{35}$

(b) Height $=\dfrac{\left(90\sqrt{5}-76\sqrt{7}\right)}{\left(12-2\sqrt{35}\right)}\times\dfrac{\left(12+2\sqrt{35}\right)}{\left(12+2\sqrt{35}\right)}$

$$=\frac{1080\sqrt{5}+180\sqrt{175}-912\sqrt{7}-152\sqrt{245}}{144-4\times 35}$$

Some students did not or could not simply from the above line to the following line.

$$=\frac{1080\sqrt{5}+180\times 5\sqrt{7}-912\sqrt{7}-1064\sqrt{5}}{4}$$

$$=\frac{16\sqrt{5}-12\sqrt{7}}{4}=4\sqrt{5}-3\sqrt{7}$$

(c) **Total surface area** $=12-2\sqrt{35}+4\left(\sqrt{7}-\sqrt{5}\right)\left(4\sqrt{5}-3\sqrt{7}\right)$

It is an opened bin so there should not have $2\left(12-2\sqrt{35}\right)$. $\boxed{\text{X}}$

$$=12-2\sqrt{35}+28\sqrt{35}-164$$

$$=26\sqrt{35}-152$$

9 **Solve the following equations. (WELL-DONE)**

(a) $4^{x+1} + 8 = 33(2^x)$

(b) $7^{2x+3} \div 7^{x^2} = 1$

(a) $2^{2x+2} + 8 = 33(2^x)$

$2^{2x} \times 2^2 + 8 = 33(2^x)$

Let $y = 2^x$

$4y^2 - 33y + 8 = 0$

$(4y-1)(y-8) = 0$

$y = \frac{1}{4}. \quad y = 8$

$x = -2, \quad x = 3$

(b) $7^{2x+3-x^2} = 7^0$

$2x + 3 - x^2 = 0$

$(x-3)(x+1) = 0$

$x = 3, x = -1$

10 **(a) In a lucky draw, $(x-3)$ winners shared a sum of $\$(3x^3 - 5x^2 + 6x - 54)$ equally. Find the share of each winner.**

(b) In a given cubic polynomial $f(x)$, the coefficient of x^3 is 1 and the roots of $f(x) = 0$ are -2, 2 and *k*. When $f(x)$ is divided by $x+1$, the remainder is -6.

(i) Find the value of *k*.

(ii) Find the remainder when $f(x)$ is divided by $x^2 - 3$.

Solution

(a)

	3	-5	6	-54
3	↓	9	12	54
	3	4	18	

Answer $\$\left(3x^2+4x+18\right)$

Remark: Some students left this blank.

(b) (i) $f(x)=(x+2)(x-2)(x-k)$

Some students wrote $(x+k)$ instead of $(x-k)$

$f(-1)=-6$

$(-1+2)(-1-2)(-1-k)=-6$

$3+3k=-6$

$k=-3$

(ii) $f(x)=x^3+3x^2-4x-12$

$$
\begin{array}{r}
x+3 \\
x^2-3\overline{)\,x^3+3x^2-4x-12} \\
-x^3+0x^2+3x \\
\hline
3x^2-x-12 \\
-3x^2\quad+9 \\
\hline
-x-3
\end{array}
$$

Remainder = $-x-3$

Some substituted x = $\sqrt{3}$ **instead.**

11 **Express $\dfrac{5x^2-4x+2}{(3x-4)(x^2+1)}$ in partial fraction.**

Solution

$$\frac{5x^2-4x+2}{(3x-4)(x^2+1)}=\frac{A}{(3x-4)}+\frac{Bx+C}{x^2+1}$$

Some students wrote $x^2+1=(x+1)(x-1)$

$5x^2-4x+2 \ = \ A(x^2+1)+(3x-4)(Bx+C)$

Let $x=0$

A = 2 + 4 C---------------------------(1)

Let $x = 1$

3 = 2A – B – C ---------------------(2)

Let $x = -1$

11 = 2A + 7B – 7C------------------(3)

Sub A = 2 + 4C into eqns (2) and (3)

7C – B = -1---------------------------(4)

C = 7 – 7B ----------------------------(5)

Solving

A = 2, B = 1, C = 0

Answer $\quad \dfrac{2}{3x-4}+\dfrac{x}{x^2+1}$

Comparing coeffs:

$$5 = A + 3B$$

$$B = \frac{5 - A}{3}$$

$$-4 = -4B + 3C$$

$$C = \frac{A - 2}{4}$$

$$-4 = -4\left(\frac{5 - A}{3}\right) + \left(\frac{3A - 6}{4}\right)$$

$$A = 2, B = 1, \ C = 0$$

12 **(a)** Factorise each of the following

(i) $1000a^3 - b^3$

(ii) $3x^4 + 81x$

(b) **(i)** Sketch the graph of $y = 4e^x$.

(ii) Add the line $y = 4 + x$ to your graph.

(iii) Hence state the number of solutions of the equation

$4e^x = 4 + x$.

(a) (i) $(10a)^3 - b^3$

$= (10a - b)(100a^2 + 10ab + b^2)$

(ii) $3x(x^3 + 3^3)$

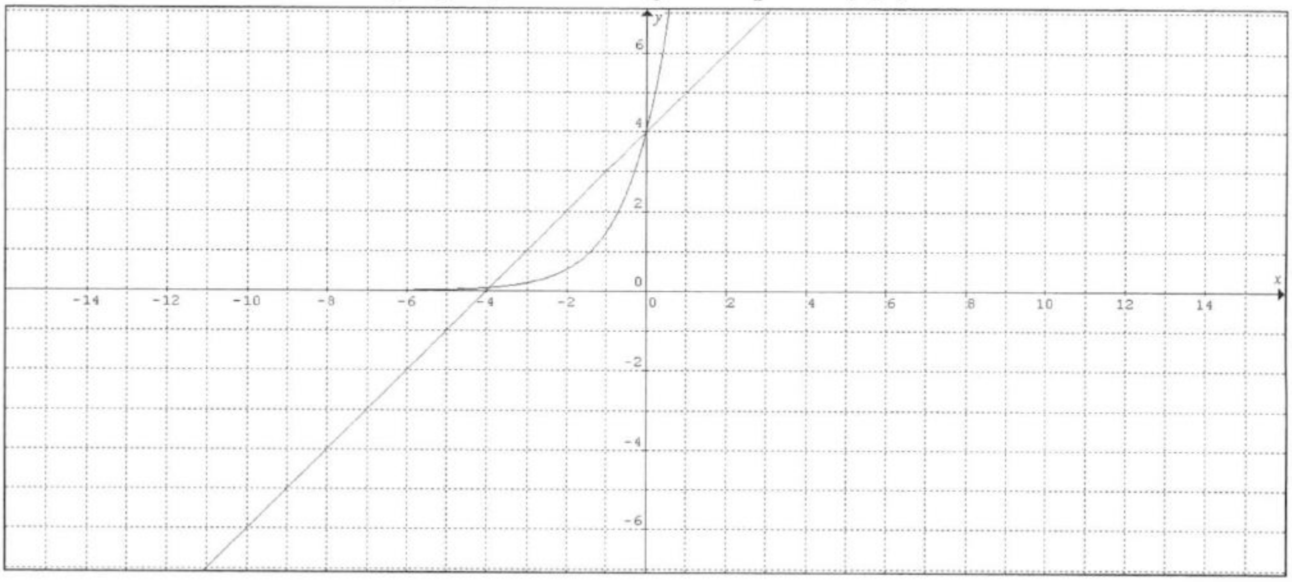
$= 3x(x + 3)(x^2 - 3x + 9)$

12a was badly done.

Many were unable to apply the cubic formulae.

b(i) shape and asymptote of

$y = 4e^x$ and the graph pass through the point (0,4)



(ii) Line $y = 4 + x$ (iii) 2 points of intersections

The two graphs were not well-drawn. Many were not able to draw the graphs to occupy both first and second quadrants of the axes. The line did not pass through the two axes.

Therefore many students were not able to obtain the 2 points of intersections.

13 (a) (i) Expand $\left(1+\dfrac{x}{4}\right)^9$ up to the first 3 terms.

(ii) Hence, given that $\left(8-2x-3x^2\right)\left(1+\dfrac{x}{4}\right)^9 = 8 + hx + kx^2 + ...,$

find the values of h and k.

(b) Evaluate the coefficient of x^7 in the binomial expansion of $\left(x^2 - \dfrac{1}{2x}\right)^{14}$.

13(a)

(i) $\left(1+\dfrac{x}{4}\right)^9 = 1 + \dfrac{9x}{4} + \dfrac{9x^2}{4} + ...$ Well-done.

(ii) $\left(8-2x-3x^2\right)\left(1+\dfrac{9x}{4}+\dfrac{9x^2}{4}+...\right)$

$= 8 + 18x + 18x^2 - 2x - \dfrac{9x^2}{2} - 3x^2 + \ldots$

$= 8 + 16x + \dfrac{21x^2}{2} + \ldots$

$\therefore\ h = 16$ and $k = \dfrac{21}{2}$

Carelessness in expansion resulting in wrong answers.

13(b) $\left(x^2 - \dfrac{1}{2x}\right)^{14}$

General term or $(r+1)^{th}$ term of the expansion

$= {}^{14}C_r \left(x^2\right)^{14-r}\left(-\dfrac{1}{2x}\right)^r$

$= {}^{14}C_r \left(-\dfrac{1}{2}\right)^r x^{28-3r}$ [M1]

For the term in x^7, $x^7 = x^{28-3r}$

$7 = 28 - 3r$

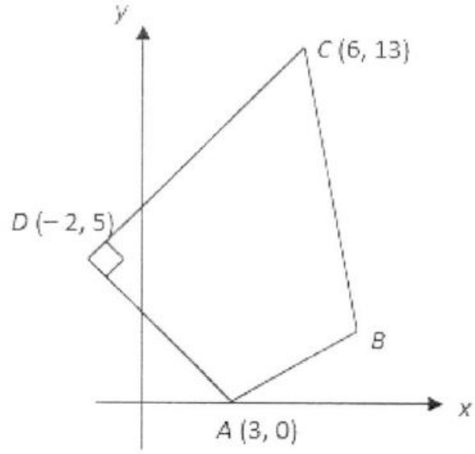
$r = 7$ [M1]

$\therefore$ Coeff of $x^7 = {}^{14}C_7\left(-\dfrac{1}{2}\right)^7 = -26\dfrac{13}{16}$ [A1]

Some students were unable to apply the general formula. They were unable to write down $\left(-\dfrac{1}{2}\right)^r$ in the working thus unable to obtain the correct answer.

14 Solutions to this equation by accurate drawing will not be accepted. The diagram shows a quadrilateral *ABCD* in which $A(3,0)$, $C(6,13)$ and $D(-2,5)$.

The equation of *AB* is $5y = 3x - 9$ and $\angle ADC = 90^0$.



Find

(i) the equation of *AD*,

(ii) <u>the perpendicular bisector of *CD*.</u>

The perpendicular bisector of *CD* passes through *B*.

(iii) Find the coordinates of *B*.

(iv) Find the area of the quadrilateral *ABCD*.

14 **(i)** gradient of $AD = -1$

$3 = -1(0) + c$

eq of AD : $y = -x + 3$

(ii) mid-point of CD = (2, 9)

Gradient of perpendicular bisector = -1

Equation of perpendicular bisector : $y - 9 = -(x - 2)$

(iii) $5(-x + 11) = 3x - 9$

$x = 8$
$y = 3$

B (8, 3)

(iv) Area $= \dfrac{1}{2}\begin{vmatrix} 3 & 8 & 6 & -2 & 3 \\ 0 & 3 & 13 & 5 & 0 \end{vmatrix} = 68 \text{ units}^2$

This question was well-done. Some students think that the point (2,9) is the perpendicular bisector.

Class	Index Number	Name

新加坡海星中学

MARIS STELLA HIGH SCHOOL

SEMESTRAL EXAMINATION ONE

SECONDARY THREE

ADDITIONAL MATHEMATICS **11 May 2016**

2 hours

Additional Materials:
Writing paper (5 sheets)

INSTRUCTIONS TO CANDIDATES

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give your answer to three significant figures. Give answer in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

This document consists of 3 printed pages and 1 blank page.

Mathematical Formulae

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

---

1. (a) Simplify $\sqrt{80} + \sqrt{180} - \dfrac{8}{\sqrt{5}+1}$. [3]

   (b) Simplify $\dfrac{2^{n+1} + 2^n}{4^{\frac{1}{2}n-1} - 2^{n-3}}$. [3]

2. (a) Solve the equation $3^{2x+1} = 10$. [2]

   (b) Given that $\log_2 x^3 = m$ and $\log_4 y = n$, find $\log_2 \sqrt{xy}$ in terms of m and n. [4]

3. Solve the equations

   (a) $2^x\left(4^{x-1}\right) = 8^{2x-1}$, [3]

   (b) $\log_5 x - \log_{25}(x+10) = \dfrac{1}{2}$. [4]

4. Sketch the graph of $y = \ln x$. Insert in your sketch an additional graph required to illustrate how a graphical solution of the equation $xe^{3x} = 1$ may be obtained. State the equation of the additional graph and the number of solutions to the equation $xe^{3x} = 1$. [6]

5. (a) Given that $\sin 110° = p$, express each of the following in terms of p.

   (i) $\sin 70°$ (ii) $\tan 20°$ [4]

   (b) Without the use of a calculator, find the exact value of $\sin\left[\cos^{-1}\left(-\dfrac{2}{3}\right)\right]$. [2]

6. Given that $\tan A = -\dfrac{5}{12}$ and that $\tan A$ and $\cos A$ have opposite signs, find the value of

   (i) $\sin(-A)$ (ii) $\sin(90° - A)$ [4]

7. The value, V dollars, of an antique is given by $V = V_0 e^{kt}$, where V_0 dollars is the initial value of the antique when it was produced, t is the time in years since it was produced and k is a constant.

(i) Find the value of k given that the value of the antique doubled after 7 years. [2]

(ii) Given further that the antique was produced in 1930 and that the value of the antique is evaluated at the beginning of every year, find the year in which its value first exceeded ten times the initial value. [3]

8. The curve $y = a\cos bx + c$, where a and b are positive integers, has an amplitude of 3 and a period of $180°$. The maximum value of y is 1.

(i) State the values of a, b and c. [3]

(ii) With the values stated in part (i), sketch the curve of $y = a\cos bx + c$ for $0° \le x \le 360°$. [3]

9. The roots of $x^2 + 3x - 6 = 0$ are α and β. The roots of another equation $x^2 - 6x + q = 0$ are $\frac{n}{\alpha^3}$ and $\frac{n}{\beta^3}$, where n and q are constants. Find the value of n and of q. [6]

10. Given that $f(x) = mx^3 - (5m-1)x^2 + (m+1)x + m^2$ is exactly divisible by $x-1$ but not by $x-4$,

(i) show that $m = 1$, [4]

(ii) using the value of m shown in part (i), solve the equation $f(x) = 0$, giving your answers correct to two decimal places where necessary. [4]

11. (a) Find the values of k if the graph of $y = (2k-1)x^2 + 2k + 4$ and the line $y = 3kx$ meet at one point only. [4]

(b) Find the range of values of h for which $(h+3)x^2 - 3x > x + h$ for all real values of x. [4]

(c) Show that $2x^2 + p = 2(x-1)$ has no real roots if $p > -\frac{3}{2}$. [3]

12. (i) Given that $2x^3 - 31x - 27 = A(x-4)(x+2)^2 + Bx + C$, find the values of the constants A, B and C. [4]

(ii) Hence or otherwise, express $\dfrac{2x^3 - 31x - 27}{(x-4)(x+2)^2}$ in partial fractions. [5]

--- End of Paper ---

2016 SECONDARY 3 ADDITIONAL MATHEMATICS SA1
MARKING SCHEME

1. (a) $\sqrt{80}+\sqrt{180}-\dfrac{8}{\sqrt{5}+1}$

$=4\sqrt{5}+6\sqrt{5}-\dfrac{8}{\sqrt{5}+1}\times\dfrac{\sqrt{5}-1}{\sqrt{5}-1}$ [M1]

$=10\sqrt{5}-\dfrac{8\left(\sqrt{5}-1\right)}{5-1}$

$=10\sqrt{5}-2\left(\sqrt{5}-1\right)$ [M1]

$=10\sqrt{5}-2\sqrt{5}+2$

$=8\sqrt{5}+2 \;\; or \;\; 2\left(4\sqrt{5}+1\right)$ [A1]

(b) $\dfrac{2^{n+1}+2^{n}}{4^{\frac{1}{2}n-1}-2^{n-3}}$

$=\dfrac{2\left(2^{n}\right)+2^{n}}{2^{-2}\left(2^{n}\right)-2^{-3}\left(2^{n}\right)}$ [M1]

$=\dfrac{2^{n}(2+1)}{2^{n}\left(\dfrac{1}{4}-\dfrac{1}{8}\right)}$ [M1]

$=24$ [A1]

2. (a) $3^{2x+1}=10$

$\lg 3^{2x+1}=\lg 10$

$(2x+1)\lg 3=1$ [M1]

$2x+1=\dfrac{1}{\lg 3}$

$x=\dfrac{1}{2}\left(\dfrac{1}{\lg 3}-1\right)$

$x=0.548 \;\; (3s.f.)$ [A1]

(b) $\log_2 x^3 = m$

$3\log_2 x = m$

$\log_2 x = \frac{1}{3}m$ [M1]

$\log_4 y = n$

$\frac{\log_2 y}{\log_2 4} = n$

$\log_2 y = 2n$ [M1]

$$\log_2 \sqrt{xy} = \frac{1}{2}\log_2 xy$$

$$= \frac{1}{2}(\log_2 x + \log_2 y) \quad [M1]$$

$$= \frac{1}{2}\left(\frac{1}{3}m + 2n\right)$$

$$= \frac{1}{6}m + n \quad [A1]$$

3. (a) $2^x\left(4^{x-1}\right) = 8^{2x-1}$

$2^x\left(2^{2x-2}\right) = 2^{6x-3}$ [M1]

$2^{3x-2} = 2^{6x-3}$

By comparing indices,

$3x - 2 = 6x - 3$ [M1]

$3x = 1$

$x = \frac{1}{3}$ [A1]

(b) $\log_5 x - \log_{25}(x+10) = \frac{1}{2}$

$\log_5 x - \frac{\log_5(x+10)}{\log_5 5^2} = \frac{1}{2}$ [M1]

$\log_5 x - \frac{\log_5(x+10)}{2} = \frac{1}{2}$ [M1]

$2\log_5 x - \log_5(x+10) = 1$

$\log_5 \frac{x^2}{x+10} = 1$

$\frac{x^2}{x+10} = 5$ [M1]

$x^2 = 5x + 50$

$x^2 - 5x - 50 = 0$

$(x-10)(x+5) = 0$

$x = 10$ *or* $x = -5$ (*rej*) [A1]

2

4. $xe^{3x} = 1$

$\ln xe^{3x} = \ln 1$

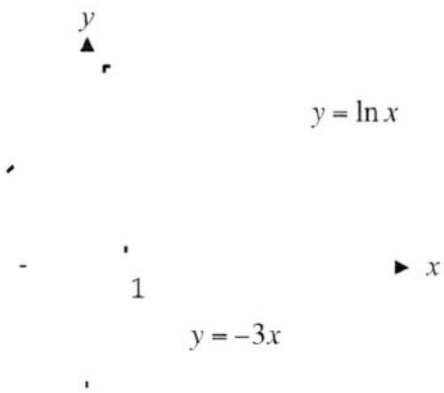
$\ln x + \ln e^{3x} = 0$ [$M1$]

$\ln x + 3x = 0$

$\ln x = -3x$

Equation of additional graph is $y = -3x$. [$A1$]

Number of solution is 1. [A1]



2 marks for graphs (1 mark each). 1 mark for labels and intercepts.

5. (ai) $\sin 70° = \sin(180° - 110°)$ [$M1$]

$= \sin 110°$

$= p$ [$A1$]

(aii) $\tan 20° = \dfrac{\sqrt{1-p^2}}{p}$ [$B2$]

(b) Let $A = \cos^{-1}\left(-\dfrac{2}{3}\right)$.

$\cos A = -\dfrac{2}{3}$ [$M1$]

$\sin A = \dfrac{\sqrt{5}}{3}$ [$A1$]

6. (i) $\sin(-A) = -\sin A$ $[M1]$

$$= -\left(-\frac{5}{13}\right)$$

$$= \frac{5}{13} \quad [A1]$$

(ii) $\sin(90° - A) = \cos A$ $[M1]$

$$= \frac{12}{13} \quad [A1]$$

7. (i) When $V = 2V_0$ and $t = 7$,

$2V_0 = V_0 e^{7k}$ $[M1]$

$e^{7k} = 2$

$k = \frac{1}{7}\ln 2$

$k = 0.0990\ (3s.f.)$ $[A1]$

(ii) When $V = 10V_0$ and $k = \frac{1}{7}\ln 2$,

$V_0 e^{\left(\frac{1}{7}\ln 2\right)t} > 10V_0$ $[M1]$

$e^{\left(\frac{1}{7}\ln 2\right)t} > 10$

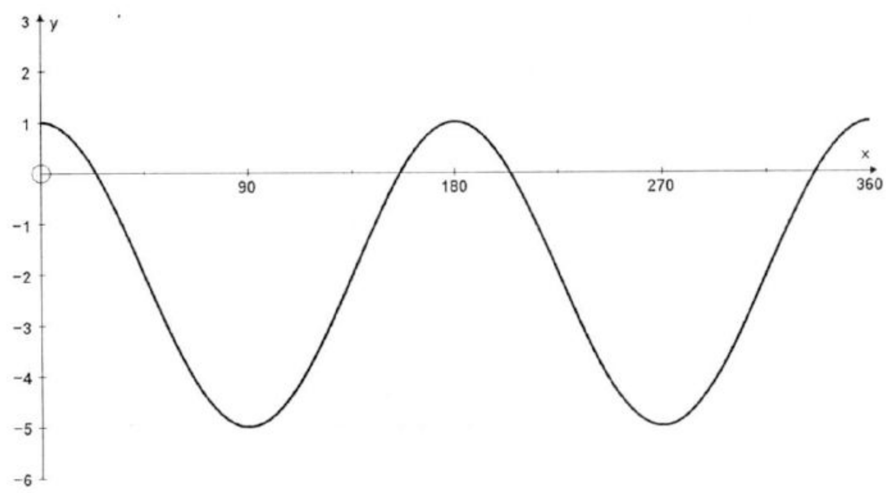
$t > \ln 10\left(\frac{7}{\ln 2}\right)$

$t > 23.253\ (5s.f.)$ $[M1]$

The year in which its value first exceeded ten times the initial value is 1954. [A1]

8. (i) $a = 3$ [B1] $b = \frac{360°}{120°}$, $b = 3$ [B1] $c = 1 - 3$, $c = -2$ [B1]

(ii) 2 marks for graph. 1 mark for labels.



4

9. $x^2+3x-6=0$

$$\alpha+\beta=-3$$
$$\alpha\beta=-6 \qquad [M1]$$

$$x^2-6x+q=0$$
$$\frac{k}{\alpha^3}+\frac{k}{\beta^3}=6 \qquad [M1]$$
$$\frac{k(\alpha^3+\beta^3)}{\alpha^3\beta^3}=6$$
$$k=\frac{6\alpha^3\beta^3}{\alpha^3+\beta^3}$$
$$=\frac{6(\alpha\beta)^3}{(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)} \qquad [M1]$$
$$=\frac{6(\alpha\beta)^3}{(\alpha+\beta)((\alpha+\beta)^2-3\alpha\beta)}$$
$$=\frac{6(-6)^3}{(-3)((-3)^2-3(-6))}$$
$$=16 \qquad [A1]$$

$$\left(\frac{k}{\alpha^3}\right)\left(\frac{k}{\beta^3}\right)=q \qquad [M1]$$
$$q=\frac{k^2}{\alpha^3\beta^3}$$
$$q=\frac{16^2}{(-6)^3}$$
$$q=-\frac{32}{27} \qquad [A1]$$

5

10. (i) $f(1)=0$

$m-(5m-1)+(m+1)+m^2=0$ [M1]

$m^2-3m+2=0$

$(m-1)(m-2)=0$

$m=1 \quad or \quad m=2$ [M1]

$f(4)\neq 0$

$64m-16(5m-1)+4(m+1)+m^2\neq 0$ [M1]

$64m-80m+16+4m+4+m^2\neq 0$

$m^2-12m+20\neq 0$

$(m-2)(m-10)\neq 0$

$m\neq 2 \quad or \quad m\neq 10$ [A1]

$\therefore \quad m=1(shown)$

(ii) $x^3-4x^2+2x+1=(x-1)(x^2+bx-1)$ [M1]

Comparing coefficients of x,

$2=-1-b$

$b=-3$

$(x-1)(x^2-3x-1)=0$ [M1]

$x=1 \quad or \quad x=\dfrac{-(-3)\pm\sqrt{(-3)2-4(1)(-1)}}{2(1)}$

$=\dfrac{3\pm\sqrt{13}}{2}$

$x=1 \quad or \quad x=-0.30\ (2d.p.) \quad or \quad x=3.30\ (2d.p.)$ [A2]

11. (a) $y=(2k-1)x^2+2k+4 \quad ---(1)$

$y=3kx \quad ---(2)$

$(1)=(2)$,

$(2k-1)x^2+2k+4=3kx$

$(2k-1)x^2-3kx+2k+4=0$ [M1]

For 1 real root,

Discriminant = 0

$(-3k)^2-4(2k-1)(2k+4)=0$ [M1]

$9k^2-4(4k^2+8k-2k-4)=0$

$9k^2-16k^2-24k+16=0$

$7k^2+24k-16=0$ [M1]

$(7k-4)(k+4)=0$

$k=\dfrac{4}{7} \quad or \quad k=-4$ [A1]

(b) $(h+3)x^2 - 3x > x + h$

$(h+3)x^2 - 4x - h > 0$

For $(h+3)x^2 - 4x - h$ to be always positive,

Discriminant < 0

$(-4)^2 - 4(h+3)(-h) < 0$ $[M1]$

$16 + 4h^2 + 12h < 0$

$h^2 + 3h + 4 < 0$ $[M1]$

$(h+1.5)^2 + 1.75 < 0$ $[M1]$

Since $(h+1.5)^2 + 1.75$ is always positive,

there are no values of h such that $(h+1.5)^2 + 1.75 < 0$. $[A1]$

(c) $2x^2 + p = 2(x-1)$

$2x^2 - 2x + p + 2 = 0$

$$\begin{aligned}\text{Discriminant} &= (-2)^2 - 4(2)(p+2) \\ &= 4 - 8p - 16 \\ &= -12 - 8p \qquad [M1]\end{aligned}$$

If $p > -\dfrac{3}{2}$, then

$2p > -3$

$-8p < 12$

$-12 - 8p < 0$ [M1]

Since discriminant < 0, $2x^2 + p = 2(x-1)$ has no real roots. [A1]

7

12. (i) $2x^3-31x-27=A(x-4)(x+2)^2+Bx+C$

Sub $x=4$,

$2(4)^3-31(4)-27=4B+C$

$4B+C=-23\ ---(1)$

Sub $x=-2$,

$2(-2)^3-31(-2)-27=-2B+C$

$-2B+C=19\ ---(2)$ [$M1$]

$(1)-(2)\quad 6B=-42$

$B=-7$ [$A1$]

Sub $B=-7$ into (1),

$4(-7)+C=-23$

$C=5$ [$A1$]

Sub $x=0$, $B=-7$, $C=5$,

$-27=A(-4)(2)^2+5$

$A=2$ [$A1$]

$\therefore\quad A=2,\ B=-7,\ C=5$

(ii) $2x^3-31x-27=2(x-4)(x+2)^2-7x+5$

$$\frac{2x^3-31x-27}{(x-4)(x+2)^2}=2+\frac{-7x+5}{(x-4)(x+2)^2}$$

Let $\dfrac{-7x+5}{(x-4)(x+2)^2}=\dfrac{P}{x-4}+\dfrac{Q}{x+2}+\dfrac{R}{(x+2)^2}$ [$M1$]

$-7x+5=P(x+2)^2+Q(x-4)(x+2)+R(x-4)$

Sub $x=-2$,

$-7(-2)+5=-6R$

$R=-\dfrac{19}{6}$ [$M1$]

Sub $x=4$,

$-7(4)+5=36P$

$P=-\dfrac{23}{36}$ [$M1$]

Sub $x=0$,

$5=4\left(-\dfrac{23}{36}\right)-8Q-4\left(-\dfrac{19}{6}\right)$

$Q=\dfrac{23}{36}$ [$M1$]

$\therefore\quad \dfrac{2x^3-31x-27}{(x-4)(x+2)^2}=2-\dfrac{23}{36(x-4)}+\dfrac{23}{36(x+2)}-\dfrac{19}{6(x+2)^2}$ [$A1$]

8

TANJONG KATONG GIRLS' SCHOOL
MID-YEAR EXAMINATION 2016
SECONDARY THREE

4047 **ADDITIONAL MATHEMATICS**

Wednesday **04 May 2016** **2 h 15 min**

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.

Write in dark blue or black pen on both sides of the paper, and use a pencil for drawing graphs and diagrams. Do not use staples, highlighters or correction fluid.

Answer **all** the questions.

Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total marks for this paper is 90.

Setter : Mrs M Loy
Markers : Mr Koh MH, Miss Yeo LS, Mrs Loy, Mr Ang WJ

This Question Paper consists of 5 printed pages, including this page.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1. Given that $Ax^3 + x^2 - 13x - 2 = (x+3)(x+B)(2x-1) + C$ for all values of x, find A, B and C. [4]

2. Given that $y = e^{\ln\sqrt{3}}$, show that $y = \sqrt{3}$.
Hence, without using a calculator, evaluate

$$\frac{e^{\ln\sqrt{3}} \times \frac{1}{2}\log_3 9}{\log_9 3}$$ [4]

3. Find the range of values of c in the exact form, for which $y = 2x + c$ meets the curve $y^2 - 2x^2 = -5$. Hence deduce the range of values of c for which there is no intersection point between the line and the curve. [5]

4. Solve $\log_{100}(1+x) = \lg 4x - \lg\sqrt{8}$. [6]

5. A chicken farm with a population of 1000 chickens was hard hit by bird flu in 2015. The spread of the bird flu is given by $S = \dfrac{1000}{1+e^{2-t}}$, where S is the number of chickens infected after t days.

   (i) Deduce the number of chickens infected with the bird flu in the long run. [1]

   (ii) Estimate the initial number of chickens infected with the bird flu, leaving your answer correct to the nearest integer. [2]

   (iii) The chickens will be culled when at least 70% of the chickens are infected. Determine when culling will take place. [3]

6. A line and a curve are represented by $27^{2x} = \frac{1}{9}(3)^y$ and $\left(9^x\right)^y = 3$ respectively.
Given that the line intersects the curve at point A and point B, find the distance between the two points, A and B. [7]

7. **(a)** A toy car moved at a speed of $\left(2+\sqrt{3}\right)$ cm per second from point M to point N. Given that the distance covered was $\left(2\sqrt{75}-1\right)$ cm, find the time taken to move from point M to point N in the form $a\sqrt{3}+b$, where a and b are constants. [4]

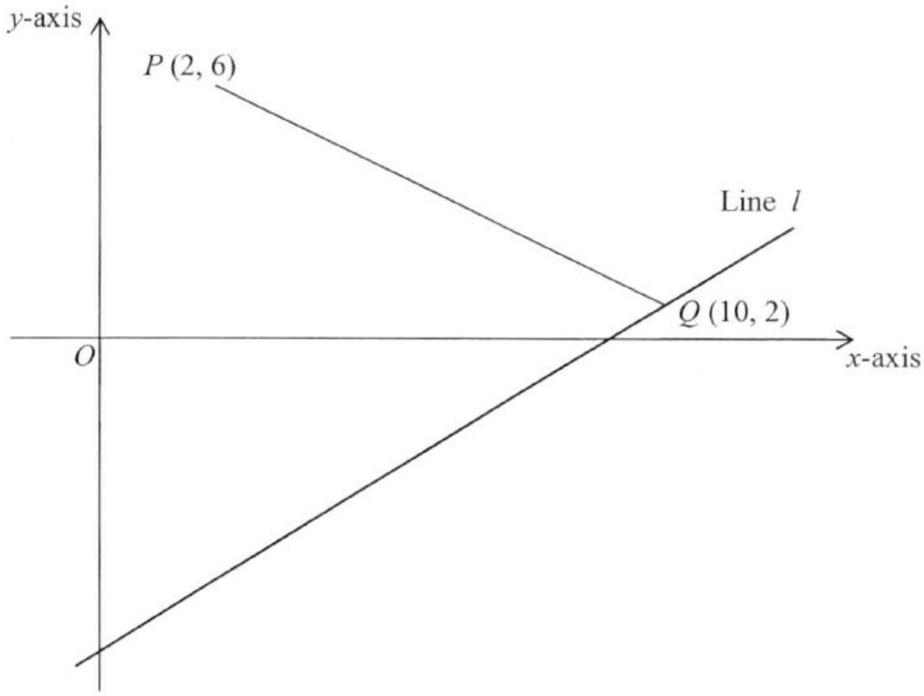
(b) Find the range of values of x that will satisfy the following inequalities, $2x+5>4$ and $6-2x^2 \geq 3+x$. [4]

8. The quadratic equation $2x^2-2x-1=0$ has roots $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$.

(i) Find the value of $\alpha^2+\beta^2$. [4]

(ii) Find the quadratic equation in x whose roots are α^3 and β^3. [4]

9.



In the diagram, PQ is a straight line joining points $P(2\ ,6)$ and $Q(10\ ,2)$.
Line l is parallel to the line $2y=x-4$ and passes through point Q.
Given that the perpendicular bisector of PQ intersects Line l at point R,

(i) find the coordinates of point R , [6]

(ii) calculate the area of the quadrilateral $PQRO$. [3]

10. The equation of a curve $y = ax^2 + 2x + 6$ can be written in the form $y = 3(x + b)^2 + c$, where a, b and c are constants.

(i) State the value of a .

Expressing $y = ax^2 + 2x + 6$ in the form $y = 3(x + b)^2 + c$, show that $b = \frac{1}{3}$ and find c . [5]

Hence,

(ii) find the greatest value of $\frac{2}{y}$. Explain your choice for the value of y, [3]

(iii) determine with explanation the number of points of intersection between the curve and the x-axis. [2]

11. The polynomial $g(x) = x^3 + ax^2 - bx - 2$ has a factor $(x + 1)$ and it leaves a remainder of 24 when divided by $(x - 2)$.

(i) Show that $a = 4$ and $b = -1$. [4]

(ii) Taking $a = 4$ and $b = -1$, solve the equation $g(x) = 0$, leaving your answers in the exact form.
Hence, find the integer value of x for which $(x-2)^3 + 4(x-2)^2 + x - 4 = 0$. [7]

12. **(a)** Solve $5^{x+1} - 2(5^{-x}) = 9$. [6]

(b) Express $\frac{4x^3 + 2x - 1}{(2x-1)(x+1)^2}$ in partial fractions. [6]

End of Paper

Suggested Answer Key

1	$A=2$, $B=-2$, $C=-8$	**2**	$2\sqrt{3}$
3	$c \le -\sqrt{5}, c \ge \sqrt{5}$ $-\sqrt{5} < c < \sqrt{5}$	**4**	$x=1$ $x=-0.5$ (rejected)
5i	1 000	**5ii**	119 (nearest integer)
5iii	2.85 days		
6	$\frac{2\sqrt{37}}{3}$		
7a	Time taken = $21\sqrt{3}-32$	**7b**	$-\frac{1}{2} < x \le 1$
8i	$\alpha^2+\beta^2=8$	**8ii**	$x^2+20x-8=0$
9i	$R(\frac{10}{3},-\frac{4}{3})$	**9ii**	Area = 38 units^2
10i	$a=3$, $c=\frac{17}{3}$	**10ii**	For greatest value of $\frac{2}{y}$, y must be min value, that is $y=c$. $\therefore \frac{2}{y}=\frac{2}{\left(\frac{17}{3}\right)}=\frac{6}{17}$
10iii	Discriminant < 0 thus there is not real roots, so there is no intersection point.		
11i	Show question	**11ii**	$x=-1,\ x=\frac{-3\pm\sqrt{17}}{2}$ Integer value of $x=1$
12a	$x=0.431$	**12b**	$2+\frac{2}{9(2x-1)}-\frac{28}{9(x+1)}+\frac{7}{3(x+1)^2}$

1. Given that $Ax^3+x^2-13x-2=(x+3)(x+B)(2x-1)+C$ for all values of x, find A , B and C . [4]

Comparing coefficient of x^3, $A = 2$ $2x^3+x^2-13x-2=(x+3)(x+B)(2x-1)+C$ Put $x = \ 3$, $2(\ 3)^3+(\ 3)^2+39\quad 2 = C$ $C = \ 8$ Put $x = 0$, $3B + C = \ 2$ $3B\quad 8 = \ 2$ $B = \ 2$	Choose appropriate value of x / expand and compare coefficients

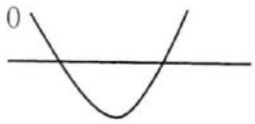
2. Given that $y=e^{\ln\sqrt{3}}$, show that $y=\sqrt{3}$.
Hence, without using a calculator, evaluate

$$\frac{e^{\ln\sqrt{3}}\times\frac{1}{2}\log_3 9}{\log_9 3}$$

[4]

$y=e^{\ln\sqrt{3}}$ $\ln y=\ln\sqrt{3}\ln e \qquad (\ln e = 1)$ $\ln y=\ln\sqrt{3}$ $\therefore y=\sqrt{3}$ (shown)	Bring ln to both sides & obtain equation
$\dfrac{e^{\ln\sqrt{3}}\times\frac{1}{2}\log_3 9}{\log_9 3}$	Put $e^{\ln\sqrt{3}}=\sqrt{3}$
$=\dfrac{\sqrt{3}\times\frac{1}{2}\log_3 3^2}{\frac{1}{2}\log_9 9}$	Apply Log Law correctly -power law -$\log_3 3 = 1$
$=\sqrt{3}\times 2$ $=2\sqrt{3}$	Answer in exact form

3. Find the range of values of c in the exact form, for which $y = 2x + c$ meets the curve $y^2 - 2x^2 = -5$. Hence deduce the range of values of c for which there is no intersection point between the line and the curve. [5]

Put $y = 2x + c$ into $y^2 - 2x^2 = -5$ $(2x+c)^2 - 2x^2 + 5 = 0$ $4x^2 + 4cx + c^2 - 2x^2 + 5 = 0$ $2x^2 + 4cx + c^2 + 5 = 0$	Combine to form a quadratic equation
For the line to meet the curve, discriminant ≥ 0 $(4c)^2 - 4(2)(c^2+5) \geq 0$ $16c^2 - 8c^2 - 40 \geq 0$ $8c^2 - 40 \geq 0$ $c^2 - 5 \geq 0$ 	Discriminant ≥ 0 *Deduct 1m if given discriminant >0*
$(c-\sqrt{5})(c+\sqrt{5}) \geq 0$ $c \leq -\sqrt{5},\ c \geq \sqrt{5}$ For no intersection points, $-\sqrt{5} < c < \sqrt{5}$	Obtain the factors

4. Solve $\log_{100}(1+x) = \lg 4x - \lg\sqrt{8}$. [6]

$\log_{100}(1+x) = \lg 4x - \lg\sqrt{8}$	Change of base to lg Apply Log Law to get $\lg\dfrac{4x}{\sqrt{8}}$ Apply Log Law to get lg $\left(\dfrac{16x^2}{8}\right)$

$\frac{\lg(1+x)}{\lg 10^2} = \lg\left(\frac{4x}{\sqrt{8}}\right)$ $\lg(1+x) = 2\lg\left(\frac{4x}{\sqrt{8}}\right)$ $\lg(1+x) = \lg\left(\frac{16x^2}{8}\right)$ $1+x = 2x^2$ $2x^2 = x+1$ $2x^2 - x - 1 = 0$ $(2x+1)(x-1) = 0$ $x = 1$, $x = -\frac{1}{2}$ (rejected) □ $\lg 4(-\frac{1}{2})$ is not defined.	Obtain factors If did not reject negative answer, no A1

5. A chicken farm with a population of 1000 chickens was hard hit by bird flu in 2015. The spread of the bird flu is given by $S = \frac{1000}{1+e^{2-t}}$, where S is the number of chickens infected after t days.

(i) Deduce the number of chickens infected with bird flu in the long run. [1]

As t becomes very large, e^{2-t} tends to 0. Number of chickens infected with flu = 1000	

(ii) Estimate the initial number of chickens infected with the bird flu, leaving your answer correct to the nearest integer. [2]

Put $t = 0$ $S = \frac{1000}{1+e^2}$ $S = 119.203$ Initial number of chickens infected with the bird flu = 119 (nearest integer)	Substitute $t = 0$

(iii) The chickens will be culled when at least 70% of the chickens are infected. Determine when culling will take place. [3]

$\dfrac{1000}{1+e^{2-t}} \geq 700$ $\dfrac{10}{7} \geq 1+e^{2-t}$ $2-t \leq -0.8473$ $t \geq 2.85$ After 2.85 days, culling will take place.	Form the linear inequality Solve linear inequality

6. A line and a curve are represented by $27^{2x} = \dfrac{1}{9}(3)^{y}$ and $(9^{x})^{y} = 3$ respectively. Given that the line intersects the curve at point A and point B, find the distance between the two points, A and B. [7]

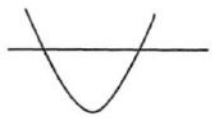
$27^{2x} = \dfrac{1}{9}(3)^{y}$ (3) $3^{6x} = 3^{-2}(3^{y})$ (2) $6x = y-2$ (1) $(9^{x})^{y} = 3$ $3^{2xy} = 3$ $2xy = 1$ From (2), $2x = \dfrac{1}{y}$ Put (3) into (1) $\dfrac{3}{y} = y-2$ $y^2-2y-3=0$ $(y-3)(y+1)=0$ $y=3$, $y=1$	 Form a linear equation Form a non-linear equation Apply elimination method to solve simultaneous equations Obtain A and B

$x=\frac{1}{6}$, $x=-\frac{1}{2}$ Distance between 2 points $=\sqrt{\left(\frac{1}{6}+\frac{1}{2}\right)^2+(3+1)^2}$ $=\sqrt{\frac{148}{9}}$ $=\frac{2\sqrt{37}}{3}$ units or 4.06 units	Apply distance formula correctly

7a. A toy car moved at a speed of $\left(2+\sqrt{3}\right)$cm per second from point M to point N.
Given that the distance covered was $\left(2\sqrt{75}-1\right)$ cm, find the time taken to move from point M to point N in the form $a\sqrt{3}+b$, where a and b are constants. [4]

Time taken $=\frac{2\sqrt{75}-1}{\left(2+\sqrt{3}\right)}$ $=\frac{\left(10\sqrt{3}-1\right)\left(2-\sqrt{3}\right)}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)}$ $=\frac{20\sqrt{3}-30-2+\sqrt{3}}{4-3}$ $=21\sqrt{3}-32$ seconds	Ratio of distance to speed Rationalise correctly Reduce denominator to 1

7b. Find the range of values of x that will satisfy the following inequalities,
$2x+5>4$ and $6-2x^2\geq 3+x$. [4]

$2x+5>4$ $2x>-1$ $x>-\frac{1}{2}$ 	Obtain $x >$ ½
$6-2x^2 \geq 3+x$ $2x^2+x-3 \leq 0$ $(2x+3)(x-1) \leq 0$ $-\frac{3}{2} \leq x \leq 1$	Solve quadratic inequality
To satisfy both inequalities: $-\frac{1}{2} < x \leq 1$	

8. The quadratic equation $2x^2-2x-1=0$ has roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

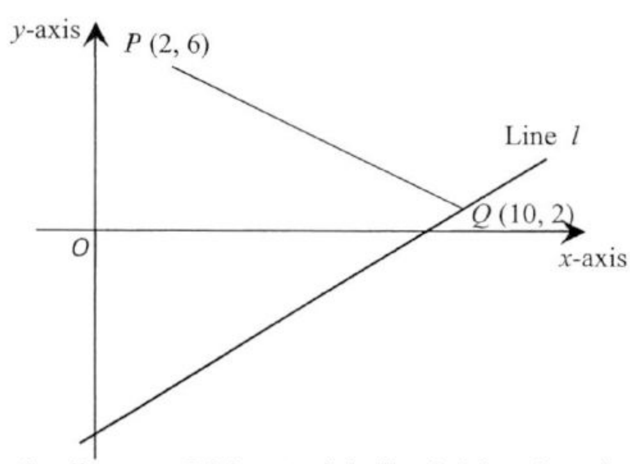
(i) Find the value of $\alpha^2+\beta^2$. [4]

Sum of roots $\frac{1}{\alpha}+\frac{1}{\beta}=1$ $\alpha+\beta=\alpha\beta$	Equate sum of roots =1
Product of roots $\frac{1}{\alpha\beta}=-\frac{1}{2}$ $\alpha\beta=-2$	Equate product of roots = $-\frac{1}{2}$
$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=(-2)^2-2(-2)$ $=8$	

(ii) Find the quadratic equation in x whose roots are α^3 and β^3. [4]

Sum of roots $=\alpha^3+\beta^3$ $=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=(-2)(8-[-2])$ $=-20$ Product of roots $=(\alpha\beta)^3$ $=(-2)^3$ $=-8$ Equation is $x^2+20x \quad 8=0$.	Equation '= 0'

9.



In the diagram, PQ is a straight line joining the points $P(2, 6)$ and $Q(10, 2)$.
Line l is parallel to the line $2y = x - 4$ and passes through point Q.
Given that the perpendicular bisector of PQ intersects Line l at point R,

(i) find the coordinates of point R, [6]

Midpoint of $PQ = (6, 4)$ Gradient of line $PQ = \frac{6-2}{2-(10)}$ $= -\frac{1}{2}$ Gradient of line perpendicular to $PQ = 2$ Equation of perpendicular bisector: $\frac{y-4}{x-6} = 2$ $y = 2x - 8$ Gradient of $RQ = \frac{1}{2}$ (given line is // to $2y = x - 4$) Equation of line RQ: $\frac{y-2}{x-10} = \frac{1}{2}$ $2y = x - 6$ Put (1) into (2) $2(2x \quad 8) = x \quad 6$ $x = \frac{10}{3}$ $y = 2\left(\frac{10}{3}\right) - 8$ $y = -\frac{4}{3}$ 4 $R = (\frac{10}{3}, -\frac{4}{3})$	Find midpt of PQ Apply gradient formula Obtain equation Obtain equation of RQ Solve simultaneous eq Obtain point R

(ii) calculate the area of the quadrilateral $PQRO$. [3]

	Area of $\otimes\ TQM = \frac{1}{2}\begin{vmatrix} 2 & 0 & \frac{10}{3} & 10 & 2 \\ 6 & 0 & \frac{-4}{3} & 2 & 6 \end{vmatrix}$ $= \frac{1}{2}\left[\left(\frac{20}{3} + 60\right) - \left(-\frac{40}{3} + 4\right)\right]$ $= 38 \text{ units}^2$	Apply 'shoe-laced' in anti-clockwise direction to find area evaluate

10. The equation of a curve $y = ax^2 + 2x + 6$ can be written in the form $y = 3(x + b)^2 + c$, where a, b and c are constants.

(i) State the value of a .

Expressing $y = ax^2 + 2x + 6$ in the form $y = 3(x + b)^2 + c$, show that $b = \frac{1}{3}$ and find c . [5]

$a = 3$ $y = a\left[x^2 + \frac{2}{a}x + \frac{6}{a}\right]$	Complete the square
$y = a\left[\left(x + \frac{2}{2a}\right)^2 + \frac{6}{a} - \left(\frac{2}{2a}\right)^2\right]$ $y = a\left(x + \frac{1}{a}\right)^2 + 6 - \frac{1}{a}$	Obtain the square term
$b = \frac{1}{a}$, $b = \frac{1}{3}$ (shown)	Able to show $b = \frac{1}{3}$
$c = 6 - \frac{1}{(3)}$, $c = \frac{17}{3}$	Obtain c

Hence,

(ii) find the greatest value of $\frac{2}{y}$. <u>Explain your choice</u> for the value of y . [3]

For y to be least, $3(x + b)^2 = 0$, $4y = c$	Explain the choice of y
Greatest value of $\frac{2}{y} = \frac{2}{(\frac{17}{3})}$ $= \frac{6}{17}$	Substitute correct y value

(iii) <u>determine with explanation</u> the number of points of intersection between the curve and the x-axis. [2]

$3x^2 + 2x + 6 = 0$	
Discriminant $= 2^2 \quad 4(3)(6)$ $= \quad 68$	Find discriminant
Since discriminant < 0, then there are no real roots. Thus there is no intersection point between the curve and the x-	Show discriminant < 0 and explain

axis. **<u>Or</u>** Put $y = 0$ (intersection with x-axis) $3x^2 + 2x + 6 = 0$ $x = \dfrac{-2 \pm \sqrt{4 - 4(3)(6)}}{2(3)}$ $x = \dfrac{-2 \pm \sqrt{-68}}{2(3)}$ No solution, then there is no intersection point between the curve and the x-axis.	solve the quadratic equation to obtain no solution explanation

11. The polynomial $g(x) = x^3 + ax^2 - bx - 2$ has a factor $(x + 1)$ and it leaves a remainder of 24 when divided by $(x \quad 2)$.

(i) Show that $a = 4$ and $b = \quad 1$. [4]

$(x + 1)$ is a factor, by Factor Thm, $g(\quad 1) = 0$	Apply Factor Thm

$1 + a + b \quad 2 = 0$ $a + b = 3 \qquad\qquad (1)$ By Remainder Thm, $g(2) = 24$ $2^3 + 4a \quad 2b \quad 2 = 24$ $2a \quad b = 9 \qquad\qquad (2)$ $(1) + (2) \quad 3a = 12$ $a = 4$ (shown) $4 + b = 3, \quad b = \quad 1$ (shown)	Obtain eqn (1) Apply Remainder Thm Obtain eqn (2)

(ii) Taking $a = 4$ and $b = \quad 1$, solve the equation $g(x) = 0$, leaving your answers in the exact form.
Hence, find the integer value of x for which $(x-2)^3 + 4(x-2)^2 + x - 4 = 0$. [7]

$g(x) = x^3 + 4x^2 + x - 2$ $x^3 + 4x^2 + x - 2 = (x+1)(x^2 + px - 2)$ where p is a constant compare coefficien t of x: $-2 + p = 1$ $p = 3$ $\therefore g(x) = (x+1)(x^2 + 3x - 2)$ $g(x) = 0, \ x = -1, \qquad x = \dfrac{-3 \pm \sqrt{9+8}}{2}$ $x = \dfrac{-3 \pm \sqrt{17}}{2}$ $(x-2)^3 + 4(x-2)^2 + x - 4 = 0$ $(x-2)^3 + 4(x-2)^2 + ((x-2) - 2 = 0$ Put $x = x - 2$ when $x = -1, \quad x - 2 = -1$ The integer va lue of $x = 1$	Obtain correct coefficient of x^2 and constant, $\quad 2$. $x = \quad 1$ apply quadratic formula to solve 2 other roots 2 correct answers Show the 2 equations are linked

12a. Solve $5^{x+1} - 2(5^{-x}) = 9$. [6]

$5^{x+1}-2(5^{-x})=9$ $5(5^x)-\frac{2}{5^x}=9$ Let $n=5^x$ $5n-\frac{2}{n}=9$ $5n^2-9n-2=0$ $(5n+1)(n-2)=0$ $n=2\quad,\quad n=-\frac{1}{5}$ $\therefore 5^x=2\qquad 5^x=-\frac{1}{5}$(rejected) $x\lg 5=\lg 2$ OR $x\ln 5=\ln 2$ $x=\frac{\lg 2}{\lg 5}$ $x=0.431$	Apply Indices Law to split the terms correctly Use substitution to form a quadratic equation Obtain $5^x=2$ & $5^x=\ \ 0.2$ Solve by bringing both sides to lg or ln

12b. Express $\frac{4x^3+2x-1}{(2x-1)(x+1)^2}$ in partial fractions. [6]

Let Let $\frac{4x^3+2x-1}{(2x-1)(x+1)^2}=2+\frac{A}{2x-1}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$, where A, B and C are constants $4x^3+2x-1=2(2x-1)(x+1)^2+A(x+1)^2+B(2x-1)(x+1)+C(2x-1)$ $x=-1,\quad -4-2-1=-3C\qquad \therefore C=\frac{7}{3}$ $x=\frac{1}{2,}\quad \frac{1}{2}+1-1=\frac{9}{4}A\qquad \therefore A=\frac{2}{9}$ $x=0,\quad -1=-2+A-B-C\qquad \therefore B=-\frac{28}{9}$ $\frac{4x^3+2x-1}{(2x-1)(x+1)^2}=2+\frac{2}{9(2x-1)}-\frac{28}{9(x+1)}+\frac{7}{3(x+1)^2}$	Obtain '2' either by inspection or long division Obtain 3 partial fractions Obtain correct values of A, B & C Express as partial fractions

CEDAR GIRLS' SECONDARY SCHOOL
End-of-Year Examination
Secondary Three

ADDITIONAL MATHEMATICS

4047
5 October 2016

2 hours 30 minutes

Additional Materials: Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper and Graph Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of **6** printed pages and **1** cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer **all** the questions.

1 The length and the width of a rectangular fish tank are $\left(\sqrt{2}+1\right)$ m and $\left(5-\sqrt{8}\right)$ m respectively. Given that the volume of the tank is 5 m^3, express the height of the tank in the form $a\sqrt{2}+b$ m, where a and b are constants. [4]

2 On the same diagram, sketch the graphs of $y^2 = 4x$ and $y = x^{\frac{3}{2}}$ for $x \geq 0$.
Find the x-coordinates of the points of intersection of the 2 graphs. [5]

3 The roots of the quadratic equation $2x^2 - 4x + 7 = 0$ are α and β.

(i) Find the value of

(a) $(\alpha+1)(\beta+1)$, [2]

(b) $\alpha^3 + \beta^3$. [2]

(ii) Find the quadratic equation whose roots are $\dfrac{\alpha^2}{\beta+1}$ and $\dfrac{\beta^2}{\alpha+1}$. [3]

4 The polynomial f(x) has degree of 4 and the coefficient of x^4 is 2. The roots of f(x) = 0 are 1, – 2 and a repeated root k, where k is a positive constant. f(x) has a remainder of 32 when divided by $x - 2$.

(i) Find the value of k. [3]

(ii) Hence, find an expression for f(x) in descending powers of x. [2]

5 Express $\dfrac{4x^3+5x-2}{x^4+x^2}$ as the sum of 3 partial fractions. [6]

6 **(a)** Sketch the graph of $y = \dfrac{2}{e^{2x}}$. [2]

(b) Solve the equation $\log_3(3x^2-6) - \log_{\sqrt{3}}(x-1) = 1$. [4]

(c) Given that $y = 2^x$, express $2^{3x-1} - 4^{3x} + 8^{x+1}$ in terms of y. [3]

7 The equation of a curve is $y = x^2 + kx + 8 - k$, where k is a constant.

(i) Find the range of values of k for which $y > 0$ for all real values of x. [3]

(ii) Given that $k = -1$, find the values of m for which the line $y = mx - 5$ is a tangent to the curve. [3]

8 (a) Find all the angles between 0 and 2π inclusive which satisfy the equation $3 + 2\sin x = 3\cos^2 x$. [4]

(b) Solve, for $0° \le x \le 360°$, the equation $\sec(2x - 50°) = 1.5$. [3]

9 (a) Find the exact value of $\cos 120° + \operatorname{cosec} 315°$. [3]

(b) Prove the identity $(\sec x - \tan x)(\sin x + 1) = \cos x$. [3]

(c) Given that $\sin x = \frac{5}{13}$ and $\tan y = -\frac{3}{4}$ and that x and y lie in the same quadrant, find the exact value of $\tan x + \cos y$. [4]

10 The function f is defined by $f(x) = 2\cos\frac{3}{2}x + 1$ for $0° \le x \le 360°$.

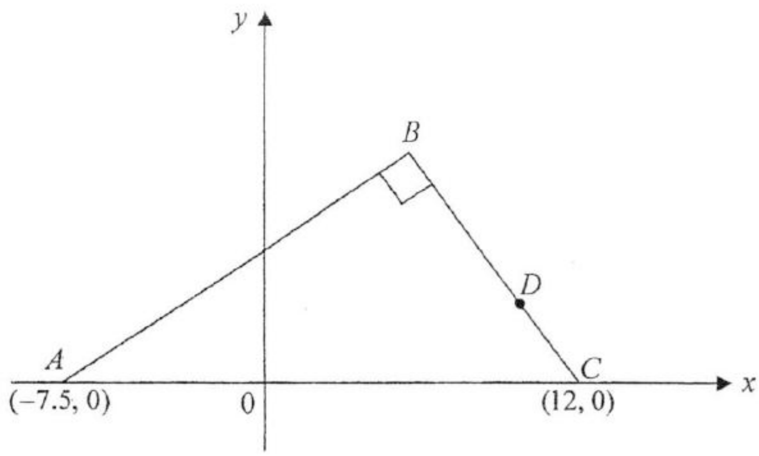
(i) State the period and amplitude of f. [2]

(ii) Find the coordinates of the maximum and minimum points of the function f. [3]

(iii) Find the x-coordinates of the points where the curve meets the x axis. [3]

(iv) Hence, sketch the graph of $y = f(x)$ for $0° \le x \le 360°$. [3]

11. **Solutions to this question by accurate drawing will not be accepted.**



The diagram above shows a right-angled triangle ABC. A is the point $(-7.5, 0)$, C is the point $(12, 0)$ and $\angle ABC$ is a right angle. The line AB is parallel to the straight line $2x-3y=10$.

(i) Find the coordinates of B. [5]

D is a point on the line BC such that the ratio of BD to DC is 2 : 1.

(ii) State the ratio of the area of triangle ABD to the area of triangle ABC. [1]

(iii) Find the coordinates of D. [2]

(iv) Calculate the area of triangle ABD. [2]

12 A circle C_1 passes through the points $X(1, 3)$ and $Y(5, 5)$, where XY is a diameter of the circle.

(i) Find the equation of the circle C_1. [3]

A second circle C_2 is a reflection of circle C_1 with $y=2$ as the line of reflection.

(ii) Find the equation of the circle C_2. [2]

A third circle C_3 has the equation $x^2+y^2-6x+8y-5=0$.

(iii) Find the coordinates of the centre and the radius of the circle C_3. [2]

(iv) Showing your working clearly, explain whether circle C_3 will intersect with circle C_1. [2]

13 Answer the whole of this question on a sheet of graph paper.

The variables x and y are related by the equation $y = \frac{a^x}{e^{3b}}$, where a and b are constants. The table below shows values of x and y.

x	1.0	1.5	2.0	2.5	3.0
y	0.05	0.22	1.00	4.48	20.09

(i) Draw a straight line graph of $\ln y$ against x, using a scale of 2 cm to represent 0.5 unit on the horizontal axis and 2 cm to represent 1 unit on the vertical axis. [2]

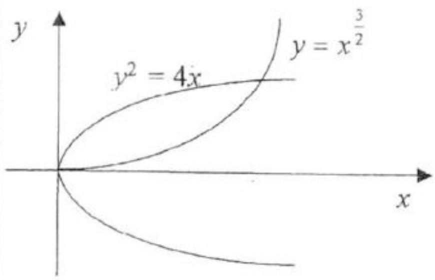
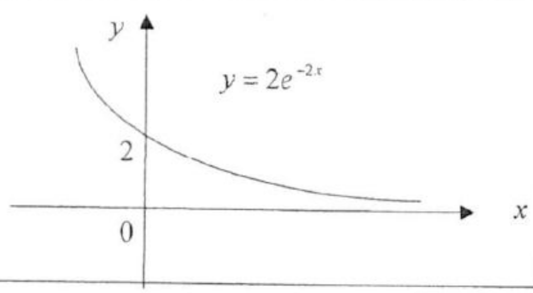
(ii) Use your graph to estimate the value of a and of b. [4]

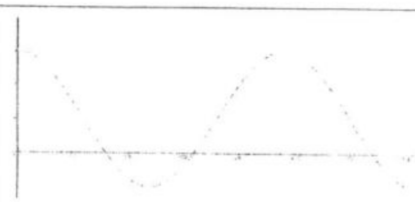
(iii) Use your graph to estimate the value of x when $y = 6$. [2]

(iv) By drawing a suitable straight line on the same diagram, find the value of x for which $(ea)^x = e^{3b-1}$. [3]

End Of Paper

CEDAR GIRLS' SECONDARY SCHOOL
SECONDARY 3 ADDITIONAL MATHEMATICS 4047
2016 End-Of-Year Examination
Answer Key

1	$\frac{15}{17}\sqrt{2}-\frac{5}{17}$
2	 $x=0, x=2, x=-2\ (rejected)$
3ia	6.5
3ib	-13
3ii	$26x^2+64x+49=0$
4i	$k=4$
4ii	$2x^4-14x^3+12x^2+64x-64$
5	$\frac{4x^3+5x-2}{x^4+x^2}=\frac{5}{x}-\frac{2}{x^2}+\frac{2-x}{x^2+1}$
6a	
6b	$x=1.5$
6c	$8\frac{1}{2}y^3-y^6$
7i	$-8<k<4$
7ii	$m=6.48,\quad m=-8.48\quad (3s.f.)$
8a	$x=0, \pi, 3.87, 5.55, 2\pi$
8b	$x=0.9°, 49.1°, 180.9°, 229.1°\quad (1\ d.p.)$

9a	$-\frac{1}{2}-\sqrt{2}$
9b	Proving
9c	$-1\frac{13}{60}$
10i	Amplitude =2 Period = 240°
10ii	Maximum points= (0°, 3), (240°, 3) Minimum points= (120°, −1), (360°, −1)
10iii	$x=80°, 160°, 320°$
10iv	
11i	$B=(6,9)$
11ii	2 : 3
11iii	$D=(10,3)$
11iv	58.5 units^2
12i	$(x-3)^2+(y-4)^2=5$
12ii	$(x-3)^2+y^2=5$
12iii	Centre = $(3,-4)$ Radius = $\sqrt{30}$ units
12iv	$r_1+r_3=7.71<8$ Hence C_3 will not intersect C_1.
13ii	$a=20.1$ (17.1 to 23.4) $b=-2$ (-1.96 to 2.04)
13iii	$x=2.6$ (2.55 to 2.65)
13iv	$x=1.25$ (1.2 to 1.3)

CHIJ ST. THERESA'S CONVENT
END-OF-YEAR EXAMINATION 2016
SECONDARY 3 EXPRESS

ADDITIONAL MATHEMATICS **4047**

05 Oct 2016
2 hours 30 minutes

Additional Material: Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your index number and name on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, staple all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Hand in questions 1 to 7 separately from questions 8 to 13

This document consists of 7 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

$R = \sqrt{a^2 + b^2}$

$\alpha = \tan^{-1}\left(\frac{b}{a}\right)$

1 Solve

(a) $4^{y-2} = 4\left(9^{3-y}\right)$. [3]

(b) $|4x - 7| = 6x$. [3]

2 (a) The curve $y = a(x+3)^b$ passes through the points $(0, 4)$, $(1, 3)$ and $(-6, k)$. Find the exact values of a, b and k. [4]

(b) Solve the equation $16^x = 66 - 4^{x-1}$. [4]

3 In the expansion of $\left(\frac{x}{2} + \frac{k}{x^2}\right)^9$ where k is a positive constant, the term independent of x is $10\frac{1}{2}$.

(i) Show that $k = 2$. [4]

(ii) With this value of k, find the coefficient of x^6 in the expansion of $\left(2x^6 - 128\right)\left(\frac{x}{2} + \frac{k}{x^2}\right)^9$. [4]

4 (i) Sketch the graph of $y = |2x - 5| - 3$, indicating clearly the coordinates of the turning point and of the points where the graph meets the x and y-axes. [4]

(ii) Hence, find the range of values of x for which $y < 0$. [1]

5 (a) Find the values of k for which the line $kx - 2y = 0$ is a tangent to the curve $y = x^2 - 3x + 4$. [4]

(b) A curve has the equation $y = kx^2 - 14x + 4k + 21$, where k is a constant. Find the range of values k for which $y > 0$ for all values of x. [4]

6 (i) Express $\dfrac{11-x}{(x-3)(x+5)}$ in partial fractions. [4]

(ii) Hence, solve the equation $\dfrac{11-x}{(x-3)(x+5)} + \dfrac{2}{x+5} = 4$. [2]

7 (a) Prove the identity $\dfrac{\sin 2\theta + \cos\theta}{\cos 2\theta - \sin\theta - 1} \equiv -\cot\theta$. [4]

(b) Given that $\sin x = -\dfrac{3}{5}$ and $\tan x > 0$, find, without using calculators, the value of

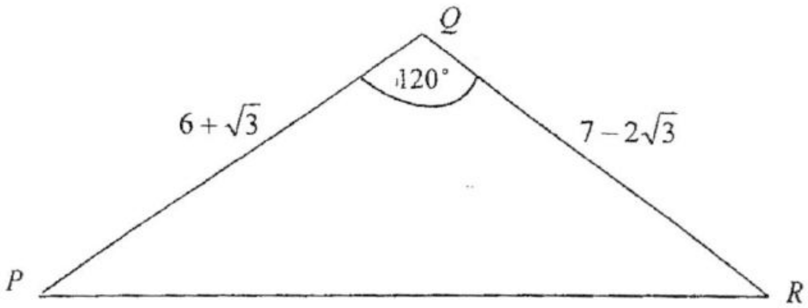
(i) $\cos x$, [1]
(ii) $\sin 2x$, [2]
(iii) $\cos\dfrac{1}{2}x$. [2]

Start Question 8 on a fresh sheet of Answer Paper.

Hand in Questions 8 to Question 13 separately from Question 1 to Question 7.

8 In triangle PQR, $PQ = 6+\sqrt{3}$ cm, $QR = 7-2\sqrt{3}$ cm and $\angle PQR = 120°$.

Express the area of PQR in the form ($p+q\sqrt{3}$), where p and q are rational numbers. . [4]

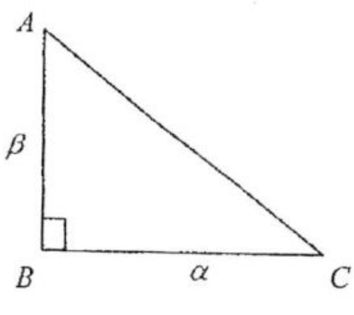


9 The roots of the quadratic equation $2x^2 - 6x + 1 = 0$ are α and β.

Find $\alpha^2 + \beta^2$ and hence, find [3]

(i) the quadratic equation whose roots are α^3 and β^3, [2]

(ii) the perimeter of a right-angled triangle ABC in the form $a+b\sqrt{2}$, if α and β represent the lengths, in cm, of the two shorter sides of the triangle as shown in the diagram below. [2]



10 The expression $f(x) = 3x^3 + ax^2 + bx - 3$, where a and b are constants, has a factor $x - 1$ and leaves a remainder of 33 when divided by $x - 4$.

(i) Find the value of a and of b. [4]

(ii) Using the values of a and b found in part **(i)**, show that $f(x)$ may be expressed in the form of $f(x) = (x-1)(3x^2 + px + q)$, where p and q are constants. [2]

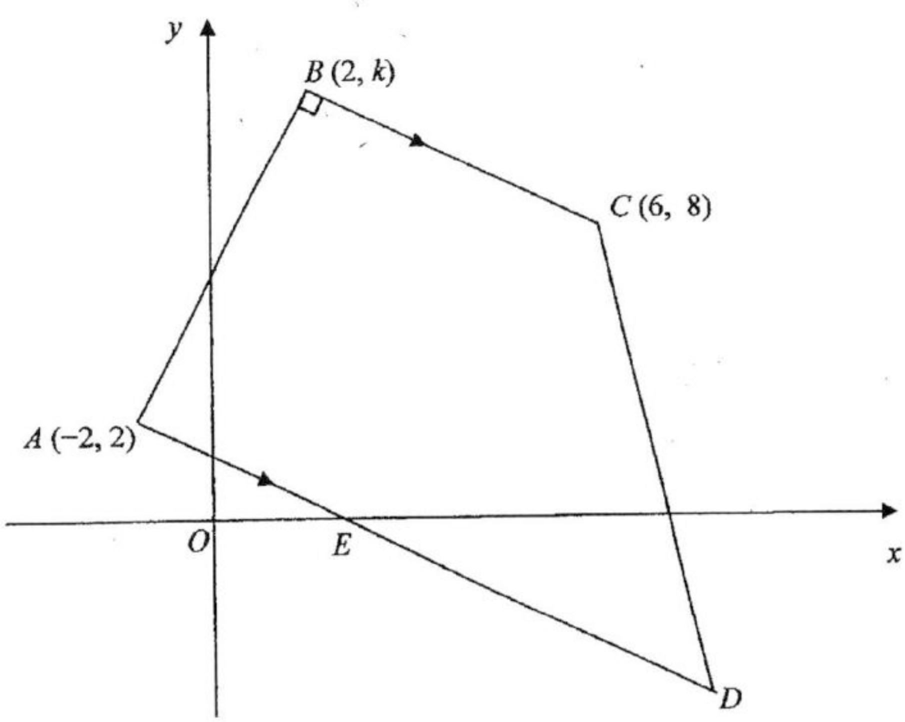
(iii) Solve $f(x) = 0$ and hence, solve the equation $\frac{3}{8}x^3 - \frac{13}{4}x^2 + \frac{13}{2}x - 3 = 0$ [5]

11 **Solutions to this question by accurate drawing will not be accepted.**

The diagram (not drawn to scale) shows a trapezium $ABCD$ in which AD is parallel to BC and AB is perpendicular to BC.

The coordinates of A, B and C are $(-2, 2)$, $(2, k)$ and $(6, 8)$ respectively.

AD cuts the x-axis at E and the gradient of CD is -3.



(i) Given that k is positive, find the value of k. [3]

(ii) Find the coordinates of E. [2]

(iii) Find the coordinates of D and hence, find the area of the trapezium $ABCD$. [4]

12 (a) Solve, for $0^\circ \le x \le 360^\circ$, the equation $2\sec^2 x = 5\tan x$. [4]

(b) Given that $y = 5\cos\theta + 2\sin\theta$, express $5\cos\theta + 2\sin\theta$ in the form of $R\cos(\theta - \alpha)$. [2]

Hence, for $0^\circ \le \theta \le 360^\circ$,

(i) state the maximum value of y and the corresponding value of θ. [2]

(ii) find the value of the acute angle θ when $y = 4$. [2]

13 (i) A curve has the equation $y = \cos 2x + 1$.

State the

(a) amplitude of the curve. [1]

(b) period of the curve. [1]

(c) maximum and minimum values of the curve. [1]

(ii) Sketch on the same diagram, the graphs of $y = \cos 2x + 1$ and $y = -2\sin x$ for the interval $0^0 \le x \le 360^0$. [4]

Hence,

(a) state the number of solutions for $\cos 2x + 1 = -2\sin x$ for $0^0 \le x \le 360^0$. [1]

(b) find the value of k given the equation $\cos 2x + k = -2\sin x$ has only one solution of x for $0^0 \le x \le 360^0$. [1]

End of Paper

(Have you checked your work?)

Sec 3 Exp. Add. Maths EOY 2016

1a) $4^{y-2} = 4(9^{3-y})$

$\frac{4^y}{16} = 4\left(\frac{9^3}{9^y}\right)$ M1

$36^y = 4(9)^3(16)$

$= 4(9)^3(4^2)$

$= (4^3)(9^3)$ M1

$= 36^3$

$\therefore \underline{\underline{y = 3}}$ A1

b) $|4x-7| = 6x$

$4x-7 = 6x$ or $4x-7 = -6x$ M1

$-7 = 2x$; $10x = 7$

$x = -3\frac{1}{2}$ (NA) A1 ; $\underline{\underline{x = \frac{7}{10}}}$ A1

6

2a) $y = a(x+3)^b$ $(0, 4)$ $(1, 3)$ $(-6, k)$

$$\left.\begin{aligned} 4 &= a(3)^b \quad —(1) \\ 3 &= a(4)^b \quad —(2) \\ k &= a(-3)^b \quad —(3) \end{aligned}\right\} \quad 1$$

$\frac{(2)}{(1)}$ $\frac{3}{4} = \left(\frac{4}{3}\right)^b$

$\left(\frac{4}{3}\right)^b = \left(\frac{4}{3}\right)^{-1}$

$\therefore b = -1$ 1

Sub $\rightarrow$ (1) $4 = a(3)^{-1}$

$12 = a$ 1

Sub $\rightarrow$ (3) $k = 12(-3)^{-1}$

$= \frac{12}{-3}$

$= -4$ 1

(4)

Ans: $a = 12$, $b = -1$, $k = -4$

b) $16^x = 66 - 4^{x-1}$

$(4^x)^2 = 66 - \frac{4^x}{4}$ 1

Let $y = 4^x$

$\therefore y^2 = 66 - \frac{y}{4}$ 1

$4y^2 + y - 264 = 0$

$(y-8)(4y+33) = 0$ 1

$y = 8$ or $y = -\frac{33}{4}$ (NA)

$2^{2x} = 2^3 \Rightarrow \underline{\underline{x = \frac{3}{2}}}$ 1

(4)

(8)

3) $\left(\frac{x}{2} + \frac{k}{x^2}\right)^9$

$$GT = {}^9C_r \left(\frac{x}{2}\right)^{9-r} \left(\frac{k}{x^2}\right)^r$$

$$= {}^9C_r \left(\frac{1}{2}\right)^{9-r} k^r x^{9-r} x^{-2r}$$

$$= {}^9C_r \left(\frac{1}{2}\right)^{9-r} k^r x^{9-3r} \quad 1$$

when $9-3r = 0$

$\Rightarrow r = 3 \quad 1$

$\therefore {}^9C_3 \left(\frac{1}{2}\right)^{9-3} k^3 = 10\frac{1}{2}$

$84\left(\frac{1}{64}\right) k^3 = \frac{21}{2}$

$k^3 = \frac{21}{2}(64)\left(\frac{1}{84}\right) \quad 1$

$= 8$

$\underline{\underline{k = 2}} \quad 1$ (4)

(ii) $(2x^6 - 128)\left(\frac{x}{2} + \frac{2}{x^2}\right)^9$

when $9 - 3r = 6 \Rightarrow 3r = 3 \Rightarrow r = 1 \quad 1$

Term in x^6 is ${}^9C_1 \left(\frac{1}{2}\right)^8 2^1 x^6$

$\frac{18}{256} x^6 \quad 1$

$\therefore (2x^6 - 128)\left(\frac{x}{2} + \frac{2}{x^2}\right)^9$

$= (2x^6 - 128)\left(\ldots\ldots + 10\frac{1}{2} + \frac{18}{256}x^6 + \ldots\right)$

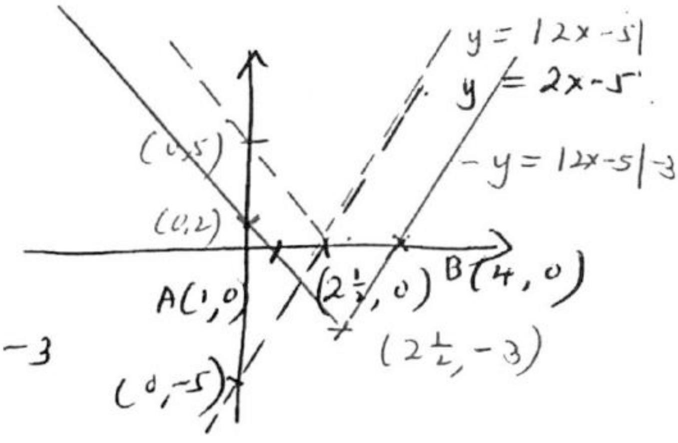
$= \ldots\ 2\left(\frac{21}{2}\right)x^6 - 128\left(\frac{18}{256}x^6\right) + \ldots$

$= 21x^6 - 9x^6 \quad 1$

$= 12x^6 \quad \therefore$ Coeff of $x^6 \underline{\underline{= 12}} \quad 1$ (4)

(8)

4 i) $y = |2x-5| - 3$



when $|2x-5| - 3 = 0$

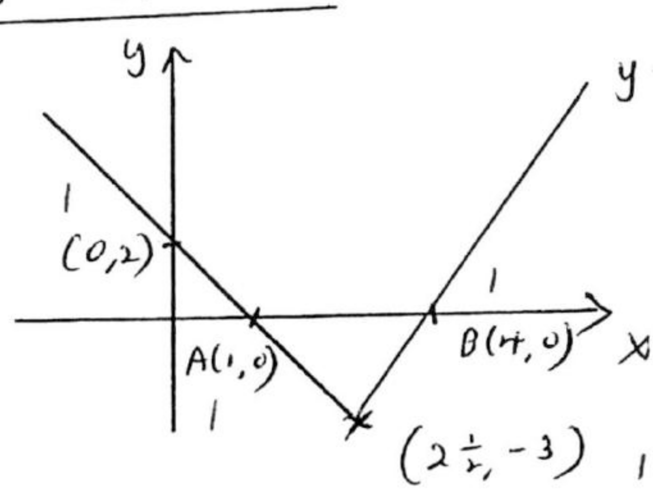
$|2x-5| = 3$

$2x-5 = 3$ or $2x-5 = -3$

$2x = 8$ $\quad$ $2x = 2$

$x = 4$ $\quad$ $x = 1$

$\therefore$ Final sketch



(4)

(ii) For $y < 0$, $\underline{\underline{1 < x < 4}}$ (1)

5 a)

$$kx - 2y = 0 \quad —①$$

$$y = x^2 - 3x + 4 \quad —②$$

Sub ② → ①

$$kx - 2(x^2 - 3x + 4) = 0$$

$$kx - 2x^2 + 6x - 8 = 0$$

$$2x^2 - kx - 6x + 8 = 0$$

$$2x^2 - (k+6)x + 8 = 0 \qquad 1$$

Since $b^2 - 4ac = 0$

$$[-(k+6)]^2 - 4(2)(8) = 0 \qquad 1$$

$$(k+6)^2 = 64$$

$$k + 6 = \pm 8 \qquad 1$$

$$k = 2 \text{ or } -14 \qquad 1$$

4

b)

$$y = kx^2 - 14x + 4k + 21$$

Graph is U shape ⇒ $k > 0$ —① 1

$$b^2 - 4ac < 0 \qquad 1$$

$$\therefore (-14)^2 - 4(k)(4k+21) < 0$$

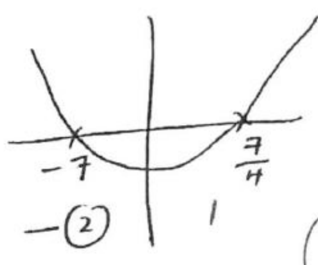
$$196 - 4k(4k+21) < 0$$

$$196 - 16k^2 - 84k < 0$$

$$16k^2 + 84k - 196 > 0$$

$$4k^2 + 21k - 49 > 0$$

$$(4k-7)(k+7) > 0$$



$$k < -7 \text{ or } k > \frac{7}{4} \quad —② \qquad 1$$

From ① & ② $k > \frac{7}{4}$ Ans. 1

S

8

4

6 i) $$\frac{11-x}{(x-3)(x+5)} = \frac{A}{x-3} + \frac{B}{x+5}$$ 1

$$= \frac{A(x+5)+B(x-3)}{(x-3)(x+5)}$$

$$\therefore\ 11-x = A(x+5)+B(x-3)$$ 1

when $x=-5$: $16 = -8B$

$$\Rightarrow B = -2$$ 1

when $x = 3$ $\quad 8 = 8A$

$$\Rightarrow A = 1$$ 1

$$\therefore \frac{11-x}{(x-3)(x+5)} = \underline{\underline{\frac{1}{x-3} - \frac{2}{x+5}}}$$ (4)

ii) $$\frac{11-x}{(x-3)(x+5)} + \frac{2}{x+5} = 4$$

$$\therefore \frac{1}{x-3} - \frac{2}{x+5} + \frac{2}{x+5} = 4$$

$$\therefore \frac{1}{x-3} = 4$$ 1

$$\frac{1}{4} = x-3$$

$$\underline{\underline{x = 3\tfrac{1}{4}}}$$ 1 (2)

(6)

7) a) To prove $\dfrac{\sin 2\theta + \cos\theta}{\cos 2\theta - \sin\theta - 1} = -\cot\theta$

Proof $LHS = \dfrac{2\sin\theta\cos\theta + \cos\theta}{1 - 2\sin^2\theta - \sin\theta - 1}$ B1, B1

$= \dfrac{\cos\theta\,(2\sin\theta + 1)}{-2\sin^2\theta - \sin\theta}$

$= \dfrac{\cos\theta\,(2\sin\theta + 1)}{-\sin\theta\,(2\sin\theta + 1)}$ 1

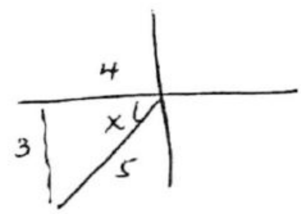
$= -\dfrac{\cos\theta}{\sin\theta}$ 1

$= -\cot\theta$ proved. (4)

b) $\sin x = -\frac{3}{5}$

$\cos x = -\frac{4}{5}$

$\tan x = \frac{3}{4}$



(i) $\cos x = -\frac{4}{5}$ 1

(ii) $\sin 2x = 2\sin x\cos x$

$= 2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right)$ 1

$= \frac{24}{25}$ 1

$\cos x = 2\cos^2\frac{1}{2}x - 1$

$\dfrac{\cos x + 1}{2} = \cos^2\frac{x}{2}$

$\dfrac{-\frac{4}{5} + 1}{2} = \cos^2\frac{x}{2}$

$\cos^2\frac{x}{2} = \frac{1}{10}$

$\cos\frac{x}{2} = \pm\frac{1}{\sqrt{10}}$ 1

Since $180^\circ < x < 270^\circ$

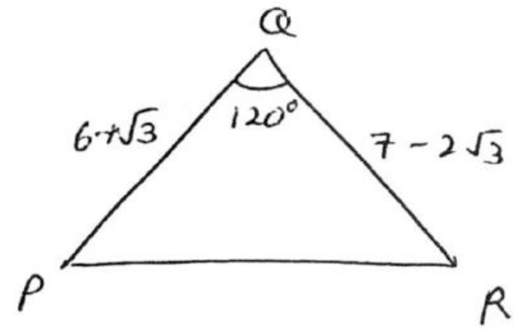
$90^\circ < \frac{x}{2} < 135^\circ$

$\Rightarrow \frac{x}{2}$ in 2nd Quad.

(5) $\therefore \cos\frac{x}{2} = -\frac{1}{\sqrt{10}}$ 1

(9)

8)



Area = '$\frac{1}{2}ab\sin C$'

$= \frac{1}{2}(6+\sqrt{3})(7-2\sqrt{3})\sin 120°$ 1

$= \frac{1}{2}(6+\sqrt{3})(7-2\sqrt{3})\frac{\sqrt{3}}{2}$

$= \frac{1}{4}(6\sqrt{3}+3)(7-2\sqrt{3})$ 1

$= \frac{1}{4}(42\sqrt{3}-36+21-6\sqrt{3})$

$= \frac{1}{4}(36\sqrt{3}-15)$ 1

$= 9\sqrt{3}-\frac{15}{4}$ 1

4

9) $2x^2 - 6x + 1 = 0$

$\alpha + \beta = 3$ 1

$\alpha\beta = \frac{1}{2}$ 1

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 9 - 1$

$= 8$ 1

(i) $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= (3)(8 - \frac{1}{2})$

$= 3(\frac{15}{2})$

$= \frac{45}{2}$ 1

$\alpha^3\beta^3 = (\frac{1}{2})^3$

$= \frac{1}{8}$

The eqn is $x^2 - \frac{45}{2}x + \frac{1}{8} = 0$ 1

(ii) $AC = \sqrt{\alpha^2 + \beta^2}$

$= \sqrt{8}$

$= 2\sqrt{2}$ 1

Perimeter $= \alpha + \beta + AC$

$= 3 + 2\sqrt{2}$ cm 1

7

10 i) $f(x) = 3x^3 + ax^2 + bx - 3$

$f(1) = 0$ — (1)

$f(4) = 33$ — (2)

From (1) $3 + a + b - 3 = 0 \Rightarrow a + b = 0$ — (3) 1

From (2) $192 + 16a + 4b - 3 = 33$

$16a + 4b = -156$

$4a + b = -39$ — (4) 1

(4) − (3) $3a = -39$

$\underline{\underline{a = -13}}$ 1

$\therefore \underline{\underline{b = 13}}$ 1

(4)

(ii) $f(x) = 3x^3 - 13x^2 + 13x - 3$

$$\begin{array}{rrrrr|l} & 3 & -13 & 13 & -3 & \\ +) & 0 & 3 & -10 & +3 & 1 \\ \hline & 3 & -10 & 3 & \boxed{0} & \end{array}$$

1

(2)

$\therefore f(x) = \underline{\underline{(x-1)(3x^2 - 10x + 3)}}$ 1

(iii) $f(x) = 0 \Rightarrow \underline{\underline{x = 1}}$ or $3x^2 - 10x + 3 = 0$

$(3x-1)(x-3) = 0$ 1

$\underline{\underline{x = \frac{1}{3} \text{ or } x = 3}}$ 1

$\frac{3}{8}x^3 - \frac{13}{4}x^2 + \frac{13}{2}x - 3 = 0$

$3\left(\frac{1}{2}x\right)^3 - 13\left(\frac{1}{2}x\right)^2 + 13\left(\frac{1}{2}x\right) - 3 = 0$ 1

let $y = \frac{x}{2}$

$\therefore 3y^3 - 13y^2 + 13y - 3 = 0$

$\therefore y = 1$ or $y = \frac{1}{3}$ or $y = 3$ 1

$\frac{x}{2} = 1$ $\quad \frac{x}{2} = \frac{1}{3}$ $\quad \frac{x}{2} = 3$

$\underline{\underline{x = 2 \quad x = \frac{2}{3} \quad x = 6}}$ 1

(5)

(11)

11) i) Grad of $AB = \frac{k-2}{2+2}$

$= \frac{k-2}{4}$

Grad of $BC = \frac{k-8}{2-6}$

$= \frac{k-8}{-4}$ 1

$\therefore \left(\frac{k-2}{4}\right)\left(\frac{k-8}{-4}\right) = -1$ 1

$(k-2)(k-8) = 16$

$k^2 - 10k + 16 - 16 = 0$

$k(k-10) = 0$

$k = 0$ or $\underline{\underline{k = 10}}$ 1 (3)

NA

(ii) Grad of $BC = \frac{10-8}{-4}$

$= -\frac{1}{2}$

Eqn of AD is $\frac{y-2}{x+2} = -\frac{1}{2}$

$2y - 4 = -x - 2$

$2y = -x + 2$ —(1) 1

when $y = 0$, $x = 2 \Rightarrow \underline{\underline{E \equiv (2, 0)}}$ 1

(2)

(iii) Eqn of CD is $\frac{y-8}{x-6} = -3$

$y - 8 = -3x + 18$

$y = -3x + 26$ —(2) 1

Sub (2) → (1)

$-6x + 52 = -x + 2$

$50 = 5x$

$x = 10$ 1

$\therefore y = -4$

$\therefore \underline{\underline{D \equiv (10, -4)}}$ 1

(9)

Area of $ABCD = \frac{1}{2}\begin{vmatrix} -2 & 10 & 6 & 2 & -2 \\ 2 & -4 & 8 & 10 & 2 \end{vmatrix}$

$= \frac{1}{2}\left| 152 + 8 \right|$ 1

$= \underline{\underline{80 \text{ unit}^2}}$ 1

(4)

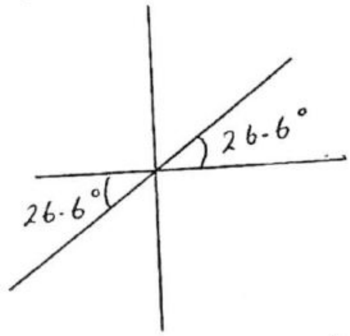
12a) $2\sec^2 x = 5\tan x$

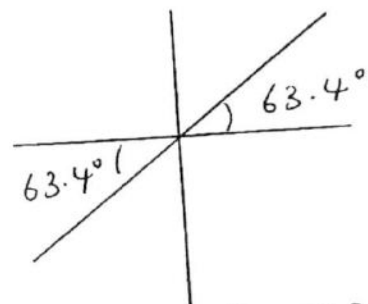
$2(1+\tan^2 x) = 5\tan x$

$2\tan^2 x - 5\tan x + 2 = 0$.

$(2\tan x - 1)(\tan x - 2) = 0$

$\tan x = \frac{1}{2}$ or $\tan x = 2$.





$x = 26.6°, 206.6°$

$x = 63.4°, 243.4°$

b) $y = 5\cos\theta + 2\sin\theta$.

$= R\cos(\theta - \alpha)$

$\tan\alpha = \frac{2}{5}$

$R = \sqrt{5^2 + 2^2}$

$\alpha = 21.8°$

$= \sqrt{29}$

$y = \sqrt{29}\cos(\theta - 21.8°)$

bi) max $y = \sqrt{29}$ when $\theta - 21.8014° = 0$.

$\theta = 21.8°$

bii) $\sqrt{29}\cos(\theta - 21.8014°) = 4$

$\cos(\theta - 21.8014°) = \frac{4}{\sqrt{29}}$

$\theta - 21.8014° = 42.0311°$

$\theta \approx 63.8°$.

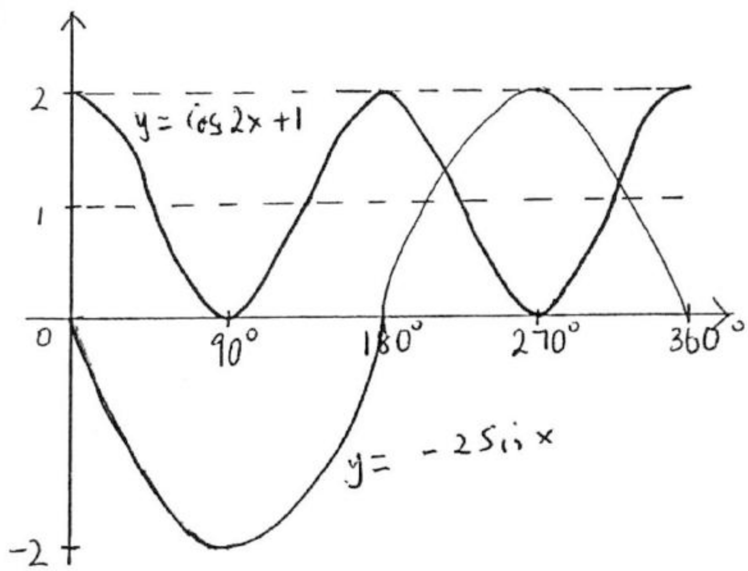
13i) $y = \cos 2x + 1$

a) Amplitude = 1

b) Period = $180°$

c) Max = 2 min = 0

ii)



a) $\cos 2x + 1 = -2\sin x$

There are $\underline{\underline{2}}$ solutions.

b) $\cos 2x + K = -2\sin x$

$K = 3$.

Name : ______________________________ Register Number : ______

Class : ______

Clementi Town Secondary School
End-of-Year Examination 2016
Secondary 3 Express

ADDITIONAL MATHEMATICS

4047
5 October 2016
2 hours 30 minutes

Additional Materials provided: Answer Paper (7 sheets)
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Do not open the booklets until you are told to do so.
Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the answer paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or in 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

This Question Paper consists of **6** printed pages, including this cover page.

[Turn over]

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer ***all*** questions on the answer paper provided.

1 Express $\dfrac{4x+7}{x^2+6x+9}$ in partial fractions. [4]

2 Find the value of m and of n such that $\dfrac{\sqrt{5}-\sqrt{3}}{2\sqrt{5}+5\sqrt{3}} = m + n\sqrt{15}$. [4]

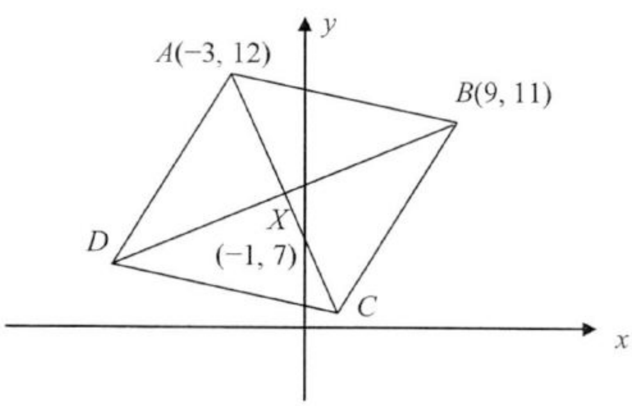
3 Find the first three terms in the expansion of $\left(3-x^2\right)^4$ in ascending powers of x.
Hence find the coefficient of x^4 in the expansion of $\left(1+x^2\right)\left(3-x^2\right)^4$. [5]

4 **(i)** Sketch the graph of $y = 1 + |3-2x|$ for $-1 \le x \le 3$. [3]

(ii) State the range of values of x for which $y < 2$. [2]

(iii) Find the range of values of m for which $1 + |3-2x| = m$ has two real roots. [1]

5 **Solutions to this question by accurate drawing will not be accepted.**



In the diagram above, $ABCD$ is a rhombus. A and B are $(-3, 12)$ and $(9, 11)$ respectively.
The diagonals of the rhombus intersect at $X(-1, 7)$.
Find

(i) the equation of line AC, [2]

(ii) coordinates of D, [2]

(iii) area of rhombus $ABCD$. [3]

[Turn over

6 The function f is defined, for $0 \leq x \leq 2\pi$, by $f(x) = 1 + 3\sin x$.

(i) State the amplitude and the period of f. [2]

(ii) Sketch the graph of $y = f(x)$. [3]

(iii) State the coordinates of the maximum point of the curve $y = f(x)$. [1]

(iv) Find the range of x when $y > 1$. [2]

7 **(a)** Given that $4x^3 - 6x^2 + ax + 3$ leaves a remainder of 7 when divided by $2x - 1$, find the value of a. [3]

(b) Given that $3x^2 - 11x + 3 = A(x-2)(x-1) + B(x-1) + C$ for all values of x, find the values of A, B and C. [5]

8 The quadratic equation $2x^2 - 3x + 4 = 0$ has roots α and β.

(i) Find the value of $\alpha^2 + \beta^2$. [3]

(ii) Find the quadratic equation whose roots are α^3 and β^3. [5]

9 **(a)** The straight line $y = 2p + 1$ intersects the curve $y = x + \dfrac{p^2}{x}$ at two distinct points. Find the range of values of p. [4]

(b) Find the range of values of k for which the straight line $y = 2x + k$ does not cut the curve $x^2 + y^2 = 20$. [5]

10 **(a)** Given that $\tan\theta = \dfrac{1}{p}$, where $180^\circ < \theta < 270^\circ$, express in terms of p,

(i) $\sin\theta$, [2]

(ii) $\cos(-\theta)$. [1]

(b) Solve, for angles between 0° and 360°, the equation $8\sin^2 x = 7$. [4]

(c) Solve, for angles between 0 and π, the equation $\tan(y - 0.2) = 1.2$. [2]

11 A circle, C, has equation $x^2 + y^2 - 10x + 6y + 9 = 0$.

(i) Find the coordinates of the centre of C and the radius of C. [2]

(ii) Give a reason why the y-axis is a tangent to C. [1]

(iii) The circle C crosses the x-axis at the point $P(1 , 0)$.
Show that the equation of the tangent to the circle C at P is $3y - 4x = -4$. [3]

(iv) Find the coordinates of the point where the circle C crosses the x-axis again. [2]

(v) Show that the point $S(6, 1)$ is inside the circle. [2]

12 **(a)** Solve $5^x = 6$. [2]

(b) Solve $e^x(2e^x - 1) = 10$. [4]

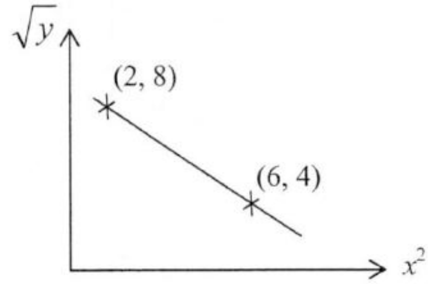
(c) Solve the simultaneous equations

$$\frac{27^x}{\sqrt{9^y}} = 3,$$

$$\log_2 x - 2 = \log_2 y .$$ [5]

[Turn over

13 **(a)**



The figure shows part of a straight line obtained by plotting $\sqrt{y}$ against x^2. The line passes through the points (2, 8) and (6, 4). Find y in terms of x. [3]

(b) **Answer this part of the question on a single sheet of graph paper.**
The table shows some experimental values of two variables, x and y, which are known to be related by the equation

$$y = \frac{a}{x} + \frac{b}{x^2}.$$

x	1.0	1.5	2.0	2.5	3.0
y	11.9	9.8	8.0	6.7	5.8

(i) Draw a straight line graph of xy against $\frac{1}{x}$, using a scale of 2 cm to represent 0.2 units on the $\frac{1}{x}$ - axis and 2 cm to represent 2 units on the xy - axis. [3]

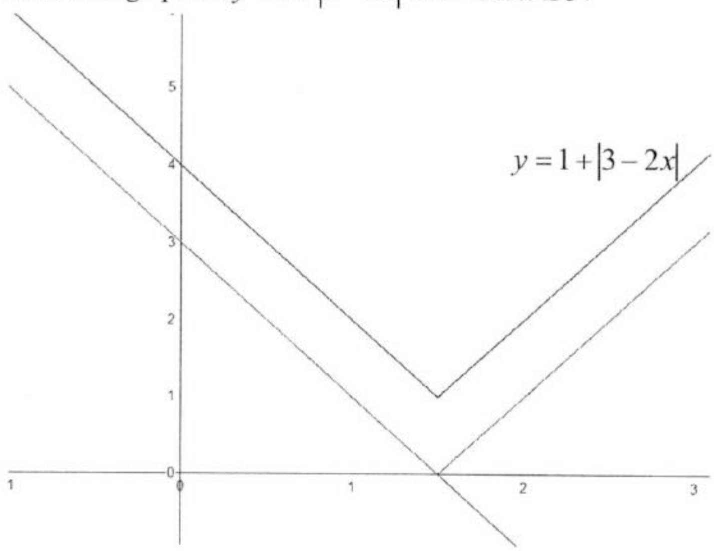
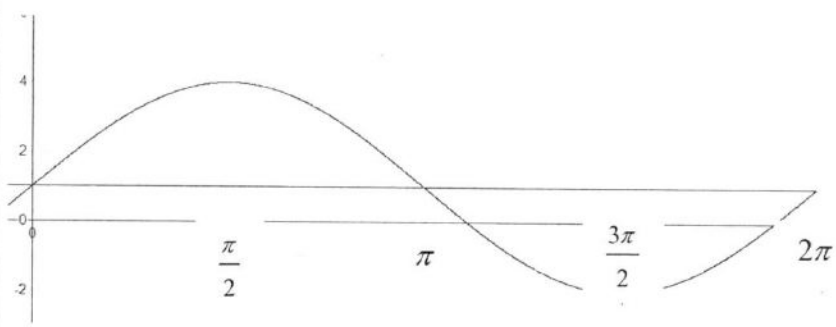
(ii) Use your graph to estimate

(a) the value of a and of b, [3]

(b) the value of x for which $y = \frac{13}{x}$. [2]

End of Paper

ANSWER SCHEME AM 3E 2016

1	$\dfrac{4}{x+3} - \dfrac{5}{(x+3)^2}$
2	$m = -\dfrac{5}{11}$ $\quad n = \dfrac{7}{55}$
3	First three terms of $\left(3 - x^2\right)^4$ $= 81 - 108x^2 + 54x^4$ Coeff $= -54$
4i	Sketch the graph of $y = 1 + \lvert 3 - 2x \rvert$ for $-1 \le x \le 3$. 
4ii	when $y < 2$, Draw $y = 2$ $\quad 1 < x < 2$
4iii	ANS $\quad m > 1$ for real roots
5i	$2y + 5x = 9$
5ii	$D = (-11, 3)$
5iii	area of rhombus $ABCD$=116 sq units
6	 (iii) max pt is $(\frac{\pi}{2}, 4)$ (iii) when $y > 1$ $\quad 0 < x < \pi$

	(draw $y = 1$)
7(a)	$a = 10$
(b)	$A = 3$
8(i)	$= -1\frac{3}{4}$
8(ii)	Required eqn is $x^2 + \frac{45}{8}x + 8 = 0$ OR $8x^2 + 45x + 64 = 0$
9a	$p > -\frac{1}{4}$
9b	$k > 10$ OR $k < -10$
10a	
(i)	$\sin\theta = -\frac{1}{\sqrt{1+p^2}}$
(ii)	$\cos(-\theta) = -\frac{p}{\sqrt{1+p^2}}$
10b	$x = 69.3°,\ 110.7°$ OR $x = 249.3°,\ 290.7°$
10c	1.08 radians
11i	$r = 5$ units
11ii	y-axis is the tangent.
11iii	$3y - 4x = -4$
11iv	(9, 0)
11v	S is in the circle.
12a	$x = 1.11$(3s.f.)
12b	$x = 0.916$(3s.f.)
12c	$y = \frac{1}{11}$ $x = 4(\frac{1}{11}) = \frac{4}{11}$
13a	$y = (10 - x^2)^2$
13b	Answers on graph paper
	END OF PAPER

O

Name	Register Number	Class

GREENRIDGE SECONDARY SCHOOL

End-of-Year Examination 2016
Secondary 3 Express

ADDITIONAL MATHEMATICS **4047/1**
Paper 1
October 2016
2 hours

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **ALL** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Setter: Mrs Goh-Kok Mei Leng

For Examiner's Use
80

This paper consists of <u>5</u> printed pages, including this cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.4$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B)$$

$$\sin A - \sin B = 2 \cos \tfrac{1}{2}(A+B) \sin \tfrac{1}{2}(A-B)$$

$$\cos A + \cos B = 2 \cos \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B)$$

$$\cos A - \cos B = -2 \sin \tfrac{1}{2}(A+B) \sin \tfrac{1}{2}(A-B)$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 The line $2y - x = 3$ meets the curve $x^2 - xy - y^2 = 1$ at the points P and Q. Show that the distance PQ can be expressed in the form $a\sqrt{b}$, where a and b are integers. [5]

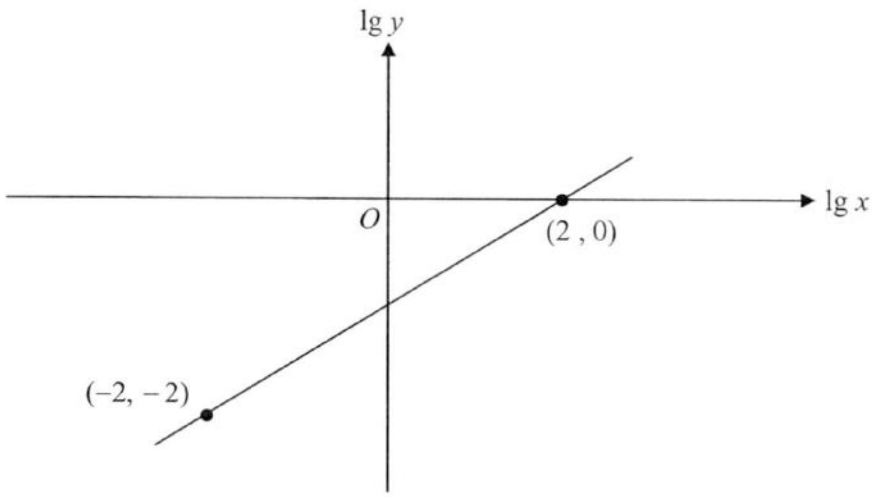
2 Simplify

(i) $(2\sqrt{3} - \sqrt{10})(3\sqrt{6} + 2\sqrt{5})$, [2]

(ii) $\sqrt[3]{16} + \sqrt[3]{250} - \sqrt[3]{\dfrac{125}{4}}$. [3]

3 Find the range of values of k for which the expression $k(x^2 + 2x + 3) - 4x - 2$ is always positive for all real values of x. [4]

4 The diagram shows part of the straight line graph drawn to represent the equation $y = ax^b$. Given that the straight line passes through (2, 0) and $(-2, -2)$, find the value of a and of b. [4]



5 $(x-2)$ is a factor of the polynomial $x^3 - 4x^2 + ax + b$, where a and b are constants. It leaves a remainder of -60 when the polynomial is divided by $x+3$.

(i) Find the value of a and of b. [4]

(ii) Factorise the polynomial completely and hence solve the equation $(x+1)^3 - 4(x+1)^2 + (x+1) + 6 = 0$. [5]

6 **(a)** Express $\dfrac{9x+6}{(2x-3)(x^2+1)}$ in partial fractions. [4]

(b) Divide $x^2 - x + 1$ by $x^2 - 5x - 6$. Hence, express $\dfrac{x^2 - x + 1}{x^2 - 5x - 6}$ in partial fractions. [5]

7 The equation $2x^2 - 3x + 4 = 0$ has roots α and β. Write down the value of $\alpha + \beta$ and $\alpha\beta$. Find the equation whose roots are $\alpha + \dfrac{1}{\alpha}$ and $\beta + \dfrac{1}{\beta}$. [7]

8 **(a)** Find, in ascending powers of x, the first 4 terms in the expansion of $\left(2x - \dfrac{1}{2}\right)^9$. Hence obtain the coefficient of x^7 in the expansion $(1-x)\left(2x - \dfrac{1}{2}\right)^9$. [4]

(b) The term independent of x in the binomial expansion of $\left(x^3 + \dfrac{a}{x^2}\right)^{10}$ is 210. Find the values of a. [4]

9 **(a)** Express $(11 + \sqrt{3}) - \left(\dfrac{13}{4 + \sqrt{3}}\right)^2$ in the form $a + b\sqrt{3}$, where a and b are integers to be determined. [4]

(b) Given that $\dfrac{9^{n+2} - 3^{2n+2}}{2^5} = 2^a 3^b$, where a and b are integers, find the value of a and express b in terms of n. [3]

10 Solve the following equations.

(i) $\log_5(x-1) + \log_5(x-2) = 2\log_5\sqrt{6}$ [3]

(ii) $\log_2 x = 4 - 3\log_x 2$ [4]

11 **(i)** Using the substitution $u = 2^x$, express the equation $8^x + 48 = 7(4^x)$ as a cubic equation in u. [2]

(ii) Show that $u = 4$ is the only integer solution of this equation. [3]

(iii) Hence find the integral value of x for $8^x + 48 = 7(4^x)$. [1]

12 The line $3y-2x=6$ meets the x-axis at A and the y-axis at B. Find

(i) the coordinates of A and B, [2]

(ii) the area of triangle OAB, where O is the origin, [1]

(iii) the line which is parallel to AB and which passes through the point $C(4,-4)$, [2]

(iv) the coordinates of the point D if $ABCD$ is a parallelogram, [2]

(v) the area of $ABCD$. [2]

End of Paper

Greenridge Secondary School
Preliminary Examination 2016
Secondary 4 Express Additional Mathematics Paper 1

Qn	Parts	Answer
1		$P(13,8)$ and $Q(-1,1)$ Dist = $7\sqrt{5}$ units
2	(i)	$8\sqrt{2}-2\sqrt{15}$
	(ii)	$\frac{9}{2}\sqrt[3]{2}$
3		$k<-2$ (NA), $k>1$
4		$a=\frac{1}{10}$, $b=\frac{1}{2}$
5	(i)	$a=1$, $b=6$
	(ii)	$f(x)=(x-2)(x-3)(x+1)$ $x=1,2,-2$
6	(a)	$\frac{6}{2x-3}-\frac{3x}{x^2+1}$
	(b)	$1+\frac{31}{7(x-6)}-\frac{3}{7(x+1)}$
7		$x^2-\frac{9}{4}x+\frac{13}{8}=0$
8	(a)	1824
	(b)	1 or -1
9	(a)	$-8+9\sqrt{3}$
	(b)	$a=-2$, $b=2n+2$
10	(i)	$x=4$ or -1 (NA)
	(ii)	8 or 2
11	(i)	$u^3-7u^2+48=0$
	(iii)	$x=2$
13	(i)	$A(-3,0)$ and $B(0,-2)$
	(ii)	3 unit^2

	(iii)	$y=\frac{2}{3}x-\frac{20}{3}$
	(iv)	$D(1,-6)$
	(v)	26 units^2

Class	Index Number	Name

新加坡海星中学

MARIS STELLA HIGH SCHOOL

SEMESTRAL EXAMINATION TWO

SECONDARY THREE

ADDITIONAL MATHEMATICS **14 October 2016**

2 hours

Additional Materials:
Writing paper (6 sheets)

INSTRUCTIONS TO CANDIDATES

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give your answer to three significant figures. Give answer in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

This document consists of 5 printed pages and 1 blank page.

1. Without using the calculator, show that $\cos\left(\frac{5\pi}{12}\right)+\cos\left(\frac{\pi}{12}\right)=\frac{\sqrt{6}}{2}$. [4]

2. Express $\log_7 x-\log_{49}(x-2)=\log_3 1$ in the form $ax^2+bx+c=0$ and hence explain why there are no real solutions to the equation. [4]

3 The length and the width of a closed rectangular tank are $\left(1+\sqrt{2}\right)$ m and $\left(3-\sqrt{8}\right)$ m respectively. If the volume of the rectangular tank is 2 m^3, find the height of the tank in the form $\left(a+b\sqrt{2}\right)m$, where a and b are integers. [4]

4. If $\frac{1}{p}=\frac{\operatorname{cosec}x-1}{\cot x}$, prove that $p=\frac{\operatorname{cosec}x+1}{\cot x}$. Hence find cos x in terms of p. [5]

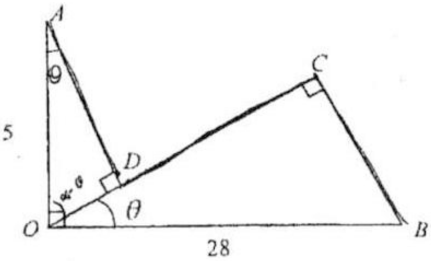
5. (i) Calculate the coordinates of the points of intersection of the graph $y=|2x-5|-3$ with the coordinates axes. [3]

(ii) Hence sketch the graph of $y=|2x-5|-3$. [2]

6. (i) Prove that $\operatorname{cosec}2\theta+\cot 2\theta=\cot\theta$ [3]

(ii) Hence, find, in radians, the angle for which $\operatorname{cosec}2\theta+\cot 2\theta=3\tan\theta$ where $0\leq\theta\leq\pi$. [3]

7.



The diagram shows three fixed points O, A and B such that OA = 15 cm and $OB=28$ cm and $\angle AOB=\angle ADO=\angle OCB=90°$.
The line OC makes an angle θ with the line OB, the angle θ can vary in such a way that the point D lies along the line OC. Given that $L=AD+DC+CB$,

(i) show that $L=\left(43\cos\theta+13\sin\theta\right)$ cm, [3]

(ii) express L in the form of $R\cos(\theta-\alpha)$, where R is positive and α is acute, [2]

(iii) find the value of θ for which L = 40 cm. [3]

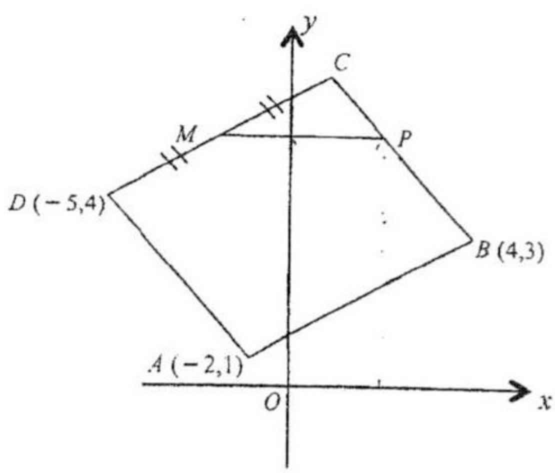
3

8. The equation of the curve is $y = kx^2 + 4x + 3 + k$, where k is a constant.
   (i) Find the range of values of k for which the curve lies completely above the x-axis. [4]
   (ii) In the case where $k = 2$, find the values of m for which the line $y = mx - 3$ is a tangent to the curve. [4]

9. (i) Show that $x - 3$ is the factor of the cubic polynomial $2x^3 - 9x^2 + 27$. Hence factorise completely $2x^3 - 9x^2 + 27$. [3]
   (ii) Express $\dfrac{(x+3)^2}{2x^3 - 9x^2 + 27}$ as the sum of 3 partial fractions. [5]

10. The function f is defined by $f(x) = 2\cos^2 x - 6\sin^2 x$.
   (i) Show that f(x) can be expressed as $4\cos 2x - 2$. [2]
   (ii) State the minimum value of f(x). [1]
   (iii) State the period of f(x). [1]
   (iv) Sketch the graph of $y = |f(x)|$ for $0 \le x \le \pi$. Given that the number of solutions of $|f(x)| = c$ is equal to 4, find the range of values of c. [3]

11. In the diagram, $ABCD$ is a parallelogram. The vertices A, B and D have coordinates $(-2,1)$, $(4, 3)$ and $(-5,4)$ respectively. M is the midpoint of CD. A line is drawn from M parallel to the x- axis to cut the side BC at P. Find
   (i) the coordinates of C and M, [3]
   (ii) the equation of the line BC, [3]
   (iii) the coordinates of P, [2]
   (iv) the area of $ABPMD$. [2]



12. (a) Sketch the graph of $y = x^{\frac{2}{3}}$. [2]

(b) A circle, C_1, has the equation $x^2 + y^2 - 6x + 8y - 24 = 0$.

(i) Find and the coordinates of the centre of C_1 and the radius of the circle. [3]

A second circle, C_2, has a diameter SR. The point R has coordinates $(2,-2)$ and the equation of the tangent to C_2 at S is $4y - 3x = 36$.

(ii) Find the equation of SR and hence, show that the coordinates of S is $(-4,6)$. [4]

(iii) Find the radius and the coordinates of the centre of C_2. [2]

- End of Paper -

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For Examiner's Use

1. $\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

*Note: $\frac{5\pi}{12} = \frac{5}{12}\times 180°$
$= 75°$
$\frac{\pi}{12} = 15°$
$\cdot$ $75° = 45° + 30°$
$\cdot$ $15° = 45° - 30°$

$\therefore \cos\left(\frac{5\pi}{12}\right) + \cos\left(\frac{\pi}{12}\right)$

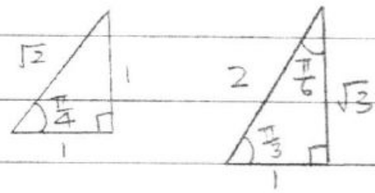
$= \left(\cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}\right) + \left(\cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}\right)$

$= 2\cos\frac{\pi}{4}\cos\frac{\pi}{6}$

$= 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{3}}{\sqrt{2}}$

$= \frac{\sqrt{6}}{2}$ // (shown)



2. $\log_7 x - \log_{49}(x-2) = \log_3 1$

$\log_7 x = 0 + \log_{49}(x-2)$

$\log_7 x = \frac{\log_7 (x-2)}{\log_7 49}$

$= \frac{\log_7 (x-2)}{\log_7 7^2}$

$= \frac{\log_7 (x-2)}{2}$

$2\log_7 x = \log_7 (x-2)$

$\log_7 x^2 = \log_7 (x-2)$

$\therefore x^2 = x - 2$

$x^2 - x + 2 = 0$ //

Discriminant $= (-1)^2 - 4(1)(2)$

$= 1 - 8$

$= -7$

Since discriminant < 0, the equation has no real solutions. //

For Examiner's Use

3. Let the height of the tank be h m.

$$(1+\sqrt{2})(3-\sqrt{8})(h) = 2$$
$$(3-\sqrt{8}+3\sqrt{2}-\sqrt{16})(h) = 2$$
$$(3-2\sqrt{2}+3\sqrt{2}-4)(h) = 2$$
$$(\sqrt{2}-1)(h) = 2$$
$$h = \frac{2}{\sqrt{2}-1}$$
$$= \frac{2(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$$
$$= \frac{2\sqrt{2}+2}{2-1}$$
$$= 2+2\sqrt{2}$$

$\therefore$ Height of tank is $(2+2\sqrt{2})$ m. //

4. $$\frac{1}{p} = \frac{\operatorname{cosec} x - 1}{\cot x}$$
$$p = \frac{\cot x}{\operatorname{cosec} x - 1}$$
$$= \frac{\cot x(\operatorname{cosec} x + 1)}{(\operatorname{cosec} x - 1)(\operatorname{cosec} x + 1)}$$
$$= \frac{\cot x(\operatorname{cosec} x + 1)}{\operatorname{cosec}^2 x - 1}$$
$$= \frac{\cot x \cdot (\operatorname{cosec} x + 1)}{\cot^2 x}$$
$$= \frac{\operatorname{cosec} x + 1}{\cot x} \quad \text{(proven)} //$$

$$\frac{1}{p} = \frac{\frac{1}{\sin x} - 1}{\frac{\cos x}{\sin x}} = \frac{1-\sin x}{\cos x}$$

$$p = \frac{\frac{1}{\sin x} + 1}{\frac{\cos x}{\sin x}} = \frac{1+\sin x}{\cos x}$$

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(cont'd)

4 $\frac{1}{p} + p = \frac{1-\sin x}{\cos x} + \frac{1+\sin x}{\cos x}$

$\frac{1}{p} + \frac{p^2}{p} = \frac{(1-\sin x) + (1+\sin x)}{\cos x}$

$\frac{1+p^2}{p} = \frac{2}{\cos x}$

$\cos x(1+p^2) = 2p$

$\therefore \cos x = \frac{2p}{1+p^2}$ //

5(i) $y = |2x-5| - 3$

When $y = 0$,

$0 = |2x-5| - 3$

$3 = |2x-5|$

$2x - 5 = 3$ or $2x - 5 = -3$

$2x = 8$ $\quad$ $2x = 2$

$x = 4$ $\quad$ $x = 1$

$\therefore$ The x-intercepts are $(4,0)$ and $(1,0)$ //

When $x = 0$,

$y = |2(0) - 5| - 3$

$= 5 - 3$

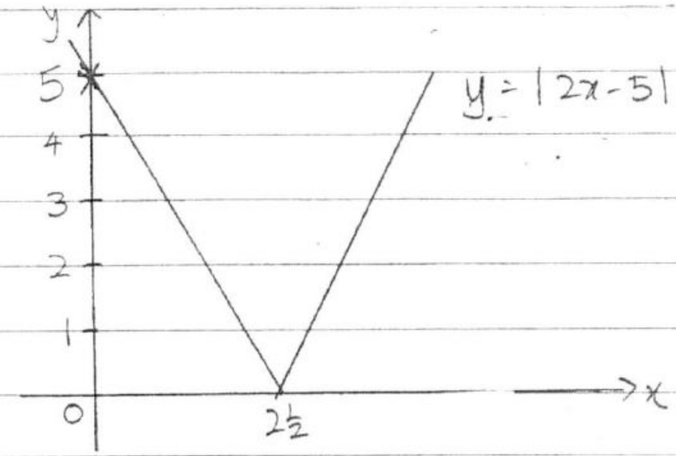
$= 2$

$\therefore$ The y-intercept is $(0,2)$ //

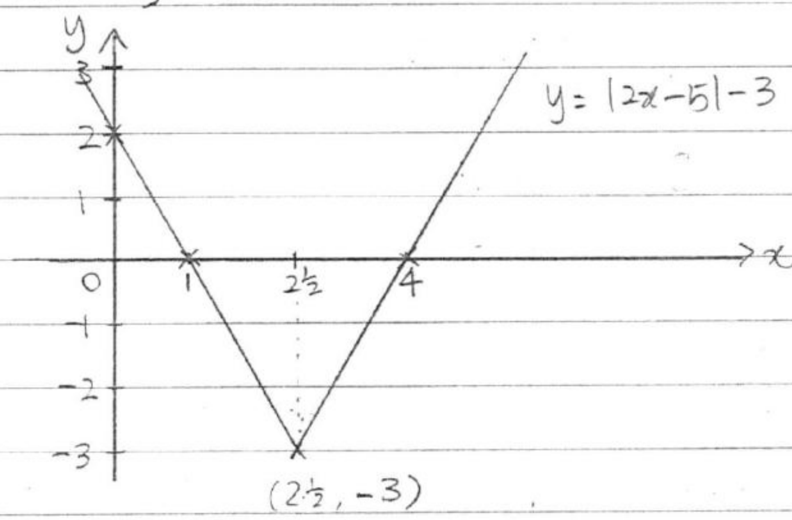
① Sketch: $y = |2x-5|$

5(ii) • When $y = 0$

$$|2x-5| = 0$$
$$2x-5 = 0$$
$$x = 2\tfrac{1}{2}$$



② sketch: $y = |2x-5| - 3$



This is the required sketch.

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6 (i) $LHS = \text{cosec}\, 2\theta + \cot 2\theta$

$= \frac{1}{\sin 2\theta} + \frac{\cos 2\theta}{\sin 2\theta}$

$= \frac{1+\cos 2\theta}{\sin 2\theta}$

$= \frac{\cos^2\theta + \sin^2\theta + \cos^2\theta - \sin^2\theta}{2\sin\theta\cos\theta}$

$= \frac{2\cos^2\theta}{2\sin\theta\cos\theta}$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta = RHS$ (proven) //

(ii) $\text{cosec}\, 2\theta + \cot 2\theta = 3\tan\theta$

$\cot\theta = 3\tan\theta$

$\frac{1}{\tan\theta} = 3\tan\theta$

$3\tan^2\theta = 1$

$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\frac{1}{\sqrt{3}}$ (θ* in quadrants 1, 2, 3 & 4)

Basic $\angle = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{\pi}{6}$

$\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$= \frac{\pi}{6}, \frac{5\pi}{6}, \underbrace{\frac{7\pi}{6}, \frac{11\pi}{6}}_{(\text{reject} \because 0<\theta<\pi)}$

$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}$ //

For Examiner's Use

7.(i)

$\cos\theta = \frac{AD}{15}$

$AD = 15\cos\theta$ cm

$\sin\theta = \frac{OD}{15}$

$OD = 15\sin\theta$ cm

$\cos\theta = \frac{OC}{28}$

$OC = 28\cos\theta$ cm

$\therefore DC = OC - OD$

$= (28\cos\theta - 15\sin\theta)$ cm

$\sin\theta = \frac{BC}{28}$

$BC = 28\sin\theta$ cm

$L = 15\cos\theta + (28\cos\theta - 15\sin\theta) + 28\sin\theta$

$= (43\cos\theta + 13\sin\theta)$ cm (shown)//

(ii)

$43\cos\theta + 13\sin\theta = R\cos(\theta - \alpha)$

$= R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$

$= R\cos\alpha\cos\theta + R\sin\alpha\sin\theta$

$R\cos\alpha = 43$ and $R\sin\alpha = 13$

$\tan\alpha = \frac{13}{43}$

$\alpha \approx 16.8214°$

$R = \sqrt{13^2 + 43^2}$

$= \sqrt{2018}$

$\therefore L \approx \sqrt{2018}\cos(\theta - 16.8°)$//

(iii)

$\sqrt{2018}\cos(\theta - 16.8214°) = 40$

$\cos(\theta - 16.8214°) = \frac{40}{\sqrt{2018}}$

Basic $\angle = \cos^{-1}\left(\frac{40}{\sqrt{2018}}\right)$

$\approx 27.072766°$

$\therefore \theta - 16.8214° = 27.072766°, \; 360° - 27.072766°$

$\theta = 43.9°, \; 349.7°$ (reject)

$\therefore \theta = 43.9°$//

For Examiner's Use

8(i) $y = kx^2 + 4x + (3+k)$

For curve to lie completely above x-axis, discriminant < 0

$$(4)^2 - 4(k)(3+k) < 0$$
$$16 - 12k - 4k^2 < 0$$
$$k^2 + 3k - 4 > 0$$
$$(k-1)(k+4) > 0.$$

$\therefore\ k < -4$ or $k > 1$. $\rightarrow \because k > 0,\ \therefore k > 1$

(ii) $y = 2x^2 + 4x + 5$ — ①
$y = mx - 3$ — ②

Sub ② into ①,

$$mx - 3 = 2x^2 + 4x + 5$$
$$2x^2 + 4x - mx + 5 + 3 = 0$$
$$2x^2 + (4-m)x + 8 = 0$$

For line to be tangent to curve, discriminant $= 0$.

$$(4-m)^2 - 4(2)(8) = 0$$
$$16 - 8m + m^2 - 64 = 0$$
$$m^2 - 8m - 48 = 0$$
$$(m+4)(m-12) = 0$$
$$\therefore\ m = -4 \text{ or } 12$$

For Examiner's Use

9(i) Let $f(x) = 2x^3 - 9x^2 + 27$.

$f(3) = 2(3)^3 - 9(3)^2 + 27$

$= 54 - 81 + 27$

$= 0$

$\therefore (x-3)$ is a factor of $f(x)$. (Shown)

$f(x) = (x-3)(2x^2 + bx - 9)$

comparing coefficient of x^2,

$-9 = -6 + b$

$b = -3$

$\therefore f(x) = (x-3)(2x^2 - 3x - 9)$

$= (x-3)(2x+3)(x-3)$

$= (x-3)^2(2x+3)$

(ii) $\dfrac{(x+3)^2}{2x^3 - 9x^2 + 27} = \dfrac{(x+3)^2}{(x-3)^2(2x+3)}$

Let $\dfrac{(x+3)^2}{(x-3)^2(2x+3)} = \dfrac{A}{x-3} + \dfrac{B}{(x-3)^2} + \dfrac{C}{2x+3}$

$(x+3)^2 = A(x-3)(2x+3) + B(2x+3) + C(x-3)^2$

When $x = 3$,

$(3+3)^2 = 0 + B[2(3)+3] + 0$

$36 = 9B$

$B = 4$

When $x = -\frac{3}{2}$

$(-\frac{3}{2}+3)^2 = 0 + 0 + C[(-\frac{3}{2}) - 3]^2$

$\frac{9}{4} = \frac{81}{4}C$

$C = \frac{1}{9}$

$\longrightarrow$

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(Cont'd)

9(ii) When $x=0$,

$$(0+3)^2 = A(-3)(3) + 4(3) + \frac{1}{9}(-3)^2$$

$$9 = -9A + 12 + 1$$

$$-4 = -9A$$

$$A = \frac{4}{9}$$

$$\therefore \frac{(x+3)^2}{2x^3-9x^2+27} = \frac{4}{9(x-3)} + \frac{4}{(x-3)^2} + \frac{1}{9(2x+3)} \quad //$$

10(i)

$$2\cos^2 x - 6\sin^2 x = 2\cos^2 x - 6(1-\cos^2 x)$$

$$= 2\cos^2 x - 6 + 6\cos^2 x$$

$$= 8\cos^2 x - 6$$

$$= 4(2\cos^2 x) - 6$$

$$= 4(\cos 2x + 1) - 6$$

$$= 4\cos 2x + 4 - 6$$

$$= 4\cos 2x - 2 \quad \text{(shown)} \quad //$$

(ii)

$$-1 \leq \cos 2x \leq 1$$

$$-4 \leq 4\cos 2x \leq 4$$

$$-6 \leq 4\cos 2x - 2 \leq 2$$

$\therefore$ Minimum value of $f(x)$ is -6 //

(iii)

$$\text{period} = \frac{2\pi}{2}$$

$$= \pi \quad //$$

$\longrightarrow$

(cont'd) 10(iv)

$y = 4\cos 2x - 2$, $0 \le x \le \pi$

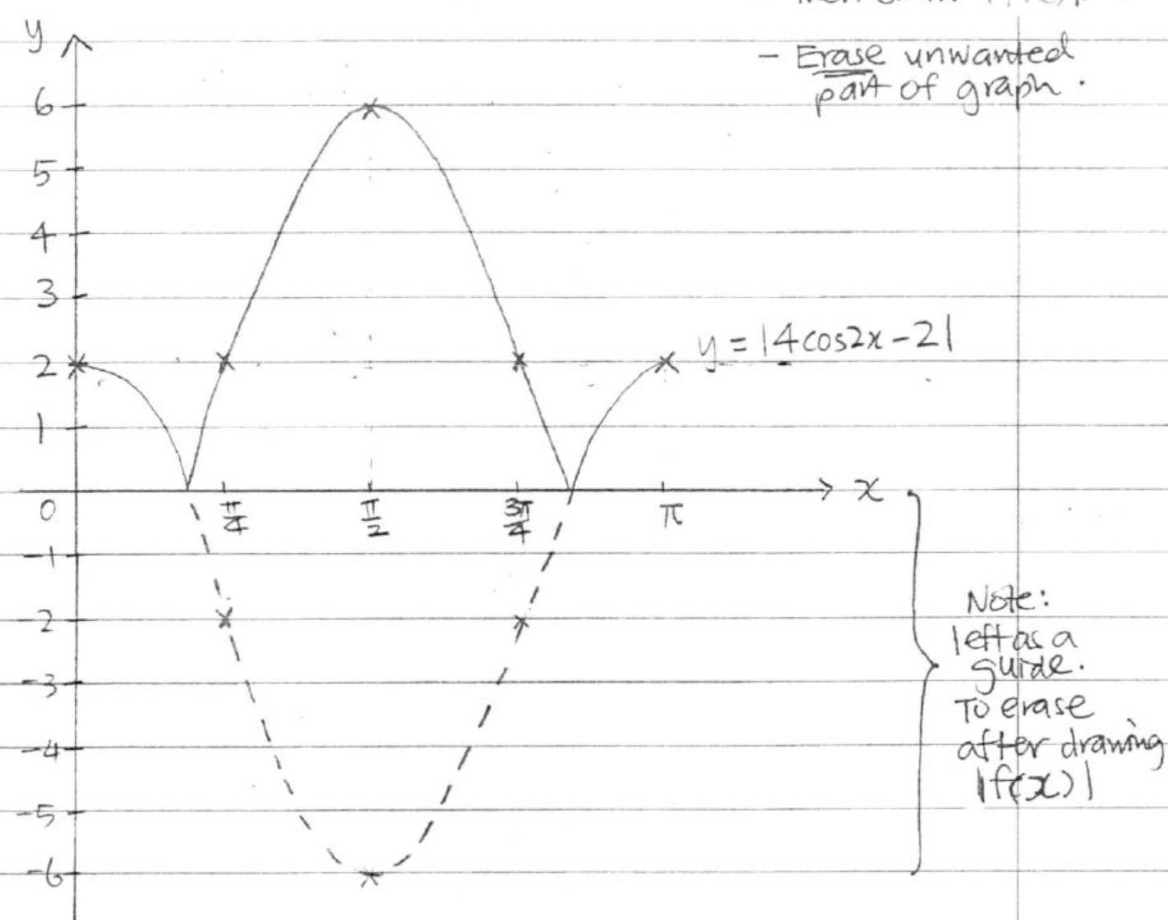
For Examiner's Use

Amplitude = 4

Period = π

Range: $-6 \le f(x) \le 2$

* Recall:
- Draw $y = 4\cos 2x - 2$ first.
- Then draw $|f(x)|$.
- Erase unwanted part of graph.



To obtain 4 solutions, the line $y = c$ must cut the $|f(x)|$ graph at 4 points.

$\therefore \quad 0 < c \le 2$ //

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1 (i) Let coordinates of C be (e, f).

midpoint of AC = midpoint of BD

$$\left(\frac{-2+e}{2}, \frac{1+f}{2}\right) = \left(\frac{4+(-5)}{2}, \frac{3+4}{2}\right)$$

$$\therefore \frac{-2+e}{2} = \frac{4+(-5)}{2} \qquad \frac{1+f}{2} = \frac{3+4}{2}$$

$$-2+e = -1 \qquad 1+f = 7$$

$$e = 1 \qquad f = 6$$

$\therefore$ coordinates of C are (1, 6).//

Midpoint of CD, M $= \left(\frac{-5+1}{2}, \frac{4+6}{2}\right)$

$= (-2, 5)$

$\therefore$ coordinates of M are (−2, 5)//

(ii) Gradient of BC $= \frac{6-3}{1-4}$

$= -1$

$$\frac{y-3}{x-4} = -1$$

$$y-3 = -(x-4)$$

$$y = -x+4+3$$

$$\therefore y = -x+7$$

Equation of BC is $y = -x+7$.//

(Cont'd)

11 (iii) Since MP is // to x-axis, equation of line MP is $y = 5$.

Sub $y = 5$ in $y = -x + 7$,

$5 = -x + 7$

$x = 2$

$\therefore$ coordinates of P are $(2, 5)$.

(iv) $\text{Area } ABPMD = \frac{1}{2}\begin{vmatrix} -2 & 4 & 2 & -2 & -5 & -2 \\ 1 & 3 & 5 & 5 & 4 & 1 \end{vmatrix}$

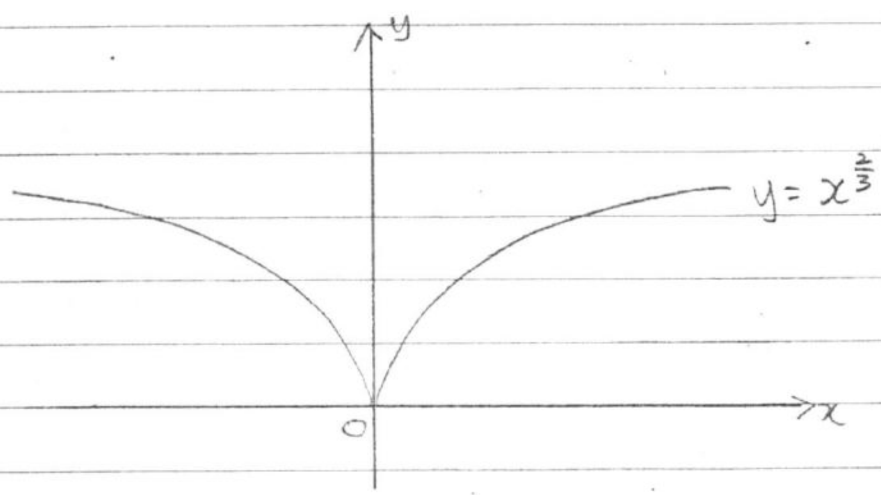
$= \frac{1}{2}\left[(-6)+20+10+(-8)+(-5)-4-6-(-10)-(-25)-(-8)\right]$

$= \frac{1}{2}[11 + 33]$

$= 22 \text{ units}^2$

12(a) $y = x^{\frac{2}{3}}$

$= \sqrt[3]{x^2}$



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For Examiner's Use

12(bi) $x^2+y^2-6x+8y-24=0$

$x^2-6x+\left(\frac{-6}{2}\right)^2-\left(\frac{-6}{2}\right)^2+y^2+8y+\left(\frac{8}{2}\right)^2-\left(\frac{8}{2}\right)^2-24=0$

$(x-3)^2-9+(y+4)^2-16-24=0$

$(x-3)^2+(y+4)^2-49=0$

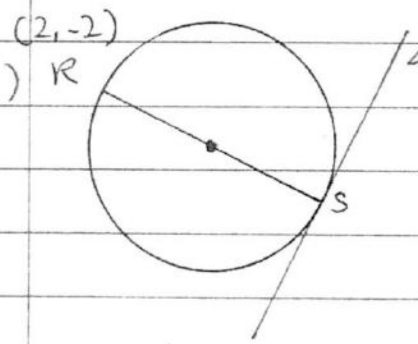
$(x-3)^2+(y+4)^2=49$

$\therefore$ coordinates of centre of C_1 are $(3,-4)$.//

Radius of $C_1 = \sqrt{49}$

$= 7$ units//

(ii)



$4y-3x=36$

$4y=3x+36$

$y=\frac{3}{4}x+9$

Gradient of tangent at $S = \frac{3}{4}$

Gradient of RS $= -\left(\frac{1}{\frac{3}{4}}\right)$

$= -\frac{4}{3}$

*Recall: tan ⊥ rad.

$\frac{y-(-2)}{x-2} = -\frac{4}{3}$

$y+2 = -\frac{4}{3}(x-2)$

$y = -\frac{4}{3}x+\frac{8}{3}-2$

$y = -\frac{4}{3}x+\frac{2}{3}$

$\therefore$ Equation of RS is $y=-\frac{4}{3}x+\frac{2}{3}$.

$\longrightarrow$

(cont'd)

For Examiner's Use

12b(ii)

$$y = \frac{3}{4}x + 9 \quad \text{— ①}$$
$$y = -\frac{4}{3}x + \frac{2}{3} \quad \text{— ②}$$

Sub. ① into ②,

$$\frac{3}{4}x + 9 = -\frac{4}{3}x + \frac{2}{3}$$
$$\frac{25}{12}x = -\frac{25}{3}$$
$$x = -4$$

Sub $x = -4$ into ①,

$$y = \frac{3}{4}(-4) + 9$$
$$y = -3 + 9$$
$$\therefore y = 6$$

Hence, coordinates of S are $(-4, 6)$. (shown)

(iii) centre of C_2 = midpoint of RS

$$= \left(\frac{2+(-4)}{2}, \frac{-2+6}{2}\right)$$
$$= (-1, 2)$$

$\therefore$ coordinates of centre of C_2 are $(-1, 2)$

Radius of $C_2 = \sqrt{(-1-2)^2 + [2-(-2)]^2}$

$= \sqrt{25}$

$= 5$ units

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NORTH VIEW SECONDARY SCHOOL
End-of-Year Examination 2016
Sec 3 Express

ADDITIONAL MATHEMATICS **4047**

11 Oct 2016

Additional Materials: Answer Paper

2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Set by: Mdm Lee YP
Vetted by: Mr Chia PC

This document consists of **6** printed pages

[Turn over

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$cosec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Express $\dfrac{7x+2}{x(x-2)^2}$ in partial fractions. [4]

2 The length and width of a rectangle are $\dfrac{2\sqrt{3}}{2-\sqrt{3}}$ cm and $\dfrac{6}{\sqrt{12}}$ cm respectively.
Find the perimeter of the rectangle, expressing your answer in the form $p+q\sqrt{3}$, where p and q are integers. [4]

3 Solve the simultaneous equations

$$3x-y=3,$$
$$2y^2=3xy+10.$$ [5]

4 By using the substitution $y=4^x$ or otherwise, solve the equation

$$4^{2x+1}=33(4^x)-8.$$ [5]

5 Given that $\dfrac{2}{\alpha}+\dfrac{2}{\beta}=-1$ and $\dfrac{4}{\alpha\beta}=\dfrac{2}{3}$, find the quadratic equation whose roots are

(i) $\dfrac{2}{\alpha}$ and $\dfrac{2}{\beta}$, [2]

(ii) α and β. [3]

6 Given that $f(x)=|x-2|+5x$,

(i) find $f(-1)$, [1]

(ii) find the value of x for which $f(x)=4$. [4]

7 Given that $P(x) = 4x^3 - 13x - 6$,

(i) find the remainder when $P(x)$ is divided by $(x+1)$, [1]

(ii) show that $P(x)$ is divisible by $(x-2)$, [1]

(iii) factorize $P(x)$ completely, [3]

(iv) hence, or otherwise, solve the equation $4x^3 = 13x + 6$. [2]

8 **(a)** Solve the quadratic inequality $(2x+1)(2x-1) > 8$. [3]

(b) **(i)** Find the range of values of m for which the equation

$$2x^2 + x + m = mx + 1$$

has no real roots. [4]

(ii) Hence state, giving a reason, what can be deduced about the curve $y = 2x^2 + x + 7$ and the line $y = 7x + 1$. [1]

9 **(a)** Find the term independent of x in the expansion of $\left(\frac{1}{2x^3} - x\right)^{12}$. [3]

(b) Find, in ascending powers of x, the first three terms in the expansion of

(i) $(2-x)^5$,

(ii) $\left(1 + \frac{1}{2}x\right)^6$.

Hence, find the coefficient of x in the expansion of $(2-x)^5\left(1 + \frac{1}{2}x\right)^6$. [5]

10 **(a)** The equation of a circle, C_1, is $x^2 + y^2 - 6x + 2ky + 17 = 0$.
Find the values of k if the radius of C_1 is $\sqrt{41}$. [4]

(b) **(i)** Another circle, C_2, has centre (2, 5).
Given that the line $x = 8$ is a tangent to C_2, find the equation of C_2. [2]

(ii) Find the possible values of c if $y = c$ is a tangent to C_2. [2]

11 **(a)** Find the value of n for which $\sin\frac{5\pi}{3} + \cot\frac{7\pi}{6} = n\sqrt{3}$. [3]

(b) Find the values of x, between $0°$ and $360°$, which satisfy

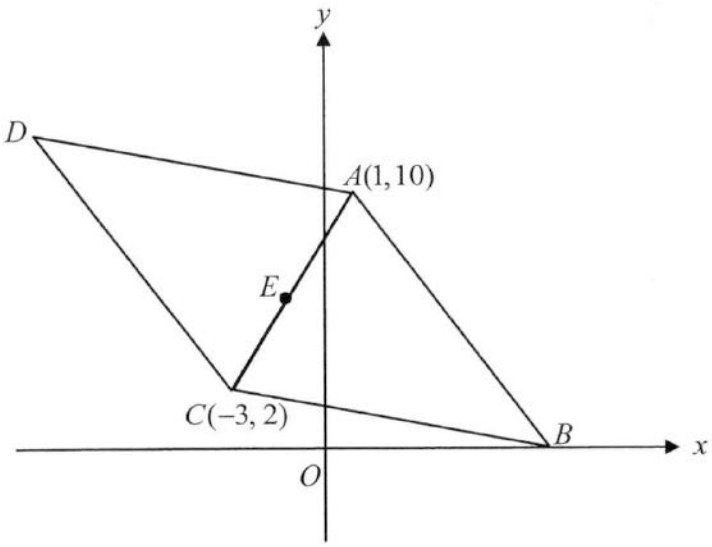
$$\sec x = -\sqrt{2}.$$ [3]

(c) Given that $\cos A = \frac{1}{\sqrt{5}}$ and A is acute, find the exact value of

(i) $\tan(90° - A)$, [2]

(ii) $\operatorname{cosec}(-A)$. [2]

12 **Solutions to this question by accurate drawing will not be accepted.**



The point $A(1,10)$ and $C(-3,2)$ are opposite vertices of a rhombus $ABCD$.
The point B lies on the x-axis and E is the midpoint of AC.

Find

(i) the coordinates of E, [2]

(ii) the equation of BD, [3]

(iii) the coordinates of B, [1]

(iv) the area of the rhombus $ABCD$. [3]

The line $px+qy=0$ is parallel to the diagonal BD.

(v) Express q in terms of p. [2]

2016, S3, NorthView, SA2, A Math / Worked Solutions

① $\frac{7x+2}{x(x-2)^2} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

$= \frac{A(x-2)^2 + Bx(x-2) + Cx}{x(x-2)^2}$

Let $x=0$
$2 = A(-2)^2$
$2 = 4A$
$A = \frac{1}{2}$

Let $x=2$
$7(2)+2 = 2C$
$16 = 2C$
$C = 8$

Let $x=3$
$7(3)+2 = A + 3B + 8(3)$
$23 = \frac{1}{2} + 3B + 24$
$3B = -1.5$
$B = -\frac{1}{2}$

Ans: $\frac{7x+2}{x(x-2)^2} = \frac{1}{2x} - \frac{1}{2(x-2)} + \frac{8}{(x-2)^2}$ ※

③ $3x - y = 3$ —— ①
$2y^2 = 3xy + 10$ —— ②
From ①, $y = 3x - 3$ —— ③
Sub ③ into ②:
$2(3x-3)^2 = 3x(3x-3) + 10$
$2(9x^2 - 18x + 9) = 9x^2 - 9x + 10$
$18x^2 - 36x + 18 = 9x^2 - 9x + 10$
$9x^2 - 27x + 8 = 0$
By quadratic formula, $x = \frac{1}{3}$; $x = \frac{8}{3}$
$\left(x = \frac{-(-27) \pm \sqrt{(-27)^2 - 4(9)(8)}}{2(9)}\right)$ $y = 3(\frac{1}{3}) - 3 = -2$; $y = 3(\frac{8}{3}) - 3 = 5$

Ans: $x = \frac{1}{3}$, $y = -2$
$x = \frac{8}{3}$, $y = 5$ ※

(4) $4^{2x+1} = 33(4^x) - 8$

Let $y = 4^x$

$4^{2x} \cdot 4 - 33(4^x) + 8 = 0$

$4(4^x)^2 - 33(4^x) + 8 = 0$

$4y^2 - 33y + 8 = 0$

By quadratic formula, $y = \frac{1}{4}$; $y = 8$

$4^x = \frac{1}{4} = 4^{-1}$; $4^x = 8$

$x = -1$; $x \lg 4 = \lg 8$

$x = \frac{\lg 8}{\lg 4} = 1.5$

Ans: $x = -1$

$x = 1.5$

⑦ $P(x) = 4x^3 - 13x - 6$

i) When divided by $(x+1)$

$R = P(-1) = 4(-1)^3 - 13(-1) - 6$

$= 3$

$$\begin{array}{r|l} & 4x^2 + 8x + 3 \\ \hline x-2 & 4x^3 + 0x^2 - 13x - 6 \\ & -(4x^3 - 8x^2) \\ \hline & \quad 8x^2 - 13x \\ & -(8x^2 - 16x) \\ \hline & \qquad 3x - 6 \\ & \qquad 3x - 6 \\ \hline & \qquad\quad 0 \end{array}$$

ii) When divided by $(x-2)$

$R = f(2) = 4(2)^3 - 13(2) - 6$

$= 0$ (shown)

iii) $P(x) = (x-2)(4x^2 + 8x + 3)$

$= (x-2)(2x+1)(2x+3)$

iv) $4x^3 = 13x + 6$

$4x^3 - 13x - 6 = 0$

$(x-2)(2x+1)(2x+3) = 0$

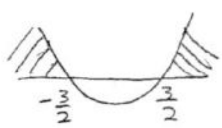
$x = 2, -\frac{1}{2}, -\frac{3}{2}$ //

⑧ a) $(2x+1)(2x-1) > 8$

$4x^2 - 1 > 8$

$4x^2 - 9 > 0$

$(2x+3)(2x-3) > 0$



$x < -\frac{3}{2}$ or $x > \frac{3}{2}$ //

8 bi) $2x^2 + x + m = mx + 1$

$2x^2 + x - mx + m - 1 = 0$

$2x^2 + (1-m)x + (m-1) = 0$

$a = 2$
$b = 1-m$
$c = m-1$

For no real roots,

$b^2 - 4ac < 0$

$(1-m)^2 - 4(2)(m-1) < 0$

$1 - 2m + m^2 - 8(m-1) < 0$

$1 - 2m + m^2 - 8m + 8 < 0$

$m^2 - 10m + 9 < 0$

$(m-1)(m-9) < 0$

$1 < m < 9$

8bii) $\left.\begin{array}{l} y = 2x^2 + x + 7 \\ y = 7x + 1 \end{array}\right\}$ $m = 7$
Curve does not intersect the line.

9i) $(2-x)^5 = 2^5 + \binom{5}{1}(2)^4(-x) + \binom{5}{2}(2)^3(-x)^2 + \cdots$

$= 32 - 80x + 80x^2 + \cdots$

ii) $\left(1 + \frac{1}{2}x\right)^6 = 1^6 + \binom{6}{1}(1)^5\left(\frac{1}{2}x\right) + \binom{6}{2}(1)^4\left(\frac{1}{2}x\right)^2 + \cdots$

$= 1 + 3x + \frac{15}{4}x^2 + \cdots$

$(2-x)^5\left(1 + \frac{1}{2}x\right)^6 \Rightarrow$ Coeff of $x = (32)(3) + (-80)(1)$

$= 16$

10a) $C_1 : x^2+y^2-6x+2ky+17=0$

$$x^2-6x+\left(\tfrac{6}{2}\right)^2+y^2+2ky+\left(\tfrac{2k}{2}\right)^2-\left(\tfrac{6}{2}\right)^2-\left(\tfrac{2k}{2}\right)^2+17=0$$

$$(x-3)^2+(y+k)^2-3^2-k^2+17=0$$

$$(x-3)^2+(y+k)^2=3^2+k^2-17$$

$$=k^2-8$$

$$=r^2$$

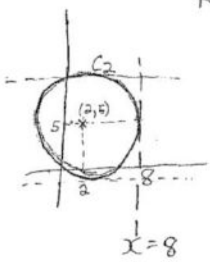
$\therefore k^2-8=r^2=41^2$

$$k^2-8=1681$$

$$k^2=1689$$

$$k=\pm\sqrt{1689} \quad \#$$

2.5980762.1

10bi) 

Radius $C_2 = 8-2=6$

Eq^n $C_2 : (x-2)^2+(y-5)^2=6^2 \quad \#$

ii) $y=c$ is tangent to C_2

$$\left.\begin{array}{l}5+6=11\\5-6=-1\end{array}\right\} \quad c=11 \text{ or } -1 \quad \#$$

11a) $\sin\frac{5\pi}{3}+\cot\frac{7\pi}{6}=n\sqrt{3}$

$\pi\text{rad}=180°$; $\sin 300° = \sin(-60°)$

$= -\sin 60°$

$= -\frac{\sqrt{3}}{2}$

$$\sin 300° + \frac{\cos 210°}{\sin 210°} = n\sqrt{3}$$

$\cos 210° = -\cos 30°$

$= -\frac{\sqrt{3}}{2}$; $\sin 210° = -\sin 30°$

$= -\frac{1}{2}$

$$-\frac{\sqrt{3}}{2}+\frac{\frac{-\sqrt{3}}{2}}{-\frac{1}{2}}=n\sqrt{3}$$

$$-\frac{\sqrt{3}}{2}+\sqrt{3}=n\sqrt{3}$$

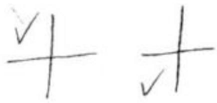
$$\frac{1}{2}\sqrt{3}=n\sqrt{3} \quad \Rightarrow \quad n=\frac{1}{2} \quad \#$$

(11b) $\sec x = -2$

$\frac{1}{\cos x} = -2$

$\cos x = -\frac{1}{2}$

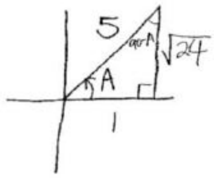
Basic $\measuredangle = \cos^{-1}\frac{1}{2} = 60^\circ$



$x = 180^\circ - 60^\circ, \quad 360^\circ - 60^\circ$

$= 120^\circ, \quad 300^\circ$ ※

c) $\cos A = \frac{1}{5}$, A acute (1^{st} quadrant)



(i) $\tan(90^\circ - A) = \frac{1}{\sqrt{24}}$

(ii) $\text{cosec}(-A) = \frac{1}{\sin(-A)} = \frac{1}{-\sin A}$

$= \frac{1}{-\frac{\sqrt{24}}{5}}$

$= -\frac{5}{\sqrt{24}}$ ※

(12i) Coord of E $= \left(\frac{1-3}{2}, \frac{10+2}{2}\right) = (-1, 6)$

ii) Grad AC $= \frac{10-2}{1-(-3)} = 2$

Grad BD $= \frac{-1}{2} = -\frac{1}{2}$

$y = -\frac{1}{2}x + C$

Sub $E(-1,6)$: $6 = -\frac{1}{2}(-1) + C$

$C = 5\frac{1}{2}$

Eq^n BD : $y = -\frac{1}{2}x + 5\frac{1}{2}$ ※

iii) Sub $y=0$ into BD : $0 = -\frac{1}{2}x + 5\frac{1}{2}$

$x = 11$

Coord of B $= (11, 0)$ ※

12 iv)

To find coord of D: Let $D = (x, y)$

$$\left(\frac{x+11}{2}, \frac{y+0}{2}\right) = (-1, 6)$$

$x + 11 = -2$; $y = 12$

$x = -13$

$D = (-13, 12)$

Area of ABCD $= \frac{1}{2}\begin{vmatrix} 1 & -13 & -3 & 11 & 1 \\ 10 & 12 & 2 & 0 & 10 \end{vmatrix}$

$= \frac{1}{2}\left[(12 - 26 - 0 + 110) - (-130 - 36 + 22 + 0)\right]$

$= 70 \text{ units}^2$ ⋇

$px + qy = 0$

$qy = -px$

$y = -\frac{p}{q}x$

v) Grad $BD = -\frac{1}{2}$

$-\frac{p}{q} = -\frac{1}{2}$

$\frac{p}{q} = \frac{1}{2}$

$2p = q$ ⋇

(2) $L = \frac{2\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$; $W = \frac{6}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$

$= \frac{2\sqrt{3}(2+\sqrt{3})}{4-3}$ $\quad = \frac{6\sqrt{12}}{12}$

$= 4\sqrt{3} + 6.$ $\quad = \frac{\sqrt{12}}{2} = \frac{2\sqrt{3}}{2}.$

Perimeter $= 2L + 2W = 2(4\sqrt{3}+6) + 2\left(\frac{2\sqrt{3}}{2}\right)$

$= 8\sqrt{3} + 12 + 2\sqrt{3}$

$= 10\sqrt{3} + 12$ cm #

(5) $\frac{2}{\alpha} + \frac{2}{\beta} = -1$; $\frac{4}{\alpha\beta} = \frac{2}{3} \Rightarrow 2\alpha\beta = 12$

$\alpha\beta = 6$

$\frac{2\beta + 2\alpha}{\alpha\beta} = -1$

$2(\alpha+\beta) = -\alpha\beta$

$\alpha+\beta = \frac{-\alpha\beta}{2} = \frac{-6}{2} = -3$

(i) Sum roots $= \frac{2}{\alpha} + \frac{2}{\beta} = -1$ (given)

product roots $= \left(\frac{2}{\alpha}\right)\left(\frac{2}{\beta}\right) = \frac{4}{\alpha\beta} = \frac{2}{3}$ (given)

Eq$^{\underline{n}}$: $x^2 - (-1)x + \frac{2}{3} = 0$

$x^2 + x + \frac{2}{3} = 0$ #

(ii) Sum roots $= \alpha+\beta = -3$

product roots $= \alpha\beta = 6$

Eq$^{\underline{n}}$: $x^2 - (-3)x + 6 = 0.$

$x^2 + 3x + 6 = 0$ #

6 $f(x) = |x-2| + 5x$

(i) $f(-1) = |-1-2| + 5(-1)$

$= |-3| - 5$

$= 3 - 5$

$= -2$ ※

(ii) $f(x) = 4$

$|x-2| + 5x = 4$

$|x-2| = 4 - 5x$

$x - 2 = 4 - 5x$; $x - 2 = -(4-5x)$

$6x = 6$ $x - 2 = 5x - 4$

$x = 1$ (reject) $4x = 2$

$x = \frac{1}{2}$ ※